Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/23-1.1.1.4a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**239**]. This is test number [23].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	98.74 (236)	1.26 (3)
Rubi	97.49 (233)	2.51 (6)
Maple	93.72 (224)	6.28 (15)
Giac	90.38 (216)	9.62 (23)
Reduce	85.77 (205)	14.23 (34)
Mupad	80.33 (192)	19.67 (47)
Fricas	74.06 (177)	25.94 (62)
Sympy	30.96 (74)	69.04 (165)
Maxima	16.74 (40)	83.26 (199)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

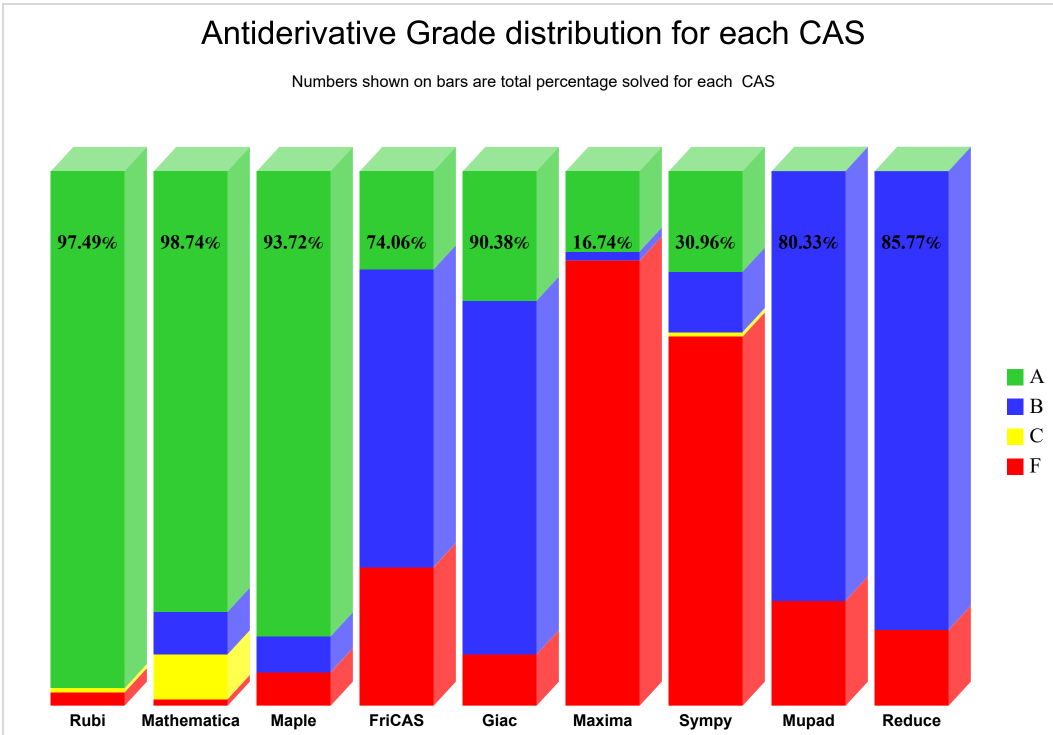
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

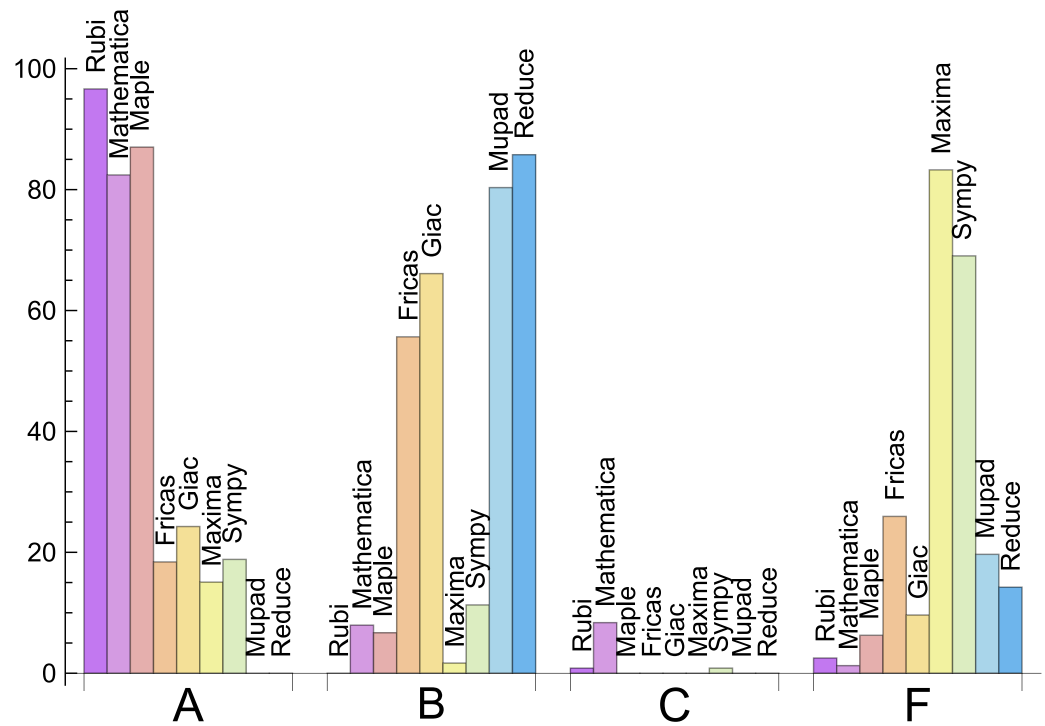
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.653	0.000	0.837	2.510
Maple	87.029	6.695	0.000	6.276
Mathematica	82.427	7.950	8.368	1.255
Giac	24.268	66.109	0.000	9.623
Sympy	18.828	11.297	0.837	69.038
Fricas	18.410	55.649	0.000	25.941
Maxima	15.063	1.674	0.000	83.264
Mupad	0.000	80.335	0.000	19.665
Reduce	0.000	85.774	0.000	14.226

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	3	100.00	0.00	0.00
Rubi	6	100.00	0.00	0.00
Maple	15	100.00	0.00	0.00
Giac	23	100.00	0.00	0.00
Reduce	34	100.00	0.00	0.00
Mupad	47	0.00	100.00	0.00
Fricas	62	24.19	75.81	0.00
Sympy	165	17.58	80.61	1.82
Maxima	199	11.56	0.00	88.44

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Giac	0.28
Rubi	0.86
Reduce	0.95
Mathematica	3.60
Fricas	7.52
Sympy	10.30
Mupad	10.61
Maple	14.32

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	392.43	1.42	307.50	1.47
Rubi	430.49	1.06	299.00	1.06
Sympy	494.20	2.15	324.50	1.91
Maple	611.61	1.43	374.00	1.14
Mathematica	1239.26	1.73	331.50	1.12
Giac	1341.26	2.90	839.00	2.30
Fricas	2271.42	6.88	1228.00	5.44
Reduce	5904.92	9.50	1732.00	6.38
Mupad	135985.03	200.59	692.00	2.08

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

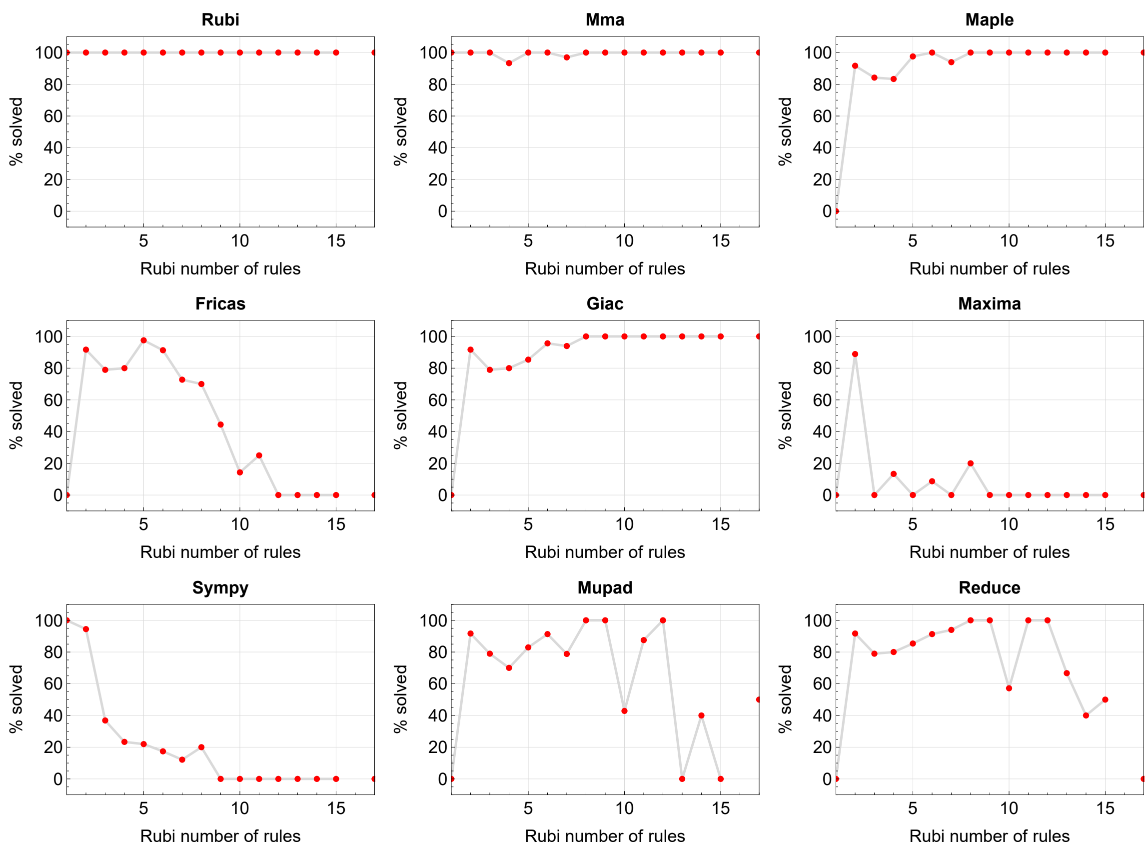


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

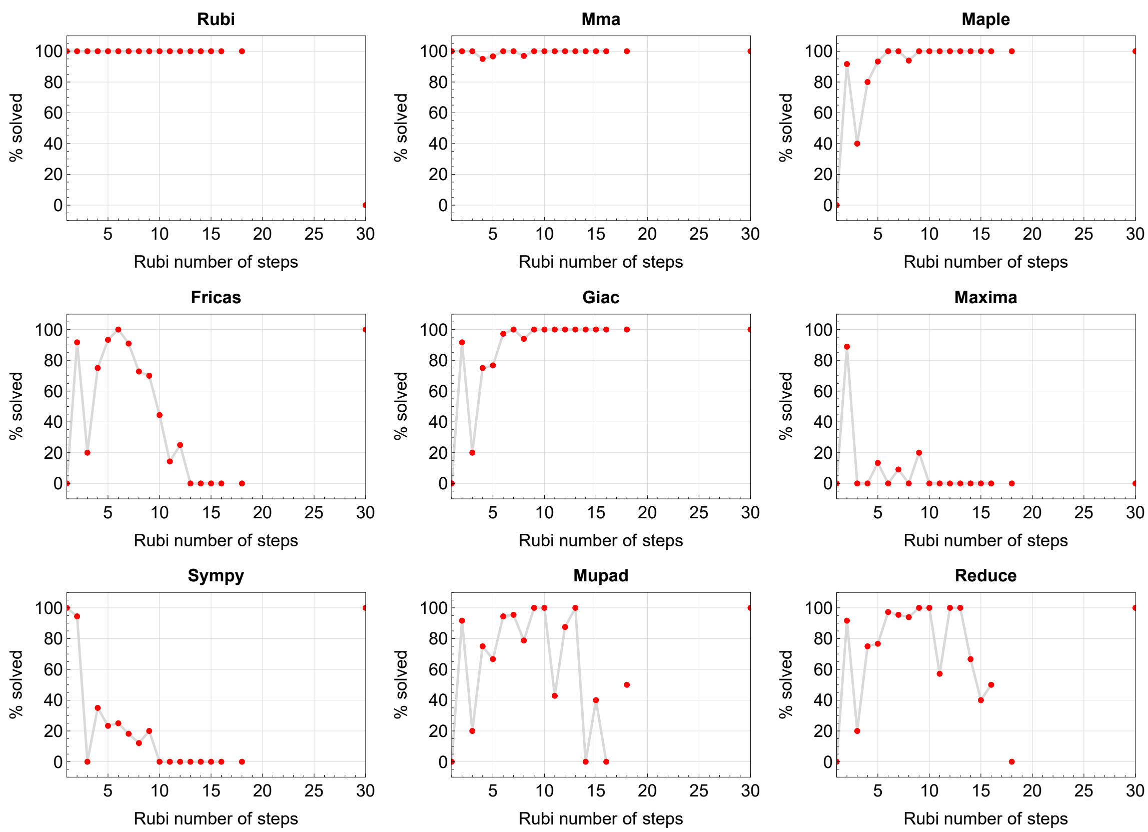


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

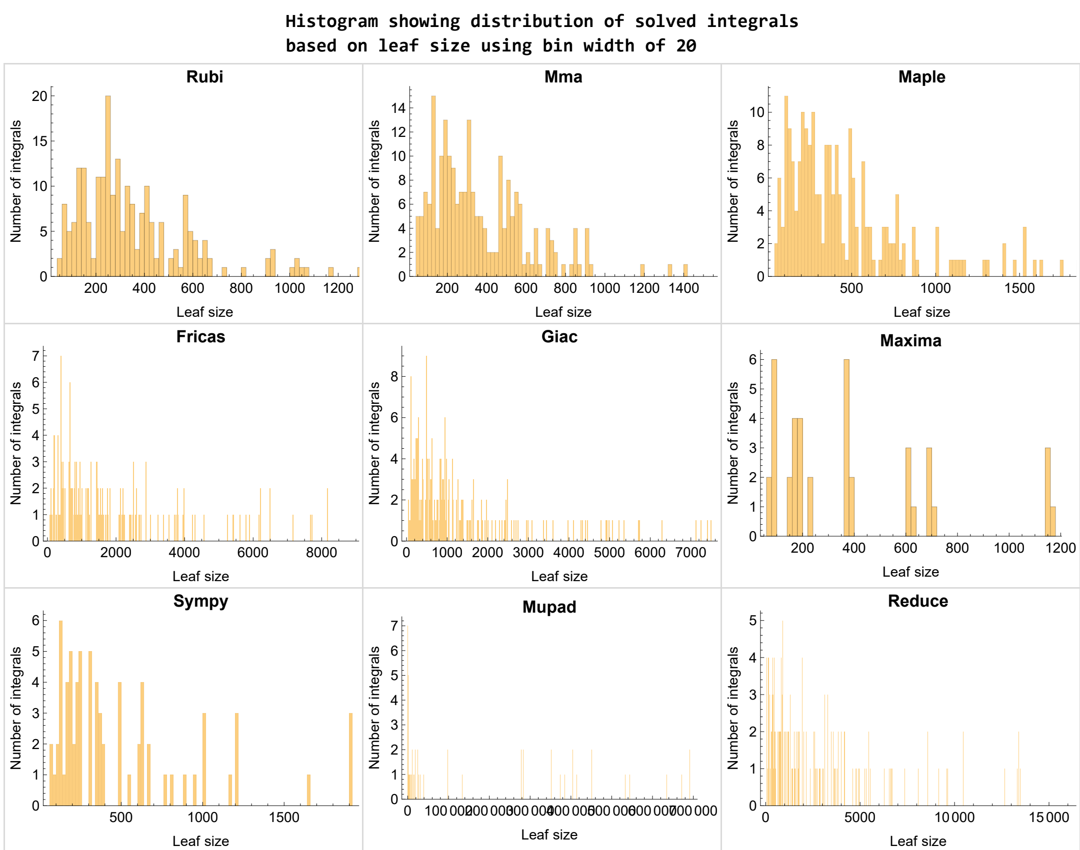


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

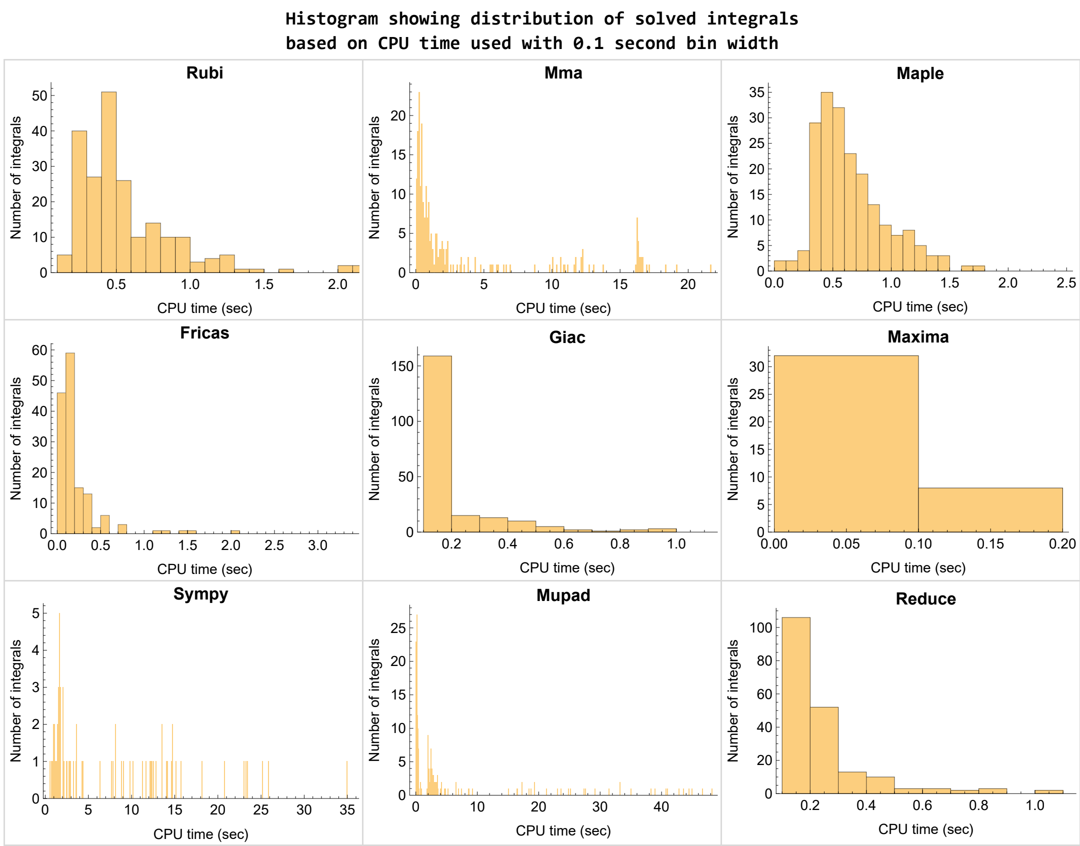


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

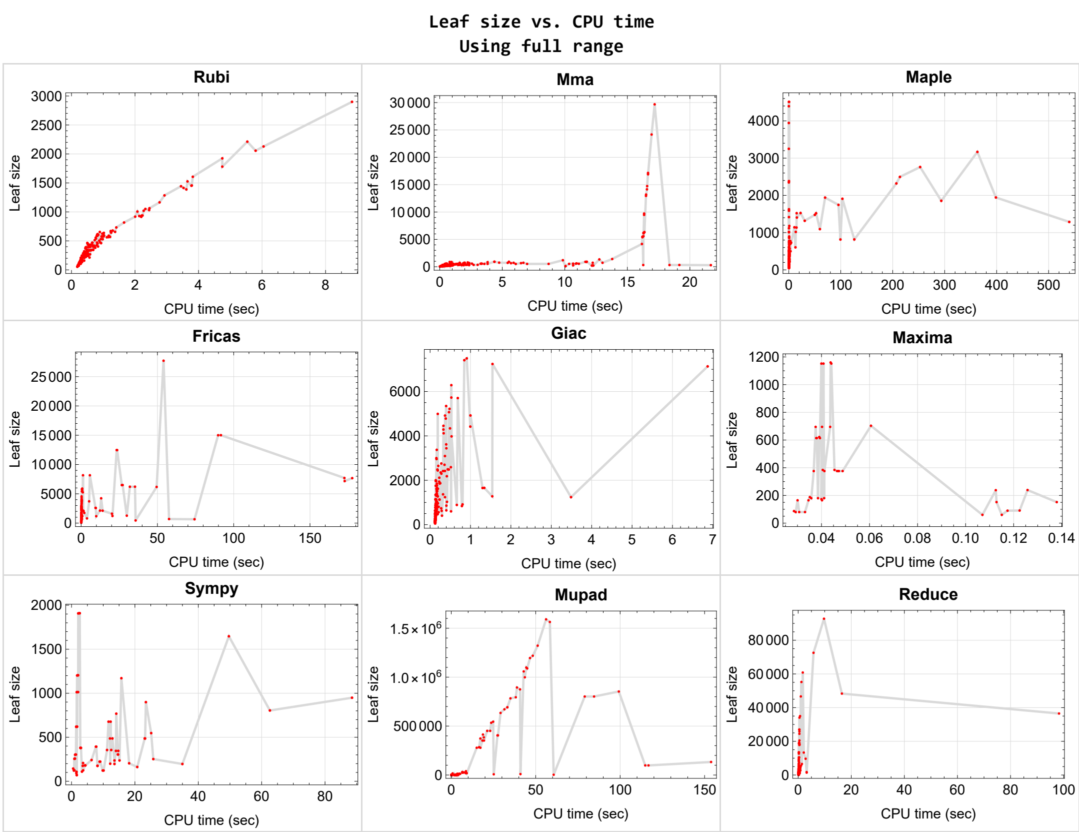


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{237, 239\}$

Mathematica $\{231, 234\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

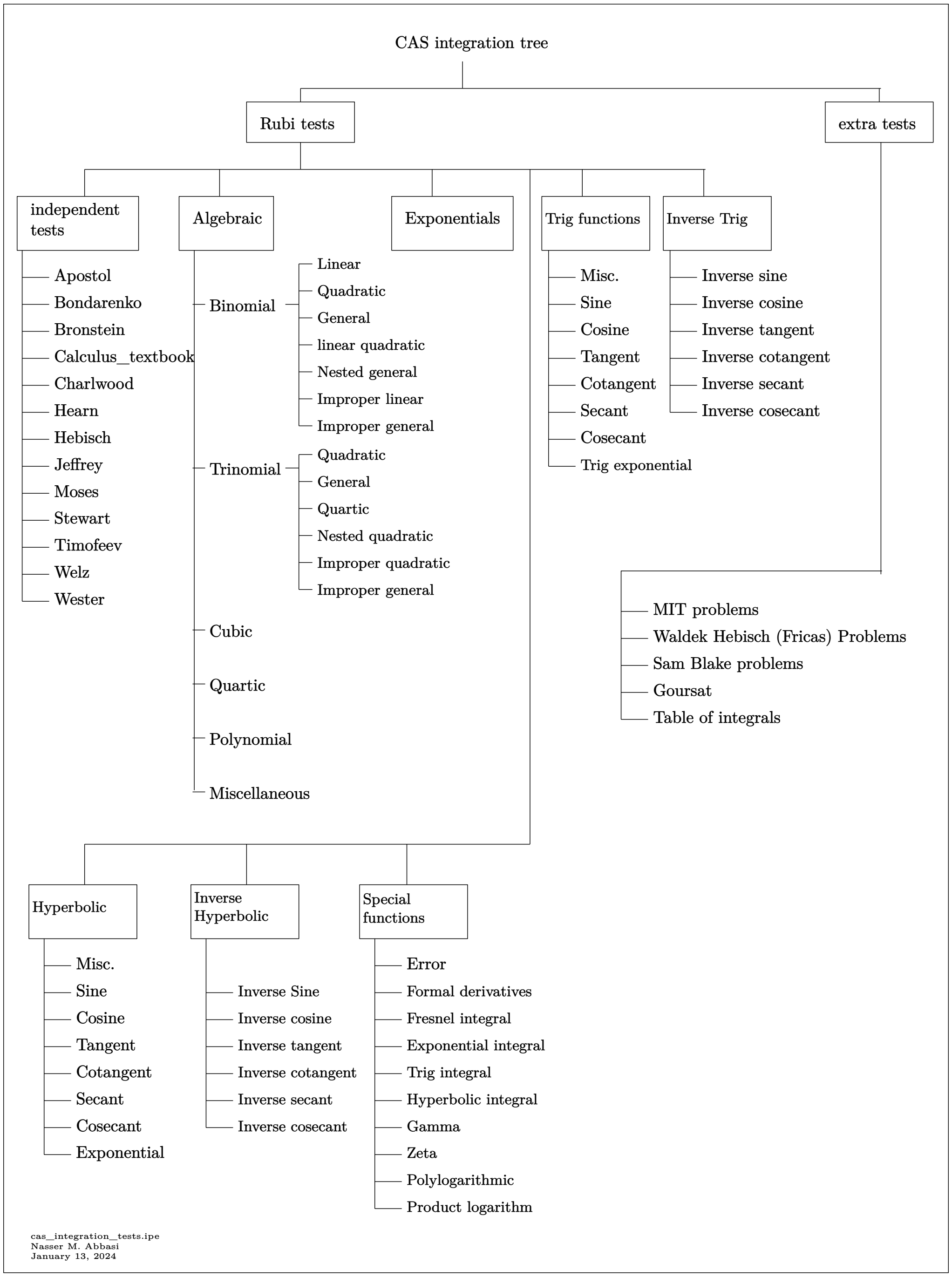
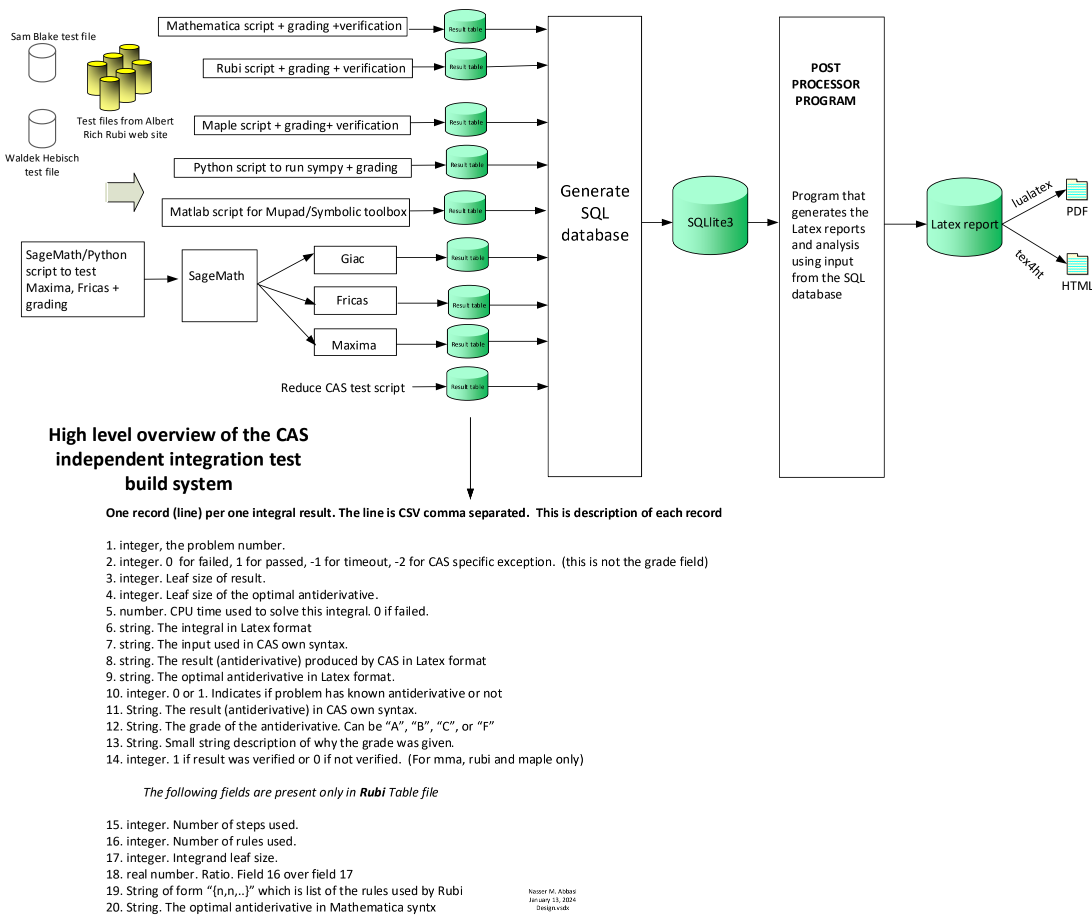


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238 }

B grade { }

C grade { 237, 239 }

F normal fail { 33, 34, 78, 79, 123, 124 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 214, 215, 216, 217, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236 }

B grade { 41, 49, 50, 57, 58, 59, 86, 94, 95, 102, 103, 104, 131, 139, 140, 147, 148, 149, 186 }

C grade { 175, 176, 183, 184, 185, 191, 192, 193, 194, 206, 207, 208, 209, 210, 211, 212, 213, 218, 223, 237 }

F normal fail { 232, 238, 239 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 190, 191, 192, 193, 194, 203, 206, 207, 208, 209, 210, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade { 177, 185, 186, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 211, 212, 213 }

C grade { }

F normal fail { 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 5, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 35, 36, 37, 45, 81, 82, 90, 105, 106, 107, 108, 109, 115, 116, 117, 125, 126, 151, 152, 153, 160, 161, 162, 197, 198, 199, 207, 208, 209, 210, 220 }

B grade { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 24, 25, 31, 32, 33, 34, 38, 39, 42, 43, 44, 46, 47, 48, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 87, 88, 89, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 132, 133, 134, 135, 136, 137, 141, 142, 143, 144, 145, 150, 154, 155, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 186, 187, 188, 189, 200, 201, 202, 203, 206, 211, 212, 213, 215, 216, 221 }

C grade { }

F normal fail { 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-1) timedout fail { 40, 41, 49, 50, 57, 58, 59, 85, 86, 94, 95, 103, 104, 130, 131, 138, 139, 140, 146, 147, 148, 149, 173, 174, 175, 176, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 204, 205, 214, 217, 218, 219, 222, 223, 224 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 11, 12, 13, 19, 20, 21, 22, 26, 27, 28, 29, 60, 61, 62, 63, 70, 71, 72, 105, 106, 107, 108, 115, 116, 117, 150, 151, 152, 153, 160, 161, 162 }

B grade { 10, 69, 114, 159 }

C grade { }

F normal fail { 206, 207, 208, 209, 210, 211, 212, 213, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 23, 24, 25, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 113, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

Giac

A grade { 6, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 44, 45, 46, 47, 48, 54, 82, 83, 90, 91, 93, 99, 107, 108, 109, 110, 117, 118, 124, 125, 126, 127, 128, 134, 135, 136, 144, 152, 153, 154, 163, 169, 170, 171, 172, 180, 216, 220, 221, 224 }

B grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 33, 34, 35, 39, 40, 41, 42, 43, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 92, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 129, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 214, 215, 217, 218, 219, 222, 223 }

C grade { }

F normal fail { 206, 207, 208, 209, 210, 211, 212, 213, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 199, 203, 204, 205, 215, 216, 217, 220, 221, 222 }

C grade { }

F normal fail { }

F(-1) timedout fail { 59, 103, 149, 175, 176, 183, 184, 185, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 5, 13, 14, 19, 20, 21, 22, 26, 27, 28, 29, 34, 35, 36, 37, 62, 63, 64, 72, 80, 81, 82, 107, 108, 109, 117, 118, 123, 124, 125, 126, 127, 151, 152, 153, 154, 161, 162, 163, 169, 170, 171, 172 }

B grade { 1, 2, 10, 11, 12, 23, 30, 33, 60, 61, 69, 70, 71, 73, 78, 79, 105, 106, 114, 115, 116, 150, 160, 168, 226, 227, 228 }

C grade { 225, 237 }

F normal fail { 24, 45, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 219, 220, 221, 222, 224 }

F(-1) timedout fail { 6, 7, 8, 9, 15, 16, 17, 18, 25, 31, 32, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 74, 75, 76, 77, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 110, 111, 112, 113, 119, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 155, 156, 157, 158, 159, 164, 165, 166, 167, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 214, 218, 223, 232, 233, 234, 235, 236, 238, 239 }

F(-2) exception fail { 229, 230, 231 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

C grade { }

F normal fail { 50, 59, 95, 104, 147, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	483	356	614	811	1013	1751	1029	364
N.S.	1	1.00	1.42	1.04	1.80	2.38	2.97	5.13	3.02	1.07
time (sec)	N/A	0.575	0.422	0.990	0.039	0.085	1.774	0.139	0.170	1.879

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	304	255	376	519	620	1147	633	259
N.S.	1	1.00	1.23	1.03	1.52	2.10	2.51	4.64	2.56	1.05
time (sec)	N/A	0.504	0.246	0.947	0.037	0.073	1.524	0.133	0.164	0.073

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	166	138	180	265	303	638	313	155
N.S.	1	1.00	1.04	0.87	1.13	1.67	1.91	4.01	1.97	0.97
time (sec)	N/A	0.333	0.122	0.823	0.041	0.066	1.207	0.122	0.162	1.944

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	61	80	123	124	261	119	80
N.S.	1	1.00	0.86	0.75	0.99	1.52	1.53	3.22	1.47	0.99
time (sec)	N/A	0.224	0.045	0.575	0.031	0.071	0.823	0.128	0.158	0.051

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	133	162	170	0	468	236	278	402	231
N.S.	1	0.85	1.04	1.09	0.00	3.00	1.51	1.78	2.58	1.48
time (sec)	N/A	0.270	0.272	0.590	0.000	0.087	13.547	0.131	0.160	1.943

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	215	186	191	0	884	0	250	989	219
N.S.	1	1.18	1.02	1.05	0.00	4.86	0.00	1.37	5.43	1.20
time (sec)	N/A	0.349	0.482	0.473	0.000	0.100	0.000	0.130	0.162	1.966

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	282	253	264	0	1420	0	497	1706	353
N.S.	1	1.15	1.03	1.08	0.00	5.80	0.00	2.03	6.96	1.44
time (sec)	N/A	0.471	0.920	0.508	0.000	0.159	0.000	0.136	0.165	2.028

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	289	374	376	0	2256	0	852	2619	644
N.S.	1	0.90	1.16	1.17	0.00	7.01	0.00	2.65	8.13	2.00
time (sec)	N/A	0.473	1.487	0.527	0.000	0.206	0.000	0.145	0.170	2.210

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	334	562	536	0	3384	0	1350	3715	862
N.S.	1	0.79	1.33	1.27	0.00	8.02	0.00	3.20	8.80	2.04
time (sec)	N/A	0.519	3.364	0.583	0.000	0.338	0.000	0.166	0.185	2.290

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	843	569	1152	1467	1909	2999	1941	692
N.S.	1	1.00	1.46	0.98	1.99	2.53	3.30	5.18	3.35	1.20
time (sec)	N/A	0.981	0.953	1.060	0.044	0.105	2.403	0.156	0.198	1.949

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	547	412	695	919	1204	2001	1225	470
N.S.	1	1.00	1.33	1.00	1.69	2.24	2.93	4.87	2.98	1.14
time (sec)	N/A	0.698	0.467	1.006	0.044	0.082	2.025	0.140	0.168	0.111

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	308	255	376	519	620	1147	633	259
N.S.	1	1.00	1.24	1.03	1.52	2.09	2.50	4.62	2.55	1.04
time (sec)	N/A	0.489	0.236	0.903	0.047	0.077	1.645	0.135	0.200	0.075

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	130	116	165	244	257	486	251	115
N.S.	1	1.00	1.03	0.92	1.31	1.94	2.04	3.86	1.99	0.91
time (sec)	N/A	0.282	0.083	0.674	0.040	0.067	1.010	0.125	0.193	1.941

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	255	277	310	0	956	486	581	865	571
N.S.	1	0.99	1.08	1.21	0.00	3.72	1.89	2.26	3.37	2.22
time (sec)	N/A	0.455	0.457	0.496	0.000	0.097	12.137	0.140	0.175	2.022

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	292	304	322	0	1738	0	482	1921	570
N.S.	1	1.18	1.23	1.30	0.00	7.01	0.00	1.94	7.75	2.30
time (sec)	N/A	0.455	0.762	0.546	0.000	0.124	0.000	0.143	0.165	2.068

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	435	465	492	0	2678	0	903	3292	639
N.S.	1	1.24	1.32	1.40	0.00	7.61	0.00	2.57	9.35	1.82
time (sec)	N/A	0.583	1.262	0.639	0.000	0.254	0.000	0.157	0.194	2.170

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	587	625	621	0	3796	0	1561	4809	931
N.S.	1	1.21	1.29	1.28	0.00	7.84	0.00	3.23	9.94	1.92
time (sec)	N/A	0.795	1.984	0.704	0.000	0.380	0.000	0.162	0.204	2.326

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	640	588	904	872	0	5434	0	2457	6626	1670
N.S.	1	0.92	1.41	1.36	0.00	8.49	0.00	3.84	10.35	2.61
time (sec)	N/A	0.815	4.377	0.794	0.000	0.595	0.000	0.187	0.244	2.838

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	242	197	214	239	646	355	330	481	413
N.S.	1	1.08	0.88	0.95	1.06	2.87	1.58	1.47	2.14	1.84
time (sec)	N/A	0.457	0.277	0.333	0.126	0.084	12.525	0.134	0.168	0.089

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	151	131	144	152	402	223	196	295	263
N.S.	1	1.03	0.89	0.98	1.03	2.73	1.52	1.33	2.01	1.79
time (sec)	N/A	0.290	0.172	0.316	0.138	0.085	9.051	0.120	0.171	1.910

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	77	81	83	91	216	122	102	153	136
N.S.	1	0.89	0.93	0.95	1.05	2.48	1.40	1.17	1.76	1.56
time (sec)	N/A	0.203	0.096	0.275	0.122	0.082	9.834	0.127	0.162	0.052

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	53	46	60	108	70	55	71	45
N.S.	1	1.02	0.98	0.85	1.11	2.00	1.30	1.02	1.31	0.83
time (sec)	N/A	0.188	0.046	0.267	0.107	0.098	1.671	0.123	0.163	0.042

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	101	103	0	443	196	109	134	2368
N.S.	1	1.06	1.00	1.02	0.00	4.39	1.94	1.08	1.33	23.45
time (sec)	N/A	0.291	0.193	0.377	0.000	0.163	13.547	0.132	0.162	2.189

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	140	123	132	0	1012	0	140	516	1827
N.S.	1	1.10	0.97	1.04	0.00	7.97	0.00	1.10	4.06	14.39
time (sec)	N/A	0.271	0.417	0.414	0.000	0.166	0.000	0.123	0.170	0.371

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	238	195	213	0	2210	0	292	1311	4852
N.S.	1	1.14	0.94	1.02	0.00	10.62	0.00	1.40	6.30	23.33
time (sec)	N/A	0.404	0.757	0.442	0.000	0.537	0.000	0.136	0.175	3.203

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	241	236	217	238	638	355	330	481	413
N.S.	1	1.08	1.06	0.97	1.07	2.86	1.59	1.48	2.16	1.85
time (sec)	N/A	0.417	0.272	0.336	0.113	0.108	11.208	0.131	0.166	0.086

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	150	157	145	152	400	223	196	295	263
N.S.	1	1.03	1.08	0.99	1.04	2.74	1.53	1.34	2.02	1.80
time (sec)	N/A	0.277	0.175	0.304	0.113	0.083	8.870	0.131	0.162	1.940

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	77	91	86	90	214	122	102	153	136
N.S.	1	0.89	1.05	0.99	1.03	2.46	1.40	1.17	1.76	1.56
time (sec)	N/A	0.199	0.115	0.269	0.117	0.091	10.106	0.123	0.165	0.052

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	53	46	60	108	70	55	71	45
N.S.	1	1.02	0.98	0.85	1.11	2.00	1.30	1.02	1.31	0.83
time (sec)	N/A	0.182	0.045	0.323	0.115	0.119	1.618	0.128	0.164	1.939

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	101	103	0	444	199	109	134	2355
N.S.	1	1.06	1.00	1.02	0.00	4.40	1.97	1.08	1.33	23.32
time (sec)	N/A	0.292	0.174	0.425	0.000	0.117	12.209	0.124	0.163	2.331

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	140	122	110	0	1002	0	137	520	1814
N.S.	1	1.09	0.95	0.86	0.00	7.83	0.00	1.07	4.06	14.17
time (sec)	N/A	0.290	0.421	0.388	0.000	0.162	0.000	0.124	0.165	2.337

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	236	194	214	0	2205	0	291	1311	4839
N.S.	1	1.15	0.95	1.04	0.00	10.76	0.00	1.42	6.40	23.60
time (sec)	N/A	0.431	0.842	0.408	0.000	0.520	0.000	0.136	0.169	3.248

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	0	456	513	0	1697	899	1050	1557	865
N.S.	1	0.00	1.07	1.20	0.00	3.97	2.11	2.46	3.65	2.03
time (sec)	N/A	0.000	0.770	0.581	0.000	0.102	23.463	0.152	0.175	2.100

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	B	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	0	277	310	0	946	486	581	865	571
N.S.	1	0.00	1.09	1.22	0.00	3.71	1.91	2.28	3.39	2.24
time (sec)	N/A	0.000	0.443	0.493	0.000	0.133	12.899	0.137	0.162	2.107

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	134	162	171	0	467	236	278	402	231
N.S.	1	0.86	1.04	1.10	0.00	2.99	1.51	1.78	2.58	1.48
time (sec)	N/A	0.279	0.254	0.441	0.000	0.105	15.139	0.137	0.156	0.136

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	95	95	103	0	214	124	122	139	108
N.S.	1	0.97	0.97	1.05	0.00	2.18	1.27	1.24	1.42	1.10
time (sec)	N/A	0.229	0.169	0.391	0.000	0.095	3.666	0.115	0.163	1.991

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	159	159	166	0	680	196	169	212	6793
N.S.	1	1.07	1.07	1.11	0.00	4.56	1.32	1.13	1.42	45.59
time (sec)	N/A	0.399	0.441	0.682	0.000	0.543	34.982	0.124	0.155	3.380

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	222	188	223	0	2100	0	254	1168	11558
N.S.	1	1.09	0.93	1.10	0.00	10.34	0.00	1.25	5.75	56.94
time (sec)	N/A	0.405	0.670	0.658	0.000	1.412	0.000	0.136	0.158	4.856

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	399	352	413	0	6490	0	743	3843	278260
N.S.	1	1.12	0.99	1.16	0.00	18.28	0.00	2.09	10.83	783.83
time (sec)	N/A	0.708	2.138	0.825	0.000	26.663	0.000	0.159	0.188	15.063

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	656	710	766	0	0	0	1939	9171	533756
N.S.	1	1.12	1.21	1.30	0.00	0.00	0.00	3.30	15.60	907.75
time (sec)	N/A	1.373	5.434	1.365	0.000	0.000	0.000	0.225	0.250	23.643

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	942	1032	5481	1522	0	0	0	4286	17810	875071
N.S.	1	1.10	5.82	1.62	0.00	0.00	0.00	4.55	18.91	928.95
time (sec)	N/A	2.444	16.223	15.162	0.000	0.000	0.000	0.330	0.446	40.732

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	474	479	513	0	2878	0	830	3133	913
N.S.	1	1.36	1.38	1.47	0.00	8.27	0.00	2.39	9.00	2.62
time (sec)	N/A	0.902	1.115	0.658	0.000	0.201	0.000	0.155	0.182	0.331

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	294	302	323	0	1741	0	484	1921	570
N.S.	1	1.19	1.22	1.31	0.00	7.05	0.00	1.96	7.78	2.31
time (sec)	N/A	0.487	0.759	0.557	0.000	0.148	0.000	0.136	0.165	0.230

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	213	187	193	0	906	0	249	989	219
N.S.	1	1.17	1.03	1.06	0.00	4.98	0.00	1.37	5.43	1.20
time (sec)	N/A	0.376	0.472	0.473	0.000	0.148	0.000	0.131	0.159	0.179

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	134	96	107	0	381	0	115	363	107
N.S.	1	1.25	0.90	1.00	0.00	3.56	0.00	1.07	3.39	1.00
time (sec)	N/A	0.331	0.277	0.418	0.000	0.091	0.000	0.128	0.156	0.107

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	223	200	226	0	2088	0	253	1168	11557
N.S.	1	1.09	0.98	1.11	0.00	10.24	0.00	1.24	5.73	56.65
time (sec)	N/A	0.431	0.599	0.596	0.000	1.566	0.000	0.137	0.165	5.274

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	334	276	306	0	8168	0	471	5334	26654
N.S.	1	1.10	0.91	1.01	0.00	26.87	0.00	1.55	17.55	87.68
time (sec)	N/A	0.573	1.899	0.672	0.000	5.732	0.000	0.187	0.199	7.458

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	569	516	574	0	14998	0	983	13393	451052
N.S.	1	1.07	0.97	1.08	0.00	28.30	0.00	1.85	25.27	851.04
time (sec)	N/A	1.215	6.984	1.155	0.000	89.796	0.000	0.179	0.267	21.280

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	872	938	6117	1143	0	0	0	2352	25836	783233
N.S.	1	1.08	7.01	1.31	0.00	0.00	0.00	2.70	29.63	898.20
time (sec)	N/A	2.236	16.299	10.829	0.000	0.000	0.000	0.263	0.487	35.042

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1368	1456	12952	1940	0	0	0	4916	29	1195213
N.S.	1	1.06	9.47	1.42	0.00	0.00	0.00	3.59	0.02	873.69
time (sec)	N/A	3.788	16.490	69.895	0.000	0.000	0.000	0.370	200.064	46.612

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	595	704	684	0	4303	0	1382	5248	1361
N.S.	1	1.42	1.68	1.63	0.00	10.25	0.00	3.29	12.50	3.24
time (sec)	N/A	1.022	1.953	0.801	0.000	0.333	0.000	0.164	0.401	0.435

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	434	462	492	0	2688	0	904	3292	639
N.S.	1	1.23	1.31	1.39	0.00	7.61	0.00	2.56	9.33	1.81
time (sec)	N/A	0.644	1.273	0.704	0.000	0.311	0.000	0.160	0.176	0.395

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	289	262	264	0	1444	0	497	1706	353
N.S.	1	1.17	1.06	1.07	0.00	5.85	0.00	2.01	6.91	1.43
time (sec)	N/A	0.502	0.935	0.530	0.000	0.202	0.000	0.135	0.162	0.277

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	144	142	136	0	736	0	234	687	222
N.S.	1	0.99	0.98	0.94	0.00	5.08	0.00	1.61	4.74	1.53
time (sec)	N/A	0.265	0.575	0.394	0.000	0.127	0.000	0.138	0.157	0.154

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	405	373	413	0	6495	0	743	3843	278260
N.S.	1	1.12	1.04	1.15	0.00	18.04	0.00	2.06	10.68	772.94
time (sec)	N/A	0.720	1.540	0.803	0.000	27.202	0.000	0.167	0.196	17.268

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	531	520	568	0	15006	0	984	13393	451051
N.S.	1	1.12	1.10	1.20	0.00	31.73	0.00	2.08	28.32	953.60
time (sec)	N/A	0.925	8.705	1.273	0.000	91.619	0.000	0.177	0.270	23.065

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	817	6193	1019	0	0	0	3442	26875	794225
N.S.	1	1.07	8.13	1.34	0.00	0.00	0.00	4.52	35.27	1042.29
time (sec)	N/A	1.655	16.286	13.435	0.000	0.000	0.000	0.403	0.482	38.208

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1204	1285	13191	1745	0	0	0	3092	48332	1218467
N.S.	1	1.07	10.96	1.45	0.00	0.00	0.00	2.57	40.14	1012.02
time (sec)	N/A	2.928	16.498	95.632	0.000	0.000	0.000	0.367	16.471	48.258

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1821	1924	24162	3167	0	0	0	5728	29	0
N.S.	1	1.06	13.27	1.74	0.00	0.00	0.00	3.15	0.02	0.00
time (sec)	N/A	4.748	16.937	362.917	0.000	0.000	0.000	0.526	200.027	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	487	356	614	998	1013	2956	1293	364
N.S.	1	1.00	1.43	1.04	1.80	2.93	2.97	8.67	3.79	1.07
time (sec)	N/A	0.621	0.470	0.942	0.038	0.124	2.078	0.160	0.163	0.124

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	311	255	376	658	620	1968	821	259
N.S.	1	1.00	1.26	1.03	1.52	2.66	2.51	7.97	3.32	1.05
time (sec)	N/A	0.512	0.252	0.947	0.047	0.093	1.765	0.149	0.150	0.072

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	169	139	180	349	303	1124	425	155
N.S.	1	1.00	1.06	0.87	1.13	2.19	1.91	7.07	2.67	0.97
time (sec)	N/A	0.346	0.136	0.844	0.039	0.078	1.384	0.136	0.161	0.059

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	61	80	172	124	473	171	80
N.S.	1	1.00	0.86	0.75	0.99	2.12	1.53	5.84	2.11	0.99
time (sec)	N/A	0.228	0.050	0.645	0.029	0.073	0.913	0.123	0.158	0.050

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	162	246	266	0	854	345	497	835	497
N.S.	1	0.83	1.26	1.36	0.00	4.36	1.76	2.54	4.26	2.54
time (sec)	N/A	0.279	0.344	0.444	0.000	0.105	14.749	0.137	0.166	0.094

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	245	287	292	0	928	0	518	1045	383
N.S.	1	0.99	1.16	1.18	0.00	3.76	0.00	2.10	4.23	1.55
time (sec)	N/A	0.366	0.638	0.494	0.000	0.141	0.000	0.139	0.164	0.302

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	313	273	275	0	1500	0	541	1779	449
N.S.	1	1.13	0.98	0.99	0.00	5.40	0.00	1.95	6.40	1.62
time (sec)	N/A	0.488	0.862	0.506	0.000	0.165	0.000	0.141	0.173	0.250

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	312	391	389	0	2236	0	881	2616	756
N.S.	1	0.94	1.18	1.18	0.00	6.76	0.00	2.66	7.90	2.28
time (sec)	N/A	0.494	1.556	0.531	0.000	0.197	0.000	0.150	0.172	0.353

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	322	557	538	0	3224	0	1320	3637	863
N.S.	1	0.78	1.35	1.30	0.00	7.79	0.00	3.19	8.79	2.08
time (sec)	N/A	0.489	3.269	0.543	0.000	0.318	0.000	0.173	0.186	0.476

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	850	569	1152	1773	1909	4992	2377	692
N.S.	1	1.00	1.47	0.98	1.99	3.06	3.30	8.62	4.11	1.20
time (sec)	N/A	0.999	0.933	1.090	0.041	0.102	2.597	0.190	0.203	2.431

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	554	412	695	1128	1204	3372	1537	470
N.S.	1	1.00	1.35	1.00	1.69	2.74	2.93	8.20	3.74	1.14
time (sec)	N/A	0.711	0.543	1.043	0.040	0.098	2.061	0.165	0.178	0.115

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	315	255	376	658	620	1968	821	259
N.S.	1	1.00	1.27	1.03	1.52	2.65	2.50	7.94	3.31	1.04
time (sec)	N/A	0.470	0.273	0.882	0.046	0.079	1.699	0.140	0.168	0.074

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	133	118	165	323	257	854	339	115
N.S.	1	1.00	1.06	0.94	1.31	2.56	2.04	6.78	2.69	0.91
time (sec)	N/A	0.271	0.101	0.645	0.035	0.073	1.083	0.134	0.157	0.063

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	284	408	480	0	1630	677	944	1635	855
N.S.	1	0.95	1.36	1.61	0.00	5.45	2.26	3.16	5.47	2.86
time (sec)	N/A	0.486	0.568	0.553	0.000	0.156	12.333	0.144	0.161	0.132

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	322	470	495	0	1822	0	941	2025	715
N.S.	1	1.00	1.46	1.54	0.00	5.68	0.00	2.93	6.31	2.23
time (sec)	N/A	0.503	0.986	0.602	0.000	0.152	0.000	0.151	0.173	2.638

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	468	505	510	0	2866	0	1040	3464	856
N.S.	1	1.11	1.20	1.21	0.00	6.79	0.00	2.46	8.21	2.03
time (sec)	N/A	0.648	1.416	0.609	0.000	0.233	0.000	0.168	0.198	2.657

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	608	674	673	0	3942	0	1653	4947	1109
N.S.	1	1.18	1.30	1.30	0.00	7.62	0.00	3.20	9.57	2.15
time (sec)	N/A	0.810	2.356	0.721	0.000	0.385	0.000	0.171	0.202	0.409

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	617	920	896	0	5410	0	2444	6644	1797
N.S.	1	0.95	1.42	1.38	0.00	8.32	0.00	3.76	10.22	2.76
time (sec)	N/A	0.822	4.365	0.779	0.000	0.566	0.000	0.189	0.237	3.127

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	0	643	774	0	2737	1171	1557	2725	1247
N.S.	1	0.00	1.37	1.65	0.00	5.84	2.50	3.32	5.81	2.66
time (sec)	N/A	0.000	0.927	0.660	0.000	0.130	15.733	0.168	0.225	0.183

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	0	412	480	0	1616	677	944	1635	855
N.S.	1	0.00	1.39	1.62	0.00	5.44	2.28	3.18	5.51	2.88
time (sec)	N/A	0.000	0.590	0.533	0.000	0.113	11.674	0.150	0.166	2.628

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	163	250	266	0	853	345	497	835	497
N.S.	1	0.83	1.28	1.36	0.00	4.35	1.76	2.54	4.26	2.54
time (sec)	N/A	0.293	0.334	0.457	0.000	0.101	14.010	0.130	0.169	0.100

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	124	138	137	0	390	175	226	323	236
N.S.	1	0.95	1.06	1.05	0.00	3.00	1.35	1.74	2.48	1.82
time (sec)	N/A	0.238	0.191	0.389	0.000	0.089	3.539	0.126	0.157	2.391

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	213	179	217	0	1213	252	307	512	13380
N.S.	1	1.12	0.94	1.14	0.00	6.35	1.32	1.61	2.68	70.05
time (sec)	N/A	0.568	0.362	0.625	0.000	20.546	25.803	0.138	0.162	3.598

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	268	232	281	0	2136	0	399	1199	18554
N.S.	1	1.09	0.94	1.14	0.00	8.65	0.00	1.62	4.85	75.12
time (sec)	N/A	0.585	1.027	0.707	0.000	12.388	0.000	0.142	0.176	9.206

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	381	357	390	0	6207	0	687	4005	283348
N.S.	1	1.09	1.03	1.12	0.00	17.84	0.00	1.97	11.51	814.22
time (sec)	N/A	0.749	2.064	0.802	0.000	31.949	0.000	0.167	0.201	16.542

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	642	719	771	0	0	0	1853	9592	544718
N.S.	1	1.11	1.24	1.33	0.00	0.00	0.00	3.20	16.57	940.79
time (sec)	N/A	1.250	5.906	1.217	0.000	0.000	0.000	0.235	0.252	24.802

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	932	1020	5574	1526	0	0	0	4118	18831	894172
N.S.	1	1.09	5.98	1.64	0.00	0.00	0.00	4.42	20.20	959.41
time (sec)	N/A	2.272	16.288	22.644	0.000	0.000	0.000	0.336	0.473	38.907

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	504	710	767	0	3008	0	1485	3285	1602
N.S.	1	1.19	1.67	1.81	0.00	7.09	0.00	3.50	7.75	3.78
time (sec)	N/A	0.873	1.591	0.758	0.000	0.218	0.000	0.165	0.196	0.476

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	324	467	496	0	1833	0	942	2025	715
N.S.	1	1.01	1.45	1.54	0.00	5.69	0.00	2.93	6.29	2.22
time (sec)	N/A	0.484	1.003	0.595	0.000	0.206	0.000	0.156	0.190	0.414

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	243	288	291	0	944	0	517	1045	383
N.S.	1	0.98	1.16	1.17	0.00	3.81	0.00	2.08	4.21	1.54
time (sec)	N/A	0.363	0.654	0.484	0.000	0.138	0.000	0.137	0.171	0.289

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	164	138	138	0	384	0	225	379	175
N.S.	1	1.12	0.95	0.95	0.00	2.63	0.00	1.54	2.60	1.20
time (sec)	N/A	0.286	0.335	0.413	0.000	0.100	0.000	0.131	0.165	0.120

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	267	230	281	0	2120	0	398	1199	18557
N.S.	1	1.08	0.93	1.14	0.00	8.58	0.00	1.61	4.85	75.13
time (sec)	N/A	0.561	1.003	0.781	0.000	14.099	0.000	0.146	0.178	8.586

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	348	339	348	0	4238	0	804	2886	38955
N.S.	1	1.10	1.08	1.10	0.00	13.45	0.00	2.55	9.16	123.67
time (sec)	N/A	0.682	2.708	0.854	0.000	13.129	0.000	0.211	0.193	8.693

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	562	510	570	0	12461	0	958	8595	97983
N.S.	1	1.08	0.98	1.10	0.00	23.96	0.00	1.84	16.53	188.43
time (sec)	N/A	1.115	6.078	1.139	0.000	23.599	0.000	0.193	0.256	114.839

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	918	6288	1136	0	0	0	2299	18011	671141
N.S.	1	1.07	7.35	1.33	0.00	0.00	0.00	2.69	21.04	784.04
time (sec)	N/A	2.005	16.363	14.802	0.000	0.000	0.000	0.279	0.551	31.474

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1359	1443	16895	1909	0	0	0	4781	29	1058660
N.S.	1	1.06	12.43	1.40	0.00	0.00	0.00	3.52	0.02	779.00
time (sec)	N/A	3.448	16.654	103.167	0.000	0.000	0.000	0.388	200.023	42.885

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	625	743	758	0	4563	0	1617	5520	1753
N.S.	1	1.27	1.50	1.53	0.00	9.24	0.00	3.27	11.17	3.55
time (sec)	N/A	0.971	2.001	0.792	0.000	0.302	0.000	0.193	1.089	0.500

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	469	501	510	0	2866	0	1040	3464	856
N.S.	1	1.11	1.18	1.21	0.00	6.78	0.00	2.46	8.19	2.02
time (sec)	N/A	0.660	1.329	0.629	0.000	0.260	0.000	0.162	0.189	0.400

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	317	275	276	0	1522	0	541	1779	449
N.S.	1	1.14	0.99	0.99	0.00	5.46	0.00	1.94	6.38	1.61
time (sec)	N/A	0.511	0.888	0.555	0.000	0.176	0.000	0.129	0.167	0.236

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	166	139	146	0	706	0	226	681	257
N.S.	1	1.09	0.91	0.96	0.00	4.64	0.00	1.49	4.48	1.69
time (sec)	N/A	0.280	0.528	0.444	0.000	0.180	0.000	0.128	0.162	2.463

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	388	365	388	0	6213	0	687	4007	283348
N.S.	1	1.10	1.03	1.10	0.00	17.60	0.00	1.95	11.35	802.69
time (sec)	N/A	0.725	1.594	0.810	0.000	35.302	0.000	0.162	0.201	16.480

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	529	527	560	0	12465	0	959	8595	97984
N.S.	1	1.08	1.07	1.14	0.00	25.39	0.00	1.95	17.51	199.56
time (sec)	N/A	0.984	3.826	1.172	0.000	23.305	0.000	0.189	0.248	116.802

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	686	732	6339	775	0	27720	0	2182	19308	133067
N.S.	1	1.07	9.24	1.13	0.00	40.41	0.00	3.18	28.15	193.98
time (sec)	N/A	1.412	16.364	2.944	0.000	54.029	0.000	0.414	0.363	153.829

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1108	1165	17142	1520	0	0	0	2698	36506	0
N.S.	1	1.05	15.47	1.37	0.00	0.00	0.00	2.44	32.95	0.00
time (sec)	N/A	2.774	16.653	52.517	0.000	0.000	0.000	0.356	98.254	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1700	1781	29680	2761	0	0	0	5220	29	1321612
N.S.	1	1.05	17.46	1.62	0.00	0.00	0.00	3.07	0.02	777.42
time (sec)	N/A	4.749	17.181	252.766	0.000	0.000	0.000	0.481	200.059	51.163

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	486	355	614	628	1012	772	769	364
N.S.	1	1.00	1.43	1.05	1.81	1.85	2.99	2.28	2.27	1.07
time (sec)	N/A	0.616	0.393	0.977	0.038	0.095	1.632	0.131	0.179	0.124

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	307	254	376	386	619	495	449	259
N.S.	1	1.00	1.25	1.04	1.53	1.58	2.53	2.02	1.83	1.06
time (sec)	N/A	0.523	0.232	0.925	0.041	0.090	1.495	0.125	0.165	0.072

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	166	139	180	186	301	268	205	155
N.S.	1	1.00	1.06	0.89	1.15	1.18	1.92	1.71	1.31	0.99
time (sec)	N/A	0.331	0.113	0.832	0.036	0.082	1.120	0.121	0.166	2.455

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	61	80	78	122	105	71	80
N.S.	1	1.00	0.86	0.77	1.01	0.99	1.54	1.33	0.90	1.01
time (sec)	N/A	0.221	0.042	0.585	0.033	0.100	0.727	0.116	0.165	0.048

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	114	112	113	0	424	180	160	342	169
N.S.	1	0.92	0.90	0.91	0.00	3.42	1.45	1.29	2.76	1.36
time (sec)	N/A	0.241	0.167	0.407	0.000	0.097	4.374	0.115	0.187	0.108

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	177	175	185	0	793	0	215	900	176
N.S.	1	1.15	1.14	1.20	0.00	5.15	0.00	1.40	5.84	1.14
time (sec)	N/A	0.314	0.461	0.437	0.000	0.120	0.000	0.130	0.177	0.209

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	249	240	248	0	1432	0	509	1691	337
N.S.	1	1.05	1.01	1.05	0.00	6.04	0.00	2.15	7.14	1.42
time (sec)	N/A	0.488	0.782	0.473	0.000	0.185	0.000	0.130	0.170	2.741

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	295	380	379	0	2380	0	922	2680	662
N.S.	1	0.90	1.16	1.15	0.00	7.23	0.00	2.80	8.15	2.01
time (sec)	N/A	0.508	1.691	0.498	0.000	0.284	0.000	0.138	0.182	0.434

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	344	563	531	0	3548	0	1376	3797	869
N.S.	1	0.79	1.29	1.22	0.00	8.16	0.00	3.16	8.73	2.00
time (sec)	N/A	0.512	2.573	0.593	0.000	0.385	0.000	0.154	0.197	3.018

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	853	568	1152	1174	1907	1344	1509	692
N.S.	1	1.00	1.48	0.98	2.00	2.03	3.31	2.33	2.62	1.20
time (sec)	N/A	1.012	0.791	1.089	0.040	0.095	2.106	0.135	0.215	0.173

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	553	411	695	712	1202	884	917	470
N.S.	1	1.00	1.35	1.00	1.70	1.74	2.94	2.16	2.24	1.15
time (sec)	N/A	0.702	0.489	1.024	0.038	0.091	1.759	0.137	0.172	0.112

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	311	254	376	386	619	495	449	259
N.S.	1	1.00	1.26	1.03	1.53	1.57	2.52	2.01	1.83	1.05
time (sec)	N/A	0.480	0.222	0.912	0.049	0.079	1.464	0.127	0.178	0.079

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	131	117	165	171	255	206	167	115
N.S.	1	1.00	1.06	0.94	1.33	1.38	2.06	1.66	1.35	0.93
time (sec)	N/A	0.280	0.083	0.674	0.030	0.072	0.903	0.127	0.186	2.482

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	235	193	192	0	872	393	382	757	328
N.S.	1	1.05	0.87	0.86	0.00	3.91	1.76	1.71	3.39	1.47
time (sec)	N/A	0.453	0.291	0.442	0.000	0.119	7.805	0.124	0.171	2.504

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	271	291	226	0	1546	0	408	1740	450
N.S.	1	1.24	1.33	1.03	0.00	7.06	0.00	1.86	7.95	2.05
time (sec)	N/A	0.454	0.699	0.530	0.000	0.129	0.000	0.133	0.172	0.221

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	400	425	448	0	2500	0	864	3133	591
N.S.	1	1.22	1.29	1.36	0.00	7.60	0.00	2.63	9.52	1.80
time (sec)	N/A	0.595	1.198	0.641	0.000	0.261	0.000	0.155	0.218	2.754

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	556	616	601	0	3808	0	1618	4777	908
N.S.	1	1.17	1.30	1.27	0.00	8.02	0.00	3.41	10.06	1.91
time (sec)	N/A	0.763	1.869	0.626	0.000	0.445	0.000	0.158	0.215	2.934

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	607	908	860	0	5612	0	2483	6708	1676
N.S.	1	0.93	1.39	1.32	0.00	8.59	0.00	3.80	10.27	2.57
time (sec)	N/A	0.822	5.531	0.763	0.000	0.723	0.000	0.180	0.247	3.416

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	B	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	0	334	319	0	1561	768	767	1402	533
N.S.	1	0.00	0.85	0.81	0.00	3.97	1.95	1.95	3.57	1.36
time (sec)	N/A	0.000	0.566	0.535	0.000	0.111	14.122	0.131	0.194	0.103

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	B	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	0	193	192	0	858	393	382	757	328
N.S.	1	0.00	0.87	0.87	0.00	3.88	1.78	1.73	3.43	1.48
time (sec)	N/A	0.000	0.312	0.444	0.000	0.098	7.695	0.128	0.175	2.469

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	114	112	112	0	421	180	160	342	169
N.S.	1	0.92	0.90	0.90	0.00	3.40	1.45	1.29	2.76	1.36
time (sec)	N/A	0.255	0.160	0.389	0.000	0.088	4.239	0.125	0.169	0.100

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	64	0	208	92	66	122	62
N.S.	1	1.00	1.00	0.86	0.00	2.81	1.24	0.89	1.65	0.84
time (sec)	N/A	0.197	0.112	0.369	0.000	0.075	1.514	0.124	0.185	0.063

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	131	101	0	971	162	113	405	3821
N.S.	1	1.00	1.00	0.77	0.00	7.41	1.24	0.86	3.09	29.17
time (sec)	N/A	0.246	0.318	0.543	0.000	0.344	20.737	0.128	0.206	4.142

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	243	195	230	0	2580	0	282	2158	24052
N.S.	1	1.16	0.93	1.10	0.00	12.34	0.00	1.35	10.33	115.08
time (sec)	N/A	0.452	0.779	0.578	0.000	9.624	0.000	0.131	0.204	6.550

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	418	363	400	0	7674	0	835	6282	351830
N.S.	1	1.15	1.00	1.10	0.00	21.14	0.00	2.30	17.31	969.23
time (sec)	N/A	0.707	2.347	0.822	0.000	177.670	0.000	0.157	0.237	19.376

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	678	724	758	0	0	0	2114	13481	634320
N.S.	1	1.13	1.21	1.27	0.00	0.00	0.00	3.53	22.51	1058.96
time (sec)	N/A	1.298	6.688	1.218	0.000	0.000	0.000	0.228	0.300	29.182

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	946	1051	4159	1405	0	0	0	4455	24455	998307
N.S.	1	1.11	4.40	1.49	0.00	0.00	0.00	4.71	25.85	1055.29
time (sec)	N/A	2.327	16.163	15.543	0.000	0.000	0.000	0.331	0.475	43.569

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	454	453	370	0	2564	0	707	2831	680
N.S.	1	1.44	1.43	1.17	0.00	8.11	0.00	2.24	8.96	2.15
time (sec)	N/A	0.819	1.279	0.582	0.000	0.204	0.000	0.145	0.233	0.272

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	271	286	223	0	1547	0	407	1740	450
N.S.	1	1.24	1.31	1.02	0.00	7.10	0.00	1.87	7.98	2.06
time (sec)	N/A	0.455	0.709	0.533	0.000	0.129	0.000	0.134	0.182	0.214

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	176	176	182	0	807	0	214	900	176
N.S.	1	1.14	1.14	1.18	0.00	5.24	0.00	1.39	5.84	1.14
time (sec)	N/A	0.338	0.437	0.466	0.000	0.108	0.000	0.130	0.178	0.192

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	99	89	0	402	0	122	358	100
N.S.	1	1.00	0.99	0.89	0.00	4.02	0.00	1.22	3.58	1.00
time (sec)	N/A	0.245	0.289	0.362	0.000	0.089	0.000	0.123	0.183	0.098

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	241	196	231	0	2596	0	281	2158	24052
N.S.	1	1.16	0.95	1.12	0.00	12.54	0.00	1.36	10.43	116.19
time (sec)	N/A	0.422	0.788	0.622	0.000	9.441	0.000	0.133	0.208	6.905

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	402	351	361	0	7176	0	840	8083	384543
N.S.	1	1.16	1.01	1.04	0.00	20.68	0.00	2.42	23.29	1108.19
time (sec)	N/A	0.675	3.032	0.871	0.000	172.737	0.000	0.174	0.245	19.301

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	670	721	709	0	0	0	1273	18732	691756
N.S.	1	1.13	1.21	1.19	0.00	0.00	0.00	2.14	31.48	1162.62
time (sec)	N/A	1.234	13.014	3.350	0.000	0.000	0.000	0.189	0.321	33.221

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	964	1063	5396	1477	0	0	0	2757	34058	1088917
N.S.	1	1.10	5.60	1.53	0.00	0.00	0.00	2.86	35.33	1129.58
time (sec)	N/A	2.455	16.206	50.119	0.000	0.000	0.000	0.278	0.503	44.957

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1482	1606	9480	2496	0	0	0	5347	55226	1563828
N.S.	1	1.08	6.40	1.68	0.00	0.00	0.00	3.61	37.26	1055.21
time (sec)	N/A	3.820	16.339	213.833	0.000	0.000	0.000	0.397	1.232	58.380

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	573	643	589	0	3979	0	1306	4932	1242
N.S.	1	1.47	1.65	1.51	0.00	10.20	0.00	3.35	12.65	3.18
time (sec)	N/A	0.959	1.625	0.786	0.000	0.340	0.000	0.151	0.225	0.476

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	395	422	451	0	2518	0	864	3133	591
N.S.	1	1.22	1.30	1.39	0.00	7.77	0.00	2.67	9.67	1.82
time (sec)	N/A	0.573	1.151	0.619	0.000	0.251	0.000	0.144	0.206	0.400

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	253	245	249	0	1442	0	509	1691	337
N.S.	1	1.06	1.03	1.04	0.00	6.03	0.00	2.13	7.08	1.41
time (sec)	N/A	0.464	0.736	0.474	0.000	0.145	0.000	0.126	0.169	0.265

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	150	142	149	0	839	0	257	735	228
N.S.	1	0.97	0.92	0.96	0.00	5.41	0.00	1.66	4.74	1.47
time (sec)	N/A	0.268	0.593	0.410	0.000	0.106	0.000	0.131	0.168	2.625

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	418	370	384	0	7703	0	835	6278	351830
N.S.	1	1.15	1.02	1.06	0.00	21.22	0.00	2.30	17.29	969.23
time (sec)	N/A	0.702	2.220	0.875	0.000	172.558	0.000	0.155	0.221	18.452

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	649	727	706	0	0	0	1274	18732	691757
N.S.	1	1.12	1.25	1.21	0.00	0.00	0.00	2.19	32.19	1188.59
time (sec)	N/A	1.242	12.230	3.364	0.000	0.000	0.000	0.189	0.326	33.228

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	922	1006	5488	1094	0	0	0	5069	29	1098262
N.S.	1	1.09	5.95	1.19	0.00	0.00	0.00	5.50	0.03	1191.17
time (sec)	N/A	2.075	16.204	59.856	0.000	0.000	0.000	0.468	200.025	44.433

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1417	1526	9721	1854	0	0	0	3617	60838	1590452
N.S.	1	1.08	6.86	1.31	0.00	0.00	0.00	2.55	42.93	1122.41
time (sec)	N/A	3.652	16.341	293.424	0.000	0.000	0.000	0.399	1.744	56.151

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2077	2212	14739	3250	0	0	0	6287	92781	0
N.S.	1	1.06	7.10	1.56	0.00	0.00	0.00	3.03	44.67	0.00
time (sec)	N/A	5.536	16.583	0.089	0.000	0.000	0.000	0.526	9.734	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	480	428	622	637	804	965	771	537
N.S.	1	1.00	1.42	1.27	1.85	1.89	2.39	2.86	2.29	1.59
time (sec)	N/A	0.599	0.415	0.391	0.039	0.125	62.598	0.148	0.274	0.152

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	302	276	384	396	486	573	451	368
N.S.	1	1.00	1.24	1.14	1.58	1.63	2.00	2.36	1.86	1.51
time (sec)	N/A	0.505	0.264	0.349	0.041	0.091	23.217	0.137	0.297	2.471

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	161	150	188	195	241	273	207	201
N.S.	1	1.00	1.04	0.97	1.21	1.26	1.55	1.76	1.34	1.30
time (sec)	N/A	0.337	0.120	0.303	0.035	0.110	6.319	0.133	0.265	0.065

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	64	87	87	107	100	72	79
N.S.	1	1.00	0.87	0.83	1.13	1.13	1.39	1.30	0.94	1.03
time (sec)	N/A	0.231	0.054	0.276	0.029	0.072	1.575	0.122	0.261	0.045

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	137	131	141	0	656	175	143	373	182
N.S.	1	1.11	1.07	1.15	0.00	5.33	1.42	1.16	3.03	1.48
time (sec)	N/A	0.279	0.224	0.405	0.000	0.097	8.188	0.127	0.274	0.143

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	229	215	209	0	1437	0	360	1089	269
N.S.	1	1.27	1.19	1.16	0.00	7.98	0.00	2.00	6.05	1.49
time (sec)	N/A	0.415	0.657	0.447	0.000	0.135	0.000	0.138	0.305	0.284

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	299	347	357	0	2506	0	607	2007	442
N.S.	1	1.08	1.25	1.28	0.00	9.01	0.00	2.18	7.22	1.59
time (sec)	N/A	0.460	1.526	0.512	0.000	0.248	0.000	0.146	0.308	2.803

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	345	527	519	0	3796	0	1033	3040	785
N.S.	1	0.92	1.41	1.39	0.00	10.15	0.00	2.76	8.13	2.10
time (sec)	N/A	0.483	2.353	0.573	0.000	0.494	0.000	0.152	0.284	0.724

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	391	747	709	0	5254	0	1508	4152	991
N.S.	1	0.82	1.57	1.49	0.00	11.01	0.00	3.16	8.70	2.08
time (sec)	N/A	0.524	4.749	0.674	0.000	1.100	0.000	0.176	0.308	3.365

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	845	736	1160	1183	0	1849	1511	1011
N.S.	1	1.00	1.47	1.28	2.02	2.06	0.00	3.22	2.63	1.76
time (sec)	N/A	0.971	0.811	0.480	0.044	0.098	0.000	0.167	0.374	0.177

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	546	479	703	721	949	1137	919	683
N.S.	1	1.00	1.34	1.18	1.73	1.77	2.33	2.79	2.26	1.68
time (sec)	N/A	0.726	0.470	0.443	0.061	0.088	88.517	0.153	0.197	0.122

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	306	276	384	396	486	573	451	368
N.S.	1	1.00	1.25	1.13	1.57	1.62	1.99	2.35	1.85	1.51
time (sec)	N/A	0.462	0.245	0.356	0.045	0.087	23.098	0.139	0.168	0.082

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	129	125	173	180	207	221	169	154
N.S.	1	1.00	1.06	1.02	1.42	1.48	1.70	1.81	1.39	1.26
time (sec)	N/A	0.292	0.094	0.313	0.040	0.083	3.636	0.125	0.246	0.056

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	224	221	231	0	1266	304	290	790	326
N.S.	1	1.29	1.28	1.34	0.00	7.32	1.76	1.68	4.57	1.88
time (sec)	N/A	0.441	0.455	0.471	0.000	0.131	14.705	0.131	0.254	0.135

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	326	331	352	0	2405	0	620	1935	601
N.S.	1	1.51	1.53	1.63	0.00	11.13	0.00	2.87	8.96	2.78
time (sec)	N/A	0.515	0.985	0.587	0.000	0.183	0.000	0.143	0.309	2.847

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	539	559	580	0	3998	0	981	3593	781
N.S.	1	1.52	1.58	1.64	0.00	11.29	0.00	2.77	10.15	2.21
time (sec)	N/A	0.770	1.945	0.681	0.000	0.390	0.000	0.151	0.287	2.934

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	604	831	862	0	5894	0	1761	5440	1085
N.S.	1	1.17	1.61	1.67	0.00	11.40	0.00	3.41	10.52	2.10
time (sec)	N/A	0.843	3.826	0.769	0.000	0.722	0.000	0.169	0.329	3.160

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	695	654	1191	1179	0	8170	0	2648	7371	1824
N.S.	1	0.94	1.71	1.70	0.00	11.76	0.00	3.81	10.61	2.62
time (sec)	N/A	0.892	9.845	1.005	0.000	1.236	0.000	0.203	0.378	4.017

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	403	381	419	0	2179	547	542	1423	512
N.S.	1	1.49	1.41	1.55	0.00	8.07	2.03	2.01	5.27	1.90
time (sec)	N/A	0.835	0.785	0.553	0.000	0.160	25.131	0.147	0.275	0.167

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	223	224	230	0	1262	304	290	790	326
N.S.	1	1.29	1.29	1.33	0.00	7.29	1.76	1.68	4.57	1.88
time (sec)	N/A	0.452	0.432	0.475	0.000	0.110	14.628	0.125	0.260	0.137

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	137	131	140	0	657	175	143	373	182
N.S.	1	1.11	1.07	1.14	0.00	5.34	1.42	1.16	3.03	1.48
time (sec)	N/A	0.287	0.211	0.395	0.000	0.094	8.195	0.127	0.281	0.121

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	77	0	377	109	87	161	96
N.S.	1	1.00	1.00	0.88	0.00	4.28	1.24	0.99	1.83	1.09
time (sec)	N/A	0.226	0.199	0.353	0.000	0.100	3.283	0.122	0.240	2.444

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	208	169	169	0	1604	204	197	909	30821
N.S.	1	1.22	0.99	0.99	0.00	9.44	1.20	1.16	5.35	181.30
time (sec)	N/A	0.404	0.669	0.572	0.000	20.211	18.177	0.139	0.289	6.502

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	344	290	277	0	0	0	623	4163	404752
N.S.	1	1.20	1.01	0.97	0.00	0.00	0.00	2.17	14.51	1410.29
time (sec)	N/A	0.685	1.750	0.720	0.000	0.000	0.000	0.158	0.303	27.358

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	580	657	680	0	0	0	1130	10467	802142
N.S.	1	1.15	1.30	1.34	0.00	0.00	0.00	2.23	20.69	1585.26
time (sec)	N/A	1.186	6.497	1.197	0.000	0.000	0.000	0.233	0.395	79.018

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	929	565	1318	0	0	0	2492	20501	0
N.S.	1	1.12	0.68	1.59	0.00	0.00	0.00	3.00	24.67	0.00
time (sec)	N/A	2.133	10.662	30.698	0.000	0.000	0.000	0.436	0.485	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1290	1414	918	2320	0	0	0	4921	34973	0
N.S.	1	1.10	0.71	1.80	0.00	0.00	0.00	3.81	27.11	0.00
time (sec)	N/A	3.527	11.755	207.014	0.000	0.000	0.000	0.999	0.717	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	471	526	514	0	3748	0	936	3035	837
N.S.	1	1.78	1.98	1.94	0.00	14.14	0.00	3.53	11.45	3.16
time (sec)	N/A	0.804	1.415	0.733	0.000	0.276	0.000	0.149	0.180	0.364

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	317	333	349	0	2423	0	623	1935	601
N.S.	1	1.45	1.53	1.60	0.00	11.11	0.00	2.86	8.88	2.76
time (sec)	N/A	0.491	0.851	0.608	0.000	0.178	0.000	0.137	0.162	0.387

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	227	216	209	0	1445	0	360	1089	269
N.S.	1	1.25	1.19	1.15	0.00	7.98	0.00	1.99	6.02	1.49
time (sec)	N/A	0.397	0.654	0.513	0.000	0.141	0.000	0.133	0.160	2.731

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	122	129	0	796	0	193	476	154
N.S.	1	1.11	0.95	1.01	0.00	6.22	0.00	1.51	3.72	1.20
time (sec)	N/A	0.274	0.401	0.431	0.000	0.101	0.000	0.126	0.162	0.118

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	345	289	275	0	0	0	618	4163	404752
N.S.	1	1.20	1.00	0.95	0.00	0.00	0.00	2.15	14.45	1405.39
time (sec)	N/A	0.661	2.350	0.791	0.000	0.000	0.000	0.157	0.202	27.665

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	565	608	391	0	0	0	1867	12668	853229
N.S.	1	1.14	1.23	0.79	0.00	0.00	0.00	3.78	25.64	1727.18
time (sec)	N/A	1.171	10.945	1.618	0.000	0.000	0.000	0.251	0.274	99.212

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	848	936	561	815	0	0	0	2417	26977	0
N.S.	1	1.10	0.66	0.96	0.00	0.00	0.00	2.85	31.81	0.00
time (sec)	N/A	2.209	10.886	99.398	0.000	0.000	0.000	0.321	0.445	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1340	1452	902	1943	0	0	0	4339	46641	0
N.S.	1	1.08	0.67	1.45	0.00	0.00	0.00	3.24	34.81	0.00
time (sec)	N/A	3.775	11.646	398.569	0.000	0.000	0.000	0.493	1.005	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1991	2129	1410	3944	0	0	0	7412	72551	0
N.S.	1	1.07	0.71	1.98	0.00	0.00	0.00	3.72	36.44	0.00
time (sec)	N/A	6.048	13.789	0.085	0.000	0.000	0.000	0.847	5.785	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	638	786	782	0	5799	0	1383	5447	1540
N.S.	1	1.66	2.04	2.03	0.00	15.06	0.00	3.59	14.15	4.00
time (sec)	N/A	1.019	2.237	1.109	0.000	0.506	0.000	0.170	0.260	0.980

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	513	562	582	0	3988	0	980	3593	781
N.S.	1	1.45	1.58	1.64	0.00	11.23	0.00	2.76	10.12	2.20
time (sec)	N/A	0.731	1.976	0.750	0.000	0.398	0.000	0.155	0.208	3.535

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	293	351	358	0	2496	0	607	2007	442
N.S.	1	1.05	1.25	1.28	0.00	8.91	0.00	2.17	7.17	1.58
time (sec)	N/A	0.442	1.445	0.584	0.000	0.238	0.000	0.147	0.185	0.387

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	188	186	193	0	1468	0	340	892	296
N.S.	1	1.04	1.03	1.07	0.00	8.11	0.00	1.88	4.93	1.64
time (sec)	N/A	0.292	0.869	0.474	0.000	0.144	0.000	0.135	0.182	0.214

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	589	649	680	0	0	0	1130	10467	802140
N.S.	1	1.14	1.26	1.32	0.00	0.00	0.00	2.19	20.28	1554.53
time (sec)	N/A	1.171	5.695	1.418	0.000	0.000	0.000	0.232	0.255	84.561

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	838	915	544	817	0	0	0	2420	26977	0
N.S.	1	1.09	0.65	0.97	0.00	0.00	0.00	2.89	32.19	0.00
time (sec)	N/A	2.196	11.122	126.088	0.000	0.000	0.000	0.367	0.450	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1292	1388	871	1287	0	0	0	7132	29	0
N.S.	1	1.07	0.67	1.00	0.00	0.00	0.00	5.52	0.02	0.00
time (sec)	N/A	3.617	11.722	539.704	0.000	0.000	0.000	6.871	200.027	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1935	2056	1338	2348	0	0	0	4418	29	0
N.S.	1	1.06	0.69	1.21	0.00	0.00	0.00	2.28	0.01	0.00
time (sec)	N/A	5.798	12.760	0.118	0.000	0.000	0.000	0.995	200.028	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2754	2899	14143	4394	0	0	0	7241	29	0
N.S.	1	1.05	5.14	1.60	0.00	0.00	0.00	2.63	0.01	0.00
time (sec)	N/A	8.832	16.568	0.124	0.000	0.000	0.000	1.545	200.025	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	461	337	4511	0	0	0	1652	29	0
N.S.	1	1.17	0.85	11.42	0.00	0.00	0.00	4.18	0.07	0.00
time (sec)	N/A	0.932	1.739	0.445	0.000	0.000	0.000	1.299	200.025	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	425	339	4511	0	0	0	1657	29	0
N.S.	1	1.07	0.86	11.39	0.00	0.00	0.00	4.18	0.07	0.00
time (sec)	N/A	0.795	1.757	0.419	0.000	0.000	0.000	1.342	200.029	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	285	461	2383	0	1746	0	2485	27	0
N.S.	1	0.59	0.96	4.97	0.00	3.65	0.00	5.19	0.06	0.00
time (sec)	N/A	0.366	1.476	0.323	0.000	2.094	0.000	0.462	200.071	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	246	332	1627	0	1190	0	1366	1398	0
N.S.	1	0.64	0.87	4.26	0.00	3.12	0.00	3.58	3.66	0.00
time (sec)	N/A	0.326	1.100	0.329	0.000	0.781	0.000	0.320	3.210	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	202	254	1019	0	774	0	559	878	2493
N.S.	1	0.73	0.91	3.67	0.00	2.78	0.00	2.01	3.16	8.97
time (sec)	N/A	0.302	10.665	0.332	0.000	0.381	0.000	0.211	0.281	60.631

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	242	201	1118	0	802	0	404	729	0
N.S.	1	1.09	0.90	5.01	0.00	3.60	0.00	1.81	3.27	0.00
time (sec)	N/A	0.338	0.483	0.332	0.000	3.844	0.000	0.294	0.249	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	189	210	1588	0	1178	0	1238	1308	0
N.S.	1	1.07	1.19	8.97	0.00	6.66	0.00	6.99	7.39	0.00
time (sec)	N/A	0.301	0.936	0.336	0.000	9.910	0.000	0.368	0.279	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	209	261	1417	0	1267	0	2589	1567	0
N.S.	1	1.01	1.27	6.88	0.00	6.15	0.00	12.57	7.61	0.00
time (sec)	N/A	0.333	0.342	0.332	0.000	29.931	0.000	0.514	0.269	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	247	223	302	0	656	0	3978	1732	528
N.S.	1	1.06	0.96	1.30	0.00	2.83	0.00	17.15	7.47	2.28
time (sec)	N/A	0.427	0.240	0.342	0.000	74.326	0.000	0.531	3.189	3.512

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	294	381	614	0	0	0	5705	2628	846
N.S.	1	0.89	1.15	1.86	0.00	0.00	0.00	17.29	7.96	2.56
time (sec)	N/A	0.469	0.380	0.348	0.000	0.000	0.000	0.685	0.364	3.687

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	343	581	1010	0	0	0	7493	3656	1177
N.S.	1	0.79	1.34	2.34	0.00	0.00	0.00	17.34	8.46	2.72
time (sec)	N/A	0.489	0.574	0.335	0.000	0.000	0.000	0.907	0.444	4.199

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	319	498	0	671	0	0	41	0
N.S.	1	1.00	1.12	1.75	0.00	2.36	0.00	0.00	0.14	0.00
time (sec)	N/A	0.408	19.158	3.335	0.000	0.128	0.000	0.000	8.750	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	210	214	219	0	310	0	0	33	0
N.S.	1	0.92	0.93	0.96	0.00	1.35	0.00	0.00	0.14	0.00
time (sec)	N/A	0.305	12.162	1.166	0.000	0.113	0.000	0.000	0.562	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	221	214	0	313	0	0	35	0
N.S.	1	1.00	1.06	1.03	0.00	1.50	0.00	0.00	0.17	0.00
time (sec)	N/A	0.302	12.186	1.305	0.000	0.115	0.000	0.000	0.578	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	219	214	0	312	0	0	96	0
N.S.	1	1.00	1.05	1.03	0.00	1.50	0.00	0.00	0.46	0.00
time (sec)	N/A	0.307	12.232	1.286	0.000	0.108	0.000	0.000	0.906	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	209	221	227	0	315	0	0	98	0
N.S.	1	1.01	1.07	1.10	0.00	1.53	0.00	0.00	0.48	0.00
time (sec)	N/A	0.293	12.279	1.342	0.000	0.116	0.000	0.000	0.888	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	139	309	604	0	1228	0	0	99	0
N.S.	1	0.96	2.13	4.17	0.00	8.47	0.00	0.00	0.68	0.00
time (sec)	N/A	0.296	16.270	12.842	0.000	0.104	0.000	0.000	9.839	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	222	312	729	0	1126	0	0	67	0
N.S.	1	1.07	1.50	3.50	0.00	5.41	0.00	0.00	0.32	0.00
time (sec)	N/A	0.369	18.370	4.535	0.000	0.114	0.000	0.000	13.022	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	289	288	0	194	0	0	45	0
N.S.	1	1.00	3.14	3.13	0.00	2.11	0.00	0.00	0.49	0.00
time (sec)	N/A	0.225	21.673	1.454	0.000	0.074	0.000	0.000	1.603	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	434	502	432	0	0	0	1026	4185	0
N.S.	1	0.97	1.12	0.96	0.00	0.00	0.00	2.29	9.34	0.00
time (sec)	N/A	0.521	2.235	1.776	0.000	0.000	0.000	0.418	0.839	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	376	472	372	0	696	0	735	2549	9140
N.S.	1	0.93	1.16	0.92	0.00	1.71	0.00	1.81	6.28	22.51
time (sec)	N/A	0.445	1.107	0.740	0.000	57.632	0.000	0.275	0.790	40.969

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	318	466	364	0	3716	0	599	1207	16501
N.S.	1	0.82	1.21	0.94	0.00	9.63	0.00	1.55	3.13	42.75
time (sec)	N/A	0.397	0.890	0.672	0.000	5.251	0.000	0.518	0.632	0.787

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	386	519	423	0	0	0	912	2870	373297
N.S.	1	0.91	1.22	1.00	0.00	0.00	0.00	2.15	6.75	878.35
time (sec)	N/A	0.445	3.567	0.724	0.000	0.000	0.000	0.804	0.843	17.237

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	458	112	495	0	0	0	1277	9649	0
N.S.	1	0.93	0.23	1.00	0.00	0.00	0.00	2.59	19.57	0.00
time (sec)	N/A	0.509	10.053	1.182	0.000	0.000	0.000	1.539	2.808	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	432	501	432	0	0	0	857	4185	0
N.S.	1	0.97	1.12	0.97	0.00	0.00	0.00	1.92	9.38	0.00
time (sec)	N/A	0.532	2.033	1.291	0.000	0.000	0.000	0.404	0.697	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	374	472	369	0	451	0	623	2549	7103
N.S.	1	0.93	1.17	0.91	0.00	1.12	0.00	1.54	6.31	17.58
time (sec)	N/A	0.442	0.958	1.460	0.000	35.700	0.000	0.286	0.635	25.159

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	320	466	364	0	6174	0	644	1207	13287
N.S.	1	0.83	1.21	0.94	0.00	15.99	0.00	1.67	3.13	34.42
time (sec)	N/A	0.404	1.879	0.647	0.000	49.491	0.000	0.401	0.553	4.678

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	392	520	423	0	0	0	838	4618	413671
N.S.	1	0.92	1.22	0.99	0.00	0.00	0.00	1.96	10.81	968.78
time (sec)	N/A	0.456	10.343	0.779	0.000	0.000	0.000	0.787	0.835	18.759

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	464	112	495	0	0	0	1239	13315	0
N.S.	1	0.94	0.23	1.00	0.00	0.00	0.00	2.50	26.90	0.00
time (sec)	N/A	0.497	10.057	1.145	0.000	0.000	0.000	3.489	2.116	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	421	565	415	0	0	0	888	6541	0
N.S.	1	0.80	1.07	0.79	0.00	0.00	0.00	1.68	12.39	0.00
time (sec)	N/A	0.587	3.333	0.904	0.000	0.000	0.000	0.661	1.419	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	49	0	0	0	144	0	129	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	2.06	0.00	1.84	0.00
time (sec)	N/A	0.209	0.109	0.000	0.000	0.000	0.516	0.000	0.166	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	49	0	0	0	379	0	129	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	5.41	0.00	1.84	0.00
time (sec)	N/A	0.226	0.002	0.000	0.000	0.000	2.874	0.000	0.195	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	70	49	0	0	0	379	0	129	0
N.S.	1	0.70	0.49	0.00	0.00	0.00	3.79	0.00	1.29	0.00
time (sec)	N/A	0.199	0.099	0.000	0.000	0.000	2.982	0.000	0.174	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	86	0	0	0	379	0	129	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	3.79	0.00	1.29	0.00
time (sec)	N/A	0.221	0.208	0.000	0.000	0.000	2.766	0.000	0.171	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	223	180	0	0	0	0	0	0	0
N.S.	1	0.92	0.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.174	0.000	0.000	0.000	0.000	0.000	0.705	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	266	195	0	0	0	0	0	0	0
N.S.	1	0.98	0.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.202	0.000	0.000	0.000	0.000	0.000	1.231	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	260	205	0	0	0	0	0	29	0
N.S.	1	0.98	0.78	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.440	0.335	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	62	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.446	0.000	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	208	0	0	0	0	0	92	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.418	0.294	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	215	0	0	0	0	0	132	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.402	0.343	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	223	0	0	0	0	0	172	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.406	0.241	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	558	577	508	0	0	0	0	0	212	0
N.S.	1	1.03	0.91	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.947	1.076	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	213	121	0	0	0	1647	0	0	0
N.S.	1	1.22	0.70	0.00	0.00	0.00	9.47	0.00	0.00	0.00
time (sec)	N/A	0.342	0.205	0.000	0.000	0.000	49.657	0.000	0.251	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	0.000	0.000	0.000	0.000	0.000	0.000	1.658	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	368	339	0	0	0	0	0	0	0	0
N.S.	1	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.509	0.000	0.000	0.000	0.000	0.000	0.000	1.723	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [104] had the largest ratio of [.5862070000000000034]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	27	0.074
2	A	2	2	1.00	27	0.074
3	A	2	2	1.00	25	0.080
4	A	2	2	1.00	20	0.100
5	A	5	4	0.85	27	0.148
6	A	5	4	1.18	27	0.148
7	A	5	4	1.15	27	0.148
8	A	5	4	0.90	27	0.148
9	A	6	5	0.79	27	0.185
10	A	2	2	1.00	29	0.069
11	A	2	2	1.00	29	0.069
12	A	2	2	1.00	27	0.074
13	A	2	2	1.00	22	0.091
14	A	7	6	0.99	29	0.207
15	A	7	6	1.18	29	0.207
16	A	7	6	1.24	29	0.207
17	A	7	6	1.21	29	0.207
18	A	7	6	0.92	29	0.207
19	A	9	8	1.08	25	0.320
20	A	7	6	1.03	25	0.240
21	A	5	4	0.89	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	1.02	18	0.222
23	A	6	5	1.06	25	0.200
24	A	6	5	1.10	25	0.200
25	A	8	7	1.14	25	0.280
26	A	9	8	1.08	25	0.320
27	A	7	6	1.03	25	0.240
28	A	5	4	0.89	23	0.174
29	A	5	4	1.02	18	0.222
30	A	7	6	1.06	25	0.240
31	A	7	6	1.09	25	0.240
32	A	9	8	1.15	25	0.320
33	F	0	0	N/A	0.000	N/A
34	F	0	0	N/A	0.000	N/A
35	A	5	4	0.86	27	0.148
36	A	5	4	0.97	22	0.182
37	A	6	5	1.07	29	0.172
38	A	6	5	1.09	29	0.172
39	A	8	7	1.12	29	0.241
40	A	10	9	1.12	29	0.310
41	A	12	11	1.10	29	0.379
42	A	9	8	1.36	29	0.276
43	A	7	6	1.19	29	0.207
44	A	5	4	1.17	27	0.148
45	A	5	4	1.25	22	0.182
46	A	6	5	1.09	29	0.172
47	A	8	7	1.10	29	0.241
48	A	10	9	1.07	29	0.310
49	A	13	12	1.08	29	0.414
50	A	15	14	1.06	29	0.483
51	A	9	8	1.42	29	0.276
52	A	7	6	1.23	29	0.207
53	A	5	4	1.17	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	0.99	22	0.182
55	A	8	7	1.12	29	0.241
56	A	11	10	1.12	29	0.345
57	A	11	10	1.07	29	0.345
58	A	13	12	1.07	29	0.414
59	A	15	14	1.06	29	0.483
60	A	2	2	1.00	27	0.074
61	A	2	2	1.00	27	0.074
62	A	2	2	1.00	25	0.080
63	A	2	2	1.00	20	0.100
64	A	6	5	0.83	27	0.185
65	A	6	5	0.99	27	0.185
66	A	6	5	1.13	27	0.185
67	A	6	5	0.94	27	0.185
68	A	6	5	0.78	27	0.185
69	A	2	2	1.00	29	0.069
70	A	2	2	1.00	29	0.069
71	A	2	2	1.00	27	0.074
72	A	2	2	1.00	22	0.091
73	A	8	7	0.95	29	0.241
74	A	9	8	1.00	29	0.276
75	A	8	7	1.11	29	0.241
76	A	8	7	1.18	29	0.241
77	A	8	7	0.95	29	0.241
78	F	0	0	N/A	0.000	N/A
79	F	0	0	N/A	0.000	N/A
80	A	6	5	0.83	27	0.185
81	A	6	5	0.95	22	0.227
82	A	8	7	1.12	29	0.241
83	A	8	7	1.09	29	0.241
84	A	8	7	1.09	29	0.241
85	A	10	9	1.11	29	0.310

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	12	11	1.09	29	0.379
87	A	10	9	1.19	29	0.310
88	A	8	7	1.01	29	0.241
89	A	6	5	0.98	27	0.185
90	A	6	5	1.12	22	0.227
91	A	8	7	1.08	29	0.241
92	A	9	8	1.10	29	0.276
93	A	12	11	1.08	29	0.379
94	A	12	11	1.07	29	0.379
95	A	15	14	1.06	29	0.483
96	A	10	9	1.27	29	0.310
97	A	8	7	1.11	29	0.241
98	A	6	5	1.14	27	0.185
99	A	6	5	1.09	22	0.227
100	A	8	7	1.10	29	0.241
101	A	10	9	1.08	29	0.310
102	A	12	11	1.07	29	0.379
103	A	15	14	1.05	29	0.483
104	A	18	17	1.05	29	0.586
105	A	2	2	1.00	27	0.074
106	A	2	2	1.00	27	0.074
107	A	2	2	1.00	25	0.080
108	A	2	2	1.00	20	0.100
109	A	4	3	0.92	27	0.111
110	A	4	3	1.15	27	0.111
111	A	4	3	1.05	27	0.111
112	A	5	4	0.90	27	0.148
113	A	6	5	0.79	27	0.185
114	A	2	2	1.00	29	0.069
115	A	2	2	1.00	29	0.069
116	A	2	2	1.00	27	0.074
117	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.05	29	0.172
119	A	7	6	1.24	29	0.207
120	A	7	6	1.22	29	0.207
121	A	7	6	1.17	29	0.207
122	A	8	7	0.93	29	0.241
123	F	0	0	N/A	0.000	N/A
124	F	0	0	N/A	0.000	N/A
125	A	4	3	0.92	27	0.111
126	A	4	3	1.00	22	0.136
127	A	4	3	1.00	29	0.103
128	A	6	5	1.16	29	0.172
129	A	8	7	1.15	29	0.241
130	A	10	9	1.13	29	0.310
131	A	12	11	1.11	29	0.379
132	A	8	7	1.44	29	0.241
133	A	6	5	1.24	29	0.172
134	A	4	3	1.14	27	0.111
135	A	4	3	1.00	22	0.136
136	A	6	5	1.16	29	0.172
137	A	7	6	1.16	29	0.207
138	A	9	8	1.13	29	0.276
139	A	12	11	1.10	29	0.379
140	A	13	12	1.08	29	0.414
141	A	8	7	1.47	29	0.241
142	A	6	5	1.22	29	0.172
143	A	4	3	1.06	27	0.111
144	A	5	4	0.97	22	0.182
145	A	8	7	1.15	29	0.241
146	A	9	8	1.12	29	0.276
147	A	11	10	1.09	29	0.345
148	A	13	12	1.08	29	0.414
149	A	15	14	1.06	29	0.483

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
150	A	2	2	1.00	27	0.074
151	A	2	2	1.00	27	0.074
152	A	2	2	1.00	25	0.080
153	A	2	2	1.00	20	0.100
154	A	4	3	1.11	27	0.111
155	A	4	3	1.27	27	0.111
156	A	5	4	1.08	27	0.148
157	A	6	5	0.92	27	0.185
158	A	7	6	0.82	27	0.222
159	A	2	2	1.00	29	0.069
160	A	2	2	1.00	29	0.069
161	A	2	2	1.00	27	0.074
162	A	2	2	1.00	22	0.091
163	A	6	5	1.29	29	0.172
164	A	6	5	1.51	29	0.172
165	A	6	5	1.52	29	0.172
166	A	7	6	1.17	29	0.207
167	A	8	7	0.94	29	0.241
168	A	8	7	1.49	29	0.241
169	A	6	5	1.29	29	0.172
170	A	4	3	1.11	27	0.111
171	A	4	3	1.00	22	0.136
172	A	6	5	1.22	29	0.172
173	A	8	7	1.20	29	0.241
174	A	10	9	1.15	29	0.310
175	A	12	11	1.12	29	0.379
176	A	14	13	1.10	29	0.448
177	A	8	7	1.78	29	0.241
178	A	6	5	1.45	29	0.172
179	A	4	3	1.25	27	0.111
180	A	5	4	1.11	22	0.182
181	A	8	7	1.20	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	9	8	1.14	29	0.276
183	A	11	10	1.10	29	0.345
184	A	14	13	1.08	29	0.448
185	A	16	15	1.07	29	0.517
186	A	8	7	1.66	29	0.241
187	A	6	5	1.45	29	0.172
188	A	5	4	1.05	27	0.148
189	A	6	5	1.04	22	0.227
190	A	10	9	1.14	29	0.310
191	A	11	10	1.09	29	0.345
192	A	14	13	1.07	29	0.448
193	A	16	15	1.06	29	0.517
194	A	18	17	1.05	29	0.586
195	A	11	10	1.17	31	0.323
196	A	11	10	1.07	31	0.323
197	A	7	6	0.59	29	0.207
198	A	6	5	0.64	29	0.172
199	A	5	4	0.73	29	0.138
200	A	5	4	1.09	29	0.138
201	A	5	4	1.07	29	0.138
202	A	5	4	1.01	29	0.138
203	A	2	2	1.06	29	0.069
204	A	3	3	0.89	29	0.103
205	A	4	4	0.79	29	0.138
206	A	6	6	1.00	33	0.182
207	A	5	5	0.92	31	0.161
208	A	5	5	1.00	32	0.156
209	A	5	5	1.00	32	0.156
210	A	5	5	1.01	33	0.152
211	A	3	3	0.96	45	0.067
212	A	5	5	1.07	39	0.128
213	A	4	4	1.00	38	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	8	7	0.97	29	0.241
215	A	7	6	0.93	29	0.207
216	A	6	5	0.82	29	0.172
217	A	7	6	0.91	29	0.207
218	A	8	7	0.93	29	0.241
219	A	8	7	0.97	29	0.241
220	A	7	6	0.93	29	0.207
221	A	6	5	0.83	29	0.172
222	A	7	6	0.92	29	0.207
223	A	8	7	0.94	29	0.241
224	A	8	7	0.80	29	0.241
225	A	1	1	1.00	30	0.033
226	A	2	2	1.00	26	0.077
227	A	2	2	0.70	35	0.057
228	A	2	2	1.00	34	0.059
229	A	3	3	0.92	21	0.143
230	A	3	3	0.98	25	0.120
231	A	4	4	0.98	29	0.138
232	A	4	4	1.00	33	0.121
233	A	4	4	1.00	34	0.118
234	A	5	5	1.00	34	0.147
235	A	3	3	1.00	34	0.088
236	A	4	4	1.03	34	0.118
237	C	8	7	1.22	35	0.200
238	A	5	4	1.00	38	0.105
239	C	8	7	0.92	38	0.184

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 341

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2(be-af)^3(de-cf)(fg-eh)(e+fx)^{3/2}}{3f^6} - \frac{2(be-af)^2(bde(4fg-5eh)-bcf(3fg-4eh)-af(dfg-2deh+cfh))(e+fx)^{5/2}}{5f^6} - \frac{2(be-af)\left(a^2df^2h+abf(3dfg-8deh+3cfh)-b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)(e+fx)^{7/2}}{7f^6} + \frac{2b(3a^2df^2h+3abf(dfg-4deh+cfh)-b^2(2de(2fg-5eh)-cf(fg-4eh)))(e+fx)^{9/2}}{9f^6} + \frac{2b^2(3adfh+b(dfg-5deh+cfh))(e+fx)^{11/2}}{11f^6} + \frac{2b^3dh(e+fx)^{13/2}}{13f^6}$$

output

```
2/3*(-a*f+b*e)^3*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(3/2)/f^6-2/5*(-a*f+b*e)^2*
(b*d*e*(-5*e*h+4*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x
+e)^(5/2)/f^6-2/7*(-a*f+b*e)*(a^2*d*f^2*h+a*b*f*(3*c*f*h-8*d*e*h+3*d*f*g)-
b^2*(2*d*e*(-5*e*h+3*f*g)-3*c*f*(-2*e*h+f*g)))*(f*x+e)^(7/2)/f^6+2/9*b*(3*
a^2*d*f^2*h+3*a*b*f*(c*f*h-4*d*e*h+d*f*g)-b^2*(2*d*e*(-5*e*h+2*f*g)-c*f*(-
4*e*h+f*g)))*(f*x+e)^(9/2)/f^6+2/11*b^2*(3*a*d*f*h+b*(c*f*h-5*d*e*h+d*f*g)
)*(f*x+e)^(11/2)/f^6+2/13*b^3*d*h*(f*x+e)^(13/2)/f^6
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.42

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2(e+fx)^{3/2}\left(429a^3f^3(7cf(5fg-2eh+3fhx)+d(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx)))+429a^2\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(429*a^3*f^3*(7*c*f*(5*f*g - 2*e*h + 3*f*h*x) + d*(8*e^
2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x))) + 429*a^2*b*f^2*(3*c*f
*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d*(-16*e^3*h +
24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x))) +
 39*a*b^2*f*(11*c*f*(-16*e^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h
*x) + 5*f^3*x^2*(9*g + 7*h*x)) + d*(128*e^4*h + 35*f^4*x^3*(11*g + 9*h*x)
+ 24*e^2*f^2*x*(11*g + 10*h*x) - 16*e^3*f*(11*g + 12*h*x) - 10*e*f^3*x^2*(
33*g + 28*h*x))) + b^3*(13*c*f*(128*e^4*h + 35*f^4*x^3*(11*g + 9*h*x) + 24
*e^2*f^2*x*(11*g + 10*h*x) - 16*e^3*f*(11*g + 12*h*x) - 10*e*f^3*x^2*(33*g
 + 28*h*x)) + d*(-1280*e^5*h + 315*f^5*x^4*(13*g + 11*h*x) + 128*e^4*f*(13
*g + 15*h*x) - 96*e^3*f^2*x*(26*g + 25*h*x) + 80*e^2*f^3*x^2*(39*g + 35*h*
x) - 70*e*f^4*x^3*(52*g + 45*h*x)))))/(45045*f^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$\Big\downarrow \text{159}$$

$$\int \left(\frac{b(e+fx)^{7/2}\left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{f^5}+\frac{(e+fx)^{5/2}(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b(e+fx)^{9/2}\left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{9f^6}-\frac{2(e+fx)^{7/2}(be-af)\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)\right)}{7f^6}+\frac{2b^2(e+fx)^{11/2}(3adfh+b(cfh-5deh+dfg))}{11f^6}-\frac{2(e+fx)^{5/2}(be-af)^2(-af(cfh-2deh+dfg)-bcf(3fg-4eh)+bde(4fg-5eh))}{5f^6}+\frac{2(e+fx)^{3/2}(be-af)^3(de-cf)(fg-eh)}{3f^6}+\frac{2b^3dh(e+fx)^{13/2}}{13f^6}$$

input
```
Int[(a + b*x)^3*(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(b*e - a*f)^3*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^6) - (2*(b*
e - a*f)^2*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(3*f*g - 4*e*h) - a*f*(d*f*g - 2
*d*e*h + c*f*h))*(e + f*x)^(5/2))/(5*f^6) - (2*(b*e - a*f)*(a^2*d*f^2*h +
a*b*f*(3*d*f*g - 8*d*e*h + 3*c*f*h) - b^2*(2*d*e*(3*f*g - 5*e*h) - 3*c*f*(
f*g - 2*e*h)))*(e + f*x)^(7/2))/(7*f^6) + (2*b*(3*a^2*d*f^2*h + 3*a*b*f*(d
*f*g - 4*d*e*h + c*f*h) - b^2*(2*d*e*(2*f*g - 5*e*h) - c*f*(f*g - 4*e*h)))
*(e + f*x)^(9/2))/(9*f^6) + (2*b^2*(3*a*d*f*h + b*(d*f*g - 5*d*e*h + c*f*h
))*(e + f*x)^(11/2))/(11*f^6) + (2*b^3*d*h*(e + f*x)^(13/2))/(13*f^6)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{2dh\,b^3(fx+e)^{\frac{13}{2}}}{13}+\frac{2\left(\left(3(af-be)b^2d+b^3(cf-de)\right)h+b^3d(-eh+fg)\right)(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left(3(af-be)^2bd+3(af-be)b^2(cf-de)\right)h+\left(3(af-\right.\right.}{9}$
default	$\frac{\frac{2dh\,b^3(fx+e)^{\frac{13}{2}}}{13}-\frac{2\left(-\left(3(af-be)b^2d+b^3(cf-de)\right)h+b^3d(eh-fg)\right)(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left(3(af-be)^2bd+3(af-be)b^2(cf-de)\right)h+\left(3(a\right.\right.}{9}$
pseudoelliptic	$-\frac{4\left(\frac{\left(-\frac{5x^3\left(\frac{9dh\,x^2}{13}+\frac{9(ch+dg)x}{11}+cg\right)b^3}{3}-\frac{45\left(\frac{7dh\,x^2}{11}+\frac{7(ch+dg)x}{9}+cg\right)a\,x^2b^2}{7}-9a^2x\left(\frac{5dh\,x^2}{9}+\frac{5(ch+dg)x}{7}+cg\right)b-5a^3\left(\frac{3dh}{7}\right.\right.}{2}\right.}{}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-3465dh\,b^3x^5f^5-12285a\,b^2d\,f^5h\,x^4-4095b^3c\,f^5h\,x^4+3150b^3de\,f^4h\,x^4-4095b^3d\,f^5g\,x^4-15015a^2bd\,f\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-3465dh\,b^3x^5f^5-12285a\,b^2d\,f^5h\,x^4-4095b^3c\,f^5h\,x^4+3150b^3de\,f^4h\,x^4-4095b^3d\,f^5g\,x^4-15015a^2bd\,f\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^6*(1/13*d*h*b^3*(f*x+e)^(13/2)+1/11*((3*(a*f-b*e)*b^2*d+b^3*(c*f-d*e))
*h+b^3*d*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*((3*(a*f-b*e)^2*b*d+3*(a*f-b*e)*b^
2*(c*f-d*e))*h+(3*(a*f-b*e)*b^2*d+b^3*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(9/2)
+1/7*(((a*f-b*e)^3*d+3*(a*f-b*e)^2*b*(c*f-d*e))*h+(3*(a*f-b*e)^2*b*d+3*(a*
f-b*e)*b^2*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*((a*f-b*e)^3*(c*f-d*e)
*h+((a*f-b*e)^3*d+3*(a*f-b*e)^2*b*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(5/2)+1/3
*(a*f-b*e)^3*(c*f-d*e)*(-e*h+f*g)*(f*x+e)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. 2(317) = 634.

Time = 0.08 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.38

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3465*b^3*d*f^6*h*x^6 + 315*(13*b^3*d*f^6*g + (b^3*d*e*f^5 + 13*(b
^3*c + 3*a*b^2*d)*f^6)*h)*x^5 + 35*(13*(b^3*d*e*f^5 + 11*(b^3*c + 3*a*b^2*
d)*f^6)*g - (10*b^3*d*e^2*f^4 - 13*(b^3*c + 3*a*b^2*d)*e*f^5 - 429*(a*b^2*
c + a^2*b*d)*f^6)*h)*x^4 - 5*(13*(8*b^3*d*e^2*f^4 - 11*(b^3*c + 3*a*b^2*d)
*e*f^5 - 297*(a*b^2*c + a^2*b*d)*f^6)*g - (80*b^3*d*e^3*f^3 - 104*(b^3*c +
 3*a*b^2*d)*e^2*f^4 + 429*(a*b^2*c + a^2*b*d)*e*f^5 + 1287*(3*a^2*b*c + a^
3*d)*f^6)*h)*x^3 + 3*(13*(16*b^3*d*e^3*f^3 - 22*(b^3*c + 3*a*b^2*d)*e^2*f^
4 + 99*(a*b^2*c + a^2*b*d)*e*f^5 + 231*(3*a^2*b*c + a^3*d)*f^6)*g - (160*b
^3*d*e^4*f^2 - 3003*a^3*c*f^6 - 208*(b^3*c + 3*a*b^2*d)*e^3*f^3 + 858*(a*b
^2*c + a^2*b*d)*e^2*f^4 - 429*(3*a^2*b*c + a^3*d)*e*f^5)*h)*x^2 + 13*(128*
b^3*d*e^5*f + 1155*a^3*c*e*f^5 - 176*(b^3*c + 3*a*b^2*d)*e^4*f^2 + 792*(a*
b^2*c + a^2*b*d)*e^3*f^3 - 462*(3*a^2*b*c + a^3*d)*e^2*f^4)*g - 2*(640*b^3
*d*e^6 + 3003*a^3*c*e^2*f^4 - 832*(b^3*c + 3*a*b^2*d)*e^5*f + 3432*(a*b^2*
c + a^2*b*d)*e^4*f^2 - 1716*(3*a^2*b*c + a^3*d)*e^3*f^3)*h - (13*(64*b^3*d
*e^4*f^2 - 1155*a^3*c*f^6 - 88*(b^3*c + 3*a*b^2*d)*e^3*f^3 + 396*(a*b^2*c
+ a^2*b*d)*e^2*f^4 - 231*(3*a^2*b*c + a^3*d)*e*f^5)*g - (640*b^3*d*e^5*f +
 3003*a^3*c*e*f^5 - 832*(b^3*c + 3*a*b^2*d)*e^4*f^2 + 3432*(a*b^2*c + a^2*
b*d)*e^3*f^3 - 1716*(3*a^2*b*c + a^3*d)*e^2*f^4)*h)*x)*sqrt(f*x + e)/f^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. 2(355) = 710.

Time = 1.77 (sec) , antiderivative size = 1013, normalized size of antiderivative = 2.97

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**3*d*h*(e + f*x)**(13/2)/(13*f**5) + (e + f*x)**(11/2)*(3*
a*b**2*d*f*h + b**3*c*f*h - 5*b**3*d*e*h + b**3*d*f*g)/(11*f**5) + (e + f*
x)**(9/2)*(3*a**2*b*d*f**2*h + 3*a*b**2*c*f**2*h - 12*a*b**2*d*e*f*h + 3*a
*b**2*d*f**2*g - 4*b**3*c*e*f*h + b**3*c*f**2*g + 10*b**3*d*e**2*h - 4*b**
3*d*e*f*g)/(9*f**5) + (e + f*x)**(7/2)*(a**3*d*f**3*h + 3*a**2*b*c*f**3*h
- 9*a**2*b*d*e*f**2*h + 3*a**2*b*d*f**3*g - 9*a*b**2*c*e*f**2*h + 3*a*b**2
*c*f**3*g + 18*a*b**2*d*e**2*f*h - 9*a*b**2*d*e*f**2*g + 6*b**3*c*e**2*f*h
 - 3*b**3*c*e*f**2*g - 10*b**3*d*e**3*h + 6*b**3*d*e**2*f*g)/(7*f**5) + (e
 + f*x)**(5/2)*(a**3*c*f**4*h - 2*a**3*d*e*f**3*h + a**3*d*f**4*g - 6*a**2
*b*c*e*f**3*h + 3*a**2*b*c*f**4*g + 9*a**2*b*d*e**2*f**2*h - 6*a**2*b*d*e*
f**3*g + 9*a*b**2*c*e**2*f**2*h - 6*a*b**2*c*e*f**3*g - 12*a*b**2*d*e**3*f
*h + 9*a*b**2*d*e**2*f**2*g - 4*b**3*c*e**3*f*h + 3*b**3*c*e**2*f**2*g + 5
*b**3*d*e**4*h - 4*b**3*d*e**3*f*g)/(5*f**5) + (e + f*x)**(3/2)*(-a**3*c*e
*f**4*h + a**3*c*f**5*g + a**3*d*e**2*f**3*h - a**3*d*e*f**4*g + 3*a**2*b*
c*e**2*f**3*h - 3*a**2*b*c*e*f**4*g - 3*a**2*b*d*e**3*f**2*h + 3*a**2*b*d*
e**2*f**3*g - 3*a*b**2*c*e**3*f**2*h + 3*a*b**2*c*e**2*f**3*g + 3*a*b**2*d
*e**4*f*h - 3*a*b**2*d*e**3*f**2*g + b**3*c*e**4*f*h - b**3*c*e**3*f**2*g
- b**3*d*e**5*h + b**3*d*e**4*f*g)/(3*f**5))/f, Ne(f, 0)), (sqrt(e)*(a**3*
c*g*x + b**3*d*h*x**6/6 + x**5*(3*a*b**2*d*h + b**3*c*h + b**3*d*g)/5 + x*
*4*(3*a**2*b*d*h + 3*a*b**2*c*h + 3*a*b**2*d*g + b**3*c*g)/4 + x**3*(a*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.80

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\left(3465\,(fx+e)^{\frac{13}{2}}b^3dh + 4095\,(b^3dfg-(5\,b^3de-(b^3c+3\,ab^2d)f)h)(fx+e)^{\frac{11}{2}} - 5005\,((4\,b^3def-(b^3c\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3465*(f*x + e)^(13/2)*b^3*d*h + 4095*(b^3*d*f*g - (5*b^3*d*e - (b
^3*c + 3*a*b^2*d)*f)*h)*(f*x + e)^(11/2) - 5005*((4*b^3*d*e*f - (b^3*c + 3
*a*b^2*d)*f^2)*g - (10*b^3*d*e^2 - 4*(b^3*c + 3*a*b^2*d)*e*f + 3*(a*b^2*c
+ a^2*b*d)*f^2)*h)*(f*x + e)^(9/2) + 6435*(3*(2*b^3*d*e^2*f - (b^3*c + 3*a
*b^2*d)*e*f^2 + (a*b^2*c + a^2*b*d)*f^3)*g - (10*b^3*d*e^3 - 6*(b^3*c + 3*
a*b^2*d)*e^2*f + 9*(a*b^2*c + a^2*b*d)*e*f^2 - (3*a^2*b*c + a^3*d)*f^3)*h)
*(f*x + e)^(7/2) - 9009*((4*b^3*d*e^3*f - 3*(b^3*c + 3*a*b^2*d)*e^2*f^2 +
6*(a*b^2*c + a^2*b*d)*e*f^3 - (3*a^2*b*c + a^3*d)*f^4)*g - (5*b^3*d*e^4 +
a^3*c*f^4 - 4*(b^3*c + 3*a*b^2*d)*e^3*f + 9*(a*b^2*c + a^2*b*d)*e^2*f^2 -
2*(3*a^2*b*c + a^3*d)*e*f^3)*h)*(f*x + e)^(5/2) + 15015*((b^3*d*e^4*f + a^
3*c*f^5 - (b^3*c + 3*a*b^2*d)*e^3*f^2 + 3*(a*b^2*c + a^2*b*d)*e^2*f^3 - (3
*a^2*b*c + a^3*d)*e*f^4)*g - (b^3*d*e^5 + a^3*c*e*f^4 - (b^3*c + 3*a*b^2*d
)*e^4*f + 3*(a*b^2*c + a^2*b*d)*e^3*f^2 - (3*a^2*b*c + a^3*d)*e^2*f^3)*h)*
(f*x + e)^(3/2))/f^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(317) = 634$.

Time = 0.14 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.13

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a^3*c*e*g + 15015*((f*x + e)^(3/2) - 3*sqrt(f
*x + e)*e)*a^3*c*g + 45045*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*b*c*e
*g/f + 15015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*d*e*g/f + 15015*((f
*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c*e*h/f + 9009*(3*(f*x + e)^(5/2) -
 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b^2*c*e*g/f^2 + 9009*(3*(f
*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*d*e*g/f
^2 + 9009*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2
)*a^2*b*c*g/f + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f
*x + e)*e^2)*a^3*d*g/f + 9009*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e +
15*sqrt(f*x + e)*e^2)*a^2*b*c*e*h/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x
+ e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*d*e*h/f^2 + 3003*(3*(f*x + e)^(5/
2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*c*h/f + 1287*(5*(f*x
 + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x
+ e)*e^3)*b^3*c*e*g/f^3 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e +
 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b^2*d*e*g/f^3 + 3861*(5*
(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(
f*x + e)*e^3)*a*b^2*c*g/f^2 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)
*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a^2*b*d*g/f^2 + 3861*(
5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqr
t(f*x + e)*e^3)*a*b^2*c*e*h/f^3 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e...
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.07

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{(e+f\,x)^{9/2}\,(2\,b^3\,c\,f^2\,g+20\,b^3\,d\,e^2\,h+6\,a\,b^2\,c\,f^2\,h+6\,a\,b^2\,d\,f^2\,g+6\,a^2\,b\,d\,f^2\,h-8\,b^3\,c\,e\,f\,h-8\,b^3\,d\,e\,f\,g}{9\,f^6}$$
$$+\frac{(e+f\,x)^{11/2}\,(2\,b^3\,c\,f\,h-10\,b^3\,d\,e\,h+2\,b^3\,d\,f\,g+6\,a\,b^2\,d\,f\,h)}{11\,f^6}$$
$$+\frac{2\,(e+f\,x)^{7/2}\,(a\,f-b\,e)\,(3\,b^2\,c\,f^2\,g+a^2\,d\,f^2\,h+10\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-}{7\,f^6}$$
$$+\frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)^2\,(a\,c\,f^2\,h+a\,d\,f^2\,g+3\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f}{5\,f^6}$$
$$+\frac{2\,b^3\,d\,h\,(e+f\,x)^{13/2}}{13\,f^6}-\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)^3\,(c\,f-d\,e)\,(e\,h-f\,g)}{3\,f^6}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^3*(c + d*x),x)
```

output
```
((e + f*x)^(9/2)*(2*b^3*c*f^2*g + 20*b^3*d*e^2*h + 6*a*b^2*c*f^2*h + 6*a*b
^2*d*f^2*g + 6*a^2*b*d*f^2*h - 8*b^3*c*e*f*h - 8*b^3*d*e*f*g - 24*a*b^2*d*
e*f*h))/(9*f^6) + ((e + f*x)^(11/2)*(2*b^3*c*f*h - 10*b^3*d*e*h + 2*b^3*d*
f*g + 6*a*b^2*d*f*h))/(11*f^6) + (2*(e + f*x)^(7/2)*(a*f - b*e)*(3*b^2*c*f
^2*g + a^2*d*f^2*h + 10*b^2*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 6*b^
2*c*e*f*h - 6*b^2*d*e*f*g - 8*a*b*d*e*f*h))/(7*f^6) + (2*(e + f*x)^(5/2)*(
a*f - b*e)^2*(a*c*f^2*h + a*d*f^2*g + 3*b*c*f^2*g + 5*b*d*e^2*h - 2*a*d*e*
f*h - 4*b*c*e*f*h - 4*b*d*e*f*g))/(5*f^6) + (2*b^3*d*h*(e + f*x)^(13/2))/(
13*f^6) - (2*(e + f*x)^(3/2)*(a*f - b*e)^3*(c*f - d*e)*(e*h - f*g))/(3*f^6
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1029, normalized size of antiderivative = 3.02

$$\int (a+bx)^3(c+dx)\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 6006*a**3*c*e**2*f**4*h + 15015*a**3*c*e*f**5*g + 300
3*a**3*c*e*f**5*h*x + 15015*a**3*c*f**6*g*x + 9009*a**3*c*f**6*h*x**2 + 34
32*a**3*d*e**3*f**3*h - 6006*a**3*d*e**2*f**4*g - 1716*a**3*d*e**2*f**4*h*
x + 3003*a**3*d*e*f**5*g*x + 1287*a**3*d*e*f**5*h*x**2 + 9009*a**3*d*f**6*
g*x**2 + 6435*a**3*d*f**6*h*x**3 + 10296*a**2*b*c*e**3*f**3*h - 18018*a**2
*b*c*e**2*f**4*g - 5148*a**2*b*c*e**2*f**4*h*x + 9009*a**2*b*c*e*f**5*g*x
+ 3861*a**2*b*c*e*f**5*h*x**2 + 27027*a**2*b*c*f**6*g*x**2 + 19305*a**2*b*
c*f**6*h*x**3 - 6864*a**2*b*d*e**4*f**2*h + 10296*a**2*b*d*e**3*f**3*g + 3
432*a**2*b*d*e**3*f**3*h*x - 5148*a**2*b*d*e**2*f**4*g*x - 2574*a**2*b*d*e
**2*f**4*h*x**2 + 3861*a**2*b*d*e*f**5*g*x**2 + 2145*a**2*b*d*e*f**5*h*x**
3 + 19305*a**2*b*d*f**6*g*x**3 + 15015*a**2*b*d*f**6*h*x**4 - 6864*a*b**2*
c*e**4*f**2*h + 10296*a*b**2*c*e**3*f**3*g + 3432*a*b**2*c*e**3*f**3*h*x -
 5148*a*b**2*c*e**2*f**4*g*x - 2574*a*b**2*c*e**2*f**4*h*x**2 + 3861*a*b**
2*c*e*f**5*g*x**2 + 2145*a*b**2*c*e*f**5*h*x**3 + 19305*a*b**2*c*f**6*g*x*
*3 + 15015*a*b**2*c*f**6*h*x**4 + 4992*a*b**2*d*e**5*f*h - 6864*a*b**2*d*e
**4*f**2*g - 2496*a*b**2*d*e**4*f**2*h*x + 3432*a*b**2*d*e**3*f**3*g*x + 1
872*a*b**2*d*e**3*f**3*h*x**2 - 2574*a*b**2*d*e**2*f**4*g*x**2 - 1560*a*b*
*2*d*e**2*f**4*h*x**3 + 2145*a*b**2*d*e*f**5*g*x**3 + 1365*a*b**2*d*e*f**5
*h*x**4 + 15015*a*b**2*d*f**6*g*x**4 + 12285*a*b**2*d*f**6*h*x**5 + 1664*b
**3*c*e**5*f*h - 2288*b**3*c*e**4*f**2*g - 832*b**3*c*e**4*f**2*h*x + 1...
```

3.2 $\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$\begin{aligned}\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx &= -\frac{2(be-af)^2(de-cf)(fg-eh)(e+fx)^{3/2}}{3f^5}\\ &+\frac{2(be-af)(bde(3fg-4eh)-bcf(2fg-3eh)-af(dfg-2deh+cfh))(e+fx)^{5/2}}{5f^5}\\ &+\frac{2(a^2df^2h+2abf(dfg-3deh+cfh)+b^2(cf(fg-3eh)-3de(fg-2eh)))\,(e+fx)^{7/2}}{7f^5}\\ &+\frac{2b(2adfh+b(dfg-4deh+cfh))(e+fx)^{9/2}}{9f^5}+\frac{2b^2dh(e+fx)^{11/2}}{11f^5}\end{aligned}$$

output
```
-2/3*(-a*f+b*e)^2*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(3/2)/f^5+2/5*(-a*f+b*e)*(
b*d*e*(-4*e*h+3*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+
e)^(5/2)/f^5+2/7*(a^2*d*f^2*h+2*a*b*f*(c*f*h-3*d*e*h+d*f*g)+b^2*(c*f*(-3*e
*h+f*g)-3*d*e*(-2*e*h+f*g)))*(f*x+e)^(7/2)/f^5+2/9*b*(2*a*d*f*h+b*(c*f*h-4
*d*e*h+d*f*g))*(f*x+e)^(9/2)/f^5+2/11*b^2*d*h*(f*x+e)^(11/2)/f^5
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.23

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2(e+fx)^{3/2}\left(33a^2f^2(7cf(5fg-2eh+3fhx)+d(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx)))+22abf(\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(33*a^2*f^2*(7*c*f*(5*f*g - 2*e*h + 3*f*h*x) + d*(8*e^2
*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x))) + 22*a*b*f*(3*c*f*(8*e^
2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d*(-16*e^3*h + 24*e^2
*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x))) + b^2*(
11*c*f*(-16*e^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x
^2*(9*g + 7*h*x)) + d*(128*e^4*h + 35*f^4*x^3*(11*g + 9*h*x) + 24*e^2*f^2*
x*(11*g + 10*h*x) - 16*e^3*f*(11*g + 12*h*x) - 10*e*f^3*x^2*(33*g + 28*h*x
)))))/(3465*f^5)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$\Big\downarrow \; 159$$
$$\int \left(\frac{(e+fx)^{5/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{f^4}+\frac{b(e+fx)^{7/2}(2adf\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{2(e+fx)^{7/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{7f^5}+\frac{2b(e+fx)^{9/2}(2adfh+b(cfh-4deh+dfg))}{9f^5}+\frac{2(e+fx)^{5/2}(be-af)(-af(cfh-2deh+dfg)-bcf(2fg-3eh)+bde(3fg-4eh))}{5f^5}-\frac{2(e+fx)^{3/2}(be-af)^2(de-cf)(fg-eh)}{3f^5}+\frac{2b^2dh(e+fx)^{11/2}}{11f^5}$$

input
```
Int[(a + b*x)^2*(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(-2*(b*e - a*f)^2*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^5) + (2*(b
*e - a*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(2*f*g - 3*e*h) - a*f*(d*f*g - 2*
d*e*h + c*f*h))*(e + f*x)^(5/2))/(5*f^5) + (2*(a^2*d*f^2*h + 2*a*b*f*(d*f*
g - 3*d*e*h + c*f*h) + b^2*(c*f*(f*g - 3*e*h) - 3*d*e*(f*g - 2*e*h)))*(e +
 f*x)^(7/2))/(7*f^5) + (2*b*(2*a*d*f*h + b*(d*f*g - 4*d*e*h + c*f*h))*(e +
 f*x)^(9/2))/(9*f^5) + (2*b^2*d*h*(e + f*x)^(11/2))/(11*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{2dh\,b^2(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left(2b(af-be)d+b^2(cf-de)\right)h+d\,b^2(-eh+fg)\right)(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(\left((af-be)^2d+2b(af-be)(cf-de)\right)h+\left(2b(af-be)\right.\right.}{7}}{}$
default	$\frac{\frac{2dh\,b^2(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left(2b(af-be)d+b^2(cf-de)\right)h+d\,b^2(eh-fg)\right)(fx+e)^{\frac{9}{2}}}{9}-\frac{2\left(-\left((af-be)^2d+2b(af-be)(cf-de)\right)h+\left(2b(af-be\right.\right.}{7}}{}$
pseudoelliptic	$-\frac{4(fx+e)^{\frac{3}{2}}\left(\left(\left(-\frac{15x^2\left(\frac{7dh\,x^2}{11}+\frac{7(ch+dg)x}{9}+cg\right)b^2}{14}-3ax\left(\frac{5dh\,x^2}{9}+\frac{5(ch+dg)x}{7}+cg\right)b-\frac{5a^2\left(\frac{3dh\,x^2}{7}+\frac{3(ch+dg)x}{5}+cg\right)}{2}\right)f\right.\right.}{}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-315dh\,b^2x^4f^4-770abd\,f^4h\,x^3-385b^2c\,f^4h\,x^3+280b^2de\,f^3h\,x^3-385b^2d\,f^4g\,x^3-495a^2d\,f^4h\,x^2-990a\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-315dh\,b^2x^4f^4-770abd\,f^4h\,x^3-385b^2c\,f^4h\,x^3+280b^2de\,f^3h\,x^3-385b^2d\,f^4g\,x^3-495a^2d\,f^4h\,x^2-990a\right.}{}$
trager	$-\frac{2\left(-315dh\,b^2f^5x^5-770abd\,f^5h\,x^4-385b^2c\,f^5h\,x^4-35b^2de\,f^4h\,x^4-385b^2d\,f^5g\,x^4-495a^2d\,f^5h\,x^3-990abc\,f^5h\,x^3-\right.}{}$
risch	$-\frac{2\left(-315dh\,b^2f^5x^5-770abd\,f^5h\,x^4-385b^2c\,f^5h\,x^4-35b^2de\,f^4h\,x^4-385b^2d\,f^5g\,x^4-495a^2d\,f^5h\,x^3-990abc\,f^5h\,x^3-\right.}{}$

input
```
int((b*x+a)^2*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^5*(1/11*d*h*b^2*(f*x+e)^(11/2)+1/9*((2*b*(a*f-b*e)*d+b^2*(c*f-d*e))*h+
d*b^2*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*(((a*f-b*e)^2*d+2*b*(a*f-b*e)*(c*f-d*e
))*h+(2*b*(a*f-b*e)*d+b^2*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*((a*f-b
*e)^2*(c*f-d*e)*h+((a*f-b*e)^2*d+2*b*(a*f-b*e)*(c*f-d*e))*(-e*h+f*g))*(f*x
+e)^(5/2)+1/3*(a*f-b*e)^2*(c*f-d*e)*(-e*h+f*g)*(f*x+e)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(229) = 458$.

Time = 0.07 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.10

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$=\frac{2\left(315\,b^2df^5hx^5+35\left(11\,b^2df^5g+\left(b^2def^4+11\left(b^2c+2\,abd\right)f^5\right)h\right)x^4+5\left(11\left(b^2def^4+9\left(b^2c+2\,abd\right)f^5\right)\right.\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/3465*(315*b^2*d*f^5*h*x^5 + 35*(11*b^2*d*f^5*g + (b^2*d*e*f^4 + 11*(b^2*
c + 2*a*b*d)*f^5)*h)*x^4 + 5*(11*(b^2*d*e*f^4 + 9*(b^2*c + 2*a*b*d)*f^5)*g
 - (8*b^2*d*e^2*f^3 - 11*(b^2*c + 2*a*b*d)*e*f^4 - 99*(2*a*b*c + a^2*d)*f^
5)*h)*x^3 - 3*(11*(2*b^2*d*e^2*f^3 - 3*(b^2*c + 2*a*b*d)*e*f^4 - 21*(2*a*b
*c + a^2*d)*f^5)*g - (16*b^2*d*e^3*f^2 + 231*a^2*c*f^5 - 22*(b^2*c + 2*a*b
*d)*e^2*f^3 + 33*(2*a*b*c + a^2*d)*e*f^4)*h)*x^2 - 11*(16*b^2*d*e^4*f - 10
5*a^2*c*e*f^4 - 24*(b^2*c + 2*a*b*d)*e^3*f^2 + 42*(2*a*b*c + a^2*d)*e^2*f^
3)*g + 2*(64*b^2*d*e^5 - 231*a^2*c*e^2*f^3 - 88*(b^2*c + 2*a*b*d)*e^4*f +
132*(2*a*b*c + a^2*d)*e^3*f^2)*h + (11*(8*b^2*d*e^3*f^2 + 105*a^2*c*f^5 -
12*(b^2*c + 2*a*b*d)*e^2*f^3 + 21*(2*a*b*c + a^2*d)*e*f^4)*g - (64*b^2*d*e
^4*f - 231*a^2*c*e*f^4 - 88*(b^2*c + 2*a*b*d)*e^3*f^2 + 132*(2*a*b*c + a^2
*d)*e^2*f^3)*h)*x)*sqrt(f*x + e)/f^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(262) = 524$.

Time = 1.52 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.51

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$$

$$= \begin{cases} \frac{2\left(\frac{b^2dh(e+fx)^{\frac{11}{2}}}{11f^4} + \frac{(e+fx)^{\frac{9}{2}}\cdot\left(2abdfh+b^2cfh-4b^2deh+b^2dfg\right)}{9f^4} + \frac{(e+fx)^{\frac{7}{2}}\left(a^2df^2h+2abcf^2h-6abdefh+2abdf^2g-3b^2cefh+b^2cf^2g+6b^2de^2h-3b^2defg\right)}{7f^4}\right.}{} \\ \sqrt{e}\left(a^2cgx + \frac{b^2dhx^5}{5} + \frac{x^4\cdot(2abdh+b^2ch+b^2dg)}{4} + \frac{x^3\left(a^2dh+2abch+2abdg+b^2cg\right)}{3} + \frac{x^2\left(a^2ch+a^2dg+2abcg\right)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**2*d*h*(e + f*x)**(11/2)/(11*f**4) + (e + f*x)**(9/2)*(2*a
*b*d*f*h + b**2*c*f*h - 4*b**2*d*e*h + b**2*d*f*g)/(9*f**4) + (e + f*x)**(
7/2)*(a**2*d*f**2*h + 2*a*b*c*f**2*h - 6*a*b*d*e*f*h + 2*a*b*d*f**2*g - 3*
b**2*c*e*f*h + b**2*c*f**2*g + 6*b**2*d*e**2*h - 3*b**2*d*e*f*g)/(7*f**4)
+ (e + f*x)**(5/2)*(a**2*c*f**3*h - 2*a**2*d*e*f**2*h + a**2*d*f**3*g - 4*
a*b*c*e*f**2*h + 2*a*b*c*f**3*g + 6*a*b*d*e**2*f*h - 4*a*b*d*e*f**2*g + 3*
b**2*c*e**2*f*h - 2*b**2*c*e*f**2*g - 4*b**2*d*e**3*h + 3*b**2*d*e**2*f*g)
/(5*f**4) + (e + f*x)**(3/2)*(-a**2*c*e*f**3*h + a**2*c*f**4*g + a**2*d*e*
*2*f**2*h - a**2*d*e*f**3*g + 2*a*b*c*e**2*f**2*h - 2*a*b*c*e*f**3*g - 2*a
*b*d*e**3*f*h + 2*a*b*d*e**2*f**2*g - b**2*c*e**3*f*h + b**2*c*e**2*f**2*g
 + b**2*d*e**4*h - b**2*d*e**3*f*g)/(3*f**4))/f, Ne(f, 0)), (sqrt(e)*(a**2
*c*g*x + b**2*d*h*x**5/5 + x**4*(2*a*b*d*h + b**2*c*h + b**2*d*g)/4 + x**3
*(a**2*d*h + 2*a*b*c*h + 2*a*b*d*g + b**2*c*g)/3 + x**2*(a**2*c*h + a**2*d
*g + 2*a*b*c*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.52

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2\left(315\,(fx+e)^{\frac{11}{2}}b^2dh+385\,(b^2dfg-(4\,b^2de-(b^2c+2\,abd)f)h)(fx+e)^{\frac{9}{2}}-495\,((3\,b^2def-(b^2c+2\,a$$

input
```
integrate((b*x+a)^2*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/3465*(315*(f*x + e)^(11/2)*b^2*d*h + 385*(b^2*d*f*g - (4*b^2*d*e - (b^2*
c + 2*a*b*d)*f)*h)*(f*x + e)^(9/2) - 495*((3*b^2*d*e*f - (b^2*c + 2*a*b*d)
*f^2)*g - (6*b^2*d*e^2 - 3*(b^2*c + 2*a*b*d)*e*f + (2*a*b*c + a^2*d)*f^2)*
h)*(f*x + e)^(7/2) + 693*((3*b^2*d*e^2*f - 2*(b^2*c + 2*a*b*d)*e*f^2 + (2*
a*b*c + a^2*d)*f^3)*g - (4*b^2*d*e^3 - a^2*c*f^3 - 3*(b^2*c + 2*a*b*d)*e^2
*f + 2*(2*a*b*c + a^2*d)*e*f^2)*h)*(f*x + e)^(5/2) - 1155*((b^2*d*e^3*f -
a^2*c*f^4 - (b^2*c + 2*a*b*d)*e^2*f^2 + (2*a*b*c + a^2*d)*e*f^3)*g - (b^2*
d*e^4 - a^2*c*e*f^3 - (b^2*c + 2*a*b*d)*e^3*f + (2*a*b*c + a^2*d)*e^2*f^2)
*h)*(f*x + e)^(3/2))/f^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. $2(229) = 458$.

Time = 0.13 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.64

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(f*x + e)*a^2*c*e*g + 1155*((f*x + e)^(3/2) - 3*sqrt(f*x
+ e)*e)*a^2*c*g + 2310*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*b*c*e*g/f +
 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*d*e*g/f + 1155*((f*x + e)^
(3/2) - 3*sqrt(f*x + e)*e)*a^2*c*e*h/f + 231*(3*(f*x + e)^(5/2) - 10*(f*x
+ e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b^2*c*e*g/f^2 + 462*(3*(f*x + e)^(5/2
) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*d*e*g/f^2 + 462*(3*(f
*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*c*g/f + 2
31*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*d
*g/f + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^
2)*a*b*c*e*h/f^2 + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt
(f*x + e)*e^2)*a^2*d*e*h/f^2 + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)
*e + 15*sqrt(f*x + e)*e^2)*a^2*c*h/f + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e
)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*d*e*g/f^3 +
 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 3
5*sqrt(f*x + e)*e^3)*b^2*c*g/f^2 + 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(
5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b*d*g/f^2 + 99*(
5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqr
t(f*x + e)*e^3)*b^2*c*e*h/f^3 + 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2
)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b*d*e*h/f^3 + 198*(
5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{(e+f\,x)^{9/2}\,(2\,b^2\,c\,f\,h-8\,b^2\,d\,e\,h+2\,b^2\,d\,f\,g+4\,a\,b\,d\,f\,h)}{9\,f^5} + \frac{(e+f\,x)^{7/2}\,(2\,b^2\,c\,f^2\,g+2\,a^2\,d\,f^2\,h+12\,b^2\,d\,e^2\,h+4\,a\,b\,c\,f^2\,h+4\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b^2\,d\,e\,f\,g}{7\,f^5} + \frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)\,(a\,c\,f^2\,h+a\,d\,f^2\,g+2\,b\,c\,f^2\,g+4\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\,h-3\,b\,d\,e\,f}{5\,f^5} + \frac{2\,b^2\,d\,h\,(e+f\,x)^{11/2}}{11\,f^5} - \frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)\,(e\,h-f\,g)}{3\,f^5}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^2*(c + d*x),x)
```

output
```
((e + f*x)^(9/2)*(2*b^2*c*f*h - 8*b^2*d*e*h + 2*b^2*d*f*g + 4*a*b*d*f*h))/
(9*f^5) + ((e + f*x)^(7/2)*(2*b^2*c*f^2*g + 2*a^2*d*f^2*h + 12*b^2*d*e^2*h
 + 4*a*b*c*f^2*h + 4*a*b*d*f^2*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 12*a*b*
d*e*f*h))/(7*f^5) + (2*(e + f*x)^(5/2)*(a*f - b*e)*(a*c*f^2*h + a*d*f^2*g
+ 2*b*c*f^2*g + 4*b*d*e^2*h - 2*a*d*e*f*h - 3*b*c*e*f*h - 3*b*d*e*f*g))/(5
*f^5) + (2*b^2*d*h*(e + f*x)^(11/2))/(11*f^5) - (2*(e + f*x)^(3/2)*(a*f -
b*e)^2*(c*f - d*e)*(e*h - f*g))/(3*f^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.56

$$\int (a+bx)^2(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\sqrt{fx+e}\,(315b^2d\,f^5h\,x^5+770abd\,f^5h\,x^4+385b^2c\,f^5h\,x^4+35b^2de\,f^4h\,x^4+385b^2d\,f^5g\,x^4+495a^2d\,f^5}{}$$

input
```
int((b*x+a)^2*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 462*a**2*c*e**2*f**3*h + 1155*a**2*c*e*f**4*g + 231*a
**2*c*e*f**4*h*x + 1155*a**2*c*f**5*g*x + 693*a**2*c*f**5*h*x**2 + 264*a**
2*d*e**3*f**2*h - 462*a**2*d*e**2*f**3*g - 132*a**2*d*e**2*f**3*h*x + 231*
a**2*d*e*f**4*g*x + 99*a**2*d*e*f**4*h*x**2 + 693*a**2*d*f**5*g*x**2 + 495
*a**2*d*f**5*h*x**3 + 528*a*b*c*e**3*f**2*h - 924*a*b*c*e**2*f**3*g - 264*
a*b*c*e**2*f**3*h*x + 462*a*b*c*e*f**4*g*x + 198*a*b*c*e*f**4*h*x**2 + 138
6*a*b*c*f**5*g*x**2 + 990*a*b*c*f**5*h*x**3 - 352*a*b*d*e**4*f*h + 528*a*b
*d*e**3*f**2*g + 176*a*b*d*e**3*f**2*h*x - 264*a*b*d*e**2*f**3*g*x - 132*a
*b*d*e**2*f**3*h*x**2 + 198*a*b*d*e*f**4*g*x**2 + 110*a*b*d*e*f**4*h*x**3
+ 990*a*b*d*f**5*g*x**3 + 770*a*b*d*f**5*h*x**4 - 176*b**2*c*e**4*f*h + 26
4*b**2*c*e**3*f**2*g + 88*b**2*c*e**3*f**2*h*x - 132*b**2*c*e**2*f**3*g*x
- 66*b**2*c*e**2*f**3*h*x**2 + 99*b**2*c*e*f**4*g*x**2 + 55*b**2*c*e*f**4*
h*x**3 + 495*b**2*c*f**5*g*x**3 + 385*b**2*c*f**5*h*x**4 + 128*b**2*d*e**5
*h - 176*b**2*d*e**4*f*g - 64*b**2*d*e**4*f*h*x + 88*b**2*d*e**3*f**2*g*x
+ 48*b**2*d*e**3*f**2*h*x**2 - 66*b**2*d*e**2*f**3*g*x**2 - 40*b**2*d*e**2
*f**3*h*x**3 + 55*b**2*d*e*f**4*g*x**3 + 35*b**2*d*e*f**4*h*x**4 + 385*b**
2*d*f**5*g*x**4 + 315*b**2*d*f**5*h*x**5))/(3465*f**5)
```

3.3 $\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2(be-af)(de-cf)(fg-eh)(e+fx)^{3/2}}{3f^4} - \frac{2(bde(2fg-3eh)-bcf(fg-2eh)-af(dfg-2deh+cfh))(e+fx)^{5/2}}{5f^4} + \frac{2(adfh+b(dfg-3deh+cfh))(e+fx)^{7/2}}{7f^4} + \frac{2bdh(e+fx)^{9/2}}{9f^4}$$

output
```
2/3*(-a*f+b*e)*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(3/2)/f^4-2/5*(b*d*e*(-3*e*h+
2*f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(5/2)/f^4+2/7
*(a*d*f*h+b*(c*f*h-3*d*e*h+d*f*g))*(f*x+e)^(7/2)/f^4+2/9*b*d*h*(f*x+e)^(9/
2)/f^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2(e+fx)^{3/2}\left(3af(7cf(5fg-2eh+3fhx)+d(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx)))+b(3cf(8e^2h\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(3*a*f*(7*c*f*(5*f*g - 2*e*h + 3*f*h*x) + d*(8*e^2*h +
3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x))) + b*(3*c*f*(8*e^2*h + 3*f^2*
x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d*(-16*e^3*h + 24*e^2*f*(g + h*x)
 - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x)))))/(315*f^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$\Big\downarrow \text{ 159}$$
$$\int \left(\frac{(e+fx)^{5/2}(adfh+b(cfh-3deh+dfg))}{f^3}+\frac{(e+fx)^{3/2}(af(cfh-2deh+dfg)+bcf(fg-2eh)-bde(2fg\right.}{f^3}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2(e+fx)^{7/2}(adfh+b(cfh-3deh+dfg))}{7f^4} - \frac{2(e+fx)^{5/2}(-af(cfh-2deh+dfg)-bcf(fg-2eh)+bde(2fg-3eh))}{5f^4} + \frac{2(e+fx)^{3/2}(be-af)(de-cf)(fg-eh)}{3f^4} + \frac{2bdh(e+fx)^{9/2}}{9f^4}$$

input
```
Int[(a + b*x)*(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^4) - (2*(b*d*
e*(2*f*g - 3*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(d*f*g - 2*d*e*h + c*f*h))*(
e + f*x)^(5/2))/(5*f^4) + (2*(a*d*f*h + b*(d*f*g - 3*d*e*h + c*f*h))*(e +
f*x)^(7/2))/(7*f^4) + (2*b*d*h*(e + f*x)^(9/2))/(9*f^4)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{4\left(\frac{\left(-3x\left(\frac{5x\left(\frac{7hx}{9}+g\right)d}{7}+c\left(\frac{5hx}{7}+g\right)\right)b-5a\left(\frac{3x\left(\frac{5hx}{7}+g\right)d}{5}+c\left(\frac{3hx}{5}+g\right)\right)\right)f^{3}}{2}+\left(\left(\left(\frac{6x\left(\frac{5hx}{6}+g\right)d}{7}+c\left(\frac{6hx}{7}+g\right)\right)b+a\left(\left(\left(6\right.\right.\right.\right.\right.}{15f^{4}}$
derivativedivides	$\frac{\frac{2dbh(fx+e)^{\frac{9}{2}}}{9}+\frac{2(((af-be)d+b(cf-de))h+db(-eh+fg))(fx+e)^{\frac{7}{2}}}{7}+\frac{2((af-be)(cf-de)h+((af-be)d+b(cf-de))(-eh+fg))(fx+e}{5}}{f^{4}}$
default	$\frac{\frac{2dbh(fx+e)^{\frac{9}{2}}}{9}+\frac{2(-((-af+be)d-b(cf-de))h-db(eh-fg))(fx+e)^{\frac{7}{2}}}{7}+\frac{2(-(-af+be)(cf-de)h+((-af+be)d-b(cf-de))(eh-fg))}{5}}{f^{4}}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-35bdh\,x^{3}f^{3}-45ad\,f^{3}h\,x^{2}-45bc\,f^{3}h\,x^{2}+30bde\,f^{2}h\,x^{2}-45bd\,f^{3}g\,x^{2}-63ac\,f^{3}hx+36ade\,f^{2}hx-63ad\,f^{3}\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-35bdh\,x^{3}f^{3}-45ad\,f^{3}h\,x^{2}-45bc\,f^{3}h\,x^{2}+30bde\,f^{2}h\,x^{2}-45bd\,f^{3}g\,x^{2}-63ac\,f^{3}hx+36ade\,f^{2}hx-63ad\,f^{3}\right.}{}$
trager	$-\frac{2\left(-35dbh\,f^{4}x^{4}-45ad\,f^{4}h\,x^{3}-45bc\,f^{4}h\,x^{3}-5bde\,f^{3}h\,x^{3}-45bd\,f^{4}g\,x^{3}-63ac\,f^{4}h\,x^{2}-9ade\,f^{3}h\,x^{2}-63ad\,f^{4}g\,x^{2}-9b\right.}{}$
risch	$-\frac{2\left(-35dbh\,f^{4}x^{4}-45ad\,f^{4}h\,x^{3}-45bc\,f^{4}h\,x^{3}-5bde\,f^{3}h\,x^{3}-45bd\,f^{4}g\,x^{3}-63ac\,f^{4}h\,x^{2}-9ade\,f^{3}h\,x^{2}-63ad\,f^{4}g\,x^{2}-9b\right.}{}$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-4/15*(1/2*(-3*x*(5/7*x*(7/9*h*x+g)*d+c*(5/7*h*x+g))*b-5*a*(3/5*x*(5/7*h*x
+g)*d+c*(3/5*h*x+g)))*f^3+((6/7*x*(5/6*h*x+g)*d+c*(6/7*h*x+g))*b+a*((6/7*h
*x+g)*d+c*h))*e*f^2-4/7*(((h*x+g)*d+c*h)*b+a*d*h)*e^2*f+8/21*b*d*e^3*h)*(f
*x+e)^(3/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.67

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$=\frac{2\left(35\,bdf^{4}hx^{4}+5\left(9\,bdf^{4}g+\left(bdef^{3}+9\left(bc+ad\right)f^{4}\right)h\right)x^{3}+3\left(3\left(bdef^{3}+7\left(bc+ad\right)f^{4}\right)g-\left(2\,bde^{2}f^{2}-\right.\right.\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/315*(35*b*d*f^4*h*x^4 + 5*(9*b*d*f^4*g + (b*d*e*f^3 + 9*(b*c + a*d)*f^4)
*h)*x^3 + 3*(3*(b*d*e*f^3 + 7*(b*c + a*d)*f^4)*g - (2*b*d*e^2*f^2 - 21*a*c
*f^4 - 3*(b*c + a*d)*e*f^3)*h)*x^2 + 3*(8*b*d*e^3*f + 35*a*c*e*f^3 - 14*(b
*c + a*d)*e^2*f^2)*g - 2*(8*b*d*e^4 + 21*a*c*e^2*f^2 - 12*(b*c + a*d)*e^3*
f)*h - (3*(4*b*d*e^2*f^2 - 35*a*c*f^4 - 7*(b*c + a*d)*e*f^3)*g - (8*b*d*e^
3*f + 21*a*c*e*f^3 - 12*(b*c + a*d)*e^2*f^2)*h)*x)*sqrt(f*x + e)/f^4
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.91

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{bdh(e+fx)^{\frac{9}{2}}}{9f^3}+\frac{(e+fx)^{\frac{7}{2}}(adfh+bcfh-3bdeh+bdfg)}{7f^3}+\frac{(e+fx)^{\frac{5}{2}}\left(acf^2h-2adefh+adf^2g-2bcefh+bcf^2g+3bde^2h-2bdefg\right)}{5f^3}+\frac{(e+fx)^{\frac{3}{2}}\left(-acef^2h+acf^3\right.}{}\right.}{f} \\ \sqrt{e}\left(acgx+\frac{bdhx^4}{4}+\frac{x^3(adh+bch+bdg)}{3}+\frac{x^2(ach+adg+bcg)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b*d*h*(e + f*x)**(9/2)/(9*f**3) + (e + f*x)**(7/2)*(a*d*f*h
+ b*c*f*h - 3*b*d*e*h + b*d*f*g)/(7*f**3) + (e + f*x)**(5/2)*(a*c*f**2*h -
 2*a*d*e*f*h + a*d*f**2*g - 2*b*c*e*f*h + b*c*f**2*g + 3*b*d*e**2*h - 2*b*
d*e*f*g)/(5*f**3) + (e + f*x)**(3/2)*(-a*c*e*f**2*h + a*c*f**3*g + a*d*e**
2*f*h - a*d*e*f**2*g + b*c*e**2*f*h - b*c*e*f**2*g - b*d*e**3*h + b*d*e**2
*f*g)/(3*f**3))/f, Ne(f, 0)), (sqrt(e)*(a*c*g*x + b*d*h*x**4/4 + x**3*(a*d
*h + b*c*h + b*d*g)/3 + x**2*(a*c*h + a*d*g + b*c*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.13

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\left(35\,(fx+e)^{\frac{9}{2}}bdh+45\,(bdfg-(3\,bde-(bc+ad)f)h)(fx+e)^{\frac{7}{2}}-63\,((2\,bdef-(bc+ad)f^2)g-(3\,bc\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/315*(35*(f*x + e)^(9/2)*b*d*h + 45*(b*d*f*g - (3*b*d*e - (b*c + a*d)*f)*
h)*(f*x + e)^(7/2) - 63*((2*b*d*e*f - (b*c + a*d)*f^2)*g - (3*b*d*e^2 + a*
c*f^2 - 2*(b*c + a*d)*e*f)*h)*(f*x + e)^(5/2) + 105*((b*d*e^2*f + a*c*f^3
- (b*c + a*d)*e*f^2)*g - (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h)*(f*x
 + e)^(3/2))/f^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(143) = 286.

Time = 0.12 (sec) , antiderivative size = 638, normalized size of antiderivative = 4.01

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(f*x + e)*a*c*e*g + 105*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*
e)*a*c*g + 105*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*b*c*e*g/f + 105*((f*x
 + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*d*e*g/f + 105*((f*x + e)^(3/2) - 3*sqrt
(f*x + e)*e)*a*c*e*h/f + 21*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15
*sqrt(f*x + e)*e^2)*b*d*e*g/f^2 + 21*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/
2)*e + 15*sqrt(f*x + e)*e^2)*b*c*g/f + 21*(3*(f*x + e)^(5/2) - 10*(f*x + e
)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*d*g/f + 21*(3*(f*x + e)^(5/2) - 10*(f*
x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*e*h/f^2 + 21*(3*(f*x + e)^(5/2)
 - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*d*e*h/f^2 + 21*(3*(f*x +
 e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c*h/f + 9*(5*(f
*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*
x + e)*e^3)*b*d*g/f^2 + 9*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(
f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*d*e*h/f^3 + 9*(5*(f*x + e)^(7
/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3
)*b*c*h/f^2 + 9*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(
3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*d*h/f^2 + (35*(f*x + e)^(9/2) - 180*(f*
x + e)^(7/2)*e + 378*(f*x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*s
qrt(f*x + e)*e^4)*b*d*h/f^3)/f
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{(e+f\,x)^{5/2}\,(2\,a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-4\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f\,g)}{5\,f^4}$$
$$+\frac{(e+f\,x)^{7/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+2\,b\,d\,f\,g)}{7\,f^4}$$
$$-\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)\,(c\,f-d\,e)\,(e\,h-f\,g)}{3\,f^4}+\frac{2\,b\,d\,h\,(e+f\,x)^{9/2}}{9\,f^4}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

output
```
((e + f*x)^(5/2)*(2*a*c*f^2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 6*b*d*e^2*h -
4*a*d*e*f*h - 4*b*c*e*f*h - 4*b*d*e*f*g))/(5*f^4) + ((e + f*x)^(7/2)*(2*a*
d*f*h + 2*b*c*f*h - 6*b*d*e*h + 2*b*d*f*g))/(7*f^4) - (2*(e + f*x)^(3/2)*(
a*f - b*e)*(c*f - d*e)*(e*h - f*g))/(3*f^4) + (2*b*d*h*(e + f*x)^(9/2))/(9
*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.97

$$\int (a+bx)(c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\sqrt{fx+e}\,(35bd\,f^4h\,x^4+45ad\,f^4h\,x^3+45bc\,f^4h\,x^3+5bde\,f^3h\,x^3+45bd\,f^4g\,x^3+63ac\,f^4h\,x^2+9ade\,f$$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)^(1/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 42*a*c*e**2*f**2*h + 105*a*c*e*f**3*g + 21*a*c*e*f**3
*h*x + 105*a*c*f**4*g*x + 63*a*c*f**4*h*x**2 + 24*a*d*e**3*f*h - 42*a*d*e*
*2*f**2*g - 12*a*d*e**2*f**2*h*x + 21*a*d*e*f**3*g*x + 9*a*d*e*f**3*h*x**2
 + 63*a*d*f**4*g*x**2 + 45*a*d*f**4*h*x**3 + 24*b*c*e**3*f*h - 42*b*c*e**2
*f**2*g - 12*b*c*e**2*f**2*h*x + 21*b*c*e*f**3*g*x + 9*b*c*e*f**3*h*x**2 +
 63*b*c*f**4*g*x**2 + 45*b*c*f**4*h*x**3 - 16*b*d*e**4*h + 24*b*d*e**3*f*g
 + 8*b*d*e**3*f*h*x - 12*b*d*e**2*f**2*g*x - 6*b*d*e**2*f**2*h*x**2 + 9*b*
d*e*f**3*g*x**2 + 5*b*d*e*f**3*h*x**3 + 45*b*d*f**4*g*x**3 + 35*b*d*f**4*h
*x**4))/(315*f**4)
```

3.4 $\int (c+dx)\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx = -\frac{2(de-cf)(fg-eh)(e+fx)^{3/2}}{3f^3} + \frac{2(dfg-2deh+cfh)(e+fx)^{5/2}}{5f^3} + \frac{2dh(e+fx)^{7/2}}{7f^3}$$

output
```
-2/3*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(3/2)/f^3+2/5*(c*f*h-2*d*e*h+d*f*g)*(f*
x+e)^(5/2)/f^3+2/7*d*h*(f*x+e)^(7/2)/f^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2(e+fx)^{3/2}\left(7cf(5fg-2eh+3fhx)+d(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx))\right)}{105f^3}$$

input
```
Integrate[(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(7*c*f*(5*f*g - 2*e*h + 3*f*h*x) + d*(8*e^2*h + 3*f^2*x
*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x))))/(105*f^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{(e+fx)^{3/2}(cfh-2deh+dfg)}{f^2}+\frac{\sqrt{e+fx}(cf-de)(fg-eh)}{f^2}+\frac{dh(e+fx)^{5/2}}{f^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(e+fx)^{5/2}(cfh-2deh+dfg)}{5f^3}-\frac{2(e+fx)^{3/2}(de-cf)(fg-eh)}{3f^3}+\frac{2dh(e+fx)^{7/2}}{7f^3}$$

input
```
Int[(c + d*x)*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(-2*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^3) + (2*(d*f*g - 2*d*e*h
 + c*f*h)*(e + f*x)^(5/2))/(5*f^3) + (2*d*h*(e + f*x)^(7/2))/(7*f^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{4\left(\frac{\left(-3x\left(\frac{5hx}{7}+g\right)d-5c\left(\frac{3hx}{5}+g\right)\right)f^{2}}{2}+\left(\left(\frac{6hx}{7}+g\right)d+ch\right)ef-\frac{4d\,e^{2}h}{7}\right)(fx+e)^{\frac{3}{2}}}{15f^{3}}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-15dh\,x^{2}f^{2}-21c\,f^{2}hx+12defhx-21d\,f^{2}gx+14cefh-35cg\,f^{2}-8d\,e^{2}h+14defg\right)}{105f^{3}}$
derivativedivides	$\frac{\frac{2dh(fx+e)^{\frac{7}{2}}}{7}+\frac{2((cf-de)h+d(-eh+fg))(fx+e)^{\frac{5}{2}}}{5}+\frac{2(cf-de)(-eh+fg)(fx+e)^{\frac{3}{2}}}{3}}{f^{3}}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-15dh\,x^{2}f^{2}-21c\,f^{2}hx+12defhx-21d\,f^{2}gx+14cefh-35cg\,f^{2}-8d\,e^{2}h+14defg\right)}{105f^{3}}$
default	$\frac{\frac{2dh(fx+e)^{\frac{7}{2}}}{7}+\frac{2(-(-cf+de)h-d(eh-fg))(fx+e)^{\frac{5}{2}}}{5}+\frac{2(-cf+de)(eh-fg)(fx+e)^{\frac{3}{2}}}{3}}{f^{3}}$
trager	$-\frac{2\left(-15dh\,f^{3}x^{3}-21c\,f^{3}h\,x^{2}-3de\,f^{2}h\,x^{2}-21d\,f^{3}g\,x^{2}-7ce\,f^{2}hx-35c\,f^{3}gx+4d\,e^{2}fhx-7de\,f^{2}gx+14c\,e^{2}fh-35ce\,f^{2}\right.}{105f^{3}}$
risch	$-\frac{2\left(-15dh\,f^{3}x^{3}-21c\,f^{3}h\,x^{2}-3de\,f^{2}h\,x^{2}-21d\,f^{3}g\,x^{2}-7ce\,f^{2}hx-35c\,f^{3}gx+4d\,e^{2}fhx-7de\,f^{2}gx+14c\,e^{2}fh-35ce\,f^{2}\right.}{105f^{3}}$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-4/15*(1/2*(-3*x*(5/7*h*x+g)*d-5*c*(3/5*h*x+g))*f^2+((6/7*h*x+g)*d+c*h)*e*
f-4/7*d*e^2*h)*(f*x+e)^(3/2)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.52

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2\left(15\,df^3hx^3+3\left(7\,df^3g+(def^2+7\,cf^3)h\right)x^2-7\left(2\,de^2f-5\,cef^2\right)g+2\left(4\,de^3-7\,ce^2f\right)h+\left(7\left(def^2+\right.\right.}{105\,f^3}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/105*(15*d*f^3*h*x^3 + 3*(7*d*f^3*g + (d*e*f^2 + 7*c*f^3)*h)*x^2 - 7*(2*d
*e^2*f - 5*c*e*f^2)*g + 2*(4*d*e^3 - 7*c*e^2*f)*h + (7*(d*e*f^2 + 5*c*f^3)
*g - (4*d*e^2*f - 7*c*e*f^2)*h)*x)*sqrt(f*x + e)/f^3
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.53

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{dh(e+fx)^{\frac{7}{2}}}{7f^2}+\frac{(e+fx)^{\frac{5}{2}}(cfh-2deh+dfg)}{5f^2}+\frac{(e+fx)^{\frac{3}{2}}\left(-cefh+cf^2g+de^2h-defg\right)}{3f^2}\right)}{f} & \text{for } f\neq 0 \\ \sqrt{e}\left(cgx+\frac{dhx^3}{3}+\frac{x^2(ch+dg)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(7/2)/(7*f**2) + (e + f*x)**(5/2)*(c*f*h - 2*
d*e*h + d*f*g)/(5*f**2) + (e + f*x)**(3/2)*(-c*e*f*h + c*f**2*g + d*e**2*h
 - d*e*f*g)/(3*f**2))/f, Ne(f, 0)), (sqrt(e)*(c*g*x + d*h*x**3/3 + x**2*(c
*h + d*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2\left(15\,(fx+e)^{\frac{7}{2}}dh + 21\,(dfg - (2\,de - cf)h)(fx+e)^{\frac{5}{2}} - 35\,((def - cf^2)g - (de^2 - cef)h)(fx+e)^{\frac{3}{2}}\right)}{105\,f^3}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/105*(15*(f*x + e)^(7/2)*d*h + 21*(d*f*g - (2*d*e - c*f)*h)*(f*x + e)^(5/
2) - 35*((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h)*(f*x + e)^(3/2))/f^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(69) = 138.

