Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/18-1.1.1.3a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**352**]. This is test number [18].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (352)	0.00 (0)
Mathematica	100.00 (352)	0.00 (0)
Reduce	98.86 (348)	1.14 (4)
Maple	98.58 (347)	1.42 (5)
Fricas	98.58 (347)	1.42 (5)
Maxima	98.58 (347)	1.42 (5)
Giac	96.31 (339)	3.69 (13)
Sympy	96.02 (338)	3.98 (14)
Mupad	90.62 (319)	9.38 (33)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

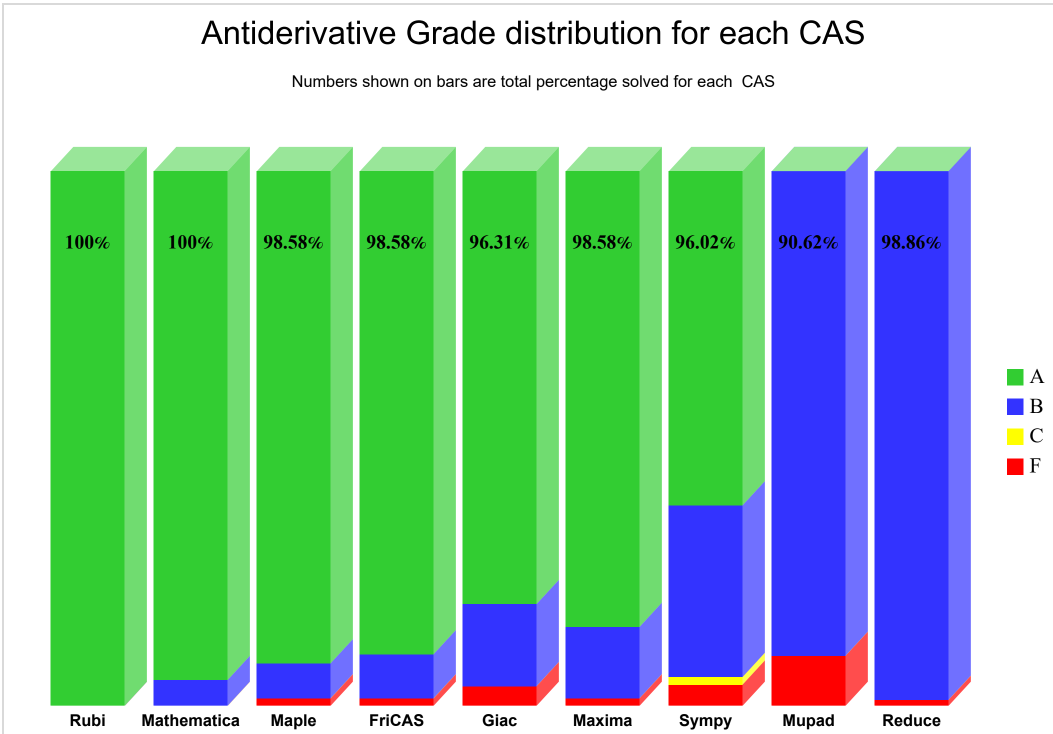
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

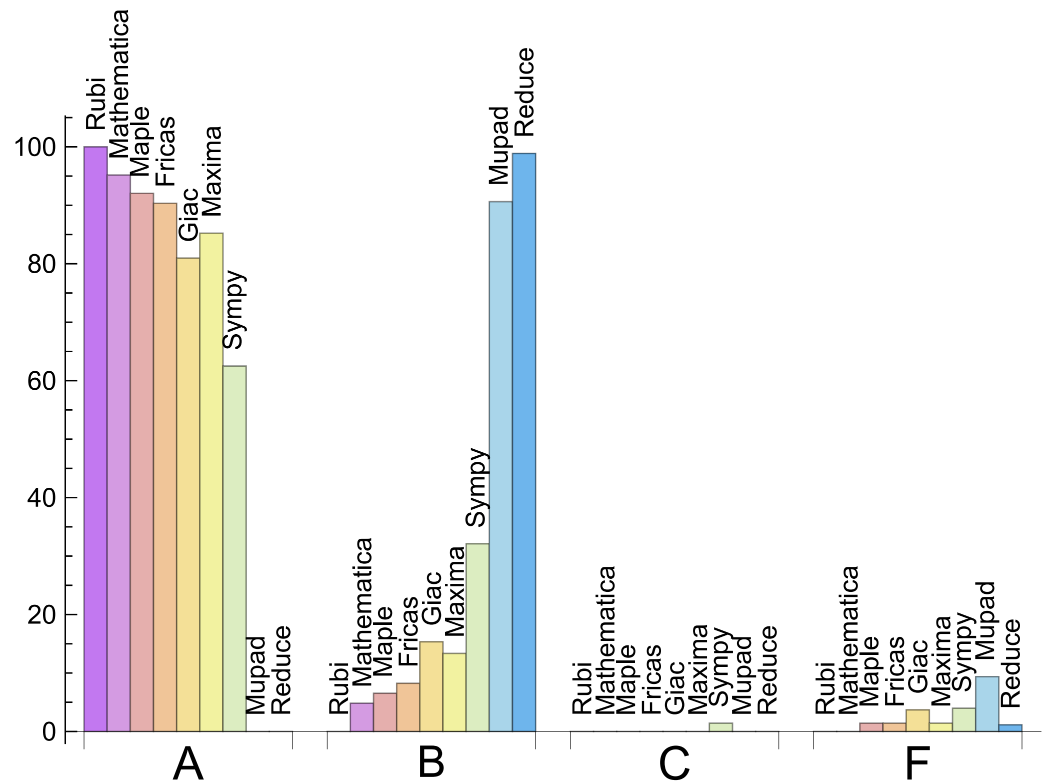
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	95.170	4.830	0.000	0.000
Maple	92.045	6.534	0.000	1.420
Fricas	90.341	8.239	0.000	1.420
Maxima	85.227	13.352	0.000	1.420
Giac	80.966	15.341	0.000	3.693
Sympy	62.500	32.102	1.420	3.977
Mupad	0.000	90.625	0.000	9.375
Reduce	0.000	98.864	0.000	1.136

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Reduce	4	100.00	0.00	0.00
Fricas	5	100.00	0.00	0.00
Maple	5	100.00	0.00	0.00
Maxima	5	100.00	0.00	0.00
Giac	13	38.46	61.54	0.00
Sympy	14	0.00	100.00	0.00
Mupad	33	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Fricas	0.08
Mathematica	0.09
Maple	0.15
Reduce	0.16
Mupad	0.17
Rubi	0.21
Giac	4.95
Sympy	11.55

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	68.56	0.80	60.00	0.68
Mathematica	91.70	1.05	75.00	0.91
Maple	92.07	1.03	75.00	0.87
Rubi	92.47	0.99	85.00	1.00
Mupad	103.25	1.18	79.00	0.97
Maxima	124.35	1.33	96.00	1.06
Giac	132.52	1.47	102.00	1.10
Fricas	144.09	1.55	116.00	1.16
Sympy	290.44	3.03	128.50	1.33

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

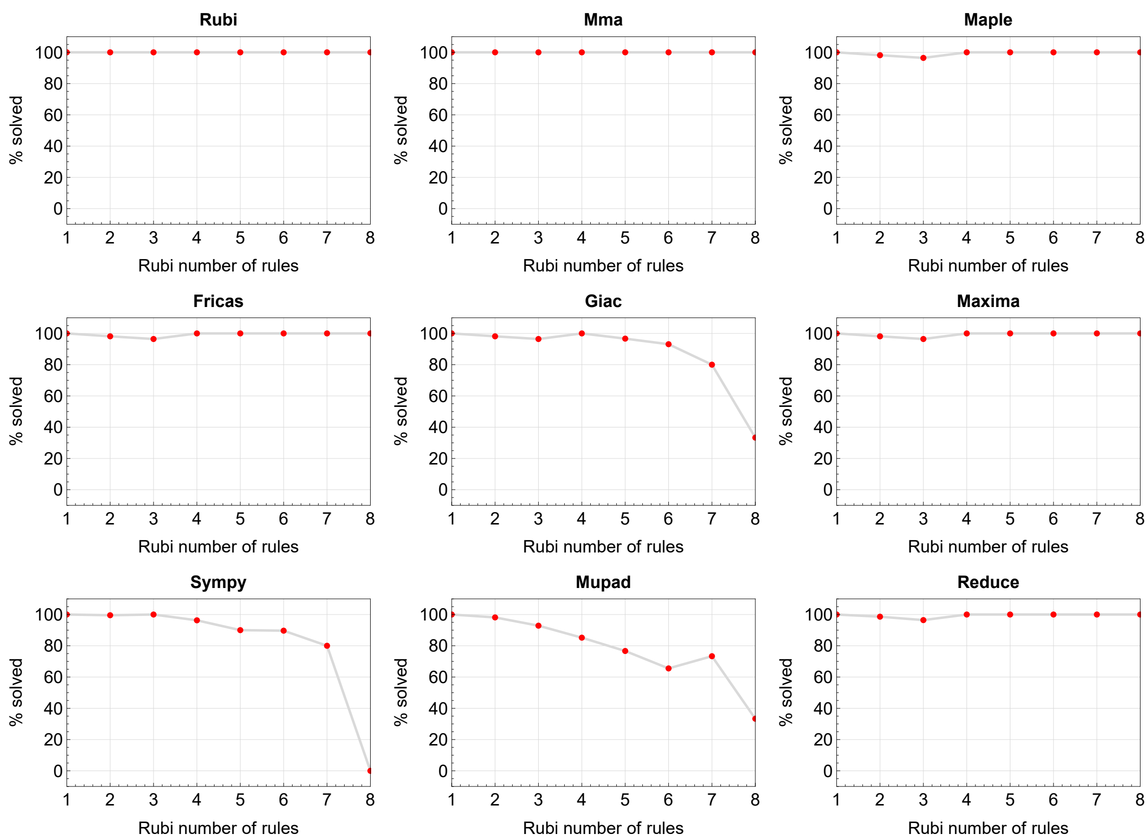


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

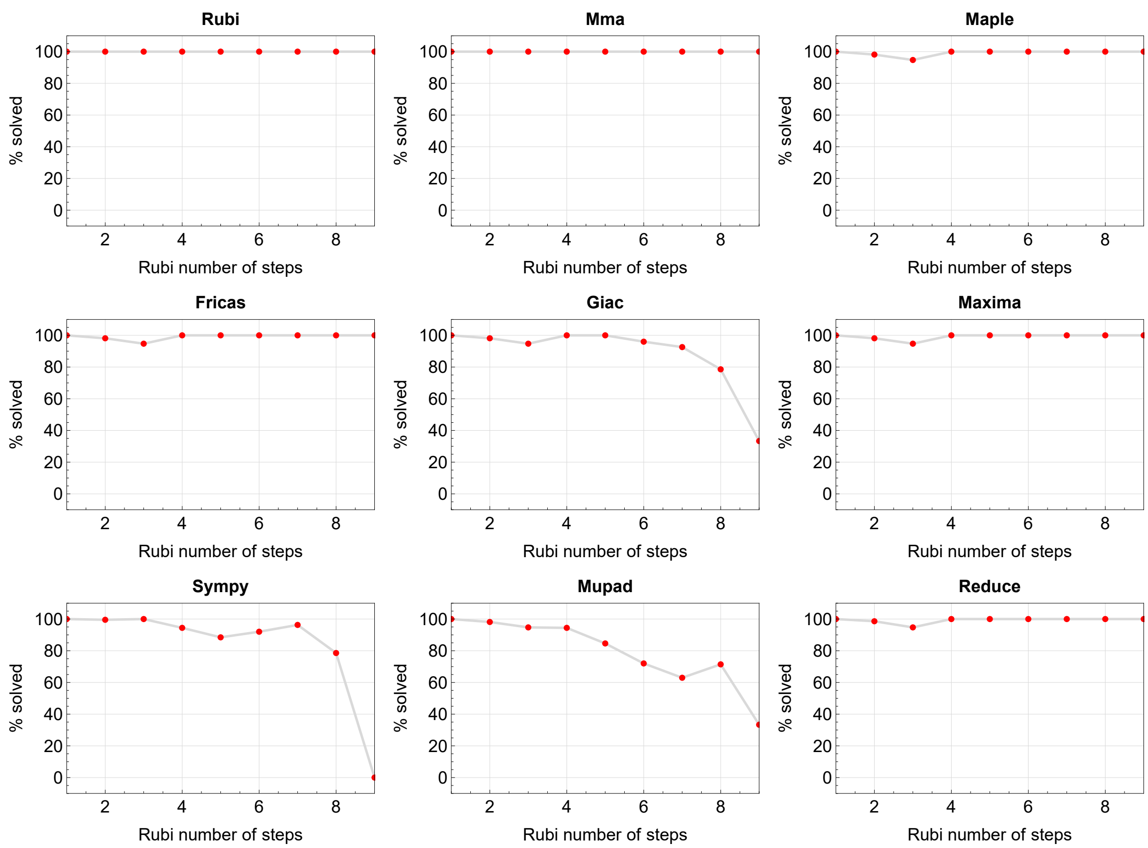


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

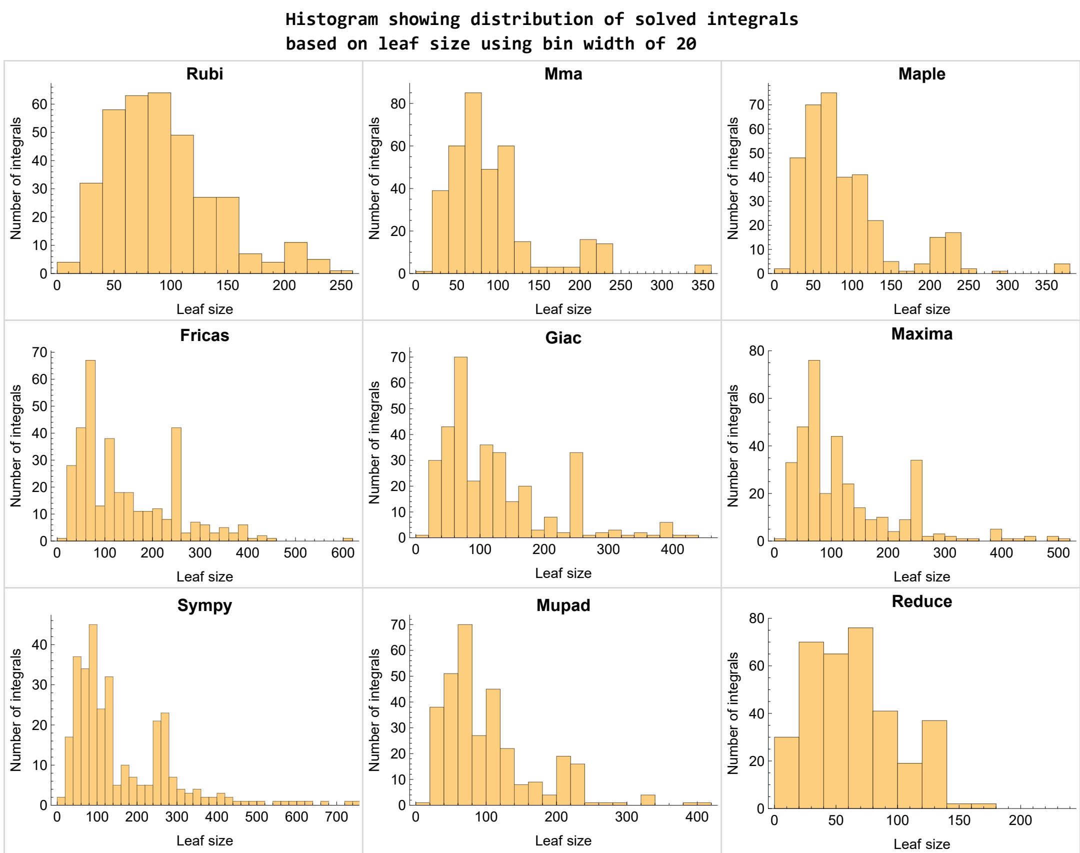


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

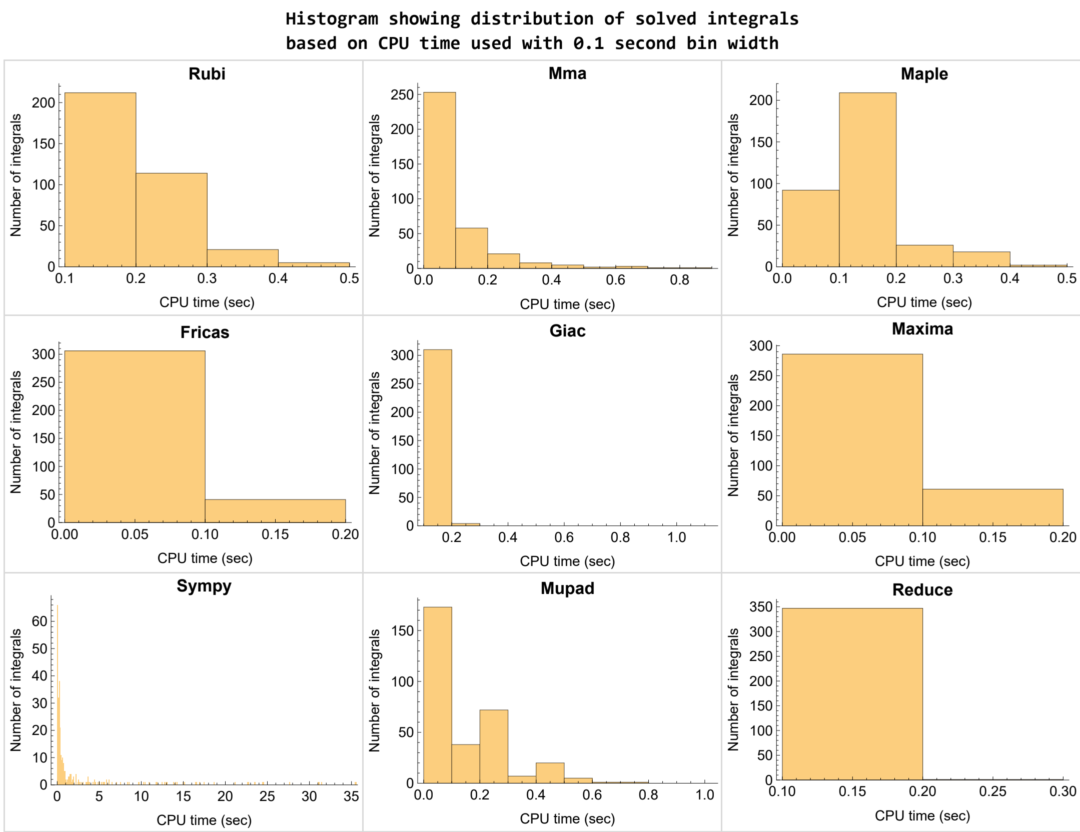


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

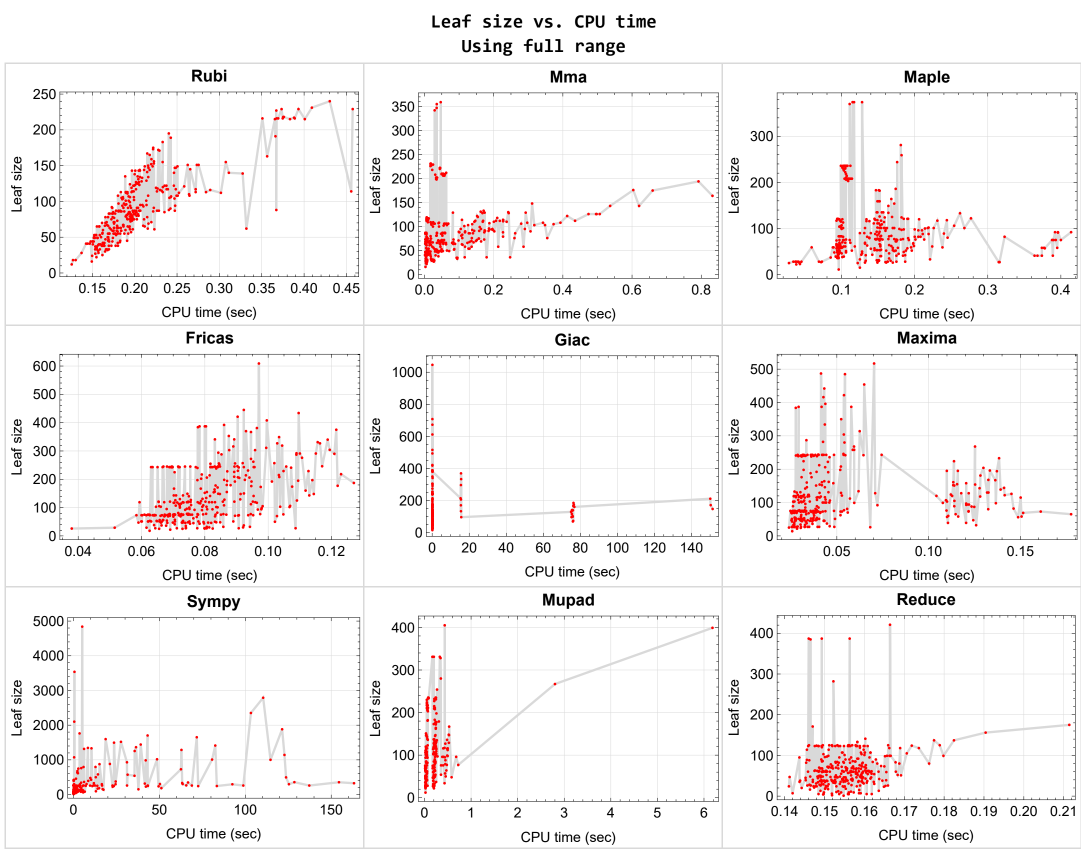


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{4, 5, 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

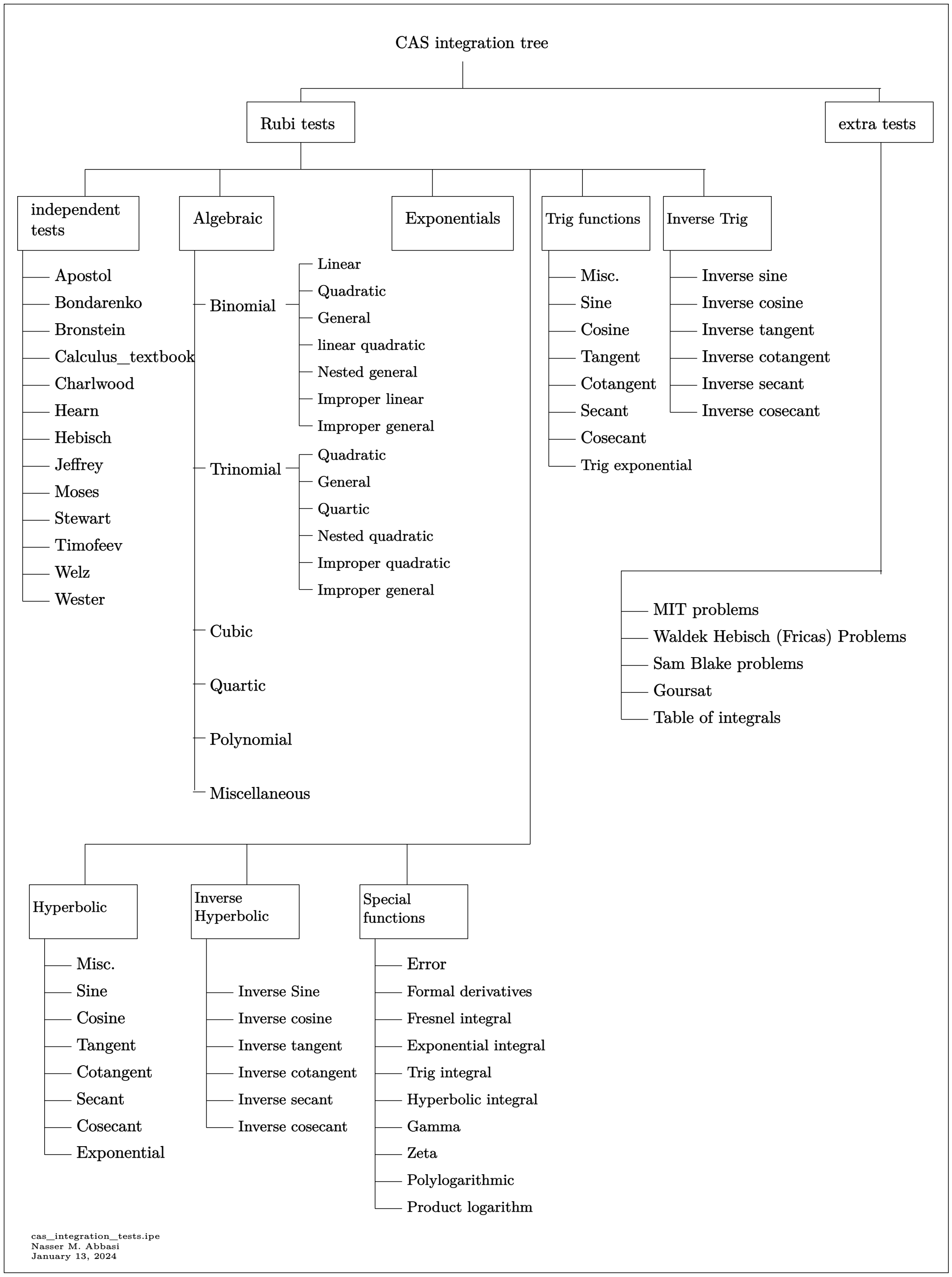
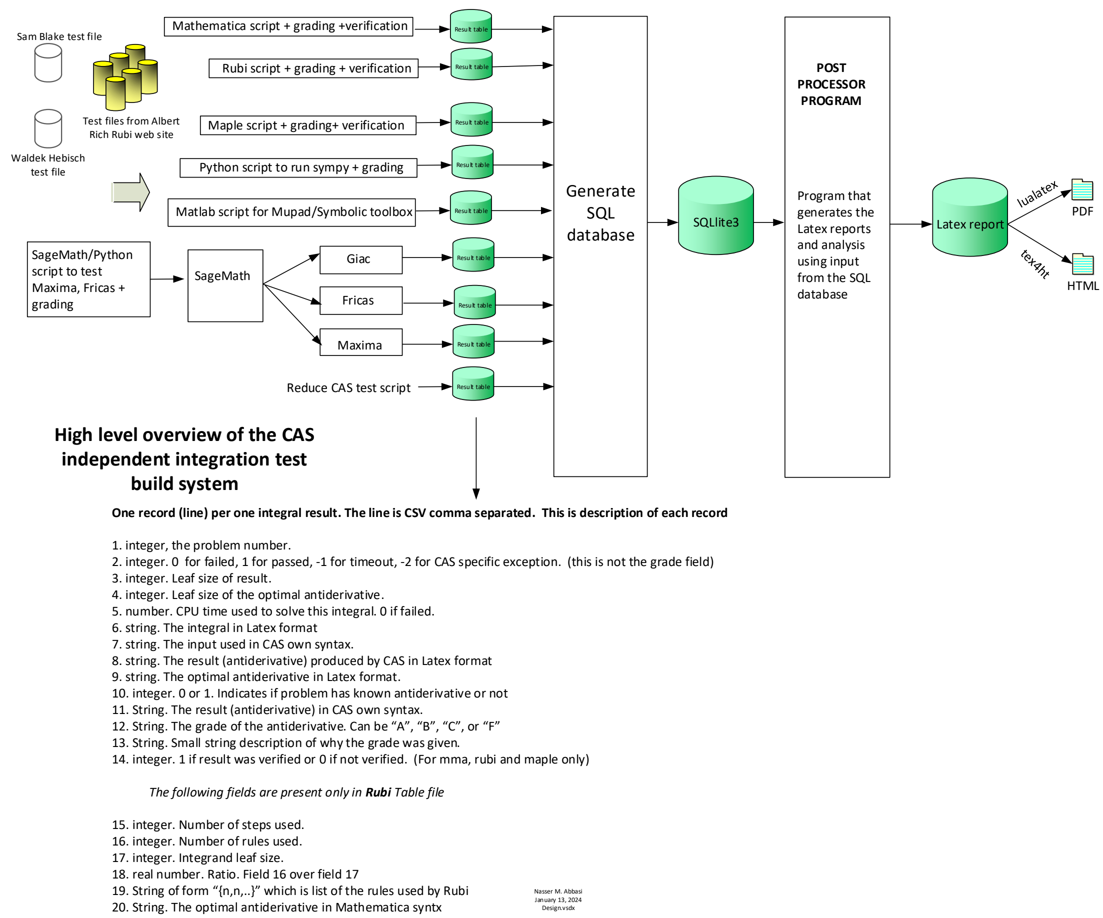


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352 }

B grade { 6, 23, 34, 44, 93, 101, 113, 114, 115, 116, 129, 130, 138, 139, 140, 141, 142 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347 }

B grade { 6, 34, 44, 77, 92, 93, 101, 113, 114, 115, 116, 129, 130, 131, 138, 139, 140, 141, 329, 330, 336, 337, 338 }

C grade { }

F normal fail { 348, 349, 350, 351, 352 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 347 }

B grade { 6, 34, 44, 77, 92, 93, 101, 113, 114, 115, 116, 129, 130, 131, 132, 138, 139, 140, 141, 142, 168, 169, 238, 248, 249, 313, 344, 345, 346 }

C grade { }

F normal fail { 348, 349, 350, 351, 352 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 294, 295, 296, 299, 300, 310, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 338, 341, 342, 343, 344, 345, 346, 347 }

B grade { 6, 34, 44, 77, 92, 93, 101, 113, 114, 115, 116, 129, 130, 131, 132, 138, 139, 140, 141, 142, 291, 292, 293, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 328, 335, 336, 337, 339, 340 }

C grade { }

F normal fail { 348, 349, 350, 351, 352 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 229, 230, 231, 232, 233, 239, 240, 241, 242, 243, 244, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329 }

B grade { 6, 34, 44, 77, 92, 93, 101, 113, 114, 115, 116, 129, 130, 131, 132, 138, 139, 140, 141, 142, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 245, 246, 247, 248, 249, 320, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347 }

C grade { }

F normal fail { 348, 349, 350, 351, 352 }

F(-1) timedout fail { 285, 286, 287, 295, 296, 297, 306, 307 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 287, 288, 291, 292, 293, 294, 302, 303, 304, 305, 313, 314, 315, 316, 317, 320, 321, 322, 323, 324, 325, 326, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347 }

C grade { }

F normal fail { }

F(-1) timedout fail { 285, 286, 289, 290, 295, 296, 297, 298, 299, 300, 301, 306, 307, 308, 309, 310, 311, 312, 318, 319, 327, 328, 329, 330, 335, 336, 337, 338, 348, 349, 350, 351, 352 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 35, 36, 37, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 143, 144, 145, 146, 147, 148, 153, 154, 155, 156, 157, 158, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 224, 225, 226, 227, 228, 229, 230, 234, 235, 236, 237, 239, 240, 241, 245, 246, 250, 251, 252, 253, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 281, 285, 286, 287, 288, 289, 290, 297, 298, 299, 307, 308, 309, 310, 318, 320, 321, 322, 323, 327, 328, 329, 330, 331 }

B grade { 6, 16, 23, 30, 34, 38, 44, 77, 83, 92, 93, 101, 102, 112, 113, 114, 115, 116, 129, 130, 131, 132, 138, 139, 140, 141, 142, 149, 150, 151, 152, 159, 160, 161, 168, 169, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 231, 232, 233, 238, 242, 243, 247, 248, 249, 254, 264, 265, 266, 273, 278, 279, 280, 282, 283, 284, 291, 292, 293, 294, 296, 300, 301, 302, 303, 304, 311, 312, 313, 314, 319, 324, 325, 326, 332, 333, 334, 337, 338, 339, 340, 341, 344, 345, 346, 347 }

C grade { 348, 349, 350, 351, 352 }

F normal fail { }

F(-1) timedout fail { 137, 244, 255, 256, 295, 305, 306, 315, 316, 317, 335, 336, 342, 343 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

C grade { }

F normal fail { 349, 350, 351, 352 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	39	47	47	49	47	38	47
N.S.	1	1.00	0.85	0.71	0.85	0.85	0.89	0.85	0.69	0.85
time (sec)	N/A	0.168	0.003	0.103	0.037	0.059	0.025	0.120	0.165	0.041

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	39	47	47	53	47	38	47
N.S.	1	1.00	0.85	0.71	0.85	0.85	0.96	0.85	0.69	0.85
time (sec)	N/A	0.170	0.002	0.095	0.039	0.066	0.023	0.123	0.155	0.029

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	37	44	44	44	44	36	44
N.S.	1	1.00	1.05	0.97	1.16	1.16	1.16	1.16	0.95	1.16
time (sec)	N/A	0.163	0.002	0.085	0.037	0.080	0.024	0.125	0.159	0.028

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	36	43	43	48	44	37	43
N.S.	1	1.00	1.02	0.77	0.91	0.91	1.02	0.94	0.79	0.91
time (sec)	N/A	0.165	0.011	0.094	0.026	0.075	0.054	0.123	0.152	0.026

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	38	45	48	44	46	40	45
N.S.	1	1.00	0.83	0.81	0.96	1.02	0.94	0.98	0.85	0.96
time (sec)	N/A	0.169	0.005	0.101	0.043	0.075	0.069	0.121	0.147	0.026

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	36	46	45	46	46	38	35
N.S.	1	1.00	2.28	2.00	2.56	2.50	2.56	2.56	2.11	1.94
time (sec)	N/A	0.127	0.005	0.096	0.027	0.076	0.082	0.120	0.156	0.027

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	45	48	44	46	40	38
N.S.	1	1.00	0.82	0.80	1.00	1.07	0.98	1.02	0.89	0.84
time (sec)	N/A	0.169	0.005	0.095	0.035	0.084	0.104	0.121	0.157	0.028

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	47	49	49	48	40	40
N.S.	1	1.00	0.84	0.78	0.94	0.98	0.98	0.96	0.80	0.80
time (sec)	N/A	0.168	0.005	0.095	0.025	0.080	0.126	0.125	0.152	0.187

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	47	47	51	47	38	46
N.S.	1	1.00	0.84	0.78	0.94	0.94	1.02	0.94	0.76	0.92
time (sec)	N/A	0.169	0.005	0.095	0.030	0.070	0.135	0.119	0.151	0.023

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	39	47	47	51	47	38	47
N.S.	1	1.00	0.85	0.71	0.85	0.85	0.93	0.85	0.69	0.85
time (sec)	N/A	0.170	0.005	0.092	0.030	0.073	0.141	0.124	0.149	0.177

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	39	47	47	51	47	38	47
N.S.	1	1.00	0.85	0.71	0.85	0.85	0.93	0.85	0.69	0.85
time (sec)	N/A	0.170	0.005	0.092	0.034	0.081	0.161	0.123	0.150	0.022

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	61	75	75	82	75	60	75
N.S.	1	1.00	1.00	0.70	0.86	0.86	0.94	0.86	0.69	0.86
time (sec)	N/A	0.205	0.004	0.097	0.026	0.070	0.025	0.116	0.150	0.018

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	61	75	75	85	75	60	75
N.S.	1	1.00	1.00	0.70	0.86	0.86	0.98	0.86	0.69	0.86
time (sec)	N/A	0.206	0.003	0.095	0.033	0.076	0.022	0.120	0.151	0.018

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	87	61	75	75	85	75	60	75
N.S.	1	1.00	1.09	0.76	0.94	0.94	1.06	0.94	0.75	0.94
time (sec)	N/A	0.205	0.003	0.098	0.038	0.075	0.025	0.127	0.149	0.017

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	85	61	75	75	80	75	60	75
N.S.	1	1.00	1.44	1.03	1.27	1.27	1.36	1.27	1.02	1.27
time (sec)	N/A	0.184	0.002	0.098	0.027	0.071	0.026	0.115	0.151	0.017

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	68	59	72	72	82	72	58	72
N.S.	1	1.00	1.79	1.55	1.89	1.89	2.16	1.89	1.53	1.89
time (sec)	N/A	0.161	0.003	0.088	0.028	0.091	0.029	0.116	0.160	0.017

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	57	70	70	78	71	59	70
N.S.	1	1.00	0.92	0.75	0.92	0.92	1.03	0.93	0.78	0.92
time (sec)	N/A	0.191	0.015	0.093	0.025	0.103	0.064	0.114	0.149	0.020

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	71	76	71	72	61	71
N.S.	1	1.00	1.00	0.79	0.97	1.04	0.97	0.99	0.84	0.97
time (sec)	N/A	0.197	0.006	0.098	0.041	0.070	0.073	0.128	0.159	0.021

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	59	73	77	78	74	62	73
N.S.	1	1.00	1.00	0.76	0.94	0.99	1.00	0.95	0.79	0.94
time (sec)	N/A	0.197	0.006	0.101	0.033	0.095	0.100	0.125	0.148	0.020

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	59	73	77	78	74	62	73
N.S.	1	1.00	1.00	0.76	0.94	0.99	1.00	0.95	0.79	0.94
time (sec)	N/A	0.215	0.006	0.096	0.052	0.074	0.124	0.126	0.150	0.027

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	57	71	77	75	72	62	60
N.S.	1	1.00	1.00	0.79	0.99	1.07	1.04	1.00	0.86	0.83
time (sec)	N/A	0.195	0.006	0.096	0.039	0.069	0.155	0.127	0.149	0.184

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	60	74	77	78	75	62	61
N.S.	1	1.00	1.00	0.76	0.94	0.97	0.99	0.95	0.78	0.77
time (sec)	N/A	0.194	0.006	0.095	0.030	0.069	0.195	0.120	0.147	0.028

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	85	61	75	75	80	75	60	74
N.S.	1	1.00	2.07	1.49	1.83	1.83	1.95	1.83	1.46	1.80
time (sec)	N/A	0.143	0.006	0.096	0.025	0.072	0.217	0.122	0.157	0.027

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	61	75	75	80	75	60	75
N.S.	1	1.00	1.00	0.73	0.89	0.89	0.95	0.89	0.71	0.89
time (sec)	N/A	0.201	0.005	0.097	0.033	0.078	0.218	0.125	0.149	0.176

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	61	75	75	80	75	60	75
N.S.	1	1.00	1.00	0.70	0.86	0.86	0.92	0.86	0.69	0.86
time (sec)	N/A	0.200	0.006	0.097	0.037	0.067	0.241	0.125	0.168	0.028

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	61	75	75	87	75	60	75
N.S.	1	1.00	0.84	0.70	0.86	0.86	1.00	0.86	0.69	0.86
time (sec)	N/A	0.207	0.003	0.099	0.030	0.062	0.028	0.121	0.150	0.018

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	61	75	75	87	75	60	75
N.S.	1	1.00	0.84	0.70	0.86	0.86	1.00	0.86	0.69	0.86
time (sec)	N/A	0.206	0.003	0.098	0.031	0.085	0.026	0.126	0.155	0.016

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	61	74	74	78	74	60	74
N.S.	1	1.00	0.85	0.76	0.92	0.92	0.98	0.92	0.75	0.92
time (sec)	N/A	0.202	0.003	0.096	0.026	0.064	0.028	0.121	0.157	0.017

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	73	61	75	75	87	75	60	75
N.S.	1	1.00	1.24	1.03	1.27	1.27	1.47	1.27	1.02	1.27
time (sec)	N/A	0.184	0.002	0.094	0.036	0.061	0.031	0.124	0.160	0.017

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	64	59	72	72	78	72	58	72
N.S.	1	1.00	1.68	1.55	1.89	1.89	2.05	1.89	1.53	1.89
time (sec)	N/A	0.163	0.002	0.089	0.036	0.064	0.030	0.118	0.156	0.017

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	58	71	71	82	72	59	71
N.S.	1	1.00	0.95	0.73	0.90	0.90	1.04	0.91	0.75	0.90
time (sec)	N/A	0.189	0.010	0.098	0.039	0.069	0.061	0.120	0.158	0.021

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	58	71	77	75	72	62	71
N.S.	1	1.00	0.81	0.77	0.95	1.03	1.00	0.96	0.83	0.95
time (sec)	N/A	0.199	0.006	0.098	0.037	0.078	0.084	0.133	0.161	0.021

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	61	75	77	82	76	62	75
N.S.	1	1.00	0.83	0.74	0.91	0.94	1.00	0.93	0.76	0.91
time (sec)	N/A	0.199	0.006	0.118	0.031	0.073	0.104	0.116	0.154	0.020

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	63	58	73	73	76	73	60	74
N.S.	1	1.00	3.50	3.22	4.06	4.06	4.22	4.06	3.33	4.11
time (sec)	N/A	0.130	0.005	0.100	0.053	0.062	0.116	0.111	0.146	0.033

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	59	73	77	78	74	62	73
N.S.	1	1.00	0.82	0.74	0.91	0.96	0.98	0.92	0.78	0.91
time (sec)	N/A	0.197	0.006	0.097	0.042	0.097	0.149	0.115	0.150	0.184

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	58	73	77	76	74	62	61
N.S.	1	1.00	0.81	0.77	0.97	1.03	1.01	0.99	0.83	0.81
time (sec)	N/A	0.195	0.006	0.096	0.025	0.086	0.174	0.120	0.161	0.028

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	61	75	77	80	76	62	62
N.S.	1	1.00	0.83	0.74	0.91	0.94	0.98	0.93	0.76	0.76
time (sec)	N/A	0.204	0.006	0.096	0.029	0.075	0.218	0.121	0.157	0.196

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	66	61	75	75	82	75	60	74
N.S.	1	1.00	1.61	1.49	1.83	1.83	2.00	1.83	1.46	1.80
time (sec)	N/A	0.147	0.005	0.096	0.031	0.087	0.243	0.122	0.155	0.026

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	73	61	75	75	82	75	60	75
N.S.	1	1.00	1.12	0.94	1.15	1.15	1.26	1.15	0.92	1.15
time (sec)	N/A	0.155	0.005	0.103	0.031	0.073	0.266	0.110	0.151	0.189

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	61	75	75	82	75	60	74
N.S.	1	1.00	0.83	0.74	0.91	0.91	1.00	0.91	0.73	0.90
time (sec)	N/A	0.196	0.005	0.105	0.027	0.075	0.255	0.123	0.161	0.028

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	61	75	75	82	75	60	75
N.S.	1	1.00	0.84	0.70	0.86	0.86	0.94	0.86	0.69	0.86
time (sec)	N/A	0.202	0.006	0.096	0.048	0.083	0.286	0.119	0.157	0.184

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	61	75	75	82	75	60	75
N.S.	1	1.00	0.84	0.70	0.86	0.86	0.94	0.86	0.69	0.86
time (sec)	N/A	0.198	0.005	0.095	0.031	0.063	0.296	0.119	0.164	0.028

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	82	102	105	109	103	84	82
N.S.	1	1.00	1.00	0.73	0.90	0.93	0.96	0.91	0.74	0.73
time (sec)	N/A	0.233	0.008	0.102	0.026	0.066	0.274	0.122	0.162	0.036

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	112	83	103	103	110	103	82	102
N.S.	1	1.00	2.73	2.02	2.51	2.51	2.68	2.51	2.00	2.49
time (sec)	N/A	0.144	0.006	0.096	0.032	0.065	0.321	0.119	0.154	0.195

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	116	83	103	103	110	103	82	103
N.S.	1	1.00	1.78	1.28	1.58	1.58	1.69	1.58	1.26	1.58
time (sec)	N/A	0.158	0.006	0.097	0.035	0.104	0.315	0.125	0.153	0.033

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	92	119	83	103	103	110	103	82	103
N.S.	1	1.03	1.34	0.93	1.16	1.16	1.24	1.16	0.92	1.16
time (sec)	N/A	0.173	0.007	0.098	0.030	0.066	0.361	0.125	0.160	0.035

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	83	103	103	110	103	82	102
N.S.	1	1.00	1.00	0.73	0.90	0.90	0.96	0.90	0.72	0.89
time (sec)	N/A	0.226	0.006	0.100	0.027	0.081	0.383	0.118	0.155	0.191

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	83	103	103	110	103	82	103
N.S.	1	1.00	1.00	0.70	0.87	0.87	0.92	0.87	0.69	0.87
time (sec)	N/A	0.225	0.006	0.099	0.026	0.079	0.382	0.124	0.145	0.035

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	24	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.73	0.85
time (sec)	N/A	0.170	0.004	0.038	0.026	0.080	0.017	0.122	0.151	0.208

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	24	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.73	0.85
time (sec)	N/A	0.167	0.004	0.037	0.040	0.091	0.019	0.118	0.150	0.212

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	24	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.73	0.85
time (sec)	N/A	0.166	0.004	0.037	0.028	0.080	0.020	0.128	0.158	0.023

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	28	27	29	29	29	24	28
N.S.	1	1.00	0.88	0.85	0.82	0.88	0.88	0.88	0.73	0.85
time (sec)	N/A	0.159	0.004	0.034	0.025	0.052	0.024	0.138	0.155	0.021

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	21	25
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.75	0.89
time (sec)	N/A	0.157	0.003	0.028	0.030	0.038	0.020	0.121	0.147	0.022

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	22	22	22	22	20	22
N.S.	1	1.00	1.00	0.92	0.92	0.92	0.92	0.92	0.83	0.92
time (sec)	N/A	0.150	0.004	0.038	0.028	0.069	0.046	0.121	0.144	0.022

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	22	26	19	23	24	22
N.S.	1	1.00	1.00	1.05	1.00	1.18	0.86	1.05	1.09	1.00
time (sec)	N/A	0.155	0.006	0.043	0.028	0.082	0.069	0.119	0.141	0.202

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	26	25	29	27	26	26	25
N.S.	1	1.00	1.04	0.96	0.93	1.07	1.00	0.96	0.96	0.93
time (sec)	N/A	0.153	0.007	0.042	0.024	0.062	0.127	0.118	0.166	0.019

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	28	27	27	31	27	24	27
N.S.	1	1.00	0.90	0.90	0.87	0.87	1.00	0.87	0.77	0.87
time (sec)	N/A	0.157	0.007	0.040	0.026	0.078	0.147	0.123	0.155	0.023

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	28	27	27	31	27	24	28
N.S.	1	1.00	0.88	0.85	0.82	0.82	0.94	0.82	0.73	0.85
time (sec)	N/A	0.153	0.006	0.041	0.027	0.081	0.190	0.128	0.147	0.021

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	28	27	27	31	27	24	28
N.S.	1	1.00	0.94	0.85	0.82	0.82	0.94	0.82	0.73	0.85
time (sec)	N/A	0.155	0.007	0.045	0.035	0.064	0.214	0.122	0.148	0.021

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	35	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.188	0.005	0.096	0.049	0.060	0.022	0.120	0.153	0.202

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	35	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.183	0.005	0.097	0.029	0.075	0.019	0.132	0.149	0.024

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	52	51	51	54	53	35	51
N.S.	1	1.00	0.91	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.180	0.007	0.097	0.031	0.073	0.022	0.125	0.144	0.026

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	52	51	51	54	53	35	51
N.S.	1	1.00	0.91	0.95	0.93	0.93	0.98	0.96	0.64	0.93
time (sec)	N/A	0.175	0.007	0.094	0.031	0.063	0.026	0.122	0.150	0.025

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	48	48	48	49	49	32	47
N.S.	1	1.00	1.21	1.26	1.26	1.26	1.29	1.29	0.84	1.24
time (sec)	N/A	0.169	0.006	0.090	0.032	0.085	0.023	0.124	0.153	0.024

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	46	46	46	46	46	31	45
N.S.	1	1.00	1.08	1.15	1.15	1.15	1.15	1.15	0.78	1.12
time (sec)	N/A	0.159	0.010	0.096	0.028	0.082	0.063	0.122	0.160	0.021

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	44	46	52	42	46	36	46
N.S.	1	1.00	0.98	1.00	1.05	1.18	0.95	1.05	0.82	1.05
time (sec)	N/A	0.170	0.015	0.096	0.043	0.065	0.094	0.126	0.155	0.027

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	43	46	53	46	47	37	46
N.S.	1	1.00	0.98	0.98	1.05	1.20	1.05	1.07	0.84	1.05
time (sec)	N/A	0.174	0.016	0.095	0.030	0.063	0.174	0.124	0.159	0.033

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	46	50	53	54	51	37	48
N.S.	1	1.00	0.98	0.94	1.02	1.08	1.10	1.04	0.76	0.98
time (sec)	N/A	0.174	0.017	0.100	0.027	0.065	0.289	0.119	0.159	0.201

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	48	51	51	56	51	35	49
N.S.	1	1.00	1.07	1.09	1.16	1.16	1.27	1.16	0.80	1.11
time (sec)	N/A	0.150	0.011	0.094	0.030	0.060	0.377	0.122	0.155	0.020

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	51	51	56	51	35	51
N.S.	1	1.00	0.91	0.87	0.93	0.93	1.02	0.93	0.64	0.93
time (sec)	N/A	0.177	0.010	0.095	0.037	0.066	0.469	0.124	0.158	0.020

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	51	51	56	51	35	51
N.S.	1	1.00	0.91	0.87	0.93	0.93	1.02	0.93	0.64	0.93
time (sec)	N/A	0.176	0.011	0.095	0.026	0.062	0.572	0.128	0.154	0.020

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	48	51	51	56	51	35	51
N.S.	1	1.00	0.91	0.87	0.93	0.93	1.02	0.93	0.64	0.93
time (sec)	N/A	0.177	0.012	0.098	0.032	0.062	0.734	0.114	0.162	0.021

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	82	77	46	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.09	1.03	0.61	0.92
time (sec)	N/A	0.211	0.007	0.103	0.030	0.058	0.029	0.122	0.153	0.175

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	80	77	46	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.07	1.03	0.61	0.92
time (sec)	N/A	0.203	0.006	0.102	0.028	0.064	0.022	0.121	0.160	0.016

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	82	77	46	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.09	1.03	0.61	0.92
time (sec)	N/A	0.199	0.007	0.097	0.039	0.072	0.022	0.115	0.159	0.016

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	74	73	73	80	76	46	68
N.S.	1	1.00	1.13	1.21	1.20	1.20	1.31	1.25	0.75	1.11
time (sec)	N/A	0.194	0.010	0.093	0.033	0.066	0.030	0.124	0.158	0.016

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	70	69	69	73	72	43	65
N.S.	1	1.00	1.76	1.84	1.82	1.82	1.92	1.89	1.13	1.71
time (sec)	N/A	0.164	0.007	0.101	0.041	0.068	0.032	0.120	0.157	0.019

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	63	69	68	68	73	70	42	63
N.S.	1	0.98	1.17	1.28	1.26	1.26	1.35	1.30	0.78	1.17
time (sec)	N/A	0.178	0.016	0.105	0.025	0.070	0.082	0.115	0.154	0.019

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	69	69	75	70	71	47	65
N.S.	1	1.00	1.03	1.06	1.06	1.15	1.08	1.09	0.72	1.00
time (sec)	N/A	0.192	0.018	0.097	0.027	0.075	0.118	0.114	0.153	0.023

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	63	69	74	68	69	47	70
N.S.	1	1.00	0.95	0.97	1.06	1.14	1.05	1.06	0.72	1.08
time (sec)	N/A	0.194	0.018	0.093	0.028	0.068	0.214	0.121	0.151	0.029

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	61	69	75	73	70	48	70
N.S.	1	1.00	1.05	0.95	1.08	1.17	1.14	1.09	0.75	1.09
time (sec)	N/A	0.194	0.024	0.098	0.026	0.073	0.410	0.121	0.154	0.037

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	58	70	64	72	75	80	73	48	71
N.S.	1	0.98	1.19	1.08	1.22	1.27	1.36	1.24	0.81	1.20
time (sec)	N/A	0.180	0.017	0.095	0.025	0.062	0.598	0.119	0.159	0.197

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	66	66	73	73	82	75	46	71
N.S.	1	1.00	1.50	1.50	1.66	1.66	1.86	1.70	1.05	1.61
time (sec)	N/A	0.148	0.014	0.093	0.032	0.060	0.711	0.120	0.155	0.026

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	66	73	73	82	75	46	73
N.S.	1	1.00	0.92	0.88	0.97	0.97	1.09	1.00	0.61	0.97
time (sec)	N/A	0.196	0.014	0.097	0.030	0.063	0.908	0.120	0.155	0.025

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	66	73	73	82	75	46	74
N.S.	1	1.00	0.92	0.88	0.97	0.97	1.09	1.00	0.61	0.99
time (sec)	N/A	0.194	0.015	0.093	0.030	0.063	1.062	0.124	0.160	0.024

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	66	73	73	82	75	46	74
N.S.	1	1.00	0.92	0.88	0.97	0.97	1.09	1.00	0.61	0.99
time (sec)	N/A	0.191	0.014	0.098	0.032	0.060	1.335	0.123	0.162	0.028

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	66	73	73	82	75	46	74
N.S.	1	1.00	0.92	0.88	0.97	0.97	1.09	1.00	0.61	0.99
time (sec)	N/A	0.198	0.015	0.098	0.033	0.072	1.526	0.120	0.160	0.025

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	133	124	68	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.14	1.06	0.58	0.91
time (sec)	N/A	0.272	0.012	0.101	0.031	0.096	0.029	0.121	0.149	0.199

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	68	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.58	0.91
time (sec)	N/A	0.249	0.010	0.094	0.027	0.074	0.036	0.118	0.149	0.025

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	117	120	119	119	134	124	68	106
N.S.	1	1.00	1.04	1.07	1.06	1.06	1.20	1.11	0.61	0.95
time (sec)	N/A	0.252	0.010	0.097	0.031	0.078	0.029	0.133	0.159	0.023

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	114	120	118	118	133	124	68	106
N.S.	1	1.00	1.31	1.38	1.36	1.36	1.53	1.43	0.78	1.22
time (sec)	N/A	0.226	0.011	0.096	0.034	0.073	0.030	0.114	0.155	0.025

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	115	121	119	119	134	125	68	107
N.S.	1	1.00	1.89	1.98	1.95	1.95	2.20	2.05	1.11	1.75
time (sec)	N/A	0.198	0.010	0.095	0.027	0.082	0.028	0.127	0.157	0.023

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	109	117	115	115	129	121	65	103
N.S.	1	1.00	2.87	3.08	3.03	3.03	3.39	3.18	1.71	2.71
time (sec)	N/A	0.172	0.010	0.093	0.039	0.079	0.034	0.124	0.158	0.032

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	77	108	115	114	114	126	118	64	101
N.S.	1	0.96	1.35	1.44	1.42	1.42	1.58	1.48	0.80	1.26
time (sec)	N/A	0.196	0.021	0.099	0.031	0.072	0.107	0.116	0.156	0.027

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	107	117	115	121	121	119	70	103
N.S.	1	1.00	1.02	1.11	1.10	1.15	1.15	1.13	0.67	0.98
time (sec)	N/A	0.241	0.026	0.098	0.041	0.095	0.140	0.126	0.155	0.188

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	113	116	121	122	119	69	108
N.S.	1	1.00	0.98	1.05	1.07	1.12	1.13	1.10	0.64	1.00
time (sec)	N/A	0.249	0.027	0.102	0.028	0.086	0.220	0.122	0.160	0.029

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	109	107	117	121	122	118	69	114
N.S.	1	1.00	1.01	0.99	1.08	1.12	1.13	1.09	0.64	1.06
time (sec)	N/A	0.251	0.018	0.100	0.029	0.089	0.405	0.125	0.157	0.185

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	106	101	116	121	122	116	70	117
N.S.	1	1.00	0.99	0.94	1.08	1.13	1.14	1.08	0.65	1.09
time (sec)	N/A	0.243	0.029	0.102	0.040	0.069	0.692	0.124	0.151	0.036

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	106	99	115	121	124	116	70	116
N.S.	1	1.00	1.02	0.95	1.11	1.16	1.19	1.12	0.67	1.12
time (sec)	N/A	0.247	0.030	0.100	0.033	0.075	1.091	0.120	0.149	0.214

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	82	109	102	118	121	131	119	70	117
N.S.	1	0.96	1.28	1.20	1.39	1.42	1.54	1.40	0.82	1.38
time (sec)	N/A	0.200	0.025	0.099	0.041	0.065	1.564	0.120	0.157	0.205

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	104	104	119	119	133	123	68	118
N.S.	1	1.00	2.36	2.36	2.70	2.70	3.02	2.80	1.55	2.68
time (sec)	N/A	0.150	0.020	0.098	0.031	0.059	1.968	0.121	0.156	0.194

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	107	104	119	119	133	123	68	120
N.S.	1	1.00	1.53	1.49	1.70	1.70	1.90	1.76	0.97	1.71
time (sec)	N/A	0.165	0.020	0.099	0.030	0.081	2.457	0.116	0.156	0.210

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	107	104	119	119	133	123	68	119
N.S.	1	1.00	0.93	0.90	1.03	1.03	1.16	1.07	0.59	1.03
time (sec)	N/A	0.232	0.020	0.098	0.035	0.067	3.019	0.120	0.149	0.036

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	107	104	119	119	133	123	68	119
N.S.	1	1.00	0.91	0.89	1.02	1.02	1.14	1.05	0.58	1.02
time (sec)	N/A	0.238	0.020	0.097	0.028	0.075	3.893	0.124	0.157	0.035

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	107	104	119	119	133	123	68	120
N.S.	1	1.00	0.91	0.89	1.02	1.02	1.14	1.05	0.58	1.03
time (sec)	N/A	0.239	0.020	0.097	0.027	0.083	4.829	0.121	0.153	0.200

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	229	236	243	243	269	245	123	211
N.S.	1	1.00	1.00	1.03	1.06	1.06	1.17	1.07	0.54	0.92
time (sec)	N/A	0.457	0.025	0.112	0.036	0.072	0.042	0.117	0.158	0.075

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	231	236	243	243	269	245	123	211
N.S.	1	1.00	1.00	1.02	1.05	1.05	1.16	1.06	0.53	0.91
time (sec)	N/A	0.409	0.019	0.108	0.043	0.063	0.039	0.123	0.158	0.221

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	235	243	243	264	244	123	210
N.S.	1	1.00	0.95	0.98	1.01	1.01	1.10	1.02	0.51	0.88
time (sec)	N/A	0.430	0.018	0.106	0.031	0.069	0.043	0.113	0.160	0.057

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	227	235	243	243	262	244	123	211
N.S.	1	1.00	1.06	1.09	1.13	1.13	1.22	1.13	0.57	0.98
time (sec)	N/A	0.401	0.019	0.104	0.032	0.082	0.041	0.124	0.153	0.055

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	228	236	242	242	264	245	123	210
N.S.	1	1.00	1.19	1.24	1.27	1.27	1.38	1.28	0.64	1.10
time (sec)	N/A	0.366	0.020	0.106	0.032	0.070	0.048	0.115	0.149	0.057

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	229	236	243	243	265	245	123	211
N.S.	1	1.00	1.40	1.45	1.49	1.49	1.63	1.50	0.75	1.29
time (sec)	N/A	0.356	0.018	0.105	0.038	0.077	0.044	0.116	0.148	0.064

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	231	236	243	243	269	245	123	211
N.S.	1	1.00	1.66	1.70	1.75	1.75	1.94	1.76	0.88	1.52
time (sec)	N/A	0.327	0.018	0.107	0.044	0.069	0.046	0.124	0.146	0.057

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	231	236	243	243	265	245	123	211
N.S.	1	1.00	2.06	2.11	2.17	2.17	2.37	2.19	1.10	1.88
time (sec)	N/A	0.302	0.017	0.104	0.054	0.070	0.043	0.122	0.156	0.057

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	226	236	242	242	262	245	123	210
N.S.	1	1.00	2.60	2.71	2.78	2.78	3.01	2.82	1.41	2.41
time (sec)	N/A	0.247	0.017	0.102	0.055	0.091	0.046	0.128	0.149	0.057

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	218	236	243	243	262	245	123	211
N.S.	1	1.00	3.57	3.87	3.98	3.98	4.30	4.02	2.02	3.46
time (sec)	N/A	0.223	0.022	0.099	0.037	0.082	0.054	0.124	0.152	0.058

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	198	232	240	240	248	241	120	208
N.S.	1	1.00	5.21	6.11	6.32	6.32	6.53	6.34	3.16	5.47
time (sec)	N/A	0.178	0.035	0.101	0.037	0.078	0.047	0.118	0.149	0.222

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	140	208	230	238	238	246	238	119	205
N.S.	1	0.95	1.41	1.55	1.61	1.61	1.66	1.61	0.80	1.39
time (sec)	N/A	0.246	0.031	0.102	0.031	0.084	0.206	0.124	0.159	0.064

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	209	236	239	245	248	239	125	207
N.S.	1	1.00	0.96	1.09	1.10	1.13	1.14	1.10	0.58	0.95
time (sec)	N/A	0.388	0.049	0.104	0.039	0.089	0.239	0.119	0.156	0.064

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	206	234	240	245	248	240	125	211
N.S.	1	1.00	0.95	1.08	1.11	1.13	1.15	1.11	0.58	0.98
time (sec)	N/A	0.368	0.054	0.105	0.032	0.083	0.344	0.137	0.156	0.212

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	208	230	241	245	250	241	125	215
N.S.	1	1.00	0.96	1.06	1.12	1.13	1.16	1.12	0.58	1.00
time (sec)	N/A	0.351	0.058	0.105	0.032	0.084	0.515	0.119	0.153	0.209

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	210	226	240	245	248	241	125	217
N.S.	1	1.00	0.98	1.05	1.12	1.14	1.15	1.12	0.58	1.01
time (sec)	N/A	0.366	0.050	0.103	0.033	0.072	0.819	0.128	0.151	0.050

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	210	222	241	245	250	241	125	221
N.S.	1	1.00	0.96	1.02	1.11	1.12	1.15	1.11	0.57	1.01
time (sec)	N/A	0.376	0.052	0.105	0.032	0.075	1.244	0.124	0.157	0.051

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	210	218	241	245	253	241	125	224
N.S.	1	1.00	0.96	1.00	1.11	1.12	1.16	1.11	0.57	1.03
time (sec)	N/A	0.375	0.054	0.106	0.033	0.106	1.860	0.121	0.150	0.052

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	210	214	241	245	253	241	125	227
N.S.	1	1.00	0.97	0.99	1.12	1.13	1.17	1.12	0.58	1.05
time (sec)	N/A	0.375	0.054	0.107	0.028	0.085	2.665	0.124	0.146	0.209

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	208	210	241	245	252	241	125	230
N.S.	1	1.00	0.96	0.97	1.12	1.13	1.17	1.12	0.58	1.06
time (sec)	N/A	0.389	0.056	0.106	0.030	0.070	3.646	0.120	0.166	0.050

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	206	205	240	245	252	240	125	232
N.S.	1	1.00	0.96	0.95	1.12	1.14	1.17	1.12	0.58	1.08
time (sec)	N/A	0.383	0.058	0.108	0.030	0.074	5.285	0.124	0.152	0.211

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	209	203	239	245	252	240	125	231
N.S.	1	1.00	0.97	0.94	1.11	1.13	1.17	1.11	0.58	1.07
time (sec)	N/A	0.369	0.059	0.109	0.029	0.067	7.095	0.116	0.179	0.244

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	145	212	206	242	245	258	243	125	232
N.S.	1	0.95	1.39	1.35	1.58	1.60	1.69	1.59	0.82	1.52
time (sec)	N/A	0.266	0.063	0.106	0.036	0.068	9.629	0.121	0.158	0.243

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	199	208	243	243	260	243	123	233
N.S.	1	1.00	4.52	4.73	5.52	5.52	5.91	5.52	2.80	5.30
time (sec)	N/A	0.152	0.034	0.104	0.042	0.063	12.452	0.119	0.159	0.078

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	71	202	208	243	243	260	243	123	235
N.S.	1	0.99	2.81	2.89	3.38	3.38	3.61	3.38	1.71	3.26
time (sec)	N/A	0.167	0.034	0.108	0.040	0.083	16.586	0.119	0.161	0.082

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	97	202	208	243	243	260	243	123	235
N.S.	1	0.96	2.00	2.06	2.41	2.41	2.57	2.41	1.22	2.33
time (sec)	N/A	0.181	0.037	0.109	0.032	0.067	22.679	0.125	0.156	0.080

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	123	202	208	243	243	260	243	123	235
N.S.	1	0.95	1.55	1.60	1.87	1.87	2.00	1.87	0.95	1.81
time (sec)	N/A	0.200	0.034	0.114	0.042	0.065	31.139	0.121	0.156	0.240

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	149	222	208	243	243	260	243	123	235
N.S.	1	0.94	1.40	1.31	1.53	1.53	1.64	1.53	0.77	1.48
time (sec)	N/A	0.212	0.045	0.109	0.036	0.082	43.195	0.124	0.148	0.246

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	175	222	208	243	243	260	243	123	235
N.S.	1	0.93	1.18	1.11	1.29	1.29	1.38	1.29	0.65	1.25
time (sec)	N/A	0.222	0.044	0.112	0.034	0.096	68.872	0.118	0.154	0.081

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	222	208	243	243	260	243	123	235
N.S.	1	1.00	0.97	0.91	1.06	1.06	1.14	1.06	0.54	1.03
time (sec)	N/A	0.393	0.044	0.110	0.029	0.092	98.836	0.133	0.160	0.241

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	220	208	243	243	260	243	123	234
N.S.	1	1.00	0.97	0.92	1.07	1.07	1.15	1.07	0.54	1.03
time (sec)	N/A	0.367	0.044	0.111	0.048	0.072	137.242	0.125	0.155	0.079

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	222	208	243	243	0	243	123	234
N.S.	1	1.00	0.97	0.91	1.06	1.06	0.00	1.06	0.54	1.02
time (sec)	N/A	0.373	0.045	0.114	0.074	0.095	0.000	0.121	0.154	0.091

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	359	374	387	387	422	389	387	331
N.S.	1	1.00	3.15	3.28	3.39	3.39	3.70	3.41	3.39	2.90
time (sec)	N/A	0.456	0.047	0.128	0.042	0.080	0.061	0.117	0.156	0.201

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	355	374	387	387	413	389	387	331
N.S.	1	1.00	4.03	4.25	4.40	4.40	4.69	4.42	4.40	3.76
time (sec)	N/A	0.367	0.035	0.117	0.029	0.078	0.077	0.119	0.149	0.329

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	347	374	387	387	408	389	387	331
N.S.	1	1.00	5.60	6.03	6.24	6.24	6.58	6.27	6.24	5.34
time (sec)	N/A	0.332	0.035	0.115	0.058	0.080	0.065	0.126	0.146	0.170

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	342	370	384	384	393	385	385	328
N.S.	1	1.00	9.00	9.74	10.11	10.11	10.34	10.13	10.13	8.63
time (sec)	N/A	0.180	0.029	0.111	0.028	0.078	0.081	0.124	0.146	0.345

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	42	11	36	36	37	36	33	36
N.S.	1	1.00	3.50	0.92	3.00	3.00	3.08	3.00	2.75	3.00
time (sec)	N/A	0.126	0.002	0.096	0.037	0.068	0.021	0.133	0.160	0.016

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	100	101	116	117	109	119	5	120
N.S.	1	1.00	0.93	0.94	1.07	1.08	1.01	1.10	0.05	1.11
time (sec)	N/A	0.265	0.024	0.132	0.030	0.070	0.185	0.120	0.161	0.028

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	82	92	94	85	94	5	94
N.S.	1	1.00	0.92	0.94	1.06	1.08	0.98	1.08	0.06	1.08
time (sec)	N/A	0.229	0.018	0.135	0.072	0.098	0.141	0.124	0.156	0.022

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	63	70	71	61	71	5	72
N.S.	1	1.00	0.92	0.95	1.06	1.08	0.92	1.08	0.08	1.09
time (sec)	N/A	0.210	0.015	0.135	0.039	0.090	0.174	0.120	0.156	0.187

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	45	47	37	45	5	46
N.S.	1	1.00	0.91	0.96	1.00	1.04	0.82	1.00	0.11	1.02
time (sec)	N/A	0.183	0.010	0.127	0.043	0.073	0.102	0.132	0.150	0.198

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	26	25	20	27	1	25
N.S.	1	1.00	1.00	1.04	1.04	1.00	0.80	1.08	0.04	1.00
time (sec)	N/A	0.162	0.006	0.132	0.068	0.066	0.090	0.117	0.155	0.027

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	30	29	28	41	31	2	28
N.S.	1	1.00	0.97	1.00	0.97	0.93	1.37	1.03	0.07	0.93
time (sec)	N/A	0.165	0.008	0.133	0.046	0.074	0.293	0.119	0.152	0.066

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	43	43	41	95	51	5	33
N.S.	1	1.00	0.98	1.00	1.00	0.95	2.21	1.19	0.12	0.77
time (sec)	N/A	0.183	0.013	0.142	0.033	0.070	0.191	0.126	0.156	0.216

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	61	63	69	131	75	5	74
N.S.	1	1.00	0.94	0.98	1.02	1.11	2.11	1.21	0.08	1.19
time (sec)	N/A	0.215	0.023	0.153	0.031	0.083	0.232	0.124	0.156	0.206

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	83	89	94	165	99	5	96
N.S.	1	1.00	0.94	0.97	1.03	1.09	1.92	1.15	0.06	1.12
time (sec)	N/A	0.222	0.035	0.153	0.026	0.082	0.359	0.124	0.162	0.054

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	100	101	112	117	189	122	5	116
N.S.	1	1.00	0.94	0.95	1.06	1.10	1.78	1.15	0.05	1.09
time (sec)	N/A	0.243	0.042	0.152	0.057	0.070	0.290	0.122	0.154	0.055

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	116	123	164	119	175	52	173
N.S.	1	1.00	0.95	1.03	1.09	1.45	1.05	1.55	0.46	1.53
time (sec)	N/A	0.284	0.041	0.134	0.034	0.083	0.275	0.118	0.158	0.199

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	90	101	140	92	144	41	115
N.S.	1	1.00	0.97	1.00	1.12	1.56	1.02	1.60	0.46	1.28
time (sec)	N/A	0.242	0.036	0.131	0.031	0.075	0.222	0.113	0.155	0.036

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	67	74	113	68	111	29	77
N.S.	1	1.00	0.96	0.97	1.07	1.64	0.99	1.61	0.42	1.12
time (sec)	N/A	0.216	0.036	0.132	0.044	0.072	0.193	0.153	0.149	0.212

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	46	53	72	44	80	17	54
N.S.	1	1.00	0.91	1.02	1.18	1.60	0.98	1.78	0.38	1.20
time (sec)	N/A	0.191	0.017	0.128	0.032	0.067	0.186	0.119	0.151	0.200

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	33	34	37	27	57	10	32
N.S.	1	1.00	0.97	1.03	1.06	1.16	0.84	1.78	0.31	1.00
time (sec)	N/A	0.168	0.008	0.125	0.030	0.066	0.096	0.123	0.153	0.025

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	42	44	62	32	55	15	39
N.S.	1	1.00	0.90	1.00	1.05	1.48	0.76	1.31	0.36	0.93
time (sec)	N/A	0.180	0.018	0.133	0.031	0.076	0.165	0.117	0.159	0.037

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	64	67	107	128	86	26	58
N.S.	1	1.00	0.86	0.98	1.03	1.65	1.97	1.32	0.40	0.89
time (sec)	N/A	0.212	0.026	0.153	0.037	0.071	0.327	0.127	0.159	0.055

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	84	99	150	184	130	43	104
N.S.	1	1.00	1.00	0.99	1.16	1.76	2.16	1.53	0.51	1.22
time (sec)	N/A	0.235	0.048	0.154	0.059	0.092	0.302	0.122	0.162	0.231

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	109	128	179	219	161	54	131
N.S.	1	1.00	0.94	0.96	1.13	1.58	1.94	1.42	0.48	1.16
time (sec)	N/A	0.272	0.058	0.155	0.071	0.088	0.456	0.130	0.162	0.254

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	116	133	197	136	125	71	147
N.S.	1	1.00	0.93	1.00	1.15	1.70	1.17	1.08	0.61	1.27
time (sec)	N/A	0.289	0.042	0.138	0.027	0.078	0.455	0.111	0.160	0.043

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	86	90	108	171	107	100	60	108
N.S.	1	1.00	0.91	0.96	1.15	1.82	1.14	1.06	0.64	1.15
time (sec)	N/A	0.248	0.036	0.140	0.029	0.084	0.501	0.134	0.162	0.049

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	75	69	85	134	83	72	46	85
N.S.	1	1.00	1.06	0.97	1.20	1.89	1.17	1.01	0.65	1.20
time (sec)	N/A	0.220	0.019	0.132	0.033	0.080	0.320	0.118	0.155	0.223

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	54	50	65	81	63	54	33	63
N.S.	1	1.00	0.98	0.91	1.18	1.47	1.15	0.98	0.60	1.15
time (sec)	N/A	0.192	0.012	0.128	0.033	0.072	0.222	0.121	0.161	0.227

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	38	38	39	24	12	39
N.S.	1	1.00	0.93	0.89	1.36	1.36	1.39	0.86	0.43	1.39
time (sec)	N/A	0.137	0.007	0.122	0.034	0.064	0.143	0.110	0.153	0.193

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	56	68	109	63	59	44	61
N.S.	1	1.00	0.93	0.98	1.19	1.91	1.11	1.04	0.77	1.07
time (sec)	N/A	0.196	0.029	0.140	0.035	0.076	0.271	0.121	0.148	0.200

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	81	85	100	187	168	99	70	87
N.S.	1	1.00	0.92	0.97	1.14	2.12	1.91	1.12	0.80	0.99
time (sec)	N/A	0.240	0.034	0.149	0.029	0.127	0.352	0.123	0.152	0.226

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	107	131	225	219	124	86	136
N.S.	1	1.00	0.93	0.97	1.19	2.05	1.99	1.13	0.78	1.24
time (sec)	N/A	0.264	0.055	0.153	0.030	0.070	0.533	0.121	0.150	0.071

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	134	164	262	262	158	97	164
N.S.	1	1.00	0.92	0.96	1.17	1.87	1.87	1.13	0.69	1.17
time (sec)	N/A	0.311	0.082	0.164	0.043	0.095	0.427	0.120	0.151	0.243

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	14	16	18	12
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	1.12	0.75
time (sec)	N/A	0.149	0.003	0.125	0.026	0.063	0.051	0.117	0.155	0.022

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	28	27	32	46	29	26	27
N.S.	1	1.00	0.79	0.72	0.69	0.82	1.18	0.74	0.67	0.69
time (sec)	N/A	0.162	0.023	0.175	0.030	0.094	0.414	0.125	0.147	0.184

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	28	27	32	46	29	26	27
N.S.	1	1.00	0.79	0.72	0.69	0.82	1.18	0.74	0.67	0.69
time (sec)	N/A	0.157	0.019	0.187	0.049	0.081	0.264	0.116	0.146	0.188

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	28	27	32	46	29	26	27
N.S.	1	1.00	0.85	0.72	0.69	0.82	1.18	0.74	0.67	0.69
time (sec)	N/A	0.157	0.019	0.163	0.027	0.086	0.213	0.129	0.149	0.189

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	28	27	30	37	29	24	27
N.S.	1	1.00	0.85	0.72	0.69	0.77	0.95	0.74	0.62	0.69
time (sec)	N/A	0.161	0.019	0.163	0.027	0.081	0.646	0.123	0.153	0.026

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	27	27	27	44	29	23	27
N.S.	1	1.00	0.84	0.73	0.73	0.73	1.19	0.78	0.62	0.73
time (sec)	N/A	0.155	0.021	0.160	0.033	0.074	0.085	0.122	0.155	0.187

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	27	26	41	29	24	27
N.S.	1	1.00	0.89	0.80	0.77	0.74	1.17	0.83	0.69	0.77
time (sec)	N/A	0.154	0.022	0.069	0.032	0.080	0.211	0.123	0.148	0.024

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	27	27	27	41	27	28	27
N.S.	1	1.00	0.80	0.77	0.77	0.77	1.17	0.77	0.80	0.77
time (sec)	N/A	0.155	0.025	0.072	0.034	0.071	0.206	0.124	0.149	0.021

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	28	27	27	46	27	28	28
N.S.	1	1.00	0.84	0.76	0.73	0.73	1.24	0.73	0.76	0.76
time (sec)	N/A	0.154	0.026	0.073	0.040	0.109	0.273	0.122	0.146	0.188

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	56	80	53	37	51
N.S.	1	1.00	0.83	0.83	0.81	0.89	1.27	0.84	0.59	0.81
time (sec)	N/A	0.180	0.035	0.196	0.031	0.066	0.505	0.122	0.152	0.194

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	56	80	53	37	51
N.S.	1	1.00	0.83	0.83	0.81	0.89	1.27	0.84	0.59	0.81
time (sec)	N/A	0.176	0.032	0.188	0.040	0.067	0.411	0.121	0.151	0.026

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	56	80	53	37	51
N.S.	1	1.00	0.83	0.83	0.81	0.89	1.27	0.84	0.59	0.81
time (sec)	N/A	0.179	0.032	0.174	0.039	0.067	0.215	0.119	0.149	0.028

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	52	51	54	66	53	35	51
N.S.	1	1.00	0.83	0.83	0.81	0.86	1.05	0.84	0.56	0.81
time (sec)	N/A	0.176	0.034	0.171	0.031	0.068	0.804	0.120	0.147	0.029

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	51	51	51	51	78	53	34	51
N.S.	1	1.00	0.84	0.84	0.84	0.84	1.28	0.87	0.56	0.84
time (sec)	N/A	0.172	0.032	0.172	0.031	0.071	0.131	0.132	0.148	0.029

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	52	51	51	75	53	35	51
N.S.	1	1.00	0.83	0.88	0.86	0.86	1.27	0.90	0.59	0.86
time (sec)	N/A	0.174	0.038	0.114	0.028	0.066	0.211	0.126	0.150	0.029

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	51	51	50	73	51	38	51
N.S.	1	1.00	0.80	0.86	0.86	0.85	1.24	0.86	0.64	0.86
time (sec)	N/A	0.175	0.039	0.106	0.028	0.102	0.225	0.123	0.149	0.032

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	48	52	51	75	52	39	52
N.S.	1	1.00	0.80	0.81	0.88	0.86	1.27	0.88	0.66	0.88
time (sec)	N/A	0.179	0.041	0.106	0.036	0.078	0.374	0.119	0.149	0.203

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	76	73	78	114	77	48	69
N.S.	1	1.00	0.84	0.89	0.86	0.92	1.34	0.91	0.56	0.81
time (sec)	N/A	0.198	0.048	0.213	0.030	0.080	0.641	0.118	0.146	0.027

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	76	73	78	114	77	48	69
N.S.	1	1.00	0.84	0.89	0.86	0.92	1.34	0.91	0.56	0.81
time (sec)	N/A	0.202	0.041	0.201	0.039	0.075	0.410	0.125	0.150	0.019

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	76	73	78	114	77	48	69
N.S.	1	1.00	0.84	0.89	0.86	0.92	1.34	0.91	0.56	0.81
time (sec)	N/A	0.193	0.040	0.175	0.027	0.075	0.245	0.125	0.148	0.019

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	76	73	76	95	77	46	69
N.S.	1	1.00	0.84	0.89	0.86	0.89	1.12	0.91	0.54	0.81
time (sec)	N/A	0.194	0.040	0.181	0.030	0.070	0.680	0.124	0.149	0.024

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	75	73	73	110	77	45	69
N.S.	1	1.00	0.84	0.90	0.88	0.88	1.33	0.93	0.54	0.83
time (sec)	N/A	0.192	0.041	0.176	0.040	0.070	0.164	0.128	0.145	0.018

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	67	76	73	73	105	77	47	69
N.S.	1	1.00	0.85	0.96	0.92	0.92	1.33	0.97	0.59	0.87
time (sec)	N/A	0.190	0.055	0.106	0.040	0.091	0.236	0.124	0.141	0.020

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	66	75	73	73	105	75	50	70
N.S.	1	1.00	0.81	0.93	0.90	0.90	1.30	0.93	0.62	0.86
time (sec)	N/A	0.191	0.062	0.111	0.038	0.094	0.254	0.124	0.149	0.205

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	65	69	74	73	105	76	50	75
N.S.	1	1.00	0.80	0.85	0.91	0.90	1.30	0.94	0.62	0.93
time (sec)	N/A	0.191	0.045	0.112	0.029	0.095	0.305	0.123	0.149	0.194

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	66	38	33	32	37	75	32	43	32
N.S.	1	1.50	0.86	0.75	0.73	0.84	1.70	0.73	0.98	0.73
time (sec)	N/A	0.169	0.040	0.221	0.126	0.092	0.242	0.124	0.159	0.186

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	123	120	124	128	276	328	139	7	151
N.S.	1	0.90	0.88	0.91	0.94	2.03	2.41	1.02	0.05	1.11
time (sec)	N/A	0.206	0.154	0.159	0.122	0.111	11.841	0.124	0.159	0.033

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	104	101	100	105	229	294	115	7	125
N.S.	1	0.92	0.89	0.88	0.93	2.03	2.60	1.02	0.06	1.11
time (sec)	N/A	0.186	0.115	0.161	0.114	0.113	3.687	0.118	0.159	0.208

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	81	76	82	180	260	91	7	101
N.S.	1	0.94	0.90	0.84	0.91	2.00	2.89	1.01	0.08	1.12
time (sec)	N/A	0.175	0.106	0.147	0.146	0.087	1.548	0.122	0.142	0.042

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	63	53	58	129	221	64	5	76
N.S.	1	0.96	0.91	0.77	0.84	1.87	3.20	0.93	0.07	1.10
time (sec)	N/A	0.165	0.062	0.149	0.151	0.091	0.794	0.126	0.148	0.052

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	102	180	39	4	37
N.S.	1	1.00	1.00	0.82	0.80	2.08	3.67	0.80	0.08	0.76
time (sec)	N/A	0.156	0.053	0.153	0.120	0.107	0.496	0.124	0.150	0.250

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	112	178	39	6	50
N.S.	1	1.00	1.00	0.82	0.80	2.29	3.63	0.80	0.12	1.02
time (sec)	N/A	0.153	0.045	0.154	0.111	0.095	0.902	0.122	0.152	0.045

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	64	54	56	141	218	55	9	54
N.S.	1	0.97	0.93	0.78	0.81	2.04	3.16	0.80	0.13	0.78
time (sec)	N/A	0.167	0.084	0.154	0.149	0.095	1.994	0.124	0.150	0.235

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	86	83	76	80	192	262	80	9	71
N.S.	1	0.96	0.92	0.84	0.89	2.13	2.91	0.89	0.10	0.79
time (sec)	N/A	0.184	0.109	0.156	0.132	0.101	5.834	0.120	0.156	0.216

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	105	103	95	103	241	299	104	9	90
N.S.	1	0.93	0.91	0.84	0.91	2.13	2.65	0.92	0.08	0.80
time (sec)	N/A	0.184	0.126	0.160	0.110	0.107	16.470	0.117	0.154	0.219

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	124	122	114	126	288	333	128	9	109
N.S.	1	0.91	0.90	0.84	0.93	2.12	2.45	0.94	0.07	0.80
time (sec)	N/A	0.194	0.152	0.158	0.145	0.085	63.255	0.128	0.157	0.253

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	128	124	139	341	986	146	68	209
N.S.	1	1.01	0.90	0.87	0.97	2.38	6.90	1.02	0.48	1.46
time (sec)	N/A	0.206	0.168	0.182	0.139	0.089	42.003	0.131	0.158	0.208

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	125	110	99	115	290	877	122	55	146
N.S.	1	1.05	0.92	0.83	0.97	2.44	7.37	1.03	0.46	1.23
time (sec)	N/A	0.203	0.144	0.179	0.150	0.121	15.101	0.126	0.154	0.036

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	106	88	77	88	231	762	95	39	107
N.S.	1	1.12	0.93	0.81	0.93	2.43	8.02	1.00	0.41	1.13
time (sec)	N/A	0.199	0.117	0.184	0.123	0.094	4.494	0.124	0.145	0.048

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	86	67	62	65	198	634	65	29	62
N.S.	1	1.18	0.92	0.85	0.89	2.71	8.68	0.89	0.40	0.85
time (sec)	N/A	0.183	0.084	0.168	0.178	0.093	2.200	0.124	0.150	0.056

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	57	58	177	615	60	25	51
N.S.	1	1.00	1.02	0.90	0.92	2.81	9.76	0.95	0.40	0.81
time (sec)	N/A	0.170	0.089	0.145	0.116	0.122	2.547	0.132	0.150	0.226

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	87	67	64	65	215	794	60	33	65
N.S.	1	1.16	0.89	0.85	0.87	2.87	10.59	0.80	0.44	0.87
time (sec)	N/A	0.183	0.082	0.165	0.115	0.103	5.810	0.119	0.152	0.219

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	106	90	77	93	257	882	85	46	81
N.S.	1	1.12	0.95	0.81	0.98	2.71	9.28	0.89	0.48	0.85
time (sec)	N/A	0.194	0.118	0.172	0.120	0.092	21.162	0.124	0.153	0.232

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	112	101	118	316	1017	110	61	103
N.S.	1	1.03	0.93	0.83	0.98	2.61	8.40	0.91	0.50	0.85
time (sec)	N/A	0.204	0.147	0.172	0.128	0.089	48.617	0.117	0.151	0.254

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	144	131	121	143	367	1142	136	72	121
N.S.	1	1.01	0.92	0.85	1.00	2.57	7.99	0.95	0.50	0.85
time (sec)	N/A	0.218	0.162	0.185	0.128	0.095	122.794	0.122	0.151	0.245

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	129	120	151	408	1652	146	99	183
N.S.	1	1.00	0.84	0.78	0.99	2.67	10.80	0.95	0.65	1.20
time (sec)	N/A	0.223	0.174	0.206	0.114	0.100	71.732	0.134	0.151	0.219

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	134	110	98	124	349	1496	119	84	143
N.S.	1	1.02	0.84	0.75	0.95	2.66	11.42	0.91	0.64	1.09
time (sec)	N/A	0.216	0.168	0.200	0.112	0.103	23.567	0.125	0.153	0.054

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	114	91	83	99	319	1333	87	67	96
N.S.	1	1.08	0.86	0.78	0.93	3.01	12.58	0.82	0.63	0.91
time (sec)	N/A	0.199	0.144	0.165	0.108	0.104	10.441	0.119	0.159	0.226

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	95	86	79	94	291	1316	82	60	84
N.S.	1	0.98	0.89	0.81	0.97	3.00	13.57	0.85	0.62	0.87
time (sec)	N/A	0.189	0.140	0.138	0.121	0.110	6.120	0.120	0.155	0.237

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	94	86	80	94	291	1345	82	59	84
N.S.	1	0.94	0.86	0.80	0.94	2.91	13.45	0.82	0.59	0.84
time (sec)	N/A	0.185	0.131	0.138	0.122	0.115	8.367	0.122	0.161	0.229

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	93	84	98	331	1598	86	71	116
N.S.	1	1.07	0.87	0.79	0.92	3.09	14.93	0.80	0.66	1.08
time (sec)	N/A	0.197	0.138	0.168	0.137	0.116	18.675	0.127	0.155	0.254

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	133	112	98	128	375	1703	108	92	114
N.S.	1	1.02	0.86	0.75	0.98	2.88	13.10	0.83	0.71	0.88
time (sec)	N/A	0.209	0.169	0.185	0.118	0.122	43.107	0.125	0.158	0.255

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	152	133	121	154	434	1882	135	107	135
N.S.	1	0.97	0.85	0.78	0.99	2.78	12.06	0.87	0.69	0.87
time (sec)	N/A	0.219	0.171	0.191	0.125	0.110	121.433	0.119	0.160	0.265

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	106	92	123	143	168	304	74	137
N.S.	1	1.00	0.70	0.61	0.81	0.95	1.11	2.01	0.49	0.91
time (sec)	N/A	0.263	0.062	0.392	0.035	0.070	0.709	0.126	0.159	0.207

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	75	100	119	139	256	63	111
N.S.	1	1.00	0.71	0.61	0.82	0.98	1.14	2.10	0.52	0.91
time (sec)	N/A	0.235	0.052	0.388	0.042	0.073	0.699	0.126	0.155	0.027

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	65	57	77	95	110	207	52	85
N.S.	1	1.00	0.68	0.60	0.81	1.00	1.16	2.18	0.55	0.89
time (sec)	N/A	0.214	0.043	0.378	0.033	0.075	0.677	0.119	0.159	0.200

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	41	54	71	82	158	40	52
N.S.	1	1.00	0.73	0.61	0.81	1.06	1.22	2.36	0.60	0.78
time (sec)	N/A	0.186	0.032	0.374	0.027	0.066	0.636	0.118	0.154	0.036

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	33	46	51	100	27	29
N.S.	1	1.00	0.71	0.64	0.79	1.10	1.21	2.38	0.64	0.69
time (sec)	N/A	0.162	0.021	0.316	0.026	0.068	0.551	0.133	0.157	0.189

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	53	46	60	108	70	55	51	45
N.S.	1	1.02	0.98	0.85	1.11	2.00	1.30	1.02	0.94	0.83
time (sec)	N/A	0.162	0.049	0.165	0.111	0.080	1.545	0.126	0.159	0.038

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	69	53	49	76	121	94	63	59	49
N.S.	1	1.17	0.90	0.83	1.29	2.05	1.59	1.07	1.00	0.83
time (sec)	N/A	0.176	0.086	0.185	0.110	0.080	13.017	0.127	0.157	0.222

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	75	68	57	120	155	151	110	73	94
N.S.	1	0.95	0.86	0.72	1.52	1.96	1.91	1.39	0.92	1.19
time (sec)	N/A	0.172	0.130	0.190	0.113	0.081	37.778	0.127	0.159	0.064

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	99	91	82	152	206	236	114	90	129
N.S.	1	0.91	0.83	0.75	1.39	1.89	2.17	1.05	0.83	1.18
time (sec)	N/A	0.194	0.185	0.200	0.113	0.081	49.494	0.128	0.163	0.236

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	128	110	102	195	255	296	176	107	178
N.S.	1	0.90	0.77	0.72	1.37	1.80	2.08	1.24	0.75	1.25
time (sec)	N/A	0.208	0.200	0.219	0.110	0.082	92.464	0.139	0.160	0.245

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	103	91	123	167	168	494	85	137
N.S.	1	1.00	0.68	0.60	0.81	1.11	1.11	3.27	0.56	0.91
time (sec)	N/A	0.273	0.065	0.396	0.032	0.070	0.809	0.120	0.158	0.197

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	75	100	143	139	422	74	111
N.S.	1	1.00	0.71	0.61	0.82	1.17	1.14	3.46	0.61	0.91
time (sec)	N/A	0.246	0.057	0.390	0.044	0.071	0.741	0.127	0.164	0.025

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	68	58	77	120	110	350	63	85
N.S.	1	1.00	0.72	0.61	0.81	1.26	1.16	3.68	0.66	0.89
time (sec)	N/A	0.209	0.049	0.374	0.030	0.065	0.733	0.133	0.148	0.035

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	41	54	95	82	275	51	52
N.S.	1	1.00	0.73	0.61	0.81	1.42	1.22	4.10	0.76	0.78
time (sec)	N/A	0.186	0.035	0.364	0.037	0.068	0.702	0.125	0.153	0.035

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	33	69	146	192	38	29
N.S.	1	1.00	0.71	0.64	0.79	1.64	3.48	4.57	0.90	0.69
time (sec)	N/A	0.164	0.025	0.315	0.033	0.067	0.208	0.128	0.150	0.024

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	66	58	73	155	87	72	72	95
N.S.	1	1.03	0.96	0.84	1.06	2.25	1.26	1.04	1.04	1.38
time (sec)	N/A	0.176	0.080	0.170	0.161	0.087	1.643	0.126	0.154	0.041

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	86	71	67	97	148	170	102	78	96
N.S.	1	1.06	0.88	0.83	1.20	1.83	2.10	1.26	0.96	1.19
time (sec)	N/A	0.188	0.098	0.198	0.120	0.091	14.034	0.128	0.152	0.044

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	72	68	130	174	228	119	84	127
N.S.	1	1.01	0.80	0.76	1.44	1.93	2.53	1.32	0.93	1.41
time (sec)	N/A	0.187	0.151	0.198	0.132	0.108	42.103	0.126	0.153	0.243

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	97	92	82	158	207	264	128	90	138
N.S.	1	0.92	0.87	0.77	1.49	1.95	2.49	1.21	0.85	1.30
time (sec)	N/A	0.187	0.174	0.205	0.120	0.090	65.628	0.127	0.149	0.060

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	122	110	100	195	256	298	176	107	177
N.S.	1	0.87	0.79	0.71	1.39	1.83	2.13	1.26	0.76	1.26
time (sec)	N/A	0.198	0.237	0.214	0.116	0.090	125.389	0.131	0.154	0.234

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	149	128	117	224	303	0	168	124	203
N.S.	1	0.89	0.77	0.70	1.34	1.81	0.00	1.01	0.74	1.22
time (sec)	N/A	0.221	0.243	0.231	0.114	0.096	0.000	0.132	0.148	0.072

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	106	92	123	192	168	708	96	137
N.S.	1	1.00	0.70	0.61	0.81	1.27	1.11	4.69	0.64	0.91
time (sec)	N/A	0.275	0.069	0.414	0.035	0.095	0.883	0.133	0.150	0.209

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	75	100	167	139	612	85	111
N.S.	1	1.00	0.71	0.61	0.82	1.37	1.14	5.02	0.70	0.91
time (sec)	N/A	0.237	0.059	0.400	0.046	0.079	0.794	0.133	0.149	0.026

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	68	58	77	143	292	516	74	85
N.S.	1	1.00	0.72	0.61	0.81	1.51	3.07	5.43	0.78	0.89
time (sec)	N/A	0.207	0.051	0.377	0.054	0.073	0.371	0.124	0.152	0.038

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	41	54	118	245	418	62	52
N.S.	1	1.00	0.73	0.61	0.81	1.76	3.66	6.24	0.93	0.78
time (sec)	N/A	0.187	0.039	0.368	0.027	0.071	0.341	0.122	0.150	0.034

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	27	33	93	194	306	49	29
N.S.	1	1.00	0.71	0.64	0.79	2.21	4.62	7.29	1.17	0.69
time (sec)	N/A	0.163	0.027	0.316	0.027	0.097	0.301	0.130	0.149	0.023

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	87	88	72	87	205	104	88	89	128
N.S.	1	1.01	1.02	0.84	1.01	2.38	1.21	1.02	1.03	1.49
time (sec)	N/A	0.178	0.101	0.175	0.125	0.091	1.697	0.123	0.146	0.199

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	102	91	88	120	202	253	137	99	100
N.S.	1	0.97	0.87	0.84	1.14	1.92	2.41	1.30	0.94	0.95
time (sec)	N/A	0.199	0.158	0.211	0.104	0.079	13.828	0.132	0.166	0.053

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	108	91	89	155	206	316	155	103	162
N.S.	1	0.92	0.78	0.76	1.32	1.76	2.70	1.32	0.88	1.38
time (sec)	N/A	0.216	0.175	0.240	0.116	0.094	41.477	0.127	0.150	0.242

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	113	96	88	168	226	348	134	101	182
N.S.	1	0.94	0.80	0.73	1.40	1.88	2.90	1.12	0.84	1.52
time (sec)	N/A	0.196	0.186	0.219	0.133	0.089	66.649	0.133	0.155	0.070

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	119	111	101	194	257	326	177	107	177
N.S.	1	0.88	0.82	0.74	1.43	1.89	2.40	1.30	0.79	1.30
time (sec)	N/A	0.196	0.227	0.226	0.137	0.102	163.279	0.135	0.159	0.255

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	144	129	118	233	304	0	181	124	217
N.S.	1	0.86	0.77	0.71	1.40	1.82	0.00	1.08	0.74	1.30
time (sec)	N/A	0.211	0.293	0.244	0.138	0.119	0.000	0.132	0.172	0.068

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	172	148	133	268	353	0	240	141	254
N.S.	1	0.86	0.74	0.67	1.35	1.77	0.00	1.21	0.71	1.28
time (sec)	N/A	0.228	0.311	0.262	0.125	0.088	0.000	0.135	0.160	0.269

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	106	92	123	120	167	142	63	137
N.S.	1	1.00	0.71	0.62	0.83	0.81	1.12	0.95	0.42	0.92
time (sec)	N/A	0.251	0.057	0.396	0.029	0.075	0.640	0.123	0.160	0.204

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	87	75	100	96	138	117	52	110
N.S.	1	1.00	0.72	0.62	0.83	0.80	1.15	0.98	0.43	0.92
time (sec)	N/A	0.233	0.059	0.393	0.044	0.074	0.602	0.130	0.168	0.026

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	68	58	77	72	109	94	41	85
N.S.	1	1.00	0.73	0.62	0.83	0.77	1.17	1.01	0.44	0.91
time (sec)	N/A	0.210	0.043	0.395	0.039	0.069	0.603	0.129	0.157	0.208

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	41	54	48	80	67	29	52
N.S.	1	1.00	0.74	0.63	0.83	0.74	1.23	1.03	0.45	0.80
time (sec)	N/A	0.187	0.031	0.389	0.039	0.073	0.562	0.114	0.151	0.200

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	39	25	53	39	16	28
N.S.	1	1.00	0.72	0.65	0.98	0.62	1.32	0.98	0.40	0.70
time (sec)	N/A	0.158	0.021	0.149	0.027	0.073	0.369	0.118	0.146	0.022

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	47	91	54	36	38	32
N.S.	1	1.00	1.00	0.88	1.18	2.28	1.35	0.90	0.95	0.80
time (sec)	N/A	0.151	0.032	0.165	0.124	0.079	0.940	0.117	0.144	0.035

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	74	108	83	57	51	41
N.S.	1	1.00	1.00	0.86	1.51	2.20	1.69	1.16	1.04	0.84
time (sec)	N/A	0.156	0.071	0.187	0.122	0.086	6.547	0.116	0.169	0.054

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	78	70	59	122	156	156	111	72	100
N.S.	1	0.93	0.83	0.70	1.45	1.86	1.86	1.32	0.86	1.19
time (sec)	N/A	0.174	0.136	0.195	0.141	0.083	16.926	0.122	0.151	0.239

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	106	93	82	161	209	245	127	90	145
N.S.	1	0.92	0.81	0.71	1.40	1.82	2.13	1.10	0.78	1.26
time (sec)	N/A	0.186	0.198	0.208	0.113	0.089	23.427	0.124	0.155	0.227

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	134	112	101	196	256	303	176	107	181
N.S.	1	0.92	0.77	0.69	1.34	1.75	2.08	1.21	0.73	1.24
time (sec)	N/A	0.206	0.218	0.218	0.133	0.084	63.520	0.126	0.155	0.066

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	106	96	131	130	163	166	52	135
N.S.	1	1.00	0.72	0.65	0.89	0.88	1.11	1.13	0.35	0.92
time (sec)	N/A	0.248	0.063	0.205	0.045	0.072	1.688	0.123	0.150	0.213

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	79	108	106	134	134	41	111
N.S.	1	1.00	0.74	0.68	0.93	0.91	1.16	1.16	0.35	0.96
time (sec)	N/A	0.230	0.053	0.191	0.039	0.072	1.450	0.134	0.157	0.031

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	58	85	82	105	102	30	83
N.S.	1	1.00	0.74	0.64	0.93	0.90	1.15	1.12	0.33	0.91
time (sec)	N/A	0.204	0.044	0.177	0.040	0.084	1.163	0.125	0.154	0.039

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	41	61	57	76	69	18	52
N.S.	1	1.00	0.75	0.65	0.97	0.90	1.21	1.10	0.29	0.83
time (sec)	N/A	0.184	0.037	0.175	0.038	0.071	0.969	0.121	0.150	0.034

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	37	35	60	34	11	25
N.S.	1	1.00	0.71	0.68	0.97	0.92	1.58	0.89	0.29	0.66
time (sec)	N/A	0.161	0.027	0.170	0.045	0.074	0.177	0.124	0.157	0.196

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	57	148	65	49	31	42
N.S.	1	1.00	1.00	0.90	1.14	2.96	1.30	0.98	0.62	0.84
time (sec)	N/A	0.160	0.065	0.178	0.110	0.085	1.966	0.134	0.156	0.053

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	63	61	107	208	224	87	51	82
N.S.	1	1.01	0.88	0.85	1.49	2.89	3.11	1.21	0.71	1.14
time (sec)	N/A	0.179	0.108	0.224	0.109	0.104	22.765	0.126	0.157	0.062

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	101	88	80	144	272	185	125	73	123
N.S.	1	0.94	0.82	0.75	1.35	2.54	1.73	1.17	0.68	1.15
time (sec)	N/A	0.193	0.173	0.245	0.141	0.095	51.138	0.125	0.157	0.068

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	129	112	106	181	327	246	165	90	172
N.S.	1	0.92	0.80	0.76	1.29	2.34	1.76	1.18	0.64	1.23
time (sec)	N/A	0.214	0.207	0.254	0.128	0.116	83.423	0.124	0.156	0.243

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	106	96	129	141	162	157	54	131
N.S.	1	1.00	0.72	0.65	0.88	0.96	1.10	1.07	0.37	0.89
time (sec)	N/A	0.247	0.068	0.210	0.052	0.082	1.735	0.122	0.161	0.214

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	86	75	106	117	133	125	42	103
N.S.	1	1.00	0.73	0.64	0.90	0.99	1.13	1.06	0.36	0.87
time (sec)	N/A	0.227	0.059	0.192	0.032	0.096	1.465	0.129	0.164	0.047

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	63	62	81	92	299	92	31	79
N.S.	1	1.00	0.69	0.68	0.89	1.01	3.29	1.01	0.34	0.87
time (sec)	N/A	0.203	0.046	0.199	0.038	0.074	0.316	0.123	0.148	0.219

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	46	41	58	69	211	55	20	52
N.S.	1	1.00	0.73	0.65	0.92	1.10	3.35	0.87	0.32	0.83
time (sec)	N/A	0.181	0.040	0.184	0.030	0.072	0.322	0.118	0.150	0.032

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	28	46	124	28	13	29
N.S.	1	1.00	0.72	0.65	0.70	1.15	3.10	0.70	0.32	0.72
time (sec)	N/A	0.163	0.028	0.161	0.029	0.074	0.295	0.122	0.164	0.021

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	64	59	69	218	83	61	57	56
N.S.	1	1.06	0.96	0.88	1.03	3.25	1.24	0.91	0.85	0.84
time (sec)	N/A	0.175	0.083	0.238	0.152	0.123	2.583	0.124	0.156	0.049

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	94	86	82	130	327	1520	90	73	103
N.S.	1	0.97	0.89	0.85	1.34	3.37	15.67	0.93	0.75	1.06
time (sec)	N/A	0.195	0.128	0.323	0.133	0.103	27.604	0.125	0.165	0.227

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	122	107	101	167	392	1287	149	92	147
N.S.	1	0.92	0.80	0.76	1.26	2.95	9.68	1.12	0.69	1.11
time (sec)	N/A	0.199	0.188	0.268	0.136	0.086	62.824	0.125	0.158	0.235

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	150	130	122	204	445	1001	200	103	198
N.S.	1	0.89	0.77	0.73	1.21	2.65	5.96	1.19	0.61	1.18
time (sec)	N/A	0.216	0.242	0.277	0.130	0.092	114.665	0.126	0.159	0.256

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	171	176	159	242	292	357	0	114	0
N.S.	1	0.89	0.92	0.83	1.26	1.52	1.86	0.00	0.59	0.00
time (sec)	N/A	0.229	0.604	0.149	0.065	0.109	128.563	0.000	0.160	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	141	126	135	198	243	296	0	95	0
N.S.	1	0.89	0.79	0.85	1.25	1.53	1.86	0.00	0.60	0.00
time (sec)	N/A	0.215	0.498	0.157	0.032	0.082	24.460	0.000	0.161	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	111	105	111	154	194	207	0	76	399
N.S.	1	0.88	0.83	0.88	1.22	1.54	1.64	0.00	0.60	3.17
time (sec)	N/A	0.191	0.373	0.144	0.041	0.087	0.823	0.000	0.159	6.183

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	83	76	88	109	143	151	104	57	96
N.S.	1	0.89	0.82	0.95	1.17	1.54	1.62	1.12	0.61	1.03
time (sec)	N/A	0.177	0.109	0.146	0.044	0.113	2.293	75.795	0.160	0.680

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	77	69	77	89	128	121	100	59	0
N.S.	1	1.12	1.00	1.12	1.29	1.86	1.75	1.45	0.86	0.00
time (sec)	N/A	0.176	0.164	0.146	0.039	0.086	2.215	75.887	0.166	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	66	78	73	131	117	97	55	0
N.S.	1	1.01	0.96	1.13	1.06	1.90	1.70	1.41	0.80	0.00
time (sec)	N/A	0.161	0.124	0.149	0.122	0.079	1.881	75.205	0.162	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	31	100	51	114	73	61	54
N.S.	1	1.00	0.68	0.58	1.89	0.96	2.15	1.38	1.15	1.02
time (sec)	N/A	0.154	0.116	0.150	0.043	0.089	3.468	0.141	0.157	0.354

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	58	53	146	76	427	107	80	76
N.S.	1	0.99	0.69	0.63	1.74	0.90	5.08	1.27	0.95	0.90
time (sec)	N/A	0.173	0.136	0.148	0.034	0.105	10.220	0.144	0.159	0.356

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	73	77	192	101	930	137	99	96
N.S.	1	0.97	0.62	0.66	1.64	0.86	7.95	1.17	0.85	0.82
time (sec)	N/A	0.189	0.173	0.150	0.039	0.084	31.097	0.156	0.167	0.381

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	143	95	101	238	125	1413	171	118	116
N.S.	1	0.95	0.63	0.67	1.59	0.83	9.42	1.14	0.79	0.77
time (sec)	N/A	0.201	0.196	0.159	0.036	0.109	82.516	0.155	0.169	0.403

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	195	194	183	280	340	0	0	133	0
N.S.	1	0.88	0.87	0.82	1.26	1.53	0.00	0.00	0.60	0.00
time (sec)	N/A	0.240	0.792	0.151	0.054	0.119	0.000	0.000	0.158	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	165	143	159	236	287	355	0	114	0
N.S.	1	0.87	0.76	0.84	1.25	1.52	1.88	0.00	0.60	0.00
time (sec)	N/A	0.218	0.620	0.152	0.046	0.109	154.520	0.000	0.146	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	135	126	135	287	246	457	0	95	0
N.S.	1	0.87	0.81	0.87	1.84	1.58	2.93	0.00	0.61	0.00
time (sec)	N/A	0.203	0.507	0.149	0.033	0.117	1.410	0.000	0.154	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	107	99	111	282	195	394	136	76	0
N.S.	1	0.87	0.80	0.90	2.29	1.59	3.20	1.11	0.62	0.00
time (sec)	N/A	0.193	0.160	0.149	0.042	0.109	8.548	75.384	0.160	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	101	90	93	129	176	255	130	82	0
N.S.	1	0.99	0.88	0.91	1.26	1.73	2.50	1.27	0.80	0.00
time (sec)	N/A	0.189	0.307	0.152	0.041	0.095	3.687	74.978	0.156	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	103	88	93	146	160	221	140	84	0
N.S.	1	1.07	0.92	0.97	1.52	1.67	2.30	1.46	0.88	0.00
time (sec)	N/A	0.185	0.227	0.148	0.043	0.112	3.708	76.001	0.155	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	91	90	102	158	178	190	117	86	0
N.S.	1	0.99	0.98	1.11	1.72	1.93	2.07	1.27	0.93	0.00
time (sec)	N/A	0.174	0.202	0.149	0.048	0.083	4.544	75.426	0.157	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	31	176	74	500	80	80	76
N.S.	1	1.00	0.68	0.58	3.32	1.40	9.43	1.51	1.51	1.43
time (sec)	N/A	0.152	0.180	0.148	0.036	0.073	12.946	0.148	0.169	0.400

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	58	53	222	100	1365	110	99	96
N.S.	1	0.99	0.69	0.63	2.64	1.19	16.25	1.31	1.18	1.14
time (sec)	N/A	0.171	0.213	0.145	0.045	0.072	36.377	0.169	0.166	0.411

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	76	77	268	126	2351	144	118	117
N.S.	1	0.97	0.65	0.66	2.29	1.08	20.09	1.23	1.01	1.00
time (sec)	N/A	0.190	0.260	0.148	0.059	0.071	103.250	0.157	0.166	0.431

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	143	95	101	314	149	0	174	137	127
N.S.	1	0.95	0.63	0.67	2.09	0.99	0.00	1.16	0.91	0.85
time (sec)	N/A	0.206	0.290	0.154	0.062	0.074	0.000	0.175	0.177	0.447

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	189	164	183	422	341	0	0	133	0
N.S.	1	0.86	0.75	0.83	1.92	1.55	0.00	0.00	0.60	0.00
time (sec)	N/A	0.243	0.833	0.147	0.054	0.083	0.000	0.000	0.158	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	159	175	159	485	294	748	0	114	0
N.S.	1	0.85	0.94	0.85	2.59	1.57	4.00	0.00	0.61	0.00
time (sec)	N/A	0.220	0.660	0.151	0.054	0.084	2.801	0.000	0.151	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	131	118	135	454	243	541	167	95	0
N.S.	1	0.85	0.77	0.88	2.95	1.58	3.51	1.08	0.62	0.00
time (sec)	N/A	0.204	0.210	0.148	0.065	0.084	35.486	76.165	0.144	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	125	112	119	487	230	478	161	101	0
N.S.	1	0.95	0.85	0.90	3.69	1.74	3.62	1.22	0.77	0.00
time (sec)	N/A	0.204	0.435	0.146	0.041	0.090	9.779	76.739	0.160	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	127	108	118	191	214	364	175	107	0
N.S.	1	0.94	0.80	0.87	1.41	1.59	2.70	1.30	0.79	0.00
time (sec)	N/A	0.205	0.343	0.149	0.045	0.093	6.131	76.465	0.162	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	127	108	117	244	214	303	186	107	0
N.S.	1	1.02	0.86	0.94	1.95	1.71	2.42	1.49	0.86	0.00
time (sec)	N/A	0.202	0.297	0.154	0.040	0.082	7.597	76.269	0.162	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	112	106	126	258	228	581	142	105	0
N.S.	1	0.96	0.91	1.08	2.21	1.95	4.97	1.21	0.90	0.00
time (sec)	N/A	0.190	0.280	0.152	0.059	0.087	14.209	75.473	0.171	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	31	258	98	1442	80	99	95
N.S.	1	1.00	0.68	0.58	4.87	1.85	27.21	1.51	1.87	1.79
time (sec)	N/A	0.159	0.247	0.151	0.059	0.098	39.098	0.177	0.180	0.438

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	58	53	304	123	2790	114	118	115
N.S.	1	0.99	0.69	0.63	3.62	1.46	33.21	1.36	1.40	1.37
time (sec)	N/A	0.174	0.286	0.153	0.054	0.079	110.392	0.168	0.174	0.487

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	76	77	350	149	0	144	137	136
N.S.	1	0.97	0.65	0.66	2.99	1.27	0.00	1.23	1.17	1.16
time (sec)	N/A	0.191	0.355	0.154	0.052	0.084	0.000	0.182	0.182	0.485

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	143	95	101	396	173	0	178	156	148
N.S.	1	0.95	0.63	0.67	2.64	1.15	0.00	1.19	1.04	0.99
time (sec)	N/A	0.205	0.105	0.162	0.044	0.098	0.000	0.173	0.190	0.521

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	173	114	125	442	198	0	208	175	167
N.S.	1	0.95	0.62	0.68	2.42	1.08	0.00	1.14	0.96	0.91
time (sec)	N/A	0.222	0.108	0.169	0.043	0.086	0.000	0.213	0.211	0.530

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	147	126	135	206	246	303	212	95	0
N.S.	1	0.92	0.79	0.85	1.30	1.55	1.91	1.33	0.60	0.00
time (sec)	N/A	0.216	0.496	0.155	0.044	0.090	50.296	150.102	0.157	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	117	108	111	160	197	245	172	76	0
N.S.	1	0.93	0.86	0.88	1.27	1.56	1.94	1.37	0.60	0.00
time (sec)	N/A	0.202	0.393	0.151	0.034	0.114	10.984	150.321	0.153	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	87	86	89	121	148	129	148	56	267
N.S.	1	0.94	0.92	0.96	1.30	1.59	1.39	1.59	0.60	2.87
time (sec)	N/A	0.181	0.262	0.151	0.053	0.114	0.917	151.369	0.158	2.801

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	67	74	75	105	110	74	37	77
N.S.	1	1.00	1.20	1.32	1.34	1.88	1.96	1.32	0.66	1.38
time (sec)	N/A	0.162	0.142	0.152	0.033	0.098	1.308	76.125	0.161	0.725

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	66	49	106	44	70	43	48
N.S.	1	1.00	1.04	1.32	0.98	2.12	0.88	1.40	0.86	0.96
time (sec)	N/A	0.152	0.085	0.152	0.039	0.107	1.311	75.941	0.160	0.582

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	30	62	30	66	65	40	34
N.S.	1	1.00	0.66	0.57	1.17	0.57	1.25	1.23	0.75	0.64
time (sec)	N/A	0.155	0.093	0.151	0.052	0.092	1.633	0.140	0.155	0.430

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	58	53	106	53	342	105	61	57
N.S.	1	0.99	0.69	0.63	1.26	0.63	4.07	1.25	0.73	0.68
time (sec)	N/A	0.170	0.117	0.152	0.046	0.073	4.402	0.137	0.162	0.442

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	76	77	152	78	796	139	80	79
N.S.	1	0.97	0.65	0.66	1.30	0.67	6.80	1.19	0.68	0.68
time (sec)	N/A	0.184	0.137	0.152	0.042	0.074	12.779	0.137	0.163	0.447

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	143	95	101	198	102	1255	169	99	99
N.S.	1	0.95	0.63	0.67	1.32	0.68	8.37	1.13	0.66	0.66
time (sec)	N/A	0.197	0.163	0.158	0.052	0.084	35.580	0.144	0.166	0.488

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	156	126	186	212	303	243	207	76	0
N.S.	1	1.02	0.82	1.22	1.39	1.98	1.59	1.35	0.50	0.00
time (sec)	N/A	0.223	0.474	0.174	0.040	0.088	72.562	15.536	0.160	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	103	164	246	252	182	171	57	0
N.S.	1	1.05	0.86	1.37	2.05	2.10	1.52	1.42	0.48	0.00
time (sec)	N/A	0.202	0.348	0.168	0.054	0.085	14.913	15.501	0.156	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	96	79	149	134	192	122	134	38	0
N.S.	1	1.16	0.95	1.80	1.61	2.31	1.47	1.61	0.46	0.00
time (sec)	N/A	0.190	0.227	0.164	0.062	0.089	5.548	15.544	0.157	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	62	121	81	154	68	97	25	0
N.S.	1	1.00	1.03	2.02	1.35	2.57	1.13	1.62	0.42	0.00
time (sec)	N/A	0.163	0.110	0.145	0.056	0.083	5.423	15.712	0.165	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	55	43	63	160	23	50
N.S.	1	1.00	0.67	0.61	1.12	0.88	1.29	3.27	0.47	1.02
time (sec)	N/A	0.152	0.095	0.167	0.027	0.078	5.685	0.162	0.165	0.455

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	78	54	52	96	67	265	215	40	77
N.S.	1	0.94	0.65	0.63	1.16	0.81	3.19	2.59	0.48	0.93
time (sec)	N/A	0.165	0.127	0.161	0.033	0.077	11.742	0.170	0.153	0.480

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	108	75	77	142	92	573	254	61	98
N.S.	1	0.95	0.66	0.68	1.25	0.81	5.03	2.23	0.54	0.86
time (sec)	N/A	0.180	0.157	0.164	0.033	0.077	31.474	0.173	0.149	0.497

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	138	94	101	188	116	1008	292	80	116
N.S.	1	0.94	0.64	0.69	1.28	0.79	6.86	1.99	0.54	0.79
time (sec)	N/A	0.200	0.188	0.170	0.036	0.078	80.669	0.185	0.176	0.498

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	183	143	281	416	421	0	370	98	0
N.S.	1	0.99	0.77	1.52	2.25	2.28	0.00	2.00	0.53	0.00
time (sec)	N/A	0.233	0.537	0.181	0.043	0.090	0.000	15.568	0.165	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	155	122	259	517	370	0	334	85	0
N.S.	1	1.02	0.80	1.70	3.40	2.43	0.00	2.20	0.56	0.00
time (sec)	N/A	0.233	0.412	0.182	0.070	0.093	0.000	15.644	0.158	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	125	103	244	334	311	729	297	68	0
N.S.	1	1.12	0.92	2.18	2.98	2.78	6.51	2.65	0.61	0.00
time (sec)	N/A	0.203	0.318	0.176	0.043	0.099	62.532	15.523	0.152	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	75	182	89	229	376	215	56	0
N.S.	1	1.05	0.91	2.22	1.09	2.79	4.59	2.62	0.68	0.00
time (sec)	N/A	0.181	0.157	0.152	0.114	0.105	24.541	15.329	0.159	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	35	30	161	50	139	130	30	64
N.S.	1	1.00	0.54	0.46	2.48	0.77	2.14	2.00	0.46	0.98
time (sec)	N/A	0.158	0.095	0.151	0.042	0.083	17.432	0.151	0.162	0.470

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	78	54	53	151	76	250	211	39	88
N.S.	1	0.96	0.67	0.65	1.86	0.94	3.09	2.60	0.48	1.09
time (sec)	N/A	0.169	0.130	0.170	0.039	0.079	43.288	0.177	0.165	0.449

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	105	70	72	130	100	495	303	55	112
N.S.	1	0.93	0.62	0.64	1.15	0.88	4.38	2.68	0.49	0.99
time (sec)	N/A	0.186	0.149	0.174	0.028	0.075	123.720	0.206	0.158	0.497

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	135	94	97	176	127	0	341	70	129
N.S.	1	0.94	0.65	0.67	1.22	0.88	0.00	2.37	0.49	0.90
time (sec)	N/A	0.189	0.195	0.182	0.031	0.076	0.000	0.226	0.153	0.524

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	167	114	121	224	152	0	380	83	147
N.S.	1	0.94	0.64	0.68	1.27	0.86	0.00	2.15	0.47	0.83
time (sec)	N/A	0.215	0.227	0.190	0.032	0.080	0.000	0.244	0.168	0.526

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	105	154	208	609	3538	1046	421	405
N.S.	1	1.00	0.68	0.99	1.34	3.93	22.83	6.75	2.72	2.61
time (sec)	N/A	0.308	0.138	0.172	0.052	0.097	0.542	0.135	0.166	0.434

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	120	162	381	2101	673	282	280
N.S.	1	1.00	0.74	0.99	1.34	3.15	17.36	5.56	2.33	2.31
time (sec)	N/A	0.258	0.099	0.134	0.046	0.096	0.417	0.134	0.152	0.351

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	90	116	217	1073	380	171	179
N.S.	1	1.00	0.80	0.99	1.27	2.38	11.79	4.18	1.88	1.97
time (sec)	N/A	0.223	0.084	0.125	0.044	0.079	0.341	0.131	0.147	0.309

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	59	70	94	418	167	88	97
N.S.	1	1.00	0.97	0.98	1.17	1.57	6.97	2.78	1.47	1.62
time (sec)	N/A	0.189	0.046	0.059	0.056	0.081	0.242	0.127	0.146	0.280

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	0	0	0	146	0	13	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	2.21	0.00	0.20	0.00
time (sec)	N/A	0.179	0.049	0.000	0.000	0.000	1.203	0.000	0.149	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	83	64	0	0	0	678	0	35	0
N.S.	1	1.06	0.82	0.00	0.00	0.00	8.69	0.00	0.45	0.00
time (sec)	N/A	0.193	0.054	0.000	0.000	0.000	2.261	0.000	0.148	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	72	0	0	0	1763	0	262	0
N.S.	1	1.01	0.80	0.00	0.00	0.00	19.59	0.00	2.91	0.00
time (sec)	N/A	0.196	0.050	0.000	0.000	0.000	3.507	0.000	0.166	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	72	0	0	0	4840	0	555	0
N.S.	1	1.01	0.80	0.00	0.00	0.00	53.78	0.00	6.17	0.00
time (sec)	N/A	0.192	0.057	0.000	0.000	0.000	5.060	0.000	0.162	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	101	81	0	0	0	80	0	0	0
N.S.	1	1.13	0.91	0.00	0.00	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.214	0.052	0.000	0.000	0.000	4.243	0.000	0.166	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [256] had the largest ratio of [.444444000000000006]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	20	0.100
2	A	2	2	1.00	18	0.111
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	20	0.100
5	A	2	2	1.00	20	0.100
6	A	1	1	1.00	20	0.050
7	A	2	2	1.00	20	0.100
8	A	2	2	1.00	20	0.100
9	A	2	2	1.00	20	0.100
10	A	2	2	1.00	20	0.100
11	A	2	2	1.00	20	0.100
12	A	2	2	1.00	20	0.100
13	A	2	2	1.00	20	0.100
14	A	2	2	1.00	20	0.100
15	A	2	2	1.00	18	0.111
16	A	2	2	1.00	17	0.118
17	A	2	2	1.00	20	0.100
18	A	2	2	1.00	20	0.100
19	A	2	2	1.00	20	0.100
20	A	2	2	1.00	20	0.100
21	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	20	0.100
23	A	3	3	1.00	20	0.150
24	A	2	2	1.00	20	0.100
25	A	2	2	1.00	20	0.100
26	A	2	2	1.00	20	0.100
27	A	2	2	1.00	20	0.100
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	18	0.111
30	A	2	2	1.00	17	0.118
31	A	2	2	1.00	20	0.100
32	A	2	2	1.00	20	0.100
33	A	2	2	1.00	20	0.100
34	A	1	1	1.00	20	0.050
35	A	2	2	1.00	20	0.100
36	A	2	2	1.00	20	0.100
37	A	2	2	1.00	20	0.100
38	A	3	3	1.00	20	0.150
39	A	4	4	1.00	20	0.200
40	A	2	2	1.00	20	0.100
41	A	2	2	1.00	20	0.100
42	A	2	2	1.00	20	0.100
43	A	2	2	1.00	20	0.100
44	A	3	3	1.00	20	0.150
45	A	4	4	1.00	20	0.200
46	A	5	5	1.03	20	0.250
47	A	2	2	1.00	20	0.100
48	A	2	2	1.00	20	0.100
49	A	2	2	1.00	14	0.143
50	A	2	2	1.00	14	0.143
51	A	2	2	1.00	14	0.143
52	A	2	2	1.00	12	0.167
53	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	14	0.143
55	A	2	2	1.00	14	0.143
56	A	2	2	1.00	14	0.143
57	A	2	2	1.00	14	0.143
58	A	2	2	1.00	14	0.143
59	A	2	2	1.00	14	0.143
60	A	2	2	1.00	16	0.125
61	A	2	2	1.00	16	0.125
62	A	2	2	1.00	16	0.125
63	A	2	2	1.00	14	0.143
64	A	2	2	1.00	13	0.154
65	A	3	3	1.00	16	0.188
66	A	2	2	1.00	16	0.125
67	A	2	2	1.00	16	0.125
68	A	2	2	1.00	16	0.125
69	A	2	2	1.00	16	0.125
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	16	0.125
72	A	2	2	1.00	16	0.125
73	A	2	2	1.00	16	0.125
74	A	2	2	1.00	16	0.125
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	14	0.143
77	A	2	2	1.00	13	0.154
78	A	3	3	0.98	16	0.188
79	A	2	2	1.00	16	0.125
80	A	2	2	1.00	16	0.125
81	A	2	2	1.00	16	0.125
82	A	3	3	0.98	16	0.188
83	A	2	2	1.00	16	0.125
84	A	2	2	1.00	16	0.125
85	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	16	0.125
87	A	2	2	1.00	16	0.125
88	A	2	2	1.00	16	0.125
89	A	2	2	1.00	16	0.125
90	A	2	2	1.00	16	0.125
91	A	2	2	1.00	16	0.125
92	A	2	2	1.00	14	0.143
93	A	2	2	1.00	13	0.154
94	A	3	3	0.96	16	0.188
95	A	2	2	1.00	16	0.125
96	A	2	2	1.00	16	0.125
97	A	2	2	1.00	16	0.125
98	A	2	2	1.00	16	0.125
99	A	2	2	1.00	16	0.125
100	A	3	3	0.96	16	0.188
101	A	2	2	1.00	16	0.125
102	A	3	3	1.00	16	0.188
103	A	2	2	1.00	16	0.125
104	A	2	2	1.00	16	0.125
105	A	2	2	1.00	16	0.125
106	A	2	2	1.00	16	0.125
107	A	2	2	1.00	16	0.125
108	A	2	2	1.00	16	0.125
109	A	2	2	1.00	16	0.125
110	A	2	2	1.00	16	0.125
111	A	2	2	1.00	16	0.125
112	A	2	2	1.00	16	0.125
113	A	2	2	1.00	16	0.125
114	A	2	2	1.00	16	0.125
115	A	2	2	1.00	14	0.143
116	A	2	2	1.00	13	0.154
117	A	3	3	0.95	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	16	0.125
119	A	2	2	1.00	16	0.125
120	A	2	2	1.00	16	0.125
121	A	2	2	1.00	16	0.125
122	A	2	2	1.00	16	0.125
123	A	2	2	1.00	16	0.125
124	A	2	2	1.00	16	0.125
125	A	2	2	1.00	16	0.125
126	A	2	2	1.00	16	0.125
127	A	2	2	1.00	16	0.125
128	A	3	3	0.95	16	0.188
129	A	2	2	1.00	16	0.125
130	A	3	3	0.99	16	0.188
131	A	4	4	0.96	16	0.250
132	A	5	5	0.95	16	0.312
133	A	6	6	0.94	16	0.375
134	A	7	7	0.93	16	0.438
135	A	2	2	1.00	16	0.125
136	A	2	2	1.00	16	0.125
137	A	2	2	1.00	16	0.125
138	A	2	2	1.00	16	0.125
139	A	2	2	1.00	16	0.125
140	A	2	2	1.00	14	0.143
141	A	2	2	1.00	13	0.154
142	A	1	1	1.00	14	0.071
143	A	2	2	1.00	16	0.125
144	A	2	2	1.00	16	0.125
145	A	2	2	1.00	16	0.125
146	A	2	2	1.00	14	0.143
147	A	2	2	1.00	13	0.154
148	A	2	2	1.00	16	0.125
149	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.00	16	0.125
151	A	2	2	1.00	16	0.125
152	A	2	2	1.00	16	0.125
153	A	2	2	1.00	16	0.125
154	A	2	2	1.00	16	0.125
155	A	2	2	1.00	16	0.125
156	A	2	2	1.00	14	0.143
157	A	2	2	1.00	13	0.154
158	A	2	2	1.00	16	0.125
159	A	2	2	1.00	16	0.125
160	A	2	2	1.00	16	0.125
161	A	2	2	1.00	16	0.125
162	A	2	2	1.00	16	0.125
163	A	2	2	1.00	16	0.125
164	A	2	2	1.00	16	0.125
165	A	2	2	1.00	14	0.143
166	A	1	1	1.00	13	0.077
167	A	2	2	1.00	16	0.125
168	A	2	2	1.00	16	0.125
169	A	2	2	1.00	16	0.125
170	A	2	2	1.00	16	0.125
171	A	2	2	1.00	12	0.167
172	A	2	2	1.00	16	0.125
173	A	2	2	1.00	16	0.125
174	A	2	2	1.00	16	0.125
175	A	2	2	1.00	16	0.125
176	A	2	2	1.00	16	0.125
177	A	2	2	1.00	16	0.125
178	A	2	2	1.00	16	0.125
179	A	2	2	1.00	16	0.125
180	A	2	2	1.00	18	0.111
181	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.00	18	0.111
183	A	2	2	1.00	18	0.111
184	A	2	2	1.00	18	0.111
185	A	2	2	1.00	18	0.111
186	A	2	2	1.00	18	0.111
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	18	0.111
189	A	2	2	1.00	18	0.111
190	A	2	2	1.00	18	0.111
191	A	2	2	1.00	18	0.111
192	A	2	2	1.00	18	0.111
193	A	2	2	1.00	18	0.111
194	A	2	2	1.00	18	0.111
195	A	2	2	1.00	18	0.111
196	A	8	7	1.50	18	0.389
197	A	8	7	0.90	18	0.389
198	A	7	6	0.92	18	0.333
199	A	6	5	0.94	18	0.278
200	A	5	4	0.96	18	0.222
201	A	4	3	1.00	18	0.167
202	A	4	3	1.00	18	0.167
203	A	5	4	0.97	18	0.222
204	A	6	5	0.96	18	0.278
205	A	7	6	0.93	18	0.333
206	A	8	7	0.91	18	0.389
207	A	8	7	1.01	18	0.389
208	A	7	6	1.05	18	0.333
209	A	6	5	1.12	18	0.278
210	A	5	4	1.18	18	0.222
211	A	4	3	1.00	18	0.167
212	A	5	4	1.16	18	0.222
213	A	6	5	1.12	18	0.278

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	7	6	1.03	18	0.333
215	A	8	7	1.01	18	0.389
216	A	8	7	1.00	18	0.389
217	A	7	6	1.02	18	0.333
218	A	6	5	1.08	18	0.278
219	A	5	4	0.98	18	0.222
220	A	5	4	0.94	18	0.222
221	A	6	5	1.07	18	0.278
222	A	7	6	1.02	18	0.333
223	A	8	7	0.97	18	0.389
224	A	2	2	1.00	18	0.111
225	A	2	2	1.00	18	0.111
226	A	2	2	1.00	18	0.111
227	A	2	2	1.00	16	0.125
228	A	2	2	1.00	15	0.133
229	A	5	4	1.02	18	0.222
230	A	5	4	1.17	18	0.222
231	A	5	4	0.95	18	0.222
232	A	6	5	0.91	18	0.278
233	A	7	6	0.90	18	0.333
234	A	2	2	1.00	18	0.111
235	A	2	2	1.00	18	0.111
236	A	2	2	1.00	18	0.111
237	A	2	2	1.00	16	0.125
238	A	2	2	1.00	15	0.133
239	A	6	5	1.03	18	0.278
240	A	6	5	1.06	18	0.278
241	A	6	5	1.01	18	0.278
242	A	6	5	0.92	18	0.278
243	A	7	6	0.87	18	0.333
244	A	8	7	0.89	18	0.389
245	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	2	2	1.00	18	0.111
247	A	2	2	1.00	18	0.111
248	A	2	2	1.00	16	0.125
249	A	2	2	1.00	15	0.133
250	A	7	6	1.01	18	0.333
251	A	7	6	0.97	18	0.333
252	A	7	6	0.92	18	0.333
253	A	7	6	0.94	18	0.333
254	A	7	6	0.88	18	0.333
255	A	8	7	0.86	18	0.389
256	A	9	8	0.86	18	0.444
257	A	2	2	1.00	18	0.111
258	A	2	2	1.00	18	0.111
259	A	2	2	1.00	18	0.111
260	A	2	2	1.00	16	0.125
261	A	2	2	1.00	15	0.133
262	A	4	3	1.00	18	0.167
263	A	4	3	1.00	18	0.167
264	A	5	4	0.93	18	0.222
265	A	6	5	0.92	18	0.278
266	A	7	6	0.92	18	0.333
267	A	2	2	1.00	18	0.111
268	A	2	2	1.00	18	0.111
269	A	2	2	1.00	18	0.111
270	A	2	2	1.00	16	0.125
271	A	2	2	1.00	15	0.133
272	A	4	3	1.00	18	0.167
273	A	5	4	1.01	18	0.222
274	A	6	5	0.94	18	0.278
275	A	7	6	0.92	18	0.333
276	A	2	2	1.00	18	0.111
277	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	2	2	1.00	18	0.111
279	A	2	2	1.00	16	0.125
280	A	2	2	1.00	15	0.133
281	A	5	4	1.06	18	0.222
282	A	6	5	0.97	18	0.278
283	A	7	6	0.92	18	0.333
284	A	8	7	0.89	18	0.389
285	A	8	7	0.89	20	0.350
286	A	7	6	0.89	20	0.300
287	A	6	5	0.88	20	0.250
288	A	5	4	0.89	20	0.200
289	A	5	4	1.12	20	0.200
290	A	5	4	1.01	20	0.200
291	A	2	2	1.00	20	0.100
292	A	3	3	0.99	20	0.150
293	A	4	4	0.97	20	0.200
294	A	5	5	0.95	20	0.250
295	A	9	8	0.88	20	0.400
296	A	8	7	0.87	20	0.350
297	A	7	6	0.87	20	0.300
298	A	6	5	0.87	20	0.250
299	A	6	5	0.99	20	0.250
300	A	6	5	1.07	20	0.250
301	A	6	5	0.99	20	0.250
302	A	2	2	1.00	20	0.100
303	A	3	3	0.99	20	0.150
304	A	4	4	0.97	20	0.200
305	A	5	5	0.95	20	0.250
306	A	9	8	0.86	20	0.400
307	A	8	7	0.85	20	0.350
308	A	7	6	0.85	20	0.300
309	A	7	6	0.95	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	7	6	0.94	20	0.300
311	A	7	6	1.02	20	0.300
312	A	7	6	0.96	20	0.300
313	A	2	2	1.00	20	0.100
314	A	3	3	0.99	20	0.150
315	A	4	4	0.97	20	0.200
316	A	5	5	0.95	20	0.250
317	A	6	6	0.95	20	0.300
318	A	7	6	0.92	20	0.300
319	A	6	5	0.93	20	0.250
320	A	5	4	0.94	20	0.200
321	A	4	3	1.00	20	0.150
322	A	4	3	1.00	20	0.150
323	A	2	2	1.00	20	0.100
324	A	3	3	0.99	20	0.150
325	A	4	4	0.97	20	0.200
326	A	5	5	0.95	20	0.250
327	A	7	6	1.02	20	0.300
328	A	6	5	1.05	20	0.250
329	A	5	4	1.16	20	0.200
330	A	4	3	1.00	20	0.150
331	A	2	2	1.00	20	0.100
332	A	3	3	0.94	20	0.150
333	A	4	4	0.95	20	0.200
334	A	5	5	0.94	20	0.250
335	A	8	7	0.99	20	0.350
336	A	7	6	1.02	20	0.300
337	A	6	5	1.12	20	0.250
338	A	5	4	1.05	20	0.200
339	A	2	2	1.00	20	0.100
340	A	3	3	0.96	20	0.150
341	A	4	4	0.93	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	5	5	0.94	20	0.250
343	A	6	6	0.94	20	0.300
344	A	2	2	1.00	18	0.111
345	A	2	2	1.00	18	0.111
346	A	2	2	1.00	18	0.111
347	A	2	2	1.00	16	0.125
348	A	2	2	1.00	18	0.111
349	A	2	2	1.06	18	0.111
350	A	2	2	1.01	18	0.111
351	A	2	2	1.01	18	0.111
352	A	3	3	1.13	18	0.167

CHAPTER 3

Listing of integrals

3.1 $\int x^2(a+bx)(ac-bcx)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^2(a+bx)(ac-bcx)^3\,dx=\frac{1}{3}a^4c^3x^3-\frac{1}{2}a^3bc^3x^4+\frac{1}{3}ab^3c^3x^6-\frac{1}{7}b^4c^3x^7$$

output
```
1/3*a^4*c^3*x^3-1/2*a^3*b*c^3*x^4+1/3*a*b^3*c^3*x^6-1/7*b^4*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(ac-bcx)^3\,dx=c^3\left(\frac{a^4x^3}{3}-\frac{1}{2}a^3bx^4+\frac{1}{3}ab^3x^6-\frac{b^4x^7}{7}\right)$$

input
```
Integrate[x^2*(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
c^3*((a^4*x^3)/3 - (a^3*b*x^4)/2 + (a*b^3*x^6)/3 - (b^4*x^7)/7)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)(ac-bcx)^3\,dx$$

$$\Big\downarrow 84$$

$$\int \left(a^4c^3x^2-2a^3bc^3x^3+2ab^3c^3x^5-b^4c^3x^6\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{1}{3}a^4c^3x^3-\frac{1}{2}a^3bc^3x^4+\frac{1}{3}ab^3c^3x^6-\frac{1}{7}b^4c^3x^7$$

input
```
Int[x^2*(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
(a^4*c^3*x^3)/3 - (a^3*b*c^3*x^4)/2 + (a*b^3*c^3*x^6)/3 - (b^4*c^3*x^7)/7
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{x^{3}\left(-6 b^{4} x^{4}+14 a\,x^{3} b^{3}-21 a^{3} b x+14 a^{4}\right) c^{3}}{42}$	39
default	$\frac{1}{3} a^{4} c^{3} x^{3}-\frac{1}{2} a^{3} b\,c^{3} x^{4}+\frac{1}{3} a\,b^{3} c^{3} x^{6}-\frac{1}{7} b^{4} c^{3} x^{7}$	48
norman	$\frac{1}{3} a^{4} c^{3} x^{3}-\frac{1}{2} a^{3} b\,c^{3} x^{4}+\frac{1}{3} a\,b^{3} c^{3} x^{6}-\frac{1}{7} b^{4} c^{3} x^{7}$	48
risch	$\frac{1}{3} a^{4} c^{3} x^{3}-\frac{1}{2} a^{3} b\,c^{3} x^{4}+\frac{1}{3} a\,b^{3} c^{3} x^{6}-\frac{1}{7} b^{4} c^{3} x^{7}$	48
parallelrisch	$\frac{1}{3} a^{4} c^{3} x^{3}-\frac{1}{2} a^{3} b\,c^{3} x^{4}+\frac{1}{3} a\,b^{3} c^{3} x^{6}-\frac{1}{7} b^{4} c^{3} x^{7}$	48
orering	$\frac{x^{3}\left(-6 b^{4} x^{4}+14 a\,x^{3} b^{3}-21 a^{3} b x+14 a^{4}\right) \left(-b c x+a c\right)^{3}}{42 \left(-b x+a\right)^{3}}$	55

input
```
int(x^2*(b*x+a)*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/42*x^3*(-6*b^4*x^4+14*a*b^3*x^3-21*a^3*b*x+14*a^4)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(ac-bcx)^3\,dx = -\frac{1}{7}\,b^{4}c^{3}x^{7} + \frac{1}{3}\,ab^{3}c^{3}x^{6} - \frac{1}{2}\,a^{3}bc^{3}x^{4} + \frac{1}{3}\,a^{4}c^{3}x^{3}$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/7*b^4*c^3*x^7 + 1/3*a*b^3*c^3*x^6 - 1/2*a^3*b*c^3*x^4 + 1/3*a^4*c^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int x^2(a+bx)(ac-bcx)^3\,dx=\frac{a^4c^3x^3}{3}-\frac{a^3bc^3x^4}{2}+\frac{ab^3c^3x^6}{3}-\frac{b^4c^3x^7}{7}$$

input
```
integrate(x**2*(b*x+a)*(-b*c*x+a*c)**3,x)
```

output
```
a**4*c**3*x**3/3 - a**3*b*c**3*x**4/2 + a*b**3*c**3*x**6/3 - b**4*c**3*x**
7/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(ac-bcx)^3\,dx=-\frac{1}{7}\,b^4c^3x^7+\frac{1}{3}\,ab^3c^3x^6-\frac{1}{2}\,a^3bc^3x^4+\frac{1}{3}\,a^4c^3x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/7*b^4*c^3*x^7 + 1/3*a*b^3*c^3*x^6 - 1/2*a^3*b*c^3*x^4 + 1/3*a^4*c^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(ac-bcx)^3\,dx=-\frac{1}{7}\,b^4c^3x^7+\frac{1}{3}\,ab^3c^3x^6-\frac{1}{2}\,a^3bc^3x^4+\frac{1}{3}\,a^4c^3x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-1/7*b^4*c^3*x^7 + 1/3*a*b^3*c^3*x^6 - 1/2*a^3*b*c^3*x^4 + 1/3*a^4*c^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(ac-bcx)^3\,dx=\frac{a^4\,c^3\,x^3}{3}-\frac{a^3\,b\,c^3\,x^4}{2}+\frac{a\,b^3\,c^3\,x^6}{3}-\frac{b^4\,c^3\,x^7}{7}$$

input
```
int(x^2*(a*c - b*c*x)^3*(a + b*x),x)
```

output
```
(a^4*c^3*x^3)/3 - (b^4*c^3*x^7)/7 - (a^3*b*c^3*x^4)/2 + (a*b^3*c^3*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)(ac-bcx)^3\,dx=\frac{c^3x^3(-6b^4x^4+14a\,b^3x^3-21a^3bx+14a^4)}{42}$$

input
```
int(x^2*(b*x+a)*(-b*c*x+a*c)^3,x)
```

output
```
(c**3*x**3*(14*a**4 - 21*a**3*b*x + 14*a*b**3*x**3 - 6*b**4*x**4))/42
```

3.2 $\int x(a+bx)(ac-bcx)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int x(a+bx)(ac-bcx)^3\,dx=\frac{1}{2}a^4c^3x^2-\frac{2}{3}a^3bc^3x^3+\frac{2}{5}ab^3c^3x^5-\frac{1}{6}b^4c^3x^6$$

output
```
1/2*a^4*c^3*x^2-2/3*a^3*b*c^3*x^3+2/5*a*b^3*c^3*x^5-1/6*b^4*c^3*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x(a+bx)(ac-bcx)^3\,dx=c^3\left(\frac{a^4x^2}{2}-\frac{2}{3}a^3bx^3+\frac{2}{5}ab^3x^5-\frac{b^4x^6}{6}\right)$$

input
```
Integrate[x*(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
c^3*((a^4*x^2)/2 - (2*a^3*b*x^3)/3 + (2*a*b^3*x^5)/5 - (b^4*x^6)/6)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)(ac-bcx)^3\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(a^4c^3x-2a^3bc^3x^2+2ab^3c^3x^4-b^4c^3x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4c^3x^2-\frac{2}{3}a^3bc^3x^3+\frac{2}{5}ab^3c^3x^5-\frac{1}{6}b^4c^3x^6$$

input
```
Int[x*(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
(a^4*c^3*x^2)/2 - (2*a^3*b*c^3*x^3)/3 + (2*a*b^3*c^3*x^5)/5 - (b^4*c^3*x^6
)/6
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
gosper	$\frac{x^2(-5b^4x^4+12a\,x^3b^3-20a^3bx+15a^4)c^3}{30}$	39
default	$\frac{1}{2}a^4c^3x^2-\frac{2}{3}a^3b\,c^3x^3+\frac{2}{5}a\,b^3c^3x^5-\frac{1}{6}b^4c^3x^6$	48
norman	$\frac{1}{2}a^4c^3x^2-\frac{2}{3}a^3b\,c^3x^3+\frac{2}{5}a\,b^3c^3x^5-\frac{1}{6}b^4c^3x^6$	48
risch	$\frac{1}{2}a^4c^3x^2-\frac{2}{3}a^3b\,c^3x^3+\frac{2}{5}a\,b^3c^3x^5-\frac{1}{6}b^4c^3x^6$	48
parallelrisch	$\frac{1}{2}a^4c^3x^2-\frac{2}{3}a^3b\,c^3x^3+\frac{2}{5}a\,b^3c^3x^5-\frac{1}{6}b^4c^3x^6$	48
orering	$\frac{x^2(-5b^4x^4+12a\,x^3b^3-20a^3bx+15a^4)(-bcx+ac)^3}{30(-bx+a)^3}$	55

input
```
int(x*(b*x+a)*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2*(-5*b^4*x^4+12*a*b^3*x^3-20*a^3*b*x+15*a^4)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x(a+bx)(ac-bcx)^3\,dx=-\frac{1}{6}\,b^4c^3x^6+\frac{2}{5}\,ab^3c^3x^5-\frac{2}{3}\,a^3bc^3x^3+\frac{1}{2}\,a^4c^3x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/6*b^4*c^3*x^6 + 2/5*a*b^3*c^3*x^5 - 2/3*a^3*b*c^3*x^3 + 1/2*a^4*c^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(a+bx)(ac-bcx)^3\,dx = \frac{a^4c^3x^2}{2} - \frac{2a^3bc^3x^3}{3} + \frac{2ab^3c^3x^5}{5} - \frac{b^4c^3x^6}{6}$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)**3,x)
```

output
```
a**4*c**3*x**2/2 - 2*a**3*b*c**3*x**3/3 + 2*a*b**3*c**3*x**5/5 - b**4*c**3
*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x(a+bx)(ac-bcx)^3\,dx = -\frac{1}{6}\,b^4c^3x^6 + \frac{2}{5}\,ab^3c^3x^5 - \frac{2}{3}\,a^3bc^3x^3 + \frac{1}{2}\,a^4c^3x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/6*b^4*c^3*x^6 + 2/5*a*b^3*c^3*x^5 - 2/3*a^3*b*c^3*x^3 + 1/2*a^4*c^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x(a+bx)(ac-bcx)^3\,dx = -\frac{1}{6}\,b^4c^3x^6 + \frac{2}{5}\,ab^3c^3x^5 - \frac{2}{3}\,a^3bc^3x^3 + \frac{1}{2}\,a^4c^3x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-1/6*b^4*c^3*x^6 + 2/5*a*b^3*c^3*x^5 - 2/3*a^3*b*c^3*x^3 + 1/2*a^4*c^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int x(a+bx)(ac-bcx)^3\,dx=\frac{a^4\,c^3\,x^2}{2}-\frac{2\,a^3\,b\,c^3\,x^3}{3}+\frac{2\,a\,b^3\,c^3\,x^5}{5}-\frac{b^4\,c^3\,x^6}{6}$$

input
```
int(x*(a*c - b*c*x)^3*(a + b*x),x)
```

output
```
(a^4*c^3*x^2)/2 - (b^4*c^3*x^6)/6 - (2*a^3*b*c^3*x^3)/3 + (2*a*b^3*c^3*x^5
)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int x(a+bx)(ac-bcx)^3\,dx=\frac{c^3x^2(-5b^4x^4+12a\,b^3x^3-20a^3bx+15a^4)}{30}$$

input
```
int(x*(b*x+a)*(-b*c*x+a*c)^3,x)
```

output
```
(c**3*x**2*(15*a**4 - 20*a**3*b*x + 12*a*b**3*x**3 - 5*b**4*x**4))/30
```

3.3 $\int (a+bx)(ac-bcx)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (a+bx)(ac-bcx)^3 \, dx = -\frac{ac^3(a-bx)^4}{2b} + \frac{c^3(a-bx)^5}{5b}$$

output
```
-1/2*a*c^3*(-b*x+a)^4/b+1/5*c^3*(-b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (a+bx)(ac-bcx)^3 \, dx = c^3\left(a^4x - a^3bx^2 + \frac{1}{2}ab^3x^4 - \frac{b^4x^5}{5}\right)$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)^3\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

input
```
Int[(a + b*x)*(a*c - b*c*x)^3,x]
```

output
```
-1/2*(a*c^3*(a - b*x)^4)/b + (c^3*(a - b*x)^5)/(5*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
gosper	$\frac{x(-2b^4x^4+5a\,x^3b^3-10a^3bx+10a^4)c^3}{10}$	37
default	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45
norman	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45
risch	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45
parallelrisch	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45
orering	$\frac{x(-2b^4x^4+5a\,x^3b^3-10a^3bx+10a^4)(-bcx+ac)^3}{10(-bx+a)^3}$	53

input
```
int((b*x+a)*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*x*(-2*b^4*x^4+5*a*b^3*x^3-10*a^3*b*x+10*a^4)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx=-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

output
```
-1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = a^4c^3x - a^3bc^3x^2 + \frac{ab^3c^3x^4}{2} - \frac{b^4c^3x^5}{5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3,x)
```

output
```
a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = -\frac{1}{5}\,b^4c^3x^5 + \frac{1}{2}\,ab^3c^3x^4 - a^3bc^3x^2 + a^4c^3x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

output
```
-1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = -\frac{1}{5}\,b^4c^3x^5 + \frac{1}{2}\,ab^3c^3x^4 - a^3bc^3x^2 + a^4c^3x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
```

output
```
-1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+bx)(ac-bcx)^3\,dx = a^4\,c^3\,x - a^3\,b\,c^3\,x^2 + \frac{a\,b^3\,c^3\,x^4}{2} - \frac{b^4\,c^3\,x^5}{5}$$

input
```
int((a*c - b*c*x)^3*(a + b*x),x)
```

output
```
a^4*c^3*x - (b^4*c^3*x^5)/5 - a^3*b*c^3*x^2 + (a*b^3*c^3*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a+bx)(ac-bcx)^3\,dx = \frac{c^3x(-2b^4x^4 + 5a\,b^3x^3 - 10a^3bx + 10a^4)}{10}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3,x)
```

output
```
(c**3*x*(10*a**4 - 10*a**3*b*x + 5*a*b**3*x**3 - 2*b**4*x**4))/10
```

3.4 $\int \frac{(a+bx)(ac-bcx)^3}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{(a+bx)(ac-bcx)^3}{x} dx = -2a^3bc^3x + \frac{2}{3}ab^3c^3x^3 - \frac{1}{4}b^4c^3x^4 + a^4c^3\log(x)$$

output
```
-2*a^3*b*c^3*x+2/3*a*b^3*c^3*x^3-1/4*b^4*c^3*x^4+a^4*c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)(ac-bcx)^3}{x} dx = c^3\left(\frac{1}{12}(19a^4 - 24a^3bx + 8ab^3x^3 - 3b^4x^4) + a^4\log(-bx)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x,x]
```

output
```
c^3*((19*a^4 - 24*a^3*b*x + 8*a*b^3*x^3 - 3*b^4*x^4)/12 + a^4*Log[-(b*x)])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x} \, dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^4c^3}{x} - 2a^3bc^3 + 2ab^3c^3x^2 - b^4c^3x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^4c^3\log(x) - 2a^3bc^3x + \frac{2}{3}ab^3c^3x^3 - \frac{1}{4}b^4c^3x^4$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x,x]
```

output
```
-2*a^3*b*c^3*x + (2*a*b^3*c^3*x^3)/3 - (b^4*c^3*x^4)/4 + a^4*c^3*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
default	$c^3\left(-\frac{b^4x^4}{4}+\frac{2a\,x^3b^3}{3}-2a^3bx+a^4\ln\left(x\right)\right)$	36
norman	$-2a^3b\,c^3x+\frac{2a\,b^3c^3x^3}{3}-\frac{b^4c^3x^4}{4}+a^4c^3\ln\left(x\right)$	44
risch	$-2a^3b\,c^3x+\frac{2a\,b^3c^3x^3}{3}-\frac{b^4c^3x^4}{4}+a^4c^3\ln\left(x\right)$	44
parallelrisch	$-2a^3b\,c^3x+\frac{2a\,b^3c^3x^3}{3}-\frac{b^4c^3x^4}{4}+a^4c^3\ln\left(x\right)$	44

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/4*b^4*x^4+2/3*a*x^3*b^3-2*a^3*b*x+a^4*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int\frac{(a+bx)(ac-bcx)^3}{x}\,dx=-\frac{1}{4}\,b^4c^3x^4+\frac{2}{3}\,ab^3c^3x^3-2\,a^3bc^3x+a^4c^3\log\left(x\right)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x,x, algorithm="fricas")
```

output
```
-1/4*b^4*c^3*x^4 + 2/3*a*b^3*c^3*x^3 - 2*a^3*b*c^3*x + a^4*c^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int\frac{(a+bx)(ac-bcx)^3}{x}\,dx=a^4c^3\log\left(x\right)-2a^3bc^3x+\frac{2ab^3c^3x^3}{3}-\frac{b^4c^3x^4}{4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x,x)
```

output
```
a**4*c**3*log(x) - 2*a**3*b*c**3*x + 2*a*b**3*c**3*x**3/3 - b**4*c**3*x**4
/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^3}{x}\,dx=-\frac{1}{4}\,b^4c^3x^4+\frac{2}{3}\,ab^3c^3x^3-2\,a^3bc^3x+a^4c^3\log\left(x\right)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x,x, algorithm="maxima")
```

output
```
-1/4*b^4*c^3*x^4 + 2/3*a*b^3*c^3*x^3 - 2*a^3*b*c^3*x + a^4*c^3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x}\,dx=-\frac{1}{4}\,b^4c^3x^4+\frac{2}{3}\,ab^3c^3x^3-2\,a^3bc^3x+a^4c^3\log\left(|x|\right)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x,x, algorithm="giac")
```

output
```
-1/4*b^4*c^3*x^4 + 2/3*a*b^3*c^3*x^3 - 2*a^3*b*c^3*x + a^4*c^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^3}{x}\,dx=a^4\,c^3\,\ln\left(x\right)-\frac{b^4\,c^3\,x^4}{4}+\frac{2\,a\,b^3\,c^3\,x^3}{3}-2\,a^3\,b\,c^3\,x$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x,x)
```

output
```
a^4*c^3*log(x) - (b^4*c^3*x^4)/4 + (2*a*b^3*c^3*x^3)/3 - 2*a^3*b*c^3*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)(ac-bcx)^3}{x}\,dx = \frac{c^3(12\log(x)\,a^4-24a^3bx+8a\,b^3x^3-3b^4x^4)}{12}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x,x)
```

output
```
(c**3*(12*log(x)*a**4 - 24*a**3*b*x + 8*a*b**3*x**3 - 3*b**4*x**4))/12
```

3.5 $\int \frac{(a+bx)(ac-bcx)^3}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2} \, dx = -\frac{a^4c^3}{x} + ab^3c^3x^2 - \frac{1}{3}b^4c^3x^3 - 2a^3bc^3\log(x)$$

output
```
-a^4*c^3/x+a*b^3*c^3*x^2-1/3*b^4*c^3*x^3-2*a^3*b*c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2} \, dx = c^3\left(-\frac{a^4}{x} + ab^3x^2 - \frac{b^4x^3}{3} - 2a^3b\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^2,x]
```

output
```
c^3*(-(a^4/x) + a*b^3*x^2 - (b^4*x^3)/3 - 2*a^3*b*Log[x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^4c^3}{x^2}-\frac{2a^3bc^3}{x}+2ab^3c^3x-b^4c^3x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4c^3}{x}-2a^3bc^3\log(x)+ab^3c^3x^2-\frac{1}{3}b^4c^3x^3$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^2,x]
```

output
```
-((a^4*c^3)/x) + a*b^3*c^3*x^2 - (b^4*c^3*x^3)/3 - 2*a^3*b*c^3*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$c^{3}\left(-\frac{b^{4} x^{3}}{3}+a\,b^{3} x^{2}-2 a^{3} b \ln\left(x\right)-\frac{a^{4}}{x}\right)$	38
risch	$-\frac{a^{4} c^{3}}{x}+a\,b^{3} c^{3} x^{2}-\frac{b^{4} c^{3} x^{3}}{3}-2 a^{3} b\,c^{3} \ln\left(x\right)$	46
norman	$\frac{a\,b^{3} c^{3} x^{3}-a^{4} c^{3}-\frac{1}{3} b^{4} c^{3} x^{4}}{x}-2 a^{3} b\,c^{3} \ln\left(x\right)$	48
parallelrisch	$-\frac{b^{4} c^{3} x^{4}-3 a\,b^{3} c^{3} x^{3}+6 a^{3} c^{3} b \ln\left(x\right) x+3 a^{4} c^{3}}{3 x}$	49

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*b^4*x^3+a*b^3*x^2-2*a^3*b*ln(x)-a^4/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx = -\frac{b^4c^3x^4 - 3\,ab^3c^3x^3 + 6\,a^3bc^3x\log\left(x\right) + 3\,a^4c^3}{3\,x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^2,x, algorithm="fricas")
```

output
```
-1/3*(b^4*c^3*x^4 - 3*a*b^3*c^3*x^3 + 6*a^3*b*c^3*x*log(x) + 3*a^4*c^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx = -\frac{a^4c^3}{x} - 2a^3bc^3\log\left(x\right) + ab^3c^3x^2 - \frac{b^4c^3x^3}{3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**2,x)
```

output
```
-a**4*c**3/x - 2*a**3*b*c**3*log(x) + a*b**3*c**3*x**2 - b**4*c**3*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx = -\frac{1}{3}\,b^4c^3x^3 + ab^3c^3x^2 - 2\,a^3bc^3\log\left(x\right) - \frac{a^4c^3}{x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^2,x, algorithm="maxima")
```

output
```
-1/3*b^4*c^3*x^3 + a*b^3*c^3*x^2 - 2*a^3*b*c^3*log(x) - a^4*c^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx = -\frac{1}{3}\,b^4c^3x^3 + ab^3c^3x^2 - 2\,a^3bc^3\log\left(|x|\right) - \frac{a^4c^3}{x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^2,x, algorithm="giac")
```

output
```
-1/3*b^4*c^3*x^3 + a*b^3*c^3*x^2 - 2*a^3*b*c^3*log(abs(x)) - a^4*c^3/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx = a\,b^3\,c^3\,x^2 - \frac{b^4\,c^3\,x^3}{3} - \frac{a^4\,c^3}{x} - 2\,a^3\,b\,c^3\,\ln\left(x\right)$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^2,x)
```

output
```
a*b^3*c^3*x^2 - (b^4*c^3*x^3)/3 - (a^4*c^3)/x - 2*a^3*b*c^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^2}\,dx = \frac{c^3(-6\log(x)\,a^3bx - 3a^4 + 3a\,b^3x^3 - b^4x^4)}{3x}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^2,x)
```

output
```
(c**3*( - 6*log(x)*a**3*b*x - 3*a**4 + 3*a*b**3*x**3 - b**4*x**4))/(3*x)
```

3.6 $\int \frac{(a+bx)(ac-bcx)^3}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3} dx = -\frac{c^3(a-bx)^4}{2x^2}$$

output
```
-1/2*c^3*(-b*x+a)^4/x^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3} dx = c^3\left(-\frac{a^4}{2x^2}+\frac{2a^3b}{x}+2ab^3x-\frac{b^4x^2}{2}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^3,x]
```

output
```
c^3*(-1/2*a^4/x^2 + (2*a^3*b)/x + 2*a*b^3*x - (b^4*x^2)/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{83\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3} \, dx$$

$$\Big\downarrow \; 83$$

$$-\frac{c^3(a-bx)^4}{2x^2}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^3,x]
```

output
```
-1/2*(c^3*(a - b*x)^4)/x^2
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

method	result	size
gosper	$-\frac{c^3(b^4x^4-4a\,x^3b^3-4a^3bx+a^4)}{2x^2}$	36
default	$c^3\left(-\frac{b^4x^2}{2}+2a\,b^3x-\frac{a^4}{2x^2}+\frac{2a^3b}{x}\right)$	38
risch	$-\frac{b^4c^3x^2}{2}+2b^3c^3ax+\frac{2a^3b\,c^3x-\frac{1}{2}a^4c^3}{x^2}$	46
parallelrisch	$-\frac{b^4c^3x^4-4a\,b^3c^3x^3-4a^3b\,c^3x+a^4c^3}{2x^2}$	46
norman	$\frac{-\frac{1}{2}a^4c^3-\frac{1}{2}b^4c^3x^4+2a\,b^3c^3x^3+2a^3b\,c^3x}{x^2}$	47
orering	$-\frac{(b^4x^4-4a\,x^3b^3-4a^3bx+a^4)(-bcx+ac)^3}{2x^2(-bx+a)^3}$	52

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^3*(b^4*x^4-4*a*b^3*x^3-4*a^3*b*x+a^4)/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3}\,dx = -\frac{b^4c^3x^4-4\,ab^3c^3x^3-4\,a^3bc^3x+a^4c^3}{2\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^3*x^4 - 4*a*b^3*c^3*x^3 - 4*a^3*b*c^3*x + a^4*c^3)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3}\,dx = 2ab^3c^3x - \frac{b^4c^3x^2}{2} - \frac{a^4c^3 - 4a^3bc^3x}{2x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**3,x)
```

output
```
2*a*b**3*c**3*x - b**4*c**3*x**2/2 - (a**4*c**3 - 4*a**3*b*c**3*x)/(2*x**2
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3}\,dx = -\frac{1}{2}\,b^4c^3x^2 + 2\,ab^3c^3x + \frac{4\,a^3bc^3x - a^4c^3}{2\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^4*c^3*x^2 + 2*a*b^3*c^3*x + 1/2*(4*a^3*b*c^3*x - a^4*c^3)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3}\,dx = -\frac{1}{2}\,b^4c^3x^2 + 2\,ab^3c^3x + \frac{4\,a^3bc^3x - a^4c^3}{2\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^3,x, algorithm="giac")
```

output
```
-1/2*b^4*c^3*x^2 + 2*a*b^3*c^3*x + 1/2*(4*a^3*b*c^3*x - a^4*c^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3}\,dx = -\frac{c^3\,(a^4-4\,a^3\,b\,x-4\,a\,b^3\,x^3+b^4\,x^4)}{2\,x^2}$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^3,x)
```

output
```
-(c^3*(a^4 + b^4*x^4 - 4*a*b^3*x^3 - 4*a^3*b*x))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx)(ac-bcx)^3}{x^3}\,dx = \frac{c^3(-b^4x^4+4a\,b^3x^3+4a^3bx-a^4)}{2x^2}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^3,x)
```

output
```
(c**3*( - a**4 + 4*a**3*b*x + 4*a*b**3*x**3 - b**4*x**4))/(2*x**2)
```

3.7 $\int \frac{(a+bx)(ac-bcx)^3}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4} dx = -\frac{a^4c^3}{3x^3} + \frac{a^3bc^3}{x^2} - b^4c^3x + 2ab^3c^3\log(x)$$

output
```
-1/3*a^4*c^3/x^3+a^3*b*c^3/x^2-b^4*c^3*x+2*a*b^3*c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4} dx = c^3\left(-\frac{a^4}{3x^3} + \frac{a^3b}{x^2} - b^4x + 2ab^3\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^4,x]
```

output
```
c^3*(-1/3*a^4/x^3 + (a^3*b)/x^2 - b^4*x + 2*a*b^3*Log[x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4}\,dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^4c^3}{x^4}-\frac{2a^3bc^3}{x^3}+\frac{2ab^3c^3}{x}-b^4c^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4c^3}{3x^3}+\frac{a^3bc^3}{x^2}+2ab^3c^3\log(x)-b^4c^3x$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^4,x]
```

output
```
-1/3*(a^4*c^3)/x^3 + (a^3*b*c^3)/x^2 - b^4*c^3*x + 2*a*b^3*c^3*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$c^3\left(-b^4x-\frac{a^4}{3x^3}+\frac{a^3b}{x^2}+2a\,b^3\ln\left(x\right)\right)$	36
risch	$-b^4c^3x+\frac{a^3b\,c^3x-\frac{1}{3}a^4c^3}{x^3}+2a\,b^3c^3\ln\left(x\right)$	44
norman	$\frac{a^3b\,c^3x-\frac{1}{3}a^4c^3-b^4c^3x^4}{x^3}+2a\,b^3c^3\ln\left(x\right)$	46
parallelrisch	$\frac{6a\,b^3c^3\ln(x)x^3-3b^4c^3x^4+3a^3b\,c^3x-a^4c^3}{3x^3}$	50

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-b^4*x-1/3*a^4/x^3+a^3*b/x^2+2*a*b^3*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int\frac{(a+bx)(ac-bcx)^3}{x^4}\,dx=-\frac{3\,b^4c^3x^4-6\,ab^3c^3x^3\log\left(x\right)-3\,a^3bc^3x+a^4c^3}{3\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b^4*c^3*x^4 - 6*a*b^3*c^3*x^3*log(x) - 3*a^3*b*c^3*x + a^4*c^3)/x^
3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4}\,dx = 2ab^3c^3\log(x) - b^4c^3x - \frac{a^4c^3 - 3a^3bc^3x}{3x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**4,x)
```

output
```
2*a*b**3*c**3*log(x) - b**4*c**3*x - (a**4*c**3 - 3*a**3*b*c**3*x)/(3*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4}\,dx = -b^4c^3x + 2\,ab^3c^3\log(x) + \frac{3\,a^3bc^3x - a^4c^3}{3\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^4,x, algorithm="maxima")
```

output
```
-b^4*c^3*x + 2*a*b^3*c^3*log(x) + 1/3*(3*a^3*b*c^3*x - a^4*c^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4}\,dx = -b^4c^3x + 2\,ab^3c^3\log(|x|) + \frac{3\,a^3bc^3x - a^4c^3}{3\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^4,x, algorithm="giac")
```

output
```
-b^4*c^3*x + 2*a*b^3*c^3*log(abs(x)) + 1/3*(3*a^3*b*c^3*x - a^4*c^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4}\,dx = -\frac{c^3\left(a^4+3\,b^4\,x^4-3\,a^3\,b\,x-6\,a\,b^3\,x^3\,\ln\left(x\right)\right)}{3\,x^3}$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^4,x)
```

output
```
-(c^3*(a^4 + 3*b^4*x^4 - 3*a^3*b*x - 6*a*b^3*x^3*log(x)))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(ac-bcx)^3}{x^4}\,dx = \frac{c^3(6\log(x)\,a\,b^3x^3-a^4+3a^3bx-3b^4x^4)}{3x^3}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^4,x)
```

output
```
(c**3*(6*log(x)*a*b**3*x**3 - a**4 + 3*a**3*b*x - 3*b**4*x**4))/(3*x**3)
```

3.8 $\int \frac{(a+bx)(ac-bcx)^3}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5} dx = -\frac{a^4c^3}{4x^4} + \frac{2a^3bc^3}{3x^3} - \frac{2ab^3c^3}{x} - b^4c^3\log(x)$$

output
```
-1/4*a^4*c^3/x^4+2/3*a^3*b*c^3/x^3-2*a*b^3*c^3/x-b^4*c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5} dx = c^3\left(-\frac{a^4}{4x^4} + \frac{2a^3b}{3x^3} - \frac{2ab^3}{x} - b^4\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^5,x]
```

output
```
c^3*(-1/4*a^4/x^4 + (2*a^3*b)/(3*x^3) - (2*a*b^3)/x - b^4*Log[x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5} \, dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^4c^3}{x^5}-\frac{2a^3bc^3}{x^4}+\frac{2ab^3c^3}{x^2}-\frac{b^4c^3}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4c^3}{4x^4}+\frac{2a^3bc^3}{3x^3}-\frac{2ab^3c^3}{x}-b^4c^3\log(x)$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^5,x]
```

output
```
-1/4*(a^4*c^3)/x^4 + (2*a^3*b*c^3)/(3*x^3) - (2*a*b^3*c^3)/x - b^4*c^3*Log
[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$c^3\left(\frac{2a^3b}{3x^3}-\frac{a^4}{4x^4}-b^4\ln\left(x\right)-\frac{2a\,b^3}{x}\right)$	39
norman	$\frac{-\frac{1}{4}a^4c^3-2a\,b^3c^3x^3+\frac{2}{3}a^3b\,c^3x}{x^4}-b^4c^3\ln\left(x\right)$	47
risch	$\frac{-\frac{1}{4}a^4c^3-2a\,b^3c^3x^3+\frac{2}{3}a^3b\,c^3x}{x^4}-b^4c^3\ln\left(x\right)$	47
parallelrisch	$-\frac{12b^4c^3\ln(x)x^4+24a\,b^3c^3x^3-8a^3b\,c^3x+3a^4c^3}{12x^4}$	50

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^3*(2/3*a^3*b/x^3-1/4*a^4/x^4-b^4*ln(x)-2*a*b^3/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5}\,dx=-\frac{12\,b^4c^3x^4\log\left(x\right)+24\,ab^3c^3x^3-8\,a^3bc^3x+3\,a^4c^3}{12\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^5,x, algorithm="fricas")
```

output
```
-1/12*(12*b^4*c^3*x^4*log(x) + 24*a*b^3*c^3*x^3 - 8*a^3*b*c^3*x + 3*a^4*c^
3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5}\,dx = -b^4c^3\log(x) - \frac{3a^4c^3 - 8a^3bc^3x + 24ab^3c^3x^3}{12x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**5,x)
```

output
```
-b**4*c**3*log(x) - (3*a**4*c**3 - 8*a**3*b*c**3*x + 24*a*b**3*c**3*x**3)/
(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5}\,dx = -b^4c^3\log(x) - \frac{24\,ab^3c^3x^3 - 8\,a^3bc^3x + 3\,a^4c^3}{12\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^5,x, algorithm="maxima")
```

output
```
-b^4*c^3*log(x) - 1/12*(24*a*b^3*c^3*x^3 - 8*a^3*b*c^3*x + 3*a^4*c^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5}\,dx = -b^4c^3\log(|x|) - \frac{24\,ab^3c^3x^3 - 8\,a^3bc^3x + 3\,a^4c^3}{12\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^5,x, algorithm="giac")
```

output
```
-b^4*c^3*log(abs(x)) - 1/12*(24*a*b^3*c^3*x^3 - 8*a^3*b*c^3*x + 3*a^4*c^3)
/x^4
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5}\,dx = -\frac{c^3\,\left(3\,a^4+24\,a\,b^3\,x^3+12\,b^4\,x^4\,\ln\left(x\right)-8\,a^3\,b\,x\right)}{12\,x^4}$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^5,x)
```

output
```
-(c^3*(3*a^4 + 24*a*b^3*x^3 + 12*b^4*x^4*log(x) - 8*a^3*b*x))/(12*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)(ac-bcx)^3}{x^5}\,dx = \frac{c^3(-12\log(x)\,b^4x^4-3a^4+8a^3bx-24a\,b^3x^3)}{12x^4}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^5,x)
```

output
```
(c**3*( - 12*log(x)*b**4*x**4 - 3*a**4 + 8*a**3*b*x - 24*a*b**3*x**3))/(12
*x**4)
```

3.9 $\int \frac{(a+bx)(ac-bcx)^3}{x^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6} dx = -\frac{a^4c^3}{5x^5} + \frac{a^3bc^3}{2x^4} - \frac{ab^3c^3}{x^2} + \frac{b^4c^3}{x}$$

output
```
-1/5*a^4*c^3/x^5+1/2*a^3*b*c^3/x^4-a*b^3*c^3/x^2+b^4*c^3/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6} dx = c^3\left(-\frac{a^4}{5x^5} + \frac{a^3b}{2x^4} - \frac{ab^3}{x^2} + \frac{b^4}{x}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^6,x]
```

output
```
c^3*(-1/5*a^4/x^5 + (a^3*b)/(2*x^4) - (a*b^3)/x^2 + b^4/x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6} \, dx$$

$$\Big\downarrow \; 84$$

$$\int \left(\frac{a^4c^3}{x^6}-\frac{2a^3bc^3}{x^5}+\frac{2ab^3c^3}{x^3}-\frac{b^4c^3}{x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^4c^3}{5x^5}+\frac{a^3bc^3}{2x^4}-\frac{ab^3c^3}{x^2}+\frac{b^4c^3}{x}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^6,x]
```

output
```
-1/5*(a^4*c^3)/x^5 + (a^3*b*c^3)/(2*x^4) - (a*b^3*c^3)/x^2 + (b^4*c^3)/x
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{c^{3}\left(-10b^{4}x^{4}+10a\,x^{3}b^{3}-5a^{3}bx+2a^{4}\right)}{10x^{5}}$	39
default	$c^{3}\left(-\frac{a^{4}}{5x^{5}}-\frac{a\,b^{3}}{x^{2}}+\frac{a^{3}b}{2x^{4}}+\frac{b^{4}}{x}\right)$	39
norman	$\frac{b^{4}c^{3}x^{4}-\frac{1}{5}a^{4}c^{3}-a\,b^{3}c^{3}x^{3}+\frac{1}{2}a^{3}b\,c^{3}x}{x^{5}}$	46
risch	$\frac{b^{4}c^{3}x^{4}-\frac{1}{5}a^{4}c^{3}-a\,b^{3}c^{3}x^{3}+\frac{1}{2}a^{3}b\,c^{3}x}{x^{5}}$	46
parallelrisch	$\frac{10b^{4}c^{3}x^{4}-10a\,b^{3}c^{3}x^{3}+5a^{3}b\,c^{3}x-2a^{4}c^{3}}{10x^{5}}$	48
orering	$-\frac{\left(-10b^{4}x^{4}+10a\,x^{3}b^{3}-5a^{3}bx+2a^{4}\right)\left(-bcx+ac\right)^{3}}{10x^{5}\left(-bx+a\right)^{3}}$	55

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/10*c^3*(-10*b^4*x^4+10*a*b^3*x^3-5*a^3*b*x+2*a^4)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6}\,dx=\frac{10\,b^4c^3x^4-10\,ab^3c^3x^3+5\,a^3bc^3x-2\,a^4c^3}{10\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^6,x, algorithm="fricas")
```

output
```
1/10*(10*b^4*c^3*x^4 - 10*a*b^3*c^3*x^3 + 5*a^3*b*c^3*x - 2*a^4*c^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6}\,dx = -\frac{2a^4c^3-5a^3bc^3x+10ab^3c^3x^3-10b^4c^3x^4}{10x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**6,x)
```

output
```
-(2*a**4*c**3 - 5*a**3*b*c**3*x + 10*a*b**3*c**3*x**3 - 10*b**4*c**3*x**4)
/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6}\,dx = \frac{10\,b^4c^3x^4-10\,ab^3c^3x^3+5\,a^3bc^3x-2\,a^4c^3}{10\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^6,x, algorithm="maxima")
```

output
```
1/10*(10*b^4*c^3*x^4 - 10*a*b^3*c^3*x^3 + 5*a^3*b*c^3*x - 2*a^4*c^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6}\,dx = \frac{10\,b^4c^3x^4-10\,ab^3c^3x^3+5\,a^3bc^3x-2\,a^4c^3}{10\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^6,x, algorithm="giac")
```

output
```
1/10*(10*b^4*c^3*x^4 - 10*a*b^3*c^3*x^3 + 5*a^3*b*c^3*x - 2*a^4*c^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6}\,dx = -\frac{\frac{a^4\,c^3}{5}-\frac{a^3\,b\,c^3\,x}{2}+a\,b^3\,c^3\,x^3-b^4\,c^3\,x^4}{x^5}$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^6,x)
```

output
```
-((a^4*c^3)/5 - b^4*c^3*x^4 + a*b^3*c^3*x^3 - (a^3*b*c^3*x)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)(ac-bcx)^3}{x^6}\,dx = \frac{c^3(10b^4x^4-10a\,b^3x^3+5a^3bx-2a^4)}{10x^5}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^6,x)
```

output
```
(c**3*( - 2*a**4 + 5*a**3*b*x - 10*a*b**3*x**3 + 10*b**4*x**4))/(10*x**5)
```

3.10 $\int \frac{(a+bx)(ac-bcx)^3}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7} dx = -\frac{a^4c^3}{6x^6} + \frac{2a^3bc^3}{5x^5} - \frac{2ab^3c^3}{3x^3} + \frac{b^4c^3}{2x^2}$$

output
```
-1/6*a^4*c^3/x^6+2/5*a^3*b*c^3/x^5-2/3*a*b^3*c^3/x^3+1/2*b^4*c^3/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7} dx = c^3\left(-\frac{a^4}{6x^6} + \frac{2a^3b}{5x^5} - \frac{2ab^3}{3x^3} + \frac{b^4}{2x^2}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^7,x]
```

output
```
c^3*(-1/6*a^4/x^6 + (2*a^3*b)/(5*x^5) - (2*a*b^3)/(3*x^3) + b^4/(2*x^2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7} \, dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^4c^3}{x^7}-\frac{2a^3bc^3}{x^6}+\frac{2ab^3c^3}{x^4}-\frac{b^4c^3}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4c^3}{6x^6}+\frac{2a^3bc^3}{5x^5}-\frac{2ab^3c^3}{3x^3}+\frac{b^4c^3}{2x^2}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^7,x]
```

output
```
-1/6*(a^4*c^3)/x^6 + (2*a^3*b*c^3)/(5*x^5) - (2*a*b^3*c^3)/(3*x^3) + (b^4*
c^3)/(2*x^2)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{c^{3}\left(-15 b^{4} x^{4}+20 a \,x^{3} b^{3}-12 a^{3} b x +5 a^{4}\right)}{30 x^{6}}$	39
default	$c^{3}\left(-\frac{2 a \,b^{3}}{3 x^{3}}+\frac{2 a^{3} b}{5 x^{5}}+\frac{b^{4}}{2 x^{2}}-\frac{a^{4}}{6 x^{6}}\right)$	40
norman	$\frac{-\frac{1}{6} a^{4} c^{3}+\frac{1}{2} b^{4} c^{3} x^{4}-\frac{2}{3} a \,b^{3} c^{3} x^{3}+\frac{2}{5} a^{3} b \,c^{3} x}{x^{6}}$	47
risch	$\frac{-\frac{1}{6} a^{4} c^{3}+\frac{1}{2} b^{4} c^{3} x^{4}-\frac{2}{3} a \,b^{3} c^{3} x^{3}+\frac{2}{5} a^{3} b \,c^{3} x}{x^{6}}$	47
parallelrisch	$\frac{15 b^{4} c^{3} x^{4}-20 a \,b^{3} c^{3} x^{3}+12 a^{3} b \,c^{3} x -5 a^{4} c^{3}}{30 x^{6}}$	48
orering	$-\frac{\left(-15 b^{4} x^{4}+20 a \,x^{3} b^{3}-12 a^{3} b x +5 a^{4}\right) \left(-b c x +a c \right)^{3}}{30 x^{6} \left(-b x +a \right)^{3}}$	55

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/30*c^3*(-15*b^4*x^4+20*a*b^3*x^3-12*a^3*b*x+5*a^4)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7}\,dx = \frac{15\,b^4c^3x^4 - 20\,ab^3c^3x^3 + 12\,a^3bc^3x - 5\,a^4c^3}{30\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^7,x, algorithm="fricas")
```

output
```
1/30*(15*b^4*c^3*x^4 - 20*a*b^3*c^3*x^3 + 12*a^3*b*c^3*x - 5*a^4*c^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7}\,dx = -\frac{5a^4c^3-12a^3bc^3x+20ab^3c^3x^3-15b^4c^3x^4}{30x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**7,x)
```

output
```
-(5*a**4*c**3 - 12*a**3*b*c**3*x + 20*a*b**3*c**3*x**3 - 15*b**4*c**3*x**4
)/(30*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7}\,dx = \frac{15\,b^4c^3x^4-20\,ab^3c^3x^3+12\,a^3bc^3x-5\,a^4c^3}{30\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^7,x, algorithm="maxima")
```

output
```
1/30*(15*b^4*c^3*x^4 - 20*a*b^3*c^3*x^3 + 12*a^3*b*c^3*x - 5*a^4*c^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7}\,dx = \frac{15\,b^4c^3x^4-20\,ab^3c^3x^3+12\,a^3bc^3x-5\,a^4c^3}{30\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^7,x, algorithm="giac")
```

output
```
1/30*(15*b^4*c^3*x^4 - 20*a*b^3*c^3*x^3 + 12*a^3*b*c^3*x - 5*a^4*c^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7}\,dx = -\frac{\frac{a^4\,c^3}{6}-\frac{2\,a^3\,b\,c^3\,x}{5}+\frac{2\,a\,b^3\,c^3\,x^3}{3}-\frac{b^4\,c^3\,x^4}{2}}{x^6}$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^7,x)
```

output
```
-((a^4*c^3)/6 - (b^4*c^3*x^4)/2 + (2*a*b^3*c^3*x^3)/3 - (2*a^3*b*c^3*x)/5)
/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(ac-bcx)^3}{x^7}\,dx = \frac{c^3(15b^4x^4-20a\,b^3x^3+12a^3bx-5a^4)}{30x^6}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^7,x)
```

output
```
(c**3*( - 5*a**4 + 12*a**3*b*x - 20*a*b**3*x**3 + 15*b**4*x**4))/(30*x**6)
```

3.11 $\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx = -\frac{a^4c^3}{7x^7}+\frac{a^3bc^3}{3x^6}-\frac{ab^3c^3}{2x^4}+\frac{b^4c^3}{3x^3}$$

output
```
-1/7*a^4*c^3/x^7+1/3*a^3*b*c^3/x^6-1/2*a*b^3*c^3/x^4+1/3*b^4*c^3/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx = c^3\left(-\frac{a^4}{7x^7}+\frac{a^3b}{3x^6}-\frac{ab^3}{2x^4}+\frac{b^4}{3x^3}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^3)/x^8,x]
```

output
```
c^3*(-1/7*a^4/x^7 + (a^3*b)/(3*x^6) - (a*b^3)/(2*x^4) + b^4/(3*x^3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^4c^3}{x^8}-\frac{2a^3bc^3}{x^7}+\frac{2ab^3c^3}{x^5}-\frac{b^4c^3}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4c^3}{7x^7}+\frac{a^3bc^3}{3x^6}-\frac{ab^3c^3}{2x^4}+\frac{b^4c^3}{3x^3}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^3)/x^8,x]
```

output
```
-1/7*(a^4*c^3)/x^7 + (a^3*b*c^3)/(3*x^6) - (a*b^3*c^3)/(2*x^4) + (b^4*c^3)
/(3*x^3)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{c^{3}\left(-14 b^{4} x^{4}+21 a \,x^{3} b^{3}-14 a^{3} b x +6 a^{4}\right)}{42 x^{7}}$	39
default	$c^{3} \left(\frac{b^{4}}{3 x^{3}}-\frac{a^{4}}{7 x^{7}}-\frac{a \,b^{3}}{2 x^{4}}+\frac{a^{3} b}{3 x^{6}}\right)$	40
norman	$\frac{-\frac{1}{7} a^{4} c^{3}+\frac{1}{3} b^{4} c^{3} x^{4}-\frac{1}{2} a \,b^{3} c^{3} x^{3}+\frac{1}{3} a^{3} b \,c^{3} x}{x^{7}}$	47
risch	$\frac{-\frac{1}{7} a^{4} c^{3}+\frac{1}{3} b^{4} c^{3} x^{4}-\frac{1}{2} a \,b^{3} c^{3} x^{3}+\frac{1}{3} a^{3} b \,c^{3} x}{x^{7}}$	47
parallelrisch	$\frac{14 b^{4} c^{3} x^{4}-21 a \,b^{3} c^{3} x^{3}+14 a^{3} b \,c^{3} x -6 a^{4} c^{3}}{42 x^{7}}$	48
orering	$-\frac{\left(-14 b^{4} x^{4}+21 a \,x^{3} b^{3}-14 a^{3} b x +6 a^{4}\right) \left(-b c x +a c \right)^{3}}{42 x^{7} \left(-b x +a \right)^{3}}$	55

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/42*c^3*(-14*b^4*x^4+21*a*b^3*x^3-14*a^3*b*x+6*a^4)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx=\frac{14\,b^4c^3x^4-21\,ab^3c^3x^3+14\,a^3bc^3x-6\,a^4c^3}{42\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^8,x, algorithm="fricas")
```

output
```
1/42*(14*b^4*c^3*x^4 - 21*a*b^3*c^3*x^3 + 14*a^3*b*c^3*x - 6*a^4*c^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8} \, dx = -\frac{6a^4c^3 - 14a^3bc^3x + 21ab^3c^3x^3 - 14b^4c^3x^4}{42x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**3/x**8,x)
```

output
```
-(6*a**4*c**3 - 14*a**3*b*c**3*x + 21*a*b**3*c**3*x**3 - 14*b**4*c**3*x**4
)/(42*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8} \, dx = \frac{14\,b^4c^3x^4 - 21\,ab^3c^3x^3 + 14\,a^3bc^3x - 6\,a^4c^3}{42\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^8,x, algorithm="maxima")
```

output
```
1/42*(14*b^4*c^3*x^4 - 21*a*b^3*c^3*x^3 + 14*a^3*b*c^3*x - 6*a^4*c^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8} \, dx = \frac{14\,b^4c^3x^4 - 21\,ab^3c^3x^3 + 14\,a^3bc^3x - 6\,a^4c^3}{42\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^3/x^8,x, algorithm="giac")
```

output
```
1/42*(14*b^4*c^3*x^4 - 21*a*b^3*c^3*x^3 + 14*a^3*b*c^3*x - 6*a^4*c^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx = -\frac{\frac{a^4\,c^3}{7}-\frac{a^3\,b\,c^3\,x}{3}+\frac{a\,b^3\,c^3\,x^3}{2}-\frac{b^4\,c^3\,x^4}{3}}{x^7}$$

input
```
int(((a*c - b*c*x)^3*(a + b*x))/x^8,x)
```

output
```
-((a^4*c^3)/7 - (b^4*c^3*x^4)/3 + (a*b^3*c^3*x^3)/2 - (a^3*b*c^3*x)/3)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(ac-bcx)^3}{x^8}\,dx = \frac{c^3(14b^4x^4-21a\,b^3x^3+14a^3bx-6a^4)}{42x^7}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^3/x^8,x)
```

output
```
(c**3*( - 6*a**4 + 14*a**3*b*x - 21*a*b**3*x**3 + 14*b**4*x**4))/(42*x**7)
```

3.12 $\int x^4(a+bx)(ac-bcx)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x^4(a+bx)(ac-bcx)^4\,dx=\frac{1}{5}a^5c^4x^5-\frac{1}{2}a^4bc^4x^6+\frac{2}{7}a^3b^2c^4x^7+\frac{1}{4}a^2b^3c^4x^8-\frac{1}{3}ab^4c^4x^9+\frac{1}{10}b^5c^4x^{10}$$

output
```
1/5*a^5*c^4*x^5-1/2*a^4*b*c^4*x^6+2/7*a^3*b^2*c^4*x^7+1/4*a^2*b^3*c^4*x^8-
1/3*a*b^4*c^4*x^9+1/10*b^5*c^4*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)(ac-bcx)^4\,dx=\frac{1}{5}a^5c^4x^5-\frac{1}{2}a^4bc^4x^6+\frac{2}{7}a^3b^2c^4x^7+\frac{1}{4}a^2b^3c^4x^8-\frac{1}{3}ab^4c^4x^9+\frac{1}{10}b^5c^4x^{10}$$

input
```
Integrate[x^4*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(a^5*c^4*x^5)/5 - (a^4*b*c^4*x^6)/2 + (2*a^3*b^2*c^4*x^7)/7 + (a^2*b^3*c^4
*x^8)/4 - (a*b^4*c^4*x^9)/3 + (b^5*c^4*x^10)/10
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)(ac-bcx)^4\,dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(a^5c^4x^4-3a^4bc^4x^5+2a^3b^2c^4x^6+2a^2b^3c^4x^7-3ab^4c^4x^8+b^5c^4x^9\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}a^5c^4x^5-\frac{1}{2}a^4bc^4x^6+\frac{2}{7}a^3b^2c^4x^7+\frac{1}{4}a^2b^3c^4x^8-\frac{1}{3}ab^4c^4x^9+\frac{1}{10}b^5c^4x^{10}$$

input
```
Int[x^4*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(a^5*c^4*x^5)/5 - (a^4*b*c^4*x^6)/2 + (2*a^3*b^2*c^4*x^7)/7 + (a^2*b^3*c^4
*x^8)/4 - (a*b^4*c^4*x^9)/3 + (b^5*c^4*x^10)/10
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{5}\left(42 b^{5} x^{5}-140 a\, b^{4} x^{4}+105 a^{2} b^{3} x^{3}+120 a^{3} b^{2} x^{2}-210 a^{4} b x +84 a^{5}\right) c^{4}}{420}$	61
default	$\frac{1}{5} a^{5} c^{4} x^{5}-\frac{1}{2} a^{4} b\, c^{4} x^{6}+\frac{2}{7} a^{3} b^{2} c^{4} x^{7}+\frac{1}{4} a^{2} b^{3} c^{4} x^{8}-\frac{1}{3} a\, b^{4} c^{4} x^{9}+\frac{1}{10} b^{5} c^{4} x^{10}$	76
norman	$\frac{1}{5} a^{5} c^{4} x^{5}-\frac{1}{2} a^{4} b\, c^{4} x^{6}+\frac{2}{7} a^{3} b^{2} c^{4} x^{7}+\frac{1}{4} a^{2} b^{3} c^{4} x^{8}-\frac{1}{3} a\, b^{4} c^{4} x^{9}+\frac{1}{10} b^{5} c^{4} x^{10}$	76
risch	$\frac{1}{5} a^{5} c^{4} x^{5}-\frac{1}{2} a^{4} b\, c^{4} x^{6}+\frac{2}{7} a^{3} b^{2} c^{4} x^{7}+\frac{1}{4} a^{2} b^{3} c^{4} x^{8}-\frac{1}{3} a\, b^{4} c^{4} x^{9}+\frac{1}{10} b^{5} c^{4} x^{10}$	76
parallelrisch	$\frac{1}{5} a^{5} c^{4} x^{5}-\frac{1}{2} a^{4} b\, c^{4} x^{6}+\frac{2}{7} a^{3} b^{2} c^{4} x^{7}+\frac{1}{4} a^{2} b^{3} c^{4} x^{8}-\frac{1}{3} a\, b^{4} c^{4} x^{9}+\frac{1}{10} b^{5} c^{4} x^{10}$	76
orering	$\frac{x^{5}\left(42 b^{5} x^{5}-140 a\, b^{4} x^{4}+105 a^{2} b^{3} x^{3}+120 a^{3} b^{2} x^{2}-210 a^{4} b x +84 a^{5}\right) \left(-b c x +a c \right)^{4}}{420 \left(-b x +a \right)^{4}}$	77

input
```
int(x^4*(b*x+a)*(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/420*x^5*(42*b^5*x^5-140*a*b^4*x^4+105*a^2*b^3*x^3+120*a^3*b^2*x^2-210*a^
4*b*x+84*a^5)*c^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^4\,dx=\frac{1}{10}\,b^5c^4x^{10}-\frac{1}{3}\,ab^4c^4x^9+\frac{1}{4}\,a^2b^3c^4x^8+\frac{2}{7}\,a^3b^2c^4x^7-\frac{1}{2}\,a^4bc^4x^6+\frac{1}{5}\,a^5c^4x^5$$

input
```
integrate(x^4*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
1/10*b^5*c^4*x^10 - 1/3*a*b^4*c^4*x^9 + 1/4*a^2*b^3*c^4*x^8 + 2/7*a^3*b^2*
c^4*x^7 - 1/2*a^4*b*c^4*x^6 + 1/5*a^5*c^4*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int x^4(a+bx)(ac-bcx)^4\,dx = \frac{a^5c^4x^5}{5} - \frac{a^4bc^4x^6}{2} + \frac{2a^3b^2c^4x^7}{7} + \frac{a^2b^3c^4x^8}{4} - \frac{ab^4c^4x^9}{3} + \frac{b^5c^4x^{10}}{10}$$

input
```
integrate(x**4*(b*x+a)*(-b*c*x+a*c)**4,x)
```

output
```
a**5*c**4*x**5/5 - a**4*b*c**4*x**6/2 + 2*a**3*b**2*c**4*x**7/7 + a**2*b**
3*c**4*x**8/4 - a*b**4*c**4*x**9/3 + b**5*c**4*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^4\,dx = \frac{1}{10}\,b^5c^4x^{10} - \frac{1}{3}\,ab^4c^4x^9 + \frac{1}{4}\,a^2b^3c^4x^8 + \frac{2}{7}\,a^3b^2c^4x^7 - \frac{1}{2}\,a^4bc^4x^6 + \frac{1}{5}\,a^5c^4x^5$$

input
```
integrate(x^4*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
1/10*b^5*c^4*x^10 - 1/3*a*b^4*c^4*x^9 + 1/4*a^2*b^3*c^4*x^8 + 2/7*a^3*b^2*
c^4*x^7 - 1/2*a^4*b*c^4*x^6 + 1/5*a^5*c^4*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^4\,dx = \frac{1}{10}\,b^5c^4x^{10} - \frac{1}{3}\,ab^4c^4x^9 + \frac{1}{4}\,a^2b^3c^4x^8 + \frac{2}{7}\,a^3b^2c^4x^7 - \frac{1}{2}\,a^4bc^4x^6 + \frac{1}{5}\,a^5c^4x^5$$

input
```
integrate(x^4*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
1/10*b^5*c^4*x^10 - 1/3*a*b^4*c^4*x^9 + 1/4*a^2*b^3*c^4*x^8 + 2/7*a^3*b^2*
c^4*x^7 - 1/2*a^4*b*c^4*x^6 + 1/5*a^5*c^4*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^4\,dx = \frac{a^5\,c^4\,x^5}{5} - \frac{a^4\,b\,c^4\,x^6}{2} + \frac{2\,a^3\,b^2\,c^4\,x^7}{7} + \frac{a^2\,b^3\,c^4\,x^8}{4} - \frac{a\,b^4\,c^4\,x^9}{3} + \frac{b^5\,c^4\,x^{10}}{10}$$

input
```
int(x^4*(a*c - b*c*x)^4*(a + b*x),x)
```

output
```
(a^5*c^4*x^5)/5 + (b^5*c^4*x^10)/10 - (a^4*b*c^4*x^6)/2 - (a*b^4*c^4*x^9)/
3 + (2*a^3*b^2*c^4*x^7)/7 + (a^2*b^3*c^4*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int x^4(a+bx)(ac-bcx)^4\,dx = \frac{c^4x^5(42b^5x^5-140a\,b^4x^4+105a^2b^3x^3+120a^3b^2x^2-210a^4bx+84a^5)}{420}$$

input
```
int(x^4*(b*x+a)*(-b*c*x+a*c)^4,x)
```

output
```
(c**4*x**5*(84*a**5 - 210*a**4*b*x + 120*a**3*b**2*x**2 + 105*a**2*b**3*x*
*3 - 140*a*b**4*x**4 + 42*b**5*x**5))/420
```

3.13 $\int x^3(a+bx)(ac-bcx)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{1}{4}a^5c^4x^4 - \frac{3}{5}a^4bc^4x^5 + \frac{1}{3}a^3b^2c^4x^6 + \frac{2}{7}a^2b^3c^4x^7 - \frac{3}{8}ab^4c^4x^8 + \frac{1}{9}b^5c^4x^9$$

output
```
1/4*a^5*c^4*x^4-3/5*a^4*b*c^4*x^5+1/3*a^3*b^2*c^4*x^6+2/7*a^2*b^3*c^4*x^7-
3/8*a*b^4*c^4*x^8+1/9*b^5*c^4*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{1}{4}a^5c^4x^4 - \frac{3}{5}a^4bc^4x^5 + \frac{1}{3}a^3b^2c^4x^6 + \frac{2}{7}a^2b^3c^4x^7 - \frac{3}{8}ab^4c^4x^8 + \frac{1}{9}b^5c^4x^9$$

input
```
Integrate[x^3*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(a^5*c^4*x^4)/4 - (3*a^4*b*c^4*x^5)/5 + (a^3*b^2*c^4*x^6)/3 + (2*a^2*b^3*c
^4*x^7)/7 - (3*a*b^4*c^4*x^8)/8 + (b^5*c^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)(ac-bcx)^4\,dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(a^5c^4x^3-3a^4bc^4x^4+2a^3b^2c^4x^5+2a^2b^3c^4x^6-3ab^4c^4x^7+b^5c^4x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^5c^4x^4-\frac{3}{5}a^4bc^4x^5+\frac{1}{3}a^3b^2c^4x^6+\frac{2}{7}a^2b^3c^4x^7-\frac{3}{8}ab^4c^4x^8+\frac{1}{9}b^5c^4x^9$$

input
```
Int[x^3*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(a^5*c^4*x^4)/4 - (3*a^4*b*c^4*x^5)/5 + (a^3*b^2*c^4*x^6)/3 + (2*a^2*b^3*c
^4*x^7)/7 - (3*a*b^4*c^4*x^8)/8 + (b^5*c^4*x^9)/9
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{4}\left(280 b^{5} x^{5}-945 a\,b^{4} x^{4}+720 a^{2} b^{3} x^{3}+840 a^{3} b^{2} x^{2}-1512 a^{4} b x +630 a^{5}\right) c^{4}}{2520}$	61
default	$\frac{1}{4} a^{5} c^{4} x^{4}-\frac{3}{5} a^{4} b\,c^{4} x^{5}+\frac{1}{3} a^{3} b^{2} c^{4} x^{6}+\frac{2}{7} a^{2} b^{3} c^{4} x^{7}-\frac{3}{8} a\,b^{4} c^{4} x^{8}+\frac{1}{9} b^{5} c^{4} x^{9}$	76
norman	$\frac{1}{4} a^{5} c^{4} x^{4}-\frac{3}{5} a^{4} b\,c^{4} x^{5}+\frac{1}{3} a^{3} b^{2} c^{4} x^{6}+\frac{2}{7} a^{2} b^{3} c^{4} x^{7}-\frac{3}{8} a\,b^{4} c^{4} x^{8}+\frac{1}{9} b^{5} c^{4} x^{9}$	76
risch	$\frac{1}{4} a^{5} c^{4} x^{4}-\frac{3}{5} a^{4} b\,c^{4} x^{5}+\frac{1}{3} a^{3} b^{2} c^{4} x^{6}+\frac{2}{7} a^{2} b^{3} c^{4} x^{7}-\frac{3}{8} a\,b^{4} c^{4} x^{8}+\frac{1}{9} b^{5} c^{4} x^{9}$	76
parallelrisch	$\frac{1}{4} a^{5} c^{4} x^{4}-\frac{3}{5} a^{4} b\,c^{4} x^{5}+\frac{1}{3} a^{3} b^{2} c^{4} x^{6}+\frac{2}{7} a^{2} b^{3} c^{4} x^{7}-\frac{3}{8} a\,b^{4} c^{4} x^{8}+\frac{1}{9} b^{5} c^{4} x^{9}$	76
orering	$\frac{x^{4}\left(280 b^{5} x^{5}-945 a\,b^{4} x^{4}+720 a^{2} b^{3} x^{3}+840 a^{3} b^{2} x^{2}-1512 a^{4} b x +630 a^{5}\right) \left(-b c x +a c \right)^{4}}{2520 \left(-b x +a \right)^{4}}$	77

input
```
int(x^3*(b*x+a)*(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2520*x^4*(280*b^5*x^5-945*a*b^4*x^4+720*a^2*b^3*x^3+840*a^3*b^2*x^2-1512
*a^4*b*x+630*a^5)*c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{1}{9}\,b^5c^4x^9 - \frac{3}{8}\,ab^4c^4x^8 + \frac{2}{7}\,a^2b^3c^4x^7 + \frac{1}{3}\,a^3b^2c^4x^6 - \frac{3}{5}\,a^4bc^4x^5 + \frac{1}{4}\,a^5c^4x^4$$

input
```
integrate(x^3*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
1/9*b^5*c^4*x^9 - 3/8*a*b^4*c^4*x^8 + 2/7*a^2*b^3*c^4*x^7 + 1/3*a^3*b^2*c^
4*x^6 - 3/5*a^4*b*c^4*x^5 + 1/4*a^5*c^4*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{a^5c^4x^4}{4} - \frac{3a^4bc^4x^5}{5} + \frac{a^3b^2c^4x^6}{3} + \frac{2a^2b^3c^4x^7}{7} - \frac{3ab^4c^4x^8}{8} + \frac{b^5c^4x^9}{9}$$

input
```
integrate(x**3*(b*x+a)*(-b*c*x+a*c)**4,x)
```

output
```
a**5*c**4*x**4/4 - 3*a**4*b*c**4*x**5/5 + a**3*b**2*c**4*x**6/3 + 2*a**2*b
**3*c**4*x**7/7 - 3*a*b**4*c**4*x**8/8 + b**5*c**4*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{1}{9}\,b^5c^4x^9 - \frac{3}{8}\,ab^4c^4x^8 + \frac{2}{7}\,a^2b^3c^4x^7 + \frac{1}{3}\,a^3b^2c^4x^6 - \frac{3}{5}\,a^4bc^4x^5 + \frac{1}{4}\,a^5c^4x^4$$

input
```
integrate(x^3*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
1/9*b^5*c^4*x^9 - 3/8*a*b^4*c^4*x^8 + 2/7*a^2*b^3*c^4*x^7 + 1/3*a^3*b^2*c^
4*x^6 - 3/5*a^4*b*c^4*x^5 + 1/4*a^5*c^4*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{1}{9}\,b^5c^4x^9 - \frac{3}{8}\,ab^4c^4x^8 + \frac{2}{7}\,a^2b^3c^4x^7 + \frac{1}{3}\,a^3b^2c^4x^6 - \frac{3}{5}\,a^4bc^4x^5 + \frac{1}{4}\,a^5c^4x^4$$

input
```
integrate(x^3*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
1/9*b^5*c^4*x^9 - 3/8*a*b^4*c^4*x^8 + 2/7*a^2*b^3*c^4*x^7 + 1/3*a^3*b^2*c^
4*x^6 - 3/5*a^4*b*c^4*x^5 + 1/4*a^5*c^4*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{a^5\,c^4\,x^4}{4} - \frac{3\,a^4\,b\,c^4\,x^5}{5} + \frac{a^3\,b^2\,c^4\,x^6}{3} + \frac{2\,a^2\,b^3\,c^4\,x^7}{7} - \frac{3\,a\,b^4\,c^4\,x^8}{8} + \frac{b^5\,c^4\,x^9}{9}$$

input
```
int(x^3*(a*c - b*c*x)^4*(a + b*x),x)
```

output
```
(a^5*c^4*x^4)/4 + (b^5*c^4*x^9)/9 - (3*a^4*b*c^4*x^5)/5 - (3*a*b^4*c^4*x^8
)/8 + (a^3*b^2*c^4*x^6)/3 + (2*a^2*b^3*c^4*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int x^3(a+bx)(ac-bcx)^4\,dx = \frac{c^4x^4(280b^5x^5-945a\,b^4x^4+720a^2b^3x^3+840a^3b^2x^2-1512a^4bx+630a^5)}{2520}$$

input
```
int(x^3*(b*x+a)*(-b*c*x+a*c)^4,x)
```

output
```
(c**4*x**4*(630*a**5 - 1512*a**4*b*x + 840*a**3*b**2*x**2 + 720*a**2*b**3*
x**3 - 945*a*b**4*x**4 + 280*b**5*x**5))/2520
```

3.14 $\int x^2(a+bx)(ac-bcx)^4\,dx$

Optimal result

Integrand size = 20, antiderivative size = 80

$$\int x^2(a+bx)(ac-bcx)^4\,dx=-\frac{2a^3c^4(a-bx)^5}{5b^3}+\frac{5a^2c^4(a-bx)^6}{6b^3}-\frac{4ac^4(a-bx)^7}{7b^3}+\frac{c^4(a-bx)^8}{8b^3}$$

output
```
-2/5*a^3*c^4*(-b*x+a)^5/b^3+5/6*a^2*c^4*(-b*x+a)^6/b^3-4/7*a*c^4*(-b*x+a)^
7/b^3+1/8*c^4*(-b*x+a)^8/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int x^2(a+bx)(ac-bcx)^4\,dx=\frac{1}{3}a^5c^4x^3-\frac{3}{4}a^4bc^4x^4+\frac{2}{5}a^3b^2c^4x^5+\frac{1}{3}a^2b^3c^4x^6-\frac{3}{7}ab^4c^4x^7+\frac{1}{8}b^5c^4x^8$$

input
```
Integrate[x^2*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(a^5*c^4*x^3)/3 - (3*a^4*b*c^4*x^4)/4 + (2*a^3*b^2*c^4*x^5)/5 + (a^2*b^3*c
^4*x^6)/3 - (3*a*b^4*c^4*x^7)/7 + (b^5*c^4*x^8)/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)(ac-bcx)^4\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{2a^3(ac-bcx)^4}{b^2}-\frac{5a^2(ac-bcx)^5}{b^2c}-\frac{(ac-bcx)^7}{b^2c^3}+\frac{4a(ac-bcx)^6}{b^2c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3c^4(a-bx)^5}{5b^3}+\frac{5a^2c^4(a-bx)^6}{6b^3}+\frac{c^4(a-bx)^8}{8b^3}-\frac{4ac^4(a-bx)^7}{7b^3}$$

input
```
Int[x^2*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(-2*a^3*c^4*(a - b*x)^5)/(5*b^3) + (5*a^2*c^4*(a - b*x)^6)/(6*b^3) - (4*a*
c^4*(a - b*x)^7)/(7*b^3) + (c^4*(a - b*x)^8)/(8*b^3)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{3}\left(105 b^{5} x^{5}-360 a \,b^{4} x^{4}+280 a^{2} b^{3} x^{3}+336 a^{3} b^{2} x^{2}-630 a^{4} b x +280 a^{5}\right) c^{4}}{840}$	61
default	$\frac{1}{8} c^{4} b^{5} x^{8}-\frac{3}{7} a \,c^{4} b^{4} x^{7}+\frac{1}{3} a^{2} c^{4} b^{3} x^{6}+\frac{2}{5} a^{3} c^{4} b^{2} x^{5}-\frac{3}{4} a^{4} c^{4} b \,x^{4}+\frac{1}{3} a^{5} c^{4} x^{3}$	76
norman	$\frac{1}{8} c^{4} b^{5} x^{8}-\frac{3}{7} a \,c^{4} b^{4} x^{7}+\frac{1}{3} a^{2} c^{4} b^{3} x^{6}+\frac{2}{5} a^{3} c^{4} b^{2} x^{5}-\frac{3}{4} a^{4} c^{4} b \,x^{4}+\frac{1}{3} a^{5} c^{4} x^{3}$	76
risch	$\frac{1}{8} c^{4} b^{5} x^{8}-\frac{3}{7} a \,c^{4} b^{4} x^{7}+\frac{1}{3} a^{2} c^{4} b^{3} x^{6}+\frac{2}{5} a^{3} c^{4} b^{2} x^{5}-\frac{3}{4} a^{4} c^{4} b \,x^{4}+\frac{1}{3} a^{5} c^{4} x^{3}$	76
parallelrisch	$\frac{1}{8} c^{4} b^{5} x^{8}-\frac{3}{7} a \,c^{4} b^{4} x^{7}+\frac{1}{3} a^{2} c^{4} b^{3} x^{6}+\frac{2}{5} a^{3} c^{4} b^{2} x^{5}-\frac{3}{4} a^{4} c^{4} b \,x^{4}+\frac{1}{3} a^{5} c^{4} x^{3}$	76
orering	$\frac{x^{3}\left(105 b^{5} x^{5}-360 a \,b^{4} x^{4}+280 a^{2} b^{3} x^{3}+336 a^{3} b^{2} x^{2}-630 a^{4} b x +280 a^{5}\right) \left(-b c x +a c \right)^{4}}{840 \left(-b x +a \right)^{4}}$	77

input
```
int(x^2*(b*x+a)*(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/840*x^3*(105*b^5*x^5-360*a*b^4*x^4+280*a^2*b^3*x^3+336*a^3*b^2*x^2-630*a
^4*b*x+280*a^5)*c^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^2(a+bx)(ac-bcx)^4\,dx=\frac{1}{8}\,b^5c^4x^8-\frac{3}{7}\,ab^4c^4x^7+\frac{1}{3}\,a^2b^3c^4x^6+\frac{2}{5}\,a^3b^2c^4x^5-\frac{3}{4}\,a^4bc^4x^4+\frac{1}{3}\,a^5c^4x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
1/8*b^5*c^4*x^8 - 3/7*a*b^4*c^4*x^7 + 1/3*a^2*b^3*c^4*x^6 + 2/5*a^3*b^2*c^
4*x^5 - 3/4*a^4*b*c^4*x^4 + 1/3*a^5*c^4*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int x^2(a+bx)(ac-bcx)^4\,dx = \frac{a^5c^4x^3}{3} - \frac{3a^4bc^4x^4}{4} + \frac{2a^3b^2c^4x^5}{5} + \frac{a^2b^3c^4x^6}{3} - \frac{3ab^4c^4x^7}{7} + \frac{b^5c^4x^8}{8}$$

input
```
integrate(x**2*(b*x+a)*(-b*c*x+a*c)**4,x)
```

output
```
a**5*c**4*x**3/3 - 3*a**4*b*c**4*x**4/4 + 2*a**3*b**2*c**4*x**5/5 + a**2*b
**3*c**4*x**6/3 - 3*a*b**4*c**4*x**7/7 + b**5*c**4*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^2(a+bx)(ac-bcx)^4\,dx = \frac{1}{8}\,b^5c^4x^8 - \frac{3}{7}\,ab^4c^4x^7 + \frac{1}{3}\,a^2b^3c^4x^6 + \frac{2}{5}\,a^3b^2c^4x^5 - \frac{3}{4}\,a^4bc^4x^4 + \frac{1}{3}\,a^5c^4x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
1/8*b^5*c^4*x^8 - 3/7*a*b^4*c^4*x^7 + 1/3*a^2*b^3*c^4*x^6 + 2/5*a^3*b^2*c^
4*x^5 - 3/4*a^4*b*c^4*x^4 + 1/3*a^5*c^4*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^2(a+bx)(ac-bcx)^4\,dx = \frac{1}{8}\,b^5c^4x^8 - \frac{3}{7}\,ab^4c^4x^7 + \frac{1}{3}\,a^2b^3c^4x^6 + \frac{2}{5}\,a^3b^2c^4x^5 - \frac{3}{4}\,a^4bc^4x^4 + \frac{1}{3}\,a^5c^4x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
1/8*b^5*c^4*x^8 - 3/7*a*b^4*c^4*x^7 + 1/3*a^2*b^3*c^4*x^6 + 2/5*a^3*b^2*c^
4*x^5 - 3/4*a^4*b*c^4*x^4 + 1/3*a^5*c^4*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^2(a+bx)(ac-bcx)^4\,dx = \frac{a^5\,c^4\,x^3}{3} - \frac{3\,a^4\,b\,c^4\,x^4}{4} + \frac{2\,a^3\,b^2\,c^4\,x^5}{5} + \frac{a^2\,b^3\,c^4\,x^6}{3} - \frac{3\,a\,b^4\,c^4\,x^7}{7} + \frac{b^5\,c^4\,x^8}{8}$$

input
```
int(x^2*(a*c - b*c*x)^4*(a + b*x),x)
```

output
```
(a^5*c^4*x^3)/3 + (b^5*c^4*x^8)/8 - (3*a^4*b*c^4*x^4)/4 - (3*a*b^4*c^4*x^7
)/7 + (2*a^3*b^2*c^4*x^5)/5 + (a^2*b^3*c^4*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^2(a+bx)(ac-bcx)^4\,dx = \frac{c^4x^3(105b^5x^5-360a\,b^4x^4+280a^2b^3x^3+336a^3b^2x^2-630a^4bx+280a^5)}{840}$$

input
```
int(x^2*(b*x+a)*(-b*c*x+a*c)^4,x)
```

output
```
(c**4*x**3*(280*a**5 - 630*a**4*b*x + 336*a**3*b**2*x**2 + 280*a**2*b**3*x
**3 - 360*a*b**4*x**4 + 105*b**5*x**5))/840
```

3.15 $\int x(a+bx)(ac-bcx)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int x(a+bx)(ac-bcx)^4\,dx=-\frac{2a^2c^4(a-bx)^5}{5b^2}+\frac{ac^4(a-bx)^6}{2b^2}-\frac{c^4(a-bx)^7}{7b^2}$$

output
```
-2/5*a^2*c^4*(-b*x+a)^5/b^2+1/2*a*c^4*(-b*x+a)^6/b^2-1/7*c^4*(-b*x+a)^7/b^
2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int x(a+bx)(ac-bcx)^4\,dx=\frac{1}{2}a^5c^4x^2-a^4bc^4x^3+\frac{1}{2}a^3b^2c^4x^4+\frac{2}{5}a^2b^3c^4x^5-\frac{1}{2}ab^4c^4x^6+\frac{1}{7}b^5c^4x^7$$

input
```
Integrate[x*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(a^5*c^4*x^2)/2 - a^4*b*c^4*x^3 + (a^3*b^2*c^4*x^4)/2 + (2*a^2*b^3*c^4*x^5
)/5 - (a*b^4*c^4*x^6)/2 + (b^5*c^4*x^7)/7
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)(ac-bcx)^4\,dx$$

$$\Big\downarrow \ 84$$

$$\int \left(\frac{2a^2(ac-bcx)^4}{b}+\frac{(ac-bcx)^6}{bc^2}-\frac{3a(ac-bcx)^5}{bc}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2c^4(a-bx)^5}{5b^2}-\frac{c^4(a-bx)^7}{7b^2}+\frac{ac^4(a-bx)^6}{2b^2}$$

input
```
Int[x*(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(-2*a^2*c^4*(a - b*x)^5)/(5*b^2) + (a*c^4*(a - b*x)^6)/(2*b^2) - (c^4*(a -
 b*x)^7)/(7*b^2)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{x^{2}\left(10 b^{5} x^{5}-35 a\, b^{4} x^{4}+28 a^{2} b^{3} x^{3}+35 a^{3} b^{2} x^{2}-70 a^{4} b x +35 a^{5}\right) c^{4}}{70}$	61
default	$\frac{1}{7} c^{4} b^{5} x^{7}-\frac{1}{2} a\, c^{4} b^{4} x^{6}+\frac{2}{5} a^{2} c^{4} b^{3} x^{5}+\frac{1}{2} a^{3} c^{4} b^{2} x^{4}-a^{4} c^{4} b\, x^{3}+\frac{1}{2} a^{5} c^{4} x^{2}$	76
norman	$\frac{1}{7} c^{4} b^{5} x^{7}-\frac{1}{2} a\, c^{4} b^{4} x^{6}+\frac{2}{5} a^{2} c^{4} b^{3} x^{5}+\frac{1}{2} a^{3} c^{4} b^{2} x^{4}-a^{4} c^{4} b\, x^{3}+\frac{1}{2} a^{5} c^{4} x^{2}$	76
risch	$\frac{1}{7} c^{4} b^{5} x^{7}-\frac{1}{2} a\, c^{4} b^{4} x^{6}+\frac{2}{5} a^{2} c^{4} b^{3} x^{5}+\frac{1}{2} a^{3} c^{4} b^{2} x^{4}-a^{4} c^{4} b\, x^{3}+\frac{1}{2} a^{5} c^{4} x^{2}$	76
parallelrisch	$\frac{1}{7} c^{4} b^{5} x^{7}-\frac{1}{2} a\, c^{4} b^{4} x^{6}+\frac{2}{5} a^{2} c^{4} b^{3} x^{5}+\frac{1}{2} a^{3} c^{4} b^{2} x^{4}-a^{4} c^{4} b\, x^{3}+\frac{1}{2} a^{5} c^{4} x^{2}$	76
orering	$\frac{x^{2}\left(10 b^{5} x^{5}-35 a\, b^{4} x^{4}+28 a^{2} b^{3} x^{3}+35 a^{3} b^{2} x^{2}-70 a^{4} b x +35 a^{5}\right) \left(-b c x +a c \right)^{4}}{70 \left(-b x +a \right)^{4}}$	77

input
```
int(x*(b*x+a)*(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/70*x^2*(10*b^5*x^5-35*a*b^4*x^4+28*a^2*b^3*x^3+35*a^3*b^2*x^2-70*a^4*b*x
+35*a^5)*c^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^4\,dx = \frac{1}{7}\,b^5c^4x^7 - \frac{1}{2}\,ab^4c^4x^6 + \frac{2}{5}\,a^2b^3c^4x^5 + \frac{1}{2}\,a^3b^2c^4x^4 - a^4bc^4x^3 + \frac{1}{2}\,a^5c^4x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
1/7*b^5*c^4*x^7 - 1/2*a*b^4*c^4*x^6 + 2/5*a^2*b^3*c^4*x^5 + 1/2*a^3*b^2*c^
4*x^4 - a^4*b*c^4*x^3 + 1/2*a^5*c^4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int x(a+bx)(ac-bcx)^4\,dx = \frac{a^5c^4x^2}{2} - a^4bc^4x^3 + \frac{a^3b^2c^4x^4}{2} + \frac{2a^2b^3c^4x^5}{5} - \frac{ab^4c^4x^6}{2} + \frac{b^5c^4x^7}{7}$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)**4,x)
```

output
```
a**5*c**4*x**2/2 - a**4*b*c**4*x**3 + a**3*b**2*c**4*x**4/2 + 2*a**2*b**3*
c**4*x**5/5 - a*b**4*c**4*x**6/2 + b**5*c**4*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^4\,dx = \frac{1}{7}\,b^5c^4x^7 - \frac{1}{2}\,ab^4c^4x^6 + \frac{2}{5}\,a^2b^3c^4x^5 + \frac{1}{2}\,a^3b^2c^4x^4 - a^4bc^4x^3 + \frac{1}{2}\,a^5c^4x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
1/7*b^5*c^4*x^7 - 1/2*a*b^4*c^4*x^6 + 2/5*a^2*b^3*c^4*x^5 + 1/2*a^3*b^2*c^
4*x^4 - a^4*b*c^4*x^3 + 1/2*a^5*c^4*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^4\,dx = \frac{1}{7}\,b^5c^4x^7 - \frac{1}{2}\,ab^4c^4x^6 + \frac{2}{5}\,a^2b^3c^4x^5 + \frac{1}{2}\,a^3b^2c^4x^4 - a^4bc^4x^3 + \frac{1}{2}\,a^5c^4x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
1/7*b^5*c^4*x^7 - 1/2*a*b^4*c^4*x^6 + 2/5*a^2*b^3*c^4*x^5 + 1/2*a^3*b^2*c^
4*x^4 - a^4*b*c^4*x^3 + 1/2*a^5*c^4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^4\,dx = \frac{a^5\,c^4\,x^2}{2} - a^4\,b\,c^4\,x^3 + \frac{a^3\,b^2\,c^4\,x^4}{2} + \frac{2\,a^2\,b^3\,c^4\,x^5}{5} - \frac{a\,b^4\,c^4\,x^6}{2} + \frac{b^5\,c^4\,x^7}{7}$$

input
```
int(x*(a*c - b*c*x)^4*(a + b*x),x)
```

output
```
(a^5*c^4*x^2)/2 + (b^5*c^4*x^7)/7 - a^4*b*c^4*x^3 - (a*b^4*c^4*x^6)/2 + (a
^3*b^2*c^4*x^4)/2 + (2*a^2*b^3*c^4*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int x(a+bx)(ac-bcx)^4\,dx = \frac{c^4x^2(10b^5x^5 - 35a\,b^4x^4 + 28a^2b^3x^3 + 35a^3b^2x^2 - 70a^4bx + 35a^5)}{70}$$

input
```
int(x*(b*x+a)*(-b*c*x+a*c)^4,x)
```

output
```
(c**4*x**2*(35*a**5 - 70*a**4*b*x + 35*a**3*b**2*x**2 + 28*a**2*b**3*x**3
- 35*a*b**4*x**4 + 10*b**5*x**5))/70
```

3.16 $\int (a+bx)(ac-bcx)^4\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (a+bx)(ac-bcx)^4\,dx=-\frac{2ac^4(a-bx)^5}{5b}+\frac{c^4(a-bx)^6}{6b}$$

output
```
-2/5*a*c^4*(-b*x+a)^5/b+1/6*c^4*(-b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int (a+bx)(ac-bcx)^4\,dx=c^4\left(a^5x-\frac{3}{2}a^4bx^2+\frac{2}{3}a^3b^2x^3+\frac{1}{2}a^2b^3x^4-\frac{3}{5}ab^4x^5+\frac{b^5x^6}{6}\right)$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
c^4*(a^5*x - (3*a^4*b*x^2)/2 + (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 - (3*a*
b^4*x^5)/5 + (b^5*x^6)/6)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)^4 \, dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(2a(ac-bcx)^4-\frac{(ac-bcx)^5}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^4(a-bx)^6}{6b}-\frac{2ac^4(a-bx)^5}{5b}$$

input
```
Int[(a + b*x)*(a*c - b*c*x)^4,x]
```

output
```
(-2*a*c^4*(a - b*x)^5)/(5*b) + (c^4*(a - b*x)^6)/(6*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
gosper	$\frac{x\left(5b^{5}x^{5}-18a\,b^{4}x^{4}+15a^{2}b^{3}x^{3}+20a^{3}b^{2}x^{2}-45a^{4}bx+30a^{5}\right)c^{4}}{30}$	59
default	$\frac{1}{6}b^{5}c^{4}x^{6}-\frac{3}{5}a\,b^{4}c^{4}x^{5}+\frac{1}{2}a^{2}b^{3}c^{4}x^{4}+\frac{2}{3}a^{3}b^{2}c^{4}x^{3}-\frac{3}{2}a^{4}b\,c^{4}x^{2}+a^{5}c^{4}x$	73
norman	$\frac{1}{6}b^{5}c^{4}x^{6}-\frac{3}{5}a\,b^{4}c^{4}x^{5}+\frac{1}{2}a^{2}b^{3}c^{4}x^{4}+\frac{2}{3}a^{3}b^{2}c^{4}x^{3}-\frac{3}{2}a^{4}b\,c^{4}x^{2}+a^{5}c^{4}x$	73
risch	$\frac{1}{6}b^{5}c^{4}x^{6}-\frac{3}{5}a\,b^{4}c^{4}x^{5}+\frac{1}{2}a^{2}b^{3}c^{4}x^{4}+\frac{2}{3}a^{3}b^{2}c^{4}x^{3}-\frac{3}{2}a^{4}b\,c^{4}x^{2}+a^{5}c^{4}x$	73
parallelrisch	$\frac{1}{6}b^{5}c^{4}x^{6}-\frac{3}{5}a\,b^{4}c^{4}x^{5}+\frac{1}{2}a^{2}b^{3}c^{4}x^{4}+\frac{2}{3}a^{3}b^{2}c^{4}x^{3}-\frac{3}{2}a^{4}b\,c^{4}x^{2}+a^{5}c^{4}x$	73
orering	$\frac{x\left(5b^{5}x^{5}-18a\,b^{4}x^{4}+15a^{2}b^{3}x^{3}+20a^{3}b^{2}x^{2}-45a^{4}bx+30a^{5}\right)\left(-bcx+ac\right)^{4}}{30\left(-bx+a\right)^{4}}$	75

input
```
int((b*x+a)*(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(5*b^5*x^5-18*a*b^4*x^4+15*a^2*b^3*x^3+20*a^3*b^2*x^2-45*a^4*b*x+30
*a^5)*c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^4\,dx = \frac{1}{6}\,b^5c^4x^6 - \frac{3}{5}\,ab^4c^4x^5 + \frac{1}{2}\,a^2b^3c^4x^4 + \frac{2}{3}\,a^3b^2c^4x^3 - \frac{3}{2}\,a^4bc^4x^2 + a^5c^4x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4,x, algorithm="fricas")
```

output
```
1/6*b^5*c^4*x^6 - 3/5*a*b^4*c^4*x^5 + 1/2*a^2*b^3*c^4*x^4 + 2/3*a^3*b^2*c^
4*x^3 - 3/2*a^4*b*c^4*x^2 + a^5*c^4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(29) = 58$.

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int (a+bx)(ac-bcx)^4\,dx = a^5c^4x - \frac{3a^4bc^4x^2}{2} + \frac{2a^3b^2c^4x^3}{3} + \frac{a^2b^3c^4x^4}{2} - \frac{3ab^4c^4x^5}{5} + \frac{b^5c^4x^6}{6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4,x)
```

output
```
a**5*c**4*x - 3*a**4*b*c**4*x**2/2 + 2*a**3*b**2*c**4*x**3/3 + a**2*b**3*c
**4*x**4/2 - 3*a*b**4*c**4*x**5/5 + b**5*c**4*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^4\,dx = \frac{1}{6}\,b^5c^4x^6 - \frac{3}{5}\,ab^4c^4x^5 + \frac{1}{2}\,a^2b^3c^4x^4 + \frac{2}{3}\,a^3b^2c^4x^3 - \frac{3}{2}\,a^4bc^4x^2 + a^5c^4x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4,x, algorithm="maxima")
```

output
```
1/6*b^5*c^4*x^6 - 3/5*a*b^4*c^4*x^5 + 1/2*a^2*b^3*c^4*x^4 + 2/3*a^3*b^2*c^
4*x^3 - 3/2*a^4*b*c^4*x^2 + a^5*c^4*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^4\,dx = \frac{1}{6}\,b^5c^4x^6 - \frac{3}{5}\,ab^4c^4x^5 + \frac{1}{2}\,a^2b^3c^4x^4 + \frac{2}{3}\,a^3b^2c^4x^3 - \frac{3}{2}\,a^4bc^4x^2 + a^5c^4x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4,x, algorithm="giac")
```

output
```
1/6*b^5*c^4*x^6 - 3/5*a*b^4*c^4*x^5 + 1/2*a^2*b^3*c^4*x^4 + 2/3*a^3*b^2*c^
4*x^3 - 3/2*a^4*b*c^4*x^2 + a^5*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^4\,dx = a^5\,c^4\,x - \frac{3\,a^4\,b\,c^4\,x^2}{2} + \frac{2\,a^3\,b^2\,c^4\,x^3}{3} + \frac{a^2\,b^3\,c^4\,x^4}{2} - \frac{3\,a\,b^4\,c^4\,x^5}{5} + \frac{b^5\,c^4\,x^6}{6}$$

input
```
int((a*c - b*c*x)^4*(a + b*x),x)
```

output
```
a^5*c^4*x + (b^5*c^4*x^6)/6 - (3*a^4*b*c^4*x^2)/2 - (3*a*b^4*c^4*x^5)/5 +
(2*a^3*b^2*c^4*x^3)/3 + (a^2*b^3*c^4*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int (a+bx)(ac-bcx)^4\,dx = \frac{c^4x(5b^5x^5 - 18a\,b^4x^4 + 15a^2b^3x^3 + 20a^3b^2x^2 - 45a^4bx + 30a^5)}{30}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4,x)
```

output
```
(c**4*x*(30*a**5 - 45*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 - 1
8*a*b**4*x**4 + 5*b**5*x**5))/30
```

3.17 $\int \frac{(a+bx)(ac-bcx)^4}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{(a+bx)(ac-bcx)^4}{x} \, dx = -3a^4bc^4x + a^3b^2c^4x^2 + \frac{2}{3}a^2b^3c^4x^3 - \frac{3}{4}ab^4c^4x^4 + \frac{1}{5}b^5c^4x^5 + a^5c^4\log(x)$$

output
```
-3*a^4*b*c^4*x+a^3*b^2*c^4*x^2+2/3*a^2*b^3*c^4*x^3-3/4*a*b^4*c^4*x^4+1/5*b
^5*c^4*x^5+a^5*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^4}{x} \, dx = \frac{1}{60}c^4\left(113a^5 - 180a^4bx + 60a^3b^2x^2 + 40a^2b^3x^3 - 45ab^4x^4 + 12b^5x^5 + 60a^5\log(-bcx)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x,x]
```

output
```
(c^4*(113*a^5 - 180*a^4*b*x + 60*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 45*a*b^4*x
^4 + 12*b^5*x^5 + 60*a^5*Log[-(b*c*x)]))/60
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x} \, dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^5c^4}{x}-3a^4bc^4+2a^3b^2c^4x+2a^2b^3c^4x^2-3ab^4c^4x^3+b^5c^4x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^5c^4\log(x)-3a^4bc^4x+a^3b^2c^4x^2+\frac{2}{3}a^2b^3c^4x^3-\frac{3}{4}ab^4c^4x^4+\frac{1}{5}b^5c^4x^5$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x,x]
```

output
```
-3*a^4*b*c^4*x + a^3*b^2*c^4*x^2 + (2*a^2*b^3*c^4*x^3)/3 - (3*a*b^4*c^4*x^
4)/4 + (b^5*c^4*x^5)/5 + a^5*c^4*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
default	$c^{4}\left(\frac{b^{5} x^{5}}{5}-\frac{3 a\, b^{4} x^{4}}{4}+\frac{2 a^{2} b^{3} x^{3}}{3}+a^{3} b^{2} x^{2}-3 a^{4} b x +a^{5} \ln \left(x \right)\right)$	57
norman	$-3 a^{4} b\, c^{4} x +a^{3} b^{2} c^{4} x^{2}+\frac{2 a^{2} b^{3} c^{4} x^{3}}{3}-\frac{3 a\, b^{4} c^{4} x^{4}}{4}+\frac{b^{5} c^{4} x^{5}}{5}+a^{5} c^{4} \ln \left(x \right)$	71
risch	$-3 a^{4} b\, c^{4} x +a^{3} b^{2} c^{4} x^{2}+\frac{2 a^{2} b^{3} c^{4} x^{3}}{3}-\frac{3 a\, b^{4} c^{4} x^{4}}{4}+\frac{b^{5} c^{4} x^{5}}{5}+a^{5} c^{4} \ln \left(x \right)$	71
parallelrisch	$-3 a^{4} b\, c^{4} x +a^{3} b^{2} c^{4} x^{2}+\frac{2 a^{2} b^{3} c^{4} x^{3}}{3}-\frac{3 a\, b^{4} c^{4} x^{4}}{4}+\frac{b^{5} c^{4} x^{5}}{5}+a^{5} c^{4} \ln \left(x \right)$	71

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x,x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/5*b^5*x^5-3/4*a*b^4*x^4+2/3*a^2*b^3*x^3+a^3*b^2*x^2-3*a^4*b*x+a^5*l
n(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^4}{x}\,dx = \frac{1}{5}\,b^5c^4x^5 - \frac{3}{4}\,ab^4c^4x^4 + \frac{2}{3}\,a^2b^3c^4x^3 + a^3b^2c^4x^2 - 3\,a^4bc^4x + a^5c^4\log\left(x\right)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x,x, algorithm="fricas")
```

output
```
1/5*b^5*c^4*x^5 - 3/4*a*b^4*c^4*x^4 + 2/3*a^2*b^3*c^4*x^3 + a^3*b^2*c^4*x^
2 - 3*a^4*b*c^4*x + a^5*c^4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)(ac-bcx)^4}{x}\,dx = a^5c^4\log(x) - 3a^4bc^4x + a^3b^2c^4x^2 + \frac{2a^2b^3c^4x^3}{3} - \frac{3ab^4c^4x^4}{4} + \frac{b^5c^4x^5}{5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x,x)
```

output
```
a**5*c**4*log(x) - 3*a**4*b*c**4*x + a**3*b**2*c**4*x**2 + 2*a**2*b**3*c**
4*x**3/3 - 3*a*b**4*c**4*x**4/4 + b**5*c**4*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^4}{x}\,dx = \frac{1}{5}\,b^5c^4x^5 - \frac{3}{4}\,ab^4c^4x^4 + \frac{2}{3}\,a^2b^3c^4x^3 + a^3b^2c^4x^2 - 3\,a^4bc^4x + a^5c^4\log(x)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x,x, algorithm="maxima")
```

output
```
1/5*b^5*c^4*x^5 - 3/4*a*b^4*c^4*x^4 + 2/3*a^2*b^3*c^4*x^3 + a^3*b^2*c^4*x^
2 - 3*a^4*b*c^4*x + a^5*c^4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(ac-bcx)^4}{x}\,dx = \frac{1}{5}\,b^5c^4x^5 - \frac{3}{4}\,ab^4c^4x^4 + \frac{2}{3}\,a^2b^3c^4x^3 + a^3b^2c^4x^2 - 3\,a^4bc^4x + a^5c^4\log(|x|)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x,x, algorithm="giac")
```

output
```
1/5*b^5*c^4*x^5 - 3/4*a*b^4*c^4*x^4 + 2/3*a^2*b^3*c^4*x^3 + a^3*b^2*c^4*x^
2 - 3*a^4*b*c^4*x + a^5*c^4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^4}{x}\,dx = \frac{b^5\,c^4\,x^5}{5} + a^5\,c^4\,\ln(x) - \frac{3\,a\,b^4\,c^4\,x^4}{4} + a^3\,b^2\,c^4\,x^2 + \frac{2\,a^2\,b^3\,c^4\,x^3}{3} - 3\,a^4\,b\,c^4\,x$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x,x)
```

output
```
(b^5*c^4*x^5)/5 + a^5*c^4*log(x) - (3*a*b^4*c^4*x^4)/4 + a^3*b^2*c^4*x^2 +
 (2*a^2*b^3*c^4*x^3)/3 - 3*a^4*b*c^4*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)(ac-bcx)^4}{x}\,dx = \frac{c^4(60\log(x)\,a^5 - 180a^4bx + 60a^3b^2x^2 + 40a^2b^3x^3 - 45a\,b^4x^4 + 12b^5x^5)}{60}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x,x)
```

output
```
(c**4*(60*log(x)*a**5 - 180*a**4*b*x + 60*a**3*b**2*x**2 + 40*a**2*b**3*x*
*3 - 45*a*b**4*x**4 + 12*b**5*x**5))/60
```

3.18 $\int \frac{(a+bx)(ac-bcx)^4}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2} dx = -\frac{a^5c^4}{x} + 2a^3b^2c^4x + a^2b^3c^4x^2 - ab^4c^4x^3 + \frac{1}{4}b^5c^4x^4 - 3a^4bc^4\log(x)$$

output
```
-a^5*c^4/x+2*a^3*b^2*c^4*x+a^2*b^3*c^4*x^2-a*b^4*c^4*x^3+1/4*b^5*c^4*x^4-3
*a^4*b*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2} dx = -\frac{a^5c^4}{x} + 2a^3b^2c^4x + a^2b^3c^4x^2 - ab^4c^4x^3 + \frac{1}{4}b^5c^4x^4 - 3a^4bc^4\log(x)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^2,x]
```

output
```
-((a^5*c^4)/x) + 2*a^3*b^2*c^4*x + a^2*b^3*c^4*x^2 - a*b^4*c^4*x^3 + (b^5*
c^4*x^4)/4 - 3*a^4*b*c^4*Log[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2} \, dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^5c^4}{x^2}-\frac{3a^4bc^4}{x}+2a^3b^2c^4+2a^2b^3c^4x-3ab^4c^4x^2+b^5c^4x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5c^4}{x}-3a^4bc^4\log(x)+2a^3b^2c^4x+a^2b^3c^4x^2-ab^4c^4x^3+\frac{1}{4}b^5c^4x^4$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^2,x]
```

output

```
-((a^5*c^4)/x) + 2*a^3*b^2*c^4*x + a^2*b^3*c^4*x^2 - a*b^4*c^4*x^3 + (b^5*
c^4*x^4)/4 - 3*a^4*b*c^4*Log[x]
```


Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result	size
default	$c^4\left(\frac{b^5x^4}{4}-a\,b^4x^3+a^2b^3x^2+2a^3b^2x-3a^4b\ln\left(x\right)-\frac{a^5}{x}\right)$	58
risch	$-\frac{a^5c^4}{x}+2a^3b^2c^4x+a^2b^3c^4x^2-a\,b^4c^4x^3+\frac{b^5c^4x^4}{4}-3a^4b\,c^4\ln\left(x\right)$	72
norman	$\frac{a^2b^3c^4x^3-a^5c^4+\frac{1}{4}b^5c^4x^5-a\,b^4c^4x^4+2a^3b^2c^4x^2}{x}-3a^4b\,c^4\ln\left(x\right)$	76
parallelrisch	$-\frac{-b^5c^4x^5+4a\,b^4c^4x^4-4a^2b^3c^4x^3+12a^4c^4b\ln(x)x-8a^3b^2c^4x^2+4a^5c^4}{4x}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^2,x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/4*b^5*x^4-a*b^4*x^3+a^2*b^3*x^2+2*a^3*b^2*x-3*a^4*b*ln(x)-a^5/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int\frac{(a+bx)(ac-bcx)^4}{x^2}\,dx=\frac{b^5c^4x^5-4\,ab^4c^4x^4+4\,a^2b^3c^4x^3+8\,a^3b^2c^4x^2-12\,a^4bc^4x\log\left(x\right)-4\,a^5c^4}{4\,x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^2,x, algorithm="fricas")
```

output
```
1/4*(b^5*c^4*x^5 - 4*a*b^4*c^4*x^4 + 4*a^2*b^3*c^4*x^3 + 8*a^3*b^2*c^4*x^2
 - 12*a^4*b*c^4*x*log(x) - 4*a^5*c^4)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2}\,dx = -\frac{a^5c^4}{x} - 3a^4bc^4\log(x) + 2a^3b^2c^4x + a^2b^3c^4x^2 - ab^4c^4x^3 + \frac{b^5c^4x^4}{4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**2,x)
```

output
```
-a**5*c**4/x - 3*a**4*b*c**4*log(x) + 2*a**3*b**2*c**4*x + a**2*b**3*c**4*
x**2 - a*b**4*c**4*x**3 + b**5*c**4*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2}\,dx = \frac{1}{4}\,b^5c^4x^4 - ab^4c^4x^3 + a^2b^3c^4x^2 + 2\,a^3b^2c^4x - 3\,a^4bc^4\log(x) - \frac{a^5c^4}{x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^2,x, algorithm="maxima")
```

output
```
1/4*b^5*c^4*x^4 - a*b^4*c^4*x^3 + a^2*b^3*c^4*x^2 + 2*a^3*b^2*c^4*x - 3*a^
4*b*c^4*log(x) - a^5*c^4/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2}\,dx = \frac{1}{4}\,b^5c^4x^4 - ab^4c^4x^3 + a^2b^3c^4x^2 + 2\,a^3b^2c^4x - 3\,a^4bc^4\log(|x|) - \frac{a^5c^4}{x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^2,x, algorithm="giac")
```

output
```
1/4*b^5*c^4*x^4 - a*b^4*c^4*x^3 + a^2*b^3*c^4*x^2 + 2*a^3*b^2*c^4*x - 3*a^
4*b*c^4*log(abs(x)) - a^5*c^4/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2}\,dx = \frac{b^5\,c^4\,x^4}{4} - \frac{a^5\,c^4}{x} + 2\,a^3\,b^2\,c^4\,x - a\,b^4\,c^4\,x^3 - 3\,a^4\,b\,c^4\,\ln(x) + a^2\,b^3\,c^4\,x^2$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^2,x)
```

output
```
(b^5*c^4*x^4)/4 - (a^5*c^4)/x + 2*a^3*b^2*c^4*x - a*b^4*c^4*x^3 - 3*a^4*b*
c^4*log(x) + a^2*b^3*c^4*x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(ac-bcx)^4}{x^2}\,dx = \frac{c^4(-12\log(x)\,a^4bx - 4a^5 + 8a^3b^2x^2 + 4a^2b^3x^3 - 4a\,b^4x^4 + b^5x^5)}{4x}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^2,x)
```

output
```
(c**4*( - 12*log(x)*a**4*b*x - 4*a**5 + 8*a**3*b**2*x**2 + 4*a**2*b**3*x**
3 - 4*a*b**4*x**4 + b**5*x**5))/(4*x)
```

3.19 $\int \frac{(a+bx)(ac-bcx)^4}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3} dx = -\frac{a^5c^4}{2x^2} + \frac{3a^4bc^4}{x} + 2a^2b^3c^4x - \frac{3}{2}ab^4c^4x^2 + \frac{1}{3}b^5c^4x^3 + 2a^3b^2c^4\log(x)$$

output
```
-1/2*a^5*c^4/x^2+3*a^4*b*c^4/x+2*a^2*b^3*c^4*x-3/2*a*b^4*c^4*x^2+1/3*b^5*c
^4*x^3+2*a^3*b^2*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3} dx = -\frac{a^5c^4}{2x^2} + \frac{3a^4bc^4}{x} + 2a^2b^3c^4x - \frac{3}{2}ab^4c^4x^2 + \frac{1}{3}b^5c^4x^3 + 2a^3b^2c^4\log(x)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^3,x]
```

output
```
-1/2*(a^5*c^4)/x^2 + (3*a^4*b*c^4)/x + 2*a^2*b^3*c^4*x - (3*a*b^4*c^4*x^2)
/2 + (b^5*c^4*x^3)/3 + 2*a^3*b^2*c^4*Log[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3} \, dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^5c^4}{x^3}-\frac{3a^4bc^4}{x^2}+\frac{2a^3b^2c^4}{x}+2a^2b^3c^4-3ab^4c^4x+b^5c^4x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5c^4}{2x^2}+\frac{3a^4bc^4}{x}+2a^3b^2c^4\log(x)+2a^2b^3c^4x-\frac{3}{2}ab^4c^4x^2+\frac{1}{3}b^5c^4x^3$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^3,x]
```

output

```
-1/2*(a^5*c^4)/x^2 + (3*a^4*b*c^4)/x + 2*a^2*b^3*c^4*x - (3*a*b^4*c^4*x^2)
/2 + (b^5*c^4*x^3)/3 + 2*a^3*b^2*c^4*Log[x]
```


Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result	size
default	$c^{4}\left(\frac{b^{5} x^{3}}{3}-\frac{3 a\,b^{4} x^{2}}{2}+2 a^{2} b^{3} x-\frac{a^{5}}{2 x^{2}}+2 a^{3} b^{2} \ln \left(x \right)+\frac{3 a^{4} b}{x}\right)$	59
risch	$\frac{b^{5} c^{4} x^{3}}{3}-\frac{3 a\,b^{4} c^{4} x^{2}}{2}+2 a^{2} b^{3} c^{4} x+\frac{3 a^{4} b\,c^{4} x-\frac{1}{2} a^{5} c^{4}}{x^{2}}+2 a^{3} b^{2} c^{4} \ln \left(x \right)$	73
norman	$\frac{-\frac{1}{2} a^{5} c^{4}+\frac{1}{3} b^{5} c^{4} x^{5}-\frac{3}{2} a\,b^{4} c^{4} x^{4}+2 a^{2} b^{3} c^{4} x^{3}+3 a^{4} b\,c^{4} x}{x^{2}}+2 a^{3} b^{2} c^{4} \ln \left(x \right)$	75
parallelrisch	$\frac{2 b^{5} c^{4} x^{5}-9 a\,b^{4} c^{4} x^{4}+12 a^{3} c^{4} b^{2} \ln \left(x \right) x^{2}+12 a^{2} b^{3} c^{4} x^{3}+18 a^{4} b\,c^{4} x-3 a^{5} c^{4}}{6 x^{2}}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/3*b^5*x^3-3/2*a*b^4*x^2+2*a^2*b^3*x-1/2*a^5/x^2+2*a^3*b^2*ln(x)+3*a
^4*b/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3}\,dx = \frac{2\,b^5c^4x^5-9\,ab^4c^4x^4+12\,a^2b^3c^4x^3+12\,a^3b^2c^4x^2\log\left(x\right)+18\,a^4bc^4x-3\,a^5c^4}{6\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^5*c^4*x^5 - 9*a*b^4*c^4*x^4 + 12*a^2*b^3*c^4*x^3 + 12*a^3*b^2*c^4
*x^2*log(x) + 18*a^4*b*c^4*x - 3*a^5*c^4)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3}\,dx = 2a^3b^2c^4\log{(x)} + 2a^2b^3c^4x - \frac{3ab^4c^4x^2}{2} + \frac{b^5c^4x^3}{3} + \frac{-a^5c^4 + 6a^4bc^4x}{2x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**3,x)
```

output
```
2*a**3*b**2*c**4*log(x) + 2*a**2*b**3*c**4*x - 3*a*b**4*c**4*x**2/2 + b**5
*c**4*x**3/3 + (-a**5*c**4 + 6*a**4*b*c**4*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3}\,dx = \frac{1}{3}\,b^5c^4x^3 - \frac{3}{2}\,ab^4c^4x^2 + 2\,a^2b^3c^4x + 2\,a^3b^2c^4\log{(x)} + \frac{6\,a^4bc^4x - a^5c^4}{2\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^3,x, algorithm="maxima")
```

output
```
1/3*b^5*c^4*x^3 - 3/2*a*b^4*c^4*x^2 + 2*a^2*b^3*c^4*x + 2*a^3*b^2*c^4*log(
x) + 1/2*(6*a^4*b*c^4*x - a^5*c^4)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3} \, dx = \frac{1}{3} b^5 c^4 x^3 - \frac{3}{2} a b^4 c^4 x^2 + 2 a^2 b^3 c^4 x + 2 a^3 b^2 c^4 \log(|x|) + \frac{6 a^4 b c^4 x - a^5 c^4}{2 x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^3,x, algorithm="giac")
```

output
```
1/3*b^5*c^4*x^3 - 3/2*a*b^4*c^4*x^2 + 2*a^2*b^3*c^4*x + 2*a^3*b^2*c^4*log(
abs(x)) + 1/2*(6*a^4*b*c^4*x - a^5*c^4)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3} \, dx = \frac{b^5 c^4 x^3}{3} - \frac{\frac{a^5 c^4}{2} - 3 a^4 b c^4 x}{x^2} + 2 a^2 b^3 c^4 x - \frac{3 a b^4 c^4 x^2}{2} + 2 a^3 b^2 c^4 \ln(x)$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^3,x)
```

output
```
(b^5*c^4*x^3)/3 - ((a^5*c^4)/2 - 3*a^4*b*c^4*x)/x^2 + 2*a^2*b^3*c^4*x - (3
*a*b^4*c^4*x^2)/2 + 2*a^3*b^2*c^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)(ac-bcx)^4}{x^3}\,dx = \frac{c^4(12\log(x)\,a^3b^2x^2-3a^5+18a^4bx+12a^2b^3x^3-9a\,b^4x^4+2b^5x^5)}{6x^2}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^3,x)
```

output
```
(c**4*(12*log(x)*a**3*b**2*x**2 - 3*a**5 + 18*a**4*b*x + 12*a**2*b**3*x**3
 - 9*a*b**4*x**4 + 2*b**5*x**5))/(6*x**2)
```

3.20 $\int \frac{(a+bx)(ac-bcx)^4}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4} dx = -\frac{a^5c^4}{3x^3} + \frac{3a^4bc^4}{2x^2} - \frac{2a^3b^2c^4}{x} - 3ab^4c^4x + \frac{1}{2}b^5c^4x^2 + 2a^2b^3c^4\log(x)$$

output
```
-1/3*a^5*c^4/x^3+3/2*a^4*b*c^4/x^2-2*a^3*b^2*c^4/x-3*a*b^4*c^4*x+1/2*b^5*c
^4*x^2+2*a^2*b^3*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4} dx = -\frac{a^5c^4}{3x^3} + \frac{3a^4bc^4}{2x^2} - \frac{2a^3b^2c^4}{x} - 3ab^4c^4x + \frac{1}{2}b^5c^4x^2 + 2a^2b^3c^4\log(x)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^4,x]
```

output
```
-1/3*(a^5*c^4)/x^3 + (3*a^4*b*c^4)/(2*x^2) - (2*a^3*b^2*c^4)/x - 3*a*b^4*c
^4*x + (b^5*c^4*x^2)/2 + 2*a^2*b^3*c^4*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4} \, dx$$
$$\Big\downarrow 84$$
$$\int \left(\frac{a^5c^4}{x^4}-\frac{3a^4bc^4}{x^3}+\frac{2a^3b^2c^4}{x^2}+\frac{2a^2b^3c^4}{x}-3ab^4c^4+b^5c^4x\right)dx$$
$$\Big\downarrow 2009$$
$$-\frac{a^5c^4}{3x^3}+\frac{3a^4bc^4}{2x^2}-\frac{2a^3b^2c^4}{x}+2a^2b^3c^4\log(x)-3ab^4c^4x+\frac{1}{2}b^5c^4x^2$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^4,x]
```

output
```
-1/3*(a^5*c^4)/x^3 + (3*a^4*b*c^4)/(2*x^2) - (2*a^3*b^2*c^4)/x - 3*a*b^4*c
^4*x + (b^5*c^4*x^2)/2 + 2*a^2*b^3*c^4*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result	size
default	$c^4\left(\frac{b^5x^2}{2}-3a\,b^4x-\frac{a^5}{3x^3}+\frac{3a^4b}{2x^2}+2a^2b^3\ln\left(x\right)-\frac{2a^3b^2}{x}\right)$	59
risch	$\frac{b^5c^4x^2}{2}-3a\,b^4c^4x+\frac{-2a^3b^2c^4x^2+\frac{3}{2}a^4b\,c^4x-\frac{1}{3}a^5c^4}{x^3}+2a^2b^3c^4\ln\left(x\right)$	73
norman	$\frac{-\frac{1}{3}a^5c^4+\frac{1}{2}b^5c^4x^5-3a\,b^4c^4x^4-2a^3b^2c^4x^2+\frac{3}{2}a^4b\,c^4x}{x^3}+2a^2b^3c^4\ln\left(x\right)$	75
parallelrisch	$\frac{3b^5c^4x^5+12a^2c^4b^3\ln(x)x^3-18a\,b^4c^4x^4-12a^3b^2c^4x^2+9a^4b\,c^4x-2a^5c^4}{6x^3}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^4*(1/2*b^5*x^2-3*a*b^4*x-1/3*a^5/x^3+3/2*a^4*b/x^2+2*a^2*b^3*ln(x)-2*a^3
*b^2/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int\frac{(a+bx)(ac-bcx)^4}{x^4}\,dx=\frac{3\,b^5c^4x^5-18\,ab^4c^4x^4+12\,a^2b^3c^4x^3\log\left(x\right)-12\,a^3b^2c^4x^2+9\,a^4bc^4x-2\,a^5c^4}{6\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^5*c^4*x^5 - 18*a*b^4*c^4*x^4 + 12*a^2*b^3*c^4*x^3*log(x) - 12*a^3
*b^2*c^4*x^2 + 9*a^4*b*c^4*x - 2*a^5*c^4)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4}\,dx = 2a^2b^3c^4\log{(x)} - 3ab^4c^4x + \frac{b^5c^4x^2}{2} + \frac{-2a^5c^4 + 9a^4bc^4x - 12a^3b^2c^4x^2}{6x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**4,x)
```

output
```
2*a**2*b**3*c**4*log(x) - 3*a*b**4*c**4*x + b**5*c**4*x**2/2 + (-2*a**5*c*
*4 + 9*a**4*b*c**4*x - 12*a**3*b**2*c**4*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4}\,dx = \frac{1}{2}\,b^5c^4x^2 - 3\,ab^4c^4x + 2\,a^2b^3c^4\log{(x)} - \frac{12\,a^3b^2c^4x^2 - 9\,a^4bc^4x + 2\,a^5c^4}{6\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^4,x, algorithm="maxima")
```

output
```
1/2*b^5*c^4*x^2 - 3*a*b^4*c^4*x + 2*a^2*b^3*c^4*log(x) - 1/6*(12*a^3*b^2*c
^4*x^2 - 9*a^4*b*c^4*x + 2*a^5*c^4)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4}\,dx = \frac{1}{2}\,b^5c^4x^2 - 3\,ab^4c^4x + 2\,a^2b^3c^4\log\left(|x|\right) - \frac{12\,a^3b^2c^4x^2 - 9\,a^4bc^4x + 2\,a^5c^4}{6\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^4,x, algorithm="giac")
```

output
```
1/2*b^5*c^4*x^2 - 3*a*b^4*c^4*x + 2*a^2*b^3*c^4*log(abs(x)) - 1/6*(12*a^3*
b^2*c^4*x^2 - 9*a^4*b*c^4*x + 2*a^5*c^4)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4}\,dx = \frac{b^5\,c^4\,x^2}{2} - \frac{\frac{a^5\,c^4}{3} - \frac{3\,a^4\,b\,c^4\,x}{2} + 2\,a^3\,b^2\,c^4\,x^2}{x^3} + 2\,a^2\,b^3\,c^4\,\ln\left(x\right) - 3\,a\,b^4\,c^4\,x$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^4,x)
```

output
```
(b^5*c^4*x^2)/2 - ((a^5*c^4)/3 + 2*a^3*b^2*c^4*x^2 - (3*a^4*b*c^4*x)/2)/x^
3 + 2*a^2*b^3*c^4*log(x) - 3*a*b^4*c^4*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)(ac-bcx)^4}{x^4}\,dx = \frac{c^4(12\log(x)\,a^2b^3x^3 - 2a^5 + 9a^4bx - 12a^3b^2x^2 - 18a\,b^4x^4 + 3b^5x^5)}{6x^3}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^4,x)
```

output
```
(c**4*(12*log(x)*a**2*b**3*x**3 - 2*a**5 + 9*a**4*b*x - 12*a**3*b**2*x**2
- 18*a*b**4*x**4 + 3*b**5*x**5))/(6*x**3)
```

3.21 $\int \frac{(a+bx)(ac-bcx)^4}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5} dx = -\frac{a^5c^4}{4x^4} + \frac{a^4bc^4}{x^3} - \frac{a^3b^2c^4}{x^2} - \frac{2a^2b^3c^4}{x} + b^5c^4x - 3ab^4c^4\log(x)$$

output
```
-1/4*a^5*c^4/x^4+a^4*b*c^4/x^3-a^3*b^2*c^4/x^2-2*a^2*b^3*c^4/x+b^5*c^4*x-3
*a*b^4*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5} dx = -\frac{a^5c^4}{4x^4} + \frac{a^4bc^4}{x^3} - \frac{a^3b^2c^4}{x^2} - \frac{2a^2b^3c^4}{x} + b^5c^4x - 3ab^4c^4\log(x)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^5,x]
```

output
```
-1/4*(a^5*c^4)/x^4 + (a^4*b*c^4)/x^3 - (a^3*b^2*c^4)/x^2 - (2*a^2*b^3*c^4)
/x + b^5*c^4*x - 3*a*b^4*c^4*Log[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5} \, dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^5c^4}{x^5}-\frac{3a^4bc^4}{x^4}+\frac{2a^3b^2c^4}{x^3}+\frac{2a^2b^3c^4}{x^2}-\frac{3ab^4c^4}{x}+b^5c^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5c^4}{4x^4}+\frac{a^4bc^4}{x^3}-\frac{a^3b^2c^4}{x^2}-\frac{2a^2b^3c^4}{x}-3ab^4c^4\log(x)+b^5c^4x$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^5,x]
```

output
```
-1/4*(a^5*c^4)/x^4 + (a^4*b*c^4)/x^3 - (a^3*b^2*c^4)/x^2 - (2*a^2*b^3*c^4)
/x + b^5*c^4*x - 3*a*b^4*c^4*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$c^4\left(b^5x+\frac{a^4b}{x^3}-\frac{a^3b^2}{x^2}-\frac{a^5}{4x^4}-3a\,b^4\ln\left(x\right)-\frac{2a^2b^3}{x}\right)$	57
risch	$b^5c^4x+\frac{-2a^2b^3c^4x^3-a^3b^2c^4x^2+a^4b\,c^4x-\frac{1}{4}a^5c^4}{x^4}-3a\,b^4c^4\ln\left(x\right)$	71
norman	$\frac{b^5c^4x^5+a^4b\,c^4x-\frac{1}{4}a^5c^4-2a^2b^3c^4x^3-a^3b^2c^4x^2}{x^4}-3a\,b^4c^4\ln\left(x\right)$	73
parallelrisch	$-\frac{12a\,c^4b^4\ln(x)x^4-4b^5c^4x^5+8a^2b^3c^4x^3+4a^3b^2c^4x^2-4a^4b\,c^4x+a^5c^4}{4x^4}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(b^5*x+a^4*b/x^3-a^3*b^2/x^2-1/4*a^5/x^4-3*a*b^4*ln(x)-2*a^2*b^3/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int\frac{(a+bx)(ac-bcx)^4}{x^5}\,dx=\frac{4\,b^5c^4x^5-12\,ab^4c^4x^4\log\left(x\right)-8\,a^2b^3c^4x^3-4\,a^3b^2c^4x^2+4\,a^4bc^4x-a^5c^4}{4\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*b^5*c^4*x^5 - 12*a*b^4*c^4*x^4*log(x) - 8*a^2*b^3*c^4*x^3 - 4*a^3*b
^2*c^4*x^2 + 4*a^4*b*c^4*x - a^5*c^4)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5}\,dx = -3ab^4c^4\log{(x)} + b^5c^4x + \frac{-a^5c^4 + 4a^4bc^4x - 4a^3b^2c^4x^2 - 8a^2b^3c^4x^3}{4x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**5,x)
```

output
```
-3*a*b**4*c**4*log(x) + b**5*c**4*x + (-a**5*c**4 + 4*a**4*b*c**4*x - 4*a*
*3*b**2*c**4*x**2 - 8*a**2*b**3*c**4*x**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5}\,dx = b^5c^4x - 3\,ab^4c^4\log{(x)} - \frac{8\,a^2b^3c^4x^3 + 4\,a^3b^2c^4x^2 - 4\,a^4bc^4x + a^5c^4}{4\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^5,x, algorithm="maxima")
```

output
```
b^5*c^4*x - 3*a*b^4*c^4*log(x) - 1/4*(8*a^2*b^3*c^4*x^3 + 4*a^3*b^2*c^4*x^
2 - 4*a^4*b*c^4*x + a^5*c^4)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5}\,dx = b^5c^4x - 3\,ab^4c^4\log\left(|x|\right) - \frac{8\,a^2b^3c^4x^3 + 4\,a^3b^2c^4x^2 - 4\,a^4bc^4x + a^5c^4}{4\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^5,x, algorithm="giac")
```

output
```
b^5*c^4*x - 3*a*b^4*c^4*log(abs(x)) - 1/4*(8*a^2*b^3*c^4*x^3 + 4*a^3*b^2*c
^4*x^2 - 4*a^4*b*c^4*x + a^5*c^4)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5}\,dx = -\frac{c^4\,\left(a^5-4\,b^5\,x^5+4\,a^3\,b^2\,x^2+8\,a^2\,b^3\,x^3-4\,a^4\,b\,x+12\,a\,b^4\,x^4\,\ln\left(x\right)\right)}{4\,x^4}$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^5,x)
```

output
```
-(c^4*(a^5 - 4*b^5*x^5 + 4*a^3*b^2*x^2 + 8*a^2*b^3*x^3 - 4*a^4*b*x + 12*a*
b^4*x^4*log(x)))/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^4}{x^5}\,dx = \frac{c^4(-12\log(x)\,a\,b^4x^4-a^5+4a^4bx-4a^3b^2x^2-8a^2b^3x^3+4b^5x^5)}{4x^4}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^5,x)
```

output
```
(c**4*( - 12*log(x)*a*b**4*x**4 - a**5 + 4*a**4*b*x - 4*a**3*b**2*x**2 - 8
*a**2*b**3*x**3 + 4*b**5*x**5))/(4*x**4)
```

3.22 $\int \frac{(a+bx)(ac-bcx)^4}{x^6} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6} \, dx = -\frac{a^5c^4}{5x^5}+\frac{3a^4bc^4}{4x^4}-\frac{2a^3b^2c^4}{3x^3}-\frac{a^2b^3c^4}{x^2}+\frac{3ab^4c^4}{x}+b^5c^4\log(x)$$

output
```
-1/5*a^5*c^4/x^5+3/4*a^4*b*c^4/x^4-2/3*a^3*b^2*c^4/x^3-a^2*b^3*c^4/x^2+3*a
*b^4*c^4/x+b^5*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6} \, dx = -\frac{a^5c^4}{5x^5}+\frac{3a^4bc^4}{4x^4}-\frac{2a^3b^2c^4}{3x^3}-\frac{a^2b^3c^4}{x^2}+\frac{3ab^4c^4}{x}+b^5c^4\log(x)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^6,x]
```

output
```
-1/5*(a^5*c^4)/x^5 + (3*a^4*b*c^4)/(4*x^4) - (2*a^3*b^2*c^4)/(3*x^3) - (a^
2*b^3*c^4)/x^2 + (3*a*b^4*c^4)/x + b^5*c^4*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6} \, dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^5c^4}{x^6}-\frac{3a^4bc^4}{x^5}+\frac{2a^3b^2c^4}{x^4}+\frac{2a^2b^3c^4}{x^3}-\frac{3ab^4c^4}{x^2}+\frac{b^5c^4}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5c^4}{5x^5}+\frac{3a^4bc^4}{4x^4}-\frac{2a^3b^2c^4}{3x^3}-\frac{a^2b^3c^4}{x^2}+\frac{3ab^4c^4}{x}+b^5c^4\log(x)$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^6,x]
```

output
```
-1/5*(a^5*c^4)/x^5 + (3*a^4*b*c^4)/(4*x^4) - (2*a^3*b^2*c^4)/(3*x^3) - (a^
2*b^3*c^4)/x^2 + (3*a*b^4*c^4)/x + b^5*c^4*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result	size
default	$c^4\left(-\frac{2a^3b^2}{3x^3}-\frac{a^5}{5x^5}-\frac{a^2b^3}{x^2}+\frac{3a^4b}{4x^4}+b^5\ln(x)+\frac{3a\,b^4}{x}\right)$	60
norman	$\frac{-\frac{1}{5}a^5c^4+3a\,b^4c^4x^4-a^2b^3c^4x^3-\frac{2}{3}a^3b^2c^4x^2+\frac{3}{4}a^4b\,c^4x}{x^5}+b^5c^4\ln(x)$	74
risch	$\frac{-\frac{1}{5}a^5c^4+3a\,b^4c^4x^4-a^2b^3c^4x^3-\frac{2}{3}a^3b^2c^4x^2+\frac{3}{4}a^4b\,c^4x}{x^5}+b^5c^4\ln(x)$	74
parallelrisch	$\frac{60c^4b^5\ln(x)x^5+180a\,b^4c^4x^4-60a^2b^3c^4x^3-40a^3b^2c^4x^2+45a^4b\,c^4x-12a^5c^4}{60x^5}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^6,x,method=_RETURNVERBOSE)
```

output
```
c^4*(-2/3*a^3*b^2/x^3-1/5*a^5/x^5-a^2*b^3/x^2+3/4*a^4*b/x^4+b^5*ln(x)+3*a*
b^4/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int\frac{(a+bx)(ac-bcx)^4}{x^6}\,dx=\frac{60\,b^5c^4x^5\log(x)+180\,ab^4c^4x^4-60\,a^2b^3c^4x^3-40\,a^3b^2c^4x^2+45\,a^4bc^4x-12\,a^5c^4}{60\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*b^5*c^4*x^5*log(x) + 180*a*b^4*c^4*x^4 - 60*a^2*b^3*c^4*x^3 - 40*
a^3*b^2*c^4*x^2 + 45*a^4*b*c^4*x - 12*a^5*c^4)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6}\,dx$$
$$= b^5c^4\log(x) + \frac{-12a^5c^4 + 45a^4bc^4x - 40a^3b^2c^4x^2 - 60a^2b^3c^4x^3 + 180ab^4c^4x^4}{60x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**6,x)
```

output
```
b**5*c**4*log(x) + (-12*a**5*c**4 + 45*a**4*b*c**4*x - 40*a**3*b**2*c**4*x
**2 - 60*a**2*b**3*c**4*x**3 + 180*a*b**4*c**4*x**4)/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6}\,dx$$
$$= b^5c^4\log(x) + \frac{180\,ab^4c^4x^4 - 60\,a^2b^3c^4x^3 - 40\,a^3b^2c^4x^2 + 45\,a^4bc^4x - 12\,a^5c^4}{60\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^6,x, algorithm="maxima")
```

output
```
b^5*c^4*log(x) + 1/60*(180*a*b^4*c^4*x^4 - 60*a^2*b^3*c^4*x^3 - 40*a^3*b^2
*c^4*x^2 + 45*a^4*b*c^4*x - 12*a^5*c^4)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6}\,dx = b^5c^4\log(|x|) + \frac{180\,ab^4c^4x^4 - 60\,a^2b^3c^4x^3 - 40\,a^3b^2c^4x^2 + 45\,a^4bc^4x - 12\,a^5c^4}{60\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^6,x, algorithm="giac")
```

output
```
b^5*c^4*log(abs(x)) + 1/60*(180*a*b^4*c^4*x^4 - 60*a^2*b^3*c^4*x^3 - 40*a^
3*b^2*c^4*x^2 + 45*a^4*b*c^4*x - 12*a^5*c^4)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6}\,dx = -\frac{c^4\left(\frac{a^5}{5} - 3\,a\,b^4\,x^4 + \frac{2\,a^3\,b^2\,x^2}{3} + a^2\,b^3\,x^3 - b^5\,x^5\,\ln(x) - \frac{3\,a^4\,b\,x}{4}\right)}{x^5}$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^6,x)
```

output
```
-(c^4*(a^5/5 - 3*a*b^4*x^4 + (2*a^3*b^2*x^2)/3 + a^2*b^3*x^3 - b^5*x^5*log
(x) - (3*a^4*b*x)/4))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)(ac-bcx)^4}{x^6}\,dx$$
$$= \frac{c^4(60\log(x)\,b^5x^5-12a^5+45a^4bx-40a^3b^2x^2-60a^2b^3x^3+180a\,b^4x^4)}{60x^5}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^6,x)
```

output
```
(c**4*(60*log(x)*b**5*x**5 - 12*a**5 + 45*a**4*b*x - 40*a**3*b**2*x**2 - 6
0*a**2*b**3*x**3 + 180*a*b**4*x**4))/(60*x**5)
```

3.23 $\int \frac{(a+bx)(ac-bcx)^4}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7} dx = -\frac{c^4(a-bx)^5}{6x^6} - \frac{7bc^4(a-bx)^5}{30ax^5}$$

output
```
-1/6*c^4*(-b*x+a)^5/x^6-7/30*b*c^4*(-b*x+a)^5/a/x^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(41) = 82.

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7} dx = -\frac{a^5c^4}{6x^6} + \frac{3a^4bc^4}{5x^5} - \frac{a^3b^2c^4}{2x^4} - \frac{2a^2b^3c^4}{3x^3} + \frac{3ab^4c^4}{2x^2} - \frac{b^5c^4}{x}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^7,x]
```

output
```
-1/6*(a^5*c^4)/x^6 + (3*a^4*b*c^4)/(5*x^5) - (a^3*b^2*c^4)/(2*x^4) - (2*a^
2*b^3*c^4)/(3*x^3) + (3*a*b^4*c^4)/(2*x^2) - (b^5*c^4)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {87, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{7}{6}b\int \frac{c^4(a-bx)^4}{x^6}dx-\frac{c^4(a-bx)^5}{6x^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{6}bc^4\int \frac{(a-bx)^4}{x^6}dx-\frac{c^4(a-bx)^5}{6x^6}$$

$$\Big\downarrow \text{48}$$

$$-\frac{c^4(a-bx)^5}{6x^6}-\frac{7bc^4(a-bx)^5}{30ax^5}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^7,x]
```

output
```
-1/6*(c^4*(a - b*x)^5)/x^6 - (7*b*c^4*(a - b*x)^5)/(30*a*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

method	result	size
gosper	$-\frac{c^{4}\left(30b^{5}x^{5}-45a\,b^{4}x^{4}+20a^{2}b^{3}x^{3}+15a^{3}b^{2}x^{2}-18a^{4}bx+5a^{5}\right)}{30x^{6}}$	61
default	$c^{4}\left(-\frac{2a^{2}b^{3}}{3x^{3}}+\frac{3a^{4}b}{5x^{5}}+\frac{3a\,b^{4}}{2x^{2}}-\frac{a^{3}b^{2}}{2x^{4}}-\frac{b^{5}}{x}-\frac{a^{5}}{6x^{6}}\right)$	62
norman	$\frac{-\frac{1}{6}a^{5}c^{4}-b^{5}c^{4}x^{5}+\frac{3}{2}a\,b^{4}c^{4}x^{4}-\frac{2}{3}a^{2}b^{3}c^{4}x^{3}-\frac{1}{2}a^{3}b^{2}c^{4}x^{2}+\frac{3}{5}a^{4}b\,c^{4}x}{x^{6}}$	75
risch	$\frac{-\frac{1}{6}a^{5}c^{4}-b^{5}c^{4}x^{5}+\frac{3}{2}a\,b^{4}c^{4}x^{4}-\frac{2}{3}a^{2}b^{3}c^{4}x^{3}-\frac{1}{2}a^{3}b^{2}c^{4}x^{2}+\frac{3}{5}a^{4}b\,c^{4}x}{x^{6}}$	75
parallelrisch	$\frac{-30b^{5}c^{4}x^{5}+45a\,b^{4}c^{4}x^{4}-20a^{2}b^{3}c^{4}x^{3}-15a^{3}b^{2}c^{4}x^{2}+18a^{4}b\,c^{4}x-5a^{5}c^{4}}{30x^{6}}$	76
orering	$-\frac{\left(30b^{5}x^{5}-45a\,b^{4}x^{4}+20a^{2}b^{3}x^{3}+15a^{3}b^{2}x^{2}-18a^{4}bx+5a^{5}\right)\left(-bcx+ac\right)^{4}}{30x^{6}\left(-bx+a\right)^{4}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/30*c^4*(30*b^5*x^5-45*a*b^4*x^4+20*a^2*b^3*x^3+15*a^3*b^2*x^2-18*a^4*b*
x+5*a^5)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}\,dx = -\frac{30\,b^{5}c^{4}x^{5}-45\,ab^{4}c^{4}x^{4}+20\,a^{2}b^{3}c^{4}x^{3}+15\,a^{3}b^{2}c^{4}x^{2}-18\,a^{4}bc^{4}x+5\,a^{5}c^{4}}{30\,x^{6}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^7,x, algorithm="fricas")
```

output
```
-1/30*(30*b^5*c^4*x^5 - 45*a*b^4*c^4*x^4 + 20*a^2*b^3*c^4*x^3 + 15*a^3*b^2
*c^4*x^2 - 18*a^4*b*c^4*x + 5*a^5*c^4)/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}\,dx = \frac{-5a^5c^4+18a^4bc^4x-15a^3b^2c^4x^2-20a^2b^3c^4x^3+45ab^4c^4x^4-30b^5c^4x^5}{30x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**7,x)
```

output
```
(-5*a**5*c**4 + 18*a**4*b*c**4*x - 15*a**3*b**2*c**4*x**2 - 20*a**2*b**3*c
**4*x**3 + 45*a*b**4*c**4*x**4 - 30*b**5*c**4*x**5)/(30*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}\,dx = -\frac{30\,b^5c^4x^5-45\,ab^4c^4x^4+20\,a^2b^3c^4x^3+15\,a^3b^2c^4x^2-18\,a^4bc^4x+5\,a^5c^4}{30\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^7,x, algorithm="maxima")
```

output
```
-1/30*(30*b^5*c^4*x^5 - 45*a*b^4*c^4*x^4 + 20*a^2*b^3*c^4*x^3 + 15*a^3*b^2
*c^4*x^2 - 18*a^4*b*c^4*x + 5*a^5*c^4)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}\,dx = -\frac{30\,b^5c^4x^5 - 45\,ab^4c^4x^4 + 20\,a^2b^3c^4x^3 + 15\,a^3b^2c^4x^2 - 18\,a^4bc^4x + 5\,a^5c^4}{30\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^7,x, algorithm="giac")
```

output
```
-1/30*(30*b^5*c^4*x^5 - 45*a*b^4*c^4*x^4 + 20*a^2*b^3*c^4*x^3 + 15*a^3*b^2
*c^4*x^2 - 18*a^4*b*c^4*x + 5*a^5*c^4)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}\,dx = -\frac{\frac{a^5\,c^4}{6} - \frac{3\,a^4\,b\,c^4\,x}{5} + \frac{a^3\,b^2\,c^4\,x^2}{2} + \frac{2\,a^2\,b^3\,c^4\,x^3}{3} - \frac{3\,a\,b^4\,c^4\,x^4}{2} + b^5\,c^4\,x^5}{x^6}$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^7,x)
```

output
```
-((a^5*c^4)/6 + b^5*c^4*x^5 - (3*a*b^4*c^4*x^4)/2 + (a^3*b^2*c^4*x^2)/2 +
(2*a^2*b^3*c^4*x^3)/3 - (3*a^4*b*c^4*x)/5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)(ac-bcx)^4}{x^7}\,dx = \frac{c^4(-30b^5x^5+45a\,b^4x^4-20a^2b^3x^3-15a^3b^2x^2+18a^4bx-5a^5)}{30x^6}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^7,x)
```

output
```
(c**4*( - 5*a**5 + 18*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 + 4
5*a*b**4*x**4 - 30*b**5*x**5))/(30*x**6)
```

3.24 $\int \frac{(a+bx)(ac-bcx)^4}{x^8} dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8} dx = -\frac{a^5c^4}{7x^7} + \frac{a^4bc^4}{2x^6} - \frac{2a^3b^2c^4}{5x^5} - \frac{a^2b^3c^4}{2x^4} + \frac{ab^4c^4}{x^3} - \frac{b^5c^4}{2x^2}$$

output
```
-1/7*a^5*c^4/x^7+1/2*a^4*b*c^4/x^6-2/5*a^3*b^2*c^4/x^5-1/2*a^2*b^3*c^4/x^4
+a*b^4*c^4/x^3-1/2*b^5*c^4/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8} dx = -\frac{a^5c^4}{7x^7} + \frac{a^4bc^4}{2x^6} - \frac{2a^3b^2c^4}{5x^5} - \frac{a^2b^3c^4}{2x^4} + \frac{ab^4c^4}{x^3} - \frac{b^5c^4}{2x^2}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^8,x]
```

output
```
-1/7*(a^5*c^4)/x^7 + (a^4*b*c^4)/(2*x^6) - (2*a^3*b^2*c^4)/(5*x^5) - (a^2*
b^3*c^4)/(2*x^4) + (a*b^4*c^4)/x^3 - (b^5*c^4)/(2*x^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx$$

$$\Big\downarrow \; 84$$

$$\int \left(\frac{a^5c^4}{x^8}-\frac{3a^4bc^4}{x^7}+\frac{2a^3b^2c^4}{x^6}+\frac{2a^2b^3c^4}{x^5}-\frac{3ab^4c^4}{x^4}+\frac{b^5c^4}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5c^4}{7x^7}+\frac{a^4bc^4}{2x^6}-\frac{2a^3b^2c^4}{5x^5}-\frac{a^2b^3c^4}{2x^4}+\frac{ab^4c^4}{x^3}-\frac{b^5c^4}{2x^2}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^8,x]
```

output
```
-1/7*(a^5*c^4)/x^7 + (a^4*b*c^4)/(2*x^6) - (2*a^3*b^2*c^4)/(5*x^5) - (a^2*
b^3*c^4)/(2*x^4) + (a*b^4*c^4)/x^3 - (b^5*c^4)/(2*x^2)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{c^{4}\left(35 b^{5} x^{5}-70 a\,b^{4} x^{4}+35 a^{2} b^{3} x^{3}+28 a^{3} b^{2} x^{2}-35 a^{4} b x +10 a^{5}\right)}{70 x^{7}}$	61
default	$c^{4}\left(\frac{a\,b^{4}}{x^{3}}-\frac{2 a^{3} b^{2}}{5 x^{5}}-\frac{b^{5}}{2 x^{2}}-\frac{a^{5}}{7 x^{7}}-\frac{a^{2} b^{3}}{2 x^{4}}+\frac{a^{4} b}{2 x^{6}}\right)$	61
norman	$\frac{a\,b^{4} c^{4} x^{4}-\frac{1}{7} a^{5} c^{4}-\frac{1}{2} b^{5} c^{4} x^{5}-\frac{1}{2} a^{2} b^{3} c^{4} x^{3}-\frac{2}{5} a^{3} b^{2} c^{4} x^{2}+\frac{1}{2} a^{4} b\,c^{4} x}{x^{7}}$	74
risch	$\frac{a\,b^{4} c^{4} x^{4}-\frac{1}{7} a^{5} c^{4}-\frac{1}{2} b^{5} c^{4} x^{5}-\frac{1}{2} a^{2} b^{3} c^{4} x^{3}-\frac{2}{5} a^{3} b^{2} c^{4} x^{2}+\frac{1}{2} a^{4} b\,c^{4} x}{x^{7}}$	74
parallelrisch	$\frac{-35 b^{5} c^{4} x^{5}+70 a\,b^{4} c^{4} x^{4}-35 a^{2} b^{3} c^{4} x^{3}-28 a^{3} b^{2} c^{4} x^{2}+35 a^{4} b\,c^{4} x -10 a^{5} c^{4}}{70 x^{7}}$	76
orering	$-\frac{\left(35 b^{5} x^{5}-70 a\,b^{4} x^{4}+35 a^{2} b^{3} x^{3}+28 a^{3} b^{2} x^{2}-35 a^{4} b x +10 a^{5}\right) \left(-b c x +a c \right)^{4}}{70 x^{7} \left(-b x +a \right)^{4}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/70*c^4*(35*b^5*x^5-70*a*b^4*x^4+35*a^2*b^3*x^3+28*a^3*b^2*x^2-35*a^4*b*
x+10*a^5)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx = -\frac{35\,b^5c^4x^5-70\,ab^4c^4x^4+35\,a^2b^3c^4x^3+28\,a^3b^2c^4x^2-35\,a^4bc^4x+10\,a^5c^4}{70\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^8,x, algorithm="fricas")
```

output
```
-1/70*(35*b^5*c^4*x^5 - 70*a*b^4*c^4*x^4 + 35*a^2*b^3*c^4*x^3 + 28*a^3*b^2
*c^4*x^2 - 35*a^4*b*c^4*x + 10*a^5*c^4)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx$$
$$= \frac{-10a^5c^4+35a^4bc^4x-28a^3b^2c^4x^2-35a^2b^3c^4x^3+70ab^4c^4x^4-35b^5c^4x^5}{70x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**8,x)
```

output
```
(-10*a**5*c**4 + 35*a**4*b*c**4*x - 28*a**3*b**2*c**4*x**2 - 35*a**2*b**3*
c**4*x**3 + 70*a*b**4*c**4*x**4 - 35*b**5*c**4*x**5)/(70*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx$$
$$= -\frac{35\,b^5c^4x^5-70\,ab^4c^4x^4+35\,a^2b^3c^4x^3+28\,a^3b^2c^4x^2-35\,a^4bc^4x+10\,a^5c^4}{70\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^8,x, algorithm="maxima")
```

output
```
-1/70*(35*b^5*c^4*x^5 - 70*a*b^4*c^4*x^4 + 35*a^2*b^3*c^4*x^3 + 28*a^3*b^2
*c^4*x^2 - 35*a^4*b*c^4*x + 10*a^5*c^4)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx = -\frac{35\,b^5c^4x^5 - 70\,ab^4c^4x^4 + 35\,a^2b^3c^4x^3 + 28\,a^3b^2c^4x^2 - 35\,a^4bc^4x + 10\,a^5c^4}{70\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^8,x, algorithm="giac")
```

output
```
-1/70*(35*b^5*c^4*x^5 - 70*a*b^4*c^4*x^4 + 35*a^2*b^3*c^4*x^3 + 28*a^3*b^2
*c^4*x^2 - 35*a^4*b*c^4*x + 10*a^5*c^4)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx = -\frac{\frac{a^5\,c^4}{7} - \frac{a^4\,b\,c^4\,x}{2} + \frac{2\,a^3\,b^2\,c^4\,x^2}{5} + \frac{a^2\,b^3\,c^4\,x^3}{2} - a\,b^4\,c^4\,x^4 + \frac{b^5\,c^4\,x^5}{2}}{x^7}$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^8,x)
```

output
```
-((a^5*c^4)/7 + (b^5*c^4*x^5)/2 - a*b^4*c^4*x^4 + (2*a^3*b^2*c^4*x^2)/5 +
(a^2*b^3*c^4*x^3)/2 - (a^4*b*c^4*x)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)(ac-bcx)^4}{x^8}\,dx = \frac{c^4(-35b^5x^5 + 70a\,b^4x^4 - 35a^2b^3x^3 - 28a^3b^2x^2 + 35a^4bx - 10a^5)}{70x^7}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^8,x)
```

output
```
(c**4*( - 10*a**5 + 35*a**4*b*x - 28*a**3*b**2*x**2 - 35*a**2*b**3*x**3 +
70*a*b**4*x**4 - 35*b**5*x**5))/(70*x**7)
```

3.25 $\int \frac{(a+bx)(ac-bcx)^4}{x^9} dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9} dx = -\frac{a^5c^4}{8x^8} + \frac{3a^4bc^4}{7x^7} - \frac{a^3b^2c^4}{3x^6} - \frac{2a^2b^3c^4}{5x^5} + \frac{3ab^4c^4}{4x^4} - \frac{b^5c^4}{3x^3}$$

output
```
-1/8*a^5*c^4/x^8+3/7*a^4*b*c^4/x^7-1/3*a^3*b^2*c^4/x^6-2/5*a^2*b^3*c^4/x^5
+3/4*a*b^4*c^4/x^4-1/3*b^5*c^4/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9} dx = -\frac{a^5c^4}{8x^8} + \frac{3a^4bc^4}{7x^7} - \frac{a^3b^2c^4}{3x^6} - \frac{2a^2b^3c^4}{5x^5} + \frac{3ab^4c^4}{4x^4} - \frac{b^5c^4}{3x^3}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^4)/x^9,x]
```

output
```
-1/8*(a^5*c^4)/x^8 + (3*a^4*b*c^4)/(7*x^7) - (a^3*b^2*c^4)/(3*x^6) - (2*a^
2*b^3*c^4)/(5*x^5) + (3*a*b^4*c^4)/(4*x^4) - (b^5*c^4)/(3*x^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9} \, dx$$

$$\Big\downarrow \; 84$$

$$\int \left(\frac{a^5c^4}{x^9}-\frac{3a^4bc^4}{x^8}+\frac{2a^3b^2c^4}{x^7}+\frac{2a^2b^3c^4}{x^6}-\frac{3ab^4c^4}{x^5}+\frac{b^5c^4}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5c^4}{8x^8}+\frac{3a^4bc^4}{7x^7}-\frac{a^3b^2c^4}{3x^6}-\frac{2a^2b^3c^4}{5x^5}+\frac{3ab^4c^4}{4x^4}-\frac{b^5c^4}{3x^3}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^4)/x^9,x]
```

output
```
-1/8*(a^5*c^4)/x^8 + (3*a^4*b*c^4)/(7*x^7) - (a^3*b^2*c^4)/(3*x^6) - (2*a^
2*b^3*c^4)/(5*x^5) + (3*a*b^4*c^4)/(4*x^4) - (b^5*c^4)/(3*x^3)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{c^{4}\left(280 b^{5} x^{5}-630 a \,b^{4} x^{4}+336 a^{2} b^{3} x^{3}+280 a^{3} b^{2} x^{2}-360 a^{4} b x +105 a^{5}\right)}{840 x^{8}}$	61
default	$c^{4}\left(-\frac{b^{5}}{3 x^{3}}-\frac{2 a^{2} b^{3}}{5 x^{5}}+\frac{3 a^{4} b}{7 x^{7}}+\frac{3 a \,b^{4}}{4 x^{4}}-\frac{a^{5}}{8 x^{8}}-\frac{a^{3} b^{2}}{3 x^{6}}\right)$	62
norman	$\frac{-\frac{1}{8} a^{5} c^{4}-\frac{1}{3} b^{5} c^{4} x^{5}+\frac{3}{4} a \,b^{4} c^{4} x^{4}-\frac{2}{5} a^{2} b^{3} c^{4} x^{3}-\frac{1}{3} a^{3} b^{2} c^{4} x^{2}+\frac{3}{7} a^{4} b \,c^{4} x}{x^{8}}$	75
risch	$\frac{-\frac{1}{8} a^{5} c^{4}-\frac{1}{3} b^{5} c^{4} x^{5}+\frac{3}{4} a \,b^{4} c^{4} x^{4}-\frac{2}{5} a^{2} b^{3} c^{4} x^{3}-\frac{1}{3} a^{3} b^{2} c^{4} x^{2}+\frac{3}{7} a^{4} b \,c^{4} x}{x^{8}}$	75
parallelrisch	$\frac{-280 b^{5} c^{4} x^{5}+630 a \,b^{4} c^{4} x^{4}-336 a^{2} b^{3} c^{4} x^{3}-280 a^{3} b^{2} c^{4} x^{2}+360 a^{4} b \,c^{4} x -105 a^{5} c^{4}}{840 x^{8}}$	76
orering	$-\frac{\left(280 b^{5} x^{5}-630 a \,b^{4} x^{4}+336 a^{2} b^{3} x^{3}+280 a^{3} b^{2} x^{2}-360 a^{4} b x +105 a^{5}\right) \left(-b c x +a c \right)^{4}}{840 x^{8} \left(-b x +a \right)^{4}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/840*c^4*(280*b^5*x^5-630*a*b^4*x^4+336*a^2*b^3*x^3+280*a^3*b^2*x^2-360*
a^4*b*x+105*a^5)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9}\,dx = -\frac{280\,b^5c^4x^5-630\,ab^4c^4x^4+336\,a^2b^3c^4x^3+280\,a^3b^2c^4x^2-360\,a^4bc^4x+105\,a^5c^4}{840\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^9,x, algorithm="fricas")
```

output
```
-1/840*(280*b^5*c^4*x^5 - 630*a*b^4*c^4*x^4 + 336*a^2*b^3*c^4*x^3 + 280*a^
3*b^2*c^4*x^2 - 360*a^4*b*c^4*x + 105*a^5*c^4)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9}\,dx = \frac{-105a^5c^4+360a^4bc^4x-280a^3b^2c^4x^2-336a^2b^3c^4x^3+630ab^4c^4x^4-280b^5c^4x^5}{840x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**4/x**9,x)
```

output
```
(-105*a**5*c**4 + 360*a**4*b*c**4*x - 280*a**3*b**2*c**4*x**2 - 336*a**2*b
**3*c**4*x**3 + 630*a*b**4*c**4*x**4 - 280*b**5*c**4*x**5)/(840*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9}\,dx = -\frac{280\,b^5c^4x^5-630\,ab^4c^4x^4+336\,a^2b^3c^4x^3+280\,a^3b^2c^4x^2-360\,a^4bc^4x+105\,a^5c^4}{840\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^9,x, algorithm="maxima")
```

output
```
-1/840*(280*b^5*c^4*x^5 - 630*a*b^4*c^4*x^4 + 336*a^2*b^3*c^4*x^3 + 280*a^
3*b^2*c^4*x^2 - 360*a^4*b*c^4*x + 105*a^5*c^4)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9}\,dx = -\frac{280\,b^5c^4x^5 - 630\,ab^4c^4x^4 + 336\,a^2b^3c^4x^3 + 280\,a^3b^2c^4x^2 - 360\,a^4bc^4x + 105\,a^5c^4}{840\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^4/x^9,x, algorithm="giac")
```

output
```
-1/840*(280*b^5*c^4*x^5 - 630*a*b^4*c^4*x^4 + 336*a^2*b^3*c^4*x^3 + 280*a^
3*b^2*c^4*x^2 - 360*a^4*b*c^4*x + 105*a^5*c^4)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9}\,dx = -\frac{\frac{a^5\,c^4}{8} - \frac{3\,a^4\,b\,c^4\,x}{7} + \frac{a^3\,b^2\,c^4\,x^2}{3} + \frac{2\,a^2\,b^3\,c^4\,x^3}{5} - \frac{3\,a\,b^4\,c^4\,x^4}{4} + \frac{b^5\,c^4\,x^5}{3}}{x^8}$$

input
```
int(((a*c - b*c*x)^4*(a + b*x))/x^9,x)
```

output
```
-((a^5*c^4)/8 + (b^5*c^4*x^5)/3 - (3*a*b^4*c^4*x^4)/4 + (a^3*b^2*c^4*x^2)/
3 + (2*a^2*b^3*c^4*x^3)/5 - (3*a^4*b*c^4*x)/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(ac-bcx)^4}{x^9}\,dx = \frac{c^4(-280b^5x^5 + 630a\,b^4x^4 - 336a^2b^3x^3 - 280a^3b^2x^2 + 360a^4bx - 105a^5)}{840x^8}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^4/x^9,x)
```

output
```
(c**4*( - 105*a**5 + 360*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**
3 + 630*a*b**4*x**4 - 280*b**5*x**5))/(840*x**8)
```

3.26 $\int x^4(a+bx)(ac-bcx)^5\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x^4(a+bx)(ac-bcx)^5\,dx=\frac{1}{5}a^6c^5x^5-\frac{2}{3}a^5bc^5x^6+\frac{5}{7}a^4b^2c^5x^7-\frac{5}{9}a^2b^4c^5x^9+\frac{2}{5}ab^5c^5x^{10}-\frac{1}{11}b^6c^5x^{11}$$

output
```
1/5*a^6*c^5*x^5-2/3*a^5*b*c^5*x^6+5/7*a^4*b^2*c^5*x^7-5/9*a^2*b^4*c^5*x^9+
2/5*a*b^5*c^5*x^10-1/11*b^6*c^5*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int x^4(a+bx)(ac-bcx)^5\,dx=c^5\left(\frac{a^6x^5}{5}-\frac{2}{3}a^5bx^6+\frac{5}{7}a^4b^2x^7-\frac{5}{9}a^2b^4x^9+\frac{2}{5}ab^5x^{10}-\frac{b^6x^{11}}{11}\right)$$

input
```
Integrate[x^4*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
c^5*((a^6*x^5)/5 - (2*a^5*b*x^6)/3 + (5*a^4*b^2*x^7)/7 - (5*a^2*b^4*x^9)/9
 + (2*a*b^5*x^10)/5 - (b^6*x^11)/11)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)(ac-bcx)^5\,dx$$

$$\Big\downarrow \ 84$$

$$\int \left(a^6c^5x^4-4a^5bc^5x^5+5a^4b^2c^5x^6-5a^2b^4c^5x^8+4ab^5c^5x^9-b^6c^5x^{10}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}a^6c^5x^5-\frac{2}{3}a^5bc^5x^6+\frac{5}{7}a^4b^2c^5x^7-\frac{5}{9}a^2b^4c^5x^9+\frac{2}{5}ab^5c^5x^{10}-\frac{1}{11}b^6c^5x^{11}$$

input
```
Int[x^4*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
(a^6*c^5*x^5)/5 - (2*a^5*b*c^5*x^6)/3 + (5*a^4*b^2*c^5*x^7)/7 - (5*a^2*b^4
*c^5*x^9)/9 + (2*a*b^5*c^5*x^10)/5 - (b^6*c^5*x^11)/11
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^{5}\left(-315 b^{6} x^{6}+1386 a\,x^{5} b^{5}-1925 a^{2} x^{4} b^{4}+2475 a^{4} x^{2} b^{2}-2310 a^{5} x b+693 a^{6}\right) c^{5}}{3465}$	61
default	$\frac{1}{5} a^{6} c^{5} x^{5}-\frac{2}{3} a^{5} b\,c^{5} x^{6}+\frac{5}{7} a^{4} b^{2} c^{5} x^{7}-\frac{5}{9} a^{2} b^{4} c^{5} x^{9}+\frac{2}{5} a\,b^{5} c^{5} x^{10}-\frac{1}{11} b^{6} c^{5} x^{11}$	76
norman	$\frac{1}{5} a^{6} c^{5} x^{5}-\frac{2}{3} a^{5} b\,c^{5} x^{6}+\frac{5}{7} a^{4} b^{2} c^{5} x^{7}-\frac{5}{9} a^{2} b^{4} c^{5} x^{9}+\frac{2}{5} a\,b^{5} c^{5} x^{10}-\frac{1}{11} b^{6} c^{5} x^{11}$	76
risch	$\frac{1}{5} a^{6} c^{5} x^{5}-\frac{2}{3} a^{5} b\,c^{5} x^{6}+\frac{5}{7} a^{4} b^{2} c^{5} x^{7}-\frac{5}{9} a^{2} b^{4} c^{5} x^{9}+\frac{2}{5} a\,b^{5} c^{5} x^{10}-\frac{1}{11} b^{6} c^{5} x^{11}$	76
parallelrisch	$\frac{1}{5} a^{6} c^{5} x^{5}-\frac{2}{3} a^{5} b\,c^{5} x^{6}+\frac{5}{7} a^{4} b^{2} c^{5} x^{7}-\frac{5}{9} a^{2} b^{4} c^{5} x^{9}+\frac{2}{5} a\,b^{5} c^{5} x^{10}-\frac{1}{11} b^{6} c^{5} x^{11}$	76
orering	$\frac{x^{5}\left(-315 b^{6} x^{6}+1386 a\,x^{5} b^{5}-1925 a^{2} x^{4} b^{4}+2475 a^{4} x^{2} b^{2}-2310 a^{5} x b+693 a^{6}\right) \left(-b c x+a c\right)^{5}}{3465 \left(-b x+a\right)^{5}}$	77

input
```
int(x^4*(b*x+a)*(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3465*x^5*(-315*b^6*x^6+1386*a*b^5*x^5-1925*a^2*b^4*x^4+2475*a^4*b^2*x^2-
2310*a^5*b*x+693*a^6)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^{4}(a+bx)(ac-bcx)^{5}\,dx=-\frac{1}{11}\,b^{6}c^{5}x^{11}+\frac{2}{5}\,ab^{5}c^{5}x^{10}-\frac{5}{9}\,a^{2}b^{4}c^{5}x^{9}+\frac{5}{7}\,a^{4}b^{2}c^{5}x^{7}-\frac{2}{3}\,a^{5}bc^{5}x^{6}+\frac{1}{5}\,a^{6}c^{5}x^{5}$$

input
```
integrate(x^4*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
-1/11*b^6*c^5*x^11 + 2/5*a*b^5*c^5*x^10 - 5/9*a^2*b^4*c^5*x^9 + 5/7*a^4*b^
2*c^5*x^7 - 2/3*a^5*b*c^5*x^6 + 1/5*a^6*c^5*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)(ac-bcx)^5\,dx = \frac{a^6c^5x^5}{5}-\frac{2a^5bc^5x^6}{3}+\frac{5a^4b^2c^5x^7}{7}-\frac{5a^2b^4c^5x^9}{9}+\frac{2ab^5c^5x^{10}}{5}-\frac{b^6c^5x^{11}}{11}$$

input
```
integrate(x**4*(b*x+a)*(-b*c*x+a*c)**5,x)
```

output
```
a**6*c**5*x**5/5 - 2*a**5*b*c**5*x**6/3 + 5*a**4*b**2*c**5*x**7/7 - 5*a**2
*b**4*c**5*x**9/9 + 2*a*b**5*c**5*x**10/5 - b**6*c**5*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^5\,dx = -\frac{1}{11}\,b^6c^5x^{11}+\frac{2}{5}\,ab^5c^5x^{10}-\frac{5}{9}\,a^2b^4c^5x^9+\frac{5}{7}\,a^4b^2c^5x^7-\frac{2}{3}\,a^5bc^5x^6+\frac{1}{5}\,a^6c^5x^5$$

input
```
integrate(x^4*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
-1/11*b^6*c^5*x^11 + 2/5*a*b^5*c^5*x^10 - 5/9*a^2*b^4*c^5*x^9 + 5/7*a^4*b^
2*c^5*x^7 - 2/3*a^5*b*c^5*x^6 + 1/5*a^6*c^5*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^5\,dx = -\frac{1}{11}b^6c^5x^{11} + \frac{2}{5}ab^5c^5x^{10} - \frac{5}{9}a^2b^4c^5x^9 + \frac{5}{7}a^4b^2c^5x^7 - \frac{2}{3}a^5bc^5x^6 + \frac{1}{5}a^6c^5x^5$$

input
```
integrate(x^4*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
-1/11*b^6*c^5*x^11 + 2/5*a*b^5*c^5*x^10 - 5/9*a^2*b^4*c^5*x^9 + 5/7*a^4*b^
2*c^5*x^7 - 2/3*a^5*b*c^5*x^6 + 1/5*a^6*c^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^4(a+bx)(ac-bcx)^5\,dx = \frac{a^6\,c^5\,x^5}{5} - \frac{2\,a^5\,b\,c^5\,x^6}{3} + \frac{5\,a^4\,b^2\,c^5\,x^7}{7} - \frac{5\,a^2\,b^4\,c^5\,x^9}{9} + \frac{2\,a\,b^5\,c^5\,x^{10}}{5} - \frac{b^6\,c^5\,x^{11}}{11}$$

input
```
int(x^4*(a*c - b*c*x)^5*(a + b*x),x)
```

output
```
(a^6*c^5*x^5)/5 - (b^6*c^5*x^11)/11 - (2*a^5*b*c^5*x^6)/3 + (2*a*b^5*c^5*x
^10)/5 + (5*a^4*b^2*c^5*x^7)/7 - (5*a^2*b^4*c^5*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int x^4(a+bx)(ac-bcx)^5\,dx$$
$$=\frac{c^5x^5(-315b^6x^6+1386a\,b^5x^5-1925a^2b^4x^4+2475a^4b^2x^2-2310a^5bx+693a^6)}{3465}$$

input
```
int(x^4*(b*x+a)*(-b*c*x+a*c)^5,x)
```

output
```
(c**5*x**5*(693*a**6 - 2310*a**5*b*x + 2475*a**4*b**2*x**2 - 1925*a**2*b**
4*x**4 + 1386*a*b**5*x**5 - 315*b**6*x**6))/3465
```

3.27 $\int x^3(a+bx)(ac-bcx)^5\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int x^3(a+bx)(ac-bcx)^5\,dx = \frac{1}{4}a^6c^5x^4-\frac{4}{5}a^5bc^5x^5+\frac{5}{6}a^4b^2c^5x^6-\frac{5}{8}a^2b^4c^5x^8+\frac{4}{9}ab^5c^5x^9-\frac{1}{10}b^6c^5x^{10}$$

output
```
1/4*a^6*c^5*x^4-4/5*a^5*b*c^5*x^5+5/6*a^4*b^2*c^5*x^6-5/8*a^2*b^4*c^5*x^8+
4/9*a*b^5*c^5*x^9-1/10*b^6*c^5*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int x^3(a+bx)(ac-bcx)^5\,dx = c^5\left(\frac{a^6x^4}{4}-\frac{4}{5}a^5bx^5+\frac{5}{6}a^4b^2x^6-\frac{5}{8}a^2b^4x^8+\frac{4}{9}ab^5x^9-\frac{b^6x^{10}}{10}\right)$$

input
```
Integrate[x^3*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
c^5*((a^6*x^4)/4 - (4*a^5*b*x^5)/5 + (5*a^4*b^2*x^6)/6 - (5*a^2*b^4*x^8)/8
 + (4*a*b^5*x^9)/9 - (b^6*x^10)/10)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)(ac-bcx)^5\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(a^6c^5x^3-4a^5bc^5x^4+5a^4b^2c^5x^5-5a^2b^4c^5x^7+4ab^5c^5x^8-b^6c^5x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^6c^5x^4-\frac{4}{5}a^5bc^5x^5+\frac{5}{6}a^4b^2c^5x^6-\frac{5}{8}a^2b^4c^5x^8+\frac{4}{9}ab^5c^5x^9-\frac{1}{10}b^6c^5x^{10}$$

input
```
Int[x^3*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
(a^6*c^5*x^4)/4 - (4*a^5*b*c^5*x^5)/5 + (5*a^4*b^2*c^5*x^6)/6 - (5*a^2*b^4
*c^5*x^8)/8 + (4*a*b^5*c^5*x^9)/9 - (b^6*c^5*x^10)/10
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x^4(-36b^6x^6+160a\,x^5b^5-225a^2x^4b^4+300a^4x^2b^2-288a^5xb+90a^6)c^5}{360}$	61
default	$\frac{1}{4}a^6c^5x^4-\frac{4}{5}a^5b\,c^5x^5+\frac{5}{6}a^4b^2c^5x^6-\frac{5}{8}a^2b^4c^5x^8+\frac{4}{9}a\,b^5c^5x^9-\frac{1}{10}b^6c^5x^{10}$	76
norman	$\frac{1}{4}a^6c^5x^4-\frac{4}{5}a^5b\,c^5x^5+\frac{5}{6}a^4b^2c^5x^6-\frac{5}{8}a^2b^4c^5x^8+\frac{4}{9}a\,b^5c^5x^9-\frac{1}{10}b^6c^5x^{10}$	76
risch	$\frac{1}{4}a^6c^5x^4-\frac{4}{5}a^5b\,c^5x^5+\frac{5}{6}a^4b^2c^5x^6-\frac{5}{8}a^2b^4c^5x^8+\frac{4}{9}a\,b^5c^5x^9-\frac{1}{10}b^6c^5x^{10}$	76
parallelrisch	$\frac{1}{4}a^6c^5x^4-\frac{4}{5}a^5b\,c^5x^5+\frac{5}{6}a^4b^2c^5x^6-\frac{5}{8}a^2b^4c^5x^8+\frac{4}{9}a\,b^5c^5x^9-\frac{1}{10}b^6c^5x^{10}$	76
orering	$\frac{x^4(-36b^6x^6+160a\,x^5b^5-225a^2x^4b^4+300a^4x^2b^2-288a^5xb+90a^6)(-bcx+ac)^5}{360(-bx+a)^5}$	77

input
```
int(x^3*(b*x+a)*(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/360*x^4*(-36*b^6*x^6+160*a*b^5*x^5-225*a^2*b^4*x^4+300*a^4*b^2*x^2-288*a
^5*b*x+90*a^6)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^5\,dx = -\frac{1}{10}\,b^6c^5x^{10}+\frac{4}{9}\,ab^5c^5x^9-\frac{5}{8}\,a^2b^4c^5x^8 +\frac{5}{6}\,a^4b^2c^5x^6-\frac{4}{5}\,a^5bc^5x^5+\frac{1}{4}\,a^6c^5x^4$$

input
```
integrate(x^3*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
-1/10*b^6*c^5*x^10 + 4/9*a*b^5*c^5*x^9 - 5/8*a^2*b^4*c^5*x^8 + 5/6*a^4*b^2
*c^5*x^6 - 4/5*a^5*b*c^5*x^5 + 1/4*a^6*c^5*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)(ac-bcx)^5\,dx = \frac{a^6c^5x^4}{4} - \frac{4a^5bc^5x^5}{5} + \frac{5a^4b^2c^5x^6}{6} - \frac{5a^2b^4c^5x^8}{8} + \frac{4ab^5c^5x^9}{9} - \frac{b^6c^5x^{10}}{10}$$

input
```
integrate(x**3*(b*x+a)*(-b*c*x+a*c)**5,x)
```

output
```
a**6*c**5*x**4/4 - 4*a**5*b*c**5*x**5/5 + 5*a**4*b**2*c**5*x**6/6 - 5*a**2
*b**4*c**5*x**8/8 + 4*a*b**5*c**5*x**9/9 - b**6*c**5*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^5\,dx = -\frac{1}{10}\,b^6c^5x^{10} + \frac{4}{9}\,ab^5c^5x^9 - \frac{5}{8}\,a^2b^4c^5x^8 + \frac{5}{6}\,a^4b^2c^5x^6 - \frac{4}{5}\,a^5bc^5x^5 + \frac{1}{4}\,a^6c^5x^4$$

input
```
integrate(x^3*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
-1/10*b^6*c^5*x^10 + 4/9*a*b^5*c^5*x^9 - 5/8*a^2*b^4*c^5*x^8 + 5/6*a^4*b^2
*c^5*x^6 - 4/5*a^5*b*c^5*x^5 + 1/4*a^6*c^5*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^5\,dx = -\frac{1}{10}\,b^6c^5x^{10} + \frac{4}{9}\,ab^5c^5x^9 - \frac{5}{8}\,a^2b^4c^5x^8 + \frac{5}{6}\,a^4b^2c^5x^6 - \frac{4}{5}\,a^5bc^5x^5 + \frac{1}{4}\,a^6c^5x^4$$

input
```
integrate(x^3*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
-1/10*b^6*c^5*x^10 + 4/9*a*b^5*c^5*x^9 - 5/8*a^2*b^4*c^5*x^8 + 5/6*a^4*b^2
*c^5*x^6 - 4/5*a^5*b*c^5*x^5 + 1/4*a^6*c^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int x^3(a+bx)(ac-bcx)^5\,dx = \frac{a^6\,c^5\,x^4}{4} - \frac{4\,a^5\,b\,c^5\,x^5}{5} + \frac{5\,a^4\,b^2\,c^5\,x^6}{6} - \frac{5\,a^2\,b^4\,c^5\,x^8}{8} + \frac{4\,a\,b^5\,c^5\,x^9}{9} - \frac{b^6\,c^5\,x^{10}}{10}$$

input
```
int(x^3*(a*c - b*c*x)^5*(a + b*x),x)
```

output
```
(a^6*c^5*x^4)/4 - (b^6*c^5*x^10)/10 - (4*a^5*b*c^5*x^5)/5 + (4*a*b^5*c^5*x
^9)/9 + (5*a^4*b^2*c^5*x^6)/6 - (5*a^2*b^4*c^5*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int x^3(a+bx)(ac-bcx)^5\,dx$$
$$=\frac{c^5x^4(-36b^6x^6+160a\,b^5x^5-225a^2b^4x^4+300a^4b^2x^2-288a^5bx+90a^6)}{360}$$

input

```
int(x^3*(b*x+a)*(-b*c*x+a*c)^5,x)
```

output

```
(c**5*x**4*(90*a**6 - 288*a**5*b*x + 300*a**4*b**2*x**2 - 225*a**2*b**4*x*
*4 + 160*a*b**5*x**5 - 36*b**6*x**6))/360
```

3.28 $\int x^2(a+bx)(ac-bcx)^5\,dx$

Optimal result

Integrand size = 20, antiderivative size = 80

$$\int x^2(a+bx)(ac-bcx)^5\,dx = -\frac{a^3c^5(a-bx)^6}{3b^3}+\frac{5a^2c^5(a-bx)^7}{7b^3}-\frac{ac^5(a-bx)^8}{2b^3}+\frac{c^5(a-bx)^9}{9b^3}$$

output
```
-1/3*a^3*c^5*(-b*x+a)^6/b^3+5/7*a^2*c^5*(-b*x+a)^7/b^3-1/2*a*c^5*(-b*x+a)^
8/b^3+1/9*c^5*(-b*x+a)^9/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(ac-bcx)^5\,dx = c^5\left(\frac{a^6x^3}{3}-a^5bx^4+a^4b^2x^5-\frac{5}{7}a^2b^4x^7+\frac{1}{2}ab^5x^8-\frac{b^6x^9}{9}\right)$$

input
```
Integrate[x^2*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
c^5*((a^6*x^3)/3 - a^5*b*x^4 + a^4*b^2*x^5 - (5*a^2*b^4*x^7)/7 + (a*b^5*x^
8)/2 - (b^6*x^9)/9)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)(ac-bcx)^5\,dx$$
$$\Big\downarrow \text{84}$$
$$\int \left(\frac{2a^3(ac-bcx)^5}{b^2}-\frac{5a^2(ac-bcx)^6}{b^2c}-\frac{(ac-bcx)^8}{b^2c^3}+\frac{4a(ac-bcx)^7}{b^2c^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^3c^5(a-bx)^6}{3b^3}+\frac{5a^2c^5(a-bx)^7}{7b^3}+\frac{c^5(a-bx)^9}{9b^3}-\frac{ac^5(a-bx)^8}{2b^3}$$

input
```
Int[x^2*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
-1/3*(a^3*c^5*(a - b*x)^6)/b^3 + (5*a^2*c^5*(a - b*x)^7)/(7*b^3) - (a*c^5*
(a - b*x)^8)/(2*b^3) + (c^5*(a - b*x)^9)/(9*b^3)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{3}\left(-14 b^{6} x^{6}+63 a\,x^{5} b^{5}-90 a^{2} x^{4} b^{4}+126 a^{4} x^{2} b^{2}-126 a^{5} x b+42 a^{6}\right) c^{5}}{126}$	61
default	$-\frac{1}{9} b^{6} c^{5} x^{9}+\frac{1}{2} a\,c^{5} b^{5} x^{8}-\frac{5}{7} a^{2} c^{5} b^{4} x^{7}+a^{4} c^{5} b^{2} x^{5}-a^{5} c^{5} b\,x^{4}+\frac{1}{3} a^{6} c^{5} x^{3}$	75
norman	$-\frac{1}{9} b^{6} c^{5} x^{9}+\frac{1}{2} a\,c^{5} b^{5} x^{8}-\frac{5}{7} a^{2} c^{5} b^{4} x^{7}+a^{4} c^{5} b^{2} x^{5}-a^{5} c^{5} b\,x^{4}+\frac{1}{3} a^{6} c^{5} x^{3}$	75
risch	$-\frac{1}{9} b^{6} c^{5} x^{9}+\frac{1}{2} a\,c^{5} b^{5} x^{8}-\frac{5}{7} a^{2} c^{5} b^{4} x^{7}+a^{4} c^{5} b^{2} x^{5}-a^{5} c^{5} b\,x^{4}+\frac{1}{3} a^{6} c^{5} x^{3}$	75
parallelrisch	$-\frac{1}{9} b^{6} c^{5} x^{9}+\frac{1}{2} a\,c^{5} b^{5} x^{8}-\frac{5}{7} a^{2} c^{5} b^{4} x^{7}+a^{4} c^{5} b^{2} x^{5}-a^{5} c^{5} b\,x^{4}+\frac{1}{3} a^{6} c^{5} x^{3}$	75
orering	$\frac{x^{3}\left(-14 b^{6} x^{6}+63 a\,x^{5} b^{5}-90 a^{2} x^{4} b^{4}+126 a^{4} x^{2} b^{2}-126 a^{5} x b+42 a^{6}\right) \left(-b c x+a c\right)^{5}}{126 \left(-b x+a\right)^{5}}$	77

input
```
int(x^2*(b*x+a)*(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/126*x^3*(-14*b^6*x^6+63*a*b^5*x^5-90*a^2*b^4*x^4+126*a^4*b^2*x^2-126*a^5
*b*x+42*a^6)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)(ac-bcx)^5\,dx = -\frac{1}{9}\,b^6c^5x^9 + \frac{1}{2}\,ab^5c^5x^8 - \frac{5}{7}\,a^2b^4c^5x^7 + a^4b^2c^5x^5 - a^5bc^5x^4 + \frac{1}{3}\,a^6c^5x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
-1/9*b^6*c^5*x^9 + 1/2*a*b^5*c^5*x^8 - 5/7*a^2*b^4*c^5*x^7 + a^4*b^2*c^5*x
^5 - a^5*b*c^5*x^4 + 1/3*a^6*c^5*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)(ac-bcx)^5\,dx = \frac{a^6c^5x^3}{3} - a^5bc^5x^4 + a^4b^2c^5x^5 - \frac{5a^2b^4c^5x^7}{7} + \frac{ab^5c^5x^8}{2} - \frac{b^6c^5x^9}{9}$$

input
```
integrate(x**2*(b*x+a)*(-b*c*x+a*c)**5,x)
```

output
```
a**6*c**5*x**3/3 - a**5*b*c**5*x**4 + a**4*b**2*c**5*x**5 - 5*a**2*b**4*c*
*5*x**7/7 + a*b**5*c**5*x**8/2 - b**6*c**5*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)(ac-bcx)^5\,dx = -\frac{1}{9}b^6c^5x^9 + \frac{1}{2}ab^5c^5x^8 - \frac{5}{7}a^2b^4c^5x^7 + a^4b^2c^5x^5 - a^5bc^5x^4 + \frac{1}{3}a^6c^5x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
-1/9*b^6*c^5*x^9 + 1/2*a*b^5*c^5*x^8 - 5/7*a^2*b^4*c^5*x^7 + a^4*b^2*c^5*x
^5 - a^5*b*c^5*x^4 + 1/3*a^6*c^5*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)(ac-bcx)^5\,dx = -\frac{1}{9}b^6c^5x^9 + \frac{1}{2}ab^5c^5x^8 - \frac{5}{7}a^2b^4c^5x^7 + a^4b^2c^5x^5 - a^5bc^5x^4 + \frac{1}{3}a^6c^5x^3$$

input
```
integrate(x^2*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
-1/9*b^6*c^5*x^9 + 1/2*a*b^5*c^5*x^8 - 5/7*a^2*b^4*c^5*x^7 + a^4*b^2*c^5*x
^5 - a^5*b*c^5*x^4 + 1/3*a^6*c^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)(ac-bcx)^5\,dx = \frac{a^6\,c^5\,x^3}{3} - a^5\,b\,c^5\,x^4 + a^4\,b^2\,c^5\,x^5 - \frac{5\,a^2\,b^4\,c^5\,x^7}{7} + \frac{a\,b^5\,c^5\,x^8}{2} - \frac{b^6\,c^5\,x^9}{9}$$

input
```
int(x^2*(a*c - b*c*x)^5*(a + b*x),x)
```

output
```
(a^6*c^5*x^3)/3 - (b^6*c^5*x^9)/9 - a^5*b*c^5*x^4 + (a*b^5*c^5*x^8)/2 + a^
4*b^2*c^5*x^5 - (5*a^2*b^4*c^5*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^2(a+bx)(ac-bcx)^5\,dx = \frac{c^5x^3(-14b^6x^6+63a\,b^5x^5-90a^2b^4x^4+126a^4b^2x^2-126a^5bx+42a^6)}{126}$$

input
```
int(x^2*(b*x+a)*(-b*c*x+a*c)^5,x)
```

output
```
(c**5*x**3*(42*a**6 - 126*a**5*b*x + 126*a**4*b**2*x**2 - 90*a**2*b**4*x**
4 + 63*a*b**5*x**5 - 14*b**6*x**6))/126
```

3.29 $\int x(a+bx)(ac-bcx)^5\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int x(a+bx)(ac-bcx)^5\,dx=-\frac{a^2c^5(a-bx)^6}{3b^2}+\frac{3ac^5(a-bx)^7}{7b^2}-\frac{c^5(a-bx)^8}{8b^2}$$

output
```
-1/3*a^2*c^5*(-b*x+a)^6/b^2+3/7*a*c^5*(-b*x+a)^7/b^2-1/8*c^5*(-b*x+a)^8/b^
2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x(a+bx)(ac-bcx)^5\,dx=c^5\left(\frac{a^6x^2}{2}-\frac{4}{3}a^5bx^3+\frac{5}{4}a^4b^2x^4-\frac{5}{6}a^2b^4x^6+\frac{4}{7}ab^5x^7-\frac{b^6x^8}{8}\right)$$

input
```
Integrate[x*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
c^5*((a^6*x^2)/2 - (4*a^5*b*x^3)/3 + (5*a^4*b^2*x^4)/4 - (5*a^2*b^4*x^6)/6
 + (4*a*b^5*x^7)/7 - (b^6*x^8)/8)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)(ac-bcx)^5\,dx$$
$$\Big\downarrow \; 84$$
$$\int \left(\frac{2a^2(ac-bcx)^5}{b}+\frac{(ac-bcx)^7}{bc^2}-\frac{3a(ac-bcx)^6}{bc}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^2c^5(a-bx)^6}{3b^2}-\frac{c^5(a-bx)^8}{8b^2}+\frac{3ac^5(a-bx)^7}{7b^2}$$

input
```
Int[x*(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
-1/3*(a^2*c^5*(a - b*x)^6)/b^2 + (3*a*c^5*(a - b*x)^7)/(7*b^2) - (c^5*(a -
 b*x)^8)/(8*b^2)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{x^2(-21b^6x^6+96a\,x^5b^5-140a^2x^4b^4+210a^4x^2b^2-224a^5xb+84a^6)c^5}{168}$	61
default	$-\frac{1}{8}b^6c^5x^8+\frac{4}{7}a\,c^5b^5x^7-\frac{5}{6}a^2c^5b^4x^6+\frac{5}{4}a^4c^5b^2x^4-\frac{4}{3}a^5c^5b\,x^3+\frac{1}{2}a^6c^5x^2$	76
norman	$-\frac{1}{8}b^6c^5x^8+\frac{4}{7}a\,c^5b^5x^7-\frac{5}{6}a^2c^5b^4x^6+\frac{5}{4}a^4c^5b^2x^4-\frac{4}{3}a^5c^5b\,x^3+\frac{1}{2}a^6c^5x^2$	76
risch	$-\frac{1}{8}b^6c^5x^8+\frac{4}{7}a\,c^5b^5x^7-\frac{5}{6}a^2c^5b^4x^6+\frac{5}{4}a^4c^5b^2x^4-\frac{4}{3}a^5c^5b\,x^3+\frac{1}{2}a^6c^5x^2$	76
parallelrisch	$-\frac{1}{8}b^6c^5x^8+\frac{4}{7}a\,c^5b^5x^7-\frac{5}{6}a^2c^5b^4x^6+\frac{5}{4}a^4c^5b^2x^4-\frac{4}{3}a^5c^5b\,x^3+\frac{1}{2}a^6c^5x^2$	76
orering	$\frac{x^2(-21b^6x^6+96a\,x^5b^5-140a^2x^4b^4+210a^4x^2b^2-224a^5xb+84a^6)(-bcx+ac)^5}{168(-bx+a)^5}$	77

input
```
int(x*(b*x+a)*(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/168*x^2*(-21*b^6*x^6+96*a*b^5*x^5-140*a^2*b^4*x^4+210*a^4*b^2*x^2-224*a^
5*b*x+84*a^6)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^5\,dx = -\frac{1}{8}\,b^6c^5x^8+\frac{4}{7}\,ab^5c^5x^7-\frac{5}{6}\,a^2b^4c^5x^6+\frac{5}{4}\,a^4b^2c^5x^4-\frac{4}{3}\,a^5bc^5x^3+\frac{1}{2}\,a^6c^5x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
-1/8*b^6*c^5*x^8 + 4/7*a*b^5*c^5*x^7 - 5/6*a^2*b^4*c^5*x^6 + 5/4*a^4*b^2*c
^5*x^4 - 4/3*a^5*b*c^5*x^3 + 1/2*a^6*c^5*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x(a+bx)(ac-bcx)^5\,dx = \frac{a^6c^5x^2}{2} - \frac{4a^5bc^5x^3}{3} + \frac{5a^4b^2c^5x^4}{4} - \frac{5a^2b^4c^5x^6}{6} + \frac{4ab^5c^5x^7}{7} - \frac{b^6c^5x^8}{8}$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)**5,x)
```

output
```
a**6*c**5*x**2/2 - 4*a**5*b*c**5*x**3/3 + 5*a**4*b**2*c**5*x**4/4 - 5*a**2
*b**4*c**5*x**6/6 + 4*a*b**5*c**5*x**7/7 - b**6*c**5*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^5\,dx = -\frac{1}{8}\,b^6c^5x^8 + \frac{4}{7}\,ab^5c^5x^7 - \frac{5}{6}\,a^2b^4c^5x^6 + \frac{5}{4}\,a^4b^2c^5x^4 - \frac{4}{3}\,a^5bc^5x^3 + \frac{1}{2}\,a^6c^5x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
-1/8*b^6*c^5*x^8 + 4/7*a*b^5*c^5*x^7 - 5/6*a^2*b^4*c^5*x^6 + 5/4*a^4*b^2*c
^5*x^4 - 4/3*a^5*b*c^5*x^3 + 1/2*a^6*c^5*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^5\,dx = -\frac{1}{8}\,b^6c^5x^8+\frac{4}{7}\,ab^5c^5x^7-\frac{5}{6}\,a^2b^4c^5x^6+\frac{5}{4}\,a^4b^2c^5x^4-\frac{4}{3}\,a^5bc^5x^3+\frac{1}{2}\,a^6c^5x^2$$

input
```
integrate(x*(b*x+a)*(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
-1/8*b^6*c^5*x^8 + 4/7*a*b^5*c^5*x^7 - 5/6*a^2*b^4*c^5*x^6 + 5/4*a^4*b^2*c
^5*x^4 - 4/3*a^5*b*c^5*x^3 + 1/2*a^6*c^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x(a+bx)(ac-bcx)^5\,dx = \frac{a^6\,c^5\,x^2}{2}-\frac{4\,a^5\,b\,c^5\,x^3}{3}+\frac{5\,a^4\,b^2\,c^5\,x^4}{4}-\frac{5\,a^2\,b^4\,c^5\,x^6}{6}+\frac{4\,a\,b^5\,c^5\,x^7}{7}-\frac{b^6\,c^5\,x^8}{8}$$

input
```
int(x*(a*c - b*c*x)^5*(a + b*x),x)
```

output
```
(a^6*c^5*x^2)/2 - (b^6*c^5*x^8)/8 - (4*a^5*b*c^5*x^3)/3 + (4*a*b^5*c^5*x^7
)/7 + (5*a^4*b^2*c^5*x^4)/4 - (5*a^2*b^4*c^5*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int x(a+bx)(ac-bcx)^5\,dx$$
$$=\frac{c^5x^2(-21b^6x^6+96a\,b^5x^5-140a^2b^4x^4+210a^4b^2x^2-224a^5bx+84a^6)}{168}$$

input
```
int(x*(b*x+a)*(-b*c*x+a*c)^5,x)
```

output
```
(c**5*x**2*(84*a**6 - 224*a**5*b*x + 210*a**4*b**2*x**2 - 140*a**2*b**4*x*
*4 + 96*a*b**5*x**5 - 21*b**6*x**6))/168
```

3.30 $\int (a+bx)(ac-bcx)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (a+bx)(ac-bcx)^5\,dx = -\frac{ac^5(a-bx)^6}{3b}+\frac{c^5(a-bx)^7}{7b}$$

output
```
-1/3*a*c^5*(-b*x+a)^6/b+1/7*c^5*(-b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int (a+bx)(ac-bcx)^5\,dx = c^5\left(a^6x-2a^5bx^2+\frac{5}{3}a^4b^2x^3-a^2b^4x^5+\frac{2}{3}ab^5x^6-\frac{b^6x^7}{7}\right)$$

input
```
Integrate[(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
c^5*(a^6*x - 2*a^5*b*x^2 + (5*a^4*b^2*x^3)/3 - a^2*b^4*x^5 + (2*a*b^5*x^6)
/3 - (b^6*x^7)/7)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(ac-bcx)^5\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(2a(ac-bcx)^5-\frac{(ac-bcx)^6}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^5(a-bx)^7}{7b}-\frac{ac^5(a-bx)^6}{3b}$$

input
```
Int[(a + b*x)*(a*c - b*c*x)^5,x]
```

output
```
-1/3*(a*c^5*(a - b*x)^6)/b + (c^5*(a - b*x)^7)/(7*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
gosper	$\frac{x\left(-3b^{6}x^{6}+14a\,x^{5}b^{5}-21a^{2}x^{4}b^{4}+35a^{4}x^{2}b^{2}-42a^{5}xb+21a^{6}\right)c^{5}}{21}$	59
default	$-\frac{1}{7}b^{6}c^{5}x^{7}+\frac{2}{3}a\,b^{5}c^{5}x^{6}-a^{2}b^{4}c^{5}x^{5}+\frac{5}{3}a^{4}b^{2}c^{5}x^{3}-2a^{5}b\,c^{5}x^{2}+a^{6}c^{5}x$	73
norman	$-\frac{1}{7}b^{6}c^{5}x^{7}+\frac{2}{3}a\,b^{5}c^{5}x^{6}-a^{2}b^{4}c^{5}x^{5}+\frac{5}{3}a^{4}b^{2}c^{5}x^{3}-2a^{5}b\,c^{5}x^{2}+a^{6}c^{5}x$	73
risch	$-\frac{1}{7}b^{6}c^{5}x^{7}+\frac{2}{3}a\,b^{5}c^{5}x^{6}-a^{2}b^{4}c^{5}x^{5}+\frac{5}{3}a^{4}b^{2}c^{5}x^{3}-2a^{5}b\,c^{5}x^{2}+a^{6}c^{5}x$	73
parallelrisch	$-\frac{1}{7}b^{6}c^{5}x^{7}+\frac{2}{3}a\,b^{5}c^{5}x^{6}-a^{2}b^{4}c^{5}x^{5}+\frac{5}{3}a^{4}b^{2}c^{5}x^{3}-2a^{5}b\,c^{5}x^{2}+a^{6}c^{5}x$	73
orering	$\frac{x\left(-3b^{6}x^{6}+14a\,x^{5}b^{5}-21a^{2}x^{4}b^{4}+35a^{4}x^{2}b^{2}-42a^{5}xb+21a^{6}\right)\left(-bcx+ac\right)^{5}}{21\left(-bx+a\right)^{5}}$	75

input
```
int((b*x+a)*(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(-3*b^6*x^6+14*a*b^5*x^5-21*a^2*b^4*x^4+35*a^4*b^2*x^2-42*a^5*b*x+2
1*a^6)*c^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^5\,dx = -\frac{1}{7}\,b^6c^5x^7+\frac{2}{3}\,ab^5c^5x^6-a^2b^4c^5x^5 + \frac{5}{3}\,a^4b^2c^5x^3-2\,a^5bc^5x^2+a^6c^5x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5,x, algorithm="fricas")
```

output
```
-1/7*b^6*c^5*x^7 + 2/3*a*b^5*c^5*x^6 - a^2*b^4*c^5*x^5 + 5/3*a^4*b^2*c^5*x
^3 - 2*a^5*b*c^5*x^2 + a^6*c^5*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int (a+bx)(ac-bcx)^5\,dx = a^6c^5x - 2a^5bc^5x^2 + \frac{5a^4b^2c^5x^3}{3} - a^2b^4c^5x^5 + \frac{2ab^5c^5x^6}{3} - \frac{b^6c^5x^7}{7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5,x)
```

output
```
a**6*c**5*x - 2*a**5*b*c**5*x**2 + 5*a**4*b**2*c**5*x**3/3 - a**2*b**4*c**
5*x**5 + 2*a*b**5*c**5*x**6/3 - b**6*c**5*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^5\,dx = -\frac{1}{7}\,b^6c^5x^7 + \frac{2}{3}\,ab^5c^5x^6 - a^2b^4c^5x^5 + \frac{5}{3}\,a^4b^2c^5x^3 - 2\,a^5bc^5x^2 + a^6c^5x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5,x, algorithm="maxima")
```

output
```
-1/7*b^6*c^5*x^7 + 2/3*a*b^5*c^5*x^6 - a^2*b^4*c^5*x^5 + 5/3*a^4*b^2*c^5*x
^3 - 2*a^5*b*c^5*x^2 + a^6*c^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^5\,dx = -\frac{1}{7}\,b^6c^5x^7 + \frac{2}{3}\,ab^5c^5x^6 - a^2b^4c^5x^5 + \frac{5}{3}\,a^4b^2c^5x^3 - 2\,a^5bc^5x^2 + a^6c^5x$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5,x, algorithm="giac")
```

output
```
-1/7*b^6*c^5*x^7 + 2/3*a*b^5*c^5*x^6 - a^2*b^4*c^5*x^5 + 5/3*a^4*b^2*c^5*x
^3 - 2*a^5*b*c^5*x^2 + a^6*c^5*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)(ac-bcx)^5\,dx = a^6\,c^5\,x - 2\,a^5\,b\,c^5\,x^2 + \frac{5\,a^4\,b^2\,c^5\,x^3}{3} - a^2\,b^4\,c^5\,x^5 + \frac{2\,a\,b^5\,c^5\,x^6}{3} - \frac{b^6\,c^5\,x^7}{7}$$

input
```
int((a*c - b*c*x)^5*(a + b*x),x)
```

output
```
a^6*c^5*x - (b^6*c^5*x^7)/7 - 2*a^5*b*c^5*x^2 + (2*a*b^5*c^5*x^6)/3 + (5*a
^4*b^2*c^5*x^3)/3 - a^2*b^4*c^5*x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int (a+bx)(ac-bcx)^5\,dx = \frac{c^5x(-3b^6x^6 + 14a\,b^5x^5 - 21a^2b^4x^4 + 35a^4b^2x^2 - 42a^5bx + 21a^6)}{21}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5,x)
```

output
```
(c**5*x*(21*a**6 - 42*a**5*b*x + 35*a**4*b**2*x**2 - 21*a**2*b**4*x**4 + 1
4*a*b**5*x**5 - 3*b**6*x**6))/21
```

3.31 $\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = -4a^5bc^5x + \frac{5}{2}a^4b^2c^5x^2 - \frac{5}{4}a^2b^4c^5x^4 + \frac{4}{5}ab^5c^5x^5 - \frac{1}{6}b^6c^5x^6 + a^6c^5\log(x)$$

output
```
-4*a^5*b*c^5*x+5/2*a^4*b^2*c^5*x^2-5/4*a^2*b^4*c^5*x^4+4/5*a*b^5*c^5*x^5-1
/6*b^6*c^5*x^6+a^6*c^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = c^5\left(\frac{127a^6}{60} - 4a^5bx + \frac{5}{2}a^4b^2x^2 - \frac{5}{4}a^2b^4x^4 + \frac{4}{5}ab^5x^5 - \frac{b^6x^6}{6} + a^6\log(-bx)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x,x]
```

output
```
c^5*((127*a^6)/60 - 4*a^5*b*x + (5*a^4*b^2*x^2)/2 - (5*a^2*b^4*x^4)/4 + (4
*a*b^5*x^5)/5 - (b^6*x^6)/6 + a^6*Log[-(b*x)])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x} \, dx$$

$$\Big\downarrow \ 84$$

$$\int \left(\frac{a^6c^5}{x} - 4a^5bc^5 + 5a^4b^2c^5x - 5a^2b^4c^5x^3 + 4ab^5c^5x^4 - b^6c^5x^5\right) dx$$

$$\Big\downarrow \ 2009$$

$$a^6c^5\log(x) - 4a^5bc^5x + \frac{5}{2}a^4b^2c^5x^2 - \frac{5}{4}a^2b^4c^5x^4 + \frac{4}{5}ab^5c^5x^5 - \frac{1}{6}b^6c^5x^6$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x,x]
```

output
```
-4*a^5*b*c^5*x + (5*a^4*b^2*c^5*x^2)/2 - (5*a^2*b^4*c^5*x^4)/4 + (4*a*b^5*
c^5*x^5)/5 - (b^6*c^5*x^6)/6 + a^6*c^5*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

method	result	size
default	$c^{5}\left(-\frac{b^{6} x^{6}}{6}+\frac{4 a\,x^{5} b^{5}}{5}-\frac{5 a^{2} x^{4} b^{4}}{4}+\frac{5 a^{4} x^{2} b^{2}}{2}-4 a^{5} x b +a^{6} \ln \left(x \right)\right)$	58
norman	$-4 a^{5} b\,c^{5} x +\frac{5 a^{4} b^{2} c^{5} x^{2}}{2}-\frac{5 a^{2} b^{4} c^{5} x^{4}}{4}+\frac{4 a\,b^{5} c^{5} x^{5}}{5}-\frac{b^{6} c^{5} x^{6}}{6}+a^{6} c^{5} \ln \left(x \right)$	72
risch	$-4 a^{5} b\,c^{5} x +\frac{5 a^{4} b^{2} c^{5} x^{2}}{2}-\frac{5 a^{2} b^{4} c^{5} x^{4}}{4}+\frac{4 a\,b^{5} c^{5} x^{5}}{5}-\frac{b^{6} c^{5} x^{6}}{6}+a^{6} c^{5} \ln \left(x \right)$	72
parallelrisch	$-4 a^{5} b\,c^{5} x +\frac{5 a^{4} b^{2} c^{5} x^{2}}{2}-\frac{5 a^{2} b^{4} c^{5} x^{4}}{4}+\frac{4 a\,b^{5} c^{5} x^{5}}{5}-\frac{b^{6} c^{5} x^{6}}{6}+a^{6} c^{5} \ln \left(x \right)$	72

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-1/6*b^6*x^6+4/5*a*x^5*b^5-5/4*a^2*x^4*b^4+5/2*a^4*x^2*b^2-4*a^5*x*b+
a^6*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = -\frac{1}{6}\,b^{6}c^{5}x^{6} + \frac{4}{5}\,ab^{5}c^{5}x^{5} - \frac{5}{4}\,a^{2}b^{4}c^{5}x^{4} + \frac{5}{2}\,a^{4}b^{2}c^{5}x^{2} - 4\,a^{5}bc^{5}x + a^{6}c^{5}\log\left(x\right)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x,x, algorithm="fricas")
```

output
```
-1/6*b^6*c^5*x^6 + 4/5*a*b^5*c^5*x^5 - 5/4*a^2*b^4*c^5*x^4 + 5/2*a^4*b^2*c
^5*x^2 - 4*a^5*b*c^5*x + a^6*c^5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = a^6c^5\log(x) - 4a^5bc^5x + \frac{5a^4b^2c^5x^2}{2} - \frac{5a^2b^4c^5x^4}{4} + \frac{4ab^5c^5x^5}{5} - \frac{b^6c^5x^6}{6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x,x)
```

output
```
a**6*c**5*log(x) - 4*a**5*b*c**5*x + 5*a**4*b**2*c**5*x**2/2 - 5*a**2*b**4
*c**5*x**4/4 + 4*a*b**5*c**5*x**5/5 - b**6*c**5*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = -\frac{1}{6}\,b^6c^5x^6 + \frac{4}{5}\,ab^5c^5x^5 - \frac{5}{4}\,a^2b^4c^5x^4 + \frac{5}{2}\,a^4b^2c^5x^2 - 4\,a^5bc^5x + a^6c^5\log(x)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x,x, algorithm="maxima")
```

output
```
-1/6*b^6*c^5*x^6 + 4/5*a*b^5*c^5*x^5 - 5/4*a^2*b^4*c^5*x^4 + 5/2*a^4*b^2*c
^5*x^2 - 4*a^5*b*c^5*x + a^6*c^5*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = -\frac{1}{6}b^6c^5x^6 + \frac{4}{5}ab^5c^5x^5 - \frac{5}{4}a^2b^4c^5x^4 + \frac{5}{2}a^4b^2c^5x^2 - 4a^5bc^5x + a^6c^5\log(|x|)$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x,x, algorithm="giac")
```

output
```
-1/6*b^6*c^5*x^6 + 4/5*a*b^5*c^5*x^5 - 5/4*a^2*b^4*c^5*x^4 + 5/2*a^4*b^2*c
^5*x^2 - 4*a^5*b*c^5*x + a^6*c^5*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx = a^6\,c^5\,\ln(x) - \frac{b^6\,c^5\,x^6}{6} + \frac{4\,a\,b^5\,c^5\,x^5}{5} + \frac{5\,a^4\,b^2\,c^5\,x^2}{2} - \frac{5\,a^2\,b^4\,c^5\,x^4}{4} - 4\,a^5\,b\,c^5\,x$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x,x)
```

output
```
a^6*c^5*log(x) - (b^6*c^5*x^6)/6 + (4*a*b^5*c^5*x^5)/5 + (5*a^4*b^2*c^5*x^
2)/2 - (5*a^2*b^4*c^5*x^4)/4 - 4*a^5*b*c^5*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)(ac-bcx)^5}{x}\,dx$$
$$= \frac{c^5(60\log(x)\,a^6 - 240a^5bx + 150a^4b^2x^2 - 75a^2b^4x^4 + 48a\,b^5x^5 - 10b^6x^6)}{60}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x,x)
```

output
```
(c**5*(60*log(x)*a**6 - 240*a**5*b*x + 150*a**4*b**2*x**2 - 75*a**2*b**4*x
**4 + 48*a*b**5*x**5 - 10*b**6*x**6))/60
```

3.32 $\int \frac{(a+bx)(ac-bcx)^5}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2} dx = -\frac{a^6c^5}{x} + 5a^4b^2c^5x - \frac{5}{3}a^2b^4c^5x^3 + ab^5c^5x^4 - \frac{1}{5}b^6c^5x^5 - 4a^5bc^5\log(x)$$

output
```
-a^6*c^5/x+5*a^4*b^2*c^5*x-5/3*a^2*b^4*c^5*x^3+a*b^5*c^5*x^4-1/5*b^6*c^5*x
^5-4*a^5*b*c^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2} dx = c^5\left(-\frac{a^6}{x} + 5a^4b^2x - \frac{5}{3}a^2b^4x^3 + ab^5x^4 - \frac{b^6x^5}{5} - 4a^5b\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^2,x]
```

output
```
c^5*(-(a^6/x) + 5*a^4*b^2*x - (5*a^2*b^4*x^3)/3 + a*b^5*x^4 - (b^6*x^5)/5
- 4*a^5*b*Log[x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2} \, dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^6c^5}{x^2} - \frac{4a^5bc^5}{x} + 5a^4b^2c^5 - 5a^2b^4c^5x^2 + 4ab^5c^5x^3 - b^6c^5x^4\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^6c^5}{x} - 4a^5bc^5\log(x) + 5a^4b^2c^5x - \frac{5}{3}a^2b^4c^5x^3 + ab^5c^5x^4 - \frac{1}{5}b^6c^5x^5$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^2,x]
```

output
```
-((a^6*c^5)/x) + 5*a^4*b^2*c^5*x - (5*a^2*b^4*c^5*x^3)/3 + a*b^5*c^5*x^4 -
 (b^6*c^5*x^5)/5 - 4*a^5*b*c^5*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result	size
default	$c^{5}\left(-\frac{b^{6} x^{5}}{5}+a\,b^{5} x^{4}-\frac{5 a^{2} b^{4} x^{3}}{3}+5 a^{4} b^{2} x -4 a^{5} b \ln \left(x \right)-\frac{a^{6}}{x}\right)$	58
risch	$-\frac{a^{6} c^{5}}{x}+5 a^{4} b^{2} c^{5} x -\frac{5 a^{2} b^{4} c^{5} x^{3}}{3}+a\,b^{5} c^{5} x^{4}-\frac{b^{6} c^{5} x^{5}}{5}-4 a^{5} b\,c^{5} \ln \left(x \right)$	72
norman	$\frac{a\,b^{5} c^{5} x^{5}-a^{6} c^{5}-\frac{1}{5} b^{6} c^{5} x^{6}-\frac{5}{3} a^{2} b^{4} c^{5} x^{4}+5 a^{4} b^{2} c^{5} x^{2}}{x}-4 a^{5} b\,c^{5} \ln \left(x \right)$	76
parallelrisch	$-\frac{3 b^{6} c^{5} x^{6}-15 a\,b^{5} c^{5} x^{5}+25 a^{2} b^{4} c^{5} x^{4}+60 a^{5} c^{5} b \ln \left(x \right) x -75 a^{4} b^{2} c^{5} x^{2}+15 a^{6} c^{5}}{15 x}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-1/5*b^6*x^5+a*b^5*x^4-5/3*a^2*b^4*x^3+5*a^4*b^2*x-4*a^5*b*ln(x)-a^6/
x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2}\,dx = -\frac{3\,b^{6}c^{5}x^{6} - 15\,ab^{5}c^{5}x^{5} + 25\,a^{2}b^{4}c^{5}x^{4} - 75\,a^{4}b^{2}c^{5}x^{2} + 60\,a^{5}bc^{5}x\log\left(x\right) + 15\,a^{6}c^{5}}{15\,x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^2,x, algorithm="fricas")
```

output
```
-1/15*(3*b^6*c^5*x^6 - 15*a*b^5*c^5*x^5 + 25*a^2*b^4*c^5*x^4 - 75*a^4*b^2*
c^5*x^2 + 60*a^5*b*c^5*x*log(x) + 15*a^6*c^5)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2}\,dx = -\frac{a^6c^5}{x} - 4a^5bc^5\log{(x)} + 5a^4b^2c^5x - \frac{5a^2b^4c^5x^3}{3} + ab^5c^5x^4 - \frac{b^6c^5x^5}{5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**2,x)
```

output
```
-a**6*c**5/x - 4*a**5*b*c**5*log(x) + 5*a**4*b**2*c**5*x - 5*a**2*b**4*c**
5*x**3/3 + a*b**5*c**5*x**4 - b**6*c**5*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2}\,dx = -\frac{1}{5}\,b^6c^5x^5 + ab^5c^5x^4 - \frac{5}{3}\,a^2b^4c^5x^3 + 5\,a^4b^2c^5x - 4\,a^5bc^5\log{(x)} - \frac{a^6c^5}{x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^2,x, algorithm="maxima")
```

output
```
-1/5*b^6*c^5*x^5 + a*b^5*c^5*x^4 - 5/3*a^2*b^4*c^5*x^3 + 5*a^4*b^2*c^5*x -
 4*a^5*b*c^5*log(x) - a^6*c^5/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2}\,dx = -\frac{1}{5}\,b^6c^5x^5 + ab^5c^5x^4 - \frac{5}{3}\,a^2b^4c^5x^3 + 5\,a^4b^2c^5x - 4\,a^5bc^5\log\left(|x|\right) - \frac{a^6c^5}{x}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^2,x, algorithm="giac")
```

output
```
-1/5*b^6*c^5*x^5 + a*b^5*c^5*x^4 - 5/3*a^2*b^4*c^5*x^3 + 5*a^4*b^2*c^5*x -
 4*a^5*b*c^5*log(abs(x)) - a^6*c^5/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2}\,dx = 5\,a^4\,b^2\,c^5\,x - \frac{b^6\,c^5\,x^5}{5} - \frac{a^6\,c^5}{x} + a\,b^5\,c^5\,x^4 - 4\,a^5\,b\,c^5\,\ln\left(x\right) - \frac{5\,a^2\,b^4\,c^5\,x^3}{3}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^2,x)
```

output
```
5*a^4*b^2*c^5*x - (b^6*c^5*x^5)/5 - (a^6*c^5)/x + a*b^5*c^5*x^4 - 4*a^5*b*
c^5*log(x) - (5*a^2*b^4*c^5*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^2}\,dx = \frac{c^5(-60\log(x)\,a^5bx-15a^6+75a^4b^2x^2-25a^2b^4x^4+15a\,b^5x^5-3b^6x^6)}{15x}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^2,x)
```

output
```
(c**5*( - 60*log(x)*a**5*b*x - 15*a**6 + 75*a**4*b**2*x**2 - 25*a**2*b**4*
x**4 + 15*a*b**5*x**5 - 3*b**6*x**6))/(15*x)
```

3.33 $\int \frac{(a+bx)(ac-bcx)^5}{x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3} \, dx = -\frac{a^6c^5}{2x^2} + \frac{4a^5bc^5}{x} - \frac{5}{2}a^2b^4c^5x^2 + \frac{4}{3}ab^5c^5x^3 - \frac{1}{4}b^6c^5x^4 + 5a^4b^2c^5\log(x)$$

output
```
-1/2*a^6*c^5/x^2+4*a^5*b*c^5/x-5/2*a^2*b^4*c^5*x^2+4/3*a*b^5*c^5*x^3-1/4*b
^6*c^5*x^4+5*a^4*b^2*c^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3} \, dx = c^5 \left(-\frac{a^6}{2x^2} + \frac{4a^5b}{x} - \frac{5}{2}a^2b^4x^2 + \frac{4}{3}ab^5x^3 - \frac{b^6x^4}{4} + 5a^4b^2\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^3,x]
```

output
```
c^5*(-1/2*a^6/x^2 + (4*a^5*b)/x - (5*a^2*b^4*x^2)/2 + (4*a*b^5*x^3)/3 - (b
^6*x^4)/4 + 5*a^4*b^2*Log[x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3} \, dx$$

$$\Big\downarrow \; 84$$

$$\int \left(\frac{a^6c^5}{x^3}-\frac{4a^5bc^5}{x^2}+\frac{5a^4b^2c^5}{x}-5a^2b^4c^5x+4ab^5c^5x^2-b^6c^5x^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^6c^5}{2x^2}+\frac{4a^5bc^5}{x}+5a^4b^2c^5\log(x)-\frac{5}{2}a^2b^4c^5x^2+\frac{4}{3}ab^5c^5x^3-\frac{1}{4}b^6c^5x^4$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^3,x]
```

output
```
-1/2*(a^6*c^5)/x^2 + (4*a^5*b*c^5)/x - (5*a^2*b^4*c^5*x^2)/2 + (4*a*b^5*c^
5*x^3)/3 - (b^6*c^5*x^4)/4 + 5*a^4*b^2*c^5*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
default	$c^{5}\left(-\frac{b^{6} x^{4}}{4}+\frac{4 a\, b^{5} x^{3}}{3}-\frac{5 a^{2} b^{4} x^{2}}{2}-\frac{a^{6}}{2 x^{2}}+5 a^{4} b^{2} \ln \left(x \right)+\frac{4 a^{5} b}{x}\right)$	61
norman	$\frac{-\frac{1}{2} a^{6} c^{5}-\frac{1}{4} b^{6} c^{5} x^{6}+\frac{4}{3} a\, b^{5} c^{5} x^{5}-\frac{5}{2} a^{2} b^{4} c^{5} x^{4}+4 a^{5} b\, c^{5} x}{x^{2}}+5 a^{4} b^{2} c^{5} \ln \left(x \right)$	75
risch	$-\frac{b^{6} c^{5} x^{4}}{4}+\frac{4 a\, b^{5} c^{5} x^{3}}{3}-\frac{5 a^{2} b^{4} c^{5} x^{2}}{2}+\frac{4 a^{5} b\, c^{5} x -\frac{1}{2} a^{6} c^{5}}{x^{2}}+5 a^{4} b^{2} c^{5} \ln \left(x \right)$	75
parallelrisch	$\frac{-3 b^{6} c^{5} x^{6}+16 a\, b^{5} c^{5} x^{5}-30 a^{2} b^{4} c^{5} x^{4}+60 a^{4} c^{5} b^{2} \ln \left(x \right) x^{2}+48 a^{5} b\, c^{5} x -6 a^{6} c^{5}}{12 x^{2}}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-1/4*b^6*x^4+4/3*a*b^5*x^3-5/2*a^2*b^4*x^2-1/2*a^6/x^2+5*a^4*b^2*ln(x
)+4*a^5*b/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3}\,dx = -\frac{3\,b^{6}c^{5}x^{6} - 16\,ab^{5}c^{5}x^{5} + 30\,a^{2}b^{4}c^{5}x^{4} - 60\,a^{4}b^{2}c^{5}x^{2}\log\left(x\right) - 48\,a^{5}bc^{5}x + 6\,a^{6}c^{5}}{12\,x^{2}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^3,x, algorithm="fricas")
```

output
```
-1/12*(3*b^6*c^5*x^6 - 16*a*b^5*c^5*x^5 + 30*a^2*b^4*c^5*x^4 - 60*a^4*b^2*
c^5*x^2*log(x) - 48*a^5*b*c^5*x + 6*a^6*c^5)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3}\,dx = 5a^4b^2c^5\log{(x)} - \frac{5a^2b^4c^5x^2}{2} + \frac{4ab^5c^5x^3}{3} - \frac{b^6c^5x^4}{4} - \frac{a^6c^5 - 8a^5bc^5x}{2x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**3,x)
```

output
```
5*a**4*b**2*c**5*log(x) - 5*a**2*b**4*c**5*x**2/2 + 4*a*b**5*c**5*x**3/3 -
 b**6*c**5*x**4/4 - (a**6*c**5 - 8*a**5*b*c**5*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3}\,dx = -\frac{1}{4}\,b^6c^5x^4 + \frac{4}{3}\,ab^5c^5x^3 - \frac{5}{2}\,a^2b^4c^5x^2 + 5\,a^4b^2c^5\log{(x)} + \frac{8\,a^5bc^5x - a^6c^5}{2\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^3,x, algorithm="maxima")
```

output
```
-1/4*b^6*c^5*x^4 + 4/3*a*b^5*c^5*x^3 - 5/2*a^2*b^4*c^5*x^2 + 5*a^4*b^2*c^5
*log(x) + 1/2*(8*a^5*b*c^5*x - a^6*c^5)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3}\,dx = -\frac{1}{4}\,b^6c^5x^4+\frac{4}{3}\,ab^5c^5x^3-\frac{5}{2}\,a^2b^4c^5x^2 + 5\,a^4b^2c^5\log\left(|x|\right)+\frac{8\,a^5bc^5x-a^6c^5}{2\,x^2}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^3,x, algorithm="giac")
```

output
```
-1/4*b^6*c^5*x^4 + 4/3*a*b^5*c^5*x^3 - 5/2*a^2*b^4*c^5*x^2 + 5*a^4*b^2*c^5
*log(abs(x)) + 1/2*(8*a^5*b*c^5*x - a^6*c^5)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3}\,dx = \frac{4\,a\,b^5\,c^5\,x^3}{3}-\frac{b^6\,c^5\,x^4}{4}-\frac{\frac{a^6\,c^5}{2}-4\,a^5\,b\,c^5\,x}{x^2} - \frac{5\,a^2\,b^4\,c^5\,x^2}{2}+5\,a^4\,b^2\,c^5\,\ln\left(x\right)$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^3,x)
```

output
```
(4*a*b^5*c^5*x^3)/3 - (b^6*c^5*x^4)/4 - ((a^6*c^5)/2 - 4*a^5*b*c^5*x)/x^2
- (5*a^2*b^4*c^5*x^2)/2 + 5*a^4*b^2*c^5*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)(ac-bcx)^5}{x^3}\,dx = \frac{c^5(60\log(x)\,a^4b^2x^2-6a^6+48a^5bx-30a^2b^4x^4+16a\,b^5x^5-3b^6x^6)}{12x^2}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^3,x)
```

output
```
(c**5*(60*log(x)*a**4*b**2*x**2 - 6*a**6 + 48*a**5*b*x - 30*a**2*b**4*x**4
 + 16*a*b**5*x**5 - 3*b**6*x**6))/(12*x**2)
```

3.34 $\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = -\frac{c^5(a-bx)^6}{3x^3}$$

output
```
-1/3*c^5*(-b*x+a)^6/x^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.50

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = c^5\left(-\frac{a^6}{3x^3}+\frac{2a^5b}{x^2}-\frac{5a^4b^2}{x}-5a^2b^4x+2ab^5x^2-\frac{b^6x^3}{3}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^4,x]
```

output
```
c^5*(-1/3*a^6/x^3 + (2*a^5*b)/x^2 - (5*a^4*b^2)/x - 5*a^2*b^4*x + 2*a*b^5*
x^2 - (b^6*x^3)/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4} \, dx$$

$$\Big\downarrow \; 83$$

$$-\frac{c^5(a-bx)^6}{3x^3}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^4,x]
```

output
```
-1/3*(c^5*(a - b*x)^6)/x^3
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(16) = 32.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

method	result	size
gosper	$-\frac{c^5(b^6x^6-6a\,x^5b^5+15a^2x^4b^4+15a^4x^2b^2-6a^5xb+a^6)}{3x^3}$	58
default	$c^5\left(-\frac{b^6x^3}{3}+2a\,b^5x^2-5a^2b^4x-\frac{a^6}{3x^3}+\frac{2a^5b}{x^2}-\frac{5a^4b^2}{x}\right)$	60
risch	$-\frac{b^6c^5x^3}{3}+2b^5c^5a\,x^2-5b^4c^5a^2x+\frac{-5a^4b^2c^5x^2+2a^5b\,c^5x-\frac{1}{3}a^6c^5}{x^3}$	74
parallelrisch	$-\frac{b^6c^5x^6-6a\,b^5c^5x^5+15a^2b^4c^5x^4+15a^4b^2c^5x^2-6a^5b\,c^5x+a^6c^5}{3x^3}$	74
orering	$-\frac{(b^6x^6-6a\,x^5b^5+15a^2x^4b^4+15a^4x^2b^2-6a^5xb+a^6)(-bcx+ac)^5}{3x^3(-bx+a)^5}$	74
norman	$\frac{-\frac{1}{3}a^6c^5-\frac{1}{3}b^6c^5x^6+2a\,b^5c^5x^5-5a^2b^4c^5x^4-5a^4b^2c^5x^2+2a^5b\,c^5x}{x^3}$	75

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^5*(b^6*x^6-6*a*b^5*x^5+15*a^2*b^4*x^4+15*a^4*b^2*x^2-6*a^5*b*x+a^6)
/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = -\frac{b^6c^5x^6-6\,ab^5c^5x^5+15\,a^2b^4c^5x^4+15\,a^4b^2c^5x^2-6\,a^5bc^5x+a^6c^5}{3\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^4,x, algorithm="fricas")
```

output
```
-1/3*(b^6*c^5*x^6 - 6*a*b^5*c^5*x^5 + 15*a^2*b^4*c^5*x^4 + 15*a^4*b^2*c^5*
x^2 - 6*a^5*b*c^5*x + a^6*c^5)/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(15) = 30.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.22

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx$$
$$= -5a^2b^4c^5x + 2ab^5c^5x^2 - \frac{b^6c^5x^3}{3} - \frac{a^6c^5 - 6a^5bc^5x + 15a^4b^2c^5x^2}{3x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**4,x)
```

output
```
-5*a**2*b**4*c**5*x + 2*a*b**5*c**5*x**2 - b**6*c**5*x**3/3 - (a**6*c**5 -
 6*a**5*b*c**5*x + 15*a**4*b**2*c**5*x**2)/(3*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(17) = 34.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = -\frac{1}{3}\,b^6c^5x^3 + 2\,ab^5c^5x^2 - 5\,a^2b^4c^5x$$
$$- \frac{15\,a^4b^2c^5x^2 - 6\,a^5bc^5x + a^6c^5}{3\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^6*c^5*x^3 + 2*a*b^5*c^5*x^2 - 5*a^2*b^4*c^5*x - 1/3*(15*a^4*b^2*c^5
*x^2 - 6*a^5*b*c^5*x + a^6*c^5)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = -\frac{1}{3}\,b^6c^5x^3 + 2\,ab^5c^5x^2 - 5\,a^2b^4c^5x - \frac{15\,a^4b^2c^5x^2 - 6\,a^5bc^5x + a^6c^5}{3\,x^3}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^4,x, algorithm="giac")
```

output
```
-1/3*b^6*c^5*x^3 + 2*a*b^5*c^5*x^2 - 5*a^2*b^4*c^5*x - 1/3*(15*a^4*b^2*c^5
*x^2 - 6*a^5*b*c^5*x + a^6*c^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 4.11

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = 2\,a\,b^5\,c^5\,x^2 - \frac{b^6\,c^5\,x^3}{3} - 5\,a^2\,b^4\,c^5\,x - \frac{\frac{a^6\,c^5}{3} - 2\,a^5\,b\,c^5\,x + 5\,a^4\,b^2\,c^5\,x^2}{x^3}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^4,x)
```

output
```
2*a*b^5*c^5*x^2 - (b^6*c^5*x^3)/3 - 5*a^2*b^4*c^5*x - ((a^6*c^5)/3 + 5*a^4
*b^2*c^5*x^2 - 2*a^5*b*c^5*x)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.33

$$\int \frac{(a+bx)(ac-bcx)^5}{x^4}\,dx = \frac{c^5(-b^6x^6+6a\,b^5x^5-15a^2b^4x^4-15a^4b^2x^2+6a^5bx-a^6)}{3x^3}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^4,x)
```

output
```
(c**5*( - a**6 + 6*a**5*b*x - 15*a**4*b**2*x**2 - 15*a**2*b**4*x**4 + 6*a*
b**5*x**5 - b**6*x**6))/(3*x**3)
```

3.35 $\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = -\frac{a^6c^5}{4x^4}+\frac{4a^5bc^5}{3x^3}-\frac{5a^4b^2c^5}{2x^2}+4ab^5c^5x-\frac{1}{2}b^6c^5x^2-5a^2b^4c^5\log(x)$$

output
```
-1/4*a^6*c^5/x^4+4/3*a^5*b*c^5/x^3-5/2*a^4*b^2*c^5/x^2+4*a*b^5*c^5*x-1/2*b
^6*c^5*x^2-5*a^2*b^4*c^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = c^5\left(-\frac{a^6}{4x^4}+\frac{4a^5b}{3x^3}-\frac{5a^4b^2}{2x^2}+4ab^5x-\frac{b^6x^2}{2}-5a^2b^4\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^5,x]
```

output
```
c^5*(-1/4*a^6/x^4 + (4*a^5*b)/(3*x^3) - (5*a^4*b^2)/(2*x^2) + 4*a*b^5*x -
(b^6*x^2)/2 - 5*a^2*b^4*Log[x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5} \, dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^6c^5}{x^5} - \frac{4a^5bc^5}{x^4} + \frac{5a^4b^2c^5}{x^3} - \frac{5a^2b^4c^5}{x} + 4ab^5c^5 - b^6c^5x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6c^5}{4x^4} + \frac{4a^5bc^5}{3x^3} - \frac{5a^4b^2c^5}{2x^2} - 5a^2b^4c^5\log(x) + 4ab^5c^5x - \frac{1}{2}b^6c^5x^2$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^5,x]
```

output
```
-1/4*(a^6*c^5)/x^4 + (4*a^5*b*c^5)/(3*x^3) - (5*a^4*b^2*c^5)/(2*x^2) + 4*a
*b^5*c^5*x - (b^6*c^5*x^2)/2 - 5*a^2*b^4*c^5*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result	size
default	$c^{5} \left(-\frac{b^{6} x^{2}}{2}+4 a \,b^{5} x +\frac{4 a^{5} b}{3 x^{3}}-\frac{5 a^{4} b^{2}}{2 x^{2}}-\frac{a^{6}}{4 x^{4}}-5 a^{2} b^{4} \ln \left(x \right)\right)$	59
risch	$-\frac{b^{6} c^{5} x^{2}}{2}+4 a \,b^{5} c^{5} x +\frac{-\frac{5}{2} a^{4} b^{2} c^{5} x^{2}+\frac{4}{3} a^{5} b \,c^{5} x -\frac{1}{4} a^{6} c^{5}}{x^{4}}-5 a^{2} b^{4} c^{5} \ln \left(x \right)$	73
norman	$\frac{-\frac{1}{4} a^{6} c^{5}-\frac{1}{2} b^{6} c^{5} x^{6}+4 a \,b^{5} c^{5} x^{5}-\frac{5}{2} a^{4} b^{2} c^{5} x^{2}+\frac{4}{3} a^{5} b \,c^{5} x}{x^{4}}-5 a^{2} b^{4} c^{5} \ln \left(x \right)$	75
parallelrisch	$-\frac{6 b^{6} c^{5} x^{6}+60 a^{2} c^{5} b^{4} \ln \left(x \right) x^{4}-48 a \,b^{5} c^{5} x^{5}+30 a^{4} b^{2} c^{5} x^{2}-16 a^{5} b \,c^{5} x +3 a^{6} c^{5}}{12 x^{4}}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-1/2*b^6*x^2+4*a*b^5*x+4/3*a^5*b/x^3-5/2*a^4*b^2/x^2-1/4*a^6/x^4-5*a^
2*b^4*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = -\frac{6\,b^{6}c^{5}x^{6} - 48\,ab^{5}c^{5}x^{5} + 60\,a^{2}b^{4}c^{5}x^{4}\log\left(x\right) + 30\,a^{4}b^{2}c^{5}x^{2} - 16\,a^{5}bc^{5}x + 3\,a^{6}c^{5}}{12\,x^{4}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*b^6*c^5*x^6 - 48*a*b^5*c^5*x^5 + 60*a^2*b^4*c^5*x^4*log(x) + 30*a
^4*b^2*c^5*x^2 - 16*a^5*b*c^5*x + 3*a^6*c^5)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = -5a^2b^4c^5\log(x) + 4ab^5c^5x - \frac{b^6c^5x^2}{2} - \frac{3a^6c^5 - 16a^5bc^5x + 30a^4b^2c^5x^2}{12x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**5,x)
```

output
```
-5*a**2*b**4*c**5*log(x) + 4*a*b**5*c**5*x - b**6*c**5*x**2/2 - (3*a**6*c*
*5 - 16*a**5*b*c**5*x + 30*a**4*b**2*c**5*x**2)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = -\frac{1}{2}\,b^6c^5x^2 + 4\,ab^5c^5x - 5\,a^2b^4c^5\log(x) - \frac{30\,a^4b^2c^5x^2 - 16\,a^5bc^5x + 3\,a^6c^5}{12\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^5,x, algorithm="maxima")
```

output
```
-1/2*b^6*c^5*x^2 + 4*a*b^5*c^5*x - 5*a^2*b^4*c^5*log(x) - 1/12*(30*a^4*b^2
*c^5*x^2 - 16*a^5*b*c^5*x + 3*a^6*c^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = -\frac{1}{2}\,b^6c^5x^2 + 4\,ab^5c^5x - 5\,a^2b^4c^5\log\left(|x|\right) - \frac{30\,a^4b^2c^5x^2 - 16\,a^5bc^5x + 3\,a^6c^5}{12\,x^4}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^5,x, algorithm="giac")
```

output
```
-1/2*b^6*c^5*x^2 + 4*a*b^5*c^5*x - 5*a^2*b^4*c^5*log(abs(x)) - 1/12*(30*a^
4*b^2*c^5*x^2 - 16*a^5*b*c^5*x + 3*a^6*c^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = 4\,a\,b^5\,c^5\,x - \frac{b^6\,c^5\,x^2}{2} - 5\,a^2\,b^4\,c^5\,\ln\left(x\right) - \frac{\frac{a^6\,c^5}{4} - \frac{4\,a^5\,b\,c^5\,x}{3} + \frac{5\,a^4\,b^2\,c^5\,x^2}{2}}{x^4}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^5,x)
```

output
```
4*a*b^5*c^5*x - (b^6*c^5*x^2)/2 - 5*a^2*b^4*c^5*log(x) - ((a^6*c^5)/4 + (5
*a^4*b^2*c^5*x^2)/2 - (4*a^5*b*c^5*x)/3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)(ac-bcx)^5}{x^5}\,dx = \frac{c^5(-60\log(x)\,a^2b^4x^4-3a^6+16a^5bx-30a^4b^2x^2+48a\,b^5x^5-6b^6x^6)}{12x^4}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^5,x)
```

output
```
(c**5*( - 60*log(x)*a**2*b**4*x**4 - 3*a**6 + 16*a**5*b*x - 30*a**4*b**2*x
**2 + 48*a*b**5*x**5 - 6*b**6*x**6))/(12*x**4)
```

3.36 $\int \frac{(a+bx)(ac-bcx)^5}{x^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6} dx = -\frac{a^6c^5}{5x^5} + \frac{a^5bc^5}{x^4} - \frac{5a^4b^2c^5}{3x^3} + \frac{5a^2b^4c^5}{x} - b^6c^5x + 4ab^5c^5\log(x)$$

output
```
-1/5*a^6*c^5/x^5+a^5*b*c^5/x^4-5/3*a^4*b^2*c^5/x^3+5*a^2*b^4*c^5/x-b^6*c^5
*x+4*a*b^5*c^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6} dx = c^5\left(-\frac{a^6}{5x^5} + \frac{a^5b}{x^4} - \frac{5a^4b^2}{3x^3} + \frac{5a^2b^4}{x} - b^6x + 4ab^5\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^6,x]
```

output
```
c^5*(-1/5*a^6/x^5 + (a^5*b)/x^4 - (5*a^4*b^2)/(3*x^3) + (5*a^2*b^4)/x - b^
6*x + 4*a*b^5*Log[x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6} \, dx$$

$$\Big\downarrow \; 84$$

$$\int \left(\frac{a^6c^5}{x^6}-\frac{4a^5bc^5}{x^5}+\frac{5a^4b^2c^5}{x^4}-\frac{5a^2b^4c^5}{x^2}+\frac{4ab^5c^5}{x}-b^6c^5\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^6c^5}{5x^5}+\frac{a^5bc^5}{x^4}-\frac{5a^4b^2c^5}{3x^3}+\frac{5a^2b^4c^5}{x}+4ab^5c^5\log(x)-b^6c^5x$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^6,x]
```

output
```
-1/5*(a^6*c^5)/x^5 + (a^5*b*c^5)/x^4 - (5*a^4*b^2*c^5)/(3*x^3) + (5*a^2*b^
4*c^5)/x - b^6*c^5*x + 4*a*b^5*c^5*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result	size
default	$c^{5}\left(-b^{6}x-\frac{5a^{4}b^{2}}{3x^{3}}-\frac{a^{6}}{5x^{5}}+\frac{a^{5}b}{x^{4}}+4a\,b^{5}\ln\left(x\right)+\frac{5a^{2}b^{4}}{x}\right)$	58
risch	$-b^{6}c^{5}x+\frac{5a^{2}b^{4}c^{5}x^{4}-\frac{5}{3}a^{4}b^{2}c^{5}x^{2}+a^{5}b\,c^{5}x-\frac{1}{5}a^{6}c^{5}}{x^{5}}+4a\,b^{5}c^{5}\ln\left(x\right)$	72
norman	$\frac{a^{5}b\,c^{5}x-\frac{1}{5}a^{6}c^{5}-b^{6}c^{5}x^{6}+5a^{2}b^{4}c^{5}x^{4}-\frac{5}{3}a^{4}b^{2}c^{5}x^{2}}{x^{5}}+4a\,b^{5}c^{5}\ln\left(x\right)$	74
parallelrisch	$\frac{60a\,c^{5}b^{5}\ln\left(x\right)x^{5}-15b^{6}c^{5}x^{6}+75a^{2}b^{4}c^{5}x^{4}-25a^{4}b^{2}c^{5}x^{2}+15a^{5}b\,c^{5}x-3a^{6}c^{5}}{15x^{5}}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
c^5*(-b^6*x-5/3*a^4*b^2/x^3-1/5*a^6/x^5+a^5*b/x^4+4*a*b^5*ln(x)+5*a^2*b^4/
x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int\frac{(a+bx)(ac-bcx)^5}{x^6}\,dx$$
$$=-\frac{15\,b^{6}c^{5}x^{6}-60\,ab^{5}c^{5}x^{5}\log\left(x\right)-75\,a^{2}b^{4}c^{5}x^{4}+25\,a^{4}b^{2}c^{5}x^{2}-15\,a^{5}bc^{5}x+3\,a^{6}c^{5}}{15\,x^{5}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^6,x, algorithm="fricas")
```

output
```
-1/15*(15*b^6*c^5*x^6 - 60*a*b^5*c^5*x^5*log(x) - 75*a^2*b^4*c^5*x^4 + 25*
a^4*b^2*c^5*x^2 - 15*a^5*b*c^5*x + 3*a^6*c^5)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6}\,dx = 4ab^5c^5\log{(x)} - b^6c^5x - \frac{3a^6c^5 - 15a^5bc^5x + 25a^4b^2c^5x^2 - 75a^2b^4c^5x^4}{15x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**6,x)
```

output
```
4*a*b**5*c**5*log(x) - b**6*c**5*x - (3*a**6*c**5 - 15*a**5*b*c**5*x + 25*
a**4*b**2*c**5*x**2 - 75*a**2*b**4*c**5*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6}\,dx = -b^6c^5x + 4\,ab^5c^5\log{(x)} + \frac{75\,a^2b^4c^5x^4 - 25\,a^4b^2c^5x^2 + 15\,a^5bc^5x - 3\,a^6c^5}{15\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^6,x, algorithm="maxima")
```

output
```
-b^6*c^5*x + 4*a*b^5*c^5*log(x) + 1/15*(75*a^2*b^4*c^5*x^4 - 25*a^4*b^2*c^
5*x^2 + 15*a^5*b*c^5*x - 3*a^6*c^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6}\,dx = -b^6c^5x + 4\,ab^5c^5\log\left(|x|\right) + \frac{75\,a^2b^4c^5x^4 - 25\,a^4b^2c^5x^2 + 15\,a^5bc^5x - 3\,a^6c^5}{15\,x^5}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^6,x, algorithm="giac")
```

output
```
-b^6*c^5*x + 4*a*b^5*c^5*log(abs(x)) + 1/15*(75*a^2*b^4*c^5*x^4 - 25*a^4*b
^2*c^5*x^2 + 15*a^5*b*c^5*x - 3*a^6*c^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6}\,dx = -\frac{c^5\left(\frac{a^6}{5} + b^6\,x^6 + \frac{5\,a^4\,b^2\,x^2}{3} - 5\,a^2\,b^4\,x^4 - a^5\,b\,x - 4\,a\,b^5\,x^5\,\ln\left(x\right)\right)}{x^5}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^6,x)
```

output
```
-(c^5*(a^6/5 + b^6*x^6 + (5*a^4*b^2*x^2)/3 - 5*a^2*b^4*x^4 - a^5*b*x - 4*a
*b^5*x^5*log(x)))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^6}\,dx = \frac{c^5(60\log(x)\,a\,b^5x^5-3a^6+15a^5bx-25a^4b^2x^2+75a^2b^4x^4-15b^6x^6)}{15x^5}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^6,x)
```

output
```
(c**5*(60*log(x)*a*b**5*x**5 - 3*a**6 + 15*a**5*b*x - 25*a**4*b**2*x**2 +
75*a**2*b**4*x**4 - 15*b**6*x**6))/(15*x**5)
```

3.37 $\int \frac{(a+bx)(ac-bcx)^5}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7} dx = -\frac{a^6c^5}{6x^6} + \frac{4a^5bc^5}{5x^5} - \frac{5a^4b^2c^5}{4x^4} + \frac{5a^2b^4c^5}{2x^2} - \frac{4ab^5c^5}{x} - b^6c^5\log(x)$$

output
```
-1/6*a^6*c^5/x^6+4/5*a^5*b*c^5/x^5-5/4*a^4*b^2*c^5/x^4+5/2*a^2*b^4*c^5/x^2
-4*a*b^5*c^5/x-b^6*c^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7} dx = c^5\left(-\frac{a^6}{6x^6} + \frac{4a^5b}{5x^5} - \frac{5a^4b^2}{4x^4} + \frac{5a^2b^4}{2x^2} - \frac{4ab^5}{x} - b^6\log(x)\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^7,x]
```

output
```
c^5*(-1/6*a^6/x^6 + (4*a^5*b)/(5*x^5) - (5*a^4*b^2)/(4*x^4) + (5*a^2*b^4)/
(2*x^2) - (4*a*b^5)/x - b^6*Log[x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7}\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^6c^5}{x^7}-\frac{4a^5bc^5}{x^6}+\frac{5a^4b^2c^5}{x^5}-\frac{5a^2b^4c^5}{x^3}+\frac{4ab^5c^5}{x^2}-\frac{b^6c^5}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6c^5}{6x^6}+\frac{4a^5bc^5}{5x^5}-\frac{5a^4b^2c^5}{4x^4}+\frac{5a^2b^4c^5}{2x^2}-\frac{4ab^5c^5}{x}-b^6c^5\log(x)$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^7,x]
```

output
```
-1/6*(a^6*c^5)/x^6 + (4*a^5*b*c^5)/(5*x^5) - (5*a^4*b^2*c^5)/(4*x^4) + (5*
a^2*b^4*c^5)/(2*x^2) - (4*a*b^5*c^5)/x - b^6*c^5*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
default	$c^{5}\left(\frac{4a^{5}b}{5x^{5}}+\frac{5a^{2}b^{4}}{2x^{2}}-\frac{5a^{4}b^{2}}{4x^{4}}-b^{6}\ln\left(x\right)-\frac{4a\,b^{5}}{x}-\frac{a^{6}}{6x^{6}}\right)$	61
norman	$\frac{-\frac{1}{6}a^{6}c^{5}-4a\,b^{5}c^{5}x^{5}+\frac{5}{2}a^{2}b^{4}c^{5}x^{4}-\frac{5}{4}a^{4}b^{2}c^{5}x^{2}+\frac{4}{5}a^{5}b\,c^{5}x}{x^{6}}-b^{6}c^{5}\ln\left(x\right)$	75
risch	$\frac{-\frac{1}{6}a^{6}c^{5}-4a\,b^{5}c^{5}x^{5}+\frac{5}{2}a^{2}b^{4}c^{5}x^{4}-\frac{5}{4}a^{4}b^{2}c^{5}x^{2}+\frac{4}{5}a^{5}b\,c^{5}x}{x^{6}}-b^{6}c^{5}\ln\left(x\right)$	75
parallelrisch	$-\frac{60b^{6}c^{5}\ln\left(x\right)x^{6}+240a\,b^{5}c^{5}x^{5}-150a^{2}b^{4}c^{5}x^{4}+75a^{4}b^{2}c^{5}x^{2}-48a^{5}b\,c^{5}x+10a^{6}c^{5}}{60x^{6}}$	78

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
c^5*(4/5*a^5*b/x^5+5/2*a^2*b^4/x^2-5/4*a^4*b^2/x^4-b^6*ln(x)-4*a*b^5/x-1/6
*a^6/x^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int\frac{(a+bx)(ac-bcx)^{5}}{x^{7}}\,dx=-\frac{60\,b^{6}c^{5}x^{6}\log\left(x\right)+240\,ab^{5}c^{5}x^{5}-150\,a^{2}b^{4}c^{5}x^{4}+75\,a^{4}b^{2}c^{5}x^{2}-48\,a^{5}bc^{5}x+10\,a^{6}c^{5}}{60\,x^{6}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^7,x, algorithm="fricas")
```

output
```
-1/60*(60*b^6*c^5*x^6*log(x) + 240*a*b^5*c^5*x^5 - 150*a^2*b^4*c^5*x^4 + 7
5*a^4*b^2*c^5*x^2 - 48*a^5*b*c^5*x + 10*a^6*c^5)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7}\,dx$$
$$= -b^6c^5\log(x) - \frac{10a^6c^5 - 48a^5bc^5x + 75a^4b^2c^5x^2 - 150a^2b^4c^5x^4 + 240ab^5c^5x^5}{60x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**7,x)
```

output
```
-b**6*c**5*log(x) - (10*a**6*c**5 - 48*a**5*b*c**5*x + 75*a**4*b**2*c**5*x
**2 - 150*a**2*b**4*c**5*x**4 + 240*a*b**5*c**5*x**5)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7}\,dx$$
$$= -b^6c^5\log(x) - \frac{240\,ab^5c^5x^5 - 150\,a^2b^4c^5x^4 + 75\,a^4b^2c^5x^2 - 48\,a^5bc^5x + 10\,a^6c^5}{60\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^7,x, algorithm="maxima")
```

output
```
-b^6*c^5*log(x) - 1/60*(240*a*b^5*c^5*x^5 - 150*a^2*b^4*c^5*x^4 + 75*a^4*b
^2*c^5*x^2 - 48*a^5*b*c^5*x + 10*a^6*c^5)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7}\,dx$$
$$= -b^6c^5\log(|x|) - \frac{240\,ab^5c^5x^5 - 150\,a^2b^4c^5x^4 + 75\,a^4b^2c^5x^2 - 48\,a^5bc^5x + 10\,a^6c^5}{60\,x^6}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^7,x, algorithm="giac")
```

output
```
-b^6*c^5*log(abs(x)) - 1/60*(240*a*b^5*c^5*x^5 - 150*a^2*b^4*c^5*x^4 + 75*
a^4*b^2*c^5*x^2 - 48*a^5*b*c^5*x + 10*a^6*c^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7}\,dx$$
$$= -\frac{c^5\left(10\,a^6 + 240\,a\,b^5\,x^5 + 75\,a^4\,b^2\,x^2 - 150\,a^2\,b^4\,x^4 + 60\,b^6\,x^6\,\ln(x) - 48\,a^5\,b\,x\right)}{60\,x^6}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^7,x)
```

output
```
-(c^5*(10*a^6 + 240*a*b^5*x^5 + 75*a^4*b^2*x^2 - 150*a^2*b^4*x^4 + 60*b^6*
x^6*log(x) - 48*a^5*b*x))/(60*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)(ac-bcx)^5}{x^7}\,dx$$
$$= \frac{c^5(-60\log(x)\,b^6x^6-10a^6+48a^5bx-75a^4b^2x^2+150a^2b^4x^4-240a\,b^5x^5)}{60x^6}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^7,x)
```

output
```
(c**5*( - 60*log(x)*b**6*x**6 - 10*a**6 + 48*a**5*b*x - 75*a**4*b**2*x**2
+ 150*a**2*b**4*x**4 - 240*a*b**5*x**5))/(60*x**6)
```

3.38 $\int \frac{(a+bx)(ac-bcx)^5}{x^8} dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx=-\frac{c^5(a-bx)^6}{7x^7}-\frac{4bc^5(a-bx)^6}{21ax^6}$$

output
```
-1/7*c^5*(-b*x+a)^6/x^7-4/21*b*c^5*(-b*x+a)^6/a/x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx=c^5\left(-\frac{a^6}{7x^7}+\frac{2a^5b}{3x^6}-\frac{a^4b^2}{x^5}+\frac{5a^2b^4}{3x^3}-\frac{2ab^5}{x^2}+\frac{b^6}{x}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^8,x]
```

output
```
c^5*(-1/7*a^6/x^7 + (2*a^5*b)/(3*x^6) - (a^4*b^2)/x^5 + (5*a^2*b^4)/(3*x^3
) - (2*a*b^5)/x^2 + b^6/x)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {87, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{8}{7}b\int \frac{c^5(a-bx)^5}{x^7}dx-\frac{c^5(a-bx)^6}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{7}bc^5\int \frac{(a-bx)^5}{x^7}dx-\frac{c^5(a-bx)^6}{7x^7}$$

$$\Big\downarrow \text{48}$$

$$-\frac{c^5(a-bx)^6}{7x^7}-\frac{4bc^5(a-bx)^6}{21ax^6}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^8,x]
```

output
```
-1/7*(c^5*(a - b*x)^6)/x^7 - (4*b*c^5*(a - b*x)^6)/(21*a*x^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

method	result	size
gosper	$-\frac{c^{5}\left(-21b^{6}x^{6}+42a\,x^{5}b^{5}-35a^{2}x^{4}b^{4}+21a^{4}x^{2}b^{2}-14a^{5}xb+3a^{6}\right)}{21x^{7}}$	61
default	$c^{5}\left(\frac{5a^{2}b^{4}}{3x^{3}}-\frac{a^{4}b^{2}}{x^{5}}-\frac{2a\,b^{5}}{x^{2}}-\frac{a^{6}}{7x^{7}}+\frac{b^{6}}{x}+\frac{2a^{5}b}{3x^{6}}\right)$	61
norman	$\frac{b^{6}c^{5}x^{6}-\frac{1}{7}a^{6}c^{5}-2a\,b^{5}c^{5}x^{5}+\frac{5}{3}a^{2}b^{4}c^{5}x^{4}-a^{4}b^{2}c^{5}x^{2}+\frac{2}{3}a^{5}b\,c^{5}x}{x^{7}}$	74
risch	$\frac{b^{6}c^{5}x^{6}-\frac{1}{7}a^{6}c^{5}-2a\,b^{5}c^{5}x^{5}+\frac{5}{3}a^{2}b^{4}c^{5}x^{4}-a^{4}b^{2}c^{5}x^{2}+\frac{2}{3}a^{5}b\,c^{5}x}{x^{7}}$	74
parallelrisch	$\frac{21b^{6}c^{5}x^{6}-42a\,b^{5}c^{5}x^{5}+35a^{2}b^{4}c^{5}x^{4}-21a^{4}b^{2}c^{5}x^{2}+14a^{5}b\,c^{5}x-3a^{6}c^{5}}{21x^{7}}$	76
orering	$-\frac{\left(-21b^{6}x^{6}+42a\,x^{5}b^{5}-35a^{2}x^{4}b^{4}+21a^{4}x^{2}b^{2}-14a^{5}xb+3a^{6}\right)\left(-bcx+ac\right)^{5}}{21x^{7}\left(-bx+a\right)^{5}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21*c^5*(-21*b^6*x^6+42*a*b^5*x^5-35*a^2*b^4*x^4+21*a^4*b^2*x^2-14*a^5*b
*x+3*a^6)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx = \frac{21\,b^6c^5x^6-42\,ab^5c^5x^5+35\,a^2b^4c^5x^4-21\,a^4b^2c^5x^2+14\,a^5bc^5x-3\,a^6c^5}{21\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^8,x, algorithm="fricas")
```

output
```
1/21*(21*b^6*c^5*x^6 - 42*a*b^5*c^5*x^5 + 35*a^2*b^4*c^5*x^4 - 21*a^4*b^2*
c^5*x^2 + 14*a^5*b*c^5*x - 3*a^6*c^5)/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx = -\frac{3a^6c^5 - 14a^5bc^5x + 21a^4b^2c^5x^2 - 35a^2b^4c^5x^4 + 42ab^5c^5x^5 - 21b^6c^5x^6}{21x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**8,x)
```

output
```
-(3*a**6*c**5 - 14*a**5*b*c**5*x + 21*a**4*b**2*c**5*x**2 - 35*a**2*b**4*c
**5*x**4 + 42*a*b**5*c**5*x**5 - 21*b**6*c**5*x**6)/(21*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx = \frac{21\,b^6c^5x^6 - 42\,ab^5c^5x^5 + 35\,a^2b^4c^5x^4 - 21\,a^4b^2c^5x^2 + 14\,a^5bc^5x - 3\,a^6c^5}{21\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^8,x, algorithm="maxima")
```

output
```
1/21*(21*b^6*c^5*x^6 - 42*a*b^5*c^5*x^5 + 35*a^2*b^4*c^5*x^4 - 21*a^4*b^2*
c^5*x^2 + 14*a^5*b*c^5*x - 3*a^6*c^5)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx = \frac{21\,b^6c^5x^6-42\,ab^5c^5x^5+35\,a^2b^4c^5x^4-21\,a^4b^2c^5x^2+14\,a^5bc^5x-3\,a^6c^5}{21\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^8,x, algorithm="giac")
```

output
```
1/21*(21*b^6*c^5*x^6 - 42*a*b^5*c^5*x^5 + 35*a^2*b^4*c^5*x^4 - 21*a^4*b^2*
c^5*x^2 + 14*a^5*b*c^5*x - 3*a^6*c^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx = -\frac{\frac{a^6\,c^5}{7}-\frac{2\,a^5\,b\,c^5\,x}{3}+a^4\,b^2\,c^5\,x^2-\frac{5\,a^2\,b^4\,c^5\,x^4}{3}+2\,a\,b^5\,c^5\,x^5-b^6\,c^5\,x^6}{x^7}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^8,x)
```

output
```
-((a^6*c^5)/7 - b^6*c^5*x^6 + 2*a*b^5*c^5*x^5 + a^4*b^2*c^5*x^2 - (5*a^2*b
^4*c^5*x^4)/3 - (2*a^5*b*c^5*x)/3)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)(ac-bcx)^5}{x^8}\,dx = \frac{c^5(21b^6x^6-42a\,b^5x^5+35a^2b^4x^4-21a^4b^2x^2+14a^5bx-3a^6)}{21x^7}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^8,x)
```

output
```
(c**5*( - 3*a**6 + 14*a**5*b*x - 21*a**4*b**2*x**2 + 35*a**2*b**4*x**4 - 4
2*a*b**5*x**5 + 21*b**6*x**6))/(21*x**7)
```

3.39 $\int \frac{(a+bx)(ac-bcx)^5}{x^9} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9} \, dx = -\frac{c^5(a-bx)^6}{8x^8} - \frac{5bc^5(a-bx)^6}{28ax^7} - \frac{5b^2c^5(a-bx)^6}{168a^2x^6}$$

output
```
-1/8*c^5*(-b*x+a)^6/x^8-5/28*b*c^5*(-b*x+a)^6/a/x^7-5/168*b^2*c^5*(-b*x+a)
^6/a^2/x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9} \, dx = c^5 \left(-\frac{a^6}{8x^8} + \frac{4a^5b}{7x^7} - \frac{5a^4b^2}{6x^6} + \frac{5a^2b^4}{4x^4} - \frac{4ab^5}{3x^3} + \frac{b^6}{2x^2} \right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^9,x]
```

output
```
c^5*(-1/8*a^6/x^8 + (4*a^5*b)/(7*x^7) - (5*a^4*b^2)/(6*x^6) + (5*a^2*b^4)/
(4*x^4) - (4*a*b^5)/(3*x^3) + b^6/(2*x^2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {87, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9} \, dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{5}{4}b\int \frac{c^5(a-bx)^5}{x^8}dx-\frac{c^5(a-bx)^6}{8x^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{4}bc^5\int \frac{(a-bx)^5}{x^8}dx-\frac{c^5(a-bx)^6}{8x^8}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{5}{4}bc^5\left(\frac{b\int \frac{(a-bx)^5}{x^7}dx}{7a}-\frac{(a-bx)^6}{7ax^7}\right)-\frac{c^5(a-bx)^6}{8x^8}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{5}{4}bc^5\left(-\frac{b(a-bx)^6}{42a^2x^6}-\frac{(a-bx)^6}{7ax^7}\right)-\frac{c^5(a-bx)^6}{8x^8}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^9,x]
```

output
```
-1/8*(c^5*(a - b*x)^6)/x^8 + (5*b*c^5*(-1/7*(a - b*x)^6/(a*x^7) - (b*(a -
b*x)^6)/(42*a^2*x^6)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{c^5(-84b^6x^6+224a\,x^5b^5-210a^2x^4b^4+140a^4x^2b^2-96a^5xb+21a^6)}{168x^8}$	61
default	$c^5\left(-\frac{4a\,b^5}{3x^3}+\frac{b^6}{2x^2}+\frac{4a^5b}{7x^7}+\frac{5a^2b^4}{4x^4}-\frac{a^6}{8x^8}-\frac{5a^4b^2}{6x^6}\right)$	62
norman	$\frac{-\frac{1}{8}a^6c^5+\frac{1}{2}b^6c^5x^6-\frac{4}{3}a\,b^5c^5x^5+\frac{5}{4}a^2b^4c^5x^4-\frac{5}{6}a^4b^2c^5x^2+\frac{4}{7}a^5b\,c^5x}{x^8}$	75
risch	$\frac{-\frac{1}{8}a^6c^5+\frac{1}{2}b^6c^5x^6-\frac{4}{3}a\,b^5c^5x^5+\frac{5}{4}a^2b^4c^5x^4-\frac{5}{6}a^4b^2c^5x^2+\frac{4}{7}a^5b\,c^5x}{x^8}$	75
parallelrisch	$\frac{84b^6c^5x^6-224a\,b^5c^5x^5+210a^2b^4c^5x^4-140a^4b^2c^5x^2+96a^5b\,c^5x-21a^6c^5}{168x^8}$	76
orering	$-\frac{(-84b^6x^6+224a\,x^5b^5-210a^2x^4b^4+140a^4x^2b^2-96a^5xb+21a^6)(-bcx+ac)^5}{168x^8(-bx+a)^5}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/168*c^5*(-84*b^6*x^6+224*a*b^5*x^5-210*a^2*b^4*x^4+140*a^4*b^2*x^2-96*a
^5*b*x+21*a^6)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9}\,dx = \frac{84\,b^6c^5x^6-224\,ab^5c^5x^5+210\,a^2b^4c^5x^4-140\,a^4b^2c^5x^2+96\,a^5bc^5x-21\,a^6c^5}{168\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^9,x, algorithm="fricas")
```

output
```
1/168*(84*b^6*c^5*x^6 - 224*a*b^5*c^5*x^5 + 210*a^2*b^4*c^5*x^4 - 140*a^4*
b^2*c^5*x^2 + 96*a^5*b*c^5*x - 21*a^6*c^5)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9}\,dx$$
$$= -\frac{21a^6c^5 - 96a^5bc^5x + 140a^4b^2c^5x^2 - 210a^2b^4c^5x^4 + 224ab^5c^5x^5 - 84b^6c^5x^6}{168x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**9,x)
```

output
```
-(21*a**6*c**5 - 96*a**5*b*c**5*x + 140*a**4*b**2*c**5*x**2 - 210*a**2*b**
4*c**5*x**4 + 224*a*b**5*c**5*x**5 - 84*b**6*c**5*x**6)/(168*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9}\,dx$$
$$= \frac{84\,b^6c^5x^6 - 224\,ab^5c^5x^5 + 210\,a^2b^4c^5x^4 - 140\,a^4b^2c^5x^2 + 96\,a^5bc^5x - 21\,a^6c^5}{168\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^9,x, algorithm="maxima")
```

output
```
1/168*(84*b^6*c^5*x^6 - 224*a*b^5*c^5*x^5 + 210*a^2*b^4*c^5*x^4 - 140*a^4*
b^2*c^5*x^2 + 96*a^5*b*c^5*x - 21*a^6*c^5)/x^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9}\,dx = \frac{84\,b^6c^5x^6 - 224\,ab^5c^5x^5 + 210\,a^2b^4c^5x^4 - 140\,a^4b^2c^5x^2 + 96\,a^5bc^5x - 21\,a^6c^5}{168\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^9,x, algorithm="giac")
```

output
```
1/168*(84*b^6*c^5*x^6 - 224*a*b^5*c^5*x^5 + 210*a^2*b^4*c^5*x^4 - 140*a^4*
b^2*c^5*x^2 + 96*a^5*b*c^5*x - 21*a^6*c^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9}\,dx = -\frac{\frac{a^6\,c^5}{8} - \frac{4\,a^5\,b\,c^5\,x}{7} + \frac{5\,a^4\,b^2\,c^5\,x^2}{6} - \frac{5\,a^2\,b^4\,c^5\,x^4}{4} + \frac{4\,a\,b^5\,c^5\,x^5}{3} - \frac{b^6\,c^5\,x^6}{2}}{x^8}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^9,x)
```

output
```
-((a^6*c^5)/8 - (b^6*c^5*x^6)/2 + (4*a*b^5*c^5*x^5)/3 + (5*a^4*b^2*c^5*x^2
)/6 - (5*a^2*b^4*c^5*x^4)/4 - (4*a^5*b*c^5*x)/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^5}{x^9}\,dx = \frac{c^5(84b^6x^6-224a\,b^5x^5+210a^2b^4x^4-140a^4b^2x^2+96a^5bx-21a^6)}{168x^8}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^9,x)
```

output
```
(c**5*( - 21*a**6 + 96*a**5*b*x - 140*a**4*b**2*x**2 + 210*a**2*b**4*x**4
- 224*a*b**5*x**5 + 84*b**6*x**6))/(168*x**8)
```

3.40 $\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = -\frac{a^6c^5}{9x^9}+\frac{a^5bc^5}{2x^8}-\frac{5a^4b^2c^5}{7x^7}+\frac{a^2b^4c^5}{x^5}-\frac{ab^5c^5}{x^4}+\frac{b^6c^5}{3x^3}$$

output
```
-1/9*a^6*c^5/x^9+1/2*a^5*b*c^5/x^8-5/7*a^4*b^2*c^5/x^7+a^2*b^4*c^5/x^5-a*b
^5*c^5/x^4+1/3*b^6*c^5/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = c^5\left(-\frac{a^6}{9x^9}+\frac{a^5b}{2x^8}-\frac{5a^4b^2}{7x^7}+\frac{a^2b^4}{x^5}-\frac{ab^5}{x^4}+\frac{b^6}{3x^3}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^10,x]
```

output
```
c^5*(-1/9*a^6/x^9 + (a^5*b)/(2*x^8) - (5*a^4*b^2)/(7*x^7) + (a^2*b^4)/x^5
- (a*b^5)/x^4 + b^6/(3*x^3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^6c^5}{x^{10}}-\frac{4a^5bc^5}{x^9}+\frac{5a^4b^2c^5}{x^8}-\frac{5a^2b^4c^5}{x^6}+\frac{4ab^5c^5}{x^5}-\frac{b^6c^5}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^6c^5}{9x^9}+\frac{a^5bc^5}{2x^8}-\frac{5a^4b^2c^5}{7x^7}+\frac{a^2b^4c^5}{x^5}-\frac{ab^5c^5}{x^4}+\frac{b^6c^5}{3x^3}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^10,x]
```

output
```
-1/9*(a^6*c^5)/x^9 + (a^5*b*c^5)/(2*x^8) - (5*a^4*b^2*c^5)/(7*x^7) + (a^2*
b^4*c^5)/x^5 - (a*b^5*c^5)/x^4 + (b^6*c^5)/(3*x^3)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{c^{5}\left(-42 b^{6} x^{6}+126 a\,x^{5} b^{5}-126 a^{2} x^{4} b^{4}+90 a^{4} x^{2} b^{2}-63 a^{5} x b+14 a^{6}\right)}{126 x^{9}}$	61
default	$c^{5}\left(\frac{b^{6}}{3 x^{3}}+\frac{a^{2} b^{4}}{x^{5}}-\frac{5 a^{4} b^{2}}{7 x^{7}}-\frac{a\,b^{5}}{x^{4}}+\frac{a^{5} b}{2 x^{8}}-\frac{a^{6}}{9 x^{9}}\right)$	61
norman	$\frac{a^{2} b^{4} c^{5} x^{4}-\frac{1}{9} a^{6} c^{5}+\frac{1}{3} b^{6} c^{5} x^{6}-a\,b^{5} c^{5} x^{5}-\frac{5}{7} a^{4} b^{2} c^{5} x^{2}+\frac{1}{2} a^{5} b\,c^{5} x}{x^{9}}$	74
risch	$\frac{a^{2} b^{4} c^{5} x^{4}-\frac{1}{9} a^{6} c^{5}+\frac{1}{3} b^{6} c^{5} x^{6}-a\,b^{5} c^{5} x^{5}-\frac{5}{7} a^{4} b^{2} c^{5} x^{2}+\frac{1}{2} a^{5} b\,c^{5} x}{x^{9}}$	74
parallelrisch	$\frac{42 b^{6} c^{5} x^{6}-126 a\,b^{5} c^{5} x^{5}+126 a^{2} b^{4} c^{5} x^{4}-90 a^{4} b^{2} c^{5} x^{2}+63 a^{5} b\,c^{5} x-14 a^{6} c^{5}}{126 x^{9}}$	76
orering	$-\frac{\left(-42 b^{6} x^{6}+126 a\,x^{5} b^{5}-126 a^{2} x^{4} b^{4}+90 a^{4} x^{2} b^{2}-63 a^{5} x b+14 a^{6}\right) \left(-b c x+a c\right)^{5}}{126 x^{9} \left(-b x+a\right)^{5}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/126*c^5*(-42*b^6*x^6+126*a*b^5*x^5-126*a^2*b^4*x^4+90*a^4*b^2*x^2-63*a^
5*b*x+14*a^6)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = \frac{42\,b^6c^5x^6-126\,ab^5c^5x^5+126\,a^2b^4c^5x^4-90\,a^4b^2c^5x^2+63\,a^5bc^5x-14\,a^6c^5}{126\,x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^10,x, algorithm="fricas")
```

output
```
1/126*(42*b^6*c^5*x^6 - 126*a*b^5*c^5*x^5 + 126*a^2*b^4*c^5*x^4 - 90*a^4*b
^2*c^5*x^2 + 63*a^5*b*c^5*x - 14*a^6*c^5)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = -\frac{14a^6c^5-63a^5bc^5x+90a^4b^2c^5x^2-126a^2b^4c^5x^4+126ab^5c^5x^5-42b^6c^5x^6}{126x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**10,x)
```

output
```
-(14*a**6*c**5 - 63*a**5*b*c**5*x + 90*a**4*b**2*c**5*x**2 - 126*a**2*b**4
*c**5*x**4 + 126*a*b**5*c**5*x**5 - 42*b**6*c**5*x**6)/(126*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = \frac{42\,b^6c^5x^6-126\,ab^5c^5x^5+126\,a^2b^4c^5x^4-90\,a^4b^2c^5x^2+63\,a^5bc^5x-14\,a^6c^5}{126\,x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^10,x, algorithm="maxima")
```

output
```
1/126*(42*b^6*c^5*x^6 - 126*a*b^5*c^5*x^5 + 126*a^2*b^4*c^5*x^4 - 90*a^4*b
^2*c^5*x^2 + 63*a^5*b*c^5*x - 14*a^6*c^5)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = \frac{42\,b^6c^5x^6 - 126\,ab^5c^5x^5 + 126\,a^2b^4c^5x^4 - 90\,a^4b^2c^5x^2 + 63\,a^5bc^5x - 14\,a^6c^5}{126\,x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^10,x, algorithm="giac")
```

output
```
1/126*(42*b^6*c^5*x^6 - 126*a*b^5*c^5*x^5 + 126*a^2*b^4*c^5*x^4 - 90*a^4*b
^2*c^5*x^2 + 63*a^5*b*c^5*x - 14*a^6*c^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = -\frac{\frac{a^6\,c^5}{9} - \frac{a^5\,b\,c^5\,x}{2} + \frac{5\,a^4\,b^2\,c^5\,x^2}{7} - a^2\,b^4\,c^5\,x^4 + a\,b^5\,c^5\,x^5 - \frac{b^6\,c^5\,x^6}{3}}{x^9}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^10,x)
```

output
```
-((a^6*c^5)/9 - (b^6*c^5*x^6)/3 + a*b^5*c^5*x^5 + (5*a^4*b^2*c^5*x^2)/7 -
a^2*b^4*c^5*x^4 - (a^5*b*c^5*x)/2)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{10}}\,dx = \frac{c^5(42b^6x^6 - 126a\,b^5x^5 + 126a^2b^4x^4 - 90a^4b^2x^2 + 63a^5bx - 14a^6)}{126x^9}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^10,x)
```

output
```
(c**5*( - 14*a**6 + 63*a**5*b*x - 90*a**4*b**2*x**2 + 126*a**2*b**4*x**4 -
 126*a*b**5*x**5 + 42*b**6*x**6))/(126*x**9)
```

3.41 $\int \frac{(a+bx)(ac-bcx)^5}{x^{11}} dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}} dx = -\frac{a^6c^5}{10x^{10}} + \frac{4a^5bc^5}{9x^9} - \frac{5a^4b^2c^5}{8x^8} + \frac{5a^2b^4c^5}{6x^6} - \frac{4ab^5c^5}{5x^5} + \frac{b^6c^5}{4x^4}$$

output
```
-1/10*a^6*c^5/x^10+4/9*a^5*b*c^5/x^9-5/8*a^4*b^2*c^5/x^8+5/6*a^2*b^4*c^5/x
^6-4/5*a*b^5*c^5/x^5+1/4*b^6*c^5/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}} dx = c^5 \left(-\frac{a^6}{10x^{10}} + \frac{4a^5b}{9x^9} - \frac{5a^4b^2}{8x^8} + \frac{5a^2b^4}{6x^6} - \frac{4ab^5}{5x^5} + \frac{b^6}{4x^4}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^11,x]
```

output
```
c^5*(-1/10*a^6/x^10 + (4*a^5*b)/(9*x^9) - (5*a^4*b^2)/(8*x^8) + (5*a^2*b^4
)/(6*x^6) - (4*a*b^5)/(5*x^5) + b^6/(4*x^4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^6c^5}{x^{11}}-\frac{4a^5bc^5}{x^{10}}+\frac{5a^4b^2c^5}{x^9}-\frac{5a^2b^4c^5}{x^7}+\frac{4ab^5c^5}{x^6}-\frac{b^6c^5}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6c^5}{10x^{10}}+\frac{4a^5bc^5}{9x^9}-\frac{5a^4b^2c^5}{8x^8}+\frac{5a^2b^4c^5}{6x^6}-\frac{4ab^5c^5}{5x^5}+\frac{b^6c^5}{4x^4}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^11,x]
```

output
```
-1/10*(a^6*c^5)/x^10 + (4*a^5*b*c^5)/(9*x^9) - (5*a^4*b^2*c^5)/(8*x^8) + (
5*a^2*b^4*c^5)/(6*x^6) - (4*a*b^5*c^5)/(5*x^5) + (b^6*c^5)/(4*x^4)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{c^{5}\left(-90 b^{6} x^{6}+288 a \,x^{5} b^{5}-300 a^{2} x^{4} b^{4}+225 a^{4} x^{2} b^{2}-160 a^{5} x b +36 a^{6}\right)}{360 x^{10}}$	61
default	$c^{5} \left(-\frac{4 a \,b^{5}}{5 x^{5}}+\frac{b^{6}}{4 x^{4}}-\frac{5 a^{4} b^{2}}{8 x^{8}}-\frac{a^{6}}{10 x^{10}}+\frac{5 a^{2} b^{4}}{6 x^{6}}+\frac{4 a^{5} b}{9 x^{9}}\right)$	62
norman	$\frac{-\frac{1}{10} a^{6} c^{5}+\frac{1}{4} b^{6} c^{5} x^{6}-\frac{4}{5} a \,b^{5} c^{5} x^{5}+\frac{5}{6} a^{2} b^{4} c^{5} x^{4}-\frac{5}{8} a^{4} b^{2} c^{5} x^{2}+\frac{4}{9} a^{5} b \,c^{5} x}{x^{10}}$	75
risch	$\frac{-\frac{1}{10} a^{6} c^{5}+\frac{1}{4} b^{6} c^{5} x^{6}-\frac{4}{5} a \,b^{5} c^{5} x^{5}+\frac{5}{6} a^{2} b^{4} c^{5} x^{4}-\frac{5}{8} a^{4} b^{2} c^{5} x^{2}+\frac{4}{9} a^{5} b \,c^{5} x}{x^{10}}$	75
parallelrisch	$\frac{90 b^{6} c^{5} x^{6}-288 a \,b^{5} c^{5} x^{5}+300 a^{2} b^{4} c^{5} x^{4}-225 a^{4} b^{2} c^{5} x^{2}+160 a^{5} b \,c^{5} x -36 a^{6} c^{5}}{360 x^{10}}$	76
orering	$-\frac{\left(-90 b^{6} x^{6}+288 a \,x^{5} b^{5}-300 a^{2} x^{4} b^{4}+225 a^{4} x^{2} b^{2}-160 a^{5} x b +36 a^{6}\right) \left(-b c x +a c \right)^{5}}{360 x^{10} \left(-b x +a \right)^{5}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/360*c^5*(-90*b^6*x^6+288*a*b^5*x^5-300*a^2*b^4*x^4+225*a^4*b^2*x^2-160*
a^5*b*x+36*a^6)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx = \frac{90\,b^6c^5x^6-288\,ab^5c^5x^5+300\,a^2b^4c^5x^4-225\,a^4b^2c^5x^2+160\,a^5bc^5x-36\,a^6c^5}{360\,x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^11,x, algorithm="fricas")
```

output
```
1/360*(90*b^6*c^5*x^6 - 288*a*b^5*c^5*x^5 + 300*a^2*b^4*c^5*x^4 - 225*a^4*
b^2*c^5*x^2 + 160*a^5*b*c^5*x - 36*a^6*c^5)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx = -\frac{36a^6c^5 - 160a^5bc^5x + 225a^4b^2c^5x^2 - 300a^2b^4c^5x^4 + 288ab^5c^5x^5 - 90b^6c^5x^6}{360x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**11,x)
```

output
```
-(36*a**6*c**5 - 160*a**5*b*c**5*x + 225*a**4*b**2*c**5*x**2 - 300*a**2*b*
*4*c**5*x**4 + 288*a*b**5*c**5*x**5 - 90*b**6*c**5*x**6)/(360*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx = \frac{90\,b^6c^5x^6 - 288\,ab^5c^5x^5 + 300\,a^2b^4c^5x^4 - 225\,a^4b^2c^5x^2 + 160\,a^5bc^5x - 36\,a^6c^5}{360\,x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^11,x, algorithm="maxima")
```

output
```
1/360*(90*b^6*c^5*x^6 - 288*a*b^5*c^5*x^5 + 300*a^2*b^4*c^5*x^4 - 225*a^4*
b^2*c^5*x^2 + 160*a^5*b*c^5*x - 36*a^6*c^5)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx = \frac{90\,b^6c^5x^6 - 288\,ab^5c^5x^5 + 300\,a^2b^4c^5x^4 - 225\,a^4b^2c^5x^2 + 160\,a^5bc^5x - 36\,a^6c^5}{360\,x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^11,x, algorithm="giac")
```

output
```
1/360*(90*b^6*c^5*x^6 - 288*a*b^5*c^5*x^5 + 300*a^2*b^4*c^5*x^4 - 225*a^4*
b^2*c^5*x^2 + 160*a^5*b*c^5*x - 36*a^6*c^5)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx = -\frac{\frac{a^6\,c^5}{10} - \frac{4\,a^5\,b\,c^5\,x}{9} + \frac{5\,a^4\,b^2\,c^5\,x^2}{8} - \frac{5\,a^2\,b^4\,c^5\,x^4}{6} + \frac{4\,a\,b^5\,c^5\,x^5}{5} - \frac{b^6\,c^5\,x^6}{4}}{x^{10}}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^11,x)
```

output
```
-((a^6*c^5)/10 - (b^6*c^5*x^6)/4 + (4*a*b^5*c^5*x^5)/5 + (5*a^4*b^2*c^5*x^
2)/8 - (5*a^2*b^4*c^5*x^4)/6 - (4*a^5*b*c^5*x)/9)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{11}}\,dx = \frac{c^5(90b^6x^6-288a\,b^5x^5+300a^2b^4x^4-225a^4b^2x^2+160a^5bx-36a^6)}{360x^{10}}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^11,x)
```

output
```
(c**5*( - 36*a**6 + 160*a**5*b*x - 225*a**4*b**2*x**2 + 300*a**2*b**4*x**4
 - 288*a*b**5*x**5 + 90*b**6*x**6))/(360*x**10)
```

3.42 $\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = -\frac{a^6c^5}{11x^{11}}+\frac{2a^5bc^5}{5x^{10}}-\frac{5a^4b^2c^5}{9x^9}+\frac{5a^2b^4c^5}{7x^7}-\frac{2ab^5c^5}{3x^6}+\frac{b^6c^5}{5x^5}$$

output
```
-1/11*a^6*c^5/x^11+2/5*a^5*b*c^5/x^10-5/9*a^4*b^2*c^5/x^9+5/7*a^2*b^4*c^5/
x^7-2/3*a*b^5*c^5/x^6+1/5*b^6*c^5/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = c^5\left(-\frac{a^6}{11x^{11}}+\frac{2a^5b}{5x^{10}}-\frac{5a^4b^2}{9x^9}+\frac{5a^2b^4}{7x^7}-\frac{2ab^5}{3x^6}+\frac{b^6}{5x^5}\right)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^5)/x^12,x]
```

output
```
c^5*(-1/11*a^6/x^11 + (2*a^5*b)/(5*x^10) - (5*a^4*b^2)/(9*x^9) + (5*a^2*b^
4)/(7*x^7) - (2*a*b^5)/(3*x^6) + b^6/(5*x^5))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx$$

$$\Big\downarrow \text{84}$$

$$\int \left(\frac{a^6c^5}{x^{12}}-\frac{4a^5bc^5}{x^{11}}+\frac{5a^4b^2c^5}{x^{10}}-\frac{5a^2b^4c^5}{x^8}+\frac{4ab^5c^5}{x^7}-\frac{b^6c^5}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6c^5}{11x^{11}}+\frac{2a^5bc^5}{5x^{10}}-\frac{5a^4b^2c^5}{9x^9}+\frac{5a^2b^4c^5}{7x^7}-\frac{2ab^5c^5}{3x^6}+\frac{b^6c^5}{5x^5}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^5)/x^12,x]
```

output
```
-1/11*(a^6*c^5)/x^11 + (2*a^5*b*c^5)/(5*x^10) - (5*a^4*b^2*c^5)/(9*x^9) +
(5*a^2*b^4*c^5)/(7*x^7) - (2*a*b^5*c^5)/(3*x^6) + (b^6*c^5)/(5*x^5)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{c^{5}\left(-693 b^{6} x^{6}+2310 a\,x^{5} b^{5}-2475 a^{2} x^{4} b^{4}+1925 a^{4} x^{2} b^{2}-1386 a^{5} x b+315 a^{6}\right)}{3465 x^{11}}$	61
default	$c^{5}\left(\frac{b^{6}}{5 x^{5}}-\frac{a^{6}}{11 x^{11}}+\frac{5 a^{2} b^{4}}{7 x^{7}}+\frac{2 a^{5} b}{5 x^{10}}-\frac{2 a\,b^{5}}{3 x^{6}}-\frac{5 a^{4} b^{2}}{9 x^{9}}\right)$	62
norman	$\frac{-\frac{1}{11} a^{6} c^{5}+\frac{1}{5} b^{6} c^{5} x^{6}-\frac{2}{3} a\,b^{5} c^{5} x^{5}+\frac{5}{7} a^{2} b^{4} c^{5} x^{4}-\frac{5}{9} a^{4} b^{2} c^{5} x^{2}+\frac{2}{5} a^{5} b\,c^{5} x}{x^{11}}$	75
risch	$\frac{-\frac{1}{11} a^{6} c^{5}+\frac{1}{5} b^{6} c^{5} x^{6}-\frac{2}{3} a\,b^{5} c^{5} x^{5}+\frac{5}{7} a^{2} b^{4} c^{5} x^{4}-\frac{5}{9} a^{4} b^{2} c^{5} x^{2}+\frac{2}{5} a^{5} b\,c^{5} x}{x^{11}}$	75
parallelrisch	$\frac{693 b^{6} c^{5} x^{6}-2310 a\,b^{5} c^{5} x^{5}+2475 a^{2} b^{4} c^{5} x^{4}-1925 a^{4} b^{2} c^{5} x^{2}+1386 a^{5} b\,c^{5} x-315 a^{6} c^{5}}{3465 x^{11}}$	76
orering	$-\frac{\left(-693 b^{6} x^{6}+2310 a\,x^{5} b^{5}-2475 a^{2} x^{4} b^{4}+1925 a^{4} x^{2} b^{2}-1386 a^{5} x b+315 a^{6}\right) \left(-b c x+a c\right)^{5}}{3465 x^{11} \left(-b x+a\right)^{5}}$	77

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/3465*c^5*(-693*b^6*x^6+2310*a*b^5*x^5-2475*a^2*b^4*x^4+1925*a^4*b^2*x^2
-1386*a^5*b*x+315*a^6)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = \frac{693\,b^6c^5x^6-2310\,ab^5c^5x^5+2475\,a^2b^4c^5x^4-1925\,a^4b^2c^5x^2+1386\,a^5bc^5x-315\,a^6c^5}{3465\,x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^12,x, algorithm="fricas")
```

output
```
1/3465*(693*b^6*c^5*x^6 - 2310*a*b^5*c^5*x^5 + 2475*a^2*b^4*c^5*x^4 - 1925
*a^4*b^2*c^5*x^2 + 1386*a^5*b*c^5*x - 315*a^6*c^5)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = -\frac{315a^6c^5 - 1386a^5bc^5x + 1925a^4b^2c^5x^2 - 2475a^2b^4c^5x^4 + 2310ab^5c^5x^5 - 693b^6c^5x^6}{3465x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**5/x**12,x)
```

output
```
-(315*a**6*c**5 - 1386*a**5*b*c**5*x + 1925*a**4*b**2*c**5*x**2 - 2475*a**
2*b**4*c**5*x**4 + 2310*a*b**5*c**5*x**5 - 693*b**6*c**5*x**6)/(3465*x**11
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = \frac{693\,b^6c^5x^6 - 2310\,ab^5c^5x^5 + 2475\,a^2b^4c^5x^4 - 1925\,a^4b^2c^5x^2 + 1386\,a^5bc^5x - 315\,a^6c^5}{3465\,x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^12,x, algorithm="maxima")
```

output
```
1/3465*(693*b^6*c^5*x^6 - 2310*a*b^5*c^5*x^5 + 2475*a^2*b^4*c^5*x^4 - 1925
*a^4*b^2*c^5*x^2 + 1386*a^5*b*c^5*x - 315*a^6*c^5)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = \frac{693\,b^6c^5x^6-2310\,ab^5c^5x^5+2475\,a^2b^4c^5x^4-1925\,a^4b^2c^5x^2+1386\,a^5bc^5x-315\,a^6c^5}{3465\,x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^5/x^12,x, algorithm="giac")
```

output
```
1/3465*(693*b^6*c^5*x^6 - 2310*a*b^5*c^5*x^5 + 2475*a^2*b^4*c^5*x^4 - 1925
*a^4*b^2*c^5*x^2 + 1386*a^5*b*c^5*x - 315*a^6*c^5)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = -\frac{\frac{a^6\,c^5}{11}-\frac{2\,a^5\,b\,c^5\,x}{5}+\frac{5\,a^4\,b^2\,c^5\,x^2}{9}-\frac{5\,a^2\,b^4\,c^5\,x^4}{7}+\frac{2\,a\,b^5\,c^5\,x^5}{3}-\frac{b^6\,c^5\,x^6}{5}}{x^{11}}$$

input
```
int(((a*c - b*c*x)^5*(a + b*x))/x^12,x)
```

output
```
-((a^6*c^5)/11 - (b^6*c^5*x^6)/5 + (2*a*b^5*c^5*x^5)/3 + (5*a^4*b^2*c^5*x^
2)/9 - (5*a^2*b^4*c^5*x^4)/7 - (2*a^5*b*c^5*x)/5)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(ac-bcx)^5}{x^{12}}\,dx = \frac{c^5(693b^6x^6-2310a\,b^5x^5+2475a^2b^4x^4-1925a^4b^2x^2+1386a^5bx-315a^6)}{3465x^{11}}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^5/x^12,x)
```

output
```
(c**5*( - 315*a**6 + 1386*a**5*b*x - 1925*a**4*b**2*x**2 + 2475*a**2*b**4*
x**4 - 2310*a*b**5*x**5 + 693*b**6*x**6))/(3465*x**11)
```

3.43 $\int \frac{(a+bx)(ac-bcx)^6}{x^8} dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8} dx = -\frac{a^7c^6}{7x^7} + \frac{5a^6bc^6}{6x^6} - \frac{9a^5b^2c^6}{5x^5} + \frac{5a^4b^3c^6}{4x^4} + \frac{5a^3b^4c^6}{3x^3} - \frac{9a^2b^5c^6}{2x^2} + \frac{5ab^6c^6}{x} + b^7c^6\log(x)$$

output
```
-1/7*a^7*c^6/x^7+5/6*a^6*b*c^6/x^6-9/5*a^5*b^2*c^6/x^5+5/4*a^4*b^3*c^6/x^4
+5/3*a^3*b^4*c^6/x^3-9/2*a^2*b^5*c^6/x^2+5*a*b^6*c^6/x+b^7*c^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8} dx = -\frac{a^7c^6}{7x^7} + \frac{5a^6bc^6}{6x^6} - \frac{9a^5b^2c^6}{5x^5} + \frac{5a^4b^3c^6}{4x^4} + \frac{5a^3b^4c^6}{3x^3} - \frac{9a^2b^5c^6}{2x^2} + \frac{5ab^6c^6}{x} + b^7c^6\log(x)$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^6)/x^8,x]
```

output
```
-1/7*(a^7*c^6)/x^7 + (5*a^6*b*c^6)/(6*x^6) - (9*a^5*b^2*c^6)/(5*x^5) + (5*
a^4*b^3*c^6)/(4*x^4) + (5*a^3*b^4*c^6)/(3*x^3) - (9*a^2*b^5*c^6)/(2*x^2) +
 (5*a*b^6*c^6)/x + b^7*c^6*Log[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8} \, dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^7c^6}{x^8}-\frac{5a^6bc^6}{x^7}+\frac{9a^5b^2c^6}{x^6}-\frac{5a^4b^3c^6}{x^5}-\frac{5a^3b^4c^6}{x^4}+\frac{9a^2b^5c^6}{x^3}-\frac{5ab^6c^6}{x^2}+\frac{b^7c^6}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7c^6}{7x^7}+\frac{5a^6bc^6}{6x^6}-\frac{9a^5b^2c^6}{5x^5}+\frac{5a^4b^3c^6}{4x^4}+\frac{5a^3b^4c^6}{3x^3}-\frac{9a^2b^5c^6}{2x^2}+\frac{5ab^6c^6}{x}+b^7c^6\log(x)$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^6)/x^8,x]
```

output

```
-1/7*(a^7*c^6)/x^7 + (5*a^6*b*c^6)/(6*x^6) - (9*a^5*b^2*c^6)/(5*x^5) + (5*
a^4*b^3*c^6)/(4*x^4) + (5*a^3*b^4*c^6)/(3*x^3) - (9*a^2*b^5*c^6)/(2*x^2) +
 (5*a*b^6*c^6)/x + b^7*c^6*Log[x]
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result	size
default	$c^{6}\left(\frac{5a^{3}b^{4}}{3x^{3}}-\frac{9a^{5}b^{2}}{5x^{5}}-\frac{9a^{2}b^{5}}{2x^{2}}-\frac{a^{7}}{7x^{7}}+\frac{5a^{4}b^{3}}{4x^{4}}+b^{7}\ln\left(x\right)+\frac{5a\,b^{6}}{x}+\frac{5a^{6}b}{6x^{6}}\right)$	82
norman	$\frac{-\frac{1}{7}a^{7}c^{6}+5a\,b^{6}c^{6}x^{6}-\frac{9}{2}a^{2}b^{5}c^{6}x^{5}+\frac{5}{3}a^{3}b^{4}c^{6}x^{4}+\frac{5}{4}a^{4}b^{3}c^{6}x^{3}-\frac{9}{5}a^{5}b^{2}c^{6}x^{2}+\frac{5}{6}a^{6}b\,c^{6}x}{x^{7}}+b^{7}c^{6}\ln\left(x\right)$	102
risch	$\frac{-\frac{1}{7}a^{7}c^{6}+5a\,b^{6}c^{6}x^{6}-\frac{9}{2}a^{2}b^{5}c^{6}x^{5}+\frac{5}{3}a^{3}b^{4}c^{6}x^{4}+\frac{5}{4}a^{4}b^{3}c^{6}x^{3}-\frac{9}{5}a^{5}b^{2}c^{6}x^{2}+\frac{5}{6}a^{6}b\,c^{6}x}{x^{7}}+b^{7}c^{6}\ln\left(x\right)$	102
parallelrisch	$\frac{420b^{7}c^{6}\ln\left(x\right)x^{7}+2100a\,b^{6}c^{6}x^{6}-1890a^{2}b^{5}c^{6}x^{5}+700a^{3}b^{4}c^{6}x^{4}+525a^{4}b^{3}c^{6}x^{3}-756a^{5}b^{2}c^{6}x^{2}+350a^{6}b\,c^{6}x-60a^{7}c^{6}}{420x^{7}}$	106

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^8,x,method=_RETURNVERBOSE)
```

output
```
c^6*(5/3*a^3*b^4/x^3-9/5*a^5*b^2/x^5-9/2*a^2*b^5/x^2-1/7*a^7/x^7+5/4*a^4*b
^3/x^4+b^7*ln(x)+5*a*b^6/x+5/6*a^6*b/x^6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int\frac{(a+bx)(ac-bcx)^{6}}{x^{8}}\,dx$$
$$=\frac{420\,b^{7}c^{6}x^{7}\log\left(x\right)+2100\,ab^{6}c^{6}x^{6}-1890\,a^{2}b^{5}c^{6}x^{5}+700\,a^{3}b^{4}c^{6}x^{4}+525\,a^{4}b^{3}c^{6}x^{3}-756\,a^{5}b^{2}c^{6}x^{2}+350\,a^{6}}{420\,x^{7}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^8,x, algorithm="fricas")
```

output
```
1/420*(420*b^7*c^6*x^7*log(x) + 2100*a*b^6*c^6*x^6 - 1890*a^2*b^5*c^6*x^5
+ 700*a^3*b^4*c^6*x^4 + 525*a^4*b^3*c^6*x^3 - 756*a^5*b^2*c^6*x^2 + 350*a^
6*b*c^6*x - 60*a^7*c^6)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8}\,dx = b^7c^6\log(x) + \frac{-60a^7c^6 + 350a^6bc^6x - 756a^5b^2c^6x^2 + 525a^4b^3c^6x^3 + 700a^3b^4c^6x^4 - 1890a^2b^5c^6x^5 + 2100ab^6c^6x^6}{420x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**6/x**8,x)
```

output
```
b**7*c**6*log(x) + (-60*a**7*c**6 + 350*a**6*b*c**6*x - 756*a**5*b**2*c**6
*x**2 + 525*a**4*b**3*c**6*x**3 + 700*a**3*b**4*c**6*x**4 - 1890*a**2*b**5
*c**6*x**5 + 2100*a*b**6*c**6*x**6)/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8}\,dx = b^7c^6\log(x) + \frac{2100\,ab^6c^6x^6 - 1890\,a^2b^5c^6x^5 + 700\,a^3b^4c^6x^4 + 525\,a^4b^3c^6x^3 - 756\,a^5b^2c^6x^2 + 350\,a^6bc^6x - 60\,a^7c^6}{420\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^8,x, algorithm="maxima")
```

output
```
b^7*c^6*log(x) + 1/420*(2100*a*b^6*c^6*x^6 - 1890*a^2*b^5*c^6*x^5 + 700*a^
3*b^4*c^6*x^4 + 525*a^4*b^3*c^6*x^3 - 756*a^5*b^2*c^6*x^2 + 350*a^6*b*c^6*
x - 60*a^7*c^6)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8}\,dx = b^7c^6\log\left(|x|\right) + \frac{2100\,ab^6c^6x^6 - 1890\,a^2b^5c^6x^5 + 700\,a^3b^4c^6x^4 + 525\,a^4b^3c^6x^3 - 756\,a^5b^2c^6x^2 + 350\,a^6bc^6x - 60\,a^7c^6}{420\,x^7}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^8,x, algorithm="giac")
```

output
```
b^7*c^6*log(abs(x)) + 1/420*(2100*a*b^6*c^6*x^6 - 1890*a^2*b^5*c^6*x^5 + 7
00*a^3*b^4*c^6*x^4 + 525*a^4*b^3*c^6*x^3 - 756*a^5*b^2*c^6*x^2 + 350*a^6*b
*c^6*x - 60*a^7*c^6)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8}\,dx = \frac{c^6\left(5\,a\,b^6\,x^6 - \frac{a^7}{7} - \frac{9\,a^5\,b^2\,x^2}{5} + \frac{5\,a^4\,b^3\,x^3}{4} + \frac{5\,a^3\,b^4\,x^4}{3} - \frac{9\,a^2\,b^5\,x^5}{2} + b^7\,x^7\,\ln\left(x\right) + \frac{5\,a^6\,b\,x}{6}\right)}{x^7}$$

input
```
int(((a*c - b*c*x)^6*(a + b*x))/x^8,x)
```

output
```
(c^6*(5*a*b^6*x^6 - a^7/7 - (9*a^5*b^2*x^2)/5 + (5*a^4*b^3*x^3)/4 + (5*a^3
*b^4*x^4)/3 - (9*a^2*b^5*x^5)/2 + b^7*x^7*log(x) + (5*a^6*b*x)/6))/x^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)(ac-bcx)^6}{x^8}\,dx$$
$$= \frac{c^6(420\log(x)\,b^7x^7-60a^7+350a^6bx-756a^5b^2x^2+525a^4b^3x^3+700a^3b^4x^4-1890a^2b^5x^5+2100a\,b^6x^6)}{420x^7}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^8,x)
```

output
```
(c**6*(420*log(x)*b**7*x**7 - 60*a**7 + 350*a**6*b*x - 756*a**5*b**2*x**2
+ 525*a**4*b**3*x**3 + 700*a**3*b**4*x**4 - 1890*a**2*b**5*x**5 + 2100*a*b
**6*x**6))/(420*x**7)
```

3.44 $\int \frac{(a+bx)(ac-bcx)^6}{x^9} dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9} dx = -\frac{c^6(a-bx)^7}{8x^8} - \frac{9bc^6(a-bx)^7}{56ax^7}$$

output
```
-1/8*c^6*(-b*x+a)^7/x^8-9/56*b*c^6*(-b*x+a)^7/a/x^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(41) = 82.