Time = 0.13 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.22

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx = \frac{2\left(105\,\sqrt{fx+e}ceg + 35\left((fx+e)^{\frac{3}{2}} - 3\,\sqrt{fx+e}e\right)cg + \frac{35\left((fx+e)^{\frac{3}{2}} - 3\,\sqrt{fx+e}e\right)deg}{f} + \frac{35\left((fx+e)^{\frac{3}{2}} - 3\,\sqrt{fx+e}e\right)c\cdots}{f}\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(f*x + e)*c*e*g + 35*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*
c*g + 35*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*d*e*g/f + 35*((f*x + e)^(3/
2) - 3*sqrt(f*x + e)*e)*c*e*h/f + 7*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2
)*e + 15*sqrt(f*x + e)*e^2)*d*g/f + 7*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3
/2)*e + 15*sqrt(f*x + e)*e^2)*d*e*h/f^2 + 7*(3*(f*x + e)^(5/2) - 10*(f*x +
 e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*h/f + 3*(5*(f*x + e)^(7/2) - 21*(f*x
 + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*d*h/f^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\,(e+f\,x)^{3/2}\,\left(15\,d\,h\,(e+f\,x)^2+35\,c\,f^2\,g+35\,d\,e^2\,h+21\,c\,f\,h\,(e+f\,x)-42\,d\,e\,h\,(e+f\,x)+21\,d\,f\,g\,(\right.}{105\,f^3}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(c + d*x),x)
```

output
```
(2*(e + f*x)^(3/2)*(15*d*h*(e + f*x)^2 + 35*c*f^2*g + 35*d*e^2*h + 21*c*f*
h*(e + f*x) - 42*d*e*h*(e + f*x) + 21*d*f*g*(e + f*x) - 35*c*e*f*h - 35*d*
e*f*g))/(105*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int (c+dx)\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\sqrt{fx+e}\,(15d\,f^3h\,x^3+21c\,f^3h\,x^2+3de\,f^2h\,x^2+21d\,f^3g\,x^2+7ce\,f^2hx+35c\,f^3gx-4d\,e^2fhx+7de}{105f^3}$$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 14*c*e**2*f*h + 35*c*e*f**2*g + 7*c*e*f**2*h*x + 35*c
*f**3*g*x + 21*c*f**3*h*x**2 + 8*d*e**3*h - 14*d*e**2*f*g - 4*d*e**2*f*h*x
 + 7*d*e*f**2*g*x + 3*d*e*f**2*h*x**2 + 21*d*f**3*g*x**2 + 15*d*f**3*h*x**
3))/(105*f**3)
```

3.5 $\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \frac{2(bc-ad)(bg-ah)\sqrt{e+fx}}{b^3} - \frac{2(d(be+af)h-bf(dg+ch))(e+fx)^{3/2}}{3b^2f^2} + \frac{2dh(e+fx)^{5/2}}{5bf^2} - \frac{2(bc-ad)\sqrt{be-af}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{7/2}}$$

output
```
2*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^3-2/3*(d*(a*f+b*e)*h-b*f*(c*h+d*g)
)*(f*x+e)^(3/2)/b^2/f^2+2/5*d*h*(f*x+e)^(5/2)/b/f^2-2*(-a*d+b*c)*(-a*f+b*e
)^(1/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$= \frac{2\sqrt{e+fx}(15a^2df^2h-5abf(3cfh+d(3fg+eh+fhx))+b^2(-d(e+fx)(-5fg+2eh-3fhx)+5cf(}{15b^3f^2}$$
$$-\frac{2(bc-ad)\sqrt{-be+af}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{7/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x),x]
```

output
```
(2*Sqrt[e + f*x]*(15*a^2*d*f^2*h - 5*a*b*f*(3*c*f*h + d*(3*f*g + e*h + f*h
*x)) + b^2*(-(d*(e + f*x)*(-5*f*g + 2*e*h - 3*f*h*x)) + 5*c*f*(3*f*g + e*h
 + f*h*x))))/(15*b^3*f^2) - (2*(b*c - a*d)*Sqrt[-(b*e) + a*f]*(b*g - a*h)*
ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$\Big\downarrow \text{164}$$
$$\frac{(bc-ad)(bg-ah)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b^2}-\frac{2(e+fx)^{3/2}(5adfh-5bf(ch+dg)+2bdeh-3bdfhx)}{15b^2f^2}$$
$$\Big\downarrow \text{60}$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{(be-af)\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b^2}-\frac{2(e+fx)^{3/2}(5adfh-5bf(ch+dg)+2bdeh-3bdfhx)}{15b^2f^2}$$

$$\Big\downarrow \; 73$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{2(be-af)\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b^2}-\frac{2(e+fx)^{3/2}(5adfh-5bf(ch+dg)+2bdeh-3bdfhx)}{15b^2f^2}$$

$$\Big\downarrow \; 221$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)}{b^2}-\frac{2(e+fx)^{3/2}(5adfh-5bf(ch+dg)+2bdeh-3bdfhx)}{15b^2f^2}$$

input
```
Int[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x),x]
```

output
```
(-2*(e + f*x)^(3/2)*(2*b*d*e*h + 5*a*d*f*h - 5*b*f*(d*g + c*h) - 3*b*d*f*h
*x))/(15*b^2*f^2) + ((b*c - a*d)*(b*g - a*h)*((2*Sqrt[e + f*x])/b - (2*Sqr
t[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b
^2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{2\left(-\left(\left(\frac{\left(\frac{(3fx-2e)d}{5}+cf\right)(fx+e)h}{3}+\left(\frac{(fx+e)d}{3}+cf\right)gf\right)b^2-a\left(\left(\frac{(fx+e)d}{3}+cf\right)h+dfg\right)fb+a^2d\,f^2h\right)\sqrt{(af-be)b}\,\sqrt{}}{\sqrt{(af-be)b}\,f^2b^3}$
risch	$\frac{2(3dh\,b^2f^2x^2-5abd\,f^2hx+5b^2c\,f^2hx+b^2defhx+5b^2d\,f^2gx+15a^2d\,f^2h-15abc\,f^2h-5abdefh-15abd\,f^2g+5b^2cefh+}{15f^2b^3}$
derivativedivides	$\frac{\frac{2\left(\frac{dh(fx+e)^{\frac{5}{2}}b^2}{5}-\frac{abdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2cfh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2deh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2dfg(fx+e)^{\frac{3}{2}}}{3}+a^2d\,f^2h\sqrt{fx+e}-abc\,f^2h\sqrt{fx+e}-abd\,f^2\right.}{b^3}}{f^2}$
default	$\frac{\frac{2\left(\frac{dh(fx+e)^{\frac{5}{2}}b^2}{5}-\frac{abdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2cfh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2deh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2dfg(fx+e)^{\frac{3}{2}}}{3}+a^2d\,f^2h\sqrt{fx+e}-abc\,f^2h\sqrt{fx+e}-abd\,f^2\right.}{b^3}}{f^2}$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/((a*f-b*e)*b)^(1/2)*(-((1/3*(1/5*(3*f*x-2*e)*d+c*f)*(f*x+e)*h+(1/3*(f*x
+e)*d+c*f)*g*f)*b^2-a*((1/3*(f*x+e)*d+c*f)*h+d*f*g)*f*b+a^2*d*f^2*h)*((a*f
-b*e)*b)^(1/2)*(f*x+e)^(1/2)+f^2*(a*h-b*g)*(a*f-b*e)*(a*d-b*c)*arctan(b*(f
*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))/f^2/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.00

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$= \left[\frac{15\left((b^2c-abd)f^2g-(abc-a^2d)f^2h\right)\sqrt{\frac{be-af}{b}}\log\left(\frac{bfx+2\,be-af-2\,\sqrt{fx+e}b\sqrt{\frac{be-af}{b}}}{bx+a}\right)+2\left(3\,b^2df^2hx^2+5\left(b^2c\right.\right.}{}\right.$$
$$-\frac{2\left(15\left((b^2c-abd)f^2g-(abc-a^2d)f^2h\right)\sqrt{-\frac{be-af}{b}}\arctan\left(-\frac{\sqrt{fx+e}b\sqrt{-\frac{be-af}{b}}}{be-af}\right)-\left(3\,b^2df^2hx^2+5\left(b^2d\right.\right.\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/15*(15*((b^2*c - a*b*d)*f^2*g - (a*b*c - a^2*d)*f^2*h)*sqrt((b*e - a*f)
/b)*log((b*f*x + 2*b*e - a*f - 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x
 + a)) + 2*(3*b^2*d*f^2*h*x^2 + 5*(b^2*d*e*f + 3*(b^2*c - a*b*d)*f^2)*g -
(2*b^2*d*e^2 - 5*(b^2*c - a*b*d)*e*f + 15*(a*b*c - a^2*d)*f^2)*h + (5*b^2*
d*f^2*g + (b^2*d*e*f + 5*(b^2*c - a*b*d)*f^2)*h)*x)*sqrt(f*x + e))/(b^3*f^
2), -2/15*(15*((b^2*c - a*b*d)*f^2*g - (a*b*c - a^2*d)*f^2*h)*sqrt(-(b*e -
 a*f)/b)*arctan(-sqrt(f*x + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) - (3*b^
2*d*f^2*h*x^2 + 5*(b^2*d*e*f + 3*(b^2*c - a*b*d)*f^2)*g - (2*b^2*d*e^2 - 5
*(b^2*c - a*b*d)*e*f + 15*(a*b*c - a^2*d)*f^2)*h + (5*b^2*d*f^2*g + (b^2*d
*e*f + 5*(b^2*c - a*b*d)*f^2)*h)*x)*sqrt(f*x + e))/(b^3*f^2)]
```

Sympy [A] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \begin{cases} \frac{2\left(\frac{dh(e+fx)^{\frac{5}{2}}}{5bf} + \frac{(e+fx)^{\frac{3}{2}}(-adfh+bcfh-bdeh+bdfg)}{3b^2f} + \frac{\sqrt{e+fx}\left(a^2dfh-abcfh-abdfg+b^2cfg\right)}{b^3} - \frac{f(ad-bc)(af-be)(ah-bg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{af-be}{b}}}\right)}{b^4\sqrt{\frac{af-be}{b}}}\right)}{f} & \text{for} \\ \sqrt{e}\left(\frac{dhx^2}{2b} + \frac{x(-adh+bch+bdg)}{b^2} + \frac{(ad-bc)(ah-bg)\left(\begin{cases} \frac{x}{a} & \text{for } b=0 \\ \frac{\log(a+bx)}{b} & \text{otherwise} \end{cases}\right)}{b^2}\right) & \text{oth} \end{cases}$$

input
```
integrate((d*x+c)*(f*x+e)**(1/2)*(h*x+g)/(b*x+a),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(5/2)/(5*b*f) + (e + f*x)**(3/2)*(-a*d*f*h +
b*c*f*h - b*d*e*h + b*d*f*g)/(3*b**2*f) + sqrt(e + f*x)*(a**2*d*f*h - a*b*
c*f*h - a*b*d*f*g + b**2*c*f*g)/b**3 - f*(a*d - b*c)*(a*f - b*e)*(a*h - b*
g)*atan(sqrt(e + f*x)/sqrt((a*f - b*e)/b))/(b**4*sqrt((a*f - b*e)/b)))/f,
Ne(f, 0)), (sqrt(e)*(d*h*x**2/(2*b) + x*(-a*d*h + b*c*h + b*d*g)/b**2 + (a
*d - b*c)*(a*h - b*g)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/b
**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(136) = 272$.

Time = 0.13 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$= \frac{2\,(b^3ceg - ab^2deg - ab^2cfg + a^2bdfg - ab^2ceh + a^2bdeh + a^2bcfh - a^3dfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{\sqrt{-b^2e+abf}b^3}$$
$$+ \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^4df^9g + 15\,\sqrt{fx+e}b^4cf^{10}g - 15\,\sqrt{fx+e}ab^3df^{10}g + 3\,(fx+e)^{\frac{5}{2}}b^4df^8h - 5\,(fx+e)^{\frac{3}{2}}\right.}{15\,b^5}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x, algorithm="giac")
```

output
```
2*(b^3*c*e*g - a*b^2*d*e*g - a*b^2*c*f*g + a^2*b*d*f*g - a*b^2*c*e*h + a^2
*b*d*e*h + a^2*b*c*f*h - a^3*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a
*b*f))/(sqrt(-b^2*e + a*b*f)*b^3) + 2/15*(5*(f*x + e)^(3/2)*b^4*d*f^9*g +
15*sqrt(f*x + e)*b^4*c*f^10*g - 15*sqrt(f*x + e)*a*b^3*d*f^10*g + 3*(f*x +
 e)^(5/2)*b^4*d*f^8*h - 5*(f*x + e)^(3/2)*b^4*d*e*f^8*h + 5*(f*x + e)^(3/2
)*b^4*c*f^9*h - 5*(f*x + e)^(3/2)*a*b^3*d*f^9*h - 15*sqrt(f*x + e)*a*b^3*c
*f^10*h + 15*sqrt(f*x + e)*a^2*b^2*d*f^10*h)/(b^5*f^10)
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx = (e+f\,x)^{3/2}\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{3\,b\,f^2}-\frac{2\,d\,h\,(a\,f^3-b\,e\,f^2)}{3\,b^2\,f^4}\right) - \sqrt{e+f\,x}\left(\frac{2\,(c\,f-d\,e)\,(e\,h-f\,g)}{b\,f^2} + \frac{(a\,f^3-b\,e\,f^2)\,\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{b\,f^2}-\frac{2\,d\,h\,(a\,f^3-b\,e\,f^2)}{b^2\,f^4}\right)}{b\,f^2}\right) + \frac{2\,d\,h\,(e+f\,x)^{5/2}}{5\,b\,f^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,1\mathrm{i}}{\sqrt{b\,e-a\,f}}\right)\,(a\,d-b\,c)\,\sqrt{b\,e-a\,f}\,(a\,h-b\,g)\,2\mathrm{i}}{b^{7/2}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x))/(a + b*x),x)
```

output
```
(e + f*x)^(3/2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(3*b*f^2) - (2*d*h*(a*f^3 -
 b*e*f^2))/(3*b^2*f^4)) - (e + f*x)^(1/2)*((2*(c*f - d*e)*(e*h - f*g))/(b*
f^2) + ((a*f^3 - b*e*f^2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(b*f^2) - (2*d*h*
(a*f^3 - b*e*f^2))/(b^2*f^4)))/(b*f^2)) + (2*d*h*(e + f*x)^(5/2))/(5*b*f^2
) + (atan((b^(1/2)*(e + f*x)^(1/2)*1i)/(b*e - a*f)^(1/2))*(a*d - b*c)*(b*e
 - a*f)^(1/2)*(a*h - b*g)*2i)/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.58

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \frac{-2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a^2d\,f^2h+2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)abc\,f^2h+2\sqrt{b}\,\sqrt{af-be}\,ata}{}$$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**2*d*f**2*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*f**2*h + 15*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*d*f**2*g - 15*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c*f*
*2*g + 15*sqrt(e + f*x)*a**2*b*d*f**2*h - 15*sqrt(e + f*x)*a*b**2*c*f**2*h
 - 5*sqrt(e + f*x)*a*b**2*d*e*f*h - 15*sqrt(e + f*x)*a*b**2*d*f**2*g - 5*s
qrt(e + f*x)*a*b**2*d*f**2*h*x + 5*sqrt(e + f*x)*b**3*c*e*f*h + 15*sqrt(e
+ f*x)*b**3*c*f**2*g + 5*sqrt(e + f*x)*b**3*c*f**2*h*x - 2*sqrt(e + f*x)*b
**3*d*e**2*h + 5*sqrt(e + f*x)*b**3*d*e*f*g + sqrt(e + f*x)*b**3*d*e*f*h*x
 + 5*sqrt(e + f*x)*b**3*d*f**2*g*x + 3*sqrt(e + f*x)*b**3*d*f**2*h*x**2))/
(15*b**4*f**2)
```

3.6 $\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2} dx = \frac{2(bdg+bch-2adh)\sqrt{e+fx}}{b^3} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{b^3(a+bx)} + \frac{2dh(e+fx)^{3/2}}{3b^2f} - \frac{(5a^2dfh+b^2(2deg+cfg+2ceh)-ab(3dfg+4deh+3cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{7/2}\sqrt{be-af}}$$

output
```
2*(-2*a*d*h+b*c*h+b*d*g)*(f*x+e)^(1/2)/b^3-(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(
1/2)/b^3/(b*x+a)+2/3*d*h*(f*x+e)^(3/2)/b^2/f-(5*a^2*d*f*h+b^2*(2*c*e*h+c*f
*g+2*d*e*g)-a*b*(3*c*f*h+4*d*e*h+3*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(
-a*f+b*e)^(1/2))/b^(7/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$=\frac{\sqrt{e+fx}(-15a^2dfh+ab(9cfh+d(9fg+2eh-10fhx))+b^2(-3cf(g-2hx)+2dx(3fg+eh+fhx)}{3b^3f(a+bx)}$$
$$+\frac{(5a^2dfh+b^2(2deg+cfg+2ceh)-ab(3dfg+4deh+3cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{7/2}\sqrt{-be+af}}$$

input
```
Integrate[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(-15*a^2*d*f*h + a*b*(9*c*f*h + d*(9*f*g + 2*e*h - 10*f*h*x
)) + b^2*(-3*c*f*(g - 2*h*x) + 2*d*x*(3*f*g + e*h + f*h*x))))/(3*b^3*f*(a
+ b*x)) + ((5*a^2*d*f*h + b^2*(2*d*e*g + c*f*g + 2*c*e*h) - a*b*(3*d*f*g +
 4*d*e*h + 3*c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(
b^(7/2)*Sqrt[-(b*e) + a*f])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {163, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \text{163}$$
$$\frac{(5a^2dfh-ab(3cfh+4deh+3dfg)+b^2(2ceh+cfg+2deg))\int \frac{\sqrt{e+fx}}{a+bx}dx}{2b^2(be-af)}-\frac{(e+fx)^{3/2}(5a^2dfh-ab(3cfh+2deh+3dfg)-2bdhx(be-af)+3b^2cfg)}{3b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(5a^2dfh-ab(3cfh+4deh+3dfg)+b^2(2ceh+cfg+2deg)\right)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{2b^2(be-af)}-\frac{(e+fx)^{3/2}\left(5a^2dfh-ab(3cfh+2deh+3dfg)-2bdhx(be-af)+3b^2cfg\right)}{3b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(5a^2dfh-ab(3cfh+4deh+3dfg)+b^2(2ceh+cfg+2deg)\right)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{2b^2(be-af)}-\frac{(e+fx)^{3/2}\left(5a^2dfh-ab(3cfh+2deh+3dfg)-2bdhx(be-af)+3b^2cfg\right)}{3b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)\left(5a^2dfh-ab(3cfh+4deh+3dfg)+b^2(2ceh+cfg+2deg)\right)}{2b^2(be-af)}-\frac{(e+fx)^{3/2}\left(5a^2dfh-ab(3cfh+2deh+3dfg)-2bdhx(be-af)+3b^2cfg\right)}{3b^2f(a+bx)(be-af)}$$

input
```
Int[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^2,x]
```

output
```
-1/3*((e + f*x)^(3/2)*(3*b^2*c*f*g + 5*a^2*d*f*h - a*b*(3*d*f*g + 2*d*e*h
+ 3*c*f*h) - 2*b*d*(b*e - a*f)*h*x))/(b^2*f*(b*e - a*f)*(a + b*x)) + ((5*a
^2*d*f*h + b^2*(2*d*e*g + c*f*g + 2*c*e*h) - a*b*(3*d*f*g + 4*d*e*h + 3*c*
f*h))*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e +
f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/(2*b^2*(b*e - a*f))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{5\left(-\left(\frac{(cfg+2e(ch+dg))b^2}{5}-\frac{3a\left(f(ch+dg)+\frac{4deh}{3}\right)b}{5}+a^2dfh\right)(bx+a)f\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\left(\frac{\left(\left(-2x\left(\frac{hx}{3}+g\right)d+c(-2h\cdots}{5}\cdots\right.\right.}{\sqrt{(af-be)b}\,f\,b^3(bx+a)}$
risch	$-\frac{2(-dhxbf+6adfh-3bcfh-bdeh-3bdfg)\sqrt{fx+e}}{3f\,b^3}+\frac{2\left(-\frac{1}{2}a^2dfh+\frac{1}{2}abcfh+\frac{1}{2}abdfg-\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{\left(5a^2dfh-3abcf\cdots\right.}{b^3\cdots}$
derivativedivides	$\frac{-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^3}+\frac{2f\left(\frac{\left(-\frac{1}{2}a^2dfh+\frac{1}{2}abcfh+\frac{1}{2}abdfg-\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{\left(5a^2df\cdots\right.}{\cdots}\right.}{\cdots}}{f}$
default	$\frac{-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^3}+\frac{2f\left(\frac{\left(-\frac{1}{2}a^2dfh+\frac{1}{2}abcfh+\frac{1}{2}abdfg-\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{\left(5a^2df\cdots\right.}{\cdots}\right.}{\cdots}}{f}$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/((a*f-b*e)*b)^(1/2)*(-(1/5*(c*f*g+2*e*(c*h+d*g))*b^2-3/5*a*(f*(c*h+d*g)
+4/3*d*e*h)*b+a^2*d*f*h)*(b*x+a)*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1
/2))+(1/5*((-2*x*(1/3*h*x+g)*d+c*(-2*h*x+g))*f-2/3*d*e*h*x)*b^2-3/5*(((-10
/9*h*x+g)*d+c*h)*f+2/9*d*e*h)*a*b+a^2*d*f*h)*(f*x+e)^(1/2)*((a*f-b*e)*b)^(
1/2))/f/b^3/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(164) = 328.

Time = 0.10 (sec) , antiderivative size = 884, normalized size of antiderivative = 4.86

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(b^2*e - a*b*f)*((2*a*b^2*d*e*f + (a*b^2*c - 3*a^2*b*d)*f^2)*g
 + (2*(a*b^2*c - 2*a^2*b*d)*e*f - (3*a^2*b*c - 5*a^3*d)*f^2)*h + ((2*b^3*d
*e*f + (b^3*c - 3*a*b^2*d)*f^2)*g + (2*(b^3*c - 2*a*b^2*d)*e*f - (3*a*b^2*
c - 5*a^2*b*d)*f^2)*h)*x)*log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)
*sqrt(f*x + e))/(b*x + a)) + 2*(2*(b^4*d*e*f - a*b^3*d*f^2)*h*x^2 - 3*((b^
4*c - 3*a*b^3*d)*e*f - (a*b^3*c - 3*a^2*b^2*d)*f^2)*g + (2*a*b^3*d*e^2 + (
9*a*b^3*c - 17*a^2*b^2*d)*e*f - 3*(3*a^2*b^2*c - 5*a^3*b*d)*f^2)*h + 2*(3*
(b^4*d*e*f - a*b^3*d*f^2)*g + (b^4*d*e^2 + 3*(b^4*c - 2*a*b^3*d)*e*f - (3*
a*b^3*c - 5*a^2*b^2*d)*f^2)*h)*x)*sqrt(f*x + e))/(a*b^5*e*f - a^2*b^4*f^2
+ (b^6*e*f - a*b^5*f^2)*x), 1/3*(3*sqrt(-b^2*e + a*b*f)*((2*a*b^2*d*e*f +
(a*b^2*c - 3*a^2*b*d)*f^2)*g + (2*(a*b^2*c - 2*a^2*b*d)*e*f - (3*a^2*b*c -
 5*a^3*d)*f^2)*h + ((2*b^3*d*e*f + (b^3*c - 3*a*b^2*d)*f^2)*g + (2*(b^3*c
- 2*a*b^2*d)*e*f - (3*a*b^2*c - 5*a^2*b*d)*f^2)*h)*x)*arctan(sqrt(-b^2*e +
 a*b*f)*sqrt(f*x + e)/(b*f*x + b*e)) + (2*(b^4*d*e*f - a*b^3*d*f^2)*h*x^2
- 3*((b^4*c - 3*a*b^3*d)*e*f - (a*b^3*c - 3*a^2*b^2*d)*f^2)*g + (2*a*b^3*d
*e^2 + (9*a*b^3*c - 17*a^2*b^2*d)*e*f - 3*(3*a^2*b^2*c - 5*a^3*b*d)*f^2)*h
 + 2*(3*(b^4*d*e*f - a*b^3*d*f^2)*g + (b^4*d*e^2 + 3*(b^4*c - 2*a*b^3*d)*e
*f - (3*a*b^3*c - 5*a^2*b^2*d)*f^2)*h)*x)*sqrt(f*x + e))/(a*b^5*e*f - a^2*
b^4*f^2 + (b^6*e*f - a*b^5*f^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \frac{(2\,b^2deg + b^2cfg - 3\,abdfg + 2\,b^2ceh - 4\,abdeh - 3\,abcfh + 5\,a^2dfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{\sqrt{-b^2e+abf}b^3} - \frac{\sqrt{fx+e}b^2cfg - \sqrt{fx+e}abdfg - \sqrt{fx+e}abcfh + \sqrt{fx+e}a^2dfh}{((fx+e)b-be+af)b^3} + \frac{2\left(3\sqrt{fx+e}b^4df^3g + (fx+e)^{\frac{3}{2}}b^4df^2h + 3\sqrt{fx+e}b^4cf^3h - 6\sqrt{fx+e}ab^3df^3h\right)}{3\,b^6f^3}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(2*b^2*d*e*g + b^2*c*f*g - 3*a*b*d*f*g + 2*b^2*c*e*h - 4*a*b*d*e*h - 3*a*b
*c*f*h + 5*a^2*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-
b^2*e + a*b*f)*b^3) - (sqrt(f*x + e)*b^2*c*f*g - sqrt(f*x + e)*a*b*d*f*g -
 sqrt(f*x + e)*a*b*c*f*h + sqrt(f*x + e)*a^2*d*f*h)/(((f*x + e)*b - b*e +
a*f)*b^3) + 2/3*(3*sqrt(f*x + e)*b^4*d*f^3*g + (f*x + e)^(3/2)*b^4*d*f^2*h
 + 3*sqrt(f*x + e)*b^4*c*f^3*h - 6*sqrt(f*x + e)*a*b^3*d*f^3*h)/(b^6*f^3)
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$= \sqrt{e+f\,x}\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{b^2\,f}-\frac{4\,d\,h\,(a\,f-b\,e)}{b^3\,f}\right)$$
$$-\frac{\sqrt{e+f\,x}\,(b^2\,c\,f\,g+a^2\,d\,f\,h-a\,b\,c\,f\,h-a\,b\,d\,f\,g)}{b^4\,(e+f\,x)-b^4\,e+a\,b^3\,f}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,(2\,b^2\,c\,e\,h+b^2\,c\,f\,g+2\,b^2\,d\,e\,g+5\,a^2\,d\,f\,h-3\,a\,b\,c\,f\,h-4\,a\,b\,d\,e\,h-3\,a\,b\,d\,f\,g)}{b^{7/2}\,\sqrt{a\,f-b\,e}}$$
$$+\frac{2\,d\,h\,(e+f\,x)^{3/2}}{3\,b^2\,f}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x))/(a + b*x)^2,x)
```

output
```
(e + f*x)^(1/2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(b^2*f) - (4*d*h*(a*f - b*e
))/(b^3*f)) - ((e + f*x)^(1/2)*(b^2*c*f*g + a^2*d*f*h - a*b*c*f*h - a*b*d*
f*g))/(b^4*(e + f*x) - b^4*e + a*b^3*f) + (atan((b^(1/2)*(e + f*x)^(1/2))/
(a*f - b*e)^(1/2))*(2*b^2*c*e*h + b^2*c*f*g + 2*b^2*d*e*g + 5*a^2*d*f*h -
3*a*b*c*f*h - 4*a*b*d*e*h - 3*a*b*d*f*g))/(b^(7/2)*(a*f - b*e)^(1/2)) + (2
*d*h*(e + f*x)^(3/2))/(3*b^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 989, normalized size of antiderivative = 5.43

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**3*d*f**2*h - 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**2*b*c*f**2*h - 12*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*e*f*h - 9*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*f**
2*g + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**2*b*d*f**2*h*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*e*f*h + 3*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*f**2*g - 9*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*
*2*c*f**2*h*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a*b**2*d*e*f*g - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*e*f*h*x - 9*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*f**2*g
*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*b**3*c*e*f*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*b**3*c*f**2*g*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*d*e*f*g*x - 15*sqrt(e +
 f*x)*a**3*b*d*f**2*h + 9*sqrt(e + f*x)*a**2*b**2*c*f**2*h + 17*sqrt(e + f
*x)*a**2*b**2*d*e*f*h + 9*sqrt(e + f*x)*a**2*b**2*d*f**2*g - 10*sqrt(e ...
```

3.7 $\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 245

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3} dx = \frac{2dh\sqrt{e+fx}}{b^3} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{2b^3(a+bx)^2} - \frac{(9a^2dfh+b^2(4deg+cfg+4ceh)-ab(5dfg+8deh+5cfh))\sqrt{e+fx}}{4b^3(be-af)(a+bx)} - \frac{(15a^2df^2h-3abf(dfg+8deh+cfh)-b^2(cf(fg-4eh)-4de(fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{4b^{7/2}(be-af)^{3/2}}$$

output
```
2*d*h*(f*x+e)^(1/2)/b^3-1/2*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/(b*x+a
)^2-1/4*(9*a^2*d*f*h+b^2*(4*c*e*h+c*f*g+4*d*e*g)-a*b*(5*c*f*h+8*d*e*h+5*d*
f*g))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)/(b*x+a)-1/4*(15*a^2*d*f^2*h-3*a*b*f*(c*
f*h+8*d*e*h+d*f*g)-b^2*(c*f*(-4*e*h+f*g)-4*d*e*(2*e*h+f*g)))*arctanh(b^(1/
2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(7/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$

$$= \frac{\frac{\sqrt{b}\sqrt{e+fx}(-15a^3dfh-b^3(cfgx+4dex(g-2hx)+2ce(g+2hx))+ab^2(-2ceh-2de(g-12hx)+dfx(5g-8hx)+cf(g+5hx))+a^2b(3cfh+d(3fg+14e}{(be-af)(a+bx)^2}}{4b^{7/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^3,x]
```

output
```
((Sqrt[b]*Sqrt[e + f*x]*(-15*a^3*d*f*h - b^3*(c*f*g*x + 4*d*e*x*(g - 2*h*x
) + 2*c*e*(g + 2*h*x)) + a*b^2*(-2*c*e*h - 2*d*e*(g - 12*h*x) + d*f*x*(5*g
 - 8*h*x) + c*f*(g + 5*h*x)) + a^2*b*(3*c*f*h + d*(3*f*g + 14*e*h - 25*f*h
*x))))/((b*e - a*f)*(a + b*x)^2) - ((15*a^2*d*f^2*h - 3*a*b*f*(d*f*g + 8*d
*e*h + c*f*h) + b^2*(4*d*e*(f*g + 2*e*h) + c*f*(-(f*g) + 4*e*h)))*ArcTan[(
Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e) + a*f)^(3/2))/(4*b^(7/
2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {162, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{162}$$

$$\frac{\left(15a^2df^2h-3abf(cfh+8deh+dfg)-\left(b^2(cf(fg-4eh)-4de(2eh+fg))\right)\right)\int \frac{\sqrt{e+fx}}{a+bx}dx}{8b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(7a^2dfh-ab(3cfh+8deh+3dfg)+b^2(4ceh-cfg+4deg)\right)-a^2b(cfh+6deh+dfg\right.}{4b^2(a+bx)^2(be-af)^2}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(15a^2df^2h-3abf(cfh+8deh+dfg)-\left(b^2(cf(fg-4eh)-4de(2eh+fg))\right)\right)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right.}{8b^2(be-af)^2}$$
$$\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(7a^2dfh-ab(3cfh+8deh+3dfg)+b^2(4ceh-cfg+4deg)\right)-a^2b(cfh+6deh+dfg\right.}{4b^2(a+bx)^2(be-af)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(15a^2df^2h-3abf(cfh+8deh+dfg)-\left(b^2(cf(fg-4eh)-4de(2eh+fg))\right)\right)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\right.}{8b^2(be-af)^2}$$
$$\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(7a^2dfh-ab(3cfh+8deh+3dfg)+b^2(4ceh-cfg+4deg)\right)-a^2b(cfh+6deh+dfg\right.}{4b^2(a+bx)^2(be-af)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)\left(15a^2df^2h-3abf(cfh+8deh+dfg)-\left(b^2(cf(fg-4eh)-4de(2eh+\right.\right.}{8b^2(be-af)^2}$$
$$\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(7a^2dfh-ab(3cfh+8deh+3dfg)+b^2(4ceh-cfg+4deg)\right)-a^2b(cfh+6deh+dfg\right.}{4b^2(a+bx)^2(be-af)^2}$$

input
```
Int[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^3,x]
```

output
```
-1/4*((e + f*x)^(3/2)*(2*b^3*c*e*g + 5*a^3*d*f*h + a*b^2*(2*d*e*g - 3*c*f*
g + 2*c*e*h) - a^2*b*(d*f*g + 6*d*e*h + c*f*h) + b*(7*a^2*d*f*h + b^2*(4*d
*e*g - c*f*g + 4*c*e*h) - a*b*(3*d*f*g + 8*d*e*h + 3*c*f*h))*x))/(b^2*(b*e
 - a*f)^2*(a + b*x)^2) + ((15*a^2*d*f^2*h - 3*a*b*f*(d*f*g + 8*d*e*h + c*f
*h) - b^2*(c*f*(f*g - 4*e*h) - 4*d*e*(f*g + 2*e*h)))*((2*Sqrt[e + f*x])/b
- (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(
3/2)))/(8*b^2*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{15\left(\frac{\left(-cg f^2+4e(ch+dg)f+8d e^2h\right)b^2}{15}-\frac{a(f(ch+dg)+8deh)fb}{5}+a^2d f^2h\right)(bx+a)^2\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{4}+\frac{15\sqrt{(af-be)b}\left(\frac{\left(cfg \cdots\right.}{\cdots}\right.}{\cdots}}{\sqrt{(a\cdots}}$
derivativedivides	$\frac{2dh\sqrt{fx+e}}{b^3}-\frac{2\left(\frac{-\frac{bf\left(9a^2dfh-5abcfh-8abdeh-5abdfg+4b^2ceh+b^2cfg+4b^2deg\right)(fx+e)^{\frac{3}{2}}}{8(af-be)}-\frac{f\left(7a^2dfh-3abcfh-8abdeh-3a\cdots\right.}{\cdots}}{((fx+e)b+af-be)^2}\cdots\right.}{\cdots}$
default	$\frac{2dh\sqrt{fx+e}}{b^3}-\frac{2\left(\frac{-\frac{bf\left(9a^2dfh-5abcfh-8abdeh-5abdfg+4b^2ceh+b^2cfg+4b^2deg\right)(fx+e)^{\frac{3}{2}}}{8(af-be)}-\frac{f\left(7a^2dfh-3abcfh-8abdeh-3a\cdots\right.}{\cdots}}{((fx+e)b+af-be)^2}\cdots\right.}{\cdots}$
risch	$\frac{2dh\sqrt{fx+e}}{b^3}-\frac{\frac{-\frac{bf\left(9a^2dfh-5abcfh-8abdeh-5abdfg+4b^2ceh+b^2cfg+4b^2deg\right)(fx+e)^{\frac{3}{2}}}{4(af-be)}-\frac{f\left(7a^2dfh-3abcfh-8abdeh-3abdf\cdots\right.}{4\cdots}}{((fx+e)b+af-be)^2}}{\cdots}$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
15/4/((a*f-b*e)*b)^(1/2)*(-(1/15*(-c*g*f^2+4*e*(c*h+d*g)*f+8*d*e^2*h)*b^2-
1/5*a*(f*(c*h+d*g)+8*d*e*h)*f*b+a^2*d*f^2*h)*(b*x+a)^2*arctan(b*(f*x+e)^(1
/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(1/15*(c*f*g*x+2*(-4*d*h*x^2+
2*(c*h+d*g)*x+c*g)*e)*b^3+2/15*a*((4*d*h*x^2+5/2*(-c*h-d*g)*x-1/2*c*g)*f+e
*(-12*d*h*x+c*h+d*g))*b^2-1/5*a^2*((-25/3*d*h*x+c*h+d*g)*f+14/3*d*e*h)*b+a
^3*d*f*h)*(f*x+e)^(1/2))/b^3/(b*x+a)^2/(a*f-b*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(223) = 446.

Time = 0.16 (sec) , antiderivative size = 1420, normalized size of antiderivative = 5.80

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(b^2*e - a*b*f)*((((4*b^4*d*e*f - (b^4*c + 3*a*b^3*d)*f^2)*g + (
8*b^4*d*e^2 + 4*(b^4*c - 6*a*b^3*d)*e*f - 3*(a*b^3*c - 5*a^2*b^2*d)*f^2)*h
)*x^2 + (4*a^2*b^2*d*e*f - (a^2*b^2*c + 3*a^3*b*d)*f^2)*g + (8*a^2*b^2*d*e
^2 + 4*(a^2*b^2*c - 6*a^3*b*d)*e*f - 3*(a^3*b*c - 5*a^4*d)*f^2)*h + 2*((4*
a*b^3*d*e*f - (a*b^3*c + 3*a^2*b^2*d)*f^2)*g + (8*a*b^3*d*e^2 + 4*(a*b^3*c
 - 6*a^2*b^2*d)*e*f - 3*(a^2*b^2*c - 5*a^3*b*d)*f^2)*h)*x)*log((b*f*x + 2*
b*e - a*f + 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*(8*(b^5*d*
e^2 - 2*a*b^4*d*e*f + a^2*b^3*d*f^2)*h*x^2 - (2*(b^5*c + a*b^4*d)*e^2 - (3
*a*b^4*c + 5*a^2*b^3*d)*e*f + (a^2*b^3*c + 3*a^3*b^2*d)*f^2)*g - (2*(a*b^4
*c - 7*a^2*b^3*d)*e^2 - (5*a^2*b^3*c - 29*a^3*b^2*d)*e*f + 3*(a^3*b^2*c -
5*a^4*b*d)*f^2)*h - ((4*b^5*d*e^2 + (b^5*c - 9*a*b^4*d)*e*f - (a*b^4*c - 5
*a^2*b^3*d)*f^2)*g + (4*(b^5*c - 6*a*b^4*d)*e^2 - (9*a*b^4*c - 49*a^2*b^3*
d)*e*f + 5*(a^2*b^3*c - 5*a^3*b^2*d)*f^2)*h)*x)*sqrt(f*x + e))/(a^2*b^6*e^
2 - 2*a^3*b^5*e*f + a^4*b^4*f^2 + (b^8*e^2 - 2*a*b^7*e*f + a^2*b^6*f^2)*x^
2 + 2*(a*b^7*e^2 - 2*a^2*b^6*e*f + a^3*b^5*f^2)*x), 1/4*(sqrt(-b^2*e + a*b
*f)*((((4*b^4*d*e*f - (b^4*c + 3*a*b^3*d)*f^2)*g + (8*b^4*d*e^2 + 4*(b^4*c
- 6*a*b^3*d)*e*f - 3*(a*b^3*c - 5*a^2*b^2*d)*f^2)*h)*x^2 + (4*a^2*b^2*d*e*
f - (a^2*b^2*c + 3*a^3*b*d)*f^2)*g + (8*a^2*b^2*d*e^2 + 4*(a^2*b^2*c - 6*a
^3*b*d)*e*f - 3*(a^3*b*c - 5*a^4*d)*f^2)*h + 2*((4*a*b^3*d*e*f - (a*b^3*c
+ 3*a^2*b^2*d)*f^2)*g + (8*a*b^3*d*e^2 + 4*(a*b^3*c - 6*a^2*b^2*d)*e*f ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(223) = 446$.

Time = 0.14 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$
$$= \frac{(4\,b^2defg - b^2cf^2g - 3\,abdf^2g + 8\,b^2de^2h + 4\,b^2cefh - 24\,abdefh - 3\,abcf^2h + 15\,a^2df^2h)\arctan\left(\frac{\sqrt{f}}{\sqrt{-b^2}}\right.}{4\,(b^4e - ab^3f)\sqrt{-b^2e + abf}}$$
$$+ \frac{2\sqrt{fx+e}dh}{b^3}$$
$$- \frac{4\,(fx+e)^{\frac{3}{2}}b^3defg - 4\sqrt{fx+e}b^3de^2fg + (fx+e)^{\frac{3}{2}}b^3cf^2g - 5\,(fx+e)^{\frac{3}{2}}ab^2df^2g + \sqrt{fx+e}b^3cef^2g}{}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(4*b^2*d*e*f*g - b^2*c*f^2*g - 3*a*b*d*f^2*g + 8*b^2*d*e^2*h + 4*b^2*c
*e*f*h - 24*a*b*d*e*f*h - 3*a*b*c*f^2*h + 15*a^2*d*f^2*h)*arctan(sqrt(f*x
+ e)*b/sqrt(-b^2*e + a*b*f))/((b^4*e - a*b^3*f)*sqrt(-b^2*e + a*b*f)) + 2*
sqrt(f*x + e)*d*h/b^3 - 1/4*(4*(f*x + e)^(3/2)*b^3*d*e*f*g - 4*sqrt(f*x +
e)*b^3*d*e^2*f*g + (f*x + e)^(3/2)*b^3*c*f^2*g - 5*(f*x + e)^(3/2)*a*b^2*d
*f^2*g + sqrt(f*x + e)*b^3*c*e*f^2*g + 7*sqrt(f*x + e)*a*b^2*d*e*f^2*g - s
qrt(f*x + e)*a*b^2*c*f^3*g - 3*sqrt(f*x + e)*a^2*b*d*f^3*g + 4*(f*x + e)^(
3/2)*b^3*c*e*f*h - 8*(f*x + e)^(3/2)*a*b^2*d*e*f*h - 4*sqrt(f*x + e)*b^3*c
*e^2*f*h + 8*sqrt(f*x + e)*a*b^2*d*e^2*f*h - 5*(f*x + e)^(3/2)*a*b^2*c*f^2
*h + 9*(f*x + e)^(3/2)*a^2*b*d*f^2*h + 7*sqrt(f*x + e)*a*b^2*c*e*f^2*h - 1
5*sqrt(f*x + e)*a^2*b*d*e*f^2*h - 3*sqrt(f*x + e)*a^2*b*c*f^3*h + 7*sqrt(f
*x + e)*a^3*d*f^3*h)/((b^4*e - a*b^3*f)*((f*x + e)*b - b*e + a*f)^2)
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,\left(b^2\,c\,f^2\,g-15\,a^2\,d\,f^2\,h-8\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g-4\,b^2\,c\,e\,f\,h-4\,b^2\,d\,e\,f\,g}{4\,b^{7/2}\,{\left(a\,f-b\,e\right)}^{3/2}}$$
$$-\frac{\sqrt{e+f\,x}\,\left(\frac{b^2\,c\,f^2\,g}{4}-\frac{7\,a^2\,d\,f^2\,h}{4}+\frac{3\,a\,b\,c\,f^2\,h}{4}+\frac{3\,a\,b\,d\,f^2\,g}{4}-b^2\,c\,e\,f\,h-b^2\,d\,e\,f\,g+2\,a\,b\,d\,e\,f\,h\right)-\frac{{\left(e+f\,x\right)}^{3/2}\,(b}{}}{b^5\,{\left(e+f\,x\right)}^2-\left(e+f\,x\right)\,\left(2\,b^5\,e-2\,a\,b^4\,f\right)+b^5\,e^2+a^2}$$
$$+\frac{2\,d\,h\,\sqrt{e+f\,x}}{b^3}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x))/(a + b*x)^3,x)
```

output
```
(atan((b^(1/2)*(e + f*x)^(1/2))/(a*f - b*e)^(1/2))*(b^2*c*f^2*g - 15*a^2*d
*f^2*h - 8*b^2*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 4*b^2*c*e*f*h - 4
*b^2*d*e*f*g + 24*a*b*d*e*f*h))/(4*b^(7/2)*(a*f - b*e)^(3/2)) - ((e + f*x)
^(1/2)*((b^2*c*f^2*g)/4 - (7*a^2*d*f^2*h)/4 + (3*a*b*c*f^2*h)/4 + (3*a*b*d
*f^2*g)/4 - b^2*c*e*f*h - b^2*d*e*f*g + 2*a*b*d*e*f*h) - ((e + f*x)^(3/2)*
(b^3*c*f^2*g - 5*a*b^2*c*f^2*h - 5*a*b^2*d*f^2*g + 9*a^2*b*d*f^2*h + 4*b^3
*c*e*f*h + 4*b^3*d*e*f*g - 8*a*b^2*d*e*f*h))/(4*(a*f - b*e)))/(b^5*(e + f*
x)^2 - (e + f*x)*(2*b^5*e - 2*a*b^4*f) + b^5*e^2 + a^2*b^3*f^2 - 2*a*b^4*e
*f) + (2*d*h*(e + f*x)^(1/2))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1706, normalized size of antiderivative = 6.96

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**4*d*f**2*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**3*b*c*f**2*h + 24*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d*e*f*h + 3*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d*
f**2*g - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**3*b*d*f**2*h*x - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*e*f*h + sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*f**2*g +
 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**2*b**2*c*f**2*h*x - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*e**2*h - 4*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*e*f*g + 48*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**2*b**2*d*e*f*h*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*f**2*g*x - 15*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*f**2*h*x**2
- 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a*b**3*c*e*f*h*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a*b**3*c*f**2*g*x + 3*sqrt(b)*sqrt(a*f - b*e)*...
```

3.8 $\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 322

$$\begin{aligned}\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx &= -\frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{3b^3(a+bx)^3}\\ &-\frac{\left(13a^2dfh+b^2(6deg+cfg+6ceh)-ab(7dfg+12deh+7cfh)\right)\sqrt{e+fx}}{12b^3(be-af)(a+bx)^2}\\ &-\frac{\left(11a^2df^2h-abf(dfg+20deh+cfh)-b^2(cf(fg-2eh)-2de(fg+4eh))\right)\sqrt{e+fx}}{8b^3(be-af)^2(a+bx)}\\ &-\frac{f\left(5a^2df^2h+abf(dfg-12deh+cfh)-b^2(2de(fg-4eh)-cf(fg-2eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{8b^{7/2}(be-af)^{5/2}}\end{aligned}$$

output

```
-1/3*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/(b*x+a)^3-1/12*(13*a^2*d*f*h+
b^2*(6*c*e*h+c*f*g+6*d*e*g)-a*b*(7*c*f*h+12*d*e*h+7*d*f*g))*(f*x+e)^(1/2)/
b^3/(-a*f+b*e)/(b*x+a)^2-1/8*(11*a^2*d*f^2*h-a*b*f*(c*f*h+20*d*e*h+d*f*g)-
b^2*(c*f*(-2*e*h+f*g)-2*d*e*(4*e*h+f*g)))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^2/(
b*x+a)-1/8*f*(5*a^2*d*f^2*h+a*b*f*(c*f*h-12*d*e*h+d*f*g)-b^2*(2*d*e*(-4*e*
h+f*g)-c*f*(-2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/
b^(7/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx =$$

$$-\sqrt{e+fx}(15a^4df^2h+a^3bf(3cfh+d(3fg-26eh+40fhx))+b^4(6dex(fgx+2e(g+2hx))+c(-3f^{\cdots}$$

$$+\frac{f(5a^2df^2h+abf(dfg-12deh+cfh)+b^2(cf(fg-2eh)+2de(-fg+4eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{8b^{7/2}(-be+af)^{5/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^4,x]
```

output
```
-1/24*(Sqrt[e + f*x]*(15*a^4*d*f^2*h + a^3*b*f*(3*c*f*h + d*(3*f*g - 26*e*
h + 40*f*h*x)) + b^4*(6*d*e*x*(f*g*x + 2*e*(g + 2*h*x)) + c*(-3*f^2*g*x^2
+ 2*e*f*x*(g + 3*h*x) + 4*e^2*(2*g + 3*h*x))) + a*b^3*(c*(4*e^2*h - 14*e*f
*(g + h*x) - f^2*x*(8*g + 3*h*x)) + d*(-3*f^2*g*x^2 + 4*e^2*(g + 6*h*x) -
2*e*f*x*(7*g + 30*h*x))) + a^2*b^2*(c*f*(3*f*g - 4*e*h + 8*f*h*x) + d*(8*e
^2*h + f^2*x*(8*g + 33*h*x) - 2*e*f*(2*g + 35*h*x)))))/(b^3*(b*e - a*f)^2*
(a + b*x)^3) + (f*(5*a^2*d*f^2*h + a*b*f*(d*f*g - 12*d*e*h + c*f*h) + b^2*
(c*f*(f*g - 2*e*h) + 2*d*e*(-(f*g) + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x
])/Sqrt[-(b*e) + a*f]])/(8*b^(7/2)*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {162, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx$$

$$\Big\downarrow \; 162$$

$$\frac{\left(5a^2df^2h+abf(cfh-12deh+dfg)-\left(b^2(2de(fg-4eh)-cf(fg-2eh))\right)\right)\int \frac{\sqrt{e+fx}}{(a+bx)^2}dx}{8b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+3bx\left(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg)\right)+a^2b(cfh-8deh+dfg)\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 51

$$\frac{\left(5a^2df^2h+abf(cfh-12deh+dfg)-\left(b^2(2de(fg-4eh)-cf(fg-2eh))\right)\right)\left(\frac{f\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2b}-\frac{\sqrt{e+fx}}{b(a+bx)}\right)}{8b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+3bx\left(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg)\right)+a^2b(cfh-8deh+dfg)\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 73

$$\frac{\left(5a^2df^2h+abf(cfh-12deh+dfg)-\left(b^2(2de(fg-4eh)-cf(fg-2eh))\right)\right)\left(\frac{\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b}-\frac{\sqrt{e+fx}}{b(a+bx)}\right)}{8b^2(be-af)^2}$$
$$\frac{(e+fx)^{3/2}\left(5a^3dfh+3bx\left(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg)\right)+a^2b(cfh-8deh+dfg)\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 221

$$\frac{\left(-\frac{f\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}\sqrt{be-af}}-\frac{\sqrt{e+fx}}{b(a+bx)}\right)\left(5a^2df^2h+abf(cfh-12deh+dfg)-\left(b^2(2de(fg-4eh)-cf(fg-2eh))\right)\right)}{8b^2(be-af)^2}$$
$$\frac{(e+fx)^{3/2}\left(5a^3dfh+3bx\left(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg)\right)+a^2b(cfh-8deh+dfg)\right.}{12b^2(a+bx)^3(be-af)^2}$$

input
```
Int[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^4,x]
```

output

```
-1/12*((e + f*x)^(3/2)*(4*b^3*c*e*g + 5*a^3*d*f*h + 2*a*b^2*(d*e*g - (7*c*
f*g)/2 + c*e*h) + a^2*b*(d*f*g - 8*d*e*h + c*f*h) + 3*b*(3*a^2*d*f*h + b^2
*(2*d*e*g - c*f*g + 2*c*e*h) - a*b*(d*f*g + 4*d*e*h + c*f*h))*x))/(b^2*(b*
e - a*f)^2*(a + b*x)^3) + ((5*a^2*d*f^2*h + a*b*f*(d*f*g - 12*d*e*h + c*f*
h) - b^2*(2*d*e*(f*g - 4*e*h) - c*f*(f*g - 2*e*h)))*(-(Sqrt[e + f*x]/(b*(a
 + b*x))) - (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(3/2)*
Sqrt[b*e - a*f])))/(8*b^2*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$-\frac{5\left(\left(\left(\left(a^{2}\left(\frac{8}{3}abx+\frac{11}{5}b^{2}x^{2}+a^{2}\right)f^{2}-\frac{26a\left(\frac{30}{13}b^{2}x^{2}+\frac{35}{13}abx+a^{2}\right)bef}{15}+\frac{8b^{2}e^{2}\left(3b^{2}x^{2}+3abx+a^{2}\right)}{15}\right)h+\frac{\left(a(3bx+a)\left(-\frac{bx}{3}+a\right)f^{2}-\frac{4(}{}\right.}{}\right.\right.\right.}{}$
derivativedivides	$2f\left(-\frac{\frac{\left(11a^{2}df^{2}h-abcf^{2}h-20abdefh-abdf^{2}g+2b^{2}cefh-b^{2}cf^{2}g+8b^{2}de^{2}h+2b^{2}defg\right)(fx+e)^{\frac{5}{2}}}{16b\left(a^{2}f^{2}-2abfe+b^{2}e^{2}\right)}+\frac{\left(5a^{2}df^{2}h+abcf^{2}h-12a\right.}{}}{((}\right.$
default	$2f\left(-\frac{\frac{\left(11a^{2}df^{2}h-abcf^{2}h-20abdefh-abdf^{2}g+2b^{2}cefh-b^{2}cf^{2}g+8b^{2}de^{2}h+2b^{2}defg\right)(fx+e)^{\frac{5}{2}}}{16b\left(a^{2}f^{2}-2abfe+b^{2}e^{2}\right)}+\frac{\left(5a^{2}df^{2}h+abcf^{2}h-12a\right.}{}}{((}\right.$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*b)^(1/2)*(-(((a^2*(8/3*a*b*x+11/5*b^2*x^2+a^2)*f^2-26/15*a*
(30/13*b^2*x^2+35/13*a*b*x+a^2)*b*e*f+8/15*b^2*e^2*(3*b^2*x^2+3*a*b*x+a^2)
)*h+1/5*(a*(3*b*x+a)*(-1/3*b*x+a)*f^2-4/3*(-3/2*b^2*x^2+7/2*a*b*x+a^2)*b*e
*f+4/3*b^2*e^2*(3*b*x+a))*b*g)*d+1/5*c*b*((a*(3*b*x+a)*(-1/3*b*x+a)*f^2-4/
3*(-3/2*b^2*x^2+7/2*a*b*x+a^2)*b*e*f+4/3*b^2*e^2*(3*b*x+a))*h+((1/3*b*x+a)
*f-2/3*b*e)*b*g*((-3*b*x+a)*f-4*b*e)))*((a*f-b*e)*b)^(1/2)*(f*x+e)^(1/2)+(
((a^2*f^2-12/5*a*b*f*e+8/5*b^2*e^2)*h+1/5*b*f*g*(a*f-2*b*e))*d+1/5*c*((a*f
-2*b*e)*h+b*f*g)*b*f)*(b*x+a)^3*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/
2)))/(b*x+a)^3/(a*f-b*e)^2/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. 2(298) = 596.

Time = 0.21 (sec) , antiderivative size = 2256, normalized size of antiderivative = 7.01

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/48*(3*(((2*b^5*d*e*f^2 - (b^5*c + a*b^4*d)*f^3)*g - (8*b^5*d*e^2*f - 2
*(b^5*c + 6*a*b^4*d)*e*f^2 + (a*b^4*c + 5*a^2*b^3*d)*f^3)*h)*x^3 + 3*((2*a
*b^4*d*e*f^2 - (a*b^4*c + a^2*b^3*d)*f^3)*g - (8*a*b^4*d*e^2*f - 2*(a*b^4*
c + 6*a^2*b^3*d)*e*f^2 + (a^2*b^3*c + 5*a^3*b^2*d)*f^3)*h)*x^2 + (2*a^3*b^
2*d*e*f^2 - (a^3*b^2*c + a^4*b*d)*f^3)*g - (8*a^3*b^2*d*e^2*f - 2*(a^3*b^2
*c + 6*a^4*b*d)*e*f^2 + (a^4*b*c + 5*a^5*d)*f^3)*h + 3*((2*a^2*b^3*d*e*f^2
 - (a^2*b^3*c + a^3*b^2*d)*f^3)*g - (8*a^2*b^3*d*e^2*f - 2*(a^2*b^3*c + 6*
a^3*b^2*d)*e*f^2 + (a^3*b^2*c + 5*a^4*b*d)*f^3)*h)*x)*sqrt(b^2*e - a*b*f)*
log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a))
 + 2*(3*((2*b^6*d*e^2*f - (b^6*c + 3*a*b^5*d)*e*f^2 + (a*b^5*c + a^2*b^4*d
)*f^3)*g + (8*b^6*d*e^3 + 2*(b^6*c - 14*a*b^5*d)*e^2*f - (3*a*b^5*c - 31*a
^2*b^4*d)*e*f^2 + (a^2*b^4*c - 11*a^3*b^3*d)*f^3)*h)*x^2 + (4*(2*b^6*c + a
*b^5*d)*e^3 - 2*(11*a*b^5*c + 4*a^2*b^4*d)*e^2*f + (17*a^2*b^4*c + 7*a^3*b
^3*d)*e*f^2 - 3*(a^3*b^3*c + a^4*b^2*d)*f^3)*g + (4*(a*b^5*c + 2*a^2*b^4*d
)*e^3 - 2*(4*a^2*b^4*c + 17*a^3*b^3*d)*e^2*f + (7*a^3*b^3*c + 41*a^4*b^2*d
)*e*f^2 - 3*(a^4*b^2*c + 5*a^5*b*d)*f^3)*h + 2*((6*b^6*d*e^3 + (b^6*c - 13
*a*b^5*d)*e^2*f - (5*a*b^5*c - 11*a^2*b^4*d)*e*f^2 + 4*(a^2*b^4*c - a^3*b^
3*d)*f^3)*g + (6*(b^6*c + 2*a*b^5*d)*e^3 - (13*a*b^5*c + 47*a^2*b^4*d)*e^2
*f + 11*(a^2*b^4*c + 5*a^3*b^3*d)*e*f^2 - 4*(a^3*b^3*c + 5*a^4*b^2*d)*f^3)
*h)*x)*sqrt(f*x + e))/(a^3*b^7*e^3 - 3*a^4*b^6*e^2*f + 3*a^5*b^5*e*f^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(298) = 596.

Time = 0.14 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.65

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/8*(2*b^2*d*e*f^2*g - b^2*c*f^3*g - a*b*d*f^3*g - 8*b^2*d*e^2*f*h + 2*b^
2*c*e*f^2*h + 12*a*b*d*e*f^2*h - a*b*c*f^3*h - 5*a^2*d*f^3*h)*arctan(sqrt(
f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*e^2 - 2*a*b^4*e*f + a^2*b^3*f^2)*sq
rt(-b^2*e + a*b*f)) - 1/24*(6*(f*x + e)^(5/2)*b^4*d*e*f^2*g - 6*sqrt(f*x +
 e)*b^4*d*e^3*f^2*g - 3*(f*x + e)^(5/2)*b^4*c*f^3*g - 3*(f*x + e)^(5/2)*a*
b^3*d*f^3*g + 8*(f*x + e)^(3/2)*b^4*c*e*f^3*g - 8*(f*x + e)^(3/2)*a*b^3*d*
e*f^3*g + 3*sqrt(f*x + e)*b^4*c*e^2*f^3*g + 15*sqrt(f*x + e)*a*b^3*d*e^2*f
^3*g - 8*(f*x + e)^(3/2)*a*b^3*c*f^4*g + 8*(f*x + e)^(3/2)*a^2*b^2*d*f^4*g
 - 6*sqrt(f*x + e)*a*b^3*c*e*f^4*g - 12*sqrt(f*x + e)*a^2*b^2*d*e*f^4*g +
3*sqrt(f*x + e)*a^2*b^2*c*f^5*g + 3*sqrt(f*x + e)*a^3*b*d*f^5*g + 24*(f*x
+ e)^(5/2)*b^4*d*e^2*f*h - 48*(f*x + e)^(3/2)*b^4*d*e^3*f*h + 24*sqrt(f*x
+ e)*b^4*d*e^4*f*h + 6*(f*x + e)^(5/2)*b^4*c*e*f^2*h - 60*(f*x + e)^(5/2)*
a*b^3*d*e*f^2*h + 144*(f*x + e)^(3/2)*a*b^3*d*e^2*f^2*h - 6*sqrt(f*x + e)*
b^4*c*e^3*f^2*h - 84*sqrt(f*x + e)*a*b^3*d*e^3*f^2*h - 3*(f*x + e)^(5/2)*a
*b^3*c*f^3*h + 33*(f*x + e)^(5/2)*a^2*b^2*d*f^3*h - 8*(f*x + e)^(3/2)*a*b^
3*c*e*f^3*h - 136*(f*x + e)^(3/2)*a^2*b^2*d*e*f^3*h + 15*sqrt(f*x + e)*a*b
^3*c*e^2*f^3*h + 111*sqrt(f*x + e)*a^2*b^2*d*e^2*f^3*h + 8*(f*x + e)^(3/2)
*a^2*b^2*c*f^4*h + 40*(f*x + e)^(3/2)*a^3*b*d*f^4*h - 12*sqrt(f*x + e)*a^2
*b^2*c*e*f^4*h - 66*sqrt(f*x + e)*a^3*b*d*e*f^4*h + 3*sqrt(f*x + e)*a^3*b*
c*f^5*h + 15*sqrt(f*x + e)*a^4*d*f^5*h)/((b^5*e^2 - 2*a*b^4*e*f + a^2*b...
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \frac{f\,\mathrm{atan}\left(\frac{\sqrt{b}\,f\,\sqrt{e+f\,x}\,\left(b^2\,c\,f^2\,g+5\,a^2\,d\,f^2\,h+8\,b^2\,d\,e^2\,h+a\,b\,c\,f^2\,h+a\,b\,d\,f^2\,g-2\,b^2\,c\,e\,f\,h-2\,b^2\,d\,e\,f\,g-12\,a\,b\,d\,e\,f\,h\right)}{\sqrt{a\,f-b\,e}\,\left(b^2\,c\,f^3\,g+5\,a^2\,d\,f^3\,h-2\,b^2\,c\,e\,f^2\,h-2\,b^2\,d\,e\,f^2\,g+8\,b^2\,d\,e^2\,f\,h+a\,b\,c\,f^3\,h+a\,b\,d\,f^3\,g-12\,a\,b\,d\,e\,f^2\,h\right)}\right)\,\left(b^2\,c\,f^2\,g+5\,a^2 \cdots\right.}{8\,b^{7/2}\,{\left(a\,f-b\,e\right)}^{5/2}} - \frac{\frac{\sqrt{e+f\,x}\,\left(b^2\,c\,f^3\,g+5\,a^2\,d\,f^3\,h-2\,b^2\,c\,e\,f^2\,h-2\,b^2\,d\,e\,f^2\,g+8\,b^2\,d\,e^2\,f\,h+a\,b\,c\,f^3\,h+a\,b\,d\,f^3\,g-12\,a\,b\,d\,e\,f^2\,h\right)}{8\,b^3} - \frac{{\left(e+f\,x\right)}^{5/2}\,\left(b^2\,c\,f^3\,g-11\,a^2 \cdots\right.}{\cdots}}{\left(e+f\,x\right)\,\left(3\,a^2\,b\,f^2-6\,a\,b^2\,e\,f+3\,b^3\,e^2\right)+b^3\,\left(e+f \cdots\right.}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x))/(a + b*x)^4,x)
```

output
```
(f*atan((b^(1/2)*f*(e + f*x)^(1/2)*(b^2*c*f^2*g + 5*a^2*d*f^2*h + 8*b^2*d*
e^2*h + a*b*c*f^2*h + a*b*d*f^2*g - 2*b^2*c*e*f*h - 2*b^2*d*e*f*g - 12*a*b
*d*e*f*h))/((a*f - b*e)^(1/2)*(b^2*c*f^3*g + 5*a^2*d*f^3*h - 2*b^2*c*e*f^2
*h - 2*b^2*d*e*f^2*g + 8*b^2*d*e^2*f*h + a*b*c*f^3*h + a*b*d*f^3*g - 12*a*
b*d*e*f^2*h)))*(b^2*c*f^2*g + 5*a^2*d*f^2*h + 8*b^2*d*e^2*h + a*b*c*f^2*h
+ a*b*d*f^2*g - 2*b^2*c*e*f*h - 2*b^2*d*e*f*g - 12*a*b*d*e*f*h))/(8*b^(7/2
)*(a*f - b*e)^(5/2)) - (((e + f*x)^(1/2)*(b^2*c*f^3*g + 5*a^2*d*f^3*h - 2*
b^2*c*e*f^2*h - 2*b^2*d*e*f^2*g + 8*b^2*d*e^2*f*h + a*b*c*f^3*h + a*b*d*f^
3*g - 12*a*b*d*e*f^2*h))/(8*b^3) - ((e + f*x)^(5/2)*(b^2*c*f^3*g - 11*a^2*
d*f^3*h - 2*b^2*c*e*f^2*h - 2*b^2*d*e*f^2*g - 8*b^2*d*e^2*f*h + a*b*c*f^3*
h + a*b*d*f^3*g + 20*a*b*d*e*f^2*h))/(8*b*(a*f - b*e)^2) + ((e + f*x)^(3/2
)*(5*a^2*d*f^3*h - b^2*c*f^3*g + 6*b^2*d*e^2*f*h + a*b*c*f^3*h + a*b*d*f^3
*g - 12*a*b*d*e*f^2*h))/(3*b^2*(a*f - b*e)))/((e + f*x)*(3*b^3*e^2 + 3*a^2
*b*f^2 - 6*a*b^2*e*f) + b^3*(e + f*x)^3 - (e + f*x)^2*(3*b^3*e - 3*a*b^2*f
) + a^3*f^3 - b^3*e^3 + 3*a*b^2*e^2*f - 3*a^2*b*e*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2619, normalized size of antiderivative = 8.13

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x)
```

output

```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**5*d*f**3*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**4*b*c*f**3*h - 36*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d*e*f**2*h + 3*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d*
f**3*g + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**4*b*d*f**3*h*x - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*e*f**2*h + 3*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*f**
3*g + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**3*b**2*c*f**3*h*x + 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e**2*f*h - 6*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e*f
**2*g - 108*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**3*b**2*d*e*f**2*h*x + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*f**3*g*x + 45*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2
*d*f**3*h*x**2 - 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**2*b**3*c*e*f**2*h*x + 9*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c*f**3*g*x + 9...
```

3.9 $\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5} dx$

Optimal result

Integrand size = 27, antiderivative size = 422

$$\begin{aligned}\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5} dx = &-\frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{4b^3(a+bx)^4} \\ &-\frac{(17a^2dfh+b^2(8deg+cfg+8ceh)-ab(9dfg+16deh+9cfh))\sqrt{e+fx}}{24b^3(be-af)(a+bx)^3} \\ &-\frac{(59a^2df^2h-abf(3dfg+112deh+3cfh)-b^2(cf(5fg-8eh)-8de(fg+6eh)))\sqrt{e+fx}}{96b^3(be-af)^2(a+bx)^2} \\ &-\frac{f(5a^2df^2h+abf(3dfg-16deh+3cfh)+b^2(cf(5fg-8eh)-8de(fg-2eh)))\sqrt{e+fx}}{64b^3(be-af)^3(a+bx)} \\ &+\frac{f^2(5a^2df^2h+abf(3dfg-16deh+3cfh)+b^2(cf(5fg-8eh)-8de(fg-2eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{64b^{7/2}(be-af)^{7/2}}\end{aligned}$$

output
```
-1/4*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/(b*x+a)^4-1/24*(17*a^2*d*f*h+
b^2*(8*c*e*h+c*f*g+8*d*e*g)-a*b*(9*c*f*h+16*d*e*h+9*d*f*g))*(f*x+e)^(1/2)/
b^3/(-a*f+b*e)/(b*x+a)^3-1/96*(59*a^2*d*f^2*h-a*b*f*(3*c*f*h+112*d*e*h+3*d
*f*g)-b^2*(c*f*(-8*e*h+5*f*g)-8*d*e*(6*e*h+f*g)))*(f*x+e)^(1/2)/b^3/(-a*f+
b*e)^2/(b*x+a)^2-1/64*f*(5*a^2*d*f^2*h+a*b*f*(3*c*f*h-16*d*e*h+3*d*f*g)+b^
2*(c*f*(-8*e*h+5*f*g)-8*d*e*(-2*e*h+f*g)))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^3/
(b*x+a)+1/64*f^2*(5*a^2*d*f^2*h+a*b*f*(3*c*f*h-16*d*e*h+3*d*f*g)+b^2*(c*f*
(-8*e*h+5*f*g)-8*d*e*(-2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*
e)^(1/2))/b^(7/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 3.36 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx$$

$$= \frac{\sqrt{b}\sqrt{e+fx}\left(-15a^5df^3h-a^4bf^2(9cfh+d(9fg-38eh+55fhx))+b^5\left(8dex\left(-3f^2gx^2+2efx(g+3hx)+4e^2(2g+3hx)\right)+c\left(15f^3gx^3+8e^2fx(g+2hx)\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^5,x]
```

output
```
((Sqrt[b]*Sqrt[e + f*x]*(-15*a^5*d*f^3*h - a^4*b*f^2*(9*c*f*h + d*(9*f*g -
 38*e*h + 55*f*h*x)) + b^5*(8*d*e*x*(-3*f^2*g*x^2 + 2*e*f*x*(g + 3*h*x) +
4*e^2*(2*g + 3*h*x)) + c*(15*f^3*g*x^3 + 8*e^2*f*x*(g + 2*h*x) + 16*e^3*(3
*g + 4*h*x) - 2*e*f^2*x^2*(5*g + 12*h*x))) + a*b^4*(d*(9*f^3*g*x^3 + 16*e^
3*(g + 4*h*x) - 8*e^2*f*x*(21*g + 22*h*x) - 2*e*f^2*x^2*(47*g + 24*h*x)) +
 c*(16*e^3*h + f^3*x^2*(55*g + 9*h*x) - 8*e^2*f*(17*g + 21*h*x) - 2*e*f^2*
x*(18*g + 47*h*x))) - a^3*b^2*f*(3*c*f*(5*f*g - 6*e*h + 11*f*h*x) + d*(24*
e^2*h - 2*e*f*(9*g + 70*h*x) + f^2*x*(33*g + 73*h*x))) + a^2*b^3*(c*f*(-40
*e^2*h + f^2*x*(73*g + 33*h*x) + 2*e*f*(59*g + 46*h*x)) + d*(16*e^3*h + 3*
f^3*x^2*(11*g + 5*h*x) - 8*e^2*f*(5*g + 13*h*x) + 2*e*f^2*x*(46*g + 99*h*x
)))))/((-(b*e) + a*f)^3*(a + b*x)^4) + (3*f^2*(5*a^2*d*f^2*h + a*b*f*(3*d*
f*g - 16*d*e*h + 3*c*f*h) + b^2*(c*f*(5*f*g - 8*e*h) + 8*d*e*(-(f*g) + 2*e
*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e) + a*f)^(
7/2))/(192*b^(7/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {162, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx$$

$\big\downarrow$ 162

$$\frac{\left(5a^2df^2h+abf(3cfh-16deh+3dfg)+b^2(cf(5fg-8eh)-8de(fg-2eh))\right)\int \frac{\sqrt{e+fx}}{(a+bx)^3}dx}{16b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(11a^2dfh-ab(3cfh+16deh+3dfg)+b^2(8ceh-5cfg+8deg)\right)-2a^2b\left(5deh-\frac{3}{2}f(c\right.\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\big\downarrow$ 51

$$\frac{\left(5a^2df^2h+abf(3cfh-16deh+3dfg)+b^2(cf(5fg-8eh)-8de(fg-2eh))\right)\left(\frac{f\int \frac{1}{(a+bx)^2\sqrt{e+fx}}dx}{4b}-\frac{\sqrt{e+fx}}{2b(a+bx)^2}\right)}{16b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(11a^2dfh-ab(3cfh+16deh+3dfg)+b^2(8ceh-5cfg+8deg)\right)-2a^2b\left(5deh-\frac{3}{2}f(c\right.\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\big\downarrow$ 52

$$\frac{\left(5a^2df^2h+abf(3cfh-16deh+3dfg)+b^2(cf(5fg-8eh)-8de(fg-2eh))\right)\left(\frac{f\left(-\frac{f\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2(be-af)}-\frac{\sqrt{e+fx}}{(a+bx)(be-af\right.}{4b}\right.}{16b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(11a^2dfh-ab(3cfh+16deh+3dfg)+b^2(8ceh-5cfg+8deg)\right)-2a^2b\left(5deh-\frac{3}{2}f(c\right.\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\big\downarrow$ 73

$$\frac{\left(5a^2df^2h+abf(3cfh-16deh+3dfg)+b^2(cf(5fg-8eh)-8de(fg-2eh))\right)\left(\frac{f\left(-\frac{\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{be-af}-\frac{\sqrt{e}}{(a+bx)}\right.}{4b}\right.}{16b^2(be-af)^2}-\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(11a^2dfh-ab(3cfh+16deh+3dfg)+b^2(8ceh-5cfg+8deg)\right)-2a^2b\left(5deh-\frac{3}{2}f(c\right.\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\big\downarrow$ 221

$$\frac{\left(\frac{f\left(\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)}{4b}-\frac{\sqrt{e+fx}}{2b(a+bx)^2}\right)(5a^2df^2h+abf(3cfh-16deh+3dfg)+b^2(cf(5fg-8eh)}{16b^2(be-af)^2}$$

$$\frac{(e+fx)^{3/2}\left(5a^3dfh+bx\left(11a^2dfh-ab(3cfh+16deh+3dfg)+b^2(8ceh-5cfg+8deg)\right)-2a^2b\left(5deh-\frac{3}{2}f(c\right.\right.}{24b^2(a+bx)^4(be-af)^2}$$

input
```
Int[((c + d*x)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^5,x]
```

output
```
-1/24*((e + f*x)^(3/2)*(6*b^3*c*e*g + 5*a^3*d*f*h + 2*a*b^2*(d*e*g - (11*c
*f*g)/2 + c*e*h) - 2*a^2*b*(5*d*e*h - (3*f*(d*g + c*h))/2) + b*(11*a^2*d*f
*h + b^2*(8*d*e*g - 5*c*f*g + 8*c*e*h) - a*b*(3*d*f*g + 16*d*e*h + 3*c*f*h
))*x))/(b^2*(b*e - a*f)^2*(a + b*x)^4) + ((5*a^2*d*f^2*h + a*b*f*(3*d*f*g
- 16*d*e*h + 3*c*f*h) + b^2*(c*f*(5*f*g - 8*e*h) - 8*d*e*(f*g - 2*e*h)))*(
-1/2*Sqrt[e + f*x]/(b*(a + b*x)^2) + (f*(-(Sqrt[e + f*x]/((b*e - a*f)*(a +
 b*x))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b
*e - a*f)^(3/2))))/(4*b)))/(16*b^2*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{\frac{5(bx+a)^4 f^2\left(\left(cg\,f^2-\frac{8e(ch+dg)f}{5}+\frac{16d\,e^2h}{5}\right)b^2+\frac{3a\left(f(ch+dg)-\frac{16deh}{3}\right)fb}{5}+a^2d\,f^2h\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{64}-\frac{5\sqrt{fx+e}\,\sqrt{(af-b}}{}}{}$
derivativedivides	$2f^2\left(-\frac{-\frac{\left(5a^2d\,f^2h+3abc\,f^2h-16abdefh+3abd\,f^2g-8b^2cefh+5b^2c\,f^2g+16b^2d\,e^2h-8b^2defg\right)(fx+e)^{\frac{7}{2}}}{128\left(a^3f^3-3a^2be\,f^2+3a\,b^2e^2f-b^3e^3\right)}+\frac{\left(73a^2d\,f^2h-33ab\right.}{}}{}\right.$
default	$2f^2\left(-\frac{-\frac{\left(5a^2d\,f^2h+3abc\,f^2h-16abdefh+3abd\,f^2g-8b^2cefh+5b^2c\,f^2g+16b^2d\,e^2h-8b^2defg\right)(fx+e)^{\frac{7}{2}}}{128\left(a^3f^3-3a^2be\,f^2+3a\,b^2e^2f-b^3e^3\right)}+\frac{\left(73a^2d\,f^2h-33ab\right.}{}}{}\right.$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/64/((a*f-b*e)*b)^(1/2)*((b*x+a)^4*f^2*((c*g*f^2-8/5*e*(c*h+d*g)*f+16/5*d
*e^2*h)*b^2+3/5*a*(f*(c*h+d*g)-16/3*d*e*h)*f*b+a^2*d*f^2*h)*arctan(b*(f*x+
e)^(1/2)/((a*f-b*e)*b)^(1/2))-(f*x+e)^(1/2)*((a*f-b*e)*b)^(1/2)*((-c*f^3*g
*x^3+2/3*x^2*(12/5*(c*h+d*g)*x+c*g)*e*f^2-8/15*x*(6*d*h*x^2+2*(c*h+d*g)*x+
c*g)*e^2*f-16/5*(2*d*h*x^2+4/3*(c*h+d*g)*x+c*g)*e^3)*b^5-16/15*a*(1/16*(9*
(c*h+d*g)*x^3+55*c*g*x^2)*f^3-9/4*(4/3*d*h*x^2+47/18*(c*h+d*g)*x+c*g)*x*e*
f^2-17/2*(22/17*d*h*x^2+21/17*(c*h+d*g)*x+c*g)*e^2*f+e^3*(4*d*h*x+c*h+d*g)
)*b^4+8/3*a^2*(1/8*(-3*d*h*x^3+33/5*(-c*h-d*g)*x^2-73/5*c*g*x)*f^3-59/20*e
*(99/59*d*h*x^2+46/59*(c*h+d*g)*x+c*g)*f^2+e^2*(13/5*d*h*x+c*h+d*g)*f-2/5*
d*e^3*h)*b^3-6/5*a^3*(1/6*(-73/3*d*h*x^2+11*(-c*h-d*g)*x-5*c*g)*f^2+e*(70/
9*d*h*x+c*h+d*g)*f-4/3*d*e^2*h)*f*b^2+3/5*a^4*((55/9*d*h*x+c*h+d*g)*f-38/9
*d*e*h)*f^2*b+a^5*d*f^3*h))/(b*x+a)^4/(a*f-b*e)^3/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. 2(398) = 796.

Time = 0.34 (sec) , antiderivative size = 3384, normalized size of antiderivative = 8.02

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(398) = 796$.

Time = 0.17 (sec) , antiderivative size = 1350, normalized size of antiderivative = 3.20

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x, algorithm="giac")
```

output
```
1/64*(8*b^2*d*e*f^3*g - 5*b^2*c*f^4*g - 3*a*b*d*f^4*g - 16*b^2*d*e^2*f^2*h
 + 8*b^2*c*e*f^3*h + 16*a*b*d*e*f^3*h - 3*a*b*c*f^4*h - 5*a^2*d*f^4*h)*arc
tan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^3 - 3*a*b^5*e^2*f + 3*a^
2*b^4*e*f^2 - a^3*b^3*f^3)*sqrt(-b^2*e + a*b*f)) + 1/192*(24*(f*x + e)^(7/
2)*b^5*d*e*f^3*g - 88*(f*x + e)^(5/2)*b^5*d*e^2*f^3*g + 40*(f*x + e)^(3/2)
*b^5*d*e^3*f^3*g + 24*sqrt(f*x + e)*b^5*d*e^4*f^3*g - 15*(f*x + e)^(7/2)*b
^5*c*f^4*g - 9*(f*x + e)^(7/2)*a*b^4*d*f^4*g + 55*(f*x + e)^(5/2)*b^5*c*e*
f^4*g + 121*(f*x + e)^(5/2)*a*b^4*d*e*f^4*g - 73*(f*x + e)^(3/2)*b^5*c*e^2
*f^4*g - 47*(f*x + e)^(3/2)*a*b^4*d*e^2*f^4*g - 15*sqrt(f*x + e)*b^5*c*e^3
*f^4*g - 81*sqrt(f*x + e)*a*b^4*d*e^3*f^4*g - 55*(f*x + e)^(5/2)*a*b^4*c*f
^5*g - 33*(f*x + e)^(5/2)*a^2*b^3*d*f^5*g + 146*(f*x + e)^(3/2)*a*b^4*c*e*
f^5*g - 26*(f*x + e)^(3/2)*a^2*b^3*d*e*f^5*g + 45*sqrt(f*x + e)*a*b^4*c*e^
2*f^5*g + 99*sqrt(f*x + e)*a^2*b^3*d*e^2*f^5*g - 73*(f*x + e)^(3/2)*a^2*b^
3*c*f^6*g + 33*(f*x + e)^(3/2)*a^3*b^2*d*f^6*g - 45*sqrt(f*x + e)*a^2*b^3*
c*e*f^6*g - 51*sqrt(f*x + e)*a^3*b^2*d*e*f^6*g + 15*sqrt(f*x + e)*a^3*b^2*
c*f^7*g + 9*sqrt(f*x + e)*a^4*b*d*f^7*g - 48*(f*x + e)^(7/2)*b^5*d*e^2*f^2
*h + 48*(f*x + e)^(5/2)*b^5*d*e^3*f^2*h + 48*(f*x + e)^(3/2)*b^5*d*e^4*f^2
*h - 48*sqrt(f*x + e)*b^5*d*e^5*f^2*h + 24*(f*x + e)^(7/2)*b^5*c*e*f^3*h +
 48*(f*x + e)^(7/2)*a*b^4*d*e*f^3*h - 88*(f*x + e)^(5/2)*b^5*c*e^2*f^3*h +
 32*(f*x + e)^(5/2)*a*b^4*d*e^2*f^3*h + 40*(f*x + e)^(3/2)*b^5*c*e^3*f^...
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 862, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx$$

$$= \frac{f^2 \operatorname{atan}\left(\frac{\sqrt{b}\,f^2\,\sqrt{e+f\,x}\,(5\,b^2\,c\,f^2\,g+5\,a^2\,d\,f^2\,h+16\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g-8\,b^2\,c\,e\,f\,h-8\,b^2\,d\,e\,f\,g-16\,a\,b\,d\,e\,f\,h)}{\sqrt{a\,f-b\,e}\,(5\,b^2\,c\,f^4\,g+5\,a^2\,d\,f^4\,h-8\,b^2\,c\,e\,f^3\,h-8\,b^2\,d\,e\,f^3\,g+16\,b^2\,d\,e^2\,f^2\,h+3\,a\,b\,c\,f^4\,h+3\,a\,b\,d\,f^4\,g-16\,a\,b\,d\,e\,f^3\,h)}\right)\,(5\,b^2\,c\,f^2}{64\,b^{7/2}\,(a\,f-}$$

$$-\frac{\frac{\sqrt{e+f\,x}\,(5\,b^2\,c\,f^4\,g+5\,a^2\,d\,f^4\,h-8\,b^2\,c\,e\,f^3\,h-8\,b^2\,d\,e\,f^3\,g+16\,b^2\,d\,e^2\,f^2\,h+3\,a\,b\,c\,f^4\,h+3\,a\,b\,d\,f^4\,g-16\,a\,b\,d\,e\,f^3\,h)}{64\,b^3}-\frac{(e+f\,x)^{7/2}\,(5\,b^2\,c\,f^4}{}}{b^4\,(e+f\,x)^4-(e+f\,x)}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x))/(a + b*x)^5,x)
```

output
```
(f^2*atan((b^(1/2)*f^2*(e + f*x)^(1/2)*(5*b^2*c*f^2*g + 5*a^2*d*f^2*h + 16
*b^2*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 8*b^2*c*e*f*h - 8*b^2*d*e*f
*g - 16*a*b*d*e*f*h))/((a*f - b*e)^(1/2)*(5*b^2*c*f^4*g + 5*a^2*d*f^4*h -
8*b^2*c*e*f^3*h - 8*b^2*d*e*f^3*g + 16*b^2*d*e^2*f^2*h + 3*a*b*c*f^4*h + 3
*a*b*d*f^4*g - 16*a*b*d*e*f^3*h))*(5*b^2*c*f^2*g + 5*a^2*d*f^2*h + 16*b^2
*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 8*b^2*c*e*f*h - 8*b^2*d*e*f*g -
 16*a*b*d*e*f*h))/(64*b^(7/2)*(a*f - b*e)^(7/2)) - (((e + f*x)^(1/2)*(5*b^
2*c*f^4*g + 5*a^2*d*f^4*h - 8*b^2*c*e*f^3*h - 8*b^2*d*e*f^3*g + 16*b^2*d*e
^2*f^2*h + 3*a*b*c*f^4*h + 3*a*b*d*f^4*g - 16*a*b*d*e*f^3*h))/(64*b^3) - (
(e + f*x)^(7/2)*(5*b^2*c*f^4*g + 5*a^2*d*f^4*h - 8*b^2*c*e*f^3*h - 8*b^2*d
*e*f^3*g + 16*b^2*d*e^2*f^2*h + 3*a*b*c*f^4*h + 3*a*b*d*f^4*g - 16*a*b*d*e
*f^3*h))/(64*(a*f - b*e)^3) + ((e + f*x)^(3/2)*(55*a^2*d*f^4*h - 73*b^2*c*
f^4*g + 40*b^2*c*e*f^3*h + 40*b^2*d*e*f^3*g + 48*b^2*d*e^2*f^2*h + 33*a*b*
c*f^4*h + 33*a*b*d*f^4*g - 176*a*b*d*e*f^3*h))/(192*b^2*(a*f - b*e)) - ((e
 + f*x)^(5/2)*(55*b^2*c*f^4*g - 73*a^2*d*f^4*h - 88*b^2*c*e*f^3*h - 88*b^2
*d*e*f^3*g + 48*b^2*d*e^2*f^2*h + 33*a*b*c*f^4*h + 33*a*b*d*f^4*g + 80*a*b
*d*e*f^3*h))/(192*b*(a*f - b*e)^2))/(b^4*(e + f*x)^4 - (e + f*x)^3*(4*b^4*
e - 4*a*b^3*f) - (e + f*x)*(4*b^4*e^3 - 4*a^3*b*f^3 + 12*a^2*b^2*e*f^2 - 1
2*a*b^3*e^2*f) + a^4*f^4 + b^4*e^4 + (e + f*x)^2*(6*b^4*e^2 + 6*a^2*b^2*f^
2 - 12*a*b^3*e*f) + 6*a^2*b^2*e^2*f^2 - 4*a*b^3*e^3*f - 4*a^3*b*e*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3715, normalized size of antiderivative = 8.80

$$\int \frac{(c+dx)\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**6*d*f**4*h + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**5*b*c*f**4*h - 48*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d*e*f**3*h + 9*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d*
f**4*g + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**5*b*d*f**4*h*x - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*e*f**3*h + 15*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*f
**4*g + 36*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**4*b**2*c*f**4*h*x + 48*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e**2*f**2*h - 24*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**
2*d*e*f**3*g - 192*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**4*b**2*d*e*f**3*h*x + 36*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*f**4*g*x + 90*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
**4*b**2*d*f**4*h*x**2 - 96*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**3*c*e*f**3*h*x + 60*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**3*c*f**...
```

3.10 $\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 579

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = -\frac{2(be-af)^3(de-cf)^2(fg-eh)(e+fx)^{3/2}}{3f^7}$$

$$+\frac{2(be-af)^2(de-cf)(bde(5fg-6eh)-bcf(3fg-4eh)-af(2dfg-3deh+cfh))(e+fx)^{5/2}}{5f^7}$$

$$-\frac{2(be-af)\left(a^2df^2(dfg-3deh+2cfh)+abf(3c^2f^2h-d^2e(8fg-15eh)+2cdf(3fg-8eh))-b^2(4c}{7f^7}$$

$$+\frac{2(a^3d^2f^3h+3a^2bdf^2(dfg-4deh+2cfh)+3ab^2f(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh))-b^3(}{9f^7}$$

$$+\frac{2b(3a^2d^2f^2h+3abdf(dfg-5deh+2cfh)+b^2(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)))\,(e+fx)}{11f^7}$$

$$+\frac{2b^2d(3adfh+b(dfg-6deh+2cfh))(e+fx)^{13/2}}{13f^7}+\frac{2b^3d^2h(e+fx)^{15/2}}{15f^7}$$

output
```
-2/3*(-a*f+b*e)^3*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(3/2)/f^7+2/5*(-a*f+b*e)
^2*(-c*f+d*e)*(b*d*e*(-6*e*h+5*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-3*d*e*
h+2*d*f*g))*(f*x+e)^(5/2)/f^7-2/7*(-a*f+b*e)*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d
*f*g)+a*b*f*(3*c^2*f^2*h-d^2*e*(-15*e*h+8*f*g)+2*c*d*f*(-8*e*h+3*f*g))-b^2
*(4*c*d*e*f*(-5*e*h+3*f*g)-5*d^2*e^2*(-3*e*h+2*f*g)-3*c^2*f^2*(-2*e*h+f*g)
))*(f*x+e)^(7/2)/f^7+2/9*(a^3*d^2*f^3*h+3*a^2*b*d*f^2*(2*c*f*h-4*d*e*h+d*f
*g)+3*a*b^2*f*(c^2*f^2*h-2*d^2*e*(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g))-b^3*
(4*c*d*e*f*(-5*e*h+2*f*g)-c^2*f^2*(-4*e*h+f*g)-10*d^2*e^2*(-2*e*h+f*g)))*(
f*x+e)^(9/2)/f^7+2/11*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(2*c*f*h-5*d*e*h+d*f*g)
+b^2*(c^2*f^2*h+2*c*d*f*(-5*e*h+f*g)-5*d^2*e*(-3*e*h+f*g)))*(f*x+e)^(11/2)
/f^7+2/13*b^2*d*(3*a*d*f*h+b*(2*c*f*h-6*d*e*h+d*f*g))*(f*x+e)^(13/2)/f^7+2
/15*b^3*d^2*h*(f*x+e)^(15/2)/f^7
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.46

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2(e+fx)^{3/2}\left(143a^3f^3(21c^2f^2(5fg-2eh+3fhx)+6cdf(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx))+\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(143*a^3*f^3*(21*c^2*f^2*(5*f*g - 2*e*h + 3*f*h*x) + 6*
c*d*f*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d^2*(-16*e
^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h
*x))) + 39*a^2*b*f^2*(33*c^2*f^2*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*
(7*g + 6*h*x)) + 22*c*d*f*(-16*e^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g
 + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x)) + d^2*(128*e^4*h + 35*f^4*x^3*(11*g +
 9*h*x) + 24*e^2*f^2*x*(11*g + 10*h*x) - 16*e^3*f*(11*g + 12*h*x) - 10*e*f
^3*x^2*(33*g + 28*h*x))) + 3*a*b^2*f*(143*c^2*f^2*(-16*e^3*h + 24*e^2*f*(g
 + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x)) + 26*c*d*f*(1
28*e^4*h + 35*f^4*x^3*(11*g + 9*h*x) + 24*e^2*f^2*x*(11*g + 10*h*x) - 16*e
^3*f*(11*g + 12*h*x) - 10*e*f^3*x^2*(33*g + 28*h*x)) + d^2*(-1280*e^5*h +
315*f^5*x^4*(13*g + 11*h*x) + 128*e^4*f*(13*g + 15*h*x) - 96*e^3*f^2*x*(26
*g + 25*h*x) + 80*e^2*f^3*x^2*(39*g + 35*h*x) - 70*e*f^4*x^3*(52*g + 45*h*
x))) + b^3*(d^2*(1024*e^6*h + 1920*e^4*f^2*x*(g + h*x) - 256*e^5*f*(5*g +
6*h*x) + 280*e^2*f^4*x^3*(10*g + 9*h*x) + 231*f^6*x^5*(15*g + 13*h*x) - 16
0*e^3*f^3*x^2*(15*g + 14*h*x) - 126*e*f^5*x^4*(25*g + 22*h*x)) + 13*c^2*f^
2*(128*e^4*h + 35*f^4*x^3*(11*g + 9*h*x) + 24*e^2*f^2*x*(11*g + 10*h*x) -
16*e^3*f*(11*g + 12*h*x) - 10*e*f^3*x^2*(33*g + 28*h*x)) + 2*c*d*f*(-1280*
e^5*h + 315*f^5*x^4*(13*g + 11*h*x) + 128*e^4*f*(13*g + 15*h*x) - 96*e^3*f
^2*x*(26*g + 25*h*x) + 80*e^2*f^3*x^2*(39*g + 35*h*x) - 70*e*f^4*x^3*(5...
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{(e+fx)^{5/2}(be-af)\left(-a^2df^2(2cfh-3deh+dfg)-abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh)\right.\right.}{f^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(e+fx)^{7/2}(be-af)\left(a^2df^2(2cfh-3deh+dfg)+abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh)\right)-\right.}{7f^7}$$
$$\frac{2b(e+fx)^{11/2}\left(3a^2d^2f^2h+3abdf(2cfh-5deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)\right)\right)}{11f^7}+$$
$$\frac{2(e+fx)^{9/2}\left(a^3d^2f^3h+3a^2bdf^2(2cfh-4deh+dfg)+3ab^2f\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)-\right.}{9f^7}$$
$$\frac{2b^2d(e+fx)^{13/2}(3adfh+b(2cfh-6deh+dfg))}{13f^7}+$$
$$\frac{2(e+fx)^{5/2}(be-af)^2(de-cf)(-af(cfh-3deh+2dfg)-bcf(3fg-4eh)+bde(5fg-6eh))}{5f^7}-$$
$$\frac{2(e+fx)^{3/2}(be-af)^3(de-cf)^2(fg-eh)}{3f^7}+\frac{2b^3d^2h(e+fx)^{15/2}}{15f^7}$$

input
```
Int[(a + b*x)^3*(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(-2*(b*e - a*f)^3*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^7) + (2*
(b*e - a*f)^2*(d*e - c*f)*(b*d*e*(5*f*g - 6*e*h) - b*c*f*(3*f*g - 4*e*h) -
 a*f*(2*d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(5/2))/(5*f^7) - (2*(b*e - a*f
)*(a^2*d*f^2*(d*f*g - 3*d*e*h + 2*c*f*h) + a*b*f*(3*c^2*f^2*h - d^2*e*(8*f
*g - 15*e*h) + 2*c*d*f*(3*f*g - 8*e*h)) - b^2*(4*c*d*e*f*(3*f*g - 5*e*h) -
 5*d^2*e^2*(2*f*g - 3*e*h) - 3*c^2*f^2*(f*g - 2*e*h)))*(e + f*x)^(7/2))/(7
*f^7) + (2*(a^3*d^2*f^3*h + 3*a^2*b*d*f^2*(d*f*g - 4*d*e*h + 2*c*f*h) + 3*
a*b^2*f*(c^2*f^2*h - 2*d^2*e*(2*f*g - 5*e*h) + 2*c*d*f*(f*g - 4*e*h)) - b^
3*(4*c*d*e*f*(2*f*g - 5*e*h) - c^2*f^2*(f*g - 4*e*h) - 10*d^2*e^2*(f*g - 2
*e*h)))*(e + f*x)^(9/2))/(9*f^7) + (2*b*(3*a^2*d^2*f^2*h + 3*a*b*d*f*(d*f*
g - 5*d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h + 2*c*d*f*(f*g - 5*e*h) - 5*d^2*e*
(f*g - 3*e*h)))*(e + f*x)^(11/2))/(11*f^7) + (2*b^2*d*(3*a*d*f*h + b*(d*f*
g - 6*d*e*h + 2*c*f*h))*(e + f*x)^(13/2))/(13*f^7) + (2*b^3*d^2*h*(e + f*x
)^(15/2))/(15*f^7)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{2h\,d^2b^3(fx+e)^{\frac{15}{2}}}{15}+\frac{2\left(\left(3(af-be)b^2d^2+2b^3d(cf-de)\right)h+b^3d^2(-eh+fg)\right)(fx+e)^{\frac{13}{2}}}{13}+\frac{2\left(\left(3(af-be)^2b\,d^2+6(af-be)b^2d(cf-de)+\right.\right.}{}}{}$
default	$\frac{\frac{2h\,d^2b^3(fx+e)^{\frac{15}{2}}}{15}-\frac{2\left(-\left(3(af-be)b^2d^2+2b^3d(cf-de)\right)h+b^3d^2(eh-fg)\right)(fx+e)^{\frac{13}{2}}}{13}-\frac{2\left(-\left(3(af-be)^2b\,d^2+6(af-be)b^2d(cf-de)\right.\right.}{}}{}$
pseudoelliptic	$-\frac{4\left(\frac{\left(-\frac{5x^3\left(3\left(\frac{1}{5}h\,x^3+\frac{3}{13}g\,x^2\right)d^2+\frac{18xc\left(\frac{11hx}{13}+g\right)d}{11}+c^2\left(\frac{9hx}{11}+g\right)\right)b^3}{3}-\frac{45a\,x^2\left(7\left(\frac{1}{13}h\,x^3+\frac{1}{11}g\,x^2\right)d^2+\frac{14xc\left(\frac{9hx}{11}+g\right)d}{9}+c^2\right.}{7}\right.}{}\right.}{}$
gosper	Expression too large to display
orering	Expression too large to display
trager	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^7*(1/15*h*d^2*b^3*(f*x+e)^(15/2)+1/13*((3*(a*f-b*e)*b^2*d^2+2*b^3*d*(c
*f-d*e))*h+b^3*d^2*(-e*h+f*g))*(f*x+e)^(13/2)+1/11*((3*(a*f-b*e)^2*b*d^2+6
*(a*f-b*e)*b^2*d*(c*f-d*e)+b^3*(c*f-d*e)^2)*h+(3*(a*f-b*e)*b^2*d^2+2*b^3*d
*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*(((a*f-b*e)^3*d^2+6*(a*f-b*e)^2
*b*d*(c*f-d*e)+3*(a*f-b*e)*b^2*(c*f-d*e)^2)*h+(3*(a*f-b*e)^2*b*d^2+6*(a*f-
b*e)*b^2*d*(c*f-d*e)+b^3*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((2*(a
*f-b*e)^3*d*(c*f-d*e)+3*(a*f-b*e)^2*b*(c*f-d*e)^2)*h+((a*f-b*e)^3*d^2+6*(a
*f-b*e)^2*b*d*(c*f-d*e)+3*(a*f-b*e)*b^2*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(
7/2)+1/5*((a*f-b*e)^3*(c*f-d*e)^2*h+(2*(a*f-b*e)^3*d*(c*f-d*e)+3*(a*f-b*e)
^2*b*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(5/2)+1/3*(a*f-b*e)^3*(c*f-d*e)^2*(-
e*h+f*g)*(f*x+e)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(548) = 1096$.

Time = 0.11 (sec) , antiderivative size = 1467, normalized size of antiderivative = 2.53

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^3*d^2*f^7*h*x^7 + 231*(15*b^3*d^2*f^7*g + (b^3*d^2*e*f^6 +
 15*(2*b^3*c*d + 3*a*b^2*d^2)*f^7)*h)*x^6 + 63*(5*(b^3*d^2*e*f^6 + 13*(2*b
^3*c*d + 3*a*b^2*d^2)*f^7)*g - (4*b^3*d^2*e^2*f^5 - 5*(2*b^3*c*d + 3*a*b^2
*d^2)*e*f^6 - 65*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^7)*h)*x^5 - 35*((
10*b^3*d^2*e^2*f^5 - 13*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^6 - 143*(b^3*c^2 + 6
*a*b^2*c*d + 3*a^2*b*d^2)*f^7)*g - (8*b^3*d^2*e^3*f^4 - 10*(2*b^3*c*d + 3*
a*b^2*d^2)*e^2*f^5 + 13*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^6 + 143*
(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^7)*h)*x^4 + 5*((80*b^3*d^2*e^3*f^4
 - 104*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^5 + 143*(b^3*c^2 + 6*a*b^2*c*d + 3*
a^2*b*d^2)*e*f^6 + 1287*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^7)*g - (64
*b^3*d^2*e^4*f^3 - 80*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^4 + 104*(b^3*c^2 + 6
*a*b^2*c*d + 3*a^2*b*d^2)*e^2*f^5 - 143*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d
^2)*e*f^6 - 1287*(3*a^2*b*c^2 + 2*a^3*c*d)*f^7)*h)*x^3 - 3*((160*b^3*d^2*e
^4*f^3 - 208*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^4 + 286*(b^3*c^2 + 6*a*b^2*c*
d + 3*a^2*b*d^2)*e^2*f^5 - 429*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^6
 - 3003*(3*a^2*b*c^2 + 2*a^3*c*d)*f^7)*g - (128*b^3*d^2*e^5*f^2 + 3003*a^3
*c^2*f^7 - 160*(2*b^3*c*d + 3*a*b^2*d^2)*e^4*f^3 + 208*(b^3*c^2 + 6*a*b^2*
c*d + 3*a^2*b*d^2)*e^3*f^4 - 286*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2
*f^5 + 429*(3*a^2*b*c^2 + 2*a^3*c*d)*e*f^6)*h)*x^2 - (1280*b^3*d^2*e^6*f -
 15015*a^3*c^2*e*f^6 - 1664*(2*b^3*c*d + 3*a*b^2*d^2)*e^5*f^2 + 2288*(b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. $2(620) = 1240$.

Time = 2.40 (sec) , antiderivative size = 1909, normalized size of antiderivative = 3.30

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**2*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**3*d**2*h*(e + f*x)**(15/2)/(15*f**6) + (e + f*x)**(13/2)*
(3*a*b**2*d**2*f*h + 2*b**3*c*d*f*h - 6*b**3*d**2*e*h + b**3*d**2*f*g)/(13
*f**6) + (e + f*x)**(11/2)*(3*a**2*b*d**2*f**2*h + 6*a*b**2*c*d*f**2*h - 1
5*a*b**2*d**2*e*f*h + 3*a*b**2*d**2*f**2*g + b**3*c**2*f**2*h - 10*b**3*c*
d*e*f*h + 2*b**3*c*d*f**2*g + 15*b**3*d**2*e**2*h - 5*b**3*d**2*e*f*g)/(11
*f**6) + (e + f*x)**(9/2)*(a**3*d**2*f**3*h + 6*a**2*b*c*d*f**3*h - 12*a**
2*b*d**2*e*f**2*h + 3*a**2*b*d**2*f**3*g + 3*a*b**2*c**2*f**3*h - 24*a*b**
2*c*d*e*f**2*h + 6*a*b**2*c*d*f**3*g + 30*a*b**2*d**2*e**2*f*h - 12*a*b**2
*d**2*e*f**2*g - 4*b**3*c**2*e*f**2*h + b**3*c**2*f**3*g + 20*b**3*c*d*e**
2*f*h - 8*b**3*c*d*e*f**2*g - 20*b**3*d**2*e**3*h + 10*b**3*d**2*e**2*f*g)
/(9*f**6) + (e + f*x)**(7/2)*(2*a**3*c*d*f**4*h - 3*a**3*d**2*e*f**3*h + a
**3*d**2*f**4*g + 3*a**2*b*c**2*f**4*h - 18*a**2*b*c*d*e*f**3*h + 6*a**2*b
*c*d*f**4*g + 18*a**2*b*d**2*e**2*f**2*h - 9*a**2*b*d**2*e*f**3*g - 9*a*b*
*2*c**2*e*f**3*h + 3*a*b**2*c**2*f**4*g + 36*a*b**2*c*d*e**2*f**2*h - 18*a
*b**2*c*d*e*f**3*g - 30*a*b**2*d**2*e**3*f*h + 18*a*b**2*d**2*e**2*f**2*g
+ 6*b**3*c**2*e**2*f**2*h - 3*b**3*c**2*e*f**3*g - 20*b**3*c*d*e**3*f*h +
12*b**3*c*d*e**2*f**2*g + 15*b**3*d**2*e**4*h - 10*b**3*d**2*e**3*f*g)/(7*
f**6) + (e + f*x)**(5/2)*(a**3*c**2*f**5*h - 4*a**3*c*d*e*f**4*h + 2*a**3*
c*d*f**5*g + 3*a**3*d**2*e**2*f**3*h - 2*a**3*d**2*e*f**4*g - 6*a**2*b*c**
2*e*f**4*h + 3*a**2*b*c**2*f**5*g + 18*a**2*b*c*d*e**2*f**3*h - 12*a**2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(548) = 1096$.

Time = 0.04 (sec) , antiderivative size = 1152, normalized size of antiderivative = 1.99

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3003*(f*x + e)^(15/2)*b^3*d^2*h + 3465*(b^3*d^2*f*g - (6*b^3*d^2*
e - (2*b^3*c*d + 3*a*b^2*d^2)*f)*h)*(f*x + e)^(13/2) - 4095*((5*b^3*d^2*e*
f - (2*b^3*c*d + 3*a*b^2*d^2)*f^2)*g - (15*b^3*d^2*e^2 - 5*(2*b^3*c*d + 3*
a*b^2*d^2)*e*f + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*h)*(f*x + e)^(
11/2) + 5005*((10*b^3*d^2*e^2*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^2 + (b^3
*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^3)*g - (20*b^3*d^2*e^3 - 10*(2*b^3*c*d
 + 3*a*b^2*d^2)*e^2*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2 - (3
*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^3)*h)*(f*x + e)^(9/2) - 6435*((10*b^
3*d^2*e^3*f - 6*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^2 + 3*(b^3*c^2 + 6*a*b^2*c
*d + 3*a^2*b*d^2)*e*f^3 - (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^4)*g - (
15*b^3*d^2*e^4 - 10*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f + 6*(b^3*c^2 + 6*a*b^2
*c*d + 3*a^2*b*d^2)*e^2*f^2 - 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^
3 + (3*a^2*b*c^2 + 2*a^3*c*d)*f^4)*h)*(f*x + e)^(7/2) + 9009*((5*b^3*d^2*e
^4*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^2 + 3*(b^3*c^2 + 6*a*b^2*c*d + 3*
a^2*b*d^2)*e^2*f^3 - 2*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^4 + (3*a^
2*b*c^2 + 2*a^3*c*d)*f^5)*g - (6*b^3*d^2*e^5 - a^3*c^2*f^5 - 5*(2*b^3*c*d
+ 3*a*b^2*d^2)*e^4*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^2 - 3
*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2*f^3 + 2*(3*a^2*b*c^2 + 2*a^3*c*
d)*e*f^4)*h)*(f*x + e)^(5/2) - 15015*((b^3*d^2*e^5*f - a^3*c^2*f^6 - (2*b^
3*c*d + 3*a*b^2*d^2)*e^4*f^2 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2999 vs. $2(548) = 1096$.

Time = 0.16 (sec) , antiderivative size = 2999, normalized size of antiderivative = 5.18

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a^3*c^2*e*g + 15015*((f*x + e)^(3/2) - 3*sqrt
(f*x + e)*e)*a^3*c^2*g + 45045*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*b
*c^2*e*g/f + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c*d*e*g/f + 1
5015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c^2*e*h/f + 9009*(3*(f*x +
e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b^2*c^2*e*g/f^2
+ 18018*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*
a^2*b*c*d*e*g/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sq
rt(f*x + e)*e^2)*a^3*d^2*e*g/f^2 + 9009*(3*(f*x + e)^(5/2) - 10*(f*x + e)^
(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*c^2*g/f + 6006*(3*(f*x + e)^(5/2) -
10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*c*d*g/f + 9009*(3*(f*x +
e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*c^2*e*h/f^2
+ 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a
^3*c*d*e*h/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(
f*x + e)*e^2)*a^3*c^2*h/f + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e
 + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^3*c^2*e*g/f^3 + 7722*(
5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqr
t(f*x + e)*e^3)*a*b^2*c*d*e*g/f^3 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e)
^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a^2*b*d^2*e*g/f^
3 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^
2 - 35*sqrt(f*x + e)*e^3)*a*b^2*c^2*g/f^2 + 7722*(5*(f*x + e)^(7/2) - 2...
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.20

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{(e+f\,x)^{11/2}\,(6\,h\,a^2\,b\,d^2\,f^2+12\,h\,a\,b^2\,c\,d\,f^2-30\,h\,a\,b^2\,d^2\,e\,f+6\,g\,a\,b^2\,d^2\,f^2+2\,h\,b^3\,c^2\,f^2-20\,h\,b^3\,c\,d\,e\,f}{11\,f^7}$$
$$+\frac{(e+f\,x)^{9/2}\,(2\,h\,a^3\,d^2\,f^3+12\,h\,a^2\,b\,c\,d\,f^3-24\,h\,a^2\,b\,d^2\,e\,f^2+6\,g\,a^2\,b\,d^2\,f^3+6\,h\,a\,b^2\,c^2\,f^3-48\,h\,a\,b^2\,c}{}$$
$$+\frac{2\,(e+f\,x)^{7/2}\,(a\,f-b\,e)\,(2\,h\,a^2\,c\,d\,f^3-3\,h\,a^2\,d^2\,e\,f^2+g\,a^2\,d^2\,f^3+3\,h\,a\,b\,c^2\,f^3-16\,h\,a\,b\,c\,d\,e\,f^2+6\,g}{}$$
$$-\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)^3\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{3\,f^7}+\frac{2\,b^3\,d^2\,h\,(e+f\,x)^{15/2}}{15\,f^7}$$
$$+\frac{2\,b^2\,d\,(e+f\,x)^{13/2}\,(3\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+b\,d\,f\,g)}{13\,f^7}$$
$$+\frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+3\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-4\,b\,c\,e\,f}{5\,f^7}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^3*(c + d*x)^2,x)
```

output
```
((e + f*x)^(11/2)*(2*b^3*c^2*f^2*h + 30*b^3*d^2*e^2*h + 4*b^3*c*d*f^2*g -
10*b^3*d^2*e*f*g + 6*a*b^2*d^2*f^2*g + 6*a^2*b*d^2*f^2*h - 20*b^3*c*d*e*f*
h + 12*a*b^2*c*d*f^2*h - 30*a*b^2*d^2*e*f*h))/(11*f^7) + ((e + f*x)^(9/2)*
(2*b^3*c^2*f^3*g + 2*a^3*d^2*f^3*h - 40*b^3*d^2*e^3*h + 6*a*b^2*c^2*f^3*h
+ 6*a^2*b*d^2*f^3*g - 8*b^3*c^2*e*f^2*h + 20*b^3*d^2*e^2*f*g + 12*a*b^2*c*
d*f^3*g + 12*a^2*b*c*d*f^3*h - 16*b^3*c*d*e*f^2*g + 40*b^3*c*d*e^2*f*h - 2
4*a*b^2*d^2*e*f^2*g + 60*a*b^2*d^2*e^2*f*h - 24*a^2*b*d^2*e*f^2*h - 48*a*b
^2*c*d*e*f^2*h))/(9*f^7) + (2*(e + f*x)^(7/2)*(a*f - b*e)*(a^2*d^2*f^3*g +
 3*b^2*c^2*f^3*g - 15*b^2*d^2*e^3*h + 3*a*b*c^2*f^3*h + 2*a^2*c*d*f^3*h -
3*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h + 10*b^2*d^2*e^2*f*g + 6*a*b*c*d*f^3
*g - 8*a*b*d^2*e*f^2*g + 15*a*b*d^2*e^2*f*h - 12*b^2*c*d*e*f^2*g + 20*b^2*
c*d*e^2*f*h - 16*a*b*c*d*e*f^2*h))/(7*f^7) - (2*(e + f*x)^(3/2)*(a*f - b*e
)^3*(c*f - d*e)^2*(e*h - f*g))/(3*f^7) + (2*b^3*d^2*h*(e + f*x)^(15/2))/(1
5*f^7) + (2*b^2*d*(e + f*x)^(13/2)*(3*a*d*f*h + 2*b*c*f*h - 6*b*d*e*h + b*
d*f*g))/(13*f^7) + (2*(e + f*x)^(5/2)*(a*f - b*e)^2*(c*f - d*e)*(a*c*f^2*h
 + 2*a*d*f^2*g + 3*b*c*f^2*g + 6*b*d*e^2*h - 3*a*d*e*f*h - 4*b*c*e*f*h - 5
*b*d*e*f*g))/(5*f^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1941, normalized size of antiderivative = 3.35

$$\int (a+bx)^3(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 6006*a**3*c**2*e**2*f**5*h + 15015*a**3*c**2*e*f**6*g
 + 3003*a**3*c**2*e*f**6*h*x + 15015*a**3*c**2*f**7*g*x + 9009*a**3*c**2*f
**7*h*x**2 + 6864*a**3*c*d*e**3*f**4*h - 12012*a**3*c*d*e**2*f**5*g - 3432
*a**3*c*d*e**2*f**5*h*x + 6006*a**3*c*d*e*f**6*g*x + 2574*a**3*c*d*e*f**6*
h*x**2 + 18018*a**3*c*d*f**7*g*x**2 + 12870*a**3*c*d*f**7*h*x**3 - 2288*a*
*3*d**2*e**4*f**3*h + 3432*a**3*d**2*e**3*f**4*g + 1144*a**3*d**2*e**3*f**
4*h*x - 1716*a**3*d**2*e**2*f**5*g*x - 858*a**3*d**2*e**2*f**5*h*x**2 + 12
87*a**3*d**2*e*f**6*g*x**2 + 715*a**3*d**2*e*f**6*h*x**3 + 6435*a**3*d**2*
f**7*g*x**3 + 5005*a**3*d**2*f**7*h*x**4 + 10296*a**2*b*c**2*e**3*f**4*h -
 18018*a**2*b*c**2*e**2*f**5*g - 5148*a**2*b*c**2*e**2*f**5*h*x + 9009*a**
2*b*c**2*e*f**6*g*x + 3861*a**2*b*c**2*e*f**6*h*x**2 + 27027*a**2*b*c**2*f
**7*g*x**2 + 19305*a**2*b*c**2*f**7*h*x**3 - 13728*a**2*b*c*d*e**4*f**3*h
+ 20592*a**2*b*c*d*e**3*f**4*g + 6864*a**2*b*c*d*e**3*f**4*h*x - 10296*a**
2*b*c*d*e**2*f**5*g*x - 5148*a**2*b*c*d*e**2*f**5*h*x**2 + 7722*a**2*b*c*d
*e*f**6*g*x**2 + 4290*a**2*b*c*d*e*f**6*h*x**3 + 38610*a**2*b*c*d*f**7*g*x
**3 + 30030*a**2*b*c*d*f**7*h*x**4 + 4992*a**2*b*d**2*e**5*f**2*h - 6864*a
**2*b*d**2*e**4*f**3*g - 2496*a**2*b*d**2*e**4*f**3*h*x + 3432*a**2*b*d**2
*e**3*f**4*g*x + 1872*a**2*b*d**2*e**3*f**4*h*x**2 - 2574*a**2*b*d**2*e**2
*f**5*g*x**2 - 1560*a**2*b*d**2*e**2*f**5*h*x**3 + 2145*a**2*b*d**2*e*f**6
*g*x**3 + 1365*a**2*b*d**2*e*f**6*h*x**4 + 15015*a**2*b*d**2*f**7*g*x**...
```

3.11 $\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 411

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \frac{2(be-af)^2(de-cf)^2(fg-eh)(e+fx)^{3/2}}{3f^6}$$
$$-\frac{2(be-af)(de-cf)(bde(4fg-5eh)-bcf(2fg-3eh)-af(2dfg-3deh+cfh))(e+fx)^{5/2}}{5f^6}$$
$$+\frac{2(a^2df^2(dfg-3deh+2cfh)+2abf(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh))+b^2(2d^2e^2(3fg-5e}{7f^6}$$
$$+\frac{2(a^2d^2f^2h+2abdf(dfg-4deh+2cfh)+b^2(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh)))\,(e+fx)^{9/}}{9f^6}$$
$$+\frac{2bd(2adfh+b(dfg-5deh+2cfh))(e+fx)^{11/2}}{11f^6}+\frac{2b^2d^2h(e+fx)^{13/2}}{13f^6}$$

output
```
2/3*(-a*f+b*e)^2*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(3/2)/f^6-2/5*(-a*f+b*e)*
(-c*f+d*e)*(b*d*e*(-5*e*h+4*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-3*d*e*h+2
*d*f*g))*(f*x+e)^(5/2)/f^6+2/7*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d*f*g)+2*a*b*f*
(c^2*f^2*h+2*c*d*f*(-3*e*h+f*g)-3*d^2*e*(-2*e*h+f*g))+b^2*(2*d^2*e^2*(-5*e
*h+3*f*g)+c^2*f^2*(-3*e*h+f*g)-6*c*d*e*f*(-2*e*h+f*g)))*(f*x+e)^(7/2)/f^6+
2/9*(a^2*d^2*f^2*h+2*a*b*d*f*(2*c*f*h-4*d*e*h+d*f*g)+b^2*(c^2*f^2*h-2*d^2*
e*(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g)))*(f*x+e)^(9/2)/f^6+2/11*b*d*(2*a*d*
f*h+b*(2*c*f*h-5*d*e*h+d*f*g))*(f*x+e)^(11/2)/f^6+2/13*b^2*d^2*h*(f*x+e)^(
13/2)/f^6
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.33

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2(e+fx)^{3/2}\left(143a^2f^2(21c^2f^2(5fg-2eh+3fhx)+6cdf(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx))+\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(143*a^2*f^2*(21*c^2*f^2*(5*f*g - 2*e*h + 3*f*h*x) + 6*
c*d*f*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d^2*(-16*e
^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h
*x))) + 26*a*b*f*(33*c^2*f^2*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g
 + 6*h*x)) + 22*c*d*f*(-16*e^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5
*h*x) + 5*f^3*x^2*(9*g + 7*h*x)) + d^2*(128*e^4*h + 35*f^4*x^3*(11*g + 9*h
*x) + 24*e^2*f^2*x*(11*g + 10*h*x) - 16*e^3*f*(11*g + 12*h*x) - 10*e*f^3*x
^2*(33*g + 28*h*x))) + b^2*(143*c^2*f^2*(-16*e^3*h + 24*e^2*f*(g + h*x) -
6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x)) + 26*c*d*f*(128*e^4*h +
 35*f^4*x^3*(11*g + 9*h*x) + 24*e^2*f^2*x*(11*g + 10*h*x) - 16*e^3*f*(11*g
 + 12*h*x) - 10*e*f^3*x^2*(33*g + 28*h*x)) + d^2*(-1280*e^5*h + 315*f^5*x^
4*(13*g + 11*h*x) + 128*e^4*f*(13*g + 15*h*x) - 96*e^3*f^2*x*(26*g + 25*h*
x) + 80*e^2*f^3*x^2*(39*g + 35*h*x) - 70*e*f^4*x^3*(52*g + 45*h*x)))))/(45
045*f^6)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{(e+fx)^{5/2}\left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-\right.\right.}{f^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{7/2}\left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-3e\right.\right.}{7f^6}$$
$$\frac{2(e+fx)^{9/2}\left(a^2d^2f^2h+2abdf(2cfh-4deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)\right)}{9f^6}+$$
$$\frac{2bd(e+fx)^{11/2}(2adfh+b(2cfh-5deh+dfg))}{11f^6}-$$
$$\frac{2(e+fx)^{5/2}(be-af)(de-cf)(-af(cfh-3deh+2dfg)-bcf(2fg-3eh)+bde(4fg-5eh))}{5f^6}+$$
$$\frac{2(e+fx)^{3/2}(be-af)^2(de-cf)^2(fg-eh)}{3f^6}+\frac{2b^2d^2h(e+fx)^{13/2}}{13f^6}$$

input
```
Int[(a + b*x)^2*(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(b*e - a*f)^2*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^6) - (2*(
b*e - a*f)*(d*e - c*f)*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(2*f*g - 3*e*h) - a*
f*(2*d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(5/2))/(5*f^6) + (2*(a^2*d*f^2*(d
*f*g - 3*d*e*h + 2*c*f*h) + 2*a*b*f*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3
*d^2*e*(f*g - 2*e*h)) + b^2*(2*d^2*e^2*(3*f*g - 5*e*h) + c^2*f^2*(f*g - 3*
e*h) - 6*c*d*e*f*(f*g - 2*e*h)))*(e + f*x)^(7/2))/(7*f^6) + (2*(a^2*d^2*f^
2*h + 2*a*b*d*f*(d*f*g - 4*d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h - 2*d^2*e*(2*
f*g - 5*e*h) + 2*c*d*f*(f*g - 4*e*h)))*(e + f*x)^(9/2))/(9*f^6) + (2*b*d*(
2*a*d*f*h + b*(d*f*g - 5*d*e*h + 2*c*f*h))*(e + f*x)^(11/2))/(11*f^6) + (2
*b^2*d^2*h*(e + f*x)^(13/2))/(13*f^6)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2h\,b^2d^2(fx+e)^{\frac{13}{2}}}{13}+\frac{2\left(\left(2b(af-be)d^2+2b^2d(cf-de)\right)h+b^2d^2(-eh+fg)\right)(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left((af-be)^2d^2+4b(af-be)d(cf-de)+b^2(cf \ldots\right.\right.}{\ldots}$
default	$\frac{2h\,b^2d^2(fx+e)^{\frac{13}{2}}}{13}-\frac{2\left(-\left(2b(af-be)d^2+2b^2d(cf-de)\right)h+b^2d^2(eh-fg)\right)(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left((af-be)^2d^2+4b(af-be)d(cf-de)+b^2(\ldots\right.\right.}{\ldots}$
pseudoelliptic	$-\frac{4(fx+e)^{\frac{3}{2}}\left(\left(\left(-\frac{15x^2\left(7\left(\frac{1}{13}h\,x^3+\frac{1}{11}g\,x^2\right)d^2+\frac{14xc\left(\frac{9hx}{11}+g\right)d}{9}+c^2\left(\frac{7hx}{9}+g\right)\right)b^2}{14}-3ax\left(\frac{5x^2\left(\frac{9hx}{11}+g\right)d^2}{9}+\frac{10xc\left(\frac{7hx}{9}+g\right) \ldots}{7}\right.\right.\right.\right.}{\ldots}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-3465h\,b^2d^2x^5f^5-8190ab\,d^2f^5h\,x^4-8190b^2cd\,f^5h\,x^4+3150b^2d^2e\,f^4h\,x^4-4095b^2d^2f^5g\,x^4-5005a^2d^2 \ldots\right.}{\ldots}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-3465h\,b^2d^2x^5f^5-8190ab\,d^2f^5h\,x^4-8190b^2cd\,f^5h\,x^4+3150b^2d^2e\,f^4h\,x^4-4095b^2d^2f^5g\,x^4-5005a^2d^2 \ldots\right.}{\ldots}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^2*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^6*(1/13*h*b^2*d^2*(f*x+e)^(13/2)+1/11*((2*b*(a*f-b*e)*d^2+2*b^2*d*(c*f
-d*e))*h+b^2*d^2*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*(((a*f-b*e)^2*d^2+4*b*(a*f
-b*e)*d*(c*f-d*e)+b^2*(c*f-d*e)^2)*h+(2*b*(a*f-b*e)*d^2+2*b^2*d*(c*f-d*e))
*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((2*(a*f-b*e)^2*d*(c*f-d*e)+2*b*(a*f-b*e)*(
c*f-d*e)^2)*h+((a*f-b*e)^2*d^2+4*b*(a*f-b*e)*d*(c*f-d*e)+b^2*(c*f-d*e)^2)*
(-e*h+f*g))*(f*x+e)^(7/2)+1/5*((a*f-b*e)^2*(c*f-d*e)^2*h+(2*(a*f-b*e)^2*d*
(c*f-d*e)+2*b*(a*f-b*e)*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(5/2)+1/3*(a*f-b*
e)^2*(c*f-d*e)^2*(-e*h+f*g)*(f*x+e)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(387) = 774$.

Time = 0.08 (sec) , antiderivative size = 919, normalized size of antiderivative = 2.24

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3465*b^2*d^2*f^6*h*x^6 + 315*(13*b^2*d^2*f^6*g + (b^2*d^2*e*f^5 +
 26*(b^2*c*d + a*b*d^2)*f^6)*h)*x^5 + 35*(13*(b^2*d^2*e*f^5 + 22*(b^2*c*d
+ a*b*d^2)*f^6)*g - (10*b^2*d^2*e^2*f^4 - 26*(b^2*c*d + a*b*d^2)*e*f^5 - 1
43*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^6)*h)*x^4 - 5*(13*(8*b^2*d^2*e^2*f^4
- 22*(b^2*c*d + a*b*d^2)*e*f^5 - 99*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^6)*g
 - (80*b^2*d^2*e^3*f^3 - 208*(b^2*c*d + a*b*d^2)*e^2*f^4 + 143*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*e*f^5 + 2574*(a*b*c^2 + a^2*c*d)*f^6)*h)*x^3 + 3*(13*
(16*b^2*d^2*e^3*f^3 - 44*(b^2*c*d + a*b*d^2)*e^2*f^4 + 33*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*e*f^5 + 462*(a*b*c^2 + a^2*c*d)*f^6)*g - (160*b^2*d^2*e^4*
f^2 - 3003*a^2*c^2*f^6 - 416*(b^2*c*d + a*b*d^2)*e^3*f^3 + 286*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*e^2*f^4 - 858*(a*b*c^2 + a^2*c*d)*e*f^5)*h)*x^2 + 13*
(128*b^2*d^2*e^5*f + 1155*a^2*c^2*e*f^5 - 352*(b^2*c*d + a*b*d^2)*e^4*f^2
+ 264*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^3 - 924*(a*b*c^2 + a^2*c*d)*e^
2*f^4)*g - 2*(640*b^2*d^2*e^6 + 3003*a^2*c^2*e^2*f^4 - 1664*(b^2*c*d + a*b
*d^2)*e^5*f + 1144*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^4*f^2 - 3432*(a*b*c^2
 + a^2*c*d)*e^3*f^3)*h - (13*(64*b^2*d^2*e^4*f^2 - 1155*a^2*c^2*f^6 - 176*
(b^2*c*d + a*b*d^2)*e^3*f^3 + 132*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^4
- 462*(a*b*c^2 + a^2*c*d)*e*f^5)*g - (640*b^2*d^2*e^5*f + 3003*a^2*c^2*e*f
^5 - 1664*(b^2*c*d + a*b*d^2)*e^4*f^2 + 1144*(b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*e^3*f^3 - 3432*(a*b*c^2 + a^2*c*d)*e^2*f^4)*h)*x)*sqrt(f*x + e)/f^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(445) = 890$.

Time = 2.02 (sec) , antiderivative size = 1204, normalized size of antiderivative = 2.93

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2*(d*x+c)**2*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**2*d**2*h*(e + f*x)**(13/2)/(13*f**5) + (e + f*x)**(11/2)*
(2*a*b*d**2*f*h + 2*b**2*c*d*f*h - 5*b**2*d**2*e*h + b**2*d**2*f*g)/(11*f*
*5) + (e + f*x)**(9/2)*(a**2*d**2*f**2*h + 4*a*b*c*d*f**2*h - 8*a*b*d**2*e
*f*h + 2*a*b*d**2*f**2*g + b**2*c**2*f**2*h - 8*b**2*c*d*e*f*h + 2*b**2*c*
d*f**2*g + 10*b**2*d**2*e**2*h - 4*b**2*d**2*e*f*g)/(9*f**5) + (e + f*x)**
(7/2)*(2*a**2*c*d*f**3*h - 3*a**2*d**2*e*f**2*h + a**2*d**2*f**3*g + 2*a*b
*c**2*f**3*h - 12*a*b*c*d*e*f**2*h + 4*a*b*c*d*f**3*g + 12*a*b*d**2*e**2*f
*h - 6*a*b*d**2*e*f**2*g - 3*b**2*c**2*e*f**2*h + b**2*c**2*f**3*g + 12*b*
*2*c*d*e**2*f*h - 6*b**2*c*d*e*f**2*g - 10*b**2*d**2*e**3*h + 6*b**2*d**2*
e**2*f*g)/(7*f**5) + (e + f*x)**(5/2)*(a**2*c**2*f**4*h - 4*a**2*c*d*e*f**
3*h + 2*a**2*c*d*f**4*g + 3*a**2*d**2*e**2*f**2*h - 2*a**2*d**2*e*f**3*g -
 4*a*b*c**2*e*f**3*h + 2*a*b*c**2*f**4*g + 12*a*b*c*d*e**2*f**2*h - 8*a*b*
c*d*e*f**3*g - 8*a*b*d**2*e**3*f*h + 6*a*b*d**2*e**2*f**2*g + 3*b**2*c**2*
e**2*f**2*h - 2*b**2*c**2*e*f**3*g - 8*b**2*c*d*e**3*f*h + 6*b**2*c*d*e**2
*f**2*g + 5*b**2*d**2*e**4*h - 4*b**2*d**2*e**3*f*g)/(5*f**5) + (e + f*x)*
*(3/2)*(-a**2*c**2*e*f**4*h + a**2*c**2*f**5*g + 2*a**2*c*d*e**2*f**3*h -
2*a**2*c*d*e*f**4*g - a**2*d**2*e**3*f**2*h + a**2*d**2*e**2*f**3*g + 2*a*
b*c**2*e**2*f**3*h - 2*a*b*c**2*e*f**4*g - 4*a*b*c*d*e**3*f**2*h + 4*a*b*c
*d*e**2*f**3*g + 2*a*b*d**2*e**4*f*h - 2*a*b*d**2*e**3*f**2*g - b**2*c**2*
e**3*f**2*h + b**2*c**2*e**2*f**3*g + 2*b**2*c*d*e**4*f*h - 2*b**2*c*d*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.69

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3465*(f*x + e)^(13/2)*b^2*d^2*h + 4095*(b^2*d^2*f*g - (5*b^2*d^2*
e - 2*(b^2*c*d + a*b*d^2)*f)*h)*(f*x + e)^(11/2) - 5005*(2*(2*b^2*d^2*e*f
- (b^2*c*d + a*b*d^2)*f^2)*g - (10*b^2*d^2*e^2 - 8*(b^2*c*d + a*b*d^2)*e*f
 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2)*h)*(f*x + e)^(9/2) + 6435*((6*b^2*
d^2*e^2*f - 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*
f^3)*g - (10*b^2*d^2*e^3 - 12*(b^2*c*d + a*b*d^2)*e^2*f + 3*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*e*f^2 - 2*(a*b*c^2 + a^2*c*d)*f^3)*h)*(f*x + e)^(7/2) -
9009*(2*(2*b^2*d^2*e^3*f - 3*(b^2*c*d + a*b*d^2)*e^2*f^2 + (b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*e*f^3 - (a*b*c^2 + a^2*c*d)*f^4)*g - (5*b^2*d^2*e^4 + a^2
*c^2*f^4 - 8*(b^2*c*d + a*b*d^2)*e^3*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*e^2*f^2 - 4*(a*b*c^2 + a^2*c*d)*e*f^3)*h)*(f*x + e)^(5/2) + 15015*((b^2*d
^2*e^4*f + a^2*c^2*f^5 - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*e^2*f^3 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*g - (b^2*d^2*e^5 + a
^2*c^2*e*f^4 - 2*(b^2*c*d + a*b*d^2)*e^4*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*e^3*f^2 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^3)*h)*(f*x + e)^(3/2))/f^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(387) = 774$.

Time = 0.14 (sec) , antiderivative size = 2001, normalized size of antiderivative = 4.87

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a^2*c^2*e*g + 15015*((f*x + e)^(3/2) - 3*sqrt
(f*x + e)*e)*a^2*c^2*g + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*b*c
^2*e*g/f + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c*d*e*g/f + 150
15*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c^2*e*h/f + 3003*(3*(f*x + e)
^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b^2*c^2*e*g/f^2 + 12
012*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*
c*d*e*g/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x
 + e)*e^2)*a^2*d^2*e*g/f^2 + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*
e + 15*sqrt(f*x + e)*e^2)*a*b*c^2*g/f + 6006*(3*(f*x + e)^(5/2) - 10*(f*x
+ e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c*d*g/f + 6006*(3*(f*x + e)^(5/2)
 - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*c^2*e*h/f^2 + 6006*(3*
(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c*d*e*h
/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e
^2)*a^2*c^2*h/f + 2574*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x
 + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*c*d*e*g/f^3 + 2574*(5*(f*x + e
)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)
*e^3)*a*b*d^2*e*g/f^3 + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 3
5*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*c^2*g/f^2 + 5148*(5*(f*x
 + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x
+ e)*e^3)*a*b*c*d*g/f^2 + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*...
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.14

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$

$$= \frac{(e+f\,x)^{7/2}\,(4\,h\,a^2\,c\,d\,f^3-6\,h\,a^2\,d^2\,e\,f^2+2\,g\,a^2\,d^2\,f^3+4\,h\,a\,b\,c^2\,f^3-24\,h\,a\,b\,c\,d\,e\,f^2+8\,g\,a\,b\,c\,d\,f^3+2}{}$$

$$+\frac{(e+f\,x)^{9/2}\,(2\,h\,a^2\,d^2\,f^2+8\,h\,a\,b\,c\,d\,f^2-16\,h\,a\,b\,d^2\,e\,f+4\,g\,a\,b\,d^2\,f^2+2\,h\,b^2\,c^2\,f^2-16\,h\,b^2\,c\,d\,e\,f+}{9\,f^6}$$

$$-\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{3\,f^6}+\frac{2\,b^2\,d^2\,h\,(e+f\,x)^{13/2}}{13\,f^6}$$

$$+\frac{2\,b\,d\,(e+f\,x)^{11/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-5\,b\,d\,e\,h+b\,d\,f\,g)}{11\,f^6}$$

$$+\frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-3\,b\,c\,e\,f}{5\,f^6}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^2*(c + d*x)^2,x)
```

output
```
((e + f*x)^(7/2)*(2*a^2*d^2*f^3*g + 2*b^2*c^2*f^3*g - 20*b^2*d^2*e^3*h + 4
*a*b*c^2*f^3*h + 4*a^2*c*d*f^3*h - 6*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h +
 12*b^2*d^2*e^2*f*g + 8*a*b*c*d*f^3*g - 12*a*b*d^2*e*f^2*g + 24*a*b*d^2*e^
2*f*h - 12*b^2*c*d*e*f^2*g + 24*b^2*c*d*e^2*f*h - 24*a*b*c*d*e*f^2*h))/(7*
f^6) + ((e + f*x)^(9/2)*(2*a^2*d^2*f^2*h + 2*b^2*c^2*f^2*h + 20*b^2*d^2*e^
2*h + 4*a*b*d^2*f^2*g + 4*b^2*c*d*f^2*g - 8*b^2*d^2*e*f*g + 8*a*b*c*d*f^2*
h - 16*a*b*d^2*e*f*h - 16*b^2*c*d*e*f*h))/(9*f^6) - (2*(e + f*x)^(3/2)*(a*
f - b*e)^2*(c*f - d*e)^2*(e*h - f*g))/(3*f^6) + (2*b^2*d^2*h*(e + f*x)^(13
/2))/(13*f^6) + (2*b*d*(e + f*x)^(11/2)*(2*a*d*f*h + 2*b*c*f*h - 5*b*d*e*h
 + b*d*f*g))/(11*f^6) + (2*(e + f*x)^(5/2)*(a*f - b*e)*(c*f - d*e)*(a*c*f^
2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 5*b*d*e^2*h - 3*a*d*e*f*h - 3*b*c*e*f*h
- 4*b*d*e*f*g))/(5*f^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1225, normalized size of antiderivative = 2.98

$$\int (a+bx)^2(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 6006*a**2*c**2*e**2*f**4*h + 15015*a**2*c**2*e*f**5*g
 + 3003*a**2*c**2*e*f**5*h*x + 15015*a**2*c**2*f**6*g*x + 9009*a**2*c**2*f
**6*h*x**2 + 6864*a**2*c*d*e**3*f**3*h - 12012*a**2*c*d*e**2*f**4*g - 3432
*a**2*c*d*e**2*f**4*h*x + 6006*a**2*c*d*e*f**5*g*x + 2574*a**2*c*d*e*f**5*
h*x**2 + 18018*a**2*c*d*f**6*g*x**2 + 12870*a**2*c*d*f**6*h*x**3 - 2288*a*
*2*d**2*e**4*f**2*h + 3432*a**2*d**2*e**3*f**3*g + 1144*a**2*d**2*e**3*f**
3*h*x - 1716*a**2*d**2*e**2*f**4*g*x - 858*a**2*d**2*e**2*f**4*h*x**2 + 12
87*a**2*d**2*e*f**5*g*x**2 + 715*a**2*d**2*e*f**5*h*x**3 + 6435*a**2*d**2*
f**6*g*x**3 + 5005*a**2*d**2*f**6*h*x**4 + 6864*a*b*c**2*e**3*f**3*h - 120
12*a*b*c**2*e**2*f**4*g - 3432*a*b*c**2*e**2*f**4*h*x + 6006*a*b*c**2*e*f*
*5*g*x + 2574*a*b*c**2*e*f**5*h*x**2 + 18018*a*b*c**2*f**6*g*x**2 + 12870*
a*b*c**2*f**6*h*x**3 - 9152*a*b*c*d*e**4*f**2*h + 13728*a*b*c*d*e**3*f**3*
g + 4576*a*b*c*d*e**3*f**3*h*x - 6864*a*b*c*d*e**2*f**4*g*x - 3432*a*b*c*d
*e**2*f**4*h*x**2 + 5148*a*b*c*d*e*f**5*g*x**2 + 2860*a*b*c*d*e*f**5*h*x**
3 + 25740*a*b*c*d*f**6*g*x**3 + 20020*a*b*c*d*f**6*h*x**4 + 3328*a*b*d**2*
e**5*f*h - 4576*a*b*d**2*e**4*f**2*g - 1664*a*b*d**2*e**4*f**2*h*x + 2288*
a*b*d**2*e**3*f**3*g*x + 1248*a*b*d**2*e**3*f**3*h*x**2 - 1716*a*b*d**2*e*
*2*f**4*g*x**2 - 1040*a*b*d**2*e**2*f**4*h*x**3 + 1430*a*b*d**2*e*f**5*g*x
**3 + 910*a*b*d**2*e*f**5*h*x**4 + 10010*a*b*d**2*f**6*g*x**4 + 8190*a*b*d
**2*f**6*h*x**5 - 2288*b**2*c**2*e**4*f**2*h + 3432*b**2*c**2*e**3*f**3...
```

3.12 $\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 248

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = -\frac{2(be-af)(de-cf)^2(fg-eh)(e+fx)^{3/2}}{3f^5} + \frac{2(de-cf)(bde(3fg-4eh)-bcf(fg-2eh)-af(2dfg-3deh+cfh))(e+fx)^{5/2}}{5f^5} + \frac{2(adf(dfg-3deh+2cfh)+b(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)))(e+fx)^{7/2}}{7f^5} + \frac{2d(adfh+b(dfg-4deh+2cfh))(e+fx)^{9/2}}{9f^5} + \frac{2bd^2h(e+fx)^{11/2}}{11f^5}$$

output
```
-2/3*(-a*f+b*e)*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(3/2)/f^5+2/5*(-c*f+d*e)*(
b*d*e*(-4*e*h+3*f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-3*d*e*h+2*d*f*g))*(f*x+
e)^(5/2)/f^5+2/7*(a*d*f*(2*c*f*h-3*d*e*h+d*f*g)+b*(c^2*f^2*h+2*c*d*f*(-3*e
*h+f*g)-3*d^2*e*(-2*e*h+f*g)))*(f*x+e)^(7/2)/f^5+2/9*d*(a*d*f*h+b*(2*c*f*h
-4*d*e*h+d*f*g))*(f*x+e)^(9/2)/f^5+2/11*b*d^2*h*(f*x+e)^(11/2)/f^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.24

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2(e+fx)^{3/2}\left(11af(21c^2f^2(5fg-2eh+3fhx)+6cdf(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx))+d^2(\cdot\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(11*a*f*(21*c^2*f^2*(5*f*g - 2*e*h + 3*f*h*x) + 6*c*d*f
*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d^2*(-16*e^3*h
+ 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x)))
 + b*(33*c^2*f^2*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) +
 22*c*d*f*(-16*e^3*h + 24*e^2*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^
3*x^2*(9*g + 7*h*x)) + d^2*(128*e^4*h + 35*f^4*x^3*(11*g + 9*h*x) + 24*e^2
*f^2*x*(11*g + 10*h*x) - 16*e^3*f*(11*g + 12*h*x) - 10*e*f^3*x^2*(33*g + 2
8*h*x)))))/(3465*f^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$

$$\Big\downarrow \text{ 159}$$

$$\int \left(\frac{(e+fx)^{5/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{f^4}+\frac{d(e+fx)^{7/2}(adf\cdots}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(e+fx)^{7/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{7f^5}+\frac{2d(e+fx)^{9/2}(adfh+b(2cfh-4deh+dfg))}{9f^5}+\frac{2(e+fx)^{5/2}(de-cf)(-af(cfh-3deh+2dfg)-bcf(fg-2eh)+bde(3fg-4eh))}{5f^5}-\frac{2(e+fx)^{3/2}(be-af)(de-cf)^2(fg-eh)}{3f^5}+\frac{2bd^2h(e+fx)^{11/2}}{11f^5}$$

input
```
Int[(a + b*x)*(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(-2*(b*e - a*f)*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^5) + (2*(d
*e - c*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(2*d*f*g - 3*
d*e*h + c*f*h))*(e + f*x)^(5/2))/(5*f^5) + (2*(a*d*f*(d*f*g - 3*d*e*h + 2*
c*f*h) + b*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3*d^2*e*(f*g - 2*e*h)))*(e
 + f*x)^(7/2))/(7*f^5) + (2*d*(a*d*f*h + b*(d*f*g - 4*d*e*h + 2*c*f*h))*(e
 + f*x)^(9/2))/(9*f^5) + (2*b*d^2*h*(e + f*x)^(11/2))/(11*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2hb\,d^2(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left((af-be)d^2+2bd(cf-de)\right)h+b\,d^2(-eh+fg)\right)(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(\left(2(af-be)d(cf-de)+b(cf-de)^2\right)h+\left((af-be)d^2\right.\right.}{7}$
default	$\frac{2hb\,d^2(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(-\left((-af+be)d^2-2bd(cf-de)\right)h-b\,d^2(eh-fg)\right)(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(-\left(2(-af+be)d(cf-de)-b(cf-de)^2\right)h+\left((-af\right.\right.}{7}$
pseudoelliptic	$-\frac{4(fx+e)^{\frac{3}{2}}\left(\left(\left(-\frac{15x^2\left(\frac{7bh\,x^2}{11}+\frac{7(ah+bg)x}{9}+ga\right)d^2}{14}-3xc\left(\frac{5bh\,x^2}{9}+\frac{5(ah+bg)x}{7}+ga\right)d-\frac{5c^2\left(\frac{3bh\,x^2}{7}+\frac{3(ah+bg)x}{5}+ga\right)}{2}\right)f\right.\right.}{}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-315hb\,d^2x^4f^4-385a\,d^2f^4h\,x^3-770bcd\,f^4h\,x^3+280b\,d^2e\,f^3h\,x^3-385b\,d^2f^4g\,x^3-990acd\,f^4h\,x^2+330a\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-315hb\,d^2x^4f^4-385a\,d^2f^4h\,x^3-770bcd\,f^4h\,x^3+280b\,d^2e\,f^3h\,x^3-385b\,d^2f^4g\,x^3-990acd\,f^4h\,x^2+330a\right.}{}$
trager	$-\frac{2\left(-315hb\,d^2f^5x^5-385a\,d^2f^5h\,x^4-770bcd\,f^5h\,x^4-35b\,d^2e\,f^4h\,x^4-385b\,d^2f^5g\,x^4-990acd\,f^5h\,x^3-55a\,d^2e\,f^4h\,x^3\right.}{}$
risch	$-\frac{2\left(-315hb\,d^2f^5x^5-385a\,d^2f^5h\,x^4-770bcd\,f^5h\,x^4-35b\,d^2e\,f^4h\,x^4-385b\,d^2f^5g\,x^4-990acd\,f^5h\,x^3-55a\,d^2e\,f^4h\,x^3\right.}{}$

input
```
int((b*x+a)*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^5*(1/11*h*b*d^2*(f*x+e)^(11/2)+1/9*(((a*f-b*e)*d^2+2*b*d*(c*f-d*e))*h+
b*d^2*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((2*(a*f-b*e)*d*(c*f-d*e)+b*(c*f-d*e)^
2)*h+((a*f-b*e)*d^2+2*b*d*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*((a*f-b
*e)*(c*f-d*e)^2*h+(2*(a*f-b*e)*d*(c*f-d*e)+b*(c*f-d*e)^2)*(-e*h+f*g))*(f*x
+e)^(5/2)+1/3*(a*f-b*e)*(c*f-d*e)^2*(-e*h+f*g)*(f*x+e)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(228) = 456$.

Time = 0.08 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.09

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$=\frac{2\left(315\,bd^2f^5hx^5+35\left(11\,bd^2f^5g+\left(bd^2ef^4+11\left(2\,bcd+ad^2\right)f^5\right)h\right)x^4+5\left(11\left(bd^2ef^4+9\left(2\,bcd+ad^2\right)f\right.\right.\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/3465*(315*b*d^2*f^5*h*x^5 + 35*(11*b*d^2*f^5*g + (b*d^2*e*f^4 + 11*(2*b*
c*d + a*d^2)*f^5)*h)*x^4 + 5*(11*(b*d^2*e*f^4 + 9*(2*b*c*d + a*d^2)*f^5)*g
 - (8*b*d^2*e^2*f^3 - 11*(2*b*c*d + a*d^2)*e*f^4 - 99*(b*c^2 + 2*a*c*d)*f^
5)*h)*x^3 - 3*(11*(2*b*d^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e*f^4 - 21*(b*c^2
 + 2*a*c*d)*f^5)*g - (16*b*d^2*e^3*f^2 + 231*a*c^2*f^5 - 22*(2*b*c*d + a*d
^2)*e^2*f^3 + 33*(b*c^2 + 2*a*c*d)*e*f^4)*h)*x^2 - 11*(16*b*d^2*e^4*f - 10
5*a*c^2*e*f^4 - 24*(2*b*c*d + a*d^2)*e^3*f^2 + 42*(b*c^2 + 2*a*c*d)*e^2*f^
3)*g + 2*(64*b*d^2*e^5 - 231*a*c^2*e^2*f^3 - 88*(2*b*c*d + a*d^2)*e^4*f +
132*(b*c^2 + 2*a*c*d)*e^3*f^2)*h + (11*(8*b*d^2*e^3*f^2 + 105*a*c^2*f^5 -
12*(2*b*c*d + a*d^2)*e^2*f^3 + 21*(b*c^2 + 2*a*c*d)*e*f^4)*g - (64*b*d^2*e
^4*f - 231*a*c^2*e*f^4 - 88*(2*b*c*d + a*d^2)*e^3*f^2 + 132*(b*c^2 + 2*a*c
*d)*e^2*f^3)*h)*x)*sqrt(f*x + e)/f^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(265) = 530$.

Time = 1.65 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.50

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \begin{cases} \frac{2\left(\frac{bd^2h(e+fx)^{\frac{11}{2}}}{11f^4}+\frac{(e+fx)^{\frac{9}{2}}\left(ad^2fh+2bcdfh-4bd^2eh+bd^2fg\right)}{9f^4}+\frac{(e+fx)^{\frac{7}{2}}\cdot\left(2acdf^2h-3ad^2efh+ad^2f^2g+bc^2f^2h-6bcdefh+2bcdf^2g+6bd^2e^2h-3bd^2efg\right.}{7f^4}\right.}{} \\ \sqrt{e}\left(ac^2gx+\frac{bd^2hx^5}{5}+\frac{x^4\left(ad^2h+2bcdh+bd^2g\right)}{4}+\frac{x^3\cdot\left(2acdh+ad^2g+bc^2h+2bcdg\right)}{3}+\frac{x^2\left(ac^2h+2acdg+bc^2g\right)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**2*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b*d**2*h*(e + f*x)**(11/2)/(11*f**4) + (e + f*x)**(9/2)*(a*d
**2*f*h + 2*b*c*d*f*h - 4*b*d**2*e*h + b*d**2*f*g)/(9*f**4) + (e + f*x)**(
7/2)*(2*a*c*d*f**2*h - 3*a*d**2*e*f*h + a*d**2*f**2*g + b*c**2*f**2*h - 6*
b*c*d*e*f*h + 2*b*c*d*f**2*g + 6*b*d**2*e**2*h - 3*b*d**2*e*f*g)/(7*f**4)
+ (e + f*x)**(5/2)*(a*c**2*f**3*h - 4*a*c*d*e*f**2*h + 2*a*c*d*f**3*g + 3*
a*d**2*e**2*f*h - 2*a*d**2*e*f**2*g - 2*b*c**2*e*f**2*h + b*c**2*f**3*g +
6*b*c*d*e**2*f*h - 4*b*c*d*e*f**2*g - 4*b*d**2*e**3*h + 3*b*d**2*e**2*f*g)
/(5*f**4) + (e + f*x)**(3/2)*(-a*c**2*e*f**3*h + a*c**2*f**4*g + 2*a*c*d*e
**2*f**2*h - 2*a*c*d*e*f**3*g - a*d**2*e**3*f*h + a*d**2*e**2*f**2*g + b*c
**2*e**2*f**2*h - b*c**2*e*f**3*g - 2*b*c*d*e**3*f*h + 2*b*c*d*e**2*f**2*g
 + b*d**2*e**4*h - b*d**2*e**3*f*g)/(3*f**4))/f, Ne(f, 0)), (sqrt(e)*(a*c*
*2*g*x + b*d**2*h*x**5/5 + x**4*(a*d**2*h + 2*b*c*d*h + b*d**2*g)/4 + x**3
*(2*a*c*d*h + a*d**2*g + b*c**2*h + 2*b*c*d*g)/3 + x**2*(a*c**2*h + 2*a*c*
d*g + b*c**2*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.52

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\left(315\,(fx+e)^{\frac{11}{2}}bd^2h+385\,(bd^2fg-(4\,bd^2e-(2\,bcd+ad^2)f)h)(fx+e)^{\frac{9}{2}}-495\,((3\,bd^2ef-(2\,bcd+\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/3465*(315*(f*x + e)^(11/2)*b*d^2*h + 385*(b*d^2*f*g - (4*b*d^2*e - (2*b*
c*d + a*d^2)*f)*h)*(f*x + e)^(9/2) - 495*((3*b*d^2*e*f - (2*b*c*d + a*d^2)
*f^2)*g - (6*b*d^2*e^2 - 3*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*
h)*(f*x + e)^(7/2) + 693*((3*b*d^2*e^2*f - 2*(2*b*c*d + a*d^2)*e*f^2 + (b*
c^2 + 2*a*c*d)*f^3)*g - (4*b*d^2*e^3 - a*c^2*f^3 - 3*(2*b*c*d + a*d^2)*e^2
*f + 2*(b*c^2 + 2*a*c*d)*e*f^2)*h)*(f*x + e)^(5/2) - 1155*((b*d^2*e^3*f -
a*c^2*f^4 - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*g - (b*d^
2*e^4 - a*c^2*e*f^3 - (2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^2*f^2)
*h)*(f*x + e)^(3/2))/f^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. $2(228) = 456$.