Time = 0.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9} dx = -\frac{a^7c^6}{8x^8} + \frac{5a^6bc^6}{7x^7} - \frac{3a^5b^2c^6}{2x^6} + \frac{a^4b^3c^6}{x^5} + \frac{5a^3b^4c^6}{4x^4} - \frac{3a^2b^5c^6}{x^3} + \frac{5ab^6c^6}{2x^2} - \frac{b^7c^6}{x}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^6)/x^9,x]
```

output
```
-1/8*(a^7*c^6)/x^8 + (5*a^6*b*c^6)/(7*x^7) - (3*a^5*b^2*c^6)/(2*x^6) + (a^
4*b^3*c^6)/x^5 + (5*a^3*b^4*c^6)/(4*x^4) - (3*a^2*b^5*c^6)/x^3 + (5*a*b^6*
c^6)/(2*x^2) - (b^7*c^6)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {87, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{9}{8}b\int \frac{c^6(a-bx)^6}{x^8}dx-\frac{c^6(a-bx)^7}{8x^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{9}{8}bc^6\int \frac{(a-bx)^6}{x^8}dx-\frac{c^6(a-bx)^7}{8x^8}$$

$$\Big\downarrow \text{48}$$

$$-\frac{c^6(a-bx)^7}{8x^8}-\frac{9bc^6(a-bx)^7}{56ax^7}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^6)/x^9,x]
```

output
```
-1/8*(c^6*(a - b*x)^7)/x^8 - (9*b*c^6*(a - b*x)^7)/(56*a*x^7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.02

method	result	size
gosper	$-\frac{c^{6}\left(56 b^{7} x^{7}-140 a \,b^{6} x^{6}+168 a^{2} b^{5} x^{5}-70 a^{3} b^{4} x^{4}-56 a^{4} b^{3} x^{3}+84 a^{5} b^{2} x^{2}-40 a^{6} b x +7 a^{7}\right)}{56 x^{8}}$	83
default	$c^{6} \left(-\frac{3 a^{2} b^{5}}{x^{3}}+\frac{a^{4} b^{3}}{x^{5}}+\frac{5 a \,b^{6}}{2 x^{2}}+\frac{5 a^{6} b}{7 x^{7}}+\frac{5 a^{3} b^{4}}{4 x^{4}}-\frac{a^{7}}{8 x^{8}}-\frac{b^{7}}{x}-\frac{3 a^{5} b^{2}}{2 x^{6}}\right)$	83
orering	$-\frac{\left(56 b^{7} x^{7}-140 a \,b^{6} x^{6}+168 a^{2} b^{5} x^{5}-70 a^{3} b^{4} x^{4}-56 a^{4} b^{3} x^{3}+84 a^{5} b^{2} x^{2}-40 a^{6} b x +7 a^{7}\right) \left(-b c x +a c \right)^{6}}{56 x^{8} \left(-b x +a \right)^{6}}$	99
norman	$\frac{a^{4} b^{3} c^{6} x^{3}-\frac{1}{8} a^{7} c^{6}-b^{7} c^{6} x^{7}+\frac{5}{2} a \,b^{6} c^{6} x^{6}-3 a^{2} b^{5} c^{6} x^{5}+\frac{5}{4} a^{3} b^{4} c^{6} x^{4}-\frac{3}{2} a^{5} b^{2} c^{6} x^{2}+\frac{5}{7} a^{6} b \,c^{6} x}{x^{8}}$	102
risch	$\frac{a^{4} b^{3} c^{6} x^{3}-\frac{1}{8} a^{7} c^{6}-b^{7} c^{6} x^{7}+\frac{5}{2} a \,b^{6} c^{6} x^{6}-3 a^{2} b^{5} c^{6} x^{5}+\frac{5}{4} a^{3} b^{4} c^{6} x^{4}-\frac{3}{2} a^{5} b^{2} c^{6} x^{2}+\frac{5}{7} a^{6} b \,c^{6} x}{x^{8}}$	102
parallelrisch	$\frac{-56 b^{7} c^{6} x^{7}+140 a \,b^{6} c^{6} x^{6}-168 a^{2} b^{5} c^{6} x^{5}+70 a^{3} b^{4} c^{6} x^{4}+56 a^{4} b^{3} c^{6} x^{3}-84 a^{5} b^{2} c^{6} x^{2}+40 a^{6} b \,c^{6} x -7 a^{7} c^{6}}{56 x^{8}}$	104

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/56*c^6*(56*b^7*x^7-140*a*b^6*x^6+168*a^2*b^5*x^5-70*a^3*b^4*x^4-56*a^4*
b^3*x^3+84*a^5*b^2*x^2-40*a^6*b*x+7*a^7)/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx = -\frac{56\,b^7c^6x^7 - 140\,ab^6c^6x^6 + 168\,a^2b^5c^6x^5 - 70\,a^3b^4c^6x^4 - 56\,a^4b^3c^6x^3 + 84\,a^5b^2c^6x^2 - 40\,a^6bc^6x + 7\,a^7c^6}{56\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^9,x, algorithm="fricas")
```

output
```
-1/56*(56*b^7*c^6*x^7 - 140*a*b^6*c^6*x^6 + 168*a^2*b^5*c^6*x^5 - 70*a^3*b
^4*c^6*x^4 - 56*a^4*b^3*c^6*x^3 + 84*a^5*b^2*c^6*x^2 - 40*a^6*b*c^6*x + 7*
a^7*c^6)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.68

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx = \frac{-7a^7c^6 + 40a^6bc^6x - 84a^5b^2c^6x^2 + 56a^4b^3c^6x^3 + 70a^3b^4c^6x^4 - 168a^2b^5c^6x^5 + 140ab^6c^6x^6 - 56b^7c^6x^7}{56x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**6/x**9,x)
```

output
```
(-7*a**7*c**6 + 40*a**6*b*c**6*x - 84*a**5*b**2*c**6*x**2 + 56*a**4*b**3*c
**6*x**3 + 70*a**3*b**4*c**6*x**4 - 168*a**2*b**5*c**6*x**5 + 140*a*b**6*c
**6*x**6 - 56*b**7*c**6*x**7)/(56*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx = -\frac{56\,b^7c^6x^7 - 140\,ab^6c^6x^6 + 168\,a^2b^5c^6x^5 - 70\,a^3b^4c^6x^4 - 56\,a^4b^3c^6x^3 + 84\,a^5b^2c^6x^2 - 40\,a^6bc^6x + 7\,a^7c^6}{56\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^9,x, algorithm="maxima")
```

output
```
-1/56*(56*b^7*c^6*x^7 - 140*a*b^6*c^6*x^6 + 168*a^2*b^5*c^6*x^5 - 70*a^3*b
^4*c^6*x^4 - 56*a^4*b^3*c^6*x^3 + 84*a^5*b^2*c^6*x^2 - 40*a^6*b*c^6*x + 7*
a^7*c^6)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx = -\frac{56\,b^7c^6x^7 - 140\,ab^6c^6x^6 + 168\,a^2b^5c^6x^5 - 70\,a^3b^4c^6x^4 - 56\,a^4b^3c^6x^3 + 84\,a^5b^2c^6x^2 - 40\,a^6bc^6x + 7\,a^7c^6}{56\,x^8}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^9,x, algorithm="giac")
```

output
```
-1/56*(56*b^7*c^6*x^7 - 140*a*b^6*c^6*x^6 + 168*a^2*b^5*c^6*x^5 - 70*a^3*b
^4*c^6*x^4 - 56*a^4*b^3*c^6*x^3 + 84*a^5*b^2*c^6*x^2 - 40*a^6*b*c^6*x + 7*
a^7*c^6)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx = -\frac{\frac{a^7\,c^6}{8}-\frac{5\,a^6\,b\,c^6\,x}{7}+\frac{3\,a^5\,b^2\,c^6\,x^2}{2}-a^4\,b^3\,c^6\,x^3-\frac{5\,a^3\,b^4\,c^6\,x^4}{4}+3\,a^2\,b^5\,c^6\,x^5-\frac{5\,a\,b^6\,c^6\,x^6}{2}+b^7\,c^6\,x^7}{x^8}$$

input
```
int(((a*c - b*c*x)^6*(a + b*x))/x^9,x)
```

output
```
-((a^7*c^6)/8 + b^7*c^6*x^7 - (5*a*b^6*c^6*x^6)/2 + (3*a^5*b^2*c^6*x^2)/2
- a^4*b^3*c^6*x^3 - (5*a^3*b^4*c^6*x^4)/4 + 3*a^2*b^5*c^6*x^5 - (5*a^6*b*c
^6*x)/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9}\,dx = \frac{c^6(-56b^7x^7+140a\,b^6x^6-168a^2b^5x^5+70a^3b^4x^4+56a^4b^3x^3-84a^5b^2x^2+40a^6bx-7a^7)}{56x^8}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^9,x)
```

output
```
(c**6*( - 7*a**7 + 40*a**6*b*x - 84*a**5*b**2*x**2 + 56*a**4*b**3*x**3 + 7
0*a**3*b**4*x**4 - 168*a**2*b**5*x**5 + 140*a*b**6*x**6 - 56*b**7*x**7))/(
56*x**8)
```

3.45 $\int \frac{(a+bx)(ac-bcx)^6}{x^{10}} dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}} dx = -\frac{c^6(a-bx)^7}{9x^9} - \frac{11bc^6(a-bx)^7}{72ax^8} - \frac{11b^2c^6(a-bx)^7}{504a^2x^7}$$

output
```
-1/9*c^6*(-b*x+a)^7/x^9-11/72*b*c^6*(-b*x+a)^7/a/x^8-11/504*b^2*c^6*(-b*x+
a)^7/a^2/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}} dx = -\frac{a^7c^6}{9x^9} + \frac{5a^6bc^6}{8x^8} - \frac{9a^5b^2c^6}{7x^7} + \frac{5a^4b^3c^6}{6x^6} + \frac{a^3b^4c^6}{x^5} - \frac{9a^2b^5c^6}{4x^4} + \frac{5ab^6c^6}{3x^3} - \frac{b^7c^6}{2x^2}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^6)/x^10,x]
```

output
```
-1/9*(a^7*c^6)/x^9 + (5*a^6*b*c^6)/(8*x^8) - (9*a^5*b^2*c^6)/(7*x^7) + (5*
a^4*b^3*c^6)/(6*x^6) + (a^3*b^4*c^6)/x^5 - (9*a^2*b^5*c^6)/(4*x^4) + (5*a*
b^6*c^6)/(3*x^3) - (b^7*c^6)/(2*x^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{11}{9}b\int \frac{c^6(a-bx)^6}{x^9}dx-\frac{c^6(a-bx)^7}{9x^9}$$

$$\Big\downarrow \; 27$$

$$\frac{11}{9}bc^6\int \frac{(a-bx)^6}{x^9}dx-\frac{c^6(a-bx)^7}{9x^9}$$

$$\Big\downarrow \; 55$$

$$\frac{11}{9}bc^6\left(\frac{b\int \frac{(a-bx)^6}{x^8}dx}{8a}-\frac{(a-bx)^7}{8ax^8}\right)-\frac{c^6(a-bx)^7}{9x^9}$$

$$\Big\downarrow \; 48$$

$$\frac{11}{9}bc^6\left(-\frac{b(a-bx)^7}{56a^2x^7}-\frac{(a-bx)^7}{8ax^8}\right)-\frac{c^6(a-bx)^7}{9x^9}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^6)/x^10,x]
```

output
```
-1/9*(c^6*(a - b*x)^7)/x^9 + (11*b*c^6*(-1/8*(a - b*x)^7/(a*x^8) - (b*(a -
b*x)^7)/(56*a^2*x^7)))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

method	result	size
gosper	$-\frac{c^6(252b^7x^7-840a\,b^6x^6+1134a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3+648a^5b^2x^2-315a^6bx+56a^7)}{504x^9}$	83
default	$c^6\left(\frac{5a\,b^6}{3x^3}+\frac{a^3b^4}{x^5}-\frac{b^7}{2x^2}-\frac{9a^5b^2}{7x^7}-\frac{9a^2b^5}{4x^4}+\frac{5a^6b}{8x^8}+\frac{5a^4b^3}{6x^6}-\frac{a^7}{9x^9}\right)$	83
orering	$-\frac{(252b^7x^7-840a\,b^6x^6+1134a^2b^5x^5-504a^3b^4x^4-420a^4b^3x^3+648a^5b^2x^2-315a^6bx+56a^7)(-bcx+ac)^6}{504x^9(-bx+a)^6}$	99
norman	$\frac{a^3b^4c^6x^4-\frac{1}{9}a^7c^6-\frac{1}{2}b^7c^6x^7+\frac{5}{3}a\,b^6c^6x^6-\frac{9}{4}a^2b^5c^6x^5+\frac{5}{6}a^4b^3c^6x^3-\frac{9}{7}a^5b^2c^6x^2+\frac{5}{8}a^6b\,c^6x}{x^9}$	102
risch	$\frac{a^3b^4c^6x^4-\frac{1}{9}a^7c^6-\frac{1}{2}b^7c^6x^7+\frac{5}{3}a\,b^6c^6x^6-\frac{9}{4}a^2b^5c^6x^5+\frac{5}{6}a^4b^3c^6x^3-\frac{9}{7}a^5b^2c^6x^2+\frac{5}{8}a^6b\,c^6x}{x^9}$	102
parallelrisch	$\frac{-252b^7c^6x^7+840a\,b^6c^6x^6-1134a^2b^5c^6x^5+504a^3b^4c^6x^4+420a^4b^3c^6x^3-648a^5b^2c^6x^2+315a^6b\,c^6x-56a^7c^6}{504x^9}$	104

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/504*c^6*(252*b^7*x^7-840*a*b^6*x^6+1134*a^2*b^5*x^5-504*a^3*b^4*x^4-420
*a^4*b^3*x^3+648*a^5*b^2*x^2-315*a^6*b*x+56*a^7)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx = -\frac{252\,b^7c^6x^7-840\,ab^6c^6x^6+1134\,a^2b^5c^6x^5-504\,a^3b^4c^6x^4-420\,a^4b^3c^6x^3+648\,a^5b^2c^6x^2-315\,a^6bc^6x+}{504\,x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^10,x, algorithm="fricas")
```

output
```
-1/504*(252*b^7*c^6*x^7 - 840*a*b^6*c^6*x^6 + 1134*a^2*b^5*c^6*x^5 - 504*a
^3*b^4*c^6*x^4 - 420*a^4*b^3*c^6*x^3 + 648*a^5*b^2*c^6*x^2 - 315*a^6*b*c^6
*x + 56*a^7*c^6)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx$$
$$= \frac{-56a^7c^6 + 315a^6bc^6x - 648a^5b^2c^6x^2 + 420a^4b^3c^6x^3 + 504a^3b^4c^6x^4 - 1134a^2b^5c^6x^5 + 840ab^6c^6x^6 - 252b}{504x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**6/x**10,x)
```

output
```
(-56*a**7*c**6 + 315*a**6*b*c**6*x - 648*a**5*b**2*c**6*x**2 + 420*a**4*b*
*3*c**6*x**3 + 504*a**3*b**4*c**6*x**4 - 1134*a**2*b**5*c**6*x**5 + 840*a*
b**6*c**6*x**6 - 252*b**7*c**6*x**7)/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx =$$
$$-\frac{252\,b^7c^6x^7 - 840\,ab^6c^6x^6 + 1134\,a^2b^5c^6x^5 - 504\,a^3b^4c^6x^4 - 420\,a^4b^3c^6x^3 + 648\,a^5b^2c^6x^2 - 315\,a^6bc^6x}{504\,x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^10,x, algorithm="maxima")
```

output
```
-1/504*(252*b^7*c^6*x^7 - 840*a*b^6*c^6*x^6 + 1134*a^2*b^5*c^6*x^5 - 504*a
^3*b^4*c^6*x^4 - 420*a^4*b^3*c^6*x^3 + 648*a^5*b^2*c^6*x^2 - 315*a^6*b*c^6
*x + 56*a^7*c^6)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx = -\frac{252\,b^7c^6x^7 - 840\,ab^6c^6x^6 + 1134\,a^2b^5c^6x^5 - 504\,a^3b^4c^6x^4 - 420\,a^4b^3c^6x^3 + 648\,a^5b^2c^6x^2 - 315\,a^6bc^6x +}{504\,x^9}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^10,x, algorithm="giac")
```

output
```
-1/504*(252*b^7*c^6*x^7 - 840*a*b^6*c^6*x^6 + 1134*a^2*b^5*c^6*x^5 - 504*a
^3*b^4*c^6*x^4 - 420*a^4*b^3*c^6*x^3 + 648*a^5*b^2*c^6*x^2 - 315*a^6*b*c^6
*x + 56*a^7*c^6)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx = -\frac{\frac{a^7\,c^6}{9} - \frac{5\,a^6\,b\,c^6\,x}{8} + \frac{9\,a^5\,b^2\,c^6\,x^2}{7} - \frac{5\,a^4\,b^3\,c^6\,x^3}{6} - a^3\,b^4\,c^6\,x^4 + \frac{9\,a^2\,b^5\,c^6\,x^5}{4} - \frac{5\,a\,b^6\,c^6\,x^6}{3} + \frac{b^7\,c^6\,x^7}{2}}{x^9}$$

input
```
int(((a*c - b*c*x)^6*(a + b*x))/x^10,x)
```

output
```
-((a^7*c^6)/9 + (b^7*c^6*x^7)/2 - (5*a*b^6*c^6*x^6)/3 + (9*a^5*b^2*c^6*x^2
)/7 - (5*a^4*b^3*c^6*x^3)/6 - a^3*b^4*c^6*x^4 + (9*a^2*b^5*c^6*x^5)/4 - (5
*a^6*b*c^6*x)/8)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{10}}\,dx$$
$$= \frac{c^6(-252b^7x^7+840a\,b^6x^6-1134a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3-648a^5b^2x^2+315a^6bx-56a^7)}{504x^9}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^10,x)
```

output
```
(c**6*( - 56*a**7 + 315*a**6*b*x - 648*a**5*b**2*x**2 + 420*a**4*b**3*x**3
 + 504*a**3*b**4*x**4 - 1134*a**2*b**5*x**5 + 840*a*b**6*x**6 - 252*b**7*x
**7))/(504*x**9)
```

3.46 $\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx = -\frac{c^6(a-bx)^7}{10x^{10}} - \frac{13bc^6(a-bx)^7}{90ax^9} - \frac{13b^2c^6(a-bx)^7}{360a^2x^8} - \frac{13b^3c^6(a-bx)^7}{2520a^3x^7}$$

output
```
-1/10*c^6*(-b*x+a)^7/x^10-13/90*b*c^6*(-b*x+a)^7/a/x^9-13/360*b^2*c^6*(-b*
x+a)^7/a^2/x^8-13/2520*b^3*c^6*(-b*x+a)^7/a^3/x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx = -\frac{a^7c^6}{10x^{10}} + \frac{5a^6bc^6}{9x^9} - \frac{9a^5b^2c^6}{8x^8} + \frac{5a^4b^3c^6}{7x^7} + \frac{5a^3b^4c^6}{6x^6} - \frac{9a^2b^5c^6}{5x^5} + \frac{5ab^6c^6}{4x^4} - \frac{b^7c^6}{3x^3}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^6)/x^11,x]
```

output
```
-1/10*(a^7*c^6)/x^10 + (5*a^6*b*c^6)/(9*x^9) - (9*a^5*b^2*c^6)/(8*x^8) + (
5*a^4*b^3*c^6)/(7*x^7) + (5*a^3*b^4*c^6)/(6*x^6) - (9*a^2*b^5*c^6)/(5*x^5)
 + (5*a*b^6*c^6)/(4*x^4) - (b^7*c^6)/(3*x^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}} \, dx$$

$$\Big\downarrow \; 87$$

$$\frac{13}{10} b \int \frac{c^6(a-bx)^6}{x^{10}}dx - \frac{c^6(a-bx)^7}{10x^{10}}$$

$$\Big\downarrow \; 27$$

$$\frac{13}{10} bc^6 \int \frac{(a-bx)^6}{x^{10}}dx - \frac{c^6(a-bx)^7}{10x^{10}}$$

$$\Big\downarrow \; 55$$

$$\frac{13}{10} bc^6 \left(\frac{2b\int \frac{(a-bx)^6}{x^9}dx}{9a} - \frac{(a-bx)^7}{9ax^9}\right) - \frac{c^6(a-bx)^7}{10x^{10}}$$

$$\Big\downarrow \; 55$$

$$\frac{13}{10} bc^6 \left(\frac{2b\left(\frac{b\int \frac{(a-bx)^6}{x^8}dx}{8a} - \frac{(a-bx)^7}{8ax^8}\right)}{9a} - \frac{(a-bx)^7}{9ax^9}\right) - \frac{c^6(a-bx)^7}{10x^{10}}$$

$$\Big\downarrow \; 48$$

$$\frac{13}{10} bc^6 \left(\frac{2b\left(-\frac{b(a-bx)^7}{56a^2x^7} - \frac{(a-bx)^7}{8ax^8}\right)}{9a} - \frac{(a-bx)^7}{9ax^9}\right) - \frac{c^6(a-bx)^7}{10x^{10}}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^6)/x^11,x]
```

output
```
-1/10*(c^6*(a - b*x)^7)/x^10 + (13*b*c^6*(-1/9*(a - b*x)^7/(a*x^9) + (2*b*
(-1/8*(a - b*x)^7/(a*x^8) - (b*(a - b*x)^7)/(56*a^2*x^7)))/(9*a)))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{c^{6}\left(840 b^{7} x^{7}-3150 a\,b^{6} x^{6}+4536 a^{2} b^{5} x^{5}-2100 a^{3} b^{4} x^{4}-1800 a^{4} b^{3} x^{3}+2835 a^{5} b^{2} x^{2}-1400 a^{6} b x+252 a^{7}\right)}{2520 x^{10}}$	83
default	$c^{6} \left(-\frac{b^{7}}{3 x^{3}}-\frac{9 a^{2} b^{5}}{5 x^{5}}+\frac{5 a^{4} b^{3}}{7 x^{7}}+\frac{5 a\,b^{6}}{4 x^{4}}-\frac{9 a^{5} b^{2}}{8 x^{8}}-\frac{a^{7}}{10 x^{10}}+\frac{5 a^{3} b^{4}}{6 x^{6}}+\frac{5 a^{6} b}{9 x^{9}}\right)$	84
orering	$-\frac{\left(840 b^{7} x^{7}-3150 a\,b^{6} x^{6}+4536 a^{2} b^{5} x^{5}-2100 a^{3} b^{4} x^{4}-1800 a^{4} b^{3} x^{3}+2835 a^{5} b^{2} x^{2}-1400 a^{6} b x+252 a^{7}\right) \left(-b c x+a c\right)^{6}}{2520 x^{10} \left(-b x+a\right)^{6}}$	99
norman	$\frac{-\frac{1}{10} a^{7} c^{6}-\frac{1}{3} b^{7} c^{6} x^{7}+\frac{5}{4} a\,b^{6} c^{6} x^{6}-\frac{9}{5} a^{2} b^{5} c^{6} x^{5}+\frac{5}{6} a^{3} b^{4} c^{6} x^{4}+\frac{5}{7} a^{4} b^{3} c^{6} x^{3}-\frac{9}{8} a^{5} b^{2} c^{6} x^{2}+\frac{5}{9} a^{6} b\,c^{6} x}{x^{10}}$	103
risch	$\frac{-\frac{1}{10} a^{7} c^{6}-\frac{1}{3} b^{7} c^{6} x^{7}+\frac{5}{4} a\,b^{6} c^{6} x^{6}-\frac{9}{5} a^{2} b^{5} c^{6} x^{5}+\frac{5}{6} a^{3} b^{4} c^{6} x^{4}+\frac{5}{7} a^{4} b^{3} c^{6} x^{3}-\frac{9}{8} a^{5} b^{2} c^{6} x^{2}+\frac{5}{9} a^{6} b\,c^{6} x}{x^{10}}$	103
parallelrisch	$\frac{-840 b^{7} c^{6} x^{7}+3150 a\,b^{6} c^{6} x^{6}-4536 a^{2} b^{5} c^{6} x^{5}+2100 a^{3} b^{4} c^{6} x^{4}+1800 a^{4} b^{3} c^{6} x^{3}-2835 a^{5} b^{2} c^{6} x^{2}+1400 a^{6} b\,c^{6} x-252 a^{7} c^{6}}{2520 x^{10}}$	104

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/2520*c^6*(840*b^7*x^7-3150*a*b^6*x^6+4536*a^2*b^5*x^5-2100*a^3*b^4*x^4-
1800*a^4*b^3*x^3+2835*a^5*b^2*x^2-1400*a^6*b*x+252*a^7)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx = -\frac{840\,b^7c^6x^7 - 3150\,ab^6c^6x^6 + 4536\,a^2b^5c^6x^5 - 2100\,a^3b^4c^6x^4 - 1800\,a^4b^3c^6x^3 + 2835\,a^5b^2c^6x^2 - 1400\,a}{2520\,x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^11,x, algorithm="fricas")
```

output
```
-1/2520*(840*b^7*c^6*x^7 - 3150*a*b^6*c^6*x^6 + 4536*a^2*b^5*c^6*x^5 - 210
0*a^3*b^4*c^6*x^4 - 1800*a^4*b^3*c^6*x^3 + 2835*a^5*b^2*c^6*x^2 - 1400*a^6
*b*c^6*x + 252*a^7*c^6)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx$$
$$= \frac{-252a^7c^6 + 1400a^6bc^6x - 2835a^5b^2c^6x^2 + 1800a^4b^3c^6x^3 + 2100a^3b^4c^6x^4 - 4536a^2b^5c^6x^5 + 3150ab^6c^6x^6}{2520x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**6/x**11,x)
```

output
```
(-252*a**7*c**6 + 1400*a**6*b*c**6*x - 2835*a**5*b**2*c**6*x**2 + 1800*a**
4*b**3*c**6*x**3 + 2100*a**3*b**4*c**6*x**4 - 4536*a**2*b**5*c**6*x**5 + 3
150*a*b**6*c**6*x**6 - 840*b**7*c**6*x**7)/(2520*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx =$$
$$-\frac{840\,b^7c^6x^7 - 3150\,ab^6c^6x^6 + 4536\,a^2b^5c^6x^5 - 2100\,a^3b^4c^6x^4 - 1800\,a^4b^3c^6x^3 + 2835\,a^5b^2c^6x^2 - 1400\,a}{2520\,x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^11,x, algorithm="maxima")
```

output
```
-1/2520*(840*b^7*c^6*x^7 - 3150*a*b^6*c^6*x^6 + 4536*a^2*b^5*c^6*x^5 - 210
0*a^3*b^4*c^6*x^4 - 1800*a^4*b^3*c^6*x^3 + 2835*a^5*b^2*c^6*x^2 - 1400*a^6
*b*c^6*x + 252*a^7*c^6)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx = -\frac{840\,b^7c^6x^7 - 3150\,ab^6c^6x^6 + 4536\,a^2b^5c^6x^5 - 2100\,a^3b^4c^6x^4 - 1800\,a^4b^3c^6x^3 + 2835\,a^5b^2c^6x^2 - 1400\,a}{2520\,x^{10}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^11,x, algorithm="giac")
```

output
```
-1/2520*(840*b^7*c^6*x^7 - 3150*a*b^6*c^6*x^6 + 4536*a^2*b^5*c^6*x^5 - 210
0*a^3*b^4*c^6*x^4 - 1800*a^4*b^3*c^6*x^3 + 2835*a^5*b^2*c^6*x^2 - 1400*a^6
*b*c^6*x + 252*a^7*c^6)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx = -\frac{\frac{a^7\,c^6}{10} - \frac{5\,a^6\,b\,c^6\,x}{9} + \frac{9\,a^5\,b^2\,c^6\,x^2}{8} - \frac{5\,a^4\,b^3\,c^6\,x^3}{7} - \frac{5\,a^3\,b^4\,c^6\,x^4}{6} + \frac{9\,a^2\,b^5\,c^6\,x^5}{5} - \frac{5\,a\,b^6\,c^6\,x^6}{4} + \frac{b^7\,c^6\,x^7}{3}}{x^{10}}$$

input
```
int(((a*c - b*c*x)^6*(a + b*x))/x^11,x)
```

output
```
-((a^7*c^6)/10 + (b^7*c^6*x^7)/3 - (5*a*b^6*c^6*x^6)/4 + (9*a^5*b^2*c^6*x^
2)/8 - (5*a^4*b^3*c^6*x^3)/7 - (5*a^3*b^4*c^6*x^4)/6 + (9*a^2*b^5*c^6*x^5)
/5 - (5*a^6*b*c^6*x)/9)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{11}}\,dx$$
$$= \frac{c^6(-840b^7x^7+3150a\,b^6x^6-4536a^2b^5x^5+2100a^3b^4x^4+1800a^4b^3x^3-2835a^5b^2x^2+1400a^6bx-252a^7)}{2520x^{10}}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^11,x)
```

output
```
(c**6*( - 252*a**7 + 1400*a**6*b*x - 2835*a**5*b**2*x**2 + 1800*a**4*b**3*
x**3 + 2100*a**3*b**4*x**4 - 4536*a**2*b**5*x**5 + 3150*a*b**6*x**6 - 840*
b**7*x**7))/(2520*x**10)
```

3.47 $\int \frac{(a+bx)(ac-bcx)^6}{x^{12}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}} \, dx = -\frac{a^7c^6}{11x^{11}} + \frac{a^6bc^6}{2x^{10}} - \frac{a^5b^2c^6}{x^9} + \frac{5a^4b^3c^6}{8x^8} + \frac{5a^3b^4c^6}{7x^7} - \frac{3a^2b^5c^6}{2x^6} + \frac{ab^6c^6}{x^5} - \frac{b^7c^6}{4x^4}$$

output
```
-1/11*a^7*c^6/x^11+1/2*a^6*b*c^6/x^10-a^5*b^2*c^6/x^9+5/8*a^4*b^3*c^6/x^8+
5/7*a^3*b^4*c^6/x^7-3/2*a^2*b^5*c^6/x^6+a*b^6*c^6/x^5-1/4*b^7*c^6/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}} \, dx = -\frac{a^7c^6}{11x^{11}} + \frac{a^6bc^6}{2x^{10}} - \frac{a^5b^2c^6}{x^9} + \frac{5a^4b^3c^6}{8x^8} + \frac{5a^3b^4c^6}{7x^7} - \frac{3a^2b^5c^6}{2x^6} + \frac{ab^6c^6}{x^5} - \frac{b^7c^6}{4x^4}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^6)/x^12,x]
```

output
```
-1/11*(a^7*c^6)/x^11 + (a^6*b*c^6)/(2*x^10) - (a^5*b^2*c^6)/x^9 + (5*a^4*b
^3*c^6)/(8*x^8) + (5*a^3*b^4*c^6)/(7*x^7) - (3*a^2*b^5*c^6)/(2*x^6) + (a*b
^6*c^6)/x^5 - (b^7*c^6)/(4*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^7c^6}{x^{12}}-\frac{5a^6bc^6}{x^{11}}+\frac{9a^5b^2c^6}{x^{10}}-\frac{5a^4b^3c^6}{x^9}-\frac{5a^3b^4c^6}{x^8}+\frac{9a^2b^5c^6}{x^7}-\frac{5ab^6c^6}{x^6}+\frac{b^7c^6}{x^5}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7c^6}{11x^{11}}+\frac{a^6bc^6}{2x^{10}}-\frac{a^5b^2c^6}{x^9}+\frac{5a^4b^3c^6}{8x^8}+\frac{5a^3b^4c^6}{7x^7}-\frac{3a^2b^5c^6}{2x^6}+\frac{ab^6c^6}{x^5}-\frac{b^7c^6}{4x^4}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^6)/x^12,x]
```

output

```
-1/11*(a^7*c^6)/x^11 + (a^6*b*c^6)/(2*x^10) - (a^5*b^2*c^6)/x^9 + (5*a^4*b
^3*c^6)/(8*x^8) + (5*a^3*b^4*c^6)/(7*x^7) - (3*a^2*b^5*c^6)/(2*x^6) + (a*b
^6*c^6)/x^5 - (b^7*c^6)/(4*x^4)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{c^{6}\left(154 b^{7} x^{7}-616 a\,b^{6} x^{6}+924 a^{2} b^{5} x^{5}-440 a^{3} b^{4} x^{4}-385 a^{4} b^{3} x^{3}+616 a^{5} b^{2} x^{2}-308 a^{6} b x +56 a^{7}\right)}{616 x^{11}}$	83
default	$c^{6} \left(\frac{a\,b^{6}}{x^{5}}-\frac{a^{7}}{11 x^{11}}+\frac{5 a^{3} b^{4}}{7 x^{7}}-\frac{b^{7}}{4 x^{4}}+\frac{5 a^{4} b^{3}}{8 x^{8}}+\frac{a^{6} b}{2 x^{10}}-\frac{3 a^{2} b^{5}}{2 x^{6}}-\frac{a^{5} b^{2}}{x^{9}}\right)$	83
orering	$-\frac{\left(154 b^{7} x^{7}-616 a\,b^{6} x^{6}+924 a^{2} b^{5} x^{5}-440 a^{3} b^{4} x^{4}-385 a^{4} b^{3} x^{3}+616 a^{5} b^{2} x^{2}-308 a^{6} b x +56 a^{7}\right) \left(-b c x +a c \right)^{6}}{616 x^{11} \left(-b x +a \right)^{6}}$	99
norman	$\frac{a\,b^{6} c^{6} x^{6}-\frac{1}{11} a^{7} c^{6}-\frac{1}{4} b^{7} c^{6} x^{7}-\frac{3}{2} a^{2} b^{5} c^{6} x^{5}+\frac{5}{7} a^{3} b^{4} c^{6} x^{4}+\frac{5}{8} a^{4} b^{3} c^{6} x^{3}-a^{5} b^{2} c^{6} x^{2}+\frac{1}{2} a^{6} b\,c^{6} x}{x^{11}}$	102
risch	$\frac{a\,b^{6} c^{6} x^{6}-\frac{1}{11} a^{7} c^{6}-\frac{1}{4} b^{7} c^{6} x^{7}-\frac{3}{2} a^{2} b^{5} c^{6} x^{5}+\frac{5}{7} a^{3} b^{4} c^{6} x^{4}+\frac{5}{8} a^{4} b^{3} c^{6} x^{3}-a^{5} b^{2} c^{6} x^{2}+\frac{1}{2} a^{6} b\,c^{6} x}{x^{11}}$	102
parallelrisch	$\frac{-154 b^{7} c^{6} x^{7}+616 a\,b^{6} c^{6} x^{6}-924 a^{2} b^{5} c^{6} x^{5}+440 a^{3} b^{4} c^{6} x^{4}+385 a^{4} b^{3} c^{6} x^{3}-616 a^{5} b^{2} c^{6} x^{2}+308 a^{6} b\,c^{6} x -56 a^{7} c^{6}}{616 x^{11}}$	104

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/616*c^6*(154*b^7*x^7-616*a*b^6*x^6+924*a^2*b^5*x^5-440*a^3*b^4*x^4-385*
a^4*b^3*x^3+616*a^5*b^2*x^2-308*a^6*b*x+56*a^7)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}\,dx = -\frac{154\,b^7c^6x^7 - 616\,ab^6c^6x^6 + 924\,a^2b^5c^6x^5 - 440\,a^3b^4c^6x^4 - 385\,a^4b^3c^6x^3 + 616\,a^5b^2c^6x^2 - 308\,a^6bc^6x + \cdots}{616\,x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^12,x, algorithm="fricas")
```

output
```
-1/616*(154*b^7*c^6*x^7 - 616*a*b^6*c^6*x^6 + 924*a^2*b^5*c^6*x^5 - 440*a^
3*b^4*c^6*x^4 - 385*a^4*b^3*c^6*x^3 + 616*a^5*b^2*c^6*x^2 - 308*a^6*b*c^6*
x + 56*a^7*c^6)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}\,dx = \frac{-56a^7c^6 + 308a^6bc^6x - 616a^5b^2c^6x^2 + 385a^4b^3c^6x^3 + 440a^3b^4c^6x^4 - 924a^2b^5c^6x^5 + 616ab^6c^6x^6 - 154b^7\cdots}{616x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**6/x**12,x)
```

output
```
(-56*a**7*c**6 + 308*a**6*b*c**6*x - 616*a**5*b**2*c**6*x**2 + 385*a**4*b*
*3*c**6*x**3 + 440*a**3*b**4*c**6*x**4 - 924*a**2*b**5*c**6*x**5 + 616*a*b
**6*c**6*x**6 - 154*b**7*c**6*x**7)/(616*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}\,dx = -\frac{154\,b^7c^6x^7 - 616\,ab^6c^6x^6 + 924\,a^2b^5c^6x^5 - 440\,a^3b^4c^6x^4 - 385\,a^4b^3c^6x^3 + 616\,a^5b^2c^6x^2 - 308\,a^6bc^6x + }{616\,x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^12,x, algorithm="maxima")
```

output
```
-1/616*(154*b^7*c^6*x^7 - 616*a*b^6*c^6*x^6 + 924*a^2*b^5*c^6*x^5 - 440*a^
3*b^4*c^6*x^4 - 385*a^4*b^3*c^6*x^3 + 616*a^5*b^2*c^6*x^2 - 308*a^6*b*c^6*
x + 56*a^7*c^6)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}\,dx = -\frac{154\,b^7c^6x^7 - 616\,ab^6c^6x^6 + 924\,a^2b^5c^6x^5 - 440\,a^3b^4c^6x^4 - 385\,a^4b^3c^6x^3 + 616\,a^5b^2c^6x^2 - 308\,a^6bc^6x + }{616\,x^{11}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^12,x, algorithm="giac")
```

output
```
-1/616*(154*b^7*c^6*x^7 - 616*a*b^6*c^6*x^6 + 924*a^2*b^5*c^6*x^5 - 440*a^
3*b^4*c^6*x^4 - 385*a^4*b^3*c^6*x^3 + 616*a^5*b^2*c^6*x^2 - 308*a^6*b*c^6*
x + 56*a^7*c^6)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}\,dx = -\frac{\frac{a^7\,c^6}{11}-\frac{a^6\,b\,c^6\,x}{2}+a^5\,b^2\,c^6\,x^2-\frac{5\,a^4\,b^3\,c^6\,x^3}{8}-\frac{5\,a^3\,b^4\,c^6\,x^4}{7}+\frac{3\,a^2\,b^5\,c^6\,x^5}{2}-a\,b^6\,c^6\,x^6+\frac{b^7\,c^6\,x^7}{4}}{x^{11}}$$

input
```
int(((a*c - b*c*x)^6*(a + b*x))/x^12,x)
```

output
```
-((a^7*c^6)/11 + (b^7*c^6*x^7)/4 - a*b^6*c^6*x^6 + a^5*b^2*c^6*x^2 - (5*a^
4*b^3*c^6*x^3)/8 - (5*a^3*b^4*c^6*x^4)/7 + (3*a^2*b^5*c^6*x^5)/2 - (a^6*b*
c^6*x)/2)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{12}}\,dx = \frac{c^6(-154b^7x^7+616a\,b^6x^6-924a^2b^5x^5+440a^3b^4x^4+385a^4b^3x^3-616a^5b^2x^2+308a^6bx-56a^7)}{616x^{11}}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^12,x)
```

output
```
(c**6*( - 56*a**7 + 308*a**6*b*x - 616*a**5*b**2*x**2 + 385*a**4*b**3*x**3
 + 440*a**3*b**4*x**4 - 924*a**2*b**5*x**5 + 616*a*b**6*x**6 - 154*b**7*x*
*7))/(616*x**11)
```

3.48 $\int \frac{(a+bx)(ac-bcx)^6}{x^{13}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 119

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}} \, dx = -\frac{a^7c^6}{12x^{12}}+\frac{5a^6bc^6}{11x^{11}}-\frac{9a^5b^2c^6}{10x^{10}}+\frac{5a^4b^3c^6}{9x^9}+\frac{5a^3b^4c^6}{8x^8}-\frac{9a^2b^5c^6}{7x^7}+\frac{5ab^6c^6}{6x^6}-\frac{b^7c^6}{5x^5}$$

output
```
-1/12*a^7*c^6/x^12+5/11*a^6*b*c^6/x^11-9/10*a^5*b^2*c^6/x^10+5/9*a^4*b^3*c
^6/x^9+5/8*a^3*b^4*c^6/x^8-9/7*a^2*b^5*c^6/x^7+5/6*a*b^6*c^6/x^6-1/5*b^7*c
^6/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}} \, dx = -\frac{a^7c^6}{12x^{12}}+\frac{5a^6bc^6}{11x^{11}}-\frac{9a^5b^2c^6}{10x^{10}}+\frac{5a^4b^3c^6}{9x^9}+\frac{5a^3b^4c^6}{8x^8}-\frac{9a^2b^5c^6}{7x^7}+\frac{5ab^6c^6}{6x^6}-\frac{b^7c^6}{5x^5}$$

input
```
Integrate[((a + b*x)*(a*c - b*c*x)^6)/x^13,x]
```

output
```
-1/12*(a^7*c^6)/x^12 + (5*a^6*b*c^6)/(11*x^11) - (9*a^5*b^2*c^6)/(10*x^10)
 + (5*a^4*b^3*c^6)/(9*x^9) + (5*a^3*b^4*c^6)/(8*x^8) - (9*a^2*b^5*c^6)/(7*
x^7) + (5*a*b^6*c^6)/(6*x^6) - (b^7*c^6)/(5*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}} \, dx$$

$$\Big\downarrow \text{ 84}$$

$$\int \left(\frac{a^7c^6}{x^{13}}-\frac{5a^6bc^6}{x^{12}}+\frac{9a^5b^2c^6}{x^{11}}-\frac{5a^4b^3c^6}{x^{10}}-\frac{5a^3b^4c^6}{x^9}+\frac{9a^2b^5c^6}{x^8}-\frac{5ab^6c^6}{x^7}+\frac{b^7c^6}{x^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7c^6}{12x^{12}}+\frac{5a^6bc^6}{11x^{11}}-\frac{9a^5b^2c^6}{10x^{10}}+\frac{5a^4b^3c^6}{9x^9}+\frac{5a^3b^4c^6}{8x^8}-\frac{9a^2b^5c^6}{7x^7}+\frac{5ab^6c^6}{6x^6}-\frac{b^7c^6}{5x^5}$$

input
```
Int[((a + b*x)*(a*c - b*c*x)^6)/x^13,x]
```

output

```
-1/12*(a^7*c^6)/x^12 + (5*a^6*b*c^6)/(11*x^11) - (9*a^5*b^2*c^6)/(10*x^10)
 + (5*a^4*b^3*c^6)/(9*x^9) + (5*a^3*b^4*c^6)/(8*x^8) - (9*a^2*b^5*c^6)/(7*
x^7) + (5*a*b^6*c^6)/(6*x^6) - (b^7*c^6)/(5*x^5)
```

Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

method	result
gosper	$-\frac{c^{6} \left(5544 b^{7} x^{7}-23100 a\,b^{6} x^{6}+35640 a^{2} b^{5} x^{5}-17325 a^{3} b^{4} x^{4}-15400 a^{4} b^{3} x^{3}+24948 a^{5} b^{2} x^{2}-12600 a^{6} b x+2310 a^{7}\right)}{27720 x^{12}}$
default	$c^{6} \left(-\frac{b^{7}}{5 x^{5}}+\frac{5 a^{6} b}{11 x^{11}}-\frac{9 a^{2} b^{5}}{7 x^{7}}+\frac{5 a^{3} b^{4}}{8 x^{8}}-\frac{9 a^{5} b^{2}}{10 x^{10}}+\frac{5 a\,b^{6}}{6 x^{6}}+\frac{5 a^{4} b^{3}}{9 x^{9}}-\frac{a^{7}}{12 x^{12}}\right)$
orering	$-\frac{\left(5544 b^{7} x^{7}-23100 a\,b^{6} x^{6}+35640 a^{2} b^{5} x^{5}-17325 a^{3} b^{4} x^{4}-15400 a^{4} b^{3} x^{3}+24948 a^{5} b^{2} x^{2}-12600 a^{6} b x+2310 a^{7}\right) \left(-b c x+a c\right)^{6}}{27720 x^{12} \left(-b x+a\right)^{6}}$
norman	$\frac{-\frac{1}{12} a^{7} c^{6}-\frac{1}{5} b^{7} c^{6} x^{7}+\frac{5}{6} a\,b^{6} c^{6} x^{6}-\frac{9}{7} a^{2} b^{5} c^{6} x^{5}+\frac{5}{8} a^{3} b^{4} c^{6} x^{4}+\frac{5}{9} a^{4} b^{3} c^{6} x^{3}-\frac{9}{10} a^{5} b^{2} c^{6} x^{2}+\frac{5}{11} a^{6} b\,c^{6} x}{x^{12}}$
risch	$\frac{-\frac{1}{12} a^{7} c^{6}-\frac{1}{5} b^{7} c^{6} x^{7}+\frac{5}{6} a\,b^{6} c^{6} x^{6}-\frac{9}{7} a^{2} b^{5} c^{6} x^{5}+\frac{5}{8} a^{3} b^{4} c^{6} x^{4}+\frac{5}{9} a^{4} b^{3} c^{6} x^{3}-\frac{9}{10} a^{5} b^{2} c^{6} x^{2}+\frac{5}{11} a^{6} b\,c^{6} x}{x^{12}}$
parallelrisch	$\frac{-5544 b^{7} c^{6} x^{7}+23100 a\,b^{6} c^{6} x^{6}-35640 a^{2} b^{5} c^{6} x^{5}+17325 a^{3} b^{4} c^{6} x^{4}+15400 a^{4} b^{3} c^{6} x^{3}-24948 a^{5} b^{2} c^{6} x^{2}+12600 a^{6} b\,c^{6} x-2310 a^{7} c^{6}}{27720 x^{12}}$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/27720*c^6*(5544*b^7*x^7-23100*a*b^6*x^6+35640*a^2*b^5*x^5-17325*a^3*b^4
*x^4-15400*a^4*b^3*x^3+24948*a^5*b^2*x^2-12600*a^6*b*x+2310*a^7)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}}\,dx = -\frac{5544\,b^7c^6x^7 - 23100\,ab^6c^6x^6 + 35640\,a^2b^5c^6x^5 - 17325\,a^3b^4c^6x^4 - 15400\,a^4b^3c^6x^3 + 24948\,a^5b^2c^6x^2 -}{27720\,x^{12}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^13,x, algorithm="fricas")
```

output
```
-1/27720*(5544*b^7*c^6*x^7 - 23100*a*b^6*c^6*x^6 + 35640*a^2*b^5*c^6*x^5 -
 17325*a^3*b^4*c^6*x^4 - 15400*a^4*b^3*c^6*x^3 + 24948*a^5*b^2*c^6*x^2 - 1
2600*a^6*b*c^6*x + 2310*a^7*c^6)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}}\,dx = \frac{-2310a^7c^6 + 12600a^6bc^6x - 24948a^5b^2c^6x^2 + 15400a^4b^3c^6x^3 + 17325a^3b^4c^6x^4 - 35640a^2b^5c^6x^5 + 2310}{27720x^{12}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)**6/x**13,x)
```

output
```
(-2310*a**7*c**6 + 12600*a**6*b*c**6*x - 24948*a**5*b**2*c**6*x**2 + 15400
*a**4*b**3*c**6*x**3 + 17325*a**3*b**4*c**6*x**4 - 35640*a**2*b**5*c**6*x*
*5 + 23100*a*b**6*c**6*x**6 - 5544*b**7*c**6*x**7)/(27720*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}}\,dx = -\frac{5544\,b^7c^6x^7-23100\,ab^6c^6x^6+35640\,a^2b^5c^6x^5-17325\,a^3b^4c^6x^4-15400\,a^4b^3c^6x^3+24948\,a^5b^2c^6x^2-}{27720\,x^{12}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^13,x, algorithm="maxima")
```

output
```
-1/27720*(5544*b^7*c^6*x^7 - 23100*a*b^6*c^6*x^6 + 35640*a^2*b^5*c^6*x^5 -
 17325*a^3*b^4*c^6*x^4 - 15400*a^4*b^3*c^6*x^3 + 24948*a^5*b^2*c^6*x^2 - 1
2600*a^6*b*c^6*x + 2310*a^7*c^6)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}}\,dx = -\frac{5544\,b^7c^6x^7-23100\,ab^6c^6x^6+35640\,a^2b^5c^6x^5-17325\,a^3b^4c^6x^4-15400\,a^4b^3c^6x^3+24948\,a^5b^2c^6x^2-}{27720\,x^{12}}$$

input
```
integrate((b*x+a)*(-b*c*x+a*c)^6/x^13,x, algorithm="giac")
```

output
```
-1/27720*(5544*b^7*c^6*x^7 - 23100*a*b^6*c^6*x^6 + 35640*a^2*b^5*c^6*x^5 -
 17325*a^3*b^4*c^6*x^4 - 15400*a^4*b^3*c^6*x^3 + 24948*a^5*b^2*c^6*x^2 - 1
2600*a^6*b*c^6*x + 2310*a^7*c^6)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}}\,dx = -\frac{\frac{a^7\,c^6}{12}-\frac{5\,a^6\,b\,c^6\,x}{11}+\frac{9\,a^5\,b^2\,c^6\,x^2}{10}-\frac{5\,a^4\,b^3\,c^6\,x^3}{9}-\frac{5\,a^3\,b^4\,c^6\,x^4}{8}+\frac{9\,a^2\,b^5\,c^6\,x^5}{7}-\frac{5\,a\,b^6\,c^6\,x^6}{6}+\frac{b^7\,c^6\,x^7}{5}}{x^{12}}$$

input
```
int(((a*c - b*c*x)^6*(a + b*x))/x^13,x)
```

output
```
-((a^7*c^6)/12 + (b^7*c^6*x^7)/5 - (5*a*b^6*c^6*x^6)/6 + (9*a^5*b^2*c^6*x^
2)/10 - (5*a^4*b^3*c^6*x^3)/9 - (5*a^3*b^4*c^6*x^4)/8 + (9*a^2*b^5*c^6*x^5
)/7 - (5*a^6*b*c^6*x)/11)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(ac-bcx)^6}{x^{13}}\,dx = \frac{c^6(-5544b^7x^7+23100a\,b^6x^6-35640a^2b^5x^5+17325a^3b^4x^4+15400a^4b^3x^3-24948a^5b^2x^2+12600a^6bx}{27720x^{12}}$$

input
```
int((b*x+a)*(-b*c*x+a*c)^6/x^13,x)
```

output
```
(c**6*( - 2310*a**7 + 12600*a**6*b*x - 24948*a**5*b**2*x**2 + 15400*a**4*b
**3*x**3 + 17325*a**3*b**4*x**4 - 35640*a**2*b**5*x**5 + 23100*a*b**6*x**6
 - 5544*b**7*x**7))/(27720*x**12)
```

3.49 $\int x^4(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int x^4(a+bx)(A+Bx)\,dx=\frac{1}{5}aAx^5+\frac{1}{6}(Ab+aB)x^6+\frac{1}{7}bBx^7$$

output
```
1/5*a*A*x^5+1/6*(A*b+B*a)*x^6+1/7*b*B*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)(A+Bx)\,dx=\frac{1}{5}aAx^5+\frac{1}{6}(Ab+aB)x^6+\frac{1}{7}bBx^7$$

input
```
Integrate[x^4*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^5)/5 + ((A*b + a*B)*x^6)/6 + (b*B*x^7)/7
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(x^5(aB+Ab)+aAx^4+bBx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{5}aAx^5+\frac{1}{7}bBx^7$$

input
```
Int[x^4*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^5)/5 + ((A*b + a*B)*x^6)/6 + (b*B*x^7)/7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{5}}{5}+\frac{\left(Ab+Ba\right)x^{6}}{6}+\frac{bB\,x^{7}}{7}$	28
orering	$\frac{x^{5}\left(30bB\,x^{2}+35Abx+35Bax+42Aa\right)}{210}$	28
norman	$\frac{bB\,x^{7}}{7}+\left(\frac{Ab}{6}+\frac{Ba}{6}\right)x^{6}+\frac{aA\,x^{5}}{5}$	29
gosper	$\frac{1}{7}bB\,x^{7}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{5}aA\,x^{5}$	30
risch	$\frac{1}{7}bB\,x^{7}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{5}aA\,x^{5}$	30
parallelrisch	$\frac{1}{7}bB\,x^{7}+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ba+\frac{1}{5}aA\,x^{5}$	30

input
```
int(x^4*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*A*x^5+1/6*(A*b+B*a)*x^6+1/7*b*B*x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^4(a+bx)(A+Bx)\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{5}\,Aax^5+\frac{1}{6}\,(Ba+Ab)x^6$$

input
```
integrate(x^4*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
1/7*B*b*x^7 + 1/5*A*a*x^5 + 1/6*(B*a + A*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^4(a+bx)(A+Bx)\,dx = \frac{Aax^5}{5} + \frac{Bbx^7}{7} + x^6\left(\frac{Ab}{6} + \frac{Ba}{6}\right)$$

input
```
integrate(x**4*(b*x+a)*(B*x+A),x)
```

output
```
A*a*x**5/5 + B*b*x**7/7 + x**6*(A*b/6 + B*a/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^4(a+bx)(A+Bx)\,dx = \frac{1}{7}\,Bbx^7 + \frac{1}{5}\,Aax^5 + \frac{1}{6}\,(Ba+Ab)x^6$$

input
```
integrate(x^4*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
1/7*B*b*x^7 + 1/5*A*a*x^5 + 1/6*(B*a + A*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^4(a+bx)(A+Bx)\,dx = \frac{1}{7}\,Bbx^7 + \frac{1}{6}\,Bax^6 + \frac{1}{6}\,Abx^6 + \frac{1}{5}\,Aax^5$$

input
```
integrate(x^4*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
1/7*B*b*x^7 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/5*A*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^4(a+bx)(A+Bx)\,dx=\frac{B\,b\,x^7}{7}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,x^6+\frac{A\,a\,x^5}{5}$$

input
```
int(x^4*(A + B*x)*(a + b*x),x)
```

output
```
x^6*((A*b)/6 + (B*a)/6) + (A*a*x^5)/5 + (B*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int x^4(a+bx)(A+Bx)\,dx=\frac{x^5(15b^2x^2+35abx+21a^2)}{105}$$

input
```
int(x^4*(b*x+a)*(B*x+A),x)
```

output
```
(x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.50 $\int x^3(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int x^3(a+bx)(A+Bx)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}(Ab+aB)x^5+\frac{1}{6}bBx^6$$

output
```
1/4*a*A*x^4+1/5*(A*b+B*a)*x^5+1/6*b*B*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)(A+Bx)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}(Ab+aB)x^5+\frac{1}{6}bBx^6$$

input
```
Integrate[x^3*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^4)/4 + ((A*b + a*B)*x^5)/5 + (b*B*x^6)/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(x^4(aB+Ab)+aAx^3+bBx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{4}aAx^4+\frac{1}{6}bBx^6$$

input
```
Int[x^3*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^4)/4 + ((A*b + a*B)*x^5)/5 + (b*B*x^6)/6
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{4}}{4}+\frac{(Ab+Ba)x^{5}}{5}+\frac{bB\,x^{6}}{6}$	28
orering	$\frac{x^{4}(10bB\,x^{2}+12Abx+12Bax+15Aa)}{60}$	28
norman	$\frac{bB\,x^{6}}{6}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+\frac{aA\,x^{4}}{4}$	29
gosper	$\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{4}aA\,x^{4}$	30
risch	$\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{4}aA\,x^{4}$	30
parallelrisch	$\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{4}aA\,x^{4}$	30

input
```
int(x^3*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*A*x^4+1/5*(A*b+B*a)*x^5+1/6*b*B*x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^{3}(a+bx)(A+Bx)\,dx=\frac{1}{6}\,Bbx^{6}+\frac{1}{4}\,Aax^{4}+\frac{1}{5}\,(Ba+Ab)x^{5}$$

input
```
integrate(x^3*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
1/6*B*b*x^6 + 1/4*A*a*x^4 + 1/5*(B*a + A*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)(A+Bx)\,dx = \frac{Aax^4}{4} + \frac{Bbx^6}{6} + x^5\left(\frac{Ab}{5} + \frac{Ba}{5}\right)$$

input
```
integrate(x**3*(b*x+a)*(B*x+A),x)
```

output
```
A*a*x**4/4 + B*b*x**6/6 + x**5*(A*b/5 + B*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^3(a+bx)(A+Bx)\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{4}\,Aax^4 + \frac{1}{5}\,(Ba+Ab)x^5$$

input
```
integrate(x^3*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
1/6*B*b*x^6 + 1/4*A*a*x^4 + 1/5*(B*a + A*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^3(a+bx)(A+Bx)\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{5}\,Bax^5 + \frac{1}{5}\,Abx^5 + \frac{1}{4}\,Aax^4$$

input
```
integrate(x^3*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
1/6*B*b*x^6 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/4*A*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^3(a+bx)(A+Bx)\,dx=\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^4}{4}$$

input
```
int(x^3*(A + B*x)*(a + b*x),x)
```

output
```
x^5*((A*b)/5 + (B*a)/5) + (A*a*x^4)/4 + (B*b*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int x^3(a+bx)(A+Bx)\,dx=\frac{x^4(10b^2x^2+24abx+15a^2)}{60}$$

input
```
int(x^3*(b*x+a)*(B*x+A),x)
```

output
```
(x**4*(15*a**2 + 24*a*b*x + 10*b**2*x**2))/60
```

3.51 $\int x^2(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}(Ab+aB)x^4+\frac{1}{5}bBx^5$$

output
```
1/3*a*A*x^3+1/4*(A*b+B*a)*x^4+1/5*b*B*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}(Ab+aB)x^4+\frac{1}{5}bBx^5$$

input
```
Integrate[x^2*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^4)/4 + (b*B*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^3(aB+Ab)+aAx^2+bBx^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{5}bBx^5$$

input
```
Int[x^2*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^4)/4 + (b*B*x^5)/5
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^3}{3}+\frac{(Ab+Ba)x^4}{4}+\frac{bB\,x^5}{5}$	28
orering	$\frac{x^3(12bB\,x^2+15Abx+15Bax+20Aa)}{60}$	28
norman	$\frac{bB\,x^5}{5}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^4+\frac{aA\,x^3}{3}$	29
gosper	$\frac{1}{5}bB\,x^5+\frac{1}{4}x^4Ab+\frac{1}{4}x^4Ba+\frac{1}{3}aA\,x^3$	30
risch	$\frac{1}{5}bB\,x^5+\frac{1}{4}x^4Ab+\frac{1}{4}x^4Ba+\frac{1}{3}aA\,x^3$	30
parallelrisch	$\frac{1}{5}bB\,x^5+\frac{1}{4}x^4Ab+\frac{1}{4}x^4Ba+\frac{1}{3}aA\,x^3$	30

input
```
int(x^2*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/4*(A*b+B*a)*x^4+1/5*b*B*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{1}{5}\,Bbx^5+\frac{1}{3}\,Aax^3+\frac{1}{4}\,(Ba+Ab)x^4$$

input
```
integrate(x^2*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
1/5*B*b*x^5 + 1/3*A*a*x^3 + 1/4*(B*a + A*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{Aax^3}{3}+\frac{Bbx^5}{5}+x^4\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

input
```
integrate(x**2*(b*x+a)*(B*x+A),x)
```

output
```
A*a*x**3/3 + B*b*x**5/5 + x**4*(A*b/4 + B*a/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{1}{5}\,Bbx^5+\frac{1}{3}\,Aax^3+\frac{1}{4}\,(Ba+Ab)x^4$$

input
```
integrate(x^2*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
1/5*B*b*x^5 + 1/3*A*a*x^3 + 1/4*(B*a + A*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{1}{5}\,Bbx^5+\frac{1}{4}\,Bax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
1/5*B*b*x^5 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^4+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x)*(a + b*x),x)
```

output
```
x^4*((A*b)/4 + (B*a)/4) + (A*a*x^3)/3 + (B*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int x^2(a+bx)(A+Bx)\,dx=\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$$

input
```
int(x^2*(b*x+a)*(B*x+A),x)
```

output
```
(x**3*(10*a**2 + 15*a*b*x + 6*b**2*x**2))/30
```

3.52 $\int x(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x(a+bx)(A+Bx)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}(Ab+aB)x^3+\frac{1}{4}bBx^4$$

output
```
1/2*a*A*x^2+1/3*(A*b+B*a)*x^3+1/4*b*B*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx)(A+Bx)\,dx=\frac{1}{12}x^2(bx(4A+3Bx)+a(6A+4Bx))$$

input
```
Integrate[x*(a + b*x)*(A + B*x),x]
```

output
```
(x^2*(b*x*(4*A + 3*B*x) + a*(6*A + 4*B*x)))/12
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^2(aB+Ab)+aAx+bBx^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{4}bBx^4$$

input
```
Int[x*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^3)/3 + (b*B*x^4)/4
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^2}{2}+\frac{(Ab+Ba)x^3}{3}+\frac{bB\,x^4}{4}$	28
orering	$\frac{x^2(3bB\,x^2+4Abx+4Bax+6Aa)}{12}$	28
norman	$\frac{bB\,x^4}{4}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+\frac{aA\,x^2}{2}$	29
gosper	$\frac{1}{4}bB\,x^4+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+\frac{1}{2}aA\,x^2$	30
risch	$\frac{1}{4}bB\,x^4+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+\frac{1}{2}aA\,x^2$	30
parallelrisch	$\frac{1}{4}bB\,x^4+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+\frac{1}{2}aA\,x^2$	30

input
```
int(x*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/3*(A*b+B*a)*x^3+1/4*b*B*x^4
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx)(A+Bx)\,dx=\frac{1}{4}x^4bB+\frac{1}{3}x^3aB+\frac{1}{3}x^3bA+\frac{1}{2}x^2aA$$

input
```
integrate(x*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
1/4*x^4*b*B + 1/3*x^3*a*B + 1/3*x^3*b*A + 1/2*x^2*a*A
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx)(A+Bx)\,dx = \frac{Aax^2}{2} + \frac{Bbx^4}{4} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

input
```
integrate(x*(b*x+a)*(B*x+A),x)
```

output
```
A*a*x**2/2 + B*b*x**4/4 + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx)(A+Bx)\,dx = \frac{1}{4}\,Bbx^4 + \frac{1}{2}\,Aax^2 + \frac{1}{3}\,(Ba+Ab)x^3$$

input
```
integrate(x*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
1/4*B*b*x^4 + 1/2*A*a*x^2 + 1/3*(B*a + A*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx)(A+Bx)\,dx = \frac{1}{4}\,Bbx^4 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 + \frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
1/4*B*b*x^4 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + 1/2*A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(a+bx)(A+Bx)\,dx=\frac{B\,b\,x^4}{4}+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^3+\frac{A\,a\,x^2}{2}$$

input
```
int(x*(A + B*x)*(a + b*x),x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + (A*a*x^2)/2 + (B*b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int x(a+bx)(A+Bx)\,dx=\frac{x^2(3b^2x^2+8abx+6a^2)}{12}$$

input
```
int(x*(b*x+a)*(B*x+A),x)
```

output
```
(x**2*(6*a**2 + 8*a*b*x + 3*b**2*x**2))/12
```

3.53 $\int (a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int (a+bx)(A+Bx)\,dx = aAx+\frac{1}{2}(Ab+aB)x^2+\frac{1}{3}bBx^3$$

output
```
a*A*x+1/2*(A*b+B*a)*x^2+1/3*b*B*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx)(A+Bx)\,dx = aAx+\frac{1}{2}(Ab+aB)x^2+\frac{1}{3}bBx^3$$

input
```
Integrate[(a + b*x)*(A + B*x),x]
```

output
```
a*A*x + ((A*b + a*B)*x^2)/2 + (b*B*x^3)/3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x(aB+Ab)+aA+bBx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aB+Ab)+aAx+\frac{1}{3}bBx^3$$

input
```
Int[(a + b*x)*(A + B*x),x]
```

output
```
a*A*x + ((A*b + a*B)*x^2)/2 + (b*B*x^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^2}{2}+\frac{bB\,x^3}{3}$	25
norman	$\frac{bB\,x^3}{3}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^2+aAx$	26
orering	$\frac{x(2bB\,x^2+3Abx+3Bax+6Aa)}{6}$	26
gosper	$\frac{1}{3}bB\,x^3+\frac{1}{2}x^2Ab+\frac{1}{2}x^2Ba+aAx$	27
risch	$\frac{1}{3}bB\,x^3+\frac{1}{2}x^2Ab+\frac{1}{2}x^2Ba+aAx$	27
parallelrisch	$\frac{1}{3}bB\,x^3+\frac{1}{2}x^2Ab+\frac{1}{2}x^2Ba+aAx$	27

input
```
int((b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*(A*b+B*a)*x^2+1/3*b*B*x^3
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx)(A+Bx)\,dx=\frac{1}{3}x^3bB+\frac{1}{2}x^2aB+\frac{1}{2}x^2bA+xaA$$

input
```
integrate((b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
1/3*x^3*b*B + 1/2*x^2*a*B + 1/2*x^2*b*A + x*a*A
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx)(A+Bx)\,dx = Aax + \frac{Bbx^3}{3} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

input
```
integrate((b*x+a)*(B*x+A),x)
```

output
```
A*a*x + B*b*x**3/3 + x**2*(A*b/2 + B*a/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)(A+Bx)\,dx = \frac{1}{3}\,Bbx^3 + Aax + \frac{1}{2}\,(Ba+Ab)x^2$$

input
```
integrate((b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
1/3*B*b*x^3 + A*a*x + 1/2*(B*a + A*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx)(A+Bx)\,dx = \frac{1}{3}\,Bbx^3 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 + Aax$$

input
```
integrate((b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
1/3*B*b*x^3 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx)(A+Bx)\,dx = \frac{B\,b\,x^3}{3} + \left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,x^2 + A\,a\,x$$

input
```
int((A + B*x)*(a + b*x),x)
```

output
```
x^2*((A*b)/2 + (B*a)/2) + A*a*x + (B*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (a+bx)(A+Bx)\,dx = \frac{x(b^2x^2 + 3abx + 3a^2)}{3}$$

input
```
int((b*x+a)*(B*x+A),x)
```

output
```
(x*(3*a**2 + 3*a*b*x + b**2*x**2))/3
```

3.54 $\int \frac{(a+bx)(A+Bx)}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{(a+bx)(A+Bx)}{x} \, dx = (Ab+aB)x + \frac{1}{2}bBx^2 + aA\log(x)$$

output
```
(A*b+B*a)*x+1/2*b*B*x^2+a*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(A+Bx)}{x} \, dx = (Ab+aB)x + \frac{1}{2}bBx^2 + aA\log(x)$$

input
```
Integrate[((a + b*x)*(A + B*x))/x,x]
```

output
```
(A*b + a*B)*x + (b*B*x^2)/2 + a*A*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{aA}{x}+aB+Ab+bBx\right) dx$$

$$\Big\downarrow \; 2009$$

$$x(aB+Ab)+aA\log(x)+\frac{1}{2}bBx^2$$

input
```
Int[((a + b*x)*(A + B*x))/x,x]
```

output
```
(A*b + a*B)*x + (b*B*x^2)/2 + a*A*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{bB\,x^{2}}{2}+Abx+Bax+aA\ln\left(x\right)$	22
risch	$\frac{bB\,x^{2}}{2}+Abx+Bax+aA\ln\left(x\right)$	22
parallelrisch	$\frac{bB\,x^{2}}{2}+Abx+Bax+aA\ln\left(x\right)$	22
norman	$\left(Ab+Ba\right)x+\frac{bB\,x^{2}}{2}+aA\ln\left(x\right)$	23

input
```
int((b*x+a)*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*B*x^2+A*b*x+B*a*x+a*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(A+Bx)}{x}\,dx=\frac{1}{2}\,Bbx^{2}+Aa\log\left(x\right)+(Ba+Ab)x$$

input
```
integrate((b*x+a)*(B*x+A)/x,x, algorithm="fricas")
```

output
```
1/2*B*b*x^2 + A*a*log(x) + (B*a + A*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(A+Bx)}{x}\,dx=Aa\log\left(x\right)+\frac{Bbx^{2}}{2}+x(Ab+Ba)$$

input
```
integrate((b*x+a)*(B*x+A)/x,x)
```

output
```
A*a*log(x) + B*b*x**2/2 + x*(A*b + B*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(A+Bx)}{x}\,dx = \frac{1}{2}\,Bbx^2 + Aa\log{(x)} + (Ba+Ab)x$$

input
```
integrate((b*x+a)*(B*x+A)/x,x, algorithm="maxima")
```

output
```
1/2*B*b*x^2 + A*a*log(x) + (B*a + A*b)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(A+Bx)}{x}\,dx = \frac{1}{2}\,Bbx^2 + Bax + Abx + Aa\log{(|x|)}$$

input
```
integrate((b*x+a)*(B*x+A)/x,x, algorithm="giac")
```

output
```
1/2*B*b*x^2 + B*a*x + A*b*x + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)(A+Bx)}{x}\,dx = x\,(A\,b+B\,a) + \frac{B\,b\,x^2}{2} + A\,a\,\ln{(x)}$$

input
```
int(((A + B*x)*(a + b*x))/x,x)
```

output
```
x*(A*b + B*a) + (B*b*x^2)/2 + A*a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(A+Bx)}{x}\,dx = \log(x)\,a^2 + 2abx + \frac{b^2x^2}{2}$$

input
```
int((b*x+a)*(B*x+A)/x,x)
```

output
```
(2*log(x)*a**2 + 4*a*b*x + b**2*x**2)/2
```

3.55 $\int \frac{(a+bx)(A+Bx)}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{(a+bx)(A+Bx)}{x^2} \, dx = -\frac{aA}{x} + bBx + (Ab + aB) \log(x)$$

output
```
-a*A/x+b*B*x+(A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(A+Bx)}{x^2} \, dx = -\frac{aA}{x} + bBx + (Ab + aB) \log(x)$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^2,x]
```

output
```
-((a*A)/x) + b*B*x + (A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^2} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{aB+Ab}{x}+\frac{aA}{x^2}+bB\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)(aB+Ab)-\frac{aA}{x}+bBx$$

input
```
Int[((a + b*x)*(A + B*x))/x^2,x]
```

output
```
-((a*A)/x) + b*B*x + (A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{aA}{x}+bBx+(Ab+Ba)\ln(x)$	23
risch	$-\frac{aA}{x}+bBx+A\ln(x)\,b+B\ln(x)\,a$	23
norman	$\frac{bB\,x^2-Aa}{x}+(Ab+Ba)\ln(x)$	27
parallelrisch	$\frac{A\ln(x)xb+B\ln(x)xa+bB\,x^2-Aa}{x}$	28

input
```
int((b*x+a)*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*A/x+b*B*x+(A*b+B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)(A+Bx)}{x^2}\,dx=\frac{Bbx^2+(Ba+Ab)x\log(x)-Aa}{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
(B*b*x^2 + (B*a + A*b)*x*log(x) - A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)(A+Bx)}{x^2}\,dx=-\frac{Aa}{x}+Bbx+(Ab+Ba)\log(x)$$

input
```
integrate((b*x+a)*(B*x+A)/x**2,x)
```

output
```
-A*a/x + B*b*x + (A*b + B*a)*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(A+Bx)}{x^2}\,dx = Bbx + (Ba+Ab)\log(x) - \frac{Aa}{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
B*b*x + (B*a + A*b)*log(x) - A*a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)(A+Bx)}{x^2}\,dx = Bbx + (Ba+Ab)\log(|x|) - \frac{Aa}{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
B*b*x + (B*a + A*b)*log(abs(x)) - A*a/x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(A+Bx)}{x^2}\,dx = \ln(x)\ (A\,b+B\,a) + B\,b\,x - \frac{A\,a}{x}$$

input
```
int(((A + B*x)*(a + b*x))/x^2,x)
```

output
```
log(x)*(A*b + B*a) + B*b*x - (A*a)/x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)(A+Bx)}{x^2}\,dx = \frac{2\log(x)\,abx - a^2 + b^2x^2}{x}$$

input
```
int((b*x+a)*(B*x+A)/x^2,x)
```

output
```
(2*log(x)*a*b*x - a**2 + b**2*x**2)/x
```

3.56 $\int \frac{(a+bx)(A+Bx)}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{(a+bx)(A+Bx)}{x^3} \, dx = -\frac{aA}{2x^2} - \frac{Ab+aB}{x} + bB\log(x)$$

output
```
-1/2*a*A/x^2-(A*b+B*a)/x+b*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)(A+Bx)}{x^3} \, dx = -\frac{aA}{2x^2} + \frac{-Ab-aB}{x} + bB\log(x)$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 + (-(A*b) - a*B)/x + b*B*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^3} \, dx$$
$$\Big\downarrow 85$$
$$\int \left(\frac{aB+Ab}{x^2}+\frac{aA}{x^3}+\frac{bB}{x}\right)dx$$
$$\Big\downarrow 2009$$
$$-\frac{aB+Ab}{x}-\frac{aA}{2x^2}+bB\log(x)$$

input
```
Int[((a + b*x)*(A + B*x))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (A*b + a*B)/x + b*B*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{aA}{2x^2}-\frac{Ab+Ba}{x}+bB\ln\left(x\right)$	26
norman	$\frac{(-Ab-Ba)x-\frac{Aa}{2}}{x^2}+bB\ln\left(x\right)$	27
risch	$\frac{(-Ab-Ba)x-\frac{Aa}{2}}{x^2}+bB\ln\left(x\right)$	27
parallelrisch	$-\frac{-2Bb\ln(x)x^2+2Abx+2Bax+Aa}{2x^2}$	29

input
```
int((b*x+a)*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*A/x^2-(A*b+B*a)/x+b*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)(A+Bx)}{x^3}\,dx=\frac{2\,Bbx^2\log\left(x\right)-Aa-2\,(Ba+Ab)x}{2\,x^2}$$

input
```
integrate((b*x+a)*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*b*x^2*log(x) - A*a - 2*(B*a + A*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(A+Bx)}{x^3}\,dx=Bb\log\left(x\right)+\frac{-Aa+x(-2Ab-2Ba)}{2x^2}$$

input
```
integrate((b*x+a)*(B*x+A)/x**3,x)
```

output
```
B*b*log(x) + (-A*a + x*(-2*A*b - 2*B*a))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(A+Bx)}{x^3}\,dx = Bb\log(x) - \frac{Aa + 2\,(Ba + Ab)x}{2\,x^2}$$

input
```
integrate((b*x+a)*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
B*b*log(x) - 1/2*(A*a + 2*(B*a + A*b)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(A+Bx)}{x^3}\,dx = Bb\log(|x|) - \frac{Aa + 2\,(Ba + Ab)x}{2\,x^2}$$

input
```
integrate((b*x+a)*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
B*b*log(abs(x)) - 1/2*(A*a + 2*(B*a + A*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(A+Bx)}{x^3}\,dx = B\,b\,\ln(x) - \frac{\frac{A\,a}{2} + x\,(A\,b + B\,a)}{x^2}$$

input
```
int(((A + B*x)*(a + b*x))/x^3,x)
```

output
```
B*b*log(x) - ((A*a)/2 + x*(A*b + B*a))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)(A+Bx)}{x^3}\,dx = \frac{2\log(x)\,b^2x^2 - a^2 - 4abx}{2x^2}$$

input
```
int((b*x+a)*(B*x+A)/x^3,x)
```

output
```
(2*log(x)*b**2*x**2 - a**2 - 4*a*b*x)/(2*x**2)
```

3.57 $\int \frac{(a+bx)(A+Bx)}{x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{(a+bx)(A+Bx)}{x^4} dx = -\frac{aA}{3x^3} - \frac{Ab+aB}{2x^2} - \frac{bB}{x}$$

output
```
-1/3*a*A/x^3-1/2*(A*b+B*a)/x^2-b*B/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(A+Bx)}{x^4} dx = -\frac{3bx(A+2Bx)+a(2A+3Bx)}{6x^3}$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^4,x]
```

output
```
-1/6*(3*b*x*(A + 2*B*x) + a*(2*A + 3*B*x))/x^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^4} \, dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{aB+Ab}{x^3}+\frac{aA}{x^4}+\frac{bB}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{aB+Ab}{2x^2}-\frac{aA}{3x^3}-\frac{bB}{x}$$

input
```
Int[((a + b*x)*(A + B*x))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (A*b + a*B)/(2*x^2) - (b*B)/x
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{6bB\,x^2+3Abx+3Bax+2Aa}{6x^3}$	28
default	$-\frac{aA}{3x^3}-\frac{Ab+Ba}{2x^2}-\frac{bB}{x}$	28
norman	$\frac{-bB\,x^2+\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x-\frac{Aa}{3}}{x^3}$	28
risch	$\frac{-bB\,x^2+\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x-\frac{Aa}{3}}{x^3}$	28
parallelrisch	$-\frac{6bB\,x^2+3Abx+3Bax+2Aa}{6x^3}$	28
orering	$-\frac{6bB\,x^2+3Abx+3Bax+2Aa}{6x^3}$	28

input
```
int((b*x+a)*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*B*b*x^2+3*A*b*x+3*B*a*x+2*A*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(A+Bx)}{x^4}\,dx = -\frac{6\,Bbx^2+2\,Aa+3\,(Ba+Ab)x}{6\,x^3}$$

input
```
integrate((b*x+a)*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(6*B*b*x^2 + 2*A*a + 3*(B*a + A*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)(A+Bx)}{x^4}\,dx = \frac{-2Aa - 6Bbx^2 + x(-3Ab - 3Ba)}{6x^3}$$

input
```
integrate((b*x+a)*(B*x+A)/x**4,x)
```

output
```
(-2*A*a - 6*B*b*x**2 + x*(-3*A*b - 3*B*a))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(A+Bx)}{x^4}\,dx = -\frac{6\,Bbx^2 + 2\,Aa + 3\,(Ba + Ab)x}{6\,x^3}$$

input
```
integrate((b*x+a)*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(6*B*b*x^2 + 2*A*a + 3*(B*a + A*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(A+Bx)}{x^4}\,dx = -\frac{6\,Bbx^2 + 3\,Bax + 3\,Abx + 2\,Aa}{6\,x^3}$$

input
```
integrate((b*x+a)*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
-1/6*(6*B*b*x^2 + 3*B*a*x + 3*A*b*x + 2*A*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)(A+Bx)}{x^4}\,dx = -\frac{B\,b\,x^2+\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,x+\frac{A\,a}{3}}{x^3}$$

input
```
int(((A + B*x)*(a + b*x))/x^4,x)
```

output
```
-((A*a)/3 + x*((A*b)/2 + (B*a)/2) + B*b*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^4}\,dx = \frac{-3b^2x^2-3abx-a^2}{3x^3}$$

input
```
int((b*x+a)*(B*x+A)/x^4,x)
```

output
```
( - a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

3.58 $\int \frac{(a+bx)(A+Bx)}{x^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \frac{(a+bx)(A+Bx)}{x^5} dx = -\frac{aA}{4x^4} - \frac{Ab+aB}{3x^3} - \frac{bB}{2x^2}$$

output
```
-1/4*a*A/x^4-1/3*(A*b+B*a)/x^3-1/2*b*B/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)(A+Bx)}{x^5} dx = -\frac{3aA+4Abx+4aBx+6bBx^2}{12x^4}$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^5,x]
```

output
```
-1/12*(3*a*A + 4*A*b*x + 4*a*B*x + 6*b*B*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^5} \, dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{aB+Ab}{x^4}+\frac{aA}{x^5}+\frac{bB}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{aB+Ab}{3x^3}-\frac{aA}{4x^4}-\frac{bB}{2x^2}$$

input
```
Int[((a + b*x)*(A + B*x))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 - (A*b + a*B)/(3*x^3) - (b*B)/(2*x^2)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{6bB\,x^{2}+4Abx+4Bax+3Aa}{12x^{4}}$	28
default	$-\frac{aA}{4x^{4}}-\frac{Ab+Ba}{3x^{3}}-\frac{bB}{2x^{2}}$	28
norman	$\frac{-\frac{bB\,x^{2}}{2}+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x-\frac{Aa}{4}}{x^{4}}$	28
risch	$\frac{-\frac{bB\,x^{2}}{2}+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x-\frac{Aa}{4}}{x^{4}}$	28
parallelrisch	$-\frac{6bB\,x^{2}+4Abx+4Bax+3Aa}{12x^{4}}$	28
orering	$-\frac{6bB\,x^{2}+4Abx+4Bax+3Aa}{12x^{4}}$	28

input
```
int((b*x+a)*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(6*B*b*x^2+4*A*b*x+4*B*a*x+3*A*a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(A+Bx)}{x^5}\,dx=-\frac{6\,Bbx^{2}+3\,Aa+4\,(Ba+Ab)x}{12\,x^{4}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(6*B*b*x^2 + 3*A*a + 4*(B*a + A*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(A+Bx)}{x^5}\,dx = \frac{-3Aa-6Bbx^2+x(-4Ab-4Ba)}{12x^4}$$

input
```
integrate((b*x+a)*(B*x+A)/x**5,x)
```

output
```
(-3*A*a - 6*B*b*x**2 + x*(-4*A*b - 4*B*a))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(A+Bx)}{x^5}\,dx = -\frac{6\,Bbx^2+3\,Aa+4\,(Ba+Ab)x}{12\,x^4}$$

input
```
integrate((b*x+a)*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(6*B*b*x^2 + 3*A*a + 4*(B*a + A*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(A+Bx)}{x^5}\,dx = -\frac{6\,Bbx^2+4\,Bax+4\,Abx+3\,Aa}{12\,x^4}$$

input
```
integrate((b*x+a)*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
-1/12*(6*B*b*x^2 + 4*B*a*x + 4*A*b*x + 3*A*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(A+Bx)}{x^5}\,dx = -\frac{\frac{B\,b\,x^2}{2}+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x+\frac{A\,a}{4}}{x^4}$$

input
```
int(((A + B*x)*(a + b*x))/x^5,x)
```

output
```
-((A*a)/4 + x*((A*b)/3 + (B*a)/3) + (B*b*x^2)/2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{x^5}\,dx = \frac{-6b^2x^2-8abx-3a^2}{12x^4}$$

input
```
int((b*x+a)*(B*x+A)/x^5,x)
```

output
```
( - 3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

3.59 $\int \frac{(a+bx)(A+Bx)}{x^6} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \frac{(a+bx)(A+Bx)}{x^6} \, dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{4x^4} - \frac{bB}{3x^3}$$

output
```
-1/5*a*A/x^5-1/4*(A*b+B*a)/x^4-1/3*b*B/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(A+Bx)}{x^6} \, dx = -\frac{5bx(3A+4Bx)+3a(4A+5Bx)}{60x^5}$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^6,x]
```

output
```
-1/60*(5*b*x*(3*A + 4*B*x) + 3*a*(4*A + 5*B*x))/x^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^6} \, dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{aB+Ab}{x^5}+\frac{aA}{x^6}+\frac{bB}{x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{aB+Ab}{4x^4}-\frac{aA}{5x^5}-\frac{bB}{3x^3}$$

input
```
Int[((a + b*x)*(A + B*x))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(4*x^4) - (b*B)/(3*x^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{20 b B \,x^{2}+15 A b x +15 B a x +12 A a}{60 x^{5}}$	28
default	$-\frac{a A}{5 x^{5}}-\frac{A b +B a}{4 x^{4}}-\frac{b B}{3 x^{3}}$	28
norman	$\frac{-\frac{b B \,x^{2}}{3}+\left(-\frac{A b}{4}-\frac{B a}{4}\right) x -\frac{A a}{5}}{x^{5}}$	28
risch	$\frac{-\frac{b B \,x^{2}}{3}+\left(-\frac{A b}{4}-\frac{B a}{4}\right) x -\frac{A a}{5}}{x^{5}}$	28
parallelrisch	$-\frac{20 b B \,x^{2}+15 A b x +15 B a x +12 A a}{60 x^{5}}$	28
orering	$-\frac{20 b B \,x^{2}+15 A b x +15 B a x +12 A a}{60 x^{5}}$	28

input
```
int((b*x+a)*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(20*B*b*x^2+15*A*b*x+15*B*a*x+12*A*a)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(A+Bx)}{x^6}\,dx=-\frac{20\,Bbx^2+12\,Aa+15\,(Ba+Ab)x}{60\,x^5}$$

input
```
integrate((b*x+a)*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(20*B*b*x^2 + 12*A*a + 15*(B*a + A*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)(A+Bx)}{x^6}\,dx = \frac{-12Aa - 20Bbx^2 + x(-15Ab - 15Ba)}{60x^5}$$

input
```
integrate((b*x+a)*(B*x+A)/x**6,x)
```

output
```
(-12*A*a - 20*B*b*x**2 + x*(-15*A*b - 15*B*a))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(A+Bx)}{x^6}\,dx = -\frac{20\,Bbx^2 + 12\,Aa + 15\,(Ba + Ab)x}{60\,x^5}$$

input
```
integrate((b*x+a)*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/60*(20*B*b*x^2 + 12*A*a + 15*(B*a + A*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)(A+Bx)}{x^6}\,dx = -\frac{20\,Bbx^2 + 15\,Bax + 15\,Abx + 12\,Aa}{60\,x^5}$$

input
```
integrate((b*x+a)*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
-1/60*(20*B*b*x^2 + 15*B*a*x + 15*A*b*x + 12*A*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)(A+Bx)}{x^6}\,dx = -\frac{\frac{B\,b\,x^2}{3} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right)\,x + \frac{A\,a}{5}}{x^5}$$

input
```
int(((A + B*x)*(a + b*x))/x^6,x)
```

output
```
-((A*a)/5 + x*((A*b)/4 + (B*a)/4) + (B*b*x^2)/3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{x^6}\,dx = \frac{-10b^2x^2 - 15abx - 6a^2}{30x^5}$$

input
```
int((b*x+a)*(B*x+A)/x^6,x)
```

output
```
( - 6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

3.60 $\int x^4(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int x^4(a+bx)^2(A+Bx)\,dx=\frac{1}{5}a^2Ax^5+\frac{1}{6}a(2Ab+aB)x^6+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{8}b^2Bx^8$$

output
```
1/5*a^2*A*x^5+1/6*a*(2*A*b+B*a)*x^6+1/7*b*(A*b+2*B*a)*x^7+1/8*b^2*B*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)^2(A+Bx)\,dx=\frac{1}{5}a^2Ax^5+\frac{1}{6}a(2Ab+aB)x^6+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{8}b^2Bx^8$$

input
```
Integrate[x^4*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^6)/6 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B
*x^8)/8
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2Ax^4+bx^6(2aB+Ab)+ax^5(aB+2Ab)+b^2Bx^7\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}a^2Ax^5+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{6}ax^6(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

input
```
Int[x^4*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^6)/6 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B
*x^8)/8
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B x^8}{8}+\frac{(b^2 A+2abB)x^7}{7}+\frac{(2abA+a^2 B)x^6}{6}+\frac{a^2 A x^5}{5}$	52
norman	$\frac{b^2 B x^8}{8}+\left(\frac{1}{7}b^2 A+\frac{2}{7}abB\right)x^7+\left(\frac{1}{3}abA+\frac{1}{6}a^2 B\right)x^6+\frac{a^2 A x^5}{5}$	52
orering	$\frac{x^5(105B b^2 x^3+120A b^2 x^2+240Bab x^2+280aAbx+140B a^2 x+168a^2 A)}{840}$	52
gosper	$\frac{1}{8}b^2 B x^8+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{3}x^6 abA+\frac{1}{6}x^6 a^2 B+\frac{1}{5}a^2 A x^5$	54
risch	$\frac{1}{8}b^2 B x^8+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{3}x^6 abA+\frac{1}{6}x^6 a^2 B+\frac{1}{5}a^2 A x^5$	54
parallelrisch	$\frac{1}{8}b^2 B x^8+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{1}{3}x^6 abA+\frac{1}{6}x^6 a^2 B+\frac{1}{5}a^2 A x^5$	54

input
```
int(x^4*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*B*x^8+1/7*(A*b^2+2*B*a*b)*x^7+1/6*(2*A*a*b+B*a^2)*x^6+1/5*a^2*A*x^
5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(a+bx)^2(A+Bx)\,dx=\frac{1}{8}Bb^2x^8+\frac{1}{5}Aa^2x^5+\frac{1}{7}\left(2Bab+Ab^2\right)x^7+\frac{1}{6}\left(Ba^2+2Aab\right)x^6$$

input
```
integrate(x^4*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
1/8*B*b^2*x^8 + 1/5*A*a^2*x^5 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/6*(B*a^2 + 2
*A*a*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^4(a+bx)^2(A+Bx)\,dx=\frac{Aa^2x^5}{5}+\frac{Bb^2x^8}{8}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ba^2}{6}\right)$$

input
```
integrate(x**4*(b*x+a)**2*(B*x+A),x)
```

output
```
A*a**2*x**5/5 + B*b**2*x**8/8 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**6*(A*a*b/
3 + B*a**2/6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(a+bx)^2(A+Bx)\,dx=\frac{1}{8}\,Bb^2x^8+\frac{1}{5}\,Aa^2x^5+\frac{1}{7}\,(2\,Bab+Ab^2)x^7+\frac{1}{6}\,(Ba^2+2\,Aab)x^6$$

input
```
integrate(x^4*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
1/8*B*b^2*x^8 + 1/5*A*a^2*x^5 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/6*(B*a^2 + 2
*A*a*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^4(a+bx)^2(A+Bx)\,dx=\frac{1}{8}\,Bb^2x^8+\frac{2}{7}\,Babx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{6}\,Ba^2x^6+\frac{1}{3}\,Aabx^6+\frac{1}{5}\,Aa^2x^5$$

input
```
integrate(x^4*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
1/8*B*b^2*x^8 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/6*B*a^2*x^6 + 1/3*A*a*b*
x^6 + 1/5*A*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(a+bx)^2(A+Bx)\,dx = x^6\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{A\,a^2\,x^5}{5}+\frac{B\,b^2\,x^8}{8}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^6*((B*a^2)/6 + (A*a*b)/3) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (A*a^2*x^5)/
5 + (B*b^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x^4(a+bx)^2(A+Bx)\,dx = \frac{x^5(35b^3x^3+120a\,b^2x^2+140a^2bx+56a^3)}{280}$$

input
```
int(x^4*(b*x+a)^2*(B*x+A),x)
```

output
```
(x**5*(56*a**3 + 140*a**2*b*x + 120*a*b**2*x**2 + 35*b**3*x**3))/280
```

3.61 $\int x^3(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int x^3(a+bx)^2(A+Bx)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{6}b(Ab+2aB)x^6+\frac{1}{7}b^2Bx^7$$

output
```
1/4*a^2*A*x^4+1/5*a*(2*A*b+B*a)*x^5+1/6*b*(A*b+2*B*a)*x^6+1/7*b^2*B*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)^2(A+Bx)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{6}b(Ab+2aB)x^6+\frac{1}{7}b^2Bx^7$$

input
```
Integrate[x^3*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*x^4)/4 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B
*x^7)/7
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2Ax^3+bx^5(2aB+Ab)+ax^4(aB+2Ab)+b^2Bx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^2Ax^4+\frac{1}{6}bx^6(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

input
```
Int[x^3*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*x^4)/4 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B
*x^7)/7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B x^7}{7}+\frac{(b^2 A+2abB)x^6}{6}+\frac{(2abA+a^2 B)x^5}{5}+\frac{a^2 A x^4}{4}$	52
norman	$\frac{b^2 B x^7}{7}+\left(\frac{1}{6}b^2 A+\frac{1}{3}abB\right)x^6+\left(\frac{2}{5}abA+\frac{1}{5}a^2 B\right)x^5+\frac{a^2 A x^4}{4}$	52
orering	$\frac{x^4(60B\,b^2x^3+70A\,b^2x^2+140Bab\,x^2+168aAbx+84B\,a^2x+105a^2A)}{420}$	52
gosper	$\frac{1}{7}b^2B\,x^7+\frac{1}{6}x^6b^2A+\frac{1}{3}x^6abB+\frac{2}{5}x^5abA+\frac{1}{5}x^5a^2B+\frac{1}{4}a^2A\,x^4$	54
risch	$\frac{1}{7}b^2B\,x^7+\frac{1}{6}x^6b^2A+\frac{1}{3}x^6abB+\frac{2}{5}x^5abA+\frac{1}{5}x^5a^2B+\frac{1}{4}a^2A\,x^4$	54
parallelrisch	$\frac{1}{7}b^2B\,x^7+\frac{1}{6}x^6b^2A+\frac{1}{3}x^6abB+\frac{2}{5}x^5abA+\frac{1}{5}x^5a^2B+\frac{1}{4}a^2A\,x^4$	54

input
```
int(x^3*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*B*x^7+1/6*(A*b^2+2*B*a*b)*x^6+1/5*(2*A*a*b+B*a^2)*x^5+1/4*a^2*A*x^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(a+bx)^2(A+Bx)\,dx=\frac{1}{7}\,Bb^2x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(2\,Bab+Ab^2)x^6+\frac{1}{5}\,(Ba^2+2\,Aab)x^5$$

input
```
integrate(x^3*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
1/7*B*b^2*x^7 + 1/4*A*a^2*x^4 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/5*(B*a^2 + 2
*A*a*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)^2(A+Bx)\,dx=\frac{Aa^2x^4}{4}+\frac{Bb^2x^7}{7}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate(x**3*(b*x+a)**2*(B*x+A),x)
```

output
```
A*a**2*x**4/4 + B*b**2*x**7/7 + x**6*(A*b**2/6 + B*a*b/3) + x**5*(2*A*a*b/
5 + B*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(a+bx)^2(A+Bx)\,dx=\frac{1}{7}\,Bb^2x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(2\,Bab+Ab^2)x^6+\frac{1}{5}\,(Ba^2+2\,Aab)x^5$$

input
```
integrate(x^3*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
1/7*B*b^2*x^7 + 1/4*A*a^2*x^4 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/5*(B*a^2 + 2
*A*a*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^3(a+bx)^2(A+Bx)\,dx=\frac{1}{7}\,Bb^2x^7+\frac{1}{3}\,Babx^6+\frac{1}{6}\,Ab^2x^6+\frac{1}{5}\,Ba^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{4}\,Aa^2x^4$$

input
```
integrate(x^3*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
1/7*B*b^2*x^7 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/5*B*a^2*x^5 + 2/5*A*a*b*
x^5 + 1/4*A*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(a+bx)^2(A+Bx)\,dx = x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right) + x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right)+\frac{A\,a^2\,x^4}{4}+\frac{B\,b^2\,x^7}{7}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/6 + (B*a*b)/3) + (A*a^2*x^4)/
4 + (B*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x^3(a+bx)^2(A+Bx)\,dx = \frac{x^4(20b^3x^3+70a\,b^2x^2+84a^2bx+35a^3)}{140}$$

input
```
int(x^3*(b*x+a)^2*(B*x+A),x)
```

output
```
(x**4*(35*a**3 + 84*a**2*b*x + 70*a*b**2*x**2 + 20*b**3*x**3))/140
```

3.62 $\int x^2(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int x^2(a+bx)^2(A+Bx)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{6}b^2Bx^6$$

output
```
1/3*a^2*A*x^3+1/4*a*(2*A*b+B*a)*x^4+1/5*b*(A*b+2*B*a)*x^5+1/6*b^2*B*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x^2(a+bx)^2(A+Bx)\,dx=\frac{1}{60}x^3\left(5a^2(4A+3Bx)+6abx(5A+4Bx)+2b^2x^2(6A+5Bx)\right)$$

input
```
Integrate[x^2*(a + b*x)^2*(A + B*x),x]
```

output
```
(x^3*(5*a^2*(4*A + 3*B*x) + 6*a*b*x*(5*A + 4*B*x) + 2*b^2*x^2*(6*A + 5*B*x
)))/60
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(a^2Ax^2+bx^4(2aB+Ab)+ax^3(aB+2Ab)+b^2Bx^5\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

input
```
Int[x^2*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B
*x^6)/6
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B x^6}{6}+\frac{(b^2 A+2abB)x^5}{5}+\frac{(2abA+a^2 B)x^4}{4}+\frac{a^2 A x^3}{3}$	52
norman	$\frac{b^2 B x^6}{6}+\left(\frac{1}{5}b^2 A+\frac{2}{5}abB\right)x^5+\left(\frac{1}{2}abA+\frac{1}{4}a^2 B\right)x^4+\frac{a^2 A x^3}{3}$	52
orering	$\frac{x^3(10B b^2 x^3+12A b^2 x^2+24Bab x^2+30aAbx+15B a^2 x+20a^2 A)}{60}$	52
gosper	$\frac{1}{6}b^2 B x^6+\frac{1}{5}x^5 b^2 A+\frac{2}{5}x^5 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{3}a^2 A x^3$	54
risch	$\frac{1}{6}b^2 B x^6+\frac{1}{5}x^5 b^2 A+\frac{2}{5}x^5 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{3}a^2 A x^3$	54
parallelrisch	$\frac{1}{6}b^2 B x^6+\frac{1}{5}x^5 b^2 A+\frac{2}{5}x^5 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{3}a^2 A x^3$	54

input
```
int(x^2*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*B*x^6+1/5*(A*b^2+2*B*a*b)*x^5+1/4*(2*A*a*b+B*a^2)*x^4+1/3*a^2*A*x^
3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx)^2(A+Bx)\,dx=\frac{1}{6}\,Bb^2x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\left(2\,Bab+Ab^2\right)x^5+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4$$

input
```
integrate(x^2*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
1/6*B*b^2*x^6 + 1/3*A*a^2*x^3 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)^2(A+Bx)\,dx = \frac{Aa^2x^3}{3} + \frac{Bb^2x^6}{6} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

input
```
integrate(x**2*(b*x+a)**2*(B*x+A),x)
```

output
```
A*a**2*x**3/3 + B*b**2*x**6/6 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**4*(A*a*b/
2 + B*a**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx)^2(A+Bx)\,dx = \frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\,Aa^2x^3 + \frac{1}{5}\,(2\,Bab + Ab^2)x^5 + \frac{1}{4}\,(Ba^2 + 2\,Aab)x^4$$

input
```
integrate(x^2*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
1/6*B*b^2*x^6 + 1/3*A*a^2*x^3 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^2(a+bx)^2(A+Bx)\,dx = \frac{1}{6}\,Bb^2x^6 + \frac{2}{5}\,Babx^5 + \frac{1}{5}\,Ab^2x^5 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{2}\,Aabx^4 + \frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
1/6*B*b^2*x^6 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/4*B*a^2*x^4 + 1/2*A*a*b*
x^4 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx)^2(A+Bx)\,dx = x^4\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^5\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^6}{6}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^4*((B*a^2)/4 + (A*a*b)/2) + x^5*((A*b^2)/5 + (2*B*a*b)/5) + (A*a^2*x^3)/
3 + (B*b^2*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x^2(a+bx)^2(A+Bx)\,dx = \frac{x^3(10b^3x^3+36a\,b^2x^2+45a^2bx+20a^3)}{60}$$

input
```
int(x^2*(b*x+a)^2*(B*x+A),x)
```

output
```
(x**3*(20*a**3 + 45*a**2*b*x + 36*a*b**2*x**2 + 10*b**3*x**3))/60
```

3.63 $\int x(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int x(a+bx)^2(A+Bx)\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{5}b^2Bx^5$$

output
```
1/2*a^2*A*x^2+1/3*a*(2*A*b+B*a)*x^3+1/4*b*(A*b+2*B*a)*x^4+1/5*b^2*B*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x(a+bx)^2(A+Bx)\,dx=\frac{1}{60}x^2\left(10a^2(3A+2Bx)+10abx(4A+3Bx)+3b^2x^2(5A+4Bx)\right)$$

input
```
Integrate[x*(a + b*x)^2*(A + B*x),x]
```

output
```
(x^2*(10*a^2*(3*A + 2*B*x) + 10*a*b*x*(4*A + 3*B*x) + 3*b^2*x^2*(5*A + 4*B
*x)))/60
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2Ax+bx^3(2aB+Ab)+ax^2(aB+2Ab)+b^2Bx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{1}{4}bx^4(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

input
```
Int[x*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B
*x^5)/5
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B\,x^5}{5}+\frac{(b^2 A+2abB)x^4}{4}+\frac{(2abA+a^2 B)x^3}{3}+\frac{a^2 A\,x^2}{2}$	52
norman	$\frac{b^2 B\,x^5}{5}+\left(\frac{1}{4}b^2 A+\frac{1}{2}abB\right)x^4+\left(\frac{2}{3}abA+\frac{1}{3}a^2 B\right)x^3+\frac{a^2 A\,x^2}{2}$	52
orering	$\frac{x^2(12B\,b^2x^3+15A\,b^2x^2+30Bab\,x^2+40aAbx+20B\,a^2x+30a^2A)}{60}$	52
gosper	$\frac{1}{5}b^2B\,x^5+\frac{1}{4}x^4b^2A+\frac{1}{2}x^4abB+\frac{2}{3}x^3abA+\frac{1}{3}x^3a^2B+\frac{1}{2}a^2A\,x^2$	54
risch	$\frac{1}{5}b^2B\,x^5+\frac{1}{4}x^4b^2A+\frac{1}{2}x^4abB+\frac{2}{3}x^3abA+\frac{1}{3}x^3a^2B+\frac{1}{2}a^2A\,x^2$	54
parallelrisch	$\frac{1}{5}b^2B\,x^5+\frac{1}{4}x^4b^2A+\frac{1}{2}x^4abB+\frac{2}{3}x^3abA+\frac{1}{3}x^3a^2B+\frac{1}{2}a^2A\,x^2$	54

input
```
int(x*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*B*x^5+1/4*(A*b^2+2*B*a*b)*x^4+1/3*(2*A*a*b+B*a^2)*x^3+1/2*a^2*A*x^
2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx)^2(A+Bx)\,dx=\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(2\,Bab+Ab^2)x^4+\frac{1}{3}\,(Ba^2+2\,Aab)x^3$$

input
```
integrate(x*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
1/5*B*b^2*x^5 + 1/2*A*a^2*x^2 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/3*(B*a^2 + 2
*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x(a+bx)^2(A+Bx)\,dx=\frac{Aa^2x^2}{2}+\frac{Bb^2x^5}{5}+x^4\left(\frac{Ab^2}{4}+\frac{Bab}{2}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate(x*(b*x+a)**2*(B*x+A),x)
```

output
```
A*a**2*x**2/2 + B*b**2*x**5/5 + x**4*(A*b**2/4 + B*a*b/2) + x**3*(2*A*a*b/
3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx)^2(A+Bx)\,dx=\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(2\,Bab+Ab^2)x^4+\frac{1}{3}\,(Ba^2+2\,Aab)x^3$$

input
```
integrate(x*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
1/5*B*b^2*x^5 + 1/2*A*a^2*x^2 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/3*(B*a^2 + 2
*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(a+bx)^2(A+Bx)\,dx=\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\,Babx^4+\frac{1}{4}\,Ab^2x^4+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
1/5*B*b^2*x^5 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/3*B*a^2*x^3 + 2/3*A*a*b*
x^3 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx)^2(A+Bx)\,dx = x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^5}{5}$$

input
```
int(x*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^4*((A*b^2)/4 + (B*a*b)/2) + (A*a^2*x^2)/
2 + (B*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int x(a+bx)^2(A+Bx)\,dx = \frac{x^2(4b^3x^3+15a\,b^2x^2+20a^2bx+10a^3)}{20}$$

input
```
int(x*(b*x+a)^2*(B*x+A),x)
```

output
```
(x**2*(10*a**3 + 20*a**2*b*x + 15*a*b**2*x**2 + 4*b**3*x**3))/20
```

3.64 $\int (a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int (a+bx)^2(A+Bx)\,dx=\frac{(Ab-aB)(a+bx)^3}{3b^2}+\frac{B(a+bx)^4}{4b^2}$$

output
```
1/3*(A*b-B*a)*(b*x+a)^3/b^2+1/4*B*(b*x+a)^4/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (a+bx)^2(A+Bx)\,dx=\frac{1}{12}x\left(6a^2(2A+Bx)+4abx(3A+2Bx)+b^2x^2(4A+3Bx)\right)$$

input
```
Integrate[(a + b*x)^2*(A + B*x),x]
```

output
```
(x*(6*a^2*(2*A + B*x) + 4*a*b*x*(3*A + 2*B*x) + b^2*x^2*(4*A + 3*B*x)))/12
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^2(Ab-aB)}{b}+\frac{B(a+bx)^3}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^3(Ab-aB)}{3b^2}+\frac{B(a+bx)^4}{4b^2}$$

input
```
Int[(a + b*x)^2*(A + B*x),x]
```

output
```
((A*b - a*B)*(a + b*x)^3)/(3*b^2) + (B*(a + b*x)^4)/(4*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{b^2 B x^4}{4}+\left(\frac{1}{3}b^2 A+\frac{2}{3}abB\right)x^3+\left(abA+\frac{1}{2}a^2 B\right)x^2+a^2 A x$	48
default	$\frac{b^2 B x^4}{4}+\frac{(b^2 A+2abB)x^3}{3}+\frac{(2abA+a^2 B)x^2}{2}+a^2 A x$	49
gosper	$\frac{1}{4}b^2 B x^4+\frac{1}{3}x^3 b^2 A+\frac{2}{3}x^3 abB+x^2 abA+\frac{1}{2}x^2 a^2 B+a^2 A x$	50
risch	$\frac{1}{4}b^2 B x^4+\frac{1}{3}x^3 b^2 A+\frac{2}{3}x^3 abB+x^2 abA+\frac{1}{2}x^2 a^2 B+a^2 A x$	50
parallelrisch	$\frac{1}{4}b^2 B x^4+\frac{1}{3}x^3 b^2 A+\frac{2}{3}x^3 abB+x^2 abA+\frac{1}{2}x^2 a^2 B+a^2 A x$	50
orering	$\frac{x(3B b^2 x^3+4A b^2 x^2+8Bab x^2+12aAbx+6B a^2 x+12a^2 A)}{12}$	50

input
```
int((b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*B*x^4+(1/3*b^2*A+2/3*a*b*B)*x^3+(a*b*A+1/2*a^2*B)*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(A+Bx)\,dx=\frac{1}{4}\,Bb^2x^4+Aa^2x+\frac{1}{3}\,(2\,Bab+Ab^2)x^3+\frac{1}{2}\,(Ba^2+2\,Aab)x^2$$

input
```
integrate((b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
1/4*B*b^2*x^4 + A*a^2*x + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/2*(B*a^2 + 2*A*a*b
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+bx)^2(A+Bx)\,dx = Aa^2x + \frac{Bb^2x^4}{4} + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

input
```
integrate((b*x+a)**2*(B*x+A),x)
```

output
```
A*a**2*x + B*b**2*x**4/4 + x**3*(A*b**2/3 + 2*B*a*b/3) + x**2*(A*a*b + B*a
**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(A+Bx)\,dx = \frac{1}{4}\,Bb^2x^4 + Aa^2x + \frac{1}{3}\,(2\,Bab + Ab^2)x^3 + \frac{1}{2}\,(Ba^2 + 2\,Aab)x^2$$

input
```
integrate((b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
1/4*B*b^2*x^4 + A*a^2*x + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/2*(B*a^2 + 2*A*a*b
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+bx)^2(A+Bx)\,dx = \frac{1}{4}\,Bb^2x^4 + \frac{2}{3}\,Babx^3 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{2}\,Ba^2x^2 + Aabx^2 + Aa^2x$$

input
```
integrate((b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
1/4*B*b^2*x^4 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + 1/2*B*a^2*x^2 + A*a*b*x^2
+ A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (a+bx)^2(A+Bx)\,dx = x^2\left(\frac{B\,a^2}{2}+A\,b\,a\right)+x^3\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{B\,b^2\,x^4}{4}+A\,a^2\,x$$

input
```
int((A + B*x)*(a + b*x)^2,x)
```

output
```
x^2*((B*a^2)/2 + A*a*b) + x^3*((A*b^2)/3 + (2*B*a*b)/3) + (B*b^2*x^4)/4 +
A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int (a+bx)^2(A+Bx)\,dx = \frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$$

input
```
int((b*x+a)^2*(B*x+A),x)
```

output
```
(x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/4
```

3.65 $\int \frac{(a+bx)^2(A+Bx)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{(a+bx)^2(A+Bx)}{x} \, dx = 2aAbx + \frac{1}{2}Ab^2x^2 + \frac{B(a+bx)^3}{3b} + a^2A\log(x)$$

output
```
2*a*A*b*x+1/2*A*b^2*x^2+1/3*B*(b*x+a)^3/b+a^2*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2(A+Bx)}{x} \, dx = a^2Bx + abx(2A+Bx) + \frac{1}{6}b^2x^2(3A+2Bx) + a^2A\log(x)$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x,x]
```

output
```
a^2*B*x + a*b*x*(2*A + B*x) + (b^2*x^2*(3*A + 2*B*x))/6 + a^2*A*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x} \, dx$$

$$\Big\downarrow \text{ 90}$$

$$A\int \frac{(a+bx)^2}{x}dx+\frac{B(a+bx)^3}{3b}$$

$$\Big\downarrow \text{ 49}$$

$$A\int \left(\frac{a^2}{x}+2ba+b^2x\right)dx+\frac{B(a+bx)^3}{3b}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(a^2\log(x)+2abx+\frac{b^2x^2}{2}\right)+\frac{B(a+bx)^3}{3b}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x,x]
```

output
```
(B*(a + b*x)^3)/(3*b) + A*(2*a*b*x + (b^2*x^2)/2 + a^2*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{B\,b^{2}x^{3}}{3}+\frac{A\,b^{2}x^{2}}{2}+Bab\,x^{2}+2aAbx+B\,a^{2}x+a^{2}A\ln\left(x\right)$	46
norman	$\left(\frac{1}{2}b^{2}A+abB\right)x^{2}+\left(2abA+a^{2}B\right)x+\frac{B\,b^{2}x^{3}}{3}+a^{2}A\ln\left(x\right)$	46
risch	$\frac{B\,b^{2}x^{3}}{3}+\frac{A\,b^{2}x^{2}}{2}+Bab\,x^{2}+2aAbx+B\,a^{2}x+a^{2}A\ln\left(x\right)$	46
parallelrisch	$\frac{B\,b^{2}x^{3}}{3}+\frac{A\,b^{2}x^{2}}{2}+Bab\,x^{2}+2aAbx+B\,a^{2}x+a^{2}A\ln\left(x\right)$	46

input
```
int((b*x+a)^2*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*b^2*x^3+1/2*A*b^2*x^2+B*a*b*x^2+2*a*A*b*x+B*a^2*x+a^2*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int\frac{(a+bx)^2(A+Bx)}{x}\,dx=\frac{1}{3}\,Bb^2x^3+Aa^2\log\left(x\right)+\frac{1}{2}\,\left(2\,Bab+Ab^2\right)x^2+\left(Ba^2+2\,Aab\right)x$$

input
```
integrate((b*x+a)^2*(B*x+A)/x,x, algorithm="fricas")
```

output
```
1/3*B*b^2*x^3 + A*a^2*log(x) + 1/2*(2*B*a*b + A*b^2)*x^2 + (B*a^2 + 2*A*a*
b)*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(A+Bx)}{x}\,dx = Aa^2\log{(x)} + \frac{Bb^2x^3}{3} + x^2\left(\frac{Ab^2}{2} + Bab\right) + x\left(2Aab + Ba^2\right)$$

input
```
integrate((b*x+a)**2*(B*x+A)/x,x)
```

output
```
A*a**2*log(x) + B*b**2*x**3/3 + x**2*(A*b**2/2 + B*a*b) + x*(2*A*a*b + B*a
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(A+Bx)}{x}\,dx = \frac{1}{3}\,Bb^2x^3 + Aa^2\log{(x)} + \frac{1}{2}\,\left(2\,Bab + Ab^2\right)x^2 + \left(Ba^2 + 2\,Aab\right)x$$

input
```
integrate((b*x+a)^2*(B*x+A)/x,x, algorithm="maxima")
```

output
```
1/3*B*b^2*x^3 + A*a^2*log(x) + 1/2*(2*B*a*b + A*b^2)*x^2 + (B*a^2 + 2*A*a*
b)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(A+Bx)}{x}\,dx = \frac{1}{3}\,Bb^2x^3 + Babx^2 + \frac{1}{2}\,Ab^2x^2 + Ba^2x + 2\,Aabx + Aa^2\log{(|x|)}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x,x, algorithm="giac")
```

output
```
1/3*B*b^2*x^3 + B*a*b*x^2 + 1/2*A*b^2*x^2 + B*a^2*x + 2*A*a*b*x + A*a^2*lo
g(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^2(A+Bx)}{x}\,dx = x^2\left(\frac{A\,b^2}{2}+B\,a\,b\right)+x\,\left(B\,a^2+2\,A\,b\,a\right)+\frac{B\,b^2\,x^3}{3}+A\,a^2\,\ln\left(x\right)$$

input
```
int(((A + B*x)*(a + b*x)^2)/x,x)
```

output
```
x^2*((A*b^2)/2 + B*a*b) + x*(B*a^2 + 2*A*a*b) + (B*b^2*x^3)/3 + A*a^2*log(
x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^2(A+Bx)}{x}\,dx = \log(x)\,a^3+3a^2bx+\frac{3a\,b^2x^2}{2}+\frac{b^3x^3}{3}$$

input
```
int((b*x+a)^2*(B*x+A)/x,x)
```

output
```
(6*log(x)*a**3 + 18*a**2*b*x + 9*a*b**2*x**2 + 2*b**3*x**3)/6
```

3.66 $\int \frac{(a+bx)^2(A+Bx)}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx)^2(A+Bx)}{x^2} \, dx = -\frac{a^2A}{x} + b(Ab+2aB)x + \frac{1}{2}b^2Bx^2 + a(2Ab+aB)\log(x)$$

output
```
-a^2*A/x+b*(A*b+2*B*a)*x+1/2*b^2*B*x^2+a*(2*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2(A+Bx)}{x^2} \, dx = -\frac{a^2A}{x} + 2abBx + \frac{1}{2}b^2x(2A+Bx) + a(2Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^2,x]
```

output
```
-((a^2*A)/x) + 2*a*b*B*x + (b^2*x*(2*A + B*x))/2 + a*(2*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^2} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^2}+\frac{a(aB+2Ab)}{x}+b(2aB+Ab)+b^2Bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x}+bx(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{2}b^2Bx^2$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^2,x]
```

output
```
-((a^2*A)/x) + b*(A*b + 2*a*B)*x + (b^2*B*x^2)/2 + a*(2*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{b^2 B\,x^2}{2}+A\,b^2 x+2Babx+a(2Ab+Ba)\ln\left(x\right)-\frac{a^2 A}{x}$	44
risch	$\frac{b^2 B\,x^2}{2}+A\,b^2 x+2Babx-\frac{a^2 A}{x}+2A\ln\left(x\right)ab+B\ln\left(x\right)a^2$	46
norman	$\frac{(b^2 A+2abB)x^2-a^2 A+\frac{B\,b^2 x^3}{2}}{x}+\left(2abA+a^2 B\right)\ln\left(x\right)$	51
parallelrisch	$\frac{B\,b^2 x^3+4A\ln(x)xab+2A\,b^2 x^2+2B\ln(x)x\,a^2+4Bab\,x^2-2a^2 A}{2x}$	55

input
```
int((b*x+a)^2*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2*B*x^2+A*b^2*x+2*B*a*b*x+a*(2*A*b+B*a)*ln(x)-a^2*A/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int\frac{(a+bx)^2(A+Bx)}{x^2}\,dx =\frac{Bb^2x^3-2\,Aa^2+2\,(2\,Bab+Ab^2)x^2+2\,(Ba^2+2\,Aab)x\log\left(x\right)}{2\,x}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/2*(B*b^2*x^3 - 2*A*a^2 + 2*(2*B*a*b + A*b^2)*x^2 + 2*(B*a^2 + 2*A*a*b)*x
*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx = -\frac{Aa^2}{x} + \frac{Bb^2x^2}{2} + a(2Ab+Ba)\log(x) + x\left(Ab^2+2Bab\right)$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**2,x)
```

output
```
-A*a**2/x + B*b**2*x**2/2 + a*(2*A*b + B*a)*log(x) + x*(A*b**2 + 2*B*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx = \frac{1}{2}\,Bb^2x^2 - \frac{Aa^2}{x} + \left(2\,Bab+Ab^2\right)x + \left(Ba^2+2\,Aab\right)\log(x)$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/2*B*b^2*x^2 - A*a^2/x + (2*B*a*b + A*b^2)*x + (B*a^2 + 2*A*a*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx = \frac{1}{2}\,Bb^2x^2 + 2\,Babx + Ab^2x - \frac{Aa^2}{x} + \left(Ba^2+2\,Aab\right)\log(|x|)$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/2*B*b^2*x^2 + 2*B*a*b*x + A*b^2*x - A*a^2/x + (B*a^2 + 2*A*a*b)*log(abs(
x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx = \ln(x)\,\left(B\,a^2+2\,A\,b\,a\right)+x\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{A\,a^2}{x}+\frac{B\,b^2\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^2,x)
```

output
```
log(x)*(B*a^2 + 2*A*a*b) + x*(A*b^2 + 2*B*a*b) - (A*a^2)/x + (B*b^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx = \frac{6\log(x)\,a^2bx-2a^3+6a\,b^2x^2+b^3x^3}{2x}$$

input
```
int((b*x+a)^2*(B*x+A)/x^2,x)
```

output
```
(6*log(x)*a**2*b*x - 2*a**3 + 6*a*b**2*x**2 + b**3*x**3)/(2*x)
```

3.67 $\int \frac{(a+bx)^2(A+Bx)}{x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx)^2(A+Bx)}{x^3} \, dx = -\frac{a^2A}{2x^2} - \frac{a(2Ab+aB)}{x} + b^2Bx + b(Ab+2aB)\log(x)$$

output
```
-1/2*a^2*A/x^2-a*(2*A*b+B*a)/x+b^2*B*x+b*(A*b+2*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2(A+Bx)}{x^3} \, dx = -\frac{2aAb}{x} + b^2Bx - \frac{a^2(A+2Bx)}{2x^2} + b(Ab+2aB)\log(x)$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^3,x]
```

output
```
(-2*a*A*b)/x + b^2*B*x - (a^2*(A + 2*B*x))/(2*x^2) + b*(A*b + 2*a*B)*Log[x
]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^3} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^2A}{x^3}+\frac{a(aB+2Ab)}{x^2}+\frac{b(2aB+Ab)}{x}+b^2B\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2A}{2x^2}-\frac{a(aB+2Ab)}{x}+b\log(x)(2aB+Ab)+b^2Bx$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a*(2*A*b + a*B))/x + b^2*B*x + b*(A*b + 2*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{a^2 A}{2x^2}-\frac{a(2Ab+Ba)}{x}+b^2 Bx+b(Ab+2Ba)\ln(x)$	43
risch	$b^2 Bx+\frac{(-2abA-a^2 B)x-\frac{a^2 A}{2}}{x^2}+A\ln(x)\,b^2+2B\ln(x)\,ab$	47
norman	$\frac{(-2abA-a^2 B)x+B\,b^2 x^3-\frac{a^2 A}{2}}{x^2}+(b^2 A+2abB)\ln(x)$	49
parallelrisch	$\frac{2A\ln(x)x^2 b^2+4B\ln(x)x^2 ab+2B\,b^2 x^3-4aAbx-2B\,a^2 x-a^2 A}{2x^2}$	56

input
```
int((b*x+a)^2*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*A/x^2-a*(2*A*b+B*a)/x+b^2*B*x+b*(A*b+2*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx = \frac{2\,Bb^2x^3+2\,(2\,Bab+Ab^2)x^2\log(x)-Aa^2-2\,(Ba^2+2\,Aab)x}{2\,x^2}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*b^2*x^3 + 2*(2*B*a*b + A*b^2)*x^2*log(x) - A*a^2 - 2*(B*a^2 + 2*A
*a*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx = Bb^2x + b(Ab+2Ba)\log(x) + \frac{-Aa^2 + x(-4Aab-2Ba^2)}{2x^2}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**3,x)
```

output
```
B*b**2*x + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x*(-4*A*a*b - 2*B*a**2))/(2
*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx = Bb^2x + \left(2\,Bab + Ab^2\right)\log(x) - \frac{Aa^2 + 2\,(Ba^2 + 2\,Aab)x}{2\,x^2}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
B*b^2*x + (2*B*a*b + A*b^2)*log(x) - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*x)/x
^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx = Bb^2x + \left(2\,Bab + Ab^2\right)\log(|x|) - \frac{Aa^2 + 2\,(Ba^2 + 2\,Aab)x}{2\,x^2}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
B*b^2*x + (2*B*a*b + A*b^2)*log(abs(x)) - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)
*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx = \ln(x)\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{\frac{A\,a^2}{2}+x\,\left(B\,a^2+2\,A\,b\,a\right)}{x^2}+B\,b^2\,x$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^3,x)
```

output
```
log(x)*(A*b^2 + 2*B*a*b) - ((A*a^2)/2 + x*(B*a^2 + 2*A*a*b))/x^2 + B*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx = \frac{6\log(x)\,a\,b^2x^2-a^3-6a^2bx+2b^3x^3}{2x^2}$$

input
```
int((b*x+a)^2*(B*x+A)/x^3,x)
```

output
```
(6*log(x)*a*b**2*x**2 - a**3 - 6*a**2*b*x + 2*b**3*x**3)/(2*x**2)
```

3.68 $\int \frac{(a+bx)^2(A+Bx)}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{(a+bx)^2(A+Bx)}{x^4} dx = -\frac{a^2A}{3x^3} - \frac{a(2Ab+aB)}{2x^2} - \frac{b(Ab+2aB)}{x} + b^2B\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a*(2*A*b+B*a)/x^2-b*(A*b+2*B*a)/x+b^2*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2(A+Bx)}{x^4} dx = -\frac{6Ab^2x^2 + 6abx(A+2Bx) + a^2(2A+3Bx)}{6x^3} + b^2B\log(x)$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^4,x]
```

output
```
-1/6*(6*A*b^2*x^2 + 6*a*b*x*(A + 2*B*x) + a^2*(2*A + 3*B*x))/x^3 + b^2*B*L
og[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^2 A}{x^4}+\frac{a(aB+2Ab)}{x^3}+\frac{b(2aB+Ab)}{x^2}+\frac{b^2 B}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2 A}{3x^3}-\frac{a(aB+2Ab)}{2x^2}-\frac{b(2aB+Ab)}{x}+b^2 B\log(x)$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/(2*x^2) - (b*(A*b + 2*a*B))/x + b^2*B
*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{2} A}{3 x^{3}}-\frac{a (2 A b+B a)}{2 x^{2}}-\frac{b (A b+2 B a)}{x}+b^{2} B \ln \left(x \right)$	46
norman	$\frac{\left(-a b A-\frac{1}{2} a^{2} B\right) x+\left(-b^{2} A-2 a b B\right) x^{2}-\frac{a^{2} A}{3}}{x^{3}}+b^{2} B \ln \left(x \right)$	50
risch	$\frac{\left(-a b A-\frac{1}{2} a^{2} B\right) x+\left(-b^{2} A-2 a b B\right) x^{2}-\frac{a^{2} A}{3}}{x^{3}}+b^{2} B \ln \left(x \right)$	50
parallelrisch	$-\frac{-6 b^{2} B \ln \left(x \right) x^{3}+6 A \,b^{2} x^{2}+12 B a b \,x^{2}+6 a A b x +3 B \,a^{2} x +2 a^{2} A}{6 x^{3}}$	54

input
```
int((b*x+a)^2*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*A/x^3-1/2*a*(2*A*b+B*a)/x^2-b*(A*b+2*B*a)/x+b^2*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx = \frac{6\,Bb^2x^3\log\left(x\right) - 2\,Aa^2 - 6\,(2\,Bab + Ab^2)x^2 - 3\,(Ba^2 + 2\,Aab)x}{6\,x^3}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*B*b^2*x^3*log(x) - 2*A*a^2 - 6*(2*B*a*b + A*b^2)*x^2 - 3*(B*a^2 + 2
*A*a*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx = Bb^2\log{(x)} + \frac{-2Aa^2 + x^2(-6Ab^2 - 12Bab) + x(-6Aab - 3Ba^2)}{6x^3}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**4,x)
```

output
```
B*b**2*log(x) + (-2*A*a**2 + x**2*(-6*A*b**2 - 12*B*a*b) + x*(-6*A*a*b - 3
*B*a**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx = Bb^2\log{(x)} - \frac{2\,Aa^2 + 6\,(2\,Bab + Ab^2)x^2 + 3\,(Ba^2 + 2\,Aab)x}{6\,x^3}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
B*b^2*log(x) - 1/6*(2*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 3*(B*a^2 + 2*A*a*b
)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx = Bb^2\log{(|x|)} - \frac{2\,Aa^2 + 6\,(2\,Bab + Ab^2)x^2 + 3\,(Ba^2 + 2\,Aab)x}{6\,x^3}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
B*b^2*log(abs(x)) - 1/6*(2*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 3*(B*a^2 + 2*
A*a*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx = B\,b^2\,\ln(x) - \frac{x^2\,(A\,b^2+2\,B\,a\,b)+\frac{A\,a^2}{3}+x\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)}{x^3}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^4,x)
```

output
```
B*b^2*log(x) - (x^2*(A*b^2 + 2*B*a*b) + (A*a^2)/3 + x*((B*a^2)/2 + A*a*b))
/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx = \frac{6\log(x)\,b^3x^3-2a^3-9a^2bx-18a\,b^2x^2}{6x^3}$$

input
```
int((b*x+a)^2*(B*x+A)/x^4,x)
```

output
```
(6*log(x)*b**3*x**3 - 2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

3.69 $\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx=-\frac{A(a+bx)^3}{4ax^4}+\frac{(Ab-4aB)(a+bx)^3}{12a^2x^3}$$

output
```
-1/4*A*(b*x+a)^3/a/x^4+1/12*(A*b-4*B*a)*(b*x+a)^3/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx=-\frac{6b^2x^2(A+2Bx)+4abx(2A+3Bx)+a^2(3A+4Bx)}{12x^4}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^5,x]
```

output
```
-1/12*(6*b^2*x^2*(A + 2*B*x) + 4*a*b*x*(2*A + 3*B*x) + a^2*(3*A + 4*B*x))/
x^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^5} \, dx$$
$$\Big\downarrow \; 87$$
$$-\frac{(Ab-4aB)\int \frac{(a+bx)^2}{x^4}dx}{4a}-\frac{A(a+bx)^3}{4ax^4}$$
$$\Big\downarrow \; 48$$
$$\frac{(a+bx)^3(Ab-4aB)}{12a^2x^3}-\frac{A(a+bx)^3}{4ax^4}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^5,x]
```

output
```
-1/4*(A*(a + b*x)^3)/(a*x^4) + ((A*b - 4*a*B)*(a + b*x)^3)/(12*a^2*x^3)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{a(2Ab+Ba)}{3x^3}-\frac{b(Ab+2Ba)}{2x^2}-\frac{a^2A}{4x^4}-\frac{b^2B}{x}$	48
norman	$\frac{-B\,b^2x^3+\left(-\frac{1}{2}b^2A-abB\right)x^2+\left(-\frac{2}{3}abA-\frac{1}{3}a^2B\right)x-\frac{a^2A}{4}}{x^4}$	51
risch	$\frac{-B\,b^2x^3+\left(-\frac{1}{2}b^2A-abB\right)x^2+\left(-\frac{2}{3}abA-\frac{1}{3}a^2B\right)x-\frac{a^2A}{4}}{x^4}$	51
gosper	$-\frac{12B\,b^2x^3+6A\,b^2x^2+12Bab\,x^2+8aAbx+4B\,a^2x+3a^2A}{12x^4}$	52
parallelrisch	$-\frac{12B\,b^2x^3+6A\,b^2x^2+12Bab\,x^2+8aAbx+4B\,a^2x+3a^2A}{12x^4}$	52
orering	$-\frac{12B\,b^2x^3+6A\,b^2x^2+12Bab\,x^2+8aAbx+4B\,a^2x+3a^2A}{12x^4}$	52

input
```
int((b*x+a)^2*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*(2*A*b+B*a)/x^3-1/2*b*(A*b+2*B*a)/x^2-1/4*a^2*A/x^4-b^2*B/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx = -\frac{12\,Bb^2x^3+3\,Aa^2+6\,(2\,Bab+Ab^2)x^2+4\,(Ba^2+2\,Aab)x}{12\,x^4}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(12*B*b^2*x^3 + 3*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 4*(B*a^2 + 2*A*a
*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx = \frac{-3Aa^2-12Bb^2x^3+x^2(-6Ab^2-12Bab)+x(-8Aab-4Ba^2)}{12x^4}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**5,x)
```

output
```
(-3*A*a**2 - 12*B*b**2*x**3 + x**2*(-6*A*b**2 - 12*B*a*b) + x*(-8*A*a*b -
4*B*a**2))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx = -\frac{12\,Bb^2x^3+3\,Aa^2+6\,(2\,Bab+Ab^2)x^2+4\,(Ba^2+2\,Aab)x}{12\,x^4}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(12*B*b^2*x^3 + 3*A*a^2 + 6*(2*B*a*b + A*b^2)*x^2 + 4*(B*a^2 + 2*A*a
*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx = -\frac{12\,Bb^2x^3+12\,Babx^2+6\,Ab^2x^2+4\,Ba^2x+8\,Aabx+3\,Aa^2}{12\,x^4}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
-1/12*(12*B*b^2*x^3 + 12*B*a*b*x^2 + 6*A*b^2*x^2 + 4*B*a^2*x + 8*A*a*b*x +
 3*A*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx = -\frac{x^2\left(\frac{A\,b^2}{2}+B\,a\,b\right)+\frac{A\,a^2}{4}+x\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+B\,b^2\,x^3}{x^4}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^5,x)
```

output
```
-(x^2*((A*b^2)/2 + B*a*b) + (A*a^2)/4 + x*((B*a^2)/3 + (2*A*a*b)/3) + B*b^
2*x^3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx = \frac{-4b^3x^3-6a\,b^2x^2-4a^2bx-a^3}{4x^4}$$

input
```
int((b*x+a)^2*(B*x+A)/x^5,x)
```

output
```
( - a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

3.70 $\int \frac{(a+bx)^2(A+Bx)}{x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{(a+bx)^2(A+Bx)}{x^6} \, dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{4x^4} - \frac{b(Ab+2aB)}{3x^3} - \frac{b^2B}{2x^2}$$

output
```
-1/5*a^2*A/x^5-1/4*a*(2*A*b+B*a)/x^4-1/3*b*(A*b+2*B*a)/x^3-1/2*b^2*B/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2(A+Bx)}{x^6} \, dx = -\frac{10b^2x^2(2A+3Bx)+10abx(3A+4Bx)+3a^2(4A+5Bx)}{60x^5}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^6,x]
```

output
```
-1/60*(10*b^2*x^2*(2*A + 3*B*x) + 10*a*b*x*(3*A + 4*B*x) + 3*a^2*(4*A + 5*
B*x))/x^5
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^6} \, dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a(aB+2Ab)}{x^5}+\frac{b(2aB+Ab)}{x^4}+\frac{b^2B}{x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{2x^2}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/(3*x^3) -
 (b^2*B)/(2*x^2)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^2 A}{5x^5}-\frac{a(2Ab+Ba)}{4x^4}-\frac{b(Ab+2Ba)}{3x^3}-\frac{b^2 B}{2x^2}$	48
norman	$\frac{-\frac{B b^2 x^3}{2}+\left(-\frac{1}{3}b^2 A-\frac{2}{3}abB\right)x^2+\left(-\frac{1}{2}abA-\frac{1}{4}a^2 B\right)x-\frac{a^2 A}{5}}{x^5}$	51
risch	$\frac{-\frac{B b^2 x^3}{2}+\left(-\frac{1}{3}b^2 A-\frac{2}{3}abB\right)x^2+\left(-\frac{1}{2}abA-\frac{1}{4}a^2 B\right)x-\frac{a^2 A}{5}}{x^5}$	51
gosper	$-\frac{30B b^2 x^3+20A b^2 x^2+40Bab x^2+30aAbx+15B a^2 x+12a^2 A}{60x^5}$	52
parallelrisch	$-\frac{30B b^2 x^3+20A b^2 x^2+40Bab x^2+30aAbx+15B a^2 x+12a^2 A}{60x^5}$	52
orering	$-\frac{30B b^2 x^3+20A b^2 x^2+40Bab x^2+30aAbx+15B a^2 x+12a^2 A}{60x^5}$	52

input
```
int((b*x+a)^2*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2*A/x^5-1/4*a*(2*A*b+B*a)/x^4-1/3*b*(A*b+2*B*a)/x^3-1/2*b^2*B/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}\,dx = -\frac{30\,Bb^2x^3+12\,Aa^2+20\,(2\,Bab+Ab^2)x^2+15\,(Ba^2+2\,Aab)x}{60\,x^5}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(30*B*b^2*x^3 + 12*A*a^2 + 20*(2*B*a*b + A*b^2)*x^2 + 15*(B*a^2 + 2*
A*a*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}\,dx = \frac{-12Aa^2 - 30Bb^2x^3 + x^2(-20Ab^2 - 40Bab) + x(-30Aab - 15Ba^2)}{60x^5}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**6,x)
```

output
```
(-12*A*a**2 - 30*B*b**2*x**3 + x**2*(-20*A*b**2 - 40*B*a*b) + x*(-30*A*a*b
 - 15*B*a**2))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}\,dx = -\frac{30\,Bb^2x^3 + 12\,Aa^2 + 20\,(2\,Bab + Ab^2)x^2 + 15\,(Ba^2 + 2\,Aab)x}{60\,x^5}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/60*(30*B*b^2*x^3 + 12*A*a^2 + 20*(2*B*a*b + A*b^2)*x^2 + 15*(B*a^2 + 2*
A*a*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}\,dx = -\frac{30\,Bb^2x^3+40\,Babx^2+20\,Ab^2x^2+15\,Ba^2x+30\,Aabx+12\,Aa^2}{60\,x^5}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
-1/60*(30*B*b^2*x^3 + 40*B*a*b*x^2 + 20*A*b^2*x^2 + 15*B*a^2*x + 30*A*a*b*
x + 12*A*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}\,dx = -\frac{x^2\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{A\,a^2}{5}+x\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+\frac{B\,b^2\,x^3}{2}}{x^5}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^6,x)
```

output
```
-(x^2*((A*b^2)/3 + (2*B*a*b)/3) + (A*a^2)/5 + x*((B*a^2)/4 + (A*a*b)/2) +
(B*b^2*x^3)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2(A+Bx)}{x^6}\,dx = \frac{-10b^3x^3-20a\,b^2x^2-15a^2bx-4a^3}{20x^5}$$

input
```
int((b*x+a)^2*(B*x+A)/x^6,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

3.71 $\int \frac{(a+bx)^2(A+Bx)}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{(a+bx)^2(A+Bx)}{x^7} dx = -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{4x^4} - \frac{b^2B}{3x^3}$$

output
```
-1/6*a^2*A/x^6-1/5*a*(2*A*b+B*a)/x^5-1/4*b*(A*b+2*B*a)/x^4-1/3*b^2*B/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2(A+Bx)}{x^7} dx = -\frac{5b^2x^2(3A+4Bx)+6abx(4A+5Bx)+2a^2(5A+6Bx)}{60x^6}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^7,x]
```

output
```
-1/60*(5*b^2*x^2*(3*A + 4*B*x) + 6*a*b*x*(4*A + 5*B*x) + 2*a^2*(5*A + 6*B*
x))/x^6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^7} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^7}+\frac{a(aB+2Ab)}{x^6}+\frac{b(2aB+Ab)}{x^5}+\frac{b^2B}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{4x^4}-\frac{b^2B}{3x^3}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^7,x]
```

output
```
-1/6*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(4*x^4) -
 (b^2*B)/(3*x^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{2} A}{6 x^{6}}-\frac{a(2 A b+B a)}{5 x^{5}}-\frac{b(A b+2 B a)}{4 x^{4}}-\frac{b^{2} B}{3 x^{3}}$	48
norman	$\frac{-\frac{B\,b^{2} x^{3}}{3}+\left(-\frac{1}{4} b^{2} A-\frac{1}{2} a b B\right) x^{2}+\left(-\frac{2}{5} a b A-\frac{1}{5} a^{2} B\right) x-\frac{a^{2} A}{6}}{x^{6}}$	51
risch	$\frac{-\frac{B\,b^{2} x^{3}}{3}+\left(-\frac{1}{4} b^{2} A-\frac{1}{2} a b B\right) x^{2}+\left(-\frac{2}{5} a b A-\frac{1}{5} a^{2} B\right) x-\frac{a^{2} A}{6}}{x^{6}}$	51
gosper	$-\frac{20 B\,b^{2} x^{3}+15 A\,b^{2} x^{2}+30 B a b\,x^{2}+24 a A b x+12 B\,a^{2} x+10 a^{2} A}{60 x^{6}}$	52
parallelrisch	$-\frac{20 B\,b^{2} x^{3}+15 A\,b^{2} x^{2}+30 B a b\,x^{2}+24 a A b x+12 B\,a^{2} x+10 a^{2} A}{60 x^{6}}$	52
orering	$-\frac{20 B\,b^{2} x^{3}+15 A\,b^{2} x^{2}+30 B a b\,x^{2}+24 a A b x+12 B\,a^{2} x+10 a^{2} A}{60 x^{6}}$	52

input
```
int((b*x+a)^2*(B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*A/x^6-1/5*a*(2*A*b+B*a)/x^5-1/4*b*(A*b+2*B*a)/x^4-1/3*b^2*B/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^7}\,dx = -\frac{20\,Bb^2x^3+10\,Aa^2+15\,(2\,Bab+Ab^2)x^2+12\,(Ba^2+2\,Aab)x}{60\,x^6}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(20*B*b^2*x^3 + 10*A*a^2 + 15*(2*B*a*b + A*b^2)*x^2 + 12*(B*a^2 + 2*
A*a*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^2(A+Bx)}{x^7}\,dx = \frac{-10Aa^2-20Bb^2x^3+x^2(-15Ab^2-30Bab)+x(-24Aab-12Ba^2)}{60x^6}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**7,x)
```

output
```
(-10*A*a**2 - 20*B*b**2*x**3 + x**2*(-15*A*b**2 - 30*B*a*b) + x*(-24*A*a*b
 - 12*B*a**2))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^7}\,dx = -\frac{20\,Bb^2x^3+10\,Aa^2+15\,(2\,Bab+Ab^2)x^2+12\,(Ba^2+2\,Aab)x}{60\,x^6}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^7,x, algorithm="maxima")
```

output
```
-1/60*(20*B*b^2*x^3 + 10*A*a^2 + 15*(2*B*a*b + A*b^2)*x^2 + 12*(B*a^2 + 2*
A*a*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^7} \, dx = -\frac{20\,Bb^2x^3 + 30\,Babx^2 + 15\,Ab^2x^2 + 12\,Ba^2x + 24\,Aabx + 10\,Aa^2}{60\,x^6}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^7,x, algorithm="giac")
```

output
```
-1/60*(20*B*b^2*x^3 + 30*B*a*b*x^2 + 15*A*b^2*x^2 + 12*B*a^2*x + 24*A*a*b*
x + 10*A*a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^7} \, dx = -\frac{x^2\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2}{6}+x\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+\frac{B\,b^2\,x^3}{3}}{x^6}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^7,x)
```

output
```
-(x^2*((A*b^2)/4 + (B*a*b)/2) + (A*a^2)/6 + x*((B*a^2)/5 + (2*A*a*b)/5) +
(B*b^2*x^3)/3)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2(A+Bx)}{x^7} \, dx = \frac{-20b^3x^3 - 45a\,b^2x^2 - 36a^2bx - 10a^3}{60x^6}$$

input
```
int((b*x+a)^2*(B*x+A)/x^7,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

3.72 $\int \frac{(a+bx)^2(A+Bx)}{x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{(a+bx)^2(A+Bx)}{x^8} dx = -\frac{a^2A}{7x^7} - \frac{a(2Ab+aB)}{6x^6} - \frac{b(Ab+2aB)}{5x^5} - \frac{b^2B}{4x^4}$$

output
```
-1/7*a^2*A/x^7-1/6*a*(2*A*b+B*a)/x^6-1/5*b*(A*b+2*B*a)/x^5-1/4*b^2*B/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^2(A+Bx)}{x^8} dx = -\frac{21b^2x^2(4A+5Bx)+28abx(5A+6Bx)+10a^2(6A+7Bx)}{420x^7}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^8,x]
```

output
```
-1/420*(21*b^2*x^2*(4*A + 5*B*x) + 28*a*b*x*(5*A + 6*B*x) + 10*a^2*(6*A + 
7*B*x))/x^7
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^2A}{x^8}+\frac{a(aB+2Ab)}{x^7}+\frac{b(2aB+Ab)}{x^6}+\frac{b^2B}{x^5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{6x^6}-\frac{b(2aB+Ab)}{5x^5}-\frac{b^2B}{4x^4}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^8,x]
```

output
```
-1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(6*x^6) - (b*(A*b + 2*a*B))/(5*x^5) -
 (b^2*B)/(4*x^4)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^2A}{7x^7}-\frac{a(2Ab+Ba)}{6x^6}-\frac{b(Ab+2Ba)}{5x^5}-\frac{b^2B}{4x^4}$	48
norman	$\frac{-\frac{B\,b^2x^3}{4}+\left(-\frac{1}{5}b^2A-\frac{2}{5}abB\right)x^2+\left(-\frac{1}{3}abA-\frac{1}{6}a^2B\right)x-\frac{a^2A}{7}}{x^7}$	51
risch	$\frac{-\frac{B\,b^2x^3}{4}+\left(-\frac{1}{5}b^2A-\frac{2}{5}abB\right)x^2+\left(-\frac{1}{3}abA-\frac{1}{6}a^2B\right)x-\frac{a^2A}{7}}{x^7}$	51
gosper	$-\frac{105B\,b^2x^3+84A\,b^2x^2+168Bab\,x^2+140aAbx+70B\,a^2x+60a^2A}{420x^7}$	52
parallelrisch	$-\frac{105B\,b^2x^3+84A\,b^2x^2+168Bab\,x^2+140aAbx+70B\,a^2x+60a^2A}{420x^7}$	52
orering	$-\frac{105B\,b^2x^3+84A\,b^2x^2+168Bab\,x^2+140aAbx+70B\,a^2x+60a^2A}{420x^7}$	52

input
```
int((b*x+a)^2*(B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2*A/x^7-1/6*a*(2*A*b+B*a)/x^6-1/5*b*(A*b+2*B*a)/x^5-1/4*b^2*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx = -\frac{105\,Bb^2x^3+60\,Aa^2+84\,(2\,Bab+Ab^2)x^2+70\,(Ba^2+2\,Aab)x}{420\,x^7}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^8,x, algorithm="fricas")
```

output
```
-1/420*(105*B*b^2*x^3 + 60*A*a^2 + 84*(2*B*a*b + A*b^2)*x^2 + 70*(B*a^2 +
2*A*a*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx = \frac{-60Aa^2-105Bb^2x^3+x^2(-84Ab^2-168Bab)+x(-140Aab-70Ba^2)}{420x^7}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**8,x)
```

output
```
(-60*A*a**2 - 105*B*b**2*x**3 + x**2*(-84*A*b**2 - 168*B*a*b) + x*(-140*A*
a*b - 70*B*a**2))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx = -\frac{105\,Bb^2x^3+60\,Aa^2+84\,(2\,Bab+Ab^2)x^2+70\,(Ba^2+2\,Aab)x}{420\,x^7}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^8,x, algorithm="maxima")
```

output
```
-1/420*(105*B*b^2*x^3 + 60*A*a^2 + 84*(2*B*a*b + A*b^2)*x^2 + 70*(B*a^2 +
2*A*a*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx = -\frac{105\,Bb^2x^3+168\,Babx^2+84\,Ab^2x^2+70\,Ba^2x+140\,Aabx+60\,Aa^2}{420\,x^7}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^8,x, algorithm="giac")
```

output
```
-1/420*(105*B*b^2*x^3 + 168*B*a*b*x^2 + 84*A*b^2*x^2 + 70*B*a^2*x + 140*A*
a*b*x + 60*A*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx = -\frac{x^2\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+\frac{A\,a^2}{7}+x\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+\frac{B\,b^2\,x^3}{4}}{x^7}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^8,x)
```

output
```
-(x^2*((A*b^2)/5 + (2*B*a*b)/5) + (A*a^2)/7 + x*((B*a^2)/6 + (A*a*b)/3) +
(B*b^2*x^3)/4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2(A+Bx)}{x^8}\,dx = \frac{-35b^3x^3-84a\,b^2x^2-70a^2bx-20a^3}{140x^7}$$

input
```
int((b*x+a)^2*(B*x+A)/x^8,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

3.73 $\int x^4(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x^4(a+bx)^3(A+Bx)\,dx=\frac{1}{5}a^3Ax^5+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{7}ab(Ab+aB)x^7+\frac{1}{8}b^2(Ab+3aB)x^8+\frac{1}{9}b^3Bx^9$$

output
```
1/5*a^3*A*x^5+1/6*a^2*(3*A*b+B*a)*x^6+3/7*a*b*(A*b+B*a)*x^7+1/8*b^2*(A*b+3
*B*a)*x^8+1/9*b^3*B*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)^3(A+Bx)\,dx=\frac{1}{5}a^3Ax^5+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{7}ab(Ab+aB)x^7+\frac{1}{8}b^2(Ab+3aB)x^8+\frac{1}{9}b^3Bx^9$$

input
```
Integrate[x^4*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*x^5)/5 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*b*(A*b + a*B)*x^7)/7 + (b
^2*(A*b + 3*a*B)*x^8)/8 + (b^3*B*x^9)/9
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(a^3Ax^4+a^2x^5(aB+3Ab)+b^2x^7(3aB+Ab)+3abx^6(aB+Ab)+b^3Bx^8\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}a^3Ax^5+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{8}b^2x^8(3aB+Ab)+\frac{3}{7}abx^7(aB+Ab)+\frac{1}{9}b^3Bx^9$$

input
```
Int[x^4*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*x^5)/5 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*b*(A*b + a*B)*x^7)/7 + (b
^2*(A*b + 3*a*B)*x^8)/8 + (b^3*B*x^9)/9
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	s
norman	$\frac{b^3 B x^9}{9}+\left(\frac{1}{8} b^3 A+\frac{3}{8} a b^2 B\right) x^8+\left(\frac{3}{7} a b^2 A+\frac{3}{7} a^2 b B\right) x^7+\left(\frac{1}{2} a^2 b A+\frac{1}{6} a^3 B\right) x^6+\frac{a^3 A x^5}{5}$	7
default	$\frac{b^3 B x^9}{9}+\frac{\left(b^3 A+3 a b^2 B\right) x^8}{8}+\frac{\left(3 a b^2 A+3 a^2 b B\right) x^7}{7}+\frac{\left(3 a^2 b A+a^3 B\right) x^6}{6}+\frac{a^3 A x^5}{5}$	7
orering	$\frac{x^5\left(280 B b^3 x^4+315 A b^3 x^3+945 B a b^2 x^3+1080 a A b^2 x^2+1080 B a^2 b x^2+1260 a^2 A b x+420 B a^3 x+504 a^3 A\right)}{2520}$	7
gosper	$\frac{1}{9} b^3 B x^9+\frac{1}{8} x^8 b^3 A+\frac{3}{8} x^8 a b^2 B+\frac{3}{7} x^7 a b^2 A+\frac{3}{7} x^7 a^2 b B+\frac{1}{2} x^6 a^2 b A+\frac{1}{6} x^6 a^3 B+\frac{1}{5} a^3 A x^5$	7
risch	$\frac{1}{9} b^3 B x^9+\frac{1}{8} x^8 b^3 A+\frac{3}{8} x^8 a b^2 B+\frac{3}{7} x^7 a b^2 A+\frac{3}{7} x^7 a^2 b B+\frac{1}{2} x^6 a^2 b A+\frac{1}{6} x^6 a^3 B+\frac{1}{5} a^3 A x^5$	7
parallelrisch	$\frac{1}{9} b^3 B x^9+\frac{1}{8} x^8 b^3 A+\frac{3}{8} x^8 a b^2 B+\frac{3}{7} x^7 a b^2 A+\frac{3}{7} x^7 a^2 b B+\frac{1}{2} x^6 a^2 b A+\frac{1}{6} x^6 a^3 B+\frac{1}{5} a^3 A x^5$	7

input
```
int(x^4*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^3*B*x^9+(1/8*b^3*A+3/8*a*b^2*B)*x^8+(3/7*a*b^2*A+3/7*a^2*b*B)*x^7+(1
/2*a^2*b*A+1/6*a^3*B)*x^6+1/5*a^3*A*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^4(a+bx)^3(A+Bx)\,dx = \frac{1}{9}\,Bb^3x^9+\frac{1}{5}\,Aa^3x^5+\frac{1}{8}\,\left(3\,Bab^2+Ab^3\right)x^8 + \frac{3}{7}\,\left(Ba^2b+Aab^2\right)x^7+\frac{1}{6}\,\left(Ba^3+3\,Aa^2b\right)x^6$$

input
```
integrate(x^4*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
1/9*B*b^3*x^9 + 1/5*A*a^3*x^5 + 1/8*(3*B*a*b^2 + A*b^3)*x^8 + 3/7*(B*a^2*b
 + A*a*b^2)*x^7 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int x^4(a+bx)^3(A+Bx)\,dx = \frac{Aa^3x^5}{5} + \frac{Bb^3x^9}{9} + x^8\left(\frac{Ab^3}{8} + \frac{3Bab^2}{8}\right) + x^7 \cdot \left(\frac{3Aab^2}{7} + \frac{3Ba^2b}{7}\right) + x^6\left(\frac{Aa^2b}{2} + \frac{Ba^3}{6}\right)$$

input
```
integrate(x**4*(b*x+a)**3*(B*x+A),x)
```

output
```
A*a**3*x**5/5 + B*b**3*x**9/9 + x**8*(A*b**3/8 + 3*B*a*b**2/8) + x**7*(3*A
*a*b**2/7 + 3*B*a**2*b/7) + x**6*(A*a**2*b/2 + B*a**3/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^4(a+bx)^3(A+Bx)\,dx = \frac{1}{9}\,Bb^3x^9 + \frac{1}{5}\,Aa^3x^5 + \frac{1}{8}\,(3\,Bab^2 + Ab^3)x^8 + \frac{3}{7}\,(Ba^2b + Aab^2)x^7 + \frac{1}{6}\,(Ba^3 + 3\,Aa^2b)x^6$$

input
```
integrate(x^4*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
1/9*B*b^3*x^9 + 1/5*A*a^3*x^5 + 1/8*(3*B*a*b^2 + A*b^3)*x^8 + 3/7*(B*a^2*b
 + A*a*b^2)*x^7 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^4(a+bx)^3(A+Bx)\,dx = \frac{1}{9}Bb^3x^9 + \frac{3}{8}Bab^2x^8 + \frac{1}{8}Ab^3x^8 + \frac{3}{7}Ba^2bx^7 + \frac{3}{7}Aab^2x^7 + \frac{1}{6}Ba^3x^6 + \frac{1}{2}Aa^2bx^6 + \frac{1}{5}Aa^3x^5$$

input
```
integrate(x^4*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
1/9*B*b^3*x^9 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*B*a^2*b*x^7 + 3/7*A*
a*b^2*x^7 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*A*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int x^4(a+bx)^3(A+Bx)\,dx = x^6\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+x^8\left(\frac{A\,b^3}{8}+\frac{3\,B\,a\,b^2}{8}\right) + \frac{A\,a^3\,x^5}{5}+\frac{B\,b^3\,x^9}{9}+\frac{3\,a\,b\,x^7\,(A\,b+B\,a)}{7}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^6*((B*a^3)/6 + (A*a^2*b)/2) + x^8*((A*b^3)/8 + (3*B*a*b^2)/8) + (A*a^3*x
^5)/5 + (B*b^3*x^9)/9 + (3*a*b*x^7*(A*b + B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int x^4(a+bx)^3(A+Bx)\,dx = \frac{x^5(70b^4x^4+315a\,b^3x^3+540a^2b^2x^2+420a^3bx+126a^4)}{630}$$

input
```
int(x^4*(b*x+a)^3*(B*x+A),x)
```

output
```
(x**5*(126*a**4 + 420*a**3*b*x + 540*a**2*b**2*x**2 + 315*a*b**3*x**3 + 70
*b**4*x**4))/630
```

3.74 $\int x^3(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x^3(a+bx)^3(A+Bx)\,dx=\frac{1}{4}a^3Ax^4+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{1}{2}ab(Ab+aB)x^6+\frac{1}{7}b^2(Ab+3aB)x^7+\frac{1}{8}b^3Bx^8$$

output
```
1/4*a^3*A*x^4+1/5*a^2*(3*A*b+B*a)*x^5+1/2*a*b*(A*b+B*a)*x^6+1/7*b^2*(A*b+3
*B*a)*x^7+1/8*b^3*B*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^3(a+bx)^3(A+Bx)\,dx=\frac{1}{4}a^3Ax^4+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{1}{2}ab(Ab+aB)x^6+\frac{1}{7}b^2(Ab+3aB)x^7+\frac{1}{8}b^3Bx^8$$

input
```
Integrate[x^3*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^5)/5 + (a*b*(A*b + a*B)*x^6)/2 + (b^2
*(A*b + 3*a*B)*x^7)/7 + (b^3*B*x^8)/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{85, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^3Ax^3+a^2x^4(aB+3Ab)+b^2x^6(3aB+Ab)+3abx^5(aB+Ab)+b^3Bx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{1}{7}b^2x^7(3aB+Ab)+\frac{1}{2}abx^6(aB+Ab)+\frac{1}{8}b^3Bx^8$$

input
```
Int[x^3*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^5)/5 + (a*b*(A*b + a*B)*x^6)/2 + (b^2
*(A*b + 3*a*B)*x^7)/7 + (b^3*B*x^8)/8
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	s
norman	$\frac{b^3 B x^8}{8}+\left(\frac{1}{7}b^3 A+\frac{3}{7}a\,b^2 B\right)x^7+\left(\frac{1}{2}a\,b^2 A+\frac{1}{2}a^2 bB\right)x^6+\left(\frac{3}{5}a^2 bA+\frac{1}{5}a^3 B\right)x^5+\frac{a^3 A\,x^4}{4}$	7
default	$\frac{b^3 B\,x^8}{8}+\frac{\left(b^3 A+3a\,b^2 B\right)x^7}{7}+\frac{\left(3a\,b^2 A+3a^2 bB\right)x^6}{6}+\frac{\left(3a^2 bA+a^3 B\right)x^5}{5}+\frac{a^3 A\,x^4}{4}$	7
orering	$\frac{x^4\left(35B\,b^3 x^4+40A\,b^3 x^3+120Ba\,b^2 x^3+140aA\,b^2 x^2+140B\,a^2 b\,x^2+168a^2 Abx+56B\,a^3 x+70a^3 A\right)}{280}$	7
gosper	$\frac{1}{8}b^3 B\,x^8+\frac{1}{7}x^7 b^3 A+\frac{3}{7}x^7 a\,b^2 B+\frac{1}{2}x^6 a\,b^2 A+\frac{1}{2}x^6 a^2 bB+\frac{3}{5}x^5 a^2 bA+\frac{1}{5}x^5 a^3 B+\frac{1}{4}a^3 A\,x^4$	7
risch	$\frac{1}{8}b^3 B\,x^8+\frac{1}{7}x^7 b^3 A+\frac{3}{7}x^7 a\,b^2 B+\frac{1}{2}x^6 a\,b^2 A+\frac{1}{2}x^6 a^2 bB+\frac{3}{5}x^5 a^2 bA+\frac{1}{5}x^5 a^3 B+\frac{1}{4}a^3 A\,x^4$	7
parallelrisch	$\frac{1}{8}b^3 B\,x^8+\frac{1}{7}x^7 b^3 A+\frac{3}{7}x^7 a\,b^2 B+\frac{1}{2}x^6 a\,b^2 A+\frac{1}{2}x^6 a^2 bB+\frac{3}{5}x^5 a^2 bA+\frac{1}{5}x^5 a^3 B+\frac{1}{4}a^3 A\,x^4$	7

input
```
int(x^3*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^3*B*x^8+(1/7*b^3*A+3/7*a*b^2*B)*x^7+(1/2*a*b^2*A+1/2*a^2*b*B)*x^6+(3
/5*a^2*b*A+1/5*a^3*B)*x^5+1/4*a^3*A*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^3(a+bx)^3(A+Bx)\,dx=\frac{1}{8}\,Bb^3x^8+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\,(3\,Bab^2+Ab^3)x^7+\frac{1}{2}\,(Ba^2b+Aab^2)x^6+\frac{1}{5}\,(Ba^3+3\,Aa^2b)x^5$$

input
```
integrate(x^3*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
1/8*B*b^3*x^8 + 1/4*A*a^3*x^4 + 1/7*(3*B*a*b^2 + A*b^3)*x^7 + 1/2*(B*a^2*b
 + A*a*b^2)*x^6 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int x^3(a+bx)^3(A+Bx)\,dx = \frac{Aa^3x^4}{4} + \frac{Bb^3x^8}{8} + x^7\left(\frac{Ab^3}{7} + \frac{3Bab^2}{7}\right) + x^6\left(\frac{Aab^2}{2} + \frac{Ba^2b}{2}\right) + x^5\cdot\left(\frac{3Aa^2b}{5} + \frac{Ba^3}{5}\right)$$

input
```
integrate(x**3*(b*x+a)**3*(B*x+A),x)
```

output
```
A*a**3*x**4/4 + B*b**3*x**8/8 + x**7*(A*b**3/7 + 3*B*a*b**2/7) + x**6*(A*a
*b**2/2 + B*a**2*b/2) + x**5*(3*A*a**2*b/5 + B*a**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^3(a+bx)^3(A+Bx)\,dx = \frac{1}{8}\,Bb^3x^8 + \frac{1}{4}\,Aa^3x^4 + \frac{1}{7}\,(3\,Bab^2 + Ab^3)x^7 + \frac{1}{2}\,(Ba^2b + Aab^2)x^6 + \frac{1}{5}\,(Ba^3 + 3\,Aa^2b)x^5$$

input
```
integrate(x^3*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
1/8*B*b^3*x^8 + 1/4*A*a^3*x^4 + 1/7*(3*B*a*b^2 + A*b^3)*x^7 + 1/2*(B*a^2*b
 + A*a*b^2)*x^6 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^3(a+bx)^3(A+Bx)\,dx = \frac{1}{8}\,Bb^3x^8 + \frac{3}{7}\,Bab^2x^7 + \frac{1}{7}\,Ab^3x^7 + \frac{1}{2}\,Ba^2bx^6 + \frac{1}{2}\,Aab^2x^6 + \frac{1}{5}\,Ba^3x^5 + \frac{3}{5}\,Aa^2bx^5 + \frac{1}{4}\,Aa^3x^4$$

input
```
integrate(x^3*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
1/8*B*b^3*x^8 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*B*a^2*b*x^6 + 1/2*A*
a*b^2*x^6 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/4*A*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int x^3(a+bx)^3(A+Bx)\,dx = x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+x^7\left(\frac{A\,b^3}{7}+\frac{3\,B\,a\,b^2}{7}\right) + \frac{A\,a^3\,x^4}{4}+\frac{B\,b^3\,x^8}{8}+\frac{a\,b\,x^6\,(A\,b+B\,a)}{2}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^5*((B*a^3)/5 + (3*A*a^2*b)/5) + x^7*((A*b^3)/7 + (3*B*a*b^2)/7) + (A*a^3
*x^4)/4 + (B*b^3*x^8)/8 + (a*b*x^6*(A*b + B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int x^3(a+bx)^3(A+Bx)\,dx=\frac{x^4(35b^4x^4+160a\,b^3x^3+280a^2b^2x^2+224a^3bx+70a^4)}{280}$$

input
```
int(x^3*(b*x+a)^3*(B*x+A),x)
```

output
```
(x**4*(70*a**4 + 224*a**3*b*x + 280*a**2*b**2*x**2 + 160*a*b**3*x**3 + 35*
b**4*x**4))/280
```

3.75 $\int x^2(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x^2(a+bx)^3(A+Bx)\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{3}{5}ab(Ab+aB)x^5+\frac{1}{6}b^2(Ab+3aB)x^6+\frac{1}{7}b^3Bx^7$$

output
```
1/3*a^3*A*x^3+1/4*a^2*(3*A*b+B*a)*x^4+3/5*a*b*(A*b+B*a)*x^5+1/6*b^2*(A*b+3
*B*a)*x^6+1/7*b^3*B*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^2(a+bx)^3(A+Bx)\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{3}{5}ab(Ab+aB)x^5+\frac{1}{6}b^2(Ab+3aB)x^6+\frac{1}{7}b^3Bx^7$$

input
```
Integrate[x^2*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^4)/4 + (3*a*b*(A*b + a*B)*x^5)/5 + (b
^2*(A*b + 3*a*B)*x^6)/6 + (b^3*B*x^7)/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \ 85$$

$$\int \left(a^3Ax^2+a^2x^3(aB+3Ab)+b^2x^5(3aB+Ab)+3abx^4(aB+Ab)+b^3Bx^6\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{6}b^2x^6(3aB+Ab)+\frac{3}{5}abx^5(aB+Ab)+\frac{1}{7}b^3Bx^7$$

input
```
Int[x^2*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^4)/4 + (3*a*b*(A*b + a*B)*x^5)/5 + (b
^2*(A*b + 3*a*B)*x^6)/6 + (b^3*B*x^7)/7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	s
norman	$\frac{b^{3} B \,x^{7}}{7}+\left(\frac{1}{6} b^{3} A +\frac{1}{2} a \,b^{2} B \right) x^{6}+\left(\frac{3}{5} a \,b^{2} A +\frac{3}{5} a^{2} b B \right) x^{5}+\left(\frac{3}{4} a^{2} b A +\frac{1}{4} a^{3} B \right) x^{4}+\frac{a^{3} A \,x^{3}}{3}$	7
default	$\frac{b^{3} B \,x^{7}}{7}+\frac{\left(b^{3} A +3 a \,b^{2} B \right) x^{6}}{6}+\frac{\left(3 a \,b^{2} A +3 a^{2} b B \right) x^{5}}{5}+\frac{\left(3 a^{2} b A +a^{3} B \right) x^{4}}{4}+\frac{a^{3} A \,x^{3}}{3}$	7
orering	$\frac{x^{3} \left(60 B \,b^{3} x^{4}+70 A \,b^{3} x^{3}+210 B a \,b^{2} x^{3}+252 a A \,b^{2} x^{2}+252 B \,a^{2} b \,x^{2}+315 a^{2} A b x +105 B \,a^{3} x +140 a^{3} A \right)}{420}$	7
gosper	$\frac{1}{7} b^{3} B \,x^{7}+\frac{1}{6} x^{6} b^{3} A +\frac{1}{2} x^{6} a \,b^{2} B +\frac{3}{5} x^{5} a \,b^{2} A +\frac{3}{5} x^{5} a^{2} b B +\frac{3}{4} x^{4} a^{2} b A +\frac{1}{4} x^{4} a^{3} B +\frac{1}{3} a^{3} A \,x^{3}$	7
risch	$\frac{1}{7} b^{3} B \,x^{7}+\frac{1}{6} x^{6} b^{3} A +\frac{1}{2} x^{6} a \,b^{2} B +\frac{3}{5} x^{5} a \,b^{2} A +\frac{3}{5} x^{5} a^{2} b B +\frac{3}{4} x^{4} a^{2} b A +\frac{1}{4} x^{4} a^{3} B +\frac{1}{3} a^{3} A \,x^{3}$	7
parallelrisch	$\frac{1}{7} b^{3} B \,x^{7}+\frac{1}{6} x^{6} b^{3} A +\frac{1}{2} x^{6} a \,b^{2} B +\frac{3}{5} x^{5} a \,b^{2} A +\frac{3}{5} x^{5} a^{2} b B +\frac{3}{4} x^{4} a^{2} b A +\frac{1}{4} x^{4} a^{3} B +\frac{1}{3} a^{3} A \,x^{3}$	7

input
```
int(x^2*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*B*x^7+(1/6*b^3*A+1/2*a*b^2*B)*x^6+(3/5*a*b^2*A+3/5*a^2*b*B)*x^5+(3
/4*a^2*b*A+1/4*a^3*B)*x^4+1/3*a^3*A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^2(a+bx)^3(A+Bx)\,dx = \frac{1}{7}\,Bb^3x^7+\frac{1}{3}\,Aa^3x^3+\frac{1}{6}\,\left(3\,Bab^2+Ab^3\right)x^6 + \frac{3}{5}\,\left(Ba^2b+Aab^2\right)x^5+\frac{1}{4}\,\left(Ba^3+3\,Aa^2b\right)x^4$$

input
```
integrate(x^2*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
1/7*B*b^3*x^7 + 1/3*A*a^3*x^3 + 1/6*(3*B*a*b^2 + A*b^3)*x^6 + 3/5*(B*a^2*b
 + A*a*b^2)*x^5 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int x^2(a+bx)^3(A+Bx)\,dx = \frac{Aa^3x^3}{3} + \frac{Bb^3x^7}{7} + x^6\left(\frac{Ab^3}{6} + \frac{Bab^2}{2}\right) + x^5 \cdot \left(\frac{3Aab^2}{5} + \frac{3Ba^2b}{5}\right) + x^4 \cdot \left(\frac{3Aa^2b}{4} + \frac{Ba^3}{4}\right)$$

input
```
integrate(x**2*(b*x+a)**3*(B*x+A),x)
```

output
```
A*a**3*x**3/3 + B*b**3*x**7/7 + x**6*(A*b**3/6 + B*a*b**2/2) + x**5*(3*A*a
*b**2/5 + 3*B*a**2*b/5) + x**4*(3*A*a**2*b/4 + B*a**3/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^2(a+bx)^3(A+Bx)\,dx = \frac{1}{7}\,Bb^3x^7 + \frac{1}{3}\,Aa^3x^3 + \frac{1}{6}\,(3\,Bab^2 + Ab^3)x^6 + \frac{3}{5}\,(Ba^2b + Aab^2)x^5 + \frac{1}{4}\,(Ba^3 + 3\,Aa^2b)x^4$$

input
```
integrate(x^2*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
1/7*B*b^3*x^7 + 1/3*A*a^3*x^3 + 1/6*(3*B*a*b^2 + A*b^3)*x^6 + 3/5*(B*a^2*b
 + A*a*b^2)*x^5 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^2(a+bx)^3(A+Bx)\,dx = \frac{1}{7}Bb^3x^7 + \frac{1}{2}Bab^2x^6 + \frac{1}{6}Ab^3x^6 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{4}Ba^3x^4 + \frac{3}{4}Aa^2bx^4 + \frac{1}{3}Aa^3x^3$$

input
```
integrate(x^2*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
1/7*B*b^3*x^7 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 3/5*B*a^2*b*x^5 + 3/5*A*
a*b^2*x^5 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/3*A*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int x^2(a+bx)^3(A+Bx)\,dx = x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+x^6\left(\frac{A\,b^3}{6}+\frac{B\,a\,b^2}{2}\right) + \frac{A\,a^3\,x^3}{3}+\frac{B\,b^3\,x^7}{7}+\frac{3\,a\,b\,x^5\,(A\,b+B\,a)}{5}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^4*((B*a^3)/4 + (3*A*a^2*b)/4) + x^6*((A*b^3)/6 + (B*a*b^2)/2) + (A*a^3*x
^3)/3 + (B*b^3*x^7)/7 + (3*a*b*x^5*(A*b + B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int x^2(a+bx)^3(A+Bx)\,dx = \frac{x^3(15b^4x^4+70a\,b^3x^3+126a^2b^2x^2+105a^3bx+35a^4)}{105}$$

input
```
int(x^2*(b*x+a)^3*(B*x+A),x)
```

output
```
(x**3*(35*a**4 + 105*a**3*b*x + 126*a**2*b**2*x**2 + 70*a*b**3*x**3 + 15*b
**4*x**4))/105
```

3.76 $\int x(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int x(a+bx)^3(A+Bx)\,dx=-\frac{a(Ab-aB)(a+bx)^4}{4b^3}+\frac{(Ab-2aB)(a+bx)^5}{5b^3}+\frac{B(a+bx)^6}{6b^3}$$

output
```
-1/4*a*(A*b-B*a)*(b*x+a)^4/b^3+1/5*(A*b-2*B*a)*(b*x+a)^5/b^3+1/6*B*(b*x+a)
^6/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int x(a+bx)^3(A+Bx)\,dx=\frac{1}{60}x^2\left(10a^3(3A+2Bx)+15a^2bx(4A+3Bx)+9ab^2x^2(5A+4Bx)+2b^3x^3(6A+5Bx)\right)$$

input
```
Integrate[x*(a + b*x)^3*(A + B*x),x]
```

output
```
(x^2*(10*a^3*(3*A + 2*B*x) + 15*a^2*b*x*(4*A + 3*B*x) + 9*a*b^2*x^2*(5*A +
 4*B*x) + 2*b^3*x^3*(6*A + 5*B*x)))/60
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{(a+bx)^4(Ab-2aB)}{b^2}+\frac{a(a+bx)^3(aB-Ab)}{b^2}+\frac{B(a+bx)^5}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^5(Ab-2aB)}{5b^3}-\frac{a(a+bx)^4(Ab-aB)}{4b^3}+\frac{B(a+bx)^6}{6b^3}$$

input
```
Int[x*(a + b*x)^3*(A + B*x),x]
```

output
```
-1/4*(a*(A*b - a*B)*(a + b*x)^4)/b^3 + ((A*b - 2*a*B)*(a + b*x)^5)/(5*b^3)
 + (B*(a + b*x)^6)/(6*b^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

method	result	si
norman	$\frac{b^{3} B \,x^{6}}{6}+\left(\frac{1}{5} b^{3} A +\frac{3}{5} a \,b^{2} B \right) x^{5}+\left(\frac{3}{4} a \,b^{2} A +\frac{3}{4} a^{2} b B \right) x^{4}+\left(a^{2} b A +\frac{1}{3} a^{3} B \right) x^{3}+\frac{a^{3} A \,x^{2}}{2}$	74
default	$\frac{b^{3} B \,x^{6}}{6}+\frac{\left(b^{3} A +3 a \,b^{2} B \right) x^{5}}{5}+\frac{\left(3 a \,b^{2} A +3 a^{2} b B \right) x^{4}}{4}+\frac{\left(3 a^{2} b A +a^{3} B \right) x^{3}}{3}+\frac{a^{3} A \,x^{2}}{2}$	76
orering	$\frac{x^{2} \left(10 B \,b^{3} x^{4}+12 A \,b^{3} x^{3}+36 B a \,b^{2} x^{3}+45 a A \,b^{2} x^{2}+45 B \,a^{2} b \,x^{2}+60 a^{2} A b x +20 B \,a^{3} x +30 a^{3} A \right)}{60}$	76
gosper	$\frac{1}{6} b^{3} B \,x^{6}+\frac{1}{5} x^{5} b^{3} A +\frac{3}{5} x^{5} a \,b^{2} B +\frac{3}{4} x^{4} a \,b^{2} A +\frac{3}{4} x^{4} a^{2} b B +x^{3} a^{2} b A +\frac{1}{3} x^{3} a^{3} B +\frac{1}{2} a^{3} A \,x^{2}$	77
risch	$\frac{1}{6} b^{3} B \,x^{6}+\frac{1}{5} x^{5} b^{3} A +\frac{3}{5} x^{5} a \,b^{2} B +\frac{3}{4} x^{4} a \,b^{2} A +\frac{3}{4} x^{4} a^{2} b B +x^{3} a^{2} b A +\frac{1}{3} x^{3} a^{3} B +\frac{1}{2} a^{3} A \,x^{2}$	77
parallelrisch	$\frac{1}{6} b^{3} B \,x^{6}+\frac{1}{5} x^{5} b^{3} A +\frac{3}{5} x^{5} a \,b^{2} B +\frac{3}{4} x^{4} a \,b^{2} A +\frac{3}{4} x^{4} a^{2} b B +x^{3} a^{2} b A +\frac{1}{3} x^{3} a^{3} B +\frac{1}{2} a^{3} A \,x^{2}$	77

input
```
int(x*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3*B*x^6+(1/5*b^3*A+3/5*a*b^2*B)*x^5+(3/4*a*b^2*A+3/4*a^2*b*B)*x^4+(a
^2*b*A+1/3*a^3*B)*x^3+1/2*a^3*A*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int x(a+bx)^3(A+Bx)\,dx=\frac{1}{6}\,Bb^3x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\,\left(3\,Bab^2+Ab^3\right)x^5+\frac{3}{4}\,\left(Ba^2b+Aab^2\right)x^4+\frac{1}{3}\,\left(Ba^3+3\,Aa^2b\right)x^3$$

input
```
integrate(x*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
1/6*B*b^3*x^6 + 1/2*A*a^3*x^2 + 1/5*(3*B*a*b^2 + A*b^3)*x^5 + 3/4*(B*a^2*b
 + A*a*b^2)*x^4 + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int x(a+bx)^3(A+Bx)\,dx = \frac{Aa^3x^2}{2} + \frac{Bb^3x^6}{6} + x^5\left(\frac{Ab^3}{5} + \frac{3Bab^2}{5}\right) + x^4 \cdot \left(\frac{3Aab^2}{4} + \frac{3Ba^2b}{4}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

input
```
integrate(x*(b*x+a)**3*(B*x+A),x)
```

output
```
A*a**3*x**2/2 + B*b**3*x**6/6 + x**5*(A*b**3/5 + 3*B*a*b**2/5) + x**4*(3*A
*a*b**2/4 + 3*B*a**2*b/4) + x**3*(A*a**2*b + B*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int x(a+bx)^3(A+Bx)\,dx = \frac{1}{6}\,Bb^3x^6 + \frac{1}{2}\,Aa^3x^2 + \frac{1}{5}\,(3\,Bab^2 + Ab^3)x^5 + \frac{3}{4}\,(Ba^2b + Aab^2)x^4 + \frac{1}{3}\,(Ba^3 + 3\,Aa^2b)x^3$$

input
```
integrate(x*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
1/6*B*b^3*x^6 + 1/2*A*a^3*x^2 + 1/5*(3*B*a*b^2 + A*b^3)*x^5 + 3/4*(B*a^2*b
 + A*a*b^2)*x^4 + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int x(a+bx)^3(A+Bx)\,dx = \frac{1}{6}Bb^3x^6 + \frac{3}{5}Bab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{4}Ba^2bx^4 + \frac{3}{4}Aab^2x^4 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + \frac{1}{2}Aa^3x^2$$

input
```
integrate(x*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
1/6*B*b^3*x^6 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/4*B*a^2*b*x^4 + 3/4*A*
a*b^2*x^4 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + 1/2*A*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int x(a+bx)^3(A+Bx)\,dx = x^3\left(\frac{B\,a^3}{3} + A\,b\,a^2\right) + x^5\left(\frac{A\,b^3}{5} + \frac{3\,B\,a\,b^2}{5}\right) + \frac{A\,a^3\,x^2}{2} + \frac{B\,b^3\,x^6}{6} + \frac{3\,a\,b\,x^4\,(A\,b+B\,a)}{4}$$

input
```
int(x*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^3*((B*a^3)/3 + A*a^2*b) + x^5*((A*b^3)/5 + (3*B*a*b^2)/5) + (A*a^3*x^2)/
2 + (B*b^3*x^6)/6 + (3*a*b*x^4*(A*b + B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int x(a+bx)^3(A+Bx)\,dx=\frac{x^2(5b^4x^4+24a\,b^3x^3+45a^2b^2x^2+40a^3bx+15a^4)}{30}$$

input
```
int(x*(b*x+a)^3*(B*x+A),x)
```

output
```
(x**2*(15*a**4 + 40*a**3*b*x + 45*a**2*b**2*x**2 + 24*a*b**3*x**3 + 5*b**4
*x**4))/30
```

3.77 $\int (a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int (a+bx)^3(A+Bx)\,dx=\frac{(Ab-aB)(a+bx)^4}{4b^2}+\frac{B(a+bx)^5}{5b^2}$$

output
```
1/4*(A*b-B*a)*(b*x+a)^4/b^2+1/5*B*(b*x+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int (a+bx)^3(A+Bx)\,dx=a^3Ax+\frac{1}{2}a^2(3Ab+aB)x^2+ab(Ab+aB)x^3+\frac{1}{4}b^2(Ab+3aB)x^4+\frac{1}{5}b^3Bx^5$$

input
```
Integrate[(a + b*x)^3*(A + B*x),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^2)/2 + a*b*(A*b + a*B)*x^3 + (b^2*(A*b + 3*
a*B)*x^4)/4 + (b^3*B*x^5)/5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(a+bx)^3(Ab-aB)}{b}+\frac{B(a+bx)^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^4(Ab-aB)}{4b^2}+\frac{B(a+bx)^5}{5b^2}$$

input
```
Int[(a + b*x)^3*(A + B*x),x]
```

output
```
((A*b - a*B)*(a + b*x)^4)/(4*b^2) + (B*(a + b*x)^5)/(5*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

method	result	size
norman	$\frac{b^3 B x^5}{5} + \left(\frac{1}{4} b^3 A + \frac{3}{4} a b^2 B\right) x^4 + \left(a b^2 A + a^2 b B\right) x^3 + \left(\frac{3}{2} a^2 b A + \frac{1}{2} a^3 B\right) x^2 + a^3 A x$	70
gosper	$\frac{1}{5} b^3 B x^5 + \frac{1}{4} x^4 b^3 A + \frac{3}{4} x^4 a b^2 B + A a b^2 x^3 + B a^2 b x^3 + \frac{3}{2} x^2 a^2 b A + \frac{1}{2} x^2 a^3 B + a^3 A x$	73
default	$\frac{b^3 B x^5}{5} + \frac{\left(b^3 A + 3 a b^2 B\right) x^4}{4} + \frac{\left(3 a b^2 A + 3 a^2 b B\right) x^3}{3} + \frac{\left(3 a^2 b A + a^3 B\right) x^2}{2} + a^3 A x$	73
risch	$\frac{1}{5} b^3 B x^5 + \frac{1}{4} x^4 b^3 A + \frac{3}{4} x^4 a b^2 B + A a b^2 x^3 + B a^2 b x^3 + \frac{3}{2} x^2 a^2 b A + \frac{1}{2} x^2 a^3 B + a^3 A x$	73
parallelrisch	$\frac{1}{5} b^3 B x^5 + \frac{1}{4} x^4 b^3 A + \frac{3}{4} x^4 a b^2 B + A a b^2 x^3 + B a^2 b x^3 + \frac{3}{2} x^2 a^2 b A + \frac{1}{2} x^2 a^3 B + a^3 A x$	73
orering	$\frac{x \left(4 B b^3 x^4 + 5 A b^3 x^3 + 15 B a b^2 x^3 + 20 a A b^2 x^2 + 20 B a^2 b x^2 + 30 a^2 A b x + 10 B a^3 x + 20 a^3 A\right)}{20}$	74

input
```
int((b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*b^3*B*x^5+(1/4*b^3*A+3/4*a*b^2*B)*x^4+(A*a*b^2+B*a^2*b)*x^3+(3/2*a^2*b
*A+1/2*a^3*B)*x^2+a^3*A*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int (a+bx)^3(A+Bx)\,dx = \frac{1}{5}\,Bb^3x^5 + Aa^3x + \frac{1}{4}\left(3\,Bab^2 + Ab^3\right)x^4 + \left(Ba^2b + Aab^2\right)x^3 + \frac{1}{2}\left(Ba^3 + 3\,Aa^2b\right)x^2$$

input
```
integrate((b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
1/5*B*b^3*x^5 + A*a^3*x + 1/4*(3*B*a*b^2 + A*b^3)*x^4 + (B*a^2*b + A*a*b^2
)*x^3 + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int (a+bx)^3(A+Bx)\,dx = Aa^3x + \frac{Bb^3x^5}{5} + x^4\left(\frac{Ab^3}{4} + \frac{3Bab^2}{4}\right) + x^3\left(Aab^2 + Ba^2b\right) + x^2\cdot\left(\frac{3Aa^2b}{2} + \frac{Ba^3}{2}\right)$$

input
```
integrate((b*x+a)**3*(B*x+A),x)
```

output
```
A*a**3*x + B*b**3*x**5/5 + x**4*(A*b**3/4 + 3*B*a*b**2/4) + x**3*(A*a*b**2
 + B*a**2*b) + x**2*(3*A*a**2*b/2 + B*a**3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int (a+bx)^3(A+Bx)\,dx = \frac{1}{5}\,Bb^3x^5 + Aa^3x + \frac{1}{4}\,\left(3\,Bab^2 + Ab^3\right)x^4 + \left(Ba^2b + Aab^2\right)x^3 + \frac{1}{2}\,\left(Ba^3 + 3\,Aa^2b\right)x^2$$

input
```
integrate((b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
1/5*B*b^3*x^5 + A*a^3*x + 1/4*(3*B*a*b^2 + A*b^3)*x^4 + (B*a^2*b + A*a*b^2
)*x^3 + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int (a+bx)^3(A+Bx)\,dx = \frac{1}{5}\,Bb^3x^5 + \frac{3}{4}\,Bab^2x^4 + \frac{1}{4}\,Ab^3x^4 + Ba^2bx^3 + Aab^2x^3 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{2}\,Aa^2bx^2 + Aa^3x$$

input
```
integrate((b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
1/5*B*b^3*x^5 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + B*a^2*b*x^3 + A*a*b^2*x^
3 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (a+bx)^3(A+Bx)\,dx = x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+x^4\left(\frac{A\,b^3}{4}+\frac{3\,B\,a\,b^2}{4}\right) + \frac{B\,b^3\,x^5}{5} + A\,a^3\,x + a\,b\,x^3\,(A\,b+B\,a)$$

input
```
int((A + B*x)*(a + b*x)^3,x)
```

output
```
x^2*((B*a^3)/2 + (3*A*a^2*b)/2) + x^4*((A*b^3)/4 + (3*B*a*b^2)/4) + (B*b^3
*x^5)/5 + A*a^3*x + a*b*x^3*(A*b + B*a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int (a+bx)^3(A+Bx)\,dx = \frac{x(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4)}{5}$$

input
```
int((b*x+a)^3*(B*x+A),x)
```

output
```
(x*(5*a**4 + 10*a**3*b*x + 10*a**2*b**2*x**2 + 5*a*b**3*x**3 + b**4*x**4))
/5
```

3.78 $\int \frac{(a+bx)^3(A+Bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = 3a^2Abx + \frac{3}{2}aAb^2x^2 + \frac{1}{3}Ab^3x^3 + \frac{B(a+bx)^4}{4b} + a^3A\log(x)$$

output
```
3*a^2*A*b*x+3/2*a*A*b^2*x^2+1/3*A*b^3*x^3+1/4*B*(b*x+a)^4/b+a^3*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = \frac{1}{12}x\big(12a^3B + 18a^2b(2A+Bx) + 6ab^2x(3A+2Bx) + b^3x^2(4A+3Bx)\big) + a^3A\log(x)$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x,x]
```

output
```
(x*(12*a^3*B + 18*a^2*b*(2*A + B*x) + 6*a*b^2*x*(3*A + 2*B*x) + b^3*x^2*(4
*A + 3*B*x)))/12 + a^3*A*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x} \, dx$$

$$\Big\downarrow \text{90}$$

$$A\int \frac{(a+bx)^3}{x}dx+\frac{B(a+bx)^4}{4b}$$

$$\Big\downarrow \text{49}$$

$$A\int \left(\frac{a^3}{x}+3ba^2+3b^2xa+b^3x^2\right)dx+\frac{B(a+bx)^4}{4b}$$

$$\Big\downarrow \text{2009}$$

$$A\left(a^3\log(x)+3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}\right)+\frac{B(a+bx)^4}{4b}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x,x]
```

output
```
(B*(a + b*x)^4)/(4*b) + A*(3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3
*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

method	result	size
norman	$\left(\frac{1}{3}b^{3}A+a\,b^{2}B\right)x^{3}+\left(\frac{3}{2}a\,b^{2}A+\frac{3}{2}a^{2}bB\right)x^{2}+\left(3a^{2}bA+a^{3}B\right)x+\frac{B\,b^{3}x^{4}}{4}+a^{3}A\ln\left(x\right)$	69
default	$\frac{B\,b^{3}x^{4}}{4}+\frac{A\,b^{3}x^{3}}{3}+Ba\,b^{2}x^{3}+\frac{3aA\,b^{2}x^{2}}{2}+\frac{3B\,a^{2}b\,x^{2}}{2}+3a^{2}Abx+B\,a^{3}x+a^{3}A\ln\left(x\right)$	70
risch	$\frac{B\,b^{3}x^{4}}{4}+\frac{A\,b^{3}x^{3}}{3}+Ba\,b^{2}x^{3}+\frac{3aA\,b^{2}x^{2}}{2}+\frac{3B\,a^{2}b\,x^{2}}{2}+3a^{2}Abx+B\,a^{3}x+a^{3}A\ln\left(x\right)$	70
parallelrisch	$\frac{B\,b^{3}x^{4}}{4}+\frac{A\,b^{3}x^{3}}{3}+Ba\,b^{2}x^{3}+\frac{3aA\,b^{2}x^{2}}{2}+\frac{3B\,a^{2}b\,x^{2}}{2}+3a^{2}Abx+B\,a^{3}x+a^{3}A\ln\left(x\right)$	70

input
```
int((b*x+a)^3*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/3*b^3*A+a*b^2*B)*x^3+(3/2*a*b^2*A+3/2*a^2*b*B)*x^2+(3*A*a^2*b+B*a^3)*x+
1/4*B*b^3*x^4+a^3*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int\frac{(a+bx)^{3}(A+Bx)}{x}\,dx=\frac{1}{4}\,Bb^{3}x^{4}+Aa^{3}\log\left(x\right)+\frac{1}{3}\left(3\,Bab^{2}+Ab^{3}\right)x^{3}+\frac{3}{2}\left(Ba^{2}b+Aab^{2}\right)x^{2}+\left(Ba^{3}+3\,Aa^{2}b\right)x$$

input
```
integrate((b*x+a)^3*(B*x+A)/x,x, algorithm="fricas")
```

output
```
1/4*B*b^3*x^4 + A*a^3*log(x) + 1/3*(3*B*a*b^2 + A*b^3)*x^3 + 3/2*(B*a^2*b
+ A*a*b^2)*x^2 + (B*a^3 + 3*A*a^2*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = Aa^3\log(x) + \frac{Bb^3x^4}{4} + x^3\left(\frac{Ab^3}{3} + Bab^2\right) + x^2\cdot\left(\frac{3Aab^2}{2} + \frac{3Ba^2b}{2}\right) + x(3Aa^2b + Ba^3)$$

input
```
integrate((b*x+a)**3*(B*x+A)/x,x)
```

output
```
A*a**3*log(x) + B*b**3*x**4/4 + x**3*(A*b**3/3 + B*a*b**2) + x**2*(3*A*a*b
**2/2 + 3*B*a**2*b/2) + x*(3*A*a**2*b + B*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = \frac{1}{4}\,Bb^3x^4 + Aa^3\log(x) + \frac{1}{3}\,(3\,Bab^2 + Ab^3)x^3 + \frac{3}{2}\,(Ba^2b + Aab^2)x^2 + (Ba^3 + 3\,Aa^2b)x$$

input
```
integrate((b*x+a)^3*(B*x+A)/x,x, algorithm="maxima")
```

output
```
1/4*B*b^3*x^4 + A*a^3*log(x) + 1/3*(3*B*a*b^2 + A*b^3)*x^3 + 3/2*(B*a^2*b
+ A*a*b^2)*x^2 + (B*a^3 + 3*A*a^2*b)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = \frac{1}{4}\,Bb^3x^4 + Bab^2x^3 + \frac{1}{3}\,Ab^3x^3 + \frac{3}{2}\,Ba^2bx^2 + \frac{3}{2}\,Aab^2x^2 + Ba^3x + 3\,Aa^2bx + Aa^3\log\left(|x|\right)$$

input
```
integrate((b*x+a)^3*(B*x+A)/x,x, algorithm="giac")
```

output
```
1/4*B*b^3*x^4 + B*a*b^2*x^3 + 1/3*A*b^3*x^3 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^
2*x^2 + B*a^3*x + 3*A*a^2*b*x + A*a^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = x\left(B\,a^3+3\,A\,b\,a^2\right) + x^3\left(\frac{A\,b^3}{3}+B\,a\,b^2\right) + \frac{B\,b^3\,x^4}{4} + A\,a^3\,\ln\left(x\right) + \frac{3\,a\,b\,x^2\,\left(A\,b+B\,a\right)}{2}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x,x)
```

output
```
x*(B*a^3 + 3*A*a^2*b) + x^3*((A*b^3)/3 + B*a*b^2) + (B*b^3*x^4)/4 + A*a^3*
log(x) + (3*a*b*x^2*(A*b + B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^3(A+Bx)}{x}\,dx = \log(x)\,a^4 + 4a^3bx + 3a^2b^2x^2 + \frac{4a\,b^3x^3}{3} + \frac{b^4x^4}{4}$$

input
```
int((b*x+a)^3*(B*x+A)/x,x)
```

output
```
(12*log(x)*a**4 + 48*a**3*b*x + 36*a**2*b**2*x**2 + 16*a*b**3*x**3 + 3*b**
4*x**4)/12
```

3.79 $\int \frac{(a+bx)^3(A+Bx)}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{(a+bx)^3(A+Bx)}{x^2} dx = -\frac{a^3A}{x} + 3ab(Ab+aB)x + \frac{1}{2}b^2(Ab+3aB)x^2 + \frac{1}{3}b^3Bx^3 + a^2(3Ab+aB)\log(x)$$

output
```
-a^3*A/x+3*a*b*(A*b+B*a)*x+1/2*b^2*(A*b+3*B*a)*x^2+1/3*b^3*B*x^3+a^2*(3*A*
b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^3(A+Bx)}{x^2} dx = -\frac{a^3A}{x} + 3ab(Ab+aB)x + \frac{1}{2}b^2(Ab+3aB)x^2 + \frac{1}{3}b^3Bx^3 + \left(3a^2Ab+a^3B\right)\log(x)$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^2,x]
```

output
```
-((a^3*A)/x) + 3*a*b*(A*b + a*B)*x + (b^2*(A*b + 3*a*B)*x^2)/2 + (b^3*B*x^
3)/3 + (3*a^2*A*b + a^3*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^2} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^3 A}{x^2}+\frac{a^2(aB+3Ab)}{x}+b^2x(3aB+Ab)+3ab(aB+Ab)+b^3Bx^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3A}{x}+a^2\log(x)(aB+3Ab)+\frac{1}{2}b^2x^2(3aB+Ab)+3abx(aB+Ab)+\frac{1}{3}b^3Bx^3$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^2,x]
```

output
```
-((a^3*A)/x) + 3*a*b*(A*b + a*B)*x + (b^2*(A*b + 3*a*B)*x^2)/2 + (b^3*B*x^
3)/3 + a^2*(3*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{b^3 B x^3}{3}+\frac{A b^3 x^2}{2}+\frac{3 B a b^2 x^2}{2}+3 a b^2 A x+3 a^2 b B x+a^2 (3 A b+B a) \ln(x)-\frac{a^3 A}{x}$	69
risch	$\frac{b^3 B x^3}{3}+\frac{A b^3 x^2}{2}+\frac{3 B a b^2 x^2}{2}+3 a b^2 A x+3 a^2 b B x-\frac{a^3 A}{x}+3 A \ln(x) a^2 b+B \ln(x) a^3$	71
norman	$\frac{(\frac{1}{2} b^3 A+\frac{3}{2} a b^2 B) x^3+(3 a b^2 A+3 a^2 b B) x^2-a^3 A+\frac{B b^3 x^4}{3}}{x}+(3 a^2 b A+a^3 B) \ln(x)$	75
parallelrisch	$\frac{2 B b^3 x^4+3 A b^3 x^3+9 B a b^2 x^3+18 A \ln(x) x a^2 b+18 a A b^2 x^2+6 B \ln(x) x a^3+18 B a^2 b x^2-6 a^3 A}{6 x}$	80

input
```
int((b*x+a)^3*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3*B*x^3+1/2*A*b^3*x^2+3/2*B*a*b^2*x^2+3*a*b^2*A*x+3*a^2*b*B*x+a^2*(3
*A*b+B*a)*ln(x)-a^3*A/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^3(A+Bx)}{x^2}\,dx = \frac{2\,Bb^3x^4-6\,Aa^3+3\,(3\,Bab^2+Ab^3)x^3+18\,(Ba^2b+Aab^2)x^2+6\,(Ba^3+3\,Aa^2b)x\log(x)}{6\,x}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*x^4 - 6*A*a^3 + 3*(3*B*a*b^2 + A*b^3)*x^3 + 18*(B*a^2*b + A*a
*b^2)*x^2 + 6*(B*a^3 + 3*A*a^2*b)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3(A+Bx)}{x^2}\,dx = -\frac{Aa^3}{x} + \frac{Bb^3x^3}{3} + a^2\cdot(3Ab+Ba)\log(x) + x^2\left(\frac{Ab^3}{2} + \frac{3Bab^2}{2}\right) + x(3Aab^2 + 3Ba^2b)$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**2,x)
```

output
```
-A*a**3/x + B*b**3*x**3/3 + a**2*(3*A*b + B*a)*log(x) + x**2*(A*b**3/2 + 3
*B*a*b**2/2) + x*(3*A*a*b**2 + 3*B*a**2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3(A+Bx)}{x^2}\,dx = \frac{1}{3}Bb^3x^3 - \frac{Aa^3}{x} + \frac{1}{2}\left(3Bab^2 + Ab^3\right)x^2 + 3\left(Ba^2b + Aab^2\right)x + \left(Ba^3 + 3Aa^2b\right)\log(x)$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/3*B*b^3*x^3 - A*a^3/x + 1/2*(3*B*a*b^2 + A*b^3)*x^2 + 3*(B*a^2*b + A*a*b
^2)*x + (B*a^3 + 3*A*a^2*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^2}\,dx = \frac{1}{3}Bb^3x^3 + \frac{3}{2}Bab^2x^2 + \frac{1}{2}Ab^3x^2 + 3Ba^2bx + 3Aab^2x - \frac{Aa^3}{x} + \left(Ba^3 + 3Aa^2b\right)\log(|x|)$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/3*B*b^3*x^3 + 3/2*B*a*b^2*x^2 + 1/2*A*b^3*x^2 + 3*B*a^2*b*x + 3*A*a*b^2*
x - A*a^3/x + (B*a^3 + 3*A*a^2*b)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3(A+Bx)}{x^2}\,dx = x^2\left(\frac{A\,b^3}{2}+\frac{3\,B\,a\,b^2}{2}\right)+\ln\left(x\right)\,\left(B\,a^3+3\,A\,b\,a^2\right) - \frac{A\,a^3}{x}+\frac{B\,b^3\,x^3}{3}+3\,a\,b\,x\,(A\,b+B\,a)$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^2,x)
```

output
```
x^2*((A*b^3)/2 + (3*B*a*b^2)/2) + log(x)*(B*a^3 + 3*A*a^2*b) - (A*a^3)/x +
 (B*b^3*x^3)/3 + 3*a*b*x*(A*b + B*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^3(A+Bx)}{x^2}\,dx = \frac{12\log(x)\,a^3bx-3a^4+18a^2b^2x^2+6a\,b^3x^3+b^4x^4}{3x}$$

input
```
int((b*x+a)^3*(B*x+A)/x^2,x)
```

output
```
(12*log(x)*a**3*b*x - 3*a**4 + 18*a**2*b**2*x**2 + 6*a*b**3*x**3 + b**4*x*
*4)/(3*x)
```

3.80 $\int \frac{(a+bx)^3(A+Bx)}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{(a+bx)^3(A+Bx)}{x^3} \, dx = -\frac{a^3A}{2x^2} - \frac{a^2(3Ab+aB)}{x} + b^2(Ab+3aB)x + \frac{1}{2}b^3Bx^2 + 3ab(Ab+aB)\log(x)$$

output
```
-1/2*a^3*A/x^2-a^2*(3*A*b+B*a)/x+b^2*(A*b+3*B*a)*x+1/2*b^3*B*x^2+3*a*b*(A*
b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^3(A+Bx)}{x^3} \, dx = \frac{1}{2}\left(-\frac{6a^2Ab}{x} + 6ab^2Bx + b^3x(2A+Bx) - \frac{a^3(A+2Bx)}{x^2} + 6ab(Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^3,x]
```

output
```
((-6*a^2*A*b)/x + 6*a*b^2*B*x + b^3*x*(2*A + B*x) - (a^3*(A + 2*B*x))/x^2
+ 6*a*b*(A*b + a*B)*Log[x])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^3} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3A}{x^3}+\frac{a^2(aB+3Ab)}{x^2}+b^2(3aB+Ab)+\frac{3ab(aB+Ab)}{x}+b^3Bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{2x^2}-\frac{a^2(aB+3Ab)}{x}+b^2x(3aB+Ab)+3ab\log(x)(aB+Ab)+\frac{1}{2}b^3Bx^2$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^3,x]
```

output
```
-1/2*(a^3*A)/x^2 - (a^2*(3*A*b + a*B))/x + b^2*(A*b + 3*a*B)*x + (b^3*B*x^
2)/2 + 3*a*b*(A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{b^3 B x^2}{2}+A b^3 x+3 B a b^2 x-\frac{a^3 A}{2 x^2}+3 a b(A b+B a) \ln(x)-\frac{a^2(3 A b+B a)}{x}$	63
risch	$\frac{b^3 B x^2}{2}+A b^3 x+3 B a b^2 x+\frac{(-3 a^2 b A-a^3 B) x-\frac{a^3 A}{2}}{x^2}+3 A \ln(x) a b^2+3 B \ln(x) a^2 b$	70
norman	$\frac{(b^3 A+3 a b^2 B) x^3+(-3 a^2 b A-a^3 B) x-\frac{a^3 A}{2}+\frac{B b^3 x^4}{2}}{x^2}+(3 a b^2 A+3 a^2 b B) \ln(x)$	73
parallelrisch	$\frac{B b^3 x^4+6 A \ln(x) x^2 a b^2+2 A b^3 x^3+6 B \ln(x) x^2 a^2 b+6 B a b^2 x^3-6 a^2 A b x-2 B a^3 x-a^3 A}{2 x^2}$	79

input
```
int((b*x+a)^3*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^3*B*x^2+A*b^3*x+3*B*a*b^2*x-1/2*a^3*A/x^2+3*a*b*(A*b+B*a)*ln(x)-a^2*
(3*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^3(A+Bx)}{x^3}\,dx = \frac{Bb^3x^4-Aa^3+2\,(3\,Bab^2+Ab^3)x^3+6\,(Ba^2b+Aab^2)x^2\log(x)-2\,(Ba^3+3\,Aa^2b)x}{2\,x^2}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/2*(B*b^3*x^4 - A*a^3 + 2*(3*B*a*b^2 + A*b^3)*x^3 + 6*(B*a^2*b + A*a*b^2)
*x^2*log(x) - 2*(B*a^3 + 3*A*a^2*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^3(A+Bx)}{x^3}\,dx = \frac{Bb^3x^2}{2} + 3ab(Ab+Ba)\log{(x)} + x\left(Ab^3+3Bab^2\right) + \frac{-Aa^3+x(-6Aa^2b-2Ba^3)}{2x^2}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**3,x)
```

output
```
B*b**3*x**2/2 + 3*a*b*(A*b + B*a)*log(x) + x*(A*b**3 + 3*B*a*b**2) + (-A*a
**3 + x*(-6*A*a**2*b - 2*B*a**3))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3(A+Bx)}{x^3}\,dx = \frac{1}{2}\,Bb^3x^2 + \left(3\,Bab^2+Ab^3\right)x + 3\,\left(Ba^2b+Aab^2\right)\log{(x)} - \frac{Aa^3+2\,(Ba^3+3\,Aa^2b)x}{2\,x^2}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/2*B*b^3*x^2 + (3*B*a*b^2 + A*b^3)*x + 3*(B*a^2*b + A*a*b^2)*log(x) - 1/2
*(A*a^3 + 2*(B*a^3 + 3*A*a^2*b)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3(A+Bx)}{x^3}\,dx = \frac{1}{2}\,Bb^3x^2 + 3\,Bab^2x + Ab^3x + 3\left(Ba^2b + Aab^2\right)\log\left(|x|\right) - \frac{Aa^3 + 2\,(Ba^3 + 3\,Aa^2b)x}{2\,x^2}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/2*B*b^3*x^2 + 3*B*a*b^2*x + A*b^3*x + 3*(B*a^2*b + A*a*b^2)*log(abs(x))
- 1/2*(A*a^3 + 2*(B*a^3 + 3*A*a^2*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3(A+Bx)}{x^3}\,dx = \ln\left(x\right)\,\left(3\,B\,a^2\,b + 3\,A\,a\,b^2\right) - \frac{x\,(B\,a^3 + 3\,A\,b\,a^2) + \frac{A\,a^3}{2}}{x^2} + x\,\left(A\,b^3 + 3\,B\,a\,b^2\right) + \frac{B\,b^3\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^3,x)
```

output
```
log(x)*(3*A*a*b^2 + 3*B*a^2*b) - (x*(B*a^3 + 3*A*a^2*b) + (A*a^3)/2)/x^2 +
 x*(A*b^3 + 3*B*a*b^2) + (B*b^3*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^3(A+Bx)}{x^3}\,dx = \frac{12\log(x)\,a^2b^2x^2 - a^4 - 8a^3bx + 8a\,b^3x^3 + b^4x^4}{2x^2}$$

input
```
int((b*x+a)^3*(B*x+A)/x^3,x)
```

output
```
(12*log(x)*a**2*b**2*x**2 - a**4 - 8*a**3*b*x + 8*a*b**3*x**3 + b**4*x**4)
/(2*x**2)
```

3.81 $\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = -\frac{a^3A}{3x^3} - \frac{a^2(3Ab+aB)}{2x^2} - \frac{3ab(Ab+aB)}{x} + b^3Bx + b^2(Ab+3aB)\log(x)$$

output
```
-1/3*a^3*A/x^3-1/2*a^2*(3*A*b+B*a)/x^2-3*a*b*(A*b+B*a)/x+b^3*B*x+b^2*(A*b+
3*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = -\frac{18aAb^2x^2 - 6b^3Bx^4 + 9a^2bx(A+2Bx) + a^3(2A+3Bx)}{6x^3} + b^2(Ab+3aB)\log(x)$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^4,x]
```

output
```
-1/6*(18*a*A*b^2*x^2 - 6*b^3*B*x^4 + 9*a^2*b*x*(A + 2*B*x) + a^3*(2*A + 3*
B*x))/x^3 + b^2*(A*b + 3*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^3A}{x^4}+\frac{a^2(aB+3Ab)}{x^3}+\frac{b^2(3aB+Ab)}{x}+\frac{3ab(aB+Ab)}{x^2}+b^3B\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3A}{3x^3}-\frac{a^2(aB+3Ab)}{2x^2}+b^2\log(x)(3aB+Ab)-\frac{3ab(aB+Ab)}{x}+b^3Bx$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^4,x]
```

output
```
-1/3*(a^3*A)/x^3 - (a^2*(3*A*b + a*B))/(2*x^2) - (3*a*b*(A*b + a*B))/x + b
^3*B*x + b^2*(A*b + 3*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a^{3} A}{3 x^{3}}-\frac{a^{2} (3 A b+B a)}{2 x^{2}}-\frac{3 a b (A b+B a)}{x}+b^{3} B x+b^{2} (A b+3 B a) \ln \left(x \right)$	61
risch	$b^{3} B x +\frac{\left(-3 a \,b^{2} A -3 a^{2} b B \right) x^{2}+\left(-\frac{3}{2} a^{2} b A -\frac{1}{2} a^{3} B \right) x -\frac{a^{3} A}{3}}{x^{3}}+A \ln \left(x \right) b^{3}+3 B \ln \left(x \right) a \,b^{2}$	70
norman	$\frac{\left(-\frac{3}{2} a^{2} b A -\frac{1}{2} a^{3} B \right) x +\left(-3 a \,b^{2} A -3 a^{2} b B \right) x^{2}+B \,b^{3} x^{4}-\frac{a^{3} A}{3}}{x^{3}}+\left(b^{3} A +3 a \,b^{2} B \right) \ln \left(x \right)$	72
parallelrisch	$\frac{6 A \ln \left(x \right) x^{3} b^{3}+18 B \ln \left(x \right) x^{3} a \,b^{2}+6 B \,b^{3} x^{4}-18 a A \,b^{2} x^{2}-18 B \,a^{2} b \,x^{2}-9 a^{2} A b x -3 B \,a^{3} x -2 a^{3} A}{6 x^{3}}$	80

input
```
int((b*x+a)^3*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3*A/x^3-1/2*a^2*(3*A*b+B*a)/x^2-3*a*b*(A*b+B*a)/x+b^3*B*x+b^2*(A*b+
3*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = \frac{6\,Bb^3x^4+6\,(3\,Bab^2+Ab^3)x^3\log{(x)}-2\,Aa^3-18\,(Ba^2b+Aab^2)x^2-3\,(Ba^3+3\,Aa^2b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*B*b^3*x^4 + 6*(3*B*a*b^2 + A*b^3)*x^3*log(x) - 2*A*a^3 - 18*(B*a^2*
b + A*a*b^2)*x^2 - 3*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = Bb^3x + b^2(Ab+3Ba)\log(x) + \frac{-2Aa^3 + x^2(-18Aab^2 - 18Ba^2b) + x(-9Aa^2b - 3Ba^3)}{6x^3}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**4,x)
```

output
```
B*b**3*x + b**2*(A*b + 3*B*a)*log(x) + (-2*A*a**3 + x**2*(-18*A*a*b**2 - 1
8*B*a**2*b) + x*(-9*A*a**2*b - 3*B*a**3))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = Bb^3x + \left(3\,Bab^2 + Ab^3\right)\log(x) - \frac{2\,Aa^3 + 18\,(Ba^2b + Aab^2)x^2 + 3\,(Ba^3 + 3\,Aa^2b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
B*b^3*x + (3*B*a*b^2 + A*b^3)*log(x) - 1/6*(2*A*a^3 + 18*(B*a^2*b + A*a*b^
2)*x^2 + 3*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = Bb^3x + \left(3\,Bab^2 + Ab^3\right)\log\left(|x|\right) - \frac{2\,Aa^3 + 18\,(Ba^2b + Aab^2)x^2 + 3\,(Ba^3 + 3\,Aa^2b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
B*b^3*x + (3*B*a*b^2 + A*b^3)*log(abs(x)) - 1/6*(2*A*a^3 + 18*(B*a^2*b + A
*a*b^2)*x^2 + 3*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = \ln\left(x\right)\,\left(A\,b^3 + 3\,B\,a\,b^2\right) - \frac{x^2\,\left(3\,B\,a^2\,b + 3\,A\,a\,b^2\right) + x\,\left(\frac{B\,a^3}{2} + \frac{3\,A\,b\,a^2}{2}\right) + \frac{A\,a^3}{3}}{x^3} + B\,b^3\,x$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^4,x)
```

output
```
log(x)*(A*b^3 + 3*B*a*b^2) - (x^2*(3*A*a*b^2 + 3*B*a^2*b) + x*((B*a^3)/2 +
 (3*A*a^2*b)/2) + (A*a^3)/3)/x^3 + B*b^3*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)^3(A+Bx)}{x^4}\,dx = \frac{12\log(x)\,a\,b^3x^3 - a^4 - 6a^3bx - 18a^2b^2x^2 + 3b^4x^4}{3x^3}$$

input
```
int((b*x+a)^3*(B*x+A)/x^4,x)
```

output
```
(12*log(x)*a*b**3*x**3 - a**4 - 6*a**3*b*x - 18*a**2*b**2*x**2 + 3*b**4*x*
*4)/(3*x**3)
```

3.82 $\int \frac{(a+bx)^3(A+Bx)}{x^5} dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{(a+bx)^3(A+Bx)}{x^5} dx = -\frac{a^3B}{3x^3} - \frac{3a^2bB}{2x^2} - \frac{3ab^2B}{x} - \frac{A(a+bx)^4}{4ax^4} + b^3B\log(x)$$

output
```
-1/3*a^3*B/x^3-3/2*a^2*b*B/x^2-3*a*b^2*B/x-1/4*A*(b*x+a)^4/a/x^4+b^3*B*ln(
x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^3(A+Bx)}{x^5} dx = -\frac{12Ab^3x^3 + 18ab^2x^2(A+2Bx) + 6a^2bx(2A+3Bx) + a^3(3A+4Bx) - 12b^3Bx^4\log(x)}{12x^4}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^5,x]
```

output
```
-1/12*(12*A*b^3*x^3 + 18*a*b^2*x^2*(A + 2*B*x) + 6*a^2*b*x*(2*A + 3*B*x) +
 a^3*(3*A + 4*B*x) - 12*b^3*B*x^4*Log[x])/x^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^5}dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{(a+bx)^3}{x^4}dx-\frac{A(a+bx)^4}{4ax^4}$$

$$\Big\downarrow \text{49}$$

$$B\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^3}+\frac{3b^2a}{x^2}+\frac{b^3}{x}\right)dx-\frac{A(a+bx)^4}{4ax^4}$$

$$\Big\downarrow \text{2009}$$

$$B\left(-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\right)-\frac{A(a+bx)^4}{4ax^4}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^5,x]
```

output
```
-1/4*(A*(a + b*x)^4)/(a*x^4) + B*(-1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*
b^2)/x + b^3*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{a^{2}(3Ab+Ba)}{3x^{3}}-\frac{3ab(Ab+Ba)}{2x^{2}}-\frac{a^{3}A}{4x^{4}}+b^{3}B\ln\left(x\right)-\frac{b^{2}(Ab+3Ba)}{x}$	64
norman	$\frac{\left(-\frac{3}{2}a\,b^{2}A-\frac{3}{2}a^{2}bB\right)x^{2}+\left(-a^{2}bA-\frac{1}{3}a^{3}B\right)x+\left(-b^{3}A-3a\,b^{2}B\right)x^{3}-\frac{a^{3}A}{4}}{x^{4}}+b^{3}B\ln\left(x\right)$	73
risch	$\frac{\left(-\frac{3}{2}a\,b^{2}A-\frac{3}{2}a^{2}bB\right)x^{2}+\left(-a^{2}bA-\frac{1}{3}a^{3}B\right)x+\left(-b^{3}A-3a\,b^{2}B\right)x^{3}-\frac{a^{3}A}{4}}{x^{4}}+b^{3}B\ln\left(x\right)$	73
parallelrisch	$-\frac{-12b^{3}B\ln\left(x\right)x^{4}+12A\,b^{3}x^{3}+36Ba\,b^{2}x^{3}+18aA\,b^{2}x^{2}+18B\,a^{2}b\,x^{2}+12a^{2}Abx+4B\,a^{3}x+3a^{3}A}{12x^{4}}$	78

input
```
int((b*x+a)^3*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*(3*A*b+B*a)/x^3-3/2*a*b*(A*b+B*a)/x^2-1/4*a^3*A/x^4+b^3*B*ln(x)-b
^2*(A*b+3*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int\frac{(a+bx)^{3}(A+Bx)}{x^{5}}\,dx$$
$$=\frac{12\,Bb^{3}x^{4}\log\left(x\right)-3\,Aa^{3}-12\,(3\,Bab^{2}+Ab^{3})x^{3}-18\,(Ba^{2}b+Aab^{2})x^{2}-4\,(Ba^{3}+3\,Aa^{2}b)x}{12\,x^{4}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*B*b^3*x^4*log(x) - 3*A*a^3 - 12*(3*B*a*b^2 + A*b^3)*x^3 - 18*(B*a
^2*b + A*a*b^2)*x^2 - 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^3(A+Bx)}{x^5}\,dx = Bb^3\log{(x)} + \frac{-3Aa^3 + x^3(-12Ab^3 - 36Bab^2) + x^2(-18Aab^2 - 18Ba^2b) + x(-12Aa^2b - 4Ba^3)}{12x^4}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**5,x)
```

output
```
B*b**3*log(x) + (-3*A*a**3 + x**3*(-12*A*b**3 - 36*B*a*b**2) + x**2*(-18*A
*a*b**2 - 18*B*a**2*b) + x*(-12*A*a**2*b - 4*B*a**3))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^3(A+Bx)}{x^5}\,dx = Bb^3\log{(x)} - \frac{3\,Aa^3 + 12\,(3\,Bab^2 + Ab^3)x^3 + 18\,(Ba^2b + Aab^2)x^2 + 4\,(Ba^3 + 3\,Aa^2b)x}{12\,x^4}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
B*b^3*log(x) - 1/12*(3*A*a^3 + 12*(3*B*a*b^2 + A*b^3)*x^3 + 18*(B*a^2*b +
A*a*b^2)*x^2 + 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)^3(A+Bx)}{x^5}\,dx = Bb^3\log(|x|) - \frac{3\,Aa^3 + 12\,(3\,Bab^2 + Ab^3)x^3 + 18\,(Ba^2b + Aab^2)x^2 + 4\,(Ba^3 + 3\,Aa^2b)x}{12\,x^4}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
B*b^3*log(abs(x)) - 1/12*(3*A*a^3 + 12*(3*B*a*b^2 + A*b^3)*x^3 + 18*(B*a^2
*b + A*a*b^2)*x^2 + 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^3(A+Bx)}{x^5}\,dx = B\,b^3\,\ln(x) - \frac{x^2\left(\frac{3\,B\,a^2\,b}{2} + \frac{3\,A\,a\,b^2}{2}\right) + x\left(\frac{B\,a^3}{3} + A\,b\,a^2\right) + \frac{A\,a^3}{4} + x^3\left(A\,b^3 + 3\,B\,a\,b^2\right)}{x^4}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^5,x)
```

output
```
B*b^3*log(x) - (x^2*((3*A*a*b^2)/2 + (3*B*a^2*b)/2) + x*((B*a^3)/3 + A*a^2
*b) + (A*a^3)/4 + x^3*(A*b^3 + 3*B*a*b^2))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3(A+Bx)}{x^5}\,dx = \frac{12\log(x)\,b^4x^4 - 3a^4 - 16a^3bx - 36a^2b^2x^2 - 48a\,b^3x^3}{12x^4}$$

input
```
int((b*x+a)^3*(B*x+A)/x^5,x)
```

output
```
(12*log(x)*b**4*x**4 - 3*a**4 - 16*a**3*b*x - 36*a**2*b**2*x**2 - 48*a*b**
3*x**3)/(12*x**4)
```

3.83 $\int \frac{(a+bx)^3(A+Bx)}{x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx)^3(A+Bx)}{x^6} dx = -\frac{A(a+bx)^4}{5ax^5} + \frac{(Ab-5aB)(a+bx)^4}{20a^2x^4}$$

output
```
-1/5*A*(b*x+a)^4/a/x^5+1/20*(A*b-5*B*a)*(b*x+a)^4/a^2/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx)^3(A+Bx)}{x^6} dx = -\frac{10b^3x^3(A+2Bx)+10ab^2x^2(2A+3Bx)+5a^2bx(3A+4Bx)+a^3(4A+5Bx)}{20x^5}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^6,x]
```

output
```
-1/20*(10*b^3*x^3*(A + 2*B*x) + 10*a*b^2*x^2*(2*A + 3*B*x) + 5*a^2*b*x*(3*
A + 4*B*x) + a^3*(4*A + 5*B*x))/x^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^6} \, dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(Ab-5aB)\int \frac{(a+bx)^3}{x^5}dx}{5a}-\frac{A(a+bx)^4}{5ax^5}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(a+bx)^4(Ab-5aB)}{20a^2x^4}-\frac{A(a+bx)^4}{5ax^5}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^6,x]
```

output
```
-1/5*(A*(a + b*x)^4)/(a*x^5) + ((A*b - 5*a*B)*(a + b*x)^4)/(20*a^2*x^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{ab(Ab+Ba)}{x^{3}}-\frac{a^{3}A}{5x^{5}}-\frac{b^{2}(Ab+3Ba)}{2x^{2}}-\frac{a^{2}(3Ab+Ba)}{4x^{4}}-\frac{b^{3}B}{x}$	66
norman	$\frac{-B\,b^{3}x^{4}+\left(-\frac{1}{2}b^{3}A-\frac{3}{2}a\,b^{2}B\right)x^{3}+\left(-a\,b^{2}A-a^{2}bB\right)x^{2}+\left(-\frac{3}{4}a^{2}bA-\frac{1}{4}a^{3}B\right)x-\frac{a^{3}A}{5}}{x^{5}}$	74
risch	$\frac{-B\,b^{3}x^{4}+\left(-\frac{1}{2}b^{3}A-\frac{3}{2}a\,b^{2}B\right)x^{3}+\left(-a\,b^{2}A-a^{2}bB\right)x^{2}+\left(-\frac{3}{4}a^{2}bA-\frac{1}{4}a^{3}B\right)x-\frac{a^{3}A}{5}}{x^{5}}$	74
gosper	$-\frac{20B\,b^{3}x^{4}+10A\,b^{3}x^{3}+30Ba\,b^{2}x^{3}+20aA\,b^{2}x^{2}+20B\,a^{2}b\,x^{2}+15a^{2}Abx+5B\,a^{3}x+4a^{3}A}{20x^{5}}$	76
parallelrisch	$-\frac{20B\,b^{3}x^{4}+10A\,b^{3}x^{3}+30Ba\,b^{2}x^{3}+20aA\,b^{2}x^{2}+20B\,a^{2}b\,x^{2}+15a^{2}Abx+5B\,a^{3}x+4a^{3}A}{20x^{5}}$	76
orering	$-\frac{20B\,b^{3}x^{4}+10A\,b^{3}x^{3}+30Ba\,b^{2}x^{3}+20aA\,b^{2}x^{2}+20B\,a^{2}b\,x^{2}+15a^{2}Abx+5B\,a^{3}x+4a^{3}A}{20x^{5}}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-a*b*(A*b+B*a)/x^3-1/5*a^3*A/x^5-1/2*b^2*(A*b+3*B*a)/x^2-1/4*a^2*(3*A*b+B*
a)/x^4-b^3*B/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^3(A+Bx)}{x^6}\,dx = -\frac{20\,Bb^3x^4+4\,Aa^3+10\,(3\,Bab^2+Ab^3)x^3+20\,(Ba^2b+Aab^2)x^2+5\,(Ba^3+3\,Aa^2b)x}{20\,x^5}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
-1/20*(20*B*b^3*x^4 + 4*A*a^3 + 10*(3*B*a*b^2 + A*b^3)*x^3 + 20*(B*a^2*b +
 A*a*b^2)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.71 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^3(A+Bx)}{x^6}\,dx = \frac{-4Aa^3 - 20Bb^3x^4 + x^3(-10Ab^3 - 30Bab^2) + x^2(-20Aab^2 - 20Ba^2b) + x(-15Aa^2b - 5Ba^3)}{20x^5}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**6,x)
```

output
```
(-4*A*a**3 - 20*B*b**3*x**4 + x**3*(-10*A*b**3 - 30*B*a*b**2) + x**2*(-20*
A*a*b**2 - 20*B*a**2*b) + x*(-15*A*a**2*b - 5*B*a**3))/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^3(A+Bx)}{x^6}\,dx = -\frac{20\,Bb^3x^4 + 4\,Aa^3 + 10\,(3\,Bab^2 + Ab^3)x^3 + 20\,(Ba^2b + Aab^2)x^2 + 5\,(Ba^3 + 3\,Aa^2b)x}{20\,x^5}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/20*(20*B*b^3*x^4 + 4*A*a^3 + 10*(3*B*a*b^2 + A*b^3)*x^3 + 20*(B*a^2*b +
 A*a*b^2)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^3(A+Bx)}{x^6}\,dx = -\frac{20\,Bb^3x^4+30\,Bab^2x^3+10\,Ab^3x^3+20\,Ba^2bx^2+20\,Aab^2x^2+5\,Ba^3x+15\,Aa^2bx+4\,Aa^3}{20\,x^5}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
-1/20*(20*B*b^3*x^4 + 30*B*a*b^2*x^3 + 10*A*b^3*x^3 + 20*B*a^2*b*x^2 + 20*
A*a*b^2*x^2 + 5*B*a^3*x + 15*A*a^2*b*x + 4*A*a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^3(A+Bx)}{x^6}\,dx = -\frac{x^2\,(B\,a^2\,b+A\,a\,b^2)+x\,\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+\frac{A\,a^3}{5}+x^3\,\left(\frac{A\,b^3}{2}+\frac{3\,B\,a\,b^2}{2}\right)+B\,b^3\,x^4}{x^5}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^6,x)
```

output
```
-(x^2*(A*a*b^2 + B*a^2*b) + x*((B*a^3)/4 + (3*A*a^2*b)/4) + (A*a^3)/5 + x^
3*((A*b^3)/2 + (3*B*a*b^2)/2) + B*b^3*x^4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^3(A+Bx)}{x^6}\,dx = \frac{-5b^4x^4-10a\,b^3x^3-10a^2b^2x^2-5a^3bx-a^4}{5x^5}$$

input
```
int((b*x+a)^3*(B*x+A)/x^6,x)
```

output
```
( - a**4 - 5*a**3*b*x - 10*a**2*b**2*x**2 - 10*a*b**3*x**3 - 5*b**4*x**4)/
(5*x**5)
```

3.84 $\int \frac{(a+bx)^3(A+Bx)}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{(a+bx)^3(A+Bx)}{x^7} dx = -\frac{a^3A}{6x^6} - \frac{a^2(3Ab+aB)}{5x^5} - \frac{3ab(Ab+aB)}{4x^4} - \frac{b^2(Ab+3aB)}{3x^3} - \frac{b^3B}{2x^2}$$

output
```
-1/6*a^3*A/x^6-1/5*a^2*(3*A*b+B*a)/x^5-3/4*a*b*(A*b+B*a)/x^4-1/3*b^2*(A*b+
3*B*a)/x^3-1/2*b^3*B/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3(A+Bx)}{x^7} dx = -\frac{10b^3x^3(2A+3Bx)+15ab^2x^2(3A+4Bx)+9a^2bx(4A+5Bx)+2a^3(5A+6Bx)}{60x^6}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^7,x]
```

output
```
-1/60*(10*b^3*x^3*(2*A + 3*B*x) + 15*a*b^2*x^2*(3*A + 4*B*x) + 9*a^2*b*x*(
4*A + 5*B*x) + 2*a^3*(5*A + 6*B*x))/x^6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{a^3A}{x^7}+\frac{a^2(aB+3Ab)}{x^6}+\frac{b^2(3aB+Ab)}{x^4}+\frac{3ab(aB+Ab)}{x^5}+\frac{b^3B}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^3A}{6x^6}-\frac{a^2(aB+3Ab)}{5x^5}-\frac{b^2(3aB+Ab)}{3x^3}-\frac{3ab(aB+Ab)}{4x^4}-\frac{b^3B}{2x^2}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^7,x]
```

output
```
-1/6*(a^3*A)/x^6 - (a^2*(3*A*b + a*B))/(5*x^5) - (3*a*b*(A*b + a*B))/(4*x^
4) - (b^2*(A*b + 3*a*B))/(3*x^3) - (b^3*B)/(2*x^2)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^3 A}{6x^6}-\frac{a^2(3Ab+Ba)}{5x^5}-\frac{3ab(Ab+Ba)}{4x^4}-\frac{b^2(Ab+3Ba)}{3x^3}-\frac{b^3 B}{2x^2}$	66
norman	$\frac{-\frac{B\,b^3x^4}{2}+(-\frac{1}{3}b^3A-a\,b^2B)x^3+(-\frac{3}{4}a\,b^2A-\frac{3}{4}a^2bB)x^2+(-\frac{3}{5}a^2bA-\frac{1}{5}a^3B)x-\frac{a^3A}{6}}{x^6}$	74
risch	$\frac{-\frac{B\,b^3x^4}{2}+(-\frac{1}{3}b^3A-a\,b^2B)x^3+(-\frac{3}{4}a\,b^2A-\frac{3}{4}a^2bB)x^2+(-\frac{3}{5}a^2bA-\frac{1}{5}a^3B)x-\frac{a^3A}{6}}{x^6}$	74
gosper	$-\frac{30B\,b^3x^4+20A\,b^3x^3+60Ba\,b^2x^3+45aA\,b^2x^2+45B\,a^2b\,x^2+36a^2Abx+12B\,a^3x+10a^3A}{60x^6}$	76
parallelrisch	$-\frac{30B\,b^3x^4+20A\,b^3x^3+60Ba\,b^2x^3+45aA\,b^2x^2+45B\,a^2b\,x^2+36a^2Abx+12B\,a^3x+10a^3A}{60x^6}$	76
orering	$-\frac{30B\,b^3x^4+20A\,b^3x^3+60Ba\,b^2x^3+45aA\,b^2x^2+45B\,a^2b\,x^2+36a^2Abx+12B\,a^3x+10a^3A}{60x^6}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^3*A/x^6-1/5*a^2*(3*A*b+B*a)/x^5-3/4*a*b*(A*b+B*a)/x^4-1/3*b^2*(A*b+
3*B*a)/x^3-1/2*b^3*B/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx = -\frac{30\,Bb^3x^4+10\,Aa^3+20\,(3\,Bab^2+Ab^3)x^3+45\,(Ba^2b+Aab^2)x^2+12\,(Ba^3+3\,Aa^2b)x}{60\,x^6}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(30*B*b^3*x^4 + 10*A*a^3 + 20*(3*B*a*b^2 + A*b^3)*x^3 + 45*(B*a^2*b
+ A*a*b^2)*x^2 + 12*(B*a^3 + 3*A*a^2*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx = \frac{-10Aa^3 - 30Bb^3x^4 + x^3(-20Ab^3 - 60Bab^2) + x^2(-45Aab^2 - 45Ba^2b) + x(-36Aa^2b - 12Ba^3)}{60x^6}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**7,x)
```

output
```
(-10*A*a**3 - 30*B*b**3*x**4 + x**3*(-20*A*b**3 - 60*B*a*b**2) + x**2*(-45
*A*a*b**2 - 45*B*a**2*b) + x*(-36*A*a**2*b - 12*B*a**3))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx = -\frac{30\,Bb^3x^4 + 10\,Aa^3 + 20\,(3\,Bab^2 + Ab^3)x^3 + 45\,(Ba^2b + Aab^2)x^2 + 12\,(Ba^3 + 3\,Aa^2b)x}{60\,x^6}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^7,x, algorithm="maxima")
```

output
```
-1/60*(30*B*b^3*x^4 + 10*A*a^3 + 20*(3*B*a*b^2 + A*b^3)*x^3 + 45*(B*a^2*b
+ A*a*b^2)*x^2 + 12*(B*a^3 + 3*A*a^2*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx = -\frac{30\,Bb^3x^4+60\,Bab^2x^3+20\,Ab^3x^3+45\,Ba^2bx^2+45\,Aab^2x^2+12\,Ba^3x+36\,Aa^2bx+10\,Aa^3}{60\,x^6}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^7,x, algorithm="giac")
```

output
```
-1/60*(30*B*b^3*x^4 + 60*B*a*b^2*x^3 + 20*A*b^3*x^3 + 45*B*a^2*b*x^2 + 45*
A*a*b^2*x^2 + 12*B*a^3*x + 36*A*a^2*b*x + 10*A*a^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx = -\frac{x^2\left(\frac{3\,B\,a^2\,b}{4}+\frac{3\,A\,a\,b^2}{4}\right)+x\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+\frac{A\,a^3}{6}+x^3\left(\frac{A\,b^3}{3}+B\,a\,b^2\right)+\frac{B\,b^3\,x^4}{2}}{x^6}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^7,x)
```

output
```
-(x^2*((3*A*a*b^2)/4 + (3*B*a^2*b)/4) + x*((B*a^3)/5 + (3*A*a^2*b)/5) + (A
*a^3)/6 + x^3*((A*b^3)/3 + B*a*b^2) + (B*b^3*x^4)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx)^3(A+Bx)}{x^7}\,dx = \frac{-15b^4x^4 - 40a\,b^3x^3 - 45a^2b^2x^2 - 24a^3bx - 5a^4}{30x^6}$$

input
```
int((b*x+a)^3*(B*x+A)/x^7,x)
```

output
```
( - 5*a**4 - 24*a**3*b*x - 45*a**2*b**2*x**2 - 40*a*b**3*x**3 - 15*b**4*x*
*4)/(30*x**6)
```

3.85 $\int \frac{(a+bx)^3(A+Bx)}{x^8} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{(a+bx)^3(A+Bx)}{x^8} \, dx = -\frac{a^3A}{7x^7} - \frac{a^2(3Ab+aB)}{6x^6} - \frac{3ab(Ab+aB)}{5x^5} - \frac{b^2(Ab+3aB)}{4x^4} - \frac{b^3B}{3x^3}$$

output
```
-1/7*a^3*A/x^7-1/6*a^2*(3*A*b+B*a)/x^6-3/5*a*b*(A*b+B*a)/x^5-1/4*b^2*(A*b+
3*B*a)/x^4-1/3*b^3*B/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3(A+Bx)}{x^8} \, dx = -\frac{35b^3x^3(3A+4Bx)+63ab^2x^2(4A+5Bx)+42a^2bx(5A+6Bx)+10a^3(6A+7Bx)}{420x^7}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^8,x]
```

output
```
-1/420*(35*b^3*x^3*(3*A + 4*B*x) + 63*a*b^2*x^2*(4*A + 5*B*x) + 42*a^2*b*x
*(5*A + 6*B*x) + 10*a^3*(6*A + 7*B*x))/x^7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^8} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3A}{x^8}+\frac{a^2(aB+3Ab)}{x^7}+\frac{b^2(3aB+Ab)}{x^5}+\frac{3ab(aB+Ab)}{x^6}+\frac{b^3B}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{7x^7}-\frac{a^2(aB+3Ab)}{6x^6}-\frac{b^2(3aB+Ab)}{4x^4}-\frac{3ab(aB+Ab)}{5x^5}-\frac{b^3B}{3x^3}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^8,x]
```

output
```
-1/7*(a^3*A)/x^7 - (a^2*(3*A*b + a*B))/(6*x^6) - (3*a*b*(A*b + a*B))/(5*x^
5) - (b^2*(A*b + 3*a*B))/(4*x^4) - (b^3*B)/(3*x^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3} A}{7 x^{7}}-\frac{a^{2} \left(3 A b+B a\right)}{6 x^{6}}-\frac{3 a b \left(A b+B a\right)}{5 x^{5}}-\frac{b^{2} \left(A b+3 B a\right)}{4 x^{4}}-\frac{b^{3} B}{3 x^{3}}$	66
norman	$\frac{-\frac{B\,b^{3} x^{4}}{3}+\left(-\frac{1}{4} b^{3} A-\frac{3}{4} a\,b^{2} B\right) x^{3}+\left(-\frac{3}{5} a\,b^{2} A-\frac{3}{5} a^{2} b B\right) x^{2}+\left(-\frac{1}{2} a^{2} b A-\frac{1}{6} a^{3} B\right) x-\frac{a^{3} A}{7}}{x^{7}}$	74
risch	$\frac{-\frac{B\,b^{3} x^{4}}{3}+\left(-\frac{1}{4} b^{3} A-\frac{3}{4} a\,b^{2} B\right) x^{3}+\left(-\frac{3}{5} a\,b^{2} A-\frac{3}{5} a^{2} b B\right) x^{2}+\left(-\frac{1}{2} a^{2} b A-\frac{1}{6} a^{3} B\right) x-\frac{a^{3} A}{7}}{x^{7}}$	74
gosper	$-\frac{140 B\,b^{3} x^{4}+105 A\,b^{3} x^{3}+315 B a\,b^{2} x^{3}+252 a A\,b^{2} x^{2}+252 B\,a^{2} b\,x^{2}+210 a^{2} A b x+70 B\,a^{3} x+60 a^{3} A}{420 x^{7}}$	76
parallelrisch	$-\frac{140 B\,b^{3} x^{4}+105 A\,b^{3} x^{3}+315 B a\,b^{2} x^{3}+252 a A\,b^{2} x^{2}+252 B\,a^{2} b\,x^{2}+210 a^{2} A b x+70 B\,a^{3} x+60 a^{3} A}{420 x^{7}}$	76
orering	$-\frac{140 B\,b^{3} x^{4}+105 A\,b^{3} x^{3}+315 B a\,b^{2} x^{3}+252 a A\,b^{2} x^{2}+252 B\,a^{2} b\,x^{2}+210 a^{2} A b x+70 B\,a^{3} x+60 a^{3} A}{420 x^{7}}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^3*A/x^7-1/6*a^2*(3*A*b+B*a)/x^6-3/5*a*b*(A*b+B*a)/x^5-1/4*b^2*(A*b+
3*B*a)/x^4-1/3*b^3*B/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^8}\,dx = -\frac{140\,Bb^3x^4+60\,Aa^3+105\,(3\,Bab^2+Ab^3)x^3+252\,(Ba^2b+Aab^2)x^2+70\,(Ba^3+3\,Aa^2b)x}{420\,x^7}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^8,x, algorithm="fricas")
```

output
```
-1/420*(140*B*b^3*x^4 + 60*A*a^3 + 105*(3*B*a*b^2 + A*b^3)*x^3 + 252*(B*a^
2*b + A*a*b^2)*x^2 + 70*(B*a^3 + 3*A*a^2*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^8}\,dx = \frac{-60Aa^3 - 140Bb^3x^4 + x^3(-105Ab^3 - 315Bab^2) + x^2(-252Aab^2 - 252Ba^2b) + x(-210Aa^2b - 70Ba^3)}{420x^7}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**8,x)
```

output
```
(-60*A*a**3 - 140*B*b**3*x**4 + x**3*(-105*A*b**3 - 315*B*a*b**2) + x**2*(
-252*A*a*b**2 - 252*B*a**2*b) + x*(-210*A*a**2*b - 70*B*a**3))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^8}\,dx = -\frac{140\,Bb^3x^4 + 60\,Aa^3 + 105\,(3\,Bab^2 + Ab^3)x^3 + 252\,(Ba^2b + Aab^2)x^2 + 70\,(Ba^3 + 3\,Aa^2b)x}{420\,x^7}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^8,x, algorithm="maxima")
```

output
```
-1/420*(140*B*b^3*x^4 + 60*A*a^3 + 105*(3*B*a*b^2 + A*b^3)*x^3 + 252*(B*a^
2*b + A*a*b^2)*x^2 + 70*(B*a^3 + 3*A*a^2*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3(A+Bx)}{x^8}\,dx = -\frac{140\,Bb^3x^4+315\,Bab^2x^3+105\,Ab^3x^3+252\,Ba^2bx^2+252\,Aab^2x^2+70\,Ba^3x+210\,Aa^2bx+60\,Aa^3}{420\,x^7}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^8,x, algorithm="giac")
```

output
```
-1/420*(140*B*b^3*x^4 + 315*B*a*b^2*x^3 + 105*A*b^3*x^3 + 252*B*a^2*b*x^2
+ 252*A*a*b^2*x^2 + 70*B*a^3*x + 210*A*a^2*b*x + 60*A*a^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^3(A+Bx)}{x^8}\,dx = -\frac{x^2\left(\frac{3\,B\,a^2\,b}{5}+\frac{3\,A\,a\,b^2}{5}\right)+x\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+\frac{A\,a^3}{7}+x^3\left(\frac{A\,b^3}{4}+\frac{3\,B\,a\,b^2}{4}\right)+\frac{B\,b^3\,x^4}{3}}{x^7}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^8,x)
```

output
```
-(x^2*((3*A*a*b^2)/5 + (3*B*a^2*b)/5) + x*((B*a^3)/6 + (A*a^2*b)/2) + (A*a
^3)/7 + x^3*((A*b^3)/4 + (3*B*a*b^2)/4) + (B*b^3*x^4)/3)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx)^3(A+Bx)}{x^8}\,dx = \frac{-35b^4x^4-105a\,b^3x^3-126a^2b^2x^2-70a^3bx-15a^4}{105x^7}$$

input
```
int((b*x+a)^3*(B*x+A)/x^8,x)
```

output
```
( - 15*a**4 - 70*a**3*b*x - 126*a**2*b**2*x**2 - 105*a*b**3*x**3 - 35*b**4
*x**4)/(105*x**7)
```

3.86 $\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = -\frac{a^3A}{8x^8}-\frac{a^2(3Ab+aB)}{7x^7}-\frac{ab(Ab+aB)}{2x^6}-\frac{b^2(Ab+3aB)}{5x^5}-\frac{b^3B}{4x^4}$$

output
```
-1/8*a^3*A/x^8-1/7*a^2*(3*A*b+B*a)/x^7-1/2*a*b*(A*b+B*a)/x^6-1/5*b^2*(A*b+
3*B*a)/x^5-1/4*b^3*B/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx$$
$$= -\frac{14b^3x^3(4A+5Bx)+28ab^2x^2(5A+6Bx)+20a^2bx(6A+7Bx)+5a^3(7A+8Bx)}{280x^8}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^9,x]
```

output
```
-1/280*(14*b^3*x^3*(4*A + 5*B*x) + 28*a*b^2*x^2*(5*A + 6*B*x) + 20*a^2*b*x
*(6*A + 7*B*x) + 5*a^3*(7*A + 8*B*x))/x^8
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^9} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3A}{x^9}+\frac{a^2(aB+3Ab)}{x^8}+\frac{b^2(3aB+Ab)}{x^6}+\frac{3ab(aB+Ab)}{x^7}+\frac{b^3B}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{8x^8}-\frac{a^2(aB+3Ab)}{7x^7}-\frac{b^2(3aB+Ab)}{5x^5}-\frac{ab(aB+Ab)}{2x^6}-\frac{b^3B}{4x^4}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^9,x]
```

output
```
-1/8*(a^3*A)/x^8 - (a^2*(3*A*b + a*B))/(7*x^7) - (a*b*(A*b + a*B))/(2*x^6)
 - (b^2*(A*b + 3*a*B))/(5*x^5) - (b^3*B)/(4*x^4)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^3A}{8x^8}-\frac{a^2(3Ab+Ba)}{7x^7}-\frac{ab(Ab+Ba)}{2x^6}-\frac{b^2(Ab+3Ba)}{5x^5}-\frac{b^3B}{4x^4}$	66
norman	$\frac{-\frac{B\,b^3x^4}{4}+(-\frac{1}{5}b^3A-\frac{3}{5}a\,b^2B)x^3+(-\frac{1}{2}a\,b^2A-\frac{1}{2}a^2bB)x^2+(-\frac{3}{7}a^2bA-\frac{1}{7}a^3B)x-\frac{a^3A}{8}}{x^8}$	74
risch	$\frac{-\frac{B\,b^3x^4}{4}+(-\frac{1}{5}b^3A-\frac{3}{5}a\,b^2B)x^3+(-\frac{1}{2}a\,b^2A-\frac{1}{2}a^2bB)x^2+(-\frac{3}{7}a^2bA-\frac{1}{7}a^3B)x-\frac{a^3A}{8}}{x^8}$	74
gosper	$-\frac{70B\,b^3x^4+56A\,b^3x^3+168Ba\,b^2x^3+140aA\,b^2x^2+140B\,a^2b\,x^2+120a^2Abx+40B\,a^3x+35a^3A}{280x^8}$	76
parallelrisch	$-\frac{70B\,b^3x^4+56A\,b^3x^3+168Ba\,b^2x^3+140aA\,b^2x^2+140B\,a^2b\,x^2+120a^2Abx+40B\,a^3x+35a^3A}{280x^8}$	76
orering	$-\frac{70B\,b^3x^4+56A\,b^3x^3+168Ba\,b^2x^3+140aA\,b^2x^2+140B\,a^2b\,x^2+120a^2Abx+40B\,a^3x+35a^3A}{280x^8}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^3*A/x^8-1/7*a^2*(3*A*b+B*a)/x^7-1/2*a*b*(A*b+B*a)/x^6-1/5*b^2*(A*b+
3*B*a)/x^5-1/4*b^3*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = -\frac{70\,Bb^3x^4+35\,Aa^3+56\,(3\,Bab^2+Ab^3)x^3+140\,(Ba^2b+Aab^2)x^2+40\,(Ba^3+3\,Aa^2b)x}{280\,x^8}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^9,x, algorithm="fricas")
```

output
```
-1/280*(70*B*b^3*x^4 + 35*A*a^3 + 56*(3*B*a*b^2 + A*b^3)*x^3 + 140*(B*a^2*
b + A*a*b^2)*x^2 + 40*(B*a^3 + 3*A*a^2*b)*x)/x^8
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = \frac{-35Aa^3-70Bb^3x^4+x^3(-56Ab^3-168Bab^2)+x^2(-140Aab^2-140Ba^2b)+x(-120Aa^2b-40Ba^3)}{280x^8}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**9,x)
```

output
```
(-35*A*a**3 - 70*B*b**3*x**4 + x**3*(-56*A*b**3 - 168*B*a*b**2) + x**2*(-1
40*A*a*b**2 - 140*B*a**2*b) + x*(-120*A*a**2*b - 40*B*a**3))/(280*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = -\frac{70\,Bb^3x^4+35\,Aa^3+56\,(3\,Bab^2+Ab^3)x^3+140\,(Ba^2b+Aab^2)x^2+40\,(Ba^3+3\,Aa^2b)x}{280\,x^8}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^9,x, algorithm="maxima")
```

output
```
-1/280*(70*B*b^3*x^4 + 35*A*a^3 + 56*(3*B*a*b^2 + A*b^3)*x^3 + 140*(B*a^2*
b + A*a*b^2)*x^2 + 40*(B*a^3 + 3*A*a^2*b)*x)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = -\frac{70\,Bb^3x^4+168\,Bab^2x^3+56\,Ab^3x^3+140\,Ba^2bx^2+140\,Aab^2x^2+40\,Ba^3x+120\,Aa^2bx+35\,Aa^3}{280\,x^8}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^9,x, algorithm="giac")
```

output
```
-1/280*(70*B*b^3*x^4 + 168*B*a*b^2*x^3 + 56*A*b^3*x^3 + 140*B*a^2*b*x^2 +
140*A*a*b^2*x^2 + 40*B*a^3*x + 120*A*a^2*b*x + 35*A*a^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = -\frac{x^2\left(\frac{B\,a^2\,b}{2}+\frac{A\,a\,b^2}{2}\right)+x\left(\frac{B\,a^3}{7}+\frac{3\,A\,b\,a^2}{7}\right)+\frac{A\,a^3}{8}+x^3\left(\frac{A\,b^3}{5}+\frac{3\,B\,a\,b^2}{5}\right)+\frac{B\,b^3\,x^4}{4}}{x^8}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^9,x)
```

output
```
-(x^2*((A*a*b^2)/2 + (B*a^2*b)/2) + x*((B*a^3)/7 + (3*A*a^2*b)/7) + (A*a^3
)/8 + x^3*((A*b^3)/5 + (3*B*a*b^2)/5) + (B*b^3*x^4)/4)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx)^3(A+Bx)}{x^9}\,dx = \frac{-70b^4x^4-224a\,b^3x^3-280a^2b^2x^2-160a^3bx-35a^4}{280x^8}$$

input
```
int((b*x+a)^3*(B*x+A)/x^9,x)
```

output
```
( - 35*a**4 - 160*a**3*b*x - 280*a**2*b**2*x**2 - 224*a*b**3*x**3 - 70*b**
4*x**4)/(280*x**8)
```

3.87 $\int \frac{(a+bx)^3(A+Bx)}{x^{10}} dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}} dx = -\frac{a^3A}{9x^9} - \frac{a^2(3Ab+aB)}{8x^8} - \frac{3ab(Ab+aB)}{7x^7} - \frac{b^2(Ab+3aB)}{6x^6} - \frac{b^3B}{5x^5}$$

output
```
-1/9*a^3*A/x^9-1/8*a^2*(3*A*b+B*a)/x^8-3/7*a*b*(A*b+B*a)/x^7-1/6*b^2*(A*b+
3*B*a)/x^6-1/5*b^3*B/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}} dx = -\frac{84b^3x^3(5A+6Bx)+180ab^2x^2(6A+7Bx)+135a^2bx(7A+8Bx)+35a^3(8A+9Bx)}{2520x^9}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^10,x]
```

output
```
-1/2520*(84*b^3*x^3*(5*A + 6*B*x) + 180*a*b^2*x^2*(6*A + 7*B*x) + 135*a^2*
b*x*(7*A + 8*B*x) + 35*a^3*(8*A + 9*B*x))/x^9
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3A}{x^{10}}+\frac{a^2(aB+3Ab)}{x^9}+\frac{b^2(3aB+Ab)}{x^7}+\frac{3ab(aB+Ab)}{x^8}+\frac{b^3B}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{9x^9}-\frac{a^2(aB+3Ab)}{8x^8}-\frac{b^2(3aB+Ab)}{6x^6}-\frac{3ab(aB+Ab)}{7x^7}-\frac{b^3B}{5x^5}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^10,x]
```

output
```
-1/9*(a^3*A)/x^9 - (a^2*(3*A*b + a*B))/(8*x^8) - (3*a*b*(A*b + a*B))/(7*x^
7) - (b^2*(A*b + 3*a*B))/(6*x^6) - (b^3*B)/(5*x^5)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3} A}{9 x^{9}}-\frac{a^{2} \left(3 A b +B a \right)}{8 x^{8}}-\frac{3 a b \left(A b +B a \right)}{7 x^{7}}-\frac{b^{2} \left(A b +3 B a \right)}{6 x^{6}}-\frac{b^{3} B}{5 x^{5}}$	66
norman	$\frac{-\frac{B \,b^{3} x^{4}}{5}+\left(-\frac{1}{6} b^{3} A -\frac{1}{2} a \,b^{2} B \right) x^{3}+\left(-\frac{3}{7} a \,b^{2} A -\frac{3}{7} a^{2} b B \right) x^{2}+\left(-\frac{3}{8} a^{2} b A -\frac{1}{8} a^{3} B \right) x -\frac{a^{3} A}{9}}{x^{9}}$	74
risch	$\frac{-\frac{B \,b^{3} x^{4}}{5}+\left(-\frac{1}{6} b^{3} A -\frac{1}{2} a \,b^{2} B \right) x^{3}+\left(-\frac{3}{7} a \,b^{2} A -\frac{3}{7} a^{2} b B \right) x^{2}+\left(-\frac{3}{8} a^{2} b A -\frac{1}{8} a^{3} B \right) x -\frac{a^{3} A}{9}}{x^{9}}$	74
gosper	$-\frac{504 B \,b^{3} x^{4}+420 A \,b^{3} x^{3}+1260 B a \,b^{2} x^{3}+1080 a A \,b^{2} x^{2}+1080 B \,a^{2} b \,x^{2}+945 a^{2} A b x +315 B \,a^{3} x +280 a^{3} A}{2520 x^{9}}$	76
parallelrisch	$-\frac{504 B \,b^{3} x^{4}+420 A \,b^{3} x^{3}+1260 B a \,b^{2} x^{3}+1080 a A \,b^{2} x^{2}+1080 B \,a^{2} b \,x^{2}+945 a^{2} A b x +315 B \,a^{3} x +280 a^{3} A}{2520 x^{9}}$	76
orering	$-\frac{504 B \,b^{3} x^{4}+420 A \,b^{3} x^{3}+1260 B a \,b^{2} x^{3}+1080 a A \,b^{2} x^{2}+1080 B \,a^{2} b \,x^{2}+945 a^{2} A b x +315 B \,a^{3} x +280 a^{3} A}{2520 x^{9}}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^3*A/x^9-1/8*a^2*(3*A*b+B*a)/x^8-3/7*a*b*(A*b+B*a)/x^7-1/6*b^2*(A*b+
3*B*a)/x^6-1/5*b^3*B/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx = -\frac{504\,Bb^3x^4+280\,Aa^3+420\,(3\,Bab^2+Ab^3)x^3+1080\,(Ba^2b+Aab^2)x^2+315\,(Ba^3+3\,Aa^2b)x}{2520\,x^9}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^10,x, algorithm="fricas")
```

output
```
-1/2520*(504*B*b^3*x^4 + 280*A*a^3 + 420*(3*B*a*b^2 + A*b^3)*x^3 + 1080*(B
*a^2*b + A*a*b^2)*x^2 + 315*(B*a^3 + 3*A*a^2*b)*x)/x^9
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx = \frac{-280Aa^3 - 504Bb^3x^4 + x^3(-420Ab^3 - 1260Bab^2) + x^2(-1080Aab^2 - 1080Ba^2b) + x(-945Aa^2b - 315}{2520x^9}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**10,x)
```

output
```
(-280*A*a**3 - 504*B*b**3*x**4 + x**3*(-420*A*b**3 - 1260*B*a*b**2) + x**2
*(-1080*A*a*b**2 - 1080*B*a**2*b) + x*(-945*A*a**2*b - 315*B*a**3))/(2520*
x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx = -\frac{504\,Bb^3x^4 + 280\,Aa^3 + 420\,(3\,Bab^2 + Ab^3)x^3 + 1080\,(Ba^2b + Aab^2)x^2 + 315\,(Ba^3 + 3\,Aa^2b)x}{2520\,x^9}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^10,x, algorithm="maxima")
```

output
```
-1/2520*(504*B*b^3*x^4 + 280*A*a^3 + 420*(3*B*a*b^2 + A*b^3)*x^3 + 1080*(B
*a^2*b + A*a*b^2)*x^2 + 315*(B*a^3 + 3*A*a^2*b)*x)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx = -\frac{504\,Bb^3x^4 + 1260\,Bab^2x^3 + 420\,Ab^3x^3 + 1080\,Ba^2bx^2 + 1080\,Aab^2x^2 + 315\,Ba^3x + 945\,Aa^2bx + 280}{2520\,x^9}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^10,x, algorithm="giac")
```

output
```
-1/2520*(504*B*b^3*x^4 + 1260*B*a*b^2*x^3 + 420*A*b^3*x^3 + 1080*B*a^2*b*x
^2 + 1080*A*a*b^2*x^2 + 315*B*a^3*x + 945*A*a^2*b*x + 280*A*a^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx = -\frac{x^2\left(\frac{3\,B\,a^2\,b}{7}+\frac{3\,A\,a\,b^2}{7}\right)+x\left(\frac{B\,a^3}{8}+\frac{3\,A\,b\,a^2}{8}\right)+\frac{A\,a^3}{9}+x^3\left(\frac{A\,b^3}{6}+\frac{B\,a\,b^2}{2}\right)+\frac{B\,b^3\,x^4}{5}}{x^9}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^10,x)
```

output
```
-(x^2*((3*A*a*b^2)/7 + (3*B*a^2*b)/7) + x*((B*a^3)/8 + (3*A*a^2*b)/8) + (A
*a^3)/9 + x^3*((A*b^3)/6 + (B*a*b^2)/2) + (B*b^3*x^4)/5)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx)^3(A+Bx)}{x^{10}}\,dx = \frac{-126b^4x^4 - 420a\,b^3x^3 - 540a^2b^2x^2 - 315a^3bx - 70a^4}{630x^9}$$

input
```
int((b*x+a)^3*(B*x+A)/x^10,x)
```

output
```
( - 70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b*
*4*x**4)/(630*x**9)
```

3.88 $\int x^5(a+bx)^5(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{1}{6}a^5Ax^6+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{1}{2}ab^3(Ab+2aB)x^{10} + \frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{12}b^5Bx^{12}$$

output
```
1/6*a^5*A*x^6+1/7*a^4*(5*A*b+B*a)*x^7+5/8*a^3*b*(2*A*b+B*a)*x^8+10/9*a^2*b
^2*(A*b+B*a)*x^9+1/2*a*b^3*(A*b+2*B*a)*x^10+1/11*b^4*(A*b+5*B*a)*x^11+1/12
*b^5*B*x^12
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{1}{6}a^5Ax^6+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{1}{2}ab^3(Ab+2aB)x^{10} + \frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{12}b^5Bx^{12}$$

input
```
Integrate[x^5*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^6)/6 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*
b + 5*a*B)*x^11)/11 + (b^5*B*x^12)/12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+bx)^5(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^5Ax^5+a^4x^6(aB+5Ab)+5a^3bx^7(aB+2Ab)+10a^2b^2x^8(aB+Ab)+b^4x^{10}(5aB+Ab)+5ab^3x^9(2aB+A$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^5Ax^6+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{12}b^5Bx^{12}$$

input
```
Int[x^5*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^6)/6 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*
b + 5*a*B)*x^11)/11 + (b^5*B*x^12)/12
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{b^5 B x^{12}}{12}+\left(\frac{1}{11} b^5 A+\frac{5}{11} a b^4 B\right) x^{11}+\left(\frac{1}{2} a b^4 A+a^2 b^3 B\right) x^{10}+\left(\frac{10}{9} a^2 b^3 A+\frac{10}{9} a^3 b^2 B\right) x^9+\left(\frac{5}{4} a^3\right.$
default	$\frac{b^5 B x^{12}}{12}+\frac{\left(b^5 A+5 a b^4 B\right) x^{11}}{11}+\frac{\left(5 a b^4 A+10 a^2 b^3 B\right) x^{10}}{10}+\frac{\left(10 a^2 b^3 A+10 a^3 b^2 B\right) x^9}{9}+\frac{\left(10 a^3 b^2 A+5 a^4 b B\right) x^8}{8}+\frac{(5 a^4 b}{}$
orering	$\frac{x^6 \left(462 B b^5 x^6+504 A b^5 x^5+2520 B a b^4 x^5+2772 a A b^4 x^4+5544 B a^2 b^3 x^4+6160 a^2 A b^3 x^3+6160 B a^3 b^2 x^3+6930 a^3 A b^2 x^2+346\right.}{5544}$
gosper	$\frac{1}{12} b^5 B x^{12}+\frac{1}{11} x^{11} b^5 A+\frac{5}{11} x^{11} a b^4 B+\frac{1}{2} x^{10} a b^4 A+x^{10} a^2 b^3 B+\frac{10}{9} x^9 a^2 b^3 A+\frac{10}{9} x^9 a^3 b^2 B+$
risch	$\frac{1}{12} b^5 B x^{12}+\frac{1}{11} x^{11} b^5 A+\frac{5}{11} x^{11} a b^4 B+\frac{1}{2} x^{10} a b^4 A+x^{10} a^2 b^3 B+\frac{10}{9} x^9 a^2 b^3 A+\frac{10}{9} x^9 a^3 b^2 B+$
parallelrisch	$\frac{1}{12} b^5 B x^{12}+\frac{1}{11} x^{11} b^5 A+\frac{5}{11} x^{11} a b^4 B+\frac{1}{2} x^{10} a b^4 A+x^{10} a^2 b^3 B+\frac{10}{9} x^9 a^2 b^3 A+\frac{10}{9} x^9 a^3 b^2 B+$

input
```
int(x^5*(b*x+a)^5*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^5*B*x^12+(1/11*b^5*A+5/11*a*b^4*B)*x^11+(1/2*a*b^4*A+a^2*b^3*B)*x^1
0+(10/9*a^2*b^3*A+10/9*a^3*b^2*B)*x^9+(5/4*a^3*b^2*A+5/8*a^4*b*B)*x^8+(5/7
*a^4*b*A+1/7*a^5*B)*x^7+1/6*a^5*A*x^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{1}{12}\,Bb^5x^{12} + \frac{1}{6}\,Aa^5x^6 + \frac{1}{11}\,\left(5\,Bab^4 + Ab^5\right)x^{11} + \frac{1}{2}\,\left(2\,Ba^2b^3 + Aab^4\right)x^{10} + \frac{10}{9}\,\left(Ba^3b^2 + Aa^2b^3\right)x^9 + \frac{5}{8}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^8 + \frac{1}{7}\,\left(Ba^5 + 5\,Aa^4b\right)x^7$$

input
```
integrate(x^5*(b*x+a)^5*(B*x+A),x, algorithm="fricas")
```

output
```
1/12*B*b^5*x^12 + 1/6*A*a^5*x^6 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 1/2*(2*B
*a^2*b^3 + A*a*b^4)*x^10 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/8*(B*a^4*b
 + 2*A*a^3*b^2)*x^8 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{Aa^5x^6}{6} + \frac{Bb^5x^{12}}{12} + x^{11}\left(\frac{Ab^5}{11} + \frac{5Bab^4}{11}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^9\cdot\left(\frac{10Aa^2b^3}{9} + \frac{10Ba^3b^2}{9}\right) + x^8\cdot\left(\frac{5Aa^3b^2}{4} + \frac{5Ba^4b}{8}\right) + x^7\cdot\left(\frac{5Aa^4b}{7} + \frac{Ba^5}{7}\right)$$

input
```
integrate(x**5*(b*x+a)**5*(B*x+A),x)
```

output
```
A*a**5*x**6/6 + B*b**5*x**12/12 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**1
0*(A*a*b**4/2 + B*a**2*b**3) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2/9)
+ x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{1}{12}\,Bb^5x^{12}+\frac{1}{6}\,Aa^5x^6+\frac{1}{11}\,\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{1}{2}\,\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{10}{9}\,\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{8}\,\left(Ba^4b+2\,Aa^3b^2\right)x^8+\frac{1}{7}\,\left(Ba^5+5\,Aa^4b\right)x^7$$

input
```
integrate(x^5*(b*x+a)^5*(B*x+A),x, algorithm="maxima")
```

output
```
1/12*B*b^5*x^12 + 1/6*A*a^5*x^6 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 1/2*(2*B
*a^2*b^3 + A*a*b^4)*x^10 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/8*(B*a^4*b
 + 2*A*a^3*b^2)*x^8 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{1}{12}\,Bb^5x^{12}+\frac{5}{11}\,Bab^4x^{11}+\frac{1}{11}\,Ab^5x^{11}+Ba^2b^3x^{10}+\frac{1}{2}\,Aab^4x^{10}+\frac{10}{9}\,Ba^3b^2x^9+\frac{10}{9}\,Aa^2b^3x^9+\frac{5}{8}\,Ba^4bx^8+\frac{5}{4}\,Aa^3b^2x^8+\frac{1}{7}\,Ba^5x^7+\frac{5}{7}\,Aa^4bx^7+\frac{1}{6}\,Aa^5x^6$$

input
```
integrate(x^5*(b*x+a)^5*(B*x+A),x, algorithm="giac")
```

output
```
1/12*B*b^5*x^12 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + B*a^2*b^3*x^10 + 1
/2*A*a*b^4*x^10 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/8*B*a^4*b*x^
8 + 5/4*A*a^3*b^2*x^8 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/6*A*a^5*x^6
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^5(a+bx)^5(A+Bx)\,dx = x^7\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^{11}\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)+\frac{A\,a^5\,x^6}{6}+\frac{B\,b^5\,x^{12}}{12}+\frac{10\,a^2\,b^2\,x^9\,(A\,b+B\,a)}{9}+\frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8}+\frac{a\,b^3\,x^{10}\,(A\,b+2\,B\,a)}{2}$$

input
```
int(x^5*(A + B*x)*(a + b*x)^5,x)
```

output
```
x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^11*((A*b^5)/11 + (5*B*a*b^4)/11) + (A*
a^5*x^6)/6 + (B*b^5*x^12)/12 + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x
^8*(2*A*b + B*a))/8 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

$$\int x^5(a+bx)^5(A+Bx)\,dx = \frac{x^6(462b^6x^6+3024a\,b^5x^5+8316a^2b^4x^4+12320a^3b^3x^3+10395a^4b^2x^2+4752a^5bx+924a^6)}{5544}$$

input
```
int(x^5*(b*x+a)^5*(B*x+A),x)
```

output
```
(x**6*(924*a**6 + 4752*a**5*b*x + 10395*a**4*b**2*x**2 + 12320*a**3*b**3*x
**3 + 8316*a**2*b**4*x**4 + 3024*a*b**5*x**5 + 462*b**6*x**6))/5544
```

3.89 $\int x^4(a+bx)^5(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{1}{5}a^5Ax^5+\frac{1}{6}a^4(5Ab+aB)x^6+\frac{5}{7}a^3b(2Ab+aB)x^7 + \frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{9}ab^3(Ab+2aB)x^9 + \frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{11}b^5Bx^{11}$$

output
```
1/5*a^5*A*x^5+1/6*a^4*(5*A*b+B*a)*x^6+5/7*a^3*b*(2*A*b+B*a)*x^7+5/4*a^2*b^
2*(A*b+B*a)*x^8+5/9*a*b^3*(A*b+2*B*a)*x^9+1/10*b^4*(A*b+5*B*a)*x^10+1/11*b
^5*B*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{1}{5}a^5Ax^5+\frac{1}{6}a^4(5Ab+aB)x^6+\frac{5}{7}a^3b(2Ab+aB)x^7 + \frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{9}ab^3(Ab+2aB)x^9 + \frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{11}b^5Bx^{11}$$

input
```
Integrate[x^4*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B)*x^7)/7
+ (5*a^2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*
b + 5*a*B)*x^10)/10 + (b^5*B*x^11)/11
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^5(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^5Ax^4+a^4x^5(aB+5Ab)+5a^3bx^6(aB+2Ab)+10a^2b^2x^7(aB+Ab)+b^4x^9(5aB+Ab)+5ab^3x^8(2aB+A$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^5Ax^5+\frac{1}{6}a^4x^6(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

input
```
Int[x^4*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B)*x^7)/7
+ (5*a^2*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*
b + 5*a*B)*x^10)/10 + (b^5*B*x^11)/11
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{b^5 B\,x^{11}}{11}+\left(\frac{1}{10}b^5 A+\frac{1}{2}a\,b^4 B\right)x^{10}+\left(\frac{5}{9}a\,b^4 A+\frac{10}{9}a^2 b^3 B\right)x^9+\left(\frac{5}{4}a^2 b^3 A+\frac{5}{4}a^3 b^2 B\right)x^8+\left(\frac{10}{7}a^3\right.$
default	$\frac{b^5 B\,x^{11}}{11}+\frac{(b^5 A+5a\,b^4 B)x^{10}}{10}+\frac{(5a\,b^4 A+10a^2 b^3 B)x^9}{9}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^8}{8}+\frac{(10a^3 b^2 A+5a^4 bB)x^7}{7}+\frac{(5a^4 b}{}$
orering	$\frac{x^5(1260B\,b^5 x^6+1386A\,b^5 x^5+6930Ba\,b^4 x^5+7700aA\,b^4 x^4+15400B\,a^2 b^3 x^4+17325a^2 A\,b^3 x^3+17325B\,a^3 b^2 x^3+19800a^3 A\,b^2}{13860}$
gosper	$\frac{1}{11}b^5 B\,x^{11}+\frac{1}{10}x^{10}b^5 A+\frac{1}{2}x^{10}a\,b^4 B+\frac{5}{9}x^9 a\,b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{5}{4}x^8 a^2 b^3 A+\frac{5}{4}x^8 a^3 b^2 B+\frac{10}{7}$
risch	$\frac{1}{11}b^5 B\,x^{11}+\frac{1}{10}x^{10}b^5 A+\frac{1}{2}x^{10}a\,b^4 B+\frac{5}{9}x^9 a\,b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{5}{4}x^8 a^2 b^3 A+\frac{5}{4}x^8 a^3 b^2 B+\frac{10}{7}$
parallelrisch	$\frac{1}{11}b^5 B\,x^{11}+\frac{1}{10}x^{10}b^5 A+\frac{1}{2}x^{10}a\,b^4 B+\frac{5}{9}x^9 a\,b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{5}{4}x^8 a^2 b^3 A+\frac{5}{4}x^8 a^3 b^2 B+\frac{10}{7}$

input
```
int(x^4*(b*x+a)^5*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^5*B*x^11+(1/10*b^5*A+1/2*a*b^4*B)*x^10+(5/9*a*b^4*A+10/9*a^2*b^3*B)
*x^9+(5/4*a^2*b^3*A+5/4*a^3*b^2*B)*x^8+(10/7*a^3*b^2*A+5/7*a^4*b*B)*x^7+(5
/6*a^4*b*A+1/6*a^5*B)*x^6+1/5*a^5*A*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{1}{11}Bb^5x^{11} + \frac{1}{5}Aa^5x^5 + \frac{1}{10}\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{9}\left(2\,Ba^2b^3 + Aab^4\right)x^9 + \frac{5}{4}\left(Ba^3b^2 + Aa^2b^3\right)x^8 + \frac{5}{7}\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + \frac{1}{6}\left(Ba^5 + 5\,Aa^4b\right)x^6$$

input
```
integrate(x^4*(b*x+a)^5*(B*x+A),x, algorithm="fricas")
```

output
```
1/11*B*b^5*x^11 + 1/5*A*a^5*x^5 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/9*(2*B
*a^2*b^3 + A*a*b^4)*x^9 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 5/7*(B*a^4*b +
 2*A*a^3*b^2)*x^7 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{Aa^5x^5}{5} + \frac{Bb^5x^{11}}{11} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^9 \cdot \left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^8 \cdot \left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^7 \cdot \left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^6 \cdot \left(\frac{5Aa^4b}{6} + \frac{Ba^5}{6}\right)$$

input
```
integrate(x**4*(b*x+a)**5*(B*x+A),x)
```

output
```
A*a**5*x**5/5 + B*b**5*x**11/11 + x**10*(A*b**5/10 + B*a*b**4/2) + x**9*(5
*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4)
 + x**7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{1}{11}\,Bb^5x^{11} + \frac{1}{5}\,Aa^5x^5 + \frac{1}{10}\,\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{9}\,\left(2\,Ba^2b^3 + Aab^4\right)x^9 + \frac{5}{4}\,\left(Ba^3b^2 + Aa^2b^3\right)x^8 + \frac{5}{7}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + \frac{1}{6}\,\left(Ba^5 + 5\,Aa^4b\right)x^6$$

input
```
integrate(x^4*(b*x+a)^5*(B*x+A),x, algorithm="maxima")
```

output
```
1/11*B*b^5*x^11 + 1/5*A*a^5*x^5 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/9*(2*B
*a^2*b^3 + A*a*b^4)*x^9 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 5/7*(B*a^4*b +
 2*A*a^3*b^2)*x^7 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{1}{11}\,Bb^5x^{11} + \frac{1}{2}\,Bab^4x^{10} + \frac{1}{10}\,Ab^5x^{10} + \frac{10}{9}\,Ba^2b^3x^9 + \frac{5}{9}\,Aab^4x^9 + \frac{5}{4}\,Ba^3b^2x^8 + \frac{5}{4}\,Aa^2b^3x^8 + \frac{5}{7}\,Ba^4bx^7 + \frac{10}{7}\,Aa^3b^2x^7 + \frac{1}{6}\,Ba^5x^6 + \frac{5}{6}\,Aa^4bx^6 + \frac{1}{5}\,Aa^5x^5$$

input
```
integrate(x^4*(b*x+a)^5*(B*x+A),x, algorithm="giac")
```

output
```
1/11*B*b^5*x^11 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + 5/7*B*a^4*b*x^
7 + 10/7*A*a^3*b^2*x^7 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/5*A*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^4(a+bx)^5(A+Bx)\,dx = x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{10}\left(\frac{A\,b^5}{10}+\frac{B\,a\,b^4}{2}\right)+\frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{11}}{11}+\frac{5\,a^2\,b^2\,x^8\,(A\,b+B\,a)}{4}+\frac{5\,a^3\,b\,x^7\,(2\,A\,b+B\,a)}{7}+\frac{5\,a\,b^3\,x^9\,(A\,b+2\,B\,a)}{9}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^5,x)
```

output
```
x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^10*((A*b^5)/10 + (B*a*b^4)/2) + (A*a^5
*x^5)/5 + (B*b^5*x^11)/11 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + (5*a^3*b*x^7*(
2*A*b + B*a))/7 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

$$\int x^4(a+bx)^5(A+Bx)\,dx = \frac{x^5(210b^6x^6+1386a\,b^5x^5+3850a^2b^4x^4+5775a^3b^3x^3+4950a^4b^2x^2+2310a^5bx+462a^6)}{2310}$$

input
```
int(x^4*(b*x+a)^5*(B*x+A),x)
```

output
```
(x**5*(462*a**6 + 2310*a**5*b*x + 4950*a**4*b**2*x**2 + 5775*a**3*b**3*x**
3 + 3850*a**2*b**4*x**4 + 1386*a*b**5*x**5 + 210*b**6*x**6))/2310
```

3.90 $\int x^3(a+bx)^5(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int x^3(a+bx)^5(A+Bx)\,dx=-\frac{a^3(Ab-aB)(a+bx)^6}{6b^5}+\frac{a^2(3Ab-4aB)(a+bx)^7}{7b^5}-\frac{3a(Ab-2aB)(a+bx)^8}{8b^5}+\frac{(Ab-4aB)(a+bx)^9}{9b^5}+\frac{B(a+bx)^{10}}{10b^5}$$

output
```
-1/6*a^3*(A*b-B*a)*(b*x+a)^6/b^5+1/7*a^2*(3*A*b-4*B*a)*(b*x+a)^7/b^5-3/8*a
*(A*b-2*B*a)*(b*x+a)^8/b^5+1/9*(A*b-4*B*a)*(b*x+a)^9/b^5+1/10*B*(b*x+a)^10
/b^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int x^3(a+bx)^5(A+Bx)\,dx=\frac{1}{4}a^5Ax^4+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{6}a^3b(2Ab+aB)x^6+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{8}ab^3(Ab+2aB)x^8+\frac{1}{9}b^4(Ab+5aB)x^9+\frac{1}{10}b^5Bx^{10}$$

input
```
Integrate[x^3*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^6)/6
+ (10*a^2*b^2*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A
*b + 5*a*B)*x^9)/9 + (b^5*B*x^10)/10
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^5(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3(a+bx)^5(aB-Ab)}{b^4} - \frac{a^2(a+bx)^6(4aB-3Ab)}{b^4} + \frac{(a+bx)^8(Ab-4aB)}{b^4} + \frac{3a(a+bx)^7(2aB-Ab)}{b^4} + \frac{B(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(a+bx)^6(Ab-aB)}{6b^5} + \frac{a^2(a+bx)^7(3Ab-4aB)}{7b^5} + \frac{(a+bx)^9(Ab-4aB)}{9b^5} - \frac{3a(a+bx)^8(Ab-2aB)}{8b^5} + \frac{B(a+bx)^{10}}{10b^5}$$

input
```
Int[x^3*(a + b*x)^5*(A + B*x),x]
```

output
```
-1/6*(a^3*(A*b - a*B)*(a + b*x)^6)/b^5 + (a^2*(3*A*b - 4*a*B)*(a + b*x)^7)
/(7*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x)^8)/(8*b^5) + ((A*b - 4*a*B)*(a + b
*x)^9)/(9*b^5) + (B*(a + b*x)^10)/(10*b^5)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
norman	$\frac{b^5 B x^{10}}{10}+\left(\frac{1}{9} b^5 A+\frac{5}{9} a b^4 B\right) x^9+\left(\frac{5}{8} a b^4 A+\frac{5}{4} a^2 b^3 B\right) x^8+\left(\frac{10}{7} a^2 b^3 A+\frac{10}{7} a^3 b^2 B\right) x^7+\left(\frac{5}{3} a^3 b^2\right.$
default	$\frac{b^5 B x^{10}}{10}+\frac{\left(b^5 A+5 a b^4 B\right) x^9}{9}+\frac{\left(5 a b^4 A+10 a^2 b^3 B\right) x^8}{8}+\frac{\left(10 a^2 b^3 A+10 a^3 b^2 B\right) x^7}{7}+\frac{\left(10 a^3 b^2 A+5 a^4 b B\right) x^6}{6}+\frac{(5 a^4 b A}{}$
orering	$\frac{x^4\left(252 B b^5 x^6+280 A b^5 x^5+1400 B a b^4 x^5+1575 a A b^4 x^4+3150 B a^2 b^3 x^4+3600 a^2 A b^3 x^3+3600 B a^3 b^2 x^3+4200 a^3 A b^2 x^2+210\right.}{2520}$
gosper	$\frac{1}{10} b^5 B x^{10}+\frac{1}{9} x^9 b^5 A+\frac{5}{9} x^9 a b^4 B+\frac{5}{8} x^8 a b^4 A+\frac{5}{4} x^8 a^2 b^3 B+\frac{10}{7} x^7 a^2 b^3 A+\frac{10}{7} x^7 a^3 b^2 B+\frac{5}{3} x^6$
risch	$\frac{1}{10} b^5 B x^{10}+\frac{1}{9} x^9 b^5 A+\frac{5}{9} x^9 a b^4 B+\frac{5}{8} x^8 a b^4 A+\frac{5}{4} x^8 a^2 b^3 B+\frac{10}{7} x^7 a^2 b^3 A+\frac{10}{7} x^7 a^3 b^2 B+\frac{5}{3} x^6$
parallelrisch	$\frac{1}{10} b^5 B x^{10}+\frac{1}{9} x^9 b^5 A+\frac{5}{9} x^9 a b^4 B+\frac{5}{8} x^8 a b^4 A+\frac{5}{4} x^8 a^2 b^3 B+\frac{10}{7} x^7 a^2 b^3 A+\frac{10}{7} x^7 a^3 b^2 B+\frac{5}{3} x^6$

input
```
int(x^3*(b*x+a)^5*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^5*B*x^10+(1/9*b^5*A+5/9*a*b^4*B)*x^9+(5/8*a*b^4*A+5/4*a^2*b^3*B)*x^
8+(10/7*a^2*b^3*A+10/7*a^3*b^2*B)*x^7+(5/3*a^3*b^2*A+5/6*a^4*b*B)*x^6+(a^4
*b*A+1/5*a^5*B)*x^5+1/4*a^5*A*x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int x^3(a+bx)^5(A+Bx)\,dx = \frac{1}{10}Bb^5x^{10} + \frac{1}{4}Aa^5x^4 + \frac{1}{9}\left(5\,Bab^4 + Ab^5\right)x^9 + \frac{5}{8}\left(2\,Ba^2b^3 + Aab^4\right)x^8 + \frac{10}{7}\left(Ba^3b^2 + Aa^2b^3\right)x^7 + \frac{5}{6}\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + \frac{1}{5}\left(Ba^5 + 5\,Aa^4b\right)x^5$$

input
```
integrate(x^3*(b*x+a)^5*(B*x+A),x, algorithm="fricas")
```

output
```
1/10*B*b^5*x^10 + 1/4*A*a^5*x^4 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/8*(2*B*a
^2*b^3 + A*a*b^4)*x^8 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int x^3(a+bx)^5(A+Bx)\,dx = \frac{Aa^5x^4}{4} + \frac{Bb^5x^{10}}{10} + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^8 \cdot \left(\frac{5Aab^4}{8} + \frac{5Ba^2b^3}{4}\right) + x^7 \cdot \left(\frac{10Aa^2b^3}{7} + \frac{10Ba^3b^2}{7}\right) + x^6 \cdot \left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6}\right) + x^5\left(Aa^4b + \frac{Ba^5}{5}\right)$$

input
```
integrate(x**3*(b*x+a)**5*(B*x+A),x)
```

output
```
A*a**5*x**4/4 + B*b**5*x**10/10 + x**9*(A*b**5/9 + 5*B*a*b**4/9) + x**8*(5
*A*a*b**4/8 + 5*B*a**2*b**3/4) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7
) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**5*(A*a**4*b + B*a**5/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int x^3(a+bx)^5(A+Bx)\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{1}{4}\,Aa^5x^4 + \frac{1}{9}\,\left(5\,Bab^4 + Ab^5\right)x^9 + \frac{5}{8}\,\left(2\,Ba^2b^3 + Aab^4\right)x^8 + \frac{10}{7}\,\left(Ba^3b^2 + Aa^2b^3\right)x^7 + \frac{5}{6}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + \frac{1}{5}\,\left(Ba^5 + 5\,Aa^4b\right)x^5$$

input
```
integrate(x^3*(b*x+a)^5*(B*x+A),x, algorithm="maxima")
```

output
```
1/10*B*b^5*x^10 + 1/4*A*a^5*x^4 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/8*(2*B*a
^2*b^3 + A*a*b^4)*x^8 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int x^3(a+bx)^5(A+Bx)\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{5}{9}\,Bab^4x^9 + \frac{1}{9}\,Ab^5x^9 + \frac{5}{4}\,Ba^2b^3x^8 + \frac{5}{8}\,Aab^4x^8 + \frac{10}{7}\,Ba^3b^2x^7 + \frac{10}{7}\,Aa^2b^3x^7 + \frac{5}{6}\,Ba^4bx^6 + \frac{5}{3}\,Aa^3b^2x^6 + \frac{1}{5}\,Ba^5x^5 + Aa^4bx^5 + \frac{1}{4}\,Aa^5x^4$$

input
```
integrate(x^3*(b*x+a)^5*(B*x+A),x, algorithm="giac")
```

output
```
1/10*B*b^5*x^10 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/4*B*a^2*b^3*x^8 + 5/
8*A*a*b^4*x^8 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/6*B*a^4*b*x^6
+ 5/3*A*a^3*b^2*x^6 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/4*A*a^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int x^3(a+bx)^5(A+Bx)\,dx = x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^9\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{10}}{10}+\frac{10\,a^2\,b^2\,x^7\,(A\,b+B\,a)}{7}+\frac{5\,a^3\,b\,x^6\,(2\,A\,b+B\,a)}{6}+\frac{5\,a\,b^3\,x^8\,(A\,b+2\,B\,a)}{8}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^5,x)
```

output
```
x^5*((B*a^5)/5 + A*a^4*b) + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (A*a^5*x^4)/
4 + (B*b^5*x^10)/10 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^6*(2*A*b
 + B*a))/6 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int x^3(a+bx)^5(A+Bx)\,dx = \frac{x^4(84b^6x^6+560a\,b^5x^5+1575a^2b^4x^4+2400a^3b^3x^3+2100a^4b^2x^2+1008a^5bx+210a^6)}{840}$$

input
```
int(x^3*(b*x+a)^5*(B*x+A),x)
```

output
```
(x**4*(210*a**6 + 1008*a**5*b*x + 2100*a**4*b**2*x**2 + 2400*a**3*b**3*x**
3 + 1575*a**2*b**4*x**4 + 560*a*b**5*x**5 + 84*b**6*x**6))/840
```

3.91 $\int x^2(a+bx)^5(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int x^2(a+bx)^5(A+Bx)\,dx=\frac{a^2(Ab-aB)(a+bx)^6}{6b^4}-\frac{a(2Ab-3aB)(a+bx)^7}{7b^4}+\frac{(Ab-3aB)(a+bx)^8}{8b^4}+\frac{B(a+bx)^9}{9b^4}$$

output
```
1/6*a^2*(A*b-B*a)*(b*x+a)^6/b^4-1/7*a*(2*A*b-3*B*a)*(b*x+a)^7/b^4+1/8*(A*b
-3*B*a)*(b*x+a)^8/b^4+1/9*B*(b*x+a)^9/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int x^2(a+bx)^5(A+Bx)\,dx=\frac{1}{3}a^5Ax^3+\frac{1}{4}a^4(5Ab+aB)x^4+a^3b(2Ab+aB)x^5+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{9}b^5Bx^9$$

input
```
Integrate[x^2*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^4)/4 + a^3*b*(2*A*b + a*B)*x^5 + (5*a
^2*b^2*(A*b + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*
a*B)*x^8)/8 + (b^5*B*x^9)/9
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^5(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(-\frac{a^2(a+bx)^5(aB-Ab)}{b^3}+\frac{(a+bx)^7(Ab-3aB)}{b^3}+\frac{a(a+bx)^6(3aB-2Ab)}{b^3}+\frac{B(a+bx)^8}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^6(Ab-aB)}{6b^4}+\frac{(a+bx)^8(Ab-3aB)}{8b^4}-\frac{a(a+bx)^7(2Ab-3aB)}{7b^4}+\frac{B(a+bx)^9}{9b^4}$$

input
```
Int[x^2*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^2*(A*b - a*B)*(a + b*x)^6)/(6*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x)^7)/(7
*b^4) + ((A*b - 3*a*B)*(a + b*x)^8)/(8*b^4) + (B*(a + b*x)^9)/(9*b^4)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

method	result
norman	$\frac{b^5 B x^9}{9}+\left(\frac{1}{8}b^5 A+\frac{5}{8}a b^4 B\right)x^8+\left(\frac{5}{7}a b^4 A+\frac{10}{7}a^2 b^3 B\right)x^7+\left(\frac{5}{3}a^2 b^3 A+\frac{5}{3}a^3 b^2 B\right)x^6+(2a^3 b^2 A$
default	$\frac{b^5 B x^9}{9}+\frac{(b^5 A+5a b^4 B)x^8}{8}+\frac{(5a b^4 A+10a^2 b^3 B)x^7}{7}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^6}{6}+\frac{(10a^3 b^2 A+5a^4 b B)x^5}{5}+\frac{(5a^4 bA+$
orering	$\frac{x^3(56B b^5 x^6+63A b^5 x^5+315Ba b^4 x^5+360aA b^4 x^4+720B a^2 b^3 x^4+840a^2 A b^3 x^3+840B a^3 b^2 x^3+1008a^3 A b^2 x^2+504B a^4 b x$}{504}$
gosper	$\frac{1}{9}b^5 B x^9+\frac{1}{8}x^8 b^5 A+\frac{5}{8}x^8 a b^4 B+\frac{5}{7}x^7 a b^4 A+\frac{10}{7}x^7 a^2 b^3 B+\frac{5}{3}x^6 a^2 b^3 A+\frac{5}{3}x^6 a^3 b^2 B+2A a^3 b$
risch	$\frac{1}{9}b^5 B x^9+\frac{1}{8}x^8 b^5 A+\frac{5}{8}x^8 a b^4 B+\frac{5}{7}x^7 a b^4 A+\frac{10}{7}x^7 a^2 b^3 B+\frac{5}{3}x^6 a^2 b^3 A+\frac{5}{3}x^6 a^3 b^2 B+2A a^3 b$
parallelrisch	$\frac{1}{9}b^5 B x^9+\frac{1}{8}x^8 b^5 A+\frac{5}{8}x^8 a b^4 B+\frac{5}{7}x^7 a b^4 A+\frac{10}{7}x^7 a^2 b^3 B+\frac{5}{3}x^6 a^2 b^3 A+\frac{5}{3}x^6 a^3 b^2 B+2A a^3 b$

input
```
int(x^2*(b*x+a)^5*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5*B*x^9+(1/8*b^5*A+5/8*a*b^4*B)*x^8+(5/7*a*b^4*A+10/7*a^2*b^3*B)*x^7
+(5/3*a^2*b^3*A+5/3*a^3*b^2*B)*x^6+(2*A*a^3*b^2+B*a^4*b)*x^5+(5/4*a^4*b*A+
1/4*a^5*B)*x^4+1/3*a^5*A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int x^2(a+bx)^5(A+Bx)\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{1}{3}\,Aa^5x^3 + \frac{1}{8}\,\left(5\,Bab^4 + Ab^5\right)x^8 + \frac{5}{7}\,\left(2\,Ba^2b^3 + Aab^4\right)x^7 + \frac{5}{3}\,\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \left(Ba^4b + 2\,Aa^3b^2\right)x^5 + \frac{1}{4}\,\left(Ba^5 + 5\,Aa^4b\right)x^4$$

input
```
integrate(x^2*(b*x+a)^5*(B*x+A),x, algorithm="fricas")
```

output
```
1/9*B*b^5*x^9 + 1/3*A*a^5*x^3 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/7*(2*B*a^2
*b^3 + A*a*b^4)*x^7 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + (B*a^4*b + 2*A*a^3
*b^2)*x^5 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.53

$$\int x^2(a+bx)^5(A+Bx)\,dx = \frac{Aa^5x^3}{3} + \frac{Bb^5x^9}{9} + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^7 \cdot \left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^6 \cdot \left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x^5 \cdot \left(2Aa^3b^2 + Ba^4b\right) + x^4 \cdot \left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

input
```
integrate(x**2*(b*x+a)**5*(B*x+A),x)
```

output
```
A*a**5*x**3/3 + B*b**5*x**9/9 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**7*(5*A
*a*b**4/7 + 10*B*a**2*b**3/7) + x**6*(5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) +
 x**5*(2*A*a**3*b**2 + B*a**4*b) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int x^2(a+bx)^5(A+Bx)\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{1}{3}\,Aa^5x^3 + \frac{1}{8}\,\left(5\,Bab^4 + Ab^5\right)x^8 + \frac{5}{7}\,\left(2\,Ba^2b^3 + Aab^4\right)x^7 + \frac{5}{3}\,\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \left(Ba^4b + 2\,Aa^3b^2\right)x^5 + \frac{1}{4}\,\left(Ba^5 + 5\,Aa^4b\right)x^4$$

input
```
integrate(x^2*(b*x+a)^5*(B*x+A),x, algorithm="maxima")
```

output
```
1/9*B*b^5*x^9 + 1/3*A*a^5*x^3 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/7*(2*B*a^2
*b^3 + A*a*b^4)*x^7 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + (B*a^4*b + 2*A*a^3
*b^2)*x^5 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int x^2(a+bx)^5(A+Bx)\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{5}{8}\,Bab^4x^8 + \frac{1}{8}\,Ab^5x^8 + \frac{10}{7}\,Ba^2b^3x^7 + \frac{5}{7}\,Aab^4x^7 + \frac{5}{3}\,Ba^3b^2x^6 + \frac{5}{3}\,Aa^2b^3x^6 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{4}\,Ba^5x^4 + \frac{5}{4}\,Aa^4bx^4 + \frac{1}{3}\,Aa^5x^3$$

input
```
integrate(x^2*(b*x+a)^5*(B*x+A),x, algorithm="giac")
```

output
```
1/9*B*b^5*x^9 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 10/7*B*a^2*b^3*x^7 + 5/7
*A*a*b^4*x^7 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + B*a^4*b*x^5 + 2*A*a
^3*b^2*x^5 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + 1/3*A*a^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int x^2(a+bx)^5(A+Bx)\,dx = x^4\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)+x^8\left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right)+\frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^9}{9}+\frac{5\,a^2\,b^2\,x^6\,(A\,b+B\,a)}{3}+a^3\,b\,x^5\,(2\,A\,b+B\,a)+\frac{5\,a\,b^3\,x^7\,(A\,b+2\,B\,a)}{7}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^5,x)
```

output
```
x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^8*((A*b^5)/8 + (5*B*a*b^4)/8) + (A*a^5
*x^3)/3 + (B*b^5*x^9)/9 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + a^3*b*x^5*(2*A*b
 + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

$$\int x^2(a+bx)^5(A+Bx)\,dx = \frac{x^3(28b^6x^6+189a\,b^5x^5+540a^2b^4x^4+840a^3b^3x^3+756a^4b^2x^2+378a^5bx+84a^6)}{252}$$

input
```
int(x^2*(b*x+a)^5*(B*x+A),x)
```

output
```
(x**3*(84*a**6 + 378*a**5*b*x + 756*a**4*b**2*x**2 + 840*a**3*b**3*x**3 +
540*a**2*b**4*x**4 + 189*a*b**5*x**5 + 28*b**6*x**6))/252
```

3.92 $\int x(a+bx)^5(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int x(a+bx)^5(A+Bx)\,dx=-\frac{a(Ab-aB)(a+bx)^6}{6b^3}+\frac{(Ab-2aB)(a+bx)^7}{7b^3}+\frac{B(a+bx)^8}{8b^3}$$

output
```
-1/6*a*(A*b-B*a)*(b*x+a)^6/b^3+1/7*(A*b-2*B*a)*(b*x+a)^7/b^3+1/8*B*(b*x+a)
^8/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.89

$$\int x(a+bx)^5(A+Bx)\,dx=\frac{1}{2}a^5Ax^2+\frac{1}{3}a^4(5Ab+aB)x^3+\frac{5}{4}a^3b(2Ab+aB)x^4+2a^2b^2(Ab+aB)x^5+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{8}b^5Bx^8$$

input
```
Integrate[x*(a + b*x)^5*(A + B*x),x]
```

output
```
(a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^4)/4
+ 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b +
5*a*B)*x^7)/7 + (b^5*B*x^8)/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^5(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{(a+bx)^6(Ab-2aB)}{b^2}+\frac{a(a+bx)^5(aB-Ab)}{b^2}+\frac{B(a+bx)^7}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^7(Ab-2aB)}{7b^3}-\frac{a(a+bx)^6(Ab-aB)}{6b^3}+\frac{B(a+bx)^8}{8b^3}$$

input
```
Int[x*(a + b*x)^5*(A + B*x),x]
```

output
```
-1/6*(a*(A*b - a*B)*(a + b*x)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x)^7)/(7*b^3)
 + (B*(a + b*x)^8)/(8*b^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

method	result
norman	$\frac{b^5 B x^8}{8}+\left(\frac{1}{7}b^5 A+\frac{5}{7}a\,b^4 B\right)x^7+\left(\frac{5}{6}a\,b^4 A+\frac{5}{3}a^2 b^3 B\right)x^6+\left(2a^2 b^3 A+2a^3 b^2 B\right)x^5+\left(\frac{5}{2}a^3 b^2 A\right.$
default	$\frac{b^5 B x^8}{8}+\frac{\left(b^5 A+5a\,b^4 B\right)x^7}{7}+\frac{\left(5a\,b^4 A+10a^2 b^3 B\right)x^6}{6}+\frac{\left(10a^2 b^3 A+10a^3 b^2 B\right)x^5}{5}+\frac{\left(10a^3 b^2 A+5a^4 bB\right)x^4}{4}+\frac{\left(5a^4 bA\right.}{}$
orering	$\frac{x^2\left(21B\,b^5 x^6+24A\,b^5 x^5+120Ba\,b^4 x^5+140aA\,b^4 x^4+280B\,a^2 b^3 x^4+336a^2 A\,b^3 x^3+336B\,a^3 b^2 x^3+420a^3 A\,b^2 x^2+210B\,a^4 b\,x^2\right.}{168}$
gosper	$\frac{1}{8}b^5 B\,x^8+\frac{1}{7}x^7 b^5 A+\frac{5}{7}x^7 a\,b^4 B+\frac{5}{6}x^6 a\,b^4 A+\frac{5}{3}x^6 a^2 b^3 B+2A\,a^2 b^3 x^5+2B\,a^3 b^2 x^5+\frac{5}{2}x^4 a^3 b$
risch	$\frac{1}{8}b^5 B\,x^8+\frac{1}{7}x^7 b^5 A+\frac{5}{7}x^7 a\,b^4 B+\frac{5}{6}x^6 a\,b^4 A+\frac{5}{3}x^6 a^2 b^3 B+2A\,a^2 b^3 x^5+2B\,a^3 b^2 x^5+\frac{5}{2}x^4 a^3 b$
parallelrisch	$\frac{1}{8}b^5 B\,x^8+\frac{1}{7}x^7 b^5 A+\frac{5}{7}x^7 a\,b^4 B+\frac{5}{6}x^6 a\,b^4 A+\frac{5}{3}x^6 a^2 b^3 B+2A\,a^2 b^3 x^5+2B\,a^3 b^2 x^5+\frac{5}{2}x^4 a^3 b$

input
```
int(x*(b*x+a)^5*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5*B*x^8+(1/7*b^5*A+5/7*a*b^4*B)*x^7+(5/6*a*b^4*A+5/3*a^2*b^3*B)*x^6+
(2*A*a^2*b^3+2*B*a^3*b^2)*x^5+(5/2*a^3*b^2*A+5/4*a^4*b*B)*x^4+(5/3*a^4*b*A
+1/3*a^5*B)*x^3+1/2*a^5*A*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(56) = 112$.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int x(a+bx)^5(A+Bx)\,dx = \frac{1}{8}\,Bb^5x^8 + \frac{1}{2}\,Aa^5x^2 + \frac{1}{7}\,(5\,Bab^4 + Ab^5)x^7 + \frac{5}{6}\,(2\,Ba^2b^3 + Aab^4)x^6 + 2\,(Ba^3b^2 + Aa^2b^3)x^5 + \frac{5}{4}\,(Ba^4b + 2\,Aa^3b^2)x^4 + \frac{1}{3}\,(Ba^5 + 5\,Aa^4b)x^3$$

input
```
integrate(x*(b*x+a)^5*(B*x+A),x, algorithm="fricas")
```

output
```
1/8*B*b^5*x^8 + 1/2*A*a^5*x^2 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/6*(2*B*a^2
*b^3 + A*a*b^4)*x^6 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/4*(B*a^4*b + 2*A*a
^3*b^2)*x^4 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(54) = 108$.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.20

$$\int x(a+bx)^5(A+Bx)\,dx = \frac{Aa^5x^2}{2} + \frac{Bb^5x^8}{8} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^6 \cdot \left(\frac{5Aab^4}{6} + \frac{5Ba^2b^3}{3}\right) + x^5 \cdot (2Aa^2b^3 + 2Ba^3b^2) + x^4 \cdot \left(\frac{5Aa^3b^2}{2} + \frac{5Ba^4b}{4}\right) + x^3 \cdot \left(\frac{5Aa^4b}{3} + \frac{Ba^5}{3}\right)$$

input
```
integrate(x*(b*x+a)**5*(B*x+A),x)
```

output
```
A*a**5*x**2/2 + B*b**5*x**8/8 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**6*(5*A
*a*b**4/6 + 5*B*a**2*b**3/3) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**4
*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int x(a+bx)^5(A+Bx)\,dx = \frac{1}{8}\,Bb^5x^8 + \frac{1}{2}\,Aa^5x^2 + \frac{1}{7}\,\left(5\,Bab^4 + Ab^5\right)x^7 + \frac{5}{6}\,\left(2\,Ba^2b^3 + Aab^4\right)x^6 + 2\,\left(Ba^3b^2 + Aa^2b^3\right)x^5 + \frac{5}{4}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + \frac{1}{3}\,\left(Ba^5 + 5\,Aa^4b\right)x^3$$

input
```
integrate(x*(b*x+a)^5*(B*x+A),x, algorithm="maxima")
```

output
```
1/8*B*b^5*x^8 + 1/2*A*a^5*x^2 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/6*(2*B*a^2
*b^3 + A*a*b^4)*x^6 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/4*(B*a^4*b + 2*A*a
^3*b^2)*x^4 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int x(a+bx)^5(A+Bx)\,dx = \frac{1}{8}\,Bb^5x^8 + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + \frac{5}{3}\,Ba^2b^3x^6 + \frac{5}{6}\,Aab^4x^6 + 2\,Ba^3b^2x^5 + 2\,Aa^2b^3x^5 + \frac{5}{4}\,Ba^4bx^4 + \frac{5}{2}\,Aa^3b^2x^4 + \frac{1}{3}\,Ba^5x^3 + \frac{5}{3}\,Aa^4bx^3 + \frac{1}{2}\,Aa^5x^2$$

input
```
integrate(x*(b*x+a)^5*(B*x+A),x, algorithm="giac")
```

output
```
1/8*B*b^5*x^8 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/3*B*a^2*b^3*x^6 + 5/6*
A*a*b^4*x^6 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/4*B*a^4*b*x^4 + 5/2*A*
a^3*b^2*x^4 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + 1/2*A*a^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int x(a+bx)^5(A+Bx)\,dx = x^3\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^7\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right) + \frac{A\,a^5\,x^2}{2}+\frac{B\,b^5\,x^8}{8}+2\,a^2\,b^2\,x^5\,(A\,b+B\,a) + \frac{5\,a^3\,b\,x^4\,(2\,A\,b+B\,a)}{4}+\frac{5\,a\,b^3\,x^6\,(A\,b+2\,B\,a)}{6}$$

input
```
int(x*(A + B*x)*(a + b*x)^5,x)
```

output
```
x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^7*((A*b^5)/7 + (5*B*a*b^4)/7) + (A*a^5
*x^2)/2 + (B*b^5*x^8)/8 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^4*(2*A*b
+ B*a))/4 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int x(a+bx)^5(A+Bx)\,dx = \frac{x^2(7b^6x^6+48a\,b^5x^5+140a^2b^4x^4+224a^3b^3x^3+210a^4b^2x^2+112a^5bx+28a^6)}{56}$$

input
```
int(x*(b*x+a)^5*(B*x+A),x)
```

output
```
(x**2*(28*a**6 + 112*a**5*b*x + 210*a**4*b**2*x**2 + 224*a**3*b**3*x**3 +
140*a**2*b**4*x**4 + 48*a*b**5*x**5 + 7*b**6*x**6))/56
```

3.93 $\int (a+bx)^5(A+Bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int (a+bx)^5(A+Bx)\,dx=\frac{(Ab-aB)(a+bx)^6}{6b^2}+\frac{B(a+bx)^7}{7b^2}$$

output
```
1/6*(A*b-B*a)*(b*x+a)^6/b^2+1/7*B*(b*x+a)^7/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(38) = 76.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\begin{aligned}\int (a+bx)^5(A+Bx)\,dx &= a^5Ax+\frac{1}{2}a^4(5Ab+aB)x^2+\frac{5}{3}a^3b(2Ab+aB)x^3\\&\quad+\frac{5}{2}a^2b^2(Ab+aB)x^4+ab^3(Ab+2aB)x^5\\&\quad+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{7}b^5Bx^7\end{aligned}$$

input
```
Integrate[(a + b*x)^5*(A + B*x),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^2)/2 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a
^2*b^2*(A*b + a*B)*x^4)/2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x
^6)/6 + (b^5*B*x^7)/7
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^5(A+Bx)\,dx$$

$$\Big\downarrow 49$$

$$\int \left(\frac{(a+bx)^5(Ab-aB)}{b}+\frac{B(a+bx)^6}{b}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{(a+bx)^6(Ab-aB)}{6b^2}+\frac{B(a+bx)^7}{7b^2}$$

input
```
Int[(a + b*x)^5*(A + B*x),x]
```

output
```
((A*b - a*B)*(a + b*x)^6)/(6*b^2) + (B*(a + b*x)^7)/(7*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.08

method	result
norman	$\frac{b^5 B\,x^7}{7}+\left(\frac{1}{6}b^5A+\frac{5}{6}a\,b^4B\right)x^6+\left(a\,b^4A+2a^2b^3B\right)x^5+\left(\frac{5}{2}a^2b^3A+\frac{5}{2}a^3b^2B\right)x^4+\left(\frac{10}{3}a^3b^2A\right.$
default	$\frac{b^5 B\,x^7}{7}+\frac{(b^5A+5a\,b^4B)x^6}{6}+\frac{(5a\,b^4A+10a^2b^3B)x^5}{5}+\frac{(10a^2b^3A+10a^3b^2B)x^4}{4}+\frac{(10a^3b^2A+5a^4bB)x^3}{3}+\frac{(5a^4bA+}{}$
gosper	$\frac{1}{7}b^5B\,x^7+\frac{1}{6}x^6b^5A+\frac{5}{6}x^6a\,b^4B+Aa\,b^4x^5+2B\,a^2b^3x^5+\frac{5}{2}x^4a^2b^3A+\frac{5}{2}x^4a^3b^2B+\frac{10}{3}x^3a^3b^2$
risch	$\frac{1}{7}b^5B\,x^7+\frac{1}{6}x^6b^5A+\frac{5}{6}x^6a\,b^4B+Aa\,b^4x^5+2B\,a^2b^3x^5+\frac{5}{2}x^4a^2b^3A+\frac{5}{2}x^4a^3b^2B+\frac{10}{3}x^3a^3b^2$
parallelrisch	$\frac{1}{7}b^5B\,x^7+\frac{1}{6}x^6b^5A+\frac{5}{6}x^6a\,b^4B+Aa\,b^4x^5+2B\,a^2b^3x^5+\frac{5}{2}x^4a^2b^3A+\frac{5}{2}x^4a^3b^2B+\frac{10}{3}x^3a^3b^2$
orering	$\frac{x\left(6B\,b^5x^6+7A\,b^5x^5+35Ba\,b^4x^5+42aA\,b^4x^4+84B\,a^2b^3x^4+105a^2A\,b^3x^3+105B\,a^3b^2x^3+140a^3A\,b^2x^2+70B\,a^4b\,x^2+105a^4\right.}{42}$

input
```
int((b*x+a)^5*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^5*B*x^7+(1/6*b^5*A+5/6*a*b^4*B)*x^6+(A*a*b^4+2*B*a^2*b^3)*x^5+(5/2*a
^2*b^3*A+5/2*a^3*b^2*B)*x^4+(10/3*a^3*b^2*A+5/3*a^4*b*B)*x^3+(5/2*a^4*b*A+
1/2*a^5*B)*x^2+a^5*A*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(34) = 68.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.03

$$\begin{aligned}\int (a+bx)^5(A+Bx)\,dx = {} & \frac{1}{7}\,Bb^5x^7+Aa^5x+\frac{1}{6}\left(5\,Bab^4+Ab^5\right)x^6 \\ & +\left(2\,Ba^2b^3+Aab^4\right)x^5+\frac{5}{2}\left(Ba^3b^2+Aa^2b^3\right)x^4 \\ & +\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2\end{aligned}$$

input
```
integrate((b*x+a)^5*(B*x+A),x, algorithm="fricas")
```

output
```
1/7*B*b^5*x^7 + A*a^5*x + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + (2*B*a^2*b^3 + A*a
*b^4)*x^5 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*
x^3 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int (a+bx)^5(A+Bx)\,dx = Aa^5x + \frac{Bb^5x^7}{7} + x^6\left(\frac{Ab^5}{6} + \frac{5Bab^4}{6}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^4\cdot\left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x^3\cdot\left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right) + x^2\cdot\left(\frac{5Aa^4b}{2} + \frac{Ba^5}{2}\right)$$

input
```
integrate((b*x+a)**5*(B*x+A),x)
```

output
```
A*a**5*x + B*b**5*x**7/7 + x**6*(A*b**5/6 + 5*B*a*b**4/6) + x**5*(A*a*b**4
 + 2*B*a**2*b**3) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x**3*(10*A*
a**3*b**2/3 + 5*B*a**4*b/3) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.03

$$\int (a+bx)^5(A+Bx)\,dx = \frac{1}{7}\,Bb^5x^7 + Aa^5x + \frac{1}{6}\,(5\,Bab^4 + Ab^5)x^6 + (2\,Ba^2b^3 + Aab^4)x^5 + \frac{5}{2}\,(Ba^3b^2 + Aa^2b^3)x^4 + \frac{5}{3}\,(Ba^4b + 2\,Aa^3b^2)x^3 + \frac{1}{2}\,(Ba^5 + 5\,Aa^4b)x^2$$

input
```
integrate((b*x+a)^5*(B*x+A),x, algorithm="maxima")
```

output
```
1/7*B*b^5*x^7 + A*a^5*x + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + (2*B*a^2*b^3 + A*a
*b^4)*x^5 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*
x^3 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(34) = 68.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int (a+bx)^5(A+Bx)\,dx = \frac{1}{7}Bb^5x^7 + \frac{5}{6}Bab^4x^6 + \frac{1}{6}Ab^5x^6 + 2\,Ba^2b^3x^5 + Aab^4x^5 + \frac{5}{2}Ba^3b^2x^4 + \frac{5}{2}Aa^2b^3x^4 + \frac{5}{3}Ba^4bx^3 + \frac{10}{3}Aa^3b^2x^3 + \frac{1}{2}Ba^5x^2 + \frac{5}{2}Aa^4bx^2 + Aa^5x$$

input
```
integrate((b*x+a)^5*(B*x+A),x, algorithm="giac")
```

output
```
1/7*B*b^5*x^7 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 2*B*a^2*b^3*x^5 + A*a*b^
4*x^5 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5/3*B*a^4*b*x^3 + 10/3*A*a
^3*b^2*x^3 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 + A*a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.71

$$\int (a+bx)^5(A+Bx)\,dx = x^2\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)+x^6\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right) + \frac{B\,b^5\,x^7}{7} + A\,a^5\,x + \frac{5\,a^2\,b^2\,x^4\,(A\,b+B\,a)}{2} + \frac{5\,a^3\,b\,x^3\,(2\,A\,b+B\,a)}{3} + a\,b^3\,x^5\,(A\,b+2\,B\,a)$$

input
```
int((A + B*x)*(a + b*x)^5,x)
```

output
```
x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^6*((A*b^5)/6 + (5*B*a*b^4)/6) + (B*b^5
*x^7)/7 + A*a^5*x + (5*a^2*b^2*x^4*(A*b + B*a))/2 + (5*a^3*b*x^3*(2*A*b +
B*a))/3 + a*b^3*x^5*(A*b + 2*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (a+bx)^5(A+Bx)\,dx = \frac{x(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6)}{7}$$

input
```
int((b*x+a)^5*(B*x+A),x)
```

output
```
(x*(7*a**6 + 21*a**5*b*x + 35*a**4*b**2*x**2 + 35*a**3*b**3*x**3 + 21*a**2
*b**4*x**4 + 7*a*b**5*x**5 + b**6*x**6))/7
```

3.94 $\int \frac{(a+bx)^5(A+Bx)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int \frac{(a+bx)^5(A+Bx)}{x} \, dx = 5a^4Abx + 5a^3Ab^2x^2 + \frac{10}{3}a^2Ab^3x^3 + \frac{5}{4}aAb^4x^4 + \frac{1}{5}Ab^5x^5 + \frac{B(a+bx)^6}{6b} + a^5A\log(x)$$

output
```
5*a^4*A*b*x+5*a^3*A*b^2*x^2+10/3*a^2*A*b^3*x^3+5/4*a*A*b^4*x^4+1/5*A*b^5*x
^5+1/6*B*(b*x+a)^6/b+a^5*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^5(A+Bx)}{x} \, dx = a^4(5Ab+aB)x + \frac{5}{2}a^3b(2Ab+aB)x^2 + \frac{10}{3}a^2b^2(Ab+aB)x^3 + \frac{5}{4}ab^3(Ab+2aB)x^4 + \frac{1}{5}b^4(Ab+5aB)x^5 + \frac{1}{6}b^5Bx^6 + a^5A\log(x)$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x,x]
```

output
```
a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (10*a^2*b^2*(A*b + a
*B)*x^3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^5)/5 + (
b^5*B*x^6)/6 + a^5*A*Log[x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x} \, dx$$

$$\Big\downarrow \text{ 90}$$

$$A\int \frac{(a+bx)^5}{x}dx+\frac{B(a+bx)^6}{6b}$$

$$\Big\downarrow \text{ 49}$$

$$A\int \left(\frac{a^5}{x}+5ba^4+10b^2xa^3+10b^3x^2a^2+5b^4x^3a+b^5x^4\right)dx+\frac{B(a+bx)^6}{6b}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}\right)+\frac{B(a+bx)^6}{6b}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x,x]
```

output
```
(B*(a + b*x)^6)/(6*b) + A*(5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3
+ (5*a*b^4*x^4)/4 + (b^5*x^5)/5 + a^5*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

method	result
norman	$\left(\frac{1}{5}b^5A+a\,b^4B\right)x^5+\left(\frac{5}{4}a\,b^4A+\frac{5}{2}a^2b^3B\right)x^4+\left(\frac{10}{3}a^2b^3A+\frac{10}{3}a^3b^2B\right)x^3+\left(5a^3b^2A+\frac{5}{2}a^4bB\right.$
default	$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^5}{5}+Ba\,b^4x^5+\frac{5aA\,b^4x^4}{4}+\frac{5B\,a^2b^3x^4}{2}+\frac{10a^2A\,b^3x^3}{3}+\frac{10B\,a^3b^2x^3}{3}+5a^3A\,b^2x^2+\frac{5B\,a^4b}{2}$
risch	$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^5}{5}+Ba\,b^4x^5+\frac{5aA\,b^4x^4}{4}+\frac{5B\,a^2b^3x^4}{2}+\frac{10a^2A\,b^3x^3}{3}+\frac{10B\,a^3b^2x^3}{3}+5a^3A\,b^2x^2+\frac{5B\,a^4b}{2}$
parallelrisch	$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^5}{5}+Ba\,b^4x^5+\frac{5aA\,b^4x^4}{4}+\frac{5B\,a^2b^3x^4}{2}+\frac{10a^2A\,b^3x^3}{3}+\frac{10B\,a^3b^2x^3}{3}+5a^3A\,b^2x^2+\frac{5B\,a^4b}{2}$

input
```
int((b*x+a)^5*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/5*b^5*A+a*b^4*B)*x^5+(5/4*a*b^4*A+5/2*a^2*b^3*B)*x^4+(10/3*a^2*b^3*A+10
/3*a^3*b^2*B)*x^3+(5*a^3*b^2*A+5/2*a^4*b*B)*x^2+(5*A*a^4*b+B*a^5)*x+1/6*B*
b^5*x^6+a^5*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^5(A+Bx)}{x}\,dx = \frac{1}{6}\,Bb^5x^6 + Aa^5\log(x) + \frac{1}{5}\left(5\,Bab^4 + Ab^5\right)x^5 + \frac{5}{4}\left(2\,Ba^2b^3 + Aab^4\right)x^4 + \frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^3 + \frac{5}{2}\left(Ba^4b + 2\,Aa^3b^2\right)x^2 + \left(Ba^5 + 5\,Aa^4b\right)x$$

input
```
integrate((b*x+a)^5*(B*x+A)/x,x, algorithm="fricas")
```

output
```
1/6*B*b^5*x^6 + A*a^5*log(x) + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/4*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^5(A+Bx)}{x}\,dx = Aa^5\log(x) + \frac{Bb^5x^6}{6} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^4 \cdot \left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x^3 \cdot \left(\frac{10Aa^2b^3}{3} + \frac{10Ba^3b^2}{3}\right) + x^2 \cdot \left(5Aa^3b^2 + \frac{5Ba^4b}{2}\right) + x\left(5Aa^4b + Ba^5\right)$$

input
```
integrate((b*x+a)**5*(B*x+A)/x,x)
```

output
```
A*a**5*log(x) + B*b**5*x**6/6 + x**5*(A*b**5/5 + B*a*b**4) + x**4*(5*A*a*b
**4/4 + 5*B*a**2*b**3/2) + x**3*(10*A*a**2*b**3/3 + 10*B*a**3*b**2/3) + x*
*2*(5*A*a**3*b**2 + 5*B*a**4*b/2) + x*(5*A*a**4*b + B*a**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^5(A+Bx)}{x}\,dx = \frac{1}{6}\,Bb^5x^6 + Aa^5\log(x) + \frac{1}{5}\,(5\,Bab^4 + Ab^5)x^5 + \frac{5}{4}\,(2\,Ba^2b^3 + Aab^4)x^4 + \frac{10}{3}\,(Ba^3b^2 + Aa^2b^3)x^3 + \frac{5}{2}\,(Ba^4b + 2\,Aa^3b^2)x^2 + (Ba^5 + 5\,Aa^4b)x$$

input
```
integrate((b*x+a)^5*(B*x+A)/x,x, algorithm="maxima")
```

output
```
1/6*B*b^5*x^6 + A*a^5*log(x) + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/4*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^5(A+Bx)}{x}\,dx = \frac{1}{6}\,Bb^5x^6 + Bab^4x^5 + \frac{1}{5}\,Ab^5x^5 + \frac{5}{2}\,Ba^2b^3x^4 + \frac{5}{4}\,Aab^4x^4 + \frac{10}{3}\,Ba^3b^2x^3 + \frac{10}{3}\,Aa^2b^3x^3 + \frac{5}{2}\,Ba^4bx^2 + 5\,Aa^3b^2x^2 + Ba^5x + 5\,Aa^4bx + Aa^5\log(|x|)$$

input
```
integrate((b*x+a)^5*(B*x+A)/x,x, algorithm="giac")
```

output
```
1/6*B*b^5*x^6 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5/2*B*a^2*b^3*x^4 + 5/4*A*a*
b^4*x^4 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5/2*B*a^4*b*x^2 + 5*A*
a^3*b^2*x^2 + B*a^5*x + 5*A*a^4*b*x + A*a^5*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^5(A+Bx)}{x}\,dx = x\left(B\,a^5+5\,A\,b\,a^4\right)+x^5\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+\frac{B\,b^5\,x^6}{6}+A\,a^5\,\ln\left(x\right)+\frac{10\,a^2\,b^2\,x^3\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^2\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^3\,x^4\,\left(A\,b+2\,B\,a\right)}{4}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x,x)
```

output
```
x*(B*a^5 + 5*A*a^4*b) + x^5*((A*b^5)/5 + B*a*b^4) + (B*b^5*x^6)/6 + A*a^5*
log(x) + (10*a^2*b^2*x^3*(A*b + B*a))/3 + (5*a^3*b*x^2*(2*A*b + B*a))/2 +
(5*a*b^3*x^4*(A*b + 2*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^5(A+Bx)}{x}\,dx = \log(x)\,a^6+6a^5bx+\frac{15a^4b^2x^2}{2}+\frac{20a^3b^3x^3}{3}+\frac{15a^2b^4x^4}{4}+\frac{6a\,b^5x^5}{5}+\frac{b^6x^6}{6}$$

input
```
int((b*x+a)^5*(B*x+A)/x,x)
```

output
```
(60*log(x)*a**6 + 360*a**5*b*x + 450*a**4*b**2*x**2 + 400*a**3*b**3*x**3 +
 225*a**2*b**4*x**4 + 72*a*b**5*x**5 + 10*b**6*x**6)/60
```

3.95 $\int \frac{(a+bx)^5(A+Bx)}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{(a+bx)^5(A+Bx)}{x^2} \, dx = -\frac{a^5A}{x} + 5a^3b(2Ab+aB)x + 5a^2b^2(Ab+aB)x^2 + \frac{5}{3}ab^3(Ab+2aB)x^3 + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{5}b^5Bx^5 + a^4(5Ab+aB)\log(x)$$

output
```
-a^5*A/x+5*a^3*b*(2*A*b+B*a)*x+5*a^2*b^2*(A*b+B*a)*x^2+5/3*a*b^3*(A*b+2*B*
a)*x^3+1/4*b^4*(A*b+5*B*a)*x^4+1/5*b^5*B*x^5+a^4*(5*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^2} \, dx = -\frac{a^5A}{x} + 5a^3b(2Ab+aB)x + 5a^2b^2(Ab+aB)x^2 + \frac{5}{3}ab^3(Ab+2aB)x^3 + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{5}b^5Bx^5 + \left(5a^4Ab+a^5B\right)\log(x)$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^2,x]
```

output
```
-((a^5*A)/x) + 5*a^3*b*(2*A*b + a*B)*x + 5*a^2*b^2*(A*b + a*B)*x^2 + (5*a*
b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*x^5)/5 + (5*
a^4*A*b + a^5*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^2} \, dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^5A}{x^2}+\frac{a^4(aB+5Ab)}{x}+5a^3b(aB+2Ab)+10a^2b^2x(aB+Ab)+b^4x^3(5aB+Ab)+5ab^3x^2(2aB+Ab)+b^5$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5A}{x}+a^4\log(x)(aB+5Ab)+5a^3bx(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^2,x]
```

output
```
-((a^5*A)/x) + 5*a^3*b*(2*A*b + a*B)*x + 5*a^2*b^2*(A*b + a*B)*x^2 + (5*a*
b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*x^5)/5 + a^4
*(5*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

method	result
default	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+10 a^3 b^2 A x+5 a^4$
risch	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+10 a^3 b^2 A x+5 a^4$
norman	$\frac{\left(\frac{1}{4} b^5 A+\frac{5}{4} a b^4 B\right) x^5+\left(\frac{5}{3} a b^4 A+\frac{10}{3} a^2 b^3 B\right) x^4+\left(5 a^2 b^3 A+5 a^3 b^2 B\right) x^3+\left(10 a^3 b^2 A+5 a^4 b B\right) x^2-a^5 A+\frac{B b^5 x^6}{5}}{x}+\left(5 a^4 b A+\right.$
parallelrisch	$\frac{12 B b^5 x^6+15 A b^5 x^5+75 B a b^4 x^5+100 a A b^4 x^4+200 B a^2 b^3 x^4+300 a^2 A b^3 x^3+300 B a^3 b^2 x^3+300 A \ln(x) x a^4 b+600 a^3 A b^2 x^2+}{60 x}$

input
```
int((b*x+a)^5*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^5*B*x^5+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3
*x^3+5*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2+10*a^3*b^2*A*x+5*a^4*b*B*x+a^4*(5*A*b
+B*a)*ln(x)-a^5*A/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx = \frac{12\,Bb^5x^6 - 60\,Aa^5 + 15\,(5\,Bab^4 + Ab^5)x^5 + 100\,(2\,Ba^2b^3 + Aab^4)x^4 + 300\,(Ba^3b^2 + Aa^2b^3)x^3 + 300\,(B}{60\,x}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/60*(12*B*b^5*x^6 - 60*A*a^5 + 15*(5*B*a*b^4 + A*b^5)*x^5 + 100*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 300*(B*a^4*b + 2*A*
a^3*b^2)*x^2 + 60*(B*a^5 + 5*A*a^4*b)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx = & -\frac{Aa^5}{x} + \frac{Bb^5x^5}{5} + a^4\cdot(5Ab+Ba)\log(x) \\ & + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x^3\cdot\left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) \\ & + x^2\cdot\left(5Aa^2b^3 + 5Ba^3b^2\right) + x\left(10Aa^3b^2 + 5Ba^4b\right)\end{aligned}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**2,x)
```

output
```
-A*a**5/x + B*b**5*x**5/5 + a**4*(5*A*b + B*a)*log(x) + x**4*(A*b**5/4 + 5
*B*a*b**4/4) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + x**2*(5*A*a**2*b**
3 + 5*B*a**3*b**2) + x*(10*A*a**3*b**2 + 5*B*a**4*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx = \frac{1}{5}\,Bb^5x^5 - \frac{Aa^5}{x} + \frac{1}{4}\left(5\,Bab^4 + Ab^5\right)x^4 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^3 + 5\left(Ba^3b^2 + Aa^2b^3\right)x^2 + 5\left(Ba^4b + 2\,Aa^3b^2\right)x + \left(Ba^5 + 5\,Aa^4b\right)\log\left(x\right)$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/5*B*b^5*x^5 - A*a^5/x + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5/3*(2*B*a^2*b^3 +
 A*a*b^4)*x^3 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 + 5*(B*a^4*b + 2*A*a^3*b^2)*
x + (B*a^5 + 5*A*a^4*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx = \frac{1}{5}\,Bb^5x^5 + \frac{5}{4}\,Bab^4x^4 + \frac{1}{4}\,Ab^5x^4 + \frac{10}{3}\,Ba^2b^3x^3 + \frac{5}{3}\,Aab^4x^3 + 5\,Ba^3b^2x^2 + 5\,Aa^2b^3x^2 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{Aa^5}{x} + \left(Ba^5 + 5\,Aa^4b\right)\log\left(|x|\right)$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/5*B*b^5*x^5 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10/3*B*a^2*b^3*x^3 + 5/3
*A*a*b^4*x^3 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 + 5*B*a^4*b*x + 10*A*a^3*
b^2*x - A*a^5/x + (B*a^5 + 5*A*a^4*b)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx = x^4\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+\ln\left(x\right)\,\left(B\,a^5+5\,A\,b\,a^4\right) - \frac{A\,a^5}{x}+\frac{B\,b^5\,x^5}{5}+5\,a^2\,b^2\,x^2\,\left(A\,b+B\,a\right) + 5\,a^3\,b\,x\,\left(2\,A\,b+B\,a\right)+\frac{5\,a\,b^3\,x^3\,\left(A\,b+2\,B\,a\right)}{3}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^2,x)
```

output
```
x^4*((A*b^5)/4 + (5*B*a*b^4)/4) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/x +
 (B*b^5*x^5)/5 + 5*a^2*b^2*x^2*(A*b + B*a) + 5*a^3*b*x*(2*A*b + B*a) + (5*
a*b^3*x^3*(A*b + 2*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx = \frac{60\log(x)\,a^5bx-10a^6+150a^4b^2x^2+100a^3b^3x^3+50a^2b^4x^4+15a\,b^5x^5+2b^6x^6}{10x}$$

input
```
int((b*x+a)^5*(B*x+A)/x^2,x)
```

output
```
(60*log(x)*a**5*b*x - 10*a**6 + 150*a**4*b**2*x**2 + 100*a**3*b**3*x**3 +
50*a**2*b**4*x**4 + 15*a*b**5*x**5 + 2*b**6*x**6)/(10*x)
```

3.96 $\int \frac{(a+bx)^5(A+Bx)}{x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{(a+bx)^5(A+Bx)}{x^3} dx = -\frac{a^5A}{2x^2} - \frac{a^4(5Ab+aB)}{x} + 10a^2b^2(Ab+aB)x + \frac{5}{2}ab^3(Ab+2aB)x^2 + \frac{1}{3}b^4(Ab+5aB)x^3 + \frac{1}{4}b^5Bx^4 + 5a^3b(2Ab+aB)\log(x)$$

output
```
-1/2*a^5*A/x^2-a^4*(5*A*b+B*a)/x+10*a^2*b^2*(A*b+B*a)*x+5/2*a*b^3*(A*b+2*B
*a)*x^2+1/3*b^4*(A*b+5*B*a)*x^3+1/4*b^5*B*x^4+5*a^3*b*(2*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^5(A+Bx)}{x^3} dx = -\frac{5a^4Ab}{x} + 10a^3b^2Bx + 5a^2b^3x(2A+Bx) - \frac{a^5(A+2Bx)}{2x^2} + \frac{5}{6}ab^4x^2(3A+2Bx) + \frac{1}{12}b^5x^3(4A+3Bx) + 5a^3b(2Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^3,x]
```

output
```
(-5*a^4*A*b)/x + 10*a^3*b^2*B*x + 5*a^2*b^3*x*(2*A + B*x) - (a^5*(A + 2*B*
x))/(2*x^2) + (5*a*b^4*x^2*(3*A + 2*B*x))/6 + (b^5*x^3*(4*A + 3*B*x))/12 +
 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^3} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^5A}{x^3}+\frac{a^4(aB+5Ab)}{x^2}+\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2(aB+Ab)+b^4x^2(5aB+Ab)+5ab^3x(2aB+Ab)+b^5E\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5A}{2x^2}-\frac{a^4(aB+5Ab)}{x}+5a^3b\log(x)(aB+2Ab)+10a^2b^2x(aB+Ab)+\frac{1}{3}b^4x^3(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^3,x]
```

output
```
-1/2*(a^5*A)/x^2 - (a^4*(5*A*b + a*B))/x + 10*a^2*b^2*(A*b + a*B)*x + (5*a
*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a*B)*x^3)/3 + (b^5*B*x^4)/4 + 5*
a^3*b*(2*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^{5} B\,x^{4}}{4}+\frac{A\,b^{5} x^{3}}{3}+\frac{5 B a\,b^{4} x^{3}}{3}+\frac{5 A a\,b^{4} x^{2}}{2}+5 B\,a^{2} b^{3} x^{2}+10 A\,a^{2} b^{3} x+10 B\,a^{3} b^{2} x-\frac{a^{5} A}{2 x^{2}}+5 a^{3} b (2 A$
risch	$\frac{b^{5} B\,x^{4}}{4}+\frac{A\,b^{5} x^{3}}{3}+\frac{5 B a\,b^{4} x^{3}}{3}+\frac{5 A a\,b^{4} x^{2}}{2}+5 B\,a^{2} b^{3} x^{2}+10 A\,a^{2} b^{3} x+10 B\,a^{3} b^{2} x+\frac{\left(-5 a^{4} b A-a^{5} B\right) x-}{x^{2}}$
norman	$\frac{\left(\frac{1}{3} b^{5} A+\frac{5}{3} a\,b^{4} B\right) x^{5}+\left(\frac{5}{2} a\,b^{4} A+5 a^{2} b^{3} B\right) x^{4}+\left(10 a^{2} b^{3} A+10 a^{3} b^{2} B\right) x^{3}+\left(-5 a^{4} b A-a^{5} B\right) x-\frac{a^{5} A}{2}+\frac{B\,b^{5} x^{6}}{4}}{x^{2}}+(10 a^{3} b^{2} A+$
parallelrisch	$\frac{3 B\,b^{5} x^{6}+4 A\,b^{5} x^{5}+20 B a\,b^{4} x^{5}+30 a A\,b^{4} x^{4}+60 B\,a^{2} b^{3} x^{4}+120 A \ln (x) x^{2} a^{3} b^{2}+120 a^{2} A\,b^{3} x^{3}+60 B \ln (x) x^{2} a^{4} b+120 B\,a^{3} b^{2} x^{3}-}{12 x^{2}}$

input
```
int((b*x+a)^5*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^5*B*x^4+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^
2+10*A*a^2*b^3*x+10*B*a^3*b^2*x-1/2*a^5*A/x^2+5*a^3*b*(2*A*b+B*a)*ln(x)-a^
4*(5*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx = \frac{3\,Bb^5x^6 - 6\,Aa^5 + 4\,(5\,Bab^4 + Ab^5)x^5 + 30\,(2\,Ba^2b^3 + Aab^4)x^4 + 120\,(Ba^3b^2 + Aa^2b^3)x^3 + 60\,(Ba^4b + }{12\,x^2}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*B*b^5*x^6 - 6*A*a^5 + 4*(5*B*a*b^4 + A*b^5)*x^5 + 30*(2*B*a^2*b^3
+ A*a*b^4)*x^4 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 60*(B*a^4*b + 2*A*a^3*b
^2)*x^2*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx = & \frac{Bb^5x^4}{4} + 5a^3b(2Ab+Ba)\log(x) \\ & + x^3\left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right) + x^2\cdot\left(\frac{5Aab^4}{2} + 5Ba^2b^3\right) \\ & + x\left(10Aa^2b^3 + 10Ba^3b^2\right) + \frac{-Aa^5 + x(-10Aa^4b - 2Ba^5)}{2x^2}\end{aligned}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**3,x)
```

output
```
B*b**5*x**4/4 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**
4/3) + x**2*(5*A*a*b**4/2 + 5*B*a**2*b**3) + x*(10*A*a**2*b**3 + 10*B*a**3
*b**2) + (-A*a**5 + x*(-10*A*a**4*b - 2*B*a**5))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx = \frac{1}{4}\,Bb^5x^4 + \frac{1}{3}\left(5\,Bab^4 + Ab^5\right)x^3 + \frac{5}{2}\left(2\,Ba^2b^3 + Aab^4\right)x^2 + 10\left(Ba^3b^2 + Aa^2b^3\right)x + 5\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(x\right) - \frac{Aa^5 + 2\,(Ba^5 + 5\,Aa^4b)x}{2\,x^2}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/4*B*b^5*x^4 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*
x^2 + 10*(B*a^3*b^2 + A*a^2*b^3)*x + 5*(B*a^4*b + 2*A*a^3*b^2)*log(x) - 1/
2*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx = \frac{1}{4}\,Bb^5x^4 + \frac{5}{3}\,Bab^4x^3 + \frac{1}{3}\,Ab^5x^3 + 5\,Ba^2b^3x^2 + \frac{5}{2}\,Aab^4x^2 + 10\,Ba^3b^2x + 10\,Aa^2b^3x + 5\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(|x|\right) - \frac{Aa^5 + 2\,(Ba^5 + 5\,Aa^4b)x}{2\,x^2}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/4*B*b^5*x^4 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*B*a^2*b^3*x^2 + 5/2*A*
a*b^4*x^2 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x + 5*(B*a^4*b + 2*A*a^3*b^2)*lo
g(abs(x)) - 1/2*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx = \ln(x)\ \left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{x\,\left(B\,a^5+5\,A\,b\,a^4\right)+\frac{A\,a^5}{2}}{x^2}+x^3\,\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+\frac{B\,b^5\,x^4}{4}+10\,a^2\,b^2\,x\,\left(A\,b+B\,a\right)+\frac{5\,a\,b^3\,x^2\,\left(A\,b+2\,B\,a\right)}{2}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^3,x)
```

output
```
log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - (x*(B*a^5 + 5*A*a^4*b) + (A*a^5)/2)/x^
2 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^4)/4 + 10*a^2*b^2*x*(A*b +
B*a) + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx = \frac{60\log(x)\,a^4b^2x^2-2a^6-24a^5bx+80a^3b^3x^3+30a^2b^4x^4+8a\,b^5x^5+b^6x^6}{4x^2}$$

input
```
int((b*x+a)^5*(B*x+A)/x^3,x)
```

output
```
(60*log(x)*a**4*b**2*x**2 - 2*a**6 - 24*a**5*b*x + 80*a**3*b**3*x**3 + 30*
a**2*b**4*x**4 + 8*a*b**5*x**5 + b**6*x**6)/(4*x**2)
```

3.97 $\int \frac{(a+bx)^5(A+Bx)}{x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{(a+bx)^5(A+Bx)}{x^4} \, dx = -\frac{a^5A}{3x^3} - \frac{a^4(5Ab+aB)}{2x^2} - \frac{5a^3b(2Ab+aB)}{x} + 5ab^3(Ab + 2aB)x + \frac{1}{2}b^4(Ab+5aB)x^2 + \frac{1}{3}b^5Bx^3 + 10a^2b^2(Ab+aB)\log(x)$$

output
```
-1/3*a^5*A/x^3-1/2*a^4*(5*A*b+B*a)/x^2-5*a^3*b*(2*A*b+B*a)/x+5*a*b^3*(A*b+
2*B*a)*x+1/2*b^4*(A*b+5*B*a)*x^2+1/3*b^5*B*x^3+10*a^2*b^2*(A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^5(A+Bx)}{x^4} \, dx = \frac{-60a^3Ab^2x^2 + 60a^2b^3Bx^4 + 15ab^4x^4(2A+Bx) - 15a^4bx(A+2Bx) + b^5x^5(3A+2Bx) - a^5(2A+3Bx}{6x^3}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^4,x]
```

output
```
(-60*a^3*A*b^2*x^2 + 60*a^2*b^3*B*x^4 + 15*a*b^4*x^4*(2*A + B*x) - 15*a^4*
b*x*(A + 2*B*x) + b^5*x^5*(3*A + 2*B*x) - a^5*(2*A + 3*B*x) + 60*a^2*b^2*(
A*b + a*B)*x^3*Log[x])/(6*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^5A}{x^4}+\frac{a^4(aB+5Ab)}{x^3}+\frac{5a^3b(aB+2Ab)}{x^2}+\frac{10a^2b^2(aB+Ab)}{x}+b^4x(5aB+Ab)+5ab^3(2aB+Ab)+b^5Bx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{3x^3}-\frac{a^4(aB+5Ab)}{2x^2}-\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{3}b^5Bx^3$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^4,x]
```

output
```
-1/3*(a^5*A)/x^3 - (a^4*(5*A*b + a*B))/(2*x^2) - (5*a^3*b*(2*A*b + a*B))/x
 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^3)/3 + 1
0*a^2*b^2*(A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
default	$\frac{b^5 B\,x^3}{3}+\frac{A\,b^5x^2}{2}+\frac{5Ba\,b^4x^2}{2}+5Aa\,b^4x+10B\,a^2b^3x-\frac{a^5A}{3x^3}-\frac{a^4(5Ab+Ba)}{2x^2}+10a^2b^2(Ab+Ba)\ln$
risch	$\frac{b^5 B\,x^3}{3}+\frac{A\,b^5x^2}{2}+\frac{5Ba\,b^4x^2}{2}+5Aa\,b^4x+10B\,a^2b^3x+\frac{(-10a^3b^2A-5a^4bB)x^2+(-\frac{5}{2}a^4bA-\frac{1}{2}a^5B)x-\frac{a^5A}{3}}{x^3}$
norman	$\frac{(\frac{1}{2}b^5A+\frac{5}{2}a\,b^4B)x^5+(-\frac{5}{2}a^4bA-\frac{1}{2}a^5B)x+(5a\,b^4A+10a^2b^3B)x^4+(-10a^3b^2A-5a^4bB)x^2-\frac{a^5A}{3}+\frac{B\,b^5x^6}{3}}{x^3}+(10a^2b^3A$
parallelrisch	$\frac{2B\,b^5x^6+3A\,b^5x^5+15Ba\,b^4x^5+60A\ln(x)x^3a^2b^3+30aA\,b^4x^4+60B\ln(x)x^3a^3b^2+60B\,a^2b^3x^4-60a^3A\,b^2x^2-30B\,a^4b\,x^2-15}{6x^3}$

input
```
int((b*x+a)^5*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^5*B*x^3+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2+5*A*a*b^4*x+10*B*a^2*b^3*x-1/3
*a^5*A/x^3-1/2*a^4*(5*A*b+B*a)/x^2+10*a^2*b^2*(A*b+B*a)*ln(x)-5*a^3*b*(2*A
*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx$$
$$= \frac{2\,Bb^5x^6 - 2\,Aa^5 + 3\,(5\,Bab^4 + Ab^5)x^5 + 30\,(2\,Ba^2b^3 + Aab^4)x^4 + 60\,(Ba^3b^2 + Aa^2b^3)x^3\log(x) - 30\,(B}{6\,x^3}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^5*x^6 - 2*A*a^5 + 3*(5*B*a*b^4 + A*b^5)*x^5 + 30*(2*B*a^2*b^3 +
 A*a*b^4)*x^4 + 60*(B*a^3*b^2 + A*a^2*b^3)*x^3*log(x) - 30*(B*a^4*b + 2*A*
a^3*b^2)*x^2 - 3*(B*a^5 + 5*A*a^4*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx = \frac{Bb^5x^3}{3} + 10a^2b^2(Ab+Ba)\log(x) + x^2\left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right) + x\left(5Aab^4 + 10Ba^2b^3\right) + \frac{-2Aa^5 + x^2(-60Aa^3b^2 - 30Ba^4b) + x(-15Aa^4b - 3Ba^5)}{6x^3}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**4,x)
```

output
```
B*b**5*x**3/3 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**2*(A*b**5/2 + 5*B*a*b
**4/2) + x*(5*A*a*b**4 + 10*B*a**2*b**3) + (-2*A*a**5 + x**2*(-60*A*a**3*b
**2 - 30*B*a**4*b) + x*(-15*A*a**4*b - 3*B*a**5))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx = \frac{1}{3}\,Bb^5x^3 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)x^2 + 5\left(2\,Ba^2b^3 + Aab^4\right)x + 10\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x\right) - \frac{2\,Aa^5 + 30\,(Ba^4b + 2\,Aa^3b^2)x^2 + 3\,(Ba^5 + 5\,Aa^4b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/3*B*b^5*x^3 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 + 5*(2*B*a^2*b^3 + A*a*b^4)*x
+ 10*(B*a^3*b^2 + A*a^2*b^3)*log(x) - 1/6*(2*A*a^5 + 30*(B*a^4*b + 2*A*a^3
*b^2)*x^2 + 3*(B*a^5 + 5*A*a^4*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx = \frac{1}{3}\,Bb^5x^3 + \frac{5}{2}\,Bab^4x^2 + \frac{1}{2}\,Ab^5x^2 + 10\,Ba^2b^3x + 5\,Aab^4x + 10\left(Ba^3b^2 + Aa^2b^3\right)\log\left(|x|\right) - \frac{2\,Aa^5 + 30\,(Ba^4b + 2\,Aa^3b^2)x^2 + 3\,(Ba^5 + 5\,Aa^4b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/3*B*b^5*x^3 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 + 10*B*a^2*b^3*x + 5*A*a*b
^4*x + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/6*(2*A*a^5 + 30*(B*a^4*b
 + 2*A*a^3*b^2)*x^2 + 3*(B*a^5 + 5*A*a^4*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx = x^2\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right) - \frac{x\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)+\frac{A\,a^5}{3}+x^2\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)}{x^3} + \ln\left(x\right)\,\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right) + \frac{B\,b^5\,x^3}{3}+5\,a\,b^3\,x\,\left(A\,b+2\,B\,a\right)$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^4,x)
```

output
```
x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - (x*((B*a^5)/2 + (5*A*a^4*b)/2) + (A*a^5)
/3 + x^2*(10*A*a^3*b^2 + 5*B*a^4*b))/x^3 + log(x)*(10*A*a^2*b^3 + 10*B*a^3
*b^2) + (B*b^5*x^3)/3 + 5*a*b^3*x*(A*b + 2*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx = \frac{60\log(x)\,a^3b^3x^3-a^6-9a^5bx-45a^4b^2x^2+45a^2b^4x^4+9a\,b^5x^5+b^6x^6}{3x^3}$$

input
```
int((b*x+a)^5*(B*x+A)/x^4,x)
```

output
```
(60*log(x)*a**3*b**3*x**3 - a**6 - 9*a**5*b*x - 45*a**4*b**2*x**2 + 45*a**
2*b**4*x**4 + 9*a*b**5*x**5 + b**6*x**6)/(3*x**3)
```

3.98 $\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{2x^2}-\frac{10a^2b^2(Ab+aB)}{x}+b^4(Ab+5aB)x+\frac{1}{2}b^5Bx^2+5ab^3(Ab+2aB)\log(x)$$

output
```
-1/4*a^5*A/x^4-1/3*a^4*(5*A*b+B*a)/x^3-5/2*a^3*b*(2*A*b+B*a)/x^2-10*a^2*b^
2*(A*b+B*a)/x+b^4*(A*b+5*B*a)*x+1/2*b^5*B*x^2+5*a*b^3*(A*b+2*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = -\frac{10a^2Ab^3}{x}+5ab^4Bx+\frac{1}{2}b^5x(2A+Bx)-\frac{5a^3b^2(A+2Bx)}{x^2}-\frac{5a^4b(2A+3Bx)}{6x^3}-\frac{a^5(3A+4Bx)}{12x^4}+5ab^3(Ab+2aB)\log(x)$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^5,x]
```

output
```
(-10*a^2*A*b^3)/x + 5*a*b^4*B*x + (b^5*x*(2*A + B*x))/2 - (5*a^3*b^2*(A +
2*B*x))/x^2 - (5*a^4*b*(2*A + 3*B*x))/(6*x^3) - (a^5*(3*A + 4*B*x))/(12*x^
4) + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^5} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^5A}{x^5}+\frac{a^4(aB+5Ab)}{x^4}+\frac{5a^3b(aB+2Ab)}{x^3}+\frac{10a^2b^2(aB+Ab)}{x^2}+b^4(5aB+Ab)+\frac{5ab^3(2aB+Ab)}{x}+b^5Bx\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{10a^2b^2(aB+Ab)}{x}+b^4x(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{2}b^5Bx^2$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^5,x]
```

output
```
-1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/(
2*x^2) - (10*a^2*b^2*(A*b + a*B))/x + b^4*(A*b + 5*a*B)*x + (b^5*B*x^2)/2
+ 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^5 B x^2}{2}+A b^5 x+5 B a b^4 x-\frac{a^4 (5 A b+B a)}{3 x^3}-\frac{5 a^3 b (2 A b+B a)}{2 x^2}-\frac{a^5 A}{4 x^4}+5 a b^3 (A b+2 B a) \ln (x)-\frac{10a}{}$
risch	$\frac{b^5 B x^2}{2}+A b^5 x+5 B a b^4 x+\frac{(-10 a^2 b^3 A-10 a^3 b^2 B) x^3+(-5 a^3 b^2 A-\frac{5}{2} a^4 b B) x^2+(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B) x-\frac{a^5 A}{4}}{x^4}+5A$
norman	$\frac{(-5 a^3 b^2 A-\frac{5}{2} a^4 b B) x^2+(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B) x+(b^5 A+5 a b^4 B) x^5+(-10 a^2 b^3 A-10 a^3 b^2 B) x^3-\frac{a^5 A}{4}+\frac{B b^5 x^6}{2}}{x^4}+(5 a b^4 A+$
parallelrisch	$\frac{6 B b^5 x^6+60 A \ln (x) x^4 a b^4+12 A b^5 x^5+120 B \ln (x) x^4 a^2 b^3+60 B a b^4 x^5-120 a^2 A b^3 x^3-120 B a^3 b^2 x^3-60 a^3 A b^2 x^2-30 B a^4 b x}{12 x^4}$

input
```
int((b*x+a)^5*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^5*B*x^2+A*b^5*x+5*B*a*b^4*x-1/3*a^4*(5*A*b+B*a)/x^3-5/2*a^3*b*(2*A*b
+B*a)/x^2-1/4*a^5*A/x^4+5*a*b^3*(A*b+2*B*a)*ln(x)-10*a^2*b^2*(A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = \frac{6\,Bb^5x^6 - 3\,Aa^5 + 12\,(5\,Bab^4 + Ab^5)x^5 + 60\,(2\,Ba^2b^3 + Aab^4)x^4\log(x) - 120\,(Ba^3b^2 + Aa^2b^3)x^3 - 30}{12\,x^4}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
1/12*(6*B*b^5*x^6 - 3*A*a^5 + 12*(5*B*a*b^4 + A*b^5)*x^5 + 60*(2*B*a^2*b^3
 + A*a*b^4)*x^4*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 30*(B*a^4*b + 2
*A*a^3*b^2)*x^2 - 4*(B*a^5 + 5*A*a^4*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = \frac{Bb^5x^2}{2} + 5ab^3(Ab+2Ba)\log(x) + x\left(Ab^5+5Bab^4\right) + \frac{-3Aa^5 + x^3(-120Aa^2b^3 - 120Ba^3b^2) + x^2(-60Aa^3b^2 - 30Ba^4b) + x(-20Aa^4b - 4Ba^5)}{12x^4}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**5,x)
```

output
```
B*b**5*x**2/2 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x*(A*b**5 + 5*B*a*b**4) +
(-3*A*a**5 + x**3*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**2*(-60*A*a**3*
b**2 - 30*B*a**4*b) + x*(-20*A*a**4*b - 4*B*a**5))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = \frac{1}{2}\,Bb^5x^2 + \left(5\,Bab^4 + Ab^5\right)x + 5\left(2\,Ba^2b^3 + Aab^4\right)\log\left(x\right) - \frac{3\,Aa^5 + 120\,(Ba^3b^2 + Aa^2b^3)x^3 + 30\,(Ba^4b + 2\,Aa^3b^2)x^2 + 4\,(Ba^5 + 5\,Aa^4b)x}{12\,x^4}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
1/2*B*b^5*x^2 + (5*B*a*b^4 + A*b^5)*x + 5*(2*B*a^2*b^3 + A*a*b^4)*log(x) -
 1/12*(3*A*a^5 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 30*(B*a^4*b + 2*A*a^3*b
^2)*x^2 + 4*(B*a^5 + 5*A*a^4*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = \frac{1}{2}\,Bb^5x^2 + 5\,Bab^4x + Ab^5x + 5\left(2\,Ba^2b^3 + Aab^4\right)\log\left(|x|\right) - \frac{3\,Aa^5 + 120\,(Ba^3b^2 + Aa^2b^3)x^3 + 30\,(Ba^4b + 2\,Aa^3b^2)x^2 + 4\,(Ba^5 + 5\,Aa^4b)x}{12\,x^4}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
1/2*B*b^5*x^2 + 5*B*a*b^4*x + A*b^5*x + 5*(2*B*a^2*b^3 + A*a*b^4)*log(abs(
x)) - 1/12*(3*A*a^5 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 30*(B*a^4*b + 2*A*
a^3*b^2)*x^2 + 4*(B*a^5 + 5*A*a^4*b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx$$
$$= x\left(A\,b^5+5\,B\,a\,b^4\right)+\ln\left(x\right)\,\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right) - \frac{x\,\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+\frac{A\,a^5}{4}+x^2\,\left(\frac{5\,B\,a^4\,b}{2}+5\,A\,a^3\,b^2\right)+x^3\,\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)}{x^4} + \frac{B\,b^5\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^5,x)
```

output
```
x*(A*b^5 + 5*B*a*b^4) + log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - (x*((B*a^5)/3
+ (5*A*a^4*b)/3) + (A*a^5)/4 + x^2*(5*A*a^3*b^2 + (5*B*a^4*b)/2) + x^3*(10
*A*a^2*b^3 + 10*B*a^3*b^2))/x^4 + (B*b^5*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx = \frac{60\log(x)\,a^2b^4x^4-a^6-8a^5bx-30a^4b^2x^2-80a^3b^3x^3+24a\,b^5x^5+2b^6x^6}{4x^4}$$

input
```
int((b*x+a)^5*(B*x+A)/x^5,x)
```

output
```
(60*log(x)*a**2*b**4*x**4 - a**6 - 8*a**5*b*x - 30*a**4*b**2*x**2 - 80*a**
3*b**3*x**3 + 24*a*b**5*x**5 + 2*b**6*x**6)/(4*x**4)
```

3.99 $\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{5a^2b^2(Ab+aB)}{x^2} - \frac{5ab^3(Ab+2aB)}{x}+b^5Bx+b^4(Ab+5aB)\log(x)$$

output
```
-1/5*a^5*A/x^5-1/4*a^4*(5*A*b+B*a)/x^4-5/3*a^3*b*(2*A*b+B*a)/x^3-5*a^2*b^2
*(A*b+B*a)/x^2-5*a*b^3*(A*b+2*B*a)/x+b^5*B*x+b^4*(A*b+5*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = -\frac{5aAb^4}{x}+b^5Bx-\frac{5a^2b^3(A+2Bx)}{x^2}-\frac{5a^3b^2(2A+3Bx)}{3x^3} - \frac{5a^4b(3A+4Bx)}{12x^4}-\frac{a^5(4A+5Bx)}{20x^5}+b^4(Ab+5aB)\log(x)$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^6,x]
```

output
```
(-5*a*A*b^4)/x + b^5*B*x - (5*a^2*b^3*(A + 2*B*x))/x^2 - (5*a^3*b^2*(2*A +
 3*B*x))/(3*x^3) - (5*a^4*b*(3*A + 4*B*x))/(12*x^4) - (a^5*(4*A + 5*B*x))/
(20*x^5) + b^4*(A*b + 5*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^6} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^5A}{x^6}+\frac{a^4(aB+5Ab)}{x^5}+\frac{5a^3b(aB+2Ab)}{x^4}+\frac{10a^2b^2(aB+Ab)}{x^3}+\frac{b^4(5aB+Ab)}{x}+\frac{5ab^3(2aB+Ab)}{x^2}+b^5B\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{5a^2b^2(aB+Ab)}{x^2}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+b^5Bx$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/(
3*x^3) - (5*a^2*b^2*(A*b + a*B))/x^2 - (5*a*b^3*(A*b + 2*a*B))/x + b^5*B*x
 + b^4*(A*b + 5*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a^{5} A}{5 x^{5}}-\frac{a^{4} (5 A b +B a)}{4 x^{4}}-\frac{5 a^{3} b (2 A b +B a)}{3 x^{3}}-\frac{5 a^{2} b^{2} (A b +B a)}{x^{2}}-\frac{5 a \,b^{3} (A b +2 B a)}{x}+b^{5} B x +b^{4} (A b +5 B a) \ln$
risch	$b^{5} B x +\frac{\left(-5 a \,b^{4} A -10 a^{2} b^{3} B \right) x^{4}+\left(-5 a^{2} b^{3} A -5 a^{3} b^{2} B \right) x^{3}+\left(-\frac{10}{3} a^{3} b^{2} A -\frac{5}{3} a^{4} b B \right) x^{2}+\left(-\frac{5}{4} a^{4} b A -\frac{1}{4} a^{5} B \right) x -\frac{a^{5} A}{5}}{x^{5}}+A \ln$
norman	$\frac{\left(-\frac{10}{3} a^{3} b^{2} A -\frac{5}{3} a^{4} b B \right) x^{2}+\left(-\frac{5}{4} a^{4} b A -\frac{1}{4} a^{5} B \right) x +\left(-5 a \,b^{4} A -10 a^{2} b^{3} B \right) x^{4}+\left(-5 a^{2} b^{3} A -5 a^{3} b^{2} B \right) x^{3}+B \,b^{5} x^{6}-\frac{a^{5} A}{5}}{x^{5}}+(b^{5} A$
parallelrisch	$\frac{60 A \ln (x) x^{5} b^{5}+300 B \ln (x) x^{5} a \,b^{4}+60 B \,b^{5} x^{6}-300 a A \,b^{4} x^{4}-600 B \,a^{2} b^{3} x^{4}-300 a^{2} A \,b^{3} x^{3}-300 B \,a^{3} b^{2} x^{3}-200 a^{3} A \,b^{2} x^{2}-100 B}{60 x^{5}}$

input
```
int((b*x+a)^5*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^5*A/x^5-1/4*a^4*(5*A*b+B*a)/x^4-5/3*a^3*b*(2*A*b+B*a)/x^3-5*a^2*b^2
*(A*b+B*a)/x^2-5*a*b^3*(A*b+2*B*a)/x+b^5*B*x+b^4*(A*b+5*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = \frac{60\,Bb^5x^6 + 60\,(5\,Bab^4 + Ab^5)x^5\log(x) - 12\,Aa^5 - 300\,(2\,Ba^2b^3 + Aab^4)x^4 - 300\,(Ba^3b^2 + Aa^2b^3)x^3 -}{60\,x^5}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^5*x^6 + 60*(5*B*a*b^4 + A*b^5)*x^5*log(x) - 12*A*a^5 - 300*(2
*B*a^2*b^3 + A*a*b^4)*x^4 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 100*(B*a^4*b
 + 2*A*a^3*b^2)*x^2 - 15*(B*a^5 + 5*A*a^4*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = Bb^5x + b^4(Ab+5Ba)\log(x) + \frac{-12Aa^5 + x^4(-300Aab^4 - 600Ba^2b^3) + x^3(-300Aa^2b^3 - 300Ba^3b^2) + x^2(-200Aa^3b^2 - 100Ba^4b) +}{60x^5}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**6,x)
```

output
```
B*b**5*x + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**4*(-300*A*a*b**4 -
 600*B*a**2*b**3) + x**3*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**2*(-200
*A*a**3*b**2 - 100*B*a**4*b) + x*(-75*A*a**4*b - 15*B*a**5))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = Bb^5x + \left(5\,Bab^4 + Ab^5\right)\log\left(x\right)$$
$$-\frac{12\,Aa^5 + 300\,(2\,Ba^2b^3 + Aab^4)x^4 + 300\,(Ba^3b^2 + Aa^2b^3)x^3 + 100\,(Ba^4b + 2\,Aa^3b^2)x^2 + 15\,(Ba^5 + 5\,}{60\,x^5}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
B*b^5*x + (5*B*a*b^4 + A*b^5)*log(x) - 1/60*(12*A*a^5 + 300*(2*B*a^2*b^3 +
 A*a*b^4)*x^4 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 100*(B*a^4*b + 2*A*a^3*b
^2)*x^2 + 15*(B*a^5 + 5*A*a^4*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = Bb^5x + \left(5\,Bab^4 + Ab^5\right)\log\left(|x|\right)$$
$$-\frac{12\,Aa^5 + 300\,(2\,Ba^2b^3 + Aab^4)x^4 + 300\,(Ba^3b^2 + Aa^2b^3)x^3 + 100\,(Ba^4b + 2\,Aa^3b^2)x^2 + 15\,(Ba^5 + 5\,}{60\,x^5}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
B*b^5*x + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/60*(12*A*a^5 + 300*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 100*(B*a^4*b + 2*A*
a^3*b^2)*x^2 + 15*(B*a^5 + 5*A*a^4*b)*x)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx = \ln(x)\,\left(A\,b^5+5\,B\,a\,b^4\right)$$
$$-\frac{x\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)+\frac{A\,a^5}{5}+x^4\,(10\,B\,a^2\,b^3+5\,A\,a\,b^4)+x^2\left(\frac{5\,B\,a^4\,b}{3}+\frac{10\,A\,a^3\,b^2}{3}\right)+x^3\,(5\,B\,a^3\,b^2+5\,A\,a^2\,b}{x^5}$$
$$+B\,b^5\,x$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^6,x)
```

output
```
log(x)*(A*b^5 + 5*B*a*b^4) - (x*((B*a^5)/4 + (5*A*a^4*b)/4) + (A*a^5)/5 +
x^4*(10*B*a^2*b^3 + 5*A*a*b^4) + x^2*((10*A*a^3*b^2)/3 + (5*B*a^4*b)/3) +
x^3*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^5 + B*b^5*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx$$
$$=\frac{60\log(x)\,a\,b^5x^5-2a^6-15a^5bx-50a^4b^2x^2-100a^3b^3x^3-150a^2b^4x^4+10b^6x^6}{10x^5}$$

input
```
int((b*x+a)^5*(B*x+A)/x^6,x)
```

output
```
(60*log(x)*a*b**5*x**5 - 2*a**6 - 15*a**5*b*x - 50*a**4*b**2*x**2 - 100*a*
*3*b**3*x**3 - 150*a**2*b**4*x**4 + 10*b**6*x**6)/(10*x**5)
```

3.100 $\int \frac{(a+bx)^5(A+Bx)}{x^7} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{(a+bx)^5(A+Bx)}{x^7} \, dx = -\frac{a^5B}{5x^5} - \frac{5a^4bB}{4x^4} - \frac{10a^3b^2B}{3x^3} - \frac{5a^2b^3B}{x^2} - \frac{5ab^4B}{x} - \frac{A(a+bx)^6}{6ax^6} + b^5B\log(x)$$

output
```
-1/5*a^5*B/x^5-5/4*a^4*b*B/x^4-10/3*a^3*b^2*B/x^3-5*a^2*b^3*B/x^2-5*a*b^4*
B/x-1/6*A*(b*x+a)^6/a/x^6+b^5*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^5(A+Bx)}{x^7} \, dx = -\frac{60Ab^5x^5 + 150ab^4x^4(A+2Bx) + 100a^2b^3x^3(2A+3Bx) + 50a^3b^2x^2(3A+4Bx) + 15a^4bx(4A+5Bx}{60x^6}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^7,x]
```

output
```
-1/60*(60*A*b^5*x^5 + 150*a*b^4*x^4*(A + 2*B*x) + 100*a^2*b^3*x^3*(2*A + 3
*B*x) + 50*a^3*b^2*x^2*(3*A + 4*B*x) + 15*a^4*b*x*(4*A + 5*B*x) + 2*a^5*(5
*A + 6*B*x) - 60*b^5*B*x^6*Log[x])/x^6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{(a+bx)^5}{x^6}dx-\frac{A(a+bx)^6}{6ax^6}$$

$$\Big\downarrow \text{49}$$

$$B\int \left(\frac{a^5}{x^6}+\frac{5ba^4}{x^5}+\frac{10b^2a^3}{x^4}+\frac{10b^3a^2}{x^3}+\frac{5b^4a}{x^2}+\frac{b^5}{x}\right)dx-\frac{A(a+bx)^6}{6ax^6}$$

$$\Big\downarrow \text{2009}$$

$$B\left(-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\right)-\frac{A(a+bx)^6}{6ax^6}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^7,x]
```

output
```
-1/6*(A*(a + b*x)^6)/(a*x^6) + B*(-1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a
^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 - (5*a*b^4)/x + b^5*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

method	result
default	$-\frac{10a^2b^2(Ab+Ba)}{3x^3}-\frac{a^4(5Ab+Ba)}{5x^5}-\frac{5a\,b^3(Ab+2Ba)}{2x^2}-\frac{5a^3b(2Ab+Ba)}{4x^4}+b^5B\ln\left(x\right)-\frac{b^4(Ab+5Ba)}{x}-\frac{a^5A}{6x^6}$
norman	$\frac{\left(-\frac{5}{2}a\,b^4A-5a^2b^3B\right)x^4+\left(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B\right)x^3+\left(-\frac{5}{2}a^3b^2A-\frac{5}{4}a^4bB\right)x^2+\left(-a^4bA-\frac{1}{5}a^5B\right)x+\left(-b^5A-5a\,b^4B\right)x^5-\frac{a^5A}{6}}{x^6}$
risch	$\frac{\left(-\frac{5}{2}a\,b^4A-5a^2b^3B\right)x^4+\left(-\frac{10}{3}a^2b^3A-\frac{10}{3}a^3b^2B\right)x^3+\left(-\frac{5}{2}a^3b^2A-\frac{5}{4}a^4bB\right)x^2+\left(-a^4bA-\frac{1}{5}a^5B\right)x+\left(-b^5A-5a\,b^4B\right)x^5-\frac{a^5A}{6}}{x^6}$
parallelrisch	$-\frac{-60b^5B\ln(x)x^6+60A\,b^5x^5+300Ba\,b^4x^5+150aA\,b^4x^4+300B\,a^2b^3x^4+200a^2A\,b^3x^3+200B\,a^3b^2x^3+150a^3A\,b^2x^2+75B\,a^4}{60x^6}$

input
```
int((b*x+a)^5*(B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-10/3*a^2*b^2*(A*b+B*a)/x^3-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/
x^2-5/4*a^3*b*(2*A*b+B*a)/x^4+b^5*B*ln(x)-b^4*(A*b+5*B*a)/x-1/6*a^5*A/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx$$
$$= \frac{60\,Bb^5x^6\log\left(x\right) - 10\,Aa^5 - 60\,(5\,Bab^4 + Ab^5)x^5 - 150\,(2\,Ba^2b^3 + Aab^4)x^4 - 200\,(Ba^3b^2 + Aa^2b^3)x^3 -}{60\,x^6}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*B*b^5*x^6*log(x) - 10*A*a^5 - 60*(5*B*a*b^4 + A*b^5)*x^5 - 150*(2
*B*a^2*b^3 + A*a*b^4)*x^4 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^3 - 75*(B*a^4*b
+ 2*A*a^3*b^2)*x^2 - 12*(B*a^5 + 5*A*a^4*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx = Bb^5\log\left(x\right)$$
$$+ \frac{-10Aa^5 + x^5(-60Ab^5 - 300Bab^4) + x^4(-150Aab^4 - 300Ba^2b^3) + x^3(-200Aa^2b^3 - 200Ba^3b^2) + x^2(}{60x^6}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**7,x)
```

output
```
B*b**5*log(x) + (-10*A*a**5 + x**5*(-60*A*b**5 - 300*B*a*b**4) + x**4*(-15
0*A*a*b**4 - 300*B*a**2*b**3) + x**3*(-200*A*a**2*b**3 - 200*B*a**3*b**2)
+ x**2*(-150*A*a**3*b**2 - 75*B*a**4*b) + x*(-60*A*a**4*b - 12*B*a**5))/(6
0*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx = Bb^5\log(x) - \frac{10\,Aa^5 + 60\,(5\,Bab^4 + Ab^5)x^5 + 150\,(2\,Ba^2b^3 + Aab^4)x^4 + 200\,(Ba^3b^2 + Aa^2b^3)x^3 + 75\,(Ba^4b + 2\,Aa}{60\,x^6}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^7,x, algorithm="maxima")
```

output
```
B*b^5*log(x) - 1/60*(10*A*a^5 + 60*(5*B*a*b^4 + A*b^5)*x^5 + 150*(2*B*a^2*
b^3 + A*a*b^4)*x^4 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 75*(B*a^4*b + 2*A*a
^3*b^2)*x^2 + 12*(B*a^5 + 5*A*a^4*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx = Bb^5\log(|x|) - \frac{10\,Aa^5 + 60\,(5\,Bab^4 + Ab^5)x^5 + 150\,(2\,Ba^2b^3 + Aab^4)x^4 + 200\,(Ba^3b^2 + Aa^2b^3)x^3 + 75\,(Ba^4b + 2\,Aa}{60\,x^6}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^7,x, algorithm="giac")
```

output
```
B*b^5*log(abs(x)) - 1/60*(10*A*a^5 + 60*(5*B*a*b^4 + A*b^5)*x^5 + 150*(2*B
*a^2*b^3 + A*a*b^4)*x^4 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 75*(B*a^4*b +
2*A*a^3*b^2)*x^2 + 12*(B*a^5 + 5*A*a^4*b)*x)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx = B\,b^5\,\ln(x) - \frac{x\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+\frac{A\,a^5}{6}+x^4\left(5\,B\,a^2\,b^3+\frac{5\,A\,a\,b^4}{2}\right)+x^2\left(\frac{5\,B\,a^4\,b}{4}+\frac{5\,A\,a^3\,b^2}{2}\right)+x^5\left(A\,b^5+5\,B\,a\,b^4\right)+x^3}{x^6}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^7,x)
```

output
```
B*b^5*log(x) - (x*((B*a^5)/5 + A*a^4*b) + (A*a^5)/6 + x^4*(5*B*a^2*b^3 + (
5*A*a*b^4)/2) + x^2*((5*A*a^3*b^2)/2 + (5*B*a^4*b)/4) + x^5*(A*b^5 + 5*B*a
*b^4) + x^3*((10*A*a^2*b^3)/3 + (10*B*a^3*b^2)/3))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx = \frac{60\log(x)\,b^6x^6-10a^6-72a^5bx-225a^4b^2x^2-400a^3b^3x^3-450a^2b^4x^4-360a\,b^5x^5}{60x^6}$$

input
```
int((b*x+a)^5*(B*x+A)/x^7,x)
```

output
```
(60*log(x)*b**6*x**6 - 10*a**6 - 72*a**5*b*x - 225*a**4*b**2*x**2 - 400*a*
*3*b**3*x**3 - 450*a**2*b**4*x**4 - 360*a*b**5*x**5)/(60*x**6)
```

3.101 $\int \frac{(a+bx)^5(A+Bx)}{x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx)^5(A+Bx)}{x^8} dx = -\frac{A(a+bx)^6}{7ax^7} + \frac{(Ab-7aB)(a+bx)^6}{42a^2x^6}$$

output
```
-1/7*A*(b*x+a)^6/a/x^7+1/42*(A*b-7*B*a)*(b*x+a)^6/a^2/x^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(44) = 88.

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx)^5(A+Bx)}{x^8} dx = -\frac{21b^5x^5(A+2Bx) + 35ab^4x^4(2A+3Bx) + 35a^2b^3x^3(3A+4Bx) + 21a^3b^2x^2(4A+5Bx) + 7a^4bx(5A}{42x^7}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^8,x]
```

output
```
-1/42*(21*b^5*x^5*(A + 2*B*x) + 35*a*b^4*x^4*(2*A + 3*B*x) + 35*a^2*b^3*x^
3*(3*A + 4*B*x) + 21*a^3*b^2*x^2*(4*A + 5*B*x) + 7*a^4*b*x*(5*A + 6*B*x) +
 a^5*(6*A + 7*B*x))/x^7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^8} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-7aB)\int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{A(a+bx)^6}{7ax^7}$$

$$\Big\downarrow \text{48}$$

$$\frac{(a+bx)^6(Ab-7aB)}{42a^2x^6}-\frac{A(a+bx)^6}{7ax^7}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^8,x]
```

output
```
-1/7*(A*(a + b*x)^6)/(a*x^7) + ((A*b - 7*a*B)*(a + b*x)^6)/(42*a^2*x^6)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

method	result
default	$-\frac{5a\,b^{3}(Ab+2Ba)}{3x^{3}}-\frac{a^{3}b(2Ab+Ba)}{x^{5}}-\frac{b^{4}(Ab+5Ba)}{2x^{2}}-\frac{a^{5}A}{7x^{7}}-\frac{5a^{2}b^{2}(Ab+Ba)}{2x^{4}}-\frac{b^{5}B}{x}-\frac{a^{4}(5Ab+Ba)}{6x^{6}}$
norman	$\frac{-B\,b^{5}x^{6}+\left(-\frac{1}{2}b^{5}A-\frac{5}{2}a\,b^{4}B\right)x^{5}+\left(-\frac{5}{3}a\,b^{4}A-\frac{10}{3}a^{2}b^{3}B\right)x^{4}+\left(-\frac{5}{2}a^{2}b^{3}A-\frac{5}{2}a^{3}b^{2}B\right)x^{3}+\left(-2a^{3}b^{2}A-a^{4}bB\right)x^{2}+\left(-\frac{5}{6}a^{4}bA-\frac{1}{6}a\cdots\right.}{x^{7}}$
risch	$\frac{-B\,b^{5}x^{6}+\left(-\frac{1}{2}b^{5}A-\frac{5}{2}a\,b^{4}B\right)x^{5}+\left(-\frac{5}{3}a\,b^{4}A-\frac{10}{3}a^{2}b^{3}B\right)x^{4}+\left(-\frac{5}{2}a^{2}b^{3}A-\frac{5}{2}a^{3}b^{2}B\right)x^{3}+\left(-2a^{3}b^{2}A-a^{4}bB\right)x^{2}+\left(-\frac{5}{6}a^{4}bA-\frac{1}{6}a\cdots\right.}{x^{7}}$
gosper	$-\frac{42B\,b^{5}x^{6}+21A\,b^{5}x^{5}+105Ba\,b^{4}x^{5}+70aA\,b^{4}x^{4}+140B\,a^{2}b^{3}x^{4}+105a^{2}A\,b^{3}x^{3}+105B\,a^{3}b^{2}x^{3}+84a^{3}A\,b^{2}x^{2}+42B\,a^{4}b\,x^{2}+35a\cdots}{42x^{7}}$
parallelrisch	$-\frac{42B\,b^{5}x^{6}+21A\,b^{5}x^{5}+105Ba\,b^{4}x^{5}+70aA\,b^{4}x^{4}+140B\,a^{2}b^{3}x^{4}+105a^{2}A\,b^{3}x^{3}+105B\,a^{3}b^{2}x^{3}+84a^{3}A\,b^{2}x^{2}+42B\,a^{4}b\,x^{2}+35a\cdots}{42x^{7}}$
orering	$-\frac{42B\,b^{5}x^{6}+21A\,b^{5}x^{5}+105Ba\,b^{4}x^{5}+70aA\,b^{4}x^{4}+140B\,a^{2}b^{3}x^{4}+105a^{2}A\,b^{3}x^{3}+105B\,a^{3}b^{2}x^{3}+84a^{3}A\,b^{2}x^{2}+42B\,a^{4}b\,x^{2}+35a\cdots}{42x^{7}}$

input
```
int((b*x+a)^5*(B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-5/3*a*b^3*(A*b+2*B*a)/x^3-a^3*b*(2*A*b+B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2-1
/7*a^5*A/x^7-5/2*a^2*b^2*(A*b+B*a)/x^4-b^5*B/x-1/6*a^4*(5*A*b+B*a)/x^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.70

$$\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx = -\frac{42\,Bb^5x^6 + 6\,Aa^5 + 21\,(5\,Bab^4 + Ab^5)x^5 + 70\,(2\,Ba^2b^3 + Aab^4)x^4 + 105\,(Ba^3b^2 + Aa^2b^3)x^3 + 42\,(Ba}{42\,x^7}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^8,x, algorithm="fricas")
```

output
```
-1/42*(42*B*b^5*x^6 + 6*A*a^5 + 21*(5*B*a*b^4 + A*b^5)*x^5 + 70*(2*B*a^2*b
^3 + A*a*b^4)*x^4 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^2 + 7*(B*a^5 + 5*A*a^4*b)*x)/x^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(37) = 74$.

Time = 1.97 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx = \frac{-6Aa^5 - 42Bb^5x^6 + x^5(-21Ab^5 - 105Bab^4) + x^4(-70Aab^4 - 140Ba^2b^3) + x^3(-105Aa^2b^3 - 105Ba^3b^2}{42x^7}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**8,x)
```

output
```
(-6*A*a**5 - 42*B*b**5*x**6 + x**5*(-21*A*b**5 - 105*B*a*b**4) + x**4*(-70
*A*a*b**4 - 140*B*a**2*b**3) + x**3*(-105*A*a**2*b**3 - 105*B*a**3*b**2) +
 x**2*(-84*A*a**3*b**2 - 42*B*a**4*b) + x*(-35*A*a**4*b - 7*B*a**5))/(42*x
**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(41) = 82$.

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.70

$$\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx = -\frac{42\,Bb^5x^6 + 6\,Aa^5 + 21\,(5\,Bab^4 + Ab^5)x^5 + 70\,(2\,Ba^2b^3 + Aab^4)x^4 + 105\,(Ba^3b^2 + Aa^2b^3)x^3 + 42\,(Ba}{42\,x^7}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^8,x, algorithm="maxima")
```

output
```
-1/42*(42*B*b^5*x^6 + 6*A*a^5 + 21*(5*B*a*b^4 + A*b^5)*x^5 + 70*(2*B*a^2*b
^3 + A*a*b^4)*x^4 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^2 + 7*(B*a^5 + 5*A*a^4*b)*x)/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.80

$$\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx = -\frac{42\,Bb^5x^6 + 105\,Bab^4x^5 + 21\,Ab^5x^5 + 140\,Ba^2b^3x^4 + 70\,Aab^4x^4 + 105\,Ba^3b^2x^3 + 105\,Aa^2b^3x^3 + 42\,B}{42\,x^7}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^8,x, algorithm="giac")
```

output
```
-1/42*(42*B*b^5*x^6 + 105*B*a*b^4*x^5 + 21*A*b^5*x^5 + 140*B*a^2*b^3*x^4 +
 70*A*a*b^4*x^4 + 105*B*a^3*b^2*x^3 + 105*A*a^2*b^3*x^3 + 42*B*a^4*b*x^2 +
 84*A*a^3*b^2*x^2 + 7*B*a^5*x + 35*A*a^4*b*x + 6*A*a^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.68

$$\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx = -\frac{x\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+\frac{A\,a^5}{7}+x^2\,\left(B\,a^4\,b+2\,A\,a^3\,b^2\right)+x^4\,\left(\frac{10\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{3}\right)+x^5\,\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+x^3\,(}{x^7}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^8,x)
```

output
```
-(x*((B*a^5)/6 + (5*A*a^4*b)/6) + (A*a^5)/7 + x^2*(2*A*a^3*b^2 + B*a^4*b)
+ x^4*((10*B*a^2*b^3)/3 + (5*A*a*b^4)/3) + x^5*((A*b^5)/2 + (5*B*a*b^4)/2)
 + x^3*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2) + B*b^5*x^6)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx = \frac{-7b^6x^6-21a\,b^5x^5-35a^2b^4x^4-35a^3b^3x^3-21a^4b^2x^2-7a^5bx-a^6}{7x^7}$$

input
```
int((b*x+a)^5*(B*x+A)/x^8,x)
```

output
```
( - a**6 - 7*a**5*b*x - 21*a**4*b**2*x**2 - 35*a**3*b**3*x**3 - 35*a**2*b*
*4*x**4 - 21*a*b**5*x**5 - 7*b**6*x**6)/(7*x**7)
```

3.102 $\int \frac{(a+bx)^5(A+Bx)}{x^9} dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{(a+bx)^5(A+Bx)}{x^9} dx = -\frac{A(a+bx)^6}{8ax^8} + \frac{(Ab-4aB)(a+bx)^6}{28a^2x^7} - \frac{b(Ab-4aB)(a+bx)^6}{168a^3x^6}$$

output
```
-1/8*A*(b*x+a)^6/a/x^8+1/28*(A*b-4*B*a)*(b*x+a)^6/a^2/x^7-1/168*b*(A*b-4*B
*a)*(b*x+a)^6/a^3/x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^5(A+Bx)}{x^9} dx = -\frac{28b^5x^5(2A+3Bx) + 70ab^4x^4(3A+4Bx) + 84a^2b^3x^3(4A+5Bx) + 56a^3b^2x^2(5A+6Bx) + 20a^4bx(6}{168x^8}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^9,x]
```

output
```
-1/168*(28*b^5*x^5*(2*A + 3*B*x) + 70*a*b^4*x^4*(3*A + 4*B*x) + 84*a^2*b^3
*x^3*(4*A + 5*B*x) + 56*a^3*b^2*x^2*(5*A + 6*B*x) + 20*a^4*b*x*(6*A + 7*B*
x) + 3*a^5*(7*A + 8*B*x))/x^8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^9} \, dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(Ab-4aB)\int \frac{(a+bx)^5}{x^8}dx}{4a}-\frac{A(a+bx)^6}{8ax^8}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(Ab-4aB)\left(-\frac{b\int \frac{(a+bx)^5}{x^7}dx}{7a}-\frac{(a+bx)^6}{7ax^7}\right)}{4a}-\frac{A(a+bx)^6}{8ax^8}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}\right)(Ab-4aB)}{4a}-\frac{A(a+bx)^6}{8ax^8}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^9,x]
```

output
```
-1/8*(A*(a + b*x)^6)/(a*x^8) - ((A*b - 4*a*B)*(-1/7*(a + b*x)^6/(a*x^7) +
(b*(a + b*x)^6)/(42*a^2*x^6)))/(4*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

method	result
default	$-\frac{b^4(Ab+5Ba)}{3x^3}-\frac{2a^2b^2(Ab+Ba)}{x^5}-\frac{b^5B}{2x^2}-\frac{a^4(5Ab+Ba)}{7x^7}-\frac{5a\,b^3(Ab+2Ba)}{4x^4}-\frac{a^5A}{8x^8}-\frac{5a^3b(2Ab+Ba)}{6x^6}$
norman	$\frac{-\frac{B\,b^5x^6}{2}+\left(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B\right)x^5+\left(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B\right)x^4+\left(-2a^2b^3A-2a^3b^2B\right)x^3+\left(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB\right)x^2+\left(-\frac{5}{7}a^4bA-\frac{1}{7}\right.}{x^8}$
risch	$\frac{-\frac{B\,b^5x^6}{2}+\left(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B\right)x^5+\left(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B\right)x^4+\left(-2a^2b^3A-2a^3b^2B\right)x^3+\left(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB\right)x^2+\left(-\frac{5}{7}a^4bA-\frac{1}{7}\right.}{x^8}$
gosper	$-\frac{84B\,b^5x^6+56A\,b^5x^5+280Ba\,b^4x^5+210aA\,b^4x^4+420B\,a^2b^3x^4+336a^2A\,b^3x^3+336B\,a^3b^2x^3+280a^3A\,b^2x^2+140B\,a^4b\,x^2+}{168x^8}$
parallelrisch	$-\frac{84B\,b^5x^6+56A\,b^5x^5+280Ba\,b^4x^5+210aA\,b^4x^4+420B\,a^2b^3x^4+336a^2A\,b^3x^3+336B\,a^3b^2x^3+280a^3A\,b^2x^2+140B\,a^4b\,x^2+}{168x^8}$
orering	$-\frac{84B\,b^5x^6+56A\,b^5x^5+280Ba\,b^4x^5+210aA\,b^4x^4+420B\,a^2b^3x^4+336a^2A\,b^3x^3+336B\,a^3b^2x^3+280a^3A\,b^2x^2+140B\,a^4b\,x^2+}{168x^8}$

input
```
int((b*x+a)^5*(B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^4*(A*b+5*B*a)/x^3-2*a^2*b^2*(A*b+B*a)/x^5-1/2*b^5*B/x^2-1/7*a^4*(5*
A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-1/8*a^5*A/x^8-5/6*a^3*b*(2*A*b+B*a)
/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx = -\frac{84\,Bb^5x^6+21\,Aa^5+56\,(5\,Bab^4+Ab^5)x^5+210\,(2\,Ba^2b^3+Aab^4)x^4+336\,(Ba^3b^2+Aa^2b^3)x^3+140}{168\,x^8}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^9,x, algorithm="fricas")
```

output
```
-1/168*(84*B*b^5*x^6 + 21*A*a^5 + 56*(5*B*a*b^4 + A*b^5)*x^5 + 210*(2*B*a^
2*b^3 + A*a*b^4)*x^4 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^2 + 24*(B*a^5 + 5*A*a^4*b)*x)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(63) = 126.

Time = 2.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx = \frac{-21Aa^5-84Bb^5x^6+x^5(-56Ab^5-280Bab^4)+x^4(-210Aab^4-420Ba^2b^3)+x^3(-336Aa^2b^3-336Ba}{168x^8}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**9,x)
```

output
```
(-21*A*a**5 - 84*B*b**5*x**6 + x**5*(-56*A*b**5 - 280*B*a*b**4) + x**4*(-2
10*A*a*b**4 - 420*B*a**2*b**3) + x**3*(-336*A*a**2*b**3 - 336*B*a**3*b**2)
 + x**2*(-280*A*a**3*b**2 - 140*B*a**4*b) + x*(-120*A*a**4*b - 24*B*a**5))
/(168*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx = -\frac{84\,Bb^5x^6 + 21\,Aa^5 + 56\,(5\,Bab^4 + Ab^5)x^5 + 210\,(2\,Ba^2b^3 + Aab^4)x^4 + 336\,(Ba^3b^2 + Aa^2b^3)x^3 + 140}{168\,x^8}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^9,x, algorithm="maxima")
```

output
```
-1/168*(84*B*b^5*x^6 + 21*A*a^5 + 56*(5*B*a*b^4 + A*b^5)*x^5 + 210*(2*B*a^
2*b^3 + A*a*b^4)*x^4 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^2 + 24*(B*a^5 + 5*A*a^4*b)*x)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx = -\frac{84\,Bb^5x^6 + 280\,Bab^4x^5 + 56\,Ab^5x^5 + 420\,Ba^2b^3x^4 + 210\,Aab^4x^4 + 336\,Ba^3b^2x^3 + 336\,Aa^2b^3x^3 + 140}{168\,x^8}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^9,x, algorithm="giac")
```

output
```
-1/168*(84*B*b^5*x^6 + 280*B*a*b^4*x^5 + 56*A*b^5*x^5 + 420*B*a^2*b^3*x^4
+ 210*A*a*b^4*x^4 + 336*B*a^3*b^2*x^3 + 336*A*a^2*b^3*x^3 + 140*B*a^4*b*x^
2 + 280*A*a^3*b^2*x^2 + 24*B*a^5*x + 120*A*a^4*b*x + 21*A*a^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx =$$
$$-\frac{x\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+\frac{A\,a^5}{8}+x^4\left(\frac{5\,B\,a^2\,b^3}{2}+\frac{5\,A\,a\,b^4}{4}\right)+x^2\left(\frac{5\,B\,a^4\,b}{6}+\frac{5\,A\,a^3\,b^2}{3}\right)+x^5\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+x^3\,(2}{x^8}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^9,x)
```

output
```
-(x*((B*a^5)/7 + (5*A*a^4*b)/7) + (A*a^5)/8 + x^4*((5*B*a^2*b^3)/2 + (5*A*
a*b^4)/4) + x^2*((5*A*a^3*b^2)/3 + (5*B*a^4*b)/6) + x^5*((A*b^5)/3 + (5*B*
a*b^4)/3) + x^3*(2*A*a^2*b^3 + 2*B*a^3*b^2) + (B*b^5*x^6)/2)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx = \frac{-28b^6x^6-112a\,b^5x^5-210a^2b^4x^4-224a^3b^3x^3-140a^4b^2x^2-48a^5bx-7a^6}{56x^8}$$

input
```
int((b*x+a)^5*(B*x+A)/x^9,x)
```

output
```
( - 7*a**6 - 48*a**5*b*x - 140*a**4*b**2*x**2 - 224*a**3*b**3*x**3 - 210*a
**2*b**4*x**4 - 112*a*b**5*x**5 - 28*b**6*x**6)/(56*x**8)
```

3.103 $\int \frac{(a+bx)^5(A+Bx)}{x^{10}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 115

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}} \, dx = -\frac{a^5A}{9x^9} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{5a^3b(2Ab+aB)}{7x^7} - \frac{5a^2b^2(Ab+aB)}{3x^6} - \frac{ab^3(Ab+2aB)}{x^5} - \frac{b^4(Ab+5aB)}{4x^4} - \frac{b^5B}{3x^3}$$

output
```
-1/9*a^5*A/x^9-1/8*a^4*(5*A*b+B*a)/x^8-5/7*a^3*b*(2*A*b+B*a)/x^7-5/3*a^2*b
^2*(A*b+B*a)/x^6-a*b^3*(A*b+2*B*a)/x^5-1/4*b^4*(A*b+5*B*a)/x^4-1/3*b^5*B/x
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}} \, dx = -\frac{42b^5x^5(3A+4Bx) + 126ab^4x^4(4A+5Bx) + 168a^2b^3x^3(5A+6Bx) + 120a^3b^2x^2(6A+7Bx) + 45a^4b}{504x^9}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^10,x]
```

output
```
-1/504*(42*b^5*x^5*(3*A + 4*B*x) + 126*a*b^4*x^4*(4*A + 5*B*x) + 168*a^2*b
^3*x^3*(5*A + 6*B*x) + 120*a^3*b^2*x^2*(6*A + 7*B*x) + 45*a^4*b*x*(7*A + 8
*B*x) + 7*a^5*(8*A + 9*B*x))/x^9
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^5A}{x^{10}}+\frac{a^4(aB+5Ab)}{x^9}+\frac{5a^3b(aB+2Ab)}{x^8}+\frac{10a^2b^2(aB+Ab)}{x^7}+\frac{b^4(5aB+Ab)}{x^5}+\frac{5ab^3(2aB+Ab)}{x^6}+\frac{b^5B}{x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{3x^6}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{3x^3}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^10,x]
```

output
```
-1/9*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*b + a*B))/(
7*x^7) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (a*b^3*(A*b + 2*a*B))/x^5 - (b^
4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/(3*x^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^{5} A}{9 x^{9}}-\frac{a^{4} (5 A b+B a)}{8 x^{8}}-\frac{5 a^{3} b (2 A b+B a)}{7 x^{7}}-\frac{5 a^{2} b^{2} (A b+B a)}{3 x^{6}}-\frac{a\, b^{3} (A b+2 B a)}{x^{5}}-\frac{b^{4} (A b+5 B a)}{4 x^{4}}-\frac{b^{5} B}{3 x^{3}}$
norman	$\frac{-\frac{B\, b^{5} x^{6}}{3}+\left(-\frac{1}{4} b^{5} A-\frac{5}{4} a\, b^{4} B\right) x^{5}+\left(-a\, b^{4} A-2 a^{2} b^{3} B\right) x^{4}+\left(-\frac{5}{3} a^{2} b^{3} A-\frac{5}{3} a^{3} b^{2} B\right) x^{3}+\left(-\frac{10}{7} a^{3} b^{2} A-\frac{5}{7} a^{4} b B\right) x^{2}+\left(-\frac{5}{8} a^{4} b A-\frac{1}{8} a\right.}{x^{9}}$
risch	$\frac{-\frac{B\, b^{5} x^{6}}{3}+\left(-\frac{1}{4} b^{5} A-\frac{5}{4} a\, b^{4} B\right) x^{5}+\left(-a\, b^{4} A-2 a^{2} b^{3} B\right) x^{4}+\left(-\frac{5}{3} a^{2} b^{3} A-\frac{5}{3} a^{3} b^{2} B\right) x^{3}+\left(-\frac{10}{7} a^{3} b^{2} A-\frac{5}{7} a^{4} b B\right) x^{2}+\left(-\frac{5}{8} a^{4} b A-\frac{1}{8} a\right.}{x^{9}}$
gosper	$-\frac{168 B\, b^{5} x^{6}+126 A\, b^{5} x^{5}+630 B a\, b^{4} x^{5}+504 a A\, b^{4} x^{4}+1008 B\, a^{2} b^{3} x^{4}+840 a^{2} A\, b^{3} x^{3}+840 B\, a^{3} b^{2} x^{3}+720 a^{3} A\, b^{2} x^{2}+360 B\, a^{4} b}{504 x^{9}}$
parallelrisch	$-\frac{168 B\, b^{5} x^{6}+126 A\, b^{5} x^{5}+630 B a\, b^{4} x^{5}+504 a A\, b^{4} x^{4}+1008 B\, a^{2} b^{3} x^{4}+840 a^{2} A\, b^{3} x^{3}+840 B\, a^{3} b^{2} x^{3}+720 a^{3} A\, b^{2} x^{2}+360 B\, a^{4} b}{504 x^{9}}$
orering	$-\frac{168 B\, b^{5} x^{6}+126 A\, b^{5} x^{5}+630 B a\, b^{4} x^{5}+504 a A\, b^{4} x^{4}+1008 B\, a^{2} b^{3} x^{4}+840 a^{2} A\, b^{3} x^{3}+840 B\, a^{3} b^{2} x^{3}+720 a^{3} A\, b^{2} x^{2}+360 B\, a^{4} b}{504 x^{9}}$

input
```
int((b*x+a)^5*(B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^5*A/x^9-1/8*a^4*(5*A*b+B*a)/x^8-5/7*a^3*b*(2*A*b+B*a)/x^7-5/3*a^2*b
^2*(A*b+B*a)/x^6-a*b^3*(A*b+2*B*a)/x^5-1/4*b^4*(A*b+5*B*a)/x^4-1/3*b^5*B/x
^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx = -\frac{168\,Bb^5x^6 + 56\,Aa^5 + 126\,(5\,Bab^4 + Ab^5)x^5 + 504\,(2\,Ba^2b^3 + Aab^4)x^4 + 840\,(Ba^3b^2 + Aa^2b^3)x^3 + 36}{504\,x^9}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^10,x, algorithm="fricas")
```

output
```
-1/504*(168*B*b^5*x^6 + 56*A*a^5 + 126*(5*B*a*b^4 + A*b^5)*x^5 + 504*(2*B*
a^2*b^3 + A*a*b^4)*x^4 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 360*(B*a^4*b +
2*A*a^3*b^2)*x^2 + 63*(B*a^5 + 5*A*a^4*b)*x)/x^9
```

Sympy [A] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx = \frac{-56Aa^5 - 168Bb^5x^6 + x^5(-126Ab^5 - 630Bab^4) + x^4(-504Aab^4 - 1008Ba^2b^3) + x^3(-840Aa^2b^3 - 840}{504x^9}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**10,x)
```

output
```
(-56*A*a**5 - 168*B*b**5*x**6 + x**5*(-126*A*b**5 - 630*B*a*b**4) + x**4*(
-504*A*a*b**4 - 1008*B*a**2*b**3) + x**3*(-840*A*a**2*b**3 - 840*B*a**3*b*
*2) + x**2*(-720*A*a**3*b**2 - 360*B*a**4*b) + x*(-315*A*a**4*b - 63*B*a**
5))/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx = -\frac{168\,Bb^5x^6 + 56\,Aa^5 + 126\,(5\,Bab^4 + Ab^5)x^5 + 504\,(2\,Ba^2b^3 + Aab^4)x^4 + 840\,(Ba^3b^2 + Aa^2b^3)x^3 + 36}{504\,x^9}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^10,x, algorithm="maxima")
```

output
```
-1/504*(168*B*b^5*x^6 + 56*A*a^5 + 126*(5*B*a*b^4 + A*b^5)*x^5 + 504*(2*B*
a^2*b^3 + A*a*b^4)*x^4 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 360*(B*a^4*b +
2*A*a^3*b^2)*x^2 + 63*(B*a^5 + 5*A*a^4*b)*x)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx = -\frac{168\,Bb^5x^6 + 630\,Bab^4x^5 + 126\,Ab^5x^5 + 1008\,Ba^2b^3x^4 + 504\,Aab^4x^4 + 840\,Ba^3b^2x^3 + 840\,Aa^2b^3x^3 +}{504\,x^9}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^10,x, algorithm="giac")
```

output
```
-1/504*(168*B*b^5*x^6 + 630*B*a*b^4*x^5 + 126*A*b^5*x^5 + 1008*B*a^2*b^3*x
^4 + 504*A*a*b^4*x^4 + 840*B*a^3*b^2*x^3 + 840*A*a^2*b^3*x^3 + 360*B*a^4*b
*x^2 + 720*A*a^3*b^2*x^2 + 63*B*a^5*x + 315*A*a^4*b*x + 56*A*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx = -\frac{x\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+\frac{A\,a^5}{9}+x^4\,\left(2\,B\,a^2\,b^3+A\,a\,b^4\right)+x^2\,\left(\frac{5\,B\,a^4\,b}{7}+\frac{10\,A\,a^3\,b^2}{7}\right)+x^5\,\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^3\,\left(\right.}{x^9}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^10,x)
```

output
```
-(x*((B*a^5)/8 + (5*A*a^4*b)/8) + (A*a^5)/9 + x^4*(2*B*a^2*b^3 + A*a*b^4)
+ x^2*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^5*((A*b^5)/4 + (5*B*a*b^4)/4)
 + x^3*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3) + (B*b^5*x^6)/3)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx = \frac{-84b^6x^6-378a\,b^5x^5-756a^2b^4x^4-840a^3b^3x^3-540a^4b^2x^2-189a^5bx-28a^6}{252x^9}$$

input
```
int((b*x+a)^5*(B*x+A)/x^10,x)
```

output
```
( - 28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756
*a**2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

3.104 $\int \frac{(a+bx)^5(A+Bx)}{x^{11}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\begin{aligned}\int \frac{(a+bx)^5(A+Bx)}{x^{11}} \, dx = &-\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{9x^9}\\ &-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{10a^2b^2(Ab+aB)}{7x^7}\\ &-\frac{5ab^3(Ab+2aB)}{6x^6}-\frac{b^4(Ab+5aB)}{5x^5}-\frac{b^5B}{4x^4}\end{aligned}$$

output
```
-1/10*a^5*A/x^10-1/9*a^4*(5*A*b+B*a)/x^9-5/8*a^3*b*(2*A*b+B*a)/x^8-10/7*a^
2*b^2*(A*b+B*a)/x^7-5/6*a*b^3*(A*b+2*B*a)/x^6-1/5*b^4*(A*b+5*B*a)/x^5-1/4*
b^5*B/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}} \, dx = -\frac{126b^5x^5(4A+5Bx)+420ab^4x^4(5A+6Bx)+600a^2b^3x^3(6A+7Bx)+450a^3b^2x^2(7A+8Bx)+175a}{2520x^{10}}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^11,x]
```

output
```
-1/2520*(126*b^5*x^5*(4*A + 5*B*x) + 420*a*b^4*x^4*(5*A + 6*B*x) + 600*a^2
*b^3*x^3*(6*A + 7*B*x) + 450*a^3*b^2*x^2*(7*A + 8*B*x) + 175*a^4*b*x*(8*A
+ 9*B*x) + 28*a^5*(9*A + 10*B*x))/x^10
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^5A}{x^{11}}+\frac{a^4(aB+5Ab)}{x^{10}}+\frac{5a^3b(aB+2Ab)}{x^9}+\frac{10a^2b^2(aB+Ab)}{x^8}+\frac{b^4(5aB+Ab)}{x^6}+\frac{5ab^3(2aB+Ab)}{x^7}+\frac{b^5B}{x^5}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{6x^6}-\frac{b^5B}{4x^4}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^11,x]
```

output
```
-1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))
/(8*x^8) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(6*x
^6) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(4*x^4)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{5}A}{10x^{10}}-\frac{a^{4}(5Ab+Ba)}{9x^{9}}-\frac{5a^{3}b(2Ab+Ba)}{8x^{8}}-\frac{10a^{2}b^{2}(Ab+Ba)}{7x^{7}}-\frac{5a\,b^{3}(Ab+2Ba)}{6x^{6}}-\frac{b^{4}(Ab+5Ba)}{5x^{5}}-\frac{b^{5}B}{4x^{4}}$
norman	$\frac{-\frac{B\,b^{5}x^{6}}{4}+\left(-\frac{1}{5}b^{5}A-a\,b^{4}B\right)x^{5}+\left(-\frac{5}{6}a\,b^{4}A-\frac{5}{3}a^{2}b^{3}B\right)x^{4}+\left(-\frac{10}{7}a^{2}b^{3}A-\frac{10}{7}a^{3}b^{2}B\right)x^{3}+\left(-\frac{5}{4}a^{3}b^{2}A-\frac{5}{8}a^{4}bB\right)x^{2}+\left(-\frac{5}{9}a^{4}bA-\right.}{x^{10}}$
risch	$\frac{-\frac{B\,b^{5}x^{6}}{4}+\left(-\frac{1}{5}b^{5}A-a\,b^{4}B\right)x^{5}+\left(-\frac{5}{6}a\,b^{4}A-\frac{5}{3}a^{2}b^{3}B\right)x^{4}+\left(-\frac{10}{7}a^{2}b^{3}A-\frac{10}{7}a^{3}b^{2}B\right)x^{3}+\left(-\frac{5}{4}a^{3}b^{2}A-\frac{5}{8}a^{4}bB\right)x^{2}+\left(-\frac{5}{9}a^{4}bA-\right.}{x^{10}}$
gosper	$-\frac{630B\,b^{5}x^{6}+504A\,b^{5}x^{5}+2520Ba\,b^{4}x^{5}+2100aA\,b^{4}x^{4}+4200B\,a^{2}b^{3}x^{4}+3600a^{2}A\,b^{3}x^{3}+3600B\,a^{3}b^{2}x^{3}+3150a^{3}A\,b^{2}x^{2}+1575}{2520x^{10}}$
parallelrisch	$-\frac{630B\,b^{5}x^{6}+504A\,b^{5}x^{5}+2520Ba\,b^{4}x^{5}+2100aA\,b^{4}x^{4}+4200B\,a^{2}b^{3}x^{4}+3600a^{2}A\,b^{3}x^{3}+3600B\,a^{3}b^{2}x^{3}+3150a^{3}A\,b^{2}x^{2}+1575}{2520x^{10}}$
orering	$-\frac{630B\,b^{5}x^{6}+504A\,b^{5}x^{5}+2520Ba\,b^{4}x^{5}+2100aA\,b^{4}x^{4}+4200B\,a^{2}b^{3}x^{4}+3600a^{2}A\,b^{3}x^{3}+3600B\,a^{3}b^{2}x^{3}+3150a^{3}A\,b^{2}x^{2}+1575}{2520x^{10}}$

input
```
int((b*x+a)^5*(B*x+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^5*A/x^10-1/9*a^4*(5*A*b+B*a)/x^9-5/8*a^3*b*(2*A*b+B*a)/x^8-10/7*a^
2*b^2*(A*b+B*a)/x^7-5/6*a*b^3*(A*b+2*B*a)/x^6-1/5*b^4*(A*b+5*B*a)/x^5-1/4*
b^5*B/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx = -\frac{630\,Bb^5x^6+252\,Aa^5+504\,(5\,Bab^4+Ab^5)x^5+2100\,(2\,Ba^2b^3+Aab^4)x^4+3600\,(Ba^3b^2+Aa^2b^3)x^3}{2520\,x^{10}}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^11,x, algorithm="fricas")
```

output
```
-1/2520*(630*B*b^5*x^6 + 252*A*a^5 + 504*(5*B*a*b^4 + A*b^5)*x^5 + 2100*(2
*B*a^2*b^3 + A*a*b^4)*x^4 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^2 + 280*(B*a^5 + 5*A*a^4*b)*x)/x^10
```

Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx = \frac{-252Aa^5-630Bb^5x^6+x^5(-504Ab^5-2520Bab^4)+x^4(-2100Aab^4-4200Ba^2b^3)+x^3(-3600Aa^2b^3}{2520x^{10}}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**11,x)
```

output
```
(-252*A*a**5 - 630*B*b**5*x**6 + x**5*(-504*A*b**5 - 2520*B*a*b**4) + x**4
*(-2100*A*a*b**4 - 4200*B*a**2*b**3) + x**3*(-3600*A*a**2*b**3 - 3600*B*a*
*3*b**2) + x**2*(-3150*A*a**3*b**2 - 1575*B*a**4*b) + x*(-1400*A*a**4*b -
280*B*a**5))/(2520*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx =$$
$$-\frac{630\,Bb^5x^6 + 252\,Aa^5 + 504\,(5\,Bab^4 + Ab^5)x^5 + 2100\,(2\,Ba^2b^3 + Aab^4)x^4 + 3600\,(Ba^3b^2 + Aa^2b^3)x^3 +$$
$$2520\,x^{10}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^11,x, algorithm="maxima")
```

output
```
-1/2520*(630*B*b^5*x^6 + 252*A*a^5 + 504*(5*B*a*b^4 + A*b^5)*x^5 + 2100*(2
*B*a^2*b^3 + A*a*b^4)*x^4 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^2 + 280*(B*a^5 + 5*A*a^4*b)*x)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx =$$
$$-\frac{630\,Bb^5x^6 + 2520\,Bab^4x^5 + 504\,Ab^5x^5 + 4200\,Ba^2b^3x^4 + 2100\,Aab^4x^4 + 3600\,Ba^3b^2x^3 + 3600\,Aa^2b^3x$$
$$2520\,x^{10}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^11,x, algorithm="giac")
```

output
```
-1/2520*(630*B*b^5*x^6 + 2520*B*a*b^4*x^5 + 504*A*b^5*x^5 + 4200*B*a^2*b^3
*x^4 + 2100*A*a*b^4*x^4 + 3600*B*a^3*b^2*x^3 + 3600*A*a^2*b^3*x^3 + 1575*B
*a^4*b*x^2 + 3150*A*a^3*b^2*x^2 + 280*B*a^5*x + 1400*A*a^4*b*x + 252*A*a^5
)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx = -\frac{x\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+\frac{A\,a^5}{10}+x^4\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^2\left(\frac{5\,B\,a^4\,b}{8}+\frac{5\,A\,a^3\,b^2}{4}\right)+x^5\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^3\left(\right.}{x^{10}}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^11,x)
```

output
```
-(x*((B*a^5)/9 + (5*A*a^4*b)/9) + (A*a^5)/10 + x^4*((5*B*a^2*b^3)/3 + (5*A
*a*b^4)/6) + x^2*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^5*((A*b^5)/5 + B*a*
b^4) + x^3*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7) + (B*b^5*x^6)/4)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx = \frac{-210b^6x^6-1008a\,b^5x^5-2100a^2b^4x^4-2400a^3b^3x^3-1575a^4b^2x^2-560a^5bx-84a^6}{840x^{10}}$$

input
```
int((b*x+a)^5*(B*x+A)/x^11,x)
```

output
```
( - 84*a**6 - 560*a**5*b*x - 1575*a**4*b**2*x**2 - 2400*a**3*b**3*x**3 - 2
100*a**2*b**4*x**4 - 1008*a*b**5*x**5 - 210*b**6*x**6)/(840*x**10)
```

3.105 $\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(5Ab+aB)}{10x^{10}} - \frac{5a^3b(2Ab+aB)}{9x^9} - \frac{5a^2b^2(Ab+aB)}{4x^8} - \frac{5ab^3(Ab+2aB)}{7x^7} - \frac{b^4(Ab+5aB)}{6x^6} - \frac{b^5B}{5x^5}$$

output
```
-1/11*a^5*A/x^11-1/10*a^4*(5*A*b+B*a)/x^10-5/9*a^3*b*(2*A*b+B*a)/x^9-5/4*a
^2*b^2*(A*b+B*a)/x^8-5/7*a*b^3*(A*b+2*B*a)/x^7-1/6*b^4*(A*b+5*B*a)/x^6-1/5
*b^5*B/x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = -\frac{462b^5x^5(5A+6Bx)+1650ab^4x^4(6A+7Bx)+2475a^2b^3x^3(7A+8Bx)+1925a^3b^2x^2(8A+9Bx)+7}{13860x^{11}}$$

input
```
Integrate[((a + b*x)^5*(A + B*x))/x^12,x]
```

output
```
-1/13860*(462*b^5*x^5*(5*A + 6*B*x) + 1650*a*b^4*x^4*(6*A + 7*B*x) + 2475*
a^2*b^3*x^3*(7*A + 8*B*x) + 1925*a^3*b^2*x^2*(8*A + 9*B*x) + 770*a^4*b*x*(
9*A + 10*B*x) + 126*a^5*(10*A + 11*B*x))/x^11
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^5A}{x^{12}}+\frac{a^4(aB+5Ab)}{x^{11}}+\frac{5a^3b(aB+2Ab)}{x^{10}}+\frac{10a^2b^2(aB+Ab)}{x^9}+\frac{b^4(5aB+Ab)}{x^7}+\frac{5ab^3(2aB+Ab)}{x^8}+\frac{b^5B}{x^6}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{6x^6}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{5x^5}$$

input
```
Int[((a + b*x)^5*(A + B*x))/x^12,x]
```

output
```
-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B
))/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (5*a*b^3*(A*b + 2*a*B))/(7*
x^7) - (b^4*(A*b + 5*a*B))/(6*x^6) - (b^5*B)/(5*x^5)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{5} A}{11 x^{11}}-\frac{a^{4}(5 A b+B a)}{10 x^{10}}-\frac{5 a^{3} b(2 A b+B a)}{9 x^{9}}-\frac{5 a^{2} b^{2}(A b+B a)}{4 x^{8}}-\frac{5 a\, b^{3}(A b+2 B a)}{7 x^{7}}-\frac{b^{4}(A b+5 B a)}{6 x^{6}}-\frac{b^{5} B}{5 x^{5}}$
norman	$\frac{-\frac{B\, b^{5} x^{6}}{5}+\left(-\frac{1}{6} b^{5} A-\frac{5}{6} a\, b^{4} B\right) x^{5}+\left(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B\right) x^{4}+\left(-\frac{5}{4} a^{2} b^{3} A-\frac{5}{4} a^{3} b^{2} B\right) x^{3}+\left(-\frac{10}{9} a^{3} b^{2} A-\frac{5}{9} a^{4} b B\right) x^{2}+\left(-\frac{1}{2} a^{4} b A-\right.}{x^{11}}$
risch	$\frac{-\frac{B\, b^{5} x^{6}}{5}+\left(-\frac{1}{6} b^{5} A-\frac{5}{6} a\, b^{4} B\right) x^{5}+\left(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B\right) x^{4}+\left(-\frac{5}{4} a^{2} b^{3} A-\frac{5}{4} a^{3} b^{2} B\right) x^{3}+\left(-\frac{10}{9} a^{3} b^{2} A-\frac{5}{9} a^{4} b B\right) x^{2}+\left(-\frac{1}{2} a^{4} b A-\right.}{x^{11}}$
gosper	$-\frac{2772 B\, b^{5} x^{6}+2310 A\, b^{5} x^{5}+11550 B a\, b^{4} x^{5}+9900 a A\, b^{4} x^{4}+19800 B\, a^{2} b^{3} x^{4}+17325 a^{2} A\, b^{3} x^{3}+17325 B\, a^{3} b^{2} x^{3}+15400 a^{3} A\, b^{2}}{13860 x^{11}}$
parallelrisch	$-\frac{2772 B\, b^{5} x^{6}+2310 A\, b^{5} x^{5}+11550 B a\, b^{4} x^{5}+9900 a A\, b^{4} x^{4}+19800 B\, a^{2} b^{3} x^{4}+17325 a^{2} A\, b^{3} x^{3}+17325 B\, a^{3} b^{2} x^{3}+15400 a^{3} A\, b^{2}}{13860 x^{11}}$
orering	$-\frac{2772 B\, b^{5} x^{6}+2310 A\, b^{5} x^{5}+11550 B a\, b^{4} x^{5}+9900 a A\, b^{4} x^{4}+19800 B\, a^{2} b^{3} x^{4}+17325 a^{2} A\, b^{3} x^{3}+17325 B\, a^{3} b^{2} x^{3}+15400 a^{3} A\, b^{2}}{13860 x^{11}}$

input
```
int((b*x+a)^5*(B*x+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^5*A/x^11-1/10*a^4*(5*A*b+B*a)/x^10-5/9*a^3*b*(2*A*b+B*a)/x^9-5/4*a
^2*b^2*(A*b+B*a)/x^8-5/7*a*b^3*(A*b+2*B*a)/x^7-1/6*b^4*(A*b+5*B*a)/x^6-1/5
*b^5*B/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = -\frac{2772\,Bb^5x^6 + 1260\,Aa^5 + 2310\,(5\,Bab^4 + Ab^5)x^5 + 9900\,(2\,Ba^2b^3 + Aab^4)x^4 + 17325\,(Ba^3b^2 + Aa^2b^3}{13860\,x^{11}}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^12,x, algorithm="fricas")
```

output
```
-1/13860*(2772*B*b^5*x^6 + 1260*A*a^5 + 2310*(5*B*a*b^4 + A*b^5)*x^5 + 990
0*(2*B*a^2*b^3 + A*a*b^4)*x^4 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^2 + 1386*(B*a^5 + 5*A*a^4*b)*x)/x^11
```

Sympy [A] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = \frac{-1260Aa^5 - 2772Bb^5x^6 + x^5(-2310Ab^5 - 11550Bab^4) + x^4(-9900Aab^4 - 19800Ba^2b^3) + x^3(-17325}{13860x^{11}}$$

input
```
integrate((b*x+a)**5*(B*x+A)/x**12,x)
```

output
```
(-1260*A*a**5 - 2772*B*b**5*x**6 + x**5*(-2310*A*b**5 - 11550*B*a*b**4) +
x**4*(-9900*A*a*b**4 - 19800*B*a**2*b**3) + x**3*(-17325*A*a**2*b**3 - 173
25*B*a**3*b**2) + x**2*(-15400*A*a**3*b**2 - 7700*B*a**4*b) + x*(-6930*A*a
**4*b - 1386*B*a**5))/(13860*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = -\frac{2772\,Bb^5x^6 + 1260\,Aa^5 + 2310\,(5\,Bab^4 + Ab^5)x^5 + 9900\,(2\,Ba^2b^3 + Aab^4)x^4 + 17325\,(Ba^3b^2 + Aa^2b^3}{13860\,x^{11}}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^12,x, algorithm="maxima")
```

output
```
-1/13860*(2772*B*b^5*x^6 + 1260*A*a^5 + 2310*(5*B*a*b^4 + A*b^5)*x^5 + 990
0*(2*B*a^2*b^3 + A*a*b^4)*x^4 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^2 + 1386*(B*a^5 + 5*A*a^4*b)*x)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = -\frac{2772\,Bb^5x^6 + 11550\,Bab^4x^5 + 2310\,Ab^5x^5 + 19800\,Ba^2b^3x^4 + 9900\,Aab^4x^4 + 17325\,Ba^3b^2x^3 + 17325}{13860\,x^{11}}$$

input
```
integrate((b*x+a)^5*(B*x+A)/x^12,x, algorithm="giac")
```

output
```
-1/13860*(2772*B*b^5*x^6 + 11550*B*a*b^4*x^5 + 2310*A*b^5*x^5 + 19800*B*a^
2*b^3*x^4 + 9900*A*a*b^4*x^4 + 17325*B*a^3*b^2*x^3 + 17325*A*a^2*b^3*x^3 +
 7700*B*a^4*b*x^2 + 15400*A*a^3*b^2*x^2 + 1386*B*a^5*x + 6930*A*a^4*b*x +
1260*A*a^5)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = -\frac{x\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+\frac{A\,a^5}{11}+x^4\left(\frac{10\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{7}\right)+x^2\left(\frac{5\,B\,a^4\,b}{9}+\frac{10\,A\,a^3\,b^2}{9}\right)+x^5\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^3\left(\right.}{x^{11}}$$

input
```
int(((A + B*x)*(a + b*x)^5)/x^12,x)
```

output
```
-(x*((B*a^5)/10 + (A*a^4*b)/2) + (A*a^5)/11 + x^4*((10*B*a^2*b^3)/7 + (5*A
*a*b^4)/7) + x^2*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^5*((A*b^5)/6 + (5*
B*a*b^4)/6) + x^3*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4) + (B*b^5*x^6)/5)/x^1
1
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx = \frac{-462b^6x^6-2310a\,b^5x^5-4950a^2b^4x^4-5775a^3b^3x^3-3850a^4b^2x^2-1386a^5bx-210a^6}{2310x^{11}}$$

input
```
int((b*x+a)^5*(B*x+A)/x^12,x)
```

output
```
( - 210*a**6 - 1386*a**5*b*x - 3850*a**4*b**2*x**2 - 5775*a**3*b**3*x**3 -
 4950*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 462*b**6*x**6)/(2310*x**11)
```

3.106 $\int x^{10}(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int x^{10}(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{11}a^{10}Ax^{11}+\frac{1}{12}a^9(10Ab+aB)x^{12}\\ &+\frac{5}{13}a^8b(9Ab+2aB)x^{13}+\frac{15}{14}a^7b^2(8Ab+3aB)x^{14}\\ &+2a^6b^3(7Ab+4aB)x^{15}+\frac{21}{8}a^5b^4(6Ab+5aB)x^{16}\\ &+\frac{42}{17}a^4b^5(5Ab+6aB)x^{17}+\frac{5}{3}a^3b^6(4Ab+7aB)x^{18}\\ &+\frac{15}{19}a^2b^7(3Ab+8aB)x^{19}+\frac{1}{4}ab^8(2Ab+9aB)x^{20}\\ &+\frac{1}{21}b^9(Ab+10aB)x^{21}+\frac{1}{22}b^{10}Bx^{22}\end{aligned}$$

output
```
1/11*a^10*A*x^11+1/12*a^9*(10*A*b+B*a)*x^12+5/13*a^8*b*(9*A*b+2*B*a)*x^13+
15/14*a^7*b^2*(8*A*b+3*B*a)*x^14+2*a^6*b^3*(7*A*b+4*B*a)*x^15+21/8*a^5*b^4
*(6*A*b+5*B*a)*x^16+42/17*a^4*b^5*(5*A*b+6*B*a)*x^17+5/3*a^3*b^6*(4*A*b+7*
B*a)*x^18+15/19*a^2*b^7*(3*A*b+8*B*a)*x^19+1/4*a*b^8*(2*A*b+9*B*a)*x^20+1/
21*b^9*(A*b+10*B*a)*x^21+1/22*b^10*B*x^22
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00

$$\int x^{10}(a+bx)^{10}(A+Bx)\,dx = \frac{1}{11}a^{10}Ax^{11}+\frac{1}{12}a^9(10Ab+aB)x^{12} + \frac{5}{13}a^8b(9Ab+2aB)x^{13}+\frac{15}{14}a^7b^2(8Ab+3aB)x^{14} + 2a^6b^3(7Ab+4aB)x^{15}+\frac{21}{8}a^5b^4(6Ab+5aB)x^{16} + \frac{42}{17}a^4b^5(5Ab+6aB)x^{17}+\frac{5}{3}a^3b^6(4Ab+7aB)x^{18} + \frac{15}{19}a^2b^7(3Ab+8aB)x^{19}+\frac{1}{4}ab^8(2Ab+9aB)x^{20} + \frac{1}{21}b^9(Ab+10aB)x^{21}+\frac{1}{22}b^{10}Bx^{22}$$

input
```
Integrate[x^10*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^11)/11 + (a^9*(10*A*b + a*B)*x^12)/12 + (5*a^8*b*(9*A*b + 2*a*B)
*x^13)/13 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^14)/14 + 2*a^6*b^3*(7*A*b + 4*a*
B)*x^15 + (21*a^5*b^4*(6*A*b + 5*a*B)*x^16)/8 + (42*a^4*b^5*(5*A*b + 6*a*B
)*x^17)/17 + (5*a^3*b^6*(4*A*b + 7*a*B)*x^18)/3 + (15*a^2*b^7*(3*A*b + 8*a
*B)*x^19)/19 + (a*b^8*(2*A*b + 9*a*B)*x^20)/4 + (b^9*(A*b + 10*a*B)*x^21)/
21 + (b^10*B*x^22)/22
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \ 85$$

$$\int \left(a^{10}Ax^{10} + a^9x^{11}(aB+10Ab) + 5a^8bx^{12}(2aB+9Ab) + 15a^7b^2x^{13}(3aB+8Ab) + 30a^6b^3x^{14}(4aB+7Ab) + 4\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{11}a^{10}Ax^{11} + \frac{1}{12}a^9x^{12}(aB+10Ab) + \frac{5}{13}a^8bx^{13}(2aB+9Ab) + \frac{15}{14}a^7b^2x^{14}(3aB+8Ab) + 2a^6b^3x^{15}(4aB+7Ab) + \frac{21}{8}a^5b^4x^{16}(5aB+6Ab) + \frac{42}{17}a^4b^5x^{17}(6aB+5Ab) + \frac{5}{3}a^3b^6x^{18}(7aB+4Ab) + \frac{15}{19}a^2b^7x^{19}(8aB+3Ab) + \frac{1}{21}b^9x^{21}(10aB+Ab) + \frac{1}{4}ab^8x^{20}(9aB+2Ab) + \frac{1}{22}b^{10}Bx^{22}$$

input

```
Int[x^10*(a + b*x)^10*(A + B*x),x]
```

output

```
(a^10*A*x^11)/11 + (a^9*(10*A*b + a*B)*x^12)/12 + (5*a^8*b*(9*A*b + 2*a*B)
*x^13)/13 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^14)/14 + 2*a^6*b^3*(7*A*b + 4*a*
B)*x^15 + (21*a^5*b^4*(6*A*b + 5*a*B)*x^16)/8 + (42*a^4*b^5*(5*A*b + 6*a*B
)*x^17)/17 + (5*a^3*b^6*(4*A*b + 7*a*B)*x^18)/3 + (15*a^2*b^7*(3*A*b + 8*a
*B)*x^19)/19 + (a*b^8*(2*A*b + 9*a*B)*x^20)/4 + (b^9*(A*b + 10*a*B)*x^21)/
21 + (b^10*B*x^22)/22
```

Defintions of rubi rules used

rule 85

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^{10}A\,x^{11}}{11}+\left(\frac{5}{6}a^{9}bA+\frac{1}{12}a^{10}B\right)x^{12}+\left(\frac{45}{13}a^{8}b^{2}A+\frac{10}{13}a^{9}bB\right)x^{13}+\left(\frac{60}{7}a^{7}b^{3}A+\frac{45}{14}a^{8}b^{2}B\right)x^{14}+$
default	$\frac{b^{10}B\,x^{22}}{22}+\frac{\left(b^{10}A+10a\,b^{9}B\right)x^{21}}{21}+\frac{\left(10a\,b^{9}A+45a^{2}b^{8}B\right)x^{20}}{20}+\frac{\left(45a^{2}b^{8}A+120a^{3}b^{7}B\right)x^{19}}{19}+\frac{\left(120a^{3}b^{7}A+210a^{4}b^{6}B\right)x}{18}$
orering	$x^{11}\left(352716B\,b^{10}x^{11}+369512A\,b^{10}x^{10}+3695120Ba\,b^{9}x^{10}+3879876aA\,b^{9}x^{9}+17459442B\,a^{2}b^{8}x^{9}+18378360a^{2}A\,b^{8}x^{8}+4900\right.$
gosper	$\frac{1}{11}a^{10}A\,x^{11}+\frac{5}{6}x^{12}a^{9}bA+\frac{1}{12}x^{12}a^{10}B+\frac{45}{13}x^{13}a^{8}b^{2}A+\frac{10}{13}x^{13}a^{9}bB+\frac{60}{7}x^{14}a^{7}b^{3}A+\frac{45}{14}x^{14}a^{8}b^{2}$
risch	$\frac{1}{11}a^{10}A\,x^{11}+\frac{5}{6}x^{12}a^{9}bA+\frac{1}{12}x^{12}a^{10}B+\frac{45}{13}x^{13}a^{8}b^{2}A+\frac{10}{13}x^{13}a^{9}bB+\frac{60}{7}x^{14}a^{7}b^{3}A+\frac{45}{14}x^{14}a^{8}b^{2}$
parallelrisch	$\frac{1}{11}a^{10}A\,x^{11}+\frac{5}{6}x^{12}a^{9}bA+\frac{1}{12}x^{12}a^{10}B+\frac{45}{13}x^{13}a^{8}b^{2}A+\frac{10}{13}x^{13}a^{9}bB+\frac{60}{7}x^{14}a^{7}b^{3}A+\frac{45}{14}x^{14}a^{8}b^{2}$

input
```
int(x^10*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*a^10*A*x^11+(5/6*a^9*b*A+1/12*a^10*B)*x^12+(45/13*a^8*b^2*A+10/13*a^9
*b*B)*x^13+(60/7*a^7*b^3*A+45/14*a^8*b^2*B)*x^14+(14*A*a^6*b^4+8*B*a^7*b^3
)*x^15+(63/4*a^5*b^5*A+105/8*a^6*b^4*B)*x^16+(210/17*a^4*b^6*A+252/17*a^5*
b^5*B)*x^17+(20/3*a^3*b^7*A+35/3*a^4*b^6*B)*x^18+(45/19*a^2*b^8*A+120/19*a
^3*b^7*B)*x^19+(1/2*a*b^9*A+9/4*a^2*b^8*B)*x^20+(1/21*b^10*A+10/21*a*b^9*B
)*x^21+1/22*b^10*B*x^22
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$
\begin{aligned}
\int x^{10}(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{22}\,Bb^{10}x^{22}+\frac{1}{11}\,Aa^{10}x^{11}+\frac{1}{21}\,\left(10\,Bab^{9}+Ab^{10}\right)x^{21} \\
&\quad+\frac{1}{4}\,\left(9\,Ba^{2}b^{8}+2\,Aab^{9}\right)x^{20}+\frac{15}{19}\,\left(8\,Ba^{3}b^{7}+3\,Aa^{2}b^{8}\right)x^{19} \\
&\quad+\frac{5}{3}\,\left(7\,Ba^{4}b^{6}+4\,Aa^{3}b^{7}\right)x^{18}+\frac{42}{17}\,\left(6\,Ba^{5}b^{5}+5\,Aa^{4}b^{6}\right)x^{17} \\
&\quad+\frac{21}{8}\,\left(5\,Ba^{6}b^{4}+6\,Aa^{5}b^{5}\right)x^{16} \\
&\quad+2\,\left(4\,Ba^{7}b^{3}+7\,Aa^{6}b^{4}\right)x^{15}+\frac{15}{14}\,\left(3\,Ba^{8}b^{2}+8\,Aa^{7}b^{3}\right)x^{14} \\
&\quad+\frac{5}{13}\,\left(2\,Ba^{9}b+9\,Aa^{8}b^{2}\right)x^{13}+\frac{1}{12}\,\left(Ba^{10}+10\,Aa^{9}b\right)x^{12}
\end{aligned}
$$

input
```
integrate(x^10*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/22*B*b^10*x^22 + 1/11*A*a^10*x^11 + 1/21*(10*B*a*b^9 + A*b^10)*x^21 + 1/
4*(9*B*a^2*b^8 + 2*A*a*b^9)*x^20 + 15/19*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^19
+ 5/3*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^18 + 42/17*(6*B*a^5*b^5 + 5*A*a^4*b^6)
*x^17 + 21/8*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^16 + 2*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^15 + 15/14*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^14 + 5/13*(2*B*a^9*b + 9*A*
a^8*b^2)*x^13 + 1/12*(B*a^10 + 10*A*a^9*b)*x^12
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.17

$$\int x^{10}(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^{11}}{11} + \frac{Bb^{10}x^{22}}{22} + x^{21}\left(\frac{Ab^{10}}{21} + \frac{10Bab^9}{21}\right) + x^{20}\left(\frac{Aab^9}{2} + \frac{9Ba^2b^8}{4}\right) + x^{19}\cdot\left(\frac{45Aa^2b^8}{19} + \frac{120Ba^3b^7}{19}\right) + x^{18}\cdot\left(\frac{20Aa^3b^7}{3} + \frac{35Ba^4b^6}{3}\right) + x^{17}\cdot\left(\frac{210Aa^4b^6}{17} + \frac{252Ba^5b^5}{17}\right) + x^{16}\cdot\left(\frac{63Aa^5b^5}{4} + \frac{105Ba^6b^4}{8}\right) + x^{15}\cdot\left(14Aa^6b^4 + 8Ba^7b^3\right) + x^{14}\cdot\left(\frac{60Aa^7b^3}{7} + \frac{45Ba^8b^2}{14}\right) + x^{13}\cdot\left(\frac{45Aa^8b^2}{13} + \frac{10Ba^9b}{13}\right) + x^{12}\cdot\left(\frac{5Aa^9b}{6} + \frac{Ba^{10}}{12}\right)$$

input
```
integrate(x**10*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**11/11 + B*b**10*x**22/22 + x**21*(A*b**10/21 + 10*B*a*b**9/21)
+ x**20*(A*a*b**9/2 + 9*B*a**2*b**8/4) + x**19*(45*A*a**2*b**8/19 + 120*B*
a**3*b**7/19) + x**18*(20*A*a**3*b**7/3 + 35*B*a**4*b**6/3) + x**17*(210*A
*a**4*b**6/17 + 252*B*a**5*b**5/17) + x**16*(63*A*a**5*b**5/4 + 105*B*a**6
*b**4/8) + x**15*(14*A*a**6*b**4 + 8*B*a**7*b**3) + x**14*(60*A*a**7*b**3/
7 + 45*B*a**8*b**2/14) + x**13*(45*A*a**8*b**2/13 + 10*B*a**9*b/13) + x**1
2*(5*A*a**9*b/6 + B*a**10/12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\begin{aligned}\int x^{10}(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{22}Bb^{10}x^{22} + \frac{1}{11}Aa^{10}x^{11} + \frac{1}{21}\left(10\,Bab^9 + Ab^{10}\right)x^{21} \\ &+ \frac{1}{4}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{20} + \frac{15}{19}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^{19} \\ &+ \frac{5}{3}\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^{18} + \frac{42}{17}\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^{17} \\ &+ \frac{21}{8}\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^{16} \\ &+ 2\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^{15} + \frac{15}{14}\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^{14} \\ &+ \frac{5}{13}\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^{13} + \frac{1}{12}\left(Ba^{10} + 10\,Aa^9b\right)x^{12}\end{aligned}$$

input
```
integrate(x^10*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/22*B*b^10*x^22 + 1/11*A*a^10*x^11 + 1/21*(10*B*a*b^9 + A*b^10)*x^21 + 1/
4*(9*B*a^2*b^8 + 2*A*a*b^9)*x^20 + 15/19*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^19
+ 5/3*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^18 + 42/17*(6*B*a^5*b^5 + 5*A*a^4*b^6)
*x^17 + 21/8*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^16 + 2*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^15 + 15/14*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^14 + 5/13*(2*B*a^9*b + 9*A*
a^8*b^2)*x^13 + 1/12*(B*a^10 + 10*A*a^9*b)*x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07

$$\int x^{10}(a+bx)^{10}(A+Bx)\,dx = \frac{1}{22}Bb^{10}x^{22} + \frac{10}{21}Bab^9x^{21} + \frac{1}{21}Ab^{10}x^{21} + \frac{9}{4}Ba^2b^8x^{20} + \frac{1}{2}Aab^9x^{20} + \frac{120}{19}Ba^3b^7x^{19} + \frac{45}{19}Aa^2b^8x^{19} + \frac{35}{3}Ba^4b^6x^{18} + \frac{20}{3}Aa^3b^7x^{18} + \frac{252}{17}Ba^5b^5x^{17} + \frac{210}{17}Aa^4b^6x^{17} + \frac{105}{8}Ba^6b^4x^{16} + \frac{63}{4}Aa^5b^5x^{16} + 8Ba^7b^3x^{15} + 14Aa^6b^4x^{15} + \frac{45}{14}Ba^8b^2x^{14} + \frac{60}{7}Aa^7b^3x^{14} + \frac{10}{13}Ba^9bx^{13} + \frac{45}{13}Aa^8b^2x^{13} + \frac{1}{12}Ba^{10}x^{12} + \frac{5}{6}Aa^9bx^{12} + \frac{1}{11}Aa^{10}x^{11}$$

input
```
integrate(x^10*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/22*B*b^10*x^22 + 10/21*B*a*b^9*x^21 + 1/21*A*b^10*x^21 + 9/4*B*a^2*b^8*x
^20 + 1/2*A*a*b^9*x^20 + 120/19*B*a^3*b^7*x^19 + 45/19*A*a^2*b^8*x^19 + 35
/3*B*a^4*b^6*x^18 + 20/3*A*a^3*b^7*x^18 + 252/17*B*a^5*b^5*x^17 + 210/17*A
*a^4*b^6*x^17 + 105/8*B*a^6*b^4*x^16 + 63/4*A*a^5*b^5*x^16 + 8*B*a^7*b^3*x
^15 + 14*A*a^6*b^4*x^15 + 45/14*B*a^8*b^2*x^14 + 60/7*A*a^7*b^3*x^14 + 10/
13*B*a^9*b*x^13 + 45/13*A*a^8*b^2*x^13 + 1/12*B*a^10*x^12 + 5/6*A*a^9*b*x^
12 + 1/11*A*a^10*x^11
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.92

$$\int x^{10}(a+bx)^{10}(A+Bx)\,dx = x^{12}\left(\frac{B\,a^{10}}{12}+\frac{5\,A\,b\,a^9}{6}\right)+x^{21}\left(\frac{A\,b^{10}}{21}+\frac{10\,B\,a\,b^9}{21}\right) + \frac{A\,a^{10}\,x^{11}}{11}+\frac{B\,b^{10}\,x^{22}}{22}+\frac{15\,a^7\,b^2\,x^{14}\,(8\,A\,b+3\,B\,a)}{14} + 2\,a^6\,b^3\,x^{15}\,(7\,A\,b+4\,B\,a)+\frac{21\,a^5\,b^4\,x^{16}\,(6\,A\,b+5\,B\,a)}{8} + \frac{42\,a^4\,b^5\,x^{17}\,(5\,A\,b+6\,B\,a)}{17} + \frac{5\,a^3\,b^6\,x^{18}\,(4\,A\,b+7\,B\,a)}{3}+\frac{15\,a^2\,b^7\,x^{19}\,(3\,A\,b+8\,B\,a)}{19} + \frac{5\,a^8\,b\,x^{13}\,(9\,A\,b+2\,B\,a)}{13}+\frac{a\,b^8\,x^{20}\,(2\,A\,b+9\,B\,a)}{4}$$

input
```
int(x^10*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^12*((B*a^10)/12 + (5*A*a^9*b)/6) + x^21*((A*b^10)/21 + (10*B*a*b^9)/21)
+ (A*a^10*x^11)/11 + (B*b^10*x^22)/22 + (15*a^7*b^2*x^14*(8*A*b + 3*B*a))/
14 + 2*a^6*b^3*x^15*(7*A*b + 4*B*a) + (21*a^5*b^4*x^16*(6*A*b + 5*B*a))/8
+ (42*a^4*b^5*x^17*(5*A*b + 6*B*a))/17 + (5*a^3*b^6*x^18*(4*A*b + 7*B*a))/
3 + (15*a^2*b^7*x^19*(3*A*b + 8*B*a))/19 + (5*a^8*b*x^13*(9*A*b + 2*B*a))/
13 + (a*b^8*x^20*(2*A*b + 9*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int x^{10}(a+bx)^{10}(A+Bx)\,dx = \frac{x^{11}(352716b^{11}x^{11}+4064632a\,b^{10}x^{10}+21339318a^2b^9x^9+67387320a^3b^8x^8+142262120a^4b^7x^7+210882}{}$$

input
```
int(x^10*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**11*(705432*a**11 + 7113106*a**10*b*x + 32829720*a**9*b**2*x**2 + 91454
220*a**8*b**3*x**3 + 170714544*a**7*b**4*x**4 + 224062839*a**6*b**5*x**5 +
 210882672*a**5*b**6*x**6 + 142262120*a**4*b**7*x**7 + 67387320*a**3*b**8*
x**8 + 21339318*a**2*b**9*x**9 + 4064632*a*b**10*x**10 + 352716*b**11*x**1
1))/7759752
```

3.107 $\int x^9(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 231

$$\begin{aligned}\int x^9(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{10}a^{10}Ax^{10}+\frac{1}{11}a^9(10Ab+aB)x^{11}\\&+\frac{5}{12}a^8b(9Ab+2aB)x^{12}+\frac{15}{13}a^7b^2(8Ab+3aB)x^{13}\\&+\frac{15}{7}a^6b^3(7Ab+4aB)x^{14}+\frac{14}{5}a^5b^4(6Ab+5aB)x^{15}\\&+\frac{21}{8}a^4b^5(5Ab+6aB)x^{16}+\frac{30}{17}a^3b^6(4Ab+7aB)x^{17}\\&+\frac{5}{6}a^2b^7(3Ab+8aB)x^{18}+\frac{5}{19}ab^8(2Ab+9aB)x^{19}\\&+\frac{1}{20}b^9(Ab+10aB)x^{20}+\frac{1}{21}b^{10}Bx^{21}\end{aligned}$$

output
```
1/10*a^10*A*x^10+1/11*a^9*(10*A*b+B*a)*x^11+5/12*a^8*b*(9*A*b+2*B*a)*x^12+
15/13*a^7*b^2*(8*A*b+3*B*a)*x^13+15/7*a^6*b^3*(7*A*b+4*B*a)*x^14+14/5*a^5*
b^4*(6*A*b+5*B*a)*x^15+21/8*a^4*b^5*(5*A*b+6*B*a)*x^16+30/17*a^3*b^6*(4*A*
b+7*B*a)*x^17+5/6*a^2*b^7*(3*A*b+8*B*a)*x^18+5/19*a*b^8*(2*A*b+9*B*a)*x^19
+1/20*b^9*(A*b+10*B*a)*x^20+1/21*b^10*B*x^21
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = \frac{1}{10}a^{10}Ax^{10}+\frac{1}{11}a^9(10Ab+aB)x^{11} + \frac{5}{12}a^8b(9Ab+2aB)x^{12}+\frac{15}{13}a^7b^2(8Ab+3aB)x^{13} + \frac{15}{7}a^6b^3(7Ab+4aB)x^{14}+\frac{14}{5}a^5b^4(6Ab+5aB)x^{15} + \frac{21}{8}a^4b^5(5Ab+6aB)x^{16}+\frac{30}{17}a^3b^6(4Ab+7aB)x^{17} + \frac{5}{6}a^2b^7(3Ab+8aB)x^{18}+\frac{5}{19}ab^8(2Ab+9aB)x^{19} + \frac{1}{20}b^9(Ab+10aB)x^{20}+\frac{1}{21}b^{10}Bx^{21}$$

input
```
Integrate[x^9*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^10)/10 + (a^9*(10*A*b + a*B)*x^11)/11 + (5*a^8*b*(9*A*b + 2*a*B)
*x^12)/12 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^13)/13 + (15*a^6*b^3*(7*A*b + 4*
a*B)*x^14)/7 + (14*a^5*b^4*(6*A*b + 5*a*B)*x^15)/5 + (21*a^4*b^5*(5*A*b +
6*a*B)*x^16)/8 + (30*a^3*b^6*(4*A*b + 7*a*B)*x^17)/17 + (5*a^2*b^7*(3*A*b
+ 8*a*B)*x^18)/6 + (5*a*b^8*(2*A*b + 9*a*B)*x^19)/19 + (b^9*(A*b + 10*a*B)
*x^20)/20 + (b^10*B*x^21)/21
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^{10}Ax^9 + a^9x^{10}(aB+10Ab) + 5a^8bx^{11}(2aB+9Ab) + 15a^7b^2x^{12}(3aB+8Ab) + 30a^6b^3x^{13}(4aB+7Ab) + 4\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a^{10}Ax^{10} + \frac{1}{11}a^9x^{11}(aB+10Ab) + \frac{5}{12}a^8bx^{12}(2aB+9Ab) + \frac{15}{13}a^7b^2x^{13}(3aB+8Ab) + \frac{15}{7}a^6b^3x^{14}(4aB+7Ab) + \frac{14}{5}a^5b^4x^{15}(5aB+6Ab) + \frac{21}{8}a^4b^5x^{16}(6aB+5Ab) + \frac{30}{17}a^3b^6x^{17}(7aB+4Ab) + \frac{5}{6}a^2b^7x^{18}(8aB+3Ab) + \frac{1}{20}b^9x^{20}(10aB+Ab) + \frac{5}{19}ab^8x^{19}(9aB+2Ab) + \frac{1}{21}b^{10}Bx^{21}$$

input
```
Int[x^9*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^10)/10 + (a^9*(10*A*b + a*B)*x^11)/11 + (5*a^8*b*(9*A*b + 2*a*B)
*x^12)/12 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^13)/13 + (15*a^6*b^3*(7*A*b + 4*
a*B)*x^14)/7 + (14*a^5*b^4*(6*A*b + 5*a*B)*x^15)/5 + (21*a^4*b^5*(5*A*b +
6*a*B)*x^16)/8 + (30*a^3*b^6*(4*A*b + 7*a*B)*x^17)/17 + (5*a^2*b^7*(3*A*b
+ 8*a*B)*x^18)/6 + (5*a*b^8*(2*A*b + 9*a*B)*x^19)/19 + (b^9*(A*b + 10*a*B)
*x^20)/20 + (b^10*B*x^21)/21
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^{10}A\,x^{10}}{10}+\left(\frac{10}{11}a^9bA+\frac{1}{11}a^{10}B\right)x^{11}+\left(\frac{15}{4}a^8b^2A+\frac{5}{6}a^9bB\right)x^{12}+\left(\frac{120}{13}a^7b^3A+\frac{45}{13}a^8b^2B\right)x^{13}+$
default	$\frac{b^{10}B\,x^{21}}{21}+\frac{(b^{10}A+10a\,b^9B)x^{20}}{20}+\frac{(10a\,b^9A+45a^2b^8B)x^{19}}{19}+\frac{(45a^2b^8A+120a^3b^7B)x^{18}}{18}+\frac{(120a^3b^7A+210a^4b^6B)x}{17}$
orering	$\frac{x^{10}(1847560B\,b^{10}x^{11}+1939938A\,b^{10}x^{10}+19399380Ba\,b^9x^{10}+20420400aA\,b^9x^9+91891800B\,a^2b^8x^9+96996900a^2A\,b^8x^8+}{}$
gosper	$\frac{1}{10}a^{10}A\,x^{10}+\frac{10}{11}x^{11}a^9bA+\frac{1}{11}x^{11}a^{10}B+\frac{15}{4}x^{12}a^8b^2A+\frac{5}{6}x^{12}a^9bB+\frac{120}{13}x^{13}a^7b^3A+\frac{45}{13}x^{13}a^8b$
risch	$\frac{1}{10}a^{10}A\,x^{10}+\frac{10}{11}x^{11}a^9bA+\frac{1}{11}x^{11}a^{10}B+\frac{15}{4}x^{12}a^8b^2A+\frac{5}{6}x^{12}a^9bB+\frac{120}{13}x^{13}a^7b^3A+\frac{45}{13}x^{13}a^8b$
parallelrisch	$\frac{1}{10}a^{10}A\,x^{10}+\frac{10}{11}x^{11}a^9bA+\frac{1}{11}x^{11}a^{10}B+\frac{15}{4}x^{12}a^8b^2A+\frac{5}{6}x^{12}a^9bB+\frac{120}{13}x^{13}a^7b^3A+\frac{45}{13}x^{13}a^8b$

input
```
int(x^9*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*a^10*A*x^10+(10/11*a^9*b*A+1/11*a^10*B)*x^11+(15/4*a^8*b^2*A+5/6*a^9*
b*B)*x^12+(120/13*a^7*b^3*A+45/13*a^8*b^2*B)*x^13+(15*a^6*b^4*A+60/7*a^7*b
^3*B)*x^14+(84/5*a^5*b^5*A+14*a^6*b^4*B)*x^15+(105/8*a^4*b^6*A+63/4*a^5*b^
5*B)*x^16+(120/17*a^3*b^7*A+210/17*a^4*b^6*B)*x^17+(5/2*a^2*b^8*A+20/3*a^3
*b^7*B)*x^18+(10/19*a*b^9*A+45/19*a^2*b^8*B)*x^19+(1/20*b^10*A+1/2*a*b^9*B
)*x^20+1/21*b^10*B*x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.05

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = \frac{1}{21}\,Bb^{10}x^{21} + \frac{1}{10}\,Aa^{10}x^{10} + \frac{1}{20}\,\left(10\,Bab^9 + Ab^{10}\right)x^{20} + \frac{5}{19}\,\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{19} + \frac{5}{6}\,\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^{18} + \frac{30}{17}\,\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^{17} + \frac{21}{8}\,\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^{16} + \frac{14}{5}\,\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^{15} + \frac{15}{7}\,\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^{14} + \frac{15}{13}\,\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^{13} + \frac{5}{12}\,\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^{12} + \frac{1}{11}\,\left(Ba^{10} + 10\,Aa^9b\right)x^{11}$$

input

```
integrate(x^9*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output

```
1/21*B*b^10*x^21 + 1/10*A*a^10*x^10 + 1/20*(10*B*a*b^9 + A*b^10)*x^20 + 5/
19*(9*B*a^2*b^8 + 2*A*a*b^9)*x^19 + 5/6*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^18 +
 30/17*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^17 + 21/8*(6*B*a^5*b^5 + 5*A*a^4*b^6)
*x^16 + 14/5*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^15 + 15/7*(4*B*a^7*b^3 + 7*A*a^
6*b^4)*x^14 + 15/13*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^13 + 5/12*(2*B*a^9*b + 9
*A*a^8*b^2)*x^12 + 1/11*(B*a^10 + 10*A*a^9*b)*x^11
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.16

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^{10}}{10} + \frac{Bb^{10}x^{21}}{21} + x^{20}\left(\frac{Ab^{10}}{20} + \frac{Bab^9}{2}\right) + x^{19} \cdot \left(\frac{10Aab^9}{19} + \frac{45Ba^2b^8}{19}\right) + x^{18} \cdot \left(\frac{5Aa^2b^8}{2} + \frac{20Ba^3b^7}{3}\right) + x^{17} \cdot \left(\frac{120Aa^3b^7}{17} + \frac{210Ba^4b^6}{17}\right) + x^{16} \cdot \left(\frac{105Aa^4b^6}{8} + \frac{63Ba^5b^5}{4}\right) + x^{15} \cdot \left(\frac{84Aa^5b^5}{5} + 14Ba^6b^4\right) + x^{14} \cdot \left(15Aa^6b^4 + \frac{60Ba^7b^3}{7}\right) + x^{13} \cdot \left(\frac{120Aa^7b^3}{13} + \frac{45Ba^8b^2}{13}\right) + x^{12} \cdot \left(\frac{15Aa^8b^2}{4} + \frac{5Ba^9b}{6}\right) + x^{11} \cdot \left(\frac{10Aa^9b}{11} + \frac{Ba^{10}}{11}\right)$$

input
```
integrate(x**9*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**10/10 + B*b**10*x**21/21 + x**20*(A*b**10/20 + B*a*b**9/2) + x*
*19*(10*A*a*b**9/19 + 45*B*a**2*b**8/19) + x**18*(5*A*a**2*b**8/2 + 20*B*a
**3*b**7/3) + x**17*(120*A*a**3*b**7/17 + 210*B*a**4*b**6/17) + x**16*(105
*A*a**4*b**6/8 + 63*B*a**5*b**5/4) + x**15*(84*A*a**5*b**5/5 + 14*B*a**6*b
**4) + x**14*(15*A*a**6*b**4 + 60*B*a**7*b**3/7) + x**13*(120*A*a**7*b**3/
13 + 45*B*a**8*b**2/13) + x**12*(15*A*a**8*b**2/4 + 5*B*a**9*b/6) + x**11*
(10*A*a**9*b/11 + B*a**10/11)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.05

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = \frac{1}{21}\,Bb^{10}x^{21} + \frac{1}{10}\,Aa^{10}x^{10} + \frac{1}{20}\,\left(10\,Bab^9 + Ab^{10}\right)x^{20} + \frac{5}{19}\,\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{19} + \frac{5}{6}\,\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^{18} + \frac{30}{17}\,\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^{17} + \frac{21}{8}\,\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^{16} + \frac{14}{5}\,\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^{15} + \frac{15}{7}\,\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^{14} + \frac{15}{13}\,\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^{13} + \frac{5}{12}\,\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^{12} + \frac{1}{11}\,\left(Ba^{10} + 10\,Aa^9b\right)x^{11}$$

input

```
integrate(x^9*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output

```
1/21*B*b^10*x^21 + 1/10*A*a^10*x^10 + 1/20*(10*B*a*b^9 + A*b^10)*x^20 + 5/
19*(9*B*a^2*b^8 + 2*A*a*b^9)*x^19 + 5/6*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^18 +
 30/17*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^17 + 21/8*(6*B*a^5*b^5 + 5*A*a^4*b^6)
*x^16 + 14/5*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^15 + 15/7*(4*B*a^7*b^3 + 7*A*a^
6*b^4)*x^14 + 15/13*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^13 + 5/12*(2*B*a^9*b + 9
*A*a^8*b^2)*x^12 + 1/11*(B*a^10 + 10*A*a^9*b)*x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.06

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = \frac{1}{21}Bb^{10}x^{21} + \frac{1}{2}Bab^9x^{20} + \frac{1}{20}Ab^{10}x^{20} + \frac{45}{19}Ba^2b^8x^{19} + \frac{10}{19}Aab^9x^{19} + \frac{20}{3}Ba^3b^7x^{18} + \frac{5}{2}Aa^2b^8x^{18} + \frac{210}{17}Ba^4b^6x^{17} + \frac{120}{17}Aa^3b^7x^{17} + \frac{63}{4}Ba^5b^5x^{16} + \frac{105}{8}Aa^4b^6x^{16} + 14\,Ba^6b^4x^{15} + \frac{84}{5}Aa^5b^5x^{15} + \frac{60}{7}Ba^7b^3x^{14} + 15\,Aa^6b^4x^{14} + \frac{45}{13}Ba^8b^2x^{13} + \frac{120}{13}Aa^7b^3x^{13} + \frac{5}{6}Ba^9bx^{12} + \frac{15}{4}Aa^8b^2x^{12} + \frac{1}{11}Ba^{10}x^{11} + \frac{10}{11}Aa^9bx^{11} + \frac{1}{10}Aa^{10}x^{10}$$

input
```
integrate(x^9*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/21*B*b^10*x^21 + 1/2*B*a*b^9*x^20 + 1/20*A*b^10*x^20 + 45/19*B*a^2*b^8*x
^19 + 10/19*A*a*b^9*x^19 + 20/3*B*a^3*b^7*x^18 + 5/2*A*a^2*b^8*x^18 + 210/
17*B*a^4*b^6*x^17 + 120/17*A*a^3*b^7*x^17 + 63/4*B*a^5*b^5*x^16 + 105/8*A*
a^4*b^6*x^16 + 14*B*a^6*b^4*x^15 + 84/5*A*a^5*b^5*x^15 + 60/7*B*a^7*b^3*x^
14 + 15*A*a^6*b^4*x^14 + 45/13*B*a^8*b^2*x^13 + 120/13*A*a^7*b^3*x^13 + 5/
6*B*a^9*b*x^12 + 15/4*A*a^8*b^2*x^12 + 1/11*B*a^10*x^11 + 10/11*A*a^9*b*x^
11 + 1/10*A*a^10*x^10
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.91

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = x^{11}\left(\frac{B\,a^{10}}{11}+\frac{10\,A\,b\,a^9}{11}\right)+x^{20}\left(\frac{A\,b^{10}}{20}+\frac{B\,a\,b^9}{2}\right) + \frac{A\,a^{10}\,x^{10}}{10}+\frac{B\,b^{10}\,x^{21}}{21}+\frac{15\,a^7\,b^2\,x^{13}\,(8\,A\,b+3\,B\,a)}{13} + \frac{15\,a^6\,b^3\,x^{14}\,(7\,A\,b+4\,B\,a)}{7} + \frac{14\,a^5\,b^4\,x^{15}\,(6\,A\,b+5\,B\,a)}{5}+\frac{21\,a^4\,b^5\,x^{16}\,(5\,A\,b+6\,B\,a)}{8} + \frac{30\,a^3\,b^6\,x^{17}\,(4\,A\,b+7\,B\,a)}{17}+\frac{5\,a^2\,b^7\,x^{18}\,(3\,A\,b+8\,B\,a)}{6} + \frac{5\,a^8\,b\,x^{12}\,(9\,A\,b+2\,B\,a)}{12}+\frac{5\,a\,b^8\,x^{19}\,(2\,A\,b+9\,B\,a)}{19}$$

input
```
int(x^9*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^11*((B*a^10)/11 + (10*A*a^9*b)/11) + x^20*((A*b^10)/20 + (B*a*b^9)/2) +
(A*a^10*x^10)/10 + (B*b^10*x^21)/21 + (15*a^7*b^2*x^13*(8*A*b + 3*B*a))/13
 + (15*a^6*b^3*x^14*(7*A*b + 4*B*a))/7 + (14*a^5*b^4*x^15*(6*A*b + 5*B*a))
/5 + (21*a^4*b^5*x^16*(5*A*b + 6*B*a))/8 + (30*a^3*b^6*x^17*(4*A*b + 7*B*a
))/17 + (5*a^2*b^7*x^18*(3*A*b + 8*B*a))/6 + (5*a^8*b*x^12*(9*A*b + 2*B*a)
)/12 + (5*a*b^8*x^19*(2*A*b + 9*B*a))/19
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.53

$$\int x^9(a+bx)^{10}(A+Bx)\,dx = \frac{x^{10}(167960b^{11}x^{11}+1939938a\,b^{10}x^{10}+10210200a^2b^9x^9+32332300a^3b^8x^8+68468400a^4b^7x^7+1018467\cdots}{}$$

input
```
int(x^9*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**10*(352716*a**11 + 3527160*a**10*b*x + 16166150*a**9*b**2*x**2 + 44767
800*a**8*b**3*x**3 + 83140200*a**7*b**4*x**4 + 108636528*a**6*b**5*x**5 +
101846745*a**5*b**6*x**6 + 68468400*a**4*b**7*x**7 + 32332300*a**3*b**8*x*
*8 + 10210200*a**2*b**9*x**9 + 1939938*a*b**10*x**10 + 167960*b**11*x**11)
)/3527160
```

3.108 $\int x^8(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 240

$$\int x^8(a+bx)^{10}(A+Bx)\,dx=\frac{a^8(Ab-aB)(a+bx)^{11}}{11b^{10}}-\frac{a^7(8Ab-9aB)(a+bx)^{12}}{12b^{10}}+\frac{4a^6(7Ab-9aB)(a+bx)^{13}}{13b^{10}}-\frac{2a^5(2Ab-3aB)(a+bx)^{14}}{b^{10}}+\frac{14a^4(5Ab-9aB)(a+bx)^{15}}{15b^{10}}-\frac{7a^3(4Ab-9aB)(a+bx)^{16}}{8b^{10}}+\frac{28a^2(Ab-3aB)(a+bx)^{17}}{17b^{10}}-\frac{2a(2Ab-9aB)(a+bx)^{18}}{9b^{10}}+\frac{(Ab-9aB)(a+bx)^{19}}{19b^{10}}+\frac{B(a+bx)^{20}}{20b^{10}}$$

output
```
1/11*a^8*(A*b-B*a)*(b*x+a)^11/b^10-1/12*a^7*(8*A*b-9*B*a)*(b*x+a)^12/b^10+
4/13*a^6*(7*A*b-9*B*a)*(b*x+a)^13/b^10-2*a^5*(2*A*b-3*B*a)*(b*x+a)^14/b^10
+14/15*a^4*(5*A*b-9*B*a)*(b*x+a)^15/b^10-7/8*a^3*(4*A*b-9*B*a)*(b*x+a)^16/
b^10+28/17*a^2*(A*b-3*B*a)*(b*x+a)^17/b^10-2/9*a*(2*A*b-9*B*a)*(b*x+a)^18/
b^10+1/19*(A*b-9*B*a)*(b*x+a)^19/b^10+1/20*B*(b*x+a)^20/b^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int x^8(a+bx)^{10}(A+Bx)\,dx=\frac{1}{9}a^{10}Ax^9+\frac{1}{10}a^9(10Ab+aB)x^{10}+\frac{5}{11}a^8b(9Ab+2aB)x^{11}+\frac{5}{4}a^7b^2(8Ab+3aB)x^{12}+\frac{30}{13}a^6b^3(7Ab+4aB)x^{13}+3a^5b^4(6Ab+5aB)x^{14}+\frac{14}{5}a^4b^5(5Ab+6aB)x^{15}+\frac{15}{8}a^3b^6(4Ab+7aB)x^{16}+\frac{15}{17}a^2b^7(3Ab+8aB)x^{17}+\frac{5}{18}ab^8(2Ab+9aB)x^{18}+\frac{1}{19}b^9(Ab+10aB)x^{19}+\frac{1}{20}b^{10}Bx^{20}$$

input
```
Integrate[x^8*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^9)/9 + (a^9*(10*A*b + a*B)*x^10)/10 + (5*a^8*b*(9*A*b + 2*a*B)*x
^11)/11 + (5*a^7*b^2*(8*A*b + 3*a*B)*x^12)/4 + (30*a^6*b^3*(7*A*b + 4*a*B)
*x^13)/13 + 3*a^5*b^4*(6*A*b + 5*a*B)*x^14 + (14*a^4*b^5*(5*A*b + 6*a*B)*x
^15)/5 + (15*a^3*b^6*(4*A*b + 7*a*B)*x^16)/8 + (15*a^2*b^7*(3*A*b + 8*a*B)
*x^17)/17 + (5*a*b^8*(2*A*b + 9*a*B)*x^18)/18 + (b^9*(A*b + 10*a*B)*x^19)/
19 + (b^10*B*x^20)/20
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(-\frac{a^8(a+bx)^{10}(aB-Ab)}{b^9}+\frac{a^7(a+bx)^{11}(9aB-8Ab)}{b^9}-\frac{4a^6(a+bx)^{12}(9aB-7Ab)}{b^9}+\frac{28a^5(a+bx)^{13}(3aB}{b^9}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8(a+bx)^{11}(Ab-aB)}{11b^{10}}-\frac{a^7(a+bx)^{12}(8Ab-9aB)}{12b^{10}}+\frac{4a^6(a+bx)^{13}(7Ab-9aB)}{13b^{10}}-\frac{2a^5(a+bx)^{14}(2Ab-3aB)}{b^{10}}+\frac{14a^4(a+bx)^{15}(5Ab-9aB)}{15b^{10}}-\frac{7a^3(a+bx)^{16}(4Ab-9aB)}{8b^{10}}+\frac{28a^2(a+bx)^{17}(Ab-3aB)}{17b^{10}}+\frac{(a+bx)^{19}(Ab-9aB)}{19b^{10}}-\frac{2a(a+bx)^{18}(2Ab-9aB)}{9b^{10}}+\frac{B(a+bx)^{20}}{20b^{10}}$$

input
```
Int[x^8*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^8*(A*b - a*B)*(a + b*x)^11)/(11*b^10) - (a^7*(8*A*b - 9*a*B)*(a + b*x)^
12)/(12*b^10) + (4*a^6*(7*A*b - 9*a*B)*(a + b*x)^13)/(13*b^10) - (2*a^5*(2
*A*b - 3*a*B)*(a + b*x)^14)/b^10 + (14*a^4*(5*A*b - 9*a*B)*(a + b*x)^15)/(
15*b^10) - (7*a^3*(4*A*b - 9*a*B)*(a + b*x)^16)/(8*b^10) + (28*a^2*(A*b -
3*a*B)*(a + b*x)^17)/(17*b^10) - (2*a*(2*A*b - 9*a*B)*(a + b*x)^18)/(9*b^1
0) + ((A*b - 9*a*B)*(a + b*x)^19)/(19*b^10) + (B*(a + b*x)^20)/(20*b^10)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a^{10}A\,x^9}{9}+\left(a^9bA+\frac{1}{10}a^{10}B\right)x^{10}+\left(\frac{45}{11}a^8b^2A+\frac{10}{11}a^9bB\right)x^{11}+\left(10a^7b^3A+\frac{15}{4}a^8b^2B\right)x^{12}+($
default	$\frac{b^{10}B\,x^{20}}{20}+\frac{(b^{10}A+10a\,b^9B)x^{19}}{19}+\frac{(10a\,b^9A+45a^2b^8B)x^{18}}{18}+\frac{(45a^2b^8A+120a^3b^7B)x^{17}}{17}+\frac{(120a^3b^7A+210a^4b^6B)x}{16}$
orering	$\frac{x^9(831402B\,b^{10}x^{11}+875160A\,b^{10}x^{10}+8751600Ba\,b^9x^{10}+9237800aA\,b^9x^9+41570100B\,a^2b^8x^9+44015400a^2A\,b^8x^8+11737}{}$
gosper	$\frac{1}{9}a^{10}A\,x^9+x^{10}a^9bA+\frac{1}{10}x^{10}a^{10}B+\frac{45}{11}x^{11}a^8b^2A+\frac{10}{11}x^{11}a^9bB+10x^{12}a^7b^3A+\frac{15}{4}x^{12}a^8b^2B$
risch	$\frac{1}{9}a^{10}A\,x^9+x^{10}a^9bA+\frac{1}{10}x^{10}a^{10}B+\frac{45}{11}x^{11}a^8b^2A+\frac{10}{11}x^{11}a^9bB+10x^{12}a^7b^3A+\frac{15}{4}x^{12}a^8b^2B$
parallelrisch	$\frac{1}{9}a^{10}A\,x^9+x^{10}a^9bA+\frac{1}{10}x^{10}a^{10}B+\frac{45}{11}x^{11}a^8b^2A+\frac{10}{11}x^{11}a^9bB+10x^{12}a^7b^3A+\frac{15}{4}x^{12}a^8b^2B$

input
```
int(x^8*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*a^10*A*x^9+(a^9*b*A+1/10*a^10*B)*x^10+(45/11*a^8*b^2*A+10/11*a^9*b*B)*
x^11+(10*a^7*b^3*A+15/4*a^8*b^2*B)*x^12+(210/13*a^6*b^4*A+120/13*a^7*b^3*B
)*x^13+(18*A*a^5*b^5+15*B*a^6*b^4)*x^14+(14*a^4*b^6*A+84/5*a^5*b^5*B)*x^15
+(15/2*a^3*b^7*A+105/8*a^4*b^6*B)*x^16+(45/17*a^2*b^8*A+120/17*a^3*b^7*B)*
x^17+(5/9*a*b^9*A+5/2*a^2*b^8*B)*x^18+(1/19*b^10*A+10/19*a*b^9*B)*x^19+1/2
0*b^10*B*x^20
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01

$$\begin{aligned}\int x^8(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{20}\,Bb^{10}x^{20}+\frac{1}{9}\,Aa^{10}x^9+\frac{1}{19}\,(10\,Bab^9+Ab^{10})x^{19}\\&\quad+\frac{5}{18}\,(9\,Ba^2b^8+2\,Aab^9)x^{18}+\frac{15}{17}\,(8\,Ba^3b^7+3\,Aa^2b^8)x^{17}\\&\quad+\frac{15}{8}\,(7\,Ba^4b^6+4\,Aa^3b^7)x^{16}\\&\quad+\frac{14}{5}\,(6\,Ba^5b^5+5\,Aa^4b^6)x^{15}+3\,(5\,Ba^6b^4+6\,Aa^5b^5)x^{14}\\&\quad+\frac{30}{13}\,(4\,Ba^7b^3+7\,Aa^6b^4)x^{13}+\frac{5}{4}\,(3\,Ba^8b^2+8\,Aa^7b^3)x^{12}\\&\quad+\frac{5}{11}\,(2\,Ba^9b+9\,Aa^8b^2)x^{11}+\frac{1}{10}\,(Ba^{10}+10\,Aa^9b)x^{10}\end{aligned}$$

input
```
integrate(x^8*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/20*B*b^10*x^20 + 1/9*A*a^10*x^9 + 1/19*(10*B*a*b^9 + A*b^10)*x^19 + 5/18
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^18 + 15/17*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^17 +
 15/8*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^16 + 14/5*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^15 + 3*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^14 + 30/13*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^13 + 5/4*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^12 + 5/11*(2*B*a^9*b + 9*A*a^
8*b^2)*x^11 + 1/10*(B*a^10 + 10*A*a^9*b)*x^10
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.10

$$\int x^8(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^9}{9} + \frac{Bb^{10}x^{20}}{20} + x^{19}\left(\frac{Ab^{10}}{19} + \frac{10Bab^9}{19}\right) + x^{18} \cdot \left(\frac{5Aab^9}{9} + \frac{5Ba^2b^8}{2}\right) + x^{17} \cdot \left(\frac{45Aa^2b^8}{17} + \frac{120Ba^3b^7}{17}\right) + x^{16} \cdot \left(\frac{15Aa^3b^7}{2} + \frac{105Ba^4b^6}{8}\right) + x^{15} \cdot \left(14Aa^4b^6 + \frac{84Ba^5b^5}{5}\right) + x^{14} \cdot \left(18Aa^5b^5 + 15Ba^6b^4\right) + x^{13} \cdot \left(\frac{210Aa^6b^4}{13} + \frac{120Ba^7b^3}{13}\right) + x^{12} \cdot \left(10Aa^7b^3 + \frac{15Ba^8b^2}{4}\right) + x^{11} \cdot \left(\frac{45Aa^8b^2}{11} + \frac{10Ba^9b}{11}\right) + x^{10}\left(Aa^9b + \frac{Ba^{10}}{10}\right)$$

input
```
integrate(x**8*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**9/9 + B*b**10*x**20/20 + x**19*(A*b**10/19 + 10*B*a*b**9/19) +
x**18*(5*A*a*b**9/9 + 5*B*a**2*b**8/2) + x**17*(45*A*a**2*b**8/17 + 120*B*
a**3*b**7/17) + x**16*(15*A*a**3*b**7/2 + 105*B*a**4*b**6/8) + x**15*(14*A
*a**4*b**6 + 84*B*a**5*b**5/5) + x**14*(18*A*a**5*b**5 + 15*B*a**6*b**4) +
 x**13*(210*A*a**6*b**4/13 + 120*B*a**7*b**3/13) + x**12*(10*A*a**7*b**3 +
 15*B*a**8*b**2/4) + x**11*(45*A*a**8*b**2/11 + 10*B*a**9*b/11) + x**10*(A
*a**9*b + B*a**10/10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01

$$\int x^8(a+bx)^{10}(A+Bx)\,dx = \frac{1}{20}\,Bb^{10}x^{20} + \frac{1}{9}\,Aa^{10}x^9 + \frac{1}{19}\,\left(10\,Bab^9 + Ab^{10}\right)x^{19} + \frac{5}{18}\,\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{18} + \frac{15}{17}\,\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^{17} + \frac{15}{8}\,\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^{16} + \frac{14}{5}\,\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^{15} + 3\,\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^{14} + \frac{30}{13}\,\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^{13} + \frac{5}{4}\,\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^{12} + \frac{5}{11}\,\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^{11} + \frac{1}{10}\,\left(Ba^{10} + 10\,Aa^9b\right)x^{10}$$

input
```
integrate(x^8*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/20*B*b^10*x^20 + 1/9*A*a^10*x^9 + 1/19*(10*B*a*b^9 + A*b^10)*x^19 + 5/18
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^18 + 15/17*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^17 +
 15/8*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^16 + 14/5*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^15 + 3*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^14 + 30/13*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^13 + 5/4*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^12 + 5/11*(2*B*a^9*b + 9*A*a^
8*b^2)*x^11 + 1/10*(B*a^10 + 10*A*a^9*b)*x^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int x^8(a+bx)^{10}(A+Bx)\,dx = \frac{1}{20}Bb^{10}x^{20} + \frac{10}{19}Bab^9x^{19} + \frac{1}{19}Ab^{10}x^{19} + \frac{5}{2}Ba^2b^8x^{18} + \frac{5}{9}Aab^9x^{18} + \frac{120}{17}Ba^3b^7x^{17} + \frac{45}{17}Aa^2b^8x^{17} + \frac{105}{8}Ba^4b^6x^{16} + \frac{15}{2}Aa^3b^7x^{16} + \frac{84}{5}Ba^5b^5x^{15} + 14\,Aa^4b^6x^{15} + 15\,Ba^6b^4x^{14} + 18\,Aa^5b^5x^{14} + \frac{120}{13}Ba^7b^3x^{13} + \frac{210}{13}Aa^6b^4x^{13} + \frac{15}{4}Ba^8b^2x^{12} + 10\,Aa^7b^3x^{12} + \frac{10}{11}Ba^9bx^{11} + \frac{45}{11}Aa^8b^2x^{11} + \frac{1}{10}Ba^{10}x^{10} + Aa^9bx^{10} + \frac{1}{9}Aa^{10}x^9$$

input
```
integrate(x^8*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/20*B*b^10*x^20 + 10/19*B*a*b^9*x^19 + 1/19*A*b^10*x^19 + 5/2*B*a^2*b^8*x
^18 + 5/9*A*a*b^9*x^18 + 120/17*B*a^3*b^7*x^17 + 45/17*A*a^2*b^8*x^17 + 10
5/8*B*a^4*b^6*x^16 + 15/2*A*a^3*b^7*x^16 + 84/5*B*a^5*b^5*x^15 + 14*A*a^4*
b^6*x^15 + 15*B*a^6*b^4*x^14 + 18*A*a^5*b^5*x^14 + 120/13*B*a^7*b^3*x^13 +
 210/13*A*a^6*b^4*x^13 + 15/4*B*a^8*b^2*x^12 + 10*A*a^7*b^3*x^12 + 10/11*B
*a^9*b*x^11 + 45/11*A*a^8*b^2*x^11 + 1/10*B*a^10*x^10 + A*a^9*b*x^10 + 1/9
*A*a^10*x^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.88

$$\int x^8(a+bx)^{10}(A+Bx)\,dx = x^{10}\left(\frac{B\,a^{10}}{10}+A\,b\,a^9\right)+x^{19}\left(\frac{A\,b^{10}}{19}+\frac{10\,B\,a\,b^9}{19}\right) + \frac{A\,a^{10}\,x^9}{9}+\frac{B\,b^{10}\,x^{20}}{20}+\frac{5\,a^7\,b^2\,x^{12}\,(8\,A\,b+3\,B\,a)}{4} + \frac{30\,a^6\,b^3\,x^{13}\,(7\,A\,b+4\,B\,a)}{13} + 3\,a^5\,b^4\,x^{14}\,(6\,A\,b+5\,B\,a)+\frac{14\,a^4\,b^5\,x^{15}\,(5\,A\,b+6\,B\,a)}{5} + \frac{15\,a^3\,b^6\,x^{16}\,(4\,A\,b+7\,B\,a)}{8}+\frac{15\,a^2\,b^7\,x^{17}\,(3\,A\,b+8\,B\,a)}{17} + \frac{5\,a^8\,b\,x^{11}\,(9\,A\,b+2\,B\,a)}{11}+\frac{5\,a\,b^8\,x^{18}\,(2\,A\,b+9\,B\,a)}{18}$$

input
```
int(x^8*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^10*((B*a^10)/10 + A*a^9*b) + x^19*((A*b^10)/19 + (10*B*a*b^9)/19) + (A*a
^10*x^9)/9 + (B*b^10*x^20)/20 + (5*a^7*b^2*x^12*(8*A*b + 3*B*a))/4 + (30*a
^6*b^3*x^13*(7*A*b + 4*B*a))/13 + 3*a^5*b^4*x^14*(6*A*b + 5*B*a) + (14*a^4
*b^5*x^15*(5*A*b + 6*B*a))/5 + (15*a^3*b^6*x^16*(4*A*b + 7*B*a))/8 + (15*a
^2*b^7*x^17*(3*A*b + 8*B*a))/17 + (5*a^8*b*x^11*(9*A*b + 2*B*a))/11 + (5*a
*b^8*x^18*(2*A*b + 9*B*a))/18
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int x^8(a+bx)^{10}(A+Bx)\,dx = \frac{x^9(75582b^{11}x^{11}+875160a\,b^{10}x^{10}+4618900a^2b^9x^9+14671800a^3b^8x^8+31177575a^4b^7x^7+46558512a^5b}{15}$$

input
```
int(x^8*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**9*(167960*a**11 + 1662804*a**10*b*x + 7558200*a**9*b**2*x**2 + 2078505
0*a**8*b**3*x**3 + 38372400*a**7*b**4*x**4 + 49884120*a**6*b**5*x**5 + 465
58512*a**5*b**6*x**6 + 31177575*a**4*b**7*x**7 + 14671800*a**3*b**8*x**8 +
 4618900*a**2*b**9*x**9 + 875160*a*b**10*x**10 + 75582*b**11*x**11))/15116
40
```

3.109 $\int x^7(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 215

$$\int x^7(a+bx)^{10}(A+Bx)\,dx = -\frac{a^7(Ab-aB)(a+bx)^{11}}{11b^9}+\frac{a^6(7Ab-8aB)(a+bx)^{12}}{12b^9} - \frac{7a^5(3Ab-4aB)(a+bx)^{13}}{13b^9} + \frac{a^4(5Ab-8aB)(a+bx)^{14}}{2b^9}-\frac{7a^3(Ab-2aB)(a+bx)^{15}}{3b^9} + \frac{7a^2(3Ab-8aB)(a+bx)^{16}}{16b^9}-\frac{7a(Ab-4aB)(a+bx)^{17}}{17b^9} + \frac{(Ab-8aB)(a+bx)^{18}}{18b^9}+\frac{B(a+bx)^{19}}{19b^9}$$

output
```
-1/11*a^7*(A*b-B*a)*(b*x+a)^11/b^9+1/12*a^6*(7*A*b-8*B*a)*(b*x+a)^12/b^9-7
/13*a^5*(3*A*b-4*B*a)*(b*x+a)^13/b^9+1/2*a^4*(5*A*b-8*B*a)*(b*x+a)^14/b^9-
7/3*a^3*(A*b-2*B*a)*(b*x+a)^15/b^9+7/16*a^2*(3*A*b-8*B*a)*(b*x+a)^16/b^9-7
/17*a*(A*b-4*B*a)*(b*x+a)^17/b^9+1/18*(A*b-8*B*a)*(b*x+a)^18/b^9+1/19*B*(b
*x+a)^19/b^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06

$$\int x^7(a+bx)^{10}(A+Bx)\,dx=\frac{1}{8}a^{10}Ax^8+\frac{1}{9}a^9(10Ab+aB)x^9+\frac{1}{2}a^8b(9Ab+2aB)x^{10}+\frac{15}{11}a^7b^2(8Ab+3aB)x^{11}+\frac{5}{2}a^6b^3(7Ab+4aB)x^{12}+\frac{42}{13}a^5b^4(6Ab+5aB)x^{13}+3a^4b^5(5Ab+6aB)x^{14}+2a^3b^6(4Ab+7aB)x^{15}+\frac{15}{16}a^2b^7(3Ab+8aB)x^{16}+\frac{5}{17}ab^8(2Ab+9aB)x^{17}+\frac{1}{18}b^9(Ab+10aB)x^{18}+\frac{1}{19}b^{10}Bx^{19}$$

input
```
Integrate[x^7*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^8)/8 + (a^9*(10*A*b + a*B)*x^9)/9 + (a^8*b*(9*A*b + 2*a*B)*x^10)
/2 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^11)/11 + (5*a^6*b^3*(7*A*b + 4*a*B)*x^1
2)/2 + (42*a^5*b^4*(6*A*b + 5*a*B)*x^13)/13 + 3*a^4*b^5*(5*A*b + 6*a*B)*x^
14 + 2*a^3*b^6*(4*A*b + 7*a*B)*x^15 + (15*a^2*b^7*(3*A*b + 8*a*B)*x^16)/16
 + (5*a*b^8*(2*A*b + 9*a*B)*x^17)/17 + (b^9*(A*b + 10*a*B)*x^18)/18 + (b^1
0*B*x^19)/19
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^7(a+bx)^{10}(aB-Ab)}{b^8} - \frac{a^6(a+bx)^{11}(8aB-7Ab)}{b^8} + \frac{7a^5(a+bx)^{12}(4aB-3Ab)}{b^8} - \frac{7a^4(a+bx)^{13}(8aB-}{b^8} \right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^7(a+bx)^{11}(Ab-aB)}{11b^9} + \frac{a^6(a+bx)^{12}(7Ab-8aB)}{12b^9} - \frac{7a^5(a+bx)^{13}(3Ab-4aB)}{13b^9} + \frac{a^4(a+bx)^{14}(5Ab-8aB)}{2b^9} - \frac{7a^3(a+bx)^{15}(Ab-2aB)}{3b^9} + \frac{7a^2(a+bx)^{16}(3Ab-8aB)}{16b^9} + \frac{(a+bx)^{18}(Ab-8aB)}{18b^9} - \frac{7a(a+bx)^{17}(Ab-4aB)}{17b^9} + \frac{B(a+bx)^{19}}{19b^9}$$

input
```
Int[x^7*(a + b*x)^10*(A + B*x),x]
```

output
```
-1/11*(a^7*(A*b - a*B)*(a + b*x)^11)/b^9 + (a^6*(7*A*b - 8*a*B)*(a + b*x)^
12)/(12*b^9) - (7*a^5*(3*A*b - 4*a*B)*(a + b*x)^13)/(13*b^9) + (a^4*(5*A*b
 - 8*a*B)*(a + b*x)^14)/(2*b^9) - (7*a^3*(A*b - 2*a*B)*(a + b*x)^15)/(3*b^
9) + (7*a^2*(3*A*b - 8*a*B)*(a + b*x)^16)/(16*b^9) - (7*a*(A*b - 4*a*B)*(a
 + b*x)^17)/(17*b^9) + ((A*b - 8*a*B)*(a + b*x)^18)/(18*b^9) + (B*(a + b*x
)^19)/(19*b^9)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.09

method	result
norman	$\frac{a^{10}Ax^8}{8}+\left(\frac{10}{9}a^9bA+\frac{1}{9}a^{10}B\right)x^9+\left(\frac{9}{2}a^8b^2A+a^9bB\right)x^{10}+\left(\frac{120}{11}a^7b^3A+\frac{45}{11}a^8b^2B\right)x^{11}+\left(\frac{35}{2}\right.$
default	$\frac{b^{10}Bx^{19}}{19}+\frac{(b^{10}A+10ab^9B)x^{18}}{18}+\frac{(10ab^9A+45a^2b^8B)x^{17}}{17}+\frac{(45a^2b^8A+120a^3b^7B)x^{16}}{16}+\frac{(120a^3b^7A+210a^4b^6B)x}{15}$
orering	$x^8(350064Bb^{10}x^{11}+369512Ab^{10}x^{10}+3695120Bab^9x^{10}+3912480aAb^9x^9+17606160Ba^2b^8x^9+18706545a^2Ab^8x^8+49884$
gosper	$\frac{1}{8}a^{10}Ax^8+\frac{10}{9}x^9a^9bA+\frac{1}{9}x^9a^{10}B+\frac{9}{2}x^{10}a^8b^2A+x^{10}a^9bB+\frac{120}{11}x^{11}a^7b^3A+\frac{45}{11}x^{11}a^8b^2B+$
risch	$\frac{1}{8}a^{10}Ax^8+\frac{10}{9}x^9a^9bA+\frac{1}{9}x^9a^{10}B+\frac{9}{2}x^{10}a^8b^2A+x^{10}a^9bB+\frac{120}{11}x^{11}a^7b^3A+\frac{45}{11}x^{11}a^8b^2B+$
parallelrisch	$\frac{1}{8}a^{10}Ax^8+\frac{10}{9}x^9a^9bA+\frac{1}{9}x^9a^{10}B+\frac{9}{2}x^{10}a^8b^2A+x^{10}a^9bB+\frac{120}{11}x^{11}a^7b^3A+\frac{45}{11}x^{11}a^8b^2B+$

input
```
int(x^7*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*a^10*A*x^8+(10/9*a^9*b*A+1/9*a^10*B)*x^9+(9/2*a^8*b^2*A+a^9*b*B)*x^10+
(120/11*a^7*b^3*A+45/11*a^8*b^2*B)*x^11+(35/2*a^6*b^4*A+10*a^7*b^3*B)*x^12
+(252/13*a^5*b^5*A+210/13*a^6*b^4*B)*x^13+(15*A*a^4*b^6+18*B*a^5*b^5)*x^14
+(8*A*a^3*b^7+14*B*a^4*b^6)*x^15+(45/16*a^2*b^8*A+15/2*a^3*b^7*B)*x^16+(10
/17*a*b^9*A+45/17*a^2*b^8*B)*x^17+(1/18*b^10*A+5/9*a*b^9*B)*x^18+1/19*b^10
*B*x^19
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.13

$$\begin{aligned}\int x^7(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{19}Bb^{10}x^{19}+\frac{1}{8}Aa^{10}x^8+\frac{1}{18}\left(10Bab^9+Ab^{10}\right)x^{18}\\&+\frac{5}{17}\left(9Ba^2b^8+2Aab^9\right)x^{17}+\frac{15}{16}\left(8Ba^3b^7+3Aa^2b^8\right)x^{16}\\&+2\left(7Ba^4b^6+4Aa^3b^7\right)x^{15}+3\left(6Ba^5b^5+5Aa^4b^6\right)x^{14}\\&+\frac{42}{13}\left(5Ba^6b^4+6Aa^5b^5\right)x^{13}\\&+\frac{5}{2}\left(4Ba^7b^3+7Aa^6b^4\right)x^{12}+\frac{15}{11}\left(3Ba^8b^2+8Aa^7b^3\right)x^{11}\\&+\frac{1}{2}\left(2Ba^9b+9Aa^8b^2\right)x^{10}+\frac{1}{9}\left(Ba^{10}+10Aa^9b\right)x^9\end{aligned}$$

input
```
integrate(x^7*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/19*B*b^10*x^19 + 1/8*A*a^10*x^8 + 1/18*(10*B*a*b^9 + A*b^10)*x^18 + 5/17
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^17 + 15/16*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^16 +
 2*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^15 + 3*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^14 +
 42/13*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^13 + 5/2*(4*B*a^7*b^3 + 7*A*a^6*b^4)*
x^12 + 15/11*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^11 + 1/2*(2*B*a^9*b + 9*A*a^8*b
^2)*x^10 + 1/9*(B*a^10 + 10*A*a^9*b)*x^9
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.22

$$\int x^7(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^8}{8} + \frac{Bb^{10}x^{19}}{19} + x^{18}\left(\frac{Ab^{10}}{18} + \frac{5Bab^9}{9}\right) + x^{17} \cdot \left(\frac{10Aab^9}{17} + \frac{45Ba^2b^8}{17}\right) + x^{16} \cdot \left(\frac{45Aa^2b^8}{16} + \frac{15Ba^3b^7}{2}\right) + x^{15} \cdot \left(8Aa^3b^7 + 14Ba^4b^6\right) + x^{14} \cdot \left(15Aa^4b^6 + 18Ba^5b^5\right) + x^{13} \cdot \left(\frac{252Aa^5b^5}{13} + \frac{210Ba^6b^4}{13}\right) + x^{12} \cdot \left(\frac{35Aa^6b^4}{2} + 10Ba^7b^3\right) + x^{11} \cdot \left(\frac{120Aa^7b^3}{11} + \frac{45Ba^8b^2}{11}\right) + x^{10} \cdot \left(\frac{9Aa^8b^2}{2} + Ba^9b\right) + x^9 \cdot \left(\frac{10Aa^9b}{9} + \frac{Ba^{10}}{9}\right)$$

input
```
integrate(x**7*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**8/8 + B*b**10*x**19/19 + x**18*(A*b**10/18 + 5*B*a*b**9/9) + x*
*17*(10*A*a*b**9/17 + 45*B*a**2*b**8/17) + x**16*(45*A*a**2*b**8/16 + 15*B
*a**3*b**7/2) + x**15*(8*A*a**3*b**7 + 14*B*a**4*b**6) + x**14*(15*A*a**4*
b**6 + 18*B*a**5*b**5) + x**13*(252*A*a**5*b**5/13 + 210*B*a**6*b**4/13) +
 x**12*(35*A*a**6*b**4/2 + 10*B*a**7*b**3) + x**11*(120*A*a**7*b**3/11 + 4
5*B*a**8*b**2/11) + x**10*(9*A*a**8*b**2/2 + B*a**9*b) + x**9*(10*A*a**9*b
/9 + B*a**10/9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.13

$$\int x^7(a+bx)^{10}(A+Bx)\,dx = \frac{1}{19}\,Bb^{10}x^{19} + \frac{1}{8}\,Aa^{10}x^8 + \frac{1}{18}\,(10\,Bab^9 + Ab^{10})x^{18} + \frac{5}{17}\,(9\,Ba^2b^8 + 2\,Aab^9)x^{17} + \frac{15}{16}\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^{16} + 2\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^{15} + 3\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^{14} + \frac{42}{13}\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^{13} + \frac{5}{2}\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^{12} + \frac{15}{11}\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^{11} + \frac{1}{2}\,(2\,Ba^9b + 9\,Aa^8b^2)x^{10} + \frac{1}{9}\,(Ba^{10} + 10\,Aa^9b)x^9$$

input
```
integrate(x^7*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/19*B*b^10*x^19 + 1/8*A*a^10*x^8 + 1/18*(10*B*a*b^9 + A*b^10)*x^18 + 5/17
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^17 + 15/16*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^16 +
 2*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^15 + 3*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^14 +
 42/13*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^13 + 5/2*(4*B*a^7*b^3 + 7*A*a^6*b^4)*
x^12 + 15/11*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^11 + 1/2*(2*B*a^9*b + 9*A*a^8*b
^2)*x^10 + 1/9*(B*a^10 + 10*A*a^9*b)*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.13

$$\int x^7(a+bx)^{10}(A+Bx)\,dx = \frac{1}{19}\,Bb^{10}x^{19} + \frac{5}{9}\,Bab^9x^{18} + \frac{1}{18}\,Ab^{10}x^{18} + \frac{45}{17}\,Ba^2b^8x^{17} + \frac{10}{17}\,Aab^9x^{17} + \frac{15}{2}\,Ba^3b^7x^{16} + \frac{45}{16}\,Aa^2b^8x^{16} + 14\,Ba^4b^6x^{15} + 8\,Aa^3b^7x^{15} + 18\,Ba^5b^5x^{14} + 15\,Aa^4b^6x^{14} + \frac{210}{13}\,Ba^6b^4x^{13} + \frac{252}{13}\,Aa^5b^5x^{13} + 10\,Ba^7b^3x^{12} + \frac{35}{2}\,Aa^6b^4x^{12} + \frac{45}{11}\,Ba^8b^2x^{11} + \frac{120}{11}\,Aa^7b^3x^{11} + Ba^9bx^{10} + \frac{9}{2}\,Aa^8b^2x^{10} + \frac{1}{9}\,Ba^{10}x^9 + \frac{10}{9}\,Aa^9bx^9 + \frac{1}{8}\,Aa^{10}x^8$$

input
```
integrate(x^7*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/19*B*b^10*x^19 + 5/9*B*a*b^9*x^18 + 1/18*A*b^10*x^18 + 45/17*B*a^2*b^8*x
^17 + 10/17*A*a*b^9*x^17 + 15/2*B*a^3*b^7*x^16 + 45/16*A*a^2*b^8*x^16 + 14
*B*a^4*b^6*x^15 + 8*A*a^3*b^7*x^15 + 18*B*a^5*b^5*x^14 + 15*A*a^4*b^6*x^14
 + 210/13*B*a^6*b^4*x^13 + 252/13*A*a^5*b^5*x^13 + 10*B*a^7*b^3*x^12 + 35/
2*A*a^6*b^4*x^12 + 45/11*B*a^8*b^2*x^11 + 120/11*A*a^7*b^3*x^11 + B*a^9*b*
x^10 + 9/2*A*a^8*b^2*x^10 + 1/9*B*a^10*x^9 + 10/9*A*a^9*b*x^9 + 1/8*A*a^10
*x^8
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.98

$$\int x^7(a+bx)^{10}(A+Bx)\,dx = x^9\left(\frac{B\,a^{10}}{9}+\frac{10\,A\,b\,a^9}{9}\right)+x^{18}\left(\frac{A\,b^{10}}{18}+\frac{5\,B\,a\,b^9}{9}\right)+\frac{A\,a^{10}\,x^8}{8}+\frac{B\,b^{10}\,x^{19}}{19}+\frac{15\,a^7\,b^2\,x^{11}\,(8\,A\,b+3\,B\,a)}{11}+\frac{5\,a^6\,b^3\,x^{12}\,(7\,A\,b+4\,B\,a)}{2}+\frac{42\,a^5\,b^4\,x^{13}\,(6\,A\,b+5\,B\,a)}{13}+3\,a^4\,b^5\,x^{14}\,(5\,A\,b+6\,B\,a)+2\,a^3\,b^6\,x^{15}\,(4\,A\,b+7\,B\,a)+\frac{15\,a^2\,b^7\,x^{16}\,(3\,A\,b+8\,B\,a)}{16}+\frac{a^8\,b\,x^{10}\,(9\,A\,b+2\,B\,a)}{2}+\frac{5\,a\,b^8\,x^{17}\,(2\,A\,b+9\,B\,a)}{17}$$

input
```
int(x^7*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^9*((B*a^10)/9 + (10*A*a^9*b)/9) + x^18*((A*b^10)/18 + (5*B*a*b^9)/9) + (
A*a^10*x^8)/8 + (B*b^10*x^19)/19 + (15*a^7*b^2*x^11*(8*A*b + 3*B*a))/11 +
(5*a^6*b^3*x^12*(7*A*b + 4*B*a))/2 + (42*a^5*b^4*x^13*(6*A*b + 5*B*a))/13
+ 3*a^4*b^5*x^14*(5*A*b + 6*B*a) + 2*a^3*b^6*x^15*(4*A*b + 7*B*a) + (15*a^
2*b^7*x^16*(3*A*b + 8*B*a))/16 + (a^8*b*x^10*(9*A*b + 2*B*a))/2 + (5*a*b^8
*x^17*(2*A*b + 9*B*a))/17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.57

$$\int x^7(a+bx)^{10}(A+Bx)\,dx$$
$$=\frac{x^8(31824b^{11}x^{11}+369512a\,b^{10}x^{10}+1956240a^2b^9x^9+6235515a^3b^8x^8+13302432a^4b^7x^7+19953648a^5b^6\ldots}{604\ldots}$$

input
```
int(x^7*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**8*(75582*a**11 + 739024*a**10*b*x + 3325608*a**9*b**2*x**2 + 9069840*a
**8*b**3*x**3 + 16628040*a**7*b**4*x**4 + 21488544*a**6*b**5*x**5 + 199536
48*a**5*b**6*x**6 + 13302432*a**4*b**7*x**7 + 6235515*a**3*b**8*x**8 + 195
6240*a**2*b**9*x**9 + 369512*a*b**10*x**10 + 31824*b**11*x**11))/604656
```

3.110 $\int x^6(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 191

$$\int x^6(a+bx)^{10}(A+Bx)\,dx = \frac{a^6(Ab-aB)(a+bx)^{11}}{11b^8}-\frac{a^5(6Ab-7aB)(a+bx)^{12}}{12b^8}+\frac{3a^4(5Ab-7aB)(a+bx)^{13}}{13b^8}-\frac{5a^3(4Ab-7aB)(a+bx)^{14}}{14b^8}+\frac{a^2(3Ab-7aB)(a+bx)^{15}}{3b^8}-\frac{3a(2Ab-7aB)(a+bx)^{16}}{16b^8}+\frac{(Ab-7aB)(a+bx)^{17}}{17b^8}+\frac{B(a+bx)^{18}}{18b^8}$$

output
```
1/11*a^6*(A*b-B*a)*(b*x+a)^11/b^8-1/12*a^5*(6*A*b-7*B*a)*(b*x+a)^12/b^8+3/
13*a^4*(5*A*b-7*B*a)*(b*x+a)^13/b^8-5/14*a^3*(4*A*b-7*B*a)*(b*x+a)^14/b^8+
1/3*a^2*(3*A*b-7*B*a)*(b*x+a)^15/b^8-3/16*a*(2*A*b-7*B*a)*(b*x+a)^16/b^8+1
/17*(A*b-7*B*a)*(b*x+a)^17/b^8+1/18*B*(b*x+a)^18/b^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.19

$$\int x^6(a+bx)^{10}(A+Bx)\,dx = \frac{1}{7}a^{10}Ax^7 + \frac{1}{8}a^9(10Ab+aB)x^8 + \frac{5}{9}a^8b(9Ab+2aB)x^9 + \frac{3}{2}a^7b^2(8Ab+3aB)x^{10} + \frac{30}{11}a^6b^3(7Ab+4aB)x^{11} + \frac{7}{2}a^5b^4(6Ab+5aB)x^{12} + \frac{42}{13}a^4b^5(5Ab+6aB)x^{13} + \frac{15}{7}a^3b^6(4Ab+7aB)x^{14} + a^2b^7(3Ab+8aB)x^{15} + \frac{5}{16}ab^8(2Ab+9aB)x^{16} + \frac{1}{17}b^9(Ab+10aB)x^{17} + \frac{1}{18}b^{10}Bx^{18}$$

input
```
Integrate[x^6*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^7)/7 + (a^9*(10*A*b + a*B)*x^8)/8 + (5*a^8*b*(9*A*b + 2*a*B)*x^9
)/9 + (3*a^7*b^2*(8*A*b + 3*a*B)*x^10)/2 + (30*a^6*b^3*(7*A*b + 4*a*B)*x^1
1)/11 + (7*a^5*b^4*(6*A*b + 5*a*B)*x^12)/2 + (42*a^4*b^5*(5*A*b + 6*a*B)*x
^13)/13 + (15*a^3*b^6*(4*A*b + 7*a*B)*x^14)/7 + a^2*b^7*(3*A*b + 8*a*B)*x^
15 + (5*a*b^8*(2*A*b + 9*a*B)*x^16)/16 + (b^9*(A*b + 10*a*B)*x^17)/17 + (b
^10*B*x^18)/18
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(-\frac{a^6(a+bx)^{10}(aB-Ab)}{b^7}+\frac{a^5(a+bx)^{11}(7aB-6Ab)}{b^7}-\frac{3a^4(a+bx)^{12}(7aB-5Ab)}{b^7}+\frac{5a^3(a+bx)^{13}(7aB-}{b^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6(a+bx)^{11}(Ab-aB)}{11b^8}-\frac{a^5(a+bx)^{12}(6Ab-7aB)}{12b^8}+\frac{3a^4(a+bx)^{13}(5Ab-7aB)}{13b^8}-\frac{5a^3(a+bx)^{14}(4Ab-7aB)}{14b^8}+\frac{a^2(a+bx)^{15}(3Ab-7aB)}{3b^8}+\frac{(a+bx)^{17}(Ab-7aB)}{17b^8}-\frac{3a(a+bx)^{16}(2Ab-7aB)}{16b^8}+\frac{B(a+bx)^{18}}{18b^8}$$

input
```
Int[x^6*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^6*(A*b - a*B)*(a + b*x)^11)/(11*b^8) - (a^5*(6*A*b - 7*a*B)*(a + b*x)^1
2)/(12*b^8) + (3*a^4*(5*A*b - 7*a*B)*(a + b*x)^13)/(13*b^8) - (5*a^3*(4*A*
b - 7*a*B)*(a + b*x)^14)/(14*b^8) + (a^2*(3*A*b - 7*a*B)*(a + b*x)^15)/(3*
b^8) - (3*a*(2*A*b - 7*a*B)*(a + b*x)^16)/(16*b^8) + ((A*b - 7*a*B)*(a + b
*x)^17)/(17*b^8) + (B*(a + b*x)^18)/(18*b^8)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.24

method	result
norman	$\frac{a^{10}A\,x^{7}}{7}+\left(\frac{5}{4}a^{9}bA+\frac{1}{8}a^{10}B\right)x^{8}+\left(5a^{8}b^{2}A+\frac{10}{9}a^{9}bB\right)x^{9}+\left(12a^{7}b^{3}A+\frac{9}{2}a^{8}b^{2}B\right)x^{10}+\left(\frac{210}{11}a\right.$
default	$\frac{b^{10}B\,x^{18}}{18}+\frac{(b^{10}A+10a\,b^{9}B)x^{17}}{17}+\frac{(10a\,b^{9}A+45a^{2}b^{8}B)x^{16}}{16}+\frac{(45a^{2}b^{8}A+120a^{3}b^{7}B)x^{15}}{15}+\frac{(120a^{3}b^{7}A+210a^{4}b^{6}B)x}{14}$
orering	$\frac{x^{7}(136136B\,b^{10}x^{11}+144144A\,b^{10}x^{10}+1441440Ba\,b^{9}x^{10}+1531530aA\,b^{9}x^{9}+6891885B\,a^{2}b^{8}x^{9}+7351344a^{2}A\,b^{8}x^{8}+1960358}{}$
gosper	$\frac{1}{7}a^{10}A\,x^{7}+\frac{5}{4}x^{8}a^{9}bA+\frac{1}{8}x^{8}a^{10}B+5x^{9}a^{8}b^{2}A+\frac{10}{9}x^{9}a^{9}bB+12x^{10}a^{7}b^{3}A+\frac{9}{2}x^{10}a^{8}b^{2}B+\frac{210}{11}$
risch	$\frac{1}{7}a^{10}A\,x^{7}+\frac{5}{4}x^{8}a^{9}bA+\frac{1}{8}x^{8}a^{10}B+5x^{9}a^{8}b^{2}A+\frac{10}{9}x^{9}a^{9}bB+12x^{10}a^{7}b^{3}A+\frac{9}{2}x^{10}a^{8}b^{2}B+\frac{210}{11}$
parallelrisch	$\frac{1}{7}a^{10}A\,x^{7}+\frac{5}{4}x^{8}a^{9}bA+\frac{1}{8}x^{8}a^{10}B+5x^{9}a^{8}b^{2}A+\frac{10}{9}x^{9}a^{9}bB+12x^{10}a^{7}b^{3}A+\frac{9}{2}x^{10}a^{8}b^{2}B+\frac{210}{11}$

input
```
int(x^6*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*a^10*A*x^7+(5/4*a^9*b*A+1/8*a^10*B)*x^8+(5*a^8*b^2*A+10/9*a^9*b*B)*x^9
+(12*a^7*b^3*A+9/2*a^8*b^2*B)*x^10+(210/11*a^6*b^4*A+120/11*a^7*b^3*B)*x^1
1+(21*a^5*b^5*A+35/2*a^6*b^4*B)*x^12+(210/13*a^4*b^6*A+252/13*a^5*b^5*B)*x
^13+(60/7*a^3*b^7*A+15*a^4*b^6*B)*x^14+(3*A*a^2*b^8+8*B*a^3*b^7)*x^15+(5/8
*a*b^9*A+45/16*a^2*b^8*B)*x^16+(1/17*b^10*A+10/17*a*b^9*B)*x^17+1/18*b^10*
B*x^18
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.27

$$\begin{aligned}\int x^6(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{18}\,Bb^{10}x^{18}+\frac{1}{7}\,Aa^{10}x^{7}+\frac{1}{17}\left(10\,Bab^{9}+Ab^{10}\right)x^{17}\\&\quad+\frac{5}{16}\left(9\,Ba^{2}b^{8}+2\,Aab^{9}\right)x^{16}+\left(8\,Ba^{3}b^{7}+3\,Aa^{2}b^{8}\right)x^{15}\\&\quad+\frac{15}{7}\left(7\,Ba^{4}b^{6}+4\,Aa^{3}b^{7}\right)x^{14}\\&\quad+\frac{42}{13}\left(6\,Ba^{5}b^{5}+5\,Aa^{4}b^{6}\right)x^{13}+\frac{7}{2}\left(5\,Ba^{6}b^{4}+6\,Aa^{5}b^{5}\right)x^{12}\\&\quad+\frac{30}{11}\left(4\,Ba^{7}b^{3}+7\,Aa^{6}b^{4}\right)x^{11}+\frac{3}{2}\left(3\,Ba^{8}b^{2}+8\,Aa^{7}b^{3}\right)x^{10}\\&\quad+\frac{5}{9}\left(2\,Ba^{9}b+9\,Aa^{8}b^{2}\right)x^{9}+\frac{1}{8}\left(Ba^{10}+10\,Aa^{9}b\right)x^{8}\end{aligned}$$

input
```
integrate(x^6*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/18*B*b^10*x^18 + 1/7*A*a^10*x^7 + 1/17*(10*B*a*b^9 + A*b^10)*x^17 + 5/16
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^16 + (8*B*a^3*b^7 + 3*A*a^2*b^8)*x^15 + 15/7*
(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^14 + 42/13*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^13
+ 7/2*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^12 + 30/11*(4*B*a^7*b^3 + 7*A*a^6*b^4)
*x^11 + 3/2*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^10 + 5/9*(2*B*a^9*b + 9*A*a^8*b^
2)*x^9 + 1/8*(B*a^10 + 10*A*a^9*b)*x^8
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.38

$$\int x^6(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^7}{7} + \frac{Bb^{10}x^{18}}{18} + x^{17}\left(\frac{Ab^{10}}{17} + \frac{10Bab^9}{17}\right) + x^{16} \cdot \left(\frac{5Aab^9}{8} + \frac{45Ba^2b^8}{16}\right) + x^{15} \cdot \left(3Aa^2b^8 + 8Ba^3b^7\right) + x^{14} \cdot \left(\frac{60Aa^3b^7}{7} + 15Ba^4b^6\right) + x^{13} \cdot \left(\frac{210Aa^4b^6}{13} + \frac{252Ba^5b^5}{13}\right) + x^{12} \cdot \left(21Aa^5b^5 + \frac{35Ba^6b^4}{2}\right) + x^{11} \cdot \left(\frac{210Aa^6b^4}{11} + \frac{120Ba^7b^3}{11}\right) + x^{10} \cdot \left(12Aa^7b^3 + \frac{9Ba^8b^2}{2}\right) + x^9 \cdot \left(5Aa^8b^2 + \frac{10Ba^9b}{9}\right) + x^8 \cdot \left(\frac{5Aa^9b}{4} + \frac{Ba^{10}}{8}\right)$$

input
```
integrate(x**6*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**7/7 + B*b**10*x**18/18 + x**17*(A*b**10/17 + 10*B*a*b**9/17) +
x**16*(5*A*a*b**9/8 + 45*B*a**2*b**8/16) + x**15*(3*A*a**2*b**8 + 8*B*a**3
*b**7) + x**14*(60*A*a**3*b**7/7 + 15*B*a**4*b**6) + x**13*(210*A*a**4*b**
6/13 + 252*B*a**5*b**5/13) + x**12*(21*A*a**5*b**5 + 35*B*a**6*b**4/2) + x
**11*(210*A*a**6*b**4/11 + 120*B*a**7*b**3/11) + x**10*(12*A*a**7*b**3 + 9
*B*a**8*b**2/2) + x**9*(5*A*a**8*b**2 + 10*B*a**9*b/9) + x**8*(5*A*a**9*b/
4 + B*a**10/8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.27

$$\int x^6(a+bx)^{10}(A+Bx)\,dx = \frac{1}{18}Bb^{10}x^{18} + \frac{1}{7}Aa^{10}x^7 + \frac{1}{17}(10\,Bab^9 + Ab^{10})x^{17} + \frac{5}{16}(9\,Ba^2b^8 + 2\,Aab^9)x^{16} + (8\,Ba^3b^7 + 3\,Aa^2b^8)x^{15} + \frac{15}{7}(7\,Ba^4b^6 + 4\,Aa^3b^7)x^{14} + \frac{42}{13}(6\,Ba^5b^5 + 5\,Aa^4b^6)x^{13} + \frac{7}{2}(5\,Ba^6b^4 + 6\,Aa^5b^5)x^{12} + \frac{30}{11}(4\,Ba^7b^3 + 7\,Aa^6b^4)x^{11} + \frac{3}{2}(3\,Ba^8b^2 + 8\,Aa^7b^3)x^{10} + \frac{5}{9}(2\,Ba^9b + 9\,Aa^8b^2)x^9 + \frac{1}{8}(Ba^{10} + 10\,Aa^9b)x^8$$

input

```
integrate(x^6*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output

```
1/18*B*b^10*x^18 + 1/7*A*a^10*x^7 + 1/17*(10*B*a*b^9 + A*b^10)*x^17 + 5/16
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^16 + (8*B*a^3*b^7 + 3*A*a^2*b^8)*x^15 + 15/7*
(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^14 + 42/13*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^13
+ 7/2*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^12 + 30/11*(4*B*a^7*b^3 + 7*A*a^6*b^4)
*x^11 + 3/2*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^10 + 5/9*(2*B*a^9*b + 9*A*a^8*b^
2)*x^9 + 1/8*(B*a^10 + 10*A*a^9*b)*x^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.28

$$\int x^6(a+bx)^{10}(A+Bx)\,dx = \frac{1}{18}Bb^{10}x^{18} + \frac{10}{17}Bab^9x^{17} + \frac{1}{17}Ab^{10}x^{17} + \frac{45}{16}Ba^2b^8x^{16} + \frac{5}{8}Aab^9x^{16} + 8\,Ba^3b^7x^{15} + 3\,Aa^2b^8x^{15} + 15\,Ba^4b^6x^{14} + \frac{60}{7}Aa^3b^7x^{14} + \frac{252}{13}Ba^5b^5x^{13} + \frac{210}{13}Aa^4b^6x^{13} + \frac{35}{2}Ba^6b^4x^{12} + 21\,Aa^5b^5x^{12} + \frac{120}{11}Ba^7b^3x^{11} + \frac{210}{11}Aa^6b^4x^{11} + \frac{9}{2}Ba^8b^2x^{10} + 12\,Aa^7b^3x^{10} + \frac{10}{9}Ba^9bx^9 + 5\,Aa^8b^2x^9 + \frac{1}{8}Ba^{10}x^8 + \frac{5}{4}Aa^9bx^8 + \frac{1}{7}Aa^{10}x^7$$

input
```
integrate(x^6*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/18*B*b^10*x^18 + 10/17*B*a*b^9*x^17 + 1/17*A*b^10*x^17 + 45/16*B*a^2*b^8
*x^16 + 5/8*A*a*b^9*x^16 + 8*B*a^3*b^7*x^15 + 3*A*a^2*b^8*x^15 + 15*B*a^4*
b^6*x^14 + 60/7*A*a^3*b^7*x^14 + 252/13*B*a^5*b^5*x^13 + 210/13*A*a^4*b^6*
x^13 + 35/2*B*a^6*b^4*x^12 + 21*A*a^5*b^5*x^12 + 120/11*B*a^7*b^3*x^11 + 2
10/11*A*a^6*b^4*x^11 + 9/2*B*a^8*b^2*x^10 + 12*A*a^7*b^3*x^10 + 10/9*B*a^9
*b*x^9 + 5*A*a^8*b^2*x^9 + 1/8*B*a^10*x^8 + 5/4*A*a^9*b*x^8 + 1/7*A*a^10*x
^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.10

$$\int x^6(a+bx)^{10}(A+Bx)\,dx = x^8\left(\frac{B\,a^{10}}{8}+\frac{5\,A\,b\,a^9}{4}\right)+x^{17}\left(\frac{A\,b^{10}}{17}+\frac{10\,B\,a\,b^9}{17}\right)+\frac{A\,a^{10}\,x^7}{7}+\frac{B\,b^{10}\,x^{18}}{18}+\frac{3\,a^7\,b^2\,x^{10}\,(8\,A\,b+3\,B\,a)}{2}+\frac{30\,a^6\,b^3\,x^{11}\,(7\,A\,b+4\,B\,a)}{11}+\frac{7\,a^5\,b^4\,x^{12}\,(6\,A\,b+5\,B\,a)}{2}+\frac{42\,a^4\,b^5\,x^{13}\,(5\,A\,b+6\,B\,a)}{13}+\frac{15\,a^3\,b^6\,x^{14}\,(4\,A\,b+7\,B\,a)}{7}+a^2\,b^7\,x^{15}\,(3\,A\,b+8\,B\,a)+\frac{5\,a^8\,b\,x^9\,(9\,A\,b+2\,B\,a)}{9}+\frac{5\,a\,b^8\,x^{16}\,(2\,A\,b+9\,B\,a)}{16}$$

input
```
int(x^6*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^8*((B*a^10)/8 + (5*A*a^9*b)/4) + x^17*((A*b^10)/17 + (10*B*a*b^9)/17) +
(A*a^10*x^7)/7 + (B*b^10*x^18)/18 + (3*a^7*b^2*x^10*(8*A*b + 3*B*a))/2 + (
30*a^6*b^3*x^11*(7*A*b + 4*B*a))/11 + (7*a^5*b^4*x^12*(6*A*b + 5*B*a))/2 +
 (42*a^4*b^5*x^13*(5*A*b + 6*B*a))/13 + (15*a^3*b^6*x^14*(4*A*b + 7*B*a))/
7 + a^2*b^7*x^15*(3*A*b + 8*B*a) + (5*a^8*b*x^9*(9*A*b + 2*B*a))/9 + (5*a*
b^8*x^16*(2*A*b + 9*B*a))/16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int x^6(a+bx)^{10}(A+Bx)\,dx$$
$$= \frac{x^7(12376b^{11}x^{11}+144144a\,b^{10}x^{10}+765765a^2b^9x^9+2450448a^3b^8x^8+5250960a^4b^7x^7+7916832a^5b^6x^6+}{222768}$$

input
```
int(x^6*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**7*(31824*a**11 + 306306*a**10*b*x + 1361360*a**9*b**2*x**2 + 3675672*a
**8*b**3*x**3 + 6683040*a**7*b**4*x**4 + 8576568*a**6*b**5*x**5 + 7916832*
a**5*b**6*x**6 + 5250960*a**4*b**7*x**7 + 2450448*a**3*b**8*x**8 + 765765*
a**2*b**9*x**9 + 144144*a*b**10*x**10 + 12376*b**11*x**11))/222768
```

3.111 $\int x^5(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 163

$$\begin{aligned}\int x^5(a+bx)^{10}(A+Bx)\,dx = &-\frac{a^5(Ab-aB)(a+bx)^{11}}{11b^7}+\frac{a^4(5Ab-6aB)(a+bx)^{12}}{12b^7}\\ &-\frac{5a^3(2Ab-3aB)(a+bx)^{13}}{13b^7}\\ &+\frac{5a^2(Ab-2aB)(a+bx)^{14}}{7b^7}-\frac{a(Ab-3aB)(a+bx)^{15}}{3b^7}\\ &+\frac{(Ab-6aB)(a+bx)^{16}}{16b^7}+\frac{B(a+bx)^{17}}{17b^7}\end{aligned}$$

output
```
-1/11*a^5*(A*b-B*a)*(b*x+a)^11/b^7+1/12*a^4*(5*A*b-6*B*a)*(b*x+a)^12/b^7-5
/13*a^3*(2*A*b-3*B*a)*(b*x+a)^13/b^7+5/7*a^2*(A*b-2*B*a)*(b*x+a)^14/b^7-1/
3*a*(A*b-3*B*a)*(b*x+a)^15/b^7+1/16*(A*b-6*B*a)*(b*x+a)^16/b^7+1/17*B*(b*x
+a)^17/b^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.40

$$\int x^5(a+bx)^{10}(A+Bx)\,dx = \frac{1}{6}a^{10}Ax^6+\frac{1}{7}a^9(10Ab+aB)x^7+\frac{5}{8}a^8b(9Ab+2aB)x^8+\frac{5}{3}a^7b^2(8Ab+3aB)x^9+3a^6b^3(7Ab+4aB)x^{10}+\frac{42}{11}a^5b^4(6Ab+5aB)x^{11}+\frac{7}{2}a^4b^5(5Ab+6aB)x^{12}+\frac{30}{13}a^3b^6(4Ab+7aB)x^{13}+\frac{15}{14}a^2b^7(3Ab+8aB)x^{14}+\frac{1}{3}ab^8(2Ab+9aB)x^{15}+\frac{1}{16}b^9(Ab+10aB)x^{16}+\frac{1}{17}b^{10}Bx^{17}$$

input
```
Integrate[x^5*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^6)/6 + (a^9*(10*A*b + a*B)*x^7)/7 + (5*a^8*b*(9*A*b + 2*a*B)*x^8
)/8 + (5*a^7*b^2*(8*A*b + 3*a*B)*x^9)/3 + 3*a^6*b^3*(7*A*b + 4*a*B)*x^10 +
 (42*a^5*b^4*(6*A*b + 5*a*B)*x^11)/11 + (7*a^4*b^5*(5*A*b + 6*a*B)*x^12)/2
 + (30*a^3*b^6*(4*A*b + 7*a*B)*x^13)/13 + (15*a^2*b^7*(3*A*b + 8*a*B)*x^14
)/14 + (a*b^8*(2*A*b + 9*a*B)*x^15)/3 + (b^9*(A*b + 10*a*B)*x^16)/16 + (b^
10*B*x^17)/17
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^5(a+bx)^{10}(aB-Ab)}{b^6} - \frac{a^4(a+bx)^{11}(6aB-5Ab)}{b^6} + \frac{5a^3(a+bx)^{12}(3aB-2Ab)}{b^6} - \frac{10a^2(a+bx)^{13}(2aB-}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5(a+bx)^{11}(Ab-aB)}{11b^7} + \frac{a^4(a+bx)^{12}(5Ab-6aB)}{12b^7} - \frac{5a^3(a+bx)^{13}(2Ab-3aB)}{13b^7} + \frac{5a^2(a+bx)^{14}(Ab-2aB)}{7b^7} + \frac{(a+bx)^{16}(Ab-6aB)}{16b^7} - \frac{a(a+bx)^{15}(Ab-3aB)}{3b^7} + \frac{B(a+bx)^{17}}{17b^7}$$

input
```
Int[x^5*(a + b*x)^10*(A + B*x),x]
```

output
```
-1/11*(a^5*(A*b - a*B)*(a + b*x)^11)/b^7 + (a^4*(5*A*b - 6*a*B)*(a + b*x)^
12)/(12*b^7) - (5*a^3*(2*A*b - 3*a*B)*(a + b*x)^13)/(13*b^7) + (5*a^2*(A*b
 - 2*a*B)*(a + b*x)^14)/(7*b^7) - (a*(A*b - 3*a*B)*(a + b*x)^15)/(3*b^7) +
 ((A*b - 6*a*B)*(a + b*x)^16)/(16*b^7) + (B*(a + b*x)^17)/(17*b^7)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.45

method	result
norman	$\frac{a^{10}Ax^6}{6}+\left(\frac{10}{7}a^9bA+\frac{1}{7}a^{10}B\right)x^7+\left(\frac{45}{8}a^8b^2A+\frac{5}{4}a^9bB\right)x^8+\left(\frac{40}{3}a^7b^3A+5a^8b^2B\right)x^9+(21a^6$
default	$\frac{b^{10}Bx^{17}}{17}+\frac{(b^{10}A+10ab^9B)x^{16}}{16}+\frac{(10ab^9A+45a^2b^8B)x^{15}}{15}+\frac{(45a^2b^8A+120a^3b^7B)x^{14}}{14}+\frac{(120a^3b^7A+210a^4b^6B)x}{13}$
orering	$\frac{x^6(48048Bb^{10}x^{11}+51051Ab^{10}x^{10}+510510Bab^9x^{10}+544544aAb^9x^9+2450448Ba^2b^8x^9+2625480a^2Ab^8x^8+7001280Ba^3}{}$
gosper	$\frac{1}{6}a^{10}Ax^6+\frac{10}{7}x^7a^9bA+\frac{1}{7}x^7a^{10}B+\frac{45}{8}x^8a^8b^2A+\frac{5}{4}x^8a^9bB+\frac{40}{3}x^9a^7b^3A+5x^9a^8b^2B+21A$
risch	$\frac{1}{6}a^{10}Ax^6+\frac{10}{7}x^7a^9bA+\frac{1}{7}x^7a^{10}B+\frac{45}{8}x^8a^8b^2A+\frac{5}{4}x^8a^9bB+\frac{40}{3}x^9a^7b^3A+5x^9a^8b^2B+21A$
parallelrisch	$\frac{1}{6}a^{10}Ax^6+\frac{10}{7}x^7a^9bA+\frac{1}{7}x^7a^{10}B+\frac{45}{8}x^8a^8b^2A+\frac{5}{4}x^8a^9bB+\frac{40}{3}x^9a^7b^3A+5x^9a^8b^2B+21A$

input
```
int(x^5*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*a^10*A*x^6+(10/7*a^9*b*A+1/7*a^10*B)*x^7+(45/8*a^8*b^2*A+5/4*a^9*b*B)*
x^8+(40/3*a^7*b^3*A+5*a^8*b^2*B)*x^9+(21*A*a^6*b^4+12*B*a^7*b^3)*x^10+(252
/11*a^5*b^5*A+210/11*a^6*b^4*B)*x^11+(35/2*a^4*b^6*A+21*a^5*b^5*B)*x^12+(1
20/13*a^3*b^7*A+210/13*a^4*b^6*B)*x^13+(45/14*a^2*b^8*A+60/7*a^3*b^7*B)*x^
14+(2/3*a*b^9*A+3*a^2*b^8*B)*x^15+(1/16*b^10*A+5/8*a*b^9*B)*x^16+1/17*b^10
*B*x^17
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.49

$$\begin{aligned}\int x^5(a+bx)^{10}(A+Bx)\,dx=&\ \frac{1}{17}Bb^{10}x^{17}+\frac{1}{6}Aa^{10}x^6+\frac{1}{16}\left(10\,Bab^9+Ab^{10}\right)x^{16}\\&+\frac{1}{3}\left(9\,Ba^2b^8+2\,Aab^9\right)x^{15}+\frac{15}{14}\left(8\,Ba^3b^7+3\,Aa^2b^8\right)x^{14}\\&+\frac{30}{13}\left(7\,Ba^4b^6+4\,Aa^3b^7\right)x^{13}\\&+\frac{7}{2}\left(6\,Ba^5b^5+5\,Aa^4b^6\right)x^{12}+\frac{42}{11}\left(5\,Ba^6b^4+6\,Aa^5b^5\right)x^{11}\\&+3\left(4\,Ba^7b^3+7\,Aa^6b^4\right)x^{10}+\frac{5}{3}\left(3\,Ba^8b^2+8\,Aa^7b^3\right)x^9\\&+\frac{5}{8}\left(2\,Ba^9b+9\,Aa^8b^2\right)x^8+\frac{1}{7}\left(Ba^{10}+10\,Aa^9b\right)x^7\end{aligned}$$

input
```
integrate(x^5*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/17*B*b^10*x^17 + 1/6*A*a^10*x^6 + 1/16*(10*B*a*b^9 + A*b^10)*x^16 + 1/3*
(9*B*a^2*b^8 + 2*A*a*b^9)*x^15 + 15/14*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^14 +
30/13*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^13 + 7/2*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x
^12 + 42/11*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^11 + 3*(4*B*a^7*b^3 + 7*A*a^6*b^
4)*x^10 + 5/3*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^9 + 5/8*(2*B*a^9*b + 9*A*a^8*b
^2)*x^8 + 1/7*(B*a^10 + 10*A*a^9*b)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.63

$$\int x^5(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^6}{6} + \frac{Bb^{10}x^{17}}{17} + x^{16}\left(\frac{Ab^{10}}{16} + \frac{5Bab^9}{8}\right) + x^{15} \cdot \left(\frac{2Aab^9}{3} + 3Ba^2b^8\right) + x^{14} \cdot \left(\frac{45Aa^2b^8}{14} + \frac{60Ba^3b^7}{7}\right) + x^{13} \cdot \left(\frac{120Aa^3b^7}{13} + \frac{210Ba^4b^6}{13}\right) + x^{12} \cdot \left(\frac{35Aa^4b^6}{2} + 21Ba^5b^5\right) + x^{11} \cdot \left(\frac{252Aa^5b^5}{11} + \frac{210Ba^6b^4}{11}\right) + x^{10} \cdot \left(21Aa^6b^4 + 12Ba^7b^3\right) + x^9 \cdot \left(\frac{40Aa^7b^3}{3} + 5Ba^8b^2\right) + x^8 \cdot \left(\frac{45Aa^8b^2}{8} + \frac{5Ba^9b}{4}\right) + x^7 \cdot \left(\frac{10Aa^9b}{7} + \frac{Ba^{10}}{7}\right)$$

input
```
integrate(x**5*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**6/6 + B*b**10*x**17/17 + x**16*(A*b**10/16 + 5*B*a*b**9/8) + x*
*15*(2*A*a*b**9/3 + 3*B*a**2*b**8) + x**14*(45*A*a**2*b**8/14 + 60*B*a**3*
b**7/7) + x**13*(120*A*a**3*b**7/13 + 210*B*a**4*b**6/13) + x**12*(35*A*a*
*4*b**6/2 + 21*B*a**5*b**5) + x**11*(252*A*a**5*b**5/11 + 210*B*a**6*b**4/
11) + x**10*(21*A*a**6*b**4 + 12*B*a**7*b**3) + x**9*(40*A*a**7*b**3/3 + 5
*B*a**8*b**2) + x**8*(45*A*a**8*b**2/8 + 5*B*a**9*b/4) + x**7*(10*A*a**9*b
/7 + B*a**10/7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.49

$$\int x^5(a+bx)^{10}(A+Bx)\,dx = \frac{1}{17}\,Bb^{10}x^{17} + \frac{1}{6}\,Aa^{10}x^6 + \frac{1}{16}\,(10\,Bab^9 + Ab^{10})x^{16} + \frac{1}{3}\,(9\,Ba^2b^8 + 2\,Aab^9)x^{15} + \frac{15}{14}\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^{14} + \frac{30}{13}\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^{13} + \frac{7}{2}\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^{12} + \frac{42}{11}\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^{11} + 3\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^{10} + \frac{5}{3}\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^9 + \frac{5}{8}\,(2\,Ba^9b + 9\,Aa^8b^2)x^8 + \frac{1}{7}\,(Ba^{10} + 10\,Aa^9b)x^7$$

input

```
integrate(x^5*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output

```
1/17*B*b^10*x^17 + 1/6*A*a^10*x^6 + 1/16*(10*B*a*b^9 + A*b^10)*x^16 + 1/3*
(9*B*a^2*b^8 + 2*A*a*b^9)*x^15 + 15/14*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^14 +
30/13*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^13 + 7/2*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x
^12 + 42/11*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^11 + 3*(4*B*a^7*b^3 + 7*A*a^6*b^
4)*x^10 + 5/3*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^9 + 5/8*(2*B*a^9*b + 9*A*a^8*b
^2)*x^8 + 1/7*(B*a^10 + 10*A*a^9*b)*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.50

$$\int x^5(a+bx)^{10}(A+Bx)\,dx = \frac{1}{17}\,Bb^{10}x^{17} + \frac{5}{8}\,Bab^9x^{16} + \frac{1}{16}\,Ab^{10}x^{16} + 3\,Ba^2b^8x^{15} + \frac{2}{3}\,Aab^9x^{15} + \frac{60}{7}\,Ba^3b^7x^{14} + \frac{45}{14}\,Aa^2b^8x^{14} + \frac{210}{13}\,Ba^4b^6x^{13} + \frac{120}{13}\,Aa^3b^7x^{13} + 21\,Ba^5b^5x^{12} + \frac{35}{2}\,Aa^4b^6x^{12} + \frac{210}{11}\,Ba^6b^4x^{11} + \frac{252}{11}\,Aa^5b^5x^{11} + 12\,Ba^7b^3x^{10} + 21\,Aa^6b^4x^{10} + 5\,Ba^8b^2x^9 + \frac{40}{3}\,Aa^7b^3x^9 + \frac{5}{4}\,Ba^9bx^8 + \frac{45}{8}\,Aa^8b^2x^8 + \frac{1}{7}\,Ba^{10}x^7 + \frac{10}{7}\,Aa^9bx^7 + \frac{1}{6}\,Aa^{10}x^6$$

input
```
integrate(x^5*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/17*B*b^10*x^17 + 5/8*B*a*b^9*x^16 + 1/16*A*b^10*x^16 + 3*B*a^2*b^8*x^15
+ 2/3*A*a*b^9*x^15 + 60/7*B*a^3*b^7*x^14 + 45/14*A*a^2*b^8*x^14 + 210/13*B
*a^4*b^6*x^13 + 120/13*A*a^3*b^7*x^13 + 21*B*a^5*b^5*x^12 + 35/2*A*a^4*b^6
*x^12 + 210/11*B*a^6*b^4*x^11 + 252/11*A*a^5*b^5*x^11 + 12*B*a^7*b^3*x^10
+ 21*A*a^6*b^4*x^10 + 5*B*a^8*b^2*x^9 + 40/3*A*a^7*b^3*x^9 + 5/4*B*a^9*b*x
^8 + 45/8*A*a^8*b^2*x^8 + 1/7*B*a^10*x^7 + 10/7*A*a^9*b*x^7 + 1/6*A*a^10*x
^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int x^5(a+bx)^{10}(A+Bx)\,dx = x^7\left(\frac{B\,a^{10}}{7}+\frac{10\,A\,b\,a^9}{7}\right)+x^{16}\left(\frac{A\,b^{10}}{16}+\frac{5\,B\,a\,b^9}{8}\right)+\frac{A\,a^{10}\,x^6}{6}+\frac{B\,b^{10}\,x^{17}}{17}+\frac{5\,a^7\,b^2\,x^9\,(8\,A\,b+3\,B\,a)}{3}+3\,a^6\,b^3\,x^{10}\,(7\,A\,b+4\,B\,a)+\frac{42\,a^5\,b^4\,x^{11}\,(6\,A\,b+5\,B\,a)}{11}+\frac{7\,a^4\,b^5\,x^{12}\,(5\,A\,b+6\,B\,a)}{2}+\frac{30\,a^3\,b^6\,x^{13}\,(4\,A\,b+7\,B\,a)}{13}+\frac{15\,a^2\,b^7\,x^{14}\,(3\,A\,b+8\,B\,a)}{14}+\frac{5\,a^8\,b\,x^8\,(9\,A\,b+2\,B\,a)}{8}+\frac{a\,b^8\,x^{15}\,(2\,A\,b+9\,B\,a)}{3}$$

input
```
int(x^5*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^7*((B*a^10)/7 + (10*A*a^9*b)/7) + x^16*((A*b^10)/16 + (5*B*a*b^9)/8) + (
A*a^10*x^6)/6 + (B*b^10*x^17)/17 + (5*a^7*b^2*x^9*(8*A*b + 3*B*a))/3 + 3*a
^6*b^3*x^10*(7*A*b + 4*B*a) + (42*a^5*b^4*x^11*(6*A*b + 5*B*a))/11 + (7*a^
4*b^5*x^12*(5*A*b + 6*B*a))/2 + (30*a^3*b^6*x^13*(4*A*b + 7*B*a))/13 + (15
*a^2*b^7*x^14*(3*A*b + 8*B*a))/14 + (5*a^8*b*x^8*(9*A*b + 2*B*a))/8 + (a*b
^8*x^15*(2*A*b + 9*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int x^5(a+bx)^{10}(A+Bx)\,dx$$
$$=\frac{x^6(4368b^{11}x^{11}+51051a\,b^{10}x^{10}+272272a^2b^9x^9+875160a^3b^8x^8+1884960a^4b^7x^7+2858856a^5b^6x^6+31}{74256}$$

input
```
int(x^5*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**6*(12376*a**11 + 116688*a**10*b*x + 510510*a**9*b**2*x**2 + 1361360*a*
*8*b**3*x**3 + 2450448*a**7*b**4*x**4 + 3118752*a**6*b**5*x**5 + 2858856*a
**5*b**6*x**6 + 1884960*a**4*b**7*x**7 + 875160*a**3*b**8*x**8 + 272272*a*
*2*b**9*x**9 + 51051*a*b**10*x**10 + 4368*b**11*x**11))/74256
```

3.112 $\int x^4(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int x^4(a+bx)^{10}(A+Bx)\,dx = \frac{a^4(Ab-aB)(a+bx)^{11}}{11b^6}-\frac{a^3(4Ab-5aB)(a+bx)^{12}}{12b^6}+\frac{2a^2(3Ab-5aB)(a+bx)^{13}}{13b^6}-\frac{a(2Ab-5aB)(a+bx)^{14}}{7b^6}+\frac{(Ab-5aB)(a+bx)^{15}}{15b^6}+\frac{B(a+bx)^{16}}{16b^6}$$

output
```
1/11*a^4*(A*b-B*a)*(b*x+a)^11/b^6-1/12*a^3*(4*A*b-5*B*a)*(b*x+a)^12/b^6+2/
13*a^2*(3*A*b-5*B*a)*(b*x+a)^13/b^6-1/7*a*(2*A*b-5*B*a)*(b*x+a)^14/b^6+1/1
5*(A*b-5*B*a)*(b*x+a)^15/b^6+1/16*B*(b*x+a)^16/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.66

$$\int x^4(a+bx)^{10}(A+Bx)\,dx = \frac{1}{5}a^{10}Ax^5+\frac{1}{6}a^9(10Ab+aB)x^6+\frac{5}{7}a^8b(9Ab+2aB)x^7+\frac{15}{8}a^7b^2(8Ab+3aB)x^8+\frac{10}{3}a^6b^3(7Ab+4aB)x^9+\frac{21}{5}a^5b^4(6Ab+5aB)x^{10}+\frac{42}{11}a^4b^5(5Ab+6aB)x^{11}+\frac{5}{2}a^3b^6(4Ab+7aB)x^{12}+\frac{15}{13}a^2b^7(3Ab+8aB)x^{13}+\frac{5}{14}ab^8(2Ab+9aB)x^{14}+\frac{1}{15}b^9(Ab+10aB)x^{15}+\frac{1}{16}b^{10}Bx^{16}$$

input
```
Integrate[x^4*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^5)/5 + (a^9*(10*A*b + a*B)*x^6)/6 + (5*a^8*b*(9*A*b + 2*a*B)*x^7
)/7 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^8)/8 + (10*a^6*b^3*(7*A*b + 4*a*B)*x^9
)/3 + (21*a^5*b^4*(6*A*b + 5*a*B)*x^10)/5 + (42*a^4*b^5*(5*A*b + 6*a*B)*x^
11)/11 + (5*a^3*b^6*(4*A*b + 7*a*B)*x^12)/2 + (15*a^2*b^7*(3*A*b + 8*a*B)*
x^13)/13 + (5*a*b^8*(2*A*b + 9*a*B)*x^14)/14 + (b^9*(A*b + 10*a*B)*x^15)/1
5 + (b^10*B*x^16)/16
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(-\frac{a^4(a+bx)^{10}(aB-Ab)}{b^5} + \frac{a^3(a+bx)^{11}(5aB-4Ab)}{b^5} - \frac{2a^2(a+bx)^{12}(5aB-3Ab)}{b^5} + \frac{(a+bx)^{14}(Ab-5a\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4(a+bx)^{11}(Ab-aB)}{11b^6} - \frac{a^3(a+bx)^{12}(4Ab-5aB)}{12b^6} + \frac{2a^2(a+bx)^{13}(3Ab-5aB)}{13b^6} + \frac{(a+bx)^{15}(Ab-5aB)}{15b^6} - \frac{a(a+bx)^{14}(2Ab-5aB)}{7b^6} + \frac{B(a+bx)^{16}}{16b^6}$$

input
```
Int[x^4*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^4*(A*b - a*B)*(a + b*x)^11)/(11*b^6) - (a^3*(4*A*b - 5*a*B)*(a + b*x)^1
2)/(12*b^6) + (2*a^2*(3*A*b - 5*a*B)*(a + b*x)^13)/(13*b^6) - (a*(2*A*b -
5*a*B)*(a + b*x)^14)/(7*b^6) + ((A*b - 5*a*B)*(a + b*x)^15)/(15*b^6) + (B*
(a + b*x)^16)/(16*b^6)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.70

method	result
norman	$\frac{a^{10}A\,x^5}{5}+\left(\frac{5}{3}a^9bA+\frac{1}{6}a^{10}B\right)x^6+\left(\frac{45}{7}a^8b^2A+\frac{10}{7}a^9bB\right)x^7+\left(15a^7b^3A+\frac{45}{8}a^8b^2B\right)x^8+\left(\frac{70}{3}a\right.$
default	$\frac{b^{10}B\,x^{16}}{16}+\frac{(b^{10}A+10a\,b^9B)x^{15}}{15}+\frac{(10a\,b^9A+45a^2b^8B)x^{14}}{14}+\frac{(45a^2b^8A+120a^3b^7B)x^{13}}{13}+\frac{(120a^3b^7A+210a^4b^6B)x}{12}$
orering	$\frac{x^5(15015B\,b^{10}x^{11}+16016A\,b^{10}x^{10}+160160Ba\,b^9x^{10}+171600aA\,b^9x^9+772200B\,a^2b^8x^9+831600a^2A\,b^8x^8+2217600B\,a^3b^7}{}$
gosper	$\frac{1}{5}a^{10}A\,x^5+\frac{5}{3}x^6a^9bA+\frac{1}{6}x^6a^{10}B+\frac{45}{7}x^7a^8b^2A+\frac{10}{7}x^7a^9bB+15x^8a^7b^3A+\frac{45}{8}x^8a^8b^2B+\frac{70}{3}$
risch	$\frac{1}{5}a^{10}A\,x^5+\frac{5}{3}x^6a^9bA+\frac{1}{6}x^6a^{10}B+\frac{45}{7}x^7a^8b^2A+\frac{10}{7}x^7a^9bB+15x^8a^7b^3A+\frac{45}{8}x^8a^8b^2B+\frac{70}{3}$
parallelrisch	$\frac{1}{5}a^{10}A\,x^5+\frac{5}{3}x^6a^9bA+\frac{1}{6}x^6a^{10}B+\frac{45}{7}x^7a^8b^2A+\frac{10}{7}x^7a^9bB+15x^8a^7b^3A+\frac{45}{8}x^8a^8b^2B+\frac{70}{3}$

input
```
int(x^4*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^10*A*x^5+(5/3*a^9*b*A+1/6*a^10*B)*x^6+(45/7*a^8*b^2*A+10/7*a^9*b*B)*
x^7+(15*a^7*b^3*A+45/8*a^8*b^2*B)*x^8+(70/3*a^6*b^4*A+40/3*a^7*b^3*B)*x^9+
(126/5*a^5*b^5*A+21*a^6*b^4*B)*x^10+(210/11*a^4*b^6*A+252/11*a^5*b^5*B)*x^
11+(10*a^3*b^7*A+35/2*a^4*b^6*B)*x^12+(45/13*a^2*b^8*A+120/13*a^3*b^7*B)*x
^13+(5/7*a*b^9*A+45/14*a^2*b^8*B)*x^14+(1/15*b^10*A+2/3*a*b^9*B)*x^15+1/16
*b^10*B*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.75

$$\int x^4(a+bx)^{10}(A+Bx)\,dx=\frac{1}{16}\,Bb^{10}x^{16}+\frac{1}{5}\,Aa^{10}x^5+\frac{1}{15}\,(10\,Bab^9+Ab^{10})x^{15}+\frac{5}{14}\,(9\,Ba^2b^8+2\,Aab^9)x^{14}+\frac{15}{13}\,(8\,Ba^3b^7+3\,Aa^2b^8)x^{13}+\frac{5}{2}\,(7\,Ba^4b^6+4\,Aa^3b^7)x^{12}+\frac{42}{11}\,(6\,Ba^5b^5+5\,Aa^4b^6)x^{11}+\frac{21}{5}\,(5\,Ba^6b^4+6\,Aa^5b^5)x^{10}+\frac{10}{3}\,(4\,Ba^7b^3+7\,Aa^6b^4)x^9+\frac{15}{8}\,(3\,Ba^8b^2+8\,Aa^7b^3)x^8+\frac{5}{7}\,(2\,Ba^9b+9\,Aa^8b^2)x^7+\frac{1}{6}\,(Ba^{10}+10\,Aa^9b)x^6$$

input
```
integrate(x^4*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/16*B*b^10*x^16 + 1/5*A*a^10*x^5 + 1/15*(10*B*a*b^9 + A*b^10)*x^15 + 5/14
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^14 + 15/13*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^13 +
 5/2*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^12 + 42/11*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^11 + 21/5*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^10 + 10/3*(4*B*a^7*b^3 + 7*A*a^6
*b^4)*x^9 + 15/8*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^8 + 5/7*(2*B*a^9*b + 9*A*a^
8*b^2)*x^7 + 1/6*(B*a^10 + 10*A*a^9*b)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(131) = 262$.

Time = 0.05 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.94

$$\int x^4(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^5}{5} + \frac{Bb^{10}x^{16}}{16} + x^{15}\left(\frac{Ab^{10}}{15} + \frac{2Bab^9}{3}\right) + x^{14} \cdot \left(\frac{5Aab^9}{7} + \frac{45Ba^2b^8}{14}\right) + x^{13} \cdot \left(\frac{45Aa^2b^8}{13} + \frac{120Ba^3b^7}{13}\right) + x^{12} \cdot \left(10Aa^3b^7 + \frac{35Ba^4b^6}{2}\right) + x^{11} \cdot \left(\frac{210Aa^4b^6}{11} + \frac{252Ba^5b^5}{11}\right) + x^{10} \cdot \left(\frac{126Aa^5b^5}{5} + 21Ba^6b^4\right) + x^9 \cdot \left(\frac{70Aa^6b^4}{3} + \frac{40Ba^7b^3}{3}\right) + x^8 \cdot \left(15Aa^7b^3 + \frac{45Ba^8b^2}{8}\right) + x^7 \cdot \left(\frac{45Aa^8b^2}{7} + \frac{10Ba^9b}{7}\right) + x^6 \cdot \left(\frac{5Aa^9b}{3} + \frac{Ba^{10}}{6}\right)$$

input
```
integrate(x**4*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**5/5 + B*b**10*x**16/16 + x**15*(A*b**10/15 + 2*B*a*b**9/3) + x*
*14*(5*A*a*b**9/7 + 45*B*a**2*b**8/14) + x**13*(45*A*a**2*b**8/13 + 120*B*
a**3*b**7/13) + x**12*(10*A*a**3*b**7 + 35*B*a**4*b**6/2) + x**11*(210*A*a
**4*b**6/11 + 252*B*a**5*b**5/11) + x**10*(126*A*a**5*b**5/5 + 21*B*a**6*b
**4) + x**9*(70*A*a**6*b**4/3 + 40*B*a**7*b**3/3) + x**8*(15*A*a**7*b**3 +
 45*B*a**8*b**2/8) + x**7*(45*A*a**8*b**2/7 + 10*B*a**9*b/7) + x**6*(5*A*a
**9*b/3 + B*a**10/6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.75

$$\int x^4(a+bx)^{10}(A+Bx)\,dx = \frac{1}{16}\,Bb^{10}x^{16} + \frac{1}{5}\,Aa^{10}x^5 + \frac{1}{15}\,(10\,Bab^9 + Ab^{10})x^{15} + \frac{5}{14}\,(9\,Ba^2b^8 + 2\,Aab^9)x^{14} + \frac{15}{13}\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^{13} + \frac{5}{2}\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^{12} + \frac{42}{11}\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^{11} + \frac{21}{5}\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^{10} + \frac{10}{3}\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^9 + \frac{15}{8}\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^8 + \frac{5}{7}\,(2\,Ba^9b + 9\,Aa^8b^2)x^7 + \frac{1}{6}\,(Ba^{10} + 10\,Aa^9b)x^6$$

input
```
integrate(x^4*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/16*B*b^10*x^16 + 1/5*A*a^10*x^5 + 1/15*(10*B*a*b^9 + A*b^10)*x^15 + 5/14
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^14 + 15/13*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^13 +
 5/2*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^12 + 42/11*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^11 + 21/5*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^10 + 10/3*(4*B*a^7*b^3 + 7*A*a^6
*b^4)*x^9 + 15/8*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^8 + 5/7*(2*B*a^9*b + 9*A*a^
8*b^2)*x^7 + 1/6*(B*a^10 + 10*A*a^9*b)*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.76

$$\int x^4(a+bx)^{10}(A+Bx)\,dx = \frac{1}{16}\,Bb^{10}x^{16} + \frac{2}{3}\,Bab^9x^{15} + \frac{1}{15}\,Ab^{10}x^{15} + \frac{45}{14}\,Ba^2b^8x^{14} + \frac{5}{7}\,Aab^9x^{14} + \frac{120}{13}\,Ba^3b^7x^{13} + \frac{45}{13}\,Aa^2b^8x^{13} + \frac{35}{2}\,Ba^4b^6x^{12} + 10\,Aa^3b^7x^{12} + \frac{252}{11}\,Ba^5b^5x^{11} + \frac{210}{11}\,Aa^4b^6x^{11} + 21\,Ba^6b^4x^{10} + \frac{126}{5}\,Aa^5b^5x^{10} + \frac{40}{3}\,Ba^7b^3x^9 + \frac{70}{3}\,Aa^6b^4x^9 + \frac{45}{8}\,Ba^8b^2x^8 + 15\,Aa^7b^3x^8 + \frac{10}{7}\,Ba^9bx^7 + \frac{45}{7}\,Aa^8b^2x^7 + \frac{1}{6}\,Ba^{10}x^6 + \frac{5}{3}\,Aa^9bx^6 + \frac{1}{5}\,Aa^{10}x^5$$

input
```
integrate(x^4*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/16*B*b^10*x^16 + 2/3*B*a*b^9*x^15 + 1/15*A*b^10*x^15 + 45/14*B*a^2*b^8*x
^14 + 5/7*A*a*b^9*x^14 + 120/13*B*a^3*b^7*x^13 + 45/13*A*a^2*b^8*x^13 + 35
/2*B*a^4*b^6*x^12 + 10*A*a^3*b^7*x^12 + 252/11*B*a^5*b^5*x^11 + 210/11*A*a
^4*b^6*x^11 + 21*B*a^6*b^4*x^10 + 126/5*A*a^5*b^5*x^10 + 40/3*B*a^7*b^3*x^
9 + 70/3*A*a^6*b^4*x^9 + 45/8*B*a^8*b^2*x^8 + 15*A*a^7*b^3*x^8 + 10/7*B*a^
9*b*x^7 + 45/7*A*a^8*b^2*x^7 + 1/6*B*a^10*x^6 + 5/3*A*a^9*b*x^6 + 1/5*A*a^
10*x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.52

$$\int x^4(a+bx)^{10}(A+Bx)\,dx = x^6\left(\frac{B\,a^{10}}{6}+\frac{5\,A\,b\,a^9}{3}\right)+x^{15}\left(\frac{A\,b^{10}}{15}+\frac{2\,B\,a\,b^9}{3}\right)+\frac{A\,a^{10}\,x^5}{5}+\frac{B\,b^{10}\,x^{16}}{16}+\frac{15\,a^7\,b^2\,x^8\,(8\,A\,b+3\,B\,a)}{8}+\frac{10\,a^6\,b^3\,x^9\,(7\,A\,b+4\,B\,a)}{3}+\frac{21\,a^5\,b^4\,x^{10}\,(6\,A\,b+5\,B\,a)}{5}+\frac{42\,a^4\,b^5\,x^{11}\,(5\,A\,b+6\,B\,a)}{11}+\frac{5\,a^3\,b^6\,x^{12}\,(4\,A\,b+7\,B\,a)}{2}+\frac{15\,a^2\,b^7\,x^{13}\,(3\,A\,b+8\,B\,a)}{13}+\frac{5\,a^8\,b\,x^7\,(9\,A\,b+2\,B\,a)}{7}+\frac{5\,a\,b^8\,x^{14}\,(2\,A\,b+9\,B\,a)}{14}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^6*((B*a^10)/6 + (5*A*a^9*b)/3) + x^15*((A*b^10)/15 + (2*B*a*b^9)/3) + (A
*a^10*x^5)/5 + (B*b^10*x^16)/16 + (15*a^7*b^2*x^8*(8*A*b + 3*B*a))/8 + (10
*a^6*b^3*x^9*(7*A*b + 4*B*a))/3 + (21*a^5*b^4*x^10*(6*A*b + 5*B*a))/5 + (4
2*a^4*b^5*x^11*(5*A*b + 6*B*a))/11 + (5*a^3*b^6*x^12*(4*A*b + 7*B*a))/2 +
(15*a^2*b^7*x^13*(3*A*b + 8*B*a))/13 + (5*a^8*b*x^7*(9*A*b + 2*B*a))/7 + (
5*a*b^8*x^14*(2*A*b + 9*B*a))/14
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int x^4(a+bx)^{10}(A+Bx)\,dx$$
$$= \frac{x^5(1365b^{11}x^{11}+16016a\,b^{10}x^{10}+85800a^2b^9x^9+277200a^3b^8x^8+600600a^4b^7x^7+917280a^5b^6x^6+10090}{21840}$$

input

```
int(x^4*(b*x+a)^10*(B*x+A),x)
```

output

```
(x**5*(4368*a**11 + 40040*a**10*b*x + 171600*a**9*b**2*x**2 + 450450*a**8*
b**3*x**3 + 800800*a**7*b**4*x**4 + 1009008*a**6*b**5*x**5 + 917280*a**5*b
**6*x**6 + 600600*a**4*b**7*x**7 + 277200*a**3*b**8*x**8 + 85800*a**2*b**9
*x**9 + 16016*a*b**10*x**10 + 1365*b**11*x**11))/21840
```

3.113 $\int x^3(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = -\frac{a^3(Ab-aB)(a+bx)^{11}}{11b^5}+\frac{a^2(3Ab-4aB)(a+bx)^{12}}{12b^5}-\frac{3a(Ab-2aB)(a+bx)^{13}}{13b^5}+\frac{(Ab-4aB)(a+bx)^{14}}{14b^5}+\frac{B(a+bx)^{15}}{15b^5}$$

output
```
-1/11*a^3*(A*b-B*a)*(b*x+a)^11/b^5+1/12*a^2*(3*A*b-4*B*a)*(b*x+a)^12/b^5-3
/13*a*(A*b-2*B*a)*(b*x+a)^13/b^5+1/14*(A*b-4*B*a)*(b*x+a)^14/b^5+1/15*B*(b
*x+a)^15/b^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 231 vs. 2(112) = 224.

Time = 0.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.06

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = \frac{1}{4}a^{10}Ax^4 + \frac{1}{5}a^9(10Ab+aB)x^5 + \frac{5}{6}a^8b(9Ab+2aB)x^6 + \frac{15}{7}a^7b^2(8Ab+3aB)x^7 + \frac{15}{4}a^6b^3(7Ab+4aB)x^8 + \frac{14}{3}a^5b^4(6Ab+5aB)x^9 + \frac{21}{5}a^4b^5(5Ab+6aB)x^{10} + \frac{30}{11}a^3b^6(4Ab+7aB)x^{11} + \frac{5}{4}a^2b^7(3Ab+8aB)x^{12} + \frac{5}{13}ab^8(2Ab+9aB)x^{13} + \frac{1}{14}b^9(Ab+10aB)x^{14} + \frac{1}{15}b^{10}Bx^{15}$$

input
```
Integrate[x^3*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^4)/4 + (a^9*(10*A*b + a*B)*x^5)/5 + (5*a^8*b*(9*A*b + 2*a*B)*x^6
)/6 + (15*a^7*b^2*(8*A*b + 3*a*B)*x^7)/7 + (15*a^6*b^3*(7*A*b + 4*a*B)*x^8
)/4 + (14*a^5*b^4*(6*A*b + 5*a*B)*x^9)/3 + (21*a^4*b^5*(5*A*b + 6*a*B)*x^1
0)/5 + (30*a^3*b^6*(4*A*b + 7*a*B)*x^11)/11 + (5*a^2*b^7*(3*A*b + 8*a*B)*x
^12)/4 + (5*a*b^8*(2*A*b + 9*a*B)*x^13)/13 + (b^9*(A*b + 10*a*B)*x^14)/14
+ (b^10*B*x^15)/15
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^3(a+bx)^{10}(aB-Ab)}{b^4} - \frac{a^2(a+bx)^{11}(4aB-3Ab)}{b^4} + \frac{(a+bx)^{13}(Ab-4aB)}{b^4} + \frac{3a(a+bx)^{12}(2aB-Ab)}{b^4} + \right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3(a+bx)^{11}(Ab-aB)}{11b^5} + \frac{a^2(a+bx)^{12}(3Ab-4aB)}{12b^5} + \frac{(a+bx)^{14}(Ab-4aB)}{14b^5} - \frac{3a(a+bx)^{13}(Ab-2aB)}{13b^5} + \frac{B(a+bx)^{15}}{15b^5}$$

input
```
Int[x^3*(a + b*x)^10*(A + B*x),x]
```

output
```
-1/11*(a^3*(A*b - a*B)*(a + b*x)^11)/b^5 + (a^2*(3*A*b - 4*a*B)*(a + b*x)^
12)/(12*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x)^13)/(13*b^5) + ((A*b - 4*a*B)*
(a + b*x)^14)/(14*b^5) + (B*(a + b*x)^15)/(15*b^5)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(102) = 204$.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.11

method	result
norman	$\frac{a^{10}Ax^4}{4}+\left(2a^9bA+\frac{1}{5}a^{10}B\right)x^5+\left(\frac{15}{2}a^8b^2A+\frac{5}{3}a^9bB\right)x^6+\left(\frac{120}{7}a^7b^3A+\frac{45}{7}a^8b^2B\right)x^7+\left(\frac{105}{4}\right.$
default	$\frac{b^{10}Bx^{15}}{15}+\frac{(b^{10}A+10a\,b^9B)x^{14}}{14}+\frac{(10a\,b^9A+45a^2b^8B)x^{13}}{13}+\frac{(45a^2b^8A+120a^3b^7B)x^{12}}{12}+\frac{(120a^3b^7A+210a^4b^6B)x}{11}$
orering	$\frac{x^4(4004Bb^{10}x^{11}+4290Ab^{10}x^{10}+42900Ba\,b^9x^{10}+46200aAb^9x^9+207900Ba^2b^8x^9+225225a^2Ab^8x^8+600600Ba^3b^7x^8+65}{}$
gosper	$\frac{1}{4}a^{10}Ax^4+2x^5a^9bA+\frac{1}{5}x^5a^{10}B+\frac{15}{2}x^6a^8b^2A+\frac{5}{3}x^6a^9bB+\frac{120}{7}x^7a^7b^3A+\frac{45}{7}x^7a^8b^2B+\frac{105}{4}$
risch	$\frac{1}{4}a^{10}Ax^4+2x^5a^9bA+\frac{1}{5}x^5a^{10}B+\frac{15}{2}x^6a^8b^2A+\frac{5}{3}x^6a^9bB+\frac{120}{7}x^7a^7b^3A+\frac{45}{7}x^7a^8b^2B+\frac{105}{4}$
parallelrisch	$\frac{1}{4}a^{10}Ax^4+2x^5a^9bA+\frac{1}{5}x^5a^{10}B+\frac{15}{2}x^6a^8b^2A+\frac{5}{3}x^6a^9bB+\frac{120}{7}x^7a^7b^3A+\frac{45}{7}x^7a^8b^2B+\frac{105}{4}$

input
```
int(x^3*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^10*A*x^4+(2*a^9*b*A+1/5*a^10*B)*x^5+(15/2*a^8*b^2*A+5/3*a^9*b*B)*x^6
+(120/7*a^7*b^3*A+45/7*a^8*b^2*B)*x^7+(105/4*a^6*b^4*A+15*a^7*b^3*B)*x^8+(
28*a^5*b^5*A+70/3*a^6*b^4*B)*x^9+(21*a^4*b^6*A+126/5*a^5*b^5*B)*x^10+(120/
11*a^3*b^7*A+210/11*a^4*b^6*B)*x^11+(15/4*a^2*b^8*A+10*a^3*b^7*B)*x^12+(10
/13*a*b^9*A+45/13*a^2*b^8*B)*x^13+(1/14*b^10*A+5/7*a*b^9*B)*x^14+1/15*b^10
*B*x^15
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(104) = 208.

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.17

$$\begin{aligned}\int x^3(a+bx)^{10}(A+Bx)\,dx &= \frac{1}{15}Bb^{10}x^{15}+\frac{1}{4}Aa^{10}x^4+\frac{1}{14}\left(10\,Bab^9+Ab^{10}\right)x^{14}\\&\quad+\frac{5}{13}\left(9\,Ba^2b^8+2\,Aab^9\right)x^{13}+\frac{5}{4}\left(8\,Ba^3b^7+3\,Aa^2b^8\right)x^{12}\\&\quad+\frac{30}{11}\left(7\,Ba^4b^6+4\,Aa^3b^7\right)x^{11}\\&\quad+\frac{21}{5}\left(6\,Ba^5b^5+5\,Aa^4b^6\right)x^{10}+\frac{14}{3}\left(5\,Ba^6b^4+6\,Aa^5b^5\right)x^9\\&\quad+\frac{15}{4}\left(4\,Ba^7b^3+7\,Aa^6b^4\right)x^8+\frac{15}{7}\left(3\,Ba^8b^2+8\,Aa^7b^3\right)x^7\\&\quad+\frac{5}{6}\left(2\,Ba^9b+9\,Aa^8b^2\right)x^6+\frac{1}{5}\left(Ba^{10}+10\,Aa^9b\right)x^5\end{aligned}$$

input
```
integrate(x^3*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/15*B*b^10*x^15 + 1/4*A*a^10*x^4 + 1/14*(10*B*a*b^9 + A*b^10)*x^14 + 5/13
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^13 + 5/4*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^12 + 3
0/11*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^11 + 21/5*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x
^10 + 14/3*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^9 + 15/4*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^8 + 15/7*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^7 + 5/6*(2*B*a^9*b + 9*A*a^8*
b^2)*x^6 + 1/5*(B*a^10 + 10*A*a^9*b)*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(105) = 210$.

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.37

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^4}{4} + \frac{Bb^{10}x^{15}}{15} + x^{14}\left(\frac{Ab^{10}}{14} + \frac{5Bab^9}{7}\right) + x^{13} \cdot\left(\frac{10Aab^9}{13} + \frac{45Ba^2b^8}{13}\right) + x^{12}\cdot\left(\frac{15Aa^2b^8}{4} + 10Ba^3b^7\right) + x^{11} \cdot\left(\frac{120Aa^3b^7}{11} + \frac{210Ba^4b^6}{11}\right) + x^{10}\cdot\left(21Aa^4b^6 + \frac{126Ba^5b^5}{5}\right) + x^9\cdot\left(28Aa^5b^5 + \frac{70Ba^6b^4}{3}\right) + x^8 \cdot\left(\frac{105Aa^6b^4}{4} + 15Ba^7b^3\right) + x^7\cdot\left(\frac{120Aa^7b^3}{7} + \frac{45Ba^8b^2}{7}\right) + x^6\cdot\left(\frac{15Aa^8b^2}{2} + \frac{5Ba^9b}{3}\right) + x^5\cdot\left(2Aa^9b + \frac{Ba^{10}}{5}\right)$$

input
```
integrate(x**3*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**4/4 + B*b**10*x**15/15 + x**14*(A*b**10/14 + 5*B*a*b**9/7) + x*
*13*(10*A*a*b**9/13 + 45*B*a**2*b**8/13) + x**12*(15*A*a**2*b**8/4 + 10*B*
a**3*b**7) + x**11*(120*A*a**3*b**7/11 + 210*B*a**4*b**6/11) + x**10*(21*A
*a**4*b**6 + 126*B*a**5*b**5/5) + x**9*(28*A*a**5*b**5 + 70*B*a**6*b**4/3)
 + x**8*(105*A*a**6*b**4/4 + 15*B*a**7*b**3) + x**7*(120*A*a**7*b**3/7 + 4
5*B*a**8*b**2/7) + x**6*(15*A*a**8*b**2/2 + 5*B*a**9*b/3) + x**5*(2*A*a**9
*b + B*a**10/5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(104) = 208$.

Time = 0.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.17

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = \frac{1}{15}\,Bb^{10}x^{15} + \frac{1}{4}\,Aa^{10}x^4 + \frac{1}{14}\,(10\,Bab^9 + Ab^{10})x^{14} + \frac{5}{13}\,(9\,Ba^2b^8 + 2\,Aab^9)x^{13} + \frac{5}{4}\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^{12} + \frac{30}{11}\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^{11} + \frac{21}{5}\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^{10} + \frac{14}{3}\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^9 + \frac{15}{4}\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^8 + \frac{15}{7}\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^7 + \frac{5}{6}\,(2\,Ba^9b + 9\,Aa^8b^2)x^6 + \frac{1}{5}\,(Ba^{10} + 10\,Aa^9b)x^5$$

input
```
integrate(x^3*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/15*B*b^10*x^15 + 1/4*A*a^10*x^4 + 1/14*(10*B*a*b^9 + A*b^10)*x^14 + 5/13
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^13 + 5/4*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^12 + 3
0/11*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^11 + 21/5*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x
^10 + 14/3*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^9 + 15/4*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^8 + 15/7*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^7 + 5/6*(2*B*a^9*b + 9*A*a^8*
b^2)*x^6 + 1/5*(B*a^10 + 10*A*a^9*b)*x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(104) = 208$.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.19

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = \frac{1}{15}Bb^{10}x^{15} + \frac{5}{7}Bab^9x^{14} + \frac{1}{14}Ab^{10}x^{14} + \frac{45}{13}Ba^2b^8x^{13} + \frac{10}{13}Aab^9x^{13} + 10\,Ba^3b^7x^{12} + \frac{15}{4}Aa^2b^8x^{12} + \frac{210}{11}Ba^4b^6x^{11} + \frac{120}{11}Aa^3b^7x^{11} + \frac{126}{5}Ba^5b^5x^{10} + 21\,Aa^4b^6x^{10} + \frac{70}{3}Ba^6b^4x^9 + 28\,Aa^5b^5x^9 + 15\,Ba^7b^3x^8 + \frac{105}{4}Aa^6b^4x^8 + \frac{45}{7}Ba^8b^2x^7 + \frac{120}{7}Aa^7b^3x^7 + \frac{5}{3}Ba^9bx^6 + \frac{15}{2}Aa^8b^2x^6 + \frac{1}{5}Ba^{10}x^5 + 2\,Aa^9bx^5 + \frac{1}{4}Aa^{10}x^4$$

input
```
integrate(x^3*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/15*B*b^10*x^15 + 5/7*B*a*b^9*x^14 + 1/14*A*b^10*x^14 + 45/13*B*a^2*b^8*x
^13 + 10/13*A*a*b^9*x^13 + 10*B*a^3*b^7*x^12 + 15/4*A*a^2*b^8*x^12 + 210/1
1*B*a^4*b^6*x^11 + 120/11*A*a^3*b^7*x^11 + 126/5*B*a^5*b^5*x^10 + 21*A*a^4
*b^6*x^10 + 70/3*B*a^6*b^4*x^9 + 28*A*a^5*b^5*x^9 + 15*B*a^7*b^3*x^8 + 105
/4*A*a^6*b^4*x^8 + 45/7*B*a^8*b^2*x^7 + 120/7*A*a^7*b^3*x^7 + 5/3*B*a^9*b*
x^6 + 15/2*A*a^8*b^2*x^6 + 1/5*B*a^10*x^5 + 2*A*a^9*b*x^5 + 1/4*A*a^10*x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.88

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = x^5\left(\frac{B\,a^{10}}{5}+2\,A\,b\,a^9\right)+x^{14}\left(\frac{A\,b^{10}}{14}+\frac{5\,B\,a\,b^9}{7}\right)+\frac{A\,a^{10}\,x^4}{4}+\frac{B\,b^{10}\,x^{15}}{15}+\frac{15\,a^7\,b^2\,x^7\,(8\,A\,b+3\,B\,a)}{7}+\frac{15\,a^6\,b^3\,x^8\,(7\,A\,b+4\,B\,a)}{4}+\frac{14\,a^5\,b^4\,x^9\,(6\,A\,b+5\,B\,a)}{3}+\frac{21\,a^4\,b^5\,x^{10}\,(5\,A\,b+6\,B\,a)}{5}+\frac{30\,a^3\,b^6\,x^{11}\,(4\,A\,b+7\,B\,a)}{11}+\frac{5\,a^2\,b^7\,x^{12}\,(3\,A\,b+8\,B\,a)}{4}+\frac{5\,a^8\,b\,x^6\,(9\,A\,b+2\,B\,a)}{6}+\frac{5\,a\,b^8\,x^{13}\,(2\,A\,b+9\,B\,a)}{13}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^5*((B*a^10)/5 + 2*A*a^9*b) + x^14*((A*b^10)/14 + (5*B*a*b^9)/7) + (A*a^1
0*x^4)/4 + (B*b^10*x^15)/15 + (15*a^7*b^2*x^7*(8*A*b + 3*B*a))/7 + (15*a^6
*b^3*x^8*(7*A*b + 4*B*a))/4 + (14*a^5*b^4*x^9*(6*A*b + 5*B*a))/3 + (21*a^4
*b^5*x^10*(5*A*b + 6*B*a))/5 + (30*a^3*b^6*x^11*(4*A*b + 7*B*a))/11 + (5*a
^2*b^7*x^12*(3*A*b + 8*B*a))/4 + (5*a^8*b*x^6*(9*A*b + 2*B*a))/6 + (5*a*b^
8*x^13*(2*A*b + 9*B*a))/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10

$$\int x^3(a+bx)^{10}(A+Bx)\,dx = \frac{x^4(364b^{11}x^{11}+4290a\,b^{10}x^{10}+23100a^2b^9x^9+75075a^3b^8x^8+163800a^4b^7x^7+252252a^5b^6x^6+280280a^6}{5460}$$

input
```
int(x^3*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**4*(1365*a**11 + 12012*a**10*b*x + 50050*a**9*b**2*x**2 + 128700*a**8*b
**3*x**3 + 225225*a**7*b**4*x**4 + 280280*a**6*b**5*x**5 + 252252*a**5*b**
6*x**6 + 163800*a**4*b**7*x**7 + 75075*a**3*b**8*x**8 + 23100*a**2*b**9*x*
*9 + 4290*a*b**10*x**10 + 364*b**11*x**11))/5460
```

3.114 $\int x^2(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = \frac{a^2(Ab-aB)(a+bx)^{11}}{11b^4} - \frac{a(2Ab-3aB)(a+bx)^{12}}{12b^4} + \frac{(Ab-3aB)(a+bx)^{13}}{13b^4} + \frac{B(a+bx)^{14}}{14b^4}$$

output

```
1/11*a^2*(A*b-B*a)*(b*x+a)^11/b^4-1/12*a*(2*A*b-3*B*a)*(b*x+a)^12/b^4+1/13
*(A*b-3*B*a)*(b*x+a)^13/b^4+1/14*B*(b*x+a)^14/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(87) = 174$.

Time = 0.02 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.60

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = \frac{1}{3}a^{10}Ax^3+\frac{1}{4}a^9(10Ab+aB)x^4 + a^8b(9Ab+2aB)x^5+\frac{5}{2}a^7b^2(8Ab+3aB)x^6 + \frac{30}{7}a^6b^3(7Ab+4aB)x^7+\frac{21}{4}a^5b^4(6Ab+5aB)x^8 + \frac{14}{3}a^4b^5(5Ab+6aB)x^9+3a^3b^6(4Ab+7aB)x^{10} + \frac{15}{11}a^2b^7(3Ab+8aB)x^{11}+\frac{5}{12}ab^8(2Ab+9aB)x^{12} + \frac{1}{13}b^9(Ab+10aB)x^{13}+\frac{1}{14}b^{10}Bx^{14}$$

input
```
Integrate[x^2*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*A*x^3)/3 + (a^9*(10*A*b + a*B)*x^4)/4 + a^8*b*(9*A*b + 2*a*B)*x^5 +
(5*a^7*b^2*(8*A*b + 3*a*B)*x^6)/2 + (30*a^6*b^3*(7*A*b + 4*a*B)*x^7)/7 + (
21*a^5*b^4*(6*A*b + 5*a*B)*x^8)/4 + (14*a^4*b^5*(5*A*b + 6*a*B)*x^9)/3 + 3
*a^3*b^6*(4*A*b + 7*a*B)*x^10 + (15*a^2*b^7*(3*A*b + 8*a*B)*x^11)/11 + (5*
a*b^8*(2*A*b + 9*a*B)*x^12)/12 + (b^9*(A*b + 10*a*B)*x^13)/13 + (b^10*B*x^
14)/14
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow 85$$

$$\int \left(-\frac{a^2(a+bx)^{10}(aB-Ab)}{b^3} + \frac{(a+bx)^{12}(Ab-3aB)}{b^3} + \frac{a(a+bx)^{11}(3aB-2Ab)}{b^3} + \frac{B(a+bx)^{13}}{b^3} \right) dx$$

$$\Big\downarrow 2009$$

$$\frac{a^2(a+bx)^{11}(Ab-aB)}{11b^4} + \frac{(a+bx)^{13}(Ab-3aB)}{13b^4} - \frac{a(a+bx)^{12}(2Ab-3aB)}{12b^4} + \frac{B(a+bx)^{14}}{14b^4}$$

input
```
Int[x^2*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^2*(A*b - a*B)*(a + b*x)^11)/(11*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x)^12)
/(12*b^4) + ((A*b - 3*a*B)*(a + b*x)^13)/(13*b^4) + (B*(a + b*x)^14)/(14*b
^4)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(79) = 158$.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.71

method	result
norman	$\frac{a^{10}Ax^3}{3}+\left(\frac{5}{2}a^9bA+\frac{1}{4}a^{10}B\right)x^4+\left(9a^8b^2A+2a^9bB\right)x^5+\left(20a^7b^3A+\frac{15}{2}a^8b^2B\right)x^6+\left(30a^6b\right.$
default	$\frac{b^{10}Bx^{14}}{14}+\frac{\left(b^{10}A+10a\,b^9B\right)x^{13}}{13}+\frac{\left(10a\,b^9A+45a^2b^8B\right)x^{12}}{12}+\frac{\left(45a^2b^8A+120a^3b^7B\right)x^{11}}{11}+\frac{\left(120a^3b^7A+210a^4b^6B\right)x}{10}$
orering	$\frac{x^3\left(858B\,b^{10}x^{11}+924A\,b^{10}x^{10}+9240Ba\,b^9x^{10}+10010aA\,b^9x^9+45045B\,a^2b^8x^9+49140a^2A\,b^8x^8+131040B\,a^3b^7x^8+144144a\right.}{}$
gosper	$\frac{1}{3}a^{10}A\,x^3+\frac{5}{2}x^4a^9bA+\frac{1}{4}x^4a^{10}B+9A\,a^8b^2x^5+2B\,a^9b\,x^5+20x^6a^7b^3A+\frac{15}{2}x^6a^8b^2B+30x$
risch	$\frac{1}{3}a^{10}A\,x^3+\frac{5}{2}x^4a^9bA+\frac{1}{4}x^4a^{10}B+9A\,a^8b^2x^5+2B\,a^9b\,x^5+20x^6a^7b^3A+\frac{15}{2}x^6a^8b^2B+30x$
parallelrisch	$\frac{1}{3}a^{10}A\,x^3+\frac{5}{2}x^4a^9bA+\frac{1}{4}x^4a^{10}B+9A\,a^8b^2x^5+2B\,a^9b\,x^5+20x^6a^7b^3A+\frac{15}{2}x^6a^8b^2B+30x$

input
```
int(x^2*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^10*A*x^3+(5/2*a^9*b*A+1/4*a^10*B)*x^4+(9*A*a^8*b^2+2*B*a^9*b)*x^5+(2
0*a^7*b^3*A+15/2*a^8*b^2*B)*x^6+(30*a^6*b^4*A+120/7*a^7*b^3*B)*x^7+(63/2*a
^5*b^5*A+105/4*a^6*b^4*B)*x^8+(70/3*a^4*b^6*A+28*a^5*b^5*B)*x^9+(12*A*a^3*
b^7+21*B*a^4*b^6)*x^10+(45/11*a^2*b^8*A+120/11*a^3*b^7*B)*x^11+(5/6*a*b^9*
A+15/4*a^2*b^8*B)*x^12+(1/13*b^10*A+10/13*a*b^9*B)*x^13+1/14*b^10*B*x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.78

$$\begin{aligned}\int x^2(a+bx)^{10}(A+Bx)\,dx =& \frac{1}{14}\,Bb^{10}x^{14}+\frac{1}{3}\,Aa^{10}x^3+\frac{1}{13}\,\left(10\,Bab^9+Ab^{10}\right)x^{13}\\&+\frac{5}{12}\,\left(9\,Ba^2b^8+2\,Aab^9\right)x^{12}+\frac{15}{11}\,\left(8\,Ba^3b^7+3\,Aa^2b^8\right)x^{11}\\&+3\,\left(7\,Ba^4b^6+4\,Aa^3b^7\right)x^{10}+\frac{14}{3}\,\left(6\,Ba^5b^5+5\,Aa^4b^6\right)x^9\\&+\frac{21}{4}\,\left(5\,Ba^6b^4+6\,Aa^5b^5\right)x^8\\&+\frac{30}{7}\,\left(4\,Ba^7b^3+7\,Aa^6b^4\right)x^7+\frac{5}{2}\,\left(3\,Ba^8b^2+8\,Aa^7b^3\right)x^6\\&+\left(2\,Ba^9b+9\,Aa^8b^2\right)x^5+\frac{1}{4}\,\left(Ba^{10}+10\,Aa^9b\right)x^4\end{aligned}$$

input
```
integrate(x^2*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/14*B*b^10*x^14 + 1/3*A*a^10*x^3 + 1/13*(10*B*a*b^9 + A*b^10)*x^13 + 5/12
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^12 + 15/11*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^11 +
 3*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^10 + 14/3*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^9
 + 21/4*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^8 + 30/7*(4*B*a^7*b^3 + 7*A*a^6*b^4)
*x^7 + 5/2*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^6 + (2*B*a^9*b + 9*A*a^8*b^2)*x^5
 + 1/4*(B*a^10 + 10*A*a^9*b)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(80) = 160$.

Time = 0.05 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.01

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^3}{3} + \frac{Bb^{10}x^{14}}{14} + x^{13}\left(\frac{Ab^{10}}{13} + \frac{10Bab^9}{13}\right) + x^{12} \cdot \left(\frac{5Aab^9}{6} + \frac{15Ba^2b^8}{4}\right) + x^{11} \cdot \left(\frac{45Aa^2b^8}{11} + \frac{120Ba^3b^7}{11}\right) + x^{10} \cdot \left(12Aa^3b^7 + 21Ba^4b^6\right) + x^9 \cdot \left(\frac{70Aa^4b^6}{3} + 28Ba^5b^5\right) + x^8 \cdot \left(\frac{63Aa^5b^5}{2} + \frac{105Ba^6b^4}{4}\right) + x^7 \cdot \left(30Aa^6b^4 + \frac{120Ba^7b^3}{7}\right) + x^6 \cdot \left(20Aa^7b^3 + \frac{15Ba^8b^2}{2}\right) + x^5 \cdot \left(9Aa^8b^2 + 2Ba^9b\right) + x^4 \cdot \left(\frac{5Aa^9b}{2} + \frac{Ba^{10}}{4}\right)$$

input
```
integrate(x**2*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**3/3 + B*b**10*x**14/14 + x**13*(A*b**10/13 + 10*B*a*b**9/13) +
x**12*(5*A*a*b**9/6 + 15*B*a**2*b**8/4) + x**11*(45*A*a**2*b**8/11 + 120*B
*a**3*b**7/11) + x**10*(12*A*a**3*b**7 + 21*B*a**4*b**6) + x**9*(70*A*a**4
*b**6/3 + 28*B*a**5*b**5) + x**8*(63*A*a**5*b**5/2 + 105*B*a**6*b**4/4) +
x**7*(30*A*a**6*b**4 + 120*B*a**7*b**3/7) + x**6*(20*A*a**7*b**3 + 15*B*a*
*8*b**2/2) + x**5*(9*A*a**8*b**2 + 2*B*a**9*b) + x**4*(5*A*a**9*b/2 + B*a*
*10/4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.

Time = 0.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.78

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = \frac{1}{14}Bb^{10}x^{14} + \frac{1}{3}Aa^{10}x^3 + \frac{1}{13}\left(10\,Bab^9 + Ab^{10}\right)x^{13} + \frac{5}{12}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{12} + \frac{15}{11}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^{11} + 3\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^{10} + \frac{14}{3}\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^9 + \frac{21}{4}\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^8 + \frac{30}{7}\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^7 + \frac{5}{2}\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^6 + \left(2\,Ba^9b + 9\,Aa^8b^2\right)x^5 + \frac{1}{4}\left(Ba^{10} + 10\,Aa^9b\right)x^4$$

input
```
integrate(x^2*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/14*B*b^10*x^14 + 1/3*A*a^10*x^3 + 1/13*(10*B*a*b^9 + A*b^10)*x^13 + 5/12
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^12 + 15/11*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^11 +
 3*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^10 + 14/3*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^9
 + 21/4*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^8 + 30/7*(4*B*a^7*b^3 + 7*A*a^6*b^4)
*x^7 + 5/2*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^6 + (2*B*a^9*b + 9*A*a^8*b^2)*x^5
 + 1/4*(B*a^10 + 10*A*a^9*b)*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.82

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = \frac{1}{14}Bb^{10}x^{14} + \frac{10}{13}Bab^9x^{13} + \frac{1}{13}Ab^{10}x^{13} + \frac{15}{4}Ba^2b^8x^{12} + \frac{5}{6}Aab^9x^{12} + \frac{120}{11}Ba^3b^7x^{11} + \frac{45}{11}Aa^2b^8x^{11} + 21\,Ba^4b^6x^{10} + 12\,Aa^3b^7x^{10} + 28\,Ba^5b^5x^9 + \frac{70}{3}Aa^4b^6x^9 + \frac{105}{4}Ba^6b^4x^8 + \frac{63}{2}Aa^5b^5x^8 + \frac{120}{7}Ba^7b^3x^7 + 30\,Aa^6b^4x^7 + \frac{15}{2}Ba^8b^2x^6 + 20\,Aa^7b^3x^6 + 2\,Ba^9bx^5 + 9\,Aa^8b^2x^5 + \frac{1}{4}Ba^{10}x^4 + \frac{5}{2}Aa^9bx^4 + \frac{1}{3}Aa^{10}x^3$$

input
```
integrate(x^2*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/14*B*b^10*x^14 + 10/13*B*a*b^9*x^13 + 1/13*A*b^10*x^13 + 15/4*B*a^2*b^8*
x^12 + 5/6*A*a*b^9*x^12 + 120/11*B*a^3*b^7*x^11 + 45/11*A*a^2*b^8*x^11 + 2
1*B*a^4*b^6*x^10 + 12*A*a^3*b^7*x^10 + 28*B*a^5*b^5*x^9 + 70/3*A*a^4*b^6*x
^9 + 105/4*B*a^6*b^4*x^8 + 63/2*A*a^5*b^5*x^8 + 120/7*B*a^7*b^3*x^7 + 30*A
*a^6*b^4*x^7 + 15/2*B*a^8*b^2*x^6 + 20*A*a^7*b^3*x^6 + 2*B*a^9*b*x^5 + 9*A
*a^8*b^2*x^5 + 1/4*B*a^10*x^4 + 5/2*A*a^9*b*x^4 + 1/3*A*a^10*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.41

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = x^4\left(\frac{B\,a^{10}}{4}+\frac{5\,A\,b\,a^9}{2}\right)+x^{13}\left(\frac{A\,b^{10}}{13}+\frac{10\,B\,a\,b^9}{13}\right)+\frac{A\,a^{10}\,x^3}{3}+\frac{B\,b^{10}\,x^{14}}{14}+\frac{5\,a^7\,b^2\,x^6\,(8\,A\,b+3\,B\,a)}{2}+\frac{30\,a^6\,b^3\,x^7\,(7\,A\,b+4\,B\,a)}{7}+\frac{21\,a^5\,b^4\,x^8\,(6\,A\,b+5\,B\,a)}{4}+\frac{14\,a^4\,b^5\,x^9\,(5\,A\,b+6\,B\,a)}{3}+3\,a^3\,b^6\,x^{10}\,(4\,A\,b+7\,B\,a)+\frac{15\,a^2\,b^7\,x^{11}\,(3\,A\,b+8\,B\,a)}{11}+a^8\,b\,x^5\,(9\,A\,b+2\,B\,a)+\frac{5\,a\,b^8\,x^{12}\,(2\,A\,b+9\,B\,a)}{12}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^4*((B*a^10)/4 + (5*A*a^9*b)/2) + x^13*((A*b^10)/13 + (10*B*a*b^9)/13) +
(A*a^10*x^3)/3 + (B*b^10*x^14)/14 + (5*a^7*b^2*x^6*(8*A*b + 3*B*a))/2 + (3
0*a^6*b^3*x^7*(7*A*b + 4*B*a))/7 + (21*a^5*b^4*x^8*(6*A*b + 5*B*a))/4 + (1
4*a^4*b^5*x^9*(5*A*b + 6*B*a))/3 + 3*a^3*b^6*x^10*(4*A*b + 7*B*a) + (15*a^
2*b^7*x^11*(3*A*b + 8*B*a))/11 + a^8*b*x^5*(9*A*b + 2*B*a) + (5*a*b^8*x^12
*(2*A*b + 9*B*a))/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int x^2(a+bx)^{10}(A+Bx)\,dx = \frac{x^3(78b^{11}x^{11}+924a\,b^{10}x^{10}+5005a^2b^9x^9+16380a^3b^8x^8+36036a^4b^7x^7+56056a^5b^6x^6+63063a^6b^5x^5+}{1092}$$

input
```
int(x^2*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**3*(364*a**11 + 3003*a**10*b*x + 12012*a**9*b**2*x**2 + 30030*a**8*b**3
*x**3 + 51480*a**7*b**4*x**4 + 63063*a**6*b**5*x**5 + 56056*a**5*b**6*x**6
 + 36036*a**4*b**7*x**7 + 16380*a**3*b**8*x**8 + 5005*a**2*b**9*x**9 + 924
*a*b**10*x**10 + 78*b**11*x**11))/1092
```

3.115 $\int x(a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int x(a+bx)^{10}(A+Bx)\,dx = -\frac{a(Ab-aB)(a+bx)^{11}}{11b^3} + \frac{(Ab-2aB)(a+bx)^{12}}{12b^3} + \frac{B(a+bx)^{13}}{13b^3}$$

output
```
-1/11*a*(A*b-B*a)*(b*x+a)^11/b^3+1/12*(A*b-2*B*a)*(b*x+a)^12/b^3+1/13*B*(b
*x+a)^13/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. $2(61) = 122$.

Time = 0.02 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.57

$$\int x(a+bx)^{10}(A+Bx)\,dx = \frac{1}{6}a^{10}x^2(3A+2Bx)+\frac{5}{6}a^9bx^3(4A+3Bx) + \frac{9}{4}a^8b^2x^4(5A+4Bx)+4a^7b^3x^5(6A+5Bx) + 5a^6b^4x^6(7A+6Bx)+\frac{9}{2}a^5b^5x^7(8A+7Bx) + \frac{35}{12}a^4b^6x^8(9A+8Bx)+\frac{4}{3}a^3b^7x^9(10A+9Bx) + \frac{9}{22}a^2b^8x^{10}(11A+10Bx)+\frac{5}{66}ab^9x^{11}(12A+11Bx) + \frac{1}{156}b^{10}x^{12}(13A+12Bx)$$

input
```
Integrate[x*(a + b*x)^10*(A + B*x),x]
```

output
```
(a^10*x^2*(3*A + 2*B*x))/6 + (5*a^9*b*x^3*(4*A + 3*B*x))/6 + (9*a^8*b^2*x^
4*(5*A + 4*B*x))/4 + 4*a^7*b^3*x^5*(6*A + 5*B*x) + 5*a^6*b^4*x^6*(7*A + 6*
B*x) + (9*a^5*b^5*x^7*(8*A + 7*B*x))/2 + (35*a^4*b^6*x^8*(9*A + 8*B*x))/12
 + (4*a^3*b^7*x^9*(10*A + 9*B*x))/3 + (9*a^2*b^8*x^10*(11*A + 10*B*x))/22
+ (5*a*b^9*x^11*(12*A + 11*B*x))/66 + (b^10*x^12*(13*A + 12*B*x))/156
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{(a+bx)^{11}(Ab-2aB)}{b^2}+\frac{a(a+bx)^{10}(aB-Ab)}{b^2}+\frac{B(a+bx)^{12}}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^{12}(Ab-2aB)}{12b^3}-\frac{a(a+bx)^{11}(Ab-aB)}{11b^3}+\frac{B(a+bx)^{13}}{13b^3}$$

input
```
Int[x*(a + b*x)^10*(A + B*x),x]
```

output
```
-1/11*(a*(A*b - a*B)*(a + b*x)^11)/b^3 + ((A*b - 2*a*B)*(a + b*x)^12)/(12*
b^3) + (B*(a + b*x)^13)/(13*b^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.87

method	result
norman	$\frac{b^{10}B\,x^{13}}{13}+\left(\frac{1}{12}b^{10}A+\frac{5}{6}a\,b^{9}B\right)x^{12}+\left(\frac{10}{11}a\,b^{9}A+\frac{45}{11}a^{2}b^{8}B\right)x^{11}+\left(\frac{9}{2}a^{2}b^{8}A+12a^{3}b^{7}B\right)x^{10}+$
default	$\frac{b^{10}B\,x^{13}}{13}+\frac{(b^{10}A+10a\,b^{9}B)x^{12}}{12}+\frac{(10a\,b^{9}A+45a^{2}b^{8}B)x^{11}}{11}+\frac{(45a^{2}b^{8}A+120a^{3}b^{7}B)x^{10}}{10}+\frac{(120a^{3}b^{7}A+210a^{4}b^{6}B)x}{9}$
orering	$x^{2}\left(132B\,b^{10}x^{11}+143A\,b^{10}x^{10}+1430Ba\,b^{9}x^{10}+1560aA\,b^{9}x^{9}+7020B\,a^{2}b^{8}x^{9}+7722a^{2}A\,b^{8}x^{8}+20592B\,a^{3}b^{7}x^{8}+22880a^{3}A\,b^{7}\right.$
gosper	$\frac{1}{13}b^{10}B\,x^{13}+\frac{1}{12}x^{12}b^{10}A+\frac{5}{6}x^{12}a\,b^{9}B+\frac{10}{11}x^{11}a\,b^{9}A+\frac{45}{11}x^{11}a^{2}b^{8}B+\frac{9}{2}x^{10}a^{2}b^{8}A+12x^{10}a^{3}b^{7}$
risch	$\frac{1}{13}b^{10}B\,x^{13}+\frac{1}{12}x^{12}b^{10}A+\frac{5}{6}x^{12}a\,b^{9}B+\frac{10}{11}x^{11}a\,b^{9}A+\frac{45}{11}x^{11}a^{2}b^{8}B+\frac{9}{2}x^{10}a^{2}b^{8}A+12x^{10}a^{3}b^{7}$
parallelrisch	$\frac{1}{13}b^{10}B\,x^{13}+\frac{1}{12}x^{12}b^{10}A+\frac{5}{6}x^{12}a\,b^{9}B+\frac{10}{11}x^{11}a\,b^{9}A+\frac{45}{11}x^{11}a^{2}b^{8}B+\frac{9}{2}x^{10}a^{2}b^{8}A+12x^{10}a^{3}b^{7}$

input
```
int(x*(b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^10*B*x^13+(1/12*b^10*A+5/6*a*b^9*B)*x^12+(10/11*a*b^9*A+45/11*a^2*b
^8*B)*x^11+(9/2*a^2*b^8*A+12*a^3*b^7*B)*x^10+(40/3*a^3*b^7*A+70/3*a^4*b^6*
B)*x^9+(105/4*a^4*b^6*A+63/2*a^5*b^5*B)*x^8+(36*A*a^5*b^5+30*B*a^6*b^4)*x^
7+(35*A*a^6*b^4+20*B*a^7*b^3)*x^6+(24*A*a^7*b^3+9*B*a^8*b^2)*x^5+(45/4*a^8
*b^2*A+5/2*a^9*b*B)*x^4+(10/3*a^9*b*A+1/3*a^10*B)*x^3+1/2*a^10*A*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(56) = 112$.

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.98

$$
\begin{aligned}
\int x(a+bx)^{10}(A+Bx)\,dx = {} & \frac{1}{13}\,Bb^{10}x^{13}+\frac{1}{2}\,Aa^{10}x^{2}+\frac{1}{12}\,(10\,Bab^{9}+Ab^{10})x^{12} \\
& +\frac{5}{11}\,(9\,Ba^{2}b^{8}+2\,Aab^{9})x^{11}+\frac{3}{2}\,(8\,Ba^{3}b^{7}+3\,Aa^{2}b^{8})x^{10} \\
& +\frac{10}{3}\,(7\,Ba^{4}b^{6}+4\,Aa^{3}b^{7})x^{9} \\
& +\frac{21}{4}\,(6\,Ba^{5}b^{5}+5\,Aa^{4}b^{6})x^{8}+6\,(5\,Ba^{6}b^{4}+6\,Aa^{5}b^{5})x^{7} \\
& +5\,(4\,Ba^{7}b^{3}+7\,Aa^{6}b^{4})x^{6}+3\,(3\,Ba^{8}b^{2}+8\,Aa^{7}b^{3})x^{5} \\
& +\frac{5}{4}\,(2\,Ba^{9}b+9\,Aa^{8}b^{2})x^{4}+\frac{1}{3}\,(Ba^{10}+10\,Aa^{9}b)x^{3}
\end{aligned}
$$

input
```
integrate(x*(b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output
```
1/13*B*b^10*x^13 + 1/2*A*a^10*x^2 + 1/12*(10*B*a*b^9 + A*b^10)*x^12 + 5/11
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^11 + 3/2*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^10 + 1
0/3*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^9 + 21/4*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^8
 + 6*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^7 + 5*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^6 +
 3*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^5 + 5/4*(2*B*a^9*b + 9*A*a^8*b^2)*x^4 + 1
/3*(B*a^10 + 10*A*a^9*b)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(53) = 106$.

Time = 0.05 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.30

$$\int x(a+bx)^{10}(A+Bx)\,dx = \frac{Aa^{10}x^2}{2} + \frac{Bb^{10}x^{13}}{13} + x^{12}\left(\frac{Ab^{10}}{12} + \frac{5Bab^9}{6}\right) + x^{11} \cdot \left(\frac{10Aab^9}{11} + \frac{45Ba^2b^8}{11}\right) + x^{10} \cdot \left(\frac{9Aa^2b^8}{2} + 12Ba^3b^7\right) + x^9 \cdot \left(\frac{40Aa^3b^7}{3} + \frac{70Ba^4b^6}{3}\right) + x^8 \cdot \left(\frac{105Aa^4b^6}{4} + \frac{63Ba^5b^5}{2}\right) + x^7 \cdot \left(36Aa^5b^5 + 30Ba^6b^4\right) + x^6 \cdot \left(35Aa^6b^4 + 20Ba^7b^3\right) + x^5 \cdot \left(24Aa^7b^3 + 9Ba^8b^2\right) + x^4 \cdot \left(\frac{45Aa^8b^2}{4} + \frac{5Ba^9b}{2}\right) + x^3 \cdot \left(\frac{10Aa^9b}{3} + \frac{Ba^{10}}{3}\right)$$

input
```
integrate(x*(b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x**2/2 + B*b**10*x**13/13 + x**12*(A*b**10/12 + 5*B*a*b**9/6) + x*
*11*(10*A*a*b**9/11 + 45*B*a**2*b**8/11) + x**10*(9*A*a**2*b**8/2 + 12*B*a
**3*b**7) + x**9*(40*A*a**3*b**7/3 + 70*B*a**4*b**6/3) + x**8*(105*A*a**4*
b**6/4 + 63*B*a**5*b**5/2) + x**7*(36*A*a**5*b**5 + 30*B*a**6*b**4) + x**6
*(35*A*a**6*b**4 + 20*B*a**7*b**3) + x**5*(24*A*a**7*b**3 + 9*B*a**8*b**2)
 + x**4*(45*A*a**8*b**2/4 + 5*B*a**9*b/2) + x**3*(10*A*a**9*b/3 + B*a**10/
3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(56) = 112$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.98

$$\int x(a+bx)^{10}(A+Bx)\,dx = \frac{1}{13}\,Bb^{10}x^{13} + \frac{1}{2}\,Aa^{10}x^2 + \frac{1}{12}\,(10\,Bab^9 + Ab^{10})x^{12} + \frac{5}{11}\,(9\,Ba^2b^8 + 2\,Aab^9)x^{11} + \frac{3}{2}\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^{10} + \frac{10}{3}\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^9 + \frac{21}{4}\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^8 + 6\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^7 + 5\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^6 + 3\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^5 + \frac{5}{4}\,(2\,Ba^9b + 9\,Aa^8b^2)x^4 + \frac{1}{3}\,(Ba^{10} + 10\,Aa^9b)x^3$$

input
```
integrate(x*(b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/13*B*b^10*x^13 + 1/2*A*a^10*x^2 + 1/12*(10*B*a*b^9 + A*b^10)*x^12 + 5/11
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^11 + 3/2*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^10 + 1
0/3*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^9 + 21/4*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^8
 + 6*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^7 + 5*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^6 +
 3*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^5 + 5/4*(2*B*a^9*b + 9*A*a^8*b^2)*x^4 + 1
/3*(B*a^10 + 10*A*a^9*b)*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.02

$$\int x(a+bx)^{10}(A+Bx)\,dx = \frac{1}{13}Bb^{10}x^{13} + \frac{5}{6}Bab^9x^{12} + \frac{1}{12}Ab^{10}x^{12} + \frac{45}{11}Ba^2b^8x^{11} + \frac{10}{11}Aab^9x^{11} + 12\,Ba^3b^7x^{10} + \frac{9}{2}Aa^2b^8x^{10} + \frac{70}{3}Ba^4b^6x^9 + \frac{40}{3}Aa^3b^7x^9 + \frac{63}{2}Ba^5b^5x^8 + \frac{105}{4}Aa^4b^6x^8 + 30\,Ba^6b^4x^7 + 36\,Aa^5b^5x^7 + 20\,Ba^7b^3x^6 + 35\,Aa^6b^4x^6 + 9\,Ba^8b^2x^5 + 24\,Aa^7b^3x^5 + \frac{5}{2}Ba^9bx^4 + \frac{45}{4}Aa^8b^2x^4 + \frac{1}{3}Ba^{10}x^3 + \frac{10}{3}Aa^9bx^3 + \frac{1}{2}Aa^{10}x^2$$

input
```
integrate(x*(b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/13*B*b^10*x^13 + 5/6*B*a*b^9*x^12 + 1/12*A*b^10*x^12 + 45/11*B*a^2*b^8*x
^11 + 10/11*A*a*b^9*x^11 + 12*B*a^3*b^7*x^10 + 9/2*A*a^2*b^8*x^10 + 70/3*B
*a^4*b^6*x^9 + 40/3*A*a^3*b^7*x^9 + 63/2*B*a^5*b^5*x^8 + 105/4*A*a^4*b^6*x
^8 + 30*B*a^6*b^4*x^7 + 36*A*a^5*b^5*x^7 + 20*B*a^7*b^3*x^6 + 35*A*a^6*b^4
*x^6 + 9*B*a^8*b^2*x^5 + 24*A*a^7*b^3*x^5 + 5/2*B*a^9*b*x^4 + 45/4*A*a^8*b
^2*x^4 + 1/3*B*a^10*x^3 + 10/3*A*a^9*b*x^3 + 1/2*A*a^10*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.46

$$\int x(a+bx)^{10}(A+Bx)\,dx = x^3\left(\frac{B\,a^{10}}{3}+\frac{10\,A\,b\,a^9}{3}\right)+x^{12}\left(\frac{A\,b^{10}}{12}+\frac{5\,B\,a\,b^9}{6}\right)+\frac{A\,a^{10}\,x^2}{2}+\frac{B\,b^{10}\,x^{13}}{13}+3\,a^7\,b^2\,x^5\,(8\,A\,b+3\,B\,a)+5\,a^6\,b^3\,x^6\,(7\,A\,b+4\,B\,a)+6\,a^5\,b^4\,x^7\,(6\,A\,b+5\,B\,a)+\frac{21\,a^4\,b^5\,x^8\,(5\,A\,b+6\,B\,a)}{4}+\frac{10\,a^3\,b^6\,x^9\,(4\,A\,b+7\,B\,a)}{3}+\frac{3\,a^2\,b^7\,x^{10}\,(3\,A\,b+8\,B\,a)}{2}+\frac{5\,a^8\,b\,x^4\,(9\,A\,b+2\,B\,a)}{4}+\frac{5\,a\,b^8\,x^{11}\,(2\,A\,b+9\,B\,a)}{11}$$

input
```
int(x*(A + B*x)*(a + b*x)^10,x)
```

output
```
x^3*((B*a^10)/3 + (10*A*a^9*b)/3) + x^12*((A*b^10)/12 + (5*B*a*b^9)/6) + (
A*a^10*x^2)/2 + (B*b^10*x^13)/13 + 3*a^7*b^2*x^5*(8*A*b + 3*B*a) + 5*a^6*b
^3*x^6*(7*A*b + 4*B*a) + 6*a^5*b^4*x^7*(6*A*b + 5*B*a) + (21*a^4*b^5*x^8*(
5*A*b + 6*B*a))/4 + (10*a^3*b^6*x^9*(4*A*b + 7*B*a))/3 + (3*a^2*b^7*x^10*(
3*A*b + 8*B*a))/2 + (5*a^8*b*x^4*(9*A*b + 2*B*a))/4 + (5*a*b^8*x^11*(2*A*b
 + 9*B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int x(a+bx)^{10}(A+Bx)\,dx = \frac{x^2(12b^{11}x^{11}+143a\,b^{10}x^{10}+780a^2b^9x^9+2574a^3b^8x^8+5720a^4b^7x^7+9009a^5b^6x^6+10296a^6b^5x^5+8580}{156}$$

input
```
int(x*(b*x+a)^10*(B*x+A),x)
```

output
```
(x**2*(78*a**11 + 572*a**10*b*x + 2145*a**9*b**2*x**2 + 5148*a**8*b**3*x**
3 + 8580*a**7*b**4*x**4 + 10296*a**6*b**5*x**5 + 9009*a**5*b**6*x**6 + 572
0*a**4*b**7*x**7 + 2574*a**3*b**8*x**8 + 780*a**2*b**9*x**9 + 143*a*b**10*
x**10 + 12*b**11*x**11))/156
```

3.116 $\int (a+bx)^{10}(A+Bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int (a+bx)^{10}(A+Bx)\,dx=\frac{(Ab-aB)(a+bx)^{11}}{11b^2}+\frac{B(a+bx)^{12}}{12b^2}$$

output
```
1/11*(A*b-B*a)*(b*x+a)^11/b^2+1/12*B*(b*x+a)^12/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.21

$$\begin{aligned}\int (a+bx)^{10}(A+Bx)\,dx=\frac{1}{132}x\big(&66a^{10}(2A+Bx)+220a^9bx(3A+2Bx)\\&+495a^8b^2x^2(4A+3Bx)+792a^7b^3x^3(5A+4Bx)\\&+924a^6b^4x^4(6A+5Bx)+792a^5b^5x^5(7A+6Bx)\\&+495a^4b^6x^6(8A+7Bx)+220a^3b^7x^7(9A+8Bx)\\&+66a^2b^8x^8(10A+9Bx)+12ab^9x^9(11A+10Bx)\\&+b^{10}x^{10}(12A+11Bx)\big)\end{aligned}$$

input
```
Integrate[(a + b*x)^10*(A + B*x),x]
```

output
```
(x*(66*a^10*(2*A + B*x) + 220*a^9*b*x*(3*A + 2*B*x) + 495*a^8*b^2*x^2*(4*A
 + 3*B*x) + 792*a^7*b^3*x^3*(5*A + 4*B*x) + 924*a^6*b^4*x^4*(6*A + 5*B*x)
+ 792*a^5*b^5*x^5*(7*A + 6*B*x) + 495*a^4*b^6*x^6*(8*A + 7*B*x) + 220*a^3*
b^7*x^7*(9*A + 8*B*x) + 66*a^2*b^8*x^8*(10*A + 9*B*x) + 12*a*b^9*x^9*(11*A
 + 10*B*x) + b^10*x^10*(12*A + 11*B*x)))/132
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{10}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{(a+bx)^{10}(Ab-aB)}{b}+\frac{B(a+bx)^{11}}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^{11}(Ab-aB)}{11b^2}+\frac{B(a+bx)^{12}}{12b^2}$$

input
```
Int[(a + b*x)^10*(A + B*x),x]
```

output
```
((A*b - a*B)*(a + b*x)^11)/(11*b^2) + (B*(a + b*x)^12)/(12*b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 6.11

method	result
norman	$\frac{b^{10} B\,x^{12}}{12}+\left(\frac{1}{11} b^{10} A+\frac{10}{11} a\,b^{9} B\right) x^{11}+\left(a\,b^{9} A+\frac{9}{2} a^{2} b^{8} B\right) x^{10}+\left(5 a^{2} b^{8} A+\frac{40}{3} a^{3} b^{7} B\right) x^{9}+(15$
default	$\frac{b^{10} B\,x^{12}}{12}+\frac{\left(b^{10} A+10 a\,b^{9} B\right) x^{11}}{11}+\frac{\left(10 a\,b^{9} A+45 a^{2} b^{8} B\right) x^{10}}{10}+\frac{\left(45 a^{2} b^{8} A+120 a^{3} b^{7} B\right) x^{9}}{9}+\frac{\left(120 a^{3} b^{7} A+210 a^{4} b^{6} B\right) x^{8}}{8}$
gosper	$\frac{1}{12} b^{10} B\,x^{12}+\frac{1}{11} x^{11} b^{10} A+\frac{10}{11} x^{11} a\,b^{9} B+x^{10} a\,b^{9} A+\frac{9}{2} x^{10} a^{2} b^{8} B+5 x^{9} a^{2} b^{8} A+\frac{40}{3} x^{9} a^{3} b^{7} B+$
risch	$\frac{1}{12} b^{10} B\,x^{12}+\frac{1}{11} x^{11} b^{10} A+\frac{10}{11} x^{11} a\,b^{9} B+x^{10} a\,b^{9} A+\frac{9}{2} x^{10} a^{2} b^{8} B+5 x^{9} a^{2} b^{8} A+\frac{40}{3} x^{9} a^{3} b^{7} B+$
parallelrisch	$\frac{1}{12} b^{10} B\,x^{12}+\frac{1}{11} x^{11} b^{10} A+\frac{10}{11} x^{11} a\,b^{9} B+x^{10} a\,b^{9} A+\frac{9}{2} x^{10} a^{2} b^{8} B+5 x^{9} a^{2} b^{8} A+\frac{40}{3} x^{9} a^{3} b^{7} B+$
orering	$x(11 B\,b^{10} x^{11}+12 A\,b^{10} x^{10}+120 B a\,b^{9} x^{10}+132 a A\,b^{9} x^{9}+594 B\,a^{2} b^{8} x^{9}+660 a^{2} A\,b^{8} x^{8}+1760 B\,a^{3} b^{7} x^{8}+1980 a^{3} A\,b^{7} x^{7}+3465 B$

input
```
int((b*x+a)^10*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^10*B*x^12+(1/11*b^10*A+10/11*a*b^9*B)*x^11+(a*b^9*A+9/2*a^2*b^8*B)*
x^10+(5*a^2*b^8*A+40/3*a^3*b^7*B)*x^9+(15*a^3*b^7*A+105/4*a^4*b^6*B)*x^8+(
30*A*a^4*b^6+36*B*a^5*b^5)*x^7+(42*A*a^5*b^5+35*B*a^6*b^4)*x^6+(42*A*a^6*b
^4+24*B*a^7*b^3)*x^5+(30*a^7*b^3*A+45/4*a^8*b^2*B)*x^4+(15*a^8*b^2*A+10/3*
a^9*b*B)*x^3+(5*a^9*b*A+1/2*a^10*B)*x^2+a^10*A*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 6.32

$$\int (a+bx)^{10}(A+Bx)\,dx = \frac{1}{12}Bb^{10}x^{12} + Aa^{10}x + \frac{1}{11}\left(10\,Bab^9 + Ab^{10}\right)x^{11} + \frac{1}{2}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{10} + \frac{5}{3}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^9 + \frac{15}{4}\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^8 + 6\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^7 + 7\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^6 + 6\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^5 + \frac{15}{4}\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^4 + \frac{5}{3}\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^3 + \frac{1}{2}\left(Ba^{10} + 10\,Aa^9b\right)x^2$$

input

```
integrate((b*x+a)^10*(B*x+A),x, algorithm="fricas")
```

output

```
1/12*B*b^10*x^12 + A*a^10*x + 1/11*(10*B*a*b^9 + A*b^10)*x^11 + 1/2*(9*B*a
^2*b^8 + 2*A*a*b^9)*x^10 + 5/3*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^9 + 15/4*(7*B
*a^4*b^6 + 4*A*a^3*b^7)*x^8 + 6*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^7 + 7*(5*B*a
^6*b^4 + 6*A*a^5*b^5)*x^6 + 6*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^5 + 15/4*(3*B*
a^8*b^2 + 8*A*a^7*b^3)*x^4 + 5/3*(2*B*a^9*b + 9*A*a^8*b^2)*x^3 + 1/2*(B*a^
10 + 10*A*a^9*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(32) = 64$.

Time = 0.05 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.53

$$\int (a+bx)^{10}(A+Bx)\,dx = Aa^{10}x + \frac{Bb^{10}x^{12}}{12} + x^{11}\left(\frac{Ab^{10}}{11} + \frac{10Bab^9}{11}\right) + x^{10}\left(Aab^9 + \frac{9Ba^2b^8}{2}\right) + x^9\cdot\left(5Aa^2b^8 + \frac{40Ba^3b^7}{3}\right) + x^8\cdot\left(15Aa^3b^7 + \frac{105Ba^4b^6}{4}\right) + x^7\cdot\left(30Aa^4b^6 + 36Ba^5b^5\right) + x^6\cdot\left(42Aa^5b^5 + 35Ba^6b^4\right) + x^5\cdot\left(42Aa^6b^4 + 24Ba^7b^3\right) + x^4\cdot\left(30Aa^7b^3 + \frac{45Ba^8b^2}{4}\right) + x^3\cdot\left(15Aa^8b^2 + \frac{10Ba^9b}{3}\right) + x^2\cdot\left(5Aa^9b + \frac{Ba^{10}}{2}\right)$$

input
```
integrate((b*x+a)**10*(B*x+A),x)
```

output
```
A*a**10*x + B*b**10*x**12/12 + x**11*(A*b**10/11 + 10*B*a*b**9/11) + x**10
*(A*a*b**9 + 9*B*a**2*b**8/2) + x**9*(5*A*a**2*b**8 + 40*B*a**3*b**7/3) +
x**8*(15*A*a**3*b**7 + 105*B*a**4*b**6/4) + x**7*(30*A*a**4*b**6 + 36*B*a*
*5*b**5) + x**6*(42*A*a**5*b**5 + 35*B*a**6*b**4) + x**5*(42*A*a**6*b**4 +
 24*B*a**7*b**3) + x**4*(30*A*a**7*b**3 + 45*B*a**8*b**2/4) + x**3*(15*A*a
**8*b**2 + 10*B*a**9*b/3) + x**2*(5*A*a**9*b + B*a**10/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 6.32

$$\int (a+bx)^{10}(A+Bx)\,dx = \frac{1}{12}Bb^{10}x^{12} + Aa^{10}x + \frac{1}{11}\left(10\,Bab^9 + Ab^{10}\right)x^{11} + \frac{1}{2}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^{10} + \frac{5}{3}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^9 + \frac{15}{4}\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^8 + 6\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^7 + 7\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^6 + 6\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^5 + \frac{15}{4}\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^4 + \frac{5}{3}\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^3 + \frac{1}{2}\left(Ba^{10} + 10\,Aa^9b\right)x^2$$

input
```
integrate((b*x+a)^10*(B*x+A),x, algorithm="maxima")
```

output
```
1/12*B*b^10*x^12 + A*a^10*x + 1/11*(10*B*a*b^9 + A*b^10)*x^11 + 1/2*(9*B*a
^2*b^8 + 2*A*a*b^9)*x^10 + 5/3*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^9 + 15/4*(7*B
*a^4*b^6 + 4*A*a^3*b^7)*x^8 + 6*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^7 + 7*(5*B*a
^6*b^4 + 6*A*a^5*b^5)*x^6 + 6*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^5 + 15/4*(3*B*
a^8*b^2 + 8*A*a^7*b^3)*x^4 + 5/3*(2*B*a^9*b + 9*A*a^8*b^2)*x^3 + 1/2*(B*a^
10 + 10*A*a^9*b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.34

$$\int (a+bx)^{10}(A+Bx)\,dx = \frac{1}{12}Bb^{10}x^{12} + \frac{10}{11}Bab^9x^{11} + \frac{1}{11}Ab^{10}x^{11} + \frac{9}{2}Ba^2b^8x^{10} + Aab^9x^{10} + \frac{40}{3}Ba^3b^7x^9 + 5\,Aa^2b^8x^9 + \frac{105}{4}Ba^4b^6x^8 + 15\,Aa^3b^7x^8 + 36\,Ba^5b^5x^7 + 30\,Aa^4b^6x^7 + 35\,Ba^6b^4x^6 + 42\,Aa^5b^5x^6 + 24\,Ba^7b^3x^5 + 42\,Aa^6b^4x^5 + \frac{45}{4}Ba^8b^2x^4 + 30\,Aa^7b^3x^4 + \frac{10}{3}Ba^9bx^3 + 15\,Aa^8b^2x^3 + \frac{1}{2}Ba^{10}x^2 + 5\,Aa^9bx^2 + Aa^{10}x$$

input
```
integrate((b*x+a)^10*(B*x+A),x, algorithm="giac")
```

output
```
1/12*B*b^10*x^12 + 10/11*B*a*b^9*x^11 + 1/11*A*b^10*x^11 + 9/2*B*a^2*b^8*x
^10 + A*a*b^9*x^10 + 40/3*B*a^3*b^7*x^9 + 5*A*a^2*b^8*x^9 + 105/4*B*a^4*b^
6*x^8 + 15*A*a^3*b^7*x^8 + 36*B*a^5*b^5*x^7 + 30*A*a^4*b^6*x^7 + 35*B*a^6*
b^4*x^6 + 42*A*a^5*b^5*x^6 + 24*B*a^7*b^3*x^5 + 42*A*a^6*b^4*x^5 + 45/4*B*
a^8*b^2*x^4 + 30*A*a^7*b^3*x^4 + 10/3*B*a^9*b*x^3 + 15*A*a^8*b^2*x^3 + 1/2
*B*a^10*x^2 + 5*A*a^9*b*x^2 + A*a^10*x
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 5.47

$$\int (a+bx)^{10}(A+Bx)\,dx = x^2\left(\frac{B\,a^{10}}{2}+5\,A\,b\,a^9\right)+x^{11}\left(\frac{A\,b^{10}}{11}+\frac{10\,B\,a\,b^9}{11}\right)+\frac{B\,b^{10}\,x^{12}}{12}+A\,a^{10}\,x+\frac{15\,a^7\,b^2\,x^4\,(8\,A\,b+3\,B\,a)}{4}+6\,a^6\,b^3\,x^5\,(7\,A\,b+4\,B\,a)+7\,a^5\,b^4\,x^6\,(6\,A\,b+5\,B\,a)+6\,a^4\,b^5\,x^7\,(5\,A\,b+6\,B\,a)+\frac{15\,a^3\,b^6\,x^8\,(4\,A\,b+7\,B\,a)}{4}+\frac{5\,a^2\,b^7\,x^9\,(3\,A\,b+8\,B\,a)}{3}+\frac{5\,a^8\,b\,x^3\,(9\,A\,b+2\,B\,a)}{3}+\frac{a\,b^8\,x^{10}\,(2\,A\,b+9\,B\,a)}{2}$$

input
```
int((A + B*x)*(a + b*x)^10,x)
```

output
```
x^2*((B*a^10)/2 + 5*A*a^9*b) + x^11*((A*b^10)/11 + (10*B*a*b^9)/11) + (B*b
^10*x^12)/12 + A*a^10*x + (15*a^7*b^2*x^4*(8*A*b + 3*B*a))/4 + 6*a^6*b^3*x
^5*(7*A*b + 4*B*a) + 7*a^5*b^4*x^6*(6*A*b + 5*B*a) + 6*a^4*b^5*x^7*(5*A*b
+ 6*B*a) + (15*a^3*b^6*x^8*(4*A*b + 7*B*a))/4 + (5*a^2*b^7*x^9*(3*A*b + 8*
B*a))/3 + (5*a^8*b*x^3*(9*A*b + 2*B*a))/3 + (a*b^8*x^10*(2*A*b + 9*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.16

$$\int (a+bx)^{10}(A+Bx)\,dx$$
$$=\frac{x(b^{11}x^{11}+12a\,b^{10}x^{10}+66a^2b^9x^9+220a^3b^8x^8+495a^4b^7x^7+792a^5b^6x^6+924a^6b^5x^5+792a^7b^4x^4+495}{12}$$

input
```
int((b*x+a)^10*(B*x+A),x)
```

output
```
(x*(12*a**11 + 66*a**10*b*x + 220*a**9*b**2*x**2 + 495*a**8*b**3*x**3 + 79
2*a**7*b**4*x**4 + 924*a**6*b**5*x**5 + 792*a**5*b**6*x**6 + 495*a**4*b**7
*x**7 + 220*a**3*b**8*x**8 + 66*a**2*b**9*x**9 + 12*a*b**10*x**10 + b**11*
x**11))/12
```

3.117 $\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx &= 10a^9Abx+\frac{45}{2}a^8Ab^2x^2+40a^7Ab^3x^3+\frac{105}{2}a^6Ab^4x^4\\&\quad+\frac{252}{5}a^5Ab^5x^5+35a^4Ab^6x^6+\frac{120}{7}a^3Ab^7x^7+\frac{45}{8}a^2Ab^8x^8\\&\quad+\frac{10}{9}aAb^9x^9+\frac{1}{10}Ab^{10}x^{10}+\frac{B(a+bx)^{11}}{11b}+a^{10}A\log(x)\end{aligned}$$

output
```
10*a^9*A*b*x+45/2*a^8*A*b^2*x^2+40*a^7*A*b^3*x^3+105/2*a^6*A*b^4*x^4+252/5
*a^5*A*b^5*x^5+35*a^4*A*b^6*x^6+120/7*a^3*A*b^7*x^7+45/8*a^2*A*b^8*x^8+10/
9*a*A*b^9*x^9+1/10*A*b^10*x^10+1/11*B*(b*x+a)^11/b+a^10*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = a^{10}Bx + 5a^9bx(2A+Bx) + \frac{15}{2}a^8b^2x^2(3A+2Bx) + 10a^7b^3x^3(4A+3Bx) + \frac{21}{2}a^6b^4x^4(5A+4Bx) + \frac{42}{5}a^5b^5x^5(6A+5Bx) + 5a^4b^6x^6(7A+6Bx) + \frac{15}{7}a^3b^7x^7(8A+7Bx) + \frac{5}{8}a^2b^8x^8(9A+8Bx) + \frac{1}{9}ab^9x^9(10A+9Bx) + \frac{1}{110}b^{10}x^{10}(11A+10Bx) + a^{10}A\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x,x]
```

output
```
a^10*B*x + 5*a^9*b*x*(2*A + B*x) + (15*a^8*b^2*x^2*(3*A + 2*B*x))/2 + 10*a
^7*b^3*x^3*(4*A + 3*B*x) + (21*a^6*b^4*x^4*(5*A + 4*B*x))/2 + (42*a^5*b^5*
x^5*(6*A + 5*B*x))/5 + 5*a^4*b^6*x^6*(7*A + 6*B*x) + (15*a^3*b^7*x^7*(8*A
+ 7*B*x))/7 + (5*a^2*b^8*x^8*(9*A + 8*B*x))/8 + (a*b^9*x^9*(10*A + 9*B*x))
/9 + (b^10*x^10*(11*A + 10*B*x))/110 + a^10*A*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx$$

$$\Big\downarrow \; 90$$

$$A\int \frac{(a+bx)^{10}}{x}dx+\frac{B(a+bx)^{11}}{11b}$$

$\big\downarrow$ 49

$$A\int \left(\frac{a^{10}}{x}+10ba^9+45b^2xa^8+120b^3x^2a^7+210b^4x^3a^6+252b^5x^4a^5+210b^6x^5a^4+120b^7x^6a^3+45b^8x^7a^2+10\right.$$
$$\frac{B(a+bx)^{11}}{11b}$$

$\big\downarrow$ 2009

$$A\left(a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8\right.$$
$$\frac{B(a+bx)^{11}}{11b}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x,x]
```

output
```
(B*(a + b*x)^11)/(11*b) + A*(10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*
x^3 + (105*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^
3*b^7*x^7)/7 + (45*a^2*b^8*x^8)/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^
10*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.55

method	result
norman	$\left(\frac{1}{10}b^{10}A+a\,b^{9}B\right)x^{10}+\left(\frac{10}{9}a\,b^{9}A+5a^{2}b^{8}B\right)x^{9}+\left(\frac{45}{8}a^{2}b^{8}A+15a^{3}b^{7}B\right)x^{8}+\left(\frac{120}{7}a^{3}b^{7}A+\right.$
default	$\frac{B\,b^{10}x^{11}}{11}+\frac{A\,b^{10}x^{10}}{10}+Ba\,b^{9}x^{10}+\frac{10aA\,b^{9}x^{9}}{9}+5B\,a^{2}b^{8}x^{9}+\frac{45a^{2}A\,b^{8}x^{8}}{8}+15B\,a^{3}b^{7}x^{8}+\frac{120a^{3}A\,b^{7}x^{7}}{7}$
risch	$\frac{B\,b^{10}x^{11}}{11}+\frac{A\,b^{10}x^{10}}{10}+Ba\,b^{9}x^{10}+\frac{10aA\,b^{9}x^{9}}{9}+5B\,a^{2}b^{8}x^{9}+\frac{45a^{2}A\,b^{8}x^{8}}{8}+15B\,a^{3}b^{7}x^{8}+\frac{120a^{3}A\,b^{7}x^{7}}{7}$
parallelrisch	$\frac{B\,b^{10}x^{11}}{11}+\frac{A\,b^{10}x^{10}}{10}+Ba\,b^{9}x^{10}+\frac{10aA\,b^{9}x^{9}}{9}+5B\,a^{2}b^{8}x^{9}+\frac{45a^{2}A\,b^{8}x^{8}}{8}+15B\,a^{3}b^{7}x^{8}+\frac{120a^{3}A\,b^{7}x^{7}}{7}$

input
```
int((b*x+a)^10*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/10*b^10*A+a*b^9*B)*x^10+(10/9*a*b^9*A+5*a^2*b^8*B)*x^9+(45/8*a^2*b^8*A+
15*a^3*b^7*B)*x^8+(120/7*a^3*b^7*A+30*a^4*b^6*B)*x^7+(252/5*a^5*b^5*A+42*a
^6*b^4*B)*x^5+(105/2*a^6*b^4*A+30*a^7*b^3*B)*x^4+(45/2*a^8*b^2*A+5*a^9*b*B
)*x^2+(35*A*a^4*b^6+42*B*a^5*b^5)*x^6+(40*A*a^7*b^3+15*B*a^8*b^2)*x^3+(10*
A*a^9*b+B*a^10)*x+1/11*B*b^10*x^11+a^10*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.61

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = {} & \frac{1}{11}\,Bb^{10}x^{11}+Aa^{10}\log\left(x\right)+\frac{1}{10}\left(10\,Bab^{9}+Ab^{10}\right)x^{10}\\ & +\frac{5}{9}\left(9\,Ba^{2}b^{8}+2\,Aab^{9}\right)x^{9}+\frac{15}{8}\left(8\,Ba^{3}b^{7}+3\,Aa^{2}b^{8}\right)x^{8}\\ & +\frac{30}{7}\left(7\,Ba^{4}b^{6}+4\,Aa^{3}b^{7}\right)x^{7}\\ & +7\left(6\,Ba^{5}b^{5}+5\,Aa^{4}b^{6}\right)x^{6}+\frac{42}{5}\left(5\,Ba^{6}b^{4}+6\,Aa^{5}b^{5}\right)x^{5}\\ & +\frac{15}{2}\left(4\,Ba^{7}b^{3}+7\,Aa^{6}b^{4}\right)x^{4}+5\left(3\,Ba^{8}b^{2}+8\,Aa^{7}b^{3}\right)x^{3}\\ & +\frac{5}{2}\left(2\,Ba^{9}b+9\,Aa^{8}b^{2}\right)x^{2}+\left(Ba^{10}+10\,Aa^{9}b\right)x\end{aligned}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x,x, algorithm="fricas")
```

output
```
1/11*B*b^10*x^11 + A*a^10*log(x) + 1/10*(10*B*a*b^9 + A*b^10)*x^10 + 5/9*(
9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 15/8*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 30/7
*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 7*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 42/
5*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 15/2*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 +
 5*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 5/2*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + (
B*a^10 + 10*A*a^9*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = Aa^{10}\log(x) + \frac{Bb^{10}x^{11}}{11} + x^{10}\left(\frac{Ab^{10}}{10} + Bab^9\right) + x^9 \cdot \left(\frac{10Aab^9}{9} + 5Ba^2b^8\right) + x^8 \cdot \left(\frac{45Aa^2b^8}{8} + 15Ba^3b^7\right) + x^7 \cdot \left(\frac{120Aa^3b^7}{7} + 30Ba^4b^6\right) + x^6 \cdot \left(35Aa^4b^6 + 42Ba^5b^5\right) + x^5 \cdot \left(\frac{252Aa^5b^5}{5} + 42Ba^6b^4\right) + x^4 \cdot \left(\frac{105Aa^6b^4}{2} + 30Ba^7b^3\right) + x^3 \cdot \left(40Aa^7b^3 + 15Ba^8b^2\right) + x^2 \cdot \left(\frac{45Aa^8b^2}{2} + 5Ba^9b\right) + x\left(10Aa^9b + Ba^{10}\right)$$

input
```
integrate((b*x+a)**10*(B*x+A)/x,x)
```

output
```
A*a**10*log(x) + B*b**10*x**11/11 + x**10*(A*b**10/10 + B*a*b**9) + x**9*(
10*A*a*b**9/9 + 5*B*a**2*b**8) + x**8*(45*A*a**2*b**8/8 + 15*B*a**3*b**7)
+ x**7*(120*A*a**3*b**7/7 + 30*B*a**4*b**6) + x**6*(35*A*a**4*b**6 + 42*B*
a**5*b**5) + x**5*(252*A*a**5*b**5/5 + 42*B*a**6*b**4) + x**4*(105*A*a**6*
b**4/2 + 30*B*a**7*b**3) + x**3*(40*A*a**7*b**3 + 15*B*a**8*b**2) + x**2*(
45*A*a**8*b**2/2 + 5*B*a**9*b) + x*(10*A*a**9*b + B*a**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = \frac{1}{11}\,Bb^{10}x^{11} + Aa^{10}\log(x) + \frac{1}{10}\left(10\,Bab^9 + Ab^{10}\right)x^{10} + \frac{5}{9}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^9 + \frac{15}{8}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^8 + \frac{30}{7}\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^7 + 7\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^6 + \frac{42}{5}\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^5 + \frac{15}{2}\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^4 + 5\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^3 + \frac{5}{2}\left(2\,Ba^9b + 9\,Aa^8b^2\right)x^2 + \left(Ba^{10} + 10\,Aa^9b\right)x$$

input

```
integrate((b*x+a)^10*(B*x+A)/x,x, algorithm="maxima")
```

output

```
1/11*B*b^10*x^11 + A*a^10*log(x) + 1/10*(10*B*a*b^9 + A*b^10)*x^10 + 5/9*(
9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 15/8*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 30/7
*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 7*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 42/
5*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 15/2*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 +
 5*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 5/2*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + (
B*a^10 + 10*A*a^9*b)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = \frac{1}{11}\,Bb^{10}x^{11} + Bab^9x^{10} + \frac{1}{10}\,Ab^{10}x^{10} + 5\,Ba^2b^8x^9 + \frac{10}{9}\,Aab^9x^9 + 15\,Ba^3b^7x^8 + \frac{45}{8}\,Aa^2b^8x^8 + 30\,Ba^4b^6x^7 + \frac{120}{7}\,Aa^3b^7x^7 + 42\,Ba^5b^5x^6 + 35\,Aa^4b^6x^6 + 42\,Ba^6b^4x^5 + \frac{252}{5}\,Aa^5b^5x^5 + 30\,Ba^7b^3x^4 + \frac{105}{2}\,Aa^6b^4x^4 + 15\,Ba^8b^2x^3 + 40\,Aa^7b^3x^3 + 5\,Ba^9bx^2 + \frac{45}{2}\,Aa^8b^2x^2 + Ba^{10}x + 10\,Aa^9bx + Aa^{10}\log(|x|)$$

input
```
integrate((b*x+a)^10*(B*x+A)/x,x, algorithm="giac")
```

output
```
1/11*B*b^10*x^11 + B*a*b^9*x^10 + 1/10*A*b^10*x^10 + 5*B*a^2*b^8*x^9 + 10/
9*A*a*b^9*x^9 + 15*B*a^3*b^7*x^8 + 45/8*A*a^2*b^8*x^8 + 30*B*a^4*b^6*x^7 +
 120/7*A*a^3*b^7*x^7 + 42*B*a^5*b^5*x^6 + 35*A*a^4*b^6*x^6 + 42*B*a^6*b^4*
x^5 + 252/5*A*a^5*b^5*x^5 + 30*B*a^7*b^3*x^4 + 105/2*A*a^6*b^4*x^4 + 15*B*
a^8*b^2*x^3 + 40*A*a^7*b^3*x^3 + 5*B*a^9*b*x^2 + 45/2*A*a^8*b^2*x^2 + B*a^
10*x + 10*A*a^9*b*x + A*a^10*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = x\left(B\,a^{10}+10\,A\,b\,a^9\right)+x^{10}\left(\frac{A\,b^{10}}{10}+B\,a\,b^9\right) + \frac{B\,b^{10}\,x^{11}}{11}+A\,a^{10}\,\ln\left(x\right)+5\,a^7\,b^2\,x^3\,\left(8\,A\,b+3\,B\,a\right) + \frac{15\,a^6\,b^3\,x^4\,\left(7\,A\,b+4\,B\,a\right)}{2} + \frac{42\,a^5\,b^4\,x^5\,\left(6\,A\,b+5\,B\,a\right)}{5}+7\,a^4\,b^5\,x^6\,\left(5\,A\,b+6\,B\,a\right) + \frac{30\,a^3\,b^6\,x^7\,\left(4\,A\,b+7\,B\,a\right)}{7}+\frac{15\,a^2\,b^7\,x^8\,\left(3\,A\,b+8\,B\,a\right)}{8} + \frac{5\,a^8\,b\,x^2\,\left(9\,A\,b+2\,B\,a\right)}{2}+\frac{5\,a\,b^8\,x^9\,\left(2\,A\,b+9\,B\,a\right)}{9}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x,x)
```

output
```
x*(B*a^10 + 10*A*a^9*b) + x^10*((A*b^10)/10 + B*a*b^9) + (B*b^10*x^11)/11
+ A*a^10*log(x) + 5*a^7*b^2*x^3*(8*A*b + 3*B*a) + (15*a^6*b^3*x^4*(7*A*b +
 4*B*a))/2 + (42*a^5*b^4*x^5*(6*A*b + 5*B*a))/5 + 7*a^4*b^5*x^6*(5*A*b + 6
*B*a) + (30*a^3*b^6*x^7*(4*A*b + 7*B*a))/7 + (15*a^2*b^7*x^8*(3*A*b + 8*B*
a))/8 + (5*a^8*b*x^2*(9*A*b + 2*B*a))/2 + (5*a*b^8*x^9*(2*A*b + 9*B*a))/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{10}(A+Bx)}{x}\,dx = \log(x)\,a^{11}+11a^{10}bx+\frac{55a^9b^2x^2}{2}+55a^8b^3x^3+\frac{165a^7b^4x^4}{2}+\frac{462a^6b^5x^5}{5}+77a^5b^6x^6+\frac{330a^4b^7x^7}{7}+\frac{165a^3b^8x^8}{8}+\frac{55a^2b^9x^9}{9}+\frac{11a\,b^{10}x^{10}}{10}+\frac{b^{11}x^{11}}{11}$$

input
```
int((b*x+a)^10*(B*x+A)/x,x)
```

output
```
(27720*log(x)*a**11 + 304920*a**10*b*x + 762300*a**9*b**2*x**2 + 1524600*a
**8*b**3*x**3 + 2286900*a**7*b**4*x**4 + 2561328*a**6*b**5*x**5 + 2134440*
a**5*b**6*x**6 + 1306800*a**4*b**7*x**7 + 571725*a**3*b**8*x**8 + 169400*a
**2*b**9*x**9 + 30492*a*b**10*x**10 + 2520*b**11*x**11)/27720
```

3.118 $\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 217

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx = & -\frac{a^{10}A}{x}+5a^8b(9Ab+2aB)x+\frac{15}{2}a^7b^2(8Ab+3aB)x^2\\ & +10a^6b^3(7Ab+4aB)x^3+\frac{21}{2}a^5b^4(6Ab+5aB)x^4\\ & +\frac{42}{5}a^4b^5(5Ab+6aB)x^5+5a^3b^6(4Ab+7aB)x^6\\ & +\frac{15}{7}a^2b^7(3Ab+8aB)x^7+\frac{5}{8}ab^8(2Ab+9aB)x^8\\ & +\frac{1}{9}b^9(Ab+10aB)x^9+\frac{1}{10}b^{10}Bx^{10}+a^9(10Ab+aB)\log(x)\end{aligned}$$

output
```
-a^10*A/x+5*a^8*b*(9*A*b+2*B*a)*x+15/2*a^7*b^2*(8*A*b+3*B*a)*x^2+10*a^6*b^
3*(7*A*b+4*B*a)*x^3+21/2*a^5*b^4*(6*A*b+5*B*a)*x^4+42/5*a^4*b^5*(5*A*b+6*B
*a)*x^5+5*a^3*b^6*(4*A*b+7*B*a)*x^6+15/7*a^2*b^7*(3*A*b+8*B*a)*x^7+5/8*a*b
^8*(2*A*b+9*B*a)*x^8+1/9*b^9*(A*b+10*B*a)*x^9+1/10*b^10*B*x^10+a^9*(10*A*b
+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx = -\frac{a^{10}A}{x} + 10a^9bBx + \frac{45}{2}a^8b^2x(2A+Bx) + 20a^7b^3x^2(3A+2Bx) + \frac{35}{2}a^6b^4x^3(4A+3Bx) + \frac{63}{5}a^5b^5x^4(5A+4Bx) + 7a^4b^6x^5(6A+5Bx) + \frac{20}{7}a^3b^7x^6(7A+6Bx) + \frac{45}{56}a^2b^8x^7(8A+7Bx) + \frac{5}{36}ab^9x^8(9A+8Bx) + \frac{1}{90}b^{10}x^9(10A+9Bx) + a^9(10Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^2,x]
```

output
```
-((a^10*A)/x) + 10*a^9*b*B*x + (45*a^8*b^2*x*(2*A + B*x))/2 + 20*a^7*b^3*x
^2*(3*A + 2*B*x) + (35*a^6*b^4*x^3*(4*A + 3*B*x))/2 + (63*a^5*b^5*x^4*(5*A
 + 4*B*x))/5 + 7*a^4*b^6*x^5*(6*A + 5*B*x) + (20*a^3*b^7*x^6*(7*A + 6*B*x)
)/7 + (45*a^2*b^8*x^7*(8*A + 7*B*x))/56 + (5*a*b^9*x^8*(9*A + 8*B*x))/36 +
 (b^10*x^9*(10*A + 9*B*x))/90 + a^9*(10*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^{10}A}{x^2} + \frac{a^9(aB+10Ab)}{x} + 5a^8b(2aB+9Ab) + 15a^7b^2x(3aB+8Ab) + 30a^6b^3x^2(4aB+7Ab) + 42a^5b^4x^3(5a$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^{10}A}{x} + a^9\log(x)(aB+10Ab) + 5a^8bx(2aB+9Ab) + \frac{15}{2}a^7b^2x^2(3aB+8Ab) + 10a^6b^3x^3(4aB+7Ab) + \frac{21}{2}a^5b^4x^4(5aB+6Ab) + \frac{42}{5}a^4b^5x^5(6aB+5Ab) + 5a^3b^6x^6(7aB+4Ab) + \frac{15}{7}a^2b^7x^7(8aB+3Ab) + \frac{1}{9}b^9x^9(10aB+Ab) + \frac{5}{8}ab^8x^8(9aB+2Ab) + \frac{1}{10}b^{10}Bx^{10}$$

input

```
Int[((a + b*x)^10*(A + B*x))/x^2,x]
```

output

```
-((a^10*A)/x) + 5*a^8*b*(9*A*b + 2*a*B)*x + (15*a^7*b^2*(8*A*b + 3*a*B)*x^
2)/2 + 10*a^6*b^3*(7*A*b + 4*a*B)*x^3 + (21*a^5*b^4*(6*A*b + 5*a*B)*x^4)/2
 + (42*a^4*b^5*(5*A*b + 6*a*B)*x^5)/5 + 5*a^3*b^6*(4*A*b + 7*a*B)*x^6 + (1
5*a^2*b^7*(3*A*b + 8*a*B)*x^7)/7 + (5*a*b^8*(2*A*b + 9*a*B)*x^8)/8 + (b^9*
(A*b + 10*a*B)*x^9)/9 + (b^10*B*x^10)/10 + a^9*(10*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 85

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.09

method	result
norman	$\frac{\left(\frac{1}{9} b^{10} A+\frac{10}{9} a\, b^{9} B\right) x^{10}+\left(\frac{5}{4} a\, b^{9} A+\frac{45}{8} a^{2} b^{8} B\right) x^{9}+\left(\frac{45}{7} a^{2} b^{8} A+\frac{120}{7} a^{3} b^{7} B\right) x^{8}+\left(42 a^{4} b^{6} A+\frac{252}{5} a^{5} b^{5} B\right) x^{6}+\left(63 a^{5} b^{5} A+\frac{105}{2} a^{6} b^{4}\right.}{}$
default	$\frac{b^{10} B\, x^{10}}{10}+\frac{A\, b^{10} x^{9}}{9}+\frac{10 B a\, b^{9} x^{9}}{9}+\frac{5 A a\, b^{9} x^{8}}{4}+\frac{45 B\, a^{2} b^{8} x^{8}}{8}+\frac{45 A\, a^{2} b^{8} x^{7}}{7}+\frac{120 B\, a^{3} b^{7} x^{7}}{7}+20 A\, a^{3} b^{7} x^{6}+$
risch	$\frac{b^{10} B\, x^{10}}{10}+\frac{A\, b^{10} x^{9}}{9}+\frac{10 B a\, b^{9} x^{9}}{9}+\frac{5 A a\, b^{9} x^{8}}{4}+\frac{45 B\, a^{2} b^{8} x^{8}}{8}+\frac{45 A\, a^{2} b^{8} x^{7}}{7}+\frac{120 B\, a^{3} b^{7} x^{7}}{7}+20 A\, a^{3} b^{7} x^{6}+$
parallelrisch	$\frac{252 B\, b^{10} x^{11}+280 A\, b^{10} x^{10}+2800 B a\, b^{9} x^{10}+3150 a A\, b^{9} x^{9}+14175 B\, a^{2} b^{8} x^{9}+16200 a^{2} A\, b^{8} x^{8}+43200 B\, a^{3} b^{7} x^{8}+50400 a^{3} A\, b^{7} x}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((1/9*b^10*A+10/9*a*b^9*B)*x^10+(5/4*a*b^9*A+45/8*a^2*b^8*B)*x^9+(45/7*a^2
*b^8*A+120/7*a^3*b^7*B)*x^8+(42*a^4*b^6*A+252/5*a^5*b^5*B)*x^6+(63*a^5*b^5
*A+105/2*a^6*b^4*B)*x^5+(60*a^7*b^3*A+45/2*a^8*b^2*B)*x^3+(20*A*a^3*b^7+35
*B*a^4*b^6)*x^7+(70*A*a^6*b^4+40*B*a^7*b^3)*x^4+(45*A*a^8*b^2+10*B*a^9*b)*
x^2-a^10*A+1/10*B*b^10*x^11)/x+(10*A*a^9*b+B*a^10)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx$$
$$= \frac{252\,Bb^{10}x^{11}-2520\,Aa^{10}+280\,(10\,Bab^{9}+Ab^{10})x^{10}+1575\,(9\,Ba^{2}b^{8}+2\,Aab^{9})x^{9}+5400\,(8\,Ba^{3}b^{7}+3\,A}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/2520*(252*B*b^10*x^11 - 2520*A*a^10 + 280*(10*B*a*b^9 + A*b^10)*x^10 + 1
575*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 5400*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 +
 12600*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 21168*(6*B*a^5*b^5 + 5*A*a^4*b^6)
*x^6 + 26460*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 25200*(4*B*a^7*b^3 + 7*A*a^
6*b^4)*x^4 + 18900*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 12600*(2*B*a^9*b + 9*
A*a^8*b^2)*x^2 + 2520*(B*a^10 + 10*A*a^9*b)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx = -\frac{Aa^{10}}{x} + \frac{Bb^{10}x^{10}}{10} + a^9\cdot(10Ab+Ba)\log(x) + x^9\left(\frac{Ab^{10}}{9}+\frac{10Bab^9}{9}\right) + x^8\cdot\left(\frac{5Aab^9}{4}+\frac{45Ba^2b^8}{8}\right) + x^7\cdot\left(\frac{45Aa^2b^8}{7}+\frac{120Ba^3b^7}{7}\right) + x^6\cdot\left(20Aa^3b^7+35Ba^4b^6\right) + x^5\cdot\left(42Aa^4b^6+\frac{252Ba^5b^5}{5}\right) + x^4\cdot\left(63Aa^5b^5+\frac{105Ba^6b^4}{2}\right) + x^3\cdot\left(70Aa^6b^4+40Ba^7b^3\right) + x^2\cdot\left(60Aa^7b^3+\frac{45Ba^8b^2}{2}\right) + x\left(45Aa^8b^2+10Ba^9b\right)$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**2,x)
```

output
```
-A*a**10/x + B*b**10*x**10/10 + a**9*(10*A*b + B*a)*log(x) + x**9*(A*b**10
/9 + 10*B*a*b**9/9) + x**8*(5*A*a*b**9/4 + 45*B*a**2*b**8/8) + x**7*(45*A*
a**2*b**8/7 + 120*B*a**3*b**7/7) + x**6*(20*A*a**3*b**7 + 35*B*a**4*b**6)
+ x**5*(42*A*a**4*b**6 + 252*B*a**5*b**5/5) + x**4*(63*A*a**5*b**5 + 105*B
*a**6*b**4/2) + x**3*(70*A*a**6*b**4 + 40*B*a**7*b**3) + x**2*(60*A*a**7*b
**3 + 45*B*a**8*b**2/2) + x*(45*A*a**8*b**2 + 10*B*a**9*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx = \frac{1}{10}\,Bb^{10}x^{10} - \frac{Aa^{10}}{x} + \frac{1}{9}\left(10\,Bab^9 + Ab^{10}\right)x^9 + \frac{5}{8}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^8 + \frac{15}{7}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^7 + 5\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^6 + \frac{42}{5}\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^5 + \frac{21}{2}\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^4 + 10\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^3 + \frac{15}{2}\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x^2 + 5\left(2\,Ba^9b + 9\,Aa^8b^2\right)x + \left(Ba^{10} + 10\,Aa^9b\right)\log\left(x\right)$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/10*B*b^10*x^10 - A*a^10/x + 1/9*(10*B*a*b^9 + A*b^10)*x^9 + 5/8*(9*B*a^2
*b^8 + 2*A*a*b^9)*x^8 + 15/7*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^7 + 5*(7*B*a^4*
b^6 + 4*A*a^3*b^7)*x^6 + 42/5*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^5 + 21/2*(5*B*
a^6*b^4 + 6*A*a^5*b^5)*x^4 + 10*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^3 + 15/2*(3*
B*a^8*b^2 + 8*A*a^7*b^3)*x^2 + 5*(2*B*a^9*b + 9*A*a^8*b^2)*x + (B*a^10 + 1
0*A*a^9*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx = \frac{1}{10}\,Bb^{10}x^{10} + \frac{10}{9}\,Bab^9x^9 + \frac{1}{9}\,Ab^{10}x^9 + \frac{45}{8}\,Ba^2b^8x^8 + \frac{5}{4}\,Aab^9x^8 + \frac{120}{7}\,Ba^3b^7x^7 + \frac{45}{7}\,Aa^2b^8x^7 + 35\,Ba^4b^6x^6 + 20\,Aa^3b^7x^6 + \frac{252}{5}\,Ba^5b^5x^5 + 42\,Aa^4b^6x^5 + \frac{105}{2}\,Ba^6b^4x^4 + 63\,Aa^5b^5x^4 + 40\,Ba^7b^3x^3 + 70\,Aa^6b^4x^3 + \frac{45}{2}\,Ba^8b^2x^2 + 60\,Aa^7b^3x^2 + 10\,Ba^9bx + 45\,Aa^8b^2x - \frac{Aa^{10}}{x} + \left(Ba^{10} + 10\,Aa^9b\right)\log\left(|x|\right)$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/10*B*b^10*x^10 + 10/9*B*a*b^9*x^9 + 1/9*A*b^10*x^9 + 45/8*B*a^2*b^8*x^8
+ 5/4*A*a*b^9*x^8 + 120/7*B*a^3*b^7*x^7 + 45/7*A*a^2*b^8*x^7 + 35*B*a^4*b^
6*x^6 + 20*A*a^3*b^7*x^6 + 252/5*B*a^5*b^5*x^5 + 42*A*a^4*b^6*x^5 + 105/2*
B*a^6*b^4*x^4 + 63*A*a^5*b^5*x^4 + 40*B*a^7*b^3*x^3 + 70*A*a^6*b^4*x^3 + 4
5/2*B*a^8*b^2*x^2 + 60*A*a^7*b^3*x^2 + 10*B*a^9*b*x + 45*A*a^8*b^2*x - A*a
^10/x + (B*a^10 + 10*A*a^9*b)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx = x^9\left(\frac{A\,b^{10}}{9}+\frac{10\,B\,a\,b^9}{9}\right)+\ln(x)\,\left(B\,a^{10}+10\,A\,b\,a^9\right) - \frac{A\,a^{10}}{x}+\frac{B\,b^{10}\,x^{10}}{10}+\frac{15\,a^7\,b^2\,x^2\,(8\,A\,b+3\,B\,a)}{2} + 10\,a^6\,b^3\,x^3\,(7\,A\,b+4\,B\,a)+\frac{21\,a^5\,b^4\,x^4\,(6\,A\,b+5\,B\,a)}{2} + \frac{42\,a^4\,b^5\,x^5\,(5\,A\,b+6\,B\,a)}{5} + 5\,a^3\,b^6\,x^6\,(4\,A\,b+7\,B\,a)+\frac{15\,a^2\,b^7\,x^7\,(3\,A\,b+8\,B\,a)}{7} + 5\,a^8\,b\,x\,(9\,A\,b+2\,B\,a)+\frac{5\,a\,b^8\,x^8\,(2\,A\,b+9\,B\,a)}{8}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^2,x)
```

output
```
x^9*((A*b^10)/9 + (10*B*a*b^9)/9) + log(x)*(B*a^10 + 10*A*a^9*b) - (A*a^10
)/x + (B*b^10*x^10)/10 + (15*a^7*b^2*x^2*(8*A*b + 3*B*a))/2 + 10*a^6*b^3*x
^3*(7*A*b + 4*B*a) + (21*a^5*b^4*x^4*(6*A*b + 5*B*a))/2 + (42*a^4*b^5*x^5*
(5*A*b + 6*B*a))/5 + 5*a^3*b^6*x^6*(4*A*b + 7*B*a) + (15*a^2*b^7*x^7*(3*A*
b + 8*B*a))/7 + 5*a^8*b*x*(9*A*b + 2*B*a) + (5*a*b^8*x^8*(2*A*b + 9*B*a))/
8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^2}\,dx$$
$$= \frac{27720\log(x)\,a^{10}bx - 2520a^{11} + 138600a^9b^2x^2 + 207900a^8b^3x^3 + 277200a^7b^4x^4 + 291060a^6b^5x^5 + 23284}{2520x}$$

input
```
int((b*x+a)^10*(B*x+A)/x^2,x)
```

output
```
(27720*log(x)*a**10*b*x - 2520*a**11 + 138600*a**9*b**2*x**2 + 207900*a**8
*b**3*x**3 + 277200*a**7*b**4*x**4 + 291060*a**6*b**5*x**5 + 232848*a**5*b
**6*x**6 + 138600*a**4*b**7*x**7 + 59400*a**3*b**8*x**8 + 17325*a**2*b**9*
x**9 + 3080*a*b**10*x**10 + 252*b**11*x**11)/(2520*x)
```

3.119 $\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 216

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx = &-\frac{a^{10}A}{2x^2}-\frac{a^9(10Ab+aB)}{x}+15a^7b^2(8Ab+3aB)x\\&+15a^6b^3(7Ab+4aB)x^2+14a^5b^4(6Ab+5aB)x^3\\&+\frac{21}{2}a^4b^5(5Ab+6aB)x^4+6a^3b^6(4Ab+7aB)x^5\\&+\frac{5}{2}a^2b^7(3Ab+8aB)x^6+\frac{5}{7}ab^8(2Ab+9aB)x^7\\&+\frac{1}{8}b^9(Ab+10aB)x^8+\frac{1}{9}b^{10}Bx^9+5a^8b(9Ab+2aB)\log(x)\end{aligned}$$

output
```
-1/2*a^10*A/x^2-a^9*(10*A*b+B*a)/x+15*a^7*b^2*(8*A*b+3*B*a)*x+15*a^6*b^3*(
7*A*b+4*B*a)*x^2+14*a^5*b^4*(6*A*b+5*B*a)*x^3+21/2*a^4*b^5*(5*A*b+6*B*a)*x
^4+6*a^3*b^6*(4*A*b+7*B*a)*x^5+5/2*a^2*b^7*(3*A*b+8*B*a)*x^6+5/7*a*b^8*(2*
A*b+9*B*a)*x^7+1/8*b^9*(A*b+10*B*a)*x^8+1/9*b^10*B*x^9+5*a^8*b*(9*A*b+2*B*
a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx = -\frac{10a^9Ab}{x}+45a^8b^2Bx+60a^7b^3x(2A+Bx)-\frac{a^{10}(A+2Bx)}{2x^2}+35a^6b^4x^2(3A+2Bx)+21a^5b^5x^3(4A+3Bx)+\frac{21}{2}a^4b^6x^4(5A+4Bx)+4a^3b^7x^5(6A+5Bx)+\frac{15}{14}a^2b^8x^6(7A+6Bx)+\frac{5}{28}ab^9x^7(8A+7Bx)+\frac{1}{72}b^{10}x^8(9A+8Bx)+5a^8b(9Ab+2aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^3,x]
```

output
```
(-10*a^9*A*b)/x + 45*a^8*b^2*B*x + 60*a^7*b^3*x*(2*A + B*x) - (a^10*(A + 2
*B*x))/(2*x^2) + 35*a^6*b^4*x^2*(3*A + 2*B*x) + 21*a^5*b^5*x^3*(4*A + 3*B*
x) + (21*a^4*b^6*x^4*(5*A + 4*B*x))/2 + 4*a^3*b^7*x^5*(6*A + 5*B*x) + (15*
a^2*b^8*x^6*(7*A + 6*B*x))/14 + (5*a*b^9*x^7*(8*A + 7*B*x))/28 + (b^10*x^8
*(9*A + 8*B*x))/72 + 5*a^8*b*(9*A*b + 2*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^3}+\frac{a^9(aB+10Ab)}{x^2}+\frac{5a^8b(2aB+9Ab)}{x}+15a^7b^2(3aB+8Ab)+30a^6b^3x(4aB+7Ab)+42a^5b^4x^2(5aB\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{2x^2}-\frac{a^9(aB+10Ab)}{x}+5a^8b\log(x)(2aB+9Ab)+15a^7b^2x(3aB+8Ab)+15a^6b^3x^2(4aB+7Ab)+14a^5b^4x^3(5aB+6Ab)+\frac{21}{2}a^4b^5x^4(6aB+5Ab)+6a^3b^6x^5(7aB+4Ab)+\frac{5}{2}a^2b^7x^6(8aB+3Ab)+\frac{1}{8}b^9x^8(10aB+Ab)+\frac{5}{7}ab^8x^7(9aB+2Ab)+\frac{1}{9}b^{10}Bx^9$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^3,x]
```

output
```
-1/2*(a^10*A)/x^2 - (a^9*(10*A*b + a*B))/x + 15*a^7*b^2*(8*A*b + 3*a*B)*x
+ 15*a^6*b^3*(7*A*b + 4*a*B)*x^2 + 14*a^5*b^4*(6*A*b + 5*a*B)*x^3 + (21*a^
4*b^5*(5*A*b + 6*a*B)*x^4)/2 + 6*a^3*b^6*(4*A*b + 7*a*B)*x^5 + (5*a^2*b^7*
(3*A*b + 8*a*B)*x^6)/2 + (5*a*b^8*(2*A*b + 9*a*B)*x^7)/7 + (b^9*(A*b + 10*
a*B)*x^8)/8 + (b^10*B*x^9)/9 + 5*a^8*b*(9*A*b + 2*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.08

method	result
default	$\frac{b^{10} B x^9}{9}+\frac{A b^{10} x^8}{8}+\frac{5 B a b^9 x^8}{4}+\frac{10 A a b^9 x^7}{7}+\frac{45 B a^2 b^8 x^7}{7}+\frac{15 A a^2 b^8 x^6}{2}+20 B a^3 b^7 x^6+24 A a^3 b^7 x^5+$
norman	$\frac{(\frac{1}{8} b^{10} A+\frac{5}{4} a b^9 B) x^{10}+(\frac{10}{7} a b^9 A+\frac{45}{7} a^2 b^8 B) x^9+(\frac{15}{2} a^2 b^8 A+20 a^3 b^7 B) x^8+(\frac{105}{2} a^4 b^6 A+63 a^5 b^5 B) x^6+(24 a^3 b^7 A+42 a^4 b^6 B)}{x^2}$
risch	$\frac{b^{10} B x^9}{9}+\frac{A b^{10} x^8}{8}+\frac{5 B a b^9 x^8}{4}+\frac{10 A a b^9 x^7}{7}+\frac{45 B a^2 b^8 x^7}{7}+\frac{15 A a^2 b^8 x^6}{2}+20 B a^3 b^7 x^6+24 A a^3 b^7 x^5+$
parallelrisch	$56 B b^{10} x^{11}+63 A b^{10} x^{10}+630 B a b^9 x^{10}+720 a A b^9 x^9+3240 B a^2 b^8 x^9+3780 a^2 A b^8 x^8+10080 B a^3 b^7 x^8+12096 a^3 A b^7 x^7+211$

input
```
int((b*x+a)^10*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^10*B*x^9+1/8*A*b^10*x^8+5/4*B*a*b^9*x^8+10/7*A*a*b^9*x^7+45/7*B*a^2*
b^8*x^7+15/2*A*a^2*b^8*x^6+20*B*a^3*b^7*x^6+24*A*a^3*b^7*x^5+42*B*a^4*b^6*
x^5+105/2*A*a^4*b^6*x^4+63*B*a^5*b^5*x^4+84*A*a^5*b^5*x^3+70*B*a^6*b^4*x^3
+105*A*a^6*b^4*x^2+60*B*a^7*b^3*x^2+120*a^7*b^3*A*x+45*a^8*b^2*B*x-1/2*a^1
0*A/x^2+5*a^8*b*(9*A*b+2*B*a)*ln(x)-a^9*(10*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx$$
$$=\frac{56\,Bb^{10}x^{11}-252\,Aa^{10}+63\,(10\,Bab^9+Ab^{10})x^{10}+360\,(9\,Ba^2b^8+2\,Aab^9)x^9+1260\,(8\,Ba^3b^7+3\,Aa^2b^8}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/504*(56*B*b^10*x^11 - 252*A*a^10 + 63*(10*B*a*b^9 + A*b^10)*x^10 + 360*(
9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 1260*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 3024
*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 5292*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 +
7056*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 7560*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^
4 + 7560*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 2520*(2*B*a^9*b + 9*A*a^8*b^2)*
x^2*log(x) - 504*(B*a^10 + 10*A*a^9*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx = \frac{Bb^{10}x^9}{9} + 5a^8b(9Ab+2Ba)\log(x) + x^8\left(\frac{Ab^{10}}{8} + \frac{5Bab^9}{4}\right) + x^7\cdot\left(\frac{10Aab^9}{7} + \frac{45Ba^2b^8}{7}\right) + x^6\cdot\left(\frac{15Aa^2b^8}{2} + 20Ba^3b^7\right) + x^5\cdot\left(24Aa^3b^7 + 42Ba^4b^6\right) + x^4\cdot\left(\frac{105Aa^4b^6}{2} + 63Ba^5b^5\right) + x^3\cdot\left(84Aa^5b^5 + 70Ba^6b^4\right) + x^2\cdot\left(105Aa^6b^4 + 60Ba^7b^3\right) + x\left(120Aa^7b^3 + 45Ba^8b^2\right) + \frac{-Aa^{10} + x(-20Aa^9b - 2Ba^{10})}{2x^2}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**3,x)
```

output
```
B*b**10*x**9/9 + 5*a**8*b*(9*A*b + 2*B*a)*log(x) + x**8*(A*b**10/8 + 5*B*a
*b**9/4) + x**7*(10*A*a*b**9/7 + 45*B*a**2*b**8/7) + x**6*(15*A*a**2*b**8/
2 + 20*B*a**3*b**7) + x**5*(24*A*a**3*b**7 + 42*B*a**4*b**6) + x**4*(105*A
*a**4*b**6/2 + 63*B*a**5*b**5) + x**3*(84*A*a**5*b**5 + 70*B*a**6*b**4) +
x**2*(105*A*a**6*b**4 + 60*B*a**7*b**3) + x*(120*A*a**7*b**3 + 45*B*a**8*b
**2) + (-A*a**10 + x*(-20*A*a**9*b - 2*B*a**10))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx = \frac{1}{9}\,Bb^{10}x^9 + \frac{1}{8}\left(10\,Bab^9 + Ab^{10}\right)x^8 + \frac{5}{7}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^7 + \frac{5}{2}\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^6 + 6\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^5 + \frac{21}{2}\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x^4 + 14\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)x^3 + 15\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)x^2 + 15\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)x + 5\left(2\,Ba^9b + 9\,Aa^8b^2\right)\log(x) - \frac{Aa^{10} + 2\,(Ba^{10} + 10\,Aa^9b)x}{2\,x^2}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/9*B*b^10*x^9 + 1/8*(10*B*a*b^9 + A*b^10)*x^8 + 5/7*(9*B*a^2*b^8 + 2*A*a*
b^9)*x^7 + 5/2*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^6 + 6*(7*B*a^4*b^6 + 4*A*a^3*
b^7)*x^5 + 21/2*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^4 + 14*(5*B*a^6*b^4 + 6*A*a^
5*b^5)*x^3 + 15*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^2 + 15*(3*B*a^8*b^2 + 8*A*a^
7*b^3)*x + 5*(2*B*a^9*b + 9*A*a^8*b^2)*log(x) - 1/2*(A*a^10 + 2*(B*a^10 +
10*A*a^9*b)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx &= \frac{1}{9}\,Bb^{10}x^9 + \frac{5}{4}\,Bab^9x^8 + \frac{1}{8}\,Ab^{10}x^8 + \frac{45}{7}\,Ba^2b^8x^7 \\ &\quad + \frac{10}{7}\,Aab^9x^7 + 20\,Ba^3b^7x^6 + \frac{15}{2}\,Aa^2b^8x^6 + 42\,Ba^4b^6x^5 \\ &\quad + 24\,Aa^3b^7x^5 + 63\,Ba^5b^5x^4 + \frac{105}{2}\,Aa^4b^6x^4 + 70\,Ba^6b^4x^3 \\ &\quad + 84\,Aa^5b^5x^3 + 60\,Ba^7b^3x^2 + 105\,Aa^6b^4x^2 + 45\,Ba^8b^2x \\ &\quad + 120\,Aa^7b^3x + 5\left(2\,Ba^9b + 9\,Aa^8b^2\right)\log\left(|x|\right) \\ &\quad - \frac{Aa^{10} + 2\,(Ba^{10} + 10\,Aa^9b)x}{2\,x^2}\end{aligned}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/9*B*b^10*x^9 + 5/4*B*a*b^9*x^8 + 1/8*A*b^10*x^8 + 45/7*B*a^2*b^8*x^7 + 1
0/7*A*a*b^9*x^7 + 20*B*a^3*b^7*x^6 + 15/2*A*a^2*b^8*x^6 + 42*B*a^4*b^6*x^5
 + 24*A*a^3*b^7*x^5 + 63*B*a^5*b^5*x^4 + 105/2*A*a^4*b^6*x^4 + 70*B*a^6*b^
4*x^3 + 84*A*a^5*b^5*x^3 + 60*B*a^7*b^3*x^2 + 105*A*a^6*b^4*x^2 + 45*B*a^8
*b^2*x + 120*A*a^7*b^3*x + 5*(2*B*a^9*b + 9*A*a^8*b^2)*log(abs(x)) - 1/2*(
A*a^10 + 2*(B*a^10 + 10*A*a^9*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx = \ln(x)\,\left(10\,B\,a^9\,b+45\,A\,a^8\,b^2\right) - \frac{x\,(B\,a^{10}+10\,A\,b\,a^9)+\frac{A\,a^{10}}{2}}{x^2} + x^8\left(\frac{A\,b^{10}}{8}+\frac{5\,B\,a\,b^9}{4}\right) + \frac{B\,b^{10}\,x^9}{9} + 15\,a^6\,b^3\,x^2\,(7\,A\,b+4\,B\,a) + 14\,a^5\,b^4\,x^3\,(6\,A\,b+5\,B\,a) + \frac{21\,a^4\,b^5\,x^4\,(5\,A\,b+6\,B\,a)}{2} + 6\,a^3\,b^6\,x^5\,(4\,A\,b+7\,B\,a) + \frac{5\,a^2\,b^7\,x^6\,(3\,A\,b+8\,B\,a)}{2} + 15\,a^7\,b^2\,x\,(8\,A\,b+3\,B\,a) + \frac{5\,a\,b^8\,x^7\,(2\,A\,b+9\,B\,a)}{7}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^3,x)
```

output
```
log(x)*(45*A*a^8*b^2 + 10*B*a^9*b) - (x*(B*a^10 + 10*A*a^9*b) + (A*a^10)/2
)/x^2 + x^8*((A*b^10)/8 + (5*B*a*b^9)/4) + (B*b^10*x^9)/9 + 15*a^6*b^3*x^2
*(7*A*b + 4*B*a) + 14*a^5*b^4*x^3*(6*A*b + 5*B*a) + (21*a^4*b^5*x^4*(5*A*b
 + 6*B*a))/2 + 6*a^3*b^6*x^5*(4*A*b + 7*B*a) + (5*a^2*b^7*x^6*(3*A*b + 8*B
*a))/2 + 15*a^7*b^2*x*(8*A*b + 3*B*a) + (5*a*b^8*x^7*(2*A*b + 9*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^3}\,dx = \frac{27720\log(x)\,a^9b^2x^2-252a^{11}-5544a^{10}bx+83160a^8b^3x^3+83160a^7b^4x^4+77616a^6b^5x^5+58212a^5b^6x^6}{504x^2}$$

input
```
int((b*x+a)^10*(B*x+A)/x^3,x)
```

output
```
(27720*log(x)*a**9*b**2*x**2 - 252*a**11 - 5544*a**10*b*x + 83160*a**8*b**
3*x**3 + 83160*a**7*b**4*x**4 + 77616*a**6*b**5*x**5 + 58212*a**5*b**6*x**
6 + 33264*a**4*b**7*x**7 + 13860*a**3*b**8*x**8 + 3960*a**2*b**9*x**9 + 69
3*a*b**10*x**10 + 56*b**11*x**11)/(504*x**2)
```

3.120 $\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 216

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = -\frac{a^{10}A}{3x^3}-\frac{a^9(10Ab+aB)}{2x^2}-\frac{5a^8b(9Ab+2aB)}{x} + 30a^6b^3(7Ab+4aB)x+21a^5b^4(6Ab+5aB)x^2 + 14a^4b^5(5Ab+6aB)x^3+\frac{15}{2}a^3b^6(4Ab+7aB)x^4 + 3a^2b^7(3Ab+8aB)x^5+\frac{5}{6}ab^8(2Ab+9aB)x^6 + \frac{1}{7}b^9(Ab+10aB)x^7+\frac{1}{8}b^{10}Bx^8+15a^7b^2(8Ab+3aB)\log(x)$$

output

```
-1/3*a^10*A/x^3-1/2*a^9*(10*A*b+B*a)/x^2-5*a^8*b*(9*A*b+2*B*a)/x+30*a^6*b^
3*(7*A*b+4*B*a)*x+21*a^5*b^4*(6*A*b+5*B*a)*x^2+14*a^4*b^5*(5*A*b+6*B*a)*x^
3+15/2*a^3*b^6*(4*A*b+7*B*a)*x^4+3*a^2*b^7*(3*A*b+8*B*a)*x^5+5/6*a*b^8*(2*
A*b+9*B*a)*x^6+1/7*b^9*(A*b+10*B*a)*x^7+1/8*b^10*B*x^8+15*a^7*b^2*(8*A*b+3
*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = -\frac{45a^8Ab^2}{x}+120a^7b^3Bx+105a^6b^4x(2A+Bx) -\frac{5a^9b(A+2Bx)}{x^2}+42a^5b^5x^2(3A+2Bx)-\frac{a^{10}(2A+3Bx)}{6x^3} +\frac{35}{2}a^4b^6x^3(4A+3Bx)+6a^3b^7x^4(5A+4Bx) +\frac{3}{2}a^2b^8x^5(6A+5Bx)+\frac{5}{21}ab^9x^6(7A+6Bx) +\frac{1}{56}b^{10}x^7(8A+7Bx)+15a^7b^2(8Ab+3aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^4,x]
```

output
```
(-45*a^8*A*b^2)/x + 120*a^7*b^3*B*x + 105*a^6*b^4*x*(2*A + B*x) - (5*a^9*b
*(A + 2*B*x))/x^2 + 42*a^5*b^5*x^2*(3*A + 2*B*x) - (a^10*(2*A + 3*B*x))/(6
*x^3) + (35*a^4*b^6*x^3*(4*A + 3*B*x))/2 + 6*a^3*b^7*x^4*(5*A + 4*B*x) + (
3*a^2*b^8*x^5*(6*A + 5*B*x))/2 + (5*a*b^9*x^6*(7*A + 6*B*x))/21 + (b^10*x^
7*(8*A + 7*B*x))/56 + 15*a^7*b^2*(8*A*b + 3*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^4}+\frac{a^9(aB+10Ab)}{x^3}+\frac{5a^8b(2aB+9Ab)}{x^2}+\frac{15a^7b^2(3aB+8Ab)}{x}+30a^6b^3(4aB+7Ab)+42a^5b^4x(5aB\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{3x^3}-\frac{a^9(aB+10Ab)}{2x^2}-\frac{5a^8b(2aB+9Ab)}{x}+15a^7b^2\log(x)(3aB+8Ab)+30a^6b^3x(4aB+7Ab)+21a^5b^4x^2(5aB+6Ab)+14a^4b^5x^3(6aB+5Ab)+\frac{15}{2}a^3b^6x^4(7aB+4Ab)+3a^2b^7x^5(8aB+3Ab)+\frac{1}{7}b^9x^7(10aB+Ab)+\frac{5}{6}ab^8x^6(9aB+2Ab)+\frac{1}{8}b^{10}Bx^8$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^4,x]
```

output
```
-1/3*(a^10*A)/x^3 - (a^9*(10*A*b + a*B))/(2*x^2) - (5*a^8*b*(9*A*b + 2*a*B
))/x + 30*a^6*b^3*(7*A*b + 4*a*B)*x + 21*a^5*b^4*(6*A*b + 5*a*B)*x^2 + 14*
a^4*b^5*(5*A*b + 6*a*B)*x^3 + (15*a^3*b^6*(4*A*b + 7*a*B)*x^4)/2 + 3*a^2*b
^7*(3*A*b + 8*a*B)*x^5 + (5*a*b^8*(2*A*b + 9*a*B)*x^6)/6 + (b^9*(A*b + 10*
a*B)*x^7)/7 + (b^10*B*x^8)/8 + 15*a^7*b^2*(8*A*b + 3*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^{10} B\,x^{8}}{8}+\frac{A\,b^{10} x^{7}}{7}+\frac{10 B a\,b^{9} x^{7}}{7}+\frac{5 A a\,b^{9} x^{6}}{3}+\frac{15 B\,a^{2} b^{8} x^{6}}{2}+9 A\,a^{2} b^{8} x^{5}+24 B\,a^{3} b^{7} x^{5}+30 A\,a^{3} b^{7} x^{4}$
norman	$\frac{\left(\frac{1}{7} b^{10} A+\frac{10}{7} a\,b^{9} B\right) x^{10}+\left(\frac{5}{3} a\,b^{9} A+\frac{15}{2} a^{2} b^{8} B\right) x^{9}+\left(30 a^{3} b^{7} A+\frac{105}{2} a^{4} b^{6} B\right) x^{7}+\left(-5 a^{9} b A-\frac{1}{2} a^{10} B\right) x+\left(9 a^{2} b^{8} A+24 a^{3} b^{7} B\right) x^{8}+}{}$
risch	$\frac{b^{10} B\,x^{8}}{8}+\frac{A\,b^{10} x^{7}}{7}+\frac{10 B a\,b^{9} x^{7}}{7}+\frac{5 A a\,b^{9} x^{6}}{3}+\frac{15 B\,a^{2} b^{8} x^{6}}{2}+9 A\,a^{2} b^{8} x^{5}+24 B\,a^{3} b^{7} x^{5}+30 A\,a^{3} b^{7} x^{4}$
parallelrisch	$\frac{21 B\,b^{10} x^{11}+24 A\,b^{10} x^{10}+240 B a\,b^{9} x^{10}+280 a A\,b^{9} x^{9}+1260 B\,a^{2} b^{8} x^{9}+1512 a^{2} A\,b^{8} x^{8}+4032 B\,a^{3} b^{7} x^{8}+5040 a^{3} A\,b^{7} x^{7}+8820 B}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^10*B*x^8+1/7*A*b^10*x^7+10/7*B*a*b^9*x^7+5/3*A*a*b^9*x^6+15/2*B*a^2*
b^8*x^6+9*A*a^2*b^8*x^5+24*B*a^3*b^7*x^5+30*A*a^3*b^7*x^4+105/2*B*a^4*b^6*
x^4+70*A*a^4*b^6*x^3+84*B*a^5*b^5*x^3+126*A*a^5*b^5*x^2+105*B*a^6*b^4*x^2+
210*A*a^6*b^4*x+120*B*a^7*b^3*x-1/3*a^10*A/x^3-1/2*a^9*(10*A*b+B*a)/x^2+15
*a^7*b^2*(8*A*b+3*B*a)*ln(x)-5*a^8*b*(9*A*b+2*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = \frac{21\,Bb^{10}x^{11}-56\,Aa^{10}+24\,(10\,Bab^9+Ab^{10})x^{10}+140\,(9\,Ba^2b^8+2\,Aab^9)x^9+504\,(8\,Ba^3b^7+3\,Aa^2b^8)x}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/168*(21*B*b^10*x^11 - 56*A*a^10 + 24*(10*B*a*b^9 + A*b^10)*x^10 + 140*(9
*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 504*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 1260*(
7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 2352*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 35
28*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 5040*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4
+ 2520*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3*log(x) - 840*(2*B*a^9*b + 9*A*a^8*b
^2)*x^2 - 84*(B*a^10 + 10*A*a^9*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = \frac{Bb^{10}x^8}{8} + 15a^7b^2\cdot(8Ab+3Ba)\log(x) + x^7\left(\frac{Ab^{10}}{7}+\frac{10Bab^9}{7}\right) + x^6\cdot\left(\frac{5Aab^9}{3}+\frac{15Ba^2b^8}{2}\right) + x^5\cdot(9Aa^2b^8+24Ba^3b^7) + x^4\cdot\left(30Aa^3b^7+\frac{105Ba^4b^6}{2}\right) + x^3\cdot(70Aa^4b^6+84Ba^5b^5) + x^2\cdot(126Aa^5b^5+105Ba^6b^4) + x(210Aa^6b^4+120Ba^7b^3) + \frac{-2Aa^{10}+x^2(-270Aa^8b^2-60Ba^9b)+x(-30Aa^9b-3Ba^{10})}{6x^3}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**4,x)
```

output
```
B*b**10*x**8/8 + 15*a**7*b**2*(8*A*b + 3*B*a)*log(x) + x**7*(A*b**10/7 + 1
0*B*a*b**9/7) + x**6*(5*A*a*b**9/3 + 15*B*a**2*b**8/2) + x**5*(9*A*a**2*b*
*8 + 24*B*a**3*b**7) + x**4*(30*A*a**3*b**7 + 105*B*a**4*b**6/2) + x**3*(7
0*A*a**4*b**6 + 84*B*a**5*b**5) + x**2*(126*A*a**5*b**5 + 105*B*a**6*b**4)
 + x*(210*A*a**6*b**4 + 120*B*a**7*b**3) + (-2*A*a**10 + x**2*(-270*A*a**8
*b**2 - 60*B*a**9*b) + x*(-30*A*a**9*b - 3*B*a**10))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = \frac{1}{8}Bb^{10}x^8 + \frac{1}{7}(10\,Bab^9+Ab^{10})x^7 + \frac{5}{6}(9\,Ba^2b^8+2\,Aab^9)x^6 + 3\,(8\,Ba^3b^7+3\,Aa^2b^8)x^5 + \frac{15}{2}(7\,Ba^4b^6+4\,Aa^3b^7)x^4 + 14\,(6\,Ba^5b^5+5\,Aa^4b^6)x^3 + 21\,(5\,Ba^6b^4+6\,Aa^5b^5)x^2 + 30\,(4\,Ba^7b^3+7\,Aa^6b^4)x + 15\,(3\,Ba^8b^2+8\,Aa^7b^3)\log(x) - \frac{2\,Aa^{10}+30\,(2\,Ba^9b+9\,Aa^8b^2)x^2+3\,(Ba^{10}+10\,Aa^9b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/8*B*b^10*x^8 + 1/7*(10*B*a*b^9 + A*b^10)*x^7 + 5/6*(9*B*a^2*b^8 + 2*A*a*
b^9)*x^6 + 3*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^5 + 15/2*(7*B*a^4*b^6 + 4*A*a^3
*b^7)*x^4 + 14*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^3 + 21*(5*B*a^6*b^4 + 6*A*a^5
*b^5)*x^2 + 30*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x + 15*(3*B*a^8*b^2 + 8*A*a^7*b
^3)*log(x) - 1/6*(2*A*a^10 + 30*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 3*(B*a^10
+ 10*A*a^9*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = \frac{1}{8}Bb^{10}x^8 + \frac{10}{7}Bab^9x^7 + \frac{1}{7}Ab^{10}x^7 + \frac{15}{2}Ba^2b^8x^6 + \frac{5}{3}Aab^9x^6 + 24\,Ba^3b^7x^5 + 9\,Aa^2b^8x^5 + \frac{105}{2}Ba^4b^6x^4 + 30\,Aa^3b^7x^4 + 84\,Ba^5b^5x^3 + 70\,Aa^4b^6x^3 + 105\,Ba^6b^4x^2 + 126\,Aa^5b^5x^2 + 120\,Ba^7b^3x + 210\,Aa^6b^4x + 15\left(3\,Ba^8b^2 + 8\,Aa^7b^3\right)\log\left(|x|\right) - \frac{2\,Aa^{10} + 30\,(2\,Ba^9b + 9\,Aa^8b^2)x^2 + 3\,(Ba^{10} + 10\,Aa^9b)x}{6\,x^3}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/8*B*b^10*x^8 + 10/7*B*a*b^9*x^7 + 1/7*A*b^10*x^7 + 15/2*B*a^2*b^8*x^6 +
5/3*A*a*b^9*x^6 + 24*B*a^3*b^7*x^5 + 9*A*a^2*b^8*x^5 + 105/2*B*a^4*b^6*x^4
 + 30*A*a^3*b^7*x^4 + 84*B*a^5*b^5*x^3 + 70*A*a^4*b^6*x^3 + 105*B*a^6*b^4*
x^2 + 126*A*a^5*b^5*x^2 + 120*B*a^7*b^3*x + 210*A*a^6*b^4*x + 15*(3*B*a^8*
b^2 + 8*A*a^7*b^3)*log(abs(x)) - 1/6*(2*A*a^10 + 30*(2*B*a^9*b + 9*A*a^8*b
^2)*x^2 + 3*(B*a^10 + 10*A*a^9*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = x^7\left(\frac{A\,b^{10}}{7}+\frac{10\,B\,a\,b^9}{7}\right) - \frac{x\left(\frac{B\,a^{10}}{2}+5\,A\,b\,a^9\right)+\frac{A\,a^{10}}{3}+x^2\left(10\,B\,a^9\,b+45\,A\,a^8\,b^2\right)}{x^3} + \ln(x)\,\left(45\,B\,a^8\,b^2+120\,A\,a^7\,b^3\right)+\frac{B\,b^{10}\,x^8}{8} + 21\,a^5\,b^4\,x^2\,(6\,A\,b+5\,B\,a)+14\,a^4\,b^5\,x^3\,(5\,A\,b+6\,B\,a) + \frac{15\,a^3\,b^6\,x^4\,(4\,A\,b+7\,B\,a)}{2}+3\,a^2\,b^7\,x^5\,(3\,A\,b+8\,B\,a) + 30\,a^6\,b^3\,x\,(7\,A\,b+4\,B\,a)+\frac{5\,a\,b^8\,x^6\,(2\,A\,b+9\,B\,a)}{6}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^4,x)
```

output
```
x^7*((A*b^10)/7 + (10*B*a*b^9)/7) - (x*((B*a^10)/2 + 5*A*a^9*b) + (A*a^10)
/3 + x^2*(45*A*a^8*b^2 + 10*B*a^9*b))/x^3 + log(x)*(120*A*a^7*b^3 + 45*B*a
^8*b^2) + (B*b^10*x^8)/8 + 21*a^5*b^4*x^2*(6*A*b + 5*B*a) + 14*a^4*b^5*x^3
*(5*A*b + 6*B*a) + (15*a^3*b^6*x^4*(4*A*b + 7*B*a))/2 + 3*a^2*b^7*x^5*(3*A
*b + 8*B*a) + 30*a^6*b^3*x*(7*A*b + 4*B*a) + (5*a*b^8*x^6*(2*A*b + 9*B*a))
/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^4}\,dx = \frac{27720\log(x)\,a^8b^3x^3-56a^{11}-924a^{10}bx-9240a^9b^2x^2+55440a^7b^4x^4+38808a^6b^5x^5+25872a^5b^6x^6+1}{168x^3}$$

input
```
int((b*x+a)^10*(B*x+A)/x^4,x)
```

output

```
(27720*log(x)*a**8*b**3*x**3 - 56*a**11 - 924*a**10*b*x - 9240*a**9*b**2*x
**2 + 55440*a**7*b**4*x**4 + 38808*a**6*b**5*x**5 + 25872*a**5*b**6*x**6 +
 13860*a**4*b**7*x**7 + 5544*a**3*b**8*x**8 + 1540*a**2*b**9*x**9 + 264*a*
b**10*x**10 + 21*b**11*x**11)/(168*x**3)
```

3.121 $\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 215

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = &-\frac{a^{10}A}{4x^4}-\frac{a^9(10Ab+aB)}{3x^3}-\frac{5a^8b(9Ab+2aB)}{2x^2}\\&-\frac{15a^7b^2(8Ab+3aB)}{x}+42a^5b^4(6Ab+5aB)x\\&+21a^4b^5(5Ab+6aB)x^2+10a^3b^6(4Ab+7aB)x^3\\&+\frac{15}{4}a^2b^7(3Ab+8aB)x^4+ab^8(2Ab+9aB)x^5\\&+\frac{1}{6}b^9(Ab+10aB)x^6+\frac{1}{7}b^{10}Bx^7+30a^6b^3(7Ab+4aB)\log(x)\end{aligned}$$

output
```
-1/4*a^10*A/x^4-1/3*a^9*(10*A*b+B*a)/x^3-5/2*a^8*b*(9*A*b+2*B*a)/x^2-15*a^
7*b^2*(8*A*b+3*B*a)/x+42*a^5*b^4*(6*A*b+5*B*a)*x+21*a^4*b^5*(5*A*b+6*B*a)*
x^2+10*a^3*b^6*(4*A*b+7*B*a)*x^3+15/4*a^2*b^7*(3*A*b+8*B*a)*x^4+a*b^8*(2*A
*b+9*B*a)*x^5+1/6*b^9*(A*b+10*B*a)*x^6+1/7*b^10*B*x^7+30*a^6*b^3*(7*A*b+4*
B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = -\frac{120a^7Ab^3}{x} + 210a^6b^4Bx + 126a^5b^5x(2A+Bx) - \frac{45a^8b^2(A+2Bx)}{2x^2} + 35a^4b^6x^2(3A+2Bx) - \frac{5a^9b(2A+3Bx)}{3x^3} + 10a^3b^7x^3(4A+3Bx) - \frac{a^{10}(3A+4Bx)}{12x^4} + \frac{9}{4}a^2b^8x^4(5A+4Bx) + \frac{1}{3}ab^9x^5(6A+5Bx) + \frac{1}{42}b^{10}x^6(7A+6Bx) + 30a^6b^3(7Ab+4aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^5,x]
```

output
```
(-120*a^7*A*b^3)/x + 210*a^6*b^4*B*x + 126*a^5*b^5*x*(2*A + B*x) - (45*a^8
*b^2*(A + 2*B*x))/(2*x^2) + 35*a^4*b^6*x^2*(3*A + 2*B*x) - (5*a^9*b*(2*A +
 3*B*x))/(3*x^3) + 10*a^3*b^7*x^3*(4*A + 3*B*x) - (a^10*(3*A + 4*B*x))/(12
*x^4) + (9*a^2*b^8*x^4*(5*A + 4*B*x))/4 + (a*b^9*x^5*(6*A + 5*B*x))/3 + (b
^10*x^6*(7*A + 6*B*x))/42 + 30*a^6*b^3*(7*A*b + 4*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^5} + \frac{a^9(aB+10Ab)}{x^4} + \frac{5a^8b(2aB+9Ab)}{x^3} + \frac{15a^7b^2(3aB+8Ab)}{x^2} + \frac{30a^6b^3(4aB+7Ab)}{x} + 42a^5b^4(5aB+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{4x^4}-\frac{a^9(aB+10Ab)}{3x^3}-\frac{5a^8b(2aB+9Ab)}{2x^2}-\frac{15a^7b^2(3aB+8Ab)}{x}+30a^6b^3\log(x)(4aB+7Ab)+42a^5b^4x(5aB+6Ab)+21a^4b^5x^2(6aB+5Ab)+10a^3b^6x^3(7aB+4Ab)+\frac{15}{4}a^2b^7x^4(8aB+3Ab)+\frac{1}{6}b^9x^6(10aB+Ab)+ab^8x^5(9aB+2Ab)+\frac{1}{7}b^{10}Bx^7$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^5,x]
```

output
```
-1/4*(a^10*A)/x^4 - (a^9*(10*A*b + a*B))/(3*x^3) - (5*a^8*b*(9*A*b + 2*a*B
))/(2*x^2) - (15*a^7*b^2*(8*A*b + 3*a*B))/x + 42*a^5*b^4*(6*A*b + 5*a*B)*x
 + 21*a^4*b^5*(5*A*b + 6*a*B)*x^2 + 10*a^3*b^6*(4*A*b + 7*a*B)*x^3 + (15*a
^2*b^7*(3*A*b + 8*a*B)*x^4)/4 + a*b^8*(2*A*b + 9*a*B)*x^5 + (b^9*(A*b + 10
*a*B)*x^6)/6 + (b^10*B*x^7)/7 + 30*a^6*b^3*(7*A*b + 4*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^{10} B x^7}{7}+\frac{A b^{10} x^6}{6}+\frac{5 B a b^9 x^6}{3}+2 A a b^9 x^5+9 B a^2 b^8 x^5+\frac{45 A a^2 b^8 x^4}{4}+30 B a^3 b^7 x^4+40 A a^3 b^7 x^3$
norman	$\frac{(\frac{1}{6} b^{10} A+\frac{5}{3} a b^9 B) x^{10}+(\frac{45}{4} a^2 b^8 A+30 a^3 b^7 B) x^8+(-\frac{45}{2} a^8 b^2 A-5 a^9 b B) x^2+(-\frac{10}{3} a^9 b A-\frac{1}{3} a^{10} B) x+(2 a b^9 A+9 a^2 b^8 B) x^9+(4}{}$
risch	$\frac{b^{10} B x^7}{7}+\frac{A b^{10} x^6}{6}+\frac{5 B a b^9 x^6}{3}+2 A a b^9 x^5+9 B a^2 b^8 x^5+\frac{45 A a^2 b^8 x^4}{4}+30 B a^3 b^7 x^4+40 A a^3 b^7 x^3$
parallelrisch	$\frac{12 B b^{10} x^{11}+14 A b^{10} x^{10}+140 B a b^9 x^{10}+168 a A b^9 x^9+756 B a^2 b^8 x^9+945 a^2 A b^8 x^8+2520 B a^3 b^7 x^8+3360 a^3 A b^7 x^7+5880 B a}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^10*B*x^7+1/6*A*b^10*x^6+5/3*B*a*b^9*x^6+2*A*a*b^9*x^5+9*B*a^2*b^8*x^
5+45/4*A*a^2*b^8*x^4+30*B*a^3*b^7*x^4+40*A*a^3*b^7*x^3+70*B*a^4*b^6*x^3+10
5*A*a^4*b^6*x^2+126*B*a^5*b^5*x^2+252*A*a^5*b^5*x+210*B*a^6*b^4*x-1/3*a^9*
(10*A*b+B*a)/x^3-5/2*a^8*b*(9*A*b+2*B*a)/x^2-1/4*a^10*A/x^4+30*a^6*b^3*(7*
A*b+4*B*a)*ln(x)-15*a^7*b^2*(8*A*b+3*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx$$
$$=\frac{12\,Bb^{10}x^{11}-21\,Aa^{10}+14\,(10\,Bab^9+Ab^{10})x^{10}+84\,(9\,Ba^2b^8+2\,Aab^9)x^9+315\,(8\,Ba^3b^7+3\,Aa^2b^8)x^8}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
1/84*(12*B*b^10*x^11 - 21*A*a^10 + 14*(10*B*a*b^9 + A*b^10)*x^10 + 84*(9*B
*a^2*b^8 + 2*A*a*b^9)*x^9 + 315*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 840*(7*B
*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 1764*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 3528*
(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 2520*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4*log
(x) - 1260*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 210*(2*B*a^9*b + 9*A*a^8*b^2)
*x^2 - 28*(B*a^10 + 10*A*a^9*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = \frac{Bb^{10}x^7}{7} + 30a^6b^3\cdot(7Ab+4Ba)\log{(x)} + x^6\left(\frac{Ab^{10}}{6}+\frac{5Bab^9}{3}\right) + x^5\cdot\left(2Aab^9+9Ba^2b^8\right) + x^4\cdot\left(\frac{45Aa^2b^8}{4}+30Ba^3b^7\right) + x^3\cdot\left(40Aa^3b^7+70Ba^4b^6\right) + x^2\cdot\left(105Aa^4b^6+126Ba^5b^5\right) + x\left(252Aa^5b^5+210Ba^6b^4\right) + \frac{-3Aa^{10}+x^3(-1440Aa^7b^3-540Ba^8b^2)+x^2(-270Aa^8b^2-60Ba^9b)+x(-40Aa^9b-4Ba^{10})}{12x^4}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**5,x)
```

output
```
B*b**10*x**7/7 + 30*a**6*b**3*(7*A*b + 4*B*a)*log(x) + x**6*(A*b**10/6 + 5
*B*a*b**9/3) + x**5*(2*A*a*b**9 + 9*B*a**2*b**8) + x**4*(45*A*a**2*b**8/4
+ 30*B*a**3*b**7) + x**3*(40*A*a**3*b**7 + 70*B*a**4*b**6) + x**2*(105*A*a
**4*b**6 + 126*B*a**5*b**5) + x*(252*A*a**5*b**5 + 210*B*a**6*b**4) + (-3*
A*a**10 + x**3*(-1440*A*a**7*b**3 - 540*B*a**8*b**2) + x**2*(-270*A*a**8*b
**2 - 60*B*a**9*b) + x*(-40*A*a**9*b - 4*B*a**10))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = \frac{1}{7}\,Bb^{10}x^7 + \frac{1}{6}\,\left(10\,Bab^9+Ab^{10}\right)x^6 + \left(9\,Ba^2b^8+2\,Aab^9\right)x^5 + \frac{15}{4}\,\left(8\,Ba^3b^7+3\,Aa^2b^8\right)x^4 + 10\,\left(7\,Ba^4b^6+4\,Aa^3b^7\right)x^3 + 21\,\left(6\,Ba^5b^5+5\,Aa^4b^6\right)x^2 + 42\,\left(5\,Ba^6b^4+6\,Aa^5b^5\right)x + 30\,\left(4\,Ba^7b^3+7\,Aa^6b^4\right)\log{(x)} - \frac{3\,Aa^{10}+180\,(3\,Ba^8b^2+8\,Aa^7b^3)x^3+30\,(2\,Ba^9b+9\,Aa^8b^2)x^2+4\,(Ba^{10}+10\,Aa^9b)x}{12\,x^4}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
1/7*B*b^10*x^7 + 1/6*(10*B*a*b^9 + A*b^10)*x^6 + (9*B*a^2*b^8 + 2*A*a*b^9)
*x^5 + 15/4*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^4 + 10*(7*B*a^4*b^6 + 4*A*a^3*b^
7)*x^3 + 21*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^2 + 42*(5*B*a^6*b^4 + 6*A*a^5*b^
5)*x + 30*(4*B*a^7*b^3 + 7*A*a^6*b^4)*log(x) - 1/12*(3*A*a^10 + 180*(3*B*a
^8*b^2 + 8*A*a^7*b^3)*x^3 + 30*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 4*(B*a^10 +
 10*A*a^9*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = \frac{1}{7}\,Bb^{10}x^7 + \frac{5}{3}\,Bab^9x^6 + \frac{1}{6}\,Ab^{10}x^6 + 9\,Ba^2b^8x^5 + 2\,Aab^9x^5 + 30\,Ba^3b^7x^4 + \frac{45}{4}\,Aa^2b^8x^4 + 70\,Ba^4b^6x^3 + 40\,Aa^3b^7x^3 + 126\,Ba^5b^5x^2 + 105\,Aa^4b^6x^2 + 210\,Ba^6b^4x + 252\,Aa^5b^5x + 30\left(4\,Ba^7b^3 + 7\,Aa^6b^4\right)\log\left(|x|\right) - \frac{3\,Aa^{10} + 180\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^3 + 30\,(2\,Ba^9b + 9\,Aa^8b^2)x^2 + 4\,(Ba^{10} + 10\,Aa^9b)x}{12\,x^4}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
1/7*B*b^10*x^7 + 5/3*B*a*b^9*x^6 + 1/6*A*b^10*x^6 + 9*B*a^2*b^8*x^5 + 2*A*
a*b^9*x^5 + 30*B*a^3*b^7*x^4 + 45/4*A*a^2*b^8*x^4 + 70*B*a^4*b^6*x^3 + 40*
A*a^3*b^7*x^3 + 126*B*a^5*b^5*x^2 + 105*A*a^4*b^6*x^2 + 210*B*a^6*b^4*x +
252*A*a^5*b^5*x + 30*(4*B*a^7*b^3 + 7*A*a^6*b^4)*log(abs(x)) - 1/12*(3*A*a
^10 + 180*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 30*(2*B*a^9*b + 9*A*a^8*b^2)*x
^2 + 4*(B*a^10 + 10*A*a^9*b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = x^6\left(\frac{A\,b^{10}}{6}+\frac{5\,B\,a\,b^9}{3}\right) - \frac{x\left(\frac{B\,a^{10}}{3}+\frac{10\,A\,b\,a^9}{3}\right)+\frac{A\,a^{10}}{4}+x^2\left(5\,B\,a^9\,b+\frac{45\,A\,a^8\,b^2}{2}\right)+x^3\left(45\,B\,a^8\,b^2+120\,A\,a^7\,b^3\right)}{x^4} + \ln(x)\,\left(120\,B\,a^7\,b^3+210\,A\,a^6\,b^4\right)+\frac{B\,b^{10}\,x^7}{7}+21\,a^4\,b^5\,x^2\,(5\,A\,b+6\,B\,a) + 10\,a^3\,b^6\,x^3\,(4\,A\,b+7\,B\,a)+\frac{15\,a^2\,b^7\,x^4\,(3\,A\,b+8\,B\,a)}{4} + 42\,a^5\,b^4\,x\,(6\,A\,b+5\,B\,a)+a\,b^8\,x^5\,(2\,A\,b+9\,B\,a)$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^5,x)
```

output
```
x^6*((A*b^10)/6 + (5*B*a*b^9)/3) - (x*((B*a^10)/3 + (10*A*a^9*b)/3) + (A*a
^10)/4 + x^2*((45*A*a^8*b^2)/2 + 5*B*a^9*b) + x^3*(120*A*a^7*b^3 + 45*B*a^
8*b^2))/x^4 + log(x)*(210*A*a^6*b^4 + 120*B*a^7*b^3) + (B*b^10*x^7)/7 + 21
*a^4*b^5*x^2*(5*A*b + 6*B*a) + 10*a^3*b^6*x^3*(4*A*b + 7*B*a) + (15*a^2*b^
7*x^4*(3*A*b + 8*B*a))/4 + 42*a^5*b^4*x*(6*A*b + 5*B*a) + a*b^8*x^5*(2*A*b
 + 9*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^5}\,dx = \frac{27720\log(x)\,a^7b^4x^4-21a^{11}-308a^{10}bx-2310a^9b^2x^2-13860a^8b^3x^3+38808a^6b^5x^5+19404a^5b^6x^6+9}{84x^4}$$

input
```
int((b*x+a)^10*(B*x+A)/x^5,x)
```

output

```
(27720*log(x)*a**7*b**4*x**4 - 21*a**11 - 308*a**10*b*x - 2310*a**9*b**2*x
**2 - 13860*a**8*b**3*x**3 + 38808*a**6*b**5*x**5 + 19404*a**5*b**6*x**6 +
 9240*a**4*b**7*x**7 + 3465*a**3*b**8*x**8 + 924*a**2*b**9*x**9 + 154*a*b*
*10*x**10 + 12*b**11*x**11)/(84*x**4)
```

3.122 $\int \frac{(a+bx)^{10}(A+Bx)}{x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 218

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^6} \, dx = & -\frac{a^{10}A}{5x^5} - \frac{a^9(10Ab+aB)}{4x^4} - \frac{5a^8b(9Ab+2aB)}{3x^3} \\ & - \frac{15a^7b^2(8Ab+3aB)}{2x^2} - \frac{30a^6b^3(7Ab+4aB)}{x} \\ & + 42a^4b^5(5Ab+6aB)x + 15a^3b^6(4Ab+7aB)x^2 \\ & + 5a^2b^7(3Ab+8aB)x^3 + \frac{5}{4}ab^8(2Ab+9aB)x^4 \\ & + \frac{1}{5}b^9(Ab+10aB)x^5 + \frac{1}{6}b^{10}Bx^6 + 42a^5b^4(6Ab+5aB)\log(x)\end{aligned}$$

output
```
-1/5*a^10*A/x^5-1/4*a^9*(10*A*b+B*a)/x^4-5/3*a^8*b*(9*A*b+2*B*a)/x^3-15/2*
a^7*b^2*(8*A*b+3*B*a)/x^2-30*a^6*b^3*(7*A*b+4*B*a)/x+42*a^4*b^5*(5*A*b+6*B
*a)*x+15*a^3*b^6*(4*A*b+7*B*a)*x^2+5*a^2*b^7*(3*A*b+8*B*a)*x^3+5/4*a*b^8*(
2*A*b+9*B*a)*x^4+1/5*b^9*(A*b+10*B*a)*x^5+1/6*b^10*B*x^6+42*a^5*b^4*(6*A*b
+5*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx = -\frac{210a^6Ab^4}{x}+252a^5b^5Bx+105a^4b^6x(2A+Bx) -\frac{60a^7b^3(A+2Bx)}{x^2}+20a^3b^7x^2(3A+2Bx) -\frac{15a^8b^2(2A+3Bx)}{2x^3}+\frac{15}{4}a^2b^8x^3(4A+3Bx) -\frac{5a^9b(3A+4Bx)}{6x^4}+\frac{1}{2}ab^9x^4(5A+4Bx)-\frac{a^{10}(4A+5Bx)}{20x^5} +\frac{1}{30}b^{10}x^5(6A+5Bx)+42a^5b^4(6Ab+5aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^6,x]
```

output
```
(-210*a^6*A*b^4)/x + 252*a^5*b^5*B*x + 105*a^4*b^6*x*(2*A + B*x) - (60*a^7
*b^3*(A + 2*B*x))/x^2 + 20*a^3*b^7*x^2*(3*A + 2*B*x) - (15*a^8*b^2*(2*A +
3*B*x))/(2*x^3) + (15*a^2*b^8*x^3*(4*A + 3*B*x))/4 - (5*a^9*b*(3*A + 4*B*x
))/(6*x^4) + (a*b^9*x^4*(5*A + 4*B*x))/2 - (a^10*(4*A + 5*B*x))/(20*x^5) +
 (b^10*x^5*(6*A + 5*B*x))/30 + 42*a^5*b^4*(6*A*b + 5*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{a^{10}A}{x^6}+\frac{a^9(aB+10Ab)}{x^5}+\frac{5a^8b(2aB+9Ab)}{x^4}+\frac{15a^7b^2(3aB+8Ab)}{x^3}+\frac{30a^6b^3(4aB+7Ab)}{x^2}+\frac{42a^5b^4(5aB+}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{5x^5}-\frac{a^9(aB+10Ab)}{4x^4}-\frac{5a^8b(2aB+9Ab)}{3x^3}-\frac{15a^7b^2(3aB+8Ab)}{2x^2}-\frac{30a^6b^3(4aB+7Ab)}{x}+42a^5b^4\log(x)(5aB+6Ab)+42a^4b^5x(6aB+5Ab)+15a^3b^6x^2(7aB+4Ab)+5a^2b^7x^3(8aB+3Ab)+\frac{1}{5}b^9x^5(10aB+Ab)+\frac{5}{4}ab^8x^4(9aB+2Ab)+\frac{1}{6}b^{10}Bx^6$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^6,x]
```

output
```
-1/5*(a^10*A)/x^5 - (a^9*(10*A*b + a*B))/(4*x^4) - (5*a^8*b*(9*A*b + 2*a*B
))/(3*x^3) - (15*a^7*b^2*(8*A*b + 3*a*B))/(2*x^2) - (30*a^6*b^3*(7*A*b + 4
*a*B))/x + 42*a^4*b^5*(5*A*b + 6*a*B)*x + 15*a^3*b^6*(4*A*b + 7*a*B)*x^2 +
 5*a^2*b^7*(3*A*b + 8*a*B)*x^3 + (5*a*b^8*(2*A*b + 9*a*B)*x^4)/4 + (b^9*(A
*b + 10*a*B)*x^5)/5 + (b^10*B*x^6)/6 + 42*a^5*b^4*(6*A*b + 5*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02

method	result
default	$\frac{b^{10}Bx^6}{6}+\frac{Ab^{10}x^5}{5}+2Ba\,b^9x^5+\frac{5Aa\,b^9x^4}{2}+\frac{45B\,a^2b^8x^4}{4}+15A\,a^2b^8x^3+40B\,a^3b^7x^3+60A\,a^3b^7x$
norman	$\frac{(\frac{1}{5}b^{10}A+2a\,b^9B)x^{10}+(\frac{5}{2}a\,b^9A+\frac{45}{4}a^2b^8B)x^9+(-60a^7b^3A-\frac{45}{2}a^8b^2B)x^3+(-15a^8b^2A-\frac{10}{3}a^9bB)x^2+(-\frac{5}{2}a^9bA-\frac{1}{4}a^{10}B)x}{}$
risch	$\frac{b^{10}Bx^6}{6}+\frac{Ab^{10}x^5}{5}+2Ba\,b^9x^5+\frac{5Aa\,b^9x^4}{2}+\frac{45B\,a^2b^8x^4}{4}+15A\,a^2b^8x^3+40B\,a^3b^7x^3+60A\,a^3b^7x$
parallelrisch	$\frac{10B\,b^{10}x^{11}+12A\,b^{10}x^{10}+120Ba\,b^9x^{10}+150aA\,b^9x^9+675B\,a^2b^8x^9+900a^2A\,b^8x^8+2400B\,a^3b^7x^8+3600a^3A\,b^7x^7+6300B\,a}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^10*B*x^6+1/5*A*b^10*x^5+2*B*a*b^9*x^5+5/2*A*a*b^9*x^4+45/4*B*a^2*b^8
*x^4+15*A*a^2*b^8*x^3+40*B*a^3*b^7*x^3+60*A*a^3*b^7*x^2+105*B*a^4*b^6*x^2+
210*A*a^4*b^6*x+252*B*a^5*b^5*x-5/3*a^8*b*(9*A*b+2*B*a)/x^3-1/5*a^10*A/x^5
-15/2*a^7*b^2*(8*A*b+3*B*a)/x^2-1/4*a^9*(10*A*b+B*a)/x^4+42*a^5*b^4*(6*A*b
+5*B*a)*ln(x)-30*a^6*b^3*(7*A*b+4*B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx$$
$$=\frac{10\,Bb^{10}x^{11}-12\,Aa^{10}+12\,(10\,Bab^9+Ab^{10})x^{10}+75\,(9\,Ba^2b^8+2\,Aab^9)x^9+300\,(8\,Ba^3b^7+3\,Aa^2b^8)x^8}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
1/60*(10*B*b^10*x^11 - 12*A*a^10 + 12*(10*B*a*b^9 + A*b^10)*x^10 + 75*(9*B
*a^2*b^8 + 2*A*a*b^9)*x^9 + 300*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 900*(7*B
*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 2520*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 2520*
(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5*log(x) - 1800*(4*B*a^7*b^3 + 7*A*a^6*b^4)*
x^4 - 450*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 100*(2*B*a^9*b + 9*A*a^8*b^2)*
x^2 - 15*(B*a^10 + 10*A*a^9*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx$$
$$= \frac{Bb^{10}x^6}{6} + 42a^5b^4\cdot(6Ab+5Ba)\log(x) + x^5\left(\frac{Ab^{10}}{5}+2Bab^9\right) + x^4\cdot\left(\frac{5Aab^9}{2}+\frac{45Ba^2b^8}{4}\right)$$
$$+ x^3\cdot\left(15Aa^2b^8+40Ba^3b^7\right) + x^2\cdot\left(60Aa^3b^7+105Ba^4b^6\right) + x\left(210Aa^4b^6+252Ba^5b^5\right)$$
$$+ \frac{-12Aa^{10} + x^4(-12600Aa^6b^4 - 7200Ba^7b^3) + x^3(-3600Aa^7b^3 - 1350Ba^8b^2) + x^2(-900Aa^8b^2 - 200A}{60x^5}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**6,x)
```

output
```
B*b**10*x**6/6 + 42*a**5*b**4*(6*A*b + 5*B*a)*log(x) + x**5*(A*b**10/5 + 2
*B*a*b**9) + x**4*(5*A*a*b**9/2 + 45*B*a**2*b**8/4) + x**3*(15*A*a**2*b**8
 + 40*B*a**3*b**7) + x**2*(60*A*a**3*b**7 + 105*B*a**4*b**6) + x*(210*A*a*
*4*b**6 + 252*B*a**5*b**5) + (-12*A*a**10 + x**4*(-12600*A*a**6*b**4 - 720
0*B*a**7*b**3) + x**3*(-3600*A*a**7*b**3 - 1350*B*a**8*b**2) + x**2*(-900*
A*a**8*b**2 - 200*B*a**9*b) + x*(-150*A*a**9*b - 15*B*a**10))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx$$
$$= \frac{1}{6}Bb^{10}x^6 + \frac{1}{5}\left(10\,Bab^9 + Ab^{10}\right)x^5 + \frac{5}{4}\left(9\,Ba^2b^8 + 2\,Aab^9\right)x^4 + 5\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)x^3$$
$$+ 15\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)x^2 + 42\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)x + 42\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)\log(x)$$
$$- \frac{12\,Aa^{10} + 1800\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^4 + 450\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^3 + 100\,(2\,Ba^9b + 9\,Aa^8b^2)x^2 + 15\,(}{60\,x^5}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
1/6*B*b^10*x^6 + 1/5*(10*B*a*b^9 + A*b^10)*x^5 + 5/4*(9*B*a^2*b^8 + 2*A*a*
b^9)*x^4 + 5*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^3 + 15*(7*B*a^4*b^6 + 4*A*a^3*b
^7)*x^2 + 42*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x + 42*(5*B*a^6*b^4 + 6*A*a^5*b^5
)*log(x) - 1/60*(12*A*a^10 + 1800*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 450*(3
*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 100*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 15*(B*
a^10 + 10*A*a^9*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx = \frac{1}{6}\,Bb^{10}x^6 + 2\,Bab^9x^5 + \frac{1}{5}\,Ab^{10}x^5 + \frac{45}{4}\,Ba^2b^8x^4 + \frac{5}{2}\,Aab^9x^4 + 40\,Ba^3b^7x^3 + 15\,Aa^2b^8x^3 + 105\,Ba^4b^6x^2 + 60\,Aa^3b^7x^2 + 252\,Ba^5b^5x + 210\,Aa^4b^6x + 42\left(5\,Ba^6b^4 + 6\,Aa^5b^5\right)\log\left(|x|\right) - \frac{12\,Aa^{10} + 1800\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^4 + 450\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^3 + 100\,(2\,Ba^9b + 9\,Aa^8b^2)x^2 + 15\,(}{60\,x^5}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
1/6*B*b^10*x^6 + 2*B*a*b^9*x^5 + 1/5*A*b^10*x^5 + 45/4*B*a^2*b^8*x^4 + 5/2
*A*a*b^9*x^4 + 40*B*a^3*b^7*x^3 + 15*A*a^2*b^8*x^3 + 105*B*a^4*b^6*x^2 + 6
0*A*a^3*b^7*x^2 + 252*B*a^5*b^5*x + 210*A*a^4*b^6*x + 42*(5*B*a^6*b^4 + 6*
A*a^5*b^5)*log(abs(x)) - 1/60*(12*A*a^10 + 1800*(4*B*a^7*b^3 + 7*A*a^6*b^4
)*x^4 + 450*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 100*(2*B*a^9*b + 9*A*a^8*b^2
)*x^2 + 15*(B*a^10 + 10*A*a^9*b)*x)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx = x^5\left(\frac{A\,b^{10}}{5}+2\,B\,a\,b^9\right) - \frac{x\left(\frac{B\,a^{10}}{4}+\frac{5\,A\,b\,a^9}{2}\right)+\frac{A\,a^{10}}{5}+x^2\left(\frac{10\,B\,a^9\,b}{3}+15\,A\,a^8\,b^2\right)+x^3\left(\frac{45\,B\,a^8\,b^2}{2}+60\,A\,a^7\,b^3\right)+x^4\,(120\,B\,a^7\,b^3+}{x^5} + \ln(x)\,\left(210\,B\,a^6\,b^4+252\,A\,a^5\,b^5\right)+\frac{B\,b^{10}\,x^6}{6}+15\,a^3\,b^6\,x^2\,(4\,A\,b+7\,B\,a) + 5\,a^2\,b^7\,x^3\,(3\,A\,b+8\,B\,a)+42\,a^4\,b^5\,x\,(5\,A\,b+6\,B\,a)+\frac{5\,a\,b^8\,x^4\,(2\,A\,b+9\,B\,a)}{4}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^6,x)
```

output
```
x^5*((A*b^10)/5 + 2*B*a*b^9) - (x*((B*a^10)/4 + (5*A*a^9*b)/2) + (A*a^10)/
5 + x^2*(15*A*a^8*b^2 + (10*B*a^9*b)/3) + x^3*(60*A*a^7*b^3 + (45*B*a^8*b^
2)/2) + x^4*(210*A*a^6*b^4 + 120*B*a^7*b^3))/x^5 + log(x)*(252*A*a^5*b^5 +
 210*B*a^6*b^4) + (B*b^10*x^6)/6 + 15*a^3*b^6*x^2*(4*A*b + 7*B*a) + 5*a^2*
b^7*x^3*(3*A*b + 8*B*a) + 42*a^4*b^5*x*(5*A*b + 6*B*a) + (5*a*b^8*x^4*(2*A
*b + 9*B*a))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^6}\,dx = \frac{27720\log(x)\,a^6b^5x^5-12a^{11}-165a^{10}bx-1100a^9b^2x^2-4950a^8b^3x^3-19800a^7b^4x^4+27720a^5b^6x^6+99}{60x^5}$$

input
```
int((b*x+a)^10*(B*x+A)/x^6,x)
```

output
```
(27720*log(x)*a**6*b**5*x**5 - 12*a**11 - 165*a**10*b*x - 1100*a**9*b**2*x
**2 - 4950*a**8*b**3*x**3 - 19800*a**7*b**4*x**4 + 27720*a**5*b**6*x**6 +
9900*a**4*b**7*x**7 + 3300*a**3*b**8*x**8 + 825*a**2*b**9*x**9 + 132*a*b**
10*x**10 + 10*b**11*x**11)/(60*x**5)
```

3.123 $\int \frac{(a+bx)^{10}(A+Bx)}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 218

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^7} dx = &-\frac{a^{10}A}{6x^6} - \frac{a^9(10Ab+aB)}{5x^5} - \frac{5a^8b(9Ab+2aB)}{4x^4} \\ &- \frac{5a^7b^2(8Ab+3aB)}{x^3} - \frac{15a^6b^3(7Ab+4aB)}{x^2} \\ &- \frac{42a^5b^4(6Ab+5aB)}{x} + 30a^3b^6(4Ab+7aB)x \\ &+ \frac{15}{2}a^2b^7(3Ab+8aB)x^2 + \frac{5}{3}ab^8(2Ab+9aB)x^3 \\ &+ \frac{1}{4}b^9(Ab+10aB)x^4 + \frac{1}{5}b^{10}Bx^5 + 42a^4b^5(5Ab+6aB)\log(x)\end{aligned}$$

output
```
-1/6*a^10*A/x^6-1/5*a^9*(10*A*b+B*a)/x^5-5/4*a^8*b*(9*A*b+2*B*a)/x^4-5*a^7
*b^2*(8*A*b+3*B*a)/x^3-15*a^6*b^3*(7*A*b+4*B*a)/x^2-42*a^5*b^4*(6*A*b+5*B*
a)/x+30*a^3*b^6*(4*A*b+7*B*a)*x+15/2*a^2*b^7*(3*A*b+8*B*a)*x^2+5/3*a*b^8*(
2*A*b+9*B*a)*x^3+1/4*b^9*(A*b+10*B*a)*x^4+1/5*b^10*B*x^5+42*a^4*b^5*(5*A*b
+6*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx = -\frac{252a^5Ab^5}{x} + 210a^4b^6Bx + 60a^3b^7x(2A+Bx) - \frac{105a^6b^4(A+2Bx)}{x^2} + \frac{15}{2}a^2b^8x^2(3A+2Bx) - \frac{20a^7b^3(2A+3Bx)}{x^3} + \frac{5}{6}ab^9x^3(4A+3Bx) - \frac{15a^8b^2(3A+4Bx)}{4x^4} + \frac{1}{20}b^{10}x^4(5A+4Bx) - \frac{a^9b(4A+5Bx)}{2x^5} - \frac{a^{10}(5A+6Bx)}{30x^6} + 42a^4b^5(5Ab+6aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^7,x]
```

output
```
(-252*a^5*A*b^5)/x + 210*a^4*b^6*B*x + 60*a^3*b^7*x*(2*A + B*x) - (105*a^6
*b^4*(A + 2*B*x))/x^2 + (15*a^2*b^8*x^2*(3*A + 2*B*x))/2 - (20*a^7*b^3*(2*
A + 3*B*x))/x^3 + (5*a*b^9*x^3*(4*A + 3*B*x))/6 - (15*a^8*b^2*(3*A + 4*B*x
))/(4*x^4) + (b^10*x^4*(5*A + 4*B*x))/20 - (a^9*b*(4*A + 5*B*x))/(2*x^5) -
 (a^10*(5*A + 6*B*x))/(30*x^6) + 42*a^4*b^5*(5*A*b + 6*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^{10}A}{x^7}+\frac{a^9(aB+10Ab)}{x^6}+\frac{5a^8b(2aB+9Ab)}{x^5}+\frac{15a^7b^2(3aB+8Ab)}{x^4}+\frac{30a^6b^3(4aB+7Ab)}{x^3}+\frac{42a^5b^4(5aB+\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^{10}A}{6x^6}-\frac{a^9(aB+10Ab)}{5x^5}-\frac{5a^8b(2aB+9Ab)}{4x^4}-\frac{5a^7b^2(3aB+8Ab)}{x^3}-\frac{15a^6b^3(4aB+7Ab)}{x^2}-\frac{42a^5b^4(5aB+6Ab)}{x}+42a^4b^5\log(x)(6aB+5Ab)+30a^3b^6x(7aB+4Ab)+\frac{15}{2}a^2b^7x^2(8aB+3Ab)+\frac{1}{4}b^9x^4(10aB+Ab)+\frac{5}{3}ab^8x^3(9aB+2Ab)+\frac{1}{5}b^{10}Bx^5$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^7,x]
```

output
```
-1/6*(a^10*A)/x^6 - (a^9*(10*A*b + a*B))/(5*x^5) - (5*a^8*b*(9*A*b + 2*a*B
))/(4*x^4) - (5*a^7*b^2*(8*A*b + 3*a*B))/x^3 - (15*a^6*b^3*(7*A*b + 4*a*B)
)/x^2 - (42*a^5*b^4*(6*A*b + 5*a*B))/x + 30*a^3*b^6*(4*A*b + 7*a*B)*x + (1
5*a^2*b^7*(3*A*b + 8*a*B)*x^2)/2 + (5*a*b^8*(2*A*b + 9*a*B)*x^3)/3 + (b^9*
(A*b + 10*a*B)*x^4)/4 + (b^10*B*x^5)/5 + 42*a^4*b^5*(5*A*b + 6*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^{10}B\,x^5}{5}+\frac{A\,b^{10}x^4}{4}+\frac{5Ba\,b^9x^4}{2}+\frac{10Aa\,b^9x^3}{3}+15B\,a^2b^8x^3+\frac{45A\,a^2b^8x^2}{2}+60B\,a^3b^7x^2+120A\,a^3b^7x$
norman	$\frac{(\frac{1}{4}b^{10}A+\frac{5}{2}a\,b^9B)x^{10}+(\frac{10}{3}a\,b^9A+15a^2b^8B)x^9+(\frac{45}{2}a^2b^8A+60a^3b^7B)x^8+(-\frac{45}{4}a^8b^2A-\frac{5}{2}a^9bB)x^2+(-2a^9bA-\frac{1}{5}a^{10}B)x+}{}$
risch	$\frac{b^{10}B\,x^5}{5}+\frac{A\,b^{10}x^4}{4}+\frac{5Ba\,b^9x^4}{2}+\frac{10Aa\,b^9x^3}{3}+15B\,a^2b^8x^3+\frac{45A\,a^2b^8x^2}{2}+60B\,a^3b^7x^2+120A\,a^3b^7x$
parallelrisch	$\frac{12B\,b^{10}x^{11}+15A\,b^{10}x^{10}+150Ba\,b^9x^{10}+200aA\,b^9x^9+900B\,a^2b^8x^9+1350a^2A\,b^8x^8+3600B\,a^3b^7x^8+12600A\ln(x)x^6a^4b^6+7}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^10*B*x^5+1/4*A*b^10*x^4+5/2*B*a*b^9*x^4+10/3*A*a*b^9*x^3+15*B*a^2*b^
8*x^3+45/2*A*a^2*b^8*x^2+60*B*a^3*b^7*x^2+120*A*a^3*b^7*x+210*B*a^4*b^6*x-
5*a^7*b^2*(8*A*b+3*B*a)/x^3-1/5*a^9*(10*A*b+B*a)/x^5-15*a^6*b^3*(7*A*b+4*B
*a)/x^2-5/4*a^8*b*(9*A*b+2*B*a)/x^4+42*a^4*b^5*(5*A*b+6*B*a)*ln(x)-42*a^5*
b^4*(6*A*b+5*B*a)/x-1/6*a^10*A/x^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx$$
$$=\frac{12\,Bb^{10}x^{11}-10\,Aa^{10}+15\,(10\,Bab^9+Ab^{10})x^{10}+100\,(9\,Ba^2b^8+2\,Aab^9)x^9+450\,(8\,Ba^3b^7+3\,Aa^2b^8)x^8}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^7,x, algorithm="fricas")
```

output
```
1/60*(12*B*b^10*x^11 - 10*A*a^10 + 15*(10*B*a*b^9 + A*b^10)*x^10 + 100*(9*
B*a^2*b^8 + 2*A*a*b^9)*x^9 + 450*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 1800*(7
*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 2520*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6*log(x
) - 2520*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 - 900*(4*B*a^7*b^3 + 7*A*a^6*b^4)
*x^4 - 300*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 75*(2*B*a^9*b + 9*A*a^8*b^2)*
x^2 - 12*(B*a^10 + 10*A*a^9*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx = \frac{Bb^{10}x^5}{5} + 42a^4b^5\cdot(5Ab+6Ba)\log(x) + x^4\left(\frac{Ab^{10}}{4}+\frac{5Bab^9}{2}\right) + x^3\cdot\left(\frac{10Aab^9}{3}+15Ba^2b^8\right) + x^2\cdot\left(\frac{45Aa^2b^8}{2}+60Ba^3b^7\right) + x\left(120Aa^3b^7+210Ba^4b^6\right) + \frac{-10Aa^{10}+x^5(-15120Aa^5b^5-12600Ba^6b^4)+x^4(-6300Aa^6b^4-3600Ba^7b^3)+x^3(-2400Aa^7b^3-90}{60x^6}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**7,x)
```

output
```
B*b**10*x**5/5 + 42*a**4*b**5*(5*A*b + 6*B*a)*log(x) + x**4*(A*b**10/4 + 5
*B*a*b**9/2) + x**3*(10*A*a*b**9/3 + 15*B*a**2*b**8) + x**2*(45*A*a**2*b**
8/2 + 60*B*a**3*b**7) + x*(120*A*a**3*b**7 + 210*B*a**4*b**6) + (-10*A*a**
10 + x**5*(-15120*A*a**5*b**5 - 12600*B*a**6*b**4) + x**4*(-6300*A*a**6*b*
*4 - 3600*B*a**7*b**3) + x**3*(-2400*A*a**7*b**3 - 900*B*a**8*b**2) + x**2
*(-675*A*a**8*b**2 - 150*B*a**9*b) + x*(-120*A*a**9*b - 12*B*a**10))/(60*x
**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx = \frac{1}{5}\,Bb^{10}x^5 + \frac{1}{4}\,(10\,Bab^9 + Ab^{10})x^4 + \frac{5}{3}\,(9\,Ba^2b^8 + 2\,Aab^9)x^3 + \frac{15}{2}\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^2 + 30\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x + 42\,(6\,Ba^5b^5 + 5\,Aa^4b^6)\log(x) - \frac{10\,Aa^{10} + 2520\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^5 + 900\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^4 + 300\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^3 + 75}{60\,x^6}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^7,x, algorithm="maxima")
```

output
```
1/5*B*b^10*x^5 + 1/4*(10*B*a*b^9 + A*b^10)*x^4 + 5/3*(9*B*a^2*b^8 + 2*A*a*
b^9)*x^3 + 15/2*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^2 + 30*(7*B*a^4*b^6 + 4*A*a^
3*b^7)*x + 42*(6*B*a^5*b^5 + 5*A*a^4*b^6)*log(x) - 1/60*(10*A*a^10 + 2520*
(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 900*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 30
0*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 75*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 12*
(B*a^10 + 10*A*a^9*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx = \frac{1}{5}\,Bb^{10}x^5 + \frac{5}{2}\,Bab^9x^4 + \frac{1}{4}\,Ab^{10}x^4 + 15\,Ba^2b^8x^3 + \frac{10}{3}\,Aab^9x^3 + 60\,Ba^3b^7x^2 + \frac{45}{2}\,Aa^2b^8x^2 + 210\,Ba^4b^6x + 120\,Aa^3b^7x + 42\left(6\,Ba^5b^5 + 5\,Aa^4b^6\right)\log\left(|x|\right) - \frac{10\,Aa^{10} + 2520\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^5 + 900\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^4 + 300\,(3\,Ba^8b^2 + 8\,Aa^7b^3)x^3 + 75}{60\,x^6}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^7,x, algorithm="giac")
```

output
```
1/5*B*b^10*x^5 + 5/2*B*a*b^9*x^4 + 1/4*A*b^10*x^4 + 15*B*a^2*b^8*x^3 + 10/
3*A*a*b^9*x^3 + 60*B*a^3*b^7*x^2 + 45/2*A*a^2*b^8*x^2 + 210*B*a^4*b^6*x +
120*A*a^3*b^7*x + 42*(6*B*a^5*b^5 + 5*A*a^4*b^6)*log(abs(x)) - 1/60*(10*A*
a^10 + 2520*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 900*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^4 + 300*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 75*(2*B*a^9*b + 9*A*a^8*b^
2)*x^2 + 12*(B*a^10 + 10*A*a^9*b)*x)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx = x^4\left(\frac{A\,b^{10}}{4}+\frac{5\,B\,a\,b^9}{2}\right) - \frac{x\left(\frac{B\,a^{10}}{5}+2\,A\,b\,a^9\right)+\frac{A\,a^{10}}{6}+x^2\left(\frac{5\,B\,a^9\,b}{2}+\frac{45\,A\,a^8\,b^2}{4}\right)+x^3\left(15\,B\,a^8\,b^2+40\,A\,a^7\,b^3\right)+x^4\left(60\,B\,a^7\,b^3+1\right.}{x^6} + \ln(x)\left(252\,B\,a^5\,b^5+210\,A\,a^4\,b^6\right)+\frac{B\,b^{10}\,x^5}{5}+\frac{15\,a^2\,b^7\,x^2\,(3\,A\,b+8\,B\,a)}{2} + 30\,a^3\,b^6\,x\,(4\,A\,b+7\,B\,a)+\frac{5\,a\,b^8\,x^3\,(2\,A\,b+9\,B\,a)}{3}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^7,x)
```

output
```
x^4*((A*b^10)/4 + (5*B*a*b^9)/2) - (x*((B*a^10)/5 + 2*A*a^9*b) + (A*a^10)/
6 + x^2*((45*A*a^8*b^2)/4 + (5*B*a^9*b)/2) + x^3*(40*A*a^7*b^3 + 15*B*a^8*
b^2) + x^4*(105*A*a^6*b^4 + 60*B*a^7*b^3) + x^5*(252*A*a^5*b^5 + 210*B*a^6
*b^4))/x^6 + log(x)*(210*A*a^4*b^6 + 252*B*a^5*b^5) + (B*b^10*x^5)/5 + (15
*a^2*b^7*x^2*(3*A*b + 8*B*a))/2 + 30*a^3*b^6*x*(4*A*b + 7*B*a) + (5*a*b^8*
x^3*(2*A*b + 9*B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^7}\,dx = \frac{27720\log(x)\,a^5b^6x^6-10a^{11}-132a^{10}bx-825a^9b^2x^2-3300a^8b^3x^3-9900a^7b^4x^4-27720a^6b^5x^5+1980}{60x^6}$$

input
```
int((b*x+a)^10*(B*x+A)/x^7,x)
```

output
```
(27720*log(x)*a**5*b**6*x**6 - 10*a**11 - 132*a**10*b*x - 825*a**9*b**2*x**
*2 - 3300*a**8*b**3*x**3 - 9900*a**7*b**4*x**4 - 27720*a**6*b**5*x**5 + 19
800*a**4*b**7*x**7 + 4950*a**3*b**8*x**8 + 1100*a**2*b**9*x**9 + 165*a*b**
10*x**10 + 12*b**11*x**11)/(60*x**6)
```