Time = 0.13 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.62

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(f*x + e)*a*c^2*e*g + 1155*((f*x + e)^(3/2) - 3*sqrt(f*x
+ e)*e)*a*c^2*g + 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*b*c^2*e*g/f +
 2310*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*c*d*e*g/f + 1155*((f*x + e)^
(3/2) - 3*sqrt(f*x + e)*e)*a*c^2*e*h/f + 462*(3*(f*x + e)^(5/2) - 10*(f*x
+ e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*d*e*g/f^2 + 231*(3*(f*x + e)^(5/2
) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*d^2*e*g/f^2 + 231*(3*(f
*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c^2*g/f + 4
62*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c*d
*g/f + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^
2)*b*c^2*e*h/f^2 + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt
(f*x + e)*e^2)*a*c*d*e*h/f^2 + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)
*e + 15*sqrt(f*x + e)*e^2)*a*c^2*h/f + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e
)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*d^2*e*g/f^3 +
 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 -
35*sqrt(f*x + e)*e^3)*b*c*d*g/f^2 + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(
5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*d^2*g/f^2 + 198*
(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sq
rt(f*x + e)*e^3)*b*c*d*e*h/f^3 + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2
)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*d^2*e*h/f^3 + 99*(5
*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*s...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.04

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{(e+f\,x)^{9/2}\,(2\,a\,d^2\,f\,h-8\,b\,d^2\,e\,h+2\,b\,d^2\,f\,g+4\,b\,c\,d\,f\,h)}{9\,f^5}$$
$$+\frac{(e+f\,x)^{7/2}\,(2\,a\,d^2\,f^2\,g+2\,b\,c^2\,f^2\,h+12\,b\,d^2\,e^2\,h+4\,a\,c\,d\,f^2\,h+4\,b\,c\,d\,f^2\,g-6\,a\,d^2\,e\,f\,h-6\,b\,d^2\,e\,f\,g}{7\,f^5}$$
$$+\frac{2\,(e+f\,x)^{5/2}\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+b\,c\,f^2\,g+4\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-2\,b\,c\,e\,f\,h-3\,b\,d\,e\,f}{5\,f^5}$$
$$+\frac{2\,b\,d^2\,h\,(e+f\,x)^{11/2}}{11\,f^5}-\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{3\,f^5}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(a + b*x)*(c + d*x)^2,x)
```

output
```
((e + f*x)^(9/2)*(2*a*d^2*f*h - 8*b*d^2*e*h + 2*b*d^2*f*g + 4*b*c*d*f*h))/
(9*f^5) + ((e + f*x)^(7/2)*(2*a*d^2*f^2*g + 2*b*c^2*f^2*h + 12*b*d^2*e^2*h
 + 4*a*c*d*f^2*h + 4*b*c*d*f^2*g - 6*a*d^2*e*f*h - 6*b*d^2*e*f*g - 12*b*c*
d*e*f*h))/(7*f^5) + (2*(e + f*x)^(5/2)*(c*f - d*e)*(a*c*f^2*h + 2*a*d*f^2*
g + b*c*f^2*g + 4*b*d*e^2*h - 3*a*d*e*f*h - 2*b*c*e*f*h - 3*b*d*e*f*g))/(5
*f^5) + (2*b*d^2*h*(e + f*x)^(11/2))/(11*f^5) - (2*(e + f*x)^(3/2)*(a*f -
b*e)*(c*f - d*e)^2*(e*h - f*g))/(3*f^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.55

$$\int (a+bx)(c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\sqrt{fx+e}\,(315b\,d^2f^5h\,x^5+385a\,d^2f^5h\,x^4+770bcd\,f^5h\,x^4+35b\,d^2e\,f^4h\,x^4+385b\,d^2f^5g\,x^4+990acd\,f^5}{}$$

input
```
int((b*x+a)*(d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 462*a*c**2*e**2*f**3*h + 1155*a*c**2*e*f**4*g + 231*a
*c**2*e*f**4*h*x + 1155*a*c**2*f**5*g*x + 693*a*c**2*f**5*h*x**2 + 528*a*c
*d*e**3*f**2*h - 924*a*c*d*e**2*f**3*g - 264*a*c*d*e**2*f**3*h*x + 462*a*c
*d*e*f**4*g*x + 198*a*c*d*e*f**4*h*x**2 + 1386*a*c*d*f**5*g*x**2 + 990*a*c
*d*f**5*h*x**3 - 176*a*d**2*e**4*f*h + 264*a*d**2*e**3*f**2*g + 88*a*d**2*
e**3*f**2*h*x - 132*a*d**2*e**2*f**3*g*x - 66*a*d**2*e**2*f**3*h*x**2 + 99
*a*d**2*e*f**4*g*x**2 + 55*a*d**2*e*f**4*h*x**3 + 495*a*d**2*f**5*g*x**3 +
 385*a*d**2*f**5*h*x**4 + 264*b*c**2*e**3*f**2*h - 462*b*c**2*e**2*f**3*g
- 132*b*c**2*e**2*f**3*h*x + 231*b*c**2*e*f**4*g*x + 99*b*c**2*e*f**4*h*x*
*2 + 693*b*c**2*f**5*g*x**2 + 495*b*c**2*f**5*h*x**3 - 352*b*c*d*e**4*f*h
+ 528*b*c*d*e**3*f**2*g + 176*b*c*d*e**3*f**2*h*x - 264*b*c*d*e**2*f**3*g*
x - 132*b*c*d*e**2*f**3*h*x**2 + 198*b*c*d*e*f**4*g*x**2 + 110*b*c*d*e*f**
4*h*x**3 + 990*b*c*d*f**5*g*x**3 + 770*b*c*d*f**5*h*x**4 + 128*b*d**2*e**5
*h - 176*b*d**2*e**4*f*g - 64*b*d**2*e**4*f*h*x + 88*b*d**2*e**3*f**2*g*x
+ 48*b*d**2*e**3*f**2*h*x**2 - 66*b*d**2*e**2*f**3*g*x**2 - 40*b*d**2*e**2
*f**3*h*x**3 + 55*b*d**2*e*f**4*g*x**3 + 35*b*d**2*e*f**4*h*x**4 + 385*b*d
**2*f**5*g*x**4 + 315*b*d**2*f**5*h*x**5))/(3465*f**5)
```

3.13 $\int (c+dx)^2 \sqrt{e+fx}(g+hx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int (c+dx)^2 \sqrt{e+fx}(g+hx)\,dx = \frac{2(de-cf)^2(fg-eh)(e+fx)^{3/2}}{3f^4} - \frac{2(de-cf)(2dfg-3deh+cfh)(e+fx)^{5/2}}{5f^4} + \frac{2d(dfg-3deh+2cfh)(e+fx)^{7/2}}{7f^4} + \frac{2d^2h(e+fx)^{9/2}}{9f^4}$$

output
```
2/3*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(3/2)/f^4-2/5*(-c*f+d*e)*(c*f*h-3*d*e*
h+2*d*f*g)*(f*x+e)^(5/2)/f^4+2/7*d*(2*c*f*h-3*d*e*h+d*f*g)*(f*x+e)^(7/2)/f
^4+2/9*d^2*h*(f*x+e)^(9/2)/f^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$=\frac{2(e+fx)^{3/2}\left(21c^2f^2(5fg-2eh+3fhx)+6cdf(8e^2h+3f^2x(7g+5hx)-2ef(7g+6hx))+d^2(-16e^3\right.}{315f^4}$$

input
```
Integrate[(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output
```
(2*(e + f*x)^(3/2)*(21*c^2*f^2*(5*f*g - 2*e*h + 3*f*h*x) + 6*c*d*f*(8*e^2*
h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + d^2*(-16*e^3*h + 24*e^2
*f*(g + h*x) - 6*e*f^2*x*(6*g + 5*h*x) + 5*f^3*x^2*(9*g + 7*h*x))))/(315*f
^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$\Big\downarrow \text{86}$$
$$\int \left(\frac{d(e+fx)^{5/2}(2cfh-3deh+dfg)}{f^3}+\frac{(e+fx)^{3/2}(cf-de)(cfh-3deh+2dfg)}{f^3}+\frac{\sqrt{e+fx}(cf-de)^2(fg-\right.}{f^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{2d(e+fx)^{7/2}(2cfh-3deh+dfg)}{7f^4}-\frac{2(e+fx)^{5/2}(de-cf)(cfh-3deh+2dfg)}{5f^4}+\frac{2(e+fx)^{3/2}(de-cf)^2(fg-eh)}{3f^4}+\frac{2d^2h(e+fx)^{9/2}}{9f^4}$$

input

```
Int[(c + d*x)^2*Sqrt[e + f*x]*(g + h*x),x]
```

output

```
(2*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(3/2))/(3*f^4) - (2*(d*e - c*f)*(2*
d*f*g - 3*d*e*h + c*f*h)*(e + f*x)^(5/2))/(5*f^4) + (2*d*(d*f*g - 3*d*e*h
+ 2*c*f*h)*(e + f*x)^(7/2))/(7*f^4) + (2*d^2*h*(e + f*x)^(9/2))/(9*f^4)
```

Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{4\left(\left(\left(-\frac{15x^{2}\left(\frac{7hx}{9}+g\right)d^{2}}{14}-3xc\left(\frac{5hx}{7}+g\right)d-\frac{5c^{2}\left(\frac{3hx}{5}+g\right)}{2}\right)f^{3}+\left(\frac{6x\left(\frac{5hx}{6}+g\right)d^{2}}{7}+2c\left(\frac{6hx}{7}+g\right)d+h\,c^{2}\right)e\,f^{2}-\frac{8e^{2}\left(\frac{(hx+}{2}\right.}{}\right.\right.}{15f^{4}}$
derivativedivides	$\frac{\frac{2d^{2}h(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(2d(cf-de)h+d^{2}(-eh+fg)\right)(fx+e)^{\frac{7}{2}}}{7}+\frac{2\left((cf-de)^{2}h+2d(cf-de)(-eh+fg)\right)(fx+e)^{\frac{5}{2}}}{5}+\frac{2(cf-de)^{2}(-eh+fg)(}{3}}{f^{4}}$
default	$\frac{\frac{2d^{2}h(fx+e)^{\frac{9}{2}}}{9}-\frac{2\left(-2d(cf-de)h+d^{2}(eh-fg)\right)(fx+e)^{\frac{7}{2}}}{7}-\frac{2\left(-(cf-de)^{2}h+2d(cf-de)(eh-fg)\right)(fx+e)^{\frac{5}{2}}}{5}-\frac{2(cf-de)^{2}(eh-fg)(f}{3}}{f^{4}}$
gosper	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-35d^{2}h\,x^{3}f^{3}-90cd\,f^{3}h\,x^{2}+30d^{2}e\,f^{2}h\,x^{2}-45d^{2}f^{3}g\,x^{2}-63c^{2}f^{3}hx+72cde\,f^{2}hx-126cd\,f^{3}gx-24d^{2}e^{2}f\right.}{315f^{4}}$
orering	$-\frac{2(fx+e)^{\frac{3}{2}}\left(-35d^{2}h\,x^{3}f^{3}-90cd\,f^{3}h\,x^{2}+30d^{2}e\,f^{2}h\,x^{2}-45d^{2}f^{3}g\,x^{2}-63c^{2}f^{3}hx+72cde\,f^{2}hx-126cd\,f^{3}gx-24d^{2}e^{2}f\right.}{315f^{4}}$
trager	$-\frac{2\left(-35d^{2}h\,f^{4}x^{4}-90cd\,f^{4}h\,x^{3}-5d^{2}e\,f^{3}h\,x^{3}-45d^{2}f^{4}g\,x^{3}-63c^{2}f^{4}h\,x^{2}-18cde\,f^{3}h\,x^{2}-126cd\,f^{4}g\,x^{2}+6d^{2}e^{2}f^{2}h\,x^{2}-\right.}{}$
risch	$-\frac{2\left(-35d^{2}h\,f^{4}x^{4}-90cd\,f^{4}h\,x^{3}-5d^{2}e\,f^{3}h\,x^{3}-45d^{2}f^{4}g\,x^{3}-63c^{2}f^{4}h\,x^{2}-18cde\,f^{3}h\,x^{2}-126cd\,f^{4}g\,x^{2}+6d^{2}e^{2}f^{2}h\,x^{2}-\right.}{}$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-4/15*((-15/14*x^2*(7/9*h*x+g)*d^2-3*x*c*(5/7*h*x+g)*d-5/2*c^2*(3/5*h*x+g)
)*f^3+(6/7*x*(5/6*h*x+g)*d^2+2*c*(6/7*h*x+g)*d+h*c^2)*e*f^2-8/7*e^2*(1/2*(
h*x+g)*d+c*h)*d*f+8/21*d^2*e^3*h)*(f*x+e)^(3/2)/f^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(110) = 220$.

Time = 0.07 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.94

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\left(35\,d^2f^4hx^4+5\left(9\,d^2f^4g+(d^2ef^3+18\,cdf^4)h\right)x^3+3\left(3\left(d^2ef^3+14\,cdf^4\right)g-\left(2\,d^2e^2f^2-6\,cdef^3-21\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/315*(35*d^2*f^4*h*x^4 + 5*(9*d^2*f^4*g + (d^2*e*f^3 + 18*c*d*f^4)*h)*x^3
 + 3*(3*(d^2*e*f^3 + 14*c*d*f^4)*g - (2*d^2*e^2*f^2 - 6*c*d*e*f^3 - 21*c^2
*f^4)*h)*x^2 + 3*(8*d^2*e^3*f - 28*c*d*e^2*f^2 + 35*c^2*e*f^3)*g - 2*(8*d^
2*e^4 - 24*c*d*e^3*f + 21*c^2*e^2*f^2)*h - (3*(4*d^2*e^2*f^2 - 14*c*d*e*f^
3 - 35*c^2*f^4)*g - (8*d^2*e^3*f - 24*c*d*e^2*f^2 + 21*c^2*e*f^3)*h)*x)*sq
rt(f*x + e)/f^4
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.04

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{9}{2}}}{9f^3}+\frac{(e+fx)^{\frac{7}{2}}\cdot\left(2cdfh-3d^2eh+d^2fg\right)}{7f^3}+\frac{(e+fx)^{\frac{5}{2}}\left(c^2f^2h-4cdefh+2cdf^2g+3d^2e^2h-2d^2efg\right)}{5f^3}+\frac{(e+fx)^{\frac{3}{2}}\left(-c^2ef^2h+c^2f^3g+2cde^2fh-2cde\right.}{3f^3}\right.}{f} \\ \sqrt{e}\left(c^2gx+\frac{d^2hx^4}{4}+\frac{x^3\cdot(2cdh+d^2g)}{3}+\frac{x^2\left(c^2h+2cdg\right)}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(1/2)*(h*x+g),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(9/2)/(9*f**3) + (e + f*x)**(7/2)*(2*c*d*f
*h - 3*d**2*e*h + d**2*f*g)/(7*f**3) + (e + f*x)**(5/2)*(c**2*f**2*h - 4*c
*d*e*f*h + 2*c*d*f**2*g + 3*d**2*e**2*h - 2*d**2*e*f*g)/(5*f**3) + (e + f*
x)**(3/2)*(-c**2*e*f**2*h + c**2*f**3*g + 2*c*d*e**2*f*h - 2*c*d*e*f**2*g
- d**2*e**3*h + d**2*e**2*f*g)/(3*f**3))/f, Ne(f, 0)), (sqrt(e)*(c**2*g*x
+ d**2*h*x**4/4 + x**3*(2*c*d*h + d**2*g)/3 + x**2*(c**2*h + 2*c*d*g)/2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.31

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \frac{2\left(35\,(fx+e)^{\frac{9}{2}}d^2h+45\,(d^2fg-(3\,d^2e-2\,cdf)h)(fx+e)^{\frac{7}{2}}-63\,(2\,(d^2ef-cdf^2)g-(3\,d^2e^2-4\,cdef\right.}{315\,f^4}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/315*(35*(f*x + e)^(9/2)*d^2*h + 45*(d^2*f*g - (3*d^2*e - 2*c*d*f)*h)*(f*
x + e)^(7/2) - 63*(2*(d^2*e*f - c*d*f^2)*g - (3*d^2*e^2 - 4*c*d*e*f + c^2*
f^2)*h)*(f*x + e)^(5/2) + 105*((d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*g - (d^
2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*h)*(f*x + e)^(3/2))/f^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.86

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \frac{2\left(315\sqrt{fx+e}c^2eg+105\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)c^2g+\frac{210\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)cdeg}{f}+\frac{105\left((fx+e)^{\frac{3}{2}}-3\sqrt{f\right.}{f}\right.}{}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(f*x + e)*c^2*e*g + 105*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*
e)*c^2*g + 210*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*c*d*e*g/f + 105*((f*x
 + e)^(3/2) - 3*sqrt(f*x + e)*e)*c^2*e*h/f + 21*(3*(f*x + e)^(5/2) - 10*(f
*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*d^2*e*g/f^2 + 42*(3*(f*x + e)^(5/2
) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*d*g/f + 42*(3*(f*x + e)
^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*d*e*h/f^2 + 21*(3*
(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c^2*h/f + 9
*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*s
qrt(f*x + e)*e^3)*d^2*g/f^2 + 9*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e
+ 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*d^2*e*h/f^3 + 18*(5*(f*x
+ e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x +
 e)*e^3)*c*d*h/f^2 + (35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*
x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*d^2*h/
f^3)/f
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx = \frac{(e+f\,x)^{7/2}\,(2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h)}{7\,f^4} + \frac{2\,(e+f\,x)^{5/2}\,(c\,f-d\,e)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g)}{5\,f^4} - \frac{2\,(e+f\,x)^{3/2}\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{3\,f^4} + \frac{2\,d^2\,h\,(e+f\,x)^{9/2}}{9\,f^4}$$

input
```
int((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^2,x)
```

output
```
((e + f*x)^(7/2)*(2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h))/(7*f^4) + (2*(e + f*
x)^(5/2)*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(5*f^4) - (2*(e + f*x)^(
3/2)*(c*f - d*e)^2*(e*h - f*g))/(3*f^4) + (2*d^2*h*(e + f*x)^(9/2))/(9*f^4
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.99

$$\int (c+dx)^2\sqrt{e+fx}(g+hx)\,dx$$
$$= \frac{2\sqrt{fx+e}\,(35d^2f^4h\,x^4+90cd\,f^4h\,x^3+5d^2e\,f^3h\,x^3+45d^2f^4g\,x^3+63c^2f^4h\,x^2+18cde\,f^3h\,x^2+126cd\,f$$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 42*c**2*e**2*f**2*h + 105*c**2*e*f**3*g + 21*c**2*e*f
**3*h*x + 105*c**2*f**4*g*x + 63*c**2*f**4*h*x**2 + 48*c*d*e**3*f*h - 84*c
*d*e**2*f**2*g - 24*c*d*e**2*f**2*h*x + 42*c*d*e*f**3*g*x + 18*c*d*e*f**3*
h*x**2 + 126*c*d*f**4*g*x**2 + 90*c*d*f**4*h*x**3 - 16*d**2*e**4*h + 24*d*
*2*e**3*f*g + 8*d**2*e**3*f*h*x - 12*d**2*e**2*f**2*g*x - 6*d**2*e**2*f**2
*h*x**2 + 9*d**2*e*f**3*g*x**2 + 5*d**2*e*f**3*h*x**3 + 45*d**2*f**4*g*x**
3 + 35*d**2*f**4*h*x**4))/(315*f**4)
```

3.14 $\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx} dx$

Optimal result

Integrand size = 29, antiderivative size = 257

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx &= \frac{2(bc-ad)^2(bg-ah)\sqrt{e+fx}}{b^4}\\
&+\frac{2(a^2d^2f^2h-abdf(dfg-deh+2cfh)+b^2(c^2f^2h-d^2e(fg-eh)+2cdf(fg-eh)))\,(e+fx)^{3/2}}{3b^3f^3}\\
&-\frac{2d(adfh-b(dfg-2deh+2cfh))(e+fx)^{5/2}}{5b^2f^3}+\frac{2d^2h(e+fx)^{7/2}}{7bf^3}\\
&-\frac{2(bc-ad)^2\sqrt{be-af}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{9/2}}\end{aligned}$$

output
```
2*(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^4+2/3*(a^2*d^2*f^2*h-a*b*d*f*(2*
c*f*h-d*e*h+d*f*g)+b^2*(c^2*f^2*h-d^2*e*(-e*h+f*g)+2*c*d*f*(-e*h+f*g)))*(f
*x+e)^(3/2)/b^3/f^3-2/5*d*(a*d*f*h-b*(2*c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(5/2
)/b^2/f^3+2/7*d^2*h*(f*x+e)^(7/2)/b/f^3-2*(-a*d+b*c)^2*(-a*f+b*e)^(1/2)*(-
a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$= \frac{2\sqrt{e+fx}(-105a^3d^2f^3h+35a^2bdf^2(6cfh+d(3fg+eh+fhx))-7ab^2f(15c^2f^2h-d^2(e+fx)(-5fg}{}$$
$$-\frac{2(bc-ad)^2\sqrt{-be+af}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{9/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x),x]
```

output
```
(2*Sqrt[e + f*x]*(-105*a^3*d^2*f^3*h + 35*a^2*b*d*f^2*(6*c*f*h + d*(3*f*g
+ e*h + f*h*x)) - 7*a*b^2*f*(15*c^2*f^2*h - d^2*(e + f*x)*(-5*f*g + 2*e*h
- 3*f*h*x) + 10*c*d*f*(3*f*g + e*h + f*h*x)) + b^3*(35*c^2*f^2*(3*f*g + e*
h + f*h*x) + 14*c*d*f*(e + f*x)*(5*f*g - 2*e*h + 3*f*h*x) + d^2*(e + f*x)*
(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)))))/(105*b^4*f^3) -
 (2*(b*c - a*d)^2*Sqrt[-(b*e) + a*f]*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e +
f*x])/Sqrt[-(b*e) + a*f]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$\Big\downarrow \text{170}$$
$$\frac{2\int \frac{(c+dx)\sqrt{e+fx}(7bcfg-4adeh-3acfh-(7adfh-b(7dfg-4deh+4cfh))x)}{2(a+bx)}dx}{7bf}+\frac{2h(c+dx)^2(e+fx)^{3/2}}{7bf}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(7bcfg-4adeh-3acfh-(7adfh-b(7dfg-4deh+4cfh))x)}{a+bx}dx}{7bf}+\frac{2h(c+dx)^2(e+fx)^{3/2}}{7bf}$$

$$\Big\downarrow \; 164$$

$$\frac{\frac{7f(bc-ad)^2(bg-ah)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b^2}+\frac{2(e+fx)^{3/2}(35a^2d^2f^2h-3bdfx(7adfh-b(4cfh-4deh+7dfg))-7abdf(10cfh-2deh+5dfg)+2b^2(10c^2f^2h+\cdots}{15b^2f^2}}{7bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{3/2}}{7bf}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{7f(bc-ad)^2(bg-ah)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b^2}+\frac{2(e+fx)^{3/2}(35a^2d^2f^2h-3bdfx(7adfh-b(4cfh-4deh+7dfg))-7abdf(10cfh-2\cdots}{15b^2f^2}}{7bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{3/2}}{7bf}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{7f(bc-ad)^2(bg-ah)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b^2}+\frac{2(e+fx)^{3/2}(35a^2d^2f^2h-3bdfx(7adfh-b(4cfh-4deh+7dfg))-7abdf(\cdots}{15\cdots}}{7bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{3/2}}{7bf}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2(e+fx)^{3/2}(35a^2d^2f^2h-3bdfx(7adfh-b(4cfh-4deh+7dfg))-7abdf(10cfh-2deh+5dfg)+2b^2(10c^2f^2h+7cdf(5fg-2eh)+d^2(-e)(7fg-4eh))\cdots}{15b^2f^2}}{7bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{3/2}}{7bf}$$

input
```
Int[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x),x]
```

output
```
(2*h*(c + d*x)^2*(e + f*x)^(3/2))/(7*b*f) + ((2*(e + f*x)^(3/2)*(35*a^2*d^
2*f^2*h - 7*a*b*d*f*(5*d*f*g - 2*d*e*h + 10*c*f*h) + 2*b^2*(10*c^2*f^2*h -
 d^2*e*(7*f*g - 4*e*h) + 7*c*d*f*(5*f*g - 2*e*h)) - 3*b*d*f*(7*a*d*f*h - b
*(7*d*f*g - 4*d*e*h + 4*c*f*h))*x))/(15*b^2*f^2) + (7*(b*c - a*d)^2*f*(b*g
 - a*h)*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e
+ f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b^2)/(7*b*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$\frac{-2\left(\left(\left(\left(-\frac{8\left(\frac{15}{8}f^2x^2-\frac{3}{2}efx+e^2\right)(fx+e)b^3}{105}-\frac{2a\left(-\frac{3fx}{2}+e\right)f(fx+e)b^2}{15}-\frac{a^2f^2(fx+e)b}{3}+a^3f^3\right)d^2-2c\left(-\frac{2(fx+e)\left(-\frac{3fx}{2}+e\right)}{15}\right.\right.\right.\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{d^2h(fx+e)^{\frac{7}{2}}b^3}{7}+\frac{a\,b^2d^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3cdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{2b^3d^2eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3d^2fg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2b\,d^2f^2h(fx+e)^{\frac{3}{2}}}{3}+2a\right.}{}$
default	$-\frac{2\left(-\frac{d^2h(fx+e)^{\frac{7}{2}}b^3}{7}+\frac{a\,b^2d^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3cdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{2b^3d^2eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3d^2fg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2b\,d^2f^2h(fx+e)^{\frac{3}{2}}}{3}+2a\right.}{}$
risch	$-\frac{2\left(-15d^2h\,b^3f^3x^3+21a\,b^2d^2f^3h\,x^2-42b^3cd\,f^3h\,x^2-3b^3d^2e\,f^2h\,x^2-21b^3d^2f^3g\,x^2-35a^2b\,d^2f^3hx+70a\,b^2cd\,f^3hx+\right.}{}$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/((a*f-b*e)*b)^(1/2)*(-((((-8/105*(15/8*f^2*x^2-3/2*e*f*x+e^2)*(f*x+e)*b^3
-2/15*a*(-3/2*f*x+e)*f*(f*x+e)*b^2-1/3*a^2*f^2*(f*x+e)*b+a^3*f^3)*d^2-2*c*
(-2/15*(f*x+e)*(-3/2*f*x+e)*b^2-1/3*a*f*(f*x+e)*b+a^2*f^2)*b*f*d+c^2*(1/3*
(-f*x-e)*b+a*f)*b^2*f^2)*h-b*g*f*((-2/15*(f*x+e)*(-3/2*f*x+e)*b^2-1/3*a*f*
(f*x+e)*b+a^2*f^2)*d^2-2*c*(1/3*(-f*x-e)*b+a*f)*b*f*d+b^2*c^2*f^2))*((a*f-
b*e)*b)^(1/2)*(f*x+e)^(1/2)+f^3*(a*d-b*c)^2*(a*h-b*g)*(a*f-b*e)*arctan(b*(
f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))/f^3/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(233) = 466$.

Time = 0.10 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.72

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/105*(105*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^3*g - (a*b^2*c^2 - 2*a
^2*b*c*d + a^3*d^2)*f^3*h)*sqrt((b*e - a*f)/b)*log((b*f*x + 2*b*e - a*f +
2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a)) - 2*(15*b^3*d^2*f^3*h*x^
3 + 3*(7*b^3*d^2*f^3*g + (b^3*d^2*e*f^2 + 7*(2*b^3*c*d - a*b^2*d^2)*f^3)*h
)*x^2 - 7*(2*b^3*d^2*e^2*f - 5*(2*b^3*c*d - a*b^2*d^2)*e*f^2 - 15*(b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*f^3)*g + (8*b^3*d^2*e^3 - 14*(2*b^3*c*d - a*b^
2*d^2)*e^2*f + 35*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*e*f^2 - 105*(a*b^2*c
^2 - 2*a^2*b*c*d + a^3*d^2)*f^3)*h + (7*(b^3*d^2*e*f^2 + 5*(2*b^3*c*d - a*
b^2*d^2)*f^3)*g - (4*b^3*d^2*e^2*f - 7*(2*b^3*c*d - a*b^2*d^2)*e*f^2 - 35*
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(b^4*f^3), -
2/105*(105*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^3*g - (a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2)*f^3*h)*sqrt(-(b*e - a*f)/b)*arctan(-sqrt(f*x + e)*b*sqrt
(-(b*e - a*f)/b)/(b*e - a*f)) - (15*b^3*d^2*f^3*h*x^3 + 3*(7*b^3*d^2*f^3*g
 + (b^3*d^2*e*f^2 + 7*(2*b^3*c*d - a*b^2*d^2)*f^3)*h)*x^2 - 7*(2*b^3*d^2*e
^2*f - 5*(2*b^3*c*d - a*b^2*d^2)*e*f^2 - 15*(b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*f^3)*g + (8*b^3*d^2*e^3 - 14*(2*b^3*c*d - a*b^2*d^2)*e^2*f + 35*(b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*e*f^2 - 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3
*d^2)*f^3)*h + (7*(b^3*d^2*e*f^2 + 5*(2*b^3*c*d - a*b^2*d^2)*f^3)*g - (4*b
^3*d^2*e^2*f - 7*(2*b^3*c*d - a*b^2*d^2)*e*f^2 - 35*(b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(b^4*f^3)]
```

Sympy [A] (verification not implemented)

Time = 12.14 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{7}{2}}}{7bf^2} + \frac{(e+fx)^{\frac{5}{2}}\left(-ad^2fh+2bcdfh-2bd^2eh+bd^2fg\right)}{5b^2f^2} + \frac{(e+fx)^{\frac{3}{2}}\left(a^2d^2f^2h-2abcdf^2h+abd^2efh-abd^2f^2g+b^2c^2f^2h-2b^2cdefh+2b^2cdf^2g+b^2c\right.}{3b^3f^2}\right.}{f} \\ \sqrt{e}\left(\frac{d^2hx^3}{3b} + \frac{x^2\left(-ad^2h+2bcdh+bd^2g\right)}{2b^2} + \frac{x\left(a^2d^2h-2abcdh-abd^2g+b^2c^2h+2b^2cdg\right)}{b^3} - \frac{(ad-bc)^2(ah-bg)\left(\left(\begin{cases}\frac{x}{a} & \text{for } b = \\ \frac{\log(a+bx)}{b} & \text{otherw}\end{cases}\right.\right.}{b^3}\right. \end{cases}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(1/2)*(h*x+g)/(b*x+a),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(7/2)/(7*b*f**2) + (e + f*x)**(5/2)*(-a*d*
*2*f*h + 2*b*c*d*f*h - 2*b*d**2*e*h + b*d**2*f*g)/(5*b**2*f**2) + (e + f*x
)**(3/2)*(a**2*d**2*f**2*h - 2*a*b*c*d*f**2*h + a*b*d**2*e*f*h - a*b*d**2*
f**2*g + b**2*c**2*f**2*h - 2*b**2*c*d*e*f*h + 2*b**2*c*d*f**2*g + b**2*d*
*2*e**2*h - b**2*d**2*e*f*g)/(3*b**3*f**2) + sqrt(e + f*x)*(-a**3*d**2*f*h
 + 2*a**2*b*c*d*f*h + a**2*b*d**2*f*g - a*b**2*c**2*f*h - 2*a*b**2*c*d*f*g
 + b**3*c**2*f*g)/b**4 + f*(a*d - b*c)**2*(a*f - b*e)*(a*h - b*g)*atan(sqr
t(e + f*x)/sqrt((a*f - b*e)/b))/(b**5*sqrt((a*f - b*e)/b)))/f, Ne(f, 0)),
(sqrt(e)*(d**2*h*x**3/(3*b) + x**2*(-a*d**2*h + 2*b*c*d*h + b*d**2*g)/(2*b
**2) + x*(a**2*d**2*h - 2*a*b*c*d*h - a*b*d**2*g + b**2*c**2*h + 2*b**2*c*
d*g)/b**3 - (a*d - b*c)**2*(a*h - b*g)*Piecewise((x/a, Eq(b, 0)), (log(a +
 b*x)/b, True))/b**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(233) = 466$.

Time = 0.14 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.26

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$
$$= \frac{2\,(b^4c^2eg - 2\,ab^3cdeg + a^2b^2d^2eg - ab^3c^2fg + 2\,a^2b^2cdfg - a^3bd^2fg - ab^3c^2eh + 2\,a^2b^2cdeh - a^3bd^2eh +}{\sqrt{-b^2e+abf}b^4}$$
$$+ \frac{2\left(21\,(fx+e)^{\frac{5}{2}}b^6d^2f^{19}g - 35\,(fx+e)^{\frac{3}{2}}b^6d^2ef^{19}g + 70\,(fx+e)^{\frac{3}{2}}b^6cdf^{20}g - 35\,(fx+e)^{\frac{3}{2}}ab^5d^2f^{20}g +\right.}{}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x, algorithm="giac")
```

output
```
2*(b^4*c^2*e*g - 2*a*b^3*c*d*e*g + a^2*b^2*d^2*e*g - a*b^3*c^2*f*g + 2*a^2
*b^2*c*d*f*g - a^3*b*d^2*f*g - a*b^3*c^2*e*h + 2*a^2*b^2*c*d*e*h - a^3*b*d
^2*e*h + a^2*b^2*c^2*f*h - 2*a^3*b*c*d*f*h + a^4*d^2*f*h)*arctan(sqrt(f*x
+ e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^4) + 2/105*(21*(f*x +
 e)^(5/2)*b^6*d^2*f^19*g - 35*(f*x + e)^(3/2)*b^6*d^2*e*f^19*g + 70*(f*x +
 e)^(3/2)*b^6*c*d*f^20*g - 35*(f*x + e)^(3/2)*a*b^5*d^2*f^20*g + 105*sqrt(
f*x + e)*b^6*c^2*f^21*g - 210*sqrt(f*x + e)*a*b^5*c*d*f^21*g + 105*sqrt(f*
x + e)*a^2*b^4*d^2*f^21*g + 15*(f*x + e)^(7/2)*b^6*d^2*f^18*h - 42*(f*x +
e)^(5/2)*b^6*d^2*e*f^18*h + 35*(f*x + e)^(3/2)*b^6*d^2*e^2*f^18*h + 42*(f*
x + e)^(5/2)*b^6*c*d*f^19*h - 21*(f*x + e)^(5/2)*a*b^5*d^2*f^19*h - 70*(f*
x + e)^(3/2)*b^6*c*d*e*f^19*h + 35*(f*x + e)^(3/2)*a*b^5*d^2*e*f^19*h + 35
*(f*x + e)^(3/2)*b^6*c^2*f^20*h - 70*(f*x + e)^(3/2)*a*b^5*c*d*f^20*h + 35
*(f*x + e)^(3/2)*a^2*b^4*d^2*f^20*h - 105*sqrt(f*x + e)*a*b^5*c^2*f^21*h +
 210*sqrt(f*x + e)*a^2*b^4*c*d*f^21*h - 105*sqrt(f*x + e)*a^3*b^3*d^2*f^21
*h)/(b^7*f^21)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.22

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx$$

$$= (e+f\,x)^{5/2}\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{5\,b\,f^3}-\frac{2\,d^2\,h\,\left(a\,f^4-b\,e\,f^3\right)}{5\,b^2\,f^6}\right)$$

$$-\sqrt{e+f\,x}\left(\frac{2\,{\left(c\,f-d\,e\right)}^2\,\left(e\,h-f\,g\right)}{b\,f^3}-\frac{\left(\frac{\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b\,f^3}-\frac{2\,d^2\,h\,\left(a\,f^4-b\,e\,f^3\right)}{b^2\,f^6}\right)\,\left(a\,f^4-b\,e\,f^3\right)}{b\,f^3}-\frac{2\,\left(c\,f-d\,e\right)\,\left(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g\right)}{b\,f^3}\right)\,\left(a\,f^4-b\,e\,f^3\right)}{b\,f^3}\right)$$

$$-(e+f\,x)^{3/2}\left(\frac{\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b\,f^3}-\frac{2\,d^2\,h\,\left(a\,f^4-b\,e\,f^3\right)}{b^2\,f^6}\right)\,\left(a\,f^4-b\,e\,f^3\right)}{3\,b\,f^3}-\frac{2\,\left(c\,f-d\,e\right)\,\left(c\,f\,h-3\,d\,e\,h+\right.}{3\,b\,f^3}\right.$$

$$+\frac{2\,d^2\,h\,{\left(e+f\,x\right)}^{7/2}}{7\,b\,f^3}$$

$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,{\left(a\,d-b\,c\right)}^2\,\sqrt{b\,e-a\,f}\,\left(a\,h-b\,g\right)\,1\mathrm{i}}{b^4\,c^2\,e\,g+a^4\,d^2\,f\,h-a\,b^3\,c^2\,e\,h-a\,b^3\,c^2\,f\,g-a^3\,b\,d^2\,e\,h-a^3\,b\,d^2\,f\,g+a^2\,b^2\,d^2\,e\,g+a^2\,b^2\,c^2\,f\,h-2\,a\,b^3\,c\,d\,e\,g-2\,a^3\,b\,c\,d\,f\,h+2\,a^2\,b^2\,c\,d\,e\,h}\right.}{b^{9/2}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^2)/(a + b*x),x)
```

output
```
(e + f*x)^(5/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(5*b*f^3) - (2*d^2*h*
(a*f^4 - b*e*f^3))/(5*b^2*f^6)) - (e + f*x)^(1/2)*((2*(c*f - d*e)^2*(e*h -
 f*g))/(b*f^3) - (((((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b*f^3) - (2*d^2*
h*(a*f^4 - b*e*f^3))/(b^2*f^6))*(a*f^4 - b*e*f^3))/(b*f^3) - (2*(c*f - d*e
)*(c*f*h - 3*d*e*h + 2*d*f*g))/(b*f^3))*(a*f^4 - b*e*f^3))/(b*f^3)) - (e +
 f*x)^(3/2)*((((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b*f^3) - (2*d^2*h*(a*f
^4 - b*e*f^3))/(b^2*f^6))*(a*f^4 - b*e*f^3))/(3*b*f^3) - (2*(c*f - d*e)*(c
*f*h - 3*d*e*h + 2*d*f*g))/(3*b*f^3)) + (2*d^2*h*(e + f*x)^(7/2))/(7*b*f^3
) + (atan((b^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)^2*(b*e - a*f)^(1/2)*(a*h -
b*g)*1i)/(b^4*c^2*e*g + a^4*d^2*f*h - a*b^3*c^2*e*h - a*b^3*c^2*f*g - a^3*
b*d^2*e*h - a^3*b*d^2*f*g + a^2*b^2*d^2*e*g + a^2*b^2*c^2*f*h - 2*a*b^3*c*
d*e*g - 2*a^3*b*c*d*f*h + 2*a^2*b^2*c*d*e*h + 2*a^2*b^2*c*d*f*g))*(a*d - b
*c)^2*(b*e - a*f)^(1/2)*(a*h - b*g)*2i)/b^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 865, normalized size of antiderivative = 3.37

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a),x)
```

output
```
(2*(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**3*d**2*f**3*h - 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d*f**3*h - 105*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**2*f**3*g
+ 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a*b**2*c**2*f**3*h + 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d*f**3*g - 105*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c**2*f**3*g -
105*sqrt(e + f*x)*a**3*b*d**2*f**3*h + 210*sqrt(e + f*x)*a**2*b**2*c*d*f**
3*h + 35*sqrt(e + f*x)*a**2*b**2*d**2*e*f**2*h + 105*sqrt(e + f*x)*a**2*b*
*2*d**2*f**3*g + 35*sqrt(e + f*x)*a**2*b**2*d**2*f**3*h*x - 105*sqrt(e + f
*x)*a*b**3*c**2*f**3*h - 70*sqrt(e + f*x)*a*b**3*c*d*e*f**2*h - 210*sqrt(e
 + f*x)*a*b**3*c*d*f**3*g - 70*sqrt(e + f*x)*a*b**3*c*d*f**3*h*x + 14*sqrt
(e + f*x)*a*b**3*d**2*e**2*f*h - 35*sqrt(e + f*x)*a*b**3*d**2*e*f**2*g - 7
*sqrt(e + f*x)*a*b**3*d**2*e*f**2*h*x - 35*sqrt(e + f*x)*a*b**3*d**2*f**3*
g*x - 21*sqrt(e + f*x)*a*b**3*d**2*f**3*h*x**2 + 35*sqrt(e + f*x)*b**4*c**
2*e*f**2*h + 105*sqrt(e + f*x)*b**4*c**2*f**3*g + 35*sqrt(e + f*x)*b**4*c*
*2*f**3*h*x - 28*sqrt(e + f*x)*b**4*c*d*e**2*f*h + 70*sqrt(e + f*x)*b**4*c
*d*e*f**2*g + 14*sqrt(e + f*x)*b**4*c*d*e*f**2*h*x + 70*sqrt(e + f*x)*b**4
*c*d*f**3*g*x + 42*sqrt(e + f*x)*b**4*c*d*f**3*h*x**2 + 8*sqrt(e + f*x)...
```

3.15 $\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 248

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx &= \frac{2(bc-ad)(2bdg+bch-3adh)\sqrt{e+fx}}{b^4}-\frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{b^4(a+bx)}\\ &\quad-\frac{2d(2adfh-b(dfg-deh+2cfh))(e+fx)^{3/2}}{3b^3f^2}+\frac{2d^2h(e+fx)^{5/2}}{5b^2f^2}\\ &\quad-\frac{(bc-ad)\left(7a^2dfh+b^2(4deg+cfg+2ceh)-ab(5dfg+6deh+3cfh)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{9/2}\sqrt{be-af}}\end{aligned}$$

output
```
2*(-a*d+b*c)*(-3*a*d*h+b*c*h+2*b*d*g)*(f*x+e)^(1/2)/b^4-(-a*d+b*c)^2*(-a*h
+b*g)*(f*x+e)^(1/2)/b^4/(b*x+a)-2/3*d*(2*a*d*f*h-b*(2*c*f*h-d*e*h+d*f*g))*
(f*x+e)^(3/2)/b^3/f^2+2/5*d^2*h*(f*x+e)^(5/2)/b^2/f^2-(-a*d+b*c)*(7*a^2*d*
f*h+b^2*(2*c*e*h+c*f*g+4*d*e*g)-a*b*(3*c*f*h+6*d*e*h+5*d*f*g))*arctanh(b^(
1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(9/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$= \frac{\sqrt{e+fx}(105a^3d^2f^2h-5a^2bdf(30cfh+d(15fg+4eh-14fhx))+b^3(-15c^2f^2(g-2hx)+20cdfx(3f$$
$$+\frac{(bc-ad)\left(7a^2dfh+b^2(4deg+cfg+2ceh)-ab(5dfg+6deh+3cfh)\right)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{9/2}\sqrt{-be+af}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(105*a^3*d^2*f^2*h - 5*a^2*b*d*f*(30*c*f*h + d*(15*f*g + 4*
e*h - 14*f*h*x)) + b^3*(-15*c^2*f^2*(g - 2*h*x) + 20*c*d*f*x*(3*f*g + e*h
+ f*h*x) + 2*d^2*x*(e + f*x)*(5*f*g - 2*e*h + 3*f*h*x)) + a*b^2*(45*c^2*f^
2*h + 10*c*d*f*(9*f*g + 2*e*h - 10*f*h*x) - 2*d^2*(2*e^2*h + f^2*x*(25*g +
 7*h*x) + e*f*(-5*g + 9*h*x)))))/(15*b^4*f^2*(a + b*x)) + ((b*c - a*d)*(7*
a^2*d*f*h + b^2*(4*d*e*g + c*f*g + 2*c*e*h) - a*b*(5*d*f*g + 6*d*e*h + 3*c
*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(9/2)*Sqrt[-
(b*e) + a*f])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {166, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg+bcfg+2bceh-4adeh-3acfh+d(5bfg+2beh-7afh)x)}{2(a+bx)}dx}{b(be-af)}-\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg+bcfg+2bceh-4adeh-3acfh+d(5bfg+2beh-7afh)x)}{a+bx}dx}{2b(be-af)}-\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 164

$$\frac{\frac{(bc-ad)(7a^2dfh-ab(3cfh+6deh+5dfg)+b^2(2ceh+cfg+4deg))\int \frac{\sqrt{e+fx}}{a+bx}dx}{b^2}+\frac{2d(e+fx)^{3/2}(35a^2df^2h-abf(50cfh+16deh+25dfg)+3bdfx(-7}{15b^2f^2}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 60

$$\frac{\frac{(bc-ad)(7a^2dfh-ab(3cfh+6deh+5dfg)+b^2(2ceh+cfg+4deg))\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b^2}+\frac{2d(e+fx)^{3/2}(35a^2df^2h-abf(50cfh}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 73

$$\frac{\frac{(bc-ad)(7a^2dfh-ab(3cfh+6deh+5dfg)+b^2(2ceh+cfg+4deg))\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b^2}+\frac{2d(e+fx)^{3/2}(35a^2df^2h-ab}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 221

$$\frac{\frac{(bc-ad)\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)(7a^2dfh-ab(3cfh+6deh+5dfg)+b^2(2ceh+cfg+4deg))}{b^2}+\frac{2d(e+fx)^{3/2}(35a^2df^2h-abf(}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

input
```
Int[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^2,x]
```

output
```
-(((b*g - a*h)*(c + d*x)^2*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x))) + (
(2*d*(e + f*x)^(3/2)*(35*a^2*d*f^2*h - a*b*f*(25*d*f*g + 16*d*e*h + 50*c*f
*h) + 2*b^2*(d*e*(5*f*g - 2*e*h) + 5*c*f*(3*f*g + 2*e*h)) + 3*b*d*f*(5*b*f
*g + 2*b*e*h - 7*a*f*h)*x))/(15*b^2*f^2) + ((b*c - a*d)*(7*a^2*d*f*h + b^2
*(4*d*e*g + c*f*g + 2*c*e*h) - a*b*(5*d*f*g + 6*d*e*h + 3*c*f*h))*((2*Sqrt
[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e
 - a*f]])/b^(3/2)))/b^2)/(2*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{7\left(\left(\frac{(cfg+2e(ch+2dg))b^2}{7}-\frac{3a\left(\left(ch+\frac{5dg}{3}\right)f+2deh\right)b}{7}+a^2dfh\right)(ad-bc)(bx+a)f^2\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)-\left(\left(\left(\frac{2x^2\left(\frac{3hx}{5}\cdots\right.\right.\right.\right.}{3}\cdots\right.}{\cdots}$
risch	$\frac{2(3d^2hx^2b^2f^2-10abd^2f^2hx+10b^2cdf^2hx+b^2d^2efhx+5b^2d^2f^2gx+45a^2d^2f^2h-60abcdf^2h-10abd^2efh-30abd^2f\cdots}{15f^2b^4}$
derivativedivides	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}-\frac{2abd^2fh(fx+e)^{\frac{3}{2}}}{3}+\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-4abcdf^2h\sqrt{\cdots}\right.}{b^4}$
default	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}-\frac{2abd^2fh(fx+e)^{\frac{3}{2}}}{3}+\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-4abcdf^2h\sqrt{\cdots}\right.}{b^4}$

input

```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-7/((a*f-b*e)*b)^(1/2)*((1/7*(c*f*g+2*e*(c*h+2*d*g))*b^2-3/7*a*((c*h+5/3*d
*g)*f+2*d*e*h)*b+a^2*d*f*h)*(a*d-b*c)*(b*x+a)*f^2*arctan(b*(f*x+e)^(1/2)/(
(a*f-b*e)*b)^(1/2))-(1/7*((2/3*x^2*(3/5*h*x+g)*d^2+4*x*c*(1/3*h*x+g)*d-c^2
*(-2*h*x+g))*f^2+4/3*x*(1/2*(1/5*h*x+g)*d+c*h)*d*e*f-4/15*d^2*e^2*h*x)*b^3
+3/7*a*((-10/9*(7/25*h*x+g)*x*d^2+2*c*(-10/9*h*x+g)*d+h*c^2)*f^2+4/9*d*(1/
2*(-9/5*h*x+g)*d+c*h)*e*f-4/45*d^2*e^2*h)*b^2-10/7*a^2*d*(((-7/15*h*x+1/2*
g)*d+c*h)*f+2/15*d*e*h)*f*b+a^3*d^2*f^2*h)*(f*x+e)^(1/2)*((a*f-b*e)*b)^(1/
2))/f^2/b^4/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(226) = 452$.

Time = 0.12 (sec) , antiderivative size = 1738, normalized size of antiderivative = 7.01

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*sqrt(b^2*e - a*b*f)*((4*(a*b^3*c*d - a^2*b^2*d^2)*e*f^2 + (a*b^
3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^2)*f^3)*g + (2*(a*b^3*c^2 - 4*a^2*b^2*c*
d + 3*a^3*b*d^2)*e*f^2 - (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 7*a^4*d^2)*f^3)*h
 + ((4*(b^4*c*d - a*b^3*d^2)*e*f^2 + (b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^
2)*f^3)*g + (2*(b^4*c^2 - 4*a*b^3*c*d + 3*a^2*b^2*d^2)*e*f^2 - (3*a*b^3*c^
2 - 10*a^2*b^2*c*d + 7*a^3*b*d^2)*f^3)*h)*x)*log((b*f*x + 2*b*e - a*f + 2*
sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*(6*(b^5*d^2*e*f^2 - a*b^
4*d^2*f^3)*h*x^3 + 2*(5*(b^5*d^2*e*f^2 - a*b^4*d^2*f^3)*g + (b^5*d^2*e^2*f
 + 2*(5*b^5*c*d - 4*a*b^4*d^2)*e*f^2 - (10*a*b^4*c*d - 7*a^2*b^3*d^2)*f^3)
*h)*x^2 + 5*(2*a*b^4*d^2*e^2*f - (3*b^5*c^2 - 18*a*b^4*c*d + 17*a^2*b^3*d^
2)*e*f^2 + 3*(a*b^4*c^2 - 6*a^2*b^3*c*d + 5*a^3*b^2*d^2)*f^3)*g - (4*a*b^4
*d^2*e^3 - 4*(5*a*b^4*c*d - 4*a^2*b^3*d^2)*e^2*f - 5*(9*a*b^4*c^2 - 34*a^2
*b^3*c*d + 25*a^3*b^2*d^2)*e*f^2 + 15*(3*a^2*b^3*c^2 - 10*a^3*b^2*c*d + 7*
a^4*b*d^2)*f^3)*h + 2*(5*(b^5*d^2*e^2*f + 6*(b^5*c*d - a*b^4*d^2)*e*f^2 -
(6*a*b^4*c*d - 5*a^2*b^3*d^2)*f^3)*g - (2*b^5*d^2*e^3 - (10*b^5*c*d - 7*a*
b^4*d^2)*e^2*f - (15*b^5*c^2 - 60*a*b^4*c*d + 44*a^2*b^3*d^2)*e*f^2 + 5*(3
*a*b^4*c^2 - 10*a^2*b^3*c*d + 7*a^3*b^2*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(a*
b^6*e*f^2 - a^2*b^5*f^3 + (b^7*e*f^2 - a*b^6*f^3)*x), 1/15*(15*sqrt(-b^2*e
 + a*b*f)*((4*(a*b^3*c*d - a^2*b^2*d^2)*e*f^2 + (a*b^3*c^2 - 6*a^2*b^2*c*d
 + 5*a^3*b*d^2)*f^3)*g + (2*(a*b^3*c^2 - 4*a^2*b^2*c*d + 3*a^3*b*d^2)*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(226) = 452.

Time = 0.14 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.94

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$= \frac{(4\,b^3cdeg - 4\,ab^2d^2eg + b^3c^2fg - 6\,ab^2cdfg + 5\,a^2bd^2fg + 2\,b^3c^2eh - 8\,ab^2cdeh + 6\,a^2bd^2eh - 3\,ab^2c^2fh}{\sqrt{-b^2e+abf}b^4}$$
$$- \frac{\sqrt{fx+e}b^3c^2fg - 2\sqrt{fx+e}ab^2cdfg + \sqrt{fx+e}a^2bd^2fg - \sqrt{fx+e}ab^2c^2fh + 2\sqrt{fx+e}a^2bcdfh - \sqrt{}}{((fx+e)b - be + af)b^4}$$
$$+ \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^8d^2f^9g + 30\sqrt{fx+e}b^8cdf^{10}g - 30\sqrt{fx+e}ab^7d^2f^{10}g + 3\,(fx+e)^{\frac{5}{2}}b^8d^2f^8h - 5\,(fx+\right.}{}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(4*b^3*c*d*e*g - 4*a*b^2*d^2*e*g + b^3*c^2*f*g - 6*a*b^2*c*d*f*g + 5*a^2*b
*d^2*f*g + 2*b^3*c^2*e*h - 8*a*b^2*c*d*e*h + 6*a^2*b*d^2*e*h - 3*a*b^2*c^2
*f*h + 10*a^2*b*c*d*f*h - 7*a^3*d^2*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*
e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^4) - (sqrt(f*x + e)*b^3*c^2*f*g - 2*sq
rt(f*x + e)*a*b^2*c*d*f*g + sqrt(f*x + e)*a^2*b*d^2*f*g - sqrt(f*x + e)*a*
b^2*c^2*f*h + 2*sqrt(f*x + e)*a^2*b*c*d*f*h - sqrt(f*x + e)*a^3*d^2*f*h)/(
((f*x + e)*b - b*e + a*f)*b^4) + 2/15*(5*(f*x + e)^(3/2)*b^8*d^2*f^9*g + 3
0*sqrt(f*x + e)*b^8*c*d*f^10*g - 30*sqrt(f*x + e)*a*b^7*d^2*f^10*g + 3*(f*
x + e)^(5/2)*b^8*d^2*f^8*h - 5*(f*x + e)^(3/2)*b^8*d^2*e*f^8*h + 10*(f*x +
 e)^(3/2)*b^8*c*d*f^9*h - 10*(f*x + e)^(3/2)*a*b^7*d^2*f^9*h + 15*sqrt(f*x
 + e)*b^8*c^2*f^10*h - 60*sqrt(f*x + e)*a*b^7*c*d*f^10*h + 45*sqrt(f*x + e
)*a^2*b^6*d^2*f^10*h)/(b^10*f^10)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$
$$= (e+f\,x)^{3/2}\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{3\,b^2\,f^2}-\frac{4\,d^2\,h\,(a\,f-b\,e)}{3\,b^3\,f^2}\right)$$
$$-\sqrt{e+f\,x}\left(\frac{2\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b^2\,f^2}-\frac{4\,d^2\,h\,(a\,f-b\,e)}{b^3\,f^2}\right)(a\,f-b\,e)}{b}\right.$$
$$\left.-\frac{2\,(c\,f-d\,e)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g)}{b^2\,f^2}+\frac{2\,d^2\,h\,(a\,f-b\,e)^2}{b^4\,f^2}\right)$$
$$-\frac{\sqrt{e+f\,x}\,(-f\,h\,a^3\,d^2+2\,f\,h\,a^2\,b\,c\,d+f\,g\,a^2\,b\,d^2-f\,h\,a\,b^2\,c^2-2\,f\,g\,a\,b^2\,c\,d+f\,g\,b^3\,c^2)}{b^5\,(e+f\,x)-b^5\,e+a\,b^4\,f}$$
$$+\frac{2\,d^2\,h\,(e+f\,x)^{5/2}}{5\,b^2\,f^2}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,(a\,d-b\,c)\,(2\,b^2\,c\,e\,h+b^2\,c\,f\,g+4\,b^2\,d\,e\,g+7\,a^2\,d\,f\,h-3\,a\,b\,c\,f\,h-6\,a\,b\,d\,e\,h-5\,a\,b\,d\,f\,g)}{\sqrt{a\,f-b\,e}\,(2\,b^3\,c^2\,e\,h+b^3\,c^2\,f\,g-7\,a^3\,d^2\,f\,h-4\,a\,b^2\,d^2\,e\,g-3\,a\,b^2\,c^2\,f\,h+6\,a^2\,b\,d^2\,e\,h+5\,a^2\,b\,d^2\,f\,g+4\,b^3\,c\,d\,e\,g-8\,a\,b^2\,c\,d\,e\,h-6\,a\,b^2\,c\,d}\right.}{b^{9/2}\,\sqrt{}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^2,x)
```

output
```
(e + f*x)^(3/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(3*b^2*f^2) - (4*d^2*
h*(a*f - b*e))/(3*b^3*f^2)) - (e + f*x)^(1/2)*((2*((2*d^2*f*g - 6*d^2*e*h
+ 4*c*d*f*h)/(b^2*f^2) - (4*d^2*h*(a*f - b*e))/(b^3*f^2))*(a*f - b*e))/b -
 (2*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(b^2*f^2) + (2*d^2*h*(a*f - b
*e)^2)/(b^4*f^2)) - ((e + f*x)^(1/2)*(b^3*c^2*f*g - a^3*d^2*f*h - a*b^2*c^
2*f*h + a^2*b*d^2*f*g - 2*a*b^2*c*d*f*g + 2*a^2*b*c*d*f*h))/(b^5*(e + f*x)
 - b^5*e + a*b^4*f) + (2*d^2*h*(e + f*x)^(5/2))/(5*b^2*f^2) + (atan((b^(1/
2)*(e + f*x)^(1/2)*(a*d - b*c)*(2*b^2*c*e*h + b^2*c*f*g + 4*b^2*d*e*g + 7*
a^2*d*f*h - 3*a*b*c*f*h - 6*a*b*d*e*h - 5*a*b*d*f*g))/((a*f - b*e)^(1/2)*(
2*b^3*c^2*e*h + b^3*c^2*f*g - 7*a^3*d^2*f*h - 4*a*b^2*d^2*e*g - 3*a*b^2*c^
2*f*h + 6*a^2*b*d^2*e*h + 5*a^2*b*d^2*f*g + 4*b^3*c*d*e*g - 8*a*b^2*c*d*e*
h - 6*a*b^2*c*d*f*g + 10*a^2*b*c*d*f*h)))*(a*d - b*c)*(2*b^2*c*e*h + b^2*c
*f*g + 4*b^2*d*e*g + 7*a^2*d*f*h - 3*a*b*c*f*h - 6*a*b*d*e*h - 5*a*b*d*f*g
))/(b^(9/2)*(a*f - b*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1921, normalized size of antiderivative = 7.75

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**4*d**2*f**3*h + 150*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d*f**3*h + 90*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*e*f**2*h
 + 75*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*b*d**2*f**3*g - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*f**3*h*x - 45*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**2*f*
*3*h - 120*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**2*b**2*c*d*e*f**2*h - 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*d*f**3*g + 150*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**
2*c*d*f**3*h*x - 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f**2*g + 90*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f**2*h*x
 + 75*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**2*b**2*d**2*f**3*g*x + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c**2*e*f**2*h + 15*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c**2*f
**3*g - 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt...
```

3.16 $\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 352

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \frac{2d(bdg+2bch-3adh)\sqrt{e+fx}}{b^4} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{2b^4(a+bx)^2} - \frac{(bc-ad)\left(13a^2dfh+b^2(8deg+cfg+4ceh)-ab(9dfg+12deh+5cfh)\right)\sqrt{e+fx}}{4b^4(be-af)(a+bx)} + \frac{2d^2h(e+fx)^{3/2}}{3b^3f} + \frac{(35a^3d^2f^2h-15a^2bdf(dfg+4deh+2cfh)-b^3(8d^2e^2g-c^2f(fg-4eh)+8cde(fg+2eh))+3ab^2(c\ldots}{4b^{9/2}(be-af)^{3/2}}$$

output
```
2*d*(-3*a*d*h+2*b*c*h+b*d*g)*(f*x+e)^(1/2)/b^4-1/2*(-a*d+b*c)^2*(-a*h+b*g)
*(f*x+e)^(1/2)/b^4/(b*x+a)^2-1/4*(-a*d+b*c)*(13*a^2*d*f*h+b^2*(4*c*e*h+c*f
*g+8*d*e*g)-a*b*(5*c*f*h+12*d*e*h+9*d*f*g))*(f*x+e)^(1/2)/b^4/(-a*f+b*e)/(
b*x+a)+2/3*d^2*h*(f*x+e)^(3/2)/b^3/f+1/4*(35*a^3*d^2*f^2*h-15*a^2*b*d*f*(2
*c*f*h+4*d*e*h+d*f*g)-b^3*(8*d^2*e^2*g-c^2*f*(-4*e*h+f*g)+8*c*d*e*(2*e*h+f
*g))+3*a*b^2*(c^2*f^2*h+8*d^2*e*(e*h+f*g)+2*c*d*f*(8*e*h+f*g)))*arctanh(b^
(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(9/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$
$$= \frac{\sqrt{e+fx}(105a^4d^2f^2h-5a^3bdf(18cfh+d(9fg+22eh-35fhx))+b^4(-24cdefx(g-2hx)-3c^2f(2eg$$
$$-\frac{(-35a^3d^2f^2h+15a^2bdf(dfg+4deh+2cfh)+b^3(8d^2e^2g+8cde(fg+2eh)+c^2f(-fg+4eh))-3a}{4b^{9/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(105*a^4*d^2*f^2*h - 5*a^3*b*d*f*(18*c*f*h + d*(9*f*g + 22*
e*h - 35*f*h*x)) + b^4*(-24*c*d*e*f*x*(g - 2*h*x) - 3*c^2*f*(2*e*g + f*g*x
 + 4*e*h*x) + 8*d^2*e*x^2*(3*f*g + e*h + f*h*x)) + a*b^3*(6*c*d*f*(-2*e*(g
 - 12*h*x) + f*x*(5*g - 8*h*x)) + 3*c^2*f*(-2*e*h + f*(g + 5*h*x)) - 8*d^2
*x*(-2*e^2*h + f^2*x*(3*g + h*x) + e*f*(-9*g + 8*h*x))) + a^2*b^2*(9*c^2*f
^2*h + 6*c*d*f*(3*f*g + 14*e*h - 25*f*h*x) + d^2*(8*e^2*h + 2*e*f*(21*g -
94*h*x) + f^2*x*(-75*g + 56*h*x))))))/(12*b^4*f*(b*e - a*f)*(a + b*x)^2) -
((-35*a^3*d^2*f^2*h + 15*a^2*b*d*f*(d*f*g + 4*d*e*h + 2*c*f*h) + b^3*(8*d^
2*e^2*g + 8*c*d*e*(f*g + 2*e*h) + c^2*f*(-(f*g) + 4*e*h)) - 3*a*b^2*(c^2*f
^2*h + 8*d^2*e*(f*g + e*h) + 2*c*d*f*(f*g + 8*e*h)))*ArcTan[(Sqrt[b]*Sqrt[
e + f*x])/Sqrt[-(b*e) + a*f]])/(4*b^(9/2)*(-(b*e) + a*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 163, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg-bcfg+4bceh-4adeh-3acfh+d(3bfg+4beh-7afh)x)}{2(a+bx)^2}dx}{2b(be-af)}-\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg-bcfg+4bceh-4adeh-3acfh+d(3bfg+4beh-7afh)x)}{(a+bx)^2}dx}{4b(be-af)}-\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \; 163$$

$$\frac{\frac{(e+fx)^{3/2}(35a^3d^2f^2h-a^2bdf(30cfh+46deh+15dfg)+ab^2(9c^2f^2h+6cdf(6eh+fg)+2d^2e(4eh+9fg))+2bd^2x(be-af)(-7afh+4beh+3bfg)-3b}{3b^2f(a+bx)(be-af)}}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(e+fx)^{3/2}(35a^3d^2f^2h-a^2bdf(30cfh+46deh+15dfg)+ab^2(9c^2f^2h+6cdf(6eh+fg)+2d^2e(4eh+9fg))+2bd^2x(be-af)(-7afh+4beh+3bfg)-3b}{3b^2f(a+bx)(be-af)}}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(e+fx)^{3/2}(35a^3d^2f^2h-a^2bdf(30cfh+46deh+15dfg)+ab^2(9c^2f^2h+6cdf(6eh+fg)+2d^2e(4eh+9fg))+2bd^2x(be-af)(-7afh+4beh+3bfg)-3b}{3b^2f(a+bx)(be-af)}}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(e+fx)^{3/2}\left(35a^3d^2f^2h-a^2bdf(30cfh+46deh+15dfg)+ab^2\left(9c^2f^2h+6cdf(6eh+fg)+2d^2e(4eh+9fg)\right)+2bd^2x(be-af)(-7afh+4beh+3bfg)-3b}{3b^2f(a+bx)(be-af)}}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

input
```
Int[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^3,x]
```

output
```
-1/2*((b*g - a*h)*(c + d*x)^2*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x)^2)
 + (((e + f*x)^(3/2)*(35*a^3*d^2*f^2*h - 3*b^3*c*f*(4*d*e*g - c*f*g + 4*c*
e*h) - a^2*b*d*f*(15*d*f*g + 46*d*e*h + 30*c*f*h) + a*b^2*(9*c^2*f^2*h + 2
*d^2*e*(9*f*g + 4*e*h) + 6*c*d*f*(f*g + 6*e*h)) + 2*b*d^2*(b*e - a*f)*(3*b
*f*g + 4*b*e*h - 7*a*f*h)*x))/(3*b^2*f*(b*e - a*f)*(a + b*x)) - ((35*a^3*d
^2*f^2*h - 15*a^2*b*d*f*(d*f*g + 4*d*e*h + 2*c*f*h) - b^3*(8*d^2*e^2*g - c
^2*f*(f*g - 4*e*h) + 8*c*d*e*(f*g + 2*e*h)) + 3*a*b^2*(c^2*f^2*h + 8*d^2*e
*(f*g + e*h) + 2*c*d*f*(f*g + 8*e*h)))*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e
- a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/(2*b^2*
(b*e - a*f)))/(4*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.40

method	result
pseudoelliptic	$-\frac{35\left(-\left(\left(\frac{\left(g f^{2} c^{2}-4 c e (c h+2 d g) f-16\left(c h+\frac{d g}{2}\right) d e^{2}\right) b^{3}}{35}+\frac{3\left(\left(h c^{2}+2 c d g\right) f^{2}+8\left(2 c d h+d^{2} g\right) e f+8 d^{2} e^{2} h\right) a b^{2}}{35}-\frac{6 a^{2}\left(\left(c h+\frac{d g}{2}\right) f+\cdots\right.}{7}\cdots\right.\right.}{\cdots}$
risch	$-\frac{2 d(-h x d b f+9 a d f h-6 b c f h-b d e h-3 b d f g) \sqrt{f x+e}}{3 f\, b^{4}}+\frac{-\frac{b f\left(13 a^{3} d^{2} f h-18 a^{2} b c d f h-12 a^{2} b\, d^{2} e h-9 a^{2} b\, d^{2} f g+5 a\, b^{2} c^{2} f h+1\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$\frac{-\frac{2 d\left(-\frac{d h (f x+e)^{\frac{3}{2}} b}{3}+3 a d f h \sqrt{f x+e}-2 b c f h \sqrt{f x+e}-b d f g \sqrt{f x+e}\right)}{b^{4}}+\frac{2 f\left(-\frac{b f\left(13 a^{3} d^{2} f h-18 a^{2} b c d f h-12 a^{2} b\, d^{2} e h-9 a^{2} b\, d^{2} f g+5 a\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{2 d\left(-\frac{d h (f x+e)^{\frac{3}{2}} b}{3}+3 a d f h \sqrt{f x+e}-2 b c f h \sqrt{f x+e}-b d f g \sqrt{f x+e}\right)}{b^{4}}+\frac{2 f\left(-\frac{b f\left(13 a^{3} d^{2} f h-18 a^{2} b c d f h-12 a^{2} b\, d^{2} e h-9 a^{2} b\, d^{2} f g+5 a\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-35/4/((a*f-b*e)*b)^(1/2)*(-(1/35*(g*f^2*c^2-4*c*e*(c*h+2*d*g)*f-16*(c*h+1
/2*d*g)*d*e^2)*b^3+3/35*((c^2*h+2*c*d*g)*f^2+8*(2*c*d*h+d^2*g)*e*f+8*d^2*e
^2*h)*a*b^2-6/7*a^2*((c*h+1/2*d*g)*f+2*d*e*h)*d*f*b+a^3*d^2*f^2*h)*(b*x+a)
^2*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+(1/35*(-c^2*f^2*g*x-2*(4*
(-1/3*h*x^3-g*x^2)*d^2+4*c*x*(-2*h*x+g)*d+c^2*(2*h*x+g))*e*f+8/3*d^2*e^2*h
*x^2)*b^4-2/35*a*((4*(1/3*h*x^3+g*x^2)*d^2-5*x*(-8/5*h*x+g)*c*d-1/2*c^2*(5
*h*x+g))*f^2+(-12*x*(-8/9*h*x+g)*d^2+2*c*(-12*h*x+g)*d+h*c^2)*e*f-8/3*d^2*
e^2*h*x)*b^3+3/35*a^2*((1/3*(56/3*h*x^2-25*g*x)*d^2+2*c*(-25/3*h*x+g)*d+h*
c^2)*f^2+28/3*((-47/21*h*x+1/2*g)*d+c*h)*d*e*f+8/9*d^2*e^2*h)*b^2-6/7*a^3*
d*((1/2*(-35/9*h*x+g)*d+c*h)*f+11/9*d*e*h)*f*b+a^4*d^2*f^2*h)*(f*x+e)^(1/2
)*((a*f-b*e)*b)^(1/2))/(b*x+a)^2/b^4/(a*f-b*e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1332 vs. $2(326) = 652$.

Time = 0.25 (sec) , antiderivative size = 2678, normalized size of antiderivative = 7.61

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(b^2*e - a*b*f)*(((8*b^5*d^2*e^2*f + 8*(b^5*c*d - 3*a*b^4*d^2
)*e*f^2 - (b^5*c^2 + 6*a*b^4*c*d - 15*a^2*b^3*d^2)*f^3)*g + (8*(2*b^5*c*d
- 3*a*b^4*d^2)*e^2*f + 4*(b^5*c^2 - 12*a*b^4*c*d + 15*a^2*b^3*d^2)*e*f^2 -
 (3*a*b^4*c^2 - 30*a^2*b^3*c*d + 35*a^3*b^2*d^2)*f^3)*h)*x^2 + (8*a^2*b^3*
d^2*e^2*f + 8*(a^2*b^3*c*d - 3*a^3*b^2*d^2)*e*f^2 - (a^2*b^3*c^2 + 6*a^3*b
^2*c*d - 15*a^4*b*d^2)*f^3)*g + (8*(2*a^2*b^3*c*d - 3*a^3*b^2*d^2)*e^2*f +
 4*(a^2*b^3*c^2 - 12*a^3*b^2*c*d + 15*a^4*b*d^2)*e*f^2 - (3*a^3*b^2*c^2 -
30*a^4*b*c*d + 35*a^5*d^2)*f^3)*h + 2*((8*a*b^4*d^2*e^2*f + 8*(a*b^4*c*d -
 3*a^2*b^3*d^2)*e*f^2 - (a*b^4*c^2 + 6*a^2*b^3*c*d - 15*a^3*b^2*d^2)*f^3)*
g + (8*(2*a*b^4*c*d - 3*a^2*b^3*d^2)*e^2*f + 4*(a*b^4*c^2 - 12*a^2*b^3*c*d
 + 15*a^3*b^2*d^2)*e*f^2 - (3*a^2*b^3*c^2 - 30*a^3*b^2*c*d + 35*a^4*b*d^2)
*f^3)*h)*x)*log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e)
)/(b*x + a)) + 2*(8*(b^6*d^2*e^2*f - 2*a*b^5*d^2*e*f^2 + a^2*b^4*d^2*f^3)*
h*x^3 + 8*(3*(b^6*d^2*e^2*f - 2*a*b^5*d^2*e*f^2 + a^2*b^4*d^2*f^3)*g + (b^
6*d^2*e^3 + 3*(2*b^6*c*d - 3*a*b^5*d^2)*e^2*f - 3*(4*a*b^5*c*d - 5*a^2*b^4
*d^2)*e*f^2 + (6*a^2*b^4*c*d - 7*a^3*b^3*d^2)*f^3)*h)*x^2 - 3*(2*(b^6*c^2
+ 2*a*b^5*c*d - 7*a^2*b^4*d^2)*e^2*f - (3*a*b^5*c^2 + 10*a^2*b^4*c*d - 29*
a^3*b^3*d^2)*e*f^2 + (a^2*b^4*c^2 + 6*a^3*b^3*c*d - 15*a^4*b^2*d^2)*f^3)*g
 + (8*a^2*b^4*d^2*e^3 - 2*(3*a*b^5*c^2 - 42*a^2*b^4*c*d + 59*a^3*b^3*d^2)*
e^2*f + (15*a^2*b^4*c^2 - 174*a^3*b^3*c*d + 215*a^4*b^2*d^2)*e*f^2 - 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(326) = 652$.

Time $= 0.16$ (sec) , antiderivative size $= 903$, normalized size of antiderivative $= 2.57$

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g + 8*b^3*c*d*e*f*g - 24*a*b^2*d^2*e*f*g - b^3*c^2*f^2*
g - 6*a*b^2*c*d*f^2*g + 15*a^2*b*d^2*f^2*g + 16*b^3*c*d*e^2*h - 24*a*b^2*d
^2*e^2*h + 4*b^3*c^2*e*f*h - 48*a*b^2*c*d*e*f*h + 60*a^2*b*d^2*e*f*h - 3*a
*b^2*c^2*f^2*h + 30*a^2*b*c*d*f^2*h - 35*a^3*d^2*f^2*h)*arctan(sqrt(f*x +
e)*b/sqrt(-b^2*e + a*b*f))/((b^5*e - a*b^4*f)*sqrt(-b^2*e + a*b*f)) - 1/4*
(8*(f*x + e)^(3/2)*b^4*c*d*e*f*g - 8*(f*x + e)^(3/2)*a*b^3*d^2*e*f*g - 8*s
qrt(f*x + e)*b^4*c*d*e^2*f*g + 8*sqrt(f*x + e)*a*b^3*d^2*e^2*f*g + (f*x +
e)^(3/2)*b^4*c^2*f^2*g - 10*(f*x + e)^(3/2)*a*b^3*c*d*f^2*g + 9*(f*x + e)^
(3/2)*a^2*b^2*d^2*f^2*g + sqrt(f*x + e)*b^4*c^2*e*f^2*g + 14*sqrt(f*x + e)
*a*b^3*c*d*e*f^2*g - 15*sqrt(f*x + e)*a^2*b^2*d^2*e*f^2*g - sqrt(f*x + e)*
a*b^3*c^2*f^3*g - 6*sqrt(f*x + e)*a^2*b^2*c*d*f^3*g + 7*sqrt(f*x + e)*a^3*
b*d^2*f^3*g + 4*(f*x + e)^(3/2)*b^4*c^2*e*f*h - 16*(f*x + e)^(3/2)*a*b^3*c
*d*e*f*h + 12*(f*x + e)^(3/2)*a^2*b^2*d^2*e*f*h - 4*sqrt(f*x + e)*b^4*c^2*
e^2*f*h + 16*sqrt(f*x + e)*a*b^3*c*d*e^2*f*h - 12*sqrt(f*x + e)*a^2*b^2*d^
2*e^2*f*h - 5*(f*x + e)^(3/2)*a*b^3*c^2*f^2*h + 18*(f*x + e)^(3/2)*a^2*b^2
*c*d*f^2*h - 13*(f*x + e)^(3/2)*a^3*b*d^2*f^2*h + 7*sqrt(f*x + e)*a*b^3*c^
2*e*f^2*h - 30*sqrt(f*x + e)*a^2*b^2*c*d*e*f^2*h + 23*sqrt(f*x + e)*a^3*b*
d^2*e*f^2*h - 3*sqrt(f*x + e)*a^2*b^2*c^2*f^3*h + 14*sqrt(f*x + e)*a^3*b*c
*d*f^3*h - 11*sqrt(f*x + e)*a^4*d^2*f^3*h)/((b^5*e - a*b^4*f)*((f*x + e)*b
 - b*e + a*f)^2) + 2/3*(3*sqrt(f*x + e)*b^6*d^2*f^3*g + (f*x + e)^(3/2)...
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx$$
$$= \sqrt{e+f\,x}\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b^3\,f}-\frac{6\,d^2\,h\,(a\,f-b\,e)}{b^4\,f}\right)$$
$$-\frac{\sqrt{e+f\,x}\left(\frac{11\,h\,a^3\,d^2\,f^2}{4}-\frac{7\,h\,a^2\,b\,c\,d\,f^2}{2}-\frac{7\,g\,a^2\,b\,d^2\,f^2}{4}-3\,e\,h\,a^2\,b\,d^2\,f+\frac{3\,h\,a\,b^2\,c^2\,f^2}{4}+\frac{3\,g\,a\,b^2\,c\,d\,f^2}{2}+4\,e\,h\,a\,b^2\,c\,a \cdots\right.}{\cdots}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,(35\,h\,a^3\,d^2\,f^2-30\,h\,a^2\,b\,c\,d\,f^2-60\,h\,a^2\,b\,d^2\,e\,f-15\,g\,a^2\,b\,d^2\,f^2+3\,h\,a\,b^2\,c^2\,f^2+48\,h \cdots}{\cdots}$$
$$+\frac{2\,d^2\,h\,(e+f\,x)^{3/2}}{3\,b^3\,f}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^3,x)
```

output
```
(e + f*x)^(1/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b^3*f) - (6*d^2*h*(a
*f - b*e))/(b^4*f)) - ((e + f*x)^(1/2)*((b^3*c^2*f^2*g)/4 + (11*a^3*d^2*f^
2*h)/4 - b^3*c^2*e*f*h + (3*a*b^2*c^2*f^2*h)/4 - (7*a^2*b*d^2*f^2*g)/4 - 2
*b^3*c*d*e*f*g + (3*a*b^2*c*d*f^2*g)/2 - (7*a^2*b*c*d*f^2*h)/2 + 2*a*b^2*d
^2*e*f*g - 3*a^2*b*d^2*e*f*h + 4*a*b^2*c*d*e*f*h) - ((e + f*x)^(3/2)*(b^4*
c^2*f^2*g + 4*b^4*c^2*e*f*h - 5*a*b^3*c^2*f^2*h - 13*a^3*b*d^2*f^2*h + 9*a
^2*b^2*d^2*f^2*g + 8*b^4*c*d*e*f*g - 10*a*b^3*c*d*f^2*g - 8*a*b^3*d^2*e*f*
g + 18*a^2*b^2*c*d*f^2*h + 12*a^2*b^2*d^2*e*f*h - 16*a*b^3*c*d*e*f*h))/(4*
(a*f - b*e)))/(b^6*(e + f*x)^2 - (e + f*x)*(2*b^6*e - 2*a*b^5*f) + b^6*e^2
 + a^2*b^4*f^2 - 2*a*b^5*e*f) + (atan((b^(1/2)*(e + f*x)^(1/2))/(a*f - b*e
)^(1/2))*(b^3*c^2*f^2*g - 8*b^3*d^2*e^2*g + 35*a^3*d^2*f^2*h - 16*b^3*c*d*
e^2*h - 4*b^3*c^2*e*f*h + 3*a*b^2*c^2*f^2*h + 24*a*b^2*d^2*e^2*h - 15*a^2*
b*d^2*f^2*g - 8*b^3*c*d*e*f*g + 6*a*b^2*c*d*f^2*g - 30*a^2*b*c*d*f^2*h + 2
4*a*b^2*d^2*e*f*g - 60*a^2*b*d^2*e*f*h + 48*a*b^2*c*d*e*f*h))/(4*b^(9/2)*(
a*f - b*e)^(3/2)) + (2*d^2*h*(e + f*x)^(3/2))/(3*b^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3292, normalized size of antiderivative = 9.35

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3,x)
```

output

```
(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**5*d**2*f**3*h - 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d*f**3*h - 180*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*e*f**2*h -
45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**4*b*d**2*f**3*g + 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*f**3*h*x + 9*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*f**3*h
 + 144*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**3*b**2*c*d*e*f**2*h + 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*f**3*g - 180*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*
d*f**3*h*x + 72*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**3*b**2*d**2*e**2*f*h + 72*sqrt(b)*sqrt(a*f - b*e)*atan(
(sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*e*f**2*g - 360
*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**3*b**2*d**2*e*f**2*h*x - 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*f**3*g*x + 105*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*
*2*f**3*h*x**2 - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr...
```

3.17 $\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 484

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \frac{2d^2h\sqrt{e+fx}}{b^4} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{3b^4(a+bx)^3}$$
$$- \frac{(bc-ad)\left(19a^2dfh+b^2(12deg+cfg+6ceh)-ab(13dfg+18deh+7cfh)\right)\sqrt{e+fx}}{12b^4(be-af)(a+bx)^2}$$
$$+ \frac{(29a^3d^2f^2h-a^2bdf(11dfg+54deh+22cfh)-b^3(8d^2e^2g-c^2f(fg-2eh)+4cde(fg+4eh))+ab^2(}{8b^4(be-af)^2(a+bx)}$$
$$+ \frac{(35a^3d^2f^3h-5a^2bdf^2(dfg+18deh+2cfh)+b^3(4cdef(fg-4eh)-c^2f^2(fg-2eh)-8d^2e^2(fg+2}{8b^{9/2}(be-af)^{5/2}}$$

output
```
2*d^2*h*(f*x+e)^(1/2)/b^4-1/3*(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^4/(b
*x+a)^3-1/12*(-a*d+b*c)*(19*a^2*d*f*h+b^2*(6*c*e*h+c*f*g+12*d*e*g)-a*b*(7*
c*f*h+18*d*e*h+13*d*f*g))*(f*x+e)^(1/2)/b^4/(-a*f+b*e)/(b*x+a)^2+1/8*(29*a
^3*d^2*f^2*h-a^2*b*d*f*(22*c*f*h+54*d*e*h+11*d*f*g)-b^3*(8*d^2*e^2*g-c^2*f
*(-2*e*h+f*g)+4*c*d*e*(4*e*h+f*g))+a*b^2*(c^2*f^2*h+4*d^2*e*(6*e*h+5*f*g)+
2*c*d*f*(20*e*h+f*g)))*(f*x+e)^(1/2)/b^4/(-a*f+b*e)^2/(b*x+a)+1/8*(35*a^3*
d^2*f^3*h-5*a^2*b*d*f^2*(2*c*f*h+18*d*e*h+d*f*g)+b^3*(4*c*d*e*f*(-4*e*h+f*
g)-c^2*f^2*(-2*e*h+f*g)-8*d^2*e^2*(2*e*h+f*g))-a*b^2*f*(c^2*f^2*h+2*c*d*f*
(-12*e*h+f*g)-12*d^2*e*(6*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b
*e)^(1/2))/b^(9/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx$$
$$= \frac{\sqrt{e+fx}(105a^5d^2f^2h-5a^4bdf(6cfh+d(3fg+40eh-56fhx))-b^5(24d^2e^2x^2(g-2hx)+12cdex(fgx$$
$$+\frac{(-35a^3d^2f^3h+5a^2bdf^2(dfg+18deh+2cfh)+b^3(c^2f^2(fg-2eh)+8d^2e^2(fg+2eh)+4cdef(-fg}{8b^{9/2}(-be+af)^{5/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^4,x]
```

output
```
(Sqrt[e + f*x]*(105*a^5*d^2*f^2*h - 5*a^4*b*d*f*(6*c*f*h + d*(3*f*g + 40*e
*h - 56*f*h*x)) - b^5*(24*d^2*e^2*x^2*(g - 2*h*x) + 12*c*d*e*x*(f*g*x + 2*
e*(g + 2*h*x)) + c^2*(-3*f^2*g*x^2 + 2*e*f*x*(g + 3*h*x) + 4*e^2*(2*g + 3*
h*x))) + a*b^4*(12*d^2*e*x*(-2*e*(g - 9*h*x) + f*x*(5*g - 8*h*x)) + c^2*(-
4*e^2*h + 14*e*f*(g + h*x) + f^2*x*(8*g + 3*h*x)) + 2*c*d*(3*f^2*g*x^2 - 4
*e^2*(g + 6*h*x) + 2*e*f*x*(7*g + 30*h*x))) - a^2*b^3*(c^2*f*(3*f*g - 4*e*
h + 8*f*h*x) + 2*c*d*(8*e^2*h + f^2*x*(8*g + 33*h*x) - 2*e*f*(2*g + 35*h*x
)) + d^2*(4*e^2*(2*g - 63*h*x) + 3*f^2*x^2*(11*g - 16*h*x) + 10*e*f*x*(-7*
g + 45*h*x))) + a^3*b^2*(-3*c^2*f^2*h - 2*c*d*f*(3*f*g - 26*e*h + 40*f*h*x
) + d^2*(92*e^2*h + e*f*(26*g - 538*h*x) + f^2*x*(-40*g + 231*h*x)))))/(24
*b^4*(b*e - a*f)^2*(a + b*x)^3) + ((-35*a^3*d^2*f^3*h + 5*a^2*b*d*f^2*(d*f
*g + 18*d*e*h + 2*c*f*h) + b^3*(c^2*f^2*(f*g - 2*e*h) + 8*d^2*e^2*(f*g + 2
*e*h) + 4*c*d*e*f*(-(f*g) + 4*e*h)) + a*b^2*f*(c^2*f^2*h + 2*c*d*f*(f*g -
12*e*h) - 12*d^2*e*(f*g + 6*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b
*e) + a*f]])/(8*b^(9/2)*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 162, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx$$

$$\Big\downarrow \text{ 166}$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg-3bcfg+6bceh-4adeh-3acfh+d(bfg+6beh-7afh)x)}{2(a+bx)^3}dx}{3b(be-af)} - \frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg-3bcfg+6bceh-4adeh-3acfh+d(bfg+6beh-7afh)x)}{(a+bx)^3}dx}{6b(be-af)} - \frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{ 162}$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h-a^3bdf(10cfh+76deh+5dfg)-a^2b^2\left(9c^2f^2h+2cdf(fg-10eh)-2d^2e(22eh+5fg)\right)+bx\left(49a^3d^2f^2h-a^2bdf(30cfh+110deh\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h-a^3bdf(10cfh+76deh+5dfg)-a^2b^2\left(9c^2f^2h+2cdf(fg-10eh)-2d^2e(22eh+5fg)\right)+bx\left(49a^3d^2f^2h-a^2bdf(30cfh+110deh\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h-a^3bdf(10cfh+76deh+5dfg)-a^2b^2\left(9c^2f^2h+2cdf(fg-10eh)-2d^2e(22eh+5fg)\right)+bx\left(49a^3d^2f^2h-a^2bdf(30cfh+110deh\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h-a^3bdf(10cfh+76deh+5dfg)-a^2b^2\left(9c^2f^2h+2cdf(fg-10eh)-2d^2e(22eh+5fg)\right)+bx\left(49a^3d^2f^2h-a^2bdf(30cfh+110deh\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

input
```
Int[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^4,x]
```

output
```
-1/3*((b*g - a*h)*(c + d*x)^2*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x)^3)
 + (((e + f*x)^(3/2)*(35*a^4*d^2*f^2*h - 2*b^4*c*e*(4*d*e*g - 3*c*f*g + 6*
c*e*h) - a^3*b*d*f*(5*d*f*g + 76*d*e*h + 10*c*f*h) - a*b^3*(8*d^2*e^2*g +
3*c^2*f*(3*f*g - 8*e*h) - 16*c*d*e*(f*g - e*h)) - a^2*b^2*(9*c^2*f^2*h + 2
*c*d*f*(f*g - 10*e*h) - 2*d^2*e*(5*f*g + 22*e*h)) + b*(49*a^3*d^2*f^2*h -
a^2*b*d*f*(7*d*f*g + 110*d*e*h + 30*c*f*h) - b^3*(16*d^2*e^2*g - 12*c*d*e*
(f*g - 4*e*h) + 3*c^2*f*(f*g - 2*e*h)) - a*b^2*(3*c^2*f^2*h + 6*c*d*f*(f*g
 - 12*e*h) - 4*d^2*e*(5*f*g + 16*e*h)))*x))/(4*b^2*(b*e - a*f)^2*(a + b*x)
^2) - (3*(35*a^3*d^2*f^3*h - 5*a^2*b*d*f^2*(d*f*g + 18*d*e*h + 2*c*f*h) +
b^3*(4*c*d*e*f*(f*g - 4*e*h) - c^2*f^2*(f*g - 2*e*h) - 8*d^2*e^2*(f*g + 2*
e*h)) - a*b^2*f*(c^2*f^2*h + 2*c*d*f*(f*g - 12*e*h) - 12*d^2*e*(f*g + 6*e*
h)))*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f
*x])/Sqrt[b*e - a*f]])/b^(3/2)))/(8*b^2*(b*e - a*f)^2))/(6*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$-\frac{35\left(\left(\frac{\left(-c^2 g f^3+2ce(ch+2dg)f^2-16\left(ch+\frac{dg}{2}\right)d e^2 f-16d^2e^3h\right)b^3}{35}-\frac{\left(\left(h c^2+2cdg\right)f^2-24\left(ch+\frac{dg}{2}\right)def-72d^2e^2h\right)afb^2}{35}-\frac{2a^2}{}\right.\right.}{}$
derivativedivides	$\frac{2d^2h\sqrt{fx+e}}{b^4}-\frac{2\left(-\frac{b^2f\left(29a^3d^2f^2h-22a^2bcdf^2h-54a^2bd^2efh-11a^2bd^2f^2g+ab^2c^2f^2h+40ab^2cdefh+2ab^2cdf^2g+24ab^2\right.}{16\left(a^2f^2-2abfe+b^2\right.}\right.}{}$
default	$\frac{2d^2h\sqrt{fx+e}}{b^4}-\frac{2\left(-\frac{b^2f\left(29a^3d^2f^2h-22a^2bcdf^2h-54a^2bd^2efh-11a^2bd^2f^2g+ab^2c^2f^2h+40ab^2cdefh+2ab^2cdf^2g+24ab^2\right.}{16\left(a^2f^2-2abfe+b^2\right.}\right.}{}$
risch	$\frac{2d^2h\sqrt{fx+e}}{b^4}-\frac{-\frac{b^2f\left(29a^3d^2f^2h-22a^2bcdf^2h-54a^2bd^2efh-11a^2bd^2f^2g+ab^2c^2f^2h+40ab^2cdefh+2ab^2cdf^2g+24ab^2d^2\right.}{8\left(a^2f^2-2abfe+b^2e^2\right)}}{}$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-35/8/((a*f-b*e)*b)^(1/2)*((1/35*(-c^2*g*f^3+2*c*e*(c*h+2*d*g)*f^2-16*(c*h
+1/2*d*g)*d*e^2*f-16*d^2*e^3*h)*b^3-1/35*((c^2*h+2*c*d*g)*f^2-24*(c*h+1/2*
d*g)*d*e*f-72*d^2*e^2*h)*a*f*b^2-2/7*a^2*((c*h+1/2*d*g)*f+9*d*e*h)*d*f^2*b
+a^3*d^2*f^3*h)*(b*x+a)^3*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-(1/3
5*(c^2*f^2*g*x^2-2/3*x*(6*d*g*x+c*(3*h*x+g))*c*e*f-8/3*(3*(-2*h*x^3+g*x^2)
*d^2+3*c*x*(2*h*x+g)*d+c^2*(3/2*h*x+g))*e^2)*b^5-4/105*(-2*x*c*(3/4*d*g*x+
c*(3/8*h*x+g))*f^2-7/2*(30/7*x^2*(-8/5*h*x+g)*d^2+2*x*c*(30/7*h*x+g)*d+c^2
*(h*x+g))*e*f+e^2*(6*x*(-9*h*x+g)*d^2+2*c*(6*h*x+g)*d+h*c^2))*a*b^4+4/105*
a^2*((3*(4*h*x^3-11/4*g*x^2)*d^2-4*x*c*(33/8*h*x+g)*d-3/4*(8/3*h*x+g)*c^2)
*f^2+(35/2*(-45/7*h*x+g)*x*d^2+2*c*(35/2*h*x+g)*d+h*c^2)*e*f-4*d*(1/2*(-63
/2*h*x+g)*d+c*h)*e^2)*b^3-1/35*a^3*(((-77*h*x^2+40/3*g*x)*d^2+2*(40/3*h*x+
g)*c*d+h*c^2)*f^2-52/3*d*e*(1/2*(-269/13*h*x+g)*d+c*h)*f-92/3*d^2*e^2*h)*b
^2-2/7*a^4*d*(((-28/3*h*x+1/2*g)*d+c*h)*f+20/3*d*e*h)*f*b+a^5*d^2*f^2*h)*(
f*x+e)^(1/2)*((a*f-b*e)*b)^(1/2))/(a*f-b*e)^2/b^4/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(458) = 916$.

Time = 0.38 (sec) , antiderivative size = 3796, normalized size of antiderivative = 7.84

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(458) = 916$.

Time = 0.16 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.23

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*(8*b^3*d^2*e^2*f*g - 4*b^3*c*d*e*f^2*g - 12*a*b^2*d^2*e*f^2*g + b^3*c^
2*f^3*g + 2*a*b^2*c*d*f^3*g + 5*a^2*b*d^2*f^3*g + 16*b^3*d^2*e^3*h + 16*b^
3*c*d*e^2*f*h - 72*a*b^2*d^2*e^2*f*h - 2*b^3*c^2*e*f^2*h - 24*a*b^2*c*d*e*
f^2*h + 90*a^2*b*d^2*e*f^2*h + a*b^2*c^2*f^3*h + 10*a^2*b*c*d*f^3*h - 35*a
^3*d^2*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^2 - 2*a
*b^5*e*f + a^2*b^4*f^2)*sqrt(-b^2*e + a*b*f)) + 2*sqrt(f*x + e)*d^2*h/b^4
- 1/24*(24*(f*x + e)^(5/2)*b^5*d^2*e^2*f*g - 48*(f*x + e)^(3/2)*b^5*d^2*e^
3*f*g + 24*sqrt(f*x + e)*b^5*d^2*e^4*f*g + 12*(f*x + e)^(5/2)*b^5*c*d*e*f^
2*g - 60*(f*x + e)^(5/2)*a*b^4*d^2*e*f^2*g + 144*(f*x + e)^(3/2)*a*b^4*d^2
*e^2*f^2*g - 12*sqrt(f*x + e)*b^5*c*d*e^3*f^2*g - 84*sqrt(f*x + e)*a*b^4*d
^2*e^3*f^2*g - 3*(f*x + e)^(5/2)*b^5*c^2*f^3*g - 6*(f*x + e)^(5/2)*a*b^4*c
*d*f^3*g + 33*(f*x + e)^(5/2)*a^2*b^3*d^2*f^3*g + 8*(f*x + e)^(3/2)*b^5*c^
2*e*f^3*g - 16*(f*x + e)^(3/2)*a*b^4*c*d*e*f^3*g - 136*(f*x + e)^(3/2)*a^2
*b^3*d^2*e*f^3*g + 3*sqrt(f*x + e)*b^5*c^2*e^2*f^3*g + 30*sqrt(f*x + e)*a*
b^4*c*d*e^2*f^3*g + 111*sqrt(f*x + e)*a^2*b^3*d^2*e^2*f^3*g - 8*(f*x + e)^
(3/2)*a*b^4*c^2*f^4*g + 16*(f*x + e)^(3/2)*a^2*b^3*c*d*f^4*g + 40*(f*x + e
)^(3/2)*a^3*b^2*d^2*f^4*g - 6*sqrt(f*x + e)*a*b^4*c^2*e*f^4*g - 24*sqrt(f*
x + e)*a^2*b^3*c*d*e*f^4*g - 66*sqrt(f*x + e)*a^3*b^2*d^2*e*f^4*g + 3*sqrt
(f*x + e)*a^2*b^3*c^2*f^5*g + 6*sqrt(f*x + e)*a^3*b^2*c*d*f^5*g + 15*sqrt(
f*x + e)*a^4*b*d^2*f^5*g + 48*(f*x + e)^(5/2)*b^5*c*d*e^2*f*h - 72*(f*x...
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^4,x)
```

output
```
(2*d^2*h*(e + f*x)^(1/2))/b^4 - ((e + f*x)^(1/2)*((b^3*c^2*f^3*g)/8 - (19*
a^3*d^2*f^3*h)/8 + (a*b^2*c^2*f^3*h)/8 + (5*a^2*b*d^2*f^3*g)/8 - (b^3*c^2*
e*f^2*h)/4 + b^3*d^2*e^2*f*g + (a*b^2*c*d*f^3*g)/4 + (5*a^2*b*c*d*f^3*h)/4
 - (b^3*c*d*e*f^2*g)/2 + 2*b^3*c*d*e^2*f*h - (3*a*b^2*d^2*e*f^2*g)/2 - 3*a
*b^2*d^2*e^2*f*h + (21*a^2*b*d^2*e*f^2*h)/4 - 3*a*b^2*c*d*e*f^2*h) - ((e +
 f*x)^(5/2)*(b^5*c^2*f^3*g + a*b^4*c^2*f^3*h - 2*b^5*c^2*e*f^2*h - 8*b^5*d
^2*e^2*f*g - 11*a^2*b^3*d^2*f^3*g + 29*a^3*b^2*d^2*f^3*h - 54*a^2*b^3*d^2*
e*f^2*h + 2*a*b^4*c*d*f^3*g - 4*b^5*c*d*e*f^2*g - 16*b^5*c*d*e^2*f*h - 22*
a^2*b^3*c*d*f^3*h + 20*a*b^4*d^2*e*f^2*g + 24*a*b^4*d^2*e^2*f*h + 40*a*b^4
*c*d*e*f^2*h))/(8*(a*f - b*e)^2) + ((e + f*x)^(3/2)*(a*b^3*c^2*f^3*h - b^4
*c^2*f^3*g - 17*a^3*b*d^2*f^3*h + 6*b^4*d^2*e^2*f*g + 5*a^2*b^2*d^2*f^3*g
+ 36*a^2*b^2*d^2*e*f^2*h + 2*a*b^3*c*d*f^3*g + 12*b^4*c*d*e^2*f*h + 10*a^2
*b^2*c*d*f^3*h - 12*a*b^3*d^2*e*f^2*g - 18*a*b^3*d^2*e^2*f*h - 24*a*b^3*c*
d*e*f^2*h))/(3*(a*f - b*e)))/(b^7*(e + f*x)^3 - (e + f*x)^2*(3*b^7*e - 3*a
*b^6*f) + (e + f*x)*(3*b^7*e^2 + 3*a^2*b^5*f^2 - 6*a*b^6*e*f) - b^7*e^3 +
a^3*b^4*f^3 - 3*a^2*b^5*e*f^2 + 3*a*b^6*e^2*f) + (atan((b^(1/2)*(e + f*x)^
(1/2))/(a*f - b*e)^(1/2))*(b^3*c^2*f^3*g - 35*a^3*d^2*f^3*h + 16*b^3*d^2*e
^3*h + a*b^2*c^2*f^3*h + 5*a^2*b*d^2*f^3*g - 2*b^3*c^2*e*f^2*h + 8*b^3*d^2
*e^2*f*g + 2*a*b^2*c*d*f^3*g + 10*a^2*b*c*d*f^3*h - 4*b^3*c*d*e*f^2*g + 16
*b^3*c*d*e^2*f*h - 12*a*b^2*d^2*e*f^2*g - 72*a*b^2*d^2*e^2*f*h + 90*a^2...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4809, normalized size of antiderivative = 9.94

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**6*d**2*f**3*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c*d*f**3*h + 270*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**2*e*f**2*h
 + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**5*b*d**2*f**3*g - 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**2*f**3*h*x + 3*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c**2*f**
3*h - 72*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**4*b**2*c*d*e*f**2*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*d*f**3*g + 90*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*
d*f**3*h*x - 216*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**4*b**2*d**2*e**2*f*h - 36*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*e*f**2*g + 81
0*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**4*b**2*d**2*e*f**2*h*x + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*f**3*g*x - 315*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d
**2*f**3*h*x**2 - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr...
```

3.18 $\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 640

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = -\frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{4b^4(a+bx)^4}$$
$$-\frac{(bc-ad)\left(25a^2dfh+b^2(16deg+cfg+8ceh)-ab(17dfg+24deh+9cfh)\right)\sqrt{e+fx}}{24b^4(be-af)(a+bx)^3}$$
$$+\frac{(163a^3d^2f^2h-a^2bdf(59dfg+312deh+118cfh)-b^3(48d^2e^2g-c^2f(5fg-8eh)+16cde(fg+6eh)\ldots}{96b^4(be-af)^2(a+bx)^2}$$
$$+\frac{(93a^3d^2f^3h-a^2bdf^2(5dfg+264deh+10cfh)-b^3(c^2f^2(5fg-8eh)-16cdef(fg-2eh)+16d^2e^2(\ldots}{64b^4(be-af)^3(a+bx)\ldots}$$
$$+\frac{f(35a^3d^2f^3h+5a^2bdf^2(dfg-24deh+2cfh)+ab^2f(3c^2f^2h+2cdf(3fg-16eh)-16d^2e(fg-9eh)\ldots}{64b^{9/2}(be-af)^{7/\ldots}}$$

output
```
-1/4*(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^4/(b*x+a)^4-1/24*(-a*d+b*c)*(
25*a^2*d*f*h+b^2*(8*c*e*h+c*f*g+16*d*e*g)-a*b*(9*c*f*h+24*d*e*h+17*d*f*g))
*(f*x+e)^(1/2)/b^4/(-a*f+b*e)/(b*x+a)^3+1/96*(163*a^3*d^2*f^2*h-a^2*b*d*f*
(118*c*f*h+312*d*e*h+59*d*f*g)-b^3*(48*d^2*e^2*g-c^2*f*(-8*e*h+5*f*g)+16*c
*d*e*(6*e*h+f*g))+a*b^2*(3*c^2*f^2*h+16*d^2*e*(9*e*h+7*f*g)+2*c*d*f*(112*e
*h+3*f*g)))*(f*x+e)^(1/2)/b^4/(-a*f+b*e)^2/(b*x+a)^2+1/64*(93*a^3*d^2*f^3*
h-a^2*b*d*f^2*(10*c*f*h+264*d*e*h+5*d*f*g)-b^3*(c^2*f^2*(-8*e*h+5*f*g)-16*
c*d*e*f*(-2*e*h+f*g)+16*d^2*e^2*(4*e*h+f*g))-a*b^2*f*(3*c^2*f^2*h+2*c*d*f*
(-16*e*h+3*f*g)-16*d^2*e*(15*e*h+f*g)))*(f*x+e)^(1/2)/b^4/(-a*f+b*e)^3/(b*
x+a)+1/64*f*(35*a^3*d^2*f^3*h+5*a^2*b*d*f^2*(2*c*f*h-24*d*e*h+d*f*g)+a*b^2
*f*(3*c^2*f^2*h+2*c*d*f*(-16*e*h+3*f*g)-16*d^2*e*(-9*e*h+f*g))+b^3*(c^2*f^
2*(-8*e*h+5*f*g)+16*d^2*e^2*(-4*e*h+f*g)-16*c*d*e*f*(-2*e*h+f*g)))*arctanh
(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(9/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 4.38 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx$$

$$= \frac{\sqrt{b}\sqrt{e+fx}\left(-105a^6d^2f^3h-5a^5bdf^2(6cfh+d(3fg-58eh+77fhx))+b^6\left(48d^2e^2x^2(fgx+2e(g+2hx))+16cdex\left(-3f^2gx^2+2efx(g+3hx)+4e^2(2g\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^5,x]
```

output
```
((Sqrt[b]*Sqrt[e + f*x]*(-105*a^6*d^2*f^3*h - 5*a^5*b*d*f^2*(6*c*f*h + d*(
3*f*g - 58*e*h + 77*f*h*x)) + b^6*(48*d^2*e^2*x^2*(f*g*x + 2*e*(g + 2*h*x)
) + 16*c*d*e*x*(-3*f^2*g*x^2 + 2*e*f*x*(g + 3*h*x) + 4*e^2*(2*g + 3*h*x))
+ c^2*(15*f^3*g*x^3 + 8*e^2*f*x*(g + 2*h*x) + 16*e^3*(3*g + 4*h*x) - 2*e*f
^2*x^2*(5*g + 12*h*x))) + a*b^5*(2*c*d*(9*f^3*g*x^3 + 16*e^3*(g + 4*h*x) -
 8*e^2*f*x*(21*g + 22*h*x) - 2*e*f^2*x^2*(47*g + 24*h*x)) + 16*d^2*e*x*(-3
*f^2*g*x^2 + 2*e^2*(2*g + 9*h*x) - e*f*x*(11*g + 45*h*x)) + c^2*(16*e^3*h
+ f^3*x^2*(55*g + 9*h*x) - 8*e^2*f*(17*g + 21*h*x) - 2*e*f^2*x*(18*g + 47*
h*x))) + a^2*b^4*(d^2*(15*f^3*g*x^3 + 198*e*f^2*x^2*(g + 4*h*x) + 16*e^3*(
g + 12*h*x) - 104*e^2*f*x*(g + 12*h*x)) + c^2*f*(-40*e^2*h + f^2*x*(73*g +
 33*h*x) + 2*e*f*(59*g + 46*h*x)) + 2*c*d*(16*e^3*h + 3*f^3*x^2*(11*g + 5*
h*x) - 8*e^2*f*(5*g + 13*h*x) + 2*e*f^2*x*(46*g + 99*h*x))) - a^4*b^2*f*(9
*c^2*f^2*h + 2*c*d*f*(9*f*g - 38*e*h + 55*f*h*x) + d^2*(248*e^2*h + f^2*x*
(55*g + 511*h*x) - 2*e*f*(19*g + 534*h*x))) + a^3*b^3*(-3*c^2*f^2*(5*f*g -
 6*e*h + 11*f*h*x) - 2*c*d*f*(24*e^2*h - 2*e*f*(9*g + 70*h*x) + f^2*x*(33*
g + 73*h*x)) + d^2*(48*e^3*h - 8*e^2*f*(3*g + 115*h*x) - f^3*x^2*(73*g + 2
79*h*x) + 2*e*f^2*x*(70*g + 713*h*x)))))/((-(b*e) + a*f)^3*(a + b*x)^4) +
(3*f*(35*a^3*d^2*f^3*h + 5*a^2*b*d*f^2*(d*f*g - 24*d*e*h + 2*c*f*h) + b^3*
(c^2*f^2*(5*f*g - 8*e*h) + 16*d^2*e^2*(f*g - 4*e*h) + 16*c*d*e*f*(-(f*g) +
 2*e*h)) + a*b^2*f*(3*c^2*f^2*h + 2*c*d*f*(3*f*g - 16*e*h) + 16*d^2*e*(...
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 162, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg-5bcfg+8bceh-4adeh-3acfh-d(bfg-8beh+7afh)x)}{2(a+bx)^4}dx}{4b(be-af)}-\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)\sqrt{e+fx}(4bdeg-5bcfg+8bceh-4adeh-3acfh-d(bfg-8beh+7afh)x)}{(a+bx)^4}dx}{8b(be-af)}-\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \; 162$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h+a^3bdf(10cfh-92deh+5dfg)-3a^2b^2\left(7c^2f^2h-2cdf(fg-4eh)+4d^2e(fg-6eh)\right)+3bx\left(21a^3d^2f^2h+a^2bdf(-10cfh-56deh-\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \; 51$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h+a^3bdf(10cfh-92deh+5dfg)-3a^2b^2\left(7c^2f^2h-2cdf(fg-4eh)+4d^2e(fg-6eh)\right)+3bx\left(21a^3d^2f^2h+a^2bdf(-10cfh-56deh-\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \; 73$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h+a^3bdf(10cfh-92deh+5dfg)-3a^2b^2\left(7c^2f^2h-2cdf(fg-4eh)+4d^2e(fg-6eh)\right)+3bx\left(21a^3d^2f^2h+a^2bdf(-10cfh-56deh-\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{(e+fx)^{3/2}\left(35a^4d^2f^2h+a^3bdf(10cfh-92deh+5dfg)-3a^2b^2\left(7c^2f^2h-2cdf(fg-4eh)+4d^2e(fg-6eh)\right)+3bx\left(21a^3d^2f^2h+a^2bdf(-10cfh-56deh-\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

input
```
Int[((c + d*x)^2*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^5,x]
```

output
```
-1/4*((b*g - a*h)*(c + d*x)^2*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x)^4)
 + (((e + f*x)^(3/2)*(35*a^4*d^2*f^2*h - 4*b^4*c*e*(4*d*e*g - 5*c*f*g + 8*
c*e*h) + a^3*b*d*f*(5*d*f*g - 92*d*e*h + 10*c*f*h) - 3*a^2*b^2*(7*c^2*f^2*
h + 4*d^2*e*(f*g - 6*e*h) - 2*c*d*f*(f*g - 4*e*h)) - a*b^3*(8*d^2*e^2*g +
c^2*f*(35*f*g - 68*e*h) - 8*c*d*e*(5*f*g - 2*e*h)) + 3*b*(21*a^3*d^2*f^2*h
 + a^2*b*d*f*(3*d*f*g - 56*d*e*h - 10*c*f*h) + a*b^2*(40*d^2*e^2*h - 3*c^2
*f^2*h - 2*c*d*f*(3*f*g - 16*e*h)) - b^3*(8*d^2*e^2*g + c^2*f*(5*f*g - 8*e
*h) - 16*c*d*e*(f*g - 2*e*h)))*x))/(12*b^2*(b*e - a*f)^2*(a + b*x)^3) - ((
35*a^3*d^2*f^3*h + 5*a^2*b*d*f^2*(d*f*g - 24*d*e*h + 2*c*f*h) + a*b^2*f*(3
*c^2*f^2*h + 2*c*d*f*(3*f*g - 16*e*h) - 16*d^2*e*(f*g - 9*e*h)) + b^3*(c^2
*f^2*(5*f*g - 8*e*h) + 16*d^2*e^2*(f*g - 4*e*h) - 16*c*d*e*f*(f*g - 2*e*h)
))*(-(Sqrt[e + f*x]/(b*(a + b*x))) - (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sq
rt[b*e - a*f]])/(b^(3/2)*Sqrt[b*e - a*f])))/(8*b^2*(b*e - a*f)^2))/(8*b*(b
*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.36

method	result	size
pseudoelliptic	Expression too large to display	872
derivativedivides	Expression too large to display	1181
default	Expression too large to display	1181

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
35/64*((1/7*(c^2*g*f^3-8/5*c*e*(c*h+2*d*g)*f^2+32/5*(c*h+1/2*d*g)*d*e^2*f-
64/5*d^2*e^3*h)*b^3+3/35*((c^2*h+2*c*d*g)*f^2-32/3*(c*h+1/2*d*g)*d*e*f+48*
d^2*e^2*h)*a*f*b^2+2/7*a^2*d*((c*h+1/2*d*g)*f-12*d*e*h)*f^2*b+a^3*d^2*f^3*
h)*(b*x+a)^4*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-(f*x+e)^(1/2)*(
(a*f-b*e)*b)^(1/2)*(1/7*(-c^2*f^3*g*x^3+2/3*(24/5*d*g*x+c*(12/5*h*x+g))*x^
2*c*e*f^2-8/15*x*(6*d^2*g*x^2+4*c*x*(3*h*x+g)*d+c^2*(2*h*x+g))*e^2*f-16/5*
e^3*(2*(2*h*x^3+g*x^2)*d^2+8/3*x*c*(3/2*h*x+g)*d+c^2*(4/3*h*x+g)))*b^6-16/
105*a*(55/16*x^2*c*(18/55*d*g*x+c*(9/55*h*x+g))*f^3-9/4*(4/3*d^2*g*x^2+47/
9*x*c*(24/47*h*x+g)*d+c^2*(47/18*h*x+g))*x*e*f^2-17/2*(22/17*x^2*(45/11*h*
x+g)*d^2+42/17*(22/21*h*x+g)*x*c*d+c^2*(21/17*h*x+g))*e^2*f+(2*(9*h*x^2+2*
g*x)*d^2+2*c*(4*h*x+g)*d+h*c^2)*e^3)*b^5+8/21*a^2*(-73/40*(15/73*d^2*g*x^2
+66/73*x*c*(5/11*h*x+g)*d+c^2*(33/73*h*x+g))*x*f^3-59/20*(99/59*x^2*(4*h*x
+g)*d^2+92/59*x*c*(99/46*h*x+g)*d+c^2*(46/59*h*x+g))*e*f^2+(13/5*x*(12*h*x
+g)*d^2+2*c*(13/5*h*x+g)*d+h*c^2)*e^2*f-4/5*((6*h*x+1/2*g)*d+c*h)*d*e^3)*b
^4-6/35*a^3*((1/2*(-31*h*x^3-73/9*g*x^2)*d^2-11/3*x*(73/33*h*x+g)*c*d-5/6*
c^2*(11/5*h*x+g))*f^3+(1/9*(713*h*x^2+70*g*x)*d^2+2*c*(70/9*h*x+g)*d+h*c^2
)*e*f^2-8/3*(1/2*(115/3*h*x+g)*d+c*h)*d*e^2*f+8/3*d^2*e^3*h)*b^3+3/35*a^4*
((1/9*(511*h*x^2+55*g*x)*d^2+2*(55/9*h*x+g)*c*d+h*c^2)*f^2-76/9*((267/19*h
*x+1/2*g)*d+c*h)*d*e*f+248/9*d^2*e^2*h)*f*b^2+2/7*a^5*d*((1/2*(77/3*h*x+g)
*d+c*h)*f-29/3*d*e*h)*f^2*b+a^6*d^2*f^3*h))/((a*f-b*e)*b)^(1/2)/(b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2710 vs. $2(612) = 1224$.

Time = 0.60 (sec) , antiderivative size = 5434, normalized size of antiderivative = 8.49

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(612) = 1224$.

Time = 0.19 (sec) , antiderivative size = 2457, normalized size of antiderivative = 3.84

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/64*(16*b^3*d^2*e^2*f^2*g - 16*b^3*c*d*e*f^3*g - 16*a*b^2*d^2*e*f^3*g +
5*b^3*c^2*f^4*g + 6*a*b^2*c*d*f^4*g + 5*a^2*b*d^2*f^4*g - 64*b^3*d^2*e^3*f
*h + 32*b^3*c*d*e^2*f^2*h + 144*a*b^2*d^2*e^2*f^2*h - 8*b^3*c^2*e*f^3*h -
32*a*b^2*c*d*e*f^3*h - 120*a^2*b*d^2*e*f^3*h + 3*a*b^2*c^2*f^4*h + 10*a^2*
b*c*d*f^4*h + 35*a^3*d^2*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f
))/((b^7*e^3 - 3*a*b^6*e^2*f + 3*a^2*b^5*e*f^2 - a^3*b^4*f^3)*sqrt(-b^2*e
+ a*b*f)) - 1/192*(48*(f*x + e)^(7/2)*b^6*d^2*e^2*f^2*g - 48*(f*x + e)^(5/
2)*b^6*d^2*e^3*f^2*g - 48*(f*x + e)^(3/2)*b^6*d^2*e^4*f^2*g + 48*sqrt(f*x
+ e)*b^6*d^2*e^5*f^2*g - 48*(f*x + e)^(7/2)*b^6*c*d*e*f^3*g - 48*(f*x + e)
^(7/2)*a*b^5*d^2*e*f^3*g + 176*(f*x + e)^(5/2)*b^6*c*d*e^2*f^3*g - 32*(f*x
 + e)^(5/2)*a*b^5*d^2*e^2*f^3*g - 80*(f*x + e)^(3/2)*b^6*c*d*e^3*f^3*g + 2
72*(f*x + e)^(3/2)*a*b^5*d^2*e^3*f^3*g - 48*sqrt(f*x + e)*b^6*c*d*e^4*f^3*
g - 192*sqrt(f*x + e)*a*b^5*d^2*e^4*f^3*g + 15*(f*x + e)^(7/2)*b^6*c^2*f^4
*g + 18*(f*x + e)^(7/2)*a*b^5*c*d*f^4*g + 15*(f*x + e)^(7/2)*a^2*b^4*d^2*f
^4*g - 55*(f*x + e)^(5/2)*b^6*c^2*e*f^4*g - 242*(f*x + e)^(5/2)*a*b^5*c*d*
e*f^4*g + 153*(f*x + e)^(5/2)*a^2*b^4*d^2*e*f^4*g + 73*(f*x + e)^(3/2)*b^6
*c^2*e^2*f^4*g + 94*(f*x + e)^(3/2)*a*b^5*c*d*e^2*f^4*g - 455*(f*x + e)^(3
/2)*a^2*b^4*d^2*e^2*f^4*g + 15*sqrt(f*x + e)*b^6*c^2*e^3*f^4*g + 162*sqrt(
f*x + e)*a*b^5*c*d*e^3*f^4*g + 303*sqrt(f*x + e)*a^2*b^4*d^2*e^3*f^4*g + 5
5*(f*x + e)^(5/2)*a*b^5*c^2*f^5*g + 66*(f*x + e)^(5/2)*a^2*b^4*c*d*f^5*...
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 1670, normalized size of antiderivative = 2.61

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^5,x)
```

output
```
(f*atan((b^(1/2)*f*(e + f*x)^(1/2)*(5*b^3*c^2*f^3*g + 35*a^3*d^2*f^3*h - 6
4*b^3*d^2*e^3*h + 3*a*b^2*c^2*f^3*h + 5*a^2*b*d^2*f^3*g - 8*b^3*c^2*e*f^2*
h + 16*b^3*d^2*e^2*f*g + 6*a*b^2*c*d*f^3*g + 10*a^2*b*c*d*f^3*h - 16*b^3*c
*d*e*f^2*g + 32*b^3*c*d*e^2*f*h - 16*a*b^2*d^2*e*f^2*g + 144*a*b^2*d^2*e^2
*f*h - 120*a^2*b*d^2*e*f^2*h - 32*a*b^2*c*d*e*f^2*h))/((a*f - b*e)^(1/2)*(
5*b^3*c^2*f^4*g + 35*a^3*d^2*f^4*h + 3*a*b^2*c^2*f^4*h + 5*a^2*b*d^2*f^4*g
 - 8*b^3*c^2*e*f^3*h - 64*b^3*d^2*e^3*f*h + 16*b^3*d^2*e^2*f^2*g + 144*a*b
^2*d^2*e^2*f^2*h + 6*a*b^2*c*d*f^4*g + 10*a^2*b*c*d*f^4*h - 16*b^3*c*d*e*f
^3*g - 16*a*b^2*d^2*e*f^3*g - 120*a^2*b*d^2*e*f^3*h + 32*b^3*c*d*e^2*f^2*h
 - 32*a*b^2*c*d*e*f^3*h)))*(5*b^3*c^2*f^3*g + 35*a^3*d^2*f^3*h - 64*b^3*d^
2*e^3*h + 3*a*b^2*c^2*f^3*h + 5*a^2*b*d^2*f^3*g - 8*b^3*c^2*e*f^2*h + 16*b
^3*d^2*e^2*f*g + 6*a*b^2*c*d*f^3*g + 10*a^2*b*c*d*f^3*h - 16*b^3*c*d*e*f^2
*g + 32*b^3*c*d*e^2*f*h - 16*a*b^2*d^2*e*f^2*g + 144*a*b^2*d^2*e^2*f*h - 1
20*a^2*b*d^2*e*f^2*h - 32*a*b^2*c*d*e*f^2*h))/(64*b^(9/2)*(a*f - b*e)^(7/2
)) - (((e + f*x)^(1/2)*(5*b^3*c^2*f^4*g + 35*a^3*d^2*f^4*h + 3*a*b^2*c^2*f
^4*h + 5*a^2*b*d^2*f^4*g - 8*b^3*c^2*e*f^3*h - 64*b^3*d^2*e^3*f*h + 16*b^3
*d^2*e^2*f^2*g + 144*a*b^2*d^2*e^2*f^2*h + 6*a*b^2*c*d*f^4*g + 10*a^2*b*c*
d*f^4*h - 16*b^3*c*d*e*f^3*g - 16*a*b^2*d^2*e*f^3*g - 120*a^2*b*d^2*e*f^3*
h + 32*b^3*c*d*e^2*f^2*h - 32*a*b^2*c*d*e*f^3*h))/(64*b^4) - ((e + f*x)^(7
/2)*(5*b^3*c^2*f^4*g - 93*a^3*d^2*f^4*h + 3*a*b^2*c^2*f^4*h + 5*a^2*b*d...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6626, normalized size of antiderivative = 10.35

$$\int \frac{(c+dx)^2\sqrt{e+fx}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5,x)
```

output
```
(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**7*d**2*f**4*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d*f**4*h - 360*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*e*f**3*h +
15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**6*b*d**2*f**4*g + 420*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*f**4*h*x + 9*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*f**4*h
 - 96*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**5*b**2*c*d*e*f**3*h + 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d*f**4*g + 120*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d
*f**4*h*x + 432*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**5*b**2*d**2*e**2*f**2*h - 48*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*e*f**3*g -
1440*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**5*b**2*d**2*e*f**3*h*x + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*f**4*g*x + 630*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**
2*d**2*f**4*h*x**2 - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/...
```

3.19 $\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 225

$$\begin{aligned}\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx &= 2a^3e\sqrt{c+dx} \\ &+ \frac{2(a^3d^3f+b^3c^2(de-cf)-3ab^2cd(de-cf)+3a^2bd^2(de-cf))(c+dx)^{3/2}}{3d^4} \\ &+ \frac{2b(3a^2d^2f-b^2c(2de-3cf)+3abd(de-2cf))(c+dx)^{5/2}}{5d^4} \\ &+ \frac{2b^2(bde-3bcf+3adf)(c+dx)^{7/2}}{7d^4} \\ &+ \frac{2b^3f(c+dx)^{9/2}}{9d^4} - 2a^3\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\end{aligned}$$

output
```
2*a^3*e*(d*x+c)^(1/2)+2/3*(a^3*d^3*f+b^3*c^2*(-c*f+d*e)-3*a*b^2*c*d*(-c*f+
d*e)+3*a^2*b*d^2*(-c*f+d*e))*(d*x+c)^(3/2)/d^4+2/5*b*(3*a^2*d^2*f-b^2*c*(-
3*c*f+2*d*e)+3*a*b*d*(-2*c*f+d*e))*(d*x+c)^(5/2)/d^4+2/7*b^2*(3*a*d*f-3*b*
c*f+b*d*e)*(d*x+c)^(7/2)/d^4+2/9*b^3*f*(d*x+c)^(9/2)/d^4-2*a^3*c^(1/2)*e*a
rctanh((d*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \frac{2\sqrt{c+dx}(105a^3d^3(3de+cf+dfx)+63a^2bd^2(c+dx)(5de-2cf+3dfx)+9ab^2d(c+dx)\left(8c^2f+3d^2\right.}{3}$$
$$-2a^3\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*Sqrt[c + d*x]*(105*a^3*d^3*(3*d*e + c*f + d*f*x) + 63*a^2*b*d^2*(c + d*
x)*(5*d*e - 2*c*f + 3*d*f*x) + 9*a*b^2*d*(c + d*x)*(8*c^2*f + 3*d^2*x*(7*e
 + 5*f*x) - 2*c*d*(7*e + 6*f*x)) - b^3*(c + d*x)*(16*c^3*f - 24*c^2*d*(e +
 f*x) + 6*c*d^2*x*(6*e + 5*f*x) - 5*d^3*x^2*(9*e + 7*f*x))))/(315*d^4) - 2
*a^3*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {170, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$\Big\downarrow \ 170$$
$$\frac{2\int \frac{3(a+bx)^2\sqrt{c+dx}(3ade+(3bde-2bcf+2adf)x)}{2x}dx}{9d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{(a+bx)^2\sqrt{c+dx}(3ade+(3bde-2bcf+2adf)x)}{x}dx}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \ 170$$

$$\frac{\frac{2\int \frac{(a+bx)\sqrt{c+dx}\left(21a^2ed^2+\left(21abd^2e-4(bc-ad)(3bde-2bcf+2adf)\right)x\right)}{2x}dx}{7d}+\frac{2(a+bx)^2(c+dx)^{3/2}(2adf-2bcf+3bde)}{7d}}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{(a+bx)\sqrt{c+dx}\left(21a^2ed^2+\left(21abd^2e-4(bc-ad)(3bde-2bcf+2adf)\right)x\right)}{x}dx}{7d}+\frac{2(a+bx)^2(c+dx)^{3/2}(2adf-2bcf+3bde)}{7d}}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \ 164$$

$$\frac{\frac{21a^3d^2e\int \frac{\sqrt{c+dx}}{x}dx+\frac{2(c+dx)^{3/2}\left(40a^3d^3f+6a^2bd^2(45de-16cf)-18ab^2cd(7de-4cf)+3bdx\left(21abd^2e-4(bc-ad)(2adf-2bcf+3bde)\right)+8b^3c^2(3de-2cf)\right)}{15d^2}}{7d}+\cdots}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{21a^3d^2e\left(c\int \frac{1}{x\sqrt{c+dx}}dx+2\sqrt{c+dx}\right)+\frac{2(c+dx)^{3/2}\left(40a^3d^3f+6a^2bd^2(45de-16cf)-18ab^2cd(7de-4cf)+3bdx\left(21abd^2e-4(bc-ad)(2adf-2bcf+3bde)\right)+8b^3c^2\cdots\right.}{15d^2}}{7d}\cdots}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{21a^3d^2e\left(\frac{2c\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{d}+2\sqrt{c+dx}\right)+\frac{2(c+dx)^{3/2}\left(40a^3d^3f+6a^2bd^2(45de-16cf)-18ab^2cd(7de-4cf)+3bdx\left(21abd^2e-4(bc-ad)(2adf-2bcf+3bde)\cdots\right.\right.}{15d^2}}{7d}\cdots}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{21a^3d^2e\left(2\sqrt{c+dx}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+\frac{2(c+dx)^{3/2}\left(40a^3d^3f+6a^2bd^2(45de-16cf)-18ab^2cd(7de-4cf)+3bdx\left(21abd^2e-4(bc-ad)(2adf-2bcf+3bd\cdots\right.\right.}{15d^2}}{7d}\cdots}{3d}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

input
```
Int[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + ((2*(3*b*d*e - 2*b*c*f + 2*a*d*f
)*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + ((2*(c + d*x)^(3/2)*(40*a^3*d^3*f +
 6*a^2*b*d^2*(45*d*e - 16*c*f) - 18*a*b^2*c*d*(7*d*e - 4*c*f) + 8*b^3*c^2*
(3*d*e - 2*c*f) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a*d)*(3*b*d*e - 2*b*c*f +
 2*a*d*f))*x))/(15*d^2) + 21*a^3*d^2*e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*ArcTan
h[Sqrt[c + d*x]/Sqrt[c]]))/(7*d))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{-2a^3\sqrt{c}\,d^4 e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)+\frac{2\left(3\left(\frac{\left(\frac{7fx}{9}+e\right)x^3b^3}{7}+\frac{3a\left(\frac{5fx}{7}+e\right)x^2b^2}{5}+a^2x\left(\frac{3fx}{5}+e\right)b+a^3\left(\frac{fx}{3}+e\right)\right)d^4+\left(\frac{3x^2\left(\frac{5fx}{9}+e\right)b^{\cdots}}{35}\cdots\right)\right)}{\cdots}}{\cdots}$
derivativedivides	$\frac{\frac{2f\,b^3(xd+c)^{\frac{9}{2}}}{9}+\frac{6a\,b^2df(xd+c)^{\frac{7}{2}}}{7}-\frac{6b^3cf(xd+c)^{\frac{7}{2}}}{7}+\frac{2b^3de(xd+c)^{\frac{7}{2}}}{7}+\frac{6a^2b\,d^2f(xd+c)^{\frac{5}{2}}}{5}-\frac{12a\,b^2cdf(xd+c)^{\frac{5}{2}}}{5}+\frac{6a\,b^2d^2e(xd+c)^{\frac{5}{2}}}{5}\cdots}{\cdots}$
default	$\frac{\frac{2f\,b^3(xd+c)^{\frac{9}{2}}}{9}+\frac{6a\,b^2df(xd+c)^{\frac{7}{2}}}{7}-\frac{6b^3cf(xd+c)^{\frac{7}{2}}}{7}+\frac{2b^3de(xd+c)^{\frac{7}{2}}}{7}+\frac{6a^2b\,d^2f(xd+c)^{\frac{5}{2}}}{5}-\frac{12a\,b^2cdf(xd+c)^{\frac{5}{2}}}{5}+\frac{6a\,b^2d^2e(xd+c)^{\frac{5}{2}}}{5}\cdots}{\cdots}$

input
```
int((b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-3*a^3*c^(1/2)*d^4*e*arctanh((d*x+c)^(1/2)/c^(1/2))+(3*(1/7*(7/9*f*x+
e)*x^3*b^3+3/5*a*(5/7*f*x+e)*x^2*b^2+a^2*x*(3/5*f*x+e)*b+a^3*(1/3*f*x+e))*
d^4+(3/35*x^2*(5/9*f*x+e)*b^3+3/5*a*(3/7*f*x+e)*x*b^2+3*a^2*(1/5*f*x+e)*b+
f*a^3)*c*d^3-6/5*c^2*(2/21*(1/2*f*x+e)*x*b^2+a*(2/7*f*x+e)*b+a^2*f)*b*d^2+
24/35*c^3*(1/3*(1/3*f*x+e)*b+a*f)*b^2*d-16/105*b^3*c^4*f)*(d*x+c)^(1/2))/d
^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \left[\frac{315\,a^3\sqrt{c}d^4e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,(35\,b^3d^4fx^4+5\,(9\,b^3d^4e+(b^3cd^3+27\,ab^2d^4)f)x^3+3\,(3\,(b^3cd^3\right.$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/315*(315*a^3*sqrt(c)*d^4*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
 + 2*(35*b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^
3 + 3*(3*(b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 6
3*a^2*b*d^4)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3
+ 105*a^3*d^4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*
a^3*c*d^3)*f - (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*
b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*
x + c))/d^4, 2/315*(315*a^3*sqrt(-c)*d^4*e*arctan(sqrt(-c)/sqrt(d*x + c))
+ (35*b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3 +
 3*(3*(b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a
^2*b*d^4)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 1
05*a^3*d^4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3
*c*d^3)*f - (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3
*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x +
 c))/d^4]
```

Sympy [A] (verification not implemented)

Time = 12.53 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \begin{cases} \frac{2a^3ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2a^3e\sqrt{c+dx} + \frac{2b^3f(c+dx)^{\frac{9}{2}}}{9d^4} + \frac{2(c+dx)^{\frac{7}{2}}\cdot(3ab^2df-3b^3cf+b^3de)}{7d^4} + \frac{2(c+dx)^{\frac{5}{2}}\cdot(3a^2bd^2f-6ab^2cdf+3ab^2}{5d^4} \\ \sqrt{c}\left(a^3e\log{(x)} + a^3fx + 3a^2bex + \frac{b^3fx^4}{4} + \frac{x^3\cdot(3ab^2f+b^3e)}{3} + \frac{x^2\cdot(3a^2bf+3ab^2e)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**3*(d*x+c)**(1/2)*(f*x+e)/x,x)
```

output
```
Piecewise((2*a**3*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**3*e*sqr
t(c + d*x) + 2*b**3*f*(c + d*x)**(9/2)/(9*d**4) + 2*(c + d*x)**(7/2)*(3*a*
b**2*d*f - 3*b**3*c*f + b**3*d*e)/(7*d**4) + 2*(c + d*x)**(5/2)*(3*a**2*b*
d**2*f - 6*a*b**2*c*d*f + 3*a*b**2*d**2*e + 3*b**3*c**2*f - 2*b**3*c*d*e)/
(5*d**4) + 2*(c + d*x)**(3/2)*(a**3*d**3*f - 3*a**2*b*c*d**2*f + 3*a**2*b*
d**3*e + 3*a*b**2*c**2*d*f - 3*a*b**2*c*d**2*e - b**3*c**3*f + b**3*c**2*d
*e)/(3*d**4), Ne(d, 0)), (sqrt(c)*(a**3*e*log(x) + a**3*f*x + 3*a**2*b*e*x
 + b**3*f*x**4/4 + x**3*(3*a*b**2*f + b**3*e)/3 + x**2*(3*a**2*b*f + 3*a*b
**2*e)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = a^3\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)$$
$$+ \frac{2\left(315\sqrt{dx+c}a^3d^4e + 35(dx+c)^{\frac{9}{2}}b^3f + 45(b^3de - 3(b^3c-ab^2d)f)(dx+c)^{\frac{7}{2}} - 63((2b^3cd - 3ab^2d\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="maxima")
```

output
```
a^3*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2
/315*(315*sqrt(d*x + c)*a^3*d^4*e + 35*(d*x + c)^(9/2)*b^3*f + 45*(b^3*d*e
 - 3*(b^3*c - a*b^2*d)*f)*(d*x + c)^(7/2) - 63*((2*b^3*c*d - 3*a*b^2*d^2)*
e - 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f)*(d*x + c)^(5/2) + 105*((b^3*c
^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*e - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*f)*(d*x + c)^(3/2))/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,a^3ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(45\,(dx+c)^{\frac{7}{2}}b^3d^{33}e - 126\,(dx+c)^{\frac{5}{2}}b^3cd^{33}e + 105\,(dx+c)^{\frac{3}{2}}b^3c^2d^{33}e + 189\,(dx+c)^{\frac{5}{2}}ab^2d^{34}e - 315\,(\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a^3*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/315*(45*(d*x + c)^(7
/2)*b^3*d^33*e - 126*(d*x + c)^(5/2)*b^3*c*d^33*e + 105*(d*x + c)^(3/2)*b^
3*c^2*d^33*e + 189*(d*x + c)^(5/2)*a*b^2*d^34*e - 315*(d*x + c)^(3/2)*a*b^
2*c*d^34*e + 315*(d*x + c)^(3/2)*a^2*b*d^35*e + 315*sqrt(d*x + c)*a^3*d^36
*e + 35*(d*x + c)^(9/2)*b^3*d^32*f - 135*(d*x + c)^(7/2)*b^3*c*d^32*f + 18
9*(d*x + c)^(5/2)*b^3*c^2*d^32*f - 105*(d*x + c)^(3/2)*b^3*c^3*d^32*f + 13
5*(d*x + c)^(7/2)*a*b^2*d^33*f - 378*(d*x + c)^(5/2)*a*b^2*c*d^33*f + 315*
(d*x + c)^(3/2)*a*b^2*c^2*d^33*f + 189*(d*x + c)^(5/2)*a^2*b*d^34*f - 315*
(d*x + c)^(3/2)*a^2*b*c*d^34*f + 105*(d*x + c)^(3/2)*a^3*d^35*f)/d^36
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \left(c\left(c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b(ad-bc)(adf-2bcf+bde)}{d^4}\right)+\frac{2(ad-bc)^2(adf-4bcf+3bde)}{d^4}\right)-\frac{2(ad-bc)^3(cf-de)}{d^4}\right)\sqrt{c+dx}$$
$$+\left(\frac{c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b(ad-bc)(adf-2bcf+bde)}{d^4}\right)}{3}+\frac{2(ad-bc)^2(adf-4bcf+3bde)}{3d^4}\right)(c+dx)^{3/2}$$
$$+\left(\frac{2b^3de-8b^3cf+6ab^2df}{7d^4}+\frac{2b^3cf}{7d^4}\right)(c+dx)^{7/2}$$
$$+\left(\frac{c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)}{5}+\frac{6b(ad-bc)(adf-2bcf+bde)}{5d^4}\right)(c+dx)^{5/2}$$
$$+\frac{2b^3f(c+dx)^{9/2}}{9d^4}+a^3\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+dx}\,1\mathrm{i}}{\sqrt{c}}\right)2\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)^3*(c + d*x)^(1/2))/x,x)
```

output
```
(c*(c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) + (
6*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4) + (2*(a*d - b*c)^2*(a*d*f
- 4*b*c*f + 3*b*d*e))/d^4) - (2*(a*d - b*c)^3*(c*f - d*e))/d^4)*(c + d*x)^
(1/2) + ((c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^
4) + (6*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4))/3 + (2*(a*d - b*c)^
2*(a*d*f - 4*b*c*f + 3*b*d*e))/(3*d^4))*(c + d*x)^(3/2) + ((2*b^3*d*e - 8*
b^3*c*f + 6*a*b^2*d*f)/(7*d^4) + (2*b^3*c*f)/(7*d^4))*(c + d*x)^(7/2) + ((
c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4))/5 + (6*b*
(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/(5*d^4))*(c + d*x)^(5/2) + a^3*c^(1
/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*b^3*f*(c + d*x)^(9/2))/(9
*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx)^3\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \frac{630\sqrt{dx+c}\,a^3d^4e - 32\sqrt{dx+c}\,b^3c^4f + 210\sqrt{dx+c}\,a^3c\,d^3f + 210\sqrt{dx+c}\,a^3d^4fx + 48\sqrt{dx+c}\,b^3c^3de}{}$$

input
```
int((b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)/x,x)
```

output
```
(210*sqrt(c + d*x)*a**3*c*d**3*f + 630*sqrt(c + d*x)*a**3*d**4*e + 210*sqr
t(c + d*x)*a**3*d**4*f*x - 252*sqrt(c + d*x)*a**2*b*c**2*d**2*f + 630*sqrt
(c + d*x)*a**2*b*c*d**3*e + 126*sqrt(c + d*x)*a**2*b*c*d**3*f*x + 630*sqrt
(c + d*x)*a**2*b*d**4*e*x + 378*sqrt(c + d*x)*a**2*b*d**4*f*x**2 + 144*sqr
t(c + d*x)*a*b**2*c**3*d*f - 252*sqrt(c + d*x)*a*b**2*c**2*d**2*e - 72*sqr
t(c + d*x)*a*b**2*c**2*d**2*f*x + 126*sqrt(c + d*x)*a*b**2*c*d**3*e*x + 54
*sqrt(c + d*x)*a*b**2*c*d**3*f*x**2 + 378*sqrt(c + d*x)*a*b**2*d**4*e*x**2
 + 270*sqrt(c + d*x)*a*b**2*d**4*f*x**3 - 32*sqrt(c + d*x)*b**3*c**4*f + 4
8*sqrt(c + d*x)*b**3*c**3*d*e + 16*sqrt(c + d*x)*b**3*c**3*d*f*x - 24*sqrt
(c + d*x)*b**3*c**2*d**2*e*x - 12*sqrt(c + d*x)*b**3*c**2*d**2*f*x**2 + 18
*sqrt(c + d*x)*b**3*c*d**3*e*x**2 + 10*sqrt(c + d*x)*b**3*c*d**3*f*x**3 +
90*sqrt(c + d*x)*b**3*d**4*e*x**3 + 70*sqrt(c + d*x)*b**3*d**4*f*x**4 + 31
5*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a**3*d**4*e - 315*sqrt(c)*log(sqrt(
c + d*x) + sqrt(c))*a**3*d**4*e)/(315*d**4)
```

3.20 $\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = 2a^2e\sqrt{c+dx}+\frac{2(a^2d^2f-b^2c(de-cf)+2abd(de-cf))\,(c+dx)^{3/2}}{3d^3} + \frac{2b(bde-2bcf+2adf)(c+dx)^{5/2}}{5d^3}+\frac{2b^2f(c+dx)^{7/2}}{7d^3}-2a^2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

output
```
2*a^2*e*(d*x+c)^(1/2)+2/3*(a^2*d^2*f-b^2*c*(-c*f+d*e)+2*a*b*d*(-c*f+d*e))*
(d*x+c)^(3/2)/d^3+2/5*b*(2*a*d*f-2*b*c*f+b*d*e)*(d*x+c)^(5/2)/d^3+2/7*b^2*
f*(d*x+c)^(7/2)/d^3-2*a^2*c^(1/2)*e*arctanh((d*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\sqrt{c+dx}(35a^2d^2(3de+cf+dfx)+14abd(c+dx)(5de-2cf+3dfx)+b^2(c+dx)\left(8c^2f+3d^2x(7e+\right.}{105d^3} - 2a^2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*Sqrt[c + d*x]*(35*a^2*d^2*(3*d*e + c*f + d*f*x) + 14*a*b*d*(c + d*x)*(5
*d*e - 2*c*f + 3*d*f*x) + b^2*(c + d*x)*(8*c^2*f + 3*d^2*x*(7*e + 5*f*x) -
 2*c*d*(7*e + 6*f*x))))/(105*d^3) - 2*a^2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/
Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$$

$$\Big\downarrow \text{170}$$

$$\frac{2\int \frac{(a+bx)\sqrt{c+dx}(7ade+(7bde-4bcf+4adf)x)}{2x}dx}{7d}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)\sqrt{c+dx}(7ade+(7bde-4bcf+4adf)x)}{x}dx}{7d}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

$$\Big\downarrow \text{164}$$

$$\frac{7a^2de\int \frac{\sqrt{c+dx}}{x}dx+\frac{2(c+dx)^{3/2}(20a^2d^2f+3bdx(4adf-4bcf+7bde)+14abd(5de-2cf)-2b^2c(7de-4cf))}{15d^2}}{7d}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

$$\Big\downarrow \text{60}$$

$$\frac{7a^2de\left(c\int \frac{1}{x\sqrt{c+dx}}dx+2\sqrt{c+dx}\right)+\frac{2(c+dx)^{3/2}(20a^2d^2f+3bdx(4adf-4bcf+7bde)+14abd(5de-2cf)-2b^2c(7de-4cf))}{15d^2}}{7d}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

$$\Big\downarrow \; 73$$

$$\frac{7a^2de\left(\frac{2c\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{d}+2\sqrt{c+dx}\right)+\frac{2(c+dx)^{3/2}(20a^2d^2f+3bdx(4adf-4bcf+7bde)+14abd(5de-2cf)-2b^2c(7de-4cf))}{15d^2}}{7d}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

$$\Big\downarrow \; 221$$

$$\frac{7a^2de\left(2\sqrt{c+dx}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+\frac{2(c+dx)^{3/2}(20a^2d^2f+3bdx(4adf-4bcf+7bde)+14abd(5de-2cf)-2b^2c(7de-4cf))}{15d^2}}{7d}+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

input
```
Int[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + ((2*(c + d*x)^(3/2)*(20*a^2*d^2*
f - 2*b^2*c*(7*d*e - 4*c*f) + 14*a*b*d*(5*d*e - 2*c*f) + 3*b*d*(7*b*d*e -
4*b*c*f + 4*a*d*f)*x))/(15*d^2) + 7*a^2*d*e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]]))/(7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{-2a^2\sqrt{c}\,d^3e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)+\frac{2\left(\left(\frac{3\left(\frac{5fx}{7}+e\right)x^2b^2}{5}+2ax\left(\frac{3fx}{5}+e\right)b+3a^2\left(\frac{fx}{3}+e\right)\right)d^3+c\left(\frac{\left(\frac{3fx}{7}+e\right)x\,b^2}{5}+2a\left(\frac{fx}{5}+e\right)b+a^2\right.}{3}}{d^3}$
derivativedivides	$\frac{\frac{2b^2f(xd+c)^{\frac{7}{2}}}{7}+\frac{4abdf(xd+c)^{\frac{5}{2}}}{5}-\frac{4b^2cf(xd+c)^{\frac{5}{2}}}{5}+\frac{2b^2de(xd+c)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(xd+c)^{\frac{3}{2}}}{3}-\frac{4abcdf(xd+c)^{\frac{3}{2}}}{3}+\frac{4ab\,d^2e(xd+c)^{\frac{3}{2}}}{3}+2b^2}{d^3}$
default	$\frac{\frac{2b^2f(xd+c)^{\frac{7}{2}}}{7}+\frac{4abdf(xd+c)^{\frac{5}{2}}}{5}-\frac{4b^2cf(xd+c)^{\frac{5}{2}}}{5}+\frac{2b^2de(xd+c)^{\frac{5}{2}}}{5}+\frac{2a^2d^2f(xd+c)^{\frac{3}{2}}}{3}-\frac{4abcdf(xd+c)^{\frac{3}{2}}}{3}+\frac{4ab\,d^2e(xd+c)^{\frac{3}{2}}}{3}+2b^2}{d^3}$

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-3*a^2*c^(1/2)*d^3*e*arctanh((d*x+c)^(1/2)/c^(1/2))+((3/5*(5/7*f*x+e)
*x^2*b^2+2*a*x*(3/5*f*x+e)*b+3*a^2*(1/3*f*x+e))*d^3+c*(1/5*(3/7*f*x+e)*x*b
^2+2*a*(1/5*f*x+e)*b+a^2*f)*d^2-4/5*((1/7*f*x+1/2*e)*b+a*f)*c^2*b*d+8/35*b
^2*c^3*f)*(d*x+c)^(1/2))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$$

$$= \left[\frac{105\,a^2\sqrt{c}d^3e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,(15\,b^2d^3fx^3+3\,(7\,b^2d^3e+(b^2cd^2+14\,abd^3)f)x^2-7\,(2\,b^2c^2d-}{}\right.$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/105*(105*a^2*sqrt(c)*d^3*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
 + 2*(15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2*c*d^2 + 14*a*b*d^3)*f)*x^2
- 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e + (8*b^2*c^3 - 28*a*b*c^2*
d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3)*e - (4*b^2*c^2*d - 14*a*
b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3, 2/105*(105*a^2*sqrt(-c)*d^
3*e*arctan(sqrt(-c)/sqrt(d*x + c)) + (15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e +
(b^2*c*d^2 + 14*a*b*d^3)*f)*x^2 - 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d
^3)*e + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a
*b*d^3)*e - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))
/d^3]
```

Sympy [A] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \begin{cases} \frac{2a^2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2a^2e\sqrt{c+dx} + \frac{2b^2f(c+dx)^{\frac{7}{2}}}{7d^3} + \frac{2(c+dx)^{\frac{5}{2}}\cdot(2abdf-2b^2cf+b^2de)}{5d^3} + \frac{2(c+dx)^{\frac{3}{2}}(a^2d^2f-2abcdf+2abd^2e+ \\ \sqrt{c}\left(a^2e\log(x)+a^2fx+2abex+\frac{b^2fx^3}{3}+\frac{x^2\cdot(2abf+b^2e)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)**(1/2)*(f*x+e)/x,x)
```

output
```
Piecewise((2*a**2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**2*e*sqr
t(c + d*x) + 2*b**2*f*(c + d*x)**(7/2)/(7*d**3) + 2*(c + d*x)**(5/2)*(2*a*
b*d*f - 2*b**2*c*f + b**2*d*e)/(5*d**3) + 2*(c + d*x)**(3/2)*(a**2*d**2*f
- 2*a*b*c*d*f + 2*a*b*d**2*e + b**2*c**2*f - b**2*c*d*e)/(3*d**3), Ne(d, 0
)), (sqrt(c)*(a**2*e*log(x) + a**2*f*x + 2*a*b*e*x + b**2*f*x**3/3 + x**2*
(2*a*b*f + b**2*e)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = a^2\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)$$
$$+\frac{2\left(105\sqrt{dx+c}a^2d^3e+15\,(dx+c)^{\frac{7}{2}}b^2f+21\,(b^2de-2\,(b^2c-abd)f)(dx+c)^{\frac{5}{2}}-35\,((b^2cd-2\,abd^2)e\right.}{105\,d^3}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="maxima")
```

output
```
a^2*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2
/105*(105*sqrt(d*x + c)*a^2*d^3*e + 15*(d*x + c)^(7/2)*b^2*f + 21*(b^2*d*e
 - 2*(b^2*c - a*b*d)*f)*(d*x + c)^(5/2) - 35*((b^2*c*d - 2*a*b*d^2)*e - (b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*f)*(d*x + c)^(3/2))/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,a^2ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$
$$+\frac{2\left(21\,(dx+c)^{\frac{5}{2}}b^2d^{19}e-35\,(dx+c)^{\frac{3}{2}}b^2cd^{19}e+70\,(dx+c)^{\frac{3}{2}}abd^{20}e+105\sqrt{dx+c}a^2d^{21}e+15\,(dx+c)\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a^2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/105*(21*(d*x + c)^(5
/2)*b^2*d^19*e - 35*(d*x + c)^(3/2)*b^2*c*d^19*e + 70*(d*x + c)^(3/2)*a*b*
d^20*e + 105*sqrt(d*x + c)*a^2*d^21*e + 15*(d*x + c)^(7/2)*b^2*d^18*f - 42
*(d*x + c)^(5/2)*b^2*c*d^18*f + 35*(d*x + c)^(3/2)*b^2*c^2*d^18*f + 42*(d*
x + c)^(5/2)*a*b*d^19*f - 70*(d*x + c)^(3/2)*a*b*c*d^19*f + 35*(d*x + c)^(
3/2)*a^2*d^20*f)/d^21
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{5\,d^3}+\frac{2\,b^2\,c\,f}{5\,d^3}\right)\,{\left(c+d\,x\right)}^{5/2} + \left(c\,\left(c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)+\frac{2\,\left(a\,d-b\,c\right)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{d^3}\right) - \frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(c\,f-d\,e\right)}{d^3}\right)\,\sqrt{c+d\,x} + \left(\frac{c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)}{3} + \frac{2\,\left(a\,d-b\,c\right)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2} + \frac{2\,b^2\,f\,{\left(c+d\,x\right)}^{7/2}}{7\,d^3}+a^2\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,2{}\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)^2*(c + d*x)^(1/2))/x,x)
```

output
```
((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/(5*d^3) + (2*b^2*c*f)/(5*d^3))*(c + d
*x)^(5/2) + (c*(c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d^3 + (2*b^2*c*f)/d
^3) + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/d^3) - (2*(a*d - b*c)^2*
(c*f - d*e))/d^3)*(c + d*x)^(1/2) + ((c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*
f)/d^3 + (2*b^2*c*f)/d^3))/3 + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))
/(3*d^3))*(c + d*x)^(3/2) + a^2*c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2
))*2i + (2*b^2*f*(c + d*x)^(7/2))/(7*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx)^2\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{70\sqrt{dx+c}\,a^2 c\,d^2 f+210\sqrt{dx+c}\,a^2 d^3 e+70\sqrt{dx+c}\,a^2 d^3 f x-56\sqrt{dx+c}\,ab\,c^2 df+140\sqrt{dx+c}\,abc\,d^2 e}{}$$

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)/x,x)
```

output
```
(70*sqrt(c + d*x)*a**2*c*d**2*f + 210*sqrt(c + d*x)*a**2*d**3*e + 70*sqrt(
c + d*x)*a**2*d**3*f*x - 56*sqrt(c + d*x)*a*b*c**2*d*f + 140*sqrt(c + d*x)
*a*b*c*d**2*e + 28*sqrt(c + d*x)*a*b*c*d**2*f*x + 140*sqrt(c + d*x)*a*b*d*
*3*e*x + 84*sqrt(c + d*x)*a*b*d**3*f*x**2 + 16*sqrt(c + d*x)*b**2*c**3*f -
 28*sqrt(c + d*x)*b**2*c**2*d*e - 8*sqrt(c + d*x)*b**2*c**2*d*f*x + 14*sqr
t(c + d*x)*b**2*c*d**2*e*x + 6*sqrt(c + d*x)*b**2*c*d**2*f*x**2 + 42*sqrt(
c + d*x)*b**2*d**3*e*x**2 + 30*sqrt(c + d*x)*b**2*d**3*f*x**3 + 105*sqrt(c
)*log(sqrt(c + d*x) - sqrt(c))*a**2*d**3*e - 105*sqrt(c)*log(sqrt(c + d*x)
 + sqrt(c))*a**2*d**3*e)/(105*d**3)
```

3.21 $\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = 2ae\sqrt{c+dx}+\frac{2(bde-bcf+adf)(c+dx)^{3/2}}{3d^2} + \frac{2bf(c+dx)^{5/2}}{5d^2}-2a\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

output
```
2*a*e*(d*x+c)^(1/2)+2/3*(a*d*f-b*c*f+b*d*e)*(d*x+c)^(3/2)/d^2+2/5*b*f*(d*x
+c)^(5/2)/d^2-2*a*c^(1/2)*e*arctanh((d*x+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\sqrt{c+dx}(-b(c+dx)(-5de+2cf-3dfx)+5ad(3de+cf+dfx))}{15d^2} - 2a\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*Sqrt[c + d*x]*(-(b*(c + d*x)*(-5*d*e + 2*c*f - 3*d*f*x)) + 5*a*d*(3*d*e
 + c*f + d*f*x)))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$$

$$\Big\downarrow \text{164}$$

$$ae\int \frac{\sqrt{c+dx}}{x}dx-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}$$

$$\Big\downarrow \text{60}$$

$$ae\left(c\int \frac{1}{x\sqrt{c+dx}}dx+2\sqrt{c+dx}\right)-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}$$

$$\Big\downarrow \text{73}$$

$$ae\left(\frac{2c\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{d}+2\sqrt{c+dx}\right)-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}$$

$$\Big\downarrow \text{221}$$

$$ae\left(2\sqrt{c+dx}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}$$

input
```
Int[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(-2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d*f*x))/(15*d^2) + a*
e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$\frac{-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)+\frac{2\sqrt{xd+c}\left(\left(x\left(\frac{3fx}{5}+e\right)b+3a\left(\frac{fx}{3}+e\right)\right)d^{2}+c\left(\left(\frac{fx}{5}+e\right)b+af\right)d-\frac{2b\,c^{2}f}{5}\right)}{3}}{d^{2}}$	83
derivativedivides	$\frac{\frac{2fb(xd+c)^{\frac{5}{2}}}{5}+\frac{2adf(xd+c)^{\frac{3}{2}}}{3}-\frac{2bcf(xd+c)^{\frac{3}{2}}}{3}+\frac{2bde(xd+c)^{\frac{3}{2}}}{3}+2a\,d^{2}e\sqrt{xd+c}-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{d^{2}}$	89
default	$\frac{\frac{2fb(xd+c)^{\frac{5}{2}}}{5}+\frac{2adf(xd+c)^{\frac{3}{2}}}{3}-\frac{2bcf(xd+c)^{\frac{3}{2}}}{3}+\frac{2bde(xd+c)^{\frac{3}{2}}}{3}+2a\,d^{2}e\sqrt{xd+c}-2a\sqrt{c}\,d^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{d^{2}}$	89

input
```
int((b*x+a)*(d*x+c)^(1/2)*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(-3*a*c^(1/2)*d^2*e*arctanh((d*x+c)^(1/2)/c^(1/2))+(d*x+c)^(1/2)*((x*(
3/5*f*x+e)*b+3*a*(1/3*f*x+e))*d^2+c*((1/5*f*x+e)*b+a*f)*d-2/5*b*c^2*f))/d^
2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$=\left[\frac{15\,a\sqrt{c}d^{2}e\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2\,(3\,bd^{2}fx^{2}+5\,(bcd+3\,ad^{2})e-(2\,bc^{2}-5\,acd)f+(5\,bd^{2}e+(bcd+}{15\,d^{2}}\right.$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*a*sqrt(c)*d^2*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2
*(3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (2*b*c^2 - 5*a*c*d)*f + (5*b*d^2
*e + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/d^2, 2/15*(15*a*sqrt(-c)*d^2*e
*arctan(sqrt(-c)/sqrt(d*x + c)) + (3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e -
 (2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c
))/d^2]
```

Sympy [A] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \begin{cases} \frac{2ace\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2ae\sqrt{c+dx} + \frac{2bf(c+dx)^{\frac{5}{2}}}{5d^2} + \frac{2(c+dx)^{\frac{3}{2}}(adf-bcf+bde)}{3d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(ae\log{(x)} + afx + bex + \frac{bfx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**(1/2)*(f*x+e)/x,x)
```

output
```
Piecewise((2*a*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a*e*sqrt(c +
d*x) + 2*b*f*(c + d*x)**(5/2)/(5*d**2) + 2*(c + d*x)**(3/2)*(a*d*f - b*c*f
 + b*d*e)/(3*d**2), Ne(d, 0)), (sqrt(c)*(a*e*log(x) + a*f*x + b*e*x + b*f*
x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = a\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + \frac{2\left(15\sqrt{dx+c}ad^2e + 3(dx+c)^{\frac{5}{2}}bf + 5(bde-(bc-ad)f)(dx+c)^{\frac{3}{2}}\right)}{15\,d^2}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="maxima")
```

output
```
a*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/1
5*(15*sqrt(d*x + c)*a*d^2*e + 3*(d*x + c)^(5/2)*b*f + 5*(b*d*e - (b*c - a*
d)*f)*(d*x + c)^(3/2))/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,ace\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2\left(5\,(dx+c)^{\frac{3}{2}}bd^9e + 15\,\sqrt{dx+c}ad^{10}e + 3\,(dx+c)^{\frac{5}{2}}bd^8f - 5\,(dx+c)^{\frac{3}{2}}bcd^8f + 5\,(dx+c)^{\frac{3}{2}}ad^9f\right)}{15\,d^{10}}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/15*(5*(d*x + c)^(3/2)*
b*d^9*e + 15*sqrt(d*x + c)*a*d^10*e + 3*(d*x + c)^(5/2)*b*d^8*f - 5*(d*x +
 c)^(3/2)*b*c*d^8*f + 5*(d*x + c)^(3/2)*a*d^9*f)/d^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx = \left(c\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{d^2}+\frac{2\,b\,c\,f}{d^2}\right) - \frac{2\,(a\,d-b\,c)\,(c\,f-d\,e)}{d^2}\right)\sqrt{c+d\,x} + \left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{3\,d^2}+\frac{2\,b\,c\,f}{3\,d^2}\right)(c+d\,x)^{3/2} + \frac{2\,b\,f\,(c+d\,x)^{5/2}}{5\,d^2} + a\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)*(c + d*x)^(1/2))/x,x)
```

output
```
(c*((2*a*d*f - 4*b*c*f + 2*b*d*e)/d^2 + (2*b*c*f)/d^2) - (2*(a*d - b*c)*(c
*f - d*e))/d^2)*(c + d*x)^(1/2) + ((2*a*d*f - 4*b*c*f + 2*b*d*e)/(3*d^2) +
 (2*b*c*f)/(3*d^2))*(c + d*x)^(3/2) + (2*b*f*(c + d*x)^(5/2))/(5*d^2) + a*
c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \frac{10\sqrt{dx+c}\,acdf+30\sqrt{dx+c}\,a\,d^2e+10\sqrt{dx+c}\,a\,d^2fx-4\sqrt{dx+c}\,b\,c^2f+10\sqrt{dx+c}\,bcde+2\sqrt{dx+}}{}$$

input
```
int((b*x+a)*(d*x+c)^(1/2)*(f*x+e)/x,x)
```

output
```
(10*sqrt(c + d*x)*a*c*d*f + 30*sqrt(c + d*x)*a*d**2*e + 10*sqrt(c + d*x)*a
*d**2*f*x - 4*sqrt(c + d*x)*b*c**2*f + 10*sqrt(c + d*x)*b*c*d*e + 2*sqrt(c
 + d*x)*b*c*d*f*x + 10*sqrt(c + d*x)*b*d**2*e*x + 6*sqrt(c + d*x)*b*d**2*f
*x**2 + 15*sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a*d**2*e - 15*sqrt(c)*log(
sqrt(c + d*x) + sqrt(c))*a*d**2*e)/(15*d**2)
```

3.22 $\int \frac{\sqrt{c+dx}(e+fx)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{\sqrt{c+dx}(e+fx)}{x} \, dx = 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} - 2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

output
```
2*e*(d*x+c)^(1/2)+2/3*f*(d*x+c)^(3/2)/d-2*c^(1/2)*e*arctanh((d*x+c)^(1/2)/
c^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{c+dx}(e+fx)}{x} \, dx = \frac{2\sqrt{c+dx}(3de+cf+dfx)}{3d} - 2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*Sqrt[c + d*x]*(3*d*e + c*f + d*f*x))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx$$

$$\Big\downarrow \text{ 90}$$

$$e\int \frac{\sqrt{c+dx}}{x}dx+\frac{2f(c+dx)^{3/2}}{3d}$$

$$\Big\downarrow \text{ 60}$$

$$e\left(c\int \frac{1}{x\sqrt{c+dx}}dx+2\sqrt{c+dx}\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

$$\Big\downarrow \text{ 73}$$

$$e\left(\frac{2c\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{d}+2\sqrt{c+dx}\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

$$\Big\downarrow \text{ 221}$$

$$e\left(2\sqrt{c+dx}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

output
```
(2*f*(c + d*x)^(3/2))/(3*d) + e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{2 f (x d+c)^{\frac{3}{2}}}{3}+2 d e \sqrt{x d+c}-2 \sqrt{c}\, d e \,\operatorname{arctanh}\left(\frac{\sqrt{x d+c}}{\sqrt{c}}\right)}{d}$	46
default	$\frac{\frac{2 f (x d+c)^{\frac{3}{2}}}{3}+2 d e \sqrt{x d+c}-2 \sqrt{c}\, d e \,\operatorname{arctanh}\left(\frac{\sqrt{x d+c}}{\sqrt{c}}\right)}{d}$	46
pseudoelliptic	$\frac{-6 \sqrt{c}\, d e \,\operatorname{arctanh}\left(\frac{\sqrt{x d+c}}{\sqrt{c}}\right)+2 \left(\left(f x+3 e\right) d+c f\right) \sqrt{x d+c}}{3 d}$	48

input
```
int((d*x+c)^(1/2)*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/3*f*(d*x+c)^(3/2)+d*e*(d*x+c)^(1/2)-c^(1/2)*d*e*arctanh((d*x+c)^(1/
2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \left[\frac{3\sqrt{c}de\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\,(dfx+3\,de+cf)\sqrt{dx+c}}{3\,d}, \frac{2\left(3\sqrt{-c}de\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx+c}}\right)+(dfx+3\,d\right.}{3\,d}\right.$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(c)*d*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(d*f*
x + 3*d*e + c*f)*sqrt(d*x + c))/d, 2/3*(3*sqrt(-c)*d*e*arctan(sqrt(-c)/sqr
t(d*x + c)) + (d*f*x + 3*d*e + c*f)*sqrt(d*x + c))/d]
```

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \begin{cases} \frac{2ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2e\sqrt{c+dx} + \frac{2f(c+dx)^{\frac{3}{2}}}{3d} & \text{for } d \neq 0 \\ \sqrt{c}(e\log(fx)+fx) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)/x,x)
```

output
```
Piecewise((2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*e*sqrt(c + d*x)
 + 2*f*(c + d*x)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)*(e*log(f*x) + f*x), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(3\sqrt{dx+c}de+(dx+c)^{\frac{3}{2}}f\right)}{3\,d}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="maxima")
```

output
```
sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/3*(
3*sqrt(d*x + c)*d*e + (d*x + c)^(3/2)*f)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\left(3\sqrt{dx+c}d^3e+(dx+c)^{\frac{3}{2}}d^2f\right)}{3\,d^3}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + 2/3*(3*sqrt(d*x + c)*d^3*e
 + (d*x + c)^(3/2)*d^2*f)/d^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx = 2\,e\sqrt{c+d\,x}+\frac{2\,f\,(c+d\,x)^{3/2}}{3\,d}+\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

input
```
int(((e + f*x)*(c + d*x)^(1/2))/x,x)
```

output
```
2*e*(c + d*x)^(1/2) + c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2
*f*(c + d*x)^(3/2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{c+dx}(e+fx)}{x}\,dx$$
$$= \frac{2\sqrt{dx+c}\,cf + 6\sqrt{dx+c}\,de + 2\sqrt{dx+c}\,dfx + 3\sqrt{c}\log\left(\sqrt{dx+c} - \sqrt{c}\right)de - 3\sqrt{c}\log\left(\sqrt{dx+c} + \sqrt{c}\right)d}{3d}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)/x,x)
```

output
```
(2*sqrt(c + d*x)*c*f + 6*sqrt(c + d*x)*d*e + 2*sqrt(c + d*x)*d*f*x + 3*sqr
t(c)*log(sqrt(c + d*x) - sqrt(c))*d*e - 3*sqrt(c)*log(sqrt(c + d*x) + sqrt
(c))*d*e)/(3*d)
```

3.23 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2f\sqrt{c+dx}}{b}-\frac{2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2\sqrt{bc-ad}(be-af)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

output
```
2*f*(d*x+c)^(1/2)/b-2*c^(1/2)*e*arctanh((d*x+c)^(1/2)/c^(1/2))/a+2*(-a*d+b
*c)^(1/2)*(-a*f+b*e)*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(
3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2f\sqrt{c+dx}}{b}+\frac{2\sqrt{-bc+ad}(be-af)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

output
```
(2*f*Sqrt[c + d*x])/b + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*ArcTan[(Sqrt[b]*
Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(a*b^(3/2)) - (2*Sqrt[c]*e*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)} \, dx$$

$$\Big\downarrow \; 171$$

$$\frac{2\int \frac{bce+(bde+bcf-adf)x}{2x(a+bx)\sqrt{c+dx}}dx}{b}+\frac{2f\sqrt{c+dx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{bce+(bde+bcf-adf)x}{x(a+bx)\sqrt{c+dx}}dx}{b}+\frac{2f\sqrt{c+dx}}{b}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{bce\int \frac{1}{x\sqrt{c+dx}}dx}{a}-\frac{(bc-ad)(be-af)\int \frac{1}{(a+bx)\sqrt{c+dx}}dx}{a}}{b}+\frac{2f\sqrt{c+dx}}{b}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2bce\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{ad}-\frac{2(bc-ad)(be-af)\int \frac{1}{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{ad}}{b}+\frac{2f\sqrt{c+dx}}{b}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2\sqrt{bc-ad}(be-af)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2b\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}}{b}+\frac{2f\sqrt{c+dx}}{b}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

output
```
(2*f*Sqrt[c + d*x])/b + ((-2*b*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a
 + (2*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c
 - a*d]])/(a*Sqrt[b]))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{2f\sqrt{xd+c}}{b}+\frac{2(-a^2df+abcf+abde-ce\,b^2)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{ab\sqrt{(ad-bc)b}}-\frac{2\sqrt{c}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{a}$	103
default	$\frac{2f\sqrt{xd+c}}{b}+\frac{2(-a^2df+abcf+abde-ce\,b^2)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{ab\sqrt{(ad-bc)b}}-\frac{2\sqrt{c}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{a}$	103
pseudoelliptic	$\frac{-2(af-be)(ad-bc)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+2\left(-\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)be+\sqrt{xd+c}\,af\right)\sqrt{(ad-bc)b}}{ab\sqrt{(ad-bc)b}}$	105

input
```
int((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*f*(d*x+c)^(1/2)/b+2*(-a^2*d*f+a*b*c*f+a*b*d*e-b^2*c*e)/a/b/((a*d-b*c)*b)
^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2*c^(1/2)*e*arctanh((d*
x+c)^(1/2)/c^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 4.39

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx$$
$$=\left[\frac{b\sqrt{c}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}af-(be-af)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab},\ \frac{b\sqrt{c}e\ldots}{}\right.$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a),x, algorithm="fricas")
```

output
```
[(b*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x +
c)*a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), (b*sqrt(c)*e*log((d*x
- 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c)*a*f + 2*(b*e - a*f)*
sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)))/(a*b), (2*b*sqrt(-c)*e*arctan(sqrt(-c)/sqrt(d*x + c)) + 2*sqrt(d*x +
 c)*a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqr
t(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), 2*(b*sqrt(-c)*e*arcta
n(sqrt(-c)/sqrt(d*x + c)) + sqrt(d*x + c)*a*f + (b*e - a*f)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(88) = 176$.