3.124 $\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 216

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = & -\frac{a^{10}A}{7x^7}-\frac{a^9(10Ab+aB)}{6x^6}-\frac{a^8b(9Ab+2aB)}{x^5} \\ & -\frac{15a^7b^2(8Ab+3aB)}{4x^4}-\frac{10a^6b^3(7Ab+4aB)}{x^3} \\ & -\frac{21a^5b^4(6Ab+5aB)}{x^2}-\frac{42a^4b^5(5Ab+6aB)}{x} \\ & +15a^2b^7(3Ab+8aB)x+\frac{5}{2}ab^8(2Ab+9aB)x^2 \\ & +\frac{1}{3}b^9(Ab+10aB)x^3+\frac{1}{4}b^{10}Bx^4+30a^3b^6(4Ab+7aB)\log(x)\end{aligned}$$

output
```
-1/7*a^10*A/x^7-1/6*a^9*(10*A*b+B*a)/x^6-a^8*b*(9*A*b+2*B*a)/x^5-15/4*a^7*
b^2*(8*A*b+3*B*a)/x^4-10*a^6*b^3*(7*A*b+4*B*a)/x^3-21*a^5*b^4*(6*A*b+5*B*a
)/x^2-42*a^4*b^5*(5*A*b+6*B*a)/x+15*a^2*b^7*(3*A*b+8*B*a)*x+5/2*a*b^8*(2*A
*b+9*B*a)*x^2+1/3*b^9*(A*b+10*B*a)*x^3+1/4*b^10*B*x^4+30*a^3*b^6*(4*A*b+7*
B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = -\frac{210a^4Ab^6}{x}+120a^3b^7Bx+\frac{45}{2}a^2b^8x(2A+Bx) - \frac{126a^5b^5(A+2Bx)}{x^2}+\frac{5}{3}ab^9x^2(3A+2Bx) - \frac{35a^6b^4(2A+3Bx)}{x^3}+\frac{1}{12}b^{10}x^3(4A+3Bx) - \frac{10a^7b^3(3A+4Bx)}{x^4}-\frac{9a^8b^2(4A+5Bx)}{4x^5}-\frac{a^9b(5A+6Bx)}{3x^6} - \frac{a^{10}(6A+7Bx)}{42x^7}+30a^3b^6(4Ab+7aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^8,x]
```

output
```
(-210*a^4*A*b^6)/x + 120*a^3*b^7*B*x + (45*a^2*b^8*x*(2*A + B*x))/2 - (126
*a^5*b^5*(A + 2*B*x))/x^2 + (5*a*b^9*x^2*(3*A + 2*B*x))/3 - (35*a^6*b^4*(2
*A + 3*B*x))/x^3 + (b^10*x^3*(4*A + 3*B*x))/12 - (10*a^7*b^3*(3*A + 4*B*x)
)/x^4 - (9*a^8*b^2*(4*A + 5*B*x))/(4*x^5) - (a^9*b*(5*A + 6*B*x))/(3*x^6)
- (a^10*(6*A + 7*B*x))/(42*x^7) + 30*a^3*b^6*(4*A*b + 7*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^{10}A}{x^8}+\frac{a^9(aB+10Ab)}{x^7}+\frac{5a^8b(2aB+9Ab)}{x^6}+\frac{15a^7b^2(3aB+8Ab)}{x^5}+\frac{30a^6b^3(4aB+7Ab)}{x^4}+\frac{42a^5b^4(5aB+}{x^3}\right.$$

$$\downarrow \text{2009}$$

$$-\frac{a^{10}A}{7x^7}-\frac{a^9(aB+10Ab)}{6x^6}-\frac{a^8b(2aB+9Ab)}{x^5}-\frac{15a^7b^2(3aB+8Ab)}{4x^4}-\frac{10a^6b^3(4aB+7Ab)}{x^3}-\frac{21a^5b^4(5aB+6Ab)}{x^2}-\frac{42a^4b^5(6aB+5Ab)}{x}+30a^3b^6\log(x)(7aB+4Ab)+15a^2b^7x(8aB+3Ab)+\frac{1}{3}b^9x^3(10aB+Ab)+\frac{5}{2}ab^8x^2(9aB+2Ab)+\frac{1}{4}b^{10}Bx^4$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^8,x]
```

output
```
-1/7*(a^10*A)/x^7 - (a^9*(10*A*b + a*B))/(6*x^6) - (a^8*b*(9*A*b + 2*a*B))
/x^5 - (15*a^7*b^2*(8*A*b + 3*a*B))/(4*x^4) - (10*a^6*b^3*(7*A*b + 4*a*B))
/x^3 - (21*a^5*b^4*(6*A*b + 5*a*B))/x^2 - (42*a^4*b^5*(5*A*b + 6*a*B))/x +
 15*a^2*b^7*(3*A*b + 8*a*B)*x + (5*a*b^8*(2*A*b + 9*a*B)*x^2)/2 + (b^9*(A*
b + 10*a*B)*x^3)/3 + (b^10*B*x^4)/4 + 30*a^3*b^6*(4*A*b + 7*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99

method	result
default	$\frac{b^{10}B\,x^4}{4}+\frac{A\,b^{10}x^3}{3}+\frac{10Ba\,b^9x^3}{3}+5Aa\,b^9x^2+\frac{45B\,a^2b^8x^2}{2}+45A\,a^2b^8x+120B\,a^3b^7x-\frac{10a^6b^3(7Ab+}{x^3}$
risch	$\frac{b^{10}B\,x^4}{4}+\frac{A\,b^{10}x^3}{3}+\frac{10Ba\,b^9x^3}{3}+5Aa\,b^9x^2+\frac{45B\,a^2b^8x^2}{2}+45A\,a^2b^8x+120B\,a^3b^7x+\frac{(-210a^4b^6A}{}$
norman	$(\frac{1}{3}b^{10}A+\frac{10}{3}a\,b^9B)x^{10}+(5a\,b^9A+\frac{45}{2}a^2b^8B)x^9+(-30a^7b^3A-\frac{45}{4}a^8b^2B)x^3+(-\frac{5}{3}a^9bA-\frac{1}{6}a^{10}B)x+(45a^2b^8A+120a^3b^7B)x$
parallelrisch	$21B\,b^{10}x^{11}+28A\,b^{10}x^{10}+280Ba\,b^9x^{10}+420aA\,b^9x^9+1890B\,a^2b^8x^9+10080A\ln(x)x^7a^3b^7+3780a^2A\,b^8x^8+17640B\ln(x)x^7$

input
```
int((b*x+a)^10*(B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^10*B*x^4+1/3*A*b^10*x^3+10/3*B*a*b^9*x^3+5*A*a*b^9*x^2+45/2*B*a^2*b^
8*x^2+45*A*a^2*b^8*x+120*B*a^3*b^7*x-10*a^6*b^3*(7*A*b+4*B*a)/x^3-a^8*b*(9
*A*b+2*B*a)/x^5-21*a^5*b^4*(6*A*b+5*B*a)/x^2-1/7*a^10*A/x^7-15/4*a^7*b^2*(
8*A*b+3*B*a)/x^4+30*a^3*b^6*(4*A*b+7*B*a)*ln(x)-42*a^4*b^5*(5*A*b+6*B*a)/x
-1/6*a^9*(10*A*b+B*a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx$$
$$=\frac{21\,Bb^{10}x^{11}-12\,Aa^{10}+28\,(10\,Bab^9+Ab^{10})x^{10}+210\,(9\,Ba^2b^8+2\,Aab^9)x^9+1260\,(8\,Ba^3b^7+3\,Aa^2b^8)}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^8,x, algorithm="fricas")
```

output
```
1/84*(21*B*b^10*x^11 - 12*A*a^10 + 28*(10*B*a*b^9 + A*b^10)*x^10 + 210*(9*
B*a^2*b^8 + 2*A*a*b^9)*x^9 + 1260*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 2520*(
7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7*log(x) - 3528*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x
^6 - 1764*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 - 840*(4*B*a^7*b^3 + 7*A*a^6*b^4
)*x^4 - 315*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 84*(2*B*a^9*b + 9*A*a^8*b^2)
*x^2 - 14*(B*a^10 + 10*A*a^9*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = \frac{Bb^{10}x^4}{4} + 30a^3b^6\cdot(4Ab+7Ba)\log(x) + x^3\left(\frac{Ab^{10}}{3}+\frac{10Bab^9}{3}\right) + x^2\cdot\left(5Aab^9+\frac{45Ba^2b^8}{2}\right) + x\left(45Aa^2b^8+120Ba^3b^7\right) + \frac{-12Aa^{10}+x^6(-17640Aa^4b^6-21168Ba^5b^5)+x^5(-10584Aa^5b^5-8820Ba^6b^4)+x^4(-5880Aa^6b^4-3\ldots}{84x\ldots}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**8,x)
```

output
```
B*b**10*x**4/4 + 30*a**3*b**6*(4*A*b + 7*B*a)*log(x) + x**3*(A*b**10/3 + 1
0*B*a*b**9/3) + x**2*(5*A*a*b**9 + 45*B*a**2*b**8/2) + x*(45*A*a**2*b**8 +
 120*B*a**3*b**7) + (-12*A*a**10 + x**6*(-17640*A*a**4*b**6 - 21168*B*a**5
*b**5) + x**5*(-10584*A*a**5*b**5 - 8820*B*a**6*b**4) + x**4*(-5880*A*a**6
*b**4 - 3360*B*a**7*b**3) + x**3*(-2520*A*a**7*b**3 - 945*B*a**8*b**2) + x
**2*(-756*A*a**8*b**2 - 168*B*a**9*b) + x*(-140*A*a**9*b - 14*B*a**10))/(8
4*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = \frac{1}{4}Bb^{10}x^4 + \frac{1}{3}\left(10\,Bab^9+Ab^{10}\right)x^3 + \frac{5}{2}\left(9\,Ba^2b^8+2\,Aab^9\right)x^2 + 15\left(8\,Ba^3b^7+3\,Aa^2b^8\right)x + 30\left(7\,Ba^4b^6+4\,Aa^3b^7\right)\log(x) - \frac{12\,Aa^{10}+3528\,(6\,Ba^5b^5+5\,Aa^4b^6)x^6+1764\,(5\,Ba^6b^4+6\,Aa^5b^5)x^5+840\,(4\,Ba^7b^3+7\,Aa^6b^4)x^4+3\ldots}{84\,x^7}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^8,x, algorithm="maxima")
```

output
```
1/4*B*b^10*x^4 + 1/3*(10*B*a*b^9 + A*b^10)*x^3 + 5/2*(9*B*a^2*b^8 + 2*A*a*
b^9)*x^2 + 15*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x + 30*(7*B*a^4*b^6 + 4*A*a^3*b^
7)*log(x) - 1/84*(12*A*a^10 + 3528*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 1764*
(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 840*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 31
5*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 84*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 14*
(B*a^10 + 10*A*a^9*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = \frac{1}{4}\,Bb^{10}x^4 + \frac{10}{3}\,Bab^9x^3 + \frac{1}{3}\,Ab^{10}x^3 + \frac{45}{2}\,Ba^2b^8x^2 + 5\,Aab^9x^2 + 120\,Ba^3b^7x + 45\,Aa^2b^8x + 30\,\left(7\,Ba^4b^6 + 4\,Aa^3b^7\right)\log\left(|x|\right) - \frac{12\,Aa^{10} + 3528\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^6 + 1764\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^5 + 840\,(4\,Ba^7b^3 + 7\,Aa^6b^4)x^4 + 3}{84\,x^7}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^8,x, algorithm="giac")
```

output
```
1/4*B*b^10*x^4 + 10/3*B*a*b^9*x^3 + 1/3*A*b^10*x^3 + 45/2*B*a^2*b^8*x^2 +
5*A*a*b^9*x^2 + 120*B*a^3*b^7*x + 45*A*a^2*b^8*x + 30*(7*B*a^4*b^6 + 4*A*a
^3*b^7)*log(abs(x)) - 1/84*(12*A*a^10 + 3528*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x
^6 + 1764*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 840*(4*B*a^7*b^3 + 7*A*a^6*b^4
)*x^4 + 315*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 84*(2*B*a^9*b + 9*A*a^8*b^2)
*x^2 + 14*(B*a^10 + 10*A*a^9*b)*x)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = x^3\left(\frac{A\,b^{10}}{3} + \frac{10\,B\,a\,b^9}{3}\right) - \frac{x\left(\frac{B\,a^{10}}{6} + \frac{5\,A\,b\,a^9}{3}\right) + \frac{A\,a^{10}}{7} + x^2\,(2\,B\,a^9\,b + 9\,A\,a^8\,b^2) + x^3\left(\frac{45\,B\,a^8\,b^2}{4} + 30\,A\,a^7\,b^3\right) + x^4\,(40\,B\,a^7\,b^3 + 70}{x^7} + \ln\left(x\right)\,\left(210\,B\,a^4\,b^6 + 120\,A\,a^3\,b^7\right) + \frac{B\,b^{10}\,x^4}{4} + 15\,a^2\,b^7\,x\,(3\,A\,b + 8\,B\,a) + \frac{5\,a\,b^8\,x^2\,(2\,A\,b + 9\,B\,a)}{2}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^8,x)
```

output
```
x^3*((A*b^10)/3 + (10*B*a*b^9)/3) - (x*((B*a^10)/6 + (5*A*a^9*b)/3) + (A*a
^10)/7 + x^2*(9*A*a^8*b^2 + 2*B*a^9*b) + x^3*(30*A*a^7*b^3 + (45*B*a^8*b^2
)/4) + x^4*(70*A*a^6*b^4 + 40*B*a^7*b^3) + x^5*(126*A*a^5*b^5 + 105*B*a^6*
b^4) + x^6*(210*A*a^4*b^6 + 252*B*a^5*b^5))/x^7 + log(x)*(120*A*a^3*b^7 +
210*B*a^4*b^6) + (B*b^10*x^4)/4 + 15*a^2*b^7*x*(3*A*b + 8*B*a) + (5*a*b^8*
x^2*(2*A*b + 9*B*a))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^8}\,dx = \frac{27720\log(x)\,a^4b^7x^7 - 12a^{11} - 154a^{10}bx - 924a^9b^2x^2 - 3465a^8b^3x^3 - 9240a^7b^4x^4 - 19404a^6b^5x^5 - 3880}{84x^7}$$

input
```
int((b*x+a)^10*(B*x+A)/x^8,x)
```

output
```
(27720*log(x)*a**4*b**7*x**7 - 12*a**11 - 154*a**10*b*x - 924*a**9*b**2*x*
*2 - 3465*a**8*b**3*x**3 - 9240*a**7*b**4*x**4 - 19404*a**6*b**5*x**5 - 38
808*a**5*b**6*x**6 + 13860*a**3*b**8*x**8 + 2310*a**2*b**9*x**9 + 308*a*b*
*10*x**10 + 21*b**11*x**11)/(84*x**7)
```

3.125 $\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 216

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx = -\frac{a^{10}A}{8x^8}-\frac{a^9(10Ab+aB)}{7x^7}-\frac{5a^8b(9Ab+2aB)}{6x^6} -\frac{3a^7b^2(8Ab+3aB)}{x^5}-\frac{15a^6b^3(7Ab+4aB)}{2x^4} -\frac{14a^5b^4(6Ab+5aB)}{x^3}-\frac{21a^4b^5(5Ab+6aB)}{x^2} -\frac{30a^3b^6(4Ab+7aB)}{x}+5ab^8(2Ab+9aB)x +\frac{1}{2}b^9(Ab+10aB)x^2+\frac{1}{3}b^{10}Bx^3+15a^2b^7(3Ab+8aB)\log(x)$$

output
```
-1/8*a^10*A/x^8-1/7*a^9*(10*A*b+B*a)/x^7-5/6*a^8*b*(9*A*b+2*B*a)/x^6-3*a^7
*b^2*(8*A*b+3*B*a)/x^5-15/2*a^6*b^3*(7*A*b+4*B*a)/x^4-14*a^5*b^4*(6*A*b+5*
B*a)/x^3-21*a^4*b^5*(5*A*b+6*B*a)/x^2-30*a^3*b^6*(4*A*b+7*B*a)/x+5*a*b^8*(
2*A*b+9*B*a)*x+1/2*b^9*(A*b+10*B*a)*x^2+1/3*b^10*B*x^3+15*a^2*b^7*(3*A*b+8
*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx = -\frac{120a^3Ab^7}{x}+45a^2b^8Bx+5ab^9x(2A+Bx) -\frac{105a^4b^6(A+2Bx)}{x^2}+\frac{1}{6}b^{10}x^2(3A+2Bx) -\frac{42a^5b^5(2A+3Bx)}{x^3}-\frac{35a^6b^4(3A+4Bx)}{2x^4} -\frac{6a^7b^3(4A+5Bx)}{x^5}-\frac{3a^8b^2(5A+6Bx)}{2x^6}-\frac{5a^9b(6A+7Bx)}{21x^7} -\frac{a^{10}(7A+8Bx)}{56x^8}+15a^2b^7(3Ab+8aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^9,x]
```

output
```
(-120*a^3*A*b^7)/x + 45*a^2*b^8*B*x + 5*a*b^9*x*(2*A + B*x) - (105*a^4*b^6
*(A + 2*B*x))/x^2 + (b^10*x^2*(3*A + 2*B*x))/6 - (42*a^5*b^5*(2*A + 3*B*x)
)/x^3 - (35*a^6*b^4*(3*A + 4*B*x))/(2*x^4) - (6*a^7*b^3*(4*A + 5*B*x))/x^5
 - (3*a^8*b^2*(5*A + 6*B*x))/(2*x^6) - (5*a^9*b*(6*A + 7*B*x))/(21*x^7) -
(a^10*(7*A + 8*B*x))/(56*x^8) + 15*a^2*b^7*(3*A*b + 8*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^9}+\frac{a^9(aB+10Ab)}{x^8}+\frac{5a^8b(2aB+9Ab)}{x^7}+\frac{15a^7b^2(3aB+8Ab)}{x^6}+\frac{30a^6b^3(4aB+7Ab)}{x^5}+\frac{42a^5b^4(5aB+}{x^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{8x^8}-\frac{a^9(aB+10Ab)}{7x^7}-\frac{5a^8b(2aB+9Ab)}{6x^6}-\frac{3a^7b^2(3aB+8Ab)}{x^5}-\frac{15a^6b^3(4aB+7Ab)}{2x^4}-\frac{14a^5b^4(5aB+6Ab)}{x^3}-\frac{21a^4b^5(6aB+5Ab)}{x^2}-\frac{30a^3b^6(7aB+4Ab)}{x}+15a^2b^7\log(x)(8aB+3Ab)+\frac{1}{2}b^9x^2(10aB+Ab)+5ab^8x(9aB+2Ab)+\frac{1}{3}b^{10}Bx^3$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^9,x]
```

output
```
-1/8*(a^10*A)/x^8 - (a^9*(10*A*b + a*B))/(7*x^7) - (5*a^8*b*(9*A*b + 2*a*B
))/(6*x^6) - (3*a^7*b^2*(8*A*b + 3*a*B))/x^5 - (15*a^6*b^3*(7*A*b + 4*a*B)
)/(2*x^4) - (14*a^5*b^4*(6*A*b + 5*a*B))/x^3 - (21*a^4*b^5*(5*A*b + 6*a*B)
)/x^2 - (30*a^3*b^6*(4*A*b + 7*a*B))/x + 5*a*b^8*(2*A*b + 9*a*B)*x + (b^9*
(A*b + 10*a*B)*x^2)/2 + (b^10*B*x^3)/3 + 15*a^2*b^7*(3*A*b + 8*a*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^{10}Bx^3}{3}+\frac{Ab^{10}x^2}{2}+5Bab^9x^2+10Aab^9x+45Ba^2b^8x-\frac{14a^5b^4(6Ab+5Ba)}{x^3}-\frac{3a^7b^2(8Ab+3Ba)}{x^5}-2$
risch	$\frac{b^{10}Bx^3}{3}+\frac{Ab^{10}x^2}{2}+5Bab^9x^2+10Aab^9x+45Ba^2b^8x+\frac{(-120a^3b^7A-210a^4b^6B)x^7+(-105a^4b^6A-126}{}$
norman	$\frac{(\frac{1}{2}b^{10}A+5ab^9B)x^{10}+(-\frac{105}{2}a^6b^4A-30a^7b^3B)x^4+(-\frac{15}{2}a^8b^2A-\frac{5}{3}a^9bB)x^2+(-\frac{10}{7}a^9bA-\frac{1}{7}a^{10}B)x+(10ab^9A+45a^2b^8B)x}{}$
parallelrisch	$\frac{56Bb^{10}x^{11}+84Ab^{10}x^{10}+840Bab^9x^{10}+7560A\ln(x)x^8a^2b^8+1680aAb^9x^9+20160B\ln(x)x^8a^3b^7+7560Ba^2b^8x^9-20160a^3A}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^10*B*x^3+1/2*A*b^10*x^2+5*B*a*b^9*x^2+10*A*a*b^9*x+45*B*a^2*b^8*x-14
*a^5*b^4*(6*A*b+5*B*a)/x^3-3*a^7*b^2*(8*A*b+3*B*a)/x^5-21*a^4*b^5*(5*A*b+6
*B*a)/x^2-1/7*a^9*(10*A*b+B*a)/x^7-15/2*a^6*b^3*(7*A*b+4*B*a)/x^4-1/8*a^10
*A/x^8+15*a^2*b^7*(3*A*b+8*B*a)*ln(x)-30*a^3*b^6*(4*A*b+7*B*a)/x-5/6*a^8*b
*(9*A*b+2*B*a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx = \frac{56\,Bb^{10}x^{11}-21\,Aa^{10}+84\,(10\,Bab^9+Ab^{10})x^{10}+840\,(9\,Ba^2b^8+2\,Aab^9)x^9+2520\,(8\,Ba^3b^7+3\,Aa^2b^8)}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^9,x, algorithm="fricas")
```

output
```
1/168*(56*B*b^10*x^11 - 21*A*a^10 + 84*(10*B*a*b^9 + A*b^10)*x^10 + 840*(9
*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 2520*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8*log(x)
- 5040*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 - 3528*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^6 - 2352*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 - 1260*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^4 - 504*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 140*(2*B*a^9*b + 9*A*a^8*b
^2)*x^2 - 24*(B*a^10 + 10*A*a^9*b)*x)/x^8
```

Sympy [A] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx$$
$$= \frac{Bb^{10}x^3}{3} + 15a^2b^7\cdot(3Ab+8Ba)\log(x) + x^2\left(\frac{Ab^{10}}{2}+5Bab^9\right) + x\left(10Aab^9+45Ba^2b^8\right)$$
$$+ \frac{-21Aa^{10} + x^7(-20160Aa^3b^7 - 35280Ba^4b^6) + x^6(-17640Aa^4b^6 - 21168Ba^5b^5) + x^5(-14112Aa^5b^5}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**9,x)
```

output
```
B*b**10*x**3/3 + 15*a**2*b**7*(3*A*b + 8*B*a)*log(x) + x**2*(A*b**10/2 + 5
*B*a*b**9) + x*(10*A*a*b**9 + 45*B*a**2*b**8) + (-21*A*a**10 + x**7*(-2016
0*A*a**3*b**7 - 35280*B*a**4*b**6) + x**6*(-17640*A*a**4*b**6 - 21168*B*a*
*5*b**5) + x**5*(-14112*A*a**5*b**5 - 11760*B*a**6*b**4) + x**4*(-8820*A*a
**6*b**4 - 5040*B*a**7*b**3) + x**3*(-4032*A*a**7*b**3 - 1512*B*a**8*b**2)
 + x**2*(-1260*A*a**8*b**2 - 280*B*a**9*b) + x*(-240*A*a**9*b - 24*B*a**10
))/(168*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx = \frac{1}{3}\,Bb^{10}x^3 + \frac{1}{2}\left(10\,Bab^9 + Ab^{10}\right)x^2$$
$$+ 5\left(9\,Ba^2b^8 + 2\,Aab^9\right)x + 15\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)\log(x)$$
$$- \frac{21\,Aa^{10} + 5040\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^7 + 3528\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^6 + 2352\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^5 +}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^9,x, algorithm="maxima")
```

output
```
1/3*B*b^10*x^3 + 1/2*(10*B*a*b^9 + A*b^10)*x^2 + 5*(9*B*a^2*b^8 + 2*A*a*b^
9)*x + 15*(8*B*a^3*b^7 + 3*A*a^2*b^8)*log(x) - 1/168*(21*A*a^10 + 5040*(7*
B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 3528*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 2352
*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 1260*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 +
504*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 140*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 +
24*(B*a^10 + 10*A*a^9*b)*x)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx = \frac{1}{3}\,Bb^{10}x^3 + 5\,Bab^9x^2 + \frac{1}{2}\,Ab^{10}x^2 + 45\,Ba^2b^8x + 10\,Aab^9x + 15\left(8\,Ba^3b^7 + 3\,Aa^2b^8\right)\log\left(|x|\right) - \frac{21\,Aa^{10} + 5040\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^7 + 3528\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^6 + 2352\,(5\,Ba^6b^4 + 6\,Aa^5b^5)x^5 +}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^9,x, algorithm="giac")
```

output
```
1/3*B*b^10*x^3 + 5*B*a*b^9*x^2 + 1/2*A*b^10*x^2 + 45*B*a^2*b^8*x + 10*A*a*
b^9*x + 15*(8*B*a^3*b^7 + 3*A*a^2*b^8)*log(abs(x)) - 1/168*(21*A*a^10 + 50
40*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 3528*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6
+ 2352*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 1260*(4*B*a^7*b^3 + 7*A*a^6*b^4)*
x^4 + 504*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 140*(2*B*a^9*b + 9*A*a^8*b^2)*
x^2 + 24*(B*a^10 + 10*A*a^9*b)*x)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx = x^2\left(\frac{A\,b^{10}}{2} + 5\,B\,a\,b^9\right) - \frac{x\left(\frac{B\,a^{10}}{7} + \frac{10\,A\,b\,a^9}{7}\right) + \frac{A\,a^{10}}{8} + x^2\left(\frac{5\,B\,a^9\,b}{3} + \frac{15\,A\,a^8\,b^2}{2}\right) + x^3\left(9\,B\,a^8\,b^2 + 24\,A\,a^7\,b^3\right) + x^4\left(30\,B\,a^7\,b^3 + 10\right.}{x} + \ln\left(x\right)\,\left(120\,B\,a^3\,b^7 + 45\,A\,a^2\,b^8\right) + \frac{B\,b^{10}\,x^3}{3} + 5\,a\,b^8\,x\,\left(2\,A\,b + 9\,B\,a\right)$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^9,x)
```

output
```
x^2*((A*b^10)/2 + 5*B*a*b^9) - (x*((B*a^10)/7 + (10*A*a^9*b)/7) + (A*a^10)
/8 + x^2*((15*A*a^8*b^2)/2 + (5*B*a^9*b)/3) + x^3*(24*A*a^7*b^3 + 9*B*a^8*
b^2) + x^4*((105*A*a^6*b^4)/2 + 30*B*a^7*b^3) + x^5*(84*A*a^5*b^5 + 70*B*a
^6*b^4) + x^6*(105*A*a^4*b^6 + 126*B*a^5*b^5) + x^7*(120*A*a^3*b^7 + 210*B
*a^4*b^6))/x^8 + log(x)*(45*A*a^2*b^8 + 120*B*a^3*b^7) + (B*b^10*x^3)/3 +
5*a*b^8*x*(2*A*b + 9*B*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^9}\,dx$$
$$= \frac{27720\log(x)\,a^3b^8x^8 - 21a^{11} - 264a^{10}bx - 1540a^9b^2x^2 - 5544a^8b^3x^3 - 13860a^7b^4x^4 - 25872a^6b^5x^5 - 38}{168x^8}$$

input
```
int((b*x+a)^10*(B*x+A)/x^9,x)
```

output
```
(27720*log(x)*a**3*b**8*x**8 - 21*a**11 - 264*a**10*b*x - 1540*a**9*b**2*x
**2 - 5544*a**8*b**3*x**3 - 13860*a**7*b**4*x**4 - 25872*a**6*b**5*x**5 -
38808*a**5*b**6*x**6 - 55440*a**4*b**7*x**7 + 9240*a**2*b**9*x**9 + 924*a*
b**10*x**10 + 56*b**11*x**11)/(168*x**8)
```

3.126 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 215

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx &= -\frac{a^{10}A}{9x^9}-\frac{a^9(10Ab+aB)}{8x^8}-\frac{5a^8b(9Ab+2aB)}{7x^7}\\ &\quad-\frac{5a^7b^2(8Ab+3aB)}{2x^6}-\frac{6a^6b^3(7Ab+4aB)}{x^5}\\ &\quad-\frac{21a^5b^4(6Ab+5aB)}{2x^4}-\frac{14a^4b^5(5Ab+6aB)}{x^3}\\ &\quad-\frac{15a^3b^6(4Ab+7aB)}{x^2}-\frac{15a^2b^7(3Ab+8aB)}{x}\\ &\quad+b^9(Ab+10aB)x+\frac{1}{2}b^{10}Bx^2+5ab^8(2Ab+9aB)\log(x)\end{aligned}$$

output

```
-1/9*a^10*A/x^9-1/8*a^9*(10*A*b+B*a)/x^8-5/7*a^8*b*(9*A*b+2*B*a)/x^7-5/2*a
^7*b^2*(8*A*b+3*B*a)/x^6-6*a^6*b^3*(7*A*b+4*B*a)/x^5-21/2*a^5*b^4*(6*A*b+5
*B*a)/x^4-14*a^4*b^5*(5*A*b+6*B*a)/x^3-15*a^3*b^6*(4*A*b+7*B*a)/x^2-15*a^2
*b^7*(3*A*b+8*B*a)/x+b^9*(A*b+10*B*a)*x+1/2*b^10*B*x^2+5*a*b^8*(2*A*b+9*B*
a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx = -\frac{45a^2Ab^8}{x}+10ab^9Bx+\frac{1}{2}b^{10}x(2A+Bx)-\frac{60a^3b^7(A+2Bx)}{x^2} -\frac{35a^4b^6(2A+3Bx)}{x^3}-\frac{21a^5b^5(3A+4Bx)}{x^4} -\frac{21a^6b^4(4A+5Bx)}{2x^5}-\frac{4a^7b^3(5A+6Bx)}{x^6} -\frac{15a^8b^2(6A+7Bx)}{14x^7}-\frac{5a^9b(7A+8Bx)}{28x^8} -\frac{a^{10}(8A+9Bx)}{72x^9}+5ab^8(2Ab+9aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^10,x]
```

output
```
(-45*a^2*A*b^8)/x + 10*a*b^9*B*x + (b^10*x*(2*A + B*x))/2 - (60*a^3*b^7*(A
 + 2*B*x))/x^2 - (35*a^4*b^6*(2*A + 3*B*x))/x^3 - (21*a^5*b^5*(3*A + 4*B*x
))/x^4 - (21*a^6*b^4*(4*A + 5*B*x))/(2*x^5) - (4*a^7*b^3*(5*A + 6*B*x))/x^
6 - (15*a^8*b^2*(6*A + 7*B*x))/(14*x^7) - (5*a^9*b*(7*A + 8*B*x))/(28*x^8)
 - (a^10*(8*A + 9*B*x))/(72*x^9) + 5*a*b^8*(2*A*b + 9*a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^{10}A}{x^{10}}+\frac{a^9(aB+10Ab)}{x^9}+\frac{5a^8b(2aB+9Ab)}{x^8}+\frac{15a^7b^2(3aB+8Ab)}{x^7}+\frac{30a^6b^3(4aB+7Ab)}{x^6}+\frac{42a^5b^4(5aB+}{x^5}\right.$$

$$\downarrow \text{2009}$$

$$-\frac{a^{10}A}{9x^9}-\frac{a^9(aB+10Ab)}{8x^8}-\frac{5a^8b(2aB+9Ab)}{7x^7}-\frac{5a^7b^2(3aB+8Ab)}{2x^6}-\frac{6a^6b^3(4aB+7Ab)}{x^5}-\frac{21a^5b^4(5aB+6Ab)}{2x^4}-\frac{14a^4b^5(6aB+5Ab)}{x^3}-\frac{15a^3b^6(7aB+4Ab)}{x^2}-\frac{15a^2b^7(8aB+3Ab)}{x}+b^9x(10aB+Ab)+5ab^8\log(x)(9aB+2Ab)+\frac{1}{2}b^{10}Bx^2$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^10,x]
```

output
```
-1/9*(a^10*A)/x^9 - (a^9*(10*A*b + a*B))/(8*x^8) - (5*a^8*b*(9*A*b + 2*a*B
))/(7*x^7) - (5*a^7*b^2*(8*A*b + 3*a*B))/(2*x^6) - (6*a^6*b^3*(7*A*b + 4*a
*B))/x^5 - (21*a^5*b^4*(6*A*b + 5*a*B))/(2*x^4) - (14*a^4*b^5*(5*A*b + 6*a
*B))/x^3 - (15*a^3*b^6*(4*A*b + 7*a*B))/x^2 - (15*a^2*b^7*(3*A*b + 8*a*B))
/x + b^9*(A*b + 10*a*B)*x + (b^10*B*x^2)/2 + 5*a*b^8*(2*A*b + 9*a*B)*Log[x
]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.95

method	result
default	$\frac{b^{10}B\,x^2}{2}+A\,b^{10}x+10Ba\,b^9x-\frac{14a^4b^5(5Ab+6Ba)}{x^3}-\frac{6a^6b^3(7Ab+4Ba)}{x^5}-\frac{15a^3b^6(4Ab+7Ba)}{x^2}-\frac{5a^8b(9Ab+2}{7x^7}$
risch	$\frac{b^{10}B\,x^2}{2}+A\,b^{10}x+10Ba\,b^9x+\frac{(-45a^2b^8A-120a^3b^7B)x^8+(-60a^3b^7A-105a^4b^6B)x^7+(-70a^4b^6A-84a^5b^5B)x}{}$
norman	$\frac{\left(-63a^5b^5A-\frac{105}{2}a^6b^4B\right)x^5+\left(-20a^7b^3A-\frac{15}{2}a^8b^2B\right)x^3+\left(-\frac{45}{7}a^8b^2A-\frac{10}{7}a^9bB\right)x^2+\left(-\frac{5}{4}a^9bA-\frac{1}{8}a^{10}B\right)x+\left(b^{10}A+10a\,b^9B\right)}{}$
parallelrisch	$\frac{252B\,b^{10}x^{11}+5040A\ln(x)x^9a\,b^9+504A\,b^{10}x^{10}+22680B\ln(x)x^9a^2b^8+5040Ba\,b^9x^{10}-22680a^2A\,b^8x^8-60480B\,a^3b^7x^8-302}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^10*B*x^2+A*b^10*x+10*B*a*b^9*x-14*a^4*b^5*(5*A*b+6*B*a)/x^3-6*a^6*b^
3*(7*A*b+4*B*a)/x^5-15*a^3*b^6*(4*A*b+7*B*a)/x^2-5/7*a^8*b*(9*A*b+2*B*a)/x
^7-21/2*a^5*b^4*(6*A*b+5*B*a)/x^4-1/8*a^9*(10*A*b+B*a)/x^8+5*a*b^8*(2*A*b+
9*B*a)*ln(x)-15*a^2*b^7*(3*A*b+8*B*a)/x-5/2*a^7*b^2*(8*A*b+3*B*a)/x^6-1/9*
a^10*A/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx$$
$$=\frac{252\,Bb^{10}x^{11}-56\,Aa^{10}+504\,(10\,Bab^9+Ab^{10})x^{10}+2520\,(9\,Ba^2b^8+2\,Aab^9)x^9\log(x)-7560\,(8\,Ba^3b^7}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^10,x, algorithm="fricas")
```

output
```
1/504*(252*B*b^10*x^11 - 56*A*a^10 + 504*(10*B*a*b^9 + A*b^10)*x^10 + 2520
*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9*log(x) - 7560*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x
^8 - 7560*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 - 7056*(6*B*a^5*b^5 + 5*A*a^4*b^
6)*x^6 - 5292*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 - 3024*(4*B*a^7*b^3 + 7*A*a^
6*b^4)*x^4 - 1260*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 360*(2*B*a^9*b + 9*A*a
^8*b^2)*x^2 - 63*(B*a^10 + 10*A*a^9*b)*x)/x^9
```

Sympy [A] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx = \frac{Bb^{10}x^2}{2} + 5ab^8\cdot(2Ab+9Ba)\log(x) + x\left(Ab^{10}+10Bab^9\right) + \frac{-56Aa^{10} + x^8(-22680Aa^2b^8 - 60480Ba^3b^7) + x^7(-30240Aa^3b^7 - 52920Ba^4b^6) + x^6(-35280Aa^4b^6 - \cdots}{\cdots}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**10,x)
```

output
```
B*b**10*x**2/2 + 5*a*b**8*(2*A*b + 9*B*a)*log(x) + x*(A*b**10 + 10*B*a*b**
9) + (-56*A*a**10 + x**8*(-22680*A*a**2*b**8 - 60480*B*a**3*b**7) + x**7*(
-30240*A*a**3*b**7 - 52920*B*a**4*b**6) + x**6*(-35280*A*a**4*b**6 - 42336
*B*a**5*b**5) + x**5*(-31752*A*a**5*b**5 - 26460*B*a**6*b**4) + x**4*(-211
68*A*a**6*b**4 - 12096*B*a**7*b**3) + x**3*(-10080*A*a**7*b**3 - 3780*B*a*
*8*b**2) + x**2*(-3240*A*a**8*b**2 - 720*B*a**9*b) + x*(-630*A*a**9*b - 63
*B*a**10))/(504*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx = \frac{1}{2}Bb^{10}x^2 + \left(10\,Bab^9 + Ab^{10}\right)x + 5\left(9\,Ba^2b^8 + 2\,Aab^9\right)\log(x) - \frac{56\,Aa^{10} + 7560\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^8 + 7560\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^7 + 7056\,(6\,Ba^5b^5 + 5\,Aa^4b^6)x^6 + \cdots}{\cdots}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^10,x, algorithm="maxima")
```

output
```
1/2*B*b^10*x^2 + (10*B*a*b^9 + A*b^10)*x + 5*(9*B*a^2*b^8 + 2*A*a*b^9)*log
(x) - 1/504*(56*A*a^10 + 7560*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 7560*(7*B*
a^4*b^6 + 4*A*a^3*b^7)*x^7 + 7056*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 5292*(
5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 3024*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 12
60*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 360*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 6
3*(B*a^10 + 10*A*a^9*b)*x)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx = \frac{1}{2}\,Bb^{10}x^2+10\,Bab^9x+Ab^{10}x+5\left(9\,Ba^2b^8+2\,Aab^9\right)\log\left(|x|\right) - \frac{56\,Aa^{10}+7560\,(8\,Ba^3b^7+3\,Aa^2b^8)x^8+7560\,(7\,Ba^4b^6+4\,Aa^3b^7)x^7+7056\,(6\,Ba^5b^5+5\,Aa^4b^6)x^6+\cdots}{\cdots}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^10,x, algorithm="giac")
```

output
```
1/2*B*b^10*x^2 + 10*B*a*b^9*x + A*b^10*x + 5*(9*B*a^2*b^8 + 2*A*a*b^9)*log
(abs(x)) - 1/504*(56*A*a^10 + 7560*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 7560*
(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 7056*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 5
292*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 3024*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4
 + 1260*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 360*(2*B*a^9*b + 9*A*a^8*b^2)*x^
2 + 63*(B*a^10 + 10*A*a^9*b)*x)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx = x\left(A\,b^{10}+10\,B\,a\,b^9\right)+\ln\left(x\right)\,\left(45\,B\,a^2\,b^8+10\,A\,a\,b^9\right) - \frac{x\left(\frac{B\,a^{10}}{8}+\frac{5\,A\,b\,a^9}{4}\right)+\frac{A\,a^{10}}{9}+x^2\left(\frac{10\,B\,a^9\,b}{7}+\frac{45\,A\,a^8\,b^2}{7}\right)+x^3\left(\frac{15\,B\,a^8\,b^2}{2}+20\,A\,a^7\,b^3\right)+x^4\left(24\,B\,a^7\,b^3+42\cdots\right.}{\cdots} + \frac{B\,b^{10}\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^10,x)
```

output
```
x*(A*b^10 + 10*B*a*b^9) + log(x)*(45*B*a^2*b^8 + 10*A*a*b^9) - (x*((B*a^10
)/8 + (5*A*a^9*b)/4) + (A*a^10)/9 + x^2*((45*A*a^8*b^2)/7 + (10*B*a^9*b)/7
) + x^3*(20*A*a^7*b^3 + (15*B*a^8*b^2)/2) + x^4*(42*A*a^6*b^4 + 24*B*a^7*b
^3) + x^6*(70*A*a^4*b^6 + 84*B*a^5*b^5) + x^7*(60*A*a^3*b^7 + 105*B*a^4*b^
6) + x^8*(45*A*a^2*b^8 + 120*B*a^3*b^7) + x^5*(63*A*a^5*b^5 + (105*B*a^6*b
^4)/2))/x^9 + (B*b^10*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{10}}\,dx$$
$$= \frac{27720\log(x)\,a^2b^9x^9 - 56a^{11} - 693a^{10}bx - 3960a^9b^2x^2 - 13860a^8b^3x^3 - 33264a^7b^4x^4 - 58212a^6b^5x^5 - 7}{504x^9}$$

input
```
int((b*x+a)^10*(B*x+A)/x^10,x)
```

output
```
(27720*log(x)*a**2*b**9*x**9 - 56*a**11 - 693*a**10*b*x - 3960*a**9*b**2*x
**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 - 58212*a**6*b**5*x**5 -
 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 83160*a**3*b**8*x**8 + 5544
*a*b**10*x**10 + 252*b**11*x**11)/(504*x**9)
```

3.127 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 216

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = & -\frac{a^{10}A}{10x^{10}}-\frac{a^9(10Ab+aB)}{9x^9}-\frac{5a^8b(9Ab+2aB)}{8x^8}\\ & -\frac{15a^7b^2(8Ab+3aB)}{7x^7}-\frac{5a^6b^3(7Ab+4aB)}{x^6}\\ & -\frac{42a^5b^4(6Ab+5aB)}{5x^5}-\frac{21a^4b^5(5Ab+6aB)}{2x^4}\\ & -\frac{10a^3b^6(4Ab+7aB)}{x^3}-\frac{15a^2b^7(3Ab+8aB)}{2x^2}\\ & -\frac{5ab^8(2Ab+9aB)}{x}+b^{10}Bx+b^9(Ab+10aB)\log(x)\end{aligned}$$

output
```
-1/10*a^10*A/x^10-1/9*a^9*(10*A*b+B*a)/x^9-5/8*a^8*b*(9*A*b+2*B*a)/x^8-15/
7*a^7*b^2*(8*A*b+3*B*a)/x^7-5*a^6*b^3*(7*A*b+4*B*a)/x^6-42/5*a^5*b^4*(6*A*
b+5*B*a)/x^5-21/2*a^4*b^5*(5*A*b+6*B*a)/x^4-10*a^3*b^6*(4*A*b+7*B*a)/x^3-1
5/2*a^2*b^7*(3*A*b+8*B*a)/x^2-5*a*b^8*(2*A*b+9*B*a)/x+b^10*B*x+b^9*(A*b+10
*B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = -\frac{10aAb^9}{x}+b^{10}Bx-\frac{45a^2b^8(A+2Bx)}{2x^2}-\frac{20a^3b^7(2A+3Bx)}{x^3} -\frac{35a^4b^6(3A+4Bx)}{2x^4}-\frac{63a^5b^5(4A+5Bx)}{5x^5} -\frac{7a^6b^4(5A+6Bx)}{x^6}-\frac{20a^7b^3(6A+7Bx)}{7x^7} -\frac{45a^8b^2(7A+8Bx)}{56x^8}-\frac{5a^9b(8A+9Bx)}{36x^9} -\frac{a^{10}(9A+10Bx)}{90x^{10}}+b^9(Ab+10aB)\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^11,x]
```

output
```
(-10*a*A*b^9)/x + b^10*B*x - (45*a^2*b^8*(A + 2*B*x))/(2*x^2) - (20*a^3*b^
7*(2*A + 3*B*x))/x^3 - (35*a^4*b^6*(3*A + 4*B*x))/(2*x^4) - (63*a^5*b^5*(4
*A + 5*B*x))/(5*x^5) - (7*a^6*b^4*(5*A + 6*B*x))/x^6 - (20*a^7*b^3*(6*A +
7*B*x))/(7*x^7) - (45*a^8*b^2*(7*A + 8*B*x))/(56*x^8) - (5*a^9*b*(8*A + 9*
B*x))/(36*x^9) - (a^10*(9*A + 10*B*x))/(90*x^10) + b^9*(A*b + 10*a*B)*Log[
x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^{11}} + \frac{a^9(aB+10Ab)}{x^{10}} + \frac{5a^8b(2aB+9Ab)}{x^9} + \frac{15a^7b^2(3aB+8Ab)}{x^8} + \frac{30a^6b^3(4aB+7Ab)}{x^7} + \frac{42a^5b^4(5aB+}{x^6}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^{10}A}{10x^{10}} - \frac{a^9(aB+10Ab)}{9x^9} - \frac{5a^8b(2aB+9Ab)}{8x^8} - \frac{15a^7b^2(3aB+8Ab)}{7x^7} - \frac{5a^6b^3(4aB+7Ab)}{x^6} - \frac{42a^5b^4(5aB+6Ab)}{5x^5} - \frac{21a^4b^5(6aB+5Ab)}{2x^4} - \frac{10a^3b^6(7aB+4Ab)}{x^3} - \frac{15a^2b^7(8aB+3Ab)}{2x^2} + b^9\log(x)(10aB+Ab) - \frac{5ab^8(9aB+2Ab)}{x} + b^{10}Bx$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^11,x]
```

output
```
-1/10*(a^10*A)/x^10 - (a^9*(10*A*b + a*B))/(9*x^9) - (5*a^8*b*(9*A*b + 2*a
*B))/(8*x^8) - (15*a^7*b^2*(8*A*b + 3*a*B))/(7*x^7) - (5*a^6*b^3*(7*A*b +
4*a*B))/x^6 - (42*a^5*b^4*(6*A*b + 5*a*B))/(5*x^5) - (21*a^4*b^5*(5*A*b +
6*a*B))/(2*x^4) - (10*a^3*b^6*(4*A*b + 7*a*B))/x^3 - (15*a^2*b^7*(3*A*b +
8*a*B))/(2*x^2) - (5*a*b^8*(2*A*b + 9*a*B))/x + b^10*B*x + b^9*(A*b + 10*a
*B)*Log[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a^{10}A}{10x^{10}}-\frac{a^{9}(10Ab+Ba)}{9x^{9}}-\frac{5a^{8}b(9Ab+2Ba)}{8x^{8}}-\frac{15a^{7}b^{2}(8Ab+3Ba)}{7x^{7}}-\frac{5a^{6}b^{3}(7Ab+4Ba)}{x^{6}}-\frac{42a^{5}b^{4}(6Ab+5Ba)}{5x^{5}}-$
risch	$b^{10}Bx+\frac{(-10a\,b^{9}A-45a^{2}b^{8}B)x^{9}+(-\frac{45}{2}a^{2}b^{8}A-60a^{3}b^{7}B)x^{8}+(-40a^{3}b^{7}A-70a^{4}b^{6}B)x^{7}+(-\frac{105}{2}a^{4}b^{6}A-63a^{5}b^{5}B)x^{6}+}{}$
norman	$\frac{(-\frac{45}{2}a^{2}b^{8}A-60a^{3}b^{7}B)x^{8}+(-\frac{105}{2}a^{4}b^{6}A-63a^{5}b^{5}B)x^{6}+(-\frac{252}{5}a^{5}b^{5}A-42a^{6}b^{4}B)x^{5}+(-\frac{120}{7}a^{7}b^{3}A-\frac{45}{7}a^{8}b^{2}B)x^{3}+(-\frac{45}{8}a^{8}}{}$
parallelrisch	$\frac{2520A\ln(x)x^{10}b^{10}+25200B\ln(x)x^{10}a\,b^{9}+2520B\,b^{10}x^{11}-25200aA\,b^{9}x^{9}-113400B\,a^{2}b^{8}x^{9}-56700a^{2}A\,b^{8}x^{8}-151200B\,a^{3}b^{7}}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^10*A/x^10-1/9*a^9*(10*A*b+B*a)/x^9-5/8*a^8*b*(9*A*b+2*B*a)/x^8-15/
7*a^7*b^2*(8*A*b+3*B*a)/x^7-5*a^6*b^3*(7*A*b+4*B*a)/x^6-42/5*a^5*b^4*(6*A*
b+5*B*a)/x^5-21/2*a^4*b^5*(5*A*b+6*B*a)/x^4-10*a^3*b^6*(4*A*b+7*B*a)/x^3-1
5/2*a^2*b^7*(3*A*b+8*B*a)/x^2-5*a*b^8*(2*A*b+9*B*a)/x+b^10*B*x+b^9*(A*b+10
*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = \frac{2520\,Bb^{10}x^{11}+2520\,(10\,Bab^{9}+Ab^{10})x^{10}\log(x)-252\,Aa^{10}-12600\,(9\,Ba^{2}b^{8}+2\,Aab^{9})x^{9}-18900\,(8\,B}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^11,x, algorithm="fricas")
```

output
```
1/2520*(2520*B*b^10*x^11 + 2520*(10*B*a*b^9 + A*b^10)*x^10*log(x) - 252*A*
a^10 - 12600*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 - 18900*(8*B*a^3*b^7 + 3*A*a^2*
b^8)*x^8 - 25200*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 - 26460*(6*B*a^5*b^5 + 5*
A*a^4*b^6)*x^6 - 21168*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 - 12600*(4*B*a^7*b^
3 + 7*A*a^6*b^4)*x^4 - 5400*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 1575*(2*B*a^
9*b + 9*A*a^8*b^2)*x^2 - 280*(B*a^10 + 10*A*a^9*b)*x)/x^10
```

Sympy [A] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = Bb^{10}x + b^9(Ab+10Ba)\log{(x)} + \frac{-252Aa^{10} + x^9(-25200Aab^9 - 113400Ba^2b^8) + x^8(-56700Aa^2b^8 - 151200Ba^3b^7) + x^7(-100800Aa^3}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**11,x)
```

output
```
B*b**10*x + b**9*(A*b + 10*B*a)*log(x) + (-252*A*a**10 + x**9*(-25200*A*a*
b**9 - 113400*B*a**2*b**8) + x**8*(-56700*A*a**2*b**8 - 151200*B*a**3*b**7
) + x**7*(-100800*A*a**3*b**7 - 176400*B*a**4*b**6) + x**6*(-132300*A*a**4
*b**6 - 158760*B*a**5*b**5) + x**5*(-127008*A*a**5*b**5 - 105840*B*a**6*b*
*4) + x**4*(-88200*A*a**6*b**4 - 50400*B*a**7*b**3) + x**3*(-43200*A*a**7*
b**3 - 16200*B*a**8*b**2) + x**2*(-14175*A*a**8*b**2 - 3150*B*a**9*b) + x*
(-2800*A*a**9*b - 280*B*a**10))/(2520*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = Bb^{10}x + \left(10\,Bab^9 + Ab^{10}\right)\log{(x)} - \frac{252\,Aa^{10} + 12600\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 18900\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^8 + 25200\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^7}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^11,x, algorithm="maxima")
```

output
```
B*b^10*x + (10*B*a*b^9 + A*b^10)*log(x) - 1/2520*(252*A*a^10 + 12600*(9*B*
a^2*b^8 + 2*A*a*b^9)*x^9 + 18900*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 25200*(
7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 26460*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 2
1168*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 12600*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x
^4 + 5400*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 1575*(2*B*a^9*b + 9*A*a^8*b^2)
*x^2 + 280*(B*a^10 + 10*A*a^9*b)*x)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = Bb^{10}x + \left(10\,Bab^9 + Ab^{10}\right)\log\left(|x|\right) - \frac{252\,Aa^{10} + 12600\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 18900\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x^8 + 25200\,(7\,Ba^4b^6 + 4\,Aa^3b^7)x^7}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^11,x, algorithm="giac")
```

output
```
B*b^10*x + (10*B*a*b^9 + A*b^10)*log(abs(x)) - 1/2520*(252*A*a^10 + 12600*
(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 18900*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 25
200*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 26460*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^
6 + 21168*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 12600*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^4 + 5400*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 1575*(2*B*a^9*b + 9*A*a^8
*b^2)*x^2 + 280*(B*a^10 + 10*A*a^9*b)*x)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx = \ln(x)\,\left(A\,b^{10} + 10\,B\,a\,b^9\right) - \frac{x\left(\frac{B\,a^{10}}{9} + \frac{10\,A\,b\,a^9}{9}\right) + \frac{A\,a^{10}}{10} + x^2\left(\frac{5\,B\,a^9\,b}{4} + \frac{45\,A\,a^8\,b^2}{8}\right) + x^9\left(45\,B\,a^2\,b^8 + 10\,A\,a\,b^9\right) + x^4\left(20\,B\,a^7\,b^3 + 35\right.}{} + B\,b^{10}\,x$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^11,x)
```

output
```
log(x)*(A*b^10 + 10*B*a*b^9) - (x*((B*a^10)/9 + (10*A*a^9*b)/9) + (A*a^10)
/10 + x^2*((45*A*a^8*b^2)/8 + (5*B*a^9*b)/4) + x^9*(45*B*a^2*b^8 + 10*A*a*
b^9) + x^4*(35*A*a^6*b^4 + 20*B*a^7*b^3) + x^8*((45*A*a^2*b^8)/2 + 60*B*a^
3*b^7) + x^7*(40*A*a^3*b^7 + 70*B*a^4*b^6) + x^6*((105*A*a^4*b^6)/2 + 63*B
*a^5*b^5) + x^3*((120*A*a^7*b^3)/7 + (45*B*a^8*b^2)/7) + x^5*((252*A*a^5*b
^5)/5 + 42*B*a^6*b^4))/x^10 + B*b^10*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{11}}\,dx$$
$$= \frac{27720\log(x)\,a\,b^{10}x^{10} - 252a^{11} - 3080a^{10}bx - 17325a^9b^2x^2 - 59400a^8b^3x^3 - 138600a^7b^4x^4 - 232848a^6b}{2520x^{10}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^11,x)
```

output
```
(27720*log(x)*a*b**10*x**10 - 252*a**11 - 3080*a**10*b*x - 17325*a**9*b**2
*x**2 - 59400*a**8*b**3*x**3 - 138600*a**7*b**4*x**4 - 232848*a**6*b**5*x*
*5 - 291060*a**5*b**6*x**6 - 277200*a**4*b**7*x**7 - 207900*a**3*b**8*x**8
 - 138600*a**2*b**9*x**9 + 2520*b**11*x**11)/(2520*x**10)
```

3.128 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}} dx$

Optimal result

Integrand size = 16, antiderivative size = 153

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}} dx = -\frac{a^{10}B}{10x^{10}} - \frac{10a^9bB}{9x^9} - \frac{45a^8b^2B}{8x^8} - \frac{120a^7b^3B}{7x^7} - \frac{35a^6b^4B}{x^6} - \frac{252a^5b^5B}{5x^5} - \frac{105a^4b^6B}{2x^4} - \frac{40a^3b^7B}{x^3} - \frac{45a^2b^8B}{2x^2} - \frac{10ab^9B}{x} - \frac{A(a+bx)^{11}}{11ax^{11}} + b^{10}B\log(x)$$

output
```
-1/10*a^10*B/x^10-10/9*a^9*b*B/x^9-45/8*a^8*b^2*B/x^8-120/7*a^7*b^3*B/x^7-
35*a^6*b^4*B/x^6-252/5*a^5*b^5*B/x^5-105/2*a^4*b^6*B/x^4-40*a^3*b^7*B/x^3-
45/2*a^2*b^8*B/x^2-10*a*b^9*B/x-1/11*A*(b*x+a)^11/a/x^11+b^10*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx = -\frac{Ab^{10}}{x} - \frac{5ab^9(A+2Bx)}{x^2} - \frac{15a^2b^8(2A+3Bx)}{2x^3} - \frac{10a^3b^7(3A+4Bx)}{x^4} - \frac{21a^4b^6(4A+5Bx)}{2x^5} - \frac{42a^5b^5(5A+6Bx)}{5x^6} - \frac{5a^6b^4(6A+7Bx)}{x^7} - \frac{15a^7b^3(7A+8Bx)}{7x^8} - \frac{5a^8b^2(8A+9Bx)}{8x^9} - \frac{a^9b(9A+10Bx)}{9x^{10}} - \frac{a^{10}(10A+11Bx)}{110x^{11}} + b^{10}B\log(x)$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^12,x]
```

output
```
-((A*b^10)/x) - (5*a*b^9*(A + 2*B*x))/x^2 - (15*a^2*b^8*(2*A + 3*B*x))/(2*
x^3) - (10*a^3*b^7*(3*A + 4*B*x))/x^4 - (21*a^4*b^6*(4*A + 5*B*x))/(2*x^5)
 - (42*a^5*b^5*(5*A + 6*B*x))/(5*x^6) - (5*a^6*b^4*(6*A + 7*B*x))/x^7 - (1
5*a^7*b^3*(7*A + 8*B*x))/(7*x^8) - (5*a^8*b^2*(8*A + 9*B*x))/(8*x^9) - (a^
9*b*(9*A + 10*B*x))/(9*x^10) - (a^10*(10*A + 11*B*x))/(110*x^11) + b^10*B*
Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx$$

$$\Big\downarrow \; 87$$

$$B\int \frac{(a+bx)^{10}}{x^{11}}dx-\frac{A(a+bx)^{11}}{11ax^{11}}$$

$$\Big\downarrow \text{ 49}$$

$$B\int \left(\frac{a^{10}}{x^{11}}+\frac{10ba^9}{x^{10}}+\frac{45b^2a^8}{x^9}+\frac{120b^3a^7}{x^8}+\frac{210b^4a^6}{x^7}+\frac{252b^5a^5}{x^6}+\frac{210b^6a^4}{x^5}+\frac{120b^7a^3}{x^4}+\frac{45b^8a^2}{x^3}+\frac{10b^9a}{x^2}+\frac{b^{10}}{x}\right)$$
$$\frac{A(a+bx)^{11}}{11ax^{11}}$$

$$\Big\downarrow \text{ 2009}$$

$$B\left(-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log\right.$$
$$\frac{A(a+bx)^{11}}{11ax^{11}}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^12,x]
```

output
```
-1/11*(A*(a + b*x)^11)/(a*x^11) + B*(-1/10*a^10/x^10 - (10*a^9*b)/(9*x^9)
- (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a
^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4) - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/
(2*x^2) - (10*a*b^9)/x + b^10*Log[x])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.35

method	result
default	$-\frac{5a^2b^7(3Ab+8Ba)}{x^3}-\frac{42a^4b^5(5Ab+6Ba)}{5x^5}-\frac{a^{10}A}{11x^{11}}-\frac{5ab^8(2Ab+9Ba)}{2x^2}-\frac{30a^6b^3(7Ab+4Ba)}{7x^7}-\frac{15a^3b^6(4Ab+7Ba)}{2x^4}$
norman	$\frac{(-5ab^9A-\frac{45}{2}a^2b^8B)x^9+(-30a^3b^7A-\frac{105}{2}a^4b^6B)x^7+(-42a^4b^6A-\frac{252}{5}a^5b^5B)x^6+(-30a^6b^4A-\frac{120}{7}a^7b^3B)x^4+(-15a^7b^3}{}$
risch	$\frac{(-5ab^9A-\frac{45}{2}a^2b^8B)x^9+(-30a^3b^7A-\frac{105}{2}a^4b^6B)x^7+(-42a^4b^6A-\frac{252}{5}a^5b^5B)x^6+(-30a^6b^4A-\frac{120}{7}a^7b^3B)x^4+(-15a^7b^3}{}$
parallelrisch	$-\frac{-27720b^{10}B\ln(x)x^{11}+27720Ab^{10}x^{10}+277200Bab^9x^{10}+138600aAb^9x^9+623700Ba^2b^8x^9+415800a^2Ab^8x^8+1108800B}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-5*a^2*b^7*(3*A*b+8*B*a)/x^3-42/5*a^4*b^5*(5*A*b+6*B*a)/x^5-1/11*a^10*A/x^
11-5/2*a*b^8*(2*A*b+9*B*a)/x^2-30/7*a^6*b^3*(7*A*b+4*B*a)/x^7-15/2*a^3*b^6
*(4*A*b+7*B*a)/x^4-15/8*a^7*b^2*(8*A*b+3*B*a)/x^8+b^10*B*ln(x)-1/10*a^9*(1
0*A*b+B*a)/x^10-b^9*(A*b+10*B*a)/x-7*a^5*b^4*(6*A*b+5*B*a)/x^6-5/9*a^8*b*(
9*A*b+2*B*a)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx = \frac{27720\,Bb^{10}x^{11}\log(x)-2520\,Aa^{10}-27720\,(10\,Bab^9+Ab^{10})x^{10}-69300\,(9\,Ba^2b^8+2\,Aab^9)x^9-138600}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^12,x, algorithm="fricas")
```

output
```
1/27720*(27720*B*b^10*x^11*log(x) - 2520*A*a^10 - 27720*(10*B*a*b^9 + A*b^
10)*x^10 - 69300*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 - 138600*(8*B*a^3*b^7 + 3*A
*a^2*b^8)*x^8 - 207900*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 - 232848*(6*B*a^5*b
^5 + 5*A*a^4*b^6)*x^6 - 194040*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 - 118800*(4
*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 - 51975*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 - 15
400*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 - 2772*(B*a^10 + 10*A*a^9*b)*x)/x^11
```

Sympy [A] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx = Bb^{10}\log{(x)} + \frac{-2520Aa^{10} + x^{10}(-27720Ab^{10} - 277200Bab^9) + x^9(-138600Aab^9 - 623700Ba^2b^8) + x^8(-415800Aa}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**12,x)
```

output
```
B*b**10*log(x) + (-2520*A*a**10 + x**10*(-27720*A*b**10 - 277200*B*a*b**9)
 + x**9*(-138600*A*a*b**9 - 623700*B*a**2*b**8) + x**8*(-415800*A*a**2*b**
8 - 1108800*B*a**3*b**7) + x**7*(-831600*A*a**3*b**7 - 1455300*B*a**4*b**6
) + x**6*(-1164240*A*a**4*b**6 - 1397088*B*a**5*b**5) + x**5*(-1164240*A*a
**5*b**5 - 970200*B*a**6*b**4) + x**4*(-831600*A*a**6*b**4 - 475200*B*a**7
*b**3) + x**3*(-415800*A*a**7*b**3 - 155925*B*a**8*b**2) + x**2*(-138600*A
*a**8*b**2 - 30800*B*a**9*b) + x*(-27720*A*a**9*b - 2772*B*a**10))/(27720*
x**11)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx = Bb^{10}\log{(x)} - \frac{2520\,Aa^{10} + 27720\,(10\,Bab^9 + Ab^{10})x^{10} + 69300\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 138600\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^12,x, algorithm="maxima")
```

output
```
B*b^10*log(x) - 1/27720*(2520*A*a^10 + 27720*(10*B*a*b^9 + A*b^10)*x^10 +
69300*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 138600*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x
^8 + 207900*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 232848*(6*B*a^5*b^5 + 5*A*a^
4*b^6)*x^6 + 194040*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 118800*(4*B*a^7*b^3
+ 7*A*a^6*b^4)*x^4 + 51975*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 15400*(2*B*a^
9*b + 9*A*a^8*b^2)*x^2 + 2772*(B*a^10 + 10*A*a^9*b)*x)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx = Bb^{10}\log(|x|) - \frac{2520\,Aa^{10} + 27720\,(10\,Bab^9 + Ab^{10})x^{10} + 69300\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 138600\,(8\,Ba^3b^7 + 3\,Aa^2b^8)x}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^12,x, algorithm="giac")
```

output
```
B*b^10*log(abs(x)) - 1/27720*(2520*A*a^10 + 27720*(10*B*a*b^9 + A*b^10)*x^
10 + 69300*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 138600*(8*B*a^3*b^7 + 3*A*a^2*b
^8)*x^8 + 207900*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 232848*(6*B*a^5*b^5 + 5
*A*a^4*b^6)*x^6 + 194040*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 118800*(4*B*a^7
*b^3 + 7*A*a^6*b^4)*x^4 + 51975*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 15400*(2
*B*a^9*b + 9*A*a^8*b^2)*x^2 + 2772*(B*a^10 + 10*A*a^9*b)*x)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx = B\,b^{10}\,\ln(x) - \frac{x\left(\frac{B\,a^{10}}{10} + A\,b\,a^9\right) + \frac{A\,a^{10}}{11} + x^2\left(\frac{10\,B\,a^9\,b}{9} + 5\,A\,a^8\,b^2\right) + x^9\left(\frac{45\,B\,a^2\,b^8}{2} + 5\,A\,a\,b^9\right) + x^{10}\left(A\,b^{10} + 10\,B\,a\,b\right.}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^12,x)
```

output
```
B*b^10*log(x) - (x*((B*a^10)/10 + A*a^9*b) + (A*a^10)/11 + x^2*(5*A*a^8*b^
2 + (10*B*a^9*b)/9) + x^9*((45*B*a^2*b^8)/2 + 5*A*a*b^9) + x^10*(A*b^10 +
10*B*a*b^9) + x^8*(15*A*a^2*b^8 + 40*B*a^3*b^7) + x^3*(15*A*a^7*b^3 + (45*
B*a^8*b^2)/8) + x^5*(42*A*a^5*b^5 + 35*B*a^6*b^4) + x^7*(30*A*a^3*b^7 + (1
05*B*a^4*b^6)/2) + x^4*(30*A*a^6*b^4 + (120*B*a^7*b^3)/7) + x^6*(42*A*a^4*
b^6 + (252*B*a^5*b^5)/5))/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{12}}\,dx$$
$$= \frac{27720\log(x)\,b^{11}x^{11} - 2520a^{11} - 30492a^{10}bx - 169400a^9b^2x^2 - 571725a^8b^3x^3 - 1306800a^7b^4x^4 - 21344}{27720x^{11}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^12,x)
```

output
```
(27720*log(x)*b**11*x**11 - 2520*a**11 - 30492*a**10*b*x - 169400*a**9*b**
2*x**2 - 571725*a**8*b**3*x**3 - 1306800*a**7*b**4*x**4 - 2134440*a**6*b**
5*x**5 - 2561328*a**5*b**6*x**6 - 2286900*a**4*b**7*x**7 - 1524600*a**3*b*
*8*x**8 - 762300*a**2*b**9*x**9 - 304920*a*b**10*x**10)/(27720*x**11)
```

3.129 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = -\frac{A(a+bx)^{11}}{12ax^{12}} + \frac{(Ab-12aB)(a+bx)^{11}}{132a^2x^{11}}$$

output
```
-1/12*A*(b*x+a)^11/a/x^12+1/132*(A*b-12*B*a)*(b*x+a)^11/a^2/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(44) = 88.

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.52

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = -\frac{66b^{10}x^{10}(A+2Bx)+220ab^9x^9(2A+3Bx)+495a^2b^8x^8(3A+4Bx)+792a^3b^7x^7(4A+5Bx)+924a}{}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^13,x]
```

output
```
-1/132*(66*b^10*x^10*(A + 2*B*x) + 220*a*b^9*x^9*(2*A + 3*B*x) + 495*a^2*b
^8*x^8*(3*A + 4*B*x) + 792*a^3*b^7*x^7*(4*A + 5*B*x) + 924*a^4*b^6*x^6*(5*
A + 6*B*x) + 792*a^5*b^5*x^5*(6*A + 7*B*x) + 495*a^6*b^4*x^4*(7*A + 8*B*x)
 + 220*a^7*b^3*x^3*(8*A + 9*B*x) + 66*a^8*b^2*x^2*(9*A + 10*B*x) + 12*a^9*
b*x*(10*A + 11*B*x) + a^10*(11*A + 12*B*x))/x^12
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-12aB)\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{A(a+bx)^{11}}{12ax^{12}}$$

$$\Big\downarrow \text{48}$$

$$\frac{(a+bx)^{11}(Ab-12aB)}{132a^2x^{11}}-\frac{A(a+bx)^{11}}{12ax^{12}}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^13,x]
```

output
```
-1/12*(A*(a + b*x)^11)/(a*x^12) + ((A*b - 12*a*B)*(a + b*x)^11)/(132*a^2*x
^11)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.73

method	result
default	$-\frac{5a\,b^{8}(2Ab+9Ba)}{3x^{3}}-\frac{6a^{3}b^{6}(4Ab+7Ba)}{x^{5}}-\frac{a^{9}(10Ab+Ba)}{11x^{11}}-\frac{b^{9}(Ab+10Ba)}{2x^{2}}-\frac{6a^{5}b^{4}(6Ab+5Ba)}{x^{7}}-\frac{15a^{2}b^{7}(3Ab+8B}{4x^{4}}$
norman	$-B\,b^{10}x^{11}+(-\frac{1}{2}b^{10}A-5a\,b^{9}B)x^{10}+(-\frac{10}{3}a\,b^{9}A-15a^{2}b^{8}B)x^{9}+(-\frac{45}{4}a^{2}b^{8}A-30a^{3}b^{7}B)x^{8}+(-24a^{3}b^{7}A-42a^{4}b^{6}B)x^{7}+(-3$
risch	$-B\,b^{10}x^{11}+(-\frac{1}{2}b^{10}A-5a\,b^{9}B)x^{10}+(-\frac{10}{3}a\,b^{9}A-15a^{2}b^{8}B)x^{9}+(-\frac{45}{4}a^{2}b^{8}A-30a^{3}b^{7}B)x^{8}+(-24a^{3}b^{7}A-42a^{4}b^{6}B)x^{7}+(-3$
gosper	$-\,132B\,b^{10}x^{11}+66A\,b^{10}x^{10}+660Ba\,b^{9}x^{10}+440aA\,b^{9}x^{9}+1980B\,a^{2}b^{8}x^{9}+1485a^{2}A\,b^{8}x^{8}+3960B\,a^{3}b^{7}x^{8}+3168a^{3}A\,b^{7}x^{7}+55$
parallelrisch	$-\,132B\,b^{10}x^{11}+66A\,b^{10}x^{10}+660Ba\,b^{9}x^{10}+440aA\,b^{9}x^{9}+1980B\,a^{2}b^{8}x^{9}+1485a^{2}A\,b^{8}x^{8}+3960B\,a^{3}b^{7}x^{8}+3168a^{3}A\,b^{7}x^{7}+55$
orering	$-\,132B\,b^{10}x^{11}+66A\,b^{10}x^{10}+660Ba\,b^{9}x^{10}+440aA\,b^{9}x^{9}+1980B\,a^{2}b^{8}x^{9}+1485a^{2}A\,b^{8}x^{8}+3960B\,a^{3}b^{7}x^{8}+3168a^{3}A\,b^{7}x^{7}+55$

input
```
int((b*x+a)^10*(B*x+A)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-5/3*a*b^8*(2*A*b+9*B*a)/x^3-6*a^3*b^6*(4*A*b+7*B*a)/x^5-1/11*a^9*(10*A*b+
B*a)/x^11-1/2*b^9*(A*b+10*B*a)/x^2-6*a^5*b^4*(6*A*b+5*B*a)/x^7-15/4*a^2*b^
7*(3*A*b+8*B*a)/x^4-15/4*a^6*b^3*(7*A*b+4*B*a)/x^8-1/2*a^8*b*(9*A*b+2*B*a)
/x^10-b^10*B/x-7*a^4*b^5*(5*A*b+6*B*a)/x^6-5/3*a^7*b^2*(8*A*b+3*B*a)/x^9-1
/12*a^10*A/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 5.52

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = -\frac{132\,Bb^{10}x^{11} + 11\,Aa^{10} + 66\,(10\,Bab^9 + Ab^{10})x^{10} + 220\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 495\,(8\,Ba^3b^7 + 3\,Aa^2b}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^13,x, algorithm="fricas")
```

output
```
-1/132*(132*B*b^10*x^11 + 11*A*a^10 + 66*(10*B*a*b^9 + A*b^10)*x^10 + 220*
(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 495*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 792*
(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 924*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 79
2*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 495*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 +
220*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 66*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 1
2*(B*a^10 + 10*A*a^9*b)*x)/x^12
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(37) = 74$.

Time = 12.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.91

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = \frac{-11Aa^{10} - 132Bb^{10}x^{11} + x^{10}(-66Ab^{10} - 660Bab^9) + x^9(-440Aab^9 - 1980Ba^2b^8) + x^8(-1485Aa^2b^8 -}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**13,x)
```

output
```
(-11*A*a**10 - 132*B*b**10*x**11 + x**10*(-66*A*b**10 - 660*B*a*b**9) + x*
*9*(-440*A*a*b**9 - 1980*B*a**2*b**8) + x**8*(-1485*A*a**2*b**8 - 3960*B*a
**3*b**7) + x**7*(-3168*A*a**3*b**7 - 5544*B*a**4*b**6) + x**6*(-4620*A*a*
*4*b**6 - 5544*B*a**5*b**5) + x**5*(-4752*A*a**5*b**5 - 3960*B*a**6*b**4)
+ x**4*(-3465*A*a**6*b**4 - 1980*B*a**7*b**3) + x**3*(-1760*A*a**7*b**3 -
660*B*a**8*b**2) + x**2*(-594*A*a**8*b**2 - 132*B*a**9*b) + x*(-120*A*a**9
*b - 12*B*a**10))/(132*x**12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 5.52

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = -\frac{132\,Bb^{10}x^{11} + 11\,Aa^{10} + 66\,(10\,Bab^9 + Ab^{10})x^{10} + 220\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 495\,(8\,Ba^3b^7 + 3\,Aa^2b}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^13,x, algorithm="maxima")
```

output
```
-1/132*(132*B*b^10*x^11 + 11*A*a^10 + 66*(10*B*a*b^9 + A*b^10)*x^10 + 220*
(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 495*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 + 792*
(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 924*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 79
2*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 495*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 +
220*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 66*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 1
2*(B*a^10 + 10*A*a^9*b)*x)/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 5.52

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = -\frac{132\,Bb^{10}x^{11} + 660\,Bab^9x^{10} + 66\,Ab^{10}x^{10} + 1980\,Ba^2b^8x^9 + 440\,Aab^9x^9 + 3960\,Ba^3b^7x^8 + 1485\,Aa^2b^8}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^13,x, algorithm="giac")
```

output
```
-1/132*(132*B*b^10*x^11 + 660*B*a*b^9*x^10 + 66*A*b^10*x^10 + 1980*B*a^2*b
^8*x^9 + 440*A*a*b^9*x^9 + 3960*B*a^3*b^7*x^8 + 1485*A*a^2*b^8*x^8 + 5544*
B*a^4*b^6*x^7 + 3168*A*a^3*b^7*x^7 + 5544*B*a^5*b^5*x^6 + 4620*A*a^4*b^6*x
^6 + 3960*B*a^6*b^4*x^5 + 4752*A*a^5*b^5*x^5 + 1980*B*a^7*b^3*x^4 + 3465*A
*a^6*b^4*x^4 + 660*B*a^8*b^2*x^3 + 1760*A*a^7*b^3*x^3 + 132*B*a^9*b*x^2 +
594*A*a^8*b^2*x^2 + 12*B*a^10*x + 120*A*a^9*b*x + 11*A*a^10)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.30

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{11}+\frac{10\,A\,b\,a^9}{11}\right)+\frac{A\,a^{10}}{12}+x^2\left(B\,a^9\,b+\frac{9\,A\,a^8\,b^2}{2}\right)+x^9\left(15\,B\,a^2\,b^8+\frac{10\,A\,a\,b^9}{3}\right)+x^{10}\left(\frac{A\,b^{10}}{2}+5\,B\,a\,b^9\right.}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^13,x)
```

output
```
-(x*((B*a^10)/11 + (10*A*a^9*b)/11) + (A*a^10)/12 + x^2*((9*A*a^8*b^2)/2 +
 B*a^9*b) + x^9*(15*B*a^2*b^8 + (10*A*a*b^9)/3) + x^10*((A*b^10)/2 + 5*B*a
*b^9) + x^3*((40*A*a^7*b^3)/3 + 5*B*a^8*b^2) + x^5*(36*A*a^5*b^5 + 30*B*a^
6*b^4) + x^7*(24*A*a^3*b^7 + 42*B*a^4*b^6) + x^6*(35*A*a^4*b^6 + 42*B*a^5*
b^5) + x^8*((45*A*a^2*b^8)/4 + 30*B*a^3*b^7) + x^4*((105*A*a^6*b^4)/4 + 15
*B*a^7*b^3) + B*b^10*x^11)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.80

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{13}}\,dx = \frac{-12b^{11}x^{11}-66a\,b^{10}x^{10}-220a^2b^9x^9-495a^3b^8x^8-792a^4b^7x^7-924a^5b^6x^6-792a^6b^5x^5-495a^7b^4x^4-}{12x^{12}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^13,x)
```

output
```
( - a**11 - 12*a**10*b*x - 66*a**9*b**2*x**2 - 220*a**8*b**3*x**3 - 495*a*
*7*b**4*x**4 - 792*a**6*b**5*x**5 - 924*a**5*b**6*x**6 - 792*a**4*b**7*x**
7 - 495*a**3*b**8*x**8 - 220*a**2*b**9*x**9 - 66*a*b**10*x**10 - 12*b**11*
x**11)/(12*x**12)
```

3.130 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = -\frac{A(a+bx)^{11}}{13ax^{13}}+\frac{(2Ab-13aB)(a+bx)^{11}}{156a^2x^{12}} - \frac{b(2Ab-13aB)(a+bx)^{11}}{1716a^3x^{11}}$$

output
```
-1/13*A*(b*x+a)^11/a/x^13+1/156*(2*A*b-13*B*a)*(b*x+a)^11/a^2/x^12-1/1716*
b*(2*A*b-13*B*a)*(b*x+a)^11/a^3/x^11
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 202 vs. $2(72) = 144$.

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.81

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = -\frac{286b^{10}x^{10}(2A+3Bx)+1430ab^9x^9(3A+4Bx)+3861a^2b^8x^8(4A+5Bx)+6864a^3b^7x^7(5A+6Bx)+}{}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^14,x]
```

output
```
-1/1716*(286*b^10*x^10*(2*A + 3*B*x) + 1430*a*b^9*x^9*(3*A + 4*B*x) + 3861
*a^2*b^8*x^8*(4*A + 5*B*x) + 6864*a^3*b^7*x^7*(5*A + 6*B*x) + 8580*a^4*b^6
*x^6*(6*A + 7*B*x) + 7722*a^5*b^5*x^5*(7*A + 8*B*x) + 5005*a^6*b^4*x^4*(8*
A + 9*B*x) + 2288*a^7*b^3*x^3*(9*A + 10*B*x) + 702*a^8*b^2*x^2*(10*A + 11*
B*x) + 130*a^9*b*x*(11*A + 12*B*x) + 11*a^10*(12*A + 13*B*x))/x^13
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}} \, dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(2Ab-13aB)\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{A(a+bx)^{11}}{13ax^{13}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(2Ab-13aB)\left(-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{A(a+bx)^{11}}{13ax^{13}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)(2Ab-13aB)}{13a}-\frac{A(a+bx)^{11}}{13ax^{13}}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^14,x]
```

output
```
-1/13*(A*(a + b*x)^11)/(a*x^13) - ((2*A*b - 13*a*B)*(-1/12*(a + b*x)^11/(a
*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(66) = 132$.

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.89

method	result
default	$-\frac{b^9(Ab+10Ba)}{3x^3}-\frac{3a^2b^7(3Ab+8Ba)}{x^5}-\frac{5a^8b(9Ab+2Ba)}{11x^{11}}-\frac{b^{10}B}{2x^2}-\frac{6a^4b^5(5Ab+6Ba)}{x^7}-\frac{5a\,b^8(2Ab+9Ba)}{4x^4}-\frac{21a^5}{}$
norman	$\frac{-\frac{B\,b^{10}x^{11}}{2}+\left(-\frac{1}{3}b^{10}A-\frac{10}{3}a\,b^9B\right)x^{10}+\left(-\frac{5}{2}a\,b^9A-\frac{45}{4}a^2b^8B\right)x^9+\left(-9a^2b^8A-24a^3b^7B\right)x^8+\left(-20a^3b^7A-35a^4b^6B\right)x^7+(-3}{}$
risch	$\frac{-\frac{B\,b^{10}x^{11}}{2}+\left(-\frac{1}{3}b^{10}A-\frac{10}{3}a\,b^9B\right)x^{10}+\left(-\frac{5}{2}a\,b^9A-\frac{45}{4}a^2b^8B\right)x^9+\left(-9a^2b^8A-24a^3b^7B\right)x^8+\left(-20a^3b^7A-35a^4b^6B\right)x^7+(-3}{}$
gosper	$-\frac{858B\,b^{10}x^{11}+572A\,b^{10}x^{10}+5720Ba\,b^9x^{10}+4290aA\,b^9x^9+19305B\,a^2b^8x^9+15444a^2A\,b^8x^8+41184B\,a^3b^7x^8+34320a^3A\,b}{}$
parallelrisch	$-\frac{858B\,b^{10}x^{11}+572A\,b^{10}x^{10}+5720Ba\,b^9x^{10}+4290aA\,b^9x^9+19305B\,a^2b^8x^9+15444a^2A\,b^8x^8+41184B\,a^3b^7x^8+34320a^3A\,b}{}$
orering	$-\frac{858B\,b^{10}x^{11}+572A\,b^{10}x^{10}+5720Ba\,b^9x^{10}+4290aA\,b^9x^9+19305B\,a^2b^8x^9+15444a^2A\,b^8x^8+41184B\,a^3b^7x^8+34320a^3A\,b}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^9*(A*b+10*B*a)/x^3-3*a^2*b^7*(3*A*b+8*B*a)/x^5-5/11*a^8*b*(9*A*b+2*
B*a)/x^11-1/2*b^10*B/x^2-6*a^4*b^5*(5*A*b+6*B*a)/x^7-5/4*a*b^8*(2*A*b+9*B*
a)/x^4-21/4*a^5*b^4*(6*A*b+5*B*a)/x^8-1/13*a^10*A/x^13-3/2*a^7*b^2*(8*A*b+
3*B*a)/x^10-5*a^3*b^6*(4*A*b+7*B*a)/x^6-10/3*a^6*b^3*(7*A*b+4*B*a)/x^9-1/1
2*a^9*(10*A*b+B*a)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.38

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = -\frac{858\,Bb^{10}x^{11} + 132\,Aa^{10} + 572\,(10\,Bab^9 + Ab^{10})x^{10} + 2145\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 5148\,(8\,Ba^3b^7 + 3}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^14,x, algorithm="fricas")
```

output
```
-1/1716*(858*B*b^10*x^11 + 132*A*a^10 + 572*(10*B*a*b^9 + A*b^10)*x^10 + 2
145*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 5148*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 +
 8580*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 10296*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^6 + 9009*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 5720*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^4 + 2574*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 780*(2*B*a^9*b + 9*A*a^8*
b^2)*x^2 + 143*(B*a^10 + 10*A*a^9*b)*x)/x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(65) = 130$.

Time = 16.59 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.61

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = \frac{-132Aa^{10} - 858Bb^{10}x^{11} + x^{10}(-572Ab^{10} - 5720Bab^9) + x^9(-4290Aab^9 - 19305Ba^2b^8) + x^8(-15444A}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**14,x)
```

output
```
(-132*A*a**10 - 858*B*b**10*x**11 + x**10*(-572*A*b**10 - 5720*B*a*b**9) +
 x**9*(-4290*A*a*b**9 - 19305*B*a**2*b**8) + x**8*(-15444*A*a**2*b**8 - 41
184*B*a**3*b**7) + x**7*(-34320*A*a**3*b**7 - 60060*B*a**4*b**6) + x**6*(-
51480*A*a**4*b**6 - 61776*B*a**5*b**5) + x**5*(-54054*A*a**5*b**5 - 45045*
B*a**6*b**4) + x**4*(-40040*A*a**6*b**4 - 22880*B*a**7*b**3) + x**3*(-2059
2*A*a**7*b**3 - 7722*B*a**8*b**2) + x**2*(-7020*A*a**8*b**2 - 1560*B*a**9*
b) + x*(-1430*A*a**9*b - 143*B*a**10))/(1716*x**13)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(66) = 132$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.38

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = -\frac{858\,Bb^{10}x^{11} + 132\,Aa^{10} + 572\,(10\,Bab^9 + Ab^{10})x^{10} + 2145\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 5148\,(8\,Ba^3b^7 + 3}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^14,x, algorithm="maxima")
```

output
```
-1/1716*(858*B*b^10*x^11 + 132*A*a^10 + 572*(10*B*a*b^9 + A*b^10)*x^10 + 2
145*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 5148*(8*B*a^3*b^7 + 3*A*a^2*b^8)*x^8 +
 8580*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 10296*(6*B*a^5*b^5 + 5*A*a^4*b^6)*
x^6 + 9009*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 5720*(4*B*a^7*b^3 + 7*A*a^6*b
^4)*x^4 + 2574*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 780*(2*B*a^9*b + 9*A*a^8*
b^2)*x^2 + 143*(B*a^10 + 10*A*a^9*b)*x)/x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(66) = 132$.

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.38

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = -\frac{858\,Bb^{10}x^{11} + 5720\,Bab^9x^{10} + 572\,Ab^{10}x^{10} + 19305\,Ba^2b^8x^9 + 4290\,Aab^9x^9 + 41184\,Ba^3b^7x^8 + 1544}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^14,x, algorithm="giac")
```

output
```
-1/1716*(858*B*b^10*x^11 + 5720*B*a*b^9*x^10 + 572*A*b^10*x^10 + 19305*B*a
^2*b^8*x^9 + 4290*A*a*b^9*x^9 + 41184*B*a^3*b^7*x^8 + 15444*A*a^2*b^8*x^8
+ 60060*B*a^4*b^6*x^7 + 34320*A*a^3*b^7*x^7 + 61776*B*a^5*b^5*x^6 + 51480*
A*a^4*b^6*x^6 + 45045*B*a^6*b^4*x^5 + 54054*A*a^5*b^5*x^5 + 22880*B*a^7*b^
3*x^4 + 40040*A*a^6*b^4*x^4 + 7722*B*a^8*b^2*x^3 + 20592*A*a^7*b^3*x^3 + 1
560*B*a^9*b*x^2 + 7020*A*a^8*b^2*x^2 + 143*B*a^10*x + 1430*A*a^9*b*x + 132
*A*a^10)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.26

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{12}+\frac{5\,A\,b\,a^9}{6}\right)+\frac{A\,a^{10}}{13}+x^9\left(\frac{45\,B\,a^2\,b^8}{4}+\frac{5\,A\,a\,b^9}{2}\right)+x^2\left(\frac{10\,B\,a^9\,b}{11}+\frac{45\,A\,a^8\,b^2}{11}\right)+x^{10}\left(\frac{A\,b^{10}}{3}+\frac{10\,B\,a\,b^9}{3}\right)}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^14,x)
```

output
```
-(x*((B*a^10)/12 + (5*A*a^9*b)/6) + (A*a^10)/13 + x^9*((45*B*a^2*b^8)/4 +
(5*A*a*b^9)/2) + x^2*((45*A*a^8*b^2)/11 + (10*B*a^9*b)/11) + x^10*((A*b^10
)/3 + (10*B*a*b^9)/3) + x^3*(12*A*a^7*b^3 + (9*B*a^8*b^2)/2) + x^8*(9*A*a^
2*b^8 + 24*B*a^3*b^7) + x^7*(20*A*a^3*b^7 + 35*B*a^4*b^6) + x^6*(30*A*a^4*
b^6 + 36*B*a^5*b^5) + x^4*((70*A*a^6*b^4)/3 + (40*B*a^7*b^3)/3) + x^5*((63
*A*a^5*b^5)/2 + (105*B*a^6*b^4)/4) + (B*b^10*x^11)/2)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{14}}\,dx$$
$$= \frac{-78b^{11}x^{11} - 572a\,b^{10}x^{10} - 2145a^2b^9x^9 - 5148a^3b^8x^8 - 8580a^4b^7x^7 - 10296a^5b^6x^6 - 9009a^6b^5x^5 - 5720}{156x^{13}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^14,x)
```

output
```
( - 12*a**11 - 143*a**10*b*x - 780*a**9*b**2*x**2 - 2574*a**8*b**3*x**3 -
5720*a**7*b**4*x**4 - 9009*a**6*b**5*x**5 - 10296*a**5*b**6*x**6 - 8580*a*
*4*b**7*x**7 - 5148*a**3*b**8*x**8 - 2145*a**2*b**9*x**9 - 572*a*b**10*x**
10 - 78*b**11*x**11)/(156*x**13)
```

3.131 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = -\frac{A(a+bx)^{11}}{14ax^{14}}+\frac{(3Ab-14aB)(a+bx)^{11}}{182a^2x^{13}} -\frac{b(3Ab-14aB)(a+bx)^{11}}{1092a^3x^{12}}+\frac{b^2(3Ab-14aB)(a+bx)^{11}}{12012a^4x^{11}}$$

output
```
-1/14*A*(b*x+a)^11/a/x^14+1/182*(3*A*b-14*B*a)*(b*x+a)^11/a^2/x^13-1/1092*
b*(3*A*b-14*B*a)*(b*x+a)^11/a^3/x^12+1/12012*b^2*(3*A*b-14*B*a)*(b*x+a)^11
/a^4/x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = -\frac{1001b^{10}x^{10}(3A+4Bx)+6006ab^9x^9(4A+5Bx)+18018a^2b^8x^8(5A+6Bx)+34320a^3b^7x^7(6A+7Bx)}{}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^15,x]
```

output
```
-1/12012*(1001*b^10*x^10*(3*A + 4*B*x) + 6006*a*b^9*x^9*(4*A + 5*B*x) + 18
018*a^2*b^8*x^8*(5*A + 6*B*x) + 34320*a^3*b^7*x^7*(6*A + 7*B*x) + 45045*a^
4*b^6*x^6*(7*A + 8*B*x) + 42042*a^5*b^5*x^5*(8*A + 9*B*x) + 28028*a^6*b^4*
x^4*(9*A + 10*B*x) + 13104*a^7*b^3*x^3*(10*A + 11*B*x) + 4095*a^8*b^2*x^2*
(11*A + 12*B*x) + 770*a^9*b*x*(12*A + 13*B*x) + 66*a^10*(13*A + 14*B*x))/x
^14
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}} \, dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(3Ab-14aB)\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{A(a+bx)^{11}}{14ax^{14}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(3Ab-14aB)\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{A(a+bx)^{11}}{14ax^{14}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(3Ab-14aB)\left(-\frac{2b\left(-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{A(a+bx)^{11}}{14ax^{14}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)(3Ab-14aB)}{14a}-\frac{A(a+bx)^{11}}{14ax^{14}}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^15,x]
```

output
```
-1/14*(A*(a + b*x)^11)/(a*x^14) - ((3*A*b - 14*a*B)*(-1/13*(a + b*x)^11/(a
*x^13) - (2*b*(-1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^1
1)))/(13*a)))/(14*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(93) = 186$.

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.06

method	result
default	$-\frac{b^{10}B}{3x^3}-\frac{a\,b^8(2Ab+9Ba)}{x^5}-\frac{15a^7b^2(8Ab+3Ba)}{11x^{11}}-\frac{30a^3b^6(4Ab+7Ba)}{7x^7}-\frac{b^9(Ab+10Ba)}{4x^4}-\frac{21a^4b^5(5Ab+6Ba)}{4x^8}-$ a
norman	$-\frac{a^{10}A}{14}+(-\frac{10}{13}a^9bA-\frac{1}{13}a^{10}B)x+(-\frac{15}{4}a^8b^2A-\frac{5}{6}a^9bB)x^2+(-\frac{120}{11}a^7b^3A-\frac{45}{11}a^8b^2B)x^3+(-21a^6b^4A-12a^7b^3B)x^4+(-28a$
risch	$-\frac{a^{10}A}{14}+(-\frac{10}{13}a^9bA-\frac{1}{13}a^{10}B)x+(-\frac{15}{4}a^8b^2A-\frac{5}{6}a^9bB)x^2+(-\frac{120}{11}a^7b^3A-\frac{45}{11}a^8b^2B)x^3+(-21a^6b^4A-12a^7b^3B)x^4+(-28a$
gosper	$-$ $4004B\,b^{10}x^{11}+3003A\,b^{10}x^{10}+30030Ba\,b^9x^{10}+24024aA\,b^9x^9+108108B\,a^2b^8x^9+90090a^2A\,b^8x^8+240240B\,a^3b^7x^8+2059$
parallelrisch	$-$ $4004B\,b^{10}x^{11}+3003A\,b^{10}x^{10}+30030Ba\,b^9x^{10}+24024aA\,b^9x^9+108108B\,a^2b^8x^9+90090a^2A\,b^8x^8+240240B\,a^3b^7x^8+2059$
orering	$-$ $4004B\,b^{10}x^{11}+3003A\,b^{10}x^{10}+30030Ba\,b^9x^{10}+24024aA\,b^9x^9+108108B\,a^2b^8x^9+90090a^2A\,b^8x^8+240240B\,a^3b^7x^8+2059$

input

```
int((b*x+a)^10*(B*x+A)/x^15,x,method=_RETURNVERBOSE)
```

output

```
-1/3*b^10*B/x^3-a*b^8*(2*A*b+9*B*a)/x^5-15/11*a^7*b^2*(8*A*b+3*B*a)/x^11-3
0/7*a^3*b^6*(4*A*b+7*B*a)/x^7-1/4*b^9*(A*b+10*B*a)/x^4-21/4*a^4*b^5*(5*A*b
+6*B*a)/x^8-1/13*a^9*(10*A*b+B*a)/x^13-3*a^6*b^3*(7*A*b+4*B*a)/x^10-1/14*a
^10*A/x^14-5/2*a^2*b^7*(3*A*b+8*B*a)/x^6-14/3*a^5*b^4*(6*A*b+5*B*a)/x^9-5/
12*a^8*b*(9*A*b+2*B*a)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx =$$

$$-\frac{4004\,Bb^{10}x^{11}+858\,Aa^{10}+3003\,(10\,Bab^9+Ab^{10})x^{10}+12012\,(9\,Ba^2b^8+2\,Aab^9)x^9+30030\,(8\,Ba^3b^7}{}$$

input

```
integrate((b*x+a)^10*(B*x+A)/x^15,x, algorithm="fricas")
```

output
```
-1/12012*(4004*B*b^10*x^11 + 858*A*a^10 + 3003*(10*B*a*b^9 + A*b^10)*x^10
+ 12012*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 30030*(8*B*a^3*b^7 + 3*A*a^2*b^8)*
x^8 + 51480*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 63063*(6*B*a^5*b^5 + 5*A*a^4
*b^6)*x^6 + 56056*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 36036*(4*B*a^7*b^3 + 7
*A*a^6*b^4)*x^4 + 16380*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 5005*(2*B*a^9*b
+ 9*A*a^8*b^2)*x^2 + 924*(B*a^10 + 10*A*a^9*b)*x)/x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(95) = 190.

Time = 22.68 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.57

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = \frac{-858Aa^{10} - 4004Bb^{10}x^{11} + x^{10}(-3003Ab^{10} - 30030Bab^9) + x^9(-24024Aab^9 - 108108Ba^2b^8) + x^8(-9$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**15,x)
```

output
```
(-858*A*a**10 - 4004*B*b**10*x**11 + x**10*(-3003*A*b**10 - 30030*B*a*b**9
) + x**9*(-24024*A*a*b**9 - 108108*B*a**2*b**8) + x**8*(-90090*A*a**2*b**8
 - 240240*B*a**3*b**7) + x**7*(-205920*A*a**3*b**7 - 360360*B*a**4*b**6) +
 x**6*(-315315*A*a**4*b**6 - 378378*B*a**5*b**5) + x**5*(-336336*A*a**5*b*
*5 - 280280*B*a**6*b**4) + x**4*(-252252*A*a**6*b**4 - 144144*B*a**7*b**3)
 + x**3*(-131040*A*a**7*b**3 - 49140*B*a**8*b**2) + x**2*(-45045*A*a**8*b*
*2 - 10010*B*a**9*b) + x*(-9240*A*a**9*b - 924*B*a**10))/(12012*x**14)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(93) = 186.

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = -\frac{4004\,Bb^{10}x^{11} + 858\,Aa^{10} + 3003\,(10\,Bab^9 + Ab^{10})x^{10} + 12012\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 30030\,(8\,Ba^3b^7$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^15,x, algorithm="maxima")
```

output
```
-1/12012*(4004*B*b^10*x^11 + 858*A*a^10 + 3003*(10*B*a*b^9 + A*b^10)*x^10
+ 12012*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 30030*(8*B*a^3*b^7 + 3*A*a^2*b^8)*
x^8 + 51480*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 63063*(6*B*a^5*b^5 + 5*A*a^4
*b^6)*x^6 + 56056*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 36036*(4*B*a^7*b^3 + 7
*A*a^6*b^4)*x^4 + 16380*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 5005*(2*B*a^9*b
+ 9*A*a^8*b^2)*x^2 + 924*(B*a^10 + 10*A*a^9*b)*x)/x^14
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = -\frac{4004\,Bb^{10}x^{11} + 30030\,Bab^9x^{10} + 3003\,Ab^{10}x^{10} + 108108\,Ba^2b^8x^9 + 24024\,Aab^9x^9 + 240240\,Ba^3b^7x^8 + \cdots}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^15,x, algorithm="giac")
```

output
```
-1/12012*(4004*B*b^10*x^11 + 30030*B*a*b^9*x^10 + 3003*A*b^10*x^10 + 10810
8*B*a^2*b^8*x^9 + 24024*A*a*b^9*x^9 + 240240*B*a^3*b^7*x^8 + 90090*A*a^2*b
^8*x^8 + 360360*B*a^4*b^6*x^7 + 205920*A*a^3*b^7*x^7 + 378378*B*a^5*b^5*x^
6 + 315315*A*a^4*b^6*x^6 + 280280*B*a^6*b^4*x^5 + 336336*A*a^5*b^5*x^5 + 1
44144*B*a^7*b^3*x^4 + 252252*A*a^6*b^4*x^4 + 49140*B*a^8*b^2*x^3 + 131040*
A*a^7*b^3*x^3 + 10010*B*a^9*b*x^2 + 45045*A*a^8*b^2*x^2 + 924*B*a^10*x + 9
240*A*a^9*b*x + 858*A*a^10)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.33

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{13}+\frac{10\,A\,b\,a^9}{13}\right)+\frac{A\,a^{10}}{14}+x^9\left(9\,B\,a^2\,b^8+2\,A\,a\,b^9\right)+x^2\left(\frac{5\,B\,a^9\,b}{6}+\frac{15\,A\,a^8\,b^2}{4}\right)+x^{10}\left(\frac{A\,b^{10}}{4}+\frac{5\,B\,a\,b^9}{2}\right)}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^15,x)
```

output
```
-(x*((B*a^10)/13 + (10*A*a^9*b)/13) + (A*a^10)/14 + x^9*(9*B*a^2*b^8 + 2*A
*a*b^9) + x^2*((15*A*a^8*b^2)/4 + (5*B*a^9*b)/6) + x^10*((A*b^10)/4 + (5*B
*a*b^9)/2) + x^4*(21*A*a^6*b^4 + 12*B*a^7*b^3) + x^8*((15*A*a^2*b^8)/2 + 2
0*B*a^3*b^7) + x^5*(28*A*a^5*b^5 + (70*B*a^6*b^4)/3) + x^7*((120*A*a^3*b^7
)/7 + 30*B*a^4*b^6) + x^6*((105*A*a^4*b^6)/4 + (63*B*a^5*b^5)/2) + x^3*((1
20*A*a^7*b^3)/11 + (45*B*a^8*b^2)/11) + (B*b^10*x^11)/3)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}}\,dx = \frac{-364b^{11}x^{11}-3003a\,b^{10}x^{10}-12012a^2b^9x^9-30030a^3b^8x^8-51480a^4b^7x^7-63063a^5b^6x^6-56056a^6b^5x^5}{1092x^{14}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^15,x)
```

output
```
( - 78*a**11 - 924*a**10*b*x - 5005*a**9*b**2*x**2 - 16380*a**8*b**3*x**3
- 36036*a**7*b**4*x**4 - 56056*a**6*b**5*x**5 - 63063*a**5*b**6*x**6 - 514
80*a**4*b**7*x**7 - 30030*a**3*b**8*x**8 - 12012*a**2*b**9*x**9 - 3003*a*b
**10*x**10 - 364*b**11*x**11)/(1092*x**14)
```

3.132 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = -\frac{A(a+bx)^{11}}{15ax^{15}}+\frac{(4Ab-15aB)(a+bx)^{11}}{210a^2x^{14}}-\frac{b(4Ab-15aB)(a+bx)^{11}}{910a^3x^{13}}+\frac{b^2(4Ab-15aB)(a+bx)^{11}}{5460a^4x^{12}}-\frac{b^3(4Ab-15aB)(a+bx)^{11}}{60060a^5x^{11}}$$

output
```
-1/15*A*(b*x+a)^11/a/x^15+1/210*(4*A*b-15*B*a)*(b*x+a)^11/a^2/x^14-1/910*b
*(4*A*b-15*B*a)*(b*x+a)^11/a^3/x^13+1/5460*b^2*(4*A*b-15*B*a)*(b*x+a)^11/a
^4/x^12-1/60060*b^3*(4*A*b-15*B*a)*(b*x+a)^11/a^5/x^11
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = -\frac{3003b^{10}x^{10}(4A+5Bx)+20020ab^9x^9(5A+6Bx)+64350a^2b^8x^8(6A+7Bx)+128700a^3b^7x^7(7A+8}{}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^16,x]
```

output
```
-1/60060*(3003*b^10*x^10*(4*A + 5*B*x) + 20020*a*b^9*x^9*(5*A + 6*B*x) + 6
4350*a^2*b^8*x^8*(6*A + 7*B*x) + 128700*a^3*b^7*x^7*(7*A + 8*B*x) + 175175
*a^4*b^6*x^6*(8*A + 9*B*x) + 168168*a^5*b^5*x^5*(9*A + 10*B*x) + 114660*a^
6*b^4*x^4*(10*A + 11*B*x) + 54600*a^7*b^3*x^3*(11*A + 12*B*x) + 17325*a^8*
b^2*x^2*(12*A + 13*B*x) + 3300*a^9*b*x*(13*A + 14*B*x) + 286*a^10*(14*A +
15*B*x))/x^15
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(4Ab-15aB)\int \frac{(a+bx)^{10}}{x^{15}}dx}{15a}-\frac{A(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(4Ab-15aB)\left(-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{A(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(4Ab-15aB)\left(-\frac{3b\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{A(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(4Ab-15aB)\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{A(a+bx)^{11}}{15ax^{15}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)(4Ab-15aB)}{15a}-\frac{A(a+bx)^{11}}{15ax^{15}}$$

input

```
Int[((a + b*x)^10*(A + B*x))/x^16,x]
```

output

```
-1/15*(A*(a + b*x)^11)/(a*x^15) - ((4*A*b - 15*a*B)*(-1/14*(a + b*x)^11/(a
*x^14) - (3*b*(-1/13*(a + b*x)^11/(a*x^13) - (2*b*(-1/12*(a + b*x)^11/(a*x
^12) + (b*(a + b*x)^11)/(132*a^2*x^11)))/(13*a)))/(14*a)))/(15*a)
```

Defintions of rubi rules used

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.60

method	result
default	$-\frac{b^{9}(Ab+10Ba)}{5x^{5}}-\frac{30a^{6}b^{3}(7Ab+4Ba)}{11x^{11}}-\frac{a^{10}A}{15x^{15}}-\frac{15a^{2}b^{7}(3Ab+8Ba)}{7x^{7}}-\frac{b^{10}B}{4x^{4}}-\frac{15a^{3}b^{6}(4Ab+7Ba)}{4x^{8}}-\frac{5a^{8}b(9Ab+}{13x^{13}}$
norman	$\frac{-\frac{a^{10}A}{15}+\left(-\frac{5}{7}a^{9}bA-\frac{1}{14}a^{10}B\right)x+\left(-\frac{45}{13}a^{8}b^{2}A-\frac{10}{13}a^{9}bB\right)x^{2}+\left(-10a^{7}b^{3}A-\frac{15}{4}a^{8}b^{2}B\right)x^{3}+\left(-\frac{210}{11}a^{6}b^{4}A-\frac{120}{11}a^{7}b^{3}B\right)x^{4}+\left(-\frac{12}{5}\right.}{}$
risch	$\frac{-\frac{a^{10}A}{15}+\left(-\frac{5}{7}a^{9}bA-\frac{1}{14}a^{10}B\right)x+\left(-\frac{45}{13}a^{8}b^{2}A-\frac{10}{13}a^{9}bB\right)x^{2}+\left(-10a^{7}b^{3}A-\frac{15}{4}a^{8}b^{2}B\right)x^{3}+\left(-\frac{210}{11}a^{6}b^{4}A-\frac{120}{11}a^{7}b^{3}B\right)x^{4}+\left(-\frac{12}{5}\right.}{}$
gosper	$-\frac{15015Bb^{10}x^{11}+12012Ab^{10}x^{10}+120120Bab^{9}x^{10}+100100aAb^{9}x^{9}+450450Ba^{2}b^{8}x^{9}+386100a^{2}Ab^{8}x^{8}+1029600Ba^{3}b^{7}x}{}$
parallelrisch	$-\frac{15015Bb^{10}x^{11}+12012Ab^{10}x^{10}+120120Bab^{9}x^{10}+100100aAb^{9}x^{9}+450450Ba^{2}b^{8}x^{9}+386100a^{2}Ab^{8}x^{8}+1029600Ba^{3}b^{7}x}{}$
orering	$-\frac{15015Bb^{10}x^{11}+12012Ab^{10}x^{10}+120120Bab^{9}x^{10}+100100aAb^{9}x^{9}+450450Ba^{2}b^{8}x^{9}+386100a^{2}Ab^{8}x^{8}+1029600Ba^{3}b^{7}x}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^9*(A*b+10*B*a)/x^5-30/11*a^6*b^3*(7*A*b+4*B*a)/x^11-1/15*a^10*A/x^1
5-15/7*a^2*b^7*(3*A*b+8*B*a)/x^7-1/4*b^10*B/x^4-15/4*a^3*b^6*(4*A*b+7*B*a)
/x^8-5/13*a^8*b*(9*A*b+2*B*a)/x^13-21/5*a^5*b^4*(6*A*b+5*B*a)/x^10-1/14*a^
9*(10*A*b+B*a)/x^14-5/6*a*b^8*(2*A*b+9*B*a)/x^6-14/3*a^4*b^5*(5*A*b+6*B*a)
/x^9-5/4*a^7*b^2*(8*A*b+3*B*a)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(120) = 240$.

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = -\frac{15015\,Bb^{10}x^{11} + 4004\,Aa^{10} + 12012\,(10\,Bab^9 + Ab^{10})x^{10} + 50050\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 128700\,(8\,B}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^16,x, algorithm="fricas")
```

output
```
-1/60060*(15015*B*b^10*x^11 + 4004*A*a^10 + 12012*(10*B*a*b^9 + A*b^10)*x^
10 + 50050*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 128700*(8*B*a^3*b^7 + 3*A*a^2*b
^8)*x^8 + 225225*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 280280*(6*B*a^5*b^5 + 5
*A*a^4*b^6)*x^6 + 252252*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 163800*(4*B*a^7
*b^3 + 7*A*a^6*b^4)*x^4 + 75075*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 23100*(2
*B*a^9*b + 9*A*a^8*b^2)*x^2 + 4290*(B*a^10 + 10*A*a^9*b)*x)/x^15
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(124) = 248$.

Time = 31.14 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = \frac{-4004Aa^{10} - 15015Bb^{10}x^{11} + x^{10}(-12012Ab^{10} - 120120Bab^9) + x^9(-100100Aab^9 - 450450Ba^2b^8) +}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**16,x)
```

output
```
(-4004*A*a**10 - 15015*B*b**10*x**11 + x**10*(-12012*A*b**10 - 120120*B*a*
b**9) + x**9*(-100100*A*a*b**9 - 450450*B*a**2*b**8) + x**8*(-386100*A*a**
2*b**8 - 1029600*B*a**3*b**7) + x**7*(-900900*A*a**3*b**7 - 1576575*B*a**4
*b**6) + x**6*(-1401400*A*a**4*b**6 - 1681680*B*a**5*b**5) + x**5*(-151351
2*A*a**5*b**5 - 1261260*B*a**6*b**4) + x**4*(-1146600*A*a**6*b**4 - 655200
*B*a**7*b**3) + x**3*(-600600*A*a**7*b**3 - 225225*B*a**8*b**2) + x**2*(-2
07900*A*a**8*b**2 - 46200*B*a**9*b) + x*(-42900*A*a**9*b - 4290*B*a**10))/
(60060*x**15)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(120) = 240$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = -\frac{15015\,Bb^{10}x^{11} + 4004\,Aa^{10} + 12012\,(10\,Bab^9 + Ab^{10})x^{10} + 50050\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 128700\,(8\,B}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^16,x, algorithm="maxima")
```

output
```
-1/60060*(15015*B*b^10*x^11 + 4004*A*a^10 + 12012*(10*B*a*b^9 + A*b^10)*x^
10 + 50050*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 128700*(8*B*a^3*b^7 + 3*A*a^2*b
^8)*x^8 + 225225*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 280280*(6*B*a^5*b^5 + 5
*A*a^4*b^6)*x^6 + 252252*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 163800*(4*B*a^7
*b^3 + 7*A*a^6*b^4)*x^4 + 75075*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 23100*(2
*B*a^9*b + 9*A*a^8*b^2)*x^2 + 4290*(B*a^10 + 10*A*a^9*b)*x)/x^15
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(120) = 240$.

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = -\frac{15015\,Bb^{10}x^{11} + 120120\,Bab^9x^{10} + 12012\,Ab^{10}x^{10} + 450450\,Ba^2b^8x^9 + 100100\,Aab^9x^9 + 1029600\,Ba}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^16,x, algorithm="giac")
```

output
```
-1/60060*(15015*B*b^10*x^11 + 120120*B*a*b^9*x^10 + 12012*A*b^10*x^10 + 45
0450*B*a^2*b^8*x^9 + 100100*A*a*b^9*x^9 + 1029600*B*a^3*b^7*x^8 + 386100*A
*a^2*b^8*x^8 + 1576575*B*a^4*b^6*x^7 + 900900*A*a^3*b^7*x^7 + 1681680*B*a^
5*b^5*x^6 + 1401400*A*a^4*b^6*x^6 + 1261260*B*a^6*b^4*x^5 + 1513512*A*a^5*
b^5*x^5 + 655200*B*a^7*b^3*x^4 + 1146600*A*a^6*b^4*x^4 + 225225*B*a^8*b^2*
x^3 + 600600*A*a^7*b^3*x^3 + 46200*B*a^9*b*x^2 + 207900*A*a^8*b^2*x^2 + 42
90*B*a^10*x + 42900*A*a^9*b*x + 4004*A*a^10)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{14}+\frac{5\,A\,b\,a^9}{7}\right)+\frac{A\,a^{10}}{15}+x^9\left(\frac{15\,B\,a^2\,b^8}{2}+\frac{5\,A\,a\,b^9}{3}\right)+x^2\left(\frac{10\,B\,a^9\,b}{13}+\frac{45\,A\,a^8\,b^2}{13}\right)+x^{10}\left(\frac{A\,b^{10}}{5}+2\,B\,a\,b^9\right)}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^16,x)
```

output
```
-(x*((B*a^10)/14 + (5*A*a^9*b)/7) + (A*a^10)/15 + x^9*((15*B*a^2*b^8)/2 +
(5*A*a*b^9)/3) + x^2*((45*A*a^8*b^2)/13 + (10*B*a^9*b)/13) + x^10*((A*b^10
)/5 + 2*B*a*b^9) + x^3*(10*A*a^7*b^3 + (15*B*a^8*b^2)/4) + x^6*((70*A*a^4*
b^6)/3 + 28*B*a^5*b^5) + x^7*(15*A*a^3*b^7 + (105*B*a^4*b^6)/4) + x^5*((12
6*A*a^5*b^5)/5 + 21*B*a^6*b^4) + x^8*((45*A*a^2*b^8)/7 + (120*B*a^3*b^7)/7
) + x^4*((210*A*a^6*b^4)/11 + (120*B*a^7*b^3)/11) + (B*b^10*x^11)/4)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}}\,dx = \frac{-1365b^{11}x^{11}-12012a\,b^{10}x^{10}-50050a^2b^9x^9-128700a^3b^8x^8-225225a^4b^7x^7-280280a^5b^6x^6-252252}{5460x^{15}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^16,x)
```

output
```
( - 364*a**11 - 4290*a**10*b*x - 23100*a**9*b**2*x**2 - 75075*a**8*b**3*x*
*3 - 163800*a**7*b**4*x**4 - 252252*a**6*b**5*x**5 - 280280*a**5*b**6*x**6
 - 225225*a**4*b**7*x**7 - 128700*a**3*b**8*x**8 - 50050*a**2*b**9*x**9 -
12012*a*b**10*x**10 - 1365*b**11*x**11)/(5460*x**15)
```

3.133 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 159

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx = -\frac{A(a+bx)^{11}}{16ax^{16}} + \frac{(5Ab-16aB)(a+bx)^{11}}{240a^2x^{15}} - \frac{b(5Ab-16aB)(a+bx)^{11}}{840a^3x^{14}} + \frac{b^2(5Ab-16aB)(a+bx)^{11}}{3640a^4x^{13}} - \frac{b^3(5Ab-16aB)(a+bx)^{11}}{21840a^5x^{12}} + \frac{b^4(5Ab-16aB)(a+bx)^{11}}{240240a^6x^{11}}$$

output
```
-1/16*A*(b*x+a)^11/a/x^16+1/240*(5*A*b-16*B*a)*(b*x+a)^11/a^2/x^15-1/840*b
*(5*A*b-16*B*a)*(b*x+a)^11/a^3/x^14+1/3640*b^2*(5*A*b-16*B*a)*(b*x+a)^11/a
^4/x^13-1/21840*b^3*(5*A*b-16*B*a)*(b*x+a)^11/a^5/x^12+1/240240*b^4*(5*A*b
-16*B*a)*(b*x+a)^11/a^6/x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx = -\frac{b^{10}(5A+6Bx)}{30x^6}-\frac{5ab^9(6A+7Bx)}{21x^7}-\frac{45a^2b^8(7A+8Bx)}{56x^8}-\frac{5a^3b^7(8A+9Bx)}{3x^9}-\frac{7a^4b^6(9A+10Bx)}{3x^{10}}-\frac{126a^5b^5(10A+11Bx)}{55x^{11}}-\frac{35a^6b^4(11A+12Bx)}{22x^{12}}-\frac{10a^7b^3(12A+13Bx)}{13x^{13}}-\frac{45a^8b^2(13A+14Bx)}{182x^{14}}-\frac{a^9b(14A+15Bx)}{21x^{15}}-\frac{a^{10}(15A+16Bx)}{240x^{16}}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^17,x]
```

output
```
-1/30*(b^10*(5*A + 6*B*x))/x^6 - (5*a*b^9*(6*A + 7*B*x))/(21*x^7) - (45*a^
2*b^8*(7*A + 8*B*x))/(56*x^8) - (5*a^3*b^7*(8*A + 9*B*x))/(3*x^9) - (7*a^4
*b^6*(9*A + 10*B*x))/(3*x^10) - (126*a^5*b^5*(10*A + 11*B*x))/(55*x^11) -
(35*a^6*b^4*(11*A + 12*B*x))/(22*x^12) - (10*a^7*b^3*(12*A + 13*B*x))/(13*
x^13) - (45*a^8*b^2*(13*A + 14*B*x))/(182*x^14) - (a^9*b*(14*A + 15*B*x))/
(21*x^15) - (a^10*(15*A + 16*B*x))/(240*x^16)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {87, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(5Ab-16aB)\int \frac{(a+bx)^{10}}{x^{16}}dx}{16a}-\frac{A(a+bx)^{11}}{16ax^{16}}$$

$\downarrow$ 55

$$-\frac{(5Ab-16aB)\left(-\frac{4b\int \frac{(a+bx)^{10}}{x^{15}}dx}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{A(a+bx)^{11}}{16ax^{16}}$$

$\downarrow$ 55

$$-\frac{(5Ab-16aB)\left(-\frac{4b\left(-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{A(a+bx)^{11}}{16ax^{16}}$$

$\downarrow$ 55

$$-\frac{(5Ab-16aB)\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\int \frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{A(a+bx)^{11}}{16ax^{16}}$$

$\downarrow$ 55

$$-\frac{(5Ab-16aB)\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{A(a+bx)^{11}}{16ax^{16}}$$

$\Big\downarrow$ 48

$$-\frac{\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)(5Ab-16aB)}{16a}-\frac{A(a+bx)^{11}}{16ax^{16}}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^17,x]
```

output
```
-1/16*(A*(a + b*x)^11)/(a*x^16) - ((5*A*b - 16*a*B)*(-1/15*(a + b*x)^11/(a
*x^15) - (4*b*(-1/14*(a + b*x)^11/(a*x^14) - (3*b*(-1/13*(a + b*x)^11/(a*x
^13) - (2*b*(-1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
))/(13*a)))/(14*a)))/(15*a)))/(16*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.31

method	result
default	$-\frac{b^{10}B}{5x^5}-\frac{42a^5b^4(6Ab+5Ba)}{11x^{11}}-\frac{a^9(10Ab+Ba)}{15x^{15}}-\frac{5a\,b^8(2Ab+9Ba)}{7x^7}-\frac{15a^2b^7(3Ab+8Ba)}{8x^8}-\frac{15a^7b^2(8Ab+3Ba)}{13x^{13}}-$
norman	$\frac{-\frac{a^{10}A}{16}+(-\frac{2}{3}a^9bA-\frac{1}{15}a^{10}B)x+(-\frac{45}{14}a^8b^2A-\frac{5}{7}a^9bB)x^2+(-\frac{120}{13}a^7b^3A-\frac{45}{13}a^8b^2B)x^3+(-\frac{35}{2}a^6b^4A-10a^7b^3B)x^4+(-\frac{252}{11}}{}$
risch	$\frac{-\frac{a^{10}A}{16}+(-\frac{2}{3}a^9bA-\frac{1}{15}a^{10}B)x+(-\frac{45}{14}a^8b^2A-\frac{5}{7}a^9bB)x^2+(-\frac{120}{13}a^7b^3A-\frac{45}{13}a^8b^2B)x^3+(-\frac{35}{2}a^6b^4A-10a^7b^3B)x^4+(-\frac{252}{11}}{}$
gosper	$-\frac{48048B\,b^{10}x^{11}+40040A\,b^{10}x^{10}+400400Ba\,b^9x^{10}+343200aA\,b^9x^9+1544400B\,a^2b^8x^9+1351350a^2A\,b^8x^8+3603600B\,a^3b}{}$
parallelrisch	$-\frac{48048B\,b^{10}x^{11}+40040A\,b^{10}x^{10}+400400Ba\,b^9x^{10}+343200aA\,b^9x^9+1544400B\,a^2b^8x^9+1351350a^2A\,b^8x^8+3603600B\,a^3b}{}$
orering	$-\frac{48048B\,b^{10}x^{11}+40040A\,b^{10}x^{10}+400400Ba\,b^9x^{10}+343200aA\,b^9x^9+1544400B\,a^2b^8x^9+1351350a^2A\,b^8x^8+3603600B\,a^3b}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^10*B/x^5-42/11*a^5*b^4*(6*A*b+5*B*a)/x^11-1/15*a^9*(10*A*b+B*a)/x^1
5-5/7*a*b^8*(2*A*b+9*B*a)/x^7-15/8*a^2*b^7*(3*A*b+8*B*a)/x^8-15/13*a^7*b^2
*(8*A*b+3*B*a)/x^13-21/5*a^4*b^5*(5*A*b+6*B*a)/x^10-5/14*a^8*b*(9*A*b+2*B*
a)/x^14-1/6*b^9*(A*b+10*B*a)/x^6-10/3*a^3*b^6*(4*A*b+7*B*a)/x^9-1/16*a^10*
A/x^16-5/2*a^6*b^3*(7*A*b+4*B*a)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx = -\frac{48048\,Bb^{10}x^{11}+15015\,Aa^{10}+40040\,(10\,Bab^9+Ab^{10})x^{10}+171600\,(9\,Ba^2b^8+2\,Aab^9)x^9+450450\,(8}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^17,x, algorithm="fricas")
```

output
```
-1/240240*(48048*B*b^10*x^11 + 15015*A*a^10 + 40040*(10*B*a*b^9 + A*b^10)*
x^10 + 171600*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 450450*(8*B*a^3*b^7 + 3*A*a^
2*b^8)*x^8 + 800800*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 1009008*(6*B*a^5*b^5
 + 5*A*a^4*b^6)*x^6 + 917280*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 600600*(4*B
*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 277200*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 858
00*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 16016*(B*a^10 + 10*A*a^9*b)*x)/x^16
```

Sympy [A] (verification not implemented)

Time = 43.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx$$
$$= \frac{-15015Aa^{10} - 48048Bb^{10}x^{11} + x^{10}(-40040Ab^{10} - 400400Bab^9) + x^9(-343200Aab^9 - 1544400Ba^2b^8)$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**17,x)
```

output
```
(-15015*A*a**10 - 48048*B*b**10*x**11 + x**10*(-40040*A*b**10 - 400400*B*a
*b**9) + x**9*(-343200*A*a*b**9 - 1544400*B*a**2*b**8) + x**8*(-1351350*A*
a**2*b**8 - 3603600*B*a**3*b**7) + x**7*(-3203200*A*a**3*b**7 - 5605600*B*
a**4*b**6) + x**6*(-5045040*A*a**4*b**6 - 6054048*B*a**5*b**5) + x**5*(-55
03680*A*a**5*b**5 - 4586400*B*a**6*b**4) + x**4*(-4204200*A*a**6*b**4 - 24
02400*B*a**7*b**3) + x**3*(-2217600*A*a**7*b**3 - 831600*B*a**8*b**2) + x*
*2*(-772200*A*a**8*b**2 - 171600*B*a**9*b) + x*(-160160*A*a**9*b - 16016*B
*a**10))/(240240*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx =$$
$$-\frac{48048\,Bb^{10}x^{11} + 15015\,Aa^{10} + 40040\,(10\,Bab^9 + Ab^{10})x^{10} + 171600\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 450450\,(8$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^17,x, algorithm="maxima")
```

output
```
-1/240240*(48048*B*b^10*x^11 + 15015*A*a^10 + 40040*(10*B*a*b^9 + A*b^10)*
x^10 + 171600*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 450450*(8*B*a^3*b^7 + 3*A*a^
2*b^8)*x^8 + 800800*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 1009008*(6*B*a^5*b^5
 + 5*A*a^4*b^6)*x^6 + 917280*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 600600*(4*B
*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 277200*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3 + 858
00*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 16016*(B*a^10 + 10*A*a^9*b)*x)/x^16
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx =$$
$$-\frac{48048\,Bb^{10}x^{11} + 400400\,Bab^{9}x^{10} + 40040\,Ab^{10}x^{10} + 1544400\,Ba^{2}b^{8}x^{9} + 343200\,Aab^{9}x^{9} + 3603600\,B}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^17,x, algorithm="giac")
```

output
```
-1/240240*(48048*B*b^10*x^11 + 400400*B*a*b^9*x^10 + 40040*A*b^10*x^10 + 1
544400*B*a^2*b^8*x^9 + 343200*A*a*b^9*x^9 + 3603600*B*a^3*b^7*x^8 + 135135
0*A*a^2*b^8*x^8 + 5605600*B*a^4*b^6*x^7 + 3203200*A*a^3*b^7*x^7 + 6054048*
B*a^5*b^5*x^6 + 5045040*A*a^4*b^6*x^6 + 4586400*B*a^6*b^4*x^5 + 5503680*A*
a^5*b^5*x^5 + 2402400*B*a^7*b^3*x^4 + 4204200*A*a^6*b^4*x^4 + 831600*B*a^8
*b^2*x^3 + 2217600*A*a^7*b^3*x^3 + 171600*B*a^9*b*x^2 + 772200*A*a^8*b^2*x
^2 + 16016*B*a^10*x + 160160*A*a^9*b*x + 15015*A*a^10)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx =$$
$$-\frac{x\left(\frac{B\,a^{10}}{15}+\frac{2\,A\,b\,a^{9}}{3}\right)+\frac{A\,a^{10}}{16}+x^{2}\left(\frac{5\,B\,a^{9}\,b}{7}+\frac{45\,A\,a^{8}\,b^{2}}{14}\right)+x^{9}\left(\frac{45\,B\,a^{2}\,b^{8}}{7}+\frac{10\,A\,a\,b^{9}}{7}\right)+x^{10}\left(\frac{A\,b^{10}}{6}+\frac{5\,B\,a\,b^{9}}{3}\right)+}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^17,x)
```

output
```
-(x*((B*a^10)/15 + (2*A*a^9*b)/3) + (A*a^10)/16 + x^2*((45*A*a^8*b^2)/14 +
 (5*B*a^9*b)/7) + x^9*((45*B*a^2*b^8)/7 + (10*A*a*b^9)/7) + x^10*((A*b^10)
/6 + (5*B*a*b^9)/3) + x^4*((35*A*a^6*b^4)/2 + 10*B*a^7*b^3) + x^8*((45*A*a
^2*b^8)/8 + 15*B*a^3*b^7) + x^7*((40*A*a^3*b^7)/3 + (70*B*a^4*b^6)/3) + x^
6*(21*A*a^4*b^6 + (126*B*a^5*b^5)/5) + x^3*((120*A*a^7*b^3)/13 + (45*B*a^8
*b^2)/13) + x^5*((252*A*a^5*b^5)/11 + (210*B*a^6*b^4)/11) + (B*b^10*x^11)/
5)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}}\,dx$$
$$= \frac{-4368b^{11}x^{11} - 40040a\,b^{10}x^{10} - 171600a^2b^9x^9 - 450450a^3b^8x^8 - 800800a^4b^7x^7 - 1009008a^5b^6x^6 - 9172}{21840x^{16}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^17,x)
```

output
```
( - 1365*a**11 - 16016*a**10*b*x - 85800*a**9*b**2*x**2 - 277200*a**8*b**3
*x**3 - 600600*a**7*b**4*x**4 - 917280*a**6*b**5*x**5 - 1009008*a**5*b**6*
x**6 - 800800*a**4*b**7*x**7 - 450450*a**3*b**8*x**8 - 171600*a**2*b**9*x*
*9 - 40040*a*b**10*x**10 - 4368*b**11*x**11)/(21840*x**16)
```

3.134 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 188

$$\begin{aligned}\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = &-\frac{A(a+bx)^{11}}{17ax^{17}}+\frac{(6Ab-17aB)(a+bx)^{11}}{272a^2x^{16}}\\&-\frac{b(6Ab-17aB)(a+bx)^{11}}{816a^3x^{15}}+\frac{b^2(6Ab-17aB)(a+bx)^{11}}{2856a^4x^{14}}\\&-\frac{b^3(6Ab-17aB)(a+bx)^{11}}{12376a^5x^{13}}+\frac{b^4(6Ab-17aB)(a+bx)^{11}}{74256a^6x^{12}}\\&-\frac{b^5(6Ab-17aB)(a+bx)^{11}}{816816a^7x^{11}}\end{aligned}$$

output
```
-1/17*A*(b*x+a)^11/a/x^17+1/272*(6*A*b-17*B*a)*(b*x+a)^11/a^2/x^16-1/816*b
*(6*A*b-17*B*a)*(b*x+a)^11/a^3/x^15+1/2856*b^2*(6*A*b-17*B*a)*(b*x+a)^11/a
^4/x^14-1/12376*b^3*(6*A*b-17*B*a)*(b*x+a)^11/a^5/x^13+1/74256*b^4*(6*A*b-
17*B*a)*(b*x+a)^11/a^6/x^12-1/816816*b^5*(6*A*b-17*B*a)*(b*x+a)^11/a^7/x^1
1
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = -\frac{b^{10}(6A+7Bx)}{42x^7}-\frac{5ab^9(7A+8Bx)}{28x^8}-\frac{5a^2b^8(8A+9Bx)}{8x^9}-\frac{4a^3b^7(9A+10Bx)}{3x^{10}}-\frac{21a^4b^6(10A+11Bx)}{11x^{11}}-\frac{21a^5b^5(11A+12Bx)}{11x^{12}}-\frac{35a^6b^4(12A+13Bx)}{26x^{13}}-\frac{60a^7b^3(13A+14Bx)}{91x^{14}}-\frac{3a^8b^2(14A+15Bx)}{14x^{15}}-\frac{a^9b(15A+16Bx)}{24x^{16}}-\frac{a^{10}(16A+17Bx)}{272x^{17}}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^18,x]
```

output
```
-1/42*(b^10*(6*A + 7*B*x))/x^7 - (5*a*b^9*(7*A + 8*B*x))/(28*x^8) - (5*a^2
*b^8*(8*A + 9*B*x))/(8*x^9) - (4*a^3*b^7*(9*A + 10*B*x))/(3*x^10) - (21*a^
4*b^6*(10*A + 11*B*x))/(11*x^11) - (21*a^5*b^5*(11*A + 12*B*x))/(11*x^12)
- (35*a^6*b^4*(12*A + 13*B*x))/(26*x^13) - (60*a^7*b^3*(13*A + 14*B*x))/(9
1*x^14) - (3*a^8*b^2*(14*A + 15*B*x))/(14*x^15) - (a^9*b*(15*A + 16*B*x))/
(24*x^16) - (a^10*(16*A + 17*B*x))/(272*x^17)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {87, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(6Ab-17aB)\int \frac{(a+bx)^{10}}{x^{17}}dx}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

$\downarrow$ 55

$$-\frac{(6Ab-17aB)\left(-\frac{5b\int \frac{(a+bx)^{10}}{x^{16}}dx}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

$\downarrow$ 55

$$-\frac{(6Ab-17aB)\left(-\frac{5b\left(-\frac{4b\int \frac{(a+bx)^{10}}{x^{15}}dx}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

$\downarrow$ 55

$$-\frac{(6Ab-17aB)\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\int \frac{(a+bx)^{10}}{x^{14}}dx}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

$\downarrow$ 55

$$-\frac{(6Ab-17aB)\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\int\frac{(a+bx)^{10}}{x^{13}}dx}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(6Ab-17aB)\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{(a+bx)^{10}}{x^{12}}dx}{12a}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

$$\downarrow 48$$

$$-\frac{\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}\right)}{13a}-\frac{(a+bx)^{11}}{13ax^{13}}\right)}{14a}-\frac{(a+bx)^{11}}{14ax^{14}}\right)}{15a}-\frac{(a+bx)^{11}}{15ax^{15}}\right)}{16a}-\frac{(a+bx)^{11}}{16ax^{16}}\right)(6Ab-17aB)}{17a}-\frac{A(a+bx)^{11}}{17ax^{17}}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^18,x]
```

output
```
-1/17*(A*(a + b*x)^11)/(a*x^17) - ((6*A*b - 17*a*B)*(-1/16*(a + b*x)^11/(a
*x^16) - (5*b*(-1/15*(a + b*x)^11/(a*x^15) - (4*b*(-1/14*(a + b*x)^11/(a*x
^14) - (3*b*(-1/13*(a + b*x)^11/(a*x^13) - (2*b*(-1/12*(a + b*x)^11/(a*x^1
2) + (b*(a + b*x)^11)/(132*a^2*x^11)))/(13*a)))/(14*a)))/(15*a)))/(16*a)))
/(17*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.11

method	result
default	$-\frac{42a^4b^5(5Ab+6Ba)}{11x^{11}}-\frac{a^8b(9Ab+2Ba)}{3x^{15}}-\frac{b^9(Ab+10Ba)}{7x^7}-\frac{5a\,b^8(2Ab+9Ba)}{8x^8}-\frac{30a^6b^3(7Ab+4Ba)}{13x^{13}}-\frac{3a^3b^6(4Ab+7}{x^{10}}$
norman	$-\frac{a^{10}A}{17}+(-\frac{5}{8}a^9bA-\frac{1}{16}a^{10}B)x+(-3a^8b^2A-\frac{2}{3}a^9bB)x^2+(-\frac{60}{7}a^7b^3A-\frac{45}{14}a^8b^2B)x^3+(-\frac{210}{13}a^6b^4A-\frac{120}{13}a^7b^3B)x^4+(-21a^5$
risch	$-\frac{a^{10}A}{17}+(-\frac{5}{8}a^9bA-\frac{1}{16}a^{10}B)x+(-3a^8b^2A-\frac{2}{3}a^9bB)x^2+(-\frac{60}{7}a^7b^3A-\frac{45}{14}a^8b^2B)x^3+(-\frac{210}{13}a^6b^4A-\frac{120}{13}a^7b^3B)x^4+(-21a^5$
gosper	$-\frac{136136B\,b^{10}x^{11}+116688A\,b^{10}x^{10}+1166880Ba\,b^9x^{10}+1021020aA\,b^9x^9+4594590B\,a^2b^8x^9+4084080a^2A\,b^8x^8+10890880}{}$
parallelrisch	$-\frac{136136B\,b^{10}x^{11}+116688A\,b^{10}x^{10}+1166880Ba\,b^9x^{10}+1021020aA\,b^9x^9+4594590B\,a^2b^8x^9+4084080a^2A\,b^8x^8+10890880}{}$
orering	$-\frac{136136B\,b^{10}x^{11}+116688A\,b^{10}x^{10}+1166880Ba\,b^9x^{10}+1021020aA\,b^9x^9+4594590B\,a^2b^8x^9+4084080a^2A\,b^8x^8+10890880}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-42/11*a^4*b^5*(5*A*b+6*B*a)/x^11-1/3*a^8*b*(9*A*b+2*B*a)/x^15-1/7*b^9*(A*
b+10*B*a)/x^7-5/8*a*b^8*(2*A*b+9*B*a)/x^8-30/13*a^6*b^3*(7*A*b+4*B*a)/x^13
-3*a^3*b^6*(4*A*b+7*B*a)/x^10-15/14*a^7*b^2*(8*A*b+3*B*a)/x^14-1/17*a^10*A
/x^17-1/6*b^10*B/x^6-5/3*a^2*b^7*(3*A*b+8*B*a)/x^9-1/16*a^9*(10*A*b+B*a)/x
^16-7/2*a^5*b^4*(6*A*b+5*B*a)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = -\frac{136136\,Bb^{10}x^{11} + 48048\,Aa^{10} + 116688\,(10\,Bab^9 + Ab^{10})x^{10} + 510510\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 136136}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^18,x, algorithm="fricas")
```

output
```
-1/816816*(136136*B*b^10*x^11 + 48048*A*a^10 + 116688*(10*B*a*b^9 + A*b^10
)*x^10 + 510510*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 1361360*(8*B*a^3*b^7 + 3*A
*a^2*b^8)*x^8 + 2450448*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 3118752*(6*B*a^5
*b^5 + 5*A*a^4*b^6)*x^6 + 2858856*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 188496
0*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 875160*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3
 + 272272*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 51051*(B*a^10 + 10*A*a^9*b)*x)/x
^17
```

Sympy [A] (verification not implemented)

Time = 68.87 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = \frac{-48048Aa^{10} - 136136Bb^{10}x^{11} + x^{10}(-116688Ab^{10} - 1166880Bab^9) + x^9(-1021020Aab^9 - 4594590Ba}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**18,x)
```

output
```
(-48048*A*a**10 - 136136*B*b**10*x**11 + x**10*(-116688*A*b**10 - 1166880*
B*a*b**9) + x**9*(-1021020*A*a*b**9 - 4594590*B*a**2*b**8) + x**8*(-408408
0*A*a**2*b**8 - 10890880*B*a**3*b**7) + x**7*(-9801792*A*a**3*b**7 - 17153
136*B*a**4*b**6) + x**6*(-15593760*A*a**4*b**6 - 18712512*B*a**5*b**5) + x
**5*(-17153136*A*a**5*b**5 - 14294280*B*a**6*b**4) + x**4*(-13194720*A*a**
6*b**4 - 7539840*B*a**7*b**3) + x**3*(-7001280*A*a**7*b**3 - 2625480*B*a**
8*b**2) + x**2*(-2450448*A*a**8*b**2 - 544544*B*a**9*b) + x*(-510510*A*a**
9*b - 51051*B*a**10))/(816816*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = -\frac{136136\,Bb^{10}x^{11} + 48048\,Aa^{10} + 116688\,(10\,Bab^9 + Ab^{10})x^{10} + 510510\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 136136}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^18,x, algorithm="maxima")
```

output
```
-1/816816*(136136*B*b^10*x^11 + 48048*A*a^10 + 116688*(10*B*a*b^9 + A*b^10
)*x^10 + 510510*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 1361360*(8*B*a^3*b^7 + 3*A
*a^2*b^8)*x^8 + 2450448*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 3118752*(6*B*a^5
*b^5 + 5*A*a^4*b^6)*x^6 + 2858856*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 188496
0*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 875160*(3*B*a^8*b^2 + 8*A*a^7*b^3)*x^3
 + 272272*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 51051*(B*a^10 + 10*A*a^9*b)*x)/x
^17
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = -\frac{136136\,Bb^{10}x^{11}+1166880\,Bab^9x^{10}+116688\,Ab^{10}x^{10}+4594590\,Ba^2b^8x^9+1021020\,Aab^9x^9+10890\ldots}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^18,x, algorithm="giac")
```

output
```
-1/816816*(136136*B*b^10*x^11 + 1166880*B*a*b^9*x^10 + 116688*A*b^10*x^10
+ 4594590*B*a^2*b^8*x^9 + 1021020*A*a*b^9*x^9 + 10890880*B*a^3*b^7*x^8 + 4
084080*A*a^2*b^8*x^8 + 17153136*B*a^4*b^6*x^7 + 9801792*A*a^3*b^7*x^7 + 18
712512*B*a^5*b^5*x^6 + 15593760*A*a^4*b^6*x^6 + 14294280*B*a^6*b^4*x^5 + 1
7153136*A*a^5*b^5*x^5 + 7539840*B*a^7*b^3*x^4 + 13194720*A*a^6*b^4*x^4 + 2
625480*B*a^8*b^2*x^3 + 7001280*A*a^7*b^3*x^3 + 544544*B*a^9*b*x^2 + 245044
8*A*a^8*b^2*x^2 + 51051*B*a^10*x + 510510*A*a^9*b*x + 48048*A*a^10)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{16}+\frac{5\,A\,b\,a^9}{8}\right)+\frac{A\,a^{10}}{17}+x^2\left(\frac{2\,B\,a^9\,b}{3}+3\,A\,a^8\,b^2\right)+x^9\left(\frac{45\,B\,a^2\,b^8}{8}+\frac{5\,A\,a\,b^9}{4}\right)+x^{10}\left(\frac{A\,b^{10}}{7}+\frac{10\,B\,a\,b^9}{7}\right)+\ldots}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^18,x)
```

output
```
-(x*((B*a^10)/16 + (5*A*a^9*b)/8) + (A*a^10)/17 + x^2*(3*A*a^8*b^2 + (2*B*
a^9*b)/3) + x^9*((45*B*a^2*b^8)/8 + (5*A*a*b^9)/4) + x^10*((A*b^10)/7 + (1
0*B*a*b^9)/7) + x^7*(12*A*a^3*b^7 + 21*B*a^4*b^6) + x^8*(5*A*a^2*b^8 + (40
*B*a^3*b^7)/3) + x^5*(21*A*a^5*b^5 + (35*B*a^6*b^4)/2) + x^3*((60*A*a^7*b^
3)/7 + (45*B*a^8*b^2)/14) + x^4*((210*A*a^6*b^4)/13 + (120*B*a^7*b^3)/13)
+ x^6*((210*A*a^4*b^6)/11 + (252*B*a^5*b^5)/11) + (B*b^10*x^11)/6)/x^17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}}\,dx = \frac{-12376b^{11}x^{11} - 116688a\,b^{10}x^{10} - 510510a^2b^9x^9 - 1361360a^3b^8x^8 - 2450448a^4b^7x^7 - 3118752a^5b^6x^6 -}{74256x^{17}}$$

input
```
int((b*x+a)^10*(B*x+A)/x^18,x)
```

output
```
( - 4368*a**11 - 51051*a**10*b*x - 272272*a**9*b**2*x**2 - 875160*a**8*b**
3*x**3 - 1884960*a**7*b**4*x**4 - 2858856*a**6*b**5*x**5 - 3118752*a**5*b*
*6*x**6 - 2450448*a**4*b**7*x**7 - 1361360*a**3*b**8*x**8 - 510510*a**2*b*
*9*x**9 - 116688*a*b**10*x**10 - 12376*b**11*x**11)/(74256*x**17)
```

3.135 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}} dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}} dx = -\frac{a^{10}A}{18x^{18}} - \frac{a^9(10Ab+aB)}{17x^{17}} - \frac{5a^8b(9Ab+2aB)}{16x^{16}} - \frac{a^7b^2(8Ab+3aB)}{x^{15}} - \frac{15a^6b^3(7Ab+4aB)}{7x^{14}} - \frac{42a^5b^4(6Ab+5aB)}{13x^{13}} - \frac{7a^4b^5(5Ab+6aB)}{2x^{12}} - \frac{30a^3b^6(4Ab+7aB)}{11x^{11}} - \frac{3a^2b^7(3Ab+8aB)}{2x^{10}} - \frac{5ab^8(2Ab+9aB)}{9x^9} - \frac{b^9(Ab+10aB)}{8x^8} - \frac{b^{10}B}{7x^7}$$

output
```
-1/18*a^10*A/x^18-1/17*a^9*(10*A*b+B*a)/x^17-5/16*a^8*b*(9*A*b+2*B*a)/x^16
-a^7*b^2*(8*A*b+3*B*a)/x^15-15/7*a^6*b^3*(7*A*b+4*B*a)/x^14-42/13*a^5*b^4*
(6*A*b+5*B*a)/x^13-7/2*a^4*b^5*(5*A*b+6*B*a)/x^12-30/11*a^3*b^6*(4*A*b+7*B
*a)/x^11-3/2*a^2*b^7*(3*A*b+8*B*a)/x^10-5/9*a*b^8*(2*A*b+9*B*a)/x^9-1/8*b^
9*(A*b+10*B*a)/x^8-1/7*b^10*B/x^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx = -\frac{b^{10}(7A+8Bx)}{56x^8} - \frac{5ab^9(8A+9Bx)}{36x^9} - \frac{a^2b^8(9A+10Bx)}{2x^{10}} - \frac{12a^3b^7(10A+11Bx)}{11x^{11}} - \frac{35a^4b^6(11A+12Bx)}{22x^{12}} - \frac{21a^5b^5(12A+13Bx)}{13x^{13}} - \frac{15a^6b^4(13A+14Bx)}{13x^{14}} - \frac{4a^7b^3(14A+15Bx)}{7x^{15}} - \frac{3a^8b^2(15A+16Bx)}{16x^{16}} - \frac{5a^9b(16A+17Bx)}{136x^{17}} - \frac{a^{10}(17A+18Bx)}{306x^{18}}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^19,x]
```

output
```
-1/56*(b^10*(7*A + 8*B*x))/x^8 - (5*a*b^9*(8*A + 9*B*x))/(36*x^9) - (a^2*b
^8*(9*A + 10*B*x))/(2*x^10) - (12*a^3*b^7*(10*A + 11*B*x))/(11*x^11) - (35
*a^4*b^6*(11*A + 12*B*x))/(22*x^12) - (21*a^5*b^5*(12*A + 13*B*x))/(13*x^1
3) - (15*a^6*b^4*(13*A + 14*B*x))/(13*x^14) - (4*a^7*b^3*(14*A + 15*B*x))/
(7*x^15) - (3*a^8*b^2*(15*A + 16*B*x))/(16*x^16) - (5*a^9*b*(16*A + 17*B*x
))/(136*x^17) - (a^10*(17*A + 18*B*x))/(306*x^18)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^{19}}+\frac{a^9(aB+10Ab)}{x^{18}}+\frac{5a^8b(2aB+9Ab)}{x^{17}}+\frac{15a^7b^2(3aB+8Ab)}{x^{16}}+\frac{30a^6b^3(4aB+7Ab)}{x^{15}}+\frac{42a^5b^4(5aB+$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{10}A}{18x^{18}}-\frac{a^9(aB+10Ab)}{17x^{17}}-\frac{5a^8b(2aB+9Ab)}{16x^{16}}-\frac{a^7b^2(3aB+8Ab)}{x^{15}}-\frac{15a^6b^3(4aB+7Ab)}{7x^{14}}-\frac{42a^5b^4(5aB+6Ab)}{13x^{13}}-\frac{7a^4b^5(6aB+5Ab)}{2x^{12}}-\frac{30a^3b^6(7aB+4Ab)}{11x^{11}}-\frac{3a^2b^7(8aB+3Ab)}{2x^{10}}-\frac{b^9(10aB+Ab)}{8x^8}-\frac{5ab^8(9aB+2Ab)}{9x^9}-\frac{b^{10}B}{7x^7}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^19,x]
```

output
```
-1/18*(a^10*A)/x^18 - (a^9*(10*A*b + a*B))/(17*x^17) - (5*a^8*b*(9*A*b + 2
*a*B))/(16*x^16) - (a^7*b^2*(8*A*b + 3*a*B))/x^15 - (15*a^6*b^3*(7*A*b + 4
*a*B))/(7*x^14) - (42*a^5*b^4*(6*A*b + 5*a*B))/(13*x^13) - (7*a^4*b^5*(5*A
*b + 6*a*B))/(2*x^12) - (30*a^3*b^6*(4*A*b + 7*a*B))/(11*x^11) - (3*a^2*b^
7*(3*A*b + 8*a*B))/(2*x^10) - (5*a*b^8*(2*A*b + 9*a*B))/(9*x^9) - (b^9*(A*
b + 10*a*B))/(8*x^8) - (b^10*B)/(7*x^7)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^{10}A}{18x^{18}}-\frac{a^{9}(10Ab+Ba)}{17x^{17}}-\frac{5a^{8}b(9Ab+2Ba)}{16x^{16}}-\frac{a^{7}b^{2}(8Ab+3Ba)}{x^{15}}-\frac{15a^{6}b^{3}(7Ab+4Ba)}{7x^{14}}-\frac{42a^{5}b^{4}(6Ab+5Ba)}{13x^{13}}-7a$
norman	$-\frac{a^{10}A}{18}+\left(-\frac{10}{17}a^{9}bA-\frac{1}{17}a^{10}B\right)x+\left(-\frac{45}{16}a^{8}b^{2}A-\frac{5}{8}a^{9}bB\right)x^{2}+\left(-8a^{7}b^{3}A-3a^{8}b^{2}B\right)x^{3}+\left(-15a^{6}b^{4}A-\frac{60}{7}a^{7}b^{3}B\right)x^{4}+\left(-\frac{252}{13}a^{5}b\right.$
risch	$-\frac{a^{10}A}{18}+\left(-\frac{10}{17}a^{9}bA-\frac{1}{17}a^{10}B\right)x+\left(-\frac{45}{16}a^{8}b^{2}A-\frac{5}{8}a^{9}bB\right)x^{2}+\left(-8a^{7}b^{3}A-3a^{8}b^{2}B\right)x^{3}+\left(-15a^{6}b^{4}A-\frac{60}{7}a^{7}b^{3}B\right)x^{4}+\left(-\frac{252}{13}a^{5}b\right.$
gosper	$-350064B\,b^{10}x^{11}+306306A\,b^{10}x^{10}+3063060Ba\,b^{9}x^{10}+2722720aA\,b^{9}x^{9}+12252240B\,a^{2}b^{8}x^{9}+11027016a^{2}A\,b^{8}x^{8}+294053$
parallelrisch	$-350064B\,b^{10}x^{11}+306306A\,b^{10}x^{10}+3063060Ba\,b^{9}x^{10}+2722720aA\,b^{9}x^{9}+12252240B\,a^{2}b^{8}x^{9}+11027016a^{2}A\,b^{8}x^{8}+294053$
orering	$-350064B\,b^{10}x^{11}+306306A\,b^{10}x^{10}+3063060Ba\,b^{9}x^{10}+2722720aA\,b^{9}x^{9}+12252240B\,a^{2}b^{8}x^{9}+11027016a^{2}A\,b^{8}x^{8}+294053$

input
```
int((b*x+a)^10*(B*x+A)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*a^10*A/x^18-1/17*a^9*(10*A*b+B*a)/x^17-5/16*a^8*b*(9*A*b+2*B*a)/x^16
-a^7*b^2*(8*A*b+3*B*a)/x^15-15/7*a^6*b^3*(7*A*b+4*B*a)/x^14-42/13*a^5*b^4*
(6*A*b+5*B*a)/x^13-7/2*a^4*b^5*(5*A*b+6*B*a)/x^12-30/11*a^3*b^6*(4*A*b+7*B
*a)/x^11-3/2*a^2*b^7*(3*A*b+8*B*a)/x^10-5/9*a*b^8*(2*A*b+9*B*a)/x^9-1/8*b^
9*(A*b+10*B*a)/x^8-1/7*b^10*B/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx =$$
$$-\,350064\,Bb^{10}x^{11}+136136\,Aa^{10}+306306\,(10\,Bab^{9}+Ab^{10})x^{10}+1361360\,(9\,Ba^{2}b^{8}+2\,Aab^{9})x^{9}+3675$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^19,x, algorithm="fricas")
```

output
```
-1/2450448*(350064*B*b^10*x^11 + 136136*A*a^10 + 306306*(10*B*a*b^9 + A*b^
10)*x^10 + 1361360*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 3675672*(8*B*a^3*b^7 +
3*A*a^2*b^8)*x^8 + 6683040*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 8576568*(6*B*
a^5*b^5 + 5*A*a^4*b^6)*x^6 + 7916832*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 525
0960*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 2450448*(3*B*a^8*b^2 + 8*A*a^7*b^3)
*x^3 + 765765*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 144144*(B*a^10 + 10*A*a^9*b)
*x)/x^18
```

Sympy [A] (verification not implemented)

Time = 98.84 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx = \frac{-136136Aa^{10} - 350064Bb^{10}x^{11} + x^{10}(-306306Ab^{10} - 3063060Bab^9) + x^9(-2722720Aab^9 - 12252240\ldots}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**19,x)
```

output
```
(-136136*A*a**10 - 350064*B*b**10*x**11 + x**10*(-306306*A*b**10 - 3063060
*B*a*b**9) + x**9*(-2722720*A*a*b**9 - 12252240*B*a**2*b**8) + x**8*(-1102
7016*A*a**2*b**8 - 29405376*B*a**3*b**7) + x**7*(-26732160*A*a**3*b**7 - 4
6781280*B*a**4*b**6) + x**6*(-42882840*A*a**4*b**6 - 51459408*B*a**5*b**5)
 + x**5*(-47500992*A*a**5*b**5 - 39584160*B*a**6*b**4) + x**4*(-36756720*A
*a**6*b**4 - 21003840*B*a**7*b**3) + x**3*(-19603584*A*a**7*b**3 - 7351344
*B*a**8*b**2) + x**2*(-6891885*A*a**8*b**2 - 1531530*B*a**9*b) + x*(-14414
40*A*a**9*b - 144144*B*a**10))/(2450448*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx = -\frac{350064\,Bb^{10}x^{11} + 136136\,Aa^{10} + 306306\,(10\,Bab^9 + Ab^{10})x^{10} + 1361360\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 3675\ldots}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^19,x, algorithm="maxima")
```

output
```
-1/2450448*(350064*B*b^10*x^11 + 136136*A*a^10 + 306306*(10*B*a*b^9 + A*b^
10)*x^10 + 1361360*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 3675672*(8*B*a^3*b^7 +
3*A*a^2*b^8)*x^8 + 6683040*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 8576568*(6*B*
a^5*b^5 + 5*A*a^4*b^6)*x^6 + 7916832*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 + 525
0960*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 2450448*(3*B*a^8*b^2 + 8*A*a^7*b^3)
*x^3 + 765765*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 144144*(B*a^10 + 10*A*a^9*b)
*x)/x^18
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx =$$
$$-\frac{350064\,Bb^{10}x^{11}+3063060\,Bab^{9}x^{10}+306306\,Ab^{10}x^{10}+12252240\,Ba^{2}b^{8}x^{9}+2722720\,Aab^{9}x^{9}+2940\ldots}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^19,x, algorithm="giac")
```

output
```
-1/2450448*(350064*B*b^10*x^11 + 3063060*B*a*b^9*x^10 + 306306*A*b^10*x^10
 + 12252240*B*a^2*b^8*x^9 + 2722720*A*a*b^9*x^9 + 29405376*B*a^3*b^7*x^8 +
 11027016*A*a^2*b^8*x^8 + 46781280*B*a^4*b^6*x^7 + 26732160*A*a^3*b^7*x^7
+ 51459408*B*a^5*b^5*x^6 + 42882840*A*a^4*b^6*x^6 + 39584160*B*a^6*b^4*x^5
 + 47500992*A*a^5*b^5*x^5 + 21003840*B*a^7*b^3*x^4 + 36756720*A*a^6*b^4*x^
4 + 7351344*B*a^8*b^2*x^3 + 19603584*A*a^7*b^3*x^3 + 1531530*B*a^9*b*x^2 +
 6891885*A*a^8*b^2*x^2 + 144144*B*a^10*x + 1441440*A*a^9*b*x + 136136*A*a^
10)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{17}+\frac{10\,A\,b\,a^9}{17}\right)+\frac{A\,a^{10}}{18}+x^9\left(5\,B\,a^2\,b^8+\frac{10\,A\,a\,b^9}{9}\right)+x^2\left(\frac{5\,B\,a^9\,b}{8}+\frac{45\,A\,a^8\,b^2}{16}\right)+x^{10}\left(\frac{A\,b^{10}}{8}+\frac{5\,B\,a\,b^9}{4}\right)$$

input

```
int(((A + B*x)*(a + b*x)^10)/x^19,x)
```

output

```
-(x*((B*a^10)/17 + (10*A*a^9*b)/17) + (A*a^10)/18 + x^9*(5*B*a^2*b^8 + (10
*A*a*b^9)/9) + x^2*((45*A*a^8*b^2)/16 + (5*B*a^9*b)/8) + x^10*((A*b^10)/8
+ (5*B*a*b^9)/4) + x^3*(8*A*a^7*b^3 + 3*B*a^8*b^2) + x^8*((9*A*a^2*b^8)/2
+ 12*B*a^3*b^7) + x^6*((35*A*a^4*b^6)/2 + 21*B*a^5*b^5) + x^4*(15*A*a^6*b^
4 + (60*B*a^7*b^3)/7) + x^7*((120*A*a^3*b^7)/11 + (210*B*a^4*b^6)/11) + x^
5*((252*A*a^5*b^5)/13 + (210*B*a^6*b^4)/13) + (B*b^10*x^11)/7)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{19}}\,dx = \frac{-31824b^{11}x^{11}-306306a\,b^{10}x^{10}-1361360a^2b^9x^9-3675672a^3b^8x^8-6683040a^4b^7x^7-8576568a^5b^6x^6-}{222768x^1}$$

input

```
int((b*x+a)^10*(B*x+A)/x^19,x)
```

output

```
( - 12376*a**11 - 144144*a**10*b*x - 765765*a**9*b**2*x**2 - 2450448*a**8*
b**3*x**3 - 5250960*a**7*b**4*x**4 - 7916832*a**6*b**5*x**5 - 8576568*a**5
*b**6*x**6 - 6683040*a**4*b**7*x**7 - 3675672*a**3*b**8*x**8 - 1361360*a**
2*b**9*x**9 - 306306*a*b**10*x**10 - 31824*b**11*x**11)/(222768*x**18)
```

3.136 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx = -\frac{a^{10}A}{19x^{19}} - \frac{a^9(10Ab+aB)}{18x^{18}} - \frac{5a^8b(9Ab+2aB)}{17x^{17}} - \frac{15a^7b^2(8Ab+3aB)}{16x^{16}} - \frac{2a^6b^3(7Ab+4aB)}{x^{15}} - \frac{3a^5b^4(6Ab+5aB)}{x^{14}} - \frac{42a^4b^5(5Ab+6aB)}{13x^{13}} - \frac{5a^3b^6(4Ab+7aB)}{2x^{12}} - \frac{15a^2b^7(3Ab+8aB)}{11x^{11}} - \frac{ab^8(2Ab+9aB)}{2x^{10}} - \frac{b^9(Ab+10aB)}{9x^9} - \frac{b^{10}B}{8x^8}$$

output
```
-1/19*a^10*A/x^19-1/18*a^9*(10*A*b+B*a)/x^18-5/17*a^8*b*(9*A*b+2*B*a)/x^17
-15/16*a^7*b^2*(8*A*b+3*B*a)/x^16-2*a^6*b^3*(7*A*b+4*B*a)/x^15-3*a^5*b^4*(
6*A*b+5*B*a)/x^14-42/13*a^4*b^5*(5*A*b+6*B*a)/x^13-5/2*a^3*b^6*(4*A*b+7*B*
a)/x^12-15/11*a^2*b^7*(3*A*b+8*B*a)/x^11-1/2*a*b^8*(2*A*b+9*B*a)/x^10-1/9*
b^9*(A*b+10*B*a)/x^9-1/8*b^10*B/x^8
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx = -\frac{b^{10}(8A+9Bx)}{72x^9}-\frac{ab^9(9A+10Bx)}{9x^{10}}-\frac{9a^2b^8(10A+11Bx)}{22x^{11}} -\frac{10a^3b^7(11A+12Bx)}{11x^{12}}-\frac{35a^4b^6(12A+13Bx)}{26x^{13}} -\frac{18a^5b^5(13A+14Bx)}{13x^{14}}-\frac{a^6b^4(14A+15Bx)}{x^{15}} -\frac{a^7b^3(15A+16Bx)}{2x^{16}}-\frac{45a^8b^2(16A+17Bx)}{272x^{17}} -\frac{5a^9b(17A+18Bx)}{153x^{18}}-\frac{a^{10}(18A+19Bx)}{342x^{19}}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^20,x]
```

output
```
-1/72*(b^10*(8*A + 9*B*x))/x^9 - (a*b^9*(9*A + 10*B*x))/(9*x^10) - (9*a^2*
b^8*(10*A + 11*B*x))/(22*x^11) - (10*a^3*b^7*(11*A + 12*B*x))/(11*x^12) -
(35*a^4*b^6*(12*A + 13*B*x))/(26*x^13) - (18*a^5*b^5*(13*A + 14*B*x))/(13*
x^14) - (a^6*b^4*(14*A + 15*B*x))/x^15 - (a^7*b^3*(15*A + 16*B*x))/(2*x^16
) - (45*a^8*b^2*(16*A + 17*B*x))/(272*x^17) - (5*a^9*b*(17*A + 18*B*x))/(1
53*x^18) - (a^10*(18*A + 19*B*x))/(342*x^19)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^{10}A}{x^{20}} + \frac{a^9(aB+10Ab)}{x^{19}} + \frac{5a^8b(2aB+9Ab)}{x^{18}} + \frac{15a^7b^2(3aB+8Ab)}{x^{17}} + \frac{30a^6b^3(4aB+7Ab)}{x^{16}} + \frac{42a^5b^4(5aB+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{19x^{19}} - \frac{a^9(aB+10Ab)}{18x^{18}} - \frac{5a^8b(2aB+9Ab)}{17x^{17}} - \frac{15a^7b^2(3aB+8Ab)}{16x^{16}} - \frac{2a^6b^3(4aB+7Ab)}{x^{15}} - \frac{3a^5b^4(5aB+6Ab)}{x^{14}} - \frac{42a^4b^5(6aB+5Ab)}{13x^{13}} - \frac{5a^3b^6(7aB+4Ab)}{2x^{12}} - \frac{15a^2b^7(8aB+3Ab)}{11x^{11}} - \frac{b^9(10aB+Ab)}{9x^9} - \frac{ab^8(9aB+2Ab)}{2x^{10}} - \frac{b^{10}B}{8x^8}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^20,x]
```

output
```
-1/19*(a^10*A)/x^19 - (a^9*(10*A*b + a*B))/(18*x^18) - (5*a^8*b*(9*A*b + 2
*a*B))/(17*x^17) - (15*a^7*b^2*(8*A*b + 3*a*B))/(16*x^16) - (2*a^6*b^3*(7*
A*b + 4*a*B))/x^15 - (3*a^5*b^4*(6*A*b + 5*a*B))/x^14 - (42*a^4*b^5*(5*A*b
 + 6*a*B))/(13*x^13) - (5*a^3*b^6*(4*A*b + 7*a*B))/(2*x^12) - (15*a^2*b^7*
(3*A*b + 8*a*B))/(11*x^11) - (a*b^8*(2*A*b + 9*a*B))/(2*x^10) - (b^9*(A*b
+ 10*a*B))/(9*x^9) - (b^10*B)/(8*x^8)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{10}A}{19x^{19}}-\frac{a^{9}(10Ab+Ba)}{18x^{18}}-\frac{5a^{8}b(9Ab+2Ba)}{17x^{17}}-\frac{15a^{7}b^{2}(8Ab+3Ba)}{16x^{16}}-\frac{2a^{6}b^{3}(7Ab+4Ba)}{x^{15}}-\frac{3a^{5}b^{4}(6Ab+5Ba)}{x^{14}}-42$
norman	$\frac{-\frac{a^{10}A}{19}+\left(-\frac{5}{9}a^{9}bA-\frac{1}{18}a^{10}B\right)x+\left(-\frac{45}{17}a^{8}b^{2}A-\frac{10}{17}a^{9}bB\right)x^{2}+\left(-\frac{15}{2}a^{7}b^{3}A-\frac{45}{16}a^{8}b^{2}B\right)x^{3}+\left(-14a^{6}b^{4}A-8a^{7}b^{3}B\right)x^{4}+(-18a^{5}b}{}$
risch	$\frac{-\frac{a^{10}A}{19}+\left(-\frac{5}{9}a^{9}bA-\frac{1}{18}a^{10}B\right)x+\left(-\frac{45}{17}a^{8}b^{2}A-\frac{10}{17}a^{9}bB\right)x^{2}+\left(-\frac{15}{2}a^{7}b^{3}A-\frac{45}{16}a^{8}b^{2}B\right)x^{3}+\left(-14a^{6}b^{4}A-8a^{7}b^{3}B\right)x^{4}+(-18a^{5}b}{}$
gosper	$-\frac{831402Bb^{10}x^{11}+739024Ab^{10}x^{10}+7390240Bab^{9}x^{10}+6651216aAb^{9}x^{9}+29930472Ba^{2}b^{8}x^{9}+27209520a^{2}Ab^{8}x^{8}+725587}{}$
parallelrisch	$-\frac{831402Bb^{10}x^{11}+739024Ab^{10}x^{10}+7390240Bab^{9}x^{10}+6651216aAb^{9}x^{9}+29930472Ba^{2}b^{8}x^{9}+27209520a^{2}Ab^{8}x^{8}+725587}{}$
orering	$-\frac{831402Bb^{10}x^{11}+739024Ab^{10}x^{10}+7390240Bab^{9}x^{10}+6651216aAb^{9}x^{9}+29930472Ba^{2}b^{8}x^{9}+27209520a^{2}Ab^{8}x^{8}+725587}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/19*a^10*A/x^19-1/18*a^9*(10*A*b+B*a)/x^18-5/17*a^8*b*(9*A*b+2*B*a)/x^17
-15/16*a^7*b^2*(8*A*b+3*B*a)/x^16-2*a^6*b^3*(7*A*b+4*B*a)/x^15-3*a^5*b^4*(
6*A*b+5*B*a)/x^14-42/13*a^4*b^5*(5*A*b+6*B*a)/x^13-5/2*a^3*b^6*(4*A*b+7*B*
a)/x^12-15/11*a^2*b^7*(3*A*b+8*B*a)/x^11-1/2*a*b^8*(2*A*b+9*B*a)/x^10-1/9*
b^9*(A*b+10*B*a)/x^9-1/8*b^10*B/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx = -\frac{831402\,Bb^{10}x^{11}+350064\,Aa^{10}+739024\,(10\,Bab^{9}+Ab^{10})x^{10}+3325608\,(9\,Ba^{2}b^{8}+2\,Aab^{9})x^{9}+9069}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^20,x, algorithm="fricas")
```

output
```
-1/6651216*(831402*B*b^10*x^11 + 350064*A*a^10 + 739024*(10*B*a*b^9 + A*b^
10)*x^10 + 3325608*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 9069840*(8*B*a^3*b^7 +
3*A*a^2*b^8)*x^8 + 16628040*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 21488544*(6*
B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 19953648*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 +
13302432*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 6235515*(3*B*a^8*b^2 + 8*A*a^7*
b^3)*x^3 + 1956240*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 369512*(B*a^10 + 10*A*a
^9*b)*x)/x^19
```

Sympy [A] (verification not implemented)

Time = 137.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx$$
$$= \frac{-350064Aa^{10} - 831402Bb^{10}x^{11} + x^{10}(-739024Ab^{10} - 7390240Bab^9) + x^9(-6651216Aab^9 - 29930472B}{}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**20,x)
```

output
```
(-350064*A*a**10 - 831402*B*b**10*x**11 + x**10*(-739024*A*b**10 - 7390240
*B*a*b**9) + x**9*(-6651216*A*a*b**9 - 29930472*B*a**2*b**8) + x**8*(-2720
9520*A*a**2*b**8 - 72558720*B*a**3*b**7) + x**7*(-66512160*A*a**3*b**7 - 1
16396280*B*a**4*b**6) + x**6*(-107442720*A*a**4*b**6 - 128931264*B*a**5*b*
*5) + x**5*(-119721888*A*a**5*b**5 - 99768240*B*a**6*b**4) + x**4*(-931170
24*A*a**6*b**4 - 53209728*B*a**7*b**3) + x**3*(-49884120*A*a**7*b**3 - 187
06545*B*a**8*b**2) + x**2*(-17606160*A*a**8*b**2 - 3912480*B*a**9*b) + x*(
-3695120*A*a**9*b - 369512*B*a**10))/(6651216*x**19)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx =$$
$$-\frac{831402\,Bb^{10}x^{11} + 350064\,Aa^{10} + 739024\,(10\,Bab^9 + Ab^{10})x^{10} + 3325608\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 906}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^20,x, algorithm="maxima")
```

output
```
-1/6651216*(831402*B*b^10*x^11 + 350064*A*a^10 + 739024*(10*B*a*b^9 + A*b^
10)*x^10 + 3325608*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 9069840*(8*B*a^3*b^7 +
3*A*a^2*b^8)*x^8 + 16628040*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 21488544*(6*
B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 19953648*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^5 +
13302432*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 6235515*(3*B*a^8*b^2 + 8*A*a^7*
b^3)*x^3 + 1956240*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 369512*(B*a^10 + 10*A*a
^9*b)*x)/x^19
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx = -\frac{831402\,Bb^{10}x^{11} + 7390240\,Bab^9x^{10} + 739024\,Ab^{10}x^{10} + 29930472\,Ba^2b^8x^9 + 6651216\,Aab^9x^9 + 7255\cdots}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^20,x, algorithm="giac")
```

output
```
-1/6651216*(831402*B*b^10*x^11 + 7390240*B*a*b^9*x^10 + 739024*A*b^10*x^10
 + 29930472*B*a^2*b^8*x^9 + 6651216*A*a*b^9*x^9 + 72558720*B*a^3*b^7*x^8 +
 27209520*A*a^2*b^8*x^8 + 116396280*B*a^4*b^6*x^7 + 66512160*A*a^3*b^7*x^7
 + 128931264*B*a^5*b^5*x^6 + 107442720*A*a^4*b^6*x^6 + 99768240*B*a^6*b^4*
x^5 + 119721888*A*a^5*b^5*x^5 + 53209728*B*a^7*b^3*x^4 + 93117024*A*a^6*b^
4*x^4 + 18706545*B*a^8*b^2*x^3 + 49884120*A*a^7*b^3*x^3 + 3912480*B*a^9*b*
x^2 + 17606160*A*a^8*b^2*x^2 + 369512*B*a^10*x + 3695120*A*a^9*b*x + 35006
4*A*a^10)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{18}+\frac{5\,A\,b\,a^9}{9}\right)+\frac{A\,a^{10}}{19}+x^9\left(\frac{9\,B\,a^2\,b^8}{2}+A\,a\,b^9\right)+x^2\left(\frac{10\,B\,a^9\,b}{17}+\frac{45\,A\,a^8\,b^2}{17}\right)+x^{10}\left(\frac{A\,b^{10}}{9}+\frac{10\,B\,a\,b^9}{9}\right)+}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^20,x)
```

output
```
-(x*((B*a^10)/18 + (5*A*a^9*b)/9) + (A*a^10)/19 + x^9*((9*B*a^2*b^8)/2 + A
*a*b^9) + x^2*((45*A*a^8*b^2)/17 + (10*B*a^9*b)/17) + x^10*((A*b^10)/9 + (
10*B*a*b^9)/9) + x^4*(14*A*a^6*b^4 + 8*B*a^7*b^3) + x^5*(18*A*a^5*b^5 + 15
*B*a^6*b^4) + x^7*(10*A*a^3*b^7 + (35*B*a^4*b^6)/2) + x^3*((15*A*a^7*b^3)/
2 + (45*B*a^8*b^2)/16) + x^8*((45*A*a^2*b^8)/11 + (120*B*a^3*b^7)/11) + x^
6*((210*A*a^4*b^6)/13 + (252*B*a^5*b^5)/13) + (B*b^10*x^11)/8)/x^19
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{20}}\,dx = \frac{-75582b^{11}x^{11}-739024a\,b^{10}x^{10}-3325608a^2b^9x^9-9069840a^3b^8x^8-16628040a^4b^7x^7-21488544a^5b^6x^6}{604656}$$

input
```
int((b*x+a)^10*(B*x+A)/x^20,x)
```

output
```
( - 31824*a**11 - 369512*a**10*b*x - 1956240*a**9*b**2*x**2 - 6235515*a**8
*b**3*x**3 - 13302432*a**7*b**4*x**4 - 19953648*a**6*b**5*x**5 - 21488544*
a**5*b**6*x**6 - 16628040*a**4*b**7*x**7 - 9069840*a**3*b**8*x**8 - 332560
8*a**2*b**9*x**9 - 739024*a*b**10*x**10 - 75582*b**11*x**11)/(604656*x**19
)
```

3.137 $\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = -\frac{a^{10}A}{20x^{20}}-\frac{a^9(10Ab+aB)}{19x^{19}}-\frac{5a^8b(9Ab+2aB)}{18x^{18}}-\frac{15a^7b^2(8Ab+3aB)}{17x^{17}}-\frac{15a^6b^3(7Ab+4aB)}{8x^{16}}-\frac{14a^5b^4(6Ab+5aB)}{5x^{15}}-\frac{3a^4b^5(5Ab+6aB)}{x^{14}}-\frac{30a^3b^6(4Ab+7aB)}{13x^{13}}-\frac{5a^2b^7(3Ab+8aB)}{4x^{12}}-\frac{5ab^8(2Ab+9aB)}{11x^{11}}-\frac{b^9(Ab+10aB)}{10x^{10}}-\frac{b^{10}B}{9x^9}$$

output
```
-1/20*a^10*A/x^20-1/19*a^9*(10*A*b+B*a)/x^19-5/18*a^8*b*(9*A*b+2*B*a)/x^18
-15/17*a^7*b^2*(8*A*b+3*B*a)/x^17-15/8*a^6*b^3*(7*A*b+4*B*a)/x^16-14/5*a^5
*b^4*(6*A*b+5*B*a)/x^15-3*a^4*b^5*(5*A*b+6*B*a)/x^14-30/13*a^3*b^6*(4*A*b+
7*B*a)/x^13-5/4*a^2*b^7*(3*A*b+8*B*a)/x^12-5/11*a*b^8*(2*A*b+9*B*a)/x^11-1
/10*b^9*(A*b+10*B*a)/x^10-1/9*b^10*B/x^9
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = -\frac{b^{10}(9A+10Bx)}{90x^{10}} - \frac{ab^9(10A+11Bx)}{11x^{11}} - \frac{15a^2b^8(11A+12Bx)}{44x^{12}} - \frac{10a^3b^7(12A+13Bx)}{13x^{13}} - \frac{15a^4b^6(13A+14Bx)}{13x^{14}} - \frac{6a^5b^5(14A+15Bx)}{5x^{15}} - \frac{7a^6b^4(15A+16Bx)}{8x^{16}} - \frac{15a^7b^3(16A+17Bx)}{34x^{17}} - \frac{5a^8b^2(17A+18Bx)}{34x^{18}} - \frac{5a^9b(18A+19Bx)}{171x^{19}} - \frac{a^{10}(19A+20Bx)}{380x^{20}}$$

input
```
Integrate[((a + b*x)^10*(A + B*x))/x^21,x]
```

output
```
-1/90*(b^10*(9*A + 10*B*x))/x^10 - (a*b^9*(10*A + 11*B*x))/(11*x^11) - (15
*a^2*b^8*(11*A + 12*B*x))/(44*x^12) - (10*a^3*b^7*(12*A + 13*B*x))/(13*x^1
3) - (15*a^4*b^6*(13*A + 14*B*x))/(13*x^14) - (6*a^5*b^5*(14*A + 15*B*x))/
(5*x^15) - (7*a^6*b^4*(15*A + 16*B*x))/(8*x^16) - (15*a^7*b^3*(16*A + 17*B
*x))/(34*x^17) - (5*a^8*b^2*(17*A + 18*B*x))/(34*x^18) - (5*a^9*b*(18*A +
19*B*x))/(171*x^19) - (a^10*(19*A + 20*B*x))/(380*x^20)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^{10}A}{x^{21}} + \frac{a^9(aB+10Ab)}{x^{20}} + \frac{5a^8b(2aB+9Ab)}{x^{19}} + \frac{15a^7b^2(3aB+8Ab)}{x^{18}} + \frac{30a^6b^3(4aB+7Ab)}{x^{17}} + \frac{42a^5b^4(5aB+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{10}A}{20x^{20}} - \frac{a^9(aB+10Ab)}{19x^{19}} - \frac{5a^8b(2aB+9Ab)}{18x^{18}} - \frac{15a^7b^2(3aB+8Ab)}{17x^{17}} - \frac{15a^6b^3(4aB+7Ab)}{8x^{16}} - \frac{14a^5b^4(5aB+6Ab)}{5x^{15}} - \frac{3a^4b^5(6aB+5Ab)}{x^{14}} - \frac{30a^3b^6(7aB+4Ab)}{13x^{13}} - \frac{5a^2b^7(8aB+3Ab)}{4x^{12}} - \frac{b^9(10aB+Ab)}{10x^{10}} - \frac{5ab^8(9aB+2Ab)}{11x^{11}} - \frac{b^{10}B}{9x^9}$$

input
```
Int[((a + b*x)^10*(A + B*x))/x^21,x]
```

output
```
-1/20*(a^10*A)/x^20 - (a^9*(10*A*b + a*B))/(19*x^19) - (5*a^8*b*(9*A*b + 2
*a*B))/(18*x^18) - (15*a^7*b^2*(8*A*b + 3*a*B))/(17*x^17) - (15*a^6*b^3*(7
*A*b + 4*a*B))/(8*x^16) - (14*a^5*b^4*(6*A*b + 5*a*B))/(5*x^15) - (3*a^4*b
^5*(5*A*b + 6*a*B))/x^14 - (30*a^3*b^6*(4*A*b + 7*a*B))/(13*x^13) - (5*a^2
*b^7*(3*A*b + 8*a*B))/(4*x^12) - (5*a*b^8*(2*A*b + 9*a*B))/(11*x^11) - (b^
9*(A*b + 10*a*B))/(10*x^10) - (b^10*B)/(9*x^9)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^{10}A}{20x^{20}}-\frac{a^{9}(10Ab+Ba)}{19x^{19}}-\frac{5a^{8}b(9Ab+2Ba)}{18x^{18}}-\frac{15a^{7}b^{2}(8Ab+3Ba)}{17x^{17}}-\frac{15a^{6}b^{3}(7Ab+4Ba)}{8x^{16}}-\frac{14a^{5}b^{4}(6Ab+5Ba)}{5x^{15}}-$
norman	$-\frac{a^{10}A}{20}+\left(-\frac{10}{19}a^{9}bA-\frac{1}{19}a^{10}B\right)x+\left(-\frac{5}{2}a^{8}b^{2}A-\frac{5}{9}a^{9}bB\right)x^{2}+\left(-\frac{120}{17}a^{7}b^{3}A-\frac{45}{17}a^{8}b^{2}B\right)x^{3}+\left(-\frac{105}{8}a^{6}b^{4}A-\frac{15}{2}a^{7}b^{3}B\right)x^{4}+\left(-\frac{84}{5}\right.$
risch	$-\frac{a^{10}A}{20}+\left(-\frac{10}{19}a^{9}bA-\frac{1}{19}a^{10}B\right)x+\left(-\frac{5}{2}a^{8}b^{2}A-\frac{5}{9}a^{9}bB\right)x^{2}+\left(-\frac{120}{17}a^{7}b^{3}A-\frac{45}{17}a^{8}b^{2}B\right)x^{3}+\left(-\frac{105}{8}a^{6}b^{4}A-\frac{15}{2}a^{7}b^{3}B\right)x^{4}+\left(-\frac{84}{5}\right.$
gosper	$-\frac{1847560Bb^{10}x^{11}+1662804Ab^{10}x^{10}+16628040Bab^{9}x^{10}+15116400aAb^{9}x^{9}+68023800Ba^{2}b^{8}x^{9}+62355150a^{2}Ab^{8}x^{8}+16}{}$
parallelrisch	$-\frac{1847560Bb^{10}x^{11}+1662804Ab^{10}x^{10}+16628040Bab^{9}x^{10}+15116400aAb^{9}x^{9}+68023800Ba^{2}b^{8}x^{9}+62355150a^{2}Ab^{8}x^{8}+16}{}$
orering	$-\frac{1847560Bb^{10}x^{11}+1662804Ab^{10}x^{10}+16628040Bab^{9}x^{10}+15116400aAb^{9}x^{9}+68023800Ba^{2}b^{8}x^{9}+62355150a^{2}Ab^{8}x^{8}+16}{}$

input
```
int((b*x+a)^10*(B*x+A)/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/20*a^10*A/x^20-1/19*a^9*(10*A*b+B*a)/x^19-5/18*a^8*b*(9*A*b+2*B*a)/x^18
-15/17*a^7*b^2*(8*A*b+3*B*a)/x^17-15/8*a^6*b^3*(7*A*b+4*B*a)/x^16-14/5*a^5
*b^4*(6*A*b+5*B*a)/x^15-3*a^4*b^5*(5*A*b+6*B*a)/x^14-30/13*a^3*b^6*(4*A*b+
7*B*a)/x^13-5/4*a^2*b^7*(3*A*b+8*B*a)/x^12-5/11*a*b^8*(2*A*b+9*B*a)/x^11-1
/10*b^9*(A*b+10*B*a)/x^10-1/9*b^10*B/x^9
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = -\frac{1847560\,Bb^{10}x^{11}+831402\,Aa^{10}+1662804\,(10\,Bab^{9}+Ab^{10})x^{10}+7558200\,(9\,Ba^{2}b^{8}+2\,Aab^{9})x^{9}+20}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^21,x, algorithm="fricas")
```

output
```
-1/16628040*(1847560*B*b^10*x^11 + 831402*A*a^10 + 1662804*(10*B*a*b^9 + A
*b^10)*x^10 + 7558200*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 20785050*(8*B*a^3*b^
7 + 3*A*a^2*b^8)*x^8 + 38372400*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 49884120
*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 46558512*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^
5 + 31177575*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 14671800*(3*B*a^8*b^2 + 8*A
*a^7*b^3)*x^3 + 4618900*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 875160*(B*a^10 + 1
0*A*a^9*b)*x)/x^20
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**10*(B*x+A)/x**21,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = -\frac{1847560\,Bb^{10}x^{11} + 831402\,Aa^{10} + 1662804\,(10\,Bab^9 + Ab^{10})x^{10} + 7558200\,(9\,Ba^2b^8 + 2\,Aab^9)x^9 + 20}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^21,x, algorithm="maxima")
```

output
```
-1/16628040*(1847560*B*b^10*x^11 + 831402*A*a^10 + 1662804*(10*B*a*b^9 + A
*b^10)*x^10 + 7558200*(9*B*a^2*b^8 + 2*A*a*b^9)*x^9 + 20785050*(8*B*a^3*b^
7 + 3*A*a^2*b^8)*x^8 + 38372400*(7*B*a^4*b^6 + 4*A*a^3*b^7)*x^7 + 49884120
*(6*B*a^5*b^5 + 5*A*a^4*b^6)*x^6 + 46558512*(5*B*a^6*b^4 + 6*A*a^5*b^5)*x^
5 + 31177575*(4*B*a^7*b^3 + 7*A*a^6*b^4)*x^4 + 14671800*(3*B*a^8*b^2 + 8*A
*a^7*b^3)*x^3 + 4618900*(2*B*a^9*b + 9*A*a^8*b^2)*x^2 + 875160*(B*a^10 + 1
0*A*a^9*b)*x)/x^20
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = -\frac{1847560\,Bb^{10}x^{11} + 16628040\,Bab^{9}x^{10} + 1662804\,Ab^{10}x^{10} + 68023800\,Ba^{2}b^{8}x^{9} + 15116400\,Aab^{9}x^{9} +}{}$$

input
```
integrate((b*x+a)^10*(B*x+A)/x^21,x, algorithm="giac")
```

output
```
-1/16628040*(1847560*B*b^10*x^11 + 16628040*B*a*b^9*x^10 + 1662804*A*b^10*
x^10 + 68023800*B*a^2*b^8*x^9 + 15116400*A*a*b^9*x^9 + 166280400*B*a^3*b^7
*x^8 + 62355150*A*a^2*b^8*x^8 + 268606800*B*a^4*b^6*x^7 + 153489600*A*a^3*
b^7*x^7 + 299304720*B*a^5*b^5*x^6 + 249420600*A*a^4*b^6*x^6 + 232792560*B*
a^6*b^4*x^5 + 279351072*A*a^5*b^5*x^5 + 124710300*B*a^7*b^3*x^4 + 21824302
5*A*a^6*b^4*x^4 + 44015400*B*a^8*b^2*x^3 + 117374400*A*a^7*b^3*x^3 + 92378
00*B*a^9*b*x^2 + 41570100*A*a^8*b^2*x^2 + 875160*B*a^10*x + 8751600*A*a^9*
b*x + 831402*A*a^10)/x^20
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx = -\frac{x\left(\frac{B\,a^{10}}{19}+\frac{10\,A\,b\,a^{9}}{19}\right)+\frac{A\,a^{10}}{20}+x^{2}\left(\frac{5\,B\,a^{9}\,b}{9}+\frac{5\,A\,a^{8}\,b^{2}}{2}\right)+x^{9}\left(\frac{45\,B\,a^{2}\,b^{8}}{11}+\frac{10\,A\,a\,b^{9}}{11}\right)+x^{10}\left(\frac{A\,b^{10}}{10}+B\,a\,b^{9}\right)+}{}$$

input
```
int(((A + B*x)*(a + b*x)^10)/x^21,x)
```

output
```
-(x*((B*a^10)/19 + (10*A*a^9*b)/19) + (A*a^10)/20 + x^2*((5*A*a^8*b^2)/2 +
 (5*B*a^9*b)/9) + x^9*((45*B*a^2*b^8)/11 + (10*A*a*b^9)/11) + x^10*((A*b^1
0)/10 + B*a*b^9) + x^8*((15*A*a^2*b^8)/4 + 10*B*a^3*b^7) + x^6*(15*A*a^4*b
^6 + 18*B*a^5*b^5) + x^5*((84*A*a^5*b^5)/5 + 14*B*a^6*b^4) + x^4*((105*A*a
^6*b^4)/8 + (15*B*a^7*b^3)/2) + x^3*((120*A*a^7*b^3)/17 + (45*B*a^8*b^2)/1
7) + x^7*((120*A*a^3*b^7)/13 + (210*B*a^4*b^6)/13) + (B*b^10*x^11)/9)/x^20
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{21}}\,dx$$
$$= \frac{-167960b^{11}x^{11} - 1662804a\,b^{10}x^{10} - 7558200a^2b^9x^9 - 20785050a^3b^8x^8 - 38372400a^4b^7x^7 - 49884120a^5}{1511}$$

input
```
int((b*x+a)^10*(B*x+A)/x^21,x)
```

output
```
( - 75582*a**11 - 875160*a**10*b*x - 4618900*a**9*b**2*x**2 - 14671800*a**
8*b**3*x**3 - 31177575*a**7*b**4*x**4 - 46558512*a**6*b**5*x**5 - 49884120
*a**5*b**6*x**6 - 38372400*a**4*b**7*x**7 - 20785050*a**3*b**8*x**8 - 7558
200*a**2*b**9*x**9 - 1662804*a*b**10*x**10 - 167960*b**11*x**11)/(1511640*
x**20)
```

3.138 $\int x^3(a+bx)(c+dx)^{16}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int x^3(a+bx)(c+dx)^{16}\,dx=\frac{c^3(bc-ad)(c+dx)^{17}}{17d^5}-\frac{c^2(4bc-3ad)(c+dx)^{18}}{18d^5}+\frac{3c(2bc-ad)(c+dx)^{19}}{19d^5}-\frac{(4bc-ad)(c+dx)^{20}}{20d^5}+\frac{b(c+dx)^{21}}{21d^5}$$

output
```
1/17*c^3*(-a*d+b*c)*(d*x+c)^17/d^5-1/18*c^2*(-3*a*d+4*b*c)*(d*x+c)^18/d^5+
3/19*c*(-a*d+2*b*c)*(d*x+c)^19/d^5-1/20*(-a*d+4*b*c)*(d*x+c)^20/d^5+1/21*b
*(d*x+c)^21/d^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(114) = 228.

Time = 0.05 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.15

$$\int x^3(a+bx)(c+dx)^{16}\,dx = \frac{1}{4}ac^{16}x^4+\frac{1}{5}c^{15}(bc+16ad)x^5+\frac{4}{3}c^{14}d(2bc+15ad)x^6+\frac{40}{7}c^{13}d^2(3bc+14ad)x^7+\frac{35}{2}c^{12}d^3(4bc+13ad)x^8+\frac{364}{9}c^{11}d^4(5bc+12ad)x^9+\frac{364}{5}c^{10}d^5(6bc+11ad)x^{10}+104c^9d^6(7bc+10ad)x^{11}+\frac{715}{6}c^8d^7(8bc+9ad)x^{12}+110c^7d^8(9bc+8ad)x^{13}+\frac{572}{7}c^6d^9(10bc+7ad)x^{14}+\frac{728}{15}c^5d^{10}(11bc+6ad)x^{15}+\frac{91}{4}c^4d^{11}(12bc+5ad)x^{16}+\frac{140}{17}c^3d^{12}(13bc+4ad)x^{17}+\frac{20}{9}c^2d^{13}(14bc+3ad)x^{18}+\frac{8}{19}cd^{14}(15bc+2ad)x^{19}+\frac{1}{20}d^{15}(16bc+ad)x^{20}+\frac{1}{21}bd^{16}x^{21}$$

input
```
Integrate[x^3*(a + b*x)*(c + d*x)^16,x]
```

output
```
(a*c^16*x^4)/4 + (c^15*(b*c + 16*a*d)*x^5)/5 + (4*c^14*d*(2*b*c + 15*a*d)*
x^6)/3 + (40*c^13*d^2*(3*b*c + 14*a*d)*x^7)/7 + (35*c^12*d^3*(4*b*c + 13*a
*d)*x^8)/2 + (364*c^11*d^4*(5*b*c + 12*a*d)*x^9)/9 + (364*c^10*d^5*(6*b*c
+ 11*a*d)*x^10)/5 + 104*c^9*d^6*(7*b*c + 10*a*d)*x^11 + (715*c^8*d^7*(8*b*
c + 9*a*d)*x^12)/6 + 110*c^7*d^8*(9*b*c + 8*a*d)*x^13 + (572*c^6*d^9*(10*b
*c + 7*a*d)*x^14)/7 + (728*c^5*d^10*(11*b*c + 6*a*d)*x^15)/15 + (91*c^4*d^
11*(12*b*c + 5*a*d)*x^16)/4 + (140*c^3*d^12*(13*b*c + 4*a*d)*x^17)/17 + (2
0*c^2*d^13*(14*b*c + 3*a*d)*x^18)/9 + (8*c*d^14*(15*b*c + 2*a*d)*x^19)/19
+ (d^15*(16*b*c + a*d)*x^20)/20 + (b*d^16*x^21)/21
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)(c+dx)^{16}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{c^3(c+dx)^{16}(bc-ad)}{d^4}-\frac{c^2(c+dx)^{17}(4bc-3ad)}{d^4}+\frac{(c+dx)^{19}(ad-4bc)}{d^4}+\frac{3c(c+dx)^{18}(2bc-ad)}{d^4}+\frac{b(c+\cdots}{d\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3(c+dx)^{17}(bc-ad)}{17d^5}-\frac{c^2(c+dx)^{18}(4bc-3ad)}{18d^5}-\frac{(c+dx)^{20}(4bc-ad)}{20d^5}+\frac{3c(c+dx)^{19}(2bc-ad)}{19d^5}+\frac{b(c+dx)^{21}}{21d^5}$$

input
```
Int[x^3*(a + b*x)*(c + d*x)^16,x]
```

output
```
(c^3*(b*c - a*d)*(c + d*x)^17)/(17*d^5) - (c^2*(4*b*c - 3*a*d)*(c + d*x)^1
8)/(18*d^5) + (3*c*(2*b*c - a*d)*(c + d*x)^19)/(19*d^5) - ((4*b*c - a*d)*(
c + d*x)^20)/(20*d^5) + (b*(c + d*x)^21)/(21*d^5)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(104) = 208$.

Time = 0.13 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.28

method	result
norman	$\frac{a\,c^{16}x^{4}}{4}+\left(\frac{16}{5}a\,c^{15}d+\frac{1}{5}b\,c^{16}\right)x^{5}+\left(20a\,c^{14}d^{2}+\frac{8}{3}b\,c^{15}d\right)x^{6}+\left(80a\,c^{13}d^{3}+\frac{120}{7}b\,c^{14}d^{2}\right)x^{7}+\big($
default	$\frac{b\,d^{16}x^{21}}{21}+\frac{\left(a\,d^{16}+16bc\,d^{15}\right)x^{20}}{20}+\frac{\left(16ac\,d^{15}+120b\,c^{2}d^{14}\right)x^{19}}{19}+\frac{\left(120a\,c^{2}d^{14}+560b\,c^{3}d^{13}\right)x^{18}}{18}+\frac{(560a\,c^{3}d^{13}+1820b\,c}{17}$
orering	$x^{4}\left(19380b\,d^{16}x^{17}+20349a\,d^{16}x^{16}+325584bc\,d^{15}x^{16}+342720ac\,d^{15}x^{15}+2570400b\,c^{2}d^{14}x^{15}+2713200a\,c^{2}d^{14}x^{14}+12661600b\right.$
gosper	$\frac{1}{4}a\,c^{16}x^{4}+80x^{7}a\,c^{13}d^{3}+\frac{120}{7}x^{7}b\,c^{14}d^{2}+\frac{455}{2}x^{8}a\,c^{12}d^{4}+70x^{8}b\,c^{13}d^{3}+\frac{1456}{3}x^{9}a\,c^{11}d^{5}+\frac{1820}{9}x$
risch	$\frac{1}{4}a\,c^{16}x^{4}+80x^{7}a\,c^{13}d^{3}+\frac{120}{7}x^{7}b\,c^{14}d^{2}+\frac{455}{2}x^{8}a\,c^{12}d^{4}+70x^{8}b\,c^{13}d^{3}+\frac{1456}{3}x^{9}a\,c^{11}d^{5}+\frac{1820}{9}x$
parallelrisch	$\frac{1}{4}a\,c^{16}x^{4}+80x^{7}a\,c^{13}d^{3}+\frac{120}{7}x^{7}b\,c^{14}d^{2}+\frac{455}{2}x^{8}a\,c^{12}d^{4}+70x^{8}b\,c^{13}d^{3}+\frac{1456}{3}x^{9}a\,c^{11}d^{5}+\frac{1820}{9}x$

input

```
int(x^3*(b*x+a)*(d*x+c)^16,x,method=_RETURNVERBOSE)
```

output

```
1/4*a*c^16*x^4+(16/5*a*c^15*d+1/5*b*c^16)*x^5+(20*a*c^14*d^2+8/3*b*c^15*d)
*x^6+(80*a*c^13*d^3+120/7*b*c^14*d^2)*x^7+(455/2*a*c^12*d^4+70*b*c^13*d^3)
*x^8+(1456/3*a*c^11*d^5+1820/9*b*c^12*d^4)*x^9+(4004/5*a*c^10*d^6+2184/5*b
*c^11*d^5)*x^10+(1040*a*c^9*d^7+728*b*c^10*d^6)*x^11+(2145/2*a*c^8*d^8+286
0/3*b*c^9*d^7)*x^12+(880*a*c^7*d^9+990*b*c^8*d^8)*x^13+(572*a*c^6*d^10+572
0/7*b*c^7*d^9)*x^14+(1456/5*a*c^5*d^11+8008/15*b*c^6*d^10)*x^15+(455/4*a*c
^4*d^12+273*b*c^5*d^11)*x^16+(560/17*a*c^3*d^13+1820/17*b*c^4*d^12)*x^17+(
20/3*a*c^2*d^14+280/9*b*c^3*d^13)*x^18+(16/19*a*c*d^15+120/19*b*c^2*d^14)*
x^19+(1/20*a*d^16+4/5*b*c*d^15)*x^20+1/21*b*d^16*x^21
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(104) = 208$.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.39

$$\int x^3(a+bx)(c+dx)^{16}\,dx = \frac{1}{21}\,bd^{16}x^{21} + \frac{1}{4}\,ac^{16}x^4 + \frac{1}{20}\,\left(16\,bcd^{15} + ad^{16}\right)x^{20} + \frac{8}{19}\,\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{19} + \frac{20}{9}\,\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{18} + \frac{140}{17}\,\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{17} + \frac{91}{4}\,\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{16} + \frac{728}{15}\,\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{15} + \frac{572}{7}\,\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{14} + 110\,\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{13} + \frac{715}{6}\,\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^{12} + 104\,\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^{11} + \frac{364}{5}\,\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^{10} + \frac{364}{9}\,\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^9 + \frac{35}{2}\,\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^8 + \frac{40}{7}\,\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^7 + \frac{4}{3}\,\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^6 + \frac{1}{5}\,\left(bc^{16} + 16\,ac^{15}d\right)x^5$$

input
```
integrate(x^3*(b*x+a)*(d*x+c)^16,x, algorithm="fricas")
```

output

```
1/21*b*d^16*x^21 + 1/4*a*c^16*x^4 + 1/20*(16*b*c*d^15 + a*d^16)*x^20 + 8/1
9*(15*b*c^2*d^14 + 2*a*c*d^15)*x^19 + 20/9*(14*b*c^3*d^13 + 3*a*c^2*d^14)*
x^18 + 140/17*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^17 + 91/4*(12*b*c^5*d^11 +
5*a*c^4*d^12)*x^16 + 728/15*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^15 + 572/7*(1
0*b*c^7*d^9 + 7*a*c^6*d^10)*x^14 + 110*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^13 +
715/6*(8*b*c^9*d^7 + 9*a*c^8*d^8)*x^12 + 104*(7*b*c^10*d^6 + 10*a*c^9*d^7)
*x^11 + 364/5*(6*b*c^11*d^5 + 11*a*c^10*d^6)*x^10 + 364/9*(5*b*c^12*d^4 +
12*a*c^11*d^5)*x^9 + 35/2*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^8 + 40/7*(3*b*c
^14*d^2 + 14*a*c^13*d^3)*x^7 + 4/3*(2*b*c^15*d + 15*a*c^14*d^2)*x^6 + 1/5*
(b*c^16 + 16*a*c^15*d)*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(104) = 208$.

Time = 0.06 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.70

$$\int x^3(a+bx)(c+dx)^{16}\,dx = \frac{ac^{16}x^4}{4} + \frac{bd^{16}x^{21}}{21} + x^{20}\left(\frac{ad^{16}}{20} + \frac{4bcd^{15}}{5}\right) + x^{19} \cdot \left(\frac{16acd^{15}}{19} + \frac{120bc^2d^{14}}{19}\right) + x^{18} \cdot \left(\frac{20ac^2d^{14}}{3} + \frac{280bc^3d^{13}}{9}\right) + x^{17} \cdot \left(\frac{560ac^3d^{13}}{17} + \frac{1820bc^4d^{12}}{17}\right) + x^{16} \cdot \left(\frac{455ac^4d^{12}}{4} + 273bc^5d^{11}\right) + x^{15} \cdot \left(\frac{1456ac^5d^{11}}{5} + \frac{8008bc^6d^{10}}{15}\right) + x^{14} \cdot \left(572ac^6d^{10} + \frac{5720bc^7d^9}{7}\right) + x^{13} \cdot \left(880ac^7d^9 + 990bc^8d^8\right) + x^{12} \cdot \left(\frac{2145ac^8d^8}{2} + \frac{2860bc^9d^7}{3}\right) + x^{11} \cdot \left(1040ac^9d^7 + 728bc^{10}d^6\right) + x^{10} \cdot \left(\frac{4004ac^{10}d^6}{5} + \frac{2184bc^{11}d^5}{5}\right) + x^9 \cdot \left(\frac{1456ac^{11}d^5}{3} + \frac{1820bc^{12}d^4}{9}\right) + x^8 \cdot \left(\frac{455ac^{12}d^4}{2} + 70bc^{13}d^3\right) + x^7 \cdot \left(80ac^{13}d^3 + \frac{120bc^{14}d^2}{7}\right) + x^6 \cdot \left(20ac^{14}d^2 + \frac{8bc^{15}d}{3}\right) + x^5 \cdot \left(\frac{16ac^{15}d}{5} + \frac{bc^{16}}{5}\right)$$

input

```
integrate(x**3*(b*x+a)*(d*x+c)**16,x)
```

output
```
a*c**16*x**4/4 + b*d**16*x**21/21 + x**20*(a*d**16/20 + 4*b*c*d**15/5) + x
**19*(16*a*c*d**15/19 + 120*b*c**2*d**14/19) + x**18*(20*a*c**2*d**14/3 +
280*b*c**3*d**13/9) + x**17*(560*a*c**3*d**13/17 + 1820*b*c**4*d**12/17) +
 x**16*(455*a*c**4*d**12/4 + 273*b*c**5*d**11) + x**15*(1456*a*c**5*d**11/
5 + 8008*b*c**6*d**10/15) + x**14*(572*a*c**6*d**10 + 5720*b*c**7*d**9/7)
+ x**13*(880*a*c**7*d**9 + 990*b*c**8*d**8) + x**12*(2145*a*c**8*d**8/2 +
2860*b*c**9*d**7/3) + x**11*(1040*a*c**9*d**7 + 728*b*c**10*d**6) + x**10*
(4004*a*c**10*d**6/5 + 2184*b*c**11*d**5/5) + x**9*(1456*a*c**11*d**5/3 +
1820*b*c**12*d**4/9) + x**8*(455*a*c**12*d**4/2 + 70*b*c**13*d**3) + x**7*
(80*a*c**13*d**3 + 120*b*c**14*d**2/7) + x**6*(20*a*c**14*d**2 + 8*b*c**15
*d/3) + x**5*(16*a*c**15*d/5 + b*c**16/5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(104) = 208$.

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.39

$$\begin{aligned}\int x^3(a+bx)(c+dx)^{16}\,dx = {} & \frac{1}{21}\,bd^{16}x^{21}+\frac{1}{4}\,ac^{16}x^4+\frac{1}{20}\left(16\,bcd^{15}+ad^{16}\right)x^{20}\\
&+\frac{8}{19}\left(15\,bc^2d^{14}+2\,acd^{15}\right)x^{19}\\
&+\frac{20}{9}\left(14\,bc^3d^{13}+3\,ac^2d^{14}\right)x^{18}\\
&+\frac{140}{17}\left(13\,bc^4d^{12}+4\,ac^3d^{13}\right)x^{17}\\
&+\frac{91}{4}\left(12\,bc^5d^{11}+5\,ac^4d^{12}\right)x^{16}\\
&+\frac{728}{15}\left(11\,bc^6d^{10}+6\,ac^5d^{11}\right)x^{15}\\
&+\frac{572}{7}\left(10\,bc^7d^9+7\,ac^6d^{10}\right)x^{14}+110\left(9\,bc^8d^8+8\,ac^7d^9\right)x^{13}\\
&+\frac{715}{6}\left(8\,bc^9d^7+9\,ac^8d^8\right)x^{12}+104\left(7\,bc^{10}d^6+10\,ac^9d^7\right)x^{11}\\
&+\frac{364}{5}\left(6\,bc^{11}d^5+11\,ac^{10}d^6\right)x^{10}\\
&+\frac{364}{9}\left(5\,bc^{12}d^4+12\,ac^{11}d^5\right)x^9\\
&+\frac{35}{2}\left(4\,bc^{13}d^3+13\,ac^{12}d^4\right)x^8+\frac{40}{7}\left(3\,bc^{14}d^2+14\,ac^{13}d^3\right)x^7\\
&+\frac{4}{3}\left(2\,bc^{15}d+15\,ac^{14}d^2\right)x^6+\frac{1}{5}\left(bc^{16}+16\,ac^{15}d\right)x^5\end{aligned}$$

input
```
integrate(x^3*(b*x+a)*(d*x+c)^16,x, algorithm="maxima")
```

output
```
1/21*b*d^16*x^21 + 1/4*a*c^16*x^4 + 1/20*(16*b*c*d^15 + a*d^16)*x^20 + 8/1
9*(15*b*c^2*d^14 + 2*a*c*d^15)*x^19 + 20/9*(14*b*c^3*d^13 + 3*a*c^2*d^14)*
x^18 + 140/17*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^17 + 91/4*(12*b*c^5*d^11 +
5*a*c^4*d^12)*x^16 + 728/15*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^15 + 572/7*(1
0*b*c^7*d^9 + 7*a*c^6*d^10)*x^14 + 110*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^13 +
715/6*(8*b*c^9*d^7 + 9*a*c^8*d^8)*x^12 + 104*(7*b*c^10*d^6 + 10*a*c^9*d^7)
*x^11 + 364/5*(6*b*c^11*d^5 + 11*a*c^10*d^6)*x^10 + 364/9*(5*b*c^12*d^4 +
12*a*c^11*d^5)*x^9 + 35/2*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^8 + 40/7*(3*b*c
^14*d^2 + 14*a*c^13*d^3)*x^7 + 4/3*(2*b*c^15*d + 15*a*c^14*d^2)*x^6 + 1/5*
(b*c^16 + 16*a*c^15*d)*x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(104) = 208$.

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.41

$$\begin{aligned}\int x^3(a+bx)(c+dx)^{16}\,dx = {} & \frac{1}{21}\,bd^{16}x^{21} + \frac{4}{5}\,bcd^{15}x^{20} + \frac{1}{20}\,ad^{16}x^{20} + \frac{120}{19}\,bc^2d^{14}x^{19} \\ & + \frac{16}{19}\,acd^{15}x^{19} + \frac{280}{9}\,bc^3d^{13}x^{18} + \frac{20}{3}\,ac^2d^{14}x^{18} \\ & + \frac{1820}{17}\,bc^4d^{12}x^{17} + \frac{560}{17}\,ac^3d^{13}x^{17} + 273\,bc^5d^{11}x^{16} \\ & + \frac{455}{4}\,ac^4d^{12}x^{16} + \frac{8008}{15}\,bc^6d^{10}x^{15} + \frac{1456}{5}\,ac^5d^{11}x^{15} \\ & + \frac{5720}{7}\,bc^7d^9x^{14} + 572\,ac^6d^{10}x^{14} + 990\,bc^8d^8x^{13} \\ & + 880\,ac^7d^9x^{13} + \frac{2860}{3}\,bc^9d^7x^{12} + \frac{2145}{2}\,ac^8d^8x^{12} \\ & + 728\,bc^{10}d^6x^{11} + 1040\,ac^9d^7x^{11} + \frac{2184}{5}\,bc^{11}d^5x^{10} \\ & + \frac{4004}{5}\,ac^{10}d^6x^{10} + \frac{1820}{9}\,bc^{12}d^4x^9 + \frac{1456}{3}\,ac^{11}d^5x^9 \\ & + 70\,bc^{13}d^3x^8 + \frac{455}{2}\,ac^{12}d^4x^8 + \frac{120}{7}\,bc^{14}d^2x^7 + 80\,ac^{13}d^3x^7 \\ & + \frac{8}{3}\,bc^{15}dx^6 + 20\,ac^{14}d^2x^6 + \frac{1}{5}\,bc^{16}x^5 + \frac{16}{5}\,ac^{15}dx^5 + \frac{1}{4}\,ac^{16}x^4\end{aligned}$$

input
```
integrate(x^3*(b*x+a)*(d*x+c)^16,x, algorithm="giac")
```

output
```
1/21*b*d^16*x^21 + 4/5*b*c*d^15*x^20 + 1/20*a*d^16*x^20 + 120/19*b*c^2*d^1
4*x^19 + 16/19*a*c*d^15*x^19 + 280/9*b*c^3*d^13*x^18 + 20/3*a*c^2*d^14*x^1
8 + 1820/17*b*c^4*d^12*x^17 + 560/17*a*c^3*d^13*x^17 + 273*b*c^5*d^11*x^16
 + 455/4*a*c^4*d^12*x^16 + 8008/15*b*c^6*d^10*x^15 + 1456/5*a*c^5*d^11*x^1
5 + 5720/7*b*c^7*d^9*x^14 + 572*a*c^6*d^10*x^14 + 990*b*c^8*d^8*x^13 + 880
*a*c^7*d^9*x^13 + 2860/3*b*c^9*d^7*x^12 + 2145/2*a*c^8*d^8*x^12 + 728*b*c^
10*d^6*x^11 + 1040*a*c^9*d^7*x^11 + 2184/5*b*c^11*d^5*x^10 + 4004/5*a*c^10
*d^6*x^10 + 1820/9*b*c^12*d^4*x^9 + 1456/3*a*c^11*d^5*x^9 + 70*b*c^13*d^3*
x^8 + 455/2*a*c^12*d^4*x^8 + 120/7*b*c^14*d^2*x^7 + 80*a*c^13*d^3*x^7 + 8/
3*b*c^15*d*x^6 + 20*a*c^14*d^2*x^6 + 1/5*b*c^16*x^5 + 16/5*a*c^15*d*x^5 +
1/4*a*c^16*x^4
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.90

$$\int x^3(a+bx)(c+dx)^{16}\,dx = x^5\left(\frac{b\,c^{16}}{5}+\frac{16\,a\,d\,c^{15}}{5}\right)+x^{20}\left(\frac{a\,d^{16}}{20}+\frac{4\,b\,c\,d^{15}}{5}\right)+\frac{a\,c^{16}\,x^4}{4}+\frac{b\,d^{16}\,x^{21}}{21}+\frac{4\,c^{14}\,d\,x^6\,(15\,a\,d+2\,b\,c)}{3}+\frac{8\,c\,d^{14}\,x^{19}\,(2\,a\,d+15\,b\,c)}{19}+\frac{40\,c^{13}\,d^2\,x^7\,(14\,a\,d+3\,b\,c)}{7}+\frac{35\,c^{12}\,d^3\,x^8\,(13\,a\,d+4\,b\,c)}{2}+\frac{364\,c^{11}\,d^4\,x^9\,(12\,a\,d+5\,b\,c)}{9}+\frac{364\,c^{10}\,d^5\,x^{10}\,(11\,a\,d+6\,b\,c)}{5}+104\,c^9\,d^6\,x^{11}\,(10\,a\,d+7\,b\,c)+\frac{715\,c^8\,d^7\,x^{12}\,(9\,a\,d+8\,b\,c)}{6}+110\,c^7\,d^8\,x^{13}\,(8\,a\,d+9\,b\,c)+\frac{572\,c^6\,d^9\,x^{14}\,(7\,a\,d+10\,b\,c)}{7}+\frac{728\,c^5\,d^{10}\,x^{15}\,(6\,a\,d+11\,b\,c)}{15}+\frac{91\,c^4\,d^{11}\,x^{16}\,(5\,a\,d+12\,b\,c)}{4}+\frac{140\,c^3\,d^{12}\,x^{17}\,(4\,a\,d+13\,b\,c)}{17}+\frac{20\,c^2\,d^{13}\,x^{18}\,(3\,a\,d+14\,b\,c)}{9}$$

input
```
int(x^3*(a + b*x)*(c + d*x)^16,x)
```

output
```
x^5*((b*c^16)/5 + (16*a*c^15*d)/5) + x^20*((a*d^16)/20 + (4*b*c*d^15)/5) +
 (a*c^16*x^4)/4 + (b*d^16*x^21)/21 + (4*c^14*d*x^6*(15*a*d + 2*b*c))/3 + (
8*c*d^14*x^19*(2*a*d + 15*b*c))/19 + (40*c^13*d^2*x^7*(14*a*d + 3*b*c))/7
+ (35*c^12*d^3*x^8*(13*a*d + 4*b*c))/2 + (364*c^11*d^4*x^9*(12*a*d + 5*b*c
))/9 + (364*c^10*d^5*x^10*(11*a*d + 6*b*c))/5 + 104*c^9*d^6*x^11*(10*a*d +
 7*b*c) + (715*c^8*d^7*x^12*(9*a*d + 8*b*c))/6 + 110*c^7*d^8*x^13*(8*a*d +
 9*b*c) + (572*c^6*d^9*x^14*(7*a*d + 10*b*c))/7 + (728*c^5*d^10*x^15*(6*a*
d + 11*b*c))/15 + (91*c^4*d^11*x^16*(5*a*d + 12*b*c))/4 + (140*c^3*d^12*x^
17*(4*a*d + 13*b*c))/17 + (20*c^2*d^13*x^18*(3*a*d + 14*b*c))/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.39

$$\int x^3(a+bx)(c+dx)^{16}\,dx$$
$$= \frac{x^4(19380b\,d^{16}x^{17} + 20349a\,d^{16}x^{16} + 325584bc\,d^{15}x^{16} + 342720ac\,d^{15}x^{15} + 2570400b\,c^2d^{14}x^{15} + 2713200a}{}$$

input
```
int(x^3*(b*x+a)*(d*x+c)^16,x)
```

output
```
(x**4*(101745*a*c**16 + 1302336*a*c**15*d*x + 8139600*a*c**14*d**2*x**2 +
32558400*a*c**13*d**3*x**3 + 92587950*a*c**12*d**4*x**4 + 197520960*a*c**1
1*d**5*x**5 + 325909584*a*c**10*d**6*x**6 + 423259200*a*c**9*d**7*x**7 + 4
36486050*a*c**8*d**8*x**8 + 358142400*a*c**7*d**9*x**9 + 232792560*a*c**6*
d**10*x**10 + 118512576*a*c**5*d**11*x**11 + 46293975*a*c**4*d**12*x**12 +
 13406400*a*c**3*d**13*x**13 + 2713200*a*c**2*d**14*x**14 + 342720*a*c*d**
15*x**15 + 20349*a*d**16*x**16 + 81396*b*c**16*x + 1085280*b*c**15*d*x**2
+ 6976800*b*c**14*d**2*x**3 + 28488600*b*c**13*d**3*x**4 + 82300400*b*c**1
2*d**4*x**5 + 177768864*b*c**11*d**5*x**6 + 296281440*b*c**10*d**6*x**7 +
387987600*b*c**9*d**7*x**8 + 402910200*b*c**8*d**8*x**9 + 332560800*b*c**7
*d**9*x**10 + 217273056*b*c**6*d**10*x**11 + 111105540*b*c**5*d**11*x**12
+ 43570800*b*c**4*d**12*x**13 + 12661600*b*c**3*d**13*x**14 + 2570400*b*c*
*2*d**14*x**15 + 325584*b*c*d**15*x**16 + 19380*b*d**16*x**17))/406980
```

3.139 $\int x^2(a+bx)(c+dx)^{16}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int x^2(a+bx)(c+dx)^{16}\,dx=-\frac{c^2(bc-ad)(c+dx)^{17}}{17d^4}+\frac{c(3bc-2ad)(c+dx)^{18}}{18d^4}-\frac{(3bc-ad)(c+dx)^{19}}{19d^4}+\frac{b(c+dx)^{20}}{20d^4}$$

output
```
-1/17*c^2*(-a*d+b*c)*(d*x+c)^17/d^4+1/18*c*(-2*a*d+3*b*c)*(d*x+c)^18/d^4-1
/19*(-a*d+3*b*c)*(d*x+c)^19/d^4+1/20*b*(d*x+c)^20/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 355 vs. 2(88) = 176.

Time = 0.04 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.03

$$\int x^2(a+bx)(c+dx)^{16}\,dx = \frac{1}{3}ac^{16}x^3+\frac{1}{4}c^{15}(bc+16ad)x^4+\frac{8}{5}c^{14}d(2bc+15ad)x^5 + \frac{20}{3}c^{13}d^2(3bc+14ad)x^6+20c^{12}d^3(4bc+13ad)x^7 + \frac{91}{2}c^{11}d^4(5bc+12ad)x^8+\frac{728}{9}c^{10}d^5(6bc+11ad)x^9 + \frac{572}{5}c^9d^6(7bc+10ad)x^{10}+130c^8d^7(8bc+9ad)x^{11} + \frac{715}{6}c^7d^8(9bc+8ad)x^{12}+88c^6d^9(10bc+7ad)x^{13} + 52c^5d^{10}(11bc+6ad)x^{14}+\frac{364}{15}c^4d^{11}(12bc+5ad)x^{15} + \frac{35}{4}c^3d^{12}(13bc+4ad)x^{16}+\frac{40}{17}c^2d^{13}(14bc+3ad)x^{17} + \frac{4}{9}cd^{14}(15bc+2ad)x^{18}+\frac{1}{19}d^{15}(16bc+ad)x^{19}+\frac{1}{20}bd^{16}x^{20}$$

input
```
Integrate[x^2*(a + b*x)*(c + d*x)^16,x]
```

output
```
(a*c^16*x^3)/3 + (c^15*(b*c + 16*a*d)*x^4)/4 + (8*c^14*d*(2*b*c + 15*a*d)*
x^5)/5 + (20*c^13*d^2*(3*b*c + 14*a*d)*x^6)/3 + 20*c^12*d^3*(4*b*c + 13*a*
d)*x^7 + (91*c^11*d^4*(5*b*c + 12*a*d)*x^8)/2 + (728*c^10*d^5*(6*b*c + 11*
a*d)*x^9)/9 + (572*c^9*d^6*(7*b*c + 10*a*d)*x^10)/5 + 130*c^8*d^7*(8*b*c +
 9*a*d)*x^11 + (715*c^7*d^8*(9*b*c + 8*a*d)*x^12)/6 + 88*c^6*d^9*(10*b*c +
 7*a*d)*x^13 + 52*c^5*d^10*(11*b*c + 6*a*d)*x^14 + (364*c^4*d^11*(12*b*c +
 5*a*d)*x^15)/15 + (35*c^3*d^12*(13*b*c + 4*a*d)*x^16)/4 + (40*c^2*d^13*(1
4*b*c + 3*a*d)*x^17)/17 + (4*c*d^14*(15*b*c + 2*a*d)*x^18)/9 + (d^15*(16*b
*c + a*d)*x^19)/19 + (b*d^16*x^20)/20
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)(c+dx)^{16}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(-\frac{c^2(c+dx)^{16}(bc-ad)}{d^3}+\frac{(c+dx)^{18}(ad-3bc)}{d^3}+\frac{c(c+dx)^{17}(3bc-2ad)}{d^3}+\frac{b(c+dx)^{19}}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2(c+dx)^{17}(bc-ad)}{17d^4}-\frac{(c+dx)^{19}(3bc-ad)}{19d^4}+\frac{c(c+dx)^{18}(3bc-2ad)}{18d^4}+\frac{b(c+dx)^{20}}{20d^4}$$

input
```
Int[x^2*(a + b*x)*(c + d*x)^16,x]
```

output
```
-1/17*(c^2*(b*c - a*d)*(c + d*x)^17)/d^4 + (c*(3*b*c - 2*a*d)*(c + d*x)^18
)/(18*d^4) - ((3*b*c - a*d)*(c + d*x)^19)/(19*d^4) + (b*(c + d*x)^20)/(20*
d^4)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(80) = 160.

Time = 0.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 4.25

method	result
norman	$\frac{a\,c^{16}x^{3}}{3}+\left(4a\,c^{15}d+\frac{1}{4}b\,c^{16}\right)x^{4}+\left(24a\,c^{14}d^{2}+\frac{16}{5}b\,c^{15}d\right)x^{5}+\left(\frac{280}{3}a\,c^{13}d^{3}+20b\,c^{14}d^{2}\right)x^{6}+(2$
default	$\frac{b\,d^{16}x^{20}}{20}+\frac{(a\,d^{16}+16bc\,d^{15})x^{19}}{19}+\frac{(16ac\,d^{15}+120b\,c^{2}d^{14})x^{18}}{18}+\frac{(120a\,c^{2}d^{14}+560b\,c^{3}d^{13})x^{17}}{17}+\frac{(560a\,c^{3}d^{13}+1820b\,c}{16}$
orering	$\frac{x^{3}(2907b\,d^{16}x^{17}+3060a\,d^{16}x^{16}+48960bc\,d^{15}x^{16}+51680ac\,d^{15}x^{15}+387600b\,c^{2}d^{14}x^{15}+410400a\,c^{2}d^{14}x^{14}+1915200b\,c^{3}d^{13}x}{}$
gosper	$880b\,c^{7}d^{9}x^{13}+312a\,c^{5}d^{11}x^{14}+572b\,c^{6}d^{10}x^{14}+1170a\,c^{8}d^{8}x^{11}+1040b\,c^{9}d^{7}x^{11}+616a\,c^{6}d^{1}$
risch	$880b\,c^{7}d^{9}x^{13}+312a\,c^{5}d^{11}x^{14}+572b\,c^{6}d^{10}x^{14}+1170a\,c^{8}d^{8}x^{11}+1040b\,c^{9}d^{7}x^{11}+616a\,c^{6}d^{1}$
parallelrisch	$880b\,c^{7}d^{9}x^{13}+312a\,c^{5}d^{11}x^{14}+572b\,c^{6}d^{10}x^{14}+1170a\,c^{8}d^{8}x^{11}+1040b\,c^{9}d^{7}x^{11}+616a\,c^{6}d^{1}$

input
```
int(x^2*(b*x+a)*(d*x+c)^16,x,method=_RETURNVERBOSE)
```

output
```
1/3*a*c^16*x^3+(4*a*c^15*d+1/4*b*c^16)*x^4+(24*a*c^14*d^2+16/5*b*c^15*d)*x
^5+(280/3*a*c^13*d^3+20*b*c^14*d^2)*x^6+(260*a*c^12*d^4+80*b*c^13*d^3)*x^7
+(546*a*c^11*d^5+455/2*b*c^12*d^4)*x^8+(8008/9*a*c^10*d^6+1456/3*b*c^11*d^
5)*x^9+(1144*a*c^9*d^7+4004/5*b*c^10*d^6)*x^10+(1170*a*c^8*d^8+1040*b*c^9*
d^7)*x^11+(2860/3*a*c^7*d^9+2145/2*b*c^8*d^8)*x^12+(616*a*c^6*d^10+880*b*c
^7*d^9)*x^13+(312*a*c^5*d^11+572*b*c^6*d^10)*x^14+(364/3*a*c^4*d^12+1456/5
*b*c^5*d^11)*x^15+(35*a*c^3*d^13+455/4*b*c^4*d^12)*x^16+(120/17*a*c^2*d^14
+560/17*b*c^3*d^13)*x^17+(8/9*a*c*d^15+20/3*b*c^2*d^14)*x^18+(1/19*a*d^16+
16/19*b*c*d^15)*x^19+1/20*b*d^16*x^20
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 4.40

$$\int x^2(a+bx)(c+dx)^{16}\,dx = \frac{1}{20}\,bd^{16}x^{20} + \frac{1}{3}\,ac^{16}x^3 + \frac{1}{19}\,\left(16\,bcd^{15} + ad^{16}\right)x^{19} + \frac{4}{9}\,\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{18} + \frac{40}{17}\,\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{17} + \frac{35}{4}\,\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{16} + \frac{364}{15}\,\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{15} + 52\,\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{14} + 88\,\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{13} + \frac{715}{6}\,\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{12} + 130\,\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^{11} + \frac{572}{5}\,\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^{10} + \frac{728}{9}\,\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^9 + \frac{91}{2}\,\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^8 + 20\,\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^7 + \frac{20}{3}\,\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^6 + \frac{8}{5}\,\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^5 + \frac{1}{4}\,\left(bc^{16} + 16\,ac^{15}d\right)x^4$$

input
```
integrate(x^2*(b*x+a)*(d*x+c)^16,x, algorithm="fricas")
```

output
```
1/20*b*d^16*x^20 + 1/3*a*c^16*x^3 + 1/19*(16*b*c*d^15 + a*d^16)*x^19 + 4/9
*(15*b*c^2*d^14 + 2*a*c*d^15)*x^18 + 40/17*(14*b*c^3*d^13 + 3*a*c^2*d^14)*
x^17 + 35/4*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^16 + 364/15*(12*b*c^5*d^11 +
5*a*c^4*d^12)*x^15 + 52*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^14 + 88*(10*b*c^7
*d^9 + 7*a*c^6*d^10)*x^13 + 715/6*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^12 + 130*(
8*b*c^9*d^7 + 9*a*c^8*d^8)*x^11 + 572/5*(7*b*c^10*d^6 + 10*a*c^9*d^7)*x^10
 + 728/9*(6*b*c^11*d^5 + 11*a*c^10*d^6)*x^9 + 91/2*(5*b*c^12*d^4 + 12*a*c^
11*d^5)*x^8 + 20*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^7 + 20/3*(3*b*c^14*d^2 +
 14*a*c^13*d^3)*x^6 + 8/5*(2*b*c^15*d + 15*a*c^14*d^2)*x^5 + 1/4*(b*c^16 +
 16*a*c^15*d)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.69

$$\int x^2(a+bx)(c+dx)^{16}\,dx = \frac{ac^{16}x^3}{3} + \frac{bd^{16}x^{20}}{20} + x^{19}\left(\frac{ad^{16}}{19} + \frac{16bcd^{15}}{19}\right) + x^{18} \cdot \left(\frac{8acd^{15}}{9} + \frac{20bc^2d^{14}}{3}\right) + x^{17} \cdot \left(\frac{120ac^2d^{14}}{17} + \frac{560bc^3d^{13}}{17}\right) + x^{16} \cdot \left(35ac^3d^{13} + \frac{455bc^4d^{12}}{4}\right) + x^{15} \cdot \left(\frac{364ac^4d^{12}}{3} + \frac{1456bc^5d^{11}}{5}\right) + x^{14} \cdot \left(312ac^5d^{11} + 572bc^6d^{10}\right) + x^{13} \cdot \left(616ac^6d^{10} + 880bc^7d^9\right) + x^{12} \cdot \left(\frac{2860ac^7d^9}{3} + \frac{2145bc^8d^8}{2}\right) + x^{11} \cdot \left(1170ac^8d^8 + 1040bc^9d^7\right) + x^{10} \cdot \left(1144ac^9d^7 + \frac{4004bc^{10}d^6}{5}\right) + x^9 \cdot \left(\frac{8008ac^{10}d^6}{9} + \frac{1456bc^{11}d^5}{3}\right) + x^8 \cdot \left(546ac^{11}d^5 + \frac{455bc^{12}d^4}{2}\right) + x^7 \cdot \left(260ac^{12}d^4 + 80bc^{13}d^3\right) + x^6 \cdot \left(\frac{280ac^{13}d^3}{3} + 20bc^{14}d^2\right) + x^5 \cdot \left(24ac^{14}d^2 + \frac{16bc^{15}d}{5}\right) + x^4 \cdot \left(4ac^{15}d + \frac{bc^{16}}{4}\right)$$

input
```
integrate(x**2*(b*x+a)*(d*x+c)**16,x)
```

output
```
a*c**16*x**3/3 + b*d**16*x**20/20 + x**19*(a*d**16/19 + 16*b*c*d**15/19) +
 x**18*(8*a*c*d**15/9 + 20*b*c**2*d**14/3) + x**17*(120*a*c**2*d**14/17 +
560*b*c**3*d**13/17) + x**16*(35*a*c**3*d**13 + 455*b*c**4*d**12/4) + x**1
5*(364*a*c**4*d**12/3 + 1456*b*c**5*d**11/5) + x**14*(312*a*c**5*d**11 + 5
72*b*c**6*d**10) + x**13*(616*a*c**6*d**10 + 880*b*c**7*d**9) + x**12*(286
0*a*c**7*d**9/3 + 2145*b*c**8*d**8/2) + x**11*(1170*a*c**8*d**8 + 1040*b*c
**9*d**7) + x**10*(1144*a*c**9*d**7 + 4004*b*c**10*d**6/5) + x**9*(8008*a*
c**10*d**6/9 + 1456*b*c**11*d**5/3) + x**8*(546*a*c**11*d**5 + 455*b*c**12
*d**4/2) + x**7*(260*a*c**12*d**4 + 80*b*c**13*d**3) + x**6*(280*a*c**13*d
**3/3 + 20*b*c**14*d**2) + x**5*(24*a*c**14*d**2 + 16*b*c**15*d/5) + x**4*
(4*a*c**15*d + b*c**16/4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(80) = 160$.

Time = 0.03 (sec) , antiderivative size = 387, normalized size of antiderivative = 4.40

$$\int x^2(a+bx)(c+dx)^{16}\,dx = \frac{1}{20}\,bd^{16}x^{20} + \frac{1}{3}\,ac^{16}x^3 + \frac{1}{19}\left(16\,bcd^{15} + ad^{16}\right)x^{19} + \frac{4}{9}\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{18} + \frac{40}{17}\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{17} + \frac{35}{4}\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{16} + \frac{364}{15}\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{15} + 52\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{14} + 88\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{13} + \frac{715}{6}\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{12} + 130\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^{11} + \frac{572}{5}\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^{10} + \frac{728}{9}\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^9 + \frac{91}{2}\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^8 + 20\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^7 + \frac{20}{3}\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^6 + \frac{8}{5}\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^5 + \frac{1}{4}\left(bc^{16} + 16\,ac^{15}d\right)x^4$$

input
```
integrate(x^2*(b*x+a)*(d*x+c)^16,x, algorithm="maxima")
```

output
```
1/20*b*d^16*x^20 + 1/3*a*c^16*x^3 + 1/19*(16*b*c*d^15 + a*d^16)*x^19 + 4/9
*(15*b*c^2*d^14 + 2*a*c*d^15)*x^18 + 40/17*(14*b*c^3*d^13 + 3*a*c^2*d^14)*
x^17 + 35/4*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^16 + 364/15*(12*b*c^5*d^11 +
5*a*c^4*d^12)*x^15 + 52*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^14 + 88*(10*b*c^7
*d^9 + 7*a*c^6*d^10)*x^13 + 715/6*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^12 + 130*(
8*b*c^9*d^7 + 9*a*c^8*d^8)*x^11 + 572/5*(7*b*c^10*d^6 + 10*a*c^9*d^7)*x^10
 + 728/9*(6*b*c^11*d^5 + 11*a*c^10*d^6)*x^9 + 91/2*(5*b*c^12*d^4 + 12*a*c^
11*d^5)*x^8 + 20*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^7 + 20/3*(3*b*c^14*d^2 +
 14*a*c^13*d^3)*x^6 + 8/5*(2*b*c^15*d + 15*a*c^14*d^2)*x^5 + 1/4*(b*c^16 +
 16*a*c^15*d)*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.42

$$\begin{aligned}\int x^2(a+bx)(c+dx)^{16}\,dx =& \frac{1}{20}\,bd^{16}x^{20} + \frac{16}{19}\,bcd^{15}x^{19} + \frac{1}{19}\,ad^{16}x^{19} + \frac{20}{3}\,bc^2d^{14}x^{18} \\ &+ \frac{8}{9}\,acd^{15}x^{18} + \frac{560}{17}\,bc^3d^{13}x^{17} + \frac{120}{17}\,ac^2d^{14}x^{17} \\ &+ \frac{455}{4}\,bc^4d^{12}x^{16} + 35\,ac^3d^{13}x^{16} + \frac{1456}{5}\,bc^5d^{11}x^{15} \\ &+ \frac{364}{3}\,ac^4d^{12}x^{15} + 572\,bc^6d^{10}x^{14} + 312\,ac^5d^{11}x^{14} \\ &+ 880\,bc^7d^9x^{13} + 616\,ac^6d^{10}x^{13} + \frac{2145}{2}\,bc^8d^8x^{12} \\ &+ \frac{2860}{3}\,ac^7d^9x^{12} + 1040\,bc^9d^7x^{11} + 1170\,ac^8d^8x^{11} \\ &+ \frac{4004}{5}\,bc^{10}d^6x^{10} + 1144\,ac^9d^7x^{10} + \frac{1456}{3}\,bc^{11}d^5x^9 \\ &+ \frac{8008}{9}\,ac^{10}d^6x^9 + \frac{455}{2}\,bc^{12}d^4x^8 + 546\,ac^{11}d^5x^8 \\ &+ 80\,bc^{13}d^3x^7 + 260\,ac^{12}d^4x^7 + 20\,bc^{14}d^2x^6 + \frac{280}{3}\,ac^{13}d^3x^6 \\ &+ \frac{16}{5}\,bc^{15}dx^5 + 24\,ac^{14}d^2x^5 + \frac{1}{4}\,bc^{16}x^4 + 4\,ac^{15}dx^4 + \frac{1}{3}\,ac^{16}x^3\end{aligned}$$

input
```
integrate(x^2*(b*x+a)*(d*x+c)^16,x, algorithm="giac")
```

output
```
1/20*b*d^16*x^20 + 16/19*b*c*d^15*x^19 + 1/19*a*d^16*x^19 + 20/3*b*c^2*d^1
4*x^18 + 8/9*a*c*d^15*x^18 + 560/17*b*c^3*d^13*x^17 + 120/17*a*c^2*d^14*x^
17 + 455/4*b*c^4*d^12*x^16 + 35*a*c^3*d^13*x^16 + 1456/5*b*c^5*d^11*x^15 +
 364/3*a*c^4*d^12*x^15 + 572*b*c^6*d^10*x^14 + 312*a*c^5*d^11*x^14 + 880*b
*c^7*d^9*x^13 + 616*a*c^6*d^10*x^13 + 2145/2*b*c^8*d^8*x^12 + 2860/3*a*c^7
*d^9*x^12 + 1040*b*c^9*d^7*x^11 + 1170*a*c^8*d^8*x^11 + 4004/5*b*c^10*d^6*
x^10 + 1144*a*c^9*d^7*x^10 + 1456/3*b*c^11*d^5*x^9 + 8008/9*a*c^10*d^6*x^9
 + 455/2*b*c^12*d^4*x^8 + 546*a*c^11*d^5*x^8 + 80*b*c^13*d^3*x^7 + 260*a*c
^12*d^4*x^7 + 20*b*c^14*d^2*x^6 + 280/3*a*c^13*d^3*x^6 + 16/5*b*c^15*d*x^5
 + 24*a*c^14*d^2*x^5 + 1/4*b*c^16*x^4 + 4*a*c^15*d*x^4 + 1/3*a*c^16*x^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.76

$$\int x^2(a+bx)(c+dx)^{16}\,dx = x^4\left(\frac{b\,c^{16}}{4}+4\,a\,d\,c^{15}\right)+x^{19}\left(\frac{a\,d^{16}}{19}+\frac{16\,b\,c\,d^{15}}{19}\right)+\frac{a\,c^{16}\,x^3}{3}+\frac{b\,d^{16}\,x^{20}}{20}+\frac{8\,c^{14}\,d\,x^5\,(15\,a\,d+2\,b\,c)}{5}+\frac{4\,c\,d^{14}\,x^{18}\,(2\,a\,d+15\,b\,c)}{9}+\frac{20\,c^{13}\,d^2\,x^6\,(14\,a\,d+3\,b\,c)}{3}+20\,c^{12}\,d^3\,x^7\,(13\,a\,d+4\,b\,c)+\frac{91\,c^{11}\,d^4\,x^8\,(12\,a\,d+5\,b\,c)}{2}+\frac{728\,c^{10}\,d^5\,x^9\,(11\,a\,d+6\,b\,c)}{9}+\frac{572\,c^9\,d^6\,x^{10}\,(10\,a\,d+7\,b\,c)}{5}+130\,c^8\,d^7\,x^{11}\,(9\,a\,d+8\,b\,c)+\frac{715\,c^7\,d^8\,x^{12}\,(8\,a\,d+9\,b\,c)}{6}+88\,c^6\,d^9\,x^{13}\,(7\,a\,d+10\,b\,c)+52\,c^5\,d^{10}\,x^{14}\,(6\,a\,d+11\,b\,c)+\frac{364\,c^4\,d^{11}\,x^{15}\,(5\,a\,d+12\,b\,c)}{15}+\frac{35\,c^3\,d^{12}\,x^{16}\,(4\,a\,d+13\,b\,c)}{4}+\frac{40\,c^2\,d^{13}\,x^{17}\,(3\,a\,d+14\,b\,c)}{17}$$

input
```
int(x^2*(a + b*x)*(c + d*x)^16,x)
```

output
```
x^4*((b*c^16)/4 + 4*a*c^15*d) + x^19*((a*d^16)/19 + (16*b*c*d^15)/19) + (a
*c^16*x^3)/3 + (b*d^16*x^20)/20 + (8*c^14*d*x^5*(15*a*d + 2*b*c))/5 + (4*c
*d^14*x^18*(2*a*d + 15*b*c))/9 + (20*c^13*d^2*x^6*(14*a*d + 3*b*c))/3 + 20
*c^12*d^3*x^7*(13*a*d + 4*b*c) + (91*c^11*d^4*x^8*(12*a*d + 5*b*c))/2 + (7
28*c^10*d^5*x^9*(11*a*d + 6*b*c))/9 + (572*c^9*d^6*x^10*(10*a*d + 7*b*c))/
5 + 130*c^8*d^7*x^11*(9*a*d + 8*b*c) + (715*c^7*d^8*x^12*(8*a*d + 9*b*c))/
6 + 88*c^6*d^9*x^13*(7*a*d + 10*b*c) + 52*c^5*d^10*x^14*(6*a*d + 11*b*c) +
 (364*c^4*d^11*x^15*(5*a*d + 12*b*c))/15 + (35*c^3*d^12*x^16*(4*a*d + 13*b
*c))/4 + (40*c^2*d^13*x^17*(3*a*d + 14*b*c))/17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 387, normalized size of antiderivative = 4.40

$$\int x^2(a+bx)(c+dx)^{16}\,dx = \frac{x^3(2907b\,d^{16}x^{17}+3060a\,d^{16}x^{16}+48960bc\,d^{15}x^{16}+51680ac\,d^{15}x^{15}+387600b\,c^2d^{14}x^{15}+410400a\,c^2d^{14}x}{}$$

input
```
int(x^2*(b*x+a)*(d*x+c)^16,x)
```

output
```
(x**3*(19380*a*c**16 + 232560*a*c**15*d*x + 1395360*a*c**14*d**2*x**2 + 54
26400*a*c**13*d**3*x**3 + 15116400*a*c**12*d**4*x**4 + 31744440*a*c**11*d*
*5*x**5 + 51731680*a*c**10*d**6*x**6 + 66512160*a*c**9*d**7*x**7 + 6802380
0*a*c**8*d**8*x**8 + 55426800*a*c**7*d**9*x**9 + 35814240*a*c**6*d**10*x**
10 + 18139680*a*c**5*d**11*x**11 + 7054320*a*c**4*d**12*x**12 + 2034900*a*
c**3*d**13*x**13 + 410400*a*c**2*d**14*x**14 + 51680*a*c*d**15*x**15 + 306
0*a*d**16*x**16 + 14535*b*c**16*x + 186048*b*c**15*d*x**2 + 1162800*b*c**1
4*d**2*x**3 + 4651200*b*c**13*d**3*x**4 + 13226850*b*c**12*d**4*x**5 + 282
17280*b*c**11*d**5*x**6 + 46558512*b*c**10*d**6*x**7 + 60465600*b*c**9*d**
7*x**8 + 62355150*b*c**8*d**8*x**9 + 51163200*b*c**7*d**9*x**10 + 33256080
*b*c**6*d**10*x**11 + 16930368*b*c**5*d**11*x**12 + 6613425*b*c**4*d**12*x
**13 + 1915200*b*c**3*d**13*x**14 + 387600*b*c**2*d**14*x**15 + 48960*b*c*
d**15*x**16 + 2907*b*d**16*x**17))/58140
```

3.140 $\int x(a+bx)(c+dx)^{16}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int x(a+bx)(c+dx)^{16}\,dx=\frac{c(bc-ad)(c+dx)^{17}}{17d^3}-\frac{(2bc-ad)(c+dx)^{18}}{18d^3}+\frac{b(c+dx)^{19}}{19d^3}$$

output
```
1/17*c*(-a*d+b*c)*(d*x+c)^17/d^3-1/18*(-a*d+2*b*c)*(d*x+c)^18/d^3+1/19*b*(
d*x+c)^19/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 347 vs. $2(62) = 124$.

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.60

$$\begin{aligned}\int x(a+bx)(c+dx)^{16}\,dx = {} & \frac{1}{2}ac^{16}x^2+\frac{1}{3}c^{15}(bc+16ad)x^3+2c^{14}d(2bc+15ad)x^4 \\ & +8c^{13}d^2(3bc+14ad)x^5+\frac{70}{3}c^{12}d^3(4bc+13ad)x^6 \\ & +52c^{11}d^4(5bc+12ad)x^7+91c^{10}d^5(6bc+11ad)x^8 \\ & +\frac{1144}{9}c^9d^6(7bc+10ad)x^9+143c^8d^7(8bc+9ad)x^{10} \\ & +130c^7d^8(9bc+8ad)x^{11}+\frac{286}{3}c^6d^9(10bc+7ad)x^{12} \\ & +56c^5d^{10}(11bc+6ad)x^{13}+26c^4d^{11}(12bc+5ad)x^{14} \\ & +\frac{28}{3}c^3d^{12}(13bc+4ad)x^{15}+\frac{5}{2}c^2d^{13}(14bc+3ad)x^{16} \\ & +\frac{8}{17}cd^{14}(15bc+2ad)x^{17}+\frac{1}{18}d^{15}(16bc+ad)x^{18}+\frac{1}{19}bd^{16}x^{19}\end{aligned}$$

input
```
Integrate[x*(a + b*x)*(c + d*x)^16,x]
```

output
```
(a*c^16*x^2)/2 + (c^15*(b*c + 16*a*d)*x^3)/3 + 2*c^14*d*(2*b*c + 15*a*d)*x
^4 + 8*c^13*d^2*(3*b*c + 14*a*d)*x^5 + (70*c^12*d^3*(4*b*c + 13*a*d)*x^6)/
3 + 52*c^11*d^4*(5*b*c + 12*a*d)*x^7 + 91*c^10*d^5*(6*b*c + 11*a*d)*x^8 +
(1144*c^9*d^6*(7*b*c + 10*a*d)*x^9)/9 + 143*c^8*d^7*(8*b*c + 9*a*d)*x^10 +
 130*c^7*d^8*(9*b*c + 8*a*d)*x^11 + (286*c^6*d^9*(10*b*c + 7*a*d)*x^12)/3
+ 56*c^5*d^10*(11*b*c + 6*a*d)*x^13 + 26*c^4*d^11*(12*b*c + 5*a*d)*x^14 +
(28*c^3*d^12*(13*b*c + 4*a*d)*x^15)/3 + (5*c^2*d^13*(14*b*c + 3*a*d)*x^16)
/2 + (8*c*d^14*(15*b*c + 2*a*d)*x^17)/17 + (d^15*(16*b*c + a*d)*x^18)/18 +
 (b*d^16*x^19)/19
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)(c+dx)^{16}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{(c+dx)^{17}(ad-2bc)}{d^2}+\frac{c(c+dx)^{16}(bc-ad)}{d^2}+\frac{b(c+dx)^{18}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^{18}(2bc-ad)}{18d^3}+\frac{c(c+dx)^{17}(bc-ad)}{17d^3}+\frac{b(c+dx)^{19}}{19d^3}$$

input
```
Int[x*(a + b*x)*(c + d*x)^16,x]
```

output
```
(c*(b*c - a*d)*(c + d*x)^17)/(17*d^3) - ((2*b*c - a*d)*(c + d*x)^18)/(18*d
^3) + (b*(c + d*x)^19)/(19*d^3)
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 6.03

method	result
norman	$\frac{a\,c^{16}x^2}{2}+\left(\frac{16}{3}a\,c^{15}d+\frac{1}{3}b\,c^{16}\right)x^3+\left(30a\,c^{14}d^2+4b\,c^{15}d\right)x^4+\left(112a\,c^{13}d^3+24b\,c^{14}d^2\right)x^5+\left(\right.$
default	$\frac{b\,d^{16}x^{19}}{19}+\frac{\left(a\,d^{16}+16bc\,d^{15}\right)x^{18}}{18}+\frac{\left(16ac\,d^{15}+120b\,c^2d^{14}\right)x^{17}}{17}+\frac{\left(120a\,c^2d^{14}+560b\,c^3d^{13}\right)x^{16}}{16}+\frac{\left(560a\,c^3d^{13}+1820b\,c\right.}{15}$
orering	$\frac{x^2\left(306b\,d^{16}x^{17}+323a\,d^{16}x^{16}+5168bc\,d^{15}x^{16}+5472ac\,d^{15}x^{15}+41040b\,c^2d^{14}x^{15}+43605a\,c^2d^{14}x^{14}+203490b\,c^3d^{13}x^{14}+2170\right.}{}$
gosper	$30a\,c^{14}d^2x^4+4b\,c^{15}d\,x^4+112a\,c^{13}d^3x^5+24b\,c^{14}d^2x^5+624a\,c^{11}d^5x^7+260b\,c^{12}d^4x^7+100$
risch	$30a\,c^{14}d^2x^4+4b\,c^{15}d\,x^4+112a\,c^{13}d^3x^5+24b\,c^{14}d^2x^5+624a\,c^{11}d^5x^7+260b\,c^{12}d^4x^7+100$
parallelrisch	$30a\,c^{14}d^2x^4+4b\,c^{15}d\,x^4+112a\,c^{13}d^3x^5+24b\,c^{14}d^2x^5+624a\,c^{11}d^5x^7+260b\,c^{12}d^4x^7+100$

input

```
int(x*(b*x+a)*(d*x+c)^16,x,method=_RETURNVERBOSE)
```

output

```
1/2*a*c^16*x^2+(16/3*a*c^15*d+1/3*b*c^16)*x^3+(30*a*c^14*d^2+4*b*c^15*d)*x
^4+(112*a*c^13*d^3+24*b*c^14*d^2)*x^5+(910/3*a*c^12*d^4+280/3*b*c^13*d^3)*
x^6+(624*a*c^11*d^5+260*b*c^12*d^4)*x^7+(1001*a*c^10*d^6+546*b*c^11*d^5)*x
^8+(11440/9*a*c^9*d^7+8008/9*b*c^10*d^6)*x^9+(1287*a*c^8*d^8+1144*b*c^9*d^
7)*x^10+(1040*a*c^7*d^9+1170*b*c^8*d^8)*x^11+(2002/3*a*c^6*d^10+2860/3*b*c
^7*d^9)*x^12+(336*a*c^5*d^11+616*b*c^6*d^10)*x^13+(130*a*c^4*d^12+312*b*c^
5*d^11)*x^14+(112/3*a*c^3*d^13+364/3*b*c^4*d^12)*x^15+(15/2*a*c^2*d^14+35*
b*c^3*d^13)*x^16+(16/17*a*c*d^15+120/17*b*c^2*d^14)*x^17+(1/18*a*d^16+8/9*
b*c*d^15)*x^18+1/19*b*d^16*x^19
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(56) = 112$.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 6.24

$$\int x(a+bx)(c+dx)^{16}\,dx = \frac{1}{19}bd^{16}x^{19} + \frac{1}{2}ac^{16}x^2 + \frac{1}{18}\left(16\,bcd^{15} + ad^{16}\right)x^{18} + \frac{8}{17}\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{17} + \frac{5}{2}\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{16} + \frac{28}{3}\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{15} + 26\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{14} + 56\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{13} + \frac{286}{3}\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{12} + 130\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{11} + 143\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^{10} + \frac{1144}{9}\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^9 + 91\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^8 + 52\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^7 + \frac{70}{3}\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^6 + 8\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^5 + 2\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^4 + \frac{1}{3}\left(bc^{16} + 16\,ac^{15}d\right)x^3$$

input

```
integrate(x*(b*x+a)*(d*x+c)^16,x, algorithm="fricas")
```

output

```
1/19*b*d^16*x^19 + 1/2*a*c^16*x^2 + 1/18*(16*b*c*d^15 + a*d^16)*x^18 + 8/1
7*(15*b*c^2*d^14 + 2*a*c*d^15)*x^17 + 5/2*(14*b*c^3*d^13 + 3*a*c^2*d^14)*x
^16 + 28/3*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^15 + 26*(12*b*c^5*d^11 + 5*a*c
^4*d^12)*x^14 + 56*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^13 + 286/3*(10*b*c^7*d
^9 + 7*a*c^6*d^10)*x^12 + 130*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^11 + 143*(8*b*
c^9*d^7 + 9*a*c^8*d^8)*x^10 + 1144/9*(7*b*c^10*d^6 + 10*a*c^9*d^7)*x^9 + 9
1*(6*b*c^11*d^5 + 11*a*c^10*d^6)*x^8 + 52*(5*b*c^12*d^4 + 12*a*c^11*d^5)*x
^7 + 70/3*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^6 + 8*(3*b*c^14*d^2 + 14*a*c^13
*d^3)*x^5 + 2*(2*b*c^15*d + 15*a*c^14*d^2)*x^4 + 1/3*(b*c^16 + 16*a*c^15*d
)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(53) = 106$.

Time = 0.06 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.58

$$\int x(a+bx)(c+dx)^{16}\,dx = \frac{ac^{16}x^2}{2} + \frac{bd^{16}x^{19}}{19} + x^{18}\left(\frac{ad^{16}}{18} + \frac{8bcd^{15}}{9}\right) + x^{17} \cdot \left(\frac{16acd^{15}}{17} + \frac{120bc^2d^{14}}{17}\right) + x^{16} \cdot \left(\frac{15ac^2d^{14}}{2} + 35bc^3d^{13}\right) + x^{15} \cdot \left(\frac{112ac^3d^{13}}{3} + \frac{364bc^4d^{12}}{3}\right) + x^{14} \cdot \left(130ac^4d^{12} + 312bc^5d^{11}\right) + x^{13} \cdot \left(336ac^5d^{11} + 616bc^6d^{10}\right) + x^{12} \cdot \left(\frac{2002ac^6d^{10}}{3} + \frac{2860bc^7d^9}{3}\right) + x^{11} \cdot \left(1040ac^7d^9 + 1170bc^8d^8\right) + x^{10} \cdot \left(1287ac^8d^8 + 1144bc^9d^7\right) + x^9 \cdot \left(\frac{11440ac^9d^7}{9} + \frac{8008bc^{10}d^6}{9}\right) + x^8 \cdot \left(1001ac^{10}d^6 + 546bc^{11}d^5\right) + x^7 \cdot \left(624ac^{11}d^5 + 260bc^{12}d^4\right) + x^6 \cdot \left(\frac{910ac^{12}d^4}{3} + \frac{280bc^{13}d^3}{3}\right) + x^5 \cdot \left(112ac^{13}d^3 + 24bc^{14}d^2\right) + x^4 \cdot \left(30ac^{14}d^2 + 4bc^{15}d\right) + x^3 \cdot \left(\frac{16ac^{15}d}{3} + \frac{bc^{16}}{3}\right)$$

input
```
integrate(x*(b*x+a)*(d*x+c)**16,x)
```

output
```
a*c**16*x**2/2 + b*d**16*x**19/19 + x**18*(a*d**16/18 + 8*b*c*d**15/9) + x
**17*(16*a*c*d**15/17 + 120*b*c**2*d**14/17) + x**16*(15*a*c**2*d**14/2 +
35*b*c**3*d**13) + x**15*(112*a*c**3*d**13/3 + 364*b*c**4*d**12/3) + x**14
*(130*a*c**4*d**12 + 312*b*c**5*d**11) + x**13*(336*a*c**5*d**11 + 616*b*c
**6*d**10) + x**12*(2002*a*c**6*d**10/3 + 2860*b*c**7*d**9/3) + x**11*(104
0*a*c**7*d**9 + 1170*b*c**8*d**8) + x**10*(1287*a*c**8*d**8 + 1144*b*c**9*
d**7) + x**9*(11440*a*c**9*d**7/9 + 8008*b*c**10*d**6/9) + x**8*(1001*a*c*
*10*d**6 + 546*b*c**11*d**5) + x**7*(624*a*c**11*d**5 + 260*b*c**12*d**4)
+ x**6*(910*a*c**12*d**4/3 + 280*b*c**13*d**3/3) + x**5*(112*a*c**13*d**3
+ 24*b*c**14*d**2) + x**4*(30*a*c**14*d**2 + 4*b*c**15*d) + x**3*(16*a*c**
15*d/3 + b*c**16/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(56) = 112$.

Time = 0.06 (sec) , antiderivative size = 387, normalized size of antiderivative = 6.24

$$\int x(a+bx)(c+dx)^{16}\,dx = \frac{1}{19}\,bd^{16}x^{19} + \frac{1}{2}\,ac^{16}x^2 + \frac{1}{18}\,\left(16\,bcd^{15} + ad^{16}\right)x^{18} + \frac{8}{17}\,\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{17} + \frac{5}{2}\,\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{16} + \frac{28}{3}\,\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{15} + 26\,\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{14} + 56\,\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{13} + \frac{286}{3}\,\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{12} + 130\,\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{11} + 143\,\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^{10} + \frac{1144}{9}\,\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^9 + 91\,\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^8 + 52\,\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^7 + \frac{70}{3}\,\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^6 + 8\,\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^5 + 2\,\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^4 + \frac{1}{3}\,\left(bc^{16} + 16\,ac^{15}d\right)x^3$$

input

```
integrate(x*(b*x+a)*(d*x+c)^16,x, algorithm="maxima")
```

output

```
1/19*b*d^16*x^19 + 1/2*a*c^16*x^2 + 1/18*(16*b*c*d^15 + a*d^16)*x^18 + 8/1
7*(15*b*c^2*d^14 + 2*a*c*d^15)*x^17 + 5/2*(14*b*c^3*d^13 + 3*a*c^2*d^14)*x
^16 + 28/3*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^15 + 26*(12*b*c^5*d^11 + 5*a*c
^4*d^12)*x^14 + 56*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^13 + 286/3*(10*b*c^7*d
^9 + 7*a*c^6*d^10)*x^12 + 130*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^11 + 143*(8*b*
c^9*d^7 + 9*a*c^8*d^8)*x^10 + 1144/9*(7*b*c^10*d^6 + 10*a*c^9*d^7)*x^9 + 9
1*(6*b*c^11*d^5 + 11*a*c^10*d^6)*x^8 + 52*(5*b*c^12*d^4 + 12*a*c^11*d^5)*x
^7 + 70/3*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^6 + 8*(3*b*c^14*d^2 + 14*a*c^13
*d^3)*x^5 + 2*(2*b*c^15*d + 15*a*c^14*d^2)*x^4 + 1/3*(b*c^16 + 16*a*c^15*d
)*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 389, normalized size of antiderivative = 6.27

$$\int x(a+bx)(c+dx)^{16}\,dx = \frac{1}{19}bd^{16}x^{19} + \frac{8}{9}bcd^{15}x^{18} + \frac{1}{18}ad^{16}x^{18} + \frac{120}{17}bc^2d^{14}x^{17} + \frac{16}{17}acd^{15}x^{17} + 35bc^3d^{13}x^{16} + \frac{15}{2}ac^2d^{14}x^{16} + \frac{364}{3}bc^4d^{12}x^{15} + \frac{112}{3}ac^3d^{13}x^{15} + 312bc^5d^{11}x^{14} + 130ac^4d^{12}x^{14} + 616bc^6d^{10}x^{13} + 336ac^5d^{11}x^{13} + \frac{2860}{3}bc^7d^9x^{12} + \frac{2002}{3}ac^6d^{10}x^{12} + 1170bc^8d^8x^{11} + 1040ac^7d^9x^{11} + 1144bc^9d^7x^{10} + 1287ac^8d^8x^{10} + \frac{8008}{9}bc^{10}d^6x^9 + \frac{11440}{9}ac^9d^7x^9 + 546bc^{11}d^5x^8 + 1001ac^{10}d^6x^8 + 260bc^{12}d^4x^7 + 624ac^{11}d^5x^7 + \frac{280}{3}bc^{13}d^3x^6 + \frac{910}{3}ac^{12}d^4x^6 + 24bc^{14}d^2x^5 + 112ac^{13}d^3x^5 + 4bc^{15}dx^4 + 30ac^{14}d^2x^4 + \frac{1}{3}bc^{16}x^3 + \frac{16}{3}ac^{15}dx^3 + \frac{1}{2}ac^{16}x^2$$

input
```
integrate(x*(b*x+a)*(d*x+c)^16,x, algorithm="giac")
```

output
```
1/19*b*d^16*x^19 + 8/9*b*c*d^15*x^18 + 1/18*a*d^16*x^18 + 120/17*b*c^2*d^1
4*x^17 + 16/17*a*c*d^15*x^17 + 35*b*c^3*d^13*x^16 + 15/2*a*c^2*d^14*x^16 +
 364/3*b*c^4*d^12*x^15 + 112/3*a*c^3*d^13*x^15 + 312*b*c^5*d^11*x^14 + 130
*a*c^4*d^12*x^14 + 616*b*c^6*d^10*x^13 + 336*a*c^5*d^11*x^13 + 2860/3*b*c^
7*d^9*x^12 + 2002/3*a*c^6*d^10*x^12 + 1170*b*c^8*d^8*x^11 + 1040*a*c^7*d^9
*x^11 + 1144*b*c^9*d^7*x^10 + 1287*a*c^8*d^8*x^10 + 8008/9*b*c^10*d^6*x^9
+ 11440/9*a*c^9*d^7*x^9 + 546*b*c^11*d^5*x^8 + 1001*a*c^10*d^6*x^8 + 260*b
*c^12*d^4*x^7 + 624*a*c^11*d^5*x^7 + 280/3*b*c^13*d^3*x^6 + 910/3*a*c^12*d
^4*x^6 + 24*b*c^14*d^2*x^5 + 112*a*c^13*d^3*x^5 + 4*b*c^15*d*x^4 + 30*a*c^
14*d^2*x^4 + 1/3*b*c^16*x^3 + 16/3*a*c^15*d*x^3 + 1/2*a*c^16*x^2
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.34

$$\int x(a+bx)(c+dx)^{16}\,dx = x^3\left(\frac{b\,c^{16}}{3}+\frac{16\,a\,d\,c^{15}}{3}\right)+x^{18}\left(\frac{a\,d^{16}}{18}+\frac{8\,b\,c\,d^{15}}{9}\right)+\frac{a\,c^{16}\,x^2}{2}+\frac{b\,d^{16}\,x^{19}}{19}+2\,c^{14}\,d\,x^4\,(15\,a\,d+2\,b\,c)+\frac{8\,c\,d^{14}\,x^{17}\,(2\,a\,d+15\,b\,c)}{17}+8\,c^{13}\,d^2\,x^5\,(14\,a\,d+3\,b\,c)+\frac{70\,c^{12}\,d^3\,x^6\,(13\,a\,d+4\,b\,c)}{3}+52\,c^{11}\,d^4\,x^7\,(12\,a\,d+5\,b\,c)+91\,c^{10}\,d^5\,x^8\,(11\,a\,d+6\,b\,c)+\frac{1144\,c^9\,d^6\,x^9\,(10\,a\,d+7\,b\,c)}{9}+143\,c^8\,d^7\,x^{10}\,(9\,a\,d+8\,b\,c)+130\,c^7\,d^8\,x^{11}\,(8\,a\,d+9\,b\,c)+\frac{286\,c^6\,d^9\,x^{12}\,(7\,a\,d+10\,b\,c)}{3}+56\,c^5\,d^{10}\,x^{13}\,(6\,a\,d+11\,b\,c)+26\,c^4\,d^{11}\,x^{14}\,(5\,a\,d+12\,b\,c)+\frac{28\,c^3\,d^{12}\,x^{15}\,(4\,a\,d+13\,b\,c)}{3}+\frac{5\,c^2\,d^{13}\,x^{16}\,(3\,a\,d+14\,b\,c)}{2}$$

input
```
int(x*(a + b*x)*(c + d*x)^16,x)
```

output
```
x^3*((b*c^16)/3 + (16*a*c^15*d)/3) + x^18*((a*d^16)/18 + (8*b*c*d^15)/9) +
 (a*c^16*x^2)/2 + (b*d^16*x^19)/19 + 2*c^14*d*x^4*(15*a*d + 2*b*c) + (8*c*
d^14*x^17*(2*a*d + 15*b*c))/17 + 8*c^13*d^2*x^5*(14*a*d + 3*b*c) + (70*c^1
2*d^3*x^6*(13*a*d + 4*b*c))/3 + 52*c^11*d^4*x^7*(12*a*d + 5*b*c) + 91*c^10
*d^5*x^8*(11*a*d + 6*b*c) + (1144*c^9*d^6*x^9*(10*a*d + 7*b*c))/9 + 143*c^
8*d^7*x^10*(9*a*d + 8*b*c) + 130*c^7*d^8*x^11*(8*a*d + 9*b*c) + (286*c^6*d
^9*x^12*(7*a*d + 10*b*c))/3 + 56*c^5*d^10*x^13*(6*a*d + 11*b*c) + 26*c^4*d
^11*x^14*(5*a*d + 12*b*c) + (28*c^3*d^12*x^15*(4*a*d + 13*b*c))/3 + (5*c^2
*d^13*x^16*(3*a*d + 14*b*c))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 387, normalized size of antiderivative = 6.24

$$\int x(a+bx)(c+dx)^{16}\,dx$$
$$= \frac{x^2(306b\,d^{16}x^{17}+323a\,d^{16}x^{16}+5168bc\,d^{15}x^{16}+5472ac\,d^{15}x^{15}+41040b\,c^2d^{14}x^{15}+43605a\,c^2d^{14}x^{14}+20}{}$$

input
```
int(x*(b*x+a)*(d*x+c)^16,x)
```

output
```
(x**2*(2907*a*c**16 + 31008*a*c**15*d*x + 174420*a*c**14*d**2*x**2 + 65116
8*a*c**13*d**3*x**3 + 1763580*a*c**12*d**4*x**4 + 3627936*a*c**11*d**5*x**
5 + 5819814*a*c**10*d**6*x**6 + 7390240*a*c**9*d**7*x**7 + 7482618*a*c**8*
d**8*x**8 + 6046560*a*c**7*d**9*x**9 + 3879876*a*c**6*d**10*x**10 + 195350
4*a*c**5*d**11*x**11 + 755820*a*c**4*d**12*x**12 + 217056*a*c**3*d**13*x**
13 + 43605*a*c**2*d**14*x**14 + 5472*a*c*d**15*x**15 + 323*a*d**16*x**16 +
 1938*b*c**16*x + 23256*b*c**15*d*x**2 + 139536*b*c**14*d**2*x**3 + 542640
*b*c**13*d**3*x**4 + 1511640*b*c**12*d**4*x**5 + 3174444*b*c**11*d**5*x**6
 + 5173168*b*c**10*d**6*x**7 + 6651216*b*c**9*d**7*x**8 + 6802380*b*c**8*d
**8*x**9 + 5542680*b*c**7*d**9*x**10 + 3581424*b*c**6*d**10*x**11 + 181396
8*b*c**5*d**11*x**12 + 705432*b*c**4*d**12*x**13 + 203490*b*c**3*d**13*x**
14 + 41040*b*c**2*d**14*x**15 + 5168*b*c*d**15*x**16 + 306*b*d**16*x**17))
/5814
```

3.141 $\int (a+bx)(c+dx)^{16}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int (a+bx)(c+dx)^{16}\,dx = -\frac{(bc-ad)(c+dx)^{17}}{17d^2}+\frac{b(c+dx)^{18}}{18d^2}$$

output
```
-1/17*(-a*d+b*c)*(d*x+c)^17/d^2+1/18*b*(d*x+c)^18/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 342, normalized size of antiderivative = 9.00

$$\int (a+bx)(c+dx)^{16}\,dx = ac^{16}x+\frac{1}{2}c^{15}(bc+16ad)x^2+\frac{8}{3}c^{14}d(2bc+15ad)x^3 +10c^{13}d^2(3bc+14ad)x^4+28c^{12}d^3(4bc+13ad)x^5 +\frac{182}{3}c^{11}d^4(5bc+12ad)x^6+104c^{10}d^5(6bc+11ad)x^7 +143c^9d^6(7bc+10ad)x^8+\frac{1430}{9}c^8d^7(8bc+9ad)x^9 +143c^7d^8(9bc+8ad)x^{10}+104c^6d^9(10bc+7ad)x^{11} +\frac{182}{3}c^5d^{10}(11bc+6ad)x^{12}+28c^4d^{11}(12bc+5ad)x^{13} +10c^3d^{12}(13bc+4ad)x^{14}+\frac{8}{3}c^2d^{13}(14bc+3ad)x^{15} +\frac{1}{2}cd^{14}(15bc+2ad)x^{16}+\frac{1}{17}d^{15}(16bc+ad)x^{17}+\frac{1}{18}bd^{16}x^{18}$$

input
```
Integrate[(a + b*x)*(c + d*x)^16,x]
```

output
```
a*c^16*x + (c^15*(b*c + 16*a*d)*x^2)/2 + (8*c^14*d*(2*b*c + 15*a*d)*x^3)/3
 + 10*c^13*d^2*(3*b*c + 14*a*d)*x^4 + 28*c^12*d^3*(4*b*c + 13*a*d)*x^5 + (
182*c^11*d^4*(5*b*c + 12*a*d)*x^6)/3 + 104*c^10*d^5*(6*b*c + 11*a*d)*x^7 +
 143*c^9*d^6*(7*b*c + 10*a*d)*x^8 + (1430*c^8*d^7*(8*b*c + 9*a*d)*x^9)/9 +
 143*c^7*d^8*(9*b*c + 8*a*d)*x^10 + 104*c^6*d^9*(10*b*c + 7*a*d)*x^11 + (1
82*c^5*d^10*(11*b*c + 6*a*d)*x^12)/3 + 28*c^4*d^11*(12*b*c + 5*a*d)*x^13 +
 10*c^3*d^12*(13*b*c + 4*a*d)*x^14 + (8*c^2*d^13*(14*b*c + 3*a*d)*x^15)/3
+ (c*d^14*(15*b*c + 2*a*d)*x^16)/2 + (d^15*(16*b*c + a*d)*x^17)/17 + (b*d^
16*x^18)/18
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^{16}\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(c+dx)^{16}(ad-bc)}{d}+\frac{b(c+dx)^{17}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(c+dx)^{18}}{18d^2}-\frac{(c+dx)^{17}(bc-ad)}{17d^2}$$

input
```
Int[(a + b*x)*(c + d*x)^16,x]
```

output
```
-1/17*((b*c - a*d)*(c + d*x)^17)/d^2 + (b*(c + d*x)^18)/(18*d^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(34) = 68.

Time = 0.11 (sec) , antiderivative size = 370, normalized size of antiderivative = 9.74

method	result
norman	$a\,c^{16}x+\left(8a\,c^{15}d+\frac{1}{2}b\,c^{16}\right)x^{2}+\left(40a\,c^{14}d^{2}+\frac{16}{3}b\,c^{15}d\right)x^{3}+\left(140a\,c^{13}d^{3}+30b\,c^{14}d^{2}\right)x^{4}+($
default	$\frac{b\,d^{16}x^{18}}{18}+\frac{\left(a\,d^{16}+16bc\,d^{15}\right)x^{17}}{17}+\frac{\left(16ac\,d^{15}+120b\,c^{2}d^{14}\right)x^{16}}{16}+\frac{\left(120a\,c^{2}d^{14}+560b\,c^{3}d^{13}\right)x^{15}}{15}+\frac{\left(560a\,c^{3}d^{13}+1820b\,c\right.}{14}$
gosper	$\frac{16}{17}x^{17}bc\,d^{15}+140a\,c^{13}d^{3}x^{4}+8x^{2}a\,c^{15}d+30b\,c^{14}d^{2}x^{4}+364a\,c^{12}d^{4}x^{5}+112b\,c^{13}d^{3}x^{5}+114$
risch	$\frac{16}{17}x^{17}bc\,d^{15}+140a\,c^{13}d^{3}x^{4}+8x^{2}a\,c^{15}d+30b\,c^{14}d^{2}x^{4}+364a\,c^{12}d^{4}x^{5}+112b\,c^{13}d^{3}x^{5}+114$
parallelrisch	$\frac{16}{17}x^{17}bc\,d^{15}+140a\,c^{13}d^{3}x^{4}+8x^{2}a\,c^{15}d+30b\,c^{14}d^{2}x^{4}+364a\,c^{12}d^{4}x^{5}+112b\,c^{13}d^{3}x^{5}+114$
orering	$\frac{x\left(17b\,d^{16}x^{17}+18a\,d^{16}x^{16}+288bc\,d^{15}x^{16}+306ac\,d^{15}x^{15}+2295b\,c^{2}d^{14}x^{15}+2448a\,c^{2}d^{14}x^{14}+11424b\,c^{3}d^{13}x^{14}+12240a\,c^{3}d^{13}x\right.}{}$

input
```
int((b*x+a)*(d*x+c)^16,x,method=_RETURNVERBOSE)
```

output
```
a*c^16*x+(8*a*c^15*d+1/2*b*c^16)*x^2+(40*a*c^14*d^2+16/3*b*c^15*d)*x^3+(14
0*a*c^13*d^3+30*b*c^14*d^2)*x^4+(364*a*c^12*d^4+112*b*c^13*d^3)*x^5+(728*a
*c^11*d^5+910/3*b*c^12*d^4)*x^6+(1144*a*c^10*d^6+624*b*c^11*d^5)*x^7+(1430
*a*c^9*d^7+1001*b*c^10*d^6)*x^8+(1430*a*c^8*d^8+11440/9*b*c^9*d^7)*x^9+(11
44*a*c^7*d^9+1287*b*c^8*d^8)*x^10+(728*a*c^6*d^10+1040*b*c^7*d^9)*x^11+(36
4*a*c^5*d^11+2002/3*b*c^6*d^10)*x^12+(140*a*c^4*d^12+336*b*c^5*d^11)*x^13+
(40*a*c^3*d^13+130*b*c^4*d^12)*x^14+(8*a*c^2*d^14+112/3*b*c^3*d^13)*x^15+(
a*c*d^15+15/2*b*c^2*d^14)*x^16+(1/17*a*d^16+16/17*b*c*d^15)*x^17+1/18*b*d^
16*x^18
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 384, normalized size of antiderivative = 10.11

$$\int (a+bx)(c+dx)^{16}\,dx = \frac{1}{18}\,bd^{16}x^{18} + ac^{16}x + \frac{1}{17}\,\left(16\,bcd^{15} + ad^{16}\right)x^{17} + \frac{1}{2}\,\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{16} + \frac{8}{3}\,\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{15} + 10\,\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{14} + 28\,\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{13} + \frac{182}{3}\,\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{12} + 104\,\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{11} + 143\,\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{10} + \frac{1430}{9}\,\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^9 + 143\,\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^8 + 104\,\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^7 + \frac{182}{3}\,\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^6 + 28\,\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^5 + 10\,\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^4 + \frac{8}{3}\,\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^3 + \frac{1}{2}\,\left(bc^{16} + 16\,ac^{15}d\right)x^2$$

input
```
integrate((b*x+a)*(d*x+c)^16,x, algorithm="fricas")
```

output
```
1/18*b*d^16*x^18 + a*c^16*x + 1/17*(16*b*c*d^15 + a*d^16)*x^17 + 1/2*(15*b
*c^2*d^14 + 2*a*c*d^15)*x^16 + 8/3*(14*b*c^3*d^13 + 3*a*c^2*d^14)*x^15 + 1
0*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^14 + 28*(12*b*c^5*d^11 + 5*a*c^4*d^12)*
x^13 + 182/3*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^12 + 104*(10*b*c^7*d^9 + 7*a
*c^6*d^10)*x^11 + 143*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^10 + 1430/9*(8*b*c^9*d
^7 + 9*a*c^8*d^8)*x^9 + 143*(7*b*c^10*d^6 + 10*a*c^9*d^7)*x^8 + 104*(6*b*c
^11*d^5 + 11*a*c^10*d^6)*x^7 + 182/3*(5*b*c^12*d^4 + 12*a*c^11*d^5)*x^6 +
28*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^5 + 10*(3*b*c^14*d^2 + 14*a*c^13*d^3)*
x^4 + 8/3*(2*b*c^15*d + 15*a*c^14*d^2)*x^3 + 1/2*(b*c^16 + 16*a*c^15*d)*x^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 393, normalized size of antiderivative = 10.34

$$\int (a+bx)(c+dx)^{16}\,dx = ac^{16}x + \frac{bd^{16}x^{18}}{18} + x^{17}\left(\frac{ad^{16}}{17} + \frac{16bcd^{15}}{17}\right) + x^{16}\left(acd^{15} + \frac{15bc^2d^{14}}{2}\right) + x^{15}\cdot\left(8ac^2d^{14} + \frac{112bc^3d^{13}}{3}\right) + x^{14}\cdot\left(40ac^3d^{13} + 130bc^4d^{12}\right) + x^{13}\cdot\left(140ac^4d^{12} + 336bc^5d^{11}\right) + x^{12}\cdot\left(364ac^5d^{11} + \frac{2002bc^6d^{10}}{3}\right) + x^{11}\cdot\left(728ac^6d^{10} + 1040bc^7d^9\right) + x^{10}\cdot\left(1144ac^7d^9 + 1287bc^8d^8\right) + x^9\cdot\left(1430ac^8d^8 + \frac{11440bc^9d^7}{9}\right) + x^8\cdot\left(1430ac^9d^7 + 1001bc^{10}d^6\right) + x^7\cdot\left(1144ac^{10}d^6 + 624bc^{11}d^5\right) + x^6\cdot\left(728ac^{11}d^5 + \frac{910bc^{12}d^4}{3}\right) + x^5\cdot\left(364ac^{12}d^4 + 112bc^{13}d^3\right) + x^4\cdot\left(140ac^{13}d^3 + 30bc^{14}d^2\right) + x^3\cdot\left(40ac^{14}d^2 + \frac{16bc^{15}d}{3}\right) + x^2\cdot\left(8ac^{15}d + \frac{bc^{16}}{2}\right)$$

input

```
integrate((b*x+a)*(d*x+c)**16,x)
```

output

```
a*c**16*x + b*d**16*x**18/18 + x**17*(a*d**16/17 + 16*b*c*d**15/17) + x**1
6*(a*c*d**15 + 15*b*c**2*d**14/2) + x**15*(8*a*c**2*d**14 + 112*b*c**3*d**
13/3) + x**14*(40*a*c**3*d**13 + 130*b*c**4*d**12) + x**13*(140*a*c**4*d**
12 + 336*b*c**5*d**11) + x**12*(364*a*c**5*d**11 + 2002*b*c**6*d**10/3) +
x**11*(728*a*c**6*d**10 + 1040*b*c**7*d**9) + x**10*(1144*a*c**7*d**9 + 12
87*b*c**8*d**8) + x**9*(1430*a*c**8*d**8 + 11440*b*c**9*d**7/9) + x**8*(14
30*a*c**9*d**7 + 1001*b*c**10*d**6) + x**7*(1144*a*c**10*d**6 + 624*b*c**1
1*d**5) + x**6*(728*a*c**11*d**5 + 910*b*c**12*d**4/3) + x**5*(364*a*c**12
*d**4 + 112*b*c**13*d**3) + x**4*(140*a*c**13*d**3 + 30*b*c**14*d**2) + x*
*3*(40*a*c**14*d**2 + 16*b*c**15*d/3) + x**2*(8*a*c**15*d + b*c**16/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 384, normalized size of antiderivative = 10.11

$$\int (a+bx)(c+dx)^{16}\,dx = \frac{1}{18}bd^{16}x^{18} + ac^{16}x + \frac{1}{17}\left(16\,bcd^{15} + ad^{16}\right)x^{17} + \frac{1}{2}\left(15\,bc^2d^{14} + 2\,acd^{15}\right)x^{16} + \frac{8}{3}\left(14\,bc^3d^{13} + 3\,ac^2d^{14}\right)x^{15} + 10\left(13\,bc^4d^{12} + 4\,ac^3d^{13}\right)x^{14} + 28\left(12\,bc^5d^{11} + 5\,ac^4d^{12}\right)x^{13} + \frac{182}{3}\left(11\,bc^6d^{10} + 6\,ac^5d^{11}\right)x^{12} + 104\left(10\,bc^7d^9 + 7\,ac^6d^{10}\right)x^{11} + 143\left(9\,bc^8d^8 + 8\,ac^7d^9\right)x^{10} + \frac{1430}{9}\left(8\,bc^9d^7 + 9\,ac^8d^8\right)x^9 + 143\left(7\,bc^{10}d^6 + 10\,ac^9d^7\right)x^8 + 104\left(6\,bc^{11}d^5 + 11\,ac^{10}d^6\right)x^7 + \frac{182}{3}\left(5\,bc^{12}d^4 + 12\,ac^{11}d^5\right)x^6 + 28\left(4\,bc^{13}d^3 + 13\,ac^{12}d^4\right)x^5 + 10\left(3\,bc^{14}d^2 + 14\,ac^{13}d^3\right)x^4 + \frac{8}{3}\left(2\,bc^{15}d + 15\,ac^{14}d^2\right)x^3 + \frac{1}{2}\left(bc^{16} + 16\,ac^{15}d\right)x^2$$

input
```
integrate((b*x+a)*(d*x+c)^16,x, algorithm="maxima")
```

output
```
1/18*b*d^16*x^18 + a*c^16*x + 1/17*(16*b*c*d^15 + a*d^16)*x^17 + 1/2*(15*b
*c^2*d^14 + 2*a*c*d^15)*x^16 + 8/3*(14*b*c^3*d^13 + 3*a*c^2*d^14)*x^15 + 1
0*(13*b*c^4*d^12 + 4*a*c^3*d^13)*x^14 + 28*(12*b*c^5*d^11 + 5*a*c^4*d^12)*
x^13 + 182/3*(11*b*c^6*d^10 + 6*a*c^5*d^11)*x^12 + 104*(10*b*c^7*d^9 + 7*a
*c^6*d^10)*x^11 + 143*(9*b*c^8*d^8 + 8*a*c^7*d^9)*x^10 + 1430/9*(8*b*c^9*d
^7 + 9*a*c^8*d^8)*x^9 + 143*(7*b*c^10*d^6 + 10*a*c^9*d^7)*x^8 + 104*(6*b*c
^11*d^5 + 11*a*c^10*d^6)*x^7 + 182/3*(5*b*c^12*d^4 + 12*a*c^11*d^5)*x^6 +
28*(4*b*c^13*d^3 + 13*a*c^12*d^4)*x^5 + 10*(3*b*c^14*d^2 + 14*a*c^13*d^3)*
x^4 + 8/3*(2*b*c^15*d + 15*a*c^14*d^2)*x^3 + 1/2*(b*c^16 + 16*a*c^15*d)*x^
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 10.13

$$\int (a+bx)(c+dx)^{16}\,dx = \frac{1}{18}\,bd^{16}x^{18} + \frac{16}{17}\,bcd^{15}x^{17} + \frac{1}{17}\,ad^{16}x^{17} + \frac{15}{2}\,bc^{2}d^{14}x^{16} + acd^{15}x^{16} + \frac{112}{3}\,bc^{3}d^{13}x^{15} + 8\,ac^{2}d^{14}x^{15} + 130\,bc^{4}d^{12}x^{14} + 40\,ac^{3}d^{13}x^{14} + 336\,bc^{5}d^{11}x^{13} + 140\,ac^{4}d^{12}x^{13} + \frac{2002}{3}\,bc^{6}d^{10}x^{12} + 364\,ac^{5}d^{11}x^{12} + 1040\,bc^{7}d^{9}x^{11} + 728\,ac^{6}d^{10}x^{11} + 1287\,bc^{8}d^{8}x^{10} + 1144\,ac^{7}d^{9}x^{10} + \frac{11440}{9}\,bc^{9}d^{7}x^{9} + 1430\,ac^{8}d^{8}x^{9} + 1001\,bc^{10}d^{6}x^{8} + 1430\,ac^{9}d^{7}x^{8} + 624\,bc^{11}d^{5}x^{7} + 1144\,ac^{10}d^{6}x^{7} + \frac{910}{3}\,bc^{12}d^{4}x^{6} + 728\,ac^{11}d^{5}x^{6} + 112\,bc^{13}d^{3}x^{5} + 364\,ac^{12}d^{4}x^{5} + 30\,bc^{14}d^{2}x^{4} + 140\,ac^{13}d^{3}x^{4} + \frac{16}{3}\,bc^{15}dx^{3} + 40\,ac^{14}d^{2}x^{3} + \frac{1}{2}\,bc^{16}x^{2} + 8\,ac^{15}dx^{2} + ac^{16}x$$

input
```
integrate((b*x+a)*(d*x+c)^16,x, algorithm="giac")
```

output
```
1/18*b*d^16*x^18 + 16/17*b*c*d^15*x^17 + 1/17*a*d^16*x^17 + 15/2*b*c^2*d^1
4*x^16 + a*c*d^15*x^16 + 112/3*b*c^3*d^13*x^15 + 8*a*c^2*d^14*x^15 + 130*b
*c^4*d^12*x^14 + 40*a*c^3*d^13*x^14 + 336*b*c^5*d^11*x^13 + 140*a*c^4*d^12
*x^13 + 2002/3*b*c^6*d^10*x^12 + 364*a*c^5*d^11*x^12 + 1040*b*c^7*d^9*x^11
 + 728*a*c^6*d^10*x^11 + 1287*b*c^8*d^8*x^10 + 1144*a*c^7*d^9*x^10 + 11440
/9*b*c^9*d^7*x^9 + 1430*a*c^8*d^8*x^9 + 1001*b*c^10*d^6*x^8 + 1430*a*c^9*d
^7*x^8 + 624*b*c^11*d^5*x^7 + 1144*a*c^10*d^6*x^7 + 910/3*b*c^12*d^4*x^6 +
 728*a*c^11*d^5*x^6 + 112*b*c^13*d^3*x^5 + 364*a*c^12*d^4*x^5 + 30*b*c^14*
d^2*x^4 + 140*a*c^13*d^3*x^4 + 16/3*b*c^15*d*x^3 + 40*a*c^14*d^2*x^3 + 1/2
*b*c^16*x^2 + 8*a*c^15*d*x^2 + a*c^16*x
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 328, normalized size of antiderivative = 8.63

$$\int (a+bx)(c+dx)^{16}\,dx = x^2\left(\frac{b\,c^{16}}{2}+8\,a\,d\,c^{15}\right)+x^{17}\left(\frac{a\,d^{16}}{17}+\frac{16\,b\,c\,d^{15}}{17}\right)+\frac{b\,d^{16}\,x^{18}}{18} + a\,c^{16}\,x+\frac{8\,c^{14}\,d\,x^3\,(15\,a\,d+2\,b\,c)}{3}+\frac{c\,d^{14}\,x^{16}\,(2\,a\,d+15\,b\,c)}{2} + 10\,c^{13}\,d^2\,x^4\,(14\,a\,d+3\,b\,c)+28\,c^{12}\,d^3\,x^5\,(13\,a\,d+4\,b\,c) + \frac{182\,c^{11}\,d^4\,x^6\,(12\,a\,d+5\,b\,c)}{3}+104\,c^{10}\,d^5\,x^7\,(11\,a\,d+6\,b\,c) + 143\,c^9\,d^6\,x^8\,(10\,a\,d+7\,b\,c)+\frac{1430\,c^8\,d^7\,x^9\,(9\,a\,d+8\,b\,c)}{9} + 143\,c^7\,d^8\,x^{10}\,(8\,a\,d+9\,b\,c)+104\,c^6\,d^9\,x^{11}\,(7\,a\,d+10\,b\,c) + \frac{182\,c^5\,d^{10}\,x^{12}\,(6\,a\,d+11\,b\,c)}{3}+28\,c^4\,d^{11}\,x^{13}\,(5\,a\,d+12\,b\,c) + 10\,c^3\,d^{12}\,x^{14}\,(4\,a\,d+13\,b\,c)+\frac{8\,c^2\,d^{13}\,x^{15}\,(3\,a\,d+14\,b\,c)}{3}$$

input
```
int((a + b*x)*(c + d*x)^16,x)
```

output
```
x^2*((b*c^16)/2 + 8*a*c^15*d) + x^17*((a*d^16)/17 + (16*b*c*d^15)/17) + (b
*d^16*x^18)/18 + a*c^16*x + (8*c^14*d*x^3*(15*a*d + 2*b*c))/3 + (c*d^14*x^
16*(2*a*d + 15*b*c))/2 + 10*c^13*d^2*x^4*(14*a*d + 3*b*c) + 28*c^12*d^3*x^
5*(13*a*d + 4*b*c) + (182*c^11*d^4*x^6*(12*a*d + 5*b*c))/3 + 104*c^10*d^5*
x^7*(11*a*d + 6*b*c) + 143*c^9*d^6*x^8*(10*a*d + 7*b*c) + (1430*c^8*d^7*x^
9*(9*a*d + 8*b*c))/9 + 143*c^7*d^8*x^10*(8*a*d + 9*b*c) + 104*c^6*d^9*x^11
*(7*a*d + 10*b*c) + (182*c^5*d^10*x^12*(6*a*d + 11*b*c))/3 + 28*c^4*d^11*x
^13*(5*a*d + 12*b*c) + 10*c^3*d^12*x^14*(4*a*d + 13*b*c) + (8*c^2*d^13*x^1
5*(3*a*d + 14*b*c))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 385, normalized size of antiderivative = 10.13

$$\int (a+bx)(c+dx)^{16}\,dx$$
$$= \frac{x(17b\,d^{16}x^{17}+18a\,d^{16}x^{16}+288bc\,d^{15}x^{16}+306ac\,d^{15}x^{15}+2295b\,c^2d^{14}x^{15}+2448a\,c^2d^{14}x^{14}+11424b\,c^3d}{}$$

input
```
int((b*x+a)*(d*x+c)^16,x)
```

output
```
(x*(306*a*c**16 + 2448*a*c**15*d*x + 12240*a*c**14*d**2*x**2 + 42840*a*c**
13*d**3*x**3 + 111384*a*c**12*d**4*x**4 + 222768*a*c**11*d**5*x**5 + 35006
4*a*c**10*d**6*x**6 + 437580*a*c**9*d**7*x**7 + 437580*a*c**8*d**8*x**8 +
350064*a*c**7*d**9*x**9 + 222768*a*c**6*d**10*x**10 + 111384*a*c**5*d**11*
x**11 + 42840*a*c**4*d**12*x**12 + 12240*a*c**3*d**13*x**13 + 2448*a*c**2*
d**14*x**14 + 306*a*c*d**15*x**15 + 18*a*d**16*x**16 + 153*b*c**16*x + 163
2*b*c**15*d*x**2 + 9180*b*c**14*d**2*x**3 + 34272*b*c**13*d**3*x**4 + 9282
0*b*c**12*d**4*x**5 + 190944*b*c**11*d**5*x**6 + 306306*b*c**10*d**6*x**7
+ 388960*b*c**9*d**7*x**8 + 393822*b*c**8*d**8*x**9 + 318240*b*c**7*d**9*x
**10 + 204204*b*c**6*d**10*x**11 + 102816*b*c**5*d**11*x**12 + 39780*b*c**
4*d**12*x**13 + 11424*b*c**3*d**13*x**14 + 2295*b*c**2*d**14*x**15 + 288*b
*c*d**15*x**16 + 17*b*d**16*x**17))/306
```

3.142 $\int x^2(2+x)^5(2+3x)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int x^2(2+x)^5(2+3x)\,dx=\frac{1}{3}x^3(2+x)^6$$

output
```
1/3*x^3*(2+x)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int x^2(2+x)^5(2+3x)\,dx=\frac{64x^3}{3}+64x^4+80x^5+\frac{160x^6}{3}+20x^7+4x^8+\frac{x^9}{3}$$

input
```
Integrate[x^2*(2 + x)^5*(2 + 3*x),x]
```

output
```
(64*x^3)/3 + 64*x^4 + 80*x^5 + (160*x^6)/3 + 20*x^7 + 4*x^8 + x^9/3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(x+2)^5(3x+2)\,dx$$

$$\Big\downarrow \, 83$$

$$\frac{1}{3}x^3(x+2)^6$$

input
```
Int[x^2*(2 + x)^5*(2 + 3*x),x]
```

output
```
(x^3*(2 + x)^6)/3
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
orering	$\frac{x^3(2+x)^6}{3}$	11
gosper	$\frac{1}{3}x^9+4x^8+20x^7+\frac{160}{3}x^6+80x^5+64x^4+\frac{64}{3}x^3$	37
default	$\frac{1}{3}x^9+4x^8+20x^7+\frac{160}{3}x^6+80x^5+64x^4+\frac{64}{3}x^3$	37
norman	$\frac{1}{3}x^9+4x^8+20x^7+\frac{160}{3}x^6+80x^5+64x^4+\frac{64}{3}x^3$	37
risch	$\frac{1}{3}x^9+4x^8+20x^7+\frac{160}{3}x^6+80x^5+64x^4+\frac{64}{3}x^3$	37
parallelrisch	$\frac{1}{3}x^9+4x^8+20x^7+\frac{160}{3}x^6+80x^5+64x^4+\frac{64}{3}x^3$	37

input
```
int(x^2*(2+x)^5*(2+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*(2+x)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int x^2(2+x)^5(2+3x)\,dx = \frac{1}{3}\,x^9+4\,x^8+20\,x^7+\frac{160}{3}\,x^6+80\,x^5+64\,x^4+\frac{64}{3}\,x^3$$

input
```
integrate(x^2*(2+x)^5*(2+3*x),x, algorithm="fricas")
```

output
```
1/3*x^9 + 4*x^8 + 20*x^7 + 160/3*x^6 + 80*x^5 + 64*x^4 + 64/3*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int x^2(2+x)^5(2+3x)\,dx = \frac{x^9}{3}+4x^8+20x^7+\frac{160x^6}{3}+80x^5+64x^4+\frac{64x^3}{3}$$

input
```
integrate(x**2*(2+x)**5*(2+3*x),x)
```

output
```
x**9/3 + 4*x**8 + 20*x**7 + 160*x**6/3 + 80*x**5 + 64*x**4 + 64*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int x^2(2+x)^5(2+3x)\,dx = \frac{1}{3}\,x^9 + 4\,x^8 + 20\,x^7 + \frac{160}{3}\,x^6 + 80\,x^5 + 64\,x^4 + \frac{64}{3}\,x^3$$

input
```
integrate(x^2*(2+x)^5*(2+3*x),x, algorithm="maxima")
```

output
```
1/3*x^9 + 4*x^8 + 20*x^7 + 160/3*x^6 + 80*x^5 + 64*x^4 + 64/3*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int x^2(2+x)^5(2+3x)\,dx = \frac{1}{3}\,x^9 + 4\,x^8 + 20\,x^7 + \frac{160}{3}\,x^6 + 80\,x^5 + 64\,x^4 + \frac{64}{3}\,x^3$$

input
```
integrate(x^2*(2+x)^5*(2+3*x),x, algorithm="giac")
```

output
```
1/3*x^9 + 4*x^8 + 20*x^7 + 160/3*x^6 + 80*x^5 + 64*x^4 + 64/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int x^2(2+x)^5(2+3x)\,dx = \frac{x^9}{3} + 4\,x^8 + 20\,x^7 + \frac{160\,x^6}{3} + 80\,x^5 + 64\,x^4 + \frac{64\,x^3}{3}$$

input
```
int(x^2*(3*x + 2)*(x + 2)^5,x)
```

output
```
(64*x^3)/3 + 64*x^4 + 80*x^5 + (160*x^6)/3 + 20*x^7 + 4*x^8 + x^9/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int x^2(2+x)^5(2+3x)\,dx = \frac{x^3(x^6+12x^5+60x^4+160x^3+240x^2+192x+64)}{3}$$

input
```
int(x^2*(2+x)^5*(2+3*x),x)
```

output
```
(x**3*(x**6 + 12*x**5 + 60*x**4 + 160*x**3 + 240*x**2 + 192*x + 64))/3
```

3.143 $\int \frac{x^4(A+Bx)}{a+bx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{x^4(A+Bx)}{a+bx} \, dx = -\frac{a^3(Ab-aB)x}{b^5} + \frac{a^2(Ab-aB)x^2}{2b^4} - \frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^5}{5b} + \frac{a^4(Ab-aB)\log(a+bx)}{b^6}$$

output
```
-a^3*(A*b-B*a)*x/b^5+1/2*a^2*(A*b-B*a)*x^2/b^4-1/3*a*(A*b-B*a)*x^3/b^3+1/4
*(A*b-B*a)*x^4/b^2+1/5*B*x^5/b+a^4*(A*b-B*a)*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{x^4(A+Bx)}{a+bx} \, dx = \frac{bx(60a^4B - 30a^3b(2A+Bx) + 10a^2b^2x(3A+2Bx) - 5ab^3x^2(4A+3Bx) + 3b^4x^3(5A+4Bx)) - 60a^4(}{60b^6}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x),x]
```

output
```
(b*x*(60*a^4*B - 30*a^3*b*(2*A + B*x) + 10*a^2*b^2*x*(3*A + 2*B*x) - 5*a*b
^3*x^2*(4*A + 3*B*x) + 3*b^4*x^3*(5*A + 4*B*x)) - 60*a^4*(-(A*b) + a*B)*Lo
g[a + b*x])/(60*b^6)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{a+bx} \, dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)}+\frac{a^3(aB-Ab)}{b^5}-\frac{a^2x(aB-Ab)}{b^4}+\frac{ax^2(aB-Ab)}{b^3}+\frac{x^3(Ab-aB)}{b^2}+\frac{Bx^4}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4(Ab-aB)\log(a+bx)}{b^6}-\frac{a^3x(Ab-aB)}{b^5}+\frac{a^2x^2(Ab-aB)}{2b^4}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^5}{5b}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x),x]
```

output
```
-((a^3*(A*b - a*B)*x)/b^5) + (a^2*(A*b - a*B)*x^2)/(2*b^4) - (a*(A*b - a*B
)*x^3)/(3*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^5)/(5*b) + (a^4*(A*b - a
*B)*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

method	result
norman	$-\frac{a^{3}(Ab-Ba)x}{b^{5}}+\frac{a^{2}(Ab-Ba)x^{2}}{2b^{4}}-\frac{a(Ab-Ba)x^{3}}{3b^{3}}+\frac{(Ab-Ba)x^{4}}{4b^{2}}+\frac{B\,x^{5}}{5b}+\frac{a^{4}(Ab-Ba)\ln(bx+a)}{b^{6}}$
default	$-\frac{-\frac{1}{5}B\,b^{4}x^{5}-\frac{1}{4}A\,b^{4}x^{4}+\frac{1}{4}Ba\,b^{3}x^{4}+\frac{1}{3}Aa\,b^{3}x^{3}-\frac{1}{3}B\,a^{2}b^{2}x^{3}-\frac{1}{2}A\,a^{2}b^{2}x^{2}+\frac{1}{2}B\,a^{3}b\,x^{2}+A\,a^{3}bx-B\,a^{4}x}{b^{5}}+\frac{a^{4}(Ab-Ba)\ln(bx\cdots}{b^{6}}$
risch	$\frac{B\,x^{5}}{5b}+\frac{A\,x^{4}}{4b}-\frac{Ba\,x^{4}}{4b^{2}}-\frac{Aa\,x^{3}}{3b^{2}}+\frac{B\,a^{2}x^{3}}{3b^{3}}+\frac{A\,a^{2}x^{2}}{2b^{3}}-\frac{B\,a^{3}x^{2}}{2b^{4}}-\frac{A\,a^{3}x}{b^{4}}+\frac{B\,a^{4}x}{b^{5}}+\frac{a^{4}\ln(bx+a)A}{b^{5}}-\frac{a^{5}\ln(bx\cdots}{b^{6}}$
parallelrisch	$\frac{12b^{5}B\,x^{5}+15A\,b^{5}x^{4}-15Ba\,b^{4}x^{4}-20Aa\,b^{4}x^{3}+20B\,a^{2}b^{3}x^{3}+30A\,a^{2}b^{3}x^{2}-30B\,a^{3}b^{2}x^{2}+60A\ln(bx+a)a^{4}b-60a^{3}b^{2}Ax-60B\ln\cdots}{60b^{6}}$

input
```
int(x^4*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-a^3*(A*b-B*a)*x/b^5+1/2*a^2*(A*b-B*a)*x^2/b^4-1/3*a*(A*b-B*a)*x^3/b^3+1/4
*(A*b-B*a)*x^4/b^2+1/5*B*x^5/b+a^4*(A*b-B*a)*ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{x^4(A+Bx)}{a+bx}\,dx = \frac{12\,Bb^5x^5-15\,(Bab^4-Ab^5)x^4+20\,(Ba^2b^3-Aab^4)x^3-30\,(Ba^3b^2-Aa^2b^3)x^2+60\,(Ba^4b-Aa^3b^2)x\cdots}{60\,b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
1/60*(12*B*b^5*x^5 - 15*(B*a*b^4 - A*b^5)*x^4 + 20*(B*a^2*b^3 - A*a*b^4)*x
^3 - 30*(B*a^3*b^2 - A*a^2*b^3)*x^2 + 60*(B*a^4*b - A*a^3*b^2)*x - 60*(B*a
^5 - A*a^4*b)*log(b*x + a))/b^6
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{x^4(A+Bx)}{a+bx}\,dx = \frac{Bx^5}{5b} - \frac{a^4(-Ab+Ba)\log{(a+bx)}}{b^6} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x^3\left(-\frac{Aa}{3b^2} + \frac{Ba^2}{3b^3}\right) + x^2\left(\frac{Aa^2}{2b^3} - \frac{Ba^3}{2b^4}\right) + x\left(-\frac{Aa^3}{b^4} + \frac{Ba^4}{b^5}\right)$$

input
```
integrate(x**4*(B*x+A)/(b*x+a),x)
```

output
```
B*x**5/(5*b) - a**4*(-A*b + B*a)*log(a + b*x)/b**6 + x**4*(A/(4*b) - B*a/(
4*b**2)) + x**3*(-A*a/(3*b**2) + B*a**2/(3*b**3)) + x**2*(A*a**2/(2*b**3)
- B*a**3/(2*b**4)) + x*(-A*a**3/b**4 + B*a**4/b**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx)}{a+bx}\,dx = \frac{12\,Bb^4x^5 - 15\,(Bab^3 - Ab^4)x^4 + 20\,(Ba^2b^2 - Aab^3)x^3 - 30\,(Ba^3b - Aa^2b^2)x^2 + 60\,(Ba^4 - Aa^3b)x}{60\,b^5} - \frac{(Ba^5 - Aa^4b)\log{(bx+a)}}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
1/60*(12*B*b^4*x^5 - 15*(B*a*b^3 - A*b^4)*x^4 + 20*(B*a^2*b^2 - A*a*b^3)*x
^3 - 30*(B*a^3*b - A*a^2*b^2)*x^2 + 60*(B*a^4 - A*a^3*b)*x)/b^5 - (B*a^5 -
 A*a^4*b)*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{x^4(A+Bx)}{a+bx}\,dx = \frac{12\,Bb^4x^5 - 15\,Bab^3x^4 + 15\,Ab^4x^4 + 20\,Ba^2b^2x^3 - 20\,Aab^3x^3 - 30\,Ba^3bx^2 + 30\,Aa^2b^2x^2 + 60\,Ba^4x - 60}{60\,b^5} - \frac{(Ba^5 - Aa^4b)\log(|bx+a|)}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
1/60*(12*B*b^4*x^5 - 15*B*a*b^3*x^4 + 15*A*b^4*x^4 + 20*B*a^2*b^2*x^3 - 20
*A*a*b^3*x^3 - 30*B*a^3*b*x^2 + 30*A*a^2*b^2*x^2 + 60*B*a^4*x - 60*A*a^3*b
*x)/b^5 - (B*a^5 - A*a^4*b)*log(abs(b*x + a))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{x^4(A+Bx)}{a+bx}\,dx = x^4\left(\frac{A}{4\,b} - \frac{B\,a}{4\,b^2}\right) - \frac{\ln(a+b\,x)\,\left(B\,a^5 - A\,a^4\,b\right)}{b^6} + \frac{B\,x^5}{5\,b} + \frac{a^2\,x^2\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{2\,b^2} - \frac{a\,x^3\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{3\,b} - \frac{a^3\,x\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{b^3}$$

input
```
int((x^4*(A + B*x))/(a + b*x),x)
```

output
```
x^4*(A/(4*b) - (B*a)/(4*b^2)) - (log(a + b*x)*(B*a^5 - A*a^4*b))/b^6 + (B*
x^5)/(5*b) + (a^2*x^2*(A/b - (B*a)/b^2))/(2*b^2) - (a*x^3*(A/b - (B*a)/b^2
))/(3*b) - (a^3*x*(A/b - (B*a)/b^2))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.05

$$\int \frac{x^4(A+Bx)}{a+bx}\,dx=\frac{x^5}{5}$$

input
```
int(x^4*(B*x+A)/(b*x+a),x)
```

output
```
x**5/5
```

3.144 $\int \frac{x^3(A+Bx)}{a+bx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = \frac{a^2(Ab-aB)x}{b^4} - \frac{a(Ab-aB)x^2}{2b^3} + \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^4}{4b} - \frac{a^3(Ab-aB)\log(a+bx)}{b^5}$$

output
```
a^2*(A*b-B*a)*x/b^4-1/2*a*(A*b-B*a)*x^2/b^3+1/3*(A*b-B*a)*x^3/b^2+1/4*B*x^
4/b-a^3*(A*b-B*a)*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = \frac{bx(-12a^3B+6a^2b(2A+Bx)-2ab^2x(3A+2Bx)+b^3x^2(4A+3Bx))+12a^3(-Ab+aB)\log(a+bx)}{12b^5}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x),x]
```

output
```
(b*x*(-12*a^3*B + 6*a^2*b*(2*A + B*x) - 2*a*b^2*x*(3*A + 2*B*x) + b^3*x^2*
(4*A + 3*B*x)) + 12*a^3*(-(A*b) + a*B)*Log[a + b*x])/(12*b^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)}-\frac{a^2(aB-Ab)}{b^4}+\frac{ax(aB-Ab)}{b^3}+\frac{x^2(Ab-aB)}{b^2}+\frac{Bx^3}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3(Ab-aB)\log(a+bx)}{b^5}+\frac{a^2x(Ab-aB)}{b^4}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^4}{4b}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x),x]
```

output
```
(a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b - a*B)*x^3)/
(3*b^2) + (B*x^4)/(4*b) - (a^3*(A*b - a*B)*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	si
norman	$\frac{a^2(Ab-Ba)x}{b^4}-\frac{a(Ab-Ba)x^2}{2b^3}+\frac{(Ab-Ba)x^3}{3b^2}+\frac{B\,x^4}{4b}-\frac{a^3(Ab-Ba)\ln(bx+a)}{b^5}$	8
default	$\frac{\frac{1}{4}B\,b^3x^4+\frac{1}{3}A\,b^3x^3-\frac{1}{3}Ba\,b^2x^3-\frac{1}{2}aA\,b^2x^2+\frac{1}{2}B\,a^2b\,x^2+a^2Abx-B\,a^3x}{b^4}-\frac{a^3(Ab-Ba)\ln(bx+a)}{b^5}$	9
risch	$\frac{B\,x^4}{4b}+\frac{A\,x^3}{3b}-\frac{Ba\,x^3}{3b^2}-\frac{aA\,x^2}{2b^2}+\frac{B\,a^2x^2}{2b^3}+\frac{a^2Ax}{b^3}-\frac{B\,a^3x}{b^4}-\frac{a^3\ln(bx+a)A}{b^4}+\frac{a^4\ln(bx+a)B}{b^5}$	1
parallelrisch	$-\frac{-3B\,x^4b^4-4A\,x^3b^4+4B\,x^3a\,b^3+6A\,x^2a\,b^3-6B\,x^2a^2b^2+12A\ln(bx+a)a^3b-12Ax\,a^2b^2-12B\ln(bx+a)a^4+12Bx\,a^3b}{12b^5}$	1

input
```
int(x^3*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
a^2*(A*b-B*a)*x/b^4-1/2*a*(A*b-B*a)*x^2/b^3+1/3*(A*b-B*a)*x^3/b^2+1/4*B*x^
4/b-a^3*(A*b-B*a)*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx$$
$$=\frac{3\,Bb^4x^4-4\,(Bab^3-Ab^4)x^3+6\,(Ba^2b^2-Aab^3)x^2-12\,(Ba^3b-Aa^2b^2)x+12\,(Ba^4-Aa^3b)\log\left(bx+\right.}{12\,b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*B*b^4*x^4 - 4*(B*a*b^3 - A*b^4)*x^3 + 6*(B*a^2*b^2 - A*a*b^3)*x^2
- 12*(B*a^3*b - A*a^2*b^2)*x + 12*(B*a^4 - A*a^3*b)*log(b*x + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = \frac{Bx^4}{4b} + \frac{a^3(-Ab+Ba)\log{(a+bx)}}{b^5} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right) + x^2\left(-\frac{Aa}{2b^2} + \frac{Ba^2}{2b^3}\right) + x\left(\frac{Aa^2}{b^3} - \frac{Ba^3}{b^4}\right)$$

input
```
integrate(x**3*(B*x+A)/(b*x+a),x)
```

output
```
B*x**4/(4*b) + a**3*(-A*b + B*a)*log(a + b*x)/b**5 + x**3*(A/(3*b) - B*a/(
3*b**2)) + x**2*(-A*a/(2*b**2) + B*a**2/(2*b**3)) + x*(A*a**2/b**3 - B*a**
3/b**4)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = \frac{3\,Bb^3x^4 - 4\,(Bab^2 - Ab^3)x^3 + 6\,(Ba^2b - Aab^2)x^2 - 12\,(Ba^3 - Aa^2b)x}{12\,b^4} + \frac{(Ba^4 - Aa^3b)\log{(bx+a)}}{b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(3*B*b^3*x^4 - 4*(B*a*b^2 - A*b^3)*x^3 + 6*(B*a^2*b - A*a*b^2)*x^2 -
12*(B*a^3 - A*a^2*b)*x)/b^4 + (B*a^4 - A*a^3*b)*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = \frac{3\,Bb^3x^4 - 4\,Bab^2x^3 + 4\,Ab^3x^3 + 6\,Ba^2bx^2 - 6\,Aab^2x^2 - 12\,Ba^3x + 12\,Aa^2bx}{12\,b^4} + \frac{(Ba^4 - Aa^3b)\log(|bx+a|)}{b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*B*b^3*x^4 - 4*B*a*b^2*x^3 + 4*A*b^3*x^3 + 6*B*a^2*b*x^2 - 6*A*a*b^
2*x^2 - 12*B*a^3*x + 12*A*a^2*b*x)/b^4 + (B*a^4 - A*a^3*b)*log(abs(b*x + a
))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = x^3\left(\frac{A}{3\,b} - \frac{B\,a}{3\,b^2}\right) + \frac{\ln(a+b\,x)\,\left(B\,a^4 - A\,a^3\,b\right)}{b^5} + \frac{B\,x^4}{4\,b} - \frac{a\,x^2\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{2\,b} + \frac{a^2\,x\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{b^2}$$

input
```
int((x^3*(A + B*x))/(a + b*x),x)
```

output
```
x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x)*(B*a^4 - A*a^3*b))/b^5 + (B*
x^4)/(4*b) - (a*x^2*(A/b - (B*a)/b^2))/(2*b) + (a^2*x*(A/b - (B*a)/b^2))/b
^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.06

$$\int \frac{x^3(A+Bx)}{a+bx}\,dx = \frac{x^4}{4}$$

input

```
int(x^3*(B*x+A)/(b*x+a),x)
```

output

```
x**4/4
```

3.145 $\int \frac{x^2(A+Bx)}{a+bx} dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{x^2(A+Bx)}{a+bx} dx = -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^3}{3b} + \frac{a^2(Ab-aB)\log(a+bx)}{b^4}$$

output
```
-a*(A*b-B*a)*x/b^3+1/2*(A*b-B*a)*x^2/b^2+1/3*B*x^3/b+a^2*(A*b-B*a)*ln(b*x+
a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx)}{a+bx} dx = \frac{bx(6a^2B-3ab(2A+Bx)+b^2x(3A+2Bx))+6a^2(Ab-aB)\log(a+bx)}{6b^4}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x),x]
```

output
```
(b*x*(6*a^2*B - 3*a*b*(2*A + B*x) + b^2*x*(3*A + 2*B*x)) + 6*a^2*(A*b - a*
B)*Log[a + b*x])/(6*b^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{a+bx}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)}+\frac{a(aB-Ab)}{b^3}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(Ab-aB)\log(a+bx)}{b^4}-\frac{ax(Ab-aB)}{b^3}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^3}{3b}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x),x]
```

output
```
-((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^2)/(2*b^2) + (B*x^3)/(3*b) + (a^
2*(A*b - a*B)*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
norman	$-\frac{a(Ab-Ba)x}{b^{3}}+\frac{(Ab-Ba)x^{2}}{2b^{2}}+\frac{B\,x^{3}}{3b}+\frac{a^{2}(Ab-Ba)\ln(bx+a)}{b^{4}}$	63
default	$-\frac{-\frac{1}{3}B\,b^{2}x^{3}-\frac{1}{2}A\,b^{2}x^{2}+\frac{1}{2}Bab\,x^{2}+aAbx-B\,a^{2}x}{b^{3}}+\frac{a^{2}(Ab-Ba)\ln(bx+a)}{b^{4}}$	67
risch	$\frac{B\,x^{3}}{3b}+\frac{A\,x^{2}}{2b}-\frac{Ba\,x^{2}}{2b^{2}}-\frac{aAx}{b^{2}}+\frac{B\,a^{2}x}{b^{3}}+\frac{a^{2}\ln(bx+a)A}{b^{3}}-\frac{a^{3}\ln(bx+a)B}{b^{4}}$	76
parallelrisch	$\frac{2b^{3}B\,x^{3}+3A\,x^{2}b^{3}-3B\,x^{2}a\,b^{2}+6A\ln(bx+a)a^{2}b-6Axa\,b^{2}-6B\ln(bx+a)a^{3}+6Bx\,a^{2}b}{6b^{4}}$	76

input
```
int(x^2*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-a*(A*b-B*a)*x/b^3+1/2*(A*b-B*a)*x^2/b^2+1/3*B*x^3/b+a^2*(A*b-B*a)*ln(b*x+
a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int\frac{x^2(A+Bx)}{a+bx}\,dx$$
$$=\frac{2\,Bb^3x^3-3\,(Bab^2-Ab^3)x^2+6\,(Ba^2b-Aab^2)x-6\,(Ba^3-Aa^2b)\log\left(bx+a\right)}{6\,b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*x^3 - 3*(B*a*b^2 - A*b^3)*x^2 + 6*(B*a^2*b - A*a*b^2)*x - 6*(
B*a^3 - A*a^2*b)*log(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx)}{a+bx}\,dx = \frac{Bx^3}{3b} - \frac{a^2(-Ab+Ba)\log{(a+bx)}}{b^4} + x^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right) + x\left(-\frac{Aa}{b^2}+\frac{Ba^2}{b^3}\right)$$

input
```
integrate(x**2*(B*x+A)/(b*x+a),x)
```

output
```
B*x**3/(3*b) - a**2*(-A*b + B*a)*log(a + b*x)/b**4 + x**2*(A/(2*b) - B*a/(
2*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx)}{a+bx}\,dx = \frac{2\,Bb^2x^3 - 3\,(Bab-Ab^2)x^2 + 6\,(Ba^2-Aab)x}{6\,b^3} - \frac{(Ba^3-Aa^2b)\log{(bx+a)}}{b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(2*B*b^2*x^3 - 3*(B*a*b - A*b^2)*x^2 + 6*(B*a^2 - A*a*b)*x)/b^3 - (B*a
^3 - A*a^2*b)*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{x^2(A+Bx)}{a+bx}\,dx = \frac{2\,Bb^2x^3 - 3\,Babx^2 + 3\,Ab^2x^2 + 6\,Ba^2x - 6\,Aabx}{6\,b^3} - \frac{(Ba^3 - Aa^2b)\log(|bx+a|)}{b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
1/6*(2*B*b^2*x^3 - 3*B*a*b*x^2 + 3*A*b^2*x^2 + 6*B*a^2*x - 6*A*a*b*x)/b^3
- (B*a^3 - A*a^2*b)*log(abs(b*x + a))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx)}{a+bx}\,dx = x^2\left(\frac{A}{2\,b} - \frac{B\,a}{2\,b^2}\right) - \frac{\ln(a+b\,x)\,\left(B\,a^3 - A\,a^2\,b\right)}{b^4} + \frac{B\,x^3}{3\,b} - \frac{a\,x\,\left(\frac{A}{b} - \frac{B\,a}{b^2}\right)}{b}$$

input
```
int((x^2*(A + B*x))/(a + b*x),x)
```

output
```
x^2*(A/(2*b) - (B*a)/(2*b^2)) - (log(a + b*x)*(B*a^3 - A*a^2*b))/b^4 + (B*
x^3)/(3*b) - (a*x*(A/b - (B*a)/b^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.08

$$\int \frac{x^2(A+Bx)}{a+bx}\,dx = \frac{x^3}{3}$$

input
```
int(x^2*(B*x+A)/(b*x+a),x)
```

output
```
x**3/3
```

3.146 $\int \frac{x(A+Bx)}{a+bx} dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{x(A+Bx)}{a+bx} dx = \frac{(Ab-aB)x}{b^2} + \frac{Bx^2}{2b} - \frac{a(Ab-aB)\log(a+bx)}{b^3}$$

output
```
(A*b-B*a)*x/b^2+1/2*B*x^2/b-a*(A*b-B*a)*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx)}{a+bx} dx = \frac{bx(2Ab-2aB+bBx)+2a(-Ab+aB)\log(a+bx)}{2b^3}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x),x]
```

output
```
(b*x*(2*A*b - 2*a*B + b*B*x) + 2*a*(-(A*b) + a*B)*Log[a + b*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{a+bx}\,dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{a(aB-Ab)}{b^2(a+bx)}+\frac{Ab-aB}{b^2}+\frac{Bx}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a(Ab-aB)\log(a+bx)}{b^3}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^2}{2b}$$

input
```
Int[(x*(A + B*x))/(a + b*x),x]
```

output
```
((A*b - a*B)*x)/b^2 + (B*x^2)/(2*b) - (a*(A*b - a*B)*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\frac{1}{2}bB\,x^2+Abx-Bax}{b^2}-\frac{a(Ab-Ba)\ln(bx+a)}{b^3}$	43
norman	$\frac{(Ab-Ba)x}{b^2}+\frac{B\,x^2}{2b}-\frac{a(Ab-Ba)\ln(bx+a)}{b^3}$	44
risch	$\frac{B\,x^2}{2b}+\frac{Ax}{b}-\frac{Bax}{b^2}-\frac{a\ln(bx+a)A}{b^2}+\frac{a^2\ln(bx+a)B}{b^3}$	52
parallelrisch	$-\frac{-b^2B\,x^2+2A\ln(bx+a)ab-2A\,b^2x-2B\ln(bx+a)a^2+2Babx}{2b^3}$	52

input
```
int(x*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*b*B*x^2+A*b*x-B*a*x)-a*(A*b-B*a)*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx)}{a+bx}\,dx=\frac{Bb^2x^2-2\,(Bab-Ab^2)x+2\,(Ba^2-Aab)\log(bx+a)}{2\,b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(B*b^2*x^2 - 2*(B*a*b - A*b^2)*x + 2*(B*a^2 - A*a*b)*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{x(A+Bx)}{a+bx}\,dx=\frac{Bx^2}{2b}+\frac{a(-Ab+Ba)\log(a+bx)}{b^3}+x\left(\frac{A}{b}-\frac{Ba}{b^2}\right)$$

input
```
integrate(x*(B*x+A)/(b*x+a),x)
```

output
```
B*x**2/(2*b) + a*(-A*b + B*a)*log(a + b*x)/b**3 + x*(A/b - B*a/b**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx)}{a+bx}\,dx = \frac{Bbx^2-2\,(Ba-Ab)x}{2\,b^2}+\frac{(Ba^2-Aab)\log{(bx+a)}}{b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 - 2*(B*a - A*b)*x)/b^2 + (B*a^2 - A*a*b)*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx)}{a+bx}\,dx = \frac{Bbx^2-2\,Bax+2\,Abx}{2\,b^2}+\frac{(Ba^2-Aab)\log{(|bx+a|)}}{b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(B*b*x^2 - 2*B*a*x + 2*A*b*x)/b^2 + (B*a^2 - A*a*b)*log(abs(b*x + a))/
b^3
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{x(A+Bx)}{a+bx}\,dx = x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^2}{2\,b}+\frac{\ln{(a+b\,x)}\,\left(B\,a^2-A\,a\,b\right)}{b^3}$$

input
```
int((x*(A + B*x))/(a + b*x),x)
```

output
```
x*(A/b - (B*a)/b^2) + (B*x^2)/(2*b) + (log(a + b*x)*(B*a^2 - A*a*b))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.11

$$\int \frac{x(A+Bx)}{a+bx}\,dx = \frac{x^2}{2}$$

input
```
int(x*(B*x+A)/(b*x+a),x)
```

output
```
x**2/2
```

3.147 $\int \frac{A+Bx}{a+bx} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{A+Bx}{a+bx} \, dx = \frac{Bx}{b} + \frac{(Ab-aB)\log(a+bx)}{b^2}$$

output
```
B*x/b+(A*b-B*a)*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a+bx} \, dx = \frac{Bx}{b} + \frac{(Ab-aB)\log(a+bx)}{b^2}$$

input
```
Integrate[(A + B*x)/(a + b*x),x]
```

output
```
(B*x)/b + ((A*b - a*B)*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{a+bx}\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{Ab-aB}{b(a+bx)}+\frac{B}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ab-aB)\log(a+bx)}{b^2}+\frac{Bx}{b}$$

input
```
Int[(A + B*x)/(a + b*x),x]
```

output
```
(B*x)/b + ((A*b - a*B)*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{Bx}{b}+\frac{(Ab-Ba)\ln(bx+a)}{b^2}$	26
norman	$\frac{Bx}{b}+\frac{(Ab-Ba)\ln(bx+a)}{b^2}$	26
parallelrisch	$\frac{A\ln(bx+a)b-B\ln(bx+a)a+bBx}{b^2}$	29
risch	$\frac{Bx}{b}+\frac{\ln(bx+a)A}{b}-\frac{\ln(bx+a)Ba}{b^2}$	32

input
```
int((B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
B*x/b+(A*b-B*a)*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a+bx}\,dx=\frac{Bbx-(Ba-Ab)\log{(bx+a)}}{b^2}$$

input
```
integrate((B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
(B*b*x - (B*a - A*b)*log(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{a+bx}\,dx=\frac{Bx}{b}-\frac{(-Ab+Ba)\log{(a+bx)}}{b^2}$$

input
```
integrate((B*x+A)/(b*x+a),x)
```

output
```
B*x/b - (-A*b + B*a)*log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{a+bx}\,dx=\frac{Bx}{b}-\frac{(Ba-Ab)\log{(bx+a)}}{b^2}$$

input
```
integrate((B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
B*x/b - (B*a - A*b)*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{a+bx}\,dx=\frac{Bx}{b}-\frac{(Ba-Ab)\log{(|bx+a|)}}{b^2}$$

input
```
integrate((B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
B*x/b - (B*a - A*b)*log(abs(b*x + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a+bx}\,dx=\frac{B\,x}{b}+\frac{\ln{(a+b\,x)}\,\left(A\,b-B\,a\right)}{b^2}$$

input
```
int((A + B*x)/(a + b*x),x)
```

output
```
(B*x)/b + (log(a + b*x)*(A*b - B*a))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{A+Bx}{a+bx}\,dx=x$$

input
```
int((B*x+A)/(b*x+a),x)
```

output
```
x
```

3.148 $\int \frac{A+Bx}{x(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{A+Bx}{x(a+bx)}\,dx=\frac{A\log(x)}{a}-\frac{(Ab-aB)\log(a+bx)}{ab}$$

output
```
A*ln(x)/a-(A*b-B*a)*ln(b*x+a)/a/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x(a+bx)}\,dx=\frac{A\log(x)}{a}+\frac{(-Ab+aB)\log(a+bx)}{ab}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x)),x]
```

output
```
(A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x])/(a*b)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(a+bx)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{aB-Ab}{a(a+bx)}+\frac{A}{ax}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log(a+bx)}{ab}$$

input
```
Int[(A + B*x)/(x*(a + b*x)),x]
```

output
```
(A*Log[x])/a - ((A*b - a*B)*Log[a + b*x])/(a*b)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{A\ln(x)}{a}+\frac{(-Ab+Ba)\ln(bx+a)}{ab}$	30
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(bx+a)}{ab}$	31
parallelrisch	$\frac{A\ln(x)b-A\ln(bx+a)b+B\ln(bx+a)a}{ab}$	33
risch	$-\frac{\ln(bx+a)A}{a}+\frac{\ln(bx+a)B}{b}+\frac{A\ln(-x)}{a}$	34

input
```
int((B*x+A)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a+(-A*b+B*a)/a/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x(a+bx)}\,dx=\frac{Ab\log(x)+(Ba-Ab)\log(bx+a)}{ab}$$

input
```
integrate((B*x+A)/x/(b*x+a),x, algorithm="fricas")
```

output
```
(A*b*log(x) + (B*a - A*b)*log(b*x + a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{x(a+bx)}\,dx=\frac{A\log(x)}{a}+\frac{(-Ab+Ba)\log\left(x+\frac{-Aa+\frac{a(-Ab+Ba)}{b}}{-2Ab+Ba}\right)}{ab}$$

input
```
integrate((B*x+A)/x/(b*x+a),x)
```

output
```
A*log(x)/a + (-A*b + B*a)*log(x + (-A*a + a*(-A*b + B*a)/b)/(-2*A*b + B*a)
)/(a*b)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x(a+bx)}\,dx = \frac{A\log{(x)}}{a} + \frac{(Ba-Ab)\log{(bx+a)}}{ab}$$

input
```
integrate((B*x+A)/x/(b*x+a),x, algorithm="maxima")
```

output
```
A*log(x)/a + (B*a - A*b)*log(b*x + a)/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x(a+bx)}\,dx = \frac{A\log{(|x|)}}{a} + \frac{(Ba-Ab)\log{(|bx+a|)}}{ab}$$

input
```
integrate((B*x+A)/x/(b*x+a),x, algorithm="giac")
```

output
```
A*log(abs(x))/a + (B*a - A*b)*log(abs(b*x + a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x(a+bx)}\,dx = \frac{A\,\ln{(x)}}{a} - \ln{(a+b\,x)}\,\left(\frac{A}{a}-\frac{B}{b}\right)$$

input
```
int((A + B*x)/(x*(a + b*x)),x)
```

output
```
(A*log(x))/a - log(a + b*x)*(A/a - B/b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.07

$$\int \frac{A+Bx}{x(a+bx)}\,dx = \log(x)$$

input
```
int((B*x+A)/x/(b*x+a),x)
```

output
```
log(x)
```

3.149 $\int \frac{A+Bx}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx=-\frac{A}{ax}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log(a+bx)}{a^2}$$

output
```
-A/a/x-(A*b-B*a)*ln(x)/a^2+(A*b-B*a)*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx=-\frac{A}{ax}+\frac{(-Ab+aB)\log(x)}{a^2}+\frac{(Ab-aB)\log(a+bx)}{a^2}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x)),x]
```

output
```
-(A/(a*x)) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{aB-Ab}{a^2x}-\frac{b(aB-Ab)}{a^2(a+bx)}+\frac{A}{ax^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(x)(Ab-aB)}{a^2}+\frac{(Ab-aB)\log(a+bx)}{a^2}-\frac{A}{ax}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{A}{ax}+\frac{(-Ab+Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln(bx+a)}{a^2}$	43
norman	$-\frac{A}{ax}-\frac{(Ab-Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln(bx+a)}{a^2}$	44
parallelrisch	$-\frac{A\ln(x)xb-A\ln(bx+a)xb-B\ln(x)xa+B\ln(bx+a)xa+Aa}{a^2x}$	47
risch	$-\frac{A}{ax}-\frac{\ln(x)Ab}{a^2}+\frac{\ln(x)B}{a}+\frac{\ln(-bx-a)Ab}{a^2}-\frac{\ln(-bx-a)B}{a}$	57

input
```
int((B*x+A)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-A/a/x+1/a^2*(-A*b+B*a)*ln(x)+(A*b-B*a)*ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx=-\frac{(Ba-Ab)x\log(bx+a)-(Ba-Ab)x\log(x)+Aa}{a^2x}$$

input
```
integrate((B*x+A)/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
-((B*a - A*b)*x*log(b*x + a) - (B*a - A*b)*x*log(x) + A*a)/(a^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(34) = 68.

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx=-\frac{A}{ax}+\frac{(-Ab+Ba)\log\left(x+\frac{-Aab+Ba^2-a(-Ab+Ba)}{-2Ab^2+2Bab}\right)}{a^2}-\frac{(-Ab+Ba)\log\left(x+\frac{-Aab+Ba^2+a(-Ab+Ba)}{-2Ab^2+2Bab}\right)}{a^2}$$

input
```
integrate((B*x+A)/x**2/(b*x+a),x)
```

output
```
-A/(a*x) + (-A*b + B*a)*log(x + (-A*a*b + B*a**2 - a*(-A*b + B*a))/(-2*A*b
**2 + 2*B*a*b))/a**2 - (-A*b + B*a)*log(x + (-A*a*b + B*a**2 + a*(-A*b + B
*a))/(-2*A*b**2 + 2*B*a*b))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx = -\frac{(Ba-Ab)\log(bx+a)}{a^2}+\frac{(Ba-Ab)\log(x)}{a^2}-\frac{A}{ax}$$

input
```
integrate((B*x+A)/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
-(B*a - A*b)*log(b*x + a)/a^2 + (B*a - A*b)*log(x)/a^2 - A/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx = \frac{(Ba-Ab)\log(|x|)}{a^2}-\frac{A}{ax}-\frac{(Bab-Ab^2)\log(|bx+a|)}{a^2b}$$

input
```
integrate((B*x+A)/x^2/(b*x+a),x, algorithm="giac")
```

output
```
(B*a - A*b)*log(abs(x))/a^2 - A/(a*x) - (B*a*b - A*b^2)*log(abs(b*x + a))/
(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)\,\left(A\,b-B\,a\right)}{a^2}-\frac{A}{a\,x}$$

input
```
int((A + B*x)/(x^2*(a + b*x)),x)
```

output
```
(2*atanh((2*b*x)/a + 1)*(A*b - B*a))/a^2 - A/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx}{x^2(a+bx)}\,dx = -\frac{1}{x}$$

input
```
int((B*x+A)/x^2/(b*x+a),x)
```

output
```
( - 1)/x
```

3.150 $\int \frac{A+Bx}{x^3(a+bx)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{A+Bx}{x^3(a+bx)} \, dx = -\frac{A}{2ax^2} + \frac{Ab-aB}{a^2x} + \frac{b(Ab-aB)\log(x)}{a^3} - \frac{b(Ab-aB)\log(a+bx)}{a^3}$$

output
```
-1/2*A/a/x^2+(A*b-B*a)/a^2/x+b*(A*b-B*a)*ln(x)/a^3-b*(A*b-B*a)*ln(b*x+a)/a
^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^3(a+bx)} \, dx = \frac{-\frac{a(aA-2Abx+2aBx)}{x^2} + 2b(Ab-aB)\log(x) + 2b(-Ab+aB)\log(a+bx)}{2a^3}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x)),x]
```

output
```
(-((a*(a*A - 2*A*b*x + 2*a*B*x))/x^2) + 2*b*(A*b - a*B)*Log[x] + 2*b*(-(A*
b) + a*B)*Log[a + b*x])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(a+bx)} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{b^2(aB-Ab)}{a^3(a+bx)}-\frac{b(aB-Ab)}{a^3x}+\frac{aB-Ab}{a^2x^2}+\frac{A}{ax^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\log(x)(Ab-aB)}{a^3}-\frac{b(Ab-aB)\log(a+bx)}{a^3}+\frac{Ab-aB}{a^2x}-\frac{A}{2ax^2}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x)),x]
```

output
```
-1/2*A/(a*x^2) + (A*b - a*B)/(a^2*x) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A*
b - a*B)*Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{\frac{(Ab-Ba)x}{a^2}-\frac{A}{2a}}{x^2}+\frac{b(Ab-Ba)\ln(x)}{a^3}-\frac{b(Ab-Ba)\ln(bx+a)}{a^3}$	61
default	$-\frac{A}{2a\,x^2}-\frac{-Ab+Ba}{x\,a^2}+\frac{b(Ab-Ba)\ln(x)}{a^3}-\frac{b(Ab-Ba)\ln(bx+a)}{a^3}$	62
risch	$\frac{\frac{(Ab-Ba)x}{a^2}-\frac{A}{2a}}{x^2}-\frac{b^2\ln(bx+a)A}{a^3}+\frac{b\ln(bx+a)B}{a^2}+\frac{b^2\ln(-x)A}{a^3}-\frac{b\ln(-x)B}{a^2}$	76
parallelrisch	$\frac{2A\ln(x)x^2b^2-2A\ln(bx+a)x^2b^2-2B\ln(x)x^2ab+2B\ln(bx+a)x^2ab+2aAbx-2B\,a^2x-a^2A}{2a^3x^2}$	79

input
```
int((B*x+A)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
((A*b-B*a)/a^2*x-1/2*A/a)/x^2+b*(A*b-B*a)*ln(x)/a^3-b*(A*b-B*a)*ln(b*x+a)/
a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{x^3(a+bx)}\,dx = \frac{2\,(Bab-Ab^2)x^2\log{(bx+a)}-2\,(Bab-Ab^2)x^2\log{(x)}-Aa^2-2\,(Ba^2-Aab)x}{2\,a^3x^2}$$

input
```
integrate((B*x+A)/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*(B*a*b - A*b^2)*x^2*log(b*x + a) - 2*(B*a*b - A*b^2)*x^2*log(x) - A
*a^2 - 2*(B*a^2 - A*a*b)*x)/(a^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

$$\int \frac{A+Bx}{x^3(a+bx)}\,dx = \frac{-Aa+x(2Ab-2Ba)}{2a^2x^2} - \frac{b(-Ab+Ba)\log\left(x+\frac{-Aab^2+Ba^2b-ab(-Ab+Ba)}{-2Ab^3+2Bab^2}\right)}{a^3} + \frac{b(-Ab+Ba)\log\left(x+\frac{-Aab^2+Ba^2b+ab(-Ab+Ba)}{-2Ab^3+2Bab^2}\right)}{a^3}$$

input
```
integrate((B*x+A)/x**3/(b*x+a),x)
```

output
```
(-A*a + x*(2*A*b - 2*B*a))/(2*a**2*x**2) - b*(-A*b + B*a)*log(x + (-A*a*b*
*2 + B*a**2*b - a*b*(-A*b + B*a))/(-2*A*b**3 + 2*B*a*b**2))/a**3 + b*(-A*b
 + B*a)*log(x + (-A*a*b**2 + B*a**2*b + a*b*(-A*b + B*a))/(-2*A*b**3 + 2*B
*a*b**2))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x^3(a+bx)}\,dx = \frac{(Bab-Ab^2)\log(bx+a)}{a^3} - \frac{(Bab-Ab^2)\log(x)}{a^3} - \frac{Aa+2\,(Ba-Ab)x}{2\,a^2x^2}$$

input
```
integrate((B*x+A)/x^3/(b*x+a),x, algorithm="maxima")
```

output
```
(B*a*b - A*b^2)*log(b*x + a)/a^3 - (B*a*b - A*b^2)*log(x)/a^3 - 1/2*(A*a +
 2*(B*a - A*b)*x)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^3(a+bx)}\,dx = -\frac{(Bab-Ab^2)\log(|x|)}{a^3}+\frac{(Bab^2-Ab^3)\log(|bx+a|)}{a^3b} - \frac{Aa^2+2\,(Ba^2-Aab)x}{2\,a^3x^2}$$

input
```
integrate((B*x+A)/x^3/(b*x+a),x, algorithm="giac")
```

output
```
-(B*a*b - A*b^2)*log(abs(x))/a^3 + (B*a*b^2 - A*b^3)*log(abs(b*x + a))/(a^
3*b) - 1/2*(A*a^2 + 2*(B*a^2 - A*a*b)*x)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^3(a+bx)}\,dx = -\frac{\frac{A}{2\,a}-\frac{x\,(A\,b-B\,a)}{a^2}}{x^2} - \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(A\,b-B\,a)\,(a+2\,b\,x)}{a\,(A\,b^2-B\,a\,b)}\right)\,(A\,b-B\,a)}{a^3}$$

input
```
int((A + B*x)/(x^3*(a + b*x)),x)
```

output
```
- (A/(2*a) - (x*(A*b - B*a))/a^2)/x^2 - (2*b*atanh((b*(A*b - B*a)*(a + 2*b
*x))/(a*(A*b^2 - B*a*b)))*(A*b - B*a))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.08

$$\int \frac{A+Bx}{x^3(a+bx)}\,dx = -\frac{1}{2x^2}$$

input
```
int((B*x+A)/x^3/(b*x+a),x)
```

output

```
( - 1)/(2*x**2)
```

3.151 $\int \frac{A+Bx}{x^4(a+bx)} dx$

Optimal result

Integrand size = 16, antiderivative size = 86

$$\int \frac{A+Bx}{x^4(a+bx)} dx = -\frac{A}{3ax^3} + \frac{Ab-aB}{2a^2x^2} - \frac{b(Ab-aB)}{a^3x} - \frac{b^2(Ab-aB)\log(x)}{a^4} + \frac{b^2(Ab-aB)\log(a+bx)}{a^4}$$

output
```
-1/3*A/a/x^3+1/2*(A*b-B*a)/a^2/x^2-b*(A*b-B*a)/a^3/x-b^2*(A*b-B*a)*ln(x)/a
^4+b^2*(A*b-B*a)*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^4(a+bx)} dx = \frac{\frac{a(-6Ab^2x^2+3abx(A+2Bx)-a^2(2A+3Bx))}{x^3} + 6b^2(-Ab+aB)\log(x) + 6b^2(Ab-aB)\log(a+bx)}{6a^4}$$

input
```
Integrate[(A + B*x)/(x^4*(a + b*x)),x]
```

output
```
((a*(-6*A*b^2*x^2 + 3*a*b*x*(A + 2*B*x) - a^2*(2*A + 3*B*x)))/x^3 + 6*b^2*
(-(A*b) + a*B)*Log[x] + 6*b^2*(A*b - a*B)*Log[a + b*x])/(6*a^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4(a+bx)} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{b^3(aB-Ab)}{a^4(a+bx)}+\frac{b^2(aB-Ab)}{a^4x}-\frac{b(aB-Ab)}{a^3x^2}+\frac{aB-Ab}{a^2x^3}+\frac{A}{ax^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\log(x)(Ab-aB)}{a^4}+\frac{b^2(Ab-aB)\log(a+bx)}{a^4}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{3ax^3}$$

input
```
Int[(A + B*x)/(x^4*(a + b*x)),x]
```

output
```
-1/3*A/(a*x^3) + (A*b - a*B)/(2*a^2*x^2) - (b*(A*b - a*B))/(a^3*x) - (b^2*
(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{3a\,x^{3}}-\frac{-Ab+Ba}{2a^{2}x^{2}}-\frac{b(Ab-Ba)}{a^{3}x}-\frac{b^{2}(Ab-Ba)\ln(x)}{a^{4}}+\frac{b^{2}(Ab-Ba)\ln(bx+a)}{a^{4}}$
norman	$\frac{-\frac{A}{3a}+\frac{(Ab-Ba)x}{2a^{2}}-\frac{(Ab-Ba)b\,x^{2}}{a^{3}}}{x^{3}}+\frac{b^{2}(Ab-Ba)\ln(bx+a)}{a^{4}}-\frac{b^{2}(Ab-Ba)\ln(x)}{a^{4}}$
risch	$\frac{-\frac{A}{3a}+\frac{(Ab-Ba)x}{2a^{2}}-\frac{(Ab-Ba)b\,x^{2}}{a^{3}}}{x^{3}}+\frac{b^{3}\ln(-bx-a)A}{a^{4}}-\frac{b^{2}\ln(-bx-a)B}{a^{3}}-\frac{b^{3}\ln(x)A}{a^{4}}+\frac{b^{2}\ln(x)B}{a^{3}}$
parallelrisch	$-\frac{6A\ln(x)x^{3}b^{3}-6A\ln(bx+a)x^{3}b^{3}-6B\ln(x)x^{3}a\,b^{2}+6B\ln(bx+a)x^{3}a\,b^{2}+6aA\,b^{2}x^{2}-6B\,a^{2}b\,x^{2}-3a^{2}Abx+3B\,a^{3}x+2a^{3}A}{6a^{4}x^{3}}$

input
```
int((B*x+A)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a/x^3-1/2*(-A*b+B*a)/a^2/x^2-b*(A*b-B*a)/a^3/x-b^2*(A*b-B*a)*ln(x)/
a^4+b^2*(A*b-B*a)*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^4(a+bx)}\,dx = -\frac{6\,(Bab^{2}-Ab^{3})x^{3}\log\left(bx+a\right)-6\,(Bab^{2}-Ab^{3})x^{3}\log\left(x\right)+2\,Aa^{3}-6\,(Ba^{2}b-Aab^{2})x^{2}+3\,(Ba^{3}-A}{6\,a^{4}x^{3}}$$

input
```
integrate((B*x+A)/x^4/(b*x+a),x, algorithm="fricas")
```

output
```
-1/6*(6*(B*a*b^2 - A*b^3)*x^3*log(b*x + a) - 6*(B*a*b^2 - A*b^3)*x^3*log(x
) + 2*A*a^3 - 6*(B*a^2*b - A*a*b^2)*x^2 + 3*(B*a^3 - A*a^2*b)*x)/(a^4*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(75) = 150$.

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx}{x^4(a+bx)}\,dx = \frac{-2Aa^2 + x^2(-6Ab^2 + 6Bab) + x(3Aab - 3Ba^2)}{6a^3x^3} + \frac{b^2(-Ab+Ba)\log\left(x + \frac{-Aab^3+Ba^2b^2-ab^2(-Ab+Ba)}{-2Ab^4+2Bab^3}\right)}{a^4} - \frac{b^2(-Ab+Ba)\log\left(x + \frac{-Aab^3+Ba^2b^2+ab^2(-Ab+Ba)}{-2Ab^4+2Bab^3}\right)}{a^4}$$

input
```
integrate((B*x+A)/x**4/(b*x+a),x)
```

output
```
(-2*A*a**2 + x**2*(-6*A*b**2 + 6*B*a*b) + x*(3*A*a*b - 3*B*a**2))/(6*a**3*
x**3) + b**2*(-A*b + B*a)*log(x + (-A*a*b**3 + B*a**2*b**2 - a*b**2*(-A*b
+ B*a))/(-2*A*b**4 + 2*B*a*b**3))/a**4 - b**2*(-A*b + B*a)*log(x + (-A*a*b
**3 + B*a**2*b**2 + a*b**2*(-A*b + B*a))/(-2*A*b**4 + 2*B*a*b**3))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^4(a+bx)}\,dx = -\frac{(Bab^2 - Ab^3)\log(bx+a)}{a^4} + \frac{(Bab^2 - Ab^3)\log(x)}{a^4} - \frac{2\,Aa^2 - 6\,(Bab - Ab^2)x^2 + 3\,(Ba^2 - Aab)x}{6\,a^3x^3}$$

input
```
integrate((B*x+A)/x^4/(b*x+a),x, algorithm="maxima")
```

output
```
-(B*a*b^2 - A*b^3)*log(b*x + a)/a^4 + (B*a*b^2 - A*b^3)*log(x)/a^4 - 1/6*(
2*A*a^2 - 6*(B*a*b - A*b^2)*x^2 + 3*(B*a^2 - A*a*b)*x)/(a^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{x^4(a+bx)}\,dx = \frac{(Bab^2-Ab^3)\log(|x|)}{a^4} - \frac{(Bab^3-Ab^4)\log(|bx+a|)}{a^4b} - \frac{2\,Aa^3-6\,(Ba^2b-Aab^2)x^2+3\,(Ba^3-Aa^2b)x}{6\,a^4x^3}$$

input
```
integrate((B*x+A)/x^4/(b*x+a),x, algorithm="giac")
```

output
```
(B*a*b^2 - A*b^3)*log(abs(x))/a^4 - (B*a*b^3 - A*b^4)*log(abs(b*x + a))/(a
^4*b) - 1/6*(2*A*a^3 - 6*(B*a^2*b - A*a*b^2)*x^2 + 3*(B*a^3 - A*a^2*b)*x)/
(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x^4(a+bx)}\,dx = \frac{2\,b^2\,\mathrm{atanh}\left(\frac{b^2\,(A\,b-B\,a)\,(a+2\,b\,x)}{a\,(A\,b^3-B\,a\,b^2)}\right)\,(A\,b-B\,a)}{a^4} - \frac{\frac{A}{3\,a}-\frac{x\,(A\,b-B\,a)}{2\,a^2}+\frac{b\,x^2\,(A\,b-B\,a)}{a^3}}{x^3}$$

input
```
int((A + B*x)/(x^4*(a + b*x)),x)
```

output
```
(2*b^2*atanh((b^2*(A*b - B*a)*(a + 2*b*x))/(a*(A*b^3 - B*a*b^2)))*(A*b - B
*a))/a^4 - (A/(3*a) - (x*(A*b - B*a))/(2*a^2) + (b*x^2*(A*b - B*a))/a^3)/x
^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx}{x^4(a+bx)}\,dx=-\frac{1}{3x^3}$$

input
```
int((B*x+A)/x^4/(b*x+a),x)
```

output
```
( - 1)/(3*x**3)
```

3.152 $\int \frac{A+Bx}{x^5(a+bx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = -\frac{A}{4ax^4}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{b^2(Ab-aB)}{a^4x}+\frac{b^3(Ab-aB)\log(x)}{a^5}-\frac{b^3(Ab-aB)\log(a+bx)}{a^5}$$

output
```
-1/4*A/a/x^4+1/3*(A*b-B*a)/a^2/x^3-1/2*b*(A*b-B*a)/a^3/x^2+b^2*(A*b-B*a)/a
^4/x+b^3*(A*b-B*a)*ln(x)/a^5-b^3*(A*b-B*a)*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = \frac{\frac{a(12Ab^3x^3-6ab^2x^2(A+2Bx)+2a^2bx(2A+3Bx)-a^3(3A+4Bx))}{x^4}+12b^3(Ab-aB)\log(x)-12b^3(Ab-aB)\log(a+bx)}{12a^5}$$

input
```
Integrate[(A + B*x)/(x^5*(a + b*x)),x]
```

output
```
((a*(12*A*b^3*x^3 - 6*a*b^2*x^2*(A + 2*B*x) + 2*a^2*b*x*(2*A + 3*B*x) - a^
3*(3*A + 4*B*x)))/x^4 + 12*b^3*(A*b - a*B)*Log[x] - 12*b^3*(A*b - a*B)*Log
[a + b*x])/(12*a^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{b^4(aB-Ab)}{a^5(a+bx)}-\frac{b^3(aB-Ab)}{a^5x}+\frac{b^2(aB-Ab)}{a^4x^2}-\frac{b(aB-Ab)}{a^3x^3}+\frac{aB-Ab}{a^2x^4}+\frac{A}{ax^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\log(x)(Ab-aB)}{a^5}-\frac{b^3(Ab-aB)\log(a+bx)}{a^5}+\frac{b^2(Ab-aB)}{a^4x}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{4ax^4}$$

input
```
Int[(A + B*x)/(x^5*(a + b*x)),x]
```

output
```
-1/4*A/(a*x^4) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(2*a^3*x^2) + (
b^2*(A*b - a*B))/(a^4*x) + (b^3*(A*b - a*B)*Log[x])/a^5 - (b^3*(A*b - a*B)
*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

method	result
default	$-\frac{A}{4a\,x^4}-\frac{-Ab+Ba}{3x^3a^2}+\frac{b^3(Ab-Ba)\ln(x)}{a^5}-\frac{b(Ab-Ba)}{2a^3x^2}+\frac{b^2(Ab-Ba)}{a^4x}-\frac{b^3(Ab-Ba)\ln(bx+a)}{a^5}$
norman	$\frac{\frac{(Ab-Ba)b^2x^3}{a^4}-\frac{A}{4a}+\frac{(Ab-Ba)x}{3a^2}-\frac{(Ab-Ba)b\,x^2}{2a^3}}{x^4}+\frac{b^3(Ab-Ba)\ln(x)}{a^5}-\frac{b^3(Ab-Ba)\ln(bx+a)}{a^5}$
risch	$\frac{\frac{(Ab-Ba)b^2x^3}{a^4}-\frac{A}{4a}+\frac{(Ab-Ba)x}{3a^2}-\frac{(Ab-Ba)b\,x^2}{2a^3}}{x^4}-\frac{b^4\ln(bx+a)A}{a^5}+\frac{b^3\ln(bx+a)B}{a^4}+\frac{b^4\ln(-x)A}{a^5}-\frac{b^3\ln(-x)B}{a^4}$
parallelrisch	$\frac{12A\ln(x)x^4b^4-12A\ln(bx+a)x^4b^4-12B\ln(x)x^4a\,b^3+12B\ln(bx+a)x^4a\,b^3+12Aa\,b^3x^3-12B\,a^2b^2x^3-6A\,a^2b^2x^2+6B\,a^3b\,x^2}{12a^5x^4}$

input
```
int((B*x+A)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a/x^4-1/3*(-A*b+B*a)/x^3/a^2+b^3*(A*b-B*a)*ln(x)/a^5-1/2*b*(A*b-B*a
)/a^3/x^2+b^2*(A*b-B*a)/a^4/x-b^3*(A*b-B*a)*ln(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int\frac{A+Bx}{x^5(a+bx)}\,dx$$
$$=\frac{12\,(Bab^3-Ab^4)x^4\log(bx+a)-12\,(Bab^3-Ab^4)x^4\log(x)-3\,Aa^4-12\,(Ba^2b^2-Aab^3)x^3+6\,(Ba^3b-}{12\,a^5x^4}$$

input
```
integrate((B*x+A)/x^5/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(12*(B*a*b^3 - A*b^4)*x^4*log(b*x + a) - 12*(B*a*b^3 - A*b^4)*x^4*log
(x) - 3*A*a^4 - 12*(B*a^2*b^2 - A*a*b^3)*x^3 + 6*(B*a^3*b - A*a^2*b^2)*x^2
 - 4*(B*a^4 - A*a^3*b)*x)/(a^5*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(94) = 188$.

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = \frac{-3Aa^3+x^3\cdot(12Ab^3-12Bab^2)+x^2(-6Aab^2+6Ba^2b)+x(4Aa^2b-4Ba^3)}{12a^4x^4} - \frac{b^3(-Ab+Ba)\log\left(x+\frac{-Aab^4+Ba^2b^3-ab^3(-Ab+Ba)}{-2Ab^5+2Bab^4}\right)}{a^5} + \frac{b^3(-Ab+Ba)\log\left(x+\frac{-Aab^4+Ba^2b^3+ab^3(-Ab+Ba)}{-2Ab^5+2Bab^4}\right)}{a^5}$$

input
```
integrate((B*x+A)/x**5/(b*x+a),x)
```

output
```
(-3*A*a**3 + x**3*(12*A*b**3 - 12*B*a*b**2) + x**2*(-6*A*a*b**2 + 6*B*a**2
*b) + x*(4*A*a**2*b - 4*B*a**3))/(12*a**4*x**4) - b**3*(-A*b + B*a)*log(x
+ (-A*a*b**4 + B*a**2*b**3 - a*b**3*(-A*b + B*a))/(-2*A*b**5 + 2*B*a*b**4)
)/a**5 + b**3*(-A*b + B*a)*log(x + (-A*a*b**4 + B*a**2*b**3 + a*b**3*(-A*b
 + B*a))/(-2*A*b**5 + 2*B*a*b**4))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = \frac{(Bab^3-Ab^4)\log(bx+a)}{a^5} - \frac{(Bab^3-Ab^4)\log(x)}{a^5} - \frac{3\,Aa^3+12\,(Bab^2-Ab^3)x^3-6\,(Ba^2b-Aab^2)x^2+4\,(Ba^3-Aa^2b)x}{12\,a^4x^4}$$

input
```
integrate((B*x+A)/x^5/(b*x+a),x, algorithm="maxima")
```

output
```
(B*a*b^3 - A*b^4)*log(b*x + a)/a^5 - (B*a*b^3 - A*b^4)*log(x)/a^5 - 1/12*(
3*A*a^3 + 12*(B*a*b^2 - A*b^3)*x^3 - 6*(B*a^2*b - A*a*b^2)*x^2 + 4*(B*a^3
- A*a^2*b)*x)/(a^4*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = -\frac{(Bab^3-Ab^4)\log(|x|)}{a^5} + \frac{(Bab^4-Ab^5)\log(|bx+a|)}{a^5b} - \frac{3\,Aa^4+12\,(Ba^2b^2-Aab^3)x^3-6\,(Ba^3b-Aa^2b^2)x^2+4\,(Ba^4-Aa^3b)x}{12\,a^5x^4}$$

input
```
integrate((B*x+A)/x^5/(b*x+a),x, algorithm="giac")
```

output
```
-(B*a*b^3 - A*b^4)*log(abs(x))/a^5 + (B*a*b^4 - A*b^5)*log(abs(b*x + a))/(
a^5*b) - 1/12*(3*A*a^4 + 12*(B*a^2*b^2 - A*a*b^3)*x^3 - 6*(B*a^3*b - A*a^2
*b^2)*x^2 + 4*(B*a^4 - A*a^3*b)*x)/(a^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = -\frac{\frac{A}{4\,a}-\frac{x\,(A\,b-B\,a)}{3\,a^2}-\frac{b^2\,x^3\,(A\,b-B\,a)}{a^4}+\frac{b\,x^2\,(A\,b-B\,a)}{2\,a^3}}{x^4} - \frac{2\,b^3\,\mathrm{atanh}\left(\frac{b^3\,(A\,b-B\,a)\,(a+2\,b\,x)}{a\,(A\,b^4-B\,a\,b^3)}\right)\,(A\,b-B\,a)}{a^5}$$

input
```
int((A + B*x)/(x^5*(a + b*x)),x)
```

output
```
- (A/(4*a) - (x*(A*b - B*a))/(3*a^2) - (b^2*x^3*(A*b - B*a))/a^4 + (b*x^2*
(A*b - B*a))/(2*a^3))/x^4 - (2*b^3*atanh((b^3*(A*b - B*a)*(a + 2*b*x))/(a*
(A*b^4 - B*a*b^3)))*(A*b - B*a))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.05

$$\int \frac{A+Bx}{x^5(a+bx)}\,dx = -\frac{1}{4x^4}$$

input
```
int((B*x+A)/x^5/(b*x+a),x)
```

output
```
( - 1)/(4*x**4)
```

3.153 $\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^2}{2b^4} + \frac{(Ab-2aB)x^3}{3b^3} + \frac{Bx^4}{4b^2} - \frac{a^4(Ab-aB)}{b^6(a+bx)} - \frac{a^3(4Ab-5aB)\log(a+bx)}{b^6}$$

output
```
a^2*(3*A*b-4*B*a)*x/b^5-1/2*a*(2*A*b-3*B*a)*x^2/b^4+1/3*(A*b-2*B*a)*x^3/b^
3+1/4*B*x^4/b^2-a^4*(A*b-B*a)/b^6/(b*x+a)-a^3*(4*A*b-5*B*a)*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{-12a^2b(-3Ab+4aB)x + 6ab^2(-2Ab+3aB)x^2 + 4b^3(Ab-2aB)x^3 + 3b^4Bx^4 + \frac{12a^4(-Ab+aB)}{a+bx} + 12a^3(-}{12b^6}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x)^2,x]
```

output
```
(-12*a^2*b*(-3*A*b + 4*a*B)*x + 6*a*b^2*(-2*A*b + 3*a*B)*x^2 + 4*b^3*(A*b
- 2*a*B)*x^3 + 3*b^4*B*x^4 + (12*a^4*(-(A*b) + a*B))/(a + b*x) + 12*a^3*(-
4*A*b + 5*a*B)*Log[a + b*x])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)^2}+\frac{a^3(5aB-4Ab)}{b^5(a+bx)}-\frac{a^2(4aB-3Ab)}{b^5}+\frac{ax(3aB-2Ab)}{b^4}+\frac{x^2(Ab-2aB)}{b^3}+\frac{Bx^3}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4(Ab-aB)}{b^6(a+bx)}-\frac{a^3(4Ab-5aB)\log(a+bx)}{b^6}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^4}{4b^2}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x)^2,x]
```

output
```
(a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*
a*B)*x^3)/(3*b^3) + (B*x^4)/(4*b^2) - (a^4*(A*b - a*B))/(b^6*(a + b*x)) -
(a^3*(4*A*b - 5*a*B)*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{4} B\, b^3 x^4+\frac{1}{3} A\, b^3 x^3-\frac{2}{3} B a\, b^2 x^3-a A\, b^2 x^2+\frac{3}{2} B\, a^2 b\, x^2+3 a^2 A b x-4 B\, a^3 x}{b^5}-\frac{a^4 (A b-B a)}{b^6 (b x+a)}-\frac{a^3 (4 A b-5 B a) \ln (b x+a)}{b^6}$
norman	$\frac{\frac{B\, x^5}{4 b}-\frac{a \left(4 A\, a^3 b-5 B\, a^4\right)}{b^6}+\frac{(4 A b-5 B a) x^4}{12 b^2}-\frac{a (4 A b-5 B a) x^3}{6 b^3}+\frac{a^2 (4 A b-5 B a) x^2}{2 b^4}}{b x+a}-\frac{a^3 (4 A b-5 B a) \ln (b x+a)}{b^6}$
risch	$\frac{B\, x^4}{4 b^2}+\frac{A\, x^3}{3 b^2}-\frac{2 B a\, x^3}{3 b^3}-\frac{a A\, x^2}{b^3}+\frac{3 B\, a^2 x^2}{2 b^4}+\frac{3 a^2 A x}{b^4}-\frac{4 B\, a^3 x}{b^5}-\frac{a^4 A}{b^5 (b x+a)}+\frac{a^5 B}{b^6 (b x+a)}-\frac{4 a^3 \ln (b x+a) A}{b^5}+$
parallelrisch	$-\frac{-3 b^5 B\, x^5-4 A\, b^5 x^4+5 B a\, b^4 x^4+8 A a\, b^4 x^3-10 B\, a^2 b^3 x^3+48 A \ln (b x+a) x\, a^3 b^2-24 A\, a^2 b^3 x^2-60 B \ln (b x+a) x\, a^4 b+30 B\, a^3 b}{12 b^6 (b x+a)}$

input
```
int(x^4*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/4*B*b^3*x^4+1/3*A*b^3*x^3-2/3*B*a*b^2*x^3-a*A*b^2*x^2+3/2*B*a^2*b
*x^2+3*a^2*A*b*x-4*B*a^3*x)-a^4*(A*b-B*a)/b^6/(b*x+a)-a^3*(4*A*b-5*B*a)*ln
(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{3\,Bb^5x^5 + 12\,Ba^5 - 12\,Aa^4b - (5\,Bab^4 - 4\,Ab^5)x^4 + 2\,(5\,Ba^2b^3 - 4\,Aab^4)x^3 - 6\,(5\,Ba^3b^2 - 4\,Aa^2b^3)x^2}{12\,(b^7x + ab^6)}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*B*b^5*x^5 + 12*B*a^5 - 12*A*a^4*b - (5*B*a*b^4 - 4*A*b^5)*x^4 + 2*
(5*B*a^2*b^3 - 4*A*a*b^4)*x^3 - 6*(5*B*a^3*b^2 - 4*A*a^2*b^3)*x^2 - 12*(4*
B*a^4*b - 3*A*a^3*b^2)*x + 12*(5*B*a^5 - 4*A*a^4*b + (5*B*a^4*b - 4*A*a^3*
b^2)*x)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{Bx^4}{4b^2} + \frac{a^3(-4Ab+5Ba)\log{(a+bx)}}{b^6} + x^3\left(\frac{A}{3b^2} - \frac{2Ba}{3b^3}\right) + x^2\left(-\frac{Aa}{b^3} + \frac{3Ba^2}{2b^4}\right) + x\left(\frac{3Aa^2}{b^4} - \frac{4Ba^3}{b^5}\right) + \frac{-Aa^4b + Ba^5}{ab^6 + b^7x}$$

input
```
integrate(x**4*(B*x+A)/(b*x+a)**2,x)
```

output
```
B*x**4/(4*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x)/b**6 + x**3*(A/(3*b**
2) - 2*B*a/(3*b**3)) + x**2*(-A*a/b**3 + 3*B*a**2/(2*b**4)) + x*(3*A*a**2/
b**4 - 4*B*a**3/b**5) + (-A*a**4*b + B*a**5)/(a*b**6 + b**7*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{Ba^5-Aa^4b}{b^7x+ab^6} + \frac{3\,Bb^3x^4-4\,(2\,Bab^2-Ab^3)x^3+6\,(3\,Ba^2b-2\,Aab^2)x^2-12\,(4\,Ba^3-3\,Aa^2b)x}{12\,b^5} + \frac{(5\,Ba^4-4\,Aa^3b)\log(bx+a)}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(B*a^5 - A*a^4*b)/(b^7*x + a*b^6) + 1/12*(3*B*b^3*x^4 - 4*(2*B*a*b^2 - A*b
^3)*x^3 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^2 - 12*(4*B*a^3 - 3*A*a^2*b)*x)/b^5
+ (5*B*a^4 - 4*A*a^3*b)*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.55

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{(bx+a)^4\left(3\,B-\frac{4\,(5\,Bab-Ab^2)}{(bx+a)b}+\frac{12\,(5\,Ba^2b^2-2\,Aab^3)}{(bx+a)^2b^2}-\frac{24\,(5\,Ba^3b^3-3\,Aa^2b^4)}{(bx+a)^3b^3}\right)}{12\,b^6} - \frac{(5\,Ba^4-4\,Aa^3b)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^6} + \frac{\frac{Ba^5b^4}{bx+a}-\frac{Aa^4b^5}{bx+a}}{b^{10}}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/12*(b*x + a)^4*(3*B - 4*(5*B*a*b - A*b^2)/((b*x + a)*b) + 12*(5*B*a^2*b^
2 - 2*A*a*b^3)/((b*x + a)^2*b^2) - 24*(5*B*a^3*b^3 - 3*A*a^2*b^4)/((b*x +
a)^3*b^3))/b^6 - (5*B*a^4 - 4*A*a^3*b)*log(abs(b*x + a)/((b*x + a)^2*abs(b
)))/b^6 + (B*a^5*b^4/(b*x + a) - A*a^4*b^5/(b*x + a))/b^10
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.53

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = x\left(\frac{2\,a\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}\right) + x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right) - x^2\left(\frac{a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{2\,b^4}\right) + \frac{\ln(a+b\,x)\,\left(5\,B\,a^4-4\,A\,a^3\,b\right)}{b^6}+\frac{B\,x^4}{4\,b^2}+\frac{B\,a^5-A\,a^4\,b}{b\,\left(x\,b^6+a\,b^5\right)}$$

input
```
int((x^4*(A + B*x))/(a + b*x)^2,x)
```

output
```
x*((2*a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (
2*B*a)/b^3))/b^2) + x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) - x^2*((a*(A/b^2 - (
2*B*a)/b^3))/b + (B*a^2)/(2*b^4)) + (log(a + b*x)*(5*B*a^4 - 4*A*a^3*b))/b
^6 + (B*x^4)/(4*b^2) + (B*a^5 - A*a^4*b)/(b*(a*b^5 + b^6*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.46

$$\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx = \frac{12\log(bx+a)\,a^4-12a^3bx+6a^2b^2x^2-4a\,b^3x^3+3b^4x^4}{12b^5}$$

input
```
int(x^4*(B*x+A)/(b*x+a)^2,x)
```

output
```
(12*log(a + b*x)*a**4 - 12*a**3*b*x + 6*a**2*b**2*x**2 - 4*a*b**3*x**3 + 3
*b**4*x**4)/(12*b**5)
```

3.154 $\int \frac{x^3(A+Bx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \frac{x^3(A+Bx)}{(a+bx)^2} \, dx = -\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^3}{3b^2}+\frac{a^3(Ab-aB)}{b^5(a+bx)}+\frac{a^2(3Ab-4aB)\log(a+bx)}{b^5}$$

output
```
-a*(2*A*b-3*B*a)*x/b^4+1/2*(A*b-2*B*a)*x^2/b^3+1/3*B*x^3/b^2+a^3*(A*b-B*a)
/b^5/(b*x+a)+a^2*(3*A*b-4*B*a)*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx)}{(a+bx)^2} \, dx = \frac{6ab(-2Ab+3aB)x+3b^2(Ab-2aB)x^2+2b^3Bx^3+\frac{6a^3(Ab-aB)}{a+bx}+6a^2(3Ab-4aB)\log(a+bx)}{6b^5}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x)^2,x]
```

output
```
(6*a*b*(-2*A*b + 3*a*B)*x + 3*b^2*(A*b - 2*a*B)*x^2 + 2*b^3*B*x^3 + (6*a^3
*(A*b - a*B))/(a + b*x) + 6*a^2*(3*A*b - 4*a*B)*Log[a + b*x])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)^2}-\frac{a^2(4aB-3Ab)}{b^4(a+bx)}+\frac{a(3aB-2Ab)}{b^4}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^2}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3(Ab-aB)}{b^5(a+bx)}+\frac{a^2(3Ab-4aB)\log(a+bx)}{b^5}-\frac{ax(2Ab-3aB)}{b^4}+\frac{x^2(Ab-2aB)}{2b^3}+\frac{Bx^3}{3b^2}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x)^2,x]
```

output
```
-((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^3)/(3*b^
2) + (a^3*(A*b - a*B))/(b^5*(a + b*x)) + (a^2*(3*A*b - 4*a*B)*Log[a + b*x]
)/b^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result
default	$-\frac{-\frac{1}{3} B\, b^{2} x^{3}-\frac{1}{2} A\, b^{2} x^{2}+B a b\, x^{2}+2 a A b x -3 B\, a^{2} x}{b^{4}}+\frac{a^{3} \left(A b -B a \right)}{b^{5} \left(b x +a \right)}+\frac{a^{2} \left(3 A b -4 B a \right) \ln \left(b x +a \right)}{b^{5}}$
norman	$\frac{\frac{a \left(3 a^{2} b A -4 a^{3} B \right)}{b^{5}}+\frac{B\, x^{4}}{3 b}+\frac{\left(3 A b -4 B a \right) x^{3}}{6 b^{2}}-\frac{a \left(3 A b -4 B a \right) x^{2}}{2 b^{3}}}{b x +a}+\frac{a^{2} \left(3 A b -4 B a \right) \ln \left(b x +a \right)}{b^{5}}$
risch	$\frac{B\, x^{3}}{3 b^{2}}+\frac{A\, x^{2}}{2 b^{2}}-\frac{B a\, x^{2}}{b^{3}}-\frac{2 a A x}{b^{3}}+\frac{3 B\, a^{2} x}{b^{4}}+\frac{a^{3} A}{b^{4} \left(b x +a \right)}-\frac{a^{4} B}{b^{5} \left(b x +a \right)}+\frac{3 a^{2} \ln \left(b x +a \right) A}{b^{4}}-\frac{4 a^{3} \ln \left(b x +a \right) B}{b^{5}}$
parallelrisch	$\frac{2 B\, x^{4} b^{4}+3 A\, x^{3} b^{4}-4 B\, x^{3} a\, b^{3}+18 A \ln \left(b x +a \right) x\, a^{2} b^{2}-9 A\, x^{2} a\, b^{3}-24 B \ln \left(b x +a \right) x\, a^{3} b +12 B\, x^{2} a^{2} b^{2}+18 A \ln \left(b x +a \right) a^{3} b -24 B \ln}{6 b^{5} \left(b x +a \right)}$

input
```
int(x^3*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/3*B*b^2*x^3-1/2*A*b^2*x^2+B*a*b*x^2+2*a*A*b*x-3*B*a^2*x)+a^3*(A
*b-B*a)/b^5/(b*x+a)+a^2*(3*A*b-4*B*a)*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx$$
$$= \frac{2\,Bb^4x^4 - 6\,Ba^4 + 6\,Aa^3b - (4\,Bab^3 - 3\,Ab^4)x^3 + 3\,(4\,Ba^2b^2 - 3\,Aab^3)x^2 + 6\,(3\,Ba^3b - 2\,Aa^2b^2)x - 6\,(}{6\,(b^6x + ab^5)}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^4*x^4 - 6*B*a^4 + 6*A*a^3*b - (4*B*a*b^3 - 3*A*b^4)*x^3 + 3*(4*
B*a^2*b^2 - 3*A*a*b^3)*x^2 + 6*(3*B*a^3*b - 2*A*a^2*b^2)*x - 6*(4*B*a^4 -
3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx = \frac{Bx^3}{3b^2} - \frac{a^2(-3Ab+4Ba)\log{(a+bx)}}{b^5} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{Aa^3b - Ba^4}{ab^5 + b^6x}$$

input
```
integrate(x**3*(B*x+A)/(b*x+a)**2,x)
```

output
```
B*x**3/(3*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x)/b**5 + x**2*(A/(2*b**
2) - B*a/b**3) + x*(-2*A*a/b**3 + 3*B*a**2/b**4) + (A*a**3*b - B*a**4)/(a*
b**5 + b**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx = -\frac{Ba^4 - Aa^3b}{b^6x + ab^5} + \frac{2\,Bb^2x^3 - 3\,(2\,Bab - Ab^2)x^2 + 6\,(3\,Ba^2 - 2\,Aab)x}{6\,b^4} - \frac{(4\,Ba^3 - 3\,Aa^2b)\log\left(bx + a\right)}{b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(B*a^4 - A*a^3*b)/(b^6*x + a*b^5) + 1/6*(2*B*b^2*x^3 - 3*(2*B*a*b - A*b^2
)*x^2 + 6*(3*B*a^2 - 2*A*a*b)*x)/b^4 - (4*B*a^3 - 3*A*a^2*b)*log(b*x + a)/
b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx = \frac{(bx+a)^3\left(2\,B - \frac{3\,(4\,Bab - Ab^2)}{(bx+a)b} + \frac{18\,(2\,Ba^2b^2 - Aab^3)}{(bx+a)^2b^2}\right)}{6\,b^5} + \frac{(4\,Ba^3 - 3\,Aa^2b)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5} - \frac{\frac{Ba^4b^3}{bx+a} - \frac{Aa^3b^4}{bx+a}}{b^8}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*(b*x + a)^3*(2*B - 3*(4*B*a*b - A*b^2)/((b*x + a)*b) + 18*(2*B*a^2*b^2
 - A*a*b^3)/((b*x + a)^2*b^2))/b^5 + (4*B*a^3 - 3*A*a^2*b)*log(abs(b*x + a
)/((b*x + a)^2*abs(b)))/b^5 - (B*a^4*b^3/(b*x + a) - A*a^3*b^4/(b*x + a))/
b^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx = x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)-x\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)-\frac{\ln\left(a+b\,x\right)\,\left(4\,B\,a^3-3\,A\,a^2\,b\right)}{b^5}+\frac{B\,x^3}{3\,b^2}-\frac{B\,a^4-A\,a^3\,b}{b\,\left(x\,b^5+a\,b^4\right)}$$

input
```
int((x^3*(A + B*x))/(a + b*x)^2,x)
```

output
```
x^2*(A/(2*b^2) - (B*a)/b^3) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b
^4) - (log(a + b*x)*(4*B*a^3 - 3*A*a^2*b))/b^5 + (B*x^3)/(3*b^2) - (B*a^4
- A*a^3*b)/(b*(a*b^4 + b^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.46

$$\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx = \frac{-6\log(bx+a)\,a^3+6a^2bx-3a\,b^2x^2+2b^3x^3}{6b^4}$$

input
```
int(x^3*(B*x+A)/(b*x+a)^2,x)
```

output
```
( - 6*log(a + b*x)*a**3 + 6*a**2*b*x - 3*a*b**2*x**2 + 2*b**3*x**3)/(6*b**
4)
```

3.155 $\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = \frac{(Ab-2aB)x}{b^3}+\frac{Bx^2}{2b^2}-\frac{a^2(Ab-aB)}{b^4(a+bx)}-\frac{a(2Ab-3aB)\log(a+bx)}{b^4}$$

output
```
(A*b-2*B*a)*x/b^3+1/2*B*x^2/b^2-a^2*(A*b-B*a)/b^4/(b*x+a)-a*(2*A*b-3*B*a)*
ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = \frac{2b(Ab-2aB)x+b^2Bx^2+\frac{2a^2(-Ab+aB)}{a+bx}+2a(-2Ab+3aB)\log(a+bx)}{2b^4}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x)^2,x]
```

output
```
(2*b*(A*b - 2*a*B)*x + b^2*B*x^2 + (2*a^2*(-(A*b) + a*B))/(a + b*x) + 2*a*
(-2*A*b + 3*a*B)*Log[a + b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 86$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)^2}+\frac{a(3aB-2Ab)}{b^3(a+bx)}+\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2(Ab-aB)}{b^4(a+bx)}-\frac{a(2Ab-3aB)\log(a+bx)}{b^4}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^2}{2b^2}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x)^2,x]
```

output
```
((A*b - 2*a*B)*x)/b^3 + (B*x^2)/(2*b^2) - (a^2*(A*b - a*B))/(b^4*(a + b*x)
) - (a*(2*A*b - 3*a*B)*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{1}{2}bB\,x^2+Abx-2Bax}{b^3}-\frac{a^2(Ab-Ba)}{b^4(bx+a)}-\frac{a(2Ab-3Ba)\ln(bx+a)}{b^4}$
norman	$\frac{\frac{B\,x^3}{2b}-\frac{a\left(2abA-3a^2B\right)}{b^4}+\frac{(2Ab-3Ba)x^2}{2b^2}}{bx+a}-\frac{a(2Ab-3Ba)\ln(bx+a)}{b^4}$
risch	$\frac{B\,x^2}{2b^2}+\frac{Ax}{b^2}-\frac{2Bax}{b^3}-\frac{a^2A}{b^3(bx+a)}+\frac{a^3B}{b^4(bx+a)}-\frac{2a\ln(bx+a)A}{b^3}+\frac{3a^2\ln(bx+a)B}{b^4}$
parallelrisch	$-\frac{-b^3B\,x^3+4A\ln(bx+a)xa\,b^2-2A\,x^2b^3-6B\ln(bx+a)x\,a^2b+3B\,x^2a\,b^2+4A\ln(bx+a)a^2b-6B\ln(bx+a)a^3+4a^2bA-6a^3B}{2b^4(bx+a)}$

input
```
int(x^2*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/2*b*B*x^2+A*b*x-2*B*a*x)-a^2*(A*b-B*a)/b^4/(b*x+a)-a*(2*A*b-3*B*a
)*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx$$
$$=\frac{Bb^3x^3+2\,Ba^3-2\,Aa^2b-(3\,Bab^2-2\,Ab^3)x^2-2\,(2\,Ba^2b-Aab^2)x+2\,(3\,Ba^3-2\,Aa^2b+(3\,Ba^2b-2}{2\,(b^5x+ab^4)}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(B*b^3*x^3 + 2*B*a^3 - 2*A*a^2*b - (3*B*a*b^2 - 2*A*b^3)*x^2 - 2*(2*B*
a^2*b - A*a*b^2)*x + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x)*l
og(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = \frac{Bx^2}{2b^2} + \frac{a(-2Ab+3Ba)\log{(a+bx)}}{b^4} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{-Aa^2b+Ba^3}{ab^4+b^5x}$$

input
```
integrate(x**2*(B*x+A)/(b*x+a)**2,x)
```

output
```
B*x**2/(2*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x)/b**4 + x*(A/b**2 - 2*B*a
/b**3) + (-A*a**2*b + B*a**3)/(a*b**4 + b**5*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = \frac{Ba^3-Aa^2b}{b^5x+ab^4} + \frac{Bbx^2-2\,(2\,Ba-Ab)x}{2\,b^3} + \frac{(3\,Ba^2-2\,Aab)\log{(bx+a)}}{b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(B*a^3 - A*a^2*b)/(b^5*x + a*b^4) + 1/2*(B*b*x^2 - 2*(2*B*a - A*b)*x)/b^3
+ (3*B*a^2 - 2*A*a*b)*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = \frac{(bx+a)^2\left(B-\frac{2\,(3\,Bab-Ab^2)}{(bx+a)b}\right)}{2\,b^4} - \frac{(3\,Ba^2-2\,Aab)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} + \frac{\frac{Ba^3b^2}{bx+a}-\frac{Aa^2b^3}{bx+a}}{b^6}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*x + a)^2*(B - 2*(3*B*a*b - A*b^2)/((b*x + a)*b))/b^4 - (3*B*a^2 - 2
*A*a*b)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4 + (B*a^3*b^2/(b*x + a)
- A*a^2*b^3/(b*x + a))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)+\frac{B\,x^2}{2\,b^2}+\frac{B\,a^3-A\,a^2\,b}{b\,\left(x\,b^4+a\,b^3\right)} + \frac{\ln\left(a+b\,x\right)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{b^4}$$

input
```
int((x^2*(A + B*x))/(a + b*x)^2,x)
```

output
```
x*(A/b^2 - (2*B*a)/b^3) + (B*x^2)/(2*b^2) + (B*a^3 - A*a^2*b)/(b*(a*b^3 +
b^4*x)) + (log(a + b*x)*(3*B*a^2 - 2*A*a*b))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.42

$$\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx = \frac{2\log(bx+a)\,a^2-2abx+b^2x^2}{2b^3}$$

input
```
int(x^2*(B*x+A)/(b*x+a)^2,x)
```

output
```
(2*log(a + b*x)*a**2 - 2*a*b*x + b**2*x**2)/(2*b**3)
```

3.156 $\int \frac{x(A+Bx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{Bx}{b^2}+\frac{a(Ab-aB)}{b^3(a+bx)}+\frac{(Ab-2aB)\log(a+bx)}{b^3}$$

output
```
B*x/b^2+a*(A*b-B*a)/b^3/(b*x+a)+(A*b-2*B*a)*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{bBx+\frac{a(Ab-aB)}{a+bx}+(Ab-2aB)\log(a+bx)}{b^3}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x)^2,x]
```

output
```
(b*B*x + (a*(A*b - a*B))/(a + b*x) + (A*b - 2*a*B)*Log[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{Ab-2aB}{b^2(a+bx)}+\frac{a(aB-Ab)}{b^2(a+bx)^2}+\frac{B}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(Ab-aB)}{b^3(a+bx)}+\frac{(Ab-2aB)\log(a+bx)}{b^3}+\frac{Bx}{b^2}$$

input
```
Int[(x*(A + B*x))/(a + b*x)^2,x]
```

output
```
(B*x)/b^2 + (a*(A*b - a*B))/(b^3*(a + b*x)) + ((A*b - 2*a*B)*Log[a + b*x])
/b^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{Bx}{b^2}+\frac{a(Ab-Ba)}{b^3(bx+a)}+\frac{(Ab-2Ba)\ln(bx+a)}{b^3}$	46
norman	$\frac{\frac{B\,x^2}{b}+\frac{a(Ab-2Ba)}{b^3}}{bx+a}+\frac{(Ab-2Ba)\ln(bx+a)}{b^3}$	50
risch	$\frac{Bx}{b^2}+\frac{aA}{b^2(bx+a)}-\frac{a^2B}{b^3(bx+a)}+\frac{\ln(bx+a)A}{b^2}-\frac{2\ln(bx+a)Ba}{b^3}$	61
parallelrisch	$\frac{A\ln(bx+a)x\,b^2-2B\ln(bx+a)xab+b^2B\,x^2+A\ln(bx+a)ab-2B\ln(bx+a)a^2+abA-2a^2B}{b^3(bx+a)}$	77

input
```
int(x*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^2+a*(A*b-B*a)/b^3/(b*x+a)+(A*b-2*B*a)*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{Bb^2x^2+Babx-Ba^2+Aab-(2\,Ba^2-Aab+(2\,Bab-Ab^2)x)\log(bx+a)}{b^4x+ab^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(B*b^2*x^2 + B*a*b*x - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (2*B*a*b - A*b^2
)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{Bx}{b^2} + \frac{Aab - Ba^2}{ab^3 + b^4x} - \frac{(-Ab + 2Ba)\log(a+bx)}{b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a)**2,x)
```

output
```
B*x/b**2 + (A*a*b - B*a**2)/(a*b**3 + b**4*x) - (-A*b + 2*B*a)*log(a + b*x
)/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = -\frac{Ba^2 - Aab}{b^4x + ab^3} + \frac{Bx}{b^2} - \frac{(2\,Ba - Ab)\log(bx+a)}{b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(B*a^2 - A*a*b)/(b^4*x + a*b^3) + B*x/b^2 - (2*B*a - A*b)*log(b*x + a)/b^
3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.78

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{\frac{(bx+a)B}{b^2} + \frac{(2\,Ba - Ab)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx+a} - \frac{Aab^2}{bx+a}}{b^3}}{b}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
((b*x + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^
2 - (B*a^2*b/(b*x + a) - A*a*b^2/(b*x + a))/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{B\,x}{b^2} - \frac{B\,a^2 - A\,a\,b}{b\,\left(x\,b^3 + a\,b^2\right)} + \frac{\ln\left(a+b\,x\right)\,\left(A\,b - 2\,B\,a\right)}{b^3}$$

input
```
int((x*(A + B*x))/(a + b*x)^2,x)
```

output
```
(B*x)/b^2 - (B*a^2 - A*a*b)/(b*(a*b^2 + b^3*x)) + (log(a + b*x)*(A*b - 2*B
*a))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.38

$$\int \frac{x(A+Bx)}{(a+bx)^2}\,dx = \frac{-\log(bx+a)\,a + bx}{b^2}$$

input
```
int(x*(B*x+A)/(b*x+a)^2,x)
```

output
```
( - log(a + b*x)*a + b*x)/b**2
```

3.157 $\int \frac{A+Bx}{(a+bx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{A+Bx}{(a+bx)^2} dx = -\frac{Ab-aB}{b^2(a+bx)} + \frac{B\log(a+bx)}{b^2}$$

output
```
-(A*b-B*a)/b^2/(b*x+a)+B*ln(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{(a+bx)^2} dx = \frac{-Ab+aB}{b^2(a+bx)} + \frac{B\log(a+bx)}{b^2}$$

input
```
Integrate[(A + B*x)/(a + b*x)^2,x]
```

output
```
(-(A*b) + a*B)/(b^2*(a + b*x)) + (B*Log[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{B\log(a+bx)}{b^2}-\frac{Ab-aB}{b^2(a+bx)}$$

input
```
Int[(A + B*x)/(a + b*x)^2,x]
```

output
```
-((A*b - a*B)/(b^2*(a + b*x))) + (B*Log[a + b*x])/b^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{Ab-Ba}{b^2(bx+a)}+\frac{B\ln(bx+a)}{b^2}$	33
norman	$-\frac{Ab-Ba}{b^2(bx+a)}+\frac{B\ln(bx+a)}{b^2}$	33
risch	$-\frac{A}{b(bx+a)}+\frac{Ba}{b^2(bx+a)}+\frac{B\ln(bx+a)}{b^2}$	39
parallelrisch	$-\frac{-B\ln(bx+a)xb-B\ln(bx+a)a+Ab-Ba}{b^2(bx+a)}$	42

input
```
int((B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(A*b-B*a)/b^2/(b*x+a)+B*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{(a+bx)^2}\,dx=\frac{Ba-Ab+(Bbx+Ba)\log(bx+a)}{b^3x+ab^2}$$

input
```
integrate((B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
(B*a - A*b + (B*b*x + B*a)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{(a+bx)^2}\,dx=\frac{B\log(a+bx)}{b^2}+\frac{-Ab+Ba}{ab^2+b^3x}$$

input
```
integrate((B*x+A)/(b*x+a)**2,x)
```

output
```
B*log(a + b*x)/b**2 + (-A*b + B*a)/(a*b**2 + b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{(a+bx)^2}\,dx=\frac{Ba-Ab}{b^3x+ab^2}+\frac{B\log\left(bx+a\right)}{b^2}$$

input
```
integrate((B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(B*a - A*b)/(b^3*x + a*b^2) + B*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx}{(a+bx)^2}\,dx=-\frac{B\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)}{b}-\frac{A}{(bx+a)b}$$

input
```
integrate((B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-B*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b - A/((b*
x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{(a+bx)^2} dx = \frac{B \ln(a+bx)}{b^2} - \frac{Ab-Ba}{b^2 (a+bx)}$$

input
```
int((A + B*x)/(a + b*x)^2,x)
```

output
```
(B*log(a + b*x))/b^2 - (A*b - B*a)/(b^2*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx}{(a+bx)^2} dx = \frac{\log(bx+a)}{b}$$

input
```
int((B*x+A)/(b*x+a)^2,x)
```

output
```
log(a + b*x)/b
```

3.158 $\int \frac{A+Bx}{x(a+bx)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{A+Bx}{x(a+bx)^2} \, dx = \frac{Ab-aB}{ab(a+bx)} + \frac{A\log(x)}{a^2} - \frac{A\log(a+bx)}{a^2}$$

output
```
(A*b-B*a)/a/b/(b*x+a)+A*ln(x)/a^2-A*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x(a+bx)^2} \, dx = \frac{\frac{a(Ab-aB)}{b(a+bx)} + A\log(x) - A\log(a+bx)}{a^2}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x)^2),x]
```

output
```
((a*(A*b - a*B))/(b*(a + b*x)) + A*Log[x] - A*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{Ab}{a^2(a+bx)}+\frac{A}{a^2x}+\frac{aB-Ab}{a(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\log(a+bx)}{a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{ab(a+bx)}$$

input
```
Int[(A + B*x)/(x*(a + b*x)^2),x]
```

output
```
(A*b - a*B)/(a*b*(a + b*x)) + (A*Log[x])/a^2 - (A*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
norman	$-\frac{(Ab-Ba)x}{a^2(bx+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln(bx+a)}{a^2}$	42
default	$\frac{A\ln(x)}{a^2}-\frac{-Ab+Ba}{ab(bx+a)}-\frac{A\ln(bx+a)}{a^2}$	44
risch	$\frac{A}{a(bx+a)}-\frac{B}{(bx+a)b}+\frac{A\ln(-x)}{a^2}-\frac{A\ln(bx+a)}{a^2}$	48
parallelrisch	$\frac{A\ln(x)xb-A\ln(bx+a)xb+aA\ln(x)-A\ln(bx+a)a-Abx+Bax}{a^2(bx+a)}$	54

input
```
int((B*x+A)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-(A*b-B*a)/a^2*x/(b*x+a)+A*ln(x)/a^2-A*ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx = -\frac{Ba^2-Aab+(Ab^2x+Aab)\log(bx+a)-(Ab^2x+Aab)\log(x)}{a^2b^2x+a^3b}$$

input
```
integrate((B*x+A)/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(B*a^2 - A*a*b + (A*b^2*x + A*a*b)*log(b*x + a) - (A*b^2*x + A*a*b)*log(x
))/(a^2*b^2*x + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx=\frac{A\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^2}+\frac{Ab-Ba}{a^2b+ab^2x}$$

input
```
integrate((B*x+A)/x/(b*x+a)**2,x)
```

output
```
A*(log(x) - log(a/b + x))/a**2 + (A*b - B*a)/(a**2*b + a*b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx=-\frac{Ba-Ab}{ab^2x+a^2b}-\frac{A\log{(bx+a)}}{a^2}+\frac{A\log{(x)}}{a^2}$$

input
```
integrate((B*x+A)/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(B*a - A*b)/(a*b^2*x + a^2*b) - A*log(b*x + a)/a^2 + A*log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx=b\left(\frac{A\log{\left(\left|-\frac{a}{bx+a}+1\right|\right)}}{a^2b}-\frac{\frac{Ba}{bx+a}-\frac{Ab}{bx+a}}{ab^2}\right)$$

input
```
integrate((B*x+A)/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
b*(A*log(abs(-a/(b*x + a) + 1))/(a^2*b) - (B*a/(b*x + a) - A*b/(b*x + a))/
(a*b^2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx = \frac{A\,b-B\,a}{a\,b\,\left(a+b\,x\right)}-\frac{2\,A\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

input
```
int((A + B*x)/(x*(a + b*x)^2),x)
```

output
```
(A*b - B*a)/(a*b*(a + b*x)) - (2*A*atanh((2*b*x)/a + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx}{x(a+bx)^2}\,dx = \frac{-\log(bx+a)+\log(x)}{a}$$

input
```
int((B*x+A)/x/(b*x+a)^2,x)
```

output
```
( - log(a + b*x) + log(x))/a
```

3.159 $\int \frac{A+Bx}{x^2(a+bx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = -\frac{A}{a^2x}-\frac{Ab-aB}{a^2(a+bx)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log(a+bx)}{a^3}$$

output
```
-A/a^2/x-(A*b-B*a)/a^2/(b*x+a)-(2*A*b-B*a)*ln(x)/a^3+(2*A*b-B*a)*ln(b*x+a)
/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = \frac{-\frac{aA}{x}+\frac{a(-Ab+aB)}{a+bx}+(-2Ab+aB)\log(x)+(2Ab-aB)\log(a+bx)}{a^3}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x)^2),x]
```

output
```
(-((a*A)/x) + (a*(-(A*b) + a*B))/(a + b*x) + (-2*A*b + a*B)*Log[x] + (2*A*
b - a*B)*Log[a + b*x])/a^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{aB-2Ab}{a^3x}-\frac{b(aB-2Ab)}{a^3(a+bx)}-\frac{b(aB-Ab)}{a^2(a+bx)^2}+\frac{A}{a^2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(x)(2Ab-aB)}{a^3}+\frac{(2Ab-aB)\log(a+bx)}{a^3}-\frac{Ab-aB}{a^2(a+bx)}-\frac{A}{a^2x}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x)^2),x]
```

output
```
-(A/(a^2*x)) - (A*b - a*B)/(a^2*(a + b*x)) - ((2*A*b - a*B)*Log[x])/a^3 +
((2*A*b - a*B)*Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result
default	$-\frac{A}{a^2 x}+\frac{(-2Ab+Ba)\ln(x)}{a^3}+\frac{(2Ab-Ba)\ln(bx+a)}{a^3}-\frac{Ab-Ba}{a^2(bx+a)}$
norman	$\frac{\frac{b(2Ab-Ba)x^2}{a^3}-\frac{A}{a}}{x(bx+a)}+\frac{(2Ab-Ba)\ln(bx+a)}{a^3}-\frac{(2Ab-Ba)\ln(x)}{a^3}$
risch	$\frac{-\frac{(2Ab-Ba)x}{a^2}-\frac{A}{a}}{x(bx+a)}+\frac{2\ln(-bx-a)Ab}{a^3}-\frac{\ln(-bx-a)B}{a^2}-\frac{2\ln(x)Ab}{a^3}+\frac{\ln(x)B}{a^2}$
parallelrisch	$-\frac{2A\ln(x)x^2b^3-2A\ln(bx+a)x^2b^3-B\ln(x)x^2a\,b^2+B\ln(bx+a)x^2a\,b^2+2A\ln(x)xa\,b^2-2A\ln(bx+a)xa\,b^2-B\ln(x)x\,a^2b+B\ln}{a^3bx(bx+a)}$

input
```
int((B*x+A)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x+(-2*A*b+B*a)/a^3*ln(x)+(2*A*b-B*a)*ln(b*x+a)/a^3-(A*b-B*a)/a^2/(b
*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = -\frac{Aa^2-(Ba^2-2\,Aab)x+((Bab-2\,Ab^2)x^2+(Ba^2-2\,Aab)x)\log(bx+a)-((Bab-2\,Ab^2)x^2+(B}{a^3bx^2+a^4x}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(A*a^2 - (B*a^2 - 2*A*a*b)*x + ((B*a*b - 2*A*b^2)*x^2 + (B*a^2 - 2*A*a*b)
*x)*log(b*x + a) - ((B*a*b - 2*A*b^2)*x^2 + (B*a^2 - 2*A*a*b)*x)*log(x))/(
a^3*b*x^2 + a^4*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(54) = 108$.

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = \frac{-Aa+x(-2Ab+Ba)}{a^3x+a^2bx^2} + \frac{(-2Ab+Ba)\log\left(x+\frac{-2Aab+Ba^2-a(-2Ab+Ba)}{-4Ab^2+2Bab}\right)}{a^3} - \frac{(-2Ab+Ba)\log\left(x+\frac{-2Aab+Ba^2+a(-2Ab+Ba)}{-4Ab^2+2Bab}\right)}{a^3}$$

input
```
integrate((B*x+A)/x**2/(b*x+a)**2,x)
```

output
```
(-A*a + x*(-2*A*b + B*a))/(a**3*x + a**2*b*x**2) + (-2*A*b + B*a)*log(x +
(-2*A*a*b + B*a**2 - a*(-2*A*b + B*a))/(-4*A*b**2 + 2*B*a*b))/a**3 - (-2*A
*b + B*a)*log(x + (-2*A*a*b + B*a**2 + a*(-2*A*b + B*a))/(-4*A*b**2 + 2*B*
a*b))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = -\frac{Aa-(Ba-2\,Ab)x}{a^2bx^2+a^3x} - \frac{(Ba-2\,Ab)\log(bx+a)}{a^3} + \frac{(Ba-2\,Ab)\log(x)}{a^3}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(A*a - (B*a - 2*A*b)*x)/(a^2*b*x^2 + a^3*x) - (B*a - 2*A*b)*log(b*x + a)/
a^3 + (B*a - 2*A*b)*log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = \frac{Ab}{a^3\left(\frac{a}{bx+a}-1\right)} + \frac{(Bab-2\,Ab^2)\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3b} + \frac{\frac{Bab^2}{bx+a}-\frac{Ab^3}{bx+a}}{a^2b^2}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
A*b/(a^3*(a/(b*x + a) - 1)) + (B*a*b - 2*A*b^2)*log(abs(-a/(b*x + a) + 1))
/(a^3*b) + (B*a*b^2/(b*x + a) - A*b^3/(b*x + a))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)\,(2\,A\,b-B\,a)}{a^3} - \frac{\frac{A}{a}+\frac{x\,(2\,A\,b-B\,a)}{a^2}}{b\,x^2+a\,x}$$

input
```
int((A + B*x)/(x^2*(a + b*x)^2),x)
```

output
```
(2*atanh((2*b*x)/a + 1)*(2*A*b - B*a))/a^3 - (A/a + (x*(2*A*b - B*a))/a^2)
/(a*x + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx}{x^2(a+bx)^2}\,dx = \frac{\log(bx+a)\,bx-\log(x)\,bx-a}{a^2x}$$

input
```
int((B*x+A)/x^2/(b*x+a)^2,x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.160 $\int \frac{A+Bx}{x^3(a+bx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = -\frac{A}{2a^2x^2}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)}{a^3(a+bx)} + \frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log(a+bx)}{a^4}$$

output
```
-1/2*A/a^2/x^2+(2*A*b-B*a)/a^3/x+b*(A*b-B*a)/a^3/(b*x+a)+b*(3*A*b-2*B*a)*l
n(x)/a^4-b*(3*A*b-2*B*a)*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = \frac{-\frac{a(-6Ab^2x^2+a^2(A+2Bx)+abx(-3A+4Bx))}{x^2(a+bx)}+2b(3Ab-2aB)\log(x)+2b(-3Ab+2aB)\log(a+bx)}{2a^4}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x)^2),x]
```

output
```
(-((a*(-6*A*b^2*x^2 + a^2*(A + 2*B*x) + a*b*x*(-3*A + 4*B*x)))/(x^2*(a + b
*x))) + 2*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(-3*A*b + 2*a*B)*Log[a + b*x])/(2
*a^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{b^2(2aB-3Ab)}{a^4(a+bx)}-\frac{b(2aB-3Ab)}{a^4x}+\frac{b^2(aB-Ab)}{a^3(a+bx)^2}+\frac{aB-2Ab}{a^3x^2}+\frac{A}{a^2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{b(3Ab-2aB)\log(a+bx)}{a^4}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)}{a^3(a+bx)}-\frac{A}{2a^2x^2}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x)^2),x]
```

output
```
-1/2*A/(a^2*x^2) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B))/(a^3*(a + b*x))
 + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{2a^2x^2}-\frac{-2Ab+Ba}{x\,a^3}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln(bx+a)}{a^4}+\frac{b(Ab-Ba)}{a^3(bx+a)}$
norman	$\frac{-\frac{A}{2a}+\frac{(3Ab-2Ba)x}{2a^2}-\frac{b\left(3b^2A-2abB\right)x^3}{a^4}}{x^2(bx+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln(bx+a)}{a^4}$
risch	$\frac{\frac{b(3Ab-2Ba)x^2}{a^3}+\frac{(3Ab-2Ba)x}{2a^2}-\frac{A}{2a}}{x^2(bx+a)}+\frac{3b^2\ln(-x)A}{a^4}-\frac{2b\ln(-x)B}{a^3}-\frac{3b^2\ln(bx+a)A}{a^4}+\frac{2b\ln(bx+a)B}{a^3}$
parallelrisch	$\frac{6A\ln(x)x^3b^3-6A\ln(bx+a)x^3b^3-4B\ln(x)x^3a\,b^2+4B\ln(bx+a)x^3a\,b^2+6A\ln(x)x^2a\,b^2-6A\ln(bx+a)x^2a\,b^2-6A\,b^3x^3-4B\ln}{2a^4x^2(bx+a)}$

input
```
int((B*x+A)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2-(-2*A*b+B*a)/x/a^3+b*(3*A*b-2*B*a)*ln(x)/a^4-b*(3*A*b-2*B*a
)*ln(b*x+a)/a^4+b*(A*b-B*a)/a^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = -\frac{Aa^3+2\,(2\,Ba^2b-3\,Aab^2)x^2+(2\,Ba^3-3\,Aa^2b)x-2\,((2\,Bab^2-3\,Ab^3)x^3+(2\,Ba^2b-3\,Aab^2)x^2)\,\mathrm{lo}}{2\,(a^4bx^3+a^5x^2)}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(A*a^3 + 2*(2*B*a^2*b - 3*A*a*b^2)*x^2 + (2*B*a^3 - 3*A*a^2*b)*x - 2*
((2*B*a*b^2 - 3*A*b^3)*x^3 + (2*B*a^2*b - 3*A*a*b^2)*x^2)*log(b*x + a) + 2
*((2*B*a*b^2 - 3*A*b^3)*x^3 + (2*B*a^2*b - 3*A*a*b^2)*x^2)*log(x))/(a^4*b*
x^3 + a^5*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(80) = 160$.

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = \frac{-Aa^2+x^2\cdot(6Ab^2-4Bab)+x(3Aab-2Ba^2)}{2a^4x^2+2a^3bx^3} - \frac{b(-3Ab+2Ba)\log\left(x+\frac{-3Aab^2+2Ba^2b-ab(-3Ab+2Ba)}{-6Ab^3+4Bab^2}\right)}{a^4} + \frac{b(-3Ab+2Ba)\log\left(x+\frac{-3Aab^2+2Ba^2b+ab(-3Ab+2Ba)}{-6Ab^3+4Bab^2}\right)}{a^4}$$

input
```
integrate((B*x+A)/x**3/(b*x+a)**2,x)
```

output
```
(-A*a**2 + x**2*(6*A*b**2 - 4*B*a*b) + x*(3*A*a*b - 2*B*a**2))/(2*a**4*x**
2 + 2*a**3*b*x**3) - b*(-3*A*b + 2*B*a)*log(x + (-3*A*a*b**2 + 2*B*a**2*b
- a*b*(-3*A*b + 2*B*a))/(-6*A*b**3 + 4*B*a*b**2))/a**4 + b*(-3*A*b + 2*B*a
)*log(x + (-3*A*a*b**2 + 2*B*a**2*b + a*b*(-3*A*b + 2*B*a))/(-6*A*b**3 + 4
*B*a*b**2))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = -\frac{Aa^2+2\,(2\,Bab-3\,Ab^2)x^2+(2\,Ba^2-3\,Aab)x}{2\,(a^3bx^3+a^4x^2)} + \frac{(2\,Bab-3\,Ab^2)\log{(bx+a)}}{a^4} - \frac{(2\,Bab-3\,Ab^2)\log{(x)}}{a^4}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(A*a^2 + 2*(2*B*a*b - 3*A*b^2)*x^2 + (2*B*a^2 - 3*A*a*b)*x)/(a^3*b*x^
3 + a^4*x^2) + (2*B*a*b - 3*A*b^2)*log(b*x + a)/a^4 - (2*B*a*b - 3*A*b^2)*
log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = -\frac{(2\,Bab^2-3\,Ab^3)\log{\left(\left|-\frac{a}{bx+a}+1\right|\right)}}{a^4b} - \frac{\frac{Bab^4}{bx+a}-\frac{Ab^5}{bx+a}}{a^3b^3} - \frac{2\,Bab-5\,Ab^2-\frac{2\,(Ba^2b^2-3\,Aab^3)}{(bx+a)b}}{2\,a^4\left(\frac{a}{bx+a}-1\right)^2}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*B*a*b^2 - 3*A*b^3)*log(abs(-a/(b*x + a) + 1))/(a^4*b) - (B*a*b^4/(b*x
+ a) - A*b^5/(b*x + a))/(a^3*b^3) - 1/2*(2*B*a*b - 5*A*b^2 - 2*(B*a^2*b^2
- 3*A*a*b^3)/((b*x + a)*b))/(a^4*(a/(b*x + a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = \frac{\frac{x\,(3\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{2\,a}+\frac{b\,x^2\,(3\,A\,b-2\,B\,a)}{a^3}}{b\,x^3+a\,x^2} - \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(3\,A\,b-2\,B\,a)\,(a+2\,b\,x)}{a\,(3\,A\,b^2-2\,B\,a\,b)}\right)\,(3\,A\,b-2\,B\,a)}{a^4}$$

input
```
int((A + B*x)/(x^3*(a + b*x)^2),x)
```

output
```
((x*(3*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (b*x^2*(3*A*b - 2*B*a))/a^3)/(a*x
^2 + b*x^3) - (2*b*atanh((b*(3*A*b - 2*B*a)*(a + 2*b*x))/(a*(3*A*b^2 - 2*B
*a*b)))*(3*A*b - 2*B*a))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx}{x^3(a+bx)^2}\,dx = \frac{-2\log(bx+a)\,b^2x^2+2\log(x)\,b^2x^2-a^2+2abx}{2a^3x^2}$$

input
```
int((B*x+A)/x^3/(b*x+a)^2,x)
```

output
```
( - 2*log(a + b*x)*b**2*x**2 + 2*log(x)*b**2*x**2 - a**2 + 2*a*b*x)/(2*a**
3*x**2)
```

3.161 $\int \frac{A+Bx}{x^4(a+bx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{2a^3x^2}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)}{a^4(a+bx)} - \frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(4Ab-3aB)\log(a+bx)}{a^5}$$

output
```
-1/3*A/a^2/x^3+1/2*(2*A*b-B*a)/a^3/x^2-b*(3*A*b-2*B*a)/a^4/x-b^2*(A*b-B*a)
/a^4/(b*x+a)-b^2*(4*A*b-3*B*a)*ln(x)/a^5+b^2*(4*A*b-3*B*a)*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = \frac{-\frac{2a^3A}{x^3}-\frac{3a^2(-2Ab+aB)}{x^2}+\frac{6ab(-3Ab+2aB)}{x}+\frac{6ab^2(-Ab+aB)}{a+bx}+6b^2(-4Ab+3aB)\log(x)+6b^2(4Ab-3aB)\log(}{6a^5}$$

input
```
Integrate[(A + B*x)/(x^4*(a + b*x)^2),x]
```

output
```
((-2*a^3*A)/x^3 - (3*a^2*(-2*A*b + a*B))/x^2 + (6*a*b*(-3*A*b + 2*a*B))/x
+ (6*a*b^2*(-(A*b) + a*B))/(a + b*x) + 6*b^2*(-4*A*b + 3*a*B)*Log[x] + 6*b
^2*(4*A*b - 3*a*B)*Log[a + b*x])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx$$

$$\Big\downarrow \ 86$$

$$\int \left(-\frac{b^3(3aB-4Ab)}{a^5(a+bx)}+\frac{b^2(3aB-4Ab)}{a^5x}-\frac{b^3(aB-Ab)}{a^4(a+bx)^2}-\frac{b(2aB-3Ab)}{a^4x^2}+\frac{aB-2Ab}{a^3x^3}+\frac{A}{a^2x^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{b^2\log(x)(4Ab-3aB)}{a^5}+\frac{b^2(4Ab-3aB)\log(a+bx)}{a^5}-\frac{b^2(Ab-aB)}{a^4(a+bx)}-\frac{b(3Ab-2aB)}{a^4x}+\frac{2Ab-aB}{2a^3x^2}-\frac{A}{3a^2x^3}$$

input
```
Int[(A + B*x)/(x^4*(a + b*x)^2),x]
```

output
```
-1/3*A/(a^2*x^3) + (2*A*b - a*B)/(2*a^3*x^2) - (b*(3*A*b - 2*a*B))/(a^4*x)
 - (b^2*(A*b - a*B))/(a^4*(a + b*x)) - (b^2*(4*A*b - 3*a*B)*Log[x])/a^5 +
(b^2*(4*A*b - 3*a*B)*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

method	result
default	$-\frac{A}{3a^2x^3}-\frac{-2Ab+Ba}{2x^2a^3}-\frac{b(3Ab-2Ba)}{a^4x}-\frac{b^2(4Ab-3Ba)\ln(x)}{a^5}+\frac{b^2(4Ab-3Ba)\ln(bx+a)}{a^5}-\frac{b^2(Ab-Ba)}{a^4(bx+a)}$
norman	$\frac{\frac{b\left(4b^3A-3a\,b^2B\right)x^4}{a^5}-\frac{A}{3a}+\frac{(4Ab-3Ba)x}{6a^2}-\frac{b(4Ab-3Ba)x^2}{2a^3}}{x^3(bx+a)}+\frac{b^2(4Ab-3Ba)\ln(bx+a)}{a^5}-\frac{b^2(4Ab-3Ba)\ln(x)}{a^5}$
risch	$\frac{-\frac{b^2(4Ab-3Ba)x^3}{a^4}-\frac{b(4Ab-3Ba)x^2}{2a^3}+\frac{(4Ab-3Ba)x}{6a^2}-\frac{A}{3a}}{x^3(bx+a)}+\frac{4b^3\ln(-bx-a)A}{a^5}-\frac{3b^2\ln(-bx-a)B}{a^4}-\frac{4b^3\ln(x)A}{a^5}+\frac{3b^2\ln(x)}{a^4}$
parallelrisch	$-\frac{24A\ln(x)x^4b^4-24A\ln(bx+a)x^4b^4-18B\ln(x)x^4a\,b^3+18B\ln(bx+a)x^4a\,b^3+24A\ln(x)x^3a\,b^3-24A\ln(bx+a)x^3a\,b^3-24A\,b}{6a^5x^3(bx+a}$

input
```
int((B*x+A)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^2/x^3-1/2*(-2*A*b+B*a)/x^2/a^3-b*(3*A*b-2*B*a)/a^4/x-b^2*(4*A*b-3
*B*a)*ln(x)/a^5+b^2*(4*A*b-3*B*a)*ln(b*x+a)/a^5-b^2*(A*b-B*a)/a^4/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = -\frac{2\,Aa^4 - 6\,(3\,Ba^2b^2 - 4\,Aab^3)x^3 - 3\,(3\,Ba^3b - 4\,Aa^2b^2)x^2 + (3\,Ba^4 - 4\,Aa^3b)x + 6\,((3\,Bab^3 - 4\,Ab^4)\cdots}{6\,(a^5bx^4 + \cdots}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*A*a^4 - 6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 4*A*a^2*b
^2)*x^2 + (3*B*a^4 - 4*A*a^3*b)*x + 6*((3*B*a*b^3 - 4*A*b^4)*x^4 + (3*B*a^
2*b^2 - 4*A*a*b^3)*x^3)*log(b*x + a) - 6*((3*B*a*b^3 - 4*A*b^4)*x^4 + (3*B
*a^2*b^2 - 4*A*a*b^3)*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(104) = 208.

Time = 0.46 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{A+Bx}{x^4(a+bx)^2}\,dx &= \frac{-2Aa^3 + x^3(-24Ab^3 + 18Bab^2) + x^2(-12Aab^2 + 9Ba^2b) + x(4Aa^2b - 3Ba^3)}{6a^5x^3 + 6a^4bx^4} \\ &\quad + \frac{b^2(-4Ab + 3Ba)\log\left(x + \frac{-4Aab^3 + 3Ba^2b^2 - ab^2(-4Ab+3Ba)}{-8Ab^4 + 6Bab^3}\right)}{a^5} \\ &\quad - \frac{b^2(-4Ab + 3Ba)\log\left(x + \frac{-4Aab^3 + 3Ba^2b^2 + ab^2(-4Ab+3Ba)}{-8Ab^4 + 6Bab^3}\right)}{a^5}\end{aligned}$$

input
```
integrate((B*x+A)/x**4/(b*x+a)**2,x)
```

output
```
(-2*A*a**3 + x**3*(-24*A*b**3 + 18*B*a*b**2) + x**2*(-12*A*a*b**2 + 9*B*a*
*2*b) + x*(4*A*a**2*b - 3*B*a**3))/(6*a**5*x**3 + 6*a**4*b*x**4) + b**2*(-
4*A*b + 3*B*a)*log(x + (-4*A*a*b**3 + 3*B*a**2*b**2 - a*b**2*(-4*A*b + 3*B
*a))/(-8*A*b**4 + 6*B*a*b**3))/a**5 - b**2*(-4*A*b + 3*B*a)*log(x + (-4*A*
a*b**3 + 3*B*a**2*b**2 + a*b**2*(-4*A*b + 3*B*a))/(-8*A*b**4 + 6*B*a*b**3)
)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = -\frac{2\,Aa^3-6\,(3\,Bab^2-4\,Ab^3)x^3-3\,(3\,Ba^2b-4\,Aab^2)x^2+(3\,Ba^3-4\,Aa^2b)x}{6\,(a^4bx^4+a^5x^3)} - \frac{(3\,Bab^2-4\,Ab^3)\log(bx+a)}{a^5} + \frac{(3\,Bab^2-4\,Ab^3)\log(x)}{a^5}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*A*a^3 - 6*(3*B*a*b^2 - 4*A*b^3)*x^3 - 3*(3*B*a^2*b - 4*A*a*b^2)*x^
2 + (3*B*a^3 - 4*A*a^2*b)*x)/(a^4*b*x^4 + a^5*x^3) - (3*B*a*b^2 - 4*A*b^3)
*log(b*x + a)/a^5 + (3*B*a*b^2 - 4*A*b^3)*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = \frac{(3\,Bab^3-4\,Ab^4)\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^5b} + \frac{\frac{Bab^6}{bx+a}-\frac{Ab^7}{bx+a}}{a^4b^4} - \frac{15\,Bab^2-26\,Ab^3-\frac{3\,(11\,Ba^2b^3-20\,Aab^4)}{(bx+a)b}+\frac{18\,(Ba^3b^4-2\,Aa^2b^5)}{(bx+a)^2b^2}}{6\,a^5\left(\frac{a}{bx+a}-1\right)^3}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^2,x, algorithm="giac")
```

output
```
(3*B*a*b^3 - 4*A*b^4)*log(abs(-a/(b*x + a) + 1))/(a^5*b) + (B*a*b^6/(b*x +
 a) - A*b^7/(b*x + a))/(a^4*b^4) - 1/6*(15*B*a*b^2 - 26*A*b^3 - 3*(11*B*a^
2*b^3 - 20*A*a*b^4)/((b*x + a)*b) + 18*(B*a^3*b^4 - 2*A*a^2*b^5)/((b*x + a
)^2*b^2))/(a^5*(a/(b*x + a) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = \frac{2\,b^2\,\mathrm{atanh}\left(\frac{b^2\,(4\,A\,b-3\,B\,a)\,(a+2\,b\,x)}{a\,(4\,A\,b^3-3\,B\,a\,b^2)}\right)\,(4\,A\,b-3\,B\,a)}{a^5} - \frac{\frac{A}{3\,a}-\frac{x\,(4\,A\,b-3\,B\,a)}{6\,a^2}+\frac{b^2\,x^3\,(4\,A\,b-3\,B\,a)}{a^4}+\frac{b\,x^2\,(4\,A\,b-3\,B\,a)}{2\,a^3}}{b\,x^4+a\,x^3}$$

input
```
int((A + B*x)/(x^4*(a + b*x)^2),x)
```

output
```
(2*b^2*atanh((b^2*(4*A*b - 3*B*a)*(a + 2*b*x))/(a*(4*A*b^3 - 3*B*a*b^2)))*
(4*A*b - 3*B*a))/a^5 - (A/(3*a) - (x*(4*A*b - 3*B*a))/(6*a^2) + (b^2*x^3*(
4*A*b - 3*B*a))/a^4 + (b*x^2*(4*A*b - 3*B*a))/(2*a^3))/(a*x^3 + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{x^4(a+bx)^2}\,dx = \frac{6\log(bx+a)\,b^3x^3-6\log(x)\,b^3x^3-2a^3+3a^2bx-6a\,b^2x^2}{6a^4x^3}$$

input
```
int((B*x+A)/x^4/(b*x+a)^2,x)
```

output
```
(6*log(a + b*x)*b**3*x**3 - 6*log(x)*b**3*x**3 - 2*a**3 + 3*a**2*b*x - 6*a
*b**2*x**2)/(6*a**4*x**3)
```

3.162 $\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = -\frac{3a(Ab-2aB)x}{b^5}+\frac{(Ab-3aB)x^2}{2b^4}+\frac{Bx^3}{3b^3}-\frac{a^4(Ab-aB)}{2b^6(a+bx)^2}+\frac{a^3(4Ab-5aB)}{b^6(a+bx)}+\frac{2a^2(3Ab-5aB)\log(a+bx)}{b^6}$$

output
```
-3*a*(A*b-2*B*a)*x/b^5+1/2*(A*b-3*B*a)*x^2/b^4+1/3*B*x^3/b^3-1/2*a^4*(A*b-
B*a)/b^6/(b*x+a)^2+a^3*(4*A*b-5*B*a)/b^6/(b*x+a)+2*a^2*(3*A*b-5*B*a)*ln(b*
x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = \frac{18ab(-Ab+2aB)x+3b^2(Ab-3aB)x^2+2b^3Bx^3+\frac{3a^4(-Ab+aB)}{(a+bx)^2}+\frac{6a^3(4Ab-5aB)}{a+bx}-12a^2(-3Ab+5aB)\log}{6b^6}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x)^3,x]
```

output
```
(18*a*b*(-(A*b) + 2*a*B)*x + 3*b^2*(A*b - 3*a*B)*x^2 + 2*b^3*B*x^3 + (3*a^
4*(-(A*b) + a*B))/(a + b*x)^2 + (6*a^3*(4*A*b - 5*a*B))/(a + b*x) - 12*a^2
*(-3*A*b + 5*a*B)*Log[a + b*x])/(6*b^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)^3}+\frac{a^3(5aB-4Ab)}{b^5(a+bx)^2}-\frac{2a^2(5aB-3Ab)}{b^5(a+bx)}+\frac{3a(2aB-Ab)}{b^5}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^2}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4(Ab-aB)}{2b^6(a+bx)^2}+\frac{a^3(4Ab-5aB)}{b^6(a+bx)}+\frac{2a^2(3Ab-5aB)\log(a+bx)}{b^6}-\frac{3ax(Ab-2aB)}{b^5}+\frac{x^2(Ab-3aB)}{2b^4}+\frac{Bx^3}{3b^3}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x)^3,x]
```

output
```
(-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^3)/(3*b^3)
 - (a^4*(A*b - a*B))/(2*b^6*(a + b*x)^2) + (a^3*(4*A*b - 5*a*B))/(b^6*(a +
 b*x)) + (2*a^2*(3*A*b - 5*a*B)*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

method	result
default	$-\frac{-\frac{1}{3}B\,b^{2}x^{3}-\frac{1}{2}A\,b^{2}x^{2}+\frac{3}{2}Bab\,x^{2}+3aAbx-6B\,a^{2}x}{b^{5}}-\frac{a^{4}(Ab-Ba)}{2b^{6}(bx+a)^{2}}+\frac{a^{3}(4Ab-5Ba)}{b^{6}(bx+a)}+\frac{2a^{2}(3Ab-5Ba)\ln(bx+a)}{b^{6}}$
norman	$\frac{\frac{a^{2}\left(9a^{2}bA-15a^{3}B\right)}{b^{6}}+\frac{B\,x^{5}}{3b}+\frac{(3Ab-5Ba)x^{4}}{6b^{2}}-\frac{2a(3Ab-5Ba)x^{3}}{3b^{3}}+\frac{2a\left(6a^{2}bA-10a^{3}B\right)x}{b^{5}}}{(bx+a)^{2}}+\frac{2a^{2}(3Ab-5Ba)\ln(bx+a)}{b^{6}}$
risch	$\frac{B\,x^{3}}{3b^{3}}+\frac{A\,x^{2}}{2b^{3}}-\frac{3Ba\,x^{2}}{2b^{4}}-\frac{3aAx}{b^{4}}+\frac{6B\,a^{2}x}{b^{5}}+\frac{(4A\,a^{3}b-5B\,a^{4})x+\frac{a^{4}(7Ab-9Ba)}{2b}}{b^{5}(bx+a)^{2}}+\frac{6a^{2}\ln(bx+a)A}{b^{5}}-\frac{10a^{3}\ln(bx+a}{b^{6}}$
parallelrisch	$\frac{2b^{5}B\,x^{5}+3A\,b^{5}x^{4}-5Ba\,b^{4}x^{4}+36A\ln(bx+a)x^{2}a^{2}b^{3}-12Aa\,b^{4}x^{3}-60B\ln(bx+a)x^{2}a^{3}b^{2}+20B\,a^{2}b^{3}x^{3}+72A\ln(bx+a)x\,a^{3}b^{2}-}{6b^{6}(bx+a)^{2}}$

input
```
int(x^4*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/3*B*b^2*x^3-1/2*A*b^2*x^2+3/2*B*a*b*x^2+3*a*A*b*x-6*B*a^2*x)-1/
2*a^4*(A*b-B*a)/b^6/(b*x+a)^2+a^3*(4*A*b-5*B*a)/b^6/(b*x+a)+2*a^2*(3*A*b-5
*B*a)*ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = \frac{2\,Bb^5x^5 - 27\,Ba^5 + 21\,Aa^4b - (5\,Bab^4 - 3\,Ab^5)x^4 + 4\,(5\,Ba^2b^3 - 3\,Aab^4)x^3 + 3\,(21\,Ba^3b^2 - 11\,Aa^2b^3)x}{6\,(b^8x^2 +}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*B*b^5*x^5 - 27*B*a^5 + 21*A*a^4*b - (5*B*a*b^4 - 3*A*b^5)*x^4 + 4*(
5*B*a^2*b^3 - 3*A*a*b^4)*x^3 + 3*(21*B*a^3*b^2 - 11*A*a^2*b^3)*x^2 + 6*(B*
a^4*b + A*a^3*b^2)*x - 12*(5*B*a^5 - 3*A*a^4*b + (5*B*a^3*b^2 - 3*A*a^2*b^
3)*x^2 + 2*(5*B*a^4*b - 3*A*a^3*b^2)*x)*log(b*x + a))/(b^8*x^2 + 2*a*b^7*x
 + a^2*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = \frac{Bx^3}{3b^3} - \frac{2a^2(-3Ab+5Ba)\log{(a+bx)}}{b^6} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + x\left(-\frac{3Aa}{b^4} + \frac{6Ba^2}{b^5}\right) + \frac{7Aa^4b - 9Ba^5 + x(8Aa^3b^2 - 10Ba^4b)}{2a^2b^6 + 4ab^7x + 2b^8x^2}$$

input
```
integrate(x**4*(B*x+A)/(b*x+a)**3,x)
```

output
```
B*x**3/(3*b**3) - 2*a**2*(-3*A*b + 5*B*a)*log(a + b*x)/b**6 + x**2*(A/(2*b
**3) - 3*B*a/(2*b**4)) + x*(-3*A*a/b**4 + 6*B*a**2/b**5) + (7*A*a**4*b - 9
*B*a**5 + x*(8*A*a**3*b**2 - 10*B*a**4*b))/(2*a**2*b**6 + 4*a*b**7*x + 2*b
**8*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = -\frac{9\,Ba^5-7\,Aa^4b+2\,(5\,Ba^4b-4\,Aa^3b^2)x}{2\,(b^8x^2+2\,ab^7x+a^2b^6)} + \frac{2\,Bb^2x^3-3\,(3\,Bab-Ab^2)x^2+18\,(2\,Ba^2-Aab)x}{6\,b^5} - \frac{2\,(5\,Ba^3-3\,Aa^2b)\log(bx+a)}{b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(9*B*a^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x)/(b^8*x^2 + 2*a*
b^7*x + a^2*b^6) + 1/6*(2*B*b^2*x^3 - 3*(3*B*a*b - A*b^2)*x^2 + 18*(2*B*a^
2 - A*a*b)*x)/b^5 - 2*(5*B*a^3 - 3*A*a^2*b)*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = -\frac{2\,(5\,Ba^3-3\,Aa^2b)\log(|bx+a|)}{b^6} - \frac{9\,Ba^5-7\,Aa^4b+2\,(5\,Ba^4b-4\,Aa^3b^2)x}{2\,(bx+a)^2b^6} + \frac{2\,Bb^6x^3-9\,Bab^5x^2+3\,Ab^6x^2+36\,Ba^2b^4x-18\,Aab^5x}{6\,b^9}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*(5*B*a^3 - 3*A*a^2*b)*log(abs(b*x + a))/b^6 - 1/2*(9*B*a^5 - 7*A*a^4*b
+ 2*(5*B*a^4*b - 4*A*a^3*b^2)*x)/((b*x + a)^2*b^6) + 1/6*(2*B*b^6*x^3 - 9*
B*a*b^5*x^2 + 3*A*b^6*x^2 + 36*B*a^2*b^4*x - 18*A*a*b^5*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.27

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right)-x\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)-\frac{x\,\left(5\,B\,a^4-4\,A\,a^3\,b\right)+\frac{9\,B\,a^5-7\,A\,a^4\,b}{2\,b}}{a^2\,b^5+2\,a\,b^6\,x+b^7\,x^2}-\frac{\ln\left(a+b\,x\right)\,\left(10\,B\,a^3-6\,A\,a^2\,b\right)}{b^6}+\frac{B\,x^3}{3\,b^3}$$

input
```
int((x^4*(A + B*x))/(a + b*x)^3,x)
```

output
```
x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) - x*((3*a*(A/b^3 - (3*B*a)/b^4))/b + (3*
B*a^2)/b^5) - (x*(5*B*a^4 - 4*A*a^3*b) + (9*B*a^5 - 7*A*a^4*b)/(2*b))/(a^2
*b^5 + b^7*x^2 + 2*a*b^6*x) - (log(a + b*x)*(10*B*a^3 - 6*A*a^2*b))/b^6 +
(B*x^3)/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx = \frac{-12\log(bx+a)\,a^4-12\log(bx+a)\,a^3bx+12a^3bx+6a^2b^2x^2-2a\,b^3x^3+b^4x^4}{3b^5\,(bx+a)}$$

input
```
int(x^4*(B*x+A)/(b*x+a)^3,x)
```

output
```
( - 12*log(a + b*x)*a**4 - 12*log(a + b*x)*a**3*b*x + 12*a**3*b*x + 6*a**2
*b**2*x**2 - 2*a*b**3*x**3 + b**4*x**4)/(3*b**5*(a + b*x))
```

3.163 $\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 94

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{(Ab-3aB)x}{b^4}+\frac{Bx^2}{2b^3}+\frac{a^3(Ab-aB)}{2b^5(a+bx)^2} - \frac{a^2(3Ab-4aB)}{b^5(a+bx)}-\frac{3a(Ab-2aB)\log(a+bx)}{b^5}$$

output
```
(A*b-3*B*a)*x/b^4+1/2*B*x^2/b^3+1/2*a^3*(A*b-B*a)/b^5/(b*x+a)^2-a^2*(3*A*b
-4*B*a)/b^5/(b*x+a)-3*a*(A*b-2*B*a)*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{2b(Ab-3aB)x+b^2Bx^2+\frac{a^3(Ab-aB)}{(a+bx)^2}+\frac{2a^2(-3Ab+4aB)}{a+bx}+6a(-Ab+2aB)\log(a+bx)}{2b^5}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x)^3,x]
```

output
```
(2*b*(A*b - 3*a*B)*x + b^2*B*x^2 + (a^3*(A*b - a*B))/(a + b*x)^2 + (2*a^2*
(-3*A*b + 4*a*B))/(a + b*x) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx)^3} dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)^3}-\frac{a^2(4aB-3Ab)}{b^4(a+bx)^2}+\frac{3a(2aB-Ab)}{b^4(a+bx)}+\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3(Ab-aB)}{2b^5(a+bx)^2}-\frac{a^2(3Ab-4aB)}{b^5(a+bx)}-\frac{3a(Ab-2aB)\log(a+bx)}{b^5}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^2}{2b^3}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x)^3,x]
```

output
```
((A*b - 3*a*B)*x)/b^4 + (B*x^2)/(2*b^3) + (a^3*(A*b - a*B))/(2*b^5*(a + b*
x)^2) - (a^2*(3*A*b - 4*a*B))/(b^5*(a + b*x)) - (3*a*(A*b - 2*a*B)*Log[a +
 b*x])/b^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

method	result
default	$\frac{\frac{1}{2} b B\,x^{2}+A b x-3 B a x}{b^{4}}+\frac{a^{3} \left(A b-B a\right)}{2 b^{5} \left(b x+a\right)^{2}}-\frac{a^{2} \left(3 A b-4 B a\right)}{b^{5} \left(b x+a\right)}-\frac{3 a \left(A b-2 B a\right) \ln \left(b x+a\right)}{b^{5}}$
norman	$\frac{\frac{\left(A b-2 B a\right) x^{3}}{b^{2}}+\frac{B\,x^{4}}{2 b}-\frac{a^{2} \left(9 a b A-18 a^{2} B\right)}{2 b^{5}}-\frac{2 a \left(3 a b A-6 a^{2} B\right) x}{b^{4}}}{\left(b x+a\right)^{2}}-\frac{3 a \left(A b-2 B a\right) \ln \left(b x+a\right)}{b^{5}}$
risch	$\frac{B\,x^{2}}{2 b^{3}}+\frac{A x}{b^{3}}-\frac{3 B a x}{b^{4}}+\frac{\left(-3 a^{2} b A+4 a^{3} B\right) x-\frac{a^{3} \left(5 A b-7 B a\right)}{2 b}}{b^{4} \left(b x+a\right)^{2}}-\frac{3 a \ln \left(b x+a\right) A}{b^{4}}+\frac{6 a^{2} \ln \left(b x+a\right) B}{b^{5}}$
parallelrisch	$-\frac{-B\,x^{4} b^{4}+6 A \ln \left(b x+a\right) x^{2} a\,b^{3}-2 A\,x^{3} b^{4}-12 B \ln \left(b x+a\right) x^{2} a^{2} b^{2}+4 B\,x^{3} a\,b^{3}+12 A \ln \left(b x+a\right) x\,a^{2} b^{2}-24 B \ln \left(b x+a\right) x\,a^{3} b+6 A}{2 b^{5} \left(b x+a\right)^{2}}$

input
```
int(x^3*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/2*b*B*x^2+A*b*x-3*B*a*x)+1/2*a^3*(A*b-B*a)/b^5/(b*x+a)^2-a^2*(3*A
*b-4*B*a)/b^5/(b*x+a)-3*a*(A*b-2*B*a)*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.82

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx$$
$$= \frac{Bb^4x^4 + 7\,Ba^4 - 5\,Aa^3b - 2\,(2\,Bab^3 - Ab^4)x^3 - (11\,Ba^2b^2 - 4\,Aab^3)x^2 + 2\,(Ba^3b - 2\,Aa^2b^2)x + 6\,(2\,B}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(B*b^4*x^4 + 7*B*a^4 - 5*A*a^3*b - 2*(2*B*a*b^3 - A*b^4)*x^3 - (11*B*a
^2*b^2 - 4*A*a*b^3)*x^2 + 2*(B*a^3*b - 2*A*a^2*b^2)*x + 6*(2*B*a^4 - A*a^3
*b + (2*B*a^2*b^2 - A*a*b^3)*x^2 + 2*(2*B*a^3*b - A*a^2*b^2)*x)*log(b*x +
a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{Bx^2}{2b^3} + \frac{3a(-Ab+2Ba)\log(a+bx)}{b^5} + x\left(\frac{A}{b^3} - \frac{3Ba}{b^4}\right) + \frac{-5Aa^3b + 7Ba^4 + x(-6Aa^2b^2 + 8Ba^3b)}{2a^2b^5 + 4ab^6x + 2b^7x^2}$$

input
```
integrate(x**3*(B*x+A)/(b*x+a)**3,x)
```

output
```
B*x**2/(2*b**3) + 3*a*(-A*b + 2*B*a)*log(a + b*x)/b**5 + x*(A/b**3 - 3*B*a
/b**4) + (-5*A*a**3*b + 7*B*a**4 + x*(-6*A*a**2*b**2 + 8*B*a**3*b))/(2*a**
2*b**5 + 4*a*b**6*x + 2*b**7*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x}{2\,(b^7x^2+2\,ab^6x+a^2b^5)} + \frac{Bbx^2-2\,(3\,Ba-Ab)x}{2\,b^4} + \frac{3\,(2\,Ba^2-Aab)\log\left(bx+a\right)}{b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x)/(b^7*x^2 + 2*a*b
^6*x + a^2*b^5) + 1/2*(B*b*x^2 - 2*(3*B*a - A*b)*x)/b^4 + 3*(2*B*a^2 - A*a
*b)*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{3\,(2\,Ba^2-Aab)\log\left(|bx+a|\right)}{b^5} + \frac{Bb^3x^2-6\,Bab^2x+2\,Ab^3x}{2\,b^6} + \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x}{2\,(bx+a)^2b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
3*(2*B*a^2 - A*a*b)*log(abs(b*x + a))/b^5 + 1/2*(B*b^3*x^2 - 6*B*a*b^2*x +
 2*A*b^3*x)/b^6 + 1/2*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x
)/((b*x + a)^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{x\,(4\,B\,a^3-3\,A\,a^2\,b)+\frac{7\,B\,a^4-5\,A\,a^3\,b}{2\,b}}{a^2\,b^4+2\,a\,b^5\,x+b^6\,x^2}+x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)+\frac{B\,x^2}{2\,b^3}+\frac{\ln(a+b\,x)\,\left(6\,B\,a^2-3\,A\,a\,b\right)}{b^5}$$

input
```
int((x^3*(A + B*x))/(a + b*x)^3,x)
```

output
```
(x*(4*B*a^3 - 3*A*a^2*b) + (7*B*a^4 - 5*A*a^3*b)/(2*b))/(a^2*b^4 + b^6*x^2
 + 2*a*b^5*x) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^2)/(2*b^3) + (log(a + b*x)*
(6*B*a^2 - 3*A*a*b))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

$$\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx = \frac{6\log(bx+a)\,a^3+6\log(bx+a)\,a^2bx-6a^2bx-3a\,b^2x^2+b^3x^3}{2b^4\,(bx+a)}$$

input
```
int(x^3*(B*x+A)/(b*x+a)^3,x)
```

output
```
(6*log(a + b*x)*a**3 + 6*log(a + b*x)*a**2*b*x - 6*a**2*b*x - 3*a*b**2*x**
2 + b**3*x**3)/(2*b**4*(a + b*x))
```

3.164 $\int \frac{x^2(A+Bx)}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{x^2(A+Bx)}{(a+bx)^3} \, dx = \frac{Bx}{b^3} - \frac{a^2(Ab-aB)}{2b^4(a+bx)^2} + \frac{a(2Ab-3aB)}{b^4(a+bx)} + \frac{(Ab-3aB)\log(a+bx)}{b^4}$$

output
```
B*x/b^3-1/2*a^2*(A*b-B*a)/b^4/(b*x+a)^2+a*(2*A*b-3*B*a)/b^4/(b*x+a)+(A*b-3
*B*a)*ln(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx)}{(a+bx)^3} \, dx = \frac{Bx}{b^3} + \frac{-a^2Ab+a^3B}{2b^4(a+bx)^2} + \frac{2aAb-3a^2B}{b^4(a+bx)} + \frac{(Ab-3aB)\log(a+bx)}{b^4}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x)^3,x]
```

output
```
(B*x)/b^3 + (-(a^2*A*b) + a^3*B)/(2*b^4*(a + b*x)^2) + (2*a*A*b - 3*a^2*B)
/(b^4*(a + b*x)) + ((A*b - 3*a*B)*Log[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx)^3} \, dx$$

$$\Big\downarrow \; 86$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)^3}+\frac{a(3aB-2Ab)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}+\frac{B}{b^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2(Ab-aB)}{2b^4(a+bx)^2}+\frac{a(2Ab-3aB)}{b^4(a+bx)}+\frac{(Ab-3aB)\log(a+bx)}{b^4}+\frac{Bx}{b^3}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x)^3,x]
```

output
```
(B*x)/b^3 - (a^2*(A*b - a*B))/(2*b^4*(a + b*x)^2) + (a*(2*A*b - 3*a*B))/(b
^4*(a + b*x)) + ((A*b - 3*a*B)*Log[a + b*x])/b^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{B\,x^3}{b}+\frac{a^2(3Ab-9Ba)}{2b^4}+\frac{2a(Ab-3Ba)x}{b^3}}{(bx+a)^2}+\frac{(Ab-3Ba)\ln(bx+a)}{b^4}$
default	$\frac{Bx}{b^3}-\frac{a^2(Ab-Ba)}{2b^4(bx+a)^2}+\frac{a(2Ab-3Ba)}{b^4(bx+a)}+\frac{(Ab-3Ba)\ln(bx+a)}{b^4}$
risch	$\frac{Bx}{b^3}+\frac{(2abA-3a^2B)x+\frac{a^2(3Ab-5Ba)}{2b}}{b^3(bx+a)^2}+\frac{\ln(bx+a)A}{b^3}-\frac{3\ln(bx+a)Ba}{b^4}$
parallelrisch	$\frac{2A\ln(bx+a)x^2b^3-6B\ln(bx+a)x^2a\,b^2+2b^3B\,x^3+4A\ln(bx+a)xa\,b^2-12B\ln(bx+a)x\,a^2b+2A\ln(bx+a)a^2b+4Axa\,b^2-6B\ln(}{2b^4(bx+a)^2}$

input
```
int(x^2*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(B*x^3/b+1/2*a^2*(3*A*b-9*B*a)/b^4+2*a*(A*b-3*B*a)/b^3*x)/(b*x+a)^2+(A*b-3
*B*a)*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx$$
$$=\frac{2\,Bb^3x^3+4\,Bab^2x^2-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x-2\,(3\,Ba^3-Aa^2b+(3\,Bab^2-Ab^3)x^2+2\,(}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*b^3*x^3 + 4*B*a*b^2*x^2 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*
b^2)*x - 2*(3*B*a^3 - A*a^2*b + (3*B*a*b^2 - A*b^3)*x^2 + 2*(3*B*a^2*b - A
*a*b^2)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx = \frac{Bx}{b^3} + \frac{3Aa^2b-5Ba^3+x(4Aab^2-6Ba^2b)}{2a^2b^4+4ab^5x+2b^6x^2} - \frac{(-Ab+3Ba)\log{(a+bx)}}{b^4}$$

input
```
integrate(x**2*(B*x+A)/(b*x+a)**3,x)
```

output
```
B*x/b**3 + (3*A*a**2*b - 5*B*a**3 + x*(4*A*a*b**2 - 6*B*a**2*b))/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) - (-A*b + 3*B*a)*log(a + b*x)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx = -\frac{5\,Ba^3-3\,Aa^2b+2\,(3\,Ba^2b-2\,Aab^2)x}{2\,(b^6x^2+2\,ab^5x+a^2b^4)} + \frac{Bx}{b^3} - \frac{(3\,Ba-Ab)\log{(bx+a)}}{b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x)/(b^6*x^2 + 2*a*b^
5*x + a^2*b^4) + B*x/b^3 - (3*B*a - A*b)*log(b*x + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx = \frac{Bx}{b^3} - \frac{(3\,Ba - Ab)\log(|bx+a|)}{b^4} - \frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x}{2\,(bx+a)^2b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
B*x/b^3 - (3*B*a - A*b)*log(abs(b*x + a))/b^4 - 1/2*(5*B*a^3 - 3*A*a^2*b +
 2*(3*B*a^2*b - 2*A*a*b^2)*x)/((b*x + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx = \frac{B\,x}{b^3} - \frac{\frac{5\,B\,a^3-3\,A\,a^2\,b}{2\,b} + x\,(3\,B\,a^2 - 2\,A\,a\,b)}{a^2\,b^3 + 2\,a\,b^4\,x + b^5\,x^2} + \frac{\ln(a+b\,x)\,(A\,b - 3\,B\,a)}{b^4}$$

input
```
int((x^2*(A + B*x))/(a + b*x)^3,x)
```

output
```
(B*x)/b^3 - ((5*B*a^3 - 3*A*a^2*b)/(2*b) + x*(3*B*a^2 - 2*A*a*b))/(a^2*b^3
 + b^5*x^2 + 2*a*b^4*x) + (log(a + b*x)*(A*b - 3*B*a))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx = \frac{-2\log(bx+a)\,a^2 - 2\log(bx+a)\,abx + 2abx + b^2x^2}{b^3\,(bx+a)}$$

input
```
int(x^2*(B*x+A)/(b*x+a)^3,x)
```

output
```
( - 2*log(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + 2*a*b*x + b**2*x**2)/(b**
3*(a + b*x))
```

3.165 $\int \frac{x(A+Bx)}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{x(A+Bx)}{(a+bx)^3} \, dx = \frac{a(Ab-aB)}{2b^3(a+bx)^2} - \frac{Ab-2aB}{b^3(a+bx)} + \frac{B\log(a+bx)}{b^3}$$

output
```
1/2*a*(A*b-B*a)/b^3/(b*x+a)^2-(A*b-2*B*a)/b^3/(b*x+a)+B*ln(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx)}{(a+bx)^3} \, dx = \frac{3a^2B - 2Ab^2x - ab(A-4Bx) + 2B(a+bx)^2\log(a+bx)}{2b^3(a+bx)^2}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x)^3,x]
```

output
```
(3*a^2*B - 2*A*b^2*x - a*b*(A - 4*B*x) + 2*B*(a + b*x)^2*Log[a + b*x])/(2*
b^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{a(aB-Ab)}{b^2(a+bx)^3}+\frac{B}{b^2(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab-2aB}{b^3(a+bx)}+\frac{a(Ab-aB)}{2b^3(a+bx)^2}+\frac{B\log(a+bx)}{b^3}$$

input
```
Int[(x*(A + B*x))/(a + b*x)^3,x]
```

output
```
(a*(A*b - a*B))/(2*b^3*(a + b*x)^2) - (A*b - 2*a*B)/(b^3*(a + b*x)) + (B*L
og[a + b*x])/b^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{2b^3}-\frac{(Ab-2Ba)x}{b^2}}{(bx+a)^2}+\frac{B\ln(bx+a)}{b^3}$	50
risch	$\frac{-\frac{a(Ab-3Ba)}{2b^3}-\frac{(Ab-2Ba)x}{b^2}}{(bx+a)^2}+\frac{B\ln(bx+a)}{b^3}$	50
default	$\frac{a(Ab-Ba)}{2b^3(bx+a)^2}-\frac{Ab-2Ba}{b^3(bx+a)}+\frac{B\ln(bx+a)}{b^3}$	54
parallelrisch	$-\frac{-2B\ln(bx+a)x^2b^2-4B\ln(bx+a)xab+2A\,b^2x-2B\ln(bx+a)a^2-4Babx+abA-3a^2B}{2b^3(bx+a)^2}$	76

input
```
int(x*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a*(A*b-3*B*a)/b^3-(A*b-2*B*a)/b^2*x)/(b*x+a)^2+B*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx = \frac{3\,Ba^2-Aab+2\,(2\,Bab-Ab^2)x+2\,(Bb^2x^2+2\,Babx+Ba^2)\log{(bx+a)}}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x + 2*(B*b^2*x^2 + 2*B*a*b*x +
B*a^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx = \frac{B\log{(a+bx)}}{b^3} + \frac{-Aab+3Ba^2+x(-2Ab^2+4Bab)}{2a^2b^3+4ab^4x+2b^5x^2}$$

input
```
integrate(x*(B*x+A)/(b*x+a)**3,x)
```

output
```
B*log(a + b*x)/b**3 + (-A*a*b + 3*B*a**2 + x*(-2*A*b**2 + 4*B*a*b))/(2*a**
2*b**3 + 4*a*b**4*x + 2*b**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx = \frac{3\,Ba^2-Aab+2\,(2\,Bab-Ab^2)x}{2\,(b^5x^2+2\,ab^4x+a^2b^3)} + \frac{B\log{(bx+a)}}{b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x)/(b^5*x^2 + 2*a*b^4*x + a^2*b
^3) + B*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx = \frac{B\log{(|bx+a|)}}{b^3} + \frac{2\,(2\,Ba-Ab)x+\frac{3\,Ba^2-Aab}{b}}{2\,(bx+a)^2b^2}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
B*log(abs(b*x + a))/b^3 + 1/2*(2*(2*B*a - A*b)*x + (3*B*a^2 - A*a*b)/b)/((
b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx = \frac{\frac{3\,B\,a^2-A\,a\,b}{2\,b^3}-\frac{x\,(A\,b-2\,B\,a)}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{B\,\ln\left(a+b\,x\right)}{b^3}$$

input
```
int((x*(A + B*x))/(a + b*x)^3,x)
```

output
```
((3*B*a^2 - A*a*b)/(2*b^3) - (x*(A*b - 2*B*a))/b^2)/(a^2 + b^2*x^2 + 2*a*b
*x) + (B*log(a + b*x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int \frac{x(A+Bx)}{(a+bx)^3}\,dx = \frac{\log(bx+a)\,a+\log(bx+a)\,bx-bx}{b^2\,(bx+a)}$$

input
```
int(x*(B*x+A)/(b*x+a)^3,x)
```

output
```
(log(a + b*x)*a + log(a + b*x)*b*x - b*x)/(b**2*(a + b*x))
```

3.166 $\int \frac{A+Bx}{(a+bx)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{A+Bx}{(a+bx)^3} dx = -\frac{(A+Bx)^2}{2(Ab-aB)(a+bx)^2}$$

output
```
-1/2*(B*x+A)^2/(A*b-B*a)/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{(a+bx)^3} dx = -\frac{Ab+B(a+2bx)}{2b^2(a+bx)^2}$$

input
```
Integrate[(A + B*x)/(a + b*x)^3,x]
```

output
```
-1/2*(A*b + B*(a + 2*b*x))/(b^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^3}\,dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(A+Bx)^2}{2(a+bx)^2(Ab-aB)}$$

input
```
Int[(A + B*x)/(a + b*x)^3,x]
```

output
```
-1/2*(A + B*x)^2/((A*b - a*B)*(a + b*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2bBx+Ab+Ba}{2b^{2}(bx+a)^{2}}$	25
parallelrisch	$-\frac{2bBx+Ab+Ba}{2b^{2}(bx+a)^{2}}$	25
orering	$-\frac{2bBx+Ab+Ba}{2b^{2}(bx+a)^{2}}$	25
norman	$\frac{-\frac{Bx}{b}-\frac{Ab+Ba}{2b^{2}}}{(bx+a)^{2}}$	29
risch	$\frac{-\frac{Bx}{b}-\frac{Ab+Ba}{2b^{2}}}{(bx+a)^{2}}$	29
default	$-\frac{Ab-Ba}{2b^{2}(bx+a)^{2}}-\frac{B}{(bx+a)b^{2}}$	35

input
```
int((B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*B*b*x+A*b+B*a)/b^2/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx}{(a+bx)^3}\,dx=-\frac{2\,Bbx+Ba+Ab}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate((B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*B*b*x + B*a + A*b)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx}{(a+bx)^3}\,dx = \frac{-Ab-Ba-2Bbx}{2a^2b^2+4ab^3x+2b^4x^2}$$

input
```
integrate((B*x+A)/(b*x+a)**3,x)
```

output
```
(-A*b - B*a - 2*B*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx}{(a+bx)^3}\,dx = -\frac{2\,Bbx+Ba+Ab}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate((B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*B*b*x + B*a + A*b)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(a+bx)^3}\,dx = -\frac{2\,Bbx+Ba+Ab}{2\,(bx+a)^2b^2}$$

input
```
integrate((B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*(2*B*b*x + B*a + A*b)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx}{(a+bx)^3}\,dx = -\frac{\frac{A\,b+B\,a}{2\,b^2}+\frac{B\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int((A + B*x)/(a + b*x)^3,x)
```

output
```
-((A*b + B*a)/(2*b^2) + (B*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx}{(a+bx)^3}\,dx = \frac{x}{a\,(bx+a)}$$

input
```
int((B*x+A)/(b*x+a)^3,x)
```

output
```
x/(a*(a + b*x))
```

3.167 $\int \frac{A+Bx}{x(a+bx)^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{A+Bx}{x(a+bx)^3} \, dx = \frac{Ab-aB}{2ab(a+bx)^2} + \frac{A}{a^2(a+bx)} + \frac{A\log(x)}{a^3} - \frac{A\log(a+bx)}{a^3}$$

output
```
1/2*(A*b-B*a)/a/b/(b*x+a)^2+A/a^2/(b*x+a)+A*ln(x)/a^3-A*ln(b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x(a+bx)^3} \, dx = \frac{\frac{a(3aAb-a^2B+2Ab^2x)}{b(a+bx)^2} + 2A\log(x) - 2A\log(a+bx)}{2a^3}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x)^3),x]
```

output
```
((a*(3*a*A*b - a^2*B + 2*A*b^2*x))/(b*(a + b*x)^2) + 2*A*Log[x] - 2*A*Log[
a + b*x])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{Ab}{a^3(a+bx)}+\frac{A}{a^3x}-\frac{Ab}{a^2(a+bx)^2}+\frac{aB-Ab}{a(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\log(a+bx)}{a^3}+\frac{A\log(x)}{a^3}+\frac{A}{a^2(a+bx)}+\frac{Ab-aB}{2ab(a+bx)^2}$$

input
```
Int[(A + B*x)/(x*(a + b*x)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*(a + b*x)^2) + A/(a^2*(a + b*x)) + (A*Log[x])/a^3 - (A*
Log[a + b*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

method	result
default	$\frac{A\ln(x)}{a^3}-\frac{-Ab+Ba}{2ab(bx+a)^2}-\frac{A\ln(bx+a)}{a^3}+\frac{A}{a^2(bx+a)}$
risch	$\frac{\frac{Abx}{a^2}+\frac{3Ab-Ba}{2ab}}{(bx+a)^2}+\frac{A\ln(-x)}{a^3}-\frac{A\ln(bx+a)}{a^3}$
norman	$\frac{-\frac{(2Ab-Ba)x}{a^2}-\frac{b(3Ab-Ba)x^2}{2a^3}}{(bx+a)^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln(bx+a)}{a^3}$
parallelrisch	$\frac{2A\ln(x)x^2b^2-2A\ln(bx+a)x^2b^2+4A\ln(x)xab-4A\ln(bx+a)xab-3A\,b^2x^2+Bab\,x^2+2a^2A\ln(x)-2A\ln(bx+a)a^2-4aAbx+2}{2a^3(bx+a)^2}$

input
```
int((B*x+A)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^3-1/2*(-A*b+B*a)/a/b/(b*x+a)^2-A*ln(b*x+a)/a^3+A/a^2/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx = \frac{2\,Aab^2x-Ba^3+3\,Aa^2b-2\,(Ab^3x^2+2\,Aab^2x+Aa^2b)\log{(bx+a)}+2\,(Ab^3x^2+2\,Aab^2x+Aa^2b)\log{(}}{2\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}$$

input
```
integrate((B*x+A)/x/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*A*a*b^2*x - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*
b)*log(b*x + a) + 2*(A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*b)*log(x))/(a^3*b^3*x
^2 + 2*a^4*b^2*x + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx = \frac{A\left(\log{(x)} - \log{\left(\frac{a}{b} + x\right)}\right)}{a^3} + \frac{3Aab + 2Ab^2x - Ba^2}{2a^4b + 4a^3b^2x + 2a^2b^3x^2}$$

input
```
integrate((B*x+A)/x/(b*x+a)**3,x)
```

output
```
A*(log(x) - log(a/b + x))/a**3 + (3*A*a*b + 2*A*b**2*x - B*a**2)/(2*a**4*b
 + 4*a**3*b**2*x + 2*a**2*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx = \frac{2\,Ab^2x - Ba^2 + 3\,Aab}{2\,(a^2b^3x^2 + 2\,a^3b^2x + a^4b)} - \frac{A\log{(bx+a)}}{a^3} + \frac{A\log{(x)}}{a^3}$$

input
```
integrate((B*x+A)/x/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(2*A*b^2*x - B*a^2 + 3*A*a*b)/(a^2*b^3*x^2 + 2*a^3*b^2*x + a^4*b) - A*
log(b*x + a)/a^3 + A*log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx = -\frac{A\log{(|bx+a|)}}{a^3} + \frac{A\log{(|x|)}}{a^3} + \frac{2\,Aab^2x - Ba^3 + 3\,Aa^2b}{2\,(bx+a)^2a^3b}$$

input
```
integrate((B*x+A)/x/(b*x+a)^3,x, algorithm="giac")
```

output
```
-A*log(abs(b*x + a))/a^3 + A*log(abs(x))/a^3 + 1/2*(2*A*a*b^2*x - B*a^3 +
3*A*a^2*b)/((b*x + a)^2*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx = \frac{\frac{3\,A\,b-B\,a}{2\,a\,b}+\frac{A\,b\,x}{a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}-\frac{2\,A\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

input
```
int((A + B*x)/(x*(a + b*x)^3),x)
```

output
```
((3*A*b - B*a)/(2*a*b) + (A*b*x)/a^2)/(a^2 + b^2*x^2 + 2*a*b*x) - (2*A*ata
nh((2*b*x)/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{x(a+bx)^3}\,dx = \frac{-\log(bx+a)\,a-\log(bx+a)\,bx+\log(x)\,a+\log(x)\,bx-bx}{a^2\,(bx+a)}$$

input
```
int((B*x+A)/x/(b*x+a)^3,x)
```

output
```
( - log(a + b*x)*a - log(a + b*x)*b*x + log(x)*a + log(x)*b*x - b*x)/(a**2
*(a + b*x))
```

3.168 $\int \frac{A+Bx}{x^2(a+bx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = -\frac{A}{a^3x}-\frac{Ab-aB}{2a^2(a+bx)^2}-\frac{2Ab-aB}{a^3(a+bx)} -\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log(a+bx)}{a^4}$$

output
```
-A/a^3/x-1/2*(A*b-B*a)/a^2/(b*x+a)^2-(2*A*b-B*a)/a^3/(b*x+a)-(3*A*b-B*a)*l
n(x)/a^4+(3*A*b-B*a)*ln(b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = \frac{-\frac{2aA}{x}+\frac{a^2(-Ab+aB)}{(a+bx)^2}+\frac{2a(-2Ab+aB)}{a+bx}+2(-3Ab+aB)\log(x)+2(3Ab-aB)\log(a+bx)}{2a^4}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x)^3),x]
```

output
```
((-2*a*A)/x + (a^2*(-(A*b) + a*B))/(a + b*x)^2 + (2*a*(-2*A*b + a*B))/(a +
 b*x) + 2*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(a+bx)^3} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{aB-3Ab}{a^4x}-\frac{b(aB-3Ab)}{a^4(a+bx)}-\frac{b(aB-2Ab)}{a^3(a+bx)^2}+\frac{A}{a^3x^2}-\frac{b(aB-Ab)}{a^2(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(x)(3Ab-aB)}{a^4}+\frac{(3Ab-aB)\log(a+bx)}{a^4}-\frac{2Ab-aB}{a^3(a+bx)}-\frac{A}{a^3x}-\frac{Ab-aB}{2a^2(a+bx)^2}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x)^3),x]
```

output
```
-(A/(a^3*x)) - (A*b - a*B)/(2*a^2*(a + b*x)^2) - (2*A*b - a*B)/(a^3*(a + b
*x)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x])/a^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{a^{3} x}+\frac{\left(-3 A b +B a \right) \ln \left(x \right)}{a^{4}}-\frac{2 A b -B a}{a^{3} \left(b x +a \right)}+\frac{\left(3 A b -B a \right) \ln \left(b x +a \right)}{a^{4}}-\frac{A b -B a}{2 a^{2} \left(b x +a \right)^{2}}$
norman	$\frac{-\frac{A}{a}+\frac{2 b \left(3 A b -B a \right) x^{2}}{a^{3}}+\frac{b^{2} \left(9 A b -3 B a \right) x^{3}}{2 a^{4}}}{x \left(b x +a \right)^{2}}+\frac{\left(3 A b -B a \right) \ln \left(b x +a \right)}{a^{4}}-\frac{\left(3 A b -B a \right) \ln \left(x \right)}{a^{4}}$
risch	$\frac{-\frac{b \left(3 A b -B a \right) x^{2}}{a^{3}}-\frac{3 \left(3 A b -B a \right) x}{2 a^{2}}-\frac{A}{a}}{x \left(b x +a \right)^{2}}-\frac{3 \ln \left(x \right) A b}{a^{4}}+\frac{\ln \left(x \right) B}{a^{3}}+\frac{3 \ln \left(-b x -a \right) A b}{a^{4}}-\frac{\ln \left(-b x -a \right) B}{a^{3}}$
parallelrisch	$-\frac{6 A \ln \left(x \right) x^{3} b^{3}-6 A \ln \left(b x +a \right) x^{3} b^{3}-2 B \ln \left(x \right) x^{3} a \,b^{2}+2 B \ln \left(b x +a \right) x^{3} a \,b^{2}+12 A \ln \left(x \right) x^{2} a \,b^{2}-12 A \ln \left(b x +a \right) x^{2} a \,b^{2}-9 A \,b^{3} x^{3}-\cdots}{\cdots}$

input
```
int((B*x+A)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-A/a^3/x+(-3*A*b+B*a)/a^4*ln(x)-(2*A*b-B*a)/a^3/(b*x+a)+(3*A*b-B*a)*ln(b*x
+a)/a^4-1/2*(A*b-B*a)/a^2/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(82) = 164.

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = -\frac{2\,Aa^3 - 2\,(Ba^2b - 3\,Aab^2)x^2 - 3\,(Ba^3 - 3\,Aa^2b)x + 2\,((Bab^2 - 3\,Ab^3)x^3 + 2\,(Ba^2b - 3\,Aab^2)x^2 + (\cdots}{2\,(a^4b^2x^3 + 2\cdots}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*A*a^3 - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - 3*(B*a^3 - 3*A*a^2*b)*x + 2*
((B*a*b^2 - 3*A*b^3)*x^3 + 2*(B*a^2*b - 3*A*a*b^2)*x^2 + (B*a^3 - 3*A*a^2*
b)*x)*log(b*x + a) - 2*((B*a*b^2 - 3*A*b^3)*x^3 + 2*(B*a^2*b - 3*A*a*b^2)*
x^2 + (B*a^3 - 3*A*a^2*b)*x)*log(x))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.

Time = 0.35 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = \frac{-2Aa^2 + x^2(-6Ab^2 + 2Bab) + x(-9Aab + 3Ba^2)}{2a^5x + 4a^4bx^2 + 2a^3b^2x^3} + \frac{(-3Ab+Ba)\log\left(x + \frac{-3Aab+Ba^2-a(-3Ab+Ba)}{-6Ab^2+2Bab}\right)}{a^4} - \frac{(-3Ab+Ba)\log\left(x + \frac{-3Aab+Ba^2+a(-3Ab+Ba)}{-6Ab^2+2Bab}\right)}{a^4}$$

input
```
integrate((B*x+A)/x**2/(b*x+a)**3,x)
```

output
```
(-2*A*a**2 + x**2*(-6*A*b**2 + 2*B*a*b) + x*(-9*A*a*b + 3*B*a**2))/(2*a**5
*x + 4*a**4*b*x**2 + 2*a**3*b**2*x**3) + (-3*A*b + B*a)*log(x + (-3*A*a*b
+ B*a**2 - a*(-3*A*b + B*a))/(-6*A*b**2 + 2*B*a*b))/a**4 - (-3*A*b + B*a)*
log(x + (-3*A*a*b + B*a**2 + a*(-3*A*b + B*a))/(-6*A*b**2 + 2*B*a*b))/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = -\frac{2\,Aa^2 - 2\,(Bab - 3\,Ab^2)x^2 - 3\,(Ba^2 - 3\,Aab)x}{2\,(a^3b^2x^3 + 2\,a^4bx^2 + a^5x)} - \frac{(Ba - 3\,Ab)\log(bx+a)}{a^4} + \frac{(Ba - 3\,Ab)\log(x)}{a^4}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*A*a^2 - 2*(B*a*b - 3*A*b^2)*x^2 - 3*(B*a^2 - 3*A*a*b)*x)/(a^3*b^2*
x^3 + 2*a^4*b*x^2 + a^5*x) - (B*a - 3*A*b)*log(b*x + a)/a^4 + (B*a - 3*A*b
)*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = \frac{(Ba-3\,Ab)\log(|x|)}{a^4} - \frac{(Bab-3\,Ab^2)\log(|bx+a|)}{a^4b} - \frac{2\,Aa^3-2\,(Ba^2b-3\,Aab^2)x^2-3\,(Ba^3-3\,Aa^2b)x}{2\,(bx+a)^2a^4x}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^3,x, algorithm="giac")
```

output
```
(B*a - 3*A*b)*log(abs(x))/a^4 - (B*a*b - 3*A*b^2)*log(abs(b*x + a))/(a^4*b
) - 1/2*(2*A*a^3 - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - 3*(B*a^3 - 3*A*a^2*b)*x)/
((b*x + a)^2*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)\,(3\,A\,b-B\,a)}{a^4} - \frac{\frac{A}{a}+\frac{3\,x\,(3\,A\,b-B\,a)}{2\,a^2}+\frac{b\,x^2\,(3\,A\,b-B\,a)}{a^3}}{a^2\,x+2\,a\,b\,x^2+b^2\,x^3}$$

input
```
int((A + B*x)/(x^2*(a + b*x)^3),x)
```

output
```
(2*atanh((2*b*x)/a + 1)*(3*A*b - B*a))/a^4 - (A/a + (3*x*(3*A*b - B*a))/(2
*a^2) + (b*x^2*(3*A*b - B*a))/a^3)/(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^2(a+bx)^3}\,dx = \frac{2\log(bx+a)\,abx+2\log(bx+a)\,b^2x^2-2\log(x)\,abx-2\log(x)\,b^2x^2-a^2+2b^2x^2}{a^3x\,(bx+a)}$$

input
```
int((B*x+A)/x^2/(b*x+a)^3,x)
```

output
```
(2*log(a + b*x)*a*b*x + 2*log(a + b*x)*b**2*x**2 - 2*log(x)*a*b*x - 2*log(
x)*b**2*x**2 - a**2 + 2*b**2*x**2)/(a**3*x*(a + b*x))
```

3.169 $\int \frac{A+Bx}{x^3(a+bx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx=-\frac{A}{2a^3x^2}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)}{2a^3(a+bx)^2}+\frac{b(3Ab-2aB)}{a^4(a+bx)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{3b(2Ab-aB)\log(a+bx)}{a^5}$$

output
```
-1/2*A/a^3/x^2+(3*A*b-B*a)/a^4/x+1/2*b*(A*b-B*a)/a^3/(b*x+a)^2+b*(3*A*b-2*
B*a)/a^4/(b*x+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-3*b*(2*A*b-B*a)*ln(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx =\frac{-\frac{a(-12Ab^3x^3+6ab^2x^2(-3A+Bx)+a^3(A+2Bx)+a^2bx(-4A+9Bx))}{x^2(a+bx)^2}+6b(2Ab-aB)\log(x)+6b(-2Ab+aB)\log(a+b}{2a^5}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x)^3),x]
```

output
```
(-((a*(-12*A*b^3*x^3 + 6*a*b^2*x^2*(-3*A + B*x) + a^3*(A + 2*B*x) + a^2*b*
x*(-4*A + 9*B*x)))/(x^2*(a + b*x)^2)) + 6*b*(2*A*b - a*B)*Log[x] + 6*b*(-2
*A*b + a*B)*Log[a + b*x])/(2*a^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{3b^2(aB-2Ab)}{a^5(a+bx)}-\frac{3b(aB-2Ab)}{a^5x}+\frac{b^2(2aB-3Ab)}{a^4(a+bx)^2}+\frac{aB-3Ab}{a^4x^2}+\frac{b^2(aB-Ab)}{a^3(a+bx)^3}+\frac{A}{a^3x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{3b(2Ab-aB)\log(a+bx)}{a^5}+\frac{3Ab-aB}{a^4x}+\frac{b(3Ab-2aB)}{a^4(a+bx)}+\frac{b(Ab-aB)}{2a^3(a+bx)^2}-\frac{A}{2a^3x^2}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x)^3),x]
```

output
```
-1/2*A/(a^3*x^2) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B))/(2*a^3*(a + b*x
)^2) + (b*(3*A*b - 2*a*B))/(a^4*(a + b*x)) + (3*b*(2*A*b - a*B)*Log[x])/a^
5 - (3*b*(2*A*b - a*B)*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{2a^{3}x^{2}}-\frac{-3Ab+Ba}{x\,a^{4}}+\frac{3b(2Ab-Ba)\ln(x)}{a^{5}}-\frac{3b(2Ab-Ba)\ln(bx+a)}{a^{5}}+\frac{b(3Ab-2Ba)}{a^{4}(bx+a)}+\frac{b(Ab-Ba)}{2a^{3}(bx+a)^{2}}$
norman	$\frac{\frac{(2Ab-Ba)x}{a^{2}}-\frac{A}{2a}-\frac{2b\left(6b^{2}A-3abB\right)x^{3}}{a^{4}}-\frac{b^{2}\left(18b^{2}A-9abB\right)x^{4}}{2a^{5}}}{x^{2}(bx+a)^{2}}+\frac{3b(2Ab-Ba)\ln(x)}{a^{5}}-\frac{3b(2Ab-Ba)\ln(bx+a)}{a^{5}}$
risch	$\frac{\frac{3b^{2}(2Ab-Ba)x^{3}}{a^{4}}+\frac{9b(2Ab-Ba)x^{2}}{2a^{3}}+\frac{(2Ab-Ba)x}{a^{2}}-\frac{A}{2a}}{x^{2}(bx+a)^{2}}+\frac{6b^{2}\ln(-x)A}{a^{5}}-\frac{3b\ln(-x)B}{a^{4}}-\frac{6b^{2}\ln(bx+a)A}{a^{5}}+\frac{3b\ln(bx+a)B}{a^{4}}$
parallelrisch	$\frac{12A\ln(x)x^{4}b^{4}-12A\ln(bx+a)x^{4}b^{4}-6B\ln(x)x^{4}a\,b^{3}+6B\ln(bx+a)x^{4}a\,b^{3}+24A\ln(x)x^{3}a\,b^{3}-24A\ln(bx+a)x^{3}a\,b^{3}-18A\,b^{4}x^{4}-}{}$

input
```
int((B*x+A)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^3/x^2-(-3*A*b+B*a)/x/a^4+3*b*(2*A*b-B*a)*ln(x)/a^5-3*b*(2*A*b-B*a
)*ln(b*x+a)/a^5+b*(3*A*b-2*B*a)/a^4/(b*x+a)+1/2*b*(A*b-B*a)/a^3/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(105) = 210$.

Time = 0.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.05

$$\int \frac{A + Bx}{x^3(a+bx)^3}\,dx = -\frac{Aa^4 + 6\,(Ba^2b^2 - 2\,Aab^3)x^3 + 9\,(Ba^3b - 2\,Aa^2b^2)x^2 + 2\,(Ba^4 - 2\,Aa^3b)x - 6\,((Bab^3 - 2\,Ab^4)x^4 + 2}{}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(A*a^4 + 6*(B*a^2*b^2 - 2*A*a*b^3)*x^3 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^
2 + 2*(B*a^4 - 2*A*a^3*b)*x - 6*((B*a*b^3 - 2*A*b^4)*x^4 + 2*(B*a^2*b^2 -
2*A*a*b^3)*x^3 + (B*a^3*b - 2*A*a^2*b^2)*x^2)*log(b*x + a) + 6*((B*a*b^3 -
 2*A*b^4)*x^4 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^3 + (B*a^3*b - 2*A*a^2*b^2)*x^
2)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(104) = 208$.

Time = 0.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{A + Bx}{x^3(a+bx)^3}\,dx &= \frac{-Aa^3 + x^3\cdot(12Ab^3 - 6Bab^2) + x^2\cdot(18Aab^2 - 9Ba^2b) + x(4Aa^2b - 2Ba^3)}{2a^6x^2 + 4a^5bx^3 + 2a^4b^2x^4} \\ &\quad - \frac{3b(-2Ab + Ba)\log\left(x + \frac{-6Aab^2 + 3Ba^2b - 3ab(-2Ab+Ba)}{-12Ab^3 + 6Bab^2}\right)}{a^5} \\ &\quad + \frac{3b(-2Ab + Ba)\log\left(x + \frac{-6Aab^2 + 3Ba^2b + 3ab(-2Ab+Ba)}{-12Ab^3 + 6Bab^2}\right)}{a^5}\end{aligned}$$

input
```
integrate((B*x+A)/x**3/(b*x+a)**3,x)
```

output
```
(-A*a**3 + x**3*(12*A*b**3 - 6*B*a*b**2) + x**2*(18*A*a*b**2 - 9*B*a**2*b)
 + x*(4*A*a**2*b - 2*B*a**3))/(2*a**6*x**2 + 4*a**5*b*x**3 + 2*a**4*b**2*x
**4) - 3*b*(-2*A*b + B*a)*log(x + (-6*A*a*b**2 + 3*B*a**2*b - 3*a*b*(-2*A*
b + B*a))/(-12*A*b**3 + 6*B*a*b**2))/a**5 + 3*b*(-2*A*b + B*a)*log(x + (-6
*A*a*b**2 + 3*B*a**2*b + 3*a*b*(-2*A*b + B*a))/(-12*A*b**3 + 6*B*a*b**2))/
a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx = -\frac{Aa^3+6\,(Bab^2-2\,Ab^3)x^3+9\,(Ba^2b-2\,Aab^2)x^2+2\,(Ba^3-2\,Aa^2b)x}{2\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)} + \frac{3\,(Bab-2\,Ab^2)\log(bx+a)}{a^5} - \frac{3\,(Bab-2\,Ab^2)\log(x)}{a^5}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(A*a^3 + 6*(B*a*b^2 - 2*A*b^3)*x^3 + 9*(B*a^2*b - 2*A*a*b^2)*x^2 + 2*
(B*a^3 - 2*A*a^2*b)*x)/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2) + 3*(B*a*b -
2*A*b^2)*log(b*x + a)/a^5 - 3*(B*a*b - 2*A*b^2)*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx = -\frac{3\,(Bab-2\,Ab^2)\log(|x|)}{a^5} + \frac{3\,(Bab^2-2\,Ab^3)\log(|bx+a|)}{a^5b} - \frac{6\,Bab^2x^3-12\,Ab^3x^3+9\,Ba^2bx^2-18\,Aab^2x^2+2\,Ba^3x-4\,Aa^2bx+Aa^3}{2\,(bx^2+ax)^2a^4}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^3,x, algorithm="giac")
```

output
```
-3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + 3*(B*a*b^2 - 2*A*b^3)*log(abs(b*x +
 a))/(a^5*b) - 1/2*(6*B*a*b^2*x^3 - 12*A*b^3*x^3 + 9*B*a^2*b*x^2 - 18*A*a*
b^2*x^2 + 2*B*a^3*x - 4*A*a^2*b*x + A*a^3)/((b*x^2 + a*x)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx=\frac{\frac{x\,(2\,A\,b-B\,a)}{a^2}-\frac{A}{2\,a}+\frac{3\,b^2\,x^3\,(2\,A\,b-B\,a)}{a^4}+\frac{9\,b\,x^2\,(2\,A\,b-B\,a)}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4}-\frac{6\,b\,\mathrm{atanh}\left(\frac{3\,b\,(2\,A\,b-B\,a)\,(a+2\,b\,x)}{a\,(6\,A\,b^2-3\,B\,a\,b)}\right)\,(2\,A\,b-B\,a)}{a^5}$$

input
```
int((A + B*x)/(x^3*(a + b*x)^3),x)
```

output
```
((x*(2*A*b - B*a))/a^2 - A/(2*a) + (3*b^2*x^3*(2*A*b - B*a))/a^4 + (9*b*x^
2*(2*A*b - B*a))/(2*a^3))/(a^2*x^2 + b^2*x^4 + 2*a*b*x^3) - (6*b*atanh((3*
b*(2*A*b - B*a)*(a + 2*b*x))/(a*(6*A*b^2 - 3*B*a*b)))*(2*A*b - B*a))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x^3(a+bx)^3}\,dx=\frac{-6\log(bx+a)\,a\,b^2x^2-6\log(bx+a)\,b^3x^3+6\log(x)\,a\,b^2x^2+6\log(x)\,b^3x^3-a^3+3a^2bx-6b^3x^3}{2a^4x^2\,(bx+a)}$$

input
```
int((B*x+A)/x^3/(b*x+a)^3,x)
```

output
```
( - 6*log(a + b*x)*a*b**2*x**2 - 6*log(a + b*x)*b**3*x**3 + 6*log(x)*a*b**
2*x**2 + 6*log(x)*b**3*x**3 - a**3 + 3*a**2*b*x - 6*b**3*x**3)/(2*a**4*x**
2*(a + b*x))
```

3.170 $\int \frac{A+Bx}{x^4(a+bx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 140

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{2a^4x^2}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)}{2a^4(a+bx)^2} -\frac{b^2(4Ab-3aB)}{a^5(a+bx)}-\frac{2b^2(5Ab-3aB)\log(x)}{a^6} +\frac{2b^2(5Ab-3aB)\log(a+bx)}{a^6}$$

output
```
-1/3*A/a^3/x^3+1/2*(3*A*b-B*a)/a^4/x^2-3*b*(2*A*b-B*a)/a^5/x-1/2*b^2*(A*b-
B*a)/a^4/(b*x+a)^2-b^2*(4*A*b-3*B*a)/a^5/(b*x+a)-2*b^2*(5*A*b-3*B*a)*ln(x)
/a^6+2*b^2*(5*A*b-3*B*a)*ln(b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = \frac{\frac{a(-60Ab^4x^4+18ab^3x^3(-5A+2Bx)-a^4(2A+3Bx)+a^3bx(5A+12Bx)+2a^2b^2x^2(-10A+27Bx))}{x^3(a+bx)^2}+12b^2(-5Ab+3aB)\log(x)+12}{6a^6}$$

input
```
Integrate[(A + B*x)/(x^4*(a + b*x)^3),x]
```

output
```
((a*(-60*A*b^4*x^4 + 18*a*b^3*x^3*(-5*A + 2*B*x) - a^4*(2*A + 3*B*x) + a^3
*b*x*(5*A + 12*B*x) + 2*a^2*b^2*x^2*(-10*A + 27*B*x)))/(x^3*(a + b*x)^2) +
 12*b^2*(-5*A*b + 3*a*B)*Log[x] + 12*b^2*(5*A*b - 3*a*B)*Log[a + b*x])/(6*
a^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx$$

$$\Big\downarrow \; 86$$

$$\int \left(-\frac{2b^3(3aB-5Ab)}{a^6(a+bx)}+\frac{2b^2(3aB-5Ab)}{a^6x}-\frac{b^3(3aB-4Ab)}{a^5(a+bx)^2}-\frac{3b(aB-2Ab)}{a^5x^2}-\frac{b^3(aB-Ab)}{a^4(a+bx)^3}+\frac{aB-3Ab}{a^4x^3}+\frac{}{a}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}+\frac{2b^2(5Ab-3aB)\log(a+bx)}{a^6}-\frac{b^2(4Ab-3aB)}{a^5(a+bx)}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)}{2a^4(a+bx)^2}+\frac{3Ab-aB}{2a^4x^2}-\frac{A}{3a^3x^3}$$

input
```
Int[(A + B*x)/(x^4*(a + b*x)^3),x]
```

output
```
-1/3*A/(a^3*x^3) + (3*A*b - a*B)/(2*a^4*x^2) - (3*b*(2*A*b - a*B))/(a^5*x)
 - (b^2*(A*b - a*B))/(2*a^4*(a + b*x)^2) - (b^2*(4*A*b - 3*a*B))/(a^5*(a +
b*x)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])/a^6 + (2*b^2*(5*A*b - 3*a*B)*Log[a
+ b*x])/a^6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

method	result
default	$-\frac{A}{3a^3x^3}-\frac{-3Ab+Ba}{2x^2a^4}-\frac{3b(2Ab-Ba)}{a^5x}-\frac{2b^2(5Ab-3Ba)\ln(x)}{a^6}-\frac{b^2(4Ab-3Ba)}{a^5(bx+a)}-\frac{b^2(Ab-Ba)}{2a^4(bx+a)^2}+\frac{2b^2(5Ab-3Ba)\,\mathrm{l}}{a^6}$
norman	$\frac{-\frac{A}{3a}+\frac{(5Ab-3Ba)x}{6a^2}-\frac{2b(5Ab-3Ba)x^2}{3a^3}-\frac{2\left(5b^5A-3a\,b^4B\right)x^4}{a^5b}-\frac{\left(15b^5A-9a\,b^4B\right)x^3}{a^4b^2}}{x^3(bx+a)^2}-\frac{2b^2(5Ab-3Ba)\ln(x)}{a^6}+\frac{2b^2(5Ab-3Ba)\,\mathrm{l}}{a^6}$
risch	$\frac{-\frac{2b^3(5Ab-3Ba)x^4}{a^5}-\frac{3b^2(5Ab-3Ba)x^3}{a^4}-\frac{2b(5Ab-3Ba)x^2}{3a^3}+\frac{(5Ab-3Ba)x}{6a^2}-\frac{A}{3a}}{x^3(bx+a)^2}+\frac{10b^3\ln(-bx-a)A}{a^6}-\frac{6b^2\ln(-bx-a)B}{a^5}-10$
parallelrisch	$-\frac{60A\ln(x)x^5b^7-60A\ln(bx+a)x^5b^7-36B\ln(x)x^5a\,b^6+36B\ln(bx+a)x^5a\,b^6+120A\ln(x)x^4a\,b^6-120A\ln(bx+a)x^4a\,b^6-72}{}$

input
```
int((B*x+A)/x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^3/x^3-1/2*(-3*A*b+B*a)/x^2/a^4-3*b*(2*A*b-B*a)/a^5/x-2*b^2*(5*A*b
-3*B*a)*ln(x)/a^6-b^2*(4*A*b-3*B*a)/a^5/(b*x+a)-1/2*b^2*(A*b-B*a)/a^4/(b*x
+a)^2+2*b^2*(5*A*b-3*B*a)*ln(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.87

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = -\frac{2\,Aa^5 - 12\,(3\,Ba^2b^3 - 5\,Aab^4)x^4 - 18\,(3\,Ba^3b^2 - 5\,Aa^2b^3)x^3 - 4\,(3\,Ba^4b - 5\,Aa^3b^2)x^2 + (3\,Ba^5 - 5\,A}{}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*A*a^5 - 12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^4 - 18*(3*B*a^3*b^2 - 5*A*a
^2*b^3)*x^3 - 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^2 + (3*B*a^5 - 5*A*a^4*b)*x +
12*((3*B*a*b^4 - 5*A*b^5)*x^5 + 2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^4 + (3*B*a^3
*b^2 - 5*A*a^2*b^3)*x^3)*log(b*x + a) - 12*((3*B*a*b^4 - 5*A*b^5)*x^5 + 2*
(3*B*a^2*b^3 - 5*A*a*b^4)*x^4 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^3)*log(x))/(
a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \frac{A+Bx}{x^4(a+bx)^3}\,dx &= \frac{-2Aa^4 + x^4(-60Ab^4 + 36Bab^3) + x^3(-90Aab^3 + 54Ba^2b^2) + x^2(-20Aa^2b^2 + 12Ba^3b) + x(5Aa^3b - 3B}{6a^7x^3 + 12a^6bx^4 + 6a^5b^2x^5} \\ &+ \frac{2b^2(-5Ab + 3Ba)\log\left(x + \frac{-10Aab^3 + 6Ba^2b^2 - 2ab^2(-5Ab+3Ba)}{-20Ab^4 + 12Bab^3}\right)}{a^6} \\ &- \frac{2b^2(-5Ab + 3Ba)\log\left(x + \frac{-10Aab^3 + 6Ba^2b^2 + 2ab^2(-5Ab+3Ba)}{-20Ab^4 + 12Bab^3}\right)}{a^6}\end{aligned}$$

input
```
integrate((B*x+A)/x**4/(b*x+a)**3,x)
```

output

```
(-2*A*a**4 + x**4*(-60*A*b**4 + 36*B*a*b**3) + x**3*(-90*A*a*b**3 + 54*B*a
**2*b**2) + x**2*(-20*A*a**2*b**2 + 12*B*a**3*b) + x*(5*A*a**3*b - 3*B*a**
4))/(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 2*b**2*(-5*A*b + 3
*B*a)*log(x + (-10*A*a*b**3 + 6*B*a**2*b**2 - 2*a*b**2*(-5*A*b + 3*B*a))/(
-20*A*b**4 + 12*B*a*b**3))/a**6 - 2*b**2*(-5*A*b + 3*B*a)*log(x + (-10*A*a
*b**3 + 6*B*a**2*b**2 + 2*a*b**2*(-5*A*b + 3*B*a))/(-20*A*b**4 + 12*B*a*b*
*3))/a**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = -\frac{2\,Aa^4-12\,(3\,Bab^3-5\,Ab^4)x^4-18\,(3\,Ba^2b^2-5\,Aab^3)x^3-4\,(3\,Ba^3b-5\,Aa^2b^2)x^2+(3\,Ba^4-5\,Aa^3}{6\,(a^5b^2x^5+2\,a^6bx^4+a^7x^3)} - \frac{2\,(3\,Bab^2-5\,Ab^3)\log(bx+a)}{a^6}+\frac{2\,(3\,Bab^2-5\,Ab^3)\log(x)}{a^6}$$

input

```
integrate((B*x+A)/x^4/(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/6*(2*A*a^4 - 12*(3*B*a*b^3 - 5*A*b^4)*x^4 - 18*(3*B*a^2*b^2 - 5*A*a*b^3
)*x^3 - 4*(3*B*a^3*b - 5*A*a^2*b^2)*x^2 + (3*B*a^4 - 5*A*a^3*b)*x)/(a^5*b^
2*x^5 + 2*a^6*b*x^4 + a^7*x^3) - 2*(3*B*a*b^2 - 5*A*b^3)*log(b*x + a)/a^6
+ 2*(3*B*a*b^2 - 5*A*b^3)*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = \frac{2\,(3\,Bab^2-5\,Ab^3)\log(|x|)}{a^6}-\frac{2\,(3\,Bab^3-5\,Ab^4)\log(|bx+a|)}{a^6b} - \frac{2\,Aa^5-12\,(3\,Ba^2b^3-5\,Aab^4)x^4-18\,(3\,Ba^3b^2-5\,Aa^2b^3)x^3-4\,(3\,Ba^4b-5\,Aa^3b^2)x^2+(3\,Ba^5-5\,A}{6\,(bx+a)^2a^6x^3}$$

input

```
integrate((B*x+A)/x^4/(b*x+a)^3,x, algorithm="giac")
```

output
```
2*(3*B*a*b^2 - 5*A*b^3)*log(abs(x))/a^6 - 2*(3*B*a*b^3 - 5*A*b^4)*log(abs(
b*x + a))/(a^6*b) - 1/6*(2*A*a^5 - 12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^4 - 18*(
3*B*a^3*b^2 - 5*A*a^2*b^3)*x^3 - 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^2 + (3*B*a^
5 - 5*A*a^4*b)*x)/((b*x + a)^2*a^6*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = \frac{4\,b^2\,\mathrm{atanh}\left(\frac{2\,b^2\,(5\,A\,b-3\,B\,a)\,(a+2\,b\,x)}{a\,(10\,A\,b^3-6\,B\,a\,b^2)}\right)\,(5\,A\,b-3\,B\,a)}{a^6} - \frac{\frac{A}{3\,a}-\frac{x\,(5\,A\,b-3\,B\,a)}{6\,a^2}+\frac{3\,b^2\,x^3\,(5\,A\,b-3\,B\,a)}{a^4}+\frac{2\,b^3\,x^4\,(5\,A\,b-3\,B\,a)}{a^5}+\frac{2\,b\,x^2\,(5\,A\,b-3\,B\,a)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5}$$

input
```
int((A + B*x)/(x^4*(a + b*x)^3),x)
```

output
```
(4*b^2*atanh((2*b^2*(5*A*b - 3*B*a)*(a + 2*b*x))/(a*(10*A*b^3 - 6*B*a*b^2)
))*(5*A*b - 3*B*a))/a^6 - (A/(3*a) - (x*(5*A*b - 3*B*a))/(6*a^2) + (3*b^2*
x^3*(5*A*b - 3*B*a))/a^4 + (2*b^3*x^4*(5*A*b - 3*B*a))/a^5 + (2*b*x^2*(5*A
*b - 3*B*a))/(3*a^3))/(a^2*x^3 + b^2*x^5 + 2*a*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{x^4(a+bx)^3}\,dx = \frac{12\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,b^4x^4-12\log(x)\,a\,b^3x^3-12\log(x)\,b^4x^4-a^4+2a^3bx-6a^2b^2x^2+}{3a^5x^3\,(bx+a)}$$

input
```
int((B*x+A)/x^4/(b*x+a)^3,x)
```

output
```
(12*log(a + b*x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a*b**
3*x**3 - 12*log(x)*b**4*x**4 - a**4 + 2*a**3*b*x - 6*a**2*b**2*x**2 + 12*b
**4*x**4)/(3*a**5*x**3*(a + b*x))
```

3.171 $\int \frac{1+x}{(-1+x)x^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \frac{1+x}{(-1+x)x^2} \, dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

output
```
1/x+2*ln(1-x)-2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(-1+x)x^2} \, dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

input
```
Integrate[(1 + x)/((-1 + x)*x^2),x]
```

output
```
x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x-1)x^2} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{1}{x^2}-\frac{2}{x}+\frac{2}{x-1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

input
```
Int[(1 + x)/((-1 + x)*x^2),x]
```

output
```
x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{x}-2\ln\left(x\right)+2\ln\left(-1+x\right)$	15
norman	$\frac{1}{x}-2\ln\left(x\right)+2\ln\left(-1+x\right)$	15
risch	$\frac{1}{x}-2\ln\left(x\right)+2\ln\left(-1+x\right)$	15
parallelrisch	$-\frac{2\ln(x)x-2\ln(-1+x)x-1}{x}$	20
meijerg	$\frac{1}{x}-2\ln\left(x\right)-2i\pi+2\ln\left(1-x\right)$	21

input
```
int((1+x)/(-1+x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x-2*ln(x)+2*ln(-1+x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1+x}{(-1+x)x^2}\,dx=\frac{2\,x\log\left(x-1\right)-2\,x\log\left(x\right)+1}{x}$$

input
```
integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
```

output
```
(2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{(-1+x)x^2}\,dx = -2\log{(x)} + 2\log{(x-1)} + \frac{1}{x}$$

input
```
integrate((1+x)/(-1+x)/x**2,x)
```

output
```
-2*log(x) + 2*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} + 2\,\log{(x-1)} - 2\,\log{(x)}$$

input
```
integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
```

output
```
1/x + 2*log(x - 1) - 2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} + 2\,\log{(|x-1|)} - 2\,\log{(|x|)}$$

input
```
integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
```

output
```
1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} - 4\,\mathrm{atanh}(2\,x-1)$$

input
```
int((x + 1)/(x^2*(x - 1)),x)
```

output
```
1/x - 4*atanh(2*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{2\log(x-1)\,x - 2\log(x)\,x + 1}{x}$$

input
```
int((1+x)/(-1+x)/x^2,x)
```

output
```
(2*log(x - 1)*x - 2*log(x)*x + 1)/x
```

3.172 $\int x^{7/2}(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int x^{7/2}(a+bx)(A+Bx)\,dx=\frac{2}{9}aAx^{9/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{13}bBx^{13/2}$$

output
```
2/9*a*A*x^(9/2)+2/11*(A*b+B*a)*x^(11/2)+2/13*b*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{7/2}(a+bx)(A+Bx)\,dx=\frac{2x^{9/2}(143aA+117Abx+117aBx+99bBx^2)}{1287}$$

input
```
Integrate[x^(7/2)*(a + b*x)*(A + B*x),x]
```

output
```
(2*x^(9/2)*(143*a*A + 117*A*b*x + 117*a*B*x + 99*b*B*x^2))/1287
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(x^{9/2}(aB+Ab)+aAx^{7/2}+bBx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{13}bBx^{13/2}$$

input
```
Int[x^(7/2)*(a + b*x)*(A + B*x),x]
```

output
```
(2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{9}{2}}(99bB\,x^{2}+117Abx+117Bax+143Aa)}{1287}$	28
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
trager	$\frac{2x^{\frac{9}{2}}(99bB\,x^{2}+117Abx+117Bax+143Aa)}{1287}$	28
risch	$\frac{2x^{\frac{9}{2}}(99bB\,x^{2}+117Abx+117Bax+143Aa)}{1287}$	28
orering	$\frac{2x^{\frac{9}{2}}(99bB\,x^{2}+117Abx+117Bax+143Aa)}{1287}$	28

input
```
int(x^(7/2)*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/1287*x^(9/2)*(99*B*b*x^2+117*A*b*x+117*B*a*x+143*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{7/2}(a+bx)(A+Bx)\,dx=\frac{2}{1287}\left(99\,Bbx^{6}+143\,Aax^{4}+117\,(Ba+Ab)x^{5}\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
2/1287*(99*B*b*x^6 + 143*A*a*x^4 + 117*(B*a + A*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{7/2}(a+bx)(A+Bx)\,dx = \frac{2Aax^{\frac{9}{2}}}{9} + \frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Bax^{\frac{11}{2}}}{11} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(7/2)*(b*x+a)*(B*x+A),x)
```

output
```
2*A*a*x**(9/2)/9 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(13/
2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}(a+bx)(A+Bx)\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{9}\,Aax^{\frac{9}{2}} + \frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
2/13*B*b*x^(13/2) + 2/9*A*a*x^(9/2) + 2/11*(B*a + A*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{7/2}(a+bx)(A+Bx)\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{11}\,Bax^{\frac{11}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}} + \frac{2}{9}\,Aax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
2/13*B*b*x^(13/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/9*A*a*x^(9/2
)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}(a+bx)(A+Bx)\,dx = \frac{2\,x^{9/2}\,(143\,A\,a+117\,A\,b\,x+117\,B\,a\,x+99\,B\,b\,x^2)}{1287}$$

input
```
int(x^(7/2)*(A + B*x)*(a + b*x),x)
```

output
```
(2*x^(9/2)*(143*A*a + 117*A*b*x + 117*B*a*x + 99*B*b*x^2))/1287
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int x^{7/2}(a+bx)(A+Bx)\,dx = \frac{2\sqrt{x}\,x^4(99b^2x^2+234abx+143a^2)}{1287}$$

input
```
int(x^(7/2)*(b*x+a)*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**4*(143*a**2 + 234*a*b*x + 99*b**2*x**2))/1287
```

3.173 $\int x^{5/2}(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int x^{5/2}(a+bx)(A+Bx)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{11}bBx^{11/2}$$

output
```
2/7*a*A*x^(7/2)+2/9*(A*b+B*a)*x^(9/2)+2/11*b*B*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}(a+bx)(A+Bx)\,dx=\frac{2}{693}x^{7/2}\left(99aA+77Abx+77aBx+63bBx^2\right)$$

input
```
Integrate[x^(5/2)*(a + b*x)*(A + B*x),x]
```

output
```
(2*x^(7/2)*(99*a*A + 77*A*b*x + 77*a*B*x + 63*b*B*x^2))/693
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^{7/2}(aB+Ab)+aAx^{5/2}+bBx^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{11}bBx^{11/2}$$

input
```
Int[x^(5/2)*(a + b*x)*(A + B*x),x]
```

output
```
(2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(63bB\,x^{2}+77Abx+77Bax+99Aa\right)}{693}$	28
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{11}{2}}}{11}$	28
trager	$\frac{2x^{\frac{7}{2}}\left(63bB\,x^{2}+77Abx+77Bax+99Aa\right)}{693}$	28
risch	$\frac{2x^{\frac{7}{2}}\left(63bB\,x^{2}+77Abx+77Bax+99Aa\right)}{693}$	28
orering	$\frac{2x^{\frac{7}{2}}\left(63bB\,x^{2}+77Abx+77Bax+99Aa\right)}{693}$	28

input
```
int(x^(5/2)*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/693*x^(7/2)*(63*B*b*x^2+77*A*b*x+77*B*a*x+99*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}(a+bx)(A+Bx)\,dx=\frac{2}{693}\left(63\,Bbx^{5}+99\,Aax^{3}+77\,(Ba+Ab)x^{4}\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
2/693*(63*B*b*x^5 + 99*A*a*x^3 + 77*(B*a + A*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}(a+bx)(A+Bx)\,dx = \frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Bax^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(5/2)*(b*x+a)*(B*x+A),x)
```

output
```
2*A*a*x**(7/2)/7 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(11/2)/1
1
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx)(A+Bx)\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}} + \frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
2/11*B*b*x^(11/2) + 2/7*A*a*x^(7/2) + 2/9*(B*a + A*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}(a+bx)(A+Bx)\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{9}\,Bax^{\frac{9}{2}} + \frac{2}{9}\,Abx^{\frac{9}{2}} + \frac{2}{7}\,Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
2/11*B*b*x^(11/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/7*A*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx)(A+Bx)\,dx=\frac{2\,x^{7/2}\,\left(99\,A\,a+77\,A\,b\,x+77\,B\,a\,x+63\,B\,b\,x^2\right)}{693}$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x),x)
```

output
```
(2*x^(7/2)*(99*A*a + 77*A*b*x + 77*B*a*x + 63*B*b*x^2))/693
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int x^{5/2}(a+bx)(A+Bx)\,dx=\frac{2\sqrt{x}\,x^3(63b^2x^2+154abx+99a^2)}{693}$$

input
```
int(x^(5/2)*(b*x+a)*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**3*(99*a**2 + 154*a*b*x + 63*b**2*x**2))/693
```

3.174 $\int x^{3/2}(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{9}bBx^{9/2}$$

output
```
2/5*a*A*x^(5/2)+2/7*(A*b+B*a)*x^(7/2)+2/9*b*B*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2}{315}x^{5/2}(9a(7A+5Bx)+5bx(9A+7Bx))$$

input
```
Integrate[x^(3/2)*(a + b*x)*(A + B*x),x]
```

output
```
(2*x^(5/2)*(9*a*(7*A + 5*B*x) + 5*b*x*(9*A + 7*B*x)))/315
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^{5/2}(aB+Ab)+aAx^{3/2}+bBx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{9}bBx^{9/2}$$

input
```
Int[x^(3/2)*(a + b*x)*(A + B*x),x]
```

output
```
(2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}(35bB\,x^{2}+45Abx+45Bax+63Aa)}{315}$	28
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{9}{2}}}{9}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{9}{2}}}{9}$	28
trager	$\frac{2x^{\frac{5}{2}}(35bB\,x^{2}+45Abx+45Bax+63Aa)}{315}$	28
risch	$\frac{2x^{\frac{5}{2}}(35bB\,x^{2}+45Abx+45Bax+63Aa)}{315}$	28
orering	$\frac{2x^{\frac{5}{2}}(35bB\,x^{2}+45Abx+45Bax+63Aa)}{315}$	28

input
```
int(x^(3/2)*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(5/2)*(35*B*b*x^2+45*A*b*x+45*B*a*x+63*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2}{315}\left(35\,Bbx^{4}+63\,Aax^{2}+45\,(Ba+Ab)x^{3}\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
2/315*(35*B*b*x^4 + 63*A*a*x^2 + 45*(B*a + A*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bax^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

input
```
integrate(x**(3/2)*(b*x+a)*(B*x+A),x)
```

output
```
2*A*a*x**(5/2)/5 + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
2/9*B*b*x^(9/2) + 2/5*A*a*x^(5/2) + 2/7*(B*a + A*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{7}\,Bax^{\frac{7}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
2/9*B*b*x^(9/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2/5*A*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2\,x^{5/2}\,\left(63\,A\,a+45\,A\,b\,x+45\,B\,a\,x+35\,B\,b\,x^2\right)}{315}$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x),x)
```

output
```
(2*x^(5/2)*(63*A*a + 45*A*b*x + 45*B*a*x + 35*B*b*x^2))/315
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int x^{3/2}(a+bx)(A+Bx)\,dx=\frac{2\sqrt{x}\,x^2(35b^2x^2+90abx+63a^2)}{315}$$

input
```
int(x^(3/2)*(b*x+a)*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**2*(63*a**2 + 90*a*b*x + 35*b**2*x**2))/315
```

3.175 $\int \sqrt{x}(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx=\frac{2}{3}aAx^{3/2}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{7}bBx^{7/2}$$

output
```
2/3*a*A*x^(3/2)+2/5*(A*b+B*a)*x^(5/2)+2/7*b*B*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx=\frac{2}{105}x^{3/2}(7a(5A+3Bx)+3bx(7A+5Bx))$$

input
```
Integrate[Sqrt[x]*(a + b*x)*(A + B*x),x]
```

output
```
(2*x^(3/2)*(7*a*(5*A + 3*B*x) + 3*b*x*(7*A + 5*B*x)))/105
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^{3/2}(aB+Ab)+aA\sqrt{x}+bBx^{5/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}x^{5/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{7}bBx^{7/2}$$

input
```
Int[Sqrt[x]*(a + b*x)*(A + B*x),x]
```

output
```
(2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(15bB\,x^{2}+21Abx+21Bax+35Aa\right)}{105}$	28
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2bB\,x^{\frac{7}{2}}}{7}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2bB\,x^{\frac{7}{2}}}{7}$	28
trager	$\frac{2x^{\frac{3}{2}}\left(15bB\,x^{2}+21Abx+21Bax+35Aa\right)}{105}$	28
risch	$\frac{2x^{\frac{3}{2}}\left(15bB\,x^{2}+21Abx+21Bax+35Aa\right)}{105}$	28
orering	$\frac{2x^{\frac{3}{2}}\left(15bB\,x^{2}+21Abx+21Bax+35Aa\right)}{105}$	28

input
```
int(x^(1/2)*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/105*x^(3/2)*(15*B*b*x^2+21*A*b*x+21*B*a*x+35*A*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx=\frac{2}{105}\left(15\,Bbx^{3}+35\,Aax+21\,(Ba+Ab)x^{2}\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
2/105*(15*B*b*x^3 + 35*A*a*x + 21*(B*a + A*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx = \frac{2Aax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{7}{2}}}{7} + \frac{2x^{\frac{5}{2}}(Ab+Ba)}{5}$$

input
```
integrate(x**(1/2)*(b*x+a)*(B*x+A),x)
```

output
```
2*A*a*x**(3/2)/3 + 2*B*b*x**(7/2)/7 + 2*x**(5/2)*(A*b + B*a)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{3}\,Aax^{\frac{3}{2}} + \frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
2/7*B*b*x^(7/2) + 2/3*A*a*x^(3/2) + 2/5*(B*a + A*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}} + \frac{2}{5}\,Bax^{\frac{5}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}} + \frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
2/7*B*b*x^(7/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) + 2/3*A*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx = \frac{2\,x^{3/2}\,(35\,A\,a+21\,A\,b\,x+21\,B\,a\,x+15\,B\,b\,x^2)}{105}$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x),x)
```

output
```
(2*x^(3/2)*(35*A*a + 21*A*b*x + 21*B*a*x + 15*B*b*x^2))/105
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx)(A+Bx)\,dx = \frac{2\sqrt{x}\,x(15b^2x^2+42abx+35a^2)}{105}$$

input
```
int(x^(1/2)*(b*x+a)*(B*x+A),x)
```

output
```
(2*sqrt(x)*x*(35*a**2 + 42*a*b*x + 15*b**2*x**2))/105
```

3.176 $\int \frac{(a+bx)(A+Bx)}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}} \, dx = 2aA\sqrt{x} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{5}bBx^{5/2}$$

output
```
2*a*A*x^(1/2)+2/3*(A*b+B*a)*x^(3/2)+2/5*b*B*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}} \, dx = \frac{2}{15}\sqrt{x}(5a(3A+Bx)+bx(5A+3Bx))$$

input
```
Integrate[((a + b*x)*(A + B*x))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(5*a*(3*A + B*x) + b*x*(5*A + 3*B*x)))/15
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\sqrt{x}(aB+Ab)+\frac{aA}{\sqrt{x}}+bBx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}x^{3/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{5}bBx^{5/2}$$

input
```
Int[((a + b*x)*(A + B*x))/Sqrt[x],x]
```

output
```
2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(5/2))/5
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
trager	$\left(\frac{2}{5}bB\,x^{2}+\frac{2}{3}Abx+\frac{2}{3}Bax+2Aa\right)\sqrt{x}$	27
gosper	$\frac{2\sqrt{x}\,(3bB\,x^{2}+5Abx+5Bax+15Aa)}{15}$	28
derivativedivides	$2aA\sqrt{x}+\frac{2(Ab+Ba)x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{5}{2}}}{5}$	28
default	$2aA\sqrt{x}+\frac{2(Ab+Ba)x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{5}{2}}}{5}$	28
risch	$\frac{2\sqrt{x}\,(3bB\,x^{2}+5Abx+5Bax+15Aa)}{15}$	28
orering	$\frac{2\sqrt{x}\,(3bB\,x^{2}+5Abx+5Bax+15Aa)}{15}$	28

input
```
int((b*x+a)*(B*x+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5*b*B*x^2+2/3*A*b*x+2/3*B*a*x+2*A*a)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx=\frac{2}{15}\left(3\,Bbx^{2}+15\,Aa+5\,(Ba+Ab)x\right)\sqrt{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b*x^2 + 15*A*a + 5*(B*a + A*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx = 2Aa\sqrt{x} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x+a)*(B*x+A)/x**(1/2),x)
```

output
```
2*A*a*sqrt(x) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,Aa\sqrt{x} + \frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/5*B*b*x^(5/2) + 2*A*a*sqrt(x) + 2/3*(B*a + A*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{5}\,Bbx^{\frac{5}{2}} + \frac{2}{3}\,Bax^{\frac{3}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/5*B*b*x^(5/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) + 2*A*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(15\,A\,a+5\,A\,b\,x+5\,B\,a\,x+3\,B\,b\,x^2)}{15}$$

input
```
int(((A + B*x)*(a + b*x))/x^(1/2),x)
```

output
```
(2*x^(1/2)*(15*A*a + 5*A*b*x + 5*B*a*x + 3*B*b*x^2))/15
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(3b^2x^2+10abx+15a^2)}{15}$$

input
```
int((b*x+a)*(B*x+A)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(15*a**2 + 10*a*b*x + 3*b**2*x**2))/15
```

3.177 $\int \frac{(a+bx)(A+Bx)}{x^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}} \, dx = -\frac{2aA}{\sqrt{x}} + 2(Ab+aB)\sqrt{x} + \frac{2}{3}bBx^{3/2}$$

output
```
-2*a*A/x^(1/2)+2*(A*b+B*a)*x^(1/2)+2/3*b*B*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}} \, dx = -\frac{2(3aA-3Abx-3aBx-bBx^2)}{3\sqrt{x}}$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^(3/2),x]
```

output
```
(-2*(3*a*A - 3*A*b*x - 3*a*B*x - b*B*x^2))/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{aB+Ab}{\sqrt{x}}+\frac{aA}{x^{3/2}}+bB\sqrt{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{3}bBx^{3/2}$$

input
```
Int[((a + b*x)*(A + B*x))/x^(3/2),x]
```

output
```
(-2*a*A)/Sqrt[x] + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(3/2))/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{2\left(-bB\,x^{2}-3Abx-3Bax+3Aa\right)}{3\sqrt{x}}$	28
trager	$-\frac{2\left(-bB\,x^{2}-3Abx-3Bax+3Aa\right)}{3\sqrt{x}}$	28
risch	$-\frac{2\left(-bB\,x^{2}-3Abx-3Bax+3Aa\right)}{3\sqrt{x}}$	28
orering	$-\frac{2\left(-bB\,x^{2}-3Abx-3Bax+3Aa\right)}{3\sqrt{x}}$	28
derivativedivides	$\frac{2bB\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{\sqrt{x}}$	30

input
```
int((b*x+a)*(B*x+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-B*b*x^2-3*A*b*x-3*B*a*x+3*A*a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}}\,dx = \frac{2\left(Bbx^{2}-3\,Aa+3\,(Ba+Ab)x\right)}{3\,\sqrt{x}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/3*(B*b*x^2 - 3*A*a + 3*(B*a + A*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}}\,dx = -\frac{2Aa}{\sqrt{x}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x+a)*(B*x+A)/x**(3/2),x)
```

output
```
-2*A*a/sqrt(x) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}}\,dx = \frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\,Aa}{\sqrt{x}} + 2\,(Ba+Ab)\sqrt{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/3*B*b*x^(3/2) - 2*A*a/sqrt(x) + 2*(B*a + A*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}}\,dx = \frac{2}{3}\,Bbx^{\frac{3}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{\sqrt{x}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/3*B*b*x^(3/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2*A*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}}\,dx = \frac{6\,A\,b\,x-6\,A\,a+6\,B\,a\,x+2\,B\,b\,x^2}{3\,\sqrt{x}}$$

input
```
int(((A + B*x)*(a + b*x))/x^(3/2),x)
```

output
```
(6*A*b*x - 6*A*a + 6*B*a*x + 2*B*b*x^2)/(3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)(A+Bx)}{x^{3/2}}\,dx = \frac{\frac{2}{3}b^2x^2+4abx-2a^2}{\sqrt{x}}$$

input
```
int((b*x+a)*(B*x+A)/x^(3/2),x)
```

output
```
(2*( - 3*a**2 + 6*a*b*x + b**2*x**2))/(3*sqrt(x))
```

3.178 $\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = -\frac{2aA}{3x^{3/2}} - \frac{2(Ab+aB)}{\sqrt{x}} + 2bB\sqrt{x}$$

output
```
-2/3*a*A/x^(3/2)-2*(A*b+B*a)/x^(1/2)+2*b*B*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = -\frac{2(3bx(A-Bx)+a(A+3Bx))}{3x^{3/2}}$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^(5/2),x]
```

output
```
(-2*(3*b*x*(A - B*x) + a*(A + 3*B*x)))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{aB+Ab}{x^{3/2}}+\frac{aA}{x^{5/2}}+\frac{bB}{\sqrt{x}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{3x^{3/2}}+2bB\sqrt{x}$$

input
```
Int[((a + b*x)*(A + B*x))/x^(5/2),x]
```

output
```
(-2*a*A)/(3*x^(3/2)) - (2*(A*b + a*B))/Sqrt[x] + 2*b*B*Sqrt[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(-3bB\,x^2+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	27
trager	$-\frac{2(-3bB\,x^2+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	27
risch	$-\frac{2(-3bB\,x^2+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	27
orering	$-\frac{2(-3bB\,x^2+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	27
derivativedivides	$-\frac{2aA}{3x^{\frac{3}{2}}}-\frac{2(Ab+Ba)}{\sqrt{x}}+2bB\sqrt{x}$	28
default	$-\frac{2aA}{3x^{\frac{3}{2}}}-\frac{2(Ab+Ba)}{\sqrt{x}}+2bB\sqrt{x}$	28

input
```
int((b*x+a)*(B*x+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*B*b*x^2+3*A*b*x+3*B*a*x+A*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx=\frac{2\,(3\,Bbx^2-Aa-3\,(Ba+Ab)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*B*b*x^2 - A*a - 3*(B*a + A*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = -\frac{2Aa}{3x^{\frac{3}{2}}} - \frac{2Ab}{\sqrt{x}} - \frac{2Ba}{\sqrt{x}} + 2Bb\sqrt{x}$$

input
```
integrate((b*x+a)*(B*x+A)/x**(5/2),x)
```

output
```
-2*A*a/(3*x**(3/2)) - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + 2*B*b*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = 2\,Bb\sqrt{x} - \frac{2\,(Aa+3\,(Ba+Ab)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2*B*b*sqrt(x) - 2/3*(A*a + 3*(B*a + A*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = 2\,Bb\sqrt{x} - \frac{2\,(3\,Bax+3\,Abx+Aa)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(5/2),x, algorithm="giac")
```

output
```
2*B*b*sqrt(x) - 2/3*(3*B*a*x + 3*A*b*x + A*a)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = -\frac{2\,A\,a+6\,A\,b\,x+6\,B\,a\,x-6\,B\,b\,x^2}{3\,x^{3/2}}$$

input
```
int(((A + B*x)*(a + b*x))/x^(5/2),x)
```

output
```
-(2*A*a + 6*A*b*x + 6*B*a*x - 6*B*b*x^2)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)(A+Bx)}{x^{5/2}}\,dx = \frac{2b^2x^2-4abx-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((b*x+a)*(B*x+A)/x^(5/2),x)
```

output
```
(2*( - a**2 - 6*a*b*x + 3*b**2*x**2))/(3*sqrt(x)*x)
```

3.179 $\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = -\frac{2aA}{5x^{5/2}} - \frac{2(Ab+aB)}{3x^{3/2}} - \frac{2bB}{\sqrt{x}}$$

output
```
-2/5*a*A/x^(5/2)-2/3*(A*b+B*a)/x^(3/2)-2*b*B/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = -\frac{2(3aA+5Abx+5aBx+15bBx^2)}{15x^{5/2}}$$

input
```
Integrate[((a + b*x)*(A + B*x))/x^(7/2),x]
```

output
```
(-2*(3*a*A + 5*A*b*x + 5*a*B*x + 15*b*B*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx$$
$$\Big\downarrow \text{85}$$
$$\int \left(\frac{aB+Ab}{x^{5/2}}+\frac{aA}{x^{7/2}}+\frac{bB}{x^{3/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2(aB+Ab)}{3x^{3/2}}-\frac{2aA}{5x^{5/2}}-\frac{2bB}{\sqrt{x}}$$

input
```
Int[((a + b*x)*(A + B*x))/x^(7/2),x]
```

output
```
(-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/(3*x^(3/2)) - (2*b*B)/Sqrt[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(15bB\,x^2+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	28
derivativedivides	$-\frac{2aA}{5x^{\frac{5}{2}}}-\frac{2(Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2bB}{\sqrt{x}}$	28
default	$-\frac{2aA}{5x^{\frac{5}{2}}}-\frac{2(Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2bB}{\sqrt{x}}$	28
trager	$-\frac{2(15bB\,x^2+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	28
risch	$-\frac{2(15bB\,x^2+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	28
orering	$-\frac{2(15bB\,x^2+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	28

input
```
int((b*x+a)*(B*x+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*B*b*x^2+5*A*b*x+5*B*a*x+3*A*a)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx=-\frac{2\,(15\,Bbx^2+3\,Aa+5\,(Ba+Ab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(15*B*b*x^2 + 3*A*a + 5*(B*a + A*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = -\frac{2Aa}{5x^{\frac{5}{2}}} - \frac{2Ab}{3x^{\frac{3}{2}}} - \frac{2Ba}{3x^{\frac{3}{2}}} - \frac{2Bb}{\sqrt{x}}$$

input
```
integrate((b*x+a)*(B*x+A)/x**(7/2),x)
```

output
```
-2*A*a/(5*x**(5/2)) - 2*A*b/(3*x**(3/2)) - 2*B*a/(3*x**(3/2)) - 2*B*b/sqrt
(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = -\frac{2\,(15\,Bbx^2 + 3\,Aa + 5\,(Ba+Ab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/15*(15*B*b*x^2 + 3*A*a + 5*(B*a + A*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = -\frac{2\,(15\,Bbx^2 + 5\,Bax + 5\,Abx + 3\,Aa)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)*(B*x+A)/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*B*b*x^2 + 5*B*a*x + 5*A*b*x + 3*A*a)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = -\frac{2\,B\,b\,x^2 + \left(\frac{2\,A\,b}{3} + \frac{2\,B\,a}{3}\right)\,x + \frac{2\,A\,a}{5}}{x^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x))/x^(7/2),x)
```

output
```
-((2*A*a)/5 + x*((2*A*b)/3 + (2*B*a)/3) + 2*B*b*x^2)/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)(A+Bx)}{x^{7/2}}\,dx = \frac{-2b^2x^2 - \frac{4}{3}abx - \frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((b*x+a)*(B*x+A)/x^(7/2),x)
```

output
```
(2*( - 3*a**2 - 10*a*b*x - 15*b**2*x**2))/(15*sqrt(x)*x**2)
```

3.180 $\int x^{7/2}(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{15}b^2Bx^{15/2}$$

output
```
2/9*a^2*A*x^(9/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+
2/15*b^2*B*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx=\frac{2x^{9/2}(65a^2(11A+9Bx)+90abx(13A+11Bx)+33b^2x^2(15A+13Bx))}{6435}$$

input
```
Integrate[x^(7/2)*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*x^(9/2)*(65*a^2*(11*A + 9*B*x) + 90*a*b*x*(13*A + 11*B*x) + 33*b^2*x^2*
(15*A + 13*B*x)))/6435
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2Ax^{7/2}+bx^{11/2}(2aB+Ab)+ax^{9/2}(aB+2Ab)+b^2Bx^{13/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

input
```
Int[x^(7/2)*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)
*x^(13/2))/13 + (2*b^2*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{9}{2}}\left(429B\,b^{2}x^{3}+495A\,b^{2}x^{2}+990Bab\,x^{2}+1170aAbx+585B\,a^{2}x+715a^{2}A\right)}{6435}$	52
derivativedivides	$\frac{2b^{2}B\,x^{\frac{15}{2}}}{15}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{2}A\,x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^{2}B\,x^{\frac{15}{2}}}{15}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{2}A\,x^{\frac{9}{2}}}{9}$	52
trager	$\frac{2x^{\frac{9}{2}}\left(429B\,b^{2}x^{3}+495A\,b^{2}x^{2}+990Bab\,x^{2}+1170aAbx+585B\,a^{2}x+715a^{2}A\right)}{6435}$	52
risch	$\frac{2x^{\frac{9}{2}}\left(429B\,b^{2}x^{3}+495A\,b^{2}x^{2}+990Bab\,x^{2}+1170aAbx+585B\,a^{2}x+715a^{2}A\right)}{6435}$	52
orering	$\frac{2x^{\frac{9}{2}}\left(429B\,b^{2}x^{3}+495A\,b^{2}x^{2}+990Bab\,x^{2}+1170aAbx+585B\,a^{2}x+715a^{2}A\right)}{6435}$	52

input
```
int(x^(7/2)*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/6435*x^(9/2)*(429*B*b^2*x^3+495*A*b^2*x^2+990*B*a*b*x^2+1170*A*a*b*x+585
*B*a^2*x+715*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx=\frac{2}{6435}\left(429\,Bb^2x^7+715\,Aa^2x^4+495\left(2\,Bab+Ab^2\right)x^6+585\left(Ba^2+2\,Aab\right)x^5\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
2/6435*(429*B*b^2*x^7 + 715*A*a^2*x^4 + 495*(2*B*a*b + A*b^2)*x^6 + 585*(B
*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx = \frac{2Aa^2x^{\frac{9}{2}}}{9} + \frac{4Aabx^{\frac{11}{2}}}{11} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(7/2)*(b*x+a)**2*(B*x+A),x)
```

output
```
2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(13/2)/13 + 2*B*a
**2*x**(11/2)/11 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{2}{9}\,Aa^2x^{\frac{9}{2}} + \frac{2}{13}\,\left(2\,Bab + Ab^2\right)x^{\frac{13}{2}} + \frac{2}{11}\,\left(Ba^2 + 2\,Aab\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
2/15*B*b^2*x^(15/2) + 2/9*A*a^2*x^(9/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2)
+ 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{13}\,Babx^{\frac{13}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{11}\,Ba^2x^{\frac{11}{2}} + \frac{4}{11}\,Aabx^{\frac{11}{2}} + \frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
2/15*B*b^2*x^(15/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/11*B*a
^2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/9*A*a^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx = x^{11/2}\left(\frac{2\,B\,a^2}{11} + \frac{4\,A\,b\,a}{11}\right) + x^{13/2}\left(\frac{2\,A\,b^2}{13} + \frac{4\,B\,a\,b}{13}\right) + \frac{2\,A\,a^2\,x^{9/2}}{9} + \frac{2\,B\,b^2\,x^{15/2}}{15}$$

input
```
int(x^(7/2)*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b
)/13) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int x^{7/2}(a+bx)^2(A+Bx)\,dx = \frac{2\sqrt{x}\,x^4(429b^3x^3+1485a\,b^2x^2+1755a^2bx+715a^3)}{6435}$$

input
```
int(x^(7/2)*(b*x+a)^2*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**4*(715*a**3 + 1755*a**2*b*x + 1485*a*b**2*x**2 + 429*b**3*x*
*3))/6435
```

3.181 $\int x^{5/2}(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{7}a^2Ax^{7/2} + \frac{2}{9}a(2Ab+aB)x^{9/2} + \frac{2}{11}b(Ab+2aB)x^{11/2} + \frac{2}{13}b^2Bx^{13/2}$$

output
```
2/7*a^2*A*x^(7/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/
13*b^2*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = \frac{2x^{7/2}(143a^2(9A+7Bx)+182abx(11A+9Bx)+63b^2x^2(13A+11Bx))}{9009}$$

input
```
Integrate[x^(5/2)*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*x^(7/2)*(143*a^2*(9*A + 7*B*x) + 182*a*b*x*(11*A + 9*B*x) + 63*b^2*x^2*
(13*A + 11*B*x)))/9009
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2Ax^{5/2}+bx^{9/2}(2aB+Ab)+ax^{7/2}(aB+2Ab)+b^2Bx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

input
```
Int[x^(5/2)*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x
^(11/2))/11 + (2*b^2*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(693B\,b^{2}x^{3}+819A\,b^{2}x^{2}+1638Bab\,x^{2}+2002aAbx+1001B\,a^{2}x+1287a^{2}A\right)}{9009}$	52
derivativedivides	$\frac{2b^{2}B\,x^{\frac{13}{2}}}{13}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{11}{2}}}{11}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2a^{2}A\,x^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^{2}B\,x^{\frac{13}{2}}}{13}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{11}{2}}}{11}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2a^{2}A\,x^{\frac{7}{2}}}{7}$	52
trager	$\frac{2x^{\frac{7}{2}}\left(693B\,b^{2}x^{3}+819A\,b^{2}x^{2}+1638Bab\,x^{2}+2002aAbx+1001B\,a^{2}x+1287a^{2}A\right)}{9009}$	52
risch	$\frac{2x^{\frac{7}{2}}\left(693B\,b^{2}x^{3}+819A\,b^{2}x^{2}+1638Bab\,x^{2}+2002aAbx+1001B\,a^{2}x+1287a^{2}A\right)}{9009}$	52
orering	$\frac{2x^{\frac{7}{2}}\left(693B\,b^{2}x^{3}+819A\,b^{2}x^{2}+1638Bab\,x^{2}+2002aAbx+1001B\,a^{2}x+1287a^{2}A\right)}{9009}$	52

input
```
int(x^(5/2)*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/9009*x^(7/2)*(693*B*b^2*x^3+819*A*b^2*x^2+1638*B*a*b*x^2+2002*A*a*b*x+10
01*B*a^2*x+1287*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx=\frac{2}{9009}\left(693\,Bb^2x^6+1287\,Aa^2x^3+819\left(2\,Bab+Ab^2\right)x^5+1001\left(Ba^2+2\,Aab\right)x^4\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
2/9009*(693*B*b^2*x^6 + 1287*A*a^2*x^3 + 819*(2*B*a*b + A*b^2)*x^5 + 1001*
(B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = \frac{2Aa^2x^{\frac{7}{2}}}{7} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{11}{2}}}{11} + \frac{2Bb^2x^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(5/2)*(b*x+a)**2*(B*x+A),x)
```

output
```
2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(11/2)/11 + 2*B*a**
2*x**(9/2)/9 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}} + \frac{2}{11}\,\left(2\,Bab + Ab^2\right)x^{\frac{11}{2}} + \frac{2}{9}\,\left(Ba^2 + 2\,Aab\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
2/13*B*b^2*x^(13/2) + 2/7*A*a^2*x^(7/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2)
+ 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{11}\,Babx^{\frac{11}{2}} + \frac{2}{11}\,Ab^2x^{\frac{11}{2}} + \frac{2}{9}\,Ba^2x^{\frac{9}{2}} + \frac{4}{9}\,Aabx^{\frac{9}{2}} + \frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
2/13*B*b^2*x^(13/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/9*B*a^
2*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = x^{9/2}\left(\frac{2\,B\,a^2}{9} + \frac{4\,A\,b\,a}{9}\right) + x^{11/2}\left(\frac{2\,A\,b^2}{11} + \frac{4\,B\,a\,b}{11}\right) + \frac{2\,A\,a^2\,x^{7/2}}{7} + \frac{2\,B\,b^2\,x^{13/2}}{13}$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/1
1) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int x^{5/2}(a+bx)^2(A+Bx)\,dx = \frac{2\sqrt{x}\,x^3(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3)}{3003}$$

input
```
int(x^(5/2)*(b*x+a)^2*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**3*(429*a**3 + 1001*a**2*b*x + 819*a*b**2*x**2 + 231*b**3*x**
3))/3003
```

3.182 $\int x^{3/2}(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{11}b^2Bx^{11/2}$$

output
```
2/5*a^2*A*x^(5/2)+2/7*a*(2*A*b+B*a)*x^(7/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/11
*b^2*B*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx=\frac{2x^{5/2}(99a^2(7A+5Bx)+110abx(9A+7Bx)+35b^2x^2(11A+9Bx))}{3465}$$

input
```
Integrate[x^(3/2)*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*x^(5/2)*(99*a^2*(7*A + 5*B*x) + 110*a*b*x*(9*A + 7*B*x) + 35*b^2*x^2*(1
1*A + 9*B*x)))/3465
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2Ax^{3/2}+bx^{7/2}(2aB+Ab)+ax^{5/2}(aB+2Ab)+b^2Bx^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

input
```
Int[x^(3/2)*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x
^(9/2))/9 + (2*b^2*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(315B\,b^{2}x^{3}+385A\,b^{2}x^{2}+770Bab\,x^{2}+990aAbx+495B\,a^{2}x+693a^{2}A\right)}{3465}$	52
derivativedivides	$\frac{2b^{2}B\,x^{\frac{11}{2}}}{11}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{9}{2}}}{9}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{7}{2}}}{7}+\frac{2a^{2}A\,x^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^{2}B\,x^{\frac{11}{2}}}{11}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{9}{2}}}{9}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{7}{2}}}{7}+\frac{2a^{2}A\,x^{\frac{5}{2}}}{5}$	52
trager	$\frac{2x^{\frac{5}{2}}\left(315B\,b^{2}x^{3}+385A\,b^{2}x^{2}+770Bab\,x^{2}+990aAbx+495B\,a^{2}x+693a^{2}A\right)}{3465}$	52
risch	$\frac{2x^{\frac{5}{2}}\left(315B\,b^{2}x^{3}+385A\,b^{2}x^{2}+770Bab\,x^{2}+990aAbx+495B\,a^{2}x+693a^{2}A\right)}{3465}$	52
orering	$\frac{2x^{\frac{5}{2}}\left(315B\,b^{2}x^{3}+385A\,b^{2}x^{2}+770Bab\,x^{2}+990aAbx+495B\,a^{2}x+693a^{2}A\right)}{3465}$	52

input
```
int(x^(3/2)*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x^(5/2)*(315*B*b^2*x^3+385*A*b^2*x^2+770*B*a*b*x^2+990*A*a*b*x+495*
B*a^2*x+693*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx=\frac{2}{3465}\left(315\,Bb^2x^5+693\,Aa^2x^2+385\left(2\,Bab+Ab^2\right)x^4+495\left(Ba^2+2\,Aab\right)x^3\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^2*x^5 + 693*A*a^2*x^2 + 385*(2*B*a*b + A*b^2)*x^4 + 495*(B
*a^2 + 2*A*a*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx = \frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{7}{2}}}{7}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(3/2)*(b*x+a)**2*(B*x+A),x)
```

output
```
2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*
x**(7/2)/7 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}+\frac{2}{9}\,\left(2\,Bab+Ab^2\right)x^{\frac{9}{2}}+\frac{2}{7}\,\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
2/11*B*b^2*x^(11/2) + 2/5*A*a^2*x^(5/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) +
2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx = \frac{2}{11}\,Bb^2x^{\frac{11}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{7}\,Ba^2x^{\frac{7}{2}} + \frac{4}{7}\,Aabx^{\frac{7}{2}} + \frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
2/11*B*b^2*x^(11/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/7*B*a^2*x^
(7/2) + 4/7*A*a*b*x^(7/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx = x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right) + x^{9/2}\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right) + \frac{2\,A\,a^2\,x^{5/2}}{5} + \frac{2\,B\,b^2\,x^{11/2}}{11}$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9)
+ (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int x^{3/2}(a+bx)^2(A+Bx)\,dx = \frac{2\sqrt{x}\,x^2(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3)}{1155}$$

input
```
int(x^(3/2)*(b*x+a)^2*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**2*(231*a**3 + 495*a**2*b*x + 385*a*b**2*x**2 + 105*b**3*x**3
))/1155
```

3.183 $\int \sqrt{x}(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2}{5}a(2Ab+aB)x^{5/2} + \frac{2}{7}b(Ab+2aB)x^{7/2} + \frac{2}{9}b^2Bx^{9/2}$$

output
```
2/3*a^2*A*x^(3/2)+2/5*a*(2*A*b+B*a)*x^(5/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/9*
b^2*B*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = \frac{2}{315}x^{3/2}\left(21a^2(5A+3Bx)+18abx(7A+5Bx) + 5b^2x^2(9A+7Bx)\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*x^(3/2)*(21*a^2*(5*A + 3*B*x) + 18*a*b*x*(7*A + 5*B*x) + 5*b^2*x^2*(9*A
 + 7*B*x)))/315
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^2A\sqrt{x}+bx^{5/2}(2aB+Ab)+ax^{3/2}(aB+2Ab)+b^2Bx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

input
```
Int[Sqrt[x]*(a + b*x)^2*(A + B*x),x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x
^(7/2))/7 + (2*b^2*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(35B\,b^{2}x^{3}+45A\,b^{2}x^{2}+90Bab\,x^{2}+126aAbx+63B\,a^{2}x+105a^{2}A\right)}{315}$	52
derivativedivides	$\frac{2b^{2}B\,x^{\frac{9}{2}}}{9}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{7}{2}}}{7}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{5}{2}}}{5}+\frac{2a^{2}A\,x^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^{2}B\,x^{\frac{9}{2}}}{9}+\frac{2\left(b^{2}A+2abB\right)x^{\frac{7}{2}}}{7}+\frac{2\left(2abA+a^{2}B\right)x^{\frac{5}{2}}}{5}+\frac{2a^{2}A\,x^{\frac{3}{2}}}{3}$	52
trager	$\frac{2x^{\frac{3}{2}}\left(35B\,b^{2}x^{3}+45A\,b^{2}x^{2}+90Bab\,x^{2}+126aAbx+63B\,a^{2}x+105a^{2}A\right)}{315}$	52
risch	$\frac{2x^{\frac{3}{2}}\left(35B\,b^{2}x^{3}+45A\,b^{2}x^{2}+90Bab\,x^{2}+126aAbx+63B\,a^{2}x+105a^{2}A\right)}{315}$	52
orering	$\frac{2x^{\frac{3}{2}}\left(35B\,b^{2}x^{3}+45A\,b^{2}x^{2}+90Bab\,x^{2}+126aAbx+63B\,a^{2}x+105a^{2}A\right)}{315}$	52

input
```
int(x^(1/2)*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(3/2)*(35*B*b^2*x^3+45*A*b^2*x^2+90*B*a*b*x^2+126*A*a*b*x+63*B*a^2
*x+105*A*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx$$
$$=\frac{2}{315}\left(35\,Bb^2x^4+105\,Aa^2x+45\left(2\,Bab+Ab^2\right)x^3+63\left(Ba^2+2\,Aab\right)x^2\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^2*x^4 + 105*A*a^2*x + 45*(2*B*a*b + A*b^2)*x^3 + 63*(B*a^2 +
 2*A*a*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = \frac{2Aa^2x^{\frac{3}{2}}}{3} + \frac{2Bb^2x^{\frac{9}{2}}}{9} + \frac{2x^{\frac{7}{2}}(Ab^2+2Bab)}{7} + \frac{2x^{\frac{5}{2}}\cdot(2Aab+Ba^2)}{5}$$

input
```
integrate(x**(1/2)*(b*x+a)**2*(B*x+A),x)
```

output
```
2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(9/2)/9 + 2*x**(7/2)*(A*b**2 + 2*B*a*b)/
7 + 2*x**(5/2)*(2*A*a*b + B*a**2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = \frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}} + \frac{2}{7}\,\left(2\,Bab + Ab^2\right)x^{\frac{7}{2}} + \frac{2}{5}\,\left(Ba^2 + 2\,Aab\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
2/9*B*b^2*x^(9/2) + 2/3*A*a^2*x^(3/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2/
5*(B*a^2 + 2*A*a*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = \frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{4}{7}\,Babx^{\frac{7}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + \frac{2}{5}\,Ba^2x^{\frac{5}{2}} + \frac{4}{5}\,Aabx^{\frac{5}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
2/9*B*b^2*x^(9/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/5*B*a^2*x^(5
/2) + 4/5*A*a*b*x^(5/2) + 2/3*A*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = x^{5/2}\left(\frac{2\,B\,a^2}{5} + \frac{4\,A\,b\,a}{5}\right) + x^{7/2}\left(\frac{2\,A\,b^2}{7} + \frac{4\,B\,a\,b}{7}\right) + \frac{2\,A\,a^2\,x^{3/2}}{3} + \frac{2\,B\,b^2\,x^{9/2}}{9}$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x)^2,x)
```

output
```
x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7)
+ (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \sqrt{x}(a+bx)^2(A+Bx)\,dx = \frac{2\sqrt{x}\,x(35b^3x^3+135a\,b^2x^2+189a^2bx+105a^3)}{315}$$

input
```
int(x^(1/2)*(b*x+a)^2*(B*x+A),x)
```

output
```
(2*sqrt(x)*x*(105*a**3 + 189*a**2*b*x + 135*a*b**2*x**2 + 35*b**3*x**3))/3
15
```

3.184 $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{5}b(Ab+2aB)x^{5/2}+\frac{2}{7}b^2Bx^{7/2}$$

output
```
2*a^2*A*x^(1/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/5*b*(A*b+2*B*a)*x^(5/2)+2/7*b^
2*B*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{105}\sqrt{x}\big(35a^2(3A+Bx)+14abx(5A+3Bx)+3b^2x^2(7A+5Bx)\big)$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(35*a^2*(3*A + B*x) + 14*a*b*x*(5*A + 3*B*x) + 3*b^2*x^2*(7*A +
 5*B*x)))/105
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^2A}{\sqrt{x}}+bx^{3/2}(2aB+Ab)+a\sqrt{x}(aB+2Ab)+b^2Bx^{5/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$2a^2A\sqrt{x}+\frac{2}{5}bx^{5/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{7}b^2Bx^{7/2}$$

input
```
Int[((a + b*x)^2*(A + B*x))/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(5/
2))/5 + (2*b^2*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result	size
trager	$\left(\frac{2}{7}B\,b^{2}x^{3}+\frac{2}{5}A\,b^{2}x^{2}+\frac{4}{5}Bab\,x^{2}+\frac{4}{3}aAbx+\frac{2}{3}B\,a^{2}x+2a^{2}A\right)\sqrt{x}$	51
gosper	$\frac{2\sqrt{x}\,(15B\,b^{2}x^{3}+21A\,b^{2}x^{2}+42Bab\,x^{2}+70aAbx+35B\,a^{2}x+105a^{2}A)}{105}$	52
derivativedivides	$\frac{2b^{2}B\,x^{\frac{7}{2}}}{7}+\frac{2(b^{2}A+2abB)x^{\frac{5}{2}}}{5}+\frac{2(2abA+a^{2}B)x^{\frac{3}{2}}}{3}+2a^{2}A\sqrt{x}$	52
default	$\frac{2b^{2}B\,x^{\frac{7}{2}}}{7}+\frac{2(b^{2}A+2abB)x^{\frac{5}{2}}}{5}+\frac{2(2abA+a^{2}B)x^{\frac{3}{2}}}{3}+2a^{2}A\sqrt{x}$	52
risch	$\frac{2\sqrt{x}\,(15B\,b^{2}x^{3}+21A\,b^{2}x^{2}+42Bab\,x^{2}+70aAbx+35B\,a^{2}x+105a^{2}A)}{105}$	52
orering	$\frac{2\sqrt{x}\,(15B\,b^{2}x^{3}+21A\,b^{2}x^{2}+42Bab\,x^{2}+70aAbx+35B\,a^{2}x+105a^{2}A)}{105}$	52

input
```
int((b*x+a)^2*(B*x+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*B*b^2*x^3+2/5*A*b^2*x^2+4/5*B*a*b*x^2+4/3*a*A*b*x+2/3*B*a^2*x+2*a^2*A
)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{105}\left(15\,Bb^{2}x^{3}+105\,Aa^{2}+21\left(2\,Bab+Ab^{2}\right)x^{2}+35\left(Ba^{2}+2\,Aab\right)x\right)\sqrt{x}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^2*x^3 + 105*A*a^2 + 21*(2*B*a*b + A*b^2)*x^2 + 35*(B*a^2 + 2
*A*a*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = 2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{5}{2}}}{5} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{5}{2}}}{5} + \frac{2Bb^2x^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(5/2)/5 + 2*B*a**2*x**
(3/2)/3 + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + 2\,Aa^2\sqrt{x} + \frac{2}{5}\,\left(2\,Bab + Ab^2\right)x^{\frac{5}{2}} + \frac{2}{3}\,\left(Ba^2 + 2\,Aab\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/7*B*b^2*x^(7/2) + 2*A*a^2*sqrt(x) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) + 2/3*
(B*a^2 + 2*A*a*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{4}{5}\,Babx^{\frac{5}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/7*B*b^2*x^(7/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2/3*B*a^2*x^(3
/2) + 4/3*A*a*b*x^(3/2) + 2*A*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right) + x^{5/2}\left(\frac{2\,A\,b^2}{5}+\frac{4\,B\,a\,b}{5}\right) + 2\,A\,a^2\,\sqrt{x} + \frac{2\,B\,b^2\,x^{7/2}}{7}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^(1/2),x)
```

output
```
x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(5/2)*((2*A*b^2)/5 + (4*B*a*b)/5)
+ 2*A*a^2*x^(1/2) + (2*B*b^2*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(5b^3x^3+21a\,b^2x^2+35a^2bx+35a^3)}{35}$$

input
```
int((b*x+a)^2*(B*x+A)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(35*a**3 + 35*a**2*b*x + 21*a*b**2*x**2 + 5*b**3*x**3))/35
```

3.185 $\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx=-\frac{2a^2A}{\sqrt{x}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{3}b(Ab+2aB)x^{3/2}+\frac{2}{5}b^2Bx^{5/2}$$

output
```
-2*a^2*A/x^(1/2)+2*a*(2*A*b+B*a)*x^(1/2)+2/3*b*(A*b+2*B*a)*x^(3/2)+2/5*b^2
*B*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx=\frac{-30a^2(A-Bx)+20abx(3A+Bx)+2b^2x^2(5A+3Bx)}{15\sqrt{x}}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^(3/2),x]
```

output
```
(-30*a^2*(A - B*x) + 20*a*b*x*(3*A + B*x) + 2*b^2*x^2*(5*A + 3*B*x))/(15*S
qrt[x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}} \, dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^2A}{x^{3/2}}+\frac{a(aB+2Ab)}{\sqrt{x}}+b\sqrt{x}(2aB+Ab)+b^2Bx^{3/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}bx^{3/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{5}b^2Bx^{5/2}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^(3/2),x]
```

output
```
(-2*a^2*A)/Sqrt[x] + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(3/2
))/3 + (2*b^2*B*x^(5/2))/5
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{2(-3B\,b^2x^3-5A\,b^2x^2-10Bab\,x^2-30aAbx-15B\,a^2x+15a^2A)}{15\sqrt{x}}$	52
trager	$-\frac{2(-3B\,b^2x^3-5A\,b^2x^2-10Bab\,x^2-30aAbx-15B\,a^2x+15a^2A)}{15\sqrt{x}}$	52
risch	$-\frac{2(-3B\,b^2x^3-5A\,b^2x^2-10Bab\,x^2-30aAbx-15B\,a^2x+15a^2A)}{15\sqrt{x}}$	52
orering	$-\frac{2(-3B\,b^2x^3-5A\,b^2x^2-10Bab\,x^2-30aAbx-15B\,a^2x+15a^2A)}{15\sqrt{x}}$	52
derivativedivides	$\frac{2b^2B\,x^{\frac{5}{2}}}{5}+\frac{2A\,b^2x^{\frac{3}{2}}}{3}+\frac{4Bab\,x^{\frac{3}{2}}}{3}+4abA\sqrt{x}+2a^2B\sqrt{x}-\frac{2a^2A}{\sqrt{x}}$	54
default	$\frac{2b^2B\,x^{\frac{5}{2}}}{5}+\frac{2A\,b^2x^{\frac{3}{2}}}{3}+\frac{4Bab\,x^{\frac{3}{2}}}{3}+4abA\sqrt{x}+2a^2B\sqrt{x}-\frac{2a^2A}{\sqrt{x}}$	54

input
```
int((b*x+a)^2*(B*x+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^3-5*A*b^2*x^2-10*B*a*b*x^2-30*A*a*b*x-15*B*a^2*x+15*A*a^
2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx=\frac{2\,(3\,Bb^2x^3-15\,Aa^2+5\,(2\,Bab+Ab^2)x^2+15\,(Ba^2+2\,Aab)x)}{15\,\sqrt{x}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^2*x^3 - 15*A*a^2 + 5*(2*B*a*b + A*b^2)*x^2 + 15*(B*a^2 + 2*A*a
*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}} + 4Aab\sqrt{x} + \frac{2Ab^2x^{\frac{3}{2}}}{3} + 2Ba^2\sqrt{x} + \frac{4Babx^{\frac{3}{2}}}{3} + \frac{2Bb^2x^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(3/2)/3 + 2*B*a**2*sqrt(
x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx = \frac{2}{5}\,Bb^2x^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}} + \frac{2}{3}\left(2\,Bab + Ab^2\right)x^{\frac{3}{2}} + 2\left(Ba^2 + 2\,Aab\right)\sqrt{x}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/5*B*b^2*x^(5/2) - 2*A*a^2/sqrt(x) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) + 2*(B
*a^2 + 2*A*a*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx = \frac{2}{5}\,Bb^2x^{\frac{5}{2}} + \frac{4}{3}\,Babx^{\frac{3}{2}} + \frac{2}{3}\,Ab^2x^{\frac{3}{2}} + 2\,Ba^2\sqrt{x} + 4\,Aab\sqrt{x} - \frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/5*B*b^2*x^(5/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) + 2*B*a^2*sqrt(x
) + 4*A*a*b*sqrt(x) - 2*A*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx = \sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right) + x^{3/2}\left(\frac{2\,A\,b^2}{3}+\frac{4\,B\,a\,b}{3}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,b^2\,x^{5/2}}{5}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^(3/2),x)
```

output
```
x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(3/2)*((2*A*b^2)/3 + (4*B*a*b)/3) - (2*A*a
^2)/x^(1/2) + (2*B*b^2*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)^2(A+Bx)}{x^{3/2}}\,dx = \frac{\frac{2}{5}b^3x^3+2a\,b^2x^2+6a^2bx-2a^3}{\sqrt{x}}$$

input
```
int((b*x+a)^2*(B*x+A)/x^(3/2),x)
```

output
```
(2*( - 5*a**3 + 15*a**2*b*x + 5*a*b**2*x**2 + b**3*x**3))/(5*sqrt(x))
```

3.186 $\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}} - \frac{2a(2Ab+aB)}{\sqrt{x}} + 2b(Ab+2aB)\sqrt{x} + \frac{2}{3}b^2Bx^{3/2}$$

output
```
-2/3*a^2*A/x^(3/2)-2*a*(2*A*b+B*a)/x^(1/2)+2*b*(A*b+2*B*a)*x^(1/2)+2/3*b^2
*B*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = \frac{2(6abx(-A+Bx)+b^2x^2(3A+Bx)-a^2(A+3Bx))}{3x^{3/2}}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^(5/2),x]
```

output
```
(2*(6*a*b*x*(-A + B*x) + b^2*x^2*(3*A + B*x) - a^2*(A + 3*B*x)))/(3*x^(3/2
))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^{5/2}}+\frac{a(aB+2Ab)}{x^{3/2}}+\frac{b(2aB+Ab)}{\sqrt{x}}+b^2B\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{3x^{3/2}}-\frac{2a(aB+2Ab)}{\sqrt{x}}+2b\sqrt{x}(2aB+Ab)+\frac{2}{3}b^2Bx^{3/2}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + 2*b*(A*b + 2*a*B)*S
qrt[x] + (2*b^2*B*x^(3/2))/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2\left(-B\,b^{2} x^{3}-3 A\,b^{2} x^{2}-6 B a b\,x^{2}+6 a A b x+3 B\,a^{2} x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
derivativedivides	$\frac{2 b^{2} B\,x^{\frac{3}{2}}}{3}+2 b^{2} A \sqrt{x}+4 a b B \sqrt{x}-\frac{2 a \left(2 A b+B a\right)}{\sqrt{x}}-\frac{2 a^{2} A}{3 x^{\frac{3}{2}}}$	51
default	$\frac{2 b^{2} B\,x^{\frac{3}{2}}}{3}+2 b^{2} A \sqrt{x}+4 a b B \sqrt{x}-\frac{2 a \left(2 A b+B a\right)}{\sqrt{x}}-\frac{2 a^{2} A}{3 x^{\frac{3}{2}}}$	51
trager	$-\frac{2\left(-B\,b^{2} x^{3}-3 A\,b^{2} x^{2}-6 B a b\,x^{2}+6 a A b x+3 B\,a^{2} x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
risch	$-\frac{2\left(-B\,b^{2} x^{3}-3 A\,b^{2} x^{2}-6 B a b\,x^{2}+6 a A b x+3 B\,a^{2} x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
orering	$-\frac{2\left(-B\,b^{2} x^{3}-3 A\,b^{2} x^{2}-6 B a b\,x^{2}+6 a A b x+3 B\,a^{2} x+a^{2} A\right)}{3 x^{\frac{3}{2}}}$	51

input
```
int((b*x+a)^2*(B*x+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-B*b^2*x^3-3*A*b^2*x^2-6*B*a*b*x^2+6*A*a*b*x+3*B*a^2*x+A*a^2)/x^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = \frac{2\,(Bb^2x^3 - Aa^2 + 3\,(2\,Bab + Ab^2)x^2 - 3\,(Ba^2 + 2\,Aab)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*b^2*x^3 - A*a^2 + 3*(2*B*a*b + A*b^2)*x^2 - 3*(B*a^2 + 2*A*a*b)*x)/
x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}} - \frac{4Aab}{\sqrt{x}} + 2Ab^2\sqrt{x} - \frac{2Ba^2}{\sqrt{x}} + 4Bab\sqrt{x} + \frac{2Bb^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**(5/2),x)
```

output
```
-2*A*a**2/(3*x**(3/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*sqrt(x) - 2*B*a**2/sqr
t(x) + 4*B*a*b*sqrt(x) + 2*B*b**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = \frac{2}{3}\,Bb^2x^{\frac{3}{2}} + 2\left(2\,Bab + Ab^2\right)\sqrt{x} - \frac{2\,(Aa^2 + 3\,(Ba^2 + 2\,Aab)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*b^2*x^(3/2) + 2*(2*B*a*b + A*b^2)*sqrt(x) - 2/3*(A*a^2 + 3*(B*a^2 +
2*A*a*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = \frac{2}{3}\,Bb^2x^{\frac{3}{2}} + 4\,Bab\sqrt{x} + 2\,Ab^2\sqrt{x} - \frac{2\,(3\,Ba^2x + 6\,Aabx + Aa^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*B*b^2*x^(3/2) + 4*B*a*b*sqrt(x) + 2*A*b^2*sqrt(x) - 2/3*(3*B*a^2*x + 6
*A*a*b*x + A*a^2)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = -\frac{6\,B\,a^2\,x+2\,A\,a^2-12\,B\,a\,b\,x^2+12\,A\,a\,b\,x-2\,B\,b^2\,x^3-6\,A\,b^2\,x^2}{3\,x^{3/2}}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^(5/2),x)
```

output
```
-(2*A*a^2 - 6*A*b^2*x^2 - 2*B*b^2*x^3 + 6*B*a^2*x - 12*B*a*b*x^2 + 12*A*a*
b*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx)^2(A+Bx)}{x^{5/2}}\,dx = \frac{\frac{2}{3}b^3x^3+6a\,b^2x^2-6a^2bx-\frac{2}{3}a^3}{\sqrt{x}\,x}$$

input
```
int((b*x+a)^2*(B*x+A)/x^(5/2),x)
```

output
```
(2*( - a**3 - 9*a**2*b*x + 9*a*b**2*x**2 + b**3*x**3))/(3*sqrt(x)*x)
```

3.187 $\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = -\frac{2a^2A}{5x^{5/2}} - \frac{2a(2Ab+aB)}{3x^{3/2}} - \frac{2b(Ab+2aB)}{\sqrt{x}} + 2b^2B\sqrt{x}$$

output
```
-2/5*a^2*A/x^(5/2)-2/3*a*(2*A*b+B*a)/x^(3/2)-2*b*(A*b+2*B*a)/x^(1/2)+2*b^2
*B*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = -\frac{2(15b^2x^2(A-Bx)+10abx(A+3Bx)+a^2(3A+5Bx))}{15x^{5/2}}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/x^(7/2),x]
```

output
```
(-2*(15*b^2*x^2*(A - B*x) + 10*a*b*x*(A + 3*B*x) + a^2*(3*A + 5*B*x)))/(15
*x^(5/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^2A}{x^{7/2}}+\frac{a(aB+2Ab)}{x^{5/2}}+\frac{b(2aB+Ab)}{x^{3/2}}+\frac{b^2B}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{5x^{5/2}}-\frac{2a(aB+2Ab)}{3x^{3/2}}-\frac{2b(2aB+Ab)}{\sqrt{x}}+2b^2B\sqrt{x}$$

input
```
Int[((a + b*x)^2*(A + B*x))/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/(3*x^(3/2)) - (2*b*(A*b + 2*a
*B))/Sqrt[x] + 2*b^2*B*Sqrt[x]
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2b(Ab+2Ba)}{\sqrt{x}}+2b^2B\sqrt{x}$	48
default	$-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2b(Ab+2Ba)}{\sqrt{x}}+2b^2B\sqrt{x}$	48
gosper	$-\frac{2(-15B\,b^2x^3+15A\,b^2x^2+30Bab\,x^2+10aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}}$	52
trager	$-\frac{2(-15B\,b^2x^3+15A\,b^2x^2+30Bab\,x^2+10aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}}$	52
risch	$-\frac{2(-15B\,b^2x^3+15A\,b^2x^2+30Bab\,x^2+10aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}}$	52
orering	$-\frac{2(-15B\,b^2x^3+15A\,b^2x^2+30Bab\,x^2+10aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}}$	52

input
```
int((b*x+a)^2*(B*x+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^2*A/x^(5/2)-2/3*a*(2*A*b+B*a)/x^(3/2)-2*b*(A*b+2*B*a)/x^(1/2)+2*b^2
*B*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = \frac{2\,(15\,Bb^2x^3-3\,Aa^2-15\,(2\,Bab+Ab^2)x^2-5\,(Ba^2+2\,Aab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(15*B*b^2*x^3 - 3*A*a^2 - 15*(2*B*a*b + A*b^2)*x^2 - 5*(B*a^2 + 2*A*a
*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}} - \frac{4Aab}{3x^{\frac{3}{2}}} - \frac{2Ab^2}{\sqrt{x}} - \frac{2Ba^2}{3x^{\frac{3}{2}}} - \frac{4Bab}{\sqrt{x}} + 2Bb^2\sqrt{x}$$

input
```
integrate((b*x+a)**2*(B*x+A)/x**(7/2),x)
```

output
```
-2*A*a**2/(5*x**(5/2)) - 4*A*a*b/(3*x**(3/2)) - 2*A*b**2/sqrt(x) - 2*B*a**
2/(3*x**(3/2)) - 4*B*a*b/sqrt(x) + 2*B*b**2*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = 2\,Bb^2\sqrt{x} - \frac{2\,(3\,Aa^2 + 15\,(2\,Bab + Ab^2)x^2 + 5\,(Ba^2 + 2\,Aab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2*B*b^2*sqrt(x) - 2/15*(3*A*a^2 + 15*(2*B*a*b + A*b^2)*x^2 + 5*(B*a^2 + 2*
A*a*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = 2\,Bb^2\sqrt{x} - \frac{2\,(30\,Babx^2 + 15\,Ab^2x^2 + 5\,Ba^2x + 10\,Aabx + 3\,Aa^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^2*(B*x+A)/x^(7/2),x, algorithm="giac")
```

output
```
2*B*b^2*sqrt(x) - 2/15*(30*B*a*b*x^2 + 15*A*b^2*x^2 + 5*B*a^2*x + 10*A*a*b
*x + 3*A*a^2)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = 2\,B\,b^2\,\sqrt{x} - \frac{x^2\,\left(2\,A\,b^2+4\,B\,a\,b\right)+\frac{2\,A\,a^2}{5}+x\,\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)}{x^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x)^2)/x^(7/2),x)
```

output
```
2*B*b^2*x^(1/2) - (x^2*(2*A*b^2 + 4*B*a*b) + (2*A*a^2)/5 + x*((2*B*a^2)/3
+ (4*A*a*b)/3))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx)^2(A+Bx)}{x^{7/2}}\,dx = \frac{2b^3x^3-6a\,b^2x^2-2a^2bx-\frac{2}{5}a^3}{\sqrt{x}\,x^2}$$

input
```
int((b*x+a)^2*(B*x+A)/x^(7/2),x)
```

output
```
(2*( - a**3 - 5*a**2*b*x - 15*a*b**2*x**2 + 5*b**3*x**3))/(5*sqrt(x)*x**2)
```

3.188 $\int x^{7/2}(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{17}b^3Bx^{17/2}$$

output
```
2/9*a^3*A*x^(9/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/13*a*b*(A*b+B*a)*x^(13/2
)+2/15*b^2*(A*b+3*B*a)*x^(15/2)+2/17*b^3*B*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx=\frac{2x^{9/2}(1105a^3(11A+9Bx)+2295a^2bx(13A+11Bx)+1683ab^2x^2(15A+13Bx)+429b^3x^3(17}{109395}$$

input
```
Integrate[x^(7/2)*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*x^(9/2)*(1105*a^3*(11*A + 9*B*x) + 2295*a^2*b*x*(13*A + 11*B*x) + 1683*
a*b^2*x^2*(15*A + 13*B*x) + 429*b^3*x^3*(17*A + 15*B*x)))/109395
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(a^3Ax^{7/2}+a^2x^{9/2}(aB+3Ab)+b^2x^{13/2}(3aB+Ab)+3abx^{11/2}(aB+Ab)+b^3Bx^{15/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

input
```
Int[x^(7/2)*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*
B)*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(17/2))/1
7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{9}{2}}\left(6435B\,b^{3}x^{4}+7293A\,b^{3}x^{3}+21879Ba\,b^{2}x^{3}+25245aA\,b^{2}x^{2}+25245B\,a^{2}b\,x^{2}+29835a^{2}Abx+9945B\,a^{3}x+12155a^{3}A\right.}{109395}$
derivativedivides	$\frac{2b^{3}B\,x^{\frac{17}{2}}}{17}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{3}A\,x^{\frac{9}{2}}}{9}$
default	$\frac{2b^{3}B\,x^{\frac{17}{2}}}{17}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{11}{2}}}{11}+\frac{2a^{3}A\,x^{\frac{9}{2}}}{9}$
trager	$\frac{2x^{\frac{9}{2}}\left(6435B\,b^{3}x^{4}+7293A\,b^{3}x^{3}+21879Ba\,b^{2}x^{3}+25245aA\,b^{2}x^{2}+25245B\,a^{2}b\,x^{2}+29835a^{2}Abx+9945B\,a^{3}x+12155a^{3}A\right.}{109395}$
risch	$\frac{2x^{\frac{9}{2}}\left(6435B\,b^{3}x^{4}+7293A\,b^{3}x^{3}+21879Ba\,b^{2}x^{3}+25245aA\,b^{2}x^{2}+25245B\,a^{2}b\,x^{2}+29835a^{2}Abx+9945B\,a^{3}x+12155a^{3}A\right.}{109395}$
orering	$\frac{2x^{\frac{9}{2}}\left(6435B\,b^{3}x^{4}+7293A\,b^{3}x^{3}+21879Ba\,b^{2}x^{3}+25245aA\,b^{2}x^{2}+25245B\,a^{2}b\,x^{2}+29835a^{2}Abx+9945B\,a^{3}x+12155a^{3}A\right.}{109395}$

input
```
int(x^(7/2)*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/109395*x^(9/2)*(6435*B*b^3*x^4+7293*A*b^3*x^3+21879*B*a*b^2*x^3+25245*A*
a*b^2*x^2+25245*B*a^2*b*x^2+29835*A*a^2*b*x+9945*B*a^3*x+12155*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{109395}\left(6435\,Bb^3x^8+12155\,Aa^3x^4+7293\left(3\,Bab^2+Ab^3\right)x^7+25245\left(Ba^2b+Aab^2\right)x^6+994\right.$$

input
```
integrate(x^(7/2)*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
2/109395*(6435*B*b^3*x^8 + 12155*A*a^3*x^4 + 7293*(3*B*a*b^2 + A*b^3)*x^7
+ 25245*(B*a^2*b + A*a*b^2)*x^6 + 9945*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx = \frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Bb^3x^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(7/2)*(b*x+a)**3*(B*x+A),x)
```

output
```
2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(13/2)/13 +
2*A*b**3*x**(15/2)/15 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(13/2)/13 +
2*B*a*b**2*x**(15/2)/5 + 2*B*b**3*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{17}\,Bb^3x^{\frac{17}{2}} + \frac{2}{9}\,Aa^3x^{\frac{9}{2}} + \frac{2}{15}\,\left(3\,Bab^2 + Ab^3\right)x^{\frac{15}{2}} + \frac{6}{13}\,\left(Ba^2b + Aab^2\right)x^{\frac{13}{2}} + \frac{2}{11}\,\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
2/17*B*b^3*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2
) + 6/13*(B*a^2*b + A*a*b^2)*x^(13/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{17}\,Bb^3x^{\frac{17}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{6}{13}\,Ba^2bx^{\frac{13}{2}} + \frac{6}{13}\,Aab^2x^{\frac{13}{2}} + \frac{2}{11}\,Ba^3x^{\frac{11}{2}} + \frac{6}{11}\,Aa^2bx^{\frac{11}{2}} + \frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
2/17*B*b^3*x^(17/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*
b*x^(11/2) + 2/9*A*a^3*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx = x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right) + x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{17/2}}{17}+\frac{6\,a\,b\,x^{13/2}\,(A\,b+B\,a)}{13}$$

input
```
int(x^(7/2)*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(15/2)*((2*A*b^3)/15 + (2*B*a
*b^2)/5) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(17/2))/17 + (6*a*b*x^(13/2)*(
A*b + B*a))/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int x^{7/2}(a+bx)^3(A+Bx)\,dx = \frac{2\sqrt{x}\,x^4(6435b^4x^4+29172a\,b^3x^3+50490a^2b^2x^2+39780a^3bx+12155a^4)}{109395}$$

input
```
int(x^(7/2)*(b*x+a)^3*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**4*(12155*a**4 + 39780*a**3*b*x + 50490*a**2*b**2*x**2 + 2917
2*a*b**3*x**3 + 6435*b**4*x**4))/109395
```

3.189 $\int x^{5/2}(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{15}b^3Bx^{15/2}$$

output
```
2/7*a^3*A*x^(7/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+
2/13*b^2*(A*b+3*B*a)*x^(13/2)+2/15*b^3*B*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx=\frac{2x^{7/2}(715a^3(9A+7Bx)+1365a^2bx(11A+9Bx)+945ab^2x^2(13A+11Bx)+231b^3x^3(15A+}{45045}$$

input
```
Integrate[x^(5/2)*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*x^(7/2)*(715*a^3*(9*A + 7*B*x) + 1365*a^2*b*x*(11*A + 9*B*x) + 945*a*b^
2*x^2*(13*A + 11*B*x) + 231*b^3*x^3*(15*A + 13*B*x)))/45045
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{85, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^3Ax^{5/2}+a^2x^{7/2}(aB+3Ab)+b^2x^{11/2}(3aB+Ab)+3abx^{9/2}(aB+Ab)+b^3Bx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

input
```
Int[x^(5/2)*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)
*x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(15/2))/15
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{7}{2}}\left(3003Bb^3x^4+3465Ab^3x^3+10395Bab^2x^3+12285aAb^2x^2+12285Ba^2bx^2+15015a^2Abx+5005Ba^3x+6435a^3A\right)}{45045}$
derivativedivides	$\frac{2b^3Bx^{\frac{15}{2}}}{15}+\frac{2(b^3A+3ab^2B)x^{\frac{13}{2}}}{13}+\frac{2(3ab^2A+3a^2bB)x^{\frac{11}{2}}}{11}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{7}{2}}}{7}$
default	$\frac{2b^3Bx^{\frac{15}{2}}}{15}+\frac{2(b^3A+3ab^2B)x^{\frac{13}{2}}}{13}+\frac{2(3ab^2A+3a^2bB)x^{\frac{11}{2}}}{11}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{7}{2}}}{7}$
trager	$\frac{2x^{\frac{7}{2}}\left(3003Bb^3x^4+3465Ab^3x^3+10395Bab^2x^3+12285aAb^2x^2+12285Ba^2bx^2+15015a^2Abx+5005Ba^3x+6435a^3A\right)}{45045}$
risch	$\frac{2x^{\frac{7}{2}}\left(3003Bb^3x^4+3465Ab^3x^3+10395Bab^2x^3+12285aAb^2x^2+12285Ba^2bx^2+15015a^2Abx+5005Ba^3x+6435a^3A\right)}{45045}$
orering	$\frac{2x^{\frac{7}{2}}\left(3003Bb^3x^4+3465Ab^3x^3+10395Bab^2x^3+12285aAb^2x^2+12285Ba^2bx^2+15015a^2Abx+5005Ba^3x+6435a^3A\right)}{45045}$

input
```
int(x^(5/2)*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/45045*x^(7/2)*(3003*B*b^3*x^4+3465*A*b^3*x^3+10395*B*a*b^2*x^3+12285*A*a
*b^2*x^2+12285*B*a^2*b*x^2+15015*A*a^2*b*x+5005*B*a^3*x+6435*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx=\frac{2}{45045}\left(3003\,Bb^3x^7+6435\,Aa^3x^3+3465\,\left(3\,Bab^2+Ab^3\right)x^6+12285\,\left(Ba^2b+Aab^2\right)x^5+5005\,\left(\right.\right.$$

input
```
integrate(x^(5/2)*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*b^3*x^7 + 6435*A*a^3*x^3 + 3465*(3*B*a*b^2 + A*b^3)*x^6 +
12285*(B*a^2*b + A*a*b^2)*x^5 + 5005*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx = \frac{2Aa^3x^{\frac{7}{2}}}{7} + \frac{2Aa^2bx^{\frac{9}{2}}}{3} + \frac{6Aab^2x^{\frac{11}{2}}}{11} + \frac{2Ab^3x^{\frac{13}{2}}}{13} + \frac{2Ba^3x^{\frac{9}{2}}}{9} + \frac{6Ba^2bx^{\frac{11}{2}}}{11} + \frac{6Bab^2x^{\frac{13}{2}}}{13} + \frac{2Bb^3x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(5/2)*(b*x+a)**3*(B*x+A),x)
```

output
```
2*A*a**3*x**(7/2)/7 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a*b**2*x**(11/2)/11 + 2*
A*b**3*x**(13/2)/13 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x**(11/2)/11 + 6*B*
a*b**2*x**(13/2)/13 + 2*B*b**3*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}} + \frac{2}{13}\,\left(3\,Bab^2 + Ab^3\right)x^{\frac{13}{2}} + \frac{6}{11}\,\left(Ba^2b + Aab^2\right)x^{\frac{11}{2}} + \frac{2}{9}\,\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
2/15*B*b^3*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2
) + 6/11*(B*a^2*b + A*a*b^2)*x^(11/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{6}{13}\,Bab^2x^{\frac{13}{2}} + \frac{2}{13}\,Ab^3x^{\frac{13}{2}} + \frac{6}{11}\,Ba^2bx^{\frac{11}{2}} + \frac{6}{11}\,Aab^2x^{\frac{11}{2}} + \frac{2}{9}\,Ba^3x^{\frac{9}{2}} + \frac{2}{3}\,Aa^2bx^{\frac{9}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
2/15*B*b^3*x^(15/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/11*B
*a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*
x^(9/2) + 2/7*A*a^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx = x^{9/2}\left(\frac{2\,B\,a^3}{9} + \frac{2\,A\,b\,a^2}{3}\right) + x^{13/2}\left(\frac{2\,A\,b^3}{13} + \frac{6\,B\,a\,b^2}{13}\right) + \frac{2\,A\,a^3\,x^{7/2}}{7} + \frac{2\,B\,b^3\,x^{15/2}}{15} + \frac{6\,a\,b\,x^{11/2}\,(A\,b+B\,a)}{11}$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^
2)/13) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(15/2))/15 + (6*a*b*x^(11/2)*(A*
b + B*a))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int x^{5/2}(a+bx)^3(A+Bx)\,dx = \frac{2\sqrt{x}\,x^3(3003b^4x^4+13860a\,b^3x^3+24570a^2b^2x^2+20020a^3bx+6435a^4)}{45045}$$

input
```
int(x^(5/2)*(b*x+a)^3*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**3*(6435*a**4 + 20020*a**3*b*x + 24570*a**2*b**2*x**2 + 13860
*a*b**3*x**3 + 3003*b**4*x**4))/45045
```

3.190 $\int x^{3/2}(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{11}b^2(Ab+3aB)x^{11/2}+\frac{2}{13}b^3Bx^{13/2}$$

output
```
2/5*a^3*A*x^(5/2)+2/7*a^2*(3*A*b+B*a)*x^(7/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2/
11*b^2*(A*b+3*B*a)*x^(11/2)+2/13*b^3*B*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx=\frac{2x^{5/2}(429a^3(7A+5Bx)+715a^2bx(9A+7Bx)+455ab^2x^2(11A+9Bx)+105b^3x^3(13A+11B}{15015}$$

input
```
Integrate[x^(3/2)*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*x^(5/2)*(429*a^3*(7*A + 5*B*x) + 715*a^2*b*x*(9*A + 7*B*x) + 455*a*b^2*
x^2*(11*A + 9*B*x) + 105*b^3*x^3*(13*A + 11*B*x)))/15015
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^3Ax^{3/2}+a^2x^{5/2}(aB+3Ab)+b^2x^{9/2}(3aB+Ab)+3abx^{7/2}(aB+Ab)+b^3Bx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

input
```
Int[x^(3/2)*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (2*a*b*(A*b + a*B)
*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*B*x^(13/2))/13
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	s
gosper	$\frac{2x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005aA\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435a^{2}Abx+2145B\,a^{3}x+3003a^{3}A\right)}{15015}$	7
derivativedivides	$\frac{2b^{3}B\,x^{\frac{13}{2}}}{13}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{11}{2}}}{11}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{7}{2}}}{7}+\frac{2a^{3}A\,x^{\frac{5}{2}}}{5}$	7
default	$\frac{2b^{3}B\,x^{\frac{13}{2}}}{13}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{11}{2}}}{11}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{7}{2}}}{7}+\frac{2a^{3}A\,x^{\frac{5}{2}}}{5}$	7
trager	$\frac{2x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005aA\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435a^{2}Abx+2145B\,a^{3}x+3003a^{3}A\right)}{15015}$	7
risch	$\frac{2x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005aA\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435a^{2}Abx+2145B\,a^{3}x+3003a^{3}A\right)}{15015}$	7
orering	$\frac{2x^{\frac{5}{2}}\left(1155B\,b^{3}x^{4}+1365A\,b^{3}x^{3}+4095Ba\,b^{2}x^{3}+5005aA\,b^{2}x^{2}+5005B\,a^{2}b\,x^{2}+6435a^{2}Abx+2145B\,a^{3}x+3003a^{3}A\right)}{15015}$	7

input
```
int(x^(3/2)*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/15015*x^(5/2)*(1155*B*b^3*x^4+1365*A*b^3*x^3+4095*B*a*b^2*x^3+5005*A*a*b
^2*x^2+5005*B*a^2*b*x^2+6435*A*a^2*b*x+2145*B*a^3*x+3003*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx=\frac{2}{15015}\left(1155\,Bb^3x^6+3003\,Aa^3x^2+1365\,\left(3\,Bab^2+Ab^3\right)x^5+5005\,\left(Ba^2b+Aab^2\right)x^4+2145\,(\ldots$$

input
```
integrate(x^(3/2)*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
2/15015*(1155*B*b^3*x^6 + 3003*A*a^3*x^2 + 1365*(3*B*a*b^2 + A*b^3)*x^5 +
5005*(B*a^2*b + A*a*b^2)*x^4 + 2145*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx = \frac{2Aa^3x^{\frac{5}{2}}}{5} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{6Bab^2x^{\frac{11}{2}}}{11} + \frac{2Bb^3x^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(3/2)*(b*x+a)**3*(B*x+A),x)
```

output
```
2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(7/2)/7 + 2*A*a*b**2*x**(9/2)/3 + 2*A*
b**3*x**(11/2)/11 + 2*B*a**3*x**(7/2)/7 + 2*B*a**2*b*x**(9/2)/3 + 6*B*a*b*
*2*x**(11/2)/11 + 2*B*b**3*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{13}Bb^3x^{\frac{13}{2}} + \frac{2}{5}Aa^3x^{\frac{5}{2}} + \frac{2}{11}\left(3Bab^2 + Ab^3\right)x^{\frac{11}{2}} + \frac{2}{3}\left(Ba^2b + Aab^2\right)x^{\frac{9}{2}} + \frac{2}{7}\left(Ba^3 + 3Aa^2b\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
2/13*B*b^3*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/11*(3*B*a*b^2 + A*b^3)*x^(11/2
) + 2/3*(B*a^2*b + A*a*b^2)*x^(9/2) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx = \frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{6}{11}\,Bab^2x^{\frac{11}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}} + \frac{2}{3}\,Ba^2bx^{\frac{9}{2}} + \frac{2}{3}\,Aab^2x^{\frac{9}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + \frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
2/13*B*b^3*x^(13/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2) + 2/3*B*
a^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^(7
/2) + 2/5*A*a^3*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx = x^{7/2}\left(\frac{2\,B\,a^3}{7} + \frac{6\,A\,b\,a^2}{7}\right) + x^{11/2}\left(\frac{2\,A\,b^3}{11} + \frac{6\,B\,a\,b^2}{11}\right) + \frac{2\,A\,a^3\,x^{5/2}}{5} + \frac{2\,B\,b^3\,x^{13/2}}{13} + \frac{2\,a\,b\,x^{9/2}\,(A\,b+B\,a)}{3}$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(11/2)*((2*A*b^3)/11 + (6*B*a*b^
2)/11) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(13/2))/13 + (2*a*b*x^(9/2)*(A*b
 + B*a))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int x^{3/2}(a+bx)^3(A+Bx)\,dx=\frac{2\sqrt{x}\,x^2(1155b^4x^4+5460a\,b^3x^3+10010a^2b^2x^2+8580a^3bx+3003a^4)}{15015}$$

input
```
int(x^(3/2)*(b*x+a)^3*(B*x+A),x)
```

output
```
(2*sqrt(x)*x**2*(3003*a**4 + 8580*a**3*b*x + 10010*a**2*b**2*x**2 + 5460*a
*b**3*x**3 + 1155*b**4*x**4))/15015
```

3.191 $\int \sqrt{x}(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = \frac{2}{3}a^3Ax^{3/2}+\frac{2}{5}a^2(3Ab+aB)x^{5/2} + \frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{9}b^2(Ab+3aB)x^{9/2}+\frac{2}{11}b^3Bx^{11/2}$$

output
```
2/3*a^3*A*x^(3/2)+2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/
9*b^2*(A*b+3*B*a)*x^(9/2)+2/11*b^3*B*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = \frac{2x^{3/2}(231a^3(5A+3Bx)+297a^2bx(7A+5Bx)+165ab^2x^2(9A+7Bx)+35b^3x^3(11A+9Bx))}{3465}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*x^(3/2)*(231*a^3*(5*A + 3*B*x) + 297*a^2*b*x*(7*A + 5*B*x) + 165*a*b^2*
x^2*(9*A + 7*B*x) + 35*b^3*x^3*(11*A + 9*B*x)))/3465
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^3A\sqrt{x}+a^2x^{3/2}(aB+3Ab)+b^2x^{7/2}(3aB+Ab)+3abx^{5/2}(aB+Ab)+b^3Bx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

input
```
Int[Sqrt[x]*(a + b*x)^3*(A + B*x),x]
```

output
```
(2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)
*x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*x^(11/2))/11
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485aA\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079a^{2}Abx+693B\,a^{3}x+1155a^{3}A\right)}{3465}$	76
derivativedivides	$\frac{2b^{3}B\,x^{\frac{11}{2}}}{11}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{7}{2}}}{7}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{5}{2}}}{5}+\frac{2a^{3}A\,x^{\frac{3}{2}}}{3}$	76
default	$\frac{2b^{3}B\,x^{\frac{11}{2}}}{11}+\frac{2\left(b^{3}A+3a\,b^{2}B\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3a\,b^{2}A+3a^{2}bB\right)x^{\frac{7}{2}}}{7}+\frac{2\left(3a^{2}bA+a^{3}B\right)x^{\frac{5}{2}}}{5}+\frac{2a^{3}A\,x^{\frac{3}{2}}}{3}$	76
trager	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485aA\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079a^{2}Abx+693B\,a^{3}x+1155a^{3}A\right)}{3465}$	76
risch	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485aA\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079a^{2}Abx+693B\,a^{3}x+1155a^{3}A\right)}{3465}$	76
orering	$\frac{2x^{\frac{3}{2}}\left(315B\,b^{3}x^{4}+385A\,b^{3}x^{3}+1155Ba\,b^{2}x^{3}+1485aA\,b^{2}x^{2}+1485B\,a^{2}b\,x^{2}+2079a^{2}Abx+693B\,a^{3}x+1155a^{3}A\right)}{3465}$	76

input
```
int(x^(1/2)*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x^(3/2)*(315*B*b^3*x^4+385*A*b^3*x^3+1155*B*a*b^2*x^3+1485*A*a*b^2*
x^2+1485*B*a^2*b*x^2+2079*A*a^2*b*x+693*B*a^3*x+1155*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx$$
$$=\frac{2}{3465}\left(315\,Bb^3x^5+1155\,Aa^3x+385\left(3\,Bab^2+Ab^3\right)x^4+1485\left(Ba^2b+Aab^2\right)x^3+693\left(Ba^3+3\,Aa^2b\right)x\right.$$

input
```
integrate(x^(1/2)*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^3*x^5 + 1155*A*a^3*x + 385*(3*B*a*b^2 + A*b^3)*x^4 + 1485*
(B*a^2*b + A*a*b^2)*x^3 + 693*(B*a^3 + 3*A*a^2*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = \frac{2Aa^3x^{\frac{3}{2}}}{3} + \frac{2Bb^3x^{\frac{11}{2}}}{11} + \frac{2x^{\frac{9}{2}}(Ab^3+3Bab^2)}{9} + \frac{2x^{\frac{7}{2}}\cdot(3Aab^2+3Ba^2b)}{7} + \frac{2x^{\frac{5}{2}}\cdot(3Aa^2b+Ba^3)}{5}$$

input
```
integrate(x**(1/2)*(b*x+a)**3*(B*x+A),x)
```

output
```
2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(11/2)/11 + 2*x**(9/2)*(A*b**3 + 3*B*a*b
**2)/9 + 2*x**(7/2)*(3*A*a*b**2 + 3*B*a**2*b)/7 + 2*x**(5/2)*(3*A*a**2*b +
 B*a**3)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}} + \frac{2}{9}\,\left(3\,Bab^2 + Ab^3\right)x^{\frac{9}{2}} + \frac{6}{7}\,\left(Ba^2b + Aab^2\right)x^{\frac{7}{2}} + \frac{2}{5}\,\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
2/11*B*b^3*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/9*(3*B*a*b^2 + A*b^3)*x^(9/2)
+ 6/7*(B*a^2*b + A*a*b^2)*x^(7/2) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{2}{3}\,Bab^2x^{\frac{9}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{6}{7}\,Ba^2bx^{\frac{7}{2}} + \frac{6}{7}\,Aab^2x^{\frac{7}{2}} + \frac{2}{5}\,Ba^3x^{\frac{5}{2}} + \frac{6}{5}\,Aa^2bx^{\frac{5}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
2/11*B*b^3*x^(11/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 6/7*B*a^2*
b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5/2)
+ 2/3*A*a^3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+x^{9/2}\left(\frac{2\,A\,b^3}{9}+\frac{2\,B\,a\,b^2}{3}\right) + \frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{11/2}}{11}+\frac{6\,a\,b\,x^{7/2}\,(A\,b+B\,a)}{7}$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x)^3,x)
```

output
```
x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(9/2)*((2*A*b^3)/9 + (2*B*a*b^2)
/3) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(11/2))/11 + (6*a*b*x^(7/2)*(A*b +
B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.54

$$\int \sqrt{x}(a+bx)^3(A+Bx)\,dx = \frac{2\sqrt{x}\,x(315b^4x^4+1540a\,b^3x^3+2970a^2b^2x^2+2772a^3bx+1155a^4)}{3465}$$

input
```
int(x^(1/2)*(b*x+a)^3*(B*x+A),x)
```

output
```
(2*sqrt(x)*x*(1155*a**4 + 2772*a**3*b*x + 2970*a**2*b**2*x**2 + 1540*a*b**
3*x**3 + 315*b**4*x**4))/3465
```

3.192 $\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = 2a^3A\sqrt{x}+\frac{2}{3}a^2(3Ab+aB)x^{3/2} + \frac{6}{5}ab(Ab+aB)x^{5/2}+\frac{2}{7}b^2(Ab+3aB)x^{7/2}+\frac{2}{9}b^3Bx^{9/2}$$

output
```
2*a^3*A*x^(1/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+6/5*a*b*(A*b+B*a)*x^(5/2)+2/7*
b^2*(A*b+3*B*a)*x^(7/2)+2/9*b^3*B*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{315}\sqrt{x}\big(105a^3(3A+Bx)+63a^2bx(5A+3Bx) + 27ab^2x^2(7A+5Bx)+5b^3x^3(9A+7Bx)\big)$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(105*a^3*(3*A + B*x) + 63*a^2*b*x*(5*A + 3*B*x) + 27*a*b^2*x^2*
(7*A + 5*B*x) + 5*b^3*x^3*(9*A + 7*B*x)))/315
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{a^3A}{\sqrt{x}}+a^2\sqrt{x}(aB+3Ab)+b^2x^{5/2}(3aB+Ab)+3abx^{3/2}(aB+Ab)+b^3Bx^{7/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$2a^3A\sqrt{x}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{9}b^3Bx^{9/2}$$

input
```
Int[((a + b*x)^3*(A + B*x))/Sqrt[x],x]
```

output
```
2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*b + a*B)*x^(
5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^(9/2))/9
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result
trager	$\left(\frac{2}{9}B\,b^3x^4+\frac{2}{7}A\,b^3x^3+\frac{6}{7}Ba\,b^2x^3+\frac{6}{5}aA\,b^2x^2+\frac{6}{5}B\,a^2b\,x^2+2a^2Abx+\frac{2}{3}B\,a^3x+2a^3A\right.$
gosper	$\frac{2\sqrt{x}\,(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189aA\,b^2x^2+189B\,a^2b\,x^2+315a^2Abx+105B\,a^3x+315a^3A)}{315}$
derivativedivides	$\frac{2b^3B\,x^{\frac{9}{2}}}{9}+\frac{2(b^3A+3a\,b^2B)x^{\frac{7}{2}}}{7}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{5}{2}}}{5}+\frac{2(3a^2bA+a^3B)x^{\frac{3}{2}}}{3}+2a^3A\sqrt{x}$
default	$\frac{2b^3B\,x^{\frac{9}{2}}}{9}+\frac{2(b^3A+3a\,b^2B)x^{\frac{7}{2}}}{7}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{5}{2}}}{5}+\frac{2(3a^2bA+a^3B)x^{\frac{3}{2}}}{3}+2a^3A\sqrt{x}$
risch	$\frac{2\sqrt{x}\,(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189aA\,b^2x^2+189B\,a^2b\,x^2+315a^2Abx+105B\,a^3x+315a^3A)}{315}$
orering	$\frac{2\sqrt{x}\,(35B\,b^3x^4+45A\,b^3x^3+135Ba\,b^2x^3+189aA\,b^2x^2+189B\,a^2b\,x^2+315a^2Abx+105B\,a^3x+315a^3A)}{315}$

input
```
int((b*x+a)^3*(B*x+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/9*B*b^3*x^4+2/7*A*b^3*x^3+6/7*B*a*b^2*x^3+6/5*a*A*b^2*x^2+6/5*B*a^2*b*x
^2+2*a^2*A*b*x+2/3*B*a^3*x+2*a^3*A)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx$$
$$=\frac{2}{315}\left(35\,Bb^3x^4+315\,Aa^3+45\left(3\,Bab^2+Ab^3\right)x^3+189\left(Ba^2b+Aab^2\right)x^2+105\left(Ba^3+3\,Aa^2b\right)x\right)\sqrt{x}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^3*x^4 + 315*A*a^3 + 45*(3*B*a*b^2 + A*b^3)*x^3 + 189*(B*a^2*
b + A*a*b^2)*x^2 + 105*(B*a^3 + 3*A*a^2*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x} + 2Aa^2bx^{\frac{3}{2}} + \frac{6Aab^2x^{\frac{5}{2}}}{5} + \frac{2Ab^3x^{\frac{7}{2}}}{7} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{6Ba^2bx^{\frac{5}{2}}}{5} + \frac{6Bab^2x^{\frac{7}{2}}}{7} + \frac{2Bb^3x^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**(1/2),x)
```

output
```
2*A*a**3*sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a*b**2*x**(5/2)/5 + 2*A*b**3*
x**(7/2)/7 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(5/2)/5 + 6*B*a*b**2*x**(
7/2)/7 + 2*B*b**3*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{9}\,Bb^3x^{\frac{9}{2}} + 2\,Aa^3\sqrt{x} + \frac{2}{7}\left(3\,Bab^2 + Ab^3\right)x^{\frac{7}{2}} + \frac{6}{5}\left(Ba^2b + Aab^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/9*B*b^3*x^(9/2) + 2*A*a^3*sqrt(x) + 2/7*(3*B*a*b^2 + A*b^3)*x^(7/2) + 6/
5*(B*a^2*b + A*a*b^2)*x^(5/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = \frac{2}{9}\,Bb^3x^{\frac{9}{2}} + \frac{6}{7}\,Bab^2x^{\frac{7}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}} + \frac{6}{5}\,Ba^2bx^{\frac{5}{2}} + \frac{6}{5}\,Aab^2x^{\frac{5}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} + 2\,Aa^3\sqrt{x}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/9*B*b^3*x^(9/2) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 6/5*B*a^2*b*
x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) + 2*
A*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,B\,a^3}{3} + 2\,A\,b\,a^2\right) + x^{7/2}\left(\frac{2\,A\,b^3}{7} + \frac{6\,B\,a\,b^2}{7}\right) + 2\,A\,a^3\,\sqrt{x} + \frac{2\,B\,b^3\,x^{9/2}}{9} + \frac{6\,a\,b\,x^{5/2}\,(A\,b + B\,a)}{5}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^(1/2),x)
```

output
```
x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(7/2)*((2*A*b^3)/7 + (6*B*a*b^2)/7)
+ 2*A*a^3*x^(1/2) + (2*B*b^3*x^(9/2))/9 + (6*a*b*x^(5/2)*(A*b + B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx)^3(A+Bx)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(35b^4x^4+180a\,b^3x^3+378a^2b^2x^2+420a^3bx+315a^4)}{315}$$

input
```
int((b*x+a)^3*(B*x+A)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(315*a**4 + 420*a**3*b*x + 378*a**2*b**2*x**2 + 180*a*b**3*x**3
 + 35*b**4*x**4))/315
```

3.193 $\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 79

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}} + 2a^2(3Ab+aB)\sqrt{x} + 2ab(Ab+aB)x^{3/2} + \frac{2}{5}b^2(Ab+3aB)x^{5/2} + \frac{2}{7}b^3Bx^{7/2}$$

output
```
-2*a^3*A/x^(1/2)+2*a^2*(3*A*b+B*a)*x^(1/2)+2*a*b*(A*b+B*a)*x^(3/2)+2/5*b^2
*(A*b+3*B*a)*x^(5/2)+2/7*b^3*B*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = \frac{2(-35a^3(A-Bx)+35a^2bx(3A+Bx)+7ab^2x^2(5A+3Bx)+b^3x^3(7A+5B}{35\sqrt{x}}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^(3/2),x]
```

output
```
(2*(-35*a^3*(A - B*x) + 35*a^2*b*x*(3*A + B*x) + 7*a*b^2*x^2*(5*A + 3*B*x)
 + b^3*x^3*(7*A + 5*B*x)))/(35*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3A}{x^{3/2}}+\frac{a^2(aB+3Ab)}{\sqrt{x}}+b^2x^{3/2}(3aB+Ab)+3ab\sqrt{x}(aB+Ab)+b^3Bx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{\sqrt{x}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{5}b^2x^{5/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{7}b^3Bx^{7/2}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^(3/2),x]
```

output
```
(-2*a^3*A)/Sqrt[x] + 2*a^2*(3*A*b + a*B)*Sqrt[x] + 2*a*b*(A*b + a*B)*x^(3/
2) + (2*b^2*(A*b + 3*a*B)*x^(5/2))/5 + (2*b^3*B*x^(7/2))/7
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{2(-5B\,b^3x^4-7A\,b^3x^3-21Ba\,b^2x^3-35aA\,b^2x^2-35B\,a^2b\,x^2-105a^2Abx-35B\,a^3x+35a^3A)}{35\sqrt{x}}$
trager	$-\frac{2(-5B\,b^3x^4-7A\,b^3x^3-21Ba\,b^2x^3-35aA\,b^2x^2-35B\,a^2b\,x^2-105a^2Abx-35B\,a^3x+35a^3A)}{35\sqrt{x}}$
risch	$-\frac{2(-5B\,b^3x^4-7A\,b^3x^3-21Ba\,b^2x^3-35aA\,b^2x^2-35B\,a^2b\,x^2-105a^2Abx-35B\,a^3x+35a^3A)}{35\sqrt{x}}$
orering	$-\frac{2(-5B\,b^3x^4-7A\,b^3x^3-21Ba\,b^2x^3-35aA\,b^2x^2-35B\,a^2b\,x^2-105a^2Abx-35B\,a^3x+35a^3A)}{35\sqrt{x}}$
derivativedivides	$\frac{2b^3B\,x^{\frac{7}{2}}}{7}+\frac{2A\,b^3x^{\frac{5}{2}}}{5}+\frac{6Ba\,b^2x^{\frac{5}{2}}}{5}+2Aa\,b^2x^{\frac{3}{2}}+2B\,a^2b\,x^{\frac{3}{2}}+6a^2bA\sqrt{x}+2a^3B\sqrt{x}-\frac{2a^3A}{\sqrt{x}}$
default	$\frac{2b^3B\,x^{\frac{7}{2}}}{7}+\frac{2A\,b^3x^{\frac{5}{2}}}{5}+\frac{6Ba\,b^2x^{\frac{5}{2}}}{5}+2Aa\,b^2x^{\frac{3}{2}}+2B\,a^2b\,x^{\frac{3}{2}}+6a^2bA\sqrt{x}+2a^3B\sqrt{x}-\frac{2a^3A}{\sqrt{x}}$

input
```
int((b*x+a)^3*(B*x+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-5*B*b^3*x^4-7*A*b^3*x^3-21*B*a*b^2*x^3-35*A*a*b^2*x^2-35*B*a^2*b*x
^2-105*A*a^2*b*x-35*B*a^3*x+35*A*a^3)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = \frac{2\,(5\,Bb^3x^4-35\,Aa^3+7\,(3\,Bab^2+Ab^3)x^3+35\,(Ba^2b+Aab^2)x^2+35\,(Ba^3+}{35\,\sqrt{x}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*b^3*x^4 - 35*A*a^3 + 7*(3*B*a*b^2 + A*b^3)*x^3 + 35*(B*a^2*b + A
*a*b^2)*x^2 + 35*(B*a^3 + 3*A*a^2*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = -\frac{2Aa^3}{\sqrt{x}} + 6Aa^2b\sqrt{x} + 2Aab^2x^{\frac{3}{2}} + \frac{2Ab^3x^{\frac{5}{2}}}{5} + 2Ba^3\sqrt{x} + 2Ba^2bx^{\frac{3}{2}} + \frac{6Bab^2x^{\frac{5}{2}}}{5} + \frac{2Bb^3x^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**(3/2),x)
```

output
```
-2*A*a**3/sqrt(x) + 6*A*a**2*b*sqrt(x) + 2*A*a*b**2*x**(3/2) + 2*A*b**3*x*
*(5/2)/5 + 2*B*a**3*sqrt(x) + 2*B*a**2*b*x**(3/2) + 6*B*a*b**2*x**(5/2)/5
+ 2*B*b**3*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = \frac{2}{7}\,Bb^3x^{\frac{7}{2}} - \frac{2\,Aa^3}{\sqrt{x}} + \frac{2}{5}\left(3\,Bab^2 + Ab^3\right)x^{\frac{5}{2}} + 2\left(Ba^2b + Aab^2\right)x^{\frac{3}{2}} + 2\left(Ba^3 + 3\,Aa^2b\right)\sqrt{x}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/7*B*b^3*x^(7/2) - 2*A*a^3/sqrt(x) + 2/5*(3*B*a*b^2 + A*b^3)*x^(5/2) + 2*
(B*a^2*b + A*a*b^2)*x^(3/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = \frac{2}{7}\,Bb^3x^{\frac{7}{2}} + \frac{6}{5}\,Bab^2x^{\frac{5}{2}} + \frac{2}{5}\,Ab^3x^{\frac{5}{2}} + 2\,Ba^2bx^{\frac{3}{2}} + 2\,Aab^2x^{\frac{3}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{\sqrt{x}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/7*B*b^3*x^(7/2) + 6/5*B*a*b^2*x^(5/2) + 2/5*A*b^3*x^(5/2) + 2*B*a^2*b*x^
(3/2) + 2*A*a*b^2*x^(3/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2*A*a^3/
sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = \sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right) + x^{5/2}\left(\frac{2\,A\,b^3}{5}+\frac{6\,B\,a\,b^2}{5}\right) - \frac{2\,A\,a^3}{\sqrt{x}} + \frac{2\,B\,b^3\,x^{7/2}}{7} + 2\,a\,b\,x^{3/2}\,(A\,b+B\,a)$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^(3/2),x)
```

output
```
x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(5/2)*((2*A*b^3)/5 + (6*B*a*b^2)/5) - (2
*A*a^3)/x^(1/2) + (2*B*b^3*x^(7/2))/7 + 2*a*b*x^(3/2)*(A*b + B*a)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)^3(A+Bx)}{x^{3/2}}\,dx = \frac{\frac{2}{7}b^4x^4+\frac{8}{5}a\,b^3x^3+4a^2b^2x^2+8a^3bx-2a^4}{\sqrt{x}}$$

input
```
int((b*x+a)^3*(B*x+A)/x^(3/2),x)
```

output
```
(2*( - 35*a**4 + 140*a**3*b*x + 70*a**2*b**2*x**2 + 28*a*b**3*x**3 + 5*b**
4*x**4))/(35*sqrt(x))
```

3.194 $\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} - \frac{2a^2(3Ab+aB)}{\sqrt{x}} + 6ab(Ab+aB)\sqrt{x} + \frac{2}{3}b^2(Ab+3aB)x^{3/2} + \frac{2}{5}b^3Bx^{5/2}$$

output
```
-2/3*a^3*A/x^(3/2)-2*a^2*(3*A*b+B*a)/x^(1/2)+6*a*b*(A*b+B*a)*x^(1/2)+2/3*b
^2*(A*b+3*B*a)*x^(3/2)+2/5*b^3*B*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = \frac{2(45a^2bx(-A+Bx)+15ab^2x^2(3A+Bx)-5a^3(A+3Bx)+b^3x^3(5A+3Bx}{15x^{3/2}}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^(5/2),x]
```

output
```
(2*(45*a^2*b*x*(-A + B*x) + 15*a*b^2*x^2*(3*A + B*x) - 5*a^3*(A + 3*B*x) +
 b^3*x^3*(5*A + 3*B*x)))/(15*x^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{a^3A}{x^{5/2}}+\frac{a^2(aB+3Ab)}{x^{3/2}}+b^2\sqrt{x}(3aB+Ab)+\frac{3ab(aB+Ab)}{\sqrt{x}}+b^3Bx^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3A}{3x^{3/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}(3aB+Ab)+6ab\sqrt{x}(aB+Ab)+\frac{2}{5}b^3Bx^{5/2}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^(5/2),x]
```

output
```
(-2*a^3*A)/(3*x^(3/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 6*a*b*(A*b + a*B)
*Sqrt[x] + (2*b^2*(A*b + 3*a*B)*x^(3/2))/3 + (2*b^3*B*x^(5/2))/5
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2b^{3}B\,x^{\frac{5}{2}}}{5}+\frac{2A\,b^{3}x^{\frac{3}{2}}}{3}+2Ba\,b^{2}x^{\frac{3}{2}}+6a\,b^{2}A\sqrt{x}+6a^{2}bB\sqrt{x}-\frac{2a^{2}(3Ab+Ba)}{\sqrt{x}}-\frac{2a^{3}A}{3x^{\frac{3}{2}}}$	75
default	$\frac{2b^{3}B\,x^{\frac{5}{2}}}{5}+\frac{2A\,b^{3}x^{\frac{3}{2}}}{3}+2Ba\,b^{2}x^{\frac{3}{2}}+6a\,b^{2}A\sqrt{x}+6a^{2}bB\sqrt{x}-\frac{2a^{2}(3Ab+Ba)}{\sqrt{x}}-\frac{2a^{3}A}{3x^{\frac{3}{2}}}$	75
gosper	$-\frac{2(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45aA\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45a^{2}Abx+15B\,a^{3}x+5a^{3}A)}{15x^{\frac{3}{2}}}$	76
trager	$-\frac{2(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45aA\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45a^{2}Abx+15B\,a^{3}x+5a^{3}A)}{15x^{\frac{3}{2}}}$	76
risch	$-\frac{2(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45aA\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45a^{2}Abx+15B\,a^{3}x+5a^{3}A)}{15x^{\frac{3}{2}}}$	76
orering	$-\frac{2(-3B\,b^{3}x^{4}-5A\,b^{3}x^{3}-15Ba\,b^{2}x^{3}-45aA\,b^{2}x^{2}-45B\,a^{2}b\,x^{2}+45a^{2}Abx+15B\,a^{3}x+5a^{3}A)}{15x^{\frac{3}{2}}}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b^3*B*x^(5/2)+2/3*A*b^3*x^(3/2)+2*B*a*b^2*x^(3/2)+6*a*b^2*A*x^(1/2)+6*
a^2*b*B*x^(1/2)-2*a^2*(3*A*b+B*a)/x^(1/2)-2/3*a^3*A/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{3}(A+Bx)}{x^{5/2}}\,dx=\frac{2\,(3\,Bb^{3}x^{4}-5\,Aa^{3}+5\,(3\,Bab^{2}+Ab^{3})x^{3}+45\,(Ba^{2}b+Aab^{2})x^{2}-15\,(Ba^{3}+3}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^3*x^4 - 5*A*a^3 + 5*(3*B*a*b^2 + A*b^3)*x^3 + 45*(B*a^2*b + A*
a*b^2)*x^2 - 15*(B*a^3 + 3*A*a^2*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = -\frac{2Aa^3}{3x^{\frac{3}{2}}} - \frac{6Aa^2b}{\sqrt{x}} + 6Aab^2\sqrt{x} + \frac{2Ab^3x^{\frac{3}{2}}}{3} - \frac{2Ba^3}{\sqrt{x}} + 6Ba^2b\sqrt{x} + 2Bab^2x^{\frac{3}{2}} + \frac{2Bb^3x^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**(5/2),x)
```

output
```
-2*A*a**3/(3*x**(3/2)) - 6*A*a**2*b/sqrt(x) + 6*A*a*b**2*sqrt(x) + 2*A*b**
3*x**(3/2)/3 - 2*B*a**3/sqrt(x) + 6*B*a**2*b*sqrt(x) + 2*B*a*b**2*x**(3/2)
 + 2*B*b**3*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = \frac{2}{5}\,Bb^3x^{\frac{5}{2}} + \frac{2}{3}\left(3\,Bab^2 + Ab^3\right)x^{\frac{3}{2}} + 6\left(Ba^2b + Aab^2\right)\sqrt{x} - \frac{2\,(Aa^3 + 3\,(Ba^3 + 3\,Aa^2b)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/5*B*b^3*x^(5/2) + 2/3*(3*B*a*b^2 + A*b^3)*x^(3/2) + 6*(B*a^2*b + A*a*b^2
)*sqrt(x) - 2/3*(A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = \frac{2}{5}\,Bb^3x^{\frac{5}{2}} + 2\,Bab^2x^{\frac{3}{2}} + \frac{2}{3}\,Ab^3x^{\frac{3}{2}} + 6\,Ba^2b\sqrt{x} + 6\,Aab^2\sqrt{x} - \frac{2\,(3\,Ba^3x + 9\,Aa^2bx + Aa^3)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/5*B*b^3*x^(5/2) + 2*B*a*b^2*x^(3/2) + 2/3*A*b^3*x^(3/2) + 6*B*a^2*b*sqrt
(x) + 6*A*a*b^2*sqrt(x) - 2/3*(3*B*a^3*x + 9*A*a^2*b*x + A*a^3)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,A\,b^3}{3} + 2\,B\,a\,b^2\right) - \frac{x\,(2\,B\,a^3 + 6\,A\,b\,a^2) + \frac{2\,A\,a^3}{3}}{x^{3/2}} + \frac{2\,B\,b^3\,x^{5/2}}{5} + 6\,a\,b\,\sqrt{x}\,(A\,b + B\,a)$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^(5/2),x)
```

output
```
x^(3/2)*((2*A*b^3)/3 + 2*B*a*b^2) - (x*(2*B*a^3 + 6*A*a^2*b) + (2*A*a^3)/3
)/x^(3/2) + (2*B*b^3*x^(5/2))/5 + 6*a*b*x^(1/2)*(A*b + B*a)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^3(A+Bx)}{x^{5/2}}\,dx = \frac{\frac{2}{5}b^4x^4+\frac{8}{3}a\,b^3x^3+12a^2b^2x^2-8a^3bx-\frac{2}{3}a^4}{\sqrt{x}\,x}$$

input
```
int((b*x+a)^3*(B*x+A)/x^(5/2),x)
```

output
```
(2*( - 5*a**4 - 60*a**3*b*x + 90*a**2*b**2*x**2 + 20*a*b**3*x**3 + 3*b**4*
x**4))/(15*sqrt(x)*x)
```

3.195 $\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = -\frac{2a^3A}{5x^{5/2}} - \frac{2a^2(3Ab+aB)}{3x^{3/2}} - \frac{6ab(Ab+aB)}{\sqrt{x}} + 2b^2(Ab+3aB)\sqrt{x} + \frac{2}{3}b^3Bx^{3/2}$$

output
```
-2/5*a^3*A/x^(5/2)-2/3*a^2*(3*A*b+B*a)/x^(3/2)-6*a*b*(A*b+B*a)/x^(1/2)+2*b
^2*(A*b+3*B*a)*x^(1/2)+2/3*b^3*B*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = -\frac{2(45ab^2x^2(A-Bx) - 5b^3x^3(3A+Bx) + 15a^2bx(A+3Bx) + a^3(3A+5Bx))}{15x^{5/2}}$$

input
```
Integrate[((a + b*x)^3*(A + B*x))/x^(7/2),x]
```

output
```
(-2*(45*a*b^2*x^2*(A - B*x) - 5*b^3*x^3*(3*A + B*x) + 15*a^2*b*x*(A + 3*B*
x) + a^3*(3*A + 5*B*x)))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(aB+3Ab)}{x^{5/2}}+\frac{b^2(3aB+Ab)}{\sqrt{x}}+\frac{3ab(aB+Ab)}{x^{3/2}}+b^3B\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{3x^{3/2}}+2b^2\sqrt{x}(3aB+Ab)-\frac{6ab(aB+Ab)}{\sqrt{x}}+\frac{2}{3}b^3Bx^{3/2}$$

input
```
Int[((a + b*x)^3*(A + B*x))/x^(7/2),x]
```

output
```
(-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/(3*x^(3/2)) - (6*a*b*(A*b +
 a*B))/Sqrt[x] + 2*b^2*(A*b + 3*a*B)*Sqrt[x] + (2*b^3*B*x^(3/2))/3
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b^3 B x^{\frac{3}{2}}}{3}+2A b^3\sqrt{x}+6Ba b^2\sqrt{x}-\frac{6ab(Ab+Ba)}{\sqrt{x}}-\frac{2a^2(3Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2a^3A}{5x^{\frac{5}{2}}}$	69
default	$\frac{2b^3 B x^{\frac{3}{2}}}{3}+2A b^3\sqrt{x}+6Ba b^2\sqrt{x}-\frac{6ab(Ab+Ba)}{\sqrt{x}}-\frac{2a^2(3Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2a^3A}{5x^{\frac{5}{2}}}$	69
gosper	$-\frac{2(-5B b^3x^4-15A b^3x^3-45Ba b^2x^3+45aA b^2x^2+45B a^2b x^2+15a^2Abx+5B a^3x+3a^3A)}{15x^{\frac{5}{2}}}$	76
trager	$-\frac{2(-5B b^3x^4-15A b^3x^3-45Ba b^2x^3+45aA b^2x^2+45B a^2b x^2+15a^2Abx+5B a^3x+3a^3A)}{15x^{\frac{5}{2}}}$	76
risch	$-\frac{2(-5B b^3x^4-15A b^3x^3-45Ba b^2x^3+45aA b^2x^2+45B a^2b x^2+15a^2Abx+5B a^3x+3a^3A)}{15x^{\frac{5}{2}}}$	76
orering	$-\frac{2(-5B b^3x^4-15A b^3x^3-45Ba b^2x^3+45aA b^2x^2+45B a^2b x^2+15a^2Abx+5B a^3x+3a^3A)}{15x^{\frac{5}{2}}}$	76

input
```
int((b*x+a)^3*(B*x+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^3*B*x^(3/2)+2*A*b^3*x^(1/2)+6*B*a*b^2*x^(1/2)-6*a*b*(A*b+B*a)/x^(1/2
)-2/3*a^2*(3*A*b+B*a)/x^(3/2)-2/5*a^3*A/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = \frac{2\,(5\,Bb^3x^4 - 3\,Aa^3 + 15\,(3\,Bab^2 + Ab^3)x^3 - 45\,(Ba^2b + Aab^2)x^2 - 5\,(Ba^3 + 3}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*B*b^3*x^4 - 3*A*a^3 + 15*(3*B*a*b^2 + A*b^3)*x^3 - 45*(B*a^2*b + A
*a*b^2)*x^2 - 5*(B*a^3 + 3*A*a^2*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = -\frac{2Aa^3}{5x^{\frac{5}{2}}} - \frac{2Aa^2b}{x^{\frac{3}{2}}} - \frac{6Aab^2}{\sqrt{x}} + 2Ab^3\sqrt{x} - \frac{2Ba^3}{3x^{\frac{3}{2}}} - \frac{6Ba^2b}{\sqrt{x}} + 6Bab^2\sqrt{x} + \frac{2Bb^3x^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x+a)**3*(B*x+A)/x**(7/2),x)
```

output
```
-2*A*a**3/(5*x**(5/2)) - 2*A*a**2*b/x**(3/2) - 6*A*a*b**2/sqrt(x) + 2*A*b*
*3*sqrt(x) - 2*B*a**3/(3*x**(3/2)) - 6*B*a**2*b/sqrt(x) + 6*B*a*b**2*sqrt(
x) + 2*B*b**3*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = \frac{2}{3}\,Bb^3x^{\frac{3}{2}} + 2\left(3\,Bab^2 + Ab^3\right)\sqrt{x} - \frac{2\,(3\,Aa^3 + 45\,(Ba^2b + Aab^2)x^2 + 5\,(Ba^3 + 3\,Aa^2b)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*B*b^3*x^(3/2) + 2*(3*B*a*b^2 + A*b^3)*sqrt(x) - 2/15*(3*A*a^3 + 45*(B*
a^2*b + A*a*b^2)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = \frac{2}{3}\,Bb^3x^{\frac{3}{2}} + 6\,Bab^2\sqrt{x} + 2\,Ab^3\sqrt{x} - \frac{2\,(45\,Ba^2bx^2 + 45\,Aab^2x^2 + 5\,Ba^3x + 15\,Aa^2bx + 3\,Aa^3)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^3*(B*x+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/3*B*b^3*x^(3/2) + 6*B*a*b^2*sqrt(x) + 2*A*b^3*sqrt(x) - 2/15*(45*B*a^2*b
*x^2 + 45*A*a*b^2*x^2 + 5*B*a^3*x + 15*A*a^2*b*x + 3*A*a^3)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = -\frac{10\,B\,a^3\,x + 6\,A\,a^3 + 90\,B\,a^2\,b\,x^2 + 30\,A\,a^2\,b\,x - 90\,B\,a\,b^2\,x^3 + 90\,A\,a\,b^2\,x^2 - 10\,B\,b^3\,x^4 - 30\,A\,b^3\,x^3}{15\,x^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x)^3)/x^(7/2),x)
```

output
```
-(6*A*a^3 - 30*A*b^3*x^3 - 10*B*b^3*x^4 + 10*B*a^3*x + 30*A*a^2*b*x + 90*A
*a*b^2*x^2 + 90*B*a^2*b*x^2 - 90*B*a*b^2*x^3)/(15*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^3(A+Bx)}{x^{7/2}}\,dx = \frac{\frac{2}{3}b^4x^4 + 8a\,b^3x^3 - 12a^2b^2x^2 - \frac{8}{3}a^3bx - \frac{2}{5}a^4}{\sqrt{x}\,x^2}$$

input
```
int((b*x+a)^3*(B*x+A)/x^(7/2),x)
```

output
```
(2*( - 3*a**4 - 20*a**3*b*x - 90*a**2*b**2*x**2 + 60*a*b**3*x**3 + 5*b**4*
x**4))/(15*sqrt(x)*x**2)
```

3.196 $\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = -450\sqrt{x}+72x^{3/2}-\frac{54x^{5/2}}{5}-\frac{125\sqrt{x}}{1+x}+575\arctan\left(\sqrt{x}\right)$$

output
```
-450*x^(1/2)+72*x^(3/2)-54/5*x^(5/2)-125*x^(1/2)/(1+x)+575*arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = -\frac{\sqrt{x}(2875+1890x-306x^2+54x^3)}{5(1+x)}+575\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[((2 - 3*x)^3*Sqrt[x])/(1 + x)^2,x]
```

output
```
-1/5*(Sqrt[x]*(2875 + 1890*x - 306*x^2 + 54*x^3))/(1 + x) + 575*ArcTan[Sqr
t[x]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {108, 27, 170, 27, 164, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-3x)^3\sqrt{x}}{(x+1)^2}dx$$

$$\Big\downarrow \text{ 108}$$

$$\int \frac{(2-21x)(2-3x)^2}{2\sqrt{x}(x+1)}dx-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{(2-21x)(2-3x)^2}{\sqrt{x}(x+1)}dx-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

$$\Big\downarrow \text{ 170}$$

$$\frac{1}{2}\left(\frac{2}{5}\int \frac{(62-513x)(2-3x)}{2\sqrt{x}(x+1)}dx-\frac{42}{5}(2-3x)^2\sqrt{x}\right)-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{5}\int \frac{(62-513x)(2-3x)}{\sqrt{x}(x+1)}dx-\frac{42}{5}(2-3x)^2\sqrt{x}\right)-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

$$\Big\downarrow \text{ 164}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(2875\int \frac{1}{\sqrt{x}(x+1)}dx-6(917-171x)\sqrt{x}\right)-\frac{42}{5}(2-3x)^2\sqrt{x}\right)-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(5750\int \frac{1}{x+1}d\sqrt{x}-6(917-171x)\sqrt{x}\right)-\frac{42}{5}(2-3x)^2\sqrt{x}\right)-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(5750\arctan\left(\sqrt{x}\right)-6(917-171x)\sqrt{x}\right)-\frac{42}{5}(2-3x)^2\sqrt{x}\right)-\frac{(2-3x)^3\sqrt{x}}{x+1}$$

input
```
Int[((2 - 3*x)^3*Sqrt[x])/(1 + x)^2,x]
```

output
```
-(((2 - 3*x)^3*Sqrt[x])/(1 + x)) + ((-42*(2 - 3*x)^2*Sqrt[x])/5 + (-6*(917
 - 171*x)*Sqrt[x] + 5750*ArcTan[Sqrt[x]])/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-450\sqrt{x}+72x^{\frac{3}{2}}-\frac{54x^{\frac{5}{2}}}{5}-\frac{125\sqrt{x}}{1+x}+575\arctan\left(\sqrt{x}\right)$
default	$-450\sqrt{x}+72x^{\frac{3}{2}}-\frac{54x^{\frac{5}{2}}}{5}-\frac{125\sqrt{x}}{1+x}+575\arctan\left(\sqrt{x}\right)$
risch	$-\frac{\left(54x^{3}-306x^{2}+1890x+2875\right)\sqrt{x}}{5(1+x)}+575\arctan\left(\sqrt{x}\right)$
trager	$-\frac{\left(54x^{3}-306x^{2}+1890x+2875\right)\sqrt{x}}{5(1+x)}-\frac{575\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{x}-x+1}{1+x}\right)}{2}$
meijerg	$-\frac{8\sqrt{x}}{1+x}+575\arctan\left(\sqrt{x}\right)-\frac{36\sqrt{x}\,(10x+15)}{5(1+x)}-\frac{18\sqrt{x}\,(-14x^{2}+70x+105)}{7(1+x)}-\frac{3\sqrt{x}\,(18x^{3}-42x^{2}+210x+3}{5(1+x)}$

input
```
int((2-3*x)^3*x^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-450*x^(1/2)+72*x^(3/2)-54/5*x^(5/2)-125*x^(1/2)/(1+x)+575*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = \frac{2875\,(x+1)\arctan\left(\sqrt{x}\right)-(54\,x^3-306\,x^2+1890\,x+2875)\sqrt{x}}{5\,(x+1)}$$

input
```
integrate((2-3*x)^3*x^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
1/5*(2875*(x + 1)*arctan(sqrt(x)) - (54*x^3 - 306*x^2 + 1890*x + 2875)*sqr
t(x))/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = -\frac{54x^{\frac{7}{2}}}{5x+5}+\frac{306x^{\frac{5}{2}}}{5x+5}-\frac{1890x^{\frac{3}{2}}}{5x+5}-\frac{2875\sqrt{x}}{5x+5}+\frac{2875x\,\mathrm{atan}\left(\sqrt{x}\right)}{5x+5}+\frac{2875\,\mathrm{atan}\left(\sqrt{x}\right)}{5x+5}$$

input
```
integrate((2-3*x)**3*x**(1/2)/(1+x)**2,x)
```

output
```
-54*x**(7/2)/(5*x + 5) + 306*x**(5/2)/(5*x + 5) - 1890*x**(3/2)/(5*x + 5)
- 2875*sqrt(x)/(5*x + 5) + 2875*x*atan(sqrt(x))/(5*x + 5) + 2875*atan(sqrt
(x))/(5*x + 5)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = -\frac{54}{5}\,x^{\frac{5}{2}}+72\,x^{\frac{3}{2}}-450\,\sqrt{x}-\frac{125\,\sqrt{x}}{x+1}+575\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate((2-3*x)^3*x^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
-54/5*x^(5/2) + 72*x^(3/2) - 450*sqrt(x) - 125*sqrt(x)/(x + 1) + 575*arcta
n(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = -\frac{54}{5}\,x^{\frac{5}{2}} + 72\,x^{\frac{3}{2}} - 450\,\sqrt{x} - \frac{125\,\sqrt{x}}{x+1} + 575\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate((2-3*x)^3*x^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
-54/5*x^(5/2) + 72*x^(3/2) - 450*sqrt(x) - 125*sqrt(x)/(x + 1) + 575*arcta
n(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = 575\,\mathrm{atan}\left(\sqrt{x}\right) - \frac{125\,\sqrt{x}}{x+1} - 450\,\sqrt{x} + 72\,x^{3/2} - \frac{54\,x^{5/2}}{5}$$

input
```
int(-(x^(1/2)*(3*x - 2)^3)/(x + 1)^2,x)
```

output
```
575*atan(x^(1/2)) - (125*x^(1/2))/(x + 1) - 450*x^(1/2) + 72*x^(3/2) - (54
*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(2-3x)^3\sqrt{x}}{(1+x)^2}\,dx = \frac{2875atan(\sqrt{x})\,x+2875atan(\sqrt{x})-54\sqrt{x}\,x^3+306\sqrt{x}\,x^2-1890\sqrt{x}\,x-2875\sqrt{x}}{5x+5}$$

input
```
int((2-3*x)^3*x^(1/2)/(1+x)^2,x)
```

output
```
(2875*atan(sqrt(x))*x + 2875*atan(sqrt(x)) - 54*sqrt(x)*x**3 + 306*sqrt(x)
*x**2 - 1890*sqrt(x)*x - 2875*sqrt(x))/(5*(x + 1))
```

3.197 $\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = -\frac{2a^3(Ab-aB)\sqrt{x}}{b^5}+\frac{2a^2(Ab-aB)x^{3/2}}{3b^4} -\frac{2a(Ab-aB)x^{5/2}}{5b^3}+\frac{2(Ab-aB)x^{7/2}}{7b^2}+\frac{2Bx^{9/2}}{9b}+\frac{2a^{7/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}}$$

output
```
-2*a^3*(A*b-B*a)*x^(1/2)/b^5+2/3*a^2*(A*b-B*a)*x^(3/2)/b^4-2/5*a*(A*b-B*a)
*x^(5/2)/b^3+2/7*(A*b-B*a)*x^(7/2)/b^2+2/9*B*x^(9/2)/b+2*a^(7/2)*(A*b-B*a)
*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = \frac{2\sqrt{x}(315a^4B-105a^3b(3A+Bx)+21a^2b^2x(5A+3Bx)-9ab^3x^2(7A+5Bx)+5b}{315b^5} -\frac{2a^{7/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a + b*x),x]
```

output
```
(2*Sqrt[x]*(315*a^4*B - 105*a^3*b*(3*A + B*x) + 21*a^2*b^2*x*(5*A + 3*B*x)
 - 9*a*b^3*x^2*(7*A + 5*B*x) + 5*b^4*x^3*(9*A + 7*B*x)))/(315*b^5) - (2*a^
(7/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(11/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {90, 60, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(Ab-aB)\int \frac{x^{7/2}}{a+bx}dx}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\int \frac{x^{5/2}}{a+bx}dx}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\big\downarrow$ 60

$$\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\big\downarrow$ 73

$$\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\big\downarrow$ 218

$$\frac{(Ab-aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a + b*x),x]
```

output
```
(2*B*x^(9/2))/(9*b) + ((A*b - a*B)*((2*x^(7/2))/(7*b) - (a*((2*x^(5/2))/(5
*b) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[
b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/b))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{2\left(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}+45B\,x^{3}a\,b^{3}+63A\,x^{2}a\,b^{3}-63B\,x^{2}a^{2}b^{2}-105Ax\,a^{2}b^{2}+105Bx\,a^{3}b+315A\,a^{3}b-315B\,a^{4}\right)\sqrt{x}}{315b^{5}}$
derivativedivides	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}b^{4}}{9}-\frac{A\,b^{4}x^{\frac{7}{2}}}{7}+\frac{Ba\,b^{3}x^{\frac{7}{2}}}{7}+\frac{Aa\,b^{3}x^{\frac{5}{2}}}{5}-\frac{B\,a^{2}b^{2}x^{\frac{5}{2}}}{5}-\frac{A\,a^{2}b^{2}x^{\frac{3}{2}}}{3}+\frac{B\,a^{3}b\,x^{\frac{3}{2}}}{3}+A\,a^{3}b\sqrt{x}-B\,a^{4}\sqrt{x}\right)}{b^{5}}+\frac{2a^{4}(Ab-$
default	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}b^{4}}{9}-\frac{A\,b^{4}x^{\frac{7}{2}}}{7}+\frac{Ba\,b^{3}x^{\frac{7}{2}}}{7}+\frac{Aa\,b^{3}x^{\frac{5}{2}}}{5}-\frac{B\,a^{2}b^{2}x^{\frac{5}{2}}}{5}-\frac{A\,a^{2}b^{2}x^{\frac{3}{2}}}{3}+\frac{B\,a^{3}b\,x^{\frac{3}{2}}}{3}+A\,a^{3}b\sqrt{x}-B\,a^{4}\sqrt{x}\right)}{b^{5}}+\frac{2a^{4}(Ab-$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-35*B*b^4*x^4-45*A*b^4*x^3+45*B*a*b^3*x^3+63*A*a*b^3*x^2-63*B*a^2*
b^2*x^2-105*A*a^2*b^2*x+105*B*a^3*b*x+315*A*a^3*b-315*B*a^4)*x^(1/2)/b^5+2
*a^4*(A*b-B*a)/b^5/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.03

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = \left[-\frac{315\,(Ba^4-Aa^3b)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(35\,Bb^4x^4+315\,Ba^4-315\,Aa^3b-\cdots}{315\,b^5}\right.$$

$$-\frac{2\left(315\,(Ba^4-Aa^3b)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(35\,Bb^4x^4+315\,Ba^4-315\,Aa^3b-45\,(Bab^3-Ab^4)x^3+6\cdots\right.}{315\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/315*(315*(B*a^4 - A*a^3*b)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b
) - a)/(b*x + a)) - 2*(35*B*b^4*x^4 + 315*B*a^4 - 315*A*a^3*b - 45*(B*a*b^
3 - A*b^4)*x^3 + 63*(B*a^2*b^2 - A*a*b^3)*x^2 - 105*(B*a^3*b - A*a^2*b^2)*
x)*sqrt(x))/b^5, -2/315*(315*(B*a^4 - A*a^3*b)*sqrt(a/b)*arctan(b*sqrt(x)*
sqrt(a/b)/a) - (35*B*b^4*x^4 + 315*B*a^4 - 315*A*a^3*b - 45*(B*a*b^3 - A*b
^4)*x^3 + 63*(B*a^2*b^2 - A*a*b^3)*x^2 - 105*(B*a^3*b - A*a^2*b^2)*x)*sqrt
(x))/b^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(131) = 262.

Time = 11.84 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.41

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{9}{2}}}{9}\right) \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ \frac{Aa^4\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^5\sqrt{-\frac{a}{b}}}-\frac{Aa^4\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^5\sqrt{-\frac{a}{b}}}-\frac{2Aa^3\sqrt{x}}{b^4}+\frac{2Aa^2x^{\frac{3}{2}}}{3b^3}-\frac{2Aax^{\frac{5}{2}}}{5b^2}+\frac{2Ax^{\frac{7}{2}}}{7b}-\frac{Ba^5\log(\cdots}{b^6\cdots} \end{cases}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b*x+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(7/2)/7 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((2*A*x**(7/2)/7 + 2*B*x*
*(9/2)/9)/b, Eq(a, 0)), (A*a**4*log(sqrt(x) - sqrt(-a/b))/(b**5*sqrt(-a/b)
) - A*a**4*log(sqrt(x) + sqrt(-a/b))/(b**5*sqrt(-a/b)) - 2*A*a**3*sqrt(x)/
b**4 + 2*A*a**2*x**(3/2)/(3*b**3) - 2*A*a*x**(5/2)/(5*b**2) + 2*A*x**(7/2)
/(7*b) - B*a**5*log(sqrt(x) - sqrt(-a/b))/(b**6*sqrt(-a/b)) + B*a**5*log(s
qrt(x) + sqrt(-a/b))/(b**6*sqrt(-a/b)) + 2*B*a**4*sqrt(x)/b**5 - 2*B*a**3*
x**(3/2)/(3*b**4) + 2*B*a**2*x**(5/2)/(5*b**3) - 2*B*a*x**(7/2)/(7*b**2) +
 2*B*x**(9/2)/(9*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = -\frac{2\,(Ba^5 - Aa^4b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}$$
$$+\frac{2\left(35\,Bb^4x^{\frac{9}{2}} - 45\,(Bab^3 - Ab^4)x^{\frac{7}{2}} + 63\,(Ba^2b^2 - Aab^3)x^{\frac{5}{2}} - 105\,(Ba^3b - Aa^2b^2)x^{\frac{3}{2}} + 315\,(Ba^4 - Aa^3b)\right.}{315\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*(B*a^5 - A*a^4*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) + 2/315*(
35*B*b^4*x^(9/2) - 45*(B*a*b^3 - A*b^4)*x^(7/2) + 63*(B*a^2*b^2 - A*a*b^3)
*x^(5/2) - 105*(B*a^3*b - A*a^2*b^2)*x^(3/2) + 315*(B*a^4 - A*a^3*b)*sqrt(
x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = -\frac{2\,(Ba^5 - Aa^4b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{2\left(35\,Bb^8x^{\frac{9}{2}} - 45\,Bab^7x^{\frac{7}{2}} + 45\,Ab^8x^{\frac{7}{2}} + 63\,Ba^2b^6x^{\frac{5}{2}} - 63\,Aab^7x^{\frac{5}{2}} - 105\,Ba^3b^5x^{\frac{3}{2}} + 105\,Aa^2b^6x^{\frac{3}{2}} + 315\right.}{315\,b^9}$$

input

```
integrate(x^(7/2)*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output

```
-2*(B*a^5 - A*a^4*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) + 2/315*(
35*B*b^8*x^(9/2) - 45*B*a*b^7*x^(7/2) + 45*A*b^8*x^(7/2) + 63*B*a^2*b^6*x^
(5/2) - 63*A*a*b^7*x^(5/2) - 105*B*a^3*b^5*x^(3/2) + 105*A*a^2*b^6*x^(3/2)
 + 315*B*a^4*b^4*sqrt(x) - 315*A*a^3*b^5*sqrt(x))/b^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = x^{7/2}\left(\frac{2\,A}{7\,b} - \frac{2\,B\,a}{7\,b^2}\right) + \frac{2\,B\,x^{9/2}}{9\,b} + \frac{a^2\,x^{3/2}\,\left(\frac{2\,A}{b} - \frac{2\,B\,a}{b^2}\right)}{3\,b^2} - \frac{a^3\,\sqrt{x}\,\left(\frac{2\,A}{b} - \frac{2\,B\,a}{b^2}\right)}{b^3} - \frac{2\,a^{7/2}\,\mathrm{atan}\left(\frac{a^{7/2}\,\sqrt{b}\,\sqrt{x}\,(A\,b-B\,a)}{B\,a^5-A\,a^4\,b}\right)\,(A\,b-B\,a)}{b^{11/2}} - \frac{a\,x^{5/2}\,\left(\frac{2\,A}{b} - \frac{2\,B\,a}{b^2}\right)}{5\,b}$$

input

```
int((x^(7/2)*(A + B*x))/(a + b*x),x)
```

output

```
x^(7/2)*((2*A)/(7*b) - (2*B*a)/(7*b^2)) + (2*B*x^(9/2))/(9*b) + (a^2*x^(3/
2)*((2*A)/b - (2*B*a)/b^2))/(3*b^2) - (a^3*x^(1/2)*((2*A)/b - (2*B*a)/b^2)
)/b^3 - (2*a^(7/2)*atan((a^(7/2)*b^(1/2)*x^(1/2)*(A*b - B*a))/(B*a^5 - A*a
^4*b))*(A*b - B*a))/b^(11/2) - (a*x^(5/2)*((2*A)/b - (2*B*a)/b^2))/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.05

$$\int \frac{x^{7/2}(A+Bx)}{a+bx}\,dx = \frac{2\sqrt{x}\,x^4}{9}$$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a),x)
```

output
```
(2*sqrt(x)*x**4)/9
```

3.198 $\int \frac{x^{5/2}(A+Bx)}{a+bx} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int \frac{x^{5/2}(A+Bx)}{a+bx} \, dx = \frac{2a^2(Ab-aB)\sqrt{x}}{b^4} - \frac{2a(Ab-aB)x^{3/2}}{3b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2a^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
2*a^2*(A*b-B*a)*x^(1/2)/b^4-2/3*a*(A*b-B*a)*x^(3/2)/b^3+2/5*(A*b-B*a)*x^(5
/2)/b^2+2/7*B*x^(7/2)/b-2*a^(5/2)*(A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2)
)/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{x^{5/2}(A+Bx)}{a+bx} \, dx = \frac{2\sqrt{x}(-105a^3B + 35a^2b(3A+Bx) - 7ab^2x(5A+3Bx) + 3b^3x^2(7A+5Bx))}{105b^4} + \frac{2a^{5/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x),x]
```

output
```
(2*Sqrt[x]*(-105*a^3*B + 35*a^2*b*(3*A + B*x) - 7*a*b^2*x*(5*A + 3*B*x) +
3*b^3*x^2*(7*A + 5*B*x)))/(105*b^4) + (2*a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {90, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx$$

$$\Big\downarrow \; 90$$

$$\frac{(Ab-aB)\int \frac{x^{5/2}}{a+bx}dx}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 60$$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 60$$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \; 60$$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 73

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 218

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x),x]
```

output
```
(2*B*x^(7/2))/(7*b) + ((A*b - a*B)*((2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3
*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^
(3/2)))/b))/b))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
risch	$\frac{2\left(15 b^{3} B \,x^{3}+21 A \,x^{2} b^{3}-21 B \,x^{2} a \,b^{2}-35 A x a \,b^{2}+35 B x \,a^{2} b +105 a^{2} b A -105 a^{3} B \right) \sqrt{x}}{105 b^{4}}-\frac{2 a^{3} \left(A b -B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{b^{4} \sqrt{a b}}$
derivativedivides	$\frac{\frac{2 b^{3} B \,x^{\frac{7}{2}}}{7}+\frac{2 A \,b^{3} x^{\frac{5}{2}}}{5}-\frac{2 B a \,b^{2} x^{\frac{5}{2}}}{5}-\frac{2 A a \,b^{2} x^{\frac{3}{2}}}{3}+\frac{2 B \,a^{2} b \,x^{\frac{3}{2}}}{3}+2 a^{2} b A \sqrt{x}-2 a^{3} B \sqrt{x}}{b^{4}}-\frac{2 a^{3} \left(A b -B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{b^{4} \sqrt{a b}}$
default	$\frac{\frac{2 b^{3} B \,x^{\frac{7}{2}}}{7}+\frac{2 A \,b^{3} x^{\frac{5}{2}}}{5}-\frac{2 B a \,b^{2} x^{\frac{5}{2}}}{5}-\frac{2 A a \,b^{2} x^{\frac{3}{2}}}{3}+\frac{2 B \,a^{2} b \,x^{\frac{3}{2}}}{3}+2 a^{2} b A \sqrt{x}-2 a^{3} B \sqrt{x}}{b^{4}}-\frac{2 a^{3} \left(A b -B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{b^{4} \sqrt{a b}}$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*B*b^3*x^3+21*A*b^3*x^2-21*B*a*b^2*x^2-35*A*a*b^2*x+35*B*a^2*b*x+
105*A*a^2*b-105*B*a^3)*x^(1/2)/b^4-2*a^3*(A*b-B*a)/b^4/(a*b)^(1/2)*arctan(
b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.03

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx = \left[-\frac{105\,(Ba^3-Aa^2b)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(15\,Bb^3x^3-105\,Ba^3+105\,Aa\right.}{105\,b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/105*(105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b
) - a)/(b*x + a)) - 2*(15*B*b^3*x^3 - 105*B*a^3 + 105*A*a^2*b - 21*(B*a*b^
2 - A*b^3)*x^2 + 35*(B*a^2*b - A*a*b^2)*x)*sqrt(x))/b^4, 2/105*(105*(B*a^3
 - A*a^2*b)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (15*B*b^3*x^3 - 105*
B*a^3 + 105*A*a^2*b - 21*(B*a*b^2 - A*b^3)*x^2 + 35*(B*a^2*b - A*a*b^2)*x)
*sqrt(x))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(109) = 218$.

Time = 3.69 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.60

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{5}{2}}}{5} + \frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7} + \frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5} + \frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ -\frac{Aa^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}} + \frac{Aa^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}} + \frac{2Aa^2\sqrt{x}}{b^3} - \frac{2Aax^{\frac{3}{2}}}{3b^2} + \frac{2Ax^{\frac{5}{2}}}{5b} + \frac{Ba^4\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^5\sqrt{-\frac{a}{b}}} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b*x+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(
7/2)/7)/b, Eq(a, 0)), (-A*a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt(-a/b))
 + A*a**3*log(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*A*a**2*sqrt(x)/b
**3 - 2*A*a*x**(3/2)/(3*b**2) + 2*A*x**(5/2)/(5*b) + B*a**4*log(sqrt(x) -
sqrt(-a/b))/(b**5*sqrt(-a/b)) - B*a**4*log(sqrt(x) + sqrt(-a/b))/(b**5*sqr
t(-a/b)) - 2*B*a**3*sqrt(x)/b**4 + 2*B*a**2*x**(3/2)/(3*b**3) - 2*B*a*x**(
5/2)/(5*b**2) + 2*B*x**(7/2)/(7*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx = \frac{2\,(Ba^4 - Aa^3b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(15\,Bb^3x^{\frac{7}{2}} - 21\,(Bab^2 - Ab^3)x^{\frac{5}{2}} + 35\,(Ba^2b - Aab^2)x^{\frac{3}{2}} - 105\,(Ba^3 - Aa^2b)\sqrt{x}\right)}{105\,b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
2*(B*a^4 - A*a^3*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(1
5*B*b^3*x^(7/2) - 21*(B*a*b^2 - A*b^3)*x^(5/2) + 35*(B*a^2*b - A*a*b^2)*x^
(3/2) - 105*(B*a^3 - A*a^2*b)*sqrt(x))/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx = \frac{2\left(Ba^4-Aa^3b\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(15\,Bb^6x^{\frac{7}{2}}-21\,Bab^5x^{\frac{5}{2}}+21\,Ab^6x^{\frac{5}{2}}+35\,Ba^2b^4x^{\frac{3}{2}}-35\,Aab^5x^{\frac{3}{2}}-105\,Ba^3b^3\sqrt{x}+105\,Aa^2b^4\sqrt{x}\right)}{105\,b^7}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
2*(B*a^4 - A*a^3*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(1
5*B*b^6*x^(7/2) - 21*B*a*b^5*x^(5/2) + 21*A*b^6*x^(5/2) + 35*B*a^2*b^4*x^(
3/2) - 35*A*a*b^5*x^(3/2) - 105*B*a^3*b^3*sqrt(x) + 105*A*a^2*b^4*sqrt(x))
/b^7
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx = x^{5/2}\left(\frac{2\,A}{5\,b}-\frac{2\,B\,a}{5\,b^2}\right)+\frac{2\,B\,x^{7/2}}{7\,b}+\frac{a^2\,\sqrt{x}\,\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)}{b^2} + \frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,\sqrt{x}\,(A\,b-B\,a)}{B\,a^4-A\,a^3\,b}\right)\,(A\,b-B\,a)}{b^{9/2}}-\frac{a\,x^{3/2}\,\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)}{3\,b}$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x),x)
```

output
```
x^(5/2)*((2*A)/(5*b) - (2*B*a)/(5*b^2)) + (2*B*x^(7/2))/(7*b) + (a^2*x^(1/
2)*((2*A)/b - (2*B*a)/b^2))/b^2 + (2*a^(5/2)*atan((a^(5/2)*b^(1/2)*x^(1/2)
*(A*b - B*a))/(B*a^4 - A*a^3*b))*(A*b - B*a))/b^(9/2) - (a*x^(3/2)*((2*A)/
b - (2*B*a)/b^2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.06

$$\int \frac{x^{5/2}(A+Bx)}{a+bx}\,dx=\frac{2\sqrt{x}\,x^3}{7}$$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a),x)
```

output
```
(2*sqrt(x)*x**3)/7
```

3.199 $\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = -\frac{2a(Ab-aB)\sqrt{x}}{b^3}+\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{5/2}}{5b}+\frac{2a^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
-2*a*(A*b-B*a)*x^(1/2)/b^3+2/3*(A*b-B*a)*x^(3/2)/b^2+2/5*B*x^(5/2)/b+2*a^(
3/2)*(A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = \frac{2\sqrt{x}(15a^2B-5ab(3A+Bx)+b^2x(5A+3Bx))}{15b^3}-\frac{2a^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x),x]
```

output
```
(2*Sqrt[x]*(15*a^2*B - 5*a*b*(3*A + B*x) + b^2*x*(5*A + 3*B*x)))/(15*b^3)
- (2*a^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {90, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx$$

$$\Big\downarrow \; 90$$

$$\frac{(Ab-aB)\int \frac{x^{3/2}}{a+bx}dx}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 73$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \; 218$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x),x]
```

output
```
(2*B*x^(5/2))/(5*b) + ((A*b - a*B)*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b
- (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{2\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+5Babx+15abA-15a^{2}B\right)\sqrt{x}}{15b^{3}}+\frac{2a^{2}\left(Ab-Ba\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}$	76
derivativedivides	$-\frac{2\left(-\frac{b^{2}B\,x^{\frac{5}{2}}}{5}-\frac{A\,b^{2}x^{\frac{3}{2}}}{3}+\frac{Bab\,x^{\frac{3}{2}}}{3}+abA\sqrt{x}-a^{2}B\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(Ab-Ba\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}$	82
default	$-\frac{2\left(-\frac{b^{2}B\,x^{\frac{5}{2}}}{5}-\frac{A\,b^{2}x^{\frac{3}{2}}}{3}+\frac{Bab\,x^{\frac{3}{2}}}{3}+abA\sqrt{x}-a^{2}B\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(Ab-Ba\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}$	82

input
```
int(x^(3/2)*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+5*B*a*b*x+15*A*a*b-15*B*a^2)*x^(1/2)/b^3+2*a
^2*(A*b-B*a)/b^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = \left[-\frac{15\,(Ba^{2}-Aab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(3\,Bb^{2}x^{2}+15\,Ba^{2}-15\,Aab-5\right.$$

$$\left.-\frac{2\left(15\,(Ba^{2}-Aab)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,Bb^{2}x^{2}+15\,Ba^{2}-15\,Aab-5\,(Bab-Ab^{2})x)\sqrt{x}\right)}{15\,b^{3}}\right]$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/15*(15*(B*a^2 - A*a*b)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) -
a)/(b*x + a)) - 2*(3*B*b^2*x^2 + 15*B*a^2 - 15*A*a*b - 5*(B*a*b - A*b^2)*x
)*sqrt(x))/b^3, -2/15*(15*(B*a^2 - A*a*b)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(
a/b)/a) - (3*B*b^2*x^2 + 15*B*a^2 - 15*A*a*b - 5*(B*a*b - A*b^2)*x)*sqrt(x
))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(87) = 174$.

Time = 1.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.89

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3} + \frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5} + \frac{2Bx^{\frac{7}{2}}}{7}}{a} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3} + \frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{Aa^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}} - \frac{Aa^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}} - \frac{2Aa\sqrt{x}}{b^2} + \frac{2Ax^{\frac{3}{2}}}{3b} - \frac{Ba^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}} + \frac{Ba^3\log\left(\right.}{b^4} \end{cases}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b*x+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(5/2)/5 + 2*B*x**(7/2)/7)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(
5/2)/5)/b, Eq(a, 0)), (A*a**2*log(sqrt(x) - sqrt(-a/b))/(b**3*sqrt(-a/b))
- A*a**2*log(sqrt(x) + sqrt(-a/b))/(b**3*sqrt(-a/b)) - 2*A*a*sqrt(x)/b**2
+ 2*A*x**(3/2)/(3*b) - B*a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt(-a/b))
+ B*a**3*log(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*B*a**2*sqrt(x)/b*
*3 - 2*B*a*x**(3/2)/(3*b**2) + 2*B*x**(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = -\frac{2\,(Ba^3-Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,Bb^2x^{\frac{5}{2}}-5\,(Bab-Ab^2)x^{\frac{3}{2}}+15\,(Ba^2-Aab)\sqrt{x}\right)}{15\,b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*(B*a^3 - A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3
*B*b^2*x^(5/2) - 5*(B*a*b - A*b^2)*x^(3/2) + 15*(B*a^2 - A*a*b)*sqrt(x))/b
^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = -\frac{2\,(Ba^3-Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(3\,Bb^4x^{\frac{5}{2}}-5\,Bab^3x^{\frac{3}{2}}+5\,Ab^4x^{\frac{3}{2}}+15\,Ba^2b^2\sqrt{x}-15\,Aab^3\sqrt{x}\right)}{15\,b^5}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
-2*(B*a^3 - A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3
*B*b^4*x^(5/2) - 5*B*a*b^3*x^(3/2) + 5*A*b^4*x^(3/2) + 15*B*a^2*b^2*sqrt(x
) - 15*A*a*b^3*sqrt(x))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{5/2}}{5\,b} - \frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,\sqrt{x}\,(A\,b-B\,a)}{B\,a^3-A\,a^2\,b}\right)\,(A\,b-B\,a)}{b^{7/2}}-\frac{a\,\sqrt{x}\,\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)}{b}$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x),x)
```

output
```
x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(5/2))/(5*b) - (2*a^(3/2)
*atan((a^(3/2)*b^(1/2)*x^(1/2)*(A*b - B*a))/(B*a^3 - A*a^2*b))*(A*b - B*a)
)/b^(7/2) - (a*x^(1/2)*((2*A)/b - (2*B*a)/b^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.08

$$\int \frac{x^{3/2}(A+Bx)}{a+bx}\,dx = \frac{2\sqrt{x}\,x^2}{5}$$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a),x)
```

output
```
(2*sqrt(x)*x**2)/5
```

3.200 $\int \frac{\sqrt{x}(A+Bx)}{a+bx} dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx=\frac{2(Ab-aB)\sqrt{x}}{b^2}+\frac{2Bx^{3/2}}{3b}-\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
2*(A*b-B*a)*x^(1/2)/b^2+2/3*B*x^(3/2)/b-2*a^(1/2)*(A*b-B*a)*arctan(b^(1/2)
*x^(1/2)/a^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx=\frac{2\sqrt{x}(3Ab-3aB+bBx)}{3b^2}+\frac{2\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x),x]
```

output
```
(2*Sqrt[x]*(3*A*b - 3*a*B + b*B*x))/(3*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {90, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{x}}{a+bx}dx}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{73}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{218}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x),x]
```

output
```
(2*B*x^(3/2))/(3*b) + ((A*b - a*B)*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2(bBx+3Ab-3Ba)\sqrt{x}}{3b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2bB\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$	58
default	$\frac{\frac{2bB\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$	58

input
```
int(x^(1/2)*(B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-3*B*a)*x^(1/2)/b^2-2*a*(A*b-B*a)/b^2/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx = \left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(Bbx-3\,Ba+3\,Ab)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,(Ba-Ab)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\cdots}{\cdots}\right.\right.}{\cdots}\right.$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/3*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*
x + a)) - 2*(B*b*x - 3*B*a + 3*A*b)*sqrt(x))/b^2, 2/3*(3*(B*a - A*b)*sqrt(
a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (B*b*x - 3*B*a + 3*A*b)*sqrt(x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(65) = 130$.

Time = 0.79 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x} + \frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ -\frac{Aa\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{Aa\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{2A\sqrt{x}}{b} + \frac{Ba^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}} - \frac{Ba^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}} - \frac{2Ba\sqrt{x}}{b^2} + \frac{2Bx^{\frac{3}{2}}}{3b} \end{cases}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b*x+a),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(3/2)/3 + 2*B*x**(5/2)/5)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(3/2)/3
)/b, Eq(a, 0)), (-A*a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b)) + A*a*lo
g(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*A*sqrt(x)/b + B*a**2*log(sqr
t(x) - sqrt(-a/b))/(b**3*sqrt(-a/b)) - B*a**2*log(sqrt(x) + sqrt(-a/b))/(b
**3*sqrt(-a/b)) - 2*B*a*sqrt(x)/b**2 + 2*B*x**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(Bbx^{\frac{3}{2}}-3\,(Ba-Ab)\sqrt{x}\right)}{3\,b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
2*(B*a^2 - A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(B*b*x
^(3/2) - 3*(B*a - A*b)*sqrt(x))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(Bb^2x^{\frac{3}{2}}-3\,Bab\sqrt{x}+3\,Ab^2\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
2*(B*a^2 - A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(B*b^2
*x^(3/2) - 3*B*a*b*sqrt(x) + 3*A*b^2*sqrt(x))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx = \sqrt{x}\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)+\frac{2\,B\,x^{3/2}}{3\,b} + \frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(A\,b-B\,a)}{B\,a^2-A\,a\,b}\right)\,(A\,b-B\,a)}{b^{5/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x),x)
```

output
```
x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(3/2))/(3*b) + (2*a^(1/2)*atan((a
^(1/2)*b^(1/2)*x^(1/2)*(A*b - B*a))/(B*a^2 - A*a*b))*(A*b - B*a))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx}\,dx = \frac{2\sqrt{x}\,x}{3}$$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a),x)
```

output
```
(2*sqrt(x)*x)/3
```

3.201 $\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx=\frac{2B\sqrt{x}}{b}+\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
2*B*x^(1/2)/b+2*(A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx=\frac{2B\sqrt{x}}{b}-\frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x)),x]
```

output
```
(2*B*Sqrt[x])/b - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sq
rt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {90, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}+\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2(Ab-aB)\int \frac{1}{a+bx}d\sqrt{x}}{b}+\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{2B\sqrt{x}}{b}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x)),x]
```

output
```
(2*B*Sqrt[x])/b + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[
a]*b^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	40
default	$\frac{2B\sqrt{x}}{b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	40
risch	$\frac{2B\sqrt{x}}{b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	40

input
```
int((B*x+A)/x^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b+2*(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx = \left[\frac{2\,Bab\sqrt{x}+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab^2}, \frac{2\left(Bab\sqrt{x}+(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)\right)}{ab^2}\right]$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[(2*B*a*b*sqrt(x) + (B*a - A*b)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^2), 2*(B*a*b*sqrt(x) + (B*a - A*b)*sqrt(a*b)*arctan
(sqrt(a*b)/(b*sqrt(x))))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(46) = 92$.

Time $= 0.50$ (sec) , antiderivative size $= 180$, normalized size of antiderivative $= 3.67$

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+2B\sqrt{x}\right) & \text{for } a=0 \wedge b=0 \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} & \text{for } b=0 \\ \frac{-\frac{2A}{\sqrt{x}}+2B\sqrt{x}}{b} & \text{for } a=0 \\ \frac{A\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{A\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{Ba\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}}+\frac{Ba\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}}+\frac{2B\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/x**(1/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2*A*s
qrt(x) + 2*B*x**(3/2)/3)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))/b, Eq
(a, 0)), (A*log(sqrt(x) - sqrt(-a/b))/(b*sqrt(-a/b)) - A*log(sqrt(x) + sqr
t(-a/b))/(b*sqrt(-a/b)) - B*a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b))
+ B*a*log(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*B*sqrt(x)/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx = \frac{2\,B\sqrt{x}}{b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*B*sqrt(x)/b - 2*(B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx = \frac{2\,B\sqrt{x}}{b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b - 2*(B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx = \frac{2\,B\,\sqrt{x}}{b} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b-B\,a)}{\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x)),x)
```

output
```
(2*B*x^(1/2))/b + (2*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b - B*a))/(a^(1/2)
*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.08

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)}\,dx = 2\sqrt{x}$$

input
```
int((B*x+A)/x^(1/2)/(b*x+a),x)
```

output
```
2*sqrt(x)
```

3.202 $\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx=-\frac{2A}{a\sqrt{x}}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

output
```
-2*A/a/x^(1/2)-2*(A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx=-\frac{2A}{a\sqrt{x}}+\frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x)),x]
```

output
```
(-2*A)/(a*Sqrt[x]) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/
(a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2(Ab-aB)\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{2A}{a\sqrt{x}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x)),x]
```

output
```
(-2*A)/(a*Sqrt[x]) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(a^
(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2A}{a\sqrt{x}}+\frac{2(-Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	40
default	$-\frac{2A}{a\sqrt{x}}+\frac{2(-Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	40
risch	$-\frac{2A}{a\sqrt{x}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	40

input
```
int((B*x+A)/x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*A/a/x^(1/2)+2*(-A*b+B*a)/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx = \left[-\frac{2\,Aab\sqrt{x}-(Ba-Ab)\sqrt{-ab}x\log\left(\frac{bx-a+2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{a^2bx},\right.$$
$$\left.-\frac{2\left(Aab\sqrt{x}+(Ba-Ab)\sqrt{ab}x\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)\right)}{a^2bx}\right]$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[-(2*A*a*b*sqrt(x) - (B*a - A*b)*sqrt(-a*b)*x*log((b*x - a + 2*sqrt(-a*b)*
sqrt(x))/(b*x + a)))/(a^2*b*x), -2*(A*a*b*sqrt(x) + (B*a - A*b)*sqrt(a*b)*
x*arctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(48) = 96$.

Time = 0.90 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.63

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}} - \frac{2B}{\sqrt{x}}\right) & \text{for } a \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}} - \frac{2B}{\sqrt{x}}}{b} & \text{for } a \\ \frac{-\frac{2A}{\sqrt{x}} + 2B\sqrt{x}}{a} & \text{for } b \\ -\frac{A\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}} + \frac{A\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}} - \frac{2A}{a\sqrt{x}} + \frac{B\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{B\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} & \text{othe} \end{cases}$$

input
```
integrate((B*x+A)/x**(3/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x
))/a, Eq(b, 0)), (-A*log(sqrt(x) - sqrt(-a/b))/(a*sqrt(-a/b)) + A*log(sqrt
(x) + sqrt(-a/b))/(a*sqrt(-a/b)) - 2*A/(a*sqrt(x)) + B*log(sqrt(x) - sqrt(
-a/b))/(b*sqrt(-a/b)) - B*log(sqrt(x) + sqrt(-a/b))/(b*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx = \frac{2\,(Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2\,A}{a\sqrt{x}}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*(B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2*A/(a*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx = \frac{2\,(Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2\,A}{a\sqrt{x}}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*(B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2*A/(a*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx = \frac{2\,B\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\,A}{a\,\sqrt{x}}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x)),x)
```

output
```
(2*B*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2)) - (2*A*b^(1/2)*ata
n((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2) - (2*A)/(a*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx}{x^{3/2}(a+bx)}\,dx=-\frac{2}{\sqrt{x}}$$

input
```
int((B*x+A)/x^(3/2)/(b*x+a),x)
```

output
```
( - 2)/sqrt(x)
```

3.203 $\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx=-\frac{2A}{3ax^{3/2}}+\frac{2(Ab-aB)}{a^2\sqrt{x}}+\frac{2\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-2/3*A/a/x^(3/2)+2*(A*b-B*a)/a^2/x^(1/2)+2*b^(1/2)*(A*b-B*a)*arctan(b^(1/2
)*x^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx=-\frac{2(aA-3Abx+3aBx)}{3a^2x^{3/2}}-\frac{2\sqrt{b}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x)),x]
```

output
```
(-2*(a*A - 3*A*b*x + 3*a*B*x))/(3*a^2*x^(3/2)) - (2*Sqrt[b]*(-(A*b) + a*B)
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{(Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{(Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)) - ((A*b - a*B)*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2(-3Abx+3Bax+Aa)}{3a^2x^{\frac{3}{2}}}+\frac{2(Ab-Ba)b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	54
derivativedivides	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(-Ab+Ba)}{a^2\sqrt{x}}+\frac{2(Ab-Ba)b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	57
default	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(-Ab+Ba)}{a^2\sqrt{x}}+\frac{2(Ab-Ba)b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	57

input
```
int((B*x+A)/x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*A*b*x+3*B*a*x+A*a)/a^2/x^(3/2)+2*(A*b-B*a)/a^2*b/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx = \left[-\frac{3\,(Ba-Ab)x^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(Aa+3\,(Ba-Ab)x)\sqrt{x}}{3\,a^2x^2},\right.$$
$$\left.-\frac{2\left(3\,(Ba-Ab)x^2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(Aa+3\,(Ba-Ab)x)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/3*(3*(B*a - A*b)*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)
/(b*x + a)) + 2*(A*a + 3*(B*a - A*b)*x)*sqrt(x))/(a^2*x^2), -2/3*(3*(B*a -
 A*b)*x^2*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (A*a + 3*(B*a - A*b)*x)*sq
rt(x))/(a^2*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(65) = 130$.

Time = 1.99 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.16

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}}{a} \\ -\frac{2A}{3ax^{\frac{3}{2}}}+\frac{Ab\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}}-\frac{Ab\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}}+\frac{2Ab}{a^2\sqrt{x}}-\frac{B\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}}+\frac{B\log\left(\sqrt{x}+\sqrt{-\right.}}{a\sqrt{-\frac{a}{b}}} \end{cases}$$

input
```
integrate((B*x+A)/x**(5/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2
)) - 2*B/sqrt(x))/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + A*b*log(sqrt(x) - s
qrt(-a/b))/(a**2*sqrt(-a/b)) - A*b*log(sqrt(x) + sqrt(-a/b))/(a**2*sqrt(-a
/b)) + 2*A*b/(a**2*sqrt(x)) - B*log(sqrt(x) - sqrt(-a/b))/(a*sqrt(-a/b)) +
 B*log(sqrt(x) + sqrt(-a/b))/(a*sqrt(-a/b)) - 2*B/(a*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx = -\frac{2\,(Bab-Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}-\frac{2\,(Aa+3\,(Ba-Ab)x)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*(B*a*b - A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/3*(A*a
+ 3*(B*a - A*b)*x)/(a^2*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx = -\frac{2\,(Bab-Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{2\,(3\,Bax-3\,Abx+Aa)}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a),x, algorithm="giac")
```

output
```
-2*(B*a*b - A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/3*(3*B*
a*x - 3*A*b*x + A*a)/(a^2*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx = \frac{2\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\,(A\,b-B\,a)}{a^{5/2}} - \frac{\frac{2\,A}{3\,a}-\frac{2\,x\,(A\,b-B\,a)}{a^2}}{x^{3/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x)),x)
```

output
```
(2*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b - B*a))/a^(5/2) - ((2*A)/(
3*a) - (2*x*(A*b - B*a))/a^2)/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx}{x^{5/2}(a+bx)}\,dx = -\frac{2}{3\sqrt{x}\,x}$$

input
```
int((B*x+A)/x^(5/2)/(b*x+a),x)
```

output
```
( - 2)/(3*sqrt(x)*x)
```

3.204 $\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx=-\frac{2A}{5ax^{5/2}}+\frac{2(Ab-aB)}{3a^2x^{3/2}}-\frac{2b(Ab-aB)}{a^3\sqrt{x}}-\frac{2b^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-2/5*A/a/x^(5/2)+2/3*(A*b-B*a)/a^2/x^(3/2)-2*b*(A*b-B*a)/a^3/x^(1/2)-2*b^(
3/2)*(A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx=-\frac{2(15Ab^2x^2-5abx(A+3Bx)+a^2(3A+5Bx))}{15a^3x^{5/2}}+\frac{2b^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a + b*x)),x]
```

output
```
(-2*(15*A*b^2*x^2 - 5*a*b*x*(A + 3*B*x) + a^2*(3*A + 5*B*x)))/(15*a^3*x^(5
/2)) + (2*b^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a + b*x)),x]
```

output
```
(-2*A)/(5*a*x^(5/2)) - ((A*b - a*B)*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x])
 - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{3a^2x^{\frac{3}{2}}}-\frac{2b(Ab-Ba)}{a^3\sqrt{x}}-\frac{2b^2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	76
default	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{3a^2x^{\frac{3}{2}}}-\frac{2b(Ab-Ba)}{a^3\sqrt{x}}-\frac{2b^2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	76
risch	$-\frac{2(15A\,b^2x^2-15Bab\,x^2-5aAbx+5B\,a^2x+3a^2A)}{15a^3x^{\frac{5}{2}}}-\frac{2b^2(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	79

input
```
int((B*x+A)/x^(7/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a/x^(5/2)-2/3*(-A*b+B*a)/a^2/x^(3/2)-2*b*(A*b-B*a)/a^3/x^(1/2)-2*b^
2*(A*b-B*a)/a^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx = \left[-\frac{15\,(Bab-Ab^2)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,Aa^2-15\,(Bab-Ab^2)x^2+5}{15\,a^3x^3}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/15*(15*(B*a*b - A*b^2)*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a
) - a)/(b*x + a)) + 2*(3*A*a^2 - 15*(B*a*b - A*b^2)*x^2 + 5*(B*a^2 - A*a*b
)*x)*sqrt(x))/(a^3*x^3), 2/15*(15*(B*a*b - A*b^2)*x^3*sqrt(b/a)*arctan(sqr
t(x)*sqrt(b/a)) - (3*A*a^2 - 15*(B*a*b - A*b^2)*x^2 + 5*(B*a^2 - A*a*b)*x)
*sqrt(x))/(a^3*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(87) = 174$.

Time = 5.83 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.91

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}}{b} \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{a} \\ -\frac{2A}{5ax^{\frac{5}{2}}}+\frac{2Ab}{3a^2x^{\frac{3}{2}}}-\frac{Ab^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}}+\frac{Ab^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}}-\frac{2Ab^2}{a^3\sqrt{x}}-\frac{2B}{3ax^{\frac{3}{2}}}+\frac{Bb\log\left(\sqrt{x}-\sqrt{-\right.}}{a^2\sqrt{-\frac{a}{b}}} \end{cases}$$

input
```
integrate((B*x+A)/x**(7/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2)))/b, Eq(a, 0)), ((-2*A/(5*x**(5/2
)) - 2*B/(3*x**(3/2)))/a, Eq(b, 0)), (-2*A/(5*a*x**(5/2)) + 2*A*b/(3*a**2*
x**(3/2)) - A*b**2*log(sqrt(x) - sqrt(-a/b))/(a**3*sqrt(-a/b)) + A*b**2*lo
g(sqrt(x) + sqrt(-a/b))/(a**3*sqrt(-a/b)) - 2*A*b**2/(a**3*sqrt(x)) - 2*B/
(3*a*x**(3/2)) + B*b*log(sqrt(x) - sqrt(-a/b))/(a**2*sqrt(-a/b)) - B*b*log
(sqrt(x) + sqrt(-a/b))/(a**2*sqrt(-a/b)) + 2*B*b/(a**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx = \frac{2\,(Bab^2-Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\,(3\,Aa^2-15\,(Bab-Ab^2)x^2+5\,(Ba^2-Aab)x)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*(B*a*b^2 - A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(3*
A*a^2 - 15*(B*a*b - A*b^2)*x^2 + 5*(B*a^2 - A*a*b)*x)/(a^3*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx = \frac{2\,(Bab^2-Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{2\,(15\,Babx^2-15\,Ab^2x^2-5\,Ba^2x+5\,Aabx-3\,Aa^2)}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*(B*a*b^2 - A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + 2/15*(15
*B*a*b*x^2 - 15*A*b^2*x^2 - 5*B*a^2*x + 5*A*a*b*x - 3*A*a^2)/(a^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx = -\frac{\frac{2A}{5a}-\frac{2x(Ab-Ba)}{3a^2}+\frac{2bx^2(Ab-Ba)}{a^3}}{x^{5/2}}-\frac{2b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)(Ab-Ba)}{a^{7/2}}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x)),x)
```

output
```
- ((2*A)/(5*a) - (2*x*(A*b - B*a))/(3*a^2) + (2*b*x^2*(A*b - B*a))/a^3)/x^
(5/2) - (2*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b - B*a))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx}{x^{7/2}(a+bx)}\,dx = -\frac{2}{5\sqrt{x}\,x^2}$$

input
```
int((B*x+A)/x^(7/2)/(b*x+a),x)
```

output
```
( - 2)/(5*sqrt(x)*x**2)
```

3.205 $\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = -\frac{2A}{7ax^{7/2}}+\frac{2(Ab-aB)}{5a^2x^{5/2}}-\frac{2b(Ab-aB)}{3a^3x^{3/2}}+\frac{2b^2(Ab-aB)}{a^4\sqrt{x}}+\frac{2b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-2/7*A/a/x^(7/2)+2/5*(A*b-B*a)/a^2/x^(5/2)-2/3*b*(A*b-B*a)/a^3/x^(3/2)+2*b
^2*(A*b-B*a)/a^4/x^(1/2)+2*b^(5/2)*(A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2
))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = \frac{210Ab^3x^3-70ab^2x^2(A+3Bx)+14a^2bx(3A+5Bx)-6a^3(5A+7Bx)}{105a^4x^{7/2}}+\frac{2b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*(a + b*x)),x]
```

output
```
(210*A*b^3*x^3 - 70*a*b^2*x^2*(A + 3*B*x) + 14*a^2*b*x*(3*A + 5*B*x) - 6*a
^3*(5*A + 7*B*x))/(105*a^4*x^(7/2)) + (2*b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[
b]*Sqrt[x])/Sqrt[a]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^{7/2}(a+bx)}dx}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \; 218$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2A}{7ax^{7/2}}$$

input
```
Int[(A + B*x)/(x^(9/2)*(a + b*x)),x]
```

output
```
(-2*A)/(7*a*x^(7/2)) - ((A*b - a*B)*(-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2
)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^
(3/2)))/a))/a))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2(Ab-Ba)b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}-\frac{2A}{7a\,x^{\frac{7}{2}}}-\frac{2(-Ab+Ba)}{5a^2x^{\frac{5}{2}}}-\frac{2b(Ab-Ba)}{3a^3x^{\frac{3}{2}}}+\frac{2b^2(Ab-Ba)}{a^4\sqrt{x}}$
default	$\frac{2(Ab-Ba)b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}-\frac{2A}{7a\,x^{\frac{7}{2}}}-\frac{2(-Ab+Ba)}{5a^2x^{\frac{5}{2}}}-\frac{2b(Ab-Ba)}{3a^3x^{\frac{3}{2}}}+\frac{2b^2(Ab-Ba)}{a^4\sqrt{x}}$
risch	$-\frac{2\left(-105A\,b^3x^3+105Ba\,b^2x^3+35aA\,b^2x^2-35B\,a^2b\,x^2-21a^2Abx+21B\,a^3x+15a^3A\right)}{105a^4x^{\frac{7}{2}}}+\frac{2(Ab-Ba)b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}$

input
```
int((B*x+A)/x^(9/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(A*b-B*a)/a^4*b^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))-2/7*A/a/x^(7
/2)-2/5*(-A*b+B*a)/a^2/x^(5/2)-2/3*b*(A*b-B*a)/a^3/x^(3/2)+2*b^2*(A*b-B*a)
/a^4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = \left[-\frac{105\,(Bab^2-Ab^3)x^4\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,Aa^3+105\,(Bab^2-Ab^3)x\right.$$

$$-\frac{2\left(105\,(Bab^2-Ab^3)x^4\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(15\,Aa^3+105\,(Bab^2-Ab^3)x^3-35\,(Ba^2b-Aab^2)x^2+2\right.}{105\,a^4x^4}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/105*(105*(B*a*b^2 - A*b^3)*x^4*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(
-b/a) - a)/(b*x + a)) + 2*(15*A*a^3 + 105*(B*a*b^2 - A*b^3)*x^3 - 35*(B*a^
2*b - A*a*b^2)*x^2 + 21*(B*a^3 - A*a^2*b)*x)*sqrt(x))/(a^4*x^4), -2/105*(1
05*(B*a*b^2 - A*b^3)*x^4*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (15*A*a^3 +
 105*(B*a*b^2 - A*b^3)*x^3 - 35*(B*a^2*b - A*a*b^2)*x^2 + 21*(B*a^3 - A*a^
2*b)*x)*sqrt(x))/(a^4*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(109) = 218$.

Time = 16.47 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{b} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}}{a} \\ -\frac{2A}{7ax^{\frac{7}{2}}}+\frac{2Ab}{5a^2x^{\frac{5}{2}}}-\frac{2Ab^2}{3a^3x^{\frac{3}{2}}}+\frac{Ab^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a^4\sqrt{-\frac{a}{b}}}-\frac{Ab^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a^4\sqrt{-\frac{a}{b}}}+\frac{2Ab^3}{a^4\sqrt{x}}-\frac{2B}{5ax^{\frac{5}{2}}}+\frac{2B}{3a^2x} \end{cases}$$

input
```
integrate((B*x+A)/x**(9/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2)))/b, Eq(a, 0)), ((-2*A/(7*x**(7/2
)) - 2*B/(5*x**(5/2)))/a, Eq(b, 0)), (-2*A/(7*a*x**(7/2)) + 2*A*b/(5*a**2*
x**(5/2)) - 2*A*b**2/(3*a**3*x**(3/2)) + A*b**3*log(sqrt(x) - sqrt(-a/b))/
(a**4*sqrt(-a/b)) - A*b**3*log(sqrt(x) + sqrt(-a/b))/(a**4*sqrt(-a/b)) + 2
*A*b**3/(a**4*sqrt(x)) - 2*B/(5*a*x**(5/2)) + 2*B*b/(3*a**2*x**(3/2)) - B*
b**2*log(sqrt(x) - sqrt(-a/b))/(a**3*sqrt(-a/b)) + B*b**2*log(sqrt(x) + sq
rt(-a/b))/(a**3*sqrt(-a/b)) - 2*B*b**2/(a**3*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = -\frac{2\left(Bab^3-Ab^4\right)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{2\left(15\,Aa^3+105\,(Bab^2-Ab^3)x^3-35\,(Ba^2b-Aab^2)x^2+21\,(Ba^3-Aa^2b)x\right)}{105\,a^4x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-2*(B*a*b^3 - A*b^4)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - 2/105*(
15*A*a^3 + 105*(B*a*b^2 - A*b^3)*x^3 - 35*(B*a^2*b - A*a*b^2)*x^2 + 21*(B*
a^3 - A*a^2*b)*x)/(a^4*x^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = -\frac{2\,(Bab^3-Ab^4)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{2\,(105\,Bab^2x^3-105\,Ab^3x^3-35\,Ba^2bx^2+35\,Aab^2x^2+21\,Ba^3x-21\,Aa^2bx+15\,Aa^3)}{105\,a^4x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a),x, algorithm="giac")
```

output
```
-2*(B*a*b^3 - A*b^4)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - 2/105*(
105*B*a*b^2*x^3 - 105*A*b^3*x^3 - 35*B*a^2*b*x^2 + 35*A*a*b^2*x^2 + 21*B*a
^3*x - 21*A*a^2*b*x + 15*A*a^3)/(a^4*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx = \frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{9/2}} - \frac{\frac{2\,A}{7\,a}-\frac{2\,x\,\left(A\,b-B\,a\right)}{5\,a^2}-\frac{2\,b^2\,x^3\,\left(A\,b-B\,a\right)}{a^4}+\frac{2\,b\,x^2\,\left(A\,b-B\,a\right)}{3\,a^3}}{x^{7/2}}$$

input
```
int((A + B*x)/(x^(9/2)*(a + b*x)),x)
```

output
```
(2*b^(5/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b - B*a))/a^(9/2) - ((2*A)/(
7*a) - (2*x*(A*b - B*a))/(5*a^2) - (2*b^2*x^3*(A*b - B*a))/a^4 + (2*b*x^2*
(A*b - B*a))/(3*a^3))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.08

$$\int \frac{A+Bx}{x^{9/2}(a+bx)}\,dx=-\frac{2}{7\sqrt{x}\,x^3}$$

input
```
int((B*x+A)/x^(9/2)/(b*x+a),x)
```

output
```
( - 2)/(7*sqrt(x)*x**3)
```

3.206 $\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx=-\frac{2A}{9ax^{9/2}}+\frac{2(Ab-aB)}{7a^2x^{7/2}}-\frac{2b(Ab-aB)}{5a^3x^{5/2}}+\frac{2b^2(Ab-aB)}{3a^4x^{3/2}}-\frac{2b^3(Ab-aB)}{a^5\sqrt{x}}-\frac{2b^{7/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{11/2}}$$

output
```
-2/9*A/a/x^(9/2)+2/7*(A*b-B*a)/a^2/x^(7/2)-2/5*b*(A*b-B*a)/a^3/x^(5/2)+2/3
*b^2*(A*b-B*a)/a^4/x^(3/2)-2*b^3*(A*b-B*a)/a^5/x^(1/2)-2*b^(7/2)*(A*b-B*a)
*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx=-\frac{2(315Ab^4x^4-105ab^3x^3(A+3Bx)+21a^2b^2x^2(3A+5Bx)-9a^3bx(5A+7Bx)+5a^4(7A+9Bx))}{315a^5x^{9/2}}+\frac{2b^{7/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(A + B*x)/(x^(11/2)*(a + b*x)),x]
```

output
```
(-2*(315*A*b^4*x^4 - 105*a*b^3*x^3*(A + 3*B*x) + 21*a^2*b^2*x^2*(3*A + 5*B
*x) - 9*a^3*b*x*(5*A + 7*B*x) + 5*a^4*(7*A + 9*B*x)))/(315*a^5*x^(9/2)) +
(2*b^(7/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(11/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {87, 61, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^{9/2}(a+bx)}dx}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^{7/2}(a+bx)}dx}{a}-\frac{2}{7ax^{7/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$\downarrow$ 61

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$\downarrow$ 73

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

$\downarrow$ 218

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{a}-\frac{2A}{9ax^{9/2}}$$

input
```
Int[(A + B*x)/(x^(11/2)*(a + b*x)),x]
```

output
```
(-2*A)/(9*a*x^(9/2)) - ((A*b - a*B)*(-2/(7*a*x^(7/2)) - (b*(-2/(5*a*x^(5/2
)) - (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b
]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a))/a))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{2A}{9a\,x^{\frac{9}{2}}}-\frac{2(-Ab+Ba)}{7a^{2}x^{\frac{7}{2}}}-\frac{2b^{3}(Ab-Ba)}{a^{5}\sqrt{x}}-\frac{2b(Ab-Ba)}{5a^{3}x^{\frac{5}{2}}}+\frac{2b^{2}(Ab-Ba)}{3a^{4}x^{\frac{3}{2}}}-\frac{2b^{4}(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{5}\sqrt{ab}}$
default	$-\frac{2A}{9a\,x^{\frac{9}{2}}}-\frac{2(-Ab+Ba)}{7a^{2}x^{\frac{7}{2}}}-\frac{2b^{3}(Ab-Ba)}{a^{5}\sqrt{x}}-\frac{2b(Ab-Ba)}{5a^{3}x^{\frac{5}{2}}}+\frac{2b^{2}(Ab-Ba)}{3a^{4}x^{\frac{3}{2}}}-\frac{2b^{4}(Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{5}\sqrt{ab}}$
risch	$-\frac{2\left(315A\,b^{4}x^{4}-315Ba\,b^{3}x^{4}-105Aa\,b^{3}x^{3}+105B\,a^{2}b^{2}x^{3}+63A\,a^{2}b^{2}x^{2}-63B\,a^{3}b\,x^{2}-45A\,a^{3}bx+45B\,a^{4}x+35A\,a^{4}\right)}{315a^{5}x^{\frac{9}{2}}}-$

input
```
int((B*x+A)/x^(11/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/9*A/a/x^(9/2)-2/7*(-A*b+B*a)/a^2/x^(7/2)-2*b^3*(A*b-B*a)/a^5/x^(1/2)-2/
5*b*(A*b-B*a)/a^3/x^(5/2)+2/3*b^2*(A*b-B*a)/a^4/x^(3/2)-2*b^4*(A*b-B*a)/a^
5/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx = \left[-\frac{315\,(Bab^3-Ab^4)x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(35\,Aa^4-315\,(Bab^3-Ab^4)\cdots}{315\,a^5\cdots}\right.$$

input
```
integrate((B*x+A)/x^(11/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/315*(315*(B*a*b^3 - A*b^4)*x^5*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(
-b/a) - a)/(b*x + a)) + 2*(35*A*a^4 - 315*(B*a*b^3 - A*b^4)*x^4 + 105*(B*a
^2*b^2 - A*a*b^3)*x^3 - 63*(B*a^3*b - A*a^2*b^2)*x^2 + 45*(B*a^4 - A*a^3*b
)*x)*sqrt(x))/(a^5*x^5), 2/315*(315*(B*a*b^3 - A*b^4)*x^5*sqrt(b/a)*arctan
(sqrt(x)*sqrt(b/a)) - (35*A*a^4 - 315*(B*a*b^3 - A*b^4)*x^4 + 105*(B*a^2*b
^2 - A*a*b^3)*x^3 - 63*(B*a^3*b - A*a^2*b^2)*x^2 + 45*(B*a^4 - A*a^3*b)*x)
*sqrt(x))/(a^5*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(131) = 262$.

Time = 63.25 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.45

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{9x^{\frac{9}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{9x^{\frac{9}{2}}}}{b} \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{a} \\ -\frac{2A}{9ax^{\frac{9}{2}}}+\frac{2Ab}{7a^2x^{\frac{7}{2}}}-\frac{2Ab^2}{5a^3x^{\frac{5}{2}}}+\frac{2Ab^3}{3a^4x^{\frac{3}{2}}}-\frac{Ab^4\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a^5\sqrt{-\frac{a}{b}}}+\frac{Ab^4\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a^5\sqrt{-\frac{a}{b}}}-\frac{2Ab^4}{a^5\sqrt{x}}-\frac{2\cdots}{7a\cdots} \end{cases}$$

input
```
integrate((B*x+A)/x**(11/2)/(b*x+a),x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(9*x**(9/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/(11*x**(11/2)) - 2*B/(9*x**(9/2)))/b, Eq(a, 0)), ((-2*A/(9*x**
(9/2)) - 2*B/(7*x**(7/2)))/a, Eq(b, 0)), (-2*A/(9*a*x**(9/2)) + 2*A*b/(7*a
**2*x**(7/2)) - 2*A*b**2/(5*a**3*x**(5/2)) + 2*A*b**3/(3*a**4*x**(3/2)) -
A*b**4*log(sqrt(x) - sqrt(-a/b))/(a**5*sqrt(-a/b)) + A*b**4*log(sqrt(x) +
sqrt(-a/b))/(a**5*sqrt(-a/b)) - 2*A*b**4/(a**5*sqrt(x)) - 2*B/(7*a*x**(7/2
)) + 2*B*b/(5*a**2*x**(5/2)) - 2*B*b**2/(3*a**3*x**(3/2)) + B*b**3*log(sqr
t(x) - sqrt(-a/b))/(a**4*sqrt(-a/b)) - B*b**3*log(sqrt(x) + sqrt(-a/b))/(a
**4*sqrt(-a/b)) + 2*B*b**3/(a**4*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx = \frac{2\,(Bab^4-Ab^5)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{2\,(35\,Aa^4-315\,(Bab^3-Ab^4)x^4+105\,(Ba^2b^2-Aab^3)x^3-63\,(Ba^3b-Aa^2b^2)x^2+45\,(Ba^4-Aa^3b)x)}{315\,a^5x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)/x^(11/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*(B*a*b^4 - A*b^5)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - 2/315*(3
5*A*a^4 - 315*(B*a*b^3 - A*b^4)*x^4 + 105*(B*a^2*b^2 - A*a*b^3)*x^3 - 63*(
B*a^3*b - A*a^2*b^2)*x^2 + 45*(B*a^4 - A*a^3*b)*x)/(a^5*x^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx = \frac{2\,(Bab^4-Ab^5)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} + \frac{2\,(315\,Bab^3x^4-315\,Ab^4x^4-105\,Ba^2b^2x^3+105\,Aab^3x^3+63\,Ba^3bx^2-63\,Aa^2b^2x^2-45\,Ba^4x+45\,Aa^3b}{315\,a^5x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)/x^(11/2)/(b*x+a),x, algorithm="giac")
```

output
```
2*(B*a*b^4 - A*b^5)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) + 2/315*(3
15*B*a*b^3*x^4 - 315*A*b^4*x^4 - 105*B*a^2*b^2*x^3 + 105*A*a*b^3*x^3 + 63*
B*a^3*b*x^2 - 63*A*a^2*b^2*x^2 - 45*B*a^4*x + 45*A*a^3*b*x - 35*A*a^4)/(a^
5*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx = -\frac{\frac{2A}{9a}-\frac{2x(Ab-Ba)}{7a^2}-\frac{2b^2x^3(Ab-Ba)}{3a^4}+\frac{2b^3x^4(Ab-Ba)}{a^5}+\frac{2bx^2(Ab-Ba)}{5a^3}}{x^{9/2}} - \frac{2b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)(Ab-Ba)}{a^{11/2}}$$

input
```
int((A + B*x)/(x^(11/2)*(a + b*x)),x)
```

output
```
- ((2*A)/(9*a) - (2*x*(A*b - B*a))/(7*a^2) - (2*b^2*x^3*(A*b - B*a))/(3*a^
4) + (2*b^3*x^4*(A*b - B*a))/a^5 + (2*b*x^2*(A*b - B*a))/(5*a^3))/x^(9/2)
- (2*b^(7/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b - B*a))/a^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.07

$$\int \frac{A+Bx}{x^{11/2}(a+bx)}\,dx = -\frac{2}{9\sqrt{x}\,x^4}$$

input
```
int((B*x+A)/x^(11/2)/(b*x+a),x)
```

output
```
( - 2)/(9*sqrt(x)*x**4)
```

3.207 $\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{2a^2(3Ab-4aB)\sqrt{x}}{b^5} - \frac{2a(2Ab-3aB)x^{3/2}}{3b^4} + \frac{2(Ab-2aB)x^{5/2}}{5b^3} + \frac{2Bx^{7/2}}{7b^2} + \frac{a^3(Ab-aB)\sqrt{x}}{b^5(a+bx)} - \frac{a^{5/2}(7Ab-9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}}$$

output
```
2*a^2*(3*A*b-4*B*a)*x^(1/2)/b^5-2/3*a*(2*A*b-3*B*a)*x^(3/2)/b^4+2/5*(A*b-2
*B*a)*x^(5/2)/b^3+2/7*B*x^(7/2)/b^2+a^3*(A*b-B*a)*x^(1/2)/b^5/(b*x+a)-a^(5
/2)*(7*A*b-9*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{\sqrt{x}(-945a^4B + 105a^3b(7A-6Bx) + 6b^4x^3(7A+5Bx) + 14a^2b^2x(35A+9Bx) -}{105b^5(a+bx)} + \frac{a^{5/2}(-7Ab+9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[x]*(-945*a^4*B + 105*a^3*b*(7*A - 6*B*x) + 6*b^4*x^3*(7*A + 5*B*x) +
 14*a^2*b^2*x*(35*A + 9*B*x) - 2*a*b^3*x^2*(49*A + 27*B*x)))/(105*b^5*(a +
 b*x)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(1
1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {87, 60, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\int \frac{x^{7/2}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\int \frac{x^{5/2}}{a+bx}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \; 60$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \; 73$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \; 218$$

$$\frac{x^{9/2}(Ab-aB)}{ab(a+bx)}-\frac{(7Ab-9aB)\left(\frac{2x^{7/2}}{7b}-\frac{a\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a + b*x)^2,x]
```

output
```
((A*b - a*B)*x^(9/2))/(a*b*(a + b*x)) - ((7*A*b - 9*a*B)*((2*x^(7/2))/(7*b
) - (a*((2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2
*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2\left(15b^3Bx^3+21Ax^2b^3-42Bx^2ab^2-70Axab^2+105Bxa^2b+315a^2bA-420a^3B\right)\sqrt{x}}{105b^5}-\frac{a^3\left(\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-\cdots}{\cdots}\right.}{b^5}$
derivativedivides	$\frac{\frac{2b^3Bx^{\frac{7}{2}}}{7}+\frac{2Ab^3x^{\frac{5}{2}}}{5}-\frac{4Bab^2x^{\frac{5}{2}}}{5}-\frac{4Aab^2x^{\frac{3}{2}}}{3}+2Ba^2bx^{\frac{3}{2}}+6a^2bA\sqrt{x}-8a^3B\sqrt{x}}{b^5}-\frac{2a^3\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-9Ba)\,\mathrm{ar}\cdots}{2\sqrt{\cdots}}\right.}{b^5}$
default	$\frac{\frac{2b^3Bx^{\frac{7}{2}}}{7}+\frac{2Ab^3x^{\frac{5}{2}}}{5}-\frac{4Bab^2x^{\frac{5}{2}}}{5}-\frac{4Aab^2x^{\frac{3}{2}}}{3}+2Ba^2bx^{\frac{3}{2}}+6a^2bA\sqrt{x}-8a^3B\sqrt{x}}{b^5}-\frac{2a^3\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-9Ba)\,\mathrm{ar}\cdots}{2\sqrt{\cdots}}\right.}{b^5}$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*B*b^3*x^3+21*A*b^3*x^2-42*B*a*b^2*x^2-70*A*a*b^2*x+105*B*a^2*b*x
+315*A*a^2*b-420*B*a^3)*x^(1/2)/b^5-1/b^5*a^3*(2*(-1/2*A*b+1/2*B*a)*x^(1/2
)/(b*x+a)+(7*A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.38

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \left[-\frac{105\,(9\,Ba^4-7\,Aa^3b+(9\,Ba^3b-7\,Aa^2b^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(30\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/210*(105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(-a/b
)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(30*B*b^4*x^4 - 94
5*B*a^4 + 735*A*a^3*b - 6*(9*B*a*b^3 - 7*A*b^4)*x^3 + 14*(9*B*a^2*b^2 - 7*
A*a*b^3)*x^2 - 70*(9*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(x))/(b^6*x + a*b^5), 1
/105*(105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) + (30*B*b^4*x^4 - 945*B*a^4 + 735*A*a^3*b - 6*
(9*B*a*b^3 - 7*A*b^4)*x^3 + 14*(9*B*a^2*b^2 - 7*A*a*b^3)*x^2 - 70*(9*B*a^3
*b - 7*A*a^2*b^2)*x)*sqrt(x))/(b^6*x + a*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(143) = 286.

Time = 42.00 (sec) , antiderivative size = 986, normalized size of antiderivative = 6.90

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(11/2)/11)/a**2, Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B
*x**(7/2)/7)/b**2, Eq(a, 0)), (-735*A*a**4*b*log(sqrt(x) - sqrt(-a/b))/(21
0*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 735*A*a**4*b*log(sqrt(x) +
sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 1470*A*a**3*
b**2*sqrt(x)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) -
735*A*a**3*b**2*x*log(sqrt(x) - sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b
**7*x*sqrt(-a/b)) + 735*A*a**3*b**2*x*log(sqrt(x) + sqrt(-a/b))/(210*a*b**
6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 980*A*a**2*b**3*x**(3/2)*sqrt(-a/b
)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) - 196*A*a*b**4*x**(5/2)*
sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 84*A*b**5*x**
(7/2)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) + 945*B*a
**5*log(sqrt(x) - sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/
b)) - 945*B*a**5*log(sqrt(x) + sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b*
*7*x*sqrt(-a/b)) - 1890*B*a**4*b*sqrt(x)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b)
 + 210*b**7*x*sqrt(-a/b)) + 945*B*a**4*b*x*log(sqrt(x) - sqrt(-a/b))/(210*
a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) - 945*B*a**4*b*x*log(sqrt(x) +
sqrt(-a/b))/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) - 1260*B*a**3*
b**2*x**(3/2)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*x*sqrt(-a/b)) +
 252*B*a**2*b**3*x**(5/2)*sqrt(-a/b)/(210*a*b**6*sqrt(-a/b) + 210*b**7*...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = -\frac{(Ba^4-Aa^3b)\sqrt{x}}{b^6x+ab^5} + \frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{2\left(15\,Bb^3x^{\frac{7}{2}}-21\,(2\,Bab^2-Ab^3)x^{\frac{5}{2}}+35\,(3\,Ba^2b-2\,Aab^2)x^{\frac{3}{2}}-105\,(4\,Ba^3-3\,Aa^2b)\sqrt{x}\right)}{105\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(B*a^4 - A*a^3*b)*sqrt(x)/(b^6*x + a*b^5) + (9*B*a^4 - 7*A*a^3*b)*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) + 2/105*(15*B*b^3*x^(7/2) - 21*(2*B*a
*b^2 - A*b^3)*x^(5/2) + 35*(3*B*a^2*b - 2*A*a*b^2)*x^(3/2) - 105*(4*B*a^3
- 3*A*a^2*b)*sqrt(x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} - \frac{Ba^4\sqrt{x}-Aa^3b\sqrt{x}}{(bx+a)b^5} + \frac{2\left(15\,Bb^{12}x^{\frac{7}{2}}-42\,Bab^{11}x^{\frac{5}{2}}+21\,Ab^{12}x^{\frac{5}{2}}+105\,Ba^2b^{10}x^{\frac{3}{2}}-70\,Aab^{11}x^{\frac{3}{2}}-420\,Ba^3b^9\sqrt{x}+315\,Aa^2b^{10}\sqrt{x}\right.}{105\,b^{14}}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(9*B*a^4 - 7*A*a^3*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) - (B*a^4
*sqrt(x) - A*a^3*b*sqrt(x))/((b*x + a)*b^5) + 2/105*(15*B*b^12*x^(7/2) - 4
2*B*a*b^11*x^(5/2) + 21*A*b^12*x^(5/2) + 105*B*a^2*b^10*x^(3/2) - 70*A*a*b
^11*x^(3/2) - 420*B*a^3*b^9*sqrt(x) + 315*A*a^2*b^10*sqrt(x))/b^14
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.46

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \sqrt{x}\left(\frac{2\,a\left(\frac{2\,a\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{b}+\frac{2\,B\,a^2}{b^4}\right)}{b}-\frac{a^2\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{b^2}\right) + x^{5/2}\left(\frac{2\,A}{5\,b^2}-\frac{4\,B\,a}{5\,b^3}\right) - x^{3/2}\left(\frac{2\,a\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{3\,b}+\frac{2\,B\,a^2}{3\,b^4}\right)+\frac{2\,B\,x^{7/2}}{7\,b^2}-\frac{\sqrt{x}\,(B\,a^4-A\,a^3\,b)}{x\,b^6+a\,b^5}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,\sqrt{x}\,(7\,A\,b-9\,B\,a)}{9\,B\,a^4-7\,A\,a^3\,b}\right)\,(7\,A\,b}{b^{11/2}}$$

input
```
int((x^(7/2)*(A + B*x))/(a + b*x)^2,x)
```

output
```
x^(1/2)*((2*a*((2*a*((2*A)/b^2 - (4*B*a)/b^3))/b + (2*B*a^2)/b^4))/b - (a^
2*((2*A)/b^2 - (4*B*a)/b^3))/b^2) + x^(5/2)*((2*A)/(5*b^2) - (4*B*a)/(5*b^
3)) - x^(3/2)*((2*a*((2*A)/b^2 - (4*B*a)/b^3))/(3*b) + (2*B*a^2)/(3*b^4))
+ (2*B*x^(7/2))/(7*b^2) - (x^(1/2)*(B*a^4 - A*a^3*b))/(a*b^5 + b^6*x) + (a
^(5/2)*atan((a^(5/2)*b^(1/2)*x^(1/2)*(7*A*b - 9*B*a))/(9*B*a^4 - 7*A*a^3*b
))*(7*A*b - 9*B*a))/b^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^3 - 2\sqrt{x}\,a^3b + \frac{2\sqrt{x}\,a^2b^2x}{3} - \frac{2\sqrt{x}\,a\,b^3x^2}{5} + \frac{2\sqrt{x}\,b^4x^3}{7}}{b^5}$$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a)^2,x)
```

output
```
(2*(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 - 105*sqr
t(x)*a**3*b + 35*sqrt(x)*a**2*b**2*x - 21*sqrt(x)*a*b**3*x**2 + 15*sqrt(x)
*b**4*x**3))/(105*b**5)
```

3.208 $\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = -\frac{2a(2Ab-3aB)\sqrt{x}}{b^4}+\frac{2(Ab-2aB)x^{3/2}}{3b^3}+\frac{2Bx^{5/2}}{5b^2}-\frac{a^2(Ab-aB)\sqrt{x}}{b^4(a+bx)}+\frac{a^{3/2}(5Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
-2*a*(2*A*b-3*B*a)*x^(1/2)/b^4+2/3*(A*b-2*B*a)*x^(3/2)/b^3+2/5*B*x^(5/2)/b
^2-a^2*(A*b-B*a)*x^(1/2)/b^4/(b*x+a)+a^(3/2)*(5*A*b-7*B*a)*arctan(b^(1/2)*
x^(1/2)/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{\sqrt{x}(105a^3B+2b^3x^2(5A+3Bx)-2ab^2x(25A+7Bx)+a^2(-75Ab+70bBx))}{15b^4(a+bx)}-\frac{a^{3/2}(-5Ab+7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[x]*(105*a^3*B + 2*b^3*x^2*(5*A + 3*B*x) - 2*a*b^2*x*(25*A + 7*B*x) +
 a^2*(-75*A*b + 70*b*B*x)))/(15*b^4*(a + b*x)) - (a^(3/2)*(-5*A*b + 7*a*B)
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\int \frac{x^{5/2}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$\Big\downarrow$ 73

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{2ab}$$

$\Big\downarrow$ 218

$$\frac{x^{7/2}(Ab-aB)}{ab(a+bx)}-\frac{(5Ab-7aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x)^2,x]
```

output
```
((A*b - a*B)*x^(7/2))/(a*b*(a + b*x)) - ((5*A*b - 7*a*B)*((2*x^(5/2))/(5*b
) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

method	result	s
risch	$-\frac{2\left(-3 b^{2} B \,x^{2}-5 A \,b^{2} x +10 B a b x +30 a b A -45 a^{2} B \right) \sqrt{x}}{15 b^{4}}+\frac{a^{2} \left(\frac{2 \left(-\frac{A b}{2}+\frac{B a}{2}\right) \sqrt{x}}{b x +a}+\frac{\left(5 A b -7 B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{\sqrt{a b}}\right)}{b^{4}}$	9
derivativedivides	$-\frac{2 \left(-\frac{b^{2} B \,x^{\frac{5}{2}}}{5}-\frac{A \,b^{2} x^{\frac{3}{2}}}{3}+\frac{2 B a b \,x^{\frac{3}{2}}}{3}+2 a b A \sqrt{x}-3 a^{2} B \sqrt{x}\right)}{b^{4}}+\frac{2 a^{2} \left(\frac{\left(-\frac{A b}{2}+\frac{B a}{2}\right) \sqrt{x}}{b x +a}+\frac{\left(5 A b -7 B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{4}}$	1
default	$-\frac{2 \left(-\frac{b^{2} B \,x^{\frac{5}{2}}}{5}-\frac{A \,b^{2} x^{\frac{3}{2}}}{3}+\frac{2 B a b \,x^{\frac{3}{2}}}{3}+2 a b A \sqrt{x}-3 a^{2} B \sqrt{x}\right)}{b^{4}}+\frac{2 a^{2} \left(\frac{\left(-\frac{A b}{2}+\frac{B a}{2}\right) \sqrt{x}}{b x +a}+\frac{\left(5 A b -7 B a \right) \arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{4}}$	1

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+10*B*a*b*x+30*A*a*b-45*B*a^2)*x^(1/2)/b^4+1/
b^4*a^2*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/(b*x+a)+(5*A*b-7*B*a)/(a*b)^(1/2)*ar
ctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.44

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = \left[-\frac{15\,(7\,Ba^3-5\,Aa^2b+(7\,Ba^2b-5\,Aab^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(6\,Bb^3 \cdots}{30\,(b^5x \cdots}\right.$$

$$-\frac{15\,(7\,Ba^3-5\,Aa^2b+(7\,Ba^2b-5\,Aab^2)x)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(6\,Bb^3x^3+105\,Ba^3-75\,Aa^2b-2\,(7 \cdots}{15\,(b^5x+ab^4)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x)*sqrt(-a/b)*lo
g((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(6*B*b^3*x^3 + 105*B*a
^3 - 75*A*a^2*b - 2*(7*B*a*b^2 - 5*A*b^3)*x^2 + 10*(7*B*a^2*b - 5*A*a*b^2)
*x)*sqrt(x))/(b^5*x + a*b^4), -1/15*(15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b
- 5*A*a*b^2)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - (6*B*b^3*x^3 + 1
05*B*a^3 - 75*A*a^2*b - 2*(7*B*a*b^2 - 5*A*b^3)*x^2 + 10*(7*B*a^2*b - 5*A*
a*b^2)*x)*sqrt(x))/(b^5*x + a*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(117) = 234$.

Time = 15.10 (sec) , antiderivative size = 877, normalized size of antiderivative = 7.37

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x
**(5/2)/5)/b**2, Eq(a, 0)), (75*A*a**3*b*log(sqrt(x) - sqrt(-a/b))/(30*a*b
**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) - 75*A*a**3*b*log(sqrt(x) + sqrt(-a
/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) - 150*A*a**2*b**2*sqrt(
x)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) + 75*A*a**2*b*
*2*x*log(sqrt(x) - sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b
)) - 75*A*a**2*b**2*x*log(sqrt(x) + sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30
*b**6*x*sqrt(-a/b)) - 100*A*a*b**3*x**(3/2)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/
b) + 30*b**6*x*sqrt(-a/b)) + 20*A*b**4*x**(5/2)*sqrt(-a/b)/(30*a*b**5*sqrt
(-a/b) + 30*b**6*x*sqrt(-a/b)) - 105*B*a**4*log(sqrt(x) - sqrt(-a/b))/(30*
a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) + 105*B*a**4*log(sqrt(x) + sqrt(
-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) + 210*B*a**3*b*sqrt(x
)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)) - 105*B*a**3*b*
x*log(sqrt(x) - sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6*x*sqrt(-a/b))
+ 105*B*a**3*b*x*log(sqrt(x) + sqrt(-a/b))/(30*a*b**5*sqrt(-a/b) + 30*b**6
*x*sqrt(-a/b)) + 140*B*a**2*b**2*x**(3/2)*sqrt(-a/b)/(30*a*b**5*sqrt(-a/b)
 + 30*b**6*x*sqrt(-a/b)) - 28*B*a*b**3*x**(5/2)*sqrt(-a/b)/(30*a*b**5*sqrt
(-a/b) + 30*b**6*x*sqrt(-a/b)) + 12*B*b**4*x**(7/2)*sqrt(-a/b)/(30*a*b**5*
sqrt(-a/b) + 30*b**6*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{(Ba^3-Aa^2b)\sqrt{x}}{b^5x+ab^4} - \frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{2\left(3\,Bb^2x^{\frac{5}{2}}-5\,(2\,Bab-Ab^2)x^{\frac{3}{2}}+15\,(3\,Ba^2-2\,Aab)\sqrt{x}\right)}{15\,b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(B*a^3 - A*a^2*b)*sqrt(x)/(b^5*x + a*b^4) - (7*B*a^3 - 5*A*a^2*b)*arctan(b
*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/15*(3*B*b^2*x^(5/2) - 5*(2*B*a*b -
 A*b^2)*x^(3/2) + 15*(3*B*a^2 - 2*A*a*b)*sqrt(x))/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = -\frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{Ba^3\sqrt{x}-Aa^2b\sqrt{x}}{(bx+a)b^4} + \frac{2\left(3\,Bb^8x^{\frac{5}{2}}-10\,Bab^7x^{\frac{3}{2}}+5\,Ab^8x^{\frac{3}{2}}+45\,Ba^2b^6\sqrt{x}-30\,Aab^7\sqrt{x}\right)}{15\,b^{10}}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-(7*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + (B*a^
3*sqrt(x) - A*a^2*b*sqrt(x))/((b*x + a)*b^4) + 2/15*(3*B*b^8*x^(5/2) - 10*
B*a*b^7*x^(3/2) + 5*A*b^8*x^(3/2) + 45*B*a^2*b^6*sqrt(x) - 30*A*a*b^7*sqrt
(x))/b^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.23

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b^2}-\frac{4\,B\,a}{3\,b^3}\right)-\sqrt{x}\left(\frac{2\,a\,\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)}{b}+\frac{2\,B\,a^2}{b^4}\right) + \frac{2\,B\,x^{5/2}}{5\,b^2}+\frac{\sqrt{x}\,(B\,a^3-A\,a^2\,b)}{x\,b^5+a\,b^4}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,\sqrt{x}\,(5\,A\,b-7\,B\,a)}{7\,B\,a^3-5\,A\,a^2\,b}\right)\,(5\,A\,b-7\,B\,a)}{b^{9/2}}$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x)^2,x)
```

output
```
x^(3/2)*((2*A)/(3*b^2) - (4*B*a)/(3*b^3)) - x^(1/2)*((2*a*((2*A)/b^2 - (4*
B*a)/b^3))/b + (2*B*a^2)/b^4) + (2*B*x^(5/2))/(5*b^2) + (x^(1/2)*(B*a^3 -
A*a^2*b))/(a*b^4 + b^5*x) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x^(1/2)*(5*A*b
- 7*B*a))/(7*B*a^3 - 5*A*a^2*b))*(5*A*b - 7*B*a))/b^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.46

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+2\sqrt{x}\,a^2b-\frac{2\sqrt{x}\,a\,b^2x}{3}+\frac{2\sqrt{x}\,b^3x^2}{5}}{b^4}$$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^2,x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 15*sq
rt(x)*a**2*b - 5*sqrt(x)*a*b**2*x + 3*sqrt(x)*b**3*x**2))/(15*b**4)
```

3.209 $\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2} \, dx = \frac{2(Ab-2aB)\sqrt{x}}{b^3} + \frac{2Bx^{3/2}}{3b^2} + \frac{a(Ab-aB)\sqrt{x}}{b^3(a+bx)} - \frac{\sqrt{a}(3Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
2*(A*b-2*B*a)*x^(1/2)/b^3+2/3*B*x^(3/2)/b^2+a*(A*b-B*a)*x^(1/2)/b^3/(b*x+a
)-a^(1/2)*(3*A*b-5*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2} \, dx = \frac{\sqrt{x}(-15a^2B + ab(9A-10Bx) + 2b^2x(3A+Bx))}{3b^3(a+bx)} + \frac{\sqrt{a}(-3Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[x]*(-15*a^2*B + a*b*(9*A - 10*B*x) + 2*b^2*x*(3*A + B*x)))/(3*b^3*(a
 + b*x)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^
(7/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\int \frac{x^{3/2}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2ab}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^{5/2}(Ab-aB)}{ab(a+bx)}-\frac{(3Ab-5aB)\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2ab}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x)^2,x]
```

output
```
((A*b - a*B)*x^(5/2))/(a*b*(a + b*x)) - ((3*A*b - 5*a*B)*((2*x^(3/2))/(3*b
) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3
/2)))/b))/(2*a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{2(bBx+3Ab-6Ba)\sqrt{x}}{3b^3}-\frac{a\left(\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^3}$	77
derivativedivides	$\frac{\frac{2bB\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^3}-\frac{2a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	82
default	$\frac{\frac{2bB\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^3}-\frac{2a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	82

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-6*B*a)*x^(1/2)/b^3-a/b^3*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/(b
*x+a)+(3*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.43

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}\,dx = \left[-\frac{3\,(5\,Ba^2-3\,Aab+(5\,Bab-3\,Ab^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)-2\,(2\,Bb^2x^2 -}{6\,(b^4x+ab^3)}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x)*sqrt(-a/b)*log((b*x -
 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) - 2*(2*B*b^2*x^2 - 15*B*a^2 + 9*A*
a*b - 2*(5*B*a*b - 3*A*b^2)*x)*sqrt(x))/(b^4*x + a*b^3), 1/3*(3*(5*B*a^2 -
 3*A*a*b + (5*B*a*b - 3*A*b^2)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (2*B*b^2*x^2 - 15*B*a^2 + 9*A*a*b - 2*(5*B*a*b - 3*A*b^2)*x)*sqrt(x))/(b
^4*x + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(90) = 180$.

Time = 4.49 (sec) , antiderivative size = 762, normalized size of antiderivative = 8.02

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(7/2)/7)/a**2, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(3/2
)/3)/b**2, Eq(a, 0)), (-9*A*a**2*b*log(sqrt(x) - sqrt(-a/b))/(6*a*b**4*sqr
t(-a/b) + 6*b**5*x*sqrt(-a/b)) + 9*A*a**2*b*log(sqrt(x) + sqrt(-a/b))/(6*a
*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 18*A*a*b**2*sqrt(x)*sqrt(-a/b)/(
6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 9*A*a*b**2*x*log(sqrt(x) - sq
rt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 9*A*a*b**2*x*log(s
qrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 12*A*b*
*3*x**(3/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 15*B*
a**3*log(sqrt(x) - sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b))
 - 15*B*a**3*log(sqrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqr
t(-a/b)) - 30*B*a**2*b*sqrt(x)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*
sqrt(-a/b)) + 15*B*a**2*b*x*log(sqrt(x) - sqrt(-a/b))/(6*a*b**4*sqrt(-a/b)
 + 6*b**5*x*sqrt(-a/b)) - 15*B*a**2*b*x*log(sqrt(x) + sqrt(-a/b))/(6*a*b**
4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 20*B*a*b**2*x**(3/2)*sqrt(-a/b)/(6*a
*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 4*B*b**3*x**(5/2)*sqrt(-a/b)/(6*
a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}\,dx = -\frac{(Ba^2-Aab)\sqrt{x}}{b^4x+ab^3} + \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{2\left(Bbx^{\frac{3}{2}}-3\,(2\,Ba-Ab)\sqrt{x}\right)}{3\,b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(B*a^2 - A*a*b)*sqrt(x)/(b^4*x + a*b^3) + (5*B*a^2 - 3*A*a*b)*arctan(b*sq
rt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/3*(B*b*x^(3/2) - 3*(2*B*a - A*b)*sqrt
(x))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{Ba^2\sqrt{x}-Aab\sqrt{x}}{(bx+a)b^3} + \frac{2\left(Bb^4x^{\frac{3}{2}}-6\,Bab^3\sqrt{x}+3\,Ab^4\sqrt{x}\right)}{3\,b^6}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(5*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - (B*a^2*s
qrt(x) - A*a*b*sqrt(x))/((b*x + a)*b^3) + 2/3*(B*b^4*x^(3/2) - 6*B*a*b^3*s
qrt(x) + 3*A*b^4*sqrt(x))/b^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}\,dx = \sqrt{x}\left(\frac{2\,A}{b^2}-\frac{4\,B\,a}{b^3}\right)-\frac{\sqrt{x}\,(B\,a^2-A\,a\,b)}{x\,b^4+a\,b^3} + \frac{2\,B\,x^{3/2}}{3\,b^2}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(3\,A\,b-5\,B\,a)}{5\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-5\,B\,a)}{b^{7/2}}$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x)^2,x)
```

output
```
x^(1/2)*((2*A)/b^2 - (4*B*a)/b^3) - (x^(1/2)*(B*a^2 - A*a*b))/(a*b^3 + b^4
*x) + (2*B*x^(3/2))/(3*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x^(1/2)*(3*A*
b - 5*B*a))/(5*B*a^2 - 3*A*a*b))*(3*A*b - 5*B*a))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-2\sqrt{x}\,ab+\frac{2\sqrt{x}\,b^2x}{3}}{b^3}$$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^2,x)
```

output
```
(2*(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(x)*a*
b + sqrt(x)*b**2*x))/(3*b**3)
```

3.210 $\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2} dx = \frac{2B\sqrt{x}}{b^2} - \frac{(Ab-aB)\sqrt{x}}{b^2(a+bx)} + \frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

output
```
2*B*x^(1/2)/b^2-(A*b-B*a)*x^(1/2)/b^2/(b*x+a)+(A*b-3*B*a)*arctan(b^(1/2)*x
^(1/2)/a^(1/2))/a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2} dx = \frac{\sqrt{x}(-Ab+3aB+2bBx)}{b^2(a+bx)} + \frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[x]*(-(A*b) + 3*a*B + 2*b*B*x))/(b^2*(a + b*x)) + ((A*b - 3*a*B)*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {87, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\int \frac{\sqrt{x}}{a+bx}dx}{2ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2ab}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2ab}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^{3/2}(Ab-aB)}{ab(a+bx)}-\frac{(Ab-3aB)\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2ab}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x)^2,x]
```

output
```
((A*b - a*B)*x^(3/2))/(a*b*(a + b*x)) - ((A*b - 3*a*B)*((2*Sqrt[x])/b - (2
*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/(2*a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	62
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	63
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	63

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b^2+1/b^2*(2*(-1/2*A*b+1/2*B*a)*x^(1/2)/(b*x+a)+(A*b-3*B*a)/(a
*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}\,dx$$
$$=\left[\frac{(3\,Ba^2-Aab+(3\,Bab-Ab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(2\,Bab^2x+3\,Ba^2b-Aab^2)\sqrt{x}}{2\,(ab^4x+a^2b^3)},\frac{(3\,Ba\right.$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*((3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*
sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(2*B*a*b^2*x + 3*B*a^2*b - A*a*b^2)*sqr
t(x))/(a*b^4*x + a^2*b^3), ((3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x)*sqrt(a
*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (2*B*a*b^2*x + 3*B*a^2*b - A*a*b^2)*sq
rt(x))/(a*b^4*x + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(66) = 132$.

Time = 2.20 (sec) , antiderivative size = 634, normalized size of antiderivative = 8.68

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+2B\sqrt{x}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{a^2} \\ \frac{-\frac{2A}{\sqrt{x}}+2B\sqrt{x}}{b^2} \\ \frac{Aab\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{Aab\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{2Ab^2\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{Ab^2x\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{Ab^2x\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{3}{2} \end{cases}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2*A*x
**(3/2)/3 + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))
/b**2, Eq(a, 0)), (A*a*b*log(sqrt(x) - sqrt(-a/b))/(2*a*b**3*sqrt(-a/b) +
2*b**4*x*sqrt(-a/b)) - A*a*b*log(sqrt(x) + sqrt(-a/b))/(2*a*b**3*sqrt(-a/b
) + 2*b**4*x*sqrt(-a/b)) - 2*A*b**2*sqrt(x)*sqrt(-a/b)/(2*a*b**3*sqrt(-a/b
) + 2*b**4*x*sqrt(-a/b)) + A*b**2*x*log(sqrt(x) - sqrt(-a/b))/(2*a*b**3*sq
rt(-a/b) + 2*b**4*x*sqrt(-a/b)) - A*b**2*x*log(sqrt(x) + sqrt(-a/b))/(2*a*
b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*B*a**2*log(sqrt(x) - sqrt(-a/b)
)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*B*a**2*log(sqrt(x) + sqr
t(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 6*B*a*b*sqrt(x)*sqr
t(-a/b)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*B*a*b*x*log(sqrt(x
) - sqrt(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*B*a*b*x*lo
g(sqrt(x) + sqrt(-a/b))/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 4*B*
b**2*x**(3/2)*sqrt(-a/b)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}\,dx = \frac{(Ba-Ab)\sqrt{x}}{b^3x+ab^2} + \frac{2\,B\sqrt{x}}{b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
(B*a - A*b)*sqrt(x)/(b^3*x + a*b^2) + 2*B*sqrt(x)/b^2 - (3*B*a - A*b)*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}\,dx = \frac{2\,B\sqrt{x}}{b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{(bx+a)b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^2 - (3*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2
) + (B*a*sqrt(x) - A*b*sqrt(x))/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}\,dx = \frac{2\,B\,\sqrt{x}}{b^2} - \frac{\sqrt{x}\,(A\,b-B\,a)}{x\,b^3+a\,b^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b-3\,B\,a)}{\sqrt{a}\,b^{5/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x)^2,x)
```

output
```
(2*B*x^(1/2))/b^2 - (x^(1/2)*(A*b - B*a))/(a*b^2 + b^3*x) + (atan((b^(1/2)
*x^(1/2))/a^(1/2))*(A*b - 3*B*a))/(a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^2}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)+2\sqrt{x}\,b}{b^2}$$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^2,x)
```

output
```
(2*( - sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))) + sqrt(x)*b))/b
**2
```

3.211 $\int \frac{A+Bx}{\sqrt{x}(a+bx)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2} dx = \frac{(Ab-aB)\sqrt{x}}{ab(a+bx)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

output
```
(A*b-B*a)*x^(1/2)/a/b/(b*x+a)+(A*b+B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^
(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2} dx = -\frac{(-Ab+aB)\sqrt{x}}{ab(a+bx)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x)^2),x]
```

output
```
-(((-(A*b) + a*B)*Sqrt[x])/(a*b*(a + b*x))) + ((A*b + a*B)*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB+Ab)\int \frac{1}{\sqrt{x}(a+bx)}dx}{2ab}+\frac{\sqrt{x}(Ab-aB)}{ab(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{(aB+Ab)\int \frac{1}{a+bx}d\sqrt{x}}{ab}+\frac{\sqrt{x}(Ab-aB)}{ab(a+bx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}+\frac{\sqrt{x}(Ab-aB)}{ab(a+bx)}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x)^2),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(a*b*(a + b*x)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/(a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{ab(bx+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{ab\sqrt{ab}}$	57
default	$\frac{(Ab-Ba)\sqrt{x}}{ab(bx+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{ab\sqrt{ab}}$	57

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(A*b-B*a)*x^(1/2)/a/b/(b*x+a)+(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(
a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx = \left[-\frac{(Ba^2+Aab+(Bab+Ab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(Ba^2b-Aab^2)\sqrt{x}}{2\,(a^2b^3x+a^3b^2)},\right.$$
$$\left.-\frac{(Ba^2+Aab+(Bab+Ab^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(Ba^2b-Aab^2)\sqrt{x}}{a^2b^3x+a^3b^2}\right]$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqr
t(-a*b)*sqrt(x))/(b*x + a)) + 2*(B*a^2*b - A*a*b^2)*sqrt(x))/(a^2*b^3*x +
a^3*b^2), -((B*a^2 + A*a*b + (B*a*b + A*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)
/(b*sqrt(x))) + (B*a^2*b - A*a*b^2)*sqrt(x))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(53) = 106.

Time = 2.55 (sec) , antiderivative size = 615, normalized size of antiderivative = 9.76

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{a^2} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}}{b^2} \\ \frac{Aab\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}}-\frac{Aab\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}}+\frac{2Ab^2\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}}+\frac{Ab^2x\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x\sqrt{-\frac{a}{b}}}-\frac{Ab^2x\log\left(\sqrt{x}+\sqrt{\right.}{2a^2b^2\sqrt{-\frac{a}{b}}+2ab^3x} \end{cases}$$

input
```
integrate((B*x+A)/x**(1/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*sqrt(x) + 2*B*x**(3/2)/3)/a**2, Eq(b, 0)), ((-2*A/(3*x**(3/2)) - 2*B/s
qrt(x))/b**2, Eq(a, 0)), (A*a*b*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b**2*sqr
t(-a/b) + 2*a*b**3*x*sqrt(-a/b)) - A*a*b*log(sqrt(x) + sqrt(-a/b))/(2*a**2
*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) + 2*A*b**2*sqrt(x)*sqrt(-a/b)/(2
*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) + A*b**2*x*log(sqrt(x) - sq
rt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) - A*b**2*x*log(
sqrt(x) + sqrt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)) + B
*a**2*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(
-a/b)) - B*a**2*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b**2*sqrt(-a/b) + 2*a*b*
*3*x*sqrt(-a/b)) - 2*B*a*b*sqrt(x)*sqrt(-a/b)/(2*a**2*b**2*sqrt(-a/b) + 2*
a*b**3*x*sqrt(-a/b)) + B*a*b*x*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b**2*sqrt
(-a/b) + 2*a*b**3*x*sqrt(-a/b)) - B*a*b*x*log(sqrt(x) + sqrt(-a/b))/(2*a**
2*b**2*sqrt(-a/b) + 2*a*b**3*x*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx = -\frac{(Ba-Ab)\sqrt{x}}{ab^2x+a^2b} + \frac{(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}ab}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(B*a - A*b)*sqrt(x)/(a*b^2*x + a^2*b) + (B*a + A*b)*arctan(b*sqrt(x)/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}ab} - \frac{Ba\sqrt{x}-Ab\sqrt{x}}{(bx+a)ab}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) - (B*a*sqrt(x) - A
*b*sqrt(x))/((b*x + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,\left(A\,b+B\,a\right)}{a^{3/2}\,b^{3/2}}+\frac{\sqrt{x}\,\left(A\,b-B\,a\right)}{a\,b\,\left(a+b\,x\right)}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x)^2),x)
```

output
```
(atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b + B*a))/(a^(3/2)*b^(3/2)) + (x^(1/2)
*(A*b - B*a))/(a*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^2}\,dx=\frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^2,x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.212 $\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 75

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx=-\frac{2A}{a^2\sqrt{x}}-\frac{(Ab-aB)\sqrt{x}}{a^2(a+bx)}-\frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

output
```
-2*A/a^2/x^(1/2)-(A*b-B*a)*x^(1/2)/a^2/(b*x+a)-(3*A*b-B*a)*arctan(b^(1/2)*
x^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx=\frac{-2aA-3Abx+aBx}{a^2\sqrt{x}(a+bx)}+\frac{(-3Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x)^2),x]
```

output
```
(-2*a*A - 3*A*b*x + a*B*x)/(a^2*Sqrt[x]*(a + b*x)) + ((-3*A*b + a*B)*ArcTa
n[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {87, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(3Ab-aB)\int \frac{1}{x^{3/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

$$\Big\downarrow \ 61$$

$$\frac{(3Ab-aB)\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

$$\Big\downarrow \ 73$$

$$\frac{(3Ab-aB)\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

$$\Big\downarrow \ 218$$

$$\frac{(3Ab-aB)\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2ab}+\frac{Ab-aB}{ab\sqrt{x}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x)^2),x]
```

output
```
(A*b - a*B)/(a*b*Sqrt[x]*(a + b*x)) + ((3*A*b - a*B)*(-2/(a*Sqrt[x]) - (2*
Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/(2*a*b)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{2A}{a^2\sqrt{x}}$	64
default	$-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{2A}{a^2\sqrt{x}}$	64
risch	$-\frac{2A}{a^2\sqrt{x}}-\frac{\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a^2}$	64

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*((1/2*A*b-1/2*B*a)*x^(1/2)/(b*x+a)+1/2*(3*A*b-B*a)/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))-2*A/a^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx = \left[\frac{((Bab-3\,Ab^2)x^2+(Ba^2-3\,Aab)x)\sqrt{-ab}\log\left(\frac{bx-a+2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)-2\,(2\,Aa^2b-(Ba}{2\,(a^3b^2x^2+a^4bx)}\right.$$
$$\left.-\frac{((Bab-3\,Ab^2)x^2+(Ba^2-3\,Aab)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(2\,Aa^2b-(Ba^2b-3\,Aab^2)x)\sqrt{x}}{a^3b^2x^2+a^4bx}\right]$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(((B*a*b - 3*A*b^2)*x^2 + (B*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x -
a + 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2
)*x)*sqrt(x))/(a^3*b^2*x^2 + a^4*b*x), -(((B*a*b - 3*A*b^2)*x^2 + (B*a^2 -
 3*A*a*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (2*A*a^2*b - (B*a^2
*b - 3*A*a*b^2)*x)*sqrt(x))/(a^3*b^2*x^2 + a^4*b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(68) = 136$.

Time = 5.81 (sec) , antiderivative size = 794, normalized size of antiderivative = 10.59

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(3/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/sqrt(x) + 2*B*sqrt(x))/a**2, Eq(b, 0)), ((-2*A/(5*x**(5/2)) - 2*
B/(3*x**(3/2)))/b**2, Eq(a, 0)), (-3*A*a*b*sqrt(x)*log(sqrt(x) - sqrt(-a/b
))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + 3*A*a
*b*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2
*b**2*x**(3/2)*sqrt(-a/b)) - 4*A*a*b*sqrt(-a/b)/(2*a**3*b*sqrt(x)*sqrt(-a/
b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) - 3*A*b**2*x**(3/2)*log(sqrt(x) - sq
rt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b))
+ 3*A*b**2*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b)
 + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) - 6*A*b**2*x*sqrt(-a/b)/(2*a**3*b*sqrt
(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + B*a**2*sqrt(x)*log(sqr
t(x) - sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqr
t(-a/b)) - B*a**2*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt
(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + B*a*b*x**(3/2)*log(sqrt(x) - s
qrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b))
 - B*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(2*a**3*b*sqrt(x)*sqrt(-a/b) +
 2*a**2*b**2*x**(3/2)*sqrt(-a/b)) + 2*B*a*b*x*sqrt(-a/b)/(2*a**3*b*sqrt(x)
*sqrt(-a/b) + 2*a**2*b**2*x**(3/2)*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx = -\frac{2\,Aa - (Ba - 3\,Ab)x}{a^2bx^{\frac{3}{2}} + a^3\sqrt{x}} + \frac{(Ba - 3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*A*a - (B*a - 3*A*b)*x)/(a^2*b*x^(3/2) + a^3*sqrt(x)) + (B*a - 3*A*b)*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx = \frac{(Ba-3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{Bax-3\,Abx-2\,Aa}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(B*a - 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + (B*a*x - 3*A*b
*x - 2*A*a)/((b*x^(3/2) + a*sqrt(x))*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx = -\frac{\frac{2\,A}{a}+\frac{x\,(3\,A\,b-B\,a)}{a^2}}{a\,\sqrt{x}+b\,x^{3/2}} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(3\,A\,b-B\,a)}{a^{5/2}\,\sqrt{b}}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x)^2),x)
```

output
```
- ((2*A)/a + (x*(3*A*b - B*a))/a^2)/(a*x^(1/2) + b*x^(3/2)) - (atan((b^(1/
2)*x^(1/2))/a^(1/2))*(3*A*b - B*a))/(a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^2}\,dx=\frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)-2a}{\sqrt{x}\,a^2}$$

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^2,x)
```

output
```
( - 2*(sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a))) + a))/(
sqrt(x)*a**2)
```

3.213 $\int \frac{A+Bx}{x^{5/2}(a+bx)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2} \, dx = -\frac{2A}{3a^2x^{3/2}} + \frac{2(2Ab-aB)}{a^3\sqrt{x}} + \frac{b(Ab-aB)\sqrt{x}}{a^3(a+bx)} + \frac{\sqrt{b}(5Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-2/3*A/a^2/x^(3/2)+2*(2*A*b-B*a)/a^3/x^(1/2)+b*(A*b-B*a)*x^(1/2)/a^3/(b*x+
a)+b^(1/2)*(5*A*b-3*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2} \, dx = \frac{15Ab^2x^2+abx(10A-9Bx)-2a^2(A+3Bx)}{3a^3x^{3/2}(a+bx)} + \frac{\sqrt{b}(5Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x)^2),x]
```

output
```
(15*A*b^2*x^2 + a*b*x*(10*A - 9*B*x) - 2*a^2*(A + 3*B*x))/(3*a^3*x^(3/2)*(
a + b*x)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^
(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(5Ab-3aB)\int \frac{1}{x^{5/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(5Ab-3aB)\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{3/2}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x)^2),x]
```

output
```
(A*b - a*B)/(a*b*x^(3/2)*(a + b*x)) + ((5*A*b - 3*a*B)*(-2/(3*a*x^(3/2)) -
 (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2
)))/a))/(2*a*b)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{2(-6Abx+3Bax+Aa)}{3a^3x^{\frac{3}{2}}}+\frac{b\left(\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^3}$	77
derivativedivides	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2(-2Ab+Ba)}{a^3\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	81
default	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2(-2Ab+Ba)}{a^3\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	81

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-6*A*b*x+3*B*a*x+A*a)/a^3/x^(3/2)+1/a^3*b*(2*(1/2*A*b-1/2*B*a)*x^(1/
2)/(b*x+a)+(5*A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.71

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}\,dx = \left[-\frac{3\,((3\,Bab-5\,Ab^2)x^3+(3\,Ba^2-5\,Aab)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(2\,Aa}{6\,(a^3bx^3+a^4x^2)}\right.$$

$$-\frac{3\,((3\,Bab-5\,Ab^2)x^3+(3\,Ba^2-5\,Aab)x^2)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(2\,Aa^2+3\,(3\,Bab-5\,Ab^2)x^2+2\,(3\,B}{3\,(a^3bx^3+a^4x^2)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*((3*B*a*b - 5*A*b^2)*x^3 + (3*B*a^2 - 5*A*a*b)*x^2)*sqrt(-b/a)*lo
g((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(2*A*a^2 + 3*(3*B*a*b
- 5*A*b^2)*x^2 + 2*(3*B*a^2 - 5*A*a*b)*x)*sqrt(x))/(a^3*b*x^3 + a^4*x^2),
-1/3*(3*((3*B*a*b - 5*A*b^2)*x^3 + (3*B*a^2 - 5*A*a*b)*x^2)*sqrt(b/a)*arct
an(sqrt(x)*sqrt(b/a)) + (2*A*a^2 + 3*(3*B*a*b - 5*A*b^2)*x^2 + 2*(3*B*a^2
- 5*A*a*b)*x)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(90) = 180$.

Time = 21.16 (sec) , antiderivative size = 882, normalized size of antiderivative = 9.28

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(5/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2))
 - 2*B/(5*x**(5/2)))/b**2, Eq(a, 0)), (-4*A*a**2*sqrt(-a/b)/(6*a**4*x**(3/
2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*A*a*b*x**(3/2)*log(sqrt
(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/
b)) - 15*A*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a
/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 20*A*a*b*x*sqrt(-a/b)/(6*a**4*x**(3/
2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*A*b**2*x**(5/2)*log(sqr
t(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a
/b)) - 15*A*b**2*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(
-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 30*A*b**2*x**2*sqrt(-a/b)/(6*a**4*
x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 9*B*a**2*x**(3/2)*lo
g(sqrt(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sq
rt(-a/b)) + 9*B*a**2*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*s
qrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 12*B*a**2*x*sqrt(-a/b)/(6*a**4
*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 9*B*a*b*x**(5/2)*lo
g(sqrt(x) - sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sq
rt(-a/b)) + 9*B*a*b*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sq
rt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 18*B*a*b*x**2*sqrt(-a/b)/(6*a**
4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}\,dx = -\frac{2\,Aa^2+3\,(3\,Bab-5\,Ab^2)x^2+2\,(3\,Ba^2-5\,Aab)x}{3\left(a^3bx^{\frac{5}{2}}+a^4x^{\frac{3}{2}}\right)} - \frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(2*A*a^2 + 3*(3*B*a*b - 5*A*b^2)*x^2 + 2*(3*B*a^2 - 5*A*a*b)*x)/(a^3*
b*x^(5/2) + a^4*x^(3/2)) - (3*B*a*b - 5*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))
/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}\,dx = -\frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{Bab\sqrt{x}-Ab^2\sqrt{x}}{(bx+a)a^3} - \frac{2\,(3\,Bax-6\,Abx+Aa)}{3\,a^3x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-(3*B*a*b - 5*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - (B*a*b*
sqrt(x) - A*b^2*sqrt(x))/((b*x + a)*a^3) - 2/3*(3*B*a*x - 6*A*b*x + A*a)/(
a^3*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}\,dx = \frac{\frac{2\,x\,(5\,A\,b-3\,B\,a)}{3\,a^2}-\frac{2\,A}{3\,a}+\frac{b\,x^2\,(5\,A\,b-3\,B\,a)}{a^3}}{a\,x^{3/2}+b\,x^{5/2}} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(5\,A\,b-3\,B\,a)}{a^{7/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x)^2),x)
```

output
```
((2*x*(5*A*b - 3*B*a))/(3*a^2) - (2*A)/(3*a) + (b*x^2*(5*A*b - 3*B*a))/a^3
)/(a*x^(3/2) + b*x^(5/2)) + (b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(5*A*
b - 3*B*a))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^2}\,dx = \frac{2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-\frac{2a^2}{3}+2abx}{\sqrt{x}\,a^3x}$$

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^2,x)
```

output
```
(2*(3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - a*
*2 + 3*a*b*x))/(3*sqrt(x)*a**3*x)
```

3.214 $\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = -\frac{2A}{5a^2x^{5/2}}+\frac{2(2Ab-aB)}{3a^3x^{3/2}}-\frac{2b(3Ab-2aB)}{a^4\sqrt{x}}-\frac{b^2(Ab-aB)\sqrt{x}}{a^4(a+bx)}-\frac{b^{3/2}(7Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-2/5*A/a^2/x^(5/2)+2/3*(2*A*b-B*a)/a^3/x^(3/2)-2*b*(3*A*b-2*B*a)/a^4/x^(1/
2)-b^2*(A*b-B*a)*x^(1/2)/a^4/(b*x+a)-b^(3/2)*(7*A*b-5*B*a)*arctan(b^(1/2)*
x^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = \frac{-105Ab^3x^3-2a^3(3A+5Bx)+5ab^2x^2(-14A+15Bx)+2a^2bx(7A+25Bx)}{15a^4x^{5/2}(a+bx)}+\frac{b^{3/2}(-7Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a + b*x)^2),x]
```

output
```
(-105*A*b^3*x^3 - 2*a^3*(3*A + 5*B*x) + 5*a*b^2*x^2*(-14*A + 15*B*x) + 2*a
^2*b*x*(7*A + 25*B*x))/(15*a^4*x^(5/2)*(a + b*x)) + (b^(3/2)*(-7*A*b + 5*a
*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(7Ab-5aB)\int \frac{1}{x^{7/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{(7Ab-5aB)\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{5/2}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a + b*x)^2),x]
```

output
```
(A*b - a*B)/(a*b*x^(5/2)*(a + b*x)) + ((7*A*b - 5*a*B)*(-2/(5*a*x^(5/2)) -
 (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a*b)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2b^2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2(-2Ab+Ba)}{3a^3x^{\frac{3}{2}}}-\frac{2b(3Ab-2Ba)}{a^4\sqrt{x}}$	101
default	$-\frac{2b^2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2(-2Ab+Ba)}{3a^3x^{\frac{3}{2}}}-\frac{2b(3Ab-2Ba)}{a^4\sqrt{x}}$	101
risch	$-\frac{2(45A\,b^2x^2-30Bab\,x^2-10aAbx+5B\,a^2x+3a^2A)}{15a^4x^{\frac{5}{2}}}-\frac{b^2\left(\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^4}$	103

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^4*b^2*((1/2*A*b-1/2*B*a)*x^(1/2)/(b*x+a)+1/2*(7*A*b-5*B*a)/(a*b)^(1/2
)*arctan(b*x^(1/2)/(a*b)^(1/2)))-2/5*A/a^2/x^(5/2)-2/3*(-2*A*b+B*a)/a^3/x^
(3/2)-2*b*(3*A*b-2*B*a)/a^4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.61

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = \left[-\frac{15\left((5\,Bab^2-7\,Ab^3)x^4+(5\,Ba^2b-7\,Aab^2)x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(\cdots}{30\,(a^4b\cdots}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*((5*B*a*b^2 - 7*A*b^3)*x^4 + (5*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(
-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(6*A*a^3 - 15*
(5*B*a*b^2 - 7*A*b^3)*x^3 - 10*(5*B*a^2*b - 7*A*a*b^2)*x^2 + 2*(5*B*a^3 -
7*A*a^2*b)*x)*sqrt(x))/(a^4*b*x^4 + a^5*x^3), 1/15*(15*((5*B*a*b^2 - 7*A*b
^3)*x^4 + (5*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a))
 - (6*A*a^3 - 15*(5*B*a*b^2 - 7*A*b^3)*x^3 - 10*(5*B*a^2*b - 7*A*a*b^2)*x^
2 + 2*(5*B*a^3 - 7*A*a^2*b)*x)*sqrt(x))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(117) = 234$.

Time = 48.62 (sec) , antiderivative size = 1017, normalized size of antiderivative = 8.40

$$\int \frac{A + Bx}{x^{7/2}(a + bx)^2}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(7/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2)))/a**2, Eq(b, 0)), ((-2*A/(9*x**(
9/2)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-12*A*a**3*sqrt(-a/b)/(30*a**5
*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 28*A*a**2*b*x*sqrt
(-a/b)/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) - 105
*A*a*b**2*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b)
+ 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 105*A*a*b**2*x**(5/2)*log(sqrt(x) + sqr
t(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) - 1
40*A*a*b**2*x**2*sqrt(-a/b)/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7
/2)*sqrt(-a/b)) - 105*A*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(30*a**5*x
**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 105*A*b**3*x**(7/2)*
log(sqrt(x) + sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2
)*sqrt(-a/b)) - 210*A*b**3*x**3*sqrt(-a/b)/(30*a**5*x**(5/2)*sqrt(-a/b) +
30*a**4*b*x**(7/2)*sqrt(-a/b)) - 20*B*a**3*x*sqrt(-a/b)/(30*a**5*x**(5/2)*
sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 75*B*a**2*b*x**(5/2)*log(sqr
t(x) - sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(
-a/b)) - 75*B*a**2*b*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(30*a**5*x**(5/2)*
sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 100*B*a**2*b*x**2*sqrt(-a/b)
/(30*a**5*x**(5/2)*sqrt(-a/b) + 30*a**4*b*x**(7/2)*sqrt(-a/b)) + 75*B*a*b*
*2*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(30*a**5*x**(5/2)*sqrt(-a/b) + 30...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = -\frac{6\,Aa^3-15\,(5\,Bab^2-7\,Ab^3)x^3-10\,(5\,Ba^2b-7\,Aab^2)x^2+2\,(5\,Ba^3-7\,Aa^2b)x}{15\,\left(a^4bx^{\frac{7}{2}}+a^5x^{\frac{5}{2}}\right)} + \frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/15*(6*A*a^3 - 15*(5*B*a*b^2 - 7*A*b^3)*x^3 - 10*(5*B*a^2*b - 7*A*a*b^2)
*x^2 + 2*(5*B*a^3 - 7*A*a^2*b)*x)/(a^4*b*x^(7/2) + a^5*x^(5/2)) + (5*B*a*b
^2 - 7*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = \frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{Bab^2\sqrt{x}-Ab^3\sqrt{x}}{(bx+a)a^4} + \frac{2\,(30\,Babx^2-45\,Ab^2x^2-5\,Ba^2x+10\,Aabx-3\,Aa^2)}{15\,a^4x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(5*B*a*b^2 - 7*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + (B*a*b
^2*sqrt(x) - A*b^3*sqrt(x))/((b*x + a)*a^4) + 2/15*(30*B*a*b*x^2 - 45*A*b^
2*x^2 - 5*B*a^2*x + 10*A*a*b*x - 3*A*a^2)/(a^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = -\frac{\frac{2A}{5a}-\frac{2x\,(7\,A\,b-5\,B\,a)}{15\,a^2}+\frac{b^2\,x^3\,(7\,A\,b-5\,B\,a)}{a^4}+\frac{2\,b\,x^2\,(7\,A\,b-5\,B\,a)}{3\,a^3}}{a\,x^{5/2}+b\,x^{7/2}} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(7\,A\,b-5\,B\,a)}{a^{9/2}}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x)^2),x)
```

output
```
- ((2*A)/(5*a) - (2*x*(7*A*b - 5*B*a))/(15*a^2) + (b^2*x^3*(7*A*b - 5*B*a)
)/a^4 + (2*b*x^2*(7*A*b - 5*B*a))/(3*a^3))/(a*x^(5/2) + b*x^(7/2)) - (b^(3
/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(7*A*b - 5*B*a))/a^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^2}\,dx = \frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-\frac{2a^3}{5}+\frac{2a^2bx}{3}-2a\,b^2x^2}{\sqrt{x}\,a^4x^2}$$

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^2,x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2
*x**2 - 3*a**3 + 5*a**2*b*x - 15*a*b**2*x**2))/(15*sqrt(x)*a**4*x**2)
```

3.215 $\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = -\frac{2A}{7a^2x^{7/2}} + \frac{2(2Ab-aB)}{5a^3x^{5/2}} - \frac{2b(3Ab-2aB)}{3a^4x^{3/2}} + \frac{2b^2(4Ab-3aB)}{a^5\sqrt{x}} + \frac{b^3(Ab-aB)\sqrt{x}}{a^5(a+bx)} + \frac{b^{5/2}(9Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{11/2}}$$

output
```
-2/7*A/a^2/x^(7/2)+2/5*(2*A*b-B*a)/a^3/x^(5/2)-2/3*b*(3*A*b-2*B*a)/a^4/x^(
3/2)+2*b^2*(4*A*b-3*B*a)/a^5/x^(1/2)+b^3*(A*b-B*a)*x^(1/2)/a^5/(b*x+a)+b^(
5/2)*(9*A*b-7*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = \frac{945Ab^4x^4+105ab^3x^3(6A-7Bx)-6a^4(5A+7Bx)-14a^2b^2x^2(9A+35Bx)+2a^3}{105a^5x^{7/2}(a+bx)} + \frac{b^{5/2}(9Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*(a + b*x)^2),x]
```

output
```
(945*A*b^4*x^4 + 105*a*b^3*x^3*(6*A - 7*B*x) - 6*a^4*(5*A + 7*B*x) - 14*a^
2*b^2*x^2*(9*A + 35*B*x) + 2*a^3*b*x*(27*A + 49*B*x))/(105*a^5*x^(7/2)*(a
+ b*x)) + (b^(5/2)*(9*A*b - 7*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(1
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {87, 61, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(9Ab-7aB)\int \frac{1}{x^{9/2}(a+bx)}dx}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(9Ab-7aB)\left(-\frac{b\int \frac{1}{x^{7/2}(a+bx)}dx}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$$\Big\downarrow \text{61}$$

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 61

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 73

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

$\Big\downarrow$ 218

$$\frac{(9Ab-7aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{a}-\frac{2}{7ax^{7/2}}\right)}{2ab}+\frac{Ab-aB}{abx^{7/2}(a+bx)}$$

input
```
Int[(A + B*x)/(x^(9/2)*(a + b*x)^2),x]
```

output
```
(A*b - a*B)/(a*b*x^(7/2)*(a + b*x)) + ((9*A*b - 7*a*B)*(-2/(7*a*x^(7/2)) -
 (b*(-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqr
t[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a))/a))/(2*a*b)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{2A}{7a^{2}x^{\frac{7}{2}}}-\frac{2(-2Ab+Ba)}{5a^{3}x^{\frac{5}{2}}}-\frac{2b(3Ab-2Ba)}{3a^{4}x^{\frac{3}{2}}}+\frac{2b^{2}(4Ab-3Ba)}{a^{5}\sqrt{x}}+\frac{2b^{3}\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(9Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right.}{a^{5}}$
default	$-\frac{2A}{7a^{2}x^{\frac{7}{2}}}-\frac{2(-2Ab+Ba)}{5a^{3}x^{\frac{5}{2}}}-\frac{2b(3Ab-2Ba)}{3a^{4}x^{\frac{3}{2}}}+\frac{2b^{2}(4Ab-3Ba)}{a^{5}\sqrt{x}}+\frac{2b^{3}\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(9Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right.}{a^{5}}$
risch	$-\frac{2\left(-420A\,b^{3}x^{3}+315Ba\,b^{2}x^{3}+105aA\,b^{2}x^{2}-70B\,a^{2}b\,x^{2}-42a^{2}Abx+21B\,a^{3}x+15a^{3}A\right)}{105a^{5}x^{\frac{7}{2}}}+\frac{b^{3}\left(\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)\sqrt{x}}{bx+a}+\frac{(9Ab\right.}{}}{a^{5}}$

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7*A/a^2/x^(7/2)-2/5*(-2*A*b+B*a)/a^3/x^(5/2)-2/3*b*(3*A*b-2*B*a)/a^4/x^
(3/2)+2*b^2*(4*A*b-3*B*a)/a^5/x^(1/2)+2/a^5*b^3*((1/2*A*b-1/2*B*a)*x^(1/2)
/(b*x+a)+1/2*(9*A*b-7*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = \left[-\frac{105\left((7\,Bab^3-9\,Ab^4)x^5+(7\,Ba^2b^2-9\,Aab^3)x^4\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\right.$$

$$-\frac{105\left((7\,Bab^3-9\,Ab^4)x^5+(7\,Ba^2b^2-9\,Aab^3)x^4\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(30\,Aa^4+105\,(7\,Bab^3-9\,Ab^4)}{105\,(a^5bx^5+a^6x^4)}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/210*(105*((7*B*a*b^3 - 9*A*b^4)*x^5 + (7*B*a^2*b^2 - 9*A*a*b^3)*x^4)*s
qrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(30*A*a^4
+ 105*(7*B*a*b^3 - 9*A*b^4)*x^4 + 70*(7*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 14*(7
*B*a^3*b - 9*A*a^2*b^2)*x^2 + 6*(7*B*a^4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b*x
^5 + a^6*x^4), -1/105*(105*((7*B*a*b^3 - 9*A*b^4)*x^5 + (7*B*a^2*b^2 - 9*A
*a*b^3)*x^4)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) + (30*A*a^4 + 105*(7*B*a*
b^3 - 9*A*b^4)*x^4 + 70*(7*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 14*(7*B*a^3*b - 9*
A*a^2*b^2)*x^2 + 6*(7*B*a^4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b*x^5 + a^6*x^4)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(143) = 286$.

Time = 122.79 (sec) , antiderivative size = 1142, normalized size of antiderivative = 7.99

$$\int \frac{A + Bx}{x^{9/2}(a + bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(9/2)/(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(9*x**(9/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2)))/a**2, Eq(b, 0)), ((-2*A/(11*x
**(11/2)) - 2*B/(9*x**(9/2)))/b**2, Eq(a, 0)), (-60*A*a**4*sqrt(-a/b)/(210
*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/b)) + 108*A*a**3*b
*x*sqrt(-a/b)/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/
b)) - 252*A*a**2*b**2*x**2*sqrt(-a/b)/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*
a**5*b*x**(9/2)*sqrt(-a/b)) + 945*A*a*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/
b))/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/b)) - 945*
A*a*b**3*x**(7/2)*log(sqrt(x) + sqrt(-a/b))/(210*a**6*x**(7/2)*sqrt(-a/b)
+ 210*a**5*b*x**(9/2)*sqrt(-a/b)) + 1260*A*a*b**3*x**3*sqrt(-a/b)/(210*a**
6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/b)) + 945*A*b**4*x**(9
/2)*log(sqrt(x) - sqrt(-a/b))/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x
**(9/2)*sqrt(-a/b)) - 945*A*b**4*x**(9/2)*log(sqrt(x) + sqrt(-a/b))/(210*a
**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/b)) + 1890*A*b**4*x*
*4*sqrt(-a/b)/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/
b)) - 84*B*a**4*x*sqrt(-a/b)/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x*
*(9/2)*sqrt(-a/b)) + 196*B*a**3*b*x**2*sqrt(-a/b)/(210*a**6*x**(7/2)*sqrt(
-a/b) + 210*a**5*b*x**(9/2)*sqrt(-a/b)) - 735*B*a**2*b**2*x**(7/2)*log(sqr
t(x) - sqrt(-a/b))/(210*a**6*x**(7/2)*sqrt(-a/b) + 210*a**5*b*x**(9/2)*sqr
t(-a/b)) + 735*B*a**2*b**2*x**(7/2)*log(sqrt(x) + sqrt(-a/b))/(210*a**6...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = -\frac{30\,Aa^4+105\,(7\,Bab^3-9\,Ab^4)x^4+70\,(7\,Ba^2b^2-9\,Aab^3)x^3-14\,(7\,Ba^3b-9\,Aa^2b^2)x^2+6\,(7\,Ba^4-9\,A}{105\left(a^5bx^{\frac{9}{2}}+a^6x^{\frac{7}{2}}\right)} - \frac{(7\,Bab^3-9\,Ab^4)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/105*(30*A*a^4 + 105*(7*B*a*b^3 - 9*A*b^4)*x^4 + 70*(7*B*a^2*b^2 - 9*A*a
*b^3)*x^3 - 14*(7*B*a^3*b - 9*A*a^2*b^2)*x^2 + 6*(7*B*a^4 - 9*A*a^3*b)*x)/
(a^5*b*x^(9/2) + a^6*x^(7/2)) - (7*B*a*b^3 - 9*A*b^4)*arctan(b*sqrt(x)/sqr
t(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = -\frac{(7\,Bab^3-9\,Ab^4)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{Bab^3\sqrt{x}-Ab^4\sqrt{x}}{(bx+a)a^5} - \frac{2\,(315\,Bab^2x^3-420\,Ab^3x^3-70\,Ba^2bx^2+105\,Aab^2x^2+21\,Ba^3x-42\,Aa^2bx+15\,Aa^3)}{105\,a^5x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-(7*B*a*b^3 - 9*A*b^4)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - (B*a*
b^3*sqrt(x) - A*b^4*sqrt(x))/((b*x + a)*a^5) - 2/105*(315*B*a*b^2*x^3 - 42
0*A*b^3*x^3 - 70*B*a^2*b*x^2 + 105*A*a*b^2*x^2 + 21*B*a^3*x - 42*A*a^2*b*x
 + 15*A*a^3)/(a^5*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = \frac{\frac{2\,x\,(9\,A\,b-7\,B\,a)}{35\,a^2}-\frac{2\,A}{7\,a}+\frac{2\,b^2\,x^3\,(9\,A\,b-7\,B\,a)}{3\,a^4}+\frac{b^3\,x^4\,(9\,A\,b-7\,B\,a)}{a^5}-\frac{2\,b\,x^2\,(9\,A\,b-7\,B\,a)}{15\,a^3}}{a\,x^{7/2}+b\,x^{9/2}} + \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(9\,A\,b-7\,B\,a)}{a^{11/2}}$$

input
```
int((A + B*x)/(x^(9/2)*(a + b*x)^2),x)
```

output
```
((2*x*(9*A*b - 7*B*a))/(35*a^2) - (2*A)/(7*a) + (2*b^2*x^3*(9*A*b - 7*B*a)
)/(3*a^4) + (b^3*x^4*(9*A*b - 7*B*a))/a^5 - (2*b*x^2*(9*A*b - 7*B*a))/(15*
a^3))/(a*x^(7/2) + b*x^(9/2)) + (b^(5/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(
9*A*b - 7*B*a))/a^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^2}\,dx = \frac{2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^3x^3-\frac{2a^4}{7}+\frac{2a^3bx}{5}-\frac{2a^2b^2x^2}{3}+2a\,b^3x^3}{\sqrt{x}\,a^5x^3}$$

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^2,x)
```

output
```
(2*(105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**3*x
**3 - 15*a**4 + 21*a**3*b*x - 35*a**2*b**2*x**2 + 105*a*b**3*x**3))/(105*s
qrt(x)*a**5*x**3)
```

3.216 $\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = -\frac{a(7Ab-13aB)\sqrt{x}}{b^5}+\frac{(7Ab-15aB)x^{3/2}}{6b^4}+\frac{2Bx^{5/2}}{5b^3} -\frac{(Ab-aB)x^{7/2}}{2b^2(a+bx)^2}-\frac{a^2(7Ab-11aB)\sqrt{x}}{4b^5(a+bx)}+\frac{7a^{3/2}(5Ab-9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{11/2}}$$

output
```
-a*(7*A*b-13*B*a)*x^(1/2)/b^5+1/6*(7*A*b-15*B*a)*x^(3/2)/b^4+2/5*B*x^(5/2)
/b^3-1/2*(A*b-B*a)*x^(7/2)/b^2/(b*x+a)^2-1/4*a^2*(7*A*b-11*B*a)*x^(1/2)/b^
5/(b*x+a)+7/4*a^(3/2)*(5*A*b-9*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(11/
2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{\sqrt{x}(945a^4B-525a^3b(A-3Bx)+8b^4x^3(5A+3Bx)-8ab^3x^2(35A+9Bx)+7a^2b}{60b^5(a+bx)^2} -\frac{7a^{3/2}(-5Ab+9aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a + b*x)^3,x]
```

output
```
(Sqrt[x]*(945*a^4*B - 525*a^3*b*(A - 3*B*x) + 8*b^4*x^3*(5*A + 3*B*x) - 8*
a*b^3*x^2*(35*A + 9*B*x) + 7*a^2*b^2*x*(-125*A + 72*B*x)))/(60*b^5*(a + b*
x)^2) - (7*a^(3/2)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*
b^(11/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {87, 51, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\int \frac{x^{7/2}}{(a+bx)^2}dx}{4ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\int \frac{x^{5/2}}{a+bx}dx}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\int\frac{\sqrt{x}}{a+bx}dx}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}$$

$\downarrow$ 60

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}$$

$\downarrow$ 73

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}$$

$\downarrow$ 218

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(5Ab-9aB)\left(\frac{7\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{b}\right)}{2b}-\frac{x^{7/2}}{b(a+bx)}\right)}{4ab}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a + b*x)^3,x]
```

output
```
((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x)^2) - ((5*A*b - 9*a*B)*(-(x^(7/2)/(b
*(a + b*x))) + (7*((2*x^(5/2))/(5*b) - (a*((2*x^(3/2))/(3*b) - (a*((2*Sqrt
[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/b))/(
2*b)))/(4*a*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{2\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+15Babx+45abA-90a^{2}B\right)\sqrt{x}}{15b^{5}}+\frac{a^{2}\left(\frac{2\left(-\frac{13}{8}b^{2}A+\frac{17}{8}abB\right)x^{\frac{3}{2}}-\frac{a(11Ab-15Ba)\sqrt{x}}{4}}{(bx+a)^{2}}+\frac{7(5Ab-9Ba}{}\right.}{b^{5}}$
derivativedivides	$-\frac{2\left(-\frac{b^{2}B\,x^{\frac{5}{2}}}{5}-\frac{A\,b^{2}x^{\frac{3}{2}}}{3}+Bab\,x^{\frac{3}{2}}+3abA\sqrt{x}-6a^{2}B\sqrt{x}\right)}{b^{5}}+\frac{2a^{2}\left(\frac{\left(-\frac{13}{8}b^{2}A+\frac{17}{8}abB\right)x^{\frac{3}{2}}-\frac{a(11Ab-15Ba)\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{7(5Ab-}{}\right.}{b^{5}}$
default	$-\frac{2\left(-\frac{b^{2}B\,x^{\frac{5}{2}}}{5}-\frac{A\,b^{2}x^{\frac{3}{2}}}{3}+Bab\,x^{\frac{3}{2}}+3abA\sqrt{x}-6a^{2}B\sqrt{x}\right)}{b^{5}}+\frac{2a^{2}\left(\frac{\left(-\frac{13}{8}b^{2}A+\frac{17}{8}abB\right)x^{\frac{3}{2}}-\frac{a(11Ab-15Ba)\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{7(5Ab-}{}\right.}{b^{5}}$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*b^2*x^2-5*A*b^2*x+15*B*a*b*x+45*A*a*b-90*B*a^2)*x^(1/2)/b^5+a^
2/b^5*(2*((-13/8*b^2*A+17/8*a*b*B)*x^(3/2)-1/8*a*(11*A*b-15*B*a)*x^(1/2))/
(b*x+a)^2+7/4*(5*A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.67

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = \left[-\frac{105\,(9\,Ba^4 - 5\,Aa^3b + (9\,Ba^2b^2 - 5\,Aab^3)x^2 + 2\,(9\,Ba^3b - 5\,Aa^2b^2)x)\sqrt{-\frac{a}{b}}\log\right.$$

$$-\frac{105\,(9\,Ba^4 - 5\,Aa^3b + (9\,Ba^2b^2 - 5\,Aab^3)x^2 + 2\,(9\,Ba^3b - 5\,Aa^2b^2)x)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right) - (24\,Bb^4x^4}{60\,(b^7x^2 + 2}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/120*(105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 2*(9*B
*a^3*b - 5*A*a^2*b^2)*x)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)
/(b*x + a)) - 2*(24*B*b^4*x^4 + 945*B*a^4 - 525*A*a^3*b - 8*(9*B*a*b^3 - 5
*A*b^4)*x^3 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 175*(9*B*a^3*b - 5*A*a^2*
b^2)*x)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/60*(105*(9*B*a^4 - 5*
A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x)*s
qrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - (24*B*b^4*x^4 + 945*B*a^4 - 525*A
*a^3*b - 8*(9*B*a*b^3 - 5*A*b^4)*x^3 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^2 +
175*(9*B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(150) = 300$.