Time = 13.55 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx$$
$$= \begin{cases} \frac{2f\sqrt{c+dx}}{b} + \frac{2ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{2(ad-bc)(af-be)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}} \\ \sqrt{c}\left(\left(-f+\frac{be}{2a}\right)\left(\frac{2a\left(\begin{cases} -\frac{\frac{1}{x}+\frac{b}{2a}}{b} & \text{for } a=0 \\ \frac{\log\left(2a\left(\frac{1}{x}+\frac{b}{2a}\right)-b\right)}{2a} & \text{otherwise} \end{cases}\right)}{b} - \frac{2a\left(\begin{cases} \frac{\frac{1}{x}+\frac{b}{2a}}{b} & \text{for } a=0 \\ \frac{\log\left(2a\left(\frac{1}{x}+\frac{b}{2a}\right)+b\right)}{2a} & \text{otherwise} \end{cases}\right)}{b}\right) - \frac{e\log\left(\frac{a}{x\cdots}\right)}{2a\cdots} \right.\end{cases}$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)/x/(b*x+a),x)
```

output
```
Piecewise((2*f*sqrt(c + d*x)/b + 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/(a*sqr
t(-c)) - 2*(a*d - b*c)*(a*f - b*e)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))
/(a*b**2*sqrt((a*d - b*c)/b)), Ne(d, 0)), (sqrt(c)*((-f + b*e/(2*a))*(2*a*
Piecewise((-(1/x + b/(2*a))/b, Eq(a, 0)), (log(2*a*(1/x + b/(2*a)) - b)/(2
*a), True))/b - 2*a*Piecewise(((1/x + b/(2*a))/b, Eq(a, 0)), (log(2*a*(1/x
 + b/(2*a)) + b)/(2*a), True))/b) - e*log(a/x**2 + b/x)/(2*a)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}} + \frac{2\,\sqrt{dx+c}f}{b} - \frac{2\,(b^2ce-abde-abcf+a^2df)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ab}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a),x, algorithm="giac")
```

output
```
2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/(a*sqrt(-c)) + 2*sqrt(d*x + c)*f/b -
2*(b^2*c*e - a*b*d*e - a*b*c*f + a^2*d*f)*arctan(sqrt(d*x + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 2368, normalized size of antiderivative = 23.45

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)),x)
```

output
```
(2*f*(c + d*x)^(1/2))/b - (c^(1/2)*e*atan(((c^(1/2)*e*((8*(c + d*x)^(1/2)*
(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2
*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2
*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3
*c^2*d^2*f))/b + (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1
/2))/(a*b)))/a)*1i)/a + (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*
b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 -
2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*
d^3*e*f))/b - (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8
*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a)*1i)
/a)/((16*(b^3*c^2*d^3*e^3 - a*b^2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^
2*e^2*f - 3*a*b^2*c^2*d^3*e^2*f - a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*
f^2 + 2*a^2*b*c*d^4*e^2*f))/b - (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^
2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^
2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^
2*b^2*c*d^3*e*f))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f)
)/b + (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b))
)/a))/a + (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 +
2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*
e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{-2\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)af+2\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)be+2\sqrt{dx+c}\,abf+\sqrt{c}\log\left(\sqrt{dx\right.}}{a\,b^2}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a),x)
```

output
```
( - 2*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b
*c)))*a*f + 2*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt
(a*d - b*c)))*b*e + 2*sqrt(c + d*x)*a*b*f + sqrt(c)*log(sqrt(c + d*x) - sq
rt(c))*b**2*e - sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*b**2*e)/(a*b**2)
```

3.24 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{(2b^2ce-ad(be+af))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}$$

output
```
(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)-2*c^(1/2)*e*arctanh((d*x+c)^(1/2)/c^(
1/2))/a^2+(2*b^2*c*e-a*d*(a*f+b*e))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*
c)^(1/2))/a^2/b^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{\frac{a(be-af)\sqrt{c+dx}}{b(a+bx)} + \frac{(-2b^2ce+abde+a^2df)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}} - 2\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

output
```
((a*(b*e - a*f)*Sqrt[c + d*x])/(b*(a + b*x)) + ((-2*b^2*c*e + a*b*d*e + a^
2*d*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(3/2)*Sqrt[-
(b*c) + a*d]) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2} \, dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}-\frac{\int -\frac{2bce+d(be+af)x}{2x(a+bx)\sqrt{c+dx}}dx}{ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2bce+d(be+af)x}{x(a+bx)\sqrt{c+dx}}dx}{2ab}+\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{2bce\int \frac{1}{x\sqrt{c+dx}}dx}{a}-\frac{\left(2b^2ce-ad(af+be)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}dx}{a}}{2ab}+\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{4bce\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{ad}-\frac{2\left(2b^2ce-ad(af+be)\right)\int \frac{1}{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{ad}}{2ab}+\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)(2b^2ce-ad(af+be))}{a\sqrt{b}\sqrt{bc-ad}}-\frac{4b\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}}{2ab}+\frac{\sqrt{c+dx}(be-af)}{ab(a+bx)}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) + ((-4*b*Sqrt[c]*e*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/a + (2*(2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(a*Sqrt[b]*Sqrt[b*c - a*d]))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-(bx+a)\left(a^2df+abde-2ce\,b^2\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+\left(2be\sqrt{c}\,(bx+a)\,\mathrm{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)+a\sqrt{xd+c}\,(af-be)\right)\sqrt{(ad-}}{\sqrt{(ad-bc)b}\,(bx+a)\,b\,a^2}$
derivativedivides	$2d\left(-\frac{e\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{d\,a^2}+\frac{-\frac{ad(af-be)\sqrt{xd+c}}{2b((xd+c)b+ad-bc)}+\frac{\left(a^2df+abde-2ce\,b^2\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}}{a^2d}\right)$
default	$2d\left(-\frac{e\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{d\,a^2}+\frac{-\frac{ad(af-be)\sqrt{xd+c}}{2b((xd+c)b+ad-bc)}+\frac{\left(a^2df+abde-2ce\,b^2\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}}{a^2d}\right)$

input
```
int((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)*b)^(1/2)*(-(b*x+a)*(a^2*d*f+a*b*d*e-2*b^2*c*e)*arctan(b*(d*x
+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(2*b*e*c^(1/2)*(b*x+a)*arctanh((d*x+c)^(1/2
)/c^(1/2))+a*(d*x+c)^(1/2)*(a*f-b*e))*((a*d-b*c)*b)^(1/2))/(b*x+a)/b/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(109) = 218$.

Time = 0.17 (sec) , antiderivative size = 1012, normalized size of antiderivative = 7.97

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d
)*e)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*
d)*sqrt(d*x + c))/(b*x + a)) + 2*((b^4*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b
^2*d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*((a*b^3*
c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c - a^
4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e
+ (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(
-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + ((b^4*c - a*b^3*d)*e*x + (a
*b^3*c - a^2*b^2*d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x
) + ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^
3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), 1/2*(4*((b^4*c - a*b^3*d
)*e*x + (a*b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x + c)) +
 (a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*
x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sq
rt(d*x + c))/(b*x + a)) + 2*((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*
d)*f)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x),
((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*
x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x +
 b*c)) + 2*((b^4*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arct
an(sqrt(-c)/sqrt(d*x + c)) + ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)}{x\,(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)/x/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)/(x*(a + b*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}} - \frac{(2\,b^2ce-abde-a^2df)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx+c}bde-\sqrt{dx+c}adf}{((dx+c)b-bc+ad)ab}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/(a^2*sqrt(-c)) - (2*b^2*c*e - a*b*d*e
 - a^2*d*f)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*a^2*b) + (sqrt(d*x + c)*b*d*e - sqrt(d*x + c)*a*d*f)/(((d*x + c)*b -
b*c + a*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1827, normalized size of antiderivative = 14.39

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^2),x)
```

output
```
(atan((((((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) + ((4*a^5*b
^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*
f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*
c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) + (
2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a
^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a
*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b
^3*d)) - (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) - ((4*a^5
*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*
d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d -
b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) -
 (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2
*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*
(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3
*b^3*d)))/((4*(a*b^2*c*d^4*e^3 - 2*b^3*c^2*d^3*e^3 + a^3*c*d^4*e*f^2 - 2*a
*b^2*c^2*d^3*e^2*f + 2*a^2*b*c*d^4*e^2*f))/(a^3*b) + (((((2*(2*a^4*b^3*c*d
^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) + ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-
b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a
^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c
*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) + (2*(c + d*x)^(1/2)*(a^4*d^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 516, normalized size of antiderivative = 4.06

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^3df + \sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2bde + \sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right.}{}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))
*a**3*d*f + sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a
*d - b*c)))*a**2*b*d*e + sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(s
qrt(b)*sqrt(a*d - b*c)))*a**2*b*d*f*x - 2*sqrt(b)*sqrt(a*d - b*c)*atan((sq
rt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**2*c*e + sqrt(b)*sqrt(a*d -
b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**2*d*e*x - 2*sq
rt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b*
*3*c*e*x - sqrt(c + d*x)*a**3*b*d*f + sqrt(c + d*x)*a**2*b**2*c*f + sqrt(c
 + d*x)*a**2*b**2*d*e - sqrt(c + d*x)*a*b**3*c*e + sqrt(c)*log(sqrt(c + d*
x) - sqrt(c))*a**2*b**2*d*e - sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a*b**3*
c*e + sqrt(c)*log(sqrt(c + d*x) - sqrt(c))*a*b**3*d*e*x - sqrt(c)*log(sqrt
(c + d*x) - sqrt(c))*b**4*c*e*x - sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a**
2*b**2*d*e + sqrt(c)*log(sqrt(c + d*x) + sqrt(c))*a*b**3*c*e - sqrt(c)*log
(sqrt(c + d*x) + sqrt(c))*a*b**3*d*e*x + sqrt(c)*log(sqrt(c + d*x) + sqrt(
c))*b**4*c*e*x)/(a**2*b**2*(a**2*d - a*b*c + a*b*d*x - b**2*c*x))
```

3.25 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{(4b^2ce-3abde-a^2df)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}e\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}$$

output
```
1/2*(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)^2+1/4*(-a^2*d*f-3*a*b*d*e+4*b^2*c
*e)*(d*x+c)^(1/2)/a^2/b/(-a*d+b*c)/(b*x+a)-2*c^(1/2)*e*arctanh((d*x+c)^(1/
2)/c^(1/2))/a^3+1/4*(a^3*d^2*f+3*a^2*b*d^2*e-12*a*b^2*c*d*e+8*b^3*c^2*e)*a
rctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/b^(3/2)/(-a*d+b*c)^(3/2
)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx$$
$$= \frac{\frac{a\sqrt{c+dx}(a^3df+4b^3cex+3ab^2e(2c-dx)-a^2b(5de+2cf+dfx))}{b(bc-ad)(a+bx)^2} + \frac{(8b^3c^2e-12ab^2cde+3a^2bd^2e+a^3d^2f)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}(-bc+ad)^{3/2}} - 8\sqrt{c}e\operatorname{arctan}}{4a^3}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

output
```
((a*Sqrt[c + d*x]*(a^3*d*f + 4*b^3*c*e*x + 3*a*b^2*e*(2*c - d*x) - a^2*b*(
5*d*e + 2*c*f + d*f*x)))/(b*(b*c - a*d)*(a + b*x)^2) + ((8*b^3*c^2*e - 12*
a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[-(b*c) + a*d]])/(b^(3/2)*(-(b*c) + a*d)^(3/2)) - 8*Sqrt[c]*e*ArcTanh[Sq
rt[c + d*x]/Sqrt[c]])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {166, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx$$
$$\Big\downarrow \text{ 166}$$
$$\frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2} - \frac{\int -\frac{4bce+d(3be+af)x}{2x(a+bx)^2\sqrt{c+dx}}dx}{2ab}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{4bce+d(3be+af)x}{x(a+bx)^2\sqrt{c+dx}}dx}{4ab} + \frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{\int \frac{8bc(bc-ad)e+d\left(4b^2ce-ad(3be+af)\right)x}{2x(a+bx)\sqrt{c+dx}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx}(a^2(-d)f-3abde+4b^2ce)}{a(a+bx)(bc-ad)}}{4ab}+\frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{8bc(bc-ad)e+d\left(4b^2ce-ad(3be+af)\right)x}{x(a+bx)\sqrt{c+dx}}dx}{2a(bc-ad)}+\frac{\sqrt{c+dx}(a^2(-d)f-3abde+4b^2ce)}{a(a+bx)(bc-ad)}}{4ab}+\frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{\frac{8bce(bc-ad)\int \frac{1}{x\sqrt{c+dx}}dx}{a}-\frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}dx}{a}}{2a(bc-ad)}+\frac{\sqrt{c+dx}(a^2(-d)f-3abde+4b^2ce)}{a(a+bx)(bc-ad)}}{4ab}+\frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{16bce(bc-ad)\int \frac{1}{\frac{c+dx}{d}-\frac{c}{d}}d\sqrt{c+dx}}{ad}-\frac{2\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\int \frac{1}{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{ad}}{2a(bc-ad)}+\frac{\sqrt{c+dx}(a^2(-d)f-3abde+4b^2ce)}{a(a+bx)(bc-ad)}}{4ab}+\frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\sqrt{c+dx}(a^2(-d)f-3abde+4b^2ce)}{a(a+bx)(bc-ad)}+\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)}{a\sqrt{b}\sqrt{bc-ad}}-\frac{16b\sqrt{c}e(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}}{2a(bc-ad)}}{4ab}+\frac{\sqrt{c+dx}(be-af)}{2ab(a+bx)^2}$$

input

```
Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + (((4*b^2*c*e - 3*a*b*d*e
 - a^2*d*f)*Sqrt[c + d*x])/(a*(b*c - a*d)*(a + b*x)) + ((-16*b*Sqrt[c]*(b*
c - a*d)*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2*(8*b^3*c^2*e - 12*a*b^2*
c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(a*Sqrt[b]*Sqrt[b*c - a*d]))/(2*a*(b*c - a*d)))/(4*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{(bx+a)^2\left(a^3d^2f+3a^2b\,d^2e-12a\,b^2cde+8b^3c^2e\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{4}+2\sqrt{(ad-bc)b}\left((bx+a)^2\left(c^{\frac{3}{2}}b-ad\sqrt{c}\right)be\,\operatorname{arctanh}\left(\frac{\sqrt{x}}{}\right)\right)}{\sqrt{(ad-bc)b}\,(bx+a)^2(ad-bc)a^3b}$
derivativedivides	$2d^2\left(-\frac{e\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{d^2a^3}+\frac{\frac{ad\left(a^2df+3abde-4ce\,b^2\right)(xd+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(a^2df-5abde+4ce\,b^2\right)ad\sqrt{xd+c}}{8b}}{((xd+c)b+ad-bc)^2\,a^3d^2}+\frac{\left(a^3d^2f+3a^2b\,d^2\right)}{}\right)$
default	$2d^2\left(-\frac{e\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{xd+c}}{\sqrt{c}}\right)}{d^2a^3}+\frac{\frac{ad\left(a^2df+3abde-4ce\,b^2\right)(xd+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(a^2df-5abde+4ce\,b^2\right)ad\sqrt{xd+c}}{8b}}{((xd+c)b+ad-bc)^2\,a^3d^2}+\frac{\left(a^3d^2f+3a^2b\,d^2\right)}{}\right)$

input
```
int((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/((a*d-b*c)*b)^(1/2)*(1/8*(b*x+a)^2*(a^3*d^2*f+3*a^2*b*d^2*e-12*a*b^2*c*d
*e+8*b^3*c^2*e)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^
(1/2)*((b*x+a)^2*(c^(3/2)*b-a*d*c^(1/2))*b*e*arctanh((d*x+c)^(1/2)/c^(1/2)
)-1/8*(a^3*d*f-2*(5/2*d*e+f*(1/2*x*d+c))*b*a^2+6*a*(-1/2*x*d+c)*e*b^2+4*b^
3*c*e*x)*a*(d*x+c)^(1/2)))/(b*x+a)^2/(a*d-b*c)/a^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(182) = 364$.

Time = 0.54 (sec) , antiderivative size = 2210, normalized size of antiderivative = 10.62

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/8*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3
*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b
*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^2*c -
 a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b
*x + a)) - 8*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 -
 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2
*d^2)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*((6*a^2*
b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b^2*c
*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e - (a^
3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d
 + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*
b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*d^2*f + (a^3*b^2*d^2
*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 -
12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b
^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*
b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - 4*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^
2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2
- 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(
c) + 2*c)/x) - ((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^
3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)/x/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = -\frac{(8\,b^3c^2e-12\,ab^2cde+3\,a^2bd^2e+a^3d^2f)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^3b^2c-a^4bd)\sqrt{-b^2c+abd}} + \frac{2\,ce\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{a^3\sqrt{-c}} + \frac{4\,(dx+c)^{\frac{3}{2}}b^3cde-4\,\sqrt{dx+c}b^3c^2de-3\,(dx+c)^{\frac{3}{2}}ab^2d^2e+9\,\sqrt{dx+c}ab^2cd^2e-5\,\sqrt{dx+c}a^2bd^3e-(d}{4\,(a^2b^2c-a^3bd)((dx+c)b-bc+ad)^2}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*arctan(sqr
t(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c - a^4*b*d)*sqrt(-b^2*c + a*
b*d)) + 2*c*e*arctan(sqrt(d*x + c)/sqrt(-c))/(a^3*sqrt(-c)) + 1/4*(4*(d*x
+ c)^(3/2)*b^3*c*d*e - 4*sqrt(d*x + c)*b^3*c^2*d*e - 3*(d*x + c)^(3/2)*a*b
^2*d^2*e + 9*sqrt(d*x + c)*a*b^2*c*d^2*e - 5*sqrt(d*x + c)*a^2*b*d^3*e - (
d*x + c)^(3/2)*a^2*b*d^2*f - sqrt(d*x + c)*a^2*b*c*d^2*f + sqrt(d*x + c)*a
^3*d^3*f)/((a^2*b^2*c - a^3*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 4852, normalized size of antiderivative = 23.33

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^3),x)
```

output
```
(c^(1/2)*e*atan(((c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6
*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 3
20*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*
a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + (c^(1/
2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b
^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d
) + (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a
^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2
*a^5*b^2*c*d))))/a^3)*1i)/a^3 + (c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2
+ 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*
c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^
2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2
*c*d)) - (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*
d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2
- 2*a^7*b^2*c*d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^
4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 +
a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3)*1i)/a^3)/(((a^5*c*d^6*e*f^2)/4 - 12*a
^2*b^3*c^2*d^5*e^3 - 8*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2
*c*d^6*e^3)/4 + 2*a^2*b^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4
*b*c*d^6*e^2*f)/2)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1311, normalized size of antiderivative = 6.30

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)/x/(b*x+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))
*a**5*d**2*f + 3*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*a**4*b*d**2*e + 2*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c +
d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**2*f*x - 12*sqrt(b)*sqrt(a*d -
 b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d*e +
6*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c))
)*a**3*b**2*d**2*e*x + sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqr
t(b)*sqrt(a*d - b*c)))*a**3*b**2*d**2*f*x**2 + 8*sqrt(b)*sqrt(a*d - b*c)*a
tan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c**2*e - 24*sqr
t(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**
2*b**3*c*d*e*x + 3*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*a**2*b**3*d**2*e*x**2 + 16*sqrt(b)*sqrt(a*d - b*c)*atan
((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c**2*e*x - 12*sqrt(b)
*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*
c*d*e*x**2 + 8*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqr
t(a*d - b*c)))*b**5*c**2*e*x**2 - sqrt(c + d*x)*a**5*b*d**2*f + 3*sqrt(c +
 d*x)*a**4*b**2*c*d*f + 5*sqrt(c + d*x)*a**4*b**2*d**2*e + sqrt(c + d*x)*a
**4*b**2*d**2*f*x - 2*sqrt(c + d*x)*a**3*b**3*c**2*f - 11*sqrt(c + d*x)*a*
*3*b**3*c*d*e - sqrt(c + d*x)*a**3*b**3*c*d*f*x + 3*sqrt(c + d*x)*a**3*b**
3*d**2*e*x + 6*sqrt(c + d*x)*a**2*b**4*c**2*e - 7*sqrt(c + d*x)*a**2*b*...
```

3.26 $\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 223

$$\begin{aligned}\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x} dx &= 2c^3e\sqrt{a+bx} \\ &\quad - \frac{2(a^3d^3f+3ab^2cd(de+cf)-b^3c^2(3de+cf)-a^2bd^2(de+3cf))(a+bx)^{3/2}}{3b^4} \\ &\quad + \frac{2d(3a^2d^2f+3b^2c(de+cf)-2abd(de+3cf))(a+bx)^{5/2}}{5b^4} \\ &\quad + \frac{2d^2(bde+3bcf-3adf)(a+bx)^{7/2}}{7b^4} \\ &\quad + \frac{2d^3f(a+bx)^{9/2}}{9b^4} - 2\sqrt{a}c^3e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

output
```
2*c^3*e*(b*x+a)^(1/2)-2/3*(a^3*d^3*f+3*a*b^2*c*d*(c*f+d*e)-b^3*c^2*(c*f+3*
d*e)-a^2*b*d^2*(3*c*f+d*e))*(b*x+a)^(3/2)/b^4+2/5*d*(3*a^2*d^2*f+3*b^2*c*(
c*f+d*e)-2*a*b*d*(3*c*f+d*e))*(b*x+a)^(5/2)/b^4+2/7*d^2*(-3*a*d*f+3*b*c*f+
b*d*e)*(b*x+a)^(7/2)/b^4+2/9*d^3*f*(b*x+a)^(9/2)/b^4-2*a^(1/2)*c^3*e*arcta
nh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$= \frac{2\sqrt{a+bx}(-16a^4d^3f+8a^3bd^2(3de+9cf+dfx)-6a^2b^2d(21c^2f+d^2x(2e+fx)+3cd(7e+2fx))+ab\ldots}{}$$
$$-2\sqrt{a}c^3e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

output
```
(2*Sqrt[a + b*x]*(-16*a^4*d^3*f + 8*a^3*b*d^2*(3*d*e + 9*c*f + d*f*x) - 6*
a^2*b^2*d*(21*c^2*f + d^2*x*(2*e + f*x) + 3*c*d*(7*e + 2*f*x)) + a*b^3*(10
5*c^3*f + 63*c^2*d*(5*e + f*x) + 9*c*d^2*x*(7*e + 3*f*x) + d^3*x^2*(9*e +
5*f*x)) + b^4*(105*c^3*(3*e + f*x) + 63*c^2*d*x*(5*e + 3*f*x) + 27*c*d^2*x
^2*(7*e + 5*f*x) + 5*d^3*x^3*(9*e + 7*f*x))))/(315*b^4) - 2*Sqrt[a]*c^3*e*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {170, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$\Big\downarrow \text{170}$$
$$\frac{2\int \frac{3\sqrt{a+bx}(c+dx)^2(3bce+(3bde+2bcf-2adf)x)}{2x}dx}{9b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a+bx}(c+dx)^2(3bce+(3bde+2bcf-2adf)x)}{x}dx}{3b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

$$\Big\downarrow \ 170$$

$$\frac{\frac{2\int \frac{\sqrt{a+bx}(c+dx)\left(21b^2ec^2+\left(21cdeb^2+4(bc-ad)(3bde+2bcf-2adf)\right)x\right)}{2x}dx}{7b}+\frac{2(a+bx)^{3/2}(c+dx)^2(-2adf+2bcf+3bde)}{7b}}{3b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\sqrt{a+bx}(c+dx)\left(21b^2ec^2+\left(21cdeb^2+4(bc-ad)(3bde+2bcf-2adf)\right)x\right)}{x}dx}{7b}+\frac{2(a+bx)^{3/2}(c+dx)^2(-2adf+2bcf+3bde)}{7b}}{3b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

$$\Big\downarrow \ 164$$

$$\frac{\frac{21b^2c^3e\int \frac{\sqrt{a+bx}}{x}dx-\frac{2(a+bx)^{3/2}\left(16a^3d^3f-24a^2bd^2(3cf+de)-3bdx\left(4(bc-ad)(-2adf+2bcf+3bde)+21b^2cde\right)+6ab^2cd(16cf+21de)-10b^3c^2(4cf+27de)\right)}{15b^2}}{7b}}{3b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{21b^2c^3e\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}\left(16a^3d^3f-24a^2bd^2(3cf+de)-3bdx\left(4(bc-ad)(-2adf+2bcf+3bde)+21b^2cde\right)+6ab^2cd(16cf+21de)-10b^3c^2\right)}{15b^2}}{7b}}{3b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{21b^2c^3e\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)-\frac{2(a+bx)^{3/2}\left(16a^3d^3f-24a^2bd^2(3cf+de)-3bdx\left(4(bc-ad)(-2adf+2bcf+3bde)+21b^2cde\right)+6ab^2cd(16cf+21d\right)}{15b^2}}{7b}}{3b}+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{21b^2c^3e\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)-\frac{2(a+bx)^{3/2}\left(16a^3d^3f-24a^2bd^2(3cf+de)-3bdx\left(4(bc-ad)(-2adf+2bcf+3bde)+21b^2cde\right)+6ab^2cd(16cf+21}{15b^2}}{7b}}{3b}$$

$$\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

input
```
Int[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

output
```
(2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) + ((2*(3*b*d*e + 2*b*c*f - 2*a*d*f
)*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + ((-2*(a + b*x)^(3/2)*(16*a^3*d^3*f
- 24*a^2*b*d^2*(d*e + 3*c*f) - 10*b^3*c^2*(27*d*e + 4*c*f) + 6*a*b^2*c*d*(
21*d*e + 16*c*f) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*d)*(3*b*d*e + 2*b*c*f
- 2*a*d*f))*x))/(15*b^2) + 21*b^2*c^3*e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]))/(7*b))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{-2\sqrt{a}\, b^4 c^3 e \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{32\left(\frac{9\left(-5\left(\frac{7fx}{9}+e\right)x^3d^3-21c\left(\frac{5fx}{7}+e\right)x^2d^2-35c^2x\left(\frac{3fx}{5}+e\right)d-35c^3\left(\frac{fx}{3}+e\right)\right)b^4}{16}-\frac{105a\left(\frac{3x}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$
derivativedivides	$\frac{\frac{2d^3f(bx+a)^{\frac{9}{2}}}{9}-\frac{6a\,d^3f(bx+a)^{\frac{7}{2}}}{7}+\frac{6bc\,d^2f(bx+a)^{\frac{7}{2}}}{7}+\frac{2b\,d^3e(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2d^3f(bx+a)^{\frac{5}{2}}}{5}-\frac{12abc\,d^2f(bx+a)^{\frac{5}{2}}}{5}-\frac{4ab\,d^3e(bx+a)^{\frac{5}{2}}}{5}\cdots}{\cdots}$
default	$\frac{\frac{2d^3f(bx+a)^{\frac{9}{2}}}{9}-\frac{6a\,d^3f(bx+a)^{\frac{7}{2}}}{7}+\frac{6bc\,d^2f(bx+a)^{\frac{7}{2}}}{7}+\frac{2b\,d^3e(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2d^3f(bx+a)^{\frac{5}{2}}}{5}-\frac{12abc\,d^2f(bx+a)^{\frac{5}{2}}}{5}-\frac{4ab\,d^3e(bx+a)^{\frac{5}{2}}}{5}\cdots}{\cdots}$

input
```
int((b*x+a)^(1/2)*(d*x+c)^3*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/315*(-315*a^(1/2)*b^4*c^3*e*arctanh((b*x+a)^(1/2)/a^(1/2))-16*(9/16*(-5*
(7/9*f*x+e)*x^3*d^3-21*c*(5/7*f*x+e)*x^2*d^2-35*c^2*x*(3/5*f*x+e)*d-35*c^3
*(1/3*f*x+e))*b^4-105/16*a*(3/35*x^2*(5/9*f*x+e)*d^3+3/5*c*(3/7*f*x+e)*x*d
^2+3*c^2*(1/5*f*x+e)*d+c^3*f)*b^3+63/8*a^2*(2/21*(1/2*f*x+e)*x*d^2+c*(2/7*
f*x+e)*d+c^2*f)*d*b^2-9/2*a^3*d^2*(1/3*(1/3*f*x+e)*d+c*f)*b+a^4*d^3*f)*(b*
x+a)^(1/2))/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \left[\frac{315\sqrt{a}b^4c^3e\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(35\,b^4d^3fx^4+5\left(9\,b^4d^3e+\left(27\,b^4cd^2+ab^3d^3\right)f\right)x^3+3\left(3\left(21\,b^4c\right.\right.\right.}{}\right.$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^3*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/315*(315*sqrt(a)*b^4*c^3*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 + 2*(35*b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^
3 + 3*(3*(21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*
a^2*b^2*d^3)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2
+ 8*a^3*b*d^3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 1
6*a^4*d^3)*f + (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (10
5*b^4*c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*
x + a))/b^4, 2/315*(315*sqrt(-a)*b^4*c^3*e*arctan(sqrt(-a)/sqrt(b*x + a))
+ (35*b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3 +
 3*(3*(21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2
*b^2*d^3)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8
*a^3*b*d^3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a
^4*d^3)*f + (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b
^4*c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x +
 a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 11.21 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$= \begin{cases} \frac{2ac^3e\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2c^3e\sqrt{a+bx} + \frac{2d^3f(a+bx)^{\frac{9}{2}}}{9b^4} + \frac{2(a+bx)^{\frac{7}{2}}(-3ad^3f+3bcd^2f+bd^3e)}{7b^4} + \frac{2(a+bx)^{\frac{5}{2}}\cdot(3a^2d^3f-6abcd^2f-2a}{5b^4} \\ \sqrt{a}\left(c^3e\log{(x)} + c^3fx + 3c^2dex + \frac{d^3fx^4}{4} + \frac{x^3\cdot(3cd^2f+d^3e)}{3} + \frac{x^2\cdot(3c^2df+3cd^2e)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**3*(f*x+e)/x,x)
```

output
```
Piecewise((2*a*c**3*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**3*e*sqr
t(a + b*x) + 2*d**3*f*(a + b*x)**(9/2)/(9*b**4) + 2*(a + b*x)**(7/2)*(-3*a
*d**3*f + 3*b*c*d**2*f + b*d**3*e)/(7*b**4) + 2*(a + b*x)**(5/2)*(3*a**2*d
**3*f - 6*a*b*c*d**2*f - 2*a*b*d**3*e + 3*b**2*c**2*d*f + 3*b**2*c*d**2*e)
/(5*b**4) + 2*(a + b*x)**(3/2)*(-a**3*d**3*f + 3*a**2*b*c*d**2*f + a**2*b*
d**3*e - 3*a*b**2*c**2*d*f - 3*a*b**2*c*d**2*e + b**3*c**3*f + 3*b**3*c**2
*d*e)/(3*b**4), Ne(b, 0)), (sqrt(a)*(c**3*e*log(x) + c**3*f*x + 3*c**2*d*e
*x + d**3*f*x**4/4 + x**3*(3*c*d**2*f + d**3*e)/3 + x**2*(3*c**2*d*f + 3*c
*d**2*e)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \sqrt{a}c^3e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)$$
$$+ \frac{2\left(315\sqrt{bx+a}b^4c^3e + 35(bx+a)^{\frac{9}{2}}d^3f + 45(bd^3e+3(bcd^2-ad^3)f)(bx+a)^{\frac{7}{2}} + 63((3b^2cd^2-2abd}{}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^3*(f*x+e)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*c^3*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2
/315*(315*sqrt(b*x + a)*b^4*c^3*e + 35*(b*x + a)^(9/2)*d^3*f + 45*(b*d^3*e
 + 3*(b*c*d^2 - a*d^3)*f)*(b*x + a)^(7/2) + 63*((3*b^2*c*d^2 - 2*a*b*d^3)*
e + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f)*(b*x + a)^(5/2) + 105*((3*b^3
*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*f)*(b*x + a)^(3/2))/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx = \frac{2\,ac^3e\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(315\,\sqrt{bx+a}b^{36}c^3e+315\,(bx+a)^{\frac{3}{2}}b^{35}c^2de+189\,(bx+a)^{\frac{5}{2}}b^{34}cd^2e-315\,(bx+a)^{\frac{3}{2}}ab^{34}cd^2e+45\,(b$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^3*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a*c^3*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/315*(315*sqrt(b*x +
a)*b^36*c^3*e + 315*(b*x + a)^(3/2)*b^35*c^2*d*e + 189*(b*x + a)^(5/2)*b^3
4*c*d^2*e - 315*(b*x + a)^(3/2)*a*b^34*c*d^2*e + 45*(b*x + a)^(7/2)*b^33*d
^3*e - 126*(b*x + a)^(5/2)*a*b^33*d^3*e + 105*(b*x + a)^(3/2)*a^2*b^33*d^3
*e + 105*(b*x + a)^(3/2)*b^35*c^3*f + 189*(b*x + a)^(5/2)*b^34*c^2*d*f - 3
15*(b*x + a)^(3/2)*a*b^34*c^2*d*f + 135*(b*x + a)^(7/2)*b^33*c*d^2*f - 378
*(b*x + a)^(5/2)*a*b^33*c*d^2*f + 315*(b*x + a)^(3/2)*a^2*b^33*c*d^2*f + 3
5*(b*x + a)^(9/2)*b^32*d^3*f - 135*(b*x + a)^(7/2)*a*b^32*d^3*f + 189*(b*x
 + a)^(5/2)*a^2*b^32*d^3*f - 105*(b*x + a)^(3/2)*a^3*b^32*d^3*f)/b^36
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$= \left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{7\,b^4}+\frac{2\,a\,d^3\,f}{7\,b^4}\right)\,(a+b\,x)^{7/2}$$
$$+\left(\frac{a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)}{5} - \frac{6\,d\,(a\,d-b\,c)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{5\,b^4}\right)\,(a+b\,x)^{5/2}$$
$$+\left(a\,\left(a\,\left(a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)-\frac{6\,d\,(a\,d-b\,c)\,\left(b\,c\,f-2\,a\,d\,f+b\,d\,e\right)}{b^4}\right)+\frac{2\,(a\,d\right.\right.\right.$$
$$+\left(\frac{a\,\left(a\,\left(\frac{2\,b\,d^3\,e-8\,a\,d^3\,f+6\,b\,c\,d^2\,f}{b^4}+\frac{2\,a\,d^3\,f}{b^4}\right)-\frac{6\,d\,(a\,d-b\,c)\,(b\,c\,f-2\,a\,d\,f+b\,d\,e)}{b^4}\right)}{3}+\frac{2\,(a\,d-b\,c)^2\,(b\,c\,f-4\,a\,d\,f+3\,b\right.$$
$$+\frac{2\,d^3\,f\,(a+b\,x)^{9/2}}{9\,b^4}+\sqrt{a}\,c^3\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^3)/x,x)
```

output
```
((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/(7*b^4) + (2*a*d^3*f)/(7*b^4))*(a +
 b*x)^(7/2) + ((a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)
/b^4))/5 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/(5*b^4))*(a + b*x)^
(5/2) + (a*(a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/
b^4) - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4) + (2*(a*d - b*c)^2
*(b*c*f - 4*a*d*f + 3*b*d*e))/b^4) + (2*(a*d - b*c)^3*(a*f - b*e))/b^4)*(a
 + b*x)^(1/2) + ((a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d
^3*f)/b^4) - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4))/3 + (2*(a*d
 - b*c)^2*(b*c*f - 4*a*d*f + 3*b*d*e))/(3*b^4))*(a + b*x)^(3/2) + a^(1/2)*
c^3*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*d^3*f*(a + b*x)^(9/2))/(9
*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{a+bx}(c+dx)^3(e+fx)}{x}\,dx$$
$$= \frac{90\sqrt{bx+a}\,b^4d^3e\,x^3+70\sqrt{bx+a}\,b^4d^3f\,x^4+315\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^4c^3e-315\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{\right.}\right.}{}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^3*(f*x+e)/x,x)
```

output
```
( - 32*sqrt(a + b*x)*a**4*d**3*f + 144*sqrt(a + b*x)*a**3*b*c*d**2*f + 48*
sqrt(a + b*x)*a**3*b*d**3*e + 16*sqrt(a + b*x)*a**3*b*d**3*f*x - 252*sqrt(
a + b*x)*a**2*b**2*c**2*d*f - 252*sqrt(a + b*x)*a**2*b**2*c*d**2*e - 72*sq
rt(a + b*x)*a**2*b**2*c*d**2*f*x - 24*sqrt(a + b*x)*a**2*b**2*d**3*e*x - 1
2*sqrt(a + b*x)*a**2*b**2*d**3*f*x**2 + 210*sqrt(a + b*x)*a*b**3*c**3*f +
630*sqrt(a + b*x)*a*b**3*c**2*d*e + 126*sqrt(a + b*x)*a*b**3*c**2*d*f*x +
126*sqrt(a + b*x)*a*b**3*c*d**2*e*x + 54*sqrt(a + b*x)*a*b**3*c*d**2*f*x**
2 + 18*sqrt(a + b*x)*a*b**3*d**3*e*x**2 + 10*sqrt(a + b*x)*a*b**3*d**3*f*x
**3 + 630*sqrt(a + b*x)*b**4*c**3*e + 210*sqrt(a + b*x)*b**4*c**3*f*x + 63
0*sqrt(a + b*x)*b**4*c**2*d*e*x + 378*sqrt(a + b*x)*b**4*c**2*d*f*x**2 + 3
78*sqrt(a + b*x)*b**4*c*d**2*e*x**2 + 270*sqrt(a + b*x)*b**4*c*d**2*f*x**3
 + 90*sqrt(a + b*x)*b**4*d**3*e*x**3 + 70*sqrt(a + b*x)*b**4*d**3*f*x**4 +
 315*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c**3*e - 315*sqrt(a)*log(sq
rt(a + b*x) + sqrt(a))*b**4*c**3*e)/(315*b**4)
```

3.27 $\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x} \, dx = 2c^2e\sqrt{a+bx} + \frac{2(a^2d^2f + b^2c(2de+cf) - abd(de+2cf))\,(a+bx)^{3/2}}{3b^3} + \frac{2d(bde+2bcf-2adf)(a+bx)^{5/2}}{5b^3} + \frac{2d^2f(a+bx)^{7/2}}{7b^3} - 2\sqrt{a}c^2e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*c^2*e*(b*x+a)^(1/2)+2/3*(a^2*d^2*f+b^2*c*(c*f+2*d*e)-a*b*d*(2*c*f+d*e))*
(b*x+a)^(3/2)/b^3+2/5*d*(-2*a*d*f+2*b*c*f+b*d*e)*(b*x+a)^(5/2)/b^3+2/7*d^2
*f*(b*x+a)^(7/2)/b^3-2*a^(1/2)*c^2*e*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx$$
$$= \frac{2\sqrt{a+bx}(8a^3d^2f-2a^2bd(7de+14cf+2dfx)+ab^2(35c^2f+14cd(5e+fx)+d^2x(7e+3fx))+b^3(35c}{105b^3}$$
$$-2\sqrt{a}c^2e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

output
```
(2*Sqrt[a + b*x]*(8*a^3*d^2*f - 2*a^2*b*d*(7*d*e + 14*c*f + 2*d*f*x) + a*b
^2*(35*c^2*f + 14*c*d*(5*e + f*x) + d^2*x*(7*e + 3*f*x)) + b^3*(35*c^2*(3*
e + f*x) + 14*c*d*x*(5*e + 3*f*x) + 3*d^2*x^2*(7*e + 5*f*x))))/(105*b^3) -
 2*Sqrt[a]*c^2*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx$$

$$\Big\downarrow \ 170$$

$$\frac{2\int \frac{\sqrt{a+bx}(c+dx)(7bce+(7bde+4bcf-4adf)x)}{2x}dx}{7b}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{a+bx}(c+dx)(7bce+(7bde+4bcf-4adf)x)}{x}dx}{7b}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \textbf{164}$$

$$\frac{\frac{7bc^2e\int \frac{\sqrt{a+bx}}{x}dx+\frac{2(a+bx)^{3/2}(8a^2d^2f+3bdx(-4adf+4bcf+7bde)-14abd(2cf+de)+10b^2c(2cf+7de))}{15b^2}}{7b}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}}{}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{7bc^2e\left(a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}\right)+\frac{2(a+bx)^{3/2}(8a^2d^2f+3bdx(-4adf+4bcf+7bde)-14abd(2cf+de)+10b^2c(2cf+7de))}{15b^2}}{7b}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{7bc^2e\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}\right)+\frac{2(a+bx)^{3/2}(8a^2d^2f+3bdx(-4adf+4bcf+7bde)-14abd(2cf+de)+10b^2c(2cf+7de))}{15b^2}}{7b}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{2(a+bx)^{3/2}(8a^2d^2f+3bdx(-4adf+4bcf+7bde)-14abd(2cf+de)+10b^2c(2cf+7de))}{15b^2}+7bc^2e\left(2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{7b}+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

input
```
Int[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

output
```
(2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + ((2*(a + b*x)^(3/2)*(8*a^2*d^2*f
 - 14*a*b*d*(d*e + 2*c*f) + 10*b^2*c*(7*d*e + 2*c*f) + 3*b*d*(7*b*d*e + 4*
b*c*f - 4*a*d*f)*x))/(15*b^2) + 7*b*c^2*e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-210\sqrt{a}\,b^{3}c^{2}e\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+16\sqrt{bx+a}\left(\left(\frac{21\left(\frac{5fx}{7}+e\right)x^{2}d^{2}}{8}+\frac{35cx\left(\frac{3fx}{5}+e\right)d}{4}+\frac{105c^{2}\left(\frac{fx}{3}+e\right)}{8}\right)b^{3}+\frac{35\left(\frac{\left(\frac{3fx}{7}+e\right)x}{5}\right.}{}\right.}{105b^{3}}$
derivativedivides	$\frac{\frac{2d^{2}f(bx+a)^{\frac{7}{2}}}{7}-\frac{4a\,d^{2}f(bx+a)^{\frac{5}{2}}}{5}+\frac{4bcdf(bx+a)^{\frac{5}{2}}}{5}+\frac{2b\,d^{2}e(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}d^{2}f(bx+a)^{\frac{3}{2}}}{3}-\frac{4abcdf(bx+a)^{\frac{3}{2}}}{3}-\frac{2ab\,d^{2}e(bx+a)^{\frac{3}{2}}}{3}+2b}{b^{3}}$
default	$\frac{\frac{2d^{2}f(bx+a)^{\frac{7}{2}}}{7}-\frac{4a\,d^{2}f(bx+a)^{\frac{5}{2}}}{5}+\frac{4bcdf(bx+a)^{\frac{5}{2}}}{5}+\frac{2b\,d^{2}e(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}d^{2}f(bx+a)^{\frac{3}{2}}}{3}-\frac{4abcdf(bx+a)^{\frac{3}{2}}}{3}-\frac{2ab\,d^{2}e(bx+a)^{\frac{3}{2}}}{3}+2b}{b^{3}}$

input
```
int((b*x+a)^(1/2)*(d*x+c)^2*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
1/105*(-210*a^(1/2)*b^3*c^2*e*arctanh((b*x+a)^(1/2)/a^(1/2))+16*(b*x+a)^(1
/2)*((21/8*(5/7*f*x+e)*x^2*d^2+35/4*c*x*(3/5*f*x+e)*d+105/8*c^2*(1/3*f*x+e
))*b^3+35/8*(1/5*(3/7*f*x+e)*x*d^2+2*c*(1/5*f*x+e)*d+c^2*f)*a*b^2-7/2*a^2*
d*((1/7*f*x+1/2*e)*d+c*f)*b+a^3*d^2*f))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx$$
$$=\left[\frac{105\sqrt{a}b^{3}c^{2}e\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15b^{3}d^{2}fx^{3}+3\left(7b^{3}d^{2}e+\left(14b^{3}cd+ab^{2}d^{2}\right)f\right)x^{2}+7\left(15b^{3}c^{2}+\right.\right.}{}\right.$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^2*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/105*(105*sqrt(a)*b^3*c^2*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 + 2*(15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*b^3*c*d + a*b^2*d^2)*f)*x^2
+ 7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e + (35*a*b^2*c^2 - 28*a^2*b
*c*d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2)*e + (35*b^3*c^2 + 14*a*b
^2*c*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3, 2/105*(105*sqrt(-a)*b^3*c^
2*e*arctan(sqrt(-a)/sqrt(b*x + a)) + (15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e +
(14*b^3*c*d + a*b^2*d^2)*f)*x^2 + 7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d
^2)*e + (35*a*b^2*c^2 - 28*a^2*b*c*d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b
^2*d^2)*e + (35*b^3*c^2 + 14*a*b^2*c*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))
/b^3]
```

Sympy [A] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \begin{cases} \frac{2ac^2e\,\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2c^2e\sqrt{a+bx} + \frac{2d^2f(a+bx)^{\frac{7}{2}}}{7b^3} + \frac{2(a+bx)^{\frac{5}{2}}(-2ad^2f+2bcdf+bd^2e)}{5b^3} + \frac{2(a+bx)^{\frac{3}{2}}(a^2d^2f-2abcdf-abd^2e+}{3b^3} \\ \sqrt{a}\left(c^2e\log(x)+c^2fx+2cdex+\frac{d^2fx^3}{3}+\frac{x^2\cdot(2cdf+d^2e)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**2*(f*x+e)/x,x)
```

output
```
Piecewise((2*a*c**2*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**2*e*sqr
t(a + b*x) + 2*d**2*f*(a + b*x)**(7/2)/(7*b**3) + 2*(a + b*x)**(5/2)*(-2*a
*d**2*f + 2*b*c*d*f + b*d**2*e)/(5*b**3) + 2*(a + b*x)**(3/2)*(a**2*d**2*f
 - 2*a*b*c*d*f - a*b*d**2*e + b**2*c**2*f + 2*b**2*c*d*e)/(3*b**3), Ne(b,
0)), (sqrt(a)*(c**2*e*log(x) + c**2*f*x + 2*c*d*e*x + d**2*f*x**3/3 + x**2
*(2*c*d*f + d**2*e)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \sqrt{a}c^2e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2\left(105\sqrt{bx+a}b^3c^2e+15\,(bx+a)^{\frac{7}{2}}d^2f+21\,(bd^2e+2\,(bcd-ad^2)f)(bx+a)^{\frac{5}{2}}+35\,((2\,b^2cd-abd^2)e\right.}{105\,b^3}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^2*(f*x+e)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*c^2*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2
/105*(105*sqrt(b*x + a)*b^3*c^2*e + 15*(b*x + a)^(7/2)*d^2*f + 21*(b*d^2*e
 + 2*(b*c*d - a*d^2)*f)*(b*x + a)^(5/2) + 35*((2*b^2*c*d - a*b*d^2)*e + (b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*f)*(b*x + a)^(3/2))/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \frac{2\,ac^2e\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(105\sqrt{bx+a}b^{21}c^2e+70\,(bx+a)^{\frac{3}{2}}b^{20}cde+21\,(bx+a)^{\frac{5}{2}}b^{19}d^2e-35\,(bx+a)^{\frac{3}{2}}ab^{19}d^2e+35\,(bx+a)\right.}{}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^2*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a*c^2*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/105*(105*sqrt(b*x +
a)*b^21*c^2*e + 70*(b*x + a)^(3/2)*b^20*c*d*e + 21*(b*x + a)^(5/2)*b^19*d^
2*e - 35*(b*x + a)^(3/2)*a*b^19*d^2*e + 35*(b*x + a)^(3/2)*b^20*c^2*f + 42
*(b*x + a)^(5/2)*b^19*c*d*f - 70*(b*x + a)^(3/2)*a*b^19*c*d*f + 15*(b*x +
a)^(7/2)*b^18*d^2*f - 42*(b*x + a)^(5/2)*a*b^18*d^2*f + 35*(b*x + a)^(3/2)
*a^2*b^18*d^2*f)/b^21
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{5\,b^3}+\frac{2\,a\,d^2\,f}{5\,b^3}\right)\,{\left(a+b\,x\right)}^{5/2} + \left(a\,\left(a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{b^3}\right) - \frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(a\,f-b\,e\right)}{b^3}\right)\,\sqrt{a+b\,x} + \left(\frac{a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)}{3} - \frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{3\,b^3}\right)\,{\left(a+b\,x\right)}^{3/2} + \frac{2\,d^2\,f\,{\left(a+b\,x\right)}^{7/2}}{7\,b^3}+\sqrt{a}\,c^2\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^2)/x,x)
```

output
```
((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/(5*b^3) + (2*a*d^2*f)/(5*b^3))*(a + b
*x)^(5/2) + (a*(a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b^3 + (2*a*d^2*f)/b
^3) - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/b^3) - (2*(a*d - b*c)^2*
(a*f - b*e))/b^3)*(a + b*x)^(1/2) + ((a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*
f)/b^3 + (2*a*d^2*f)/b^3))/3 - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))
/(3*b^3))*(a + b*x)^(3/2) + a^(1/2)*c^2*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2
))*2i + (2*d^2*f*(a + b*x)^(7/2))/(7*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+bx}(c+dx)^2(e+fx)}{x}\,dx = \frac{16\sqrt{bx+a}\,a^3d^2f-56\sqrt{bx+a}\,a^2bcdf-28\sqrt{bx+a}\,a^2b\,d^2e-8\sqrt{bx+a}\,a^2b\,d^2fx+70\sqrt{bx+a}\,a\,b^2c^2f+\cdots}{}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^2*(f*x+e)/x,x)
```

output
```
(16*sqrt(a + b*x)*a**3*d**2*f - 56*sqrt(a + b*x)*a**2*b*c*d*f - 28*sqrt(a
+ b*x)*a**2*b*d**2*e - 8*sqrt(a + b*x)*a**2*b*d**2*f*x + 70*sqrt(a + b*x)*
a*b**2*c**2*f + 140*sqrt(a + b*x)*a*b**2*c*d*e + 28*sqrt(a + b*x)*a*b**2*c
*d*f*x + 14*sqrt(a + b*x)*a*b**2*d**2*e*x + 6*sqrt(a + b*x)*a*b**2*d**2*f*
x**2 + 210*sqrt(a + b*x)*b**3*c**2*e + 70*sqrt(a + b*x)*b**3*c**2*f*x + 14
0*sqrt(a + b*x)*b**3*c*d*e*x + 84*sqrt(a + b*x)*b**3*c*d*f*x**2 + 42*sqrt(
a + b*x)*b**3*d**2*e*x**2 + 30*sqrt(a + b*x)*b**3*d**2*f*x**3 + 105*sqrt(a
)*log(sqrt(a + b*x) - sqrt(a))*b**3*c**2*e - 105*sqrt(a)*log(sqrt(a + b*x)
 + sqrt(a))*b**3*c**2*e)/(105*b**3)
```

3.28 $\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x} dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x} dx = 2ce\sqrt{a+bx} + \frac{2(bde+bcf-adf)(a+bx)^{3/2}}{3b^2} + \frac{2df(a+bx)^{5/2}}{5b^2} - 2\sqrt{a}ce\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*c*e*(b*x+a)^(1/2)+2/3*(-a*d*f+b*c*f+b*d*e)*(b*x+a)^(3/2)/b^2+2/5*d*f*(b*
x+a)^(5/2)/b^2-2*a^(1/2)*c*e*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x} dx = \frac{2\sqrt{a+bx}(15b^2ce+5bde(a+bx)+5bcf(a+bx)-5adf(a+bx)+3df(a+bx)^2)}{15b^2} - 2\sqrt{a}ce\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

output
```
(2*Sqrt[a + b*x]*(15*b^2*c*e + 5*b*d*e*(a + b*x) + 5*b*c*f*(a + b*x) - 5*a
*d*f*(a + b*x) + 3*d*f*(a + b*x)^2))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$

$$\Big\downarrow \text{164}$$

$$ce\int \frac{\sqrt{a+bx}}{x}dx - \frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}$$

$$\Big\downarrow \text{60}$$

$$ce\left(a\int \frac{1}{x\sqrt{a+bx}}dx + 2\sqrt{a+bx}\right) - \frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}$$

$$\Big\downarrow \text{73}$$

$$ce\left(\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b} + 2\sqrt{a+bx}\right) - \frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}$$

$$\Big\downarrow \text{221}$$

$$ce\left(2\sqrt{a+bx} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right) - \frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}$$

input
```
Int[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

output
```
(-2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d*f*x))/(15*b^2) + c*
e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{-2\sqrt{a}\,b^{2}ce\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{4\sqrt{bx+a}\left(\frac{5\left(-x\left(\frac{3fx}{5}+e\right)d-3c\left(\frac{fx}{3}+e\right)\right)b^{2}}{2}-\frac{5a\left(\left(\frac{fx}{5}+e\right)d+cf\right)b}{2}+a^{2}df\right)}{15}}{b^{2}}$	86
derivativedivides	$\frac{\frac{2df(bx+a)^{\frac{5}{2}}}{5}-\frac{2adf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bcf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bde(bx+a)^{\frac{3}{2}}}{3}+2b^{2}ce\sqrt{bx+a}-2\sqrt{a}\,b^{2}ce\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^{2}}$	89
default	$\frac{\frac{2df(bx+a)^{\frac{5}{2}}}{5}-\frac{2adf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bcf(bx+a)^{\frac{3}{2}}}{3}+\frac{2bde(bx+a)^{\frac{3}{2}}}{3}+2b^{2}ce\sqrt{bx+a}-2\sqrt{a}\,b^{2}ce\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^{2}}$	89

input
```
int((b*x+a)^(1/2)*(d*x+c)*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/15*(-15*a^(1/2)*b^2*c*e*arctanh((b*x+a)^(1/2)/a^(1/2))-2*(b*x+a)^(1/2)*(
5/2*(-x*(3/5*f*x+e)*d-3*c*(1/3*f*x+e))*b^2-5/2*a*((1/5*f*x+e)*d+c*f)*b+a^2
*d*f))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$
$$=\left[\frac{15\sqrt{a}b^{2}ce\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3b^{2}dfx^{2}+5\left(3b^{2}c+abd\right)e+\left(5abc-2a^{2}d\right)f+\left(5b^{2}de+\left(5b^{2}c\right.\right.\right.}{15b^{2}}\right.$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*sqrt(a)*b^2*c*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (5*a*b*c - 2*a^2*d)*f + (5*b^2*d
*e + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/b^2, 2/15*(15*sqrt(-a)*b^2*c*e
*arctan(sqrt(-a)/sqrt(b*x + a)) + (3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e +
 (5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a
))/b^2]
```

Sympy [A] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$
$$=\begin{cases} \frac{2ace\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2ce\sqrt{a+bx} + \frac{2df(a+bx)^{\frac{5}{2}}}{5b^2} + \frac{2(a+bx)^{\frac{3}{2}}(-adf+bcf+bde)}{3b^2} & \text{for } b \neq 0 \\ \sqrt{a}\left(ce\log(x) + cfx + dex + \frac{dfx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)*(f*x+e)/x,x)
```

output
```
Piecewise((2*a*c*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c*e*sqrt(a +
b*x) + 2*d*f*(a + b*x)**(5/2)/(5*b**2) + 2*(a + b*x)**(3/2)*(-a*d*f + b*c*
f + b*d*e)/(3*b**2), Ne(b, 0)), (sqrt(a)*(c*e*log(x) + c*f*x + d*e*x + d*f
*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx$$
$$=\sqrt{a}ce\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)$$
$$+\frac{2\left(15\sqrt{bx+a}b^2ce + 3(bx+a)^{\frac{5}{2}}df + 5(bde + (bc-ad)f)(bx+a)^{\frac{3}{2}}\right)}{15\,b^2}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)*(f*x+e)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*c*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/1
5*(15*sqrt(b*x + a)*b^2*c*e + 3*(b*x + a)^(5/2)*d*f + 5*(b*d*e + (b*c - a*
d)*f)*(b*x + a)^(3/2))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx = \frac{2\,ace\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(15\,\sqrt{bx+a}b^{10}ce+5\,(bx+a)^{\frac{3}{2}}b^9de+5\,(bx+a)^{\frac{3}{2}}b^9cf+3\,(bx+a)^{\frac{5}{2}}b^8df-5\,(bx+a)^{\frac{3}{2}}ab^8df\right)}{15\,b^{10}}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a*c*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/15*(15*sqrt(b*x + a)*b
^10*c*e + 5*(b*x + a)^(3/2)*b^9*d*e + 5*(b*x + a)^(3/2)*b^9*c*f + 3*(b*x +
 a)^(5/2)*b^8*d*f - 5*(b*x + a)^(3/2)*a*b^8*d*f)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx = \left(a\left(\frac{2\,b\,c\,f-4\,a\,d\,f+2\,b\,d\,e}{b^2}+\frac{2\,a\,d\,f}{b^2}\right)+\frac{2\,(a\,d-b\,c)\,(a\,f-b\,e)}{b^2}\right)\sqrt{a+b\,x} + \left(\frac{2\,b\,c\,f-4\,a\,d\,f+2\,b\,d\,e}{3\,b^2}+\frac{2\,a\,d\,f}{3\,b^2}\right)(a+b\,x)^{3/2} + \frac{2\,d\,f\,(a+b\,x)^{5/2}}{5\,b^2} + \sqrt{a}\,c\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x))/x,x)
```

output
```
(a*((2*b*c*f - 4*a*d*f + 2*b*d*e)/b^2 + (2*a*d*f)/b^2) + (2*(a*d - b*c)*(a
*f - b*e))/b^2)*(a + b*x)^(1/2) + ((2*b*c*f - 4*a*d*f + 2*b*d*e)/(3*b^2) +
 (2*a*d*f)/(3*b^2))*(a + b*x)^(3/2) + (2*d*f*(a + b*x)^(5/2))/(5*b^2) + a^
(1/2)*c*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{a+bx}(c+dx)(e+fx)}{x}\,dx = \frac{-4\sqrt{bx+a}\,a^2df+10\sqrt{bx+a}\,abcf+10\sqrt{bx+a}\,abde+2\sqrt{bx+a}\,abdfx+30\sqrt{bx+a}\,b^2ce+10\sqrt{bx+}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)*(f*x+e)/x,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*d*f + 10*sqrt(a + b*x)*a*b*c*f + 10*sqrt(a + b*x)
*a*b*d*e + 2*sqrt(a + b*x)*a*b*d*f*x + 30*sqrt(a + b*x)*b**2*c*e + 10*sqrt
(a + b*x)*b**2*c*f*x + 10*sqrt(a + b*x)*b**2*d*e*x + 6*sqrt(a + b*x)*b**2*
d*f*x**2 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*c*e - 15*sqrt(a)*l
og(sqrt(a + b*x) + sqrt(a))*b**2*c*e)/(15*b**2)
```

3.29 $\int \frac{\sqrt{a+bx}(e+fx)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{\sqrt{a+bx}(e+fx)}{x} dx = 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} - 2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output
```
2*e*(b*x+a)^(1/2)+2/3*f*(b*x+a)^(3/2)/b-2*a^(1/2)*e*arctanh((b*x+a)^(1/2)/
a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx}(e+fx)}{x} dx = \frac{2\sqrt{a+bx}(3be+af+bfx)}{3b} - 2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

output
```
(2*Sqrt[a + b*x]*(3*b*e + a*f + b*f*x))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x} \, dx$$
$$\Big\downarrow \text{ 90}$$
$$e \int \frac{\sqrt{a+bx}}{x}dx + \frac{2f(a+bx)^{3/2}}{3b}$$
$$\Big\downarrow \text{ 60}$$
$$e \left(a \int \frac{1}{x\sqrt{a+bx}}dx + 2\sqrt{a+bx}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$
$$\Big\downarrow \text{ 73}$$
$$e \left(\frac{2a \int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b} + 2\sqrt{a+bx}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$
$$\Big\downarrow \text{ 221}$$
$$e \left(2\sqrt{a+bx} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

input
```
Int[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

output
```
(2*f*(a + b*x)^(3/2))/(3*b) + e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{2f(bx+a)^{\frac{3}{2}}}{3}+2be\sqrt{bx+a}-2\sqrt{a}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46
default	$\frac{\frac{2f(bx+a)^{\frac{3}{2}}}{3}+2be\sqrt{bx+a}-2\sqrt{a}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b}$	46
pseudoelliptic	$\frac{-6\sqrt{a}\,be\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2((fx+3e)b+af)\sqrt{bx+a}}{3b}$	48

input
```
int((b*x+a)^(1/2)*(f*x+e)/x,x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/3*f*(b*x+a)^(3/2)+b*e*(b*x+a)^(1/2)-a^(1/2)*b*e*arctanh((b*x+a)^(1/
2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \left[\frac{3\sqrt{a}be\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(bfx+3be+af)\sqrt{bx+a}}{3b}, \frac{2\left(3\sqrt{-a}be\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx+a}}\right)+(bfx+3\right.}{3b}\right.$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(a)*b*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(b*f*
x + 3*b*e + a*f)*sqrt(b*x + a))/b, 2/3*(3*sqrt(-a)*b*e*arctan(sqrt(-a)/sqr
t(b*x + a)) + (b*f*x + 3*b*e + a*f)*sqrt(b*x + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \begin{cases} \frac{2ae\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2e\sqrt{a+bx} + \frac{2f(a+bx)^{\frac{3}{2}}}{3b} & \text{for } b \neq 0 \\ \sqrt{a}(e\log(fx)+fx) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(f*x+e)/x,x)
```

output
```
Piecewise((2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*e*sqrt(a + b*x)
 + 2*f*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*(e*log(f*x) + f*x), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \sqrt{a}e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2\left(3\sqrt{bx+a}be+(bx+a)^{\frac{3}{2}}f\right)}{3\,b}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(
3*sqrt(b*x + a)*b*e + (b*x + a)^(3/2)*f)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\left(3\sqrt{bx+a}b^3e+(bx+a)^{\frac{3}{2}}b^2f\right)}{3\,b^3}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x,x, algorithm="giac")
```

output
```
2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(3*sqrt(b*x + a)*b^3*e
 + (b*x + a)^(3/2)*b^2*f)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = 2\,e\sqrt{a+b\,x} + \frac{2\,f\,(a+b\,x)^{3/2}}{3\,b} + \sqrt{a}\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2))/x,x)
```

output
```
2*e*(a + b*x)^(1/2) + a^(1/2)*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2
*f*(a + b*x)^(3/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx = \frac{2\sqrt{bx+a}\,af+6\sqrt{bx+a}\,be+2\sqrt{bx+a}\,bfx+3\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)be-3\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b}{3b}$$

input
```
int((b*x+a)^(1/2)*(f*x+e)/x,x)
```

output
```
(2*sqrt(a + b*x)*a*f + 6*sqrt(a + b*x)*b*e + 2*sqrt(a + b*x)*b*f*x + 3*sqr
t(a)*log(sqrt(a + b*x) - sqrt(a))*b*e - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt
(a))*b*e)/(3*b)
```

3.30 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx=\frac{2f\sqrt{a+bx}}{d}+\frac{2\sqrt{bc-ad}(de-cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

output
```
2*f*(b*x+a)^(1/2)/d+2*(-a*d+b*c)^(1/2)*(-c*f+d*e)*arctan(d^(1/2)*(b*x+a)^(
1/2)/(-a*d+b*c)^(1/2))/c/d^(3/2)-2*a^(1/2)*e*arctanh((b*x+a)^(1/2)/a^(1/2)
)/c
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx=\frac{2f\sqrt{a+bx}}{d}-\frac{2\sqrt{bc-ad}(-de+cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

input
```
Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

output
```
(2*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d]*
Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {171, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)} \, dx$$

$$\Big\downarrow \text{171}$$

$$\frac{2\int \frac{ade+(bde-bcf+adf)x}{2x\sqrt{a+bx}(c+dx)}dx}{d}+\frac{2f\sqrt{a+bx}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ade+(bde-bcf+adf)x}{x\sqrt{a+bx}(c+dx)}dx}{d}+\frac{2f\sqrt{a+bx}}{d}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{(bc-ad)(de-cf)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}+\frac{ade\int \frac{1}{x\sqrt{a+bx}}dx}{c}}{d}+\frac{2f\sqrt{a+bx}}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(bc-ad)(de-cf)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}+\frac{2ade\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}}{d}+\frac{2f\sqrt{a+bx}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2ade\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}+\frac{2\sqrt{bc-ad}(de-cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{d}+\frac{2f\sqrt{a+bx}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\sqrt{bc-ad}(de-cf)\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2\sqrt{a}de\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}}{d}+\frac{2f\sqrt{a+bx}}{d}$$

input
```
Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

output
```
(2*f*Sqrt[a + b*x])/d + ((2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*Sqrt[a]*d*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c)/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{2f\sqrt{bx+a}}{d}-\frac{2\left(acdf-a\,d^{2}e-b\,c^{2}f+bcde\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{dc\sqrt{(ad-bc)d}}-\frac{2\sqrt{a}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{c}$	103
default	$\frac{2f\sqrt{bx+a}}{d}-\frac{2\left(acdf-a\,d^{2}e-b\,c^{2}f+bcde\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{dc\sqrt{(ad-bc)d}}-\frac{2\sqrt{a}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{c}$	103
pseudoelliptic	$\frac{-2(cf-de)(ad-bc)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)+2\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)de+\sqrt{bx+a}\,cf\right)\sqrt{(ad-bc)d}}{dc\sqrt{(ad-bc)d}}$	105

input
```
int((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*f*(b*x+a)^(1/2)/d-2/d*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/((a*d-b*c)*d)^
(1/2)*arctanh(d*(b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2))-2*a^(1/2)*e*arctanh((b*
x+a)^(1/2)/a^(1/2))/c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.40

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a}de\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}cf-(de-cf)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{bdx-bc+2ad-2\sqrt{bx+a}d\sqrt{-\frac{bc-ad}{d}}}{dx+c}\right)}{cd}, -\right.$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c),x, algorithm="fricas")
```

output
```
[(sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x +
a)*c*f - (d*e - c*f)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqr
t(b*x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), (2*sqrt(-a)*d*e*arct
an(sqrt(-a)/sqrt(b*x + a)) + 2*sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt(-(b*c
- a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(b*x + a)*d*sqrt(-(b*c - a*d)/d
))/(d*x + c)))/(c*d), (sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*
a)/x) + 2*sqrt(b*x + a)*c*f - 2*(d*e - c*f)*sqrt((b*c - a*d)/d)*arctan(-sq
rt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d), 2*(sqrt(-a)*d*e*arc
tan(sqrt(-a)/sqrt(b*x + a)) + sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt((b*c -
a*d)/d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(88) = 176$.

Time = 12.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \begin{cases} \frac{2ae\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{c\sqrt{-a}} + \frac{2f\sqrt{a+bx}}{d} + \frac{2(ad-bc)(cf-de)\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-\frac{ad-bc}{d}}}\right)}{cd^2\sqrt{-\frac{ad-bc}{d}}} \\ \sqrt{a}\left(\left(-f+\frac{de}{2c}\right)\left(\frac{2c\left(\begin{cases}-\frac{\frac{1}{x}+\frac{d}{2c}}{d} & \text{for } c=0\\ \frac{\log\left(2c\left(\frac{1}{x}+\frac{d}{2c}\right)-d\right)}{2c} & \text{otherwise}\end{cases}\right)}{d} - \frac{2c\left(\begin{cases}\frac{\frac{1}{x}+\frac{d}{2c}}{d} & \text{for } c=0\\ \frac{\log\left(2c\left(\frac{1}{x}+\frac{d}{2c}\right)+d\right)}{2c} & \text{otherwise}\end{cases}\right)}{d}\right) - \frac{e\log\left(\frac{c}{x^2}\right.}{2c}\right. \end{cases}$$

input
```
integrate((b*x+a)**(1/2)*(f*x+e)/x/(d*x+c),x)
```

output
```
Piecewise((2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/(c*sqrt(-a)) + 2*f*sqrt(a +
b*x)/d + 2*(a*d - b*c)*(c*f - d*e)*atan(sqrt(a + b*x)/sqrt(-(a*d - b*c)/d)
)/(c*d**2*sqrt(-(a*d - b*c)/d)), Ne(b, 0)), (sqrt(a)*((-f + d*e/(2*c))*(2*
c*Piecewise((-(1/x + d/(2*c))/d, Eq(c, 0)), (log(2*c*(1/x + d/(2*c)) - d)/
(2*c), True))/d - 2*c*Piecewise(((1/x + d/(2*c))/d, Eq(c, 0)), (log(2*c*(1
/x + d/(2*c)) + d)/(2*c), True))/d) - e*log(c/x**2 + d/x)/(2*c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c} + \frac{2\,\sqrt{bx+a}f}{d} + \frac{2\left(bcde-ad^2e-bc^2f+acdf\right)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}cd}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c),x, algorithm="giac")
```

output
```
2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c) + 2*sqrt(b*x + a)*f/d +
2*(b*c*d*e - a*d^2*e - b*c^2*f + a*c*d*f)*arctan(sqrt(b*x + a)*d/sqrt(b*c*
d - a*d^2))/(sqrt(b*c*d - a*d^2)*c*d)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 2355, normalized size of antiderivative = 23.32

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)),x)
```

output
```
(2*f*(a + b*x)^(1/2))/d - (a^(1/2)*e*atan(((a^(1/2)*e*((8*(a + b*x)^(1/2)*
(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2
*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2
*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2
*c^2*d^3*f))/d + (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1
/2))/(c*d)))/c)*1i)/c + (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^
2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 -
2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*
d^3*e*f))/d - (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8
*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c)*1i)
/c)/((16*(a^2*b^3*d^3*e^3 - a*b^4*c*d^2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^
3*e^2*f - 3*a^2*b^3*c*d^2*e^2*f + 2*a^2*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*
f^2 + 2*a*b^4*c^2*d*e^2*f))/d - (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^
2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^
2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^
2*b^2*c*d^3*e*f))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f)
)/d + (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d))
)/c))/c + (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2
+ b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*
e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$$
$$= \frac{-2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)cf+2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)de+2\sqrt{bx+a}\,cdf+\sqrt{a}\,\mathrm{lo}}{c\,d^2}$$

input
```
int((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c),x)
```

output
```
( - 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a
*d + b*c)))*c*f + 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqr
t(d)*sqrt( - a*d + b*c)))*d*e + 2*sqrt(a + b*x)*c*d*f + sqrt(a)*log(sqrt(a
 + b*x) - sqrt(a))*d**2*e - sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*d**2*e)/(
c*d**2)
```

3.31 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx = \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{(2ad^2e-bc(de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

output
```
(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)-(2*a*d^2*e-b*c*(c*f+d*e))*arctan(d^(1
/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(3/2)/(-a*d+b*c)^(1/2)-2*a^(1/2)
*e*arctanh((b*x+a)^(1/2)/a^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx = \frac{\frac{c(de-cf)\sqrt{a+bx}}{d(c+dx)} + \frac{(-2ad^2e+bc(de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} - 2\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

input
```
Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

output
```
((c*(d*e - c*f)*Sqrt[a + b*x])/(d*(c + d*x)) + ((-2*a*d^2*e + b*c*(d*e + c
*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c -
a*d]) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {166, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} \, dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)} - \frac{\int -\frac{2ade+b(de+cf)x}{2x\sqrt{a+bx}(c+dx)}dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2ade+b(de+cf)x}{x\sqrt{a+bx}(c+dx)}dx}{2cd} + \frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{2ade\int \frac{1}{x\sqrt{a+bx}}dx}{c} - \frac{(2ad^2e-bc(cf+de))\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{2cd} + \frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{4ade\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc} - \frac{2(2ad^2e-bc(cf+de))\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{2cd} + \frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{4ade\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)(2ad^2e-bc(cf+de))}{c\sqrt{d}\sqrt{bc-ad}}}{2cd}+\frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)(2ad^2e-bc(cf+de))}{c\sqrt{d}\sqrt{bc-ad}}-\frac{4\sqrt{a}de\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}}{2cd}+\frac{\sqrt{a+bx}(de-cf)}{cd(c+dx)}$$

input
```
Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

output
```
((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) + ((-2*(2*a*d^2*e - b*c*(d*e +
 c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]*Sqrt[b*
c - a*d]) - (4*Sqrt[a]*d*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c)/(2*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{-2\sqrt{a}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{-\frac{c(cf-de)\sqrt{bx+a}}{xd+c}+\frac{\left(2a\,d^{2}e-b\,c^{2}f-bcde\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{\sqrt{(ad-bc)d}}}{d}}{c^{2}}$	110
derivativedivides	$2b\left(\frac{\frac{bc(cf-de)\sqrt{bx+a}}{2d(-d(bx+a)+ad-bc)}+\frac{\left(2a\,d^{2}e-b\,c^{2}f-bcde\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{2d\sqrt{(ad-bc)d}}}{b\,c^{2}}-\frac{e\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^{2}}\right)$	137
default	$2b\left(\frac{\frac{bc(cf-de)\sqrt{bx+a}}{2d(-d(bx+a)+ad-bc)}+\frac{\left(2a\,d^{2}e-b\,c^{2}f-bcde\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)}{2d\sqrt{(ad-bc)d}}}{b\,c^{2}}-\frac{e\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^{2}}\right)$	137

input
```
int((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-2*a^(1/2)*e*arctanh((b*x+a)^(1/2)/a^(1/2))+1/d*(-c*(c*f-d*e)*(b*x+
a)^(1/2)/(d*x+c)+(2*a*d^2*e-b*c^2*f-b*c*d*e)/((a*d-b*c)*d)^(1/2)*arctanh(d
*(b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(110) = 220$.

Time = 0.16 (sec) , antiderivative size = 1002, normalized size of antiderivative = 7.83

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*((b*c^3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^
3)*e)*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a
*d^2)*sqrt(b*x + a))/(d*x + c)) - 2*((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 -
a*c*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*((b*c
^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x + a))/(b*c^4*d^2 -
 a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), 1/2*(4*((b*c*d^3 - a*d^4)*e*x + (
b*c^2*d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x + a)) - (b*c^3*f
 + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(-
b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x
+ a))/(d*x + c)) + 2*((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*s
qrt(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^
3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqr
t(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) -
 ((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(a)*log((b*x - 2*sq
rt(b*x + a)*sqrt(a) + 2*a)/x) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^
2*d^2)*f)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*
x), -((b*c^3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3
)*e)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*
x + a*d)) - 2*((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(-a)*a
rctan(sqrt(-a)/sqrt(b*x + a)) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(1/2)*(f*x+e)/x/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^2} + \frac{(bcde-2\,ad^2e+bc^2f)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}c^2d} + \frac{\sqrt{bx+a}bde-\sqrt{bx+a}bcf}{(bc+(bx+a)d-ad)cd}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^2,x, algorithm="giac")
```

output
```
2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c^2) + (b*c*d*e - 2*a*d^2*e
 + b*c^2*f)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^
2)*c^2*d) + (sqrt(b*x + a)*b*d*e - sqrt(b*x + a)*b*c*f)/((b*c + (b*x + a)*
d - a*d)*c*d)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 1814, normalized size of antiderivative = 14.17

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^2),x)
```

output
```
(atan((((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) + ((4*b^3*c
^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f
 - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c)
)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) + (2*
(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4
*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d
- b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^
3)) - (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) - ((4*b^3*c^
5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f
- 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))
^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) - (2*(
a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*
c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d -
 b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3
)))/((4*(a*b^4*c*d^2*e^3 - 2*a^2*b^3*d^3*e^3 + a*b^4*c^3*e*f^2 - 2*a^2*b^3
*c*d^2*e^2*f + 2*a*b^4*c^2*d*e^2*f))/(c^3*d) + (((((2*(2*a*b^3*c^4*d^3*e -
 2*a*b^3*c^5*d^2*f))/(c^3*d) + ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*
d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a
*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c
*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) + (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.06

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2}\,dx$$
$$= \frac{2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)ac\,d^2e+2\sqrt{d}\,\sqrt{-ad+bc}\,atan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{d}\,\sqrt{-ad+bc}}\right)a\,d^3ex-\sqrt{d}\,\sqrt{-ad+bc}\,c}{}$$

input
```
int((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^2,x)
```

output
```
(2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
+ b*c)))*a*c*d**2*e + 2*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/
(sqrt(d)*sqrt( - a*d + b*c)))*a*d**3*e*x - sqrt(d)*sqrt( - a*d + b*c)*atan
((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c**3*f - sqrt(d)*sqrt(
- a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c**2*d
*e - sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*
d + b*c)))*b*c**2*d*f*x - sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d
)/(sqrt(d)*sqrt( - a*d + b*c)))*b*c*d**2*e*x - sqrt(a + b*x)*a*c**2*d**2*f
 + sqrt(a + b*x)*a*c*d**3*e + sqrt(a + b*x)*b*c**3*d*f - sqrt(a + b*x)*b*c
**2*d**2*e + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*c*d**3*e + sqrt(a)*log
(sqrt(a + b*x) - sqrt(a))*a*d**4*e*x - sqrt(a)*log(sqrt(a + b*x) - sqrt(a)
)*b*c**2*d**2*e - sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c*d**3*e*x - sqrt
(a)*log(sqrt(a + b*x) + sqrt(a))*a*c*d**3*e - sqrt(a)*log(sqrt(a + b*x) +
sqrt(a))*a*d**4*e*x + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*c**2*d**2*e +
 sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*c*d**3*e*x)/(c**2*d**2*(a*c*d + a*
d**2*x - b*c**2 - b*c*d*x))
```

3.32 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 205

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{(4ad^2e-bc(3de+cf))\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{(12abcd^2e-8a^2d^3e-b^2c^2(3de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}} - \frac{2\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}$$

output
```
1/2*(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)^2-1/4*(4*a*d^2*e-b*c*(c*f+3*d*e))
*(b*x+a)^(1/2)/c^2/d/(-a*d+b*c)/(d*x+c)-1/4*(12*a*b*c*d^2*e-8*a^2*d^3*e-b^
2*c^2*(c*f+3*d*e))*arctan(d^(1/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3
/2)/(-a*d+b*c)^(3/2)-2*a^(1/2)*e*arctanh((b*x+a)^(1/2)/a^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx$$
$$= \frac{\frac{c\sqrt{a+bx}(2ad(3cde-c^2f+2d^2ex)+bc(c^2f-3d^2ex-cd(5e+fx)))}{d(-bc+ad)(c+dx)^2}+\frac{(-12abcd^2e+8a^2d^3e+b^2c^2(3de+cf))\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}(bc-ad)^{3/2}}-8\sqrt{a}e\mathrm{arc}}{4c^3}$$

input
```
Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

output
```
((c*Sqrt[a + b*x]*(2*a*d*(3*c*d*e - c^2*f + 2*d^2*e*x) + b*c*(c^2*f - 3*d^
2*e*x - c*d*(5*e + f*x))))/(d*(-(b*c) + a*d)*(c + d*x)^2) + ((-12*a*b*c*d^
2*e + 8*a^2*d^3*e + b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/
Sqrt[b*c - a*d]])/(d^(3/2)*(b*c - a*d)^(3/2)) - 8*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {166, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx$$
$$\Big\downarrow \text{166}$$
$$\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}-\frac{\int -\frac{4ade+b(3de+cf)x}{2x\sqrt{a+bx}(c+dx)^2}dx}{2cd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{4ade+b(3de+cf)x}{x\sqrt{a+bx}(c+dx)^2}dx}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

$$\Big\downarrow \ 168$$

$$-\frac{\frac{\int -\frac{8ad(bc-ad)e-b\left(4ad^2e-bc(3de+cf)\right)x}{2x\sqrt{a+bx}(c+dx)}dx}{c(bc-ad)}-\frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{c(c+dx)(bc-ad)}}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{8ad(bc-ad)e-b\left(4ad^2e-bc(3de+cf)\right)x}{x\sqrt{a+bx}(c+dx)}dx}{2c(bc-ad)}-\frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{c(c+dx)(bc-ad)}}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{\frac{8ade(bc-ad)\int \frac{1}{x\sqrt{a+bx}}dx}{c}-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\int \frac{1}{\sqrt{a+bx}(c+dx)}dx}{c}}{2c(bc-ad)}-\frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{c(c+dx)(bc-ad)}}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{16ade(bc-ad)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d(a+bx)}{b}}d\sqrt{a+bx}}{bc}}{2c(bc-ad)}-\frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{c(c+dx)(bc-ad)}}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{16ade(bc-ad)\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{bc}-\frac{2\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)}{c\sqrt{d}\sqrt{bc-ad}}}{2c(bc-ad)}-\frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{c(c+dx)(bc-ad)}}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{-\frac{2\arctan\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)}{c\sqrt{d}\sqrt{bc-ad}}-\frac{16\sqrt{a}de\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)(bc-ad)}{c}}{2c(bc-ad)}-\frac{\sqrt{a+bx}(4ad^2e-bc(cf+3de))}{c(c+dx)(bc-ad)}}{4cd}+\frac{\sqrt{a+bx}(de-cf)}{2cd(c+dx)^2}$$

input
```
Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

output
```
((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) + (-(((4*a*d^2*e - b*c*(3*
d*e + c*f))*Sqrt[a + b*x])/(c*(b*c - a*d)*(c + d*x))) + ((-2*(12*a*b*c*d^2
*e - 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/S
qrt[b*c - a*d]])/(c*Sqrt[d]*Sqrt[b*c - a*d]) - (16*Sqrt[a]*d*(b*c - a*d)*e
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c)/(2*c*(b*c - a*d)))/(4*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{2\left(-(xd+c)^2\left(a^2d^3e-\frac{3}{2}abcd^2e+\frac{1}{8}b^2c^3f+\frac{3}{8}b^2c^2de\right)\operatorname{arctanh}\left(\frac{d\sqrt{bx+a}}{\sqrt{(ad-bc)d}}\right)+\left((xd+c)^2\left(a^{\frac{3}{2}}d-bc\sqrt{a}\right)de\ \operatorname{arctanh}\left(\frac{\surd}{}\right.\right.\right.}{\sqrt{(ad-bc)d}\,(ad-bc)d(xd+c)^2}$
derivativedivides	$2b^2\left(-\frac{e\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2c^3}+\frac{\frac{\frac{bc\left(4a\,d^2e-b\,c^2f-3bcde\right)(bx+a)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(4a\,d^2e+b\,c^2f-5bcde\right)bc\sqrt{bx+a}}{8d}}{(-d(bx+a)+ad-bc)^2}+\frac{\left(8a^2d^3e-12abc\right.}{}}{c^3b^2}\right.$
default	$2b^2\left(-\frac{e\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2c^3}+\frac{\frac{\frac{bc\left(4a\,d^2e-b\,c^2f-3bcde\right)(bx+a)^{\frac{3}{2}}}{8ad-8bc}-\frac{\left(4a\,d^2e+b\,c^2f-5bcde\right)bc\sqrt{bx+a}}{8d}}{(-d(bx+a)+ad-bc)^2}+\frac{\left(8a^2d^3e-12abc\right.}{}}{c^3b^2}\right.$

input
```
int((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/((a*d-b*c)*d)^(1/2)*(-(d*x+c)^2*(a^2*d^3*e-3/2*a*b*c*d^2*e+1/8*b^2*c^3*
f+3/8*b^2*c^2*d*e)*arctanh(d*(b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2))+((d*x+c)^2
*(a^(3/2)*d-b*c*a^(1/2))*d*e*arctanh((b*x+a)^(1/2)/a^(1/2))+1/4*c*(-2*a*d^
3*e*x-3*(-1/2*b*x+a)*c*e*d^2+(1/2*(f*x+5*e)*b+a*f)*c^2*d-1/2*b*c^3*f)*(b*x
+a)^(1/2))*((a*d-b*c)*d)^(1/2))/(a*d-b*c)/d/(d*x+c)^2/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(179) = 358$.

Time = 0.52 (sec) , antiderivative size = 2205, normalized size of antiderivative = 10.76

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^3,x, algorithm="fricas")
```

output

```
[1/8*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*
d^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^
4*d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d +
 a*d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(
d*x + c)) + 8*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^
3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^
2*d^4)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*((5*b^2
*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2
+ 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b
^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^
3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2
*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), 1/8*(16*((b^2*c^2*d^4 - 2*a*b*
c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x +
 (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(-a)/s
qrt(b*x + a)) + (b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^
4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e +
 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqr
t(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b
*x + a))/(d*x + c)) + 2*((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*
e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(1/2)*(f*x+e)/x/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \frac{\left(3\,b^2c^2de-12\,abcd^2e+8\,a^2d^3e+b^2c^3f\right)\arctan\left(\frac{\sqrt{bx+a}d}{\sqrt{bcd-ad^2}}\right)}{4\,(bc^4d-ac^3d^2)\sqrt{bcd-ad^2}} + \frac{2\,ae\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^3} + \frac{5\,\sqrt{bx+a}b^3c^2de+3\,(bx+a)^{\frac{3}{2}}b^2cd^2e-9\,\sqrt{bx+a}ab^2cd^2e-4\,(bx+a)^{\frac{3}{2}}abd^3e+4\,\sqrt{bx+a}a^2bd^3e-\sqrt{}}{4\,(bc^3d-ac^2d^2)(bc+(bx+a)d-ad)^2}$$

input
```
integrate((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(3*b^2*c^2*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e + b^2*c^3*f)*arctan(sqrt
(b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^4*d - a*c^3*d^2)*sqrt(b*c*d - a*d^2
)) + 2*a*e*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*c^3) + 1/4*(5*sqrt(b*x
 + a)*b^3*c^2*d*e + 3*(b*x + a)^(3/2)*b^2*c*d^2*e - 9*sqrt(b*x + a)*a*b^2*
c*d^2*e - 4*(b*x + a)^(3/2)*a*b*d^3*e + 4*sqrt(b*x + a)*a^2*b*d^3*e - sqrt
(b*x + a)*b^3*c^3*f + (b*x + a)^(3/2)*b^2*c^2*d*f + sqrt(b*x + a)*a*b^2*c^
2*d*f)/((b*c^3*d - a*c^2*d^2)*(b*c + (b*x + a)*d - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 4839, normalized size of antiderivative = 23.60

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^3),x)
```

output
```
(atan((((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4
*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e
^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f
 - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) -
((d^3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b
^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c
^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^
2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 25
6*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6
*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5
*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*
b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*
d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(
a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) + ((d^3*(a
*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 +
9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c
^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4
*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*(a*d - b*
c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e +
4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1311, normalized size of antiderivative = 6.40

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(f*x+e)/x/(d*x+c)^3,x)
```

output
```
(8*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d
+ b*c)))*a**2*c**2*d**3*e + 16*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b
*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**2*c*d**4*e*x + 8*sqrt(d)*sqrt( - a
*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a**2*d**5*e
*x**2 - 12*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt
( - a*d + b*c)))*a*b*c**3*d**2*e - 24*sqrt(d)*sqrt( - a*d + b*c)*atan((sqr
t(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a*b*c**2*d**3*e*x - 12*sqrt(d)
*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*a
*b*c*d**4*e*x**2 + sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt
(d)*sqrt( - a*d + b*c)))*b**2*c**5*f + 3*sqrt(d)*sqrt( - a*d + b*c)*atan((
sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**2*c**4*d*e + 2*sqrt(d)*s
qrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**
2*c**4*d*f*x + 6*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d
)*sqrt( - a*d + b*c)))*b**2*c**3*d**2*e*x + sqrt(d)*sqrt( - a*d + b*c)*ata
n((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d + b*c)))*b**2*c**3*d**2*f*x**2 +
3*sqrt(d)*sqrt( - a*d + b*c)*atan((sqrt(a + b*x)*d)/(sqrt(d)*sqrt( - a*d +
 b*c)))*b**2*c**2*d**3*e*x**2 - 2*sqrt(a + b*x)*a**2*c**3*d**3*f + 6*sqrt(
a + b*x)*a**2*c**2*d**4*e + 4*sqrt(a + b*x)*a**2*c*d**5*e*x + 3*sqrt(a + b
*x)*a*b*c**4*d**2*f - 11*sqrt(a + b*x)*a*b*c**3*d**3*e - sqrt(a + b*x)*a*b
*c**3*d**3*f*x - 7*sqrt(a + b*x)*a*b*c**2*d**4*e*x - sqrt(a + b*x)*b**2...
```

3.33 $\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 427

$$
\begin{aligned}
\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx &= -\frac{2(bc-ad)^3(dg-ch)\sqrt{e+fx}}{d^5} \\
&+ \frac{2(a^3d^3f^3h+3a^2bd^2f^2(dfg-deh-cfh)+3ab^2df(c^2f^2h-d^2e(fg-eh)-cdf(fg-eh))-b^3(c^3f^3h}{3d^4f^4} \\
&+ \frac{2b(3a^2d^2f^2h+3abdf(dfg-2deh-cfh)+b^2(c^2f^2h-d^2e(2fg-3eh)-cdf(fg-2eh)))\,(e+fx)^{5/2}}{5d^3f^4} \\
&+ \frac{2b^2(3adfh+b(dfg-3deh-cfh))(e+fx)^{7/2}}{7d^2f^4} + \frac{2b^3h(e+fx)^{9/2}}{9df^4} \\
&+ \frac{2(bc-ad)^3\sqrt{de-cf}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{11/2}}
\end{aligned}
$$

output

```
-2*(-a*d+b*c)^3*(-c*h+d*g)*(f*x+e)^(1/2)/d^5+2/3*(a^3*d^3*f^3*h+3*a^2*b*d^
2*f^2*(-c*f*h-d*e*h+d*f*g)+3*a*b^2*d*f*(c^2*f^2*h-d^2*e*(-e*h+f*g)-c*d*f*(
-e*h+f*g))-b^3*(c^3*f^3*h-d^3*e^2*(-e*h+f*g)-c*d^2*e*f*(-e*h+f*g)-c^2*d*f^
2*(-e*h+f*g)))*(f*x+e)^(3/2)/d^4/f^4+2/5*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(-c*
f*h-2*d*e*h+d*f*g)+b^2*(c^2*f^2*h-d^2*e*(-3*e*h+2*f*g)-c*d*f*(-2*e*h+f*g))
)*(f*x+e)^(5/2)/d^3/f^4+2/7*b^2*(3*a*d*f*h+b*(-c*f*h-3*d*e*h+d*f*g))*(f*x+
e)^(7/2)/d^2/f^4+2/9*b^3*h*(f*x+e)^(9/2)/d/f^4+2*(-a*d+b*c)^3*(-c*f+d*e)^(
1/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(11/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= \frac{2\sqrt{e+fx}(105a^3d^3f^3(-3cfh+d(3fg+eh+fhx))+63a^2bd^2f^2(15c^2f^2h-d^2(e+fx)(-5fg+2eh-}{}$$
$$-\frac{2(-bc+ad)^3\sqrt{-de+cf}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{11/2}}$$

input
```
Integrate[((a + b*x)^3*Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(105*a^3*d^3*f^3*(-3*c*f*h + d*(3*f*g + e*h + f*h*x)) + 6
3*a^2*b*d^2*f^2*(15*c^2*f^2*h - d^2*(e + f*x)*(-5*f*g + 2*e*h - 3*f*h*x) -
 5*c*d*f*(3*f*g + e*h + f*h*x)) + 9*a*b^2*d*f*(-105*c^3*f^3*h + 35*c^2*d*f
^2*(3*f*g + e*h + f*h*x) - 7*c*d^2*f*(e + f*x)*(5*f*g - 2*e*h + 3*f*h*x) +
 d^3*(e + f*x)*(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x))) +
b^3*(315*c^4*f^4*h - 105*c^3*d*f^3*(3*f*g + e*h + f*h*x) + 21*c^2*d^2*f^2*
(e + f*x)*(5*f*g - 2*e*h + 3*f*h*x) - 3*c*d^3*f*(e + f*x)*(8*e^2*h + 3*f^2
*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) - d^4*(e + f*x)*(16*e^3*h - 24*e^2
*f*(g + h*x) + 6*e*f^2*x*(6*g + 5*h*x) - 5*f^3*x^2*(9*g + 7*h*x)))))/(315*
d^5*f^4) - (2*(-(b*c) + a*d)^3*Sqrt[-(d*e) + c*f]*(d*g - c*h)*ArcTan[(Sqrt
[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(11/2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$\Big\downarrow \; 170$$
$$\frac{2\int \frac{3(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{2(c+dx)}dx}{9df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 27$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}-\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(3adfg-2bceh-acfh+(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2\sqrt{e+fx}(2bceh-af(3dg-ch)-(2adfh+b(3dfg-2deh-3cfh))x)}{c+dx}dx}{3df}+\frac{2h(a+bx)^3(e+fx)^{3/2}}{9df}$$

input
```
Int[((a + b*x)^3*Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$-\frac{2\left(-f^4(ad-bc)^3(ch-dg)(cf-de)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\left(\left(\left(-\frac{x^3\left(\frac{7hx}{9}+g\right)b^3}{7}-\frac{3a\,x^2\left(\frac{5hx}{7}+g\right)b^2}{5}-a^2x\left(\frac{3hx}{5}+g\right)b-\right.\right.\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/((c*f-d*e)*d)^(1/2)*(-f^4*(a*d-b*c)^3*(c*h-d*g)*(c*f-d*e)*arctan(d*(f*x
+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(((-1/7*x^3*(7/9*h*x+g)*b^3-3/5*a*x^2*(5/7*
h*x+g)*b^2-a^2*x*(3/5*h*x+g)*b-a^3*(1/3*h*x+g))*d^4+c*(1/5*x^2*(5/7*h*x+g)
*b^3+a*x*(3/5*h*x+g)*b^2+3*a^2*(1/3*h*x+g)*b+h*a^3)*d^3-3*c^2*(1/9*x*(3/5*
h*x+g)*b^2+a*(1/3*h*x+g)*b+a^2*h)*b*d^2+3*(1/3*(1/3*h*x+g)*b+a*h)*c^3*b^2*
d-c^4*h*b^3)*f^4-1/3*((3/35*x^2*(5/9*h*x+g)*b^3+3/5*a*(3/7*h*x+g)*x*b^2+3*
a^2*(1/5*h*x+g)*b+h*a^3)*d^3-3*c*b*(1/15*(3/7*h*x+g)*x*b^2+a*(1/5*h*x+g)*b
+a^2*h)*d^2+3*c^2*b^2*(1/3*(1/5*h*x+g)*b+a*h)*d-b^3*c^3*h)*d*e*f^3+2/5*d^2
*b*((2/21*x*(1/2*h*x+g)*b^2+a*(2/7*h*x+g)*b+a^2*h)*d^2-c*(1/3*(2/7*h*x+g)*
b+a*h)*b*d+1/3*b^2*c^2*h)*e^2*f^2-8/35*d^3*((1/3*(1/3*h*x+g)*b+a*h)*d-1/3*
b*c*h)*b^2*e^3*f+16/315*b^3*d^4*e^4*h)*(f*x+e)^(1/2)*((c*f-d*e)*d)^(1/2))/
f^4/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(399) = 798$.

Time = 0.10 (sec) , antiderivative size = 1697, normalized size of antiderivative = 3.97

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/315*(315*((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*f^4*g
 - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*f^4*h)*sqrt((d*
e - c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/
d))/(d*x + c)) + 2*(35*b^3*d^4*f^4*h*x^4 + 5*(9*b^3*d^4*f^4*g + (b^3*d^4*e
*f^3 - 9*(b^3*c*d^3 - 3*a*b^2*d^4)*f^4)*h)*x^3 + 3*(3*(b^3*d^4*e*f^3 - 7*(
b^3*c*d^3 - 3*a*b^2*d^4)*f^4)*g - (2*b^3*d^4*e^2*f^2 + 3*(b^3*c*d^3 - 3*a*
b^2*d^4)*e*f^3 - 21*(b^3*c^2*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*f^4)*h)*x^
2 + 3*(8*b^3*d^4*e^3*f + 14*(b^3*c*d^3 - 3*a*b^2*d^4)*e^2*f^2 + 35*(b^3*c^
2*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*e*f^3 - 105*(b^3*c^3*d - 3*a*b^2*c^2*
d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*f^4)*g - (16*b^3*d^4*e^4 + 24*(b^3*c*d^3 -
3*a*b^2*d^4)*e^3*f + 42*(b^3*c^2*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*e^2*f^
2 + 105*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*e*f^3 - 31
5*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*f^4)*h - (3*(4*b
^3*d^4*e^2*f^2 + 7*(b^3*c*d^3 - 3*a*b^2*d^4)*e*f^3 - 35*(b^3*c^2*d^2 - 3*a
*b^2*c*d^3 + 3*a^2*b*d^4)*f^4)*g - (8*b^3*d^4*e^3*f + 12*(b^3*c*d^3 - 3*a*
b^2*d^4)*e^2*f^2 + 21*(b^3*c^2*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*e*f^3 -
105*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*f^4)*h)*x)*sqr
t(f*x + e))/(d^5*f^4), 2/315*(315*((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*
c*d^3 - a^3*d^4)*f^4*g - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*
c*d^3)*f^4*h)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(d*e -...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(418) = 836$.

Time = 23.46 (sec) , antiderivative size = 899, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(f*x+e)**(1/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(b**3*h*(e + f*x)**(9/2)/(9*d*f**3) + (e + f*x)**(7/2)*(3*a*b
**2*d*f*h - b**3*c*f*h - 3*b**3*d*e*h + b**3*d*f*g)/(7*d**2*f**3) + (e + f
*x)**(5/2)*(3*a**2*b*d**2*f**2*h - 3*a*b**2*c*d*f**2*h - 6*a*b**2*d**2*e*f
*h + 3*a*b**2*d**2*f**2*g + b**3*c**2*f**2*h + 2*b**3*c*d*e*f*h - b**3*c*d
*f**2*g + 3*b**3*d**2*e**2*h - 2*b**3*d**2*e*f*g)/(5*d**3*f**3) + (e + f*x
)**(3/2)*(a**3*d**3*f**3*h - 3*a**2*b*c*d**2*f**3*h - 3*a**2*b*d**3*e*f**2
*h + 3*a**2*b*d**3*f**3*g + 3*a*b**2*c**2*d*f**3*h + 3*a*b**2*c*d**2*e*f**
2*h - 3*a*b**2*c*d**2*f**3*g + 3*a*b**2*d**3*e**2*f*h - 3*a*b**2*d**3*e*f*
*2*g - b**3*c**3*f**3*h - b**3*c**2*d*e*f**2*h + b**3*c**2*d*f**3*g - b**3
*c*d**2*e**2*f*h + b**3*c*d**2*e*f**2*g - b**3*d**3*e**3*h + b**3*d**3*e**
2*f*g)/(3*d**4*f**3) + sqrt(e + f*x)*(-a**3*c*d**3*f*h + a**3*d**4*f*g + 3
*a**2*b*c**2*d**2*f*h - 3*a**2*b*c*d**3*f*g - 3*a*b**2*c**3*d*f*h + 3*a*b*
*2*c**2*d**2*f*g + b**3*c**4*f*h - b**3*c**3*d*f*g)/d**5 + f*(a*d - b*c)**
3*(c*f - d*e)*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**6*sq
rt((c*f - d*e)/d)))/f, Ne(f, 0)), (sqrt(e)*(b**3*h*x**4/(4*d) + x**3*(3*a*
b**2*d*h - b**3*c*h + b**3*d*g)/(3*d**2) + x**2*(3*a**2*b*d**2*h - 3*a*b**
2*c*d*h + 3*a*b**2*d**2*g + b**3*c**2*h - b**3*c*d*g)/(2*d**3) + x*(a**3*d
**3*h - 3*a**2*b*c*d**2*h + 3*a**2*b*d**3*g + 3*a*b**2*c**2*d*h - 3*a*b**2
*c*d**2*g - b**3*c**3*h + b**3*c**2*d*g)/d**4 - (a*d - b*c)**3*(c*h - d*g)
*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(399) = 798$.

Time = 0.15 (sec) , antiderivative size = 1050, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output

```
-2*(b^3*c^3*d^2*e*g - 3*a*b^2*c^2*d^3*e*g + 3*a^2*b*c*d^4*e*g - a^3*d^5*e*
g - b^3*c^4*d*f*g + 3*a*b^2*c^3*d^2*f*g - 3*a^2*b*c^2*d^3*f*g + a^3*c*d^4*
f*g - b^3*c^4*d*e*h + 3*a*b^2*c^3*d^2*e*h - 3*a^2*b*c^2*d^3*e*h + a^3*c*d^
4*e*h + b^3*c^5*f*h - 3*a*b^2*c^4*d*f*h + 3*a^2*b*c^3*d^2*f*h - a^3*c^2*d^
3*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*
d^5) + 2/315*(45*(f*x + e)^(7/2)*b^3*d^8*f^33*g - 126*(f*x + e)^(5/2)*b^3*
d^8*e*f^33*g + 105*(f*x + e)^(3/2)*b^3*d^8*e^2*f^33*g - 63*(f*x + e)^(5/2)
*b^3*c*d^7*f^34*g + 189*(f*x + e)^(5/2)*a*b^2*d^8*f^34*g + 105*(f*x + e)^(
3/2)*b^3*c*d^7*e*f^34*g - 315*(f*x + e)^(3/2)*a*b^2*d^8*e*f^34*g + 105*(f*
x + e)^(3/2)*b^3*c^2*d^6*f^35*g - 315*(f*x + e)^(3/2)*a*b^2*c*d^7*f^35*g +
 315*(f*x + e)^(3/2)*a^2*b*d^8*f^35*g - 315*sqrt(f*x + e)*b^3*c^3*d^5*f^36
*g + 945*sqrt(f*x + e)*a*b^2*c^2*d^6*f^36*g - 945*sqrt(f*x + e)*a^2*b*c*d^
7*f^36*g + 315*sqrt(f*x + e)*a^3*d^8*f^36*g + 35*(f*x + e)^(9/2)*b^3*d^8*f
^32*h - 135*(f*x + e)^(7/2)*b^3*d^8*e*f^32*h + 189*(f*x + e)^(5/2)*b^3*d^8
*e^2*f^32*h - 105*(f*x + e)^(3/2)*b^3*d^8*e^3*f^32*h - 45*(f*x + e)^(7/2)*
b^3*c*d^7*f^33*h + 135*(f*x + e)^(7/2)*a*b^2*d^8*f^33*h + 126*(f*x + e)^(5
/2)*b^3*c*d^7*e*f^33*h - 378*(f*x + e)^(5/2)*a*b^2*d^8*e*f^33*h - 105*(f*x
 + e)^(3/2)*b^3*c*d^7*e^2*f^33*h + 315*(f*x + e)^(3/2)*a*b^2*d^8*e^2*f^33*
h + 63*(f*x + e)^(5/2)*b^3*c^2*d^6*f^34*h - 189*(f*x + e)^(5/2)*a*b^2*c*d^
7*f^34*h + 189*(f*x + e)^(5/2)*a^2*b*d^8*f^34*h - 105*(f*x + e)^(3/2)*b...
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^3)/(c + d*x),x)
```

output
```
(e + f*x)^(7/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(7*d*f^4) - (2*b^3*
h*(c*f^5 - d*e*f^4))/(7*d^2*f^8)) - (e + f*x)^(1/2)*((2*(a*f - b*e)^3*(e*h
 - f*g))/(d*f^4) + ((c*f^5 - d*e*f^4)*(((c*f^5 - d*e*f^4)*((((2*b^3*f*g -
8*b^3*e*h + 6*a*b^2*f*h)/(d*f^4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*
(c*f^5 - d*e*f^4))/(d*f^4) - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(
d*f^4)))/(d*f^4) + (2*(a*f - b*e)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(d*f^4)))
/(d*f^4)) - (e + f*x)^(5/2)*((((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d*f^
4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5 - d*e*f^4))/(5*d*f^4) -
 (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(5*d*f^4)) + (e + f*x)^(3/2)*
(((c*f^5 - d*e*f^4)*((((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d*f^4) - (2*
b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5 - d*e*f^4))/(d*f^4) - (6*b*(a*f
 - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d*f^4)))/(3*d*f^4) + (2*(a*f - b*e)^2*
(a*f*h - 4*b*e*h + 3*b*f*g))/(3*d*f^4)) + (2*b^3*h*(e + f*x)^(9/2))/(9*d*f
^4) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)^3*(d*e - c*f)^(1/2)*(c*h
- d*g)*1i)/(a^3*d^5*e*g - b^3*c^5*f*h - a^3*c*d^4*e*h - a^3*c*d^4*f*g + b^
3*c^4*d*e*h + b^3*c^4*d*f*g - b^3*c^3*d^2*e*g + a^3*c^2*d^3*f*h - 3*a^2*b*
c*d^4*e*g + 3*a*b^2*c^4*d*f*h + 3*a*b^2*c^2*d^3*e*g - 3*a*b^2*c^3*d^2*e*h
- 3*a*b^2*c^3*d^2*f*g + 3*a^2*b*c^2*d^3*e*h + 3*a^2*b*c^2*d^3*f*g - 3*a^2*
b*c^3*d^2*f*h))*(a*d - b*c)^3*(d*e - c*f)^(1/2)*(c*h - d*g)*2i)/d^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1557, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x)
```

output

```
(2*(315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*a**3*c*d**3*f**4*h - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**4*f**4*g - 945*sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**2*f
**4*h + 945*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c
*f - d*e)))*a**2*b*c*d**3*f**4*g + 945*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(
e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c**3*d*f**4*h - 945*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c
**2*d**2*f**4*g - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt
(d)*sqrt(c*f - d*e)))*b**3*c**4*f**4*h + 315*sqrt(d)*sqrt(c*f - d*e)*atan(
(sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**3*c**3*d*f**4*g - 315*sqrt
(e + f*x)*a**3*c*d**4*f**4*h + 105*sqrt(e + f*x)*a**3*d**5*e*f**3*h + 315*
sqrt(e + f*x)*a**3*d**5*f**4*g + 105*sqrt(e + f*x)*a**3*d**5*f**4*h*x + 94
5*sqrt(e + f*x)*a**2*b*c**2*d**3*f**4*h - 315*sqrt(e + f*x)*a**2*b*c*d**4*
e*f**3*h - 945*sqrt(e + f*x)*a**2*b*c*d**4*f**4*g - 315*sqrt(e + f*x)*a**2
*b*c*d**4*f**4*h*x - 126*sqrt(e + f*x)*a**2*b*d**5*e**2*f**2*h + 315*sqrt(
e + f*x)*a**2*b*d**5*e*f**3*g + 63*sqrt(e + f*x)*a**2*b*d**5*e*f**3*h*x +
315*sqrt(e + f*x)*a**2*b*d**5*f**4*g*x + 189*sqrt(e + f*x)*a**2*b*d**5*f**
4*h*x**2 - 945*sqrt(e + f*x)*a*b**2*c**3*d**2*f**4*h + 315*sqrt(e + f*x)*a
*b**2*c**2*d**3*e*f**3*h + 945*sqrt(e + f*x)*a*b**2*c**2*d**3*f**4*g + ...
```

3.34 $\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 255

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \frac{2(bc-ad)^2(dg-ch)\sqrt{e+fx}}{d^4} + \frac{2(d(2abdf^2g+a^2df^2h-b^2e(de+2cf)h)-b(de+cf)(2adfh+b(dfg-2deh-cfh)))(e+fx)^{3/2}}{3d^3f^3} + \frac{2b(2adfh+b(dfg-2deh-cfh))(e+fx)^{5/2}}{5d^2f^3} + \frac{2b^2h(e+fx)^{7/2}}{7df^3} - \frac{2(bc-ad)^2\sqrt{de-cf}(dg-ch)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{9/2}}$$

output
```
2*(-a*d+b*c)^2*(-c*h+d*g)*(f*x+e)^(1/2)/d^4+2/3*(d*(2*a*b*d*f^2*g+a^2*d*f^
2*h-b^2*e*(2*c*f+d*e)*h)-b*(c*f+d*e)*(2*a*d*f*h+b*(-c*f*h-2*d*e*h+d*f*g)))
*(f*x+e)^(3/2)/d^3/f^3+2/5*b*(2*a*d*f*h+b*(-c*f*h-2*d*e*h+d*f*g))*(f*x+e)^
(5/2)/d^2/f^3+2/7*b^2*h*(f*x+e)^(7/2)/d/f^3-2*(-a*d+b*c)^2*(-c*f+d*e)^(1/2
)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= \frac{2\sqrt{e+fx}(35a^2d^2f^2(-3cfh+d(3fg+eh+fhx))+14abdf(15c^2f^2h-d^2(e+fx)(-5fg+2eh-3fh}{}$$
$$-\frac{2(bc-ad)^2\sqrt{-de+cf}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{9/2}}$$

input
```
Integrate[((a + b*x)^2*Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(35*a^2*d^2*f^2*(-3*c*f*h + d*(3*f*g + e*h + f*h*x)) + 14
*a*b*d*f*(15*c^2*f^2*h - d^2*(e + f*x)*(-5*f*g + 2*e*h - 3*f*h*x) - 5*c*d*
f*(3*f*g + e*h + f*h*x)) + b^2*(-105*c^3*f^3*h + 35*c^2*d*f^2*(3*f*g + e*h
 + f*h*x) - 7*c*d^2*f*(e + f*x)*(5*f*g - 2*e*h + 3*f*h*x) + d^3*(e + f*x)*
(8*e^2*h + 3*f^2*x*(7*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)))))/(105*d^4*f^3) -
 (2*(b*c - a*d)^2*Sqrt[-(d*e) + c*f]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e +
f*x])/Sqrt[-(d*e) + c*f]])/d^(9/2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$\Big\downarrow \; 170$$
$$\frac{2\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{2(c+dx)}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$
$$\Big\downarrow \; 27$$
$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)\sqrt{e+fx}(4bceh-af(7dg-3ch)-(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}+\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2(e+fx)^{3/2}}{7df}-\frac{\int -\frac{(a+bx)\sqrt{e+fx}(7adfg-4bceh-3acfh+(4adfh+b(7dfg-4deh-7cfh))x)}{c+dx}dx}{7df}$$

input
```
Int[((a + b*x)^2*Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-2\sqrt{(cf-de)d}\left(\left(\left(\left(-\frac{8(fx+e)\left(\frac{15}{8}f^2x^2-\frac{3}{2}efx+e^2\right)d^3}{105}-\frac{2c(fx+e)f\left(-\frac{3fx}{2}+e\right)d^2}{15}-\frac{c^2f^2(fx+e)d}{3}+c^3f^3\right)b^2-2a\left(-\frac{2(fx+\cdots}{}\right.\right.\right.\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{h\,b^2(fx+e)^{\frac{7}{2}}d^3}{7}-\frac{2ab\,d^3fh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^2c\,d^2fh(fx+e)^{\frac{5}{2}}}{5}+\frac{2b^2d^3eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^2d^3fg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2d^3f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{2ab\cdots}{}\right.}{}$
default	$-\frac{2\left(-\frac{h\,b^2(fx+e)^{\frac{7}{2}}d^3}{7}-\frac{2ab\,d^3fh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^2c\,d^2fh(fx+e)^{\frac{5}{2}}}{5}+\frac{2b^2d^3eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^2d^3fg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2d^3f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{2ab\cdots}{}\right.}{}$
risch	$-\frac{2\left(-15h\,b^2d^3f^3x^3-42ab\,d^3f^3h\,x^2+21b^2c\,d^2f^3h\,x^2-3b^2d^3e\,f^2h\,x^2-21b^2d^3f^3g\,x^2-35a^2d^3f^3hx+70abc\,d^2f^3hx-1\cdots\right.}{}$

input
```
int((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/((c*f-d*e)*d)^(1/2)*(-((c*f-d*e)*d)^(1/2)*(((-8/105*(f*x+e)*(15/8*f^2*x^
2-3/2*e*f*x+e^2)*d^3-2/15*c*(f*x+e)*f*(-3/2*f*x+e)*d^2-1/3*c^2*f^2*(f*x+e)
*d+c^3*f^3)*b^2-2*a*(-2/15*(f*x+e)*(-3/2*f*x+e)*d^2-1/3*c*f*(f*x+e)*d+c^2*
f^2)*d*f*b+a^2*d^2*(1/3*(-f*x-e)*d+c*f)*f^2)*h-((-2/15*(f*x+e)*(-3/2*f*x+e
)*d^2-1/3*c*f*(f*x+e)*d+c^2*f^2)*b^2-2*a*d*(1/3*(-f*x-e)*d+c*f)*f*b+a^2*d^
2*f^2)*d*g*f)*(f*x+e)^(1/2)+f^3*(a*d-b*c)^2*(c*h-d*g)*(c*f-d*e)*arctan(d*(
f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))/f^3/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(231) = 462$.

Time = 0.13 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.71

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/105*(105*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^3*g - (b^2*c^3 - 2*a*b
*c^2*d + a^2*c*d^2)*f^3*h)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f +
2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)) - 2*(15*b^2*d^3*f^3*h*x^
3 + 3*(7*b^2*d^3*f^3*g + (b^2*d^3*e*f^2 - 7*(b^2*c*d^2 - 2*a*b*d^3)*f^3)*h
)*x^2 - 7*(2*b^2*d^3*e^2*f + 5*(b^2*c*d^2 - 2*a*b*d^3)*e*f^2 - 15*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)*f^3)*g + (8*b^2*d^3*e^3 + 14*(b^2*c*d^2 - 2*a*
b*d^3)*e^2*f + 35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*e*f^2 - 105*(b^2*c^3
 - 2*a*b*c^2*d + a^2*c*d^2)*f^3)*h + (7*(b^2*d^3*e*f^2 - 5*(b^2*c*d^2 - 2*
a*b*d^3)*f^3)*g - (4*b^2*d^3*e^2*f + 7*(b^2*c*d^2 - 2*a*b*d^3)*e*f^2 - 35*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^3)*h)*x)*sqrt(f*x + e))/(d^4*f^3), -
2/105*(105*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^3*g - (b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)*f^3*h)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt
(-(d*e - c*f)/d)/(d*e - c*f)) - (15*b^2*d^3*f^3*h*x^3 + 3*(7*b^2*d^3*f^3*g
 + (b^2*d^3*e*f^2 - 7*(b^2*c*d^2 - 2*a*b*d^3)*f^3)*h)*x^2 - 7*(2*b^2*d^3*e
^2*f + 5*(b^2*c*d^2 - 2*a*b*d^3)*e*f^2 - 15*(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*f^3)*g + (8*b^2*d^3*e^3 + 14*(b^2*c*d^2 - 2*a*b*d^3)*e^2*f + 35*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*e*f^2 - 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2)*f^3)*h + (7*(b^2*d^3*e*f^2 - 5*(b^2*c*d^2 - 2*a*b*d^3)*f^3)*g - (4*b
^2*d^3*e^2*f + 7*(b^2*c*d^2 - 2*a*b*d^3)*e*f^2 - 35*(b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*f^3)*h)*x)*sqrt(f*x + e))/(d^4*f^3)]
```

Sympy [A] (verification not implemented)

Time = 12.90 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.91

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{b^2h(e+fx)^{\frac{7}{2}}}{7df^2} + \frac{(e+fx)^{\frac{5}{2}}\cdot\left(2abdfh-b^2cfh-2b^2deh+b^2dfg\right)}{5d^2f^2} + \frac{(e+fx)^{\frac{3}{2}}\left(a^2d^2f^2h-2abcdf^2h-2abd^2efh+2abd^2f^2g+b^2c^2f^2h+b^2cdefh-b^2cdf^2g+b^2d^2 \cdots\right.}{3d^3f^2} \cdots\right.}{f} \\ \sqrt{e}\left(\frac{b^2hx^3}{3d} + \frac{x^2\cdot\left(2abdh-b^2ch+b^2dg\right)}{2d^2} + \frac{x\left(a^2d^2h-2abcdh+2abd^2g+b^2c^2h-b^2cdg\right)}{d^3} - \frac{(ad-bc)^2(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d = \cdots \\ \frac{\log(c+dx)}{d} & \text{otherw}\cdots\end{cases}\right.}{d^3} \cdots\right. \end{cases}$$

input
```
integrate((b*x+a)**2*(f*x+e)**(1/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(b**2*h*(e + f*x)**(7/2)/(7*d*f**2) + (e + f*x)**(5/2)*(2*a*b
*d*f*h - b**2*c*f*h - 2*b**2*d*e*h + b**2*d*f*g)/(5*d**2*f**2) + (e + f*x)
**(3/2)*(a**2*d**2*f**2*h - 2*a*b*c*d*f**2*h - 2*a*b*d**2*e*f*h + 2*a*b*d*
*2*f**2*g + b**2*c**2*f**2*h + b**2*c*d*e*f*h - b**2*c*d*f**2*g + b**2*d**
2*e**2*h - b**2*d**2*e*f*g)/(3*d**3*f**2) + sqrt(e + f*x)*(-a**2*c*d**2*f*
h + a**2*d**3*f*g + 2*a*b*c**2*d*f*h - 2*a*b*c*d**2*f*g - b**2*c**3*f*h +
b**2*c**2*d*f*g)/d**4 + f*(a*d - b*c)**2*(c*f - d*e)*(c*h - d*g)*atan(sqrt
(e + f*x)/sqrt((c*f - d*e)/d))/(d**5*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), (
sqrt(e)*(b**2*h*x**3/(3*d) + x**2*(2*a*b*d*h - b**2*c*h + b**2*d*g)/(2*d**
2) + x*(a**2*d**2*h - 2*a*b*c*d*h + 2*a*b*d**2*g + b**2*c**2*h - b**2*c*d*
g)/d**3 - (a*d - b*c)**2*(c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d
*x)/d, True))/d**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(231) = 462$.

Time = 0.14 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= \frac{2\,(b^2c^2d^2eg - 2\,abcd^3eg + a^2d^4eg - b^2c^3dfg + 2\,abc^2d^2fg - a^2cd^3fg - b^2c^3deh + 2\,abc^2d^2eh - a^2cd^3eh -}{\sqrt{-d^2e+cdf}\,d^4}$$
$$+ \frac{2\left(21\,(fx+e)^{\frac{5}{2}}b^2d^6f^{19}g - 35\,(fx+e)^{\frac{3}{2}}b^2d^6ef^{19}g - 35\,(fx+e)^{\frac{3}{2}}b^2cd^5f^{20}g + 70\,(fx+e)^{\frac{3}{2}}abd^6f^{20}g +\right.}{}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^2*c^2*d^2*e*g - 2*a*b*c*d^3*e*g + a^2*d^4*e*g - b^2*c^3*d*f*g + 2*a*b
*c^2*d^2*f*g - a^2*c*d^3*f*g - b^2*c^3*d*e*h + 2*a*b*c^2*d^2*e*h - a^2*c*d
^3*e*h + b^2*c^4*f*h - 2*a*b*c^3*d*f*h + a^2*c^2*d^2*f*h)*arctan(sqrt(f*x
+ e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^4) + 2/105*(21*(f*x +
 e)^(5/2)*b^2*d^6*f^19*g - 35*(f*x + e)^(3/2)*b^2*d^6*e*f^19*g - 35*(f*x +
 e)^(3/2)*b^2*c*d^5*f^20*g + 70*(f*x + e)^(3/2)*a*b*d^6*f^20*g + 105*sqrt(
f*x + e)*b^2*c^2*d^4*f^21*g - 210*sqrt(f*x + e)*a*b*c*d^5*f^21*g + 105*sqr
t(f*x + e)*a^2*d^6*f^21*g + 15*(f*x + e)^(7/2)*b^2*d^6*f^18*h - 42*(f*x +
e)^(5/2)*b^2*d^6*e*f^18*h + 35*(f*x + e)^(3/2)*b^2*d^6*e^2*f^18*h - 21*(f*
x + e)^(5/2)*b^2*c*d^5*f^19*h + 42*(f*x + e)^(5/2)*a*b*d^6*f^19*h + 35*(f*
x + e)^(3/2)*b^2*c*d^5*e*f^19*h - 70*(f*x + e)^(3/2)*a*b*d^6*e*f^19*h + 35
*(f*x + e)^(3/2)*b^2*c^2*d^4*f^20*h - 70*(f*x + e)^(3/2)*a*b*c*d^5*f^20*h
+ 35*(f*x + e)^(3/2)*a^2*d^6*f^20*h - 105*sqrt(f*x + e)*b^2*c^3*d^3*f^21*h
 + 210*sqrt(f*x + e)*a*b*c^2*d^4*f^21*h - 105*sqrt(f*x + e)*a^2*c*d^5*f^21
*h)/(d^7*f^21)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= (e+f\,x)^{5/2}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{5\,d\,f^3}-\frac{2\,b^2\,h\,(c\,f^4-d\,e\,f^3)}{5\,d^2\,f^6}\right)$$
$$-\sqrt{e+f\,x}\left(\frac{2\,(a\,f-b\,e)^2\,(e\,h-f\,g)}{d\,f^3}-\frac{\left(\frac{\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d\,f^3}-\frac{2\,b^2\,h\,\left(c\,f^4-d\,e\,f^3\right)}{d^2\,f^6}\right)(c\,f^4-d\,e\,f^3)}{d\,f^3}-\frac{2\,(a\,f-b\,e)\,(a\,f\,h-3\,b\,e\,h+2\,b\,f\,g)}{d\,f^3}\right)(c\,f^4-d\,e\,f^3)}{d\,f^3}\right)$$
$$-(e+f\,x)^{3/2}\left(\frac{\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d\,f^3}-\frac{2\,b^2\,h\,(c\,f^4-d\,e\,f^3)}{d^2\,f^6}\right)(c\,f^4-d\,e\,f^3)}{3\,d\,f^3}-\frac{2\,(a\,f-b\,e)\,(a\,f\,h-3\,b\,e\,h+\right.$$
$$+\frac{2\,b^2\,h\,(e+f\,x)^{7/2}}{7\,d\,f^3}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)^2\,\sqrt{d\,e-c\,f}\,(c\,h-d\,g)\,1\mathrm{i}}{a^2\,d^4\,e\,g+b^2\,c^4\,f\,h-a^2\,c\,d^3\,e\,h-a^2\,c\,d^3\,f\,g-b^2\,c^3\,d\,e\,h-b^2\,c^3\,d\,f\,g+b^2\,c^2\,d^2\,e\,g+a^2\,c^2\,d^2\,f\,h-2\,a\,b\,c\,d^3\,e\,g-2\,a\,b\,c^3\,d\,f\,h+2\,a\,b\,c^2\,d^2\,e\,h}\right.}{d^{9/2}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^2)/(c + d*x),x)
```

output
```
(e + f*x)^(5/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(5*d*f^3) - (2*b^2*h*
(c*f^4 - d*e*f^3))/(5*d^2*f^6)) - (e + f*x)^(1/2)*((2*(a*f - b*e)^2*(e*h -
 f*g))/(d*f^3) - (((((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d*f^3) - (2*b^2*
h*(c*f^4 - d*e*f^3))/(d^2*f^6))*(c*f^4 - d*e*f^3))/(d*f^3) - (2*(a*f - b*e
)*(a*f*h - 3*b*e*h + 2*b*f*g))/(d*f^3))*(c*f^4 - d*e*f^3))/(d*f^3)) - (e +
 f*x)^(3/2)*((((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d*f^3) - (2*b^2*h*(c*f
^4 - d*e*f^3))/(d^2*f^6))*(c*f^4 - d*e*f^3))/(3*d*f^3) - (2*(a*f - b*e)*(a
*f*h - 3*b*e*h + 2*b*f*g))/(3*d*f^3)) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*
d - b*c)^2*(d*e - c*f)^(1/2)*(c*h - d*g)*1i)/(a^2*d^4*e*g + b^2*c^4*f*h -
a^2*c*d^3*e*h - a^2*c*d^3*f*g - b^2*c^3*d*e*h - b^2*c^3*d*f*g + b^2*c^2*d^
2*e*g + a^2*c^2*d^2*f*h - 2*a*b*c*d^3*e*g - 2*a*b*c^3*d*f*h + 2*a*b*c^2*d^
2*e*h + 2*a*b*c^2*d^2*f*g))*(a*d - b*c)^2*(d*e - c*f)^(1/2)*(c*h - d*g)*2i
)/d^(9/2) + (2*b^2*h*(e + f*x)^(7/2))/(7*d*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 865, normalized size of antiderivative = 3.39

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x)
```

output

```
(2*(105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*a**2*c*d**2*f**3*h - 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**3*f**3*g - 210*sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d*f**3*h
+ 210*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a*b*c*d**2*f**3*g + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**3*f**3*h - 105*sqrt(d)*sqrt(c*f - d*
e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**2*d*f**3*g -
105*sqrt(e + f*x)*a**2*c*d**3*f**3*h + 35*sqrt(e + f*x)*a**2*d**4*e*f**2*h
 + 105*sqrt(e + f*x)*a**2*d**4*f**3*g + 35*sqrt(e + f*x)*a**2*d**4*f**3*h*
x + 210*sqrt(e + f*x)*a*b*c**2*d**2*f**3*h - 70*sqrt(e + f*x)*a*b*c*d**3*e
*f**2*h - 210*sqrt(e + f*x)*a*b*c*d**3*f**3*g - 70*sqrt(e + f*x)*a*b*c*d**
3*f**3*h*x - 28*sqrt(e + f*x)*a*b*d**4*e**2*f*h + 70*sqrt(e + f*x)*a*b*d**
4*e*f**2*g + 14*sqrt(e + f*x)*a*b*d**4*e*f**2*h*x + 70*sqrt(e + f*x)*a*b*d
**4*f**3*g*x + 42*sqrt(e + f*x)*a*b*d**4*f**3*h*x**2 - 105*sqrt(e + f*x)*b
**2*c**3*d*f**3*h + 35*sqrt(e + f*x)*b**2*c**2*d**2*e*f**2*h + 105*sqrt(e
+ f*x)*b**2*c**2*d**2*f**3*g + 35*sqrt(e + f*x)*b**2*c**2*d**2*f**3*h*x +
14*sqrt(e + f*x)*b**2*c*d**3*e**2*f*h - 35*sqrt(e + f*x)*b**2*c*d**3*e*f**
2*g - 7*sqrt(e + f*x)*b**2*c*d**3*e*f**2*h*x - 35*sqrt(e + f*x)*b**2*c*d**
3*f**3*g*x - 21*sqrt(e + f*x)*b**2*c*d**3*f**3*h*x**2 + 8*sqrt(e + f*x)...
```

3.35 $\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx = -\frac{2(bc-ad)(dg-ch)\sqrt{e+fx}}{d^3} - \frac{2(b(de+cf)h-df(bg+ah))(e+fx)^{3/2}}{3d^2f^2} + \frac{2bh(e+fx)^{5/2}}{5df^2} + \frac{2(bc-ad)\sqrt{de-cf}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}}$$

output

```
-2*(-a*d+b*c)*(-c*h+d*g)*(f*x+e)^(1/2)/d^3-2/3*(b*(c*f+d*e)*h-d*f*(a*h+b*g
))*(f*x+e)^(3/2)/d^2/f^2+2/5*b*h*(f*x+e)^(5/2)/d/f^2+2*(-a*d+b*c)*(-c*f+d*
e)^(1/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2
)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= \frac{2\sqrt{e+fx}(5adf(-3cfh+d(3fg+eh+fhx))+b(15c^2f^2h-d^2(e+fx)(-5fg+2eh-3fhx)-5cdf(}{15d^3f^2}$$
$$-\frac{2(-bc+ad)\sqrt{-de+cf}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}}$$

input
```
Integrate[((a + b*x)*Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(5*a*d*f*(-3*c*f*h + d*(3*f*g + e*h + f*h*x)) + b*(15*c^2
*f^2*h - d^2*(e + f*x)*(-5*f*g + 2*e*h - 3*f*h*x) - 5*c*d*f*(3*f*g + e*h +
 f*h*x))))/(15*d^3*f^2) - (2*(-(b*c) + a*d)*Sqrt[-(d*e) + c*f]*(d*g - c*h)
*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(7/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$\Big\downarrow \; 164$$
$$-\frac{(bc-ad)(dg-ch)\int \frac{\sqrt{e+fx}}{c+dx}dx}{d^2}-\frac{2(e+fx)^{3/2}(-5df(ah+bg)+5bcfh+2bdeh-3bdfhx)}{15d^2f^2}$$
$$\Big\downarrow \; 60$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{d^2}-\frac{2(e+fx)^{3/2}(-5df(ah+bg)+5bcfh+2bdeh-3bdfhx)}{15d^2f^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{d^2}-\frac{2(e+fx)^{3/2}(-5df(ah+bg)+5bcfh+2bdeh-3bdfhx)}{15d^2f^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d^2}-\frac{2(e+fx)^{3/2}(-5df(ah+bg)+5bcfh+2bdeh-3bdfhx)}{15d^2f^2}$$

input
```
Int[((a + b*x)*Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
(-2*(e + f*x)^(3/2)*(2*b*d*e*h + 5*b*c*f*h - 5*d*f*(b*g + a*h) - 3*b*d*f*h
*x))/(15*d^2*f^2) - ((b*c - a*d)*(d*g - c*h)*((2*Sqrt[e + f*x])/d - (2*Sqr
t[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d
^2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{2\Bigg(\bigg(\Big(\Big(-\frac{(fx+e)\left(\frac{(3fx-2e)b}{5}+af\right)h}{3}-\left(\frac{(fx+e)b}{3}+af\right)gf\Big)d^2+c\left(\left(\frac{(fx+e)b}{3}+af\right)h+bfg\right)fd-b\,c^2f^2h\bigg)\sqrt{(cf-de)d}\,\sqrt{}}{\sqrt{(cf-de)d}\,f^2d^3}$
risch	$-\frac{2(-3hb\,d^2f^2x^2-5a\,d^2f^2hx+5bcd\,f^2hx-b\,d^2efhx-5b\,d^2f^2gx+15acd\,f^2h-5a\,d^2efh-15a\,d^2f^2g-15b\,c^2f^2h+5bca}{15f^2d^3}$
derivativedivides	$-\frac{2\Big(-\frac{hb(fx+e)^{\frac{5}{2}}d^2}{5}-\frac{a\,d^2fh(fx+e)^{\frac{3}{2}}}{3}+\frac{bcdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b\,d^2eh(fx+e)^{\frac{3}{2}}}{3}-\frac{b\,d^2fg(fx+e)^{\frac{3}{2}}}{3}+acd\,f^2h\sqrt{fx+e}-a\,d^2f^2g\sqrt{fx+e}-b}{d^3}$
default	$-\frac{2\Big(-\frac{hb(fx+e)^{\frac{5}{2}}d^2}{5}-\frac{a\,d^2fh(fx+e)^{\frac{3}{2}}}{3}+\frac{bcdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b\,d^2eh(fx+e)^{\frac{3}{2}}}{3}-\frac{b\,d^2fg(fx+e)^{\frac{3}{2}}}{3}+acd\,f^2h\sqrt{fx+e}-a\,d^2f^2g\sqrt{fx+e}-b}{d^3}$

input
```
int((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/((c*f-d*e)*d)^(1/2)*(((-1/3*(f*x+e)*(1/5*(3*f*x-2*e)*b+a*f)*h-(1/3*(f*x
+e)*b+a*f)*g*f)*d^2+c*((1/3*(f*x+e)*b+a*f)*h+b*f*g)*f*d-b*c^2*f^2*h)*((c*f
-d*e)*d)^(1/2)*(f*x+e)^(1/2)-f^2*(c*h-d*g)*(c*f-d*e)*(a*d-b*c)*arctan(d*(f
*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))/f^2/d^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.99

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= \left[\frac{15\left((bcd-ad^2)f^2g-(bc^2-acd)f^2h\right)\sqrt{\frac{de-cf}{d}}\log\left(\frac{dfx+2\,de-cf+2\,\sqrt{fx+e}d\sqrt{\frac{de-cf}{d}}}{dx+c}\right)+2\left(3\,bd^2f^2hx^2+5\,(bd\right.}{}\right.$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/15*(15*((b*c*d - a*d^2)*f^2*g - (b*c^2 - a*c*d)*f^2*h)*sqrt((d*e - c*f)
/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x
 + c)) + 2*(3*b*d^2*f^2*h*x^2 + 5*(b*d^2*e*f - 3*(b*c*d - a*d^2)*f^2)*g -
(2*b*d^2*e^2 + 5*(b*c*d - a*d^2)*e*f - 15*(b*c^2 - a*c*d)*f^2)*h + (5*b*d^
2*f^2*g + (b*d^2*e*f - 5*(b*c*d - a*d^2)*f^2)*h)*x)*sqrt(f*x + e))/(d^3*f^
2), 2/15*(15*((b*c*d - a*d^2)*f^2*g - (b*c^2 - a*c*d)*f^2*h)*sqrt(-(d*e -
c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(d*e - c*f)/d)/(d*e - c*f)) + (3*b*d
^2*f^2*h*x^2 + 5*(b*d^2*e*f - 3*(b*c*d - a*d^2)*f^2)*g - (2*b*d^2*e^2 + 5*
(b*c*d - a*d^2)*e*f - 15*(b*c^2 - a*c*d)*f^2)*h + (5*b*d^2*f^2*g + (b*d^2*
e*f - 5*(b*c*d - a*d^2)*f^2)*h)*x)*sqrt(f*x + e))/(d^3*f^2)]
```

Sympy [A] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \begin{cases} \frac{2\left(\frac{bh(e+fx)^{\frac{5}{2}}}{5df}+\frac{(e+fx)^{\frac{3}{2}}(adfh-bcfh-bdeh+bdfg)}{3d^2f}+\frac{\sqrt{e+fx}\left(-acdfh+ad^2fg+bc^2fh-bcdfg\right)}{d^3}+\frac{f(ad-bc)(cf-de)(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^4\sqrt{\frac{cf-de}{d}}}\right)}{f} & \text{for} \\ \sqrt{e}\left(\frac{bhx^2}{2d}+\frac{x(adh-bch+bdg)}{d^2}-\frac{(ad-bc)(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d^2}\right) & \text{oth} \end{cases}$$

input
```
integrate((b*x+a)*(f*x+e)**(1/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(b*h*(e + f*x)**(5/2)/(5*d*f) + (e + f*x)**(3/2)*(a*d*f*h - b
*c*f*h - b*d*e*h + b*d*f*g)/(3*d**2*f) + sqrt(e + f*x)*(-a*c*d*f*h + a*d**
2*f*g + b*c**2*f*h - b*c*d*f*g)/d**3 + f*(a*d - b*c)*(c*f - d*e)*(c*h - d*
g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**4*sqrt((c*f - d*e)/d)))/f,
Ne(f, 0)), (sqrt(e)*(b*h*x**2/(2*d) + x*(a*d*h - b*c*h + b*d*g)/d**2 - (a*
d - b*c)*(c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d*
*2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx = -\frac{2\,(bcd^2eg-ad^3eg-bc^2dfg+acd^2fg-bc^2deh+acd^2eh+bc^3fh-ac^2dfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,d^3} + \frac{2\left(5\,(fx+e)^{\frac{3}{2}}bd^4f^9g-15\,\sqrt{fx+e}bcd^3f^{10}g+15\,\sqrt{fx+e}ad^4f^{10}g+3\,(fx+e)^{\frac{5}{2}}bd^4f^8h-5\,(fx+e)\right.}{15\,d}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b*c*d^2*e*g - a*d^3*e*g - b*c^2*d*f*g + a*c*d^2*f*g - b*c^2*d*e*h + a*
c*d^2*e*h + b*c^3*f*h - a*c^2*d*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e +
c*d*f))/(sqrt(-d^2*e + c*d*f)*d^3) + 2/15*(5*(f*x + e)^(3/2)*b*d^4*f^9*g -
 15*sqrt(f*x + e)*b*c*d^3*f^10*g + 15*sqrt(f*x + e)*a*d^4*f^10*g + 3*(f*x
+ e)^(5/2)*b*d^4*f^8*h - 5*(f*x + e)^(3/2)*b*d^4*e*f^8*h - 5*(f*x + e)^(3/
2)*b*c*d^3*f^9*h + 5*(f*x + e)^(3/2)*a*d^4*f^9*h + 15*sqrt(f*x + e)*b*c^2*
d^2*f^10*h - 15*sqrt(f*x + e)*a*c*d^3*f^10*h)/(d^5*f^10)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= (e+fx)^{3/2}\left(\frac{2afh-4beh+2bfg}{3df^2}-\frac{2bh(cf^3-def^2)}{3d^2f^4}\right) - \sqrt{e+fx}\left(\frac{2(af-be)(eh-fg)}{df^2} + \frac{(cf^3-def^2)\left(\frac{2afh-4beh+2bfg}{df^2}-\frac{2bh(cf^3-def^2)}{d^2f^4}\right)}{df^2}\right) + \frac{2bh(e+fx)^{5/2}}{5df^2} - \frac{\mathrm{atan}\left(\frac{\sqrt{d}\sqrt{e+fx}\,1\mathrm{i}}{\sqrt{de-cf}}\right)(ad-bc)\sqrt{de-cf}\,(ch-dg)\,2\mathrm{i}}{d^{7/2}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x))/(c + d*x),x)
```

output
```
(e + f*x)^(3/2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(3*d*f^2) - (2*b*h*(c*f^3 -
 d*e*f^2))/(3*d^2*f^4)) - (e + f*x)^(1/2)*((2*(a*f - b*e)*(e*h - f*g))/(d*
f^2) + ((c*f^3 - d*e*f^2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(d*f^2) - (2*b*h*
(c*f^3 - d*e*f^2))/(d^2*f^4)))/(d*f^2)) + (2*b*h*(e + f*x)^(5/2))/(5*d*f^2
) - (atan((d^(1/2)*(e + f*x)^(1/2)*1i)/(d*e - c*f)^(1/2))*(a*d - b*c)*(d*e
 - c*f)^(1/2)*(c*h - d*g)*2i)/d^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$= \frac{2\sqrt{d}\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\sqrt{cf-de}}\right)acdf^2h - 2\sqrt{d}\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\sqrt{cf-de}}\right)ad^2f^2g - 2\sqrt{d}\sqrt{cf-de}\,ata}{}$$

input
```
int((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c),x)
```

output
```
(2*(15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a*c*d*f**2*h - 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*a*d**2*f**2*g - 15*sqrt(d)*sqrt(c*f - d*e)*atan((
sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*f**2*h + 15*sqrt(d)*sqr
t(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d*f**2*
g - 15*sqrt(e + f*x)*a*c*d**2*f**2*h + 5*sqrt(e + f*x)*a*d**3*e*f*h + 15*s
qrt(e + f*x)*a*d**3*f**2*g + 5*sqrt(e + f*x)*a*d**3*f**2*h*x + 15*sqrt(e +
 f*x)*b*c**2*d*f**2*h - 5*sqrt(e + f*x)*b*c*d**2*e*f*h - 15*sqrt(e + f*x)*
b*c*d**2*f**2*g - 5*sqrt(e + f*x)*b*c*d**2*f**2*h*x - 2*sqrt(e + f*x)*b*d*
*3*e**2*h + 5*sqrt(e + f*x)*b*d**3*e*f*g + sqrt(e + f*x)*b*d**3*e*f*h*x +
5*sqrt(e + f*x)*b*d**3*f**2*g*x + 3*sqrt(e + f*x)*b*d**3*f**2*h*x**2))/(15
*d**4*f**2)
```

3.36 $\int \frac{\sqrt{e+fx}(g+hx)}{c+dx} dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx} dx = \frac{2(dg-ch)\sqrt{e+fx}}{d^2} + \frac{2h(e+fx)^{3/2}}{3df} - \frac{2\sqrt{de-cf}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}}$$

output
```
2*(-c*h+d*g)*(f*x+e)^(1/2)/d^2+2/3*h*(f*x+e)^(3/2)/d/f-2*(-c*f+d*e)^(1/2)*
(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx} dx = \frac{2\sqrt{e+fx}(-3cfh+d(3fg+eh+fhx))}{3d^2f} - \frac{2\sqrt{-de+cf}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(-3*c*f*h + d*(3*f*g + e*h + f*h*x)))/(3*d^2*f) - (2*Sqrt
[-(d*e) + c*f]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*
f]])/d^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$

$$\Big\downarrow \; 90$$

$$\frac{(dg-ch)\int \frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2h(e+fx)^{3/2}}{3df}$$

$$\Big\downarrow \; 60$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2h(e+fx)^{3/2}}{3df}$$

$$\Big\downarrow \; 73$$

$$\frac{(dg-ch)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2h(e+fx)^{3/2}}{3df}$$

$$\Big\downarrow \; 221$$

$$\frac{(dg-ch)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2h(e+fx)^{3/2}}{3df}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/(c + d*x),x]
```

output
```
(2*h*(e + f*x)^(3/2))/(3*d*f) + ((d*g - c*h)*((2*Sqrt[e + f*x])/d - (2*Sqr
t[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{2(-fdhx+3cfh-deh-3dfg)\sqrt{fx+e}}{3f\,d^{2}}+\frac{2(c^{2}fh-cdeh-cdfg+d^{2}eg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d^{2}\sqrt{(cf-de)d}}$	103
pseudoelliptic	$-\frac{2\left(-f(ch-dg)(cf-de)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\left(\left(\left(-\frac{hx}{3}-g\right)f-\frac{eh}{3}\right)d+cfh\right)\sqrt{fx+e}\,\sqrt{(cf-de)d}\right)}{\sqrt{(cf-de)d}\,f\,d^{2}}$	105
derivativedivides	$\frac{-\frac{2\left(-\frac{h(fx+e)^{\frac{3}{2}}d}{3}+cfh\sqrt{fx+e}-dfg\sqrt{fx+e}\right)}{d^{2}}+\frac{2f\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d^{2}\sqrt{(cf-de)d}}}{f}$	111
default	$\frac{-\frac{2\left(-\frac{h(fx+e)^{\frac{3}{2}}d}{3}+cfh\sqrt{fx+e}-dfg\sqrt{fx+e}\right)}{d^{2}}+\frac{2f\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d^{2}\sqrt{(cf-de)d}}}{f}$	111

input
```
int((f*x+e)^(1/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-d*f*h*x+3*c*f*h-d*e*h-3*d*f*g)*(f*x+e)^(1/2)/f/d^2+2*(c^2*f*h-c*d*e
*h-c*d*f*g+d^2*e*g)/d^2/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d
*e)*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx$$
$$=\left[-\frac{3\,(dfg-cfh)\sqrt{\frac{de-cf}{d}}\log\left(\frac{dfx+2\,de-cf+2\,\sqrt{fx+e}d\sqrt{\frac{de-cf}{d}}}{dx+c}\right)-2\,(dfhx+3\,dfg+(de-3\,cf)h)\sqrt{fx+e}}{3\,d^{2}f},\right.$$
$$\left.-\frac{2\left(3\,(dfg-cfh)\sqrt{-\frac{de-cf}{d}}\arctan\left(-\frac{\sqrt{fx+e}d\sqrt{-\frac{de-cf}{d}}}{de-cf}\right)-(dfhx+3\,dfg+(de-3\,cf)h)\sqrt{fx+e}\right)}{3\,d^{2}f}\right]$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/3*(3*(d*f*g - c*f*h)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*
sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)) - 2*(d*f*h*x + 3*d*f*g + (
d*e - 3*c*f)*h)*sqrt(f*x + e))/(d^2*f), -2/3*(3*(d*f*g - c*f*h)*sqrt(-(d*e
 - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(d*e - c*f)/d)/(d*e - c*f)) - (d*
f*h*x + 3*d*f*g + (d*e - 3*c*f)*h)*sqrt(f*x + e))/(d^2*f)]
```

Sympy [A] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \begin{cases} \frac{2\left(\frac{h(e+fx)^{\frac{3}{2}}}{3d}+\frac{\sqrt{e+fx}(-cfh+dfg)}{d^2}+\frac{f(cf-de)(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^3\sqrt{\frac{cf-de}{d}}}\right)}{f} & \text{for } f\neq 0 \\ \sqrt{e}\left(\frac{hx}{d}-\frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(h*(e + f*x)**(3/2)/(3*d) + sqrt(e + f*x)*(-c*f*h + d*f*g)/d*
*2 + f*(c*f - d*e)*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d*
*3*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), (sqrt(e)*(h*x/d - (c*h - d*g)*Piece
wise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \frac{2\,(d^2eg-cdfg-cdeh+c^2fh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,d^2} + \frac{2\left(3\sqrt{fx+e}d^2f^3g+(fx+e)^{\frac{3}{2}}d^2f^2h-3\sqrt{fx+e}cdf^3h\right)}{3\,d^3f^3}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
2*(d^2*e*g - c*d*f*g - c*d*e*h + c^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2
*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^2) + 2/3*(3*sqrt(f*x + e)*d^2*f^3*g +
 (f*x + e)^(3/2)*d^2*f^2*h - 3*sqrt(f*x + e)*c*d*f^3*h)/(d^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \frac{2\,h\,(e+f\,x)^{3/2}}{3\,d\,f} - \sqrt{e+f\,x}\left(\frac{2\,e\,h-2\,f\,g}{d\,f}+\frac{2\,h\,(c\,f^2-d\,e\,f)}{d^2\,f^2}\right) + \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{d\,e-c\,f}}\right)\,\sqrt{d\,e-c\,f}\,(c\,h-d\,g)}{d^{5/2}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/(c + d*x),x)
```

output
```
(2*h*(e + f*x)^(3/2))/(3*d*f) - (e + f*x)^(1/2)*((2*e*h - 2*f*g)/(d*f) + (
2*h*(c*f^2 - d*e*f))/(d^2*f^2)) + (2*atanh((d^(1/2)*(e + f*x)^(1/2))/(d*e
- c*f)^(1/2))*(d*e - c*f)^(1/2)*(c*h - d*g))/d^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{e+fx}(g+hx)}{c+dx}\,dx = \frac{2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cfh-2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)dfg-2\sqrt{fx+e}\,cdfh+\frac{2\sqrt{fx+e}\,d^2}{3}}{d^3f}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(d*x+c),x)
```

output
```
(2*(3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*c*f*h - 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sq
rt(c*f - d*e)))*d*f*g - 3*sqrt(e + f*x)*c*d*f*h + sqrt(e + f*x)*d**2*e*h +
 3*sqrt(e + f*x)*d**2*f*g + sqrt(e + f*x)*d**2*f*h*x))/(3*d**3*f)
```

3.37 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 149

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{2h\sqrt{e+fx}}{bd} - \frac{2\sqrt{be-af}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(bc-ad)} + \frac{2\sqrt{de-cf}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(bc-ad)}$$

output
```
2*h*(f*x+e)^(1/2)/b/d-2*(-a*f+b*e)^(1/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e
)^(1/2)/(-a*f+b*e)^(1/2))/b^(3/2)/(-a*d+b*c)+2*(-c*f+d*e)^(1/2)*(-c*h+d*g)
*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{-2d^{3/2}\sqrt{-be+af}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right) + 2\sqrt{b}\left(\sqrt{d}(bc-ad)h\sqrt{e+fx} + b\sqrt{-de+cf}(dg-ch)\,a\right.}{b^{3/2}d^{3/2}(bc-ad)}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(-2*d^(3/2)*Sqrt[-(b*e) + a*f]*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/
Sqrt[-(b*e) + a*f]] + 2*Sqrt[b]*(Sqrt[d]*(b*c - a*d)*h*Sqrt[e + f*x] + b*S
qrt[-(d*e) + c*f]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) +
 c*f]]))/(b^(3/2)*d^(3/2)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{171}$$

$$\frac{2\int \frac{bdeg-acfh-(adfh-b(dfg+deh-cfh))x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd}+\frac{2h\sqrt{e+fx}}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bdeg-acfh-(adfh-b(dfg+deh-cfh))x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd}+\frac{2h\sqrt{e+fx}}{bd}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{d(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{b(de-cf)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{bd}+\frac{2h\sqrt{e+fx}}{bd}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2d(be-af)(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2b(de-cf)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{bd}+\frac{2h\sqrt{e+fx}}{bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2b\sqrt{de-cf}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)}-\frac{2d\sqrt{be-af}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)}}{bd}+\frac{2h\sqrt{e+fx}}{bd}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(2*h*Sqrt[e + f*x])/(b*d) + ((-2*d*Sqrt[b*e - a*f]*(b*g - a*h)*ArcTanh[(Sq
rt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)) + (2*b*Sqrt[d
*e - c*f]*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(S
qrt[d]*(b*c - a*d)))/(b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{2h\sqrt{fx+e}}{bd}+\frac{2\left(-a^{2}fh+abeh+abfg-b^{2}eg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)b\sqrt{(af-be)b}}+\frac{2\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d(ad-bc)\sqrt{(cf-de)d}}$
default	$\frac{2h\sqrt{fx+e}}{bd}+\frac{2\left(-a^{2}fh+abeh+abfg-b^{2}eg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)b\sqrt{(af-be)b}}+\frac{2\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d(ad-bc)\sqrt{(cf-de)d}}$
risch	$\frac{2h\sqrt{fx+e}}{bd}-\frac{\frac{2d\left(a^{2}fh-abeh-abfg+b^{2}eg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)\sqrt{(af-be)b}}-\frac{2b\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)\sqrt{(cf-de)d}}}{bd}$
pseudoelliptic	$\frac{-2(ah-bg)d\sqrt{(cf-de)d}\,(af-be)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+2\sqrt{(af-be)b}\left(b(ch-dg)(cf-de)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+(ad-\cdots\right.}{bd(ad-bc)\sqrt{(af-be)b}\,\sqrt{(cf-de)d}}$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*h*(f*x+e)^(1/2)/b/d+2*(-a^2*f*h+a*b*e*h+a*b*f*g-b^2*e*g)/(a*d-b*c)/b/((a
*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+2/d*(c^2*f*h-
c*d*e*h-c*d*f*g+d^2*e*g)/(a*d-b*c)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1
/2)/((c*f-d*e)*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 680, normalized size of antiderivative = 4.56

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx$$
$$= \left[\frac{2\,(bc-ad)\sqrt{fx+e}h+(bdg-adh)\sqrt{\frac{be-af}{b}}\log\left(\frac{bfx+2\,be-af-2\,\sqrt{fx+e}b\sqrt{\frac{be-af}{b}}}{bx+a}\right)+(bdg-bch)\sqrt{\frac{de-cf}{d}}\log}{b^2cd-abd^2}\right.$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
[(2*(b*c - a*d)*sqrt(f*x + e)*h + (b*d*g - a*d*h)*sqrt((b*e - a*f)/b)*log(
(b*f*x + 2*b*e - a*f - 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a)) +
 (b*d*g - b*c*h)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x
 + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)))/(b^2*c*d - a*b*d^2), (2*(b*c - a*
d)*sqrt(f*x + e)*h - 2*(b*d*g - a*d*h)*sqrt(-(b*e - a*f)/b)*arctan(-sqrt(f
*x + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) + (b*d*g - b*c*h)*sqrt((d*e -
c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/
(d*x + c)))/(b^2*c*d - a*b*d^2), (2*(b*c - a*d)*sqrt(f*x + e)*h + 2*(b*d*g
 - b*c*h)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(d*e - c*f)/d
)/(d*e - c*f)) + (b*d*g - a*d*h)*sqrt((b*e - a*f)/b)*log((b*f*x + 2*b*e -
a*f - 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a)))/(b^2*c*d - a*b*d^
2), 2*((b*c - a*d)*sqrt(f*x + e)*h - (b*d*g - a*d*h)*sqrt(-(b*e - a*f)/b)*
arctan(-sqrt(f*x + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) + (b*d*g - b*c*h
)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(d*e - c*f)/d)/(d*e -
 c*f)))/(b^2*c*d - a*b*d^2)]
```

Sympy [A] (verification not implemented)

Time = 34.98 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \begin{cases} \frac{2\left(\frac{f(cf-de)(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^2\sqrt{\frac{cf-de}{d}}(ad-bc)}+\frac{fh\sqrt{e+fx}}{bd}-\frac{f(af-be)(ah-bg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{af-be}{b}}}\right)}{b^2\sqrt{\frac{af-be}{b}}(ad-bc)}\right)}{f} & \text{for } f\neq 0 \\ \sqrt{e}\left(\frac{(ah-bg)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{ad-bc}-\frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{ad-bc}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
Piecewise((2*(f*(c*f - d*e)*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e
)/d))/(d**2*sqrt((c*f - d*e)/d)*(a*d - b*c)) + f*h*sqrt(e + f*x)/(b*d) - f
*(a*f - b*e)*(a*h - b*g)*atan(sqrt(e + f*x)/sqrt((a*f - b*e)/b))/(b**2*sqr
t((a*f - b*e)/b)*(a*d - b*c)))/f, Ne(f, 0)), (sqrt(e)*((a*h - b*g)*Piecewi
se((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/(a*d - b*c) - (c*h - d*g)*Piec
ewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/(a*d - b*c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{2\left(b^2eg-abfg-abeh+a^2fh\right)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^2c-abd)\sqrt{-b^2e+abf}} - \frac{2\left(d^2eg-cdfg-cdeh+c^2fh\right)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(bcd-ad^2)\sqrt{-d^2e+cdf}} + \frac{2\sqrt{fx+e}h}{bd}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^2*e*g - a*b*f*g - a*b*e*h + a^2*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2
*e + a*b*f))/((b^2*c - a*b*d)*sqrt(-b^2*e + a*b*f)) - 2*(d^2*e*g - c*d*f*g
 - c*d*e*h + c^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b*c*d
 - a*d^2)*sqrt(-d^2*e + c*d*f)) + 2*sqrt(f*x + e)*h/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 6793, normalized size of antiderivative = 45.59

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)*(c + d*x)),x)
```

output
```
(2*h*(e + f*x)^(1/2))/(b*d) - (atan(((((((8*(a*b^4*c^3*d^2*f^4*h + a^3*b^2
*c*d^4*f^4*h - a^3*b^2*d^5*e*f^3*h - b^5*c^3*d^2*e*f^3*h - 2*a^2*b^3*c^2*d
^3*f^4*h + a^2*b^3*d^5*e^2*f^2*h + b^5*c^2*d^3*e^2*f^2*h - 2*a*b^4*c*d^4*e
^2*f^2*h + a*b^4*c^2*d^3*e*f^3*h + a^2*b^3*c*d^4*e*f^3*h))/(b*d) - (8*(e +
 f*x)^(1/2)*(a*h - b*g)*(-b^3*(a*f - b*e))^(1/2)*(a*b^5*c^2*d^4*f^3 - b^6*
c^3*d^3*f^3 - a^3*b^3*d^6*f^3 + a^2*b^4*c*d^5*f^3 + 2*a^2*b^4*d^6*e*f^2 +
2*b^6*c^2*d^4*e*f^2 - 4*a*b^5*c*d^5*e*f^2))/(b*d*(b^4*c - a*b^3*d)))*(a*h
- b*g)*(-b^3*(a*f - b*e))^(1/2))/(b^4*c - a*b^3*d) + (8*(e + f*x)^(1/2)*(a
^4*d^4*f^4*h^2 + b^4*c^4*f^4*h^2 + a^2*b^2*d^4*f^4*g^2 + b^4*c^2*d^2*f^4*g
^2 + 2*b^4*d^4*e^2*f^2*g^2 - 2*a*b^3*d^4*e*f^3*g^2 - 2*a^3*b*d^4*e*f^3*h^2
 - 2*b^4*c*d^3*e*f^3*g^2 - 2*b^4*c^3*d*e*f^3*h^2 - 2*a^3*b*d^4*f^4*g*h - 2
*b^4*c^3*d*f^4*g*h + a^2*b^2*d^4*e^2*f^2*h^2 + b^4*c^2*d^2*e^2*f^2*h^2 - 2
*a*b^3*d^4*e^2*f^2*g*h + 4*a^2*b^2*d^4*e*f^3*g*h - 2*b^4*c*d^3*e^2*f^2*g*h
 + 4*b^4*c^2*d^2*e*f^3*g*h))/(b*d))*(a*h - b*g)*(-b^3*(a*f - b*e))^(1/2)*1
i)/(b^4*c - a*b^3*d) - (((((8*(a*b^4*c^3*d^2*f^4*h + a^3*b^2*c*d^4*f^4*h -
 a^3*b^2*d^5*e*f^3*h - b^5*c^3*d^2*e*f^3*h - 2*a^2*b^3*c^2*d^3*f^4*h + a^2
*b^3*d^5*e^2*f^2*h + b^5*c^2*d^3*e^2*f^2*h - 2*a*b^4*c*d^4*e^2*f^2*h + a*b
^4*c^2*d^3*e*f^3*h + a^2*b^3*c*d^4*e*f^3*h))/(b*d) + (8*(e + f*x)^(1/2)*(a
*h - b*g)*(-b^3*(a*f - b*e))^(1/2)*(a*b^5*c^2*d^4*f^3 - b^6*c^3*d^3*f^3 -
a^3*b^3*d^6*f^3 + a^2*b^4*c*d^5*f^3 + 2*a^2*b^4*d^6*e*f^2 + 2*b^6*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a\,d^2h+2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)b\,d^2g+2\sqrt{d}\,\sqrt{cf-de}\,atan\Big(\frac{}{\sqrt{}}}{b^2d^2\,(ad-bc}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
(2*( - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a*d**2*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*b*d**2*g + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d
)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c*h - sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*d*g + sqrt(e + f*x)*a*b*d**2*
h - sqrt(e + f*x)*b**2*c*d*h))/(b**2*d**2*(a*d - b*c))
```

3.38 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)} dx$

Optimal result

Integrand size = 29, antiderivative size = 203

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = -\frac{(bg-ah)\sqrt{e+fx}}{b(bc-ad)(a+bx)} - \frac{(a^2dfh+abf(dg-3ch)-b^2(2deg-cfg-2ceh))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(bc-ad)^2\sqrt{be-af}} - \frac{2\sqrt{de-cf}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)^2}$$

output
```
-(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)-(a^2*d*f*h+a*b*f*(-3*c*h+d*
g)-b^2*(-2*c*e*h-c*f*g+2*d*e*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^
(1/2))/b^(3/2)/(-a*d+b*c)^2/(-a*f+b*e)^(1/2)-2*(-c*f+d*e)^(1/2)*(-c*h+d*g)
*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \frac{\frac{(bc-ad)(-bg+ah)\sqrt{e+fx}}{b(a+bx)}+\frac{(a^2dfh+abf(dg-3ch)+b^2(-2deg+cfg+2ceh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{3/2}\sqrt{-be+af}}-\frac{2\sqrt{-de+cf}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}}}{(bc-ad)^2}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^2*(c + d*x)),x]
```

output
```
(((b*c - a*d)*(-(b*g) + a*h)*Sqrt[e + f*x])/(b*(a + b*x)) + ((a^2*d*f*h +
a*b*f*(d*g - 3*c*h) + b^2*(-2*d*e*g + c*f*g + 2*c*e*h))*ArcTan[(Sqrt[b]*Sq
rt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(3/2)*Sqrt[-(b*e) + a*f]) - (2*Sqrt[-
(d*e) + c*f]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]
])/Sqrt[d])/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{acfh+b(2deg-cfg-2ceh)+f(bdg-2bch+adh)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2bdeg+acfh-bc(fg+2eh)+f(bdg-2bch+adh)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$-\frac{-\frac{\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{2b(de-cf)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{2\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{4b(de-cf)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{\sqrt{b}(bc-ad)\sqrt{be-af}}+\frac{4b\sqrt{de-cf}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^2*(c + d*x)),x]
```

output
```
-(((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x))) - ((2*(a^2*d*f*h
+ a*b*f*(d*g - 3*c*h) - b^2*(2*d*e*g - c*f*g - 2*c*e*h))*ArcTanh[(Sqrt[b]*
Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f]) + (
4*b*Sqrt[d*e - c*f]*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e -
 c*f]])/(Sqrt[d]*(b*c - a*d)))/(2*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.10

method	result
derivativedivides	$2f\left(\frac{(cf-de)(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f(ad-bc)^2\sqrt{(cf-de)d}}-\frac{\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2b((fx+e)b+af-be)}-\frac{\left(a^2dfh-3abcfh+abdfg+2b^2ceh+b^2c\right.}{2b\sqrt{(af}}}{(ad-bc)^2f}\right.$
default	$2f\left(\frac{(cf-de)(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f(ad-bc)^2\sqrt{(cf-de)d}}-\frac{\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2b((fx+e)b+af-be)}-\frac{\left(a^2dfh-3abcfh+abdfg+2b^2ceh+b^2c\right.}{2b\sqrt{(af}}}{(ad-bc)^2f}\right.$
pseudoelliptic	$-\frac{-(bx+a)\left((2ceh+cfg-2deg)b^2-3a\left(ch-\frac{dg}{3}\right)fb+a^2dfh\right)\sqrt{(cf-de)d}\,\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\sqrt{(af-be)b}\left(-2b(ch-\right.}{\sqrt{(af-be)b}\,\sqrt{(cf-de)d}\,b(bx+a)(ad-}$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*f*((c*f-d*e)/f*(c*h-d*g)/(a*d-b*c)^2/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e
)^(1/2)/((c*f-d*e)*d)^(1/2))-1/(a*d-b*c)^2/f*(1/2*f*(a^2*d*h-a*b*c*h-a*b*d
*g+b^2*c*g)/b*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)-1/2*(a^2*d*f*h-3*a*b*c*f*h
+a*b*d*f*g+2*b^2*c*e*h+b^2*c*f*g-2*b^2*d*e*g)/b/((a*f-b*e)*b)^(1/2)*arctan
(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(181) = 362$.