Time = 71.73 (sec) , antiderivative size = 1652, normalized size of antiderivative = 10.80

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(11/2)/11)/a**3, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B
*x**(5/2)/5)/b**3, Eq(a, 0)), (525*A*a**4*b*log(sqrt(x) - sqrt(-a/b))/(120
*a**2*b**6*sqrt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b)
) - 525*A*a**4*b*log(sqrt(x) + sqrt(-a/b))/(120*a**2*b**6*sqrt(-a/b) + 240
*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b)) - 1050*A*a**3*b**2*sqrt(x
)*sqrt(-a/b)/(120*a**2*b**6*sqrt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 120*b**
8*x**2*sqrt(-a/b)) + 1050*A*a**3*b**2*x*log(sqrt(x) - sqrt(-a/b))/(120*a**
2*b**6*sqrt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b)) -
1050*A*a**3*b**2*x*log(sqrt(x) + sqrt(-a/b))/(120*a**2*b**6*sqrt(-a/b) + 2
40*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b)) - 1750*A*a**2*b**3*x**(
3/2)*sqrt(-a/b)/(120*a**2*b**6*sqrt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 120*
b**8*x**2*sqrt(-a/b)) + 525*A*a**2*b**3*x**2*log(sqrt(x) - sqrt(-a/b))/(12
0*a**2*b**6*sqrt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b
)) - 525*A*a**2*b**3*x**2*log(sqrt(x) + sqrt(-a/b))/(120*a**2*b**6*sqrt(-a
/b) + 240*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b)) - 560*A*a*b**4*x
**(5/2)*sqrt(-a/b)/(120*a**2*b**6*sqrt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 1
20*b**8*x**2*sqrt(-a/b)) + 80*A*b**5*x**(7/2)*sqrt(-a/b)/(120*a**2*b**6*sq
rt(-a/b) + 240*a*b**7*x*sqrt(-a/b) + 120*b**8*x**2*sqrt(-a/b)) - 945*B*a**
5*log(sqrt(x) - sqrt(-a/b))/(120*a**2*b**6*sqrt(-a/b) + 240*a*b**7*x*sq...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{(17\,Ba^3b-13\,Aa^2b^2)x^{\frac{3}{2}}+(15\,Ba^4-11\,Aa^3b)\sqrt{x}}{4\,(b^7x^2+2\,ab^6x+a^2b^5)} - \frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^5} + \frac{2\left(3\,Bb^2x^{\frac{5}{2}}-5\,(3\,Bab-Ab^2)x^{\frac{3}{2}}+45\,(2\,Ba^2-Aab)\sqrt{x}\right)}{15\,b^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*((17*B*a^3*b - 13*A*a^2*b^2)*x^(3/2) + (15*B*a^4 - 11*A*a^3*b)*sqrt(x)
)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) - 7/4*(9*B*a^3 - 5*A*a^2*b)*arctan(b*sqr
t(x)/sqrt(a*b))/(sqrt(a*b)*b^5) + 2/15*(3*B*b^2*x^(5/2) - 5*(3*B*a*b - A*b
^2)*x^(3/2) + 45*(2*B*a^2 - A*a*b)*sqrt(x))/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = -\frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^5} + \frac{17\,Ba^3bx^{\frac{3}{2}}-13\,Aa^2b^2x^{\frac{3}{2}}+15\,Ba^4\sqrt{x}-11\,Aa^3b\sqrt{x}}{4\,(bx+a)^2b^5} + \frac{2\left(3\,Bb^{12}x^{\frac{5}{2}}-15\,Bab^{11}x^{\frac{3}{2}}+5\,Ab^{12}x^{\frac{3}{2}}+90\,Ba^2b^{10}\sqrt{x}-45\,Aab^{11}\sqrt{x}\right)}{15\,b^{15}}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-7/4*(9*B*a^3 - 5*A*a^2*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) + 1
/4*(17*B*a^3*b*x^(3/2) - 13*A*a^2*b^2*x^(3/2) + 15*B*a^4*sqrt(x) - 11*A*a^
3*b*sqrt(x))/((b*x + a)^2*b^5) + 2/15*(3*B*b^12*x^(5/2) - 15*B*a*b^11*x^(3
/2) + 5*A*b^12*x^(3/2) + 90*B*a^2*b^10*sqrt(x) - 45*A*a*b^11*sqrt(x))/b^15
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.20

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = x^{3/2}\left(\frac{2A}{3b^3}-\frac{2Ba}{b^4}\right) - \frac{x^{3/2}\left(\frac{13Aa^2b^2}{4}-\frac{17Ba^3b}{4}\right)-\sqrt{x}\left(\frac{15Ba^4}{4}-\frac{11Aa^3b}{4}\right)}{a^2b^5+2ab^6x+b^7x^2} - \sqrt{x}\left(\frac{3a\left(\frac{2A}{b^3}-\frac{6Ba}{b^4}\right)}{b}+\frac{6Ba^2}{b^5}\right)+\frac{2Bx^{5/2}}{5b^3}-\frac{7a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\sqrt{b}\sqrt{x}\,(5Ab-9Ba)}{9Ba^3-5Aa^2b}\right)(5Ab-9Ba)}{4b^{11/2}}$$

input
```
int((x^(7/2)*(A + B*x))/(a + b*x)^3,x)
```

output
```
x^(3/2)*((2*A)/(3*b^3) - (2*B*a)/b^4) - (x^(3/2)*((13*A*a^2*b^2)/4 - (17*B
*a^3*b)/4) - x^(1/2)*((15*B*a^4)/4 - (11*A*a^3*b)/4))/(a^2*b^5 + b^7*x^2 +
 2*a*b^6*x) - x^(1/2)*((3*a*((2*A)/b^3 - (6*B*a)/b^4))/b + (6*B*a^2)/b^5)
+ (2*B*x^(5/2))/(5*b^3) - (7*a^(3/2)*atan((a^(3/2)*b^(1/2)*x^(1/2)*(5*A*b
- 9*B*a))/(9*B*a^3 - 5*A*a^2*b))*(5*A*b - 9*B*a))/(4*b^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{-105\sqrt{b}\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\sqrt{a}}\right)a^3-105\sqrt{b}\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\sqrt{a}}\right)a^2bx+105\sqrt{x}\,a^3b+70\sqrt{x}\,a^2}{15b^5\,(bx+a)}$$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a)^3,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**3 - 105*sqr
t(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2*b*x + 105*sqrt(x)*a*
*3*b + 70*sqrt(x)*a**2*b**2*x - 14*sqrt(x)*a*b**3*x**2 + 6*sqrt(x)*b**4*x*
*3)/(15*b**5*(a + b*x))
```

3.217 $\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{(5Ab-13aB)\sqrt{x}}{2b^4}+\frac{2Bx^{3/2}}{3b^3}-\frac{(Ab-aB)x^{5/2}}{2b^2(a+bx)^2}+\frac{a(5Ab-9aB)\sqrt{x}}{4b^4(a+bx)}-\frac{5\sqrt{a}(3Ab-7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

output
```
1/2*(5*A*b-13*B*a)*x^(1/2)/b^4+2/3*B*x^(3/2)/b^3-1/2*(A*b-B*a)*x^(5/2)/b^2
/(b*x+a)^2+1/4*a*(5*A*b-9*B*a)*x^(1/2)/b^4/(b*x+a)-5/4*a^(1/2)*(3*A*b-7*B*
a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{\sqrt{x}(-105a^3B+ab^2x(75A-56Bx)+5a^2b(9A-35Bx)+8b^3x^2(3A+Bx))}{12b^4(a+bx)^2}+\frac{5\sqrt{a}(-3Ab+7aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x)^3,x]
```

output
```
(Sqrt[x]*(-105*a^3*B + a*b^2*x*(75*A - 56*B*x) + 5*a^2*b*(9*A - 35*B*x) +
8*b^3*x^2*(3*A + B*x)))/(12*b^4*(a + b*x)^2) + (5*Sqrt[a]*(-3*A*b + 7*a*B)
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3} \, dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\int \frac{x^{5/2}}{(a+bx)^2}dx}{4ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\int \frac{x^{3/2}}{a+bx}dx}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}dx}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}$$

$\Big\downarrow$ 73

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}$$

$\Big\downarrow$ 218

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(3Ab-7aB)\left(\frac{5\left(\frac{2x^{3/2}}{3b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}\right)}{2b}-\frac{x^{5/2}}{b(a+bx)}\right)}{4ab}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x)^3,x]
```

output
```
((A*b - a*B)*x^(7/2))/(2*a*b*(a + b*x)^2) - ((3*A*b - 7*a*B)*(-(x^(5/2)/(b
*(a + b*x))) + (5*((2*x^(3/2))/(3*b) - (a*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)))/b))/(2*b)))/(4*a*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2(bBx+3Ab-9Ba)\sqrt{x}}{3b^4}-\frac{a\left(\frac{2\left(-\frac{9}{8}b^2A+\frac{13}{8}abB\right)x^{\frac{3}{2}}-\frac{a(7Ab-11Ba)\sqrt{x}}{4}}{(bx+a)^2}+\frac{5(3Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^4}$	98
derivativedivides	$\frac{\frac{2bBx^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-6Ba\sqrt{x}}{b^4}-\frac{2a\left(\frac{\left(-\frac{9}{8}b^2A+\frac{13}{8}abB\right)x^{\frac{3}{2}}-\frac{a(7Ab-11Ba)\sqrt{x}}{8}}{(bx+a)^2}+\frac{5(3Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	102
default	$\frac{\frac{2bBx^{\frac{3}{2}}}{3}+2Ab\sqrt{x}-6Ba\sqrt{x}}{b^4}-\frac{2a\left(\frac{\left(-\frac{9}{8}b^2A+\frac{13}{8}abB\right)x^{\frac{3}{2}}-\frac{a(7Ab-11Ba)\sqrt{x}}{8}}{(bx+a)^2}+\frac{5(3Ab-7Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	102

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*b*x+3*A*b-9*B*a)*x^(1/2)/b^4-a/b^4*(2*((-9/8*b^2*A+13/8*a*b*B)*x^(3
/2)-1/8*a*(7*A*b-11*B*a)*x^(1/2))/(b*x+a)^2+5/4*(3*A*b-7*B*a)/(a*b)^(1/2)*
arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.66

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = \left[-\frac{15\,(7\,Ba^3-3\,Aa^2b+(7\,Bab^2-3\,Ab^3)x^2+2\,(7\,Ba^2b-3\,Aab^2)x)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-}{}\right.}{24}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/24*(15*(7*B*a^3 - 3*A*a^2*b + (7*B*a*b^2 - 3*A*b^3)*x^2 + 2*(7*B*a^2*b
 - 3*A*a*b^2)*x)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) - 2*(8*B*b^3*x^3 - 105*B*a^3 + 45*A*a^2*b - 8*(7*B*a*b^2 - 3*A*b^3)*x^
2 - 25*(7*B*a^2*b - 3*A*a*b^2)*x)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
, 1/12*(15*(7*B*a^3 - 3*A*a^2*b + (7*B*a*b^2 - 3*A*b^3)*x^2 + 2*(7*B*a^2*b
 - 3*A*a*b^2)*x)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*B*b^3*x^3 -
105*B*a^3 + 45*A*a^2*b - 8*(7*B*a*b^2 - 3*A*b^3)*x^2 - 25*(7*B*a^2*b - 3*A
*a*b^2)*x)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1496 vs. $2(128) = 256$.

Time = 23.57 (sec) , antiderivative size = 1496, normalized size of antiderivative = 11.42

$$\int \frac{x^{5/2}(A + Bx)}{(a + bx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(7/2)/7 + 2*B*x**(9/2)/9)/a**3, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(3/2
)/3)/b**3, Eq(a, 0)), (-45*A*a**3*b*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**
5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 45*A*a*
*3*b*log(sqrt(x) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt
(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 90*A*a**2*b**2*sqrt(x)*sqrt(-a/b)/(24*
a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) -
 90*A*a**2*b**2*x*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*
a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 90*A*a**2*b**2*x*log(sqrt
(x) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b
**7*x**2*sqrt(-a/b)) + 150*A*a*b**3*x**(3/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt
(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 45*A*a*b**3*x
**2*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(
-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 45*A*a*b**3*x**2*log(sqrt(x) + sqrt(-a/
b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(
-a/b)) + 48*A*b**4*x**(5/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**
6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 105*B*a**4*log(sqrt(x) - sqrt(
-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sq
rt(-a/b)) - 105*B*a**4*log(sqrt(x) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b)
+ 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*B*a**3*b*sqrt...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = -\frac{(13\,Ba^2b-9\,Aab^2)x^{\frac{3}{2}}+(11\,Ba^3-7\,Aa^2b)\sqrt{x}}{4\,(b^6x^2+2\,ab^5x+a^2b^4)} + \frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4} + \frac{2\left(Bbx^{\frac{3}{2}}-3\,(3\,Ba-Ab)\sqrt{x}\right)}{3\,b^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*((13*B*a^2*b - 9*A*a*b^2)*x^(3/2) + (11*B*a^3 - 7*A*a^2*b)*sqrt(x))/(
b^6*x^2 + 2*a*b^5*x + a^2*b^4) + 5/4*(7*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/
sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(B*b*x^(3/2) - 3*(3*B*a - A*b)*sqrt(x))/b
^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4} - \frac{13\,Ba^2bx^{\frac{3}{2}}-9\,Aab^2x^{\frac{3}{2}}+11\,Ba^3\sqrt{x}-7\,Aa^2b\sqrt{x}}{4\,(bx+a)^2b^4} + \frac{2\left(Bb^6x^{\frac{3}{2}}-9\,Bab^5\sqrt{x}+3\,Ab^6\sqrt{x}\right)}{3\,b^9}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
5/4*(7*B*a^2 - 3*A*a*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*
(13*B*a^2*b*x^(3/2) - 9*A*a*b^2*x^(3/2) + 11*B*a^3*sqrt(x) - 7*A*a^2*b*sqr
t(x))/((b*x + a)^2*b^4) + 2/3*(B*b^6*x^(3/2) - 9*B*a*b^5*sqrt(x) + 3*A*b^6
*sqrt(x))/b^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{x^{3/2}\left(\frac{9\,A\,a\,b^2}{4}-\frac{13\,B\,a^2\,b}{4}\right)-\sqrt{x}\left(\frac{11\,B\,a^3}{4}-\frac{7\,A\,a^2\,b}{4}\right)}{a^2\,b^4+2\,a\,b^5\,x+b^6\,x^2} + \sqrt{x}\left(\frac{2\,A}{b^3}-\frac{6\,B\,a}{b^4}\right) + \frac{2\,B\,x^{3/2}}{3\,b^3} + \frac{5\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(3\,A\,b-7\,B\,a)}{7\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-7\,B\,a)}{4\,b^{9/2}}$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x)^3,x)
```

output
```
(x^(3/2)*((9*A*a*b^2)/4 - (13*B*a^2*b)/4) - x^(1/2)*((11*B*a^3)/4 - (7*A*a
^2*b)/4))/(a^2*b^4 + b^6*x^2 + 2*a*b^5*x) + x^(1/2)*((2*A)/b^3 - (6*B*a)/b
^4) + (2*B*x^(3/2))/(3*b^3) + (5*a^(1/2)*atan((a^(1/2)*b^(1/2)*x^(1/2)*(3*
A*b - 7*B*a))/(7*B*a^2 - 3*A*a*b))*(3*A*b - 7*B*a))/(4*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx-15\sqrt{x}\,a^2b-10\sqrt{x}\,a\,b^2x+}{3b^4\,(bx+a)}$$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a**2 + 15*sqrt(b)*
sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x - 15*sqrt(x)*a**2*b - 10
*sqrt(x)*a*b**2*x + 2*sqrt(x)*b**3*x**2)/(3*b**4*(a + b*x))
```

3.218 $\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3} dx = \frac{2B\sqrt{x}}{b^3} - \frac{(Ab-aB)x^{3/2}}{2b^2(a+bx)^2} - \frac{(3Ab-7aB)\sqrt{x}}{4b^3(a+bx)} + \frac{3(Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{7/2}}$$

output
```
2*B*x^(1/2)/b^3-1/2*(A*b-B*a)*x^(3/2)/b^2/(b*x+a)^2-1/4*(3*A*b-7*B*a)*x^(1
/2)/b^3/(b*x+a)+3/4*(A*b-5*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^
(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3} dx = \frac{\sqrt{x}(15a^2B + b^2x(-5A+8Bx) + a(-3Ab+25bBx))}{4b^3(a+bx)^2} + \frac{3(Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x)^3,x]
```

output
```
(Sqrt[x]*(15*a^2*B + b^2*x*(-5*A + 8*B*x) + a*(-3*A*b + 25*b*B*x)))/(4*b^3
*(a + b*x)^2) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqr
t[a]*b^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = $\{87, 51, 60, 73, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\int \frac{x^{3/2}}{(a+bx)^2}dx}{4ab}$$

$$\Big\downarrow \text{51}$$

$$\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\int \frac{\sqrt{x}}{a+bx}dx}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}dx}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{a+bx}d\sqrt{x}}{b}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}$$

$\Big\downarrow$ 218

$$\frac{x^{5/2}(Ab-aB)}{2ab(a+bx)^2}-\frac{(Ab-5aB)\left(\frac{3\left(\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^{3/2}}{b(a+bx)}\right)}{4ab}$$

input

```
Int[(x^(3/2)*(A + B*x))/(a + b*x)^3,x]
```

output

```
((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*x)^2) - ((A*b - 5*a*B)*(-(x^(3/2)/(b*(
a + b*x))) + (3*((2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/b^(3/2)))/(2*b)))/(4*a*b)
```

Defintions of rubi rules used

rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{3}{2}}-\frac{a(3Ab-7Ba)\sqrt{x}}{8}\right)}{(bx+a)^2}+\frac{3(Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{b^3}$	83
default	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{3}{2}}-\frac{a(3Ab-7Ba)\sqrt{x}}{8}\right)}{(bx+a)^2}+\frac{3(Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{b^3}$	83
risch	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^{\frac{3}{2}}-\frac{a(3Ab-7Ba)\sqrt{x}}{4}}{(bx+a)^2}+\frac{3(Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{b^3}$	83

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b^3+2/b^3*(((-5/8*b^2*A+9/8*a*b*B)*x^(3/2)-1/8*a*(3*A*b-7*B*a)
*x^(1/2))/(b*x+a)^2+3/8*(A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.01

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}\,dx = \left[\frac{3\,(5\,Ba^3 - Aa^2b + (5\,Bab^2 - Ab^3)x^2 + 2\,(5\,Ba^2b - Aab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-a}}{bx+a}\right.}{8\,(ab^6x^2 + 2\,a^2b^5x +}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(5*B*a^3 - A*a^2*b + (5*B*a*b^2 - A*b^3)*x^2 + 2*(5*B*a^2*b - A*a*
b^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(8*
B*a*b^3*x^2 + 15*B*a^3*b - 3*A*a^2*b^2 + 5*(5*B*a^2*b^2 - A*a*b^3)*x)*sqrt
(x))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/4*(3*(5*B*a^3 - A*a^2*b + (5*B
*a*b^2 - A*b^3)*x^2 + 2*(5*B*a^2*b - A*a*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (8*B*a*b^3*x^2 + 15*B*a^3*b - 3*A*a^2*b^2 + 5*(5*B*a^2*b^
2 - A*a*b^3)*x)*sqrt(x))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. 2(104) = 208.

Time = 10.44 (sec) , antiderivative size = 1333, normalized size of antiderivative = 12.58

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b*x+a)**3,x)
```

output

```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2*A*x
**(5/2)/5 + 2*B*x**(7/2)/7)/a**3, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))
/b**3, Eq(a, 0)), (3*A*a**2*b*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(
-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 3*A*a**2*b*log(
sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8
*b**6*x**2*sqrt(-a/b)) - 6*A*a*b**2*sqrt(x)*sqrt(-a/b)/(8*a**2*b**4*sqrt(-
a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 6*A*a*b**2*x*log
(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) +
8*b**6*x**2*sqrt(-a/b)) - 6*A*a*b**2*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b
**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 10*A*b
**3*x**(3/2)*sqrt(-a/b)/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) +
 8*b**6*x**2*sqrt(-a/b)) + 3*A*b**3*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2
*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 3*A*
b**3*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*
sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 15*B*a**3*log(sqrt(x) - sqrt(-a/b))
/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)
) + 15*B*a**3*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**
5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 30*B*a**2*b*sqrt(x)*sqrt(-a/b)/
(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b))
 - 30*B*a**2*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{(9\,Bab-5\,Ab^2)x^{\frac{3}{2}}+(7\,Ba^2-3\,Aab)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)} + \frac{2\,B\sqrt{x}}{b^3} - \frac{3\,(5\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3}$$

input

```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/4*((9*B*a*b - 5*A*b^2)*x^(3/2) + (7*B*a^2 - 3*A*a*b)*sqrt(x))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + 2*B*sqrt(x)/b^3 - 3/4*(5*B*a - A*b)*arctan(b*sqrt(
x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{2\,B\sqrt{x}}{b^3} - \frac{3\,(5\,Ba-Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} + \frac{9\,Babx^{\frac{3}{2}}-5\,Ab^2x^{\frac{3}{2}}+7\,Ba^2\sqrt{x}-3\,Aab\sqrt{x}}{4\,(bx+a)^2b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^3 - 3/4*(5*B*a - A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^3) + 1/4*(9*B*a*b*x^(3/2) - 5*A*b^2*x^(3/2) + 7*B*a^2*sqrt(x) - 3*A*a*b
*sqrt(x))/((b*x + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{\sqrt{x}\left(\frac{7\,B\,a^2}{4}-\frac{3\,A\,a\,b}{4}\right)-x^{3/2}\left(\frac{5\,A\,b^2}{4}-\frac{9\,B\,a\,b}{4}\right)}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2} + \frac{2\,B\,\sqrt{x}}{b^3} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b-5\,B\,a)}{4\,\sqrt{a}\,b^{7/2}}$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x)^3,x)
```

output
```
(x^(1/2)*((7*B*a^2)/4 - (3*A*a*b)/4) - x^(3/2)*((5*A*b^2)/4 - (9*B*a*b)/4)
)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) + (2*B*x^(1/2))/b^3 + (3*atan((b^(1/2)*x
^(1/2))/a^(1/2))*(A*b - 5*B*a))/(4*a^(1/2)*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^3}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx+3\sqrt{x}\,ab+2\sqrt{x}\,b^2x}{b^3\,(bx+a)}$$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^3,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(b)*sq
rt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x + 3*sqrt(x)*a*b + 2*sqrt(x)*
b**2*x)/(b**3*(a + b*x))
```

3.219 $\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 97

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3} dx = -\frac{(Ab-aB)\sqrt{x}}{2b^2(a+bx)^2} + \frac{(Ab-5aB)\sqrt{x}}{4ab^2(a+bx)} + \frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{5/2}}$$

output
```
-1/2*(A*b-B*a)*x^(1/2)/b^2/(b*x+a)^2+1/4*(A*b-5*B*a)*x^(1/2)/a/b^2/(b*x+a)
+1/4*(A*b+3*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3} dx = -\frac{\sqrt{x}(aAb+3a^2B-Ab^2x+5abBx)}{4ab^2(a+bx)^2} + \frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x)^3,x]
```

output
```
-1/4*(Sqrt[x]*(a*A*b + 3*a^2*B - A*b^2*x + 5*a*b*B*x))/(a*b^2*(a + b*x)^2)
 + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(3/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(3aB+Ab)\int \frac{\sqrt{x}}{(a+bx)^2}dx}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 51$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{b}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{(3aB+Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}\right)}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab(a+bx)^2}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x)^3,x]
```

output
```
((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x)^2) + ((A*b + 3*a*B)*(-(Sqrt[x]/(b*(
a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2))))/(4*a*b)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{(Ab-5Ba)x^{\frac{3}{2}}}{4ab}-\frac{(Ab+3Ba)\sqrt{x}}{4b^{2}}}{(bx+a)^{2}}+\frac{(Ab+3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}a\sqrt{ab}}$	79
default	$\frac{\frac{(Ab-5Ba)x^{\frac{3}{2}}}{4ab}-\frac{(Ab+3Ba)\sqrt{x}}{4b^{2}}}{(bx+a)^{2}}+\frac{(Ab+3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}a\sqrt{ab}}$	79

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*(A*b-5*B*a)/a/b*x^(3/2)-1/8*(A*b+3*B*a)/b^2*x^(1/2))/(b*x+a)^2+1/4*
(A*b+3*B*a)/b^2/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}\,dx$$
$$= \left[-\frac{(3\,Ba^3 + Aa^2b + (3\,Bab^2 + Ab^3)x^2 + 2\,(3\,Ba^2b + Aab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right) + 2\,(3\,Ba^3b + \right.}{8\,(a^2b^5x^2 + 2\,a^3b^4x + a^4b^3)}$$
$$-\frac{(3\,Ba^3 + Aa^2b + (3\,Bab^2 + Ab^3)x^2 + 2\,(3\,Ba^2b + Aab^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right) + (3\,Ba^3b + Aa^2b^2 + (5}{4\,(a^2b^5x^2 + 2\,a^3b^4x + a^4b^3)}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*((3*B*a^3 + A*a^2*b + (3*B*a*b^2 + A*b^3)*x^2 + 2*(3*B*a^2*b + A*a*b
^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(3*B
*a^3*b + A*a^2*b^2 + (5*B*a^2*b^2 - A*a*b^3)*x)*sqrt(x))/(a^2*b^5*x^2 + 2*
a^3*b^4*x + a^4*b^3), -1/4*((3*B*a^3 + A*a^2*b + (3*B*a*b^2 + A*b^3)*x^2 +
 2*(3*B*a^2*b + A*a*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (3*B
*a^3*b + A*a^2*b^2 + (5*B*a^2*b^2 - A*a*b^3)*x)*sqrt(x))/(a^2*b^5*x^2 + 2*
a^3*b^4*x + a^4*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. 2(90) = 180.

Time = 6.12 (sec) , antiderivative size = 1316, normalized size of antiderivative = 13.57

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(3/2)/3 + 2*B*x**(5/2)/5)/a**3, Eq(b, 0)), ((-2*A/(3*x**(3/2)) - 2*
B/sqrt(x))/b**3, Eq(a, 0)), (A*a**2*b*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b*
*3*sqrt(-a/b) + 16*a**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b)) - A*
a**2*b*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**3*sqrt(-a/b) + 16*a**2*b**4*x*
sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b)) - 2*A*a*b**2*sqrt(x)*sqrt(-a/b)/(8*
a**3*b**3*sqrt(-a/b) + 16*a**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b
)) + 2*A*a*b**2*x*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**3*sqrt(-a/b) + 16*a
**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b)) - 2*A*a*b**2*x*log(sqrt(
x) + sqrt(-a/b))/(8*a**3*b**3*sqrt(-a/b) + 16*a**2*b**4*x*sqrt(-a/b) + 8*a
*b**5*x**2*sqrt(-a/b)) + 2*A*b**3*x**(3/2)*sqrt(-a/b)/(8*a**3*b**3*sqrt(-a
/b) + 16*a**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b)) + A*b**3*x**2*
log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**3*sqrt(-a/b) + 16*a**2*b**4*x*sqrt(-a
/b) + 8*a*b**5*x**2*sqrt(-a/b)) - A*b**3*x**2*log(sqrt(x) + sqrt(-a/b))/(8
*a**3*b**3*sqrt(-a/b) + 16*a**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/
b)) + 3*B*a**3*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**3*sqrt(-a/b) + 16*a**2
*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b)) - 3*B*a**3*log(sqrt(x) + sq
rt(-a/b))/(8*a**3*b**3*sqrt(-a/b) + 16*a**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x
**2*sqrt(-a/b)) - 6*B*a**2*b*sqrt(x)*sqrt(-a/b)/(8*a**3*b**3*sqrt(-a/b) +
16*a**2*b**4*x*sqrt(-a/b) + 8*a*b**5*x**2*sqrt(-a/b)) + 6*B*a**2*b*x*lo...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}\,dx = -\frac{(5\,Bab-Ab^2)x^{\frac{3}{2}}+(3\,Ba^2+Aab)\sqrt{x}}{4\,(ab^4x^2+2\,a^2b^3x+a^3b^2)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*((5*B*a*b - A*b^2)*x^(3/2) + (3*B*a^2 + A*a*b)*sqrt(x))/(a*b^4*x^2 +
2*a^2*b^3*x + a^3*b^2) + 1/4*(3*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sq
rt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}\,dx = \frac{(3\,Ba+Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}ab^2} - \frac{5\,Babx^{\frac{3}{2}} - Ab^2x^{\frac{3}{2}} + 3\,Ba^2\sqrt{x} + Aab\sqrt{x}}{4\,(bx+a)^2ab^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(3*B*a + A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/4*(5*B
*a*b*x^(3/2) - A*b^2*x^(3/2) + 3*B*a^2*sqrt(x) + A*a*b*sqrt(x))/((b*x + a)
^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(A\,b+3\,B\,a)}{4\,a^{3/2}\,b^{5/2}} - \frac{\frac{\sqrt{x}\,(A\,b+3\,B\,a)}{4\,b^2} - \frac{x^{3/2}\,(A\,b-5\,B\,a)}{4\,a\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x)^3,x)
```

output
```
(atan((b^(1/2)*x^(1/2))/a^(1/2))*(A*b + 3*B*a))/(4*a^(3/2)*b^(5/2)) - ((x^
(1/2)*(A*b + 3*B*a))/(4*b^2) - (x^(3/2)*(A*b - 5*B*a))/(4*a*b))/(a^2 + b^2
*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-\sqrt{x}\,ab}{a\,b^2\,(bx+a)}$$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - sqrt(x)*a*b)/(a*b**2*(a + b*x))
```

3.220 $\int \frac{A+Bx}{\sqrt{x}(a+bx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3} dx = \frac{(Ab-aB)\sqrt{x}}{2ab(a+bx)^2} + \frac{(3Ab+aB)\sqrt{x}}{4a^2b(a+bx)} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}b^{3/2}}$$

output
```
1/2*(A*b-B*a)*x^(1/2)/a/b/(b*x+a)^2+1/4*(3*A*b+B*a)*x^(1/2)/a^2/b/(b*x+a)+
1/4*(3*A*b+B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3} dx$$
$$= -\frac{\sqrt{x}(-5aAb+a^2B-3Ab^2x-abBx)}{4a^2b(a+bx)^2} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x)^3),x]
```

output
```
-1/4*(Sqrt[x]*(-5*a*A*b + a^2*B - 3*A*b^2*x - a*b*B*x))/(a^2*b*(a + b*x)^2
) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(aB+3Ab)\int \frac{1}{\sqrt{x}(a+bx)^2}dx}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 52$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{1}{\sqrt{x}(a+bx)}dx}{2a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{1}{a+bx}d\sqrt{x}}{a}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{(aB+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}\right)}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab(a+bx)^2}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x)^3),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x)^2) + ((3*A*b + a*B)*(Sqrt[x]/(a*(a
+ b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])))/(4*a*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{(3Ab+Ba)x^{\frac{3}{2}}}{4a^2}+\frac{(5Ab-Ba)\sqrt{x}}{4ab}}{(bx+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^2b\sqrt{ab}}$	80
default	$\frac{\frac{(3Ab+Ba)x^{\frac{3}{2}}}{4a^2}+\frac{(5Ab-Ba)\sqrt{x}}{4ab}}{(bx+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^2b\sqrt{ab}}$	80

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*(3*A*b+B*a)/a^2*x^(3/2)+1/8*(5*A*b-B*a)/a/b*x^(1/2))/(b*x+a)^2+1/4*
(3*A*b+B*a)/a^2/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.91

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}\,dx$$
$$= \left[-\frac{(Ba^3+3\,Aa^2b+(Bab^2+3\,Ab^3)x^2+2\,(Ba^2b+3\,Aab^2)x)\sqrt{-ab}\log\left(\frac{bx-a-2\,\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\,(Ba^3b-5\,}{8\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)}\right.$$
$$-\frac{(Ba^3+3\,Aa^2b+(Bab^2+3\,Ab^3)x^2+2\,(Ba^2b+3\,Aab^2)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(Ba^3b-5\,Aa^2b^2-(B}{4\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*((B*a^3 + 3*A*a^2*b + (B*a*b^2 + 3*A*b^3)*x^2 + 2*(B*a^2*b + 3*A*a*b
^2)*x)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) + 2*(B*a
^3*b - 5*A*a^2*b^2 - (B*a^2*b^2 + 3*A*a*b^3)*x)*sqrt(x))/(a^3*b^4*x^2 + 2*
a^4*b^3*x + a^5*b^2), -1/4*((B*a^3 + 3*A*a^2*b + (B*a*b^2 + 3*A*b^3)*x^2 +
 2*(B*a^2*b + 3*A*a*b^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (B*a
^3*b - 5*A*a^2*b^2 - (B*a^2*b^2 + 3*A*a*b^3)*x)*sqrt(x))/(a^3*b^4*x^2 + 2*
a^4*b^3*x + a^5*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(90) = 180$.

Time = 8.37 (sec) , antiderivative size = 1345, normalized size of antiderivative = 13.45

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(1/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*sqrt(x) + 2*B*x**(3/2)/3)/a**3, Eq(b, 0)), ((-2*A/(5*x**(5/2)) -
2*B/(3*x**(3/2)))/b**3, Eq(a, 0)), (3*A*a**2*b*log(sqrt(x) - sqrt(-a/b))/(
8*a**4*b**2*sqrt(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt
(-a/b)) - 3*A*a**2*b*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b**2*sqrt(-a/b) + 1
6*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt(-a/b)) + 10*A*a*b**2*sqrt
(x)*sqrt(-a/b)/(8*a**4*b**2*sqrt(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8*a**
2*b**4*x**2*sqrt(-a/b)) + 6*A*a*b**2*x*log(sqrt(x) - sqrt(-a/b))/(8*a**4*b
**2*sqrt(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt(-a/b))
- 6*A*a*b**2*x*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b**2*sqrt(-a/b) + 16*a**3
*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt(-a/b)) + 6*A*b**3*x**(3/2)*sqrt
(-a/b)/(8*a**4*b**2*sqrt(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x
**2*sqrt(-a/b)) + 3*A*b**3*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**4*b**2*sqr
t(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt(-a/b)) - 3*A*b
**3*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b**2*sqrt(-a/b) + 16*a**3*b**3*
x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt(-a/b)) + B*a**3*log(sqrt(x) - sqrt(-a
/b))/(8*a**4*b**2*sqrt(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**
2*sqrt(-a/b)) - B*a**3*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b**2*sqrt(-a/b) +
 16*a**3*b**3*x*sqrt(-a/b) + 8*a**2*b**4*x**2*sqrt(-a/b)) - 2*B*a**2*b*sqr
t(x)*sqrt(-a/b)/(8*a**4*b**2*sqrt(-a/b) + 16*a**3*b**3*x*sqrt(-a/b) + 8...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}\,dx = \frac{(Bab+3\,Ab^2)x^{\frac{3}{2}}-(Ba^2-5\,Aab)\sqrt{x}}{4\,(a^2b^3x^2+2\,a^3b^2x+a^4b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*((B*a*b + 3*A*b^2)*x^(3/2) - (B*a^2 - 5*A*a*b)*sqrt(x))/(a^2*b^3*x^2 +
 2*a^3*b^2*x + a^4*b) + 1/4*(B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqr
t(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}\,dx$$
$$=\frac{(Ba+3\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2b}+\frac{Babx^{\frac{3}{2}}+3\,Ab^2x^{\frac{3}{2}}-Ba^2\sqrt{x}+5\,Aab\sqrt{x}}{4\,(bx+a)^2a^2b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(B*a + 3*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/4*(B*a
*b*x^(3/2) + 3*A*b^2*x^(3/2) - B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x + a)
^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}\,dx=\frac{\frac{x^{3/2}\,(3\,A\,b+B\,a)}{4\,a^2}+\frac{\sqrt{x}\,(5\,A\,b-B\,a)}{4\,a\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(3\,A\,b+B\,a)}{4\,a^{5/2}\,b^{3/2}}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x)^3),x)
```

output
```
((x^(3/2)*(3*A*b + B*a))/(4*a^2) + (x^(1/2)*(5*A*b - B*a))/(4*a*b))/(a^2 +
 b^2*x^2 + 2*a*b*x) + (atan((b^(1/2)*x^(1/2))/a^(1/2))*(3*A*b + B*a))/(4*a
^(5/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx+\sqrt{x}\,ab}{a^2b\,(bx+a)}$$

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*a
tan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x + sqrt(x)*a*b)/(a**2*b*(a + b*x))
```

3.221 $\int \frac{A+Bx}{x^{3/2}(a+bx)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3} dx = -\frac{2A}{a^3\sqrt{x}} - \frac{(Ab-aB)\sqrt{x}}{2a^2(a+bx)^2} - \frac{(7Ab-3aB)\sqrt{x}}{4a^3(a+bx)} - \frac{3(5Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

output
```
-2*A/a^3/x^(1/2)-1/2*(A*b-B*a)*x^(1/2)/a^2/(b*x+a)^2-1/4*(7*A*b-3*B*a)*x^(
1/2)/a^3/(b*x+a)-3/4*(5*A*b-B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(7/2)/b
^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3} dx = \frac{-15Ab^2x^2 + abx(-25A+3Bx) + a^2(-8A+5Bx)}{4a^3\sqrt{x}(a+bx)^2} + \frac{3(-5Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x)^3),x]
```

output
```
(-15*A*b^2*x^2 + a*b*x*(-25*A + 3*B*x) + a^2*(-8*A + 5*B*x))/(4*a^3*Sqrt[x
]*(a + b*x)^2) + (3*(-5*A*b + a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a
^(7/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(5Ab-aB)\int \frac{1}{x^{3/2}(a+bx)^2}dx}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{(5Ab-aB)\left(\frac{3\int \frac{1}{x^{3/2}(a+bx)}dx}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(5Ab-aB)\left(\frac{3\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{2a}+\frac{1}{a\sqrt{x}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}(a+bx)^2}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x)^2) + ((5*A*b - a*B)*(1/(a*Sqrt[x]*(a
+ b*x)) + (3*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]
])/a^(3/2)))/(2*a)))/(4*a*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{3}{2}}+\frac{a(9Ab-5Ba)\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	84
default	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{3}{2}}+\frac{a(9Ab-5Ba)\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	84
risch	$-\frac{2A}{a^{3}\sqrt{x}}-\frac{\frac{2\left(\frac{7}{8}b^{2}A-\frac{3}{8}abB\right)x^{\frac{3}{2}}+\frac{a(9Ab-5Ba)\sqrt{x}}{4}}{(bx+a)^{2}}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{a^{3}}$	85

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^3/x^(1/2)-2/a^3*(((7/8*b^2*A-3/8*a*b*B)*x^(3/2)+1/8*a*(9*A*b-5*B*a)
*x^(1/2))/(b*x+a)^2+3/8*(5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.09

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}\,dx = \left[\frac{3\left((Bab^2-5\,Ab^3)x^3+2\,(Ba^2b-5\,Aab^2)x^2+(Ba^3-5\,Aa^2b)x\right)\sqrt{-ab}\log\left(\frac{bx-a+2}{bx}\right.}{8\,(a^4b^3x^3+2\,a^5b^2x}\right.$$

$$-\frac{3\left((Bab^2-5\,Ab^3)x^3+2\,(Ba^2b-5\,Aab^2)x^2+(Ba^3-5\,Aa^2b)x\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(8\,Aa^3b-3\,(Ba^2b^2}{4\,(a^4b^3x^3+2\,a^5b^2x^2+a^6bx)}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*((B*a*b^2 - 5*A*b^3)*x^3 + 2*(B*a^2*b - 5*A*a*b^2)*x^2 + (B*a^3 -
5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x - a + 2*sqrt(-a*b)*sqrt(x))/(b*x + a)) -
 2*(8*A*a^3*b - 3*(B*a^2*b^2 - 5*A*a*b^3)*x^2 - 5*(B*a^3*b - 5*A*a^2*b^2)*
x)*sqrt(x))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x), -1/4*(3*((B*a*b^2 - 5
*A*b^3)*x^3 + 2*(B*a^2*b - 5*A*a*b^2)*x^2 + (B*a^3 - 5*A*a^2*b)*x)*sqrt(a*
b)*arctan(sqrt(a*b)/(b*sqrt(x))) + (8*A*a^3*b - 3*(B*a^2*b^2 - 5*A*a*b^3)*
x^2 - 5*(B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 +
 a^6*b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(105) = 210$.

Time = 18.67 (sec) , antiderivative size = 1598, normalized size of antiderivative = 14.93

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(3/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/sqrt(x) + 2*B*sqrt(x))/a**3, Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*
B/(5*x**(5/2)))/b**3, Eq(a, 0)), (-15*A*a**2*b*sqrt(x)*log(sqrt(x) - sqrt(
-a/b))/(8*a**5*b*sqrt(x)*sqrt(-a/b) + 16*a**4*b**2*x**(3/2)*sqrt(-a/b) + 8
*a**3*b**3*x**(5/2)*sqrt(-a/b)) + 15*A*a**2*b*sqrt(x)*log(sqrt(x) + sqrt(-
a/b))/(8*a**5*b*sqrt(x)*sqrt(-a/b) + 16*a**4*b**2*x**(3/2)*sqrt(-a/b) + 8*
a**3*b**3*x**(5/2)*sqrt(-a/b)) - 16*A*a**2*b*sqrt(-a/b)/(8*a**5*b*sqrt(x)*
sqrt(-a/b) + 16*a**4*b**2*x**(3/2)*sqrt(-a/b) + 8*a**3*b**3*x**(5/2)*sqrt(
-a/b)) - 30*A*a*b**2*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(8*a**5*b*sqrt(x)*
sqrt(-a/b) + 16*a**4*b**2*x**(3/2)*sqrt(-a/b) + 8*a**3*b**3*x**(5/2)*sqrt(
-a/b)) + 30*A*a*b**2*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(8*a**5*b*sqrt(x)*
sqrt(-a/b) + 16*a**4*b**2*x**(3/2)*sqrt(-a/b) + 8*a**3*b**3*x**(5/2)*sqrt(
-a/b)) - 50*A*a*b**2*x*sqrt(-a/b)/(8*a**5*b*sqrt(x)*sqrt(-a/b) + 16*a**4*b
**2*x**(3/2)*sqrt(-a/b) + 8*a**3*b**3*x**(5/2)*sqrt(-a/b)) - 15*A*b**3*x**
(5/2)*log(sqrt(x) - sqrt(-a/b))/(8*a**5*b*sqrt(x)*sqrt(-a/b) + 16*a**4*b**
2*x**(3/2)*sqrt(-a/b) + 8*a**3*b**3*x**(5/2)*sqrt(-a/b)) + 15*A*b**3*x**(5
/2)*log(sqrt(x) + sqrt(-a/b))/(8*a**5*b*sqrt(x)*sqrt(-a/b) + 16*a**4*b**2*
x**(3/2)*sqrt(-a/b) + 8*a**3*b**3*x**(5/2)*sqrt(-a/b)) - 30*A*b**3*x**2*sq
rt(-a/b)/(8*a**5*b*sqrt(x)*sqrt(-a/b) + 16*a**4*b**2*x**(3/2)*sqrt(-a/b) +
 8*a**3*b**3*x**(5/2)*sqrt(-a/b)) + 3*B*a**3*sqrt(x)*log(sqrt(x) - sqrt...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}\,dx = -\frac{8\,Aa^2-3\,(Bab-5\,Ab^2)x^2-5\,(Ba^2-5\,Aab)x}{4\left(a^3b^2x^{\frac{5}{2}}+2\,a^4bx^{\frac{3}{2}}+a^5\sqrt{x}\right)} + \frac{3\,(Ba-5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(8*A*a^2 - 3*(B*a*b - 5*A*b^2)*x^2 - 5*(B*a^2 - 5*A*a*b)*x)/(a^3*b^2*
x^(5/2) + 2*a^4*b*x^(3/2) + a^5*sqrt(x)) + 3/4*(B*a - 5*A*b)*arctan(b*sqrt
(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}\,dx = \frac{3\,(Ba-5\,Ab)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3} - \frac{2\,A}{a^3\sqrt{x}} + \frac{3\,Babx^{\frac{3}{2}}-7\,Ab^2x^{\frac{3}{2}}+5\,Ba^2\sqrt{x}-9\,Aab\sqrt{x}}{4\,(bx+a)^2a^3}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
3/4*(B*a - 5*A*b)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*A/(a^3*s
qrt(x)) + 1/4*(3*B*a*b*x^(3/2) - 7*A*b^2*x^(3/2) + 5*B*a^2*sqrt(x) - 9*A*a
*b*sqrt(x))/((b*x + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}\,dx = -\frac{\frac{2\,A}{a}+\frac{5\,x\,(5\,A\,b-B\,a)}{4\,a^2}+\frac{3\,b\,x^2\,(5\,A\,b-B\,a)}{4\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{5/2}+2\,a\,b\,x^{3/2}} - \frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{b}\,\sqrt{x}\,(5\,A\,b-B\,a)}{\sqrt{a}\,(15\,A\,b-3\,B\,a)}\right)\,(5\,A\,b-B\,a)}{4\,a^{7/2}\,\sqrt{b}}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x)^3),x)
```

output
```
- ((2*A)/a + (5*x*(5*A*b - B*a))/(4*a^2) + (3*b*x^2*(5*A*b - B*a))/(4*a^3)
)/(a^2*x^(1/2) + b^2*x^(5/2) + 2*a*b*x^(3/2)) - (3*atan((3*b^(1/2)*x^(1/2)
*(5*A*b - B*a))/(a^(1/2)*(15*A*b - 3*B*a)))*(5*A*b - B*a))/(4*a^(7/2)*b^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^3}\,dx = \frac{-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)bx-2a^2-3abx}{\sqrt{x}\,a^3\,(bx+a)}$$

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^3,x)
```

output
```
( - 3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a - 3*sq
rt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*x - 2*a**2 - 3
*a*b*x)/(sqrt(x)*a**3*(a + b*x))
```

3.222 $\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx=-\frac{2A}{3a^3x^{3/2}}+\frac{2(3Ab-aB)}{a^4\sqrt{x}}+\frac{b(Ab-aB)\sqrt{x}}{2a^3(a+bx)^2}+\frac{b(11Ab-7aB)\sqrt{x}}{4a^4(a+bx)}+\frac{5\sqrt{b}(7Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
-2/3*A/a^3/x^(3/2)+2*(3*A*b-B*a)/a^4/x^(1/2)+1/2*b*(A*b-B*a)*x^(1/2)/a^3/(
b*x+a)^2+1/4*b*(11*A*b-7*B*a)*x^(1/2)/a^4/(b*x+a)+5/4*b^(1/2)*(7*A*b-3*B*a
)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx=\frac{105Ab^3x^3+a^2bx(56A-75Bx)+5ab^2x^2(35A-9Bx)-8a^3(A+3Bx)}{12a^4x^{3/2}(a+bx)^2}+\frac{5\sqrt{b}(7Ab-3aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x)^3),x]
```

output
```
(105*A*b^3*x^3 + a^2*b*x*(56*A - 75*B*x) + 5*a*b^2*x^2*(35*A - 9*B*x) - 8*
a^3*(A + 3*B*x))/(12*a^4*x^(3/2)*(a + b*x)^2) + (5*Sqrt[b]*(7*A*b - 3*a*B)
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(7Ab-3aB)\int \frac{1}{x^{5/2}(a+bx)^2}dx}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{(7Ab-3aB)\left(\frac{5\int \frac{1}{x^{5/2}(a+bx)}dx}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\int \frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}$$

$\downarrow$ 73

$$\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\left(-\frac{2b\int\frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}$$

$\downarrow$ 218

$$\frac{(7Ab-3aB)\left(\frac{5\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{2a}+\frac{1}{ax^{3/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx)^2}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x)^2) + ((7*A*b - 3*a*B)*(1/(a*x^(3/2)*(
a + b*x)) + (5*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/(2*a)))/(4*a*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{2(-9Abx+3Bax+Aa)}{3a^{4}x^{\frac{3}{2}}}+\frac{b\left(\frac{2\left(\frac{11}{8}b^{2}A-\frac{7}{8}abB\right)x^{\frac{3}{2}}+\frac{a(13Ab-9Ba)\sqrt{x}}{4}}{(bx+a)^{2}}+\frac{5(7Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^{4}}$	98
derivativedivides	$-\frac{2A}{3a^{3}x^{\frac{3}{2}}}-\frac{2(-3Ab+Ba)}{a^{4}\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{11}{8}b^{2}A-\frac{7}{8}abB\right)x^{\frac{3}{2}}+\frac{a(13Ab-9Ba)\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{5(7Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	101
default	$-\frac{2A}{3a^{3}x^{\frac{3}{2}}}-\frac{2(-3Ab+Ba)}{a^{4}\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{11}{8}b^{2}A-\frac{7}{8}abB\right)x^{\frac{3}{2}}+\frac{a(13Ab-9Ba)\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{5(7Ab-3Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	101

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-9*A*b*x+3*B*a*x+A*a)/a^4/x^(3/2)+1/a^4*b*(2*((11/8*b^2*A-7/8*a*b*B)
*x^(3/2)+1/8*a*(13*A*b-9*B*a)*x^(1/2))/(b*x+a)^2+5/4*(7*A*b-3*B*a)/(a*b)^(
1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.88

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx = \left[-\frac{15\left((3\,Bab^2-7\,Ab^3)x^4+2\,(3\,Ba^2b-7\,Aab^2)x^3+(3\,Ba^3-7\,Aa^2b)x^2\right)\sqrt{-\frac{b}{a}}\log}{2}\right.$$

$$-\frac{15\left((3\,Bab^2-7\,Ab^3)x^4+2\,(3\,Ba^2b-7\,Aab^2)x^3+(3\,Ba^3-7\,Aa^2b)x^2\right)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{x}\sqrt{\frac{b}{a}}\right)+(8\,Aa^3+}{12\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/24*(15*((3*B*a*b^2 - 7*A*b^3)*x^4 + 2*(3*B*a^2*b - 7*A*a*b^2)*x^3 + (3
*B*a^3 - 7*A*a^2*b)*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)
/(b*x + a)) + 2*(8*A*a^3 + 15*(3*B*a*b^2 - 7*A*b^3)*x^3 + 25*(3*B*a^2*b -
7*A*a*b^2)*x^2 + 8*(3*B*a^3 - 7*A*a^2*b)*x)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*
b*x^3 + a^6*x^2), -1/12*(15*((3*B*a*b^2 - 7*A*b^3)*x^4 + 2*(3*B*a^2*b - 7*
A*a*b^2)*x^3 + (3*B*a^3 - 7*A*a^2*b)*x^2)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/
a)) + (8*A*a^3 + 15*(3*B*a*b^2 - 7*A*b^3)*x^3 + 25*(3*B*a^2*b - 7*A*a*b^2)
*x^2 + 8*(3*B*a^3 - 7*A*a^2*b)*x)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^
6*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. $2(124) = 248$.

Time = 43.11 (sec) , antiderivative size = 1703, normalized size of antiderivative = 13.10

$$\int \frac{A + Bx}{x^{5/2}(a + bx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(5/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/a**3, Eq(b, 0)), ((-2*A/(9*x**(9/2))
 - 2*B/(7*x**(7/2)))/b**3, Eq(a, 0)), (-16*A*a**3*sqrt(-a/b)/(24*a**6*x**(
3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sq
rt(-a/b)) + 105*A*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3
/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqr
t(-a/b)) - 105*A*a**2*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/
2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt
(-a/b)) + 112*A*a**2*b*x*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5
*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 210*A*a*b**2*
x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*
b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) - 210*A*a*b**2*x
**(5/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b
*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 350*A*a*b**2*x*
*2*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b)
 + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 105*A*b**3*x**(7/2)*log(sqrt(x) - s
qrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) +
24*a**4*b**2*x**(7/2)*sqrt(-a/b)) - 105*A*b**3*x**(7/2)*log(sqrt(x) + sqrt
(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*
a**4*b**2*x**(7/2)*sqrt(-a/b)) + 210*A*b**3*x**3*sqrt(-a/b)/(24*a**6*x**...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx = -\frac{8\,Aa^3+15\,(3\,Bab^2-7\,Ab^3)x^3+25\,(3\,Ba^2b-7\,Aab^2)x^2+8\,(3\,Ba^3-7\,Aa^2b)x}{12\left(a^4b^2x^{\frac{7}{2}}+2\,a^5bx^{\frac{5}{2}}+a^6x^{\frac{3}{2}}\right)} - \frac{5\,(3\,Bab-7\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/12*(8*A*a^3 + 15*(3*B*a*b^2 - 7*A*b^3)*x^3 + 25*(3*B*a^2*b - 7*A*a*b^2)
*x^2 + 8*(3*B*a^3 - 7*A*a^2*b)*x)/(a^4*b^2*x^(7/2) + 2*a^5*b*x^(5/2) + a^6
*x^(3/2)) - 5/4*(3*B*a*b - 7*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx = -\frac{5\,(3\,Bab-7\,Ab^2)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4} - \frac{2\,(3\,Bax-9\,Abx+Aa)}{3\,a^4x^{\frac{3}{2}}} - \frac{7\,Bab^2x^{\frac{3}{2}}-11\,Ab^3x^{\frac{3}{2}}+9\,Ba^2b\sqrt{x}-13\,Aab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-5/4*(3*B*a*b - 7*A*b^2)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - 2/3
*(3*B*a*x - 9*A*b*x + A*a)/(a^4*x^(3/2)) - 1/4*(7*B*a*b^2*x^(3/2) - 11*A*b
^3*x^(3/2) + 9*B*a^2*b*sqrt(x) - 13*A*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx = \frac{\frac{2\,x\,(7\,A\,b-3\,B\,a)}{3\,a^2}-\frac{2\,A}{3\,a}+\frac{5\,b^2\,x^3\,(7\,A\,b-3\,B\,a)}{4\,a^4}+\frac{25\,b\,x^2\,(7\,A\,b-3\,B\,a)}{12\,a^3}}{a^2\,x^{3/2}+b^2\,x^{7/2}+2\,a\,b\,x^{5/2}} + \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(7\,A\,b-3\,B\,a)}{4\,a^{9/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x)^3),x)
```

output
```
((2*x*(7*A*b - 3*B*a))/(3*a^2) - (2*A)/(3*a) + (5*b^2*x^3*(7*A*b - 3*B*a))
/(4*a^4) + (25*b*x^2*(7*A*b - 3*B*a))/(12*a^3))/(a^2*x^(3/2) + b^2*x^(7/2)
 + 2*a*b*x^(5/2)) + (5*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2))*(7*A*b - 3*
B*a))/(4*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^3}\,dx = \frac{15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)abx+15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-2a^3+10a^2bx+1}{3\sqrt{x}\,a^4x\,(bx+a)}$$

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^3,x)
```

output
```
(15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b*x + 15
*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b**2*x**2 - 2
*a**3 + 10*a**2*b*x + 15*a*b**2*x**2)/(3*sqrt(x)*a**4*x*(a + b*x))
```

3.223 $\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 156

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = -\frac{2A}{5a^3x^{5/2}} + \frac{2(3Ab-aB)}{3a^4x^{3/2}} - \frac{6b(2Ab-aB)}{a^5\sqrt{x}} - \frac{b^2(Ab-aB)\sqrt{x}}{2a^4(a+bx)^2} - \frac{b^2(15Ab-11aB)\sqrt{x}}{4a^5(a+bx)} - \frac{7b^{3/2}(9Ab-5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{11/2}}$$

output
```
-2/5*A/a^3/x^(5/2)+2/3*(3*A*b-B*a)/a^4/x^(3/2)-6*b*(2*A*b-B*a)/a^5/x^(1/2)
-1/2*b^2*(A*b-B*a)*x^(1/2)/a^4/(b*x+a)^2-1/4*b^2*(15*A*b-11*B*a)*x^(1/2)/a
^5/(b*x+a)-7/4*b^(3/2)*(9*A*b-5*B*a)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(11
/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = \frac{-945Ab^4x^4+525ab^3x^3(-3A+Bx)-8a^4(3A+5Bx)+8a^3bx(9A+35Bx)+7a^2b}{60a^5x^{5/2}(a+bx)^2} + \frac{7b^{3/2}(-9Ab+5aB)\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{11/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a + b*x)^3),x]
```

output
```
(-945*A*b^4*x^4 + 525*a*b^3*x^3*(-3*A + B*x) - 8*a^4*(3*A + 5*B*x) + 8*a^3
*b*x*(9*A + 35*B*x) + 7*a^2*b^2*x^2*(-72*A + 125*B*x))/(60*a^5*x^(5/2)*(a
+ b*x)^2) + (7*b^(3/2)*(-9*A*b + 5*a*B)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])
/(4*a^(11/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {87, 52, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3} \, dx$$

$$\Big\downarrow \; 87$$

$$\frac{(9Ab-5aB)\int \frac{1}{x^{7/2}(a+bx)^2}dx}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

$$\Big\downarrow \; 52$$

$$\frac{(9Ab-5aB)\left(\frac{7\int \frac{1}{x^{7/2}(a+bx)}dx}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

$$\Big\downarrow \; 61$$

$$\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\int \frac{1}{x^{5/2}(a+bx)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

$$\Big\downarrow \; 61$$

$$\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\int\frac{1}{x^{3/2}(a+bx)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

$\downarrow$ 61

$$\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

$\downarrow$ 73

$$\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{2b\int \frac{1}{a+bx}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

$\downarrow$ 218

$$\frac{(9Ab-5aB)\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{2a}+\frac{1}{ax^{5/2}(a+bx)}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}(a+bx)^2}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a + b*x)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*x^(5/2)*(a + b*x)^2) + ((9*A*b - 5*a*B)*(1/(a*x^(5/2)*(
a + b*x)) + (7*(-2/(5*a*x^(5/2)) - (b*(-2/(3*a*x^(3/2)) - (b*(-2/(a*Sqrt[x
]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a)
))/(4*a*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{2b^2\left(\frac{\left(\frac{15}{8}b^2A-\frac{11}{8}abB\right)x^{\frac{3}{2}}+\frac{a(17Ab-13Ba)\sqrt{x}}{8}}{(bx+a)^2}+\frac{7(9Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2(-3Ab+Ba)}{3a^4x^{\frac{3}{2}}}-\frac{6b(2A\cdots}{a^5\cdots}$
default	$-\frac{2b^2\left(\frac{\left(\frac{15}{8}b^2A-\frac{11}{8}abB\right)x^{\frac{3}{2}}+\frac{a(17Ab-13Ba)\sqrt{x}}{8}}{(bx+a)^2}+\frac{7(9Ab-5Ba)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2(-3Ab+Ba)}{3a^4x^{\frac{3}{2}}}-\frac{6b(2A\cdots}{a^5\cdots}$
risch	$-\frac{2(90Ab^2x^2-45Babx^2-15aAbx+5Ba^2x+3a^2A)}{15a^5x^{\frac{5}{2}}}-\frac{b^2\left(\frac{2\left(\frac{15}{8}b^2A-\frac{11}{8}abB\right)x^{\frac{3}{2}}+\frac{a(17Ab-13Ba)\sqrt{x}}{4}}{(bx+a)^2}+\frac{7(9Ab-5Ba)\,\mathrm{arc}\cdots}{4\sqrt{a\cdots}}\right)}{a^5}$

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/a^5*b^2*(((15/8*b^2*A-11/8*a*b*B)*x^(3/2)+1/8*a*(17*A*b-13*B*a)*x^(1/2)
)/(b*x+a)^2+7/8*(9*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))-2
/5*A/a^3/x^(5/2)-2/3*(-3*A*b+B*a)/a^4/x^(3/2)-6*b*(2*A*b-B*a)/a^5/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = \left[-\frac{105\left((5\,Bab^3-9\,Ab^4)x^5+2\,(5\,Ba^2b^2-9\,Aab^3)x^4+(5\,Ba^3b-9\,Aa^2b^2)x^3\right)\sqrt{-\frac{b}{a}}\cdots}{\cdots}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/120*(105*((5*B*a*b^3 - 9*A*b^4)*x^5 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^4
+ (5*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b
/a) - a)/(b*x + a)) + 2*(24*A*a^4 - 105*(5*B*a*b^3 - 9*A*b^4)*x^4 - 175*(5
*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 56*(5*B*a^3*b - 9*A*a^2*b^2)*x^2 + 8*(5*B*a^
4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3), 1/60*(10
5*((5*B*a*b^3 - 9*A*b^4)*x^5 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^4 + (5*B*a^3*
b - 9*A*a^2*b^2)*x^3)*sqrt(b/a)*arctan(sqrt(x)*sqrt(b/a)) - (24*A*a^4 - 10
5*(5*B*a*b^3 - 9*A*b^4)*x^4 - 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^3 - 56*(5*B*
a^3*b - 9*A*a^2*b^2)*x^2 + 8*(5*B*a^4 - 9*A*a^3*b)*x)*sqrt(x))/(a^5*b^2*x^
5 + 2*a^6*b*x^4 + a^7*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1882 vs. $2(150) = 300$.

Time = 121.43 (sec) , antiderivative size = 1882, normalized size of antiderivative = 12.06

$$\int \frac{A + Bx}{x^{7/2}(a + bx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(7/2)/(b*x+a)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(9*x**(9/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2)))/a**3, Eq(b, 0)), ((-2*A/(11*x
**(11/2)) - 2*B/(9*x**(9/2)))/b**3, Eq(a, 0)), (-48*A*a**4*sqrt(-a/b)/(120
*a**7*x**(5/2)*sqrt(-a/b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2
*x**(9/2)*sqrt(-a/b)) + 144*A*a**3*b*x*sqrt(-a/b)/(120*a**7*x**(5/2)*sqrt(
-a/b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(-a/b)
) - 945*A*a**2*b**2*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(120*a**7*x**(5/2)*
sqrt(-a/b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(
-a/b)) + 945*A*a**2*b**2*x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(120*a**7*x**(
5/2)*sqrt(-a/b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*
sqrt(-a/b)) - 1008*A*a**2*b**2*x**2*sqrt(-a/b)/(120*a**7*x**(5/2)*sqrt(-a/
b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(-a/b)) -
 1890*A*a*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(120*a**7*x**(5/2)*sqrt(
-a/b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(-a/b)
) + 1890*A*a*b**3*x**(7/2)*log(sqrt(x) + sqrt(-a/b))/(120*a**7*x**(5/2)*sq
rt(-a/b) + 240*a**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(-a
/b)) - 3150*A*a*b**3*x**3*sqrt(-a/b)/(120*a**7*x**(5/2)*sqrt(-a/b) + 240*a
**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(-a/b)) - 945*A*b**
4*x**(9/2)*log(sqrt(x) - sqrt(-a/b))/(120*a**7*x**(5/2)*sqrt(-a/b) + 240*a
**6*b*x**(7/2)*sqrt(-a/b) + 120*a**5*b**2*x**(9/2)*sqrt(-a/b)) + 945*A*...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = -\frac{24\,Aa^4 - 105\,(5\,Bab^3 - 9\,Ab^4)x^4 - 175\,(5\,Ba^2b^2 - 9\,Aab^3)x^3 - 56\,(5\,Ba^3b - 9\,Aa^2b^2)x^2 + 8\,(5\,Ba^4 - 9\,A}{60\left(a^5b^2x^{\frac{9}{2}} + 2\,a^6bx^{\frac{7}{2}} + a^7x^{\frac{5}{2}}\right)} + \frac{7\,(5\,Bab^2 - 9\,Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^5}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/60*(24*A*a^4 - 105*(5*B*a*b^3 - 9*A*b^4)*x^4 - 175*(5*B*a^2*b^2 - 9*A*a
*b^3)*x^3 - 56*(5*B*a^3*b - 9*A*a^2*b^2)*x^2 + 8*(5*B*a^4 - 9*A*a^3*b)*x)/
(a^5*b^2*x^(9/2) + 2*a^6*b*x^(7/2) + a^7*x^(5/2)) + 7/4*(5*B*a*b^2 - 9*A*b
^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = \frac{7\,(5\,Bab^2-9\,Ab^3)\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^5} + \frac{11\,Bab^3x^{\frac{3}{2}}-15\,Ab^4x^{\frac{3}{2}}+13\,Ba^2b^2\sqrt{x}-17\,Aab^3\sqrt{x}}{4\,(bx+a)^2a^5} + \frac{2\,(45\,Babx^2-90\,Ab^2x^2-5\,Ba^2x+15\,Aabx-3\,Aa^2)}{15\,a^5x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
7/4*(5*B*a*b^2 - 9*A*b^3)*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/
4*(11*B*a*b^3*x^(3/2) - 15*A*b^4*x^(3/2) + 13*B*a^2*b^2*sqrt(x) - 17*A*a*b
^3*sqrt(x))/((b*x + a)^2*a^5) + 2/15*(45*B*a*b*x^2 - 90*A*b^2*x^2 - 5*B*a^
2*x + 15*A*a*b*x - 3*A*a^2)/(a^5*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = -\frac{\frac{2\,A}{5\,a}-\frac{2\,x\,(9\,A\,b-5\,B\,a)}{15\,a^2}+\frac{35\,b^2\,x^3\,(9\,A\,b-5\,B\,a)}{12\,a^4}+\frac{7\,b^3\,x^4\,(9\,A\,b-5\,B\,a)}{4\,a^5}+\frac{14\,b\,x^2\,(9\,A\,b-5\,B\,a)}{15\,a^3}}{a^2\,x^{5/2}+b^2\,x^{9/2}+2\,a\,b\,x^{7/2}} - \frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{11/2}}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x)^3),x)
```

output
```
- ((2*A)/(5*a) - (2*x*(9*A*b - 5*B*a))/(15*a^2) + (35*b^2*x^3*(9*A*b - 5*B
*a))/(12*a^4) + (7*b^3*x^4*(9*A*b - 5*B*a))/(4*a^5) + (14*b*x^2*(9*A*b - 5
*B*a))/(15*a^3))/(a^2*x^(5/2) + b^2*x^(9/2) + 2*a*b*x^(7/2)) - (7*b^(3/2)*
atan((b^(1/2)*x^(1/2))/a^(1/2))*(9*A*b - 5*B*a))/(4*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^3}\,dx = \frac{-105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right) a\,b^2x^2 - 105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,b}{\sqrt{b}\,\sqrt{a}}\right) b^3x^3 - 6a^4 + 14a^3}{15\sqrt{x}\,a^5x^2\,(bx+a)}$$

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^3,x)
```

output
```
( - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*a*b**2
*x**2 - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*b)/(sqrt(b)*sqrt(a)))*b*
*3*x**3 - 6*a**4 + 14*a**3*b*x - 70*a**2*b**2*x**2 - 105*a*b**3*x**3)/(15*
sqrt(x)*a**5*x**2*(a + b*x))
```

3.224 $\int x^4\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 151

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx = \frac{2a^4(Ab-aB)(a+bx)^{3/2}}{3b^6} - \frac{2a^3(4Ab-5aB)(a+bx)^{5/2}}{5b^6} + \frac{4a^2(3Ab-5aB)(a+bx)^{7/2}}{7b^6} - \frac{4a(2Ab-5aB)(a+bx)^{9/2}}{9b^6} + \frac{2(Ab-5aB)(a+bx)^{11/2}}{11b^6} + \frac{2B(a+bx)^{13/2}}{13b^6}$$

output
```
2/3*a^4*(A*b-B*a)*(b*x+a)^(3/2)/b^6-2/5*a^3*(4*A*b-5*B*a)*(b*x+a)^(5/2)/b^
6+4/7*a^2*(3*A*b-5*B*a)*(b*x+a)^(7/2)/b^6-4/9*a*(2*A*b-5*B*a)*(b*x+a)^(9/2
)/b^6+2/11*(A*b-5*B*a)*(b*x+a)^(11/2)/b^6+2/13*B*(b*x+a)^(13/2)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.70

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$
$$=\frac{2(a+bx)^{3/2}\left(-1280a^5B+315b^5x^4(13A+11Bx)+128a^4b(13A+15Bx)-96a^3b^2x(26A+25Bx)+80\right.}{45045b^6}$$

input
```
Integrate[x^4*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*(a + b*x)^(3/2)*(-1280*a^5*B + 315*b^5*x^4*(13*A + 11*B*x) + 128*a^4*b*
(13*A + 15*B*x) - 96*a^3*b^2*x*(26*A + 25*B*x) + 80*a^2*b^3*x^2*(39*A + 35
*B*x) - 70*a*b^4*x^3*(52*A + 45*B*x)))/(45045*b^6)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int\left(-\frac{a^4\sqrt{a+bx}(aB-Ab)}{b^5}+\frac{a^3(a+bx)^{3/2}(5aB-4Ab)}{b^5}-\frac{2a^2(a+bx)^{5/2}(5aB-3Ab)}{b^5}+\frac{(a+bx)^{9/2}(Ab-5a}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4(a+bx)^{3/2}(Ab-aB)}{3b^6}-\frac{2a^3(a+bx)^{5/2}(4Ab-5aB)}{5b^6}+\frac{4a^2(a+bx)^{7/2}(3Ab-5aB)}{7b^6}+\frac{2(a+bx)^{11/2}(Ab-5aB)}{11b^6}-\frac{4a(a+bx)^{9/2}(2Ab-5aB)}{9b^6}+\frac{2B(a+bx)^{13/2}}{13b^6}$$

input
```
Int[x^4*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*a^4*(A*b - a*B)*(a + b*x)^(3/2))/(3*b^6) - (2*a^3*(4*A*b - 5*a*B)*(a +
b*x)^(5/2))/(5*b^6) + (4*a^2*(3*A*b - 5*a*B)*(a + b*x)^(7/2))/(7*b^6) - (4
*a*(2*A*b - 5*a*B)*(a + b*x)^(9/2))/(9*b^6) + (2*(A*b - 5*a*B)*(a + b*x)^(
11/2))/(11*b^6) + (2*B*(a + b*x)^(13/2))/(13*b^6)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{256 (\frac{315 (\frac{11 B x}{13}+A) x^4 b^5}{128} - \frac{35 a (\frac{45 B x}{52}+A) x^3 b^4}{16} + \frac{15 a^2 (\frac{35 B x}{39}+A) x^2 b^3}{8} - \frac{3 a^3 (\frac{25 B x}{26}+A) x b^2}{2} + a^4 (\frac{15 B x}{13}+A) b - \frac{10 a^5 B}{13}) (b}{3465 b^6}$
gosper	$\frac{2(bx+a)^{\frac{3}{2}} (3465 b^5 B x^5+4095 A b^5 x^4-3150 B a b^4 x^4-3640 A a b^4 x^3+2800 B a^2 b^3 x^3+3120 A a^2 b^3 x^2-2400 B a^3 b^2 x^2-24}{45045 b^6}$
orering	$\frac{2(bx+a)^{\frac{3}{2}} (3465 b^5 B x^5+4095 A b^5 x^4-3150 B a b^4 x^4-3640 A a b^4 x^3+2800 B a^2 b^3 x^3+3120 A a^2 b^3 x^2-2400 B a^3 b^2 x^2-24}{45045 b^6}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{13}{2}}}{13}+\frac{2(Ab-5Ba)(bx+a)^{\frac{11}{2}}}{11}+\frac{2(6a^2B-4a(Ab-Ba))(bx+a)^{\frac{9}{2}}}{9}+\frac{2(-4a^3B+6a^2(Ab-Ba))(bx+a)^{\frac{7}{2}}}{7}+\frac{2(B a^4-4a^3(Ab-}{5}}{b^6}$
default	$\frac{\frac{2B(bx+a)^{\frac{13}{2}}}{13}+\frac{2(Ab-5Ba)(bx+a)^{\frac{11}{2}}}{11}+\frac{2(6a^2B-4a(Ab-Ba))(bx+a)^{\frac{9}{2}}}{9}+\frac{2(-4a^3B+6a^2(Ab-Ba))(bx+a)^{\frac{7}{2}}}{7}+\frac{2(B a^4-4a^3(Ab-}{5}}{b^6}$
trager	$\frac{2(3465 B b^6 x^6+4095 A b^6 x^5+315 B a b^5 x^5+455 A a b^5 x^4-350 B a^2 b^4 x^4-520 A a^2 b^4 x^3+400 B a^3 b^3 x^3+624 A a^3 b^3 x^2-48}{45045 b^6}$
risch	$\frac{2(3465 B b^6 x^6+4095 A b^6 x^5+315 B a b^5 x^5+455 A a b^5 x^4-350 B a^2 b^4 x^4-520 A a^2 b^4 x^3+400 B a^3 b^3 x^3+624 A a^3 b^3 x^2-48}{45045 b^6}$

input
```
int(x^4*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
256/3465*(315/128*(11/13*B*x+A)*x^4*b^5-35/16*a*(45/52*B*x+A)*x^3*b^4+15/8
*a^2*(35/39*B*x+A)*x^2*b^3-3/2*a^3*(25/26*B*x+A)*x*b^2+a^4*(15/13*B*x+A)*b
-10/13*a^5*B)*(b*x+a)^(3/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{2\,(3465\,Bb^6x^6 - 1280\,Ba^6 + 1664\,Aa^5b + 315\,(Bab^5 + 13\,Ab^6)x^5 - 35\,(10\,Ba^2b^4 - 13\,Aab^5)x^4 + 40\,(10\,\ldots}{45045\,b^6}$$

input
```
integrate(x^4*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3465*B*b^6*x^6 - 1280*B*a^6 + 1664*A*a^5*b + 315*(B*a*b^5 + 13*A*
b^6)*x^5 - 35*(10*B*a^2*b^4 - 13*A*a*b^5)*x^4 + 40*(10*B*a^3*b^3 - 13*A*a^
2*b^4)*x^3 - 48*(10*B*a^4*b^2 - 13*A*a^3*b^3)*x^2 + 64*(10*B*a^5*b - 13*A*
a^4*b^2)*x)*sqrt(b*x + a)/b^6
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$
$$= \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{13}{2}}}{13b} + \frac{(a+bx)^{\frac{11}{2}}(Ab-5Ba)}{11b} + \frac{(a+bx)^{\frac{9}{2}}\left(-4Aab+10Ba^2\right)}{9b} + \frac{(a+bx)^{\frac{7}{2}}\cdot\left(6Aa^2b-10Ba^3\right)}{7b} + \frac{(a+bx)^{\frac{5}{2}}\left(-4Aa^3b+5Ba^4\right)}{5b} + \frac{(a+bx)^{\frac{3}{2}}\left(Aa^4b-Ba^5\right)}{3b} \ldots\right)}{b^5} \\ \sqrt{a}\left(\frac{Ax^5}{5} + \frac{Bx^6}{6}\right) \end{cases}$$

input
```
integrate(x**4*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(13/2)/(13*b) + (a + b*x)**(11/2)*(A*b - 5*B*a)
/(11*b) + (a + b*x)**(9/2)*(-4*A*a*b + 10*B*a**2)/(9*b) + (a + b*x)**(7/2)
*(6*A*a**2*b - 10*B*a**3)/(7*b) + (a + b*x)**(5/2)*(-4*A*a**3*b + 5*B*a**4
)/(5*b) + (a + b*x)**(3/2)*(A*a**4*b - B*a**5)/(3*b))/b**5, Ne(b, 0)), (sq
rt(a)*(A*x**5/5 + B*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$
$$=\frac{2\left(3465\,(bx+a)^{\frac{13}{2}}B-4095\,(5\,Ba-Ab)(bx+a)^{\frac{11}{2}}+10010\,(5\,Ba^2-2\,Aab)(bx+a)^{\frac{9}{2}}-12870\,(5\,Ba^3-\right.}{45045\,b^6}$$

input
```
integrate(x^4*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3465*(b*x + a)^(13/2)*B - 4095*(5*B*a - A*b)*(b*x + a)^(11/2) + 1
0010*(5*B*a^2 - 2*A*a*b)*(b*x + a)^(9/2) - 12870*(5*B*a^3 - 3*A*a^2*b)*(b*
x + a)^(7/2) + 9009*(5*B*a^4 - 4*A*a^3*b)*(b*x + a)^(5/2) - 15015*(B*a^5 -
 A*a^4*b)*(b*x + a)^(3/2))/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(128) = 256.

Time = 0.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.01

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$
$$=\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)Aa}{b^4}+\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)\right.}{}\right.}{}$$

input
```
integrate(x^4*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(
5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A*a/b^4 + 65*(
63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 13
86*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)
*B*a/b^5 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)
^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)*A/b^4 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a
 + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5
/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*B/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx = \frac{(20\,B\,a^2-8\,A\,a\,b)\,\left(a+b\,x\right)^{9/2}}{9\,b^6}+\frac{2\,B\,\left(a+b\,x\right)^{13/2}}{13\,b^6}+\frac{(2\,A\,b-10\,B\,a)\,\left(a+b\,x\right)^{11/2}}{11\,b^6}-\frac{(2\,B\,a^5-2\,A\,a^4\,b)\,\left(a+b\,x\right)^{3/2}}{3\,b^6}+\frac{(10\,B\,a^4-8\,A\,a^3\,b)\,\left(a+b\,x\right)^{5/2}}{5\,b^6}-\frac{(20\,B\,a^3-12\,A\,a^2\,b)\,\left(a+b\,x\right)^{7/2}}{7\,b^6}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
((20*B*a^2 - 8*A*a*b)*(a + b*x)^(9/2))/(9*b^6) + (2*B*(a + b*x)^(13/2))/(1
3*b^6) + ((2*A*b - 10*B*a)*(a + b*x)^(11/2))/(11*b^6) - ((2*B*a^5 - 2*A*a^
4*b)*(a + b*x)^(3/2))/(3*b^6) + ((10*B*a^4 - 8*A*a^3*b)*(a + b*x)^(5/2))/(
5*b^6) - ((20*B*a^3 - 12*A*a^2*b)*(a + b*x)^(7/2))/(7*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.49

$$\int x^4\sqrt{a+bx}(A+Bx)\,dx$$
$$=\frac{2\sqrt{bx+a}\,(1155b^6x^6+1470a\,b^5x^5+35a^2b^4x^4-40a^3b^3x^3+48a^4b^2x^2-64a^5bx+128a^6)}{15015b^5}$$

input
```
int(x^4*(b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**6 - 64*a**5*b*x + 48*a**4*b**2*x**2 - 40*a**3*b**
3*x**3 + 35*a**2*b**4*x**4 + 1470*a*b**5*x**5 + 1155*b**6*x**6))/(15015*b*
*5)
```

3.225 $\int x^3\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx = -\frac{2a^3(Ab-aB)(a+bx)^{3/2}}{3b^5}+\frac{2a^2(3Ab-4aB)(a+bx)^{5/2}}{5b^5} - \frac{6a(Ab-2aB)(a+bx)^{7/2}}{7b^5} + \frac{2(Ab-4aB)(a+bx)^{9/2}}{9b^5}+\frac{2B(a+bx)^{11/2}}{11b^5}$$

output
```
-2/3*a^3*(A*b-B*a)*(b*x+a)^(3/2)/b^5+2/5*a^2*(3*A*b-4*B*a)*(b*x+a)^(5/2)/b
^5-6/7*a*(A*b-2*B*a)*(b*x+a)^(7/2)/b^5+2/9*(A*b-4*B*a)*(b*x+a)^(9/2)/b^5+2
/11*B*(b*x+a)^(11/2)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx = \frac{2(a+bx)^{3/2}\left(128a^4B+35b^4x^3(11A+9Bx)+24a^2b^2x(11A+10Bx)-16a^3b(11A+12Bx)-10ab^3x^2(\right.}{3465b^5}$$

input
```
Integrate[x^3*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*(a + b*x)^(3/2)*(128*a^4*B + 35*b^4*x^3*(11*A + 9*B*x) + 24*a^2*b^2*x*(
11*A + 10*B*x) - 16*a^3*b*(11*A + 12*B*x) - 10*a*b^3*x^2*(33*A + 28*B*x)))
/(3465*b^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{a^3\sqrt{a+bx}(aB-Ab)}{b^4}-\frac{a^2(a+bx)^{3/2}(4aB-3Ab)}{b^4}+\frac{(a+bx)^{7/2}(Ab-4aB)}{b^4}+\frac{3a(a+bx)^{5/2}(2aB-Ab)}{b^4}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^3(a+bx)^{3/2}(Ab-aB)}{3b^5}+\frac{2a^2(a+bx)^{5/2}(3Ab-4aB)}{5b^5}+\frac{2(a+bx)^{9/2}(Ab-4aB)}{9b^5}-\frac{6a(a+bx)^{7/2}(Ab-2aB)}{7b^5}+\frac{2B(a+bx)^{11/2}}{11b^5}$$

input
```
Int[x^3*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(-2*a^3*(A*b - a*B)*(a + b*x)^(3/2))/(3*b^5) + (2*a^2*(3*A*b - 4*a*B)*(a +
 b*x)^(5/2))/(5*b^5) - (6*a*(A*b - 2*a*B)*(a + b*x)^(7/2))/(7*b^5) + (2*(A
*b - 4*a*B)*(a + b*x)^(9/2))/(9*b^5) + (2*B*(a + b*x)^(11/2))/(11*b^5)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{32\left(-\frac{35\left(\frac{9Bx}{11}+A\right)x^3b^4}{16}+\frac{15\left(\frac{28Bx}{33}+A\right)a\,x^2b^3}{8}-\frac{3a^2\left(\frac{10Bx}{11}+A\right)x\,b^2}{2}+a^3\left(\frac{12Bx}{11}+A\right)b-\frac{8B\,a^4}{11}\right)(bx+a)^{\frac{3}{2}}}{315b^5}$
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-315B\,x^4b^4-385A\,x^3b^4+280B\,x^3a\,b^3+330A\,x^2a\,b^3-240B\,x^2a^2b^2-264Ax\,a^2b^2+192Bx\,a^3b+176A\,a^3b\right.}{3465b^5}$
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-315B\,x^4b^4-385A\,x^3b^4+280B\,x^3a\,b^3+330A\,x^2a\,b^3-240B\,x^2a^2b^2-264Ax\,a^2b^2+192Bx\,a^3b+176A\,a^3b\right.}{3465b^5}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-4Ba)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(3a^2B-3a(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(-a^3B+3a^2(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^3(Ab-Ba)(bx+a)}{3}}{b^5}$
default	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}-\frac{2(-Ab+4Ba)(bx+a)^{\frac{9}{2}}}{9}-\frac{2\left(-3a^2B+3a(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}-\frac{2\left(a^3B-3a^2(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^3(Ab-Ba)(bx+}{3}}{b^5}$
trager	$-\frac{2\left(-315b^5B\,x^5-385A\,b^5x^4-35Ba\,b^4x^4-55Aa\,b^4x^3+40B\,a^2b^3x^3+66A\,a^2b^3x^2-48B\,a^3b^2x^2-88a^3b^2Ax+64a^4bBx\right.}{3465b^5}$
risch	$-\frac{2\left(-315b^5B\,x^5-385A\,b^5x^4-35Ba\,b^4x^4-55Aa\,b^4x^3+40B\,a^2b^3x^3+66A\,a^2b^3x^2-48B\,a^3b^2x^2-88a^3b^2Ax+64a^4bBx\right.}{3465b^5}$

input
```
int(x^3*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-32/315*(-35/16*(9/11*B*x+A)*x^3*b^4+15/8*(28/33*B*x+A)*a*x^2*b^3-3/2*a^2*
(10/11*B*x+A)*x*b^2+a^3*(12/11*B*x+A)*b-8/11*B*a^4)*(b*x+a)^(3/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx = \frac{2\,(315\,Bb^5x^5+128\,Ba^5-176\,Aa^4b+35\,(Bab^4+11\,Ab^5)x^4-5\,(8\,Ba^2b^3-11\,Aab^4)x^3+6\,(8\,Ba^3b^2-1}{3465\,b^5}$$

input
```
integrate(x^3*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^5*x^5 + 128*B*a^5 - 176*A*a^4*b + 35*(B*a*b^4 + 11*A*b^5)*
x^4 - 5*(8*B*a^2*b^3 - 11*A*a*b^4)*x^3 + 6*(8*B*a^3*b^2 - 11*A*a^2*b^3)*x^
2 - 8*(8*B*a^4*b - 11*A*a^3*b^2)*x)*sqrt(b*x + a)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{11}{2}}}{11b}+\frac{(a+bx)^{\frac{9}{2}}(Ab-4Ba)}{9b}+\frac{(a+bx)^{\frac{7}{2}}\left(-3Aab+6Ba^2\right)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(3Aa^2b-4Ba^3\right)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\left(-Aa^3b+Ba^4\right)}{3b}\right)}{b^4} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(11/2)/(11*b) + (a + b*x)**(9/2)*(A*b - 4*B*a)/
(9*b) + (a + b*x)**(7/2)*(-3*A*a*b + 6*B*a**2)/(7*b) + (a + b*x)**(5/2)*(3
*A*a**2*b - 4*B*a**3)/(5*b) + (a + b*x)**(3/2)*(-A*a**3*b + B*a**4)/(3*b))
/b**4, Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx$$
$$=\frac{2\left(315\,(bx+a)^{\frac{11}{2}}B-385\,(4\,Ba-Ab)(bx+a)^{\frac{9}{2}}+1485\,(2\,Ba^2-Aab)(bx+a)^{\frac{7}{2}}-693\,(4\,Ba^3-3\,Aa^2b)\right.}{3465\,b^5}$$

input
```
integrate(x^3*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/3465*(315*(b*x + a)^(11/2)*B - 385*(4*B*a - A*b)*(b*x + a)^(9/2) + 1485*
(2*B*a^2 - A*a*b)*(b*x + a)^(7/2) - 693*(4*B*a^3 - 3*A*a^2*b)*(b*x + a)^(5
/2) + 1155*(B*a^4 - A*a^3*b)*(b*x + a)^(3/2))/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(104) = 208.

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.10

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx$$
$$=\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)Aa}{b^3}+\frac{11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+\right.}{b^4}\right.}{}$$

input
```
integrate(x^3*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)*A*a/b^3 + 11*(35*(b*x + a)^(9/2) - 180*(b*x +
a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*B*a/b^4 + 11*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 3
78*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
A/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/
2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*
x + a)*a^5)*B/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx = \frac{(12\,B\,a^2-6\,A\,a\,b)\,\left(a+b\,x\right)^{7/2}}{7\,b^5}+\frac{2\,B\,{\left(a+b\,x\right)}^{11/2}}{11\,b^5}+\frac{(2\,A\,b-8\,B\,a)\,\left(a+b\,x\right)^{9/2}}{9\,b^5}+\frac{(2\,B\,a^4-2\,A\,a^3\,b)\,\left(a+b\,x\right)^{3/2}}{3\,b^5}-\frac{(8\,B\,a^3-6\,A\,a^2\,b)\,\left(a+b\,x\right)^{5/2}}{5\,b^5}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
((12*B*a^2 - 6*A*a*b)*(a + b*x)^(7/2))/(7*b^5) + (2*B*(a + b*x)^(11/2))/(1
1*b^5) + ((2*A*b - 8*B*a)*(a + b*x)^(9/2))/(9*b^5) + ((2*B*a^4 - 2*A*a^3*b
)*(a + b*x)^(3/2))/(3*b^5) - ((8*B*a^3 - 6*A*a^2*b)*(a + b*x)^(5/2))/(5*b^
5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int x^3\sqrt{a+bx}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(105b^5x^5+140a\,b^4x^4+5a^2b^3x^3-6a^3b^2x^2+8a^4bx-16a^5)}{1155b^4}$$

input
```
int(x^3*(b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**5 + 8*a**4*b*x - 6*a**3*b**2*x**2 + 5*a**2*b**3
*x**3 + 140*a*b**4*x**4 + 105*b**5*x**5))/(1155*b**4)
```

3.226 $\int x^2\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx = \frac{2a^2(Ab-aB)(a+bx)^{3/2}}{3b^4}-\frac{2a(2Ab-3aB)(a+bx)^{5/2}}{5b^4}+\frac{2(Ab-3aB)(a+bx)^{7/2}}{7b^4}+\frac{2B(a+bx)^{9/2}}{9b^4}$$

output
```
2/3*a^2*(A*b-B*a)*(b*x+a)^(3/2)/b^4-2/5*a*(2*A*b-3*B*a)*(b*x+a)^(5/2)/b^4+
2/7*(A*b-3*B*a)*(b*x+a)^(7/2)/b^4+2/9*B*(b*x+a)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx = \frac{2(a+bx)^{3/2}\left(-16a^3B+24a^2b(A+Bx)-6ab^2x(6A+5Bx)+5b^3x^2(9A+7Bx)\right)}{315b^4}$$

input
```
Integrate[x^2*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*(a + b*x)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x) - 6*a*b^2*x*(6*A + 5*B*
x) + 5*b^3*x^2*(9*A + 7*B*x)))/(315*b^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^2\sqrt{a+bx}(aB-Ab)}{b^3}+\frac{(a+bx)^{5/2}(Ab-3aB)}{b^3}+\frac{a(a+bx)^{3/2}(3aB-2Ab)}{b^3}+\frac{B(a+bx)^{7/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(a+bx)^{3/2}(Ab-aB)}{3b^4}+\frac{2(a+bx)^{7/2}(Ab-3aB)}{7b^4}-\frac{2a(a+bx)^{5/2}(2Ab-3aB)}{5b^4}+\frac{2B(a+bx)^{9/2}}{9b^4}$$

input
```
Int[x^2*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*a^2*(A*b - a*B)*(a + b*x)^(3/2))/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*
x)^(5/2))/(5*b^4) + (2*(A*b - 3*a*B)*(a + b*x)^(7/2))/(7*b^4) + (2*B*(a +
b*x)^(9/2))/(9*b^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$\frac{16(bx+a)^{\frac{3}{2}}\left(\frac{15\left(\frac{7Bx}{9}+A\right)x^{2}b^{3}}{8}-\frac{3a\left(\frac{5Bx}{6}+A\right)x\,b^{2}}{2}+a^{2}(Bx+A)b-\frac{2a^{3}B}{3}\right)}{105b^{4}}$	57
gosper	$\frac{2(bx+a)^{\frac{3}{2}}\left(35b^{3}B\,x^{3}+45A\,x^{2}b^{3}-30B\,x^{2}a\,b^{2}-36Axa\,b^{2}+24Bx\,a^{2}b+24a^{2}bA-16a^{3}B\right)}{315b^{4}}$	71
orering	$\frac{2(bx+a)^{\frac{3}{2}}\left(35b^{3}B\,x^{3}+45A\,x^{2}b^{3}-30B\,x^{2}a\,b^{2}-36Axa\,b^{2}+24Bx\,a^{2}b+24a^{2}bA-16a^{3}B\right)}{315b^{4}}$	71
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-3Ba)(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(Ab-Ba)(bx+a)^{\frac{3}{2}}}{3}}{b^{4}}$	80
default	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-3Ba)(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(Ab-Ba)(bx+a)^{\frac{3}{2}}}{3}}{b^{4}}$	80
trager	$\frac{2\left(35B\,x^{4}b^{4}+45A\,x^{3}b^{4}+5B\,x^{3}a\,b^{3}+9A\,x^{2}a\,b^{3}-6B\,x^{2}a^{2}b^{2}-12Ax\,a^{2}b^{2}+8Bx\,a^{3}b+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{bx+a}}{315b^{4}}$	95
risch	$\frac{2\left(35B\,x^{4}b^{4}+45A\,x^{3}b^{4}+5B\,x^{3}a\,b^{3}+9A\,x^{2}a\,b^{3}-6B\,x^{2}a^{2}b^{2}-12Ax\,a^{2}b^{2}+8Bx\,a^{3}b+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{bx+a}}{315b^{4}}$	95

input
```
int(x^2*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
16/105*(b*x+a)^(3/2)*(15/8*(7/9*B*x+A)*x^2*b^3-3/2*a*(5/6*B*x+A)*x*b^2+a^2
*(B*x+A)*b-2/3*a^3*B)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx = \frac{2\left(35\,Bb^4x^4-16\,Ba^4+24\,Aa^3b+5\,(Bab^3+9\,Ab^4)x^3-3\,(2\,Ba^2b^2-3\,Aab^3)x^2+4\,(2\,Ba^3b-3\,Aa^2b^2)\right.}{315\,b^4}$$

input
```
integrate(x^2*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^4*x^4 - 16*B*a^4 + 24*A*a^3*b + 5*(B*a*b^3 + 9*A*b^4)*x^3 -
3*(2*B*a^2*b^2 - 3*A*a*b^3)*x^2 + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x)*sqrt(b*x
+ a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{9}{2}}}{9b}+\frac{(a+bx)^{\frac{7}{2}}(Ab-3Ba)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\left(-2Aab+3Ba^2\right)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\left(Aa^2b-Ba^3\right)}{3b}\right)}{b^3} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(9/2)/(9*b) + (a + b*x)**(7/2)*(A*b - 3*B*a)/(7
*b) + (a + b*x)**(5/2)*(-2*A*a*b + 3*B*a**2)/(5*b) + (a + b*x)**(3/2)*(A*a
**2*b - B*a**3)/(3*b))/b**3, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**4/4), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{2\left(35\,(bx+a)^{\frac{9}{2}}B - 45\,(3\,Ba - Ab)(bx+a)^{\frac{7}{2}} + 63\,(3\,Ba^2 - 2\,Aab)(bx+a)^{\frac{5}{2}} - 105\,(Ba^3 - Aa^2b)(bx+\right.}{315\,b^4}$$

input
```
integrate(x^2*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/315*(35*(b*x + a)^(9/2)*B - 45*(3*B*a - A*b)*(b*x + a)^(7/2) + 63*(3*B*a
^2 - 2*A*a*b)*(b*x + a)^(5/2) - 105*(B*a^3 - A*a^2*b)*(b*x + a)^(3/2))/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.18

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)Aa}{b^2} + \frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)Ba}{b^3} + \frac{9\left(5\,(bx+a)^{\frac{7}{2}}-2\right.}{}\right.}{315\,b}$$

input
```
integrate(x^2*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*A*a/b^2 + 9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*B*a/b^3 + 9*(5*(b*x + a)^(7/2) - 21*(b*x +
a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*A/b^2 + (35*(b
*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*B/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx = \frac{(6\,B\,a^2-4\,A\,a\,b)\,\left(a+b\,x\right)^{5/2}}{5\,b^4}+\frac{2\,B\,\left(a+b\,x\right)^{9/2}}{9\,b^4}+\frac{(2\,A\,b-6\,B\,a)\,\left(a+b\,x\right)^{7/2}}{7\,b^4}-\frac{(2\,B\,a^3-2\,A\,a^2\,b)\,\left(a+b\,x\right)^{3/2}}{3\,b^4}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
((6*B*a^2 - 4*A*a*b)*(a + b*x)^(5/2))/(5*b^4) + (2*B*(a + b*x)^(9/2))/(9*b
^4) + ((2*A*b - 6*B*a)*(a + b*x)^(7/2))/(7*b^4) - ((2*B*a^3 - 2*A*a^2*b)*(
a + b*x)^(3/2))/(3*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int x^2\sqrt{a+bx}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)}{315b^3}$$

input
```
int(x^2*(b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**4 - 4*a**3*b*x + 3*a**2*b**2*x**2 + 50*a*b**3*x**3
+ 35*b**4*x**4))/(315*b**3)
```

3.227 $\int x\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int x\sqrt{a+bx}(A+Bx)\,dx = -\frac{2a(Ab-aB)(a+bx)^{3/2}}{3b^3} + \frac{2(Ab-2aB)(a+bx)^{5/2}}{5b^3} + \frac{2B(a+bx)^{7/2}}{7b^3}$$

output
```
-2/3*a*(A*b-B*a)*(b*x+a)^(3/2)/b^3+2/5*(A*b-2*B*a)*(b*x+a)^(5/2)/b^3+2/7*B
*(b*x+a)^(7/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x\sqrt{a+bx}(A+Bx)\,dx = \frac{2(a+bx)^{3/2}\left(8a^2B+3b^2x(7A+5Bx)-2ab(7A+6Bx)\right)}{105b^3}$$

input
```
Integrate[x*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*(a + b*x)^(3/2)*(8*a^2*B + 3*b^2*x*(7*A + 5*B*x) - 2*a*b*(7*A + 6*B*x))
)/(105*b^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{(a+bx)^{3/2}(Ab-2aB)}{b^2}+\frac{a\sqrt{a+bx}(aB-Ab)}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2(a+bx)^{5/2}(Ab-2aB)}{5b^3}-\frac{2a(a+bx)^{3/2}(Ab-aB)}{3b^3}+\frac{2B(a+bx)^{7/2}}{7b^3}$$

input
```
Int[x*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(-2*a*(A*b - a*B)*(a + b*x)^(3/2))/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x)^(5
/2))/(5*b^3) + (2*B*(a + b*x)^(7/2))/(7*b^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{4(bx+a)^{\frac{3}{2}}\left(-\frac{3\left(\frac{5Bx}{7}+A\right)x\,b^{2}}{2}+a\left(\frac{6Bx}{7}+A\right)b-\frac{4a^{2}B}{7}\right)}{15b^{3}}$	41
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-15b^{2}B\,x^{2}-21A\,b^{2}x+12Babx+14abA-8a^{2}B\right)}{105b^{3}}$	47
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-15b^{2}B\,x^{2}-21A\,b^{2}x+12Babx+14abA-8a^{2}B\right)}{105b^{3}}$	47
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2(Ab-2Ba)(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(Ab-Ba)(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	52
default	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2(Ab-2Ba)(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(Ab-Ba)(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	52
trager	$-\frac{2\left(-15b^{3}B\,x^{3}-21A\,x^{2}b^{3}-3B\,x^{2}a\,b^{2}-7Axa\,b^{2}+4Bx\,a^{2}b+14a^{2}bA-8a^{3}B\right)\sqrt{bx+a}}{105b^{3}}$	71
risch	$-\frac{2\left(-15b^{3}B\,x^{3}-21A\,x^{2}b^{3}-3B\,x^{2}a\,b^{2}-7Axa\,b^{2}+4Bx\,a^{2}b+14a^{2}bA-8a^{3}B\right)\sqrt{bx+a}}{105b^{3}}$	71

input
```
int(x*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-4/15*(b*x+a)^(3/2)*(-3/2*(5/7*B*x+A)*x*b^2+a*(6/7*B*x+A)*b-4/7*a^2*B)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int x\sqrt{a+bx}(A+Bx)\,dx = \frac{2\,(15\,Bb^3x^3+8\,Ba^3-14\,Aa^2b+3\,(Bab^2+7\,Ab^3)x^2-(4\,Ba^2b-7\,Aab^2)x)\sqrt{bx+a}}{105\,b^3}$$

input
```
integrate(x*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^3*x^3 + 8*B*a^3 - 14*A*a^2*b + 3*(B*a*b^2 + 7*A*b^3)*x^2 - (
4*B*a^2*b - 7*A*a*b^2)*x)*sqrt(b*x + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int x\sqrt{a+bx}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{7}{2}}}{7b}+\frac{(a+bx)^{\frac{5}{2}}(Ab-2Ba)}{5b}+\frac{(a+bx)^{\frac{3}{2}}(-Aab+Ba^2)}{3b}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(7/2)/(7*b) + (a + b*x)**(5/2)*(A*b - 2*B*a)/(5
*b) + (a + b*x)**(3/2)*(-A*a*b + B*a**2)/(3*b))/b**2, Ne(b, 0)), (sqrt(a)*
(A*x**2/2 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x\sqrt{a+bx}(A+Bx)\,dx = \frac{2\left(15\,(bx+a)^{\frac{7}{2}}B-21\,(2\,Ba-Ab)(bx+a)^{\frac{5}{2}}+35\,(Ba^2-Aab)(bx+a)^{\frac{3}{2}}\right)}{105\,b^3}$$

input
```
integrate(x*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/105*(15*(b*x + a)^(7/2)*B - 21*(2*B*a - A*b)*(b*x + a)^(5/2) + 35*(B*a^2
 - A*a*b)*(b*x + a)^(3/2))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.36

$$\int x\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Aa}{b}+\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)Ba}{b^2}+\frac{7\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)A}{b}+\right.}{105\,b}$$

input
```
integrate(x*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*A*a/b + 7*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*a/b^2 + 7*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*A/b + 3*(5*(b*x + a)
^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*
a^3)*B/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{2\,{\left(a+b\,x\right)}^{3/2}\,\left(35\,B\,a^2+15\,B\,{\left(a+b\,x\right)}^2-35\,A\,a\,b+21\,A\,b\,\left(a+b\,x\right)-42\,B\,a\,\left(a+b\,x\right)\right)}{105\,b^3}$$

input
```
int(x*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(3/2)*(35*B*a^2 + 15*B*(a + b*x)^2 - 35*A*a*b + 21*A*b*(a + b
*x) - 42*B*a*(a + b*x)))/(105*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int x\sqrt{a+bx}(A+Bx)\,dx=\frac{2\sqrt{bx+a}\,(5b^3x^3+8a\,b^2x^2+a^2bx-2a^3)}{35b^2}$$

input
```
int(x*(b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**3 + a**2*b*x + 8*a*b**2*x**2 + 5*b**3*x**3))/(35
*b**2)
```

3.228 $\int \sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{a+bx}(A+Bx)\,dx = \frac{2(Ab-aB)(a+bx)^{3/2}}{3b^2}+\frac{2B(a+bx)^{5/2}}{5b^2}$$

output
```
2/3*(A*b-B*a)*(b*x+a)^(3/2)/b^2+2/5*B*(b*x+a)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \sqrt{a+bx}(A+Bx)\,dx = \frac{2(a+bx)^{3/2}(5Ab-2aB+3bBx)}{15b^2}$$

input
```
Integrate[Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*(a + b*x)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x))/(15*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{\sqrt{a+bx}(Ab-aB)}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{3/2}(Ab-aB)}{3b^2}+\frac{2B(a+bx)^{5/2}}{5b^2}$$

input
```
Int[Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(2*(A*b - a*B)*(a + b*x)^(3/2))/(3*b^2) + (2*B*(a + b*x)^(5/2))/(5*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}(3bBx+5Ab-2Ba)}{15b^2}$	27
orering	$\frac{2(bx+a)^{\frac{3}{2}}(3bBx+5Ab-2Ba)}{15b^2}$	27
pseudoelliptic	$\frac{2((3Bx+5A)b-2Ba)(bx+a)^{\frac{3}{2}}}{15b^2}$	28
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2(Ab-Ba)(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	34
default	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2(Ab-Ba)(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	34
trager	$\frac{2(3b^2B\,x^2+5A\,b^2x+Babx+5abA-2a^2B)\sqrt{bx+a}}{15b^2}$	46
risch	$\frac{2(3b^2B\,x^2+5A\,b^2x+Babx+5abA-2a^2B)\sqrt{bx+a}}{15b^2}$	46

input
```
int((b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x+a)^(3/2)*(3*B*b*x+5*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \sqrt{a+bx}(A+Bx)\,dx = \frac{2\,(3\,Bb^2x^2-2\,Ba^2+5\,Aab+(Bab+5\,Ab^2)x)\sqrt{bx+a}}{15\,b^2}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^2*x^2 - 2*B*a^2 + 5*A*a*b + (B*a*b + 5*A*b^2)*x)*sqrt(b*x + a)
/b^2
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \sqrt{a+bx}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{5}{2}}}{5b}+\frac{(a+bx)^{\frac{3}{2}}(Ab-Ba)}{3b}\right)}{b} & \text{for } b\neq 0 \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(5/2)/(5*b) + (a + b*x)**(3/2)*(A*b - B*a)/(3*b
))/b, Ne(b, 0)), (sqrt(a)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \sqrt{a+bx}(A+Bx)\,dx = \frac{2\left(3\,(bx+a)^{\frac{5}{2}}B-5\,(Ba-Ab)(bx+a)^{\frac{3}{2}}\right)}{15\,b^2}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/15*(3*(b*x + a)^(5/2)*B - 5*(B*a - A*b)*(b*x + a)^(3/2))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.38

$$\int \sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{2\left(15\sqrt{bx+a}Aa+5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)A+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Ba}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx}\right.}{b}\right.}{15\,b}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*x + a)*A*a + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*A + 5
*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*B*a/b + (3*(b*x + a)^(5/2) - 10*(b*
x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx}(A+Bx)\,dx=\frac{2\,{\left(a+b\,x\right)}^{3/2}\,\left(5\,A\,b-5\,B\,a+3\,B\,\left(a+b\,x\right)\right)}{15\,b^2}$$

input
```
int((A + B*x)*(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(3/2)*(5*A*b - 5*B*a + 3*B*(a + b*x)))/(15*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \sqrt{a+bx}(A+Bx)\,dx=\frac{2\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5b}$$

input
```
int((b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b)
```

3.229 $\int \frac{\sqrt{a+bx}(A+Bx)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x} \, dx = 2A\sqrt{a+bx} + \frac{2B(a+bx)^{3/2}}{3b} - 2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*A*(b*x+a)^(1/2)+2/3*B*(b*x+a)^(3/2)/b-2*a^(1/2)*A*arctanh((b*x+a)^(1/2)/
a^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x} \, dx = \frac{2\sqrt{a+bx}(3Ab + B(a+bx))}{3b} - 2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x,x]
```

output
```
(2*Sqrt[a + b*x]*(3*A*b + B*(a + b*x)))/(3*b) - 2*Sqrt[a]*A*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x} \, dx$$

$$\Big\downarrow \text{90}$$

$$A \int \frac{\sqrt{a+bx}}{x}dx+\frac{2B(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{60}$$

$$A\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2B(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{73}$$

$$A\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2B(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{221}$$

$$A\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2B(a+bx)^{3/2}}{3b}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x,x]
```

output
```
(2*B*(a + b*x)^(3/2))/(3*b) + A*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-2A\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46
default	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-2A\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46
pseudoelliptic	$\frac{-2A\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\left(\left(\frac{Bx}{3}+A\right)b+\frac{Ba}{3}\right)\sqrt{bx+a}}{b}$	47

input
```
int((b*x+a)^(1/2)*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/3*B*(b*x+a)^(3/2)+A*b*(b*x+a)^(1/2)-A*a^(1/2)*b*arctanh((b*x+a)^(1/
2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx = \left[\frac{3\,A\sqrt{a}b\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(Bbx+Ba+3\,Ab)\sqrt{bx+a}}{3\,b}, \frac{2\left(3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(Bbx+\right.}{3\,b}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*A*sqrt(a)*b*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(B*b*
x + B*a + 3*A*b)*sqrt(b*x + a))/b, 2/3*(3*A*sqrt(-a)*b*arctan(sqrt(-a)/sqr
t(b*x + a)) + (B*b*x + B*a + 3*A*b)*sqrt(b*x + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx = \begin{cases} \frac{2Aa\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2A\sqrt{a+bx} + \frac{2B(a+bx)^{\frac{3}{2}}}{3b} & \text{for } b \neq 0 \\ \sqrt{a}(A\log(Bx)+Bx) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x,x)
```

output
```
Piecewise((2*A*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*A*sqrt(a + b*x)
 + 2*B*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*(A*log(B*x) + B*x), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx = A\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left((bx+a)^{\frac{3}{2}}B+3\sqrt{bx+a}Ab\right)}{3\,b}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x,x, algorithm="maxima")
```

output
```
A*sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(
(b*x + a)^(3/2)*B + 3*sqrt(b*x + a)*A*b)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx = \frac{2\,Aa\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\left((bx+a)^{\frac{3}{2}}Bb^2+3\sqrt{bx+a}Ab^3\right)}{3\,b^3}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x,x, algorithm="giac")
```

output
```
2*A*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*((b*x + a)^(3/2)*B*b^2
 + 3*sqrt(b*x + a)*A*b^3)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx = 2\,A\,\sqrt{a+b\,x}+\frac{2\,B\,{(a+b\,x)}^{3/2}}{3\,b}+A\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x,x)
```

output
```
2*A*(a + b*x)^(1/2) + A*a^(1/2)*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2
*B*(a + b*x)^(3/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx = \frac{8\sqrt{bx+a}\,a}{3} + \frac{2\sqrt{bx+a}\,bx}{3} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x,x)
```

output
```
(8*sqrt(a + b*x)*a + 2*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x) - s
qrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a)/3
```

3.230 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 59

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2} dx = 2B\sqrt{a+bx} - \frac{A\sqrt{a+bx}}{x} - \frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
2*B*(b*x+a)^(1/2)-A*(b*x+a)^(1/2)/x-(A*b+2*B*a)*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2} dx = \frac{\sqrt{a+bx}(-A+2Bx)}{x} - \frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^2,x]
```

output
```
(Sqrt[a + b*x]*(-A + 2*B*x))/x - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2} \, dx$$

$$\Big\downarrow \ 87$$

$$\frac{(2aB+Ab)\int \frac{\sqrt{a+bx}}{x}dx}{2a}-\frac{A(a+bx)^{3/2}}{ax}$$

$$\Big\downarrow \ 60$$

$$\frac{(2aB+Ab)\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)}{2a}-\frac{A(a+bx)^{3/2}}{ax}$$

$$\Big\downarrow \ 73$$

$$\frac{(2aB+Ab)\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)}{2a}-\frac{A(a+bx)^{3/2}}{ax}$$

$$\Big\downarrow \ 221$$

$$\frac{(2aB+Ab)\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{2a}-\frac{A(a+bx)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^2,x]
```

output
```
-((A*(a + b*x)^(3/2))/(a*x)) + ((A*b + 2*a*B)*(2*Sqrt[a + b*x] - 2*Sqrt[a]
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(2*a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{(Ab+2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x+\sqrt{a}\,(-2Bx+A)\sqrt{bx+a}}{\sqrt{a}\,x}$	49
derivativedivides	$2B\sqrt{bx+a}-\frac{A\sqrt{bx+a}}{x}-\frac{(Ab+2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	50
default	$2B\sqrt{bx+a}-\frac{A\sqrt{bx+a}}{x}-\frac{(Ab+2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	50
risch	$2B\sqrt{bx+a}-\frac{A\sqrt{bx+a}}{x}-\frac{(Ab+2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	50

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((A*b+2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))*x+a^(1/2)*(-2*B*x+A)*(b*x+a)^
(1/2))/a^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx$$
$$=\left[\frac{(2\,Ba+Ab)\sqrt{a}x\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,Bax-Aa)\sqrt{bx+a}}{2\,ax},\frac{(2\,Ba+Ab)\sqrt{-a}x\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)}{ax}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/2*((2*B*a + A*b)*sqrt(a)*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 + 2*(2*B*a*x - A*a)*sqrt(b*x + a))/(a*x), ((2*B*a + A*b)*sqrt(-a)*x*arcta
n(sqrt(-a)/sqrt(b*x + a)) + (2*B*a*x - A*a)*sqrt(b*x + a))/(a*x)]
```

Sympy [A] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{Ab\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} + B\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx} & \text{for } b \neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**2,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - A*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))
)/sqrt(a) + B*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*sqr
t(a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx = \frac{1}{2}\,b\left(\frac{4\sqrt{bx+a}B}{b} + \frac{(2\,Ba+Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}b} - \frac{2\sqrt{bx+a}A}{bx}\right)$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*(4*sqrt(b*x + a)*B/b + (2*B*a + A*b)*log((sqrt(b*x + a) - sqrt(a))/(
sqrt(b*x + a) + sqrt(a)))/(sqrt(a)*b) - 2*sqrt(b*x + a)*A/(b*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx$$
$$= b\left(\frac{2\sqrt{bx+a}B}{b} + \frac{(2\,Ba+Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b} - \frac{\sqrt{bx+a}A}{bx}\right)$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
b*(2*sqrt(b*x + a)*B/b + (2*B*a + A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqr
t(-a)*b) - sqrt(b*x + a)*A/(b*x))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx = 2\,B\,\sqrt{a+b\,x} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(A\,b+2\,B\,a)}{\sqrt{a}} - \frac{A\,\sqrt{a+b\,x}}{x}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^2,x)
```

output
```
2*B*(a + b*x)^(1/2) - (atanh((a + b*x)^(1/2)/a^(1/2))*(A*b + 2*B*a))/a^(1/
2) - (A*(a + b*x)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx$$
$$= \frac{-2\sqrt{bx+a}\,a+4\sqrt{bx+a}\,bx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2x}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^2,x)
```

output
```
( - 2*sqrt(a + b*x)*a + 4*sqrt(a + b*x)*b*x + 3*sqrt(a)*log(sqrt(a + b*x)
- sqrt(a))*b*x - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*x)
```

3.231 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 79

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3} dx = -\frac{A\sqrt{a+bx}}{2x^2} - \frac{(Ab+4aB)\sqrt{a+bx}}{4ax} + \frac{b(Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
-1/2*A*(b*x+a)^(1/2)/x^2-1/4*(A*b+4*B*a)*(b*x+a)^(1/2)/a/x+1/4*b*(A*b-4*B*
a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3} dx = -\frac{\sqrt{a+bx}(Abx+2a(A+2Bx))}{4ax^2} + \frac{b(Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^3,x]
```

output
```
-1/4*(Sqrt[a + b*x]*(A*b*x + 2*a*(A + 2*B*x)))/(a*x^2) + (b*(A*b - 4*a*B)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-4aB)\int \frac{\sqrt{a+bx}}{x^2}dx}{4a}-\frac{A(a+bx)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-4aB)\left(\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}\right)}{4a}-\frac{A(a+bx)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(Ab-4aB)\left(\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}\right)}{4a}-\frac{A(a+bx)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(Ab-4aB)\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}\right)}{4a}-\frac{A(a+bx)^{3/2}}{2ax^2}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^3,x]
```

output
```
-1/2*(A*(a + b*x)^(3/2))/(a*x^2) - ((A*b - 4*a*B)*(-(Sqrt[a + b*x]/x) - (b
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]))/(4*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(Abx+4Bax+2Aa)}{4x^2a}+\frac{b(Ab-4Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	57
pseudoelliptic	$-\frac{-b\,x^2(Ab-4Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\left((4Bx+2A)a^{\frac{3}{2}}+A\sqrt{a}\,bx\right)\sqrt{bx+a}}{4a^{\frac{3}{2}}x^2}$	64
derivativedivides	$2b\left(-\frac{\frac{(Ab+4Ba)(bx+a)^{\frac{3}{2}}}{8a}+\left(-\frac{Ba}{2}+\frac{Ab}{8}\right)\sqrt{bx+a}}{b^2x^2}+\frac{(Ab-4Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	76
default	$2b\left(-\frac{\frac{(Ab+4Ba)(bx+a)^{\frac{3}{2}}}{8a}+\left(-\frac{Ba}{2}+\frac{Ab}{8}\right)\sqrt{bx+a}}{b^2x^2}+\frac{(Ab-4Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	76

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(A*b*x+4*B*a*x+2*A*a)/x^2/a+1/4*b*(A*b-4*B*a)*arctanh((
b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx$$
$$= \left[-\frac{(4\,Bab-Ab^2)\sqrt{a}x^2\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,Aa^2+(4\,Ba^2+Aab)x)\sqrt{bx+a}}{8\,a^2x^2},\ \frac{(4\,Bab-Ab^2)\sqrt{-}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*((4*B*a*b - A*b^2)*sqrt(a)*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) +
2*a)/x) + 2*(2*A*a^2 + (4*B*a^2 + A*a*b)*x)*sqrt(b*x + a))/(a^2*x^2), 1/4*
((4*B*a*b - A*b^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x + a)) - (2*A*a^2
+ (4*B*a^2 + A*a*b)*x)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(66) = 132$.

Time = 37.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx = -\frac{Aa}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3A\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Ab^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**3,x)
```

output
```
-A*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*A*sqrt(b)/(4*x**(3/2)*sqrt
(a/(b*x) + 1)) - A*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + A*b**2*asinh
(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x) + 1)/sqr
t(x) - B*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx = -\frac{1}{8}\,b^2\left(\frac{2\left((4\,Ba+Ab)(bx+a)^{\frac{3}{2}}-(4\,Ba^2-Aab)\sqrt{bx+a}\right)}{(bx+a)^2ab-2\,(bx+a)a^2b+a^3b} - \frac{(4\,Ba-Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}b}\right)$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
-1/8*b^2*(2*((4*B*a + A*b)*(b*x + a)^(3/2) - (4*B*a^2 - A*a*b)*sqrt(b*x +
a))/((b*x + a)^2*a*b - 2*(b*x + a)*a^2*b + a^3*b) - (4*B*a - A*b)*log((sqr
t(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(3/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx$$
$$= \frac{\frac{(4\,Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{4\,(bx+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx+a}Ba^2b^2+(bx+a)^{\frac{3}{2}}Ab^3+\sqrt{bx+a}Aab^3}{ab^2x^2}}{4\,b}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/4*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) - (4*
(b*x + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x + a)*B*a^2*b^2 + (b*x + a)^(3/2)*A*b^
3 + sqrt(b*x + a)*A*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(A\,b-4\,B\,a)}{4\,a^{3/2}} - \frac{\left(\frac{A\,b^2}{4}-B\,a\,b\right)\,\sqrt{a+b\,x}+\frac{(A\,b^2+4\,B\,a\,b)\,(a+b\,x)^{3/2}}{4\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^3,x)
```

output
```
(b*atanh((a + b*x)^(1/2)/a^(1/2))*(A*b - 4*B*a))/(4*a^(3/2)) - (((A*b^2)/4
 - B*a*b)*(a + b*x)^(1/2) + ((A*b^2 + 4*B*a*b)*(a + b*x)^(3/2))/(4*a))/((a
 + b*x)^2 - 2*a*(a + b*x) + a^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx$$
$$=\frac{-4\sqrt{bx+a}\,a^2-10\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a\,x^2}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 10*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
 b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x
**2)/(8*a*x**2)
```

3.232 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4} dx = -\frac{A\sqrt{a+bx}}{3x^3} - \frac{(Ab+6aB)\sqrt{a+bx}}{12ax^2} + \frac{b(Ab-2aB)\sqrt{a+bx}}{8a^2x} - \frac{b^2(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/3*A*(b*x+a)^(1/2)/x^3-1/12*(A*b+6*B*a)*(b*x+a)^(1/2)/a/x^2+1/8*b*(A*b-2
*B*a)*(b*x+a)^(1/2)/a^2/x-1/8*b^2*(A*b-2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2
))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4} dx = \frac{\sqrt{a+bx}(3Ab^2x^2-2abx(A+3Bx)-4a^2(2A+3Bx))}{24a^2x^3} + \frac{b^2(-Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^4,x]
```

output
```
(Sqrt[a + b*x]*(3*A*b^2*x^2 - 2*a*b*x*(A + 3*B*x) - 4*a^2*(2*A + 3*B*x)))/
(24*a^2*x^3) + (b^2*(-(A*b) + 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^
(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {87, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-2aB)\int \frac{\sqrt{a+bx}}{x^3}dx}{2a}-\frac{A(a+bx)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-2aB)\left(\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{2x^2}\right)}{2a}-\frac{A(a+bx)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(Ab-2aB)\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)}{2a}-\frac{A(a+bx)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(Ab-2aB)\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)}{2a}-\frac{A(a+bx)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(Ab-2aB)\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)}{2a}-\frac{A(a+bx)^{3/2}}{3ax^3}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^4,x]
```

output
```
-1/3*(A*(a + b*x)^(3/2))/(a*x^3) - ((A*b - 2*a*B)*(-1/2*Sqrt[a + b*x]/x^2
+ (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2))
)/4))/(2*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3A\,b^{2}x^{2}+6Bab\,x^{2}+2aAbx+12B\,a^{2}x+8a^{2}A)}{24x^{3}a^{2}}-\frac{b^{2}(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$	82
pseudoelliptic	$-\frac{\frac{3b^{2}x^{3}(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8}+\left(\frac{bx(3Bx+A)a^{\frac{3}{2}}}{4}+\left(\frac{3Bx}{2}+A\right)a^{\frac{5}{2}}-\frac{3A\sqrt{a}\,b^{2}x^{2}}{8}\right)\sqrt{bx+a}}{3a^{\frac{5}{2}}x^{3}}$	82
derivativedivides	$2b^{2}\left(-\frac{-\frac{(Ab-2Ba)(bx+a)^{\frac{5}{2}}}{16a^{2}}+\frac{Ab(bx+a)^{\frac{3}{2}}}{6a}+\left(\frac{Ab}{16}-\frac{Ba}{8}\right)\sqrt{bx+a}}{b^{3}x^{3}}-\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	92
default	$2b^{2}\left(-\frac{-\frac{(Ab-2Ba)(bx+a)^{\frac{5}{2}}}{16a^{2}}+\frac{Ab(bx+a)^{\frac{3}{2}}}{6a}+\left(\frac{Ab}{16}-\frac{Ba}{8}\right)\sqrt{bx+a}}{b^{3}x^{3}}-\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	92

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(-3*A*b^2*x^2+6*B*a*b*x^2+2*A*a*b*x+12*B*a^2*x+8*A*a^2
)/x^3/a^2-1/8*b^2*(A*b-2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4}\,dx$$

$$= \left[-\frac{3\,(2\,Bab^{2}-Ab^{3})\sqrt{a}x^{3}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,Aa^{3}+3\,(2\,Ba^{2}b-Aab^{2})x^{2}+2\,(6\,Ba^{3}+Aa^{2}b)x}{48\,a^{3}x^{3}}\right.$$

$$-\frac{3\,(2\,Bab^{2}-Ab^{3})\sqrt{-a}x^{3}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(8\,Aa^{3}+3\,(2\,Ba^{2}b-Aab^{2})x^{2}+2\,(6\,Ba^{3}+Aa^{2}b)x)\sqrt{bx+}}{24\,a^{3}x^{3}}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^4,x, algorithm="fricas")
```

output

```
[-1/48*(3*(2*B*a*b^2 - A*b^3)*sqrt(a)*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(8*A*a^3 + 3*(2*B*a^2*b - A*a*b^2)*x^2 + 2*(6*B*a^3 + A*a
^2*b)*x)*sqrt(b*x + a))/(a^3*x^3), -1/24*(3*(2*B*a*b^2 - A*b^3)*sqrt(-a)*x
^3*arctan(sqrt(-a)/sqrt(b*x + a)) + (8*A*a^3 + 3*(2*B*a^2*b - A*a*b^2)*x^2
 + 2*(6*B*a^3 + A*a^2*b)*x)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(97) = 194$.

Time = 49.49 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4}\,dx = -\frac{Aa}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5A\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{Ab^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{Ab^{\frac{5}{2}}}{8a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}} - \frac{Ba}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3B\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Bb^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

input

```
integrate((b*x+a)**(1/2)*(B*x+A)/x**4,x)
```

output

```
-A*a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*A*sqrt(b)/(12*x**(5/2)*sqr
t(a/(b*x) + 1)) + A*b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + A*b**(5/2
)/(8*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - A*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(
x)))/(8*a**(5/2)) - B*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*B*sqrt(
b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - B*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) +
 1)) + B*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4}\,dx = -\frac{1}{48}\,b^3\left(\frac{2\left(8\,(bx+a)^{\frac{3}{2}}Aab+3\,(2\,Ba-Ab)(bx+a)^{\frac{5}{2}}-3\,(2\,Ba^3-Aa^2b)\sqrt{bx+a}\right)}{(bx+a)^3a^2b-3\,(bx+a)^2a^3b+3\,(bx+a)a^4b-a^5b}+\frac{3\,(2\,Ba-Ab)\,\mathrm{lo}}{a^{\frac{5}{2}}}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
-1/48*b^3*(2*(8*(b*x + a)^(3/2)*A*a*b + 3*(2*B*a - A*b)*(b*x + a)^(5/2) -
3*(2*B*a^3 - A*a^2*b)*sqrt(b*x + a))/((b*x + a)^3*a^2*b - 3*(b*x + a)^2*a^
3*b + 3*(b*x + a)*a^4*b - a^5*b) + 3*(2*B*a - A*b)*log((sqrt(b*x + a) - sq
rt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(5/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4}\,dx = -\frac{1}{24}\,b^3\left(\frac{3\,(2\,Ba-Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b}+\frac{6\,(bx+a)^{\frac{5}{2}}Ba-6\,\sqrt{bx+a}Ba^3-3\,(bx+a)^{\frac{5}{2}}Ab+8\,(bx+a}{a^2b^4x^3}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
-1/24*b^3*(3*(2*B*a - A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*b)
 + (6*(b*x + a)^(5/2)*B*a - 6*sqrt(b*x + a)*B*a^3 - 3*(b*x + a)^(5/2)*A*b
+ 8*(b*x + a)^(3/2)*A*a*b + 3*sqrt(b*x + a)*A*a^2*b)/(a^2*b^4*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4} dx = \frac{\left(\frac{A b^3}{8}-\frac{B a b^2}{4}\right) \sqrt{a+b x}-\frac{\left(A b^3-2 B a b^2\right) (a+b x)^{5/2}}{8 a^2}+\frac{A b^3 (a+b x)^{3/2}}{3 a}}{3 a (a+b x)^2-3 a^2 (a+b x)-(a+b x)^3+a^3} - \frac{b^2 \operatorname{atanh}\left(\frac{\sqrt{a+b x}}{\sqrt{a}}\right) (A b-2 B a)}{8 a^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^4,x)
```

output
```
(((A*b^3)/8 - (B*a*b^2)/4)*(a + b*x)^(1/2) - ((A*b^3 - 2*B*a*b^2)*(a + b*x
)^(5/2))/(8*a^2) + (A*b^3*(a + b*x)^(3/2))/(3*a))/(3*a*(a + b*x)^2 - 3*a^2
*(a + b*x) - (a + b*x)^3 + a^3) - (b^2*atanh((a + b*x)^(1/2)/a^(1/2))*(A*b
 - 2*B*a))/(8*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^4} dx = \frac{-16\sqrt{bx+a}\, a^3 - 28\sqrt{bx+a}\, a^2bx - 6\sqrt{bx+a}\, a b^2x^2 - 3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right) b^3x^3 + 3\sqrt{a}\log\left(\sqrt{bx+\right.}{48a^2x^3}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 28*sqrt(a + b*x)*a**2*b*x - 6*sqrt(a + b*x)*a*
b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 3*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**2*x**3)
```

3.233 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5} dx = -\frac{A\sqrt{a+bx}}{4x^4} - \frac{(Ab+8aB)\sqrt{a+bx}}{24ax^3} + \frac{b(5Ab-8aB)\sqrt{a+bx}}{96a^2x^2} - \frac{b^2(5Ab-8aB)\sqrt{a+bx}}{64a^3x} + \frac{b^3(5Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{7/2}}$$

output
```
-1/4*A*(b*x+a)^(1/2)/x^4-1/24*(A*b+8*B*a)*(b*x+a)^(1/2)/a/x^3+1/96*b*(5*A*
b-8*B*a)*(b*x+a)^(1/2)/a^2/x^2-1/64*b^2*(5*A*b-8*B*a)*(b*x+a)^(1/2)/a^3/x+
1/64*b^3*(5*A*b-8*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5} dx = -\frac{\sqrt{a+bx}(15Ab^3x^3+8a^2bx(A+2Bx)+16a^3(3A+4Bx)-2ab^2x^2(5A+12Bx))}{192a^3x^4} + \frac{b^3(5Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^5,x]
```

output
```
-1/192*(Sqrt[a + b*x]*(15*A*b^3*x^3 + 8*a^2*b*x*(A + 2*B*x) + 16*a^3*(3*A
+ 4*B*x) - 2*a*b^2*x^2*(5*A + 12*B*x)))/(a^3*x^4) + (b^3*(5*A*b - 8*a*B)*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(7/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(5Ab-8aB)\int \frac{\sqrt{a+bx}}{x^4}dx}{8a}-\frac{A(a+bx)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(5Ab-8aB)\left(\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{3/2}}{4ax^4}$$

$\downarrow$ 221

$$-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{3/2}}{4ax^4}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^5,x]
```

output
```
-1/4*(A*(a + b*x)^(3/2))/(a*x^4) - ((5*A*b - 8*a*B)*(-1/3*Sqrt[a + b*x]/x^
3 + (b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/6))/(8*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{\left(-\frac{15}{8}A\,b^4+3Ba\,b^3\right)x^4\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\,\left(-\frac{5\left(\frac{12Bx}{5}+A\right)b^2x^2a^{\frac{3}{2}}}{4}+bx(2Bx+A)a^{\frac{5}{2}}+(8Bx+6A)a^{\frac{7}{2}}+\frac{15A\sqrt{}}{8}\right.}{24a^{\frac{7}{2}}x^4}$
risch	$-\frac{\sqrt{bx+a}\,(15A\,b^3x^3-24Ba\,b^2x^3-10aA\,b^2x^2+16B\,a^2b\,x^2+8a^2Abx+64B\,a^3x+48a^3A)}{192x^4a^3}+\frac{b^3(5Ab-8Ba)\,\mathrm{arctanh}\left(\right.}{64a^{\frac{7}{2}}}$
derivativedivides	$2b^3\left(-\frac{\frac{(5Ab-8Ba)(bx+a)^{\frac{7}{2}}}{128a^3}-\frac{11(5Ab-8Ba)(bx+a)^{\frac{5}{2}}}{384a^2}+\frac{(73Ab-40Ba)(bx+a)^{\frac{3}{2}}}{384a}+\left(\frac{5Ab}{128}-\frac{Ba}{16}\right)\sqrt{bx+a}}{b^4x^4}+\frac{(5Ab-8Ba)\,\mathrm{arct}}{128a}\right.$
default	$2b^3\left(-\frac{\frac{(5Ab-8Ba)(bx+a)^{\frac{7}{2}}}{128a^3}-\frac{11(5Ab-8Ba)(bx+a)^{\frac{5}{2}}}{384a^2}+\frac{(73Ab-40Ba)(bx+a)^{\frac{3}{2}}}{384a}+\left(\frac{5Ab}{128}-\frac{Ba}{16}\right)\sqrt{bx+a}}{b^4x^4}+\frac{(5Ab-8Ba)\,\mathrm{arct}}{128a}\right.$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*((-15/8*A*b^4+3*B*a*b^3)*x^4*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^
(1/2)*(-5/4*(12/5*B*x+A)*b^2*x^2*a^(3/2)+b*x*(2*B*x+A)*a^(5/2)+(8*B*x+6*A)
*a^(7/2)+15/8*A*a^(1/2)*b^3*x^3))/a^(7/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx$$
$$= \left[-\frac{3\,(8\,Bab^3-5\,Ab^4)\sqrt{a}x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(48\,Aa^4-3\,(8\,Ba^2b^2-5\,Aab^3)x^3+2\,(8\,Ba^3b-5}{384\,a^4x^4}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(a)*x^4*log((b*x + 2*sqrt(b*x + a)*sq
rt(a) + 2*a)/x) + 2*(48*A*a^4 - 3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^3 + 2*(8*B*a
^3*b - 5*A*a^2*b^2)*x^2 + 8*(8*B*a^4 + A*a^3*b)*x)*sqrt(b*x + a))/(a^4*x^4
), 1/192*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x +
a)) - (48*A*a^4 - 3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^3 + 2*(8*B*a^3*b - 5*A*a^2
*b^2)*x^2 + 8*(8*B*a^4 + A*a^3*b)*x)*sqrt(b*x + a))/(a^4*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(131) = 262$.

Time = 92.46 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx = -\frac{Aa}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{7A\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{Ab^{\frac{3}{2}}}{96ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Ab^{\frac{5}{2}}}{192a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Ab^{\frac{7}{2}}}{64a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{7}{2}}} - \frac{Ba}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5B\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{Bb^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{Bb^{\frac{5}{2}}}{8a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**5,x)
```

output
```
-A*a/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 7*A*sqrt(b)/(24*x**(7/2)*sqr
t(a/(b*x) + 1)) + A*b**(3/2)/(96*a*x**(5/2)*sqrt(a/(b*x) + 1)) - 5*A*b**(5
/2)/(192*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*A*b**(7/2)/(64*a**3*sqrt(x)*
sqrt(a/(b*x) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(7/2
)) - B*a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*B*sqrt(b)/(12*x**(5/2)
*sqrt(a/(b*x) + 1)) + B*b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + B*b**
(5/2)/(8*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - B*b**3*asinh(sqrt(a)/(sqrt(b)*s
qrt(x)))/(8*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx = \frac{1}{384}\,b^4\left(\frac{2\left(3\,(8\,Ba-5\,Ab)(bx+a)^{\frac{7}{2}} - 11\,(8\,Ba^2-5\,Aab)(bx+a)^{\frac{5}{2}} + (40\,Ba^3-73\,Aa^2b)(bx+a)^{\frac{3}{2}} + 3\right.}{(bx+a)^4a^3b - 4\,(bx+a)^3a^4b + 6\,(bx+a)^2a^5b - 4\,(bx+a)a^6b + a^7}\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
1/384*b^4*(2*(3*(8*B*a - 5*A*b)*(b*x + a)^(7/2) - 11*(8*B*a^2 - 5*A*a*b)*(
b*x + a)^(5/2) + (40*B*a^3 - 73*A*a^2*b)*(b*x + a)^(3/2) + 3*(8*B*a^4 - 5*
A*a^3*b)*sqrt(b*x + a))/((b*x + a)^4*a^3*b - 4*(b*x + a)^3*a^4*b + 6*(b*x
+ a)^2*a^5*b - 4*(b*x + a)*a^6*b + a^7*b) + 3*(8*B*a - 5*A*b)*log((sqrt(b*
x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(7/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx = \frac{\frac{3\,(8\,Bab^4-5\,Ab^5)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{24\,(bx+a)^{\frac{7}{2}}Bab^4-88\,(bx+a)^{\frac{5}{2}}Ba^2b^4+40\,(bx+a)^{\frac{3}{2}}Ba^3b^4+24\,\sqrt{bx+a}Ba^4b^4-15\,(bx+a)^{\frac{7}{2}}Ab^5+55\,(bx+}{a^3b^4x^4}}{192\,b}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
1/192*(3*(8*B*a*b^4 - 5*A*b^5)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^
3) + (24*(b*x + a)^(7/2)*B*a*b^4 - 88*(b*x + a)^(5/2)*B*a^2*b^4 + 40*(b*x
+ a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x + a)*B*a^4*b^4 - 15*(b*x + a)^(7/2)*A*b
^5 + 55*(b*x + a)^(5/2)*A*a*b^5 - 73*(b*x + a)^(3/2)*A*a^2*b^5 - 15*sqrt(b
*x + a)*A*a^3*b^5)/(a^3*b^4*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx = \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(5\,A\,b-8\,B\,a)}{64\,a^{7/2}} - \frac{\left(\frac{5\,A\,b^4}{64}-\frac{B\,a\,b^3}{8}\right)\sqrt{a+b\,x} - \frac{11\,(5\,A\,b^4-8\,B\,a\,b^3)\,(a+b\,x)^{5/2}}{192\,a^2} + \frac{(5\,A\,b^4-8\,B\,a\,b^3)\,(a+b\,x)^{7/2}}{64\,a^3} + \frac{(73\,A\,b^4-40\,B\,a\,b^3)\,(a+b\,x)^{3/2}}{192\,a}}{(a+b\,x)^4-4\,a^3\,(a+b\,x)-4\,a\,(a+b\,x)^3+6\,a^2\,(a+b\,x)^2+a^4}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^5,x)
```

output
```
(b^3*atanh((a + b*x)^(1/2)/a^(1/2))*(5*A*b - 8*B*a))/(64*a^(7/2)) - (((5*A
*b^4)/64 - (B*a*b^3)/8)*(a + b*x)^(1/2) - (11*(5*A*b^4 - 8*B*a*b^3)*(a + b
*x)^(5/2))/(192*a^2) + ((5*A*b^4 - 8*B*a*b^3)*(a + b*x)^(7/2))/(64*a^3) +
((73*A*b^4 - 40*B*a*b^3)*(a + b*x)^(3/2))/(192*a))/((a + b*x)^4 - 4*a^3*(a
 + b*x) - 4*a*(a + b*x)^3 + 6*a^2*(a + b*x)^2 + a^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^5}\,dx$$
$$= \frac{-32\sqrt{bx+a}\,a^4-48\sqrt{bx+a}\,a^3bx-4\sqrt{bx+a}\,a^2b^2x^2+6\sqrt{bx+a}\,a\,b^3x^3+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^4x}{128a^3x^4}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^5,x)
```

output
```
( - 32*sqrt(a + b*x)*a**4 - 48*sqrt(a + b*x)*a**3*b*x - 4*sqrt(a + b*x)*a*
*2*b**2*x**2 + 6*sqrt(a + b*x)*a*b**3*x**3 + 3*sqrt(a)*log(sqrt(a + b*x) -
 sqrt(a))*b**4*x**4 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4)/(1
28*a**3*x**4)
```

3.234 $\int x^4(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 151

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \frac{2a^4(Ab-aB)(a+bx)^{5/2}}{5b^6} - \frac{2a^3(4Ab-5aB)(a+bx)^{7/2}}{7b^6} + \frac{4a^2(3Ab-5aB)(a+bx)^{9/2}}{9b^6} - \frac{4a(2Ab-5aB)(a+bx)^{11/2}}{11b^6} + \frac{2(Ab-5aB)(a+bx)^{13/2}}{13b^6} + \frac{2B(a+bx)^{15/2}}{15b^6}$$

output
```
2/5*a^4*(A*b-B*a)*(b*x+a)^(5/2)/b^6-2/7*a^3*(4*A*b-5*B*a)*(b*x+a)^(7/2)/b^
6+4/9*a^2*(3*A*b-5*B*a)*(b*x+a)^(9/2)/b^6-4/11*a*(2*A*b-5*B*a)*(b*x+a)^(11
/2)/b^6+2/13*(A*b-5*B*a)*(b*x+a)^(13/2)/b^6+2/15*B*(b*x+a)^(15/2)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \frac{2(a+bx)^{5/2}\left(-256a^5B+1680a^2b^3x^2(A+Bx)+128a^4b(3A+5Bx)-160a^3b^2x(6A+7Bx)-\right.}{45045b^6}$$

input
```
Integrate[x^4*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(5/2)*(-256*a^5*B + 1680*a^2*b^3*x^2*(A + B*x) + 128*a^4*b*(3
*A + 5*B*x) - 160*a^3*b^2*x*(6*A + 7*B*x) - 210*a*b^4*x^3*(12*A + 11*B*x)
+ 231*b^5*x^4*(15*A + 13*B*x)))/(45045*b^6)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^4(a+bx)^{3/2}(aB-Ab)}{b^5}+\frac{a^3(a+bx)^{5/2}(5aB-4Ab)}{b^5}-\frac{2a^2(a+bx)^{7/2}(5aB-3Ab)}{b^5}+\frac{(a+bx)^{11/2}(Ab-}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4(a+bx)^{5/2}(Ab-aB)}{5b^6}-\frac{2a^3(a+bx)^{7/2}(4Ab-5aB)}{7b^6}+\frac{4a^2(a+bx)^{9/2}(3Ab-5aB)}{9b^6}+\frac{2(a+bx)^{13/2}(Ab-5aB)}{13b^6}-\frac{4a(a+bx)^{11/2}(2Ab-5aB)}{11b^6}+\frac{2B(a+bx)^{15/2}}{15b^6}$$

input
```
Int[x^4*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*a^4*(A*b - a*B)*(a + b*x)^(5/2))/(5*b^6) - (2*a^3*(4*A*b - 5*a*B)*(a +
b*x)^(7/2))/(7*b^6) + (4*a^2*(3*A*b - 5*a*B)*(a + b*x)^(9/2))/(9*b^6) - (4
*a*(2*A*b - 5*a*B)*(a + b*x)^(11/2))/(11*b^6) + (2*(A*b - 5*a*B)*(a + b*x)
^(13/2))/(13*b^6) + (2*B*(a + b*x)^(15/2))/(15*b^6)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{256\left(\frac{1155\left(\frac{13Bx}{15}+A\right)x^4b^5}{128}-\frac{105a\left(\frac{11Bx}{12}+A\right)x^3b^4}{16}+\frac{35a^2x^2(Bx+A)b^3}{8}-\frac{5a^3\left(\frac{7Bx}{6}+A\right)x\,b^2}{2}+a^4\left(\frac{5Bx}{3}+A\right)b-\frac{2a^5B}{3}\right)(bx+a}{15015b^6}$
gosper	$\frac{2(bx+a)^{\frac{5}{2}}(3003b^5Bx^5+3465Ab^5x^4-2310Bab^4x^4-2520Aab^4x^3+1680Ba^2b^3x^3+1680Aa^2b^3x^2-1120Ba^3b^2x^2-96}{45045b^6}$
orering	$\frac{2(bx+a)^{\frac{5}{2}}(3003b^5Bx^5+3465Ab^5x^4-2310Bab^4x^4-2520Aab^4x^3+1680Ba^2b^3x^3+1680Aa^2b^3x^2-1120Ba^3b^2x^2-96}{45045b^6}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{15}{2}}}{15}+\frac{2(Ab-5Ba)(bx+a)^{\frac{13}{2}}}{13}+\frac{2\left(6a^2B-4a(Ab-Ba)\right)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(-4a^3B+6a^2(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(Ba^4-4a^3(Ab}{7}}{b^6}$
default	$\frac{\frac{2B(bx+a)^{\frac{15}{2}}}{15}+\frac{2(Ab-5Ba)(bx+a)^{\frac{13}{2}}}{13}+\frac{2\left(6a^2B-4a(Ab-Ba)\right)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(-4a^3B+6a^2(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(Ba^4-4a^3(Ab}{7}}{b^6}$
trager	$\frac{2(3003Bb^7x^7+3465Ab^7x^6+3696Bab^6x^6+4410Aab^6x^5+63Ba^2b^5x^5+105Aa^2b^5x^4-70Ba^3b^4x^4-120Aa^3b^4x^3+80}{45045b^6}$
risch	$\frac{2(3003Bb^7x^7+3465Ab^7x^6+3696Bab^6x^6+4410Aab^6x^5+63Ba^2b^5x^5+105Aa^2b^5x^4-70Ba^3b^4x^4-120Aa^3b^4x^3+80}{45045b^6}$

input
```
int(x^4*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
256/15015*(1155/128*(13/15*B*x+A)*x^4*b^5-105/16*a*(11/12*B*x+A)*x^3*b^4+3
5/8*a^2*x^2*(B*x+A)*b^3-5/2*a^3*(7/6*B*x+A)*x*b^2+a^4*(5/3*B*x+A)*b-2/3*a^
5*B)*(b*x+a)^(5/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\,(3003\,Bb^7x^7 - 256\,Ba^7 + 384\,Aa^6b + 231\,(16\,Bab^6 + 15\,Ab^7)x^6 + 63\,(Ba^2b^5 + 70\,Aab^6)x^5 - 3$$

input
```
integrate(x^4*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*b^7*x^7 - 256*B*a^7 + 384*A*a^6*b + 231*(16*B*a*b^6 + 15*A
*b^7)*x^6 + 63*(B*a^2*b^5 + 70*A*a*b^6)*x^5 - 35*(2*B*a^3*b^4 - 3*A*a^2*b^
5)*x^4 + 40*(2*B*a^4*b^3 - 3*A*a^3*b^4)*x^3 - 48*(2*B*a^5*b^2 - 3*A*a^4*b^
3)*x^2 + 64*(2*B*a^6*b - 3*A*a^5*b^2)*x)*sqrt(b*x + a)/b^6
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{15}{2}}}{15b} + \frac{(a+bx)^{\frac{13}{2}}(Ab-5Ba)}{13b} + \frac{(a+bx)^{\frac{11}{2}}\left(-4Aab+10Ba^2\right)}{11b} + \frac{(a+bx)^{\frac{9}{2}}\cdot\left(6Aa^2b-10Ba^3\right)}{9b} + \frac{(a+bx)^{\frac{7}{2}}\left(-4Aa^3b+5Ba^4\right)}{7b} + \frac{(a+bx)^{\frac{5}{2}}}{}\right.}{b^5} \\ a^{\frac{3}{2}}\left(\frac{Ax^5}{5} + \frac{Bx^6}{6}\right) \end{cases}$$

input
```
integrate(x**4*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(15/2)/(15*b) + (a + b*x)**(13/2)*(A*b - 5*B*a)
/(13*b) + (a + b*x)**(11/2)*(-4*A*a*b + 10*B*a**2)/(11*b) + (a + b*x)**(9/
2)*(6*A*a**2*b - 10*B*a**3)/(9*b) + (a + b*x)**(7/2)*(-4*A*a**3*b + 5*B*a*
*4)/(7*b) + (a + b*x)**(5/2)*(A*a**4*b - B*a**5)/(5*b))/b**5, Ne(b, 0)), (
a**(3/2)*(A*x**5/5 + B*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(3003\,(bx+a)^{\frac{15}{2}}B - 3465\,(5\,Ba - Ab)(bx+a)^{\frac{13}{2}} + 8190\,(5\,Ba^2 - 2\,Aab)(bx+a)^{\frac{11}{2}} - 10010\right.}{4504}$$

input
```
integrate(x^4*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3003*(b*x + a)^(15/2)*B - 3465*(5*B*a - A*b)*(b*x + a)^(13/2) + 8
190*(5*B*a^2 - 2*A*a*b)*(b*x + a)^(11/2) - 10010*(5*B*a^3 - 3*A*a^2*b)*(b*
x + a)^(9/2) + 6435*(5*B*a^4 - 4*A*a^3*b)*(b*x + a)^(7/2) - 9009*(B*a^5 -
A*a^4*b)*(b*x + a)^(5/2))/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(128) = 256.

Time = 0.12 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.27

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(
5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A*a^2/b^4 + 65
*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 -
1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^
5)*B*a^2/b^5 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
 + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 69
3*sqrt(b*x + a)*a^5)*A*a/b^4 + 30*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(
11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x
+ a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*B*a/b^
5 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9
/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x
+ a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*A/b^4 + 7*(429*(b*x + a)^(15/2) -
 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9
/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b
*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*B/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \frac{(20\,B\,a^2-8\,A\,a\,b)\,\left(a+b\,x\right)^{11/2}}{11\,b^6}+\frac{2\,B\,\left(a+b\,x\right)^{15/2}}{15\,b^6} + \frac{(2\,A\,b-10\,B\,a)\,\left(a+b\,x\right)^{13/2}}{13\,b^6}-\frac{(2\,B\,a^5-2\,A\,a^4\,b)\,\left(a+b\,x\right)^{5/2}}{5\,b^6} + \frac{(10\,B\,a^4-8\,A\,a^3\,b)\,\left(a+b\,x\right)^{7/2}}{7\,b^6}-\frac{(20\,B\,a^3-12\,A\,a^2\,b)\,\left(a+b\,x\right)^{9/2}}{9\,b^6}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
((20*B*a^2 - 8*A*a*b)*(a + b*x)^(11/2))/(11*b^6) + (2*B*(a + b*x)^(15/2))/
(15*b^6) + ((2*A*b - 10*B*a)*(a + b*x)^(13/2))/(13*b^6) - ((2*B*a^5 - 2*A*
a^4*b)*(a + b*x)^(5/2))/(5*b^6) + ((10*B*a^4 - 8*A*a^3*b)*(a + b*x)^(7/2))
/(7*b^6) - ((20*B*a^3 - 12*A*a^2*b)*(a + b*x)^(9/2))/(9*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int x^4(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(3003b^7x^7+7161a\,b^6x^6+4473a^2b^5x^5+35a^3b^4x^4-40a^4b^3x^3+48a^5b^2x^2-64a^6bx+}{45045b^5}$$

input
```
int(x^4*(b*x+a)^(3/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**7 - 64*a**6*b*x + 48*a**5*b**2*x**2 - 40*a**4*b**
3*x**3 + 35*a**3*b**4*x**4 + 4473*a**2*b**5*x**5 + 7161*a*b**6*x**6 + 3003
*b**7*x**7))/(45045*b**5)
```

3.235 $\int x^3(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx=-\frac{2a^3(Ab-aB)(a+bx)^{5/2}}{5b^5}+\frac{2a^2(3Ab-4aB)(a+bx)^{7/2}}{7b^5}-\frac{2a(Ab-2aB)(a+bx)^{9/2}}{3b^5}+\frac{2(Ab-4aB)(a+bx)^{11/2}}{11b^5}+\frac{2B(a+bx)^{13/2}}{13b^5}$$

output
```
-2/5*a^3*(A*b-B*a)*(b*x+a)^(5/2)/b^5+2/7*a^2*(3*A*b-4*B*a)*(b*x+a)^(7/2)/b
^5-2/3*a*(A*b-2*B*a)*(b*x+a)^(9/2)/b^5+2/11*(A*b-4*B*a)*(b*x+a)^(11/2)/b^5
+2/13*B*(b*x+a)^(13/2)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx=\frac{2(a+bx)^{5/2}\left(128a^4B+105b^4x^3(13A+11Bx)-70ab^3x^2(13A+12Bx)+40a^2b^2x(13A+14B\right.}{15015b^5}$$

input
```
Integrate[x^3*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(5/2)*(128*a^4*B + 105*b^4*x^3*(13*A + 11*B*x) - 70*a*b^3*x^2
*(13*A + 12*B*x) + 40*a^2*b^2*x*(13*A + 14*B*x) - 16*a^3*b*(13*A + 20*B*x)
))/(15015*b^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{a^3(a+bx)^{3/2}(aB-Ab)}{b^4}-\frac{a^2(a+bx)^{5/2}(4aB-3Ab)}{b^4}+\frac{(a+bx)^{9/2}(Ab-4aB)}{b^4}+\frac{3a(a+bx)^{7/2}(2aB-A\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3(a+bx)^{5/2}(Ab-aB)}{5b^5}+\frac{2a^2(a+bx)^{7/2}(3Ab-4aB)}{7b^5}+\frac{2(a+bx)^{11/2}(Ab-4aB)}{11b^5}-\frac{2a(a+bx)^{9/2}(Ab-2aB)}{3b^5}+\frac{2B(a+bx)^{13/2}}{13b^5}$$

input
```
Int[x^3*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(-2*a^3*(A*b - a*B)*(a + b*x)^(5/2))/(5*b^5) + (2*a^2*(3*A*b - 4*a*B)*(a +
 b*x)^(7/2))/(7*b^5) - (2*a*(A*b - 2*a*B)*(a + b*x)^(9/2))/(3*b^5) + (2*(A
*b - 4*a*B)*(a + b*x)^(11/2))/(11*b^5) + (2*B*(a + b*x)^(13/2))/(13*b^5)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{32\left(-\frac{105\left(\frac{11Bx}{13}+A\right)x^{3}b^{4}}{16}+\frac{35a\left(\frac{12Bx}{13}+A\right)x^{2}b^{3}}{8}-\frac{5a^{2}x\left(\frac{14Bx}{13}+A\right)b^{2}}{2}+a^{3}\left(\frac{20Bx}{13}+A\right)b-\frac{8Ba^{4}}{13}\right)(bx+a)^{\frac{5}{2}}}{1155b^{5}}$
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-1155Bx^{4}b^{4}-1365Ax^{3}b^{4}+840Bx^{3}ab^{3}+910Ax^{2}ab^{3}-560Bx^{2}a^{2}b^{2}-520Axa^{2}b^{2}+320Bxa^{3}b+208A\cdots\right.}{15015b^{5}}$
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-1155Bx^{4}b^{4}-1365Ax^{3}b^{4}+840Bx^{3}ab^{3}+910Ax^{2}ab^{3}-560Bx^{2}a^{2}b^{2}-520Axa^{2}b^{2}+320Bxa^{3}b+208A\cdots\right.}{15015b^{5}}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{13}{2}}}{13}+\frac{2(Ab-4Ba)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(3a^{2}B-3a(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(-a^{3}B+3a^{2}(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^{3}(Ab-Ba)(bx+\cdots}{5}}{b^{5}}$
default	$\frac{\frac{2B(bx+a)^{\frac{13}{2}}}{13}-\frac{2(-Ab+4Ba)(bx+a)^{\frac{11}{2}}}{11}-\frac{2\left(-3a^{2}B+3a(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}-\frac{2\left(a^{3}B-3a^{2}(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^{3}(Ab-Ba)(bx\cdots}{5}}{b^{5}}$
trager	$-\frac{2\left(-1155Bb^{6}x^{6}-1365Ab^{6}x^{5}-1470Bab^{5}x^{5}-1820Aab^{5}x^{4}-35Ba^{2}b^{4}x^{4}-65Aa^{2}b^{4}x^{3}+40Ba^{3}b^{3}x^{3}+78Aa^{3}b^{3}x^{2}-\cdots\right.}{15015b^{5}}$
risch	$-\frac{2\left(-1155Bb^{6}x^{6}-1365Ab^{6}x^{5}-1470Bab^{5}x^{5}-1820Aab^{5}x^{4}-35Ba^{2}b^{4}x^{4}-65Aa^{2}b^{4}x^{3}+40Ba^{3}b^{3}x^{3}+78Aa^{3}b^{3}x^{2}-\cdots\right.}{15015b^{5}}$

input
```
int(x^3*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-32/1155*(-105/16*(11/13*B*x+A)*x^3*b^4+35/8*a*(12/13*B*x+A)*x^2*b^3-5/2*a
^2*x*(14/13*B*x+A)*b^2+a^3*(20/13*B*x+A)*b-8/13*B*a^4)*(b*x+a)^(5/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.17

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\,(1155\,Bb^6x^6+128\,Ba^6-208\,Aa^5b+105\,(14\,Bab^5+13\,Ab^6)x^5+35\,(Ba^2b^4+52\,Aab^5)x^4-}{15015\,b}$$

input
```
integrate(x^3*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/15015*(1155*B*b^6*x^6 + 128*B*a^6 - 208*A*a^5*b + 105*(14*B*a*b^5 + 13*A
*b^6)*x^5 + 35*(B*a^2*b^4 + 52*A*a*b^5)*x^4 - 5*(8*B*a^3*b^3 - 13*A*a^2*b^
4)*x^3 + 6*(8*B*a^4*b^2 - 13*A*a^3*b^3)*x^2 - 8*(8*B*a^5*b - 13*A*a^4*b^2)
*x)*sqrt(b*x + a)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{13}{2}}}{13b}+\frac{(a+bx)^{\frac{11}{2}}(Ab-4Ba)}{11b}+\frac{(a+bx)^{\frac{9}{2}}\left(-3Aab+6Ba^2\right)}{9b}+\frac{(a+bx)^{\frac{7}{2}}\cdot\left(3Aa^2b-4Ba^3\right)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\left(-Aa^3b+Ba^4\right)}{5b}\right)}{b^4} & \text{for } b\neq \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right) & \text{otherw} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(13/2)/(13*b) + (a + b*x)**(11/2)*(A*b - 4*B*a)
/(11*b) + (a + b*x)**(9/2)*(-3*A*a*b + 6*B*a**2)/(9*b) + (a + b*x)**(7/2)*
(3*A*a**2*b - 4*B*a**3)/(7*b) + (a + b*x)**(5/2)*(-A*a**3*b + B*a**4)/(5*b
))/b**4, Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(1155\,(bx+a)^{\frac{13}{2}}B-1365\,(4\,Ba-Ab)(bx+a)^{\frac{11}{2}}+5005\,(2\,Ba^2-Aab)(bx+a)^{\frac{9}{2}}-2145\,(4\,B\right.}{15015\,b^5}$$

input
```
integrate(x^3*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/15015*(1155*(b*x + a)^(13/2)*B - 1365*(4*B*a - A*b)*(b*x + a)^(11/2) + 5
005*(2*B*a^2 - A*a*b)*(b*x + a)^(9/2) - 2145*(4*B*a^3 - 3*A*a^2*b)*(b*x +
a)^(7/2) + 3003*(B*a^4 - A*a^3*b)*(b*x + a)^(5/2))/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(104) = 208.

Time = 0.13 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.46

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(\frac{1287\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)Aa^2}{b^3}+\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-\right.}{b^4}\right.}{}$$

input
```
integrate(x^3*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/45045*(1287*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*A*a^2/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315
*sqrt(b*x + a)*a^4)*B*a^2/b^4 + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7
/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x +
 a)*a^4)*A*a/b^3 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*
(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4
- 693*sqrt(b*x + a)*a^5)*B*a/b^4 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)
^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x
+ a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*A/b^3 + 15*(231*(b*x + a)^(13/2) -
 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)
*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x
 + a)*a^6)*B/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx = \frac{\left(12\,B\,a^2-6\,A\,a\,b\right)\,{\left(a+b\,x\right)}^{9/2}}{9\,b^5} + \frac{2\,B\,{\left(a+b\,x\right)}^{13/2}}{13\,b^5} + \frac{\left(2\,A\,b-8\,B\,a\right)\,{\left(a+b\,x\right)}^{11/2}}{11\,b^5} + \frac{\left(2\,B\,a^4-2\,A\,a^3\,b\right)\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5} - \frac{\left(8\,B\,a^3-6\,A\,a^2\,b\right)\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
((12*B*a^2 - 6*A*a*b)*(a + b*x)^(9/2))/(9*b^5) + (2*B*(a + b*x)^(13/2))/(1
3*b^5) + ((2*A*b - 8*B*a)*(a + b*x)^(11/2))/(11*b^5) + ((2*B*a^4 - 2*A*a^3
*b)*(a + b*x)^(5/2))/(5*b^5) - ((8*B*a^3 - 6*A*a^2*b)*(a + b*x)^(7/2))/(7*
b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.61

$$\int x^3(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(231b^6x^6+567a\,b^5x^5+371a^2b^4x^4+5a^3b^3x^3-6a^4b^2x^2+8a^5bx-16a^6)}{3003b^4}$$

input
```
int(x^3*(b*x+a)^(3/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**6 + 8*a**5*b*x - 6*a**4*b**2*x**2 + 5*a**3*b**3
*x**3 + 371*a**2*b**4*x**4 + 567*a*b**5*x**5 + 231*b**6*x**6))/(3003*b**4)
```

3.236 $\int x^2(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx=\frac{2a^2(Ab-aB)(a+bx)^{5/2}}{5b^4}-\frac{2a(2Ab-3aB)(a+bx)^{7/2}}{7b^4}+\frac{2(Ab-3aB)(a+bx)^{9/2}}{9b^4}+\frac{2B(a+bx)^{11/2}}{11b^4}$$

output
```
2/5*a^2*(A*b-B*a)*(b*x+a)^(5/2)/b^4-2/7*a*(2*A*b-3*B*a)*(b*x+a)^(7/2)/b^4+
2/9*(A*b-3*B*a)*(b*x+a)^(9/2)/b^4+2/11*B*(b*x+a)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx=\frac{2(a+bx)^{5/2}\left(-48a^3B+35b^3x^2(11A+9Bx)+8a^2b(11A+15Bx)-10ab^2x(22A+21Bx)\right)}{3465b^4}$$

input
```
Integrate[x^2*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(5/2)*(-48*a^3*B + 35*b^3*x^2*(11*A + 9*B*x) + 8*a^2*b*(11*A
+ 15*B*x) - 10*a*b^2*x*(22*A + 21*B*x)))/(3465*b^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^2(a+bx)^{3/2}(aB-Ab)}{b^3}+\frac{(a+bx)^{7/2}(Ab-3aB)}{b^3}+\frac{a(a+bx)^{5/2}(3aB-2Ab)}{b^3}+\frac{B(a+bx)^{9/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(a+bx)^{5/2}(Ab-aB)}{5b^4}+\frac{2(a+bx)^{9/2}(Ab-3aB)}{9b^4}-\frac{2a(a+bx)^{7/2}(2Ab-3aB)}{7b^4}+\frac{2B(a+bx)^{11/2}}{11b^4}$$

input
```
Int[x^2*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*a^2*(A*b - a*B)*(a + b*x)^(5/2))/(5*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*
x)^(7/2))/(7*b^4) + (2*(A*b - 3*a*B)*(a + b*x)^(9/2))/(9*b^4) + (2*B*(a +
b*x)^(11/2))/(11*b^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{16\left(\frac{35\left(\frac{9Bx}{11}+A\right)x^{2}b^{3}}{8}-\frac{5a\left(\frac{21Bx}{22}+A\right)x\,b^{2}}{2}+a^{2}\left(\frac{15Bx}{11}+A\right)b-\frac{6a^{3}B}{11}\right)(bx+a)^{\frac{5}{2}}}{315b^{4}}$
gosper	$\frac{2(bx+a)^{\frac{5}{2}}\left(315b^{3}B\,x^{3}+385A\,x^{2}b^{3}-210B\,x^{2}a\,b^{2}-220Axa\,b^{2}+120Bx\,a^{2}b+88a^{2}bA-48a^{3}B\right)}{3465b^{4}}$
orering	$\frac{2(bx+a)^{\frac{5}{2}}\left(315b^{3}B\,x^{3}+385A\,x^{2}b^{3}-210B\,x^{2}a\,b^{2}-220Axa\,b^{2}+120Bx\,a^{2}b+88a^{2}bA-48a^{3}B\right)}{3465b^{4}}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-3Ba)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^{2}(Ab-Ba)(bx+a)^{\frac{5}{2}}}{5}}{b^{4}}$
default	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-3Ba)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^{2}(Ab-Ba)(bx+a)^{\frac{5}{2}}}{5}}{b^{4}}$
trager	$\frac{2(315b^{5}B\,x^{5}+385A\,b^{5}x^{4}+420Ba\,b^{4}x^{4}+550Aa\,b^{4}x^{3}+15B\,a^{2}b^{3}x^{3}+33A\,a^{2}b^{3}x^{2}-18B\,a^{3}b^{2}x^{2}-44a^{3}b^{2}Ax+24a^{4}bBx+}{3465b^{4}}$
risch	$\frac{2(315b^{5}B\,x^{5}+385A\,b^{5}x^{4}+420Ba\,b^{4}x^{4}+550Aa\,b^{4}x^{3}+15B\,a^{2}b^{3}x^{3}+33A\,a^{2}b^{3}x^{2}-18B\,a^{3}b^{2}x^{2}-44a^{3}b^{2}Ax+24a^{4}bBx+}{3465b^{4}}$

input
```
int(x^2*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
16/315*(35/8*(9/11*B*x+A)*x^2*b^3-5/2*a*(21/22*B*x+A)*x*b^2+a^2*(15/11*B*x
+A)*b-6/11*a^3*B)*(b*x+a)^(5/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\,(315\,Bb^5x^5 - 48\,Ba^5 + 88\,Aa^4b + 35\,(12\,Bab^4 + 11\,Ab^5)x^4 + 5\,(3\,Ba^2b^3 + 110\,Aab^4)x^3 - 3\,(6$$
$$3465\,b^4$$

input
```
integrate(x^2*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^5*x^5 - 48*B*a^5 + 88*A*a^4*b + 35*(12*B*a*b^4 + 11*A*b^5)
*x^4 + 5*(3*B*a^2*b^3 + 110*A*a*b^4)*x^3 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*
x^2 + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x)*sqrt(b*x + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{11}{2}}}{11b} + \frac{(a+bx)^{\frac{9}{2}}(Ab-3Ba)}{9b} + \frac{(a+bx)^{\frac{7}{2}}\left(-2Aab+3Ba^2\right)}{7b} + \frac{(a+bx)^{\frac{5}{2}}\left(Aa^2b-Ba^3\right)}{5b}\right)}{b^3} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(11/2)/(11*b) + (a + b*x)**(9/2)*(A*b - 3*B*a)/
(9*b) + (a + b*x)**(7/2)*(-2*A*a*b + 3*B*a**2)/(7*b) + (a + b*x)**(5/2)*(A
*a**2*b - B*a**3)/(5*b))/b**3, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**4/4),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx=\frac{2\left(315\,(bx+a)^{\frac{11}{2}}B-385\,(3\,Ba-Ab)(bx+a)^{\frac{9}{2}}+495\,(3\,Ba^2-2\,Aab)(bx+a)^{\frac{7}{2}}-693\,(Ba^3-\right.}{3465\,b^4}$$

input
```
integrate(x^2*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/3465*(315*(b*x + a)^(11/2)*B - 385*(3*B*a - A*b)*(b*x + a)^(9/2) + 495*(
3*B*a^2 - 2*A*a*b)*(b*x + a)^(7/2) - 693*(B*a^3 - A*a^2*b)*(b*x + a)^(5/2)
)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(80) = 160.

Time = 0.13 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.68

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx=\frac{2\left(\frac{231\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)Aa^2}{b^2}+\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)Ba^2}{b^3}+\right.}{}$$

input
```
integrate(x^2*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*A*a^2/b^2 + 99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*B*a^2/b^3 + 198*(5*(b*x + a)^(7/2) - 2
1*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*A*a/b
^2 + 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*B*a/b^3 + 11*(35*(b
*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A/b^2 + 5*(63*(b*x + a)^(11/2) -
385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3
 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*B/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx = \frac{\left(6\,B\,a^2-4\,A\,a\,b\right)\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}+\frac{2\,B\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}+\frac{\left(2\,A\,b-6\,B\,a\right)\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}-\frac{\left(2\,B\,a^3-2\,A\,a^2\,b\right)\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
((6*B*a^2 - 4*A*a*b)*(a + b*x)^(7/2))/(7*b^4) + (2*B*(a + b*x)^(11/2))/(11
*b^4) + ((2*A*b - 6*B*a)*(a + b*x)^(9/2))/(9*b^4) - ((2*B*a^3 - 2*A*a^2*b)
*(a + b*x)^(5/2))/(5*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int x^2(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5)}{693b^3}$$

input
```
int(x^2*(b*x+a)^(3/2)*(B*x+A),x)
```

output

```
(2*sqrt(a + b*x)*(8*a**5 - 4*a**4*b*x + 3*a**3*b**2*x**2 + 113*a**2*b**3*x
**3 + 161*a*b**4*x**4 + 63*b**5*x**5))/(693*b**3)
```

3.237 $\int x(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = -\frac{2a(Ab-aB)(a+bx)^{5/2}}{5b^3} + \frac{2(Ab-2aB)(a+bx)^{7/2}}{7b^3} + \frac{2B(a+bx)^{9/2}}{9b^3}$$

output
```
-2/5*a*(A*b-B*a)*(b*x+a)^(5/2)/b^3+2/7*(A*b-2*B*a)*(b*x+a)^(7/2)/b^3+2/9*B
*(b*x+a)^(9/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \frac{2(a+bx)^{5/2}\,(8a^2B+5b^2x(9A+7Bx)-2ab(9A+10Bx))}{315b^3}$$

input
```
Integrate[x*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(5/2)*(8*a^2*B + 5*b^2*x*(9*A + 7*B*x) - 2*a*b*(9*A + 10*B*x)
))/(315*b^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{(a+bx)^{5/2}(Ab-2aB)}{b^2}+\frac{a(a+bx)^{3/2}(aB-Ab)}{b^2}+\frac{B(a+bx)^{7/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(a+bx)^{7/2}(Ab-2aB)}{7b^3}-\frac{2a(a+bx)^{5/2}(Ab-aB)}{5b^3}+\frac{2B(a+bx)^{9/2}}{9b^3}$$

input
```
Int[x*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(-2*a*(A*b - a*B)*(a + b*x)^(5/2))/(5*b^3) + (2*(A*b - 2*a*B)*(a + b*x)^(7
/2))/(7*b^3) + (2*B*(a + b*x)^(9/2))/(9*b^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	si
pseudoelliptic	$-\frac{4\left(-\frac{5\left(\frac{7Bx}{9}+A\right)x\,b^{2}}{2}+a\left(\frac{10Bx}{9}+A\right)b-\frac{4a^{2}B}{9}\right)(bx+a)^{\frac{5}{2}}}{35b^{3}}$	4
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-35b^{2}B\,x^{2}-45A\,b^{2}x+20Babx+18abA-8a^{2}B\right)}{315b^{3}}$	4
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-35b^{2}B\,x^{2}-45A\,b^{2}x+20Babx+18abA-8a^{2}B\right)}{315b^{3}}$	4
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-2Ba)(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(Ab-Ba)(bx+a)^{\frac{5}{2}}}{5}}{b^{3}}$	5
default	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-2Ba)(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(Ab-Ba)(bx+a)^{\frac{5}{2}}}{5}}{b^{3}}$	5
trager	$-\frac{2\left(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}-50B\,x^{3}a\,b^{3}-72A\,x^{2}a\,b^{3}-3B\,x^{2}a^{2}b^{2}-9Ax\,a^{2}b^{2}+4Bx\,a^{3}b+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{bx+a}}{315b^{3}}$	9
risch	$-\frac{2\left(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}-50B\,x^{3}a\,b^{3}-72A\,x^{2}a\,b^{3}-3B\,x^{2}a^{2}b^{2}-9Ax\,a^{2}b^{2}+4Bx\,a^{3}b+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{bx+a}}{315b^{3}}$	9

input
```
int(x*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-4/35*(-5/2*(7/9*B*x+A)*x*b^2+a*(10/9*B*x+A)*b-4/9*a^2*B)*(b*x+a)^(5/2)/b^
3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(35\,Bb^{4}x^{4}+8\,Ba^{4}-18\,Aa^{3}b+5\,(10\,Bab^{3}+9\,Ab^{4})x^{3}+3\,(Ba^{2}b^{2}+24\,Aab^{3})x^{2}-(4\,Ba^{3}b-9\right.}{315\,b^{3}}$$

input
```
integrate(x*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^4*x^4 + 8*B*a^4 - 18*A*a^3*b + 5*(10*B*a*b^3 + 9*A*b^4)*x^3
+ 3*(B*a^2*b^2 + 24*A*a*b^3)*x^2 - (4*B*a^3*b - 9*A*a^2*b^2)*x)*sqrt(b*x +
 a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{9}{2}}}{9b}+\frac{(a+bx)^{\frac{7}{2}}(Ab-2Ba)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\left(-Aab+Ba^2\right)}{5b}\right)}{b^2} & \text{for } b\neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(9/2)/(9*b) + (a + b*x)**(7/2)*(A*b - 2*B*a)/(7
*b) + (a + b*x)**(5/2)*(-A*a*b + B*a**2)/(5*b))/b**2, Ne(b, 0)), (a**(3/2)
*(A*x**2/2 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(35\,(bx+a)^{\frac{9}{2}}B-45\,(2\,Ba-Ab)(bx+a)^{\frac{7}{2}}+63\,(Ba^2-Aab)(bx+a)^{\frac{5}{2}}\right)}{315\,b^3}$$

input
```
integrate(x*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/315*(35*(b*x + a)^(9/2)*B - 45*(2*B*a - A*b)*(b*x + a)^(7/2) + 63*(B*a^2
 - A*a*b)*(b*x + a)^(5/2))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 4.10

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Aa^2}{b}+\frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)Ba^2}{b^2}+\frac{42\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\right.}{b}\right.}{}$$

input
```
integrate(x*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*A*a^2/b + 21*(3*(b*x + a)
^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*a^2/b^2 + 42*(3*(b
*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*A*a/b + 18*(5
*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt
(b*x + a)*a^3)*B*a/b^2 + 9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*
(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*A/b + (35*(b*x + a)^(9/2) - 18
0*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 +
315*sqrt(b*x + a)*a^4)*B/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\,{\left(a+b\,x\right)}^{5/2}\,\left(63\,B\,a^2+35\,B\,{\left(a+b\,x\right)}^2-63\,A\,a\,b+45\,A\,b\,\left(a+b\,x\right)-90\,B\,a\,\left(a+b\,x\right)\right)}{315\,b^3}$$

input
```
int(x*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^(5/2)*(63*B*a^2 + 35*B*(a + b*x)^2 - 63*A*a*b + 45*A*b*(a + b
*x) - 90*B*a*(a + b*x)))/(315*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int x(a+bx)^{3/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(7b^4x^4+19a\,b^3x^3+15a^2b^2x^2+a^3bx-2a^4)}{63b^2}$$

input
```
int(x*(b*x+a)^(3/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**4 + a**3*b*x + 15*a**2*b**2*x**2 + 19*a*b**3*x**
3 + 7*b**4*x**4))/(63*b**2)
```

3.238 $\int (a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int (a+bx)^{3/2}(A+Bx)\,dx=\frac{2(Ab-aB)(a+bx)^{5/2}}{5b^2}+\frac{2B(a+bx)^{7/2}}{7b^2}$$

output
```
2/5*(A*b-B*a)*(b*x+a)^(5/2)/b^2+2/7*B*(b*x+a)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int (a+bx)^{3/2}(A+Bx)\,dx=\frac{2(a+bx)^{5/2}(7Ab-2aB+5bBx)}{35b^2}$$

input
```
Integrate[(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x))/(35*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{(a+bx)^{3/2}(Ab-aB)}{b}+\frac{B(a+bx)^{5/2}}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2(a+bx)^{5/2}(Ab-aB)}{5b^2}+\frac{2B(a+bx)^{7/2}}{7b^2}$$

input
```
Int[(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(2*(A*b - a*B)*(a + b*x)^(5/2))/(5*b^2) + (2*B*(a + b*x)^(7/2))/(7*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}(5bBx+7Ab-2Ba)}{35b^2}$	27
orering	$\frac{2(bx+a)^{\frac{5}{2}}(5bBx+7Ab-2Ba)}{35b^2}$	27
pseudoelliptic	$\frac{2((5Bx+7A)b-2Ba)(bx+a)^{\frac{5}{2}}}{35b^2}$	28
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2(Ab-Ba)(bx+a)^{\frac{5}{2}}}{5}}{b^2}$	34
default	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2(Ab-Ba)(bx+a)^{\frac{5}{2}}}{5}}{b^2}$	34
trager	$\frac{2(5b^3Bx^3+7Ax^2b^3+8Bx^2ab^2+14Axab^2+Bxa^2b+7a^2bA-2a^3B)\sqrt{bx+a}}{35b^2}$	70
risch	$\frac{2(5b^3Bx^3+7Ax^2b^3+8Bx^2ab^2+14Axab^2+Bxa^2b+7a^2bA-2a^3B)\sqrt{bx+a}}{35b^2}$	70

input
```
int((b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/35*(b*x+a)^(5/2)*(5*B*b*x+7*A*b-2*B*a)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int (a+bx)^{3/2}(A+Bx)\,dx=\frac{2\,(5\,Bb^3x^3-2\,Ba^3+7\,Aa^2b+(8\,Bab^2+7\,Ab^3)x^2+(Ba^2b+14\,Aab^2)x)\sqrt{bx+a}}{35\,b^2}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/35*(5*B*b^3*x^3 - 2*B*a^3 + 7*A*a^2*b + (8*B*a*b^2 + 7*A*b^3)*x^2 + (B*a
^2*b + 14*A*a*b^2)*x)*sqrt(b*x + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(41) = 82$.

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int (a+bx)^{3/2}(A+Bx)\,dx = \begin{cases} \frac{2Aa^2\sqrt{a+bx}}{5b} + \frac{4Aax\sqrt{a+bx}}{5} + \frac{2Abx^2\sqrt{a+bx}}{5} - \frac{4Ba^3\sqrt{a+bx}}{35b^2} + \frac{2Ba^2x\sqrt{a+bx}}{35b} + \frac{16Bax^2\sqrt{a+bx}}{35} + \frac{2Bbx^3\sqrt{a+bx}}{7} \\ a^{\frac{3}{2}}\left(Ax + \frac{Bx^2}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A),x)
```

output
```
Piecewise((2*A*a**2*sqrt(a + b*x)/(5*b) + 4*A*a*x*sqrt(a + b*x)/5 + 2*A*b*
x**2*sqrt(a + b*x)/5 - 4*B*a**3*sqrt(a + b*x)/(35*b**2) + 2*B*a**2*x*sqrt(
a + b*x)/(35*b) + 16*B*a*x**2*sqrt(a + b*x)/35 + 2*B*b*x**3*sqrt(a + b*x)/
7, Ne(b, 0)), (a**(3/2)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int (a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(5\,(bx+a)^{\frac{7}{2}}B - 7\,(Ba-Ab)(bx+a)^{\frac{5}{2}}\right)}{35\,b^2}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/35*(5*(b*x + a)^(7/2)*B - 7*(B*a - A*b)*(b*x + a)^(5/2))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 4.57

$$\int (a+bx)^{3/2}(A+Bx)\,dx = \frac{2\left(105\sqrt{bx+a}Aa^2 + 70\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)Aa + \frac{35\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)Ba^2}{b} + 7\left(3(bx+\right.\right.}{}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(b*x + a)*A*a^2 + 70*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*
A*a + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*B*a^2/b + 7*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*A + 14*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*a/b + 3*(5*(b*x + a)^(
7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^
3)*B/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int (a+bx)^{3/2}(A+Bx)\,dx = \frac{2\,{\left(a+b\,x\right)}^{5/2}\,\left(7\,A\,b-7\,B\,a+5\,B\,\left(a+b\,x\right)\right)}{35\,b^2}$$

input
```
int((A + B*x)*(a + b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^(5/2)*(7*A*b - 7*B*a + 5*B*(a + b*x)))/(35*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (a+bx)^{3/2}(A+Bx)\,dx=\frac{2\sqrt{bx+a}\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}{7b}$$

input
```
int((b*x+a)^(3/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/(7*b)
```

3.239 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x} \, dx = 2aA\sqrt{a+bx} + \frac{2}{3}A(a+bx)^{3/2} + \frac{2B(a+bx)^{5/2}}{5b} - 2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*a*A*(b*x+a)^(1/2)+2/3*A*(b*x+a)^(3/2)+2/5*B*(b*x+a)^(5/2)/b-2*a^(3/2)*A*
arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x} \, dx = \frac{2\sqrt{a+bx}(15aAb + 5Ab(a+bx) + 3B(a+bx)^2)}{15b} - 2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x,x]
```

output
```
(2*Sqrt[a + b*x]*(15*a*A*b + 5*A*b*(a + b*x) + 3*B*(a + b*x)^2))/(15*b) -
2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}dx$$

$$\Big\downarrow \text{ 90}$$

$$A\int \frac{(a+bx)^{3/2}}{x}dx+\frac{2B(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \text{ 60}$$

$$A\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2B(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \text{ 60}$$

$$A\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2B(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \text{ 73}$$

$$A\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2B(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \text{ 221}$$

$$A\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2B(a+bx)^{5/2}}{5b}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x,x]
```

output
```
(2*B*(a + b*x)^(5/2))/(5*b) + A*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x
] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}+2Aab\sqrt{bx+a}-2A\,a^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	58
default	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}+2Aab\sqrt{bx+a}-2A\,a^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	58
pseudoelliptic	$\frac{-2A\,a^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{8\sqrt{bx+a}\left(\frac{\left(\frac{3Bx}{5}+A\right)x\,b^{2}}{4}+a\left(\frac{3Bx}{10}+A\right)b+\frac{3a^{2}B}{20}\right)}{3}}{b}$	63

input
```
int((b*x+a)^(3/2)*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/5*B*(b*x+a)^(5/2)+1/3*A*b*(b*x+a)^(3/2)+A*a*b*(b*x+a)^(1/2)-A*a^(3/
2)*b*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx = \left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(3\,Bb^{2}x^{2}+3\,Ba^{2}+20\,Aab+(6\,Bab+}{15\,b}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*A*a^(3/2)*b*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3*
B*b^2*x^2 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x)*sqrt(b*x + a))/b,
2/15*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x + a)) + (3*B*b^2*x^2 + 3*
B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x)*sqrt(b*x + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx = \begin{cases} \frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa\sqrt{a+bx} + \frac{2A(a+bx)^{\frac{3}{2}}}{3} + \frac{2B(a+bx)^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}(A\log(Bx) + Bx) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x,x)
```

output
```
Piecewise((2*A*a**2*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*A*a*sqrt(a +
 b*x) + 2*A*(a + b*x)**(3/2)/3 + 2*B*(a + b*x)**(5/2)/(5*b), Ne(b, 0)), (a
**(3/2)*(A*log(B*x) + B*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx = Aa^{\frac{3}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}B + 5\,(bx+a)^{\frac{3}{2}}Ab + 15\,\sqrt{bx+a}Aab\right)}{15\,b}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x,x, algorithm="maxima")
```

output
```
A*a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/15*
(3*(b*x + a)^(5/2)*B + 5*(b*x + a)^(3/2)*A*b + 15*sqrt(b*x + a)*A*a*b)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx = \frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}Bb^4+5\,(bx+a)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx+a}Aab^5\right)}{15\,b^5}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x,x, algorithm="giac")
```

output
```
2*A*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/15*(3*(b*x + a)^(5/2)*
B*b^4 + 5*(b*x + a)^(3/2)*A*b^5 + 15*sqrt(b*x + a)*A*a*b^5)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx = \left(\frac{2\,A\,b-2\,B\,a}{3\,b} + \frac{2\,B\,a}{3\,b}\right)\,{\left(a+b\,x\right)}^{3/2} + \frac{2\,B\,{\left(a+b\,x\right)}^{5/2}}{5\,b} + a\,\left(\frac{2\,A\,b-2\,B\,a}{b}+\frac{2\,B\,a}{b}\right)\,\sqrt{a+b\,x} + A\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x,x)
```

output
```
((2*A*b - 2*B*a)/(3*b) + (2*B*a)/(3*b))*(a + b*x)^(3/2) + A*a^(3/2)*atan((
(a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*B*(a + b*x)^(5/2))/(5*b) + a*((2*A*b
- 2*B*a)/b + (2*B*a)/b)*(a + b*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx = \frac{46\sqrt{bx+a}\,a^2}{15} + \frac{22\sqrt{bx+a}\,abx}{15} + \frac{2\sqrt{bx+a}\,b^2x^2}{5} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2 - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a^2$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x,x)
```

output
```
(46*sqrt(a + b*x)*a**2 + 22*sqrt(a + b*x)*a*b*x + 6*sqrt(a + b*x)*b**2*x**
2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2 - 15*sqrt(a)*log(sqrt(a +
 b*x) + sqrt(a))*a**2)/15
```

3.240 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2} dx = 2(Ab+aB)\sqrt{a+bx} - \frac{aA\sqrt{a+bx}}{x} + \frac{2}{3}B(a+bx)^{3/2} - \sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*(A*b+B*a)*(b*x+a)^(1/2)-a*A*(b*x+a)^(1/2)/x+2/3*B*(b*x+a)^(3/2)-a^(1/2)*
(3*A*b+2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2} dx = \frac{\sqrt{a+bx}(2bx(3A+Bx)+a(-3A+8Bx))}{3x} - \sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^2,x]
```

output
```
(Sqrt[a + b*x]*(2*b*x*(3*A + B*x) + a*(-3*A + 8*B*x)))/(3*x) - Sqrt[a]*(3*
A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(2aB+3Ab)\int \frac{(a+bx)^{3/2}}{x}dx}{2a}-\frac{A(a+bx)^{5/2}}{ax}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2aB+3Ab)\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)}{2a}-\frac{A(a+bx)^{5/2}}{ax}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2aB+3Ab)\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)}{2a}-\frac{A(a+bx)^{5/2}}{ax}$$

$$\Big\downarrow \text{73}$$

$$\frac{(2aB+3Ab)\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)}{2a}-\frac{A(a+bx)^{5/2}}{ax}$$

$$\Big\downarrow \text{221}$$

$$\frac{(2aB+3Ab)\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)}{2a}-\frac{A(a+bx)^{5/2}}{ax}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^2,x]
```

output
```
-((A*(a + b*x)^(5/2))/(a*x)) + ((3*A*b + 2*a*B)*((2*(a + b*x)^(3/2))/3 + a
*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(2*a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{3\left(ax\left(Ab+\frac{2Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{2\left(\left(\frac{4Bx}{3}-\frac{A}{2}\right)a^{\frac{3}{2}}+bx\sqrt{a}\left(\frac{Bx}{3}+A\right)\right)\sqrt{bx+a}}{3}\right)}{\sqrt{a}\,x}$
risch	$-\frac{aA\sqrt{bx+a}}{x}+\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}+2Ba\sqrt{bx+a}-\sqrt{a}\,(3Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.$
derivativedivides	$\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}+2Ba\sqrt{bx+a}-2a\left(\frac{A\sqrt{bx+a}}{2x}+\frac{(3Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$
default	$\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}+2Ba\sqrt{bx+a}-2a\left(\frac{A\sqrt{bx+a}}{2x}+\frac{(3Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3*(a*x*(A*b+2/3*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))-2/3*((4/3*B*x-1/2*A)*
a^(3/2)+b*x*a^(1/2)*(1/3*B*x+A))*(b*x+a)^(1/2))/a^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx=\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,Bbx^2-3\,Aa+2\,(4\,Ba+}{6\,x}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(2*B*a + 3*A*b)*sqrt(a)*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a
)/x) + 2*(2*B*b*x^2 - 3*A*a + 2*(4*B*a + 3*A*b)*x)*sqrt(b*x + a))/x, 1/3*(
3*(2*B*a + 3*A*b)*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x + a)) + (2*B*b*x^2 -
 3*A*a + 2*(4*B*a + 3*A*b)*x)*sqrt(b*x + a))/x]
```

Sympy [A] (verification not implemented)

Time = 14.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx = -A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} + Ab\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx} & \text{for } b\neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + Ba\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx} & \text{for } b\neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + Bb\left(\begin{cases}\frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**2,x)
```

output
```
-A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - A*a*sqrt(b)*sqrt(a/(b*x) +
 1)/sqrt(x) + A*b*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2
*sqrt(a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True)) + B*a*Piecewise((2*a*at
an(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b*x), Ne(b, 0)), (sqrt(a)
*log(x), True)) + B*b*Piecewise((2*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqr
t(a)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx = \frac{1}{6}\left(\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{b} - \frac{6\,\sqrt{bx+a}Aa}{bx} + \frac{4\left((bx+a)^{\frac{3}{2}}B+\right.}{}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/6*(3*(2*B*a + 3*A*b)*sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a
) + sqrt(a)))/b - 6*sqrt(b*x + a)*A*a/(b*x) + 4*((b*x + a)^(3/2)*B + 3*(B*
a + A*b)*sqrt(b*x + a))/b)*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx =$$
$$-\frac{1}{3}b\left(\frac{3\sqrt{bx+a}Aa}{bx}-\frac{3\left(2Ba^2+3Aab\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b}-\frac{2\left((bx+a)^{\frac{3}{2}}Bb^2+3\sqrt{bx+a}Bab^2+3\sqrt{bx+\right.}{b^3}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
-1/3*b*(3*sqrt(b*x + a)*A*a/(b*x) - 3*(2*B*a^2 + 3*A*a*b)*arctan(sqrt(b*x
+ a)/sqrt(-a))/(sqrt(-a)*b) - 2*((b*x + a)^(3/2)*B*b^2 + 3*sqrt(b*x + a)*B
*a*b^2 + 3*sqrt(b*x + a)*A*b^3)/b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx = (2\,A\,b+2\,B\,a)\,\sqrt{a+b\,x}+\frac{2\,B\,{(a+b\,x)}^{3/2}}{3}$$
$$+2\,\mathrm{atan}\left(\frac{2\,(3\,A\,b+2\,B\,a)\,\sqrt{-\frac{a}{4}}\,\sqrt{a+b\,x}}{2\,B\,a^2+3\,A\,b\,a}\right)\,(3\,A\,b+2\,B\,a)\,\sqrt{-\frac{a}{4}}-\frac{A\,a\,\sqrt{a+b\,x}}{x}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^2,x)
```

output
```
(2*A*b + 2*B*a)*(a + b*x)^(1/2) + (2*B*(a + b*x)^(3/2))/3 + 2*atan((2*(3*A
*b + 2*B*a)*(-a/4)^(1/2)*(a + b*x)^(1/2))/(2*B*a^2 + 3*A*a*b))*(3*A*b + 2*
B*a)*(-a/4)^(1/2) - (A*a*(a + b*x)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx = \frac{-6\sqrt{bx+a}\,a^2+28\sqrt{bx+a}\,abx+4\sqrt{bx+a}\,b^2x^2+15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right.}{6x}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^2,x)
```

output
```
( - 6*sqrt(a + b*x)*a**2 + 28*sqrt(a + b*x)*a*b*x + 4*sqrt(a + b*x)*b**2*x
**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*x - 15*sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))*a*b*x)/(6*x)
```

3.241 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3} dx = 2bB\sqrt{a+bx} - \frac{(3Ab+4aB)\sqrt{a+bx}}{4x} - \frac{A(a+bx)^{3/2}}{2x^2} - \frac{3b(Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
2*b*B*(b*x+a)^(1/2)-1/4*(3*A*b+4*B*a)*(b*x+a)^(1/2)/x-1/2*A*(b*x+a)^(3/2)/
x^2-3/4*b*(A*b+4*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3} dx = -\frac{\sqrt{a+bx}(bx(5A-8Bx)+2a(A+2Bx))}{4x^2} - \frac{3b(Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^3,x]
```

output
```
-1/4*(Sqrt[a + b*x]*(b*x*(5*A - 8*B*x) + 2*a*(A + 2*B*x)))/x^2 - (3*b*(A*b
 + 4*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(4aB+Ab)\int \frac{(a+bx)^{3/2}}{x^2}dx}{4a}-\frac{A(a+bx)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{(4aB+Ab)\left(\frac{3}{2}b\int \frac{\sqrt{a+bx}}{x}dx-\frac{(a+bx)^{3/2}}{x}\right)}{4a}-\frac{A(a+bx)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{(4aB+Ab)\left(\frac{3}{2}b\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)}{4a}-\frac{A(a+bx)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{(4aB+Ab)\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)}{4a}-\frac{A(a+bx)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{(4aB+Ab)\left(\frac{3}{2}b\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)-\frac{(a+bx)^{3/2}}{x}\right)}{4a}-\frac{A(a+bx)^{5/2}}{2ax^2}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^3,x]
```

output
```
-1/2*(A*(a + b*x)^(5/2))/(a*x^2) + ((A*b + 4*a*B)*(-((a + b*x)^(3/2)/x) +
(3*b*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/2))/(4*
a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$-\frac{3\left(b\,x^{2}(Ab+4Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{5\left(\frac{2(2Bx+A)a^{\frac{3}{2}}}{5}+bx\sqrt{a}\,\left(-\frac{8Bx}{5}+A\right)\right)\sqrt{bx+a}}{3}\right)}{4\sqrt{a}\,x^{2}}$	68
risch	$-\frac{\sqrt{bx+a}\,(5Abx+4Bax+2Aa)}{4x^{2}}+\frac{b\left(16B\sqrt{bx+a}-\frac{2(3Ab+12Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{8}$	69
derivativedivides	$2b\left(B\sqrt{bx+a}-\frac{\left(\frac{5Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(-\frac{1}{2}a^{2}B-\frac{3}{8}abA\right)\sqrt{bx+a}}{b^{2}x^{2}}-\frac{3(Ab+4Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	85
default	$2b\left(B\sqrt{bx+a}-\frac{\left(\frac{5Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(-\frac{1}{2}a^{2}B-\frac{3}{8}abA\right)\sqrt{bx+a}}{b^{2}x^{2}}-\frac{3(Ab+4Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	85

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b*x^2*(A*b+4*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+5/3*(2/5*(2*B*x+A)*
a^(3/2)+b*x*a^(1/2)*(-8/5*B*x+A))*(b*x+a)^(1/2))/a^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx = \left[\frac{3\,(4\,Bab+Ab^2)\sqrt{a}x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,Babx^2-2\,Aa^2-(4\,Ba}{8\,ax^2}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(8*B*a*b*x^2 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x)*sqrt(b*x + a)
)/(a*x^2), 1/4*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x
+ a)) + (8*B*a*b*x^2 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x)*sqrt(b*x + a))/(a*
x^2)]
```

Sympy [A] (verification not implemented)

Time = 42.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.53

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx = -\frac{Aa^2}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3Aa\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{Ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} - B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} + Bb\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx} & \text{for } b \neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**3,x)
```

output
```
-A*a**2/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*A*a*sqrt(b)/(4*x**(3/2)
*sqrt(a/(b*x) + 1)) - A*b**(3/2)*sqrt(a/(b*x) + 1)/sqrt(x) - A*b**(3/2)/(4
*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4
*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - B*a*sqrt(b)*sqr
t(a/(b*x) + 1)/sqrt(x) + B*b*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/s
qrt(-a) + 2*sqrt(a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx = \frac{1}{8}\,b^2\left(\frac{16\,\sqrt{bx+a}B}{b} + \frac{3\,(4\,Ba+Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}b} - \frac{2\left((4\,Ba+5\,Ab)(bx\right.}{(bx+a)}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/8*b^2*(16*sqrt(b*x + a)*B/b + 3*(4*B*a + A*b)*log((sqrt(b*x + a) - sqrt(
a))/(sqrt(b*x + a) + sqrt(a)))/(sqrt(a)*b) - 2*((4*B*a + 5*A*b)*(b*x + a)^
(3/2) - (4*B*a^2 + 3*A*a*b)*sqrt(b*x + a))/((b*x + a)^2*b - 2*(b*x + a)*a*
b + a^2*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx = \frac{8\,\sqrt{bx+a}Bb^2 + \frac{3\,(4\,Bab^2+Ab^3)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\,(bx+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx+a}Ba^2b^2+5\,(bx+a)}{b^2x^2}}{4\,b}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/4*(8*sqrt(b*x + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x + a)/sq
rt(-a))/sqrt(-a) - (4*(b*x + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x + a)*B*a^2*b^2
+ 5*(b*x + a)^(3/2)*A*b^3 - 3*sqrt(b*x + a)*A*a*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx = \frac{\left(B\,a^2\,b+\frac{3\,A\,a\,b^2}{4}\right)\,\sqrt{a+b\,x}-\left(\frac{5\,A\,b^2}{4}+B\,a\,b\right)\,{\left(a+b\,x\right)}^{3/2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}$$
$$+2\,B\,b\,\sqrt{a+b\,x}-\frac{3\,b\,\mathrm{atanh}\left(\frac{3\,b\,\left(A\,b+4\,B\,a\right)\,\sqrt{a+b\,x}}{2\,\sqrt{a}\,\left(\frac{3\,A\,b^2}{2}+6\,B\,a\,b\right)}\right)\,\left(A\,b+4\,B\,a\right)}{4\,\sqrt{a}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^3,x)
```

output
```
(((3*A*a*b^2)/4 + B*a^2*b)*(a + b*x)^(1/2) - ((5*A*b^2)/4 + B*a*b)*(a + b*
x)^(3/2))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + 2*B*b*(a + b*x)^(1/2) - (3
*b*atanh((3*b*(A*b + 4*B*a)*(a + b*x)^(1/2))/(2*a^(1/2)*((3*A*b^2)/2 + 6*B
*a*b)))*(A*b + 4*B*a))/(4*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx = \frac{-4\sqrt{bx+a}\,a^2-18\sqrt{bx+a}\,abx+16\sqrt{bx+a}\,b^2x^2+15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{}\right.}{8x^2}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 18*sqrt(a + b*x)*a*b*x + 16*sqrt(a + b*x)*b**2*
x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt(a)*log(
sqrt(a + b*x) + sqrt(a))*b**2*x**2)/(8*x**2)
```

3.242 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4} \, dx = -\frac{(Ab+2aB)\sqrt{a+bx}}{4x^2} - \frac{b(Ab+10aB)\sqrt{a+bx}}{8ax} - \frac{A(a+bx)^{3/2}}{3x^3} + \frac{b^2(Ab-6aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
-1/4*(A*b+2*B*a)*(b*x+a)^(1/2)/x^2-1/8*b*(A*b+10*B*a)*(b*x+a)^(1/2)/a/x-1/
3*A*(b*x+a)^(3/2)/x^3+1/8*b^2*(A*b-6*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a
^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4} \, dx = -\frac{\sqrt{a+bx}(3Ab^2x^2+4a^2(2A+3Bx)+2abx(7A+15Bx))}{24ax^3} + \frac{b^2(Ab-6aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^4,x]
```

output
```
-1/24*(Sqrt[a + b*x]*(3*A*b^2*x^2 + 4*a^2*(2*A + 3*B*x) + 2*a*b*x*(7*A + 1
5*B*x)))/(a*x^3) + (b^2*(A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a
^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {87, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-6aB)\int \frac{(a+bx)^{3/2}}{x^3}dx}{6a}-\frac{A(a+bx)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-6aB)\left(\frac{3}{4}b\int \frac{\sqrt{a+bx}}{x^2}dx-\frac{(a+bx)^{3/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-6aB)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(Ab-6aB)\left(\frac{3}{4}b\left(\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(Ab-6aB)\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{5/2}}{3ax^3}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^4,x]
```

output
```
-1/3*(A*(a + b*x)^(5/2))/(a*x^3) - ((A*b - 6*a*B)*(-1/2*(a + b*x)^(3/2)/x^
2 + (3*b*(-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])
)/4))/(6*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

method	result	s
risch	$-\frac{\sqrt{bx+a}\,(3A\,b^2x^2+30Bab\,x^2+14aAbx+12B\,a^2x+8a^2A)}{24x^3a}+\frac{b^2(Ab-6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}$	8
pseudoelliptic	$-\frac{-\frac{3b^2x^3(Ab-6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8}+\left(\frac{7b\left(\frac{15Bx}{7}+A\right)x\,a^{\frac{3}{2}}}{4}+\left(\frac{3Bx}{2}+A\right)a^{\frac{5}{2}}+\frac{3A\sqrt{a}\,b^2x^2}{8}\right)\sqrt{bx+a}}{3a^{\frac{3}{2}}x^3}$	8
derivativedivides	$2b^2\left(-\frac{\frac{(Ab+10Ba)(bx+a)^{\frac{5}{2}}}{16a}+\left(\frac{Ab}{6}-Ba\right)(bx+a)^{\frac{3}{2}}+\left(\frac{3}{8}a^2B-\frac{1}{16}abA\right)\sqrt{bx+a}}{b^3x^3}+\frac{(Ab-6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	9
default	$2b^2\left(-\frac{\frac{(Ab+10Ba)(bx+a)^{\frac{5}{2}}}{16a}+\left(\frac{Ab}{6}-Ba\right)(bx+a)^{\frac{3}{2}}+\left(\frac{3}{8}a^2B-\frac{1}{16}abA\right)\sqrt{bx+a}}{b^3x^3}+\frac{(Ab-6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	9

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(3*A*b^2*x^2+30*B*a*b*x^2+14*A*a*b*x+12*B*a^2*x+8*A*a^
2)/x^3/a+1/8*b^2*(A*b-6*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx = \left[-\frac{3\,(6\,Bab^2-Ab^3)\sqrt{a}x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,Aa^3+3\,(10\,Ba^2b+A}{48\,a^2x^3}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*B*a*b^2 - A*b^3)*sqrt(a)*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(8*A*a^3 + 3*(10*B*a^2*b + A*a*b^2)*x^2 + 2*(6*B*a^3 + 7*
A*a^2*b)*x)*sqrt(b*x + a))/(a^2*x^3), 1/24*(3*(6*B*a*b^2 - A*b^3)*sqrt(-a)
*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) - (8*A*a^3 + 3*(10*B*a^2*b + A*a*b^2)*
x^2 + 2*(6*B*a^3 + 7*A*a^2*b)*x)*sqrt(b*x + a))/(a^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(95) = 190$.

Time = 65.63 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.49

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx = -\frac{Aa^2}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{11Aa\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{17Ab^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Ab^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}} - \frac{Ba^2}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3Ba\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{Bb^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**4,x)
```

output
```
-A*a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*A*a*sqrt(b)/(12*x**(5/
2)*sqrt(a/(b*x) + 1)) - 17*A*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - A*
b**(5/2)/(8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + A*b**3*asinh(sqrt(a)/(sqrt(b)*s
qrt(x)))/(8*a**(3/2)) - B*a**2/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*
B*a*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - B*b**(3/2)*sqrt(a/(b*x) + 1)/
sqrt(x) - B*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*B*b**2*asinh(sqrt(a
)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx = -\frac{1}{48}b^3\left(\frac{2\left(3\,(10\,Ba+Ab)(bx+a)^{\frac{5}{2}} - 8\,(6\,Ba^2 - Aab)(bx+a)^{\frac{3}{2}} + 3\,(6\,Ba^3 - Aa^2b)\sqrt{bx+a}\right)}{(bx+a)^3ab - 3\,(bx+a)^2a^2b + 3\,(bx+a)a^3b - a^4b} - \frac{3\,(6\,Ba}{}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
-1/48*b^3*(2*(3*(10*B*a + A*b)*(b*x + a)^(5/2) - 8*(6*B*a^2 - A*a*b)*(b*x
+ a)^(3/2) + 3*(6*B*a^3 - A*a^2*b)*sqrt(b*x + a))/((b*x + a)^3*a*b - 3*(b*
x + a)^2*a^2*b + 3*(b*x + a)*a^3*b - a^4*b) - 3*(6*B*a - A*b)*log((sqrt(b*
x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(3/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx = \frac{1}{24}\,b^3\left(\frac{3\,(6\,Ba-Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab} - \frac{30\,(bx+a)^{\frac{5}{2}}Ba-48\,(bx+a)^{\frac{3}{2}}Ba^2\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/24*b^3*(3*(6*B*a - A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*b) -
(30*(b*x + a)^(5/2)*B*a - 48*(b*x + a)^(3/2)*B*a^2 + 18*sqrt(b*x + a)*B*a^
3 + 3*(b*x + a)^(5/2)*A*b + 8*(b*x + a)^(3/2)*A*a*b - 3*sqrt(b*x + a)*A*a^
2*b)/(a*b^4*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx = \frac{\left(\frac{A\,b^3}{3}-2\,B\,a\,b^2\right)\,(a+b\,x)^{3/2}+\left(\frac{3\,B\,a^2\,b^2}{4}-\frac{A\,a\,b^3}{8}\right)\,\sqrt{a+b\,x}+\frac{(A\,b^3+10\,B\,a\,b^2)\,(}{8\,a}}{3\,a\,(a+b\,x)^2-3\,a^2\,(a+b\,x)-(a+b\,x)^3+a^3} + \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(A\,b-6\,B\,a)}{8\,a^{3/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^4,x)
```

output
```
(((A*b^3)/3 - 2*B*a*b^2)*(a + b*x)^(3/2) + ((3*B*a^2*b^2)/4 - (A*a*b^3)/8)
*(a + b*x)^(1/2) + ((A*b^3 + 10*B*a*b^2)*(a + b*x)^(5/2))/(8*a))/(3*a*(a +
 b*x)^2 - 3*a^2*(a + b*x) - (a + b*x)^3 + a^3) + (b^2*atanh((a + b*x)^(1/2
)/a^(1/2))*(A*b - 6*B*a))/(8*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx = \frac{-16\sqrt{bx+a}\,a^3-52\sqrt{bx+a}\,a^2bx-66\sqrt{bx+a}\,a\,b^2x^2+15\sqrt{a}\log\left(\sqrt{bx+a}\right.}{48a\,x^3}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 52*sqrt(a + b*x)*a**2*b*x - 66*sqrt(a + b*x)*a
*b**2*x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a*x**3)
```

3.243 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5} dx = -\frac{(3Ab+8aB)\sqrt{a+bx}}{24x^3} - \frac{b(3Ab+56aB)\sqrt{a+bx}}{96ax^2} + \frac{b^2(3Ab-8aB)\sqrt{a+bx}}{64a^2x} - \frac{A(a+bx)^{3/2}}{4x^4} - \frac{b^3(3Ab-8aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{5/2}}$$

output
```
-1/24*(3*A*b+8*B*a)*(b*x+a)^(1/2)/x^3-1/96*b*(3*A*b+56*B*a)*(b*x+a)^(1/2)/
a/x^2+1/64*b^2*(3*A*b-8*B*a)*(b*x+a)^(1/2)/a^2/x-1/4*A*(b*x+a)^(3/2)/x^4-1
/64*b^3*(3*A*b-8*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5} dx = -\frac{\sqrt{a+bx}(-9Ab^3x^3+6ab^2x^2(A+4Bx)+16a^3(3A+4Bx)+8a^2bx(9A+14Bx))}{192a^2x^4} + \frac{b^3(-3Ab+8aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^5,x]
```

output
```
-1/192*(Sqrt[a + b*x]*(-9*A*b^3*x^3 + 6*a*b^2*x^2*(A + 4*B*x) + 16*a^3*(3*
A + 4*B*x) + 8*a^2*b*x*(9*A + 14*B*x)))/(a^2*x^4) + (b^3*(-3*A*b + 8*a*B)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(3Ab-8aB)\int \frac{(a+bx)^{3/2}}{x^4}dx}{8a}-\frac{A(a+bx)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(3Ab-8aB)\left(\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}dx-\frac{(a+bx)^{3/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{5/2}}{4ax^4}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^5,x]
```

output
```
-1/4*(A*(a + b*x)^(5/2))/(a*x^4) - ((3*A*b - 8*a*B)*(-1/3*(a + b*x)^(3/2)/
x^3 + (b*(-1/2*Sqrt[a + b*x]/x^2 + (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/4))/2))/(8*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{3\left(\frac{b^{3} x^{4}\left(A b-\frac{8 B a}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{8}+\sqrt{b x+a}\left(\frac{b^{2} x^{2}(4 B x+A) a^{\frac{3}{2}}}{12}+b x\left(\frac{14 B x}{9}+A\right) a^{\frac{5}{2}}+\left(\frac{8 B x}{9}+\frac{2 A}{3}\right) a^{\frac{7}{2}}-\frac{A \sqrt{a}\, b^{3} x^{3}}{8}\right)\right)}{8 a^{\frac{5}{2}} x^{4}}$
risch	$-\frac{\sqrt{b x+a}\left(-9 A b^{3} x^{3}+24 B a b^{2} x^{3}+6 a A b^{2} x^{2}+112 B a^{2} b x^{2}+72 a^{2} A b x+64 B a^{3} x+48 a^{3} A\right)}{192 x^{4} a^{2}}-\frac{b^{3}(3 A b-8 B a) \operatorname{arctanh}(}{64 a^{\frac{5}{2}}}$
derivativedivides	$2 b^{3}\left(-\frac{-\frac{(3 A b-8 B a)(b x+a)^{\frac{7}{2}}}{128 a^{2}}+\frac{(33 A b+40 B a)(b x+a)^{\frac{5}{2}}}{384 a}+\left(\frac{11 A b}{128}-\frac{11 B a}{48}\right)(b x+a)^{\frac{3}{2}}-\frac{a(3 A b-8 B a) \sqrt{b x+a}}{128}}{b^{4} x^{4}}-\frac{(3 A b-8 B a)}{}\right.$
default	$2 b^{3}\left(-\frac{-\frac{(3 A b-8 B a)(b x+a)^{\frac{7}{2}}}{128 a^{2}}+\frac{(33 A b+40 B a)(b x+a)^{\frac{5}{2}}}{384 a}+\left(\frac{11 A b}{128}-\frac{11 B a}{48}\right)(b x+a)^{\frac{3}{2}}-\frac{a(3 A b-8 B a) \sqrt{b x+a}}{128}}{b^{4} x^{4}}-\frac{(3 A b-8 B a)}{}\right.$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-3/8*(1/8*b^3*x^4*(A*b-8/3*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1/
2)*(1/12*b^2*x^2*(4*B*x+A)*a^(3/2)+b*x*(14/9*B*x+A)*a^(5/2)+(8/9*B*x+2/3*A
)*a^(7/2)-1/8*A*a^(1/2)*b^3*x^3))/a^(5/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx = \left[-\frac{3\,(8\,Bab^3-3\,Ab^4)\sqrt{a}x^4\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(48\,Aa^4+3\,(8\,Ba^2b^2 \cdots}{384\,a \cdots}\right.$$

$$-\frac{3\,(8\,Bab^3-3\,Ab^4)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(48\,Aa^4+3\,(8\,Ba^2b^2-3\,Aab^3)x^3+2\,(56\,Ba^3b+3\,Aa^2b^2)x \cdots}{192\,a^3x^4}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(a)*x^4*log((b*x - 2*sqrt(b*x + a)*sq
rt(a) + 2*a)/x) + 2*(48*A*a^4 + 3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^3 + 2*(56*B*
a^3*b + 3*A*a^2*b^2)*x^2 + 8*(8*B*a^4 + 9*A*a^3*b)*x)*sqrt(b*x + a))/(a^3*
x^4), -1/192*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*
x + a)) + (48*A*a^4 + 3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^3 + 2*(56*B*a^3*b + 3*
A*a^2*b^2)*x^2 + 8*(8*B*a^4 + 9*A*a^3*b)*x)*sqrt(b*x + a))/(a^3*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(129) = 258.

Time = 125.39 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx = -\frac{Aa^2}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Aa\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{13Ab^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{Ab^{\frac{5}{2}}}{64ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{3Ab^{\frac{7}{2}}}{64a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{5}{2}}} - \frac{Ba^2}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{11Ba\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{17Bb^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Bb^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**5,x)
```

output
```
-A*a**2/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 5*A*a*sqrt(b)/(8*x**(7/2)
*sqrt(a/(b*x) + 1)) - 13*A*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) + A*b*
*(5/2)/(64*a*x**(3/2)*sqrt(a/(b*x) + 1)) + 3*A*b**(7/2)/(64*a**2*sqrt(x)*s
qrt(a/(b*x) + 1)) - 3*A*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(5/2)
) - B*a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*B*a*sqrt(b)/(12*x**
(5/2)*sqrt(a/(b*x) + 1)) - 17*B*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) -
 B*b**(5/2)/(8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + B*b**3*asinh(sqrt(a)/(sqrt(b
)*sqrt(x)))/(8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx = -\frac{1}{384}\,b^4\left(\frac{2\left(3\,(8\,Ba-3\,Ab)(bx+a)^{\frac{7}{2}}+(40\,Ba^2+33\,Aab)(bx+a)^{\frac{5}{2}}-11\,(8\,Ba^3-3\,Aa^2b)(bx+a)^{\frac{3}{2}}+3\right.}{(bx+a)^4a^2b-4\,(bx+a)^3a^3b+6\,(bx+a)^2a^4b-4\,(bx+a)a^5b+a^6b}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
-1/384*b^4*(2*(3*(8*B*a - 3*A*b)*(b*x + a)^(7/2) + (40*B*a^2 + 33*A*a*b)*(
b*x + a)^(5/2) - 11*(8*B*a^3 - 3*A*a^2*b)*(b*x + a)^(3/2) + 3*(8*B*a^4 - 3
*A*a^3*b)*sqrt(b*x + a))/((b*x + a)^4*a^2*b - 4*(b*x + a)^3*a^3*b + 6*(b*x
 + a)^2*a^4*b - 4*(b*x + a)*a^5*b + a^6*b) + 3*(8*B*a - 3*A*b)*log((sqrt(b
*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(5/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx = -\frac{\frac{3\,(8\,Bab^4-3\,Ab^5)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{24\,(bx+a)^{\frac{7}{2}}Bab^4+40\,(bx+a)^{\frac{5}{2}}Ba^2b^4-88\,(bx+a)^{\frac{3}{2}}Ba^3b^4+24\,\sqrt{bx+a}Ba^4b^4-9\,(bx+a)^{\frac{7}{2}}Ab^5+33\,(bx+a)}{a^2b^4x^4}}{192\,b}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
-1/192*(3*(8*B*a*b^4 - 3*A*b^5)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a
^2) + (24*(b*x + a)^(7/2)*B*a*b^4 + 40*(b*x + a)^(5/2)*B*a^2*b^4 - 88*(b*x
 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x + a)*B*a^4*b^4 - 9*(b*x + a)^(7/2)*A*b
^5 + 33*(b*x + a)^(5/2)*A*a*b^5 + 33*(b*x + a)^(3/2)*A*a^2*b^5 - 9*sqrt(b*
x + a)*A*a^3*b^5)/(a^2*b^4*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx = -\frac{\left(\frac{11\,A\,b^4}{64}-\frac{11\,B\,a\,b^3}{24}\right)\,(a+b\,x)^{3/2}+\left(\frac{B\,a^2\,b^3}{8}-\frac{3\,A\,a\,b^4}{64}\right)\,\sqrt{a+b\,x}-\frac{\left(3\,A\,b^4-8\,B\,a\,b^3\right)\,(a+b\,x)^{7/2}}{64\,a^2}+\frac{\left(33\,A\,b^4+40\,B\,a\,b^3\right)\,(a}{192\,a}}{(a+b\,x)^4-4\,a^3\,(a+b\,x)-4\,a\,(a+b\,x)^3+6\,a^2\,(a+b\,x)^2+a^4} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(3\,A\,b-8\,B\,a)}{64\,a^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^5,x)
```

output
```
- (((11*A*b^4)/64 - (11*B*a*b^3)/24)*(a + b*x)^(3/2) + ((B*a^2*b^3)/8 - (3
*A*a*b^4)/64)*(a + b*x)^(1/2) - ((3*A*b^4 - 8*B*a*b^3)*(a + b*x)^(7/2))/(6
4*a^2) + ((33*A*b^4 + 40*B*a*b^3)*(a + b*x)^(5/2))/(192*a))/((a + b*x)^4 -
 4*a^3*(a + b*x) - 4*a*(a + b*x)^3 + 6*a^2*(a + b*x)^2 + a^4) - (b^3*atanh
((a + b*x)^(1/2)/a^(1/2))*(3*A*b - 8*B*a))/(64*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx = \frac{-96\sqrt{bx+a}\,a^4-272\sqrt{bx+a}\,a^3bx-236\sqrt{bx+a}\,a^2b^2x^2-30\sqrt{bx+a}\,a\,b^3x}{384a^2x^4}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^5,x)
```

output
```
( - 96*sqrt(a + b*x)*a**4 - 272*sqrt(a + b*x)*a**3*b*x - 236*sqrt(a + b*x)
*a**2*b**2*x**2 - 30*sqrt(a + b*x)*a*b**3*x**3 - 15*sqrt(a)*log(sqrt(a + b
*x) - sqrt(a))*b**4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x*
*4)/(384*a**2*x**4)
```

3.244 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 167

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6} dx = -\frac{(3Ab+10aB)\sqrt{a+bx}}{40x^4} - \frac{b(Ab+30aB)\sqrt{a+bx}}{80ax^3} + \frac{b^2(Ab-2aB)\sqrt{a+bx}}{64a^2x^2} - \frac{3b^3(Ab-2aB)\sqrt{a+bx}}{128a^3x} - \frac{A(a+bx)^{3/2}}{5x^5} + \frac{3b^4(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{7/2}}$$

output
```
-1/40*(3*A*b+10*B*a)*(b*x+a)^(1/2)/x^4-1/80*b*(A*b+30*B*a)*(b*x+a)^(1/2)/a
/x^3+1/64*b^2*(A*b-2*B*a)*(b*x+a)^(1/2)/a^2/x^2-3/128*b^3*(A*b-2*B*a)*(b*x
+a)^(1/2)/a^3/x-1/5*A*(b*x+a)^(3/2)/x^5+3/128*b^4*(A*b-2*B*a)*arctanh((b*x
+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx = -\frac{\sqrt{a+bx}(15Ab^4x^4-10ab^3x^3(A+3Bx)+4a^2b^2x^2(2A+5Bx)+32a^4(4A+5Bx)+16a^3bx(11A+15B}{640a^3x^5} + \frac{3b^4(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{7/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^6,x]
```

output
```
-1/640*(Sqrt[a + b*x]*(15*A*b^4*x^4 - 10*a*b^3*x^3*(A + 3*B*x) + 4*a^2*b^2
*x^2*(2*A + 5*B*x) + 32*a^4*(4*A + 5*B*x) + 16*a^3*b*x*(11*A + 15*B*x)))/(
a^3*x^5) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*a^(7/
2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {87, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-2aB)\int \frac{(a+bx)^{3/2}}{x^5}dx}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-2aB)\left(\frac{3}{8}b\int \frac{\sqrt{a+bx}}{x^4}dx-\frac{(a+bx)^{3/2}}{4x^4}\right)}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\int\frac{1}{x^3\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int\frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)}{2a}-\frac{A(a+bx)^{5/2}}{5ax^5}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^6,x]
```

output
```
-1/5*(A*(a + b*x)^(5/2))/(a*x^5) - ((A*b - 2*a*B)*(-1/4*(a + b*x)^(3/2)/x^
4 + (3*b*(-1/3*Sqrt[a + b*x]/x^3 + (b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(
-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a
)))/6))/8))/(2*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{-\frac{15b^4x^5(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8}+\sqrt{bx+a}\left(-\frac{5b^3x^3(3Bx+A)a^{\frac{3}{2}}}{4}+b^2x^2\left(\frac{5Bx}{2}+A\right)a^{\frac{5}{2}}+22b\left(\frac{15Bx}{11}+A\right)x\,a^{\frac{7}{2}}+(20B\right.}{80a^{\frac{7}{2}}x^5}$
derivativedivides	$2b^4\left(-\frac{\frac{3(Ab-2Ba)(bx+a)^{\frac{9}{2}}}{256a^3}-\frac{7(Ab-2Ba)(bx+a)^{\frac{7}{2}}}{128a^2}+\frac{Ab(bx+a)^{\frac{5}{2}}}{10a}+\left(\frac{7Ab}{128}-\frac{7Ba}{64}\right)(bx+a)^{\frac{3}{2}}-\frac{3a(Ab-2Ba)\sqrt{bx+a}}{256}}{b^5x^5}+\frac{3(Ab}{}\right.$
default	$2b^4\left(-\frac{\frac{3(Ab-2Ba)(bx+a)^{\frac{9}{2}}}{256a^3}-\frac{7(Ab-2Ba)(bx+a)^{\frac{7}{2}}}{128a^2}+\frac{Ab(bx+a)^{\frac{5}{2}}}{10a}+\left(\frac{7Ab}{128}-\frac{7Ba}{64}\right)(bx+a)^{\frac{3}{2}}-\frac{3a(Ab-2Ba)\sqrt{bx+a}}{256}}{b^5x^5}+\frac{3(Ab}{}\right.$
risch	$-\frac{\sqrt{bx+a}\,(15A\,b^4x^4-30Ba\,b^3x^4-10Aa\,b^3x^3+20B\,a^2b^2x^3+8A\,a^2b^2x^2+240B\,a^3b\,x^2+176A\,a^3bx+160B\,a^4x+128A\,a}{640x^5a^3}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/80*(-15/8*b^4*x^5*(A*b-2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1
/2)*(-5/4*b^3*x^3*(3*B*x+A)*a^(3/2)+b^2*x^2*(5/2*B*x+A)*a^(5/2)+22*b*(15/1
1*B*x+A)*x*a^(7/2)+(20*B*x+16*A)*a^(9/2)+15/8*A*a^(1/2)*b^4*x^4))/a^(7/2)/
x^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.81

$$\int\frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx=\left[-\frac{15\,(2\,Bab^4-Ab^5)\sqrt{a}x^5\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(128\,Aa^5-15\,(2\,Ba^2b}{}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
[-1/1280*(15*(2*B*a*b^4 - A*b^5)*sqrt(a)*x^5*log((b*x + 2*sqrt(b*x + a)*sq
rt(a) + 2*a)/x) + 2*(128*A*a^5 - 15*(2*B*a^2*b^3 - A*a*b^4)*x^4 + 10*(2*B*
a^3*b^2 - A*a^2*b^3)*x^3 + 8*(30*B*a^4*b + A*a^3*b^2)*x^2 + 16*(10*B*a^5 +
 11*A*a^4*b)*x)*sqrt(b*x + a))/(a^4*x^5), 1/640*(15*(2*B*a*b^4 - A*b^5)*sq
rt(-a)*x^5*arctan(sqrt(-a)/sqrt(b*x + a)) - (128*A*a^5 - 15*(2*B*a^2*b^3 -
 A*a*b^4)*x^4 + 10*(2*B*a^3*b^2 - A*a^2*b^3)*x^3 + 8*(30*B*a^4*b + A*a^3*b
^2)*x^2 + 16*(10*B*a^5 + 11*A*a^4*b)*x)*sqrt(b*x + a))/(a^4*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx =$$
$$-\frac{1}{1280}b^5\left(\frac{2\left(128\,(bx+a)^{\frac{5}{2}}Aa^2b-15\,(2\,Ba-Ab)(bx+a)^{\frac{9}{2}}+70\,(2\,Ba^2-Aab)(bx+a)^{\frac{7}{2}}-70\,(2\,Ba^4-\right.}{(bx+a)^5a^3b-5\,(bx+a)^4a^4b+10\,(bx+a)^3a^5b-10\,(bx+a)^2a^6b+}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/1280*b^5*(2*(128*(b*x + a)^(5/2)*A*a^2*b - 15*(2*B*a - A*b)*(b*x + a)^(
9/2) + 70*(2*B*a^2 - A*a*b)*(b*x + a)^(7/2) - 70*(2*B*a^4 - A*a^3*b)*(b*x
+ a)^(3/2) + 15*(2*B*a^5 - A*a^4*b)*sqrt(b*x + a))/((b*x + a)^5*a^3*b - 5*
(b*x + a)^4*a^4*b + 10*(b*x + a)^3*a^5*b - 10*(b*x + a)^2*a^6*b + 5*(b*x +
 a)*a^7*b - a^8*b) - 15*(2*B*a - A*b)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(
b*x + a) + sqrt(a)))/(a^(7/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx = \frac{1}{640}\,b^5\left(\frac{15\,(2\,Ba-Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}+\frac{30\,(bx+a)^{\frac{9}{2}}Ba-140\,(bx+a)^{\frac{7}{2}}B\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
1/640*b^5*(15*(2*B*a - A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*b
) + (30*(b*x + a)^(9/2)*B*a - 140*(b*x + a)^(7/2)*B*a^2 + 140*(b*x + a)^(3
/2)*B*a^4 - 30*sqrt(b*x + a)*B*a^5 - 15*(b*x + a)^(9/2)*A*b + 70*(b*x + a)
^(7/2)*A*a*b - 128*(b*x + a)^(5/2)*A*a^2*b - 70*(b*x + a)^(3/2)*A*a^3*b +
15*sqrt(b*x + a)*A*a^4*b)/(a^3*b^6*x^5))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx = \frac{\left(\frac{7\,A\,b^5}{64}-\frac{7\,B\,a\,b^4}{32}\right)\,{\left(a+b\,x\right)}^{3/2}+\left(\frac{3\,B\,a^2\,b^4}{64}-\frac{3\,A\,a\,b^5}{128}\right)\,\sqrt{a+b\,x}-\frac{7\,\left(A\,b^5-2\,B\,a\,b^4\right)}{64\,a^2}}{5\,a\,{\left(a+b\,x\right)}^4-5\,a^4\,\left(a+b\,x\right)-{\left(a+b\,x\right)}^5-10\,a^2\,\left(a+b\right.}$$
$$+\frac{3\,b^4\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(A\,b-2\,B\,a\right)}{128\,a^{7/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^6,x)
```

output
```
(((7*A*b^5)/64 - (7*B*a*b^4)/32)*(a + b*x)^(3/2) + ((3*B*a^2*b^4)/64 - (3*
A*a*b^5)/128)*(a + b*x)^(1/2) - (7*(A*b^5 - 2*B*a*b^4)*(a + b*x)^(7/2))/(6
4*a^2) + (3*(A*b^5 - 2*B*a*b^4)*(a + b*x)^(9/2))/(128*a^3) + (A*b^5*(a + b
*x)^(5/2))/(5*a))/(5*a*(a + b*x)^4 - 5*a^4*(a + b*x) - (a + b*x)^5 - 10*a^
2*(a + b*x)^3 + 10*a^3*(a + b*x)^2 + a^5) + (3*b^4*atanh((a + b*x)^(1/2)/a
^(1/2))*(A*b - 2*B*a))/(128*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx = \frac{-256\sqrt{bx+a}\,a^5-672\sqrt{bx+a}\,a^4bx-496\sqrt{bx+a}\,a^3b^2x^2-20\sqrt{bx+a}\,a^2b^3}{}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^6,x)
```

output
```
( - 256*sqrt(a + b*x)*a**5 - 672*sqrt(a + b*x)*a**4*b*x - 496*sqrt(a + b*x
)*a**3*b**2*x**2 - 20*sqrt(a + b*x)*a**2*b**3*x**3 + 30*sqrt(a + b*x)*a*b*
*4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 - 15*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**5*x**5)/(1280*a**3*x**5)
```

3.245 $\int x^4(a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 151

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \frac{2a^4(Ab-aB)(a+bx)^{7/2}}{7b^6} - \frac{2a^3(4Ab-5aB)(a+bx)^{9/2}}{9b^6} + \frac{4a^2(3Ab-5aB)(a+bx)^{11/2}}{11b^6} - \frac{4a(2Ab-5aB)(a+bx)^{13/2}}{13b^6} + \frac{2(Ab-5aB)(a+bx)^{15/2}}{15b^6} + \frac{2B(a+bx)^{17/2}}{17b^6}$$

output
```
2/7*a^4*(A*b-B*a)*(b*x+a)^(7/2)/b^6-2/9*a^3*(4*A*b-5*B*a)*(b*x+a)^(9/2)/b^
6+4/11*a^2*(3*A*b-5*B*a)*(b*x+a)^(11/2)/b^6-4/13*a*(2*A*b-5*B*a)*(b*x+a)^(
13/2)/b^6+2/15*(A*b-5*B*a)*(b*x+a)^(15/2)/b^6+2/17*B*(b*x+a)^(17/2)/b^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.70

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \frac{2(a+bx)^{7/2}\left(-1280a^5B+3003b^5x^4(17A+15Bx)+128a^4b(17A+35Bx)-224a^3b^2x(34A+\right.}{765765b^6}$$

input
```
Integrate[x^4*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(7/2)*(-1280*a^5*B + 3003*b^5*x^4*(17*A + 15*B*x) + 128*a^4*b
*(17*A + 35*B*x) - 224*a^3*b^2*x*(34*A + 45*B*x) + 336*a^2*b^3*x^2*(51*A +
 55*B*x) - 462*a*b^4*x^3*(68*A + 65*B*x)))/(765765*b^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^4(a+bx)^{5/2}(aB-Ab)}{b^5}+\frac{a^3(a+bx)^{7/2}(5aB-4Ab)}{b^5}-\frac{2a^2(a+bx)^{9/2}(5aB-3Ab)}{b^5}+\frac{(a+bx)^{13/2}(Ab-}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4(a+bx)^{7/2}(Ab-aB)}{7b^6}-\frac{2a^3(a+bx)^{9/2}(4Ab-5aB)}{9b^6}+\frac{4a^2(a+bx)^{11/2}(3Ab-5aB)}{11b^6}+\frac{2(a+bx)^{15/2}(Ab-5aB)}{15b^6}-\frac{4a(a+bx)^{13/2}(2Ab-5aB)}{13b^6}+\frac{2B(a+bx)^{17/2}}{17b^6}$$

input
```
Int[x^4*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*a^4*(A*b - a*B)*(a + b*x)^(7/2))/(7*b^6) - (2*a^3*(4*A*b - 5*a*B)*(a +
b*x)^(9/2))/(9*b^6) + (4*a^2*(3*A*b - 5*a*B)*(a + b*x)^(11/2))/(11*b^6) -
(4*a*(2*A*b - 5*a*B)*(a + b*x)^(13/2))/(13*b^6) + (2*(A*b - 5*a*B)*(a + b*
x)^(15/2))/(15*b^6) + (2*B*(a + b*x)^(17/2))/(17*b^6)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{256\left(\frac{3003\left(\frac{15Bx}{17}+A\right)x^4b^5}{128}-\frac{231a\left(\frac{65Bx}{68}+A\right)x^3b^4}{16}+\frac{63a^2x^2\left(\frac{55Bx}{51}+A\right)b^3}{8}-\frac{7a^3\left(\frac{45Bx}{34}+A\right)xb^2}{2}+a^4\left(\frac{35Bx}{17}+A\right)b-\frac{10a^5B}{17}\right)}{45045b^6}$
gosper	$\frac{2(bx+a)^{\frac{7}{2}}\left(45045b^5Bx^5+51051Ab^5x^4-30030Bab^4x^4-31416Aab^4x^3+18480Ba^2b^3x^3+17136Aa^2b^3x^2-10080Ba^3\right.}{765765b^6}$
orering	$\frac{2(bx+a)^{\frac{7}{2}}\left(45045b^5Bx^5+51051Ab^5x^4-30030Bab^4x^4-31416Aab^4x^3+18480Ba^2b^3x^3+17136Aa^2b^3x^2-10080Ba^3\right.}{765765b^6}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{17}{2}}}{17}+\frac{2(Ab-5Ba)(bx+a)^{\frac{15}{2}}}{15}+\frac{2\left(6a^2B-4a(Ab-Ba)\right)(bx+a)^{\frac{13}{2}}}{13}+\frac{2\left(-4a^3B+6a^2(Ab-Ba)\right)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(Ba^4-4a^3(A\right.}{}}{b^6}$
default	$\frac{\frac{2B(bx+a)^{\frac{17}{2}}}{17}+\frac{2(Ab-5Ba)(bx+a)^{\frac{15}{2}}}{15}+\frac{2\left(6a^2B-4a(Ab-Ba)\right)(bx+a)^{\frac{13}{2}}}{13}+\frac{2\left(-4a^3B+6a^2(Ab-Ba)\right)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(Ba^4-4a^3(A\right.}{}}{b^6}$
trager	$2\left(45045b^8Bx^8+51051Ab^8x^7+105105Bab^7x^7+121737Aab^7x^6+63525Ba^2b^6x^6+76041Aa^2b^6x^5+315Ba^3b^5x^5+595\right.$
risch	$2\left(45045b^8Bx^8+51051Ab^8x^7+105105Bab^7x^7+121737Aab^7x^6+63525Ba^2b^6x^6+76041Aa^2b^6x^5+315Ba^3b^5x^5+595\right.$

input
```
int(x^4*(b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
256/45045*(3003/128*(15/17*B*x+A)*x^4*b^5-231/16*a*(65/68*B*x+A)*x^3*b^4+6
3/8*a^2*x^2*(55/51*B*x+A)*b^3-7/2*a^3*(45/34*B*x+A)*x*b^2+a^4*(35/17*B*x+A
)*b-10/17*a^5*B)*(b*x+a)^(7/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.27

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\,(45045\,Bb^8x^8 - 1280\,Ba^8 + 2176\,Aa^7b + 3003\,(35\,Bab^7 + 17\,Ab^8)x^7 + 231\,(275\,Ba^2b^6 + 527\,A$$

input
```
integrate(x^4*(b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/765765*(45045*B*b^8*x^8 - 1280*B*a^8 + 2176*A*a^7*b + 3003*(35*B*a*b^7 +
 17*A*b^8)*x^7 + 231*(275*B*a^2*b^6 + 527*A*a*b^7)*x^6 + 63*(5*B*a^3*b^5 +
 1207*A*a^2*b^6)*x^5 - 35*(10*B*a^4*b^4 - 17*A*a^3*b^5)*x^4 + 40*(10*B*a^5
*b^3 - 17*A*a^4*b^4)*x^3 - 48*(10*B*a^6*b^2 - 17*A*a^5*b^3)*x^2 + 64*(10*B
*a^7*b - 17*A*a^6*b^2)*x)*sqrt(b*x + a)/b^6
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{17}{2}}}{17b} + \frac{(a+bx)^{\frac{15}{2}}(Ab-5Ba)}{15b} + \frac{(a+bx)^{\frac{13}{2}}\left(-4Aab+10Ba^2\right)}{13b} + \frac{(a+bx)^{\frac{11}{2}}\cdot\left(6Aa^2b-10Ba^3\right)}{11b} + \frac{(a+bx)^{\frac{9}{2}}\left(-4Aa^3b+5Ba^4\right)}{9b} + \frac{(a+bx)^{\frac{7}{2}}}{}\right.}{b^5} \\ a^{\frac{5}{2}}\left(\frac{Ax^5}{5} + \frac{Bx^6}{6}\right) \end{cases}$$

input
```
integrate(x**4*(b*x+a)**(5/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(17/2)/(17*b) + (a + b*x)**(15/2)*(A*b - 5*B*a)
/(15*b) + (a + b*x)**(13/2)*(-4*A*a*b + 10*B*a**2)/(13*b) + (a + b*x)**(11
/2)*(6*A*a**2*b - 10*B*a**3)/(11*b) + (a + b*x)**(9/2)*(-4*A*a**3*b + 5*B*
a**4)/(9*b) + (a + b*x)**(7/2)*(A*a**4*b - B*a**5)/(7*b))/b**5, Ne(b, 0)),
 (a**(5/2)*(A*x**5/5 + B*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(45045\,(bx+a)^{\frac{17}{2}}B - 51051\,(5\,Ba-Ab)(bx+a)^{\frac{15}{2}} + 117810\,(5\,Ba^2-2\,Aab)(bx+a)^{\frac{13}{2}} - 1\right.}{}$$

input
```
integrate(x^4*(b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/765765*(45045*(b*x + a)^(17/2)*B - 51051*(5*B*a - A*b)*(b*x + a)^(15/2)
+ 117810*(5*B*a^2 - 2*A*a*b)*(b*x + a)^(13/2) - 139230*(5*B*a^3 - 3*A*a^2*
b)*(b*x + a)^(11/2) + 85085*(5*B*a^4 - 4*A*a^3*b)*(b*x + a)^(9/2) - 109395
*(B*a^5 - A*a^4*b)*(b*x + a)^(7/2))/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(128) = 256.

Time = 0.13 (sec) , antiderivative size = 708, normalized size of antiderivative = 4.69

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/765765*(2431*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)
^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A*a^3/b^4 +
1105*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a
)*a^5)*B*a^3/b^5 + 3315*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990
*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4
 - 693*sqrt(b*x + a)*a^5)*A*a^2/b^4 + 765*(231*(b*x + a)^(13/2) - 1638*(b*
x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 90
09*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6
)*B*a^2/b^5 + 765*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(
b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 -
 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*A*a/b^4 + 357*(429*(b*
x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 250
25*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)
*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*B*a/b^5 + 119*(
429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^
2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a
)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*A/b^4 +
7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13
/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 87...
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \frac{(20\,B\,a^2-8\,A\,a\,b)\,(a+b\,x)^{13/2}}{13\,b^6}+\frac{2\,B\,(a+b\,x)^{17/2}}{17\,b^6}+\frac{(2\,A\,b-10\,B\,a)\,(a+b\,x)^{15/2}}{15\,b^6}-\frac{(2\,B\,a^5-2\,A\,a^4\,b)\,(a+b\,x)^{7/2}}{7\,b^6}+\frac{(10\,B\,a^4-8\,A\,a^3\,b)\,(a+b\,x)^{9/2}}{9\,b^6}-\frac{(20\,B\,a^3-12\,A\,a^2\,b)\,(a+b\,x)^{11/2}}{11\,b^6}$$

input
```
int(x^4*(A + B*x)*(a + b*x)^(5/2),x)
```

output
```
((20*B*a^2 - 8*A*a*b)*(a + b*x)^(13/2))/(13*b^6) + (2*B*(a + b*x)^(17/2))/
(17*b^6) + ((2*A*b - 10*B*a)*(a + b*x)^(15/2))/(15*b^6) - ((2*B*a^5 - 2*A*
a^4*b)*(a + b*x)^(7/2))/(7*b^6) + ((10*B*a^4 - 8*A*a^3*b)*(a + b*x)^(9/2))
/(9*b^6) - ((20*B*a^3 - 12*A*a^2*b)*(a + b*x)^(11/2))/(11*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.64

$$\int x^4(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(6435b^8x^8+22308a\,b^7x^7+26466a^2b^6x^6+10908a^3b^5x^5+35a^4b^4x^4-40a^5b^3x^3+48a}{109395b^5}$$

input
```
int(x^4*(b*x+a)^(5/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**8 - 64*a**7*b*x + 48*a**6*b**2*x**2 - 40*a**5*b**
3*x**3 + 35*a**4*b**4*x**4 + 10908*a**3*b**5*x**5 + 26466*a**2*b**6*x**6 +
 22308*a*b**7*x**7 + 6435*b**8*x**8))/(109395*b**5)
```

3.246 $\int x^3(a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = -\frac{2a^3(Ab-aB)(a+bx)^{7/2}}{7b^5} + \frac{2a^2(3Ab-4aB)(a+bx)^{9/2}}{9b^5} - \frac{6a(Ab-2aB)(a+bx)^{11/2}}{11b^5} + \frac{2(Ab-4aB)(a+bx)^{13/2}}{13b^5} + \frac{2B(a+bx)^{15/2}}{15b^5}$$

output
```
-2/7*a^3*(A*b-B*a)*(b*x+a)^(7/2)/b^5+2/9*a^2*(3*A*b-4*B*a)*(b*x+a)^(9/2)/b
^5-6/11*a*(A*b-2*B*a)*(b*x+a)^(11/2)/b^5+2/13*(A*b-4*B*a)*(b*x+a)^(13/2)/b
^5+2/15*B*(b*x+a)^(15/2)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \frac{2(a+bx)^{7/2}\left(128a^4B+168a^2b^2x(5A+6Bx)+231b^4x^3(15A+13Bx)-16a^3b(15A+28Bx)\right.}{45045b^5}$$

input
```
Integrate[x^3*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(7/2)*(128*a^4*B + 168*a^2*b^2*x*(5*A + 6*B*x) + 231*b^4*x^3*
(15*A + 13*B*x) - 16*a^3*b*(15*A + 28*B*x) - 42*a*b^3*x^2*(45*A + 44*B*x))
)/(45045*b^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{a^3(a+bx)^{5/2}(aB-Ab)}{b^4}-\frac{a^2(a+bx)^{7/2}(4aB-3Ab)}{b^4}+\frac{(a+bx)^{11/2}(Ab-4aB)}{b^4}+\frac{3a(a+bx)^{9/2}(2aB-A}{b^4}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3(a+bx)^{7/2}(Ab-aB)}{7b^5}+\frac{2a^2(a+bx)^{9/2}(3Ab-4aB)}{9b^5}+\frac{2(a+bx)^{13/2}(Ab-4aB)}{13b^5}-\frac{6a(a+bx)^{11/2}(Ab-2aB)}{11b^5}+\frac{2B(a+bx)^{15/2}}{15b^5}$$

input
```
Int[x^3*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(-2*a^3*(A*b - a*B)*(a + b*x)^(7/2))/(7*b^5) + (2*a^2*(3*A*b - 4*a*B)*(a +
 b*x)^(9/2))/(9*b^5) - (6*a*(A*b - 2*a*B)*(a + b*x)^(11/2))/(11*b^5) + (2*
(A*b - 4*a*B)*(a + b*x)^(13/2))/(13*b^5) + (2*B*(a + b*x)^(15/2))/(15*b^5)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{32\left(-\frac{231\left(\frac{13Bx}{15}+A\right)x^{3}b^{4}}{16}+\frac{63a\,x^{2}\left(\frac{44Bx}{45}+A\right)b^{3}}{8}-\frac{7a^{2}\left(\frac{6Bx}{5}+A\right)x\,b^{2}}{2}+a^{3}\left(\frac{28Bx}{15}+A\right)b-\frac{8B\,a^{4}}{15}\right)(bx+a)^{\frac{7}{2}}}{3003b^{5}}$
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-3003B\,x^{4}b^{4}-3465A\,x^{3}b^{4}+1848B\,x^{3}a\,b^{3}+1890A\,x^{2}a\,b^{3}-1008B\,x^{2}a^{2}b^{2}-840Ax\,a^{2}b^{2}+448Bx\,a^{3}b+24\cdots\right.}{45045b^{5}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-3003B\,x^{4}b^{4}-3465A\,x^{3}b^{4}+1848B\,x^{3}a\,b^{3}+1890A\,x^{2}a\,b^{3}-1008B\,x^{2}a^{2}b^{2}-840Ax\,a^{2}b^{2}+448Bx\,a^{3}b+24\cdots\right.}{45045b^{5}}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{15}{2}}}{15}+\frac{2(Ab-4Ba)(bx+a)^{\frac{13}{2}}}{13}+\frac{2\left(3a^{2}B-3a(Ab-Ba)\right)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(-a^{3}B+3a^{2}(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}-\frac{2a^{3}(Ab-Ba)(bx+\cdots}{7}}{b^{5}}$
default	$\frac{\frac{2B(bx+a)^{\frac{15}{2}}}{15}-\frac{2(-Ab+4Ba)(bx+a)^{\frac{13}{2}}}{13}-\frac{2\left(-3a^{2}B+3a(Ab-Ba)\right)(bx+a)^{\frac{11}{2}}}{11}-\frac{2\left(a^{3}B-3a^{2}(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}-\frac{2a^{3}(Ab-Ba)(b\cdots}{7}}{b^{5}}$
trager	$-\frac{2\left(-3003b^{7}B\,x^{7}-3465A\,b^{7}x^{6}-7161Ba\,b^{6}x^{6}-8505Aa\,b^{6}x^{5}-4473B\,a^{2}b^{5}x^{5}-5565A\,a^{2}b^{5}x^{4}-35B\,a^{3}b^{4}x^{4}-75A\,a^{3}b^{4}\cdots\right.}{45045b^{5}}$
risch	$-\frac{2\left(-3003b^{7}B\,x^{7}-3465A\,b^{7}x^{6}-7161Ba\,b^{6}x^{6}-8505Aa\,b^{6}x^{5}-4473B\,a^{2}b^{5}x^{5}-5565A\,a^{2}b^{5}x^{4}-35B\,a^{3}b^{4}x^{4}-75A\,a^{3}b^{4}\cdots\right.}{45045b^{5}}$

input
```
int(x^3*(b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-32/3003*(-231/16*(13/15*B*x+A)*x^3*b^4+63/8*a*x^2*(44/45*B*x+A)*b^3-7/2*a
^2*(6/5*B*x+A)*x*b^2+a^3*(28/15*B*x+A)*b-8/15*B*a^4)*(b*x+a)^(7/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\,(3003\,Bb^7x^7+128\,Ba^7-240\,Aa^6b+231\,(31\,Bab^6+15\,Ab^7)x^6+63\,(71\,Ba^2b^5+135\,Aab^6)x^5$$

input
```
integrate(x^3*(b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*b^7*x^7 + 128*B*a^7 - 240*A*a^6*b + 231*(31*B*a*b^6 + 15*A
*b^7)*x^6 + 63*(71*B*a^2*b^5 + 135*A*a*b^6)*x^5 + 35*(B*a^3*b^4 + 159*A*a^
2*b^5)*x^4 - 5*(8*B*a^4*b^3 - 15*A*a^3*b^4)*x^3 + 6*(8*B*a^5*b^2 - 15*A*a^
4*b^3)*x^2 - 8*(8*B*a^6*b - 15*A*a^5*b^2)*x)*sqrt(b*x + a)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{15}{2}}}{15b}+\frac{(a+bx)^{\frac{13}{2}}(Ab-4Ba)}{13b}+\frac{(a+bx)^{\frac{11}{2}}\left(-3Aab+6Ba^2\right)}{11b}+\frac{(a+bx)^{\frac{9}{2}}\cdot\left(3Aa^2b-4Ba^3\right)}{9b}+\frac{(a+bx)^{\frac{7}{2}}\left(-Aa^3b+Ba^4\right)}{7b}\right)}{b^4} & \text{for } b \neq \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right) & \text{otherw} \end{cases}$$

input
```
integrate(x**3*(b*x+a)**(5/2)*(B*x+A),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(15/2)/(15*b) + (a + b*x)**(13/2)*(A*b - 4*B*a)
/(13*b) + (a + b*x)**(11/2)*(-3*A*a*b + 6*B*a**2)/(11*b) + (a + b*x)**(9/2
)*(3*A*a**2*b - 4*B*a**3)/(9*b) + (a + b*x)**(7/2)*(-A*a**3*b + B*a**4)/(7
*b))/b**4, Ne(b, 0)), (a**(5/2)*(A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(3003\,(bx+a)^{\frac{15}{2}}B - 3465\,(4\,Ba-Ab)(bx+a)^{\frac{13}{2}} + 12285\,(2\,Ba^2-Aab)(bx+a)^{\frac{11}{2}} - 5005\,(4\right.}{45045\,b^5}$$

input
```
integrate(x^3*(b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3003*(b*x + a)^(15/2)*B - 3465*(4*B*a - A*b)*(b*x + a)^(13/2) + 1
2285*(2*B*a^2 - A*a*b)*(b*x + a)^(11/2) - 5005*(4*B*a^3 - 3*A*a^2*b)*(b*x
+ a)^(9/2) + 6435*(B*a^4 - A*a^3*b)*(b*x + a)^(7/2))/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(104) = 208.

Time = 0.13 (sec) , antiderivative size = 612, normalized size of antiderivative = 5.02

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/45045*(1287*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*A*a^3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315
*sqrt(b*x + a)*a^4)*B*a^3/b^4 + 429*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7
/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x +
 a)*a^4)*A*a^2/b^3 + 195*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 99
0*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)*B*a^2/b^4 + 195*(63*(b*x + a)^(11/2) - 385*(b*x
 + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*
(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*A*a/b^3 + 45*(231*(b*x + a)^(
13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a
)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*s
qrt(b*x + a)*a^6)*B*a/b^4 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/
2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a
)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*A/b^3 + 7
*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*
a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*B/b^4)
/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \frac{(12\,B\,a^2-6\,A\,a\,b)\,\left(a+b\,x\right)^{11/2}}{11\,b^5} + \frac{2\,B\,\left(a+b\,x\right)^{15/2}}{15\,b^5} + \frac{(2\,A\,b-8\,B\,a)\,\left(a+b\,x\right)^{13/2}}{13\,b^5} + \frac{(2\,B\,a^4-2\,A\,a^3\,b)\,\left(a+b\,x\right)^{7/2}}{7\,b^5} - \frac{(8\,B\,a^3-6\,A\,a^2\,b)\,\left(a+b\,x\right)^{9/2}}{9\,b^5}$$

input
```
int(x^3*(A + B*x)*(a + b*x)^(5/2),x)
```

output
```
((12*B*a^2 - 6*A*a*b)*(a + b*x)^(11/2))/(11*b^5) + (2*B*(a + b*x)^(15/2))/
(15*b^5) + ((2*A*b - 8*B*a)*(a + b*x)^(13/2))/(13*b^5) + ((2*B*a^4 - 2*A*a
^3*b)*(a + b*x)^(7/2))/(7*b^5) - ((8*B*a^3 - 6*A*a^2*b)*(a + b*x)^(9/2))/(
9*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int x^3(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(429b^7x^7+1518a\,b^6x^6+1854a^2b^5x^5+800a^3b^4x^4+5a^4b^3x^3-6a^5b^2x^2+8a^6bx-16a}{6435b^4}$$

input
```
int(x^3*(b*x+a)^(5/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**7 + 8*a**6*b*x - 6*a**5*b**2*x**2 + 5*a**4*b**3
*x**3 + 800*a**3*b**4*x**4 + 1854*a**2*b**5*x**5 + 1518*a*b**6*x**6 + 429*
b**7*x**7))/(6435*b**4)
```

3.247 $\int x^2(a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \frac{2a^2(Ab-aB)(a+bx)^{7/2}}{7b^4} - \frac{2a(2Ab-3aB)(a+bx)^{9/2}}{9b^4} + \frac{2(Ab-3aB)(a+bx)^{11/2}}{11b^4} + \frac{2B(a+bx)^{13/2}}{13b^4}$$

output
```
2/7*a^2*(A*b-B*a)*(b*x+a)^(7/2)/b^4-2/9*a*(2*A*b-3*B*a)*(b*x+a)^(9/2)/b^4+
2/11*(A*b-3*B*a)*(b*x+a)^(11/2)/b^4+2/13*B*(b*x+a)^(13/2)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \frac{2(a+bx)^{7/2}\left(-48a^3B+63b^3x^2(13A+11Bx)+8a^2b(13A+21Bx)-14ab^2x(26A+27Bx)\right)}{9009b^4}$$

input
```
Integrate[x^2*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(7/2)*(-48*a^3*B + 63*b^3*x^2*(13*A + 11*B*x) + 8*a^2*b*(13*A
 + 21*B*x) - 14*a*b^2*x*(26*A + 27*B*x)))/(9009*b^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{a^2(a+bx)^{5/2}(aB-Ab)}{b^3}+\frac{(a+bx)^{9/2}(Ab-3aB)}{b^3}+\frac{a(a+bx)^{7/2}(3aB-2Ab)}{b^3}+\frac{B(a+bx)^{11/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^2(a+bx)^{7/2}(Ab-aB)}{7b^4}+\frac{2(a+bx)^{11/2}(Ab-3aB)}{11b^4}-\frac{2a(a+bx)^{9/2}(2Ab-3aB)}{9b^4}+\frac{2B(a+bx)^{13/2}}{13b^4}$$

input
```
Int[x^2*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*a^2*(A*b - a*B)*(a + b*x)^(7/2))/(7*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*
x)^(9/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x)^(11/2))/(11*b^4) + (2*B*(a
+ b*x)^(13/2))/(13*b^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{16\left(\frac{63\left(\frac{11Bx}{13}+A\right)x^{2}b^{3}}{8}-\frac{7a\left(\frac{27Bx}{26}+A\right)x\,b^{2}}{2}+a^{2}\left(\frac{21Bx}{13}+A\right)b-\frac{6a^{3}B}{13}\right)(bx+a)^{\frac{7}{2}}}{693b^{4}}$
gosper	$\frac{2(bx+a)^{\frac{7}{2}}\left(693b^{3}B\,x^{3}+819A\,x^{2}b^{3}-378B\,x^{2}a\,b^{2}-364Axa\,b^{2}+168Bx\,a^{2}b+104a^{2}bA-48a^{3}B\right)}{9009b^{4}}$
orering	$\frac{2(bx+a)^{\frac{7}{2}}\left(693b^{3}B\,x^{3}+819A\,x^{2}b^{3}-378B\,x^{2}a\,b^{2}-364Axa\,b^{2}+168Bx\,a^{2}b+104a^{2}bA-48a^{3}B\right)}{9009b^{4}}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{13}{2}}}{13}+\frac{2(Ab-3Ba)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^{2}(Ab-Ba)(bx+a)^{\frac{7}{2}}}{7}}{b^{4}}$
default	$\frac{\frac{2B(bx+a)^{\frac{13}{2}}}{13}+\frac{2(Ab-3Ba)(bx+a)^{\frac{11}{2}}}{11}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^{2}(Ab-Ba)(bx+a)^{\frac{7}{2}}}{7}}{b^{4}}$
trager	$\frac{2(693B\,b^{6}x^{6}+819A\,b^{6}x^{5}+1701B a\,b^{5}x^{5}+2093A a\,b^{5}x^{4}+1113B\,a^{2}b^{4}x^{4}+1469A\,a^{2}b^{4}x^{3}+15B\,a^{3}b^{3}x^{3}+39A\,a^{3}b^{3}x^{2}-18}{9009b^{4}}$
risch	$\frac{2(693B\,b^{6}x^{6}+819A\,b^{6}x^{5}+1701B a\,b^{5}x^{5}+2093A a\,b^{5}x^{4}+1113B\,a^{2}b^{4}x^{4}+1469A\,a^{2}b^{4}x^{3}+15B\,a^{3}b^{3}x^{3}+39A\,a^{3}b^{3}x^{2}-18}{9009b^{4}}$

input
```
int(x^2*(b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
16/693*(63/8*(11/13*B*x+A)*x^2*b^3-7/2*a*(27/26*B*x+A)*x*b^2+a^2*(21/13*B*
x+A)*b-6/13*a^3*B)*(b*x+a)^(7/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.51

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\,(693\,Bb^6x^6 - 48\,Ba^6 + 104\,Aa^5b + 63\,(27\,Bab^5 + 13\,Ab^6)x^5 + 7\,(159\,Ba^2b^4 + 299\,Aab^5)x^4 +}{9009\,b}$$

input
```
integrate(x^2*(b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/9009*(693*B*b^6*x^6 - 48*B*a^6 + 104*A*a^5*b + 63*(27*B*a*b^5 + 13*A*b^6
)*x^5 + 7*(159*B*a^2*b^4 + 299*A*a*b^5)*x^4 + (15*B*a^3*b^3 + 1469*A*a^2*b
^4)*x^3 - 3*(6*B*a^4*b^2 - 13*A*a^3*b^3)*x^2 + 4*(6*B*a^5*b - 13*A*a^4*b^2
)*x)*sqrt(b*x + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(94) = 188.

Time = 0.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.07

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \begin{cases} \frac{16Aa^5\sqrt{a+bx}}{693b^3} - \frac{8Aa^4x\sqrt{a+bx}}{693b^2} + \frac{2Aa^3x^2\sqrt{a+bx}}{231b} + \frac{226Aa^2x^3\sqrt{a+bx}}{693} + \frac{46Aabx^4\sqrt{a+bx}}{99} + \frac{2Ab^2x^5\sqrt{a+bx}}{11} - \frac{32Ba^6}{3003} \\ a^{\frac{5}{2}}\left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right) \end{cases}$$

input
```
integrate(x**2*(b*x+a)**(5/2)*(B*x+A),x)
```

output
```
Piecewise((16*A*a**5*sqrt(a + b*x)/(693*b**3) - 8*A*a**4*x*sqrt(a + b*x)/(
693*b**2) + 2*A*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*A*a**2*x**3*sqrt(a +
 b*x)/693 + 46*A*a*b*x**4*sqrt(a + b*x)/99 + 2*A*b**2*x**5*sqrt(a + b*x)/1
1 - 32*B*a**6*sqrt(a + b*x)/(3003*b**4) + 16*B*a**5*x*sqrt(a + b*x)/(3003*
b**3) - 4*B*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*B*a**3*x**3*sqrt(a +
b*x)/(3003*b) + 106*B*a**2*x**4*sqrt(a + b*x)/429 + 54*B*a*b*x**5*sqrt(a +
 b*x)/143 + 2*B*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*(A*x**3/3
 + B*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(693\,(bx+a)^{\frac{13}{2}}B - 819\,(3\,Ba - Ab)(bx+a)^{\frac{11}{2}} + 1001\,(3\,Ba^2 - 2\,Aab)(bx+a)^{\frac{9}{2}} - 1287\,(Ba\right.}{9009\,b^4}$$

input
```
integrate(x^2*(b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/9009*(693*(b*x + a)^(13/2)*B - 819*(3*B*a - A*b)*(b*x + a)^(11/2) + 1001
*(3*B*a^2 - 2*A*a*b)*(b*x + a)^(9/2) - 1287*(B*a^3 - A*a^2*b)*(b*x + a)^(7
/2))/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(80) = 160$.

Time = 0.12 (sec) , antiderivative size = 516, normalized size of antiderivative = 5.43

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)
*a^2)*A*a^3/b^2 + 1287*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x
 + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*B*a^3/b^3 + 3861*(5*(b*x + a)^(7/2
) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*
A*a^2/b^2 + 429*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a
)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*B*a^2/b^3 +
 429*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A*a/b^2 + 195*(63*(b*x
 + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x
 + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*B*a/b^
3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*
a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x +
 a)*a^5)*A/b^2 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005
*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*B/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx = \frac{(6\,B\,a^2-4\,A\,a\,b)\,\left(a+b\,x\right)^{9/2}}{9\,b^4}+\frac{2\,B\,\left(a+b\,x\right)^{13/2}}{13\,b^4}+\frac{(2\,A\,b-6\,B\,a)\,\left(a+b\,x\right)^{11/2}}{11\,b^4}-\frac{(2\,B\,a^3-2\,A\,a^2\,b)\,\left(a+b\,x\right)^{7/2}}{7\,b^4}$$

input
```
int(x^2*(A + B*x)*(a + b*x)^(5/2),x)
```

output
```
((6*B*a^2 - 4*A*a*b)*(a + b*x)^(9/2))/(9*b^4) + (2*B*(a + b*x)^(13/2))/(13
*b^4) + ((2*A*b - 6*B*a)*(a + b*x)^(11/2))/(11*b^4) - ((2*B*a^3 - 2*A*a^2*
b)*(a + b*x)^(7/2))/(7*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

$$\int x^2(a+bx)^{5/2}(A+Bx)\,dx=\frac{2\sqrt{bx+a}\,(99b^6x^6+360a\,b^5x^5+458a^2b^4x^4+212a^3b^3x^3+3a^4b^2x^2-4a^5bx+8a^6)}{1287b^3}$$

input
```
int(x^2*(b*x+a)^(5/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**6 - 4*a**5*b*x + 3*a**4*b**2*x**2 + 212*a**3*b**3*x
**3 + 458*a**2*b**4*x**4 + 360*a*b**5*x**5 + 99*b**6*x**6))/(1287*b**3)
```

3.248 $\int x(a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int x(a+bx)^{5/2}(A+Bx)\,dx = -\frac{2a(Ab-aB)(a+bx)^{7/2}}{7b^3} + \frac{2(Ab-2aB)(a+bx)^{9/2}}{9b^3} + \frac{2B(a+bx)^{11/2}}{11b^3}$$

output
```
-2/7*a*(A*b-B*a)*(b*x+a)^(7/2)/b^3+2/9*(A*b-2*B*a)*(b*x+a)^(9/2)/b^3+2/11*
B*(b*x+a)^(11/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x(a+bx)^{5/2}(A+Bx)\,dx = \frac{2(a+bx)^{7/2}\,(8a^2B+7b^2x(11A+9Bx)-2ab(11A+14Bx))}{693b^3}$$

input
```
Integrate[x*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(7/2)*(8*a^2*B + 7*b^2*x*(11*A + 9*B*x) - 2*a*b*(11*A + 14*B*
x)))/(693*b^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{(a+bx)^{7/2}(Ab-2aB)}{b^2}+\frac{a(a+bx)^{5/2}(aB-Ab)}{b^2}+\frac{B(a+bx)^{9/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{9/2}(Ab-2aB)}{9b^3}-\frac{2a(a+bx)^{7/2}(Ab-aB)}{7b^3}+\frac{2B(a+bx)^{11/2}}{11b^3}$$

input
```
Int[x*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(-2*a*(A*b - a*B)*(a + b*x)^(7/2))/(7*b^3) + (2*(A*b - 2*a*B)*(a + b*x)^(9
/2))/(9*b^3) + (2*B*(a + b*x)^(11/2))/(11*b^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{4\left(-\frac{7\left(\frac{9Bx}{11}+A\right)x\,b^{2}}{2}+a\left(\frac{14Bx}{11}+A\right)b-\frac{4a^{2}B}{11}\right)(bx+a)^{\frac{7}{2}}}{63b^{3}}$
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-63b^{2}B\,x^{2}-77A\,b^{2}x+28Babx+22abA-8a^{2}B\right)}{693b^{3}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-63b^{2}B\,x^{2}-77A\,b^{2}x+28Babx+22abA-8a^{2}B\right)}{693b^{3}}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-2Ba)(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(Ab-Ba)(bx+a)^{\frac{7}{2}}}{7}}{b^{3}}$
default	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-2Ba)(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(Ab-Ba)(bx+a)^{\frac{7}{2}}}{7}}{b^{3}}$
trager	$-\frac{2\left(-63b^{5}B\,x^{5}-77A\,b^{5}x^{4}-161Ba\,b^{4}x^{4}-209Aa\,b^{4}x^{3}-113B\,a^{2}b^{3}x^{3}-165A\,a^{2}b^{3}x^{2}-3B\,a^{3}b^{2}x^{2}-11a^{3}b^{2}Ax+4a^{4}bBx\right.}{693b^{3}}$
risch	$-\frac{2\left(-63b^{5}B\,x^{5}-77A\,b^{5}x^{4}-161Ba\,b^{4}x^{4}-209Aa\,b^{4}x^{3}-113B\,a^{2}b^{3}x^{3}-165A\,a^{2}b^{3}x^{2}-3B\,a^{3}b^{2}x^{2}-11a^{3}b^{2}Ax+4a^{4}bBx\right.}{693b^{3}}$

input
```
int(x*(b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-4/63*(-7/2*(9/11*B*x+A)*x*b^2+a*(14/11*B*x+A)*b-4/11*a^2*B)*(b*x+a)^(7/2)
/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int x(a+bx)^{5/2}(A+Bx)\,dx=\frac{2\left(63\,Bb^{5}x^{5}+8\,Ba^{5}-22\,Aa^{4}b+7\,(23\,Bab^{4}+11\,Ab^{5})x^{4}+(113\,Ba^{2}b^{3}+209\,Aab^{4})x^{3}+3\,(Ba^{3}\right.}{693\,b^{3}}$$

input
```
integrate(x*(b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/693*(63*B*b^5*x^5 + 8*B*a^5 - 22*A*a^4*b + 7*(23*B*a*b^4 + 11*A*b^5)*x^4
 + (113*B*a^2*b^3 + 209*A*a*b^4)*x^3 + 3*(B*a^3*b^2 + 55*A*a^2*b^3)*x^2 -
(4*B*a^4*b - 11*A*a^3*b^2)*x)*sqrt(b*x + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(65) = 130$.

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.66

$$\int x(a+bx)^{5/2}(A+Bx)\,dx = \begin{cases} -\frac{4Aa^4\sqrt{a+bx}}{63b^2} + \frac{2Aa^3x\sqrt{a+bx}}{63b} + \frac{10Aa^2x^2\sqrt{a+bx}}{21} + \frac{38Aabx^3\sqrt{a+bx}}{63} + \frac{2Ab^2x^4\sqrt{a+bx}}{9} + \frac{16Ba^5\sqrt{a+bx}}{693b^3} - \frac{8Ba^4x\sqrt{}}{693b} \\ a^{\frac{5}{2}}\left(\frac{Ax^2}{2} + \frac{Bx^3}{3}\right) \end{cases}$$

input
```
integrate(x*(b*x+a)**(5/2)*(B*x+A),x)
```

output
```
Piecewise((-4*A*a**4*sqrt(a + b*x)/(63*b**2) + 2*A*a**3*x*sqrt(a + b*x)/(6
3*b) + 10*A*a**2*x**2*sqrt(a + b*x)/21 + 38*A*a*b*x**3*sqrt(a + b*x)/63 +
2*A*b**2*x**4*sqrt(a + b*x)/9 + 16*B*a**5*sqrt(a + b*x)/(693*b**3) - 8*B*a
**4*x*sqrt(a + b*x)/(693*b**2) + 2*B*a**3*x**2*sqrt(a + b*x)/(231*b) + 226
*B*a**2*x**3*sqrt(a + b*x)/693 + 46*B*a*b*x**4*sqrt(a + b*x)/99 + 2*B*b**2
*x**5*sqrt(a + b*x)/11, Ne(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(63\,(bx+a)^{\frac{11}{2}}B - 77\,(2\,Ba - Ab)(bx+a)^{\frac{9}{2}} + 99\,(Ba^2 - Aab)(bx+a)^{\frac{7}{2}}\right)}{693\,b^3}$$

input
```
integrate(x*(b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/693*(63*(b*x + a)^(11/2)*B - 77*(2*B*a - A*b)*(b*x + a)^(9/2) + 99*(B*a^
2 - A*a*b)*(b*x + a)^(7/2))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 418, normalized size of antiderivative = 6.24

$$\int x(a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(\frac{1155\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Aa^3}{b}+\frac{231\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)Ba^3}{b^2}+\frac{693\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+\right.}{b}\right.}{}$$

input
```
integrate(x*(b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/3465*(1155*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*A*a^3/b + 231*(3*(b*x +
 a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*a^3/b^2 + 693*(
3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*A*a^2/b +
 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 -
35*sqrt(b*x + a)*a^3)*B*a^2/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5
/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*A*a/b + 33*(35*(b*x
 + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*B*a/b^2 + 11*(35*(b*x + a)^(9/2) -
180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3
+ 315*sqrt(b*x + a)*a^4)*A/b + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2
)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^
(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*B/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x(a+bx)^{5/2}(A+Bx)\,dx=\frac{2\,{\left(a+b\,x\right)}^{7/2}\,\left(99\,B\,a^2+63\,B\,{\left(a+b\,x\right)}^2-99\,A\,a\,b+77\,A\,b\,\left(a+b\,x\right)-154\,B\,a\,\left(a+b\,x\right)\right)}{693\,b^3}$$

input
```
int(x*(A + B*x)*(a + b*x)^(5/2),x)
```

output
```
(2*(a + b*x)^(7/2)*(99*B*a^2 + 63*B*(a + b*x)^2 - 99*A*a*b + 77*A*b*(a + b
*x) - 154*B*a*(a + b*x)))/(693*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int x(a+bx)^{5/2}(A+Bx)\,dx=\frac{2\sqrt{bx+a}\,(9b^5x^5+34a\,b^4x^4+46a^2b^3x^3+24a^3b^2x^2+a^4bx-2a^5)}{99b^2}$$

input
```
int(x*(b*x+a)^(5/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**5 + a**4*b*x + 24*a**3*b**2*x**2 + 46*a**2*b**3*
x**3 + 34*a*b**4*x**4 + 9*b**5*x**5))/(99*b**2)
```

3.249 $\int (a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int (a+bx)^{5/2}(A+Bx)\,dx=\frac{2(Ab-aB)(a+bx)^{7/2}}{7b^2}+\frac{2B(a+bx)^{9/2}}{9b^2}$$

output
```
2/7*(A*b-B*a)*(b*x+a)^(7/2)/b^2+2/9*B*(b*x+a)^(9/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int (a+bx)^{5/2}(A+Bx)\,dx=\frac{2(a+bx)^{7/2}(9Ab-2aB+7bBx)}{63b^2}$$

input
```
Integrate[(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*(a + b*x)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x))/(63*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{(a+bx)^{5/2}(Ab-aB)}{b}+\frac{B(a+bx)^{7/2}}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(a+bx)^{7/2}(Ab-aB)}{7b^2}+\frac{2B(a+bx)^{9/2}}{9b^2}$$

input
```
Int[(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(2*(A*b - a*B)*(a + b*x)^(7/2))/(7*b^2) + (2*B*(a + b*x)^(9/2))/(9*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}(7bBx+9Ab-2Ba)}{63b^{2}}$	27
orering	$\frac{2(bx+a)^{\frac{7}{2}}(7bBx+9Ab-2Ba)}{63b^{2}}$	27
pseudoelliptic	$\frac{2((7Bx+9A)b-2Ba)(bx+a)^{\frac{7}{2}}}{63b^{2}}$	28
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-Ba)(bx+a)^{\frac{7}{2}}}{7}}{b^{2}}$	34
default	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-Ba)(bx+a)^{\frac{7}{2}}}{7}}{b^{2}}$	34
trager	$\frac{2\left(7B\,x^{4}b^{4}+9A\,x^{3}b^{4}+19B\,x^{3}a\,b^{3}+27A\,x^{2}a\,b^{3}+15B\,x^{2}a^{2}b^{2}+27Ax\,a^{2}b^{2}+Bx\,a^{3}b+9A\,a^{3}b-2B\,a^{4}\right)\sqrt{bx+a}}{63b^{2}}$	94
risch	$\frac{2\left(7B\,x^{4}b^{4}+9A\,x^{3}b^{4}+19B\,x^{3}a\,b^{3}+27A\,x^{2}a\,b^{3}+15B\,x^{2}a^{2}b^{2}+27Ax\,a^{2}b^{2}+Bx\,a^{3}b+9A\,a^{3}b-2B\,a^{4}\right)\sqrt{bx+a}}{63b^{2}}$	94

input
```
int((b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/63*(b*x+a)^(7/2)*(7*B*b*x+9*A*b-2*B*a)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(34) = 68.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.21

$$\int (a+bx)^{5/2}(A+Bx)\,dx = \frac{2\,(7\,Bb^4x^4-2\,Ba^4+9\,Aa^3b+(19\,Bab^3+9\,Ab^4)x^3+3\,(5\,Ba^2b^2+9\,Aab^3)x^2+(Ba^3b+27\,Aa}{63\,b^2}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
2/63*(7*B*b^4*x^4 - 2*B*a^4 + 9*A*a^3*b + (19*B*a*b^3 + 9*A*b^4)*x^3 + 3*(
5*B*a^2*b^2 + 9*A*a*b^3)*x^2 + (B*a^3*b + 27*A*a^2*b^2)*x)*sqrt(b*x + a)/b
^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(41) = 82$.

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.62

$$\int (a+bx)^{5/2}(A+Bx)\,dx = \begin{cases} \frac{2Aa^3\sqrt{a+bx}}{7b} + \frac{6Aa^2x\sqrt{a+bx}}{7} + \frac{6Aabx^2\sqrt{a+bx}}{7} + \frac{2Ab^2x^3\sqrt{a+bx}}{7} - \frac{4Ba^4\sqrt{a+bx}}{63b^2} + \frac{2Ba^3x\sqrt{a+bx}}{63b} + \frac{10Ba^2x^2\sqrt{a+b}}{21} \\ a^{\frac{5}{2}}\left(Ax + \frac{Bx^2}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A),x)
```

output
```
Piecewise((2*A*a**3*sqrt(a + b*x)/(7*b) + 6*A*a**2*x*sqrt(a + b*x)/7 + 6*A
*a*b*x**2*sqrt(a + b*x)/7 + 2*A*b**2*x**3*sqrt(a + b*x)/7 - 4*B*a**4*sqrt(
a + b*x)/(63*b**2) + 2*B*a**3*x*sqrt(a + b*x)/(63*b) + 10*B*a**2*x**2*sqrt
(a + b*x)/21 + 38*B*a*b*x**3*sqrt(a + b*x)/63 + 2*B*b**2*x**4*sqrt(a + b*x
)/9, Ne(b, 0)), (a**(5/2)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int (a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(7\,(bx+a)^{\frac{9}{2}}B - 9\,(Ba-Ab)(bx+a)^{\frac{7}{2}}\right)}{63\,b^2}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output
```
2/63*(7*(b*x + a)^(9/2)*B - 9*(B*a - A*b)*(b*x + a)^(7/2))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 306, normalized size of antiderivative = 7.29

$$\int (a+bx)^{5/2}(A+Bx)\,dx = \frac{2\left(315\sqrt{bx+a}Aa^3 + 315\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)Aa^2 + \frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Ba^3}{b} + 63\left(3(b\right.\right.}{}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(b*x + a)*A*a^3 + 315*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)
*A*a^2 + 105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*B*a^3/b + 63*(3*(b*x +
a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*A*a + 63*(3*(b*x +
 a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*a^2/b + 9*(5*(b
*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*
x + a)*a^3)*A + 27*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*B*a/b + (35*(b*x + a)^(9/2) - 180*(b*x
 + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*B/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int (a+bx)^{5/2}(A+Bx)\,dx = \frac{2\,{\left(a+b\,x\right)}^{7/2}\,\left(9\,A\,b-9\,B\,a+7\,B\,\left(a+b\,x\right)\right)}{63\,b^2}$$

input
```
int((A + B*x)*(a + b*x)^(5/2),x)
```

output
```
(2*(a + b*x)^(7/2)*(9*A*b - 9*B*a + 7*B*(a + b*x)))/(63*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int (a+bx)^{5/2}(A+Bx)\,dx = \frac{2\sqrt{bx+a}\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}{9b}$$

input
```
int((b*x+a)^(5/2)*(B*x+A),x)
```

output
```
(2*sqrt(a + b*x)*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b
**4*x**4))/(9*b)
```

3.250 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x} \, dx = 2a^2A\sqrt{a+bx} + \frac{2}{3}aA(a+bx)^{3/2} + \frac{2}{5}A(a+bx)^{5/2} + \frac{2B(a+bx)^{7/2}}{7b} - 2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*a^2*A*(b*x+a)^(1/2)+2/3*a*A*(b*x+a)^(3/2)+2/5*A*(b*x+a)^(5/2)+2/7*B*(b*x
+a)^(7/2)/b-2*a^(5/2)*A*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x} \, dx = \frac{2(105a^2Ab\sqrt{a+bx} + 35aAb(a+bx)^{3/2} + 21Ab(a+bx)^{5/2} + 15B(a+bx)^{7/2}}{105b} - 2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x,x]
```

output
```
(2*(105*a^2*A*b*Sqrt[a + b*x] + 35*a*A*b*(a + b*x)^(3/2) + 21*A*b*(a + b*x
)^(5/2) + 15*B*(a + b*x)^(7/2)))/(105*b) - 2*a^(5/2)*A*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}dx$$

$$\Big\downarrow \text{90}$$

$$A\int \frac{(a+bx)^{5/2}}{x}dx+\frac{2B(a+bx)^{7/2}}{7b}$$

$$\Big\downarrow \text{60}$$

$$A\left(a\int \frac{(a+bx)^{3/2}}{x}dx+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2B(a+bx)^{7/2}}{7b}$$

$$\Big\downarrow \text{60}$$

$$A\left(a\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2B(a+bx)^{7/2}}{7b}$$

$$\Big\downarrow \text{60}$$

$$A\left(a\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2B(a+bx)^{7/2}}{7b}$$

$$\Big\downarrow \text{73}$$

$$A\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2B(a+bx)^{7/2}}{7b}$$

$$\Big\downarrow \text{221}$$

$$A\left(a\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)+\frac{2B(a+bx)^{7/2}}{7b}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x,x]
```

output
```
(2*B*(a + b*x)^(7/2))/(7*b) + A*((2*(a + b*x)^(5/2))/5 + a*((2*(a + b*x)^(
3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2Ab(bx+a)^{\frac{5}{2}}}{5}+\frac{2Aab(bx+a)^{\frac{3}{2}}}{3}+2A\,a^{2}b\sqrt{bx+a}-2A\,a^{\frac{5}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	72
default	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2Ab(bx+a)^{\frac{5}{2}}}{5}+\frac{2Aab(bx+a)^{\frac{3}{2}}}{3}+2A\,a^{2}b\sqrt{bx+a}-2A\,a^{\frac{5}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	72
pseudoelliptic	$\frac{-2A\,a^{\frac{5}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{46\sqrt{bx+a}\left(\frac{3\left(\frac{5Bx}{7}+A\right)x^{2}b^{3}}{23}+\frac{11ax\left(\frac{45Bx}{77}+A\right)b^{2}}{23}+a^{2}\left(\frac{45Bx}{161}+A\right)b+\frac{15a^{3}B}{161}\right)}{15}}{b}$	80

input
```
int((b*x+a)^(5/2)*(B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/7*B*(b*x+a)^(7/2)+1/5*A*b*(b*x+a)^(5/2)+1/3*A*a*b*(b*x+a)^(3/2)+A*a
^2*b*(b*x+a)^(1/2)-A*a^(5/2)*b*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.38

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx = \left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(15\,Bb^{3}x^{3}+15\,Ba^{3}+161\,Aa^{2}b+3\,(15\right.}{105\,b}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/105*(105*A*a^(5/2)*b*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
15*B*b^3*x^3 + 15*B*a^3 + 161*A*a^2*b + 3*(15*B*a*b^2 + 7*A*b^3)*x^2 + (45
*B*a^2*b + 77*A*a*b^2)*x)*sqrt(b*x + a))/b, 2/105*(105*A*sqrt(-a)*a^2*b*ar
ctan(sqrt(-a)/sqrt(b*x + a)) + (15*B*b^3*x^3 + 15*B*a^3 + 161*A*a^2*b + 3*
(15*B*a*b^2 + 7*A*b^3)*x^2 + (45*B*a^2*b + 77*A*a*b^2)*x)*sqrt(b*x + a))/b
]
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx = \begin{cases} \frac{2Aa^3 \operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa^2\sqrt{a+bx} + \frac{2Aa(a+bx)^{\frac{3}{2}}}{3} + \frac{2A(a+bx)^{\frac{5}{2}}}{5} + \frac{2B(a+bx)^{\frac{7}{2}}}{7b} & \text{for} \\ a^{\frac{5}{2}}(A\log(Bx) + Bx) & \text{oth} \end{cases}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x,x)
```

output
```
Piecewise((2*A*a**3*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*A*a**2*sqrt(
a + b*x) + 2*A*a*(a + b*x)**(3/2)/3 + 2*A*(a + b*x)**(5/2)/5 + 2*B*(a + b*
x)**(7/2)/(7*b), Ne(b, 0)), (a**(5/2)*(A*log(B*x) + B*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx = Aa^{\frac{5}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2\left(15\,(bx+a)^{\frac{7}{2}}B + 21\,(bx+a)^{\frac{5}{2}}Ab + 35\,(bx+a)^{\frac{3}{2}}Aab + 105\,\sqrt{bx+a}Aa^2b\right)}{105\,b}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x,x, algorithm="maxima")
```

output
```
A*a^(5/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/105
*(15*(b*x + a)^(7/2)*B + 21*(b*x + a)^(5/2)*A*b + 35*(b*x + a)^(3/2)*A*a*b
 + 105*sqrt(b*x + a)*A*a^2*b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx = \frac{2\,Aa^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(15\,(bx+a)^{\frac{7}{2}}Bb^6+21\,(bx+a)^{\frac{5}{2}}Ab^7+35\,(bx+a)^{\frac{3}{2}}Aab^7+105\,\sqrt{bx+a}Aa^2b^7\right)}{105\,b^7}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x,x, algorithm="giac")
```

output
```
2*A*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/105*(15*(b*x + a)^(7/2
)*B*b^6 + 21*(b*x + a)^(5/2)*A*b^7 + 35*(b*x + a)^(3/2)*A*a*b^7 + 105*sqrt
(b*x + a)*A*a^2*b^7)/b^7
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx = \left(\frac{2\,A\,b-2\,B\,a}{5\,b}+\frac{2\,B\,a}{5\,b}\right)\,(a+b\,x)^{5/2} + a^2\left(\frac{2\,A\,b-2\,B\,a}{b}+\frac{2\,B\,a}{b}\right)\,\sqrt{a+b\,x}+\frac{2\,B\,(a+b\,x)^{7/2}}{7\,b} + \frac{a\,\left(\frac{2\,A\,b-2\,B\,a}{b}+\frac{2\,B\,a}{b}\right)\,(a+b\,x)^{3/2}}{3}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x,x)
```

output
```
((2*A*b - 2*B*a)/(5*b) + (2*B*a)/(5*b))*(a + b*x)^(5/2) + a^2*((2*A*b - 2*
B*a)/b + (2*B*a)/b)*(a + b*x)^(1/2) + A*a^(5/2)*atan(((a + b*x)^(1/2)*1i)/
a^(1/2))*2i + (2*B*(a + b*x)^(7/2))/(7*b) + (a*((2*A*b - 2*B*a)/b + (2*B*a
)/b)*(a + b*x)^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx = \frac{352\sqrt{bx+a}\,a^3}{105} + \frac{244\sqrt{bx+a}\,a^2bx}{105} + \frac{44\sqrt{bx+a}\,a\,b^2x^2}{35} + \frac{2\sqrt{bx+a}\,b^3x^3}{7} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^3 - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)a^3$$

input

```
int((b*x+a)^(5/2)*(B*x+A)/x,x)
```

output

```
(352*sqrt(a + b*x)*a**3 + 244*sqrt(a + b*x)*a**2*b*x + 132*sqrt(a + b*x)*a
*b**2*x**2 + 30*sqrt(a + b*x)*b**3*x**3 + 105*sqrt(a)*log(sqrt(a + b*x) -
sqrt(a))*a**3 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**3)/105
```

3.251 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2} dx = 2a(2Ab+aB)\sqrt{a+bx} - \frac{a^2A\sqrt{a+bx}}{x} + \frac{2}{3}(Ab+aB)(a+bx)^{3/2} + \frac{2}{5}B(a+bx)^{5/2} - a^{3/2}(5Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*a*(2*A*b+B*a)*(b*x+a)^(1/2)-a^2*A*(b*x+a)^(1/2)/x+2/3*(A*b+B*a)*(b*x+a)^
(3/2)+2/5*B*(b*x+a)^(5/2)-a^(3/2)*(5*A*b+2*B*a)*arctanh((b*x+a)^(1/2)/a^(1
/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2} dx = \frac{\sqrt{a+bx}(2b^2x^2(5A+3Bx)+2abx(35A+11Bx)+a^2(-15A+46Bx))}{15x} - a^{3/2}(5Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^2,x]
```

output
```
(Sqrt[a + b*x]*(2*b^2*x^2*(5*A + 3*B*x) + 2*a*b*x*(35*A + 11*B*x) + a^2*(-
15*A + 46*B*x)))/(15*x) - a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(2aB+5Ab)\int \frac{(a+bx)^{5/2}}{x}dx}{2a}-\frac{A(a+bx)^{7/2}}{ax}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2aB+5Ab)\left(a\int \frac{(a+bx)^{3/2}}{x}dx+\frac{2}{5}(a+bx)^{5/2}\right)}{2a}-\frac{A(a+bx)^{7/2}}{ax}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2aB+5Ab)\left(a\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)}{2a}-\frac{A(a+bx)^{7/2}}{ax}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2aB+5Ab)\left(a\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)}{2a}-\frac{A(a+bx)^{7/2}}{ax}$$

$$\Big\downarrow \text{73}$$

$$\frac{(2aB+5Ab)\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)}{2a}-\frac{A(a+bx)^{7/2}}{ax}$$

$$\Big\downarrow \text{221}$$

$$\frac{(2aB+5Ab)\left(a\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)+\frac{2}{5}(a+bx)^{5/2}\right)}{2a}-\frac{A(a+bx)^{7/2}}{ax}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^2,x]
```

output
```
-((A*(a + b*x)^(7/2))/(a*x)) + ((5*A*b + 2*a*B)*((2*(a + b*x)^(5/2))/5 + a
*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))))/(2*a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\frac{2\left(-\frac{15}{2}a^{2}bA-3a^{3}B\right)x\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{3}+\frac{2\left(7\left(\frac{11Bx}{35}+A\right)bx\,a^{\frac{3}{2}}+\left(\frac{23Bx}{5}-\frac{3A}{2}\right)a^{\frac{5}{2}}+b^{2}x^{2}\sqrt{a}\left(\frac{3Bx}{5}+A\right)\right)\sqrt{bx+a}}{3}}{\sqrt{a}\,x}$
risch	$-\frac{a^{2}A\sqrt{bx+a}}{x}+\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}+\frac{2Ba(bx+a)^{\frac{3}{2}}}{3}+4Aab\sqrt{bx+a}+2B\,a^{2}\sqrt{bx+a}-$
derivativedivides	$\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}+\frac{2Ba(bx+a)^{\frac{3}{2}}}{3}+4Aab\sqrt{bx+a}+2B\,a^{2}\sqrt{bx+a}-2a^{2}\left(\frac{A\sqrt{bx+a}}{2x}\right.$
default	$\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}+\frac{2Ba(bx+a)^{\frac{3}{2}}}{3}+4Aab\sqrt{bx+a}+2B\,a^{2}\sqrt{bx+a}-2a^{2}\left(\frac{A\sqrt{bx+a}}{2x}\right.$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*((-15/2*a^2*b*A-3*a^3*B)*x*arctanh((b*x+a)^(1/2)/a^(1/2))+(7*(11/35*B*
x+A)*b*x*a^(3/2)+(23/5*B*x-3/2*A)*a^(5/2)+b^2*x^2*a^(1/2)*(3/5*B*x+A))*(b*
x+a)^(1/2))/a^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx = \left[\frac{15\,(2\,Ba^2+5\,Aab)\sqrt{a}x\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(6\,Bb^2x^3-15\,Aa^2+2\,(1}{30\,x}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/30*(15*(2*B*a^2 + 5*A*a*b)*sqrt(a)*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a)
 + 2*a)/x) + 2*(6*B*b^2*x^3 - 15*A*a^2 + 2*(11*B*a*b + 5*A*b^2)*x^2 + 2*(2
3*B*a^2 + 35*A*a*b)*x)*sqrt(b*x + a))/x, 1/15*(15*(2*B*a^2 + 5*A*a*b)*sqrt
(-a)*x*arctan(sqrt(-a)/sqrt(b*x + a)) + (6*B*b^2*x^3 - 15*A*a^2 + 2*(11*B*
a*b + 5*A*b^2)*x^2 + 2*(23*B*a^2 + 35*A*a*b)*x)*sqrt(b*x + a))/x]
```

Sympy [A] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.41

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx &= -Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)\\ &- \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} + 2Aab\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+bx} & \text{for}\ b\neq 0\\ \sqrt{a}\log{(x)} & \text{otherwise}\end{cases}\right)\\ &+ Ab^2\left(\begin{cases}\frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)\\ &+ Ba^2\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+bx} & \text{for}\ b\neq 0\\ \sqrt{a}\log{(x)} & \text{otherwise}\end{cases}\right)\\ &+ 2Bab\left(\begin{cases}\frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases}-\frac{2a(a+bx)^{\frac{3}{2}}}{3b^2}+\frac{2(a+bx)^{\frac{5}{2}}}{5b^2} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**2,x)
```

output
```
-A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - A*a**2*sqrt(b)*sqrt(a/(b*
x) + 1)/sqrt(x) + 2*A*a*b*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt
(-a) + 2*sqrt(a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True)) + A*b**2*Piecew
ise((2*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True)) + B*a**2*Piec
ewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b*x), Ne(b,
0)), (sqrt(a)*log(x), True)) + 2*B*a*b*Piecewise((2*(a + b*x)**(3/2)/(3*b)
, Ne(b, 0)), (sqrt(a)*x, True)) + B*b**2*Piecewise((-2*a*(a + b*x)**(3/2)/
(3*b**2) + 2*(a + b*x)**(5/2)/(5*b**2), Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx = \frac{1}{30}\left(\frac{15\,(2\,Ba+5\,Ab)a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{b} - \frac{30\,\sqrt{bx+a}Aa^2}{bx} + \frac{4\left(3\,(bx+a)^{\frac{5}{2}}\right.}{}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/30*(15*(2*B*a + 5*A*b)*a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x +
 a) + sqrt(a)))/b - 30*sqrt(b*x + a)*A*a^2/(b*x) + 4*(3*(b*x + a)^(5/2)*B
+ 5*(B*a + A*b)*(b*x + a)^(3/2) + 15*(B*a^2 + 2*A*a*b)*sqrt(b*x + a))/b)*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx =$$

$$-\frac{1}{15}\left(\frac{15\,\sqrt{bx+a}Aa^2}{bx} - \frac{15\,(2\,Ba^3+5\,Aa^2b)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b} - \frac{2\left(3\,(bx+a)^{\frac{5}{2}}Bb^4+5\,(bx+a)^{\frac{3}{2}}Bab^4+\right.}{}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^2,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(b*x + a)*A*a^2/(b*x) - 15*(2*B*a^3 + 5*A*a^2*b)*arctan(sqrt
(b*x + a)/sqrt(-a))/(sqrt(-a)*b) - 2*(3*(b*x + a)^(5/2)*B*b^4 + 5*(b*x + a
)^(3/2)*B*a*b^4 + 15*sqrt(b*x + a)*B*a^2*b^4 + 5*(b*x + a)^(3/2)*A*b^5 + 3
0*sqrt(b*x + a)*A*a*b^5)/b^5)*b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx = \left(\frac{2\,A\,b}{3}+\frac{2\,B\,a}{3}\right)\,{\left(a+b\,x\right)}^{3/2} + \left(2\,a\,\left(2\,A\,b+2\,B\,a\right)-2\,B\,a^2\right)\,\sqrt{a+b\,x}+\frac{2\,B\,{\left(a+b\,x\right)}^{5/2}}{5} - \frac{A\,a^2\,\sqrt{a+b\,x}}{x}+a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(5\,A\,b+2\,B\,a\right)\,1{}\mathrm{i}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^2,x)
```

output
```
((2*A*b)/3 + (2*B*a)/3)*(a + b*x)^(3/2) + (2*a*(2*A*b + 2*B*a) - 2*B*a^2)*
(a + b*x)^(1/2) + (2*B*(a + b*x)^(5/2))/5 + a^(3/2)*atan(((a + b*x)^(1/2)*
1i)/a^(1/2))*(5*A*b + 2*B*a)*1i - (A*a^2*(a + b*x)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx = \frac{-30\sqrt{bx+a}\,a^3+232\sqrt{bx+a}\,a^2bx+64\sqrt{bx+a}\,a\,b^2x^2+12\sqrt{bx+a}\,b^3x^3+}{30x}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^2,x)
```

output
```
( - 30*sqrt(a + b*x)*a**3 + 232*sqrt(a + b*x)*a**2*b*x + 64*sqrt(a + b*x)*
a*b**2*x**2 + 12*sqrt(a + b*x)*b**3*x**3 + 105*sqrt(a)*log(sqrt(a + b*x) -
 sqrt(a))*a**2*b*x - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a**2*b*x)/(3
0*x)
```

3.252 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3} dx = \frac{1}{2}b(5Ab+8aB)\sqrt{a+bx} - \frac{a(5Ab+4aB)\sqrt{a+bx}}{4x} + \frac{2}{3}bB(a+bx)^{3/2} - \frac{A(a+bx)^{5/2}}{2x^2} - \frac{5}{4}\sqrt{a}b(3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
1/2*b*(5*A*b+8*B*a)*(b*x+a)^(1/2)-1/4*a*(5*A*b+4*B*a)*(b*x+a)^(1/2)/x+2/3*
b*B*(b*x+a)^(3/2)-1/2*A*(b*x+a)^(5/2)/x^2-5/4*a^(1/2)*b*(3*A*b+4*B*a)*arct
anh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3} dx = \frac{\sqrt{a+bx}(8b^2x^2(3A+Bx) - 6a^2(A+2Bx) + abx(-27A+56Bx))}{12x^2} - \frac{5}{4}\sqrt{a}b(3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^3,x]
```

output
```
(Sqrt[a + b*x]*(8*b^2*x^2*(3*A + B*x) - 6*a^2*(A + 2*B*x) + a*b*x*(-27*A +
 56*B*x)))/(12*x^2) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(4aB+3Ab)\int \frac{(a+bx)^{5/2}}{x^2}dx}{4a}-\frac{A(a+bx)^{7/2}}{2ax^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{(4aB+3Ab)\left(\frac{5}{2}b\int \frac{(a+bx)^{3/2}}{x}dx-\frac{(a+bx)^{5/2}}{x}\right)}{4a}-\frac{A(a+bx)^{7/2}}{2ax^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\int \frac{\sqrt{a+bx}}{x}dx+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)}{4a}-\frac{A(a+bx)^{7/2}}{2ax^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)}{4a}-\frac{A(a+bx)^{7/2}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)}{4a}-\frac{A(a+bx)^{7/2}}{2ax^2}$$

$$\Big\downarrow \; 221$$

$$\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\left(2\sqrt{a+bx}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx)^{3/2}\right)-\frac{(a+bx)^{5/2}}{x}\right)}{4a}-\frac{A(a+bx)^{7/2}}{2ax^2}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^3,x]
```

output
```
-1/2*(A*(a + b*x)^(7/2))/(a*x^2) + ((3*A*b + 4*a*B)*(-((a + b*x)^(5/2)/x)
+ (5*b*((2*(a + b*x)^(3/2))/3 + a*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])))/2))/(4*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{\frac{15ab\left(Ab+\frac{4Ba}{3}\right)x^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2}+\sqrt{bx+a}\left(-4b^2\left(\frac{Bx}{3}+A\right)x^2\sqrt{a}+\left(-\frac{28bBx^2}{3}+\left(2Ba+\frac{9Ab}{2}\right)x+Aa\right)a^{\frac{3}{2}}\right)}{2\sqrt{a}\,x^2}$
risch	$-\frac{a\sqrt{bx+a}\,(9Abx+4Bax+2Aa)}{4x^2}+\frac{b\left(\frac{16B(bx+a)^{\frac{3}{2}}}{3}+16Ab\sqrt{bx+a}+32Ba\sqrt{bx+a}-10\sqrt{a}\,(3Ab+4Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right.\right.}{8}$
derivativedivides	$2b\left(\frac{B(bx+a)^{\frac{3}{2}}}{3}+Ab\sqrt{bx+a}+2Ba\sqrt{bx+a}-a\left(\frac{\left(\frac{9Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(-\frac{7}{8}abA-\frac{1}{2}a^2B\right)\sqrt{bx+a}}{b^2x^2}+\right.\right.$
default	$2b\left(\frac{B(bx+a)^{\frac{3}{2}}}{3}+Ab\sqrt{bx+a}+2Ba\sqrt{bx+a}-a\left(\frac{\left(\frac{9Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(-\frac{7}{8}abA-\frac{1}{2}a^2B\right)\sqrt{bx+a}}{b^2x^2}+\right.\right.$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(15/2*a*b*(A*b+4/3*B*a)*x^2*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1
/2)*(-4*b^2*(1/3*B*x+A)*x^2*a^(1/2)+(-28/3*b*B*x^2+(2*B*a+9/2*A*b)*x+A*a)*
a^(3/2)))/a^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx = \left[\frac{15\,(4\,Bab+3\,Ab^2)\sqrt{a}x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,Bb^2x^3-6\,Aa^2+8\,(7\right.}{24\,x^2}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^3,x, algorithm="fricas")
```

output
```
[1/24*(15*(4*B*a*b + 3*A*b^2)*sqrt(a)*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(8*B*b^2*x^3 - 6*A*a^2 + 8*(7*B*a*b + 3*A*b^2)*x^2 - 3*(4
*B*a^2 + 9*A*a*b)*x)*sqrt(b*x + a))/x^2, 1/12*(15*(4*B*a*b + 3*A*b^2)*sqrt
(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x + a)) + (8*B*b^2*x^3 - 6*A*a^2 + 8*(7*B*
a*b + 3*A*b^2)*x^2 - 3*(4*B*a^2 + 9*A*a*b)*x)*sqrt(b*x + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 41.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.70

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx &= -\frac{7A\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} \\ &\quad - \frac{Aa^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3Aa^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{2Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{Aab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}} \\ &\quad + Ab^2\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+bx} & \text{for}\ b\neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) - Ba^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) \\ &\quad - \frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} + 2Bab\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+bx} & \text{for}\ b\neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) \\ &\quad + Bb^2\left(\begin{cases}\frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**3,x)
```

output
```
-7*A*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - A*a**3/(2*sqrt(b)*x
**(5/2)*sqrt(a/(b*x) + 1)) - 3*A*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1
)) - 2*A*a*b**(3/2)*sqrt(a/(b*x) + 1)/sqrt(x) - A*a*b**(3/2)/(4*sqrt(x)*sq
rt(a/(b*x) + 1)) + A*b**2*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt
(-a) + 2*sqrt(a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True)) - B*a**(3/2)*b*
asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - B*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x
) + 2*B*a*b*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*sqrt(
a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True)) + B*b**2*Piecewise((2*(a + b*
x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx = \frac{1}{24}\left(\frac{15\,(4\,Ba+3\,Ab)\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{b} - \frac{6\left((4\,Ba^2+9\,Aab)(bx+a)^{\frac{3}{2}} - \right.}{(bx+a)^2b-2\,(bx}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/24*(15*(4*B*a + 3*A*b)*sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x +
 a) + sqrt(a)))/b - 6*((4*B*a^2 + 9*A*a*b)*(b*x + a)^(3/2) - (4*B*a^3 + 7*
A*a^2*b)*sqrt(b*x + a))/((b*x + a)^2*b - 2*(b*x + a)*a*b + a^2*b) + 16*((b
*x + a)^(3/2)*B + 3*(2*B*a + A*b)*sqrt(b*x + a))/b)*b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx = \frac{8\,(bx+a)^{\frac{3}{2}}Bb^2 + 48\,\sqrt{bx+a}Bab^2 + 24\,\sqrt{bx+a}Ab^3 + \frac{15\,(4\,Ba^2b^2+3\,Aab^3)\arctan}{\sqrt{-a}}}{12\,b}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/12*(8*(b*x + a)^(3/2)*B*b^2 + 48*sqrt(b*x + a)*B*a*b^2 + 24*sqrt(b*x + a
)*A*b^3 + 15*(4*B*a^2*b^2 + 3*A*a*b^3)*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt
(-a) - 3*(4*(b*x + a)^(3/2)*B*a^2*b^2 - 4*sqrt(b*x + a)*B*a^3*b^2 + 9*(b*x
 + a)^(3/2)*A*a*b^3 - 7*sqrt(b*x + a)*A*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx = \left(2\,A\,b^2+4\,B\,a\,b\right)\,\sqrt{a+b\,x} - \frac{\left(B\,a^2\,b+\frac{9\,A\,a\,b^2}{4}\right)\,{\left(a+b\,x\right)}^{3/2}-\left(B\,a^3\,b+\frac{7\,A\,a^2\,b^2}{4}\right)\,\sqrt{a+b\,x}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2} + \frac{2\,B\,b\,{\left(a+b\,x\right)}^{3/2}}{3} + 2\,b\,\mathrm{atan}\left(\frac{2\,b\,\left(3\,A\,b+4\,B\,a\right)\,\sqrt{-\frac{25\,a}{64}}\,\sqrt{a+b\,x}}{5\,B\,a^2\,b+\frac{15\,A\,a\,b^2}{4}}\right)\,\left(3\,A\,b+4\,B\,a\right)\,\sqrt{-\frac{25\,a}{64}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^3,x)
```

output
```
(2*A*b^2 + 4*B*a*b)*(a + b*x)^(1/2) - (((9*A*a*b^2)/4 + B*a^2*b)*(a + b*x)
^(3/2) - ((7*A*a^2*b^2)/4 + B*a^3*b)*(a + b*x)^(1/2))/((a + b*x)^2 - 2*a*(
a + b*x) + a^2) + (2*B*b*(a + b*x)^(3/2))/3 + 2*b*atan((2*b*(3*A*b + 4*B*a
)*(-(25*a)/64)^(1/2)*(a + b*x)^(1/2))/((15*A*a*b^2)/4 + 5*B*a^2*b))*(3*A*b
 + 4*B*a)*(-(25*a)/64)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx = \frac{-12\sqrt{bx+a}\,a^3-78\sqrt{bx+a}\,a^2bx+160\sqrt{bx+a}\,a\,b^2x^2+16\sqrt{bx+a}\,b^3x^3+}{24x^2}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^3,x)
```

output
```
( - 12*sqrt(a + b*x)*a**3 - 78*sqrt(a + b*x)*a**2*b*x + 160*sqrt(a + b*x)*
a*b**2*x**2 + 16*sqrt(a + b*x)*b**3*x**3 + 105*sqrt(a)*log(sqrt(a + b*x) -
 sqrt(a))*a*b**2*x**2 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*x*
*2)/(24*x**2)
```

3.253 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = 2b^2B\sqrt{a+bx} - \frac{b(5Ab+14aB)\sqrt{a+bx}}{8x} - \frac{(5Ab+6aB)(a+bx)^{3/2}}{12x^2} - \frac{A(a+bx)^{5/2}}{3x^3} - \frac{5b^2(Ab+6aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
2*b^2*B*(b*x+a)^(1/2)-1/8*b*(5*A*b+14*B*a)*(b*x+a)^(1/2)/x-1/12*(5*A*b+6*B
*a)*(b*x+a)^(3/2)/x^2-1/3*A*(b*x+a)^(5/2)/x^3-5/8*b^2*(A*b+6*B*a)*arctanh(
(b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = -\frac{\sqrt{a+bx}(3b^2x^2(11A-16Bx)+4a^2(2A+3Bx)+2abx(13A+27Bx))}{24x^3} - \frac{5b^2(Ab+6aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^4,x]
```

output
```
-1/24*(Sqrt[a + b*x]*(3*b^2*x^2*(11*A - 16*B*x) + 4*a^2*(2*A + 3*B*x) + 2*
a*b*x*(13*A + 27*B*x)))/x^3 - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(8*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4} \, dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(6aB+Ab)\int \frac{(a+bx)^{5/2}}{x^3}dx}{6a}-\frac{A(a+bx)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{(6aB+Ab)\left(\frac{5}{4}b\int \frac{(a+bx)^{3/2}}{x^2}dx-\frac{(a+bx)^{5/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{a+bx}}{x}dx-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{60}$$

$$\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)-\frac{(a+bx)^{3/2}}{x}\right)-\frac{(a+bx)^{5/2}}{2x^2}\right)}{6a}-\frac{A(a+bx)^{7/2}}{3ax^3}$$

input

```
Int[((a + b*x)^(5/2)*(A + B*x))/x^4,x]
```

output

```
-1/3*(A*(a + b*x)^(7/2))/(a*x^3) + ((A*b + 6*a*B)*(-1/2*(a + b*x)^(5/2)/x^
2 + (5*b*(-((a + b*x)^(3/2)/x) + (3*b*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]]))/2))/4))/(6*a)
```

Defintions of rubi rules used

rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$-\frac{11\left(\frac{5b^{2}x^{3}(Ab+6Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{11}+\left(\frac{26\left(\frac{27Bx}{13}+A\right)bx\,a^{\frac{3}{2}}}{33}+\frac{4\left(Bx+\frac{2A}{3}\right)a^{\frac{5}{2}}}{11}+b^{2}x^{2}\sqrt{a}\left(-\frac{16Bx}{11}+A\right)\right)\sqrt{bx+a}\right)}{8\sqrt{a}\,x^{3}}$
risch	$-\frac{\sqrt{bx+a}\,(33A\,b^{2}x^{2}+54Bab\,x^{2}+26aAbx+12B\,a^{2}x+8a^{2}A)}{24x^{3}}+\frac{b^{2}\left(32B\sqrt{bx+a}-\frac{2(5Ab+30Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{16}$
derivativedivides	$2b^{2}\left(B\sqrt{bx+a}-\frac{\left(\frac{11Ab}{16}+\frac{9Ba}{8}\right)(bx+a)^{\frac{5}{2}}+\left(-\frac{5}{6}abA-2a^{2}B\right)(bx+a)^{\frac{3}{2}}+\left(\frac{7}{8}a^{3}B+\frac{5}{16}a^{2}bA\right)\sqrt{bx+a}}{b^{3}x^{3}}-\frac{5(Ab+6B}{}\right.$
default	$2b^{2}\left(B\sqrt{bx+a}-\frac{\left(\frac{11Ab}{16}+\frac{9Ba}{8}\right)(bx+a)^{\frac{5}{2}}+\left(-\frac{5}{6}abA-2a^{2}B\right)(bx+a)^{\frac{3}{2}}+\left(\frac{7}{8}a^{3}B+\frac{5}{16}a^{2}bA\right)\sqrt{bx+a}}{b^{3}x^{3}}-\frac{5(Ab+6B}{}\right.$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-11/8*(5/11*b^2*x^3*(A*b+6*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+(26/33*(27/
13*B*x+A)*b*x*a^(3/2)+4/11*(B*x+2/3*A)*a^(5/2)+b^2*x^2*a^(1/2)*(-16/11*B*x
+A))*(b*x+a)^(1/2))/a^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = \left[\frac{15\,(6\,Bab^2+Ab^3)\sqrt{a}x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(48\,Bab^2x^3-8\,Aa^3-3\right.}{48\,ax^3}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(15*(6*B*a*b^2 + A*b^3)*sqrt(a)*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(48*B*a*b^2*x^3 - 8*A*a^3 - 3*(18*B*a^2*b + 11*A*a*b^2)*x
^2 - 2*(6*B*a^3 + 13*A*a^2*b)*x)*sqrt(b*x + a))/(a*x^3), 1/24*(15*(6*B*a*b
^2 + A*b^3)*sqrt(-a)*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) + (48*B*a*b^2*x^3
- 8*A*a^3 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^2 - 2*(6*B*a^3 + 13*A*a^2*b)*x)*
sqrt(b*x + a))/(a*x^3)]
```

Sympy [A] (verification not implemented)

Time = 66.65 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.90

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = &-\frac{Aa^3}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{17Aa^2\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}\\ &-\frac{35Aab^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{3Ab^{\frac{5}{2}}}{8\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}\\ &-\frac{7B\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{Ba^3}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3Ba^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}\\ &-\frac{Bab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}}+Bb^2\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+bx} & \text{for } b\neq 0\\ \sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**4,x)
```

output
```
-A*a**3/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 17*A*a**2*sqrt(b)/(12*x**
(5/2)*sqrt(a/(b*x) + 1)) - 35*A*a*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1))
 - A*b**(5/2)*sqrt(a/(b*x) + 1)/sqrt(x) - 3*A*b**(5/2)/(8*sqrt(x)*sqrt(a/(
b*x) + 1)) - 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a)) - 7*B*s
qrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - B*a**3/(2*sqrt(b)*x**(5/2
)*sqrt(a/(b*x) + 1)) - 3*B*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - 2
*B*a*b**(3/2)*sqrt(a/(b*x) + 1)/sqrt(x) - B*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(
b*x) + 1)) + B*b**2*Piecewise((2*a*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) +
 2*sqrt(a + b*x), Ne(b, 0)), (sqrt(a)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = \frac{1}{48}\,b^3\left(\frac{96\,\sqrt{bx+a}B}{b}+\frac{15\,(6\,Ba+Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}b}-\frac{2\,\left(3\,(18\,Ba+11\,A\right.}{}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/48*b^3*(96*sqrt(b*x + a)*B/b + 15*(6*B*a + A*b)*log((sqrt(b*x + a) - sqr
t(a))/(sqrt(b*x + a) + sqrt(a)))/(sqrt(a)*b) - 2*(3*(18*B*a + 11*A*b)*(b*x
 + a)^(5/2) - 8*(12*B*a^2 + 5*A*a*b)*(b*x + a)^(3/2) + 3*(14*B*a^3 + 5*A*a
^2*b)*sqrt(b*x + a))/((b*x + a)^3*b - 3*(b*x + a)^2*a*b + 3*(b*x + a)*a^2*
b - a^3*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = \frac{1}{24}b^3\left(\frac{48\sqrt{bx+a}B}{b} + \frac{15\,(6\,Ba+Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b} - \frac{54\,(bx+a)^{\frac{5}{2}}Ba -\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/24*b^3*(48*sqrt(b*x + a)*B/b + 15*(6*B*a + A*b)*arctan(sqrt(b*x + a)/sqr
t(-a))/(sqrt(-a)*b) - (54*(b*x + a)^(5/2)*B*a - 96*(b*x + a)^(3/2)*B*a^2 +
 42*sqrt(b*x + a)*B*a^3 + 33*(b*x + a)^(5/2)*A*b - 40*(b*x + a)^(3/2)*A*a*
b + 15*sqrt(b*x + a)*A*a^2*b)/(b^4*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = \frac{\left(\frac{11\,A\,b^3}{8}+\frac{9\,B\,a\,b^2}{4}\right)(a+b\,x)^{5/2}+\left(\frac{7\,B\,a^3\,b^2}{4}+\frac{5\,A\,a^2\,b^3}{8}\right)\sqrt{a+b\,x}-\left(4\,B\,a^2\,b^2+\right.}{3\,a\,(a+b\,x)^2-3\,a^2\,(a+b\,x)-(a+b\,x)^3+a^3}$$
$$+2\,B\,b^2\sqrt{a+b\,x}-\frac{5\,b^2\,\mathrm{atanh}\left(\frac{5\,b^2\,(A\,b+6\,B\,a)\sqrt{a+b\,x}}{4\sqrt{a}\left(\frac{5\,A\,b^3}{4}+\frac{15\,B\,a\,b^2}{2}\right)}\right)(A\,b+6\,B\,a)}{8\sqrt{a}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^4,x)
```

output
```
(((11*A*b^3)/8 + (9*B*a*b^2)/4)*(a + b*x)^(5/2) + ((5*A*a^2*b^3)/8 + (7*B*
a^3*b^2)/4)*(a + b*x)^(1/2) - (4*B*a^2*b^2 + (5*A*a*b^3)/3)*(a + b*x)^(3/2
))/(3*a*(a + b*x)^2 - 3*a^2*(a + b*x) - (a + b*x)^3 + a^3) + 2*B*b^2*(a +
b*x)^(1/2) - (5*b^2*atanh((5*b^2*(A*b + 6*B*a)*(a + b*x)^(1/2))/(4*a^(1/2)
*((5*A*b^3)/4 + (15*B*a*b^2)/2)))*(A*b + 6*B*a))/(8*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx = \frac{-16\sqrt{bx+a}\,a^3 - 76\sqrt{bx+a}\,a^2bx - 174\sqrt{bx+a}\,a\,b^2x^2 + 96\sqrt{bx+a}\,b^3x^3 +}{48x^3}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 76*sqrt(a + b*x)*a**2*b*x - 174*sqrt(a + b*x)*
a*b**2*x**2 + 96*sqrt(a + b*x)*b**3*x**3 + 105*sqrt(a)*log(sqrt(a + b*x) -
 sqrt(a))*b**3*x**3 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/
(48*x**3)
```

3.254 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5} dx = -\frac{b(5Ab+24aB)\sqrt{a+bx}}{32x^2} - \frac{b^2(5Ab+88aB)\sqrt{a+bx}}{64ax} - \frac{(5Ab+8aB)(a+bx)^{3/2}}{24x^3} - \frac{A(a+bx)^{5/2}}{4x^4} + \frac{5b^3(Ab-8aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

output
```
-1/32*b*(5*A*b+24*B*a)*(b*x+a)^(1/2)/x^2-1/64*b^2*(5*A*b+88*B*a)*(b*x+a)^(
1/2)/a/x-1/24*(5*A*b+8*B*a)*(b*x+a)^(3/2)/x^3-1/4*A*(b*x+a)^(5/2)/x^4+5/64
*b^3*(A*b-8*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5} dx = -\frac{\sqrt{a+bx}(15Ab^3x^3+16a^3(3A+4Bx)+8a^2bx(17A+26Bx)+2ab^2x^2(59A+132Bx))}{192ax^4} + \frac{5b^3(Ab-8aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^5,x]
```

output
```
-1/192*(Sqrt[a + b*x]*(15*A*b^3*x^3 + 16*a^3*(3*A + 4*B*x) + 8*a^2*b*x*(17
*A + 26*B*x) + 2*a*b^2*x^2*(59*A + 132*B*x)))/(a*x^4) + (5*b^3*(A*b - 8*a*
B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(Ab-8aB)\int \frac{(a+bx)^{5/2}}{x^4}dx}{8a}-\frac{A(a+bx)^{7/2}}{4ax^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-8aB)\left(\frac{5}{6}b\int \frac{(a+bx)^{3/2}}{x^3}dx-\frac{(a+bx)^{5/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{7/2}}{4ax^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\int \frac{\sqrt{a+bx}}{x^2}dx-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{7/2}}{4ax^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{7/2}}{4ax^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{7/2}}{4ax^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}\right)-\frac{(a+bx)^{3/2}}{2x^2}\right)-\frac{(a+bx)^{5/2}}{3x^3}\right)}{8a}-\frac{A(a+bx)^{7/2}}{4ax^4}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^5,x]
```

output
```
-1/4*(A*(a + b*x)^(7/2))/(a*x^4) - ((A*b - 8*a*B)*(-1/3*(a + b*x)^(5/2)/x^
3 + (5*b*(-1/2*(a + b*x)^(3/2)/x^2 + (3*b*(-(Sqrt[a + b*x]/x) - (b*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]))/4))/6))/(8*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{17\left(-\frac{15 b^{3} x^{4} \left(A b - 8 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b x + a}}{\sqrt{a}}\right)}{136}+\sqrt{b x + a}\,\left(\frac{59 \left(\frac{132 B x}{59}+A\right) b^{2} x^{2} a^{\frac{3}{2}}}{68}+b x \left(\frac{26 B x}{17}+A\right) a^{\frac{5}{2}}+\frac{2 \left(4 B x + 3 A\right) a^{\frac{7}{2}}}{17}+\frac{15 A}{}\right.\right.}{24 a^{\frac{3}{2}} x^{4}}$
risch	$-\frac{\sqrt{b x + a}\,\left(15 A\,b^{3} x^{3}+264 B a\,b^{2} x^{3}+118 a A\,b^{2} x^{2}+208 B\,a^{2} b\,x^{2}+136 a^{2} A b x+64 B\,a^{3} x+48 a^{3} A\right)}{192 x^{4} a}+\frac{5 b^{3} \left(A b - 8 B a\right) \operatorname{arcta}}{64 a^{\frac{3}{2}}}$
derivativedivides	$2 b^{3} \left(-\frac{\frac{\left(5 A b + 88 B a\right) \left(b x + a\right)^{\frac{7}{2}}}{128 a}+\left(-\frac{73 B a}{48}+\frac{73 A b}{384}\right) \left(b x + a\right)^{\frac{5}{2}}-\frac{55 a \left(A b - 8 B a\right) \left(b x + a\right)^{\frac{3}{2}}}{384}+\left(-\frac{5}{16} a^{3} B+\frac{5}{128} a^{2} b A\right) \sqrt{b x + a}}{b^{4} x^{4}}+\right.$
default	$2 b^{3} \left(-\frac{\frac{\left(5 A b + 88 B a\right) \left(b x + a\right)^{\frac{7}{2}}}{128 a}+\left(-\frac{73 B a}{48}+\frac{73 A b}{384}\right) \left(b x + a\right)^{\frac{5}{2}}-\frac{55 a \left(A b - 8 B a\right) \left(b x + a\right)^{\frac{3}{2}}}{384}+\left(-\frac{5}{16} a^{3} B+\frac{5}{128} a^{2} b A\right) \sqrt{b x + a}}{b^{4} x^{4}}+\right.$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-17/24*(-15/136*b^3*x^4*(A*b-8*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)
^(1/2)*(59/68*(132/59*B*x+A)*b^2*x^2*a^(3/2)+b*x*(26/17*B*x+A)*a^(5/2)+2/1
7*(4*B*x+3*A)*a^(7/2)+15/136*A*a^(1/2)*b^3*x^3))/a^(3/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx = \left[-\frac{15\,(8\,Bab^3 - Ab^4)\sqrt{a}x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right) + 2\,(48\,Aa^4 + 3\,(88\,Ba^2b^2}{384}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[-1/384*(15*(8*B*a*b^3 - A*b^4)*sqrt(a)*x^4*log((b*x + 2*sqrt(b*x + a)*sqr
t(a) + 2*a)/x) + 2*(48*A*a^4 + 3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^3 + 2*(104*B
*a^3*b + 59*A*a^2*b^2)*x^2 + 8*(8*B*a^4 + 17*A*a^3*b)*x)*sqrt(b*x + a))/(a
^2*x^4), 1/192*(15*(8*B*a*b^3 - A*b^4)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b
*x + a)) - (48*A*a^4 + 3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^3 + 2*(104*B*a^3*b +
 59*A*a^2*b^2)*x^2 + 8*(8*B*a^4 + 17*A*a^3*b)*x)*sqrt(b*x + a))/(a^2*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(126) = 252.

Time = 163.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.40

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx = &-\frac{Aa^3}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{23Aa^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} \\ &- \frac{127Aab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{133Ab^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Ab^{\frac{7}{2}}}{64a\sqrt{x}\sqrt{\frac{a}{bx}+1}} \\ &+ \frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{3}{2}}} - \frac{Ba^3}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{17Ba^2\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} \\ &- \frac{35Bab^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{3Bb^{\frac{5}{2}}}{8\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{5Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}\end{aligned}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**5,x)
```

output
```
-A*a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*A*a**2*sqrt(b)/(24*x**
(7/2)*sqrt(a/(b*x) + 1)) - 127*A*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)
) - 133*A*b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*A*b**(7/2)/(64*a*s
qrt(x)*sqrt(a/(b*x) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*
a**(3/2)) - B*a**3/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 17*B*a**2*sqrt
(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1)) - 35*B*a*b**(3/2)/(24*x**(3/2)*sqrt(a/
(b*x) + 1)) - B*b**(5/2)*sqrt(a/(b*x) + 1)/sqrt(x) - 3*B*b**(5/2)/(8*sqrt(
x)*sqrt(a/(b*x) + 1)) - 5*B*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(
a))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx =$$

$$-\frac{1}{384}\,b^4\left(\frac{2\left(3\,(88\,Ba+5\,Ab)(bx+a)^{\frac{7}{2}}-73\,(8\,Ba^2-Aab)(bx+a)^{\frac{5}{2}}+55\,(8\,Ba^3-Aa^2b)(bx+a)^{\frac{3}{2}}-15\right.}{(bx+a)^4ab-4\,(bx+a)^3a^2b+6\,(bx+a)^2a^3b-4\,(bx+a)a^4b+a^5b}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^5,x, algorithm="maxima")
```

output
```
-1/384*b^4*(2*(3*(88*B*a + 5*A*b)*(b*x + a)^(7/2) - 73*(8*B*a^2 - A*a*b)*(
b*x + a)^(5/2) + 55*(8*B*a^3 - A*a^2*b)*(b*x + a)^(3/2) - 15*(8*B*a^4 - A*
a^3*b)*sqrt(b*x + a))/((b*x + a)^4*a*b - 4*(b*x + a)^3*a^2*b + 6*(b*x + a)
^2*a^3*b - 4*(b*x + a)*a^4*b + a^5*b) - 15*(8*B*a - A*b)*log((sqrt(b*x + a
) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(3/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx = \frac{\frac{15\,(8\,Bab^4-Ab^5)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{264\,(bx+a)^{\frac{7}{2}}Bab^4-584\,(bx+a)^{\frac{5}{2}}Ba^2b^4+440\,(bx+a)^{\frac{3}{2}}Ba^3b^4-120}{}}{192\,b}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^5,x, algorithm="giac")
```

output
```
1/192*(15*(8*B*a*b^4 - A*b^5)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a)
- (264*(b*x + a)^(7/2)*B*a*b^4 - 584*(b*x + a)^(5/2)*B*a^2*b^4 + 440*(b*x
+ a)^(3/2)*B*a^3*b^4 - 120*sqrt(b*x + a)*B*a^4*b^4 + 15*(b*x + a)^(7/2)*A*
b^5 + 73*(b*x + a)^(5/2)*A*a*b^5 - 55*(b*x + a)^(3/2)*A*a^2*b^5 + 15*sqrt(
b*x + a)*A*a^3*b^5)/(a*b^4*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx = \frac{5\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(A\,b-8\,B\,a)}{64\,a^{3/2}} - \frac{\left(\frac{73\,A\,b^4}{192}-\frac{73\,B\,a\,b^3}{24}\right)\,(a+b\,x)^{5/2}+\left(\frac{5\,A\,a^2\,b^4}{64}-\frac{5\,B\,a^3\,b^3}{8}\right)\,\sqrt{a+b\,x}+\left(\frac{55\,B\,a^2\,b^3}{24}-\frac{55\,A\,a\,b^4}{192}\right)\,(a+b\,x)^{3/2}+\frac{(5\,A}{}}{(a+b\,x)^4-4\,a^3\,(a+b\,x)-4\,a\,(a+b\,x)^3+6\,a^2\,(a+b\,x)^2+a^4}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^5,x)
```

output
```
(5*b^3*atanh((a + b*x)^(1/2)/a^(1/2))*(A*b - 8*B*a))/(64*a^(3/2)) - (((73*
A*b^4)/192 - (73*B*a*b^3)/24)*(a + b*x)^(5/2) + ((5*A*a^2*b^4)/64 - (5*B*a
^3*b^3)/8)*(a + b*x)^(1/2) + ((55*B*a^2*b^3)/24 - (55*A*a*b^4)/192)*(a + b
*x)^(3/2) + ((5*A*b^4 + 88*B*a*b^3)*(a + b*x)^(7/2))/(64*a))/((a + b*x)^4
- 4*a^3*(a + b*x) - 4*a*(a + b*x)^3 + 6*a^2*(a + b*x)^2 + a^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx = \frac{-96\sqrt{bx+a}\,a^4-400\sqrt{bx+a}\,a^3bx-652\sqrt{bx+a}\,a^2b^2x^2-558\sqrt{bx+a}\,a\,b^3}{384a\,x}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^5,x)
```

output
```
( - 96*sqrt(a + b*x)*a**4 - 400*sqrt(a + b*x)*a**3*b*x - 652*sqrt(a + b*x)
*a**2*b**2*x**2 - 558*sqrt(a + b*x)*a*b**3*x**3 + 105*sqrt(a)*log(sqrt(a +
 b*x) - sqrt(a))*b**4*x**4 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4
*x**4)/(384*a*x**4)
```

3.255 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 167

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx = -\frac{b(3Ab+22aB)\sqrt{a+bx}}{48x^3} - \frac{b^2(3Ab+118aB)\sqrt{a+bx}}{192ax^2} + \frac{b^3(3Ab-10aB)\sqrt{a+bx}}{128a^2x} - \frac{(Ab+2aB)(a+bx)^{3/2}}{8x^4} - \frac{A(a+bx)^{5/2}}{5x^5} - \frac{b^4(3Ab-10aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{5/2}}$$

output
```
-1/48*b*(3*A*b+22*B*a)*(b*x+a)^(1/2)/x^3-1/192*b^2*(3*A*b+118*B*a)*(b*x+a)
^(1/2)/a/x^2+1/128*b^3*(3*A*b-10*B*a)*(b*x+a)^(1/2)/a^2/x-1/8*(A*b+2*B*a)*
(b*x+a)^(3/2)/x^4-1/5*A*(b*x+a)^(5/2)/x^5-1/128*b^4*(3*A*b-10*B*a)*arctanh
((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx =$$
$$-\frac{\sqrt{a+bx}(-45Ab^4x^4+30ab^3x^3(A+5Bx)+96a^4(4A+5Bx)+16a^3bx(63A+85Bx)+4a^2b^2x^2(186A+}{1920a^2x^5}$$
$$+\frac{b^4(-3Ab+10aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128a^{5/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^6,x]
```

output
```
-1/1920*(Sqrt[a + b*x]*(-45*A*b^4*x^4 + 30*a*b^3*x^3*(A + 5*B*x) + 96*a^4*
(4*A + 5*B*x) + 16*a^3*b*x*(63*A + 85*B*x) + 4*a^2*b^2*x^2*(186*A + 295*B*
x)))/(a^2*x^5) + (b^4*(-3*A*b + 10*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1
28*a^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {87, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}dx$$
$$\Big\downarrow \text{87}$$
$$-\frac{(3Ab-10aB)\int \frac{(a+bx)^{5/2}}{x^5}dx}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$
$$\Big\downarrow \text{51}$$
$$-\frac{(3Ab-10aB)\left(\frac{5}{8}b\int \frac{(a+bx)^{3/2}}{x^4}dx-\frac{(a+bx)^{5/2}}{4x^4}\right)}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}dx-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}\right)-\frac{(a+bx)^{3/2}}{3x^3}\right)-\frac{(a+bx)^{5/2}}{4x^4}\right)}{10a}-\frac{A(a+bx)^{7/2}}{5ax^5}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^6,x]
```

output
```
-1/5*(A*(a + b*x)^(7/2))/(a*x^5) - ((3*A*b - 10*a*B)*(-1/4*(a + b*x)^(5/2)
/x^4 + (5*b*(-1/3*(a + b*x)^(3/2)/x^3 + (b*(-1/2*Sqrt[a + b*x]/x^2 + (b*(-
(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/4))/2
))/8))/(10*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{31\left(\frac{15b^4\left(Ab-\frac{10Ba}{3}\right)x^5 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{248}+\left(\frac{5b^3x^3(5Bx+A)a^{\frac{3}{2}}}{124}+b^2x^2\left(\frac{295Bx}{186}+A\right)a^{\frac{5}{2}}+\frac{42b\left(\frac{85Bx}{63}+A\right)x\,a^{\frac{7}{2}}}{31}+\frac{4(5Bx+}{31}\right.\right.}{80a^{\frac{5}{2}}x^5}$
risch	$-\frac{\sqrt{bx+a}\,(-45Ab^4x^4+150Ba\,b^3x^4+30Aa\,b^3x^3+1180B\,a^2b^2x^3+744A\,a^2b^2x^2+1360B\,a^3b\,x^2+1008A\,a^3bx+480B\,a^4}{1920x^5a^2}$
derivativedivides	$2b^4\left(-\frac{-\frac{(3Ab-10Ba)(bx+a)^{\frac{9}{2}}}{256a^2}+\frac{(21Ab+58Ba)(bx+a)^{\frac{7}{2}}}{384a}+\left(\frac{Ab}{10}-\frac{Ba}{3}\right)(bx+a)^{\frac{5}{2}}-\frac{7a(3Ab-10Ba)(bx+a)^{\frac{3}{2}}}{384}+\frac{a^2(3Ab-10Ba)}{256}}{b^5x^5}\right.$
default	$2b^4\left(-\frac{-\frac{(3Ab-10Ba)(bx+a)^{\frac{9}{2}}}{256a^2}+\frac{(21Ab+58Ba)(bx+a)^{\frac{7}{2}}}{384a}+\left(\frac{Ab}{10}-\frac{Ba}{3}\right)(bx+a)^{\frac{5}{2}}-\frac{7a(3Ab-10Ba)(bx+a)^{\frac{3}{2}}}{384}+\frac{a^2(3Ab-10Ba)}{256}}{b^5x^5}\right.$

input

```
int((b*x+a)^(5/2)*(B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output

```
-31/80/a^(5/2)*(15/248*b^4*(A*b-10/3*B*a)*x^5*arctanh((b*x+a)^(1/2)/a^(1/2
))+(5/124*b^3*x^3*(5*B*x+A)*a^(3/2)+b^2*x^2*(295/186*B*x+A)*a^(5/2)+42/31*
b*(85/63*B*x+A)*x*a^(7/2)+4/31*(5*B*x+4*A)*a^(9/2)-15/248*A*a^(1/2)*b^4*x^
4)*(b*x+a)^(1/2))/x^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx = \left[-\frac{15\,(10\,Bab^4-3\,Ab^5)\sqrt{a}x^5\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(384\,Aa^5+15\,(10\,B\right.$$

$$-\frac{15\,(10\,Bab^4-3\,Ab^5)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(384\,Aa^5+15\,(10\,Ba^2b^3-3\,Aab^4)x^4+10\,(118\,Ba^3b^2+3}{1920\,a^3x^5}$$

input

```
integrate((b*x+a)^(5/2)*(B*x+A)/x^6,x, algorithm="fricas")
```

output
```
[-1/3840*(15*(10*B*a*b^4 - 3*A*b^5)*sqrt(a)*x^5*log((b*x - 2*sqrt(b*x + a)
*sqrt(a) + 2*a)/x) + 2*(384*A*a^5 + 15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^4 + 10
*(118*B*a^3*b^2 + 3*A*a^2*b^3)*x^3 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x^2 +
48*(10*B*a^5 + 21*A*a^4*b)*x)*sqrt(b*x + a))/(a^3*x^5), -1/1920*(15*(10*B*
a*b^4 - 3*A*b^5)*sqrt(-a)*x^5*arctan(sqrt(-a)/sqrt(b*x + a)) + (384*A*a^5
+ 15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^4 + 10*(118*B*a^3*b^2 + 3*A*a^2*b^3)*x^3
 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x^2 + 48*(10*B*a^5 + 21*A*a^4*b)*x)*sqrt
(b*x + a))/(a^3*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx =$$

$$-\frac{1}{3840}\,b^5\left(\frac{2\left(15\,(10\,Ba-3\,Ab)(bx+a)^{\frac{9}{2}}+10\,(58\,Ba^2+21\,Aab)(bx+a)^{\frac{7}{2}}-128\,(10\,Ba^3-3\,Aa^2b)(bx+\right.}{(bx+a)^5a^2b-5\,(bx+a)^4a^3b+10\,(bx+a)^3a^4b-10\,(b}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/3840*b^5*(2*(15*(10*B*a - 3*A*b)*(b*x + a)^(9/2) + 10*(58*B*a^2 + 21*A*
a*b)*(b*x + a)^(7/2) - 128*(10*B*a^3 - 3*A*a^2*b)*(b*x + a)^(5/2) + 70*(10
*B*a^4 - 3*A*a^3*b)*(b*x + a)^(3/2) - 15*(10*B*a^5 - 3*A*a^4*b)*sqrt(b*x +
 a))/((b*x + a)^5*a^2*b - 5*(b*x + a)^4*a^3*b + 10*(b*x + a)^3*a^4*b - 10*
(b*x + a)^2*a^5*b + 5*(b*x + a)*a^6*b - a^7*b) + 15*(10*B*a - 3*A*b)*log((
sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(5/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx =$$

$$-\frac{1}{1920}b^5\left(\frac{15\,(10\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b}+\frac{150\,(bx+a)^{\frac{9}{2}}Ba+580\,(bx+a)^{\frac{7}{2}}Ba^2-1280\,(bx+a)^{\frac{5}{2}}B\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^6,x, algorithm="giac")
```

output
```
-1/1920*b^5*(15*(10*B*a - 3*A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*
a^2*b) + (150*(b*x + a)^(9/2)*B*a + 580*(b*x + a)^(7/2)*B*a^2 - 1280*(b*x
+ a)^(5/2)*B*a^3 + 700*(b*x + a)^(3/2)*B*a^4 - 150*sqrt(b*x + a)*B*a^5 - 4
5*(b*x + a)^(9/2)*A*b + 210*(b*x + a)^(7/2)*A*a*b + 384*(b*x + a)^(5/2)*A*
a^2*b - 210*(b*x + a)^(3/2)*A*a^3*b + 45*sqrt(b*x + a)*A*a^4*b)/(a^2*b^6*x
^5))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx = \frac{\left(\frac{A\,b^5}{5}-\frac{2\,B\,a\,b^4}{3}\right)\,{\left(a+b\,x\right)}^{5/2}+\left(\frac{3\,A\,a^2\,b^5}{128}-\frac{5\,B\,a^3\,b^4}{64}\right)\,\sqrt{a+b\,x}+\left(\frac{35\,B\,a^2\,b^4}{96}-\frac{7}{}\right.}{5\,a\,{\left(a+b\,x\right)}^4-5\,a^4\,\left(a+b\,x\right)-{\left(a+b\,x\right)}^5-}$$

$$-\frac{b^4\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(3\,A\,b-10\,B\,a\right)}{128\,a^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^6,x)
```

output
```
(((A*b^5)/5 - (2*B*a*b^4)/3)*(a + b*x)^(5/2) + ((3*A*a^2*b^5)/128 - (5*B*a
^3*b^4)/64)*(a + b*x)^(1/2) + ((35*B*a^2*b^4)/96 - (7*A*a*b^5)/64)*(a + b*
x)^(3/2) - ((3*A*b^5 - 10*B*a*b^4)*(a + b*x)^(9/2))/(128*a^2) + ((21*A*b^5
 + 58*B*a*b^4)*(a + b*x)^(7/2))/(192*a))/(5*a*(a + b*x)^4 - 5*a^4*(a + b*x
) - (a + b*x)^5 - 10*a^2*(a + b*x)^3 + 10*a^3*(a + b*x)^2 + a^5) - (b^4*at
anh((a + b*x)^(1/2)/a^(1/2))*(3*A*b - 10*B*a))/(128*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx = \frac{-768\sqrt{bx+a}\,a^5 - 2976\sqrt{bx+a}\,a^4bx - 4208\sqrt{bx+a}\,a^3b^2x^2 - 2420\sqrt{bx+a}}{}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^6,x)
```

output
```
( - 768*sqrt(a + b*x)*a**5 - 2976*sqrt(a + b*x)*a**4*b*x - 4208*sqrt(a + b
*x)*a**3*b**2*x**2 - 2420*sqrt(a + b*x)*a**2*b**3*x**3 - 210*sqrt(a + b*x)
*a*b**4*x**4 - 105*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 + 105*sq
rt(a)*log(sqrt(a + b*x) + sqrt(a))*b**5*x**5)/(3840*a**2*x**5)
```

3.256 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 199

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7} dx = -\frac{b(5Ab+52aB)\sqrt{a+bx}}{160x^4} - \frac{b^2(5Ab+372aB)\sqrt{a+bx}}{960ax^3} + \frac{b^3(5Ab-12aB)\sqrt{a+bx}}{768a^2x^2} - \frac{b^4(5Ab-12aB)\sqrt{a+bx}}{512a^3x} - \frac{(5Ab+12aB)(a+bx)^{3/2}}{60x^5} - \frac{A(a+bx)^{5/2}}{6x^6} + \frac{b^5(5Ab-12aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{7/2}}$$

output
```
-1/160*b*(5*A*b+52*B*a)*(b*x+a)^(1/2)/x^4-1/960*b^2*(5*A*b+372*B*a)*(b*x+a
)^(1/2)/a/x^3+1/768*b^3*(5*A*b-12*B*a)*(b*x+a)^(1/2)/a^2/x^2-1/512*b^4*(5*
A*b-12*B*a)*(b*x+a)^(1/2)/a^3/x-1/60*(5*A*b+12*B*a)*(b*x+a)^(3/2)/x^5-1/6*
A*(b*x+a)^(5/2)/x^6+1/512*b^5*(5*A*b-12*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2)
)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx =$$
$$-\frac{\sqrt{a+bx}(75Ab^5x^5+40a^2b^3x^3(A+3Bx)+256a^5(5A+6Bx)-10ab^4x^4(5A+18Bx)+48a^3b^2x^2(45A+}{7680a^3x^6}$$
$$+\frac{b^5(5Ab-12aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{7/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^7,x]
```

output
```
-1/7680*(Sqrt[a + b*x]*(75*A*b^5*x^5 + 40*a^2*b^3*x^3*(A + 3*B*x) + 256*a^
5*(5*A + 6*B*x) - 10*a*b^4*x^4*(5*A + 18*B*x) + 48*a^3*b^2*x^2*(45*A + 62*
B*x) + 64*a^4*b*x*(50*A + 63*B*x)))/(a^3*x^6) + (b^5*(5*A*b - 12*a*B)*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/(512*a^(7/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {87, 51, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}dx$$
$$\Big\downarrow \ 87$$
$$-\frac{(5Ab-12aB)\int \frac{(a+bx)^{5/2}}{x^6}dx}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$
$$\Big\downarrow \ 51$$
$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\int \frac{(a+bx)^{3/2}}{x^5}dx-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\left(\frac{3}{8}b\int\frac{\sqrt{a+bx}}{x^4}dx-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$

$$\Big\downarrow \text{51}$$

$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int\frac{1}{x^3\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int\frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}-\frac{A(a+bx)^{7/2}}{6ax^6}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(5Ab-12aB)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)-\frac{\sqrt{a+bx}}{3x^3}\right)-\frac{(a+bx)^{3/2}}{4x^4}\right)-\frac{(a+bx)^{5/2}}{5x^5}\right)}{12a}$$
$$\frac{A(a+bx)^{7/2}}{6ax^6}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^7,x]
```

output
```
-1/6*(A*(a + b*x)^(7/2))/(a*x^6) - ((5*A*b - 12*a*B)*(-1/5*(a + b*x)^(5/2)
/x^5 + (b*(-1/4*(a + b*x)^(3/2)/x^4 + (3*b*(-1/3*Sqrt[a + b*x]/x^3 + (b*(-
1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/6))/8))/2))/(12*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{-\frac{15\left(Ab-\frac{12Ba}{5}\right)b^{5}x^{6}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8}+\left(\left(\frac{192Bx}{5}+32A\right)a^{\frac{11}{2}}+\left(-\frac{5b^{3}x^{3}\left(\frac{18Bx}{5}+A\right)a^{\frac{3}{2}}}{4}+b^{2}x^{2}(3Bx+A)a^{\frac{5}{2}}+54b\left(\frac{62}{\cdots}\right.\right.\right.}{192a^{\frac{7}{2}}x^{6}}$
risch	$-\frac{\sqrt{bx+a}\,(75A\,b^{5}x^{5}-180Ba\,b^{4}x^{5}-50aA\,b^{4}x^{4}+120B\,a^{2}b^{3}x^{4}+40a^{2}A\,b^{3}x^{3}+2976B\,a^{3}b^{2}x^{3}+2160a^{3}A\,b^{2}x^{2}+4032B\,a^{4}\cdots}{7680x^{6}a^{3}}$
derivativedivides	$2b^{5}\left(-\frac{\frac{(5Ab-12Ba)(bx+a)^{\frac{11}{2}}}{1024a^{3}}-\frac{17(5Ab-12Ba)(bx+a)^{\frac{9}{2}}}{3072a^{2}}+\frac{(165Ab+116Ba)(bx+a)^{\frac{7}{2}}}{2560a}+\left(\frac{33Ab}{512}-\frac{99Ba}{640}\right)(bx+a)^{\frac{5}{2}}-\frac{17a(5Ab-\cdots}{\cdots}}{b^{6}x^{6}}\right.$
default	$2b^{5}\left(-\frac{\frac{(5Ab-12Ba)(bx+a)^{\frac{11}{2}}}{1024a^{3}}-\frac{17(5Ab-12Ba)(bx+a)^{\frac{9}{2}}}{3072a^{2}}+\frac{(165Ab+116Ba)(bx+a)^{\frac{7}{2}}}{2560a}+\left(\frac{33Ab}{512}-\frac{99Ba}{640}\right)(bx+a)^{\frac{5}{2}}-\frac{17a(5Ab-\cdots}{\cdots}}{b^{6}x^{6}}\right.$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-15/8*(A*b-12/5*B*a)*b^5*x^6*arctanh((b*x+a)^(1/2)/a^(1/2))+((192/
5*B*x+32*A)*a^(11/2)+(-5/4*b^3*x^3*(18/5*B*x+A)*a^(3/2)+b^2*x^2*(3*B*x+A)*
a^(5/2)+54*b*(62/45*B*x+A)*x*a^(7/2)+(504/5*B*x+80*A)*a^(9/2)+15/8*A*a^(1/
2)*b^4*x^4)*b*x)*(b*x+a)^(1/2))/a^(7/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx = \left[-\frac{15\,(12\,Bab^5-5\,Ab^6)\sqrt{a}x^6\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(1280\,Aa^6-15\,(12\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^7,x, algorithm="fricas")
```

output
```
[-1/15360*(15*(12*B*a*b^5 - 5*A*b^6)*sqrt(a)*x^6*log((b*x + 2*sqrt(b*x + a
)*sqrt(a) + 2*a)/x) + 2*(1280*A*a^6 - 15*(12*B*a^2*b^4 - 5*A*a*b^5)*x^5 +
10*(12*B*a^3*b^3 - 5*A*a^2*b^4)*x^4 + 8*(372*B*a^4*b^2 + 5*A*a^3*b^3)*x^3
+ 144*(28*B*a^5*b + 15*A*a^4*b^2)*x^2 + 128*(12*B*a^6 + 25*A*a^5*b)*x)*sqr
t(b*x + a))/(a^4*x^6), 1/7680*(15*(12*B*a*b^5 - 5*A*b^6)*sqrt(-a)*x^6*arct
an(sqrt(-a)/sqrt(b*x + a)) - (1280*A*a^6 - 15*(12*B*a^2*b^4 - 5*A*a*b^5)*x
^5 + 10*(12*B*a^3*b^3 - 5*A*a^2*b^4)*x^4 + 8*(372*B*a^4*b^2 + 5*A*a^3*b^3)
*x^3 + 144*(28*B*a^5*b + 15*A*a^4*b^2)*x^2 + 128*(12*B*a^6 + 25*A*a^5*b)*x
)*sqrt(b*x + a))/(a^4*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx = \frac{1}{15360}\,b^6\left(\frac{2\left(15\,(12\,Ba-5\,Ab)(bx+a)^{\frac{11}{2}}-85\,(12\,Ba^2-5\,Aab)(bx+a)^{\frac{9}{2}}-\right.}{(bx+a)^6a^3b-6\,(}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^7,x, algorithm="maxima")
```

output
```
1/15360*b^6*(2*(15*(12*B*a - 5*A*b)*(b*x + a)^(11/2) - 85*(12*B*a^2 - 5*A*
a*b)*(b*x + a)^(9/2) - 6*(116*B*a^3 + 165*A*a^2*b)*(b*x + a)^(7/2) + 198*(
12*B*a^4 - 5*A*a^3*b)*(b*x + a)^(5/2) - 85*(12*B*a^5 - 5*A*a^4*b)*(b*x + a
)^(3/2) + 15*(12*B*a^6 - 5*A*a^5*b)*sqrt(b*x + a))/((b*x + a)^6*a^3*b - 6*
(b*x + a)^5*a^4*b + 15*(b*x + a)^4*a^5*b - 20*(b*x + a)^3*a^6*b + 15*(b*x
+ a)^2*a^7*b - 6*(b*x + a)*a^8*b + a^9*b) + 15*(12*B*a - 5*A*b)*log((sqrt(
b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(7/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx = \frac{\frac{15\,(12\,Bab^6-5\,Ab^7)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{180\,(bx+a)^{\frac{11}{2}}Bab^6-1020\,(bx+a)^{\frac{9}{2}}Ba^2b^6-696\,(bx+a)^{\frac{7}{2}}Ba^3b^6}{}}{}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^7,x, algorithm="giac")
```

output
```
1/7680*(15*(12*B*a*b^6 - 5*A*b^7)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)
*a^3) + (180*(b*x + a)^(11/2)*B*a*b^6 - 1020*(b*x + a)^(9/2)*B*a^2*b^6 - 6
96*(b*x + a)^(7/2)*B*a^3*b^6 + 2376*(b*x + a)^(5/2)*B*a^4*b^6 - 1020*(b*x
+ a)^(3/2)*B*a^5*b^6 + 180*sqrt(b*x + a)*B*a^6*b^6 - 75*(b*x + a)^(11/2)*A
*b^7 + 425*(b*x + a)^(9/2)*A*a*b^7 - 990*(b*x + a)^(7/2)*A*a^2*b^7 - 990*(
b*x + a)^(5/2)*A*a^3*b^7 + 425*(b*x + a)^(3/2)*A*a^4*b^7 - 75*sqrt(b*x + a
)*A*a^5*b^7)/(a^3*b^6*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx = \frac{b^5\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(5\,A\,b-12\,B\,a)}{512\,a^{7/2}} - \frac{\left(\frac{33\,A\,b^6}{256}-\frac{99\,B\,a\,b^5}{320}\right)\,(a+b\,x)^{5/2}+\left(\frac{5\,A\,a^2\,b^6}{512}-\frac{3\,B\,a^3\,b^5}{128}\right)\,\sqrt{a+b\,x}+\left(\frac{17\,B\,a^2\,b^5}{128}-\frac{85\,A\,a\,b^6}{1536}\right)\,(a+b\,x)^{3/2}-\frac{17\,(}{}}{(a+b\,x)^6-6\,a^5\,(a+b\,x)-6\,a\,(a+b\,x)^5+15\,a^2\,(a+b\,x)^4-20}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^7,x)
```

output
```
(b^5*atanh((a + b*x)^(1/2)/a^(1/2))*(5*A*b - 12*B*a))/(512*a^(7/2)) - (((3
3*A*b^6)/256 - (99*B*a*b^5)/320)*(a + b*x)^(5/2) + ((5*A*a^2*b^6)/512 - (3
*B*a^3*b^5)/128)*(a + b*x)^(1/2) + ((17*B*a^2*b^5)/128 - (85*A*a*b^6)/1536
)*(a + b*x)^(3/2) - (17*(5*A*b^6 - 12*B*a*b^5)*(a + b*x)^(9/2))/(1536*a^2)
 + ((5*A*b^6 - 12*B*a*b^5)*(a + b*x)^(11/2))/(512*a^3) + ((165*A*b^6 + 116
*B*a*b^5)*(a + b*x)^(7/2))/(1280*a))/((a + b*x)^6 - 6*a^5*(a + b*x) - 6*a*
(a + b*x)^5 + 15*a^2*(a + b*x)^4 - 20*a^3*(a + b*x)^3 + 15*a^4*(a + b*x)^2
 + a^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^7}\,dx = \frac{-2560\sqrt{bx+a}\,a^6-9472\sqrt{bx+a}\,a^5bx-12384\sqrt{bx+a}\,a^4b^2x^2-6032\sqrt{bx+}}{}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^7,x)
```

output
```
( - 2560*sqrt(a + b*x)*a**6 - 9472*sqrt(a + b*x)*a**5*b*x - 12384*sqrt(a +
 b*x)*a**4*b**2*x**2 - 6032*sqrt(a + b*x)*a**3*b**3*x**3 - 140*sqrt(a + b*
x)*a**2*b**4*x**4 + 210*sqrt(a + b*x)*a*b**5*x**5 + 105*sqrt(a)*log(sqrt(a
 + b*x) - sqrt(a))*b**6*x**6 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*
*6*x**6)/(15360*a**3*x**6)
```

3.257 $\int \frac{x^4(A+Bx)}{\sqrt{a+bx}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 149

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}} \, dx = \frac{2a^4(Ab-aB)\sqrt{a+bx}}{b^6} - \frac{2a^3(4Ab-5aB)(a+bx)^{3/2}}{3b^6} + \frac{4a^2(3Ab-5aB)(a+bx)^{5/2}}{5b^6} - \frac{4a(2Ab-5aB)(a+bx)^{7/2}}{7b^6} + \frac{2(Ab-5aB)(a+bx)^{9/2}}{9b^6} + \frac{2B(a+bx)^{11/2}}{11b^6}$$

output
```
2*a^4*(A*b-B*a)*(b*x+a)^(1/2)/b^6-2/3*a^3*(4*A*b-5*B*a)*(b*x+a)^(3/2)/b^6+
4/5*a^2*(3*A*b-5*B*a)*(b*x+a)^(5/2)/b^6-4/7*a*(2*A*b-5*B*a)*(b*x+a)^(7/2)/
b^6+2/9*(A*b-5*B*a)*(b*x+a)^(9/2)/b^6+2/11*B*(b*x+a)^(11/2)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}} \, dx = \frac{2\sqrt{a+bx}(-1280a^5B + 128a^4b(11A+5Bx) + 35b^5x^4(11A+9Bx) - 32a^3b^2x(22A+15Bx) + 16a^2b^3x^2}{3465b^6}$$

input
```
Integrate[(x^4*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-1280*a^5*B + 128*a^4*b*(11*A + 5*B*x) + 35*b^5*x^4*(11*
A + 9*B*x) - 32*a^3*b^2*x*(22*A + 15*B*x) + 16*a^2*b^3*x^2*(33*A + 25*B*x)
 - 10*a*b^4*x^3*(44*A + 35*B*x)))/(3465*b^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5\sqrt{a+bx}}+\frac{a^3\sqrt{a+bx}(5aB-4Ab)}{b^5}-\frac{2a^2(a+bx)^{3/2}(5aB-3Ab)}{b^5}+\frac{(a+bx)^{7/2}(Ab-5aB)}{b^5}+\frac{2a(a\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sqrt{a+bx}(Ab-aB)}{b^6}-\frac{2a^3(a+bx)^{3/2}(4Ab-5aB)}{3b^6}+\frac{4a^2(a+bx)^{5/2}(3Ab-5aB)}{5b^6}+\frac{2(a+bx)^{9/2}(Ab-5aB)}{9b^6}-\frac{4a(a+bx)^{7/2}(2Ab-5aB)}{7b^6}+\frac{2B(a+bx)^{11/2}}{11b^6}$$

input
```
Int[(x^4*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(2*a^4*(A*b - a*B)*Sqrt[a + b*x])/b^6 - (2*a^3*(4*A*b - 5*a*B)*(a + b*x)^(
3/2))/(3*b^6) + (4*a^2*(3*A*b - 5*a*B)*(a + b*x)^(5/2))/(5*b^6) - (4*a*(2*
A*b - 5*a*B)*(a + b*x)^(7/2))/(7*b^6) + (2*(A*b - 5*a*B)*(a + b*x)^(9/2))/
(9*b^6) + (2*B*(a + b*x)^(11/2))/(11*b^6)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{256\left(\frac{35\left(\frac{9Bx}{11}+A\right)x^4b^5}{128}-\frac{5a\left(\frac{35Bx}{44}+A\right)x^3b^4}{16}+\frac{3a^2\left(\frac{25Bx}{33}+A\right)x^2b^3}{8}-\frac{a^3\left(\frac{15Bx}{22}+A\right)x\,b^2}{2}+a^4\left(\frac{5Bx}{11}+A\right)b-\frac{10a^5B}{11}\right)\sqrt{bx+a}}{315b^6}$
gosper	$\frac{2\sqrt{bx+a}\,(315b^5Bx^5+385Ab^5x^4-350Bab^4x^4-440Aab^4x^3+400Ba^2b^3x^3+528Aa^2b^3x^2-480Ba^3b^2x^2-704a^3b^2Ax\cdots}{3465b^6}$
trager	$\frac{2\sqrt{bx+a}\,(315b^5Bx^5+385Ab^5x^4-350Bab^4x^4-440Aab^4x^3+400Ba^2b^3x^3+528Aa^2b^3x^2-480Ba^3b^2x^2-704a^3b^2Ax\cdots}{3465b^6}$
risch	$\frac{2\sqrt{bx+a}\,(315b^5Bx^5+385Ab^5x^4-350Bab^4x^4-440Aab^4x^3+400Ba^2b^3x^3+528Aa^2b^3x^2-480Ba^3b^2x^2-704a^3b^2Ax\cdots}{3465b^6}$
orering	$\frac{2\sqrt{bx+a}\,(315b^5Bx^5+385Ab^5x^4-350Bab^4x^4-440Aab^4x^3+400Ba^2b^3x^3+528Aa^2b^3x^2-480Ba^3b^2x^2-704a^3b^2Ax\cdots}{3465b^6}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-5Ba)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(6a^2B-4a(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(-4a^3B+6a^2(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{2(Ba^4-4a^3(Ab-\cdots}{3}}{b^6}$
default	$\frac{\frac{2B(bx+a)^{\frac{11}{2}}}{11}+\frac{2(Ab-5Ba)(bx+a)^{\frac{9}{2}}}{9}+\frac{2\left(6a^2B-4a(Ab-Ba)\right)(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(-4a^3B+6a^2(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{2(Ba^4-4a^3(Ab-\cdots}{3}}{b^6}$

input
```
int(x^4*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
256/315*(35/128*(9/11*B*x+A)*x^4*b^5-5/16*a*(35/44*B*x+A)*x^3*b^4+3/8*a^2*
(25/33*B*x+A)*x^2*b^3-1/2*a^3*(15/22*B*x+A)*x*b^2+a^4*(5/11*B*x+A)*b-10/11
*a^5*B)*(b*x+a)^(1/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\,(315\,Bb^5x^5 - 1280\,Ba^5 + 1408\,Aa^4b - 35\,(10\,Bab^4 - 11\,Ab^5)x^4 + 40\,(10\,Ba^2b^3 - 11\,Aab^4)x^3 - 48\,(10}{3465\,b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*b^5*x^5 - 1280*B*a^5 + 1408*A*a^4*b - 35*(10*B*a*b^4 - 11*A*
b^5)*x^4 + 40*(10*B*a^2*b^3 - 11*A*a*b^4)*x^3 - 48*(10*B*a^3*b^2 - 11*A*a^
2*b^3)*x^2 + 64*(10*B*a^4*b - 11*A*a^3*b^2)*x)*sqrt(b*x + a)/b^6
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.12

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{11}{2}}}{11b} + \frac{(a+bx)^{\frac{9}{2}}(Ab-5Ba)}{9b} + \frac{(a+bx)^{\frac{7}{2}}\left(-4Aab+10Ba^2\right)}{7b} + \frac{(a+bx)^{\frac{5}{2}}\cdot\left(6Aa^2b-10Ba^3\right)}{5b} + \frac{(a+bx)^{\frac{3}{2}}\left(-4Aa^3b+5Ba^4\right)}{3b} + \frac{\sqrt{a+bx}\left(Aa^4b-Ba^5\right)}{b}\right)}{b^5} \\ \frac{\frac{Ax^5}{5}+\frac{Bx^6}{6}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**4*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(11/2)/(11*b) + (a + b*x)**(9/2)*(A*b - 5*B*a)/
(9*b) + (a + b*x)**(7/2)*(-4*A*a*b + 10*B*a**2)/(7*b) + (a + b*x)**(5/2)*(
6*A*a**2*b - 10*B*a**3)/(5*b) + (a + b*x)**(3/2)*(-4*A*a**3*b + 5*B*a**4)/
(3*b) + sqrt(a + b*x)*(A*a**4*b - B*a**5)/b)/b**5, Ne(b, 0)), ((A*x**5/5 +
 B*x**6/6)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(315\,(bx+a)^{\frac{11}{2}}B-385\,(5\,Ba-Ab)(bx+a)^{\frac{9}{2}}+990\,(5\,Ba^2-2\,Aab)(bx+a)^{\frac{7}{2}}-1386\,(5\,Ba^3-3\,Aa^2\right.}{3465\,b^6}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(b*x + a)^(11/2)*B - 385*(5*B*a - A*b)*(b*x + a)^(9/2) + 990*(
5*B*a^2 - 2*A*a*b)*(b*x + a)^(7/2) - 1386*(5*B*a^3 - 3*A*a^2*b)*(b*x + a)^
(5/2) + 1155*(5*B*a^4 - 4*A*a^3*b)*(b*x + a)^(3/2) - 3465*(B*a^5 - A*a^4*b
)*sqrt(b*x + a))/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(\frac{11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\right)A}{b^4}+\frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a\right.}{}\right.}{3465\,b}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(11*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/
2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A/b^4 + 5*(63*(b
*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*B/b^
5)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{(20\,B\,a^2-8\,A\,a\,b)\,(a+b\,x)^{7/2}}{7\,b^6}+\frac{2\,B\,(a+b\,x)^{11/2}}{11\,b^6}+\frac{(2\,A\,b-10\,B\,a)\,(a+b\,x)^{9/2}}{9\,b^6}-\frac{(2\,B\,a^5-2\,A\,a^4\,b)\,\sqrt{a+b\,x}}{b^6}+\frac{(10\,B\,a^4-8\,A\,a^3\,b)\,(a+b\,x)^{3/2}}{3\,b^6}-\frac{(20\,B\,a^3-12\,A\,a^2\,b)\,(a+b\,x)^{5/2}}{5\,b^6}$$

input
```
int((x^4*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
((20*B*a^2 - 8*A*a*b)*(a + b*x)^(7/2))/(7*b^6) + (2*B*(a + b*x)^(11/2))/(1
1*b^6) + ((2*A*b - 10*B*a)*(a + b*x)^(9/2))/(9*b^6) - ((2*B*a^5 - 2*A*a^4*
b)*(a + b*x)^(1/2))/b^6 + ((10*B*a^4 - 8*A*a^3*b)*(a + b*x)^(3/2))/(3*b^6)
 - ((20*B*a^3 - 12*A*a^2*b)*(a + b*x)^(5/2))/(5*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.42

$$\int \frac{x^4(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(315b^5x^5+35a\,b^4x^4-40a^2b^3x^3+48a^3b^2x^2-64a^4bx+128a^5)}{3465b^5}$$

input
```
int(x^4*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**5 - 64*a**4*b*x + 48*a**3*b**2*x**2 - 40*a**2*b**
3*x**3 + 35*a*b**4*x**4 + 315*b**5*x**5))/(3465*b**5)
```

3.258 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}} \, dx = -\frac{2a^3(Ab-aB)\sqrt{a+bx}}{b^5} + \frac{2a^2(3Ab-4aB)(a+bx)^{3/2}}{3b^5} - \frac{6a(Ab-2aB)(a+bx)^{5/2}}{5b^5} + \frac{2(Ab-4aB)(a+bx)^{7/2}}{7b^5} + \frac{2B(a+bx)^{9/2}}{9b^5}$$

output
```
-2*a^3*(A*b-B*a)*(b*x+a)^(1/2)/b^5+2/3*a^2*(3*A*b-4*B*a)*(b*x+a)^(3/2)/b^5
-6/5*a*(A*b-2*B*a)*(b*x+a)^(5/2)/b^5+2/7*(A*b-4*B*a)*(b*x+a)^(7/2)/b^5+2/9
*B*(b*x+a)^(9/2)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}} \, dx = \frac{2\sqrt{a+bx}(128a^4B + 24a^2b^2x(3A+2Bx) - 16a^3b(9A+4Bx) + 5b^4x^3(9A+7Bx) - 2ab^3x^2(27A+20E}{315b^5}$$

input
```
Integrate[(x^3*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(128*a^4*B + 24*a^2*b^2*x*(3*A + 2*B*x) - 16*a^3*b*(9*A +
 4*B*x) + 5*b^4*x^3*(9*A + 7*B*x) - 2*a*b^3*x^2*(27*A + 20*B*x)))/(315*b^5
)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4\sqrt{a+bx}}-\frac{a^2\sqrt{a+bx}(4aB-3Ab)}{b^4}+\frac{(a+bx)^{5/2}(Ab-4aB)}{b^4}+\frac{3a(a+bx)^{3/2}(2aB-Ab)}{b^4}+\frac{B(a+bx}{b^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3\sqrt{a+bx}(Ab-aB)}{b^5}+\frac{2a^2(a+bx)^{3/2}(3Ab-4aB)}{3b^5}+\frac{2(a+bx)^{7/2}(Ab-4aB)}{7b^5}-\frac{6a(a+bx)^{5/2}(Ab-2aB)}{5b^5}+\frac{2B(a+bx)^{9/2}}{9b^5}$$

input
```
Int[(x^3*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(-2*a^3*(A*b - a*B)*Sqrt[a + b*x])/b^5 + (2*a^2*(3*A*b - 4*a*B)*(a + b*x)^
(3/2))/(3*b^5) - (6*a*(A*b - 2*a*B)*(a + b*x)^(5/2))/(5*b^5) + (2*(A*b - 4
*a*B)*(a + b*x)^(7/2))/(7*b^5) + (2*B*(a + b*x)^(9/2))/(9*b^5)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{32\left(-\frac{5\left(\frac{7Bx}{9}+A\right)x^{3}b^{4}}{16}+\frac{3a\left(\frac{20Bx}{27}+A\right)x^{2}b^{3}}{8}-\frac{\left(\frac{2Bx}{3}+A\right)a^{2}x\,b^{2}}{2}+a^{3}\left(\frac{4Bx}{9}+A\right)b-\frac{8B\,a^{4}}{9}\right)\sqrt{bx+a}}{35b^{5}}$
gosper	$-\frac{2\sqrt{bx+a}\,(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}+40B\,x^{3}a\,b^{3}+54A\,x^{2}a\,b^{3}-48B\,x^{2}a^{2}b^{2}-72Ax\,a^{2}b^{2}+64Bx\,a^{3}b+144A\,a^{3}b-128B\,a}{315b^{5}}$
trager	$-\frac{2\sqrt{bx+a}\,(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}+40B\,x^{3}a\,b^{3}+54A\,x^{2}a\,b^{3}-48B\,x^{2}a^{2}b^{2}-72Ax\,a^{2}b^{2}+64Bx\,a^{3}b+144A\,a^{3}b-128B\,a}{315b^{5}}$
risch	$-\frac{2\sqrt{bx+a}\,(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}+40B\,x^{3}a\,b^{3}+54A\,x^{2}a\,b^{3}-48B\,x^{2}a^{2}b^{2}-72Ax\,a^{2}b^{2}+64Bx\,a^{3}b+144A\,a^{3}b-128B\,a}{315b^{5}}$
orering	$-\frac{2\sqrt{bx+a}\,(-35B\,x^{4}b^{4}-45A\,x^{3}b^{4}+40B\,x^{3}a\,b^{3}+54A\,x^{2}a\,b^{3}-48B\,x^{2}a^{2}b^{2}-72Ax\,a^{2}b^{2}+64Bx\,a^{3}b+144A\,a^{3}b-128B\,a}{315b^{5}}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2(Ab-4Ba)(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left(3a^{2}B-3a(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}+\frac{2\left(-a^{3}B+3a^{2}(Ab-Ba)\right)(bx+a)^{\frac{3}{2}}}{3}-2a^{3}(Ab-Ba)\sqrt{bx+}}{b^{5}}$
default	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}-\frac{2(-Ab+4Ba)(bx+a)^{\frac{7}{2}}}{7}-\frac{2\left(-3a^{2}B+3a(Ab-Ba)\right)(bx+a)^{\frac{5}{2}}}{5}-\frac{2\left(a^{3}B-3a^{2}(Ab-Ba)\right)(bx+a)^{\frac{3}{2}}}{3}-2a^{3}(Ab-Ba)\sqrt{bx}}{b^{5}}$

input
```
int(x^3*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-32/35*(-5/16*(7/9*B*x+A)*x^3*b^4+3/8*a*(20/27*B*x+A)*x^2*b^3-1/2*(2/3*B*x
+A)*a^2*x*b^2+a^3*(4/9*B*x+A)*b-8/9*B*a^4)*(b*x+a)^(1/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\,(35\,Bb^4x^4+128\,Ba^4-144\,Aa^3b-5\,(8\,Bab^3-9\,Ab^4)x^3+6\,(8\,Ba^2b^2-9\,Aab^3)x^2-8\,(8\,Ba^3b-9\,Aa}{315\,b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^4*x^4 + 128*B*a^4 - 144*A*a^3*b - 5*(8*B*a*b^3 - 9*A*b^4)*x^
3 + 6*(8*B*a^2*b^2 - 9*A*a*b^3)*x^2 - 8*(8*B*a^3*b - 9*A*a^2*b^2)*x)*sqrt(
b*x + a)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{9}{2}}}{9b}+\frac{(a+bx)^{\frac{7}{2}}(Ab-4Ba)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\left(-3Aab+6Ba^2\right)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(3Aa^2b-4Ba^3\right)}{3b}+\frac{\sqrt{a+bx}\left(-Aa^3b+Ba^4\right)}{b}\right)}{b^4} & \text{for } b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(9/2)/(9*b) + (a + b*x)**(7/2)*(A*b - 4*B*a)/(7
*b) + (a + b*x)**(5/2)*(-3*A*a*b + 6*B*a**2)/(5*b) + (a + b*x)**(3/2)*(3*A
*a**2*b - 4*B*a**3)/(3*b) + sqrt(a + b*x)*(-A*a**3*b + B*a**4)/b)/b**4, Ne
(b, 0)), ((A*x**4/4 + B*x**5/5)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(35\,(bx+a)^{\frac{9}{2}}B-45\,(4\,Ba-Ab)(bx+a)^{\frac{7}{2}}+189\,(2\,Ba^2-Aab)(bx+a)^{\frac{5}{2}}-105\,(4\,Ba^3-3\,Aa^2b)(bx\right.}{315\,b^5}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(b*x + a)^(9/2)*B - 45*(4*B*a - A*b)*(b*x + a)^(7/2) + 189*(2*B*
a^2 - A*a*b)*(b*x + a)^(5/2) - 105*(4*B*a^3 - 3*A*a^2*b)*(b*x + a)^(3/2) +
 315*(B*a^4 - A*a^3*b)*sqrt(b*x + a))/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)A}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{}\right.}{b^4}\right.}{315\,b}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)*A/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2
)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a
)*a^4)*B/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{(12\,B\,a^2-6\,A\,a\,b)\,(a+b\,x)^{5/2}}{5\,b^5}+\frac{2\,B\,(a+b\,x)^{9/2}}{9\,b^5}+\frac{(2\,A\,b-8\,B\,a)\,(a+b\,x)^{7/2}}{7\,b^5}+\frac{(2\,B\,a^4-2\,A\,a^3\,b)\,\sqrt{a+b\,x}}{b^5}-\frac{(8\,B\,a^3-6\,A\,a^2\,b)\,(a+b\,x)^{3/2}}{3\,b^5}$$

input
```
int((x^3*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
((12*B*a^2 - 6*A*a*b)*(a + b*x)^(5/2))/(5*b^5) + (2*B*(a + b*x)^(9/2))/(9*
b^5) + ((2*A*b - 8*B*a)*(a + b*x)^(7/2))/(7*b^5) + ((2*B*a^4 - 2*A*a^3*b)*
(a + b*x)^(1/2))/b^5 - ((8*B*a^3 - 6*A*a^2*b)*(a + b*x)^(3/2))/(3*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4+5a\,b^3x^3-6a^2b^2x^2+8a^3bx-16a^4)}{315b^4}$$

input
```
int(x^3*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**4 + 8*a**3*b*x - 6*a**2*b**2*x**2 + 5*a*b**3*x*
*3 + 35*b**4*x**4))/(315*b**4)
```

3.259 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 93

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}} \, dx = \frac{2a^2(Ab-aB)\sqrt{a+bx}}{b^4} - \frac{2a(2Ab-3aB)(a+bx)^{3/2}}{3b^4} + \frac{2(Ab-3aB)(a+bx)^{5/2}}{5b^4} + \frac{2B(a+bx)^{7/2}}{7b^4}$$

output
```
2*a^2*(A*b-B*a)*(b*x+a)^(1/2)/b^4-2/3*a*(2*A*b-3*B*a)*(b*x+a)^(3/2)/b^4+2/
5*(A*b-3*B*a)*(b*x+a)^(5/2)/b^4+2/7*B*(b*x+a)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}} \, dx = \frac{2\sqrt{a+bx}(-48a^3B + 8a^2b(7A+3Bx) + 3b^3x^2(7A+5Bx) - 2ab^2x(14A+9Bx))}{105b^4}$$

input
```
Integrate[(x^2*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x) + 3*b^3*x^2*(7*A + 5*B
*x) - 2*a*b^2*x*(14*A + 9*B*x)))/(105*b^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3\sqrt{a+bx}}+\frac{(a+bx)^{3/2}(Ab-3aB)}{b^3}+\frac{a\sqrt{a+bx}(3aB-2Ab)}{b^3}+\frac{B(a+bx)^{5/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^2\sqrt{a+bx}(Ab-aB)}{b^4}+\frac{2(a+bx)^{5/2}(Ab-3aB)}{5b^4}-\frac{2a(a+bx)^{3/2}(2Ab-3aB)}{3b^4}+\frac{2B(a+bx)^{7/2}}{7b^4}$$

input
```
Int[(x^2*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(2*a^2*(A*b - a*B)*Sqrt[a + b*x])/b^4 - (2*a*(2*A*b - 3*a*B)*(a + b*x)^(3/
2))/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x)^(5/2))/(5*b^4) + (2*B*(a + b*x)^(
7/2))/(7*b^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$\frac{16\left(\frac{3\left(\frac{5Bx}{7}+A\right)x^{2}b^{3}}{8}-\frac{a\left(\frac{9Bx}{14}+A\right)x\,b^{2}}{2}+a^{2}\left(\frac{3Bx}{7}+A\right)b-\frac{6a^{3}B}{7}\right)\sqrt{bx+a}}{15b^{4}}$	58
gosper	$\frac{2\sqrt{bx+a}\,\left(15b^{3}B\,x^{3}+21A\,x^{2}b^{3}-18B\,x^{2}a\,b^{2}-28Axa\,b^{2}+24Bx\,a^{2}b+56a^{2}bA-48a^{3}B\right)}{105b^{4}}$	71
trager	$\frac{2\sqrt{bx+a}\,\left(15b^{3}B\,x^{3}+21A\,x^{2}b^{3}-18B\,x^{2}a\,b^{2}-28Axa\,b^{2}+24Bx\,a^{2}b+56a^{2}bA-48a^{3}B\right)}{105b^{4}}$	71
risch	$\frac{2\sqrt{bx+a}\,\left(15b^{3}B\,x^{3}+21A\,x^{2}b^{3}-18B\,x^{2}a\,b^{2}-28Axa\,b^{2}+24Bx\,a^{2}b+56a^{2}bA-48a^{3}B\right)}{105b^{4}}$	71
orering	$\frac{2\sqrt{bx+a}\,\left(15b^{3}B\,x^{3}+21A\,x^{2}b^{3}-18B\,x^{2}a\,b^{2}-28Axa\,b^{2}+24Bx\,a^{2}b+56a^{2}bA-48a^{3}B\right)}{105b^{4}}$	71
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2(Ab-3Ba)(bx+a)^{\frac{5}{2}}}{5}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{3}{2}}}{3}+2a^{2}(Ab-Ba)\sqrt{bx+a}}{b^{4}}$	79
default	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2(Ab-3Ba)(bx+a)^{\frac{5}{2}}}{5}+\frac{2\left(a^{2}B-2a(Ab-Ba)\right)(bx+a)^{\frac{3}{2}}}{3}+2a^{2}(Ab-Ba)\sqrt{bx+a}}{b^{4}}$	79

input
```
int(x^2*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/15*(3/8*(5/7*B*x+A)*x^2*b^3-1/2*a*(9/14*B*x+A)*x*b^2+a^2*(3/7*B*x+A)*b-
6/7*a^3*B)*(b*x+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\,(15\,Bb^3x^3-48\,Ba^3+56\,Aa^2b-3\,(6\,Bab^2-7\,Ab^3)x^2+4\,(6\,Ba^2b-7\,Aab^2)x)\sqrt{bx+a}}{105\,b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^3*x^3 - 48*B*a^3 + 56*A*a^2*b - 3*(6*B*a*b^2 - 7*A*b^3)*x^2
+ 4*(6*B*a^2*b - 7*A*a*b^2)*x)*sqrt(b*x + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{7}{2}}}{7b}+\frac{(a+bx)^{\frac{5}{2}}(Ab-3Ba)}{5b}+\frac{(a+bx)^{\frac{3}{2}}(-2Aab+3Ba^2)}{3b}+\frac{\sqrt{a+bx}(Aa^2b-Ba^3)}{b}\right)}{b^3} & \text{for } b\neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(7/2)/(7*b) + (a + b*x)**(5/2)*(A*b - 3*B*a)/(5
*b) + (a + b*x)**(3/2)*(-2*A*a*b + 3*B*a**2)/(3*b) + sqrt(a + b*x)*(A*a**2
*b - B*a**3)/b)/b**3, Ne(b, 0)), ((A*x**3/3 + B*x**4/4)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(15\,(bx+a)^{\frac{7}{2}}B-21\,(3\,Ba-Ab)(bx+a)^{\frac{5}{2}}+35\,(3\,Ba^2-2\,Aab)(bx+a)^{\frac{3}{2}}-105\,(Ba^3-Aa^2b)\sqrt{bx+a}\right)}{105\,b^4}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(b*x + a)^(7/2)*B - 21*(3*B*a - A*b)*(b*x + a)^(5/2) + 35*(3*B*a
^2 - 2*A*a*b)*(b*x + a)^(3/2) - 105*(B*a^3 - A*a^2*b)*sqrt(b*x + a))/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)A}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)B}{b^3}\right)}{105\,b}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*A/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)*B/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\left(6\,B\,a^2-4\,A\,a\,b\right)\,{\left(a+b\,x\right)}^{3/2}}{3\,b^4}+\frac{2\,B\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4} + \frac{\left(2\,A\,b-6\,B\,a\right)\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}-\frac{\left(2\,B\,a^3-2\,A\,a^2\,b\right)\,\sqrt{a+b\,x}}{b^4}$$

input
```
int((x^2*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
((6*B*a^2 - 4*A*a*b)*(a + b*x)^(3/2))/(3*b^4) + (2*B*(a + b*x)^(7/2))/(7*b
^4) + ((2*A*b - 6*B*a)*(a + b*x)^(5/2))/(5*b^4) - ((2*B*a^3 - 2*A*a^2*b)*(
a + b*x)^(1/2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105b^3}$$

input
```
int(x^2*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**3 - 4*a**2*b*x + 3*a*b**2*x**2 + 15*b**3*x**3))/(10
5*b**3)
```

3.260 $\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx = -\frac{2a(Ab-aB)\sqrt{a+bx}}{b^3}+\frac{2(Ab-2aB)(a+bx)^{3/2}}{3b^3}+\frac{2B(a+bx)^{5/2}}{5b^3}$$

output
```
-2*a*(A*b-B*a)*(b*x+a)^(1/2)/b^3+2/3*(A*b-2*B*a)*(b*x+a)^(3/2)/b^3+2/5*B*(
b*x+a)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(8a^2B-2ab(5A+2Bx)+b^2x(5A+3Bx))}{15b^3}$$

input
```
Integrate[(x*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(8*a^2*B - 2*a*b*(5*A + 2*B*x) + b^2*x*(5*A + 3*B*x)))/(1
5*b^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{\sqrt{a+bx}(Ab-2aB)}{b^2}+\frac{a(aB-Ab)}{b^2\sqrt{a+bx}}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(a+bx)^{3/2}(Ab-2aB)}{3b^3}-\frac{2a\sqrt{a+bx}(Ab-aB)}{b^3}+\frac{2B(a+bx)^{5/2}}{5b^3}$$

input
```
Int[(x*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(-2*a*(A*b - a*B)*Sqrt[a + b*x])/b^3 + (2*(A*b - 2*a*B)*(a + b*x)^(3/2))/(
3*b^3) + (2*B*(a + b*x)^(5/2))/(5*b^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$-\frac{4\left(-\frac{\left(\frac{3Bx}{5}+A\right)x\,b^{2}}{2}+a\left(\frac{2Bx}{5}+A\right)b-\frac{4a^{2}B}{5}\right)\sqrt{bx+a}}{3b^{3}}$	41
gosper	$-\frac{2\sqrt{bx+a}\,\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+4Babx+10abA-8a^{2}B\right)}{15b^{3}}$	47
trager	$-\frac{2\sqrt{bx+a}\,\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+4Babx+10abA-8a^{2}B\right)}{15b^{3}}$	47
risch	$-\frac{2\sqrt{bx+a}\,\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+4Babx+10abA-8a^{2}B\right)}{15b^{3}}$	47
orering	$-\frac{2\sqrt{bx+a}\,\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+4Babx+10abA-8a^{2}B\right)}{15b^{3}}$	47
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2(Ab-2Ba)(bx+a)^{\frac{3}{2}}}{3}-2a(Ab-Ba)\sqrt{bx+a}}{b^{3}}$	52
default	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2(Ab-2Ba)(bx+a)^{\frac{3}{2}}}{3}-2a(Ab-Ba)\sqrt{bx+a}}{b^{3}}$	52

input
```
int(x*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-1/2*(3/5*B*x+A)*x*b^2+a*(2/5*B*x+A)*b-4/5*a^2*B)*(b*x+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx=\frac{2\left(3\,Bb^{2}x^{2}+8\,Ba^{2}-10\,Aab-(4\,Bab-5\,Ab^{2})x\right)\sqrt{bx+a}}{15\,b^{3}}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^2*x^2 + 8*B*a^2 - 10*A*a*b - (4*B*a*b - 5*A*b^2)*x)*sqrt(b*x +
 a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{5}{2}}}{5b}+\frac{(a+bx)^{\frac{3}{2}}(Ab-2Ba)}{3b}+\frac{\sqrt{a+bx}\left(-Aab+Ba^2\right)}{b}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(5/2)/(5*b) + (a + b*x)**(3/2)*(A*b - 2*B*a)/(3
*b) + sqrt(a + b*x)*(-A*a*b + B*a**2)/b)/b**2, Ne(b, 0)), ((A*x**2/2 + B*x
**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{2\left(3\,(bx+a)^{\frac{5}{2}}B-5\,(2\,Ba-Ab)(bx+a)^{\frac{3}{2}}+15\,(Ba^2-Aab)\sqrt{bx+a}\right)}{15\,b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*(b*x + a)^(5/2)*B - 5*(2*B*a - A*b)*(b*x + a)^(3/2) + 15*(B*a^2 -
A*a*b)*sqrt(b*x + a))/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx=\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)A}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^{2}\right)B}{b^{2}}\right)}{15\,b}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*A/b + (3*(b*x + a)^(5/2) - 1
0*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{a+b\,x}\,\left(15\,B\,a^{2}+3\,B\,{(a+b\,x)}^{2}-15\,A\,a\,b+5\,A\,b\,(a+b\,x)-10\,B\,a\,(a+b\,x)\right)}{15\,b^{3}}$$

input
```
int((x*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(1/2)*(15*B*a^2 + 3*B*(a + b*x)^2 - 15*A*a*b + 5*A*b*(a + b*x
) - 10*B*a*(a + b*x)))/(15*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{bx+a}\,(3b^{2}x^{2}+abx-2a^{2})}{15b^{2}}$$

input
```
int(x*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.261 $\int \frac{A+Bx}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx=\frac{2(Ab-aB)\sqrt{a+bx}}{b^2}+\frac{2B(a+bx)^{3/2}}{3b^2}$$

output
```
2*(A*b-B*a)*(b*x+a)^(1/2)/b^2+2/3*B*(b*x+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx=\frac{2\sqrt{a+bx}(3Ab-2aB+bBx)}{3b^2}$$

input
```
Integrate[(A + B*x)/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(3*A*b - 2*a*B + b*B*x))/(3*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2\sqrt{a+bx}(Ab-aB)}{b^2}+\frac{2B(a+bx)^{3/2}}{3b^2}$$

input
```
Int[(A + B*x)/Sqrt[a + b*x],x]
```

output
```
(2*(A*b - a*B)*Sqrt[a + b*x])/b^2 + (2*B*(a + b*x)^(3/2))/(3*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(bBx+3Ab-2Ba)}{3b^{2}}$	26
trager	$\frac{2\sqrt{bx+a}\,(bBx+3Ab-2Ba)}{3b^{2}}$	26
risch	$\frac{2\sqrt{bx+a}\,(bBx+3Ab-2Ba)}{3b^{2}}$	26
pseudoelliptic	$\frac{2\left(\left(\frac{Bx}{3}+A\right)b-\frac{2Ba}{3}\right)\sqrt{bx+a}}{b^{2}}$	26
orering	$\frac{2\sqrt{bx+a}\,(bBx+3Ab-2Ba)}{3b^{2}}$	26
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-2Ba\sqrt{bx+a}}{b^{2}}$	38
default	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-2Ba\sqrt{bx+a}}{b^{2}}$	38

input
```
int((B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(1/2)*(B*b*x+3*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx=\frac{2\,(Bbx-2\,Ba+3\,Ab)\sqrt{bx+a}}{3\,b^{2}}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(B*b*x - 2*B*a + 3*A*b)*sqrt(b*x + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \begin{cases} \frac{2A\sqrt{a+bx}+\frac{2B\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}}{b} & \text{for } b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise(((2*A*sqrt(a + b*x) + 2*B*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3
)/b)/b, Ne(b, 0)), ((A*x + B*x**2/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\left(3\sqrt{bx+a}A+\frac{\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)B}{b}\right)}{3b}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(b*x + a)*A + ((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*B/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\left(3\sqrt{bx+a}A+\frac{\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)B}{b}\right)}{3b}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(b*x + a)*A + ((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*B/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\,\sqrt{a+b\,x}\,\left(3\,A\,b-3\,B\,a+B\,\left(a+b\,x\right)\right)}{3\,b^2}$$

input
```
int((A + B*x)/(a + b*x)^(1/2),x)
```

output
```
(2*(a + b*x)^(1/2)*(3*A*b - 3*B*a + B*(a + b*x)))/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\,(bx+a)}{3b}$$

input
```
int((B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(a + b*x))/(3*b)
```

3.262 $\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \frac{2B\sqrt{a+bx}}{b} - \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
2*B*(b*x+a)^(1/2)/b-2*A*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \frac{2B\sqrt{a+bx}}{b} - \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[(A + B*x)/(x*Sqrt[a + b*x]),x]
```

output
```
(2*B*Sqrt[a + b*x])/b - (2*A*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 90}$$

$$A\int \frac{1}{x\sqrt{a+bx}}dx+\frac{2B\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2A\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+\frac{2B\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2B\sqrt{a+bx}}{b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*x)/(x*Sqrt[a + b*x]),x]
```

output
```
(2*B*Sqrt[a + b*x])/b - (2*A*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2B\sqrt{bx+a}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{b}$	35
default	$\frac{2B\sqrt{bx+a}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{b}$	35
pseudoelliptic	$\frac{-2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2B\sqrt{bx+a}\,\sqrt{a}}{b\sqrt{a}}$	38

input
```
int((B*x+A)/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(B*(b*x+a)^(1/2)-A*b/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \left[\frac{A\sqrt{a}b\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}Ba}{ab}, \frac{2\left(A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+\sqrt{bx+a}Ba\right)}{ab}\right]$$

input
```
integrate((B*x+A)/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[(A*sqrt(a)*b*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x +
a)*B*a)/(a*b), 2*(A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x + a)) + sqrt(b*x +
 a)*B*a)/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \begin{cases} \frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2B\sqrt{a+bx}}{b} & \text{for}\ b\neq 0 \\ \frac{A\log{(Bx)}+Bx}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/x/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*A*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*B*sqrt(a + b*x)/b
, Ne(b, 0)), ((A*log(B*x) + B*x)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \frac{A\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\sqrt{bx+a}B}{b}$$

input
```
integrate((B*x+A)/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
A*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) + 2*sqr
t(b*x + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\,\sqrt{bx+a}B}{b}$$

input
```
integrate((B*x+A)/x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*A*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*B/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = \frac{2\,B\,\sqrt{a+b\,x}}{b} - \frac{2\,A\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int((A + B*x)/(x*(a + b*x)^(1/2)),x)
```

output
```
(2*B*(a + b*x)^(1/2))/b - (2*A*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx = 2\sqrt{bx+a} + \sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right) - \sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)$$

input
```
int((B*x+A)/x/(b*x+a)^(1/2),x)
```

output
```
2*sqrt(a + b*x) + sqrt(a)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))
```

3.263 $\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx=-\frac{A\sqrt{a+bx}}{ax}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-A*(b*x+a)^(1/2)/a/x+(A*b-2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx=-\frac{A\sqrt{a+bx}}{ax}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^2*Sqrt[a + b*x]),x]
```

output
```
-((A*Sqrt[a + b*x])/(a*x)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
)/a^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(Ab-2aB)\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{A\sqrt{a+bx}}{ax}$$

$$\Big\downarrow \ 73$$

$$-\frac{(Ab-2aB)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}-\frac{A\sqrt{a+bx}}{ax}$$

$$\Big\downarrow \ 221$$

$$\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx}}{ax}$$

input
```
Int[(A + B*x)/(x^2*Sqrt[a + b*x]),x]
```

output
```
-((A*Sqrt[a + b*x])/(a*x)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
)/a^(3/2)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{A\sqrt{bx+a}}{ax}+\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	42
default	$-\frac{A\sqrt{bx+a}}{ax}+\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	42
risch	$-\frac{A\sqrt{bx+a}}{ax}+\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	42
pseudoelliptic	$-\frac{A\sqrt{bx+a}}{ax}+\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	42

input
```
int((B*x+A)/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x+a)^(1/2)/a/x+(A*b-2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx = \left[-\frac{(2\,Ba-Ab)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,\sqrt{bx+a}Aa}{2\,a^2x}, \frac{(2\,Ba-Ab)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)-\sqrt{bx+}}{a^2x}\right.$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((2*B*a - A*b)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x
) + 2*sqrt(b*x + a)*A*a)/(a^2*x), ((2*B*a - A*b)*sqrt(-a)*x*arctan(sqrt(-a
)/sqrt(b*x + a)) - sqrt(b*x + a)*A*a)/(a^2*x)]
```

Sympy [A] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}+B\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ \frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/x**2/(b*x+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + A*b*asinh(sqrt(a)/(sqrt(b)*sqrt
(x)))/a**(3/2) + B*Piecewise((2*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a), Ne(
b, 0)), (log(x)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx = -\frac{1}{2}\,b\left(\frac{2\sqrt{bx+a}A}{(bx+a)a-a^2} - \frac{(2\,Ba-Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}b}\right)$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*(2*sqrt(b*x + a)*A/((b*x + a)*a - a^2) - (2*B*a - A*b)*log((sqrt(b*
x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(3/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx = b\left(\frac{(2\,Ba-Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab} - \frac{\sqrt{bx+a}A}{abx}\right)$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
b*((2*B*a - A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*b) - sqrt(b*x
+ a)*A/(a*b*x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(A\,b-2\,B\,a\right)}{a^{3/2}} - \frac{A\,\sqrt{a+b\,x}}{a\,x}$$

input
```
int((A + B*x)/(x^2*(a + b*x)^(1/2)),x)
```

output
```
(atanh((a + b*x)^(1/2)/a^(1/2))*(A*b - 2*B*a))/a^(3/2) - (A*(a + b*x)^(1/2
))/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx$$
$$=\frac{-2\sqrt{bx+a}\,a+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2ax}$$

input
```
int((B*x+A)/x^2/(b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x)*a + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x - sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a*x)
```

3.264 $\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = -\frac{A\sqrt{a+bx}}{2ax^2}+\frac{(3Ab-4aB)\sqrt{a+bx}}{4a^2x}-\frac{b(3Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-1/2*A*(b*x+a)^(1/2)/a/x^2+1/4*(3*A*b-4*B*a)*(b*x+a)^(1/2)/a^2/x-1/4*b*(3*
A*b-4*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+bx}(3Abx-2a(A+2Bx))}{x^2}+b(-3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^3*Sqrt[a + b*x]),x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x]*(3*A*b*x - 2*a*(A + 2*B*x)))/x^2 + b*(-3*A*b + 4*a
*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(3Ab-4aB)\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{A\sqrt{a+bx}}{2ax^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{(3Ab-4aB)\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{A\sqrt{a+bx}}{2ax^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{(3Ab-4aB)\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{A\sqrt{a+bx}}{2ax^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{(3Ab-4aB)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{A\sqrt{a+bx}}{2ax^2}$$

input
```
Int[(A + B*x)/(x^3*Sqrt[a + b*x]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x])/(a*x^2) - ((3*A*b - 4*a*B)*(-(Sqrt[a + b*x]/(a*x))
+ (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3Abx+4Bax+2Aa)}{4a^{2}x^{2}}-\frac{b(3Ab-4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	59
pseudoelliptic	$\frac{-\frac{3b\left(Ab-\frac{4Ba}{3}\right)x^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4}+\frac{3\sqrt{bx+a}\left(\frac{2(-2Bx-A)a^{\frac{3}{2}}}{3}+A\sqrt{a}\,bx\right)}{4}}{a^{\frac{5}{2}}x^{2}}$	65
derivativedivides	$2b\left(-\frac{-\frac{(3Ab-4Ba)(bx+a)^{\frac{3}{2}}}{8a^{2}}+\frac{(5Ab-4Ba)\sqrt{bx+a}}{8a}}{b^{2}x^{2}}-\frac{(3Ab-4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}\right)$	82
default	$2b\left(-\frac{-\frac{(3Ab-4Ba)(bx+a)^{\frac{3}{2}}}{8a^{2}}+\frac{(5Ab-4Ba)\sqrt{bx+a}}{8a}}{b^{2}x^{2}}-\frac{(3Ab-4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}\right)$	82

input
```
int((B*x+A)/x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-3*A*b*x+4*B*a*x+2*A*a)/a^2/x^2-1/4*b*(3*A*b-4*B*a)*ar
ctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = \left[-\frac{(4\,Bab-3\,Ab^2)\sqrt{a}x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,Aa^2+(4\,Ba^2-3\,Aab)x)\sqrt{bx+a}}{8\,a^3x^2},\ -\frac{(4\,Bab-3\,Ab^2)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(2\,Aa^2+(4\,Ba^2-3\,Aab)x)\sqrt{bx+a}}{4\,a^3x^2}\right]$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x)*sqrt(b*x + a))/(a^3*x^2),
-1/4*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x + a)) + (2
*A*a^2 + (4*B*a^2 - 3*A*a*b)*x)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(73) = 146$.

Time = 16.93 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = -\frac{A}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{A\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{3Ab^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}} + \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x**3/(b*x+a)**(1/2),x)
```

output
```
-A/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + A*sqrt(b)/(4*a*x**(3/2)*sqrt(a
/(b*x) + 1)) + 3*A*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*A*b**2*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x) + 1
)/(a*sqrt(x)) + B*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = -\frac{1}{8}b^2\left(\frac{2\left((4Ba-3Ab)(bx+a)^{\frac{3}{2}}-(4Ba^2-5Aab)\sqrt{bx+a}\right)}{(bx+a)^2a^2b-2(bx+a)a^3b+a^4b} + \frac{(4Ba-3Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}b}\right)$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*b^2*(2*((4*B*a - 3*A*b)*(b*x + a)^(3/2) - (4*B*a^2 - 5*A*a*b)*sqrt(b*
x + a))/((b*x + a)^2*a^2*b - 2*(b*x + a)*a^3*b + a^4*b) + (4*B*a - 3*A*b)*
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(5/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = -\frac{\frac{\left(4\,Bab^2-3\,Ab^3\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{4\,(bx+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx+a}Ba^2b^2-3\,(bx+a)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx+a}Aab^3}{a^2b^2x^2}}{4\,b}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2)
+ (4*(b*x + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x + a)*B*a^2*b^2 - 3*(b*x + a)^(3/
2)*A*b^3 + 5*sqrt(b*x + a)*A*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = -\frac{\frac{\left(5\,A\,b^2-4\,B\,a\,b\right)\,\sqrt{a+b\,x}}{4\,a}-\frac{\left(3\,A\,b^2-4\,B\,a\,b\right)\,{\left(a+b\,x\right)}^{3/2}}{4\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(3\,A\,b-4\,B\,a\right)}{4\,a^{5/2}}$$

input
```
int((A + B*x)/(x^3*(a + b*x)^(1/2)),x)
```

output
```
- (((5*A*b^2 - 4*B*a*b)*(a + b*x)^(1/2))/(4*a) - ((3*A*b^2 - 4*B*a*b)*(a +
 b*x)^(3/2))/(4*a^2))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) - (b*atanh((a +
b*x)^(1/2)/a^(1/2))*(3*A*b - 4*B*a))/(4*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx = \frac{-4\sqrt{bx+a}\,a^2-2\sqrt{bx+a}\,abx-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8a^2x^2}$$

input
```
int((B*x+A)/x^3/(b*x+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 - 2*sqrt(a + b*x)*a*b*x - sqrt(a)*log(sqrt(a + b*
x) - sqrt(a))*b**2*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2)/
(8*a**2*x**2)
```

3.265 $\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 115

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx=-\frac{A\sqrt{a+bx}}{3ax^3}+\frac{(5Ab-6aB)\sqrt{a+bx}}{12a^2x^2}-\frac{b(5Ab-6aB)\sqrt{a+bx}}{8a^3x}+\frac{b^2(5Ab-6aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
-1/3*A*(b*x+a)^(1/2)/a/x^3+1/12*(5*A*b-6*B*a)*(b*x+a)^(1/2)/a^2/x^2-1/8*b*
(5*A*b-6*B*a)*(b*x+a)^(1/2)/a^3/x+1/8*b^2*(5*A*b-6*B*a)*arctanh((b*x+a)^(1
/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx=\frac{\sqrt{a+bx}(-15Ab^2x^2-4a^2(2A+3Bx)+2abx(5A+9Bx))}{24a^3x^3}+\frac{b^2(5Ab-6aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^4*Sqrt[a + b*x]),x]
```

output
```
(Sqrt[a + b*x]*(-15*A*b^2*x^2 - 4*a^2*(2*A + 3*B*x) + 2*a*b*x*(5*A + 9*B*x
)))/(24*a^3*x^3) + (b^2*(5*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8
*a^(7/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(5Ab-6aB)\int \frac{1}{x^3\sqrt{a+bx}}dx}{6a}-\frac{A\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(5Ab-6aB)\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{A\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(5Ab-6aB)\left(-\frac{3b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{A\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(5Ab-6aB)\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{A\sqrt{a+bx}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(5Ab-6aB)\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{A\sqrt{a+bx}}{3ax^3}$$

input
```
Int[(A + B*x)/(x^4*Sqrt[a + b*x]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x])/(a*x^3) - ((5*A*b - 6*a*B)*(-1/2*Sqrt[a + b*x]/(a*x
^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^
(3/2)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{-\frac{15 b^{2} x^{3}\left(A b-\frac{6 B a}{5}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{8}+\sqrt{b x+a}\left(-\frac{5 b\left(\frac{9 B x}{5}+A\right) x a^{\frac{3}{2}}}{4}+\left(\frac{3 B x}{2}+A\right) a^{\frac{5}{2}}+\frac{15 A \sqrt{a}\, b^{2} x^{2}}{8}\right)}{3 a^{\frac{7}{2}} x^{3}}$	82
risch	$-\frac{\sqrt{b x+a}\,\left(15 A\, b^{2} x^{2}-18 B a b\, x^{2}-10 a A b x+12 B\, a^{2} x+8 a^{2} A\right)}{24 a^{3} x^{3}}+\frac{b^{2}(5 A b-6 B a) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{8 a^{\frac{7}{2}}}$	83
derivativedivides	$2 b^{2}\left(-\frac{\frac{(5 A b-6 B a)(b x+a)^{\frac{5}{2}}}{16 a^{3}}-\frac{(5 A b-6 B a)(b x+a)^{\frac{3}{2}}}{6 a^{2}}+\frac{(11 A b-10 B a) \sqrt{b x+a}}{16 a}}{b^{3} x^{3}}+\frac{(5 A b-6 B a) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{16 a^{\frac{7}{2}}}\right)$	105
default	$2 b^{2}\left(-\frac{\frac{(5 A b-6 B a)(b x+a)^{\frac{5}{2}}}{16 a^{3}}-\frac{(5 A b-6 B a)(b x+a)^{\frac{3}{2}}}{6 a^{2}}+\frac{(11 A b-10 B a) \sqrt{b x+a}}{16 a}}{b^{3} x^{3}}+\frac{(5 A b-6 B a) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{16 a^{\frac{7}{2}}}\right)$	105

input
```
int((B*x+A)/x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^(7/2)*(-15/8*b^2*x^3*(A*b-6/5*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+(
b*x+a)^(1/2)*(-5/4*b*(9/5*B*x+A)*x*a^(3/2)+(3/2*B*x+A)*a^(5/2)+15/8*A*a^(1
/2)*b^2*x^2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx$$
$$= \left[-\frac{3\,(6\,Bab^2-5\,Ab^3)\sqrt{a}x^3\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(8\,Aa^3-3\,(6\,Ba^2b-5\,Aab^2)x^2+2\,(6\,Ba^3-5\,Ac\right.}{48\,a^4x^3}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*B*a*b^2 - 5*A*b^3)*sqrt(a)*x^3*log((b*x + 2*sqrt(b*x + a)*sqr
t(a) + 2*a)/x) + 2*(8*A*a^3 - 3*(6*B*a^2*b - 5*A*a*b^2)*x^2 + 2*(6*B*a^3 -
 5*A*a^2*b)*x)*sqrt(b*x + a))/(a^4*x^3), 1/24*(3*(6*B*a*b^2 - 5*A*b^3)*sqr
t(-a)*x^3*arctan(sqrt(-a)/sqrt(b*x + a)) - (8*A*a^3 - 3*(6*B*a^2*b - 5*A*a
*b^2)*x^2 + 2*(6*B*a^3 - 5*A*a^2*b)*x)*sqrt(b*x + a))/(a^4*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(105) = 210$.