Time = 1.41 (sec) , antiderivative size = 2100, normalized size of antiderivative = 10.34

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b^2*e - a*b*f)*((2*a*b^2*d*e - (a*b^2*c + a^2*b*d)*f)*g - (2*a
*b^2*c*e - (3*a^2*b*c - a^3*d)*f)*h + ((2*b^3*d*e - (b^3*c + a*b^2*d)*f)*g
 - (2*b^3*c*e - (3*a*b^2*c - a^2*b*d)*f)*h)*x)*log((b*f*x + 2*b*e - a*f -
2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*((a*b^3*d*e - a^2*b^2*
d*f)*g - (a*b^3*c*e - a^2*b^2*c*f)*h + ((b^4*d*e - a*b^3*d*f)*g - (b^4*c*e
 - a*b^3*c*f)*h)*x)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(
f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)) + 2*(((b^4*c - a*b^3*d)*e - (a*
b^3*c - a^2*b^2*d)*f)*g - ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)
*f)*h)*sqrt(f*x + e))/((a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e - (a^2*
b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*f + ((b^6*c^2 - 2*a*b^5*c*d + a^2*b
^4*d^2)*e - (a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*f)*x), -(sqrt(-b^2*e
 + a*b*f)*((2*a*b^2*d*e - (a*b^2*c + a^2*b*d)*f)*g - (2*a*b^2*c*e - (3*a^2
*b*c - a^3*d)*f)*h + ((2*b^3*d*e - (b^3*c + a*b^2*d)*f)*g - (2*b^3*c*e - (
3*a*b^2*c - a^2*b*d)*f)*h)*x)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x + e)/(b
*f*x + b*e)) + ((a*b^3*d*e - a^2*b^2*d*f)*g - (a*b^3*c*e - a^2*b^2*c*f)*h
+ ((b^4*d*e - a*b^3*d*f)*g - (b^4*c*e - a*b^3*c*f)*h)*x)*sqrt((d*e - c*f)/
d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x
+ c)) + (((b^4*c - a*b^3*d)*e - (a*b^3*c - a^2*b^2*d)*f)*g - ((a*b^3*c - a
^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*h)*sqrt(f*x + e))/((a*b^5*c^2 - 2*a
^2*b^4*c*d + a^3*b^3*d^2)*e - (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**2/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

$$\begin{aligned}&\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx\\&= -\frac{(2\,b^2deg - b^2cfg - abdfg - 2\,b^2ceh + 3\,abcfh - a^2dfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)\sqrt{-b^2e+abf}}\\&\quad + \frac{2\,(d^2eg - cdfg - cdeh + c^2fh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-d^2e+cdf}}\\&\quad - \frac{\sqrt{fx+e}bfg - \sqrt{fx+e}afh}{(b^2c-abd)((fx+e)b - be + af)}\end{aligned}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
-(2*b^2*d*e*g - b^2*c*f*g - a*b*d*f*g - 2*b^2*c*e*h + 3*a*b*c*f*h - a^2*d*
f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*sqrt(-b^2*e + a*b*f)) + 2*(d^2*e*g - c*d*f*g - c*d*e*h + c^2*
f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)*b*f*g - sqrt(f*x + e)*a*f*
h)/((b^2*c - a*b*d)*((f*x + e)*b - b*e + a*f))
```

Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 11558, normalized size of antiderivative = 56.94

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^2*(c + d*x)),x)
```

output
```
(atan(((((2*(e + f*x)^(1/2)*(a^4*d^5*f^4*h^2 + a^2*b^2*d^5*f^4*g^2 + 5*b^4
*c^2*d^3*f^4*g^2 + 8*b^4*d^5*e^2*f^2*g^2 + 4*b^4*c^4*d*f^4*h^2 + 2*a*b^3*c
*d^4*f^4*g^2 - 6*a^3*b*c*d^4*f^4*h^2 - 4*a*b^3*d^5*e*f^3*g^2 - 12*b^4*c*d^
4*e*f^3*g^2 - 8*b^4*c^3*d^2*f^4*g*h - 8*b^4*c^3*d^2*e*f^3*h^2 + 2*a^3*b*d^
5*f^4*g*h + 9*a^2*b^2*c^2*d^3*f^4*h^2 + 8*b^4*c^2*d^3*e^2*f^2*h^2 - 6*a*b^
3*c^2*d^3*f^4*g*h - 4*a^2*b^2*c*d^4*f^4*g*h - 4*a^2*b^2*d^5*e*f^3*g*h - 16
*b^4*c*d^4*e^2*f^2*g*h + 20*b^4*c^2*d^3*e*f^3*g*h - 12*a*b^3*c^2*d^3*e*f^3
*h^2 + 4*a^2*b^2*c*d^4*e*f^3*h^2 + 16*a*b^3*c*d^4*e*f^3*g*h))/(b^3*c^2 + a
^2*b*d^2 - 2*a*b^2*c*d) + ((-d*(c*f - d*e))^(1/2)*((2*(2*b^7*c^5*d^2*f^4*g
 - 8*a*b^6*c^4*d^3*f^4*g + 2*a^4*b^3*c*d^6*f^4*g - 2*a*b^6*c^5*d^2*f^4*h -
 2*a^5*b^2*c*d^6*f^4*h - 2*a^4*b^3*d^7*e*f^3*g + 2*a^5*b^2*d^7*e*f^3*h - 2
*b^7*c^4*d^3*e*f^3*g + 12*a^2*b^5*c^3*d^4*f^4*g - 8*a^3*b^4*c^2*d^5*f^4*g
+ 8*a^2*b^5*c^4*d^3*f^4*h - 12*a^3*b^4*c^3*d^4*f^4*h + 8*a^4*b^3*c^2*d^5*f
^4*h + 8*a*b^6*c^3*d^4*e*f^3*g + 8*a^3*b^4*c*d^6*e*f^3*g + 2*a*b^6*c^4*d^3
*e*f^3*h - 8*a^4*b^3*c*d^6*e*f^3*h - 12*a^2*b^5*c^2*d^5*e*f^3*g - 8*a^2*b^
5*c^3*d^4*e*f^3*h + 12*a^3*b^4*c^2*d^5*e*f^3*h))/(b^4*c^3 - a^3*b*d^3 + 3*
a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + (2*(e + f*x)^(1/2)*(-d*(c*f - d*e))^(1/2)
*(c*h - d*g)*(4*a^5*b^3*d^7*f^3 + 4*b^8*c^5*d^2*f^3 + 8*a^2*b^6*c^3*d^4*f^
3 + 8*a^3*b^5*c^2*d^5*f^3 - 12*a*b^7*c^4*d^3*f^3 - 12*a^4*b^4*c*d^6*f^3 -
8*a^4*b^4*d^7*e*f^2 - 8*b^8*c^4*d^3*e*f^2 + 32*a*b^7*c^3*d^4*e*f^2 + 32...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1168, normalized size of antiderivative = 5.75

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x)
```

output
```
(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**3*d**2*f*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**2*b*c*d*f*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**2*f*g + sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**2*f*h*x +
2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a*b**2*c*d*e*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a*b**2*c*d*f*g - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d*f*h*x - 2*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d**2*e*g
 + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a*b**2*d**2*f*g*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*b**3*c*d*e*h*x + sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c*d*f*g*x - 2*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*d**2*e*g
*x + 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**2*b**2*c*f*h - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*a**2*b**2*d*f*g - 2*sqrt(d)*sqrt(c*f - d*e)*ata
n((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**3*c*e*h + 2*sqrt(d)*sq
rt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**3*...
```

3.39 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 355

$$\begin{aligned}\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx &= -\frac{(bg-ah)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2} \\ &\quad - \frac{(a^2dfh+abf(3dg-5ch)-b^2(4deg-cfg-4ceh))\sqrt{e+fx}}{4b(bc-ad)^2(be-af)(a+bx)} \\ &\quad - \frac{(a^3d^2f^2h+3a^2bdf^2(dg-2ch)-3ab^2f(4d^2eg+c^2fh-2cd(fg+2eh))+b^3(8d^2e^2g-c^2f(fg-4eh)}{4b^{3/2}(bc-ad)^3(be-af)^{3/2}} \\ &\quad + \frac{2\sqrt{d}\sqrt{de-cf}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^3}\end{aligned}$$

output

```
-1/2*(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)^2-1/4*(a^2*d*f*h+a*b*f*
(-5*c*h+3*d*g)-b^2*(-4*c*e*h-c*f*g+4*d*e*g))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/
(-a*f+b*e)/(b*x+a)-1/4*(a^3*d^2*f^2*h+3*a^2*b*d*f^2*(-2*c*h+d*g)-3*a*b^2*f
*(4*d^2*e*g+c^2*f*h-2*c*d*(2*e*h+f*g))+b^3*(8*d^2*e^2*g-c^2*f*(-4*e*h+f*g)
-4*c*d*e*(2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(
3/2)/(-a*d+b*c)^3/(-a*f+b*e)^(3/2)+2*d^(1/2)*(-c*f+d*e)^(1/2)*(-c*h+d*g)*a
rctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx =$$

$$-\frac{\frac{(bc-ad)\sqrt{e+fx}(-a^3dfh-ab^2(6deg+cfg-2ceh-3dfgx+5cfhx)+b^3(-4degx+cfgx+2ce(g+2hx))+a^2b(-3cfh+d(5fg+2eh+fhx)))}{b(be-af)(a+bx)^2}+\frac{(}{}}{}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^3*(c + d*x)),x]
```

output
```
-1/4*(((b*c - a*d)*Sqrt[e + f*x]*(-(a^3*d*f*h) - a*b^2*(6*d*e*g + c*f*g -
2*c*e*h - 3*d*f*g*x + 5*c*f*h*x) + b^3*(-4*d*e*g*x + c*f*g*x + 2*c*e*(g +
2*h*x)) + a^2*b*(-3*c*f*h + d*(5*f*g + 2*e*h + f*h*x))))/(b*(b*e - a*f)*(a
 + b*x)^2) + ((a^3*d^2*f^2*h + 3*a^2*b*d*f^2*(d*g - 2*c*h) - 3*a*b^2*f*(4*
d^2*e*g + c^2*f*h - 2*c*d*(f*g + 2*e*h)) + b^3*(8*d^2*e^2*g - 4*c*d*e*(f*g
 + 2*e*h) + c^2*f*(-(f*g) + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-
(b*e) + a*f]])/(b^(3/2)*(-(b*e) + a*f)^(3/2)) - 8*Sqrt[d]*Sqrt[-(d*e) + c*
f]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(b*c -
a*d)^3
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{acfh+b(4deg-cfg-4ceh)+f(3bdg-4bch+adh)x}{2(a+bx)^2(c+dx)\sqrt{e+fx}}dx}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{4bdeg+acfh-bc(fg+4eh)+f(3bdg-4bch+adh)x}{(a+bx)^2(c+dx)\sqrt{e+fx}}dx}{4b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow 168$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(3dg-5ch)-b^2(-4ceh-cfg+4deg)\right)}{(a+bx)(bc-ad)(be-af)}-\frac{\int -\frac{-\left(\left(-f(fg-4eh)c^2-4de(fg+2eh)c+8d^2e^2g\right)b^2\right)+af\left(3fhc^2-d(5fg+8eh)c+8d^2eg\right)b \cdots}{2(a+bx)(c+dx)\sqrt{e+fx}}}{(bc-ad)(be-af)}}{4b(bc-ad)} \cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\int \frac{-\left(\left(-f(fg-4eh)c^2-4de(fg+2eh)c+8d^2e^2g\right)b^2\right)+af\left(3fhc^2-d(5fg+8eh)c+8d^2eg\right)b+a^2cdf^2h+df\left(dfha^2+bf(3dg-5ch)a-b^2(4deg-cfg-4ceh)\right)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}+\cdots}{4b(bc-ad)} \cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow 174$$

$$-\frac{\frac{\frac{8bd(be-af)(de-cf)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(a^3d^2f^2h+3a^2bdf^2(dg-2ch)-3ab^2f\left(c^2fh-2cd(2eh+fg)+4d^2eg\right)+b^3\left(c^2(-f)(fg-4eh)-4cde(2eh+fg)+\cdots\right.\right.}{bc-ad}}{2(bc-ad)(be-af)}\cdots}{4b(bc-ad)} \cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow 73$$

$$-\frac{\frac{\frac{16bd(be-af)(de-cf)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(a^3d^2f^2h+3a^2bdf^2(dg-2ch)-3ab^2f\left(c^2fh-2cd(2eh+fg)+4d^2eg\right)+b^3\left(c^2(-f)(fg-4eh)-4cde\cdots\right.\right.}{f(bc-ad)}}{2(bc-ad)(be-af)}\cdots}{4b(bc-ad)} \cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow 221$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(3dg-5ch)-b^2(-4ceh-cfg+4deg)\right)}{(a+bx)(bc-ad)(be-af)}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(a^3d^2f^2h+3a^2bdf^2(dg-2ch)-3ab^2f\left(c^2fh-2cd(2eh+fg)+4d^2\right.\right.}{\sqrt{b}(bc-ad)\sqrt{be-af}}}{4b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^3*(c + d*x)),x]
```

output
```
-1/2*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^2) - (((a^2*d*f*
h + a*b*f*(3*d*g - 5*c*h) - b^2*(4*d*e*g - c*f*g - 4*c*e*h))*Sqrt[e + f*x]
)/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((2*(a^3*d^2*f^2*h + 3*a^2*b*d*f^2
*(d*g - 2*c*h) - 3*a*b^2*f*(4*d^2*e*g + c^2*f*h - 2*c*d*(f*g + 2*e*h)) + b
^3*(8*d^2*e^2*g - c^2*f*(f*g - 4*e*h) - 4*c*d*e*(f*g + 2*e*h)))*ArcTanh[(S
qrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*
f]) - (16*b*Sqrt[d]*(b*e - a*f)*Sqrt[d*e - c*f]*(d*g - c*h)*ArcTanh[(Sqrt[
d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(b*c - a*d))/(2*(b*c - a*d)*(b*e - a*f
)))/(4*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{-\sqrt{(cf-de)d}\,(bx+a)^2\left(\left(f^2a^3h+3a^2b\,f^2g-12a\,b^2efg+8b^3e^2g\right)d^2-6c\left(\frac{2\left(2e^2h+efg\right)b^2}{3}+a\left(-2efh-f^2g\right)b+a^2f^2h\right.\right.}{}$
derivativedivides	$2f^2\left(\frac{(cf-de)(ch-dg)d\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f^2(ad-bc)^3\sqrt{(cf-de)d}}-\frac{-\frac{f\left(a^3d^2fh-6a^2bcdfh+3a^2b\,d^2fg+5a\,b^2c^2fh+4a\,b^2cdeh-2a\,b^2cdfg-\right.}{8(af-be)}}{}\right.$
default	$2f^2\left(\frac{(cf-de)(ch-dg)d\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f^2(ad-bc)^3\sqrt{(cf-de)d}}-\frac{-\frac{f\left(a^3d^2fh-6a^2bcdfh+3a^2b\,d^2fg+5a\,b^2c^2fh+4a\,b^2cdeh-2a\,b^2cdfg-\right.}{8(af-be)}}{}\right.$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*(-((c*f-d*e)*d)^(1/2)*(b*x+a)
^2*((a^3*f^2*h+3*a^2*b*f^2*g-12*a*b^2*e*f*g+8*b^3*e^2*g)*d^2-6*c*(2/3*(2*e
^2*h+e*f*g)*b^2+a*(-2*e*f*h-f^2*g)*b+a^2*f^2*h)*b*d-3*c^2*b^2*f*(1/3*(-4*e
*h+f*g)*b+a*f*h))*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b
)^(1/2)*(-8*b*d*(b*x+a)^2*(a*f-b*e)*(c*f-d*e)*(c*h-d*g)*arctan(d*(f*x+e)^(
1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(a*d-b*c)*(f*x+e)^(1/2)*((4*
e*g*x*b^3+6*a*(-1/2*f*x+e)*g*b^2-2*a^2*(1/2*(h*x+5*g)*f+e*h)*b+a^3*f*h)*d+
3*c*(1/3*(-f*g*x-2*e*(2*h*x+g))*b^2-2/3*a*(1/2*(-5*h*x-g)*f+e*h)*b+a^2*f*h
)*b)))/(a*f-b*e)/(a*d-b*c)^3/(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(325) = 650$.

Time = 26.66 (sec) , antiderivative size = 6490, normalized size of antiderivative = 18.28

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**3/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(325) = 650$.

Time = 0.16 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx$$
$$= \frac{(8\,b^3d^2e^2g - 4\,b^3cdefg - 12\,ab^2d^2efg - b^3c^2f^2g + 6\,ab^2cdf^2g + 3\,a^2bd^2f^2g - 8\,b^3cde^2h + 4\,b^3c^2efh + 12}{4\,(b^5c^3e - 3\,ab^4c^2de + 3\,a^2b^3cd^2e - a^3b^2d^3e - ab^4c^3f + 3\,a^2b^3c^2df - 3}$$
$$- \frac{2\,(d^3eg - cd^2fg - cd^2eh + c^2dfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{-d^2e+cdf}}$$
$$+ \frac{4\,(fx+e)^{\frac{3}{2}}b^3defg - 4\,\sqrt{fx+e}b^3de^2fg - (fx+e)^{\frac{3}{2}}b^3cf^2g - 3\,(fx+e)^{\frac{3}{2}}ab^2df^2g - \sqrt{fx+e}b^3cef^2g}{}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g - 4*b^3*c*d*e*f*g - 12*a*b^2*d^2*e*f*g - b^3*c^2*f^2*
g + 6*a*b^2*c*d*f^2*g + 3*a^2*b*d^2*f^2*g - 8*b^3*c*d*e^2*h + 4*b^3*c^2*e*
f*h + 12*a*b^2*c*d*e*f*h - 3*a*b^2*c^2*f^2*h - 6*a^2*b*c*d*f^2*h + a^3*d^2
*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*c^3*e - 3*a*b^4
*c^2*d*e + 3*a^2*b^3*c*d^2*e - a^3*b^2*d^3*e - a*b^4*c^3*f + 3*a^2*b^3*c^2
*d*f - 3*a^3*b^2*c*d^2*f + a^4*b*d^3*f)*sqrt(-b^2*e + a*b*f)) - 2*(d^3*e*g
 - c*d^2*f*g - c*d^2*e*h + c^2*d*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e +
 c*d*f))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-d^2*e
+ c*d*f)) + 1/4*(4*(f*x + e)^(3/2)*b^3*d*e*f*g - 4*sqrt(f*x + e)*b^3*d*e^2
*f*g - (f*x + e)^(3/2)*b^3*c*f^2*g - 3*(f*x + e)^(3/2)*a*b^2*d*f^2*g - sqr
t(f*x + e)*b^3*c*e*f^2*g + 9*sqrt(f*x + e)*a*b^2*d*e*f^2*g + sqrt(f*x + e)
*a*b^2*c*f^3*g - 5*sqrt(f*x + e)*a^2*b*d*f^3*g - 4*(f*x + e)^(3/2)*b^3*c*e
*f*h + 4*sqrt(f*x + e)*b^3*c*e^2*f*h + 5*(f*x + e)^(3/2)*a*b^2*c*f^2*h - (
f*x + e)^(3/2)*a^2*b*d*f^2*h - 7*sqrt(f*x + e)*a*b^2*c*e*f^2*h - sqrt(f*x
+ e)*a^2*b*d*e*f^2*h + 3*sqrt(f*x + e)*a^2*b*c*f^3*h + sqrt(f*x + e)*a^3*d
*f^3*h)/((b^4*c^2*e - 2*a*b^3*c*d*e + a^2*b^2*d^2*e - a*b^3*c^2*f + 2*a^2*
b^2*c*d*f - a^3*b*d^2*f)*((f*x + e)*b - b*e + a*f)^2)
```

Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 278260, normalized size of antiderivative = 783.83

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^3*(c + d*x)),x)
```

output
```
atan(((((8*a*b^10*c^8*d^2*f^6*g - 40*a^8*b^3*c*d^9*f^6*g + 8*a^9*b^2*c*d^9
*f^6*h + 40*a^8*b^3*d^10*e*f^5*g - 8*a^9*b^2*d^10*e*f^5*h - 8*b^11*c^8*d^2
*e*f^5*g - 88*a^2*b^9*c^7*d^3*f^6*g + 360*a^3*b^8*c^6*d^4*f^6*g - 760*a^4*
b^7*c^5*d^5*f^6*g + 920*a^5*b^6*c^4*d^6*f^6*g - 648*a^6*b^5*c^3*d^7*f^6*g
+ 248*a^7*b^4*c^2*d^8*f^6*g + 24*a^2*b^9*c^8*d^2*f^6*h - 136*a^3*b^8*c^7*d
^3*f^6*h + 312*a^4*b^7*c^6*d^4*f^6*h - 360*a^5*b^6*c^5*d^5*f^6*h + 200*a^6
*b^5*c^4*d^6*f^6*h - 24*a^7*b^4*c^3*d^7*f^6*h - 24*a^8*b^3*c^2*d^8*f^6*h +
 32*a^6*b^5*d^10*e^3*f^3*g - 72*a^7*b^4*d^10*e^2*f^4*g + 8*a^8*b^3*d^10*e^
2*f^4*h + 32*b^11*c^6*d^4*e^3*f^3*g - 24*b^11*c^7*d^3*e^2*f^4*g - 32*b^11*
c^7*d^3*e^3*f^3*h + 32*b^11*c^8*d^2*e^2*f^4*h + 480*a^2*b^9*c^4*d^6*e^3*f^
3*g + 72*a^2*b^9*c^5*d^5*e^2*f^4*g - 640*a^3*b^8*c^3*d^7*e^3*f^3*g - 600*a
^3*b^8*c^4*d^6*e^2*f^4*g + 480*a^4*b^7*c^2*d^8*e^3*f^3*g + 1080*a^4*b^7*c^
3*d^7*e^2*f^4*g - 936*a^5*b^6*c^2*d^8*e^2*f^4*g - 480*a^2*b^9*c^5*d^5*e^3*
f^3*h + 152*a^2*b^9*c^6*d^4*e^2*f^4*h + 640*a^3*b^8*c^4*d^6*e^3*f^3*h + 15
2*a^3*b^8*c^5*d^5*e^2*f^4*h - 480*a^4*b^7*c^3*d^7*e^3*f^3*h - 520*a^4*b^7*
c^4*d^6*e^2*f^4*h + 192*a^5*b^6*c^2*d^8*e^3*f^3*h + 488*a^5*b^6*c^3*d^7*e^
2*f^4*h - 184*a^6*b^5*c^2*d^8*e^2*f^4*h + 112*a*b^10*c^7*d^3*e*f^5*g - 176
*a^7*b^4*c*d^9*e*f^5*g - 56*a*b^10*c^8*d^2*e*f^5*h + 16*a^8*b^3*c*d^9*e*f^
5*h - 192*a*b^10*c^5*d^5*e^3*f^3*g + 72*a*b^10*c^6*d^4*e^2*f^4*g - 464*a^2
*b^9*c^6*d^4*e*f^5*g + 880*a^3*b^8*c^5*d^5*e*f^5*g - 800*a^4*b^7*c^4*d^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3843, normalized size of antiderivative = 10.83

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x)
```

output
```
(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**5*d**2*f**2*h - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**4*b*c*d*f**2*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*f**2*g + 2*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*
d**2*f**2*h*x - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**3*b**2*c**2*f**2*h + 12*sqrt(b)*sqrt(a*f - b*e)*atan(
(sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*e*f*h + 6*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*
b**2*c*d*f**2*g - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*f**2*h*x - 12*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*e*f*g + 6*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**3*b**2*d**2*f**2*g*x + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*f**2*h*x**2 + 4*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c**2*e*
f*h - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**2*b**3*c**2*f**2*g - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c**2*f**2*h*x - 8*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c...
```

3.40 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 588

$$
\begin{aligned}
\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx &= -\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3} \\
&- \frac{\left(a^2dfh+abf(5dg-7ch)-b^2(6deg-cfg-6ceh)\right)\sqrt{e+fx}}{12b(bc-ad)^2(be-af)(a+bx)^2} \\
&- \frac{\left(a^3d^2f^2h+a^2bdf^2(5dg-8ch)+b^3(8d^2e^2g-c^2f(fg-2eh)-2cde(fg+4eh))-ab^2f(14d^2eg+c^2f\right.}{8b(bc-ad)^3(be-af)^2(a+bx)} \\
&- \frac{\left(a^4d^3f^3h+a^3bd^2f^3(5dg-9ch)-3a^2b^2df^2(10d^2eg+3c^2fh-5cd(fg+2eh))+ab^3f(40d^3e^2g+c^3f^2\right.}{} \\
&- \frac{2d^{3/2}\sqrt{de-cf}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4}
\end{aligned}
$$

output
```
-1/3*(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)^3-1/12*(a^2*d*f*h+a*b*f
*(-7*c*h+5*d*g)-b^2*(-6*c*e*h-c*f*g+6*d*e*g))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2
/(-a*f+b*e)/(b*x+a)^2-1/8*(a^3*d^2*f^2*h+a^2*b*d*f^2*(-8*c*h+5*d*g)+b^3*(8
*d^2*e^2*g-c^2*f*(-2*e*h+f*g)-2*c*d*e*(4*e*h+f*g))-a*b^2*f*(14*d^2*e*g+c^2
*f*h-2*c*d*(7*e*h+2*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+b*e)^2/(b*x+
a)-1/8*(a^4*d^3*f^3*h+a^3*b*d^2*f^3*(-9*c*h+5*d*g)-3*a^2*b^2*d*f^2*(10*d^2
*e*g+3*c^2*f*h-5*c*d*(2*e*h+f*g))+a*b^3*f*(40*d^3*e^2*g+c^3*f^2*h-5*c^2*d*
f*(-4*e*h+f*g)-20*c*d^2*e*(2*e*h+f*g))-b^4*(16*d^3*e^3*g-2*c^2*d*e*f*(-4*e
*h+f*g)-c^3*f^2*(-2*e*h+f*g)-8*c*d^2*e^2*(2*e*h+f*g)))*arctanh(b^(1/2)*(f*
x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(3/2)/(-a*d+b*c)^4/(-a*f+b*e)^(5/2)-2*d^(3/
2)*(-c*f+d*e)^(1/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1
/2))/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx$$

$$= \frac{(bc-ad)\sqrt{e+fx}\left(3a^5d^2f^2h-a^4bdf(-24cfh+d(33fg+14eh+8fhx))+a^3b^2\left(-3c^2f^2h+d^2(2e-fx)(40fg+4eh+3fhx)+2cdf(6fg-19eh+32fhx)\right.\right.}{}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^4*(c + d*x)),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(3*a^5*d^2*f^2*h - a^4*b*d*f*(-24*c*f*h + d*(3
3*f*g + 14*e*h + 8*f*h*x)) + a^3*b^2*(-3*c^2*f^2*h + d^2*(2*e - f*x)*(40*f
*g + 4*e*h + 3*f*h*x) + 2*c*d*f*(6*f*g - 19*e*h + 32*f*h*x)) + b^5*(-24*d^
2*e^2*g*x^2 + 6*c*d*e*x*(f*g*x + 2*e*(g + 2*h*x)) - c^2*(-3*f^2*g*x^2 + 2*
e*f*x*(g + 3*h*x) + 4*e^2*(2*g + 3*h*x))) + a*b^4*(6*d^2*e*g*x*(-10*e + 7*
f*x) + c^2*(-4*e^2*h + 14*e*f*(g + h*x) + f^2*x*(8*g + 3*h*x)) + 2*c*d*(-6
*f^2*g*x^2 + 2*e^2*(7*g + 15*h*x) - e*f*x*(4*g + 21*h*x))) - a^2*b^3*(c^2*
f*(3*f*g - 4*e*h + 8*f*h*x) + d^2*g*(44*e^2 - 106*e*f*x + 15*f^2*x^2) + 2*
c*d*(-10*e^2*h + 4*f^2*x*(2*g - 3*h*x) + e*f*(23*g + 56*h*x)))))/(b*(b*e -
 a*f)^2*(a + b*x)^3) - (3*(-(a^4*d^3*f^3*h) + a^3*b*d^2*f^3*(-5*d*g + 9*c*
h) + 3*a^2*b^2*d*f^2*(10*d^2*e*g + 3*c^2*f*h - 5*c*d*(f*g + 2*e*h)) + a*b^
3*f*(-40*d^3*e^2*g - c^3*f^2*h + 5*c^2*d*f*(f*g - 4*e*h) + 20*c*d^2*e*(f*g
 + 2*e*h)) + b^4*(16*d^3*e^3*g + c^3*f^2*(-(f*g) + 2*e*h) - 8*c*d^2*e^2*(f
*g + 2*e*h) + 2*c^2*d*e*f*(-(f*g) + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x]
)/Sqrt[-(b*e) + a*f]])/(b^(3/2)*(-(b*e) + a*f)^(5/2)) - 48*d^(3/2)*Sqrt[-(
d*e) + c*f]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]]
)/(24*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {166, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{acfh+b(6deg-cfg-6ceh)+f(5bdg-6bch+adh)x}{2(a+bx)^3(c+dx)\sqrt{e+fx}}dx}{3b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{6bdeg+acfh-bc(fg+6eh)+f(5bdg-6bch+adh)x}{(a+bx)^3(c+dx)\sqrt{e+fx}}dx}{6b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(5dg-7ch)-b^2(-6ceh-cfg+6deg)\right)}{2(a+bx)^2(bc-ad)(be-af)}-\frac{\int -\frac{3\left(-\left(\left(-f(fg-2eh)c^2-2de(fg+4eh)c+8d^2e^2g\right)b^2\right)+af\left(fhc^2-d(3fg+8eh)c+8d^2eg\right)\right.}{2(a+bx)^2(c+dx)\sqrt{e+f}}}{2(bc-ad)(be-af)}}{6b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 27

$$-\frac{\frac{3\int \frac{-\left(\left(-f(fg-2eh)c^2-2de(fg+4eh)c+8d^2e^2g\right)b^2\right)+af\left(fhc^2-d(3fg+8eh)c+8d^2eg\right)b+a^2cdf^2h+df\left(dfha^2+bf(5dg-7ch)a-b^2(6deg-cfg-6ceh)\right)x}{(a+bx)^2(c+dx)\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}+}{6b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 168

$$-\frac{3\left(\frac{\sqrt{e+fx}\left(a^3d^2f^2h+a^2bdf^2(5dg-8ch)-ab^2f\left(c^2fh-2cd(7eh+2fg)+14d^2eg\right)+b^3\left(c^2(-f)(fg-2eh)-2cde(4eh+fg)+8d^2e^2g\right)\right)}{(a+bx)(bc-ad)(be-af)}-\frac{\int -\frac{\left(-f^2(fg-2eh)c^3-2d\right.}{}}{}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 27

$$-\frac{3\left(\frac{\int \frac{\left(-f^2(fg-2eh)c^3-2def(fg-4eh)c^2-8d^2e^2(fg+2eh)c+16d^3e^3g\right)b^3-af\left(f^2hc^3-2df(2fg-9eh)c^2-2d^2e(9fg+16eh)c+32d^3e^2g\right)b^2+a^2df^2\left(8fhc^2-d(1\right.}{(a+bx)(}}{2(bc-}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 174

$$-\frac{3\left(\frac{-\frac{\left(a^4d^3f^3h+a^3bd^2f^3(5dg-9ch)-3a^2b^2df^2\left(3c^2fh-5cd(2eh+fg)+10d^2eg\right)+ab^3f\left(c^3f^2h-5c^2df(fg-4eh)-20cd^2e(2eh+fg)+40d^3e^2g\right)-b^4\left(c^3\left(-f^2\right)(\right.}{bc-ad}}{2(bc-ad)(be-af)}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$-\frac{3\left(-\frac{\frac{2\left(a^4d^3f^3h+a^3bd^2f^3(5dg-9ch)-3a^2b^2df^2\left(3c^2fh-5cd(2eh+fg)+10d^2eg\right)+ab^3f\left(c^3f^2h-5c^2df(fg-4eh)-20cd^2e(2eh+fg)+40d^3e^2g\right)-b^4\left(c^3\left(-f^2\right)\right.\right.}{f(bc-ad)}}{2(bc-ad)(b}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(5dg-7ch)-b^2(-6ceh-cfg+6deg)\right)}{2(a+bx)^2(bc-ad)(be-af)}+\frac{3\left(\frac{\sqrt{e+fx}\left(a^3d^2f^2h+a^2bdf^2(5dg-8ch)-ab^2f\left(c^2fh-2cd(7eh+2fg)+14d^2eg\right)+b^3\left(c^2(-\right.\right.}{(a+bx)(bc-ad)(be-af)}\right.}{}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^4*(c + d*x)),x]
```

output
```
-1/3*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^3) - (((a^2*d*f*
h + a*b*f*(5*d*g - 7*c*h) - b^2*(6*d*e*g - c*f*g - 6*c*e*h))*Sqrt[e + f*x]
)/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (3*(((a^3*d^2*f^2*h + a^2*b*d*
f^2*(5*d*g - 8*c*h) + b^3*(8*d^2*e^2*g - c^2*f*(f*g - 2*e*h) - 2*c*d*e*(f*
g + 4*e*h)) - a*b^2*f*(14*d^2*e*g + c^2*f*h - 2*c*d*(2*f*g + 7*e*h)))*Sqrt
[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((2*(a^4*d^3*f^3*h + a^3*
b*d^2*f^3*(5*d*g - 9*c*h) - 3*a^2*b^2*d*f^2*(10*d^2*e*g + 3*c^2*f*h - 5*c*
d*(f*g + 2*e*h)) + a*b^3*f*(40*d^3*e^2*g + c^3*f^2*h - 5*c^2*d*f*(f*g - 4*
e*h) - 20*c*d^2*e*(f*g + 2*e*h)) - b^4*(16*d^3*e^3*g - 2*c^2*d*e*f*(f*g -
4*e*h) - c^3*f^2*(f*g - 2*e*h) - 8*c*d^2*e^2*(f*g + 2*e*h)))*ArcTanh[(Sqrt
[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f])
 + (32*b*d^(3/2)*(b*e - a*f)^2*Sqrt[d*e - c*f]*(d*g - c*h)*ArcTanh[(Sqrt[d
]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(b*c - a*d))/(2*(b*c - a*d)*(b*e - a*f)
)))/(4*(b*c - a*d)*(b*e - a*f)))/(6*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{-\left(\left(\left(a^4h+5a^3bg\right)f^3-30a^2b^2eg\,f^2+40a\,b^3e^2gf-16b^4e^3g\right)d^3-9c\left(a^2\left(ah-\frac{5bg}{3}\right)f^3-\frac{10\left(ah-\frac{2bg}{3}\right)abe\,f^2}{3}+\frac{40b^2\left(ah-\right.}{9}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*f-b*e)*b)^(1/2)*(-(((a^4*h+5*a^3*b*g)*f^3-30*a^2*b^2*e*g*f^2+40*a
*b^3*e^2*g*f-16*b^4*e^3*g)*d^3-9*c*(a^2*(a*h-5/3*b*g)*f^3-10/3*(a*h-2/3*b*
g)*a*b*e*f^2+40/9*b^2*(a*h-1/5*b*g)*e^2*f-16/9*b^3*e^3*h)*b*d^2-9*c^2*(a*(
a*h+5/9*b*g)*f^2-20/9*(a*h+1/10*b*g)*b*e*f+8/9*b^2*e^2*h)*b^2*f*d+c^3*((a*
h+b*g)*f-2*e*h*b)*b^3*f^2)*((c*f-d*e)*d)^(1/2)*(b*x+a)^3*arctan(b*(f*x+e)^
(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(-16*b*d^2*(b*x+a)^3*(a*f-b
*e)^2*(c*f-d*e)*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*
f-d*e)*d)^(1/2)*(a*d-b*c)*(f*x+e)^(1/2)*((a^2*(-5*b^3*g*x^2-40/3*a*x*(3/40
*h*x+g)*b^2-11*a^2*(8/33*h*x+g)*b+h*a^3)*f^2-14/3*a*(-3*b^3*g*x^2-53/7*a*b
^2*g*x-40/7*a^2*(1/40*h*x+g)*b+h*a^3)*b*e*f+8/3*(-3*b^3*g*x^2-15/2*a*b^2*g
*x+h*a^3-11/2*a^2*b*g)*b^2*e^2)*d^2+8*c*b*(a*(-1/2*b^3*g*x^2-2/3*a*x*(-3/2
*h*x+g)*b^2+1/2*a^2*(16/3*h*x+g)*b+h*a^3)*f^2-19/12*(-3/19*b^3*g*x^2+4/19*
(21/4*h*x+g)*a*x*b^2+23/19*(56/23*h*x+g)*a^2*b+h*a^3)*b*e*f+5/6*b^2*(3/5*x
*(2*h*x+g)*b^2+7/5*a*(15/7*h*x+g)*b+a^2*h)*e^2)*d-c^2*((-b^3*g*x^2-8/3*a*x
*(3/8*h*x+g)*b^2+a^2*(8/3*h*x+g)*b+h*a^3)*f^2-4/3*(-1/2*x*(3*h*x+g)*b^2+7/
2*a*(h*x+g)*b+a^2*h)*b*e*f+4/3*((3*h*x+2*g)*b+a*h)*b^2*e^2)*b^2)))/((c*f-d
*e)*d)^(1/2)/(a*f-b*e)^2/(b*x+a)^3/(a*d-b*c)^4/b
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**4/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1939 vs. $2(554) = 1108$.

Time = 0.22 (sec) , antiderivative size = 1939, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x, algorithm="giac")
```

output
```
-1/8*(16*b^4*d^3*e^3*g - 8*b^4*c*d^2*e^2*f*g - 40*a*b^3*d^3*e^2*f*g - 2*b^
4*c^2*d*e*f^2*g + 20*a*b^3*c*d^2*e*f^2*g + 30*a^2*b^2*d^3*e*f^2*g - b^4*c^
3*f^3*g + 5*a*b^3*c^2*d*f^3*g - 15*a^2*b^2*c*d^2*f^3*g - 5*a^3*b*d^3*f^3*g
 - 16*b^4*c*d^2*e^3*h + 8*b^4*c^2*d*e^2*f*h + 40*a*b^3*c*d^2*e^2*f*h + 2*b
^4*c^3*e*f^2*h - 20*a*b^3*c^2*d*e*f^2*h - 30*a^2*b^2*c*d^2*e*f^2*h - a*b^3
*c^3*f^3*h + 9*a^2*b^2*c^2*d*f^3*h + 9*a^3*b*c*d^2*f^3*h - a^4*d^3*f^3*h)*
arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^4*e^2 - 4*a*b^6*c^3*d
*e^2 + 6*a^2*b^5*c^2*d^2*e^2 - 4*a^3*b^4*c*d^3*e^2 + a^4*b^3*d^4*e^2 - 2*a
*b^6*c^4*e*f + 8*a^2*b^5*c^3*d*e*f - 12*a^3*b^4*c^2*d^2*e*f + 8*a^4*b^3*c*
d^3*e*f - 2*a^5*b^2*d^4*e*f + a^2*b^5*c^4*f^2 - 4*a^3*b^4*c^3*d*f^2 + 6*a^
4*b^3*c^2*d^2*f^2 - 4*a^5*b^2*c*d^3*f^2 + a^6*b*d^4*f^2)*sqrt(-b^2*e + a*b
*f)) + 2*(d^4*e*g - c*d^3*f*g - c*d^3*e*h + c^2*d^2*f*h)*arctan(sqrt(f*x +
 e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-d^2*e + c*d*f)) - 1/24*(24*(f*x + e)^(5/2
)*b^5*d^2*e^2*f*g - 48*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + 24*sqrt(f*x + e)*
b^5*d^2*e^4*f*g - 6*(f*x + e)^(5/2)*b^5*c*d*e*f^2*g - 42*(f*x + e)^(5/2)*a
*b^4*d^2*e*f^2*g + 144*(f*x + e)^(3/2)*a*b^4*d^2*e^2*f^2*g + 6*sqrt(f*x +
e)*b^5*c*d*e^3*f^2*g - 102*sqrt(f*x + e)*a*b^4*d^2*e^3*f^2*g - 3*(f*x + e)
^(5/2)*b^5*c^2*f^3*g + 12*(f*x + e)^(5/2)*a*b^4*c*d*f^3*g + 15*(f*x + e)^(
5/2)*a^2*b^3*d^2*f^3*g + 8*(f*x + e)^(3/2)*b^5*c^2*e*f^3*g - 16*(f*x + ...
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 533756, normalized size of antiderivative = 907.75

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)} \, dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^4*(c + d*x)),x)
```

output
```
(((e + f*x)^(3/2)*(a^3*d^2*f^3*h - b^3*c^2*f^3*g + a*b^2*c^2*f^3*h + 5*a^2
*b*d^2*f^3*g + 6*b^3*d^2*e^2*f*g + 2*a*b^2*c*d*f^3*g - 8*a^2*b*c*d*f^3*h -
 6*b^3*c*d*e^2*f*h - 12*a*b^2*d^2*e*f^2*g + 12*a*b^2*c*d*e*f^2*h))/(3*(a*d
 - b*c)*(a*f - b*e)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (f*(e + f*x)^(5/2)*
(b^4*c^2*f^2*g - 8*b^4*d^2*e^2*g + 8*b^4*c*d*e^2*h - 2*b^4*c^2*e*f*h + a*b
^3*c^2*f^2*h - a^3*b*d^2*f^2*h - 5*a^2*b^2*d^2*f^2*g + 2*b^4*c*d*e*f*g - 4
*a*b^3*c*d*f^2*g + 14*a*b^3*d^2*e*f*g + 8*a^2*b^2*c*d*f^2*h - 14*a*b^3*c*d
*e*f*h))/(8*(a*f - b*e)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2)) + ((e + f*x)^(1/2)*(b^3*c^2*f^3*g - a^3*d^2*f^3*h + a*b^2*c^2*f^3*h +
 11*a^2*b*d^2*f^3*g - 2*b^3*c^2*e*f^2*h + 8*b^3*d^2*e^2*f*g - 4*a*b^2*c*d*
f^3*g - 8*a^2*b*c*d*f^3*h + 2*b^3*c*d*e*f^2*g - 8*b^3*c*d*e^2*f*h - 18*a*b
^2*d^2*e*f^2*g + 18*a*b^2*c*d*e*f^2*h))/(8*b*(a*d - b*c)*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)))/((e + f*x)*(3*b^3*e^2 + 3*a^2*b*f^2 - 6*a*b^2*e*f) + b^3*
(e + f*x)^3 - (e + f*x)^2*(3*b^3*e - 3*a*b^2*f) + a^3*f^3 - b^3*e^3 + 3*a*
b^2*e^2*f - 3*a^2*b*e*f^2) - atan(((((176*a^12*b^3*c*d^12*f^8*g - 16*a^13*
b^2*c*d^12*f^8*h - 176*a^12*b^3*d^13*e*f^7*g + 16*a^13*b^2*d^13*e*f^7*h +
16*a^2*b^13*c^11*d^2*f^8*g - 192*a^3*b^12*c^10*d^3*f^8*g + 1136*a^4*b^11*c
^9*d^4*f^8*g - 4096*a^5*b^10*c^8*d^5*f^8*g + 9632*a^6*b^9*c^7*d^6*f^8*g -
15232*a^7*b^8*c^6*d^7*f^8*g + 16352*a^8*b^7*c^5*d^8*f^8*g - 11776*a^9*b^6*
c^4*d^9*f^8*g + 5456*a^10*b^5*c^3*d^10*f^8*g - 1472*a^11*b^4*c^2*d^11*f...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9171, normalized size of antiderivative = 15.60

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)} \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x)
```

output

```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**7*d**3*f**3*h - 27*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**2*f**3*h + 15*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**3*f**3*g + 9*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**6*b*d**3*f**3*h*x - 27*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*d*f**3*h + 90*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d**2*e*
f**2*h + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**5*b**2*c*d**2*f**3*g - 81*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d**2*f**3*h*x - 90*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*5*b**2*d**3*e*f**2*g + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**3*f**3*g*x + 9*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**3*f**3
*h*x**2 + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**4*b**3*c**3*f**3*h + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**3*c**2*d*e*f**2*h - 15*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4
*b**3*c**2*d*f**3*g - 81*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)...
```

3.41 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 942

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = -\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4}$$

$$-\frac{\left(a^2dfh+abf(7dg-9ch)-b^2(8deg-cfg-8ceh)\right)\sqrt{e+fx}}{24b(bc-ad)^2(be-af)(a+bx)^3}$$

$$-\frac{(5a^3d^2f^2h+5a^2bdf^2(7dg-10ch)+b^3(48d^2e^2g-c^2f(5fg-8eh)-8cde(fg+6eh))-ab^2f(88d^2eg}{96b(bc-ad)^3(be-af)^2(a+bx)^2}$$

$$-\frac{(5a^4d^3f^3h+5a^3bd^2f^3(7dg-11ch)-b^4(64d^3e^3g-c^3f^2(5fg-8eh)-8c^2def(fg-2eh)-16cd^2e^2(}{}$$

$$-\frac{(5a^5d^4f^4h+5a^4bd^3f^4(7dg-12ch)-10a^3b^2d^2f^3(28d^2eg+9c^2fh-14cd(fg+2eh))+10a^2b^3df^2(56}{}$$

$$+\frac{2d^{5/2}\sqrt{de-cf}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^5}$$

output
```
-1/4*(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)^4-1/24*(a^2*d*f*h+a*b*f
*(-9*c*h+7*d*g)-b^2*(-8*c*e*h-c*f*g+8*d*e*g))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2
/(-a*f+b*e)/(b*x+a)^3-1/96*(5*a^3*d^2*f^2*h+5*a^2*b*d*f^2*(-10*c*h+7*d*g)+
b^3*(48*d^2*e^2*g-c^2*f*(-8*e*h+5*f*g)-8*c*d*e*(6*e*h+f*g))-a*b^2*f*(88*d^
2*e*g+3*c^2*f*h-2*c*d*(44*e*h+9*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+
b*e)^2/(b*x+a)^2-1/64*(5*a^4*d^3*f^3*h+5*a^3*b*d^2*f^3*(-11*c*h+7*d*g)-b^4
*(64*d^3*e^3*g-c^3*f^2*(-8*e*h+5*f*g)-8*c^2*d*e*f*(-2*e*h+f*g)-16*c*d^2*e^
2*(4*e*h+f*g))+a*b^3*f*(176*d^3*e^2*g+3*c^3*f^2*h-c^2*d*f*(-48*e*h+23*f*g)
-16*c*d^2*e*(11*e*h+3*f*g))-a^2*b^2*d*f^2*(152*d^2*e*g+17*c^2*f*h-c*d*(152
*e*h+47*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^4/(-a*f+b*e)^3/(b*x+a)-1/64*(5*a
^5*d^4*f^4*h+5*a^4*b*d^3*f^4*(-12*c*h+7*d*g)-10*a^3*b^2*d^2*f^3*(28*d^2*e*
g+9*c^2*f*h-14*c*d*(2*e*h+f*g))+10*a^2*b^3*d*f^2*(56*d^3*e^2*g+2*c^3*f^2*h
-7*c^2*d*f*(-4*e*h+f*g)-28*c*d^2*e*(2*e*h+f*g))-a*b^4*f*(448*d^4*e^3*g+3*c
^4*f^3*h-56*c^2*d^2*e*f*(-4*e*h+f*g)-28*c^3*d*f^2*(-2*e*h+f*g)-224*c*d^3*e
^2*(2*e*h+f*g))+b^5*(128*d^4*e^4*g-c^4*f^3*(-8*e*h+5*f*g)-16*c^2*d^2*e^2*f
*(-4*e*h+f*g)-8*c^3*d*e*f^2*(-2*e*h+f*g)-64*c*d^3*e^3*(2*e*h+f*g)))*arctan
h(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(3/2)/(-a*d+b*c)^5/(-a*f+b*e)^
(7/2)+2*d^(5/2)*(-c*f+d*e)^(1/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/
(-c*f+d*e)^(1/2))/(-a*d+b*c)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5481 vs. $2(942) = 1884$.

Time = 16.22 (sec) , antiderivative size = 5481, normalized size of antiderivative = 5.82

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^5*(c + d*x)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 1032, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {166, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{acfh+b(8deg-cfg-8ceh)+f(7bdg-8bch+adh)x}{2(a+bx)^4(c+dx)\sqrt{e+fx}}dx}{4b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{8bdeg+acfh-bc(fg+8eh)+f(7bdg-8bch+adh)x}{(a+bx)^4(c+dx)\sqrt{e+fx}}dx}{8b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(7dg-9ch)-b^2(-8ceh-cfg+8deg)\right)}{3(a+bx)^3(bc-ad)(be-af)}-\frac{\int -\frac{-\left(\left(-f(5fg-8eh)c^2-8de(fg+6eh)c+48d^2e^2g\right)b^2\right)+af\left(3fhc^2-d(13fg+48eh)c+48d\right)}{2(a+bx)^3(c+dx)\sqrt{e}}}{3(bc-ad)(be-af}}{8b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{-\left(\left(-f(5fg-8eh)c^2-8de(fg+6eh)c+48d^2e^2g\right)b^2\right)+af\left(3fhc^2-d(13fg+48eh)c+48d^2eg\right)b+5a^2cdf^2h+5df\left(dfha^2+bf(7dg-9ch)a-b^2(8deg-cfg-8ceh)\right)x}{(a+bx)^3(c+dx)\sqrt{e+fx}}}{6(bc-ad)(be-af)}}{8b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{\frac{\sqrt{e+fx}\left(5a^3d^2f^2h+5a^2bdf^2(7dg-10ch)-ab^2f\left(3c^2fh-2cd(44eh+9fg)+88d^2eg\right)+b^3\left(c^2(-f)(5fg-8eh)-8cde(6eh+fg)+48d^2e^2g\right)\right)}{2(a+bx)^2(bc-ad)(be-af)}-\int -\frac{3\left(\left(-f^2(5fg-8eh \cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{3\int \frac{\left(-f^2(5fg-8eh)c^3-8def(fg-2eh)c^2-16d^2e^2(fg+4eh)c+64d^3e^3g\right)b^3-af\left(3f^2hc^3-2df(9fg-20eh)c^2-8d^2e(5fg+16eh)c+128d^3e^2g\right)b^2+a^2df^2\left(14fhc^2 \cdots\right.}{(a+\cdots}}{4\cdots}}{\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$\Big\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2+bf(7dg-9ch)a-b^2(8deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3+5bdf^2(7dg-10ch)a^2-b^2f\left(3fhc^2-2d(9fg+44eh)c+88d^2eg\right)a+b^3\left(-f(5fg \cdots\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{\cdots}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2+bf(7dg-9ch)a-b^2(8deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3+5bdf^2(7dg-10ch)a^2-b^2f\left(3fhc^2-2d(9fg+44eh)c+88d^2eg\right)a+b^3\left(-f(5fg \cdots\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{\cdots}$$

$\Big\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2+bf(7dg-9ch)a-b^2(8deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3+5bdf^2(7dg-10ch)a^2-b^2f\left(3fhc^2-2d(9fg+44eh)c+88d^2eg\right)a+b^3\left(-f(5fg \cdots\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{\cdots}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4}-\frac{\frac{\sqrt{e+fx}(dfha^2+bf(7dg-9ch)a-b^2(8deg-cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\frac{\sqrt{e+fx}\left(5d^2f^2ha^3+5bdf^2(7dg-10ch)a^2-b^2f\left(3fhc^2-2d(9fg+44eh)c+88d^2eg\right)a+b^3\left(-f(5fg\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}}{}$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4}-\frac{\frac{\sqrt{e+fx}(dfha^2+bf(7dg-9ch)a-b^2(8deg-cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\frac{\sqrt{e+fx}\left(5d^2f^2ha^3+5bdf^2(7dg-10ch)a^2-b^2f\left(3fhc^2-2d(9fg+44eh)c+88d^2eg\right)a+b^3\left(-f(5fg\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}}{}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^5*(c + d*x)),x]
```

output
```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^4) - (((a^2*d*f*
h + a*b*f*(7*d*g - 9*c*h) - b^2*(8*d*e*g - c*f*g - 8*c*e*h))*Sqrt[e + f*x]
)/(3*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) + (((5*a^3*d^2*f^2*h + 5*a^2*b*d
*f^2*(7*d*g - 10*c*h) + b^3*(48*d^2*e^2*g - c^2*f*(5*f*g - 8*e*h) - 8*c*d*
e*(f*g + 6*e*h)) - a*b^2*f*(88*d^2*e*g + 3*c^2*f*h - 2*c*d*(9*f*g + 44*e*h
)))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (3*(((5*a^4*d
^3*f^3*h + 5*a^3*b*d^2*f^3*(7*d*g - 11*c*h) - b^4*(64*d^3*e^3*g - c^3*f^2*
(5*f*g - 8*e*h) - 8*c^2*d*e*f*(f*g - 2*e*h) - 16*c*d^2*e^2*(f*g + 4*e*h))
+ a*b^3*f*(176*d^3*e^2*g + 3*c^3*f^2*h - c^2*d*f*(23*f*g - 48*e*h) - 16*c*
d^2*e*(3*f*g + 11*e*h)) - a^2*b^2*d*f^2*(152*d^2*e*g + 17*c^2*f*h - c*d*(4
7*f*g + 152*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((
2*(5*a^5*d^4*f^4*h + 5*a^4*b*d^3*f^4*(7*d*g - 12*c*h) - 10*a^3*b^2*d^2*f^3
*(28*d^2*e*g + 9*c^2*f*h - 14*c*d*(f*g + 2*e*h)) + 10*a^2*b^3*d*f^2*(56*d^
3*e^2*g + 2*c^3*f^2*h - 7*c^2*d*f*(f*g - 4*e*h) - 28*c*d^2*e*(f*g + 2*e*h)
) - a*b^4*f*(448*d^4*e^3*g + 3*c^4*f^3*h - 56*c^2*d^2*e*f*(f*g - 4*e*h) -
28*c^3*d*f^2*(f*g - 2*e*h) - 224*c*d^3*e^2*(f*g + 2*e*h)) + b^5*(128*d^4*e
^4*g - c^4*f^3*(5*f*g - 8*e*h) - 16*c^2*d^2*e^2*f*(f*g - 4*e*h) - 8*c^3*d*
e*f^2*(f*g - 2*e*h) - 64*c*d^3*e^3*(f*g + 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e
 + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f]) - (256*b*
d^(5/2)*(b*e - a*f)^3*Sqrt[d*e - c*f]*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 15.16 (sec) , antiderivative size = 1522, normalized size of antiderivative = 1.62

method	result	size
pseudoelliptic	Expression too large to display	1522
derivativedivides	Expression too large to display	2470
default	Expression too large to display	2470

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
5/64/((a*f-b*e)*b)^(1/2)*((-((c*f-d*e)*d)^(1/2)*(a*d-b*c)*((-(a*(a^3*d^3-1
1*a^2*b*c*d^2-17/5*a*b^2*c^2*d+3/5*b^3*c^3)*h+7*(a^3*d^3+47/35*a^2*b*c*d^2
-23/35*a*b^2*c^2*d+1/7*b^3*c^3)*b*g)*b^3*x^3-11/3*a*(a*(a^3*d^3-11*a^2*b*c
*d^2-59/55*a*b^2*c^2*d+3/5*b^3*c^3)*h+7*(a^3*d^3+389/385*a^2*b*c*d^2-23/35
*a*b^2*c^2*d+1/7*b^3*c^3)*b*g)*b^2*x^2-73/15*a^2*(a*(a^3*d^3-11*a^2*b*c*d^
2+187/73*a*b^2*c^2*d-33/73*b^3*c^3)*h+7*(a^3*d^3+251/511*a^2*b*c*d^2-37/73
*a*b^2*c^2*d+1/7*b^3*c^3)*b*g)*b*x+a^3*(a*(73/5*a^2*b*c*d^2-17/5*a*b^2*c^2
*d+3/5*b^3*c^3+a^3*d^3)*h-93/5*(a^3*d^3-47/93*a^2*b*c*d^2+23/93*a*b^2*c^2*
d-5/93*b^3*c^3)*b*g))*f^3-118/15*(228/59*(a^2*d^2+6/19*a*b*c*d-1/19*b^2*c^
2)*b^4*(c*h-d*g)*x^3-5/59*b^2*(a*(a^3*d^3-827/5*a^2*b*c*d^2-89/5*a*b^2*c^2
*d+47/5*b^3*c^3)*h+807/5*b*g*(a^3*d^3+119/807*a^2*b*c*d^2-67/807*a*b^2*c^2
*d+5/807*b^3*c^3))*x^2-18/59*a*(a*(a^3*d^3-545/9*a^2*b*c*d^2+127/9*a*b^2*c
^2*d-23/9*b^3*c^3)*h+509/9*b*g*(a^3*d^3-73/509*a^2*b*c*d^2-13/509*a*b^2*c^
2*d+9/509*b^3*c^3))*b*x+a^2*(a*(9/59*b^3*c^3+a^3*d^3+275/59*a^2*b*c*d^2-55
/59*a*b^2*c^2*d)*h-511/59*(a^3*d^3-409/511*a^2*b*c*d^2+35/73*a*b^2*c^2*d-5
9/511*b^3*c^3)*b*g))*b*e*f^2+136/15*b^2*(66/17*d*b^4*(a*d+1/11*b*c)*(c*h-d
*g)*x^3+232/17*(a^2*d^2-7/116*a*b*c*d-1/116*b^2*c^2)*b^3*(c*h-d*g)*x^2-1/1
7*(a*(a^3*d^3-293*a^2*b*c*d^2+97*a*b^2*c^2*d-21*b^3*c^3)*h+289*(a^3*d^3-91
/289*a^2*b*c*d^2+1/17*a*b^2*c^2*d+1/289*b^3*c^3)*b*g)*b*x+a*(a*(5/17*b^3*c
^3+a^3*d^3+73/17*a^2*b*c*d^2-23/17*a*b^2*c^2*d)*h-141/17*(a^3*d^3-125/1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**5/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4286 vs. 2(904) = 1808.

Time = 0.33 (sec) , antiderivative size = 4286, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x, algorithm="giac")
```

output

```
1/64*(128*b^5*d^4*e^4*g - 64*b^5*c*d^3*e^3*f*g - 448*a*b^4*d^4*e^3*f*g - 1
6*b^5*c^2*d^2*e^2*f^2*g + 224*a*b^4*c*d^3*e^2*f^2*g + 560*a^2*b^3*d^4*e^2*
f^2*g - 8*b^5*c^3*d*e*f^3*g + 56*a*b^4*c^2*d^2*e*f^3*g - 280*a^2*b^3*c*d^3
*e*f^3*g - 280*a^3*b^2*d^4*e*f^3*g - 5*b^5*c^4*f^4*g + 28*a*b^4*c^3*d*f^4*
g - 70*a^2*b^3*c^2*d^2*f^4*g + 140*a^3*b^2*c*d^3*f^4*g + 35*a^4*b*d^4*f^4*
g - 128*b^5*c*d^3*e^4*h + 64*b^5*c^2*d^2*e^3*f*h + 448*a*b^4*c*d^3*e^3*f*h
 + 16*b^5*c^3*d*e^2*f^2*h - 224*a*b^4*c^2*d^2*e^2*f^2*h - 560*a^2*b^3*c*d^
3*e^2*f^2*h + 8*b^5*c^4*e*f^3*h - 56*a*b^4*c^3*d*e*f^3*h + 280*a^2*b^3*c^2
*d^2*e*f^3*h + 280*a^3*b^2*c*d^3*e*f^3*h - 3*a*b^4*c^4*f^4*h + 20*a^2*b^3*
c^3*d*f^4*h - 90*a^3*b^2*c^2*d^2*f^4*h - 60*a^4*b*c*d^3*f^4*h + 5*a^5*d^4*
f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^9*c^5*e^3 - 5*a*b^
8*c^4*d*e^3 + 10*a^2*b^7*c^3*d^2*e^3 - 10*a^3*b^6*c^2*d^3*e^3 + 5*a^4*b^5*
c*d^4*e^3 - a^5*b^4*d^5*e^3 - 3*a*b^8*c^5*e^2*f + 15*a^2*b^7*c^4*d*e^2*f -
 30*a^3*b^6*c^3*d^2*e^2*f + 30*a^4*b^5*c^2*d^3*e^2*f - 15*a^5*b^4*c*d^4*e^
2*f + 3*a^6*b^3*d^5*e^2*f + 3*a^2*b^7*c^5*e*f^2 - 15*a^3*b^6*c^4*d*e*f^2 +
 30*a^4*b^5*c^3*d^2*e*f^2 - 30*a^5*b^4*c^2*d^3*e*f^2 + 15*a^6*b^3*c*d^4*e*
f^2 - 3*a^7*b^2*d^5*e*f^2 - a^3*b^6*c^5*f^3 + 5*a^4*b^5*c^4*d*f^3 - 10*a^5
*b^4*c^3*d^2*f^3 + 10*a^6*b^3*c^2*d^3*f^3 - 5*a^7*b^2*c*d^4*f^3 + a^8*b*d^
5*f^3)*sqrt(-b^2*e + a*b*f)) - 2*(d^5*e*g - c*d^4*f*g - c*d^4*e*h + c^2*d^
3*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^5*c^5 - 5*a*b^4...
```

Mupad [B] (verification not implemented)

Time = 40.73 (sec) , antiderivative size = 875071, normalized size of antiderivative = 928.95

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Too large to display}$$

input

```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^5*(c + d*x)),x)
```

output
```
atan(((((11904*a^16*b^3*c*d^15*f^10*g - 640*a^17*b^2*c*d^15*f^10*h - 11904
*a^16*b^3*d^16*e*f^9*g + 640*a^17*b^2*d^16*e*f^9*h - 640*a^3*b^16*c^14*d^2
*f^10*g + 9344*a^4*b^15*c^13*d^3*f^10*g - 64256*a^5*b^14*c^12*d^4*f^10*g +
 281344*a^6*b^13*c^11*d^5*f^10*g - 877440*a^7*b^12*c^10*d^6*f^10*g + 20371
20*a^8*b^11*c^9*d^7*f^10*g - 3568128*a^9*b^10*c^8*d^8*f^10*g + 4710912*a^1
0*b^9*c^7*d^9*f^10*g - 4645248*a^11*b^8*c^6*d^10*f^10*g + 3360640*a^12*b^7
*c^5*d^11*f^10*g - 1729280*a^13*b^6*c^4*d^12*f^10*g + 598784*a^14*b^5*c^3*
d^13*f^10*g - 125056*a^15*b^4*c^2*d^14*f^10*g - 384*a^4*b^15*c^14*d^2*f^10
*h + 6016*a^5*b^14*c^13*d^3*f^10*h - 48384*a^6*b^13*c^12*d^4*f^10*h + 2368
00*a^7*b^12*c^11*d^5*f^10*h - 755840*a^8*b^11*c^10*d^6*f^10*h + 1646208*a^
9*b^10*c^9*d^7*f^10*h - 2514432*a^10*b^9*c^8*d^8*f^10*h + 2723328*a^11*b^8
*c^7*d^9*f^10*h - 2079360*a^12*b^7*c^6*d^10*f^10*h + 1088640*a^13*b^6*c^5*
d^11*f^10*h - 365824*a^14*b^5*c^4*d^12*f^10*h + 66816*a^15*b^4*c^3*d^13*f^
10*h - 2944*a^16*b^3*c^2*d^14*f^10*h - 8192*a^10*b^9*d^16*e^7*f^3*g + 5120
0*a^11*b^8*d^16*e^6*f^4*g - 134144*a^12*b^7*d^16*e^5*f^5*g + 189056*a^13*b
^6*d^16*e^4*f^6*g - 151424*a^14*b^5*d^16*e^3*f^7*g + 65408*a^15*b^4*d^16*e
^2*f^8*g - 640*a^14*b^5*d^16*e^4*f^6*h + 1920*a^15*b^4*d^16*e^3*f^7*h - 19
20*a^16*b^3*d^16*e^2*f^8*h - 8192*b^19*c^10*d^6*e^7*f^3*g + 6144*b^19*c^11
*d^5*e^6*f^4*g + 1024*b^19*c^12*d^4*e^5*f^5*g + 384*b^19*c^13*d^3*e^4*f^6*
g + 640*b^19*c^14*d^2*e^3*f^7*g + 8192*b^19*c^11*d^5*e^7*f^3*h - 6144*b...
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 17810, normalized size of antiderivative = 18.91

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x)
```

output

```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**9*d**4*f**4*h - 180*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c*d**3*f**4*h + 105*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*d**4*f**4*g +
 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**8*b*d**4*f**4*h*x - 270*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c**2*d**2*f**4*h + 840*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*
c*d**3*e*f**3*h + 420*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**7*b**2*c*d**3*f**4*g - 720*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c*d**3*f**4*
h*x - 840*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**7*b**2*d**4*e*f**3*g + 420*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*d**4*f**4*g*x + 90*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*
b**2*d**4*f**4*h*x**2 + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**6*b**3*c**3*d*f**4*h + 840*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b**3*c**2*d*
*2*e*f**3*h - 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**6*b**3*c**2*d**2*f**4*g - 1080*sqrt(b)*sqrt(a*f - ...
```

3.42 $\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 348

$$\begin{aligned}
&\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx \\
&= \frac{2(bc-ad)^2(3bdg-4bch+adh)\sqrt{e+fx}}{d^5}+\frac{(bc-ad)^3(dg-ch)\sqrt{e+fx}}{d^5(c+dx)} \\
&\quad+\frac{2b(3a^2d^2f^2h+3abdf(dfg-deh-2cfh)+b^2(3c^2f^2h-d^2e(fg-eh)-2cdf(fg-eh)))\,(e+fx)^{3/2}}{3d^4f^3} \\
&\quad+\frac{2b^2(3adfh+b(dfg-2deh-2cfh))(e+fx)^{5/2}}{5d^3f^3}+\frac{2b^3h(e+fx)^{7/2}}{7d^2f^3} \\
&\quad-\frac{(bc-ad)^2\,(ad(dfg+2deh-3cfh)+b(6d^2eg+9c^2fh-cd(7fg+8eh)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{11/2}\sqrt{de-cf}}
\end{aligned}$$

output
```
2*(-a*d+b*c)^2*(a*d*h-4*b*c*h+3*b*d*g)*(f*x+e)^(1/2)/d^5+(-a*d+b*c)^3*(-c*
h+d*g)*(f*x+e)^(1/2)/d^5/(d*x+c)+2/3*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(-2*c*f*
h-d*e*h+d*f*g)+b^2*(3*c^2*f^2*h-d^2*e*(-e*h+f*g)-2*c*d*f*(-e*h+f*g)))*(f*x
+e)^(3/2)/d^4/f^3+2/5*b^2*(3*a*d*f*h+b*(-2*c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(
5/2)/d^3/f^3+2/7*b^3*h*(f*x+e)^(7/2)/d^2/f^3-(-a*d+b*c)^2*(a*d*(-3*c*f*h+2
*d*e*h+d*f*g)+b*(6*d^2*e*g+9*c^2*f*h-c*d*(8*e*h+7*f*g)))*arctanh(d^(1/2)*(
f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(11/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$= \frac{\sqrt{e+fx}(105a^3d^3f^3(-dg+3ch+2dhx)+105a^2bd^2f^2(-15c^2fh+cd(9fg+2eh-10fhx)+2d^2x(3fg}{}$$
$$+\frac{(bc-ad)^2\,(ad(dfg+2deh-3cfh)+b(6d^2eg+9c^2fh-cd(7fg+8eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{11/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)^3*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(105*a^3*d^3*f^3*(-(d*g) + 3*c*h + 2*d*h*x) + 105*a^2*b*d^2
*f^2*(-15*c^2*f*h + c*d*(9*f*g + 2*e*h - 10*f*h*x) + 2*d^2*x*(3*f*g + e*h
+ f*h*x)) + 21*a*b^2*d*f*(105*c^3*f^2*h - 5*c^2*d*f*(15*f*g + 4*e*h - 14*f
*h*x) + 2*d^3*x*(e + f*x)*(5*f*g - 2*e*h + 3*f*h*x) - 2*c*d^2*(2*e^2*h + f
^2*x*(25*g + 7*h*x) + e*f*(-5*g + 9*h*x))) + b^3*(-945*c^4*f^3*h + 105*c^3
*d*f^2*(7*f*g + 2*e*h - 6*f*h*x) + 2*d^4*x*(e + f*x)*(8*e^2*h + 3*f^2*x*(7
*g + 5*h*x) - 2*e*f*(7*g + 6*h*x)) + 14*c^2*d^2*f*(4*e^2*h + f^2*x*(35*g +
 9*h*x) + e*f*(-10*g + 13*h*x)) + 2*c*d^3*(e + f*x)*(8*e^2*h - 2*e*f*(7*g
- 8*h*x) - f^2*x*(49*g + 27*h*x)))))/(105*d^5*f^3*(c + d*x)) + ((b*c - a*d
)^2*(a*d*(d*f*g + 2*d*e*h - 3*c*f*h) + b*(6*d^2*e*g + 9*c^2*f*h - c*d*(7*f
*g + 8*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(11/2
)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {166, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$

$\big\downarrow$ 166

$$\frac{\int \frac{(a+bx)^2\sqrt{e+fx}(6be(dg-ch)+a(dfg+2deh-3cfh)+b(7dfg+2deh-9cfh)x)}{2(c+dx)}dx}{d(de-cf)}-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^2\sqrt{e+fx}(6be(dg-ch)+a(dfg+2deh-3cfh)+b(7dfg+2deh-9cfh)x)}{c+dx}dx}{2d(de-cf)}-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\big\downarrow$ 170

$$\frac{\frac{2\int -\frac{(a+bx)\sqrt{e+fx}(bc(4be+3af)(7dfg+2deh-9cfh)-7adf(6be(dg-ch)+a(dfg+2deh-3cfh))-b(adf(35dfg+22deh-57cfh)+b(2e(7fg-4eh)d^2-cf(49fg+20eh\cdots}{2(c+dx)}}{7df}\cdots}{2d(de-cf)}\cdots-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\big\downarrow$ 27

$$\frac{\frac{2b(a+bx)^2(e+fx)^{3/2}(-9cfh+2deh+7dfg)}{7df}-\frac{\int \frac{(a+bx)\sqrt{e+fx}(bc(4be+3af)(7dfg+2deh-9cfh)-7adf(6be(dg-ch)+a(dfg+2deh-3cfh))-b(adf(35dfg+2\cdots}{c+dx}}{7df}\cdots}{2d(de-cf)}\cdots-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\big\downarrow$ 164

$$\frac{\frac{2b(a+bx)^2(e+fx)^{3/2}(-9cfh+2deh+7dfg)}{7df}-\frac{-\frac{7f(bc-ad)^2(ad(-3cfh+2deh+dfg)+b(9c^2fh-cd(8eh+7fg)+6d^2eg))\int \frac{\sqrt{e+fx}}{c+dx}dx}{d^2}-\frac{2b(e+fx)^{3/2}(30a^2\cdots}{\cdots}}{\cdots}}{\cdots}-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\big\downarrow$ 60

$$\frac{\frac{2b(a+bx)^2(e+fx)^{3/2}(-9cfh+2deh+7dfg)}{7df}-\frac{-\frac{7f(bc-ad)^2\left(ad(-3cfh+2deh+dfg)+b\left(9c^2fh-cd(8eh+7fg)+6d^2eg\right)\right)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{\cdots}}{\cdots}\right)}{d^2}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2b(a+bx)^2(e+fx)^{3/2}(-9cfh+2deh+7dfg)}{7df}-\frac{-\frac{7f(bc-ad)^2\left(ad(-3cfh+2deh+dfg)+b\left(9c^2fh-cd(8eh+7fg)+6d^2eg\right)\right)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{\cdots}}{df}\cdots\right)}{d^2}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2b(a+bx)^2(e+fx)^{3/2}(-9cfh+2deh+7dfg)}{7df}-\frac{-\frac{2b(e+fx)^{3/2}\left(30a^2d^2f^2(-13cfh+6deh+7dfg)+3bdfx\left(adf(-57cfh+22deh+35dfg)+b\left(63c^2f^2h-cdf(20e\cdots\right.\right.\right.}{\cdots}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[((a + b*x)^3*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
-(((d*g - c*h)*(a + b*x)^3*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x))) + (
(2*b*(7*d*f*g + 2*d*e*h - 9*c*f*h)*(a + b*x)^2*(e + f*x)^(3/2))/(7*d*f) -
((-2*b*(e + f*x)^(3/2)*(30*a^2*d^2*f^2*(7*d*f*g + 6*d*e*h - 13*c*f*h) + 21
*a*b*d*f*(35*c^2*f^2*h + 2*d^2*e*(5*f*g - 2*e*h) - c*d*f*(25*f*g + 16*e*h)
) - b^2*(315*c^3*f^3*h + 8*c*d^2*e*f*(14*f*g - 5*e*h) + 4*d^3*e^2*(7*f*g -
 4*e*h) - 7*c^2*d*f^2*(35*f*g + 22*e*h)) + 3*b*d*f*(a*d*f*(35*d*f*g + 22*d
*e*h - 57*c*f*h) + b*(63*c^2*f^2*h + 2*d^2*e*(7*f*g - 4*e*h) - c*d*f*(49*f
*g + 20*e*h)))*x))/(15*d^2*f^2) - (7*(b*c - a*d)^2*f*(a*d*(d*f*g + 2*d*e*h
 - 3*c*f*h) + b*(6*d^2*e*g + 9*c^2*f*h - c*d*(7*f*g + 8*e*h)))*((2*Sqrt[e
+ f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e -
c*f]])/d^(3/2)))/d^2)/(7*d*f))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.47

method	result
pseudoelliptic	$\frac{-3\left(\left(-\frac{afg}{3}-\frac{2e(ah+3bg)}{3}\right)d^2+c\left(\left(ah+\frac{7bg}{3}\right)f+\frac{8ehb}{3}\right)d-3bc^2fh\right)(ad-bc)^2(xd+c)f^3\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\left(\left(\left(\frac{2x}{}\right.\right.\right.}{}$
risch	$\frac{2(15x^3hb^3d^3f^3+63ab^2d^3f^3hx^2-42b^3cd^2f^3hx^2+3b^3d^3ef^2hx^2+21b^3d^3f^3gx^2+105a^2bd^3f^3hx-210ab^2cd^2f^3hx+}{}$
derivativedivides	$\frac{2\left(\frac{hb^3(fx+e)^{\frac{7}{2}}d^3}{7}+\frac{3ab^2d^3fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3cd^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3d^3eh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^3d^3fg(fx+e)^{\frac{5}{2}}}{5}+a^2bd^3f^2h(fx+e)^{\frac{3}{2}}-2ab^2\right.}{}$
default	$\frac{2\left(\frac{hb^3(fx+e)^{\frac{7}{2}}d^3}{7}+\frac{3ab^2d^3fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3cd^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3d^3eh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^3d^3fg(fx+e)^{\frac{5}{2}}}{5}+a^2bd^3f^2h(fx+e)^{\frac{3}{2}}-2ab^2\right.}{}$

input
```
int((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/((c*f-d*e)*d)^(1/2)*(-((-1/3*a*f*g-2/3*e*(a*h+3*b*g))*d^2+c*((a*h+7/3*b*
g)*f+8/3*e*h*b)*d-3*b*c^2*f*h)*(a*d-b*c)^2*(d*x+c)*f^3*arctan(d*(f*x+e)^(1
/2)/((c*f-d*e)*d)^(1/2))+(((2/15*x^3*(5/7*h*x+g)*b^3+2/3*a*x^2*(3/5*h*x+g)
*b^2+2*a^2*x*(1/3*h*x+g)*b-1/3*a^3*(-2*h*x+g))*f^3+2/3*x*(1/15*(3/7*h*x+g)
*x*b^2+a*(1/5*h*x+g)*b+a^2*h)*b*e*f^2-4/15*x*b^2*e^2*((2/21*h*x+1/3*g)*b+a
*h)*f+16/315*b^3*e^3*h*x)*d^4+(((-6/35*h*x^3-14/45*g*x^2)*b^3-10/3*a*x*(7/
25*h*x+g)*b^2+3*a^2*(-10/9*h*x+g)*b+h*a^3)*f^3+2/3*(-3/5*(11/63*h*x+g)*x*b
^2+a*(-9/5*h*x+g)*b+a^2*h)*b*e*f^2-4/15*b^2*e^2*((-4/7*h*x+1/3*g)*b+a*h)*f
+16/315*b^3*e^3*h)*c*d^3-5*c^2*b*f*((-14/45*x*(9/35*h*x+g)*b^2+a*(-14/15*h
*x+g)*b+a^2*h)*f^2+4/15*((-13/30*h*x+1/3*g)*b+a*h)*b*e*f-8/225*b^2*e^2*h)*
d^2+7*c^3*(((-2/7*h*x+1/3*g)*b+a*h)*f+2/21*e*h*b)*b^2*f^2*d-3*b^3*c^4*f^3*
h)*((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2))/f^3/d^5/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1432 vs. $2(322) = 644$.

Time = 0.20 (sec) , antiderivative size = 2878, normalized size of antiderivative = 8.27

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/210*(105*sqrt(d^2*e - c*d*f)*((6*(b^3*c^3*d^2 - 2*a*b^2*c^2*d^3 + a^2*
b*c*d^4)*e*f^3 - (7*b^3*c^4*d - 15*a*b^2*c^3*d^2 + 9*a^2*b*c^2*d^3 - a^3*c
*d^4)*f^4)*g - (2*(4*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 6*a^2*b*c^2*d^3 - a^3*c
*d^4)*e*f^3 - 3*(3*b^3*c^5 - 7*a*b^2*c^4*d + 5*a^2*b*c^3*d^2 - a^3*c^2*d^3
)*f^4)*h + ((6*(b^3*c^2*d^3 - 2*a*b^2*c*d^4 + a^2*b*d^5)*e*f^3 - (7*b^3*c^
3*d^2 - 15*a*b^2*c^2*d^3 + 9*a^2*b*c*d^4 - a^3*d^5)*f^4)*g - (2*(4*b^3*c^3
*d^2 - 9*a*b^2*c^2*d^3 + 6*a^2*b*c*d^4 - a^3*d^5)*e*f^3 - 3*(3*b^3*c^4*d -
 7*a*b^2*c^3*d^2 + 5*a^2*b*c^2*d^3 - a^3*c*d^4)*f^4)*h)*x)*log((d*f*x + 2*
d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(30*(b^3*d
^6*e*f^3 - b^3*c*d^5*f^4)*h*x^4 + 6*(7*(b^3*d^6*e*f^3 - b^3*c*d^5*f^4)*g +
 (b^3*d^6*e^2*f^2 - (10*b^3*c*d^5 - 21*a*b^2*d^6)*e*f^3 + 3*(3*b^3*c^2*d^4
 - 7*a*b^2*c*d^5)*f^4)*h)*x^3 + 2*(7*(b^3*d^6*e^2*f^2 - (8*b^3*c*d^5 - 15*
a*b^2*d^6)*e*f^3 + (7*b^3*c^2*d^4 - 15*a*b^2*c*d^5)*f^4)*g - (4*b^3*d^6*e^
3*f + 7*(b^3*c*d^5 - 3*a*b^2*d^6)*e^2*f^2 - (74*b^3*c^2*d^4 - 168*a*b^2*c*
d^5 + 105*a^2*b*d^6)*e*f^3 + 21*(3*b^3*c^3*d^3 - 7*a*b^2*c^2*d^4 + 5*a^2*b
*c*d^5)*f^4)*h)*x^2 - 7*(4*b^3*c*d^5*e^3*f + 2*(8*b^3*c^2*d^4 - 15*a*b^2*c
*d^5)*e^2*f^2 - 5*(25*b^3*c^3*d^3 - 51*a*b^2*c^2*d^4 + 27*a^2*b*c*d^5 - 3*
a^3*d^6)*e*f^3 + 15*(7*b^3*c^4*d^2 - 15*a*b^2*c^3*d^3 + 9*a^2*b*c^2*d^4 -
a^3*c*d^5)*f^4)*g + (16*b^3*c*d^5*e^4 + 4*(10*b^3*c^2*d^4 - 21*a*b^2*c*d^5
)*e^3*f + 14*(11*b^3*c^3*d^3 - 24*a*b^2*c^2*d^4 + 15*a^2*b*c*d^5)*e^2*f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(f*x+e)**(1/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(322) = 644$.

Time = 0.16 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.39

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(6*b^3*c^2*d^2*e*g - 12*a*b^2*c*d^3*e*g + 6*a^2*b*d^4*e*g - 7*b^3*c^3*d*f*
g + 15*a*b^2*c^2*d^2*f*g - 9*a^2*b*c*d^3*f*g + a^3*d^4*f*g - 8*b^3*c^3*d*e
*h + 18*a*b^2*c^2*d^2*e*h - 12*a^2*b*c*d^3*e*h + 2*a^3*d^4*e*h + 9*b^3*c^4
*f*h - 21*a*b^2*c^3*d*f*h + 15*a^2*b*c^2*d^2*f*h - 3*a^3*c*d^3*f*h)*arctan
(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^5) + (sqrt(
f*x + e)*b^3*c^3*d*f*g - 3*sqrt(f*x + e)*a*b^2*c^2*d^2*f*g + 3*sqrt(f*x +
e)*a^2*b*c*d^3*f*g - sqrt(f*x + e)*a^3*d^4*f*g - sqrt(f*x + e)*b^3*c^4*f*h
 + 3*sqrt(f*x + e)*a*b^2*c^3*d*f*h - 3*sqrt(f*x + e)*a^2*b*c^2*d^2*f*h + s
qrt(f*x + e)*a^3*c*d^3*f*h)/(((f*x + e)*d - d*e + c*f)*d^5) + 2/105*(21*(f
*x + e)^(5/2)*b^3*d^12*f^19*g - 35*(f*x + e)^(3/2)*b^3*d^12*e*f^19*g - 70*
(f*x + e)^(3/2)*b^3*c*d^11*f^20*g + 105*(f*x + e)^(3/2)*a*b^2*d^12*f^20*g
+ 315*sqrt(f*x + e)*b^3*c^2*d^10*f^21*g - 630*sqrt(f*x + e)*a*b^2*c*d^11*f
^21*g + 315*sqrt(f*x + e)*a^2*b*d^12*f^21*g + 15*(f*x + e)^(7/2)*b^3*d^12*
f^18*h - 42*(f*x + e)^(5/2)*b^3*d^12*e*f^18*h + 35*(f*x + e)^(3/2)*b^3*d^1
2*e^2*f^18*h - 42*(f*x + e)^(5/2)*b^3*c*d^11*f^19*h + 63*(f*x + e)^(5/2)*a
*b^2*d^12*f^19*h + 70*(f*x + e)^(3/2)*b^3*c*d^11*e*f^19*h - 105*(f*x + e)^
(3/2)*a*b^2*d^12*e*f^19*h + 105*(f*x + e)^(3/2)*b^3*c^2*d^10*f^20*h - 210*
(f*x + e)^(3/2)*a*b^2*c*d^11*f^20*h + 105*(f*x + e)^(3/2)*a^2*b*d^12*f^20*
h - 420*sqrt(f*x + e)*b^3*c^3*d^9*f^21*h + 945*sqrt(f*x + e)*a*b^2*c^2*d^1
0*f^21*h - 630*sqrt(f*x + e)*a^2*b*c*d^11*f^21*h + 105*sqrt(f*x + e)*a^...
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.62

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^3)/(c + d*x)^2,x)
```

output
```
(e + f*x)^(5/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(5*d^2*f^3) - (4*b^
3*h*(c*f - d*e))/(5*d^3*f^3)) + (e + f*x)^(1/2)*((2*(c*f - d*e)*((2*((2*b^
3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d*e))/(d^3*f^
3))*(c*f - d*e))/d - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d^2*f^3)
 + (2*b^3*h*(c*f - d*e)^2)/(d^4*f^3)))/d - (((2*b^3*f*g - 8*b^3*e*h + 6*a*
b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d*e))/(d^3*f^3))*(c*f - d*e)^2)/d^2 +
 (2*(a*f - b*e)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(d^2*f^3)) - (e + f*x)^(3/2
)*((2*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d
*e))/(d^3*f^3))*(c*f - d*e))/(3*d) - (2*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b
*f*g))/(d^2*f^3) + (2*b^3*h*(c*f - d*e)^2)/(3*d^4*f^3)) - ((e + f*x)^(1/2)
*(a^3*d^4*f*g + b^3*c^4*f*h - a^3*c*d^3*f*h - b^3*c^3*d*f*g - 3*a^2*b*c*d^
3*f*g - 3*a*b^2*c^3*d*f*h + 3*a*b^2*c^2*d^2*f*g + 3*a^2*b*c^2*d^2*f*h))/(d
^6*(e + f*x) - d^6*e + c*d^5*f) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*
c)^2*(2*a*d^2*e*h + a*d^2*f*g + 6*b*d^2*e*g + 9*b*c^2*f*h - 3*a*c*d*f*h -
8*b*c*d*e*h - 7*b*c*d*f*g))/((c*f - d*e)^(1/2)*(2*a^3*d^4*e*h + a^3*d^4*f*
g + 9*b^3*c^4*f*h + 6*a^2*b*d^4*e*g - 3*a^3*c*d^3*f*h - 8*b^3*c^3*d*e*h -
7*b^3*c^3*d*f*g + 6*b^3*c^2*d^2*e*g - 12*a*b^2*c*d^3*e*g - 12*a^2*b*c*d^3*
e*h - 9*a^2*b*c*d^3*f*g - 21*a*b^2*c^3*d*f*h + 18*a*b^2*c^2*d^2*e*h + 15*a
*b^2*c^2*d^2*f*g + 15*a^2*b*c^2*d^2*f*h)))*(a*d - b*c)^2*(2*a*d^2*e*h + a*
d^2*f*g + 6*b*d^2*e*g + 9*b*c^2*f*h - 3*a*c*d*f*h - 8*b*c*d*e*h - 7*b*c...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3133, normalized size of antiderivative = 9.00

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x)
```

output
```
( - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*a**3*c**2*d**3*f**4*h + 210*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*e*f**3*h + 105*sqrt(d)*sqr
t(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4
*f**4*g - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt
(c*f - d*e)))*a**3*c*d**4*f**4*h*x + 210*sqrt(d)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**5*e*f**3*h*x + 105*sqrt(d
)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d
**5*f**4*g*x + 1575*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a**2*b*c**3*d**2*f**4*h - 1260*sqrt(d)*sqrt(c*f - d*e)
*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*e*f**3
*h - 945*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
- d*e)))*a**2*b*c**2*d**3*f**4*g + 1575*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*f**4*h*x + 630*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*
*2*b*c*d**4*e*f**3*g - 1260*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)
/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4*e*f**3*h*x - 945*sqrt(d)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4*f
**4*g*x + 630*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt
(c*f - d*e)))*a**2*b*d**5*e*f**3*g*x - 2205*sqrt(d)*sqrt(c*f - d*e)*ata...
```

3.43 $\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 247

$$\begin{aligned}\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx &= -\frac{2(bc-ad)(2bdg-3bch+adh)\sqrt{e+fx}}{d^4}-\frac{(bc-ad)^2(dg-ch)\sqrt{e+fx}}{d^4(c+dx)}\\ &\quad+\frac{2b(2adfh+b(dfg-deh-2cfh))(e+fx)^{3/2}}{3d^3f^2}+\frac{2b^2h(e+fx)^{5/2}}{5d^2f^2}\\ &\quad+\frac{(bc-ad)\left(ad(dfg+2deh-3cfh)+b(4d^2eg+7c^2fh-cd(5fg+6eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{9/2}\sqrt{de-cf}}\end{aligned}$$

output
```
-2*(-a*d+b*c)*(a*d*h-3*b*c*h+2*b*d*g)*(f*x+e)^(1/2)/d^4-(-a*d+b*c)^2*(-c*h
+d*g)*(f*x+e)^(1/2)/d^4/(d*x+c)+2/3*b*(2*a*d*f*h+b*(-2*c*f*h-d*e*h+d*f*g))
*(f*x+e)^(3/2)/d^3/f^2+2/5*b^2*h*(f*x+e)^(5/2)/d^2/f^2+(-a*d+b*c)*(a*d*(-3
*c*f*h+2*d*e*h+d*f*g)+b*(4*d^2*e*g+7*c^2*f*h-c*d*(6*e*h+5*f*g)))*arctanh(d
^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(9/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$= \frac{\sqrt{e+fx}(15a^2d^2f^2(-dg+3ch+2dhx)+10abdf(-15c^2fh+cd(9fg+2eh-10fhx)+2d^2x(3fg+eh$$
$$+\frac{(-bc+ad)\left(ad(dfg+2deh-3cfh)+b(4d^2eg+7c^2fh-cd(5fg+6eh))\right)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{9/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)^2*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(15*a^2*d^2*f^2*(-(d*g) + 3*c*h + 2*d*h*x) + 10*a*b*d*f*(-1
5*c^2*f*h + c*d*(9*f*g + 2*e*h - 10*f*h*x) + 2*d^2*x*(3*f*g + e*h + f*h*x)
) + b^2*(105*c^3*f^2*h - 5*c^2*d*f*(15*f*g + 4*e*h - 14*f*h*x) + 2*d^3*x*(
e + f*x)*(5*f*g - 2*e*h + 3*f*h*x) - 2*c*d^2*(2*e^2*h + f^2*x*(25*g + 7*h*
x) + e*f*(-5*g + 9*h*x)))))/(15*d^4*f^2*(c + d*x)) + ((-(b*c) + a*d)*(a*d*
(d*f*g + 2*d*e*h - 3*c*f*h) + b*(4*d^2*e*g + 7*c^2*f*h - c*d*(5*f*g + 6*e*
h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(9/2)*Sqrt[-(d
*e) + c*f])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {166, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)\sqrt{e+fx}(4be(dg-ch)+a(dfg+2deh-3cfh)+b(5dfg+2deh-7cfh)x)}{2(c+dx)}dx}{d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\downarrow$ 27

$$\frac{\int \frac{(a+bx)\sqrt{e+fx}(4be(dg-ch)+a(dfg+2deh-3cfh)+b(5dfg+2deh-7cfh)x)}{c+dx}dx}{2d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\downarrow$ 164

$$\frac{\frac{2b(e+fx)^{3/2}(10adf(-5cfh+2deh+3dfg)+b(35c^2f^2h-cdf(16eh+25fg)+2d^2e(5fg-2eh))+3bdfx(-7cfh+2deh+5dfg))}{15d^2f^2} - \frac{(bc-ad)(ad(-3cf\ldots}{\ldots}}{2d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\downarrow$ 60

$$\frac{\frac{2b(e+fx)^{3/2}(10adf(-5cfh+2deh+3dfg)+b(35c^2f^2h-cdf(16eh+25fg)+2d^2e(5fg-2eh))+3bdfx(-7cfh+2deh+5dfg))}{15d^2f^2} - \frac{(bc-ad)(ad(-3cf\ldots}{\ldots}}{2d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\downarrow$ 73

$$\frac{\frac{2b(e+fx)^{3/2}(10adf(-5cfh+2deh+3dfg)+b(35c^2f^2h-cdf(16eh+25fg)+2d^2e(5fg-2eh))+3bdfx(-7cfh+2deh+5dfg))}{15d^2f^2} - \frac{(bc-ad)(ad(-3cf\ldots}{\ldots}}{2d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\downarrow$ 221

$$\frac{\frac{2b(e+fx)^{3/2}(10adf(-5cfh+2deh+3dfg)+b(35c^2f^2h-cdf(16eh+25fg)+2d^2e(5fg-2eh))+3bdfx(-7cfh+2deh+5dfg))}{15d^2f^2} - \frac{(bc-ad)\left(\frac{2\sqrt{e+fx}}{d}\right.}{}}{2d(de-cf)}$$

$$\frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[((a + b*x)^2*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
-(((d*g - c*h)*(a + b*x)^2*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x))) + (
(2*b*(e + f*x)^(3/2)*(10*a*d*f*(3*d*f*g + 2*d*e*h - 5*c*f*h) + b*(35*c^2*f
^2*h + 2*d^2*e*(5*f*g - 2*e*h) - c*d*f*(25*f*g + 16*e*h)) + 3*b*d*f*(5*d*f
*g + 2*d*e*h - 7*c*f*h)*x))/(15*d^2*f^2) - ((b*c - a*d)*(a*d*(d*f*g + 2*d*
e*h - 3*c*f*h) + b*(4*d^2*e*g + 7*c^2*f*h - c*d*(5*f*g + 6*e*h)))*((2*Sqrt
[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e
 - c*f]])/d^(3/2)))/d^2)/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{-3(ad-bc)(xd+c)\left(\frac{(-afg-2e(ah+2bg))d^2}{3}+c\left(\left(ah+\frac{5bg}{3}\right)f+2ehb\right)d-\frac{7b\,c^2fh}{3}\right)f^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\sqrt{(cf-de)d}}{}$
risch	$\frac{2(3h\,b^2x^2d^2f^2+10ab\,d^2f^2hx-10b^2cd\,f^2hx+b^2d^2efhx+5b^2d^2f^2gx+15a^2d^2f^2h-60abcd\,f^2h+10ab\,d^2efh+30ab\,d^2f}{15f^2d^4}$
derivativedivides	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+\frac{2ab\,d^2fh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+a^2d^2f^2h\sqrt{fx+e}-4abcd\,f^2h\sqrt{f}\right.}{d^4}$
default	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+\frac{2ab\,d^2fh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+a^2d^2f^2h\sqrt{fx+e}-4abcd\,f^2h\sqrt{f}\right.}{d^4}$

input
```
int((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/((c*f-d*e)*d)^(1/2)*(-(a*d-b*c)*(d*x+c)*(1/3*(-a*f*g-2*e*(a*h+2*b*g))*d^
2+c*((a*h+5/3*b*g)*f+2*e*h*b)*d-7/3*b*c^2*f*h)*f^2*arctan(d*(f*x+e)^(1/2)/
((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*(1/3*((2/3*x^2*(3/5
*h*x+g)*b^2+4*a*x*(1/3*h*x+g)*b-a^2*(-2*h*x+g))*f^2+4/3*x*(1/2*(1/5*h*x+g)
*b+a*h)*b*e*f-4/15*b^2*e^2*h*x)*d^3+c*((-10/9*x*(7/25*h*x+g)*b^2+2*a*(-10/
9*h*x+g)*b+a^2*h)*f^2+4/9*(1/2*(-9/5*h*x+g)*b+a*h)*b*e*f-4/45*b^2*e^2*h)*d
^2-10/3*c^2*(((-7/15*h*x+1/2*g)*b+a*h)*f+2/15*e*h*b)*b*f*d+7/3*b^2*c^3*f^2
*h))/f^2/d^4/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(225) = 450$.

Time = 0.15 (sec) , antiderivative size = 1741, normalized size of antiderivative = 7.05

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(d^2*e - c*d*f)*((4*(b^2*c^2*d^2 - a*b*c*d^3)*e*f^2 - (5*b^2
*c^3*d - 6*a*b*c^2*d^2 + a^2*c*d^3)*f^3)*g - (2*(3*b^2*c^3*d - 4*a*b*c^2*d
^2 + a^2*c*d^3)*e*f^2 - (7*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2)*f^3)*h
 + ((4*(b^2*c*d^3 - a*b*d^4)*e*f^2 - (5*b^2*c^2*d^2 - 6*a*b*c*d^3 + a^2*d^4
)*f^3)*g - (2*(3*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*e*f^2 - (7*b^2*c^3*d
 - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*f^3)*h)*x)*log((d*f*x + 2*d*e - c*f + 2*s
qrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*(6*(b^2*d^5*e*f^2 - b^2*c
*d^4*f^3)*h*x^3 + 2*(5*(b^2*d^5*e*f^2 - b^2*c*d^4*f^3)*g + (b^2*d^5*e^2*f
 - 2*(4*b^2*c*d^4 - 5*a*b*d^5)*e*f^2 + (7*b^2*c^2*d^3 - 10*a*b*c*d^4)*f^3)*
h)*x^2 + 5*(2*b^2*c*d^4*e^2*f - (17*b^2*c^2*d^3 - 18*a*b*c*d^4 + 3*a^2*d^5
)*e*f^2 + 3*(5*b^2*c^3*d^2 - 6*a*b*c^2*d^3 + a^2*c*d^4)*f^3)*g - (4*b^2*c*
d^4*e^3 + 4*(4*b^2*c^2*d^3 - 5*a*b*c*d^4)*e^2*f - 5*(25*b^2*c^3*d^2 - 34*a
*b*c^2*d^3 + 9*a^2*c*d^4)*e*f^2 + 15*(7*b^2*c^4*d - 10*a*b*c^3*d^2 + 3*a^2
*c^2*d^3)*f^3)*h + 2*(5*(b^2*d^5*e^2*f - 6*(b^2*c*d^4 - a*b*d^5)*e*f^2 + (
5*b^2*c^2*d^3 - 6*a*b*c*d^4)*f^3)*g - (2*b^2*d^5*e^3 + (7*b^2*c*d^4 - 10*a
*b*d^5)*e^2*f - (44*b^2*c^2*d^3 - 60*a*b*c*d^4 + 15*a^2*d^5)*e*f^2 + 5*(7*
b^2*c^3*d^2 - 10*a*b*c^2*d^3 + 3*a^2*c*d^4)*f^3)*h)*x)*sqrt(f*x + e))/(c*d
^6*e*f^2 - c^2*d^5*f^3 + (d^7*e*f^2 - c*d^6*f^3)*x), -1/15*(15*sqrt(-d^2*e
 + c*d*f)*((4*(b^2*c^2*d^2 - a*b*c*d^3)*e*f^2 - (5*b^2*c^3*d - 6*a*b*c^2*d
^2 + a^2*c*d^3)*f^3)*g - (2*(3*b^2*c^3*d - 4*a*b*c^2*d^2 + a^2*c*d^3)*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(f*x+e)**(1/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(225) = 450$.

Time = 0.14 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = -\frac{(4\,b^2cd^2eg - 4\,abd^3eg - 5\,b^2c^2dfg + 6\,abcd^2fg - a^2d^3fg - 6\,b^2c^2deh + 8\,abcd^2eh - 2\,a^2d^3eh + 7\,b^2c^3f}{\sqrt{-d^2e+cdf}\,d^4} - \frac{\sqrt{fx+e}b^2c^2dfg - 2\sqrt{fx+e}abcd^2fg + \sqrt{fx+e}a^2d^3fg - \sqrt{fx+e}b^2c^3fh + 2\sqrt{fx+e}abc^2dfh - \sqrt{}}{((fx+e)d - de + cf)d^4} + \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^2d^8f^9g - 30\sqrt{fx+e}b^2cd^7f^{10}g + 30\sqrt{fx+e}abd^8f^{10}g + 3\,(fx+e)^{\frac{5}{2}}b^2d^8f^8h - 5\,(fx\right.}{}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(4*b^2*c*d^2*e*g - 4*a*b*d^3*e*g - 5*b^2*c^2*d*f*g + 6*a*b*c*d^2*f*g - a^
2*d^3*f*g - 6*b^2*c^2*d*e*h + 8*a*b*c*d^2*e*h - 2*a^2*d^3*e*h + 7*b^2*c^3*
f*h - 10*a*b*c^2*d*f*h + 3*a^2*c*d^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2
*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^4) - (sqrt(f*x + e)*b^2*c^2*d*f*g - 2
*sqrt(f*x + e)*a*b*c*d^2*f*g + sqrt(f*x + e)*a^2*d^3*f*g - sqrt(f*x + e)*b
^2*c^3*f*h + 2*sqrt(f*x + e)*a*b*c^2*d*f*h - sqrt(f*x + e)*a^2*c*d^2*f*h)/
(((f*x + e)*d - d*e + c*f)*d^4) + 2/15*(5*(f*x + e)^(3/2)*b^2*d^8*f^9*g -
30*sqrt(f*x + e)*b^2*c*d^7*f^10*g + 30*sqrt(f*x + e)*a*b*d^8*f^10*g + 3*(f
*x + e)^(5/2)*b^2*d^8*f^8*h - 5*(f*x + e)^(3/2)*b^2*d^8*e*f^8*h - 10*(f*x
+ e)^(3/2)*b^2*c*d^7*f^9*h + 10*(f*x + e)^(3/2)*a*b*d^8*f^9*h + 45*sqrt(f*
x + e)*b^2*c^2*d^6*f^10*h - 60*sqrt(f*x + e)*a*b*c*d^7*f^10*h + 15*sqrt(f*
x + e)*a^2*d^8*f^10*h)/(d^10*f^10)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.31

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$= (e+f\,x)^{3/2}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{3\,d^2\,f^2}-\frac{4\,b^2\,h\,(c\,f-d\,e)}{3\,d^3\,f^2}\right)$$
$$-\sqrt{e+f\,x}\left(\frac{2\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d^2\,f^2}-\frac{4\,b^2\,h\,(c\,f-d\,e)}{d^3\,f^2}\right)(c\,f-d\,e)}{d}\right.$$
$$\left.-\frac{2\,(a\,f-b\,e)\,(a\,f\,h-3\,b\,e\,h+2\,b\,f\,g)}{d^2\,f^2}+\frac{2\,b^2\,h\,{(c\,f-d\,e)}^2}{d^4\,f^2}\right)$$
$$-\frac{\sqrt{e+f\,x}\,(-f\,h\,a^2\,c\,d^2+f\,g\,a^2\,d^3+2\,f\,h\,a\,b\,c^2\,d-2\,f\,g\,a\,b\,c\,d^2-f\,h\,b^2\,c^3+f\,g\,b^2\,c^2\,d)}{d^5\,(e+f\,x)-d^5\,e+c\,d^4\,f}$$
$$+\frac{2\,b^2\,h\,{(e+f\,x)}^{5/2}}{5\,d^2\,f^2}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)\,(2\,a\,d^2\,e\,h+a\,d^2\,f\,g+4\,b\,d^2\,e\,g+7\,b\,c^2\,f\,h-3\,a\,c\,d\,f\,h-6\,b\,c\,d\,e\,h-5\,b\,c\,d\,f\,g)}{\sqrt{c\,f-d\,e}\,(2\,a^2\,d^3\,e\,h+a^2\,d^3\,f\,g-7\,b^2\,c^3\,f\,h-4\,b^2\,c\,d^2\,e\,g-3\,a^2\,c\,d^2\,f\,h+6\,b^2\,c^2\,d\,e\,h+5\,b^2\,c^2\,d\,f\,g+4\,a\,b\,d^3\,e\,g-8\,a\,b\,c\,d^2\,e\,h-6\,a\,b\,c\,d^2}\right.}{d^{9/2}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
(e + f*x)^(3/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(3*d^2*f^2) - (4*b^2*
h*(c*f - d*e))/(3*d^3*f^2)) - (e + f*x)^(1/2)*((2*((2*b^2*f*g - 6*b^2*e*h
+ 4*a*b*f*h)/(d^2*f^2) - (4*b^2*h*(c*f - d*e))/(d^3*f^2))*(c*f - d*e))/d -
 (2*(a*f - b*e)*(a*f*h - 3*b*e*h + 2*b*f*g))/(d^2*f^2) + (2*b^2*h*(c*f - d
*e)^2)/(d^4*f^2)) - ((e + f*x)^(1/2)*(a^2*d^3*f*g - b^2*c^3*f*h - a^2*c*d^
2*f*h + b^2*c^2*d*f*g - 2*a*b*c*d^2*f*g + 2*a*b*c^2*d*f*h))/(d^5*(e + f*x)
 - d^5*e + c*d^4*f) + (2*b^2*h*(e + f*x)^(5/2))/(5*d^2*f^2) + (atan((d^(1/
2)*(e + f*x)^(1/2)*(a*d - b*c)*(2*a*d^2*e*h + a*d^2*f*g + 4*b*d^2*e*g + 7*
b*c^2*f*h - 3*a*c*d*f*h - 6*b*c*d*e*h - 5*b*c*d*f*g))/((c*f - d*e)^(1/2)*(
2*a^2*d^3*e*h + a^2*d^3*f*g - 7*b^2*c^3*f*h - 4*b^2*c*d^2*e*g - 3*a^2*c*d^
2*f*h + 6*b^2*c^2*d*e*h + 5*b^2*c^2*d*f*g + 4*a*b*d^3*e*g - 8*a*b*c*d^2*e*
h - 6*a*b*c*d^2*f*g + 10*a*b*c^2*d*f*h)))*(a*d - b*c)*(2*a*d^2*e*h + a*d^2
*f*g + 4*b*d^2*e*g + 7*b*c^2*f*h - 3*a*c*d*f*h - 6*b*c*d*e*h - 5*b*c*d*f*g
))/(d^(9/2)*(c*f - d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1921, normalized size of antiderivative = 7.78

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x)
```

output

```
( - 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**2*c**2*d**2*f**3*h + 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*e*f**2*h + 15*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*f*
*3*g - 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*a**2*c*d**3*f**3*h*x + 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4*e*f**2*h*x + 15*sqrt(d)*sqrt
(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4*f*
*3*g*x + 150*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a*b*c**3*d*f**3*h - 120*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
+ f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*e*f**2*h - 90*sqrt(d)*s
qrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*
d**2*f**3*g + 150*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a*b*c**2*d**2*f**3*h*x + 60*sqrt(d)*sqrt(c*f - d*e)*atan
((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*e*f**2*g - 120*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*
b*c*d**3*e*f**2*h*x - 90*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*f**3*g*x + 60*sqrt(d)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*d**4*e*f**2*g*x - 10
5*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*...
```

3.44 $\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \frac{2(bdg-2bch+adh)\sqrt{e+fx}}{d^3}+\frac{(bc-ad)(dg-ch)\sqrt{e+fx}}{d^3(c+dx)}+\frac{2bh(e+fx)^{3/2}}{3d^2f}-\frac{(ad(dfg+2deh-3cfh)+b(2d^2eg+5c^2fh-cd(3fg+4eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}\sqrt{de-cf}}$$

output

```
2*(a*d*h-2*b*c*h+b*d*g)*(f*x+e)^(1/2)/d^3+(-a*d+b*c)*(-c*h+d*g)*(f*x+e)^(1
/2)/d^3/(d*x+c)+2/3*b*h*(f*x+e)^(3/2)/d^2/f-(a*d*(-3*c*f*h+2*d*e*h+d*f*g)+
b*(2*d^2*e*g+5*c^2*f*h-c*d*(4*e*h+3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(
-c*f+d*e)^(1/2))/d^(7/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$=\frac{\sqrt{e+fx}(3adf(-dg+3ch+2dhx)+b(-15c^2fh+cd(9fg+2eh-10fhx)+2d^2x(3fg+eh+fhx)))}{3d^3f(c+dx)}$$
$$+\frac{(ad(dfg+2deh-3cfh)+b(2d^2eg+5c^2fh-cd(3fg+4eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(3*a*d*f*(-(d*g) + 3*c*h + 2*d*h*x) + b*(-15*c^2*f*h + c*d*
(9*f*g + 2*e*h - 10*f*h*x) + 2*d^2*x*(3*f*g + e*h + f*h*x))))/(3*d^3*f*(c
+ d*x)) + ((a*d*(d*f*g + 2*d*e*h - 3*c*f*h) + b*(2*d^2*e*g + 5*c^2*f*h - c
*d*(3*f*g + 4*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(
d^(7/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {163, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{163}$$
$$\frac{(ad(-3cfh+2deh+dfg)+b(5c^2fh-cd(4eh+3fg)+2d^2eg))\int\frac{\sqrt{e+fx}}{c+dx}dx}{2d^2(de-cf)}-\frac{(e+fx)^{3/2}(3adf(dg-ch)-bc(-5cfh+2deh+3dfg)-2bdhx(de-cf))}{3d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(ad(-3cfh+2deh+dfg)+b\left(5c^2fh-cd(4eh+3fg)+2d^2eg\right)\right)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{2d^2(de-cf)}-\frac{(e+fx)^{3/2}(3adf(dg-ch)-bc(-5cfh+2deh+3dfg)-2bdhx(de-cf))}{3d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(ad(-3cfh+2deh+dfg)+b\left(5c^2fh-cd(4eh+3fg)+2d^2eg\right)\right)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{2d^2(de-cf)}-\frac{(e+fx)^{3/2}(3adf(dg-ch)-bc(-5cfh+2deh+3dfg)-2bdhx(de-cf))}{3d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)\left(ad(-3cfh+2deh+dfg)+b\left(5c^2fh-cd(4eh+3fg)+2d^2eg\right)\right)}{2d^2(de-cf)}-\frac{(e+fx)^{3/2}(3adf(dg-ch)-bc(-5cfh+2deh+3dfg)-2bdhx(de-cf))}{3d^2f(c+dx)(de-cf)}$$

input

```
Int[((a + b*x)*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output

```
-1/3*((e + f*x)^(3/2)*(3*a*d*f*(d*g - c*h) - b*c*(3*d*f*g + 2*d*e*h - 5*c*
f*h) - 2*b*d*(d*e - c*f)*h*x))/(d^2*f*(d*e - c*f)*(c + d*x)) + ((a*d*(d*f*
g + 2*d*e*h - 3*c*f*h) + b*(2*d^2*e*g + 5*c^2*f*h - c*d*(3*f*g + 4*e*h)))*
((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/
Sqrt[d*e - c*f]])/d^(3/2)))/(2*d^2*(d*e - c*f))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-3\left(\frac{(-afg-2(ah+bg)e)d^2}{3}+\left((ah+bg)f+\frac{4ehb}{3}\right)cd-\frac{5b\,c^2fh}{3}\right)(xd+c)f\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\sqrt{(cf-de)d}\left(\frac{\left(\left(2x\left(\frac{hx}{3}+\cdots\right)\right)\right)}{\cdots}\right)}{d^3f(xd+c)\sqrt{(cf-de)d}}$
risch	$\frac{2(hxdbf+3adfh-6bcfh+bdeh+3bdfg)\sqrt{fx+e}}{3f\,d^3}-\frac{\frac{2\left(-\frac{1}{2}acdfh+\frac{1}{2}a\,d^2fg+\frac{1}{2}b\,c^2fh-\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(3acdfh-2a\,d^2eh-a\cdots)}{\cdots}}{d^3}$
derivativedivides	$\frac{\frac{2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^3}-\frac{2f\left(\frac{\left(-\frac{1}{2}acdfh+\frac{1}{2}a\,d^2fg+\frac{1}{2}b\,c^2fh-\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(3acdfh-2\cdots)}{\cdots}\right)}{\cdots}}{f}$
default	$\frac{\frac{2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^3}-\frac{2f\left(\frac{\left(-\frac{1}{2}acdfh+\frac{1}{2}a\,d^2fg+\frac{1}{2}b\,c^2fh-\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(3acdfh-2\cdots)}{\cdots}\right)}{\cdots}}{f}$

input
```
int((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/((c*f-d*e)*d)^(1/2)*(-(1/3*(-a*f*g-2*(a*h+b*g)*e)*d^2+((a*h+b*g)*f+4/3*e
*h*b)*c*d-5/3*b*c^2*f*h)*(d*x+c)*f*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1
/2))+((c*f-d*e)*d)^(1/2)*(1/3*((2*x*(1/3*h*x+g)*b-a*(-2*h*x+g))*f+2/3*b*e*
h*x)*d^2+c*(((-10/9*h*x+g)*b+a*h)*f+2/9*e*h*b)*d-5/3*b*c^2*f*h)*(f*x+e)^(1
/2))/d^3/f/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(164) = 328.

Time = 0.15 (sec) , antiderivative size = 906, normalized size of antiderivative = 4.98

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(d^2*e - c*d*f)*((2*b*c*d^2*e*f - (3*b*c^2*d - a*c*d^2)*f^2)*
g - (2*(2*b*c^2*d - a*c*d^2)*e*f - (5*b*c^3 - 3*a*c^2*d)*f^2)*h + ((2*b*d^
3*e*f - (3*b*c*d^2 - a*d^3)*f^2)*g - (2*(2*b*c*d^2 - a*d^3)*e*f - (5*b*c^2
*d - 3*a*c*d^2)*f^2)*h)*x)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f
)*sqrt(f*x + e))/(d*x + c)) - 2*(2*(b*d^4*e*f - b*c*d^3*f^2)*h*x^2 + 3*((3
*b*c*d^3 - a*d^4)*e*f - (3*b*c^2*d^2 - a*c*d^3)*f^2)*g + (2*b*c*d^3*e^2 -
(17*b*c^2*d^2 - 9*a*c*d^3)*e*f + 3*(5*b*c^3*d - 3*a*c^2*d^2)*f^2)*h + 2*(3
*(b*d^4*e*f - b*c*d^3*f^2)*g + (b*d^4*e^2 - 3*(2*b*c*d^3 - a*d^4)*e*f + (5
*b*c^2*d^2 - 3*a*c*d^3)*f^2)*h)*x)*sqrt(f*x + e))/(c*d^5*e*f - c^2*d^4*f^2
 + (d^6*e*f - c*d^5*f^2)*x), 1/3*(3*sqrt(-d^2*e + c*d*f)*((2*b*c*d^2*e*f -
 (3*b*c^2*d - a*c*d^2)*f^2)*g - (2*(2*b*c^2*d - a*c*d^2)*e*f - (5*b*c^3 -
3*a*c^2*d)*f^2)*h + ((2*b*d^3*e*f - (3*b*c*d^2 - a*d^3)*f^2)*g - (2*(2*b*c
*d^2 - a*d^3)*e*f - (5*b*c^2*d - 3*a*c*d^2)*f^2)*h)*x)*arctan(sqrt(-d^2*e
+ c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) + (2*(b*d^4*e*f - b*c*d^3*f^2)*h*x^2
 + 3*((3*b*c*d^3 - a*d^4)*e*f - (3*b*c^2*d^2 - a*c*d^3)*f^2)*g + (2*b*c*d^
3*e^2 - (17*b*c^2*d^2 - 9*a*c*d^3)*e*f + 3*(5*b*c^3*d - 3*a*c^2*d^2)*f^2)*
h + 2*(3*(b*d^4*e*f - b*c*d^3*f^2)*g + (b*d^4*e^2 - 3*(2*b*c*d^3 - a*d^4)*
e*f + (5*b*c^2*d^2 - 3*a*c*d^3)*f^2)*h)*x)*sqrt(f*x + e))/(c*d^5*e*f - c^2
*d^4*f^2 + (d^6*e*f - c*d^5*f^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(f*x+e)**(1/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \frac{(2\,bd^2eg - 3\,bcdfg + ad^2fg - 4\,bcdeh + 2\,ad^2eh + 5\,bc^2fh - 3\,acdfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,d^3} + \frac{\sqrt{fx+e}bcdfg - \sqrt{fx+e}ad^2fg - \sqrt{fx+e}bc^2fh + \sqrt{fx+e}acdfh}{((fx+e)d - de + cf)d^3} + \frac{2\left(3\sqrt{fx+e}bd^4f^3g + (fx+e)^{\frac{3}{2}}bd^4f^2h - 6\sqrt{fx+e}bcd^3f^3h + 3\sqrt{fx+e}ad^4f^3h\right)}{3\,d^6f^3}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(2*b*d^2*e*g - 3*b*c*d*f*g + a*d^2*f*g - 4*b*c*d*e*h + 2*a*d^2*e*h + 5*b*c
^2*f*h - 3*a*c*d*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-
d^2*e + c*d*f)*d^3) + (sqrt(f*x + e)*b*c*d*f*g - sqrt(f*x + e)*a*d^2*f*g -
 sqrt(f*x + e)*b*c^2*f*h + sqrt(f*x + e)*a*c*d*f*h)/(((f*x + e)*d - d*e +
c*f)*d^3) + 2/3*(3*sqrt(f*x + e)*b*d^4*f^3*g + (f*x + e)^(3/2)*b*d^4*f^2*h
 - 6*sqrt(f*x + e)*b*c*d^3*f^3*h + 3*sqrt(f*x + e)*a*d^4*f^3*h)/(d^6*f^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$= \sqrt{e+f\,x}\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{d^2\,f}-\frac{4\,b\,h\,(c\,f-d\,e)}{d^3\,f}\right)$$
$$-\frac{\sqrt{e+f\,x}\,(a\,d^2\,f\,g+b\,c^2\,f\,h-a\,c\,d\,f\,h-b\,c\,d\,f\,g)}{d^4\,(e+f\,x)-d^4\,e+c\,d^3\,f}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,(2\,a\,d^2\,e\,h+a\,d^2\,f\,g+2\,b\,d^2\,e\,g+5\,b\,c^2\,f\,h-3\,a\,c\,d\,f\,h-4\,b\,c\,d\,e\,h-3\,b\,c\,d\,f\,g)}{d^{7/2}\,\sqrt{c\,f-d\,e}}$$
$$+\frac{2\,b\,h\,(e+f\,x)^{3/2}}{3\,d^2\,f}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x))/(c + d*x)^2,x)
```

output
```
(e + f*x)^(1/2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(d^2*f) - (4*b*h*(c*f - d*e
))/(d^3*f)) - ((e + f*x)^(1/2)*(a*d^2*f*g + b*c^2*f*h - a*c*d*f*h - b*c*d*
f*g))/(d^4*(e + f*x) - d^4*e + c*d^3*f) + (atan((d^(1/2)*(e + f*x)^(1/2))/
(c*f - d*e)^(1/2))*(2*a*d^2*e*h + a*d^2*f*g + 2*b*d^2*e*g + 5*b*c^2*f*h -
3*a*c*d*f*h - 4*b*c*d*e*h - 3*b*c*d*f*g))/(d^(7/2)*(c*f - d*e)^(1/2)) + (2
*b*h*(e + f*x)^(3/2))/(3*d^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 989, normalized size of antiderivative = 5.43

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x)
```

output
```
( - 9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a*c**2*d*f**2*h + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(
sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*e*f*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan(
(sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*f**2*g - 9*sqrt(d)*s
qrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*
f**2*h*x + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a*d**3*e*f*h*x + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x
)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**3*f**2*g*x + 15*sqrt(d)*sqrt(c*f - d*
e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*f**2*h - 12*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*
c**2*d*e*f*h - 9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*b*c**2*d*f**2*g + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d*f**2*h*x + 6*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*e*f*g
 - 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*b*c*d**2*e*f*h*x - 9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*f**2*g*x + 6*sqrt(d)*sqrt(c*f - d*e)*a
tan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*d**3*e*f*g*x + 9*sqrt(e
 + f*x)*a*c**2*d**2*f**2*h - 9*sqrt(e + f*x)*a*c*d**3*e*f*h - 3*sqrt(e + f
*x)*a*c*d**3*f**2*g + 6*sqrt(e + f*x)*a*c*d**3*f**2*h*x + 3*sqrt(e + f*...
```

3.45 $\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \frac{2h\sqrt{e+fx}}{d^2} - \frac{(dg-ch)\sqrt{e+fx}}{d^2(c+dx)} - \frac{(dfg+2deh-3cfh)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}\sqrt{de-cf}}$$

output
```
2*h*(f*x+e)^(1/2)/d^2-(-c*h+d*g)*(f*x+e)^(1/2)/d^2/(d*x+c)-(-3*c*f*h+2*d*e
*h+d*f*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-c*f+d*
e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \frac{\sqrt{e+fx}(-dg+3ch+2dhx)}{d^2(c+dx)} + \frac{(dfg+2deh-3cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}\sqrt{-de+cf}}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(-(d*g) + 3*c*h + 2*d*h*x))/(d^2*(c + d*x)) + ((d*f*g + 2*d
*e*h - 3*c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(5/
2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(-3cfh+2deh+dfg)\int \frac{\sqrt{e+fx}}{c+dx}dx}{2d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \text{60}$$

$$\frac{(-3cfh+2deh+dfg)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{2d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \text{73}$$

$$\frac{(-3cfh+2deh+dfg)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{2d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)(-3cfh+2deh+dfg)}{2d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/(c + d*x)^2,x]
```

output
```
-(((d*g - c*h)*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x))) + ((d*f*g + 2*d
*e*h - 3*c*f*h)*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]
*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$\frac{-3\left(\frac{(-2eh-fg)d}{3}+cfh\right)(xd+c)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\sqrt{(cf-de)d}\left(\frac{(2hx-g)d}{3}+ch\right)\sqrt{fx+e}}{d^{2}(xd+c)\sqrt{(cf-de)d}}$	107
derivativedivides	$\frac{2h\sqrt{fx+e}}{d^{2}}-\frac{2\left(\frac{\left(-\frac{1}{2}cfh+\frac{1}{2}dfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(3cfh-2deh-dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{2\sqrt{(cf-de)d}}\right)}{d^{2}}$	109
default	$\frac{2h\sqrt{fx+e}}{d^{2}}-\frac{2\left(\frac{\left(-\frac{1}{2}cfh+\frac{1}{2}dfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(3cfh-2deh-dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{2\sqrt{(cf-de)d}}\right)}{d^{2}}$	109
risch	$\frac{2h\sqrt{fx+e}}{d^{2}}-\frac{\frac{2\left(-\frac{1}{2}cfh+\frac{1}{2}dfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(3cfh-2deh-dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{d^{2}}$	109

input
```
int((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3*(-(1/3*(-2*e*h-f*g)*d+c*f*h)*(d*x+c)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d
)^(1/2))+((c*f-d*e)*d)^(1/2)*(1/3*(2*h*x-g)*d+c*h)*(f*x+e)^(1/2))/((c*f-d*
e)*d)^(1/2)/d^2/(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.56

$$\int\frac{\sqrt{e+fx}(g+hx)}{(c+dx)^{2}}\,dx$$
$$=\left[-\frac{(cdfg+(2\,cde-3\,c^{2}f)h+(d^{2}fg+(2\,d^{2}e-3\,cdf)h)x)\sqrt{d^{2}e-cdf}\log\left(\frac{dfx+2\,de-cf+2\,\sqrt{d^{2}e-cdf}\sqrt{fx+e}}{dx+c}\right)}{2\,(cd^{4}e-c^{2}d^{3}f+(d^{5}e-cd^{4}f)x)}\right.$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*((c*d*f*g + (2*c*d*e - 3*c^2*f)*h + (d^2*f*g + (2*d^2*e - 3*c*d*f)*h
)*x)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*
sqrt(f*x + e))/(d*x + c)) - 2*(2*(d^3*e - c*d^2*f)*h*x - (d^3*e - c*d^2*f)
*g + 3*(c*d^2*e - c^2*d*f)*h)*sqrt(f*x + e))/(c*d^4*e - c^2*d^3*f + (d^5*e
 - c*d^4*f)*x), ((c*d*f*g + (2*c*d*e - 3*c^2*f)*h + (d^2*f*g + (2*d^2*e -
3*c*d*f)*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x +
 e)/(d*f*x + d*e)) + (2*(d^3*e - c*d^2*f)*h*x - (d^3*e - c*d^2*f)*g + 3*(c
*d^2*e - c^2*d*f)*h)*sqrt(f*x + e))/(c*d^4*e - c^2*d^3*f + (d^5*e - c*d^4*
f)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \int \frac{\sqrt{e+fx}\,(g+hx)}{(c+dx)^2}\,dx$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Integral(sqrt(e + f*x)*(g + h*x)/(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \frac{2\sqrt{fx+e}h}{d^2} + \frac{(dfg+2\,deh-3\,cfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}d^2} - \frac{\sqrt{fx+e}dfg-\sqrt{fx+e}cfh}{((fx+e)d-de+cf)d^2}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
2*sqrt(f*x + e)*h/d^2 + (d*f*g + 2*d*e*h - 3*c*f*h)*arctan(sqrt(f*x + e)*d
/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^2) - (sqrt(f*x + e)*d*f*g -
 sqrt(f*x + e)*c*f*h)/(((f*x + e)*d - d*e + c*f)*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx = \frac{\sqrt{e+f\,x}\,(c\,f\,h-d\,f\,g)}{d^3\,(e+f\,x)-d^3\,e+c\,d^2\,f} + \frac{2\,h\,\sqrt{e+f\,x}}{d^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,(2\,d\,e\,h-3\,c\,f\,h+d\,f\,g)}{d^{5/2}\,\sqrt{c\,f-d\,e}}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/(c + d*x)^2,x)
```

output
```
((e + f*x)^(1/2)*(c*f*h - d*f*g))/(d^3*(e + f*x) - d^3*e + c*d^2*f) + (2*h
*(e + f*x)^(1/2))/d^2 + (atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))
*(2*d*e*h - 3*c*f*h + d*f*g))/(d^(5/2)*(c*f - d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.39

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^2}\,dx$$
$$= \frac{-3\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2fh+2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cdeh+\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{}}{\sqrt{d}}\right.}{}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^2,x)
```

output
```
( - 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*c**2*f*h + 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*c*d*e*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d
)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f*g - 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f*h*x + 2*sqrt(d)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*e*h*x + sqrt(d
)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*f
*g*x + 3*sqrt(e + f*x)*c**2*d*f*h - 3*sqrt(e + f*x)*c*d**2*e*h - sqrt(e +
f*x)*c*d**2*f*g + 2*sqrt(e + f*x)*c*d**2*f*h*x + sqrt(e + f*x)*d**3*e*g -
2*sqrt(e + f*x)*d**3*e*h*x)/(d**3*(c**2*f - c*d*e + c*d*f*x - d**2*e*x))
```

3.46 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 204

$$\begin{aligned}
&\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx \\
&= \frac{(dg-ch)\sqrt{e+fx}}{d(bc-ad)(c+dx)} - \frac{2\sqrt{be-af}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)^2} \\
&\quad - \frac{(ad(dfg+2deh-3cfh)-b(2d^2eg-cdfg-c^2fh))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(bc-ad)^2\sqrt{de-cf}}
\end{aligned}$$

output
```
(-c*h+d*g)*(f*x+e)^(1/2)/d/(-a*d+b*c)/(d*x+c)-2*(-a*f+b*e)^(1/2)*(-a*h+b*g
)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^2-(a*
d*(-3*c*f*h+2*d*e*h+d*f*g)-b*(-c^2*f*h-c*d*f*g+2*d^2*e*g))*arctanh(d^(1/2)
*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-a*d+b*c)^2/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx$$
$$= \frac{(-dg+ch)\sqrt{e+fx}}{d(-bc+ad)(c+dx)} - \frac{2\sqrt{-be+af}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{b}(bc-ad)^2}$$
$$+ \frac{(ad(dfg+2deh-3cfh)+b(-2d^2eg+cdfg+c^2fh))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{3/2}(bc-ad)^2\sqrt{-de+cf}}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)*(c + d*x)^2),x]
```

output
```
((-(d*g) + c*h)*Sqrt[e + f*x])/(d*(-(b*c) + a*d)*(c + d*x)) - (2*Sqrt[-(b*
e) + a*f]*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/
(Sqrt[b]*(b*c - a*d)^2) + ((a*d*(d*f*g + 2*d*e*h - 3*c*f*h) + b*(-2*d^2*e*
g + c*d*f*g + c^2*f*h))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]]
)/(d^(3/2)*(b*c - a*d)^2*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(bc-ad)} - \frac{\int -\frac{2bdeg-a(dfg+2deh-cfh)+f(bdg+bch-2adh)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{d(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2bdeg+acfh-ad(fg+2eh)+f(bdg+bch-2adh)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2d(bc-ad)}+\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(bc-ad)}$$

$\Big\downarrow$ 174

$$\frac{\frac{(ad(-3cfh+2deh+dfg)-b(c^2(-f)h-cdfg+2d^2eg))\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}+\frac{2d(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}}{2d(bc-ad)}+\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(bc-ad)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2(ad(-3cfh+2deh+dfg)-b(c^2(-f)h-cdfg+2d^2eg))\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}+\frac{4d(be-af)(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{2d(bc-ad)}+\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)(ad(-3cfh+2deh+dfg)-b(c^2(-f)h-cdfg+2d^2eg))}{\sqrt{d}(bc-ad)\sqrt{de-cf}}-\frac{4d\sqrt{be-af}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)}}{2d(bc-ad)}+\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)*(c + d*x)^2),x]
```

output
```
((d*g - c*h)*Sqrt[e + f*x])/(d*(b*c - a*d)*(c + d*x)) + ((-4*d*Sqrt[b*e -
a*f]*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b
]*(b*c - a*d)) - (2*(a*d*(d*f*g + 2*d*e*h - 3*c*f*h) - b*(2*d^2*e*g - c*d*
f*g - c^2*f*h))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]
*(b*c - a*d)*Sqrt[d*e - c*f]))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11

method	result
derivativedivides	$2f\left(\frac{(af-be)(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f(ad-bc)^2\sqrt{(af-be)b}}-\frac{-\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2d((fx+e)d+cf-de)}+\frac{\left(3acdfh-2a\,d^2eh-a\,d^2fg-b\,c^2fh\right.}{2d\sqrt{}}}{(ad-bc)^2f}\right.$
default	$2f\left(\frac{(af-be)(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f(ad-bc)^2\sqrt{(af-be)b}}-\frac{-\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2d((fx+e)d+cf-de)}+\frac{\left(3acdfh-2a\,d^2eh-a\,d^2fg-b\,c^2fh\right.}{2d\sqrt{}}}{(ad-bc)^2f}\right.$
pseudoelliptic	$\frac{-3\sqrt{(af-be)b}\,(xd+c)\left(\frac{2\left(beg-a\left(eh+\frac{fg}{2}\right)\right)d^2}{3}+cf\left(ah-\frac{bg}{3}\right)d-\frac{b\,c^2fh}{3}\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\sqrt{(cf-de)d}\,\left(2d(ah-bg\right.}{\sqrt{(af-be)b}\,\sqrt{(cf-de)d}\,(ad-bc)^2(xd+c}$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*f*((a*f-b*e)/f*(a*h-b*g)/(a*d-b*c)^2/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e
)^(1/2)/((a*f-b*e)*b)^(1/2))-1/(a*d-b*c)^2/f*(-1/2*f*(a*c*d*h-a*d^2*g-b*c^
2*h+b*c*d*g)/d*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(3*a*c*d*f*h-2*a*d^2*
e*h-a*d^2*f*g-b*c^2*f*h-b*c*d*f*g+2*b*d^2*e*g)/d/((c*f-d*e)*d)^(1/2)*arcta
n(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(182) = 364.

Time = 1.57 (sec) , antiderivative size = 2088, normalized size of antiderivative = 10.24

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*((b*c*d^3*e - b*c^2*d^2*f)*g - (a*c*d^3*e - a*c^2*d^2*f)*h + ((b*
d^4*e - b*c*d^3*f)*g - (a*d^4*e - a*c*d^3*f)*h)*x)*sqrt((b*e - a*f)/b)*log
((b*f*x + 2*b*e - a*f + 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a))
- sqrt(d^2*e - c*d*f)*((2*b*c*d^2*e - (b*c^2*d + a*c*d^2)*f)*g - (2*a*c*d^
2*e + (b*c^3 - 3*a*c^2*d)*f)*h + ((2*b*d^3*e - (b*c*d^2 + a*d^3)*f)*g - (2
*a*d^3*e + (b*c^2*d - 3*a*c*d^2)*f)*h)*x)*log((d*f*x + 2*d*e - c*f + 2*sqr
t(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(((b*c*d^3 - a*d^4)*e - (b*
c^2*d^2 - a*c*d^3)*f)*g - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)
*f)*h)*sqrt(f*x + e))/((b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e - (b^2*
c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*f + ((b^2*c^2*d^4 - 2*a*b*c*d^5 + a
^2*d^6)*e - (b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*f)*x), -1/2*(4*((b*c
*d^3*e - b*c^2*d^2*f)*g - (a*c*d^3*e - a*c^2*d^2*f)*h + ((b*d^4*e - b*c*d^
3*f)*g - (a*d^4*e - a*c*d^3*f)*h)*x)*sqrt(-(b*e - a*f)/b)*arctan(-sqrt(f*x
 + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) - sqrt(d^2*e - c*d*f)*((2*b*c*d^
2*e - (b*c^2*d + a*c*d^2)*f)*g - (2*a*c*d^2*e + (b*c^3 - 3*a*c^2*d)*f)*h +
 ((2*b*d^3*e - (b*c*d^2 + a*d^3)*f)*g - (2*a*d^3*e + (b*c^2*d - 3*a*c*d^2)
*f)*h)*x)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/
(d*x + c)) - 2*(((b*c*d^3 - a*d^4)*e - (b*c^2*d^2 - a*c*d^3)*f)*g - ((b*c^
2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*h)*sqrt(f*x + e))/((b^2*c^3*
d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e - (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \frac{2\,(b^2eg - abfg - abeh + a^2fh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-b^2e+abf}} - \frac{(2\,bd^2eg - bcdfg - ad^2fg - 2\,ad^2eh - bc^2fh + 3\,acdfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{-d^2e+cdf}} + \frac{\sqrt{fx+e}dfg - \sqrt{fx+e}cfh}{(bcd-ad^2)((fx+e)d - de + cf)}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
2*(b^2*e*g - a*b*f*g - a*b*e*h + a^2*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2
*e + a*b*f))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*e + a*b*f)) - (2*b
*d^2*e*g - b*c*d*f*g - a*d^2*f*g - 2*a*d^2*e*h - b*c^2*f*h + 3*a*c*d*f*h)*
arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*sqrt(-d^2*e + c*d*f)) + (sqrt(f*x + e)*d*f*g - sqrt(f*x + e)*c*f*h
)/((b*c*d - a*d^2)*((f*x + e)*d - d*e + c*f))
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 11557, normalized size of antiderivative = 56.65

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)*(c + d*x)^2),x)
```

output
```
(atan(((((2*(e + f*x)^(1/2)*(b^5*c^4*f^4*h^2 + 5*a^2*b^3*d^4*f^4*g^2 + b^5
*c^2*d^2*f^4*g^2 + 8*b^5*d^4*e^2*f^2*g^2 + 4*a^4*b*d^4*f^4*h^2 + 2*a*b^4*c
*d^3*f^4*g^2 - 6*a*b^4*c^3*d*f^4*h^2 - 12*a*b^4*d^4*e*f^3*g^2 - 4*b^5*c*d^
3*e*f^3*g^2 - 8*a^3*b^2*d^4*f^4*g*h - 8*a^3*b^2*d^4*e*f^3*h^2 + 2*b^5*c^3*
d*f^4*g*h + 9*a^2*b^3*c^2*d^2*f^4*h^2 + 8*a^2*b^3*d^4*e^2*f^2*h^2 - 4*a*b^
4*c^2*d^2*f^4*g*h - 6*a^2*b^3*c*d^3*f^4*g*h - 16*a*b^4*d^4*e^2*f^2*g*h + 2
0*a^2*b^3*d^4*e*f^3*g*h - 4*b^5*c^2*d^2*e*f^3*g*h + 4*a*b^4*c^2*d^2*e*f^3*
h^2 - 12*a^2*b^3*c*d^3*e*f^3*h^2 + 16*a*b^4*c*d^3*e*f^3*g*h))/(a^2*d^3 + b
^2*c^2*d - 2*a*b*c*d^2) + ((-d^3*(c*f - d*e))^(1/2)*((2*(2*a^5*b^2*d^7*f^4
*g + 2*a*b^6*c^4*d^3*f^4*g - 8*a^4*b^3*c*d^6*f^4*g - 2*a*b^6*c^5*d^2*f^4*h
 - 2*a^5*b^2*c*d^6*f^4*h - 2*a^4*b^3*d^7*e*f^3*g - 2*b^7*c^4*d^3*e*f^3*g +
 2*b^7*c^5*d^2*e*f^3*h - 8*a^2*b^5*c^3*d^4*f^4*g + 12*a^3*b^4*c^2*d^5*f^4*
g + 8*a^2*b^5*c^4*d^3*f^4*h - 12*a^3*b^4*c^3*d^4*f^4*h + 8*a^4*b^3*c^2*d^5
*f^4*h + 8*a*b^6*c^3*d^4*e*f^3*g + 8*a^3*b^4*c*d^6*e*f^3*g - 8*a*b^6*c^4*d
^3*e*f^3*h + 2*a^4*b^3*c*d^6*e*f^3*h - 12*a^2*b^5*c^2*d^5*e*f^3*g + 12*a^2
*b^5*c^3*d^4*e*f^3*h - 8*a^3*b^4*c^2*d^5*e*f^3*h))/(a^3*d^4 - b^3*c^3*d +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((e + f*x)^(1/2)*(-d^3*(c*f - d*e))^(1/
2)*(2*a*d^2*e*h + a*d^2*f*g - 2*b*d^2*e*g + b*c^2*f*h - 3*a*c*d*f*h + b*c*
d*f*g)*(4*a^5*b^2*d^8*f^3 + 4*b^7*c^5*d^3*f^3 + 8*a^2*b^5*c^3*d^5*f^3 + 8*
a^3*b^4*c^2*d^6*f^3 - 12*a*b^6*c^4*d^4*f^3 - 12*a^4*b^3*c*d^7*f^3 - 8*a...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1168, normalized size of antiderivative = 5.73

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x)
```

output
```
(2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a*c**2*d**2*f*h - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a*c*d**3*e*h + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c*d**3*f*h*x - 2*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*d**4*e*h*x -
2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*b*c**2*d**2*f*g + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*b*c*d**3*e*g - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c*d**3*f*g*x + 2*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*d**4*e*g*x - 3
*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a*b*c**2*d*f*h + 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a*b*c*d**2*e*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
+ f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**2*f*g - 3*sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**2*f*h*x +
 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a*b*d**3*e*h*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a*b*d**3*f*g*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
+ f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**3*f*h + sqrt(d)*sqrt(c*f - d*
e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**2*d*f*g + ...
```

3.47 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 304

$$\begin{aligned}\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2} \, dx &= -\frac{(2bdg-bch-adh)\sqrt{e+fx}}{b(bc-ad)^2(c+dx)} - \frac{(bg-ah)\sqrt{e+fx}}{b(bc-ad)(a+bx)(c+dx)} \\ &+ \frac{(a^2dfh-ab(3dfg+2deh-3cfh)+b^2(4deg-c(fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)^3\sqrt{be-af}} \\ &+ \frac{(ad(dfg+2deh-3cfh)-b(4d^2eg+c^2fh-cd(3fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)^3\sqrt{de-cf}}\end{aligned}$$

output

```
-(-a*d*h-b*c*h+2*b*d*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(d*x+c)-(-a*h+b*g)*(f
*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)/(d*x+c)+(a^2*d*f*h-a*b*(-3*c*f*h+2*d*e*h+
3*d*f*g)+b^2*(4*d*e*g-c*(2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+
b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^(1/2)+(a*d*(-3*c*f*h+2*d*e*h+d
*f*g)-b*(4*d^2*e*g+c^2*f*h-c*d*(2*e*h+3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/
2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b*c)^3/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx$$
$$= \frac{\sqrt{e+fx}(b(-cg-2dgx+chx)+a(-dg+2ch+dhx))}{(bc-ad)^2(a+bx)(c+dx)}$$
$$+ \frac{(-a^2dfh+b^2(-4deg+cfg+2ceh)+ab(3dfg+2deh-3cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{b}(bc-ad)^3\sqrt{-be+af}}$$
$$- \frac{(-ad(dfg+2deh-3cfh)+b(4d^2eg+c^2fh-cd(3fg+2eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-bc+ad)^3\sqrt{-de+cf}}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^2*(c + d*x)^2),x]
```

output
```
(Sqrt[e + f*x]*(b*(-(c*g) - 2*d*g*x + c*h*x) + a*(-(d*g) + 2*c*h + d*h*x))
)/((b*c - a*d)^2*(a + b*x)*(c + d*x)) + ((-(a^2*d*f*h) + b^2*(-4*d*e*g + c
*f*g + 2*c*e*h) + a*b*(3*d*f*g + 2*d*e*h - 3*c*f*h))*ArcTan[(Sqrt[b]*Sqrt[
e + f*x])/Sqrt[-(b*e) + a*f]])/(Sqrt[b]*(b*c - a*d)^3*Sqrt[-(b*e) + a*f])
- ((-(a*d*(d*f*g + 2*d*e*h - 3*c*f*h)) + b*(4*d^2*e*g + c^2*f*h - c*d*(3*f
*g + 2*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sqrt[d]
*(-(b*c) + a*d)^3*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx$$
$$\Big\downarrow \text{ 166}$$

$$\frac{\int -\frac{4bdeg-bcfg-2bceh-2adeh+acfh+f(3bdg-2bch-adh)x}{2(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{4bdeg-a(2de-cf)h-bc(fg+2eh)+f(3bdg-2bch-adh)x}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\big\downarrow$ 168

$$-\frac{\frac{\int \frac{b(de-cf)(4bdeg-bc(fg+2eh)-a(dfg+2deh-2cfh)+f(2bdg-bch-adh)x)}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{(bc-ad)(de-cf)}+\frac{2\sqrt{e+fx}(-adh-bch+2bdg)}{(c+dx)(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\big\downarrow$ 27

$$-\frac{\frac{b\int \frac{4bdeg-bc(fg+2eh)-a(dfg+2deh-2cfh)+f(2bdg-bch-adh)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{bc-ad}+\frac{2\sqrt{e+fx}(-adh-bch+2bdg)}{(c+dx)(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\big\downarrow$ 174

$$-\frac{\frac{b\left(\frac{\left(a^2dfh-ab(-3cfh+2deh+3dfg)+b^2(4deg-c(2eh+fg))\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}+\frac{\left(ad(-3cfh+2deh+dfg)-b\left(c^2fh-cd(2eh+3fg)+4d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}}{bc-ad}}{bc-ad}}{2b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\big\downarrow$ 73

$$-\frac{\frac{b\left(\frac{2\left(a^2dfh-ab(-3cfh+2deh+3dfg)+b^2(4deg-c(2eh+fg))\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}+\frac{2\left(ad(-3cfh+2deh+dfg)-b\left(c^2fh-cd(2eh+3fg)+4d^2eg\right)\right)\int \frac{}{c+}}{f(bc-ad)}}{bc-ad}}{2b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\big\downarrow$ 221

$$-\frac{\frac{b\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(a^2dfh-ab(-3cfh+2deh+3dfg)+b^2(4deg-c(2eh+fg))\right)}{\sqrt{b}(bc-ad)\sqrt{be-af}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(-3cfh+2deh+dfg)-b\left(c^2fh-cd(2e\right.\right.}{\sqrt{d}(bc-ad)\sqrt{de-cf}}}{bc-ad}}{2b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^2*(c + d*x)^2),x]
```

output
```
-(((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)*(c + d*x))) - ((2*(
2*b*d*g - b*c*h - a*d*h)*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)) + (b*((-2*
(a^2*d*f*h - a*b*(3*d*f*g + 2*d*e*h - 3*c*f*h) + b^2*(4*d*e*g - c*(f*g + 2
*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c -
a*d)*Sqrt[b*e - a*f]) - (2*(a*d*(d*f*g + 2*d*e*h - 3*c*f*h) - b*(4*d^2*e*g
 + c^2*f*h - c*d*(3*f*g + 2*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*
e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f])))/(b*c - a*d))/(2*b*(b*c
- a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{\left((-2ceh-cfg+4deg)b^2+3a\left(\left(-\frac{2eh}{3}-fg\right)d+cfh\right)b+a^2dfh\right)(xd+c)\sqrt{(cf-de)d}\,(bx+a)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+2\sqrt{(a\cdots}{\cdots}$
derivativedivides	$2f^2\left(-\frac{\frac{\left(-\frac{1}{2}acdfh+\frac{1}{2}a\,d^2fg+\frac{1}{2}b\,c^2fh-\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{\left(3acdfh-2a\,d^2eh-a\,d^2fg+b\,c^2fh-2bcdeh-3bcdfg+4b\,d^2eg\right)\mathrm{arcta}\cdots}{2\sqrt{(cf-de)d}}}{f^2(ad-bc)^3}\cdots\right.$
default	$2f^2\left(-\frac{\frac{\left(-\frac{1}{2}acdfh+\frac{1}{2}a\,d^2fg+\frac{1}{2}b\,c^2fh-\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{\left(3acdfh-2a\,d^2eh-a\,d^2fg+b\,c^2fh-2bcdeh-3bcdfg+4b\,d^2eg\right)\mathrm{arcta}\cdots}{2\sqrt{(cf-de)d}}}{f^2(ad-bc)^3}\cdots\right.$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*(1/2*((-2*c*e*h-c*f*g+4*d*e*g)*b
^2+3*a*((-2/3*e*h-f*g)*d+c*f*h)*b+a^2*d*f*h)*(d*x+c)*((c*f-d*e)*d)^(1/2)*(
b*x+a)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(-3
/2*(d*x+c)*((4/3*d^2*e*g+c*(-2/3*e*h-f*g)*d+1/3*c^2*f*h)*b+a*d*(1/3*(-2*e*
h-f*g)*d+c*f*h))*(b*x+a)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-
b*c)*((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((-d*g*x-1/2*c*(-h*x+g))*b+a*(1/2*(
h*x-g)*d+c*h))))/(b*x+a)/(a*d-b*c)^3/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. 2(280) = 560.