Time = 23.43 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx = -\frac{A}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{A\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Ab^{\frac{3}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Ab^{\frac{5}{2}}}{8a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}} - \frac{B}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{B\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{3Bb^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x**4/(b*x+a)**(1/2),x)
```

output
```
-A/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + A*sqrt(b)/(12*a*x**(5/2)*sqrt(
a/(b*x) + 1)) - 5*A*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*A*b*
*(5/2)/(8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*A*b**3*asinh(sqrt(a)/(sqrt(b
)*sqrt(x)))/(8*a**(7/2)) - B/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + B*sq
rt(b)/(4*a*x**(3/2)*sqrt(a/(b*x) + 1)) + 3*B*b**(3/2)/(4*a**2*sqrt(x)*sqrt
(a/(b*x) + 1)) - 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx$$
$$=\frac{1}{48}b^3\left(\frac{2\left(3\,(6\,Ba-5\,Ab)(bx+a)^{\frac{5}{2}}-8\,(6\,Ba^2-5\,Aab)(bx+a)^{\frac{3}{2}}+3\,(10\,Ba^3-11\,Aa^2b)\sqrt{bx+a}\right)}{(bx+a)^3a^3b-3\,(bx+a)^2a^4b+3\,(bx+a)a^5b-a^6b}+\cdots\right.$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/48*b^3*(2*(3*(6*B*a - 5*A*b)*(b*x + a)^(5/2) - 8*(6*B*a^2 - 5*A*a*b)*(b*
x + a)^(3/2) + 3*(10*B*a^3 - 11*A*a^2*b)*sqrt(b*x + a))/((b*x + a)^3*a^3*b
 - 3*(b*x + a)^2*a^4*b + 3*(b*x + a)*a^5*b - a^6*b) + 3*(6*B*a - 5*A*b)*lo
g((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(7/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx$$
$$=\frac{1}{24}b^3\left(\frac{3\,(6\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}+\frac{18\,(bx+a)^{\frac{5}{2}}Ba-48\,(bx+a)^{\frac{3}{2}}Ba^2+30\,\sqrt{bx+a}Ba^3-15\,(b\cdots}{a^3b^4x^3}\right.$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*b^3*(3*(6*B*a - 5*A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*b
) + (18*(b*x + a)^(5/2)*B*a - 48*(b*x + a)^(3/2)*B*a^2 + 30*sqrt(b*x + a)*
B*a^3 - 15*(b*x + a)^(5/2)*A*b + 40*(b*x + a)^(3/2)*A*a*b - 33*sqrt(b*x +
a)*A*a^2*b)/(a^3*b^4*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx = \frac{\frac{\left(5\,A\,b^3-6\,B\,a\,b^2\right)\,{\left(a+b\,x\right)}^{5/2}}{8\,a^3}-\frac{\left(5\,A\,b^3-6\,B\,a\,b^2\right)\,{\left(a+b\,x\right)}^{3/2}}{3\,a^2}+\frac{\left(11\,A\,b^3-10\,B\,a\,b^2\right)\,\sqrt{a+b\,x}}{8\,a}}{3\,a\,{\left(a+b\,x\right)}^2-3\,a^2\,\left(a+b\,x\right)-{\left(a+b\,x\right)}^3+a^3} + \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(5\,A\,b-6\,B\,a\right)}{8\,a^{7/2}}$$

input
```
int((A + B*x)/(x^4*(a + b*x)^(1/2)),x)
```

output
```
(((5*A*b^3 - 6*B*a*b^2)*(a + b*x)^(5/2))/(8*a^3) - ((5*A*b^3 - 6*B*a*b^2)*
(a + b*x)^(3/2))/(3*a^2) + ((11*A*b^3 - 10*B*a*b^2)*(a + b*x)^(1/2))/(8*a)
)/(3*a*(a + b*x)^2 - 3*a^2*(a + b*x) - (a + b*x)^3 + a^3) + (b^2*atanh((a
+ b*x)^(1/2)/a^(1/2))*(5*A*b - 6*B*a))/(8*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx = \frac{-16\sqrt{bx+a}\,a^3-4\sqrt{bx+a}\,a^2bx+6\sqrt{bx+a}\,a\,b^2x^2+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3-3\sqrt{a}\log\left(\sqrt{bx+a}\right.}{48a^3x^3}$$

input
```
int((B*x+A)/x^4/(b*x+a)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 - 4*sqrt(a + b*x)*a**2*b*x + 6*sqrt(a + b*x)*a*b
**2*x**2 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 - 3*sqrt(a)*lo
g(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**3*x**3)
```

3.266 $\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 146

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = -\frac{A\sqrt{a+bx}}{4ax^4}+\frac{(7Ab-8aB)\sqrt{a+bx}}{24a^2x^3}-\frac{5b(7Ab-8aB)\sqrt{a+bx}}{96a^3x^2}+\frac{5b^2(7Ab-8aB)\sqrt{a+bx}}{64a^4x}-\frac{5b^3(7Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{9/2}}$$

output
```
-1/4*A*(b*x+a)^(1/2)/a/x^4+1/24*(7*A*b-8*B*a)*(b*x+a)^(1/2)/a^2/x^3-5/96*b
*(7*A*b-8*B*a)*(b*x+a)^(1/2)/a^3/x^2+5/64*b^2*(7*A*b-8*B*a)*(b*x+a)^(1/2)/
a^4/x-5/64*b^3*(7*A*b-8*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}(105Ab^3x^3-16a^3(3A+4Bx)+8a^2bx(7A+10Bx)-10ab^2x^2(7A+12Bx))}{192a^4x^4}+\frac{5b^3(-7Ab+8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^5*Sqrt[a + b*x]),x]
```

output
```
(Sqrt[a + b*x]*(105*A*b^3*x^3 - 16*a^3*(3*A + 4*B*x) + 8*a^2*b*x*(7*A + 10
*B*x) - 10*a*b^2*x^2*(7*A + 12*B*x)))/(192*a^4*x^4) + (5*b^3*(-7*A*b + 8*a
*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(9/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {87, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(7Ab-8aB)\int \frac{1}{x^4\sqrt{a+bx}}dx}{8a}-\frac{A\sqrt{a+bx}}{4ax^4}$$

$$\Big\downarrow \; 52$$

$$-\frac{(7Ab-8aB)\left(-\frac{5b\int \frac{1}{x^3\sqrt{a+bx}}dx}{6a}-\frac{\sqrt{a+bx}}{3ax^3}\right)}{8a}-\frac{A\sqrt{a+bx}}{4ax^4}$$

$$\Big\downarrow \; 52$$

$$-\frac{(7Ab-8aB)\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x^2\sqrt{a+bx}}dx}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}\right)}{8a}-\frac{A\sqrt{a+bx}}{4ax^4}$$

$$\Big\downarrow \; 52$$

$$-\frac{(7Ab-8aB)\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}\right)}{8a}-\frac{A\sqrt{a+bx}}{4ax^4}$$

$$\Big\downarrow \; 73$$

$$-\frac{(7Ab-8aB)\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}\right)}{8a}-\frac{A\sqrt{a+bx}}{4ax^4}$$

$$\Big\downarrow \; 221$$

$$-\frac{(7Ab-8aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)}{4a}-\frac{\sqrt{a+bx}}{2ax^2}\right)}{6a}-\frac{\sqrt{a+bx}}{3ax^3}\right)}{8a}-\frac{A\sqrt{a+bx}}{4ax^4}$$

input
```
Int[(A + B*x)/(x^5*Sqrt[a + b*x]),x]
```

output
```
-1/4*(A*Sqrt[a + b*x])/(a*x^4) - ((7*A*b - 8*a*B)*(-1/3*Sqrt[a + b*x]/(a*x
^3) - (5*b*(-1/2*Sqrt[a + b*x]/(a*x^2) - (3*b*(-(Sqrt[a + b*x]/(a*x)) + (b
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{-\frac{35\left(Ab-\frac{8Ba}{7}\right)b^3x^4\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64}+\frac{7\sqrt{bx+a}\left(-\frac{5b^2\left(\frac{12Bx}{7}+A\right)x^2a^{\frac{3}{2}}}{4}+bx\left(\frac{10Bx}{7}+A\right)a^{\frac{5}{2}}+\frac{2(-4Bx-3A)a^{\frac{7}{2}}}{7}+\frac{15A\sqrt{a}\,b^3x^3}{8}\right.}{24}}{a^{\frac{9}{2}}x^4}$
risch	$-\frac{\sqrt{bx+a}\,(-105A\,b^3x^3+120Ba\,b^2x^3+70aA\,b^2x^2-80B\,a^2b\,x^2-56a^2Abx+64B\,a^3x+48a^3A)}{192a^4x^4}-\frac{5b^3(7Ab-8Ba)\operatorname{arct}}{64a^{\frac{9}{2}}}$
derivativedivides	$2b^3\left(-\frac{-\frac{5(7Ab-8Ba)(bx+a)^{\frac{7}{2}}}{128a^4}+\frac{55(7Ab-8Ba)(bx+a)^{\frac{5}{2}}}{384a^3}-\frac{73(7Ab-8Ba)(bx+a)^{\frac{3}{2}}}{384a^2}+\frac{(93Ab-88Ba)\sqrt{bx+a}}{128a}}{b^4x^4}-\frac{5(7Ab-8Ba)}{1}\right.$
default	$2b^3\left(-\frac{-\frac{5(7Ab-8Ba)(bx+a)^{\frac{7}{2}}}{128a^4}+\frac{55(7Ab-8Ba)(bx+a)^{\frac{5}{2}}}{384a^3}-\frac{73(7Ab-8Ba)(bx+a)^{\frac{3}{2}}}{384a^2}+\frac{(93Ab-88Ba)\sqrt{bx+a}}{128a}}{b^4x^4}-\frac{5(7Ab-8Ba)}{1}\right.$

input
```
int((B*x+A)/x^5/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7/24*(-15/8*(A*b-8/7*B*a)*b^3*x^4*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(
1/2)*(-5/4*b^2*(12/7*B*x+A)*x^2*a^(3/2)+b*x*(10/7*B*x+A)*a^(5/2)+2/7*(-4*B
*x-3*A)*a^(7/2)+15/8*A*a^(1/2)*b^3*x^3))/a^(9/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx$$
$$=\left[-\frac{15\,(8\,Bab^3-7\,Ab^4)\sqrt{a}x^4\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(48\,Aa^4+15\,(8\,Ba^2b^2-7\,Aab^3)x^3-10\,(8\,Ba^3b}{384\,a^5x^4}\right.$$
$$-\frac{15\,(8\,Bab^3-7\,Ab^4)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(48\,Aa^4+15\,(8\,Ba^2b^2-7\,Aab^3)x^3-10\,(8\,Ba^3b-7\,Aa}{192\,a^5x^4}$$

input
```
integrate((B*x+A)/x^5/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(8*B*a*b^3 - 7*A*b^4)*sqrt(a)*x^4*log((b*x - 2*sqrt(b*x + a)*s
qrt(a) + 2*a)/x) + 2*(48*A*a^4 + 15*(8*B*a^2*b^2 - 7*A*a*b^3)*x^3 - 10*(8*
B*a^3*b - 7*A*a^2*b^2)*x^2 + 8*(8*B*a^4 - 7*A*a^3*b)*x)*sqrt(b*x + a))/(a^
5*x^4), -1/192*(15*(8*B*a*b^3 - 7*A*b^4)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt
(b*x + a)) + (48*A*a^4 + 15*(8*B*a^2*b^2 - 7*A*a*b^3)*x^3 - 10*(8*B*a^3*b
- 7*A*a^2*b^2)*x^2 + 8*(8*B*a^4 - 7*A*a^3*b)*x)*sqrt(b*x + a))/(a^5*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(141) = 282$.

Time = 63.52 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = -\frac{A}{4\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{A\sqrt{b}}{24ax^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{7Ab^{\frac{3}{2}}}{96a^2x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{35Ab^{\frac{5}{2}}}{192a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{35Ab^{\frac{7}{2}}}{64a^4\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{35Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{9}{2}}} - \frac{B}{3\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{B\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Bb^{\frac{3}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5Bb^{\frac{5}{2}}}{8a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{5Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x**5/(b*x+a)**(1/2),x)
```

output
```
-A/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) + A*sqrt(b)/(24*a*x**(7/2)*sqrt(
a/(b*x) + 1)) - 7*A*b**(3/2)/(96*a**2*x**(5/2)*sqrt(a/(b*x) + 1)) + 35*A*b
**(5/2)/(192*a**3*x**(3/2)*sqrt(a/(b*x) + 1)) + 35*A*b**(7/2)/(64*a**4*sqr
t(x)*sqrt(a/(b*x) + 1)) - 35*A*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a
**(9/2)) - B/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + B*sqrt(b)/(12*a*x**(
5/2)*sqrt(a/(b*x) + 1)) - 5*B*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)
) - 5*B*b**(5/2)/(8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*B*b**3*asinh(sqrt(
a)/(sqrt(b)*sqrt(x)))/(8*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = -\frac{1}{384}\,b^4\left(\frac{2\left(15\,(8\,Ba-7\,Ab)(bx+a)^{\frac{7}{2}}-55\,(8\,Ba^2-7\,Aab)(bx+a)^{\frac{5}{2}}+73\,(8\,Ba^3-7\,Aa^2b)(bx+a)^{\frac{3}{2}}\right.}{(bx+a)^4a^4b-4\,(bx+a)^3a^5b+6\,(bx+a)^2a^6b-4\,(bx+a)a^7b}\right.$$

input
```
integrate((B*x+A)/x^5/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/384*b^4*(2*(15*(8*B*a - 7*A*b)*(b*x + a)^(7/2) - 55*(8*B*a^2 - 7*A*a*b)
*(b*x + a)^(5/2) + 73*(8*B*a^3 - 7*A*a^2*b)*(b*x + a)^(3/2) - 3*(88*B*a^4
- 93*A*a^3*b)*sqrt(b*x + a))/((b*x + a)^4*a^4*b - 4*(b*x + a)^3*a^5*b + 6*
(b*x + a)^2*a^6*b - 4*(b*x + a)*a^7*b + a^8*b) + 15*(8*B*a - 7*A*b)*log((s
qrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(9/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = -\frac{\frac{15\,(8\,Bab^4-7\,Ab^5)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{120\,(bx+a)^{\frac{7}{2}}Bab^4-440\,(bx+a)^{\frac{5}{2}}Ba^2b^4+584\,(bx+a)^{\frac{3}{2}}Ba^3b^4-264\,\sqrt{bx+a}Ba^4b^4-105\,(bx+a)^{\frac{7}{2}}Ab^5}{a^4b^4x^4}}{192\,b}$$

input
```
integrate((B*x+A)/x^5/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/192*(15*(8*B*a*b^4 - 7*A*b^5)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*
a^4) + (120*(b*x + a)^(7/2)*B*a*b^4 - 440*(b*x + a)^(5/2)*B*a^2*b^4 + 584*
(b*x + a)^(3/2)*B*a^3*b^4 - 264*sqrt(b*x + a)*B*a^4*b^4 - 105*(b*x + a)^(7
/2)*A*b^5 + 385*(b*x + a)^(5/2)*A*a*b^5 - 511*(b*x + a)^(3/2)*A*a^2*b^5 +
279*sqrt(b*x + a)*A*a^3*b^5)/(a^4*b^4*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = \frac{\frac{73\,\left(7\,A\,b^4-8\,B\,a\,b^3\right)\,{\left(a+b\,x\right)}^{3/2}}{192\,a^2}-\frac{55\,\left(7\,A\,b^4-8\,B\,a\,b^3\right)\,{\left(a+b\,x\right)}^{5/2}}{192\,a^3}+\frac{5\,\left(7\,A\,b^4-8\,B\,a\,b^3\right)\,{\left(a+b\,x\right)}^{7/2}}{64\,a^4}-\frac{\left(93\,A\,b^4-88\,B\,a\,b^3\right)\,\sqrt{a+b\,x}}{64\,a}}{{\left(a+b\,x\right)}^4-4\,a^3\,\left(a+b\,x\right)-4\,a\,{\left(a+b\,x\right)}^3+6\,a^2\,{\left(a+b\,x\right)}^2+a^4} - \frac{5\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(7\,A\,b-8\,B\,a\right)}{64\,a^{9/2}}$$

input
```
int((A + B*x)/(x^5*(a + b*x)^(1/2)),x)
```

output
```
((73*(7*A*b^4 - 8*B*a*b^3)*(a + b*x)^(3/2))/(192*a^2) - (55*(7*A*b^4 - 8*B
*a*b^3)*(a + b*x)^(5/2))/(192*a^3) + (5*(7*A*b^4 - 8*B*a*b^3)*(a + b*x)^(7
/2))/(64*a^4) - ((93*A*b^4 - 88*B*a*b^3)*(a + b*x)^(1/2))/(64*a))/((a + b*
x)^4 - 4*a^3*(a + b*x) - 4*a*(a + b*x)^3 + 6*a^2*(a + b*x)^2 + a^4) - (5*b
^3*atanh((a + b*x)^(1/2)/a^(1/2))*(7*A*b - 8*B*a))/(64*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{x^5\sqrt{a+bx}}\,dx = \frac{-96\sqrt{bx+a}\,a^4-16\sqrt{bx+a}\,a^3bx+20\sqrt{bx+a}\,a^2b^2x^2-30\sqrt{bx+a}\,a\,b^3x^3-15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{384a^4x^4}$$

input
```
int((B*x+A)/x^5/(b*x+a)^(1/2),x)
```

output
```
( - 96*sqrt(a + b*x)*a**4 - 16*sqrt(a + b*x)*a**3*b*x + 20*sqrt(a + b*x)*a
**2*b**2*x**2 - 30*sqrt(a + b*x)*a*b**3*x**3 - 15*sqrt(a)*log(sqrt(a + b*x
) - sqrt(a))*b**4*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*x**4
)/(384*a**4*x**4)
```

3.267 $\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = -\frac{2a^4(Ab-aB)}{b^6\sqrt{a+bx}} - \frac{2a^3(4Ab-5aB)\sqrt{a+bx}}{b^6} + \frac{4a^2(3Ab-5aB)(a+bx)^{3/2}}{3b^6} - \frac{4a(2Ab-5aB)(a+bx)^{5/2}}{5b^6} + \frac{2(Ab-5aB)(a+bx)^{7/2}}{7b^6} + \frac{2B(a+bx)^{9/2}}{9b^6}$$

output
```
-2*a^4*(A*b-B*a)/b^6/(b*x+a)^(1/2)-2*a^3*(4*A*b-5*B*a)*(b*x+a)^(1/2)/b^6+4
/3*a^2*(3*A*b-5*B*a)*(b*x+a)^(3/2)/b^6-4/5*a*(2*A*b-5*B*a)*(b*x+a)^(5/2)/b
^6+2/7*(A*b-5*B*a)*(b*x+a)^(7/2)/b^6+2/9*B*(b*x+a)^(9/2)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2560a^5B - 256a^4b(9A-5Bx) + 32a^2b^3x^2(9A+5Bx) - 64a^3b^2x(18A+5Bx) + 10b}{315b^6\sqrt{a+bx}}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2560*a^5*B - 256*a^4*b*(9*A - 5*B*x) + 32*a^2*b^3*x^2*(9*A + 5*B*x) - 64*
a^3*b^2*x*(18*A + 5*B*x) + 10*b^5*x^4*(9*A + 7*B*x) - 4*a*b^4*x^3*(36*A +
25*B*x))/(315*b^6*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 86}$$
$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)^{3/2}}+\frac{a^3(5aB-4Ab)}{b^5\sqrt{a+bx}}-\frac{2a^2\sqrt{a+bx}(5aB-3Ab)}{b^5}+\frac{2a(a+bx)^{3/2}(5aB-2Ab)}{b^5}+\frac{(a+bx)^{5/2}(}{b^5}\right.$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2a^4(Ab-aB)}{b^6\sqrt{a+bx}}-\frac{2a^3\sqrt{a+bx}(4Ab-5aB)}{b^6}+\frac{4a^2(a+bx)^{3/2}(3Ab-5aB)}{3b^6}+\frac{2(a+bx)^{7/2}(Ab-5aB)}{7b^6}-\frac{4a(a+bx)^{5/2}(2Ab-5aB)}{5b^6}+\frac{2B(a+bx)^{9/2}}{9b^6}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(-2*a^4*(A*b - a*B))/(b^6*Sqrt[a + b*x]) - (2*a^3*(4*A*b - 5*a*B)*Sqrt[a +
 b*x])/b^6 + (4*a^2*(3*A*b - 5*a*B)*(a + b*x)^(3/2))/(3*b^6) - (4*a*(2*A*b
 - 5*a*B)*(a + b*x)^(5/2))/(5*b^6) + (2*(A*b - 5*a*B)*(a + b*x)^(7/2))/(7*
b^6) + (2*B*(a + b*x)^(9/2))/(9*b^6)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(70B\,x^5+90A\,x^4)b^5-144\left(\frac{25Bx}{36}+A\right)a\,x^3b^4+288a^2\left(\frac{5Bx}{9}+A\right)x^2b^3-1152a^3\left(\frac{5Bx}{18}+A\right)x\,b^2-2304a^4\left(-\frac{5Bx}{9}+A\right)b+25}{315\sqrt{bx+a}\,b^6}$
gosper	$-\frac{2\left(-35b^5B\,x^5-45A\,b^5x^4+50Ba\,b^4x^4+72Aa\,b^4x^3-80B\,a^2b^3x^3-144A\,a^2b^3x^2+160B\,a^3b^2x^2+576a^3b^2Ax-640a^4b\right.}{315\sqrt{bx+a}\,b^6}$
trager	$-\frac{2\left(-35b^5B\,x^5-45A\,b^5x^4+50Ba\,b^4x^4+72Aa\,b^4x^3-80B\,a^2b^3x^3-144A\,a^2b^3x^2+160B\,a^3b^2x^2+576a^3b^2Ax-640a^4b\right.}{315\sqrt{bx+a}\,b^6}$
risch	$-\frac{2\left(-35B\,x^4b^4-45A\,x^3b^4+85B\,x^3a\,b^3+117A\,x^2a\,b^3-165B\,x^2a^2b^2-261Ax\,a^2b^2+325Bx\,a^3b+837A\,a^3b-965B\,a^4\right)}{315b^6}$
orering	$-\frac{2\left(-35b^5B\,x^5-45A\,b^5x^4+50Ba\,b^4x^4+72Aa\,b^4x^3-80B\,a^2b^3x^3-144A\,a^2b^3x^2+160B\,a^3b^2x^2+576a^3b^2Ax-640a^4b\right.}{315\sqrt{bx+a}\,b^6}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2Ab(bx+a)^{\frac{7}{2}}}{7}-\frac{10Ba(bx+a)^{\frac{7}{2}}}{7}-\frac{8Aab(bx+a)^{\frac{5}{2}}}{5}+4B\,a^2(bx+a)^{\frac{5}{2}}+4A\,a^2b(bx+a)^{\frac{3}{2}}-\frac{20B\,a^3(bx+a)^{\frac{3}{2}}}{3}-8A\,a^3b}{b^6}$
default	$\frac{\frac{2B(bx+a)^{\frac{9}{2}}}{9}+\frac{2Ab(bx+a)^{\frac{7}{2}}}{7}-\frac{10Ba(bx+a)^{\frac{7}{2}}}{7}-\frac{8Aab(bx+a)^{\frac{5}{2}}}{5}+4B\,a^2(bx+a)^{\frac{5}{2}}+4A\,a^2b(bx+a)^{\frac{3}{2}}-\frac{20B\,a^3(bx+a)^{\frac{3}{2}}}{3}-8A\,a^3b}{b^6}$

input
```
int(x^4*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((70*B*x^5+90*A*x^4)*b^5-144*(25/36*B*x+A)*a*x^3*b^4+288*a^2*(5/9*B*
x+A)*x^2*b^3-1152*a^3*(5/18*B*x+A)*x*b^2-2304*a^4*(-5/9*B*x+A)*b+2560*a^5*
B)/(b*x+a)^(1/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\,(35\,Bb^5x^5+1280\,Ba^5-1152\,Aa^4b-5\,(10\,Bab^4-9\,Ab^5)x^4+8\,(10\,Ba^2b^3-9\,Aab^4}{315\,(b^7x+ab^6)}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*b^5*x^5 + 1280*B*a^5 - 1152*A*a^4*b - 5*(10*B*a*b^4 - 9*A*b^5)
*x^4 + 8*(10*B*a^2*b^3 - 9*A*a*b^4)*x^3 - 16*(10*B*a^3*b^2 - 9*A*a^2*b^3)*
x^2 + 64*(10*B*a^4*b - 9*A*a^3*b^2)*x)*sqrt(b*x + a)/(b^7*x + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{9}{2}}}{9b}+\frac{a^4(-Ab+Ba)}{b\sqrt{a+bx}}+\frac{(a+bx)^{\frac{7}{2}}(Ab-5Ba)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\left(-4Aab+10Ba^2\right)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(6Aa^2b-10Ba^3\right)}{3b}+\frac{\sqrt{a+bx}\,(-4}{}\right.}{b^5} \\ \frac{\frac{Ax^5}{5}+\frac{Bx^6}{6}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**4*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(9/2)/(9*b) + a**4*(-A*b + B*a)/(b*sqrt(a + b*x
)) + (a + b*x)**(7/2)*(A*b - 5*B*a)/(7*b) + (a + b*x)**(5/2)*(-4*A*a*b + 1
0*B*a**2)/(5*b) + (a + b*x)**(3/2)*(6*A*a**2*b - 10*B*a**3)/(3*b) + sqrt(a
 + b*x)*(-4*A*a**3*b + 5*B*a**4)/b)/b**5, Ne(b, 0)), ((A*x**5/5 + B*x**6/6
)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(bx+a)^{\frac{9}{2}}B-45\,(5\,Ba-Ab)(bx+a)^{\frac{7}{2}}+126\,(5\,Ba^2-2\,Aab)\,(bx+a)^{\frac{5}{2}}-210\,(5\,Ba^3-3\,Aa^2b)\,(bx+a)^{\frac{3}{2}}+315\,(5\,Ba^4\cdots}{b}\right.}{315\,b^5}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(b*x + a)^(9/2)*B - 45*(5*B*a - A*b)*(b*x + a)^(7/2) + 126*(5*B
*a^2 - 2*A*a*b)*(b*x + a)^(5/2) - 210*(5*B*a^3 - 3*A*a^2*b)*(b*x + a)^(3/2
) + 315*(5*B*a^4 - 4*A*a^3*b)*sqrt(b*x + a))/b + 315*(B*a^5 - A*a^4*b)/(sq
rt(b*x + a)*b))/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.13

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\,(Ba^5-Aa^4b)}{\sqrt{bx+a}b^6} + \frac{2\left(35\,(bx+a)^{\frac{9}{2}}Bb^{48}-225\,(bx+a)^{\frac{7}{2}}Bab^{48}+630\,(bx+a)^{\frac{5}{2}}Ba^2b^{48}-1050\,(bx+a)^{\frac{3}{2}}Ba^3b^{48}+1575\,\sqrt{bx\cdots}\right.}{315\,b^{5\cdots}}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*(B*a^5 - A*a^4*b)/(sqrt(b*x + a)*b^6) + 2/315*(35*(b*x + a)^(9/2)*B*b^48
 - 225*(b*x + a)^(7/2)*B*a*b^48 + 630*(b*x + a)^(5/2)*B*a^2*b^48 - 1050*(b
*x + a)^(3/2)*B*a^3*b^48 + 1575*sqrt(b*x + a)*B*a^4*b^48 + 45*(b*x + a)^(7
/2)*A*b^49 - 252*(b*x + a)^(5/2)*A*a*b^49 + 630*(b*x + a)^(3/2)*A*a^2*b^49
 - 1260*sqrt(b*x + a)*A*a^3*b^49)/b^54
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{(20\,B\,a^2-8\,A\,a\,b)\,\left(a+b\,x\right)^{5/2}}{5\,b^6} + \frac{2\,B\,\left(a+b\,x\right)^{9/2}}{9\,b^6} + \frac{(2\,A\,b-10\,B\,a)\,\left(a+b\,x\right)^{7/2}}{7\,b^6} + \frac{2\,B\,a^5-2\,A\,a^4\,b}{b^6\,\sqrt{a+b\,x}} + \frac{(10\,B\,a^4-8\,A\,a^3\,b)\,\sqrt{a+b\,x}}{b^6} - \frac{(20\,B\,a^3-12\,A\,a^2\,b)\,\left(a+b\,x\right)^{3/2}}{3\,b^6}$$

input
```
int((x^4*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
((20*B*a^2 - 8*A*a*b)*(a + b*x)^(5/2))/(5*b^6) + (2*B*(a + b*x)^(9/2))/(9*
b^6) + ((2*A*b - 10*B*a)*(a + b*x)^(7/2))/(7*b^6) + (2*B*a^5 - 2*A*a^4*b)/
(b^6*(a + b*x)^(1/2)) + ((10*B*a^4 - 8*A*a^3*b)*(a + b*x)^(1/2))/b^6 - ((2
0*B*a^3 - 12*A*a^2*b)*(a + b*x)^(3/2))/(3*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int \frac{x^4(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$$

input
```
int(x^4*(B*x+A)/(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**4 - 64*a**3*b*x + 48*a**2*b**2*x**2 - 40*a*b**3*x
**3 + 35*b**4*x**4))/(315*b**5)
```

3.268 $\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2a^3(Ab-aB)}{b^5\sqrt{a+bx}}+\frac{2a^2(3Ab-4aB)\sqrt{a+bx}}{b^5} -\frac{2a(Ab-2aB)(a+bx)^{3/2}}{b^5}+\frac{2(Ab-4aB)(a+bx)^{5/2}}{5b^5}+\frac{2B(a+bx)^{7/2}}{7b^5}$$

output
```
2*a^3*(A*b-B*a)/b^5/(b*x+a)^(1/2)+2*a^2*(3*A*b-4*B*a)*(b*x+a)^(1/2)/b^5-2*
a*(A*b-2*B*a)*(b*x+a)^(3/2)/b^5+2/5*(A*b-4*B*a)*(b*x+a)^(5/2)/b^5+2/7*B*(b
*x+a)^(7/2)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2(-128a^4B+16a^3b(7A-4Bx)+8a^2b^2x(7A+2Bx)-2ab^3x^2(7A+4Bx)+b^4x^3(7}{35b^5\sqrt{a+bx}}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*(-128*a^4*B + 16*a^3*b*(7*A - 4*B*x) + 8*a^2*b^2*x*(7*A + 2*B*x) - 2*a*
b^3*x^2*(7*A + 4*B*x) + b^4*x^3*(7*A + 5*B*x)))/(35*b^5*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)^{3/2}}-\frac{a^2(4aB-3Ab)}{b^4\sqrt{a+bx}}+\frac{3a\sqrt{a+bx}(2aB-Ab)}{b^4}+\frac{(a+bx)^{3/2}(Ab-4aB)}{b^4}+\frac{B(a+bx)^{5/2}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3(Ab-aB)}{b^5\sqrt{a+bx}}+\frac{2a^2\sqrt{a+bx}(3Ab-4aB)}{b^5}+\frac{2(a+bx)^{5/2}(Ab-4aB)}{5b^5}-\frac{2a(a+bx)^{3/2}(Ab-2aB)}{b^5}+\frac{2B(a+bx)^{7/2}}{7b^5}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*a^3*(A*b - a*B))/(b^5*Sqrt[a + b*x]) + (2*a^2*(3*A*b - 4*a*B)*Sqrt[a +
b*x])/b^5 - (2*a*(A*b - 2*a*B)*(a + b*x)^(3/2))/b^5 + (2*(A*b - 4*a*B)*(a
+ b*x)^(5/2))/(5*b^5) + (2*B*(a + b*x)^(7/2))/(7*b^5)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{(10B\,x^4+14A\,x^3)b^4-28\left(\frac{4Bx}{7}+A\right)a\,x^2b^3+112\left(\frac{2Bx}{7}+A\right)a^2x\,b^2+224a^3\left(-\frac{4Bx}{7}+A\right)b-256B\,a^4}{35\sqrt{bx+a}\,b^5}$
gosper	$\frac{\frac{2}{7}B\,x^4b^4+\frac{2}{5}A\,x^3b^4-\frac{16}{35}B\,x^3a\,b^3-\frac{4}{5}A\,x^2a\,b^3+\frac{32}{35}B\,x^2a^2b^2+\frac{16}{5}Ax\,a^2b^2-\frac{128}{35}Bx\,a^3b+\frac{32}{5}A\,a^3b-\frac{256}{35}B\,a^4}{\sqrt{bx+a}\,b^5}$
trager	$\frac{\frac{2}{7}B\,x^4b^4+\frac{2}{5}A\,x^3b^4-\frac{16}{35}B\,x^3a\,b^3-\frac{4}{5}A\,x^2a\,b^3+\frac{32}{35}B\,x^2a^2b^2+\frac{16}{5}Ax\,a^2b^2-\frac{128}{35}Bx\,a^3b+\frac{32}{5}A\,a^3b-\frac{256}{35}B\,a^4}{\sqrt{bx+a}\,b^5}$
risch	$\frac{2(5b^3B\,x^3+7A\,x^2b^3-13B\,x^2a\,b^2-21Axa\,b^2+29Bx\,a^2b+77a^2bA-93a^3B)\sqrt{bx+a}}{35b^5}+\frac{2a^3(Ab-Ba)}{b^5\sqrt{bx+a}}$
orering	$\frac{\frac{2}{7}B\,x^4b^4+\frac{2}{5}A\,x^3b^4-\frac{16}{35}B\,x^3a\,b^3-\frac{4}{5}A\,x^2a\,b^3+\frac{32}{35}B\,x^2a^2b^2+\frac{16}{5}Ax\,a^2b^2-\frac{128}{35}Bx\,a^3b+\frac{32}{5}A\,a^3b-\frac{256}{35}B\,a^4}{\sqrt{bx+a}\,b^5}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2Ab(bx+a)^{\frac{5}{2}}}{5}-\frac{8Ba(bx+a)^{\frac{5}{2}}}{5}-2Aab(bx+a)^{\frac{3}{2}}+4B\,a^2(bx+a)^{\frac{3}{2}}+6A\,a^2b\sqrt{bx+a}-8B\,a^3\sqrt{bx+a}+\frac{2a^3(Ab-Ba}{\sqrt{bx+a}}}{b^5}$
default	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2Ab(bx+a)^{\frac{5}{2}}}{5}-\frac{8Ba(bx+a)^{\frac{5}{2}}}{5}-2Aab(bx+a)^{\frac{3}{2}}+4B\,a^2(bx+a)^{\frac{3}{2}}+6A\,a^2b\sqrt{bx+a}-8B\,a^3\sqrt{bx+a}+\frac{2a^3(Ab-Ba}{\sqrt{bx+a}}}{b^5}$

input
```
int(x^3*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*((10*B*x^4+14*A*x^3)*b^4-28*(4/7*B*x+A)*a*x^2*b^3+112*(2/7*B*x+A)*a^2
*x*b^2+224*a^3*(-4/7*B*x+A)*b-256*B*a^4)/(b*x+a)^(1/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\left(5\,Bb^4x^4 - 128\,Ba^4 + 112\,Aa^3b - (8\,Bab^3 - 7\,Ab^4)x^3 + 2\,(8\,Ba^2b^2 - 7\,Aab^3)x^2 - 8\right.}{35\,(b^6x + ab^5)}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*b^4*x^4 - 128*B*a^4 + 112*A*a^3*b - (8*B*a*b^3 - 7*A*b^4)*x^3 +
2*(8*B*a^2*b^2 - 7*A*a*b^3)*x^2 - 8*(8*B*a^3*b - 7*A*a^2*b^2)*x)*sqrt(b*x
+ a)/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{7}{2}}}{7b} - \frac{a^3(-Ab+Ba)}{b\sqrt{a+bx}} + \frac{(a+bx)^{\frac{5}{2}}(Ab-4Ba)}{5b} + \frac{(a+bx)^{\frac{3}{2}}\left(-3Aab+6Ba^2\right)}{3b} + \frac{\sqrt{a+bx}\left(3Aa^2b-4Ba^3\right)}{b}\right)}{b^4} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4} + \frac{Bx^5}{5}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(7/2)/(7*b) - a**3*(-A*b + B*a)/(b*sqrt(a + b*x
)) + (a + b*x)**(5/2)*(A*b - 4*B*a)/(5*b) + (a + b*x)**(3/2)*(-3*A*a*b + 6
*B*a**2)/(3*b) + sqrt(a + b*x)*(3*A*a**2*b - 4*B*a**3)/b)/b**4, Ne(b, 0)),
 ((A*x**4/4 + B*x**5/5)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\left(\frac{5\,(bx+a)^{\frac{7}{2}}B-7\,(4\,Ba-Ab)(bx+a)^{\frac{5}{2}}+35\,(2\,Ba^2-Aab)(bx+a)^{\frac{3}{2}}-35\,(4\,Ba^3-3\,Aa^2b)\sqrt{bx+a}}{b}-\frac{35\,(Ba^4-Aa^3b}{\sqrt{bx+a}b}\right.}{35\,b^4}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/35*((5*(b*x + a)^(7/2)*B - 7*(4*B*a - A*b)*(b*x + a)^(5/2) + 35*(2*B*a^2
 - A*a*b)*(b*x + a)^(3/2) - 35*(4*B*a^3 - 3*A*a^2*b)*sqrt(b*x + a))/b - 35
*(B*a^4 - A*a^3*b)/(sqrt(b*x + a)*b))/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = -\frac{2\,(Ba^4-Aa^3b)}{\sqrt{bx+a}b^5} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}}Bb^{30}-28\,(bx+a)^{\frac{5}{2}}Bab^{30}+70\,(bx+a)^{\frac{3}{2}}Ba^2b^{30}-140\,\sqrt{bx+a}Ba^3b^{30}+7\,(bx+a)^{\frac{5}{2}}Ab^{31}\right.}{35\,b^{35}}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*(B*a^4 - A*a^3*b)/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*B*b^30
- 28*(b*x + a)^(5/2)*B*a*b^30 + 70*(b*x + a)^(3/2)*B*a^2*b^30 - 140*sqrt(b
*x + a)*B*a^3*b^30 + 7*(b*x + a)^(5/2)*A*b^31 - 35*(b*x + a)^(3/2)*A*a*b^3
1 + 105*sqrt(b*x + a)*A*a^2*b^31)/b^35
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{(12\,B\,a^2-6\,A\,a\,b)\,(a+b\,x)^{3/2}}{3\,b^5}+\frac{2\,B\,(a+b\,x)^{7/2}}{7\,b^5} + \frac{(2\,A\,b-8\,B\,a)\,(a+b\,x)^{5/2}}{5\,b^5}-\frac{2\,B\,a^4-2\,A\,a^3\,b}{b^5\,\sqrt{a+b\,x}}-\frac{(8\,B\,a^3-6\,A\,a^2\,b)\,\sqrt{a+b\,x}}{b^5}$$

input
```
int((x^3*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
((12*B*a^2 - 6*A*a*b)*(a + b*x)^(3/2))/(3*b^5) + (2*B*(a + b*x)^(7/2))/(7*
b^5) + ((2*A*b - 8*B*a)*(a + b*x)^(5/2))/(5*b^5) - (2*B*a^4 - 2*A*a^3*b)/(
b^5*(a + b*x)^(1/2)) - ((8*B*a^3 - 6*A*a^2*b)*(a + b*x)^(1/2))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.35

$$\int \frac{x^3(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\,(5b^3x^3-6a\,b^2x^2+8a^2bx-16a^3)}{35b^4}$$

input
```
int(x^3*(B*x+A)/(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**3 + 8*a**2*b*x - 6*a*b**2*x**2 + 5*b**3*x**3))/
(35*b**4)
```

3.269 $\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = -\frac{2a^2(Ab-aB)}{b^4\sqrt{a+bx}} - \frac{2a(2Ab-3aB)\sqrt{a+bx}}{b^4} + \frac{2(Ab-3aB)(a+bx)^{3/2}}{3b^4} + \frac{2B(a+bx)^{5/2}}{5b^4}$$

output
```
-2*a^2*(A*b-B*a)/b^4/(b*x+a)^(1/2)-2*a*(2*A*b-3*B*a)*(b*x+a)^(1/2)/b^4+2/3
*(A*b-3*B*a)*(b*x+a)^(3/2)/b^4+2/5*B*(b*x+a)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2(48a^3B-8a^2b(5A-3Bx)+b^3x^2(5A+3Bx)-2ab^2x(10A+3Bx))}{15b^4\sqrt{a+bx}}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x) + b^3*x^2*(5*A + 3*B*x) - 2*a*b^2*x*(
10*A + 3*B*x)))/(15*b^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)^{3/2}}+\frac{a(3aB-2Ab)}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}(Ab-3aB)}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2(Ab-aB)}{b^4\sqrt{a+bx}}+\frac{2(a+bx)^{3/2}(Ab-3aB)}{3b^4}-\frac{2a\sqrt{a+bx}(2Ab-3aB)}{b^4}+\frac{2B(a+bx)^{5/2}}{5b^4}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(-2*a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a + b
*x])/b^4 + (2*(A*b - 3*a*B)*(a + b*x)^(3/2))/(3*b^4) + (2*B*(a + b*x)^(5/2
))/(5*b^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{16\left(-\frac{\left(\frac{3Bx}{5}+A\right)x^{2}b^{3}}{8}+\frac{a\left(\frac{3Bx}{10}+A\right)x\,b^{2}}{2}+a^{2}\left(-\frac{3Bx}{5}+A\right)b-\frac{6a^{3}B}{5}\right)}{3\sqrt{bx+a}\,b^{4}}$	58
gosper	$-\frac{2\left(-3b^{3}B\,x^{3}-5A\,x^{2}b^{3}+6B\,x^{2}a\,b^{2}+20Axa\,b^{2}-24Bx\,a^{2}b+40a^{2}bA-48a^{3}B\right)}{15\sqrt{bx+a}\,b^{4}}$	71
trager	$-\frac{2\left(-3b^{3}B\,x^{3}-5A\,x^{2}b^{3}+6B\,x^{2}a\,b^{2}+20Axa\,b^{2}-24Bx\,a^{2}b+40a^{2}bA-48a^{3}B\right)}{15\sqrt{bx+a}\,b^{4}}$	71
risch	$-\frac{2\left(-3b^{2}B\,x^{2}-5A\,b^{2}x+9Babx+25abA-33a^{2}B\right)\sqrt{bx+a}}{15b^{4}}-\frac{2a^{2}\left(Ab-Ba\right)}{b^{4}\sqrt{bx+a}}$	71
orering	$-\frac{2\left(-3b^{3}B\,x^{3}-5A\,x^{2}b^{3}+6B\,x^{2}a\,b^{2}+20Axa\,b^{2}-24Bx\,a^{2}b+40a^{2}bA-48a^{3}B\right)}{15\sqrt{bx+a}\,b^{4}}$	71
derivativedivides	$\frac{\frac{2B\left(bx+a\right)^{\frac{5}{2}}}{5}+\frac{2Ab\left(bx+a\right)^{\frac{3}{2}}}{3}-2Ba\left(bx+a\right)^{\frac{3}{2}}-4Aab\sqrt{bx+a}+6B\,a^{2}\sqrt{bx+a}-\frac{2a^{2}\left(Ab-Ba\right)}{\sqrt{bx+a}}}{b^{4}}$	84
default	$\frac{\frac{2B\left(bx+a\right)^{\frac{5}{2}}}{5}+\frac{2Ab\left(bx+a\right)^{\frac{3}{2}}}{3}-2Ba\left(bx+a\right)^{\frac{3}{2}}-4Aab\sqrt{bx+a}+6B\,a^{2}\sqrt{bx+a}-\frac{2a^{2}\left(Ab-Ba\right)}{\sqrt{bx+a}}}{b^{4}}$	84

input
```
int(x^2*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/3*(-1/8*(3/5*B*x+A)*x^2*b^3+1/2*a*(3/10*B*x+A)*x*b^2+a^2*(-3/5*B*x+A)*
b-6/5*a^3*B)/(b*x+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\left(3\,Bb^3x^3+48\,Ba^3-40\,Aa^2b-(6\,Bab^2-5\,Ab^3)x^2+4\,(6\,Ba^2b-5\,Aab^2)x\right)\sqrt{bx+}}{15\,(b^5x+ab^4)}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^3*x^3 + 48*B*a^3 - 40*A*a^2*b - (6*B*a*b^2 - 5*A*b^3)*x^2 + 4*
(6*B*a^2*b - 5*A*a*b^2)*x)*sqrt(b*x + a)/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{5}{2}}}{5b}+\frac{a^2(-Ab+Ba)}{b\sqrt{a+bx}}+\frac{(a+bx)^{\frac{3}{2}}(Ab-3Ba)}{3b}+\frac{\sqrt{a+bx}\left(-2Aab+3Ba^2\right)}{b}\right)}{b^3} & \text{for } b\neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(5/2)/(5*b) + a**2*(-A*b + B*a)/(b*sqrt(a + b*x
)) + (a + b*x)**(3/2)*(A*b - 3*B*a)/(3*b) + sqrt(a + b*x)*(-2*A*a*b + 3*B*
a**2)/b)/b**3, Ne(b, 0)), ((A*x**3/3 + B*x**4/4)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\left(\frac{3(bx+a)^{\frac{5}{2}}B-5(3Ba-Ab)(bx+a)^{\frac{3}{2}}+15(3Ba^2-2Aab)\sqrt{bx+a}}{b}+\frac{15(Ba^3-Aa^2b)}{\sqrt{bx+a}b}\right)}{15b^3}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(b*x + a)^(5/2)*B - 5*(3*B*a - A*b)*(b*x + a)^(3/2) + 15*(3*B*a^2
 - 2*A*a*b)*sqrt(b*x + a))/b + 15*(B*a^3 - A*a^2*b)/(sqrt(b*x + a)*b))/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\,(Ba^3-Aa^2b)}{\sqrt{bx+a}b^4} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}Bb^{16}-15\,(bx+a)^{\frac{3}{2}}Bab^{16}+45\,\sqrt{bx+a}Ba^2b^{16}+5\,(bx+a)^{\frac{3}{2}}Ab^{17}-30\,\sqrt{bx+a}Aab^{17}\right)}{15\,b^{20}}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*(B*a^3 - A*a^2*b)/(sqrt(b*x + a)*b^4) + 2/15*(3*(b*x + a)^(5/2)*B*b^16 -
 15*(b*x + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x + a)*B*a^2*b^16 + 5*(b*x + a)^(
3/2)*A*b^17 - 30*sqrt(b*x + a)*A*a*b^17)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{(6\,B\,a^2-4\,A\,a\,b)\,\sqrt{a+b\,x}}{b^4} + \frac{2\,B\,(a+b\,x)^{5/2}}{5\,b^4} + \frac{(2\,A\,b-6\,B\,a)\,(a+b\,x)^{3/2}}{3\,b^4} + \frac{2\,B\,a^3-2\,A\,a^2\,b}{b^4\,\sqrt{a+b\,x}}$$

input
```
int((x^2*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
((6*B*a^2 - 4*A*a*b)*(a + b*x)^(1/2))/b^4 + (2*B*(a + b*x)^(5/2))/(5*b^4)
+ ((2*A*b - 6*B*a)*(a + b*x)^(3/2))/(3*b^4) + (2*B*a^3 - 2*A*a^2*b)/(b^4*(
a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.33

$$\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$$

input
```
int(x^2*(B*x+A)/(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2 - 4*a*b*x + 3*b**2*x**2))/(15*b**3)
```

3.270 $\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{2a(Ab-aB)}{b^3\sqrt{a+bx}}+\frac{2(Ab-2aB)\sqrt{a+bx}}{b^3}+\frac{2B(a+bx)^{3/2}}{3b^3}$$

output
```
2*a*(A*b-B*a)/b^3/(b*x+a)^(1/2)+2*(A*b-2*B*a)*(b*x+a)^(1/2)/b^3+2/3*B*(b*x
+a)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{2(-8a^2B+b^2x(3A+Bx)+a(6Ab-4bBx))}{3b^3\sqrt{a+bx}}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*(-8*a^2*B + b^2*x*(3*A + B*x) + a*(6*A*b - 4*b*B*x)))/(3*b^3*Sqrt[a + b
*x])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{Ab-2aB}{b^2\sqrt{a+bx}}+\frac{a(aB-Ab)}{b^2(a+bx)^{3/2}}+\frac{B\sqrt{a+bx}}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2\sqrt{a+bx}(Ab-2aB)}{b^3}+\frac{2a(Ab-aB)}{b^3\sqrt{a+bx}}+\frac{2B(a+bx)^{3/2}}{3b^3}$$

input
```
Int[(x*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*a*(A*b - a*B))/(b^3*Sqrt[a + b*x]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x])/b^
3 + (2*B*(a + b*x)^(3/2))/(3*b^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$\frac{2\left(\frac{Bx}{3}+A\right)x\,b^{2}+4a\left(-\frac{2Bx}{3}+A\right)b-\frac{16a^{2}B}{3}}{\sqrt{bx+a}\,b^{3}}$	41
gosper	$\frac{\frac{2}{3}b^{2}B\,x^{2}+2A\,b^{2}x-\frac{8}{3}Babx+4abA-\frac{16}{3}a^{2}B}{\sqrt{bx+a}\,b^{3}}$	46
trager	$\frac{\frac{2}{3}b^{2}B\,x^{2}+2A\,b^{2}x-\frac{8}{3}Babx+4abA-\frac{16}{3}a^{2}B}{\sqrt{bx+a}\,b^{3}}$	46
orering	$\frac{\frac{2}{3}b^{2}B\,x^{2}+2A\,b^{2}x-\frac{8}{3}Babx+4abA-\frac{16}{3}a^{2}B}{\sqrt{bx+a}\,b^{3}}$	46
risch	$\frac{2(bBx+3Ab-5Ba)\sqrt{bx+a}}{3b^{3}}+\frac{2a(Ab-Ba)}{b^{3}\sqrt{bx+a}}$	48
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-4Ba\sqrt{bx+a}+\frac{2a(Ab-Ba)}{\sqrt{bx+a}}}{b^{3}}$	55
default	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-4Ba\sqrt{bx+a}+\frac{2a(Ab-Ba)}{\sqrt{bx+a}}}{b^{3}}$	55

input
```
int(x*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(1/2*(1/3*B*x+A)*x*b^2+a*(-2/3*B*x+A)*b-4/3*a^2*B)/(b*x+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\,(Bb^2x^2-8\,Ba^2+6\,Aab-(4\,Bab-3\,Ab^2)x)\sqrt{bx+a}}{3\,(b^4x+ab^3)}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(B*b^2*x^2 - 8*B*a^2 + 6*A*a*b - (4*B*a*b - 3*A*b^2)*x)*sqrt(b*x + a)/
(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{3}{2}}}{3b} - \frac{a(-Ab+Ba)}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}(Ab-2Ba)}{b}\right)}{b^2} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^3}{3}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(3/2)/(3*b) - a*(-A*b + B*a)/(b*sqrt(a + b*x))
+ sqrt(a + b*x)*(A*b - 2*B*a)/b)/b**2, Ne(b, 0)), ((A*x**2/2 + B*x**3/3)/a
**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\left(\frac{(bx+a)^{\frac{3}{2}}B - 3(2Ba-Ab)\sqrt{bx+a}}{b} - \frac{3(Ba^2-Aab)}{\sqrt{bx+a}b}\right)}{3b^2}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((b*x + a)^(3/2)*B - 3*(2*B*a - A*b)*sqrt(b*x + a))/b - 3*(B*a^2 - A*
a*b)/(sqrt(b*x + a)*b))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx = -\frac{2\,(Ba^2-Aab)}{\sqrt{bx+a}b^3} + \frac{2\left((bx+a)^{\frac{3}{2}}Bb^6-6\sqrt{bx+a}Bab^6+3\sqrt{bx+a}Ab^7\right)}{3\,b^9}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*(B*a^2 - A*a*b)/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*B*b^6 - 6*sq
rt(b*x + a)*B*a*b^6 + 3*sqrt(b*x + a)*A*b^7)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\,B\,(a+b\,x)^2-6\,B\,a^2+6\,A\,a\,b+6\,A\,b\,(a+b\,x)-12\,B\,a\,(a+b\,x)}{3\,b^3\,\sqrt{a+b\,x}}$$

input
```
int((x*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
(2*B*(a + b*x)^2 - 6*B*a^2 + 6*A*a*b + 6*A*b*(a + b*x) - 12*B*a*(a + b*x))
/(3*b^3*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.29

$$\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\,(bx-2a)}{3b^2}$$

input
```
int(x*(B*x+A)/(b*x+a)^(3/2),x)
```

output

```
(2*sqrt(a + b*x)*( - 2*a + b*x))/(3*b**2)
```

3.271 $\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx=-\frac{2(Ab-aB)}{b^2\sqrt{a+bx}}+\frac{2B\sqrt{a+bx}}{b^2}$$

output
```
(-2*A*b+2*B*a)/b^2/(b*x+a)^(1/2)+2*B*(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx=\frac{2(-Ab+2aB+bBx)}{b^2\sqrt{a+bx}}$$

input
```
Integrate[(A + B*x)/(a + b*x)^(3/2),x]
```

output
```
(2*(-(A*b) + 2*a*B + b*B*x))/(b^2*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2B\sqrt{a+bx}}{b^2}-\frac{2(Ab-aB)}{b^2\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(a + b*x)^(3/2),x]
```

output
```
(-2*(A*b - a*B))/(b^2*Sqrt[a + b*x]) + (2*B*Sqrt[a + b*x])/b^2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2(-bBx+Ab-2Ba)}{\sqrt{bx+a}\,b^2}$	26
trager	$-\frac{2(-bBx+Ab-2Ba)}{\sqrt{bx+a}\,b^2}$	26
orering	$-\frac{2(-bBx+Ab-2Ba)}{\sqrt{bx+a}\,b^2}$	26
pseudoelliptic	$\frac{(2Bx-2A)b+4Ba}{\sqrt{bx+a}\,b^2}$	27
derivativedivides	$\frac{2B\sqrt{bx+a}-\frac{2(Ab-Ba)}{\sqrt{bx+a}}}{b^2}$	33
default	$\frac{2B\sqrt{bx+a}-\frac{2(Ab-Ba)}{\sqrt{bx+a}}}{b^2}$	33
risch	$\frac{2B\sqrt{bx+a}}{b^2}-\frac{2(Ab-Ba)}{b^2\sqrt{bx+a}}$	35

input
```
int((B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(b*x+a)^(1/2)*(-B*b*x+A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx=\frac{2\,(Bbx+2\,Ba-Ab)\sqrt{bx+a}}{b^3x+ab^2}$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*(B*b*x + 2*B*a - A*b)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx = \begin{cases} -\frac{2A}{b\sqrt{a+bx}} + \frac{4Ba}{b^2\sqrt{a+bx}} + \frac{2Bx}{b\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Piecewise((-2*A/(b*sqrt(a + b*x)) + 4*B*a/(b**2*sqrt(a + b*x)) + 2*B*x/(b*
sqrt(a + b*x)), Ne(b, 0)), ((A*x + B*x**2/2)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx+a}B}{b} + \frac{Ba-Ab}{\sqrt{bx+a}b}\right)}{b}$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*(sqrt(b*x + a)*B/b + (B*a - A*b)/(sqrt(b*x + a)*b))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}B}{b^2} + \frac{2(Ba-Ab)}{\sqrt{bx+a}b^2}$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x + a)*B/b^2 + 2*(B*a - A*b)/(sqrt(b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx=\frac{4\,B\,a-2\,A\,b+2\,B\,b\,x}{b^2\,\sqrt{a+b\,x}}$$

input
```
int((A + B*x)/(a + b*x)^(3/2),x)
```

output
```
(4*B*a - 2*A*b + 2*B*b*x)/(b^2*(a + b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx=\frac{2\sqrt{bx+a}}{b}$$

input
```
int((B*x+A)/(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x))/b
```

3.272 $\int \frac{A+Bx}{x(a+bx)^{3/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 50

$$\int \frac{A+Bx}{x(a+bx)^{3/2}} \, dx = \frac{2(Ab-aB)}{ab\sqrt{a+bx}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
2*(A*b-B*a)/a/b/(b*x+a)^(1/2)-2*A*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x(a+bx)^{3/2}} \, dx = -\frac{2(-Ab+aB)}{ab\sqrt{a+bx}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x)^(3/2)),x]
```

output
```
(-2*(-(A*b) + a*B))/(a*b*Sqrt[a + b*x]) - (2*A*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{A\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2(Ab-aB)}{ab\sqrt{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2A\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2(Ab-aB)}{ab\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(Ab-aB)}{ab\sqrt{a+bx}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Int[(A + B*x)/(x*(a + b*x)^(3/2)),x]
```

output
```
(2*(A*b - a*B))/(a*b*Sqrt[a + b*x]) - (2*A*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])
/a^(3/2)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{\frac{2(Ab-Ba)}{a\sqrt{bx+a}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{b}$	45
derivativedivides	$\frac{-\frac{2(-Ab+Ba)}{a\sqrt{bx+a}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{b}$	46
default	$\frac{-\frac{2(-Ab+Ba)}{a\sqrt{bx+a}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{b}$	46

input
```
int((B*x+A)/x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*((A*b-B*a)/a/(b*x+a)^(1/2)-A*b/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx = \left[\frac{(Ab^2x+Aab)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2(Ba^2-Aab)\sqrt{bx+a}}{a^2b^2x+a^3b}, \frac{2\left((Ab^2x+Aab\right.}{}\right.$$

input
```
integrate((B*x+A)/x/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[((A*b^2*x + A*a*b)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
 2*(B*a^2 - A*a*b)*sqrt(b*x + a))/(a^2*b^2*x + a^3*b), 2*((A*b^2*x + A*a*b
)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - (B*a^2 - A*a*b)*sqrt(b*x + a))
/(a^2*b^2*x + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx = \begin{cases} \frac{2A\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{a\sqrt{-a}} - \frac{2(-Ab+Ba)}{ab\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{A\log(Bx)+Bx}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/x/(b*x+a)**(3/2),x)
```

output
```
Piecewise((2*A*atan(sqrt(a + b*x)/sqrt(-a))/(a*sqrt(-a)) - 2*(-A*b + B*a)/
(a*b*sqrt(a + b*x)), Ne(b, 0)), ((A*log(B*x) + B*x)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx = \frac{A\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2\,(Ba-Ab)}{\sqrt{bx+a}ab}$$

input
```
integrate((B*x+A)/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
A*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) - 2*(B*
a - A*b)/(sqrt(b*x + a)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{2\,(Ba-Ab)}{\sqrt{bx+a}ab}$$

input
```
integrate((B*x+A)/x/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*A*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) - 2*(B*a - A*b)/(sqrt(b*x
+ a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx = \frac{2\,(A\,b-B\,a)}{a\,b\,\sqrt{a+b\,x}} - \frac{2\,A\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((A + B*x)/(x*(a + b*x)^(3/2)),x)
```

output
```
(2*(A*b - B*a))/(a*b*(a + b*x)^(1/2)) - (2*A*atanh((a + b*x)^(1/2)/a^(1/2)
))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx = \frac{\sqrt{a}\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int((B*x+A)/x/(b*x+a)^(3/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.273 $\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx=-\frac{2(Ab-aB)}{a^2\sqrt{a+bx}}-\frac{A\sqrt{a+bx}}{a^2x}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
(-2*A*b+2*B*a)/a^2/(b*x+a)^(1/2)-A*(b*x+a)^(1/2)/a^2/x+(3*A*b-2*B*a)*arcta
nh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx=\frac{-aA-3Abx+2aBx}{a^2x\sqrt{a+bx}}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x)^(3/2)),x]
```

output
```
(-(a*A) - 3*A*b*x + 2*a*B*x)/(a^2*x*Sqrt[a + b*x]) + ((3*A*b - 2*a*B)*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(3Ab-2aB)\int \frac{1}{x(a+bx)^{3/2}}dx}{2a}-\frac{A}{ax\sqrt{a+bx}}$$

$$\Big\downarrow \ 61$$

$$-\frac{(3Ab-2aB)\left(\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{A}{ax\sqrt{a+bx}}$$

$$\Big\downarrow \ 73$$

$$-\frac{(3Ab-2aB)\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{A}{ax\sqrt{a+bx}}$$

$$\Big\downarrow \ 221$$

$$-\frac{(3Ab-2aB)\left(\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{A}{ax\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x)^(3/2)),x]
```

output
```
-(A/(a*x*Sqrt[a + b*x])) - ((3*A*b - 2*a*B)*(2/(a*Sqrt[a + b*x]) - (2*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{-\frac{A\sqrt{bx+a}}{x}+\frac{(3Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(Ab-Ba)}{\sqrt{bx+a}}}{a^2}$	61
derivativedivides	$\frac{-\frac{A\sqrt{bx+a}}{x}+\frac{(3Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{a^2}-\frac{2(Ab-Ba)}{a^2\sqrt{bx+a}}$	67
default	$\frac{-\frac{A\sqrt{bx+a}}{x}+\frac{(3Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{a^2}-\frac{2(Ab-Ba)}{a^2\sqrt{bx+a}}$	67
risch	$-\frac{A\sqrt{bx+a}}{a^2x}-\frac{-\frac{2(-2Ab+2Ba)}{\sqrt{bx+a}}-\frac{2(3Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a^2}$	68

input
```
int((B*x+A)/x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-A*(b*x+a)^(1/2)/x+(3*A*b-2*B*a)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1
/2))-2*(A*b-B*a)/(b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.89

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx = \left[-\frac{((2\,Bab-3\,Ab^2)x^2+(2\,Ba^2-3\,Aab)x)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(Aa^2-(2}{2\,(a^3bx^2+a^4x)}\right.$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(((2*B*a*b - 3*A*b^2)*x^2 + (2*B*a^2 - 3*A*a*b)*x)*sqrt(a)*log((b*x
+ 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(A*a^2 - (2*B*a^2 - 3*A*a*b)*x)*sq
rt(b*x + a))/(a^3*b*x^2 + a^4*x), (((2*B*a*b - 3*A*b^2)*x^2 + (2*B*a^2 - 3
*A*a*b)*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - (A*a^2 - (2*B*a^2 - 3
*A*a*b)*x)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(66) = 132$.

Time = 22.77 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx = A\left(-\frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}}\right) + B\left(\frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}\right)$$

input
```
integrate((B*x+A)/x**2/(b*x+a)**(3/2),x)
```

output
```
A*(-1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqr
t(a/(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)) + B*(2*a*
*3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/2) +
 a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
 + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt(1 +
 b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx = -\frac{1}{2}b\left(\frac{2\left(2\,Ba^2-2\,Aab-(2\,Ba-3\,Ab)(bx+a)\right)}{(bx+a)^{\frac{3}{2}}a^2b-\sqrt{bx+a}a^3b}-\frac{(2\,Ba-3\,Ab)\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}b}\right)$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*b*(2*(2*B*a^2 - 2*A*a*b - (2*B*a - 3*A*b)*(b*x + a))/((b*x + a)^(3/2)
*a^2*b - sqrt(b*x + a)*a^3*b) - (2*B*a - 3*A*b)*log((sqrt(b*x + a) - sqrt(
a))/(sqrt(b*x + a) + sqrt(a)))/(a^(5/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx = \frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{2\,(bx+a)Ba-2\,Ba^2-3\,(bx+a)Ab+2\,Aab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
(2*B*a - 3*A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (2*(b*x +
a)*B*a - 2*B*a^2 - 3*(b*x + a)*A*b + 2*A*a*b)/(((b*x + a)^(3/2) - sqrt(b*x
 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(3\,A\,b-2\,B\,a)}{a^{5/2}} - \frac{\frac{2\,(A\,b-B\,a)}{a}-\frac{(3\,A\,b-2\,B\,a)\,(a+b\,x)}{a^2}}{a\,\sqrt{a+b\,x}-{(a+b\,x)}^{3/2}}$$

input
```
int((A + B*x)/(x^2*(a + b*x)^(3/2)),x)
```

output
```
(atanh((a + b*x)^(1/2)/a^(1/2))*(3*A*b - 2*B*a))/a^(5/2) - ((2*(A*b - B*a)
)/a - ((3*A*b - 2*B*a)*(a + b*x))/a^2)/(a*(a + b*x)^(1/2) - (a + b*x)^(3/2
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx = \frac{-2\sqrt{bx+a}\,a-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx}{2a^2x}$$

input
```
int((B*x+A)/x^2/(b*x+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x)*a - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*x + sqrt(a)
*log(sqrt(a + b*x) + sqrt(a))*b*x)/(2*a**2*x)
```

3.274 $\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx=\frac{2b(Ab-aB)}{a^3\sqrt{a+bx}}-\frac{A\sqrt{a+bx}}{2a^2x^2}+\frac{(7Ab-4aB)\sqrt{a+bx}}{4a^3x}-\frac{3b(5Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
2*b*(A*b-B*a)/a^3/(b*x+a)^(1/2)-1/2*A*(b*x+a)^(1/2)/a^2/x^2+1/4*(7*A*b-4*B
*a)*(b*x+a)^(1/2)/a^3/x-3/4*b*(5*A*b-4*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))
/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx=\frac{15Ab^2x^2+abx(5A-12Bx)-2a^2(A+2Bx)}{4a^3x^2\sqrt{a+bx}}+\frac{3b(-5Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x)^(3/2)),x]
```

output
```
(15*A*b^2*x^2 + a*b*x*(5*A - 12*B*x) - 2*a^2*(A + 2*B*x))/(4*a^3*x^2*Sqrt[
a + b*x]) + (3*b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/
2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(5Ab-4aB)\int \frac{1}{x^2(a+bx)^{3/2}}dx}{4a}-\frac{A}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \; 52$$

$$-\frac{(5Ab-4aB)\left(-\frac{3b\int \frac{1}{x(a+bx)^{3/2}}dx}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{A}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \; 61$$

$$-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{A}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \; 73$$

$$-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{A}{2ax^2\sqrt{a+bx}}$$

$$\Big\downarrow \; 221$$

$$-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{A}{2ax^2\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x)^(3/2)),x]
```

output
```
-1/2*A/(a*x^2*Sqrt[a + b*x]) - ((5*A*b - 4*a*B)*(-(1/(a*x*Sqrt[a + b*x]))
- (3*b*(2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))
/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{bx+a}\,(-7Abx+4Bax+2Aa)}{4b\,x^{2}}-\frac{3(5Ab-4Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{2Ab-2Ba}{\sqrt{bx+a}}\right)}{a^{3}}$	80
risch	$-\frac{\sqrt{bx+a}\,(-7Abx+4Bax+2Aa)}{4a^{3}x^{2}}+\frac{b\left(-\frac{2(-8Ab+8Ba)}{\sqrt{bx+a}}-\frac{2(15Ab-12Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{8a^{3}}$	83
derivativedivides	$2b\left(-\frac{-Ab+Ba}{a^{3}\sqrt{bx+a}}-\frac{\frac{\left(-\frac{7Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(\frac{9}{8}abA-\frac{1}{2}a^{2}B\right)\sqrt{bx+a}}{b^{2}x^{2}}+\frac{3(5Ab-4Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{3}}\right)$	102
default	$2b\left(-\frac{-Ab+Ba}{a^{3}\sqrt{bx+a}}-\frac{\frac{\left(-\frac{7Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(\frac{9}{8}abA-\frac{1}{2}a^{2}B\right)\sqrt{bx+a}}{b^{2}x^{2}}+\frac{3(5Ab-4Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{3}}\right)$	102

input
```
int((B*x+A)/x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b/a^3*(-1/4*(b*x+a)^(1/2)/b*(-7*A*b*x+4*B*a*x+2*A*a)/x^2-3/4*(5*A*b-4*B*a)
/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*(A*b-B*a)/(b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx = \left[-\frac{3\left((4\,Bab^2-5\,Ab^3)x^3+(4\,Ba^2b-5\,Aab^2)x^2\right)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,(2\,A}{8\,(a^4bx^3+a^5x^2)}\right.$$

$$-\frac{3\left((4\,Bab^2-5\,Ab^3)x^3+(4\,Ba^2b-5\,Aab^2)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+\left(2\,Aa^3+3\,(4\,Ba^2b-5\,Aab^2)x^2+\right.}{4\,(a^4bx^3+a^5x^2)}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((4*B*a*b^2 - 5*A*b^3)*x^3 + (4*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(a)
*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*A*a^3 + 3*(4*B*a^2*b
- 5*A*a*b^2)*x^2 + (4*B*a^3 - 5*A*a^2*b)*x)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2), -1/4*(3*((4*B*a*b^2 - 5*A*b^3)*x^3 + (4*B*a^2*b - 5*A*a*b^2)*x^2)*
sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) + (2*A*a^3 + 3*(4*B*a^2*b - 5*A*a*
b^2)*x^2 + (4*B*a^3 - 5*A*a^2*b)*x)*sqrt(b*x + a))/(a^4*b*x^3 + a^5*x^2)]
```

Sympy [A] (verification not implemented)

Time = 51.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx = A\left(-\frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}\right) + B\left(-\frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}}\right)$$

input
```
integrate((B*x+A)/x**3/(b*x+a)**(3/2),x)
```

output
```
A*(-1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2
)*sqrt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15
*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))) + B*(-1/(a*sqrt(b)*x*
*(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a/(b*x) + 1)) + 3
*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx = -\frac{1}{8}\,b^2\left(\frac{2\left(8\,Ba^3-8\,Aa^2b+3\,(4\,Ba-5\,Ab)(bx+a)^2-5\,(4\,Ba^2-5\,Aab)(bx+a)\right)}{(bx+a)^{\frac{5}{2}}a^3b-2\,(bx+a)^{\frac{3}{2}}a^4b+\sqrt{bx+a}a^5b}+\frac{3\,(4\,Ba-5\,Ab)\log}{a^{\frac{7}{2}}b}\right.$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*b^2*(2*(8*B*a^3 - 8*A*a^2*b + 3*(4*B*a - 5*A*b)*(b*x + a)^2 - 5*(4*B*
a^2 - 5*A*a*b)*(b*x + a))/((b*x + a)^(5/2)*a^3*b - 2*(b*x + a)^(3/2)*a^4*b
 + sqrt(b*x + a)*a^5*b) + 3*(4*B*a - 5*A*b)*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/(a^(7/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx = -\frac{3\,(4\,Bab-5\,Ab^2)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3}-\frac{2\,(Bab-Ab^2)}{\sqrt{bx+a}a^3} - \frac{4\,(bx+a)^{\frac{3}{2}}Bab-4\,\sqrt{bx+a}Ba^2b-7\,(bx+a)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx+a}Aab^2}{4\,a^3b^2x^2}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-3/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2
*(B*a*b - A*b^2)/(sqrt(b*x + a)*a^3) - 1/4*(4*(b*x + a)^(3/2)*B*a*b - 4*sq
rt(b*x + a)*B*a^2*b - 7*(b*x + a)^(3/2)*A*b^2 + 9*sqrt(b*x + a)*A*a*b^2)/(
a^3*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx = \frac{\frac{2\,(A\,b^2-B\,a\,b)}{a}-\frac{5\,(5\,A\,b^2-4\,B\,a\,b)\,(a+b\,x)}{4\,a^2}+\frac{3\,(5\,A\,b^2-4\,B\,a\,b)\,(a+b\,x)^2}{4\,a^3}}{(a+b\,x)^{5/2}-2\,a\,(a+b\,x)^{3/2}+a^2\,\sqrt{a+b\,x}} - \frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(5\,A\,b-4\,B\,a)}{4\,a^{7/2}}$$

input
```
int((A + B*x)/(x^3*(a + b*x)^(3/2)),x)
```

output
```
((2*(A*b^2 - B*a*b))/a - (5*(5*A*b^2 - 4*B*a*b)*(a + b*x))/(4*a^2) + (3*(5
*A*b^2 - 4*B*a*b)*(a + b*x)^2)/(4*a^3))/((a + b*x)^(5/2) - 2*a*(a + b*x)^(
3/2) + a^2*(a + b*x)^(1/2)) - (3*b*atanh((a + b*x)^(1/2)/a^(1/2))*(5*A*b -
 4*B*a))/(4*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx = \frac{-4\sqrt{bx+a}\,a^2+6\sqrt{bx+a}\,abx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-3\sqrt{a}\log\left(\sqrt{bx+a}\right.}{8a^3x^2}$$

input
```
int((B*x+A)/x^3/(b*x+a)^(3/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2 + 6*sqrt(a + b*x)*a*b*x + 3*sqrt(a)*log(sqrt(a +
b*x) - sqrt(a))*b**2*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*x*
*2)/(8*a**3*x**2)
```

3.275 $\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = -\frac{2b^2(Ab-aB)}{a^4\sqrt{a+bx}} - \frac{A\sqrt{a+bx}}{3a^2x^3} + \frac{(11Ab-6aB)\sqrt{a+bx}}{12a^3x^2} - \frac{b(19Ab-14aB)\sqrt{a+bx}}{8a^4x} + \frac{5b^2(7Ab-6aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{9/2}}$$

output
```
-2*b^2*(A*b-B*a)/a^4/(b*x+a)^(1/2)-1/3*A*(b*x+a)^(1/2)/a^2/x^3+1/12*(11*A*
b-6*B*a)*(b*x+a)^(1/2)/a^3/x^2-1/8*b*(19*A*b-14*B*a)*(b*x+a)^(1/2)/a^4/x+5
/8*b^2*(7*A*b-6*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = \frac{-105Ab^3x^3-4a^3(2A+3Bx)+2a^2bx(7A+15Bx)+5ab^2x^2(-7A+18Bx)}{24a^4x^3\sqrt{a+bx}} + \frac{5b^2(7Ab-6aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^4*(a + b*x)^(3/2)),x]
```

output
```
(-105*A*b^3*x^3 - 4*a^3*(2*A + 3*B*x) + 2*a^2*b*x*(7*A + 15*B*x) + 5*a*b^2
*x^2*(-7*A + 18*B*x))/(24*a^4*x^3*Sqrt[a + b*x]) + (5*b^2*(7*A*b - 6*a*B)*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(7Ab-6aB)\int \frac{1}{x^3(a+bx)^{3/2}}dx}{6a}-\frac{A}{3ax^3\sqrt{a+bx}}$$

$$\Big\downarrow \ 52$$

$$-\frac{(7Ab-6aB)\left(-\frac{5b\int \frac{1}{x^2(a+bx)^{3/2}}dx}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}\right)}{6a}-\frac{A}{3ax^3\sqrt{a+bx}}$$

$$\Big\downarrow \ 52$$

$$-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x(a+bx)^{3/2}}dx}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}\right)}{6a}-\frac{A}{3ax^3\sqrt{a+bx}}$$

$$\Big\downarrow \ 61$$

$$-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{\int\frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}\right)}{6a}-\frac{A}{3ax^3\sqrt{a+bx}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{2\int\frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}\right)}{6a}-\frac{A}{3ax^3\sqrt{a+bx}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax\sqrt{a+bx}}\right)}{4a}-\frac{1}{2ax^2\sqrt{a+bx}}\right)}{6a}-\frac{A}{3ax^3\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^4*(a + b*x)^(3/2)),x]
```

output
```
-1/3*A/(a*x^3*Sqrt[a + b*x]) - ((7*A*b - 6*a*B)*(-1/2*1/(a*x^2*Sqrt[a + b*
x]) - (5*b*(-(1/(a*x*Sqrt[a + b*x])) - (3*b*(2/(a*Sqrt[a + b*x]) - (2*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{35\left(Ab-\frac{6Ba}{7}\right)\sqrt{bx+a}\,b^{2}x^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8}-\frac{35\left(-\frac{18Bx}{7}+A\right)b^{2}x^{2}a^{\frac{3}{2}}}{24}+\frac{7bx\left(\frac{15Bx}{7}+A\right)a^{\frac{5}{2}}}{12}+\frac{(-3Bx-2A)a^{\frac{7}{2}}}{6}-\frac{35A\sqrt{a}\,b^{3}x^{3}}{8}}{x^{3}a^{\frac{9}{2}}\sqrt{bx+a}}$
risch	$-\frac{\sqrt{bx+a}\,(57A\,b^{2}x^{2}-42Bab\,x^{2}-22aAbx+12B\,a^{2}x+8a^{2}A)}{24a^{4}x^{3}}-\frac{b^{2}\left(-\frac{2(-16Ab+16Ba)}{\sqrt{bx+a}}-\frac{2(35Ab-30Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right.}{16a^{4}}$
derivativedivides	$2b^{2}\left(-\frac{\frac{\left(\frac{19Ab}{16}-\frac{7Ba}{8}\right)(bx+a)^{\frac{5}{2}}+\left(-\frac{17}{6}abA+2a^{2}B\right)(bx+a)^{\frac{3}{2}}+\left(\frac{29}{16}a^{2}bA-\frac{9}{8}a^{3}B\right)\sqrt{bx+a}}{b^{3}x^{3}}+\frac{5(7Ab-6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}}{a^{4}}-\right.$
default	$2b^{2}\left(-\frac{\frac{\left(\frac{19Ab}{16}-\frac{7Ba}{8}\right)(bx+a)^{\frac{5}{2}}+\left(-\frac{17}{6}abA+2a^{2}B\right)(bx+a)^{\frac{3}{2}}+\left(\frac{29}{16}a^{2}bA-\frac{9}{8}a^{3}B\right)\sqrt{bx+a}}{b^{3}x^{3}}+\frac{5(7Ab-6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}}{a^{4}}-\right.$

input
```
int((B*x+A)/x^4/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/12/(b*x+a)^(1/2)*(15/2*(A*b-6/7*B*a)*(b*x+a)^(1/2)*b^2*x^3*arctanh((b*x+
a)^(1/2)/a^(1/2))-5/2*(-18/7*B*x+A)*b^2*x^2*a^(3/2)+b*x*(15/7*B*x+A)*a^(5/
2)+2/7*(-3*B*x-2*A)*a^(7/2)-15/2*A*a^(1/2)*b^3*x^3)/a^(9/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.34

$$\int \frac{A+Bx}{x^{4}(a+bx)^{3/2}}\,dx=\left[-\frac{15\left((6\,Bab^{3}-7\,Ab^{4})x^{4}+(6\,Ba^{2}b^{2}-7\,Aab^{3})x^{3}\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(8}{48\,(}\right.$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*((6*B*a*b^3 - 7*A*b^4)*x^4 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^3)*sqr
t(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(8*A*a^4 - 15*(6*B*a
^2*b^2 - 7*A*a*b^3)*x^3 - 5*(6*B*a^3*b - 7*A*a^2*b^2)*x^2 + 2*(6*B*a^4 - 7
*A*a^3*b)*x)*sqrt(b*x + a))/(a^5*b*x^4 + a^6*x^3), 1/24*(15*((6*B*a*b^3 -
7*A*b^4)*x^4 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^3)*sqrt(-a)*arctan(sqrt(-a)/sqr
t(b*x + a)) - (8*A*a^4 - 15*(6*B*a^2*b^2 - 7*A*a*b^3)*x^3 - 5*(6*B*a^3*b -
 7*A*a^2*b^2)*x^2 + 2*(6*B*a^4 - 7*A*a^3*b)*x)*sqrt(b*x + a))/(a^5*b*x^4 +
 a^6*x^3)]
```

Sympy [A] (verification not implemented)

Time = 83.42 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = A\left(-\frac{1}{3a\sqrt{b}x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{7\sqrt{b}}{12a^2x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{35b^{\frac{3}{2}}}{24a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{35b^{\frac{5}{2}}}{8a^4\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{9}{2}}}\right) + B\left(-\frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}\right)$$

input
```
integrate((B*x+A)/x**4/(b*x+a)**(3/2),x)
```

output
```
A*(-1/(3*a*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + 7*sqrt(b)/(12*a**2*x**(5/
2)*sqrt(a/(b*x) + 1)) - 35*b**(3/2)/(24*a**3*x**(3/2)*sqrt(a/(b*x) + 1)) -
 35*b**(5/2)/(8*a**4*sqrt(x)*sqrt(a/(b*x) + 1)) + 35*b**3*asinh(sqrt(a)/(s
qrt(b)*sqrt(x)))/(8*a**(9/2))) + B*(-1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x)
+ 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**
3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4
*a**(7/2)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = -\frac{1}{48}\,b^3\left(\frac{2\,(48\,Ba^4-48\,Aa^3b-15\,(6\,Ba-7\,Ab)(bx+a)^3+40\,(6\,Ba^2-7\,Aab)(bx+a)^2-33\,(6\,Ba^3-7\right.}{(bx+a)^{\frac{7}{2}}a^4b-3\,(bx+a)^{\frac{5}{2}}a^5b+3\,(bx+a)^{\frac{3}{2}}a^6b-\sqrt{bx+a}a^7b}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/48*b^3*(2*(48*B*a^4 - 48*A*a^3*b - 15*(6*B*a - 7*A*b)*(b*x + a)^3 + 40*
(6*B*a^2 - 7*A*a*b)*(b*x + a)^2 - 33*(6*B*a^3 - 7*A*a^2*b)*(b*x + a))/((b*
x + a)^(7/2)*a^4*b - 3*(b*x + a)^(5/2)*a^5*b + 3*(b*x + a)^(3/2)*a^6*b - s
qrt(b*x + a)*a^7*b) - 15*(6*B*a - 7*A*b)*log((sqrt(b*x + a) - sqrt(a))/(sq
rt(b*x + a) + sqrt(a)))/(a^(9/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = \frac{5\,(6\,Bab^2-7\,Ab^3)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4}+\frac{2\,(Bab^2-Ab^3)}{\sqrt{bx+a}a^4} + \frac{42\,(bx+a)^{\frac{5}{2}}Bab^2-96\,(bx+a)^{\frac{3}{2}}Ba^2b^2+54\,\sqrt{bx+a}Ba^3b^2-57\,(bx+a)^{\frac{5}{2}}Ab^3+136\,(bx+a)^{\frac{3}{2}}Aab^3-87}{24\,a^4b^3x^3}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
5/8*(6*B*a*b^2 - 7*A*b^3)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) +
2*(B*a*b^2 - A*b^3)/(sqrt(b*x + a)*a^4) + 1/24*(42*(b*x + a)^(5/2)*B*a*b^2
 - 96*(b*x + a)^(3/2)*B*a^2*b^2 + 54*sqrt(b*x + a)*B*a^3*b^2 - 57*(b*x + a
)^(5/2)*A*b^3 + 136*(b*x + a)^(3/2)*A*a*b^3 - 87*sqrt(b*x + a)*A*a^2*b^3)/
(a^4*b^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = \frac{5\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,\left(7\,A\,b-6\,B\,a\right)}{8\,a^{9/2}} - \frac{\frac{2\,\left(A\,b^3-B\,a\,b^2\right)}{a}-\frac{11\,\left(7\,A\,b^3-6\,B\,a\,b^2\right)\,\left(a+b\,x\right)}{8\,a^2}+\frac{5\,\left(7\,A\,b^3-6\,B\,a\,b^2\right)\,{\left(a+b\,x\right)}^2}{3\,a^3}-\frac{5\,\left(7\,A\,b^3-6\,B\,a\,b^2\right)\,{\left(a+b\,x\right)}^3}{8\,a^4}}{3\,a\,{\left(a+b\,x\right)}^{5/2}-{\left(a+b\,x\right)}^{7/2}+a^3\,\sqrt{a+b\,x}-3\,a^2\,{\left(a+b\,x\right)}^{3/2}}$$

input
```
int((A + B*x)/(x^4*(a + b*x)^(3/2)),x)
```

output
```
(5*b^2*atanh((a + b*x)^(1/2)/a^(1/2))*(7*A*b - 6*B*a))/(8*a^(9/2)) - ((2*(
A*b^3 - B*a*b^2))/a - (11*(7*A*b^3 - 6*B*a*b^2)*(a + b*x))/(8*a^2) + (5*(7
*A*b^3 - 6*B*a*b^2)*(a + b*x)^2)/(3*a^3) - (5*(7*A*b^3 - 6*B*a*b^2)*(a + b
*x)^3)/(8*a^4))/(3*a*(a + b*x)^(5/2) - (a + b*x)^(7/2) + a^3*(a + b*x)^(1/
2) - 3*a^2*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx = \frac{-16\sqrt{bx+a}\,a^3+20\sqrt{bx+a}\,a^2bx-30\sqrt{bx+a}\,a\,b^2x^2-15\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{48a^4x^3}$$

input
```
int((B*x+A)/x^4/(b*x+a)^(3/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3 + 20*sqrt(a + b*x)*a**2*b*x - 30*sqrt(a + b*x)*a
*b**2*x**2 - 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 15*sqrt(a
)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3)/(48*a**4*x**3)
```

3.276 $\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2a^4(Ab-aB)}{3b^6(a+bx)^{3/2}}+\frac{2a^3(4Ab-5aB)}{b^6\sqrt{a+bx}} +\frac{4a^2(3Ab-5aB)\sqrt{a+bx}}{b^6}-\frac{4a(2Ab-5aB)(a+bx)^{3/2}}{3b^6} +\frac{2(Ab-5aB)(a+bx)^{5/2}}{5b^6}+\frac{2B(a+bx)^{7/2}}{7b^6}$$

output
```
-2/3*a^4*(A*b-B*a)/b^6/(b*x+a)^(3/2)+2*a^3*(4*A*b-5*B*a)/b^6/(b*x+a)^(1/2)
+4*a^2*(3*A*b-5*B*a)*(b*x+a)^(1/2)/b^6-4/3*a*(2*A*b-5*B*a)*(b*x+a)^(3/2)/b
^6+2/5*(A*b-5*B*a)*(b*x+a)^(5/2)/b^6+2/7*B*(b*x+a)^(7/2)/b^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{-2560a^5B+256a^4b(7A-15Bx)+192a^3b^2x(14A-5Bx)+6b^5x^4(7A+5Bx)+32}{105b^6(a+bx)^{3/2}}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(-2560*a^5*B + 256*a^4*b*(7*A - 15*B*x) + 192*a^3*b^2*x*(14*A - 5*B*x) + 6
*b^5*x^4*(7*A + 5*B*x) + 32*a^2*b^3*x^2*(21*A + 5*B*x) - 4*a*b^4*x^3*(28*A
 + 15*B*x))/(105*b^6*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{a^4(aB-Ab)}{b^5(a+bx)^{5/2}}+\frac{a^3(5aB-4Ab)}{b^5(a+bx)^{3/2}}-\frac{2a^2(5aB-3Ab)}{b^5\sqrt{a+bx}}+\frac{2a\sqrt{a+bx}(5aB-2Ab)}{b^5}+\frac{(a+bx)^{3/2}(Ab-5aB)}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4(Ab-aB)}{3b^6(a+bx)^{3/2}}+\frac{2a^3(4Ab-5aB)}{b^6\sqrt{a+bx}}+\frac{4a^2\sqrt{a+bx}(3Ab-5aB)}{b^6}-\frac{4a(a+bx)^{3/2}(2Ab-5aB)}{3b^6}+\frac{2(a+bx)^{5/2}(Ab-5aB)}{5b^6}+\frac{2B(a+bx)^{7/2}}{7b^6}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(-2*a^4*(A*b - a*B))/(3*b^6*(a + b*x)^(3/2)) + (2*a^3*(4*A*b - 5*a*B))/(b^
6*Sqrt[a + b*x]) + (4*a^2*(3*A*b - 5*a*B)*Sqrt[a + b*x])/b^6 - (4*a*(2*A*b
 - 5*a*B)*(a + b*x)^(3/2))/(3*b^6) + (2*(A*b - 5*a*B)*(a + b*x)^(5/2))/(5*
b^6) + (2*B*(a + b*x)^(7/2))/(7*b^6)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(30B\,x^5+42A\,x^4)b^5-112\left(\frac{15Bx}{28}+A\right)a\,x^3b^4+672\left(\frac{5Bx}{21}+A\right)a^2x^2b^3+2688a^3x\left(-\frac{5Bx}{14}+A\right)b^2+1792a^4\left(-\frac{15Bx}{7}+A\right)b-}{105(bx+a)^{\frac{3}{2}}b^6}$
risch	$\frac{2(15b^3B\,x^3+21A\,x^2b^3-60B\,x^2a\,b^2-98Axa\,b^2+185Bx\,a^2b+511a^2bA-790a^3B)\sqrt{bx+a}}{105b^6}+\frac{2a^3(12A\,b^2x-15Babx+1}{3b^6(bx+a)}$
gosper	$\frac{\frac{2}{7}b^5B\,x^5+\frac{2}{5}A\,b^5x^4-\frac{4}{7}Ba\,b^4x^4-\frac{16}{15}Aa\,b^4x^3+\frac{32}{21}B\,a^2b^3x^3+\frac{32}{5}A\,a^2b^3x^2-\frac{64}{7}B\,a^3b^2x^2+\frac{128}{5}a^3b^2Ax-\frac{256}{7}a^4bBx+\frac{256}{15}a}{(bx+a)^{\frac{3}{2}}b^6}$
trager	$\frac{\frac{2}{7}b^5B\,x^5+\frac{2}{5}A\,b^5x^4-\frac{4}{7}Ba\,b^4x^4-\frac{16}{15}Aa\,b^4x^3+\frac{32}{21}B\,a^2b^3x^3+\frac{32}{5}A\,a^2b^3x^2-\frac{64}{7}B\,a^3b^2x^2+\frac{128}{5}a^3b^2Ax-\frac{256}{7}a^4bBx+\frac{256}{15}a}{(bx+a)^{\frac{3}{2}}b^6}$
orering	$\frac{\frac{2}{7}b^5B\,x^5+\frac{2}{5}A\,b^5x^4-\frac{4}{7}Ba\,b^4x^4-\frac{16}{15}Aa\,b^4x^3+\frac{32}{21}B\,a^2b^3x^3+\frac{32}{5}A\,a^2b^3x^2-\frac{64}{7}B\,a^3b^2x^2+\frac{128}{5}a^3b^2Ax-\frac{256}{7}a^4bBx+\frac{256}{15}a}{(bx+a)^{\frac{3}{2}}b^6}$
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2Ab(bx+a)^{\frac{5}{2}}}{5}-2Ba(bx+a)^{\frac{5}{2}}-\frac{8Aab(bx+a)^{\frac{3}{2}}}{3}+\frac{20B\,a^2(bx+a)^{\frac{3}{2}}}{3}+12A\,a^2b\sqrt{bx+a}-20B\,a^3\sqrt{bx+a}+\frac{2a^3(4Ab-}{\sqrt{bx+}}}{b^6}$
default	$\frac{\frac{2B(bx+a)^{\frac{7}{2}}}{7}+\frac{2Ab(bx+a)^{\frac{5}{2}}}{5}-2Ba(bx+a)^{\frac{5}{2}}-\frac{8Aab(bx+a)^{\frac{3}{2}}}{3}+\frac{20B\,a^2(bx+a)^{\frac{3}{2}}}{3}+12A\,a^2b\sqrt{bx+a}-20B\,a^3\sqrt{bx+a}+\frac{2a^3(4Ab-}{\sqrt{bx+}}}{b^6}$

input
```
int(x^4*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((30*B*x^5+42*A*x^4)*b^5-112*(15/28*B*x+A)*a*x^3*b^4+672*(5/21*B*x+A
)*a^2*x^2*b^3+2688*a^3*x*(-5/14*B*x+A)*b^2+1792*a^4*(-15/7*B*x+A)*b-2560*a
^5*B)/(b*x+a)^(3/2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\,(15\,Bb^5x^5-1280\,Ba^5+896\,Aa^4b-3\,(10\,Bab^4-7\,Ab^5)x^4+8\,(10\,Ba^2b^3-7\,Aab^4) \ldots}{105\,(b^8x^2+2\,ab^7x+ \ldots}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^5*x^5 - 1280*B*a^5 + 896*A*a^4*b - 3*(10*B*a*b^4 - 7*A*b^5)*
x^4 + 8*(10*B*a^2*b^3 - 7*A*a*b^4)*x^3 - 48*(10*B*a^3*b^2 - 7*A*a^2*b^3)*x
^2 - 192*(10*B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(b*x + a)/(b^8*x^2 + 2*a*b^7*x
+ a^2*b^6)
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{7}{2}}}{7b}+\frac{a^4(-Ab+Ba)}{3b(a+bx)^{\frac{3}{2}}}-\frac{a^3(-4Ab+5Ba)}{b\sqrt{a+bx}}+\frac{(a+bx)^{\frac{5}{2}}(Ab-5Ba)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\left(-4Aab+10Ba^2\right)}{3b}+\frac{\sqrt{a+bx}\left(6Aa^2b-10Ba^3 \ldots\right.}{b}\right.}{b^5} & \ldots \\ \frac{\frac{Ax^5}{5}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \ldots \end{cases}$$

input
```
integrate(x**4*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(7/2)/(7*b) + a**4*(-A*b + B*a)/(3*b*(a + b*x)*
*(3/2)) - a**3*(-4*A*b + 5*B*a)/(b*sqrt(a + b*x)) + (a + b*x)**(5/2)*(A*b
- 5*B*a)/(5*b) + (a + b*x)**(3/2)*(-4*A*a*b + 10*B*a**2)/(3*b) + sqrt(a +
b*x)*(6*A*a**2*b - 10*B*a**3)/b)/b**5, Ne(b, 0)), ((A*x**5/5 + B*x**6/6)/a
**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{15\,(bx+a)^{\frac{7}{2}}B-21\,(5\,Ba-Ab)(bx+a)^{\frac{5}{2}}+70\,(5\,Ba^2-2\,Aab)\,(bx+a)^{\frac{3}{2}}-210\,(5\,Ba^3-3\,Aa^2b)\sqrt{bx+a}}{b}+\frac{35\,(Ba^5-\dots}{\dots}\right)}{105\,b^5}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/105*((15*(b*x + a)^(7/2)*B - 21*(5*B*a - A*b)*(b*x + a)^(5/2) + 70*(5*B*
a^2 - 2*A*a*b)*(b*x + a)^(3/2) - 210*(5*B*a^3 - 3*A*a^2*b)*sqrt(b*x + a))/
b + 35*(B*a^5 - A*a^4*b - 3*(5*B*a^4 - 4*A*a^3*b)*(b*x + a))/((b*x + a)^(3
/2)*b))/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2\,(15\,(bx+a)Ba^4-Ba^5-12\,(bx+a)Aa^3b+Aa^4b)}{3\,(bx+a)^{\frac{3}{2}}b^6} + \frac{2\left(15\,(bx+a)^{\frac{7}{2}}Bb^{36}-105\,(bx+a)^{\frac{5}{2}}Bab^{36}+350\,(bx+a)^{\frac{3}{2}}Ba^2b^{36}-1050\,\sqrt{bx+a}Ba^3b^{36}+21\,(bx+a)^{\frac{5}{2}}\dots\right)}{105\,b^{42}}$$

input
```
integrate(x^4*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(15*(b*x + a)*B*a^4 - B*a^5 - 12*(b*x + a)*A*a^3*b + A*a^4*b)/((b*x +
 a)^(3/2)*b^6) + 2/105*(15*(b*x + a)^(7/2)*B*b^36 - 105*(b*x + a)^(5/2)*B*
a*b^36 + 350*(b*x + a)^(3/2)*B*a^2*b^36 - 1050*sqrt(b*x + a)*B*a^3*b^36 +
21*(b*x + a)^(5/2)*A*b^37 - 140*(b*x + a)^(3/2)*A*a*b^37 + 630*sqrt(b*x +
a)*A*a^2*b^37)/b^42
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\left(20\,B\,a^2-8\,A\,a\,b\right)\,{\left(a+b\,x\right)}^{3/2}}{3\,b^6} + \frac{2\,B\,{\left(a+b\,x\right)}^{7/2}}{7\,b^6} - \frac{\left(10\,B\,a^4-8\,A\,a^3\,b\right)\,\left(a+b\,x\right)-\frac{2\,B\,a^5}{3}+\frac{2\,A\,a^4\,b}{3}}{b^6\,{\left(a+b\,x\right)}^{3/2}} + \frac{\left(2\,A\,b-10\,B\,a\right)\,{\left(a+b\,x\right)}^{5/2}}{5\,b^6} - \frac{\left(20\,B\,a^3-12\,A\,a^2\,b\right)\,\sqrt{a+b\,x}}{b^6}$$

input
```
int((x^4*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
((20*B*a^2 - 8*A*a*b)*(a + b*x)^(3/2))/(3*b^6) + (2*B*(a + b*x)^(7/2))/(7*
b^6) - ((10*B*a^4 - 8*A*a^3*b)*(a + b*x) - (2*B*a^5)/3 + (2*A*a^4*b)/3)/(b
^6*(a + b*x)^(3/2)) + ((2*A*b - 10*B*a)*(a + b*x)^(5/2))/(5*b^6) - ((20*B*
a^3 - 12*A*a^2*b)*(a + b*x)^(1/2))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{x^4(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\frac{2}{7}b^4x^4-\frac{16}{35}a\,b^3x^3+\frac{32}{35}a^2b^2x^2-\frac{128}{35}a^3bx-\frac{256}{35}a^4}{\sqrt{bx+a}\,b^5}$$

input
```
int(x^4*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 128*a**4 - 64*a**3*b*x + 16*a**2*b**2*x**2 - 8*a*b**3*x**3 + 5*b**4
*x**4))/(35*sqrt(a + b*x)*b**5)
```

3.277 $\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 118

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2a^3(Ab-aB)}{3b^5(a+bx)^{3/2}} - \frac{2a^2(3Ab-4aB)}{b^5\sqrt{a+bx}} - \frac{6a(Ab-2aB)\sqrt{a+bx}}{b^5} + \frac{2(Ab-4aB)(a+bx)^{3/2}}{3b^5} + \frac{2B(a+bx)^{5/2}}{5b^5}$$

output
```
2/3*a^3*(A*b-B*a)/b^5/(b*x+a)^(3/2)-2*a^2*(3*A*b-4*B*a)/b^5/(b*x+a)^(1/2)-
6*a*(A*b-2*B*a)*(b*x+a)^(1/2)/b^5+2/3*(A*b-4*B*a)*(b*x+a)^(3/2)/b^5+2/5*B*
(b*x+a)^(5/2)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2(128a^4B + 24a^2b^2x(-5A+2Bx) + b^4x^3(5A+3Bx) - 2ab^3x^2(15A+4Bx) + a^3(-}{15b^5(a+bx)^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*(128*a^4*B + 24*a^2*b^2*x*(-5*A + 2*B*x) + b^4*x^3*(5*A + 3*B*x) - 2*a*
b^3*x^2*(15*A + 4*B*x) + a^3*(-80*A*b + 192*b*B*x)))/(15*b^5*(a + b*x)^(3/
2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{a^3(aB-Ab)}{b^4(a+bx)^{5/2}}-\frac{a^2(4aB-3Ab)}{b^4(a+bx)^{3/2}}+\frac{3a(2aB-Ab)}{b^4\sqrt{a+bx}}+\frac{\sqrt{a+bx}(Ab-4aB)}{b^4}+\frac{B(a+bx)^{3/2}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3(Ab-aB)}{3b^5(a+bx)^{3/2}}-\frac{2a^2(3Ab-4aB)}{b^5\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}(Ab-2aB)}{b^5}+\frac{2(a+bx)^{3/2}(Ab-4aB)}{3b^5}+\frac{2B(a+bx)^{5/2}}{5b^5}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*a^3*(A*b - a*B))/(3*b^5*(a + b*x)^(3/2)) - (2*a^2*(3*A*b - 4*a*B))/(b^5
*Sqrt[a + b*x]) - (6*a*(A*b - 2*a*B)*Sqrt[a + b*x])/b^5 + (2*(A*b - 4*a*B)
*(a + b*x)^(3/2))/(3*b^5) + (2*B*(a + b*x)^(5/2))/(5*b^5)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{32\left(-\frac{\left(\frac{3Bx}{5}+A\right)x^3b^4}{16}+\frac{3a\,x^2\left(\frac{4Bx}{15}+A\right)b^3}{8}+\frac{3a^2\left(-\frac{2Bx}{5}+A\right)x\,b^2}{2}+a^3\left(-\frac{12Bx}{5}+A\right)b-\frac{8B\,a^4}{5}\right)}{3(bx+a)^{\frac{3}{2}}b^5}$	75
risch	$-\frac{2\left(-3b^2B\,x^2-5A\,b^2x+14Babx+40abA-73a^2B\right)\sqrt{bx+a}}{15b^5}-\frac{2a^2\left(9A\,b^2x-12Babx+8abA-11a^2B\right)}{3b^5(bx+a)^{\frac{3}{2}}}$	88
gosper	$-\frac{2\left(-3B\,x^4b^4-5A\,x^3b^4+8B\,x^3a\,b^3+30A\,x^2a\,b^3-48B\,x^2a^2b^2+120Ax\,a^2b^2-192Bx\,a^3b+80A\,a^3b-128B\,a^4\right)}{15(bx+a)^{\frac{3}{2}}b^5}$	95
trager	$-\frac{2\left(-3B\,x^4b^4-5A\,x^3b^4+8B\,x^3a\,b^3+30A\,x^2a\,b^3-48B\,x^2a^2b^2+120Ax\,a^2b^2-192Bx\,a^3b+80A\,a^3b-128B\,a^4\right)}{15(bx+a)^{\frac{3}{2}}b^5}$	95
orering	$-\frac{2\left(-3B\,x^4b^4-5A\,x^3b^4+8B\,x^3a\,b^3+30A\,x^2a\,b^3-48B\,x^2a^2b^2+120Ax\,a^2b^2-192Bx\,a^3b+80A\,a^3b-128B\,a^4\right)}{15(bx+a)^{\frac{3}{2}}b^5}$	95
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}-\frac{8Ba(bx+a)^{\frac{3}{2}}}{3}-6Aab\sqrt{bx+a}+12B\,a^2\sqrt{bx+a}-\frac{2a^2(3Ab-4Ba)}{\sqrt{bx+a}}+\frac{2a^3(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^5}$	105
default	$\frac{\frac{2B(bx+a)^{\frac{5}{2}}}{5}+\frac{2Ab(bx+a)^{\frac{3}{2}}}{3}-\frac{8Ba(bx+a)^{\frac{3}{2}}}{3}-6Aab\sqrt{bx+a}+12B\,a^2\sqrt{bx+a}-\frac{2a^2(3Ab-4Ba)}{\sqrt{bx+a}}+\frac{2a^3(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^5}$	105

input
```
int(x^3*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-32/3*(-1/16*(3/5*B*x+A)*x^3*b^4+3/8*a*x^2*(4/15*B*x+A)*b^3+3/2*a^2*(-2/5*
B*x+A)*x*b^2+a^3*(-12/5*B*x+A)*b-8/5*B*a^4)/(b*x+a)^(3/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\,(3\,Bb^4x^4+128\,Ba^4-80\,Aa^3b-(8\,Bab^3-5\,Ab^4)x^3+6\,(8\,Ba^2b^2-5\,Aab^3)x^2+24}{15\,(b^7x^2+2\,ab^6x+a^2b^5)}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b^4*x^4 + 128*B*a^4 - 80*A*a^3*b - (8*B*a*b^3 - 5*A*b^4)*x^3 + 6
*(8*B*a^2*b^2 - 5*A*a*b^3)*x^2 + 24*(8*B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(b*x
+ a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{B(a+bx)^{\frac{5}{2}}}{5b}-\frac{a^3(-Ab+Ba)}{3b(a+bx)^{\frac{3}{2}}}+\frac{a^2(-3Ab+4Ba)}{b\sqrt{a+bx}}+\frac{(a+bx)^{\frac{3}{2}}(Ab-4Ba)}{3b}+\frac{\sqrt{a+bx}\left(-3Aab+6Ba^2\right)}{b}\right)}{b^4} & \text{for } b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Piecewise((2*(B*(a + b*x)**(5/2)/(5*b) - a**3*(-A*b + B*a)/(3*b*(a + b*x)*
*(3/2)) + a**2*(-3*A*b + 4*B*a)/(b*sqrt(a + b*x)) + (a + b*x)**(3/2)*(A*b
- 4*B*a)/(3*b) + sqrt(a + b*x)*(-3*A*a*b + 6*B*a**2)/b)/b**4, Ne(b, 0)), (
(A*x**4/4 + B*x**5/5)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{3\,(bx+a)^{\frac{5}{2}}B-5\,(4\,Ba-Ab)(bx+a)^{\frac{3}{2}}+45\,(2\,Ba^2-Aab)\sqrt{bx+a}}{b}-\frac{5\,(Ba^4-Aa^3b-3\,(4\,Ba^3-3\,Aa^2b)(bx+a))}{(bx+a)^{\frac{3}{2}}b}\right)}{15\,b^4}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/15*((3*(b*x + a)^(5/2)*B - 5*(4*B*a - A*b)*(b*x + a)^(3/2) + 45*(2*B*a^2
 - A*a*b)*sqrt(b*x + a))/b - 5*(B*a^4 - A*a^3*b - 3*(4*B*a^3 - 3*A*a^2*b)*
(b*x + a))/((b*x + a)^(3/2)*b))/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\,(12\,(bx+a)Ba^3-Ba^4-9\,(bx+a)Aa^2b+Aa^3b)}{3\,(bx+a)^{\frac{3}{2}}b^5} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}Bb^{20}-20\,(bx+a)^{\frac{3}{2}}Bab^{20}+90\,\sqrt{bx+a}Ba^2b^{20}+5\,(bx+a)^{\frac{3}{2}}Ab^{21}-45\,\sqrt{bx+a}Aab^{21}\right)}{15\,b^{25}}$$

input
```
integrate(x^3*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*(12*(b*x + a)*B*a^3 - B*a^4 - 9*(b*x + a)*A*a^2*b + A*a^3*b)/((b*x + a
)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2)*B*b^20 - 20*(b*x + a)^(3/2)*B*a*b^2
0 + 90*sqrt(b*x + a)*B*a^2*b^20 + 5*(b*x + a)^(3/2)*A*b^21 - 45*sqrt(b*x +
 a)*A*a*b^21)/b^25
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{(12\,B\,a^2-6\,A\,a\,b)\,\sqrt{a+b\,x}}{b^5}+\frac{2\,B\,(a+b\,x)^{5/2}}{5\,b^5} + \frac{(8\,B\,a^3-6\,A\,a^2\,b)\,(a+b\,x)-\frac{2\,B\,a^4}{3}+\frac{2\,A\,a^3\,b}{3}}{b^5\,(a+b\,x)^{3/2}}+\frac{(2\,A\,b-8\,B\,a)\,(a+b\,x)^{3/2}}{3\,b^5}$$

input
```
int((x^3*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
((12*B*a^2 - 6*A*a*b)*(a + b*x)^(1/2))/b^5 + (2*B*(a + b*x)^(5/2))/(5*b^5)
 + ((8*B*a^3 - 6*A*a^2*b)*(a + b*x) - (2*B*a^4)/3 + (2*A*a^3*b)/3)/(b^5*(a
 + b*x)^(3/2)) + ((2*A*b - 8*B*a)*(a + b*x)^(3/2))/(3*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.36

$$\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{\sqrt{bx+a}\,b^4}$$

input
```
int(x^3*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(2*(16*a**3 + 8*a**2*b*x - 2*a*b**2*x**2 + b**3*x**3))/(5*sqrt(a + b*x)*b*
*4)
```

3.278 $\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2a^2(Ab-aB)}{3b^4(a+bx)^{3/2}} + \frac{2a(2Ab-3aB)}{b^4\sqrt{a+bx}} + \frac{2(Ab-3aB)\sqrt{a+bx}}{b^4} + \frac{2B(a+bx)^{3/2}}{3b^4}$$

output
```
-2/3*a^2*(A*b-B*a)/b^4/(b*x+a)^(3/2)+2*a*(2*A*b-3*B*a)/b^4/(b*x+a)^(1/2)+2
*(A*b-3*B*a)*(b*x+a)^(1/2)/b^4+2/3*B*(b*x+a)^(3/2)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2(-16a^3B+8a^2b(A-3Bx)-6ab^2x(-2A+Bx)+b^3x^2(3A+Bx))}{3b^4(a+bx)^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x) - 6*a*b^2*x*(-2*A + B*x) + b^3*x^2*(3*
A + B*x)))/(3*b^4*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 86$$

$$\int \left(-\frac{a^2(aB-Ab)}{b^3(a+bx)^{5/2}}+\frac{a(3aB-2Ab)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^2(Ab-aB)}{3b^4(a+bx)^{3/2}}+\frac{2a(2Ab-3aB)}{b^4\sqrt{a+bx}}+\frac{2\sqrt{a+bx}(Ab-3aB)}{b^4}+\frac{2B(a+bx)^{3/2}}{3b^4}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(-2*a^2*(A*b - a*B))/(3*b^4*(a + b*x)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(b^4*
Sqrt[a + b*x]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x])/b^4 + (2*B*(a + b*x)^(3/2
))/(3*b^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$\frac{(2B\,x^3+6A\,x^2)b^3+24a\left(-\frac{Bx}{2}+A\right)x\,b^2+16a^2(-3Bx+A)b-32a^3B}{3(bx+a)^{\frac{3}{2}}b^4}$	62
risch	$\frac{2(bBx+3Ab-8Ba)\sqrt{bx+a}}{3b^4}+\frac{2a(6A\,b^2x-9Babx+5abA-8a^2B)}{3b^4(bx+a)^{\frac{3}{2}}}$	65
gosper	$\frac{\frac{2}{3}b^3B\,x^3+2A\,x^2b^3-4B\,x^2a\,b^2+8Axa\,b^2-16Bx\,a^2b+\frac{16}{3}a^2bA-\frac{32}{3}a^3B}{(bx+a)^{\frac{3}{2}}b^4}$	70
trager	$\frac{\frac{2}{3}b^3B\,x^3+2A\,x^2b^3-4B\,x^2a\,b^2+8Axa\,b^2-16Bx\,a^2b+\frac{16}{3}a^2bA-\frac{32}{3}a^3B}{(bx+a)^{\frac{3}{2}}b^4}$	70
orering	$\frac{\frac{2}{3}b^3B\,x^3+2A\,x^2b^3-4B\,x^2a\,b^2+8Axa\,b^2-16Bx\,a^2b+\frac{16}{3}a^2bA-\frac{32}{3}a^3B}{(bx+a)^{\frac{3}{2}}b^4}$	70
derivativedivides	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-6Ba\sqrt{bx+a}+\frac{2a(2Ab-3Ba)}{\sqrt{bx+a}}-\frac{2a^2(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^4}$	76
default	$\frac{\frac{2B(bx+a)^{\frac{3}{2}}}{3}+2Ab\sqrt{bx+a}-6Ba\sqrt{bx+a}+\frac{2a(2Ab-3Ba)}{\sqrt{bx+a}}-\frac{2a^2(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^4}$	76

input
```
int(x^2*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*B*x^3+6*A*x^2)*b^3+24*a*(-1/2*B*x+A)*x*b^2+16*a^2*(-3*B*x+A)*b-32*
a^3*B)/(b*x+a)^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\left(Bb^3x^3-16\,Ba^3+8\,Aa^2b-3\,(2\,Bab^2-Ab^3)x^2-12\,(2\,Ba^2b-Aab^2)x\right)\sqrt{bx+a}}{3\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*b^3*x^3 - 16*B*a^3 + 8*A*a^2*b - 3*(2*B*a*b^2 - A*b^3)*x^2 - 12*(2*
B*a^2*b - A*a*b^2)*x)*sqrt(b*x + a)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.29

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = \begin{cases} \frac{16Aa^2b}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} + \frac{24Aab^2x}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} + \frac{6Ab^3x^2}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{32Ba^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a-}} \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**2*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Piecewise((16*A*a**2*b/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) +
 24*A*a*b**2*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 6*A*b**
3*x**2/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 32*B*a**3/(3*a*
b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48*B*a**2*b*x/(3*a*b**4*sqr
t(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*B*a*b**2*x**2/(3*a*b**4*sqrt(a +
 b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*B*b**3*x**3/(3*a*b**4*sqrt(a + b*x) +
3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), ((A*x**3/3 + B*x**4/4)/a**(5/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{(bx+a)^{\frac{3}{2}}B-3\,(3\,Ba-Ab)\sqrt{bx+a}}{b} + \frac{Ba^3-Aa^2b-3\,(3\,Ba^2-2\,Aab)(bx+a)}{(bx+a)^{\frac{3}{2}}b}\right)}{3\,b^3}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(((b*x + a)^(3/2)*B - 3*(3*B*a - A*b)*sqrt(b*x + a))/b + (B*a^3 - A*a^
2*b - 3*(3*B*a^2 - 2*A*a*b)*(b*x + a))/((b*x + a)^(3/2)*b))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2\left(9\,(bx+a)Ba^2 - Ba^3 - 6\,(bx+a)Aab + Aa^2b\right)}{3\,(bx+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx+a)^{\frac{3}{2}}Bb^8 - 9\,\sqrt{bx+a}Bab^8 + 3\,\sqrt{bx+a}Ab^9\right)}{3\,b^{12}}$$

input
```
integrate(x^2*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(9*(b*x + a)*B*a^2 - B*a^3 - 6*(b*x + a)*A*a*b + A*a^2*b)/((b*x + a)^
(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*B*b^8 - 9*sqrt(b*x + a)*B*a*b^8 + 3*sqrt
(b*x + a)*A*b^9)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\,B\,a^3 + 2\,B\,(a+b\,x)^3 + 6\,A\,b\,(a+b\,x)^2 - 18\,B\,a\,(a+b\,x)^2 - 18\,B\,a^2\,(a+b\,x) - 2\,A}{3\,b^4\,(a+b\,x)^{3/2}}$$

input
```
int((x^2*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
(2*B*a^3 + 2*B*(a + b*x)^3 + 6*A*b*(a + b*x)^2 - 18*B*a*(a + b*x)^2 - 18*B
*a^2*(a + b*x) - 2*A*a^2*b + 12*A*a*b*(a + b*x))/(3*b^4*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

$$\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\frac{2}{3}b^2x^2 - \frac{8}{3}abx - \frac{16}{3}a^2}{\sqrt{bx+a}\,b^3}$$

input
```
int(x^2*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 8*a**2 - 4*a*b*x + b**2*x**2))/(3*sqrt(a + b*x)*b**3)
```

3.279 $\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2a(Ab-aB)}{3b^3(a+bx)^{3/2}} - \frac{2(Ab-2aB)}{b^3\sqrt{a+bx}} + \frac{2B\sqrt{a+bx}}{b^3}$$

output
```
2/3*a*(A*b-B*a)/b^3/(b*x+a)^(3/2)-2*(A*b-2*B*a)/b^3/(b*x+a)^(1/2)+2*B*(b*x
+a)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{16a^2B-4ab(A-6Bx)+6b^2x(-A+Bx)}{3b^3(a+bx)^{3/2}}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(16*a^2*B - 4*a*b*(A - 6*B*x) + 6*b^2*x*(-A + B*x))/(3*b^3*(a + b*x)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{a(aB-Ab)}{b^2(a+bx)^{5/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2(Ab-2aB)}{b^3\sqrt{a+bx}}+\frac{2a(Ab-aB)}{3b^3(a+bx)^{3/2}}+\frac{2B\sqrt{a+bx}}{b^3}$$

input
```
Int[(x*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*a*(A*b - a*B))/(3*b^3*(a + b*x)^(3/2)) - (2*(A*b - 2*a*B))/(b^3*Sqrt[a
+ b*x]) + (2*B*Sqrt[a + b*x])/b^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{4\left(\frac{3x(-Bx+A)b^{2}}{2}+a(-6Bx+A)b-4a^{2}B\right)}{3(bx+a)^{\frac{3}{2}}b^{3}}$	41
gosper	$-\frac{2(-3b^{2}Bx^{2}+3Ab^{2}x-12Babx+2abA-8a^{2}B)}{3(bx+a)^{\frac{3}{2}}b^{3}}$	47
trager	$-\frac{2(-3b^{2}Bx^{2}+3Ab^{2}x-12Babx+2abA-8a^{2}B)}{3(bx+a)^{\frac{3}{2}}b^{3}}$	47
orering	$-\frac{2(-3b^{2}Bx^{2}+3Ab^{2}x-12Babx+2abA-8a^{2}B)}{3(bx+a)^{\frac{3}{2}}b^{3}}$	47
derivativedivides	$\frac{2B\sqrt{bx+a}-\frac{2(Ab-2Ba)}{\sqrt{bx+a}}+\frac{2a(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^{3}}$	51
default	$\frac{2B\sqrt{bx+a}-\frac{2(Ab-2Ba)}{\sqrt{bx+a}}+\frac{2a(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^{3}}$	51
risch	$\frac{2B\sqrt{bx+a}}{b^{3}}-\frac{2(3Ab^{2}x-6Babx+2abA-5a^{2}B)}{3b^{3}(bx+a)^{\frac{3}{2}}}$	52

input
```
int(x*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(3/2*x*(-B*x+A)*b^2+a*(-6*B*x+A)*b-4*a^2*B)/(b*x+a)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\left(3\,Bb^2x^2+8\,Ba^2-2\,Aab+3\,(4\,Bab-Ab^2)x\right)\sqrt{bx+a}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*B*b^2*x^2 + 8*B*a^2 - 2*A*a*b + 3*(4*B*a*b - A*b^2)*x)*sqrt(b*x + a
)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(60) = 120$.

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.35

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \begin{cases} -\frac{4Aab}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} - \frac{6Ab^2x}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{16Ba^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{24Babx}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a}} \\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Piecewise((-4*A*a*b/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) - 6*
A*b**2*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 16*B*a**2/(3*
a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*B*a*b*x/(3*a*b**3*sqrt
(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*B*b**2*x**2/(3*a*b**3*sqrt(a + b*x
) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), ((A*x**2/2 + B*x**3/3)/a**(5/2), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\left(\frac{3\sqrt{bx+a}B}{b} - \frac{Ba^2-Aab-3\,(2\,Ba-Ab)(bx+a)}{(bx+a)^{\frac{3}{2}}b}\right)}{3\,b^2}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(b*x + a)*B/b - (B*a^2 - A*a*b - 3*(2*B*a - A*b)*(b*x + a))/((b
*x + a)^(3/2)*b))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2\sqrt{bx+a}B}{b^3} + \frac{2\left(6\left(bx+a\right)Ba - Ba^2 - 3\left(bx+a\right)Ab + Aab\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^3}$$

input
```
integrate(x*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x + a)*B/b^3 + 2/3*(6*(b*x + a)*B*a - B*a^2 - 3*(b*x + a)*A*b + A
*a*b)/((b*x + a)^(3/2)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{6\,B\,{\left(a+b\,x\right)}^2 - 2\,B\,a^2 + 2\,A\,a\,b - 6\,A\,b\,\left(a+b\,x\right) + 12\,B\,a\,\left(a+b\,x\right)}{3\,b^3\,{\left(a+b\,x\right)}^{3/2}}$$

input
```
int((x*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
(6*B*(a + b*x)^2 - 2*B*a^2 + 2*A*a*b - 6*A*b*(a + b*x) + 12*B*a*(a + b*x))
/(3*b^3*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

$$\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2bx+4a}{\sqrt{bx+a}\,b^2}$$

input
```
int(x*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(2*(2*a + b*x))/(sqrt(a + b*x)*b**2)
```

3.280 $\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab-aB)}{3b^2(a+bx)^{3/2}} - \frac{2B}{b^2\sqrt{a+bx}}$$

output
```
1/3*(-2*A*b+2*B*a)/b^2/(b*x+a)^(3/2)-2*B/b^2/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab+2aB+3bBx)}{3b^2(a+bx)^{3/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x)^(5/2),x]
```

output
```
(-2*(A*b + 2*a*B + 3*b*B*x))/(3*b^2*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(Ab-aB)}{3b^2(a+bx)^{3/2}}-\frac{2B}{b^2\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(a + b*x)^(5/2),x]
```

output
```
(-2*(A*b - a*B))/(3*b^2*(a + b*x)^(3/2)) - (2*B)/(b^2*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(3bBx+Ab+2Ba)}{3(bx+a)^{\frac{3}{2}}b^{2}}$	26
trager	$-\frac{2(3bBx+Ab+2Ba)}{3(bx+a)^{\frac{3}{2}}b^{2}}$	26
pseudoelliptic	$-\frac{2((3Bx+A)b+2Ba)}{3(bx+a)^{\frac{3}{2}}b^{2}}$	26
orering	$-\frac{2(3bBx+Ab+2Ba)}{3(bx+a)^{\frac{3}{2}}b^{2}}$	26
derivativedivides	$\frac{-\frac{2B}{\sqrt{bx+a}}-\frac{2(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^{2}}$	34
default	$\frac{-\frac{2B}{\sqrt{bx+a}}-\frac{2(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{b^{2}}$	34

input
```
int((B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x+a)^(3/2)*(3*B*b*x+A*b+2*B*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx = -\frac{2\,(3\,Bbx+2\,Ba+Ab)\sqrt{bx+a}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*B*b*x + 2*B*a + A*b)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(39) = 78$.

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.10

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx = \begin{cases} -\frac{2Ab}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} - \frac{4Ba}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} - \frac{6Bbx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input

```
integrate((B*x+A)/(b*x+a)**(5/2),x)
```

output

```
Piecewise((-2*A*b/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 4*B*
a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*B*b*x/(3*a*b**2*sq
rt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), ((A*x + B*x**2/2)/a**(5/
2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx = -\frac{2\,(3\,(bx+a)B - Ba + Ab)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

input

```
integrate((B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output

```
-2/3*(3*(b*x + a)*B - B*a + A*b)/((b*x + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx=-\frac{2\,(3\,(bx+a)B-Ba+Ab)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*(b*x + a)*B - B*a + A*b)/((b*x + a)^(3/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx=-\frac{2\,A\,b-2\,B\,a+6\,B\,(a+b\,x)}{3\,b^2\,(a+b\,x)^{3/2}}$$

input
```
int((A + B*x)/(a + b*x)^(5/2),x)
```

output
```
-(2*A*b - 2*B*a + 6*B*(a + b*x))/(3*b^2*(a + b*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx=-\frac{2}{\sqrt{bx+a}\,b}$$

input
```
int((B*x+A)/(b*x+a)^(5/2),x)
```

output
```
( - 2)/(sqrt(a + b*x)*b)
```

3.281 $\int \frac{A+Bx}{x(a+bx)^{5/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{A+Bx}{x(a+bx)^{5/2}} \, dx = \frac{2(Ab-aB)}{3ab(a+bx)^{3/2}} + \frac{2A}{a^2\sqrt{a+bx}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
2/3*(A*b-B*a)/a/b/(b*x+a)^(3/2)+2*A/a^2/(b*x+a)^(1/2)-2*A*arctanh((b*x+a)^
(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x(a+bx)^{5/2}} \, dx = -\frac{2(-aAb+a^2B-3Ab(a+bx))}{3a^2b(a+bx)^{3/2}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x)^(5/2)),x]
```

output
```
(-2*(-(a*A*b) + a^2*B - 3*A*b*(a + b*x)))/(3*a^2*b*(a + b*x)^(3/2)) - (2*A
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{A\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{A\left(\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}\right)}{a}+\frac{2(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{A\left(\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}\right)}{a}+\frac{2(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{A\left(\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{a}+\frac{2(Ab-aB)}{3ab(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x*(a + b*x)^(5/2)),x]
```

output
```
(2*(A*b - a*B))/(3*a*b*(a + b*x)^(3/2)) + (A*(2/(a*Sqrt[a + b*x]) - (2*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{3a(bx+a)^{\frac{3}{2}}}+\frac{2Ab}{a^{2}\sqrt{bx+a}}}{b}$	59
default	$\frac{-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{3a(bx+a)^{\frac{3}{2}}}+\frac{2Ab}{a^{2}\sqrt{bx+a}}}{b}$	59
pseudoelliptic	$-\frac{2\left(3Ab\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}(bx+a)^{\frac{3}{2}}+a^{\frac{5}{2}}\left(-3A\,b^{2}x-4abA+a^{2}B\right)\right)}{3(bx+a)^{\frac{3}{2}}\,b\,a^{\frac{9}{2}}}$	65

input
```
int((B*x+A)/x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-1/3*(-A*b+B*a)/a/(b*x+a)^(3/2)+A*b/a^2/(b*x+a)^(1/2)-A*b/a^(5/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.25

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx = \left[\frac{3\left(Ab^{3}x^{2}+2\,Aab^{2}x+Aa^{2}b\right)\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(3\,Aab^{2}x-Ba^{3}+4\,Aa^{2}b\right.}{3\left(a^{3}b^{3}x^{2}+2\,a^{4}b^{2}x+a^{5}b\right)}\right.$$

input
```
integrate((B*x+A)/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*b)*sqrt(a)*log((b*x - 2*sqrt(b*x
+ a)*sqrt(a) + 2*a)/x) + 2*(3*A*a*b^2*x - B*a^3 + 4*A*a^2*b)*sqrt(b*x + a)
)/(a^3*b^3*x^2 + 2*a^4*b^2*x + a^5*b), 2/3*(3*(A*b^3*x^2 + 2*A*a*b^2*x + A
*a^2*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) + (3*A*a*b^2*x - B*a^3 + 4
*A*a^2*b)*sqrt(b*x + a))/(a^3*b^3*x^2 + 2*a^4*b^2*x + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx = \begin{cases} \frac{2A}{a^2\sqrt{a+bx}} + \frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}} - \frac{2(-Ab+Ba)}{3ab(a+bx)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{A\log(Bx)+Bx}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/x/(b*x+a)**(5/2),x)
```

output
```
Piecewise((2*A/(a**2*sqrt(a + b*x)) + 2*A*atan(sqrt(a + b*x)/sqrt(-a))/(a*
*2*sqrt(-a)) - 2*(-A*b + B*a)/(3*a*b*(a + b*x)**(3/2)), Ne(b, 0)), ((A*log
(B*x) + B*x)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx = \frac{A\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{2\left(Ba^2-3\,(bx+a)Ab-Aab\right)}{3\,(bx+a)^{\frac{3}{2}}a^2b}$$

input
```
integrate((B*x+A)/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
A*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) - 2/3*(
B*a^2 - 3*(b*x + a)*A*b - A*a*b)/((b*x + a)^(3/2)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{2\,(Ba^2-3\,(bx+a)Ab-Aab)}{3\,(bx+a)^{\frac{3}{2}}a^2b}$$

input
```
integrate((B*x+A)/x/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2*A*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/3*(B*a^2 - 3*(b*x +
a)*A*b - A*a*b)/((b*x + a)^(3/2)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx = \frac{\frac{2\,(A\,b-B\,a)}{3\,a}+\frac{2\,A\,b\,(a+b\,x)}{a^2}}{b\,(a+b\,x)^{3/2}} - \frac{2\,A\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int((A + B*x)/(x*(a + b*x)^(5/2)),x)
```

output
```
((2*(A*b - B*a))/(3*a) + (2*A*b*(a + b*x))/a^2)/(b*(a + b*x)^(3/2)) - (2*A
*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx = \frac{\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)+2a}{\sqrt{bx+a}\,a^2}$$

input
```
int((B*x+A)/x/(b*x+a)^(5/2),x)
```

output
```
(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a)) - sqrt(a)*sqrt(a + b*x
)*log(sqrt(a + b*x) + sqrt(a)) + 2*a)/(sqrt(a + b*x)*a**2)
```

3.282 $\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 97

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = -\frac{2(Ab-aB)}{3a^2(a+bx)^{3/2}} - \frac{2(2Ab-aB)}{a^3\sqrt{a+bx}} - \frac{A\sqrt{a+bx}}{a^3x} + \frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
1/3*(-2*A*b+2*B*a)/a^2/(b*x+a)^(3/2)-2*(2*A*b-B*a)/a^3/(b*x+a)^(1/2)-A*(b*
x+a)^(1/2)/a^3/x+(5*A*b-2*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = \frac{-15Ab^2x^2+2abx(-10A+3Bx)+a^2(-3A+8Bx)}{3a^3x(a+bx)^{3/2}} + \frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x)^(5/2)),x]
```

output
```
(-15*A*b^2*x^2 + 2*a*b*x*(-10*A + 3*B*x) + a^2*(-3*A + 8*B*x))/(3*a^3*x*(a
 + b*x)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(5Ab-2aB)\int \frac{1}{x(a+bx)^{5/2}}dx}{2a}-\frac{A}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(5Ab-2aB)\left(\frac{\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{A}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(5Ab-2aB)\left(\frac{\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{A}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(5Ab-2aB)\left(\frac{\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{A}{ax(a+bx)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(5Ab-2aB)\left(\frac{\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{A}{ax(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x)^(5/2)),x]
```

output
```
-(A/(a*x*(a + b*x)^(3/2))) - ((5*A*b - 2*a*B)*(2/(3*a*(a + b*x)^(3/2)) + (
2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2))/a))/(2*a
)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$-\frac{-5(bx+a)^{\frac{3}{2}} x\left(Ab-\frac{2Ba}{5}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{20b\left(-\frac{3Bx}{10}+A\right)x\,a^{\frac{3}{2}}}{3}+5A\sqrt{a}\,b^2x^2+a^{\frac{5}{2}}\left(-\frac{8Bx}{3}+A\right)}{(bx+a)^{\frac{3}{2}} a^{\frac{7}{2}} x}$	82
risch	$-\frac{A\sqrt{bx+a}}{a^3x}-\frac{-\frac{2(5Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(-4Ab+2Ba)}{\sqrt{bx+a}}+\frac{4a(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}}{2a^3}$	86
derivativedivides	$\frac{-\frac{A\sqrt{bx+a}}{x}+\frac{(5Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{a^3}-\frac{2(2Ab-Ba)}{a^3\sqrt{bx+a}}-\frac{2(Ab-Ba)}{3a^2(bx+a)^{\frac{3}{2}}}$	88
default	$\frac{-\frac{A\sqrt{bx+a}}{x}+\frac{(5Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}}{a^3}-\frac{2(2Ab-Ba)}{a^3\sqrt{bx+a}}-\frac{2(Ab-Ba)}{3a^2(bx+a)^{\frac{3}{2}}}$	88

input
```
int((B*x+A)/x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-5*(b*x+a)^(3/2)*x*(A*b-2/5*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))+20/3*b*(
-3/10*B*x+A)*x*a^(3/2)+5*A*a^(1/2)*b^2*x^2+a^(5/2)*(-8/3*B*x+A))/(b*x+a)^(
3/2)/a^(7/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.37

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = \left[-\frac{3\left((2\,Bab^2-5\,Ab^3)x^3+2\,(2\,Ba^2b-5\,Aab^2)x^2+(2\,Ba^3-5\,Aa^2b)x\right)\sqrt{a}\log\left(\frac{bx+\cdots}{\cdots}\right.}{6\,(a^4b^2x^3+\cdots}\right.$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*((2*B*a*b^2 - 5*A*b^3)*x^3 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^2 + (2*B
*a^3 - 5*A*a^2*b)*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(3*A*a^3 - 3*(2*B*a^2*b - 5*A*a*b^2)*x^2 - 4*(2*B*a^3 - 5*A*a^2*b)*x)*
sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), 1/3*(3*((2*B*a*b^2 - 5
*A*b^3)*x^3 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^2 + (2*B*a^3 - 5*A*a^2*b)*x)*sqr
t(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - (3*A*a^3 - 3*(2*B*a^2*b - 5*A*a*b^2
)*x^2 - 4*(2*B*a^3 - 5*A*a^2*b)*x)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x
^2 + a^6*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1520 vs. $2(92) = 184$.

Time = 27.60 (sec) , antiderivative size = 1520, normalized size of antiderivative = 15.67

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**2/(b*x+a)**(5/2),x)
```

output
```
A*(-6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6
*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*
b**3*x**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(
1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
**3 + 6*a**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(3
9/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x
**4) - 45*a**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log
(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*
b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6
*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*
b**3*x**4) - 45*a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b
*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x*
*3*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(
6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)
*b**3*x**4) + 30*a**13*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x +
 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4))...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx =$$

$$-\frac{1}{6}\,b\left(\frac{2\,\left(2\,Ba^3-2\,Aa^2b-3\,(2\,Ba-5\,Ab)(bx+a)^2+2\,(2\,Ba^2-5\,Aab)(bx+a)\right)}{(bx+a)^{\frac{5}{2}}a^3b-(bx+a)^{\frac{3}{2}}a^4b}-\frac{3\,(2\,Ba-5\,Ab)\log}{a^{\frac{7}{2}}b}\right.$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b*(2*(2*B*a^3 - 2*A*a^2*b - 3*(2*B*a - 5*A*b)*(b*x + a)^2 + 2*(2*B*a^
2 - 5*A*a*b)*(b*x + a))/((b*x + a)^(5/2)*a^3*b - (b*x + a)^(3/2)*a^4*b) -
3*(2*B*a - 5*A*b)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
/(a^(7/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = \frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{\sqrt{bx+a}A}{a^3x} + \frac{2\,(3\,(bx+a)Ba+Ba^2-6\,(bx+a)Ab-Aab)}{3\,(bx+a)^{\frac{3}{2}}a^3}$$

input
```
integrate((B*x+A)/x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
(2*B*a - 5*A*b)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - sqrt(b*x +
 a)*A/(a^3*x) + 2/3*(3*(b*x + a)*B*a + B*a^2 - 6*(b*x + a)*A*b - A*a*b)/((
b*x + a)^(3/2)*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(5\,A\,b-2\,B\,a)}{a^{7/2}} - \frac{\frac{2\,(A\,b-B\,a)}{3\,a}+\frac{2\,(5\,A\,b-2\,B\,a)\,(a+b\,x)}{3\,a^2}-\frac{(5\,A\,b-2\,B\,a)\,{(a+b\,x)}^2}{a^3}}{a\,{(a+b\,x)}^{3/2}-{(a+b\,x)}^{5/2}}$$

input
```
int((A + B*x)/(x^2*(a + b*x)^(5/2)),x)
```

output
```
(atanh((a + b*x)^(1/2)/a^(1/2))*(5*A*b - 2*B*a))/a^(7/2) - ((2*(A*b - B*a)
)/(3*a) + (2*(5*A*b - 2*B*a)*(a + b*x))/(3*a^2) - ((5*A*b - 2*B*a)*(a + b*
x)^2)/a^3)/(a*(a + b*x)^(3/2) - (a + b*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx = \frac{-3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bx+3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bx-2a^2}{2\sqrt{bx+a}\,a^3x}$$

input

```
int((B*x+A)/x^2/(b*x+a)^(5/2),x)
```

output

```
( - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*x + 3*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b*x - 2*a**2 - 6*a*b*x)/(2*sqrt(
a + b*x)*a**3*x)
```

3.283 $\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx=\frac{2b(Ab-aB)}{3a^3(a+bx)^{3/2}}+\frac{2b(3Ab-2aB)}{a^4\sqrt{a+bx}}-\frac{A\sqrt{a+bx}}{2a^3x^2}+\frac{(11Ab-4aB)\sqrt{a+bx}}{4a^4x}-\frac{5b(7Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
2/3*b*(A*b-B*a)/a^3/(b*x+a)^(3/2)+2*b*(3*A*b-2*B*a)/a^4/(b*x+a)^(1/2)-1/2*
A*(b*x+a)^(1/2)/a^3/x^2+1/4*(11*A*b-4*B*a)*(b*x+a)^(1/2)/a^4/x-5/4*b*(7*A*
b-4*B*a)*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx=\frac{105Ab^3x^3+a^2bx(21A-80Bx)+20ab^2x^2(7A-3Bx)-6a^3(A+2Bx)}{12a^4x^2(a+bx)^{3/2}}+\frac{5b(-7Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x)^(5/2)),x]
```

output
```
(105*A*b^3*x^3 + a^2*b*x*(21*A - 80*B*x) + 20*a*b^2*x^2*(7*A - 3*B*x) - 6*
a^3*(A + 2*B*x))/(12*a^4*x^2*(a + b*x)^(3/2)) + (5*b*(-7*A*b + 4*a*B)*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {87, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(7Ab-4aB)\int \frac{1}{x^2(a+bx)^{5/2}}dx}{4a}-\frac{A}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(7Ab-4aB)\left(-\frac{5b\int \frac{1}{x(a+bx)^{5/2}}dx}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{A}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{A}{2ax^2(a+bx)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{A}{2ax^2(a+bx)^{3/2}}$$

$$\downarrow 73$$

$$-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{A}{2ax^2(a+bx)^{3/2}}$$

$$\downarrow 221$$

$$-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{A}{2ax^2(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x)^(5/2)),x]
```

output
```
-1/2*A/(a*x^2*(a + b*x)^(3/2)) - ((7*A*b - 4*a*B)*(-(1/(a*x*(a + b*x)^(3/2
))) - (5*b*(2/(3*a*(a + b*x)^(3/2)) + (2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]])/a^(3/2))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{bx+a}\,(-11Abx+4Bax+2Aa)}{4a^4x^2}+\frac{b\left(-\frac{2(35Ab-20Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(-24Ab+16Ba)}{\sqrt{bx+a}}+\frac{16a(Ab-Ba)}{3(bx+a)^{\frac{3}{2}}}\right)}{8a^4}$
pseudoelliptic	$\frac{-\frac{35(bx+a)^{\frac{3}{2}}b\left(Ab-\frac{4Ba}{7}\right)x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4}+\frac{35\left(-\frac{3Bx}{7}+A\right)b^2x^2a^{\frac{3}{2}}}{3}+\frac{7bx\left(-\frac{80Bx}{21}+A\right)a^{\frac{5}{2}}}{4}+\frac{(-2Bx-A)a^{\frac{7}{2}}}{2}+\frac{35A\sqrt{a}\,b^3x^3}{4}}{x^2a^{\frac{9}{2}}(bx+a)^{\frac{3}{2}}}$
derivativedivides	$2b\left(-\frac{-3Ab+2Ba}{a^4\sqrt{bx+a}}-\frac{-Ab+Ba}{3a^3(bx+a)^{\frac{3}{2}}}-\frac{\frac{\left(-\frac{11Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(\frac{13}{8}abA-\frac{1}{2}a^2B\right)\sqrt{bx+a}}{b^2x^2}+\frac{5(7Ab-4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+}}{\sqrt{a}}\right.}{8\sqrt{a}}}{a^4}\right.$
default	$2b\left(-\frac{-3Ab+2Ba}{a^4\sqrt{bx+a}}-\frac{-Ab+Ba}{3a^3(bx+a)^{\frac{3}{2}}}-\frac{\frac{\left(-\frac{11Ab}{8}+\frac{Ba}{2}\right)(bx+a)^{\frac{3}{2}}+\left(\frac{13}{8}abA-\frac{1}{2}a^2B\right)\sqrt{bx+a}}{b^2x^2}+\frac{5(7Ab-4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+}}{\sqrt{a}}\right.}{8\sqrt{a}}}{a^4}\right.$

input
```
int((B*x+A)/x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-11*A*b*x+4*B*a*x+2*A*a)/a^4/x^2+1/8/a^4*b*(-2*(35*A*b
-20*B*a)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-2*(-24*A*b+16*B*a)/(b*x+a)
^(1/2)+16/3*a*(A*b-B*a)/(b*x+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.95

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx=\left[-\frac{15\left((4\,Bab^3-7\,Ab^4)x^4+2\,(4\,Ba^2b^2-7\,Aab^3)x^3+(4\,Ba^3b-7\,Aa^2b^2)x^2\right)\sqrt{a}\log\right.}{}\right.$$

$$-\frac{15\left((4\,Bab^3-7\,Ab^4)x^4+2\,(4\,Ba^2b^2-7\,Aab^3)x^3+(4\,Ba^3b-7\,Aa^2b^2)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(6\,Aa^4}{12\,(a^5b^2x^4+2\,a^6bx^3+}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*((4*B*a*b^3 - 7*A*b^4)*x^4 + 2*(4*B*a^2*b^2 - 7*A*a*b^3)*x^3 +
(4*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*(6*A*a^4 + 15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^3 + 20*(4*B*a^3*b
- 7*A*a^2*b^2)*x^2 + 3*(4*B*a^4 - 7*A*a^3*b)*x)*sqrt(b*x + a))/(a^5*b^2*x^
4 + 2*a^6*b*x^3 + a^7*x^2), -1/12*(15*((4*B*a*b^3 - 7*A*b^4)*x^4 + 2*(4*B*
a^2*b^2 - 7*A*a*b^3)*x^3 + (4*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(-a)*arctan(
sqrt(-a)/sqrt(b*x + a)) + (6*A*a^4 + 15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^3 + 20
*(4*B*a^3*b - 7*A*a^2*b^2)*x^2 + 3*(4*B*a^4 - 7*A*a^3*b)*x)*sqrt(b*x + a))
/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(126) = 252$.

Time = 62.82 (sec) , antiderivative size = 1287, normalized size of antiderivative = 9.68

$$\int \frac{A + Bx}{x^3(a + bx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**3/(b*x+a)**(5/2),x)
```

output
```
A*(-6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*
x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(8
7/2)*b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 1
2*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77
*x**77/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2
)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12
*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/
2)*x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/
(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(1
55/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x)
+ 1)) - 105*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(s
qrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 1
2*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))) + B*(-6*a**17*sqrt(1
 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 +
6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*
a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 15*a*
*16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 + b*x/a) + 1)/(6
*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*
b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx =$$

$$-\frac{1}{24}\,b^2\left(\frac{2\left(8\,Ba^4-8\,Aa^3b+15\,(4\,Ba-7\,Ab)(bx+a)^3-25\,(4\,Ba^2-7\,Aab)(bx+a)^2+8\,(4\,Ba^3-7\,Aa\right.}{(bx+a)^{\frac{7}{2}}a^4b-2\,(bx+a)^{\frac{5}{2}}a^5b+(bx+a)^{\frac{3}{2}}a^6b}\right.$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*b^2*(2*(8*B*a^4 - 8*A*a^3*b + 15*(4*B*a - 7*A*b)*(b*x + a)^3 - 25*(4
*B*a^2 - 7*A*a*b)*(b*x + a)^2 + 8*(4*B*a^3 - 7*A*a^2*b)*(b*x + a))/((b*x +
 a)^(7/2)*a^4*b - 2*(b*x + a)^(5/2)*a^5*b + (b*x + a)^(3/2)*a^6*b) + 15*(4
*B*a - 7*A*b)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^
(9/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx = -\frac{5\,(4\,Bab-7\,Ab^2)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4} - \frac{2\,(6\,(bx+a)Bab+Ba^2b-9\,(bx+a)Ab^2-Aab^2)}{3\,(bx+a)^{\frac{3}{2}}a^4} - \frac{4\,(bx+a)^{\frac{3}{2}}Bab-4\,\sqrt{bx+a}Ba^2b-11\,(bx+a)^{\frac{3}{2}}Ab^2+13\,\sqrt{bx+a}Aab^2}{4\,a^4b^2x^2}$$

input
```
integrate((B*x+A)/x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-5/4*(4*B*a*b - 7*A*b^2)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) - 2
/3*(6*(b*x + a)*B*a*b + B*a^2*b - 9*(b*x + a)*A*b^2 - A*a*b^2)/((b*x + a)^
(3/2)*a^4) - 1/4*(4*(b*x + a)^(3/2)*B*a*b - 4*sqrt(b*x + a)*B*a^2*b - 11*(
b*x + a)^(3/2)*A*b^2 + 13*sqrt(b*x + a)*A*a*b^2)/(a^4*b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx = \frac{\frac{2\,(A\,b^2-B\,a\,b)}{3\,a}+\frac{2\,(7\,A\,b^2-4\,B\,a\,b)\,(a+b\,x)}{3\,a^2}-\frac{25\,(7\,A\,b^2-4\,B\,a\,b)\,(a+b\,x)^2}{12\,a^3}+\frac{5\,(7\,A\,b^2-4\,B\,a\,b)\,(a+b\,x)^3}{4\,a^4}}{(a+b\,x)^{7/2}-2\,a\,(a+b\,x)^{5/2}+a^2\,(a+b\,x)^{3/2}} - \frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(7\,A\,b-4\,B\,a)}{4\,a^{9/2}}$$

input
```
int((A + B*x)/(x^3*(a + b*x)^(5/2)),x)
```

output
```
((2*(A*b^2 - B*a*b))/(3*a) + (2*(7*A*b^2 - 4*B*a*b)*(a + b*x))/(3*a^2) - (
25*(7*A*b^2 - 4*B*a*b)*(a + b*x)^2)/(12*a^3) + (5*(7*A*b^2 - 4*B*a*b)*(a +
 b*x)^3)/(4*a^4))/((a + b*x)^(7/2) - 2*a*(a + b*x)^(5/2) + a^2*(a + b*x)^(
3/2)) - (5*b*atanh((a + b*x)^(1/2)/a^(1/2))*(7*A*b - 4*B*a))/(4*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx = \frac{15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2-}{8\sqrt{bx+a}\,a^4x^2}$$

input
```
int((B*x+A)/x^3/(b*x+a)^(5/2),x)
```

output
```
(15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 4*a**3 + 10*a**
2*b*x + 30*a*b**2*x**2)/(8*sqrt(a + b*x)*a**4*x**2)
```

3.284 $\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 168

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx = -\frac{2b^2(Ab-aB)}{3a^4(a+bx)^{3/2}} - \frac{2b^2(4Ab-3aB)}{a^5\sqrt{a+bx}} - \frac{A\sqrt{a+bx}}{3a^3x^3} + \frac{(17Ab-6aB)\sqrt{a+bx}}{12a^4x^2} - \frac{b(41Ab-22aB)\sqrt{a+bx}}{8a^5x} + \frac{35b^2(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{11/2}}$$

output
```
-2/3*b^2*(A*b-B*a)/a^4/(b*x+a)^(3/2)-2*b^2*(4*A*b-3*B*a)/a^5/(b*x+a)^(1/2)
-1/3*A*(b*x+a)^(1/2)/a^3/x^3+1/12*(17*A*b-6*B*a)*(b*x+a)^(1/2)/a^4/x^2-1/8
*b*(41*A*b-22*B*a)*(b*x+a)^(1/2)/a^5/x+35/8*b^2*(3*A*b-2*B*a)*arctanh((b*x
+a)^(1/2)/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx = \frac{-315Ab^4x^4+210ab^3x^3(-2A+Bx)-4a^4(2A+3Bx)+6a^3bx(3A+7Bx)+7a^2b^2}{24a^5x^3(a+bx)^{3/2}}$$
$$+\frac{35b^2(3Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{11/2}}$$

input
```
Integrate[(A + B*x)/(x^4*(a + b*x)^(5/2)),x]
```

output
```
(-315*A*b^4*x^4 + 210*a*b^3*x^3*(-2*A + B*x) - 4*a^4*(2*A + 3*B*x) + 6*a^3
*b*x*(3*A + 7*B*x) + 7*a^2*b^2*x^2*(-9*A + 40*B*x))/(24*a^5*x^3*(a + b*x)^
(3/2)) + (35*b^2*(3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(11/
2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {87, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(3Ab-2aB)\int \frac{1}{x^3(a+bx)^{5/2}}dx}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(3Ab-2aB)\left(-\frac{7b\int \frac{1}{x^2(a+bx)^{5/2}}dx}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}\right)}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(3Ab-2aB)\left(-\frac{7b\left(-\frac{5b\int \frac{1}{x(a+bx)^{5/2}}dx}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}\right)}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{(3Ab-2aB)\left(-\frac{7b\left(-\frac{5b\left(\frac{\int \frac{1}{x(a+bx)^{3/2}}dx}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}\right)}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{(3Ab-2aB)\left(-\frac{7b\left(-\frac{5b\left(\frac{\frac{\int \frac{1}{x\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}\right)}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{(3Ab-2aB)\left(-\frac{7b\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{ab}+\frac{2}{a\sqrt{a+bx}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}\right)}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{(3Ab-2aB)\left(-\frac{7b\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+bx}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx)^{3/2}}\right)}{2a}-\frac{1}{ax(a+bx)^{3/2}}\right)}{4a}-\frac{1}{2ax^2(a+bx)^{3/2}}\right)}{2a}-\frac{A}{3ax^3(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^4*(a + b*x)^(5/2)),x]
```

output
```
-1/3*A/(a*x^3*(a + b*x)^(3/2)) - ((3*A*b - 2*a*B)*(-1/2*1/(a*x^2*(a + b*x)
^(3/2)) - (7*b*(-(1/(a*x*(a + b*x)^(3/2))) - (5*b*(2/(3*a*(a + b*x)^(3/2))
 + (2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2))/a))/
(2*a)))/(4*a)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{105b^2\left(Ab-\frac{2Ba}{3}\right)(bx+a)^{\frac{3}{2}}x^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8}-\frac{35b^3\left(-\frac{Bx}{2}+A\right)x^3a^{\frac{3}{2}}}{2}-\frac{21b^2\left(-\frac{40Bx}{9}+A\right)x^2a^{\frac{5}{2}}}{8}+\frac{3bx\left(\frac{7Bx}{3}+A\right)a^{\frac{7}{2}}}{4}+\frac{3\left(-\frac{2B}{\cdots}\cdots\right.}{\cdots}}{x^3a^{\frac{11}{2}}(bx+a)^{\frac{3}{2}}}$
risch	$-\frac{\sqrt{bx+a}\,(123Ab^2x^2-66Babx^2-34aAbx+12Ba^2x+8a^2A)}{24a^5x^3}-\frac{b^2\left(-\frac{2(105Ab-70Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(-64Ab+48\cdots}{\sqrt{bx+a}\cdots}\cdots\right.}{16a^5}$
derivativedivides	$2b^2\left(\frac{-\frac{\left(\frac{41Ab}{16}-\frac{11Ba}{8}\right)(bx+a)^{\frac{5}{2}}+\left(-\frac{35}{6}abA+3a^2B\right)(bx+a)^{\frac{3}{2}}+\left(\frac{55}{16}a^2bA-\frac{13}{8}a^3B\right)\sqrt{bx+a}}{b^3x^3}+\frac{35(3Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}}{a^5}\cdots\right.$
default	$2b^2\left(\frac{-\frac{\left(\frac{41Ab}{16}-\frac{11Ba}{8}\right)(bx+a)^{\frac{5}{2}}+\left(-\frac{35}{6}abA+3a^2B\right)(bx+a)^{\frac{3}{2}}+\left(\frac{55}{16}a^2bA-\frac{13}{8}a^3B\right)\sqrt{bx+a}}{b^3x^3}+\frac{35(3Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}}{a^5}\cdots\right.$

input
```
int((B*x+A)/x^4/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*(35/2*b^2*(A*b-2/3*B*a)*(b*x+a)^(3/2)*x^3*arctanh((b*x+a)^(1/2)/a^(1/2
))-70/3*b^3*(-1/2*B*x+A)*x^3*a^(3/2)-7/2*b^2*(-40/9*B*x+A)*x^2*a^(5/2)+b*x
*(7/3*B*x+A)*a^(7/2)+(-2/3*B*x-4/9*A)*a^(9/2)-35/2*A*a^(1/2)*b^4*x^4)/a^(1
1/2)/(b*x+a)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx=\left[-\frac{105\left((2\,Bab^4-3\,Ab^5)x^5+2\,(2\,Ba^2b^3-3\,Aab^4)x^4+(2\,Ba^3b^2-3\,Aa^2b^3)x^3\right)\sqrt{a}\,\mathrm{l}\cdots}{\cdots}\right.$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(105*((2*B*a*b^4 - 3*A*b^5)*x^5 + 2*(2*B*a^2*b^3 - 3*A*a*b^4)*x^4 +
 (2*B*a^3*b^2 - 3*A*a^2*b^3)*x^3)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(
a) + 2*a)/x) + 2*(8*A*a^5 - 105*(2*B*a^2*b^3 - 3*A*a*b^4)*x^4 - 140*(2*B*a
^3*b^2 - 3*A*a^2*b^3)*x^3 - 21*(2*B*a^4*b - 3*A*a^3*b^2)*x^2 + 6*(2*B*a^5
- 3*A*a^4*b)*x)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3), 1/24
*(105*((2*B*a*b^4 - 3*A*b^5)*x^5 + 2*(2*B*a^2*b^3 - 3*A*a*b^4)*x^4 + (2*B*
a^3*b^2 - 3*A*a^2*b^3)*x^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - (8*A
*a^5 - 105*(2*B*a^2*b^3 - 3*A*a*b^4)*x^4 - 140*(2*B*a^3*b^2 - 3*A*a^2*b^3)
*x^3 - 21*(2*B*a^4*b - 3*A*a^3*b^2)*x^2 + 6*(2*B*a^5 - 3*A*a^4*b)*x)*sqrt(
b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(162) = 324$.

Time = 114.66 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.96

$$\int \frac{A + Bx}{x^4(a + bx)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**4/(b*x+a)**(5/2),x)
```

output
```
A*(-8*a**(133/2)*b**128*x**128/(24*a**(137/2)*b**(257/2)*x**(263/2)*sqrt(a
/(b*x) + 1) + 24*a**(135/2)*b**(259/2)*x**(265/2)*sqrt(a/(b*x) + 1)) + 18*
a**(131/2)*b**129*x**129/(24*a**(137/2)*b**(257/2)*x**(263/2)*sqrt(a/(b*x)
 + 1) + 24*a**(135/2)*b**(259/2)*x**(265/2)*sqrt(a/(b*x) + 1)) - 63*a**(12
9/2)*b**130*x**130/(24*a**(137/2)*b**(257/2)*x**(263/2)*sqrt(a/(b*x) + 1)
+ 24*a**(135/2)*b**(259/2)*x**(265/2)*sqrt(a/(b*x) + 1)) - 420*a**(127/2)*
b**131*x**131/(24*a**(137/2)*b**(257/2)*x**(263/2)*sqrt(a/(b*x) + 1) + 24*
a**(135/2)*b**(259/2)*x**(265/2)*sqrt(a/(b*x) + 1)) - 315*a**(125/2)*b**13
2*x**132/(24*a**(137/2)*b**(257/2)*x**(263/2)*sqrt(a/(b*x) + 1) + 24*a**(1
35/2)*b**(259/2)*x**(265/2)*sqrt(a/(b*x) + 1)) + 315*a**63*b**(263/2)*x**(
263/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(137/2)*b
**(257/2)*x**(263/2)*sqrt(a/(b*x) + 1) + 24*a**(135/2)*b**(259/2)*x**(265/
2)*sqrt(a/(b*x) + 1)) + 315*a**62*b**(265/2)*x**(265/2)*sqrt(a/(b*x) + 1)*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(137/2)*b**(257/2)*x**(263/2)*sqrt
(a/(b*x) + 1) + 24*a**(135/2)*b**(259/2)*x**(265/2)*sqrt(a/(b*x) + 1))) +
B*(-6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*
x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(8
7/2)*b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 1
2*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77
*x**77/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(9...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx =$$

$$-\frac{1}{48}\,b^3\left(\frac{2\left(16\,Ba^5-16\,Aa^4b-105\,(2\,Ba-3\,Ab)(bx+a)^4+280\,(2\,Ba^2-3\,Aab)(bx+a)^3-231\,(2\,Ba^3\right.}{(bx+a)^{\frac{9}{2}}a^5b-3\,(bx+a)^{\frac{7}{2}}a^6b+3\,(bx+a)^{\frac{5}{2}}a^7b-(bx}\right.$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/48*b^3*(2*(16*B*a^5 - 16*A*a^4*b - 105*(2*B*a - 3*A*b)*(b*x + a)^4 + 28
0*(2*B*a^2 - 3*A*a*b)*(b*x + a)^3 - 231*(2*B*a^3 - 3*A*a^2*b)*(b*x + a)^2
+ 48*(2*B*a^4 - 3*A*a^3*b)*(b*x + a))/((b*x + a)^(9/2)*a^5*b - 3*(b*x + a)
^(7/2)*a^6*b + 3*(b*x + a)^(5/2)*a^7*b - (b*x + a)^(3/2)*a^8*b) - 105*(2*B
*a - 3*A*b)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/(a^(1
1/2)*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx = \frac{35\,(2\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^5} + \frac{210\,(bx+a)^4Bab^2-560\,(bx+a)^3Ba^2b^2+462\,(bx+a)^2Ba^3b^2-96\,(bx+a)Ba^4b^2-16\,Ba^5b^2-315\,(bx}{24\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\right.}$$

input
```
integrate((B*x+A)/x^4/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
35/8*(2*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5) +
 1/24*(210*(b*x + a)^4*B*a*b^2 - 560*(b*x + a)^3*B*a^2*b^2 + 462*(b*x + a)
^2*B*a^3*b^2 - 96*(b*x + a)*B*a^4*b^2 - 16*B*a^5*b^2 - 315*(b*x + a)^4*A*b
^3 + 840*(b*x + a)^3*A*a*b^3 - 693*(b*x + a)^2*A*a^2*b^3 + 144*(b*x + a)*A
*a^3*b^3 + 16*A*a^4*b^3)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx = \frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)\,(3\,A\,b-2\,B\,a)}{8\,a^{11/2}} - \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{3\,a}+\frac{2\,(3\,A\,b^3-2\,B\,a\,b^2)\,(a+b\,x)}{a^2}-\frac{77\,(3\,A\,b^3-2\,B\,a\,b^2)\,(a+b\,x)^2}{8\,a^3}+\frac{35\,(3\,A\,b^3-2\,B\,a\,b^2)\,(a+b\,x)^3}{3\,a^4}-\frac{35\,(3\,A\,b^3-2\,B\,a\,b^2)\,(}{8\,a^5}}{3\,a\,(a+b\,x)^{7/2}-(a+b\,x)^{9/2}+a^3\,(a+b\,x)^{3/2}-3\,a^2\,(a+b\,x)^{5/2}}$$

input
```
int((A + B*x)/(x^4*(a + b*x)^(5/2)),x)
```

output
```
(35*b^2*atanh((a + b*x)^(1/2)/a^(1/2))*(3*A*b - 2*B*a))/(8*a^(11/2)) - ((2
*(A*b^3 - B*a*b^2))/(3*a) + (2*(3*A*b^3 - 2*B*a*b^2)*(a + b*x))/a^2 - (77*
(3*A*b^3 - 2*B*a*b^2)*(a + b*x)^2)/(8*a^3) + (35*(3*A*b^3 - 2*B*a*b^2)*(a
+ b*x)^3)/(3*a^4) - (35*(3*A*b^3 - 2*B*a*b^2)*(a + b*x)^4)/(8*a^5))/(3*a*(
a + b*x)^(7/2) - (a + b*x)^(9/2) + a^3*(a + b*x)^(3/2) - 3*a^2*(a + b*x)^(
5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x^4(a+bx)^{5/2}}\,dx = \frac{-105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^3x^3+105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^3}{48\sqrt{bx+a}\,a^5x^3}$$

input
```
int((B*x+A)/x^4/(b*x+a)^(5/2),x)
```

output
```
( - 105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*x**3 + 105
*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*x**3 - 16*a**4 +
28*a**3*b*x - 70*a**2*b**2*x**2 - 210*a*b**3*x**3)/(48*sqrt(a + b*x)*a**5*
x**3)
```

3.285 $\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 192

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx=\frac{a^3(10Ab-7aB)\sqrt{x}\sqrt{a+bx}}{128b^4}-\frac{a^2(10Ab-7aB)x^{3/2}\sqrt{a+bx}}{192b^3}+\frac{a(10Ab-7aB)x^{5/2}\sqrt{a+bx}}{240b^2}+\frac{(10Ab-7aB)x^{7/2}\sqrt{a+bx}}{40b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}-\frac{a^4(10Ab-7aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{9/2}}$$

output
```
1/128*a^3*(10*A*b-7*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^4-1/192*a^2*(10*A*b-7*B*a
)*x^(3/2)*(b*x+a)^(1/2)/b^3+1/240*a*(10*A*b-7*B*a)*x^(5/2)*(b*x+a)^(1/2)/b
^2+1/40*(10*A*b-7*B*a)*x^(7/2)*(b*x+a)^(1/2)/b+1/5*B*x^(7/2)*(b*x+a)^(3/2)
/b-1/128*a^4*(10*A*b-7*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^4B+16ab^3x^2(5A+3Bx)+96b^4x^3(5A+4Bx)+10a^3b(15A+7Bx)-}{1920b^{9/2}}$$

input
```
Integrate[x^(5/2)*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^4*B + 16*a*b^3*x^2*(5*A + 3*B*x) +
96*b^4*x^3*(5*A + 4*B*x) + 10*a^3*b*(15*A + 7*B*x) - 4*a^2*b^2*x*(25*A + 1
4*B*x)) + 300*a^4*A*b*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]
 + 210*a^5*B*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(1920*
b^(9/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {90, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(10Ab-7aB)\int x^{5/2}\sqrt{a+bx}dx}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(10Ab-7aB)\left(\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a+bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(10Ab-7aB)\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int\frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

$\Big\downarrow$ 60

$$\frac{(10Ab-7aB)\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

$\Big\downarrow$ 60

$$\frac{(10Ab-7aB)\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

$\Big\downarrow$ 65

$$\frac{(10Ab-7aB)\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

$\Big\downarrow$ 219

$$\frac{(10Ab-7aB)\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)}{10b}+\frac{Bx^{7/2}(a+bx)^{3/2}}{5b}$$

input
```
Int[x^(5/2)*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(B*x^(7/2)*(a + b*x)^(3/2))/(5*b) + ((10*A*b - 7*a*B)*((x^(7/2)*Sqrt[a + b
*x])/4 + (a*((x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a*((x^(3/2)*Sqrt[a + b*x])
/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sq
rt[a + b*x]])/b^(3/2)))/(4*b)))/(6*b)))/8))/(10*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(384B\,x^{4}b^{4}+480A\,x^{3}b^{4}+48B\,x^{3}a\,b^{3}+80A\,x^{2}a\,b^{3}-56B\,x^{2}a^{2}b^{2}-100Ax\,a^{2}b^{2}+70Bx\,a^{3}b+150A\,a^{3}b-105B\,a^{4}\right)\sqrt{x}\,\sqrt{bx+a}}{1920b^{4}}-\frac{a^{4}(10}{}$
default	$-\frac{\sqrt{x}\,\sqrt{bx+a}\,\left(-768B\,b^{\frac{9}{2}}x^{4}\sqrt{x(bx+a)}-960A\,b^{\frac{9}{2}}x^{3}\sqrt{x(bx+a)}-96Ba\,b^{\frac{7}{2}}x^{3}\sqrt{x(bx+a)}-160Aa\,b^{\frac{7}{2}}x^{2}\sqrt{x(bx+a)}+112B\,a^{2}b^{\frac{5}{2}}x^{2}\sqrt{\right.}{}$

input
```
int(x^(5/2)*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*B*b^4*x^4+480*A*b^4*x^3+48*B*a*b^3*x^3+80*A*a*b^3*x^2-56*B*a^2
*b^2*x^2-100*A*a^2*b^2*x+70*B*a^3*b*x+150*A*a^3*b-105*B*a^4)*x^(1/2)*(b*x+
a)^(1/2)/b^4-1/256*a^4*(10*A*b-7*B*a)/b^(9/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^
2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.52

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx = \left[-\frac{15\,(7\,Ba^5-10\,Aa^4b)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(384\,Bb^5x^4-105\,Ba^4b+15\right.$$

$$-\frac{15\,(7\,Ba^5-10\,Aa^4b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(384\,Bb^5x^4-105\,Ba^4b+150\,Aa^3b^2+48\,(Bab^4+10\,Ab^5)x}{1920\,b^5}$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(7*B*a^5 - 10*A*a^4*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sq
rt(b)*sqrt(x) + a) - 2*(384*B*b^5*x^4 - 105*B*a^4*b + 150*A*a^3*b^2 + 48*(
B*a*b^4 + 10*A*b^5)*x^3 - 8*(7*B*a^2*b^3 - 10*A*a*b^4)*x^2 + 10*(7*B*a^3*b
^2 - 10*A*a^2*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^5, -1/1920*(15*(7*B*a^5 - 1
0*A*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (384*B*b^5*x^
4 - 105*B*a^4*b + 150*A*a^3*b^2 + 48*(B*a*b^4 + 10*A*b^5)*x^3 - 8*(7*B*a^2
*b^3 - 10*A*a*b^4)*x^2 + 10*(7*B*a^3*b^2 - 10*A*a^2*b^3)*x)*sqrt(b*x + a)*
sqrt(x))/b^5]
```

Sympy [A] (verification not implemented)

Time = 128.56 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.86

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx = \frac{5Aa^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1+\frac{bx}{a}}} + \frac{5Aa^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1+\frac{bx}{a}}} - \frac{Aa^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}} + \frac{7A\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}} - \frac{5Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} + \frac{Abx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}} - \frac{7Ba^{\frac{9}{2}}\sqrt{x}}{128b^4\sqrt{1+\frac{bx}{a}}} - \frac{7Ba^{\frac{7}{2}}x^{\frac{3}{2}}}{384b^3\sqrt{1+\frac{bx}{a}}} + \frac{7Ba^{\frac{5}{2}}x^{\frac{5}{2}}}{960b^2\sqrt{1+\frac{bx}{a}}} - \frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}}{240b\sqrt{1+\frac{bx}{a}}} + \frac{9B\sqrt{a}x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}} + \frac{7Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}} + \frac{Bbx^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
5*A*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*A*a**(5/2)*x**(3/2)/(19
2*b**2*sqrt(1 + b*x/a)) - A*a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*A
*sqrt(a)*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*A*a**4*asinh(sqrt(b)*sqrt(x)/sq
rt(a))/(64*b**(7/2)) + A*b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a)) - 7*B*a**(
9/2)*sqrt(x)/(128*b**4*sqrt(1 + b*x/a)) - 7*B*a**(7/2)*x**(3/2)/(384*b**3*
sqrt(1 + b*x/a)) + 7*B*a**(5/2)*x**(5/2)/(960*b**2*sqrt(1 + b*x/a)) - B*a*
*(3/2)*x**(7/2)/(240*b*sqrt(1 + b*x/a)) + 9*B*sqrt(a)*x**(9/2)/(40*sqrt(1
+ b*x/a)) + 7*B*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(9/2)) + B*b*x
**(11/2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.26

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx = \frac{(bx^2+ax)^{\frac{3}{2}}Bx^2}{5\,b} - \frac{7\sqrt{bx^2+ax}Ba^3x}{64\,b^3} - \frac{7\,(bx^2+ax)^{\frac{3}{2}}Bax}{40\,b^2} + \frac{5\sqrt{bx^2+ax}Aa^2x}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}Ax}{4\,b} + \frac{7\,Ba^5\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{9}{2}}} - \frac{5\,Aa^4\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} - \frac{7\sqrt{bx^2+ax}Ba^4}{128\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}Ba^2}{48\,b^3} + \frac{5\sqrt{bx^2+ax}Aa^3}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{24\,b^2}$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a*x)^(3/2)*B*x^2/b - 7/64*sqrt(b*x^2 + a*x)*B*a^3*x/b^3 - 7/4
0*(b*x^2 + a*x)^(3/2)*B*a*x/b^2 + 5/32*sqrt(b*x^2 + a*x)*A*a^2*x/b^2 + 1/4
*(b*x^2 + a*x)^(3/2)*A*x/b + 7/256*B*a^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*
x)*sqrt(b))/b^(9/2) - 5/128*A*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(7/2) - 7/128*sqrt(b*x^2 + a*x)*B*a^4/b^4 + 7/48*(b*x^2 + a*x)^(3/2
)*B*a^2/b^3 + 5/64*sqrt(b*x^2 + a*x)*A*a^3/b^3 - 5/24*(b*x^2 + a*x)^(3/2)*
A*a/b^2
```

Giac [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx=\int x^{5/2}\,(A+B\,x)\,\sqrt{a+b\,x}\,dx$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
int(x^(5/2)*(A + B*x)*(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.59

$$\int x^{5/2}\sqrt{a+bx}(A+Bx)\,dx=\frac{15\sqrt{x}\,\sqrt{bx+a}\,a^4b-10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+176\sqrt{x}\,\sqrt{bx+a}\,a\,b^4x^3+12}{640b^4}$$

input
```
int(x^(5/2)*(b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**4*b - 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 176*sqrt(x)*sqrt(a + b*x)*a*b**4*
x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x**4 - 15*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(640*b**4)
```

3.286 $\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx=-\frac{a^2(8Ab-5aB)\sqrt{x}\sqrt{a+bx}}{64b^3}+\frac{a(8Ab-5aB)x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{(8Ab-5aB)x^{5/2}\sqrt{a+bx}}{24b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}+\frac{a^3(8Ab-5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

output
```
-1/64*a^2*(8*A*b-5*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+1/96*a*(8*A*b-5*B*a)*x^(
3/2)*(b*x+a)^(1/2)/b^2+1/24*(8*A*b-5*B*a)*x^(5/2)*(b*x+a)^(1/2)/b+1/4*B*x^
(5/2)*(b*x+a)^(3/2)/b+1/64*a^3*(8*A*b-5*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+
a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.79

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^3B+8ab^2x(2A+Bx)+16b^3x^2(4A+3Bx)-2a^2b(12A+5Bx))+6a^3(-8}{192b^{7/2}}$$

input
```
Integrate[x^(3/2)*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3*B + 8*a*b^2*x*(2*A + B*x) + 16*b^3*
x^2*(4*A + 3*B*x) - 2*a^2*b*(12*A + 5*B*x)) + 6*a^3*(-8*A*b + 5*a*B)*ArcTa
nh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(192*b^(7/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {90, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(8Ab-5aB)\int x^{3/2}\sqrt{a+bx}dx}{8b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)}{8b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)}{8b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}$$

$\Big\downarrow$ 60

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)}{8b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}$$

$\Big\downarrow$ 65

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)}{8b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}$$

$\Big\downarrow$ 219

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)}{8b}+\frac{Bx^{5/2}(a+bx)^{3/2}}{4b}$$

input
```
Int[x^(3/2)*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(B*x^(5/2)*(a + b*x)^(3/2))/(4*b) + ((8*A*b - 5*a*B)*((x^(5/2)*Sqrt[a + b*
x])/3 + (a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/
b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/6))/(8*
b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\left(-48 b^{3} B \,x^{3}-64 A \,x^{2} b^{3}-8 B \,x^{2} a \,b^{2}-16 A x a \,b^{2}+10 B x \,a^{2} b +24 a^{2} b A -15 a^{3} B \right) \sqrt{x}\, \sqrt{b x +a}}{192 b^{3}}+\frac{a^{3} \left(8 A b -5 B a \right) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right.}{128 b^{\frac{7}{2}} \sqrt{x}\, \sqrt{b x +a}}$
default	$\frac{\sqrt{x}\, \sqrt{b x +a}\, \left(96 B \,b^{\frac{7}{2}} x^{3} \sqrt{x \left(b x +a \right)}+128 A \,b^{\frac{7}{2}} x^{2} \sqrt{x \left(b x +a \right)}+16 B a \,b^{\frac{5}{2}} x^{2} \sqrt{x \left(b x +a \right)}+32 A \sqrt{x \left(b x +a \right)}\, b^{\frac{5}{2}} a x -20 B \sqrt{x \left(b x +a \right)}\, b^{\frac{3}{2}} a^{2} x +\right.}{384 b^{\frac{7}{2}} \sqrt{x \left(b x +\right.}}$

input
```
int(x^(3/2)*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*B*b^3*x^3-64*A*b^3*x^2-8*B*a*b^2*x^2-16*A*a*b^2*x+10*B*a^2*b*x
+24*A*a^2*b-15*B*a^3)*x^(1/2)*(b*x+a)^(1/2)/b^3+1/128*a^3*(8*A*b-5*B*a)/b^
(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/
(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.53

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx = \left[-\frac{3\,(5\,Ba^4-8\,Aa^3b)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(48\,Bb^4x^3+15\,Ba^3b-24\,Aa^2b}{384\,b^4}\right.$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/384*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(48*B*b^4*x^3 + 15*B*a^3*b - 24*A*a^2*b^2 + 8*(B*a*b^3
 + 8*A*b^4)*x^2 - 2*(5*B*a^2*b^2 - 8*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^
4, 1/192*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*
x + a)) + (48*B*b^4*x^3 + 15*B*a^3*b - 24*A*a^2*b^2 + 8*(B*a*b^3 + 8*A*b^4
)*x^2 - 2*(5*B*a^2*b^2 - 8*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A] (verification not implemented)

Time = 24.46 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.86

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx = -\frac{Aa^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1+\frac{bx}{a}}} - \frac{Aa^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}} + \frac{5A\sqrt{a}x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}} + \frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}} + \frac{Abx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}} + \frac{5Ba^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1+\frac{bx}{a}}} + \frac{5Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1+\frac{bx}{a}}} - \frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}} + \frac{7B\sqrt{a}x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}} - \frac{5Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} + \frac{Bbx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
-A*a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - A*a**(3/2)*x**(3/2)/(24*b*s
qrt(1 + b*x/a)) + 5*A*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + A*a**3*asinh
(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + A*b*x**(7/2)/(3*sqrt(a)*sqrt(1 +
b*x/a)) + 5*B*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*B*a**(5/2)*x*
*(3/2)/(192*b**2*sqrt(1 + b*x/a)) - B*a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x
/a)) + 7*B*sqrt(a)*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*B*a**4*asinh(sqrt(b)*
sqrt(x)/sqrt(a))/(64*b**(7/2)) + B*b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx = \frac{5\sqrt{bx^2+ax}Ba^2x}{32\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+ax}Aax}{4\,b} - \frac{5\,Ba^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \frac{Aa^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} + \frac{5\sqrt{bx^2+ax}Ba^3}{64\,b^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{24\,b^2} - \frac{\sqrt{bx^2+ax}Aa^2}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}A}{3\,b}$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
5/32*sqrt(b*x^2 + a*x)*B*a^2*x/b^2 + 1/4*(b*x^2 + a*x)^(3/2)*B*x/b - 1/4*s
qrt(b*x^2 + a*x)*A*a*x/b - 5/128*B*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/b^(7/2) + 1/16*A*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
)/b^(5/2) + 5/64*sqrt(b*x^2 + a*x)*B*a^3/b^3 - 5/24*(b*x^2 + a*x)^(3/2)*B*
a/b^2 - 1/8*sqrt(b*x^2 + a*x)*A*a^2/b^2 + 1/3*(b*x^2 + a*x)^(3/2)*A/b
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx = \int x^{3/2}\,(A+B\,x)\,\sqrt{a+b\,x}\,dx$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
int(x^(3/2)*(A + B*x)*(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int x^{3/2}\sqrt{a+bx}(A+Bx)\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^3b+2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+24\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+16\sqrt{x}\,\sqrt{bx+a}\,b^4x^3+3\sqrt{b}}{64b^3}$$

input
```
int(x^(3/2)*(b*x+a)^(1/2)*(B*x+A),x)
```

output

```
( - 3*sqrt(x)*sqrt(a + b*x)*a**3*b + 2*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
 24*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**3
 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(64*b**3
)
```

3.287 $\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx = \frac{a(2Ab-aB)\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{(2Ab-aB)x^{3/2}\sqrt{a+bx}}{4b}+\frac{Bx^{3/2}(a+bx)^{3/2}}{3b}-\frac{a^2(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}$$

output
```
1/8*a*(2*A*b-B*a)*x^(1/2)*(b*x+a)^(1/2)/b^2+1/4*(2*A*b-B*a)*x^(3/2)*(b*x+a
)^(1/2)/b+1/3*B*x^(3/2)*(b*x+a)^(3/2)/b-1/8*a^2*(2*A*b-B*a)*arctanh(b^(1/2
)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx = \frac{\sqrt{x}\sqrt{a+bx}(-3a^2B+2ab(3A+Bx)+4b^2x(3A+2Bx))}{24b^2}+\frac{a^2(-2Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}}$$

input
```
Integrate[Sqrt[x]*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-3*a^2*B + 2*a*b*(3*A + B*x) + 4*b^2*x*(3*A + 2*B*
x)))/(24*b^2) + (a^2*(-2*A*b + a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
Sqrt[a + b*x])])/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {90, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(2Ab-aB)\int \sqrt{x}\sqrt{a+bx}dx}{2b}+\frac{Bx^{3/2}(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)}{2b}+\frac{Bx^{3/2}(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)}{2b}+\frac{Bx^{3/2}(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{65}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)}{2b}+\frac{Bx^{3/2}(a+bx)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)}{2b}+\frac{Bx^{3/2}(a+bx)^{3/2}}{3b}$$

input
```
Int[Sqrt[x]*Sqrt[a + b*x]*(A + B*x),x]
```

output
```
(B*x^(3/2)*(a + b*x)^(3/2))/(3*b) + ((2*A*b - a*B)*((x^(3/2)*Sqrt[a + b*x]
)/2 + (a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
+ b*x]])/b^(3/2)))/4))/(2*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

method	result
risch	$\frac{(8b^2Bx^2+12Ab^2x+2Babx+6abA-3a^2B)\sqrt{x}\sqrt{bx+a}}{24b^2}-\frac{a^2(2Ab-Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$-\frac{\sqrt{x}\,\sqrt{bx+a}\,\left(-16B\,b^{\frac{5}{2}}x^2\sqrt{x(bx+a)}-24A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x-4B\sqrt{x(bx+a)}\,b^{\frac{3}{2}}ax+6A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b-12A\sqrt{x(bx}\right.}{48b^{\frac{5}{2}}\sqrt{x(bx+a)}}$

input
```
int(x^(1/2)*(b*x+a)^(1/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*B*b^2*x^2+12*A*b^2*x+2*B*a*b*x+6*A*a*b-3*B*a^2)*x^(1/2)*(b*x+a)^(1
/2)/b^2-1/16*a^2*(2*A*b-B*a)/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1
/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.54

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx$$

$$= \left[-\frac{3\,(Ba^3-2\,Aa^2b)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(8\,Bb^3x^2-3\,Ba^2b+6\,Aab^2+2\,(Bab^2+}{48\,b^3}\right.$$

$$-\frac{3\,(Ba^3-2\,Aa^2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(8\,Bb^3x^2-3\,Ba^2b+6\,Aab^2+2\,(Bab^2+6\,Ab^3)x)\sqrt{bx+a}\sqrt{}}{24\,b^3}$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/48*(3*(B*a^3 - 2*A*a^2*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*
sqrt(x) + a) - 2*(8*B*b^3*x^2 - 3*B*a^2*b + 6*A*a*b^2 + 2*(B*a*b^2 + 6*A*b
^3)*x)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*(B*a^3 - 2*A*a^2*b)*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (8*B*b^3*x^2 - 3*B*a^2*b + 6*A*a*b
^2 + 2*(B*a*b^2 + 6*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.64

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx$$
$$= 2A\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b}+\sqrt{a+bx}\left(\frac{a\sqrt{x}}{8b}+\frac{x^{\frac{3}{2}}}{4}\right) & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$
$$+2B\left(\begin{cases} \frac{a^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{16b^2}+\sqrt{a+bx}\left(-\frac{a^2\sqrt{x}}{16b^2}+\frac{ax^{\frac{3}{2}}}{24b}+\frac{x^{\frac{5}{2}}}{6}\right) & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{5}{2}}}{5} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**(1/2)*(b*x+a)**(1/2)*(B*x+A),x)
```

output
```
2*A*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))
/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b) + sqrt(
a + b*x)*(a*sqrt(x)/(8*b) + x**(3/2)/4), Ne(b, 0)), (sqrt(a)*x**(3/2)/3, T
rue)) + 2*B*Piecewise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*s
qrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(16*b*
*2) + sqrt(a + b*x)*(-a**2*sqrt(x)/(16*b**2) + a*x**(3/2)/(24*b) + x**(5/2
)/6), Ne(b, 0)), (sqrt(a)*x**(5/2)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx = \frac{1}{2}\sqrt{bx^2+ax}Ax - \frac{\sqrt{bx^2+ax}Bax}{4\,b} + \frac{Ba^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} - \frac{Aa^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} - \frac{\sqrt{bx^2+ax}Ba^2}{8\,b^2} + \frac{(bx^2+ax)^{\frac{3}{2}}B}{3\,b} + \frac{\sqrt{bx^2+ax}Aa}{4\,b}$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*A*x - 1/4*sqrt(b*x^2 + a*x)*B*a*x/b + 1/16*B*a^3*log
(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 1/8*A*a^2*log(2*b*x +
a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) - 1/8*sqrt(b*x^2 + a*x)*B*a^2/b^2
 + 1/3*(b*x^2 + a*x)^(3/2)*B/b + 1/4*sqrt(b*x^2 + a*x)*A*a/b
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)*(b*x+a)^(1/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.17

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{\frac{x^{11/2}\left(\frac{A\,a^2\,b^4}{2}-\frac{B\,a^3\,b^3}{4}\right)}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{11}}+\frac{x^{9/2}\left(\frac{17\,B\,a^3\,b^2}{12}+\frac{5\,A\,a^2\,b^3}{2}\right)}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{9}}-\frac{x^{7/2}\left(3\,A\,a^2\,b^2-\frac{19\,B\,a^3\,b}{2}\right)}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{7}}+\frac{x^{5/2}\left(\frac{19\,B\,a^3}{2}-3\,A\,a^2\,b\right)}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{5}}-\frac{\sqrt{x}\,(B\,a^3-2\,A\,a^2\,b)}{4\,b^2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}}{\frac{15\,b^2\,x^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}-\frac{20\,b^3\,x^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}+\frac{15\,b^4\,x^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}-\frac{6\,b^5\,x^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{10}}+\frac{b^6\,x^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{12}}-\frac{6\,b\,x}{\left(\sqrt{a+b\,x}-\sqrt{a}\right.}}$$
$$-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)\,(2\,A\,b-B\,a)}{4\,b^{5/2}}$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x)^(1/2),x)
```

output
```
((x^(11/2)*((A*a^2*b^4)/2 - (B*a^3*b^3)/4))/((a + b*x)^(1/2) - a^(1/2))^11
 + (x^(9/2)*((5*A*a^2*b^3)/2 + (17*B*a^3*b^2)/12))/((a + b*x)^(1/2) - a^(1
/2))^9 - (x^(7/2)*(3*A*a^2*b^2 - (19*B*a^3*b)/2))/((a + b*x)^(1/2) - a^(1/
2))^7 + (x^(5/2)*((19*B*a^3)/2 - 3*A*a^2*b))/((a + b*x)^(1/2) - a^(1/2))^5
 - (x^(1/2)*(B*a^3 - 2*A*a^2*b))/(4*b^2*((a + b*x)^(1/2) - a^(1/2))) + (x^
(3/2)*(17*B*a^3 + 30*A*a^2*b))/(12*b*((a + b*x)^(1/2) - a^(1/2))^3))/((15*
b^2*x^2)/((a + b*x)^(1/2) - a^(1/2))^4 - (20*b^3*x^3)/((a + b*x)^(1/2) - a
^(1/2))^6 + (15*b^4*x^4)/((a + b*x)^(1/2) - a^(1/2))^8 - (6*b^5*x^5)/((a +
 b*x)^(1/2) - a^(1/2))^10 + (b^6*x^6)/((a + b*x)^(1/2) - a^(1/2))^12 - (6*
b*x)/((a + b*x)^(1/2) - a^(1/2))^2 + 1) - (a^2*atanh((b^(1/2)*x^(1/2))/((a
 + b*x)^(1/2) - a^(1/2)))*(2*A*b - B*a))/(4*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \sqrt{x}\sqrt{a+bx}(A+Bx)\,dx$$
$$= \frac{3\sqrt{x}\,\sqrt{bx+a}\,a^2b+14\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3}{24b^2}$$

input
```
int(x^(1/2)*(b*x+a)^(1/2)*(B*x+A),x)
```

output
```
(3*sqrt(x)*sqrt(a + b*x)*a**2*b + 14*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*sq
rt(x)*sqrt(a + b*x)*b**3*x**2 - 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*a**3)/(24*b**2)
```

3.288 $\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = \frac{(4Ab-aB)\sqrt{x}\sqrt{a+bx}}{4b}+\frac{B\sqrt{x}(a+bx)^{3/2}}{2b}+\frac{a(4Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}$$

output
```
1/4*(4*A*b-B*a)*x^(1/2)*(b*x+a)^(1/2)/b+1/2*B*x^(1/2)*(b*x+a)^(3/2)/b+1/4*
a*(4*A*b-B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = \frac{\sqrt{x}\sqrt{a+bx}(4Ab+aB+2bBx)}{4b}+\frac{a(-4Ab+aB)\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{4b^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(4*A*b + a*B + 2*b*B*x))/(4*b) + (a*(-4*A*b + a*B)*
Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {90, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(4Ab-aB)\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx}{4b}+\frac{B\sqrt{x}(a+bx)^{3/2}}{2b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)}{4b}+\frac{B\sqrt{x}(a+bx)^{3/2}}{2b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(4Ab-aB)\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)}{4b}+\frac{B\sqrt{x}(a+bx)^{3/2}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(4Ab-aB)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)}{4b}+\frac{B\sqrt{x}(a+bx)^{3/2}}{2b}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/Sqrt[x],x]
```

output
```
(B*Sqrt[x]*(a + b*x)^(3/2))/(2*b) + ((4*A*b - a*B)*(Sqrt[x]*Sqrt[a + b*x]
+ (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]))/(4*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result
risch	$\frac{(2bBx+4Ab+Ba)\sqrt{x}\,\sqrt{bx+a}}{4b}+\frac{a(4Ab-Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{x}\,\Bigg(4B\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}+4A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ab+8A\,b^{\frac{3}{2}}\sqrt{x(bx+a)}-B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}+2Ba\sqrt{x(b}}{8b^{\frac{3}{2}}\sqrt{x(bx+a)}}$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*b*x+4*A*b+B*a)*x^(1/2)*(b*x+a)^(1/2)/b+1/8*a*(4*A*b-B*a)/b^(3/2)*
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = \left[-\frac{(Ba^2-4\,Aab)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(2\,Bb^2x+Bab+4\,Ab^2)\sqrt{bx+a}\sqrt{x}}{8\,b^2}, \frac{(Ba^2-\right.$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((B*a^2 - 4*A*a*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(
x) + a) - 2*(2*B*b^2*x + B*a*b + 4*A*b^2)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*
((B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (2*B*
b^2*x + B*a*b + 4*A*b^2)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = A\sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}}+\frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + 2B\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{a\sqrt{x}\sqrt{a+bx}}{8b}+\frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(1/2),x)
```

output
```
A*sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + A*a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqr
t(b) + 2*B*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*s
qrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b)
+ a*sqrt(x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a + b*x)/4, Ne(b, 0)), (sq
rt(a)*x**(3/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{bx^2+ax}Bx - \frac{Ba^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{2\sqrt{b}} + \sqrt{bx^2+ax}A + \frac{\sqrt{bx^2+ax}Ba}{4\,b}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*B*x - 1/8*B*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/b^(3/2) + 1/2*A*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sq
rt(b) + sqrt(b*x^2 + a*x)*A + 1/4*sqrt(b*x^2 + a*x)*B*a/b
```

Giac [A] (verification not implemented)

Time = 75.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = \frac{\left(\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\,(bx+a)B}{b^2}-\frac{Bab^2-4\,Ab^3}{b^4}\right)+\frac{(Ba^2-4\,Aab)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}}\right)b}{4\,|b|}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*B/b^2 - (B*a*b^2 -
 4*A*b^3)/b^4) + (B*a^2 - 4*A*a*b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((
b*x + a)*b - a*b)))/b^(3/2))*b/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = A\sqrt{x}\,\sqrt{a+b\,x}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{\sqrt{b}} + B\sqrt{x}\left(\frac{x}{2}+\frac{a}{4\,b}\right)\sqrt{a+b\,x} - \frac{B\,a^2\,\ln\left(a+2\,b\,x+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{a+b\,x}\right)}{8\,b^{3/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(1/2),x)
```

output
```
A*x^(1/2)*(a + b*x)^(1/2) + (2*A*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2
) - a^(1/2))))/b^(1/2) + B*x^(1/2)*(x/2 + a/(4*b))*(a + b*x)^(1/2) - (B*a^
2*log(a + 2*b*x + 2*b^(1/2)*x^(1/2)*(a + b*x)^(1/2)))/(8*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{x}}\,dx = \frac{5\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(1/2),x)
```

output
```
(5*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(b)*
log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b)
```

3.289 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx=-\frac{2A\sqrt{a+bx}}{\sqrt{x}}+B\sqrt{x}\sqrt{a+bx}+\frac{(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

output
```
-2*A*(b*x+a)^(1/2)/x^(1/2)+B*x^(1/2)*(b*x+a)^(1/2)+(2*A*b+B*a)*arctanh(b^(
1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx=\frac{\sqrt{a+bx}(-2A+Bx)}{\sqrt{x}}+\frac{2(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{b}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^(3/2),x]
```

output
```
(Sqrt[a + b*x]*(-2*A + B*x))/Sqrt[x] + (2*(2*A*b + a*B)*ArcTanh[(Sqrt[b]*S
qrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {87, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(aB+2Ab)\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx}{a}-\frac{2A(a+bx)^{3/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(aB+2Ab)\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)}{a}-\frac{2A(a+bx)^{3/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(aB+2Ab)\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)}{a}-\frac{2A(a+bx)^{3/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(aB+2Ab)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)}{a}-\frac{2A(a+bx)^{3/2}}{a\sqrt{x}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^(3/2),x]
```

output
```
(-2*A*(a + b*x)^(3/2))/(a*Sqrt[x]) + ((2*A*b + a*B)*(Sqrt[x]*Sqrt[a + b*x]
 + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]))/a
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-Bx+2A)}{\sqrt{x}}+\frac{\left(Ab+\frac{Ba}{2}\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	77
default	$\frac{\sqrt{bx+a}\,\left(2A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)bx+B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ax+2Bx\sqrt{x(bx+a)}\,\sqrt{b}-4A\sqrt{x(bx+a)}\,\sqrt{b}\right)}{2\sqrt{x}\,\sqrt{x(bx+a)}\,\sqrt{b}}$	118

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(1/2)*(-B*x+2*A)/x^(1/2)+(A*b+1/2*B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*
x^2+a*x)^(1/2))/b^(1/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx = \left[\frac{(Ba+2\,Ab)\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(Bbx-2\,Ab)\sqrt{bx+a}\sqrt{x}}{2\,bx}, -\frac{(Ba+2\,Ab)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(Bbx-2\,Ab)\sqrt{bx+a}\sqrt{x}}{bx}\right]$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*a + 2*A*b)*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x)
+ a) + 2*(B*b*x - 2*A*b)*sqrt(b*x + a)*sqrt(x))/(b*x), -((B*a + 2*A*b)*sqr
t(-b)*x*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (B*b*x - 2*A*b)*sqrt(b*x
+ a)*sqrt(x))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx = -\frac{2A\sqrt{a}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+2A\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2Ab\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}+B\sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}}+\frac{Ba\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(3/2),x)
```

output
```
-2*A*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*A*sqrt(b)*asinh(sqrt(b)*sqrt(x)
/sqrt(a)) - 2*A*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a)) + B*sqrt(a)*sqrt(x)*sq
rt(1 + b*x/a) + B*a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx = \frac{Ba\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,\sqrt{b}} + A\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \sqrt{bx^2+ax}B - \frac{2\,\sqrt{bx^2+ax}A}{x}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(3/2),x, algorithm="maxima")
```

output
```
1/2*B*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + A*sqrt(b)*l
og(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + sqrt(b*x^2 + a*x)*B - 2*sqrt
(b*x^2 + a*x)*A/x
```

Giac [A] (verification not implemented)

Time = 75.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx = \frac{\left(\frac{\sqrt{bx+a}\left(\frac{(bx+a)B}{b}-\frac{Bab+2\,Ab^2}{b^2}\right)}{\sqrt{(bx+a)b-ab}} - \frac{(Ba+2\,Ab)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}}\right)b^2}{|b|}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(3/2),x, algorithm="giac")
```

output
```
(sqrt(b*x + a)*((b*x + a)*B/b - (B*a*b + 2*A*b^2)/b^2)/sqrt((b*x + a)*b -
a*b) - (B*a + 2*A*b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a
*b)))/b^(3/2))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{a+b\,x}}{x^{3/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(3/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{3/2}}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,a+4\sqrt{x}\,\sqrt{bx+a}\,bx+12\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ax-9\sqrt{b}\,ax}{4x}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(3/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a + b*x)*a + 4*sqrt(x)*sqrt(a + b*x)*b*x + 12*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*x - 9*sqrt(b)*a*x)/(4*x)
```

3.290 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}} \, dx = -\frac{2B\sqrt{a+bx}}{\sqrt{x}} - \frac{2A(a+bx)^{3/2}}{3ax^{3/2}} + 2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2*B*(b*x+a)^(1/2)/x^(1/2)-2/3*A*(b*x+a)^(3/2)/a/x^(3/2)+2*b^(1/2)*B*arcta
nh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}} \, dx = -\frac{2\sqrt{a+bx}(aA+Abx+3aBx)}{3ax^{3/2}} - 2\sqrt{b}B\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^(5/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(a*A + A*b*x + 3*a*B*x))/(3*a*x^(3/2)) - 2*Sqrt[b]*B*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{\sqrt{a+bx}}{x^{3/2}}dx-\frac{2A(a+bx)^{3/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$B\left(b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2A(a+bx)^{3/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{65}$$

$$B\left(2b\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2A(a+bx)^{3/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$B\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2A(a+bx)^{3/2}}{3ax^{3/2}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^(5/2),x]
```

output
```
(-2*A*(a + b*x)^(3/2))/(3*a*x^(3/2)) + B*((-2*Sqrt[a + b*x])/Sqrt[x] + 2*S
qrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(Abx+3Bax+Aa)}{3x^{\frac{3}{2}}a}+\frac{B\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	78
default	$-\frac{\sqrt{bx+a}\,\left(-3B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ab\,x^{2}+2A\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}+6Bax\sqrt{b}\,\sqrt{x(bx+a)}+2Aa\sqrt{b}\,\sqrt{x(bx+a)}\right)}{3x^{\frac{3}{2}}\sqrt{x(bx+a)}\,a\sqrt{b}}$	112

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(A*b*x+3*B*a*x+A*a)/x^(3/2)/a+B*b^(1/2)*ln((1/2*a+b*x)/
b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx = \left[\frac{3\,Ba\sqrt{b}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(Aa+(3\,Ba+Ab)x)\sqrt{bx+a}\sqrt{x}}{3\,ax^2}, -\frac{2\left(3\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+(Aa+(3\,Ba+Ab)x)\sqrt{bx+a}\sqrt{x}\right)}{3\,ax^2}\right]$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*B*a*sqrt(b)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) -
 2*(A*a + (3*B*a + A*b)*x)*sqrt(b*x + a)*sqrt(x))/(a*x^2), -2/3*(3*B*a*sqr
t(-b)*x^2*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (A*a + (3*B*a + A*b)*x)
*sqrt(b*x + a)*sqrt(x))/(a*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}-\frac{2B\sqrt{a}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{2Bb\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(5/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*A*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a
) - 2*B*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*B*sqrt(b)*asinh(sqrt(b)*sqrt
(x)/sqrt(a)) - 2*B*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx = -\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)+\frac{2\sqrt{bx+a}}{\sqrt{x}}\right)B-\frac{2\,(bx+a)^{\frac{3}{2}}A}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(5/2),x, algorithm="maxima")
```

output
```
-(sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x))) + 2*sqrt(b*x + a)/sqrt(x))*B - 2/3*(b*x + a)^(3/2)*A/(a*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 75.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx = -\frac{2\,b^{3}\left(\frac{3\,B\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}}-\frac{\left(3\,Ba-\frac{\left(3\,Bab+Ab^{2}\right)(bx+a)}{ab}\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{3}{2}}}\right)}{3\,|b|}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*b^3*(3*B*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b
^(3/2) - (3*B*a - (3*B*a*b + A*b^2)*(b*x + a)/(a*b))*sqrt(b*x + a)/((b*x +
 a)*b - a*b)^(3/2))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{a+b\,x}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(5/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{5/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,bx}{3}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b\,x^2}{x^2}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(5/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a - 4*sqrt(x)*sqrt(a + b*x)*b*x + 3*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*x**2))/(3*x**2)
```

3.291 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}} dx = -\frac{2A(a+bx)^{3/2}}{5ax^{5/2}} + \frac{2(2Ab-5aB)(a+bx)^{3/2}}{15a^2x^{3/2}}$$

output
```
-2/5*A*(b*x+a)^(3/2)/a/x^(5/2)+2/15*(2*A*b-5*B*a)*(b*x+a)^(3/2)/a^2/x^(3/2
)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}} dx = -\frac{2(a+bx)^{3/2}(3aA-2Abx+5aBx)}{15a^2x^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^(7/2),x]
```

output
```
(-2*(a + b*x)^(3/2)*(3*a*A - 2*A*b*x + 5*a*B*x))/(15*a^2*x^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(2Ab-5aB)\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2A(a+bx)^{3/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2(a+bx)^{3/2}(2Ab-5aB)}{15a^2x^{3/2}}-\frac{2A(a+bx)^{3/2}}{5ax^{5/2}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^(7/2),x]
```

output
```
(-2*A*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (2*(2*A*b - 5*a*B)*(a + b*x)^(3/2))
/(15*a^2*x^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-2Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}a^{2}}$	31
default	$-\frac{2(bx+a)^{\frac{3}{2}}(-2Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}a^{2}}$	31
orering	$-\frac{2(bx+a)^{\frac{3}{2}}(-2Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}a^{2}}$	31
risch	$-\frac{2\sqrt{bx+a}\,(-2A\,b^{2}x^{2}+5Bab\,x^{2}+aAbx+5B\,a^{2}x+3a^{2}A)}{15x^{\frac{5}{2}}a^{2}}$	52

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-2*A*b*x+5*B*a*x+3*A*a)/x^(5/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}}\,dx = -\frac{2\,(3\,Aa^2+(5\,Bab-2\,Ab^2)x^2+(5\,Ba^2+Aab)x)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*a^2 + (5*B*a*b - 2*A*b^2)*x^2 + (5*B*a^2 + A*a*b)*x)*sqrt(b*x +
 a)/(a^2*x^(5/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.

Time = 3.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax} + \frac{4Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2} - \frac{2B\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(7/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*A*b**(3/2)*sqrt(a/(b*x) + 1)/(
15*a*x) + 4*A*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2) - 2*B*sqrt(b)*sqrt(a/(b
*x) + 1)/(3*x) - 2*B*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}}\,dx = -\frac{2\sqrt{bx^2+ax}Bb}{3\,ax} + \frac{4\sqrt{bx^2+ax}Ab^2}{15\,a^2x} - \frac{2\sqrt{bx^2+ax}B}{3\,x^2} - \frac{2\sqrt{bx^2+ax}Ab}{15\,ax^2} - \frac{2\sqrt{bx^2+ax}A}{5\,x^3}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(b*x^2 + a*x)*B*b/(a*x) + 4/15*sqrt(b*x^2 + a*x)*A*b^2/(a^2*x) -
2/3*sqrt(b*x^2 + a*x)*B/x^2 - 2/15*sqrt(b*x^2 + a*x)*A*b/(a*x^2) - 2/5*sqr
t(b*x^2 + a*x)*A/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}b\left(\frac{(5\,Bab^4-2\,Ab^5)(bx+a)}{a^2} - \frac{5\,(Ba^2b^4-Aab^5)}{a^2}\right)}{15\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(b*x + a)^(3/2)*b*((5*B*a*b^4 - 2*A*b^5)*(b*x + a)/a^2 - 5*(B*a^2*b^
4 - A*a*b^5)/a^2)/(((b*x + a)*b - a*b)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{5}-\frac{x^2\,\left(4\,A\,b^2-10\,B\,a\,b\right)}{15\,a^2}+\frac{x\,\left(10\,B\,a^2+2\,A\,b\,a\right)}{15\,a^2}\right)}{x^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(7/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/5 - (x^2*(4*A*b^2 - 10*B*a*b))/(15*a^2) + (x*(10*
B*a^2 + 2*A*a*b))/(15*a^2)))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{7/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,abx}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{5}-\frac{2\sqrt{b}\,b^2x^3}{5}}{a\,x^3}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(7/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a**2 - 2*sqrt(x)*sqrt(a + b*x)*a*b*x - sqrt(x
)*sqrt(a + b*x)*b**2*x**2 - sqrt(b)*b**2*x**3))/(5*a*x**3)
```

3.292 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = -\frac{2A(a+bx)^{3/2}}{7ax^{7/2}} + \frac{2(4Ab-7aB)(a+bx)^{3/2}}{35a^2x^{5/2}} - \frac{4b(4Ab-7aB)(a+bx)^{3/2}}{105a^3x^{3/2}}$$

output
```
-2/7*A*(b*x+a)^(3/2)/a/x^(7/2)+2/35*(4*A*b-7*B*a)*(b*x+a)^(3/2)/a^2/x^(5/2
)-4/105*b*(4*A*b-7*B*a)*(b*x+a)^(3/2)/a^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = -\frac{2(a+bx)^{3/2}\left(15a^2A-12aAbx+21a^2Bx+8Ab^2x^2-14abBx^2\right)}{105a^3x^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(3/2)*(15*a^2*A - 12*a*A*b*x + 21*a^2*B*x + 8*A*b^2*x^2 - 14
*a*b*B*x^2))/(105*a^3*x^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(4Ab-7aB)\int \frac{\sqrt{a+bx}}{x^{7/2}}dx}{7a}-\frac{2A(a+bx)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(4Ab-7aB)\left(-\frac{2b\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2A(a+bx)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)(4Ab-7aB)}{7a}-\frac{2A(a+bx)^{3/2}}{7ax^{7/2}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^(9/2),x]
```

output
```
(-2*A*(a + b*x)^(3/2))/(7*a*x^(7/2)) - ((4*A*b - 7*a*B)*((-2*(a + b*x)^(3/
2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(8A\,b^{2}x^{2}-14Bab\,x^{2}-12aAbx+21B\,a^{2}x+15a^{2}A\right)}{105x^{\frac{7}{2}}a^{3}}$	53
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(8A\,b^{2}x^{2}-14Bab\,x^{2}-12aAbx+21B\,a^{2}x+15a^{2}A\right)}{105x^{\frac{7}{2}}a^{3}}$	53
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(8A\,b^{2}x^{2}-14Bab\,x^{2}-12aAbx+21B\,a^{2}x+15a^{2}A\right)}{105x^{\frac{7}{2}}a^{3}}$	53
risch	$-\frac{2\sqrt{bx+a}\,\left(8A\,b^{3}x^{3}-14Ba\,b^{2}x^{3}-4aA\,b^{2}x^{2}+7B\,a^{2}b\,x^{2}+3a^{2}Abx+21B\,a^{3}x+15a^{3}A\right)}{105x^{\frac{7}{2}}a^{3}}$	77

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x+a)^(3/2)*(8*A*b^2*x^2-14*B*a*b*x^2-12*A*a*b*x+21*B*a^2*x+15*A*
a^2)/x^(7/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = -\frac{2\,(15\,Aa^3-2\,(7\,Bab^2-4\,Ab^3)x^3+(7\,Ba^2b-4\,Aab^2)x^2+3\,(7\,Ba^3+Aa^2b)x)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(15*A*a^3 - 2*(7*B*a*b^2 - 4*A*b^3)*x^3 + (7*B*a^2*b - 4*A*a*b^2)*x
^2 + 3*(7*B*a^3 + A*a^2*b)*x)*sqrt(b*x + a)/(a^3*x^(7/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(82) = 164$.

Time = 10.22 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.08

$$\begin{aligned}\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = &-\frac{30Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}\\ &-\frac{66Aa^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}\\ &-\frac{34Aa^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{6Aa^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}\\ &-\frac{24Aab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{16Ab^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}\\ &-\frac{2B\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2}-\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax}+\frac{4Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2}\end{aligned}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(9/2),x)
```

output
```
-30*A*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*
x**4 + 105*a**3*b**6*x**5) - 66*A*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*
a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*A*a**3*b**(
13/2)*x**2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 10
5*a**3*b**6*x**5) - 6*A*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b*
*4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*A*a*b**(17/2)*x**4
*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) - 16*A*b**(19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*
a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 2*B*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x*
*2) - 2*B*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a*x) + 4*B*b**(5/2)*sqrt(a/(b*x)
+ 1)/(15*a**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(66) = 132$.

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = \frac{4\sqrt{bx^2+ax}Bb^2}{15\,a^2x} - \frac{16\sqrt{bx^2+ax}Ab^3}{105\,a^3x} - \frac{2\sqrt{bx^2+ax}Bb}{15\,ax^2} + \frac{8\sqrt{bx^2+ax}Ab^2}{105\,a^2x^2} - \frac{2\sqrt{bx^2+ax}B}{5\,x^3} - \frac{2\sqrt{bx^2+ax}Ab}{35\,ax^3} - \frac{2\sqrt{bx^2+ax}A}{7\,x^4}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(9/2),x, algorithm="maxima")
```

output
```
4/15*sqrt(b*x^2 + a*x)*B*b^2/(a^2*x) - 16/105*sqrt(b*x^2 + a*x)*A*b^3/(a^3
*x) - 2/15*sqrt(b*x^2 + a*x)*B*b/(a*x^2) + 8/105*sqrt(b*x^2 + a*x)*A*b^2/(
a^2*x^2) - 2/5*sqrt(b*x^2 + a*x)*B/x^3 - 2/35*sqrt(b*x^2 + a*x)*A*b/(a*x^3
) - 2/7*sqrt(b*x^2 + a*x)*A/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}\left((bx+a)\left(\frac{2\,(7\,Bab^2-4\,Ab^3)(bx+a)}{a^3}-\frac{7\,(7\,Ba^2b^2-4\,Aab^3)}{a^3}\right)+\frac{35\,(Ba^3b^2-Aa^2b^3)}{a^3}\right.}{105\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(9/2),x, algorithm="giac")
```

output
```
2/105*(b*x + a)^(3/2)*((b*x + a)*(2*(7*B*a*b^2 - 4*A*b^3)*(b*x + a)/a^3 -
7*(7*B*a^2*b^2 - 4*A*a*b^3)/a^3) + 35*(B*a^3*b^2 - A*a^2*b^3)/a^3)*b^5/(((
b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A}{7}+\frac{x\,(42\,B\,a^3+6\,A\,b\,a^2)}{105\,a^3}+\frac{x^3\,(16\,A\,b^3-28\,B\,a\,b^2)}{105\,a^3}-\frac{2\,b\,x^2\,(4\,A\,b-7\,B\,a)}{105\,a^2}\right)}{x^{7/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(9/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/7 + (x*(42*B*a^3 + 6*A*a^2*b))/(105*a^3) + (x^3*(
16*A*b^3 - 28*B*a*b^2))/(105*a^3) - (2*b*x^2*(4*A*b - 7*B*a))/(105*a^2)))/
x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{4\sqrt{b}\,b^3x^4}{35}}{a^2x^4}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(9/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 - 8*sqrt(x)*sqrt(a + b*x)*a**2*b*x - s
qrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 2*sqrt(x)*sqrt(a + b*x)*b**3*x**3 - 2*s
qrt(b)*b**3*x**4))/(35*a**2*x**4)
```

3.293 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = -\frac{2A(a+bx)^{3/2}}{9ax^{9/2}} + \frac{2(2Ab-3aB)(a+bx)^{3/2}}{21a^2x^{7/2}} - \frac{8b(2Ab-3aB)(a+bx)^{3/2}}{105a^3x^{5/2}} + \frac{16b^2(2Ab-3aB)(a+bx)^{3/2}}{315a^4x^{3/2}}$$

output
```
-2/9*A*(b*x+a)^(3/2)/a/x^(9/2)+2/21*(2*A*b-3*B*a)*(b*x+a)^(3/2)/a^2/x^(7/2
)-8/105*b*(2*A*b-3*B*a)*(b*x+a)^(3/2)/a^3/x^(5/2)+16/315*b^2*(2*A*b-3*B*a)
*(b*x+a)^(3/2)/a^4/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = -\frac{2(a+bx)^{3/2}\left(-16Ab^3x^3+24ab^2x^2(A+Bx)-6a^2bx(5A+6Bx)+5a^3(7A+9Bx)\right)}{315a^4x^{9/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(3/2)*(-16*A*b^3*x^3 + 24*a*b^2*x^2*(A + B*x) - 6*a^2*b*x*(5
*A + 6*B*x) + 5*a^3*(7*A + 9*B*x)))/(315*a^4*x^(9/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(2Ab-3aB)\int \frac{\sqrt{a+bx}}{x^{9/2}}dx}{3a}-\frac{2A(a+bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(2Ab-3aB)\left(-\frac{4b\int \frac{\sqrt{a+bx}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2A(a+bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(2Ab-3aB)\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2A(a+bx)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(-\frac{4b\left(\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)(2Ab-3aB)}{3a}-\frac{2A(a+bx)^{3/2}}{9ax^{9/2}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^(11/2),x]
```

output
```
(-2*A*(a + b*x)^(3/2))/(9*a*x^(9/2)) - ((2*A*b - 3*a*B)*((-2*(a + b*x)^(3/
2))/(7*a*x^(7/2)) - (4*b*((-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b
*x)^(3/2))/(15*a^2*x^(3/2))))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16A\,b^{3}x^{3}+24Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-36B\,a^{2}b\,x^{2}-30a^{2}Abx+45B\,a^{3}x+35a^{3}A\right)}{315x^{\frac{9}{2}}a^{4}}$	77
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16A\,b^{3}x^{3}+24Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-36B\,a^{2}b\,x^{2}-30a^{2}Abx+45B\,a^{3}x+35a^{3}A\right)}{315x^{\frac{9}{2}}a^{4}}$	77
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16A\,b^{3}x^{3}+24Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-36B\,a^{2}b\,x^{2}-30a^{2}Abx+45B\,a^{3}x+35a^{3}A\right)}{315x^{\frac{9}{2}}a^{4}}$	77
risch	$-\frac{2\sqrt{bx+a}\,\left(-16A\,b^{4}x^{4}+24Ba\,b^{3}x^{4}+8Aa\,b^{3}x^{3}-12B\,a^{2}b^{2}x^{3}-6A\,a^{2}b^{2}x^{2}+9B\,a^{3}b\,x^{2}+5A\,a^{3}bx+45B\,a^{4}x+35A\,a^{4}\right)}{315x^{\frac{9}{2}}a^{4}}$	101

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(b*x+a)^(3/2)*(-16*A*b^3*x^3+24*B*a*b^2*x^3+24*A*a*b^2*x^2-36*B*a^2
*b*x^2-30*A*a^2*b*x+45*B*a^3*x+35*A*a^3)/x^(9/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = -\frac{2\,(35\,Aa^{4}+8\,(3\,Bab^{3}-2\,Ab^{4})x^{4}-4\,(3\,Ba^{2}b^{2}-2\,Aab^{3})x^{3}+3\,(3\,Ba^{3}b-2\,Aa^{2}b^{2})x^{2}+5\,(9\,Ba^{4}+Aa^{3}b)}{315\,a^{4}x^{\frac{9}{2}}}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/315*(35*A*a^4 + 8*(3*B*a*b^3 - 2*A*b^4)*x^4 - 4*(3*B*a^2*b^2 - 2*A*a*b^
3)*x^3 + 3*(3*B*a^3*b - 2*A*a^2*b^2)*x^2 + 5*(9*B*a^4 + A*a^3*b)*x)*sqrt(b
*x + a)/(a^4*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(116) = 232$.

Time = 31.10 (sec) , antiderivative size = 930, normalized size of antiderivative = 7.95

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(11/2),x)
```

output
```
-70*A*a**7*b**(19/2)*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**1
0*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 220*A*a**6*b**(21/2)
*x*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*
b**11*x**6 + 315*a**4*b**12*x**7) - 228*A*a**5*b**(23/2)*x**2*sqrt(a/(b*x)
 + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 31
5*a**4*b**12*x**7) - 80*A*a**4*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(315*a**7*
b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**
7) + 10*A*a**3*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*
a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 60*A*a**2*b
**(29/2)*x**5*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5
+ 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) + 80*A*a*b**(31/2)*x**6*sqrt(
a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x*
*6 + 315*a**4*b**12*x**7) + 32*A*b**(33/2)*x**7*sqrt(a/(b*x) + 1)/(315*a**
7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x
**7) - 30*B*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4
*b**5*x**4 + 105*a**3*b**6*x**5) - 66*B*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)
/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*B*a**
3*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 6*B*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a
**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*B*a*b**(1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(93) = 186$.

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = -\frac{16\sqrt{bx^2+ax}Bb^3}{105\,a^3x} + \frac{32\sqrt{bx^2+ax}Ab^4}{315\,a^4x} + \frac{8\sqrt{bx^2+ax}Bb^2}{105\,a^2x^2} - \frac{16\sqrt{bx^2+ax}Ab^3}{315\,a^3x^2} - \frac{2\sqrt{bx^2+ax}Bb}{35\,ax^3} + \frac{4\sqrt{bx^2+ax}Ab^2}{105\,a^2x^3} - \frac{2\sqrt{bx^2+ax}B}{7\,x^4} - \frac{2\sqrt{bx^2+ax}Ab}{63\,ax^4} - \frac{2\sqrt{bx^2+ax}A}{9\,x^5}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(11/2),x, algorithm="maxima")
```

output
```
-16/105*sqrt(b*x^2 + a*x)*B*b^3/(a^3*x) + 32/315*sqrt(b*x^2 + a*x)*A*b^4/(
a^4*x) + 8/105*sqrt(b*x^2 + a*x)*B*b^2/(a^2*x^2) - 16/315*sqrt(b*x^2 + a*x
)*A*b^3/(a^3*x^2) - 2/35*sqrt(b*x^2 + a*x)*B*b/(a*x^3) + 4/105*sqrt(b*x^2
+ a*x)*A*b^2/(a^2*x^3) - 2/7*sqrt(b*x^2 + a*x)*B/x^4 - 2/63*sqrt(b*x^2 + a
*x)*A*b/(a*x^4) - 2/9*sqrt(b*x^2 + a*x)*A/x^5
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = -\frac{2\left(\left(bx+a\right)\left(4\left(bx+a\right)\left(\frac{2\left(3\,Bab^8-2\,Ab^9\right)\left(bx+a\right)}{a^4} - \frac{9\left(3\,Ba^2b^8-2\,Aab^9\right)}{a^4}\right) + \frac{63\left(3\,Ba^3b^8-2\,Aa^2b^9\right)}{a^4}\right) - \frac{105\left(Ba^4b^8-Aa^3b^9\right)}{a^4}\right)}{315\left((bx+a)b-ab\right)^{\frac{9}{2}}|b|}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(11/2),x, algorithm="giac")
```

output
```
-2/315*((b*x + a)*(4*(b*x + a)*(2*(3*B*a*b^8 - 2*A*b^9)*(b*x + a)/a^4 - 9*
(3*B*a^2*b^8 - 2*A*a*b^9)/a^4) + 63*(3*B*a^3*b^8 - 2*A*a^2*b^9)/a^4) - 105
*(B*a^4*b^8 - A*a^3*b^9)/a^4)*(b*x + a)^(3/2)*b/(((b*x + a)*b - a*b)^(9/2)
*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{9}+\frac{x\,\left(90\,B\,a^4+10\,A\,b\,a^3\right)}{315\,a^4}-\frac{x^4\,\left(32\,A\,b^4-48\,B\,a\,b^3\right)}{315\,a^4}+\frac{8\,b^2\,x^3\,\left(2\,A\,b-3\,B\,a\right)}{315\,a^3}-\frac{2\,b\,x^2\,\left(2\,A\,b-3\,B\,a\right)}{105\,a^2}\right)}{x^{9/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(11/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/9 + (x*(90*B*a^4 + 10*A*a^3*b))/(315*a^4) - (x^4*
(32*A*b^4 - 48*B*a*b^3))/(315*a^4) + (8*b^2*x^3*(2*A*b - 3*B*a))/(315*a^3)
 - (2*b*x^2*(2*A*b - 3*B*a))/(105*a^2)))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9}-\frac{20\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{105}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{315}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b}{315}}{a^3x^5}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(11/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 50*sqrt(x)*sqrt(a + b*x)*a**3*b*x -
 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b**3*x
**3 - 8*sqrt(x)*sqrt(a + b*x)*b**4*x**4 + 8*sqrt(b)*b**4*x**5))/(315*a**3*
x**5)
```

3.294 $\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = -\frac{2A(a+bx)^{3/2}}{11ax^{11/2}} + \frac{2(8Ab-11aB)(a+bx)^{3/2}}{99a^2x^{9/2}} - \frac{4b(8Ab-11aB)(a+bx)^{3/2}}{231a^3x^{7/2}} + \frac{16b^2(8Ab-11aB)(a+bx)^{3/2}}{1155a^4x^{5/2}} - \frac{32b^3(8Ab-11aB)(a+bx)^{3/2}}{3465a^5x^{3/2}}$$

output
```
-2/11*A*(b*x+a)^(3/2)/a/x^(11/2)+2/99*(8*A*b-11*B*a)*(b*x+a)^(3/2)/a^2/x^(
9/2)-4/231*b*(8*A*b-11*B*a)*(b*x+a)^(3/2)/a^3/x^(7/2)+16/1155*b^2*(8*A*b-1
1*B*a)*(b*x+a)^(3/2)/a^4/x^(5/2)-32/3465*b^3*(8*A*b-11*B*a)*(b*x+a)^(3/2)/
a^5/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx =$$
$$-\frac{2(a+bx)^{3/2}\left(128Ab^4x^4+35a^4(9A+11Bx)+24a^2b^2x^2(10A+11Bx)-16ab^3x^3(12A+11Bx)-10a^3b\right.}{3465a^5x^{11/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(3/2)*(128*A*b^4*x^4 + 35*a^4*(9*A + 11*B*x) + 24*a^2*b^2*x^
2*(10*A + 11*B*x) - 16*a*b^3*x^3*(12*A + 11*B*x) - 10*a^3*b*x*(28*A + 33*B
*x)))/(3465*a^5*x^(11/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(8Ab-11aB)\int \frac{\sqrt{a+bx}}{x^{11/2}}dx}{11a}-\frac{2A(a+bx)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(8Ab-11aB)\left(-\frac{2b\int \frac{\sqrt{a+bx}}{x^{9/2}}dx}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}\right)}{11a}-\frac{2A(a+bx)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(8Ab-11aB)\left(-\frac{2b\left(-\frac{4b\int \frac{\sqrt{a+bx}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}\right)}{11a}-\frac{2A(a+bx)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(8Ab-11aB)\left(-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{a+bx}}{x^{5/2}}dx}{5a}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}\right)}{11a}-\frac{2A(a+bx)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(-\frac{2b\left(-\frac{4b\left(\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}\right)}{7a}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}\right)}{3a}-\frac{2(a+bx)^{3/2}}{9ax^{9/2}}\right)(8Ab-11aB)}{11a}-\frac{2A(a+bx)^{3/2}}{11ax^{11/2}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/x^(13/2),x]
```

output
```
(-2*A*(a + b*x)^(3/2))/(11*a*x^(11/2)) - ((8*A*b - 11*a*B)*((-2*(a + b*x)^
(3/2))/(9*a*x^(9/2)) - (2*b*((-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) - (4*b*((-
2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2)))
)/(7*a)))/(3*a)))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

method	result
gosper	$-\frac{2(bx+a)^{\frac{3}{2}} \left(128A\,b^{4}x^{4}-176Ba\,b^{3}x^{4}-192Aa\,b^{3}x^{3}+264B\,a^{2}b^{2}x^{3}+240A\,a^{2}b^{2}x^{2}-330B\,a^{3}b\,x^{2}-280A\,a^{3}bx+385B\,a^{4}x+315A\,a^{4}\right)}{3465x^{\frac{11}{2}}a^{5}}$
default	$-\frac{2(bx+a)^{\frac{3}{2}} \left(128A\,b^{4}x^{4}-176Ba\,b^{3}x^{4}-192Aa\,b^{3}x^{3}+264B\,a^{2}b^{2}x^{3}+240A\,a^{2}b^{2}x^{2}-330B\,a^{3}b\,x^{2}-280A\,a^{3}bx+385B\,a^{4}x+315A\,a^{4}\right)}{3465x^{\frac{11}{2}}a^{5}}$
orering	$-\frac{2(bx+a)^{\frac{3}{2}} \left(128A\,b^{4}x^{4}-176Ba\,b^{3}x^{4}-192Aa\,b^{3}x^{3}+264B\,a^{2}b^{2}x^{3}+240A\,a^{2}b^{2}x^{2}-330B\,a^{3}b\,x^{2}-280A\,a^{3}bx+385B\,a^{4}x+315A\,a^{4}\right)}{3465x^{\frac{11}{2}}a^{5}}$
risch	$-\frac{2\sqrt{bx+a}\, (128A\,b^{5}x^{5}-176Ba\,b^{4}x^{5}-64aA\,b^{4}x^{4}+88B\,a^{2}b^{3}x^{4}+48a^{2}A\,b^{3}x^{3}-66B\,a^{3}b^{2}x^{3}-40a^{3}A\,b^{2}x^{2}+55B\,a^{4}b\,x^{2}+35a^{4}Abx+38}{3465x^{\frac{11}{2}}a^{5}}$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*(b*x+a)^(3/2)*(128*A*b^4*x^4-176*B*a*b^3*x^4-192*A*a*b^3*x^3+264*B
*a^2*b^2*x^3+240*A*a^2*b^2*x^2-330*B*a^3*b*x^2-280*A*a^3*b*x+385*B*a^4*x+3
15*A*a^4)/x^(11/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = -\frac{2\,(315\,Aa^5 - 16\,(11\,Bab^4 - 8\,Ab^5)x^5 + 8\,(11\,Ba^2b^3 - 8\,Aab^4)x^4 - 6\,(11\,Ba^3b^2 - 8\,Aa^2b^3)x^3 + 5\,(11\,Ba^4b}{3465\,a^5x^{\frac{11}{2}}}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/3465*(315*A*a^5 - 16*(11*B*a*b^4 - 8*A*b^5)*x^5 + 8*(11*B*a^2*b^3 - 8*A
*a*b^4)*x^4 - 6*(11*B*a^3*b^2 - 8*A*a^2*b^3)*x^3 + 5*(11*B*a^4*b - 8*A*a^3
*b^2)*x^2 + 35*(11*B*a^5 + A*a^4*b)*x)*sqrt(b*x + a)/(a^5*x^(11/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1413 vs. 2(150) = 300.

Time = 82.52 (sec) , antiderivative size = 1413, normalized size of antiderivative = 9.42

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/x**(13/2),x)
```

output
```
-630*A*a**9*b**(33/2)*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8
*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b*
*20*x**9) - 2590*A*a**8*b**(35/2)*x*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x**
5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8
+ 3465*a**5*b**20*x**9) - 3980*A*a**7*b**(37/2)*x**2*sqrt(a/(b*x) + 1)/(34
65*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860
*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 2716*A*a**6*b**(39/2)*x**3*sqrt
(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b
**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 686*A*a**5*b**
(41/2)*x**4*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**
6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9)
- 70*A*a**4*b**(43/2)*x**5*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860
*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a*
*5*b**20*x**9) - 560*A*a**3*b**(45/2)*x**6*sqrt(a/(b*x) + 1)/(3465*a**9*b*
*16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**1
9*x**8 + 3465*a**5*b**20*x**9) - 1120*A*a**2*b**(47/2)*x**7*sqrt(a/(b*x) +
 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7
+ 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 896*A*a*b**(49/2)*x**8*s
qrt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**
7*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 256*A*b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = \frac{32\sqrt{bx^2+ax}Bb^4}{315\,a^4x} - \frac{256\sqrt{bx^2+ax}Ab^5}{3465\,a^5x} - \frac{16\sqrt{bx^2+ax}Bb^3}{315\,a^3x^2} + \frac{128\sqrt{bx^2+ax}Ab^4}{3465\,a^4x^2} + \frac{4\sqrt{bx^2+ax}Bb^2}{105\,a^2x^3} - \frac{32\sqrt{bx^2+ax}Ab^3}{1155\,a^3x^3} - \frac{2\sqrt{bx^2+ax}Bb}{63\,ax^4} + \frac{16\sqrt{bx^2+ax}Ab^2}{693\,a^2x^4} - \frac{2\sqrt{bx^2+ax}B}{9\,x^5} - \frac{2\sqrt{bx^2+ax}Ab}{99\,ax^5} - \frac{2\sqrt{bx^2+ax}A}{11\,x^6}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(13/2),x, algorithm="maxima")
```

output
```
32/315*sqrt(b*x^2 + a*x)*B*b^4/(a^4*x) - 256/3465*sqrt(b*x^2 + a*x)*A*b^5/
(a^5*x) - 16/315*sqrt(b*x^2 + a*x)*B*b^3/(a^3*x^2) + 128/3465*sqrt(b*x^2 +
 a*x)*A*b^4/(a^4*x^2) + 4/105*sqrt(b*x^2 + a*x)*B*b^2/(a^2*x^3) - 32/1155*
sqrt(b*x^2 + a*x)*A*b^3/(a^3*x^3) - 2/63*sqrt(b*x^2 + a*x)*B*b/(a*x^4) + 1
6/693*sqrt(b*x^2 + a*x)*A*b^2/(a^2*x^4) - 2/9*sqrt(b*x^2 + a*x)*B/x^5 - 2/
99*sqrt(b*x^2 + a*x)*A*b/(a*x^5) - 2/11*sqrt(b*x^2 + a*x)*A/x^6
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = \frac{2\left(\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(11\,Bab^4-8\,Ab^5)(bx+a)}{a^5}-\frac{11\,(11\,Ba^2b^4-8\,Aab^5)}{a^5}\right)+\frac{99\,(11\,Ba^3}{}\right.\right.\right.}{3465\,((bx+a)}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/x^(13/2),x, algorithm="giac")
```

output
```
2/3465*((2*(b*x + a)*(4*(b*x + a)*(2*(11*B*a*b^4 - 8*A*b^5)*(b*x + a)/a^5
- 11*(11*B*a^2*b^4 - 8*A*a*b^5)/a^5) + 99*(11*B*a^3*b^4 - 8*A*a^2*b^5)/a^5
) - 231*(11*B*a^4*b^4 - 8*A*a^3*b^5)/a^5)*(b*x + a) + 1155*(B*a^5*b^4 - A*
a^4*b^5)/a^5)*(b*x + a)^(3/2)*b^7/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A}{11}+\frac{x\,(770\,B\,a^5+70\,A\,b\,a^4)}{3465\,a^5}+\frac{x^5\,(256\,A\,b^5-352\,B\,a\,b^4)}{3465\,a^5}+\frac{4\,b^2\,x^3\,(8\,A\,b-11\,B\,a)}{1155\,a^3}-\frac{16\,b^3\,x^4\,(8\,A\,b-11\,B\,a)}{3465\,a^4}-\frac{2\,b\,x^2\,(8}{69}\right.}{x^{11/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/x^(13/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/11 + (x*(770*B*a^5 + 70*A*a^4*b))/(3465*a^5) + (x
^5*(256*A*b^5 - 352*B*a*b^4))/(3465*a^5) + (4*b^2*x^3*(8*A*b - 11*B*a))/(1
155*a^3) - (16*b^3*x^4*(8*A*b - 11*B*a))/(3465*a^4) - (2*b*x^2*(8*A*b - 11
*B*a))/(693*a^2)))/x^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx}(A+Bx)}{x^{13/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11} - \frac{8\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{33} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{231} + \frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{385} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,a}{1155}}{a^4x^6}$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/x^(13/2),x)
```

output
```
(2*( - 105*sqrt(x)*sqrt(a + b*x)*a**5 - 140*sqrt(x)*sqrt(a + b*x)*a**4*b*x
 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b
**3*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 + 16*sqrt(x)*sqrt(a + b*x)*
b**5*x**5 - 16*sqrt(b)*b**5*x**6))/(1155*a**4*x**6)
```

3.295 $\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 222

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \frac{a^4(12Ab-7aB)\sqrt{x}\sqrt{a+bx}}{512b^4} - \frac{a^3(12Ab-7aB)x^{3/2}\sqrt{a+bx}}{768b^3} + \frac{a^2(12Ab-7aB)x^{5/2}\sqrt{a+bx}}{960b^2} + \frac{11a(12Ab-7aB)x^{7/2}\sqrt{a+bx}}{480b} + \frac{1}{60}(12Ab-7aB)x^{9/2}\sqrt{a+bx} + \frac{Bx^{7/2}(a+bx)^{5/2}}{6b} - \frac{a^5(12Ab-7aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{9/2}}$$

output
```
1/512*a^4*(12*A*b-7*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^4-1/768*a^3*(12*A*b-7*B*a
)*x^(3/2)*(b*x+a)^(1/2)/b^3+1/960*a^2*(12*A*b-7*B*a)*x^(5/2)*(b*x+a)^(1/2)
/b^2+11/480*a*(12*A*b-7*B*a)*x^(7/2)*(b*x+a)^(1/2)/b+1/60*(12*A*b-7*B*a)*x
^(9/2)*(b*x+a)^(1/2)+1/6*B*x^(7/2)*(b*x+a)^(5/2)/b-1/512*a^5*(12*A*b-7*B*a
)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.87

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^5B+48a^2b^3x^2(2A+Bx)+256b^5x^4(6A+5Bx)-8a^3b^2x(15\ldots}{}$$

input
```
Integrate[x^(5/2)*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^5*B + 48*a^2*b^3*x^2*(2*A + B*x) +
256*b^5*x^4*(6*A + 5*B*x) - 8*a^3*b^2*x*(15*A + 7*B*x) + 10*a^4*b*(18*A +
7*B*x) + 64*a*b^4*x^3*(33*A + 26*B*x)) + 360*a^5*A*b*ArcTanh[(Sqrt[b]*Sqrt
[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^6*B*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sq
rt[a] + Sqrt[a + b*x])])/(7680*b^(9/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {90, 60, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(12Ab-7aB)\int x^{5/2}(a+bx)^{3/2}dx}{12b}+\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\int x^{5/2}\sqrt{a+bx}dx+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)}{12b}+\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\left(\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a+bx}}dx+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)}{12b}+\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$\big\downarrow$ 60

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int\frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)}{12b}+\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$\big\downarrow$ 60

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)}{12b}+\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$\big\downarrow$ 60

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x^{7/2}(a+bx)^{3/2}\right)}{12b}+\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$\big\downarrow$ 65

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}x\cdots}{12b}$$

$$\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

$$\Big\downarrow \ 219$$

$$\frac{(12Ab-7aB)\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)+\frac{1}{4}x^{7/2}\sqrt{a+bx}\right)+\frac{1}{5}\cdots}{12b}$$

$$\frac{Bx^{7/2}(a+bx)^{5/2}}{6b}$$

input
```
Int[x^(5/2)*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(B*x^(7/2)*(a + b*x)^(5/2))/(6*b) + ((12*A*b - 7*a*B)*((x^(7/2)*(a + b*x)^
(3/2))/5 + (3*a*((x^(7/2)*Sqrt[a + b*x])/4 + (a*((x^(5/2)*Sqrt[a + b*x])/(
3*b) - (5*a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])
/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/(6*b))
)/8))/10))/(12*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.82

method	result
risch	$\frac{(1280b^5Bx^5+1536Ab^5x^4+1664Bab^4x^4+2112Aab^4x^3+48Ba^2b^3x^3+96Aa^2b^3x^2-56Ba^3b^2x^2-120a^3b^2Ax+70a^4bBx+180a^4bA}{7680b^4}$
default	$-\frac{\sqrt{x}\sqrt{bx+a}\left(-2560Bb^{\frac{11}{2}}x^5\sqrt{x(bx+a)}-3072Ab^{\frac{11}{2}}x^4\sqrt{x(bx+a)}-3328Bab^{\frac{9}{2}}x^4\sqrt{x(bx+a)}-4224Aab^{\frac{9}{2}}x^3\sqrt{x(bx+a)}-96Ba^2b\right.}{}$

input
```
int(x^(5/2)*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7680/b^4*(1280*B*b^5*x^5+1536*A*b^5*x^4+1664*B*a*b^4*x^4+2112*A*a*b^4*x^
3+48*B*a^2*b^3*x^3+96*A*a^2*b^3*x^2-56*B*a^3*b^2*x^2-120*A*a^3*b^2*x+70*B*
a^4*b*x+180*A*a^4*b-105*B*a^5)*x^(1/2)*(b*x+a)^(1/2)-1/1024*a^5/b^(9/2)*(1
2*A*b-7*B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x
^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.53

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \left[-\frac{15\,(7\,Ba^6-12\,Aa^5b)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(1280\,Bb^6x^5-10\right.$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(7*B*a^6 - 12*A*a^5*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*s
qrt(b)*sqrt(x) + a) - 2*(1280*B*b^6*x^5 - 105*B*a^5*b + 180*A*a^4*b^2 + 12
8*(13*B*a*b^5 + 12*A*b^6)*x^4 + 48*(B*a^2*b^4 + 44*A*a*b^5)*x^3 - 8*(7*B*a
^3*b^3 - 12*A*a^2*b^4)*x^2 + 10*(7*B*a^4*b^2 - 12*A*a^3*b^3)*x)*sqrt(b*x +
 a)*sqrt(x))/b^5, -1/7680*(15*(7*B*a^6 - 12*A*a^5*b)*sqrt(-b)*arctan(sqrt(
-b)*sqrt(x)/sqrt(b*x + a)) - (1280*B*b^6*x^5 - 105*B*a^5*b + 180*A*a^4*b^2
 + 128*(13*B*a*b^5 + 12*A*b^6)*x^4 + 48*(B*a^2*b^4 + 44*A*a*b^5)*x^3 - 8*(
7*B*a^3*b^3 - 12*A*a^2*b^4)*x^2 + 10*(7*B*a^4*b^2 - 12*A*a^3*b^3)*x)*sqrt(
b*x + a)*sqrt(x))/b^5]
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.26

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = -\frac{7\sqrt{bx^2+ax}Ba^4x}{256\,b^3} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}Ba^2x}{96\,b^2} + \frac{3\sqrt{bx^2+ax}Aa^3x}{64\,b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}Bx}{6\,b} - \frac{(bx^2+ax)^{\frac{3}{2}}Aax}{8\,b} + \frac{7\,Ba^6\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{9}{2}}} - \frac{3\,Aa^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} - \frac{7\sqrt{bx^2+ax}Ba^5}{512\,b^4} + \frac{7\,(bx^2+ax)^{\frac{3}{2}}Ba^3}{192\,b^3} + \frac{3\sqrt{bx^2+ax}Aa^4}{128\,b^3} - \frac{7\,(bx^2+ax)^{\frac{5}{2}}Ba}{60\,b^2} - \frac{(bx^2+ax)^{\frac{3}{2}}Aa^2}{16\,b^2} + \frac{(bx^2+ax)^{\frac{5}{2}}A}{5\,b}$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
-7/256*sqrt(b*x^2 + a*x)*B*a^4*x/b^3 + 7/96*(b*x^2 + a*x)^(3/2)*B*a^2*x/b^
2 + 3/64*sqrt(b*x^2 + a*x)*A*a^3*x/b^2 + 1/6*(b*x^2 + a*x)^(5/2)*B*x/b - 1
/8*(b*x^2 + a*x)^(3/2)*A*a*x/b + 7/1024*B*a^6*log(2*b*x + a + 2*sqrt(b*x^2
 + a*x)*sqrt(b))/b^(9/2) - 3/256*A*a^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/b^(7/2) - 7/512*sqrt(b*x^2 + a*x)*B*a^5/b^4 + 7/192*(b*x^2 + a*x
)^(3/2)*B*a^3/b^3 + 3/128*sqrt(b*x^2 + a*x)*A*a^4/b^3 - 7/60*(b*x^2 + a*x)
^(5/2)*B*a/b^2 - 1/16*(b*x^2 + a*x)^(3/2)*A*a^2/b^2 + 1/5*(b*x^2 + a*x)^(5
/2)*A/b
```

Giac [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(5/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \int x^{5/2}\,(A+B\,x)\,\left(a+b\,x\right)^{3/2}\,dx$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
int(x^(5/2)*(A + B*x)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.60

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx)\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^5b-10\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x+8\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^2+432\sqrt{x}\,\sqrt{bx+a}}{153}$$

input
```
int(x^(5/2)*(b*x+a)^(3/2)*(B*x+A),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**5*b - 10*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a**3*b**3*x**2 + 432*sqrt(x)*sqrt(a + b*x)*a**2*b*
*4*x**3 + 640*sqrt(x)*sqrt(a + b*x)*a*b**5*x**4 + 256*sqrt(x)*sqrt(a + b*x
)*b**6*x**5 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
*6)/(1536*b**4)
```

3.296 $\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx = -\frac{3a^3(2Ab-aB)\sqrt{x}\sqrt{a+bx}}{128b^3} + \frac{a^2(2Ab-aB)x^{3/2}\sqrt{a+bx}}{64b^2} + \frac{3a(2Ab-aB)x^{5/2}\sqrt{a+bx}}{16b} + \frac{1}{8}(2Ab-aB)x^{7/2}\sqrt{a+bx} + \frac{Bx^{5/2}(a+bx)^{5/2}}{5b} + \frac{3a^4(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}$$

output
```
-3/128*a^3*(2*A*b-B*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+1/64*a^2*(2*A*b-B*a)*x^(3
/2)*(b*x+a)^(1/2)/b^2+3/16*a*(2*A*b-B*a)*x^(5/2)*(b*x+a)^(1/2)/b+1/8*(2*A*
b-B*a)*x^(7/2)*(b*x+a)^(1/2)+1/5*B*x^(5/2)*(b*x+a)^(5/2)/b+3/128*a^4*(2*A*
b-B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx = \frac{\sqrt{x}\sqrt{a+bx}(15a^4B-10a^3b(3A+Bx)+4a^2b^2x(5A+2Bx)+32b^4x^3(5A+4Bx)+}{640b^3} + \frac{3a^4(-2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{64b^{7/2}}$$

input
```
Integrate[x^(3/2)*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(15*a^4*B - 10*a^3*b*(3*A + B*x) + 4*a^2*b^2*x*(5*A
 + 2*B*x) + 32*b^4*x^3*(5*A + 4*B*x) + 16*a*b^3*x^2*(15*A + 11*B*x)))/(640
*b^3) + (3*a^4*(-2*A*b + a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a
+ b*x])])/(64*b^(7/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {90, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(2Ab-aB)\int x^{3/2}(a+bx)^{3/2}dx}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\int x^{3/2}\sqrt{a+bx}dx+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \ 60$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \ 60$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \ 60$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \ 65$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

$$\Big\downarrow \ 219$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)}{2b}+\frac{Bx^{5/2}(a+bx)^{5/2}}{5b}$$

input
```
Int[x^(3/2)*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(B*x^(5/2)*(a + b*x)^(5/2))/(5*b) + ((2*A*b - a*B)*((x^(5/2)*(a + b*x)^(3/
2))/4 + (3*a*((x^(5/2)*Sqrt[a + b*x])/3 + (a*((x^(3/2)*Sqrt[a + b*x])/(2*b
) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
+ b*x]])/b^(3/2)))/(4*b)))/6))/8))/(2*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\left(-128 B\,x^{4} b^{4}-160 A\,x^{3} b^{4}-176 B\,x^{3} a\,b^{3}-240 A\,x^{2} a\,b^{3}-8 B\,x^{2} a^{2} b^{2}-20 A x\,a^{2} b^{2}+10 B x\,a^{3} b+30 A\,a^{3} b-15 B\,a^{4}\right) \sqrt{x}\,\sqrt{b x+a}}{640 b^{3}}+\frac{3 a^{4}}{}$
default	$\frac{\sqrt{x}\,\sqrt{b x+a}\,\left(256 B\,b^{\frac{9}{2}} x^{4} \sqrt{x(b x+a)}+320 A\,b^{\frac{9}{2}} x^{3} \sqrt{x(b x+a)}+352 B a\,b^{\frac{7}{2}} x^{3} \sqrt{x(b x+a)}+480 A a\,b^{\frac{7}{2}} x^{2} \sqrt{x(b x+a)}+16 B\,a^{2} b^{\frac{5}{2}} x^{2} \sqrt{x(b a}\right.}{}$

input

```
int(x^(3/2)*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output

```
-1/640/b^3*(-128*B*b^4*x^4-160*A*b^4*x^3-176*B*a*b^3*x^3-240*A*a*b^3*x^2-8
*B*a^2*b^2*x^2-20*A*a^2*b^2*x+10*B*a^3*b*x+30*A*a^3*b-15*B*a^4)*x^(1/2)*(b
*x+a)^(1/2)+3/256*a^4/b^(7/2)*(2*A*b-B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*
x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.52

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx = \left[-\frac{15\,(Ba^{5}-2\,Aa^{4}b)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(128\,Bb^{5}x^{4}+15\,Ba^{4}}{}\right.$$

input

```
integrate(x^(3/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output

```
[-1/1280*(15*(B*a^5 - 2*A*a^4*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(128*B*b^5*x^4 + 15*B*a^4*b - 30*A*a^3*b^2 + 16*(11*B*
a*b^4 + 10*A*b^5)*x^3 + 8*(B*a^2*b^3 + 30*A*a*b^4)*x^2 - 10*(B*a^3*b^2 - 2
*A*a^2*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*(B*a^5 - 2*A*a^4*b)*s
qrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (128*B*b^5*x^4 + 15*B*a^4
*b - 30*A*a^3*b^2 + 16*(11*B*a*b^4 + 10*A*b^5)*x^3 + 8*(B*a^2*b^3 + 30*A*a
*b^4)*x^2 - 10*(B*a^3*b^2 - 2*A*a^2*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(173) = 346$.

Time = 154.52 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.88

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx = -\frac{3Aa^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1+\frac{bx}{a}}} - \frac{Aa^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}} + \frac{13Aa^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}} + \frac{5A\sqrt{a}bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}} + \frac{3Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}} + \frac{Ab^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}} + \frac{3Ba^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1+\frac{bx}{a}}} + \frac{Ba^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1+\frac{bx}{a}}} - \frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}} + \frac{23Ba^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}} + \frac{19B\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}} - \frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}} + \frac{Bb^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
-3*A*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - A*a**(5/2)*x**(3/2)/(64*
b*sqrt(1 + b*x/a)) + 13*A*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*A*sqr
t(a)*b*x**(7/2)/(8*sqrt(1 + b*x/a)) + 3*A*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(
a))/(64*b**(5/2)) + A*b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a)) + 3*B*a**(
9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + B*a**(7/2)*x**(3/2)/(128*b**2*sq
rt(1 + b*x/a)) - B*a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*B*a**(3/
2)*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*B*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*
x/a)) - 3*B*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + B*b**2*x*
*(11/2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.25

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx = \frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}Ax + \frac{3\sqrt{bx^2+ax}Ba^3x}{64\,b^2} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}Bax}{8\,b} - \frac{3\sqrt{bx^2+ax}Aa^2x}{32\,b} - \frac{3\,Ba^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,Aa^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} + \frac{3\sqrt{bx^2+ax}Ba^4}{128\,b^3} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}Ba^2}{16\,b^2} - \frac{3\sqrt{bx^2+ax}Aa^3}{64\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}}B}{5\,b} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}Aa}{8\,b}$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a*x)^(3/2)*A*x + 3/64*sqrt(b*x^2 + a*x)*B*a^3*x/b^2 - 1/8*(b*
x^2 + a*x)^(3/2)*B*a*x/b - 3/32*sqrt(b*x^2 + a*x)*A*a^2*x/b - 3/256*B*a^5*
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 3/128*A*a^4*log(2*b
*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) + 3/128*sqrt(b*x^2 + a*x)*B*
a^4/b^3 - 1/16*(b*x^2 + a*x)^(3/2)*B*a^2/b^2 - 3/64*sqrt(b*x^2 + a*x)*A*a^
3/b^2 + 1/5*(b*x^2 + a*x)^(5/2)*B/b + 1/8*(b*x^2 + a*x)^(3/2)*A*a/b
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx=\int x^{3/2}\,(A+B\,x)\,\left(a+b\,x\right)^{3/2}\,dx$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
int(x^(3/2)*(A + B*x)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.60

$$\int x^{3/2}(a+bx)^{3/2}(A+Bx)\,dx=\frac{-15\sqrt{x}\,\sqrt{bx+a}\,a^4b+10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+248\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+336\sqrt{x}\,\sqrt{bx}}{640b^3}$$

input
```
int(x^(3/2)*(b*x+a)^(3/2)*(B*x+A),x)
```

output
```
( - 15*sqrt(x)*sqrt(a + b*x)*a**4*b + 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x
 + 248*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 336*sqrt(x)*sqrt(a + b*x)*a*
b**4*x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*x**4 + 15*sqrt(b)*log((sqrt(a +
 b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(640*b**3)
```

3.297 $\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx=\frac{a^2(8Ab-3aB)\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{7a(8Ab-3aB)x^{3/2}\sqrt{a+bx}}{96b}+\frac{1}{24}(8Ab-3aB)x^{5/2}\sqrt{a+bx}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}-\frac{a^3(8Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}$$

output
```
1/64*a^2*(8*A*b-3*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^2+7/96*a*(8*A*b-3*B*a)*x^(3
/2)*(b*x+a)^(1/2)/b+1/24*(8*A*b-3*B*a)*x^(5/2)*(b*x+a)^(1/2)+1/4*B*x^(3/2)
*(b*x+a)^(5/2)/b-1/64*a^3*(8*A*b-3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1
/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-9a^3B+6a^2b(4A+Bx)+16b^3x^2(4A+3Bx)+8ab^2x(14A+9Bx))+6a^3(-}{192b^{5/2}}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-9*a^3*B + 6*a^2*b*(4*A + B*x) + 16*b^3*x^
2*(4*A + 3*B*x) + 8*a*b^2*x*(14*A + 9*B*x)) + 6*a^3*(-8*A*b + 3*a*B)*ArcTa
nh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(192*b^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {90, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(8Ab-3aB)\int \sqrt{x}(a+bx)^{3/2}dx}{8b}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\int \sqrt{x}\sqrt{a+bx}dx+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)}{8b}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)}{8b}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)}{8b}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)}{8b}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)}{8b}+\frac{Bx^{3/2}(a+bx)^{5/2}}{4b}$$

input
```
Int[Sqrt[x]*(a + b*x)^(3/2)*(A + B*x),x]
```

output
```
(B*x^(3/2)*(a + b*x)^(5/2))/(4*b) + ((8*A*b - 3*a*B)*((x^(3/2)*(a + b*x)^(
3/2))/3 + (a*((x^(3/2)*Sqrt[a + b*x])/2 + (a*((Sqrt[x]*Sqrt[a + b*x])/b -
(a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/4))/2))/(8*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
risch	$\frac{\left(48 b^{3} B\,x^{3}+64 A\,x^{2} b^{3}+72 B\,x^{2} a\,b^{2}+112 A x a\,b^{2}+6 B x\,a^{2} b+24 a^{2} b A-9 a^{3} B\right) \sqrt{x}\,\sqrt{b x+a}}{192 b^{2}}-\frac{a^{3} \left(8 A b-3 B a\right) \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right) \sqrt{}}{128 b^{\frac{5}{2}} \sqrt{x}\,\sqrt{b x+a}}$
default	$-\frac{\sqrt{x}\,\sqrt{b x+a}\,\left(-96 B\,b^{\frac{7}{2}} x^{3} \sqrt{x \left(b x+a\right)}-128 A\,b^{\frac{7}{2}} x^{2} \sqrt{x \left(b x+a\right)}-144 B a\,b^{\frac{5}{2}} x^{2} \sqrt{x \left(b x+a\right)}-224 A \sqrt{x \left(b x+a\right)}\,b^{\frac{5}{2}} a x-12 B \sqrt{x \left(b x+a\right)}\,b\right.}{384 b^{\frac{5}{2}} \sqrt{x(}}$

input
```
int(x^(1/2)*(b*x+a)^(3/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/192/b^2*(48*B*b^3*x^3+64*A*b^3*x^2+72*B*a*b^2*x^2+112*A*a*b^2*x+6*B*a^2*
b*x+24*A*a^2*b-9*B*a^3)*x^(1/2)*(b*x+a)^(1/2)-1/128*a^3/b^(5/2)*(8*A*b-3*B
*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b
*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.58

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx = \left[-\frac{3\,(3\,Ba^4-8\,Aa^3b)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(48\,Bb^4x^3-9\,Ba^3b+24\,Aa^2b^2}{384\,b^3}\right.$$

$$-\frac{3\,(3\,Ba^4-8\,Aa^3b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(48\,Bb^4x^3-9\,Ba^3b+24\,Aa^2b^2+8\,(9\,Bab^3+8\,Ab^4)x^2+2\,(3}{192\,b^3}$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/384*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(48*B*b^4*x^3 - 9*B*a^3*b + 24*A*a^2*b^2 + 8*(9*B*a*b^
3 + 8*A*b^4)*x^2 + 2*(3*B*a^2*b^2 + 56*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/192*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt
(b*x + a)) - (48*B*b^4*x^3 - 9*B*a^3*b + 24*A*a^2*b^2 + 8*(9*B*a*b^3 + 8*A
*b^4)*x^2 + 2*(3*B*a^2*b^2 + 56*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.93

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx = 2Aa\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \sqrt{a+bx}\left(\frac{a\sqrt{x}}{8b}+\frac{x^{\frac{3}{2}}}{4}\right) & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) + 2Ab\left(\begin{cases} \frac{a^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{16b^2} + \sqrt{a+bx}\left(-\frac{a^2\sqrt{x}}{16b^2}+\frac{ax^{\frac{3}{2}}}{24b}+\frac{x^{\frac{5}{2}}}{6}\right) & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{5}{2}}}{5} & \text{otherwise}\end{cases}\right) + 2Ba\left(\begin{cases} \frac{a^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{16b^2} + \sqrt{a+bx}\left(-\frac{a^2\sqrt{x}}{16b^2}+\frac{ax^{\frac{3}{2}}}{24b}+\frac{x^{\frac{5}{2}}}{6}\right) & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{5}{2}}}{5} & \text{otherwise}\end{cases}\right) + 2Bb\left(\begin{cases} -\frac{5a^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{128b^3} + \sqrt{a+bx}\left(\frac{5a^3\sqrt{x}}{128b^3}-\frac{5a^2x^{\frac{3}{2}}}{192b^2}+\frac{ax^{\frac{5}{2}}}{48b}+\frac{x^{\frac{7}{2}}}{8}\right) & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{7}{2}}}{7} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**(1/2)*(b*x+a)**(3/2)*(B*x+A),x)
```

output
```
2*A*a*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x
))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b) + sqr
t(a + b*x)*(a*sqrt(x)/(8*b) + x**(3/2)/4), Ne(b, 0)), (sqrt(a)*x**(3/2)/3,
 True)) + 2*A*b*Piecewise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2
*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(1
6*b**2) + sqrt(a + b*x)*(-a**2*sqrt(x)/(16*b**2) + a*x**(3/2)/(24*b) + x**
(5/2)/6), Ne(b, 0)), (sqrt(a)*x**(5/2)/5, True)) + 2*B*a*Piecewise((a**3*P
iecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (
sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(16*b**2) + sqrt(a + b*x)*(-a**2*sq
rt(x)/(16*b**2) + a*x**(3/2)/(24*b) + x**(5/2)/6), Ne(b, 0)), (sqrt(a)*x**
(5/2)/5, True)) + 2*B*b*Piecewise((-5*a**4*Piecewise((log(2*sqrt(b)*sqrt(a
 + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x)
, True))/(128*b**3) + sqrt(a + b*x)*(5*a**3*sqrt(x)/(128*b**3) - 5*a**2*x*
*(3/2)/(192*b**2) + a*x**(5/2)/(48*b) + x**(7/2)/8), Ne(b, 0)), (sqrt(a)*x
**(7/2)/7, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(122) = 244$.

Time = 0.03 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.84

$$\begin{aligned}\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx &= \frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}Bx + \frac{1}{2}\sqrt{bx^2+ax}Aax \\ &+ \frac{5\sqrt{bx^2+ax}Ba^2x}{32\,b} - \frac{5\,Ba^4\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} \\ &- \frac{Aa^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} + \frac{5\sqrt{bx^2+ax}Ba^3}{64\,b^2} \\ &- \frac{5\left(bx^2+ax\right)^{\frac{3}{2}}Ba}{24\,b} + \frac{\sqrt{bx^2+ax}Aa^2}{4\,b} - \frac{\sqrt{bx^2+ax}(Ba+Ab)ax}{4\,b} \\ &+ \frac{(Ba+Ab)a^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} \\ &- \frac{\sqrt{bx^2+ax}(Ba+Ab)a^2}{8\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}(Ba+Ab)}{3\,b}\end{aligned}$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a*x)^(3/2)*B*x + 1/2*sqrt(b*x^2 + a*x)*A*a*x + 5/32*sqrt(b*x^
2 + a*x)*B*a^2*x/b - 5/128*B*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(
b))/b^(5/2) - 1/8*A*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/
2) + 5/64*sqrt(b*x^2 + a*x)*B*a^3/b^2 - 5/24*(b*x^2 + a*x)^(3/2)*B*a/b + 1
/4*sqrt(b*x^2 + a*x)*A*a^2/b - 1/4*sqrt(b*x^2 + a*x)*(B*a + A*b)*a*x/b + 1
/16*(B*a + A*b)*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) -
 1/8*sqrt(b*x^2 + a*x)*(B*a + A*b)*a^2/b^2 + 1/3*(b*x^2 + a*x)^(3/2)*(B*a
+ A*b)/b
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx=\text{Timed out}$$

input
```
integrate(x^(1/2)*(b*x+a)^(3/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx=\int \sqrt{x}\,\left(A+B\,x\right)\,{\left(a+b\,x\right)}^{3/2}\,dx$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x)^(3/2),x)
```

output
```
int(x^(1/2)*(A + B*x)*(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.61

$$\int \sqrt{x}(a+bx)^{3/2}(A+Bx)\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b+118\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x+136\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2+48\sqrt{x}\,\sqrt{bx+a}\,b^4x^3-15}{192b^2}$$

input

```
int(x^(1/2)*(b*x+a)^(3/2)*(B*x+A),x)
```

output

```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b + 118*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
 136*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x**
3 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192*b
**2)
```

3.298 $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{5a(6Ab-aB)\sqrt{x}\sqrt{a+bx}}{24b} + \frac{1}{12}(6Ab-aB)x^{3/2}\sqrt{a+bx} + \frac{B\sqrt{x}(a+bx)^{5/2}}{3b} + \frac{a^2(6Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}$$

output
```
5/24*a*(6*A*b-B*a)*x^(1/2)*(b*x+a)^(1/2)/b+1/12*(6*A*b-B*a)*x^(3/2)*(b*x+a
)^(1/2)+1/3*B*x^(1/2)*(b*x+a)^(5/2)/b+1/8*a^2*(6*A*b-B*a)*arctanh(b^(1/2)*
x^(1/2)/(b*x+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{\sqrt{x}\sqrt{a+bx}(30aAb+3a^2B+12Ab^2x+14abBx+8b^2Bx^2)}{24b} + \frac{a^2(-6Ab+aB)\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{8b^{3/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/Sqrt[x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x + 14*a*b*B*x + 8*b
^2*B*x^2))/(24*b) + (a^2*(-6*A*b + a*B)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a +
b*x]])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {90, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(6Ab-aB)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx}{6b}+\frac{B\sqrt{x}(a+bx)^{5/2}}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{6b}+\frac{B\sqrt{x}(a+bx)^{5/2}}{3b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{6b}+\frac{B\sqrt{x}(a+bx)^{5/2}}{3b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{6b}+\frac{B\sqrt{x}(a+bx)^{5/2}}{3b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{6b}+\frac{B\sqrt{x}(a+bx)^{5/2}}{3b}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/Sqrt[x],x]
```

output
```
(B*Sqrt[x]*(a + b*x)^(5/2))/(3*b) + ((6*A*b - a*B)*((Sqrt[x]*(a + b*x)^(3/
2))/2 + (3*a*(Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
+ b*x]])/Sqrt[b]))/4))/(6*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(8b^2 B\,x^2+12A\,b^2x+14Babx+30abA+3a^2B)\sqrt{x}\,\sqrt{bx+a}}{24b}+\frac{a^2(6Ab-Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{x}\left(16B\,b^{\frac{5}{2}}x^2\sqrt{x(bx+a)}+24A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x+28B\sqrt{x(bx+a)}\,b^{\frac{3}{2}}ax+18A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b+60A\sqrt{x(bx+}\right.}{48b^{\frac{3}{2}}\sqrt{x(bx+a)}}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/b*(8*B*b^2*x^2+12*A*b^2*x+14*B*a*b*x+30*A*a*b+3*B*a^2)*x^(1/2)*(b*x+a
)^(1/2)+1/16*a^2/b^(3/2)*(6*A*b-B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1
/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \left[-\frac{3\,(Ba^3-6\,Aa^2b)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(8\,Bb^3x^2+3\,B}{48\,b^2}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*
sqrt(x) + a) - 2*(8*B*b^3*x^2 + 3*B*a^2*b + 30*A*a*b^2 + 2*(7*B*a*b^2 + 6*
A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)
*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (8*B*b^3*x^2 + 3*B*a^2*b + 30*A*
a*b^2 + 2*(7*B*a*b^2 + 6*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.20

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = Aa^{\frac{3}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}} + \frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + 2Ab\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) - \frac{Ba^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}} - \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}} + \frac{5B\sqrt{a}bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}} + \frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + 2Ba\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) + \frac{Bb^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(1/2),x)
```

output
```
A*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a) + A*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))
/sqrt(b) + 2*A*b*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) +
 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/
(8*b) + a*sqrt(x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a + b*x)/4, Ne(b, 0)
), (sqrt(a)*x**(3/2)/3, True)) - B*a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a))
- B*a**(3/2)*x**(3/2)/(24*sqrt(1 + b*x/a)) + 5*B*sqrt(a)*b*x**(5/2)/(12*sq
rt(1 + b*x/a)) + B*a**3*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) + 2*B*
a*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/s
qrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b) + a*sqrt(
x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a + b*x)/4, Ne(b, 0)), (sqrt(a)*x**
(3/2)/3, True)) + B*b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.29

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{bx^2+ax}Bbx^2 - \frac{5}{12}\sqrt{bx^2+ax}Bax - \frac{5\,Ba^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{3}{2}}} + \frac{Aa^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} + \frac{5\,\sqrt{bx^2+ax}Ba^2}{8\,b} + \frac{(2\,Bab+Ab^2)\sqrt{bx^2+ax}x}{2\,b} + \frac{3\,(2\,Bab+Ab^2)a^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} - \frac{(Ba^2+2\,Aab)a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} - \frac{3\,(2\,Bab+Ab^2)\sqrt{bx^2+ax}a}{4\,b^2} + \frac{(Ba^2+2\,Aab)\sqrt{bx^2+ax}}{b}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a*x)*B*b*x^2 - 5/12*sqrt(b*x^2 + a*x)*B*a*x - 5/16*B*a^3*
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + A*a^2*log(2*b*x + a
 + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 5/8*sqrt(b*x^2 + a*x)*B*a^2/b +
1/2*(2*B*a*b + A*b^2)*sqrt(b*x^2 + a*x)*x/b + 3/8*(2*B*a*b + A*b^2)*a^2*lo
g(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 1/2*(B*a^2 + 2*A*a*b)
*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) - 3/4*(2*B*a*b + A
*b^2)*sqrt(b*x^2 + a*x)*a/b^2 + (B*a^2 + 2*A*a*b)*sqrt(b*x^2 + a*x)/b
```

Giac [A] (verification not implemented)

Time = 75.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{\left(\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)B}{b^2}-\frac{Bab^2-6\,Ab^3}{b^4}\right)-\frac{3\,(Ba^2b^2-6\,A\cdots}{b^4}\right.\right.}{24\,|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(1/2),x, algorithm="giac")
```

output
```
1/24*(sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)*B/b^
2 - (B*a*b^2 - 6*A*b^3)/b^4) - 3*(B*a^2*b^2 - 6*A*a*b^3)/b^4) + 3*(B*a^3 -
 6*A*a^2*b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b^(
3/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \int \frac{(A+B\,x)\,\left(a+b\,x\right)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(1/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{33\sqrt{x}\,\sqrt{bx+a}\,a^2b+26\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2+15\sqrt{b}\log\left(\cdots\right.}{24b}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(1/2),x)
```

output
```
(33*sqrt(x)*sqrt(a + b*x)*a**2*b + 26*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b)
```

3.299 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = \frac{3}{4}(4Ab+aB)\sqrt{x}\sqrt{a+bx} - \frac{2A(a+bx)^{3/2}}{\sqrt{x}} + \frac{1}{2}B\sqrt{x}(a+bx)^{3/2} + \frac{3a(4Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}$$

output
```
3/4*(4*A*b+B*a)*x^(1/2)*(b*x+a)^(1/2)-2*A*(b*x+a)^(3/2)/x^(1/2)+1/2*B*x^(1
/2)*(b*x+a)^(3/2)+3/4*a*(4*A*b+B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = \frac{\sqrt{a+bx}(-8aA+4Abx+5aBx+2bBx^2)}{4\sqrt{x}} + \frac{3a(4Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{2\sqrt{b}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(3/2),x]
```

output
```
(Sqrt[a + b*x]*(-8*a*A + 4*A*b*x + 5*a*B*x + 2*b*B*x^2))/(4*Sqrt[x]) + (3*
a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(2*
Sqrt[b])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB+4Ab)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx}{a}-\frac{2A(a+bx)^{5/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{a}-\frac{2A(a+bx)^{5/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{a}-\frac{2A(a+bx)^{5/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{a}-\frac{2A(a+bx)^{5/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)}{a}-\frac{2A(a+bx)^{5/2}}{a\sqrt{x}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(3/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(a*Sqrt[x]) + ((4*A*b + a*B)*((Sqrt[x]*(a + b*x)^(3
/2))/2 + (3*a*(Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/Sqrt[b]))/4))/a
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{bx+a}\,(-2bB\,x^2-4Abx-5Bax+8Aa)}{4\sqrt{x}}+\frac{3a(4Ab+Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\left(4B\sqrt{x(bx+a)}\,b^{\frac{3}{2}}x^2+12Ab\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ax+8A\sqrt{x(bx+a)}\,b^{\frac{3}{2}}x+3B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2x+10B\sqrt{}\right.}{8\sqrt{x}\,\sqrt{x(bx+a)}\,\sqrt{b}}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+a)^(1/2)*(-2*B*b*x^2-4*A*b*x-5*B*a*x+8*A*a)/x^(1/2)+3/8*a*(4*A*b
+B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)*(x*(b*x+a))^(1/2)/
x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx=\left[\frac{3\,(Ba^2+4\,Aab)\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(2\,Bb^2x^2-8\,Aa}{8\,bx}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(B*a^2 + 4*A*a*b)*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sq
rt(x) + a) + 2*(2*B*b^2*x^2 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x)*sqrt(b*x +
a)*sqrt(x))/(b*x), -1/4*(3*(B*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*sq
rt(x)/sqrt(b*x + a)) - (2*B*b^2*x^2 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x)*sqr
t(b*x + a)*sqrt(x))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = -\frac{2Aa^{\frac{3}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + A\sqrt{a}b\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{2A\sqrt{a}b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}} + 3Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + Ba^{\frac{3}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}} + \frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + 2Bb\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(3/2),x)
```

output
```
-2*A*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) + A*sqrt(a)*b*sqrt(x)*sqrt(1 + b*x
/a) - 2*A*sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*
sqrt(x)/sqrt(a)) + B*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a) + B*a**2*asinh(sqrt(
b)*sqrt(x)/sqrt(a))/sqrt(b) + 2*B*b*Piecewise((-a**2*Piecewise((log(2*sqrt
(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))
/sqrt(b*x), True))/(8*b) + a*sqrt(x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a
 + b*x)/4, Ne(b, 0)), (sqrt(a)*x**(3/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = \frac{3\,Ba^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,\sqrt{b}} + \frac{3}{2}\,Aa\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{3}{4}\,\sqrt{bx^2+ax}Ba + \frac{(bx^2+ax)^{\frac{3}{2}}B}{2\,x} - \frac{3\,\sqrt{bx^2+ax}Aa}{x} + \frac{(bx^2+ax)^{\frac{3}{2}}A}{x^2}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(3/2),x, algorithm="maxima")
```

output
```
3/8*B*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + 3/2*A*a*s
qrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 3/4*sqrt(b*x^2 + a*x
)*B*a + 1/2*(b*x^2 + a*x)^(3/2)*B/x - 3*sqrt(b*x^2 + a*x)*A*a/x + (b*x^2 +
 a*x)^(3/2)*A/x^2
```

Giac [A] (verification not implemented)

Time = 74.98 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = \frac{\left(\frac{\left((bx+a)\left(\frac{2\,(bx+a)B}{b}+\frac{Bab+4\,Ab^2}{b^2}\right)-\frac{3\,\left(Ba^2b+4\,Aab^2\right)}{b^2}\right)\sqrt{bx+a}}{\sqrt{(bx+a)b-ab}}-\frac{3\,\left(Ba^2+4\,Aab\right)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\right.\right.}{b^{\frac{3}{2}}}\right.}{4\,|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(3/2),x, algorithm="giac")
```

output
```
1/4*(((b*x + a)*(2*(b*x + a)*B/b + (B*a*b + 4*A*b^2)/b^2) - 3*(B*a^2*b + 4
*A*a*b^2)/b^2)*sqrt(b*x + a)/sqrt((b*x + a)*b - a*b) - 3*(B*a^2 + 4*A*a*b)
*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b^(3/2))*b^2/a
bs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = \int \frac{(A+B\,x)\,\left(a+b\,x\right)^{3/2}}{x^{3/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(3/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{3/2}}\,dx = \frac{-8\sqrt{x}\,\sqrt{bx+a}\,a^2+9\sqrt{x}\,\sqrt{bx+a}\,abx+2\sqrt{x}\,\sqrt{bx+a}\,b^2x^2+15\sqrt{b}\log\left(\frac{\sqrt{bx\cdots}}{}\right.}{4x}$$

input

```
int((b*x+a)^(3/2)*(B*x+A)/x^(3/2),x)
```

output

```
( - 8*sqrt(x)*sqrt(a + b*x)*a**2 + 9*sqrt(x)*sqrt(a + b*x)*a*b*x + 2*sqrt(
x)*sqrt(a + b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**2*x - 10*sqrt(b)*a**2*x)/(4*x)
```

3.300 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx = -\frac{2(Ab+aB)\sqrt{a+bx}}{\sqrt{x}} + bB\sqrt{x}\sqrt{a+bx} - \frac{2A(a+bx)^{3/2}}{3x^{3/2}} + \sqrt{b}(2Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2*(A*b+B*a)*(b*x+a)^(1/2)/x^(1/2)+b*B*x^(1/2)*(b*x+a)^(1/2)-2/3*A*(b*x+a)
^(3/2)/x^(3/2)+b^(1/2)*(2*A*b+3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx = \frac{\sqrt{a+bx}(-2aA-8Abx-6aBx+3bBx^2)}{3x^{3/2}} + 2\sqrt{b}(2Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(5/2),x]
```

output
```
(Sqrt[a + b*x]*(-2*a*A - 8*A*b*x - 6*a*B*x + 3*b*B*x^2))/(3*x^(3/2)) + 2*S
qrt[b]*(2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x]
)]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {87, 57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(3aB+2Ab)\int \frac{(a+bx)^{3/2}}{x^{3/2}}dx}{3a}-\frac{2A(a+bx)^{5/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(3aB+2Ab)\left(3b\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{5/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(3aB+2Ab)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{5/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{65}$$

$$\frac{(3aB+2Ab)\left(3b\left(a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{5/2}}{3ax^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3aB+2Ab)\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{5/2}}{3ax^{3/2}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(5/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(3*a*x^(3/2)) + ((2*A*b + 3*a*B)*((-2*(a + b*x)^(3/
2))/Sqrt[x] + 3*b*(Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sq
rt[a + b*x]])/Sqrt[b])))/(3*a)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{bx+a}\,(-3bB\,x^2+8Abx+6Bax+2Aa)}{3x^{\frac{3}{2}}}+\frac{\sqrt{b}\,(2Ab+3Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\left(6A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b^2x^2+9B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ab\,x^2+6B\sqrt{x(bx+a)}\,b^{\frac{3}{2}}x^2-16A\sqrt{x(bx+a)}\,b^{\frac{3}{2}}x-12B\right.}{6x^{\frac{3}{2}}\sqrt{x(bx+a)}\,\sqrt{b}}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x+a)^(1/2)*(-3*B*b*x^2+8*A*b*x+6*B*a*x+2*A*a)/x^(3/2)+1/2*b^(1/2)*
(2*A*b+3*B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/
x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int\frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx=\left[\frac{3\,(3\,Ba+2\,Ab)\sqrt{b}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(3\,Bbx^2-2\,Aa}{6\,x^2}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(3*B*a + 2*A*b)*sqrt(b)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sq
rt(x) + a) + 2*(3*B*b*x^2 - 2*A*a - 2*(3*B*a + 4*A*b)*x)*sqrt(b*x + a)*sqr
t(x))/x^2, -1/3*(3*(3*B*a + 2*A*b)*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)/sq
rt(b*x + a)) - (3*B*b*x^2 - 2*A*a - 2*(3*B*a + 4*A*b)*x)*sqrt(b*x + a)*sqr
t(x))/x^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(95) = 190$.

Time = 3.71 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx = -\frac{2A\sqrt{a}b}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} - \frac{2Aa\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} + 2Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2Ab^2\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}} - \frac{2Ba^{\frac{3}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + B\sqrt{a}b\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{2B\sqrt{a}b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}} + 3Ba\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(5/2),x)
```

output
```
-2*A*sqrt(a)*b/(sqrt(x)*sqrt(1 + b*x/a)) - 2*A*a*sqrt(b)*sqrt(a/(b*x) + 1)
/(3*x) - 2*A*b**(3/2)*sqrt(a/(b*x) + 1)/3 + 2*A*b**(3/2)*asinh(sqrt(b)*sqr
t(x)/sqrt(a)) - 2*A*b**2*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a)) - 2*B*a**(3/2)/
(sqrt(x)*sqrt(1 + b*x/a)) + B*sqrt(a)*b*sqrt(x)*sqrt(1 + b*x/a) - 2*B*sqrt
(a)*b*sqrt(x)/sqrt(1 + b*x/a) + 3*B*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx = \frac{3}{2}\,Ba\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + Ab^{\frac{3}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{3\,\sqrt{bx^2+ax}Ba}{x} - \frac{7\,\sqrt{bx^2+ax}Ab}{3\,x} + \frac{(bx^2+ax)^{\frac{3}{2}}B}{x^2} - \frac{\sqrt{bx^2+ax}Aa}{3\,x^2} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{3\,x^3}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(5/2),x, algorithm="maxima")
```

output
```
3/2*B*a*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + A*b^(3/2)*l
og(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 3*sqrt(b*x^2 + a*x)*B*a/x -
7/3*sqrt(b*x^2 + a*x)*A*b/x + (b*x^2 + a*x)^(3/2)*B/x^2 - 1/3*sqrt(b*x^2 +
 a*x)*A*a/x^2 - 1/3*(b*x^2 + a*x)^(3/2)*A/x^3
```

Giac [A] (verification not implemented)

Time = 76.00 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx = -\frac{b^3\left(\frac{3\,(3\,Ba+2\,Ab)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}} - \frac{\left(\left(3\,(bx+a)B-\frac{4\left(3\,Ba^2b+2\,Aab^2\right)}{ab}\right)(bx+a)+\frac{3\left(3\,Ba^3b+2\,Aa^2b^2\right)}{ab}\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{3}{2}}}\right)}{3\,|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(5/2),x, algorithm="giac")
```

output
```
-1/3*b^3*(3*(3*B*a + 2*A*b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a
)*b - a*b)))/b^(3/2) - ((3*(b*x + a)*B - 4*(3*B*a^2*b + 2*A*a*b^2)/(a*b))*
(b*x + a) + 3*(3*B*a^3*b + 2*A*a^2*b^2)/(a*b))*sqrt(b*x + a)/((b*x + a)*b
- a*b)^(3/2))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx=\int \frac{(A+B\,x)\,\left(a+b\,x\right)^{3/2}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(5/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{5/2}}\,dx=\frac{-4\sqrt{x}\,\sqrt{bx+a}\,a^2-28\sqrt{x}\,\sqrt{bx+a}\,abx+6\sqrt{x}\,\sqrt{bx+a}\,b^2x^2+30\sqrt{b}\log\left(\frac{\sqrt{b}}{}\right.}{6x^2}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(5/2),x)
```

output
```
( - 4*sqrt(x)*sqrt(a + b*x)*a**2 - 28*sqrt(x)*sqrt(a + b*x)*a*b*x + 6*sqrt
(x)*sqrt(a + b*x)*b**2*x**2 + 30*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a*b*x**2 + 5*sqrt(b)*a*b*x**2)/(6*x**2)
```

3.301 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{2aB\sqrt{a+bx}}{3x^{3/2}} - \frac{8bB\sqrt{a+bx}}{3\sqrt{x}} - \frac{2A(a+bx)^{5/2}}{5ax^{5/2}} + 2b^{3/2}B\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2/3*a*B*(b*x+a)^(1/2)/x^(3/2)-8/3*b*B*(b*x+a)^(1/2)/x^(1/2)-2/5*A*(b*x+a)
^(5/2)/a/x^(5/2)+2*b^(3/2)*B*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{2\sqrt{a+bx}(3a^2A+6aAbx+5a^2Bx+3Ab^2x^2+20abBx^2)}{15ax^{5/2}} - 2b^{3/2}B\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(7/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(3*a^2*A + 6*a*A*b*x + 5*a^2*B*x + 3*A*b^2*x^2 + 20*a*b*
B*x^2))/(15*a*x^(5/2)) - 2*b^(3/2)*B*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x
]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 57, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{(a+bx)^{3/2}}{x^{5/2}}dx-\frac{2A(a+bx)^{5/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{57}$$

$$B\left(b\int \frac{\sqrt{a+bx}}{x^{3/2}}dx-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2A(a+bx)^{5/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{57}$$

$$B\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2A(a+bx)^{5/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{65}$$

$$B\left(b\left(2b\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2A(a+bx)^{5/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$B\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2A(a+bx)^{5/2}}{5ax^{5/2}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(7/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(5*a*x^(5/2)) + B*((-2*(a + b*x)^(3/2))/(3*x^(3/2))
 + b*((-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a + b*x]]))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{2\sqrt{bx+a}\,(3A\,b^{2}x^{2}+20Bab\,x^{2}+6aAbx+5B\,a^{2}x+3a^{2}A)}{15x^{\frac{5}{2}}a}+\frac{b^{\frac{3}{2}}B\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$
default	$-\frac{\sqrt{bx+a}\left(-15B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^{2}x^{3}+6A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x^{2}+40B\sqrt{x(bx+a)}\,b^{\frac{3}{2}}a\,x^{2}+12Aa\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}+10B\,a^{2}x\right.}{15x^{\frac{5}{2}}a\sqrt{x(bx+a)}\,\sqrt{b}}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(1/2)*(3*A*b^2*x^2+20*B*a*b*x^2+6*A*a*b*x+5*B*a^2*x+3*A*a^2)
/x^(5/2)/a+b^(3/2)*B*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))
^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx=\left[\frac{15\,Bab^{\frac{3}{2}}x^{3}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(3\,Aa^{2}+(20\,Bab+3\,Ab^{2}}{15\,ax^{3}}\right.$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/15*(15*B*a*b^(3/2)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)
 - 2*(3*A*a^2 + (20*B*a*b + 3*A*b^2)*x^2 + (5*B*a^2 + 6*A*a*b)*x)*sqrt(b*x
 + a)*sqrt(x))/(a*x^3), -2/15*(15*B*a*sqrt(-b)*b*x^3*arctan(sqrt(-b)*sqrt(
x)/sqrt(b*x + a)) + (3*A*a^2 + (20*B*a*b + 3*A*b^2)*x^2 + (5*B*a^2 + 6*A*a
*b)*x)*sqrt(b*x + a)*sqrt(x))/(a*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

Time = 4.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{2Aa\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{4Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{5x} - \frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{5a} - \frac{2B\sqrt{a}b}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} - \frac{2Ba\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} + 2Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2Bb^2\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(7/2),x)
```

output
```
-2*A*a*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 4*A*b**(3/2)*sqrt(a/(b*x) + 1)
/(5*x) - 2*A*b**(5/2)*sqrt(a/(b*x) + 1)/(5*a) - 2*B*sqrt(a)*b/(sqrt(x)*sqr
t(1 + b*x/a)) - 2*B*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*B*b**(3/2)*sqrt(
a/(b*x) + 1)/3 + 2*B*b**(3/2)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) - 2*B*b**2*sq
rt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(66) = 132$.

Time = 0.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = Bb^{\frac{3}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{7\,\sqrt{bx^2+ax}Bb}{3\,x} - \frac{2\,\sqrt{bx^2+ax}Ab^2}{5\,ax} - \frac{\sqrt{bx^2+ax}Ba}{3\,x^2} + \frac{\sqrt{bx^2+ax}Ab}{5\,x^2} - \frac{(bx^2+ax)^{\frac{3}{2}}B}{3\,x^3} + \frac{3\,\sqrt{bx^2+ax}Aa}{5\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{x^4}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(7/2),x, algorithm="maxima")
```

output
```
B*b^(3/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 7/3*sqrt(b*x^2 +
a*x)*B*b/x - 2/5*sqrt(b*x^2 + a*x)*A*b^2/(a*x) - 1/3*sqrt(b*x^2 + a*x)*B*a
/x^2 + 1/5*sqrt(b*x^2 + a*x)*A*b/x^2 - 1/3*(b*x^2 + a*x)^(3/2)*B/x^3 + 3/5
*sqrt(b*x^2 + a*x)*A*a/x^3 - (b*x^2 + a*x)^(3/2)*A/x^4
```

Giac [A] (verification not implemented)

Time = 75.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{2\left(15\,Bb^{\frac{3}{2}}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\frac{\left(15\,Ba^2b^4-\left(35\,Bab^4-\frac{\left(20\,Ba^2b^4+3\,Aab^5\right)(bx+a)}{a^2}\right)(bx+a)\right)\sqrt{bx+a}}{((bx+a)b-ab)^{\frac{5}{2}}}\right)}{15\,|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*B*b^(3/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*
b))) + (15*B*a^2*b^4 - (35*B*a*b^4 - (20*B*a^2*b^4 + 3*A*a*b^5)*(b*x + a)/
a^2)*(b*x + a))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(5/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = \int \frac{(A+B\,x)\,\left(a+b\,x\right)^{3/2}}{x^{7/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(7/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{7/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{22\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{46\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)b^2x^3}{x^3}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(7/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - 11*sqrt(x)*sqrt(a + b*x)*a*b*x - 23*
sqrt(x)*sqrt(a + b*x)*b**2*x**2 + 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*
sqrt(b))/sqrt(a))*b**2*x**3 + 5*sqrt(b)*b**2*x**3))/(15*x**3)
```

3.302 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}} dx = -\frac{2A(a+bx)^{5/2}}{7ax^{7/2}} + \frac{2(2Ab-7aB)(a+bx)^{5/2}}{35a^2x^{5/2}}$$

output
```
-2/7*A*(b*x+a)^(5/2)/a/x^(7/2)+2/35*(2*A*b-7*B*a)*(b*x+a)^(5/2)/a^2/x^(5/2
)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}} dx = -\frac{2(a+bx)^{5/2}(5aA-2Abx+7aBx)}{35a^2x^{7/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(9/2),x]
```

output
```
(-2*(a + b*x)^(5/2)*(5*a*A - 2*A*b*x + 7*a*B*x))/(35*a^2*x^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}} \, dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(2Ab-7aB)\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2A(a+bx)^{5/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{2(a+bx)^{5/2}(2Ab-7aB)}{35a^2x^{5/2}}-\frac{2A(a+bx)^{5/2}}{7ax^{7/2}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(9/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(7*a*x^(7/2)) + (2*(2*A*b - 7*a*B)*(a + b*x)^(5/2))
/(35*a^2*x^(5/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-2Abx+7Bax+5Aa\right)}{35x^{\frac{7}{2}}a^{2}}$	31
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-2Abx+7Bax+5Aa\right)}{35x^{\frac{7}{2}}a^{2}}$	31
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-2A\,b^{2}x^{2}+7Bab\,x^{2}+3aAbx+7B\,a^{2}x+5a^{2}A\right)}{35x^{\frac{7}{2}}a^{2}}$	53
risch	$-\frac{2\sqrt{bx+a}\,\left(-2A\,b^{3}x^{3}+7Ba\,b^{2}x^{3}+aA\,b^{2}x^{2}+14B\,a^{2}b\,x^{2}+8a^{2}Abx+7B\,a^{3}x+5a^{3}A\right)}{35x^{\frac{7}{2}}a^{2}}$	76

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(5/2)*(-2*A*b*x+7*B*a*x+5*A*a)/x^(7/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{2\,(5\,Aa^3+(7\,Bab^2-2\,Ab^3)x^3+(14\,Ba^2b+Aab^2)x^2+(7\,Ba^3+8\,Aa^2b)x)\sqrt{bx+a}}{35\,a^2x^{\frac{7}{2}}}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/35*(5*A*a^3 + (7*B*a*b^2 - 2*A*b^3)*x^3 + (14*B*a^2*b + A*a*b^2)*x^2 +
(7*B*a^3 + 8*A*a^2*b)*x)*sqrt(b*x + a)/(a^2*x^(7/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(49) = 98.

Time = 12.95 (sec) , antiderivative size = 500, normalized size of antiderivative = 9.43

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{30Aa^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{66Aa^5b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{34Aa^4b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{6Aa^3b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{24Aa^2b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{16Aab^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15ax} + \frac{4Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2} - \frac{2Ba\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{4Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{5x} - \frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{5a}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(9/2),x)
```

output
```
-30*A*a**6*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*
x**4 + 105*a**3*b**6*x**5) - 66*A*a**5*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*
a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*A*a**4*b**(
13/2)*x**2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 10
5*a**3*b**6*x**5) - 6*A*a**3*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b*
*4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*A*a**2*b**(17/2)*x
**4*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*
b**6*x**5) - 16*A*a*b**(19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 +
 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 2*A*b**(3/2)*sqrt(a/(b*x) + 1)
/(5*x**2) - 2*A*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a*x) + 4*A*b**(7/2)*sqrt(a/
(b*x) + 1)/(15*a**2) - 2*B*a*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 4*B*b**(
3/2)*sqrt(a/(b*x) + 1)/(5*x) - 2*B*b**(5/2)*sqrt(a/(b*x) + 1)/(5*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.32

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{2\sqrt{bx^2+ax}Bb^2}{5\,ax} + \frac{4\sqrt{bx^2+ax}Ab^3}{35\,a^2x} + \frac{\sqrt{bx^2+ax}Bb}{5\,x^2} - \frac{2\sqrt{bx^2+ax}Ab^2}{35\,ax^2} + \frac{3\sqrt{bx^2+ax}Ba}{5\,x^3} + \frac{3\sqrt{bx^2+ax}Ab}{70\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}B}{x^4} + \frac{3\sqrt{bx^2+ax}Aa}{14\,x^4} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{2\,x^5}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/5*sqrt(b*x^2 + a*x)*B*b^2/(a*x) + 4/35*sqrt(b*x^2 + a*x)*A*b^3/(a^2*x)
+ 1/5*sqrt(b*x^2 + a*x)*B*b/x^2 - 2/35*sqrt(b*x^2 + a*x)*A*b^2/(a*x^2) + 3
/5*sqrt(b*x^2 + a*x)*B*a/x^3 + 3/70*sqrt(b*x^2 + a*x)*A*b/x^3 - (b*x^2 + a
*x)^(3/2)*B/x^4 + 3/14*sqrt(b*x^2 + a*x)*A*a/x^4 - 1/2*(b*x^2 + a*x)^(3/2)
*A/x^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}b^5\left(\frac{(7\,Ba^2b^2-2\,Aab^3)(bx+a)}{a^3}-\frac{7\,(Ba^3b^2-Aa^2b^3)}{a^3}\right)}{35\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(9/2),x, algorithm="giac")
```

output
```
-2/35*(b*x + a)^(5/2)*b^5*((7*B*a^2*b^2 - 2*A*a*b^3)*(b*x + a)/a^3 - 7*(B*
a^3*b^2 - A*a^2*b^3)/a^3)/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A\,a}{7}+\frac{x\,\left(14\,B\,a^3+16\,A\,b\,a^2\right)}{35\,a^2}-\frac{x^3\,\left(4\,A\,b^3-14\,B\,a\,b^2\right)}{35\,a^2}+\frac{2\,b\,x^2\,\left(A\,b+14\,B\,a\right)}{35\,a}\right)}{x^{7/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(9/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a)/7 + (x*(14*B*a^3 + 16*A*a^2*b))/(35*a^2) - (x^3
*(4*A*b^3 - 14*B*a*b^2))/(35*a^2) + (2*b*x^2*(A*b + 14*B*a))/(35*a)))/x^(7
/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{7}-\frac{6\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{7}-\frac{2\sqrt{b}\,b^3x^4}{7}}{a\,x^4}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(9/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 3*s
qrt(x)*sqrt(a + b*x)*a*b**2*x**2 - sqrt(x)*sqrt(a + b*x)*b**3*x**3 - sqrt(
b)*b**3*x**4))/(7*a*x**4)
```

3.303 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{2A(a+bx)^{5/2}}{9ax^{9/2}} + \frac{2(4Ab-9aB)(a+bx)^{5/2}}{63a^2x^{7/2}} - \frac{4b(4Ab-9aB)(a+bx)^{5/2}}{315a^3x^{5/2}}$$

output
```
-2/9*A*(b*x+a)^(5/2)/a/x^(9/2)+2/63*(4*A*b-9*B*a)*(b*x+a)^(5/2)/a^2/x^(7/2
)-4/315*b*(4*A*b-9*B*a)*(b*x+a)^(5/2)/a^3/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{2(a+bx)^{5/2}\left(35a^2A-20aAbx+45a^2Bx+8Ab^2x^2-18abBx^2\right)}{315a^3x^{9/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(5/2)*(35*a^2*A - 20*a*A*b*x + 45*a^2*B*x + 8*A*b^2*x^2 - 18
*a*b*B*x^2))/(315*a^3*x^(9/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(4Ab-9aB)\int \frac{(a+bx)^{3/2}}{x^{9/2}}dx}{9a}-\frac{2A(a+bx)^{5/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(4Ab-9aB)\left(-\frac{2b\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2A(a+bx)^{5/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)(4Ab-9aB)}{9a}-\frac{2A(a+bx)^{5/2}}{9ax^{9/2}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(11/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(9*a*x^(9/2)) - ((4*A*b - 9*a*B)*((-2*(a + b*x)^(5/
2))/(7*a*x^(7/2)) + (4*b*(a + b*x)^(5/2))/(35*a^2*x^(5/2))))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}} \left(8A\,b^{2}x^{2}-18Bab\,x^{2}-20aAbx+45B\,a^{2}x+35a^{2}A\right)}{315x^{\frac{9}{2}}a^{3}}$	53
orering	$-\frac{2(bx+a)^{\frac{5}{2}} \left(8A\,b^{2}x^{2}-18Bab\,x^{2}-20aAbx+45B\,a^{2}x+35a^{2}A\right)}{315x^{\frac{9}{2}}a^{3}}$	53
default	$-\frac{2(bx+a)^{\frac{3}{2}} \left(8A\,b^{3}x^{3}-18Ba\,b^{2}x^{3}-12aA\,b^{2}x^{2}+27B\,a^{2}b\,x^{2}+15a^{2}Abx+45B\,a^{3}x+35a^{3}A\right)}{315x^{\frac{9}{2}}a^{3}}$	77
risch	$-\frac{2\sqrt{bx+a}\,\left(8A\,b^{4}x^{4}-18Ba\,b^{3}x^{4}-4Aa\,b^{3}x^{3}+9B\,a^{2}b^{2}x^{3}+3A\,a^{2}b^{2}x^{2}+72B\,a^{3}b\,x^{2}+50A\,a^{3}bx+45B\,a^{4}x+35A\,a^{4}\right)}{315x^{\frac{9}{2}}a^{3}}$	101

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(b*x+a)^(5/2)*(8*A*b^2*x^2-18*B*a*b*x^2-20*A*a*b*x+45*B*a^2*x+35*A*
a^2)/x^(9/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx =$$
$$-\frac{2\,(35\,Aa^4 - 2\,(9\,Bab^3 - 4\,Ab^4)x^4 + (9\,Ba^2b^2 - 4\,Aab^3)x^3 + 3\,(24\,Ba^3b + Aa^2b^2)x^2 + 5\,(9\,Ba^4 + 10\,Aa^3b}{315\,a^3x^{\frac{9}{2}}}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/315*(35*A*a^4 - 2*(9*B*a*b^3 - 4*A*b^4)*x^4 + (9*B*a^2*b^2 - 4*A*a*b^3)
*x^3 + 3*(24*B*a^3*b + A*a^2*b^2)*x^2 + 5*(9*B*a^4 + 10*A*a^3*b)*x)*sqrt(b
*x + a)/(a^3*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. 2(82) = 164.

Time = 36.38 (sec) , antiderivative size = 1365, normalized size of antiderivative = 16.25

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(11/2),x)
```

output
```
-70*A*a**8*b**(19/2)*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**1
0*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 220*A*a**7*b**(21/2)
*x*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*
b**11*x**6 + 315*a**4*b**12*x**7) - 228*A*a**6*b**(23/2)*x**2*sqrt(a/(b*x)
 + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 31
5*a**4*b**12*x**7) - 80*A*a**5*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(315*a**7*
b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**
7) - 30*A*a**5*b**(11/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*
b**5*x**4 + 105*a**3*b**6*x**5) + 10*A*a**4*b**(27/2)*x**4*sqrt(a/(b*x) +
1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a
**4*b**12*x**7) - 66*A*a**4*b**(13/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x
**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 60*A*a**3*b**(29/2)*x**5*
sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**
11*x**6 + 315*a**4*b**12*x**7) - 34*A*a**3*b**(15/2)*x**2*sqrt(a/(b*x) + 1
)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 80*A*a*
*2*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x
**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 6*A*a**2*b**(17/2)*x**3
*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) + 32*A*a*b**(33/2)*x**7*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 94
5*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 24*A*a...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(66) = 132$.

Time = 0.05 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.64

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = \frac{4\,\sqrt{bx^2+ax}Bb^3}{35\,a^2x} - \frac{16\,\sqrt{bx^2+ax}Ab^4}{315\,a^3x} - \frac{2\,\sqrt{bx^2+ax}Bb^2}{35\,ax^2} + \frac{8\,\sqrt{bx^2+ax}Ab^3}{315\,a^2x^2} + \frac{3\,\sqrt{bx^2+ax}Bb}{70\,x^3} - \frac{2\,\sqrt{bx^2+ax}Ab^2}{105\,ax^3} + \frac{3\,\sqrt{bx^2+ax}Ba}{14\,x^4} + \frac{\sqrt{bx^2+ax}Ab}{63\,x^4} - \frac{(bx^2+ax)^{\frac{3}{2}}B}{2\,x^5} + \frac{\sqrt{bx^2+ax}Aa}{9\,x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{3\,x^6}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(11/2),x, algorithm="maxima")
```

output
```
4/35*sqrt(b*x^2 + a*x)*B*b^3/(a^2*x) - 16/315*sqrt(b*x^2 + a*x)*A*b^4/(a^3
*x) - 2/35*sqrt(b*x^2 + a*x)*B*b^2/(a*x^2) + 8/315*sqrt(b*x^2 + a*x)*A*b^3
/(a^2*x^2) + 3/70*sqrt(b*x^2 + a*x)*B*b/x^3 - 2/105*sqrt(b*x^2 + a*x)*A*b^
2/(a*x^3) + 3/14*sqrt(b*x^2 + a*x)*B*a/x^4 + 1/63*sqrt(b*x^2 + a*x)*A*b/x^
4 - 1/2*(b*x^2 + a*x)^(3/2)*B/x^5 + 1/9*sqrt(b*x^2 + a*x)*A*a/x^5 - 1/3*(b
*x^2 + a*x)^(3/2)*A/x^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}\left((bx+a)\left(\frac{2\,(9\,Ba^2b^8-4\,Aab^9)(bx+a)}{a^4}-\frac{9\,(9\,Ba^3b^8-4\,Aa^2b^9)}{a^4}\right)+\frac{63\,(Ba^4b^8-A}{a^4}\right.}{315\,((bx+a)b-ab)^{\frac{9}{2}}\,|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(11/2),x, algorithm="giac")
```

output
```
2/315*(b*x + a)^(5/2)*((b*x + a)*(2*(9*B*a^2*b^8 - 4*A*a*b^9)*(b*x + a)/a^
4 - 9*(9*B*a^3*b^8 - 4*A*a^2*b^9)/a^4) + 63*(B*a^4*b^8 - A*a^3*b^9)/a^4)*b
/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A\,a}{9}+\frac{x\,\left(90\,B\,a^4+100\,A\,b\,a^3\right)}{315\,a^3}+\frac{x^4\,\left(16\,A\,b^4-36\,B\,a\,b^3\right)}{315\,a^3}-\frac{2\,b^2\,x^3\,\left(4\,A\,b-9\,B\,a\right)}{315\,a^2}+\frac{2\,b\,x^2\,\left(A\,b+24\,B\,a\right)}{105\,a}\right)}{x^{9/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(11/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a)/9 + (x*(90*B*a^4 + 100*A*a^3*b))/(315*a^3) + (x
^4*(16*A*b^4 - 36*B*a*b^3))/(315*a^3) - (2*b^2*x^3*(4*A*b - 9*B*a))/(315*a
^2) + (2*b*x^2*(A*b + 24*B*a))/(105*a)))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9} - \frac{38\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63} - \frac{10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{21} - \frac{2\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{63} + \frac{4\sqrt{x}\,\sqrt{bx-\cdots}}{63}}{a^2x^5}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(11/2),x)
```

output
```
(2*( - 7*sqrt(x)*sqrt(a + b*x)*a**4 - 19*sqrt(x)*sqrt(a + b*x)*a**3*b*x -
15*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - sqrt(x)*sqrt(a + b*x)*a*b**3*x**
3 + 2*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 2*sqrt(b)*b**4*x**5))/(63*a**2*x**
5)
```

3.304 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}} dx = -\frac{2A(a+bx)^{5/2}}{11ax^{11/2}} + \frac{2(6Ab-11aB)(a+bx)^{5/2}}{99a^2x^{9/2}} - \frac{8b(6Ab-11aB)(a+bx)^{5/2}}{693a^3x^{7/2}} + \frac{16b^2(6Ab-11aB)(a+bx)^{5/2}}{3465a^4x^{5/2}}$$

output
```
-2/11*A*(b*x+a)^(5/2)/a/x^(11/2)+2/99*(6*A*b-11*B*a)*(b*x+a)^(5/2)/a^2/x^(
9/2)-8/693*b*(6*A*b-11*B*a)*(b*x+a)^(5/2)/a^3/x^(7/2)+16/3465*b^2*(6*A*b-1
1*B*a)*(b*x+a)^(5/2)/a^4/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}} dx = -\frac{2(a+bx)^{5/2}\left(-48Ab^3x^3+35a^3(9A+11Bx)+8ab^2x^2(15A+11Bx)-10a^2bx(21A+22Bx)\right)}{3465a^4x^{11/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(5/2)*(-48*A*b^3*x^3 + 35*a^3*(9*A + 11*B*x) + 8*a*b^2*x^2*(
15*A + 11*B*x) - 10*a^2*b*x*(21*A + 22*B*x)))/(3465*a^4*x^(11/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(6Ab-11aB)\int \frac{(a+bx)^{3/2}}{x^{11/2}}dx}{11a}-\frac{2A(a+bx)^{5/2}}{11ax^{11/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(6Ab-11aB)\left(-\frac{4b\int \frac{(a+bx)^{3/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2A(a+bx)^{5/2}}{11ax^{11/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(6Ab-11aB)\left(-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2A(a+bx)^{5/2}}{11ax^{11/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(-\frac{4b\left(\frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)(6Ab-11aB)}{11a}-\frac{2A(a+bx)^{5/2}}{11ax^{11/2}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(13/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(11*a*x^(11/2)) - ((6*A*b - 11*a*B)*((-2*(a + b*x)^
(5/2))/(9*a*x^(9/2)) - (4*b*((-2*(a + b*x)^(5/2))/(7*a*x^(7/2)) + (4*b*(a
+ b*x)^(5/2))/(35*a^2*x^(5/2))))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

method	result
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-48A\,b^{3}x^{3}+88Ba\,b^{2}x^{3}+120aA\,b^{2}x^{2}-220B\,a^{2}b\,x^{2}-210a^{2}Abx+385B\,a^{3}x+315a^{3}A\right)}{3465x^{\frac{11}{2}}a^{4}}$
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-48A\,b^{3}x^{3}+88Ba\,b^{2}x^{3}+120aA\,b^{2}x^{2}-220B\,a^{2}b\,x^{2}-210a^{2}Abx+385B\,a^{3}x+315a^{3}A\right)}{3465x^{\frac{11}{2}}a^{4}}$
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-48A\,b^{4}x^{4}+88Ba\,b^{3}x^{4}+72Aa\,b^{3}x^{3}-132B\,a^{2}b^{2}x^{3}-90A\,a^{2}b^{2}x^{2}+165B\,a^{3}b\,x^{2}+105A\,a^{3}bx+385B\,a^{4}x+315A\,a^{4}\right)}{3465x^{\frac{11}{2}}a^{4}}$
risch	$-\frac{2\sqrt{bx+a}\,\left(-48A\,b^{5}x^{5}+88Ba\,b^{4}x^{5}+24aA\,b^{4}x^{4}-44B\,a^{2}b^{3}x^{4}-18a^{2}A\,b^{3}x^{3}+33B\,a^{3}b^{2}x^{3}+15a^{3}A\,b^{2}x^{2}+550B\,a^{4}b\,x^{2}+420a^{4}Abx+\right.}{3465x^{\frac{11}{2}}a^{4}}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*(b*x+a)^(5/2)*(-48*A*b^3*x^3+88*B*a*b^2*x^3+120*A*a*b^2*x^2-220*B*
a^2*b*x^2-210*A*a^2*b*x+385*B*a^3*x+315*A*a^3)/x^(11/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx =$$
$$-\frac{2\,(315\,Aa^5+8\,(11\,Bab^4-6\,Ab^5)x^5-4\,(11\,Ba^2b^3-6\,Aab^4)x^4+3\,(11\,Ba^3b^2-6\,Aa^2b^3)x^3+5\,(110\,Ba^4b}{3465\,a^4x^{\frac{11}{2}}}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/3465*(315*A*a^5 + 8*(11*B*a*b^4 - 6*A*b^5)*x^5 - 4*(11*B*a^2*b^3 - 6*A*
a*b^4)*x^4 + 3*(11*B*a^3*b^2 - 6*A*a^2*b^3)*x^3 + 5*(110*B*a^4*b + 3*A*a^3
*b^2)*x^2 + 35*(11*B*a^5 + 12*A*a^4*b)*x)*sqrt(b*x + a)/(a^4*x^(11/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2351 vs. $2(116) = 232$.

Time = 103.25 (sec) , antiderivative size = 2351, normalized size of antiderivative = 20.09

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(13/2),x)
```

output
```
-630*A*a**10*b**(33/2)*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**
8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b
**20*x**9) - 2590*A*a**9*b**(35/2)*x*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x*
*5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8
 + 3465*a**5*b**20*x**9) - 3980*A*a**8*b**(37/2)*x**2*sqrt(a/(b*x) + 1)/(3
465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 1386
0*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 2716*A*a**7*b**(39/2)*x**3*sqr
t(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*
b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 70*A*a**7*b**
(21/2)*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a
**5*b**11*x**6 + 315*a**4*b**12*x**7) - 686*A*a**6*b**(41/2)*x**4*sqrt(a/(
b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18
*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 220*A*a**6*b**(23/
2)*x*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**
5*b**11*x**6 + 315*a**4*b**12*x**7) - 70*A*a**5*b**(43/2)*x**5*sqrt(a/(b*x
) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x*
*7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 228*A*a**5*b**(25/2)*
x**2*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**
5*b**11*x**6 + 315*a**4*b**12*x**7) - 560*A*a**4*b**(45/2)*x**6*sqrt(a/(b*
x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(93) = 186$.

Time = 0.06 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.29

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx = -\frac{16\sqrt{bx^2+ax}Bb^4}{315\,a^3x} + \frac{32\sqrt{bx^2+ax}Ab^5}{1155\,a^4x} + \frac{8\sqrt{bx^2+ax}Bb^3}{315\,a^2x^2} - \frac{16\sqrt{bx^2+ax}Ab^4}{1155\,a^3x^2} - \frac{2\sqrt{bx^2+ax}Bb^2}{105\,ax^3} + \frac{4\sqrt{bx^2+ax}Ab^3}{385\,a^2x^3} + \frac{\sqrt{bx^2+ax}Bb}{63\,x^4} - \frac{2\sqrt{bx^2+ax}Ab^2}{231\,ax^4} + \frac{\sqrt{bx^2+ax}Ba}{9\,x^5} + \frac{\sqrt{bx^2+ax}Ab}{132\,x^5} - \frac{(bx^2+ax)^{\frac{3}{2}}B}{3\,x^6} + \frac{3\sqrt{bx^2+ax}Aa}{44\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{4\,x^7}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(13/2),x, algorithm="maxima")
```

output
```
-16/315*sqrt(b*x^2 + a*x)*B*b^4/(a^3*x) + 32/1155*sqrt(b*x^2 + a*x)*A*b^5/
(a^4*x) + 8/315*sqrt(b*x^2 + a*x)*B*b^3/(a^2*x^2) - 16/1155*sqrt(b*x^2 + a
*x)*A*b^4/(a^3*x^2) - 2/105*sqrt(b*x^2 + a*x)*B*b^2/(a*x^3) + 4/385*sqrt(b
*x^2 + a*x)*A*b^3/(a^2*x^3) + 1/63*sqrt(b*x^2 + a*x)*B*b/x^4 - 2/231*sqrt(
b*x^2 + a*x)*A*b^2/(a*x^4) + 1/9*sqrt(b*x^2 + a*x)*B*a/x^5 + 1/132*sqrt(b*
x^2 + a*x)*A*b/x^5 - 1/3*(b*x^2 + a*x)^(3/2)*B/x^6 + 3/44*sqrt(b*x^2 + a*x
)*A*a/x^6 - 1/4*(b*x^2 + a*x)^(3/2)*A/x^7
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx = -\frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(11\,Ba^2b^4-6\,Aab^5)(bx+a)}{a^5} - \frac{11\,(11\,Ba^3b^4-6\,Aa^2b^5)}{a^5}\right) + \frac{99\,(11\,Ba^4b^4-6\,Aa^3b^5)}{a^5}\right) - \frac{693\,(Ba^5b^4-A}{a^5}}{3465\,((bx+a)b-ab)^{\frac{11}{2}}\,|b|}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(13/2),x, algorithm="giac")
```

output
```
-2/3465*((b*x + a)*(4*(b*x + a)*(2*(11*B*a^2*b^4 - 6*A*a*b^5)*(b*x + a)/a^
5 - 11*(11*B*a^3*b^4 - 6*A*a^2*b^5)/a^5) + 99*(11*B*a^4*b^4 - 6*A*a^3*b^5)
/a^5) - 693*(B*a^5*b^4 - A*a^4*b^5)/a^5)*(b*x + a)^(5/2)*b^7/(((b*x + a)*b
 - a*b)^(11/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A\,a}{11}+\frac{x\,\left(770\,B\,a^5+840\,A\,b\,a^4\right)}{3465\,a^4}-\frac{x^5\,\left(96\,A\,b^5-176\,B\,a\,b^4\right)}{3465\,a^4}-\frac{2\,b^2\,x^3\,\left(6\,A\,b-11\,B\,a\right)}{1155\,a^2}+\frac{8\,b^3\,x^4\,\left(6\,A\,b-11\,B\,a\right)}{3465\,a^3}+\frac{2\,b\,x^2\,(3}{}\right.}{x^{11/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(13/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a)/11 + (x*(770*B*a^5 + 840*A*a^4*b))/(3465*a^4) -
 (x^5*(96*A*b^5 - 176*B*a*b^4))/(3465*a^4) - (2*b^2*x^3*(6*A*b - 11*B*a))/
(1155*a^2) + (8*b^3*x^4*(6*A*b - 11*B*a))/(3465*a^3) + (2*b*x^2*(3*A*b + 1
10*B*a))/(693*a)))/x^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{13/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11}-\frac{46\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{99}-\frac{226\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{693}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{231}+\frac{8\sqrt{x}\,\sqrt{b}}{}}{a^3x^6}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(13/2),x)
```

output
```
(2*( - 63*sqrt(x)*sqrt(a + b*x)*a**5 - 161*sqrt(x)*sqrt(a + b*x)*a**4*b*x
- 113*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*
b**3*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 - 8*sqrt(x)*sqrt(a + b*x)*
b**5*x**5 + 8*sqrt(b)*b**5*x**6))/(693*a**3*x**6)
```

3.305 $\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}} dx = -\frac{2A(a+bx)^{5/2}}{13ax^{13/2}} + \frac{2(8Ab-13aB)(a+bx)^{5/2}}{143a^2x^{11/2}} - \frac{4b(8Ab-13aB)(a+bx)^{5/2}}{429a^3x^{9/2}} + \frac{16b^2(8Ab-13aB)(a+bx)^{5/2}}{3003a^4x^{7/2}} - \frac{32b^3(8Ab-13aB)(a+bx)^{5/2}}{15015a^5x^{5/2}}$$

output
```
-2/13*A*(b*x+a)^(5/2)/a/x^(13/2)+2/143*(8*A*b-13*B*a)*(b*x+a)^(5/2)/a^2/x^
(11/2)-4/429*b*(8*A*b-13*B*a)*(b*x+a)^(5/2)/a^3/x^(9/2)+16/3003*b^2*(8*A*b
-13*B*a)*(b*x+a)^(5/2)/a^4/x^(7/2)-32/15015*b^3*(8*A*b-13*B*a)*(b*x+a)^(5/
2)/a^5/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx =$$
$$-\frac{2(a+bx)^{5/2}\left(128Ab^4x^4+105a^4(11A+13Bx)-70a^3bx(12A+13Bx)+40a^2b^2x^2(14A+13Bx)-16ab\right.}{15015a^5x^{13/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/x^(15/2),x]
```

output
```
(-2*(a + b*x)^(5/2)*(128*A*b^4*x^4 + 105*a^4*(11*A + 13*B*x) - 70*a^3*b*x*
(12*A + 13*B*x) + 40*a^2*b^2*x^2*(14*A + 13*B*x) - 16*a*b^3*x^3*(20*A + 13
*B*x)))/(15015*a^5*x^(13/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(8Ab-13aB)\int \frac{(a+bx)^{3/2}}{x^{13/2}}dx}{13a}-\frac{2A(a+bx)^{5/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-13aB)\left(-\frac{6b\int \frac{(a+bx)^{3/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}\right)}{13a}-\frac{2A(a+bx)^{5/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-13aB)\left(-\frac{6b\left(-\frac{4b\int \frac{(a+bx)^{3/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}\right)}{13a}-\frac{2A(a+bx)^{5/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-13aB)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{3/2}}{x^{7/2}}dx}{7a}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}\right)}{13a}-\frac{2A(a+bx)^{5/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b(a+bx)^{5/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{5/2}}{7ax^{7/2}}\right)}{9a}-\frac{2(a+bx)^{5/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{5/2}}{11ax^{11/2}}\right)(8Ab-13aB)}{13a}-\frac{2A(a+bx)^{5/2}}{13ax^{13/2}}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/x^(15/2),x]
```

output
```
(-2*A*(a + b*x)^(5/2))/(13*a*x^(13/2)) - ((8*A*b - 13*a*B)*((-2*(a + b*x)^
(5/2))/(11*a*x^(11/2)) - (6*b*((-2*(a + b*x)^(5/2))/(9*a*x^(9/2)) - (4*b*(
(-2*(a + b*x)^(5/2))/(7*a*x^(7/2)) + (4*b*(a + b*x)^(5/2))/(35*a^2*x^(5/2)
)))/(9*a)))/(11*a)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

method	result
gosper	$-\frac{2(bx+a)^{\frac{5}{2}} \left(128A\,b^4x^4-208Ba\,b^3x^4-320Aa\,b^3x^3+520B\,a^2b^2x^3+560A\,a^2b^2x^2-910B\,a^3b\,x^2-840A\,a^3bx+1365B\,a^4x+1155A\,a^4\right.}{15015x^{\frac{13}{2}}a^5}$
orering	$-\frac{2(bx+a)^{\frac{5}{2}} \left(128A\,b^4x^4-208Ba\,b^3x^4-320Aa\,b^3x^3+520B\,a^2b^2x^3+560A\,a^2b^2x^2-910B\,a^3b\,x^2-840A\,a^3bx+1365B\,a^4x+1155A\,a^4\right.}{15015x^{\frac{13}{2}}a^5}$
default	$-\frac{2(bx+a)^{\frac{3}{2}} \left(128A\,b^5x^5-208Ba\,b^4x^5-192aA\,b^4x^4+312B\,a^2b^3x^4+240a^2A\,b^3x^3-390B\,a^3b^2x^3-280a^3A\,b^2x^2+455B\,a^4b\,x^2+315a\right.}{15015x^{\frac{13}{2}}a^5}$
risch	$-\frac{2\sqrt{bx+a}\,\left(128A\,b^6x^6-208Ba\,b^5x^6-64Aa\,b^5x^5+104B\,a^2b^4x^5+48A\,a^2b^4x^4-78B\,a^3b^3x^4-40A\,a^3b^3x^3+65B\,a^4b^2x^3+35A\,a^4b^2x^2\right.}{15015x^{\frac{13}{2}}a^5}$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15015*(b*x+a)^(5/2)*(128*A*b^4*x^4-208*B*a*b^3*x^4-320*A*a*b^3*x^3+520*
B*a^2*b^2*x^3+560*A*a^2*b^2*x^2-910*B*a^3*b*x^2-840*A*a^3*b*x+1365*B*a^4*x
+1155*A*a^4)/x^(13/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx =$$
$$-\frac{2\,(1155\,Aa^6-16\,(13\,Bab^5-8\,Ab^6)x^6+8\,(13\,Ba^2b^4-8\,Aab^5)x^5-6\,(13\,Ba^3b^3-8\,Aa^2b^4)x^4+5\,(13\,Ba}{15015\,a^5x^{\frac{13}{2}}}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(15/2),x, algorithm="fricas")
```

output
```
-2/15015*(1155*A*a^6 - 16*(13*B*a*b^5 - 8*A*b^6)*x^6 + 8*(13*B*a^2*b^4 - 8
*A*a*b^5)*x^5 - 6*(13*B*a^3*b^3 - 8*A*a^2*b^4)*x^4 + 5*(13*B*a^4*b^2 - 8*A
*a^3*b^3)*x^3 + 35*(52*B*a^5*b + A*a^4*b^2)*x^2 + 105*(13*B*a^6 + 14*A*a^5
*b)*x)*sqrt(b*x + a)/(a^5*x^(13/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/x**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(120) = 240$.

Time = 0.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.09

$$\begin{aligned}\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx &= \frac{32\sqrt{bx^2+ax}Bb^5}{1155\,a^4x} - \frac{256\sqrt{bx^2+ax}Ab^6}{15015\,a^5x} \\ &- \frac{16\sqrt{bx^2+ax}Bb^4}{1155\,a^3x^2} + \frac{128\sqrt{bx^2+ax}Ab^5}{15015\,a^4x^2} + \frac{4\sqrt{bx^2+ax}Bb^3}{385\,a^2x^3} \\ &- \frac{32\sqrt{bx^2+ax}Ab^4}{5005\,a^3x^3} - \frac{2\sqrt{bx^2+ax}Bb^2}{231\,ax^4} + \frac{16\sqrt{bx^2+ax}Ab^3}{3003\,a^2x^4} \\ &+ \frac{\sqrt{bx^2+ax}Bb}{132\,x^5} - \frac{2\sqrt{bx^2+ax}Ab^2}{429\,ax^5} + \frac{3\sqrt{bx^2+ax}Ba}{44\,x^6} \\ &+ \frac{3\sqrt{bx^2+ax}Ab}{715\,x^6} - \frac{(bx^2+ax)^{\frac{3}{2}}B}{4\,x^7} + \frac{3\sqrt{bx^2+ax}Aa}{65\,x^7} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{5\,x^8}\end{aligned}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(15/2),x, algorithm="maxima")
```

output
```
32/1155*sqrt(b*x^2 + a*x)*B*b^5/(a^4*x) - 256/15015*sqrt(b*x^2 + a*x)*A*b^
6/(a^5*x) - 16/1155*sqrt(b*x^2 + a*x)*B*b^4/(a^3*x^2) + 128/15015*sqrt(b*x
^2 + a*x)*A*b^5/(a^4*x^2) + 4/385*sqrt(b*x^2 + a*x)*B*b^3/(a^2*x^3) - 32/5
005*sqrt(b*x^2 + a*x)*A*b^4/(a^3*x^3) - 2/231*sqrt(b*x^2 + a*x)*B*b^2/(a*x
^4) + 16/3003*sqrt(b*x^2 + a*x)*A*b^3/(a^2*x^4) + 1/132*sqrt(b*x^2 + a*x)*
B*b/x^5 - 2/429*sqrt(b*x^2 + a*x)*A*b^2/(a*x^5) + 3/44*sqrt(b*x^2 + a*x)*B
*a/x^6 + 3/715*sqrt(b*x^2 + a*x)*A*b/x^6 - 1/4*(b*x^2 + a*x)^(3/2)*B/x^7 +
 3/65*sqrt(b*x^2 + a*x)*A*a/x^7 - 1/5*(b*x^2 + a*x)^(3/2)*A/x^8
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx = \frac{2\left(\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(13\,Ba^2b^{12}-8\,Aab^{13})(bx+a)}{a^6} - \frac{13\,(13\,Ba^3b^{12}-8\,Aa^2b^{13})}{a^6}\right) + \right.\right.\right.}{15015\,((}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/x^(15/2),x, algorithm="giac")
```

output
```
2/15015*((2*(b*x + a)*(4*(b*x + a)*(2*(13*B*a^2*b^12 - 8*A*a*b^13)*(b*x +
a)/a^6 - 13*(13*B*a^3*b^12 - 8*A*a^2*b^13)/a^6) + 143*(13*B*a^4*b^12 - 8*A
*a^3*b^13)/a^6) - 429*(13*B*a^5*b^12 - 8*A*a^4*b^13)/a^6)*(b*x + a) + 3003
*(B*a^6*b^12 - A*a^5*b^13)/a^6)*(b*x + a)^(5/2)*b/(((b*x + a)*b - a*b)^(13
/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A\,a}{13}+x\,\left(\frac{28\,A\,b}{143}+\frac{2\,B\,a}{11}\right)+\frac{x^6\,\left(256\,A\,b^6-416\,B\,a\,b^5\right)}{15015\,a^5}-\frac{2\,b^2\,x^3\,\left(8\,A\,b-13\,B\,a\right)}{3003\,a^2}+\frac{4\,b^3\,x^4\,\left(8\,A\,b-13\,B\,a\right)}{5005\,a^3}-\frac{16\,b^4\,x^5\,(8}{150}\right.}{x^{13/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/x^(15/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a)/13 + x*((28*A*b)/143 + (2*B*a)/11) + (x^6*(256*
A*b^6 - 416*B*a*b^5))/(15015*a^5) - (2*b^2*x^3*(8*A*b - 13*B*a))/(3003*a^2
) + (4*b^3*x^4*(8*A*b - 13*B*a))/(5005*a^3) - (16*b^4*x^5*(8*A*b - 13*B*a)
)/(15015*a^4) + (2*b*x^2*(A*b + 52*B*a))/(429*a)))/x^(13/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{x^{15/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6}{13}-\frac{54\sqrt{x}\,\sqrt{bx+a}\,a^5bx}{143}-\frac{106\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x^2}{429}-\frac{10\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^3}{3003}+\frac{4\sqrt{x}\,\sqrt{}}{}}{a^4x^7}$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/x^(15/2),x)
```

output
```
(2*( - 231*sqrt(x)*sqrt(a + b*x)*a**6 - 567*sqrt(x)*sqrt(a + b*x)*a**5*b*x
 - 371*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x**2 - 5*sqrt(x)*sqrt(a + b*x)*a**3
*b**3*x**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**4*x**4 - 8*sqrt(x)*sqrt(a + b
*x)*a*b**5*x**5 + 16*sqrt(x)*sqrt(a + b*x)*b**6*x**6 - 16*sqrt(b)*b**6*x**
7))/(3003*a**4*x**7)
```

3.306 $\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 220

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx=-\frac{a^4(12Ab-5aB)\sqrt{x}\sqrt{a+bx}}{512b^3}+\frac{a^3(12Ab-5aB)x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{31a^2(12Ab-5aB)x^{5/2}\sqrt{a+bx}}{960b}+\frac{7}{160}a(12Ab-5aB)x^{7/2}\sqrt{a+bx}+\frac{1}{60}b(12Ab-5aB)x^{9/2}\sqrt{a+bx}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}+\frac{a^5(12Ab-5aB)\text{arc}}{512b^{7/2}}$$

output
```
-1/512*a^4*(12*A*b-5*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+1/768*a^3*(12*A*b-5*B*
a)*x^(3/2)*(b*x+a)^(1/2)/b^2+31/960*a^2*(12*A*b-5*B*a)*x^(5/2)*(b*x+a)^(1/
2)/b+7/160*a*(12*A*b-5*B*a)*x^(7/2)*(b*x+a)^(1/2)+1/60*b*(12*A*b-5*B*a)*x^
(9/2)*(b*x+a)^(1/2)+1/6*B*x^(5/2)*(b*x+a)^(7/2)/b+1/512*a^5*(12*A*b-5*B*a)
*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.75

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx=\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(75a^5B+40a^3b^2x(3A+Bx)+256b^5x^4(6A+5Bx)-10a^4b(18A+5$$

input
```
Integrate[x^(3/2)*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(75*a^5*B + 40*a^3*b^2*x*(3*A + B*x) + 256*
b^5*x^4*(6*A + 5*B*x) - 10*a^4*b*(18*A + 5*B*x) + 48*a^2*b^3*x^2*(62*A + 4
5*B*x) + 64*a*b^4*x^3*(63*A + 50*B*x)) + 30*a^5*(-12*A*b + 5*a*B)*ArcTanh[
(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(7680*b^(7/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {90, 60, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \; 90$$

$$\frac{(12Ab-5aB)\int x^{3/2}(a+bx)^{5/2}dx}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$$\Big\downarrow \; 60$$

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}dx+\frac{1}{5}x^{5/2}(a+bx)^{5/2}\right)}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$$\Big\downarrow \; 60$$

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\int x^{3/2}\sqrt{a+bx}dx+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}\right)}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$\big\downarrow$ 60

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a+bx}}dx+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}\right)}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$\big\downarrow$ 60

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}\right)}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$\big\downarrow$ 60

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}\right)}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$\big\downarrow$ 65

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx)^{3/2}\right)+\frac{1}{5}x^{5/2}(a+bx)^{5/2}\right)}{12b}+\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

$\big\downarrow$ 219

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)+\frac{1}{3}x^{5/2}\sqrt{a+bx}\right)+\frac{1}{4}x^{5/2}(a+bx$$

$$12b$$

$$\frac{Bx^{5/2}(a+bx)^{7/2}}{6b}$$

input
```
Int[x^(3/2)*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(B*x^(5/2)*(a + b*x)^(7/2))/(6*b) + ((12*A*b - 5*a*B)*((x^(5/2)*(a + b*x)^
(5/2))/5 + (a*((x^(5/2)*(a + b*x)^(3/2))/4 + (3*a*((x^(5/2)*Sqrt[a + b*x])
/3 + (a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b -
 (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/6))/8))/2)
)/(12*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{(-1280b^5 B x^5-1536A b^5 x^4-3200Ba b^4 x^4-4032Aa b^4 x^3-2160B a^2 b^3 x^3-2976A a^2 b^3 x^2-40B a^3 b^2 x^2-120a^3 b^2 Ax+50a^4 bBx+}{7680b^3}$
default	$\frac{\sqrt{x}\,\sqrt{bx+a}\left(2560B\, b^{\frac{11}{2}} x^5 \sqrt{x(bx+a)}+3072A\, b^{\frac{11}{2}} x^4 \sqrt{x(bx+a)}+6400Ba\, b^{\frac{9}{2}} x^4 \sqrt{x(bx+a)}+8064Aa\, b^{\frac{9}{2}} x^3 \sqrt{x(bx+a)}+4320B\, a^2 b^{\frac{7}{2}}\right.}{}$

input
```
int(x^(3/2)*(b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/b^3*(-1280*B*b^5*x^5-1536*A*b^5*x^4-3200*B*a*b^4*x^4-4032*A*a*b^4*
x^3-2160*B*a^2*b^3*x^3-2976*A*a^2*b^3*x^2-40*B*a^3*b^2*x^2-120*A*a^3*b^2*x
+50*B*a^4*b*x+180*A*a^4*b-75*B*a^5)*x^(1/2)*(b*x+a)^(1/2)+1/1024*a^5/b^(7/
2)*(12*A*b-5*B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1
/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.55

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx = \left[-\frac{15\,(5\,Ba^6-12\,Aa^5b)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(1280\,Bb^6x^5+75}{}\right.$$

input
```
integrate(x^(3/2)*(b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*s
qrt(b)*sqrt(x) + a) - 2*(1280*B*b^6*x^5 + 75*B*a^5*b - 180*A*a^4*b^2 + 128
*(25*B*a*b^5 + 12*A*b^6)*x^4 + 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^3 + 8*(5*
B*a^3*b^3 + 372*A*a^2*b^4)*x^2 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x)*sqrt(b
*x + a)*sqrt(x))/b^4, 1/7680*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(-b)*arctan(sq
rt(-b)*sqrt(x)/sqrt(b*x + a)) + (1280*B*b^6*x^5 + 75*B*a^5*b - 180*A*a^4*b
^2 + 128*(25*B*a*b^5 + 12*A*b^6)*x^4 + 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^3
 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^2 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x
)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*(b*x+a)**(5/2)*(B*x+A),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(174) = 348$.

Time = 0.05 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.92

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx = \frac{1}{6}\left(bx^2+ax\right)^{\frac{5}{2}}Bx + \frac{1}{4}\left(bx^2+ax\right)^{\frac{3}{2}}Aax - \frac{7\sqrt{bx^2+ax}Ba^4x}{256\,b^2} + \frac{7\left(bx^2+ax\right)^{\frac{3}{2}}Ba^2x}{96\,b} - \frac{3\sqrt{bx^2+ax}Aa^3x}{32\,b} + \frac{7\,Ba^6\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{1024\,b^{\frac{7}{2}}} + \frac{3\,Aa^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{5}{2}}} - \frac{7\sqrt{bx^2+ax}Ba^5}{512\,b^3} + \frac{7\left(bx^2+ax\right)^{\frac{3}{2}}Ba^3}{192\,b^2} - \frac{3\sqrt{bx^2+ax}Aa^4}{64\,b^2} - \frac{7\left(bx^2+ax\right)^{\frac{5}{2}}Ba}{60\,b} + \frac{\left(bx^2+ax\right)^{\frac{3}{2}}Aa^2}{8\,b} + \frac{3\sqrt{bx^2+ax}(Ba+Ab)a^3x}{64\,b^2} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}(Ba+Ab)ax}{8\,b} - \frac{3\,(Ba+Ab)a^5\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\sqrt{bx^2+ax}(Ba+Ab)a^4}{128\,b^3} - \frac{\left(bx^2+ax\right)^{\frac{3}{2}}(Ba+Ab)a^2}{16\,b^2} + \frac{\left(bx^2+ax\right)^{\frac{5}{2}}(Ba+Ab)}{5\,b}$$

input

```
integrate(x^(3/2)*(b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output

```
1/6*(b*x^2 + a*x)^(5/2)*B*x + 1/4*(b*x^2 + a*x)^(3/2)*A*a*x - 7/256*sqrt(b
*x^2 + a*x)*B*a^4*x/b^2 + 7/96*(b*x^2 + a*x)^(3/2)*B*a^2*x/b - 3/32*sqrt(b
*x^2 + a*x)*A*a^3*x/b + 7/1024*B*a^6*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*s
qrt(b))/b^(7/2) + 3/128*A*a^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))
/b^(5/2) - 7/512*sqrt(b*x^2 + a*x)*B*a^5/b^3 + 7/192*(b*x^2 + a*x)^(3/2)*B
*a^3/b^2 - 3/64*sqrt(b*x^2 + a*x)*A*a^4/b^2 - 7/60*(b*x^2 + a*x)^(5/2)*B*a
/b + 1/8*(b*x^2 + a*x)^(3/2)*A*a^2/b + 3/64*sqrt(b*x^2 + a*x)*(B*a + A*b)*
a^3*x/b^2 - 1/8*(b*x^2 + a*x)^(3/2)*(B*a + A*b)*a*x/b - 3/256*(B*a + A*b)*
a^5*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 3/128*sqrt(b*x^
2 + a*x)*(B*a + A*b)*a^4/b^3 - 1/16*(b*x^2 + a*x)^(3/2)*(B*a + A*b)*a^2/b^
2 + 1/5*(b*x^2 + a*x)^(5/2)*(B*a + A*b)/b
```

Giac [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)*(b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx = \int x^{3/2}\,(A+B\,x)\,{(a+b\,x)}^{5/2}\,dx$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x)^(5/2),x)
```

output
```
int(x^(3/2)*(A + B*x)*(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.60

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx)\,dx = \frac{-105\sqrt{x}\,\sqrt{bx+a}\,a^5b+70\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x+3016\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^2+6192\sqrt{x}\,\sqrt{}}{}$$

input
```
int(x^(3/2)*(b*x+a)^(5/2)*(B*x+A),x)
```

output
```
( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b + 70*sqrt(x)*sqrt(a + b*x)*a**4*b**2*
x + 3016*sqrt(x)*sqrt(a + b*x)*a**3*b**3*x**2 + 6192*sqrt(x)*sqrt(a + b*x)
*a**2*b**4*x**3 + 4736*sqrt(x)*sqrt(a + b*x)*a*b**5*x**4 + 1280*sqrt(x)*sq
rt(a + b*x)*b**6*x**5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/
sqrt(a))*a**6)/(7680*b**3)
```

3.307 $\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 187

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \frac{a^3(10Ab-3aB)\sqrt{x}\sqrt{a+bx}}{128b^2} + \frac{59a^2(10Ab-3aB)x^{3/2}\sqrt{a+bx}}{960b} + \frac{17}{240}a(10Ab-3aB)x^{5/2}\sqrt{a+bx} + \frac{1}{40}b(10Ab-3aB)x^{7/2}\sqrt{a+bx} + \frac{Bx^{3/2}(a+bx)^{7/2}}{5b} - \frac{a^4(10Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}$$

output

```
1/128*a^3*(10*A*b-3*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^2+59/960*a^2*(10*A*b-3*B*
a)*x^(3/2)*(b*x+a)^(1/2)/b+17/240*a*(10*A*b-3*B*a)*x^(5/2)*(b*x+a)^(1/2)+1
/40*b*(10*A*b-3*B*a)*x^(7/2)*(b*x+a)^(1/2)+1/5*B*x^(3/2)*(b*x+a)^(7/2)/b-1
/128*a^4*(10*A*b-3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(-45a^4B+30a^3b(5A+Bx)+96b^4x^3(5A+4Bx)+16ab^3x^2(85A+63Bx)+4}{1920b^{5/2}}$$

input
```
Integrate[Sqrt[x]*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-45*a^4*B + 30*a^3*b*(5*A + B*x) + 96*b^4*
x^3*(5*A + 4*B*x) + 16*a*b^3*x^2*(85*A + 63*B*x) + 4*a^2*b^2*x*(295*A + 18
6*B*x)) + 300*a^4*A*b*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]
 + 90*a^5*B*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(1920*b
^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {90, 60, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \; 90$$

$$\frac{(10Ab-3aB)\int \sqrt{x}(a+bx)^{5/2}dx}{10b}+\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\int \sqrt{x}(a+bx)^{3/2}dx+\frac{1}{4}x^{3/2}(a+bx)^{5/2}\right)}{10b}+\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

$$\Big\downarrow \; 60$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\int \sqrt{x}\sqrt{a+bx}dx+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}\right)}{10b}+\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

$\big\downarrow$ 60

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx+\frac{1}{2}x^{3/2}\sqrt{a+bx}\right)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\right)+\frac{1}{4}x^{3/2}(a+bx)^{5/2}\right)}{10b}+\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

$\big\downarrow$ 60

$$(10Ab-3aB)\Bigg(\frac{5}{8}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\Bigg)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\Bigg)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\Bigg)+\frac{1}{4}x^{3/2}(a+bx$$

$$10b$$

$$\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

$\big\downarrow$ 65

$$(10Ab-3aB)\Bigg(\frac{5}{8}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\Bigg)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\Bigg)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\Bigg)+\frac{1}{4}x^{3/2}($$

$$10b$$

$$\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

$\big\downarrow$ 219

$$(10Ab-3aB)\Bigg(\frac{5}{8}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{4}a\Bigg(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\Bigg)+\frac{1}{2}x^{3/2}\sqrt{a+bx}\Bigg)+\frac{1}{3}x^{3/2}(a+bx)^{3/2}\Bigg)+\frac{1}{4}x^{3/2}(a$$

$$10b$$

$$\frac{Bx^{3/2}(a+bx)^{7/2}}{5b}$$

input
```
Int[Sqrt[x]*(a + b*x)^(5/2)*(A + B*x),x]
```

output
```
(B*x^(3/2)*(a + b*x)^(7/2))/(5*b) + ((10*A*b - 3*a*B)*((x^(3/2)*(a + b*x)^
(5/2))/4 + (5*a*((x^(3/2)*(a + b*x)^(3/2))/3 + (a*((x^(3/2)*Sqrt[a + b*x])
/2 + (a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/b^(3/2)))/4))/2))/8))/(10*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

method	result
risch	$\frac{(384B\,x^4b^4+480A\,x^3b^4+1008B\,x^3a\,b^3+1360A\,x^2a\,b^3+744B\,x^2a^2b^2+1180Ax\,a^2b^2+30Bx\,a^3b+150A\,a^3b-45B\,a^4)\sqrt{x}\,\sqrt{bx+a}}{1920b^2}-$
default	$-\frac{\sqrt{x}\,\sqrt{bx+a}\left(-768B\,b^{\frac{9}{2}}x^4\sqrt{x(bx+a)}-960A\,b^{\frac{9}{2}}x^3\sqrt{x(bx+a)}-2016Ba\,b^{\frac{7}{2}}x^3\sqrt{x(bx+a)}-2720Aa\,b^{\frac{7}{2}}x^2\sqrt{x(bx+a)}-1488B\,a^2b^{\frac{5}{2}}\right.}{}$

input
```
int(x^(1/2)*(b*x+a)^(5/2)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/1920/b^2*(384*B*b^4*x^4+480*A*b^4*x^3+1008*B*a*b^3*x^3+1360*A*a*b^3*x^2+
744*B*a^2*b^2*x^2+1180*A*a^2*b^2*x+30*B*a^3*b*x+150*A*a^3*b-45*B*a^4)*x^(1
/2)*(b*x+a)^(1/2)-1/256*a^4/b^(5/2)*(10*A*b-3*B*a)*ln((1/2*a+b*x)/b^(1/2)+
(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.57

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx=\left[-\frac{15\,(3\,Ba^5-10\,Aa^4b)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(384\,Bb^5x^4-45\,Ba^4b+150}{}\right.$$

$$-\frac{15\,(3\,Ba^5-10\,Aa^4b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(384\,Bb^5x^4-45\,Ba^4b+150\,Aa^3b^2+48\,(21\,Bab^4+10\,Ab^5}{1920\,b^3}$$

input
```
integrate(x^(1/2)*(b*x+a)^(5/2)*(B*x+A),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(3*B*a^5 - 10*A*a^4*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sq
rt(b)*sqrt(x) + a) - 2*(384*B*b^5*x^4 - 45*B*a^4*b + 150*A*a^3*b^2 + 48*(2
1*B*a*b^4 + 10*A*b^5)*x^3 + 8*(93*B*a^2*b^3 + 170*A*a*b^4)*x^2 + 10*(3*B*a
^3*b^2 + 118*A*a^2*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^3, -1/1920*(15*(3*B*a^
5 - 10*A*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (384*B*b
^5*x^4 - 45*B*a^4*b + 150*A*a^3*b^2 + 48*(21*B*a*b^4 + 10*A*b^5)*x^3 + 8*(
93*B*a^2*b^3 + 170*A*a*b^4)*x^2 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x)*sqrt
(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 748, normalized size of antiderivative = 4.00

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(b*x+a)**(5/2)*(B*x+A),x)
```

output
```
2*A*a**2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqr
t(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b) +
sqrt(a + b*x)*(a*sqrt(x)/(8*b) + x**(3/2)/4), Ne(b, 0)), (sqrt(a)*x**(3/2)
/3, True)) + 4*A*a*b*Piecewise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x
) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True
))/(16*b**2) + sqrt(a + b*x)*(-a**2*sqrt(x)/(16*b**2) + a*x**(3/2)/(24*b)
+ x**(5/2)/6), Ne(b, 0)), (sqrt(a)*x**(5/2)/5, True)) + 2*A*b**2*Piecewise
((-5*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), N
e(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(128*b**3) + sqrt(a + b*
x)*(5*a**3*sqrt(x)/(128*b**3) - 5*a**2*x**(3/2)/(192*b**2) + a*x**(5/2)/(4
8*b) + x**(7/2)/8), Ne(b, 0)), (sqrt(a)*x**(7/2)/7, True)) + 2*B*a**2*Piec
ewise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b),
 Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(16*b**2) + sqrt(a + b
*x)*(-a**2*sqrt(x)/(16*b**2) + a*x**(3/2)/(24*b) + x**(5/2)/6), Ne(b, 0)),
 (sqrt(a)*x**(5/2)/5, True)) + 4*B*a*b*Piecewise((-5*a**4*Piecewise((log(2
*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqr
t(x))/sqrt(b*x), True))/(128*b**3) + sqrt(a + b*x)*(5*a**3*sqrt(x)/(128*b*
*3) - 5*a**2*x**(3/2)/(192*b**2) + a*x**(5/2)/(48*b) + x**(7/2)/8), Ne(b,
0)), (sqrt(a)*x**(7/2)/7, True)) + 2*B*b**2*Piecewise((7*a**5*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(147) = 294$.

Time = 0.05 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.59

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \frac{1}{5}\left(bx^2+ax\right)^{\frac{3}{2}}Bbx^2 - \frac{7}{40}\left(bx^2+ax\right)^{\frac{3}{2}}Bax + \frac{1}{2}\sqrt{bx^2+ax}Aa^2x - \frac{7\sqrt{bx^2+ax}Ba^3x}{64\,b} + \frac{7\,Ba^5\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{256\,b^{\frac{5}{2}}} - \frac{Aa^4\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{3}{2}}} - \frac{7\sqrt{bx^2+ax}Ba^4}{128\,b^2} + \frac{7\left(bx^2+ax\right)^{\frac{3}{2}}Ba^2}{48\,b} + \frac{\sqrt{bx^2+ax}Aa^3}{4\,b} + \frac{5\,(2\,Bab+Ab^2)\sqrt{bx^2+ax}a^2x}{32\,b^2} + \frac{(2\,Bab+Ab^2)(bx^2+ax)^{\frac{3}{2}}x}{4\,b} - \frac{(Ba^2+2\,Aab)\sqrt{bx^2+ax}ax}{4\,b} - \frac{5\,(2\,Bab+Ab^2)a^4\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{7}{2}}} + \frac{(Ba^2+2\,Aab)a^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{5}{2}}} + \frac{5\,(2\,Bab+Ab^2)\sqrt{bx^2+ax}a^3}{64\,b^3} - \frac{5\,(2\,Bab+Ab^2)(bx^2+ax)^{\frac{3}{2}}a}{24\,b^2} - \frac{(Ba^2+2\,Aab)\sqrt{bx^2+ax}a^2}{8\,b^2} + \frac{(Ba^2+2\,Aab)(bx^2+ax)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(x^(1/2)*(b*x+a)^(5/2)*(B*x+A),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a*x)^(3/2)*B*b*x^2 - 7/40*(b*x^2 + a*x)^(3/2)*B*a*x + 1/2*sqr
t(b*x^2 + a*x)*A*a^2*x - 7/64*sqrt(b*x^2 + a*x)*B*a^3*x/b + 7/256*B*a^5*lo
g(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 1/8*A*a^4*log(2*b*x +
 a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) - 7/128*sqrt(b*x^2 + a*x)*B*a^4/
b^2 + 7/48*(b*x^2 + a*x)^(3/2)*B*a^2/b + 1/4*sqrt(b*x^2 + a*x)*A*a^3/b + 5
/32*(2*B*a*b + A*b^2)*sqrt(b*x^2 + a*x)*a^2*x/b^2 + 1/4*(2*B*a*b + A*b^2)*
(b*x^2 + a*x)^(3/2)*x/b - 1/4*(B*a^2 + 2*A*a*b)*sqrt(b*x^2 + a*x)*a*x/b -
5/128*(2*B*a*b + A*b^2)*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b
^(7/2) + 1/16*(B*a^2 + 2*A*a*b)*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sq
rt(b))/b^(5/2) + 5/64*(2*B*a*b + A*b^2)*sqrt(b*x^2 + a*x)*a^3/b^3 - 5/24*(
2*B*a*b + A*b^2)*(b*x^2 + a*x)^(3/2)*a/b^2 - 1/8*(B*a^2 + 2*A*a*b)*sqrt(b*
x^2 + a*x)*a^2/b^2 + 1/3*(B*a^2 + 2*A*a*b)*(b*x^2 + a*x)^(3/2)/b
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \text{Timed out}$$

input
```
integrate(x^(1/2)*(b*x+a)^(5/2)*(B*x+A),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \int \sqrt{x}\,(A+B\,x)\,(a+b\,x)^{5/2}\,dx$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x)^(5/2),x)
```

output
```
int(x^(1/2)*(A + B*x)*(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.61

$$\int \sqrt{x}(a+bx)^{5/2}(A+Bx)\,dx = \frac{105\sqrt{x}\,\sqrt{bx+a}\,a^4b+1210\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x+2104\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^2+1488\sqrt{x}\,\sqrt{bx+a}\,a\,b}{1920b^2}$$

input
```
int(x^(1/2)*(b*x+a)^(5/2)*(B*x+A),x)
```

output
```
(105*sqrt(x)*sqrt(a + b*x)*a**4*b + 1210*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x
 + 2104*sqrt(x)*sqrt(a + b*x)*a**2*b**3*x**2 + 1488*sqrt(x)*sqrt(a + b*x)*
a*b**4*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**5*x**4 - 105*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5)/(1920*b**2)
```

3.308 $\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 154

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{11a^2(8Ab-aB)\sqrt{x}\sqrt{a+bx}}{64b} + \frac{13}{96}a(8Ab-aB)x^{3/2}\sqrt{a+bx} + \frac{1}{24}b(8Ab-aB)x^{5/2}\sqrt{a+bx} + \frac{B\sqrt{x}(a+bx)^{7/2}}{4b} + \frac{5a^3(8Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}$$

output

```
11/64*a^2*(8*A*b-B*a)*x^(1/2)*(b*x+a)^(1/2)/b+13/96*a*(8*A*b-B*a)*x^(3/2)*
(b*x+a)^(1/2)+1/24*b*(8*A*b-B*a)*x^(5/2)*(b*x+a)^(1/2)+1/4*B*x^(1/2)*(b*x+
a)^(7/2)/b+5/64*a^3*(8*A*b-B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(
3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^3B+16b^3x^2(4A+3Bx)+8ab^2x(26A+17Bx)+2a^2b(1}{192b^{3/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/Sqrt[x],x]
```

output
```
(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3*B + 16*b^3*x^2*(4*A + 3*B*x) + 8*a*
b^2*x*(26*A + 17*B*x) + 2*a^2*b*(132*A + 59*B*x)) + 15*a^3*(-8*A*b + a*B)*
Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(192*b^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {90, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(8Ab-aB)\int \frac{(a+bx)^{5/2}}{\sqrt{x}}dx}{8b}+\frac{B\sqrt{x}(a+bx)^{7/2}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{8b}+\frac{B\sqrt{x}(a+bx)^{7/2}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{8b}+\frac{B\sqrt{x}(a+bx)^{7/2}}{4b}$$

$$\Big\downarrow \; 60$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{8b}+\frac{B\sqrt{x}(a+bx)^{7/2}}{4b}$$

$$\Big\downarrow \; 65$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{8b}+\frac{B\sqrt{x}(a+bx)^{7/2}}{4b}$$

$$\Big\downarrow \; 219$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{8b}+\frac{B\sqrt{x}(a+bx)^{7/2}}{4b}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/Sqrt[x],x]
```

output
```
(B*Sqrt[x]*(a + b*x)^(7/2))/(4*b) + ((8*A*b - a*B)*((Sqrt[x]*(a + b*x)^(5/
2))/3 + (5*a*((Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a*(Sqrt[x]*Sqrt[a + b*x] +
(a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]))/4))/6))/(8*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

method	result
risch	$\frac{\left(48 b^{3} B\,x^{3}+64 A\,x^{2} b^{3}+136 B\,x^{2} a\,b^{2}+208 A x a\,b^{2}+118 B x\,a^{2} b+264 a^{2} b A+15 a^{3} B\right) \sqrt{x}\,\sqrt{b x +a}}{192 b}+\frac{5 a^{3} \left(8 A b -B a \right) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+}\right.}{128 b^{\frac{3}{2}} \sqrt{x}\,\sqrt{b x +a}}$
default	$\frac{\sqrt{b x +a}\,\sqrt{x}\,\left(96 B\,b^{\frac{7}{2}} x^{3} \sqrt{x \left(b x +a \right)}+128 A\,b^{\frac{7}{2}} x^{2} \sqrt{x \left(b x +a \right)}+272 B a\,b^{\frac{5}{2}} x^{2} \sqrt{x \left(b x +a \right)}+416 A \sqrt{x \left(b x +a \right)}\,b^{\frac{5}{2}} a x +236 B \sqrt{x \left(b x +a \right)}\,b^{\frac{3}{2}} a^{}\right.}{384 b^{\frac{3}{2}} \sqrt{x (b}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/b*(48*B*b^3*x^3+64*A*b^3*x^2+136*B*a*b^2*x^2+208*A*a*b^2*x+118*B*a^2
*b*x+264*A*a^2*b+15*B*a^3)*x^(1/2)*(b*x+a)^(1/2)+5/128*a^3/b^(3/2)*(8*A*b-
B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(
b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \Bigg[-\frac{15\,(Ba^4-8\,Aa^3b)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(48\,Bb^4x^3+}{}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b
)*sqrt(x) + a) - 2*(48*B*b^4*x^3 + 15*B*a^3*b + 264*A*a^2*b^2 + 8*(17*B*a*
b^3 + 8*A*b^4)*x^2 + 2*(59*B*a^2*b^2 + 104*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(
x))/b^2, 1/192*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sq
rt(b*x + a)) + (48*B*b^4*x^3 + 15*B*a^3*b + 264*A*a^2*b^2 + 8*(17*B*a*b^3
+ 8*A*b^4)*x^2 + 2*(59*B*a^2*b^2 + 104*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/
b^2]
```

Sympy [A] (verification not implemented)

Time = 35.49 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.51

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = Aa^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{Aa^{\frac{5}{2}}\sqrt{x}}{8\sqrt{1+\frac{bx}{a}}} - \frac{Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}} + \frac{5A\sqrt{a}b^2x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}} + \frac{9Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} + 4Aab\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) + \frac{Ab^3x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}} - \frac{11Ba^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}} - \frac{11Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}} + \frac{79Ba^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}} + \frac{23B\sqrt{a}b^2x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}} + \frac{11Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} + 2Ba^2\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) + \frac{Bb^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(1/2),x)
```

output
```
A*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a) - A*a**(5/2)*sqrt(x)/(8*sqrt(1 + b*x/a)
) - A*a**(3/2)*b*x**(3/2)/(24*sqrt(1 + b*x/a)) + 5*A*sqrt(a)*b**2*x**(5/2)
/(12*sqrt(1 + b*x/a)) + 9*A*a**3*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)
) + 4*A*a*b*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*
sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b)
 + a*sqrt(x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a + b*x)/4, Ne(b, 0)), (s
qrt(a)*x**(3/2)/3, True)) + A*b**3*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a)) -
11*B*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) - 11*B*a**(5/2)*x**(3/2)/(192
*sqrt(1 + b*x/a)) + 79*B*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*B*s
qrt(a)*b**2*x**(7/2)/(24*sqrt(1 + b*x/a)) + 11*B*a**4*asinh(sqrt(b)*sqrt(x
)/sqrt(a))/(64*b**(3/2)) + 2*B*a**2*Piecewise((-a**2*Piecewise((log(2*sqrt
(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))
/sqrt(b*x), True))/(8*b) + a*sqrt(x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a
 + b*x)/4, Ne(b, 0)), (sqrt(a)*x**(3/2)/3, True)) + B*b**3*x**(9/2)/(4*sqr
t(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(116) = 232$.