Time = 5.73 (sec) , antiderivative size = 8168, normalized size of antiderivative = 26.87

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx =$$

$$-\frac{(4\,b^2deg-b^2cfg-3\,abdfg-2\,b^2ceh-2\,abdeh+3\,abcfh+a^2dfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2e+abf}}$$
$$+\frac{(4\,bd^2eg-3\,bcdfg-ad^2fg-2\,bcdeh-2\,ad^2eh+bc^2fh+3\,acdfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-d^2e+cdf}}$$
$$-\frac{2\,(fx+e)^{\frac{3}{2}}bdfg-2\,\sqrt{fx+e}bdefg+\sqrt{fx+e}bcf^2g+\sqrt{fx+e}adf^2g-(fx+e)^{\frac{3}{2}}bcfh-(fx+e)^{\frac{3}{2}}a}{(b^2c^2-2\,abcd+a^2d^2)\big((fx+e)^2bd-2\,(fx+e)bde+bde^2+(fx+e)bcf+(}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(4*b^2*d*e*g - b^2*c*f*g - 3*a*b*d*f*g - 2*b^2*c*e*h - 2*a*b*d*e*h + 3*a*
b*c*f*h + a^2*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*e + a*b*f)) + (4*b*
d^2*e*g - 3*b*c*d*f*g - a*d^2*f*g - 2*b*c*d*e*h - 2*a*d^2*e*h + b*c^2*f*h
+ 3*a*c*d*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-d^2*e + c*d*f)) - (2*(f*x + e
)^(3/2)*b*d*f*g - 2*sqrt(f*x + e)*b*d*e*f*g + sqrt(f*x + e)*b*c*f^2*g + sq
rt(f*x + e)*a*d*f^2*g - (f*x + e)^(3/2)*b*c*f*h - (f*x + e)^(3/2)*a*d*f*h
+ sqrt(f*x + e)*b*c*e*f*h + sqrt(f*x + e)*a*d*e*f*h - 2*sqrt(f*x + e)*a*c*
f^2*h)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((f*x + e)^2*b*d - 2*(f*x + e)*b*d
*e + b*d*e^2 + (f*x + e)*b*c*f + (f*x + e)*a*d*f - b*c*e*f - a*d*e*f + a*c
*f^2))
```

Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 26654, normalized size of antiderivative = 87.68

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^2),x)
```

output
```
(((e + f*x)^(3/2)*(a*d*f*h + b*c*f*h - 2*b*d*f*g))/(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d) - ((e + f*x)^(1/2)*(a*d*f^2*g - 2*a*c*f^2*h + b*c*f^2*g + a*d*e*f
*h + b*c*e*f*h - 2*b*d*e*f*g))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/((e + f*x)
*(a*d*f + b*c*f - 2*b*d*e) + b*d*(e + f*x)^2 + a*c*f^2 + b*d*e^2 - a*d*e*f
 - b*c*e*f) - atan(((((20*a*b^8*c^6*d^3*f^4*g - 4*b^9*c^7*d^2*f^4*g - 4*a^
7*b^2*d^9*f^4*g + 20*a^6*b^3*c*d^8*f^4*g + 8*a*b^8*c^7*d^2*f^4*h + 8*a^7*b
^2*c*d^8*f^4*h + 8*a^6*b^3*d^9*e*f^3*g - 4*a^7*b^2*d^9*e*f^3*h + 8*b^9*c^6
*d^3*e*f^3*g - 4*b^9*c^7*d^2*e*f^3*h - 36*a^2*b^7*c^5*d^4*f^4*g + 20*a^3*b
^6*c^4*d^5*f^4*g + 20*a^4*b^5*c^3*d^6*f^4*g - 36*a^5*b^4*c^2*d^7*f^4*g - 4
8*a^2*b^7*c^6*d^3*f^4*h + 120*a^3*b^6*c^5*d^4*f^4*h - 160*a^4*b^5*c^4*d^5*
f^4*h + 120*a^5*b^4*c^3*d^6*f^4*h - 48*a^6*b^3*c^2*d^7*f^4*h - 48*a*b^8*c^
5*d^4*e*f^3*g - 48*a^5*b^4*c*d^8*e*f^3*g + 20*a*b^8*c^6*d^3*e*f^3*h + 20*a
^6*b^3*c*d^8*e*f^3*h + 120*a^2*b^7*c^4*d^5*e*f^3*g - 160*a^3*b^6*c^3*d^6*e
*f^3*g + 120*a^4*b^5*c^2*d^7*e*f^3*g - 36*a^2*b^7*c^5*d^4*e*f^3*h + 20*a^3
*b^6*c^4*d^5*e*f^3*h + 20*a^4*b^5*c^3*d^6*e*f^3*h - 36*a^5*b^4*c^2*d^7*e*f
^3*h)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (2*(e + f*x)^(1/2)*((4*a^
2*d^4*e^2*h^2 + a^2*d^4*f^2*g^2 + 16*b^2*d^4*e^2*g^2 + b^2*c^4*f^2*h^2 + 9
*a^2*c^2*d^2*f^2*h^2 + 4*b^2*c^2*d^2*e^2*h^2 + 9*b^2*c^2*d^2*f^2*g^2 - 8*a
*b*d^4*e*f*g^2 - 16*a*b*d^4*e^2*g*h + 4*a^2*d^4*e*f*g*h + 8*a*b*c*d^3*e...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5334, normalized size of antiderivative = 17.55

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x)
```

output

```
(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**3*c**2*d**2*f**2*h - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**3*c*d**3*e*f*h + sqrt(b)*sqrt(a*f - b*e)*atan(
(sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c*d**3*f**2*h*x - sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*d
**4*e*f*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**2*b*c**3*d*f**2*h - 5*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**2*e*f*h - 3*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b
*c**2*d**2*f**2*g + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**2*f**2*h*x + 2*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d**3*e**2*h +
 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**2*b*c*d**3*e*f*g - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d**3*e*f*h*x - 3*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d**3*f**2*g*
x + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**2*b*c*d**3*f**2*h*x**2 + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**4*e**2*h*x + 3*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**4*...
```

3.48 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 530

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx$$
$$= \frac{d(3a^2dfh-ab(11dfg+4deh-9cfh)+b^2(12deg-c(fg+8eh)))\sqrt{e+fx}}{4b(bc-ad)^3(be-af)(c+dx)}$$
$$-\frac{(bg-ah)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)}$$
$$+\frac{(a^2dfh-ab(5dfg+2deh-5cfh)+b^2(6deg-c(fg+4eh)))\sqrt{e+fx}}{4b(bc-ad)^2(be-af)(a+bx)(c+dx)}$$
$$+\frac{(3a^3d^2f^2h-3a^2bdf(5dfg+4deh-6cfh)-b^3(24d^2e^2g-c^2f(fg-4eh)-8cde(fg+2eh))+ab^2(3c}{4\sqrt{b}(bc-ad)^4(be-af)^{3/2}}$$
$$-\frac{\sqrt{d}(ad(dfg+2deh-3cfh)-b(6d^2eg+3c^2fh-cd(5fg+4eh)))\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4\sqrt{de-cf}}$$

output
```
1/4*d*(3*a^2*d*f*h-a*b*(-9*c*f*h+4*d*e*h+11*d*f*g)+b^2*(12*d*e*g-c*(8*e*h+
f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+b*e)/(d*x+c)-1/2*(-a*h+b*g)*(f*x
+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)^2/(d*x+c)+1/4*(a^2*d*f*h-a*b*(-5*c*f*h+2*d*
e*h+5*d*f*g)+b^2*(6*d*e*g-c*(4*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a
*f+b*e)/(b*x+a)/(d*x+c)+1/4*(3*a^3*d^2*f^2*h-3*a^2*b*d*f*(-6*c*f*h+4*d*e*h
+5*d*f*g)-b^3*(24*d^2*e^2*g-c^2*f*(-4*e*h+f*g)-8*c*d*e*(2*e*h+f*g))+a*b^2*
(3*c^2*f^2*h+8*d^2*e*(e*h+5*f*g)-2*c*d*f*(16*e*h+5*f*g)))*arctanh(b^(1/2)*
(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^4/(-a*f+b*e)^(3/2)-d^(1
/2)*(a*d*(-3*c*f*h+2*d*e*h+d*f*g)-b*(6*d^2*e*g+3*c^2*f*h-c*d*(4*e*h+5*f*g)
))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^4/(-c*f+d*e)
^(1/2)
```

Mathematica [A] (verified)

Time = 6.98 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx$$

$$= \frac{-\frac{(bc-ad)\sqrt{e+fx}\left(a^3df(4dg-9ch-5dhx)+b^3\left(-12d^2egx^2+c^2(2eg+fgx+4ehx)+cdx(-6eg+fgx+8ehx)\right)+a^2b\left(-3c^2fh+cd(9fg+10eh-14fh\right.\right.}{(be-af)(a+bx)^2}}{}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^3*(c + d*x)^2),x]
```

output
```
(-(((b*c - a*d)*Sqrt[e + f*x]*(a^3*d*f*(4*d*g - 9*c*h - 5*d*h*x) + b^3*(-1
2*d^2*e*g*x^2 + c^2*(2*e*g + f*g*x + 4*e*h*x) + c*d*x*(-6*e*g + f*g*x + 8*
e*h*x)) + a^2*b*(-3*c^2*f*h + c*d*(9*f*g + 10*e*h - 14*f*h*x) + d^2*(-4*e*
g + 17*f*g*x + 6*e*h*x - 3*f*h*x^2)) + a*b^2*(d^2*x*(-18*e*g + 11*f*g*x +
4*e*h*x) + c*d*(-10*e*g + 6*f*g*x + 14*e*h*x - 9*f*h*x^2) + c^2*(2*e*h - f
*(g + 5*h*x)))))/((b*e - a*f)*(a + b*x)^2*(c + d*x))) + ((3*a^3*d^2*f^2*h
- 3*a^2*b*d*f*(5*d*f*g + 4*d*e*h - 6*c*f*h) + b^3*(-24*d^2*e^2*g + c^2*f*(
f*g - 4*e*h) + 8*c*d*e*(f*g + 2*e*h)) + a*b^2*(3*c^2*f^2*h + 8*d^2*e*(5*f*
g + e*h) - 2*c*d*f*(5*f*g + 16*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[
-(b*e) + a*f]])/(Sqrt[b]*(-(b*e) + a*f)^(3/2)) + (4*Sqrt[d]*(a*d*(d*f*g +
2*d*e*h - 3*c*f*h) + b*(-6*d^2*e*g - 3*c^2*f*h + c*d*(5*f*g + 4*e*h)))*Arc
Tan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/Sqrt[-(d*e) + c*f])/(4*(b
*c - a*d)^4)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {166, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{a(2de-cf)h-b(6deg-cfg-4ceh)-f(5bdg-4bch-adh)x}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}}dx}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a(2de-cf)h-b(6deg-cfg-4ceh)-f(5bdg-4bch-adh)x}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}dx}{4b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{\sqrt{e+fx}(a^2dfh-ab(-5cfh+2deh+5dfg)+b^2(-4ceh-cfg+6deg))}{(a+bx)(c+dx)(bc-ad)(be-af)}-\frac{\int -\frac{3df(2de-cf)ha^2-b\left(2e(11fg+4eh)d^2-cf(7fg+20eh)d+3c^2f^2h\right)a+b^2\left(-f(fg\cdots\right.}{2(\cdots}}{\cdots}}{4b(bc-ad)}-$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3df(2de-cf)ha^2-b\left(2e(11fg+4eh)d^2-cf(7fg+20eh)d+3c^2f^2h\right)a+b^2\left(-f(fg-4eh)c^2-8de(fg+2eh)c+24d^2e^2g\right)+3df\left(dfha^2-b(5dfg+2deh-5cfh)a+b^2(6de\cdots\right.}{(a+bx)(c+dx)^2\sqrt{e+fx}}\cdots}{2(bc-ad)(be-af)}\cdots}{4b(bc-ad)}-$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{\int \frac{b(de-cf)\left(df(4dfg+8deh-9cfh)a^2-b\left(4e(7fg+2eh)d^2-3cf(3fg+8eh)d+3c^2f^2h\right)a+b^2\left(-f(fg-4eh)c^2-8de(fg+2eh)c+24d^2e^2g\right)+df\left(3dfha^2-b(11dfg+4de\cdots\right.\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{(bc-ad)(de-cf)}\cdots}{2(bc-ad)(be-af)\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{df(4dfg+8deh-9cfh)a^2-b\left(4e(7fg+2eh)d^2-3cf(3fg+8eh)d+3c^2f^2h\right)a+b^2\left(-f(fg-4eh)c^2-8de(fg+2eh)c+24d^2e^2g\right)+df\left(3dfha^2-b(11dfg+4deh-9cfh)\cdots\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{bc-ad}\cdots}{2(bc-ad)(be-af)\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{b\left(\frac{4d(be-af)\left(ad(-3cfh+2deh+dfg)-b\left(3c^2fh-cd(4eh+5fg)+6d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(3a^3d^2f^2h-3a^2bdf(-6cfh+4deh+5dfg)+ab^2\left(3c^2f^2h-2cdf(16e\cdots\right.\right.}{\cdots}\cdots\right.}{bc-ad}\cdots}{2(bc\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{b\left(\frac{8d(be-af)\left(ad(-3cfh+2deh+dfg)-b\left(3c^2fh-cd(4eh+5fg)+6d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(3a^3d^2f^2h-3a^2bdf(-6cfh+4deh+5dfg)+ab^2\left(3c^2f^2h\cdots\right.\right.}{\cdots}\cdots\right.}{bc-ad}\cdots}{\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\sqrt{e+fx}\left(a^2dfh-ab(-5cfh+2deh+5dfg)+b^2(-4ceh-cfg+6deg)\right)}{(a+bx)(c+dx)(bc-ad)(be-af)}+\frac{\frac{2d\sqrt{e+fx}\left(3a^2dfh-ab(-9cfh+4deh+11dfg)+b^2(-8ceh-cfg+12deg)\right)}{(c+dx)(bc-ad)}+\frac{b\left(\frac{2\text{arc}\cdots}{\cdots}\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^3*(c + d*x)^2),x]
```

output
```
-1/2*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^2*(c + d*x)) + (
((a^2*d*f*h + b^2*(6*d*e*g - c*f*g - 4*c*e*h) - a*b*(5*d*f*g + 2*d*e*h - 5
*c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)) + ((
2*d*(3*a^2*d*f*h + b^2*(12*d*e*g - c*f*g - 8*c*e*h) - a*b*(11*d*f*g + 4*d*
e*h - 9*c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)) + (b*((2*(3*a^3*d^2
*f^2*h - 3*a^2*b*d*f*(5*d*f*g + 4*d*e*h - 6*c*f*h) - b^3*(24*d^2*e^2*g - c
^2*f*(f*g - 4*e*h) - 8*c*d*e*(f*g + 2*e*h)) + a*b^2*(3*c^2*f^2*h + 8*d^2*e
*(5*f*g + e*h) - 2*c*d*f*(5*f*g + 16*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x]
)/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f]) - (8*Sqrt[d]*(b*
e - a*f)*(a*d*(d*f*g + 2*d*e*h - 3*c*f*h) - b*(6*d^2*e*g + 3*c^2*f*h - c*d
*(5*f*g + 4*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c
 - a*d)*Sqrt[d*e - c*f])))/(b*c - a*d))/(2*(b*c - a*d)*(b*e - a*f)))/(4*b*
(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{3\left(4\left(-2d^2e^2g+\frac{4c\left(eh+\frac{fg}{2}\right)ed}{3}-\frac{\left(eh-\frac{fg}{4}\right)c^2f}{3}\right)b^3+a\left(\frac{8d^2e(eh+5fg)}{3}-\frac{32c\left(eh+\frac{5fg}{16}\right)fd}{3}+c^2f^2h\right)b^2+6a^2\left(\frac{\left(-2eh-\frac{5fg}{2}\right)d}{3}+c\right.\right.}{4}$
derivativedivides	$2f^3\left(\frac{\frac{bf\left(3a^3d^2fh+2a^2bcdfh-4a^2bd^2eh-7a^2bd^2fg-5ab^2c^2fh+6ab^2cdfg+8ab^2d^2eg+4b^3c^2eh+b^3c^2fg-8b^3cdeg\right)(fx+e)^{\frac{3}{2}}}{8af-8be}}{((fx+e)b}\right.$
default	$2f^3\left(\frac{\frac{bf\left(3a^3d^2fh+2a^2bcdfh-4a^2bd^2eh-7a^2bd^2fg-5ab^2c^2fh+6ab^2cdfg+8ab^2d^2eg+4b^3c^2eh+b^3c^2fg-8b^3cdeg\right)(fx+e)^{\frac{3}{2}}}{8af-8be}}{((fx+e)b}\right.$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4/((a*f-b*e)*b)^(1/2)*((4*(-2*d^2*e^2*g+4/3*c*(e*h+1/2*f*g)*e*d-1/3*(e*h
-1/4*f*g)*c^2*f)*b^3+a*(8/3*d^2*e*(e*h+5*f*g)-32/3*c*(e*h+5/16*f*g)*f*d+c^
2*f^2*h)*b^2+6*a^2*(1/3*(-2*e*h-5/2*f*g)*d+c*f*h)*d*f*b+a^3*d^2*f^2*h)*(d*
x+c)*((c*f-d*e)*d)^(1/2)*(b*x+a)^2*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1
/2))+3*((a*f-b*e)*b)^(1/2)*(-4/3*((2*d^2*e*g-4/3*c*(e*h+5/4*f*g)*d+c^2*f*h
)*b+a*d*(1/3*(-2*e*h-f*g)*d+c*f*h))*d*(d*x+c)*(b*x+a)^2*(a*f-b*e)*arctan(d
*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((c*f-d*e)*d)^(1/2)*(f*x+e)^
(1/2)*(2/3*(2*d^2*e*g*x^2+(1/3*(-4*e*h-1/2*f*g)*x+g*e)*x*c*d-1/3*c^2*((2*e
*h+1/2*f*g)*x+g*e))*b^3-2/9*a*(((2*e*h+11/2*f*g)*x^2-9*e*g*x)*d^2-5*c*(9/1
0*h*f*x^2+1/5*(-7*e*h-3*f*g)*x+g*e)*d+c^2*(-5/2*f*h*x-1/2*f*g+e*h))*b^2+1/
3*a^2*((h*f*x^2+(-2*e*h-17/3*f*g)*x+4/3*g*e)*d^2-10/3*c*(-7/5*f*h*x+9/10*f
*g+e*h)*d+c^2*f*h)*b+a^3*d*f*(1/9*(5*h*x-4*g)*d+c*h))))/((c*f-d*e)*d)^(1/2
)/(b*x+a)^2/(a*f-b*e)/(a*d-b*c)^4/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3730 vs. $2(496) = 992$.

Time = 89.80 (sec) , antiderivative size = 14998, normalized size of antiderivative = 28.30

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(24*b^3*d^2*e^2*g - 8*b^3*c*d*e*f*g - 40*a*b^2*d^2*e*f*g - b^3*c^2*f^2
*g + 10*a*b^2*c*d*f^2*g + 15*a^2*b*d^2*f^2*g - 16*b^3*c*d*e^2*h - 8*a*b^2*
d^2*e^2*h + 4*b^3*c^2*e*f*h + 32*a*b^2*c*d*e*f*h + 12*a^2*b*d^2*e*f*h - 3*
a*b^2*c^2*f^2*h - 18*a^2*b*c*d*f^2*h - 3*a^3*d^2*f^2*h)*arctan(sqrt(f*x +
e)*b/sqrt(-b^2*e + a*b*f))/((b^5*c^4*e - 4*a*b^4*c^3*d*e + 6*a^2*b^3*c^2*d
^2*e - 4*a^3*b^2*c*d^3*e + a^4*b*d^4*e - a*b^4*c^4*f + 4*a^2*b^3*c^3*d*f -
 6*a^3*b^2*c^2*d^2*f + 4*a^4*b*c*d^3*f - a^5*d^4*f)*sqrt(-b^2*e + a*b*f))
- (6*b*d^3*e*g - 5*b*c*d^2*f*g - a*d^3*f*g - 4*b*c*d^2*e*h - 2*a*d^3*e*h +
 3*b*c^2*d*f*h + 3*a*c*d^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f
))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*sqrt(-d^2*e + c*d*f)) + (sqrt(f*x + e)*d^2*f*g - sqrt(f*x + e)*c*d*f*h)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((f*x + e)*d - d*e +
c*f)) + 1/4*(8*(f*x + e)^(3/2)*b^3*d*e*f*g - 8*sqrt(f*x + e)*b^3*d*e^2*f*g
 - (f*x + e)^(3/2)*b^3*c*f^2*g - 7*(f*x + e)^(3/2)*a*b^2*d*f^2*g - sqrt(f*
x + e)*b^3*c*e*f^2*g + 17*sqrt(f*x + e)*a*b^2*d*e*f^2*g + sqrt(f*x + e)*a*
b^2*c*f^3*g - 9*sqrt(f*x + e)*a^2*b*d*f^3*g - 4*(f*x + e)^(3/2)*b^3*c*e*f*
h - 4*(f*x + e)^(3/2)*a*b^2*d*e*f*h + 4*sqrt(f*x + e)*b^3*c*e^2*f*h + 4*sq
rt(f*x + e)*a*b^2*d*e^2*f*h + 5*(f*x + e)^(3/2)*a*b^2*c*f^2*h + 3*(f*x + e
)^(3/2)*a^2*b*d*f^2*h - 7*sqrt(f*x + e)*a*b^2*c*e*f^2*h - 9*sqrt(f*x + e)*
a^2*b*d*e*f^2*h + 3*sqrt(f*x + e)*a^2*b*c*f^3*h + 5*sqrt(f*x + e)*a^3*d...
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 451052, normalized size of antiderivative = 851.04

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^2),x)
```

output
```
- atan(((((16*a*b^12*c^10*d^2*f^6*g - 64*a^11*b^2*d^12*f^6*g + 368*a^10*b^
3*c*d^11*f^6*g + 144*a^11*b^2*c*d^11*f^6*h + 336*a^10*b^3*d^12*e*f^5*g - 8
0*a^11*b^2*d^12*e*f^5*h - 16*b^13*c^10*d^2*e*f^5*g - 272*a^2*b^11*c^9*d^3*
f^6*g + 1536*a^3*b^10*c^8*d^4*f^6*g - 4416*a^4*b^9*c^7*d^5*f^6*g + 7392*a^
5*b^8*c^6*d^6*f^6*g - 7392*a^6*b^7*c^5*d^7*f^6*g + 4032*a^7*b^6*c^4*d^8*f^
6*g - 576*a^8*b^5*c^3*d^9*f^6*g - 624*a^9*b^4*c^2*d^10*f^6*g + 48*a^2*b^11
*c^10*d^2*f^6*h - 240*a^3*b^10*c^9*d^3*f^6*h + 192*a^4*b^9*c^8*d^4*f^6*h +
 1344*a^5*b^8*c^7*d^5*f^6*h - 4704*a^6*b^7*c^6*d^6*f^6*h + 7392*a^7*b^6*c^
5*d^7*f^6*h - 6720*a^8*b^5*c^4*d^8*f^6*h + 3648*a^9*b^4*c^3*d^9*f^6*h - 11
04*a^10*b^3*c^2*d^10*f^6*h + 192*a^8*b^5*d^12*e^3*f^3*g - 464*a^9*b^4*d^12
*e^2*f^4*g - 64*a^9*b^4*d^12*e^3*f^3*h + 144*a^10*b^3*d^12*e^2*f^4*h + 192
*b^13*c^8*d^4*e^3*f^3*g - 112*b^13*c^9*d^3*e^2*f^4*g - 128*b^13*c^9*d^3*e^
3*f^3*h + 64*b^13*c^10*d^2*e^2*f^4*h + 5376*a^2*b^11*c^6*d^6*e^3*f^3*g + 5
76*a^2*b^11*c^7*d^5*e^2*f^4*g - 10752*a^3*b^10*c^5*d^7*e^3*f^3*g - 6720*a^
3*b^10*c^6*d^6*e^2*f^4*g + 13440*a^4*b^9*c^4*d^8*e^3*f^3*g + 18144*a^4*b^9
*c^5*d^7*e^2*f^4*g - 10752*a^5*b^8*c^3*d^9*e^3*f^3*g - 26208*a^5*b^8*c^4*d
^8*e^2*f^4*g + 5376*a^6*b^7*c^2*d^10*e^3*f^3*g + 22848*a^6*b^7*c^3*d^9*e^2
*f^4*g - 12096*a^7*b^6*c^2*d^10*e^2*f^4*g - 3072*a^2*b^11*c^7*d^5*e^3*f^3*
h - 1008*a^2*b^11*c^8*d^4*e^2*f^4*h + 5376*a^3*b^10*c^6*d^6*e^3*f^3*h + 55
68*a^3*b^10*c^7*d^5*e^2*f^4*h - 5376*a^4*b^9*c^5*d^7*e^3*f^3*h - 12096*...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13393, normalized size of antiderivative = 25.27

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**5*c**2*d**2*f**3*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c*d**3*e*f**2*h + 3*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c*d**3*f**3*h*x
 - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**5*d**4*e*f**2*h*x + 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**3*d*f**3*h - 30*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**2*
e*f**2*h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**4*b*c**2*d**2*f**3*g + 24*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**2*f**3*h*x + 12*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**4*b*c*d**3*e**2*f*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**3*e*f**2*g - 36*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**3*e*f*
*2*h*x - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**4*b*c*d**3*f**3*g*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**3*f**3*h*x**2 + 12*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4
*b*d**4*e**2*f*h*x + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/...
```

3.49 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 872

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx$$

$$= \frac{d(5a^3d^2f^2h - a^2bdf(27dfg + 14deh - 26cfh) - b^3(32d^2e^2g - c^2f(fg - 2eh) - 4cde(fg + 6eh)) + ab^2(c\ldots}{8b(bc - ad)^4(be - af)^2(c + dx)}$$

$$- \frac{(bg - ah)\sqrt{e + fx}}{3b(bc - ad)(a + bx)^3(c + dx)}$$

$$+ \frac{(a^2dfh - ab(7dfg + 2deh - 7cfh) + b^2(8deg - c(fg + 6eh)))\sqrt{e + fx}}{12b(bc - ad)^2(be - af)(a + bx)^2(c + dx)}$$

$$+ \frac{(5a^3d^2f^2h - 5a^2bdf(7dfg + 4deh - 8cfh) - b^3(48d^2e^2g - 3c^2f(fg - 2eh) - 2cde(5fg + 18eh)) + ab\ldots}{24b(bc - ad)^3(be - af)^2(a + bx)(c + dx)\ldots}$$

$$+ \frac{(5a^4d^3f^3h - 5a^3bd^2f^2(7dfg + 6deh - 9cfh) + b^4(64d^3e^3g - 4c^2def(fg - 4eh) - c^3f^2(fg - 2eh) - 2\ldots}{\ldots}$$

$$+ \frac{d^{3/2}(ad(dfg + 2deh - 3cfh) - b(8d^2eg + 5c^2fh - cd(7fg + 6eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc - ad)^5\sqrt{de - cf}}$$

output
```
1/8*d*(5*a^3*d^2*f^2*h-a^2*b*d*f*(-26*c*f*h+14*d*e*h+27*d*f*g)-b^3*(32*d^2
*e^2*g-c^2*f*(-2*e*h+f*g)-4*c*d*e*(6*e*h+f*g))+a*b^2*(c^2*f^2*h+4*d^2*e*(2
*e*h+15*f*g)-6*c*d*f*(8*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^4/(-a*f+b*e)
^2/(d*x+c)-1/3*(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)^3/(d*x+c)+1/1
2*(a^2*d*f*h-a*b*(-7*c*f*h+2*d*e*h+7*d*f*g)+b^2*(8*d*e*g-c*(6*e*h+f*g)))*(
f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^2/(d*x+c)+1/24*(5*a^3*d^2*f
^2*h-5*a^2*b*d*f*(-8*c*f*h+4*d*e*h+7*d*f*g)-b^3*(48*d^2*e^2*g-3*c^2*f*(-2*
e*h+f*g)-2*c*d*e*(18*e*h+5*f*g))+a*b^2*(3*c^2*f^2*h+2*d^2*e*(6*e*h+43*f*g)
-2*c*d*f*(35*e*h+8*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+b*e)^2/(b*x+a
)/(d*x+c)+1/8*(5*a^4*d^3*f^3*h-5*a^3*b*d^2*f^2*(-9*c*f*h+6*d*e*h+7*d*f*g)+
b^4*(64*d^3*e^3*g-4*c^2*d*e*f*(-4*e*h+f*g)-c^3*f^2*(-2*e*h+f*g)-24*c*d^2*e
^2*(2*e*h+f*g))-a*b^3*(c^3*f^3*h-c^2*d*f^2*(-34*e*h+7*f*g)+8*d^3*e^2*(2*e*
h+21*f*g)-8*c*d^2*e*f*(17*e*h+7*f*g))+5*a^2*b^2*d*f*(3*c^2*f^2*h+4*d^2*e*(
2*e*h+7*f*g)-c*d*f*(26*e*h+7*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*
e)^(1/2))/b^(1/2)/(-a*d+b*c)^5/(-a*f+b*e)^(5/2)+d^(3/2)*(a*d*(-3*c*f*h+2*d
*e*h+d*f*g)-b*(8*d^2*e*g+5*c^2*f*h-c*d*(6*e*h+7*f*g)))*arctanh(d^(1/2)*(f*
x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^5/(-c*f+d*e)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6117 vs. $2(872) = 1744$.

Time = 16.30 (sec) , antiderivative size = 6117, normalized size of antiderivative = 7.01

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^4*(c + d*x)^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {166, 27, 168, 27, 168, 27, 168, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int \frac{a(2de-cf)h-b(8deg-cfg-6ceh)-f(7bdg-6bch-adh)x}{2(a+bx)^3(c+dx)^2\sqrt{e+fx}}dx}{3b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a(2de-cf)h-b(8deg-cfg-6ceh)-f(7bdg-6bch-adh)x}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}dx}{6b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{\sqrt{e+fx}(a^2dfh-ab(-7cfh+2deh+7dfg)+b^2(-6ceh-cfg+8deg))}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}-\int -\frac{5df(2de-cf)ha^2-b\big(2e(23fg+6eh)d^2-cf(11fg+40eh)d+3c^2f^2h\big)a+b^2\big(-3f(}{}}{6b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{5df(2de-cf)ha^2-b\big(2e(23fg+6eh)d^2-cf(11fg+40eh)d+3c^2f^2h\big)a+b^2\big(-3f(fg-2eh)c^2-2de(5fg+18eh)c+48d^2e^2g\big)+5df\big(dfha^2-b(7dfg+2deh-7cfh)a+b^2}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}}{4(bc-ad)(be-af)}}{6b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\frac{\sqrt{e+fx}\left(5a^3d^2f^2h-5a^2bdf(-8cfh+4deh+7dfg)+ab^2\left(3c^2f^2h-2cdf(35eh+8fg)+2d^2e(6eh+43fg)\right)-b^3\left(-3c^2f(fg-2eh)-2cde(18eh+5fg)+48d^2e^2g\right)\right)}{(a+bx)(c+dx)(bc-ad)(be-af)} - \int - \cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$3\int \frac{5d^2f^2(2de-cf)ha^3-bdf\left(2e(27fg+14eh)d^2-cf(19fg+60eh)d+12c^2f^2h\right)a^2+b^2\left(8e^2(15fg+2eh)d^3-2cef(23fg+50eh)d^2-4c^2f^2(fg-7eh)d+c^3f^3h\right)a-b^3\left(- \cdots\right.}{\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+2deh-7cfh)a+b^2(8deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)} + \frac{\sqrt{e+fx}\left(5d^2f^2ha^3-5bdf(7dfg+4deh-8cfh)a^2+b^2\left(2e(43fg+6eh)d^2-2cf(8fg+35eh)\cdots\right.\right.}{(bc-ad)(be-af)(a+bx)(c\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+2deh-7cfh)a+b^2(8deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)} + \frac{\sqrt{e+fx}\left(5d^2f^2ha^3-5bdf(7dfg+4deh-8cfh)a^2+b^2\left(2e(43fg+6eh)d^2-2cf(8fg+35eh)\cdots\right.\right.}{(bc-ad)(be-af)(a+bx)(c\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+2deh-7cfh)a+b^2(8deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3-5bdf(7dfg+4deh-8cfh)a^2+b^2\left(2e(43fg+6eh)d^2-2cf(8fg+35eh)\cdots\right.\right.}{(bc-ad)(be-af)(a+bx)(c\cdots}}{\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

$\Big\downarrow$ 174

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+2deh-7cfh)a+b^2(8deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3-5bdf(7dfg+4deh-8cfh)a^2+b^2\left(2e(43fg+6eh)d^2-2cf(8fg+35eh)\cdots\right.\right.}{(bc-ad)(be-af)(a+bx)(c\cdots}}{\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

$\Big\downarrow$ 73

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+2deh-7cfh)a+b^2(8deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3-5bdf(7dfg+4deh-8cfh)a^2+b^2\left(2e(43fg+6eh)d^2-2cf(8fg+35eh)\cdots\right.\right.}{(bc-ad)(be-af)(a+bx)(c\cdots}}{\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+2deh-7cfh)a+b^2(8deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}+\frac{\sqrt{e+fx}\left(5d^2f^2ha^3-5bdf(7dfg+4deh-8cfh)a^2+b^2\left(2e(43fg+6eh)d^2-2cf(8fg+35eh)\cdots\right.\right.}{(bc-ad)(be-af)(a+bx)(c\cdots}}{\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^4*(c + d*x)^2),x]
```

output
```
-1/3*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^3*(c + d*x)) + (
((a^2*d*f*h + b^2*(8*d*e*g - c*f*g - 6*c*e*h) - a*b*(7*d*f*g + 2*d*e*h - 7
*c*f*h))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x))
+ (((5*a^3*d^2*f^2*h - 5*a^2*b*d*f*(7*d*f*g + 4*d*e*h - 8*c*f*h) - b^3*(48
*d^2*e^2*g - 3*c^2*f*(f*g - 2*e*h) - 2*c*d*e*(5*f*g + 18*e*h)) + a*b^2*(3*
c^2*f^2*h + 2*d^2*e*(43*f*g + 6*e*h) - 2*c*d*f*(8*f*g + 35*e*h)))*Sqrt[e +
 f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)) + (3*((2*d*(5*a^3*d^2
*f^2*h - a^2*b*d*f*(27*d*f*g + 14*d*e*h - 26*c*f*h) - b^3*(32*d^2*e^2*g -
c^2*f*(f*g - 2*e*h) - 4*c*d*e*(f*g + 6*e*h)) + a*b^2*(c^2*f^2*h + 4*d^2*e*
(15*f*g + 2*e*h) - 6*c*d*f*(f*g + 8*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c
+ d*x)) - (b*((-2*(5*a^4*d^3*f^3*h - 5*a^3*b*d^2*f^2*(7*d*f*g + 6*d*e*h -
9*c*f*h) + b^4*(64*d^3*e^3*g - 4*c^2*d*e*f*(f*g - 4*e*h) - c^3*f^2*(f*g -
2*e*h) - 24*c*d^2*e^2*(f*g + 2*e*h)) - a*b^3*(c^3*f^3*h - c^2*d*f^2*(7*f*g
 - 34*e*h) + 8*d^3*e^2*(21*f*g + 2*e*h) - 8*c*d^2*e*f*(7*f*g + 17*e*h)) +
5*a^2*b^2*d*f*(3*c^2*f^2*h + 4*d^2*e*(7*f*g + 2*e*h) - c*d*f*(7*f*g + 26*e
*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*
d)*Sqrt[b*e - a*f]) - (16*d^(3/2)*(b*e - a*f)^2*(a*d*(d*f*g + 2*d*e*h - 3*
c*f*h) - b*(8*d^2*e*g + 5*c^2*f*h - c*d*(7*f*g + 6*e*h)))*ArcTanh[(Sqrt[d]
*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f])))/(b*c - a
*d)))/(2*(b*c - a*d)*(b*e - a*f)))/(4*(b*c - a*d)*(b*e - a*f)))/(6*b*(b...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 10.83 (sec) , antiderivative size = 1143, normalized size of antiderivative = 1.31

method	result	size
pseudoelliptic	Expression too large to display	1143
derivativedivides	Expression too large to display	1458
default	Expression too large to display	1458

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
5/8/((a*f-b*e)*b)^(1/2)*((d*x+c)*((c*f-d*e)*d)^(1/2)*(b*x+a)^3*((64/5*d^3*
e^3*g-48/5*c*(e*h+1/2*f*g)*e^2*d^2+16/5*(e*h-1/4*f*g)*c^2*f*e*d+2/5*(e*h-1
/2*f*g)*c^3*f^2)*b^4-1/5*a*((16*e^3*h+168*e^2*f*g)*d^3+(-136*c*e^2*f*h-56*
c*e*f^2*g)*d^2+(34*c^2*e*f^2*h-7*c^2*f^3*g)*d+c^3*f^3*h)*b^3+3*a^2*d*f*((8
/3*e^2*h+28/3*e*f*g)*d^2-26/3*c*(e*h+7/26*f*g)*f*d+c^2*f^2*h)*b^2+9*a^3*d^
2*f^2*((-2/3*e*h-7/9*f*g)*d+c*f*h)*b+a^4*d^3*f^3*h)*arctan(b*(f*x+e)^(1/2)
/((a*f-b*e)*b)^(1/2))+19/5*((a*f-b*e)*b)^(1/2)*(-24/19*d^2*(d*x+c)*(b*x+a)
^3*(a*f-b*e)^2*((8/3*d^2*e*g-2*c*(e*h+7/6*f*g)*d+5/3*c^2*f*h)*b+a*d*((-2/3
*e*h-1/3*f*g)*d+c*f*h))*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b
*c)*((c*f-d*e)*d)^(1/2)*((-32/19*d^3*e^2*g*x^3-16/19*x^2*c*e*((-3/2*e*h-1/
4*f*g)*x+g*e)*d^2+16/57*x*c^2*((-3/8*e*f*h+3/16*f^2*g)*x^2+(9/4*e^2*h+5/8*
e*f*g)*x+e^2*g)*d-8/57*((3/4*e*f*h-3/8*f^2*g)*x^2+(3/2*e^2*h+1/4*e*f*g)*x+
e^2*g)*c^3)*b^5-4/57*a*(((-6*e^2*h-45*e*f*g)*x^3+60*e^2*g*x^2)*d^3+32*x*c*
((9/8*e*f*h+9/64*f^2*g)*x^2+(-3/2*e^2*h-7/8*e*f*g)*x+e^2*g)*d^2-10*(3/40*f
^2*h*x^3+(-41/20*e*f*h-1/4*f^2*g)*x^2+(23/10*e^2*h-3/20*e*f*g)*x+e^2*g)*c^
2*d+c^3*(-3/4*f^2*h*x^2+(-7/2*e*f*h-2*f^2*g)*x+e*(e*h-7/2*f*g)))*b^4+4/57*
a^2*(((-21/2*e*f*h-81/4*f^2*g)*x^3+(15*e^2*h+227/2*e*f*g)*x^2-44*e^2*g*x)*
d^3-26*(-3/4*f^2*h*x^3+(191/52*e*f*h+73/104*f^2*g)*x^2-41/26*(e*h+3/2*f*g)
*e*x+e^2*g)*c*d^2+8*c^2*(23/16*f^2*h*x^2+(-6*e*f*h-19/32*f^2*g)*x+e^2*h-2*
e*f*g)*d+c^3*f*(-2*f*h*x-3/4*f*g+e*h))*b^3-1/19*a^3*((-5*f^2*h*x^3+(106...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**4/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. $2(834) = 1668$.

Time = 0.26 (sec) , antiderivative size = 2352, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(64*b^4*d^3*e^3*g - 24*b^4*c*d^2*e^2*f*g - 168*a*b^3*d^3*e^2*f*g - 4*
b^4*c^2*d*e*f^2*g + 56*a*b^3*c*d^2*e*f^2*g + 140*a^2*b^2*d^3*e*f^2*g - b^4
*c^3*f^3*g + 7*a*b^3*c^2*d*f^3*g - 35*a^2*b^2*c*d^2*f^3*g - 35*a^3*b*d^3*f
^3*g - 48*b^4*c*d^2*e^3*h - 16*a*b^3*d^3*e^3*h + 16*b^4*c^2*d*e^2*f*h + 13
6*a*b^3*c*d^2*e^2*f*h + 40*a^2*b^2*d^3*e^2*f*h + 2*b^4*c^3*e*f^2*h - 34*a*
b^3*c^2*d*e*f^2*h - 130*a^2*b^2*c*d^2*e*f^2*h - 30*a^3*b*d^3*e*f^2*h - a*b
^3*c^3*f^3*h + 15*a^2*b^2*c^2*d*f^3*h + 45*a^3*b*c*d^2*f^3*h + 5*a^4*d^3*f
^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^5*e^2 - 5*a*b^6
*c^4*d*e^2 + 10*a^2*b^5*c^3*d^2*e^2 - 10*a^3*b^4*c^2*d^3*e^2 + 5*a^4*b^3*c
*d^4*e^2 - a^5*b^2*d^5*e^2 - 2*a*b^6*c^5*e*f + 10*a^2*b^5*c^4*d*e*f - 20*a
^3*b^4*c^3*d^2*e*f + 20*a^4*b^3*c^2*d^3*e*f - 10*a^5*b^2*c*d^4*e*f + 2*a^6
*b*d^5*e*f + a^2*b^5*c^5*f^2 - 5*a^3*b^4*c^4*d*f^2 + 10*a^4*b^3*c^3*d^2*f^
2 - 10*a^5*b^2*c^2*d^3*f^2 + 5*a^6*b*c*d^4*f^2 - a^7*d^5*f^2)*sqrt(-b^2*e
+ a*b*f)) + (8*b*d^4*e*g - 7*b*c*d^3*f*g - a*d^4*f*g - 6*b*c*d^3*e*h - 2*a
*d^4*e*h + 5*b*c^2*d^2*f*h + 3*a*c*d^3*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d
^2*e + c*d*f))/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2
*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)
*d^3*f*g - sqrt(f*x + e)*c*d^2*f*h)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((f*x + e)*d - d*e + c*f)) - 1/24*(72*(
f*x + e)^(5/2)*b^5*d^2*e^2*f*g - 144*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + ...
```

Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 783233, normalized size of antiderivative = 898.20

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^2),x)
```

output
```
atan(((((256*a^15*b^2*d^15*f^8*g - 1632*a^14*b^3*c*d^14*f^8*g - 608*a^15*b
^2*c*d^14*f^8*h - 2208*a^14*b^3*d^15*e*f^7*g + 352*a^15*b^2*d^15*e*f^7*h +
 32*a^2*b^15*c^13*d^2*f^8*g - 512*a^3*b^14*c^12*d^3*f^8*g + 4288*a^4*b^13*
c^11*d^4*f^8*g - 21504*a^5*b^12*c^10*d^5*f^8*g + 68960*a^6*b^11*c^9*d^6*f^
8*g - 148224*a^7*b^10*c^8*d^7*f^8*g + 219264*a^8*b^9*c^7*d^8*f^8*g - 22425
6*a^9*b^8*c^6*d^9*f^8*g + 154848*a^10*b^7*c^5*d^10*f^8*g - 66560*a^11*b^6*
c^4*d^11*f^8*g + 12992*a^12*b^5*c^3*d^12*f^8*g + 2048*a^13*b^4*c^2*d^13*f^
8*g + 32*a^3*b^14*c^13*d^2*f^8*h - 768*a^4*b^13*c^12*d^3*f^8*h + 5312*a^5*
b^12*c^11*d^4*f^8*h - 17920*a^6*b^11*c^10*d^5*f^8*h + 33120*a^7*b^10*c^9*d
^6*f^8*h - 29184*a^8*b^9*c^8*d^7*f^8*h - 8064*a^9*b^8*c^7*d^8*f^8*h + 5529
6*a^10*b^7*c^6*d^9*f^8*h - 72480*a^11*b^6*c^5*d^10*f^8*h + 52480*a^12*b^5*
c^4*d^11*f^8*h - 22848*a^13*b^4*c^3*d^12*f^8*h + 5632*a^14*b^3*c^2*d^13*f^
8*h - 1024*a^10*b^7*d^15*e^5*f^3*g + 4480*a^11*b^6*d^15*e^4*f^4*g - 7584*a
^12*b^5*d^15*e^3*f^5*g + 6080*a^13*b^4*d^15*e^2*f^6*g + 256*a^11*b^6*d^15*
e^5*f^3*h - 1088*a^12*b^5*d^15*e^4*f^4*h + 1760*a^13*b^4*d^15*e^3*f^5*h -
1280*a^14*b^3*d^15*e^2*f^6*h - 1024*b^17*c^10*d^5*e^5*f^3*g + 640*b^17*c^1
1*d^4*e^4*f^4*g + 96*b^17*c^12*d^3*e^3*f^5*g + 32*b^17*c^13*d^2*e^2*f^6*g
+ 768*b^17*c^11*d^4*e^5*f^3*h - 448*b^17*c^12*d^3*e^4*f^4*h - 64*b^17*c^13
*d^2*e^3*f^5*h - 46080*a^2*b^15*c^8*d^7*e^5*f^3*g - 16000*a^2*b^15*c^9*d^6
*e^4*f^4*g + 24256*a^2*b^15*c^10*d^5*e^3*f^5*g + 9792*a^2*b^15*c^11*d^4...
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 25836, normalized size of antiderivative = 29.63

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x)
```

output

```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**7*c**2*d**3*f**4*h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c*d**4*e*f**3*h + 15*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c*d**4*f**4*
h*x - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**7*d**5*e*f**3*h*x + 135*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**3*d**2*f**4*h - 225*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c
**2*d**3*e*f**3*h - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**3*f**4*g + 180*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**3*f**
4*h*x + 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**6*b*c*d**4*e**2*f**2*h + 105*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**4*e*f**3*g - 270*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*6*b*c*d**4*e*f**3*h*x - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**4*f**4*g*x + 45*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**4*f**
4*h*x**2 + 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**6*b*d**5*e**2*f**2*h*x + 105*sqrt(b)*sqrt(a*f - b*e)*a...
```

3.50 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 1368

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2} \, dx = \text{Too large to display}$$

output
```
1/64*d*(35*a^4*d^3*f^3*h-a^3*b*d^2*f^2*(-263*c*f*h+152*d*e*h+251*d*f*g)+b^
4*(320*d^3*e^3*g-c^3*f^2*(-8*e*h+5*f*g)-16*c^2*d*e*f*(-2*e*h+f*g)-16*c*d^2
*e^2*(16*e*h+3*f*g))-a*b^3*(3*c^3*f^3*h-c^2*d*f^2*(-72*e*h+31*f*g)+16*d^3*
e^2*(4*e*h+57*f*g)-16*c*d^2*e*f*(47*e*h+8*f*g))+a^2*b^2*d*f*(25*c^2*f^2*h+
16*d^2*e*(11*e*h+53*f*g)-c*d*f*(744*e*h+95*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*
c)^5/(-a*f+b*e)^3/(d*x+c)-1/4*(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a
)^4/(d*x+c)+1/24*(a^2*d*f*h-a*b*(-9*c*f*h+2*d*e*h+9*d*f*g)+b^2*(10*d*e*g-c
*(8*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^3/(d*x+c)+1
/96*(7*a^3*d^2*f^2*h-7*a^2*b*d*f*(-10*c*f*h+4*d*e*h+9*d*f*g)-b^3*(80*d^2*e
^2*g-c^2*f*(-8*e*h+5*f*g)-4*c*d*e*(16*e*h+3*f*g))+a*b^2*(3*c^2*f^2*h+4*d^2
*e*(4*e*h+37*f*g)-2*c*d*f*(62*e*h+11*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^3/(
-a*f+b*e)^2/(b*x+a)^2/(d*x+c)+1/192*(35*a^4*d^3*f^3*h-35*a^3*b*d^2*f^2*(-1
1*c*f*h+6*d*e*h+9*d*f*g)+b^4*(480*d^3*e^3*g-2*c^2*d*e*f*(-40*e*h+19*f*g)-3
*c^3*f^2*(-8*e*h+5*f*g)-16*c*d^2*e^2*(24*e*h+7*f*g))-a*b^3*(9*c^3*f^3*h-c^
2*d*f^2*(-194*e*h+83*f*g)+16*d^3*e^2*(6*e*h+83*f*g)-12*c*d^2*e*f*(92*e*h+2
5*f*g))+a^2*b^2*d*f*(69*c^2*f^2*h+2*d^2*e*(128*e*h+589*f*g)-c*d*f*(1060*e*
h+233*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^4/(-a*f+b*e)^3/(b*x+a)/(d*x+c)+1/6
4*(35*a^5*d^4*f^4*h-35*a^4*b*d^3*f^3*(-12*c*f*h+8*d*e*h+9*d*f*g)-b^5*(640*
d^4*e^4*g-c^4*f^3*(-8*e*h+5*f*g)-48*c^2*d^2*e^2*f*(-4*e*h+f*g)-16*c^3*d*e*
f^2*(-2*e*h+f*g)-256*c*d^3*e^3*(2*e*h+f*g))+70*a^3*b^2*d^2*f^2*(3*c^2*f...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12952 vs. $2(1368) = 2736$.

Time = 16.49 (sec) , antiderivative size = 12952, normalized size of antiderivative = 9.47

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^5*(c + d*x)^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.79 (sec) , antiderivative size = 1456, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {166, 27, 168, 27, 168, 27, 168, 27, 168, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{10bdeg-bcfg-8bceh-2adeh+acfh+f(9bdg-8bch-adh)x}{2(a+bx)^4(c+dx)^2\sqrt{e+fx}}dx}{4b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{10bdeg-a(2de-cf)h-bc(fg+8eh)+f(9bdg-8bch-adh)x}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}dx}{8b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{\int \frac{7df(2de-cf)ha^2-b(2e(39fg+8eh)d^2-cf(15fg+68eh)d+3c^2f^2h)a+b^2(-f(5fg-8eh)c^2-4de(3fg+16eh)c+80d^2e^2g)+7df(dfha^2-b(9dfg+2deh-9cfh)}{2(a+bx)^3(c+dx)^2\sqrt{e+fx}}}{3(bc-ad)(be-af)}}{8b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{7df(2de-cf)ha^2-b(2e(39fg+8eh)d^2-cf(15fg+68eh)d+3c^2f^2h)a+b^2(-f(5fg-8eh)c^2-4de(3fg+16eh)c+80d^2e^2g)+7df(dfha^2-b(9dfg+2deh-9cfh)}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}}{6(bc-ad)(be-af)}}{8b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{\sqrt{e+fx}\left(7a^3d^2f^2h-7a^2bdf(-10cfh+4deh+9dfg)+ab^2\left(3c^2f^2h-2cdf(62eh+11fg)+4d^2e(4eh+37fg)\right)-b^3\left(c^2(-f)(5fg-8eh)-4cde(16eh+3fg)+80d^2e^2g\right)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{35d^2f^2(2de-cf)ha^3-bdf\left(2e(219fg+88eh)d^2-cf(123fg+440eh)d+54c^2f^2h\right)a^2+b^2\left(32e^2(29fg+3eh)d^3-16cef(15fg+49eh)d^2-2c^2f^2(29fg-77eh)d+9\right.}{}}{}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$\Big\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh))\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh))\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$\Big\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh))\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$\Big\downarrow$ 25

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$-\frac{\sqrt{e+fx}(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh)))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}$$

$\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$-\frac{\sqrt{e+fx}(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh)))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}$$

$\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$-\frac{\sqrt{e+fx}(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh)))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}$$

$\downarrow$ 73

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$-\frac{\sqrt{e+fx}(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh)))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}$$

$\downarrow$ 221

$$-\frac{\sqrt{e+fx}(bg-ah)}{4b(bc-ad)(a+bx)^4(c+dx)}-$$

$$-\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+2deh-9cfh)a+b^2(10deg-c(fg+8eh))\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(7d^2f^2ha^3-7bdf(9dfg+4deh-10cfh)a^2+b^2\left(4e(37fg+4eh)d^2-2cf(11fg\right.\right.}{2(bc-ad)(be-af)(a+}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^5*(c + d*x)^2),x]
```

output
```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^4*(c + d*x)) - (
-1/3*((a^2*d*f*h - a*b*(9*d*f*g + 2*d*e*h - 9*c*f*h) + b^2*(10*d*e*g - c*(
f*g + 8*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c + d*
x)) - (((7*a^3*d^2*f^2*h - 7*a^2*b*d*f*(9*d*f*g + 4*d*e*h - 10*c*f*h) - b^
3*(80*d^2*e^2*g - c^2*f*(5*f*g - 8*e*h) - 4*c*d*e*(3*f*g + 16*e*h)) + a*b^
2*(3*c^2*f^2*h + 4*d^2*e*(37*f*g + 4*e*h) - 2*c*d*f*(11*f*g + 62*e*h)))*Sq
rt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)) + (((35*a^4
*d^3*f^3*h - 35*a^3*b*d^2*f^2*(9*d*f*g + 6*d*e*h - 11*c*f*h) + b^4*(480*d^
3*e^3*g - 2*c^2*d*e*f*(19*f*g - 40*e*h) - 3*c^3*f^2*(5*f*g - 8*e*h) - 16*c
*d^2*e^2*(7*f*g + 24*e*h)) - a*b^3*(9*c^3*f^3*h - c^2*d*f^2*(83*f*g - 194*
e*h) + 16*d^3*e^2*(83*f*g + 6*e*h) - 12*c*d^2*e*f*(25*f*g + 92*e*h)) + a^2
*b^2*d*f*(69*c^2*f^2*h + 2*d^2*e*(589*f*g + 128*e*h) - c*d*f*(233*f*g + 10
60*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)) + (
3*((2*d*(35*a^4*d^3*f^3*h - a^3*b*d^2*f^2*(251*d*f*g + 152*d*e*h - 263*c*f
*h) + b^4*(320*d^3*e^3*g - c^3*f^2*(5*f*g - 8*e*h) - 16*c^2*d*e*f*(f*g - 2
*e*h) - 16*c*d^2*e^2*(3*f*g + 16*e*h)) - a*b^3*(3*c^3*f^3*h - c^2*d*f^2*(3
1*f*g - 72*e*h) + 16*d^3*e^2*(57*f*g + 4*e*h) - 16*c*d^2*e*f*(8*f*g + 47*e
*h)) + a^2*b^2*d*f*(25*c^2*f^2*h + 16*d^2*e*(53*f*g + 11*e*h) - c*d*f*(95*
f*g + 744*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)) - (b*((-2*(35*a^5*
d^4*f^4*h - 35*a^4*b*d^3*f^3*(9*d*f*g + 8*d*e*h - 12*c*f*h) - b^5*(640*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 69.90 (sec) , antiderivative size = 1940, normalized size of antiderivative = 1.42

method	result	size
pseudoelliptic	Expression too large to display	1940
derivativedivides	Expression too large to display	2822
default	Expression too large to display	2822

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
35/64/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*((d*x+c)*((c*f-d*e)*d)^(1/2)
*(8/7*(-16*d^4*e^4*g+64/5*(e*h+1/2*f*g)*c*e^3*d^3-24/5*(e*h-1/4*f*g)*c^2*f
*e^2*d^2-4/5*(e*h-1/2*f*g)*c^3*f^2*e*d-1/5*c^4*(e*h-5/8*f*g)*f^3)*b^5+3/35
*a*(128*(1/3*e^4*h+6*e^3*f*g)*d^4-640*c*(e*h+9/20*f*g)*f*e^2*d^3+16*c^2*(1
3*e^2*f^2*h-3*e*f^3*g)*d^2+4*c^3*(20/3*e*f^3*h-3*f^4*g)*d+c^4*f^4*h)*b^4-4
/5*a^2*d*(4*(4*e^3*h+27*e^2*f*g)*d^3-96*c*(e*h+3/8*f*g)*f*e*d^2+24*c^2*(e*
h-3/16*f*g)*f^2*d+c^3*f^3*h)*f*b^3+6*a^3*d^2*(8*(1/3*e^2*h+e*f*g)*d^2+2*c*
(-4*e*f*h-f^2*g)*d+c^2*f^2*h)*f^2*b^2+12*a^4*d^3*((-2/3*e*h-3/4*f*g)*d+c*f
*h)*f^3*b+a^5*d^4*f^4*h)*(b*x+a)^4*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1
/2))+157/35*((a*f-b*e)*b)^(1/2)*(-192/157*d^3*(d*x+c)*((10/3*d^2*e*g+c*(-8
/3*e*h-3*f*g)*d+7/3*c^2*f*h)*b+a*d*(1/3*(-2*e*h-f*g)*d+c*f*h))*(b*x+a)^4*(
a*f-b*e)^3*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(16/157*(20*d^4*e^3
*g*x^4+10*x^3*c*(-3/10*f*g*x+e*(-8/5*h*x+g))*e^2*d^3-10/3*x^2*c^2*(3/10*x^
2*g*f^2+7/10*x*(-6/7*h*x+g)*e*f+e^2*(12/5*h*x+g))*e*d^2+5/3*x*c^3*(-3/16*f
^3*g*x^3-19/40*x^2*(-12/19*h*x+g)*e*f^2+3/10*x*(10/3*h*x+g)*e^2*f+e^3*(8/5
*h*x+g))*d-c^4*(5/16*f^3*g*x^3-5/24*x^2*(12/5*h*x+g)*e*f^2+1/6*e^2*x*(2*h*
x+g)*f+e^3*(4/3*h*x+g)))*b^7-16/471*a*(3*(57*e^2*f*g*x^4-70*x^3*(-2/35*h*x
+g)*e^3)*d^4-110*x^2*c*(12/55*x^2*g*f^2-56/55*x*(-141/112*h*x+g)*e*f+e^2*(
-87/55*h*x+g))*e*d^3+35*x*c^2*(-93/560*f^3*g*x^3-73/280*x^2*(-108/73*h*x+g
)*e*f^2-9/70*x*(176/9*h*x+g)*e^2*f+e^3*(86/35*h*x+g))*d^2-17*c^3*(19/13...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**5/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4916 vs. $2(1326) = 2652$.

Time = 0.37 (sec) , antiderivative size = 4916, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/64*(640*b^5*d^4*e^4*g - 256*b^5*c*d^3*e^3*f*g - 2304*a*b^4*d^4*e^3*f*g -
 48*b^5*c^2*d^2*e^2*f^2*g + 864*a*b^4*c*d^3*e^2*f^2*g + 3024*a^2*b^3*d^4*e
^2*f^2*g - 16*b^5*c^3*d*e*f^3*g + 144*a*b^4*c^2*d^2*e*f^3*g - 1008*a^2*b^3
*c*d^3*e*f^3*g - 1680*a^3*b^2*d^4*e*f^3*g - 5*b^5*c^4*f^4*g + 36*a*b^4*c^3
*d*f^4*g - 126*a^2*b^3*c^2*d^2*f^4*g + 420*a^3*b^2*c*d^3*f^4*g + 315*a^4*b
*d^4*f^4*g - 512*b^5*c*d^3*e^4*h - 128*a*b^4*d^4*e^4*h + 192*b^5*c^2*d^2*e
^3*f*h + 1920*a*b^4*c*d^3*e^3*f*h + 448*a^2*b^3*d^4*e^3*f*h + 32*b^5*c^3*d
*e^2*f^2*h - 624*a*b^4*c^2*d^2*e^2*f^2*h - 2688*a^2*b^3*c*d^3*e^2*f^2*h -
560*a^3*b^2*d^4*e^2*f^2*h + 8*b^5*c^4*e*f^3*h - 80*a*b^4*c^3*d*e*f^3*h + 6
72*a^2*b^3*c^2*d^2*e*f^3*h + 1680*a^3*b^2*c*d^3*e*f^3*h + 280*a^4*b*d^4*e*
f^3*h - 3*a*b^4*c^4*f^4*h + 28*a^2*b^3*c^3*d*f^4*h - 210*a^3*b^2*c^2*d^2*f
^4*h - 420*a^4*b*c*d^3*f^4*h - 35*a^5*d^4*f^4*h)*arctan(sqrt(f*x + e)*b/sq
rt(-b^2*e + a*b*f))/((b^9*c^6*e^3 - 6*a*b^8*c^5*d*e^3 + 15*a^2*b^7*c^4*d^2
*e^3 - 20*a^3*b^6*c^3*d^3*e^3 + 15*a^4*b^5*c^2*d^4*e^3 - 6*a^5*b^4*c*d^5*e
^3 + a^6*b^3*d^6*e^3 - 3*a*b^8*c^6*e^2*f + 18*a^2*b^7*c^5*d*e^2*f - 45*a^3
*b^6*c^4*d^2*e^2*f + 60*a^4*b^5*c^3*d^3*e^2*f - 45*a^5*b^4*c^2*d^4*e^2*f +
 18*a^6*b^3*c*d^5*e^2*f - 3*a^7*b^2*d^6*e^2*f + 3*a^2*b^7*c^6*e*f^2 - 18*a
^3*b^6*c^5*d*e*f^2 + 45*a^4*b^5*c^4*d^2*e*f^2 - 60*a^5*b^4*c^3*d^3*e*f^2 +
 45*a^6*b^3*c^2*d^4*e*f^2 - 18*a^7*b^2*c*d^5*e*f^2 + 3*a^8*b*d^6*e*f^2 - a
^3*b^6*c^6*f^3 + 6*a^4*b^5*c^5*d*f^3 - 15*a^5*b^4*c^4*d^2*f^3 + 20*a^6*...
```

Mupad [B] (verification not implemented)

Time = 46.61 (sec) , antiderivative size = 1195213, normalized size of antiderivative = 873.69

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^2),x)
```

output
```
atan(((((16384*a^19*b^2*d^18*f^10*g - 113408*a^18*b^3*c*d^17*f^10*g - 4019
2*a^19*b^2*c*d^17*f^10*h - 197888*a^18*b^3*d^18*e*f^9*g + 23808*a^19*b^2*d
^18*e*f^9*h - 1280*a^3*b^18*c^16*d^2*f^10*g + 23296*a^4*b^17*c^15*d^3*f^10
*g - 204032*a^5*b^16*c^14*d^4*f^10*g + 1180416*a^6*b^15*c^13*d^5*f^10*g -
4966656*a^7*b^14*c^12*d^6*f^10*g + 15587072*a^8*b^13*c^11*d^7*f^10*g - 367
29088*a^9*b^12*c^10*d^8*f^10*g + 65187584*a^10*b^11*c^9*d^9*f^10*g - 87174
912*a^11*b^10*c^8*d^10*f^10*g + 87372032*a^12*b^9*c^7*d^11*f^10*g - 646243
84*a^13*b^8*c^6*d^12*f^10*g + 34097408*a^14*b^7*c^5*d^13*f^10*g - 11895552
*a^15*b^6*c^4*d^14*f^10*g + 2186496*a^16*b^5*c^3*d^15*f^10*g + 58624*a^17*
b^4*c^2*d^16*f^10*g - 768*a^4*b^17*c^16*d^2*f^10*h + 15616*a^5*b^16*c^15*d
^3*f^10*h - 174848*a^6*b^15*c^14*d^4*f^10*h + 1119488*a^7*b^14*c^13*d^5*f^
10*h - 4431616*a^8*b^13*c^12*d^6*f^10*h + 11542784*a^9*b^12*c^11*d^7*f^10*
h - 20379392*a^10*b^11*c^10*d^8*f^10*h + 24135936*a^11*b^10*c^9*d^9*f^10*h
 - 17377536*a^12*b^9*c^8*d^10*f^10*h + 3708672*a^13*b^8*c^7*d^11*f^10*h +
6930176*a^14*b^7*c^6*d^12*f^10*h - 9044224*a^15*b^6*c^5*d^13*f^10*h + 5638
912*a^16*b^5*c^4*d^14*f^10*h - 2077952*a^17*b^4*c^3*d^15*f^10*h + 434944*a
^18*b^3*c^2*d^16*f^10*h - 81920*a^12*b^9*d^18*e^7*f^3*g + 520192*a^13*b^8*
d^18*e^6*f^4*g - 1392640*a^14*b^7*d^18*e^5*f^5*g + 2024704*a^15*b^6*d^18*e
^4*f^6*g - 1707776*a^16*b^5*d^18*e^3*f^7*g + 818944*a^17*b^4*d^18*e^2*f^8*
g + 16384*a^13*b^8*d^18*e^7*f^3*h - 102400*a^14*b^7*d^18*e^6*f^4*h + 26...
```

Reduce [F]

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \int \frac{\sqrt{fx+e}\,(hx+g)}{(bx+a)^5\,(dx+c)^2}dx$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x)
```

output

```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x)
```

3.51 $\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 420

$$
\begin{aligned}
&\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx \\
&= -\frac{6b(bc-ad)(bdg-2bch+adh)\sqrt{e+fx}}{d^5} + \frac{(bc-ad)^3(dg-ch)\sqrt{e+fx}}{2d^5(c+dx)^2} \\
&\quad - \frac{(bc-ad)^2\left(ad(dfg+4deh-5cfh)+b(12d^2eg+17c^2fh-cd(13fg+16eh))\right)\sqrt{e+fx}}{4d^5(de-cf)(c+dx)} \\
&\quad + \frac{2b^2(3adfh+b(dfg-deh-3cfh))(e+fx)^{3/2}}{3d^4f^2} + \frac{2b^3h(e+fx)^{5/2}}{5d^3f^2} \\
&\quad - \frac{(bc-ad)\left(a^2d^2f(dfg-4deh+3cfh)-b^2(24d^3e^2g-63c^3f^2h-12cd^2e(5fg+4eh)+7c^2df(5fg+1\right.}{4d^{11/2}(de-cf)^{3/2}}
\end{aligned}
$$

output
```
-6*b*(-a*d+b*c)*(a*d*h-2*b*c*h+b*d*g)*(f*x+e)^(1/2)/d^5+1/2*(-a*d+b*c)^3*(
-c*h+d*g)*(f*x+e)^(1/2)/d^5/(d*x+c)^2-1/4*(-a*d+b*c)^2*(a*d*(-5*c*f*h+4*d*
e*h+d*f*g)+b*(12*d^2*e*g+17*c^2*f*h-c*d*(16*e*h+13*f*g)))*(f*x+e)^(1/2)/d^
5/(-c*f+d*e)/(d*x+c)+2/3*b^2*(3*a*d*f*h+b*(-3*c*f*h-d*e*h+d*f*g))*(f*x+e)^
(3/2)/d^4/f^2+2/5*b^3*h*(f*x+e)^(5/2)/d^3/f^2-1/4*(-a*d+b*c)*(a^2*d^2*f*(3
*c*f*h-4*d*e*h+d*f*g)-b^2*(24*d^3*e^2*g-63*c^3*f^2*h-12*c*d^2*e*(4*e*h+5*f
*g)+7*c^2*d*f*(16*e*h+5*f*g))-2*a*b*d*(21*c^2*f^2*h+6*d^2*e*(2*e*h+f*g)-c*
d*f*(34*e*h+5*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(11
/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$
$$= \frac{\sqrt{e+fx}(15a^3d^3f^2(3c^2fh-d^2(fgx+2e(g+2hx))+cd(-2eh+f(g+5hx)))-45a^2bd^2f^2(15c^3fh+\cdots}{}$$
$$-\frac{(bc-ad)\left(a^2d^2f(dfg-4deh+3cfh)+b^2(-24d^3e^2g+63c^3f^2h+12cd^2e(5fg+4eh)-7c^2df(5fg+\cdots\right.}{4d^{11/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)^3*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(15*a^3*d^3*f^2*(3*c^2*f*h - d^2*(f*g*x + 2*e*(g + 2*h*x))
+ c*d*(-2*e*h + f*(g + 5*h*x))) - 45*a^2*b*d^2*f^2*(15*c^3*f*h + 4*d^3*e*x
*(g - 2*h*x) + c^2*d*(-3*f*g - 14*e*h + 25*f*h*x) + c*d^2*(2*e*(g - 12*h*x
) + f*x*(-5*g + 8*h*x))) + 15*a*b^2*d*f*(105*c^4*f^2*h - 5*c^3*d*f*(9*f*g
+ 22*e*h - 35*f*h*x) + 8*d^4*e*x^2*(3*f*g + e*h + f*h*x) - 8*c*d^3*x*(-2*e
^2*h + f^2*x*(3*g + h*x) + e*f*(-9*g + 8*h*x)) + c^2*d^2*(8*e^2*h + 2*e*f*
(21*g - 94*h*x) + f^2*x*(-75*g + 56*h*x))) - b^3*(945*c^5*f^3*h + 8*d^5*e*
x^2*(e + f*x)*(-5*f*g + 2*e*h - 3*f*h*x) - 525*c^4*d*f^2*(2*e*h + f*(g - 3
*h*x)) + 4*c^2*d^3*(4*e^3*h + e*f^2*x*(235*g - 152*h*x) - 10*e^2*f*(g - 5*
h*x) - 2*f^3*x^2*(35*g + 9*h*x)) + 8*c*d^4*x*(4*e^3*h + 10*e*f^2*x*(4*g +
h*x) + f^3*x^2*(5*g + 3*h*x) + e^2*f*(-10*g + 11*h*x)) + c^3*d^2*f*(104*e^
2*h + 2*e*f*(275*g - 896*h*x) + 7*f^2*x*(-125*g + 72*h*x)))))/(60*d^5*f^2*
(d*e - c*f)*(c + d*x)^2) - ((b*c - a*d)*(a^2*d^2*f*(d*f*g - 4*d*e*h + 3*c*
f*h) + b^2*(-24*d^3*e^2*g + 63*c^3*f^2*h + 12*c*d^2*e*(5*f*g + 4*e*h) - 7*
c^2*d*f*(5*f*g + 16*e*h)) - 2*a*b*d*(21*c^2*f^2*h + 6*d^2*e*(f*g + 2*e*h)
- c*d*f*(5*f*g + 34*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*
f]])/(4*d^(11/2)*(-(d*e) + c*f)^(3/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {166, 27, 166, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int \frac{(a+bx)^2\sqrt{e+fx}(6be(dg-ch)-a(dfg-4deh+3cfh)+b(5dfg+4deh-9cfh)x)}{2(c+dx)^2}dx}{2d(de-cf)}-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(a+bx)^2\sqrt{e+fx}(6be(dg-ch)-a(dfg-4deh+3cfh)+b(5dfg+4deh-9cfh)x)}{(c+dx)^2}dx}{4d(de-cf)}-\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \ 166$$

$$\frac{\frac{\int -\frac{(a+bx)\sqrt{e+fx}\left(2ad(be-af)(dfg-4deh+3cfh)+(4be+3af)\left(ad(dfg-4deh+3cfh)-b\left(9fhc^2-5d(fg+2eh)c+6d^2eg\right)\right)+b\left(5adf(dfg-4deh+3cfh)-b\left(8e(5fg+e\right.\right.\right.}{2(c+dx)}}{d(de-cf)}}{4d(de-cf)}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(a+bx)^2(e+fx)^{3/2}\left(ad(3cfh-4deh+dfg)-b(9c^2fh-5cd(2eh+fg)+6d^2eg)\right)}{d(c+dx)(de-cf)}-\frac{\int \frac{(a+bx)\sqrt{e+fx}\left(2ad(be-af)(dfg-4deh+3cfh)+(4be+3af)\left(ad(dfg-\right.\right.}{}}{}}{4d(de-cf)}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \ 164$$

$$\frac{\frac{(a+bx)^2(e+fx)^{3/2}(ad(3cfh-4deh+dfg)-b(9c^2fh-5cd(2eh+fg)+6d^2eg))}{d(c+dx)(de-cf)}-\frac{2b(e+fx)^{3/2}\left(30a^2d^2f^2(3cfh-4deh+dfg)+3bdfx\left(5adf(3cfh-4deh+\right.\right.}{}}{}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(a+bx)^2(e+fx)^{3/2}(ad(3cfh-4deh+dfg)-b(9c^2fh-5cd(2eh+fg)+6d^2eg))}{d(c+dx)(de-cf)}-\frac{2b(e+fx)^{3/2}\left(30a^2d^2f^2(3cfh-4deh+dfg)+3bdfx\left(5adf(3cfh-4deh+\right.\right.}{}}{}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(a+bx)^2(e+fx)^{3/2}(ad(3cfh-4deh+dfg)-b(9c^2fh-5cd(2eh+fg)+6d^2eg))}{d(c+dx)(de-cf)}-\frac{2b(e+fx)^{3/2}\left(30a^2d^2f^2(3cfh-4deh+dfg)+3bdfx\left(5adf(3cfh-4deh+\right.\right.}{}}{}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(a+bx)^2(e+fx)^{3/2}(ad(3cfh-4deh+dfg)-b(9c^2fh-5cd(2eh+fg)+6d^2eg))}{d(c+dx)(de-cf)}-\frac{2b(e+fx)^{3/2}\left(30a^2d^2f^2(3cfh-4deh+dfg)+3bdfx\left(5adf(3cfh-4deh+\right.\right.}{}}{}$$

$$\frac{(a+bx)^3(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((a + b*x)^3*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^3*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x)^2)
 + (((a*d*(d*f*g - 4*d*e*h + 3*c*f*h) - b*(6*d^2*e*g + 9*c^2*f*h - 5*c*d*(
f*g + 2*e*h)))*(a + b*x)^2*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x)) - ((
2*b*(e + f*x)^(3/2)*(30*a^2*d^2*f^2*(d*f*g - 4*d*e*h + 3*c*f*h) - 15*a*b*d
*f*(35*c^2*f^2*h + 2*d^2*e*(9*f*g + 4*e*h) - c*d*f*(15*f*g + 46*e*h)) + b^
2*(315*c^3*f^3*h - 8*d^3*e^2*(5*f*g - 2*e*h) + 2*c*d^2*e*f*(115*f*g + 44*e
*h) - 7*c^2*d*f^2*(25*f*g + 62*e*h)) + 3*b*d*f*(5*a*d*f*(d*f*g - 4*d*e*h +
 3*c*f*h) - b*(63*c^2*f^2*h + 8*d^2*e*(5*f*g + e*h) - c*d*f*(35*f*g + 76*e
*h)))*x))/(15*d^2*f^2) - ((b*c - a*d)*(a^2*d^2*f*(d*f*g - 4*d*e*h + 3*c*f*
h) - b^2*(24*d^3*e^2*g - 63*c^3*f^2*h - 12*c*d^2*e*(5*f*g + 4*e*h) + 7*c^2
*d*f*(5*f*g + 16*e*h)) - 2*a*b*d*(21*c^2*f^2*h + 6*d^2*e*(f*g + 2*e*h) - c
*d*f*(5*f*g + 34*e*h)))*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[
(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d^2)/(2*d*(d*e - c*f))
)/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.63

method	result
risch	$\frac{2b(3x^2hb^2d^2f^2+15ab\,d^2f^2hx-15b^2cd\,f^2hx+b^2d^2efhx+5b^2d^2f^2gx+45a^2d^2f^2h-135abcd\,f^2h+15ab\,d^2efh+45ab\,d^2}{15f^2d^5}$
pseudoelliptic	$-\frac{3\left(-(ad-bc)(xd+c)^2\left(\left(\left(\frac{a^2f^2g}{3}-\frac{4ae(ah+3bg)f}{3}-8b\,e^2(ah+bg)\right)d^3+c\left(a\left(ah+\frac{10bg}{3}\right)f^2+4e\left(\frac{17}{3}bha+5b^2g\right)f+16b^2e^2\right.\right.\right.}{}$
derivativedivides	$\frac{2b\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+ab\,d^2fh(fx+e)^{\frac{3}{2}}-b^2cdfh(fx+e)^{\frac{3}{2}}-\frac{b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-9abcd\,f^2h\sqrt{fx}\right.}{d^5}$
default	$\frac{2b\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+ab\,d^2fh(fx+e)^{\frac{3}{2}}-b^2cdfh(fx+e)^{\frac{3}{2}}-\frac{b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-9abcd\,f^2h\sqrt{fx}\right.}{d^5}$

input
```
int((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*b*(3*b^2*d^2*f^2*h*x^2+15*a*b*d^2*f^2*h*x-15*b^2*c*d*f^2*h*x+b^2*d^2*
e*f*h*x+5*b^2*d^2*f^2*g*x+45*a^2*d^2*f^2*h-135*a*b*c*d*f^2*h+15*a*b*d^2*e*
f*h+45*a*b*d^2*f^2*g+90*b^2*c^2*f^2*h-15*b^2*c*d*e*f*h-45*b^2*c*d*f^2*g-2*
b^2*d^2*e^2*h+5*b^2*d^2*e*f*g)*(f*x+e)^(1/2)/f^2/d^5+1/d^5*(2*a*d-2*b*c)*(
(-1/8*d*f*(5*a^2*c*d^2*f*h-4*a^2*d^3*e*h-a^2*d^3*f*g-22*a*b*c^2*d*f*h+20*a
*b*c*d^2*e*h+14*a*b*c*d^2*f*g-12*a*b*d^3*e*g+17*b^2*c^3*f*h-16*b^2*c^2*d*e
*h-13*b^2*c^2*d*f*g+12*b^2*c*d^2*e*g)/(c*f-d*e)*(f*x+e)^(3/2)-1/8*f*(3*a^2
*c*d^2*f*h-4*a^2*d^3*e*h+a^2*d^3*f*g-18*a*b*c^2*d*f*h+20*a*b*c*d^2*e*h+10*
a*b*c*d^2*f*g-12*a*b*d^3*e*g+15*b^2*c^3*f*h-16*b^2*c^2*d*e*h-11*b^2*c^2*d*
f*g+12*b^2*c*d^2*e*g)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2+1/8*(3*a^2*c*d^
2*f^2*h-4*a^2*d^3*e*f*h+a^2*d^3*f^2*g-42*a*b*c^2*d*f^2*h+68*a*b*c*d^2*e*f*
h+10*a*b*c*d^2*f^2*g-24*a*b*d^3*e^2*h-12*a*b*d^3*e*f*g+63*b^2*c^3*f^2*h-11
2*b^2*c^2*d*e*f*h-35*b^2*c^2*d*f^2*g+48*b^2*c*d^2*e^2*h+60*b^2*c*d^2*e*f*g
-24*b^2*d^3*e^2*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((
c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2145 vs. $2(390) = 780$.

Time = 0.33 (sec) , antiderivative size = 4303, normalized size of antiderivative = 10.25

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(f*x+e)**(1/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(390) = 780$.

Time = 0.16 (sec) , antiderivative size = 1382, normalized size of antiderivative = 3.29

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(24*b^3*c*d^3*e^2*g - 24*a*b^2*d^4*e^2*g - 60*b^3*c^2*d^2*e*f*g + 72*
a*b^2*c*d^3*e*f*g - 12*a^2*b*d^4*e*f*g + 35*b^3*c^3*d*f^2*g - 45*a*b^2*c^2
*d^2*f^2*g + 9*a^2*b*c*d^3*f^2*g + a^3*d^4*f^2*g - 48*b^3*c^2*d^2*e^2*h +
72*a*b^2*c*d^3*e^2*h - 24*a^2*b*d^4*e^2*h + 112*b^3*c^3*d*e*f*h - 180*a*b^
2*c^2*d^2*e*f*h + 72*a^2*b*c*d^3*e*f*h - 4*a^3*d^4*e*f*h - 63*b^3*c^4*f^2*
h + 105*a*b^2*c^3*d*f^2*h - 45*a^2*b*c^2*d^2*f^2*h + 3*a^3*c*d^3*f^2*h)*ar
ctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^6*e - c*d^5*f)*sqrt(-d^2*e
+ c*d*f)) - 1/4*(12*(f*x + e)^(3/2)*b^3*c^2*d^3*e*f*g - 24*(f*x + e)^(3/2)
*a*b^2*c*d^4*e*f*g + 12*(f*x + e)^(3/2)*a^2*b*d^5*e*f*g - 12*sqrt(f*x + e)
*b^3*c^2*d^3*e^2*f*g + 24*sqrt(f*x + e)*a*b^2*c*d^4*e^2*f*g - 12*sqrt(f*x
+ e)*a^2*b*d^5*e^2*f*g - 13*(f*x + e)^(3/2)*b^3*c^3*d^2*f^2*g + 27*(f*x +
e)^(3/2)*a*b^2*c^2*d^3*f^2*g - 15*(f*x + e)^(3/2)*a^2*b*c*d^4*f^2*g + (f*x
 + e)^(3/2)*a^3*d^5*f^2*g + 23*sqrt(f*x + e)*b^3*c^3*d^2*e*f^2*g - 45*sqrt
(f*x + e)*a*b^2*c^2*d^3*e*f^2*g + 21*sqrt(f*x + e)*a^2*b*c*d^4*e*f^2*g + s
qrt(f*x + e)*a^3*d^5*e*f^2*g - 11*sqrt(f*x + e)*b^3*c^4*d*f^3*g + 21*sqrt(
f*x + e)*a*b^2*c^3*d^2*f^3*g - 9*sqrt(f*x + e)*a^2*b*c^2*d^3*f^3*g - sqrt(
f*x + e)*a^3*c*d^4*f^3*g - 16*(f*x + e)^(3/2)*b^3*c^3*d^2*e*f*h + 36*(f*x
+ e)^(3/2)*a*b^2*c^2*d^3*e*f*h - 24*(f*x + e)^(3/2)*a^2*b*c*d^4*e*f*h + 4*
(f*x + e)^(3/2)*a^3*d^5*e*f*h + 16*sqrt(f*x + e)*b^3*c^3*d^2*e^2*f*h - 36*
sqrt(f*x + e)*a*b^2*c^2*d^3*e^2*f*h + 24*sqrt(f*x + e)*a^2*b*c*d^4*e^2*...
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1361, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^3)/(c + d*x)^3,x)
```

output
```
(e + f*x)^(3/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(3*d^3*f^2) - (2*b^
3*h*(c*f - d*e))/(d^4*f^2)) - (e + f*x)^(1/2)*((3*((2*b^3*f*g - 8*b^3*e*h
+ 6*a*b^2*f*h)/(d^3*f^2) - (6*b^3*h*(c*f - d*e))/(d^4*f^2))*(c*f - d*e))/d
 - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d^3*f^2) + (6*b^3*h*(c*f -
 d*e)^2)/(d^5*f^2)) - ((e + f*x)^(1/2)*((a^3*d^4*f^2*g)/4 - (15*b^3*c^4*f^
2*h)/4 - a^3*d^4*e*f*h + (3*a^3*c*d^3*f^2*h)/4 + (11*b^3*c^3*d*f^2*g)/4 -
(21*a*b^2*c^2*d^2*f^2*g)/4 - (21*a^2*b*c^2*d^2*f^2*h)/4 - 3*a^2*b*d^4*e*f*
g + 4*b^3*c^3*d*e*f*h + (9*a^2*b*c*d^3*f^2*g)/4 + (33*a*b^2*c^3*d*f^2*h)/4
 - 3*b^3*c^2*d^2*e*f*g - 9*a*b^2*c^2*d^2*e*f*h + 6*a*b^2*c*d^3*e*f*g + 6*a
^2*b*c*d^3*e*f*h) - ((e + f*x)^(3/2)*(a^3*d^5*f^2*g + 4*a^3*d^5*e*f*h - 5*
a^3*c*d^4*f^2*h + 17*b^3*c^4*d*f^2*h - 13*b^3*c^3*d^2*f^2*g + 27*a*b^2*c^2
*d^3*f^2*g - 39*a*b^2*c^3*d^2*f^2*h + 27*a^2*b*c^2*d^3*f^2*h + 12*a^2*b*d^
5*e*f*g - 15*a^2*b*c*d^4*f^2*g + 12*b^3*c^2*d^3*e*f*g - 16*b^3*c^3*d^2*e*f
*h + 36*a*b^2*c^2*d^3*e*f*h - 24*a*b^2*c*d^4*e*f*g - 24*a^2*b*c*d^4*e*f*h)
)/(4*(c*f - d*e)))/(d^7*(e + f*x)^2 - (e + f*x)*(2*d^7*e - 2*c*d^6*f) + d^
7*e^2 + c^2*d^5*f^2 - 2*c*d^6*e*f) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*d -
 b*c)*(a^2*d^3*f^2*g - 24*b^2*d^3*e^2*g + 63*b^2*c^3*f^2*h - 24*a*b*d^3*e^
2*h - 4*a^2*d^3*e*f*h + 3*a^2*c*d^2*f^2*h + 48*b^2*c*d^2*e^2*h - 35*b^2*c^
2*d*f^2*g - 12*a*b*d^3*e*f*g + 10*a*b*c*d^2*f^2*g - 42*a*b*c^2*d*f^2*h + 6
0*b^2*c*d^2*e*f*g - 112*b^2*c^2*d*e*f*h + 68*a*b*c*d^2*e*f*h))/((c*f - ...
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 5248, normalized size of antiderivative = 12.50

$$\int \frac{(a+bx)^3\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x)
```

output

```
(45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*a**3*c**3*d**3*f**4*h - 60*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*e*f**3*h + 15*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4
*f**4*g + 90*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a**3*c**2*d**4*f**4*h*x - 120*sqrt(d)*sqrt(c*f - d*e)*atan((s
qrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**5*e*f**3*h*x + 30*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**
3*c*d**5*f**4*g*x + 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqr
t(d)*sqrt(c*f - d*e)))*a**3*c*d**5*f**4*h*x**2 - 60*sqrt(d)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**6*e*f**3*h*x**
2 + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**3*d**6*f**4*g*x**2 - 675*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**4*d**2*f**4*h + 1080*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c
**3*d**3*e*f**3*h + 135*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sq
rt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**3*f**4*g - 1350*sqrt(d)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**3*f*
*4*h*x - 360*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a**2*b*c**2*d**4*e**2*f**2*h - 180*sqrt(d)*sqrt(c*f - d*e)...
```

3.52 $\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 353

$$\begin{aligned}
\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx
&= \frac{2b(bdg-3bch+2adh)\sqrt{e+fx}}{d^4} - \frac{(bc-ad)^2(dg-ch)\sqrt{e+fx}}{2d^4(c+dx)^2} \\
&\quad + \frac{(bc-ad)\left(ad(dfg+4deh-5cfh)+b(8d^2eg+13c^2fh-3cd(3fg+4eh))\right)\sqrt{e+fx}}{4d^4(de-cf)(c+dx)} \\
&\quad + \frac{2b^2h(e+fx)^{3/2}}{3d^3f} \\
&\quad + \frac{(a^2d^2f(dfg-4deh+3cfh)-b^2(8d^3e^2g-35c^3f^2h-24cd^2e(fg+eh)+15c^2df(fg+4eh))-2abd(1}{4d^{9/2}(de-cf)^{3/2}}
\end{aligned}$$

output
```
2*b*(2*a*d*h-3*b*c*h+b*d*g)*(f*x+e)^(1/2)/d^4-1/2*(-a*d+b*c)^2*(-c*h+d*g)*
(f*x+e)^(1/2)/d^4/(d*x+c)^2+1/4*(-a*d+b*c)*(a*d*(-5*c*f*h+4*d*e*h+d*f*g)+b
*(8*d^2*e*g+13*c^2*f*h-3*c*d*(4*e*h+3*f*g)))*(f*x+e)^(1/2)/d^4/(-c*f+d*e)/
(d*x+c)+2/3*b^2*h*(f*x+e)^(3/2)/d^3/f+1/4*(a^2*d^2*f*(3*c*f*h-4*d*e*h+d*f*
g)-b^2*(8*d^3*e^2*g-35*c^3*f^2*h-24*c*d^2*e*(e*h+f*g)+15*c^2*d*f*(4*e*h+f*
g))-2*a*b*d*(15*c^2*f^2*h+4*d^2*e*(2*e*h+f*g)-3*c*d*f*(8*e*h+f*g)))*arctan
h(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(9/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$

$$= \frac{\sqrt{e+fx}(3a^2d^2f(3c^2fh-d^2(2eg+fgx+4ehx)+cd(fg-2eh+5fhx))-6abdf(15c^3fh+4d^3ex(g-$$

$$-\frac{(-a^2d^2f(dfg-4deh+3cfh)+b^2(8d^3e^2g-35c^3f^2h-24cd^2e(fg+eh)+15c^2df(fg+4eh))+2abd}{4d^{9/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)^2*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(3*a^2*d^2*f*(3*c^2*f*h - d^2*(2*e*g + f*g*x + 4*e*h*x) + c
*d*(f*g - 2*e*h + 5*f*h*x)) - 6*a*b*d*f*(15*c^3*f*h + 4*d^3*e*x*(g - 2*h*x
) + c^2*d*(-3*f*g - 14*e*h + 25*f*h*x) + c*d^2*(2*e*(g - 12*h*x) + f*x*(-5
*g + 8*h*x))) + b^2*(105*c^4*f^2*h - 5*c^3*d*f*(9*f*g + 22*e*h - 35*f*h*x)
 + 8*d^4*e*x^2*(3*f*g + e*h + f*h*x) - 8*c*d^3*x*(-2*e^2*h + f^2*x*(3*g +
h*x) + e*f*(-9*g + 8*h*x)) + c^2*d^2*(8*e^2*h + 2*e*f*(21*g - 94*h*x) + f^
2*x*(-75*g + 56*h*x)))))/(12*d^4*f*(d*e - c*f)*(c + d*x)^2) - ((-(a^2*d^2*
f*(d*f*g - 4*d*e*h + 3*c*f*h)) + b^2*(8*d^3*e^2*g - 35*c^3*f^2*h - 24*c*d^
2*e*(f*g + e*h) + 15*c^2*d*f*(f*g + 4*e*h)) + 2*a*b*d*(15*c^2*f^2*h + 4*d^
2*e*(f*g + 2*e*h) - 3*c*d*f*(f*g + 8*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])
/Sqrt[-(d*e) + c*f]])/(4*d^(9/2)*(-(d*e) + c*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 163, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(a+bx)\sqrt{e+fx}(4be(dg-ch)-a(dfg-4deh+3cfh)+b(3dfg+4deh-7cfh)x)}{2(c+dx)^2}dx}{2d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)\sqrt{e+fx}(4be(dg-ch)-a(dfg-4deh+3cfh)+b(3dfg+4deh-7cfh)x)}{(c+dx)^2}dx}{4d(de-cf)} - \frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{163}$$

$$\frac{\frac{(e+fx)^{3/2}(3a^2d^2f(3cfh-4deh+dfg)-6abdf(5c^2fh-cd(6eh+fg)+2d^2eg)+b^2c(35c^2f^2h-cdf(46eh+15fg)+2d^2e(4eh+9fg))+2b^2dx(de-cf)(\cdots}{3d^2f(c+dx)(de-cf)}}{\cdots}$$

$$\frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{(e+fx)^{3/2}(3a^2d^2f(3cfh-4deh+dfg)-6abdf(5c^2fh-cd(6eh+fg)+2d^2eg)+b^2c(35c^2f^2h-cdf(46eh+15fg)+2d^2e(4eh+9fg))+2b^2dx(de-cf)(\cdots}{3d^2f(c+dx)(de-cf)}}{\cdots}$$

$$\frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(e+fx)^{3/2}(3a^2d^2f(3cfh-4deh+dfg)-6abdf(5c^2fh-cd(6eh+fg)+2d^2eg)+b^2c(35c^2f^2h-cdf(46eh+15fg)+2d^2e(4eh+9fg))+2b^2dx(de-cf)(\cdots}{3d^2f(c+dx)(de-cf)}}{\cdots}$$

$$\frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(e+fx)^{3/2}\left(3a^2d^2f(3cfh-4deh+dfg)-6abdf\left(5c^2fh-cd(6eh+fg)+2d^2eg\right)+b^2c\left(35c^2f^2h-cdf(46eh+15fg)+2d^2e(4eh+9fg)\right)+2b^2dx(de-cf)(\cdots}{3d^2f(c+dx)(de-cf)}}{\cdots}$$

$$\frac{(a+bx)^2(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((a + b*x)^2*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^2*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x)^2)
 + (((e + f*x)^(3/2)*(3*a^2*d^2*f*(d*f*g - 4*d*e*h + 3*c*f*h) - 6*a*b*d*f*
(2*d^2*e*g + 5*c^2*f*h - c*d*(f*g + 6*e*h)) + b^2*c*(35*c^2*f^2*h + 2*d^2*
e*(9*f*g + 4*e*h) - c*d*f*(15*f*g + 46*e*h)) + 2*b^2*d*(d*e - c*f)*(3*d*f*
g + 4*d*e*h - 7*c*f*h)*x))/(3*d^2*f*(d*e - c*f)*(c + d*x)) - ((a^2*d^2*f*(
d*f*g - 4*d*e*h + 3*c*f*h) - b^2*(8*d^3*e^2*g - 35*c^3*f^2*h - 24*c*d^2*e*
(f*g + e*h) + 15*c^2*d*f*(f*g + 4*e*h)) - 2*a*b*d*(15*c^2*f^2*h + 4*d^2*e*
(f*g + 2*e*h) - 3*c*d*f*(f*g + 8*e*h)))*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e
 - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/(2*d^2
*(d*e - c*f)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{3\left(-(xd+c)^2 f\left(\frac{\left(a^2f^2g-4ae(ah+2bg)f-16b\,e^2\left(ah+\frac{bg}{2}\right)\right)d^3}{3}+c\left(\left(a^2h+2gab\right)f^2+8\left(2bha+b^2g\right)ef+8b^2e^2h\right)d^2-10c^2\left(\cdots\right.\right.\right.}{\cdots}$
risch	$\frac{2b(hxdbf+6adfh-9bcfh+bdeh+3bdfg)\sqrt{fx+e}}{3f\,d^4}+\frac{-\frac{df\left(5a^2c\,d^2fh-4a^2d^3eh-a^2d^3fg-18ab\,c^2dfh+16abc\,d^2eh+10abc\,d^2f\cdots\right.}{4(cf-\cdots}}{\cdots}$
derivativedivides	$\frac{2b\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-3bcfh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^4}-\frac{2f\left(\frac{df\left(5a^2c\,d^2fh-4a^2d^3eh-a^2d^3fg-18ab\,c^2dfh+16abc\,d^2eh+\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{2b\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-3bcfh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^4}-\frac{2f\left(\frac{df\left(5a^2c\,d^2fh-4a^2d^3eh-a^2d^3fg-18ab\,c^2dfh+16abc\,d^2eh+\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/((c*f-d*e)*d)^(1/2)*(-(d*x+c)^2*f*(1/3*(a^2*f^2*g-4*a*e*(a*h+2*b*g)*f
-16*b*e^2*(a*h+1/2*b*g))*d^3+c*((a^2*h+2*a*b*g)*f^2+8*(2*a*b*h+b^2*g)*e*f+
8*b^2*e^2*h)*d^2-10*c^2*((a*h+1/2*b*g)*f+2*e*h*b)*b*f*d+35/3*b^2*c^3*f^2*h
)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^
(1/2)*(1/3*(-a^2*f^2*g*x-2*(4*(-1/3*h*x^3-g*x^2)*b^2+4*a*x*(-2*h*x+g)*b+a^
2*(2*h*x+g))*e*f+8/3*b^2*e^2*h*x^2)*d^4-2/3*c*((4*(1/3*h*x^3+g*x^2)*b^2-5*
a*x*(-8/5*h*x+g)*b-1/2*a^2*(5*h*x+g))*f^2+(-12*x*(-8/9*h*x+g)*b^2+2*a*(-12
*h*x+g)*b+a^2*h)*e*f-8/3*b^2*e^2*h*x)*d^3+((1/3*(56/3*h*x^2-25*g*x)*b^2+2*
a*(-25/3*h*x+g)*b+a^2*h)*f^2+28/3*((-47/21*h*x+1/2*g)*b+a*h)*b*e*f+8/9*b^2
*e^2*h)*c^2*d^2-10*c^3*((1/2*(-35/9*h*x+g)*b+a*h)*f+11/9*e*h*b)*b*f*d+35/3
*b^2*c^4*f^2*h))/(d*x+c)^2/d^4/(c*f-d*e)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. $2(327) = 654$.

Time = 0.31 (sec) , antiderivative size = 2688, normalized size of antiderivative = 7.61

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(d^2*e - c*d*f)*(((8*b^2*d^5*e^2*f - 8*(3*b^2*c*d^4 - a*b*d^5
)*e*f^2 + (15*b^2*c^2*d^3 - 6*a*b*c*d^4 - a^2*d^5)*f^3)*g - (8*(3*b^2*c*d^
4 - 2*a*b*d^5)*e^2*f - 4*(15*b^2*c^2*d^3 - 12*a*b*c*d^4 + a^2*d^5)*e*f^2 +
 (35*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a^2*c*d^4)*f^3)*h)*x^2 + (8*b^2*c^2*
d^3*e^2*f - 8*(3*b^2*c^3*d^2 - a*b*c^2*d^3)*e*f^2 + (15*b^2*c^4*d - 6*a*b*
c^3*d^2 - a^2*c^2*d^3)*f^3)*g - (8*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3)*e^2*f -
 4*(15*b^2*c^4*d - 12*a*b*c^3*d^2 + a^2*c^2*d^3)*e*f^2 + (35*b^2*c^5 - 30*
a*b*c^4*d + 3*a^2*c^3*d^2)*f^3)*h + 2*((8*b^2*c*d^4*e^2*f - 8*(3*b^2*c^2*d
^3 - a*b*c*d^4)*e*f^2 + (15*b^2*c^3*d^2 - 6*a*b*c^2*d^3 - a^2*c*d^4)*f^3)*
g - (8*(3*b^2*c^2*d^3 - 2*a*b*c*d^4)*e^2*f - 4*(15*b^2*c^3*d^2 - 12*a*b*c^
2*d^3 + a^2*c*d^4)*e*f^2 + (35*b^2*c^4*d - 30*a*b*c^3*d^2 + 3*a^2*c^2*d^3)
*f^3)*h)*x)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e)
)/(d*x + c)) + 2*(8*(b^2*d^6*e^2*f - 2*b^2*c*d^5*e*f^2 + b^2*c^2*d^4*f^3)*
h*x^3 + 8*(3*(b^2*d^6*e^2*f - 2*b^2*c*d^5*e*f^2 + b^2*c^2*d^4*f^3)*g + (b^
2*d^6*e^3 - 3*(3*b^2*c*d^5 - 2*a*b*d^6)*e^2*f + 3*(5*b^2*c^2*d^4 - 4*a*b*c
*d^5)*e*f^2 - (7*b^2*c^3*d^3 - 6*a*b*c^2*d^4)*f^3)*h)*x^2 + 3*(2*(7*b^2*c^
2*d^4 - 2*a*b*c*d^5 - a^2*d^6)*e^2*f - (29*b^2*c^3*d^3 - 10*a*b*c^2*d^4 -
3*a^2*c*d^5)*e*f^2 + (15*b^2*c^4*d^2 - 6*a*b*c^3*d^3 - a^2*c^2*d^4)*f^3)*g
 + (8*b^2*c^2*d^4*e^3 - 2*(59*b^2*c^3*d^3 - 42*a*b*c^2*d^4 + 3*a^2*c*d^5)*
e^2*f + (215*b^2*c^4*d^2 - 174*a*b*c^3*d^3 + 15*a^2*c^2*d^4)*e*f^2 - 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(f*x+e)**(1/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(327) = 654$.

Time = 0.16 (sec) , antiderivative size = 904, normalized size of antiderivative = 2.56

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(8*b^2*d^3*e^2*g - 24*b^2*c*d^2*e*f*g + 8*a*b*d^3*e*f*g + 15*b^2*c^2*d
*f^2*g - 6*a*b*c*d^2*f^2*g - a^2*d^3*f^2*g - 24*b^2*c*d^2*e^2*h + 16*a*b*d
^3*e^2*h + 60*b^2*c^2*d*e*f*h - 48*a*b*c*d^2*e*f*h + 4*a^2*d^3*e*f*h - 35*
b^2*c^3*f^2*h + 30*a*b*c^2*d*f^2*h - 3*a^2*c*d^2*f^2*h)*arctan(sqrt(f*x +
e)*d/sqrt(-d^2*e + c*d*f))/((d^5*e - c*d^4*f)*sqrt(-d^2*e + c*d*f)) + 1/4*
(8*(f*x + e)^(3/2)*b^2*c*d^3*e*f*g - 8*(f*x + e)^(3/2)*a*b*d^4*e*f*g - 8*s
qrt(f*x + e)*b^2*c*d^3*e^2*f*g + 8*sqrt(f*x + e)*a*b*d^4*e^2*f*g - 9*(f*x
+ e)^(3/2)*b^2*c^2*d^2*f^2*g + 10*(f*x + e)^(3/2)*a*b*c*d^3*f^2*g - (f*x +
 e)^(3/2)*a^2*d^4*f^2*g + 15*sqrt(f*x + e)*b^2*c^2*d^2*e*f^2*g - 14*sqrt(f
*x + e)*a*b*c*d^3*e*f^2*g - sqrt(f*x + e)*a^2*d^4*e*f^2*g - 7*sqrt(f*x + e
)*b^2*c^3*d*f^3*g + 6*sqrt(f*x + e)*a*b*c^2*d^2*f^3*g + sqrt(f*x + e)*a^2*
c*d^3*f^3*g - 12*(f*x + e)^(3/2)*b^2*c^2*d^2*e*f*h + 16*(f*x + e)^(3/2)*a*
b*c*d^3*e*f*h - 4*(f*x + e)^(3/2)*a^2*d^4*e*f*h + 12*sqrt(f*x + e)*b^2*c^2
*d^2*e^2*f*h - 16*sqrt(f*x + e)*a*b*c*d^3*e^2*f*h + 4*sqrt(f*x + e)*a^2*d^
4*e^2*f*h + 13*(f*x + e)^(3/2)*b^2*c^3*d*f^2*h - 18*(f*x + e)^(3/2)*a*b*c^
2*d^2*f^2*h + 5*(f*x + e)^(3/2)*a^2*c*d^3*f^2*h - 23*sqrt(f*x + e)*b^2*c^3
*d*e*f^2*h + 30*sqrt(f*x + e)*a*b*c^2*d^2*e*f^2*h - 7*sqrt(f*x + e)*a^2*c*
d^3*e*f^2*h + 11*sqrt(f*x + e)*b^2*c^4*f^3*h - 14*sqrt(f*x + e)*a*b*c^3*d*
f^3*h + 3*sqrt(f*x + e)*a^2*c^2*d^2*f^3*h)/((d^5*e - c*d^4*f)*((f*x + e)*d
 - d*e + c*f)^2) + 2/3*(3*sqrt(f*x + e)*b^2*d^6*f^3*g + (f*x + e)^(3/2)...
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$
$$=\sqrt{e+f\,x}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d^3\,f}-\frac{6\,b^2\,h\,(c\,f-d\,e)}{d^4\,f}\right)$$
$$-\frac{\sqrt{e+f\,x}\left(\frac{3\,h\,a^2\,c\,d^2\,f^2}{4}+\frac{g\,a^2\,d^3\,f^2}{4}-e\,h\,a^2\,d^3\,f-\frac{7\,h\,a\,b\,c^2\,d\,f^2}{2}+\frac{3\,g\,a\,b\,c\,d^2\,f^2}{2}+4\,e\,h\,a\,b\,c\,d^2\,f-2\,e\,g\,a\,b\,d^3\,f\right.}{}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(3\,h\,a^2\,c\,d^2\,f^2-4\,h\,a^2\,d^3\,e\,f+g\,a^2\,d^3\,f^2-30\,h\,a\,b\,c^2\,d\,f^2+48\,h\,a\,b\,c\,d^2\,e\,f+6\,g\,a\,b\,c\right.}{}$$
$$+\frac{2\,b^2\,h\,(e+f\,x)^{3/2}}{3\,d^3\,f}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x)^2)/(c + d*x)^3,x)
```

output
```
(e + f*x)^(1/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d^3*f) - (6*b^2*h*(c
*f - d*e))/(d^4*f)) - ((e + f*x)^(1/2)*((a^2*d^3*f^2*g)/4 + (11*b^2*c^3*f^
2*h)/4 - a^2*d^3*e*f*h + (3*a^2*c*d^2*f^2*h)/4 - (7*b^2*c^2*d*f^2*g)/4 - 2
*a*b*d^3*e*f*g + (3*a*b*c*d^2*f^2*g)/2 - (7*a*b*c^2*d*f^2*h)/2 + 2*b^2*c*d
^2*e*f*g - 3*b^2*c^2*d*e*f*h + 4*a*b*c*d^2*e*f*h) - ((e + f*x)^(3/2)*(a^2*
d^4*f^2*g + 4*a^2*d^4*e*f*h - 5*a^2*c*d^3*f^2*h - 13*b^2*c^3*d*f^2*h + 9*b
^2*c^2*d^2*f^2*g + 8*a*b*d^4*e*f*g - 10*a*b*c*d^3*f^2*g - 8*b^2*c*d^3*e*f*
g + 18*a*b*c^2*d^2*f^2*h + 12*b^2*c^2*d^2*e*f*h - 16*a*b*c*d^3*e*f*h))/(4*
(c*f - d*e)))/(d^6*(e + f*x)^2 - (e + f*x)*(2*d^6*e - 2*c*d^5*f) + d^6*e^2
 + c^2*d^4*f^2 - 2*c*d^5*e*f) + (atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e
)^(1/2))*(a^2*d^3*f^2*g - 8*b^2*d^3*e^2*g + 35*b^2*c^3*f^2*h - 16*a*b*d^3*
e^2*h - 4*a^2*d^3*e*f*h + 3*a^2*c*d^2*f^2*h + 24*b^2*c*d^2*e^2*h - 15*b^2*
c^2*d*f^2*g - 8*a*b*d^3*e*f*g + 6*a*b*c*d^2*f^2*g - 30*a*b*c^2*d*f^2*h + 2
4*b^2*c*d^2*e*f*g - 60*b^2*c^2*d*e*f*h + 48*a*b*c*d^2*e*f*h))/(4*d^(9/2)*(
c*f - d*e)^(3/2)) + (2*b^2*h*(e + f*x)^(3/2))/(3*d^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3292, normalized size of antiderivative = 9.33

$$\int \frac{(a+bx)^2\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x)
```

output

```
(9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a**2*c**3*d**2*f**3*h - 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*e*f**2*h + 3*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*f
**3*g + 18*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a**2*c**2*d**3*f**3*h*x - 24*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*e*f**2*h*x + 6*sqrt(d)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*
d**4*f**3*g*x + 9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a**2*c*d**4*f**3*h*x**2 - 12*sqrt(d)*sqrt(c*f - d*e)*ata
n((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**5*e*f**2*h*x**2 + 3
*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a**2*d**5*f**3*g*x**2 - 90*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)
/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**4*d*f**3*h + 144*sqrt(d)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*e*f**2*h
 + 18*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a*b*c**3*d**2*f**3*g - 180*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*f**3*h*x - 48*sqrt(d)*sqrt(
c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**3
*e**2*f*h - 24*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*...
```

3.53 $\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \frac{2bh\sqrt{e+fx}}{d^3}+\frac{(bc-ad)(dg-ch)\sqrt{e+fx}}{2d^3(c+dx)^2} - \frac{(ad(dfg+4deh-5cfh)+b(4d^2eg+9c^2fh-cd(5fg+8eh)))\sqrt{e+fx}}{4d^3(de-cf)(c+dx)} + \frac{(adf(dfg-4deh+3cfh)-b(15c^2f^2h+4d^2e(fg+2eh)-3cdf(fg+8eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{7/2}(de-cf)^{3/2}}$$

output

```
2*b*h*(f*x+e)^(1/2)/d^3+1/2*(-a*d+b*c)*(-c*h+d*g)*(f*x+e)^(1/2)/d^3/(d*x+c
)^2-1/4*(a*d*(-5*c*f*h+4*d*e*h+d*f*g)+b*(4*d^2*e*g+9*c^2*f*h-c*d*(8*e*h+5*
f*g)))*(f*x+e)^(1/2)/d^3/(-c*f+d*e)/(d*x+c)+1/4*(a*d*f*(3*c*f*h-4*d*e*h+d*
f*g)-b*(15*c^2*f^2*h+4*d^2*e*(2*e*h+f*g)-3*c*d*f*(8*e*h+f*g)))*arctanh(d^(
1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$

$$= \frac{\frac{\sqrt{d}\sqrt{e+fx}(ad(3c^2fh-d^2(2eg+fgx+4ehx)+cd(fg-2eh+5fhx))+b(-15c^3fh+4d^3ex(-g+2hx)+c^2d(3fg+14eh-25fhx)+cd^2(-2eg+5fgx+\cdots}{(de-cf)(c+dx)^2}}{4d^{7/2}}$$

input
```
Integrate[((a + b*x)*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
((Sqrt[d]*Sqrt[e + f*x]*(a*d*(3*c^2*f*h - d^2*(2*e*g + f*g*x + 4*e*h*x) +
c*d*(f*g - 2*e*h + 5*f*h*x)) + b*(-15*c^3*f*h + 4*d^3*e*x*(-g + 2*h*x) + c
^2*d*(3*f*g + 14*e*h - 25*f*h*x) + c*d^2*(-2*e*g + 5*f*g*x + 24*e*h*x - 8*
f*h*x^2))))/((d*e - c*f)*(c + d*x)^2) - ((-(a*d*f*(d*f*g - 4*d*e*h + 3*c*f
*h)) + b*(15*c^2*f^2*h + 4*d^2*e*(f*g + 2*e*h) - 3*c*d*f*(f*g + 8*e*h)))*A
rcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(-(d*e) + c*f)^(3/2))/(
4*d^(7/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {162, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{162}$$

$$-\frac{(adf(3cfh-4deh+dfg)-b(15c^2f^2h-3cdf(8eh+fg)+4d^2e(2eh+fg)))\int \frac{\sqrt{e+fx}}{c+dx}dx}{8d^2(de-cf)^2}-\frac{(e+fx)^{3/2}\left(-dx\left(ad(3cfh-4deh+dfg)-b\left(7c^2fh-cd(8eh+3fg)+4d^2eg\right)\right)+ad\left(c^2(-f)h-cd(3fg-2eh\cdots\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$$\Big\downarrow \; 60$$

$$-\frac{\left(adf(3cfh-4deh+dfg)-b\left(15c^2f^2h-3cdf(8eh+fg)+4d^2e(2eh+fg)\right)\right)\left(\frac{(de-cf)\int\frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{8d^2(de-cf)^2}-\frac{(e+fx)^{3/2}\left(-dx\left(ad(3cfh-4deh+dfg)-b\left(7c^2fh-cd(8eh+3fg)+4d^2eg\right)\right)+ad\left(c^2(-f)h-cd(3fg-2eh\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{\left(adf(3cfh-4deh+dfg)-b\left(15c^2f^2h-3cdf(8eh+fg)+4d^2e(2eh+fg)\right)\right)\left(\frac{2(de-cf)\int\frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\right.}{8d^2(de-cf)^2}-\frac{(e+fx)^{3/2}\left(-dx\left(ad(3cfh-4deh+dfg)-b\left(7c^2fh-cd(8eh+3fg)+4d^2eg\right)\right)+ad\left(c^2(-f)h-cd(3fg-2eh\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)\left(adf(3cfh-4deh+dfg)-b\left(15c^2f^2h-3cdf(8eh+fg)+4d^2e(2eh\right.\right.}{8d^2(de-cf)^2}-\frac{(e+fx)^{3/2}\left(-dx\left(ad(3cfh-4deh+dfg)-b\left(7c^2fh-cd(8eh+3fg)+4d^2eg\right)\right)+ad\left(c^2(-f)h-cd(3fg-2eh\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

input
```
Int[((a + b*x)*Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/4*((e + f*x)^(3/2)*(a*d*(2*d^2*e*g - c^2*f*h - c*d*(3*f*g - 2*e*h)) + b
*c*(2*d^2*e*g + 5*c^2*f*h - c*d*(f*g + 6*e*h)) - d*(a*d*(d*f*g - 4*d*e*h +
 3*c*f*h) - b*(4*d^2*e*g + 7*c^2*f*h - c*d*(3*f*g + 8*e*h)))*x))/(d^2*(d*e
 - c*f)^2*(c + d*x)^2) - ((a*d*f*(d*f*g - 4*d*e*h + 3*c*f*h) - b*(15*c^2*f
^2*h + 4*d^2*e*(f*g + 2*e*h) - 3*c*d*f*(f*g + 8*e*h)))*((2*Sqrt[e + f*x])/
d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d
^(3/2)))/(8*d^2*(d*e - c*f)^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{3\left(-(xd+c)^2\left(\frac{\left(g f^2 a-4e(ah+bg)f-8b e^2 h\right)d^2}{3}+c((ah+bg)f+8ehb)fd-5b c^2 f^2 h\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\sqrt{fx+e}\left(\cdots\right.}{\cdots}$
derivativedivides	$\frac{2bh\sqrt{fx+e}}{d^3}-\frac{2\left(\frac{\frac{df\left(5acdfh-4a d^2 eh-a d^2 fg-9b c^2 fh+8bcdeh+5bcdfg-4b d^2 eg\right)(fx+e)^{\frac{3}{2}}}{8cf-8de}+\frac{f\left(3acdfh-4a d^2 eh+a d^2 fg-7b\cdots\right.}{\cdots}}{((fx+e)d+cf-de)^2}\cdots\right.}{\cdots}$
default	$\frac{2bh\sqrt{fx+e}}{d^3}-\frac{2\left(\frac{\frac{df\left(5acdfh-4a d^2 eh-a d^2 fg-9b c^2 fh+8bcdeh+5bcdfg-4b d^2 eg\right)(fx+e)^{\frac{3}{2}}}{8cf-8de}+\frac{f\left(3acdfh-4a d^2 eh+a d^2 fg-7b\cdots\right.}{\cdots}}{((fx+e)d+cf-de)^2}\cdots\right.}{\cdots}$
risch	$\frac{2bh\sqrt{fx+e}}{d^3}+\frac{\frac{-\frac{df\left(5acdfh-4a d^2 eh-a d^2 fg-9b c^2 fh+8bcdeh+5bcdfg-4b d^2 eg\right)(fx+e)^{\frac{3}{2}}}{4(cf-de)}-\frac{f\left(3acdfh-4a d^2 eh+a d^2 fg-7b c^2\cdots\right.}{\cdots}}{((fx+e)d+cf-de)^2}}{\cdots}$

input
```
int((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/((c*f-d*e)*d)^(1/2)*(-(d*x+c)^2*(1/3*(g*f^2*a-4*e*(a*h+b*g)*f-8*b*e^2
*h)*d^2+c*((a*h+b*g)*f+8*e*h*b)*f*d-5*b*c^2*f^2*h)*arctan(d*(f*x+e)^(1/2)/
((c*f-d*e)*d)^(1/2))+(f*x+e)^(1/2)*(1/3*(-x*a*f*g-2*e*(-4*b*h*x^2+2*(a*h+b
*g)*x+g*a))*d^3-2/3*c*((4*b*h*x^2+5/2*(-a*h-b*g)*x-1/2*g*a)*f+e*(-12*b*h*x
+a*h+b*g))*d^2+c^2*((b*g+a*h-25/3*b*h*x)*f+14/3*e*h*b)*d-5*c^3*h*b*f)*((c*
f-d*e)*d)^(1/2))/d^3/(d*x+c)^2/(c*f-d*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(225) = 450.

Time = 0.20 (sec) , antiderivative size = 1444, normalized size of antiderivative = 5.85

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(d^2*e - c*d*f)*(((4*b*d^4*e*f - (3*b*c*d^3 + a*d^4)*f^2)*g + (8
*b*d^4*e^2 - 4*(6*b*c*d^3 - a*d^4)*e*f + 3*(5*b*c^2*d^2 - a*c*d^3)*f^2)*h)
*x^2 + (4*b*c^2*d^2*e*f - (3*b*c^3*d + a*c^2*d^2)*f^2)*g + (8*b*c^2*d^2*e^
2 - 4*(6*b*c^3*d - a*c^2*d^2)*e*f + 3*(5*b*c^4 - a*c^3*d)*f^2)*h + 2*((4*b
*c*d^3*e*f - (3*b*c^2*d^2 + a*c*d^3)*f^2)*g + (8*b*c*d^3*e^2 - 4*(6*b*c^2*
d^2 - a*c*d^3)*e*f + 3*(5*b*c^3*d - a*c^2*d^2)*f^2)*h)*x)*log((d*f*x + 2*d
*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*(8*(b*d^5*e
^2 - 2*b*c*d^4*e*f + b*c^2*d^3*f^2)*h*x^2 - (2*(b*c*d^4 + a*d^5)*e^2 - (5*
b*c^2*d^3 + 3*a*c*d^4)*e*f + (3*b*c^3*d^2 + a*c^2*d^3)*f^2)*g + (2*(7*b*c^
2*d^3 - a*c*d^4)*e^2 - (29*b*c^3*d^2 - 5*a*c^2*d^3)*e*f + 3*(5*b*c^4*d - a
*c^3*d^2)*f^2)*h - ((4*b*d^5*e^2 - (9*b*c*d^4 - a*d^5)*e*f + (5*b*c^2*d^3
- a*c*d^4)*f^2)*g - (4*(6*b*c*d^4 - a*d^5)*e^2 - (49*b*c^2*d^3 - 9*a*c*d^4
)*e*f + 5*(5*b*c^3*d^2 - a*c^2*d^3)*f^2)*h)*x)*sqrt(f*x + e))/(c^2*d^6*e^2
 - 2*c^3*d^5*e*f + c^4*d^4*f^2 + (d^8*e^2 - 2*c*d^7*e*f + c^2*d^6*f^2)*x^2
 + 2*(c*d^7*e^2 - 2*c^2*d^6*e*f + c^3*d^5*f^2)*x), 1/4*(sqrt(-d^2*e + c*d*
f)*(((4*b*d^4*e*f - (3*b*c*d^3 + a*d^4)*f^2)*g + (8*b*d^4*e^2 - 4*(6*b*c*d
^3 - a*d^4)*e*f + 3*(5*b*c^2*d^2 - a*c*d^3)*f^2)*h)*x^2 + (4*b*c^2*d^2*e*f
 - (3*b*c^3*d + a*c^2*d^2)*f^2)*g + (8*b*c^2*d^2*e^2 - 4*(6*b*c^3*d - a*c^
2*d^2)*e*f + 3*(5*b*c^4 - a*c^3*d)*f^2)*h + 2*((4*b*c*d^3*e*f - (3*b*c^2*d
^2 + a*c*d^3)*f^2)*g + (8*b*c*d^3*e^2 - 4*(6*b*c^2*d^2 - a*c*d^3)*e*f +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(f*x+e)**(1/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(225) = 450$.

Time = 0.13 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$
$$= \frac{(4\,bd^2efg - 3\,bcdf^2g - ad^2f^2g + 8\,bd^2e^2h - 24\,bcdefh + 4\,ad^2efh + 15\,bc^2f^2h - 3\,acdf^2h)\arctan\left(\frac{\sqrt{f}}{\sqrt{-d}}\right.}{4\,(d^4e - cd^3f)\sqrt{-d^2e + cdf}}$$
$$+ \frac{2\sqrt{fx+e}bh}{d^3}$$
$$- \frac{4\,(fx+e)^{\frac{3}{2}}bd^3efg - 4\sqrt{fx+e}bd^3e^2fg - 5\,(fx+e)^{\frac{3}{2}}bcd^2f^2g + (fx+e)^{\frac{3}{2}}ad^3f^2g + 7\sqrt{fx+e}bcd^2ef}{}$$

input
```
integrate((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(4*b*d^2*e*f*g - 3*b*c*d*f^2*g - a*d^2*f^2*g + 8*b*d^2*e^2*h - 24*b*c*
d*e*f*h + 4*a*d^2*e*f*h + 15*b*c^2*f^2*h - 3*a*c*d*f^2*h)*arctan(sqrt(f*x
+ e)*d/sqrt(-d^2*e + c*d*f))/((d^4*e - c*d^3*f)*sqrt(-d^2*e + c*d*f)) + 2*
sqrt(f*x + e)*b*h/d^3 - 1/4*(4*(f*x + e)^(3/2)*b*d^3*e*f*g - 4*sqrt(f*x +
e)*b*d^3*e^2*f*g - 5*(f*x + e)^(3/2)*b*c*d^2*f^2*g + (f*x + e)^(3/2)*a*d^3
*f^2*g + 7*sqrt(f*x + e)*b*c*d^2*e*f^2*g + sqrt(f*x + e)*a*d^3*e*f^2*g - 3
*sqrt(f*x + e)*b*c^2*d*f^3*g - sqrt(f*x + e)*a*c*d^2*f^3*g - 8*(f*x + e)^(
3/2)*b*c*d^2*e*f*h + 4*(f*x + e)^(3/2)*a*d^3*e*f*h + 8*sqrt(f*x + e)*b*c*d
^2*e^2*f*h - 4*sqrt(f*x + e)*a*d^3*e^2*f*h + 9*(f*x + e)^(3/2)*b*c^2*d*f^2
*h - 5*(f*x + e)^(3/2)*a*c*d^2*f^2*h - 15*sqrt(f*x + e)*b*c^2*d*e*f^2*h +
7*sqrt(f*x + e)*a*c*d^2*e*f^2*h + 7*sqrt(f*x + e)*b*c^3*f^3*h - 3*sqrt(f*x
 + e)*a*c^2*d*f^3*h)/((d^4*e - c*d^3*f)*((f*x + e)*d - d*e + c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(a\,d^2\,f^2\,g-15\,b\,c^2\,f^2\,h-8\,b\,d^2\,e^2\,h+3\,a\,c\,d\,f^2\,h+3\,b\,c\,d\,f^2\,g-4\,a\,d^2\,e\,f\,h-4\,b\,d^2\,e\,f}{4\,d^{7/2}\,\left(c\,f-d\,e\right)^{3/2}}$$
$$-\frac{\sqrt{e+f\,x}\,\left(\frac{a\,d^2\,f^2\,g}{4}-\frac{7\,b\,c^2\,f^2\,h}{4}+\frac{3\,a\,c\,d\,f^2\,h}{4}+\frac{3\,b\,c\,d\,f^2\,g}{4}-a\,d^2\,e\,f\,h-b\,d^2\,e\,f\,g+2\,b\,c\,d\,e\,f\,h\right)-\frac{(e+f\,x)^{3/2}\,(a}{}}{d^5\,\left(e+f\,x\right)^2-\left(e+f\,x\right)\,\left(2\,d^5\,e-2\,c\,d^4\,f\right)+d^5\,e^2+c^2}$$
$$+\frac{2\,b\,h\,\sqrt{e+f\,x}}{d^3}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(a + b*x))/(c + d*x)^3,x)
```

output
```
(atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))*(a*d^2*f^2*g - 15*b*c^2
*f^2*h - 8*b*d^2*e^2*h + 3*a*c*d*f^2*h + 3*b*c*d*f^2*g - 4*a*d^2*e*f*h - 4
*b*d^2*e*f*g + 24*b*c*d*e*f*h))/(4*d^(7/2)*(c*f - d*e)^(3/2)) - ((e + f*x)
^(1/2)*((a*d^2*f^2*g)/4 - (7*b*c^2*f^2*h)/4 + (3*a*c*d*f^2*h)/4 + (3*b*c*d
*f^2*g)/4 - a*d^2*e*f*h - b*d^2*e*f*g + 2*b*c*d*e*f*h) - ((e + f*x)^(3/2)*
(a*d^3*f^2*g - 5*a*c*d^2*f^2*h - 5*b*c*d^2*f^2*g + 9*b*c^2*d*f^2*h + 4*a*d
^3*e*f*h + 4*b*d^3*e*f*g - 8*b*c*d^2*e*f*h))/(4*(c*f - d*e)))/(d^5*(e + f*
x)^2 - (e + f*x)*(2*d^5*e - 2*c*d^4*f) + d^5*e^2 + c^2*d^3*f^2 - 2*c*d^4*e
*f) + (2*b*h*(e + f*x)^(1/2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1706, normalized size of antiderivative = 6.91

$$\int \frac{(a+bx)\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x)
```

output

```
(3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a*c**3*d*f**2*h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqr
t(d)*sqrt(c*f - d*e)))*a*c**2*d**2*e*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((s
qrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d**2*f**2*g + 6*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d
**2*f**2*h*x - 8*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a*c*d**3*e*f*h*x + 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*f**2*g*x + 3*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*f**2*
h*x**2 - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a*d**4*e*f*h*x**2 + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**4*f**2*g*x**2 - 15*sqrt(d)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**4*f**2*h + 24*
sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*
b*c**3*d*e*f*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*b*c**3*d*f**2*g - 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*d*f**2*h*x - 8*sqrt(d)*sqrt
(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d**2*
e**2*h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*b*c**2*d**2*e*f*g + 48*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e ...
```

3.54 $\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3} dx = -\frac{(dg-ch)\sqrt{e+fx}}{2d^2(c+dx)^2} - \frac{(dfg+4deh-5cfh)\sqrt{e+fx}}{4d^2(de-cf)(c+dx)} + \frac{f(dfg-4deh+3cfh)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{5/2}(de-cf)^{3/2}}$$

output
```
-1/2*(-c*h+d*g)*(f*x+e)^(1/2)/d^2/(d*x+c)^2-1/4*(-5*c*f*h+4*d*e*h+d*f*g)*(
f*x+e)^(1/2)/d^2/(-c*f+d*e)/(d*x+c)+1/4*f*(3*c*f*h-4*d*e*h+d*f*g)*arctanh(
d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3} dx = \frac{\frac{\sqrt{d}\sqrt{e+fx}(-3c^2fh+d^2(fgx+2e(g+2hx))+cd(2eh-f(g+5hx)))}{(-de+cf)(c+dx)^2} + \frac{f(dfg-4deh+3cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(-de+cf)^{3/2}}}{4d^{5/2}}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
((Sqrt[d]*Sqrt[e + f*x]*(-3*c^2*f*h + d^2*(f*g*x + 2*e*(g + 2*h*x)) + c*d*
(2*e*h - f*(g + 5*h*x))))/((-(d*e) + c*f)*(c + d*x)^2) + (f*(d*f*g - 4*d*e
*h + 3*c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(-(d*e)
+ c*f)^(3/2))/(4*d^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3} dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(3cfh-4deh+dfg)\int \frac{\sqrt{e+fx}}{(c+dx)^2}dx}{4d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{ 51}$$

$$-\frac{(3cfh-4deh+dfg)\left(\frac{f\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{2d}-\frac{\sqrt{e+fx}}{d(c+dx)}\right)}{4d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{(3cfh-4deh+dfg)\left(\frac{\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d}-\frac{\sqrt{e+fx}}{d(c+dx)}\right)}{4d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\left(-\frac{f\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}\sqrt{de-cf}}-\frac{\sqrt{e+fx}}{d(c+dx)}\right)(3cfh-4deh+dfg)}{4d(de-cf)}-\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/2*((d*g - c*h)*(e + f*x)^(3/2))/(d*(d*e - c*f)*(c + d*x)^2) - ((d*f*g -
 4*d*e*h + 3*c*f*h)*(-(Sqrt[e + f*x]/(d*(c + d*x))) - (f*ArcTanh[(Sqrt[d]*
Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(3/2)*Sqrt[d*e - c*f])))/(4*d*(d*e - c
*f))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{f\left(-\frac{\sqrt{fx+e}\,\left(5cdfhx-4d^{2}ehx-d^{2}fgx+3c^{2}fh-2cdeh+cdfg-2d^{2}eg\right)}{f(xd+c)^{2}}+\frac{(3cfh-4deh+dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}\right)}{4(cf-de)d^{2}}$	136
derivativedivides	$2f\left(-\frac{\frac{(5cfh-4deh-dfg)(fx+e)^{\frac{3}{2}}}{8(cf-de)d}+\frac{(3cfh-4deh+dfg)\sqrt{fx+e}}{8d^{2}}}{((fx+e)d+cf-de)^{2}}+\frac{(3cfh-4deh+dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{8(cf-de)d^{2}\sqrt{(cf-de)d}}\right)$	154
default	$2f\left(-\frac{\frac{(5cfh-4deh-dfg)(fx+e)^{\frac{3}{2}}}{8(cf-de)d}+\frac{(3cfh-4deh+dfg)\sqrt{fx+e}}{8d^{2}}}{((fx+e)d+cf-de)^{2}}+\frac{(3cfh-4deh+dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{8(cf-de)d^{2}\sqrt{(cf-de)d}}\right)$	154

input
```
int((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*f/(c*f-d*e)/d^2*(-(f*x+e)^(1/2)/f*(5*c*d*f*h*x-4*d^2*e*h*x-d^2*f*g*x+3
*c^2*f*h-2*c*d*e*h+c*d*f*g-2*d^2*e*g)/(d*x+c)^2+(3*c*f*h-4*d*e*h+d*f*g)/((
c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(125) = 250.

Time = 0.13 (sec) , antiderivative size = 736, normalized size of antiderivative = 5.08

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$
$$= \left[-\frac{(c^2df^2g + (d^3f^2g - (4\,d^3ef - 3\,cd^2f^2)h)x^2 - (4\,c^2def - 3\,c^3f^2)h + 2\,(cd^2f^2g - (4\,cd^2ef - 3\,c^2df^2)h}{8\,(c^2d^5e}\right.$$
$$-\frac{(c^2df^2g + (d^3f^2g - (4\,d^3ef - 3\,cd^2f^2)h)x^2 - (4\,c^2def - 3\,c^3f^2)h + 2\,(cd^2f^2g - (4\,cd^2ef - 3\,c^2df^2)h)}{4\,(c^2d^5e^2 -}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*((c^2*d*f^2*g + (d^3*f^2*g - (4*d^3*e*f - 3*c*d^2*f^2)*h)*x^2 - (4*c
^2*d*e*f - 3*c^3*f^2)*h + 2*(c*d^2*f^2*g - (4*c*d^2*e*f - 3*c^2*d*f^2)*h)*
x)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sq
rt(f*x + e))/(d*x + c)) + 2*((2*d^4*e^2 - 3*c*d^3*e*f + c^2*d^2*f^2)*g + (
2*c*d^3*e^2 - 5*c^2*d^2*e*f + 3*c^3*d*f^2)*h + ((d^4*e*f - c*d^3*f^2)*g +
(4*d^4*e^2 - 9*c*d^3*e*f + 5*c^2*d^2*f^2)*h)*x)*sqrt(f*x + e))/(c^2*d^5*e^
2 - 2*c^3*d^4*e*f + c^4*d^3*f^2 + (d^7*e^2 - 2*c*d^6*e*f + c^2*d^5*f^2)*x^
2 + 2*(c*d^6*e^2 - 2*c^2*d^5*e*f + c^3*d^4*f^2)*x), -1/4*((c^2*d*f^2*g + (
d^3*f^2*g - (4*d^3*e*f - 3*c*d^2*f^2)*h)*x^2 - (4*c^2*d*e*f - 3*c^3*f^2)*h
 + 2*(c*d^2*f^2*g - (4*c*d^2*e*f - 3*c^2*d*f^2)*h)*x)*sqrt(-d^2*e + c*d*f)
*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) + ((2*d^4*e^2 -
3*c*d^3*e*f + c^2*d^2*f^2)*g + (2*c*d^3*e^2 - 5*c^2*d^2*e*f + 3*c^3*d*f^2)
*h + ((d^4*e*f - c*d^3*f^2)*g + (4*d^4*e^2 - 9*c*d^3*e*f + 5*c^2*d^2*f^2)*
h)*x)*sqrt(f*x + e))/(c^2*d^5*e^2 - 2*c^3*d^4*e*f + c^4*d^3*f^2 + (d^7*e^2
 - 2*c*d^6*e*f + c^2*d^5*f^2)*x^2 + 2*(c*d^6*e^2 - 2*c^2*d^5*e*f + c^3*d^4
*f^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3} \, dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = -\frac{(df^2g-4\,defh+3\,cf^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{4\,(d^3e-cd^2f)\sqrt{-d^2e+cdf}} - \frac{(fx+e)^{\frac{3}{2}}d^2f^2g+\sqrt{fx+e}d^2ef^2g-\sqrt{fx+e}cdf^3g+4\,(fx+e)^{\frac{3}{2}}d^2efh-4\,\sqrt{fx+e}d^2e^2fh-5\,(fx+\cdots}{4\,(d^3e-cd^2f)((fx+e)d-de+cf)^2}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(d*f^2*g - 4*d*e*f*h + 3*c*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e
+ c*d*f))/((d^3*e - c*d^2*f)*sqrt(-d^2*e + c*d*f)) - 1/4*((f*x + e)^(3/2)*
d^2*f^2*g + sqrt(f*x + e)*d^2*e*f^2*g - sqrt(f*x + e)*c*d*f^3*g + 4*(f*x +
 e)^(3/2)*d^2*e*f*h - 4*sqrt(f*x + e)*d^2*e^2*f*h - 5*(f*x + e)^(3/2)*c*d*
f^2*h + 7*sqrt(f*x + e)*c*d*e*f^2*h - 3*sqrt(f*x + e)*c^2*f^3*h)/((d^3*e -
 c*d^2*f)*((f*x + e)*d - d*e + c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx = \frac{f\,\mathrm{atan}\left(\frac{\sqrt{d}\,f\,\sqrt{e+f\,x}\,\left(3\,c\,f\,h-4\,d\,e\,h+d\,f\,g\right)}{\sqrt{c\,f-d\,e}\,\left(3\,c\,f^2\,h+d\,f^2\,g-4\,d\,e\,f\,h\right)}\right)\,\left(3\,c\,f\,h-4\,d\,e\,h+d\,f\,g\right)}{4\,d^{5/2}\,{\left(c\,f-d\,e\right)}^{3/2}} - \frac{\frac{\sqrt{e+f\,x}\,\left(3\,c\,f^2\,h+d\,f^2\,g-4\,d\,e\,f\,h\right)}{4\,d^2}-\frac{{\left(e+f\,x\right)}^{3/2}\,\left(d\,f^2\,g-5\,c\,f^2\,h+4\,d\,e\,f\,h\right)}{4\,d\,\left(c\,f-d\,e\right)}}{d^2\,{\left(e+f\,x\right)}^2-\left(e+f\,x\right)\,\left(2\,d^2\,e-2\,c\,d\,f\right)+c^2\,f^2+d^2\,e^2-2\,c\,d\,e\,f}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/(c + d*x)^3,x)
```

output
```
(f*atan((d^(1/2)*f*(e + f*x)^(1/2)*(3*c*f*h - 4*d*e*h + d*f*g))/((c*f - d*
e)^(1/2)*(3*c*f^2*h + d*f^2*g - 4*d*e*f*h)))*(3*c*f*h - 4*d*e*h + d*f*g))/
(4*d^(5/2)*(c*f - d*e)^(3/2)) - (((e + f*x)^(1/2)*(3*c*f^2*h + d*f^2*g - 4
*d*e*f*h))/(4*d^2) - ((e + f*x)^(3/2)*(d*f^2*g - 5*c*f^2*h + 4*d*e*f*h))/(
4*d*(c*f - d*e)))/(d^2*(e + f*x)^2 - (e + f*x)*(2*d^2*e - 2*c*d*f) + c^2*f
^2 + d^2*e^2 - 2*c*d*e*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 687, normalized size of antiderivative = 4.74

$$\int \frac{\sqrt{e+fx}(g+hx)}{(c+dx)^3}\,dx$$

$$= \frac{3\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^3f^2h - 4\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2defh + \sqrt{d}\,\sqrt{cf-de}\,atan\left(\right.}{}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(d*x+c)^3,x)
```

output

```
(3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*c**3*f**2*h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*c**2*d*e*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*g + 6*sqrt(d)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*h*x - 8*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*
d**2*e*f*h*x + 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*c*d**2*f**2*g*x + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
+ f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d**2*f**2*h*x**2 - 4*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**3*e*f*h*x**
2 + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*d**3*f**2*g*x**2 - 3*sqrt(e + f*x)*c**3*d*f**2*h + 5*sqrt(e + f*x)*c**
2*d**2*e*f*h - sqrt(e + f*x)*c**2*d**2*f**2*g - 5*sqrt(e + f*x)*c**2*d**2*
f**2*h*x - 2*sqrt(e + f*x)*c*d**3*e**2*h + 3*sqrt(e + f*x)*c*d**3*e*f*g +
9*sqrt(e + f*x)*c*d**3*e*f*h*x + sqrt(e + f*x)*c*d**3*f**2*g*x - 2*sqrt(e
+ f*x)*d**4*e**2*g - 4*sqrt(e + f*x)*d**4*e**2*h*x - sqrt(e + f*x)*d**4*e*
f*g*x)/(4*d**3*(c**4*f**2 - 2*c**3*d*e*f + 2*c**3*d*f**2*x + c**2*d**2*e**
2 - 4*c**2*d**2*e*f*x + c**2*d**2*f**2*x**2 + 2*c*d**3*e**2*x - 2*c*d**3*e
*f*x**2 + d**4*e**2*x**2))
```

3.55 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 360

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3} \, dx = \frac{(dg-ch)\sqrt{e+fx}}{2d(bc-ad)(c+dx)^2} - \frac{(ad(dfg+4deh-5cfh)-b(4d^2eg-3cdfg-c^2fh))\sqrt{e+fx}}{4d(bc-ad)^2(de-cf)(c+dx)} - \frac{2\sqrt{b}\sqrt{be-af}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^3} - \frac{(a^2d^2f(dfg-4deh+3cfh)-b^2(8d^3e^2g-12cd^2efg+3c^2df^2g+c^3f^2h)+2abd(3c^2f^2h+2d^2e(fg+}{4d^{3/2}(bc-ad)^3(de-cf)^{3/2}}$$

output
```
1/2*(-c*h+d*g)*(f*x+e)^(1/2)/d/(-a*d+b*c)/(d*x+c)^2-1/4*(a*d*(-5*c*f*h+4*d
*e*h+d*f*g)-b*(-c^2*f*h-3*c*d*f*g+4*d^2*e*g))*(f*x+e)^(1/2)/d/(-a*d+b*c)^2
/(-c*f+d*e)/(d*x+c)-2*b^(1/2)*(-a*f+b*e)^(1/2)*(-a*h+b*g)*arctanh(b^(1/2)*
(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^3-1/4*(a^2*d^2*f*(3*c*f*h-4*d*e
*h+d*f*g)-b^2*(c^3*f^2*h+3*c^2*d*f^2*g-12*c*d^2*e*f*g+8*d^3*e^2*g)+2*a*b*d
*(3*c^2*f^2*h+2*d^2*e*(2*e*h+f*g)-3*c*d*f*(2*e*h+f*g)))*arctanh(d^(1/2)*(f
*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-a*d+b*c)^3/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3}\,dx$$
$$= \frac{1}{4}\left(\frac{\sqrt{e+fx}(b(c^3fh+4d^3egx+3cd^2g(2e-fx)-c^2d(5fg+2eh+fhx))+ad(3c^2fh-d^2(2eg+fgx}{d(bc-ad)^2(de-cf)(c+dx)^2}\right.$$
$$-\frac{8\sqrt{b}\sqrt{-be+af}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{(bc-ad)^3}$$
$$-\frac{(-a^2d^2f(dfg-4deh+3cfh)+b^2(8d^3e^2g-12cd^2efg+3c^2df^2g+c^3f^2h)-2abd(3c^2f^2h+2d^2e(fg}{d^{3/2}(-bc+ad)^3(-de+cf)^{3/2}}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)*(c + d*x)^3),x]
```

output
```
((Sqrt[e + f*x]*(b*(c^3*f*h + 4*d^3*e*g*x + 3*c*d^2*g*(2*e - f*x) - c^2*d*
(5*f*g + 2*e*h + f*h*x)) + a*d*(3*c^2*f*h - d^2*(2*e*g + f*g*x + 4*e*h*x)
+ c*d*(f*g - 2*e*h + 5*f*h*x))))/(d*(b*c - a*d)^2*(d*e - c*f)*(c + d*x)^2)
 - (8*Sqrt[b]*Sqrt[-(b*e) + a*f]*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x]
)/Sqrt[-(b*e) + a*f]])/(b*c - a*d)^3 - ((-(a^2*d^2*f*(d*f*g - 4*d*e*h + 3*
c*f*h)) + b^2*(8*d^3*e^2*g - 12*c*d^2*e*f*g + 3*c^2*d*f^2*g + c^3*f^2*h) -
 2*a*b*d*(3*c^2*f^2*h + 2*d^2*e*(f*g + 2*e*h) - 3*c*d*f*(f*g + 2*e*h)))*Ar
cTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(3/2)*(-(b*c) + a*d)^
3*(-(d*e) + c*f)^(3/2)))/4
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}-\frac{\int -\frac{4bdeg-a(dfg+4deh-cfh)+f(3bdg+bch-4adh)x}{2(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2d(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4bdeg+acfh-ad(fg+4eh)+f(3bdg+bch-4adh)x}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{4d(bc-ad)}+\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{\int \frac{-df(dfg-4deh+3cfh)a^2-b\left(4e(fg+2eh)d^2-cf(5fg+8eh)d+c^2f^2h\right)a+8b^2de(de-cf)g-bf\left(ad(dfg+4deh-5cfh)-b\left(-fhc^2-3dfgc+4d^2eg\right)\right)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{(bc-ad)(de-cf)}-\frac{\sqrt{e+f\cdots}}{\cdots}}{4d(bc-ad)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-df(dfg-4deh+3cfh)a^2-b\left(4e(fg+2eh)d^2-cf(5fg+8eh)d+c^2f^2h\right)a+8b^2de(de-cf)g-bf\left(ad(dfg+4deh-5cfh)-b\left(-fhc^2-3dfgc+4d^2eg\right)\right)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}-\frac{\sqrt{e+f\cdots}}{\cdots}}{4d(bc-ad)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(3c^2f^2h-3cdf(2eh+fg)+2d^2e(2eh+fg)\right)-\left(b^2\left(c^3f^2h+3c^2df^2g-12cd^2efg+8d^3e^2g\right)\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}+\frac{8bd(be-af)(bg-\cdots}{\cdots}}{2(bc-ad)(de-cf)}\cdots}{4d(bc-ad)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(3c^2f^2h-3cdf(2eh+fg)+2d^2e(2eh+fg)\right)-\left(b^2\left(c^3f^2h+3c^2df^2g-12cd^2efg+8d^3e^2g\right)\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}+\frac{16bd(be\cdots}{\cdots}}{2(bc-ad)(de-cf)}\cdots}{4d(bc-ad)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(3c^2f^2h-3cdf(2eh+fg)+2d^2e(2eh+fg)\right)-\left(b^2\left(c^3f^2h+3c^2df^2g-12cd^2efg+8d^3e^2g\right)\right)\right)}{\sqrt{d}(bc-ad)\sqrt{de-cf}}-\frac{16\sqrt{b}d\sqrt{}}{}}{2(bc-ad)(de-cf)}}{4d(bc-ad)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

input

```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)*(c + d*x)^3),x]
```

output

```
((d*g - c*h)*Sqrt[e + f*x])/(2*d*(b*c - a*d)*(c + d*x)^2) + (-(((a*d*(d*f*
g + 4*d*e*h - 5*c*f*h) - b*(4*d^2*e*g - 3*c*d*f*g - c^2*f*h))*Sqrt[e + f*x
])/((b*c - a*d)*(d*e - c*f)*(c + d*x))) + ((-16*Sqrt[b]*d*Sqrt[b*e - a*f]*
(d*e - c*f)*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/
(b*c - a*d) - (2*(a^2*d^2*f*(d*f*g - 4*d*e*h + 3*c*f*h) - b^2*(8*d^3*e^2*g
 - 12*c*d^2*e*f*g + 3*c^2*d*f^2*g + c^3*f^2*h) + 2*a*b*d*(3*c^2*f^2*h + 2*
d^2*e*(f*g + 2*e*h) - 3*c*d*f*(f*g + 2*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*
x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f]))/(2*(b*c - a*d
)*(d*e - c*f)))/(4*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{3(-\sqrt{(af-be)b}\,(xd+c)^2((-\frac{1}{3}c^3f^2h-c^2d\,f^2g+4c\,d^2efg-\frac{8}{3}d^3e^2g)b^2+2ad(\frac{2(2e^2h+efg)d^2}{3}+c(-2efh-f^2g)d+c}{}$
derivativedivides	$2f^2(-\frac{(af-be)(ah-bg)b\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f^2(ad-bc)^3\sqrt{(af-be)b}}-\frac{\frac{f(5a^2c\,d^2fh-4a^2d^3eh-a^2d^3fg-6ab\,c^2dfh+4abc\,d^2eh-2abc\,d^2fg+}{8cf-8de}}{}$
default	$2f^2(-\frac{(af-be)(ah-bg)b\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f^2(ad-bc)^3\sqrt{(af-be)b}}-\frac{\frac{f(5a^2c\,d^2fh-4a^2d^3eh-a^2d^3fg-6ab\,c^2dfh+4abc\,d^2eh-2abc\,d^2fg+}{8cf-8de}}{}$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/((c*f-d*e)*d)^(1/2)/((a*f-b*e)*b)^(1/2)*(-((a*f-b*e)*b)^(1/2)*(d*x+c)
^2*((-1/3*c^3*f^2*h-c^2*d*f^2*g+4*c*d^2*e*f*g-8/3*d^3*e^2*g)*b^2+2*a*d*(2/
3*(2*e^2*h+e*f*g)*d^2+c*(-2*e*f*h-f^2*g)*d+c^2*f^2*h)*b+a^2*d^2*(1/3*(-4*e
*h+f*g)*d+c*f*h)*f)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)
*d)^(1/2)*(8/3*b*d*(d*x+c)^2*(c*f-d*e)*(a*f-b*e)*(a*h-b*g)*arctan(b*(f*x+e
)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(a*d-b*c)*(f*x+e)^(1/2)*(
(4/3*d^3*e*g*x+2*c*(-1/2*f*x+e)*g*d^2-2/3*c^2*(1/2*(h*x+5*g)*f+e*h)*d+1/3*
c^3*f*h)*b+a*(1/3*(-f*g*x-2*e*(2*h*x+g))*d^2-2/3*c*(1/2*(-5*h*x-g)*f+e*h)*
d+c^2*f*h)*d)))/(d*x+c)^2/(a*d-b*c)^3/(c*f-d*e)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. $2(330) = 660$.

Time $= 27.20$ (sec) , antiderivative size $= 6495$, normalized size of antiderivative $=$ 18.04

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(330) = 660$.

Time = 0.17 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \frac{2\left(b^3eg-ab^2fg-ab^2eh+a^2bfh\right)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2e+abf}}$$
$$-\frac{\left(8\,b^2d^3e^2g-12\,b^2cd^2efg-4\,abd^3efg+3\,b^2c^2df^2g+6\,abcd^2f^2g-a^2d^3f^2g-8\,abd^3e^2h+12\,abcd^2efh\right.}{4\left(b^3c^3d^2e-3\,ab^2c^2d^3e+3\,a^2bcd^4e-a^3d^5e-b^3c^4df+3\,ab^2c^3d^2f\right.}$$
$$+\frac{4\,(fx+e)^{\frac{3}{2}}bd^3efg-4\,\sqrt{fx+e}bd^3e^2fg-3\,(fx+e)^{\frac{3}{2}}bcd^2f^2g-(fx+e)^{\frac{3}{2}}ad^3f^2g+9\,\sqrt{fx+e}bcd^2ef}{}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
2*(b^3*e*g - a*b^2*f*g - a*b^2*e*h + a^2*b*f*h)*arctan(sqrt(f*x + e)*b/sqr
t(-b^2*e + a*b*f))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sq
rt(-b^2*e + a*b*f)) - 1/4*(8*b^2*d^3*e^2*g - 12*b^2*c*d^2*e*f*g - 4*a*b*d^
3*e*f*g + 3*b^2*c^2*d*f^2*g + 6*a*b*c*d^2*f^2*g - a^2*d^3*f^2*g - 8*a*b*d^
3*e^2*h + 12*a*b*c*d^2*e*f*h + 4*a^2*d^3*e*f*h + b^2*c^3*f^2*h - 6*a*b*c^2
*d*f^2*h - 3*a^2*c*d^2*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))
/((b^3*c^3*d^2*e - 3*a*b^2*c^2*d^3*e + 3*a^2*b*c*d^4*e - a^3*d^5*e - b^3*c
^4*d*f + 3*a*b^2*c^3*d^2*f - 3*a^2*b*c^2*d^3*f + a^3*c*d^4*f)*sqrt(-d^2*e
+ c*d*f)) + 1/4*(4*(f*x + e)^(3/2)*b*d^3*e*f*g - 4*sqrt(f*x + e)*b*d^3*e^2
*f*g - 3*(f*x + e)^(3/2)*b*c*d^2*f^2*g - (f*x + e)^(3/2)*a*d^3*f^2*g + 9*s
qrt(f*x + e)*b*c*d^2*e*f^2*g - sqrt(f*x + e)*a*d^3*e*f^2*g - 5*sqrt(f*x +
e)*b*c^2*d*f^3*g + sqrt(f*x + e)*a*c*d^2*f^3*g - 4*(f*x + e)^(3/2)*a*d^3*e
*f*h + 4*sqrt(f*x + e)*a*d^3*e^2*f*h - (f*x + e)^(3/2)*b*c^2*d*f^2*h + 5*(
f*x + e)^(3/2)*a*c*d^2*f^2*h - sqrt(f*x + e)*b*c^2*d*e*f^2*h - 7*sqrt(f*x
+ e)*a*c*d^2*e*f^2*h + sqrt(f*x + e)*b*c^3*f^3*h + 3*sqrt(f*x + e)*a*c^2*d
*f^3*h)/((b^2*c^2*d^2*e - 2*a*b*c*d^3*e + a^2*d^4*e - b^2*c^3*d*f + 2*a*b*
c^2*d^2*f - a^2*c*d^3*f)*((f*x + e)*d - d*e + c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 278260, normalized size of antiderivative = 772.94

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3} \, dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)*(c + d*x)^3),x)
```

output
```
atan(((((8*a^8*b^2*c*d^10*f^6*g - 40*a*b^9*c^8*d^3*f^6*g + 8*a*b^9*c^9*d^2
*f^6*h - 8*a^8*b^2*d^11*e*f^5*g + 40*b^10*c^8*d^3*e*f^5*g - 8*b^10*c^9*d^2
*e*f^5*h + 248*a^2*b^8*c^7*d^4*f^6*g - 648*a^3*b^7*c^6*d^5*f^6*g + 920*a^4
*b^6*c^5*d^6*f^6*g - 760*a^5*b^5*c^4*d^7*f^6*g + 360*a^6*b^4*c^3*d^8*f^6*g
 - 88*a^7*b^3*c^2*d^9*f^6*g - 24*a^2*b^8*c^8*d^3*f^6*h - 24*a^3*b^7*c^7*d^
4*f^6*h + 200*a^4*b^6*c^6*d^5*f^6*h - 360*a^5*b^5*c^5*d^6*f^6*h + 312*a^6*
b^4*c^4*d^7*f^6*h - 136*a^7*b^3*c^3*d^8*f^6*h + 24*a^8*b^2*c^2*d^9*f^6*h +
 32*a^6*b^4*d^11*e^3*f^3*g - 24*a^7*b^3*d^11*e^2*f^4*g - 32*a^7*b^3*d^11*e
^3*f^3*h + 32*a^8*b^2*d^11*e^2*f^4*h + 32*b^10*c^6*d^5*e^3*f^3*g - 72*b^10
*c^7*d^4*e^2*f^4*g + 8*b^10*c^8*d^3*e^2*f^4*h + 480*a^2*b^8*c^4*d^7*e^3*f^
3*g - 936*a^2*b^8*c^5*d^6*e^2*f^4*g - 640*a^3*b^7*c^3*d^8*e^3*f^3*g + 1080
*a^3*b^7*c^4*d^7*e^2*f^4*g + 480*a^4*b^6*c^2*d^9*e^3*f^3*g - 600*a^4*b^6*c
^3*d^8*e^2*f^4*g + 72*a^5*b^5*c^2*d^9*e^2*f^4*g + 192*a^2*b^8*c^5*d^6*e^3*
f^3*h - 184*a^2*b^8*c^6*d^5*e^2*f^4*h - 480*a^3*b^7*c^4*d^7*e^3*f^3*h + 48
8*a^3*b^7*c^5*d^6*e^2*f^4*h + 640*a^4*b^6*c^3*d^8*e^3*f^3*h - 520*a^4*b^6*
c^4*d^7*e^2*f^4*h - 480*a^5*b^5*c^2*d^9*e^3*f^3*h + 152*a^5*b^5*c^3*d^8*e^
2*f^4*h + 152*a^6*b^4*c^2*d^9*e^2*f^4*h - 176*a*b^9*c^7*d^4*e*f^5*g + 112*
a^7*b^3*c*d^10*e*f^5*g + 16*a*b^9*c^8*d^3*e*f^5*h - 56*a^8*b^2*c*d^10*e*f^
5*h - 192*a*b^9*c^5*d^6*e^3*f^3*g + 408*a*b^9*c^6*d^5*e^2*f^4*g + 208*a^2*
b^8*c^6*d^5*e*f^5*g + 208*a^3*b^7*c^5*d^6*e*f^5*g - 800*a^4*b^6*c^4*d^7...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3843, normalized size of antiderivative = 10.68

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)(c+dx)^3} \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x)
```

output
```
( - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a*c**4*d**2*f**2*h + 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c**3*d**3*e*f*h - 16*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c**3*d**3*f**2*h*x
- 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a*c**2*d**4*e**2*h + 32*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a*c**2*d**4*e*f*h*x - 8*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c**2*d**4*f**2*h*x**
2 - 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a*c*d**5*e**2*h*x + 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c*d**5*e*f*h*x**2 - 8*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*d**6*e**2*h*x**2 +
 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*b*c**4*d**2*f**2*g - 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*b*c**3*d**3*e*f*g + 16*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c**3*d**3*f**2*g*x + 8
*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*b*c**2*d**4*e**2*g - 32*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*b*c**2*d**4*e*f*g*x + 8*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c**2*d**4*f**2*g*x**...
```

3.56 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 473

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = -\frac{(3bdg-bch-2adh)\sqrt{e+fx}}{2b(bc-ad)^2(c+dx)^2} - \frac{(bg-ah)\sqrt{e+fx}}{b(bc-ad)(a+bx)(c+dx)^2} + \frac{(ad(dfg+8deh-9cfh)-b(12d^2eg+3c^2fh-cd(11fg+4eh)))\sqrt{e+fx}}{4(bc-ad)^3(de-cf)(c+dx)} + \frac{\sqrt{b}(3a^2dfh-ab(5dfg+4deh-3cfh)+b^2(6deg-c(fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^4\sqrt{be-af}} + \frac{(a^2d^2f(dfg-4deh+3cfh)-b^2(24d^3e^2g-3c^3f^2h-8cd^2e(5fg+eh)+3c^2df(5fg+4eh))+2abd(9\cdots}{4\sqrt{d}(bc-ad)^4(de-cf)^{3/2}}$$

output
```
-1/2*(-2*a*d*h-b*c*h+3*b*d*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(d*x+c)^2-(-a*h
+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)/(d*x+c)^2+1/4*(a*d*(-9*c*f*h+8*d*
e*h+d*f*g)-b*(12*d^2*e*g+3*c^2*f*h-c*d*(4*e*h+11*f*g)))*(f*x+e)^(1/2)/(-a*
d+b*c)^3/(-c*f+d*e)/(d*x+c)+b^(1/2)*(3*a^2*d*f*h-a*b*(-3*c*f*h+4*d*e*h+5*d
*f*g)+b^2*(6*d*e*g-c*(2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e
)^(1/2))/(-a*d+b*c)^4/(-a*f+b*e)^(1/2)+1/4*(a^2*d^2*f*(3*c*f*h-4*d*e*h+d*f
*g)-b^2*(24*d^3*e^2*g-3*c^3*f^2*h-8*c*d^2*e*(e*h+5*f*g)+3*c^2*d*f*(4*e*h+5
*f*g))+2*a*b*d*(9*c^2*f^2*h+4*d^2*e*(2*e*h+f*g)-c*d*f*(16*e*h+5*f*g)))*arc
tanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b*c)^4/(-c*f+d*
e)^(3/2)
```

Mathematica [A] (verified)

Time = 8.71 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \frac{-\frac{(bc-ad)\sqrt{e+fx}\left(ab\left(-9c^3fh+d^3x(-6eg+fgx+8ehx)+c^2d(9fg+10eh-14fhx)+cd^2\left(-10eg+6fgx+14ehx-9fhx^2\right)\right)+b^2\left(-12d^3egx^2+c^3f(\cdots}{(-de+cf)(a+b\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^2*(c + d*x)^3),x]
```

output
```
(-(((b*c - a*d)*Sqrt[e + f*x]*(a*b*(-9*c^3*f*h + d^3*x*(-6*e*g + f*g*x + 8
*e*h*x) + c^2*d*(9*f*g + 10*e*h - 14*f*h*x) + c*d^2*(-10*e*g + 6*f*g*x + 1
4*e*h*x - 9*f*h*x^2)) + b^2*(-12*d^3*e*g*x^2 + c^3*f*(4*g - 5*h*x) + c*d^2
*x*(-18*e*g + 11*f*g*x + 4*e*h*x) + c^2*d*(-4*e*g + 17*f*g*x + 6*e*h*x - 3
*f*h*x^2)) + a^2*d*(-3*c^2*f*h + d^2*(f*g*x + 2*e*(g + 2*h*x)) + c*d*(2*e*
h - f*(g + 5*h*x)))))/((-(d*e) + c*f)*(a + b*x)*(c + d*x)^2)) + (4*Sqrt[b]
*(-3*a^2*d*f*h + b^2*(-6*d*e*g + c*f*g + 2*c*e*h) + a*b*(5*d*f*g + 4*d*e*h
 - 3*c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/Sqrt[-(b*
e) + a*f] + ((a^2*d^2*f*(d*f*g - 4*d*e*h + 3*c*f*h) + b^2*(-24*d^3*e^2*g +
 3*c^3*f^2*h + 8*c*d^2*e*(5*f*g + e*h) - 3*c^2*d*f*(5*f*g + 4*e*h)) + 2*a*
b*d*(9*c^2*f^2*h + 4*d^2*e*(f*g + 2*e*h) - c*d*f*(5*f*g + 16*e*h)))*ArcTan
[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sqrt[d]*(-(d*e) + c*f)^(3/2
)))/(4*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {166, 27, 168, 25, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{a(4de-cf)h-b(6deg-cfg-2ceh)-f(5bdg-2bch-3adh)x}{2(a+bx)(c+dx)^3\sqrt{e+fx}}dx}{b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(4de-cf)h-b(6deg-cfg-2ceh)-f(5bdg-2bch-3adh)x}{(a+bx)(c+dx)^3\sqrt{e+fx}}dx}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\int -\frac{b(de-cf)(12bdeg-2bc(fg+2eh)-a(dfg+8deh-3cfh)+3f(3bdg-bch-2adh)x)}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}-\frac{\sqrt{e+fx}(-2adh-bch+3bdg)}{(c+dx)^2(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{b(de-cf)(12bdeg-2bc(fg+2eh)-a(dfg+8deh-3cfh)+3f(3bdg-bch-2adh)x)}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}-\frac{\sqrt{e+fx}(-2adh-bch+3bdg)}{(c+dx)^2(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{b\int \frac{12bdeg-2bc(fg+2eh)-a(dfg+8deh-3cfh)+3f(3bdg-bch-2adh)x}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)}-\frac{\sqrt{e+fx}(-2adh-bch+3bdg)}{(c+dx)^2(bc-ad)}}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{b\left(\frac{\int -\frac{df(dfg-4deh+3cfh)a^2+b\left(8e(fg+2eh)d^2-3cf(3fg+8eh)d+9c^2f^2h\right)a-4b^2(de-cf)(6deg-c(fg+2eh))+bf\left(ad(dfg+8deh-9cfh)-b\left(3fhc^2-d(11fg+4\right.\right.}{2(a+bx)(c+dx)\sqrt{e+fx}}}{(bc-ad)(de-cf)}\right.}{2(bc-ad)}}{2b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(-\frac{\int \frac{df(dfg-4deh+3cfh)a^2+b\left(8e(fg+2eh)d^2-3cf(3fg+8eh)d+9c^2f^2h\right)a-4b^2(de-cf)(6deg-c(fg+2eh))+bf\left(ad(dfg+8deh-9cfh)-b\left(3fhc^2-d(11fg+4\right.\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{2(bc-ad)(de-cf)}\right.}{2(bc-ad)}}{2b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$-\frac{b\left(-\frac{-\frac{\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(9c^2f^2h-cdf(16eh+5fg)+4d^2e(2eh+fg)\right)-\left(b^2\left(-3c^3f^2h+3c^2df(4eh+5fg)-8cd^2e(eh+5fg)+24d^3e^2g\right)\right)\right)\int \frac{1}{(c+dx)}}{bc-ad}}{2(bc-ad)(de-cf)}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$-\frac{b\left(-\frac{-\frac{2\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(9c^2f^2h-cdf(16eh+5fg)+4d^2e(2eh+fg)\right)-\left(b^2\left(-3c^3f^2h+3c^2df(4eh+5fg)-8cd^2e(eh+5fg)+24d^3e^2g\right)\right)\right)\int \frac{}{c+\frac{d(e}{}}}{f(bc-ad)}}{2(bc-ad)(de-cf)}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$-\frac{b\left(-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(9c^2f^2h-cdf(16eh+5fg)+4d^2e(2eh+fg)\right)-\left(b^2\left(-3c^3f^2h+3c^2df(4eh+5fg)-8cd^2e(eh+5fg\right.\right.\right.}{\sqrt{d}(bc-ad)\sqrt{de-cf}}}{2(bc-ad)(de-cf)}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^2*(c + d*x)^3),x]
```

output
```
-(((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)*(c + d*x)^2)) + (-(
((3*b*d*g - b*c*h - 2*a*d*h)*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)^2)) - (
b*(-(((a*d*(d*f*g + 8*d*e*h - 9*c*f*h) - b*(12*d^2*e*g + 3*c^2*f*h - c*d*(
11*f*g + 4*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x))) - ((
8*Sqrt[b]*(d*e - c*f)*(3*a^2*d*f*h + b^2*(6*d*e*g - c*f*g - 2*c*e*h) - a*b
*(5*d*f*g + 4*d*e*h - 3*c*f*h))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e -
 a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (2*(a^2*d^2*f*(d*f*g - 4*d*e*h + 3
*c*f*h) - b^2*(24*d^3*e^2*g - 3*c^3*f^2*h - 8*c*d^2*e*(5*f*g + e*h) + 3*c^
2*d*f*(5*f*g + 4*e*h)) + 2*a*b*d*(9*c^2*f^2*h + 4*d^2*e*(f*g + 2*e*h) - c*
d*f*(5*f*g + 16*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(
Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f]))/(2*(b*c - a*d)*(d*e - c*f))))/(2*(b*
c - a*d)))/(2*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$-\frac{3\left(-\sqrt{(af-be)b}\left(\left(\left(-8b^2e^2g+\frac{16a\left(eh+\frac{fg}{2}\right)eb}{3}-\frac{4a^2f\left(eh-\frac{fg}{4}\right)}{3}\right)d^3+c\left(\frac{8e(eh+5fg)b^2}{3}-\frac{32a\left(eh+\frac{5fg}{16}\right)fb}{3}+a^2f^2h\right)d^2\right.\right.\right.}{}$
derivativedivides	$2f^3\left(-\frac{\frac{df\left(5a^2cd^2fh-4a^2d^3eh-a^2d^3fg-2abc^2dfh-6abcd^2fg+8abd^3eg-3b^2c^3fh+4b^2c^2deh+7b^2c^2dfg-8b^2cd^2eg\right)(fx+e}{8cf-8de}}{((fx+e}\right.$
default	$2f^3\left(-\frac{\frac{df\left(5a^2cd^2fh-4a^2d^3eh-a^2d^3fg-2abc^2dfh-6abcd^2fg+8abd^3eg-3b^2c^3fh+4b^2c^2deh+7b^2c^2dfg-8b^2cd^2eg\right)(fx+e}{8cf-8de}}{((fx+e}\right.$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-((a*f-b*e)*b)^(1/2)*((-8*b^2*e^2*g+16/3*a*(e*h+1/2*f*g)*e*b-4/3*a^2
*f*(e*h-1/4*f*g))*d^3+c*(8/3*e*(e*h+5*f*g)*b^2-32/3*a*(e*h+5/16*f*g)*f*b+a
^2*f^2*h)*d^2+6*c^2*((-2/3*e*h-5/6*f*g)*b+a*f*h)*b*f*d+b^2*c^3*f^2*h)*(d*x
+c)^2*(b*x+a)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1
/2)*(4*((2*b^2*e*g-4/3*a*(e*h+5/4*f*g)*b+a^2*f*h)*d+c*((-2/3*e*h-1/3*f*g)*
b+a*f*h)*b)*(d*x+c)^2*(b*x+a)*b*(c*f-d*e)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e
)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(a*d-b*c)*((4*b^2*e*g*x^2+2*a*x*((-4/3*e*h
-1/6*f*g)*x+g*e)*b-2/3*a^2*((2*e*h+1/2*f*g)*x+g*e))*d^3-2/3*c*(((2*e*h+11/
2*f*g)*x^2-9*e*g*x)*b^2-5*a*(9/10*h*f*x^2+(-7/5*e*h-3/5*f*g)*x+g*e)*b+a^2*
(-5/2*f*h*x-1/2*f*g+e*h))*d^2+c^2*((h*f*x^2+(-2*e*h-17/3*f*g)*x+4/3*g*e)*b
^2-10/3*a*(-7/5*f*h*x+9/10*f*g+e*h)*b+a^2*f*h)*d+3*c^3*b*f*((5/9*h*x-4/9*g
)*b+a*h))*(f*x+e)^(1/2)))/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)/(c*f-d*e
)/(a*d-b*c)^4/(d*x+c)^2/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3731 vs. $2(441) = 882$.

Time = 91.62 (sec) , antiderivative size = 15006, normalized size of antiderivative = 31.73

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(441) = 882$.

Time = 0.18 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(6*b^3*d*e*g - b^3*c*f*g - 5*a*b^2*d*f*g - 2*b^3*c*e*h - 4*a*b^2*d*e*h +
3*a*b^2*c*f*h + 3*a^2*b*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f)
)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*sqrt(-b^2*e + a*b*f)) + 1/4*(24*b^2*d^3*e^2*g - 40*b^2*c*d^2*e*f*g - 8*a*
b*d^3*e*f*g + 15*b^2*c^2*d*f^2*g + 10*a*b*c*d^2*f^2*g - a^2*d^3*f^2*g - 8*
b^2*c*d^2*e^2*h - 16*a*b*d^3*e^2*h + 12*b^2*c^2*d*e*f*h + 32*a*b*c*d^2*e*f
*h + 4*a^2*d^3*e*f*h - 3*b^2*c^3*f^2*h - 18*a*b*c^2*d*f^2*h - 3*a^2*c*d^2*
f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^4*d*e - 4*a*b^
3*c^3*d^2*e + 6*a^2*b^2*c^2*d^3*e - 4*a^3*b*c*d^4*e + a^4*d^5*e - b^4*c^5*
f + 4*a*b^3*c^4*d*f - 6*a^2*b^2*c^3*d^2*f + 4*a^3*b*c^2*d^3*f - a^4*c*d^4*
f)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)*b^2*f*g - sqrt(f*x + e)*a*b*f*h)
/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((f*x + e)*b - b*e +
 a*f)) - 1/4*(8*(f*x + e)^(3/2)*b*d^3*e*f*g - 8*sqrt(f*x + e)*b*d^3*e^2*f*
g - 7*(f*x + e)^(3/2)*b*c*d^2*f^2*g - (f*x + e)^(3/2)*a*d^3*f^2*g + 17*sqr
t(f*x + e)*b*c*d^2*e*f^2*g - sqrt(f*x + e)*a*d^3*e*f^2*g - 9*sqrt(f*x + e)
*b*c^2*d*f^3*g + sqrt(f*x + e)*a*c*d^2*f^3*g - 4*(f*x + e)^(3/2)*b*c*d^2*e
*f*h - 4*(f*x + e)^(3/2)*a*d^3*e*f*h + 4*sqrt(f*x + e)*b*c*d^2*e^2*f*h + 4
*sqrt(f*x + e)*a*d^3*e^2*f*h + 3*(f*x + e)^(3/2)*b*c^2*d*f^2*h + 5*(f*x +
e)^(3/2)*a*c*d^2*f^2*h - 9*sqrt(f*x + e)*b*c^2*d*e*f^2*h - 7*sqrt(f*x + e)
*a*c*d^2*e*f^2*h + 5*sqrt(f*x + e)*b*c^3*f^3*h + 3*sqrt(f*x + e)*a*c^2*...
```

Mupad [B] (verification not implemented)

Time = 23.06 (sec) , antiderivative size = 451051, normalized size of antiderivative = 953.60

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^3),x)
```

output
```
((f*(e + f*x)^(5/2)*(4*b^2*c*d^2*e*h - 12*b^2*d^3*e*g + 11*b^2*c*d^2*f*g -
 3*b^2*c^2*d*f*h + 8*a*b*d^3*e*h + a*b*d^3*f*g - 9*a*b*c*d^2*f*h))/(4*(c*f
 - d*e)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - ((e + f*x)^
(1/2)*(a^2*d^2*f^3*g - 4*b^2*c^2*f^3*g + 9*a*b*c^2*f^3*h + 3*a^2*c*d*f^3*h
 - 4*a^2*d^2*e*f^2*h - 5*b^2*c^2*e*f^2*h - 12*b^2*d^2*e^2*f*g - 9*a*b*c*d*
f^3*g + 7*a*b*d^2*e*f^2*g + 8*a*b*d^2*e^2*f*h + 17*b^2*c*d*e*f^2*g + 4*b^2
*c*d*e^2*f*h - 15*a*b*c*d*e*f^2*h))/(4*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d)) + ((e + f*x)^(3/2)*(a^2*d^3*f^3*g - 5*b^2*c^3*f^3*h - 5*a^2*c*d^
2*f^3*h + 17*b^2*c^2*d*f^3*g + 4*a^2*d^3*e*f^2*h + 24*b^2*d^3*e^2*f*g + 6*
a*b*c*d^2*f^3*g - 14*a*b*c^2*d*f^3*h - 8*a*b*d^3*e*f^2*g - 16*a*b*d^3*e^2*
f*h - 40*b^2*c*d^2*e*f^2*g - 8*b^2*c*d^2*e^2*f*h + 12*b^2*c^2*d*e*f^2*h +
32*a*b*c*d^2*e*f^2*h))/(4*(a*d - b*c)*(c*f - d*e)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)))/((e + f*x)*(b*c^2*f^2 + 3*b*d^2*e^2 + 2*a*c*d*f^2 - 2*a*d^2*e*f
- 4*b*c*d*e*f) + (e + f*x)^2*(a*d^2*f - 3*b*d^2*e + 2*b*c*d*f) + a*c^2*f^3
 - b*d^2*e^3 + b*d^2*(e + f*x)^3 + a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f
^2 + 2*b*c*d*e^2*f) - atan(((((368*a*b^11*c^10*d^3*f^6*g - 64*b^12*c^11*d^
2*f^6*g + 16*a^10*b^2*c*d^12*f^6*g + 144*a*b^11*c^11*d^2*f^6*h - 16*a^10*b
^2*d^13*e*f^5*g + 336*b^12*c^10*d^3*e*f^5*g - 80*b^12*c^11*d^2*e*f^5*h - 6
24*a^2*b^10*c^9*d^4*f^6*g - 576*a^3*b^9*c^8*d^5*f^6*g + 4032*a^4*b^8*c^7*d
^6*f^6*g - 7392*a^5*b^7*c^6*d^7*f^6*g + 7392*a^6*b^6*c^5*d^8*f^6*g - 44...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13393, normalized size of antiderivative = 28.32

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x)
```

output
```
( - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**3*c**4*d**2*f**3*h + 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**3*d**3*e*f**2*h - 24*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**3*d
**3*f**3*h*x - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**3*c**2*d**4*e**2*f*h + 48*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**2*d**4*e*f**2*h*x -
 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**3*c**2*d**4*f**3*h*x**2 - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c*d**5*e**2*f*h*x + 24*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c*d**
5*e*f**2*h*x**2 - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**3*d**6*e**2*f*h*x**2 - 12*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**5*d*f**3*h + 4
0*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**2*b*c**4*d**2*e*f**2*h + 20*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**4*d**2*f**3*g - 36*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**4*
d**2*f**3*h*x - 44*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**2*b*c**3*d**3*e**2*f*h - 40*sqrt(b)*sqrt(a*f - b*...
```

3.57 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 762

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx$$
$$= \frac{d(5a^2dfh-ab(11dfg+6deh-7cfh)+b^2(12deg-c(fg+6eh)))\sqrt{e+fx}}{4b(bc-ad)^3(be-af)(c+dx)^2}$$
$$-\frac{(bg-ah)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)^2}$$
$$+\frac{(3a^2dfh-ab(7dfg+4deh-5cfh)+b^2(8deg-c(fg+4eh)))\sqrt{e+fx}}{4b(bc-ad)^2(be-af)(a+bx)(c+dx)^2}$$
$$+\frac{d(a^2df(dfg+11deh-12cfh)+b^2(24d^2e^2g-12cde(2fg+eh)+c^2f(fg+11eh))-2ab(6c^2f^2h+6}{4(bc-ad)^4(be-af)(de-cf)(c+dx)}$$
$$+\frac{\sqrt{b}(15a^3d^2f^2h-5a^2bdf(7dfg+8deh-6cfh)-b^3(48d^2e^2g-c^2f(fg-4eh)-12cde(fg+2eh))+}{4(bc-ad)^5(be-af)^{3/2}}$$
$$-\frac{\sqrt{d}(a^2d^2f(dfg-4deh+3cfh)-b^2(48d^3e^2g-15c^3f^2h-12cd^2e(7fg+2eh)+5c^2df(7fg+8eh))+}{4(bc-ad)^5(de-cf)^{3/2}}$$

output
```
1/4*d*(5*a^2*d*f*h-a*b*(-7*c*f*h+6*d*e*h+11*d*f*g)+b^2*(12*d*e*g-c*(6*e*h+
f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+b*e)/(d*x+c)^2-1/2*(-a*h+b*g)*(f
*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)^2/(d*x+c)^2+1/4*(3*a^2*d*f*h-a*b*(-5*c*f*
h+4*d*e*h+7*d*f*g)+b^2*(8*d*e*g-c*(4*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)
^2/(-a*f+b*e)/(b*x+a)/(d*x+c)^2+1/4*d*(a^2*d*f*(-12*c*f*h+11*d*e*h+d*f*g)+
b^2*(24*d^2*e^2*g-12*c*d*e*(e*h+2*f*g)+c^2*f*(11*e*h+f*g))-2*a*b*(6*c^2*f^
2*h+6*d^2*e*(e*h+2*f*g)-c*d*f*(13*e*h+11*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^4
/(-a*f+b*e)/(-c*f+d*e)/(d*x+c)+1/4*b^(1/2)*(15*a^3*d^2*f^2*h-5*a^2*b*d*f*(
-6*c*f*h+8*d*e*h+7*d*f*g)-b^3*(48*d^2*e^2*g-c^2*f*(-4*e*h+f*g)-12*c*d*e*(2
*e*h+f*g))+a*b^2*(3*c^2*f^2*h+12*d^2*e*(2*e*h+7*f*g)-2*c*d*f*(26*e*h+7*f*g
)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^5/(-a*f+b*e
)^(3/2)-1/4*d^(1/2)*(a^2*d^2*f*(3*c*f*h-4*d*e*h+d*f*g)-b^2*(48*d^3*e^2*g-1
5*c^3*f^2*h-12*c*d^2*e*(2*e*h+7*f*g)+5*c^2*d*f*(8*e*h+7*f*g))+2*a*b*d*(15*
c^2*f^2*h+6*d^2*e*(2*e*h+f*g)-c*d*f*(26*e*h+7*f*g)))*arctanh(d^(1/2)*(f*x+
e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^5/(-c*f+d*e)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6193 vs. $2(762) = 1524$.

Time = 16.29 (sec) , antiderivative size = 6193, normalized size of antiderivative = 8.13

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^3*(c + d*x)^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {166, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{a(4de-cf)h-b(8deg-cfg-4ceh)-f(7bdg-4bch-3adh)x}{2(a+bx)^2(c+dx)^3\sqrt{e+fx}}dx}{2b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(4de-cf)h-b(8deg-cfg-4ceh)-f(7bdg-4bch-3adh)x}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}dx}{4b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\sqrt{e+fx}(3a^2dfh-ab(-5cfh+4deh+7dfg)+b^2(-4ceh-cfg+8deg))}{(a+bx)(c+dx)^2(bc-ad)(be-af)}-\int -\frac{5df(4de-cf)ha^2-b\left(4e(11fg+6eh)d^2-cf(9fg+32eh)d+3c^2f^2h\right)a+b^2\left(-f(f\right.}{}}{4b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{5df(4de-cf)ha^2-b\left(4e(11fg+6eh)d^2-cf(9fg+32eh)d+3c^2f^2h\right)a+b^2\left(-f(fg-4eh)c^2-12de(fg+2eh)c+48d^2e^2g\right)+5df\left(3dfha^2-b(7dfg+4deh-5cfh)a+b^2(8\right.}{(a+bx)(c+dx)^3\sqrt{e+fx}}}{2(bc-ad)(be-af)}}{4b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\int \frac{2b(de-cf)\left(df(2dfg+22deh-9cfh)a^2-b\left(24e(2fg+eh)d^2-cf(11fg+34eh)d+3c^2f^2h\right)a+b^2\left(-f(fg-4eh)c^2-12de(fg+2eh)c+48d^2e^2g\right)+3df\left(5dfha^2-b(11dfg\right.\right.}{(a+bx)(c+dx)^2\sqrt{e+fx}}}{2(bc-ad)(de-cf)}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\int \frac{df(2dfg+22deh-9cfh)a^2-b\left(24e(2fg+eh)d^2-cf(11fg+34eh)d+3c^2f^2h\right)a+b^2\left(-f(fg-4eh)c^2-12de(fg+2eh)c+48d^2e^2g\right)+3df\left(5dfha^2-b(11dfg+6deh-7c\right.}{(a+bx)(c+dx)^2\sqrt{e+fx}}}{bc-ad}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$\downarrow$ 168

$$b\left(\int \frac{d^2f^2(dfg-4deh+3cfh)a^3+bdf\left(e(11fg+28eh)d^2-cf(13fg+44eh)d+18c^2f^2h\right)a^2+b^2\left(-12e^2(5fg+2eh)d^3+2cef(37fg+32eh)d^2-c^2f^2(13fg+44eh)d+3c^3\right.}{}\right.$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$\downarrow$ 174

$$b\left(\frac{d(be-af)\left(a^2d^2f(3cfh-4deh+dfg)+2abd\left(15c^2f^2h-cdf(26eh+7fg)+6d^2e(2eh+fg)\right)-\left(b^2\left(-15c^3f^2h+5c^2df(8eh+7fg)-12cd^2e(2eh+7fg)+48d^3e^2g\right)\right)\right)\int}{bc-ad}\right.$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$\downarrow$ 73

$$\frac{\sqrt{e+fx}\left(3dfha^2-b(7dfg+4deh-5cfh)a+b^2(8deg-cfg-4ceh)\right)}{(bc-ad)(be-af)(a+bx)(c+dx)^2}+\frac{\frac{2d\sqrt{e+fx}\left(5dfha^2-b(11dfg+6deh-7cfh)a+b^2(12deg-cfg-6ceh)\right)}{(bc-ad)(c+dx)^2}+\frac{b\left(\frac{2d\sqrt{e+fx}\left(\right.}{}\right.}{}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)^2}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(7dfg+4deh-5cfh)a+b^2(8deg-cfg-4ceh)\right)}{(bc-ad)(be-af)(a+bx)(c+dx)^2}+\frac{\frac{2d\sqrt{e+fx}\left(5dfha^2-b(11dfg+6deh-7cfh)a+b^2(12deg-cfg-6ceh)\right)}{(bc-ad)(c+dx)^2}+\frac{b\left(\frac{2d\sqrt{e+fx}\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)^2}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^3*(c + d*x)^3),x]
```

output
```
-1/2*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2) +
 (((3*a^2*d*f*h + b^2*(8*d*e*g - c*f*g - 4*c*e*h) - a*b*(7*d*f*g + 4*d*e*h
 - 5*c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2
) + ((2*d*(5*a^2*d*f*h + b^2*(12*d*e*g - c*f*g - 6*c*e*h) - a*b*(11*d*f*g
+ 6*d*e*h - 7*c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)^2) + (b*((2*d*
(a^2*d*f*(d*f*g + 11*d*e*h - 12*c*f*h) + b^2*(24*d^2*e^2*g - 12*c*d*e*(2*f
*g + e*h) + c^2*f*(f*g + 11*e*h)) - 2*a*b*(6*c^2*f^2*h + 6*d^2*e*(2*f*g +
e*h) - c*d*f*(11*f*g + 13*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(
c + d*x)) + ((2*Sqrt[b]*(d*e - c*f)*(15*a^3*d^2*f^2*h - 5*a^2*b*d*f*(7*d*f
*g + 8*d*e*h - 6*c*f*h) - b^3*(48*d^2*e^2*g - c^2*f*(f*g - 4*e*h) - 12*c*d
*e*(f*g + 2*e*h)) + a*b^2*(3*c^2*f^2*h + 12*d^2*e*(7*f*g + 2*e*h) - 2*c*d*
f*(7*f*g + 26*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b
*c - a*d)*Sqrt[b*e - a*f]) - (2*Sqrt[d]*(b*e - a*f)*(a^2*d^2*f*(d*f*g - 4*
d*e*h + 3*c*f*h) - b^2*(48*d^3*e^2*g - 15*c^3*f^2*h - 12*c*d^2*e*(7*f*g +
2*e*h) + 5*c^2*d*f*(7*f*g + 8*e*h)) + 2*a*b*d*(15*c^2*f^2*h + 6*d^2*e*(f*g
 + 2*e*h) - c*d*f*(7*f*g + 26*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[
d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/((b*c - a*d)*(d*e - c*f))))/(b
*c - a*d))/(2*(b*c - a*d)*(b*e - a*f)))/(4*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 13.44 (sec) , antiderivative size = 1019, normalized size of antiderivative = 1.34

method	result	size
derivativedivides	Expression too large to display	1019
default	Expression too large to display	1019
pseudoelliptic	Expression too large to display	1099

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*f^4*(-d/f^4/(a*d-b*c)^5*((1/8*d*f*(5*a^2*c*d^2*f*h-4*a^2*d^3*e*h-a^2*d^3
*f*g+2*a*b*c^2*d*f*h-4*a*b*c*d^2*e*h-10*a*b*c*d^2*f*g+12*a*b*d^3*e*g-7*b^2
*c^3*f*h+8*b^2*c^2*d*e*h+11*b^2*c^2*d*f*g-12*b^2*c*d^2*e*g)/(c*f-d*e)*(f*x
+e)^(3/2)+1/8*f*(3*a^2*c*d^2*f*h-4*a^2*d^3*e*h+a^2*d^3*f*g+6*a*b*c^2*d*f*h
-4*a*b*c*d^2*e*h-14*a*b*c*d^2*f*g+12*a*b*d^3*e*g-9*b^2*c^3*f*h+8*b^2*c^2*d
*e*h+13*b^2*c^2*d*f*g-12*b^2*c*d^2*e*g)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)
^2-1/8*(3*a^2*c*d^2*f^2*h-4*a^2*d^3*e*f*h+a^2*d^3*f^2*g+30*a*b*c^2*d*f^2*h
-52*a*b*c*d^2*e*f*h-14*a*b*c*d^2*f^2*g+24*a*b*d^3*e^2*h+12*a*b*d^3*e*f*g+1
5*b^2*c^3*f^2*h-40*b^2*c^2*d*e*f*h-35*b^2*c^2*d*f^2*g+24*b^2*c*d^2*e^2*h+8
4*b^2*c*d^2*e*f*g-48*b^2*d^3*e^2*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d
*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-b/f^4/(a*d-b*c)^5*((1/8*b*f*(7*a^3*d^
2*f*h-2*a^2*b*c*d*f*h-8*a^2*b*d^2*e*h-11*a^2*b*d^2*f*g-5*a*b^2*c^2*f*h+4*a
*b^2*c*d*e*h+10*a*b^2*c*d*f*g+12*a*b^2*d^2*e*g+4*b^3*c^2*e*h+b^3*c^2*f*g-1
2*b^3*c*d*e*g)/(a*f-b*e)*(f*x+e)^(3/2)+1/8*f*(9*a^3*d^2*f*h-6*a^2*b*c*d*f*
h-8*a^2*b*d^2*e*h-13*a^2*b*d^2*f*g-3*a*b^2*c^2*f*h+4*a*b^2*c*d*e*h+14*a*b^
2*c*d*f*g+12*a*b^2*d^2*e*g+4*b^3*c^2*e*h-b^3*c^2*f*g-12*b^3*c*d*e*g)*(f*x+
e)^(1/2))/((f*x+e)*b+a*f-b*e)^2+1/8*(15*a^3*d^2*f^2*h+30*a^2*b*c*d*f^2*h-4
0*a^2*b*d^2*e*f*h-35*a^2*b*d^2*f^2*g+3*a*b^2*c^2*f^2*h-52*a*b^2*c*d*e*f*h-
14*a*b^2*c*d*f^2*g+24*a*b^2*d^2*e^2*h+84*a*b^2*d^2*e*f*g-4*b^3*c^2*e*f*h+b
^3*c^2*f^2*g+24*b^3*c*d*e^2*h+12*b^3*c*d*e*f*g-48*b^3*d^2*e^2*g)/(a*f-b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3442 vs. $2(722) = 1444$.

Time = 0.40 (sec) , antiderivative size = 3442, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(48*b^4*d^2*e^2*g - 12*b^4*c*d*e*f*g - 84*a*b^3*d^2*e*f*g - b^4*c^2*f^
2*g + 14*a*b^3*c*d*f^2*g + 35*a^2*b^2*d^2*f^2*g - 24*b^4*c*d*e^2*h - 24*a*
b^3*d^2*e^2*h + 4*b^4*c^2*e*f*h + 52*a*b^3*c*d*e*f*h + 40*a^2*b^2*d^2*e*f*
h - 3*a*b^3*c^2*f^2*h - 30*a^2*b^2*c*d*f^2*h - 15*a^3*b*d^2*f^2*h)*arctan(
sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*c^5*e - 5*a*b^5*c^4*d*e + 10*a
^2*b^4*c^3*d^2*e - 10*a^3*b^3*c^2*d^3*e + 5*a^4*b^2*c*d^4*e - a^5*b*d^5*e
- a*b^5*c^5*f + 5*a^2*b^4*c^4*d*f - 10*a^3*b^3*c^3*d^2*f + 10*a^4*b^2*c^2*
d^3*f - 5*a^5*b*c*d^4*f + a^6*d^5*f)*sqrt(-b^2*e + a*b*f)) - 1/4*(48*b^2*d
^4*e^2*g - 84*b^2*c*d^3*e*f*g - 12*a*b*d^4*e*f*g + 35*b^2*c^2*d^2*f^2*g +
14*a*b*c*d^3*f^2*g - a^2*d^4*f^2*g - 24*b^2*c*d^3*e^2*h - 24*a*b*d^4*e^2*h
 + 40*b^2*c^2*d^2*e*f*h + 52*a*b*c*d^3*e*f*h + 4*a^2*d^4*e*f*h - 15*b^2*c^
3*d*f^2*h - 30*a*b*c^2*d^2*f^2*h - 3*a^2*c*d^3*f^2*h)*arctan(sqrt(f*x + e)
*d/sqrt(-d^2*e + c*d*f))/((b^5*c^5*d*e - 5*a*b^4*c^4*d^2*e + 10*a^2*b^3*c^
3*d^3*e - 10*a^3*b^2*c^2*d^4*e + 5*a^4*b*c*d^5*e - a^5*d^6*e - b^5*c^6*f +
 5*a*b^4*c^5*d*f - 10*a^2*b^3*c^4*d^2*f + 10*a^3*b^2*c^3*d^3*f - 5*a^4*b*c
^2*d^4*f + a^5*c*d^5*f)*sqrt(-d^2*e + c*d*f)) + 1/4*(24*(f*x + e)^(7/2)*b^
4*d^4*e^2*f*g - 72*(f*x + e)^(5/2)*b^4*d^4*e^3*f*g + 72*(f*x + e)^(3/2)*b^
4*d^4*e^4*f*g - 24*sqrt(f*x + e)*b^4*d^4*e^5*f*g - 24*(f*x + e)^(7/2)*b^4*
c*d^3*e*f^2*g - 24*(f*x + e)^(7/2)*a*b^3*d^4*e*f^2*g + 108*(f*x + e)^(5/2)
*b^4*c*d^3*e^2*f^2*g + 108*(f*x + e)^(5/2)*a*b^3*d^4*e^2*f^2*g - 144*(f...
```

Mupad [B] (verification not implemented)

Time = 38.21 (sec) , antiderivative size = 794225, normalized size of antiderivative = 1042.29

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^3),x)
```

output
```
(((e + f*x)^(3/2)*(a^4*d^4*f^5*g + b^4*c^4*f^5*g - 5*a*b^3*c^4*f^5*h - 5*a
^4*c*d^3*f^5*h + 4*a^4*d^4*e*f^4*h + 4*b^4*c^4*e*f^4*h + 72*b^4*d^4*e^4*f*
g - 31*a^2*b^2*c^3*d*f^5*h - 31*a^3*b*c^2*d^2*f^5*h - 144*a*b^3*d^4*e^3*f^
2*g - 38*a^3*b*d^4*e^2*f^3*h - 144*b^4*c*d^3*e^3*f^2*g - 38*b^4*c^3*d*e^2*
f^3*h + 52*a^2*b^2*c^2*d^2*f^5*g + 85*a^2*b^2*d^4*e^2*f^3*g + 69*a^2*b^2*d
^4*e^3*f^2*h + 85*b^4*c^2*d^2*e^2*f^3*g + 69*b^4*c^2*d^2*e^3*f^2*h + 9*a*b
^3*c^3*d*f^5*g + 9*a^3*b*c*d^3*f^5*g - 13*a^3*b*d^4*e*f^4*g - 36*a*b^3*d^4
*e^4*f*h - 13*b^4*c^3*d*e*f^4*g - 36*b^4*c*d^3*e^4*f*h + 73*a*b^3*c^3*d*e*
f^4*h + 73*a^3*b*c*d^3*e*f^4*h + 262*a*b^3*c*d^3*e^2*f^3*g - 131*a*b^3*c^2
*d^2*e*f^4*g - 131*a^2*b^2*c*d^3*e*f^4*g + 150*a*b^3*c*d^3*e^3*f^2*h - 178
*a*b^3*c^2*d^2*e^2*f^3*h - 178*a^2*b^2*c*d^3*e^2*f^3*h + 134*a^2*b^2*c^2*d
^2*e*f^4*h))/(4*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)*(a^4*d^4 + b^4*c^4
 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - ((e + f*x)^(1/2)*
(a^3*d^3*f^4*g + b^3*c^3*f^4*g + 3*a*b^2*c^3*f^4*h + 3*a^3*c*d^2*f^4*h - 4
*a^3*d^3*e*f^3*h - 4*b^3*c^3*e*f^3*h + 24*b^3*d^3*e^3*f*g - 36*a*b^2*d^3*e
^2*f^2*g + 17*a^2*b*d^3*e^2*f^2*h - 36*b^3*c*d^2*e^2*f^2*g + 17*b^3*c^2*d*
e^2*f^2*h - 13*a*b^2*c^2*d*f^4*g - 13*a^2*b*c*d^2*f^4*g + 18*a^2*b*c^2*d*f
^4*h + 10*a^2*b*d^3*e*f^3*g - 12*a*b^2*d^3*e^3*f*h + 10*b^3*c^2*d*e*f^3*g
- 12*b^3*c*d^2*e^3*f*h + 52*a*b^2*c*d^2*e*f^3*g - 32*a*b^2*c^2*d*e*f^3*h -
 32*a^2*b*c*d^2*e*f^3*h + 38*a*b^2*c*d^2*e^2*f^2*h))/(4*(a^4*d^4 + b^4*...
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 26875, normalized size of antiderivative = 35.27

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**5*c**4*d**2*f**4*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**3*d**3*e*f**3*h - 30*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**3*d
**3*f**4*h*x - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**5*c**2*d**4*e**2*f**2*h + 60*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**2*d**4*e*f**3*h*
x - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**5*c**2*d**4*f**4*h*x**2 - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c*d**5*e**2*f**2*h*x + 30*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5
*c*d**5*e*f**3*h*x**2 - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**5*d**6*e**2*f**2*h*x**2 - 30*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**5*d*f
**4*h + 100*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**4*b*c**4*d**2*e*f**3*h + 35*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**4*d**2*f**4*g - 90*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*4*b*c**4*d**2*f**4*h*x - 110*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**3*d**3*e**2*f**2*h - 70*sqrt(b)...
```

3.58 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1204

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Too large to display}$$

output
```
1/24*d*(35*a^3*d^2*f^2*h-a^2*b*d*f*(-82*c*f*h+86*d*e*h+101*d*f*g)-b^3*(120
*d^2*e^2*g-3*c^2*f*(-2*e*h+f*g)-8*c*d*e*(9*e*h+2*f*g))+a*b^2*(3*c^2*f^2*h+
16*d^2*e*(3*e*h+14*f*g)-2*c*d*f*(74*e*h+11*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*
c)^4/(-a*f+b*e)^2/(d*x+c)^2-1/3*(-a*h+b*g)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x
+a)^3/(d*x+c)^2+1/12*(3*a^2*d*f*h-a*b*(-7*c*f*h+4*d*e*h+9*d*f*g)+b^2*(10*d
*e*g-c*(6*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^2/(d*
x+c)^2+1/24*(21*a^3*d^2*f^2*h-7*a^2*b*d*f*(-8*c*f*h+8*d*e*h+9*d*f*g)-b^3*(
80*d^2*e^2*g-3*c^2*f*(-2*e*h+f*g)-2*c*d*e*(24*e*h+7*f*g))+a*b^2*(3*c^2*f^2
*h+2*d^2*e*(16*e*h+73*f*g)-2*c*d*f*(49*e*h+10*f*g)))*(f*x+e)^(1/2)/b/(-a*d
+b*c)^3/(-a*f+b*e)^2/(b*x+a)/(d*x+c)^2+1/8*d*(a^3*d^2*f^2*(-29*c*f*h+27*d*
e*h+2*d*f*g)-b^3*(80*d^3*e^3*g+c^3*f^2*(-2*e*h+f*g)-12*c*d^2*e^2*(4*e*h+7*
f*g)+c^2*d*e*f*(48*e*h+5*f*g))-a*b^2*(c^3*f^3*h-4*d^3*e^2*(8*e*h+39*f*g)+2
*c*d^2*e*f*(66*e*h+79*f*g)-c^2*d*f^2*(95*e*h+8*f*g))-a^2*b*d*f*(50*c^2*f^2
*h+d^2*e*(60*e*h+77*f*g)-c*d*f*(116*e*h+71*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)
^5/(-a*f+b*e)^2/(-c*f+d*e)/(d*x+c)+1/8*b^(1/2)*(35*a^4*d^3*f^3*h-35*a^3*b*
d^2*f^2*(-3*c*f*h+4*d*e*h+3*d*f*g)+b^4*(160*d^3*e^3*g-6*c^2*d*e*f*(-4*e*h+
f*g)-c^3*f^2*(-2*e*h+f*g)-48*c*d^2*e^2*(2*e*h+f*g))-a*b^3*(c^3*f^3*h-3*c^2
*d*f^2*(-16*e*h+3*f*g)+16*d^3*e^2*(4*e*h+27*f*g)-36*c*d^2*e*f*(8*e*h+3*f*g
))+21*a^2*b^2*d*f*(c^2*f^2*h+2*d^2*e*(4*e*h+9*f*g)-c*d*f*(14*e*h+3*f*g)))*
arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^6/(-a*f+b*e)...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13191 vs. 2(1204) = 2408.

Time = 16.50 (sec) , antiderivative size = 13191, normalized size of antiderivative = 10.96

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^4*(c + d*x)^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 1285, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {166, 27, 168, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int \frac{a(4de-cf)h-b(10deg-cfg-6ceh)-3f(3bdg-2bch-adh)x}{2(a+bx)^3(c+dx)^3\sqrt{e+fx}}dx}{3b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a(4de-cf)h-b(10deg-cfg-6ceh)-3f(3bdg-2bch-adh)x}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}dx}{6b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\frac{\sqrt{e+fx}(3a^2dfh-ab(-7cfh+4deh+9dfg)+b^2(-6ceh-cfg+10deg))}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}-\int -\frac{7df(4de-cf)ha^2-b\left(4e(19fg+8eh)d^2-cf(13fg+56eh)d+3c^2f^2h\right)a+b^2\left(-3\right.}{}$$

$$6b(bc-ad)$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\int \frac{7df(4de-cf)ha^2-b\left(4e(19fg+8eh)d^2-cf(13fg+56eh)d+3c^2f^2h\right)a+b^2\left(-3f(fg-2eh)c^2-2de(7fg+24eh)c+80d^2e^2g\right)+7df\left(3dfha^2-b(9dfg+4deh-7cfh)a+b\right.}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}$$

$$4(bc-ad)(be-af)$$

$$6b(bc-ad)$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{\sqrt{e+fx}\left(21a^3d^2f^2h-7a^2bdf(-8cfh+8deh+9dfg)+ab^2\left(3c^2f^2h-2cdf(49eh+10fg)+2d^2e(16eh+73fg)\right)-b^3\left(-3c^2f(fg-2eh)-2cde(24eh+7fg)+80d^2e^2g\right)\right)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}-\frac{\int}{}}{} \cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$\downarrow$ 27

$$\int\frac{35d^2f^2(4de-cf)ha^3-bdf\left(4e(101fg+86eh)d^2-cf(89fg+392eh)d+48c^2f^2h\right)a^2+b^2\left(64e^2(14fg+3eh)d^3-2cef(127fg+312eh)d^2-6c^2f^2(2fg-19eh)d+3c^3f^3h\cdots\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(9dfg+4deh-7cfh)a+b^2(10deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}+\frac{\sqrt{e+fx}\left(21d^2f^2ha^3-7bdf(9dfg+8deh-8cfh)a^2+b^2\left(2e(73fg+16eh)d^2-2cf(10fg+\cdots\right.\right.}{(bc-ad)(be-af)(a+b\cdots}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(9dfg+4deh-7cfh)a+b^2(10deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}+\frac{\sqrt{e+fx}\left(21d^2f^2ha^3-7bdf(9dfg+8deh-8cfh)a^2+b^2\left(2e(73fg+16eh)d^2-2cf(10fg+\cdots\right.\right.}{(bc-ad)(be-af)(a+b\cdots}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(9dfg+4deh-7cfh)a+b^2(10deg-cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}+\frac{\sqrt{e+fx}\left(21d^2f^2ha^3-7bdf(9dfg+8deh-8cfh)a^2+b^2\left(2e(73fg+16eh)d^2-2cf(10fg+\cdots\right.\right.}{(bc-ad)(be-af)(a+b\cdots}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

$\big\downarrow$ 174

$$\frac{\frac{\sqrt{e+fx}(3dfha^2-b(9dfg+4deh-7cfh)a+b^2(10deg-cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}+\frac{\sqrt{e+fx}\left(21d^2f^2ha^3-7bdf(9dfg+8deh-8cfh)a^2+b^2\left(2e(73fg+16eh)d^2-2cf(10fg+\right.\right.}{(bc-ad)(be-af)(a+bx}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

$\big\downarrow$ 73

$$\frac{\frac{\sqrt{e+fx}(3dfha^2-b(9dfg+4deh-7cfh)a+b^2(10deg-cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}+\frac{\sqrt{e+fx}\left(21d^2f^2ha^3-7bdf(9dfg+8deh-8cfh)a^2+b^2\left(2e(73fg+16eh)d^2-2cf(10fg+\right.\right.}{(bc-ad)(be-af)(a+bx}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

$\big\downarrow$ 221

$$\frac{\frac{\sqrt{e+fx}(3dfha^2-b(9dfg+4deh-7cfh)a+b^2(10deg-cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}+\frac{\sqrt{e+fx}\left(21d^2f^2ha^3-7bdf(9dfg+8deh-8cfh)a^2+b^2\left(2e(73fg+16eh)d^2-2cf(10fg+\right.\right.}{(bc-ad)(be-af)(a+bx}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

input
```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^4*(c + d*x)^3),x]
```

output
```
-1/3*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^3*(c + d*x)^2) +
 (((3*a^2*d*f*h + b^2*(10*d*e*g - c*f*g - 6*c*e*h) - a*b*(9*d*f*g + 4*d*e*
h - 7*c*f*h))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d
*x)^2) + (((21*a^3*d^2*f^2*h - 7*a^2*b*d*f*(9*d*f*g + 8*d*e*h - 8*c*f*h) -
 b^3*(80*d^2*e^2*g - 3*c^2*f*(f*g - 2*e*h) - 2*c*d*e*(7*f*g + 24*e*h)) + a
*b^2*(3*c^2*f^2*h + 2*d^2*e*(73*f*g + 16*e*h) - 2*c*d*f*(10*f*g + 49*e*h))
)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2) + ((2*d*(
35*a^3*d^2*f^2*h - a^2*b*d*f*(101*d*f*g + 86*d*e*h - 82*c*f*h) - b^3*(120*
d^2*e^2*g - 3*c^2*f*(f*g - 2*e*h) - 8*c*d*e*(2*f*g + 9*e*h)) + a*b^2*(3*c^
2*f^2*h + 16*d^2*e*(14*f*g + 3*e*h) - 2*c*d*f*(11*f*g + 74*e*h)))*Sqrt[e +
 f*x])/((b*c - a*d)*(c + d*x)^2) + (3*b*((2*d*(a^3*d^2*f^2*(2*d*f*g + 27*d
*e*h - 29*c*f*h) - b^3*(80*d^3*e^3*g + c^3*f^2*(f*g - 2*e*h) - 12*c*d^2*e^
2*(7*f*g + 4*e*h) + c^2*d*e*f*(5*f*g + 48*e*h)) - a*b^2*(c^3*f^3*h - 4*d^3
*e^2*(39*f*g + 8*e*h) + 2*c*d^2*e*f*(79*f*g + 66*e*h) - c^2*d*f^2*(8*f*g +
 95*e*h)) - a^2*b*d*f*(50*c^2*f^2*h + d^2*e*(77*f*g + 60*e*h) - c*d*f*(71*
f*g + 116*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)) + ((2*
Sqrt[b]*(d*e - c*f)*(35*a^4*d^3*f^3*h - 35*a^3*b*d^2*f^2*(3*d*f*g + 4*d*e*
h - 3*c*f*h) + b^4*(160*d^3*e^3*g - 6*c^2*d*e*f*(f*g - 4*e*h) - c^3*f^2*(f
*g - 2*e*h) - 48*c*d^2*e^2*(f*g + 2*e*h)) - a*b^3*(c^3*f^3*h - 3*c^2*d*f^2
*(3*f*g - 16*e*h) + 16*d^3*e^2*(27*f*g + 4*e*h) - 36*c*d^2*e*f*(3*f*g +...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 95.63 (sec) , antiderivative size = 1745, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	Expression too large to display	1745
default	Expression too large to display	1745
pseudoelliptic	Expression too large to display	1987

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*f^5*(-d^2/f^5/(a*d-b*c)^6*((1/8*d*f*(5*a^2*c*d^2*f*h-4*a^2*d^3*e*h-a^2*d
^3*f*g+6*a*b*c^2*d*f*h-8*a*b*c*d^2*e*h-14*a*b*c*d^2*f*g+16*a*b*d^3*e*g-11*
b^2*c^3*f*h+12*b^2*c^2*d*e*h+15*b^2*c^2*d*f*g-16*b^2*c*d^2*e*g)/(c*f-d*e)*
(f*x+e)^(3/2)+1/8*(3*a^2*c*d^2*f*h-4*a^2*d^3*e*h+a^2*d^3*f*g+10*a*b*c^2*d*
f*h-8*a*b*c*d^2*e*h-18*a*b*c*d^2*f*g+16*a*b*d^3*e*g-13*b^2*c^3*f*h+12*b^2*
c^2*d*e*h+17*b^2*c^2*d*f*g-16*b^2*c*d^2*e*g)*f*(f*x+e)^(1/2))/((f*x+e)*d+c
*f-d*e)^2-1/8*(3*a^2*c*d^2*f^2*h-4*a^2*d^3*e*f*h+a^2*d^3*f^2*g+42*a*b*c^2*
d*f^2*h-72*a*b*c*d^2*e*f*h-18*a*b*c*d^2*f^2*g+32*a*b*d^3*e^2*h+16*a*b*d^3*
e*f*g+35*b^2*c^3*f^2*h-84*b^2*c^2*d*e*f*h-63*b^2*c^2*d*f^2*g+48*b^2*c*d^2*
e^2*h+144*b^2*c*d^2*e*f*g-80*b^2*d^3*e^2*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*
arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-b/f^5/(a*d-b*c)^6*((1/16*b^2*
f*(19*a^4*d^3*f^2*h+9*a^3*b*c*d^2*f^2*h-44*a^3*b*d^3*e*f*h-41*a^3*b*d^3*f^
2*g-27*a^2*b^2*c^2*d*f^2*h-6*a^2*b^2*c*d^2*e*f*h+33*a^2*b^2*c*d^2*f^2*g+24
*a^2*b^2*d^3*e^2*h+90*a^2*b^2*d^3*e*f*g-a*b^3*c^3*f^2*h+48*a*b^3*c^2*d*e*f
*h+9*a*b^3*c^2*d*f^2*g-84*a*b^3*c*d^2*e*f*g-48*a*b^3*d^3*e^2*g+2*b^4*c^3*e
*f*h-b^4*c^3*f^2*g-24*b^4*c^2*d*e^2*h-6*b^4*c^2*d*e*f*g+48*b^4*c*d^2*e^2*g
)/(a^2*f^2-2*a*b*e*f+b^2*e^2)*(f*x+e)^(5/2)+1/6*(17*a^4*d^3*f^2*h+3*a^3*b*
c*d^2*f^2*h-36*a^3*b*d^3*e*f*h-35*a^3*b*d^3*f^2*g-21*a^2*b^2*c^2*d*f^2*h+3
3*a^2*b^2*c*d^2*f^2*g+18*a^2*b^2*d^3*e^2*h+72*a^2*b^2*d^3*e*f*g+a*b^3*c^3*
f^2*h+36*a*b^3*c^2*d*e*f*h+3*a*b^3*c^2*d*f^2*g-72*a*b^3*c*d^2*e*f*g-36*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**4/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3092 vs. $2(1160) = 2320$.

Time = 0.37 (sec) , antiderivative size = 3092, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(160*b^5*d^3*e^3*g - 48*b^5*c*d^2*e^2*f*g - 432*a*b^4*d^3*e^2*f*g - 6
*b^5*c^2*d*e*f^2*g + 108*a*b^4*c*d^2*e*f^2*g + 378*a^2*b^3*d^3*e*f^2*g - b
^5*c^3*f^3*g + 9*a*b^4*c^2*d*f^3*g - 63*a^2*b^3*c*d^2*f^3*g - 105*a^3*b^2*
d^3*f^3*g - 96*b^5*c*d^2*e^3*h - 64*a*b^4*d^3*e^3*h + 24*b^5*c^2*d*e^2*f*h
 + 288*a*b^4*c*d^2*e^2*f*h + 168*a^2*b^3*d^3*e^2*f*h + 2*b^5*c^3*e*f^2*h -
 48*a*b^4*c^2*d*e*f^2*h - 294*a^2*b^3*c*d^2*e*f^2*h - 140*a^3*b^2*d^3*e*f^
2*h - a*b^4*c^3*f^3*h + 21*a^2*b^3*c^2*d*f^3*h + 105*a^3*b^2*c*d^2*f^3*h +
 35*a^4*b*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^8*c^
6*e^2 - 6*a*b^7*c^5*d*e^2 + 15*a^2*b^6*c^4*d^2*e^2 - 20*a^3*b^5*c^3*d^3*e^
2 + 15*a^4*b^4*c^2*d^4*e^2 - 6*a^5*b^3*c*d^5*e^2 + a^6*b^2*d^6*e^2 - 2*a*b
^7*c^6*e*f + 12*a^2*b^6*c^5*d*e*f - 30*a^3*b^5*c^4*d^2*e*f + 40*a^4*b^4*c^
3*d^3*e*f - 30*a^5*b^3*c^2*d^4*e*f + 12*a^6*b^2*c*d^5*e*f - 2*a^7*b*d^6*e*
f + a^2*b^6*c^6*f^2 - 6*a^3*b^5*c^5*d*f^2 + 15*a^4*b^4*c^4*d^2*f^2 - 20*a^
5*b^3*c^3*d^3*f^2 + 15*a^6*b^2*c^2*d^4*f^2 - 6*a^7*b*c*d^5*f^2 + a^8*d^6*f
^2)*sqrt(-b^2*e + a*b*f)) + 1/4*(80*b^2*d^5*e^2*g - 144*b^2*c*d^4*e*f*g -
16*a*b*d^5*e*f*g + 63*b^2*c^2*d^3*f^2*g + 18*a*b*c*d^4*f^2*g - a^2*d^5*f^2
*g - 48*b^2*c*d^4*e^2*h - 32*a*b*d^5*e^2*h + 84*b^2*c^2*d^3*e*f*h + 72*a*b
*c*d^4*e*f*h + 4*a^2*d^5*e*f*h - 35*b^2*c^3*d^2*f^2*h - 42*a*b*c^2*d^3*f^2
*h - 3*a^2*c*d^4*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^6
*c^6*d*e - 6*a*b^5*c^5*d^2*e + 15*a^2*b^4*c^4*d^3*e - 20*a^3*b^3*c^3*d^...
```

Mupad [B] (verification not implemented)

Time = 48.26 (sec) , antiderivative size = 1218467, normalized size of antiderivative = 1012.02

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^3),x)
```

output
```
atan(((((256*a^18*b^2*c*d^18*f^10*g - 256*a^18*b^2*d^19*e*f^9*g - 128*a^2*
b^18*c^17*d^2*f^10*g + 2560*a^3*b^17*c^16*d^3*f^10*g - 27776*a^4*b^16*c^15
*d^4*f^10*g + 175872*a^5*b^15*c^14*d^5*f^10*g - 700800*a^6*b^14*c^13*d^6*f
^10*g + 1866752*a^7*b^13*c^12*d^7*f^10*g - 3439744*a^8*b^12*c^11*d^8*f^10*
g + 4412672*a^9*b^11*c^10*d^9*f^10*g - 3805824*a^10*b^10*c^9*d^10*f^10*g +
 1886720*a^11*b^9*c^8*d^11*f^10*g - 35200*a^12*b^8*c^7*d^12*f^10*g - 73907
2*a^13*b^7*c^6*d^13*f^10*g + 607104*a^14*b^6*c^5*d^14*f^10*g - 258048*a^15
*b^5*c^4*d^15*f^10*g + 62080*a^16*b^4*c^3*d^16*f^10*g - 7424*a^17*b^3*c^2*
d^17*f^10*g - 128*a^3*b^17*c^17*d^2*f^10*h + 4096*a^4*b^16*c^16*d^3*f^10*h
 - 32128*a^5*b^15*c^15*d^4*f^10*h + 113408*a^6*b^14*c^14*d^5*f^10*h - 1711
36*a^7*b^13*c^13*d^6*f^10*h - 129536*a^8*b^12*c^12*d^7*f^10*h + 1170048*a^
9*b^11*c^11*d^8*f^10*h - 2728704*a^10*b^10*c^10*d^9*f^10*h + 3805824*a^11*
b^9*c^9*d^10*f^10*h - 3570688*a^12*b^8*c^8*d^11*f^10*h + 2304896*a^13*b^7*
c^7*d^12*f^10*h - 998144*a^14*b^6*c^6*d^13*f^10*h + 264832*a^15*b^5*c^5*d^
14*f^10*h - 31232*a^16*b^4*c^4*d^15*f^10*h - 2176*a^17*b^3*c^3*d^16*f^10*h
 + 768*a^18*b^2*c^2*d^17*f^10*h + 10240*a^12*b^8*d^19*e^7*f^3*g - 45568*a^
13*b^7*d^19*e^6*f^4*g + 78976*a^14*b^6*d^19*e^5*f^5*g - 65536*a^15*b^5*d^1
9*e^4*f^6*g + 24960*a^16*b^4*d^19*e^3*f^7*g - 2816*a^17*b^3*d^19*e^2*f^8*g
 - 4096*a^13*b^7*d^19*e^7*f^3*h + 17920*a^14*b^6*d^19*e^6*f^4*h - 30336*a^
15*b^5*d^19*e^5*f^5*h + 24320*a^16*b^4*d^19*e^4*f^6*h - 8832*a^17*b^3*d...
```

Reduce [B] (verification not implemented)

Time = 16.47 (sec) , antiderivative size = 48332, normalized size of antiderivative = 40.14

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**7*c**4*d**3*f**5*h + 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**3*d**4*e*f**4*h - 210*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**
3*d**4*f**5*h*x - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**7*c**2*d**5*e**2*f**3*h + 420*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**2*d**5*e*f*
*4*h*x - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**7*c**2*d**5*f**5*h*x**2 - 210*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c*d**6*e**2*f**3*h*x +
210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**7*c*d**6*e*f**4*h*x**2 - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*d**7*e**2*f**3*h*x**2 - 315*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*
b*c**5*d**2*f**5*h + 1050*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**4*d**3*e*f**4*h + 315*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**4*d**3
*f**5*g - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**6*b*c**4*d**3*f**5*h*x - 1155*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**3*d**4*e**2*f*...
```

3.59 $\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1821

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Too large to display}$$

output
```
1/192*d*(315*a^4*d^3*f^3*h-a^3*b*d^2*f^2*(-1041*c*f*h+1178*d*e*h+1123*d*f*
g)+b^4*(1440*d^3*e^3*g-2*c^2*d*e*f*(-64*e*h+31*f*g)-3*c^3*f^2*(-8*e*h+5*f*
g)-240*c*d^2*e^2*(4*e*h+f*g))-a*b^3*(9*c^3*f^3*h-c^2*d*f^2*(-266*e*h+107*f
*g)+240*d^3*e^2*(2*e*h+17*f*g)-4*c*d^2*e*f*(716*e*h+151*f*g))+a^2*b^2*d*f*
(93*c^2*f^2*h+2*d^2*e*(664*e*h+1889*f*g)-c*d*f*(2900*e*h+409*f*g)))*(f*x+e
)^(1/2)/b/(-a*d+b*c)^5/(-a*f+b*e)^3/(d*x+c)^2-1/4*(-a*h+b*g)*(f*x+e)^(1/2)
/b/(-a*d+b*c)/(b*x+a)^4/(d*x+c)^2+1/24*(3*a^2*d*f*h-a*b*(-9*c*f*h+4*d*e*h+
11*d*f*g)+b^2*(12*d*e*g-c*(8*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f
+b*e)/(b*x+a)^3/(d*x+c)^2+1/96*(27*a^3*d^2*f^2*h-9*a^2*b*d*f*(-10*c*f*h+8*
d*e*h+11*d*f*g)-b^3*(120*d^2*e^2*g-c^2*f*(-8*e*h+5*f*g)-16*c*d*e*(5*e*h+f*
g))+a*b^2*(3*c^2*f^2*h+8*d^2*e*(5*e*h+28*f*g)-2*c*d*f*(80*e*h+13*f*g)))*(f
*x+e)^(1/2)/b/(-a*d+b*c)^3/(-a*f+b*e)^2/(b*x+a)^2/(d*x+c)^2+1/192*(189*a^4
*d^3*f^3*h-63*a^3*b*d^2*f^2*(-11*c*f*h+12*d*e*h+11*d*f*g)+b^4*(960*d^3*e^3
*g-4*c^2*d*e*f*(-28*e*h+13*f*g)-3*c^3*f^2*(-8*e*h+5*f*g)-40*c*d^2*e^2*(16*
e*h+5*f*g))-a*b^3*(9*c^3*f^3*h-c^2*d*f^2*(-244*e*h+97*f*g)+40*d^3*e^2*(8*e
*h+67*f*g)-24*c*d^2*e*f*(79*e*h+21*f*g))+a^2*b^2*d*f*(87*c^2*f^2*h+4*d^2*e
*(218*e*h+607*f*g)-c*d*f*(1904*e*h+349*f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^4
/(-a*f+b*e)^3/(b*x+a)/(d*x+c)^2+1/64*d*(a^4*d^3*f^3*(-267*c*f*h+251*d*e*h+
16*d*f*g)+b^4*(960*d^4*e^4*g+c^3*d*e*f^2*(-40*e*h+19*f*g)+c^4*f^3*(-8*e*h+
5*f*g)-80*c*d^3*e^3*(8*e*h+13*f*g)+24*c^2*d^2*e^2*f*(28*e*h+3*f*g))+a*b...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24162 vs. 2(1821) = 3642.

Time = 16.94 (sec) , antiderivative size = 24162, normalized size of antiderivative = 13.27

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^5*(c + d*x)^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 1924, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {166, 27, 168, 27, 168, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{a(4de-cf)h-b(12deg-cfg-8ceh)-f(11bdg-8bch-3adh)x}{2(a+bx)^4(c+dx)^3\sqrt{e+fx}}dx}{4b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(4de-cf)h-b(12deg-cfg-8ceh)-f(11bdg-8bch-3adh)x}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}dx}{8b(bc-ad)}-\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\sqrt{e+fx}(3a^2dfh-ab(-9cfh+4deh+11dfg)+b^2(-8ceh-cfg+12deg))}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}-\frac{\int -\frac{9df(4de-cf)ha^2-b\left(4e(29fg+10eh)d^2-cf(17fg+88eh)d+3c^2f^2h\right)a+b^2\left(\right.}{}}{}}{8b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{9df(4de-cf)ha^2-b\left(4e(29fg+10eh)d^2-cf(17fg+88eh)d+3c^2f^2h\right)a+b^2\left(-f(5fg-8eh)c^2-16de(fg+5eh)c+120d^2e^2g\right)+9df\left(3dfha^2-b(11dfg+4deh-9cfh)a-\right.}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}}{6(bc-ad)(be-af)}}{8b(bc-ad)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{e+fx}\left(27a^3d^2f^2h-9a^2bdf(-10cfh+8deh+11dfg)+ab^2\left(3c^2f^2h-2cdf(80eh+13fg)+8d^2e(5eh+28fg)\right)-b^3\left(c^2(-f)(5fg-8eh)-16cde(5eh+fg)+120d^2e^2g\right)\right)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)} -$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$\downarrow$ 27

$$\int \frac{63d^2f^2(4de-cf)ha^3-bdf\left(4e(215fg+148eh)d^2-cf(167fg+784eh)d+66c^2f^2h\right)a^2+b^2\left(80e^2(23fg+4eh)d^3-8cef(49fg+167eh)d^2-2c^2f^2(31fg-94eh)d+9c^3f^3\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$\downarrow$ 168

$$\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 27

$$\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 168

$$\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\right.\right.}{2(bc-ad)(be-af)(a+}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\sqrt{e+fx}\left(3dfha^2-b(11dfg+4deh-9cfh)a+b^2(12deg-cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(27d^2f^2ha^3-9bdf(11dfg+8deh-10cfh)a^2+b^2\left(8e(28fg+5eh)d^2-2cf(13f\ldots\right.\right.}{2(bc-ad)(be-af)(a+\ldots}}{\ldots}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

input

```
Int[(Sqrt[e + f*x]*(g + h*x))/((a + b*x)^5*(c + d*x)^3),x]
```

output

```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^4*(c + d*x)^2) +
 (((3*a^2*d*f*h + b^2*(12*d*e*g - c*f*g - 8*c*e*h) - a*b*(11*d*f*g + 4*d*e
*h - 9*c*f*h))*Sqrt[e + f*x])/(3*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c +
d*x)^2) + (((27*a^3*d^2*f^2*h - 9*a^2*b*d*f*(11*d*f*g + 8*d*e*h - 10*c*f*h
) - b^3*(120*d^2*e^2*g - c^2*f*(5*f*g - 8*e*h) - 16*c*d*e*(f*g + 5*e*h)) +
 a*b^2*(3*c^2*f^2*h + 8*d^2*e*(28*f*g + 5*e*h) - 2*c*d*f*(13*f*g + 80*e*h)
))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)^2) + ((
(189*a^4*d^3*f^3*h - 63*a^3*b*d^2*f^2*(11*d*f*g + 12*d*e*h - 11*c*f*h) + b
^4*(960*d^3*e^3*g - 4*c^2*d*e*f*(13*f*g - 28*e*h) - 3*c^3*f^2*(5*f*g - 8*e
*h) - 40*c*d^2*e^2*(5*f*g + 16*e*h)) - a*b^3*(9*c^3*f^3*h - c^2*d*f^2*(97*
f*g - 244*e*h) + 40*d^3*e^2*(67*f*g + 8*e*h) - 24*c*d^2*e*f*(21*f*g + 79*e
*h)) + a^2*b^2*d*f*(87*c^2*f^2*h + 4*d^2*e*(607*f*g + 218*e*h) - c*d*f*(34
9*f*g + 1904*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c +
 d*x)^2) + ((2*d*(315*a^4*d^3*f^3*h - a^3*b*d^2*f^2*(1123*d*f*g + 1178*d*e
*h - 1041*c*f*h) + b^4*(1440*d^3*e^3*g - 2*c^2*d*e*f*(31*f*g - 64*e*h) - 3
*c^3*f^2*(5*f*g - 8*e*h) - 240*c*d^2*e^2*(f*g + 4*e*h)) - a*b^3*(9*c^3*f^3
*h - c^2*d*f^2*(107*f*g - 266*e*h) + 240*d^3*e^2*(17*f*g + 2*e*h) - 4*c*d^
2*e*f*(151*f*g + 716*e*h)) + a^2*b^2*d*f*(93*c^2*f^2*h + 2*d^2*e*(1889*f*g
 + 664*e*h) - c*d*f*(409*f*g + 2900*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c
+ d*x)^2) + (3*b*((2*d*(a^4*d^3*f^3*(16*d*f*g + 251*d*e*h - 267*c*f*h) ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 362.92 (sec) , antiderivative size = 3167, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	Expression too large to display	3167
default	Expression too large to display	3167
pseudoelliptic	Expression too large to display	3232

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*f^6*(-d^3/f^6/(a*d-b*c)^7*((1/8*d*f*(5*a^2*c*d^2*f*h-4*a^2*d^3*e*h-a^2*d
^3*f*g+10*a*b*c^2*d*f*h-12*a*b*c*d^2*e*h-18*a*b*c*d^2*f*g+20*a*b*d^3*e*g-1
5*b^2*c^3*f*h+16*b^2*c^2*d*e*h+19*b^2*c^2*d*f*g-20*b^2*c*d^2*e*g)/(c*f-d*e
)*(f*x+e)^(3/2)+1/8*f*(3*a^2*c*d^2*f*h-4*a^2*d^3*e*h+a^2*d^3*f*g+14*a*b*c^
2*d*f*h-12*a*b*c*d^2*e*h-22*a*b*c*d^2*f*g+20*a*b*d^3*e*g-17*b^2*c^3*f*h+16
*b^2*c^2*d*e*h+21*b^2*c^2*d*f*g-20*b^2*c*d^2*e*g)*(f*x+e)^(1/2))/((f*x+e)*
d+c*f-d*e)^2-1/8*(3*a^2*c*d^2*f^2*h-4*a^2*d^3*e*f*h+a^2*d^3*f^2*g+54*a*b*c
^2*d*f^2*h-92*a*b*c*d^2*e*f*h-22*a*b*c*d^2*f^2*g+40*a*b*d^3*e^2*h+20*a*b*d
^3*e*f*g+63*b^2*c^3*f^2*h-144*b^2*c^2*d*e*f*h-99*b^2*c^2*d*f^2*g+80*b^2*c*
d^2*e^2*h+220*b^2*c*d^2*e*f*g-120*b^2*d^3*e^2*g)/(c*f-d*e)/((c*f-d*e)*d)^(
1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-b/f^6/(a*d-b*c)^7*((1/12
8*b^3*f*(187*a^5*d^4*f^3*h+236*a^4*b*c*d^3*f^3*h-656*a^4*b*d^4*e*f^2*h-515
*a^4*b*d^4*f^3*g-390*a^3*b^2*c^2*d^2*f^3*h-520*a^3*b^2*c*d^3*e*f^2*h+356*a
^3*b^2*c*d^3*f^3*g+720*a^3*b^2*d^4*e^2*f*h+1704*a^3*b^2*d^4*e*f^2*g-36*a^2
*b^3*c^3*d*f^3*h+1080*a^2*b^3*c^2*d^2*e*f^2*h+198*a^2*b^3*c^2*d^2*f^3*g+43
2*a^2*b^3*c*d^3*e^2*f*h-1464*a^2*b^3*c*d^3*e*f^2*g-256*a^2*b^3*d^4*e^3*h-1
824*a^2*b^3*d^4*e^2*f*g+3*a*b^4*c^4*f^3*h+104*a*b^4*c^3*d*e*f^2*h-44*a*b^4
*c^3*d*f^3*g-1104*a*b^4*c^2*d^2*e^2*f*h-264*a*b^4*c^2*d^2*e*f^2*g-128*a*b^
4*c*d^3*e^3*h+1728*a*b^4*c*d^3*e^2*f*g+640*a*b^4*d^4*e^3*g-8*b^5*c^4*e*f^2
*h+5*b^5*c^4*f^3*g-48*b^5*c^3*d*e^2*f*h+24*b^5*c^3*d*e*f^2*g+384*b^5*c^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(1/2)*(h*x+g)/(b*x+a)**5/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5728 vs. $2(1773) = 3546$.

Time = 0.53 (sec) , antiderivative size = 5728, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/64*(1920*b^6*d^4*e^4*g - 640*b^6*c*d^3*e^3*f*g - 7040*a*b^5*d^4*e^3*f*g
- 96*b^6*c^2*d^2*e^2*f^2*g + 2112*a*b^5*c*d^3*e^2*f^2*g + 9504*a^2*b^4*d^4
*e^2*f^2*g - 24*b^6*c^3*d*e*f^3*g + 264*a*b^5*c^2*d^2*e*f^3*g - 2376*a^2*b
^4*c*d^3*e*f^3*g - 5544*a^3*b^3*d^4*e*f^3*g - 5*b^6*c^4*f^4*g + 44*a*b^5*c
^3*d*f^4*g - 198*a^2*b^4*c^2*d^2*f^4*g + 924*a^3*b^3*c*d^3*f^4*g + 1155*a^
4*b^2*d^4*f^4*g - 1280*b^6*c*d^3*e^4*h - 640*a*b^5*d^4*e^4*h + 384*b^6*c^2
*d^2*e^3*f*h + 4992*a*b^5*c*d^3*e^3*f*h + 2304*a^2*b^4*d^4*e^3*f*h + 48*b^
6*c^3*d*e^2*f^2*h - 1200*a*b^5*c^2*d^2*e^2*f^2*h - 7344*a^2*b^4*c*d^3*e^2*
f^2*h - 3024*a^3*b^3*d^4*e^2*f^2*h + 8*b^6*c^4*e*f^3*h - 104*a*b^5*c^3*d*e
*f^3*h + 1224*a^2*b^4*c^2*d^2*e*f^3*h + 4872*a^3*b^3*c*d^3*e*f^3*h + 1680*
a^4*b^2*d^4*e*f^3*h - 3*a*b^5*c^4*f^4*h + 36*a^2*b^4*c^3*d*f^4*h - 378*a^3
*b^3*c^2*d^2*f^4*h - 1260*a^4*b^2*c*d^3*f^4*h - 315*a^5*b*d^4*f^4*h)*arcta
n(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^10*c^7*e^3 - 7*a*b^9*c^6*d*e^3
 + 21*a^2*b^8*c^5*d^2*e^3 - 35*a^3*b^7*c^4*d^3*e^3 + 35*a^4*b^6*c^3*d^4*e^
3 - 21*a^5*b^5*c^2*d^5*e^3 + 7*a^6*b^4*c*d^6*e^3 - a^7*b^3*d^7*e^3 - 3*a*b
^9*c^7*e^2*f + 21*a^2*b^8*c^6*d*e^2*f - 63*a^3*b^7*c^5*d^2*e^2*f + 105*a^4
*b^6*c^4*d^3*e^2*f - 105*a^5*b^5*c^3*d^4*e^2*f + 63*a^6*b^4*c^2*d^5*e^2*f
- 21*a^7*b^3*c*d^6*e^2*f + 3*a^8*b^2*d^7*e^2*f + 3*a^2*b^8*c^7*e*f^2 - 21*
a^3*b^7*c^6*d*e*f^2 + 63*a^4*b^6*c^5*d^2*e*f^2 - 105*a^5*b^5*c^4*d^3*e*f^2
 + 105*a^6*b^4*c^3*d^4*e*f^2 - 63*a^7*b^3*c^2*d^5*e*f^2 + 21*a^8*b^2*c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{e+fx}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \int \frac{\sqrt{fx+e}\,(hx+g)}{(bx+a)^5\,(dx+c)^3}dx$$

input
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x)
```

output
```
int((f*x+e)^(1/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x)
```

3.60 $\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 341

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2(be-af)^3(de-cf)(fg-eh)(e+fx)^{5/2}}{5f^6}$$
$$-\frac{2(be-af)^2(bde(4fg-5eh)-bcf(3fg-4eh)-af(dfg-2deh+cfh))(e+fx)^{7/2}}{7f^6}$$
$$-\frac{2(be-af)\left(a^2df^2h+abf(3dfg-8deh+3cfh)-b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)(e+fx)^{9/2}}{9f^6}$$
$$+\frac{2b(3a^2df^2h+3abf(dfg-4deh+cfh)-b^2(2de(2fg-5eh)-cf(fg-4eh)))\,(e+fx)^{11/2}}{11f^6}$$
$$+\frac{2b^2(3adfh+b(dfg-5deh+cfh))(e+fx)^{13/2}}{13f^6}+\frac{2b^3dh(e+fx)^{15/2}}{15f^6}$$

output
```
2/5*(-a*f+b*e)^3*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(5/2)/f^6-2/7*(-a*f+b*e)^2*
(b*d*e*(-5*e*h+4*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x
+e)^(7/2)/f^6-2/9*(-a*f+b*e)*(a^2*d*f^2*h+a*b*f*(3*c*f*h-8*d*e*h+3*d*f*g)-
b^2*(2*d*e*(-5*e*h+3*f*g)-3*c*f*(-2*e*h+f*g)))*(f*x+e)^(9/2)/f^6+2/11*b*(3
*a^2*d*f^2*h+3*a*b*f*(c*f*h-4*d*e*h+d*f*g)-b^2*(2*d*e*(-5*e*h+2*f*g)-c*f*(
-4*e*h+f*g)))*(f*x+e)^(11/2)/f^6+2/13*b^2*(3*a*d*f*h+b*(c*f*h-5*d*e*h+d*f*
g))*(f*x+e)^(13/2)/f^6+2/15*b^3*d*h*(f*x+e)^(15/2)/f^6
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.43

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(143a^3f^3(9cf(7fg-2eh+5fhx)+d(8e^2h+5f^2x(9g+7hx)-2ef(9g+10hx))\right)}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(143*a^3*f^3*(9*c*f*(7*f*g - 2*e*h + 5*f*h*x) + d*(8*e^
2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x))) + b^3*(d*(-256*e^5*h
+ 1680*e^2*f^3*x^2*(g + h*x) + 128*e^4*f*(3*g + 5*h*x) - 160*e^3*f^2*x*(6*
g + 7*h*x) - 210*e*f^4*x^3*(12*g + 11*h*x) + 231*f^5*x^4*(15*g + 13*h*x))
+ 3*c*f*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) - 70*e*f^3*x^2*(13*g + 12
*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13*g + 20*h*x))) + 39*a^2
*b*f^2*(11*c*f*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) +
d*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*
f^2*x*(22*g + 21*h*x))) + 3*a*b^2*f*(3*d*(128*e^4*h + 105*f^4*x^3*(13*g +
11*h*x) - 70*e*f^3*x^2*(13*g + 12*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16
*e^3*f*(13*g + 20*h*x)) + 13*c*f*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) +
8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(22*g + 21*h*x)))))/(45045*f^6)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \; 159$$

$$\int \left(\frac{b(e+fx)^{9/2} \left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{f^5} + \frac{(e+fx)^{7/2}(\ldots}{}\right.$$

$\big\downarrow$ 2009

$$\frac{2b(e+fx)^{11/2} \left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{11f^6} - \frac{2(e+fx)^{9/2}(be-af)\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)\right)}{9f^6} + \frac{2b^2(e+fx)^{13/2}(3adfh+b(cfh-5deh+dfg))}{13f^6} - \frac{2(e+fx)^{7/2}(be-af)^2(-af(cfh-2deh+dfg)-bcf(3fg-4eh)+bde(4fg-5eh))}{7f^6} + \frac{2(e+fx)^{5/2}(be-af)^3(de-cf)(fg-eh)}{5f^6} + \frac{2b^3dh(e+fx)^{15/2}}{15f^6}$$

input
```
Int[(a + b*x)^3*(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(b*e - a*f)^3*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^6) - (2*(b*
e - a*f)^2*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(3*f*g - 4*e*h) - a*f*(d*f*g - 2
*d*e*h + c*f*h))*(e + f*x)^(7/2))/(7*f^6) - (2*(b*e - a*f)*(a^2*d*f^2*h +
a*b*f*(3*d*f*g - 8*d*e*h + 3*c*f*h) - b^2*(2*d*e*(3*f*g - 5*e*h) - 3*c*f*(
f*g - 2*e*h)))*(e + f*x)^(9/2))/(9*f^6) + (2*b*(3*a^2*d*f^2*h + 3*a*b*f*(d
*f*g - 4*d*e*h + c*f*h) - b^2*(2*d*e*(2*f*g - 5*e*h) - c*f*(f*g - 4*e*h)))
*(e + f*x)^(11/2))/(11*f^6) + (2*b^2*(3*a*d*f*h + b*(d*f*g - 5*d*e*h + c*f
*h))*(e + f*x)^(13/2))/(13*f^6) + (2*b^3*d*h*(e + f*x)^(15/2))/(15*f^6)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{2dh\,b^3(fx+e)^{\frac{15}{2}}}{15}+\frac{2\big(\big(3(af-be)b^2d+b^3(cf-de)\big)h+b^3d(-eh+fg)\big)(fx+e)^{\frac{13}{2}}}{13}+\frac{2\big(\big(3(af-be)^2bd+3(af-be)b^2(cf-de)\big)h+\big(3(af\cdots}{11}$
default	$\frac{2dh\,b^3(fx+e)^{\frac{15}{2}}}{15}-\frac{2\big(-\big(3(af-be)b^2d+b^3(cf-de)\big)h+b^3d(eh-fg)\big)(fx+e)^{\frac{13}{2}}}{13}-\frac{2\big(-\big(3(af-be)^2bd+3(af-be)b^2(cf-de)\big)h+\big(3(a\cdots}{11}$
pseudoelliptic	$-\frac{4(fx+e)^{\frac{5}{2}}\Bigg(\frac{\Big(7\big(-\frac{dh\,x^5}{3}+\frac{5(-ch-dg)x^4}{13}-\frac{5cg\,x^3}{11}\big)b^3-\frac{35a\,x^2\big(\frac{9dh\,x^2}{13}+\frac{9(ch+dg)x}{11}+cg\big)b^2}{3}-15a^2x\big(\frac{7dh\,x^2}{11}+\frac{7(ch+dg)x}{9}+c\cdots}{2}\cdots}{\cdots}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-3003dh\,b^3x^5f^5-10395a\,b^2d\,f^5h\,x^4-3465b^3c\,f^5h\,x^4+2310b^3de\,f^4h\,x^4-3465b^3d\,f^5g\,x^4-12285a^2bd\,f\cdots\right.}{\cdots}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-3003dh\,b^3x^5f^5-10395a\,b^2d\,f^5h\,x^4-3465b^3c\,f^5h\,x^4+2310b^3de\,f^4h\,x^4-3465b^3d\,f^5g\,x^4-12285a^2bd\,f\cdots\right.}{\cdots}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^6*(1/15*d*h*b^3*(f*x+e)^(15/2)+1/13*((3*(a*f-b*e)*b^2*d+b^3*(c*f-d*e))
*h+b^3*d*(-e*h+f*g))*(f*x+e)^(13/2)+1/11*((3*(a*f-b*e)^2*b*d+3*(a*f-b*e)*b
^2*(c*f-d*e))*h+(3*(a*f-b*e)*b^2*d+b^3*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(11/
2)+1/9*(((a*f-b*e)^3*d+3*(a*f-b*e)^2*b*(c*f-d*e))*h+(3*(a*f-b*e)^2*b*d+3*(
a*f-b*e)*b^2*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((a*f-b*e)^3*(c*f-d*
e)*h+((a*f-b*e)^3*d+3*(a*f-b*e)^2*b*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(7/2)+1
/5*(a*f-b*e)^3*(c*f-d*e)*(-e*h+f*g)*(f*x+e)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(317) = 634$.

Time = 0.12 (sec) , antiderivative size = 998, normalized size of antiderivative = 2.93

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^3*d*f^7*h*x^7 + 231*(15*b^3*d*f^7*g + (16*b^3*d*e*f^6 + 15
*(b^3*c + 3*a*b^2*d)*f^7)*h)*x^6 + 63*(5*(14*b^3*d*e*f^6 + 13*(b^3*c + 3*a
*b^2*d)*f^7)*g + (b^3*d*e^2*f^5 + 70*(b^3*c + 3*a*b^2*d)*e*f^6 + 195*(a*b^
2*c + a^2*b*d)*f^7)*h)*x^5 + 35*(3*(b^3*d*e^2*f^5 + 52*(b^3*c + 3*a*b^2*d)
*e*f^6 + 143*(a*b^2*c + a^2*b*d)*f^7)*g - (2*b^3*d*e^3*f^4 - 3*(b^3*c + 3*
a*b^2*d)*e^2*f^5 - 468*(a*b^2*c + a^2*b*d)*e*f^6 - 143*(3*a^2*b*c + a^3*d)
*f^7)*h)*x^4 - 5*(3*(8*b^3*d*e^3*f^4 - 13*(b^3*c + 3*a*b^2*d)*e^2*f^5 - 14
30*(a*b^2*c + a^2*b*d)*e*f^6 - 429*(3*a^2*b*c + a^3*d)*f^7)*g - (16*b^3*d*
e^4*f^3 + 1287*a^3*c*f^7 - 24*(b^3*c + 3*a*b^2*d)*e^3*f^4 + 117*(a*b^2*c +
 a^2*b*d)*e^2*f^5 + 1430*(3*a^2*b*c + a^3*d)*e*f^6)*h)*x^3 + 3*(3*(16*b^3*
d*e^4*f^3 + 1001*a^3*c*f^7 - 26*(b^3*c + 3*a*b^2*d)*e^3*f^4 + 143*(a*b^2*c
 + a^2*b*d)*e^2*f^5 + 1144*(3*a^2*b*c + a^3*d)*e*f^6)*g - (32*b^3*d*e^5*f^
2 - 3432*a^3*c*e*f^6 - 48*(b^3*c + 3*a*b^2*d)*e^4*f^3 + 234*(a*b^2*c + a^2
*b*d)*e^3*f^4 - 143*(3*a^2*b*c + a^3*d)*e^2*f^5)*h)*x^2 + 3*(128*b^3*d*e^6
*f + 3003*a^3*c*e^2*f^5 - 208*(b^3*c + 3*a*b^2*d)*e^5*f^2 + 1144*(a*b^2*c
+ a^2*b*d)*e^4*f^3 - 858*(3*a^2*b*c + a^3*d)*e^3*f^4)*g - 2*(128*b^3*d*e^7
 + 1287*a^3*c*e^3*f^4 - 192*(b^3*c + 3*a*b^2*d)*e^6*f + 936*(a*b^2*c + a^2
*b*d)*e^5*f^2 - 572*(3*a^2*b*c + a^3*d)*e^4*f^3)*h - (3*(64*b^3*d*e^5*f^2
- 6006*a^3*c*e*f^6 - 104*(b^3*c + 3*a*b^2*d)*e^4*f^3 + 572*(a*b^2*c + a^2*
b*d)*e^3*f^4 - 429*(3*a^2*b*c + a^3*d)*e^2*f^5)*g - (128*b^3*d*e^6*f + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(355) = 710$.

Time = 2.08 (sec) , antiderivative size = 1013, normalized size of antiderivative = 2.97

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**3*d*h*(e + f*x)**(15/2)/(15*f**5) + (e + f*x)**(13/2)*(3*
a*b**2*d*f*h + b**3*c*f*h - 5*b**3*d*e*h + b**3*d*f*g)/(13*f**5) + (e + f*
x)**(11/2)*(3*a**2*b*d*f**2*h + 3*a*b**2*c*f**2*h - 12*a*b**2*d*e*f*h + 3*
a*b**2*d*f**2*g - 4*b**3*c*e*f*h + b**3*c*f**2*g + 10*b**3*d*e**2*h - 4*b*
*3*d*e*f*g)/(11*f**5) + (e + f*x)**(9/2)*(a**3*d*f**3*h + 3*a**2*b*c*f**3*
h - 9*a**2*b*d*e*f**2*h + 3*a**2*b*d*f**3*g - 9*a*b**2*c*e*f**2*h + 3*a*b*
*2*c*f**3*g + 18*a*b**2*d*e**2*f*h - 9*a*b**2*d*e*f**2*g + 6*b**3*c*e**2*f
*h - 3*b**3*c*e*f**2*g - 10*b**3*d*e**3*h + 6*b**3*d*e**2*f*g)/(9*f**5) +
(e + f*x)**(7/2)*(a**3*c*f**4*h - 2*a**3*d*e*f**3*h + a**3*d*f**4*g - 6*a*
*2*b*c*e*f**3*h + 3*a**2*b*c*f**4*g + 9*a**2*b*d*e**2*f**2*h - 6*a**2*b*d*
e*f**3*g + 9*a*b**2*c*e**2*f**2*h - 6*a*b**2*c*e*f**3*g - 12*a*b**2*d*e**3
*f*h + 9*a*b**2*d*e**2*f**2*g - 4*b**3*c*e**3*f*h + 3*b**3*c*e**2*f**2*g +
 5*b**3*d*e**4*h - 4*b**3*d*e**3*f*g)/(7*f**5) + (e + f*x)**(5/2)*(-a**3*c
*e*f**4*h + a**3*c*f**5*g + a**3*d*e**2*f**3*h - a**3*d*e*f**4*g + 3*a**2*
b*c*e**2*f**3*h - 3*a**2*b*c*e*f**4*g - 3*a**2*b*d*e**3*f**2*h + 3*a**2*b*
d*e**2*f**3*g - 3*a*b**2*c*e**3*f**2*h + 3*a*b**2*c*e**2*f**3*g + 3*a*b**2
*d*e**4*f*h - 3*a*b**2*d*e**3*f**2*g + b**3*c*e**4*f*h - b**3*c*e**3*f**2*
g - b**3*d*e**5*h + b**3*d*e**4*f*g)/(5*f**5))/f, Ne(f, 0)), (e**(3/2)*(a*
*3*c*g*x + b**3*d*h*x**6/6 + x**5*(3*a*b**2*d*h + b**3*c*h + b**3*d*g)/5 +
 x**4*(3*a**2*b*d*h + 3*a*b**2*c*h + 3*a*b**2*d*g + b**3*c*g)/4 + x**3*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.80

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(3003\,(fx+e)^{\frac{15}{2}}b^3dh+3465\,(b^3dfg-(5\,b^3de-(b^3c+3\,ab^2d)f)h)(fx+e)^{\frac{13}{2}}-4095\,((4\,b^3de\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3003*(f*x + e)^(15/2)*b^3*d*h + 3465*(b^3*d*f*g - (5*b^3*d*e - (b
^3*c + 3*a*b^2*d)*f)*h)*(f*x + e)^(13/2) - 4095*((4*b^3*d*e*f - (b^3*c + 3
*a*b^2*d)*f^2)*g - (10*b^3*d*e^2 - 4*(b^3*c + 3*a*b^2*d)*e*f + 3*(a*b^2*c
+ a^2*b*d)*f^2)*h)*(f*x + e)^(11/2) + 5005*(3*(2*b^3*d*e^2*f - (b^3*c + 3*
a*b^2*d)*e*f^2 + (a*b^2*c + a^2*b*d)*f^3)*g - (10*b^3*d*e^3 - 6*(b^3*c + 3
*a*b^2*d)*e^2*f + 9*(a*b^2*c + a^2*b*d)*e*f^2 - (3*a^2*b*c + a^3*d)*f^3)*h
)*(f*x + e)^(9/2) - 6435*((4*b^3*d*e^3*f - 3*(b^3*c + 3*a*b^2*d)*e^2*f^2 +
 6*(a*b^2*c + a^2*b*d)*e*f^3 - (3*a^2*b*c + a^3*d)*f^4)*g - (5*b^3*d*e^4 +
 a^3*c*f^4 - 4*(b^3*c + 3*a*b^2*d)*e^3*f + 9*(a*b^2*c + a^2*b*d)*e^2*f^2 -
 2*(3*a^2*b*c + a^3*d)*e*f^3)*h)*(f*x + e)^(7/2) + 9009*((b^3*d*e^4*f + a^
3*c*f^5 - (b^3*c + 3*a*b^2*d)*e^3*f^2 + 3*(a*b^2*c + a^2*b*d)*e^2*f^3 - (3
*a^2*b*c + a^3*d)*e*f^4)*g - (b^3*d*e^5 + a^3*c*e*f^4 - (b^3*c + 3*a*b^2*d
)*e^4*f + 3*(a*b^2*c + a^2*b*d)*e^3*f^2 - (3*a^2*b*c + a^3*d)*e^2*f^3)*h)*
(f*x + e)^(5/2))/f^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2956 vs. 2(317) = 634.

Time = 0.16 (sec) , antiderivative size = 2956, normalized size of antiderivative = 8.67

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output

```
2/45045*(45045*sqrt(f*x + e)*a^3*c*e^2*g + 30030*((f*x + e)^(3/2) - 3*sqrt
(f*x + e)*e)*a^3*c*e*g + 45045*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*b
*c*e^2*g/f + 15015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*d*e^2*g/f + 1
5015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c*e^2*h/f + 3003*(3*(f*x +
e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*c*g + 9009*(3*
(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b^2*c*e^2
*g/f^2 + 9009*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)
*e^2)*a^2*b*d*e^2*g/f^2 + 18018*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e
+ 15*sqrt(f*x + e)*e^2)*a^2*b*c*e*g/f + 6006*(3*(f*x + e)^(5/2) - 10*(f*x
+ e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*d*e*g/f + 9009*(3*(f*x + e)^(5/2)
 - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*c*e^2*h/f^2 + 3003*(
3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*d*e^2
*h/f^2 + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)
*e^2)*a^3*c*e*h/f + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f
*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^3*c*e^2*g/f^3 + 3861*(5*(f*x +
 e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x +
e)*e^3)*a*b^2*d*e^2*g/f^3 + 7722*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e
 + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b^2*c*e*g/f^2 + 7722*(
5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqr
t(f*x + e)*e^3)*a^2*b*d*e*g/f^2 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e...
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.07

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{11/2}\,(2\,b^3\,c\,f^2\,g+20\,b^3\,d\,e^2\,h+6\,a\,b^2\,c\,f^2\,h+6\,a\,b^2\,d\,f^2\,g+6\,a^2\,b\,d\,f^2\,h-8\,b^3\,c\,e\,f\,h-\cdots}{11\,f^6} + \frac{(e+f\,x)^{13/2}\,(2\,b^3\,c\,f\,h-10\,b^3\,d\,e\,h+2\,b^3\,d\,f\,g+6\,a\,b^2\,d\,f\,h)}{13\,f^6} + \frac{2\,(e+f\,x)^{9/2}\,(a\,f-b\,e)\,(3\,b^2\,c\,f^2\,g+a^2\,d\,f^2\,h+10\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b\cdots}{9\,f^6} + \frac{2\,(e+f\,x)^{7/2}\,(a\,f-b\,e)^2\,(a\,c\,f^2\,h+a\,d\,f^2\,g+3\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f\,g)}{7\,f^6} + \frac{2\,b^3\,d\,h\,(e+f\,x)^{15/2}}{15\,f^6} - \frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)^3\,(c\,f-d\,e)\,(e\,h-f\,g)}{5\,f^6}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^3*(c + d*x),x)
```

output
```
((e + f*x)^(11/2)*(2*b^3*c*f^2*g + 20*b^3*d*e^2*h + 6*a*b^2*c*f^2*h + 6*a*
b^2*d*f^2*g + 6*a^2*b*d*f^2*h - 8*b^3*c*e*f*h - 8*b^3*d*e*f*g - 24*a*b^2*d
*e*f*h))/(11*f^6) + ((e + f*x)^(13/2)*(2*b^3*c*f*h - 10*b^3*d*e*h + 2*b^3*
d*f*g + 6*a*b^2*d*f*h))/(13*f^6) + (2*(e + f*x)^(9/2)*(a*f - b*e)*(3*b^2*c
*f^2*g + a^2*d*f^2*h + 10*b^2*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 6*
b^2*c*e*f*h - 6*b^2*d*e*f*g - 8*a*b*d*e*f*h))/(9*f^6) + (2*(e + f*x)^(7/2)
*(a*f - b*e)^2*(a*c*f^2*h + a*d*f^2*g + 3*b*c*f^2*g + 5*b*d*e^2*h - 2*a*d*
e*f*h - 4*b*c*e*f*h - 4*b*d*e*f*g))/(7*f^6) + (2*b^3*d*h*(e + f*x)^(15/2))
/(15*f^6) - (2*(e + f*x)^(5/2)*(a*f - b*e)^3*(c*f - d*e)*(e*h - f*g))/(5*f
^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.79

$$\int (a+bx)^3(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 2574*a**3*c*e**3*f**4*h + 9009*a**3*c*e**2*f**5*g + 1
287*a**3*c*e**2*f**5*h*x + 18018*a**3*c*e*f**6*g*x + 10296*a**3*c*e*f**6*h
*x**2 + 9009*a**3*c*f**7*g*x**2 + 6435*a**3*c*f**7*h*x**3 + 1144*a**3*d*e*
*4*f**3*h - 2574*a**3*d*e**3*f**4*g - 572*a**3*d*e**3*f**4*h*x + 1287*a**3
*d*e**2*f**5*g*x + 429*a**3*d*e**2*f**5*h*x**2 + 10296*a**3*d*e*f**6*g*x**
2 + 7150*a**3*d*e*f**6*h*x**3 + 6435*a**3*d*f**7*g*x**3 + 5005*a**3*d*f**7
*h*x**4 + 3432*a**2*b*c*e**4*f**3*h - 7722*a**2*b*c*e**3*f**4*g - 1716*a**
2*b*c*e**3*f**4*h*x + 3861*a**2*b*c*e**2*f**5*g*x + 1287*a**2*b*c*e**2*f**
5*h*x**2 + 30888*a**2*b*c*e*f**6*g*x**2 + 21450*a**2*b*c*e*f**6*h*x**3 + 1
9305*a**2*b*c*f**7*g*x**3 + 15015*a**2*b*c*f**7*h*x**4 - 1872*a**2*b*d*e**
5*f**2*h + 3432*a**2*b*d*e**4*f**3*g + 936*a**2*b*d*e**4*f**3*h*x - 1716*a
**2*b*d*e**3*f**4*g*x - 702*a**2*b*d*e**3*f**4*h*x**2 + 1287*a**2*b*d*e**2
*f**5*g*x**2 + 585*a**2*b*d*e**2*f**5*h*x**3 + 21450*a**2*b*d*e*f**6*g*x**
3 + 16380*a**2*b*d*e*f**6*h*x**4 + 15015*a**2*b*d*f**7*g*x**4 + 12285*a**2
*b*d*f**7*h*x**5 - 1872*a*b**2*c*e**5*f**2*h + 3432*a*b**2*c*e**4*f**3*g +
 936*a*b**2*c*e**4*f**3*h*x - 1716*a*b**2*c*e**3*f**4*g*x - 702*a*b**2*c*e
**3*f**4*h*x**2 + 1287*a*b**2*c*e**2*f**5*g*x**2 + 585*a*b**2*c*e**2*f**5*
h*x**3 + 21450*a*b**2*c*e*f**6*g*x**3 + 16380*a*b**2*c*e*f**6*h*x**4 + 150
15*a*b**2*c*f**7*g*x**4 + 12285*a*b**2*c*f**7*h*x**5 + 1152*a*b**2*d*e**6*
f*h - 1872*a*b**2*d*e**5*f**2*g - 576*a*b**2*d*e**5*f**2*h*x + 936*a*b*...
```

3.61 $\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = -\frac{2(be-af)^2(de-cf)(fg-eh)(e+fx)^{5/2}}{5f^5} + \frac{2(be-af)(bde(3fg-4eh)-bcf(2fg-3eh)-af(dfg-2deh+cfh))(e+fx)^{7/2}}{7f^5} + \frac{2(a^2df^2h+2abf(dfg-3deh+cfh)+b^2(cf(fg-3eh)-3de(fg-2eh)))\,(e+fx)^{9/2}}{9f^5} + \frac{2b(2adfh+b(dfg-4deh+cfh))(e+fx)^{11/2}}{11f^5} + \frac{2b^2dh(e+fx)^{13/2}}{13f^5}$$

output
```
-2/5*(-a*f+b*e)^2*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(5/2)/f^5+2/7*(-a*f+b*e)*(
b*d*e*(-4*e*h+3*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+
e)^(7/2)/f^5+2/9*(a^2*d*f^2*h+2*a*b*f*(c*f*h-3*d*e*h+d*f*g)+b^2*(c*f*(-3*e
*h+f*g)-3*d*e*(-2*e*h+f*g)))*(f*x+e)^(9/2)/f^5+2/11*b*(2*a*d*f*h+b*(c*f*h-
4*d*e*h+d*f*g))*(f*x+e)^(11/2)/f^5+2/13*b^2*d*h*(f*x+e)^(13/2)/f^5
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(143a^2f^2(9cf(7fg-2eh+5fhx)+d(8e^2h+5f^2x(9g+7hx)-2ef(9g+10hx))\right)}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(143*a^2*f^2*(9*c*f*(7*f*g - 2*e*h + 5*f*h*x) + d*(8*e^
2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x))) + 26*a*b*f*(11*c*f*(8
*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) + d*(-48*e^3*h + 35
*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(22*g + 21*
h*x))) + b^2*(3*d*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) - 70*e*f^3*x^2*
(13*g + 12*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13*g + 20*h*x))
 + 13*c*f*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x)
 - 10*e*f^2*x*(22*g + 21*h*x)))))/(45045*f^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{(e+fx)^{7/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{f^4}+\frac{b(e+fx)^{9/2}(2adf}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{9/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{9f^5}+\frac{2b(e+fx)^{11/2}(2adfh+b(cfh-4deh+dfg))}{11f^5}+\frac{2(e+fx)^{7/2}(be-af)(-af(cfh-2deh+dfg)-bcf(2fg-3eh)+bde(3fg-4eh))}{7f^5}-\frac{2(e+fx)^{5/2}(be-af)^2(de-cf)(fg-eh)}{5f^5}+\frac{2b^2dh(e+fx)^{13/2}}{13f^5}$$

input
```
Int[(a + b*x)^2*(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(-2*(b*e - a*f)^2*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^5) + (2*(b
*e - a*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(2*f*g - 3*e*h) - a*f*(d*f*g - 2*
d*e*h + c*f*h))*(e + f*x)^(7/2))/(7*f^5) + (2*(a^2*d*f^2*h + 2*a*b*f*(d*f*
g - 3*d*e*h + c*f*h) + b^2*(c*f*(f*g - 3*e*h) - 3*d*e*(f*g - 2*e*h)))*(e +
 f*x)^(9/2))/(9*f^5) + (2*b*(2*a*d*f*h + b*(d*f*g - 4*d*e*h + c*f*h))*(e +
 f*x)^(11/2))/(11*f^5) + (2*b^2*d*h*(e + f*x)^(13/2))/(13*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{2dh\,b^2(fx+e)^{\frac{13}{2}}}{13}+\frac{2\left(\left(2b(af-be)d+b^2(cf-de)\right)h+d\,b^2(-eh+fg)\right)(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left((af-be)^2d+2b(af-be)(cf-de)\right)h+\left(2b(af-be\right.\right.}{9}$
default	$\frac{\frac{2dh\,b^2(fx+e)^{\frac{13}{2}}}{13}-\frac{2\left(-\left(2b(af-be)d+b^2(cf-de)\right)h+d\,b^2(eh-fg)\right)(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left((af-be)^2d+2b(af-be)(cf-de)\right)h+\left(2b(af-\right.\right.}{9}$
pseudoelliptic	$-\frac{4\left(\left(\left(-\frac{35\left(\frac{9dh\,x^2}{13}+\frac{9(ch+dg)x}{11}+cg\right)x^2b^2}{18}-5ax\left(\frac{7dh\,x^2}{11}+\frac{7(ch+dg)x}{9}+cg\right)b-\frac{7a^2\left(\frac{5dh\,x^2}{9}+\frac{5(ch+dg)x}{7}+cg\right)}{2}\right)f^4+\left(\frac{10x(}{}\right.\right.\right.}{}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-3465dh\,b^2x^4f^4-8190abd\,f^4h\,x^3-4095b^2c\,f^4h\,x^3+2520b^2de\,f^3h\,x^3-4095b^2d\,f^4g\,x^3-5005a^2d\,f^4h\,x^2\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-3465dh\,b^2x^4f^4-8190abd\,f^4h\,x^3-4095b^2c\,f^4h\,x^3+2520b^2de\,f^3h\,x^3-4095b^2d\,f^4g\,x^3-5005a^2d\,f^4h\,x^2\right.}{}$
trager	$-\frac{2\left(-3465b^2d\,f^6h\,x^6-8190abd\,f^6h\,x^5-4095b^2c\,f^6h\,x^5-4410b^2de\,f^5h\,x^5-4095b^2d\,f^6g\,x^5-5005a^2d\,f^6h\,x^4-10010a\right.}{}$
risch	$-\frac{2\left(-3465b^2d\,f^6h\,x^6-8190abd\,f^6h\,x^5-4095b^2c\,f^6h\,x^5-4410b^2de\,f^5h\,x^5-4095b^2d\,f^6g\,x^5-5005a^2d\,f^6h\,x^4-10010a\right.}{}$

input
```
int((b*x+a)^2*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^5*(1/13*d*h*b^2*(f*x+e)^(13/2)+1/11*((2*b*(a*f-b*e)*d+b^2*(c*f-d*e))*h
+d*b^2*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*(((a*f-b*e)^2*d+2*b*(a*f-b*e)*(c*f-d
*e))*h+(2*b*(a*f-b*e)*d+b^2*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((a*f
-b*e)^2*(c*f-d*e)*h+((a*f-b*e)^2*d+2*b*(a*f-b*e)*(c*f-d*e))*(-e*h+f*g))*(f
*x+e)^(7/2)+1/5*(a*f-b*e)^2*(c*f-d*e)*(-e*h+f*g)*(f*x+e)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(229) = 458$.

Time = 0.09 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.66

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3465*b^2*d*f^6*h*x^6 + 315*(13*b^2*d*f^6*g + (14*b^2*d*e*f^5 + 13
*(b^2*c + 2*a*b*d)*f^6)*h)*x^5 + 35*(13*(12*b^2*d*e*f^5 + 11*(b^2*c + 2*a*
b*d)*f^6)*g + (3*b^2*d*e^2*f^4 + 156*(b^2*c + 2*a*b*d)*e*f^5 + 143*(2*a*b*
c + a^2*d)*f^6)*h)*x^4 + 5*(13*(3*b^2*d*e^2*f^4 + 110*(b^2*c + 2*a*b*d)*e*
f^5 + 99*(2*a*b*c + a^2*d)*f^6)*g - (24*b^2*d*e^3*f^3 - 1287*a^2*c*f^6 - 3
9*(b^2*c + 2*a*b*d)*e^2*f^4 - 1430*(2*a*b*c + a^2*d)*e*f^5)*h)*x^3 - 3*(13
*(6*b^2*d*e^3*f^3 - 231*a^2*c*f^6 - 11*(b^2*c + 2*a*b*d)*e^2*f^4 - 264*(2*
a*b*c + a^2*d)*e*f^5)*g - (48*b^2*d*e^4*f^2 + 3432*a^2*c*e*f^5 - 78*(b^2*c
 + 2*a*b*d)*e^3*f^3 + 143*(2*a*b*c + a^2*d)*e^2*f^4)*h)*x^2 - 13*(48*b^2*d
*e^5*f - 693*a^2*c*e^2*f^4 - 88*(b^2*c + 2*a*b*d)*e^4*f^2 + 198*(2*a*b*c +
 a^2*d)*e^3*f^3)*g + 2*(192*b^2*d*e^6 - 1287*a^2*c*e^3*f^3 - 312*(b^2*c +
2*a*b*d)*e^5*f + 572*(2*a*b*c + a^2*d)*e^4*f^2)*h + (13*(24*b^2*d*e^4*f^2
+ 1386*a^2*c*e*f^5 - 44*(b^2*c + 2*a*b*d)*e^3*f^3 + 99*(2*a*b*c + a^2*d)*e
^2*f^4)*g - (192*b^2*d*e^5*f - 1287*a^2*c*e^2*f^4 - 312*(b^2*c + 2*a*b*d)*
e^4*f^2 + 572*(2*a*b*c + a^2*d)*e^3*f^3)*h)*x)*sqrt(f*x + e)/f^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(262) = 524$.

Time = 1.77 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.51

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{b^2dh(e+fx)^{\frac{13}{2}}}{13f^4} + \frac{(e+fx)^{\frac{11}{2}}\cdot\left(2abdfh+b^2cfh-4b^2deh+b^2dfg\right)}{11f^4} + \frac{(e+fx)^{\frac{9}{2}}\left(a^2df^2h+2abcf^2h-6abdefh+2abdf^2g-3b^2cefh+b^2cf^2g+6b^2de \cdots\right.}{9f^4} \cdots\right.}{\cdots} \\ e^{\frac{3}{2}}\left(a^2cgx + \frac{b^2dhx^5}{5} + \frac{x^4\cdot\left(2abdh+b^2ch+b^2dg\right)}{4} + \frac{x^3\left(a^2dh+2abch+2abdg+b^2cg\right)}{3} + \frac{x^2\left(a^2ch+a^2dg+2abcg\right)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**2*d*h*(e + f*x)**(13/2)/(13*f**4) + (e + f*x)**(11/2)*(2*
a*b*d*f*h + b**2*c*f*h - 4*b**2*d*e*h + b**2*d*f*g)/(11*f**4) + (e + f*x)*
*(9/2)*(a**2*d*f**2*h + 2*a*b*c*f**2*h - 6*a*b*d*e*f*h + 2*a*b*d*f**2*g -
3*b**2*c*e*f*h + b**2*c*f**2*g + 6*b**2*d*e**2*h - 3*b**2*d*e*f*g)/(9*f**4
) + (e + f*x)**(7/2)*(a**2*c*f**3*h - 2*a**2*d*e*f**2*h + a**2*d*f**3*g -
4*a*b*c*e*f**2*h + 2*a*b*c*f**3*g + 6*a*b*d*e**2*f*h - 4*a*b*d*e*f**2*g +
3*b**2*c*e**2*f*h - 2*b**2*c*e*f**2*g - 4*b**2*d*e**3*h + 3*b**2*d*e**2*f*
g)/(7*f**4) + (e + f*x)**(5/2)*(-a**2*c*e*f**3*h + a**2*c*f**4*g + a**2*d*
e**2*f**2*h - a**2*d*e*f**3*g + 2*a*b*c*e**2*f**2*h - 2*a*b*c*e*f**3*g - 2
*a*b*d*e**3*f*h + 2*a*b*d*e**2*f**2*g - b**2*c*e**3*f*h + b**2*c*e**2*f**2
*g + b**2*d*e**4*h - b**2*d*e**3*f*g)/(5*f**4))/f, Ne(f, 0)), (e**(3/2)*(a
**2*c*g*x + b**2*d*h*x**5/5 + x**4*(2*a*b*d*h + b**2*c*h + b**2*d*g)/4 + x
**3*(a**2*d*h + 2*a*b*c*h + 2*a*b*d*g + b**2*c*g)/3 + x**2*(a**2*c*h + a**
2*d*g + 2*a*b*c*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.52

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(3465\,(fx+e)^{\frac{13}{2}}b^2dh+4095\,(b^2dfg-(4\,b^2de-(b^2c+2\,abd)f)h)(fx+e)^{\frac{11}{2}}-5005\,((3\,b^2def$$

input
```
integrate((b*x+a)^2*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3465*(f*x + e)^(13/2)*b^2*d*h + 4095*(b^2*d*f*g - (4*b^2*d*e - (b
^2*c + 2*a*b*d)*f)*h)*(f*x + e)^(11/2) - 5005*((3*b^2*d*e*f - (b^2*c + 2*a
*b*d)*f^2)*g - (6*b^2*d*e^2 - 3*(b^2*c + 2*a*b*d)*e*f + (2*a*b*c + a^2*d)*
f^2)*h)*(f*x + e)^(9/2) + 6435*((3*b^2*d*e^2*f - 2*(b^2*c + 2*a*b*d)*e*f^2
 + (2*a*b*c + a^2*d)*f^3)*g - (4*b^2*d*e^3 - a^2*c*f^3 - 3*(b^2*c + 2*a*b*
d)*e^2*f + 2*(2*a*b*c + a^2*d)*e*f^2)*h)*(f*x + e)^(7/2) - 9009*((b^2*d*e^
3*f - a^2*c*f^4 - (b^2*c + 2*a*b*d)*e^2*f^2 + (2*a*b*c + a^2*d)*e*f^3)*g -
 (b^2*d*e^4 - a^2*c*e*f^3 - (b^2*c + 2*a*b*d)*e^3*f + (2*a*b*c + a^2*d)*e^
2*f^2)*h)*(f*x + e)^(5/2))/f^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1968 vs. $2(229) = 458$.

Time = 0.15 (sec) , antiderivative size = 1968, normalized size of antiderivative = 7.97

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a^2*c*e^2*g + 30030*((f*x + e)^(3/2) - 3*sqrt
(f*x + e)*e)*a^2*c*e*g + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*b*c
*e^2*g/f + 15015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*d*e^2*g/f + 150
15*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c*e^2*h/f + 3003*(3*(f*x + e)
^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c*g + 3003*(3*(f
*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b^2*c*e^2*g/f
^2 + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2
)*a*b*d*e^2*g/f^2 + 12012*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*s
qrt(f*x + e)*e^2)*a*b*c*e*g/f + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/
2)*e + 15*sqrt(f*x + e)*e^2)*a^2*d*e*g/f + 6006*(3*(f*x + e)^(5/2) - 10*(f
*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*c*e^2*h/f^2 + 3003*(3*(f*x + e
)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*d*e^2*h/f^2 + 6
006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*
c*e*h/f + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3
/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*d*e^2*g/f^3 + 2574*(5*(f*x + e)^(7/2)
- 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^
2*c*e*g/f^2 + 5148*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e
)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b*d*e*g/f^2 + 2574*(5*(f*x + e)^(7/2
) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*
a*b*c*g/f + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + ...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{11/2}\,(2\,b^2\,c\,f\,h-8\,b^2\,d\,e\,h+2\,b^2\,d\,f\,g+4\,a\,b\,d\,f\,h)}{11\,f^5} + \frac{(e+f\,x)^{9/2}\,(2\,b^2\,c\,f^2\,g+2\,a^2\,d\,f^2\,h+12\,b^2\,d\,e^2\,h+4\,a\,b\,c\,f^2\,h+4\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b^2\,d\,e\,f\,g-1}{9\,f^5} + \frac{2\,(e+f\,x)^{7/2}\,(a\,f-b\,e)\,(a\,c\,f^2\,h+a\,d\,f^2\,g+2\,b\,c\,f^2\,g+4\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\,h-3\,b\,d\,e\,f\,g)}{7\,f^5} + \frac{2\,b^2\,d\,h\,(e+f\,x)^{13/2}}{13\,f^5} - \frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)\,(e\,h-f\,g)}{5\,f^5}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^2*(c + d*x),x)
```

output
```
((e + f*x)^(11/2)*(2*b^2*c*f*h - 8*b^2*d*e*h + 2*b^2*d*f*g + 4*a*b*d*f*h))
/(11*f^5) + ((e + f*x)^(9/2)*(2*b^2*c*f^2*g + 2*a^2*d*f^2*h + 12*b^2*d*e^2
*h + 4*a*b*c*f^2*h + 4*a*b*d*f^2*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 12*a*
b*d*e*f*h))/(9*f^5) + (2*(e + f*x)^(7/2)*(a*f - b*e)*(a*c*f^2*h + a*d*f^2*
g + 2*b*c*f^2*g + 4*b*d*e^2*h - 2*a*d*e*f*h - 3*b*c*e*f*h - 3*b*d*e*f*g))/
(7*f^5) + (2*b^2*d*h*(e + f*x)^(13/2))/(13*f^5) - (2*(e + f*x)^(5/2)*(a*f
- b*e)^2*(c*f - d*e)*(e*h - f*g))/(5*f^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 821, normalized size of antiderivative = 3.32

$$\int (a+bx)^2(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\sqrt{fx+e}\,(3465b^2d\,f^6h\,x^6+8190abd\,f^6h\,x^5+4095b^2c\,f^6h\,x^5+4410b^2de\,f^5h\,x^5+4095b^2d\,f^6g}{}$$

input
```
int((b*x+a)^2*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 2574*a**2*c*e**3*f**3*h + 9009*a**2*c*e**2*f**4*g + 1
287*a**2*c*e**2*f**4*h*x + 18018*a**2*c*e*f**5*g*x + 10296*a**2*c*e*f**5*h
*x**2 + 9009*a**2*c*f**6*g*x**2 + 6435*a**2*c*f**6*h*x**3 + 1144*a**2*d*e*
*4*f**2*h - 2574*a**2*d*e**3*f**3*g - 572*a**2*d*e**3*f**3*h*x + 1287*a**2
*d*e**2*f**4*g*x + 429*a**2*d*e**2*f**4*h*x**2 + 10296*a**2*d*e*f**5*g*x**
2 + 7150*a**2*d*e*f**5*h*x**3 + 6435*a**2*d*f**6*g*x**3 + 5005*a**2*d*f**6
*h*x**4 + 2288*a*b*c*e**4*f**2*h - 5148*a*b*c*e**3*f**3*g - 1144*a*b*c*e**
3*f**3*h*x + 2574*a*b*c*e**2*f**4*g*x + 858*a*b*c*e**2*f**4*h*x**2 + 20592
*a*b*c*e*f**5*g*x**2 + 14300*a*b*c*e*f**5*h*x**3 + 12870*a*b*c*f**6*g*x**3
 + 10010*a*b*c*f**6*h*x**4 - 1248*a*b*d*e**5*f*h + 2288*a*b*d*e**4*f**2*g
+ 624*a*b*d*e**4*f**2*h*x - 1144*a*b*d*e**3*f**3*g*x - 468*a*b*d*e**3*f**3
*h*x**2 + 858*a*b*d*e**2*f**4*g*x**2 + 390*a*b*d*e**2*f**4*h*x**3 + 14300*
a*b*d*e*f**5*g*x**3 + 10920*a*b*d*e*f**5*h*x**4 + 10010*a*b*d*f**6*g*x**4
+ 8190*a*b*d*f**6*h*x**5 - 624*b**2*c*e**5*f*h + 1144*b**2*c*e**4*f**2*g +
 312*b**2*c*e**4*f**2*h*x - 572*b**2*c*e**3*f**3*g*x - 234*b**2*c*e**3*f**
3*h*x**2 + 429*b**2*c*e**2*f**4*g*x**2 + 195*b**2*c*e**2*f**4*h*x**3 + 715
0*b**2*c*e*f**5*g*x**3 + 5460*b**2*c*e*f**5*h*x**4 + 5005*b**2*c*f**6*g*x*
*4 + 4095*b**2*c*f**6*h*x**5 + 384*b**2*d*e**6*h - 624*b**2*d*e**5*f*g - 1
92*b**2*d*e**5*f*h*x + 312*b**2*d*e**4*f**2*g*x + 144*b**2*d*e**4*f**2*h*x
**2 - 234*b**2*d*e**3*f**3*g*x**2 - 120*b**2*d*e**3*f**3*h*x**3 + 195*b...
```

3.62 $\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2(be-af)(de-cf)(fg-eh)(e+fx)^{5/2}}{5f^4} - \frac{2(bde(2fg-3eh)-bcf(fg-2eh)-af(dfg-2deh+cfh))(e+fx)^{7/2}}{7f^4} + \frac{2(adfh+b(dfg-3deh+cfh))(e+fx)^{9/2}}{9f^4} + \frac{2bdh(e+fx)^{11/2}}{11f^4}$$

output
```
2/5*(-a*f+b*e)*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(5/2)/f^4-2/7*(b*d*e*(-3*e*h+
2*f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(7/2)/f^4+2/9
*(a*d*f*h+b*(c*f*h-3*d*e*h+d*f*g))*(f*x+e)^(9/2)/f^4+2/11*b*d*h*(f*x+e)^(1
1/2)/f^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(11af(9cf(7fg-2eh+5fhx)+d(8e^2h+5f^2x(9g+7hx)-2ef(9g+10hx)))+\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(11*a*f*(9*c*f*(7*f*g - 2*e*h + 5*f*h*x) + d*(8*e^2*h +
 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x))) + b*(11*c*f*(8*e^2*h + 5*f
^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) + d*(-48*e^3*h + 35*f^3*x^2*(11
*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(22*g + 21*h*x)))))/(34
65*f^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{(e+fx)^{7/2}(adfh+b(cfh-3deh+dfg))}{f^3}+\frac{(e+fx)^{5/2}(af(cfh-2deh+dfg)+bcf(fg-2eh)-bde(2fg\right.}{f^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{9/2}(adfh+b(cfh-3deh+dfg))}{9f^4} - \frac{2(e+fx)^{7/2}(-af(cfh-2deh+dfg)-bcf(fg-2eh)+bde(2fg-3eh))}{7f^4} + \frac{2(e+fx)^{5/2}(be-af)(de-cf)(fg-eh)}{5f^4} + \frac{2bdh(e+fx)^{11/2}}{11f^4}$$

input
```
Int[(a + b*x)*(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^4) - (2*(b*d*
e*(2*f*g - 3*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(d*f*g - 2*d*e*h + c*f*h))*(
e + f*x)^(7/2))/(7*f^4) + (2*(a*d*f*h + b*(d*f*g - 3*d*e*h + c*f*h))*(e +
f*x)^(9/2))/(9*f^4) + (2*b*d*h*(e + f*x)^(11/2))/(11*f^4)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{4(fx+e)^{\frac{5}{2}}\left(\frac{\left(-5x\left(\frac{7x\left(\frac{9hx}{11}+g\right)d}{9}+c\left(\frac{7hx}{9}+g\right)\right)b-7a\left(\frac{5x\left(\frac{7hx}{9}+g\right)d}{7}+c\left(\frac{5hx}{7}+g\right)\right)\right)f^{3}}{2}+\left(\left(\left(\frac{10x\left(\frac{21hx}{22}+g\right)d}{9}+c\left(\frac{10hx}{9}+\right.\right.\right.\right.\right.}{35f^{4}}$
derivativedivides	$\frac{\frac{2dbh(fx+e)^{\frac{11}{2}}}{11}+\frac{2(((af-be)d+b(cf-de))h+db(-eh+fg))(fx+e)^{\frac{9}{2}}}{9}+\frac{2((af-be)(cf-de)h+((af-be)d+b(cf-de))(-eh+fg))(fx+}{7}}{f^{4}}$
default	$\frac{\frac{2dbh(fx+e)^{\frac{11}{2}}}{11}+\frac{2(-((-af+be)d-b(cf-de))h-db(eh-fg))(fx+e)^{\frac{9}{2}}}{9}+\frac{2(-(-af+be)(cf-de)h+((-af+be)d-b(cf-de))(eh-fg)}{7}}{f^{4}}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-315bdh\,x^{3}f^{3}-385ad\,f^{3}h\,x^{2}-385bc\,f^{3}h\,x^{2}+210bde\,f^{2}h\,x^{2}-385bd\,f^{3}g\,x^{2}-495ac\,f^{3}hx+220ade\,f^{2}hx-\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-315bdh\,x^{3}f^{3}-385ad\,f^{3}h\,x^{2}-385bc\,f^{3}h\,x^{2}+210bde\,f^{2}h\,x^{2}-385bd\,f^{3}g\,x^{2}-495ac\,f^{3}hx+220ade\,f^{2}hx-\right.}{}$
trager	$-\frac{2\left(-315bd\,f^{5}h\,x^{5}-385ad\,f^{5}h\,x^{4}-385bc\,f^{5}h\,x^{4}-420bde\,f^{4}h\,x^{4}-385bd\,f^{5}g\,x^{4}-495ac\,f^{5}h\,x^{3}-550ade\,f^{4}h\,x^{3}-495a\right.}{}$
risch	$-\frac{2\left(-315bd\,f^{5}h\,x^{5}-385ad\,f^{5}h\,x^{4}-385bc\,f^{5}h\,x^{4}-420bde\,f^{4}h\,x^{4}-385bd\,f^{5}g\,x^{4}-495ac\,f^{5}h\,x^{3}-550ade\,f^{4}h\,x^{3}-495a\right.}{}$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-4/35*(f*x+e)^(5/2)*(1/2*(-5*x*(7/9*x*(9/11*h*x+g)*d+c*(7/9*h*x+g))*b-7*a*
(5/7*x*(7/9*h*x+g)*d+c*(5/7*h*x+g)))*f^3+((10/9*x*(21/22*h*x+g)*d+c*(10/9*
h*x+g))*b+((10/9*h*x+g)*d+c*h)*a)*e*f^2-4/9*(((15/11*h*x+g)*d+c*h)*b+a*d*h
)*e^2*f+8/33*b*d*e^3*h)/f^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(143) = 286.

Time = 0.08 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.19

$$\int(a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx=\frac{2\left(315\,bdf^{5}hx^{5}+35\left(11\,bdf^{5}g+\left(12\,bdef^{4}+11\left(bc+ad\right)f^{5}\right)h\right)x^{4}+5\left(11\left(10\,bdef^{4}+9\left(bc+ad\right)\right.\right.\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/3465*(315*b*d*f^5*h*x^5 + 35*(11*b*d*f^5*g + (12*b*d*e*f^4 + 11*(b*c + a
*d)*f^5)*h)*x^4 + 5*(11*(10*b*d*e*f^4 + 9*(b*c + a*d)*f^5)*g + (3*b*d*e^2*
f^3 + 99*a*c*f^5 + 110*(b*c + a*d)*e*f^4)*h)*x^3 + 3*(11*(b*d*e^2*f^3 + 21
*a*c*f^5 + 24*(b*c + a*d)*e*f^4)*g - (6*b*d*e^3*f^2 - 264*a*c*e*f^4 - 11*(
b*c + a*d)*e^2*f^3)*h)*x^2 + 11*(8*b*d*e^4*f + 63*a*c*e^2*f^3 - 18*(b*c +
a*d)*e^3*f^2)*g - 2*(24*b*d*e^5 + 99*a*c*e^3*f^2 - 44*(b*c + a*d)*e^4*f)*h
 - (11*(4*b*d*e^3*f^2 - 126*a*c*e*f^4 - 9*(b*c + a*d)*e^2*f^3)*g - (24*b*d
*e^4*f + 99*a*c*e^2*f^3 - 44*(b*c + a*d)*e^3*f^2)*h)*x)*sqrt(f*x + e)/f^4
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.91

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{bdh(e+fx)^{\frac{11}{2}}}{11f^3}+\frac{(e+fx)^{\frac{9}{2}}(adfh+bcfh-3bdeh+bdfg)}{9f^3}+\frac{(e+fx)^{\frac{7}{2}}\left(acf^2h-2adefh+adf^2g-2bcefh+bcf^2g+3bde^2h-2bdefg\right)}{7f^3}+\frac{(e+fx)^{\frac{5}{2}}(-\cdots}{\cdots}\right)}{f} \\ e^{\frac{3}{2}}\left(acgx+\frac{bdhx^4}{4}+\frac{x^3(adh+bch+bdg)}{3}+\frac{x^2(ach+adg+bcg)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b*d*h*(e + f*x)**(11/2)/(11*f**3) + (e + f*x)**(9/2)*(a*d*f*
h + b*c*f*h - 3*b*d*e*h + b*d*f*g)/(9*f**3) + (e + f*x)**(7/2)*(a*c*f**2*h
 - 2*a*d*e*f*h + a*d*f**2*g - 2*b*c*e*f*h + b*c*f**2*g + 3*b*d*e**2*h - 2*
b*d*e*f*g)/(7*f**3) + (e + f*x)**(5/2)*(-a*c*e*f**2*h + a*c*f**3*g + a*d*e
**2*f*h - a*d*e*f**2*g + b*c*e**2*f*h - b*c*e*f**2*g - b*d*e**3*h + b*d*e*
*2*f*g)/(5*f**3))/f, Ne(f, 0)), (e**(3/2)*(a*c*g*x + b*d*h*x**4/4 + x**3*(
a*d*h + b*c*h + b*d*g)/3 + x**2*(a*c*h + a*d*g + b*c*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.13

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(315\,(fx+e)^{\frac{11}{2}}bdh+385\,(bdfg-(3\,bde-(bc+ad)f)h)(fx+e)^{\frac{9}{2}}-495\,((2\,bdef-(bc+ad$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/3465*(315*(f*x + e)^(11/2)*b*d*h + 385*(b*d*f*g - (3*b*d*e - (b*c + a*d)
*f)*h)*(f*x + e)^(9/2) - 495*((2*b*d*e*f - (b*c + a*d)*f^2)*g - (3*b*d*e^2
 + a*c*f^2 - 2*(b*c + a*d)*e*f)*h)*(f*x + e)^(7/2) + 693*((b*d*e^2*f + a*c
*f^3 - (b*c + a*d)*e*f^2)*g - (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h)
*(f*x + e)^(5/2))/f^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. 2(143) = 286.

Time = 0.14 (sec) , antiderivative size = 1124, normalized size of antiderivative = 7.07

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(f*x + e)*a*c*e^2*g + 2310*((f*x + e)^(3/2) - 3*sqrt(f*x
+ e)*e)*a*c*e*g + 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*b*c*e^2*g/f +
 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*d*e^2*g/f + 1155*((f*x + e)^
(3/2) - 3*sqrt(f*x + e)*e)*a*c*e^2*h/f + 231*(3*(f*x + e)^(5/2) - 10*(f*x
+ e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c*g + 231*(3*(f*x + e)^(5/2) - 10*(
f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*d*e^2*g/f^2 + 462*(3*(f*x + e)^
(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*e*g/f + 462*(3*(f
*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*d*e*g/f + 2
31*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*e
^2*h/f^2 + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e
)*e^2)*a*d*e^2*h/f^2 + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*
sqrt(f*x + e)*e^2)*a*c*e*h/f + 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)
*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*d*e*g/f^2 + 99*(5*(f
*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*
x + e)*e^3)*b*c*g/f + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f
*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*d*g/f + 99*(5*(f*x + e)^(7/2)
- 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*
d*e^2*h/f^3 + 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)
^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*c*e*h/f^2 + 198*(5*(f*x + e)^(7/2) -
21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a...
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{7/2}\,(2\,a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-4\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f\,g)}{7\,f^4} + \frac{(e+f\,x)^{9/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+2\,b\,d\,f\,g)}{9\,f^4} - \frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)\,(c\,f-d\,e)\,(e\,h-f\,g)}{5\,f^4} + \frac{2\,b\,d\,h\,(e+f\,x)^{11/2}}{11\,f^4}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

output
```
((e + f*x)^(7/2)*(2*a*c*f^2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 6*b*d*e^2*h -
4*a*d*e*f*h - 4*b*c*e*f*h - 4*b*d*e*f*g))/(7*f^4) + ((e + f*x)^(9/2)*(2*a*
d*f*h + 2*b*c*f*h - 6*b*d*e*h + 2*b*d*f*g))/(9*f^4) - (2*(e + f*x)^(5/2)*(
a*f - b*e)*(c*f - d*e)*(e*h - f*g))/(5*f^4) + (2*b*d*h*(e + f*x)^(11/2))/(
11*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.67

$$\int (a+bx)(c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\sqrt{fx+e}\,(315bd\,f^5h\,x^5+385ad\,f^5h\,x^4+385bc\,f^5h\,x^4+420bde\,f^4h\,x^4+385bd\,f^5g\,x^4+495ac$$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)^(3/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 198*a*c*e**3*f**2*h + 693*a*c*e**2*f**3*g + 99*a*c*e*
*2*f**3*h*x + 1386*a*c*e*f**4*g*x + 792*a*c*e*f**4*h*x**2 + 693*a*c*f**5*g
*x**2 + 495*a*c*f**5*h*x**3 + 88*a*d*e**4*f*h - 198*a*d*e**3*f**2*g - 44*a
*d*e**3*f**2*h*x + 99*a*d*e**2*f**3*g*x + 33*a*d*e**2*f**3*h*x**2 + 792*a*
d*e*f**4*g*x**2 + 550*a*d*e*f**4*h*x**3 + 495*a*d*f**5*g*x**3 + 385*a*d*f*
*5*h*x**4 + 88*b*c*e**4*f*h - 198*b*c*e**3*f**2*g - 44*b*c*e**3*f**2*h*x +
 99*b*c*e**2*f**3*g*x + 33*b*c*e**2*f**3*h*x**2 + 792*b*c*e*f**4*g*x**2 +
550*b*c*e*f**4*h*x**3 + 495*b*c*f**5*g*x**3 + 385*b*c*f**5*h*x**4 - 48*b*d
*e**5*h + 88*b*d*e**4*f*g + 24*b*d*e**4*f*h*x - 44*b*d*e**3*f**2*g*x - 18*
b*d*e**3*f**2*h*x**2 + 33*b*d*e**2*f**3*g*x**2 + 15*b*d*e**2*f**3*h*x**3 +
 550*b*d*e*f**4*g*x**3 + 420*b*d*e*f**4*h*x**4 + 385*b*d*f**5*g*x**4 + 315
*b*d*f**5*h*x**5))/(3465*f**4)
```

3.63 $\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = -\frac{2(de-cf)(fg-eh)(e+fx)^{5/2}}{5f^3} + \frac{2(dfg-2deh+cfh)(e+fx)^{7/2}}{7f^3} + \frac{2dh(e+fx)^{9/2}}{9f^3}$$

output
```
-2/5*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(5/2)/f^3+2/7*(c*f*h-2*d*e*h+d*f*g)*(f*
x+e)^(7/2)/f^3+2/9*d*h*(f*x+e)^(9/2)/f^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(9cf(7fg-2eh+5fhx)+d(8e^2h+5f^2x(9g+7hx)-2ef(9g+10hx))\right)}{315f^3}$$

input
```
Integrate[(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(9*c*f*(7*f*g - 2*e*h + 5*f*h*x) + d*(8*e^2*h + 5*f^2*x
*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x))))/(315*f^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{(e+fx)^{5/2}(cfh-2deh+dfg)}{f^2}+\frac{(e+fx)^{3/2}(cf-de)(fg-eh)}{f^2}+\frac{dh(e+fx)^{7/2}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{7/2}(cfh-2deh+dfg)}{7f^3}-\frac{2(e+fx)^{5/2}(de-cf)(fg-eh)}{5f^3}+\frac{2dh(e+fx)^{9/2}}{9f^3}$$

input
```
Int[(c + d*x)*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(-2*(d*e - c*f)*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^3) + (2*(d*f*g - 2*d*e*h
 + c*f*h)*(e + f*x)^(7/2))/(7*f^3) + (2*d*h*(e + f*x)^(9/2))/(9*f^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{4\left(\frac{\left(-5x\left(\frac{7hx}{9}+g\right)d-7c\left(\frac{5hx}{7}+g\right)\right)f^{2}}{2}+e\left(\left(\frac{10hx}{9}+g\right)d+ch\right)f-\frac{4d\,e^{2}h}{9}\right)(fx+e)^{\frac{5}{2}}}{35f^{3}}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-35dh\,x^{2}f^{2}-45c\,f^{2}hx+20defhx-45d\,f^{2}gx+18cefh-63cg\,f^{2}-8d\,e^{2}h+18defg\right)}{315f^{3}}$
derivativedivides	$\frac{\frac{2hd(fx+e)^{\frac{9}{2}}}{9}+\frac{2((cf-de)h+d(-eh+fg))(fx+e)^{\frac{7}{2}}}{7}+\frac{2(cf-de)(-eh+fg)(fx+e)^{\frac{5}{2}}}{5}}{f^{3}}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-35dh\,x^{2}f^{2}-45c\,f^{2}hx+20defhx-45d\,f^{2}gx+18cefh-63cg\,f^{2}-8d\,e^{2}h+18defg\right)}{315f^{3}}$
default	$\frac{\frac{2hd(fx+e)^{\frac{9}{2}}}{9}+\frac{2(-(-cf+de)h-d(eh-fg))(fx+e)^{\frac{7}{2}}}{7}+\frac{2(-cf+de)(eh-fg)(fx+e)^{\frac{5}{2}}}{5}}{f^{3}}$
trager	$-\frac{2\left(-35d\,f^{4}h\,x^{4}-45c\,f^{4}h\,x^{3}-50de\,f^{3}h\,x^{3}-45d\,f^{4}g\,x^{3}-72ce\,f^{3}h\,x^{2}-63c\,f^{4}g\,x^{2}-3d\,e^{2}f^{2}h\,x^{2}-72de\,f^{3}g\,x^{2}-9c\,e^{2}f \cdots\right.}{315f^{3}}$
risch	$-\frac{2\left(-35d\,f^{4}h\,x^{4}-45c\,f^{4}h\,x^{3}-50de\,f^{3}h\,x^{3}-45d\,f^{4}g\,x^{3}-72ce\,f^{3}h\,x^{2}-63c\,f^{4}g\,x^{2}-3d\,e^{2}f^{2}h\,x^{2}-72de\,f^{3}g\,x^{2}-9c\,e^{2}f \cdots\right.}{315f^{3}}$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-4/35*(1/2*(-5*x*(7/9*h*x+g)*d-7*c*(5/7*h*x+g))*f^2+e*((10/9*h*x+g)*d+c*h)
*f-4/9*d*e^2*h)*(f*x+e)^(5/2)/f^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(69) = 138$.

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.12

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(35\,df^4hx^4+5\left(9\,df^4g+\left(10\,def^3+9\,cf^4\right)h\right)x^3+3\left(3\left(8\,def^3+7\,cf^4\right)g+\left(de^2f^2+24\,cef^3\right)h\right)\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/315*(35*d*f^4*h*x^4 + 5*(9*d*f^4*g + (10*d*e*f^3 + 9*c*f^4)*h)*x^3 + 3*(
3*(8*d*e*f^3 + 7*c*f^4)*g + (d*e^2*f^2 + 24*c*e*f^3)*h)*x^2 - 9*(2*d*e^3*f
 - 7*c*e^2*f^2)*g + 2*(4*d*e^4 - 9*c*e^3*f)*h + (9*(d*e^2*f^2 + 14*c*e*f^3
)*g - (4*d*e^3*f - 9*c*e^2*f^2)*h)*x)*sqrt(f*x + e)/f^3
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.53

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{dh(e+fx)^{\frac{9}{2}}}{9f^2}+\frac{(e+fx)^{\frac{7}{2}}(cfh-2deh+dfg)}{7f^2}+\frac{(e+fx)^{\frac{5}{2}}\left(-cefh+cf^2g+de^2h-defg\right)}{5f^2}\right)}{f} & \text{for } f\neq 0 \\ e^{\frac{3}{2}}\left(cgx+\frac{dhx^3}{3}+\frac{x^2(ch+dg)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(9/2)/(9*f**2) + (e + f*x)**(7/2)*(c*f*h - 2*
d*e*h + d*f*g)/(7*f**2) + (e + f*x)**(5/2)*(-c*e*f*h + c*f**2*g + d*e**2*h
 - d*e*f*g)/(5*f**2))/f, Ne(f, 0)), (e**(3/2)*(c*g*x + d*h*x**3/3 + x**2*(
c*h + d*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(35\,(fx+e)^{\frac{9}{2}}dh+45\,(dfg-(2\,de-cf)h)(fx+e)^{\frac{7}{2}}-63\,((def-cf^2)g-(de^2-cef)h)(fx$$

$$315\,f^3$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/315*(35*(f*x + e)^(9/2)*d*h + 45*(d*f*g - (2*d*e - c*f)*h)*(f*x + e)^(7/
2) - 63*((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h)*(f*x + e)^(5/2))/f^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(69) = 138.

Time = 0.12 (sec) , antiderivative size = 473, normalized size of antiderivative = 5.84

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(315\,\sqrt{fx+e}ce^2g+210\,\left((fx+e)^{\frac{3}{2}}-3\,\sqrt{fx+e}e\right)ceg+\frac{105\,\left((fx+e)^{\frac{3}{2}}-3\,\sqrt{fx+e}e\right)de^2g}{f}+\frac{105\,\left((fx-\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(f*x + e)*c*e^2*g + 210*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*
e)*c*e*g + 105*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*d*e^2*g/f + 105*((f*x
 + e)^(3/2) - 3*sqrt(f*x + e)*e)*c*e^2*h/f + 21*(3*(f*x + e)^(5/2) - 10*(f
*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*g + 42*(3*(f*x + e)^(5/2) - 10*(
f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*d*e*g/f + 21*(3*(f*x + e)^(5/2) -
 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*d*e^2*h/f^2 + 42*(3*(f*x + e
)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*e*h/f + 9*(5*(f*x
 + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x
+ e)*e^3)*d*g/f + 18*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x +
 e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*d*e*h/f^2 + 9*(5*(f*x + e)^(7/2) - 2
1*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*c*h/f
 + (35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*x + e)^(5/2)*e^2 -
 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*d*h/f^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\,{\left(e+f\,x\right)}^{5/2}\,\left(35\,d\,h\,{\left(e+f\,x\right)}^2+63\,c\,f^2\,g+63\,d\,e^2\,h+45\,c\,f\,h\,\left(e+f\,x\right)-90\,d\,e\,h\,\left(e+f\,x\right)+\right.}{315\,f^3}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(c + d*x),x)
```

output
```
(2*(e + f*x)^(5/2)*(35*d*h*(e + f*x)^2 + 63*c*f^2*g + 63*d*e^2*h + 45*c*f*
h*(e + f*x) - 90*d*e*h*(e + f*x) + 45*d*f*g*(e + f*x) - 63*c*e*f*h - 63*d*
e*f*g))/(315*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int (c+dx)(e+fx)^{3/2}(g+hx)\,dx = \frac{2\sqrt{fx+e}\,(35d\,f^4h\,x^4+45c\,f^4h\,x^3+50de\,f^3h\,x^3+45d\,f^4g\,x^3+72ce\,f^3h\,x^2+63c\,f^4g\,x^2+3d}{}$$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 18*c*e**3*f*h + 63*c*e**2*f**2*g + 9*c*e**2*f**2*h*x
+ 126*c*e*f**3*g*x + 72*c*e*f**3*h*x**2 + 63*c*f**4*g*x**2 + 45*c*f**4*h*x
**3 + 8*d*e**4*h - 18*d*e**3*f*g - 4*d*e**3*f*h*x + 9*d*e**2*f**2*g*x + 3*
d*e**2*f**2*h*x**2 + 72*d*e*f**3*g*x**2 + 50*d*e*f**3*h*x**3 + 45*d*f**4*g
*x**3 + 35*d*f**4*h*x**4))/(315*f**3)
```

3.64 $\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 196

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \frac{2(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}}{b^4} + \frac{2(bc-ad)(bg-ah)(e+fx)^{3/2}}{3b^3} - \frac{2(d(be+af)h-bf(dg+ch))(e+fx)^{5/2}}{5b^2f^2} + \frac{2dh(e+fx)^{7/2}}{7bf^2} - \frac{2(bc-ad)(be-af)^{3/2}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{9/2}}$$

output

```
2*(-a*d+b*c)*(-a*f+b*e)*(-a*h+b*g)*(f*x+e)^(1/2)/b^4+2/3*(-a*d+b*c)*(-a*h+
b*g)*(f*x+e)^(3/2)/b^3-2/5*(d*(a*f+b*e)*h-b*f*(c*h+d*g))*(f*x+e)^(5/2)/b^2
/f^2+2/7*d*h*(f*x+e)^(7/2)/b/f^2-2*(-a*d+b*c)*(-a*f+b*e)^(3/2)*(-a*h+b*g)*
arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \frac{2\sqrt{e+fx}(-105a^3df^3h+35a^2bf^2(3cfh+d(3fg+4eh+fhx))-7a}{}$$

$$+\frac{2(bc-ad)(-be+af)^{3/2}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{9/2}}$$

input
```
Integrate[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x),x]
```

output
```
(2*Sqrt[e + f*x]*(-105*a^3*d*f^3*h + 35*a^2*b*f^2*(3*c*f*h + d*(3*f*g + 4*
e*h + f*h*x)) - 7*a*b^2*f*(5*c*f*(3*f*g + 4*e*h + f*h*x) + d*(3*e^2*h + f^
2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))) + b^3*(-3*d*(e + f*x)^2*(-7*f*g +
 2*e*h - 5*f*h*x) + 7*c*f*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h
*x)))))/(105*b^4*f^2) + (2*(b*c - a*d)*(-(b*e) + a*f)^(3/2)*(b*g - a*h)*Ar
cTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx$$

$$\Big\downarrow \text{164}$$

$$\frac{(bc-ad)(bg-ah)\int \frac{(e+fx)^{3/2}}{a+bx}dx}{b^2}-\frac{2(e+fx)^{5/2}(7adfh-7bf(ch+dg)+2bdeh-5bdfhx)}{35b^2f^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{(be-af)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}-\frac{2(e+fx)^{5/2}(7adfh-7bf(ch+dg)+2bdeh-5bdfhx)}{35b^2f^2}$$

$$\Big\downarrow \ 60$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}-\frac{2(e+fx)^{5/2}(7adfh-7bf(ch+dg)+2bdeh-5bdfhx)}{35b^2f^2}$$

$$\Big\downarrow \ 73$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{(be-af)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}-\frac{2(e+fx)^{5/2}(7adfh-7bf(ch+dg)+2bdeh-5bdfhx)}{35b^2f^2}$$

$$\Big\downarrow \ 221$$

$$\frac{(bc-ad)(bg-ah)\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}-\frac{2(e+fx)^{5/2}(7adfh-7bf(ch+dg)+2bdeh-5bdfhx)}{35b^2f^2}$$

input
```
Int[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x),x]
```

output
```
(-2*(e + f*x)^(5/2)*(2*b*d*e*h + 7*a*d*f*h - 7*b*f*(d*g + c*h) - 5*b*d*f*h
*x))/(35*b^2*f^2) + ((b*c - a*d)*(b*g - a*h)*((2*(e + f*x)^(3/2))/(3*b) +
((b*e - a*f)*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sq
rt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b))/b^2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{(af-be)b}\left(\left(\left(-\frac{\left(\frac{5}{7}dfx+cf-\frac{2}{7}de\right)(fx+e)^2b^3}{5}+\frac{4a\left(\frac{3d\,f^2x^2}{20}+\left(\frac{1}{4}c\,f^2+\frac{3}{10}def\right)x+e\left(cf+\frac{3de}{20}\right)\right)f\,b^2}{3}-a^2f^2\left(\frac{1}{3}dfx+cf\right.\right.\right.\right.\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{hd(fx+e)^{\frac{7}{2}}b^3}{7}+\frac{a\,b^2dfh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3cfh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^3deh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3dfg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2bd\,f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{a\,b^2c\,f^2h(fx\right.}{3}}{}$
default	$-\frac{2\left(-\frac{hd(fx+e)^{\frac{7}{2}}b^3}{7}+\frac{a\,b^2dfh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3cfh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^3deh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3dfg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2bd\,f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{a\,b^2c\,f^2h(fx\right.}{3}}{}$
risch	$-\frac{2\left(-15d\,f^3h\,b^3x^3+21a\,b^2d\,f^3h\,x^2-21b^3c\,f^3h\,x^2-24b^3de\,f^2h\,x^2-21b^3d\,f^3g\,x^2-35a^2bd\,f^3hx+35a\,b^2c\,f^3hx+42a\,b\right.}{}$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*(((a*f-b*e)*b)^(1/2)*((-1/5*(5/7*d*f*x+c*f-2/7*d*e)*(f*x+e)^2*b^3+4/3*a
*(3/20*d*f^2*x^2+(1/4*c*f^2+3/10*d*e*f)*x+e*(c*f+3/20*d*e))*f*b^2-a^2*f^2*
(1/3*d*f*x+c*f+4/3*d*e)*b+a^3*d*f^3)*h-((1/5*d*f^2*x^2+(2/5*d*e*f+1/3*c*f^
2)*x+1/5*d*e^2+4/3*c*e*f)*b^2-a*f*(1/3*d*f*x+c*f+4/3*d*e)*b+a^2*d*f^2)*b*g
*f)*(f*x+e)^(1/2)-f^2*(a*f-b*e)^2*(a*h-b*g)*(a*d-b*c)*arctan(b*(f*x+e)^(1/
2)/((a*f-b*e)*b)^(1/2)))/((a*f-b*e)*b)^(1/2)/f^2/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(172) = 344.

Time = 0.10 (sec) , antiderivative size = 854, normalized size of antiderivative = 4.36

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/105*(105*(((b^3*c - a*b^2*d)*e*f^2 - (a*b^2*c - a^2*b*d)*f^3)*g - ((a*
b^2*c - a^2*b*d)*e*f^2 - (a^2*b*c - a^3*d)*f^3)*h)*sqrt((b*e - a*f)/b)*log
((b*f*x + 2*b*e - a*f + 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a))
- 2*(15*b^3*d*f^3*h*x^3 + 3*(7*b^3*d*f^3*g + (8*b^3*d*e*f^2 + 7*(b^3*c - a
*b^2*d)*f^3)*h)*x^2 + 7*(3*b^3*d*e^2*f + 20*(b^3*c - a*b^2*d)*e*f^2 - 15*(
a*b^2*c - a^2*b*d)*f^3)*g - (6*b^3*d*e^3 - 21*(b^3*c - a*b^2*d)*e^2*f + 14
0*(a*b^2*c - a^2*b*d)*e*f^2 - 105*(a^2*b*c - a^3*d)*f^3)*h + (7*(6*b^3*d*e
*f^2 + 5*(b^3*c - a*b^2*d)*f^3)*g + (3*b^3*d*e^2*f + 42*(b^3*c - a*b^2*d)*
e*f^2 - 35*(a*b^2*c - a^2*b*d)*f^3)*h)*x)*sqrt(f*x + e))/(b^4*f^2), -2/105
*(105*(((b^3*c - a*b^2*d)*e*f^2 - (a*b^2*c - a^2*b*d)*f^3)*g - ((a*b^2*c -
 a^2*b*d)*e*f^2 - (a^2*b*c - a^3*d)*f^3)*h)*sqrt(-(b*e - a*f)/b)*arctan(-s
qrt(f*x + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) - (15*b^3*d*f^3*h*x^3 + 3
*(7*b^3*d*f^3*g + (8*b^3*d*e*f^2 + 7*(b^3*c - a*b^2*d)*f^3)*h)*x^2 + 7*(3*
b^3*d*e^2*f + 20*(b^3*c - a*b^2*d)*e*f^2 - 15*(a*b^2*c - a^2*b*d)*f^3)*g -
 (6*b^3*d*e^3 - 21*(b^3*c - a*b^2*d)*e^2*f + 140*(a*b^2*c - a^2*b*d)*e*f^2
 - 105*(a^2*b*c - a^3*d)*f^3)*h + (7*(6*b^3*d*e*f^2 + 5*(b^3*c - a*b^2*d)*
f^3)*g + (3*b^3*d*e^2*f + 42*(b^3*c - a*b^2*d)*e*f^2 - 35*(a*b^2*c - a^2*b
*d)*f^3)*h)*x)*sqrt(f*x + e))/(b^4*f^2)]
```

Sympy [A] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \begin{cases} \dfrac{2\left(\frac{dh(e+fx)^{\frac{7}{2}}}{7bf} + \frac{(e+fx)^{\frac{5}{2}}(-adfh+bcfh-bdeh+bdfg)}{5b^2f} + \frac{(e+fx)^{\frac{3}{2}}\left(a^2dfh-abcfh-abdfg+b^2cfg\right)}{3b^3} + \right.}{} \\ e^{\frac{3}{2}}\left(\frac{dhx^2}{2b} + \frac{x(-adh+bch+bdg)}{b^2} + \dfrac{(ad-bc)(ah-bg)\left(\begin{cases} \frac{x}{a} & \text{for } b=0 \\ \frac{\log(a+bx)}{b} & \text{otherwise} \end{cases}\right)}{b^2} \right. \end{cases}$$

input
```
integrate((d*x+c)*(f*x+e)**(3/2)*(h*x+g)/(b*x+a),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(7/2)/(7*b*f) + (e + f*x)**(5/2)*(-a*d*f*h +
b*c*f*h - b*d*e*h + b*d*f*g)/(5*b**2*f) + (e + f*x)**(3/2)*(a**2*d*f*h - a
*b*c*f*h - a*b*d*f*g + b**2*c*f*g)/(3*b**3) + sqrt(e + f*x)*(-a**3*d*f**2*
h + a**2*b*c*f**2*h + a**2*b*d*e*f*h + a**2*b*d*f**2*g - a*b**2*c*e*f*h -
a*b**2*c*f**2*g - a*b**2*d*e*f*g + b**3*c*e*f*g)/b**4 + f*(a*d - b*c)*(a*f
 - b*e)**2*(a*h - b*g)*atan(sqrt(e + f*x)/sqrt((a*f - b*e)/b))/(b**5*sqrt(
(a*f - b*e)/b)))/f, Ne(f, 0)), (e**(3/2)*(d*h*x**2/(2*b) + x*(-a*d*h + b*c
*h + b*d*g)/b**2 + (a*d - b*c)*(a*h - b*g)*Piecewise((x/a, Eq(b, 0)), (log
(a + b*x)/b, True))/b**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.54

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \frac{2\left(b^4ce^2g - ab^3de^2g - 2\,ab^3cefg + 2\,a^2b^2defg + a^2b^2cf^2g - a^3bdf^2g\right.}{}$$
$$+ \frac{2\left(21\,(fx+e)^{\frac{5}{2}}b^6df^{13}g + 35\,(fx+e)^{\frac{3}{2}}b^6cf^{14}g - 35\,(fx+e)^{\frac{3}{2}}ab^5df^{14}g + 105\,\sqrt{fx+e}b^6cef^{14}g - 105\,\sqrt{f}\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x, algorithm="giac")
```

output
```
2*(b^4*c*e^2*g - a*b^3*d*e^2*g - 2*a*b^3*c*e*f*g + 2*a^2*b^2*d*e*f*g + a^2
*b^2*c*f^2*g - a^3*b*d*f^2*g - a*b^3*c*e^2*h + a^2*b^2*d*e^2*h + 2*a^2*b^2
*c*e*f*h - 2*a^3*b*d*e*f*h - a^3*b*c*f^2*h + a^4*d*f^2*h)*arctan(sqrt(f*x
+ e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^4) + 2/105*(21*(f*x +
 e)^(5/2)*b^6*d*f^13*g + 35*(f*x + e)^(3/2)*b^6*c*f^14*g - 35*(f*x + e)^(3
/2)*a*b^5*d*f^14*g + 105*sqrt(f*x + e)*b^6*c*e*f^14*g - 105*sqrt(f*x + e)*
a*b^5*d*e*f^14*g - 105*sqrt(f*x + e)*a*b^5*c*f^15*g + 105*sqrt(f*x + e)*a^
2*b^4*d*f^15*g + 15*(f*x + e)^(7/2)*b^6*d*f^12*h - 21*(f*x + e)^(5/2)*b^6*
d*e*f^12*h + 21*(f*x + e)^(5/2)*b^6*c*f^13*h - 21*(f*x + e)^(5/2)*a*b^5*d*
f^13*h - 35*(f*x + e)^(3/2)*a*b^5*c*f^14*h + 35*(f*x + e)^(3/2)*a^2*b^4*d*
f^14*h - 105*sqrt(f*x + e)*a*b^5*c*e*f^14*h + 105*sqrt(f*x + e)*a^2*b^4*d*
e*f^14*h + 105*sqrt(f*x + e)*a^2*b^4*c*f^15*h - 105*sqrt(f*x + e)*a^3*b^3*
d*f^15*h)/(b^7*f^14)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.54

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = (e+f\,x)^{5/2}\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{5\,b\,f^2}-\frac{2\,d\,h\,(a\,f^3-b\,e\,f^2)}{5\,b^2\,f^4}\right)$$

$$-(e+f\,x)^{3/2}\left(\frac{2\,(c\,f-d\,e)\,(e\,h-f\,g)}{3\,b\,f^2}+\frac{(a\,f^3-b\,e\,f^2)\,\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{b\,f^2}-\frac{2\,d\,h\,(a\,f^3-b\,e\,f^2)}{b^2\,f^4}\right)}{3\,b\,f^2}\right)+\frac{2\,d\,h\,(e\,\cdots}{7\,b\,\cdots}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x))/(a + b*x),x)
```

output
```
(e + f*x)^(5/2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(5*b*f^2) - (2*d*h*(a*f^3 -
 b*e*f^2))/(5*b^2*f^4)) - (e + f*x)^(3/2)*((2*(c*f - d*e)*(e*h - f*g))/(3*
b*f^2) + ((a*f^3 - b*e*f^2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(b*f^2) - (2*d*
h*(a*f^3 - b*e*f^2))/(b^2*f^4)))/(3*b*f^2)) + (2*d*h*(e + f*x)^(7/2))/(7*b
*f^2) + (2*atan((b^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)*(a*f - b*e)^(3/2)*(a*
h - b*g))/(b^4*c*e^2*g + a^4*d*f^2*h - a*b^3*c*e^2*h - a*b^3*d*e^2*g - a^3
*b*c*f^2*h - a^3*b*d*f^2*g + a^2*b^2*c*f^2*g + a^2*b^2*d*e^2*h - 2*a*b^3*c
*e*f*g - 2*a^3*b*d*e*f*h + 2*a^2*b^2*c*e*f*h + 2*a^2*b^2*d*e*f*g))*(a*d -
b*c)*(a*f - b*e)^(3/2)*(a*h - b*g))/b^(9/2) + ((e + f*x)^(1/2)*((2*(c*f -
d*e)*(e*h - f*g))/(b*f^2) + ((a*f^3 - b*e*f^2)*((2*c*f*h - 4*d*e*h + 2*d*f
*g)/(b*f^2) - (2*d*h*(a*f^3 - b*e*f^2))/(b^2*f^4)))/(b*f^2))*(a*f^3 - b*e*
f^2))/(b*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 835, normalized size of antiderivative = 4.26

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x)
```

output
```
(2*(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**3*d*f**3*h - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*f**3*h - 105*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*e*f**2*h - 105*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**2*b*d*f**3*g + 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a*b**2*c*e*f**2*h + 105*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*f**3*g + 105*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2
*d*e*f**2*g - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*b**3*c*e*f**2*g - 105*sqrt(e + f*x)*a**3*b*d*f**3*h + 10
5*sqrt(e + f*x)*a**2*b**2*c*f**3*h + 140*sqrt(e + f*x)*a**2*b**2*d*e*f**2*
h + 105*sqrt(e + f*x)*a**2*b**2*d*f**3*g + 35*sqrt(e + f*x)*a**2*b**2*d*f*
*3*h*x - 140*sqrt(e + f*x)*a*b**3*c*e*f**2*h - 105*sqrt(e + f*x)*a*b**3*c*
f**3*g - 35*sqrt(e + f*x)*a*b**3*c*f**3*h*x - 21*sqrt(e + f*x)*a*b**3*d*e*
*2*f*h - 140*sqrt(e + f*x)*a*b**3*d*e*f**2*g - 42*sqrt(e + f*x)*a*b**3*d*e
*f**2*h*x - 35*sqrt(e + f*x)*a*b**3*d*f**3*g*x - 21*sqrt(e + f*x)*a*b**3*d
*f**3*h*x**2 + 21*sqrt(e + f*x)*b**4*c*e**2*f*h + 140*sqrt(e + f*x)*b**4*c
*e*f**2*g + 42*sqrt(e + f*x)*b**4*c*e*f**2*h*x + 35*sqrt(e + f*x)*b**4*c*f
**3*g*x + 21*sqrt(e + f*x)*b**4*c*f**3*h*x**2 - 6*sqrt(e + f*x)*b**4*d*...
```