Time = 0.06 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.95

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}Bb^2x^3 - \frac{7}{24}\sqrt{bx^2+ax}Babx^2 + \frac{35}{96}\sqrt{bx^2+ax}Ba^2x + \frac{35\,Ba^4\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{3}{2}}} + \frac{Aa^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} - \frac{35\,\sqrt{bx^2+ax}Ba^3}{64\,b} + \frac{(3\,Bab^2+Ab^3)\sqrt{bx^2+ax}x^2}{3\,b} - \frac{5\,(3\,Bab^2+Ab^3)\sqrt{bx^2+ax}ax}{12\,b^2} + \frac{3\,(Ba^2b+Aab^2)\sqrt{bx^2+ax}x}{2\,b} - \frac{5\,(3\,Bab^2+Ab^3)a^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} + \frac{9\,(Ba^2b+Aab^2)a^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} - \frac{(Ba^3+3\,Aa^2b)a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{5\,(3\,Bab^2+Ab^3)\sqrt{bx^2+ax}a^2}{8\,b^3} - \frac{9\,(Ba^2b+Aab^2)\sqrt{bx^2+ax}a}{4\,b^2} + \frac{(Ba^3+3\,Aa^2b)\sqrt{bx^2+ax}}{b}$$

input

```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(1/2),x, algorithm="maxima")
```

output

```
1/4*sqrt(b*x^2 + a*x)*B*b^2*x^3 - 7/24*sqrt(b*x^2 + a*x)*B*a*b*x^2 + 35/96
*sqrt(b*x^2 + a*x)*B*a^2*x + 35/128*B*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a
*x)*sqrt(b))/b^(3/2) + A*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/
sqrt(b) - 35/64*sqrt(b*x^2 + a*x)*B*a^3/b + 1/3*(3*B*a*b^2 + A*b^3)*sqrt(b
*x^2 + a*x)*x^2/b - 5/12*(3*B*a*b^2 + A*b^3)*sqrt(b*x^2 + a*x)*a*x/b^2 + 3
/2*(B*a^2*b + A*a*b^2)*sqrt(b*x^2 + a*x)*x/b - 5/16*(3*B*a*b^2 + A*b^3)*a^
3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(7/2) + 9/8*(B*a^2*b + A*
a*b^2)*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5/2) - 1/2*(B*a
^3 + 3*A*a^2*b)*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + 5
/8*(3*B*a*b^2 + A*b^3)*sqrt(b*x^2 + a*x)*a^2/b^3 - 9/4*(B*a^2*b + A*a*b^2)
*sqrt(b*x^2 + a*x)*a/b^2 + (B*a^3 + 3*A*a^2*b)*sqrt(b*x^2 + a*x)/b
```

Giac [A] (verification not implemented)

Time = 76.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{\left(\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)B}{b^2} - \frac{Bab^2-8\,Ab^3}{b^4}\right) - \frac{5\,(Ba^2b^2-8\,Aab^3)}{b^4}\right) - \frac{15\,(Ba^3b}{}\right.\right.}{}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(1/2),x, algorithm="giac")
```

output
```
1/192*((2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)*B/b^2 - (B*a*b^2 - 8*A*b^3)/
b^4) - 5*(B*a^2*b^2 - 8*A*a*b^3)/b^4) - 15*(B*a^3*b^2 - 8*A*a^2*b^3)/b^4)*
sqrt((b*x + a)*b - a*b)*sqrt(b*x + a) + 15*(B*a^4 - 8*A*a^3*b)*log(abs(-sq
rt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b^(3/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \int \frac{(A+B\,x)\,\left(a+b\,x\right)^{5/2}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(1/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{\sqrt{x}}\,dx = \frac{279\sqrt{x}\,\sqrt{bx+a}\,a^3b + 326\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x + 200\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2 + 48\sqrt{}}{192b}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(1/2),x)
```

output
```
(279*sqrt(x)*sqrt(a + b*x)*a**3*b + 326*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x
+ 200*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x*
*3 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192
*b)
```

3.309 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx = \frac{25}{24}a(6Ab+aB)\sqrt{x}\sqrt{a+bx} + \frac{5}{12}b(6Ab+aB)x^{3/2}\sqrt{a+bx} - \frac{2A(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{3}B\sqrt{x}(a+bx)^{5/2} + \frac{5a^2(6Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

output

```
25/24*a*(6*A*b+B*a)*x^(1/2)*(b*x+a)^(1/2)+5/12*b*(6*A*b+B*a)*x^(3/2)*(b*x+
a)^(1/2)-2*A*(b*x+a)^(5/2)/x^(1/2)+1/3*B*x^(1/2)*(b*x+a)^(5/2)+5/8*a^2*(6*
A*b+B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx = \frac{\sqrt{a+bx}(4b^2x^2(3A+2Bx)+2abx(27A+13Bx)+a^2(-48A+33Bx))}{24\sqrt{x}} + \frac{5a^2(6Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4\sqrt{b}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(3/2),x]
```

output
```
(Sqrt[a + b*x]*(4*b^2*x^2*(3*A + 2*B*x) + 2*a*b*x*(27*A + 13*B*x) + a^2*(-
48*A + 33*B*x)))/(24*Sqrt[x]) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*Sqrt
[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB+6Ab)\int \frac{(a+bx)^{5/2}}{\sqrt{x}}dx}{a} - \frac{2A(a+bx)^{7/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx + \frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{a} - \frac{2A(a+bx)^{7/2}}{a\sqrt{x}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{a}-\frac{2A(a+bx)^{7/2}}{a\sqrt{x}}$$

$\Big\downarrow$ 60

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{a}-\frac{2A(a+bx)^{7/2}}{a\sqrt{x}}$$

$\Big\downarrow$ 65

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{a}-\frac{2A(a+bx)^{7/2}}{a\sqrt{x}}$$

$\Big\downarrow$ 219

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}\right)}{a}-\frac{2A(a+bx)^{7/2}}{a\sqrt{x}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(3/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(a*Sqrt[x]) + ((6*A*b + a*B)*((Sqrt[x]*(a + b*x)^(5
/2))/3 + (5*a*((Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a*(Sqrt[x]*Sqrt[a + b*x] +
 (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]))/4))/6))/a
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{bx+a}\,\left(-8B\,b^{2}x^{3}-12A\,b^{2}x^{2}-26Bab\,x^{2}-54aAbx-33B\,a^{2}x+48a^{2}A\right)}{24\sqrt{x}}+\frac{5a^{2}\left(6Ab+Ba\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x\left(bx+a\right)}}{16\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\,\left(16B\sqrt{x\left(bx+a\right)}\,b^{\frac{5}{2}}x^{3}+24A\sqrt{x\left(bx+a\right)}\,b^{\frac{5}{2}}x^{2}+52B\sqrt{x\left(bx+a\right)}\,b^{\frac{3}{2}}a\,x^{2}+90Ab\ln\left(\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}x+108Aa\,b^{\frac{3}{2}}x\sqrt{}\right.}{48\sqrt{x}\,\sqrt{x\left(bx+a\right)}\,\sqrt{b}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x+a)^(1/2)*(-8*B*b^2*x^3-12*A*b^2*x^2-26*B*a*b*x^2-54*A*a*b*x-33*
B*a^2*x+48*A*a^2)/x^(1/2)+5/16*a^2*(6*A*b+B*a)*ln((1/2*a+b*x)/b^(1/2)+(b*x
^2+a*x)^(1/2))/b^(1/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx = \left[\frac{15\,(Ba^3+6\,Aa^2b)\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(8\,Bb^3x^3-48}{48\,bx}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b
)*sqrt(x) + a) + 2*(8*B*b^3*x^3 - 48*A*a^2*b + 2*(13*B*a*b^2 + 6*A*b^3)*x^
2 + 3*(11*B*a^2*b + 18*A*a*b^2)*x)*sqrt(b*x + a)*sqrt(x))/(b*x), -1/24*(15
*(B*a^3 + 6*A*a^2*b)*sqrt(-b)*x*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (
8*B*b^3*x^3 - 48*A*a^2*b + 2*(13*B*a*b^2 + 6*A*b^3)*x^2 + 3*(11*B*a^2*b +
18*A*a*b^2)*x)*sqrt(b*x + a)*sqrt(x))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 478, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx = -\frac{2Aa^{\frac{5}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + 2Aa^{\frac{3}{2}}b\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{2Aa^{\frac{3}{2}}b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}} + 4Aa^2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + 2Ab^2\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) + Ba^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{Ba^{\frac{5}{2}}\sqrt{x}}{8\sqrt{1+\frac{bx}{a}}} - \frac{Ba^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}} + \frac{5B\sqrt{a}b^2x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}} + \frac{9Ba^3\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} + 4Bab\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right) + \frac{Bb^3x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(3/2),x)
```

output
```
-2*A*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*A*a**(3/2)*b*sqrt(x)*sqrt(1 +
b*x/a) - 2*A*a**(3/2)*b*sqrt(x)/sqrt(1 + b*x/a) + 4*A*a**2*sqrt(b)*asinh(s
qrt(b)*sqrt(x)/sqrt(a)) + 2*A*b**2*Piecewise((-a**2*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/
sqrt(b*x), True))/(8*b) + a*sqrt(x)*sqrt(a + b*x)/(8*b) + x**(3/2)*sqrt(a
+ b*x)/4, Ne(b, 0)), (sqrt(a)*x**(3/2)/3, True)) + B*a**(5/2)*sqrt(x)*sqrt
(1 + b*x/a) - B*a**(5/2)*sqrt(x)/(8*sqrt(1 + b*x/a)) - B*a**(3/2)*b*x**(3/
2)/(24*sqrt(1 + b*x/a)) + 5*B*sqrt(a)*b**2*x**(5/2)/(12*sqrt(1 + b*x/a)) +
 9*B*a**3*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + 4*B*a*b*Piecewise((
-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a,
 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b) + a*sqrt(x)*sqrt(a + b
*x)/(8*b) + x**(3/2)*sqrt(a + b*x)/4, Ne(b, 0)), (sqrt(a)*x**(3/2)/3, True
)) + B*b**3*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(100) = 200$.

Time = 0.04 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.69

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx = \frac{Bb^3x^4}{3\sqrt{bx^2+ax}} - \frac{7\,Bab^2x^3}{12\sqrt{bx^2+ax}} + \frac{35\,Ba^2bx^2}{24\sqrt{bx^2+ax}} + \frac{51\,Ba^3x}{8\sqrt{bx^2+ax}} + \frac{4\,Aa^2bx}{\sqrt{bx^2+ax}} - \frac{35\,Ba^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,\sqrt{b}} - \frac{2\,Aa^3}{\sqrt{bx^2+ax}} + \frac{(4\,Bab^3+Ab^4)x^3}{2\,\sqrt{bx^2+ax}b} - \frac{5\,(4\,Bab^3+Ab^4)ax^2}{4\,\sqrt{bx^2+ax}b^2} + \frac{2\,(3\,Ba^2b^2+2\,Aab^3)x^2}{\sqrt{bx^2+ax}b} - \frac{15\,(4\,Bab^3+Ab^4)a^2x}{4\,\sqrt{bx^2+ax}b^3} + \frac{6\,(3\,Ba^2b^2+2\,Aab^3)ax}{\sqrt{bx^2+ax}b^2} - \frac{4\,(2\,Ba^3b+3\,Aa^2b^2)x}{\sqrt{bx^2+ax}b} + \frac{15\,(4\,Bab^3+Ab^4)a^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,(3\,Ba^2b^2+2\,Aab^3)a\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{5}{2}}} + \frac{2\,(2\,Ba^3b+3\,Aa^2b^2)\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*b^3*x^4/sqrt(b*x^2 + a*x) - 7/12*B*a*b^2*x^3/sqrt(b*x^2 + a*x) + 35/
24*B*a^2*b*x^2/sqrt(b*x^2 + a*x) + 51/8*B*a^3*x/sqrt(b*x^2 + a*x) + 4*A*a^
2*b*x/sqrt(b*x^2 + a*x) - 35/16*B*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*
sqrt(b))/sqrt(b) - 2*A*a^3/sqrt(b*x^2 + a*x) + 1/2*(4*B*a*b^3 + A*b^4)*x^3
/(sqrt(b*x^2 + a*x)*b) - 5/4*(4*B*a*b^3 + A*b^4)*a*x^2/(sqrt(b*x^2 + a*x)*
b^2) + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*x^2/(sqrt(b*x^2 + a*x)*b) - 15/4*(4*B*a
*b^3 + A*b^4)*a^2*x/(sqrt(b*x^2 + a*x)*b^3) + 6*(3*B*a^2*b^2 + 2*A*a*b^3)*
a*x/(sqrt(b*x^2 + a*x)*b^2) - 4*(2*B*a^3*b + 3*A*a^2*b^2)*x/(sqrt(b*x^2 +
a*x)*b) + 15/8*(4*B*a*b^3 + A*b^4)*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/b^(7/2) - 3*(3*B*a^2*b^2 + 2*A*a*b^3)*a*log(2*b*x + a + 2*sqrt(b
*x^2 + a*x)*sqrt(b))/b^(5/2) + 2*(2*B*a^3*b + 3*A*a^2*b^2)*log(2*b*x + a +
 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 76.74 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx = \frac{\left(\frac{\left(\left(2\,(bx+a)\left(\frac{4\,(bx+a)B}{b}+\frac{Bab+6\,Ab^2}{b^2}\right)+\frac{5\,\left(Ba^2b+6\,Aab^2\right)}{b^2}\right)(bx+a)-\frac{15\,\left(Ba^3b+6\,Aa^2b^2\right)}{b^2}\right)\sqrt{bx+a}}{\sqrt{(bx+a)b-ab}} - \frac{1\cdots}{\cdots}\cdots\right.}{24\,|b|}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(3/2),x, algorithm="giac")
```

output
```
1/24*(((2*(b*x + a)*(4*(b*x + a)*B/b + (B*a*b + 6*A*b^2)/b^2) + 5*(B*a^2*b
 + 6*A*a*b^2)/b^2)*(b*x + a) - 15*(B*a^3*b + 6*A*a^2*b^2)/b^2)*sqrt(b*x +
a)/sqrt((b*x + a)*b - a*b) - 15*(B*a^3 + 6*A*a^2*b)*log(abs(-sqrt(b*x + a)
*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b^(3/2))*b^2/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx=\int \frac{(A+B\,x)\,\left(a+b\,x\right)^{5/2}}{x^{3/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(3/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{3/2}}\,dx=\frac{-384\sqrt{x}\,\sqrt{bx+a}\,a^3+696\sqrt{x}\,\sqrt{bx+a}\,a^2bx+304\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2+64\sqrt{}}{192x}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(3/2),x)
```

output
```
( - 384*sqrt(x)*sqrt(a + b*x)*a**3 + 696*sqrt(x)*sqrt(a + b*x)*a**2*b*x +
304*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 64*sqrt(x)*sqrt(a + b*x)*b**3*x**3
 + 840*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*x - 525
*sqrt(b)*a**3*x)/(192*x)
```

3.310 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx=\frac{5}{4}b(4Ab+3aB)\sqrt{x}\sqrt{a+bx}-\frac{2(5Ab+3aB)(a+bx)^{3/2}}{3\sqrt{x}}+\frac{1}{2}bB\sqrt{x}(a+bx)^{3/2}-\frac{2A(a+bx)^{5/2}}{3x^{3/2}}+\frac{5}{4}a\sqrt{b}(4Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
5/4*b*(4*A*b+3*B*a)*x^(1/2)*(b*x+a)^(1/2)-2/3*(5*A*b+3*B*a)*(b*x+a)^(3/2)/
x^(1/2)+1/2*b*B*x^(1/2)*(b*x+a)^(3/2)-2/3*A*(b*x+a)^(5/2)/x^(3/2)+5/4*a*b^
(1/2)*(4*A*b+3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx=\frac{\sqrt{a+bx}(6b^2x^2(2A+Bx)-8a^2(A+3Bx)+abx(-56A+27Bx))}{12x^{3/2}}+\frac{5}{2}a\sqrt{b}(4Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(5/2),x]
```

output
```
(Sqrt[a + b*x]*(6*b^2*x^2*(2*A + B*x) - 8*a^2*(A + 3*B*x) + a*b*x*(-56*A +
 27*B*x)))/(12*x^(3/2)) + (5*a*Sqrt[b]*(4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*Sq
rt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 57, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{(3aB+4Ab)\int \frac{(a+bx)^{5/2}}{x^{3/2}}dx}{3a}-\frac{2A(a+bx)^{7/2}}{3ax^{3/2}}$$

$$\Big\downarrow \; 57$$

$$\frac{(3aB+4Ab)\left(5b\int \frac{(a+bx)^{3/2}}{\sqrt{x}}dx-\frac{2(a+bx)^{5/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{7/2}}{3ax^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{7/2}}{3ax^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{7/2}}{3ax^{3/2}}$$

$$\Big\downarrow \; 65$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\left(a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{7/2}}{3ax^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}\right)}{3a}-\frac{2A(a+bx)^{7/2}}{3ax^{3/2}}$$

input

```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(5/2),x]
```

output

```
(-2*A*(a + b*x)^(7/2))/(3*a*x^(3/2)) + ((4*A*b + 3*a*B)*((-2*(a + b*x)^(5/
2))/Sqrt[x] + 5*b*((Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a*(Sqrt[x]*Sqrt[a + b*
x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]))/4)))/(3*a)
```

Defintions of rubi rules used

rule 57

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{bx+a}\,\left(-6B\,b^{2}x^{3}-12A\,b^{2}x^{2}-27Bab\,x^{2}+56aAbx+24B\,a^{2}x+8a^{2}A\right)}{12x^{\frac{3}{2}}}+\frac{5a\sqrt{b}\,\left(4Ab+3Ba\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x\left(bx+a\right)}}{8\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\,\left(12B\sqrt{x\left(bx+a\right)}\,b^{\frac{5}{2}}x^{3}+60A\,b^{2}\ln\left(\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,x^{2}+24A\sqrt{x\left(bx+a\right)}\,b^{\frac{5}{2}}x^{2}+45Bb\ln\left(\frac{2\sqrt{x\left(bx+a\right)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}}{24x^{\frac{3}{2}}\sqrt{x\left(bx+a\right)}\,\sqrt{b}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x+a)^(1/2)*(-6*B*b^2*x^3-12*A*b^2*x^2-27*B*a*b*x^2+56*A*a*b*x+24*
B*a^2*x+8*A*a^2)/x^(3/2)+5/8*a*b^(1/2)*(4*A*b+3*B*a)*ln((1/2*a+b*x)/b^(1/2
)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx = \left[\frac{15\,(3\,Ba^2+4\,Aab)\sqrt{b}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(6\,Bb^2x^3-}{24\,x^2}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(15*(3*B*a^2 + 4*A*a*b)*sqrt(b)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt
(b)*sqrt(x) + a) + 2*(6*B*b^2*x^3 - 8*A*a^2 + 3*(9*B*a*b + 4*A*b^2)*x^2 -
8*(3*B*a^2 + 7*A*a*b)*x)*sqrt(b*x + a)*sqrt(x))/x^2, -1/12*(15*(3*B*a^2 +
4*A*a*b)*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (6*B*b^2*x^
3 - 8*A*a^2 + 3*(9*B*a*b + 4*A*b^2)*x^2 - 8*(3*B*a^2 + 7*A*a*b)*x)*sqrt(b*
x + a)*sqrt(x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.70

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx = -\frac{4Aa^{\frac{3}{2}}b}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + A\sqrt{a}b^2\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{4A\sqrt{a}b^2\sqrt{x}}{\sqrt{1+\frac{bx}{a}}} - \frac{2Aa^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{2Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} + 5Aab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2Ba^{\frac{5}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} + 2Ba^{\frac{3}{2}}b\sqrt{x}\sqrt{1+\frac{bx}{a}} - \frac{2Ba^{\frac{3}{2}}b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}} + 4Ba^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + 2Bb^2\left(\begin{cases} -\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{a\sqrt{x}\sqrt{a+bx}}{8b} + \frac{x^{\frac{3}{2}}\sqrt{a+bx}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(5/2),x)
```

output
```
-4*A*a**(3/2)*b/(sqrt(x)*sqrt(1 + b*x/a)) + A*sqrt(a)*b**2*sqrt(x)*sqrt(1
+ b*x/a) - 4*A*sqrt(a)*b**2*sqrt(x)/sqrt(1 + b*x/a) - 2*A*a**2*sqrt(b)*sqr
t(a/(b*x) + 1)/(3*x) - 2*A*a*b**(3/2)*sqrt(a/(b*x) + 1)/3 + 5*A*a*b**(3/2)
*asinh(sqrt(b)*sqrt(x)/sqrt(a)) - 2*B*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) +
 2*B*a**(3/2)*b*sqrt(x)*sqrt(1 + b*x/a) - 2*B*a**(3/2)*b*sqrt(x)/sqrt(1 +
b*x/a) + 4*B*a**2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + 2*B*b**2*Piecew
ise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b),
Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/(8*b) + a*sqrt(x)*sqrt(
a + b*x)/(8*b) + x**(3/2)*sqrt(a + b*x)/4, Ne(b, 0)), (sqrt(a)*x**(3/2)/3,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx = \frac{15}{8}\,Ba^2\sqrt{b}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + \frac{5}{2}\,Aab^{\frac{3}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{15\,\sqrt{bx^2+ax}Ba^2}{4\,x} - \frac{35\,\sqrt{bx^2+ax}Aab}{6\,x} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{4\,x^2} - \frac{5\,\sqrt{bx^2+ax}Aa^2}{6\,x^2} + \frac{(bx^2+ax)^{\frac{5}{2}}B}{2\,x^3} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{6\,x^3} + \frac{(bx^2+ax)^{\frac{5}{2}}A}{x^4}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(5/2),x, algorithm="maxima")
```

output
```
15/8*B*a^2*sqrt(b)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + 5/2*A*a*
b^(3/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 15/4*sqrt(b*x^2 + a
*x)*B*a^2/x - 35/6*sqrt(b*x^2 + a*x)*A*a*b/x + 5/4*(b*x^2 + a*x)^(3/2)*B*a
/x^2 - 5/6*sqrt(b*x^2 + a*x)*A*a^2/x^2 + 1/2*(b*x^2 + a*x)^(5/2)*B/x^3 - 5
/6*(b*x^2 + a*x)^(3/2)*A*a/x^3 + (b*x^2 + a*x)^(5/2)*A/x^4
```

Giac [A] (verification not implemented)

Time = 76.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx =$$

$$-\frac{b^3\left(\frac{15\,(3\,Ba^2+4\,Aab)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}}-\frac{\left(\left(3\left(2\,(bx+a)B+\frac{3\,Ba^2b+4\,Aab^2}{ab}\right)(bx+a)-\frac{20\left(3\,Ba^3b+4\,Aa^2b^2\right)}{ab}\right)(bx+a)+\frac{15}{}\right.}{((bx+a)b-ab)^{\frac{3}{2}}}}{12\,|b|}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(5/2),x, algorithm="giac")
```

output
```
-1/12*b^3*(15*(3*B*a^2 + 4*A*a*b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b
*x + a)*b - a*b)))/b^(3/2) - ((3*(2*(b*x + a)*B + (3*B*a^2*b + 4*A*a*b^2)/
(a*b))*(b*x + a) - 20*(3*B*a^3*b + 4*A*a^2*b^2)/(a*b))*(b*x + a) + 15*(3*B
*a^4*b + 4*A*a^3*b^2)/(a*b))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(3/2))/abs(
b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx = \int \frac{(A+B\,x)\,\left(a+b\,x\right)^{5/2}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(5/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{5/2}}\,dx = \frac{-64\sqrt{x}\,\sqrt{bx+a}\,a^3-640\sqrt{x}\,\sqrt{bx+a}\,a^2bx+312\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2+48\sqrt{x}}{96x^2}$$

input

```
int((b*x+a)^(5/2)*(B*x+A)/x^(5/2),x)
```

output

```
( - 64*sqrt(x)*sqrt(a + b*x)*a**3 - 640*sqrt(x)*sqrt(a + b*x)*a**2*b*x + 3
12*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**3*x**3
+ 840*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*x**2 +
 175*sqrt(b)*a**2*b*x**2)/(96*x**2)
```

3.311 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 125

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{2b(Ab+2aB)\sqrt{a+bx}}{\sqrt{x}}+b^2B\sqrt{x}\sqrt{a+bx} - \frac{2(Ab+aB)(a+bx)^{3/2}}{3x^{3/2}}-\frac{2A(a+bx)^{5/2}}{5x^{5/2}}+b^{3/2}(2Ab+5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2*b*(A*b+2*B*a)*(b*x+a)^(1/2)/x^(1/2)+b^2*B*x^(1/2)*(b*x+a)^(1/2)-2/3*(A*
b+B*a)*(b*x+a)^(3/2)/x^(3/2)-2/5*A*(b*x+a)^(5/2)/x^(5/2)+b^(3/2)*(2*A*b+5*
B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{\sqrt{a+bx}(b^2x^2(46A-15Bx)+2a^2(3A+5Bx)+2abx(11A+35Bx))}{15x^{5/2}} + 2b^{3/2}(2Ab+5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(7/2),x]
```

output
```
-1/15*(Sqrt[a + b*x]*(b^2*x^2*(46*A - 15*B*x) + 2*a^2*(3*A + 5*B*x) + 2*a*
b*x*(11*A + 35*B*x)))/x^(5/2) + 2*b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(Sqrt[b]
*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 57, 57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(5aB+2Ab)\int \frac{(a+bx)^{5/2}}{x^{5/2}}dx}{5a}-\frac{2A(a+bx)^{7/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\int \frac{(a+bx)^{3/2}}{x^{3/2}}dx-\frac{2(a+bx)^{5/2}}{3x^{3/2}}\right)}{5a}-\frac{2A(a+bx)^{7/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\int \frac{\sqrt{a+bx}}{\sqrt{x}}dx-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}\right)}{5a}-\frac{2A(a+bx)^{7/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}\right)}{5a}-\frac{2A(a+bx)^{7/2}}{5ax^{5/2}}$$

$$\Big\downarrow \text{65}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\left(a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}\right)}{5a}-\frac{2A(a+bx)^{7/2}}{5ax^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}+\sqrt{x}\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}}{\sqrt{x}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}\right)}{5a}-\frac{2A(a+bx)^{7/2}}{5ax^{5/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(7/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(5*a*x^(5/2)) + ((2*A*b + 5*a*B)*((-2*(a + b*x)^(5/
2))/(3*x^(3/2)) + (5*b*((-2*(a + b*x)^(3/2))/Sqrt[x] + 3*b*(Sqrt[x]*Sqrt[a
 + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b])))/3))/(5*a
)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{bx+a}\,\left(-15B\,b^{2}x^{3}+46A\,b^{2}x^{2}+70Bab\,x^{2}+22aAbx+10B\,a^{2}x+6a^{2}A\right)}{15x^{\frac{5}{2}}}+\frac{b^{\frac{3}{2}}\left(2Ab+5Ba\right)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\sqrt{bx+a}\,\left(30A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b^{3}x^{3}+75B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^{2}x^{3}+30B\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x^{3}-92A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x^{2}}{30x^{\frac{5}{2}}\sqrt{x(bx+a)}\,\sqrt{b}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x+a)^(1/2)*(-15*B*b^2*x^3+46*A*b^2*x^2+70*B*a*b*x^2+22*A*a*b*x+10
*B*a^2*x+6*A*a^2)/x^(5/2)+1/2*b^(3/2)*(2*A*b+5*B*a)*ln((1/2*a+b*x)/b^(1/2)
+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = \left[\frac{15\,(5\,Bab+2\,Ab^2)\sqrt{b}x^3\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,(15\,Bb^2x^3-}{30\,x^3}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/30*(15*(5*B*a*b + 2*A*b^2)*sqrt(b)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt
(b)*sqrt(x) + a) + 2*(15*B*b^2*x^3 - 6*A*a^2 - 2*(35*B*a*b + 23*A*b^2)*x^2
 - 2*(5*B*a^2 + 11*A*a*b)*x)*sqrt(b*x + a)*sqrt(x))/x^3, -1/15*(15*(5*B*a*
b + 2*A*b^2)*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (15*B*b
^2*x^3 - 6*A*a^2 - 2*(35*B*a*b + 23*A*b^2)*x^2 - 2*(5*B*a^2 + 11*A*a*b)*x)
*sqrt(b*x + a)*sqrt(x))/x^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(124) = 248.

Time = 7.60 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx &= -\frac{2A\sqrt{a}b^2}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}-\frac{2Aa^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2}\\ &-\frac{22Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15x}-\frac{16Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15}+2Ab^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\\ &-\frac{2Ab^3\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}-\frac{4Ba^{\frac{3}{2}}b}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+B\sqrt{a}b^2\sqrt{x}\sqrt{1+\frac{bx}{a}}-\frac{4B\sqrt{a}b^2\sqrt{x}}{\sqrt{1+\frac{bx}{a}}}\\ &-\frac{2Ba^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}+5Bab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\end{aligned}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(7/2),x)
```

output
```
-2*A*sqrt(a)*b**2/(sqrt(x)*sqrt(1 + b*x/a)) - 2*A*a**2*sqrt(b)*sqrt(a/(b*x
) + 1)/(5*x**2) - 22*A*a*b**(3/2)*sqrt(a/(b*x) + 1)/(15*x) - 16*A*b**(5/2)
*sqrt(a/(b*x) + 1)/15 + 2*A*b**(5/2)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) - 2*A*
b**3*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a)) - 4*B*a**(3/2)*b/(sqrt(x)*sqrt(1 +
b*x/a)) + B*sqrt(a)*b**2*sqrt(x)*sqrt(1 + b*x/a) - 4*B*sqrt(a)*b**2*sqrt(x
)/sqrt(1 + b*x/a) - 2*B*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*B*a*b**(3
/2)*sqrt(a/(b*x) + 1)/3 + 5*B*a*b**(3/2)*asinh(sqrt(b)*sqrt(x)/sqrt(a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(97) = 194$.

Time = 0.04 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = \frac{5}{2}\,Bab^{\frac{3}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) + Ab^{\frac{5}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{35\,\sqrt{bx^2+ax}Bab}{6\,x} - \frac{38\,\sqrt{bx^2+ax}Ab^2}{15\,x} - \frac{5\,\sqrt{bx^2+ax}Ba^2}{6\,x^2} - \frac{7\,\sqrt{bx^2+ax}Aab}{30\,x^2} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{6\,x^3} + \frac{3\,\sqrt{bx^2+ax}Aa^2}{10\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}Ab}{3\,x^3} + \frac{(bx^2+ax)^{\frac{5}{2}}B}{x^4} - \frac{(bx^2+ax)^{\frac{3}{2}}Aa}{2\,x^4} - \frac{(bx^2+ax)^{\frac{5}{2}}A}{5\,x^5}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(7/2),x, algorithm="maxima")
```

output
```
5/2*B*a*b^(3/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) + A*b^(5/2)*l
og(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 35/6*sqrt(b*x^2 + a*x)*B*a*b
/x - 38/15*sqrt(b*x^2 + a*x)*A*b^2/x - 5/6*sqrt(b*x^2 + a*x)*B*a^2/x^2 - 7
/30*sqrt(b*x^2 + a*x)*A*a*b/x^2 - 5/6*(b*x^2 + a*x)^(3/2)*B*a/x^3 + 3/10*s
qrt(b*x^2 + a*x)*A*a^2/x^3 - 1/3*(b*x^2 + a*x)^(3/2)*A*b/x^3 + (b*x^2 + a*
x)^(5/2)*B/x^4 - 1/2*(b*x^2 + a*x)^(3/2)*A*a/x^4 - 1/5*(b*x^2 + a*x)^(5/2)
*A/x^5
```

Giac [A] (verification not implemented)

Time = 76.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = -\frac{\left(\frac{15\,(5\,Bab^2+2\,Ab^3)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{\sqrt{b}}-\frac{\left(\left(\left(15\,(bx+a)Bb^4-\frac{23\,(5\,Ba^3b^6+2\,Aa^2b^7)}{a^2b^2}\right)(bx+a)+\frac{35\,(5\,Ba^4b^6+2\,Aa^3b^7)}{a^2b^2}\right)(bx \cdots\right)}{((bx+a)b-ab)^{\frac{5}{2}}}\right)}{15\,|b|}$$

input

```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(7/2),x, algorithm="giac")
```

output

```
-1/15*(15*(5*B*a*b^2 + 2*A*b^3)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x
 + a)*b - a*b)))/sqrt(b) - (((15*(b*x + a)*B*b^4 - 23*(5*B*a^3*b^6 + 2*A*a
^2*b^7)/(a^2*b^2))*(b*x + a) + 35*(5*B*a^4*b^6 + 2*A*a^3*b^7)/(a^2*b^2))*(
b*x + a) - 15*(5*B*a^5*b^6 + 2*A*a^4*b^7)/(a^2*b^2))*sqrt(b*x + a)/((b*x +
 a)*b - a*b)^(5/2))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = \int \frac{(A+B\,x)\,{(a+b\,x)}^{5/2}}{x^{7/2}}\,dx$$

input

```
int(((A + B*x)*(a + b*x)^(5/2))/x^(7/2),x)
```

output

```
int(((A + B*x)*(a + b*x)^(5/2))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{7/2}}\,dx = \frac{-24\sqrt{x}\,\sqrt{bx+a}\,a^3-128\sqrt{x}\,\sqrt{bx+a}\,a^2bx-464\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2+60\sqrt{x}}{60x^3}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(7/2),x)
```

output
```
( - 24*sqrt(x)*sqrt(a + b*x)*a**3 - 128*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 4
64*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 60*sqrt(x)*sqrt(a + b*x)*b**3*x**3
+ 420*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*x**3 +
 203*sqrt(b)*a*b**2*x**3)/(60*x**3)
```

3.312 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{2a^2B\sqrt{a+bx}}{5x^{5/2}} - \frac{22abB\sqrt{a+bx}}{15x^{3/2}} - \frac{46b^2B\sqrt{a+bx}}{15\sqrt{x}} - \frac{2A(a+bx)^{7/2}}{7ax^{7/2}} + 2b^{5/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

output
```
-2/5*a^2*B*(b*x+a)^(1/2)/x^(5/2)-22/15*a*b*B*(b*x+a)^(1/2)/x^(3/2)-46/15*b
^2*B*(b*x+a)^(1/2)/x^(1/2)-2/7*A*(b*x+a)^(7/2)/a/x^(7/2)+2*b^(5/2)*B*arcta
nh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{2\sqrt{a+bx}(15Ab^3x^3+3a^3(5A+7Bx)+a^2bx(45A+77Bx)+ab^2x^2(45A+161Bx))}{105ax^{7/2}} - 2b^{5/2}B\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(9/2),x]
```

output
```
(-2*Sqrt[a + b*x]*(15*A*b^3*x^3 + 3*a^3*(5*A + 7*B*x) + a^2*b*x*(45*A + 77
*B*x) + a*b^2*x^2*(45*A + 161*B*x)))/(105*a*x^(7/2)) - 2*b^(5/2)*B*Log[-(S
qrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 57, 57, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{(a+bx)^{5/2}}{x^{7/2}}dx-\frac{2A(a+bx)^{7/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{57}$$

$$B\left(b\int \frac{(a+bx)^{3/2}}{x^{5/2}}dx-\frac{2(a+bx)^{5/2}}{5x^{5/2}}\right)-\frac{2A(a+bx)^{7/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{57}$$

$$B\left(b\left(b\int \frac{\sqrt{a+bx}}{x^{3/2}}dx-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}\right)-\frac{2A(a+bx)^{7/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{57}$$

$$B\left(b\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}\right)-\frac{2A(a+bx)^{7/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{65}$$

$$B\left(b\left(b\left(2b\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}\right)-\frac{2A(a+bx)^{7/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 219}$$

$$B\left(b\left(b\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}\right)-\frac{2(a+bx)^{5/2}}{5x^{5/2}}\right)-\frac{2A(a+bx)^{7/2}}{7ax^{7/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(9/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(7*a*x^(7/2)) + B*((-2*(a + b*x)^(5/2))/(5*x^(5/2))
 + b*((-2*(a + b*x)^(3/2))/(3*x^(3/2)) + b*((-2*Sqrt[a + b*x])/Sqrt[x] + 2
*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{2\sqrt{bx+a}\,\left(15A\,b^{3}x^{3}+161Ba\,b^{2}x^{3}+45aA\,b^{2}x^{2}+77B\,a^{2}b\,x^{2}+45a^{2}Abx+21B\,a^{3}x+15a^{3}A\right)}{105x^{\frac{7}{2}}a}+\frac{b^{\frac{5}{2}}B\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(b}}{\sqrt{x}\,\sqrt{bx+a}}$
default	$-\frac{\sqrt{bx+a}\,\left(-105B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^{3}x^{4}+30A\sqrt{x(bx+a)}\,b^{\frac{7}{2}}x^{3}+322B\sqrt{x(bx+a)}\,b^{\frac{5}{2}}a\,x^{3}+90A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}a\,x^{2}+154}{105x^{\frac{7}{2}}a\sqrt{x(bx+a)}\,\sqrt{b}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x+a)^(1/2)*(15*A*b^3*x^3+161*B*a*b^2*x^3+45*A*a*b^2*x^2+77*B*a^2
*b*x^2+45*A*a^2*b*x+21*B*a^3*x+15*A*a^3)/x^(7/2)/a+b^(5/2)*B*ln((1/2*a+b*x
)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = \left[\frac{105\,Bab^{\frac{5}{2}}x^4\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(15\,Aa^3+(161\,Bab^2+1}{105\,a}\right.$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/105*(105*B*a*b^(5/2)*x^4*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
a) - 2*(15*A*a^3 + (161*B*a*b^2 + 15*A*b^3)*x^3 + (77*B*a^2*b + 45*A*a*b^2
)*x^2 + 3*(7*B*a^3 + 15*A*a^2*b)*x)*sqrt(b*x + a)*sqrt(x))/(a*x^4), -2/105
*(105*B*a*sqrt(-b)*b^2*x^4*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (15*A*
a^3 + (161*B*a*b^2 + 15*A*b^3)*x^3 + (77*B*a^2*b + 45*A*a*b^2)*x^2 + 3*(7*
B*a^3 + 15*A*a^2*b)*x)*sqrt(b*x + a)*sqrt(x))/(a*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(114) = 228.

Time = 14.21 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.97

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx &= -\frac{30Aa^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} \\ &- \frac{66Aa^6b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{34Aa^5b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} \\ &- \frac{6Aa^4b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{24Aa^3b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} \\ &- \frac{16Aa^2b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5} - \frac{4Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{14Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15x} \\ &- \frac{2Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{15a} - \frac{2B\sqrt{a}b^2}{\sqrt{x}\sqrt{1+\frac{bx}{a}}} - \frac{2Ba^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2} - \frac{22Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15x} \\ &- \frac{16Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15} + 2Bb^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2Bb^3\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}\end{aligned}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(9/2),x)
```

output
```
-30*A*a**7*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*
x**4 + 105*a**3*b**6*x**5) - 66*A*a**6*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*
a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*A*a**5*b**(
13/2)*x**2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 10
5*a**3*b**6*x**5) - 6*A*a**4*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b*
*4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*A*a**3*b**(17/2)*x
**4*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*
b**6*x**5) - 16*A*a**2*b**(19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**
3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 4*A*a*b**(3/2)*sqrt(a/(b*x)
 + 1)/(5*x**2) - 14*A*b**(5/2)*sqrt(a/(b*x) + 1)/(15*x) - 2*A*b**(7/2)*sqr
t(a/(b*x) + 1)/(15*a) - 2*B*sqrt(a)*b**2/(sqrt(x)*sqrt(1 + b*x/a)) - 2*B*a
**2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 22*B*a*b**(3/2)*sqrt(a/(b*x) + 1)
/(15*x) - 16*B*b**(5/2)*sqrt(a/(b*x) + 1)/15 + 2*B*b**(5/2)*asinh(sqrt(b)*
sqrt(x)/sqrt(a)) - 2*B*b**3*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(85) = 170$.

Time = 0.06 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.21

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = Bb^{\frac{5}{2}}\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right) - \frac{38\,\sqrt{bx^2+ax}Bb^2}{15\,x} - \frac{2\,\sqrt{bx^2+ax}Ab^3}{7\,ax} - \frac{7\,\sqrt{bx^2+ax}Bab}{30\,x^2} + \frac{\sqrt{bx^2+ax}Ab^2}{7\,x^2} + \frac{3\,\sqrt{bx^2+ax}Ba^2}{10\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}Bb}{3\,x^3} - \frac{3\,\sqrt{bx^2+ax}Aab}{28\,x^3} - \frac{(bx^2+ax)^{\frac{3}{2}}Ba}{2\,x^4} - \frac{15\,\sqrt{bx^2+ax}Aa^2}{28\,x^4} - \frac{(bx^2+ax)^{\frac{5}{2}}B}{5\,x^5} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{4\,x^5} - \frac{(bx^2+ax)^{\frac{5}{2}}A}{x^6}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(9/2),x, algorithm="maxima")
```

output
```
B*b^(5/2)*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)) - 38/15*sqrt(b*x^2
+ a*x)*B*b^2/x - 2/7*sqrt(b*x^2 + a*x)*A*b^3/(a*x) - 7/30*sqrt(b*x^2 + a*x
)*B*a*b/x^2 + 1/7*sqrt(b*x^2 + a*x)*A*b^2/x^2 + 3/10*sqrt(b*x^2 + a*x)*B*a
^2/x^3 - 1/3*(b*x^2 + a*x)^(3/2)*B*b/x^3 - 3/28*sqrt(b*x^2 + a*x)*A*a*b/x^
3 - 1/2*(b*x^2 + a*x)^(3/2)*B*a/x^4 - 15/28*sqrt(b*x^2 + a*x)*A*a^2/x^4 -
1/5*(b*x^2 + a*x)^(5/2)*B/x^5 + 5/4*(b*x^2 + a*x)^(3/2)*A*a/x^5 - (b*x^2 +
 a*x)^(5/2)*A/x^6
```

Giac [A] (verification not implemented)

Time = 75.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = -\frac{2\,b^5\left(\frac{105\,B\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}} - \frac{\left(105\,Ba^3b^2-\left(350\,Ba^2b^2-\left(406\,Bab^2-\frac{\left(161\,Ba^3b^3+15\,Aa^2b^4\right)(bx+a)}{a^3b}\right)(bx+a)\right)(bx+a)\right)\cdots}{((bx+a)b-ab)^{\frac{7}{2}}}\right)}{105\,|b|}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(9/2),x, algorithm="giac")
```

output
```
-2/105*b^5*(105*B*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)
))/b^(3/2) - (105*B*a^3*b^2 - (350*B*a^2*b^2 - (406*B*a*b^2 - (161*B*a^3*b
^3 + 15*A*a^2*b^4)*(b*x + a)/(a^3*b))*(b*x + a))*(b*x + a))*sqrt(b*x + a)/
((b*x + a)*b - a*b)^(7/2))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^{5/2}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(9/2),x)
```

output
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{9/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{44\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{244\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{105}-\frac{352\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{105}+2\sqrt{b}\,\mathrm{lo}}{x^4}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(9/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3 - 66*sqrt(x)*sqrt(a + b*x)*a**2*b*x -
 122*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 - 176*sqrt(x)*sqrt(a + b*x)*b**3*x*
*3 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**3*x**4
+ 56*sqrt(b)*b**3*x**4))/(105*x**4)
```

3.313 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{2A(a+bx)^{7/2}}{9ax^{9/2}} + \frac{2(2Ab-9aB)(a+bx)^{7/2}}{63a^2x^{7/2}}$$

output
```
-2/9*A*(b*x+a)^(7/2)/a/x^(9/2)+2/63*(2*A*b-9*B*a)*(b*x+a)^(7/2)/a^2/x^(7/2
)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{2(a+bx)^{7/2}(7aA-2Abx+9aBx)}{63a^2x^{9/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(11/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(7*a*A - 2*A*b*x + 9*a*B*x))/(63*a^2*x^(9/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}} \, dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(2Ab-9aB)\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2A(a+bx)^{7/2}}{9ax^{9/2}}$$

$$\Big\downarrow \; 48$$

$$\frac{2(a+bx)^{7/2}(2Ab-9aB)}{63a^2x^{7/2}}-\frac{2A(a+bx)^{7/2}}{9ax^{9/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(11/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(9*a*x^(9/2)) + (2*(2*A*b - 9*a*B)*(a + b*x)^(7/2))
/(63*a^2*x^(7/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}(-2Abx+9Bax+7Aa)}{63x^{\frac{9}{2}}a^{2}}$	31
orering	$-\frac{2(bx+a)^{\frac{7}{2}}(-2Abx+9Bax+7Aa)}{63x^{\frac{9}{2}}a^{2}}$	31
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-2A\,b^{3}x^{3}+9Ba\,b^{2}x^{3}+3aA\,b^{2}x^{2}+18B\,a^{2}b\,x^{2}+12a^{2}Abx+9B\,a^{3}x+7a^{3}A\right)}{63x^{\frac{9}{2}}a^{2}}$	77
risch	$-\frac{2\sqrt{bx+a}\,\left(-2A\,b^{4}x^{4}+9Ba\,b^{3}x^{4}+Aa\,b^{3}x^{3}+27B\,a^{2}b^{2}x^{3}+15A\,a^{2}b^{2}x^{2}+27B\,a^{3}b\,x^{2}+19A\,a^{3}bx+9B\,a^{4}x+7A\,a^{4}\right)}{63x^{\frac{9}{2}}a^{2}}$	100

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*(b*x+a)^(7/2)*(-2*A*b*x+9*B*a*x+7*A*a)/x^(9/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(41) = 82$.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx =$$
$$-\frac{2\,(7\,Aa^4+(9\,Bab^3-2\,Ab^4)x^4+(27\,Ba^2b^2+Aab^3)x^3+3\,(9\,Ba^3b+5\,Aa^2b^2)x^2+(9\,Ba^4+19\,Aa^3b)x)\sqrt{}}{63\,a^2x^{\frac{9}{2}}}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/63*(7*A*a^4 + (9*B*a*b^3 - 2*A*b^4)*x^4 + (27*B*a^2*b^2 + A*a*b^3)*x^3
+ 3*(9*B*a^3*b + 5*A*a^2*b^2)*x^2 + (9*B*a^4 + 19*A*a^3*b)*x)*sqrt(b*x + a
)/(a^2*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1442 vs. $2(49) = 98$.

Time = 39.10 (sec) , antiderivative size = 1442, normalized size of antiderivative = 27.21

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(11/2),x)
```

output
```
-70*A*a**9*b**(19/2)*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**1
0*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 220*A*a**8*b**(21/2)
*x*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*
b**11*x**6 + 315*a**4*b**12*x**7) - 228*A*a**7*b**(23/2)*x**2*sqrt(a/(b*x)
 + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 31
5*a**4*b**12*x**7) - 80*A*a**6*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(315*a**7*
b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**
7) - 60*A*a**6*b**(11/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*
b**5*x**4 + 105*a**3*b**6*x**5) + 10*A*a**5*b**(27/2)*x**4*sqrt(a/(b*x) +
1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a
**4*b**12*x**7) - 132*A*a**5*b**(13/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b**4*
x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 60*A*a**4*b**(29/2)*x**5
*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a**5*b*
*11*x**6 + 315*a**4*b**12*x**7) - 68*A*a**4*b**(15/2)*x**2*sqrt(a/(b*x) +
1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 80*A*a
**3*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*
x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 12*A*a**3*b**(17/2)*x*
*3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b
**6*x**5) + 32*A*a**2*b**(33/2)*x**7*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4
 + 945*a**6*b**10*x**5 + 945*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 258, normalized size of antiderivative = 4.87

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{2\sqrt{bx^2+ax}Bb^3}{7\,ax} + \frac{4\sqrt{bx^2+ax}Ab^4}{63\,a^2x} + \frac{\sqrt{bx^2+ax}Bb^2}{7\,x^2} - \frac{2\sqrt{bx^2+ax}Ab^3}{63\,ax^2} - \frac{3\sqrt{bx^2+ax}Bab}{28\,x^3} + \frac{\sqrt{bx^2+ax}Ab^2}{42\,x^3} - \frac{15\sqrt{bx^2+ax}Ba^2}{28\,x^4} - \frac{5\sqrt{bx^2+ax}Aab}{252\,x^4} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{4\,x^5} - \frac{5\sqrt{bx^2+ax}Aa^2}{36\,x^5} - \frac{(bx^2+ax)^{\frac{5}{2}}B}{x^6} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{12\,x^6} - \frac{(bx^2+ax)^{\frac{5}{2}}A}{2\,x^7}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(11/2),x, algorithm="maxima")
```

output
```
-2/7*sqrt(b*x^2 + a*x)*B*b^3/(a*x) + 4/63*sqrt(b*x^2 + a*x)*A*b^4/(a^2*x)
+ 1/7*sqrt(b*x^2 + a*x)*B*b^2/x^2 - 2/63*sqrt(b*x^2 + a*x)*A*b^3/(a*x^2) -
 3/28*sqrt(b*x^2 + a*x)*B*a*b/x^3 + 1/42*sqrt(b*x^2 + a*x)*A*b^2/x^3 - 15/
28*sqrt(b*x^2 + a*x)*B*a^2/x^4 - 5/252*sqrt(b*x^2 + a*x)*A*a*b/x^4 + 5/4*(
b*x^2 + a*x)^(3/2)*B*a/x^5 - 5/36*sqrt(b*x^2 + a*x)*A*a^2/x^5 - (b*x^2 + a
*x)^(5/2)*B/x^6 + 5/12*(b*x^2 + a*x)^(3/2)*A*a/x^6 - 1/2*(b*x^2 + a*x)^(5/
2)*A/x^7
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{2\,(bx+a)^{\frac{7}{2}}b\left(\frac{(9\,Ba^3b^8-2\,Aa^2b^9)(bx+a)}{a^4}-\frac{9\,(Ba^4b^8-Aa^3b^9)}{a^4}\right)}{63\,((bx+a)b-ab)^{\frac{9}{2}}|b|}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(11/2),x, algorithm="giac")
```

output
```
-2/63*(b*x + a)^(7/2)*b*((9*B*a^3*b^8 - 2*A*a^2*b^9)*(b*x + a)/a^4 - 9*(B*
a^4*b^8 - A*a^3*b^9)/a^4)/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A\,a^2}{9}+\frac{x\,(18\,B\,a^4+38\,A\,b\,a^3)}{63\,a^2}-\frac{x^4\,(4\,A\,b^4-18\,B\,a\,b^3)}{63\,a^2}+\frac{2\,b\,x^2\,(5\,A\,b+9\,B\,a)}{21}+\frac{2\,b^2\,x^3\,(A\,b+27\,B\,a)}{63\,a}\right)}{x^{9/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(11/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a^2)/9 + (x*(18*B*a^4 + 38*A*a^3*b))/(63*a^2) - (x
^4*(4*A*b^4 - 18*B*a*b^3))/(63*a^2) + (2*b*x^2*(5*A*b + 9*B*a))/21 + (2*b^
2*x^3*(A*b + 27*B*a))/(63*a)))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{11/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{9}-\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{3}-\frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^3}{9}-\frac{2\sqrt{x}\,\sqrt{bx+a}}{9}}{a\,x^5}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(11/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a**4 - 4*sqrt(x)*sqrt(a + b*x)*a**3*b*x - 6*s
qrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - 4*sqrt(x)*sqrt(a + b*x)*a*b**3*x**3
- sqrt(x)*sqrt(a + b*x)*b**4*x**4 - sqrt(b)*b**4*x**5))/(9*a*x**5)
```

3.314 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = -\frac{2A(a+bx)^{7/2}}{11ax^{11/2}} + \frac{2(4Ab-11aB)(a+bx)^{7/2}}{99a^2x^{9/2}} - \frac{4b(4Ab-11aB)(a+bx)^{7/2}}{693a^3x^{7/2}}$$

output
```
-2/11*A*(b*x+a)^(7/2)/a/x^(11/2)+2/99*(4*A*b-11*B*a)*(b*x+a)^(7/2)/a^2/x^(
9/2)-4/693*b*(4*A*b-11*B*a)*(b*x+a)^(7/2)/a^3/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = -\frac{2(a+bx)^{7/2}\left(63a^2A-28aAbx+77a^2Bx+8Ab^2x^2-22abBx^2\right)}{693a^3x^{11/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(13/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(63*a^2*A - 28*a*A*b*x + 77*a^2*B*x + 8*A*b^2*x^2 - 22
*a*b*B*x^2))/(693*a^3*x^(11/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(4Ab-11aB)\int \frac{(a+bx)^{5/2}}{x^{11/2}}dx}{11a}-\frac{2A(a+bx)^{7/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(4Ab-11aB)\left(-\frac{2b\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2A(a+bx)^{7/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)(4Ab-11aB)}{11a}-\frac{2A(a+bx)^{7/2}}{11ax^{11/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(13/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(11*a*x^(11/2)) - ((4*A*b - 11*a*B)*((-2*(a + b*x)^
(7/2))/(9*a*x^(9/2)) + (4*b*(a + b*x)^(7/2))/(63*a^2*x^(7/2))))/(11*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}} \left(8A\,b^{2}x^{2}-22Bab\,x^{2}-28aAbx+77B\,a^{2}x+63a^{2}A\right)}{693x^{\frac{11}{2}}a^{3}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}} \left(8A\,b^{2}x^{2}-22Bab\,x^{2}-28aAbx+77B\,a^{2}x+63a^{2}A\right)}{693x^{\frac{11}{2}}a^{3}}$
default	$-\frac{2(bx+a)^{\frac{3}{2}} \left(8A\,b^{4}x^{4}-22Ba\,b^{3}x^{4}-12Aa\,b^{3}x^{3}+33B\,a^{2}b^{2}x^{3}+15A\,a^{2}b^{2}x^{2}+132B\,a^{3}b\,x^{2}+98A\,a^{3}bx+77B\,a^{4}x+63A\,a^{4}\right)}{693x^{\frac{11}{2}}a^{3}}$
risch	$-\frac{2\sqrt{bx+a}\,\left(8A\,b^{5}x^{5}-22Ba\,b^{4}x^{5}-4aA\,b^{4}x^{4}+11B\,a^{2}b^{3}x^{4}+3a^{2}A\,b^{3}x^{3}+165B\,a^{3}b^{2}x^{3}+113a^{3}A\,b^{2}x^{2}+209B\,a^{4}b\,x^{2}+161a^{4}Abx+77B\right.}{693x^{\frac{11}{2}}a^{3}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*(b*x+a)^(7/2)*(8*A*b^2*x^2-22*B*a*b*x^2-28*A*a*b*x+77*B*a^2*x+63*A*
a^2)/x^(11/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = -\frac{2\,(63\,Aa^5 - 2\,(11\,Bab^4 - 4\,Ab^5)x^5 + (11\,Ba^2b^3 - 4\,Aab^4)x^4 + 3\,(55\,Ba^3b^2 + Aa^2b^3)x^3 + (209\,Ba^4b + 11}{693\,a^3x^{\frac{11}{2}}}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/693*(63*A*a^5 - 2*(11*B*a*b^4 - 4*A*b^5)*x^5 + (11*B*a^2*b^3 - 4*A*a*b^
4)*x^4 + 3*(55*B*a^3*b^2 + A*a^2*b^3)*x^3 + (209*B*a^4*b + 113*A*a^3*b^2)*
x^2 + 7*(11*B*a^5 + 23*A*a^4*b)*x)*sqrt(b*x + a)/(a^3*x^(11/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2790 vs. 2(82) = 164.

Time = 110.39 (sec) , antiderivative size = 2790, normalized size of antiderivative = 33.21

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(13/2),x)
```

output
```
-630*A*a**11*b**(33/2)*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**
8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b
**20*x**9) - 2590*A*a**10*b**(35/2)*x*sqrt(a/(b*x) + 1)/(3465*a**9*b**16*x
**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 13860*a**6*b**19*x**
8 + 3465*a**5*b**20*x**9) - 3980*A*a**9*b**(37/2)*x**2*sqrt(a/(b*x) + 1)/(
3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*x**7 + 138
60*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 2716*A*a**8*b**(39/2)*x**3*sq
rt(a/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7
*b**18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 140*A*a**8*b
**(21/2)*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945
*a**5*b**11*x**6 + 315*a**4*b**12*x**7) - 686*A*a**7*b**(41/2)*x**4*sqrt(a
/(b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**
18*x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 440*A*a**7*b**(2
3/2)*x*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a
**5*b**11*x**6 + 315*a**4*b**12*x**7) - 70*A*a**6*b**(43/2)*x**5*sqrt(a/(b
*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b**18*
x**7 + 13860*a**6*b**19*x**8 + 3465*a**5*b**20*x**9) - 456*A*a**6*b**(25/2
)*x**2*sqrt(a/(b*x) + 1)/(315*a**7*b**9*x**4 + 945*a**6*b**10*x**5 + 945*a
**5*b**11*x**6 + 315*a**4*b**12*x**7) - 560*A*a**5*b**(45/2)*x**6*sqrt(a/(
b*x) + 1)/(3465*a**9*b**16*x**5 + 13860*a**8*b**17*x**6 + 20790*a**7*b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(66) = 132$.

Time = 0.05 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = \frac{4\sqrt{bx^2+ax}Bb^4}{63\,a^2x} - \frac{16\sqrt{bx^2+ax}Ab^5}{693\,a^3x} - \frac{2\sqrt{bx^2+ax}Bb^3}{63\,ax^2} + \frac{8\sqrt{bx^2+ax}Ab^4}{693\,a^2x^2} + \frac{\sqrt{bx^2+ax}Bb^2}{42\,x^3} - \frac{2\sqrt{bx^2+ax}Ab^3}{231\,ax^3} - \frac{5\sqrt{bx^2+ax}Bab}{252\,x^4} + \frac{5\sqrt{bx^2+ax}Ab^2}{693\,x^4} - \frac{5\sqrt{bx^2+ax}Ba^2}{36\,x^5} - \frac{5\sqrt{bx^2+ax}Aab}{792\,x^5} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{12\,x^6} - \frac{5\sqrt{bx^2+ax}Aa^2}{88\,x^6} - \frac{(bx^2+ax)^{\frac{5}{2}}B}{2\,x^7} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{24\,x^7} - \frac{(bx^2+ax)^{\frac{5}{2}}A}{3\,x^8}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(13/2),x, algorithm="maxima")
```

output
```
4/63*sqrt(b*x^2 + a*x)*B*b^4/(a^2*x) - 16/693*sqrt(b*x^2 + a*x)*A*b^5/(a^3
*x) - 2/63*sqrt(b*x^2 + a*x)*B*b^3/(a*x^2) + 8/693*sqrt(b*x^2 + a*x)*A*b^4
/(a^2*x^2) + 1/42*sqrt(b*x^2 + a*x)*B*b^2/x^3 - 2/231*sqrt(b*x^2 + a*x)*A*
b^3/(a*x^3) - 5/252*sqrt(b*x^2 + a*x)*B*a*b/x^4 + 5/693*sqrt(b*x^2 + a*x)*
A*b^2/x^4 - 5/36*sqrt(b*x^2 + a*x)*B*a^2/x^5 - 5/792*sqrt(b*x^2 + a*x)*A*a
*b/x^5 + 5/12*(b*x^2 + a*x)^(3/2)*B*a/x^6 - 5/88*sqrt(b*x^2 + a*x)*A*a^2/x
^6 - 1/2*(b*x^2 + a*x)^(5/2)*B/x^7 + 5/24*(b*x^2 + a*x)^(3/2)*A*a/x^7 - 1/
3*(b*x^2 + a*x)^(5/2)*A/x^8
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = \frac{2\,(bx+a)^{\frac{7}{2}}\left((bx+a)\left(\frac{2\,(11\,Ba^3b^4-4\,Aa^2b^5)(bx+a)}{a^5}-\frac{11\,(11\,Ba^4b^4-4\,Aa^3b^5)}{a^5}\right)+\frac{99\,(Ba^5b}{}\right.}{693\,((bx+a)b-ab)^{\frac{11}{2}}\,|b|}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(13/2),x, algorithm="giac")
```

output
```
2/693*(b*x + a)^(7/2)*((b*x + a)*(2*(11*B*a^3*b^4 - 4*A*a^2*b^5)*(b*x + a)
/a^5 - 11*(11*B*a^4*b^4 - 4*A*a^3*b^5)/a^5) + 99*(B*a^5*b^4 - A*a^4*b^5)/a
^5)*b^7/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx =$$

$$-\frac{\sqrt{a+b\,x}\left(\frac{2\,A\,a^2}{11}+\frac{x\,(154\,B\,a^5+322\,A\,b\,a^4)}{693\,a^3}+\frac{x^5\,(16\,A\,b^5-44\,B\,a\,b^4)}{693\,a^3}+\frac{2\,b\,x^2\,(113\,A\,b+209\,B\,a)}{693}-\frac{2\,b^3\,x^4\,(4\,A\,b-11\,B\,a)}{693\,a^2}+\frac{2\,b^2\,x^3}{}\right.}{x^{11/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(13/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a^2)/11 + (x*(154*B*a^5 + 322*A*a^4*b))/(693*a^3)
+ (x^5*(16*A*b^5 - 44*B*a*b^4))/(693*a^3) + (2*b*x^2*(113*A*b + 209*B*a))/
693 - (2*b^3*x^4*(4*A*b - 11*B*a))/(693*a^2) + (2*b^2*x^3*(A*b + 55*B*a))/
(231*a)))/x^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{13/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^5}{11} - \frac{68\sqrt{x}\,\sqrt{bx+a}\,a^4bx}{99} - \frac{92\sqrt{x}\,\sqrt{bx+a}\,a^3b^2x^2}{99} - \frac{16\sqrt{x}\,\sqrt{bx+a}\,a^2b^3x^3}{33} - \frac{2\sqrt{x}\,\sqrt{b}}{}}{a^2x^6}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(13/2),x)
```

output
```
(2*( - 9*sqrt(x)*sqrt(a + b*x)*a**5 - 34*sqrt(x)*sqrt(a + b*x)*a**4*b*x -
46*sqrt(x)*sqrt(a + b*x)*a**3*b**2*x**2 - 24*sqrt(x)*sqrt(a + b*x)*a**2*b*
*3*x**3 - sqrt(x)*sqrt(a + b*x)*a*b**4*x**4 + 2*sqrt(x)*sqrt(a + b*x)*b**5
*x**5 - 2*sqrt(b)*b**5*x**6))/(99*a**2*x**6)
```

3.315 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx = -\frac{2A(a+bx)^{7/2}}{13ax^{13/2}}+\frac{2(6Ab-13aB)(a+bx)^{7/2}}{143a^2x^{11/2}} - \frac{8b(6Ab-13aB)(a+bx)^{7/2}}{1287a^3x^{9/2}}+\frac{16b^2(6Ab-13aB)(a+bx)^{7/2}}{9009a^4x^{7/2}}$$

output
```
-2/13*A*(b*x+a)^(7/2)/a/x^(13/2)+2/143*(6*A*b-13*B*a)*(b*x+a)^(7/2)/a^2/x^
(11/2)-8/1287*b*(6*A*b-13*B*a)*(b*x+a)^(7/2)/a^3/x^(9/2)+16/9009*b^2*(6*A*
b-13*B*a)*(b*x+a)^(7/2)/a^4/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx = -\frac{2(a+bx)^{7/2}\left(-48Ab^3x^3+63a^3(11A+13Bx)+8ab^2x^2(21A+13Bx)-14a^2bx(27A+26Bx)\right)}{9009a^4x^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(15/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(-48*A*b^3*x^3 + 63*a^3*(11*A + 13*B*x) + 8*a*b^2*x^2*
(21*A + 13*B*x) - 14*a^2*b*x*(27*A + 26*B*x)))/(9009*a^4*x^(13/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(6Ab-13aB)\int \frac{(a+bx)^{5/2}}{x^{13/2}}dx}{13a}-\frac{2A(a+bx)^{7/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(6Ab-13aB)\left(-\frac{4b\int \frac{(a+bx)^{5/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2A(a+bx)^{7/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(6Ab-13aB)\left(-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2A(a+bx)^{7/2}}{13ax^{13/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{4b\left(\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)(6Ab-13aB)}{13a}-\frac{2A(a+bx)^{7/2}}{13ax^{13/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(15/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(13*a*x^(13/2)) - ((6*A*b - 13*a*B)*((-2*(a + b*x)^
(7/2))/(11*a*x^(11/2)) - (4*b*((-2*(a + b*x)^(7/2))/(9*a*x^(9/2)) + (4*b*(
a + b*x)^(7/2))/(63*a^2*x^(7/2))))/(11*a)))/(13*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-48A\,b^{3}x^{3}+104Ba\,b^{2}x^{3}+168aA\,b^{2}x^{2}-364B\,a^{2}b\,x^{2}-378a^{2}Abx+819B\,a^{3}x+693a^{3}A\right)}{9009x^{\frac{13}{2}}a^{4}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-48A\,b^{3}x^{3}+104Ba\,b^{2}x^{3}+168aA\,b^{2}x^{2}-364B\,a^{2}b\,x^{2}-378a^{2}Abx+819B\,a^{3}x+693a^{3}A\right)}{9009x^{\frac{13}{2}}a^{4}}$
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-48A\,b^{5}x^{5}+104Ba\,b^{4}x^{5}+72aA\,b^{4}x^{4}-156B\,a^{2}b^{3}x^{4}-90a^{2}A\,b^{3}x^{3}+195B\,a^{3}b^{2}x^{3}+105a^{3}A\,b^{2}x^{2}+1274B\,a^{4}b\,x^{2}+1008\right.}{9009x^{\frac{13}{2}}a^{4}}$
risch	$-\frac{2\sqrt{bx+a}\left(-48A\,b^{6}x^{6}+104Ba\,b^{5}x^{6}+24Aa\,b^{5}x^{5}-52B\,a^{2}b^{4}x^{5}-18A\,a^{2}b^{4}x^{4}+39B\,a^{3}b^{3}x^{4}+15A\,a^{3}b^{3}x^{3}+1469B\,a^{4}b^{2}x^{3}+1113A\,a^{4}\right.}{9009x^{\frac{13}{2}}a^{4}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9009*(b*x+a)^(7/2)*(-48*A*b^3*x^3+104*B*a*b^2*x^3+168*A*a*b^2*x^2-364*B
*a^2*b*x^2-378*A*a^2*b*x+819*B*a^3*x+693*A*a^3)/x^(13/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx =$$
$$-\frac{2\,(693\,Aa^{6}+8\,(13\,Bab^{5}-6\,Ab^{6})x^{6}-4\,(13\,Ba^{2}b^{4}-6\,Aab^{5})x^{5}+3\,(13\,Ba^{3}b^{3}-6\,Aa^{2}b^{4})x^{4}+(1469\,Ba^{4}b}{9009\,a^{4}x^{\frac{13}{2}}}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(15/2),x, algorithm="fricas")
```

output
```
-2/9009*(693*A*a^6 + 8*(13*B*a*b^5 - 6*A*b^6)*x^6 - 4*(13*B*a^2*b^4 - 6*A*
a*b^5)*x^5 + 3*(13*B*a^3*b^3 - 6*A*a^2*b^4)*x^4 + (1469*B*a^4*b^2 + 15*A*a
^3*b^3)*x^3 + 7*(299*B*a^5*b + 159*A*a^4*b^2)*x^2 + 63*(13*B*a^6 + 27*A*a^
5*b)*x)*sqrt(b*x + a)/(a^4*x^(13/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(93) = 186.

Time = 0.05 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.99

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx &= -\frac{16\sqrt{bx^2+ax}Bb^5}{693\,a^3x} + \frac{32\sqrt{bx^2+ax}Ab^6}{3003\,a^4x} \\ &+ \frac{8\sqrt{bx^2+ax}Bb^4}{693\,a^2x^2} - \frac{16\sqrt{bx^2+ax}Ab^5}{3003\,a^3x^2} - \frac{2\sqrt{bx^2+ax}Bb^3}{231\,ax^3} + \frac{4\sqrt{bx^2+ax}Ab^4}{1001\,a^2x^3} \\ &+ \frac{5\sqrt{bx^2+ax}Bb^2}{693\,x^4} - \frac{10\sqrt{bx^2+ax}Ab^3}{3003\,ax^4} - \frac{5\sqrt{bx^2+ax}Bab}{792\,x^5} \\ &+ \frac{5\sqrt{bx^2+ax}Ab^2}{1716\,x^5} - \frac{5\sqrt{bx^2+ax}Ba^2}{88\,x^6} - \frac{3\sqrt{bx^2+ax}Aab}{1144\,x^6} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{24\,x^7} \\ &- \frac{3\sqrt{bx^2+ax}Aa^2}{104\,x^7} - \frac{(bx^2+ax)^{\frac{5}{2}}B}{3\,x^8} + \frac{(bx^2+ax)^{\frac{3}{2}}Aa}{8\,x^8} - \frac{(bx^2+ax)^{\frac{5}{2}}A}{4\,x^9}\end{aligned}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(15/2),x, algorithm="maxima")
```

output
```
-16/693*sqrt(b*x^2 + a*x)*B*b^5/(a^3*x) + 32/3003*sqrt(b*x^2 + a*x)*A*b^6/
(a^4*x) + 8/693*sqrt(b*x^2 + a*x)*B*b^4/(a^2*x^2) - 16/3003*sqrt(b*x^2 + a
*x)*A*b^5/(a^3*x^2) - 2/231*sqrt(b*x^2 + a*x)*B*b^3/(a*x^3) + 4/1001*sqrt(
b*x^2 + a*x)*A*b^4/(a^2*x^3) + 5/693*sqrt(b*x^2 + a*x)*B*b^2/x^4 - 10/3003
*sqrt(b*x^2 + a*x)*A*b^3/(a*x^4) - 5/792*sqrt(b*x^2 + a*x)*B*a*b/x^5 + 5/1
716*sqrt(b*x^2 + a*x)*A*b^2/x^5 - 5/88*sqrt(b*x^2 + a*x)*B*a^2/x^6 - 3/114
4*sqrt(b*x^2 + a*x)*A*a*b/x^6 + 5/24*(b*x^2 + a*x)^(3/2)*B*a/x^7 - 3/104*s
qrt(b*x^2 + a*x)*A*a^2/x^7 - 1/3*(b*x^2 + a*x)^(5/2)*B/x^8 + 1/8*(b*x^2 +
a*x)^(3/2)*A*a/x^8 - 1/4*(b*x^2 + a*x)^(5/2)*A/x^9
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx = -\frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(13\,Ba^3b^{12}-6\,Aa^2b^{13})(bx+a)}{a^6}-\frac{13\,(13\,Ba^4b^{12}-6\,Aa^3b^{13})}{a^6}\right)+\frac{143\,(13\,Ba^5b^{12}-6\,Aa^4b^{13})}{a^6}\right)-\frac{1287\,(\cdots}{}\right.}{9009\,((bx+a)b-ab)^{\frac{13}{2}}\,|b|}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(15/2),x, algorithm="giac")
```

output
```
-2/9009*((b*x + a)*(4*(b*x + a)*(2*(13*B*a^3*b^12 - 6*A*a^2*b^13)*(b*x + a
)/a^6 - 13*(13*B*a^4*b^12 - 6*A*a^3*b^13)/a^6) + 143*(13*B*a^5*b^12 - 6*A*
a^4*b^13)/a^6) - 1287*(B*a^6*b^12 - A*a^5*b^13)/a^6)*(b*x + a)^(7/2)*b/(((
b*x + a)*b - a*b)^(13/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A\,a^2}{13}+\frac{x\,(1638\,B\,a^6+3402\,A\,b\,a^5)}{9009\,a^4}-\frac{x^6\,(96\,A\,b^6-208\,B\,a\,b^5)}{9009\,a^4}+\frac{2\,b\,x^2\,(159\,A\,b+299\,B\,a)}{1287}-\frac{2\,b^3\,x^4\,(6\,A\,b-13\,B\,a)}{3003\,a^2}+\frac{8\,b\cdots}{}\right.}{x^{13/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(15/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a^2)/13 + (x*(1638*B*a^6 + 3402*A*a^5*b))/(9009*a^
4) - (x^6*(96*A*b^6 - 208*B*a*b^5))/(9009*a^4) + (2*b*x^2*(159*A*b + 299*B
*a))/1287 - (2*b^3*x^4*(6*A*b - 13*B*a))/(3003*a^2) + (8*b^4*x^5*(6*A*b -
13*B*a))/(9009*a^3) + (2*b^2*x^3*(15*A*b + 1469*B*a))/(9009*a)))/x^(13/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{15/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^6}{13} - \frac{80\sqrt{x}\,\sqrt{bx+a}\,a^5bx}{143} - \frac{916\sqrt{x}\,\sqrt{bx+a}\,a^4b^2x^2}{1287} - \frac{424\sqrt{x}\,\sqrt{bx+a}\,a^3b^3x^3}{1287} - \frac{2\sqrt{x}}{}}{a^3x^7}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(15/2),x)
```

output
```
(2*( - 99*sqrt(x)*sqrt(a + b*x)*a**6 - 360*sqrt(x)*sqrt(a + b*x)*a**5*b*x
- 458*sqrt(x)*sqrt(a + b*x)*a**4*b**2*x**2 - 212*sqrt(x)*sqrt(a + b*x)*a**
3*b**3*x**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b**4*x**4 + 4*sqrt(x)*sqrt(a +
b*x)*a*b**5*x**5 - 8*sqrt(x)*sqrt(a + b*x)*b**6*x**6 + 8*sqrt(b)*b**6*x**7
))/(1287*a**3*x**7)
```

3.316 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}} dx = -\frac{2A(a+bx)^{7/2}}{15ax^{15/2}} + \frac{2(8Ab-15aB)(a+bx)^{7/2}}{195a^2x^{13/2}} - \frac{4b(8Ab-15aB)(a+bx)^{7/2}}{715a^3x^{11/2}} + \frac{16b^2(8Ab-15aB)(a+bx)^{7/2}}{6435a^4x^{9/2}} - \frac{32b^3(8Ab-15aB)(a+bx)^{7/2}}{45045a^5x^{7/2}}$$

output
```
-2/15*A*(b*x+a)^(7/2)/a/x^(15/2)+2/195*(8*A*b-15*B*a)*(b*x+a)^(7/2)/a^2/x^
(13/2)-4/715*b*(8*A*b-15*B*a)*(b*x+a)^(7/2)/a^3/x^(11/2)+16/6435*b^2*(8*A*
b-15*B*a)*(b*x+a)^(7/2)/a^4/x^(9/2)-32/45045*b^3*(8*A*b-15*B*a)*(b*x+a)^(7
/2)/a^5/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx =$$
$$-\frac{2(a+bx)^{7/2}\left(128Ab^4x^4+168a^2b^2x^2(6A+5Bx)+231a^4(13A+15Bx)-16ab^3x^3(28A+15Bx)-42a^3\right.}{45045a^5x^{15/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(17/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(128*A*b^4*x^4 + 168*a^2*b^2*x^2*(6*A + 5*B*x) + 231*a
^4*(13*A + 15*B*x) - 16*a*b^3*x^3*(28*A + 15*B*x) - 42*a^3*b*x*(44*A + 45*
B*x)))/(45045*a^5*x^(15/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(8Ab-15aB)\int \frac{(a+bx)^{5/2}}{x^{15/2}}dx}{15a}-\frac{2A(a+bx)^{7/2}}{15ax^{15/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-15aB)\left(-\frac{6b\int \frac{(a+bx)^{5/2}}{x^{13/2}}dx}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2A(a+bx)^{7/2}}{15ax^{15/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-15aB)\left(-\frac{6b\left(-\frac{4b\int \frac{(a+bx)^{5/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2A(a+bx)^{7/2}}{15ax^{15/2}}$$

$\Big\downarrow$ 55

$$-\frac{(8Ab-15aB)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2A(a+bx)^{7/2}}{15ax^{15/2}}$$

$\Big\downarrow$ 48

$$-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)(8Ab-15aB)}{15a}-\frac{2A(a+bx)^{7/2}}{15ax^{15/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(17/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(15*a*x^(15/2)) - ((8*A*b - 15*a*B)*((-2*(a + b*x)^
(7/2))/(13*a*x^(13/2)) - (6*b*((-2*(a + b*x)^(7/2))/(11*a*x^(11/2)) - (4*b
*((-2*(a + b*x)^(7/2))/(9*a*x^(9/2)) + (4*b*(a + b*x)^(7/2))/(63*a^2*x^(7/
2))))/(11*a)))/(13*a)))/(15*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(128A\,b^{4}x^{4}-240Ba\,b^{3}x^{4}-448Aa\,b^{3}x^{3}+840B\,a^{2}b^{2}x^{3}+1008A\,a^{2}b^{2}x^{2}-1890B\,a^{3}b\,x^{2}-1848A\,a^{3}bx+3465B\,a^{4}x+3003A\right.}{45045x^{\frac{15}{2}}a^{5}}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(128A\,b^{4}x^{4}-240Ba\,b^{3}x^{4}-448Aa\,b^{3}x^{3}+840B\,a^{2}b^{2}x^{3}+1008A\,a^{2}b^{2}x^{2}-1890B\,a^{3}b\,x^{2}-1848A\,a^{3}bx+3465B\,a^{4}x+3003A\right.}{45045x^{\frac{15}{2}}a^{5}}$
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(128A\,b^{6}x^{6}-240Ba\,b^{5}x^{6}-192Aa\,b^{5}x^{5}+360B\,a^{2}b^{4}x^{5}+240A\,a^{2}b^{4}x^{4}-450B\,a^{3}b^{3}x^{4}-280A\,a^{3}b^{3}x^{3}+525B\,a^{4}b^{2}x^{3}+315\right.}{45045x^{\frac{15}{2}}a^{5}}$
risch	$-\frac{2\sqrt{bx+a}\left(128A\,b^{7}x^{7}-240Ba\,b^{6}x^{7}-64Aa\,b^{6}x^{6}+120B\,a^{2}b^{5}x^{6}+48A\,a^{2}b^{5}x^{5}-90B\,a^{3}b^{4}x^{5}-40A\,a^{3}b^{4}x^{4}+75B\,a^{4}b^{3}x^{4}+35A\,a^{4}b^{3}x^{3}\right.}{45045x^{\frac{15}{2}}a^{5}}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(17/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45045*(b*x+a)^(7/2)*(128*A*b^4*x^4-240*B*a*b^3*x^4-448*A*a*b^3*x^3+840*
B*a^2*b^2*x^3+1008*A*a^2*b^2*x^2-1890*B*a^3*b*x^2-1848*A*a^3*b*x+3465*B*a^
4*x+3003*A*a^4)/x^(15/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx =$$
$$-\frac{2\,(3003\,Aa^7 - 16\,(15\,Bab^6 - 8\,Ab^7)x^7 + 8\,(15\,Ba^2b^5 - 8\,Aab^6)x^6 - 6\,(15\,Ba^3b^4 - 8\,Aa^2b^5)x^5 + 5\,(15\,Ba}{45}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(17/2),x, algorithm="fricas")
```

output
```
-2/45045*(3003*A*a^7 - 16*(15*B*a*b^6 - 8*A*b^7)*x^7 + 8*(15*B*a^2*b^5 - 8
*A*a*b^6)*x^6 - 6*(15*B*a^3*b^4 - 8*A*a^2*b^5)*x^5 + 5*(15*B*a^4*b^3 - 8*A
*a^3*b^4)*x^4 + 35*(159*B*a^5*b^2 + A*a^4*b^3)*x^3 + 63*(135*B*a^6*b + 71*
A*a^5*b^2)*x^2 + 231*(15*B*a^7 + 31*A*a^6*b)*x)*sqrt(b*x + a)/(a^5*x^(15/2
))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(17/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(120) = 240$.

Time = 0.04 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.64

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx = \frac{32\,\sqrt{bx^2+ax}Bb^6}{3003\,a^4x} - \frac{256\,\sqrt{bx^2+ax}Ab^7}{45045\,a^5x} - \frac{16\,\sqrt{bx^2+ax}Bb^5}{3003\,a^3x^2} + \frac{128\,\sqrt{bx^2+ax}Ab^6}{45045\,a^4x^2} + \frac{4\,\sqrt{bx^2+ax}Bb^4}{1001\,a^2x^3} - \frac{32\,\sqrt{bx^2+ax}Ab^5}{15015\,a^3x^3} - \frac{10\,\sqrt{bx^2+ax}Bb^3}{3003\,ax^4} + \frac{16\,\sqrt{bx^2+ax}Ab^4}{9009\,a^2x^4} + \frac{5\,\sqrt{bx^2+ax}Bb^2}{1716\,x^5} - \frac{2\,\sqrt{bx^2+ax}Ab^3}{1287\,ax^5} - \frac{3\,\sqrt{bx^2+ax}Bab}{1144\,x^6} + \frac{\sqrt{bx^2+ax}Ab^2}{715\,x^6} - \frac{3\,\sqrt{bx^2+ax}Ba^2}{104\,x^7} - \frac{\sqrt{bx^2+ax}Aab}{780\,x^7} + \frac{(bx^2+ax)^{\frac{3}{2}}Ba}{8\,x^8} - \frac{\sqrt{bx^2+ax}Aa^2}{60\,x^8} - \frac{(bx^2+ax)^{\frac{5}{2}}B}{4\,x^9} + \frac{(bx^2+ax)^{\frac{3}{2}}Aa}{12\,x^9} - \frac{(bx^2+ax)^{\frac{5}{2}}A}{5\,x^{10}}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(17/2),x, algorithm="maxima")
```

output
```
32/3003*sqrt(b*x^2 + a*x)*B*b^6/(a^4*x) - 256/45045*sqrt(b*x^2 + a*x)*A*b^
7/(a^5*x) - 16/3003*sqrt(b*x^2 + a*x)*B*b^5/(a^3*x^2) + 128/45045*sqrt(b*x
^2 + a*x)*A*b^6/(a^4*x^2) + 4/1001*sqrt(b*x^2 + a*x)*B*b^4/(a^2*x^3) - 32/
15015*sqrt(b*x^2 + a*x)*A*b^5/(a^3*x^3) - 10/3003*sqrt(b*x^2 + a*x)*B*b^3/
(a*x^4) + 16/9009*sqrt(b*x^2 + a*x)*A*b^4/(a^2*x^4) + 5/1716*sqrt(b*x^2 +
a*x)*B*b^2/x^5 - 2/1287*sqrt(b*x^2 + a*x)*A*b^3/(a*x^5) - 3/1144*sqrt(b*x^
2 + a*x)*B*a*b/x^6 + 1/715*sqrt(b*x^2 + a*x)*A*b^2/x^6 - 3/104*sqrt(b*x^2
+ a*x)*B*a^2/x^7 - 1/780*sqrt(b*x^2 + a*x)*A*a*b/x^7 + 1/8*(b*x^2 + a*x)^(
3/2)*B*a/x^8 - 1/60*sqrt(b*x^2 + a*x)*A*a^2/x^8 - 1/4*(b*x^2 + a*x)^(5/2)*
B/x^9 + 1/12*(b*x^2 + a*x)^(3/2)*A*a/x^9 - 1/5*(b*x^2 + a*x)^(5/2)*A/x^10
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx = \frac{2\left(\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(15\,Ba^3b^6-8\,Aa^2b^7)(bx+a)}{a^7}-\frac{15\,(15\,Ba^4b^6-8\,Aa^3b^7)}{a^7}\right)+19\cdots\right.\right.\right.}{45045\,((bx\cdots}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(17/2),x, algorithm="giac")
```

output
```
2/45045*((2*(b*x + a)*(4*(b*x + a)*(2*(15*B*a^3*b^6 - 8*A*a^2*b^7)*(b*x +
a)/a^7 - 15*(15*B*a^4*b^6 - 8*A*a^3*b^7)/a^7) + 195*(15*B*a^5*b^6 - 8*A*a^
4*b^7)/a^7) - 715*(15*B*a^6*b^6 - 8*A*a^5*b^7)/a^7)*(b*x + a) + 6435*(B*a^
7*b^6 - A*a^6*b^7)/a^7)*(b*x + a)^(7/2)*b^9/(((b*x + a)*b - a*b)^(15/2)*ab
s(b))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A\,a^2}{15}+\frac{x^7\,(256\,A\,b^7-480\,B\,a\,b^6)}{45045\,a^5}+\frac{2\,a\,x\,(31\,A\,b+15\,B\,a)}{195}+\frac{2\,b\,x^2\,(71\,A\,b+135\,B\,a)}{715}-\frac{2\,b^3\,x^4\,(8\,A\,b-15\,B\,a)}{9009\,a^2}+\frac{4\,b^4\,x^5\,(\cdots}{15\cdots}\right.}{x^{15/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(17/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a^2)/15 + (x^7*(256*A*b^7 - 480*B*a*b^6))/(45045*a
^5) + (2*a*x*(31*A*b + 15*B*a))/195 + (2*b*x^2*(71*A*b + 135*B*a))/715 - (
2*b^3*x^4*(8*A*b - 15*B*a))/(9009*a^2) + (4*b^4*x^5*(8*A*b - 15*B*a))/(150
15*a^3) - (16*b^5*x^6*(8*A*b - 15*B*a))/(45045*a^4) + (2*b^2*x^3*(A*b + 15
9*B*a))/(1287*a)))/x^(15/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{17/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^7}{15} - \frac{92\sqrt{x}\,\sqrt{bx+a}\,a^6bx}{195} - \frac{412\sqrt{x}\,\sqrt{bx+a}\,a^5b^2x^2}{715} - \frac{320\sqrt{x}\,\sqrt{bx+a}\,a^4b^3x^3}{1287} - 2\sqrt{x}\,\cdots}{a^4x\cdots}$$

input

```
int((b*x+a)^(5/2)*(B*x+A)/x^(17/2),x)
```

output

```
(2*( - 429*sqrt(x)*sqrt(a + b*x)*a**7 - 1518*sqrt(x)*sqrt(a + b*x)*a**6*b*
x - 1854*sqrt(x)*sqrt(a + b*x)*a**5*b**2*x**2 - 800*sqrt(x)*sqrt(a + b*x)*
a**4*b**3*x**3 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b**4*x**4 + 6*sqrt(x)*sqrt(a
 + b*x)*a**2*b**5*x**5 - 8*sqrt(x)*sqrt(a + b*x)*a*b**6*x**6 + 16*sqrt(x)*
sqrt(a + b*x)*b**7*x**7 - 16*sqrt(b)*b**7*x**8))/(6435*a**4*x**8)
```

3.317 $\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx = -\frac{2A(a+bx)^{7/2}}{17ax^{17/2}} + \frac{2(10Ab-17aB)(a+bx)^{7/2}}{255a^2x^{15/2}} - \frac{16b(10Ab-17aB)(a+bx)^{7/2}}{3315a^3x^{13/2}} + \frac{32b^2(10Ab-17aB)(a+bx)^{7/2}}{12155a^4x^{11/2}} - \frac{128b^3(10Ab-17aB)(a+bx)^{7/2}}{109395a^5x^{9/2}} + \frac{256b^4(10Ab-17aB)(a+bx)^{7/2}}{765765a^6x^{7/2}}$$

output
```
-2/17*A*(b*x+a)^(7/2)/a/x^(17/2)+2/255*(10*A*b-17*B*a)*(b*x+a)^(7/2)/a^2/x
^(15/2)-16/3315*b*(10*A*b-17*B*a)*(b*x+a)^(7/2)/a^3/x^(13/2)+32/12155*b^2*
(10*A*b-17*B*a)*(b*x+a)^(7/2)/a^4/x^(11/2)-128/109395*b^3*(10*A*b-17*B*a)*
(b*x+a)^(7/2)/a^5/x^(9/2)+256/765765*b^4*(10*A*b-17*B*a)*(b*x+a)^(7/2)/a^6
/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx =$$
$$-\frac{2(a+bx)^{7/2}\left(-1280Ab^5x^5+3003a^5(15A+17Bx)+128ab^4x^4(35A+17Bx)-224a^2b^3x^3(45A+34Bx)\right.}{765765a^6x^{17/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x))/x^(19/2),x]
```

output
```
(-2*(a + b*x)^(7/2)*(-1280*A*b^5*x^5 + 3003*a^5*(15*A + 17*B*x) + 128*a*b^
4*x^4*(35*A + 17*B*x) - 224*a^2*b^3*x^3*(45*A + 34*B*x) + 336*a^3*b^2*x^2*
(55*A + 51*B*x) - 462*a^4*b*x*(65*A + 68*B*x)))/(765765*a^6*x^(17/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx$$
$$\Big\downarrow \; 87$$
$$-\frac{(10Ab-17aB)\int \frac{(a+bx)^{5/2}}{x^{17/2}}dx}{17a}-\frac{2A(a+bx)^{7/2}}{17ax^{17/2}}$$
$$\Big\downarrow \; 55$$
$$-\frac{(10Ab-17aB)\left(-\frac{8b\int \frac{(a+bx)^{5/2}}{x^{15/2}}dx}{15a}-\frac{2(a+bx)^{7/2}}{15ax^{15/2}}\right)}{17a}-\frac{2A(a+bx)^{7/2}}{17ax^{17/2}}$$
$$\Big\downarrow \; 55$$

$$-\frac{(10Ab-17aB)\left(-\frac{8b\left(-\frac{6b\int\frac{(a+bx)^{5/2}}{x^{13/2}}dx}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2(a+bx)^{7/2}}{15ax^{15/2}}\right)}{17a}-\frac{2A(a+bx)^{7/2}}{17ax^{17/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(10Ab-17aB)\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int\frac{(a+bx)^{5/2}}{x^{11/2}}dx}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2(a+bx)^{7/2}}{15ax^{15/2}}\right)}{17a}-\frac{2A(a+bx)^{7/2}}{17ax^{17/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(10Ab-17aB)\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{(a+bx)^{5/2}}{x^{9/2}}dx}{9a}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2(a+bx)^{7/2}}{15ax^{15/2}}\right)}{17a}-\frac{2A(a+bx)^{7/2}}{17ax^{17/2}}$$

$\downarrow$ 48

$$-\frac{\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{4b(a+bx)^{7/2}}{63a^2x^{7/2}}-\frac{2(a+bx)^{7/2}}{9ax^{9/2}}\right)}{11a}-\frac{2(a+bx)^{7/2}}{11ax^{11/2}}\right)}{13a}-\frac{2(a+bx)^{7/2}}{13ax^{13/2}}\right)}{15a}-\frac{2(a+bx)^{7/2}}{15ax^{15/2}}\right)(10Ab-17aB)}{17a}-\frac{2A(a+bx)^{7/2}}{17ax^{17/2}}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x))/x^(19/2),x]
```

output
```
(-2*A*(a + b*x)^(7/2))/(17*a*x^(17/2)) - ((10*A*b - 17*a*B)*((-2*(a + b*x)
^(7/2))/(15*a*x^(15/2)) - (8*b*((-2*(a + b*x)^(7/2))/(13*a*x^(13/2)) - (6*
b*((-2*(a + b*x)^(7/2))/(11*a*x^(11/2)) - (4*b*((-2*(a + b*x)^(7/2))/(9*a*
x^(9/2)) + (4*b*(a + b*x)^(7/2))/(63*a^2*x^(7/2))))/(11*a)))/(13*a)))/(15*
a)))/(17*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.68

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}(-1280A\,b^5x^5+2176Ba\,b^4x^5+4480aA\,b^4x^4-7616B\,a^2b^3x^4-10080a^2A\,b^3x^3+17136B\,a^3b^2x^3+18480a^3A\,b^2x^2-3141}{765765x^{\frac{17}{2}}a^6}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}(-1280A\,b^5x^5+2176Ba\,b^4x^5+4480aA\,b^4x^4-7616B\,a^2b^3x^4-10080a^2A\,b^3x^3+17136B\,a^3b^2x^3+18480a^3A\,b^2x^2-3141}{765765x^{\frac{17}{2}}a^6}$
default	$-\frac{2(bx+a)^{\frac{3}{2}}(-1280A\,b^7x^7+2176Ba\,b^6x^7+1920Aa\,b^6x^6-3264B\,a^2b^5x^6-2400A\,a^2b^5x^5+4080B\,a^3b^4x^5+2800A\,a^3b^4x^4-4760B\,a}{765765x^{\frac{17}{2}}a^6}$
risch	$-\frac{2\sqrt{bx+a}\,(-1280A\,b^8x^8+2176Ba\,b^7x^8+640Aa\,b^7x^7-1088B\,a^2b^6x^7-480A\,a^2b^6x^6+816B\,a^3b^5x^6+400A\,a^3b^5x^5-680B\,a^4b^4x^5-}{76}$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(19/2),x,method=_RETURNVERBOSE)
```

output
```
-2/765765*(b*x+a)^(7/2)*(-1280*A*b^5*x^5+2176*B*a*b^4*x^5+4480*A*a*b^4*x^4
-7616*B*a^2*b^3*x^4-10080*A*a^2*b^3*x^3+17136*B*a^3*b^2*x^3+18480*A*a^3*b^
2*x^2-31416*B*a^4*b*x^2-30030*A*a^4*b*x+51051*B*a^5*x+45045*A*a^5)/x^(17/2
)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx =$$

$$-\frac{2\,(45045\,Aa^8+128\,(17\,Bab^7-10\,Ab^8)x^8-64\,(17\,Ba^2b^6-10\,Aab^7)x^7+48\,(17\,Ba^3b^5-10\,Aa^2b^6)x^6-4}{}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(19/2),x, algorithm="fricas")
```

output
```
-2/765765*(45045*A*a^8 + 128*(17*B*a*b^7 - 10*A*b^8)*x^8 - 64*(17*B*a^2*b^
6 - 10*A*a*b^7)*x^7 + 48*(17*B*a^3*b^5 - 10*A*a^2*b^6)*x^6 - 40*(17*B*a^4*
b^4 - 10*A*a^3*b^5)*x^5 + 35*(17*B*a^5*b^3 - 10*A*a^4*b^4)*x^4 + 63*(1207*
B*a^6*b^2 + 5*A*a^5*b^3)*x^3 + 231*(527*B*a^7*b + 275*A*a^6*b^2)*x^2 + 300
3*(17*B*a^8 + 35*A*a^7*b)*x)*sqrt(b*x + a)/(a^6*x^(17/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(B*x+A)/x**(19/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(147) = 294$.

Time = 0.04 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx = &-\frac{256\,\sqrt{bx^2+ax}Bb^7}{45045\,a^5x}+\frac{512\,\sqrt{bx^2+ax}Ab^8}{153153\,a^6x}\\&+\frac{128\,\sqrt{bx^2+ax}Bb^6}{45045\,a^4x^2}-\frac{256\,\sqrt{bx^2+ax}Ab^7}{153153\,a^5x^2}-\frac{32\,\sqrt{bx^2+ax}Bb^5}{15015\,a^3x^3}\\&+\frac{64\,\sqrt{bx^2+ax}Ab^6}{51051\,a^4x^3}+\frac{16\,\sqrt{bx^2+ax}Bb^4}{9009\,a^2x^4}-\frac{160\,\sqrt{bx^2+ax}Ab^5}{153153\,a^3x^4}\\&-\frac{2\,\sqrt{bx^2+ax}Bb^3}{1287\,ax^5}+\frac{20\,\sqrt{bx^2+ax}Ab^4}{21879\,a^2x^5}+\frac{\sqrt{bx^2+ax}Bb^2}{715\,x^6}\\&-\frac{2\,\sqrt{bx^2+ax}Ab^3}{2431\,ax^6}-\frac{\sqrt{bx^2+ax}Bab}{780\,x^7}+\frac{\sqrt{bx^2+ax}Ab^2}{1326\,x^7}\\&-\frac{\sqrt{bx^2+ax}Ba^2}{60\,x^8}-\frac{\sqrt{bx^2+ax}Aab}{1428\,x^8}+\frac{(bx^2+ax)^{\frac{3}{2}}Ba}{12\,x^9}\\&-\frac{5\,\sqrt{bx^2+ax}Aa^2}{476\,x^9}-\frac{(bx^2+ax)^{\frac{5}{2}}B}{5\,x^{10}}+\frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{84\,x^{10}}-\frac{(bx^2+ax)^{\frac{5}{2}}A}{6\,x^{11}}\end{aligned}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(19/2),x, algorithm="maxima")
```

output
```
-256/45045*sqrt(b*x^2 + a*x)*B*b^7/(a^5*x) + 512/153153*sqrt(b*x^2 + a*x)*
A*b^8/(a^6*x) + 128/45045*sqrt(b*x^2 + a*x)*B*b^6/(a^4*x^2) - 256/153153*s
qrt(b*x^2 + a*x)*A*b^7/(a^5*x^2) - 32/15015*sqrt(b*x^2 + a*x)*B*b^5/(a^3*x
^3) + 64/51051*sqrt(b*x^2 + a*x)*A*b^6/(a^4*x^3) + 16/9009*sqrt(b*x^2 + a*
x)*B*b^4/(a^2*x^4) - 160/153153*sqrt(b*x^2 + a*x)*A*b^5/(a^3*x^4) - 2/1287
*sqrt(b*x^2 + a*x)*B*b^3/(a*x^5) + 20/21879*sqrt(b*x^2 + a*x)*A*b^4/(a^2*x
^5) + 1/715*sqrt(b*x^2 + a*x)*B*b^2/x^6 - 2/2431*sqrt(b*x^2 + a*x)*A*b^3/(
a*x^6) - 1/780*sqrt(b*x^2 + a*x)*B*a*b/x^7 + 1/1326*sqrt(b*x^2 + a*x)*A*b^
2/x^7 - 1/60*sqrt(b*x^2 + a*x)*B*a^2/x^8 - 1/1428*sqrt(b*x^2 + a*x)*A*a*b/
x^8 + 1/12*(b*x^2 + a*x)^(3/2)*B*a/x^9 - 5/476*sqrt(b*x^2 + a*x)*A*a^2/x^9
 - 1/5*(b*x^2 + a*x)^(5/2)*B/x^10 + 5/84*(b*x^2 + a*x)^(3/2)*A*a/x^10 - 1/
6*(b*x^2 + a*x)^(5/2)*A/x^11
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx = -\frac{2\left(\left(8\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(17\,Ba^3b^{16}-10\,Aa^2b^{17})(bx+a)}{a^8}-\frac{17\,(17\,Ba^4b^{16}-10\,Aa^3b^{17})}{a^8}\right)+\frac{255\,(17\,Ba^5b^{16}-10\,Aa^4b^{17})}{a^8}\right)\right.\right.\right.}{765765\,((bx+}$$

input
```
integrate((b*x+a)^(5/2)*(B*x+A)/x^(19/2),x, algorithm="giac")
```

output
```
-2/765765*((8*(2*(b*x + a)*(4*(b*x + a)*(2*(17*B*a^3*b^16 - 10*A*a^2*b^17)
*(b*x + a)/a^8 - 17*(17*B*a^4*b^16 - 10*A*a^3*b^17)/a^8) + 255*(17*B*a^5*b
^16 - 10*A*a^4*b^17)/a^8) - 1105*(17*B*a^6*b^16 - 10*A*a^5*b^17)/a^8)*(b*x
 + a) + 12155*(17*B*a^7*b^16 - 10*A*a^6*b^17)/a^8)*(b*x + a) - 109395*(B*a
^8*b^16 - A*a^7*b^17)/a^8)*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b)^(17/2)*a
bs(b))
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A\,a^2}{17}+\frac{2\,a\,x\,(35\,A\,b+17\,B\,a)}{255}+\frac{2\,b\,x^2\,(275\,A\,b+527\,B\,a)}{3315}-\frac{2\,b^3\,x^4\,(10\,A\,b-17\,B\,a)}{21879\,a^2}+\frac{16\,b^4\,x^5\,(10\,A\,b-17\,B\,a)}{153153\,a^3}-\frac{32\,b^5\,x^6}{}\right.}{x^{17/2}}$$

input
```
int(((A + B*x)*(a + b*x)^(5/2))/x^(19/2),x)
```

output
```
-((a + b*x)^(1/2)*((2*A*a^2)/17 + (2*a*x*(35*A*b + 17*B*a))/255 + (2*b*x^2
*(275*A*b + 527*B*a))/3315 - (2*b^3*x^4*(10*A*b - 17*B*a))/(21879*a^2) + (
16*b^4*x^5*(10*A*b - 17*B*a))/(153153*a^3) - (32*b^5*x^6*(10*A*b - 17*B*a)
)/(255255*a^4) + (128*b^6*x^7*(10*A*b - 17*B*a))/(765765*a^5) - (256*b^7*x
^8*(10*A*b - 17*B*a))/(765765*a^6) + (2*b^2*x^3*(5*A*b + 1207*B*a))/(12155
*a)))/x^(17/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^{5/2}(A+Bx)}{x^{19/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^8}{17}-\frac{104\sqrt{x}\,\sqrt{bx+a}\,a^7bx}{255}-\frac{1604\sqrt{x}\,\sqrt{bx+a}\,a^6b^2x^2}{3315}-\frac{2424\sqrt{x}\,\sqrt{bx+a}\,a^5b^3x^3}{12155}-\ldots}{}$$

input
```
int((b*x+a)^(5/2)*(B*x+A)/x^(19/2),x)
```

output
```
(2*( - 6435*sqrt(x)*sqrt(a + b*x)*a**8 - 22308*sqrt(x)*sqrt(a + b*x)*a**7*
b*x - 26466*sqrt(x)*sqrt(a + b*x)*a**6*b**2*x**2 - 10908*sqrt(x)*sqrt(a +
b*x)*a**5*b**3*x**3 - 35*sqrt(x)*sqrt(a + b*x)*a**4*b**4*x**4 + 40*sqrt(x)
*sqrt(a + b*x)*a**3*b**5*x**5 - 48*sqrt(x)*sqrt(a + b*x)*a**2*b**6*x**6 +
64*sqrt(x)*sqrt(a + b*x)*a*b**7*x**7 - 128*sqrt(x)*sqrt(a + b*x)*b**8*x**8
 + 128*sqrt(b)*b**8*x**9))/(109395*a**5*x**9)
```

3.318 $\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{5a^2(8Ab-7aB)\sqrt{x}\sqrt{a+bx}}{64b^4} - \frac{5a(8Ab-7aB)x^{3/2}\sqrt{a+bx}}{96b^3} + \frac{(8Ab-7aB)x^{5/2}\sqrt{a+bx}}{24b^2} + \frac{Bx^{7/2}\sqrt{a+bx}}{4b} - \frac{5a^3(8Ab-7aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{9/2}}$$

output
```
5/64*a^2*(8*A*b-7*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^4-5/96*a*(8*A*b-7*B*a)*x^(3
/2)*(b*x+a)^(1/2)/b^3+1/24*(8*A*b-7*B*a)*x^(5/2)*(b*x+a)^(1/2)/b^2+1/4*B*x
^(7/2)*(b*x+a)^(1/2)/b-5/64*a^3*(8*A*b-7*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x
+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.79

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\sqrt{a+bx}(-105a^3B+16b^3x^2(4A+3Bx)-8ab^2x(10A+7Bx)+10a^2b(12A+7}{192b^4} + \frac{5a^3(-8Ab+7aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{32b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-105*a^3*B + 16*b^3*x^2*(4*A + 3*B*x) - 8*a*b^2*x*
(10*A + 7*B*x) + 10*a^2*b*(12*A + 7*B*x)))/(192*b^4) + (5*a^3*(-8*A*b + 7*
a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(32*b^(9/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {90, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(8Ab-7aB)\int \frac{x^{5/2}}{\sqrt{a+bx}}dx}{8b}+\frac{Bx^{7/2}\sqrt{a+bx}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)}{8b}+\frac{Bx^{7/2}\sqrt{a+bx}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)}{8b}+\frac{Bx^{7/2}\sqrt{a+bx}}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(8Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{Bx^{7/2}\sqrt{a+bx}}{4b}$$

$\downarrow$ 65

$$\frac{(8Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{Bx^{7/2}\sqrt{a+bx}}{4b}$$

$\downarrow$ 219

$$\frac{(8Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{Bx^{7/2}\sqrt{a+bx}}{4b}$$

input
```
Int[(x^(5/2)*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(B*x^(7/2)*Sqrt[a + b*x])/(4*b) + ((8*A*b - 7*a*B)*((x^(5/2)*Sqrt[a + b*x]
)/(3*b) - (5*a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*
x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/(6*
b)))/(8*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

method	result
risch	$\frac{\left(48 b^{3} B \,x^{3}+64 A \,x^{2} b^{3}-56 B \,x^{2} a \,b^{2}-80 A x a \,b^{2}+70 B x \,a^{2} b +120 a^{2} b A -105 a^{3} B \right) \sqrt{x}\, \sqrt{b x +a}}{192 b^{4}}-\frac{5 a^{3} \left(8 A b -7 B a \right) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a}\right.}{128 b^{\frac{9}{2}} \sqrt{x}\, \sqrt{b x +a}}$
default	$-\frac{\sqrt{x}\, \sqrt{b x +a}\, \left(-96 B \,b^{\frac{7}{2}} x^{3} \sqrt{x \left(b x +a \right)}-128 A \,b^{\frac{7}{2}} x^{2} \sqrt{x \left(b x +a \right)}+112 B a \,b^{\frac{5}{2}} x^{2} \sqrt{x \left(b x +a \right)}+160 A \sqrt{x \left(b x +a \right)}\, b^{\frac{5}{2}} a x -140 B \sqrt{x \left(b x +a \right)}\right.}{384 b^{\frac{9}{2}}}$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*B*b^3*x^3+64*A*b^3*x^2-56*B*a*b^2*x^2-80*A*a*b^2*x+70*B*a^2*b*x+
120*A*a^2*b-105*B*a^3)*x^(1/2)*(b*x+a)^(1/2)/b^4-5/128*a^3*(8*A*b-7*B*a)/b
^(9/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)
/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.55

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \left[-\frac{15\,(7\,Ba^4-8\,Aa^3b)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(48\,Bb^4x^3-105\,B}{384}\right.$$

$$-\frac{15\,(7\,Ba^4-8\,Aa^3b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(48\,Bb^4x^3-105\,Ba^3b+120\,Aa^2b^2-8\,(7\,Bab^3-8\,Ab^4)x^2+}{192\,b^5}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(7*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt
(b)*sqrt(x) + a) - 2*(48*B*b^4*x^3 - 105*B*a^3*b + 120*A*a^2*b^2 - 8*(7*B*
a*b^3 - 8*A*b^4)*x^2 + 10*(7*B*a^2*b^2 - 8*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(
x))/b^5, -1/192*(15*(7*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)
/sqrt(b*x + a)) - (48*B*b^4*x^3 - 105*B*a^3*b + 120*A*a^2*b^2 - 8*(7*B*a*b
^3 - 8*A*b^4)*x^2 + 10*(7*B*a^2*b^2 - 8*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))
/b^5]
```

Sympy [A] (verification not implemented)

Time = 50.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.91

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{5Aa^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1+\frac{bx}{a}}} + \frac{5Aa^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx}{a}}} - \frac{A\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}} - \frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{Ax^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}} - \frac{35Ba^{\frac{7}{2}}\sqrt{x}}{64b^4\sqrt{1+\frac{bx}{a}}} - \frac{35Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^3\sqrt{1+\frac{bx}{a}}} + \frac{7Ba^{\frac{3}{2}}x^{\frac{5}{2}}}{96b^2\sqrt{1+\frac{bx}{a}}} - \frac{B\sqrt{a}x^{\frac{7}{2}}}{24b\sqrt{1+\frac{bx}{a}}} + \frac{35Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{9}{2}}} + \frac{Bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
5*A*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*A*a**(3/2)*x**(3/2)/(24*
b**2*sqrt(1 + b*x/a)) - A*sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*A*a*
*3*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + A*x**(7/2)/(3*sqrt(a)*sqr
t(1 + b*x/a)) - 35*B*a**(7/2)*sqrt(x)/(64*b**4*sqrt(1 + b*x/a)) - 35*B*a**
(5/2)*x**(3/2)/(192*b**3*sqrt(1 + b*x/a)) + 7*B*a**(3/2)*x**(5/2)/(96*b**2
*sqrt(1 + b*x/a)) - B*sqrt(a)*x**(7/2)/(24*b*sqrt(1 + b*x/a)) + 35*B*a**4*
asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(9/2)) + B*x**(9/2)/(4*sqrt(a)*sqrt(
1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{bx^2+ax}Bx^3}{4\,b} - \frac{7\sqrt{bx^2+ax}Bax^2}{24\,b^2} + \frac{\sqrt{bx^2+ax}Ax^2}{3\,b} + \frac{35\sqrt{bx^2+ax}Ba^2x}{96\,b^3} - \frac{5\sqrt{bx^2+ax}Aax}{12\,b^2} + \frac{35\,Ba^4\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{128\,b^{\frac{9}{2}}} - \frac{5\,Aa^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} - \frac{35\sqrt{bx^2+ax}Ba^3}{64\,b^4} + \frac{5\sqrt{bx^2+ax}Aa^2}{8\,b^3}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a*x)*B*x^3/b - 7/24*sqrt(b*x^2 + a*x)*B*a*x^2/b^2 + 1/3*s
qrt(b*x^2 + a*x)*A*x^2/b + 35/96*sqrt(b*x^2 + a*x)*B*a^2*x/b^3 - 5/12*sqrt
(b*x^2 + a*x)*A*a*x/b^2 + 35/128*B*a^4*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)
*sqrt(b))/b^(9/2) - 5/16*A*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b)
)/b^(7/2) - 35/64*sqrt(b*x^2 + a*x)*B*a^3/b^4 + 5/8*sqrt(b*x^2 + a*x)*A*a^
2/b^3
```

Giac [A] (verification not implemented)

Time = 150.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.33

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx =$$
$$-\frac{\frac{\left(\frac{105\,a^4\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{5}{2}}}-\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)}{b^3}-\frac{25\,a}{b^3}\right)+\frac{163\,a^2}{b^3}\right)-\frac{279\,a^3}{b^3}\right)\sqrt{(bx+a)b-ab}\sqrt{bx+a}\right)B|b|}{b^2}-\frac{8\left(15\,a^3\sqrt{b}\cdots\right.}{\cdots}}{192\,b}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/192*((105*a^4*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))
)/b^(5/2) - (2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 - 25*a/b^3) + 163*a
^2/b^3) - 279*a^3/b^3)*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a))*B*abs(b)/b^2
 - 8*(15*a^3*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a
*b))) + (2*(4*b*x - 9*a)*(b*x + a) + 33*a^2)*sqrt((b*x + a)*b - a*b)*sqrt(
b*x + a))*A*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{\sqrt{a+b\,x}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{15\sqrt{x}\,\sqrt{bx+a}\,a^3b - 10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x + 8\sqrt{x}\,\sqrt{bx+a}\,a\,b^3x^2 + 48\sqrt{x}\,\sqrt{bx+a}\,b}{192b^4}$$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**3*b - 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x +
8*sqrt(x)*sqrt(a + b*x)*a*b**3*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*x**3 -
 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4)/(192*b**4
)
```

3.319 $\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = -\frac{a(6Ab-5aB)\sqrt{x}\sqrt{a+bx}}{8b^3}+\frac{(6Ab-5aB)x^{3/2}\sqrt{a+bx}}{12b^2}+\frac{Bx^{5/2}\sqrt{a+bx}}{3b}+\frac{a^2(6Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

output
```
-1/8*a*(6*A*b-5*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+1/12*(6*A*b-5*B*a)*x^(3/2)*
(b*x+a)^(1/2)/b^2+1/3*B*x^(5/2)*(b*x+a)^(1/2)/b+1/8*a^2*(6*A*b-5*B*a)*arct
anh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\sqrt{a+bx}(-18aAb+15a^2B+12Ab^2x-10abBx+8b^2Bx^2)}{24b^3}-\frac{a^2(-6Ab+5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(-18*a*A*b + 15*a^2*B + 12*A*b^2*x - 10*a*b*B*x + 8
*b^2*B*x^2))/(24*b^3) - (a^2*(-6*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-
Sqrt[a] + Sqrt[a + b*x])])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {90, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(6Ab-5aB)\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}+\frac{Bx^{5/2}\sqrt{a+bx}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(6Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}+\frac{Bx^{5/2}\sqrt{a+bx}}{3b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(6Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{Bx^{5/2}\sqrt{a+bx}}{3b}$$

$$\Big\downarrow \text{65}$$

$$\frac{(6Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}+\frac{Bx^{5/2}\sqrt{a+bx}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}+\frac{Bx^{5/2}\sqrt{a+bx}}{3b}$$

input
```
Int[(x^(3/2)*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(B*x^(5/2)*Sqrt[a + b*x])/(3*b) + ((6*A*b - 5*a*B)*((x^(3/2)*Sqrt[a + b*x]
)/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/S
qrt[a + b*x]])/b^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\left(-8 b^{2} B \,x^{2}-12 A \,b^{2} x +10 B a b x +18 a b A -15 a^{2} B \right) \sqrt{x}\, \sqrt{b x +a}}{24 b^{3}}+\frac{a^{2} \left(6 A b -5 B a \right) \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b \,x^{2}+a x}\right) \sqrt{x \left(b x +a \right)}}{16 b^{\frac{7}{2}} \sqrt{x}\, \sqrt{b x +a}}$
default	$\frac{\sqrt{x}\, \sqrt{b x +a}\, \left(16 B \,b^{\frac{5}{2}} x^{2} \sqrt{x \left(b x +a \right)}+24 A \sqrt{x \left(b x +a \right)}\, b^{\frac{5}{2}} x -20 B \sqrt{x \left(b x +a \right)}\, b^{\frac{3}{2}} a x +18 A \ln \left(\frac{2 \sqrt{x \left(b x +a \right)}\, \sqrt{b}+2 b x +a}{2 \sqrt{b}}\right) a^{2} b -36 A \sqrt{x (bx+}}{48 b^{\frac{7}{2}} \sqrt{x \left(b x +a \right)}}$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*B*b^2*x^2-12*A*b^2*x+10*B*a*b*x+18*A*a*b-15*B*a^2)*x^(1/2)*(b*x+
a)^(1/2)/b^3+1/16*a^2*(6*A*b-5*B*a)/b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+
a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.56

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \left[-\frac{3\,(5\,Ba^3-6\,Aa^2b)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(8\,Bb^3x^2+15\,Ba^2b}{48\,b^4}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b
)*sqrt(x) + a) - 2*(8*B*b^3*x^2 + 15*B*a^2*b - 18*A*a*b^2 - 2*(5*B*a*b^2 -
 6*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(3*(5*B*a^3 - 6*A*a^2*b)*sqr
t(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (8*B*b^3*x^2 + 15*B*a^2*b -
 18*A*a*b^2 - 2*(5*B*a*b^2 - 6*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(117) = 234.

Time = 10.98 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.94

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = -\frac{3Aa^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1+\frac{bx}{a}}} - \frac{A\sqrt{a}x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}} + \frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{Ax^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}} + \frac{5Ba^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1+\frac{bx}{a}}} + \frac{5Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx}{a}}} - \frac{B\sqrt{a}x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}} - \frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{Bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
-3*A*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - A*sqrt(a)*x**(3/2)/(4*b*s
qrt(1 + b*x/a)) + 3*A*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + A
*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a)) + 5*B*a**(5/2)*sqrt(x)/(8*b**3*sqrt(
1 + b*x/a)) + 5*B*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 + b*x/a)) - B*sqrt(a)*
x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*B*a**3*asinh(sqrt(b)*sqrt(x)/sqrt(a))/
(8*b**(7/2)) + B*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{bx^2+ax}Bx^2}{3\,b} - \frac{5\,\sqrt{bx^2+ax}Bax}{12\,b^2} + \frac{\sqrt{bx^2+ax}Ax}{2\,b} - \frac{5\,Ba^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Aa^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{5}{2}}} + \frac{5\,\sqrt{bx^2+ax}Ba^2}{8\,b^3} - \frac{3\,\sqrt{bx^2+ax}Aa}{4\,b^2}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a*x)*B*x^2/b - 5/12*sqrt(b*x^2 + a*x)*B*a*x/b^2 + 1/2*sqr
t(b*x^2 + a*x)*A*x/b - 5/16*B*a^3*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt
(b))/b^(7/2) + 3/8*A*a^2*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(5
/2) + 5/8*sqrt(b*x^2 + a*x)*B*a^2/b^3 - 3/4*sqrt(b*x^2 + a*x)*A*a/b^2
```

Giac [A] (verification not implemented)

Time = 150.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = -\frac{\frac{6\left(3\,a^2\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)-\sqrt{(bx+a)b-ab}(2\,bx-3\,a)\sqrt{bx+a}\right)A|b|}{b^3}-\frac{\left(15\,a^3\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+(2\,(4\right.}{b}}{24\,b}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(6*(3*a^2*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b
- a*b))) - sqrt((b*x + a)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*A*abs(b)/b
^3 - (15*a^3*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a
*b))) + (2*(4*b*x - 9*a)*(b*x + a) + 33*a^2)*sqrt((b*x + a)*b - a*b)*sqrt(
b*x + a))*B*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \int \frac{x^{3/2}\,\left(A+B\,x\right)}{\sqrt{a+b\,x}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{-3\sqrt{x}\,\sqrt{bx+a}\,a^2b+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}}{\sqrt{a}}\right.}{24b^3}$$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b + 2*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*
sqrt(x)*sqrt(a + b*x)*b**3*x**2 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**3)
```

3.320 $\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}} dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}} dx = \frac{(4Ab-3aB)\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{Bx^{3/2}\sqrt{a+bx}}{2b} - \frac{a(4Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

output
```
1/4*(4*A*b-3*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^2+1/2*B*x^(3/2)*(b*x+a)^(1/2)/b-
1/4*a*(4*A*b-3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}} dx = \frac{\sqrt{x}\sqrt{a+bx}(4Ab-3aB+2bBx)}{4b^2} + \frac{a(-4Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{2b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[x]*Sqrt[a + b*x]*(4*A*b - 3*a*B + 2*b*B*x))/(4*b^2) + (a*(-4*A*b + 3
*a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {90, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(4Ab-3aB)\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}+\frac{Bx^{3/2}\sqrt{a+bx}}{2b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(4Ab-3aB)\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}+\frac{Bx^{3/2}\sqrt{a+bx}}{2b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(4Ab-3aB)\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}+\frac{Bx^{3/2}\sqrt{a+bx}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(4Ab-3aB)\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}+\frac{Bx^{3/2}\sqrt{a+bx}}{2b}$$

input
```
Int[(Sqrt[x]*(A + B*x))/Sqrt[a + b*x],x]
```

output
```
(B*x^(3/2)*Sqrt[a + b*x])/(2*b) + ((4*A*b - 3*a*B)*((Sqrt[x]*Sqrt[a + b*x]
)/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

method	result
risch	$\frac{(2bBx+4Ab-3Ba)\sqrt{x}\,\sqrt{bx+a}}{4b^{2}}-\frac{a(4Ab-3Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$-\frac{\sqrt{x}\,\sqrt{bx+a}\,\left(-4B\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}+4A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ab-8A\,b^{\frac{3}{2}}\sqrt{x(bx+a)}-3B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}+6Ba\right.}{8b^{\frac{5}{2}}\sqrt{x(bx+a)}}$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*b*x+4*A*b-3*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^2-1/8*a*(4*A*b-3*B*a)/b^
(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/
(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}\,dx$$
$$=\left[-\frac{(3\,Ba^{2}-4\,Aab)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(2\,Bb^{2}x-3\,Bab+4\,Ab^{2})\sqrt{bx+a}\sqrt{x}}{8\,b^{3}},\right.$$
$$\left.-\frac{(3\,Ba^{2}-4\,Aab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)-(2\,Bb^{2}x-3\,Bab+4\,Ab^{2})\sqrt{bx+a}\sqrt{x}}{4\,b^{3}}\right]$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((3*B*a^2 - 4*A*a*b)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqr
t(x) + a) - 2*(2*B*b^2*x - 3*B*a*b + 4*A*b^2)*sqrt(b*x + a)*sqrt(x))/b^3,
-1/4*((3*B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a))
- (2*B*b^2*x - 3*B*a*b + 4*A*b^2)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}\,dx = \begin{cases} -\frac{a\left(2A-\frac{3Ba}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise} \end{cases}\right)}{2b} + \sqrt{a+bx}\left(\frac{Bx^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\left(2A-\frac{3Ba}{2b}\right)}{2b}\right) & \text{for}\ b\neq 0 \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise((-a*(2*A - 3*B*a/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x) +
 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*log(sqrt(x))/sqrt(b*x), True))/
(2*b) + sqrt(a + b*x)*(B*x**(3/2)/(2*b) + sqrt(x)*(2*A - 3*B*a/(2*b))/(2*b
)), Ne(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(5/2)/5)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{bx^2+ax}Bx}{2\,b} + \frac{3\,Ba^2\log\left(2\,x+\frac{a}{b}+\frac{2\,\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Aa\log\left(2\,x+\frac{a}{b}+\frac{2\,\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{2\,b^{\frac{3}{2}}} - \frac{3\,\sqrt{bx^2+ax}Ba}{4\,b^2} + \frac{\sqrt{bx^2+ax}A}{b}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a*x)*B*x/b + 3/8*B*a^2*log(2*x + a/b + 2*sqrt(b*x^2 + a*x
)/sqrt(b))/b^(5/2) - 1/2*A*a*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b))/
b^(3/2) - 3/4*sqrt(b*x^2 + a*x)*B*a/b^2 + sqrt(b*x^2 + a*x)*A/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(71) = 142$.

Time = 151.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}\,dx$$
$$= \frac{\frac{4\left(a\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)+\sqrt{(bx+a)b-ab}\sqrt{bx+a}\right)A|b|}{b^2} - \frac{\left(3\,a^2\sqrt{b}\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)-\sqrt{(bx+a)b-ab}(2\,b}{b^3}}{4\,b}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(4*(a*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)
)) + sqrt((b*x + a)*b - a*b)*sqrt(b*x + a))*A*abs(b)/b^2 - (3*a^2*sqrt(b)*
log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))) - sqrt((b*x + a
)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*B*abs(b)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}\,dx$$
$$= \frac{\frac{x^{7/2}\left(2\,A\,a\,b^2-\frac{3\,B\,a^2\,b}{2}\right)}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}+\frac{x^{5/2}\left(\frac{11\,B\,a^2}{2}-2\,A\,a\,b\right)}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}-\frac{\sqrt{x}\,\left(3\,B\,a^2-4\,A\,a\,b\right)}{2\,b^2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}+\frac{x^{3/2}\left(11\,B\,a^2-4\,A\,a\,b\right)}{2\,b\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{\frac{6\,b^2\,x^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}-\frac{4\,b^3\,x^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}+\frac{b^4\,x^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}-\frac{4\,b\,x}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+1}$$
$$- \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)\,\left(4\,A\,b-3\,B\,a\right)}{2\,b^{5/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x)^(1/2),x)
```

output
```
((x^(7/2)*(2*A*a*b^2 - (3*B*a^2*b)/2))/((a + b*x)^(1/2) - a^(1/2))^7 + (x^
(5/2)*((11*B*a^2)/2 - 2*A*a*b))/((a + b*x)^(1/2) - a^(1/2))^5 - (x^(1/2)*(
3*B*a^2 - 4*A*a*b))/(2*b^2*((a + b*x)^(1/2) - a^(1/2))) + (x^(3/2)*(11*B*a
^2 - 4*A*a*b))/(2*b*((a + b*x)^(1/2) - a^(1/2))^3))/((6*b^2*x^2)/((a + b*x
)^(1/2) - a^(1/2))^4 - (4*b^3*x^3)/((a + b*x)^(1/2) - a^(1/2))^6 + (b^4*x^
4)/((a + b*x)^(1/2) - a^(1/2))^8 - (4*b*x)/((a + b*x)^(1/2) - a^(1/2))^2 +
 1) - (a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2) - a^(1/2)))*(4*A*b - 3*B
*a))/(2*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^2}$$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x - sqrt(b)*log(
(sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**2)
```

3.321 $\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx=\frac{B\sqrt{x}\sqrt{a+bx}}{b}+\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

output
```
B*x^(1/2)*(b*x+a)^(1/2)/b+(2*A*b-B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2
))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx=\frac{B\sqrt{x}\sqrt{a+bx}}{b}+\frac{2(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*Sqrt[a + b*x]),x]
```

output
```
(B*Sqrt[x]*Sqrt[a + b*x])/b + (2*(2*A*b - a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(
-Sqrt[a] + Sqrt[a + b*x])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {90, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(2Ab-aB)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}+\frac{B\sqrt{x}\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{(2Ab-aB)\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}+\frac{B\sqrt{x}\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}+\frac{B\sqrt{x}\sqrt{a+bx}}{b}$$

input
```
Int[(A + B*x)/(Sqrt[x]*Sqrt[a + b*x]),x]
```

output
```
(B*Sqrt[x]*Sqrt[a + b*x])/b + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a + b*x]])/b^(3/2)
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{B\sqrt{x}\,\sqrt{bx+a}}{b}+\frac{(2Ab-Ba)\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	74
default	$\frac{\sqrt{x}\,\sqrt{bx+a}\,\left(2A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b+2B\sqrt{b}\,\sqrt{x(bx+a)}-B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\right)}{2\sqrt{x(bx+a)}\,b^{\frac{3}{2}}}$	101

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*x^(1/2)*(b*x+a)^(1/2)/b+1/2*(2*A*b-B*a)/b^(3/2)*ln((1/2*a+b*x)/b^(1/2)+(
b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx = \left[\frac{2\sqrt{bx+a}Bb\sqrt{x}-(Ba-2\,Ab)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{2\,b^{2}},\frac{\sqrt{bx+a}Bb\sqrt{x}+(Ba-2\,Ab)\sqrt{}}{b^{2}}\right.$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(b*x + a)*B*b*sqrt(x) - (B*a - 2*A*b)*sqrt(b)*log(2*b*x + 2*sq
rt(b*x + a)*sqrt(b)*sqrt(x) + a))/b^2, (sqrt(b*x + a)*B*b*sqrt(x) + (B*a -
 2*A*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{2A\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + 2B\left(\begin{cases} -\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx}+2b\sqrt{x}\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{bx}} & \text{otherwise}\end{cases}\right)}{2b} + \frac{\sqrt{x}\sqrt{a+bx}}{2b} & \text{for } b\neq 0\\ \frac{x^{\frac{3}{2}}}{3\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/x**(1/2)/(b*x+a)**(1/2),x)
```

output
```
2*A*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b) + 2*B*Piecewise((-a*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x) + 2*b*sqrt(x))/sqrt(b), Ne(a, 0)), (sqrt(x)*lo
g(sqrt(x))/sqrt(b*x), True))/(2*b) + sqrt(x)*sqrt(a + b*x)/(2*b), Ne(b, 0)
), (x**(3/2)/(3*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx = -\frac{Ba\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+ax}B}{b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*B*a*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/b^(3/2) + A*log(2*b*
x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) + sqrt(b*x^2 + a*x)*B/b
```

Giac [A] (verification not implemented)

Time = 76.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{b\left(\frac{(Ba-2\,Ab)\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{(bx+a)b-ab}\sqrt{bx+a}B}{b^2}\right)}{|b|}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
b*((B*a - 2*A*b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))
)/b^(3/2) + sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*B/b^2)/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{B\sqrt{x}\,\sqrt{a+b\,x}}{b} - \frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{b^{3/2}} - \frac{4\,A\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
(B*x^(1/2)*(a + b*x)^(1/2))/b - (2*B*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^
(1/2) - a^(1/2))))/b^(3/2) - (4*A*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(
1/2)*x^(1/2))))/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{bx+a}\,b+\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b}$$

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b
```

3.322 $\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2A\sqrt{a+bx}}{a\sqrt{x}} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

output
```
-2*A*(b*x+a)^(1/2)/a/x^(1/2)+2*B*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^
(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2A\sqrt{a+bx}}{a\sqrt{x}} - \frac{2B\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*A*Sqrt[a + b*x])/(a*Sqrt[x]) - (2*B*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a +
b*x]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {87, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{87}$$

$$B\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx-\frac{2A\sqrt{a+bx}}{a\sqrt{x}}$$

$$\Big\downarrow \text{65}$$

$$2B\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}-\frac{2A\sqrt{a+bx}}{a\sqrt{x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}-\frac{2A\sqrt{a+bx}}{a\sqrt{x}}$$

input
```
Int[(A + B*x)/(x^(3/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*A*Sqrt[a + b*x])/(a*Sqrt[x]) + (2*B*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/Sqrt[b]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{2A\sqrt{bx+a}}{a\sqrt{x}}+\frac{B\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x(bx+a)}}{\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	66
default	$\frac{\left(B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ax-2A\sqrt{x(bx+a)}\,\sqrt{b}\right)\sqrt{bx+a}}{a\sqrt{x}\,\sqrt{x(bx+a)}\,\sqrt{b}}$	73

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A*(b*x+a)^(1/2)/a/x^(1/2)+B*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b
^(1/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.12

$$\int\frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx=\left[\frac{Ba\sqrt{b}x\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,\sqrt{bx+a}Ab\sqrt{x}}{abx},\right.$$
$$\left.-\frac{2\left(Ba\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+\sqrt{bx+a}Ab\sqrt{x}\right)}{abx}\right]$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[(B*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(
b*x + a)*A*b*sqrt(x))/(a*b*x), -2*(B*a*sqrt(-b)*x*arctan(sqrt(-b)*sqrt(x)/
sqrt(b*x + a)) + sqrt(b*x + a)*A*b*sqrt(x))/(a*b*x)]
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}+\frac{2B\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate((B*x+A)/x**(3/2)/(b*x+a)**(1/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x) + 1)/a + 2*B*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt
(b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = \frac{B\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}-\frac{2\,\sqrt{bx^2+ax}A}{ax}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
B*log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b) - 2*sqrt(b*x^2 + a*
x)*A/(a*x)
```

Giac [A] (verification not implemented)

Time = 75.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\,b^2\left(\frac{B\log\left(\left|-\sqrt{bx+a}\sqrt{b}+\sqrt{(bx+a)b-ab}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}A}{\sqrt{(bx+a)b-ab}a}\right)}{|b|}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*b^2*(B*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b^(3/
2) + sqrt(b*x + a)*A/(sqrt((b*x + a)*b - a*b)*a))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{4\,B\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}-\frac{2\,A\,\sqrt{a+b\,x}}{a\,\sqrt{x}}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x)^(1/2)),x)
```

output
```
- (4*B*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
- (2*A*(a + b*x)^(1/2))/(a*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^{3/2}\sqrt{a+bx}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)x-2\sqrt{b}\,x}{x}$$

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x) + sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*x - sqrt(b)*x))/x
```

3.323 $\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2A\sqrt{a+bx}}{3ax^{3/2}}+\frac{2(2Ab-3aB)\sqrt{a+bx}}{3a^2\sqrt{x}}$$

output
```
-2/3*A*(b*x+a)^(1/2)/a/x^(3/2)+2/3*(2*A*b-3*B*a)*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}(aA-2Abx+3aBx)}{3a^2x^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(a*A - 2*A*b*x + 3*a*B*x))/(3*a^2*x^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(2Ab-3aB)\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2A\sqrt{a+bx}}{3ax^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\sqrt{a+bx}(2Ab-3aB)}{3a^2\sqrt{x}}-\frac{2A\sqrt{a+bx}}{3ax^{3/2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*A*Sqrt[a + b*x])/(3*a*x^(3/2)) + (2*(2*A*b - 3*a*B)*Sqrt[a + b*x])/(3*
a^2*Sqrt[x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-2Abx+3Bax+Aa)}{3x^{\frac{3}{2}}a^{2}}$	30
default	$-\frac{2\sqrt{bx+a}\,(-2Abx+3Bax+Aa)}{3x^{\frac{3}{2}}a^{2}}$	30
risch	$-\frac{2\sqrt{bx+a}\,(-2Abx+3Bax+Aa)}{3x^{\frac{3}{2}}a^{2}}$	30
orering	$-\frac{2\sqrt{bx+a}\,(-2Abx+3Bax+Aa)}{3x^{\frac{3}{2}}a^{2}}$	30

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-2*A*b*x+3*B*a*x+A*a)/x^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\,(Aa+(3\,Ba-2\,Ab)x)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(A*a + (3*B*a - 2*A*b)*x)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax}+\frac{4Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^2}-\frac{2B\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}$$

input
```
integrate((B*x+A)/x**(5/2)/(b*x+a)**(1/2),x)
```

output
```
-2*A*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*A*b**(3/2)*sqrt(a/(b*x) + 1)/(3
*a**2) - 2*B*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\,\sqrt{bx^2+ax}B}{ax}+\frac{4\,\sqrt{bx^2+ax}Ab}{3\,a^2x}-\frac{2\,\sqrt{bx^2+ax}A}{3\,ax^2}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(b*x^2 + a*x)*B/(a*x) + 4/3*sqrt(b*x^2 + a*x)*A*b/(a^2*x) - 2/3*sqr
t(b*x^2 + a*x)*A/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{bx+a}b^3\left(\frac{(3\,Ba-2\,Ab)(bx+a)}{a^2}-\frac{3\,(Ba^2-Aab)}{a^2}\right)}{3\,((bx+a)b-ab)^{\frac{3}{2}}|b|}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + a)*b^3*((3*B*a - 2*A*b)*(b*x + a)/a^2 - 3*(B*a^2 - A*a*b)/
a^2)/(((b*x + a)*b - a*b)^(3/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{\left(\frac{2\,A}{3\,a}-\frac{x\,(4\,A\,b-6\,B\,a)}{3\,a^2}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x)^(1/2)),x)
```

output
```
-(((2*A)/(3*a) - (x*(4*A*b - 6*B*a))/(3*a^2))*(a + b*x)^(1/2))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{x^{5/2}\sqrt{a+bx}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}\,a-2\sqrt{x}\,\sqrt{bx+a}\,bx-2\sqrt{b}\,b\,x^2}{3a\,x^2}$$

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^(1/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a + b*x)*a + sqrt(x)*sqrt(a + b*x)*b*x + sqrt(b)*b*x**
2))/(3*a*x**2)
```

3.324 $\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx=-\frac{2A\sqrt{a+bx}}{5ax^{5/2}}+\frac{2(4Ab-5aB)\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{4b(4Ab-5aB)\sqrt{a+bx}}{15a^3\sqrt{x}}$$

output
```
-2/5*A*(b*x+a)^(1/2)/a/x^(5/2)+2/15*(4*A*b-5*B*a)*(b*x+a)^(1/2)/a^2/x^(3/2
)-4/15*b*(4*A*b-5*B*a)*(b*x+a)^(1/2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx=-\frac{2\sqrt{a+bx}(3a^2A-4aAbx+5a^2Bx+8Ab^2x^2-10abBx^2)}{15a^3x^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(3*a^2*A - 4*a*A*b*x + 5*a^2*B*x + 8*A*b^2*x^2 - 10*a*b*
B*x^2))/(15*a^3*x^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(4Ab-5aB)\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2A\sqrt{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(4Ab-5aB)\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2A\sqrt{a+bx}}{5ax^{5/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)(4Ab-5aB)}{5a}-\frac{2A\sqrt{a+bx}}{5ax^{5/2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*A*Sqrt[a + b*x])/(5*a*x^(5/2)) - ((4*A*b - 5*a*B)*((-2*Sqrt[a + b*x])/
(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(8A\,b^2x^2-10Bab\,x^2-4aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}a^3}$	53
default	$-\frac{2\sqrt{bx+a}\,(8A\,b^2x^2-10Bab\,x^2-4aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}a^3}$	53
risch	$-\frac{2\sqrt{bx+a}\,(8A\,b^2x^2-10Bab\,x^2-4aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}a^3}$	53
orering	$-\frac{2\sqrt{bx+a}\,(8A\,b^2x^2-10Bab\,x^2-4aAbx+5B\,a^2x+3a^2A)}{15x^{\frac{5}{2}}a^3}$	53

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(1/2)*(8*A*b^2*x^2-10*B*a*b*x^2-4*A*a*b*x+5*B*a^2*x+3*A*a^2)
/x^(5/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\,(3\,Aa^2-2\,(5\,Bab-4\,Ab^2)x^2+(5\,Ba^2-4\,Aab)x)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*a^2 - 2*(5*B*a*b - 4*A*b^2)*x^2 + (5*B*a^2 - 4*A*a*b)*x)*sqrt(b
*x + a)/(a^3*x^(5/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(82) = 164.

Time = 4.40 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.07

$$\begin{aligned}\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx = &-\frac{6Aa^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} \\ &-\frac{4Aa^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{6Aa^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} \\ &-\frac{24Aab^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{16Ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} \\ &-\frac{2B\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax}+\frac{4Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^2}\end{aligned}$$

input
```
integrate((B*x+A)/x**(7/2)/(b*x+a)**(1/2),x)
```

output
```
-6*A*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**
3 + 15*a**3*b**6*x**4) - 4*A*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b
**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*A*a**2*b**(13/2)*x**
2*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*
x**4) - 24*A*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a*
*4*b**5*x**3 + 15*a**3*b**6*x**4) - 16*A*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/
(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 2*B*sqrt(b)*
sqrt(a/(b*x) + 1)/(3*a*x) + 4*B*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx = \frac{4\sqrt{bx^2+ax}Bb}{3\,a^2x} - \frac{16\sqrt{bx^2+ax}Ab^2}{15\,a^3x} - \frac{2\sqrt{bx^2+ax}B}{3\,ax^2} + \frac{8\sqrt{bx^2+ax}Ab}{15\,a^2x^2} - \frac{2\sqrt{bx^2+ax}A}{5\,ax^3}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(b*x^2 + a*x)*B*b/(a^2*x) - 16/15*sqrt(b*x^2 + a*x)*A*b^2/(a^3*x)
- 2/3*sqrt(b*x^2 + a*x)*B/(a*x^2) + 8/15*sqrt(b*x^2 + a*x)*A*b/(a^2*x^2) -
 2/5*sqrt(b*x^2 + a*x)*A/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx = \frac{2\sqrt{bx+a}\left((bx+a)\left(\frac{2\,(5\,Bab^4-4\,Ab^5)(bx+a)}{a^3} - \frac{5\,(5\,Ba^2b^4-4\,Aab^5)}{a^3}\right) + \frac{15\,(Ba^3b^4-Aa^2b^5)}{a^3}\right)b}{15\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/15*sqrt(b*x + a)*((b*x + a)*(2*(5*B*a*b^4 - 4*A*b^5)*(b*x + a)/a^3 - 5*(
5*B*a^2*b^4 - 4*A*a*b^5)/a^3) + 15*(B*a^3*b^4 - A*a^2*b^5)/a^3)*b/(((b*x +
 a)*b - a*b)^(5/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx=-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{5\,a}+\frac{x^2\,\left(16\,A\,b^2-20\,B\,a\,b\right)}{15\,a^3}+\frac{x\,\left(10\,B\,a^2-8\,A\,a\,b\right)}{15\,a^3}\right)}{x^{5/2}}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x)^(1/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(5*a) + (x^2*(16*A*b^2 - 20*B*a*b))/(15*a^3) + (x
*(10*B*a^2 - 8*A*a*b))/(15*a^3)))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{x^{7/2}\sqrt{a+bx}}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,abx}{15}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}-\frac{4\sqrt{b}\,b^2x^3}{15}}{a^2x^3}$$

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 - sqrt(x)*sqrt(a + b*x)*a*b*x + 2*sqrt
(x)*sqrt(a + b*x)*b**2*x**2 - 2*sqrt(b)*b**2*x**3))/(15*a**2*x**3)
```

3.325 $\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2A\sqrt{a+bx}}{7ax^{7/2}} + \frac{2(6Ab-7aB)\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{8b(6Ab-7aB)\sqrt{a+bx}}{105a^3x^{3/2}} + \frac{16b^2(6Ab-7aB)\sqrt{a+bx}}{105a^4\sqrt{x}}$$

output
```
-2/7*A*(b*x+a)^(1/2)/a/x^(7/2)+2/35*(6*A*b-7*B*a)*(b*x+a)^(1/2)/a^2/x^(5/2
)-8/105*b*(6*A*b-7*B*a)*(b*x+a)^(1/2)/a^3/x^(3/2)+16/105*b^2*(6*A*b-7*B*a)
*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}(-48Ab^3x^3+8ab^2x^2(3A+7Bx)+3a^3(5A+7Bx)-2a^2bx(9A+14Bx))}{105a^4x^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(-48*A*b^3*x^3 + 8*a*b^2*x^2*(3*A + 7*B*x) + 3*a^3*(5*A
+ 7*B*x) - 2*a^2*b*x*(9*A + 14*B*x)))/(105*a^4*x^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(6Ab-7aB)\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx}{7a}-\frac{2A\sqrt{a+bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(6Ab-7aB)\left(-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2A\sqrt{a+bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(6Ab-7aB)\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2A\sqrt{a+bx}}{7ax^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)(6Ab-7aB)}{7a}-\frac{2A\sqrt{a+bx}}{7ax^{7/2}}$$

input
```
Int[(A + B*x)/(x^(9/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*A*Sqrt[a + b*x])/(7*a*x^(7/2)) - ((6*A*b - 7*a*B)*((-2*Sqrt[a + b*x])/
(5*a*x^(5/2)) - (4*b*((-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x
])/(3*a^2*Sqrt[x])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-48A\,b^{3}x^{3}+56Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-28B\,a^{2}b\,x^{2}-18a^{2}Abx+21B\,a^{3}x+15a^{3}A)}{105x^{\frac{7}{2}}a^{4}}$	77
default	$-\frac{2\sqrt{bx+a}\,(-48A\,b^{3}x^{3}+56Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-28B\,a^{2}b\,x^{2}-18a^{2}Abx+21B\,a^{3}x+15a^{3}A)}{105x^{\frac{7}{2}}a^{4}}$	77
risch	$-\frac{2\sqrt{bx+a}\,(-48A\,b^{3}x^{3}+56Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-28B\,a^{2}b\,x^{2}-18a^{2}Abx+21B\,a^{3}x+15a^{3}A)}{105x^{\frac{7}{2}}a^{4}}$	77
orering	$-\frac{2\sqrt{bx+a}\,(-48A\,b^{3}x^{3}+56Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-28B\,a^{2}b\,x^{2}-18a^{2}Abx+21B\,a^{3}x+15a^{3}A)}{105x^{\frac{7}{2}}a^{4}}$	77

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x+a)^(1/2)*(-48*A*b^3*x^3+56*B*a*b^2*x^3+24*A*a*b^2*x^2-28*B*a^2
*b*x^2-18*A*a^2*b*x+21*B*a^3*x+15*A*a^3)/x^(7/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2\,(15\,Aa^{3}+8\,(7\,Bab^{2}-6\,Ab^{3})x^{3}-4\,(7\,Ba^{2}b-6\,Aab^{2})x^{2}+3\,(7\,Ba^{3}-6\,Aa^{2}b)x)\sqrt{bx+a}}{105\,a^{4}x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(15*A*a^3 + 8*(7*B*a*b^2 - 6*A*b^3)*x^3 - 4*(7*B*a^2*b - 6*A*a*b^2)
*x^2 + 3*(7*B*a^3 - 6*A*a^2*b)*x)*sqrt(b*x + a)/(a^4*x^(7/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(116) = 232$.

Time = 12.78 (sec) , antiderivative size = 796, normalized size of antiderivative = 6.80

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{10Aa^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} - \frac{18Aa^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} - \frac{10Aa^4b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} + \frac{10Aa^3b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} + \frac{60Aa^2b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} + \frac{80Aab^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} + \frac{32Ab^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6} - \frac{6Ba^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{4Ba^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{6Ba^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{24Bab^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4} - \frac{16Bb^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}$$

input
```
integrate((B*x+A)/x**(9/2)/(b*x+a)**(1/2),x)
```

output
```
-10*A*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10
*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*A*a**5*b**(21/2)*x*
sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**1
1*x**5 + 35*a**4*b**12*x**6) - 10*A*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/
(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b
**12*x**6) + 10*A*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3
 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*A*
a**2*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*
x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 80*A*a*b**(29/2)*x**5*s
qrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11
*x**5 + 35*a**4*b**12*x**6) + 32*A*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a*
*7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x
**6) - 6*B*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b*
*5*x**3 + 15*a**3*b**6*x**4) - 4*B*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*
a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*B*a**2*b**(13/
2)*x**2*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3
*b**6*x**4) - 24*B*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 +
 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 16*B*b**(17/2)*x**4*sqrt(a/(b*x)
 + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{16\,\sqrt{bx^2+ax}Bb^2}{15\,a^3x} + \frac{32\,\sqrt{bx^2+ax}Ab^3}{35\,a^4x} + \frac{8\,\sqrt{bx^2+ax}Bb}{15\,a^2x^2} - \frac{16\,\sqrt{bx^2+ax}Ab^2}{35\,a^3x^2} - \frac{2\,\sqrt{bx^2+ax}B}{5\,ax^3} + \frac{12\,\sqrt{bx^2+ax}Ab}{35\,a^2x^3} - \frac{2\,\sqrt{bx^2+ax}A}{7\,ax^4}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-16/15*sqrt(b*x^2 + a*x)*B*b^2/(a^3*x) + 32/35*sqrt(b*x^2 + a*x)*A*b^3/(a^
4*x) + 8/15*sqrt(b*x^2 + a*x)*B*b/(a^2*x^2) - 16/35*sqrt(b*x^2 + a*x)*A*b^
2/(a^3*x^2) - 2/5*sqrt(b*x^2 + a*x)*B/(a*x^3) + 12/35*sqrt(b*x^2 + a*x)*A*
b/(a^2*x^3) - 2/7*sqrt(b*x^2 + a*x)*A/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx =$$
$$-\frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(7\,Bab^2-6\,Ab^3)(bx+a)}{a^4}-\frac{7\,(7\,Ba^2b^2-6\,Aab^3)}{a^4}\right)+\frac{35\,(7\,Ba^3b^2-6\,Aa^2b^3)}{a^4}\right)-\frac{105\,(Ba^4b^2-Aa^3b^3)}{a^4}\right)}{105\,((bx+a)b-ab)^{\frac{7}{2}}\,|b|}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/105*((b*x + a)*(4*(b*x + a)*(2*(7*B*a*b^2 - 6*A*b^3)*(b*x + a)/a^4 - 7*
(7*B*a^2*b^2 - 6*A*a*b^3)/a^4) + 35*(7*B*a^3*b^2 - 6*A*a^2*b^3)/a^4) - 105
*(B*a^4*b^2 - A*a^3*b^3)/a^4)*sqrt(b*x + a)*b^5/(((b*x + a)*b - a*b)^(7/2)
*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx =$$
$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{7\,a}+\frac{x\,\left(42\,B\,a^3-36\,A\,a^2\,b\right)}{105\,a^4}-\frac{x^3\,\left(96\,A\,b^3-112\,B\,a\,b^2\right)}{105\,a^4}+\frac{8\,b\,x^2\,\left(6\,A\,b-7\,B\,a\right)}{105\,a^3}\right)}{x^{7/2}}$$

input
```
int((A + B*x)/(x^(9/2)*(a + b*x)^(1/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(7*a) + (x*(42*B*a^3 - 36*A*a^2*b))/(105*a^4) - (
x^3*(96*A*b^3 - 112*B*a*b^2))/(105*a^4) + (8*b*x^2*(6*A*b - 7*B*a))/(105*a
^3)))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx}{x^{9/2}\sqrt{a+bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{105}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{105}+\frac{16\sqrt{b}\,b^3x^4}{105}}{a^3x^4}$$

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(a + b*x)*a**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*x +
4*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*b**3*x**3 +
8*sqrt(b)*b**3*x**4))/(105*a**3*x**4)
```

3.326 $\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx=-\frac{2A\sqrt{a+bx}}{9ax^{9/2}}+\frac{2(8Ab-9aB)\sqrt{a+bx}}{63a^2x^{7/2}}-\frac{4b(8Ab-9aB)\sqrt{a+bx}}{105a^3x^{5/2}}+\frac{16b^2(8Ab-9aB)\sqrt{a+bx}}{315a^4x^{3/2}}-\frac{32b^3(8Ab-9aB)\sqrt{a+bx}}{315a^5\sqrt{x}}$$

output
```
-2/9*A*(b*x+a)^(1/2)/a/x^(9/2)+2/63*(8*A*b-9*B*a)*(b*x+a)^(1/2)/a^2/x^(7/2
)-4/105*b*(8*A*b-9*B*a)*(b*x+a)^(1/2)/a^3/x^(5/2)+16/315*b^2*(8*A*b-9*B*a)
*(b*x+a)^(1/2)/a^4/x^(3/2)-32/315*b^3*(8*A*b-9*B*a)*(b*x+a)^(1/2)/a^5/x^(1
/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx=-\frac{2\sqrt{a+bx}(128Ab^4x^4+24a^2b^2x^2(2A+3Bx)-16ab^3x^3(4A+9Bx)+5a^4(7A+9Bx)-2a^3bx(20A+27}{315a^5x^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(11/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(128*A*b^4*x^4 + 24*a^2*b^2*x^2*(2*A + 3*B*x) - 16*a*b^3
*x^3*(4*A + 9*B*x) + 5*a^4*(7*A + 9*B*x) - 2*a^3*b*x*(20*A + 27*B*x)))/(31
5*a^5*x^(9/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(8Ab-9aB)\int \frac{1}{x^{9/2}\sqrt{a+bx}}dx}{9a}-\frac{2A\sqrt{a+bx}}{9ax^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-9aB)\left(-\frac{6b\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}\right)}{9a}-\frac{2A\sqrt{a+bx}}{9ax^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-9aB)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}\right)}{9a}-\frac{2A\sqrt{a+bx}}{9ax^{9/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-9aB)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}\right)}{9a}-\frac{2A\sqrt{a+bx}}{9ax^{9/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{7a}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}\right)(8Ab-9aB)}{9a}-\frac{2A\sqrt{a+bx}}{9ax^{9/2}}$$

input
```
Int[(A + B*x)/(x^(11/2)*Sqrt[a + b*x]),x]
```

output
```
(-2*A*Sqrt[a + b*x])/(9*a*x^(9/2)) - ((8*A*b - 9*a*B)*((-2*Sqrt[a + b*x])/
(7*a*x^(7/2)) - (6*b*((-2*Sqrt[a + b*x])/(5*a*x^(5/2)) - (4*b*((-2*Sqrt[a
+ b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/(5*a)))/(7*a
)))/(9*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(128A\,b^4x^4-144Ba\,b^3x^4-64Aa\,b^3x^3+72B\,a^2b^2x^3+48A\,a^2b^2x^2-54B\,a^3b\,x^2-40A\,a^3bx+45B\,a^4x+35A\,a^4)}{315x^{\frac{9}{2}}a^5}$	101
default	$-\frac{2\sqrt{bx+a}\,(128A\,b^4x^4-144Ba\,b^3x^4-64Aa\,b^3x^3+72B\,a^2b^2x^3+48A\,a^2b^2x^2-54B\,a^3b\,x^2-40A\,a^3bx+45B\,a^4x+35A\,a^4)}{315x^{\frac{9}{2}}a^5}$	101
risch	$-\frac{2\sqrt{bx+a}\,(128A\,b^4x^4-144Ba\,b^3x^4-64Aa\,b^3x^3+72B\,a^2b^2x^3+48A\,a^2b^2x^2-54B\,a^3b\,x^2-40A\,a^3bx+45B\,a^4x+35A\,a^4)}{315x^{\frac{9}{2}}a^5}$	101
orering	$-\frac{2\sqrt{bx+a}\,(128A\,b^4x^4-144Ba\,b^3x^4-64Aa\,b^3x^3+72B\,a^2b^2x^3+48A\,a^2b^2x^2-54B\,a^3b\,x^2-40A\,a^3bx+45B\,a^4x+35A\,a^4)}{315x^{\frac{9}{2}}a^5}$	101

input
```
int((B*x+A)/x^(11/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(b*x+a)^(1/2)*(128*A*b^4*x^4-144*B*a*b^3*x^4-64*A*a*b^3*x^3+72*B*a^
2*b^2*x^3+48*A*a^2*b^2*x^2-54*B*a^3*b*x^2-40*A*a^3*b*x+45*B*a^4*x+35*A*a^4
)/x^(9/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx =$$
$$-\frac{2\,(35\,Aa^4-16\,(9\,Bab^3-8\,Ab^4)x^4+8\,(9\,Ba^2b^2-8\,Aab^3)x^3-6\,(9\,Ba^3b-8\,Aa^2b^2)x^2+5\,(9\,Ba^4-8\,Aa}{315\,a^5x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)/x^(11/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(35*A*a^4 - 16*(9*B*a*b^3 - 8*A*b^4)*x^4 + 8*(9*B*a^2*b^2 - 8*A*a*b
^3)*x^3 - 6*(9*B*a^3*b - 8*A*a^2*b^2)*x^2 + 5*(9*B*a^4 - 8*A*a^3*b)*x)*sqr
t(b*x + a)/(a^5*x^(9/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(150) = 300$.

Time = 35.58 (sec) , antiderivative size = 1255, normalized size of antiderivative = 8.37

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(11/2)/(b*x+a)**(1/2),x)
```

output
```
-70*A*a**8*b**(33/2)*sqrt(a/(b*x) + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b*
*17*x**5 + 1890*a**7*b**18*x**6 + 1260*a**6*b**19*x**7 + 315*a**5*b**20*x*
*8) - 200*A*a**7*b**(35/2)*x*sqrt(a/(b*x) + 1)/(315*a**9*b**16*x**4 + 1260
*a**8*b**17*x**5 + 1890*a**7*b**18*x**6 + 1260*a**6*b**19*x**7 + 315*a**5*
b**20*x**8) - 196*A*a**6*b**(37/2)*x**2*sqrt(a/(b*x) + 1)/(315*a**9*b**16*
x**4 + 1260*a**8*b**17*x**5 + 1890*a**7*b**18*x**6 + 1260*a**6*b**19*x**7
+ 315*a**5*b**20*x**8) - 56*A*a**5*b**(39/2)*x**3*sqrt(a/(b*x) + 1)/(315*a
**9*b**16*x**4 + 1260*a**8*b**17*x**5 + 1890*a**7*b**18*x**6 + 1260*a**6*b
**19*x**7 + 315*a**5*b**20*x**8) - 70*A*a**4*b**(41/2)*x**4*sqrt(a/(b*x) +
 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**5 + 1890*a**7*b**18*x**6 + 1
260*a**6*b**19*x**7 + 315*a**5*b**20*x**8) - 560*A*a**3*b**(43/2)*x**5*sqr
t(a/(b*x) + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**5 + 1890*a**7*b**
18*x**6 + 1260*a**6*b**19*x**7 + 315*a**5*b**20*x**8) - 1120*A*a**2*b**(45
/2)*x**6*sqrt(a/(b*x) + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**5 + 1
890*a**7*b**18*x**6 + 1260*a**6*b**19*x**7 + 315*a**5*b**20*x**8) - 896*A*
a*b**(47/2)*x**7*sqrt(a/(b*x) + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*
x**5 + 1890*a**7*b**18*x**6 + 1260*a**6*b**19*x**7 + 315*a**5*b**20*x**8)
- 256*A*b**(49/2)*x**8*sqrt(a/(b*x) + 1)/(315*a**9*b**16*x**4 + 1260*a**8*
b**17*x**5 + 1890*a**7*b**18*x**6 + 1260*a**6*b**19*x**7 + 315*a**5*b**20*
x**8) - 10*B*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx = \frac{32\,\sqrt{bx^2+ax}Bb^3}{35\,a^4x} - \frac{256\,\sqrt{bx^2+ax}Ab^4}{315\,a^5x} - \frac{16\,\sqrt{bx^2+ax}Bb^2}{35\,a^3x^2} + \frac{128\,\sqrt{bx^2+ax}Ab^3}{315\,a^4x^2} + \frac{12\,\sqrt{bx^2+ax}Bb}{35\,a^2x^3} - \frac{32\,\sqrt{bx^2+ax}Ab^2}{105\,a^3x^3} - \frac{2\,\sqrt{bx^2+ax}B}{7\,ax^4} + \frac{16\,\sqrt{bx^2+ax}Ab}{63\,a^2x^4} - \frac{2\,\sqrt{bx^2+ax}A}{9\,ax^5}$$

input
```
integrate((B*x+A)/x^(11/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
32/35*sqrt(b*x^2 + a*x)*B*b^3/(a^4*x) - 256/315*sqrt(b*x^2 + a*x)*A*b^4/(a
^5*x) - 16/35*sqrt(b*x^2 + a*x)*B*b^2/(a^3*x^2) + 128/315*sqrt(b*x^2 + a*x
)*A*b^3/(a^4*x^2) + 12/35*sqrt(b*x^2 + a*x)*B*b/(a^2*x^3) - 32/105*sqrt(b*
x^2 + a*x)*A*b^2/(a^3*x^3) - 2/7*sqrt(b*x^2 + a*x)*B/(a*x^4) + 16/63*sqrt(
b*x^2 + a*x)*A*b/(a^2*x^4) - 2/9*sqrt(b*x^2 + a*x)*A/(a*x^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx = \frac{2\left(\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{2\,(9\,Bab^8-8\,Ab^9)(bx+a)}{a^5} - \frac{9\,(9\,Ba^2b^8-8\,Aab^9)}{a^5}\right) + \frac{63\,(9\,Ba^3b^8-8\,Aa^2b}{a^5}\right.\right.\right.}{315\,((bx+a)b-ab)^{\frac{9}{2}}|}$$

input
```
integrate((B*x+A)/x^(11/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*((2*(b*x + a)*(4*(b*x + a)*(2*(9*B*a*b^8 - 8*A*b^9)*(b*x + a)/a^5 -
9*(9*B*a^2*b^8 - 8*A*a*b^9)/a^5) + 63*(9*B*a^3*b^8 - 8*A*a^2*b^9)/a^5) - 1
05*(9*B*a^4*b^8 - 8*A*a^3*b^9)/a^5)*(b*x + a) + 315*(B*a^5*b^8 - A*a^4*b^9
)/a^5)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{9\,a}+\frac{x\,\left(90\,B\,a^4-80\,A\,a^3\,b\right)}{315\,a^5}+\frac{x^4\,\left(256\,A\,b^4-288\,B\,a\,b^3\right)}{315\,a^5}-\frac{16\,b^2\,x^3\,\left(8\,A\,b-9\,B\,a\right)}{315\,a^4}+\frac{4\,b\,x^2\,\left(8\,A\,b-9\,B\,a\right)}{105\,a^3}\right)}{x^{9/2}}$$

input
```
int((A + B*x)/(x^(11/2)*(a + b*x)^(1/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(9*a) + (x*(90*B*a^4 - 80*A*a^3*b))/(315*a^5) + (
x^4*(256*A*b^4 - 288*B*a*b^3))/(315*a^5) - (16*b^2*x^3*(8*A*b - 9*B*a))/(3
15*a^4) + (4*b*x^2*(8*A*b - 9*B*a))/(105*a^3)))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^{11/2}\sqrt{a+bx}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^4}{9}-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3bx}{63}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,a^2b^2x^2}{105}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,ab^3x^3}{315}+\frac{32\sqrt{x}\,\sqrt{bx+a}\,b^4x^4}{315}-}{a^4x^5}$$

input
```
int((B*x+A)/x^(11/2)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(a + b*x)*a**4 - 5*sqrt(x)*sqrt(a + b*x)*a**3*b*x +
6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*x**2 - 8*sqrt(x)*sqrt(a + b*x)*a*b**3*x*
*3 + 16*sqrt(x)*sqrt(a + b*x)*b**4*x**4 - 16*sqrt(b)*b**4*x**5))/(315*a**4
*x**5)
```

3.327 $\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = -\frac{2(Ab-aB)x^{5/2}}{b^2\sqrt{a+bx}} - \frac{5a(6Ab-7aB)\sqrt{x}\sqrt{a+bx}}{8b^4} + \frac{5(6Ab-7aB)x^{3/2}\sqrt{a+bx}}{12b^3} + \frac{Bx^{5/2}\sqrt{a+bx}}{3b^2} + \frac{5a^2(6Ab-7aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{9/2}}$$

output

```
-2*(A*b-B*a)*x^(5/2)/b^2/(b*x+a)^(1/2)-5/8*a*(6*A*b-7*B*a)*x^(1/2)*(b*x+a)
^(1/2)/b^4+5/12*(6*A*b-7*B*a)*x^(3/2)*(b*x+a)^(1/2)/b^3+1/3*B*x^(5/2)*(b*x
+a)^(1/2)/b^2+5/8*a^2*(6*A*b-7*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{\sqrt{x}(105a^3B+4b^3x^2(3A+2Bx)-2ab^2x(15A+7Bx)+a^2(-90Ab+35bBx))}{24b^4\sqrt{a+bx}} + \frac{5a^2(-6Ab+7aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{4b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(105*a^3*B + 4*b^3*x^2*(3*A + 2*B*x) - 2*a*b^2*x*(15*A + 7*B*x) +
 a^2*(-90*A*b + 35*b*B*x)))/(24*b^4*Sqrt[a + b*x]) + (5*a^2*(-6*A*b + 7*a*
B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$\frac{2x^{7/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(6Ab-7aB)\int \frac{x^{5/2}}{\sqrt{a+bx}}dx}{ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(6Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)}{ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(6Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(6Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{ab}$$

$\Big\downarrow$ 65

$$\frac{2x^{7/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(6Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)}{ab}$$

$\Big\downarrow$ 219

$$\frac{2x^{7/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(6Ab-7aB)\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)}{ab}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x^(7/2))/(a*b*Sqrt[a + b*x]) - ((6*A*b - 7*a*B)*((x^(5/2)*S
qrt[a + b*x])/(3*b) - (5*a*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]
*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))
/(4*b)))/(6*b)))/(a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(-8b^2Bx^2-12Ab^2x+22Babx+42abA-57a^2B)\sqrt{x}\,\sqrt{bx+a}}{24b^4}+\frac{a^2\left(30A\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)-\frac{35Ba\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right.}{\sqrt{b}}\right.}{16b^4\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\left(16B\,b^{\frac{7}{2}}x^3\sqrt{x(bx+a)}+24A\,b^{\frac{7}{2}}x^2\sqrt{x(bx+a)}-28Ba\,b^{\frac{5}{2}}x^2\sqrt{x(bx+a)}+90A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b^2x-60A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}a\right.}{}$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*B*b^2*x^2-12*A*b^2*x+22*B*a*b*x+42*A*a*b-57*B*a^2)*x^(1/2)*(b*x+
a)^(1/2)/b^4+1/16*a^2/b^4*(30*A*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)
^(1/2))-35*B*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)-32*(A*b-B
*a)/b/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*
x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.98

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=\left[-\frac{15\,(7\,Ba^4-6\,Aa^3b+(7\,Ba^3b-6\,Aa^2b^2)x)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x)*sqrt(b)*log
(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(8*B*b^4*x^3 + 105*B*a^3
*b - 90*A*a^2*b^2 - 2*(7*B*a*b^3 - 6*A*b^4)*x^2 + 5*(7*B*a^2*b^2 - 6*A*a*b
^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^6*x + a*b^5), 1/24*(15*(7*B*a^4 - 6*A*a^3
*b + (7*B*a^3*b - 6*A*a^2*b^2)*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*
x + a)) + (8*B*b^4*x^3 + 105*B*a^3*b - 90*A*a^2*b^2 - 2*(7*B*a*b^3 - 6*A*b
^4)*x^2 + 5*(7*B*a^2*b^2 - 6*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^6*x + a
*b^5)]
```

Sympy [A] (verification not implemented)

Time = 72.56 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.59

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = A\left(-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}}-\frac{5\sqrt{a}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right) + B\left(\frac{35a^{\frac{5}{2}}\sqrt{x}}{8b^4\sqrt{1+\frac{bx}{a}}}+\frac{35a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^3\sqrt{1+\frac{bx}{a}}}-\frac{7\sqrt{a}x^{\frac{5}{2}}}{12b^2\sqrt{1+\frac{bx}{a}}}-\frac{35a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{9}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right)$$

input
```
integrate(x**(5/2)*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
A*(-15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b
**2*sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2))
 + x**(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))) + B*(35*a**(5/2)*sqrt(x)/(8*b**
4*sqrt(1 + b*x/a)) + 35*a**(3/2)*x**(3/2)/(24*b**3*sqrt(1 + b*x/a)) - 7*sq
rt(a)*x**(5/2)/(12*b**2*sqrt(1 + b*x/a)) - 35*a**3*asinh(sqrt(b)*sqrt(x)/s
qrt(a))/(8*b**(9/2)) + x**(7/2)/(3*sqrt(a)*b*sqrt(1 + b*x/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{Bx^4}{3\sqrt{bx^2+ax}b} - \frac{7\,Bax^3}{12\sqrt{bx^2+ax}b^2} + \frac{Ax^3}{2\sqrt{bx^2+ax}b} + \frac{35\,Ba^2x^2}{24\sqrt{bx^2+ax}b^3} - \frac{5\,Aax^2}{4\sqrt{bx^2+ax}b^2} + \frac{35\,Ba^3x}{8\sqrt{bx^2+ax}b^4} - \frac{15\,Aa^2x}{4\sqrt{bx^2+ax}b^3} - \frac{35\,Ba^3\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{9}{2}}} + \frac{15\,Aa^2\log\left(2\,bx+a+2\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*x^4/(sqrt(b*x^2 + a*x)*b) - 7/12*B*a*x^3/(sqrt(b*x^2 + a*x)*b^2) + 1
/2*A*x^3/(sqrt(b*x^2 + a*x)*b) + 35/24*B*a^2*x^2/(sqrt(b*x^2 + a*x)*b^3) -
 5/4*A*a*x^2/(sqrt(b*x^2 + a*x)*b^2) + 35/8*B*a^3*x/(sqrt(b*x^2 + a*x)*b^4
) - 15/4*A*a^2*x/(sqrt(b*x^2 + a*x)*b^3) - 35/16*B*a^3*log(2*b*x + a + 2*s
qrt(b*x^2 + a*x)*sqrt(b))/b^(9/2) + 15/8*A*a^2*log(2*b*x + a + 2*sqrt(b*x^
2 + a*x)*sqrt(b))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.35

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{1}{24}\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)B|b|}{b^6} - \frac{19\,Bab^{17}|b| - 6\,Ab^{18}|b|}{b^{23}}\right.\right. + \frac{5\,(7\,Ba^3|b| - 6\,Aa^2b|b|)\log\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2\right)}{16\,b^{\frac{11}{2}}} + \frac{4\,(Ba^4|b| - Aa^3b|b|)}{\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 + ab\right)b^{\frac{9}{2}}}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)*B*abs
(b)/b^6 - (19*B*a*b^17*abs(b) - 6*A*b^18*abs(b))/b^23) + 3*(29*B*a^2*b^17*
abs(b) - 18*A*a*b^18*abs(b))/b^23) + 5/16*(7*B*a^3*abs(b) - 6*A*a^2*b*abs(
b))*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(11/2) + 4*
(B*a^4*abs(b) - A*a^3*b*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*
b - a*b))^2 + a*b)*b^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=\int \frac{x^{5/2}\,(A+B\,x)}{(a+b\,x)^{3/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{15\sqrt{x}\,\sqrt{bx+a}\,a^2b-10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x+8\sqrt{x}\,\sqrt{bx+a}\,b^3x^2-15\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\sqrt{b}}{\sqrt{a}}\right)a^3}{24b^4}$$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^(3/2),x)
```

output
```
(15*sqrt(x)*sqrt(a + b*x)*a**2*b - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*x + 8*s
qrt(x)*sqrt(a + b*x)*b**3*x**2 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*s
qrt(b))/sqrt(a))*a**3)/(24*b**4)
```

3.328 $\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 120

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=-\frac{2(Ab-aB)x^{3/2}}{b^2\sqrt{a+bx}}+\frac{3(4Ab-5aB)\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{Bx^{3/2}\sqrt{a+bx}}{2b^2}-\frac{3a(4Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}$$

output
```
-2*(A*b-B*a)*x^(3/2)/b^2/(b*x+a)^(1/2)+3/4*(4*A*b-5*B*a)*x^(1/2)*(b*x+a)^(
1/2)/b^3+1/2*B*x^(3/2)*(b*x+a)^(1/2)/b^2-3/4*a*(4*A*b-5*B*a)*arctanh(b^(1/
2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{\sqrt{x}(-15a^2B+ab(12A-5Bx)+2b^2x(2A+Bx))}{4b^3\sqrt{a+bx}}+\frac{3a(-4Ab+5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{2b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(-15*a^2*B + a*b*(12*A - 5*B*x) + 2*b^2*x*(2*A + B*x)))/(4*b^3*Sq
rt[a + b*x]) + (3*a*(-4*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
 Sqrt[a + b*x])])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {87, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{2x^{5/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(4Ab-5aB)\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(4Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(4Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{ab}$$

$$\Big\downarrow \text{65}$$

$$\frac{2x^{5/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(4Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{ab}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x^{5/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(4Ab-5aB)\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{ab}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x^(5/2))/(a*b*Sqrt[a + b*x]) - ((4*A*b - 5*a*B)*((x^(3/2)*S
qrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b
]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/(a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

method	result
risch	$\frac{(2bBx+4Ab-7Ba)\sqrt{x}\,\sqrt{bx+a}}{4b^{3}}-\frac{a\left(12A\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)-\frac{15Ba\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}-\frac{16(Ab-Ba)\sqrt{b\left(x+\frac{a}{b}\right)^{2}-\left(x+\frac{a}{b}\right.}}{b\left(x+\frac{a}{b}\right)}\right.}{8b^{3}\sqrt{x}\,\sqrt{bx+a}}$
default	$-\frac{\left(-4B\,b^{\frac{5}{2}}x^{2}\sqrt{x(bx+a)}+12A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^{2}x-8A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x-15B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}bx+10B\sqrt{\right.}{8b^{\frac{7}{2}}\sqrt{}}$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*b*x+4*A*b-7*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^3-1/8/b^3*a*(12*A*b^(1/2
)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))-15*B*a*ln((1/2*a+b*x)/b^(1/2)+
(b*x^2+a*x)^(1/2))/b^(1/2)-16*(A*b-B*a)/b/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^
(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.10

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = \left[-\frac{3\left(5\,Ba^{3}-4\,Aa^{2}b+\left(5\,Ba^{2}b-4\,Aab^{2}\right)x\right)\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-}{8\left(b^{5}x+ab^{4}\right)}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b)*log(2*b
*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(2*B*b^3*x^2 - 15*B*a^2*b +
12*A*a*b^2 - (5*B*a*b^2 - 4*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^5*x + a*b^
4), -1/4*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(-b)*arc
tan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) - (2*B*b^3*x^2 - 15*B*a^2*b + 12*A*a*b
^2 - (5*B*a*b^2 - 4*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^5*x + a*b^4)]
```

Sympy [A] (verification not implemented)

Time = 14.91 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.52

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = A\left(\frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} + \frac{x^{\frac{3}{2}}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right) + B\left(-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}} - \frac{5\sqrt{a}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}} + \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right)$$

input
```
integrate(x**(3/2)*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
A*(3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sq
rt(a))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))) + B*(-15*a**(3/2)*
sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2*sqrt(1 + b*x
/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) + x**(5/2)/(2*s
qrt(a)*b*sqrt(1 + b*x/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(94) = 188$.

Time = 0.05 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.05

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = -\frac{(bx^2+ax)^{\frac{3}{2}}Ba}{b^4x^2+2\,ab^3x+a^2b^2} - \frac{3\,\sqrt{bx^2+ax}Ba^2}{b^4x+ab^3} + \frac{(bx^2+ax)^{\frac{3}{2}}A}{b^3x^2+2\,ab^2x+a^2b} + \frac{(bx^2+ax)^{\frac{3}{2}}B}{2\,(b^3x+ab^2)} + \frac{3\,\sqrt{bx^2+ax}Aa}{b^3x+ab^2} + \frac{15\,Ba^2\log\left(2\,x+\frac{a}{b}+\frac{2\,\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Aa\log\left(2\,x+\frac{a}{b}+\frac{2\,\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{2\,b^{\frac{5}{2}}} - \frac{3\,\sqrt{bx^2+ax}Ba}{4\,b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-(b*x^2 + a*x)^(3/2)*B*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) - 3*sqrt(b*x^2 +
a*x)*B*a^2/(b^4*x + a*b^3) + (b*x^2 + a*x)^(3/2)*A/(b^3*x^2 + 2*a*b^2*x +
a^2*b) + 1/2*(b*x^2 + a*x)^(3/2)*B/(b^3*x + a*b^2) + 3*sqrt(b*x^2 + a*x)*A
*a/(b^3*x + a*b^2) + 15/8*B*a^2*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b
))/b^(7/2) - 3/2*A*a*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b))/b^(5/2)
- 3/4*sqrt(b*x^2 + a*x)*B*a/b^3
```

Giac [A] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{1}{4}\,\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\,(bx+a)B|b|}{b^5} - \frac{9\,Bab^9|b|-4\,Ab^{10}|b|}{b^{14}}\right) - \frac{3\,(5\,Ba^2|b|-4\,Aab|b|)\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{8\,b^{\frac{9}{2}}} - \frac{4\,(Ba^3|b|-Aa^2b|b|)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{7}{2}}}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*B*abs(b)/b^5 - (9*B
*a*b^9*abs(b) - 4*A*b^10*abs(b))/b^14) - 3/8*(5*B*a^2*abs(b) - 4*A*a*b*abs
(b))*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(9/2) - 4*
(B*a^3*abs(b) - A*a^2*b*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*
b - a*b))^2 + a*b)*b^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=\int \frac{x^{3/2}\,(A+B\,x)}{{(a+b\,x)}^{3/2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{-3\sqrt{x}\,\sqrt{bx+a}\,ab+2\sqrt{x}\,\sqrt{bx+a}\,b^2x+3\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2}{4b^3}$$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^(3/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(a + b*x)*a*b + 2*sqrt(x)*sqrt(a + b*x)*b**2*x + 3*sqrt(
b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2)/(4*b**3)
```

3.329 $\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx=-\frac{2(Ab-aB)\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{B\sqrt{x}\sqrt{a+bx}}{b^2}+\frac{(2Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

output
```
-2*(A*b-B*a)*x^(1/2)/b^2/(b*x+a)^(1/2)+B*x^(1/2)*(b*x+a)^(1/2)/b^2+(2*A*b-
3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{\sqrt{x}(-2Ab+3aB+bBx)}{b^2\sqrt{a+bx}}+\frac{2(2Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[x]*(-2*A*b + 3*a*B + b*B*x))/(b^2*Sqrt[a + b*x]) + (2*(2*A*b - 3*a*B
)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {87, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{2x^{3/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(2Ab-3aB)\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{ab}$$

$$\Big\downarrow \; 60$$

$$\frac{2x^{3/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(2Ab-3aB)\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{ab}$$

$$\Big\downarrow \; 65$$

$$\frac{2x^{3/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(2Ab-3aB)\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{ab}$$

$$\Big\downarrow \; 219$$

$$\frac{2x^{3/2}(Ab-aB)}{ab\sqrt{a+bx}}-\frac{(2Ab-3aB)\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{ab}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x^(3/2))/(a*b*Sqrt[a + b*x]) - ((2*A*b - 3*a*B)*((Sqrt[x]*S
qrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(
a*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(67) = 134$.

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.80

method	result
risch	$\frac{B\sqrt{x}\,\sqrt{bx+a}}{b^{2}}+\frac{\left(2A\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)-\frac{3Ba\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}-\frac{4(Ab-Ba)\sqrt{b\left(x+\frac{a}{b}\right)^{2}-\left(x+\frac{a}{b}\right)a}}{b\left(x+\frac{a}{b}\right)}\right)\sqrt{x(bx+a)}}{2b^{2}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{\left(2A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b^{2}x-3B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)abx+2B\,b^{\frac{3}{2}}x\sqrt{x(bx+a)}+2A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)ab-4A}{2\sqrt{x(bx+a)}\,b^{\frac{5}{2}}\sqrt{bx+a}}$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B*x^(1/2)*(b*x+a)^(1/2)/b^2+1/2/b^2*(2*A*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b
*x^2+a*x)^(1/2))-3*B*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)-4
*(A*b-B*a)/b/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2))*(x*(b*x+a))^(1/2)/x^(1
/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx = \left[-\frac{(3\,Ba^2-2\,Aab+(3\,Bab-2\,Ab^2)x)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(B\right.}{2\,(b^4x+ab^3)}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x)*sqrt(b)*log(2*b*x + 2*s
qrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(B*b^2*x + 3*B*a*b - 2*A*b^2)*sqrt(b
*x + a)*sqrt(x))/(b^4*x + a*b^3), ((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2
)*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (B*b^2*x + 3*B*a*b
- 2*A*b^2)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx = A\left(\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right) + B\left(\frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right)$$

input
```
integrate(x**(1/2)*(B*x+A)/(b*x+a)**(3/2),x)
```

output
```
A*(2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1
 + b*x/a))) + B*(3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt
(b)*sqrt(x)/sqrt(a))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a)))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx^2+ax}Ba}{b^3x+ab^2} - \frac{2\sqrt{bx^2+ax}A}{b^2x+ab} - \frac{3\,Ba\log\left(2\,x+\frac{a}{b}+\frac{2\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{2\,b^{\frac{5}{2}}} + \frac{A\log\left(2\,x+\frac{a}{b}+\frac{2\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+ax}B}{b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x^2 + a*x)*B*a/(b^3*x + a*b^2) - 2*sqrt(b*x^2 + a*x)*A/(b^2*x + a
*b) - 3/2*B*a*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b))/b^(5/2) + A*log
(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b))/b^(3/2) + sqrt(b*x^2 + a*x)*B/b^
2
```

Giac [A] (verification not implemented)

Time = 15.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx = \frac{\sqrt{(bx+a)b-ab}\sqrt{bx+a}B|b|}{b^4} + \frac{(3\,Ba|b|-2\,Ab|b|)\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{2\,b^{\frac{7}{2}}} + \frac{4\,(Ba^2|b|-Aab|b|)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{5}{2}}}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*B*abs(b)/b^4 + 1/2*(3*B*a*abs(b) - 2
*A*b*abs(b))*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(7
/2) + 4*(B*a^2*abs(b) - A*a*b*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x
 + a)*b - a*b))^2 + a*b)*b^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx=\int \frac{\sqrt{x}\,(A+B\,x)}{{(a+b\,x)}^{3/2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x)^(3/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{3/2}}\,dx=\frac{\sqrt{x}\,\sqrt{bx+a}\,b-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a}{b^{2}}$$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^(3/2),x)
```

output
```
(sqrt(x)*sqrt(a + b*x)*b - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a)/b**2
```

3.330 $\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx=\frac{2(Ab-aB)\sqrt{x}}{ab\sqrt{a+bx}}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

output
```
2*(A*b-B*a)*x^(1/2)/a/b/(b*x+a)^(1/2)+2*B*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^
(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx=\frac{2(Ab-aB)\sqrt{x}}{ab\sqrt{a+bx}}-\frac{2B\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

output
```
(2*(A*b - a*B)*Sqrt[x])/(a*b*Sqrt[a + b*x]) - (2*B*Log[-(Sqrt[b]*Sqrt[x])
+ Sqrt[a + b*x]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {87, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{B\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{b}+\frac{2\sqrt{x}(Ab-aB)}{ab\sqrt{a+bx}}$$

$$\Big\downarrow \; 65$$

$$\frac{2B\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}+\frac{2\sqrt{x}(Ab-aB)}{ab\sqrt{a+bx}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\sqrt{x}(Ab-aB)}{ab\sqrt{a+bx}}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

output
```
(2*(A*b - a*B)*Sqrt[x])/(a*b*Sqrt[a + b*x]) + (2*B*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[a + b*x]])/b^(3/2)
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(48) = 96$.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\left(B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)abx+2A\,b^{\frac{3}{2}}\sqrt{x(bx+a)}+B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}-2Ba\sqrt{x(bx+a)}\,\sqrt{b}\right)\sqrt{x}}{a\sqrt{x(bx+a)}\,b^{\frac{3}{2}}\sqrt{bx+a}}$	121

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(B*ln(1/2*(2*(x*(b*x+a))^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a*b*x+2*A*b^(3/2)
*(x*(b*x+a))^(1/2)+B*ln(1/2*(2*(x*(b*x+a))^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))
*a^2-2*B*a*(x*(b*x+a))^(1/2)*b^(1/2))/a*x^(1/2)/(x*(b*x+a))^(1/2)/b^(3/2)/
(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx = \left[\frac{(Babx+Ba^2)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\,(Bab-Ab^2)\sqrt{bx+a}\sqrt{x}}{ab^3x+a^2b^2}\right.$$
$$\left.-\frac{2\left((Babx+Ba^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right)+(Bab-Ab^2)\sqrt{bx+a}\sqrt{x}\right)}{ab^3x+a^2b^2}\right]$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[((B*a*b*x + B*a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
a) - 2*(B*a*b - A*b^2)*sqrt(b*x + a)*sqrt(x))/(a*b^3*x + a^2*b^2), -2*((B*
a*b*x + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (B*a*b -
A*b^2)*sqrt(b*x + a)*sqrt(x))/(a*b^3*x + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2A}{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}+B\left(\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}b\sqrt{1+\frac{bx}{a}}}\right)$$

input
```
integrate((B*x+A)/x**(1/2)/(b*x+a)**(3/2),x)
```

output
```
2*A/(a*sqrt(b)*sqrt(a/(b*x) + 1)) + B*(2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b*
*(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 + b*x/a)))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx^2+ax}A}{abx+a^2} - \frac{2\sqrt{bx^2+ax}B}{b^2x+ab} + \frac{B\log\left(2x+\frac{a}{b}+\frac{2\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x^2 + a*x)*A/(a*b*x + a^2) - 2*sqrt(b*x^2 + a*x)*B/(b^2*x + a*b)
+ B*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b))/b^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(48) = 96.

Time = 15.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx = -\frac{B\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}|b|} - \frac{4\left(Ba\sqrt{b}-Ab^{\frac{3}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)|b|}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-B*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/(sqrt(b)*abs(b
)) - 4*(B*a*sqrt(b) - A*b^(3/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)
*b - a*b))^2 + a*b)*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx=\int \frac{A+B\,x}{\sqrt{x}\,{\left(a+b\,x\right)}^{3/2}} \,d x$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(a + b*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.42

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{3/2}}\,dx=\frac{2\sqrt{b}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)}{b}$$

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^(3/2),x)
```

output
```
(2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)))/b
```

3.331 $\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2A}{a\sqrt{x}\sqrt{a+bx}} - \frac{2(2Ab-aB)\sqrt{x}}{a^2\sqrt{a+bx}}$$

output
```
-2*A/a/x^(1/2)/(b*x+a)^(1/2)-2*(2*A*b-B*a)*x^(1/2)/a^2/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2(aA+2Abx-aBx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(a*A + 2*A*b*x - a*B*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(2Ab-aB)\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}dx}{a}-\frac{2A}{a\sqrt{x}\sqrt{a+bx}}$$

$$\Big\downarrow \; 48$$

$$-\frac{2\sqrt{x}(2Ab-aB)}{a^2\sqrt{a+bx}}-\frac{2A}{a\sqrt{x}\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*A)/(a*Sqrt[x]*Sqrt[a + b*x]) - (2*(2*A*b - a*B)*Sqrt[x])/(a^2*Sqrt[a +
 b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2(2Abx-Bax+Aa)}{\sqrt{x}\,\sqrt{bx+a}\,a^{2}}$	30
default	$-\frac{2(2Abx-Bax+Aa)}{\sqrt{x}\,\sqrt{bx+a}\,a^{2}}$	30
orering	$-\frac{2(2Abx-Bax+Aa)}{\sqrt{x}\,\sqrt{bx+a}\,a^{2}}$	30
risch	$-\frac{2A\sqrt{bx+a}}{a^{2}\sqrt{x}}-\frac{2(Ab-Ba)\sqrt{x}}{a^{2}\sqrt{bx+a}}$	41

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*A*b*x-B*a*x+A*a)/x^(1/2)/(b*x+a)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2\,(Aa-(Ba-2\,Ab)x)\sqrt{bx+a}\sqrt{x}}{a^2bx^2+a^3x}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*(A*a - (B*a - 2*A*b)*x)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = A\left(-\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}+1}}\right)+\frac{2B}{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}$$

input
```
integrate((B*x+A)/x**(3/2)/(b*x+a)**(3/2),x)
```

output
```
A*(-2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
) + 2*B/(a*sqrt(b)*sqrt(a/(b*x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = \frac{2\,Bx}{\sqrt{bx^2+axa}}-\frac{4\,Abx}{\sqrt{bx^2+axa^2}}-\frac{2\,A}{\sqrt{bx^2+axa}}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
2*B*x/(sqrt(b*x^2 + a*x)*a) - 4*A*b*x/(sqrt(b*x^2 + a*x)*a^2) - 2*A/(sqrt(
b*x^2 + a*x)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(40) = 80$.

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.27

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{2\sqrt{bx+a}Ab^2}{\sqrt{(bx+a)b-ab}a^2|b|} + \frac{4\,(B^2a^2b^3-2\,ABab^4+A^2b^5)}{\left(Ba\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{3}{2}}+Ba^2b^{\frac{5}{2}}-A\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{5}{2}}-Aab^{\frac{7}{2}}\right)}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*sqrt(b*x + a)*A*b^2/(sqrt((b*x + a)*b - a*b)*a^2*abs(b)) + 4*(B^2*a^2*b
^3 - 2*A*B*a*b^4 + A^2*b^5)/((B*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*
b - a*b))^2*b^(3/2) + B*a^2*b^(5/2) - A*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x
 + a)*b - a*b))^2*b^(5/2) - A*a*b^(7/2))*a*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = -\frac{\left(\frac{2\,A}{a\,b}+\frac{x\,(4\,A\,b-2\,B\,a)}{a^2\,b}\right)\,\sqrt{a+b\,x}}{x^{3/2}+\frac{a\,\sqrt{x}}{b}}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x)^(3/2)),x)
```

output
```
-((((2*A)/(a*b) + (x*(4*A*b - 2*B*a))/(a^2*b))*(a + b*x)^(1/2))/(x^(3/2) +
(a*x^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{bx+a}-2\sqrt{b}\,x}{ax}$$

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^(3/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(a + b*x) + sqrt(b)*x))/(a*x)
```

3.332 $\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx=-\frac{2A}{3ax^{3/2}\sqrt{a+bx}}-\frac{2(4Ab-3aB)}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{4(4Ab-3aB)\sqrt{a+bx}}{3a^3\sqrt{x}}$$

output
```
-2/3*A/a/x^(3/2)/(b*x+a)^(1/2)-2/3*(4*A*b-3*B*a)/a^2/x^(1/2)/(b*x+a)^(1/2)
+4/3*(4*A*b-3*B*a)*(b*x+a)^(1/2)/a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx=-\frac{2(-8Ab^2x^2+2abx(-2A+3Bx)+a^2(A+3Bx))}{3a^3x^{3/2}\sqrt{a+bx}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(-8*A*b^2*x^2 + 2*a*b*x*(-2*A + 3*B*x) + a^2*(A + 3*B*x)))/(3*a^3*x^(3
/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(4Ab-3aB)\int \frac{1}{x^{3/2}(a+bx)^{3/2}}dx}{3a}-\frac{2A}{3ax^{3/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(4Ab-3aB)\left(\frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{x}\sqrt{a+bx}}\right)}{3a}-\frac{2A}{3ax^{3/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{2}{a\sqrt{x}\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\right)(4Ab-3aB)}{3a}-\frac{2A}{3ax^{3/2}\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)*Sqrt[a + b*x]) - ((4*A*b - 3*a*B)*(2/(a*Sqrt[x]*Sqrt[a
 + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])))/(3*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2\left(-8A\,b^{2}x^{2}+6Bab\,x^{2}-4aAbx+3B\,a^{2}x+a^{2}A\right)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^{3}}$	52
default	$-\frac{2\left(-8A\,b^{2}x^{2}+6Bab\,x^{2}-4aAbx+3B\,a^{2}x+a^{2}A\right)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^{3}}$	52
orering	$-\frac{2\left(-8A\,b^{2}x^{2}+6Bab\,x^{2}-4aAbx+3B\,a^{2}x+a^{2}A\right)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^{3}}$	52
risch	$-\frac{2\sqrt{bx+a}\,\left(-5Abx+3Bax+Aa\right)}{3a^{3}x^{\frac{3}{2}}}+\frac{2b\left(Ab-Ba\right)\sqrt{x}}{a^{3}\sqrt{bx+a}}$	55

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-8*A*b^2*x^2+6*B*a*b*x^2-4*A*a*b*x+3*B*a^2*x+A*a^2)/x^(3/2)/(b*x+a)^
(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx = -\frac{2\,(Aa^2+2\,(3\,Bab-4\,Ab^2)x^2+(3\,Ba^2-4\,Aab)x)\sqrt{bx+a}\sqrt{x}}{3\,(a^3bx^3+a^4x^2)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(A*a^2 + 2*(3*B*a*b - 4*A*b^2)*x^2 + (3*B*a^2 - 4*A*a*b)*x)*sqrt(b*x
+ a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(76) = 152.

Time = 11.74 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.19

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx = A\left(-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{}}{3a^5b^4x+6a^4b^5x}\right.$$
$$\left.+B\left(-\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}+1}}\right)\right)$$

input
```
integrate((B*x+A)/x**(5/2)/(b*x+a)**(3/2),x)
```

output
```
A*(-2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x +
6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) +
1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x*
*3*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
) + B*(-2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) +
 1)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx = -\frac{4\,Bbx}{\sqrt{bx^2+ax}a^2} + \frac{16\,Ab^2x}{3\,\sqrt{bx^2+ax}a^3} - \frac{2\,B}{\sqrt{bx^2+ax}a} + \frac{8\,Ab}{3\,\sqrt{bx^2+ax}a^2} - \frac{2\,A}{3\,\sqrt{bx^2+ax}ax}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-4*B*b*x/(sqrt(b*x^2 + a*x)*a^2) + 16/3*A*b^2*x/(sqrt(b*x^2 + a*x)*a^3) -
2*B/(sqrt(b*x^2 + a*x)*a) + 8/3*A*b/(sqrt(b*x^2 + a*x)*a^2) - 2/3*A/(sqrt(
b*x^2 + a*x)*a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(65) = 130$.

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.59

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx = -\frac{2\,\sqrt{bx+a}\left(\frac{(3\,Ba^3b^3|b|-5\,Aa^2b^4|b|)(bx+a)}{a^5b^2} - \frac{3\,(Ba^4b^3|b|-2\,Aa^3b^4|b|)}{a^5b^2}\right)}{3\,((bx+a)b-ab)^{\frac{3}{2}}} - \frac{4\,(B^2a^2b^5 - 2\,ABab^6 + A^2b^7)}{\left(Ba\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 b^{\frac{5}{2}} + Ba^2b^{\frac{7}{2}} - A\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 b^{\frac{7}{2}} - Aab^{\frac{9}{2}}\right)}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + a)*((3*B*a^3*b^3*abs(b) - 5*A*a^2*b^4*abs(b))*(b*x + a)/(a
^5*b^2) - 3*(B*a^4*b^3*abs(b) - 2*A*a^3*b^4*abs(b))/(a^5*b^2))/((b*x + a)*
b - a*b)^(3/2) - 4*(B^2*a^2*b^5 - 2*A*B*a*b^6 + A^2*b^7)/((B*a*(sqrt(b*x +
 a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(5/2) + B*a^2*b^(7/2) - A*(sqrt
(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) - A*a*b^(9/2))*a^2*
abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{3\,a\,b}+\frac{x\,\left(6\,B\,a^2-8\,A\,a\,b\right)}{3\,a^3\,b}-\frac{x^2\,\left(16\,A\,b^2-12\,B\,a\,b\right)}{3\,a^3\,b}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x)^(3/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(3*a*b) + (x*(6*B*a^2 - 8*A*a*b))/(3*a^3*b) - (x^
2*(16*A*b^2 - 12*B*a*b))/(3*a^3*b)))/(x^(5/2) + (a*x^(3/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a}{3}+\frac{4\sqrt{x}\,\sqrt{bx+a}\,bx}{3}-\frac{4\sqrt{b}\,b\,x^2}{3}}{a^2x^2}$$

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^(3/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(a + b*x)*a + 2*sqrt(x)*sqrt(a + b*x)*b*x - 2*sqrt(b)*b
*x**2))/(3*a**2*x**2)
```

3.333 $\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx=-\frac{2A}{5ax^{5/2}\sqrt{a+bx}}-\frac{2(6Ab-5aB)}{5a^2x^{3/2}\sqrt{a+bx}}+\frac{8(6Ab-5aB)\sqrt{a+bx}}{15a^3x^{3/2}}-\frac{16b(6Ab-5aB)\sqrt{a+bx}}{15a^4\sqrt{x}}$$

output
```
-2/5*A/a/x^(5/2)/(b*x+a)^(1/2)-2/5*(6*A*b-5*B*a)/a^2/x^(3/2)/(b*x+a)^(1/2)
+8/15*(6*A*b-5*B*a)*(b*x+a)^(1/2)/a^3/x^(3/2)-16/15*b*(6*A*b-5*B*a)*(b*x+a
)^(1/2)/a^4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx=-\frac{2(48Ab^3x^3+8ab^2x^2(3A-5Bx)+a^3(3A+5Bx)-2a^2bx(3A+10Bx))}{15a^4x^{5/2}\sqrt{a+bx}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(48*A*b^3*x^3 + 8*a*b^2*x^2*(3*A - 5*B*x) + a^3*(3*A + 5*B*x) - 2*a^2*
b*x*(3*A + 10*B*x)))/(15*a^4*x^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(6Ab-5aB)\int \frac{1}{x^{5/2}(a+bx)^{3/2}}dx}{5a}-\frac{2A}{5ax^{5/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(6Ab-5aB)\left(\frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{5a}-\frac{2A}{5ax^{5/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(6Ab-5aB)\left(\frac{4\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{5a}-\frac{2A}{5ax^{5/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{4\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)(6Ab-5aB)}{5a}-\frac{2A}{5ax^{5/2}\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*A)/(5*a*x^(5/2)*Sqrt[a + b*x]) - ((6*A*b - 5*a*B)*(2/(a*x^(3/2)*Sqrt[a
 + b*x]) + (4*((-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a
^2*Sqrt[x])))/a))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2\left(48A\,b^{3}x^{3}-40Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-20B\,a^{2}b\,x^{2}-6a^{2}Abx+5B\,a^{3}x+3a^{3}A\right)}{15x^{\frac{5}{2}}\sqrt{bx+a}\,a^{4}}$	77
default	$-\frac{2\left(48A\,b^{3}x^{3}-40Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-20B\,a^{2}b\,x^{2}-6a^{2}Abx+5B\,a^{3}x+3a^{3}A\right)}{15x^{\frac{5}{2}}\sqrt{bx+a}\,a^{4}}$	77
orering	$-\frac{2\left(48A\,b^{3}x^{3}-40Ba\,b^{2}x^{3}+24aA\,b^{2}x^{2}-20B\,a^{2}b\,x^{2}-6a^{2}Abx+5B\,a^{3}x+3a^{3}A\right)}{15x^{\frac{5}{2}}\sqrt{bx+a}\,a^{4}}$	77
risch	$-\frac{2\sqrt{bx+a}\,\left(33A\,b^{2}x^{2}-25Bab\,x^{2}-9aAbx+5B\,a^{2}x+3a^{2}A\right)}{15a^{4}x^{\frac{5}{2}}}-\frac{2b^{2}\left(Ab-Ba\right)\sqrt{x}}{a^{4}\sqrt{bx+a}}$	80

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(48*A*b^3*x^3-40*B*a*b^2*x^3+24*A*a*b^2*x^2-20*B*a^2*b*x^2-6*A*a^2*b
*x+5*B*a^3*x+3*A*a^3)/x^(5/2)/(b*x+a)^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{2\,(3\,Aa^{3}-8\,(5\,Bab^{2}-6\,Ab^{3})x^{3}-4\,(5\,Ba^{2}b-6\,Aab^{2})x^{2}+(5\,Ba^{3}-6\,Aa^{2}b)x)\sqrt{bx+a}\sqrt{x}}{15\,(a^{4}bx^{4}+a^{5}x^{3})}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*a^3 - 8*(5*B*a*b^2 - 6*A*b^3)*x^3 - 4*(5*B*a^2*b - 6*A*a*b^2)*x
^2 + (5*B*a^3 - 6*A*a^2*b)*x)*sqrt(b*x + a)*sqrt(x)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(110) = 220$.

Time = 31.47 (sec) , antiderivative size = 573, normalized size of antiderivative = 5.03

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx = A\left(-\frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5} - \frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+\dots}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{1\dots}}\right.$$
$$+B\left(-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3} + \frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3} + \frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3} + \frac{16\dots}{3a^5b^4x\dots}\right.$$

input
```
integrate((B*x+A)/x**(7/2)/(b*x+a)**(3/2),x)
```

output
```
A*(-2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x
**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqr
t(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4
 + 5*a**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b
**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) -
80*a*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x*
*3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b
*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a
**4*b**12*x**5)) + B*(-2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x +
6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1
)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x
**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3
) + 16*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2
+ 3*a**3*b**6*x**3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{16\,Bb^2x}{3\,\sqrt{bx^2+ax}a^3} - \frac{32\,Ab^3x}{5\,\sqrt{bx^2+ax}a^4} + \frac{8\,Bb}{3\,\sqrt{bx^2+ax}a^2} - \frac{16\,Ab^2}{5\,\sqrt{bx^2+ax}a^3} - \frac{2\,B}{3\,\sqrt{bx^2+ax}ax} + \frac{4\,Ab}{5\,\sqrt{bx^2+ax}a^2x} - \frac{2\,A}{5\,\sqrt{bx^2+ax}ax^2}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
16/3*B*b^2*x/(sqrt(b*x^2 + a*x)*a^3) - 32/5*A*b^3*x/(sqrt(b*x^2 + a*x)*a^4
) + 8/3*B*b/(sqrt(b*x^2 + a*x)*a^2) - 16/5*A*b^2/(sqrt(b*x^2 + a*x)*a^3) -
 2/3*B/(sqrt(b*x^2 + a*x)*a*x) + 4/5*A*b/(sqrt(b*x^2 + a*x)*a^2*x) - 2/5*A
/(sqrt(b*x^2 + a*x)*a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(90) = 180$.

Time = 0.17 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{2\sqrt{bx+a}\left((bx+a)\left(\frac{(25\,Ba^6b^7-33\,Aa^5b^8)(bx+a)}{a^9b^2|b|}-\frac{5\,(11\,Ba^7b^7-15\,Aa^6b^8)}{a^9b^2|b|}\right)+\frac{15\,(2\,Ba^8b^7-3\,Aa}{a^9b^2|b|}\right.}{15\,((bx+a)b-ab)^{\frac{5}{2}}}$$
$$+\frac{4\,(B^2a^2b^7-2\,ABab^8+A^2b^9)}{\left(Ba\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{7}{2}}+Ba^2b^{\frac{9}{2}}-A\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{9}{2}}-Aab^{\frac{11}{2}}\right.}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2/15*sqrt(b*x + a)*((b*x + a)*((25*B*a^6*b^7 - 33*A*a^5*b^8)*(b*x + a)/(a^
9*b^2*abs(b)) - 5*(11*B*a^7*b^7 - 15*A*a^6*b^8)/(a^9*b^2*abs(b))) + 15*(2*
B*a^8*b^7 - 3*A*a^7*b^8)/(a^9*b^2*abs(b)))/((b*x + a)*b - a*b)^(5/2) + 4*(
B^2*a^2*b^7 - 2*A*B*a*b^8 + A^2*b^9)/((B*a*(sqrt(b*x + a)*sqrt(b) - sqrt((
b*x + a)*b - a*b))^2*b^(7/2) + B*a^2*b^(9/2) - A*(sqrt(b*x + a)*sqrt(b) -
sqrt((b*x + a)*b - a*b))^2*b^(9/2) - A*a*b^(11/2))*a^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{5\,a\,b}+\frac{8\,x^2\,(6\,A\,b-5\,B\,a)}{15\,a^3}+\frac{x^3\,(96\,A\,b^3-80\,B\,a\,b^2)}{15\,a^4\,b}+\frac{x\,(10\,B\,a^3-12\,A\,a^2\,b)}{15\,a^4\,b}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x)^(3/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(5*a*b) + (8*x^2*(6*A*b - 5*B*a))/(15*a^3) + (x^3
*(96*A*b^3 - 80*B*a*b^2))/(15*a^4*b) + (x*(10*B*a^3 - 12*A*a^2*b))/(15*a^4
*b)))/(x^(7/2) + (a*x^(5/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^2}{5}+\frac{8\sqrt{x}\,\sqrt{bx+a}\,abx}{15}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,b^2x^2}{15}+\frac{16\sqrt{b}\,b^2x^3}{15}}{a^3x^3}$$

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^(3/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(a + b*x)*a**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b*x - 8*sq
rt(x)*sqrt(a + b*x)*b**2*x**2 + 8*sqrt(b)*b**2*x**3))/(15*a**3*x**3)
```

3.334 $\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 147

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = -\frac{2A}{7ax^{7/2}\sqrt{a+bx}} - \frac{2(8Ab-7aB)}{7a^2x^{5/2}\sqrt{a+bx}} + \frac{12(8Ab-7aB)\sqrt{a+bx}}{35a^3x^{5/2}} - \frac{16b(8Ab-7aB)\sqrt{a+bx}}{35a^4x^{3/2}} + \frac{32b^2(8Ab-7aB)\sqrt{a+bx}}{35a^5\sqrt{x}}$$

output
```
-2/7*A/a/x^(7/2)/(b*x+a)^(1/2)-2/7*(8*A*b-7*B*a)/a^2/x^(5/2)/(b*x+a)^(1/2)
+12/35*(8*A*b-7*B*a)*(b*x+a)^(1/2)/a^3/x^(5/2)-16/35*b*(8*A*b-7*B*a)*(b*x+
a)^(1/2)/a^4/x^(3/2)+32/35*b^2*(8*A*b-7*B*a)*(b*x+a)^(1/2)/a^5/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = -\frac{2(-128Ab^4x^4+16ab^3x^3(-4A+7Bx)+8a^2b^2x^2(2A+7Bx)-2a^3bx(4A+7Bx)+a^4(5A+7Bx))}{35a^5x^{7/2}\sqrt{a+bx}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*(-128*A*b^4*x^4 + 16*a*b^3*x^3*(-4*A + 7*B*x) + 8*a^2*b^2*x^2*(2*A + 7
*B*x) - 2*a^3*b*x*(4*A + 7*B*x) + a^4*(5*A + 7*B*x)))/(35*a^5*x^(7/2)*Sqrt
[a + b*x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(8Ab-7aB)\int \frac{1}{x^{7/2}(a+bx)^{3/2}}dx}{7a}-\frac{2A}{7ax^{7/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-7aB)\left(\frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{7a}-\frac{2A}{7ax^{7/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-7aB)\left(\frac{6\left(-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{7a}-\frac{2A}{7ax^{7/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-7aB)\left(\frac{6\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{7a}-\frac{2A}{7ax^{7/2}\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{6\left(-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)(8Ab-7aB)}{7a}-\frac{2A}{7ax^{7/2}\sqrt{a+bx}}$$

input
```
Int[(A + B*x)/(x^(9/2)*(a + b*x)^(3/2)),x]
```

output
```
(-2*A)/(7*a*x^(7/2)*Sqrt[a + b*x]) - ((8*A*b - 7*a*B)*(2/(a*x^(5/2)*Sqrt[a
 + b*x]) + (6*((-2*Sqrt[a + b*x])/(5*a*x^(5/2)) - (4*b*((-2*Sqrt[a + b*x])
/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/(5*a)))/a))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{2\left(-128 A\,b^{4} x^{4}+112 B a\,b^{3} x^{4}-64 A a\,b^{3} x^{3}+56 B\,a^{2} b^{2} x^{3}+16 A\,a^{2} b^{2} x^{2}-14 B\,a^{3} b\,x^{2}-8 A\,a^{3} b x+7 B\,a^{4} x+5 A\,a^{4}\right)}{35 x^{\frac{7}{2}} \sqrt{b x+a}\,a^{5}}$	101
default	$-\frac{2\left(-128 A\,b^{4} x^{4}+112 B a\,b^{3} x^{4}-64 A a\,b^{3} x^{3}+56 B\,a^{2} b^{2} x^{3}+16 A\,a^{2} b^{2} x^{2}-14 B\,a^{3} b\,x^{2}-8 A\,a^{3} b x+7 B\,a^{4} x+5 A\,a^{4}\right)}{35 x^{\frac{7}{2}} \sqrt{b x+a}\,a^{5}}$	101
orering	$-\frac{2\left(-128 A\,b^{4} x^{4}+112 B a\,b^{3} x^{4}-64 A a\,b^{3} x^{3}+56 B\,a^{2} b^{2} x^{3}+16 A\,a^{2} b^{2} x^{2}-14 B\,a^{3} b\,x^{2}-8 A\,a^{3} b x+7 B\,a^{4} x+5 A\,a^{4}\right)}{35 x^{\frac{7}{2}} \sqrt{b x+a}\,a^{5}}$	101
risch	$-\frac{2 \sqrt{b x+a}\,\left(-93 A\,b^{3} x^{3}+77 B a\,b^{2} x^{3}+29 a A\,b^{2} x^{2}-21 B\,a^{2} b\,x^{2}-13 a^{2} A b x+7 B\,a^{3} x+5 a^{3} A\right)}{35 a^{5} x^{\frac{7}{2}}}+\frac{2 \left(A b-B a\right) b^{3} \sqrt{x}}{a^{5} \sqrt{b x+a}}$	104

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-128*A*b^4*x^4+112*B*a*b^3*x^4-64*A*a*b^3*x^3+56*B*a^2*b^2*x^3+16*A
*a^2*b^2*x^2-14*B*a^3*b*x^2-8*A*a^3*b*x+7*B*a^4*x+5*A*a^4)/x^(7/2)/(b*x+a)
^(1/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx =$$

$$-\frac{2\,(5\,Aa^4+16\,(7\,Bab^3-8\,Ab^4)x^4+8\,(7\,Ba^2b^2-8\,Aab^3)x^3-2\,(7\,Ba^3b-8\,Aa^2b^2)x^2+(7\,Ba^4-8\,Aa^3b)}{35\,(a^5bx^5+a^6x^4)}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/35*(5*A*a^4 + 16*(7*B*a*b^3 - 8*A*b^4)*x^4 + 8*(7*B*a^2*b^2 - 8*A*a*b^3
)*x^3 - 2*(7*B*a^3*b - 8*A*a^2*b^2)*x^2 + (7*B*a^4 - 8*A*a^3*b)*x)*sqrt(b*
x + a)*sqrt(x)/(a^5*b*x^5 + a^6*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(144) = 288$.

Time = 80.67 (sec) , antiderivative size = 1008, normalized size of antiderivative = 6.86

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(9/2)/(b*x+a)**(3/2),x)
```

output
```
A*(-10*a**7*b**(33/2)*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3 + 140*a**8*b**
17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**7)
- 14*a**6*b**(35/2)*x*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3 + 140*a**8*b**
17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**7)
- 14*a**5*b**(37/2)*x**2*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3 + 140*a**8*
b**17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**
7) + 70*a**4*b**(39/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3 + 140*a*
*8*b**17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a**5*b**20*
x**7) + 560*a**3*b**(41/2)*x**4*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3 + 14
0*a**8*b**17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a**5*b*
*20*x**7) + 1120*a**2*b**(43/2)*x**5*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3
 + 140*a**8*b**17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a*
*5*b**20*x**7) + 896*a*b**(45/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**
3 + 140*a**8*b**17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a
**5*b**20*x**7) + 256*b**(47/2)*x**7*sqrt(a/(b*x) + 1)/(35*a**9*b**16*x**3
 + 140*a**8*b**17*x**4 + 210*a**7*b**18*x**5 + 140*a**6*b**19*x**6 + 35*a*
*5*b**20*x**7)) + B*(-2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2
 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*
b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 +
15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = -\frac{32\,Bb^3x}{5\,\sqrt{bx^2+ax}a^4}+\frac{256\,Ab^4x}{35\,\sqrt{bx^2+ax}a^5} - \frac{16\,Bb^2}{5\,\sqrt{bx^2+ax}a^3}+\frac{128\,Ab^3}{35\,\sqrt{bx^2+ax}a^4}+\frac{4\,Bb}{5\,\sqrt{bx^2+ax}a^2x}-\frac{32\,Ab^2}{35\,\sqrt{bx^2+ax}a^3x} - \frac{2\,B}{5\,\sqrt{bx^2+ax}ax^2}+\frac{16\,Ab}{35\,\sqrt{bx^2+ax}a^2x^2}-\frac{2\,A}{7\,\sqrt{bx^2+ax}ax^3}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
-32/5*B*b^3*x/(sqrt(b*x^2 + a*x)*a^4) + 256/35*A*b^4*x/(sqrt(b*x^2 + a*x)*
a^5) - 16/5*B*b^2/(sqrt(b*x^2 + a*x)*a^3) + 128/35*A*b^3/(sqrt(b*x^2 + a*x
)*a^4) + 4/5*B*b/(sqrt(b*x^2 + a*x)*a^2*x) - 32/35*A*b^2/(sqrt(b*x^2 + a*x
)*a^3*x) - 2/5*B/(sqrt(b*x^2 + a*x)*a*x^2) + 16/35*A*b/(sqrt(b*x^2 + a*x)*
a^2*x^2) - 2/7*A/(sqrt(b*x^2 + a*x)*a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(117) = 234$.

Time = 0.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.99

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = -\frac{2\,\Big((bx+a)\Big((bx+a)\Big(\frac{(77\,Ba^{10}b^9|b|-93\,Aa^9b^{10}|b|)(bx+a)}{a^{14}b^4}-\frac{28\,(9\,Ba^{11}b^9|b|-11\,Aa^{10}b^{10}|b|)}{a^{14}b^4}\Big)+\frac{70\,(4\,Ba^{12}b^9|b|-5\,Aa^{11}b^{10}|b|)}{a^{14}b^4}\Big)}{35\,((bx+a)b-ab)^{\frac{7}{2}}} - \frac{4\,(B^2a^2b^9-2\,ABab^{10}+A^2b^{11})}{\Big(Ba\big(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\big)^2b^{\frac{9}{2}}+Ba^2b^{\frac{11}{2}}-A\big(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\big)^2b^{\frac{11}{2}}-Aab^{\frac{13}{2}}}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2/35*((b*x + a)*((b*x + a)*((77*B*a^10*b^9*abs(b) - 93*A*a^9*b^10*abs(b))
*(b*x + a)/(a^14*b^4) - 28*(9*B*a^11*b^9*abs(b) - 11*A*a^10*b^10*abs(b))/(
a^14*b^4)) + 70*(4*B*a^12*b^9*abs(b) - 5*A*a^11*b^10*abs(b))/(a^14*b^4)) -
 35*(3*B*a^13*b^9*abs(b) - 4*A*a^12*b^10*abs(b))/(a^14*b^4))*sqrt(b*x + a)
/((b*x + a)*b - a*b)^(7/2) - 4*(B^2*a^2*b^9 - 2*A*B*a*b^10 + A^2*b^11)/((B
*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(9/2) + B*a^2*b^(
11/2) - A*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(11/2) - A
*a*b^(13/2))*a^4*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A}{7\,a\,b}+\frac{4\,x^2\,(8\,A\,b-7\,B\,a)}{35\,a^3}-\frac{x^4\,(256\,A\,b^4-224\,B\,a\,b^3)}{35\,a^5\,b}-\frac{16\,b\,x^3\,(8\,A\,b-7\,B\,a)}{35\,a^4}+\frac{x\,(14\,B\,a^4-16\,A\,a^3\,b)}{35\,a^5\,b}\right)}{x^{9/2}+\frac{a\,x^{7/2}}{b}}$$

input
```
int((A + B*x)/(x^(9/2)*(a + b*x)^(3/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(7*a*b) + (4*x^2*(8*A*b - 7*B*a))/(35*a^3) - (x^4
*(256*A*b^4 - 224*B*a*b^3))/(35*a^5*b) - (16*b*x^3*(8*A*b - 7*B*a))/(35*a^
4) + (x*(14*B*a^4 - 16*A*a^3*b))/(35*a^5*b)))/(x^(9/2) + (a*x^(7/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{3/2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{bx+a}\,a^3}{7}+\frac{12\sqrt{x}\,\sqrt{bx+a}\,a^2bx}{35}-\frac{16\sqrt{x}\,\sqrt{bx+a}\,a\,b^2x^2}{35}+\frac{32\sqrt{x}\,\sqrt{bx+a}\,b^3x^3}{35}-\frac{32\sqrt{b}\,b^3x^4}{35}}{a^4x^4}$$

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^(3/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(a + b*x)*a**3 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b*x - 8
*sqrt(x)*sqrt(a + b*x)*a*b**2*x**2 + 16*sqrt(x)*sqrt(a + b*x)*b**3*x**3 -
16*sqrt(b)*b**3*x**4))/(35*a**4*x**4)
```

3.335 $\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 185

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab-aB)x^{7/2}}{3b^2(a+bx)^{3/2}} - \frac{2(7Ab-10aB)x^{5/2}}{3b^3\sqrt{a+bx}} - \frac{35a(2Ab-3aB)\sqrt{x}\sqrt{a+bx}}{8b^5} + \frac{35(2Ab-3aB)x^{3/2}\sqrt{a+bx}}{12b^4} + \frac{Bx^{5/2}\sqrt{a+bx}}{3b^3} + \frac{35a^2(2Ab-3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{11/2}}$$

output

```
-2/3*(A*b-B*a)*x^(7/2)/b^2/(b*x+a)^(3/2)-2/3*(7*A*b-10*B*a)*x^(5/2)/b^3/(b
*x+a)^(1/2)-35/8*a*(2*A*b-3*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^5+35/12*(2*A*b-3*
B*a)*x^(3/2)*(b*x+a)^(1/2)/b^4+1/3*B*x^(5/2)*(b*x+a)^(1/2)/b^3+35/8*a^2*(2
*A*b-3*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.77

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{x}(315a^4B-210a^3b(A-2Bx)+4b^4x^3(3A+2Bx)-6ab^3x^2(7A+3Bx)+7a^2b^2}{24b^5(a+bx)^{3/2}}$$
$$+\frac{35a^2(-2Ab+3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{4b^{11/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*(315*a^4*B - 210*a^3*b*(A - 2*B*x) + 4*b^4*x^3*(3*A + 2*B*x) - 6*
a*b^3*x^2*(7*A + 3*B*x) + 7*a^2*b^2*x*(-40*A + 9*B*x)))/(24*b^5*(a + b*x)^
(3/2)) + (35*a^2*(-2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqr
t[a + b*x])])/(4*b^(11/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {87, 57, 60, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-3aB)\int \frac{x^{7/2}}{(a+bx)^{3/2}}dx}{ab}$$

$$\Big\downarrow \; 57$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-3aB)\left(\frac{7\int \frac{x^{5/2}}{\sqrt{a+bx}}dx}{b}-\frac{2x^{7/2}}{b\sqrt{a+bx}}\right)}{ab}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-3aB)\left(\frac{7\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\int\frac{x^{3/2}}{\sqrt{a+bx}}dx}{6b}\right)}{b}-\frac{2x^{7/2}}{b\sqrt{a+bx}}\right)}{ab}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-3aB)\left(\frac{7\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int\frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{6b}\right)}{b}-\frac{2x^{7/2}}{b\sqrt{a+bx}}\right)}{ab}$$

$$\Big\downarrow \ 60$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-3aB)\left(\frac{7\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{b}-\frac{2x^{7/2}}{b\sqrt{a+bx}}\right)}{ab}$$

$$\Big\downarrow \ 65$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}} - \frac{(2Ab-3aB)\left(\frac{7\left(\frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b} - \frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{6b}\right)}{b} - \frac{2x^{7/2}}{b\sqrt{a+bx}}\right)}{ab}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2x^{9/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-3aB)\left(\frac{7\left(\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{6b}\right)}{b}-\frac{2x^{7/2}}{b\sqrt{a+bx}}\right)}{ab}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x)^(3/2)) - ((2*A*b - 3*a*B)*((-2*x^
(7/2))/(b*Sqrt[a + b*x]) + (7*((x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a*((x^(3
/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/(4*b)))/(6*b)))/b))/(a*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{\left(-8b^{2}Bx^{2}-12Ab^{2}x+34Babx+66abA-123a^{2}B\right)\sqrt{x}\,\sqrt{bx+a}}{24b^{5}}+\frac{a^{2}\left(70A\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)-\frac{105Ba\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+}\right.}{\sqrt{b}}\right.}{}$
default	$\frac{\left(16B\,b^{\frac{9}{2}}x^{4}\sqrt{x(bx+a)}+24A\,b^{\frac{9}{2}}x^{3}\sqrt{x(bx+a)}-36Ba\,b^{\frac{7}{2}}x^{3}\sqrt{x(bx+a)}+210A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}b^{3}x^{2}-84Aa\,b^{\frac{7}{2}}x^{2}\sqrt{x(ba}\right.}{}$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*B*b^2*x^2-12*A*b^2*x+34*B*a*b*x+66*A*a*b-123*B*a^2)*x^(1/2)*(b*x
+a)^(1/2)/b^5+1/16*a^2/b^5*(70*A*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))-105*B*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)-32*(4*A
*b-5*B*a)/b/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2)+16*a^2*(A*b-B*a)/b^2*(2/
3/a/(x+a/b)^2*(b*(x+a/b)^2-(x+a/b)*a)^(1/2)+4/3*b/a^2/(x+a/b)*(b*(x+a/b)^2
-(x+a/b)*a)^(1/2)))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.28

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \left[-\frac{105\,(3\,Ba^5-2\,Aa^4b+(3\,Ba^3b^2-2\,Aa^2b^3)x^2+2\,(3\,Ba^4b-2\,Aa^3b^2)x)\sqrt{b}\log\left(2\right.}{}\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(105*(3*B*a^5 - 2*A*a^4*b + (3*B*a^3*b^2 - 2*A*a^2*b^3)*x^2 + 2*(3*
B*a^4*b - 2*A*a^3*b^2)*x)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt
(x) + a) - 2*(8*B*b^5*x^4 + 315*B*a^4*b - 210*A*a^3*b^2 - 6*(3*B*a*b^4 - 2
*A*b^5)*x^3 + 21*(3*B*a^2*b^3 - 2*A*a*b^4)*x^2 + 140*(3*B*a^3*b^2 - 2*A*a^
2*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^8*x^2 + 2*a*b^7*x + a^2*b^6), 1/24*(10
5*(3*B*a^5 - 2*A*a^4*b + (3*B*a^3*b^2 - 2*A*a^2*b^3)*x^2 + 2*(3*B*a^4*b -
2*A*a^3*b^2)*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (8*B*b^5
*x^4 + 315*B*a^4*b - 210*A*a^3*b^2 - 6*(3*B*a*b^4 - 2*A*b^5)*x^3 + 21*(3*B
*a^2*b^3 - 2*A*a*b^4)*x^2 + 140*(3*B*a^3*b^2 - 2*A*a^2*b^3)*x)*sqrt(b*x +
a)*sqrt(x))/(b^8*x^2 + 2*a*b^7*x + a^2*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(145) = 290$.

Time = 0.04 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.25

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{Bx^6}{3\,(bx^2+ax)^{\frac{3}{2}}b} - \frac{3\,Bax^5}{4\,(bx^2+ax)^{\frac{3}{2}}b^2} + \frac{Ax^5}{2\,(bx^2+ax)^{\frac{3}{2}}b} + \frac{35\,Ba^3x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b} + \frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2} - \frac{2\,x}{\sqrt{bx^2+ax}ab} - \frac{1}{\sqrt{bx^2+ax}b^2}\right)}{16\,b^3} - \frac{35\,Aa^2x\left(\frac{3\,x^2}{(bx^2+ax)^{\frac{3}{2}}b} + \frac{ax}{(bx^2+ax)^{\frac{3}{2}}b^2} - \frac{2\,x}{\sqrt{bx^2+ax}ab} - \frac{1}{\sqrt{bx^2+ax}b^2}\right)}{24\,b^2} + \frac{21\,Ba^2x^4}{8\,(bx^2+ax)^{\frac{3}{2}}b^3} - \frac{7\,Aax^4}{4\,(bx^2+ax)^{\frac{3}{2}}b^2} + \frac{35\,Ba^3x}{4\,\sqrt{bx^2+ax}b^5} - \frac{35\,Aa^2x}{6\,\sqrt{bx^2+ax}b^4} - \frac{105\,Ba^3\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{16\,b^{\frac{11}{2}}} + \frac{35\,Aa^2\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{8\,b^{\frac{9}{2}}} + \frac{35\,\sqrt{bx^2+ax}Ba^2}{8\,b^5} - \frac{35\,\sqrt{bx^2+ax}Aa}{12\,b^4}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*B*x^6/((b*x^2 + a*x)^(3/2)*b) - 3/4*B*a*x^5/((b*x^2 + a*x)^(3/2)*b^2)
+ 1/2*A*x^5/((b*x^2 + a*x)^(3/2)*b) + 35/16*B*a^3*x*(3*x^2/((b*x^2 + a*x)^
(3/2)*b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1
/(sqrt(b*x^2 + a*x)*b^2))/b^3 - 35/24*A*a^2*x*(3*x^2/((b*x^2 + a*x)^(3/2)*
b) + a*x/((b*x^2 + a*x)^(3/2)*b^2) - 2*x/(sqrt(b*x^2 + a*x)*a*b) - 1/(sqrt
(b*x^2 + a*x)*b^2))/b^2 + 21/8*B*a^2*x^4/((b*x^2 + a*x)^(3/2)*b^3) - 7/4*A
*a*x^4/((b*x^2 + a*x)^(3/2)*b^2) + 35/4*B*a^3*x/(sqrt(b*x^2 + a*x)*b^5) -
35/6*A*a^2*x/(sqrt(b*x^2 + a*x)*b^4) - 105/16*B*a^3*log(2*b*x + a + 2*sqrt
(b*x^2 + a*x)*sqrt(b))/b^(11/2) + 35/8*A*a^2*log(2*b*x + a + 2*sqrt(b*x^2
+ a*x)*sqrt(b))/b^(9/2) + 35/8*sqrt(b*x^2 + a*x)*B*a^2/b^5 - 35/12*sqrt(b*
x^2 + a*x)*A*a/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(145) = 290$.

Time = 15.57 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.00

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{1}{24}\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)B|b|}{b^7}-\frac{25\,Bab^{20}|b|-6\,Ab^{21}|b|}{b^{27}}\right.\right.$$
$$+\frac{35\,(3\,Ba^3|b|-2\,Aa^2b|b|)\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{16\,b^{\frac{13}{2}}}$$
$$+\frac{4\left(15\,Ba^4\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4|b|+24\,Ba^5\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b|b|-12\,Aa\cdots\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\cdots\right.\right.}$$

input
```
integrate(x^(7/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/24*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)*B*abs
(b)/b^7 - (25*B*a*b^20*abs(b) - 6*A*b^21*abs(b))/b^27) + 3*(55*B*a^2*b^20*
abs(b) - 26*A*a*b^21*abs(b))/b^27) + 35/16*(3*B*a^3*abs(b) - 2*A*a^2*b*abs
(b))*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(13/2) + 4
/3*(15*B*a^4*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*abs(b) +
24*B*a^5*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b*abs(b) - 12
*A*a^3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b*abs(b) + 13*B
*a^6*b^2*abs(b) - 18*A*a^4*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b
))^2*b^2*abs(b) - 10*A*a^5*b^3*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*
x + a)*b - a*b))^2 + a*b)^3*b^(11/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx=\int \frac{x^{7/2}\,(A+B\,x)}{{(a+b\,x)}^{5/2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.53

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx)^{5/2}}\,dx=\frac{-840\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3+525\sqrt{b}\,\sqrt{bx+a}\,a^3+840\sqrt{x}\,a^3b+280\sqrt{x}\,a}{192\sqrt{bx+a}\,b^5}$$

input
```
int(x^(7/2)*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
( - 840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a
))*a**3 + 525*sqrt(b)*sqrt(a + b*x)*a**3 + 840*sqrt(x)*a**3*b + 280*sqrt(x
)*a**2*b**2*x - 112*sqrt(x)*a*b**3*x**2 + 64*sqrt(x)*b**4*x**3)/(192*sqrt(
a + b*x)*b**5)
```

3.336 $\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab-aB)x^{5/2}}{3b^2(a+bx)^{3/2}}-\frac{2(5Ab-8aB)x^{3/2}}{3b^3\sqrt{a+bx}}+\frac{5(4Ab-7aB)\sqrt{x}\sqrt{a+bx}}{4b^4}+\frac{Bx^{3/2}\sqrt{a+bx}}{2b^3}-\frac{5a(4Ab-7aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{9/2}}$$

output
```
-2/3*(A*b-B*a)*x^(5/2)/b^2/(b*x+a)^(3/2)-2/3*(5*A*b-8*B*a)*x^(3/2)/b^3/(b*
x+a)^(1/2)+5/4*(4*A*b-7*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^4+1/2*B*x^(3/2)*(b*x+
a)^(1/2)/b^3-5/4*a*(4*A*b-7*B*a)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^
(9/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{x}(-105a^3B+ab^2x(80A-21Bx)+20a^2b(3A-7Bx)+6b^3x^2(2A+Bx))}{12b^4(a+bx)^{3/2}}+\frac{5a(-4Ab+7aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{2b^{9/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*(-105*a^3*B + a*b^2*x*(80*A - 21*B*x) + 20*a^2*b*(3*A - 7*B*x) +
6*b^3*x^2*(2*A + B*x)))/(12*b^4*(a + b*x)^(3/2)) + (5*a*(-4*A*b + 7*a*B)*A
rcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(2*b^(9/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 57, 60, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{2x^{7/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(4Ab-7aB)\int \frac{x^{5/2}}{(a+bx)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \; 57$$

$$\frac{2x^{7/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(4Ab-7aB)\left(\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}dx}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\Big\downarrow \; 60$$

$$\frac{2x^{7/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(4Ab-7aB)\left(\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\Big\downarrow \; 60$$

$$\frac{2x^{7/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(4Ab-7aB)\left(\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\Big\downarrow \text{65}$$

$$\frac{2x^{7/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(4Ab-7aB)\left(\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int\frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x^{7/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(4Ab-7aB)\left(\frac{5\left(\frac{x^{3/2}\sqrt{a+bx}}{2b}-\frac{3a\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{4b}\right)}{b}-\frac{2x^{5/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x)^(3/2)) - ((4*A*b - 7*a*B)*((-2*x^
(5/2))/(b*Sqrt[a + b*x]) + (5*((x^(3/2)*Sqrt[a + b*x])/(2*b) - (3*a*((Sqrt
[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2
)))/(4*b)))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(118) = 236$.

Time = 0.18 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(2bBx+4Ab-11Ba)\sqrt{x}\,\sqrt{bx+a}}{4b^4} - \frac{a\left(20A\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)-\frac{35Ba\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-\frac{16(3Ab-4Ba)\sqrt{b\left(x+\frac{a}{b}\right)^2-\left(x\right.}}{b\left(x+\frac{a}{b}\right)}\right.}{8b^4\sqrt{x}\,\sqrt{bx+}}$
default	$-\frac{\left(-12B\,b^{\frac{7}{2}}x^3\sqrt{x(bx+a)}+60A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^3x^2-24A\,b^{\frac{7}{2}}x^2\sqrt{x(bx+a)}-105B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2b^2x^2\right.}{}$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*b*x+4*A*b-11*B*a)*x^(1/2)*(b*x+a)^(1/2)/b^4-1/8*a/b^4*(20*A*b^(1/
2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))-35*B*a*ln((1/2*a+b*x)/b^(1/2)
+(b*x^2+a*x)^(1/2))/b^(1/2)-16*(3*A*b-4*B*a)/b/(x+a/b)*(b*(x+a/b)^2-(x+a/b
)*a)^(1/2)+8*a^2*(A*b-B*a)/b^2*(2/3/a/(x+a/b)^2*(b*(x+a/b)^2-(x+a/b)*a)^(1
/2)+4/3*b/a^2/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2)))*(x*(b*x+a))^(1/2)/x^
(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.43

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \left[-\frac{15\,(7\,Ba^4-4\,Aa^3b+(7\,Ba^2b^2-4\,Aab^3)x^2+2\,(7\,Ba^3b-4\,Aa^2b^2)x)\sqrt{b}\log\left(2\,b\right.}{}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*x^2 + 2*(7*B*a
^3*b - 4*A*a^2*b^2)*x)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x)
 + a) - 2*(6*B*b^4*x^3 - 105*B*a^3*b + 60*A*a^2*b^2 - 3*(7*B*a*b^3 - 4*A*b
^4)*x^2 - 20*(7*B*a^2*b^2 - 4*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^7*x^2
+ 2*a*b^6*x + a^2*b^5), -1/12*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*
A*a*b^3)*x^2 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-b)*arctan(sqrt(-b)*sqr
t(x)/sqrt(b*x + a)) - (6*B*b^4*x^3 - 105*B*a^3*b + 60*A*a^2*b^2 - 3*(7*B*a
*b^3 - 4*A*b^4)*x^2 - 20*(7*B*a^2*b^2 - 4*A*a*b^3)*x)*sqrt(b*x + a)*sqrt(x
))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(118) = 236$.

Time = 0.07 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.40

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{(bx^2+ax)^{\frac{5}{2}}Ba}{b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba^2}{6\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)} + \frac{5\,\sqrt{bx^2+ax}Ba^3}{6\,(b^6x^2+2\,ab^5x+a^2b^4)} + \frac{(bx^2+ax)^{\frac{5}{2}}A}{b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b} + \frac{(bx^2+ax)^{\frac{5}{2}}B}{2\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)} + \frac{5\,(bx^2+ax)^{\frac{3}{2}}Aa}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)} - \frac{5\,(bx^2+ax)^{\frac{3}{2}}Ba}{4\,(b^5x^2+2\,ab^4x+a^2b^3)} - \frac{5\,\sqrt{bx^2+ax}Aa^2}{6\,(b^5x^2+2\,ab^4x+a^2b^3)} - \frac{115\,\sqrt{bx^2+ax}Ba^2}{12\,(b^5x+ab^4)} + \frac{35\,\sqrt{bx^2+ax}Aa}{6\,(b^4x+ab^3)} + \frac{35\,Ba^2\log\left(2\,x+\frac{a}{b}+\frac{2\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{8\,b^{\frac{9}{2}}} - \frac{5\,Aa\log\left(2\,x+\frac{a}{b}+\frac{2\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-(b*x^2 + a*x)^(5/2)*B*a/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^3*b^
3*x + a^4*b^2) - 5/6*(b*x^2 + a*x)^(3/2)*B*a^2/(b^6*x^3 + 3*a*b^5*x^2 + 3*
a^2*b^4*x + a^3*b^3) + 5/6*sqrt(b*x^2 + a*x)*B*a^3/(b^6*x^2 + 2*a*b^5*x +
a^2*b^4) + (b*x^2 + a*x)^(5/2)*A/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 +
4*a^3*b^2*x + a^4*b) + 1/2*(b*x^2 + a*x)^(5/2)*B/(b^5*x^3 + 3*a*b^4*x^2 +
3*a^2*b^3*x + a^3*b^2) + 5/6*(b*x^2 + a*x)^(3/2)*A*a/(b^5*x^3 + 3*a*b^4*x^
2 + 3*a^2*b^3*x + a^3*b^2) - 5/4*(b*x^2 + a*x)^(3/2)*B*a/(b^5*x^2 + 2*a*b^
4*x + a^2*b^3) - 5/6*sqrt(b*x^2 + a*x)*A*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^
3) - 115/12*sqrt(b*x^2 + a*x)*B*a^2/(b^5*x + a*b^4) + 35/6*sqrt(b*x^2 + a*
x)*A*a/(b^4*x + a*b^3) + 35/8*B*a^2*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sq
rt(b))/b^(9/2) - 5/2*A*a*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/sqrt(b))/b^(7
/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(118) = 236$.

Time = 15.64 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.20

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{1}{4}\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2\,(bx+a)B|b|}{b^6} - \frac{13\,Bab^{11}|b| - 4\,Ab^{12}|b|}{b^{17}}\right) - \frac{5\,(7\,Ba^2|b| - 4\,Aab|b|)\log\left(\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2\right)}{8\,b^{\frac{11}{2}}} - \frac{4\left(12\,Ba^3\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^4|b| + 18\,Ba^4\left(\sqrt{bx+a}\sqrt{b} - \sqrt{(bx+a)b-ab}\right)^2 b|b| - 9\,Aa\cdots\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}\cdots\right.\right.}$$

input
```
integrate(x^(5/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/4*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*B*abs(b)/b^6 - (13*
B*a*b^11*abs(b) - 4*A*b^12*abs(b))/b^17) - 5/8*(7*B*a^2*abs(b) - 4*A*a*b*a
bs(b))*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(11/2) -
 4/3*(12*B*a^3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*abs(b)
+ 18*B*a^4*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b*abs(b) -
9*A*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b*abs(b) + 10*
B*a^5*b^2*abs(b) - 12*A*a^3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*
b))^2*b^2*abs(b) - 7*A*a^4*b^3*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*
x + a)*b - a*b))^2 + a*b)^3*b^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{{(a+b\,x)}^{5/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2 - 10\sqrt{b}\,\sqrt{bx+a}\,a^2 - 15\sqrt{x}\,a^2b - 5\sqrt{x}\,a\,b^2x +}{4\sqrt{bx+a}\,b^4}$$

input
```
int(x^(5/2)*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
**2 - 10*sqrt(b)*sqrt(a + b*x)*a**2 - 15*sqrt(x)*a**2*b - 5*sqrt(x)*a*b**2
*x + 2*sqrt(x)*b**3*x**2)/(4*sqrt(a + b*x)*b**4)
```

3.337 $\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab-aB)x^{3/2}}{3b^2(a+bx)^{3/2}} - \frac{2(Ab-2aB)\sqrt{x}}{b^3\sqrt{a+bx}} + \frac{B\sqrt{x}\sqrt{a+bx}}{b^3} + \frac{(2Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}$$

output
```
-2/3*(A*b-B*a)*x^(3/2)/b^2/(b*x+a)^(3/2)-2*(A*b-2*B*a)*x^(1/2)/b^3/(b*x+a)
^(1/2)+B*x^(1/2)*(b*x+a)^(1/2)/b^3+(2*A*b-5*B*a)*arctanh(b^(1/2)*x^(1/2)/(
b*x+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{x}(-6aAb+15a^2B-8Ab^2x+20abBx+3b^2Bx^2)}{3b^3(a+bx)^{3/2}} + \frac{2(2Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[x]*(-6*a*A*b + 15*a^2*B - 8*A*b^2*x + 20*a*b*B*x + 3*b^2*B*x^2))/(3*
b^3*(a + b*x)^(3/2)) + (2*(2*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt
[a] + Sqrt[a + b*x])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {87, 57, 60, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{2x^{5/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-5aB)\int \frac{x^{3/2}}{(a+bx)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{57}$$

$$\frac{2x^{5/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-5aB)\left(\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-5aB)\left(\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{2b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\Big\downarrow \text{65}$$

$$\frac{2x^{5/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-5aB)\left(\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

$$\downarrow \text{219}$$

$$\frac{2x^{5/2}(Ab-aB)}{3ab(a+bx)^{3/2}}-\frac{(2Ab-5aB)\left(\frac{3\left(\frac{\sqrt{x}\sqrt{a+bx}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\right)}{b}-\frac{2x^{3/2}}{b\sqrt{a+bx}}\right)}{3ab}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x)^(3/2)) - ((2*A*b - 5*a*B)*((-2*x^
(3/2))/(b*Sqrt[a + b*x]) + (3*((Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqr
t[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(90) = 180$.

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.18

method	result
risch	$\frac{B\sqrt{x}\,\sqrt{bx+a}}{b^3}+\frac{\left(2A\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)-\frac{5Ba\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}-\frac{4(2Ab-3Ba)\sqrt{b\left(x+\frac{a}{b}\right)^2-\left(x+\frac{a}{b}\right)a}}{b\left(x+\frac{a}{b}\right)}+\frac{2a^2(Ab-Ba)\left(\cdots\right)}{\cdots}\cdots\right)}{2b^3\sqrt{x}\,\sqrt{bx+a}\cdots}$
default	$\frac{\left(6A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b^3x^2-15B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^2x^2+6B\,b^{\frac{5}{2}}x^2\sqrt{x(bx+a)}+12A\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)\cdots\right.}{\cdots}$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*x^(1/2)*(b*x+a)^(1/2)/b^3+1/2/b^3*(2*A*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b
*x^2+a*x)^(1/2))-5*B*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)-4
*(2*A*b-3*B*a)/b/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2)+2*a^2*(A*b-B*a)/b^2
*(2/3/a/(x+a/b)^2*(b*(x+a/b)^2-(x+a/b)*a)^(1/2)+4/3*b/a^2/(x+a/b)*(b*(x+a/
b)^2-(x+a/b)*a)^(1/2)))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.78

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \left[-\frac{3\,(5\,Ba^3-2\,Aa^2b+(5\,Bab^2-2\,Ab^3)x^2+2\,(5\,Ba^2b-2\,Aab^2)x)\sqrt{b}\log\left(2\,bx+\right.}{6\,(b^6x^2}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*B*a^3 - 2*A*a^2*b + (5*B*a*b^2 - 2*A*b^3)*x^2 + 2*(5*B*a^2*b -
 2*A*a*b^2)*x)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) -
2*(3*B*b^3*x^2 + 15*B*a^2*b - 6*A*a*b^2 + 4*(5*B*a*b^2 - 2*A*b^3)*x)*sqrt(
b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(3*(5*B*a^3 - 2*A*a
^2*b + (5*B*a*b^2 - 2*A*b^3)*x^2 + 2*(5*B*a^2*b - 2*A*a*b^2)*x)*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (3*B*b^3*x^2 + 15*B*a^2*b - 6*A*a*
b^2 + 4*(5*B*a*b^2 - 2*A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5
*x + a^2*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(110) = 220.

Time = 62.53 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.51

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
A*(6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(
a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/
2)*x**(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a
)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 +
 b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(2
3/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2
)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sq
rt(1 + b*x/a))) + B*(-15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) +
 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**
(53/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/
2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*
x/a)) + 15*a**40*b**(45/2)*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 +
 b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(
47/2)*x**27/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)
*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/
2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*s
qrt(1 + b*x/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.98

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{(bx^2+ax)^{\frac{3}{2}}Ba}{3\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)} - \frac{\sqrt{bx^2+ax}Ba^2}{3\,(b^5x^2+2\,ab^4x+a^2b^3)} - \frac{(bx^2+ax)^{\frac{3}{2}}A}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} + \frac{(bx^2+ax)^{\frac{3}{2}}B}{b^4x^2+2\,ab^3x+a^2b^2} + \frac{\sqrt{bx^2+ax}Aa}{3\,(b^4x^2+2\,ab^3x+a^2b^2)} + \frac{16\,\sqrt{bx^2+ax}Ba}{3\,(b^4x+ab^3)} - \frac{7\,\sqrt{bx^2+ax}A}{3\,(b^3x+ab^2)} - \frac{5\,Ba\log\left(2\,x+\frac{a}{b}+\frac{2\,\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{2\,b^{\frac{7}{2}}} + \frac{A\log\left(2\,x+\frac{a}{b}+\frac{2\,\sqrt{bx^2+ax}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(b*x^2 + a*x)^(3/2)*B*a/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
) - 1/3*sqrt(b*x^2 + a*x)*B*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 1/3*(b*x
^2 + a*x)^(3/2)*A/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) + (b*x^2 +
 a*x)^(3/2)*B/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 1/3*sqrt(b*x^2 + a*x)*A*a/
(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 16/3*sqrt(b*x^2 + a*x)*B*a/(b^4*x + a*b^
3) - 7/3*sqrt(b*x^2 + a*x)*A/(b^3*x + a*b^2) - 5/2*B*a*log(2*x + a/b + 2*s
qrt(b*x^2 + a*x)/sqrt(b))/b^(7/2) + A*log(2*x + a/b + 2*sqrt(b*x^2 + a*x)/
sqrt(b))/b^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(92) = 184$.

Time = 15.52 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.65

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{(bx+a)b-ab}\sqrt{bx+a}B|b|}{b^5} + \frac{(5\,Ba|b|-2\,Ab|b|)\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{2\,b^{\frac{9}{2}}} + \frac{4\left(9\,Ba^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4|b|+12\,Ba^3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b|b|-6\,Aa\left(\right.\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\right.\right.}$$

input
```
integrate(x^(3/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*B*abs(b)/b^5 + 1/2*(5*B*a*abs(b) - 2
*A*b*abs(b))*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(9
/2) + 4/3*(9*B*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*abs
(b) + 12*B*a^3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b*abs(b
) - 6*A*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b*abs(b) + 7
*B*a^4*b^2*abs(b) - 6*A*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*
b))^2*b^2*abs(b) - 4*A*a^3*b^3*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*
x + a)*b - a*b))^2 + a*b)^3*b^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx=\int \frac{x^{3/2}\,(A+B\,x)}{{(a+b\,x)}^{5/2}} \,d x$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx)^{5/2}}\,dx=\frac{-12\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a+9\sqrt{b}\,\sqrt{bx+a}\,a+12\sqrt{x}\,ab+4\sqrt{x}\,b^{2}x}{4\sqrt{bx+a}\,b^{3}}$$

input
```
int(x^(3/2)*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)
)*a + 9*sqrt(b)*sqrt(a + b*x)*a + 12*sqrt(x)*a*b + 4*sqrt(x)*b**2*x)/(4*sq
rt(a + b*x)*b**3)
```

3.338 $\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}} \, dx = \frac{2(Ab-aB)x^{3/2}}{3ab(a+bx)^{3/2}} - \frac{2B\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

output
```
2/3*(A*b-B*a)*x^(3/2)/a/b/(b*x+a)^(3/2)-2*B*x^(1/2)/b^2/(b*x+a)^(1/2)+2*B*
arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}} \, dx = \frac{2\sqrt{x}(-3a^2B+Ab^2x-4abBx)}{3ab^2(a+bx)^{3/2}} - \frac{2B\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{b^{5/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*Sqrt[x]*(-3*a^2*B + A*b^2*x - 4*a*b*B*x))/(3*a*b^2*(a + b*x)^(3/2)) - (
2*B*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 57, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{B\int \frac{\sqrt{x}}{(a+bx)^{3/2}}dx}{b}+\frac{2x^{3/2}(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 57}$$

$$\frac{B\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{b}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}\right)}{b}+\frac{2x^{3/2}(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 65}$$

$$\frac{B\left(\frac{2\int \frac{1}{1-\frac{bx}{a+bx}}d\frac{\sqrt{x}}{\sqrt{a+bx}}}{b}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}\right)}{b}+\frac{2x^{3/2}(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2x^{3/2}(Ab-aB)}{3ab(a+bx)^{3/2}}+\frac{B\left(\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}\right)}{b}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x)^(3/2)) + (B*((-2*Sqrt[x])/(b*Sqrt
[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
 + 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
 + 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.22

method	result
default	$\frac{\left(3B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a\,b^2x^2+2A\sqrt{x(bx+a)}\,b^{\frac{5}{2}}x+6B\ln\left(\frac{2\sqrt{x(bx+a)}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^2bx-8B\sqrt{x(bx+a)}\,b^{\frac{3}{2}}ax+3B\ln\left(\frac{2\sqrt{x}\right.}{}\right.}{3a\sqrt{x(bx+a)}\,b^{\frac{5}{2}}\,(bx+a)^{\frac{3}{2}}}$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*B*ln(1/2*(2*(x*(b*x+a))^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a*b^2*x^2+2
*A*(x*(b*x+a))^(1/2)*b^(5/2)*x+6*B*ln(1/2*(2*(x*(b*x+a))^(1/2)*b^(1/2)+2*b
*x+a)/b^(1/2))*a^2*b*x-8*B*(x*(b*x+a))^(1/2)*b^(3/2)*a*x+3*B*ln(1/2*(2*(x*
(b*x+a))^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^3-6*B*(x*(b*x+a))^(1/2)*b^(1/2)
*a^2)*x^(1/2)/a/(x*(b*x+a))^(1/2)/b^(5/2)/(b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}\,dx = \left[\frac{3\,(Bab^2x^2 + 2\,Ba^2bx + Ba^3)\sqrt{b}\log\left(2\,bx + 2\,\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) - 2\,(3\,Ba^2b + (4\cdots}{3\,(ab^5x^2 + 2\,a^2b^4x + a^3b^3)}\right.$$
$$\left. -\frac{2\left(3\,(Bab^2x^2 + 2\,Ba^2bx + Ba^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}}{\sqrt{bx+a}}\right) + (3\,Ba^2b + (4\,Bab^2 - Ab^3)x)\sqrt{bx+a}\sqrt{x}\right)}{3\,(ab^5x^2 + 2\,a^2b^4x + a^3b^3)}\right]$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(B*a*b^2*x^2 + 2*B*a^2*b*x + B*a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x
 + a)*sqrt(b)*sqrt(x) + a) - 2*(3*B*a^2*b + (4*B*a*b^2 - A*b^3)*x)*sqrt(b*
x + a)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x + a^3*b^3), -2/3*(3*(B*a*b^2*x^2
+ 2*B*a^2*b*x + B*a^3)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)/sqrt(b*x + a)) + (
3*B*a^2*b + (4*B*a*b^2 - A*b^3)*x)*sqrt(b*x + a)*sqrt(x))/(a*b^5*x^2 + 2*a
^2*b^4*x + a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(78) = 156$.

Time = 24.54 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.59

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}\,dx = \frac{2Ax^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}} + B\left(\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}} + \frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}} - \frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}} - \frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}\right)$$

input
```
integrate(x**(1/2)*(B*x+A)/(b*x+a)**(5/2),x)
```

output
```
2*A*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
 + B*(6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sq
rt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(
29/2)*x**(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*
x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(
1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b*
*(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/
2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(3
9/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)
*sqrt(1 + b*x/a)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{1}{3}\,B\left(\frac{2\left(b+\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}b^{2}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}\right)+\frac{2\,Ax^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B*(2*(b + 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*b^2) + 3*log(-(sqrt
(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(5/2)) +
 2/3*A*x^(3/2)/((b*x + a)^(3/2)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(64) = 128$.

Time = 15.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}\,dx = -\frac{B|b|\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}\right)}{b^{\frac{7}{2}}} - \frac{4\left(6\,Ba\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}|b|+6\,Ba^{2}\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}b|b|-3\,A\left(\sqrt{b\cdots}\right.\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}\cdots}$$

input
```
integrate(x^(1/2)*(B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-B*abs(b)*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(7/2)
 - 4/3*(6*B*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*abs(b) +
 6*B*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b*abs(b) - 3*
A*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b*abs(b) + 4*B*a^3*b
^2*abs(b) - A*a^2*b^3*abs(b))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b
- a*b))^2 + a*b)^3*b^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}\,dx=\int \frac{\sqrt{x}\,(A+B\,x)}{{(a+b\,x)}^{5/2}} \,d x$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x)^(5/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx)^{5/2}}\,dx=\frac{2\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)-2\sqrt{b}\,\sqrt{bx+a}-2\sqrt{x}\,b}{\sqrt{bx+a}\,b^2}$$

input
```
int(x^(1/2)*(B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a)) -
 sqrt(b)*sqrt(a + b*x) - sqrt(x)*b))/(sqrt(a + b*x)*b**2)
```

3.339 $\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx=\frac{2(Ab-aB)\sqrt{x}}{3ab(a+bx)^{3/2}}+\frac{2(2Ab+aB)\sqrt{x}}{3a^2b\sqrt{a+bx}}$$

output
```
2/3*(A*b-B*a)*x^(1/2)/a/b/(b*x+a)^(3/2)+2/3*(2*A*b+B*a)*x^(1/2)/a^2/b/(b*x
+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx=\frac{2\sqrt{x}(3aA+2Abx+aBx)}{3a^2(a+bx)^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

output
```
(2*Sqrt[x]*(3*a*A + 2*A*b*x + a*B*x))/(3*a^2*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(aB+2Ab)\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}dx}{3ab}+\frac{2\sqrt{x}(Ab-aB)}{3ab(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{2\sqrt{x}(aB+2Ab)}{3a^2b\sqrt{a+bx}}+\frac{2\sqrt{x}(Ab-aB)}{3ab(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

output
```
(2*(A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x)^(3/2)) + (2*(2*A*b + a*B)*Sqrt[x]
)/(3*a^2*b*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{2\sqrt{x}\,(2Abx+Bax+3Aa)}{3(bx+a)^{\frac{3}{2}}a^{2}}$	30
default	$\frac{2\sqrt{x}\,(2Abx+Bax+3Aa)}{3(bx+a)^{\frac{3}{2}}a^{2}}$	30
orering	$\frac{2\sqrt{x}\,(2Abx+Bax+3Aa)}{3(bx+a)^{\frac{3}{2}}a^{2}}$	30

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(1/2)*(2*A*b*x+B*a*x+3*A*a)/(b*x+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{2\,(3\,Aa+(Ba+2\,Ab)x)\sqrt{bx+a}\sqrt{x}}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*A*a + (B*a + 2*A*b)*x)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b
*x + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 17.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx = A\left(\frac{6a}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}} + \frac{4bx}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}\right) + \frac{2Bx^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

input
```
integrate((B*x+A)/x**(1/2)/(b*x+a)**(5/2),x)
```

output
```
A*(6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x)
+ 1)) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a
/(b*x) + 1))) + 2*B*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*
sqrt(1 + b*x/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(53) = 106$.

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.48

$$\int \frac{A + Bx}{\sqrt{x}(a + bx)^{5/2}}\,dx = -\frac{2\sqrt{bx^2 + ax}Ba}{3\,(ab^3x^2 + 2\,a^2b^2x + a^3b)} - \frac{4\sqrt{bx^2 + ax}Ba}{3\,(a^2b^2x + a^3b)} + \frac{2\sqrt{bx^2 + ax}A}{3\,(ab^2x^2 + 2\,a^2bx + a^3)} + \frac{4\sqrt{bx^2 + ax}A}{3\,(a^2bx + a^3)} + \frac{2\sqrt{bx^2 + ax}B}{ab^2x + a^2b}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(b*x^2 + a*x)*B*a/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) - 4/3*sqrt(b*
x^2 + a*x)*B*a/(a^2*b^2*x + a^3*b) + 2/3*sqrt(b*x^2 + a*x)*A/(a*b^2*x^2 +
2*a^2*b*x + a^3) + 4/3*sqrt(b*x^2 + a*x)*A/(a^2*b*x + a^3) + 2*sqrt(b*x^2
+ a*x)*B/(a*b^2*x + a^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{A + Bx}{\sqrt{x}(a + bx)^{5/2}}\,dx = \frac{4\left(3\,B\left(\sqrt{bx + a}\sqrt{b} - \sqrt{(bx + a)b - ab}\right)^4\sqrt{b} + Ba^2b^{\frac{5}{2}} + 6\,A\left(\sqrt{bx + a}\sqrt{b} - \sqrt{(bx\right.\right.}{3\left(\left(\sqrt{bx + a}\sqrt{b} - \sqrt{(bx + a)b - ab}\right)^2 + ab\right)^3|b|}$$

input
```
integrate((B*x+A)/x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
4/3*(3*B*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + B*a
^2*b^(5/2) + 6*A*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(5/
2) + 2*A*a*b^(7/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2
+ a*b)^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{\left(\frac{x^2\,(4\,A\,b+2\,B\,a)}{3\,a^2\,b^2}+\frac{2\,A\,x}{a\,b^2}\right)\,\sqrt{a+b\,x}}{x^{5/2}+\frac{2\,a\,x^{3/2}}{b}+\frac{a^2\,\sqrt{x}}{b^2}}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x)^(5/2)),x)
```

output
```
(((x^2*(4*A*b + 2*B*a))/(3*a^2*b^2) + (2*A*x)/(a*b^2))*(a + b*x)^(1/2))/(x
^(5/2) + (2*a*x^(3/2))/b + (a^2*x^(1/2))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx}{\sqrt{x}(a+bx)^{5/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{bx+a}+2\sqrt{x}\,b}{\sqrt{bx+a}\,ab}$$

input
```
int((B*x+A)/x^(1/2)/(b*x+a)^(5/2),x)
```

output
```
(2*(sqrt(b)*sqrt(a + b*x) + sqrt(x)*b))/(sqrt(a + b*x)*a*b)
```

3.340 $\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{2A}{a\sqrt{x}(a+bx)^{3/2}} - \frac{2(4Ab-aB)\sqrt{x}}{3a^2(a+bx)^{3/2}} - \frac{4(4Ab-aB)\sqrt{x}}{3a^3\sqrt{a+bx}}$$

output
```
-2*A/a/x^(1/2)/(b*x+a)^(3/2)-2/3*(4*A*b-B*a)*x^(1/2)/a^2/(b*x+a)^(3/2)-4/3
*(4*A*b-B*a)*x^(1/2)/a^3/(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{-16Ab^2x^2-6a^2(A-Bx)+4abx(-6A+Bx)}{3a^3\sqrt{x}(a+bx)^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x)^(5/2)),x]
```

output
```
(-16*A*b^2*x^2 - 6*a^2*(A - B*x) + 4*a*b*x*(-6*A + B*x))/(3*a^3*Sqrt[x]*(a
 + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(4Ab-aB)\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}dx}{a}-\frac{2A}{a\sqrt{x}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(4Ab-aB)\left(\frac{2\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}dx}{3a}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}\right)}{a}-\frac{2A}{a\sqrt{x}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 48}$$

$$-\frac{\left(\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}\right)(4Ab-aB)}{a}-\frac{2A}{a\sqrt{x}(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*A)/(a*Sqrt[x]*(a + b*x)^(3/2)) - ((4*A*b - a*B)*((2*Sqrt[x])/(3*a*(a +
 b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2\left(8A\,b^{2}x^{2}-2Bab\,x^{2}+12aAbx-3B\,a^{2}x+3a^{2}A\right)}{3\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	53
default	$-\frac{2\left(8A\,b^{2}x^{2}-2Bab\,x^{2}+12aAbx-3B\,a^{2}x+3a^{2}A\right)}{3\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	53
orering	$-\frac{2\left(8A\,b^{2}x^{2}-2Bab\,x^{2}+12aAbx-3B\,a^{2}x+3a^{2}A\right)}{3\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	53
risch	$-\frac{2A\sqrt{bx+a}}{a^{3}\sqrt{x}}-\frac{2\left(5A\,b^{2}x-2Babx+6abA-3a^{2}B\right)\sqrt{x}}{3\left(bx+a\right)^{\frac{3}{2}}a^{3}}$	58

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(8*A*b^2*x^2-2*B*a*b*x^2+12*A*a*b*x-3*B*a^2*x+3*A*a^2)/x^(1/2)/(b*x+a
)^(3/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{2\,(3\,Aa^2-2\,(Bab-4\,Ab^2)x^2-3\,(Ba^2-4\,Aab)x)\sqrt{bx+a}\sqrt{x}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*A*a^2 - 2*(B*a*b - 4*A*b^2)*x^2 - 3*(B*a^2 - 4*A*a*b)*x)*sqrt(b*x
+ a)*sqrt(x)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(80) = 160$.

Time = 43.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.09

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = A\left(-\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}\right) + B\left(\frac{6a}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}+\frac{4bx}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}\right)$$

input
```
integrate((B*x+A)/x**(3/2)/(b*x+a)**(5/2),x)
```

output
```
A*(-6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**
3*b**6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**
4 + 6*a**4*b**5*x + 3*a**3*b**6*x**2)) + B*(6*a/(3*a**3*sqrt(b)*sqrt(a/(b*
x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)) + 4*b*x/(3*a**3*sqrt(b)*sqr
t(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(63) = 126$.

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{2\,Ba}{3\left(\sqrt{bx^2+ax}ab^2x+\sqrt{bx^2+ax}a^2b\right)} + \frac{2\,A}{3\left(\sqrt{bx^2+ax}abx+\sqrt{bx^2+ax}a^2\right)} + \frac{4\,Bx}{3\sqrt{bx^2+ax}a^2} - \frac{16\,Abx}{3\sqrt{bx^2+ax}a^3} - \frac{8\,A}{3\sqrt{bx^2+ax}a^2} + \frac{2\,B}{3\sqrt{bx^2+ax}ab}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*B*a/(sqrt(b*x^2 + a*x)*a*b^2*x + sqrt(b*x^2 + a*x)*a^2*b) + 2/3*A/(sq
rt(b*x^2 + a*x)*a*b*x + sqrt(b*x^2 + a*x)*a^2) + 4/3*B*x/(sqrt(b*x^2 + a*x
)*a^2) - 16/3*A*b*x/(sqrt(b*x^2 + a*x)*a^3) - 8/3*A/(sqrt(b*x^2 + a*x)*a^2
) + 2/3*B/(sqrt(b*x^2 + a*x)*a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.60

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{2\sqrt{bx+a}Ab^2}{\sqrt{(bx+a)b-ab}a^3|b|} + \frac{4\left(6\,Ba^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{5}{2}} - 3\,A\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{5}{2}} + 2\,Ba^3b^{\frac{7}{2}} - \right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^2|b}$$

input
```
integrate((B*x+A)/x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2*sqrt(b*x + a)*A*b^2/(sqrt((b*x + a)*b - a*b)*a^3*abs(b)) + 4/3*(6*B*a^2
*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(5/2) - 3*A*(sqrt(b
*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(5/2) + 2*B*a^3*b^(7/2) - 1
2*A*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) - 5*A*a^
2*b^(9/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*
a^2*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{a\,b^2}-\frac{x\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{3\,a^3\,b^2}+\frac{x^2\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{3\,a^3\,b^2}\right)}{x^{5/2}+\frac{2\,a\,x^{3/2}}{b}+\frac{a^2\,\sqrt{x}}{b^2}}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x)^(5/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(a*b^2) - (x*(6*B*a^2 - 24*A*a*b))/(3*a^3*b^2) +
(x^2*(16*A*b^2 - 4*B*a*b))/(3*a^3*b^2)))/(x^(5/2) + (2*a*x^(3/2))/b + (a^2
*x^(1/2))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{-4\sqrt{b}\,\sqrt{bx+a}\,x-2\sqrt{x}\,a-4\sqrt{x}\,bx}{\sqrt{bx+a}\,a^2x}$$

input
```
int((B*x+A)/x^(3/2)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 2*sqrt(b)*sqrt(a + b*x)*x - sqrt(x)*a - 2*sqrt(x)*b*x))/(sqrt(a + b
*x)*a**2*x)
```

3.341 $\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx=-\frac{2A}{3ax^{3/2}(a+bx)^{3/2}}-\frac{2(2Ab-aB)}{3a^2\sqrt{x}(a+bx)^{3/2}}-\frac{8(2Ab-aB)}{3a^3\sqrt{x}\sqrt{a+bx}}+\frac{16(2Ab-aB)\sqrt{a+bx}}{3a^4\sqrt{x}}$$

output
```
-2/3*A/a/x^(3/2)/(b*x+a)^(3/2)-2/3*(2*A*b-B*a)/a^2/x^(1/2)/(b*x+a)^(3/2)-8
/3*(2*A*b-B*a)/a^3/x^(1/2)/(b*x+a)^(1/2)+16/3*(2*A*b-B*a)*(b*x+a)^(1/2)/a^
4/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx=-\frac{2(-16Ab^3x^3-6a^2bx(A-2Bx)+8ab^2x^2(-3A+Bx)+a^3(A+3Bx))}{3a^4x^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*(-16*A*b^3*x^3 - 6*a^2*b*x*(A - 2*B*x) + 8*a*b^2*x^2*(-3*A + B*x) + a^
3*(A + 3*B*x)))/(3*a^4*x^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(2Ab-aB)\int \frac{1}{x^{3/2}(a+bx)^{5/2}}dx}{a}-\frac{2A}{3ax^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(2Ab-aB)\left(\frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}dx}{3a}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}\right)}{a}-\frac{2A}{3ax^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 55$$

$$-\frac{(2Ab-aB)\left(\frac{4\left(\frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{a}+\frac{2}{a\sqrt{x}\sqrt{a+bx}}\right)}{3a}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}\right)}{a}-\frac{2A}{3ax^{3/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \ 48$$

$$-\frac{\left(\frac{4\left(\frac{2}{a\sqrt{x}\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\right)}{3a}+\frac{2}{3a\sqrt{x}(a+bx)^{3/2}}\right)(2Ab-aB)}{a}-\frac{2A}{3ax^{3/2}(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)*(a + b*x)^(3/2)) - ((2*A*b - a*B)*(2/(3*a*Sqrt[x]*(a +
 b*x)^(3/2)) + (4*(2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sq
rt[x])))/(3*a)))/a
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(-8Abx+3Bax+Aa)}{3a^{4}x^{\frac{3}{2}}}+\frac{2b(8A\,b^{2}x-5Babx+9abA-6a^{2}B)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}a^{4}}$	72
gosper	$-\frac{2(-16A\,b^{3}x^{3}+8Ba\,b^{2}x^{3}-24aA\,b^{2}x^{2}+12B\,a^{2}b\,x^{2}-6a^{2}Abx+3B\,a^{3}x+a^{3}A)}{3x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}a^{4}}$	76
default	$-\frac{2(-16A\,b^{3}x^{3}+8Ba\,b^{2}x^{3}-24aA\,b^{2}x^{2}+12B\,a^{2}b\,x^{2}-6a^{2}Abx+3B\,a^{3}x+a^{3}A)}{3x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}a^{4}}$	76
orering	$-\frac{2(-16A\,b^{3}x^{3}+8Ba\,b^{2}x^{3}-24aA\,b^{2}x^{2}+12B\,a^{2}b\,x^{2}-6a^{2}Abx+3B\,a^{3}x+a^{3}A)}{3x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}a^{4}}$	76

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-8*A*b*x+3*B*a*x+A*a)/a^4/x^(3/2)+2/3*b*(8*A*b^2*x-5*B
*a*b*x+9*A*a*b-6*B*a^2)*x^(1/2)/(b*x+a)^(3/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx = -\frac{2\,(Aa^{3}+8\,(Bab^{2}-2\,Ab^{3})x^{3}+12\,(Ba^{2}b-2\,Aab^{2})x^{2}+3\,(Ba^{3}-2\,Aa^{2}b)x)\sqrt{bx+a}\sqrt{x}}{3\,(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2})}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(A*a^3 + 8*(B*a*b^2 - 2*A*b^3)*x^3 + 12*(B*a^2*b - 2*A*a*b^2)*x^2 + 3
*(B*a^3 - 2*A*a^2*b)*x)*sqrt(b*x + a)*sqrt(x)/(a^4*b^2*x^4 + 2*a^5*b*x^3 +
 a^6*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(110) = 220$.

Time = 123.72 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.38

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx = A\left(-\frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3\cdots}\right.$$
$$\left.+B\left(-\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}\right)$$

input
```
integrate((B*x+A)/x**(5/2)/(b*x+a)**(5/2),x)
```

output
```
A*(-2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2
+ 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x
) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**
12*x**4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**
6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**
3*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3
 + 3*a**4*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x
 + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)) + B*(-6*a**
2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b**6*x*
*2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*
a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**
4*b**5*x + 3*a**3*b**6*x**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2\,Bx}{3\,(bx^2+ax)^{\frac{3}{2}}a}-\frac{16\,Bbx}{3\,\sqrt{bx^2+ax}a^3}-\frac{4\,Abx}{3\,(bx^2+ax)^{\frac{3}{2}}a^2}+\frac{32\,Ab^2x}{3\,\sqrt{bx^2+ax}a^4}-\frac{8\,B}{3\,\sqrt{bx^2+ax}a^2}-\frac{2\,A}{3\,(bx^2+ax)^{\frac{3}{2}}a}+\frac{16\,Ab}{3\,\sqrt{bx^2+ax}a^3}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*x/((b*x^2 + a*x)^(3/2)*a) - 16/3*B*b*x/(sqrt(b*x^2 + a*x)*a^3) - 4/3
*A*b*x/((b*x^2 + a*x)^(3/2)*a^2) + 32/3*A*b^2*x/(sqrt(b*x^2 + a*x)*a^4) -
8/3*B/(sqrt(b*x^2 + a*x)*a^2) - 2/3*A/((b*x^2 + a*x)^(3/2)*a) + 16/3*A*b/(
sqrt(b*x^2 + a*x)*a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(86) = 172$.

Time = 0.21 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.68

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx = -\frac{2\sqrt{bx+a}\left(\frac{(3\,Ba^4b^3|b|-8\,Aa^3b^4|b|)(bx+a)}{a^7b^2}-\frac{3\,(Ba^5b^3|b|-3\,Aa^4b^4|b|)}{a^7b^2}\right)}{3\,((bx+a)b-ab)^{\frac{3}{2}}}$$

$$-\frac{4\left(3\,Ba\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{5}{2}}+12\,Ba^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{7}{2}}-6\,A\left(\sqrt{b}\right.\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx\right.\right.}$$

input
```
integrate((B*x+A)/x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + a)*((3*B*a^4*b^3*abs(b) - 8*A*a^3*b^4*abs(b))*(b*x + a)/(a
^7*b^2) - 3*(B*a^5*b^3*abs(b) - 3*A*a^4*b^4*abs(b))/(a^7*b^2))/((b*x + a)*
b - a*b)^(3/2) - 4/3*(3*B*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*
b))^4*b^(5/2) + 12*B*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))
^2*b^(7/2) - 6*A*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(7/
2) + 5*B*a^3*b^(9/2) - 18*A*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b -
a*b))^2*b^(9/2) - 8*A*a^2*b^(11/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x +
 a)*b - a*b))^2 + a*b)^3*a^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{3\,a\,b^2}-\frac{8\,x^2\,\left(2\,A\,b-B\,a\right)}{a^3\,b}-\frac{x^3\,\left(32\,A\,b^3-16\,B\,a\,b^2\right)}{3\,a^4\,b^2}+\frac{x\,\left(6\,B\,a^3-12\,A\,a^2\,b\right)}{3\,a^4\,b^2}\right)}{x^{7/2}+\frac{2\,a\,x^{5/2}}{b}+\frac{a^2\,x^{3/2}}{b^2}}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x)^(5/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(3*a*b^2) - (8*x^2*(2*A*b - B*a))/(a^3*b) - (x^3*
(32*A*b^3 - 16*B*a*b^2))/(3*a^4*b^2) + (x*(6*B*a^3 - 12*A*a^2*b))/(3*a^4*b
^2)))/(x^(7/2) + (2*a*x^(5/2))/b + (a^2*x^(3/2))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{-\frac{16\sqrt{b}\,\sqrt{bx+a}\,b\,x^2}{3}-\frac{2\sqrt{x}\,a^2}{3}+\frac{8\sqrt{x}\,abx}{3}+\frac{16\sqrt{x}\,b^2x^2}{3}}{\sqrt{bx+a}\,a^3x^2}$$

input
```
int((B*x+A)/x^(5/2)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 8*sqrt(b)*sqrt(a + b*x)*b*x**2 - sqrt(x)*a**2 + 4*sqrt(x)*a*b*x + 8
*sqrt(x)*b**2*x**2))/(3*sqrt(a + b*x)*a**3*x**2)
```

3.342 $\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = -\frac{2A}{5ax^{5/2}(a+bx)^{3/2}} - \frac{2(8Ab-5aB)}{15a^2x^{3/2}(a+bx)^{3/2}} - \frac{4(8Ab-5aB)}{5a^3x^{3/2}\sqrt{a+bx}} + \frac{16(8Ab-5aB)\sqrt{a+bx}}{15a^4x^{3/2}} - \frac{32b(8Ab-5aB)\sqrt{a+bx}}{15a^5\sqrt{x}}$$

output
```
-2/5*A/a/x^(5/2)/(b*x+a)^(3/2)-2/15*(8*A*b-5*B*a)/a^2/x^(3/2)/(b*x+a)^(3/2
)-4/5*(8*A*b-5*B*a)/a^3/x^(3/2)/(b*x+a)^(1/2)+16/15*(8*A*b-5*B*a)*(b*x+a)^
(1/2)/a^4/x^(3/2)-32/15*b*(8*A*b-5*B*a)*(b*x+a)^(1/2)/a^5/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = -\frac{2(128Ab^4x^4+24a^2b^2x^2(2A-5Bx)+16ab^3x^3(12A-5Bx)+a^4(3A+5Bx)-2a^3bx(4A+15Bx))}{15a^5x^{5/2}(a+bx)^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*(128*A*b^4*x^4 + 24*a^2*b^2*x^2*(2*A - 5*B*x) + 16*a*b^3*x^3*(12*A - 5
*B*x) + a^4*(3*A + 5*B*x) - 2*a^3*b*x*(4*A + 15*B*x)))/(15*a^5*x^(5/2)*(a
+ b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(8Ab-5aB)\int \frac{1}{x^{5/2}(a+bx)^{5/2}}dx}{5a}-\frac{2A}{5ax^{5/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-5aB)\left(\frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}}dx}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}\right)}{5a}-\frac{2A}{5ax^{5/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-5aB)\left(\frac{2\left(\frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}\right)}{5a}-\frac{2A}{5ax^{5/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{(8Ab-5aB)\left(\frac{2\left(\frac{4\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}\right)}{5a}-\frac{2A}{5ax^{5/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{2\left(\frac{4\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{a}+\frac{2}{ax^{3/2}\sqrt{a+bx}}\right)}{a}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}\right)(8Ab-5aB)}{5a}-\frac{2A}{5ax^{5/2}(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*A)/(5*a*x^(5/2)*(a + b*x)^(3/2)) - ((8*A*b - 5*a*B)*(2/(3*a*x^(3/2)*(a
 + b*x)^(3/2)) + (2*(2/(a*x^(3/2)*Sqrt[a + b*x]) + (4*((-2*Sqrt[a + b*x])/
(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])))/a))/a))/(5*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(73A\,b^{2}x^{2}-40Bab\,x^{2}-14aAbx+5B\,a^{2}x+3a^{2}A)}{15a^{5}x^{\frac{5}{2}}}-\frac{2b^{2}(11A\,b^{2}x-8Babx+12abA-9a^{2}B)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}a^{5}}$	97
gosper	$-\frac{2(128A\,b^{4}x^{4}-80Ba\,b^{3}x^{4}+192Aa\,b^{3}x^{3}-120B\,a^{2}b^{2}x^{3}+48A\,a^{2}b^{2}x^{2}-30B\,a^{3}b\,x^{2}-8A\,a^{3}bx+5B\,a^{4}x+3A\,a^{4})}{15x^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}a^{5}}$	101
default	$-\frac{2(128A\,b^{4}x^{4}-80Ba\,b^{3}x^{4}+192Aa\,b^{3}x^{3}-120B\,a^{2}b^{2}x^{3}+48A\,a^{2}b^{2}x^{2}-30B\,a^{3}b\,x^{2}-8A\,a^{3}bx+5B\,a^{4}x+3A\,a^{4})}{15x^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}a^{5}}$	101
orering	$-\frac{2(128A\,b^{4}x^{4}-80Ba\,b^{3}x^{4}+192Aa\,b^{3}x^{3}-120B\,a^{2}b^{2}x^{3}+48A\,a^{2}b^{2}x^{2}-30B\,a^{3}b\,x^{2}-8A\,a^{3}bx+5B\,a^{4}x+3A\,a^{4})}{15x^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}a^{5}}$	101

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(1/2)*(73*A*b^2*x^2-40*B*a*b*x^2-14*A*a*b*x+5*B*a^2*x+3*A*a^
2)/a^5/x^(5/2)-2/3*b^2*(11*A*b^2*x-8*B*a*b*x+12*A*a*b-9*B*a^2)*x^(1/2)/(b*
x+a)^(3/2)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx =$$
$$-\frac{2\,(3\,Aa^{4}-16\,(5\,Bab^{3}-8\,Ab^{4})x^{4}-24\,(5\,Ba^{2}b^{2}-8\,Aab^{3})x^{3}-6\,(5\,Ba^{3}b-8\,Aa^{2}b^{2})x^{2}+(5\,Ba^{4}-8\,Aa^{3}}{15\,(a^{5}b^{2}x^{5}+2\,a^{6}bx^{4}+a^{7}x^{3})}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*a^4 - 16*(5*B*a*b^3 - 8*A*b^4)*x^4 - 24*(5*B*a^2*b^2 - 8*A*a*b^
3)*x^3 - 6*(5*B*a^3*b - 8*A*a^2*b^2)*x^2 + (5*B*a^4 - 8*A*a^3*b)*x)*sqrt(b
*x + a)*sqrt(x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(7/2)/(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = -\frac{4\,Bbx}{3\,(bx^2+ax)^{\frac{3}{2}}a^2} + \frac{32\,Bb^2x}{3\,\sqrt{bx^2+ax}a^4} + \frac{32\,Ab^2x}{15\,(bx^2+ax)^{\frac{3}{2}}a^3} - \frac{256\,Ab^3x}{15\,\sqrt{bx^2+ax}a^5} - \frac{2\,B}{3\,(bx^2+ax)^{\frac{3}{2}}a} + \frac{16\,Bb}{3\,\sqrt{bx^2+ax}a^3} + \frac{16\,Ab}{15\,(bx^2+ax)^{\frac{3}{2}}a^2} - \frac{128\,Ab^2}{15\,\sqrt{bx^2+ax}a^4} - \frac{2\,A}{5\,(bx^2+ax)^{\frac{3}{2}}ax}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*B*b*x/((b*x^2 + a*x)^(3/2)*a^2) + 32/3*B*b^2*x/(sqrt(b*x^2 + a*x)*a^4
) + 32/15*A*b^2*x/((b*x^2 + a*x)^(3/2)*a^3) - 256/15*A*b^3*x/(sqrt(b*x^2 +
 a*x)*a^5) - 2/3*B/((b*x^2 + a*x)^(3/2)*a) + 16/3*B*b/(sqrt(b*x^2 + a*x)*a
^3) + 16/15*A*b/((b*x^2 + a*x)^(3/2)*a^2) - 128/15*A*b^2/(sqrt(b*x^2 + a*x
)*a^4) - 2/5*A/((b*x^2 + a*x)^(3/2)*a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(114) = 228$.

Time = 0.23 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.37

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = \frac{2\sqrt{bx+a}\left((bx+a)\left(\frac{(40\,Ba^8b^7-73\,Aa^7b^8)(bx+a)}{a^{12}b^2|b|}-\frac{5\,(17\,Ba^9b^7-32\,Aa^8b^8)}{a^{12}b^2|b|}\right)+\frac{45\,(Ba^{10}b^7-2\,Aa^9 \cdots}{a^{12}b^2|b|}\right.}{15\,((bx+a)b-ab)^{\frac{5}{2}}}$$
$$+\frac{4\left(6\,Ba\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{7}{2}}+18\,Ba^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{9}{2}}-9\,A\left(\sqrt{bx \cdots}\right.\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx \cdots}\right.\right.}$$

input
```
integrate((B*x+A)/x^(7/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/15*sqrt(b*x + a)*((b*x + a)*((40*B*a^8*b^7 - 73*A*a^7*b^8)*(b*x + a)/(a^
12*b^2*abs(b)) - 5*(17*B*a^9*b^7 - 32*A*a^8*b^8)/(a^12*b^2*abs(b))) + 45*(
B*a^10*b^7 - 2*A*a^9*b^8)/(a^12*b^2*abs(b)))/((b*x + a)*b - a*b)^(5/2) + 4
/3*(6*B*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(7/2) + 18
*B*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(9/2) - 9*A*(
sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(9/2) + 8*B*a^3*b^(11
/2) - 24*A*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(11/2)
- 11*A*a^2*b^(13/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2
 + a*b)^3*a^4*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = -\frac{\sqrt{a+b\,x}\,\left(\frac{2\,A}{5\,a\,b^2}+\frac{16\,x^3\,(8\,A\,b-5\,B\,a)}{5\,a^4}+\frac{4\,x^2\,(8\,A\,b-5\,B\,a)}{5\,a^3\,b}+\frac{x^4\,(256\,A\,b^4-160\,B\,a\,b^3)}{15\,a^5\,b^2}+\frac{x\,(10\,B\,a^4-16\,A\,a^3\,b)}{15\,a^5\,b^2}\right)}{x^{9/2}+\frac{2\,a\,x^{7/2}}{b}+\frac{a^2\,x^{5/2}}{b^2}}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x)^(5/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(5*a*b^2) + (16*x^3*(8*A*b - 5*B*a))/(5*a^4) + (4
*x^2*(8*A*b - 5*B*a))/(5*a^3*b) + (x^4*(256*A*b^4 - 160*B*a*b^3))/(15*a^5*
b^2) + (x*(10*B*a^4 - 16*A*a^3*b))/(15*a^5*b^2)))/(x^(9/2) + (2*a*x^(7/2))
/b + (a^2*x^(5/2))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx}{x^{7/2}(a+bx)^{5/2}}\,dx = \frac{\frac{32\sqrt{b}\,\sqrt{bx+a}\,b^2x^3}{5} - \frac{2\sqrt{x}\,a^3}{5} + \frac{4\sqrt{x}\,a^2bx}{5} - \frac{16\sqrt{x}\,a\,b^2x^2}{5} - \frac{32\sqrt{x}\,b^3x^3}{5}}{\sqrt{bx+a}\,a^4x^3}$$

input
```
int((B*x+A)/x^(7/2)/(b*x+a)^(5/2),x)
```

output
```
(2*(16*sqrt(b)*sqrt(a + b*x)*b**2*x**3 - sqrt(x)*a**3 + 2*sqrt(x)*a**2*b*x
 - 8*sqrt(x)*a*b**2*x**2 - 16*sqrt(x)*b**3*x**3))/(5*sqrt(a + b*x)*a**4*x*
*3)
```

3.343 $\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 177

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx=-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}-\frac{2(10Ab-7aB)}{21a^2x^{5/2}(a+bx)^{3/2}}-\frac{16(10Ab-7aB)}{21a^3x^{5/2}\sqrt{a+bx}}+\frac{32(10Ab-7aB)\sqrt{a+bx}}{35a^4x^{5/2}}-\frac{128b(10Ab-7aB)\sqrt{a+bx}}{105a^5x^{3/2}}+\frac{256b^2(10Ab-7aB)\sqrt{a+bx}}{105a^6\sqrt{x}}$$

output
```
-2/7*A/a/x^(7/2)/(b*x+a)^(3/2)-2/21*(10*A*b-7*B*a)/a^2/x^(5/2)/(b*x+a)^(3/
2)-16/21*(10*A*b-7*B*a)/a^3/x^(5/2)/(b*x+a)^(1/2)+32/35*(10*A*b-7*B*a)*(b*
x+a)^(1/2)/a^4/x^(5/2)-128/105*b*(10*A*b-7*B*a)*(b*x+a)^(1/2)/a^5/x^(3/2)+
256/105*b^2*(10*A*b-7*B*a)*(b*x+a)^(1/2)/a^6/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx = -\frac{2(-1280Ab^5x^5+128ab^4x^4(-15A+7Bx)+3a^5(5A+7Bx)+96a^2b^3x^3(-5A+14Bx)+16a^3b^2x^2(5A -}{105a^6x^{7/2}(a+bx)^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*(-1280*A*b^5*x^5 + 128*a*b^4*x^4*(-15*A + 7*B*x) + 3*a^5*(5*A + 7*B*x)
 + 96*a^2*b^3*x^3*(-5*A + 14*B*x) + 16*a^3*b^2*x^2*(5*A + 21*B*x) - 2*a^4*
b*x*(15*A + 28*B*x)))/(105*a^6*x^(7/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {87, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(10Ab-7aB)\int \frac{1}{x^{7/2}(a+bx)^{5/2}}dx}{7a}-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(10Ab-7aB)\left(\frac{8\int \frac{1}{x^{7/2}(a+bx)^{3/2}}dx}{3a}+\frac{2}{3ax^{5/2}(a+bx)^{3/2}}\right)}{7a}-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}$$

$$\Big\downarrow \; 55$$

$$-\frac{(10Ab-7aB)\left(\frac{8\left(\frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}dx}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{3a}+\frac{2}{3ax^{5/2}(a+bx)^{3/2}}\right)}{7a}-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}$$

$\big\downarrow$ 55

$$-\frac{(10Ab-7aB)\left(\frac{8\left(\frac{6\left(-\frac{4b\int \frac{1}{x^{5/2}\sqrt{a+bx}}dx}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{3a}+\frac{2}{3ax^{5/2}(a+bx)^{3/2}}\right)}{7a}-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}$$

$\big\downarrow$ 55

$$-\frac{(10Ab-7aB)\left(\frac{8\left(\frac{6\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^{3/2}\sqrt{a+bx}}dx}{3a}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{3a}+\frac{2}{3ax^{5/2}(a+bx)^{3/2}}\right)}{7a}-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}$$

$\big\downarrow$ 48

$$-\frac{\left(\frac{8\left(\frac{6\left(-\frac{4b\left(\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}\right)}{5a}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}\right)}{a}+\frac{2}{ax^{5/2}\sqrt{a+bx}}\right)}{3a}+\frac{2}{3ax^{5/2}(a+bx)^{3/2}}\right)(10Ab-7aB)}{7a}-\frac{2A}{7ax^{7/2}(a+bx)^{3/2}}$$

input
```
Int[(A + B*x)/(x^(9/2)*(a + b*x)^(5/2)),x]
```

output
```
(-2*A)/(7*a*x^(7/2)*(a + b*x)^(3/2)) - ((10*A*b - 7*a*B)*(2/(3*a*x^(5/2)*(
a + b*x)^(3/2)) + (8*(2/(a*x^(5/2)*Sqrt[a + b*x]) + (6*((-2*Sqrt[a + b*x])
/(5*a*x^(5/2)) - (4*b*((-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*
x])/(3*a^2*Sqrt[x])))/(5*a)))/a))/(3*a)))/(7*a)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2\sqrt{bx+a}\,(-790A\,b^3x^3+511Ba\,b^2x^3+185aA\,b^2x^2-98B\,a^2b\,x^2-60a^2Abx+21B\,a^3x+15a^3A)}{105a^6x^{\frac{7}{2}}}+\frac{2b^3(14A\,b^2x-11Babx+15abA}{3(bx+a)^{\frac{3}{2}}a^6}$
gosper	$-\frac{2(-1280A\,b^5x^5+896Ba\,b^4x^5-1920aA\,b^4x^4+1344B\,a^2b^3x^4-480a^2A\,b^3x^3+336B\,a^3b^2x^3+80a^3A\,b^2x^2-56B\,a^4b\,x^2-30a^4Abx+}{105x^{\frac{7}{2}}(bx+a)^{\frac{3}{2}}a^6}$
default	$-\frac{2(-1280A\,b^5x^5+896Ba\,b^4x^5-1920aA\,b^4x^4+1344B\,a^2b^3x^4-480a^2A\,b^3x^3+336B\,a^3b^2x^3+80a^3A\,b^2x^2-56B\,a^4b\,x^2-30a^4Abx+}{105x^{\frac{7}{2}}(bx+a)^{\frac{3}{2}}a^6}$
orering	$-\frac{2(-1280A\,b^5x^5+896Ba\,b^4x^5-1920aA\,b^4x^4+1344B\,a^2b^3x^4-480a^2A\,b^3x^3+336B\,a^3b^2x^3+80a^3A\,b^2x^2-56B\,a^4b\,x^2-30a^4Abx+}{105x^{\frac{7}{2}}(bx+a)^{\frac{3}{2}}a^6}$

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x+a)^(1/2)*(-790*A*b^3*x^3+511*B*a*b^2*x^3+185*A*a*b^2*x^2-98*B*
a^2*b*x^2-60*A*a^2*b*x+21*B*a^3*x+15*A*a^3)/a^6/x^(7/2)+2/3*b^3*(14*A*b^2*
x-11*B*a*b*x+15*A*a*b-12*B*a^2)*x^(1/2)/(b*x+a)^(3/2)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx =$$

$$-\frac{2\,(15\,Aa^5+128\,(7\,Bab^4-10\,Ab^5)x^5+192\,(7\,Ba^2b^3-10\,Aab^4)x^4+48\,(7\,Ba^3b^2-10\,Aa^2b^3)x^3-8\,(7\,B}{105\,(a^6b^2x^6+2\,a^7bx^5+a^8x^4)}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/105*(15*A*a^5 + 128*(7*B*a*b^4 - 10*A*b^5)*x^5 + 192*(7*B*a^2*b^3 - 10*
A*a*b^4)*x^4 + 48*(7*B*a^3*b^2 - 10*A*a^2*b^3)*x^3 - 8*(7*B*a^4*b - 10*A*a
^3*b^2)*x^2 + 3*(7*B*a^5 - 10*A*a^4*b)*x)*sqrt(b*x + a)*sqrt(x)/(a^6*b^2*x
^6 + 2*a^7*b*x^5 + a^8*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(9/2)/(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx &= \frac{32\,Bb^2x}{15\,(bx^2+ax)^{\frac{3}{2}}a^3}-\frac{256\,Bb^3x}{15\,\sqrt{bx^2+ax}a^5}-\frac{64\,Ab^3x}{21\,(bx^2+ax)^{\frac{3}{2}}a^4}\\&+\frac{512\,Ab^4x}{21\,\sqrt{bx^2+ax}a^6}+\frac{16\,Bb}{15\,(bx^2+ax)^{\frac{3}{2}}a^2}-\frac{128\,Bb^2}{15\,\sqrt{bx^2+ax}a^4}-\frac{32\,Ab^2}{21\,(bx^2+ax)^{\frac{3}{2}}a^3}\\&+\frac{256\,Ab^3}{21\,\sqrt{bx^2+ax}a^5}-\frac{2\,B}{5\,(bx^2+ax)^{\frac{3}{2}}ax}+\frac{4\,Ab}{7\,(bx^2+ax)^{\frac{3}{2}}a^2x}-\frac{2\,A}{7\,(bx^2+ax)^{\frac{3}{2}}ax^2}\end{aligned}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
32/15*B*b^2*x/((b*x^2 + a*x)^(3/2)*a^3) - 256/15*B*b^3*x/(sqrt(b*x^2 + a*x
)*a^5) - 64/21*A*b^3*x/((b*x^2 + a*x)^(3/2)*a^4) + 512/21*A*b^4*x/(sqrt(b*
x^2 + a*x)*a^6) + 16/15*B*b/((b*x^2 + a*x)^(3/2)*a^2) - 128/15*B*b^2/(sqrt
(b*x^2 + a*x)*a^4) - 32/21*A*b^2/((b*x^2 + a*x)^(3/2)*a^3) + 256/21*A*b^3/
(sqrt(b*x^2 + a*x)*a^5) - 2/5*B/((b*x^2 + a*x)^(3/2)*a*x) + 4/7*A*b/((b*x^
2 + a*x)^(3/2)*a^2*x) - 2/7*A/((b*x^2 + a*x)^(3/2)*a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(141) = 282$.

Time = 0.24 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx = -\frac{2\left((bx+a)\left((bx+a)\left(\frac{(511\,Ba^{13}b^9|b|-790\,Aa^{12}b^{10}|b|)(bx+a)}{a^{18}b^4}-\frac{7\,(233\,Ba^{14}b^9|b|-365\,Aa^{13}b^{10}|b|)}{a^{18}b^4}\right)+\frac{350\,(5\,Ba^{15}b^9|b|-8\,Aa^{14}b^{1}}{a^{18}b^4}\right.\right.}{105\,((bx+a)b-ab)^{\frac{7}{2}}} - \frac{4\left(9\,Ba\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4 b^{\frac{9}{2}}+24\,Ba^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2 b^{\frac{11}{2}}-12\,A\left(\sqrt{}\right.\right.}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(}\right.\right.}$$

input
```
integrate((B*x+A)/x^(9/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/105*((b*x + a)*((b*x + a)*((511*B*a^13*b^9*abs(b) - 790*A*a^12*b^10*abs
(b))*(b*x + a)/(a^18*b^4) - 7*(233*B*a^14*b^9*abs(b) - 365*A*a^13*b^10*abs
(b))/(a^18*b^4)) + 350*(5*B*a^15*b^9*abs(b) - 8*A*a^14*b^10*abs(b))/(a^18*
b^4)) - 210*(3*B*a^16*b^9*abs(b) - 5*A*a^15*b^10*abs(b))/(a^18*b^4))*sqrt(
b*x + a)/((b*x + a)*b - a*b)^(7/2) - 4/3*(9*B*a*(sqrt(b*x + a)*sqrt(b) - s
qrt((b*x + a)*b - a*b))^4*b^(9/2) + 24*B*a^2*(sqrt(b*x + a)*sqrt(b) - sqrt
((b*x + a)*b - a*b))^2*b^(11/2) - 12*A*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x
+ a)*b - a*b))^4*b^(11/2) + 11*B*a^3*b^(13/2) - 30*A*a*(sqrt(b*x + a)*sqrt
(b) - sqrt((b*x + a)*b - a*b))^2*b^(13/2) - 14*A*a^2*b^(15/2))/(((sqrt(b*x
 + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*a^5*abs(b))
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx = -\frac{\sqrt{a+b\,x}\left(\frac{2\,A}{7\,a\,b^2}-\frac{32\,x^3\,(10\,A\,b-7\,B\,a)}{35\,a^4}+\frac{16\,x^2\,(10\,A\,b-7\,B\,a)}{105\,a^3\,b}-\frac{x^5\,(2560\,A\,b^5-1792\,B\,a\,b^4)}{105\,a^6\,b^2}-\frac{128\,b\,x^4\,(10\,A\,b-7\,B\,a)}{35\,a^5}+\frac{x\,(42\,B}{10}\right.}{x^{11/2}+\frac{2\,a\,x^{9/2}}{b}+\frac{a^2\,x^{7/2}}{b^2}}$$

input
```
int((A + B*x)/(x^(9/2)*(a + b*x)^(5/2)),x)
```

output
```
-((a + b*x)^(1/2)*((2*A)/(7*a*b^2) - (32*x^3*(10*A*b - 7*B*a))/(35*a^4) +
(16*x^2*(10*A*b - 7*B*a))/(105*a^3*b) - (x^5*(2560*A*b^5 - 1792*B*a*b^4))/
(105*a^6*b^2) - (128*b*x^4*(10*A*b - 7*B*a))/(35*a^5) + (x*(42*B*a^5 - 60*
A*a^4*b))/(105*a^6*b^2)))/(x^(11/2) + (2*a*x^(9/2))/b + (a^2*x^(7/2))/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx}{x^{9/2}(a+bx)^{5/2}}\,dx = \frac{-\frac{256\sqrt{b}\,\sqrt{bx+a}\,b^3x^4}{35}-\frac{2\sqrt{x}\,a^4}{7}+\frac{16\sqrt{x}\,a^3bx}{35}-\frac{32\sqrt{x}\,a^2b^2x^2}{35}+\frac{128\sqrt{x}\,a\,b^3x^3}{35}+\frac{256\sqrt{x}\,b^4x^4}{35}}{\sqrt{bx+a}\,a^5x^4}$$

input
```
int((B*x+A)/x^(9/2)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 128*sqrt(b)*sqrt(a + b*x)*b**3*x**4 - 5*sqrt(x)*a**4 + 8*sqrt(x)*a*
*3*b*x - 16*sqrt(x)*a**2*b**2*x**2 + 64*sqrt(x)*a*b**3*x**3 + 128*sqrt(x)*
b**4*x**4))/(35*sqrt(a + b*x)*a**5*x**4)
```

3.344 $\int (ex)^m(a+bx)^4(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 155

$$\int (ex)^m(a+bx)^4(A+Bx)\,dx=\frac{a^4A(ex)^{1+m}}{e(1+m)}+\frac{a^3(4Ab+aB)(ex)^{2+m}}{e^2(2+m)}+\frac{2a^2b(3Ab+2aB)(ex)^{3+m}}{e^3(3+m)}+\frac{2ab^2(2Ab+3aB)(ex)^{4+m}}{e^4(4+m)}+\frac{b^3(Ab+4aB)(ex)^{5+m}}{e^5(5+m)}+\frac{b^4B(ex)^{6+m}}{e^6(6+m)}$$

output
```
a^4*A*(e*x)^(1+m)/e/(1+m)+a^3*(4*A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+2*a^2*b*(3
*A*b+2*B*a)*(e*x)^(3+m)/e^3/(3+m)+2*a*b^2*(2*A*b+3*B*a)*(e*x)^(4+m)/e^4/(4
+m)+b^3*(A*b+4*B*a)*(e*x)^(5+m)/e^5/(5+m)+b^4*B*(e*x)^(6+m)/e^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int (ex)^m(a+bx)^4(A+Bx)\,dx = \frac{(ex)^m\left(Bx(a+bx)^5+(-aB(1+m)+Ab(6+m))x\left(\frac{a^4}{1+m}+\frac{4a^3bx}{2+m}+\frac{6a^2b^2x^2}{3+m}+\frac{4ab^3x^3}{4+m}+\frac{b^4x^4}{5+m}\right)\right)}{b(6+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x)^4*(A + B*x),x]
```

output
```
((e*x)^m*(B*x*(a + b*x)^5 + (-(a*B*(1 + m)) + A*b*(6 + m))*x*(a^4/(1 + m)
+ (4*a^3*b*x)/(2 + m) + (6*a^2*b^2*x^2)/(3 + m) + (4*a*b^3*x^3)/(4 + m) +
(b^4*x^4)/(5 + m))))/(b*(6 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4(A+Bx)(ex)^m\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^4A(ex)^m+\frac{a^3(ex)^{m+1}(aB+4Ab)}{e}+\frac{2a^2b(ex)^{m+2}(2aB+3Ab)}{e^2}+\frac{b^3(ex)^{m+4}(4aB+Ab)}{e^4}+\frac{2ab^2(ex)^{m+3}(3}{e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4A(ex)^{m+1}}{e(m+1)}+\frac{a^3(ex)^{m+2}(aB+4Ab)}{e^2(m+2)}+\frac{2a^2b(ex)^{m+3}(2aB+3Ab)}{e^3(m+3)}+\frac{b^3(ex)^{m+5}(4aB+Ab)}{e^5(m+5)}+\frac{2ab^2(ex)^{m+4}(3aB+2Ab)}{e^4(m+4)}+\frac{b^4B(ex)^{m+6}}{e^6(m+6)}$$

input
```
Int[(e*x)^m*(a + b*x)^4*(A + B*x),x]
```

output
```
(a^4*A*(e*x)^(1 + m))/(e*(1 + m)) + (a^3*(4*A*b + a*B)*(e*x)^(2 + m))/(e^2
*(2 + m)) + (2*a^2*b*(3*A*b + 2*a*B)*(e*x)^(3 + m))/(e^3*(3 + m)) + (2*a*b
^2*(2*A*b + 3*a*B)*(e*x)^(4 + m))/(e^4*(4 + m)) + (b^3*(A*b + 4*a*B)*(e*x)
^(5 + m))/(e^5*(5 + m)) + (b^4*B*(e*x)^(6 + m))/(e^6*(6 + m))
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

method	result
norman	$\frac{A\,a^{4}x\,\mathrm{e}^{m\ln(ex)}}{1+m}+\frac{B\,b^{4}x^{6}\mathrm{e}^{m\ln(ex)}}{6+m}+\frac{a^{3}(4Ab+Ba)x^{2}\mathrm{e}^{m\ln(ex)}}{2+m}+\frac{b^{3}(Ab+4Ba)x^{5}\mathrm{e}^{m\ln(ex)}}{5+m}+\frac{2a\,b^{2}(2Ab+3Ba)x^{4}\mathrm{e}^{m\ln(\cdots}}{4+m}$
gosper	$\frac{x(B\,b^{4}m^{5}x^{5}+A\,b^{4}m^{5}x^{4}+4Ba\,b^{3}m^{5}x^{4}+15B\,b^{4}m^{4}x^{5}+4Aa\,b^{3}m^{5}x^{3}+16A\,b^{4}m^{4}x^{4}+6B\,a^{2}b^{2}m^{5}x^{3}+64Ba\,b^{3}m^{4}x^{4}+85B\,b^{4}m\cdots}{}$
risch	$\frac{x(B\,b^{4}m^{5}x^{5}+A\,b^{4}m^{5}x^{4}+4Ba\,b^{3}m^{5}x^{4}+15B\,b^{4}m^{4}x^{5}+4Aa\,b^{3}m^{5}x^{3}+16A\,b^{4}m^{4}x^{4}+6B\,a^{2}b^{2}m^{5}x^{3}+64Ba\,b^{3}m^{4}x^{4}+85B\,b^{4}m\cdots}{}$
orering	$\frac{x(B\,b^{4}m^{5}x^{5}+A\,b^{4}m^{5}x^{4}+4Ba\,b^{3}m^{5}x^{4}+15B\,b^{4}m^{4}x^{5}+4Aa\,b^{3}m^{5}x^{3}+16A\,b^{4}m^{4}x^{4}+6B\,a^{2}b^{2}m^{5}x^{3}+64Ba\,b^{3}m^{4}x^{4}+85B\,b^{4}m\cdots}{}$
parallelrisch	Expression too large to display

input
```
int((e*x)^m*(b*x+a)^4*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*a^4/(1+m)*x*exp(m*ln(e*x))+B*b^4/(6+m)*x^6*exp(m*ln(e*x))+a^3*(4*A*b+B*a
)/(2+m)*x^2*exp(m*ln(e*x))+b^3*(A*b+4*B*a)/(5+m)*x^5*exp(m*ln(e*x))+2*a*b^
2*(2*A*b+3*B*a)/(4+m)*x^4*exp(m*ln(e*x))+2*a^2*b*(3*A*b+2*B*a)/(3+m)*x^3*e
xp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(155) = 310$.

Time = 0.10 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.93

$$\int (ex)^m(a+bx)^4(A+Bx)\,dx = \frac{((Bb^4m^5+15\,Bb^4m^4+85\,Bb^4m^3+225\,Bb^4m^2+274\,Bb^4m+120\,Bb^4)x^6+((4\,Bab^3+Ab^4)m^5+576\ldots}{}$$

input
```
integrate((e*x)^m*(b*x+a)^4*(B*x+A),x, algorithm="fricas")
```

output
```
((B*b^4*m^5 + 15*B*b^4*m^4 + 85*B*b^4*m^3 + 225*B*b^4*m^2 + 274*B*b^4*m +
120*B*b^4)*x^6 + ((4*B*a*b^3 + A*b^4)*m^5 + 576*B*a*b^3 + 144*A*b^4 + 16*(
4*B*a*b^3 + A*b^4)*m^4 + 95*(4*B*a*b^3 + A*b^4)*m^3 + 260*(4*B*a*b^3 + A*b
^4)*m^2 + 324*(4*B*a*b^3 + A*b^4)*m)*x^5 + 2*((3*B*a^2*b^2 + 2*A*a*b^3)*m^
5 + 540*B*a^2*b^2 + 360*A*a*b^3 + 17*(3*B*a^2*b^2 + 2*A*a*b^3)*m^4 + 107*(
3*B*a^2*b^2 + 2*A*a*b^3)*m^3 + 307*(3*B*a^2*b^2 + 2*A*a*b^3)*m^2 + 396*(3*
B*a^2*b^2 + 2*A*a*b^3)*m)*x^4 + 2*((2*B*a^3*b + 3*A*a^2*b^2)*m^5 + 480*B*a
^3*b + 720*A*a^2*b^2 + 18*(2*B*a^3*b + 3*A*a^2*b^2)*m^4 + 121*(2*B*a^3*b +
 3*A*a^2*b^2)*m^3 + 372*(2*B*a^3*b + 3*A*a^2*b^2)*m^2 + 508*(2*B*a^3*b + 3
*A*a^2*b^2)*m)*x^3 + ((B*a^4 + 4*A*a^3*b)*m^5 + 360*B*a^4 + 1440*A*a^3*b +
 19*(B*a^4 + 4*A*a^3*b)*m^4 + 137*(B*a^4 + 4*A*a^3*b)*m^3 + 461*(B*a^4 + 4
*A*a^3*b)*m^2 + 702*(B*a^4 + 4*A*a^3*b)*m)*x^2 + (A*a^4*m^5 + 20*A*a^4*m^4
 + 155*A*a^4*m^3 + 580*A*a^4*m^2 + 1044*A*a^4*m + 720*A*a^4)*x)*(e*x)^m/(m
^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3538 vs. $2(146) = 292$.

Time = 0.54 (sec) , antiderivative size = 3538, normalized size of antiderivative = 22.83

$$\int (ex)^m (a+bx)^4 (A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x+a)**4*(B*x+A),x)
```

output
```
Piecewise(((-A*a**4/(5*x**5) - A*a**3*b/x**4 - 2*A*a**2*b**2/x**3 - 2*A*a*
b**3/x**2 - A*b**4/x - B*a**4/(4*x**4) - 4*B*a**3*b/(3*x**3) - 3*B*a**2*b*
*2/x**2 - 4*B*a*b**3/x + B*b**4*log(x))/e**6, Eq(m, -6)), ((-A*a**4/(4*x**
4) - 4*A*a**3*b/(3*x**3) - 3*A*a**2*b**2/x**2 - 4*A*a*b**3/x + A*b**4*log(
x) - B*a**4/(3*x**3) - 2*B*a**3*b/x**2 - 6*B*a**2*b**2/x + 4*B*a*b**3*log(
x) + B*b**4*x)/e**5, Eq(m, -5)), ((-A*a**4/(3*x**3) - 2*A*a**3*b/x**2 - 6*
A*a**2*b**2/x + 4*A*a*b**3*log(x) + A*b**4*x - B*a**4/(2*x**2) - 4*B*a**3*
b/x + 6*B*a**2*b**2*log(x) + 4*B*a*b**3*x + B*b**4*x**2/2)/e**4, Eq(m, -4)
), ((-A*a**4/(2*x**2) - 4*A*a**3*b/x + 6*A*a**2*b**2*log(x) + 4*A*a*b**3*x
 + A*b**4*x**2/2 - B*a**4/x + 4*B*a**3*b*log(x) + 6*B*a**2*b**2*x + 2*B*a*
b**3*x**2 + B*b**4*x**3/3)/e**3, Eq(m, -3)), ((-A*a**4/x + 4*A*a**3*b*log(
x) + 6*A*a**2*b**2*x + 2*A*a*b**3*x**2 + A*b**4*x**3/3 + B*a**4*log(x) + 4
*B*a**3*b*x + 3*B*a**2*b**2*x**2 + 4*B*a*b**3*x**3/3 + B*b**4*x**4/4)/e**2
, Eq(m, -2)), ((A*a**4*log(x) + 4*A*a**3*b*x + 3*A*a**2*b**2*x**2 + 4*A*a*
b**3*x**3/3 + A*b**4*x**4/4 + B*a**4*x + 2*B*a**3*b*x**2 + 2*B*a**2*b**2*x
**3 + B*a*b**3*x**4 + B*b**4*x**5/5)/e, Eq(m, -1)), (A*a**4*m**5*x*(e*x)**
m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 20*A
*a**4*m**4*x*(e*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 +
1764*m + 720) + 155*A*a**4*m**3*x*(e*x)**m/(m**6 + 21*m**5 + 175*m**4 + 73
5*m**3 + 1624*m**2 + 1764*m + 720) + 580*A*a**4*m**2*x*(e*x)**m/(m**6 +...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.34

$$\int (ex)^m(a+bx)^4(A+Bx)\,dx = \frac{Bb^4e^mx^6x^m}{m+6} + \frac{4\,Bab^3e^mx^5x^m}{m+5} + \frac{Ab^4e^mx^5x^m}{m+5} + \frac{6\,Ba^2b^2e^mx^4x^m}{m+4} + \frac{4\,Aab^3e^mx^4x^m}{m+4} + \frac{4\,Ba^3be^mx^3x^m}{m+3} + \frac{6\,Aa^2b^2e^mx^3x^m}{m+3} + \frac{Ba^4e^mx^2x^m}{m+2} + \frac{4\,Aa^3be^mx^2x^m}{m+2} + \frac{(ex)^{m+1}\,Aa^4}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x+a)^4*(B*x+A),x, algorithm="maxima")
```

output
```
B*b^4*e^m*x^6*x^m/(m + 6) + 4*B*a*b^3*e^m*x^5*x^m/(m + 5) + A*b^4*e^m*x^5*
x^m/(m + 5) + 6*B*a^2*b^2*e^m*x^4*x^m/(m + 4) + 4*A*a*b^3*e^m*x^4*x^m/(m +
 4) + 4*B*a^3*b*e^m*x^3*x^m/(m + 3) + 6*A*a^2*b^2*e^m*x^3*x^m/(m + 3) + B*
a^4*e^m*x^2*x^m/(m + 2) + 4*A*a^3*b*e^m*x^2*x^m/(m + 2) + (e*x)^(m + 1)*A*
a^4/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(155) = 310.

Time = 0.13 (sec) , antiderivative size = 1046, normalized size of antiderivative = 6.75

$$\int (ex)^m(a+bx)^4(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x+a)^4*(B*x+A),x, algorithm="giac")
```

output
```
((e*x)^m*B*b^4*m^5*x^6 + 4*(e*x)^m*B*a*b^3*m^5*x^5 + (e*x)^m*A*b^4*m^5*x^5
 + 15*(e*x)^m*B*b^4*m^4*x^6 + 6*(e*x)^m*B*a^2*b^2*m^5*x^4 + 4*(e*x)^m*A*a*
b^3*m^5*x^4 + 64*(e*x)^m*B*a*b^3*m^4*x^5 + 16*(e*x)^m*A*b^4*m^4*x^5 + 85*(
e*x)^m*B*b^4*m^3*x^6 + 4*(e*x)^m*B*a^3*b*m^5*x^3 + 6*(e*x)^m*A*a^2*b^2*m^5
*x^3 + 102*(e*x)^m*B*a^2*b^2*m^4*x^4 + 68*(e*x)^m*A*a*b^3*m^4*x^4 + 380*(e
*x)^m*B*a*b^3*m^3*x^5 + 95*(e*x)^m*A*b^4*m^3*x^5 + 225*(e*x)^m*B*b^4*m^2*x
^6 + (e*x)^m*B*a^4*m^5*x^2 + 4*(e*x)^m*A*a^3*b*m^5*x^2 + 72*(e*x)^m*B*a^3*
b*m^4*x^3 + 108*(e*x)^m*A*a^2*b^2*m^4*x^3 + 642*(e*x)^m*B*a^2*b^2*m^3*x^4
+ 428*(e*x)^m*A*a*b^3*m^3*x^4 + 1040*(e*x)^m*B*a*b^3*m^2*x^5 + 260*(e*x)^m
*A*b^4*m^2*x^5 + 274*(e*x)^m*B*b^4*m*x^6 + (e*x)^m*A*a^4*m^5*x + 19*(e*x)^
m*B*a^4*m^4*x^2 + 76*(e*x)^m*A*a^3*b*m^4*x^2 + 484*(e*x)^m*B*a^3*b*m^3*x^3
 + 726*(e*x)^m*A*a^2*b^2*m^3*x^3 + 1842*(e*x)^m*B*a^2*b^2*m^2*x^4 + 1228*(
e*x)^m*A*a*b^3*m^2*x^4 + 1296*(e*x)^m*B*a*b^3*m*x^5 + 324*(e*x)^m*A*b^4*m*
x^5 + 120*(e*x)^m*B*b^4*x^6 + 20*(e*x)^m*A*a^4*m^4*x + 137*(e*x)^m*B*a^4*m
^3*x^2 + 548*(e*x)^m*A*a^3*b*m^3*x^2 + 1488*(e*x)^m*B*a^3*b*m^2*x^3 + 2232
*(e*x)^m*A*a^2*b^2*m^2*x^3 + 2376*(e*x)^m*B*a^2*b^2*m*x^4 + 1584*(e*x)^m*A
*a*b^3*m*x^4 + 576*(e*x)^m*B*a*b^3*x^5 + 144*(e*x)^m*A*b^4*x^5 + 155*(e*x)
^m*A*a^4*m^3*x + 461*(e*x)^m*B*a^4*m^2*x^2 + 1844*(e*x)^m*A*a^3*b*m^2*x^2
+ 2032*(e*x)^m*B*a^3*b*m*x^3 + 3048*(e*x)^m*A*a^2*b^2*m*x^3 + 1080*(e*x)^m
*B*a^2*b^2*x^4 + 720*(e*x)^m*A*a*b^3*x^4 + 580*(e*x)^m*A*a^4*m^2*x + 70...
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.61

$$\int (ex)^m(a+bx)^4(A+Bx)\,dx = (e\,x)^m\left(\frac{A\,a^4\,x\,(m^5+20\,m^4+155\,m^3+580\,m^2+1044\,m+720)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{B\,b^4\,x^6\,(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{a^3\,x^2\,(4\,A\,b+B\,a)\,(m^5+19\,m^4+137\,m^3+461\,m^2+702\,m+360)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{b^3\,x^5\,(A\,b+4\,B\,a)\,(m^5+16\,m^4+95\,m^3+260\,m^2+324\,m+144)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{2\,a\,b^2\,x^4\,(2\,A\,b+3\,B\,a)\,(m^5+17\,m^4+107\,m^3+307\,m^2+396\,m+180)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720} + \frac{2\,a^2\,b\,x^3\,(3\,A\,b+2\,B\,a)\,(m^5+18\,m^4+121\,m^3+372\,m^2+508\,m+240)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}\right)$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x)^4,x)
```

output
```
(e*x)^m*((A*a^4*x*(1044*m + 580*m^2 + 155*m^3 + 20*m^4 + m^5 + 720))/(1764
*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (B*b^4*x^6*(274*
m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 +
 175*m^4 + 21*m^5 + m^6 + 720) + (a^3*x^2*(4*A*b + B*a)*(702*m + 461*m^2 +
 137*m^3 + 19*m^4 + m^5 + 360))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 2
1*m^5 + m^6 + 720) + (b^3*x^5*(A*b + 4*B*a)*(324*m + 260*m^2 + 95*m^3 + 16
*m^4 + m^5 + 144))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 +
 720) + (2*a*b^2*x^4*(2*A*b + 3*B*a)*(396*m + 307*m^2 + 107*m^3 + 17*m^4 +
 m^5 + 180))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)
+ (2*a^2*b*x^3*(3*A*b + 2*B*a)*(508*m + 372*m^2 + 121*m^3 + 18*m^4 + m^5 +
 240))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.72

$$\int (ex)^m (a+bx)^4 (A+Bx)\,dx$$
$$= \frac{x^m e^m x(b^5m^5x^5 + 5a\,b^4m^5x^4 + 15b^5m^4x^5 + 10a^2b^3m^5x^3 + 80a\,b^4m^4x^4 + 85b^5m^3x^5 + 10a^3b^2m^5x^2 + 170}{}$$

input
```
int((e*x)^m*(b*x+a)^4*(B*x+A),x)
```

output
```
(x**m*e**m*x*(a**5*m**5 + 20*a**5*m**4 + 155*a**5*m**3 + 580*a**5*m**2 + 1
044*a**5*m + 720*a**5 + 5*a**4*b*m**5*x + 95*a**4*b*m**4*x + 685*a**4*b*m*
*3*x + 2305*a**4*b*m**2*x + 3510*a**4*b*m*x + 1800*a**4*b*x + 10*a**3*b**2
*m**5*x**2 + 180*a**3*b**2*m**4*x**2 + 1210*a**3*b**2*m**3*x**2 + 3720*a**
3*b**2*m**2*x**2 + 5080*a**3*b**2*m*x**2 + 2400*a**3*b**2*x**2 + 10*a**2*b
**3*m**5*x**3 + 170*a**2*b**3*m**4*x**3 + 1070*a**2*b**3*m**3*x**3 + 3070*
a**2*b**3*m**2*x**3 + 3960*a**2*b**3*m*x**3 + 1800*a**2*b**3*x**3 + 5*a*b*
*4*m**5*x**4 + 80*a*b**4*m**4*x**4 + 475*a*b**4*m**3*x**4 + 1300*a*b**4*m*
*2*x**4 + 1620*a*b**4*m*x**4 + 720*a*b**4*x**4 + b**5*m**5*x**5 + 15*b**5*
m**4*x**5 + 85*b**5*m**3*x**5 + 225*b**5*m**2*x**5 + 274*b**5*m*x**5 + 120
*b**5*x**5))/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m +
720)
```

3.345 $\int (ex)^m(a+bx)^3(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int (ex)^m(a+bx)^3(A+Bx)\,dx = \frac{a^3A(ex)^{1+m}}{e(1+m)}+\frac{a^2(3Ab+aB)(ex)^{2+m}}{e^2(2+m)} + \frac{3ab(Ab+aB)(ex)^{3+m}}{e^3(3+m)} + \frac{b^2(Ab+3aB)(ex)^{4+m}}{e^4(4+m)}+\frac{b^3B(ex)^{5+m}}{e^5(5+m)}$$

output
```
a^3*A*(e*x)^(1+m)/e/(1+m)+a^2*(3*A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+3*a*b*(A*b
+B*a)*(e*x)^(3+m)/e^3/(3+m)+b^2*(A*b+3*B*a)*(e*x)^(4+m)/e^4/(4+m)+b^3*B*(e
*x)^(5+m)/e^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int (ex)^m(a+bx)^3(A+Bx)\,dx = \frac{(ex)^m\left(Bx(a+bx)^4+(-aB(1+m)+Ab(5+m))x\left(\frac{a^3}{1+m}+\frac{3a^2bx}{2+m}+\frac{3ab^2x^2}{3+m}+\frac{b^3x^3}{4+m}\right)\right)}{b(5+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x)^3*(A + B*x),x]
```

output
```
((e*x)^m*(B*x*(a + b*x)^4 + (-(a*B*(1 + m)) + A*b*(5 + m))*x*(a^3/(1 + m)
+ (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3*x^3)/(4 + m))))/(b*(5
 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(A+Bx)(ex)^m\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^3A(ex)^m+\frac{a^2(ex)^{m+1}(aB+3Ab)}{e}+\frac{b^2(ex)^{m+3}(3aB+Ab)}{e^3}+\frac{3ab(ex)^{m+2}(aB+Ab)}{e^2}+\frac{b^3B(ex)^{m+4}}{e^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3A(ex)^{m+1}}{e(m+1)}+\frac{a^2(ex)^{m+2}(aB+3Ab)}{e^2(m+2)}+\frac{b^2(ex)^{m+4}(3aB+Ab)}{e^4(m+4)}+\frac{3ab(ex)^{m+3}(aB+Ab)}{e^3(m+3)}+\frac{b^3B(ex)^{m+5}}{e^5(m+5)}$$

input
```
Int[(e*x)^m*(a + b*x)^3*(A + B*x),x]
```

output
```
(a^3*A*(e*x)^(1 + m))/(e*(1 + m)) + (a^2*(3*A*b + a*B)*(e*x)^(2 + m))/(e^2
*(2 + m)) + (3*a*b*(A*b + a*B)*(e*x)^(3 + m))/(e^3*(3 + m)) + (b^2*(A*b +
3*a*B)*(e*x)^(4 + m))/(e^4*(4 + m)) + (b^3*B*(e*x)^(5 + m))/(e^5*(5 + m))
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^2(3Ab+Ba)x^2 e^{m\ln(ex)}}{2+m}+\frac{a^3 A x\, e^{m\ln(ex)}}{1+m}+\frac{b^2(Ab+3Ba)x^4 e^{m\ln(ex)}}{4+m}+\frac{b^3 B\, x^5 e^{m\ln(ex)}}{5+m}+\frac{3ab(Ab+Ba)x^3 e^{m\ln(ex)}}{3+m}$
gosper	$x(B\,b^3m^4x^4+A\,b^3m^4x^3+3Ba\,b^2m^4x^3+10B\,b^3m^3x^4+3Aa\,b^2m^4x^2+11A\,b^3m^3x^3+3B\,a^2b\,m^4x^2+33Ba\,b^2m^3x^3+35B\,b^3m^2$
risch	$x(B\,b^3m^4x^4+A\,b^3m^4x^3+3Ba\,b^2m^4x^3+10B\,b^3m^3x^4+3Aa\,b^2m^4x^2+11A\,b^3m^3x^3+3B\,a^2b\,m^4x^2+33Ba\,b^2m^3x^3+35B\,b^3m^2$
orering	$x(B\,b^3m^4x^4+A\,b^3m^4x^3+3Ba\,b^2m^4x^3+10B\,b^3m^3x^4+3Aa\,b^2m^4x^2+11A\,b^3m^3x^3+3B\,a^2b\,m^4x^2+33Ba\,b^2m^3x^3+35B\,b^3m^2$
parallelrisch	$3B\,x^4(ex)^m a\,b^2m^4+3A\,x^3(ex)^m a\,b^2m^4+33B\,x^4(ex)^m a\,b^2m^3+3B\,x^3(ex)^m a^2b\,m^4+36A\,x^3(ex)^m a\,b^2m^3+3A\,x^2(ex)^m a^2b$

input
```
int((e*x)^m*(b*x+a)^3*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a^2*(3*A*b+B*a)/(2+m)*x^2*exp(m*ln(e*x))+a^3*A/(1+m)*x*exp(m*ln(e*x))+b^2*
(A*b+3*B*a)/(4+m)*x^4*exp(m*ln(e*x))+b^3*B/(5+m)*x^5*exp(m*ln(e*x))+3*a*b*
(A*b+B*a)/(3+m)*x^3*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(121) = 242$.

Time = 0.10 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.15

$$\int (ex)^m(a+bx)^3(A+Bx)\,dx = \frac{((Bb^3m^4+10\,Bb^3m^3+35\,Bb^3m^2+50\,Bb^3m+24\,Bb^3)x^5+((3\,Bab^2+Ab^3)m^4+90\,Bab^2+30\,Ab^3+}{}$$

input
```
integrate((e*x)^m*(b*x+a)^3*(B*x+A),x, algorithm="fricas")
```

output
```
((B*b^3*m^4 + 10*B*b^3*m^3 + 35*B*b^3*m^2 + 50*B*b^3*m + 24*B*b^3)*x^5 + (
(3*B*a*b^2 + A*b^3)*m^4 + 90*B*a*b^2 + 30*A*b^3 + 11*(3*B*a*b^2 + A*b^3)*m
^3 + 41*(3*B*a*b^2 + A*b^3)*m^2 + 61*(3*B*a*b^2 + A*b^3)*m)*x^4 + 3*((B*a^
2*b + A*a*b^2)*m^4 + 40*B*a^2*b + 40*A*a*b^2 + 12*(B*a^2*b + A*a*b^2)*m^3
+ 49*(B*a^2*b + A*a*b^2)*m^2 + 78*(B*a^2*b + A*a*b^2)*m)*x^3 + ((B*a^3 + 3
*A*a^2*b)*m^4 + 60*B*a^3 + 180*A*a^2*b + 13*(B*a^3 + 3*A*a^2*b)*m^3 + 59*(
B*a^3 + 3*A*a^2*b)*m^2 + 107*(B*a^3 + 3*A*a^2*b)*m)*x^2 + (A*a^3*m^4 + 14*
A*a^3*m^3 + 71*A*a^3*m^2 + 154*A*a^3*m + 120*A*a^3)*x)*(e*x)^m/(m^5 + 15*m
^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. $2(110) = 220$.

Time = 0.42 (sec) , antiderivative size = 2101, normalized size of antiderivative = 17.36

$$\int (ex)^m(a+bx)^3(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x+a)**3*(B*x+A),x)
```

output
```
Piecewise(((-A*a**3/(4*x**4) - A*a**2*b/x**3 - 3*A*a*b**2/(2*x**2) - A*b**
3/x - B*a**3/(3*x**3) - 3*B*a**2*b/(2*x**2) - 3*B*a*b**2/x + B*b**3*log(x)
)/e**5, Eq(m, -5)), ((-A*a**3/(3*x**3) - 3*A*a**2*b/(2*x**2) - 3*A*a*b**2/
x + A*b**3*log(x) - B*a**3/(2*x**2) - 3*B*a**2*b/x + 3*B*a*b**2*log(x) + B
*b**3*x)/e**4, Eq(m, -4)), ((-A*a**3/(2*x**2) - 3*A*a**2*b/x + 3*A*a*b**2*
log(x) + A*b**3*x - B*a**3/x + 3*B*a**2*b*log(x) + 3*B*a*b**2*x + B*b**3*x
**2/2)/e**3, Eq(m, -3)), ((-A*a**3/x + 3*A*a**2*b*log(x) + 3*A*a*b**2*x +
A*b**3*x**2/2 + B*a**3*log(x) + 3*B*a**2*b*x + 3*B*a*b**2*x**2/2 + B*b**3*
x**3/3)/e**2, Eq(m, -2)), ((A*a**3*log(x) + 3*A*a**2*b*x + 3*A*a*b**2*x**2
/2 + A*b**3*x**3/3 + B*a**3*x + 3*B*a**2*b*x**2/2 + B*a*b**2*x**3 + B*b**3
*x**4/4)/e, Eq(m, -1)), (A*a**3*m**4*x*(e*x)**m/(m**5 + 15*m**4 + 85*m**3
+ 225*m**2 + 274*m + 120) + 14*A*a**3*m**3*x*(e*x)**m/(m**5 + 15*m**4 + 85
*m**3 + 225*m**2 + 274*m + 120) + 71*A*a**3*m**2*x*(e*x)**m/(m**5 + 15*m**
4 + 85*m**3 + 225*m**2 + 274*m + 120) + 154*A*a**3*m*x*(e*x)**m/(m**5 + 15
*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 120*A*a**3*x*(e*x)**m/(m**5 +
15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 3*A*a**2*b*m**4*x**2*(e*x)**
m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 39*A*a**2*b*m**3*x
**2*(e*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 177*A*a
**2*b*m**2*x**2*(e*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 12
0) + 321*A*a**2*b*m*x**2*(e*x)**m/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 ...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int (ex)^m(a+bx)^3(A+Bx)\,dx = \frac{Bb^3e^mx^5x^m}{m+5} + \frac{3\,Bab^2e^mx^4x^m}{m+4} + \frac{Ab^3e^mx^4x^m}{m+4} + \frac{3\,Ba^2be^mx^3x^m}{m+3} + \frac{3\,Aab^2e^mx^3x^m}{m+3} + \frac{Ba^3e^mx^2x^m}{m+2} + \frac{3\,Aa^2be^mx^2x^m}{m+2} + \frac{(ex)^{m+1}\,Aa^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x+a)^3*(B*x+A),x, algorithm="maxima")
```

output
```
B*b^3*e^m*x^5*x^m/(m + 5) + 3*B*a*b^2*e^m*x^4*x^m/(m + 4) + A*b^3*e^m*x^4*
x^m/(m + 4) + 3*B*a^2*b*e^m*x^3*x^m/(m + 3) + 3*A*a*b^2*e^m*x^3*x^m/(m + 3
) + B*a^3*e^m*x^2*x^m/(m + 2) + 3*A*a^2*b*e^m*x^2*x^m/(m + 2) + (e*x)^(m +
 1)*A*a^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(121) = 242$.

Time = 0.13 (sec) , antiderivative size = 673, normalized size of antiderivative = 5.56

$$\int (ex)^m (a+bx)^3 (A+Bx)\,dx = \frac{(ex)^m Bb^3m^4x^5 + 3(ex)^m Bab^2m^4x^4 + (ex)^m Ab^3m^4x^4 + 10(ex)^m Bb^3m^3x^5 + 3(ex)^m Ba^2bm^4x^3 + 3(e}{}$$

input
```
integrate((e*x)^m*(b*x+a)^3*(B*x+A),x, algorithm="giac")
```

output
```
((e*x)^m*B*b^3*m^4*x^5 + 3*(e*x)^m*B*a*b^2*m^4*x^4 + (e*x)^m*A*b^3*m^4*x^4
 + 10*(e*x)^m*B*b^3*m^3*x^5 + 3*(e*x)^m*B*a^2*b*m^4*x^3 + 3*(e*x)^m*A*a*b^
2*m^4*x^3 + 33*(e*x)^m*B*a*b^2*m^3*x^4 + 11*(e*x)^m*A*b^3*m^3*x^4 + 35*(e*
x)^m*B*b^3*m^2*x^5 + (e*x)^m*B*a^3*m^4*x^2 + 3*(e*x)^m*A*a^2*b*m^4*x^2 + 3
6*(e*x)^m*B*a^2*b*m^3*x^3 + 36*(e*x)^m*A*a*b^2*m^3*x^3 + 123*(e*x)^m*B*a*b
^2*m^2*x^4 + 41*(e*x)^m*A*b^3*m^2*x^4 + 50*(e*x)^m*B*b^3*m*x^5 + (e*x)^m*A
*a^3*m^4*x + 13*(e*x)^m*B*a^3*m^3*x^2 + 39*(e*x)^m*A*a^2*b*m^3*x^2 + 147*(
e*x)^m*B*a^2*b*m^2*x^3 + 147*(e*x)^m*A*a*b^2*m^2*x^3 + 183*(e*x)^m*B*a*b^2
*m*x^4 + 61*(e*x)^m*A*b^3*m*x^4 + 24*(e*x)^m*B*b^3*x^5 + 14*(e*x)^m*A*a^3*
m^3*x + 59*(e*x)^m*B*a^3*m^2*x^2 + 177*(e*x)^m*A*a^2*b*m^2*x^2 + 234*(e*x)
^m*B*a^2*b*m*x^3 + 234*(e*x)^m*A*a*b^2*m*x^3 + 90*(e*x)^m*B*a*b^2*x^4 + 30
*(e*x)^m*A*b^3*x^4 + 71*(e*x)^m*A*a^3*m^2*x + 107*(e*x)^m*B*a^3*m*x^2 + 32
1*(e*x)^m*A*a^2*b*m*x^2 + 120*(e*x)^m*B*a^2*b*x^3 + 120*(e*x)^m*A*a*b^2*x^
3 + 154*(e*x)^m*A*a^3*m*x + 60*(e*x)^m*B*a^3*x^2 + 180*(e*x)^m*A*a^2*b*x^2
 + 120*(e*x)^m*A*a^3*x)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.31

$$\int (ex)^m (a+bx)^3 (A+Bx)\,dx = (e\,x)^m \left(\frac{A\,a^3\,x\,(m^4+14\,m^3+71\,m^2+154\,m+120)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{B\,b^3\,x^5\,(m^4+10\,m^3+35\,m^2+50\,m+24)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{a^2\,x^2\,(3\,A\,b+B\,a)\,(m^4+13\,m^3+59\,m^2+107\,m+60)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{b^2\,x^4\,(A\,b+3\,B\,a)\,(m^4+11\,m^3+41\,m^2+61\,m+30)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{3\,a\,b\,x^3\,(A\,b+B\,a)\,(m^4+12\,m^3+49\,m^2+78\,m+40)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120}\right)$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x)^3,x)
```

output
```
(e*x)^m*((A*a^3*x*(154*m + 71*m^2 + 14*m^3 + m^4 + 120))/(274*m + 225*m^2
+ 85*m^3 + 15*m^4 + m^5 + 120) + (B*b^3*x^5*(50*m + 35*m^2 + 10*m^3 + m^4
+ 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (a^2*x^2*(3*A*b +
 B*a)*(107*m + 59*m^2 + 13*m^3 + m^4 + 60))/(274*m + 225*m^2 + 85*m^3 + 15
*m^4 + m^5 + 120) + (b^2*x^4*(A*b + 3*B*a)*(61*m + 41*m^2 + 11*m^3 + m^4 +
 30))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (3*a*b*x^3*(A*b +
B*a)*(78*m + 49*m^2 + 12*m^3 + m^4 + 40))/(274*m + 225*m^2 + 85*m^3 + 15*m
^4 + m^5 + 120))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.33

$$\int (ex)^m (a+bx)^3 (A+Bx)\,dx = \frac{x^m e^m x(b^4m^4x^4+4a\,b^3m^4x^3+10b^4m^3x^4+6a^2b^2m^4x^2+44a\,b^3m^3x^3+35b^4m^2x^4+4a^3b\,m^4x+72a^2b^2m}{}$$

input
```
int((e*x)^m*(b*x+a)^3*(B*x+A),x)
```

output

```
(x**m*e**m*x*(a**4*m**4 + 14*a**4*m**3 + 71*a**4*m**2 + 154*a**4*m + 120*a
**4 + 4*a**3*b*m**4*x + 52*a**3*b*m**3*x + 236*a**3*b*m**2*x + 428*a**3*b*
m*x + 240*a**3*b*x + 6*a**2*b**2*m**4*x**2 + 72*a**2*b**2*m**3*x**2 + 294*
a**2*b**2*m**2*x**2 + 468*a**2*b**2*m*x**2 + 240*a**2*b**2*x**2 + 4*a*b**3
*m**4*x**3 + 44*a*b**3*m**3*x**3 + 164*a*b**3*m**2*x**3 + 244*a*b**3*m*x**
3 + 120*a*b**3*x**3 + b**4*m**4*x**4 + 10*b**4*m**3*x**4 + 35*b**4*m**2*x*
*4 + 50*b**4*m*x**4 + 24*b**4*x**4))/(m**5 + 15*m**4 + 85*m**3 + 225*m**2
+ 274*m + 120)
```

3.346 $\int (ex)^m(a+bx)^2(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int (ex)^m(a+bx)^2(A+Bx)\,dx=\frac{a^2A(ex)^{1+m}}{e(1+m)}+\frac{a(2Ab+aB)(ex)^{2+m}}{e^2(2+m)}+\frac{b(Ab+2aB)(ex)^{3+m}}{e^3(3+m)}+\frac{b^2B(ex)^{4+m}}{e^4(4+m)}$$

output
```
a^2*A*(e*x)^(1+m)/e/(1+m)+a*(2*A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+b*(A*b+2*B*a
)*(e*x)^(3+m)/e^3/(3+m)+b^2*B*(e*x)^(4+m)/e^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int (ex)^m(a+bx)^2(A+Bx)\,dx=\frac{(ex)^m\left(Bx(a+bx)^3+(-aB(1+m)+Ab(4+m))x\left(\frac{a^2}{1+m}+\frac{2abx}{2+m}+\frac{b^2x^2}{3+m}\right)\right)}{b(4+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x)^2*(A + B*x),x]
```

output
```
((e*x)^m*(B*x*(a + b*x)^3 + (-(a*B*(1 + m)) + A*b*(4 + m))*x*(a^2/(1 + m)
+ (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))))/(b*(4 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(A+Bx)(ex)^m\,dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(a^2A(ex)^m+\frac{b(ex)^{m+2}(2aB+Ab)}{e^2}+\frac{a(ex)^{m+1}(aB+2Ab)}{e}+\frac{b^2B(ex)^{m+3}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2A(ex)^{m+1}}{e(m+1)}+\frac{b(ex)^{m+3}(2aB+Ab)}{e^3(m+3)}+\frac{a(ex)^{m+2}(aB+2Ab)}{e^2(m+2)}+\frac{b^2B(ex)^{m+4}}{e^4(m+4)}$$

input
```
Int[(e*x)^m*(a + b*x)^2*(A + B*x),x]
```

output
```
(a^2*A*(e*x)^(1 + m))/(e*(1 + m)) + (a*(2*A*b + a*B)*(e*x)^(2 + m))/(e^2*(
2 + m)) + (b*(A*b + 2*a*B)*(e*x)^(3 + m))/(e^3*(3 + m)) + (b^2*B*(e*x)^(4
+ m))/(e^4*(4 + m))
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a(2Ab+Ba)x^2 e^{m\ln(ex)}}{2+m}+\frac{a^2 A x\, e^{m\ln(ex)}}{1+m}+\frac{b(Ab+2Ba)x^3 e^{m\ln(ex)}}{3+m}+\frac{b^2 B\, x^4 e^{m\ln(ex)}}{4+m}$
gosper	$x\left(B\,b^2m^3x^3+A\,b^2m^3x^2+2Bab\,m^3x^2+6B\,b^2m^2x^3+2Aab\,m^3x+7A\,b^2m^2x^2+B\,a^2m^3x+14Bab\,m^2x^2+11m\,x^3b^2B+A\,a^2m^3\right.$
risch	$x\left(B\,b^2m^3x^3+A\,b^2m^3x^2+2Bab\,m^3x^2+6B\,b^2m^2x^3+2Aab\,m^3x+7A\,b^2m^2x^2+B\,a^2m^3x+14Bab\,m^2x^2+11m\,x^3b^2B+A\,a^2m^3\right.$
orering	$x\left(B\,b^2m^3x^3+A\,b^2m^3x^2+2Bab\,m^3x^2+6B\,b^2m^2x^3+2Aab\,m^3x+7A\,b^2m^2x^2+B\,a^2m^3x+14Bab\,m^2x^2+11m\,x^3b^2B+A\,a^2m^3\right.$
parallelrisch	$B\,x^4(ex)^m b^2m^3+A\,x^3(ex)^m b^2m^3+6B\,x^4(ex)^m b^2m^2+7A\,x^3(ex)^m b^2m^2+11B\,x^4(ex)^m b^2m+B\,x^2(ex)^m a^2m^3+14A\,x^3(ex)$

input
```
int((e*x)^m*(b*x+a)^2*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*(2*A*b+B*a)/(2+m)*x^2*exp(m*ln(e*x))+a^2*A/(1+m)*x*exp(m*ln(e*x))+b*(A*b
+2*B*a)/(3+m)*x^3*exp(m*ln(e*x))+b^2*B/(4+m)*x^4*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(91) = 182.

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.38

$$\int (ex)^m(a+bx)^2(A+Bx)\,dx = \frac{((Bb^2m^3+6\,Bb^2m^2+11\,Bb^2m+6\,Bb^2)x^4+((2\,Bab+Ab^2)m^3+16\,Bab+8\,Ab^2+7\,(2\,Bab+Ab^2)m^2}{}$$

input
```
integrate((e*x)^m*(b*x+a)^2*(B*x+A),x, algorithm="fricas")
```

output
```
((B*b^2*m^3 + 6*B*b^2*m^2 + 11*B*b^2*m + 6*B*b^2)*x^4 + ((2*B*a*b + A*b^2)
*m^3 + 16*B*a*b + 8*A*b^2 + 7*(2*B*a*b + A*b^2)*m^2 + 14*(2*B*a*b + A*b^2)
*m)*x^3 + ((B*a^2 + 2*A*a*b)*m^3 + 12*B*a^2 + 24*A*a*b + 8*(B*a^2 + 2*A*a*
b)*m^2 + 19*(B*a^2 + 2*A*a*b)*m)*x^2 + (A*a^2*m^3 + 9*A*a^2*m^2 + 26*A*a^2
*m + 24*A*a^2)*x)*(e*x)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. 2(82) = 164.

Time = 0.34 (sec) , antiderivative size = 1073, normalized size of antiderivative = 11.79

$$\int (ex)^m(a+bx)^2(A+Bx)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x+a)**2*(B*x+A),x)
```

output
```
Piecewise(((-A*a**2/(3*x**3) - A*a*b/x**2 - A*b**2/x - B*a**2/(2*x**2) - 2
*B*a*b/x + B*b**2*log(x))/e**4, Eq(m, -4)), ((-A*a**2/(2*x**2) - 2*A*a*b/x
 + A*b**2*log(x) - B*a**2/x + 2*B*a*b*log(x) + B*b**2*x)/e**3, Eq(m, -3)),
 ((-A*a**2/x + 2*A*a*b*log(x) + A*b**2*x + B*a**2*log(x) + 2*B*a*b*x + B*b
**2*x**2/2)/e**2, Eq(m, -2)), ((A*a**2*log(x) + 2*A*a*b*x + A*b**2*x**2/2
+ B*a**2*x + B*a*b*x**2 + B*b**2*x**3/3)/e, Eq(m, -1)), (A*a**2*m**3*x*(e*
x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*A*a**2*m**2*x*(e*x)**m/(m
**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*A*a**2*m*x*(e*x)**m/(m**4 + 10*m
**3 + 35*m**2 + 50*m + 24) + 24*A*a**2*x*(e*x)**m/(m**4 + 10*m**3 + 35*m**
2 + 50*m + 24) + 2*A*a*b*m**3*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50
*m + 24) + 16*A*a*b*m**2*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m +
24) + 38*A*a*b*m*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24
*A*a*b*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + A*b**2*m**3*
x**3*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 7*A*b**2*m**2*x**3*
(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 14*A*b**2*m*x**3*(e*x)**
m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 8*A*b**2*x**3*(e*x)**m/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + B*a**2*m**3*x**2*(e*x)**m/(m**4 + 10*m**3
 + 35*m**2 + 50*m + 24) + 8*B*a**2*m**2*x**2*(e*x)**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 19*B*a**2*m*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 +
 50*m + 24) + 12*B*a**2*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (ex)^m(a+bx)^2(A+Bx)\,dx = \frac{Bb^2e^mx^4x^m}{m+4} + \frac{2\,Babe^mx^3x^m}{m+3} + \frac{Ab^2e^mx^3x^m}{m+3} + \frac{Ba^2e^mx^2x^m}{m+2} + \frac{2\,Aabe^mx^2x^m}{m+2} + \frac{(ex)^{m+1}\,Aa^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x+a)^2*(B*x+A),x, algorithm="maxima")
```

output
```
B*b^2*e^m*x^4*x^m/(m + 4) + 2*B*a*b*e^m*x^3*x^m/(m + 3) + A*b^2*e^m*x^3*x^
m/(m + 3) + B*a^2*e^m*x^2*x^m/(m + 2) + 2*A*a*b*e^m*x^2*x^m/(m + 2) + (e*x
)^(m + 1)*A*a^2/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.18

$$\int (ex)^m (a+bx)^2 (A+Bx)\,dx = \frac{(ex)^m Bb^2m^3x^4 + 2\,(ex)^m Babm^3x^3 + (ex)^m Ab^2m^3x^3 + 6\,(ex)^m Bb^2m^2x^4 + (ex)^m Ba^2m^3x^2 + 2\,(ex)^m \ldots}{}$$

input
```
integrate((e*x)^m*(b*x+a)^2*(B*x+A),x, algorithm="giac")
```

output
```
((e*x)^m*B*b^2*m^3*x^4 + 2*(e*x)^m*B*a*b*m^3*x^3 + (e*x)^m*A*b^2*m^3*x^3 +
 6*(e*x)^m*B*b^2*m^2*x^4 + (e*x)^m*B*a^2*m^3*x^2 + 2*(e*x)^m*A*a*b*m^3*x^2
 + 14*(e*x)^m*B*a*b*m^2*x^3 + 7*(e*x)^m*A*b^2*m^2*x^3 + 11*(e*x)^m*B*b^2*m
*x^4 + (e*x)^m*A*a^2*m^3*x + 8*(e*x)^m*B*a^2*m^2*x^2 + 16*(e*x)^m*A*a*b*m^
2*x^2 + 28*(e*x)^m*B*a*b*m*x^3 + 14*(e*x)^m*A*b^2*m*x^3 + 6*(e*x)^m*B*b^2*
x^4 + 9*(e*x)^m*A*a^2*m^2*x + 19*(e*x)^m*B*a^2*m*x^2 + 38*(e*x)^m*A*a*b*m*
x^2 + 16*(e*x)^m*B*a*b*x^3 + 8*(e*x)^m*A*b^2*x^3 + 26*(e*x)^m*A*a^2*m*x +
12*(e*x)^m*B*a^2*x^2 + 24*(e*x)^m*A*a*b*x^2 + 24*(e*x)^m*A*a^2*x)/(m^4 + 1
0*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int (ex)^m (a+bx)^2 (A+Bx)\,dx = (e\,x)^m\left(\frac{B\,b^2\,x^4\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{A\,a^2\,x\,(m^3+9\,m^2+26\,m+24)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{a\,x^2\,(2\,A\,b+B\,a)\,(m^3+8\,m^2+19\,m+12)}{m^4+10\,m^3+35\,m^2+50\,m+24} + \frac{b\,x^3\,(A\,b+2\,B\,a)\,(m^3+7\,m^2+14\,m+8)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x)^2,x)
```

output
```
(e*x)^m*((B*b^2*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (A*a^2*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (a*x^2*(2*A*b + B*a)*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 +
 10*m^3 + m^4 + 24) + (b*x^3*(A*b + 2*B*a)*(14*m + 7*m^2 + m^3 + 8))/(50*m
 + 35*m^2 + 10*m^3 + m^4 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int (ex)^m(a+bx)^2(A+Bx)\,dx$$
$$= \frac{x^m e^m x(b^3m^3x^3 + 3a\,b^2m^3x^2 + 6b^3m^2x^3 + 3a^2b\,m^3x + 21a\,b^2m^2x^2 + 11b^3m\,x^3 + a^3m^3 + 24a^2b\,m^2x + 42}{m^4 + 10m^3 + 35m^2 + 50m +}$$

input
```
int((e*x)^m*(b*x+a)^2*(B*x+A),x)
```

output
```
(x**m*e**m*x*(a**3*m**3 + 9*a**3*m**2 + 26*a**3*m + 24*a**3 + 3*a**2*b*m**
3*x + 24*a**2*b*m**2*x + 57*a**2*b*m*x + 36*a**2*b*x + 3*a*b**2*m**3*x**2
+ 21*a*b**2*m**2*x**2 + 42*a*b**2*m*x**2 + 24*a*b**2*x**2 + b**3*m**3*x**3
 + 6*b**3*m**2*x**3 + 11*b**3*m*x**3 + 6*b**3*x**3))/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24)
```

3.347 $\int (ex)^m(a+bx)(A+Bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int (ex)^m(a+bx)(A+Bx)\,dx=\frac{aA(ex)^{1+m}}{e(1+m)}+\frac{(Ab+aB)(ex)^{2+m}}{e^2(2+m)}+\frac{bB(ex)^{3+m}}{e^3(3+m)}$$

output
```
a*A*(e*x)^(1+m)/e/(1+m)+(A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+b*B*(e*x)^(3+m)/e^
3/(3+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (ex)^m(a+bx)(A+Bx)\,dx =\frac{x(ex)^m(a(3+m)(A(2+m)+B(1+m)x)+b(1+m)x(A(3+m)+B(2+m)x))}{(1+m)(2+m)(3+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x)*(A + B*x),x]
```

output
```
(x*(e*x)^m*(a*(3 + m)*(A*(2 + m) + B*(1 + m)*x) + b*(1 + m)*x*(A*(3 + m) +
 B*(2 + m)*x)))/((1 + m)*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(A+Bx)(ex)^m\,dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{(ex)^{m+1}(aB+Ab)}{e}+aA(ex)^m+\frac{bB(ex)^{m+2}}{e^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(ex)^{m+2}(aB+Ab)}{e^2(m+2)}+\frac{aA(ex)^{m+1}}{e(m+1)}+\frac{bB(ex)^{m+3}}{e^3(m+3)}$$

input
```
Int[(e*x)^m*(a + b*x)*(A + B*x),x]
```

output
```
(a*A*(e*x)^(1 + m))/(e*(1 + m)) + ((A*b + a*B)*(e*x)^(2 + m))/(e^2*(2 + m)
) + (b*B*(e*x)^(3 + m))/(e^3*(3 + m))
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result
norman	$\frac{(Ab+Ba)x^2 e^{m\ln(ex)}}{2+m}+\frac{Aax\,e^{m\ln(ex)}}{1+m}+\frac{bB\,x^3 e^{m\ln(ex)}}{3+m}$
gosper	$\frac{x(Bb\,m^2x^2+Ab\,m^2x+Ba\,m^2x+3Bbm\,x^2+Aa\,m^2+4Abmx+4Bamx+2bB\,x^2+5Aam+3Abx+3Bax+6Aa)(ex)^m}{(3+m)(2+m)(1+m)}$
risch	$\frac{x(Bb\,m^2x^2+Ab\,m^2x+Ba\,m^2x+3Bbm\,x^2+Aa\,m^2+4Abmx+4Bamx+2bB\,x^2+5Aam+3Abx+3Bax+6Aa)(ex)^m}{(3+m)(2+m)(1+m)}$
orering	$\frac{x(Bb\,m^2x^2+Ab\,m^2x+Ba\,m^2x+3Bbm\,x^2+Aa\,m^2+4Abmx+4Bamx+2bB\,x^2+5Aam+3Abx+3Bax+6Aa)(ex)^m}{(3+m)(2+m)(1+m)}$
parallelrisch	$\frac{B\,x^3(ex)^m b\,m^2+A\,x^2(ex)^m b\,m^2+3B\,x^3(ex)^m bm+B\,x^2(ex)^m a\,m^2+4A\,x^2(ex)^m bm+Ax(ex)^m a\,m^2+2B\,x^3(ex)^m b+4B\,x^2}{(3+m)(2+m)(1+m)}$

input
```
int((e*x)^m*(b*x+a)*(B*x+A),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(2+m)*x^2*exp(m*ln(e*x))+A*a/(1+m)*x*exp(m*ln(e*x))+b*B/(3+m)*x^
3*exp(m*ln(e*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.57

$$\int (ex)^m(a+bx)(A+Bx)\,dx$$
$$= \frac{((Bbm^2+3\,Bbm+2\,Bb)x^3+((Ba+Ab)m^2+3\,Ba+3\,Ab+4\,(Ba+Ab)m)x^2+(Aam^2+5\,Aam+6}{m^3+6\,m^2+11\,m+6}$$

input
```
integrate((e*x)^m*(b*x+a)*(B*x+A),x, algorithm="fricas")
```

output
```
((B*b*m^2 + 3*B*b*m + 2*B*b)*x^3 + ((B*a + A*b)*m^2 + 3*B*a + 3*A*b + 4*(B
*a + A*b)*m)*x^2 + (A*a*m^2 + 5*A*a*m + 6*A*a)*x)*(e*x)^m/(m^3 + 6*m^2 + 1
1*m + 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(51) = 102$.

Time = 0.24 (sec) , antiderivative size = 418, normalized size of antiderivative = 6.97

$$\int (ex)^m(a+bx)(A+Bx)\,dx = \begin{cases} \frac{-\frac{Aa}{2x^2}-\frac{Ab}{x}-\frac{Ba}{x}+Bb\log(x)}{e^3} \\ \frac{-\frac{Aa}{x}+Ab\log(x)+Ba\log(x)+Bbx}{e^2} \\ \frac{Aa\log(x)+Abx+Bax+\frac{Bbx^2}{2}}{e} \\ \frac{Aam^2x(ex)^m}{m^3+6m^2+11m+6}+\frac{5Aamx(ex)^m}{m^3+6m^2+11m+6}+\frac{6Aax(ex)^m}{m^3+6m^2+11m+6}+\frac{Abm^2x^2(ex)^m}{m^3+6m^2+11m+6}+\frac{4Abmx^2(ex)^m}{m^3+6m^2+11m+6}+\frac{3Abx^2(ex)^m}{m^3+6m^2+11m+6}+ \end{cases}$$

input
```
integrate((e*x)**m*(b*x+a)*(B*x+A),x)
```

output
```
Piecewise(((-A*a/(2*x**2) - A*b/x - B*a/x + B*b*log(x))/e**3, Eq(m, -3)),
((-A*a/x + A*b*log(x) + B*a*log(x) + B*b*x)/e**2, Eq(m, -2)), ((A*a*log(x)
 + A*b*x + B*a*x + B*b*x**2/2)/e, Eq(m, -1)), (A*a*m**2*x*(e*x)**m/(m**3 +
 6*m**2 + 11*m + 6) + 5*A*a*m*x*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 6*A*
a*x*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6) + A*b*m**2*x**2*(e*x)**m/(m**3 + 6
*m**2 + 11*m + 6) + 4*A*b*m*x**2*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 3*A
*b*x**2*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6) + B*a*m**2*x**2*(e*x)**m/(m**3
 + 6*m**2 + 11*m + 6) + 4*B*a*m*x**2*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6) +
 3*B*a*x**2*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6) + B*b*m**2*x**3*(e*x)**m/(
m**3 + 6*m**2 + 11*m + 6) + 3*B*b*m*x**3*(e*x)**m/(m**3 + 6*m**2 + 11*m +
6) + 2*B*b*x**3*(e*x)**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int (ex)^m(a+bx)(A+Bx)\,dx = \frac{Bbe^mx^3x^m}{m+3}+\frac{Bae^mx^2x^m}{m+2}+\frac{Abe^mx^2x^m}{m+2}+\frac{(ex)^{m+1}Aa}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x+a)*(B*x+A),x, algorithm="maxima")
```

output
```
B*b*e^m*x^3*x^m/(m + 3) + B*a*e^m*x^2*x^m/(m + 2) + A*b*e^m*x^2*x^m/(m + 2
) + (e*x)^(m + 1)*A*a/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(60) = 120$.

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.78

$$\int (ex)^m(a+bx)(A+Bx)\,dx$$
$$= \frac{(ex)^m Bbm^2x^3 + (ex)^m Bam^2x^2 + (ex)^m Abm^2x^2 + 3\,(ex)^m Bbmx^3 + (ex)^m Aam^2x + 4\,(ex)^m Bamx^2}{m^3+6\,m^2+1}$$

input
```
integrate((e*x)^m*(b*x+a)*(B*x+A),x, algorithm="giac")
```

output
```
((e*x)^m*B*b*m^2*x^3 + (e*x)^m*B*a*m^2*x^2 + (e*x)^m*A*b*m^2*x^2 + 3*(e*x)
^m*B*b*m*x^3 + (e*x)^m*A*a*m^2*x + 4*(e*x)^m*B*a*m*x^2 + 4*(e*x)^m*A*b*m*x
^2 + 2*(e*x)^m*B*b*x^3 + 5*(e*x)^m*A*a*m*x + 3*(e*x)^m*B*a*x^2 + 3*(e*x)^m
*A*b*x^2 + 6*(e*x)^m*A*a*x)/(m^3 + 6*m^2 + 11*m + 6)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int (ex)^m(a+bx)(A+Bx)\,dx = (e\,x)^m\left(\frac{x^2\,(A\,b+B\,a)\,(m^2+4\,m+3)}{m^3+6\,m^2+11\,m+6} + \frac{B\,b\,x^3\,(m^2+3\,m+2)}{m^3+6\,m^2+11\,m+6} + \frac{A\,a\,x\,(m^2+5\,m+6)}{m^3+6\,m^2+11\,m+6}\right)$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x),x)
```

output
```
(e*x)^m*((x^2*(A*b + B*a)*(4*m + m^2 + 3))/(11*m + 6*m^2 + m^3 + 6) + (B*b
*x^3*(3*m + m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (A*a*x*(5*m + m^2 + 6))/(
11*m + 6*m^2 + m^3 + 6))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

$$\int (ex)^m(a+bx)(A+Bx)\,dx$$
$$=\frac{x^m e^m x(b^2m^2x^2+2ab\,m^2x+3b^2m\,x^2+a^2m^2+8abmx+2b^2x^2+5a^2m+6abx+6a^2)}{m^3+6m^2+11m+6}$$

input
```
int((e*x)^m*(b*x+a)*(B*x+A),x)
```

output
```
(x**m*e**m*x*(a**2*m**2 + 5*a**2*m + 6*a**2 + 2*a*b*m**2*x + 8*a*b*m*x + 6
*a*b*x + b**2*m**2*x**2 + 3*b**2*m*x**2 + 2*b**2*x**2))/(m**3 + 6*m**2 + 1
1*m + 6)
```

3.348 $\int \frac{(ex)^m(A+Bx)}{a+bx} dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{(ex)^m(A+Bx)}{a+bx} dx = \frac{B(ex)^{1+m}}{be(1+m)} + \frac{(Ab-aB)(ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{abe(1+m)}$$

output
```
B*(e*x)^(1+m)/b/e/(1+m)+(A*b-B*a)*(e*x)^(1+m)*hypergeom([1, 1+m],[2+m],-b*
x/a)/a/b/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m(A+Bx)}{a+bx} dx = -\frac{x(ex)^m \left(-aB+(-Ab+aB) \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x),x]
```

output
```
-((x*(e*x)^m*(-(a*B) + (-(A*b) + a*B)*Hypergeometric2F1[1, 1 + m, 2 + m, -
((b*x)/a)]))/(a*b*(1 + m)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{a+bx}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(Ab-aB)\int \frac{(ex)^m}{a+bx}dx}{b}+\frac{B(ex)^{m+1}}{be(m+1)}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(ex)^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{abe(m+1)}+\frac{B(ex)^{m+1}}{be(m+1)}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x),x]
```

output
```
(B*(e*x)^(1 + m))/(b*e*(1 + m)) + ((A*b - a*B)*(e*x)^(1 + m)*Hypergeometri
c2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*b*e*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

Maple [F]

$$\int \frac{(ex)^m (Bx+A)}{bx+a}dx$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a),x)
```

output
```
int((e*x)^m*(B*x+A)/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx}\,dx = \int \frac{(Bx+A)(ex)^m}{bx+a}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/(b*x + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.21

$$\int \frac{(ex)^m(A+Bx)}{a+bx}\,dx = \frac{Ae^m m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)} + \frac{Ae^m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)} + \frac{Be^m m x^{m+2}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+2\right)\Gamma(m+2)}{a\Gamma(m+3)} + \frac{2Be^m x^{m+2}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+2\right)\Gamma(m+2)}{a\Gamma(m+3)}$$

input
```
integrate((e*x)**m*(B*x+A)/(b*x+a),x)
```

output
```
A*e**m*m*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)
/(a*gamma(m + 2)) + A*e**m*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m
 + 1)*gamma(m + 1)/(a*gamma(m + 2)) + B*e**m*m*x**(m + 2)*lerchphi(b*x*exp
_polar(I*pi)/a, 1, m + 2)*gamma(m + 2)/(a*gamma(m + 3)) + 2*B*e**m*x**(m +
 2)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 2)*gamma(m + 2)/(a*gamma(m + 3)
)
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx}\,dx = \int \frac{(Bx+A)(ex)^m}{bx+a}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a), x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx}\,dx = \int \frac{(Bx+A)(ex)^m}{bx+a}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{a+bx}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{a+b\,x}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x),x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.20

$$\int \frac{(ex)^m(A+Bx)}{a+bx}\,dx = \frac{x^m e^m x}{m+1}$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a),x)
```

output
```
(x**m*e**m*x)/(m + 1)
```

3.349 $\int \frac{(ex)^m(A+Bx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2} \, dx = \frac{B(ex)^{1+m}}{bem(a+bx)} + \frac{(Abm-aB(1+m))(ex)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{a}\right)}{a^2bem(1+m)}$$

output
```
B*(e*x)^(1+m)/b/e/m/(b*x+a)+(A*b*m-a*B*(1+m))*(e*x)^(1+m)*hypergeom([2, 1+
m],[2+m],-b*x/a)/a^2/b/e/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2} \, dx = \frac{x(ex)^m \left(\frac{a(Ab-aB)}{a+bx} + \frac{(-Abm+aB(1+m)) \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{1+m}\right)}{a^2b}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x)^2,x]
```

output
```
(x*(e*x)^m*((a*(A*b - a*B))/(a + b*x) + ((-(A*b*m) + a*B*(1 + m))*Hypergeo
metric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(1 + m)))/(a^2*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{(a+bx)^2}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{abe(a+bx)}-\frac{(Abm-aB(m+1))\int \frac{(ex)^m}{a+bx}dx}{ab}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{abe(a+bx)}-\frac{(ex)^{m+1}(Abm-aB(m+1))\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{a^2be(m+1)}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(a*b*e*(a + b*x)) - ((A*b*m - a*B*(1 + m))*(e*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a^2*b*e*(1 + m
))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (Bx + A)}{(bx + a)^2} dx$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a)^2,x)
```

output
```
int((e*x)^m*(B*x+A)/(b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m(A + Bx)}{(a + bx)^2} dx = \int \frac{(Bx + A)(ex)^m}{(bx + a)^2} dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 678, normalized size of antiderivative = 8.69

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x+A)/(b*x+a)**2,x)
```

output
```
A*(-a*e**m*m**2*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma
(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - a*e**m*m*x**(m + 1)*
lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2)
+ a**2*b*x*gamma(m + 2)) + a*e**m*m*x**(m + 1)*gamma(m + 1)/(a**3*gamma(m
+ 2) + a**2*b*x*gamma(m + 2)) + a*e**m*x**(m + 1)*gamma(m + 1)/(a**3*gamma
(m + 2) + a**2*b*x*gamma(m + 2)) - b*e**m*m**2*x*x**(m + 1)*lerchphi(b*x*e
xp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gam
ma(m + 2)) - b*e**m*m*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m +
1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))) + B*(-a*e**m*
m**2*x**(m + 2)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 2)*gamma(m + 2)/(a*
*3*gamma(m + 3) + a**2*b*x*gamma(m + 3)) - 3*a*e**m*m*x**(m + 2)*lerchphi(
b*x*exp_polar(I*pi)/a, 1, m + 2)*gamma(m + 2)/(a**3*gamma(m + 3) + a**2*b*
x*gamma(m + 3)) + a*e**m*m*x**(m + 2)*gamma(m + 2)/(a**3*gamma(m + 3) + a*
*2*b*x*gamma(m + 3)) - 2*a*e**m*x**(m + 2)*lerchphi(b*x*exp_polar(I*pi)/a,
 1, m + 2)*gamma(m + 2)/(a**3*gamma(m + 3) + a**2*b*x*gamma(m + 3)) + 2*a*
e**m*x**(m + 2)*gamma(m + 2)/(a**3*gamma(m + 3) + a**2*b*x*gamma(m + 3)) -
 b*e**m*m**2*x*x**(m + 2)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 2)*gamma(
m + 2)/(a**3*gamma(m + 3) + a**2*b*x*gamma(m + 3)) - 3*b*e**m*m*x*x**(m +
2)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 2)*gamma(m + 2)/(a**3*gamma(m +
3) + a**2*b*x*gamma(m + 3)) - 2*b*e**m*x*x**(m + 2)*lerchphi(b*x*exp_po...
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{(a+b\,x)^2}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x)^2,x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^2}\,dx = \frac{e^m\left(x^m-\left(\int \frac{x^m}{b\,x^2+ax}dx\right)am\right)}{bm}$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a)^2,x)
```

output
```
(e**m*(x**m - int(x**m/(a*x + b*x**2),x)*a*m))/(b*m)
```

3.350 $\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = -\frac{B(ex)^{1+m}}{be(1-m)(a+bx)^2} + \frac{(aB(1+m)+A(b-bm))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(3,1+m,2+m,-\frac{bx}{a}\right)}{a^3be(1-m)(1+m)}$$

output
```
-B*(e*x)^(1+m)/b/e/(1-m)/(b*x+a)^2+(a*B*(1+m)+A*(-b*m+b))*(e*x)^(1+m)*hype
rgeom([3, 1+m],[2+m],-b*x/a)/a^3/b/e/(1-m)/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = \frac{x(ex)^m\left(\frac{a^2(Ab-aB)}{(a+bx)^2}-\frac{(Ab(-1+m)-aB(1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{bx}{a}\right)}{1+m}\right)}{2a^3b}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x)^3,x]
```

output
```
(x*(e*x)^m*((a^2*(A*b - a*B))/(a + b*x)^2 - ((A*b*(-1 + m) - a*B*(1 + m))*
Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(1 + m)))/(2*a^3*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{(a+bx)^3}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB(m+1)+Ab(1-m))\int \frac{(ex)^m}{(a+bx)^2}dx}{2ab}+\frac{(ex)^{m+1}(Ab-aB)}{2abe(a+bx)^2}$$

$$\Big\downarrow \text{74}$$

$$\frac{(ex)^{m+1}(aB(m+1)+Ab(1-m))\,\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{bx}{a}\right)}{2a^3be(m+1)}+\frac{(ex)^{m+1}(Ab-aB)}{2abe(a+bx)^2}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x)^3,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(2*a*b*e*(a + b*x)^2) + ((A*b*(1 - m) + a*B*(1
 + m))*(e*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(2*a^
3*b*e*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (Bx+A)}{(bx+a)^3}dx$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a)^3,x)
```

output
```
int((e*x)^m*(B*x+A)/(b*x+a)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.51 (sec) , antiderivative size = 1763, normalized size of antiderivative = 19.59

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x+A)/(b*x+a)**3,x)
```

output
```
A*(a**2*e**m*m**3*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gam
ma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**
2*gamma(m + 2)) - a**2*e**m*m**2*x**(m + 1)*gamma(m + 1)/(2*a**5*gamma(m +
 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a**2*e**m
*m*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a*
*5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2))
 + a**2*e**m*m*x**(m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*g
amma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + 2*a**2*e**m*x**(m + 1)*gamm
a(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2
*gamma(m + 2)) + 2*a*b*e**m*m**3*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)
/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2)
+ 2*a**3*b**2*x**2*gamma(m + 2)) - a*b*e**m*m**2*x*x**(m + 1)*gamma(m + 1)
/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m
 + 2)) - 2*a*b*e**m*m*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m +
1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b*
*2*x**2*gamma(m + 2)) + a*b*e**m*x*x**(m + 1)*gamma(m + 1)/(2*a**5*gamma(m
 + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + b**2*e*
*m*m**3*x**2*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m
+ 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gam
ma(m + 2)) - b**2*e**m*m*x**2*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a...
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{(a+b\,x)^3}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x)^3,x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^3}\,dx$$
$$= \frac{e^m\left(x^m - \left(\int \frac{x^m}{b^2m\,x^3+2abm\,x^2-b^2x^3+a^2mx-2ab\,x^2-a^2x}dx\right)a^2m^2 + \left(\int \frac{x^m}{b^2m\,x^3+2abm\,x^2-b^2x^3+a^2mx-2ab\,x^2-a^2x}dx\right)a^2m\right.}{b\,(bmx+am}$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a)^3,x)
```

output
```
(e**m*(x**m - int(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + b*
*2*m*x**3 - b**2*x**3),x)*a**2*m**2 + int(x**m/(a**2*m*x - a**2*x + 2*a*b*
m*x**2 - 2*a*b*x**2 + b**2*m*x**3 - b**2*x**3),x)*a**2*m - int(x**m/(a**2*
m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**3 - b**2*x**3),x)*a*b
*m**2*x + int(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m
*x**3 - b**2*x**3),x)*a*b*m*x))/(b*(a*m - a + b*m*x - b*x))
```

3.351 $\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = -\frac{B(ex)^{1+m}}{be(2-m)(a+bx)^3} + \frac{(Ab(2-m)+aB(1+m))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(4,1+m,2+m,-\frac{bx}{a}\right)}{a^4be(2-m)(1+m)}$$

output
```
-B*(e*x)^(1+m)/b/e/(2-m)/(b*x+a)^3+(A*b*(2-m)+a*B*(1+m))*(e*x)^(1+m)*hyper
geom([4, 1+m],[2+m],-b*x/a)/a^4/b/e/(2-m)/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = \frac{x(ex)^m\left(\frac{a^3(Ab-aB)}{(a+bx)^3}-\frac{(Ab(-2+m)-aB(1+m))\,\text{Hypergeometric2F1}\left(3,1+m,2+m,-\frac{bx}{a}\right)}{1+m}\right)}{3a^4b}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x)^4,x]
```

output
```
(x*(e*x)^m*((a^3*(A*b - a*B))/(a + b*x)^3 - ((A*b*(-2 + m) - a*B*(1 + m))*
Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(1 + m)))/(3*a^4*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{(a+bx)^4}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(aB(m+1)+Ab(2-m))\int \frac{(ex)^m}{(a+bx)^3}dx}{3ab}+\frac{(ex)^{m+1}(Ab-aB)}{3abe(a+bx)^3}$$

$$\Big\downarrow \text{74}$$

$$\frac{(ex)^{m+1}(aB(m+1)+Ab(2-m))\,\text{Hypergeometric2F1}\left(3,m+1,m+2,-\frac{bx}{a}\right)}{3a^4be(m+1)}+\frac{(ex)^{m+1}(Ab-aB)}{3abe(a+bx)^3}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x)^4,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*(a + b*x)^3) + ((A*b*(2 - m) + a*B*(1
 + m))*(e*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(3*a^
4*b*e*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (Bx+A)}{(bx+a)^4}dx$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a)^4,x)
```

output
```
int((e*x)^m*(B*x+A)/(b*x+a)^4,x)
```

Fricas [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^4}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*
b*x + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 4840, normalized size of antiderivative = 53.78

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x+A)/(b*x+a)**4,x)
```

output
```
A*(-a**3*e**m*m**4*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*ga
mma(m + 1)/(6*a**7*gamma(m + 2) + 18*a**6*b*x*gamma(m + 2) + 18*a**5*b**2*
x**2*gamma(m + 2) + 6*a**4*b**3*x**3*gamma(m + 2)) + 2*a**3*e**m*m**3*x**(
m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(6*a**7*gamm
a(m + 2) + 18*a**6*b*x*gamma(m + 2) + 18*a**5*b**2*x**2*gamma(m + 2) + 6*a
**4*b**3*x**3*gamma(m + 2)) + a**3*e**m*m**3*x**(m + 1)*gamma(m + 1)/(6*a*
*7*gamma(m + 2) + 18*a**6*b*x*gamma(m + 2) + 18*a**5*b**2*x**2*gamma(m + 2
) + 6*a**4*b**3*x**3*gamma(m + 2)) + a**3*e**m*m**2*x**(m + 1)*lerchphi(b*
x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(6*a**7*gamma(m + 2) + 18*a**6
*b*x*gamma(m + 2) + 18*a**5*b**2*x**2*gamma(m + 2) + 6*a**4*b**3*x**3*gamm
a(m + 2)) - 3*a**3*e**m*m**2*x**(m + 1)*gamma(m + 1)/(6*a**7*gamma(m + 2)
+ 18*a**6*b*x*gamma(m + 2) + 18*a**5*b**2*x**2*gamma(m + 2) + 6*a**4*b**3*
x**3*gamma(m + 2)) - 2*a**3*e**m*m*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)
/a, 1, m + 1)*gamma(m + 1)/(6*a**7*gamma(m + 2) + 18*a**6*b*x*gamma(m + 2)
 + 18*a**5*b**2*x**2*gamma(m + 2) + 6*a**4*b**3*x**3*gamma(m + 2)) + 2*a**
3*e**m*m*x**(m + 1)*gamma(m + 1)/(6*a**7*gamma(m + 2) + 18*a**6*b*x*gamma(
m + 2) + 18*a**5*b**2*x**2*gamma(m + 2) + 6*a**4*b**3*x**3*gamma(m + 2)) +
 6*a**3*e**m*x**(m + 1)*gamma(m + 1)/(6*a**7*gamma(m + 2) + 18*a**6*b*x*ga
mma(m + 2) + 18*a**5*b**2*x**2*gamma(m + 2) + 6*a**4*b**3*x**3*gamma(m + 2
)) - 3*a**2*b*e**m*m**4*x*x**(m + 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1,...
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^4}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a)^4, x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = \int \frac{(Bx+A)(ex)^m}{(bx+a)^4}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{(a+b\,x)^4}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x)^4,x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x)^4, x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx)^4}\,dx$$
$$= \frac{e^m \left(x^m - \left(\int \frac{x^m}{b^3m\,x^4+3a\,b^2m\,x^3-2b^3x^4+3a^2bm\,x^2-6a\,b^2x^3+a^3mx-6a^2b\,x^2-2a^3x}dx\right)a^3m^2 + 2\left(\int \frac{}{b^3m\,x^4+3a\,b^2m\,x^3-2b^3x^4+3a}\right.\right.}{}$$

input
```
int((e*x)^m*(B*x+A)/(b*x+a)^4,x)
```

output
```
(e**m*(x**m - int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x
**2 + 3*a*b**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**3
*m**2 + 2*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2
+ 3*a*b**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**3*m -
 2*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b
**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**2*b*m**2*x +
 4*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b
**2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a**2*b*m*x - in
t(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b**2*m
*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a*b**2*m**2*x**2 + 2
*int(x**m/(a**3*m*x - 2*a**3*x + 3*a**2*b*m*x**2 - 6*a**2*b*x**2 + 3*a*b**
2*m*x**3 - 6*a*b**2*x**3 + b**3*m*x**4 - 2*b**3*x**4),x)*a*b**2*m*x**2))/(
b*(a**2*m - 2*a**2 + 2*a*b*m*x - 4*a*b*x + b**2*m*x**2 - 2*b**2*x**2))
```

3.352 $\int (ex)^m(a+bx)^p(A+Bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (ex)^m(a+bx)^p(A+Bx)\,dx = \frac{B(ex)^{1+m}(a+bx)^{1+p}}{be(2+m+p)} + \frac{\left(\frac{A}{a+am}-\frac{B}{b(2+m+p)}\right)(ex)^{1+m}(a+bx)^{1+p}\,\text{Hypergeometric2F1}\left(1,2+m+p,2+m,-\frac{bx}{a}\right)}{e}$$

output
```
B*(e*x)^(1+m)*(b*x+a)^(p+1)/b/e/(2+m+p)+(A/(a*m+a)-B/b/(2+m+p))*(e*x)^(1+m
)*(b*x+a)^(p+1)*hypergeom([1, 2+m+p],[2+m],-b*x/a)/e
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int (ex)^m(a+bx)^p(A+Bx)\,dx = \frac{x(ex)^m(a+bx)^p\left(B(a+bx)+\frac{(-aB(1+m)+Ab(2+m+p))\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1+m,-p,2+m,-\frac{bx}{a}\right)}{1+m}\right)}{b(2+m+p)}$$

input
```
Integrate[(e*x)^m*(a + b*x)^p*(A + B*x),x]
```

output
```
(x*(e*x)^m*(a + b*x)^p*(B*(a + b*x) + ((-(a*B*(1 + m)) + A*b*(2 + m + p))*
Hypergeometric2F1[1 + m, -p, 2 + m, -((b*x)/a)])/((1 + m)*(1 + (b*x)/a)^p)
))/(b*(2 + m + p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {90, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m(a+bx)^p\,dx$$

$$\Big\downarrow \ 90$$

$$\left(A-\frac{aB(m+1)}{b(m+p+2)}\right)\int (ex)^m(a+bx)^pdx+\frac{B(ex)^{m+1}(a+bx)^{p+1}}{be(m+p+2)}$$

$$\Big\downarrow \ 76$$

$$(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\left(A-\frac{aB(m+1)}{b(m+p+2)}\right)\int (ex)^m\left(\frac{bx}{a}+1\right)^pdx+\frac{B(ex)^{m+1}(a+bx)^{p+1}}{be(m+p+2)}$$

$$\Big\downarrow \ 74$$

$$\frac{(ex)^{m+1}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\left(A-\frac{aB(m+1)}{b(m+p+2)}\right)\text{Hypergeometric2F1}\left(m+1,-p,m+2,-\frac{bx}{a}\right)}{e(m+1)}+\frac{B(ex)^{m+1}(a+bx)^{p+1}}{be(m+p+2)}$$

input
```
Int[(e*x)^m*(a + b*x)^p*(A + B*x),x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x)^(1 + p))/(b*e*(2 + m + p)) + ((A - (a*B*(1 + m)
)/(b*(2 + m + p)))*(e*x)^(1 + m)*(a + b*x)^p*Hypergeometric2F1[1 + m, -p,
2 + m, -((b*x)/a)])/(e*(1 + m)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

Maple [F]

$$\int (ex)^m (bx + a)^p (Bx + A) \, dx$$

input
```
int((e*x)^m*(b*x+a)^p*(B*x+A),x)
```

output
```
int((e*x)^m*(b*x+a)^p*(B*x+A),x)
```

Fricas [F]

$$\int (ex)^m (a+bx)^p (A+Bx)\,dx = \int (Bx+A)(bx+a)^p (ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x+a)^p*(B*x+A),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(b*x + a)^p*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int (ex)^m (a+bx)^p (A+Bx)\,dx = \frac{A a^p e^m x^{m+1} \Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -p, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)} + \frac{B a^p e^m x^{m+2} \Gamma(m+2)\,{}_2F_1\left(\begin{matrix} -p, m+2 \\ m+3 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+3)}$$

input
```
integrate((e*x)**m*(b*x+a)**p*(B*x+A),x)
```

output
```
A*a**p*e**m*x**(m + 1)*gamma(m + 1)*hyper((-p, m + 1), (m + 2,), b*x*exp_p
olar(I*pi)/a)/gamma(m + 2) + B*a**p*e**m*x**(m + 2)*gamma(m + 2)*hyper((-p
, m + 2), (m + 3,), b*x*exp_polar(I*pi)/a)/gamma(m + 3)
```

Maxima [F]

$$\int (ex)^m(a+bx)^p(A+Bx)\,dx = \int (Bx+A)(bx+a)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x+a)^p*(B*x+A),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(a+bx)^p(A+Bx)\,dx = \int (Bx+A)(bx+a)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x+a)^p*(B*x+A),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(a+bx)^p(A+Bx)\,dx = \int (e\,x)^m\,(A+B\,x)\,(a+b\,x)^p\,dx$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x)^p,x)
```

output
```
int((e*x)^m*(A + B*x)*(a + b*x)^p, x)
```

Reduce [F]

$$\int (ex)^m (a+bx)^p (A+Bx)\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x+a)^p*(B*x+A),x)
```

output
```
(e**m*(x**m*(a + b*x)**p*a**2*p**2 + x**m*(a + b*x)**p*a**2*p + x**m*(a +
b*x)**p*a*b*m**2*x + 3*x**m*(a + b*x)**p*a*b*m*p*x + 2*x**m*(a + b*x)**p*a
*b*m*x + 2*x**m*(a + b*x)**p*a*b*p**2*x + 2*x**m*(a + b*x)**p*a*b*p*x + x*
*m*(a + b*x)**p*b**2*m**2*x**2 + 2*x**m*(a + b*x)**p*b**2*m*p*x**2 + x**m*
(a + b*x)**p*b**2*m*x**2 + x**m*(a + b*x)**p*b**2*p**2*x**2 + x**m*(a + b*
x)**p*b**2*p*x**2 - int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a
*m**2*x + 3*a*m*p**2*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a
*p*x + b*m**3*x**2 + 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6
*b*m*p*x**2 + 2*b*m*x**2 + b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a*
*3*m**4*p**2 - int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2
*x + 3*a*m*p**2*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x
+ b*m**3*x**2 + 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*
p*x**2 + 2*b*m*x**2 + b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a**3*m*
*4*p - 3*int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2*x + 3
*a*m*p**2*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x + b*m*
*3*x**2 + 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*p*x**2
 + 2*b*m*x**2 + b*p**3*x**2 + 3*b*p**2*x**2 + 2*b*p*x**2),x)*a**3*m**3*p**
3 - 6*int((x**m*(a + b*x)**p)/(a*m**3*x + 3*a*m**2*p*x + 3*a*m**2*x + 3*a*
m*p**2*x + 6*a*m*p*x + 2*a*m*x + a*p**3*x + 3*a*p**2*x + 2*a*p*x + b*m**3*
x**2 + 3*b*m**2*p*x**2 + 3*b*m**2*x**2 + 3*b*m*p**2*x**2 + 6*b*m*p*x**2...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file