3.65 $\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 247

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2} \, dx = \frac{(7a^2dfh + b^2(2deg + 3cfg + 2ceh) - ab(5dfg + 4deh + 5cfh))\sqrt{e+}}{b^4}$$
$$+ \frac{2(bdg + bch - 2adh)(e+fx)^{3/2}}{3b^3} - \frac{(bc-ad)(bg-ah)(e+fx)^{3/2}}{b^3(a+bx)} + \frac{2dh(e+fx)^{5/2}}{5b^2f}$$
$$- \frac{\sqrt{be-af}(7a^2dfh + b^2(2deg + 3cfg + 2ceh) - ab(5dfg + 4deh + 5cfh)) \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{9/2}}$$

output
```
(7*a^2*d*f*h+b^2*(2*c*e*h+3*c*f*g+2*d*e*g)-a*b*(5*c*f*h+4*d*e*h+5*d*f*g))*
(f*x+e)^(1/2)/b^4+2/3*(-2*a*d*h+b*c*h+b*d*g)*(f*x+e)^(3/2)/b^3-(-a*d+b*c)*
(-a*h+b*g)*(f*x+e)^(3/2)/b^3/(b*x+a)+2/5*d*h*(f*x+e)^(5/2)/b^2/f-(-a*f+b*e
)^(1/2)*(7*a^2*d*f*h+b^2*(2*c*e*h+3*c*f*g+2*d*e*g)-a*b*(5*c*f*h+4*d*e*h+5*
d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2} dx = \frac{\sqrt{e+fx}(105a^3df^2h - 5a^2bf(15cfh + d(15fg + 19eh - 14fhx)) + a}{}$$
$$- \frac{\sqrt{-be+af}(7a^2dfh + b^2(2deg + 3cfg + 2ceh) - ab(5dfg + 4deh + 5cfh)) \arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{9/2}}$$

input
```
Integrate[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(105*a^3*d*f^2*h - 5*a^2*b*f*(15*c*f*h + d*(15*f*g + 19*e*h
 - 14*f*h*x)) + a*b^2*(5*c*f*(9*f*g + 11*e*h - 10*f*h*x) + d*(6*e^2*h + e*
f*(55*g - 68*h*x) - 2*f^2*x*(25*g + 7*h*x))) + b^3*(2*d*x*(3*e^2*h + f^2*x
*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + 5*c*f*(2*f*x*(3*g + h*x) + e*(-3*g
+ 8*h*x)))))/(15*b^4*f*(a + b*x)) - (Sqrt[-(b*e) + a*f]*(7*a^2*d*f*h + b^2
*(2*d*e*g + 3*c*f*g + 2*c*e*h) - a*b*(5*d*f*g + 4*d*e*h + 5*c*f*h))*ArcTan
[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {163, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2} dx$$
$$\Big\downarrow \text{163}$$
$$\frac{(7a^2dfh - ab(5cfh + 4deh + 5dfg) + b^2(2ceh + 3cfg + 2deg)) \int \frac{(e+fx)^{3/2}}{a+bx} dx}{2b^2(be-af)} - \frac{(e+fx)^{5/2}\left(7a^2dfh - ab(5cfh + 2deh + 5dfg) - 2bdhx(be-af) + 5b^2cfg\right)}{5b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(7a^2dfh-ab(5cfh+4deh+5dfg)+b^2(2ceh+3cfg+2deg)\right)\left(\frac{(be-af)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{2b^2(be-af)}-\frac{(e+fx)^{5/2}\left(7a^2dfh-ab(5cfh+2deh+5dfg)-2bdhx(be-af)+5b^2cfg\right)}{5b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(7a^2dfh-ab(5cfh+4deh+5dfg)+b^2(2ceh+3cfg+2deg)\right)\left(\frac{(be-af)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{2b^2(be-af)}-\frac{(e+fx)^{5/2}\left(7a^2dfh-ab(5cfh+2deh+5dfg)-2bdhx(be-af)+5b^2cfg\right)}{5b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(7a^2dfh-ab(5cfh+4deh+5dfg)+b^2(2ceh+3cfg+2deg)\right)\left(\frac{(be-af)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{2b^2(be-af)}-\frac{(e+fx)^{5/2}\left(7a^2dfh-ab(5cfh+2deh+5dfg)-2bdhx(be-af)+5b^2cfg\right)}{5b^2f(a+bx)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)\left(7a^2dfh-ab(5cfh+4deh+5dfg)+b^2(2ceh+3cfg+2deg)\right)}{2b^2(be-af)}-\frac{(e+fx)^{5/2}\left(7a^2dfh-ab(5cfh+2deh+5dfg)-2bdhx(be-af)+5b^2cfg\right)}{5b^2f(a+bx)(be-af)}$$

input
```
Int[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^2,x]
```

output
```
-1/5*((e + f*x)^(5/2)*(5*b^2*c*f*g + 7*a^2*d*f*h - a*b*(5*d*f*g + 2*d*e*h
+ 5*c*f*h) - 2*b*d*(b*e - a*f)*h*x))/(b^2*f*(b*e - a*f)*(a + b*x)) + ((7*a
^2*d*f*h + b^2*(2*d*e*g + 3*c*f*g + 2*c*e*h) - a*b*(5*d*f*g + 4*d*e*h + 5*
c*f*h))*((2*(e + f*x)^(3/2))/(3*b) + ((b*e - a*f)*((2*Sqrt[e + f*x])/b - (
2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2
)))/b))/(2*b^2*(b*e - a*f))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$-\frac{7\left((bx+a)\left(\frac{(3cfg+2e(ch+dg))b^2}{7}-\frac{5a\left(f(ch+dg)+\frac{4deh}{5}\right)b}{7}+a^2dfh\right)f(af-be)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)-\left(\left(2x\left(\frac{x\left(\frac{3hx}{5}+g\right.}{3}\right.\right.\right.\right.}{}$
risch	$\frac{2(3dh\,x^2b^2f^2-10abd\,f^2hx+5b^2c\,f^2hx+6b^2defhx+5b^2d\,f^2gx+45a^2d\,f^2h-30abc\,f^2h-40abdefh-30abd\,f^2g+20b^2ce}{15f\,b^4}$
derivativedivides	$\frac{2\left(\frac{dh(fx+e)^{\frac{5}{2}}b^2}{5}-\frac{2abdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2cfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2dfg(fx+e)^{\frac{3}{2}}}{3}+3a^2d\,f^2h\sqrt{fx+e}-2abc\,f^2h\sqrt{fx+e}-2abdefh\sqrt{fx+e}-2ab\right.}{b^4}$
default	$\frac{2\left(\frac{dh(fx+e)^{\frac{5}{2}}b^2}{5}-\frac{2abdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2cfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2dfg(fx+e)^{\frac{3}{2}}}{3}+3a^2d\,f^2h\sqrt{fx+e}-2abc\,f^2h\sqrt{fx+e}-2abdefh\sqrt{fx+e}-2ab\right.}{b^4}$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-7*((b*x+a)*(1/7*(3*c*f*g+2*e*(c*h+d*g))*b^2-5/7*a*(f*(c*h+d*g)+4/5*d*e*h)
*b+a^2*d*f*h)*f*(a*f-b*e)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-(1/7
*(2*x*(1/3*x*(3/5*h*x+g)*d+c*(1/3*h*x+g))*f^2-(4*(-1/5*h*x^2-2/3*g*x)*d+c*
(-8/3*h*x+g))*e*f+2/5*d*e^2*h*x)*b^3+11/21*a*(1/11*(-10*x*(7/25*h*x+g)*d+9
*(-10/9*h*x+g)*c)*f^2+((-68/55*h*x+g)*d+c*h)*e*f+6/55*d*e^2*h)*b^2-5/7*a^2
*(((-14/15*h*x+g)*d+c*h)*f+19/15*d*e*h)*f*b+a^3*d*f^2*h)*((a*f-b*e)*b)^(1/
2)*(f*x+e)^(1/2))/((a*f-b*e)*b)^(1/2)/b^4/f/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(225) = 450.

Time = 0.14 (sec) , antiderivative size = 928, normalized size of antiderivative = 3.76

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/30*(15*((2*a*b^2*d*e*f + (3*a*b^2*c - 5*a^2*b*d)*f^2)*g + (2*(a*b^2*c -
 2*a^2*b*d)*e*f - (5*a^2*b*c - 7*a^3*d)*f^2)*h + ((2*b^3*d*e*f + (3*b^3*c
- 5*a*b^2*d)*f^2)*g + (2*(b^3*c - 2*a*b^2*d)*e*f - (5*a*b^2*c - 7*a^2*b*d)
*f^2)*h)*x)*sqrt((b*e - a*f)/b)*log((b*f*x + 2*b*e - a*f - 2*sqrt(f*x + e)
*b*sqrt((b*e - a*f)/b))/(b*x + a)) + 2*(6*b^3*d*f^2*h*x^3 + 2*(5*b^3*d*f^2
*g + (6*b^3*d*e*f + (5*b^3*c - 7*a*b^2*d)*f^2)*h)*x^2 - 5*((3*b^3*c - 11*a
*b^2*d)*e*f - 3*(3*a*b^2*c - 5*a^2*b*d)*f^2)*g + (6*a*b^2*d*e^2 + 5*(11*a*
b^2*c - 19*a^2*b*d)*e*f - 15*(5*a^2*b*c - 7*a^3*d)*f^2)*h + 2*(5*(4*b^3*d*
e*f + (3*b^3*c - 5*a*b^2*d)*f^2)*g + (3*b^3*d*e^2 + 2*(10*b^3*c - 17*a*b^2
*d)*e*f - 5*(5*a*b^2*c - 7*a^2*b*d)*f^2)*h)*x)*sqrt(f*x + e))/(b^5*f*x + a
*b^4*f), -1/15*(15*((2*a*b^2*d*e*f + (3*a*b^2*c - 5*a^2*b*d)*f^2)*g + (2*(
a*b^2*c - 2*a^2*b*d)*e*f - (5*a^2*b*c - 7*a^3*d)*f^2)*h + ((2*b^3*d*e*f +
(3*b^3*c - 5*a*b^2*d)*f^2)*g + (2*(b^3*c - 2*a*b^2*d)*e*f - (5*a*b^2*c - 7
*a^2*b*d)*f^2)*h)*x)*sqrt(-(b*e - a*f)/b)*arctan(-sqrt(f*x + e)*b*sqrt(-(b
*e - a*f)/b)/(b*e - a*f)) - (6*b^3*d*f^2*h*x^3 + 2*(5*b^3*d*f^2*g + (6*b^3
*d*e*f + (5*b^3*c - 7*a*b^2*d)*f^2)*h)*x^2 - 5*((3*b^3*c - 11*a*b^2*d)*e*f
 - 3*(3*a*b^2*c - 5*a^2*b*d)*f^2)*g + (6*a*b^2*d*e^2 + 5*(11*a*b^2*c - 19*
a^2*b*d)*e*f - 15*(5*a^2*b*c - 7*a^3*d)*f^2)*h + 2*(5*(4*b^3*d*e*f + (3*b^
3*c - 5*a*b^2*d)*f^2)*g + (3*b^3*d*e^2 + 2*(10*b^3*c - 17*a*b^2*d)*e*f - 5
*(5*a*b^2*c - 7*a^2*b*d)*f^2)*h)*x)*sqrt(f*x + e))/(b^5*f*x + a*b^4*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(225) = 450$.

Time = 0.14 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \frac{(2\,b^3de^2g + 3\,b^3cefg - 7\,ab^2defg - 3\,ab^2cf^2g + 5\,a^2bdf^2g + 2\,b^3ce^2h}{}$$
$$- \frac{\sqrt{fx+e}b^3cefg - \sqrt{fx+e}ab^2defg - \sqrt{fx+e}ab^2cf^2g + \sqrt{fx+e}a^2bdf^2g - \sqrt{fx+e}ab^2cefh + \sqrt{fx+}}{((fx+e)b - be + af)b^4}$$
$$+ \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^8df^5g + 15\,\sqrt{fx+e}b^8def^5g + 15\,\sqrt{fx+e}b^8cf^6g - 30\,\sqrt{fx+e}ab^7df^6g + 3\,(fx+e)^{\frac{5}{2}}b^8d\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(2*b^3*d*e^2*g + 3*b^3*c*e*f*g - 7*a*b^2*d*e*f*g - 3*a*b^2*c*f^2*g + 5*a^2
*b*d*f^2*g + 2*b^3*c*e^2*h - 4*a*b^2*d*e^2*h - 7*a*b^2*c*e*f*h + 11*a^2*b*
d*e*f*h + 5*a^2*b*c*f^2*h - 7*a^3*d*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^
2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^4) - (sqrt(f*x + e)*b^3*c*e*f*g - sq
rt(f*x + e)*a*b^2*d*e*f*g - sqrt(f*x + e)*a*b^2*c*f^2*g + sqrt(f*x + e)*a^
2*b*d*f^2*g - sqrt(f*x + e)*a*b^2*c*e*f*h + sqrt(f*x + e)*a^2*b*d*e*f*h +
sqrt(f*x + e)*a^2*b*c*f^2*h - sqrt(f*x + e)*a^3*d*f^2*h)/(((f*x + e)*b - b
*e + a*f)*b^4) + 2/15*(5*(f*x + e)^(3/2)*b^8*d*f^5*g + 15*sqrt(f*x + e)*b^
8*d*e*f^5*g + 15*sqrt(f*x + e)*b^8*c*f^6*g - 30*sqrt(f*x + e)*a*b^7*d*f^6*
g + 3*(f*x + e)^(5/2)*b^8*d*f^4*h + 5*(f*x + e)^(3/2)*b^8*c*f^5*h - 10*(f*
x + e)^(3/2)*a*b^7*d*f^5*h + 15*sqrt(f*x + e)*b^8*c*e*f^5*h - 30*sqrt(f*x
+ e)*a*b^7*d*e*f^5*h - 30*sqrt(f*x + e)*a*b^7*c*f^6*h + 45*sqrt(f*x + e)*a
^2*b^6*d*f^6*h)/(b^10*f^5)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = (e+f\,x)^{3/2}\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{3\,b^2\,f}-\frac{4\,d\,h\,(a\,f-b\,e)}{3\,b^3\,f}\right) - \sqrt{e+f\,x}\left(\frac{2\,(a\,f-b\,e)\,\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{b^2\,f}-\frac{4\,d\,h\,(a\,f-b\,e)}{b^3\,f}\right)}{b}+\frac{2\,(c\,f-d\,e)\,(e\,h-f\,g)}{b^2\,f}+\frac{2\,d\,h\,(a\,f-b\,e)^2}{b^4\,f}\right) + \frac{\sqrt{e+f\,x}\,(a^3\,d\,f^2\,h+a\,b^2\,c\,f^2\,g-a^2\,b\,c\,f^2\,h-a^2\,b\,d\,f^2\,g-b^3\,c\,e\,f\,g+a\,b^2\,c\,e\,f\,h+a\,b^2\,d\,e\,f\,g-a^2\,b\,d\,e\,f\,h)}{b^5\,(e+f\,x)-b^5\,e+a\,b^4\,f} + \frac{2\,d\,h\,(e+f\,x)^{5/2}}{5\,b^2\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,1\mathrm{i}}{\sqrt{b\,e-a\,f}}\right)\,\sqrt{b\,e-a\,f}\,(2\,b^2\,c\,e\,h+3\,b^2\,c\,f\,g+2\,b^2\,d\,e\,g+7\,a^2\,d\,f\,h-5\,a\,b\,c\,f\,h-4\,a\,b\,d\,e\,h-5\,a\,b}{b^{9/2}}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x))/(a + b*x)^2,x)
```

output
```
(e + f*x)^(3/2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(3*b^2*f) - (4*d*h*(a*f - b
*e))/(3*b^3*f)) - (e + f*x)^(1/2)*((2*(a*f - b*e)*((2*c*f*h - 4*d*e*h + 2*
d*f*g)/(b^2*f) - (4*d*h*(a*f - b*e))/(b^3*f)))/b + (2*(c*f - d*e)*(e*h - f
*g))/(b^2*f) + (2*d*h*(a*f - b*e)^2)/(b^4*f)) + ((e + f*x)^(1/2)*(a^3*d*f^
2*h + a*b^2*c*f^2*g - a^2*b*c*f^2*h - a^2*b*d*f^2*g - b^3*c*e*f*g + a*b^2*
c*e*f*h + a*b^2*d*e*f*g - a^2*b*d*e*f*h))/(b^5*(e + f*x) - b^5*e + a*b^4*f
) + (atan((b^(1/2)*(e + f*x)^(1/2)*1i)/(b*e - a*f)^(1/2))*(b*e - a*f)^(1/2
)*(2*b^2*c*e*h + 3*b^2*c*f*g + 2*b^2*d*e*g + 7*a^2*d*f*h - 5*a*b*c*f*h - 4
*a*b*d*e*h - 5*a*b*d*f*g)*1i)/b^(9/2) + (2*d*h*(e + f*x)^(5/2))/(5*b^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.23

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**3*d*f**2*h + 75*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*f**2*h + 60*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*e*f*h + 75*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b
*d*f**2*g - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**2*b*d*f**2*h*x - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*e*f*h - 45*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*f**2*g
 + 75*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a*b**2*c*f**2*h*x - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*e*f*g + 60*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*e*f*h*x + 75*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b
**2*d*f**2*g*x - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*b**3*c*e*f*h*x - 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c*f**2*g*x - 30*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*d*e*f*g*
x + 105*sqrt(e + f*x)*a**3*b*d*f**2*h - 75*sqrt(e + f*x)*a**2*b**2*c*f**2*
h - 95*sqrt(e + f*x)*a**2*b**2*d*e*f*h - 75*sqrt(e + f*x)*a**2*b**2*d*f...
```

3.66 $\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 278

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3} dx = -\frac{2(3adfh-b(dfg+deh+cfh))\sqrt{e+fx}}{b^4}$$
$$-\frac{(11a^2dfh+b^2(4deg+3cfg+4ceh)-ab(7dfg+8deh+7cfh))\sqrt{e+fx}}{4b^4(a+bx)}$$
$$+\frac{2dh(e+fx)^{3/2}}{3b^3}-\frac{(bc-ad)(bg-ah)(e+fx)^{3/2}}{2b^3(a+bx)^2}$$
$$-\frac{(35a^2df^2h-5abf(3dfg+8deh+3cfh)+b^2(4de(3fg+2eh)+3cf(fg+4eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{4b^{9/2}\sqrt{be-af}}$$

output
```
-2*(3*a*d*f*h-b*(c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)/b^4-1/4*(11*a^2*d*f*h+b
^2*(4*c*e*h+3*c*f*g+4*d*e*g)-a*b*(7*c*f*h+8*d*e*h+7*d*f*g))*(f*x+e)^(1/2)/
b^4/(b*x+a)+2/3*d*h*(f*x+e)^(3/2)/b^3-1/2*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(3
/2)/b^3/(b*x+a)^2-1/4*(35*a^2*d*f^2*h-5*a*b*f*(3*c*f*h+8*d*e*h+3*d*f*g)+b^
2*(4*d*e*(2*e*h+3*f*g)+3*c*f*(4*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(
-a*f+b*e)^(1/2))/b^(9/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \frac{\sqrt{e+fx}(-105a^3dfh+5a^2b(9cfh+d(9fg+10eh-35fhx))+ab^2(}{}$$

$$+\frac{(35a^2df^2h-5abf(3dfg+8deh+3cfh)+b^2(4de(3fg+2eh)+3cf(fg+4eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{4b^{9/2}\sqrt{-be+af}}$$

input
```
Integrate[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(-105*a^3*d*f*h + 5*a^2*b*(9*c*f*h + d*(9*f*g + 10*e*h - 35
*f*h*x)) + a*b^2*(d*f*x*(75*g - 56*h*x) + d*e*(-6*g + 88*h*x) - 3*c*(3*f*g
 + 2*e*h - 25*f*h*x)) + b^3*(-3*c*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x)) +
4*d*x*(2*f*x*(3*g + h*x) + e*(-3*g + 8*h*x)))))/(12*b^4*(a + b*x)^2) + ((3
5*a^2*d*f^2*h - 5*a*b*f*(3*d*f*g + 8*d*e*h + 3*c*f*h) + b^2*(4*d*e*(3*f*g
+ 2*e*h) + 3*c*f*(f*g + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e
) + a*f]])/(4*b^(9/2)*Sqrt[-(b*e) + a*f])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {162, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \; 162$$

$$\frac{(35a^2df^2h-5abf(3cfh+8deh+3dfg)+b^2(3cf(4eh+fg)+4de(2eh+3fg)))\int\frac{(e+fx)^{3/2}}{a+bx}dx}{8b^2(be-af)^2}-$$
$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(9a^2dfh-ab(5cfh+8deh+5dfg)+b^2(4ceh+cfg+4deg)\right)-3a^2b(cfh+2deh+df\right.}{4b^2(a+bx)^2(be-af)^2}$$

$\downarrow$ 60

$$\frac{\left(35a^2df^2h-5abf(3cfh+8deh+3dfg)+b^2(3cf(4eh+fg)+4de(2eh+3fg))\right)\left(\frac{(be-af)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b}+\frac{2(e+fx)^{3/2}}{3b}\right.}{8b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(9a^2dfh-ab(5cfh+8deh+5dfg)+b^2(4ceh+cfg+4deg)\right)-3a^2b(cfh+2deh+df\right.}{4b^2(a+bx)^2(be-af)^2}$$

$\downarrow$ 60

$$\frac{\left(35a^2df^2h-5abf(3cfh+8deh+3dfg)+b^2(3cf(4eh+fg)+4de(2eh+3fg))\right)\left(\frac{(be-af)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}\right.}{b}\right.}{8b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(9a^2dfh-ab(5cfh+8deh+5dfg)+b^2(4ceh+cfg+4deg)\right)-3a^2b(cfh+2deh+df\right.}{4b^2(a+bx)^2(be-af)^2}$$

$\downarrow$ 73

$$\frac{\left(35a^2df^2h-5abf(3cfh+8deh+3dfg)+b^2(3cf(4eh+fg)+4de(2eh+3fg))\right)\left(\frac{(be-af)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}}{bf}\right.}{b}\right.}{8b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(9a^2dfh-ab(5cfh+8deh+5dfg)+b^2(4ceh+cfg+4deg)\right)-3a^2b(cfh+2deh+df\right.}{4b^2(a+bx)^2(be-af)^2}$$

$\downarrow$ 221

$$\frac{\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)\left(35a^2df^2h-5abf(3cfh+8deh+3dfg)+b^2(3cf(4e\right.}{8b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(9a^2dfh-ab(5cfh+8deh+5dfg)+b^2(4ceh+cfg+4deg)\right)-3a^2b(cfh+2deh+df\right.}{4b^2(a+bx)^2(be-af)^2}$$

input
```
Int[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^3,x]
```

output
```
-1/4*((e + f*x)^(5/2)*(2*b^3*c*e*g + 7*a^3*d*f*h + a*b^2*(2*d*e*g - c*f*g
+ 2*c*e*h) - 3*a^2*b*(d*f*g + 2*d*e*h + c*f*h) + b*(9*a^2*d*f*h + b^2*(4*d
*e*g + c*f*g + 4*c*e*h) - a*b*(5*d*f*g + 8*d*e*h + 5*c*f*h))*x))/(b^2*(b*e
 - a*f)^2*(a + b*x)^2) + ((35*a^2*d*f^2*h - 5*a*b*f*(3*d*f*g + 8*d*e*h + 3
*c*f*h) + b^2*(4*d*e*(3*f*g + 2*e*h) + 3*c*f*(f*g + 4*e*h)))*((2*(e + f*x)
^(3/2))/(3*b) + ((b*e - a*f)*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*Arc
Tanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b))/(8*b^2*(b*e -
 a*f)^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] && !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{35\left(-\left(\frac{\left(3cgf^2+12e(ch+dg)f+8de^2h\right)b^2}{35}-\frac{3a\left(f(ch+dg)+\frac{8deh}{3}\right)fb}{7}+a^2df^2h\right)(bx+a)^2\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\sqrt{(af-\cdots}\right.}{\cdots}$
risch	$-\frac{2(-dhxbf+9adfh-3bcfh-4bdeh-3bdfg)\sqrt{fx+e}}{3b^4}+\frac{2\left(-\frac{13}{8}a^2bdf^2h+\frac{9}{8}ab^2cf^2h+ab^2defh+\frac{9}{8}ab^2df^2g-\frac{1}{2}b^3cefh-\frac{5}{8}\cdots\right.}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+3adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^4}+\frac{2\left(\left(-\frac{13}{8}a^2bdf^2h+\frac{9}{8}ab^2cf^2h+ab^2\cdots\right.\right.}{\cdots}$
default	$-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+3adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^4}+\frac{2\left(\left(-\frac{13}{8}a^2bdf^2h+\frac{9}{8}ab^2cf^2h+ab^2\cdots\right.\right.}{\cdots}$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-35/4/((a*f-b*e)*b)^(1/2)*(-(1/35*(3*c*g*f^2+12*e*(c*h+d*g)*f+8*d*e^2*h)*b
^2-3/7*a*(f*(c*h+d*g)+8/3*d*e*h)*f*b+a^2*d*f^2*h)*(b*x+a)^2*arctan(b*(f*x+
e)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(1/7*(x*(-8/15*d*h*x^2+8
/5*(-c*h-d*g)*x+c*g)*f+2/5*(-16/3*d*h*x^2+2*(c*h+d*g)*x+c*g)*e)*b^3+2/35*a
*((28/3*d*h*x^2+25/2*(-c*h-d*g)*x+3/2*c*g)*f+e*(-44/3*d*h*x+c*h+d*g))*b^2-
3/7*a^2*((-35/9*d*h*x+c*h+d*g)*f+10/9*d*e*h)*b+a^3*d*f*h)*(f*x+e)^(1/2))/(
b*x+a)^2/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(252) = 504$.

Time = 0.17 (sec) , antiderivative size = 1500, normalized size of antiderivative = 5.40

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(b^2*e - a*b*f)*((3*(4*b^4*d*e*f + (b^4*c - 5*a*b^3*d)*f^2)*g
 + (8*b^4*d*e^2 + 4*(3*b^4*c - 10*a*b^3*d)*e*f - 5*(3*a*b^3*c - 7*a^2*b^2*
d)*f^2)*h)*x^2 + 3*(4*a^2*b^2*d*e*f + (a^2*b^2*c - 5*a^3*b*d)*f^2)*g + (8*
a^2*b^2*d*e^2 + 4*(3*a^2*b^2*c - 10*a^3*b*d)*e*f - 5*(3*a^3*b*c - 7*a^4*d)
*f^2)*h + 2*(3*(4*a*b^3*d*e*f + (a*b^3*c - 5*a^2*b^2*d)*f^2)*g + (8*a*b^3*
d*e^2 + 4*(3*a*b^3*c - 10*a^2*b^2*d)*e*f - 5*(3*a^2*b^2*c - 7*a^3*b*d)*f^2
)*h)*x)*log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b
*x + a)) + 2*(8*(b^5*d*e*f - a*b^4*d*f^2)*h*x^3 + 8*(3*(b^5*d*e*f - a*b^4*
d*f^2)*g + (4*b^5*d*e^2 + (3*b^5*c - 11*a*b^4*d)*e*f - (3*a*b^4*c - 7*a^2*
b^3*d)*f^2)*h)*x^2 - 3*(2*(b^5*c + a*b^4*d)*e^2 + (a*b^4*c - 17*a^2*b^3*d)
*e*f - 3*(a^2*b^3*c - 5*a^3*b^2*d)*f^2)*g - (2*(3*a*b^4*c - 25*a^2*b^3*d)*
e^2 - (51*a^2*b^3*c - 155*a^3*b^2*d)*e*f + 15*(3*a^3*b^2*c - 7*a^4*b*d)*f^
2)*h - (3*(4*b^5*d*e^2 + (5*b^5*c - 29*a*b^4*d)*e*f - 5*(a*b^4*c - 5*a^2*b
^3*d)*f^2)*g + (4*(3*b^5*c - 22*a*b^4*d)*e^2 - (87*a*b^4*c - 263*a^2*b^3*d
)*e*f + 25*(3*a^2*b^3*c - 7*a^3*b^2*d)*f^2)*h)*x)*sqrt(f*x + e))/(a^2*b^6*
e - a^3*b^5*f + (b^8*e - a*b^7*f)*x^2 + 2*(a*b^7*e - a^2*b^6*f)*x), 1/12*(
3*sqrt(-b^2*e + a*b*f)*((3*(4*b^4*d*e*f + (b^4*c - 5*a*b^3*d)*f^2)*g + (8*
b^4*d*e^2 + 4*(3*b^4*c - 10*a*b^3*d)*e*f - 5*(3*a*b^3*c - 7*a^2*b^2*d)*f^2
)*h)*x^2 + 3*(4*a^2*b^2*d*e*f + (a^2*b^2*c - 5*a^3*b*d)*f^2)*g + (8*a^2*b^
2*d*e^2 + 4*(3*a^2*b^2*c - 10*a^3*b*d)*e*f - 5*(3*a^3*b*c - 7*a^4*d)*f^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(252) = 504.

Time = 0.14 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \frac{(12\,b^2defg+3\,b^2cf^2g-15\,abdf^2g+8\,b^2de^2h+12\,b^2cefh-40\,abde}{4\,\sqrt{-b^2e+abf}b^4}$$
$$-\frac{4\,(fx+e)^{\frac{3}{2}}b^3defg-4\,\sqrt{fx+e}b^3de^2fg+5\,(fx+e)^{\frac{3}{2}}b^3cf^2g-9\,(fx+e)^{\frac{3}{2}}ab^2df^2g-3\,\sqrt{fx+e}b^3cef^2g}{}$$
$$+\frac{2\left(3\,\sqrt{fx+e}b^6dfg+(fx+e)^{\frac{3}{2}}b^6dh+3\,\sqrt{fx+e}b^6deh+3\,\sqrt{fx+e}b^6cfh-9\,\sqrt{fx+e}ab^5dfh\right)}{3\,b^9}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(12*b^2*d*e*f*g + 3*b^2*c*f^2*g - 15*a*b*d*f^2*g + 8*b^2*d*e^2*h + 12*
b^2*c*e*f*h - 40*a*b*d*e*f*h - 15*a*b*c*f^2*h + 35*a^2*d*f^2*h)*arctan(sqr
t(f*x + e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^4) - 1/4*(4*(f*
x + e)^(3/2)*b^3*d*e*f*g - 4*sqrt(f*x + e)*b^3*d*e^2*f*g + 5*(f*x + e)^(3/
2)*b^3*c*f^2*g - 9*(f*x + e)^(3/2)*a*b^2*d*f^2*g - 3*sqrt(f*x + e)*b^3*c*e
*f^2*g + 11*sqrt(f*x + e)*a*b^2*d*e*f^2*g + 3*sqrt(f*x + e)*a*b^2*c*f^3*g
- 7*sqrt(f*x + e)*a^2*b*d*f^3*g + 4*(f*x + e)^(3/2)*b^3*c*e*f*h - 8*(f*x +
 e)^(3/2)*a*b^2*d*e*f*h - 4*sqrt(f*x + e)*b^3*c*e^2*f*h + 8*sqrt(f*x + e)*
a*b^2*d*e^2*f*h - 9*(f*x + e)^(3/2)*a*b^2*c*f^2*h + 13*(f*x + e)^(3/2)*a^2
*b*d*f^2*h + 11*sqrt(f*x + e)*a*b^2*c*e*f^2*h - 19*sqrt(f*x + e)*a^2*b*d*e
*f^2*h - 7*sqrt(f*x + e)*a^2*b*c*f^3*h + 11*sqrt(f*x + e)*a^3*d*f^3*h)/(((
f*x + e)*b - b*e + a*f)^2*b^4) + 2/3*(3*sqrt(f*x + e)*b^6*d*f*g + (f*x + e
)^(3/2)*b^6*d*h + 3*sqrt(f*x + e)*b^6*d*e*h + 3*sqrt(f*x + e)*b^6*c*f*h -
9*sqrt(f*x + e)*a*b^5*d*f*h)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \sqrt{e+f\,x}\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{b^3}\right.$$
$$\left.+\frac{2\,d\,h\,(3\,b^3\,e-3\,a\,b^2\,f)}{b^6}\right)$$
$$+\frac{\sqrt{e+f\,x}\left(\frac{7\,a^2\,b\,c\,f^3\,h}{4}-\frac{3\,a\,b^2\,c\,f^3\,g}{4}-\frac{11\,a^3\,d\,f^3\,h}{4}+\frac{7\,a^2\,b\,d\,f^3\,g}{4}+\frac{3\,b^3\,c\,e\,f^2\,g}{4}+b^3\,c\,e^2\,f\,h+b^3\,d\,e^2\,f\,g-\frac{11\,a\,b^2\,c\,e\,f^2\,h}{4}\right.}{b^6\,(e+f\,x)^2-}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,(3\,b^2\,c\,f^2\,g+35\,a^2\,d\,f^2\,h+8\,b^2\,d\,e^2\,h-15\,a\,b\,c\,f^2\,h-15\,a\,b\,d\,f^2\,g+12\,b^2\,c\,e\,f\,h+12\,b^2}{4\,b^{9/2}\,\sqrt{a\,f-b\,e}}$$
$$+\frac{2\,d\,h\,(e+f\,x)^{3/2}}{3\,b^3}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x))/(a + b*x)^3,x)
```

output
```
(e + f*x)^(1/2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/b^3 + (2*d*h*(3*b^3*e - 3*a
*b^2*f))/b^6) + ((e + f*x)^(1/2)*((7*a^2*b*c*f^3*h)/4 - (3*a*b^2*c*f^3*g)/
4 - (11*a^3*d*f^3*h)/4 + (7*a^2*b*d*f^3*g)/4 + (3*b^3*c*e*f^2*g)/4 + b^3*c
*e^2*f*h + b^3*d*e^2*f*g - (11*a*b^2*c*e*f^2*h)/4 - (11*a*b^2*d*e*f^2*g)/4
 - 2*a*b^2*d*e^2*f*h + (19*a^2*b*d*e*f^2*h)/4) - (e + f*x)^(3/2)*((5*b^3*c
*f^2*g)/4 - (9*a*b^2*c*f^2*h)/4 - (9*a*b^2*d*f^2*g)/4 + (13*a^2*b*d*f^2*h)
/4 + b^3*c*e*f*h + b^3*d*e*f*g - 2*a*b^2*d*e*f*h))/(b^6*(e + f*x)^2 - (e +
 f*x)*(2*b^6*e - 2*a*b^5*f) + b^6*e^2 + a^2*b^4*f^2 - 2*a*b^5*e*f) + (atan
((b^(1/2)*(e + f*x)^(1/2))/(a*f - b*e)^(1/2))*(3*b^2*c*f^2*g + 35*a^2*d*f^
2*h + 8*b^2*d*e^2*h - 15*a*b*c*f^2*h - 15*a*b*d*f^2*g + 12*b^2*c*e*f*h + 1
2*b^2*d*e*f*g - 40*a*b*d*e*f*h))/(4*b^(9/2)*(a*f - b*e)^(1/2)) + (2*d*h*(e
 + f*x)^(3/2))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1779, normalized size of antiderivative = 6.40

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**4*d*f**2*h - 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**3*b*c*f**2*h - 120*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d*e*f*h - 45*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d
*f**2*g + 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**3*b*d*f**2*h*x + 36*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*e*f*h + 9*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*f**
2*g - 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**2*b**2*c*f**2*h*x + 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*e**2*h + 36*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*e*f
*g - 240*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**2*b**2*d*e*f*h*x - 90*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*f**2*g*x + 105*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*f
**2*h*x**2 + 72*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a*b**3*c*e*f*h*x + 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c*f**2*g*x - 45*sqrt(b)*sq...
```

3.67 $\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4} dx$

Optimal result

Integrand size = 27, antiderivative size = 331

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4} dx = \frac{2dfh\sqrt{e+fx}}{b^4}$$
$$-\frac{(5a^2dfh+b^2(2deg+cfg+2ceh)-ab(3dfg+4deh+3cfh))\sqrt{e+fx}}{4b^4(a+bx)^2}$$
$$-\frac{(29a^2df^2h-abf(11dfg+36deh+11cfh)+b^2(2de(5fg+4eh)+cf(fg+10eh)))\sqrt{e+fx}}{8b^4(be-af)(a+bx)}$$
$$-\frac{(bc-ad)(bg-ah)(e+fx)^{3/2}}{3b^3(a+bx)^3}$$
$$-\frac{f(35a^2df^2h-5abf(dfg+12deh+cfh)-b^2(cf(fg-6eh)-6de(fg+4eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{8b^{9/2}(be-af)^{3/2}}$$

output
```
2*d*f*h*(f*x+e)^(1/2)/b^4-1/4*(5*a^2*d*f*h+b^2*(2*c*e*h+c*f*g+2*d*e*g)-a*b
*(3*c*f*h+4*d*e*h+3*d*f*g))*(f*x+e)^(1/2)/b^4/(b*x+a)^2-1/8*(29*a^2*d*f^2*
h-a*b*f*(11*c*f*h+36*d*e*h+11*d*f*g)+b^2*(2*d*e*(4*e*h+5*f*g)+c*f*(10*e*h+
f*g)))*(f*x+e)^(1/2)/b^4/(-a*f+b*e)/(b*x+a)-1/3*(-a*d+b*c)*(-a*h+b*g)*(f*x
+e)^(3/2)/b^3/(b*x+a)^3-1/8*f*(35*a^2*d*f^2*h-5*a*b*f*(c*f*h+12*d*e*h+d*f*
g)-b^2*(c*f*(-6*e*h+f*g)-6*d*e*(4*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)
/(-a*f+b*e)^(1/2))/b^(9/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx =$$
$$-\frac{\sqrt{e+fx}\left(105a^4df^2h-5a^3bf(3cfh+d(3fg+22eh-56fhx))+b^4(6dex(fx(5g-8hx)+2e(g+2hx))\right.}{}$$
$$+\frac{f\left(-35a^2df^2h+5abf(dfg+12deh+cfh)+b^2(cf(fg-6eh)-6de(fg+4eh))\right)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{8b^{9/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^4,x]
```

output
```
-1/24*(Sqrt[e + f*x]*(105*a^4*d*f^2*h - 5*a^3*b*f*(3*c*f*h + d*(3*f*g + 22
*e*h - 56*f*h*x)) + b^4*(6*d*e*x*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x)) + c
*(3*f^2*g*x^2 + 4*e^2*(2*g + 3*h*x) + 2*e*f*x*(7*g + 15*h*x))) + a*b^3*(d*
(2*e*f*x*(11*g - 126*h*x) + 4*e^2*(g + 6*h*x) + 3*f^2*x^2*(-11*g + 16*h*x)
) + c*(4*e^2*h - 2*e*f*(g - 11*h*x) - f^2*x*(8*g + 33*h*x))) + a^2*b^2*(c*
f*(-3*f*g + 8*e*h - 40*f*h*x) + d*(8*e^2*h + e*f*(8*g - 298*h*x) + f^2*x*(
-40*g + 231*h*x)))))/(b^4*(b*e - a*f)*(a + b*x)^3) + (f*(-35*a^2*d*f^2*h +
 5*a*b*f*(d*f*g + 12*d*e*h + c*f*h) + b^2*(c*f*(f*g - 6*e*h) - 6*d*e*(f*g
+ 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(8*b^(9/2)*
(-(b*e) + a*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {162, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx$$

$$\Big\downarrow \; 162$$

$$\frac{\left(35a^2df^2h-5abf(cfh+12deh+dfg)-\left(b^2(cf(fg-6eh)-6de(4eh+fg))\right)\right)\int \frac{(e+fx)^{3/2}}{(a+bx)^2}dx}{24b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(11a^2dfh-ab(5cfh+12deh+5dfg)+b^2(6ceh-cfg+6deg)\right)-a^2b(cfh+8deh+d\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 51

$$\frac{\left(35a^2df^2h-5abf(cfh+12deh+dfg)-\left(b^2(cf(fg-6eh)-6de(4eh+fg))\right)\right)\left(\frac{3f\int \frac{\sqrt{e+fx}}{a+bx}dx}{2b}-\frac{(e+fx)^{3/2}}{b(a+bx)}\right)}{24b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(11a^2dfh-ab(5cfh+12deh+5dfg)+b^2(6ceh-cfg+6deg)\right)-a^2b(cfh+8deh+d\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 60

$$\frac{\left(35a^2df^2h-5abf(cfh+12deh+dfg)-\left(b^2(cf(fg-6eh)-6de(4eh+fg))\right)\right)\left(\frac{3f\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+j}}{b}\right.}{2b}\right.}{24b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(11a^2dfh-ab(5cfh+12deh+5dfg)+b^2(6ceh-cfg+6deg)\right)-a^2b(cfh+8deh+d\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 73

$$\frac{\left(35a^2df^2h-5abf(cfh+12deh+dfg)-\left(b^2(cf(fg-6eh)-6de(4eh+fg))\right)\right)\left(\frac{3f\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}\right.}{2b}\right.}{24b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(11a^2dfh-ab(5cfh+12deh+5dfg)+b^2(6ceh-cfg+6deg)\right)-a^2b(cfh+8deh+d\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\Big\downarrow$ 221

$$\frac{\left(\frac{3f\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)}{2b}-\frac{(e+fx)^{3/2}}{b(a+bx)}\right)(35a^2df^2h-5abf(cfh+12deh+dfg)-(b^2(cf(fg-6e}{24b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}(7a^3dfh+bx(11a^2dfh-ab(5cfh+12deh+5dfg)+b^2(6ceh-cfg+6deg))-a^2b(cfh+8deh+d}{12b^2(a+bx)^3(be-af)^2}$$

input
```
Int[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^4,x]
```

output
```
-1/12*((e + f*x)^(5/2)*(4*b^3*c*e*g + 7*a^3*d*f*h + a*b^2*(2*d*e*g - 5*c*f
*g + 2*c*e*h) - a^2*b*(d*f*g + 8*d*e*h + c*f*h) + b*(11*a^2*d*f*h + b^2*(6
*d*e*g - c*f*g + 6*c*e*h) - a*b*(5*d*f*g + 12*d*e*h + 5*c*f*h))*x))/(b^2*(
b*e - a*f)^2*(a + b*x)^3) + ((35*a^2*d*f^2*h - 5*a*b*f*(d*f*g + 12*d*e*h +
 c*f*h) - b^2*(c*f*(f*g - 6*e*h) - 6*d*e*(f*g + 4*e*h)))*(-((e + f*x)^(3/2
)/(b*(a + b*x))) + (3*f*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh[
(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/(2*b)))/(24*b^2*(b*e -
 a*f)^2)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$-\frac{35\left(\left(\frac{\left(-cg\,f^{2}+6e(ch+dg)f+24d\,e^{2}h\right)b^{2}}{35}-\frac{a(f(ch+dg)+12deh)fb}{7}+a^{2}d\,f^{2}h\right)(bx+a)^{3}f\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)-\sqrt{(af-b}\right.}{}$
derivativedivides	$2f\left(\frac{dh\sqrt{fx+e}}{b^{4}}-\frac{-\frac{b^{2}\left(29a^{2}d\,f^{2}h-11abc\,f^{2}h-36abdefh-11abd\,f^{2}g+10b^{2}cefh+b^{2}c\,f^{2}g+8b^{2}d\,e^{2}h+10b^{2}defg\right)(fx+e)^{\frac{5}{2}}}{16(af-be)}-}{}\right.$
default	$2f\left(\frac{dh\sqrt{fx+e}}{b^{4}}-\frac{-\frac{b^{2}\left(29a^{2}d\,f^{2}h-11abc\,f^{2}h-36abdefh-11abd\,f^{2}g+10b^{2}cefh+b^{2}c\,f^{2}g+8b^{2}d\,e^{2}h+10b^{2}defg\right)(fx+e)^{\frac{5}{2}}}{16(af-be)}-}{}\right.$
risch	$\frac{2dfh\sqrt{fx+e}}{b^{4}}-\frac{2f\left(-\frac{b^{2}\left(29a^{2}d\,f^{2}h-11abc\,f^{2}h-36abdefh-11abd\,f^{2}g+10b^{2}cefh+b^{2}c\,f^{2}g+8b^{2}d\,e^{2}h+10b^{2}defg\right)(fx+e)^{\frac{5}{2}}}{16(af-be)}-\right.}{}$

input

```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output

```
-35/8/((a*f-b*e)*b)^(1/2)*((1/35*(-c*g*f^2+6*e*(c*h+d*g)*f+24*d*e^2*h)*b^2
-1/7*a*(f*(c*h+d*g)+12*d*e*h)*f*b+a^2*d*f^2*h)*(b*x+a)^3*f*arctan(b*(f*x+e
)^(1/2)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^(1/2)*(f*x+e)^(1/2)*(1/5*(1/7*c
*f^2*g*x^2+2/3*x*(-24/7*d*h*x^2+15/7*(c*h+d*g)*x+c*g)*e*f+8/21*(3*d*h*x^2+
3/2*(c*h+d*g)*x+c*g)*e^2)*b^4+4/105*(-2*x*(-6*d*h*x^2+33/8*(c*h+d*g)*x+c*g
)*f^2-1/2*(126*d*h*x^2+11*(-c*h-d*g)*x+c*g)*e*f+e^2*(6*d*h*x+c*h+d*g))*a*b
^3+8/105*a^2*((231/8*d*h*x^2+5*(-c*h-d*g)*x-3/8*c*g)*f^2+e*(-149/4*d*h*x+c
*h+d*g)*f+d*e^2*h)*b^2-1/7*a^3*((-56/3*d*h*x+c*h+d*g)*f+22/3*d*e*h)*f*b+a^
4*d*f^2*h))/(b*x+a)^3/b^4/(a*f-b*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(305) = 610$.

Time = 0.20 (sec) , antiderivative size = 2236, normalized size of antiderivative = 6.76

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/48*(3*(((6*b^5*d*e*f^2 - (b^5*c + 5*a*b^4*d)*f^3)*g + (24*b^5*d*e^2*f
+ 6*(b^5*c - 10*a*b^4*d)*e*f^2 - 5*(a*b^4*c - 7*a^2*b^3*d)*f^3)*h)*x^3 + 3
*((6*a*b^4*d*e*f^2 - (a*b^4*c + 5*a^2*b^3*d)*f^3)*g + (24*a*b^4*d*e^2*f +
6*(a*b^4*c - 10*a^2*b^3*d)*e*f^2 - 5*(a^2*b^3*c - 7*a^3*b^2*d)*f^3)*h)*x^2
 + (6*a^3*b^2*d*e*f^2 - (a^3*b^2*c + 5*a^4*b*d)*f^3)*g + (24*a^3*b^2*d*e^2
*f + 6*(a^3*b^2*c - 10*a^4*b*d)*e*f^2 - 5*(a^4*b*c - 7*a^5*d)*f^3)*h + 3*(
(6*a^2*b^3*d*e*f^2 - (a^2*b^3*c + 5*a^3*b^2*d)*f^3)*g + (24*a^2*b^3*d*e^2*
f + 6*(a^2*b^3*c - 10*a^3*b^2*d)*e*f^2 - 5*(a^3*b^2*c - 7*a^4*b*d)*f^3)*h)
*x)*sqrt(b^2*e - a*b*f)*log((b*f*x + 2*b*e - a*f + 2*sqrt(b^2*e - a*b*f)*s
qrt(f*x + e))/(b*x + a)) - 2*(48*(b^6*d*e^2*f - 2*a*b^5*d*e*f^2 + a^2*b^4*
d*f^3)*h*x^3 - 3*((10*b^6*d*e^2*f + (b^6*c - 21*a*b^5*d)*e*f^2 - (a*b^5*c
- 11*a^2*b^4*d)*f^3)*g + (8*b^6*d*e^3 + 2*(5*b^6*c - 46*a*b^5*d)*e^2*f - 7
*(3*a*b^5*c - 23*a^2*b^4*d)*e*f^2 + 11*(a^2*b^4*c - 7*a^3*b^3*d)*f^3)*h)*x
^2 - (4*(2*b^6*c + a*b^5*d)*e^3 - 2*(5*a*b^5*c - 2*a^2*b^4*d)*e^2*f - (a^2
*b^4*c + 23*a^3*b^3*d)*e*f^2 + 3*(a^3*b^3*c + 5*a^4*b^2*d)*f^3)*g - (4*(a*
b^5*c + 2*a^2*b^4*d)*e^3 + 2*(2*a^2*b^4*c - 59*a^3*b^3*d)*e^2*f - (23*a^3*
b^3*c - 215*a^4*b^2*d)*e*f^2 + 15*(a^4*b^2*c - 7*a^5*b*d)*f^3)*h - 2*((6*b
^6*d*e^3 + (7*b^6*c + 5*a*b^5*d)*e^2*f - (11*a*b^5*c + 31*a^2*b^4*d)*e*f^2
 + 4*(a^2*b^4*c + 5*a^3*b^3*d)*f^3)*g + (6*(b^6*c + 2*a*b^5*d)*e^3 + (5*a*
b^5*c - 161*a^2*b^4*d)*e^2*f - (31*a^2*b^4*c - 289*a^3*b^3*d)*e*f^2 + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4} dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4} dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. 2(305) = 610.

Time = 0.15 (sec) , antiderivative size = 881, normalized size of antiderivative = 2.66

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*(6*b^2*d*e*f^2*g - b^2*c*f^3*g - 5*a*b*d*f^3*g + 24*b^2*d*e^2*f*h + 6*
b^2*c*e*f^2*h - 60*a*b*d*e*f^2*h - 5*a*b*c*f^3*h + 35*a^2*d*f^3*h)*arctan(
sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*e - a*b^4*f)*sqrt(-b^2*e + a*b
*f)) + 2*sqrt(f*x + e)*d*f*h/b^4 - 1/24*(30*(f*x + e)^(5/2)*b^4*d*e*f^2*g
- 48*(f*x + e)^(3/2)*b^4*d*e^2*f^2*g + 18*sqrt(f*x + e)*b^4*d*e^3*f^2*g +
3*(f*x + e)^(5/2)*b^4*c*f^3*g - 33*(f*x + e)^(5/2)*a*b^3*d*f^3*g + 8*(f*x
+ e)^(3/2)*b^4*c*e*f^3*g + 88*(f*x + e)^(3/2)*a*b^3*d*e*f^3*g - 3*sqrt(f*x
 + e)*b^4*c*e^2*f^3*g - 51*sqrt(f*x + e)*a*b^3*d*e^2*f^3*g - 8*(f*x + e)^(
3/2)*a*b^3*c*f^4*g - 40*(f*x + e)^(3/2)*a^2*b^2*d*f^4*g + 6*sqrt(f*x + e)*
a*b^3*c*e*f^4*g + 48*sqrt(f*x + e)*a^2*b^2*d*e*f^4*g - 3*sqrt(f*x + e)*a^2
*b^2*c*f^5*g - 15*sqrt(f*x + e)*a^3*b*d*f^5*g + 24*(f*x + e)^(5/2)*b^4*d*e
^2*f*h - 48*(f*x + e)^(3/2)*b^4*d*e^3*f*h + 24*sqrt(f*x + e)*b^4*d*e^4*f*h
 + 30*(f*x + e)^(5/2)*b^4*c*e*f^2*h - 108*(f*x + e)^(5/2)*a*b^3*d*e*f^2*h
- 48*(f*x + e)^(3/2)*b^4*c*e^2*f^2*h + 240*(f*x + e)^(3/2)*a*b^3*d*e^2*f^2
*h + 18*sqrt(f*x + e)*b^4*c*e^3*f^2*h - 132*sqrt(f*x + e)*a*b^3*d*e^3*f^2*
h - 33*(f*x + e)^(5/2)*a*b^3*c*f^3*h + 87*(f*x + e)^(5/2)*a^2*b^2*d*f^3*h
+ 88*(f*x + e)^(3/2)*a*b^3*c*e*f^3*h - 328*(f*x + e)^(3/2)*a^2*b^2*d*e*f^3
*h - 51*sqrt(f*x + e)*a*b^3*c*e^2*f^3*h + 249*sqrt(f*x + e)*a^2*b^2*d*e^2*
f^3*h - 40*(f*x + e)^(3/2)*a^2*b^2*c*f^4*h + 136*(f*x + e)^(3/2)*a^3*b*d*f
^4*h + 48*sqrt(f*x + e)*a^2*b^2*c*e*f^4*h - 198*sqrt(f*x + e)*a^3*b*d*e...
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.28

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \frac{2\,d\,f\,h\,\sqrt{e+f\,x}}{b^4}$$

$$-\frac{\sqrt{e+f\,x}\,\left(\frac{a\,b^2\,c\,f^4\,g}{8}-\frac{19\,a^3\,d\,f^4\,h}{8}+\frac{5\,a^2\,b\,c\,f^4\,h}{8}+\frac{5\,a^2\,b\,d\,f^4\,g}{8}-\frac{b^3\,c\,e\,f^3\,g}{8}+b^3\,d\,e^3\,f\,h+\frac{3\,b^3\,c\,e^2\,f^2\,h}{4}+\frac{3\,b^3\,d\,e^2\,f^2\,g}{4}-\right.}{}$$

$$+\frac{f\,\mathrm{atan}\left(\frac{\sqrt{b}\,f\,\sqrt{e+f\,x}\,\left(b^2\,c\,f^2\,g-35\,a^2\,d\,f^2\,h-24\,b^2\,d\,e^2\,h+5\,a\,b\,c\,f^2\,h+5\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b^2\,d\,e\,f\,g+60\,a\,b\,d\,e\,f\,h\right)}{\sqrt{a\,f-b\,e}\,\left(b^2\,c\,f^3\,g-35\,a^2\,d\,f^3\,h-6\,b^2\,c\,e\,f^2\,h-6\,b^2\,d\,e\,f^2\,g-24\,b^2\,d\,e^2\,f\,h+5\,a\,b\,c\,f^3\,h+5\,a\,b\,d\,f^3\,g+60\,a\,b\,d\,e\,f^2\,h\right)}\right)\,\left(b^2\,c\,f^2\,g-\right.}{8\,b^{9/2}\,\left(a\,f-b\,e\right.}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x))/(a + b*x)^4,x)
```

output
```
(2*d*f*h*(e + f*x)^(1/2))/b^4 - ((e + f*x)^(1/2)*((a*b^2*c*f^4*g)/8 - (19*
a^3*d*f^4*h)/8 + (5*a^2*b*c*f^4*h)/8 + (5*a^2*b*d*f^4*g)/8 - (b^3*c*e*f^3*
g)/8 + b^3*d*e^3*f*h + (3*b^3*c*e^2*f^2*h)/4 + (3*b^3*d*e^2*f^2*g)/4 - (11
*a*b^2*c*e*f^3*h)/8 - (11*a*b^2*d*e*f^3*g)/8 + (47*a^2*b*d*e*f^3*h)/8 - (9
*a*b^2*d*e^2*f^2*h)/2) + (e + f*x)^(3/2)*((b^3*c*f^3*g)/3 + (5*a*b^2*c*f^3
*h)/3 + (5*a*b^2*d*f^3*g)/3 - (17*a^2*b*d*f^3*h)/3 - 2*b^3*c*e*f^2*h - 2*b
^3*d*e*f^2*g - 2*b^3*d*e^2*f*h + 8*a*b^2*d*e*f^2*h) - ((e + f*x)^(5/2)*(b^
4*c*f^3*g - 11*a*b^3*c*f^3*h - 11*a*b^3*d*f^3*g + 10*b^4*c*e*f^2*h + 10*b^
4*d*e*f^2*g + 8*b^4*d*e^2*f*h + 29*a^2*b^2*d*f^3*h - 36*a*b^3*d*e*f^2*h))/
(8*(a*f - b*e)))/(b^7*(e + f*x)^3 - (e + f*x)^2*(3*b^7*e - 3*a*b^6*f) + (e
 + f*x)*(3*b^7*e^2 + 3*a^2*b^5*f^2 - 6*a*b^6*e*f) - b^7*e^3 + a^3*b^4*f^3
- 3*a^2*b^5*e*f^2 + 3*a*b^6*e^2*f) + (f*atan((b^(1/2)*f*(e + f*x)^(1/2)*(b
^2*c*f^2*g - 35*a^2*d*f^2*h - 24*b^2*d*e^2*h + 5*a*b*c*f^2*h + 5*a*b*d*f^2
*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g + 60*a*b*d*e*f*h))/((a*f - b*e)^(1/2)*(
b^2*c*f^3*g - 35*a^2*d*f^3*h - 6*b^2*c*e*f^2*h - 6*b^2*d*e*f^2*g - 24*b^2*
d*e^2*f*h + 5*a*b*c*f^3*h + 5*a*b*d*f^3*g + 60*a*b*d*e*f^2*h)))*(b^2*c*f^2
*g - 35*a^2*d*f^2*h - 24*b^2*d*e^2*h + 5*a*b*c*f^2*h + 5*a*b*d*f^2*g - 6*b
^2*c*e*f*h - 6*b^2*d*e*f*g + 60*a*b*d*e*f*h))/(8*b^(9/2)*(a*f - b*e)^(3/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2616, normalized size of antiderivative = 7.90

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**5*d*f**3*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*f**3*h + 180*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d*e*f**2*h + 15*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*4*b*d*f**3*g - 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**4*b*d*f**3*h*x - 18*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*e*f**2*h + 3*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b
**2*c*f**3*g + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**3*b**2*c*f**3*h*x - 72*sqrt(b)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e**2*f*h - 18*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3
*b**2*d*e*f**2*g + 540*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e*f**2*h*x + 45*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*f**3*g*x -
315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**3*b**2*d*f**3*h*x**2 - 54*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c*e*f**2*h*x + 9*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c...
```

3.68 $\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5} dx$

Optimal result

Integrand size = 27, antiderivative size = 414

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5} dx = -\frac{(19a^2dfh+b^2(8deg+3cfg+8ceh)-ab(11dfg+16deh+11cfh))\sqrt{e+fx}}{24b^4(a+bx)^3} - \frac{(163a^2df^2h-abf(59dfg+208deh+59cfh)+b^2(8de(7fg+6eh)+cf(3fg+56eh)))\sqrt{e+fx}}{96b^4(be-af)(a+bx)^2} - \frac{f(93a^2df^2h-abf(5dfg+176deh+5cfh)-b^2(cf(3fg-8eh)-8de(fg+10eh)))\sqrt{e+fx}}{64b^4(be-af)^2(a+bx)} - \frac{(bc-ad)(bg-ah)(e+fx)^{3/2}}{4b^3(a+bx)^4} - \frac{f^2(35a^2df^2h+5abf(dfg-16deh+cfh)+b^2(cf(3fg-8eh)-8de(fg-6eh)))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{64b^{9/2}(be-af)^{5/2}}$$

output
```
-1/24*(19*a^2*d*f*h+b^2*(8*c*e*h+3*c*f*g+8*d*e*g)-a*b*(11*c*f*h+16*d*e*h+1
1*d*f*g))*(f*x+e)^(1/2)/b^4/(b*x+a)^3-1/96*(163*a^2*d*f^2*h-a*b*f*(59*c*f*
h+208*d*e*h+59*d*f*g)+b^2*(8*d*e*(6*e*h+7*f*g)+c*f*(56*e*h+3*f*g)))*(f*x+e
)^(1/2)/b^4/(-a*f+b*e)/(b*x+a)^2-1/64*f*(93*a^2*d*f^2*h-a*b*f*(5*c*f*h+176
*d*e*h+5*d*f*g)-b^2*(c*f*(-8*e*h+3*f*g)-8*d*e*(10*e*h+f*g)))*(f*x+e)^(1/2)
/b^4/(-a*f+b*e)^2/(b*x+a)-1/4*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(3/2)/b^3/(b*x
+a)^4-1/64*f^2*(35*a^2*d*f^2*h+5*a*b*f*(c*f*h-16*d*e*h+d*f*g)+b^2*(c*f*(-8
*e*h+3*f*g)-8*d*e*(-6*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^
(1/2))/b^(9/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx =$$
$$-\frac{\sqrt{e+fx}(105a^5df^3h+5a^4bf^2(3cfh+d(3fg-34eh+77fhx))+b^5(c(-9f^3gx^3+6ef^2x^2(g+4hx)+1}{}$$
$$+\frac{f^2(35a^2df^2h+5abf(dfg-16deh+cfh)+b^2(cf(3fg-8eh)+8de(-fg+6eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{64b^{9/2}(-be+af)^{5/2}}$$

input
```
Integrate[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^5,x]
```

output
```
-1/192*(Sqrt[e + f*x]*(105*a^5*d*f^3*h + 5*a^4*b*f^2*(3*c*f*h + d*(3*f*g -
 34*e*h + 77*f*h*x)) + b^5*(c*(-9*f^3*g*x^3 + 6*e*f^2*x^2*(g + 4*h*x) + 16
*e^3*(3*g + 4*h*x) + 8*e^2*f*x*(9*g + 14*h*x)) + 8*d*e*x*(3*f^2*g*x^2 + 4*
e^2*(2*g + 3*h*x) + 2*e*f*x*(7*g + 15*h*x))) + a*b^4*(c*(16*e^3*h - 8*e^2*
f*(9*g + 5*h*x) - 3*f^3*x^2*(11*g + 5*h*x) - 2*e*f^2*x*(66*g + 79*h*x)) +
d*(-15*f^3*g*x^3 + 16*e^3*(g + 4*h*x) + 8*e^2*f*x*(-5*g + 26*h*x) - 2*e*f^
2*x^2*(79*g + 264*h*x))) + a^2*b^3*(c*f*(-8*e^2*h + e*f*(6*g - 52*h*x) + f
^2*x*(33*g + 73*h*x)) + d*(16*e^3*h - 8*e^2*f*(g - 19*h*x) + f^3*x^2*(73*g
 + 279*h*x) - 2*e*f^2*x*(26*g + 421*h*x))) + a^3*b^2*f*(c*f*(9*f*g - 14*e*
h + 55*f*h*x) + d*(40*e^2*h - 2*e*f*(7*g + 314*h*x) + f^2*x*(55*g + 511*h*
x)))))/(b^4*(b*e - a*f)^2*(a + b*x)^4) + (f^2*(35*a^2*d*f^2*h + 5*a*b*f*(d
*f*g - 16*d*e*h + c*f*h) + b^2*(c*f*(3*f*g - 8*e*h) + 8*d*e*(-(f*g) + 6*e*
h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x]])/Sqrt[-(b*e) + a*f]])/(64*b^(9/2)*(-(b*
e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {162, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5} dx$$

$$\Big\downarrow \text{162}$$

$$\frac{(35a^2df^2h+5abf(cfh-16deh+dfg)+b^2(cf(3fg-8eh)-8de(fg-6eh)))\int \frac{(e+fx)^{3/2}}{(a+bx)^3}dx}{48b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(13a^2dfh-ab(5cfh+16deh+5dfg)+b^2(8ceh-3cfg+8deg)\right)+a^2b(cfh-10deh-\right.}{24b^2(a+bx)^4(be-af)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{(35a^2df^2h+5abf(cfh-16deh+dfg)+b^2(cf(3fg-8eh)-8de(fg-6eh)))\left(\frac{3f\int \frac{\sqrt{e+fx}}{(a+bx)^2}dx}{4b}-\frac{(e+fx)^{3/2}}{2b(a+bx)^2}\right)}{48b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(13a^2dfh-ab(5cfh+16deh+5dfg)+b^2(8ceh-3cfg+8deg)\right)+a^2b(cfh-10deh-\right.}{24b^2(a+bx)^4(be-af)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{(35a^2df^2h+5abf(cfh-16deh+dfg)+b^2(cf(3fg-8eh)-8de(fg-6eh)))\left(\frac{3f\left(\frac{f\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2b}-\frac{\sqrt{e+fx}}{b(a+bx)}\right)}{4b}-\right.}{48b^2(be-af)^2}-\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(13a^2dfh-ab(5cfh+16deh+5dfg)+b^2(8ceh-3cfg+8deg)\right)+a^2b(cfh-10deh-\right.}{24b^2(a+bx)^4(be-af)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(35a^2df^2h+5abf(cfh-16deh+dfg)+b^2(cf(3fg-8eh)-8de(fg-6eh))\right)\left(\frac{3f\left(\frac{\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b}-\frac{\sqrt{e+fx}}{b(a+bx)}\right.}{4b}\right.}{48b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(13a^2dfh-ab(5cfh+16deh+5dfg)+b^2(8ceh-3cfg+8deg)\right)+a^2b(cfh-10deh\right.}{24b^2(a+bx)^4(be-af)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{3f\left(-\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}\sqrt{be-af}}-\frac{\sqrt{e+fx}}{b(a+bx)}\right)}{4b}-\frac{(e+fx)^{3/2}}{2b(a+bx)^2}\right)\left(35a^2df^2h+5abf(cfh-16deh+dfg)+b^2(cf(3fg-8eh)-\right.}{48b^2(be-af)^2}$$

$$\frac{(e+fx)^{5/2}\left(7a^3dfh+bx\left(13a^2dfh-ab(5cfh+16deh+5dfg)+b^2(8ceh-3cfg+8deg)\right)+a^2b(cfh-10deh\right.}{24b^2(a+bx)^4(be-af)^2}$$

input
```
Int[((c + d*x)*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^5,x]
```

output
```
-1/24*((e + f*x)^(5/2)*(6*b^3*c*e*g + 7*a^3*d*f*h + 2*a*b^2*(d*e*g - (9*c*
f*g)/2 + c*e*h) + a^2*b*(d*f*g - 10*d*e*h + c*f*h) + b*(13*a^2*d*f*h + b^2
*(8*d*e*g - 3*c*f*g + 8*c*e*h) - a*b*(5*d*f*g + 16*d*e*h + 5*c*f*h))*x))/(
b^2*(b*e - a*f)^2*(a + b*x)^4) + ((35*a^2*d*f^2*h + 5*a*b*f*(d*f*g - 16*d*
e*h + c*f*h) + b^2*(c*f*(3*f*g - 8*e*h) - 8*d*e*(f*g - 6*e*h)))*(-1/2*(e +
 f*x)^(3/2)/(b*(a + b*x)^2) + (3*f*(-(Sqrt[e + f*x]/(b*(a + b*x))) - (f*Ar
cTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(3/2)*Sqrt[b*e - a*f]))
)/(4*b)))/(48*b^2*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$\frac{35(bx+a)^4 f^2\left(\frac{\left(3cg\,f^2-8e(ch+dg)f+48d\,e^2h\right)b^2}{35}+\frac{a(f(ch+dg)-16deh)fb}{7}+a^2d\,f^2h\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{64}-\frac{35\left(\frac{\left(-3c\,f^3g\,x^3+\right.}{}\right.}{}$
derivativedivides	$2f^2\left(-\frac{\left(93a^2d\,f^2h-5abc\,f^2h-176abdefh-5abd\,f^2g+8b^2cefh-3b^2c\,f^2g+80b^2d\,e^2h+8b^2defg\right)(fx+e)^{\frac{7}{2}}}{128b\left(a^2f^2-2abfe+b^2e^2\right)}+\frac{\left(511a^2d\,f^2h+73a\right.}{}\right.$
default	$2f^2\left(-\frac{\left(93a^2d\,f^2h-5abc\,f^2h-176abdefh-5abd\,f^2g+8b^2cefh-3b^2c\,f^2g+80b^2d\,e^2h+8b^2defg\right)(fx+e)^{\frac{7}{2}}}{128b\left(a^2f^2-2abfe+b^2e^2\right)}+\frac{\left(511a^2d\,f^2h+73a\right.}{}\right.$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
35/64/((a*f-b*e)*b)^(1/2)*((b*x+a)^4*f^2*(1/35*(3*c*g*f^2-8*e*(c*h+d*g)*f+
48*d*e^2*h)*b^2+1/7*a*(f*(c*h+d*g)-16*d*e*h)*f*b+a^2*d*f^2*h)*arctan(b*(f*
x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-(1/35*(-3*c*f^3*g*x^3+2*(4*(c*h+d*g)*x+c*g
)*x^2*e*f^2+24*x*(10/3*d*h*x^2+14/9*(c*h+d*g)*x+c*g)*e^2*f+16*(2*d*h*x^2+4
/3*(c*h+d*g)*x+c*g)*e^3)*b^5+16/105*a*(-33/16*x^2*(5/11*(c*h+d*g)*x+c*g)*f
^3-33/4*x*(4*d*h*x^2+79/66*(c*h+d*g)*x+c*g)*e*f^2-9/2*(-26/9*d*h*x^2+5/9*(
c*h+d*g)*x+c*g)*e^2*f+e^3*(4*d*h*x+c*h+d*g))*b^4-8/105*a^2*(1/8*(-279*d*h*
x^3+73*(-c*h-d*g)*x^2-33*c*g*x)*f^3-3/4*(-421/3*d*h*x^2+26/3*(-c*h-d*g)*x+
c*g)*e*f^2+e^2*(-19*d*h*x+c*h+d*g)*f-2*d*e^3*h)*b^3-2/15*a^3*f*(1/2*(-73*d
*h*x^2+55/7*(-c*h-d*g)*x-9/7*c*g)*f^2+e*(314/7*d*h*x+c*h+d*g)*f-20/7*d*e^2
*h)*b^2+1/7*a^4*((77/3*d*h*x+c*h+d*g)*f-34/3*d*e*h)*f^2*b+a^5*d*f^3*h)*((a
*f-b*e)*b)^(1/2)*(f*x+e)^(1/2))/(b*x+a)^4/(a*f-b*e)^2/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1605 vs. 2(388) = 776.

Time = 0.32 (sec) , antiderivative size = 3224, normalized size of antiderivative = 7.79

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1320 vs. $2(388) = 776$.

Time = 0.17 (sec) , antiderivative size = 1320, normalized size of antiderivative = 3.19

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/64*(8*b^2*d*e*f^3*g - 3*b^2*c*f^4*g - 5*a*b*d*f^4*g - 48*b^2*d*e^2*f^2*
h + 8*b^2*c*e*f^3*h + 80*a*b*d*e*f^3*h - 5*a*b*c*f^4*h - 35*a^2*d*f^4*h)*a
rctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^2 - 2*a*b^5*e*f + a^2*
b^4*f^2)*sqrt(-b^2*e + a*b*f)) - 1/192*(24*(f*x + e)^(7/2)*b^5*d*e*f^3*g +
 40*(f*x + e)^(5/2)*b^5*d*e^2*f^3*g - 88*(f*x + e)^(3/2)*b^5*d*e^3*f^3*g +
 24*sqrt(f*x + e)*b^5*d*e^4*f^3*g - 9*(f*x + e)^(7/2)*b^5*c*f^4*g - 15*(f*
x + e)^(7/2)*a*b^4*d*f^4*g + 33*(f*x + e)^(5/2)*b^5*c*e*f^4*g - 113*(f*x +
 e)^(5/2)*a*b^4*d*e*f^4*g + 33*(f*x + e)^(3/2)*b^5*c*e^2*f^4*g + 231*(f*x
+ e)^(3/2)*a*b^4*d*e^2*f^4*g - 9*sqrt(f*x + e)*b^5*c*e^3*f^4*g - 87*sqrt(f
*x + e)*a*b^4*d*e^3*f^4*g - 33*(f*x + e)^(5/2)*a*b^4*c*f^5*g + 73*(f*x + e
)^(5/2)*a^2*b^3*d*f^5*g - 66*(f*x + e)^(3/2)*a*b^4*c*e*f^5*g - 198*(f*x +
e)^(3/2)*a^2*b^3*d*e*f^5*g + 27*sqrt(f*x + e)*a*b^4*c*e^2*f^5*g + 117*sqrt
(f*x + e)*a^2*b^3*d*e^2*f^5*g + 33*(f*x + e)^(3/2)*a^2*b^3*c*f^6*g + 55*(f
*x + e)^(3/2)*a^3*b^2*d*f^6*g - 27*sqrt(f*x + e)*a^2*b^3*c*e*f^6*g - 69*sq
rt(f*x + e)*a^3*b^2*d*e*f^6*g + 9*sqrt(f*x + e)*a^3*b^2*c*f^7*g + 15*sqrt(
f*x + e)*a^4*b*d*f^7*g + 240*(f*x + e)^(7/2)*b^5*d*e^2*f^2*h - 624*(f*x +
e)^(5/2)*b^5*d*e^3*f^2*h + 528*(f*x + e)^(3/2)*b^5*d*e^4*f^2*h - 144*sqrt(
f*x + e)*b^5*d*e^5*f^2*h + 24*(f*x + e)^(7/2)*b^5*c*e*f^3*h - 528*(f*x + e
)^(7/2)*a*b^4*d*e*f^3*h + 40*(f*x + e)^(5/2)*b^5*c*e^2*f^3*h + 1792*(f*x +
 e)^(5/2)*a*b^4*d*e^2*f^3*h - 88*(f*x + e)^(3/2)*b^5*c*e^3*f^3*h - 1936...
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \frac{f^2 \operatorname{atan}\left(\frac{\sqrt{b}\,f^2\,\sqrt{e+f\,x}\,(3\,b^2\,c\,f^2\,g+35\,a^2\,d\,f^2\,h+48\,b^2\,d\,e^2\,h+5\,a\,b\,c\,f^2\,h+5\,a\,b\,d\,f^2\,g-8\,b^2}{\sqrt{a\,f-b\,e}\,(3\,b^2\,c\,f^4\,g+35\,a^2\,d\,f^4\,h-8\,b^2\,c\,e\,f^3\,h-8\,b^2\,d\,e\,f^3\,g+48\,b^2\,d\,e^2\,f^2\,h+5\,a\,b}\right.}{} - \frac{\frac{11\,(e+f\,x)^{3/2}\,(3\,b^2\,c\,f^4\,g+35\,a^2\,d\,f^4\,h-8\,b^2\,c\,e\,f^3\,h-8\,b^2\,d\,e\,f^3\,g+48\,b^2\,d\,e^2\,f^2\,h+5\,a\,b\,c\,f^4\,h+5\,a\,b\,d\,f^4\,g-80\,a\,b\,d\,e\,f^3\,h)}{192\,b^3} + \frac{\sqrt{e+f\,x}\,(a\,f-b\,e)}{}}{b^4\,(e+f\,x)^4-(e+f}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x))/(a + b*x)^5,x)
```

output
```
(f^2*atan((b^(1/2)*f^2*(e + f*x)^(1/2)*(3*b^2*c*f^2*g + 35*a^2*d*f^2*h + 4
8*b^2*d*e^2*h + 5*a*b*c*f^2*h + 5*a*b*d*f^2*g - 8*b^2*c*e*f*h - 8*b^2*d*e*
f*g - 80*a*b*d*e*f*h))/((a*f - b*e)^(1/2)*(3*b^2*c*f^4*g + 35*a^2*d*f^4*h
 - 8*b^2*c*e*f^3*h - 8*b^2*d*e*f^3*g + 48*b^2*d*e^2*f^2*h + 5*a*b*c*f^4*h +
 5*a*b*d*f^4*g - 80*a*b*d*e*f^3*h)))*(3*b^2*c*f^2*g + 35*a^2*d*f^2*h + 48*
b^2*d*e^2*h + 5*a*b*c*f^2*h + 5*a*b*d*f^2*g - 8*b^2*c*e*f*h - 8*b^2*d*e*f*
g - 80*a*b*d*e*f*h))/(64*b^(9/2)*(a*f - b*e)^(5/2)) - ((11*(e + f*x)^(3/2)
*(3*b^2*c*f^4*g + 35*a^2*d*f^4*h - 8*b^2*c*e*f^3*h - 8*b^2*d*e*f^3*g + 48*
b^2*d*e^2*f^2*h + 5*a*b*c*f^4*h + 5*a*b*d*f^4*g - 80*a*b*d*e*f^3*h))/(192*
b^3) + ((e + f*x)^(1/2)*(a*f - b*e)*(3*b^2*c*f^4*g + 35*a^2*d*f^4*h - 8*b^
2*c*e*f^3*h - 8*b^2*d*e*f^3*g + 48*b^2*d*e^2*f^2*h + 5*a*b*c*f^4*h + 5*a*b
*d*f^4*g - 80*a*b*d*e*f^3*h))/(64*b^4) - ((e + f*x)^(7/2)*(3*b^2*c*f^4*g -
 93*a^2*d*f^4*h - 8*b^2*c*e*f^3*h - 8*b^2*d*e*f^3*g - 80*b^2*d*e^2*f^2*h +
 5*a*b*c*f^4*h + 5*a*b*d*f^4*g + 176*a*b*d*e*f^3*h))/(64*b*(a*f - b*e)^2)
 - ((e + f*x)^(5/2)*(33*b^2*c*f^4*g - 511*a^2*d*f^4*h + 40*b^2*c*e*f^3*h +
40*b^2*d*e*f^3*g - 624*b^2*d*e^2*f^2*h - 73*a*b*c*f^4*h - 73*a*b*d*f^4*g +
 1168*a*b*d*e*f^3*h))/(192*b^2*(a*f - b*e)))/(b^4*(e + f*x)^4 - (e + f*x)^
3*(4*b^4*e - 4*a*b^3*f) - (e + f*x)*(4*b^4*e^3 - 4*a^3*b*f^3 + 12*a^2*b^2*
e*f^2 - 12*a*b^3*e^2*f) + a^4*f^4 + b^4*e^4 + (e + f*x)^2*(6*b^4*e^2 + 6*a
^2*b^2*f^2 - 12*a*b^3*e*f) + 6*a^2*b^2*e^2*f^2 - 4*a*b^3*e^3*f - 4*a^3*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3637, normalized size of antiderivative = 8.79

$$\int \frac{(c+dx)(e+fx)^{3/2}(g+hx)}{(a+bx)^5} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x)
```

output
```
(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**6*d*f**4*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**5*b*c*f**4*h - 240*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d*e*f**3*h + 15*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*
b*d*f**4*g + 420*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**5*b*d*f**4*h*x - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*e*f**3*h + 9*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2
*c*f**4*g + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**4*b**2*c*f**4*h*x + 144*sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e**2*f**2*h - 24*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
4*b**2*d*e*f**3*g - 960*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e*f**3*h*x + 60*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*f**4*g*x +
 630*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**4*b**2*d*f**4*h*x**2 - 96*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**3*c*e*f**3*h*x + 36*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**3...
```

3.69 $\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 579

$$\begin{aligned}
&\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \\
&-\frac{2(be-af)^3(de-cf)^2(fg-eh)(e+fx)^{5/2}}{5f^7} \\
&+\frac{2(be-af)^2(de-cf)(bde(5fg-6eh)-bcf(3fg-4eh)-af(2dfg-3deh+cfh))(e+fx)^{7/2}}{7f^7} \\
&-\frac{2(be-af)\left(a^2df^2(dfg-3deh+2cfh)+abf(3c^2f^2h-d^2e(8fg-15eh)+2cdf(3fg-8eh))-b^2(4cde\right.}{9f^7} \\
&+\frac{2(a^3d^2f^3h+3a^2bdf^2(dfg-4deh+2cfh)+3ab^2f(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh))-b^3(4c}{11f^7} \\
&+\frac{2b(3a^2d^2f^2h+3abdf(dfg-5deh+2cfh)+b^2(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)))\,(e+fx)^{13/2}}{13f^7} \\
&+\frac{2b^2d(3adfh+b(dfg-6deh+2cfh))(e+fx)^{15/2}}{15f^7}+\frac{2b^3d^2h(e+fx)^{17/2}}{17f^7}
\end{aligned}$$

output
```
-2/5*(-a*f+b*e)^3*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(5/2)/f^7+2/7*(-a*f+b*e)
^2*(-c*f+d*e)*(b*d*e*(-6*e*h+5*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-3*d*e*
h+2*d*f*g))*(f*x+e)^(7/2)/f^7-2/9*(-a*f+b*e)*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d
*f*g)+a*b*f*(3*c^2*f^2*h-d^2*e*(-15*e*h+8*f*g)+2*c*d*f*(-8*e*h+3*f*g))-b^2
*(4*c*d*e*f*(-5*e*h+3*f*g)-5*d^2*e^2*(-3*e*h+2*f*g)-3*c^2*f^2*(-2*e*h+f*g)
))*(f*x+e)^(9/2)/f^7+2/11*(a^3*d^2*f^3*h+3*a^2*b*d*f^2*(2*c*f*h-4*d*e*h+d*
f*g)+3*a*b^2*f*(c^2*f^2*h-2*d^2*e*(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g))-b^3
*(4*c*d*e*f*(-5*e*h+2*f*g)-c^2*f^2*(-4*e*h+f*g)-10*d^2*e^2*(-2*e*h+f*g)))*
(f*x+e)^(11/2)/f^7+2/13*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(2*c*f*h-5*d*e*h+d*f*
g)+b^2*(c^2*f^2*h+2*c*d*f*(-5*e*h+f*g)-5*d^2*e*(-3*e*h+f*g)))*(f*x+e)^(13/
2)/f^7+2/15*b^2*d*(3*a*d*f*h+b*(2*c*f*h-6*d*e*h+d*f*g))*(f*x+e)^(15/2)/f^7
+2/17*b^3*d^2*h*(f*x+e)^(17/2)/f^7
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.47

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(221a^3f^3(99c^2f^2(7fg-2eh+5fhx)+22cdf(8e^2h+5f^2x(9g+7hx)-2ef(9g+\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(221*a^3*f^3*(99*c^2*f^2*(7*f*g - 2*e*h + 5*f*h*x) + 22
*c*d*f*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) + d^2*(-48
*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*
(22*g + 21*h*x))) + 51*a^2*b*f^2*(143*c^2*f^2*(8*e^2*h + 5*f^2*x*(9*g + 7*
h*x) - 2*e*f*(9*g + 10*h*x)) + 3*d^2*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h
*x) - 70*e*f^3*x^2*(13*g + 12*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3
*f*(13*g + 20*h*x)) + 26*c*d*f*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*
e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(22*g + 21*h*x))) + 51*a*b^2*f*(d^2*(-2
56*e^5*h + 1680*e^2*f^3*x^2*(g + h*x) + 128*e^4*f*(3*g + 5*h*x) - 160*e^3*
f^2*x*(6*g + 7*h*x) - 210*e*f^4*x^3*(12*g + 11*h*x) + 231*f^5*x^4*(15*g +
13*h*x)) + 6*c*d*f*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) - 70*e*f^3*x^2
*(13*g + 12*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13*g + 20*h*x)
) + 13*c^2*f^2*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15
*h*x) - 10*e*f^2*x*(22*g + 21*h*x))) + b^3*(34*c*d*f*(-256*e^5*h + 1680*e^
2*f^3*x^2*(g + h*x) + 128*e^4*f*(3*g + 5*h*x) - 160*e^3*f^2*x*(6*g + 7*h*x
) - 210*e*f^4*x^3*(12*g + 11*h*x) + 231*f^5*x^4*(15*g + 13*h*x)) + 51*c^2*
f^2*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) - 70*e*f^3*x^2*(13*g + 12*h*x
) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13*g + 20*h*x)) + d^2*(3072*e
^6*h + 3003*f^6*x^5*(17*g + 15*h*x) - 1120*e^3*f^3*x^2*(17*g + 18*h*x) + 6
40*e^4*f^2*x*(17*g + 21*h*x) - 256*e^5*f*(17*g + 30*h*x) + 840*e^2*f^4*...
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{(e+fx)^{7/2}(be-af)\left(-a^2df^2(2cfh-3deh+dfg)-abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh\right)\right.}{f^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(e+fx)^{9/2}(be-af)\left(a^2df^2(2cfh-3deh+dfg)+abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh)\right)-\right.}{9f^7}$$
$$\frac{2b(e+fx)^{13/2}\left(3a^2d^2f^2h+3abdf(2cfh-5deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)\right)\right)}{13f^7}+$$
$$\frac{2(e+fx)^{11/2}\left(a^3d^2f^3h+3a^2bdf^2(2cfh-4deh+dfg)+3ab^2f\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)-\right.}{11f^7}$$
$$\frac{2b^2d(e+fx)^{15/2}(3adfh+b(2cfh-6deh+dfg))}{15f^7}+$$
$$\frac{2(e+fx)^{7/2}(be-af)^2(de-cf)(-af(cfh-3deh+2dfg)-bcf(3fg-4eh)+bde(5fg-6eh))}{7f^7}-$$
$$\frac{2(e+fx)^{5/2}(be-af)^3(de-cf)^2(fg-eh)}{5f^7}+\frac{2b^3d^2h(e+fx)^{17/2}}{17f^7}$$

input
```
Int[(a + b*x)^3*(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(-2*(b*e - a*f)^3*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^7) + (2*
(b*e - a*f)^2*(d*e - c*f)*(b*d*e*(5*f*g - 6*e*h) - b*c*f*(3*f*g - 4*e*h) -
 a*f*(2*d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(7/2))/(7*f^7) - (2*(b*e - a*f
)*(a^2*d*f^2*(d*f*g - 3*d*e*h + 2*c*f*h) + a*b*f*(3*c^2*f^2*h - d^2*e*(8*f
*g - 15*e*h) + 2*c*d*f*(3*f*g - 8*e*h)) - b^2*(4*c*d*e*f*(3*f*g - 5*e*h) -
 5*d^2*e^2*(2*f*g - 3*e*h) - 3*c^2*f^2*(f*g - 2*e*h)))*(e + f*x)^(9/2))/(9
*f^7) + (2*(a^3*d^2*f^3*h + 3*a^2*b*d*f^2*(d*f*g - 4*d*e*h + 2*c*f*h) + 3*
a*b^2*f*(c^2*f^2*h - 2*d^2*e*(2*f*g - 5*e*h) + 2*c*d*f*(f*g - 4*e*h)) - b^
3*(4*c*d*e*f*(2*f*g - 5*e*h) - c^2*f^2*(f*g - 4*e*h) - 10*d^2*e^2*(f*g - 2
*e*h)))*(e + f*x)^(11/2))/(11*f^7) + (2*b*(3*a^2*d^2*f^2*h + 3*a*b*d*f*(d*
f*g - 5*d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h + 2*c*d*f*(f*g - 5*e*h) - 5*d^2*
e*(f*g - 3*e*h)))*(e + f*x)^(13/2))/(13*f^7) + (2*b^2*d*(3*a*d*f*h + b*(d*
f*g - 6*d*e*h + 2*c*f*h))*(e + f*x)^(15/2))/(15*f^7) + (2*b^3*d^2*h*(e + f
*x)^(17/2))/(17*f^7)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 569, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{2h\,d^2 b^3 (fx+e)^{\frac{17}{2}}}{17}+\frac{2\big(\big(3(af-be)b^2 d^2+2b^3 d(cf-de)\big)h+b^3 d^2(-eh+fg)\big)(fx+e)^{\frac{15}{2}}}{15}+\frac{2\big(\big(3(af-be)^2 b\,d^2+6(af-be)b^2 d(cf-de)+\cdots}{\cdots}$
default	$\frac{2h\,d^2 b^3 (fx+e)^{\frac{17}{2}}}{17}-\frac{2\big(-\big(3(af-be)b^2 d^2+2b^3 d(cf-de)\big)h+b^3 d^2(eh-fg)\big)(fx+e)^{\frac{15}{2}}}{15}-\frac{2\big(-\big(3(af-be)^2 b\,d^2+6(af-be)b^2 d(cf-de)\cdots}{\cdots}$
pseudoelliptic	Expression too large to display
gosper	Expression too large to display
orering	Expression too large to display
trager	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^7*(1/17*h*d^2*b^3*(f*x+e)^(17/2)+1/15*((3*(a*f-b*e)*b^2*d^2+2*b^3*d*(c
*f-d*e))*h+b^3*d^2*(-e*h+f*g))*(f*x+e)^(15/2)+1/13*((3*(a*f-b*e)^2*b*d^2+6
*(a*f-b*e)*b^2*d*(c*f-d*e)+b^3*(c*f-d*e)^2)*h+(3*(a*f-b*e)*b^2*d^2+2*b^3*d
*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(13/2)+1/11*(((a*f-b*e)^3*d^2+6*(a*f-b*e)^
2*b*d*(c*f-d*e)+3*(a*f-b*e)*b^2*(c*f-d*e)^2)*h+(3*(a*f-b*e)^2*b*d^2+6*(a*f
-b*e)*b^2*d*(c*f-d*e)+b^3*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*((2*
(a*f-b*e)^3*d*(c*f-d*e)+3*(a*f-b*e)^2*b*(c*f-d*e)^2)*h+((a*f-b*e)^3*d^2+6*
(a*f-b*e)^2*b*d*(c*f-d*e)+3*(a*f-b*e)*b^2*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)
^(9/2)+1/7*((a*f-b*e)^3*(c*f-d*e)^2*h+(2*(a*f-b*e)^3*d*(c*f-d*e)+3*(a*f-b*
e)^2*b*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*(a*f-b*e)^3*(c*f-d*e)^2*
(-e*h+f*g)*(f*x+e)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(548) = 1096$.

Time = 0.10 (sec) , antiderivative size = 1773, normalized size of antiderivative = 3.06

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/765765*(45045*b^3*d^2*f^8*h*x^8 + 3003*(17*b^3*d^2*f^8*g + (18*b^3*d^2*e
*f^7 + 17*(2*b^3*c*d + 3*a*b^2*d^2)*f^8)*h)*x^7 + 231*(17*(16*b^3*d^2*e*f^
7 + 15*(2*b^3*c*d + 3*a*b^2*d^2)*f^8)*g + (3*b^3*d^2*e^2*f^6 + 272*(2*b^3*
c*d + 3*a*b^2*d^2)*e*f^7 + 255*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^8)*
h)*x^6 + 63*(17*(b^3*d^2*e^2*f^6 + 70*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^7 + 65
*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^8)*g - (12*b^3*d^2*e^3*f^5 - 17*(
2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^6 - 1190*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d
^2)*e*f^7 - 1105*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^8)*h)*x^5 - 35*(1
7*(2*b^3*d^2*e^3*f^5 - 3*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^6 - 156*(b^3*c^2
+ 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^7 - 143*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*
d^2)*f^8)*g - (24*b^3*d^2*e^4*f^4 - 34*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^5 +
 51*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^2*f^6 + 2652*(3*a*b^2*c^2 + 6*
a^2*b*c*d + a^3*d^2)*e*f^7 + 2431*(3*a^2*b*c^2 + 2*a^3*c*d)*f^8)*h)*x^4 +
5*(17*(16*b^3*d^2*e^4*f^4 - 24*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^5 + 39*(b^3
*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^2*f^6 + 1430*(3*a*b^2*c^2 + 6*a^2*b*c*
d + a^3*d^2)*e*f^7 + 1287*(3*a^2*b*c^2 + 2*a^3*c*d)*f^8)*g - (192*b^3*d^2*
e^5*f^3 - 21879*a^3*c^2*f^8 - 272*(2*b^3*c*d + 3*a*b^2*d^2)*e^4*f^4 + 408*
(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^5 - 663*(3*a*b^2*c^2 + 6*a^2*b
*c*d + a^3*d^2)*e^2*f^6 - 24310*(3*a^2*b*c^2 + 2*a^3*c*d)*e*f^7)*h)*x^3 -
3*(17*(32*b^3*d^2*e^5*f^3 - 3003*a^3*c^2*f^8 - 48*(2*b^3*c*d + 3*a*b^2*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. $2(620) = 1240$.

Time = 2.60 (sec) , antiderivative size = 1909, normalized size of antiderivative = 3.30

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**2*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**3*d**2*h*(e + f*x)**(17/2)/(17*f**6) + (e + f*x)**(15/2)*
(3*a*b**2*d**2*f*h + 2*b**3*c*d*f*h - 6*b**3*d**2*e*h + b**3*d**2*f*g)/(15
*f**6) + (e + f*x)**(13/2)*(3*a**2*b*d**2*f**2*h + 6*a*b**2*c*d*f**2*h - 1
5*a*b**2*d**2*e*f*h + 3*a*b**2*d**2*f**2*g + b**3*c**2*f**2*h - 10*b**3*c*
d*e*f*h + 2*b**3*c*d*f**2*g + 15*b**3*d**2*e**2*h - 5*b**3*d**2*e*f*g)/(13
*f**6) + (e + f*x)**(11/2)*(a**3*d**2*f**3*h + 6*a**2*b*c*d*f**3*h - 12*a*
*2*b*d**2*e*f**2*h + 3*a**2*b*d**2*f**3*g + 3*a*b**2*c**2*f**3*h - 24*a*b*
*2*c*d*e*f**2*h + 6*a*b**2*c*d*f**3*g + 30*a*b**2*d**2*e**2*f*h - 12*a*b**
2*d**2*e*f**2*g - 4*b**3*c**2*e*f**2*h + b**3*c**2*f**3*g + 20*b**3*c*d*e*
*2*f*h - 8*b**3*c*d*e*f**2*g - 20*b**3*d**2*e**3*h + 10*b**3*d**2*e**2*f*g
)/(11*f**6) + (e + f*x)**(9/2)*(2*a**3*c*d*f**4*h - 3*a**3*d**2*e*f**3*h +
 a**3*d**2*f**4*g + 3*a**2*b*c**2*f**4*h - 18*a**2*b*c*d*e*f**3*h + 6*a**2
*b*c*d*f**4*g + 18*a**2*b*d**2*e**2*f**2*h - 9*a**2*b*d**2*e*f**3*g - 9*a*
b**2*c**2*e*f**3*h + 3*a*b**2*c**2*f**4*g + 36*a*b**2*c*d*e**2*f**2*h - 18
*a*b**2*c*d*e*f**3*g - 30*a*b**2*d**2*e**3*f*h + 18*a*b**2*d**2*e**2*f**2*
g + 6*b**3*c**2*e**2*f**2*h - 3*b**3*c**2*e*f**3*g - 20*b**3*c*d*e**3*f*h
+ 12*b**3*c*d*e**2*f**2*g + 15*b**3*d**2*e**4*h - 10*b**3*d**2*e**3*f*g)/(
9*f**6) + (e + f*x)**(7/2)*(a**3*c**2*f**5*h - 4*a**3*c*d*e*f**4*h + 2*a**
3*c*d*f**5*g + 3*a**3*d**2*e**2*f**3*h - 2*a**3*d**2*e*f**4*g - 6*a**2*b*c
**2*e*f**4*h + 3*a**2*b*c**2*f**5*g + 18*a**2*b*c*d*e**2*f**3*h - 12*a*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(548) = 1096$.

Time = 0.04 (sec) , antiderivative size = 1152, normalized size of antiderivative = 1.99

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/765765*(45045*(f*x + e)^(17/2)*b^3*d^2*h + 51051*(b^3*d^2*f*g - (6*b^3*d
^2*e - (2*b^3*c*d + 3*a*b^2*d^2)*f)*h)*(f*x + e)^(15/2) - 58905*((5*b^3*d^
2*e*f - (2*b^3*c*d + 3*a*b^2*d^2)*f^2)*g - (15*b^3*d^2*e^2 - 5*(2*b^3*c*d
+ 3*a*b^2*d^2)*e*f + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*h)*(f*x +
e)^(13/2) + 69615*((10*b^3*d^2*e^2*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^2 +
 (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^3)*g - (20*b^3*d^2*e^3 - 10*(2*b^
3*c*d + 3*a*b^2*d^2)*e^2*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2
 - (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^3)*h)*(f*x + e)^(11/2) - 85085*
((10*b^3*d^2*e^3*f - 6*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^2 + 3*(b^3*c^2 + 6*
a*b^2*c*d + 3*a^2*b*d^2)*e*f^3 - (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^4
)*g - (15*b^3*d^2*e^4 - 10*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f + 6*(b^3*c^2 +
6*a*b^2*c*d + 3*a^2*b*d^2)*e^2*f^2 - 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^
2)*e*f^3 + (3*a^2*b*c^2 + 2*a^3*c*d)*f^4)*h)*(f*x + e)^(9/2) + 109395*((5*
b^3*d^2*e^4*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^2 + 3*(b^3*c^2 + 6*a*b^2
*c*d + 3*a^2*b*d^2)*e^2*f^3 - 2*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^
4 + (3*a^2*b*c^2 + 2*a^3*c*d)*f^5)*g - (6*b^3*d^2*e^5 - a^3*c^2*f^5 - 5*(2
*b^3*c*d + 3*a*b^2*d^2)*e^4*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^
3*f^2 - 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2*f^3 + 2*(3*a^2*b*c^2 +
 2*a^3*c*d)*e*f^4)*h)*(f*x + e)^(7/2) - 153153*((b^3*d^2*e^5*f - a^3*c^2*f
^6 - (2*b^3*c*d + 3*a*b^2*d^2)*e^4*f^2 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4992 vs. $2(548) = 1096$.

Time = 0.19 (sec) , antiderivative size = 4992, normalized size of antiderivative = 8.62

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/765765*(765765*sqrt(f*x + e)*a^3*c^2*e^2*g + 510510*((f*x + e)^(3/2) - 3
*sqrt(f*x + e)*e)*a^3*c^2*e*g + 765765*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*
e)*a^2*b*c^2*e^2*g/f + 510510*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c*
d*e^2*g/f + 255255*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c^2*e^2*h/f +
 51051*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a
^3*c^2*g + 153153*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x
+ e)*e^2)*a*b^2*c^2*e^2*g/f^2 + 306306*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(
3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*c*d*e^2*g/f^2 + 51051*(3*(f*x + e)^(5
/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*d^2*e^2*g/f^2 + 306
306*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*
b*c^2*e*g/f + 204204*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f
*x + e)*e^2)*a^3*c*d*e*g/f + 153153*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2
)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*c^2*e^2*h/f^2 + 102102*(3*(f*x + e)^(5/2
) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*c*d*e^2*h/f^2 + 10210
2*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*c^
2*e*h/f + 21879*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(
3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^3*c^2*e^2*g/f^3 + 131274*(5*(f*x + e)^(
7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^
3)*a*b^2*c*d*e^2*g/f^3 + 65637*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e +
 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a^2*b*d^2*e^2*g/f^3 + 1...
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.20

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{13/2}\,\left(6\,h\,a^2\,b\,d^2\,f^2+12\,h\,a\,b^2\,c\,d\,f^2-30\,h\,a\,b^2\,d^2\,e\,f+6\,g\,a\,b^2\,d^2\,f^2+2\,h\,b^3\,c^2\,f^2-20\,h\right.}{13\,f^7}$$
$$+\frac{(e+f\,x)^{11/2}\,\left(2\,h\,a^3\,d^2\,f^3+12\,h\,a^2\,b\,c\,d\,f^3-24\,h\,a^2\,b\,d^2\,e\,f^2+6\,g\,a^2\,b\,d^2\,f^3+6\,h\,a\,b^2\,c^2\,f^3-48\,h\,a\,b^2\,c\,d\right.}{}$$
$$+\frac{2\,(e+f\,x)^{9/2}\,(a\,f-b\,e)\,\left(2\,h\,a^2\,c\,d\,f^3-3\,h\,a^2\,d^2\,e\,f^2+g\,a^2\,d^2\,f^3+3\,h\,a\,b\,c^2\,f^3-16\,h\,a\,b\,c\,d\,e\,f^2+6\,g\,a\,b\right.}{}$$
$$-\frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)^3\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{5\,f^7}+\frac{2\,b^3\,d^2\,h\,(e+f\,x)^{17/2}}{17\,f^7}$$
$$+\frac{2\,b^2\,d\,(e+f\,x)^{15/2}\,(3\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+b\,d\,f\,g)}{15\,f^7}$$
$$+\frac{2\,(e+f\,x)^{7/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)\,\left(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+3\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h\right.}{7\,f^7}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^3*(c + d*x)^2,x)
```

output
```
((e + f*x)^(13/2)*(2*b^3*c^2*f^2*h + 30*b^3*d^2*e^2*h + 4*b^3*c*d*f^2*g -
10*b^3*d^2*e*f*g + 6*a*b^2*d^2*f^2*g + 6*a^2*b*d^2*f^2*h - 20*b^3*c*d*e*f*
h + 12*a*b^2*c*d*f^2*h - 30*a*b^2*d^2*e*f*h))/(13*f^7) + ((e + f*x)^(11/2)
*(2*b^3*c^2*f^3*g + 2*a^3*d^2*f^3*h - 40*b^3*d^2*e^3*h + 6*a*b^2*c^2*f^3*h
 + 6*a^2*b*d^2*f^3*g - 8*b^3*c^2*e*f^2*h + 20*b^3*d^2*e^2*f*g + 12*a*b^2*c
*d*f^3*g + 12*a^2*b*c*d*f^3*h - 16*b^3*c*d*e*f^2*g + 40*b^3*c*d*e^2*f*h -
24*a*b^2*d^2*e*f^2*g + 60*a*b^2*d^2*e^2*f*h - 24*a^2*b*d^2*e*f^2*h - 48*a*
b^2*c*d*e*f^2*h))/(11*f^7) + (2*(e + f*x)^(9/2)*(a*f - b*e)*(a^2*d^2*f^3*g
 + 3*b^2*c^2*f^3*g - 15*b^2*d^2*e^3*h + 3*a*b*c^2*f^3*h + 2*a^2*c*d*f^3*h
- 3*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h + 10*b^2*d^2*e^2*f*g + 6*a*b*c*d*f
^3*g - 8*a*b*d^2*e*f^2*g + 15*a*b*d^2*e^2*f*h - 12*b^2*c*d*e*f^2*g + 20*b^
2*c*d*e^2*f*h - 16*a*b*c*d*e*f^2*h))/(9*f^7) - (2*(e + f*x)^(5/2)*(a*f - b
*e)^3*(c*f - d*e)^2*(e*h - f*g))/(5*f^7) + (2*b^3*d^2*h*(e + f*x)^(17/2))/
(17*f^7) + (2*b^2*d*(e + f*x)^(15/2)*(3*a*d*f*h + 2*b*c*f*h - 6*b*d*e*h +
b*d*f*g))/(15*f^7) + (2*(e + f*x)^(7/2)*(a*f - b*e)^2*(c*f - d*e)*(a*c*f^2
*h + 2*a*d*f^2*g + 3*b*c*f^2*g + 6*b*d*e^2*h - 3*a*d*e*f*h - 4*b*c*e*f*h -
 5*b*d*e*f*g))/(7*f^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2377, normalized size of antiderivative = 4.11

$$\int (a+bx)^3(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 43758*a**3*c**2*e**3*f**5*h + 153153*a**3*c**2*e**2*f
**6*g + 21879*a**3*c**2*e**2*f**6*h*x + 306306*a**3*c**2*e*f**7*g*x + 1750
32*a**3*c**2*e*f**7*h*x**2 + 153153*a**3*c**2*f**8*g*x**2 + 109395*a**3*c*
*2*f**8*h*x**3 + 38896*a**3*c*d*e**4*f**4*h - 87516*a**3*c*d*e**3*f**5*g -
 19448*a**3*c*d*e**3*f**5*h*x + 43758*a**3*c*d*e**2*f**6*g*x + 14586*a**3*
c*d*e**2*f**6*h*x**2 + 350064*a**3*c*d*e*f**7*g*x**2 + 243100*a**3*c*d*e*f
**7*h*x**3 + 218790*a**3*c*d*f**8*g*x**3 + 170170*a**3*c*d*f**8*h*x**4 - 1
0608*a**3*d**2*e**5*f**3*h + 19448*a**3*d**2*e**4*f**4*g + 5304*a**3*d**2*
e**4*f**4*h*x - 9724*a**3*d**2*e**3*f**5*g*x - 3978*a**3*d**2*e**3*f**5*h*
x**2 + 7293*a**3*d**2*e**2*f**6*g*x**2 + 3315*a**3*d**2*e**2*f**6*h*x**3 +
 121550*a**3*d**2*e*f**7*g*x**3 + 92820*a**3*d**2*e*f**7*h*x**4 + 85085*a*
*3*d**2*f**8*g*x**4 + 69615*a**3*d**2*f**8*h*x**5 + 58344*a**2*b*c**2*e**4
*f**4*h - 131274*a**2*b*c**2*e**3*f**5*g - 29172*a**2*b*c**2*e**3*f**5*h*x
 + 65637*a**2*b*c**2*e**2*f**6*g*x + 21879*a**2*b*c**2*e**2*f**6*h*x**2 +
525096*a**2*b*c**2*e*f**7*g*x**2 + 364650*a**2*b*c**2*e*f**7*h*x**3 + 3281
85*a**2*b*c**2*f**8*g*x**3 + 255255*a**2*b*c**2*f**8*h*x**4 - 63648*a**2*b
*c*d*e**5*f**3*h + 116688*a**2*b*c*d*e**4*f**4*g + 31824*a**2*b*c*d*e**4*f
**4*h*x - 58344*a**2*b*c*d*e**3*f**5*g*x - 23868*a**2*b*c*d*e**3*f**5*h*x*
*2 + 43758*a**2*b*c*d*e**2*f**6*g*x**2 + 19890*a**2*b*c*d*e**2*f**6*h*x**3
 + 729300*a**2*b*c*d*e*f**7*g*x**3 + 556920*a**2*b*c*d*e*f**7*h*x**4 + ...
```

3.70 $\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 411

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2(be-af)^2(de-cf)^2(fg-eh)(e+fx)^{5/2}}{5f^6}$$
$$-\frac{2(be-af)(de-cf)(bde(4fg-5eh)-bcf(2fg-3eh)-af(2dfg-3deh+cfh))(e+fx)^{7/2}}{7f^6}$$
$$+\frac{2(a^2df^2(dfg-3deh+2cfh)+2abf(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh))+b^2(2d^2e^2(3fg-5eh)}{9f^6}$$
$$+\frac{2(a^2d^2f^2h+2abdf(dfg-4deh+2cfh)+b^2(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh)))\,(e+fx)^{11/2}}{11f^6}$$
$$+\frac{2bd(2adfh+b(dfg-5deh+2cfh))(e+fx)^{13/2}}{13f^6}+\frac{2b^2d^2h(e+fx)^{15/2}}{15f^6}$$

output

```
2/5*(-a*f+b*e)^2*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(5/2)/f^6-2/7*(-a*f+b*e)*
(-c*f+d*e)*(b*d*e*(-5*e*h+4*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-3*d*e*h+2
*d*f*g))*(f*x+e)^(7/2)/f^6+2/9*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d*f*g)+2*a*b*f*
(c^2*f^2*h+2*c*d*f*(-3*e*h+f*g)-3*d^2*e*(-2*e*h+f*g))+b^2*(2*d^2*e^2*(-5*e
*h+3*f*g)+c^2*f^2*(-3*e*h+f*g)-6*c*d*e*f*(-2*e*h+f*g)))*(f*x+e)^(9/2)/f^6+
2/11*(a^2*d^2*f^2*h+2*a*b*d*f*(2*c*f*h-4*d*e*h+d*f*g)+b^2*(c^2*f^2*h-2*d^2
*e*(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g)))*(f*x+e)^(11/2)/f^6+2/13*b*d*(2*a*
d*f*h+b*(2*c*f*h-5*d*e*h+d*f*g))*(f*x+e)^(13/2)/f^6+2/15*b^2*d^2*h*(f*x+e)
^(15/2)/f^6
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.35

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(13a^2f^2(99c^2f^2(7fg-2eh+5fhx)+22cdf(8e^2h+5f^2x(9g+7hx)-2ef(9g+1\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(13*a^2*f^2*(99*c^2*f^2*(7*f*g - 2*e*h + 5*f*h*x) + 22*
c*d*f*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) + d^2*(-48*
e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(
22*g + 21*h*x))) + 2*a*b*f*(143*c^2*f^2*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) -
 2*e*f*(9*g + 10*h*x)) + 3*d^2*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) -
70*e*f^3*x^2*(13*g + 12*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13
*g + 20*h*x)) + 26*c*d*f*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*
(11*g + 15*h*x) - 10*e*f^2*x*(22*g + 21*h*x))) + b^2*(d^2*(-256*e^5*h + 16
80*e^2*f^3*x^2*(g + h*x) + 128*e^4*f*(3*g + 5*h*x) - 160*e^3*f^2*x*(6*g +
7*h*x) - 210*e*f^4*x^3*(12*g + 11*h*x) + 231*f^5*x^4*(15*g + 13*h*x)) + 6*
c*d*f*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) - 70*e*f^3*x^2*(13*g + 12*h
*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13*g + 20*h*x)) + 13*c^2*f^
2*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*
f^2*x*(22*g + 21*h*x)))))/(45045*f^6)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \ 165$$

$$\int \left(\frac{(e+fx)^{7/2} \left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-\right.\right.}{f^5}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2(e+fx)^{9/2} \left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-3e\right.\right.}{9f^6}$$
$$\frac{2(e+fx)^{11/2} \left(a^2d^2f^2h+2abdf(2cfh-4deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)\right)}{11f^6}+$$
$$\frac{2bd(e+fx)^{13/2}(2adfh+b(2cfh-5deh+dfg))}{13f^6}-$$
$$\frac{2(e+fx)^{7/2}(be-af)(de-cf)(-af(cfh-3deh+2dfg)-bcf(2fg-3eh)+bde(4fg-5eh))}{7f^6}+$$
$$\frac{2(e+fx)^{5/2}(be-af)^2(de-cf)^2(fg-eh)}{5f^6}+\frac{2b^2d^2h(e+fx)^{15/2}}{15f^6}$$

input
```
Int[(a + b*x)^2*(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(b*e - a*f)^2*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^6) - (2*(
b*e - a*f)*(d*e - c*f)*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(2*f*g - 3*e*h) - a*
f*(2*d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(7/2))/(7*f^6) + (2*(a^2*d*f^2*(d
*f*g - 3*d*e*h + 2*c*f*h) + 2*a*b*f*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3
*d^2*e*(f*g - 2*e*h)) + b^2*(2*d^2*e^2*(3*f*g - 5*e*h) + c^2*f^2*(f*g - 3*
e*h) - 6*c*d*e*f*(f*g - 2*e*h)))*(e + f*x)^(9/2))/(9*f^6) + (2*(a^2*d^2*f^
2*h + 2*a*b*d*f*(d*f*g - 4*d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h - 2*d^2*e*(2*
f*g - 5*e*h) + 2*c*d*f*(f*g - 4*e*h)))*(e + f*x)^(11/2))/(11*f^6) + (2*b*d
*(2*a*d*f*h + b*(d*f*g - 5*d*e*h + 2*c*f*h))*(e + f*x)^(13/2))/(13*f^6) +
(2*b^2*d^2*h*(e + f*x)^(15/2))/(15*f^6)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{2h\,b^2d^2(fx+e)^{\frac{15}{2}}}{15}+\frac{2\left(\left(2b(af-be)d^2+2b^2d(cf-de)\right)h+b^2d^2(-eh+fg)\right)(fx+e)^{\frac{13}{2}}}{13}+\frac{2\left(\left((af-be)^2d^2+4b(af-be)d(cf-de)+b^2(cf\right.\right.}{}}{}$
default	$\frac{\frac{2h\,b^2d^2(fx+e)^{\frac{15}{2}}}{15}-\frac{2\left(-\left(2b(af-be)d^2+2b^2d(cf-de)\right)h+b^2d^2(eh-fg)\right)(fx+e)^{\frac{13}{2}}}{13}-\frac{2\left(-\left((af-be)^2d^2+4b(af-be)d(cf-de)+b^2(\right.\right.}{}}{}$
pseudoelliptic	$-\frac{4\left(\left(-\frac{35x^2\left(\left(\frac{3}{5}h\,x^3+\frac{9}{13}g\,x^2\right)d^2+\frac{18xc\left(\frac{11hx}{13}+g\right)d}{11}+c^2\left(\frac{9hx}{11}+g\right)\right)b^2}{18}-5ax\left(\left(\frac{7}{13}h\,x^3+\frac{7}{11}g\,x^2\right)d^2+\frac{14xc\left(\frac{9hx}{11}+g\right)d}{9}+c^2\left(\right.\right.\right.\right.}{}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-3003h\,b^2d^2x^5f^5-6930ab\,d^2f^5h\,x^4-6930b^2cd\,f^5h\,x^4+2310b^2d^2e\,f^4h\,x^4-3465b^2d^2f^5g\,x^4-4095a^2d^2\right.}{}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-3003h\,b^2d^2x^5f^5-6930ab\,d^2f^5h\,x^4-6930b^2cd\,f^5h\,x^4+2310b^2d^2e\,f^4h\,x^4-3465b^2d^2f^5g\,x^4-4095a^2d^2\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^2*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^6*(1/15*h*b^2*d^2*(f*x+e)^(15/2)+1/13*((2*b*(a*f-b*e)*d^2+2*b^2*d*(c*f
-d*e))*h+b^2*d^2*(-e*h+f*g))*(f*x+e)^(13/2)+1/11*(((a*f-b*e)^2*d^2+4*b*(a*
f-b*e)*d*(c*f-d*e)+b^2*(c*f-d*e)^2)*h+(2*b*(a*f-b*e)*d^2+2*b^2*d*(c*f-d*e)
)*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*((2*(a*f-b*e)^2*d*(c*f-d*e)+2*b*(a*f-b*e)
*(c*f-d*e)^2)*h+((a*f-b*e)^2*d^2+4*b*(a*f-b*e)*d*(c*f-d*e)+b^2*(c*f-d*e)^2
)*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((a*f-b*e)^2*(c*f-d*e)^2*h+(2*(a*f-b*e)^2*
d*(c*f-d*e)+2*b*(a*f-b*e)*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*(a*f-
b*e)^2*(c*f-d*e)^2*(-e*h+f*g)*(f*x+e)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(387) = 774$.

Time = 0.10 (sec) , antiderivative size = 1128, normalized size of antiderivative = 2.74

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^2*d^2*f^7*h*x^7 + 231*(15*b^2*d^2*f^7*g + 2*(8*b^2*d^2*e*f
^6 + 15*(b^2*c*d + a*b*d^2)*f^7)*h)*x^6 + 63*(10*(7*b^2*d^2*e*f^6 + 13*(b^
2*c*d + a*b*d^2)*f^7)*g + (b^2*d^2*e^2*f^5 + 140*(b^2*c*d + a*b*d^2)*e*f^6
 + 65*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^7)*h)*x^5 + 35*((3*b^2*d^2*e^2*f^5
 + 312*(b^2*c*d + a*b*d^2)*e*f^6 + 143*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^7
)*g - 2*(b^2*d^2*e^3*f^4 - 3*(b^2*c*d + a*b*d^2)*e^2*f^5 - 78*(b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*e*f^6 - 143*(a*b*c^2 + a^2*c*d)*f^7)*h)*x^4 - 5*(2*(12
*b^2*d^2*e^3*f^4 - 39*(b^2*c*d + a*b*d^2)*e^2*f^5 - 715*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*e*f^6 - 1287*(a*b*c^2 + a^2*c*d)*f^7)*g - (16*b^2*d^2*e^4*f^
3 + 1287*a^2*c^2*f^7 - 48*(b^2*c*d + a*b*d^2)*e^3*f^4 + 39*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*e^2*f^5 + 2860*(a*b*c^2 + a^2*c*d)*e*f^6)*h)*x^3 + 3*((48
*b^2*d^2*e^4*f^3 + 3003*a^2*c^2*f^7 - 156*(b^2*c*d + a*b*d^2)*e^3*f^4 + 14
3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^5 + 6864*(a*b*c^2 + a^2*c*d)*e*f^6
)*g - 2*(16*b^2*d^2*e^5*f^2 - 1716*a^2*c^2*e*f^6 - 48*(b^2*c*d + a*b*d^2)*
e^4*f^3 + 39*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^4 - 143*(a*b*c^2 + a^2*
c*d)*e^2*f^5)*h)*x^2 + (384*b^2*d^2*e^6*f + 9009*a^2*c^2*e^2*f^5 - 1248*(b
^2*c*d + a*b*d^2)*e^5*f^2 + 1144*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^4*f^3 -
 5148*(a*b*c^2 + a^2*c*d)*e^3*f^4)*g - 2*(128*b^2*d^2*e^7 + 1287*a^2*c^2*e
^3*f^4 - 384*(b^2*c*d + a*b*d^2)*e^6*f + 312*(b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*e^5*f^2 - 1144*(a*b*c^2 + a^2*c*d)*e^4*f^3)*h - (2*(96*b^2*d^2*e^5*f...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. 2(445) = 890.

Time = 2.06 (sec) , antiderivative size = 1204, normalized size of antiderivative = 2.93

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2*(d*x+c)**2*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b**2*d**2*h*(e + f*x)**(15/2)/(15*f**5) + (e + f*x)**(13/2)*
(2*a*b*d**2*f*h + 2*b**2*c*d*f*h - 5*b**2*d**2*e*h + b**2*d**2*f*g)/(13*f*
*5) + (e + f*x)**(11/2)*(a**2*d**2*f**2*h + 4*a*b*c*d*f**2*h - 8*a*b*d**2*
e*f*h + 2*a*b*d**2*f**2*g + b**2*c**2*f**2*h - 8*b**2*c*d*e*f*h + 2*b**2*c
*d*f**2*g + 10*b**2*d**2*e**2*h - 4*b**2*d**2*e*f*g)/(11*f**5) + (e + f*x)
**(9/2)*(2*a**2*c*d*f**3*h - 3*a**2*d**2*e*f**2*h + a**2*d**2*f**3*g + 2*a
*b*c**2*f**3*h - 12*a*b*c*d*e*f**2*h + 4*a*b*c*d*f**3*g + 12*a*b*d**2*e**2
*f*h - 6*a*b*d**2*e*f**2*g - 3*b**2*c**2*e*f**2*h + b**2*c**2*f**3*g + 12*
b**2*c*d*e**2*f*h - 6*b**2*c*d*e*f**2*g - 10*b**2*d**2*e**3*h + 6*b**2*d**
2*e**2*f*g)/(9*f**5) + (e + f*x)**(7/2)*(a**2*c**2*f**4*h - 4*a**2*c*d*e*f
**3*h + 2*a**2*c*d*f**4*g + 3*a**2*d**2*e**2*f**2*h - 2*a**2*d**2*e*f**3*g
 - 4*a*b*c**2*e*f**3*h + 2*a*b*c**2*f**4*g + 12*a*b*c*d*e**2*f**2*h - 8*a*
b*c*d*e*f**3*g - 8*a*b*d**2*e**3*f*h + 6*a*b*d**2*e**2*f**2*g + 3*b**2*c**
2*e**2*f**2*h - 2*b**2*c**2*e*f**3*g - 8*b**2*c*d*e**3*f*h + 6*b**2*c*d*e*
*2*f**2*g + 5*b**2*d**2*e**4*h - 4*b**2*d**2*e**3*f*g)/(7*f**5) + (e + f*x
)**(5/2)*(-a**2*c**2*e*f**4*h + a**2*c**2*f**5*g + 2*a**2*c*d*e**2*f**3*h
- 2*a**2*c*d*e*f**4*g - a**2*d**2*e**3*f**2*h + a**2*d**2*e**2*f**3*g + 2*
a*b*c**2*e**2*f**3*h - 2*a*b*c**2*e*f**4*g - 4*a*b*c*d*e**3*f**2*h + 4*a*b
*c*d*e**2*f**3*g + 2*a*b*d**2*e**4*f*h - 2*a*b*d**2*e**3*f**2*g - b**2*c**
2*e**3*f**2*h + b**2*c**2*e**2*f**3*g + 2*b**2*c*d*e**4*f*h - 2*b**2*c*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.69

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3003*(f*x + e)^(15/2)*b^2*d^2*h + 3465*(b^2*d^2*f*g - (5*b^2*d^2*
e - 2*(b^2*c*d + a*b*d^2)*f)*h)*(f*x + e)^(13/2) - 4095*(2*(2*b^2*d^2*e*f
- (b^2*c*d + a*b*d^2)*f^2)*g - (10*b^2*d^2*e^2 - 8*(b^2*c*d + a*b*d^2)*e*f
 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2)*h)*(f*x + e)^(11/2) + 5005*((6*b^2
*d^2*e^2*f - 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*f^3)*g - (10*b^2*d^2*e^3 - 12*(b^2*c*d + a*b*d^2)*e^2*f + 3*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*e*f^2 - 2*(a*b*c^2 + a^2*c*d)*f^3)*h)*(f*x + e)^(9/2) -
 6435*(2*(2*b^2*d^2*e^3*f - 3*(b^2*c*d + a*b*d^2)*e^2*f^2 + (b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*e*f^3 - (a*b*c^2 + a^2*c*d)*f^4)*g - (5*b^2*d^2*e^4 + a^
2*c^2*f^4 - 8*(b^2*c*d + a*b*d^2)*e^3*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*e^2*f^2 - 4*(a*b*c^2 + a^2*c*d)*e*f^3)*h)*(f*x + e)^(7/2) + 9009*((b^2*d
^2*e^4*f + a^2*c^2*f^5 - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*e^2*f^3 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*g - (b^2*d^2*e^5 + a
^2*c^2*e*f^4 - 2*(b^2*c*d + a*b*d^2)*e^4*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*e^3*f^2 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^3)*h)*(f*x + e)^(5/2))/f^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3372 vs. $2(387) = 774$.

Time = 0.16 (sec) , antiderivative size = 3372, normalized size of antiderivative = 8.20

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a^2*c^2*e^2*g + 30030*((f*x + e)^(3/2) - 3*sq
rt(f*x + e)*e)*a^2*c^2*e*g + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a
*b*c^2*e^2*g/f + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c*d*e^2*g
/f + 15015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c^2*e^2*h/f + 3003*(3
*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c^2*g
+ 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b
^2*c^2*e^2*g/f^2 + 12012*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sq
rt(f*x + e)*e^2)*a*b*c*d*e^2*g/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e
)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*d^2*e^2*g/f^2 + 12012*(3*(f*x + e)^(
5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*c^2*e*g/f + 12012*
(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c*d*
e*g/f + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*
e^2)*a*b*c^2*e^2*h/f^2 + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e +
15*sqrt(f*x + e)*e^2)*a^2*c*d*e^2*h/f^2 + 6006*(3*(f*x + e)^(5/2) - 10*(f*
x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c^2*e*h/f + 2574*(5*(f*x + e)^(
7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^
3)*b^2*c*d*e^2*g/f^3 + 2574*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35
*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b*d^2*e^2*g/f^3 + 2574*(5*(
f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f
*x + e)*e^3)*b^2*c^2*e*g/f^2 + 10296*(5*(f*x + e)^(7/2) - 21*(f*x + e)^...
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.14

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{9/2}\,\left(4\,h\,a^2\,c\,d\,f^3-6\,h\,a^2\,d^2\,e\,f^2+2\,g\,a^2\,d^2\,f^3+4\,h\,a\,b\,c^2\,f^3-24\,h\,a\,b\,c\,d\,e\,f^2+8\,g\,a\,b\,c\right.}{} + \frac{(e+f\,x)^{11/2}\,\left(2\,h\,a^2\,d^2\,f^2+8\,h\,a\,b\,c\,d\,f^2-16\,h\,a\,b\,d^2\,e\,f+4\,g\,a\,b\,d^2\,f^2+2\,h\,b^2\,c^2\,f^2-16\,h\,b^2\,c\,d\,e\,f+4\,g\right.}{11\,f^6} - \frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{5\,f^6} + \frac{2\,b^2\,d^2\,h\,(e+f\,x)^{15/2}}{15\,f^6} + \frac{2\,b\,d\,(e+f\,x)^{13/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-5\,b\,d\,e\,h+b\,d\,f\,g)}{13\,f^6} + \frac{2\,(e+f\,x)^{7/2}\,(a\,f-b\,e)\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\,h-\right.}{7\,f^6}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^2*(c + d*x)^2,x)
```

output
```
((e + f*x)^(9/2)*(2*a^2*d^2*f^3*g + 2*b^2*c^2*f^3*g - 20*b^2*d^2*e^3*h + 4
*a*b*c^2*f^3*h + 4*a^2*c*d*f^3*h - 6*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h +
 12*b^2*d^2*e^2*f*g + 8*a*b*c*d*f^3*g - 12*a*b*d^2*e*f^2*g + 24*a*b*d^2*e^
2*f*h - 12*b^2*c*d*e*f^2*g + 24*b^2*c*d*e^2*f*h - 24*a*b*c*d*e*f^2*h))/(9*
f^6) + ((e + f*x)^(11/2)*(2*a^2*d^2*f^2*h + 2*b^2*c^2*f^2*h + 20*b^2*d^2*e
^2*h + 4*a*b*d^2*f^2*g + 4*b^2*c*d*f^2*g - 8*b^2*d^2*e*f*g + 8*a*b*c*d*f^2
*h - 16*a*b*d^2*e*f*h - 16*b^2*c*d*e*f*h))/(11*f^6) - (2*(e + f*x)^(5/2)*(
a*f - b*e)^2*(c*f - d*e)^2*(e*h - f*g))/(5*f^6) + (2*b^2*d^2*h*(e + f*x)^(
15/2))/(15*f^6) + (2*b*d*(e + f*x)^(13/2)*(2*a*d*f*h + 2*b*c*f*h - 5*b*d*e
*h + b*d*f*g))/(13*f^6) + (2*(e + f*x)^(7/2)*(a*f - b*e)*(c*f - d*e)*(a*c*
f^2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 5*b*d*e^2*h - 3*a*d*e*f*h - 3*b*c*e*f*
h - 4*b*d*e*f*g))/(7*f^6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1537, normalized size of antiderivative = 3.74

$$\int (a+bx)^2(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 2574*a**2*c**2*e**3*f**4*h + 9009*a**2*c**2*e**2*f**5
*g + 1287*a**2*c**2*e**2*f**5*h*x + 18018*a**2*c**2*e*f**6*g*x + 10296*a**
2*c**2*e*f**6*h*x**2 + 9009*a**2*c**2*f**7*g*x**2 + 6435*a**2*c**2*f**7*h*
x**3 + 2288*a**2*c*d*e**4*f**3*h - 5148*a**2*c*d*e**3*f**4*g - 1144*a**2*c
*d*e**3*f**4*h*x + 2574*a**2*c*d*e**2*f**5*g*x + 858*a**2*c*d*e**2*f**5*h*
x**2 + 20592*a**2*c*d*e*f**6*g*x**2 + 14300*a**2*c*d*e*f**6*h*x**3 + 12870
*a**2*c*d*f**7*g*x**3 + 10010*a**2*c*d*f**7*h*x**4 - 624*a**2*d**2*e**5*f*
*2*h + 1144*a**2*d**2*e**4*f**3*g + 312*a**2*d**2*e**4*f**3*h*x - 572*a**2
*d**2*e**3*f**4*g*x - 234*a**2*d**2*e**3*f**4*h*x**2 + 429*a**2*d**2*e**2*
f**5*g*x**2 + 195*a**2*d**2*e**2*f**5*h*x**3 + 7150*a**2*d**2*e*f**6*g*x**
3 + 5460*a**2*d**2*e*f**6*h*x**4 + 5005*a**2*d**2*f**7*g*x**4 + 4095*a**2*
d**2*f**7*h*x**5 + 2288*a*b*c**2*e**4*f**3*h - 5148*a*b*c**2*e**3*f**4*g -
 1144*a*b*c**2*e**3*f**4*h*x + 2574*a*b*c**2*e**2*f**5*g*x + 858*a*b*c**2*
e**2*f**5*h*x**2 + 20592*a*b*c**2*e*f**6*g*x**2 + 14300*a*b*c**2*e*f**6*h*
x**3 + 12870*a*b*c**2*f**7*g*x**3 + 10010*a*b*c**2*f**7*h*x**4 - 2496*a*b*
c*d*e**5*f**2*h + 4576*a*b*c*d*e**4*f**3*g + 1248*a*b*c*d*e**4*f**3*h*x -
2288*a*b*c*d*e**3*f**4*g*x - 936*a*b*c*d*e**3*f**4*h*x**2 + 1716*a*b*c*d*e
**2*f**5*g*x**2 + 780*a*b*c*d*e**2*f**5*h*x**3 + 28600*a*b*c*d*e*f**6*g*x*
*3 + 21840*a*b*c*d*e*f**6*h*x**4 + 20020*a*b*c*d*f**7*g*x**4 + 16380*a*b*c
*d*f**7*h*x**5 + 768*a*b*d**2*e**6*f*h - 1248*a*b*d**2*e**5*f**2*g - 38...
```

3.71 $\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 248

$$\begin{aligned}\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx &= -\frac{2(be-af)(de-cf)^2(fg-eh)(e+fx)^{5/2}}{5f^5} \\ &+ \frac{2(de-cf)(bde(3fg-4eh)-bcf(fg-2eh)-af(2dfg-3deh+cfh))(e+fx)^{7/2}}{7f^5} \\ &+ \frac{2(adf(dfg-3deh+2cfh)+b(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)))\,(e+fx)^{9/2}}{9f^5} \\ &+ \frac{2d(adfh+b(dfg-4deh+2cfh))(e+fx)^{11/2}}{11f^5} + \frac{2bd^2h(e+fx)^{13/2}}{13f^5}\end{aligned}$$

output
```
-2/5*(-a*f+b*e)*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(5/2)/f^5+2/7*(-c*f+d*e)*(
b*d*e*(-4*e*h+3*f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-3*d*e*h+2*d*f*g))*(f*x+
e)^(7/2)/f^5+2/9*(a*d*f*(2*c*f*h-3*d*e*h+d*f*g)+b*(c^2*f^2*h+2*c*d*f*(-3*e
*h+f*g)-3*d^2*e*(-2*e*h+f*g)))*(f*x+e)^(9/2)/f^5+2/11*d*(a*d*f*h+b*(2*c*f*
h-4*d*e*h+d*f*g))*(f*x+e)^(11/2)/f^5+2/13*b*d^2*h*(f*x+e)^(13/2)/f^5
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.27

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(13af\left(99c^2f^2(7fg-2eh+5fhx)+22cdf\left(8e^2h+5f^2x(9g+7hx)-2ef(9g+10\right.\right.\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(13*a*f*(99*c^2*f^2*(7*f*g - 2*e*h + 5*f*h*x) + 22*c*d*
f*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) + d^2*(-48*e^3*
h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(22*g
 + 21*h*x))) + b*(143*c^2*f^2*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*
g + 10*h*x)) + 3*d^2*(128*e^4*h + 105*f^4*x^3*(13*g + 11*h*x) - 70*e*f^3*x
^2*(13*g + 12*h*x) + 40*e^2*f^2*x*(13*g + 14*h*x) - 16*e^3*f*(13*g + 20*h*
x)) + 26*c*d*f*(-48*e^3*h + 35*f^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15
*h*x) - 10*e*f^2*x*(22*g + 21*h*x)))))/(45045*f^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{(e+fx)^{7/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{f^4}+\frac{d(e+fx)^{9/2}(adf}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{9/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{9f^5}+\frac{2d(e+fx)^{11/2}(adfh+b(2cfh-4deh+dfg))}{11f^5}+\frac{2(e+fx)^{7/2}(de-cf)(-af(cfh-3deh+2dfg)-bcf(fg-2eh)+bde(3fg-4eh))}{7f^5}-\frac{2(e+fx)^{5/2}(be-af)(de-cf)^2(fg-eh)}{5f^5}+\frac{2bd^2h(e+fx)^{13/2}}{13f^5}$$

input
```
Int[(a + b*x)*(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(-2*(b*e - a*f)*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^5) + (2*(d
*e - c*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(2*d*f*g - 3*
d*e*h + c*f*h))*(e + f*x)^(7/2))/(7*f^5) + (2*(a*d*f*(d*f*g - 3*d*e*h + 2*
c*f*h) + b*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3*d^2*e*(f*g - 2*e*h)))*(e
 + f*x)^(9/2))/(9*f^5) + (2*d*(a*d*f*h + b*(d*f*g - 4*d*e*h + 2*c*f*h))*(e
 + f*x)^(11/2))/(11*f^5) + (2*b*d^2*h*(e + f*x)^(13/2))/(13*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2hb\,d^2(fx+e)^{\frac{13}{2}}}{13}+\frac{2(((af-be)d^2+2bd(cf-de))h+b\,d^2(-eh+fg))(fx+e)^{\frac{11}{2}}}{11}+\frac{2((2(af-be)d(cf-de)+b(cf-de)^2)h+((af-be)d}{9}$
default	$\frac{2hb\,d^2(fx+e)^{\frac{13}{2}}}{13}+\frac{2(-((-af+be)d^2-2bd(cf-de))h-b\,d^2(eh-fg))(fx+e)^{\frac{11}{2}}}{11}+\frac{2(-(2(-af+be)d(cf-de)-b(cf-de)^2)h+((-a}{9}$
pseudoelliptic	$-\frac{4(fx+e)^{\frac{5}{2}}((-\frac{35(\frac{9bh\,x^2}{13}+\frac{9(ah+bg)x}{11}+ga)x^2d^2}{18}-5(\frac{7bh\,x^2}{11}+\frac{7(ah+bg)x}{9}+ga)xcd-\frac{7c^2(\frac{5bh\,x^2}{9}+\frac{5(ah+bg)x}{7}+ga)}{2})f}{}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}(-3465hb\,d^2x^4f^4-4095a\,d^2f^4h\,x^3-8190bcd\,f^4h\,x^3+2520b\,d^2e\,f^3h\,x^3-4095b\,d^2f^4g\,x^3-10010acd\,f^4h\,x}{}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}(-3465hb\,d^2x^4f^4-4095a\,d^2f^4h\,x^3-8190bcd\,f^4h\,x^3+2520b\,d^2e\,f^3h\,x^3-4095b\,d^2f^4g\,x^3-10010acd\,f^4h\,x}{}$
trager	$-\frac{2(-3465b\,d^2f^6h\,x^6-4095a\,d^2f^6h\,x^5-8190bcd\,f^6h\,x^5-4410b\,d^2e\,f^5h\,x^5-4095b\,d^2f^6g\,x^5-10010acd\,f^6h\,x^4-5460a}{}$
risch	$-\frac{2(-3465b\,d^2f^6h\,x^6-4095a\,d^2f^6h\,x^5-8190bcd\,f^6h\,x^5-4410b\,d^2e\,f^5h\,x^5-4095b\,d^2f^6g\,x^5-10010acd\,f^6h\,x^4-5460a}{}$

input
```
int((b*x+a)*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
2/f^5*(1/13*h*b*d^2*(f*x+e)^(13/2)+1/11*(((a*f-b*e)*d^2+2*b*d*(c*f-d*e))*h
+b*d^2*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*((2*(a*f-b*e)*d*(c*f-d*e)+b*(c*f-d*e
)^2)*h+((a*f-b*e)*d^2+2*b*d*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((a*f
-b*e)*(c*f-d*e)^2*h+(2*(a*f-b*e)*d*(c*f-d*e)+b*(c*f-d*e)^2)*(-e*h+f*g))*(f
*x+e)^(7/2)+1/5*(a*f-b*e)*(c*f-d*e)^2*(-e*h+f*g)*(f*x+e)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(228) = 456$.

Time = 0.08 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.65

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/45045*(3465*b*d^2*f^6*h*x^6 + 315*(13*b*d^2*f^6*g + (14*b*d^2*e*f^5 + 13
*(2*b*c*d + a*d^2)*f^6)*h)*x^5 + 35*(13*(12*b*d^2*e*f^5 + 11*(2*b*c*d + a*
d^2)*f^6)*g + (3*b*d^2*e^2*f^4 + 156*(2*b*c*d + a*d^2)*e*f^5 + 143*(b*c^2
+ 2*a*c*d)*f^6)*h)*x^4 + 5*(13*(3*b*d^2*e^2*f^4 + 110*(2*b*c*d + a*d^2)*e*
f^5 + 99*(b*c^2 + 2*a*c*d)*f^6)*g - (24*b*d^2*e^3*f^3 - 1287*a*c^2*f^6 - 3
9*(2*b*c*d + a*d^2)*e^2*f^4 - 1430*(b*c^2 + 2*a*c*d)*e*f^5)*h)*x^3 - 3*(13
*(6*b*d^2*e^3*f^3 - 231*a*c^2*f^6 - 11*(2*b*c*d + a*d^2)*e^2*f^4 - 264*(b*
c^2 + 2*a*c*d)*e*f^5)*g - (48*b*d^2*e^4*f^2 + 3432*a*c^2*e*f^5 - 78*(2*b*c
*d + a*d^2)*e^3*f^3 + 143*(b*c^2 + 2*a*c*d)*e^2*f^4)*h)*x^2 - 13*(48*b*d^2
*e^5*f - 693*a*c^2*e^2*f^4 - 88*(2*b*c*d + a*d^2)*e^4*f^2 + 198*(b*c^2 + 2
*a*c*d)*e^3*f^3)*g + 2*(192*b*d^2*e^6 - 1287*a*c^2*e^3*f^3 - 312*(2*b*c*d
+ a*d^2)*e^5*f + 572*(b*c^2 + 2*a*c*d)*e^4*f^2)*h + (13*(24*b*d^2*e^4*f^2
+ 1386*a*c^2*e*f^5 - 44*(2*b*c*d + a*d^2)*e^3*f^3 + 99*(b*c^2 + 2*a*c*d)*e
^2*f^4)*g - (192*b*d^2*e^5*f - 1287*a*c^2*e^2*f^4 - 312*(2*b*c*d + a*d^2)*
e^4*f^2 + 572*(b*c^2 + 2*a*c*d)*e^3*f^3)*h)*x)*sqrt(f*x + e)/f^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(265) = 530.

Time = 1.70 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.50

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{bd^2h(e+fx)^{\frac{13}{2}}}{13f^4} + \frac{(e+fx)^{\frac{11}{2}}\left(ad^2fh+2bcdfh-4bd^2eh+bd^2fg\right)}{11f^4} + \frac{(e+fx)^{\frac{9}{2}}\cdot\left(2acdf^2h-3ad^2efh+ad^2f^2g+bc^2f^2h-6bcdefh+2bcdf^2g+6bd^2\right.}{9f^4}\right.}{} \\ e^{\frac{3}{2}}\left(ac^2gx + \frac{bd^2hx^5}{5} + \frac{x^4(ad^2h+2bcdh+bd^2g)}{4} + \frac{x^3\cdot(2acdh+ad^2g+bc^2h+2bcdg)}{3} + \frac{x^2(ac^2h+2acdg+bc^2g)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**2*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(b*d**2*h*(e + f*x)**(13/2)/(13*f**4) + (e + f*x)**(11/2)*(a*
d**2*f*h + 2*b*c*d*f*h - 4*b*d**2*e*h + b*d**2*f*g)/(11*f**4) + (e + f*x)*
*(9/2)*(2*a*c*d*f**2*h - 3*a*d**2*e*f*h + a*d**2*f**2*g + b*c**2*f**2*h -
6*b*c*d*e*f*h + 2*b*c*d*f**2*g + 6*b*d**2*e**2*h - 3*b*d**2*e*f*g)/(9*f**4
) + (e + f*x)**(7/2)*(a*c**2*f**3*h - 4*a*c*d*e*f**2*h + 2*a*c*d*f**3*g +
3*a*d**2*e**2*f*h - 2*a*d**2*e*f**2*g - 2*b*c**2*e*f**2*h + b*c**2*f**3*g
+ 6*b*c*d*e**2*f*h - 4*b*c*d*e*f**2*g - 4*b*d**2*e**3*h + 3*b*d**2*e**2*f*
g)/(7*f**4) + (e + f*x)**(5/2)*(-a*c**2*e*f**3*h + a*c**2*f**4*g + 2*a*c*d
*e**2*f**2*h - 2*a*c*d*e*f**3*g - a*d**2*e**3*f*h + a*d**2*e**2*f**2*g + b
*c**2*e**2*f**2*h - b*c**2*e*f**3*g - 2*b*c*d*e**3*f*h + 2*b*c*d*e**2*f**2
*g + b*d**2*e**4*h - b*d**2*e**3*f*g)/(5*f**4))/f, Ne(f, 0)), (e**(3/2)*(a
*c**2*g*x + b*d**2*h*x**5/5 + x**4*(a*d**2*h + 2*b*c*d*h + b*d**2*g)/4 + x
**3*(2*a*c*d*h + a*d**2*g + b*c**2*h + 2*b*c*d*g)/3 + x**2*(a*c**2*h + 2*a
*c*d*g + b*c**2*g)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.52

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(3465\,(fx+e)^{\frac{13}{2}}bd^2h + 4095\,(bd^2fg - (4\,bd^2e - (2\,bcd + ad^2)f)h)(fx+e)^{\frac{11}{2}} - 5005\,((3\,bd^2e \cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/45045*(3465*(f*x + e)^(13/2)*b*d^2*h + 4095*(b*d^2*f*g - (4*b*d^2*e - (2
*b*c*d + a*d^2)*f)*h)*(f*x + e)^(11/2) - 5005*((3*b*d^2*e*f - (2*b*c*d + a
*d^2)*f^2)*g - (6*b*d^2*e^2 - 3*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*
f^2)*h)*(f*x + e)^(9/2) + 6435*((3*b*d^2*e^2*f - 2*(2*b*c*d + a*d^2)*e*f^2
 + (b*c^2 + 2*a*c*d)*f^3)*g - (4*b*d^2*e^3 - a*c^2*f^3 - 3*(2*b*c*d + a*d^
2)*e^2*f + 2*(b*c^2 + 2*a*c*d)*e*f^2)*h)*(f*x + e)^(7/2) - 9009*((b*d^2*e^
3*f - a*c^2*f^4 - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*g -
 (b*d^2*e^4 - a*c^2*e*f^3 - (2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^
2*f^2)*h)*(f*x + e)^(5/2))/f^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1968 vs. $2(228) = 456$.

Time = 0.14 (sec) , antiderivative size = 1968, normalized size of antiderivative = 7.94

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a*c^2*e^2*g + 30030*((f*x + e)^(3/2) - 3*sqrt
(f*x + e)*e)*a*c^2*e*g + 15015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*b*c^2
*e^2*g/f + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*c*d*e^2*g/f + 150
15*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*c^2*e^2*h/f + 3003*(3*(f*x + e)
^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c^2*g + 6006*(3*(f
*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*d*e^2*g/f
^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2
)*a*d^2*e^2*g/f^2 + 6006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sq
rt(f*x + e)*e^2)*b*c^2*e*g/f + 12012*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/
2)*e + 15*sqrt(f*x + e)*e^2)*a*c*d*e*g/f + 3003*(3*(f*x + e)^(5/2) - 10*(f
*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c^2*e^2*h/f^2 + 6006*(3*(f*x + e
)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c*d*e^2*h/f^2 + 6
006*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c^
2*e*h/f + 1287*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3
/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*d^2*e^2*g/f^3 + 5148*(5*(f*x + e)^(7/2)
- 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*
c*d*e*g/f^2 + 2574*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e
)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*d^2*e*g/f^2 + 1287*(5*(f*x + e)^(7/2
) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*
b*c^2*g/f + 2574*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + ...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.04

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{11/2}\,(2\,a\,d^2\,f\,h-8\,b\,d^2\,e\,h+2\,b\,d^2\,f\,g+4\,b\,c\,d\,f\,h)}{11\,f^5} + \frac{(e+f\,x)^{9/2}\,(2\,a\,d^2\,f^2\,g+2\,b\,c^2\,f^2\,h+12\,b\,d^2\,e^2\,h+4\,a\,c\,d\,f^2\,h+4\,b\,c\,d\,f^2\,g-6\,a\,d^2\,e\,f\,h-6\,b\,d^2\,e\,f\,g-}{9\,f^5} + \frac{2\,(e+f\,x)^{7/2}\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+b\,c\,f^2\,g+4\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-2\,b\,c\,e\,f\,h-3\,b\,d\,e\,f\,g)}{7\,f^5} + \frac{2\,b\,d^2\,h\,(e+f\,x)^{13/2}}{13\,f^5} - \frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{5\,f^5}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(a + b*x)*(c + d*x)^2,x)
```

output
```
((e + f*x)^(11/2)*(2*a*d^2*f*h - 8*b*d^2*e*h + 2*b*d^2*f*g + 4*b*c*d*f*h))
/(11*f^5) + ((e + f*x)^(9/2)*(2*a*d^2*f^2*g + 2*b*c^2*f^2*h + 12*b*d^2*e^2
*h + 4*a*c*d*f^2*h + 4*b*c*d*f^2*g - 6*a*d^2*e*f*h - 6*b*d^2*e*f*g - 12*b*
c*d*e*f*h))/(9*f^5) + (2*(e + f*x)^(7/2)*(c*f - d*e)*(a*c*f^2*h + 2*a*d*f^
2*g + b*c*f^2*g + 4*b*d*e^2*h - 3*a*d*e*f*h - 2*b*c*e*f*h - 3*b*d*e*f*g))/
(7*f^5) + (2*b*d^2*h*(e + f*x)^(13/2))/(13*f^5) - (2*(e + f*x)^(5/2)*(a*f
- b*e)*(c*f - d*e)^2*(e*h - f*g))/(5*f^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 821, normalized size of antiderivative = 3.31

$$\int (a+bx)(c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2\sqrt{fx+e}\,(3465b\,d^2f^6h\,x^6+4095a\,d^2f^6h\,x^5+8190bcd\,f^6h\,x^5+4410b\,d^2e\,f^5h\,x^5+4095b\,d^2f^6g}{}$$

input
```
int((b*x+a)*(d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x)
```

output

```
(2*sqrt(e + f*x)*( - 2574*a*c**2*e**3*f**3*h + 9009*a*c**2*e**2*f**4*g + 1
287*a*c**2*e**2*f**4*h*x + 18018*a*c**2*e*f**5*g*x + 10296*a*c**2*e*f**5*h
*x**2 + 9009*a*c**2*f**6*g*x**2 + 6435*a*c**2*f**6*h*x**3 + 2288*a*c*d*e**
4*f**2*h - 5148*a*c*d*e**3*f**3*g - 1144*a*c*d*e**3*f**3*h*x + 2574*a*c*d*
e**2*f**4*g*x + 858*a*c*d*e**2*f**4*h*x**2 + 20592*a*c*d*e*f**5*g*x**2 + 1
4300*a*c*d*e*f**5*h*x**3 + 12870*a*c*d*f**6*g*x**3 + 10010*a*c*d*f**6*h*x*
*4 - 624*a*d**2*e**5*f*h + 1144*a*d**2*e**4*f**2*g + 312*a*d**2*e**4*f**2*
h*x - 572*a*d**2*e**3*f**3*g*x - 234*a*d**2*e**3*f**3*h*x**2 + 429*a*d**2*
e**2*f**4*g*x**2 + 195*a*d**2*e**2*f**4*h*x**3 + 7150*a*d**2*e*f**5*g*x**3
 + 5460*a*d**2*e*f**5*h*x**4 + 5005*a*d**2*f**6*g*x**4 + 4095*a*d**2*f**6*
h*x**5 + 1144*b*c**2*e**4*f**2*h - 2574*b*c**2*e**3*f**3*g - 572*b*c**2*e*
*3*f**3*h*x + 1287*b*c**2*e**2*f**4*g*x + 429*b*c**2*e**2*f**4*h*x**2 + 10
296*b*c**2*e*f**5*g*x**2 + 7150*b*c**2*e*f**5*h*x**3 + 6435*b*c**2*f**6*g*
x**3 + 5005*b*c**2*f**6*h*x**4 - 1248*b*c*d*e**5*f*h + 2288*b*c*d*e**4*f**
2*g + 624*b*c*d*e**4*f**2*h*x - 1144*b*c*d*e**3*f**3*g*x - 468*b*c*d*e**3*
f**3*h*x**2 + 858*b*c*d*e**2*f**4*g*x**2 + 390*b*c*d*e**2*f**4*h*x**3 + 14
300*b*c*d*e*f**5*g*x**3 + 10920*b*c*d*e*f**5*h*x**4 + 10010*b*c*d*f**6*g*x
**4 + 8190*b*c*d*f**6*h*x**5 + 384*b*d**2*e**6*h - 624*b*d**2*e**5*f*g - 1
92*b*d**2*e**5*f*h*x + 312*b*d**2*e**4*f**2*g*x + 144*b*d**2*e**4*f**2*h*x
**2 - 234*b*d**2*e**3*f**3*g*x**2 - 120*b*d**2*e**3*f**3*h*x**3 + 195*b...
```

3.72 $\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2(de-cf)^2(fg-eh)(e+fx)^{5/2}}{5f^4} - \frac{2(de-cf)(2dfg-3deh+cfh)(e+fx)^{7/2}}{7f^4} + \frac{2d(dfg-3deh+2cfh)(e+fx)^{9/2}}{9f^4} + \frac{2d^2h(e+fx)^{11/2}}{11f^4}$$

output
```
2/5*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(5/2)/f^4-2/7*(-c*f+d*e)*(c*f*h-3*d*e*
h+2*d*f*g)*(f*x+e)^(7/2)/f^4+2/9*d*(2*c*f*h-3*d*e*h+d*f*g)*(f*x+e)^(9/2)/f
^4+2/11*d^2*h*(f*x+e)^(11/2)/f^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2(e+fx)^{5/2}\left(99c^2f^2(7fg-2eh+5fhx)+22cdf(8e^2h+5f^2x(9g+7hx)-2ef(9g+10hx))+\right.}{3465f^4}$$

input
```
Integrate[(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(e + f*x)^(5/2)*(99*c^2*f^2*(7*f*g - 2*e*h + 5*f*h*x) + 22*c*d*f*(8*e^2
*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) + d^2*(-48*e^3*h + 35*f
^3*x^2*(11*g + 9*h*x) + 8*e^2*f*(11*g + 15*h*x) - 10*e*f^2*x*(22*g + 21*h*
x))))/(3465*f^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{d(e+fx)^{7/2}(2cfh-3deh+dfg)}{f^3}+\frac{(e+fx)^{5/2}(cf-de)(cfh-3deh+2dfg)}{f^3}+\frac{(e+fx)^{3/2}(cf-de)^2(fg}{f^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(e+fx)^{9/2}(2cfh-3deh+dfg)}{9f^4}-\frac{2(e+fx)^{7/2}(de-cf)(cfh-3deh+2dfg)}{7f^4}+\frac{2(e+fx)^{5/2}(de-cf)^2(fg-eh)}{5f^4}+\frac{2d^2h(e+fx)^{11/2}}{11f^4}$$

input
```
Int[(c + d*x)^2*(e + f*x)^(3/2)*(g + h*x),x]
```

output
```
(2*(d*e - c*f)^2*(f*g - e*h)*(e + f*x)^(5/2))/(5*f^4) - (2*(d*e - c*f)*(2*
d*f*g - 3*d*e*h + c*f*h)*(e + f*x)^(7/2))/(7*f^4) + (2*d*(d*f*g - 3*d*e*h
+ 2*c*f*h)*(e + f*x)^(9/2))/(9*f^4) + (2*d^2*h*(e + f*x)^(11/2))/(11*f^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{4\left(\left(\left(-\frac{35x^2\left(\frac{9hx}{11}+g\right)d^2}{18}-5xc\left(\frac{7hx}{9}+g\right)d-\frac{7c^2\left(\frac{5hx}{7}+g\right)}{2}\right)f^3+e\left(\frac{10x\left(\frac{21hx}{22}+g\right)d^2}{9}+2c\left(\frac{10hx}{9}+g\right)d+h\,c^2\right)f^2-\frac{8e^2d((\cdots}{\cdots}\right)\cdots}{35f^4}\cdots$
derivativedivides	$\frac{\frac{2d^2h(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(2d(cf-de)h+d^2(-eh+fg)\right)(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left((cf-de)^2h+2d(cf-de)(-eh+fg)\right)(fx+e)^{\frac{7}{2}}}{7}+\frac{2(cf-de)^2(-eh+fg)\cdots}{5}\cdots}{f^4}$
default	$\frac{\frac{2d^2h(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-2d(cf-de)h+d^2(eh-fg)\right)(fx+e)^{\frac{9}{2}}}{9}-\frac{2\left(-(cf-de)^2h+2d(cf-de)(eh-fg)\right)(fx+e)^{\frac{7}{2}}}{7}-\frac{2(cf-de)^2(eh-fg)(\cdots}{5}\cdots}{f^4}$
gosper	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-315d^2h\,x^3f^3-770cd\,f^3h\,x^2+210d^2e\,f^2h\,x^2-385d^2f^3g\,x^2-495c^2f^3hx+440cde\,f^2hx-990cd\,f^3gx-1\cdots\right)}{3465f^4}$
orering	$-\frac{2(fx+e)^{\frac{5}{2}}\left(-315d^2h\,x^3f^3-770cd\,f^3h\,x^2+210d^2e\,f^2h\,x^2-385d^2f^3g\,x^2-495c^2f^3hx+440cde\,f^2hx-990cd\,f^3gx-1\cdots\right)}{3465f^4}$
trager	$-\frac{2\left(-315d^2f^5h\,x^5-770cd\,f^5h\,x^4-420d^2e\,f^4h\,x^4-385d^2f^5g\,x^4-495c^2f^5h\,x^3-1100cde\,f^4h\,x^3-990cd\,f^5g\,x^3-15d^2\cdots\right)}{\cdots}$
risch	$-\frac{2\left(-315d^2f^5h\,x^5-770cd\,f^5h\,x^4-420d^2e\,f^4h\,x^4-385d^2f^5g\,x^4-495c^2f^5h\,x^3-1100cde\,f^4h\,x^3-990cd\,f^5g\,x^3-15d^2\cdots\right)}{\cdots}$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-4/35*((-35/18*x^2*(9/11*h*x+g)*d^2-5*x*c*(7/9*h*x+g)*d-7/2*c^2*(5/7*h*x+g
))*f^3+e*(10/9*x*(21/22*h*x+g)*d^2+2*c*(10/9*h*x+g)*d+h*c^2)*f^2-8/9*e^2*d
*((15/22*h*x+1/2*g)*d+c*h)*f+8/33*d^2*e^3*h)*(f*x+e)^(5/2)/f^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(110) = 220$.

Time = 0.07 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.56

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2\,(315\,d^2f^5hx^5 + 35\,(11\,d^2f^5g + 2\,(6\,d^2ef^4 + 11\,cdf^5)h)x^4 + 5\,(22\,(5\,d^2ef^4 + 9\,cdf^5)g + (3\,d^2e^2 \ldots}{}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="fricas")
```

output
```
2/3465*(315*d^2*f^5*h*x^5 + 35*(11*d^2*f^5*g + 2*(6*d^2*e*f^4 + 11*c*d*f^5
)*h)*x^4 + 5*(22*(5*d^2*e*f^4 + 9*c*d*f^5)*g + (3*d^2*e^2*f^3 + 220*c*d*e*
f^4 + 99*c^2*f^5)*h)*x^3 + 3*(11*(d^2*e^2*f^3 + 48*c*d*e*f^4 + 21*c^2*f^5)
*g - 2*(3*d^2*e^3*f^2 - 11*c*d*e^2*f^3 - 132*c^2*e*f^4)*h)*x^2 + 11*(8*d^2
*e^4*f - 36*c*d*e^3*f^2 + 63*c^2*e^2*f^3)*g - 2*(24*d^2*e^5 - 88*c*d*e^4*f
 + 99*c^2*e^3*f^2)*h - (22*(2*d^2*e^3*f^2 - 9*c*d*e^2*f^3 - 63*c^2*e*f^4)*
g - (24*d^2*e^4*f - 88*c*d*e^3*f^2 + 99*c^2*e^2*f^3)*h)*x)*sqrt(f*x + e)/f
^4
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.04

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{11}{2}}}{11f^3} + \frac{(e+fx)^{\frac{9}{2}}\cdot(2cdfh-3d^2eh+d^2fg)}{9f^3} + \frac{(e+fx)^{\frac{7}{2}}\left(c^2f^2h-4cdefh+2cdf^2g+3d^2e^2h-2d^2efg\right)}{7f^3} + \frac{(e+fx)^{\frac{5}{2}}\left(-c^2ef^2h+c^2f^3g+2 \ldots\right.}{}\right.}{f} \\ e^{\frac{3}{2}}\left(c^2gx + \frac{d^2hx^4}{4} + \frac{x^3\cdot(2cdh+d^2g)}{3} + \frac{x^2\left(c^2h+2cdg\right)}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(3/2)*(h*x+g),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(11/2)/(11*f**3) + (e + f*x)**(9/2)*(2*c*d
*f*h - 3*d**2*e*h + d**2*f*g)/(9*f**3) + (e + f*x)**(7/2)*(c**2*f**2*h - 4
*c*d*e*f*h + 2*c*d*f**2*g + 3*d**2*e**2*h - 2*d**2*e*f*g)/(7*f**3) + (e +
f*x)**(5/2)*(-c**2*e*f**2*h + c**2*f**3*g + 2*c*d*e**2*f*h - 2*c*d*e*f**2*
g - d**2*e**3*h + d**2*e**2*f*g)/(5*f**3))/f, Ne(f, 0)), (e**(3/2)*(c**2*g
*x + d**2*h*x**4/4 + x**3*(2*c*d*h + d**2*g)/3 + x**2*(c**2*h + 2*c*d*g)/2
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.31

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2\left(315\,(fx+e)^{\frac{11}{2}}d^2h+385\,(d^2fg-(3\,d^2e-2\,cdf)h)(fx+e)^{\frac{9}{2}}-495\,(2\,(d^2ef-cdf^2)g-(3\,d^2\right.}{}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="maxima")
```

output
```
2/3465*(315*(f*x + e)^(11/2)*d^2*h + 385*(d^2*f*g - (3*d^2*e - 2*c*d*f)*h)
*(f*x + e)^(9/2) - 495*(2*(d^2*e*f - c*d*f^2)*g - (3*d^2*e^2 - 4*c*d*e*f +
 c^2*f^2)*h)*(f*x + e)^(7/2) + 693*((d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*g
- (d^2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*h)*(f*x + e)^(5/2))/f^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 854, normalized size of antiderivative = 6.78

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(f*x + e)*c^2*e^2*g + 2310*((f*x + e)^(3/2) - 3*sqrt(f*x
+ e)*e)*c^2*e*g + 2310*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*c*d*e^2*g/f +
 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*c^2*e^2*h/f + 231*(3*(f*x + e)
^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c^2*g + 231*(3*(f*x
+ e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*d^2*e^2*g/f^2 +
924*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*d*
e*g/f + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e
^2)*c*d*e^2*h/f^2 + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqr
t(f*x + e)*e^2)*c^2*e*h/f + 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e
+ 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*d^2*e*g/f^2 + 198*(5*(f*x
 + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x
+ e)*e^3)*c*d*g/f + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x
 + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*d^2*e^2*h/f^3 + 396*(5*(f*x + e)^(
7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^
3)*c*d*e*h/f^2 + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x +
e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*c^2*h/f + 11*(35*(f*x + e)^(9/2) - 18
0*(f*x + e)^(7/2)*e + 378*(f*x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 +
315*sqrt(f*x + e)*e^4)*d^2*g/f^2 + 22*(35*(f*x + e)^(9/2) - 180*(f*x + e)^
(7/2)*e + 378*(f*x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x
 + e)*e^4)*d^2*e*h/f^3 + 22*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e...
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{(e+f\,x)^{9/2}\,(2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h)}{9\,f^4} + \frac{2\,(e+f\,x)^{7/2}\,(c\,f-d\,e)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g)}{7\,f^4} - \frac{2\,(e+f\,x)^{5/2}\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{5\,f^4} + \frac{2\,d^2\,h\,(e+f\,x)^{11/2}}{11\,f^4}$$

input
```
int((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^2,x)
```

output
```
((e + f*x)^(9/2)*(2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h))/(9*f^4) + (2*(e + f*
x)^(7/2)*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(7*f^4) - (2*(e + f*x)^(
5/2)*(c*f - d*e)^2*(e*h - f*g))/(5*f^4) + (2*d^2*h*(e + f*x)^(11/2))/(11*f
^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.69

$$\int (c+dx)^2(e+fx)^{3/2}(g+hx)\,dx = \frac{2\sqrt{fx+e}\,(315d^2f^5h\,x^5+770cd\,f^5h\,x^4+420d^2e\,f^4h\,x^4+385d^2f^5g\,x^4+495c^2f^5h\,x^3+1100cd}{}$$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g),x)
```

output
```
(2*sqrt(e + f*x)*( - 198*c**2*e**3*f**2*h + 693*c**2*e**2*f**3*g + 99*c**2
*e**2*f**3*h*x + 1386*c**2*e*f**4*g*x + 792*c**2*e*f**4*h*x**2 + 693*c**2*
f**5*g*x**2 + 495*c**2*f**5*h*x**3 + 176*c*d*e**4*f*h - 396*c*d*e**3*f**2*
g - 88*c*d*e**3*f**2*h*x + 198*c*d*e**2*f**3*g*x + 66*c*d*e**2*f**3*h*x**2
 + 1584*c*d*e*f**4*g*x**2 + 1100*c*d*e*f**4*h*x**3 + 990*c*d*f**5*g*x**3 +
 770*c*d*f**5*h*x**4 - 48*d**2*e**5*h + 88*d**2*e**4*f*g + 24*d**2*e**4*f*
h*x - 44*d**2*e**3*f**2*g*x - 18*d**2*e**3*f**2*h*x**2 + 33*d**2*e**2*f**3
*g*x**2 + 15*d**2*e**2*f**3*h*x**3 + 550*d**2*e*f**4*g*x**3 + 420*d**2*e*f
**4*h*x**4 + 385*d**2*f**5*g*x**4 + 315*d**2*f**5*h*x**5))/(3465*f**4)
```

3.73 $\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 299

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \frac{2(bc-ad)^2(be-af)(bg-ah)\sqrt{e+fx}}{b^5}$$
$$+\frac{2(bc-ad)^2(bg-ah)(e+fx)^{3/2}}{3b^4}$$
$$+\frac{2(a^2d^2f^2h-abdf(dfg-deh+2cfh)+b^2(c^2f^2h-d^2e(fg-eh)+2cdf(fg-eh)))\,(e+fx)^{5/2}}{5b^3f^3}$$
$$-\frac{2d(adfh-b(dfg-2deh+2cfh))(e+fx)^{7/2}}{7b^2f^3}+\frac{2d^2h(e+fx)^{9/2}}{9bf^3}$$
$$-\frac{2(bc-ad)^2(be-af)^{3/2}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{11/2}}$$

output
```
2*(-a*d+b*c)^2*(-a*f+b*e)*(-a*h+b*g)*(f*x+e)^(1/2)/b^5+2/3*(-a*d+b*c)^2*(-
a*h+b*g)*(f*x+e)^(3/2)/b^4+2/5*(a^2*d^2*f^2*h-a*b*d*f*(2*c*f*h-d*e*h+d*f*g
)+b^2*(c^2*f^2*h-d^2*e*(-e*h+f*g)+2*c*d*f*(-e*h+f*g)))*(f*x+e)^(5/2)/b^3/f
^3-2/7*d*(a*d*f*h-b*(2*c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(7/2)/b^2/f^3+2/9*d^2
*h*(f*x+e)^(9/2)/b/f^3-2*(-a*d+b*c)^2*(-a*f+b*e)^(3/2)*(-a*h+b*g)*arctanh(
b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \frac{2\sqrt{e+fx}(315a^4d^2f^4h-105a^3bdf^3(6cfh+d(3fg+4eh+fhx))+}{}$$
$$+\frac{2(bc-ad)^2(-be+af)^{3/2}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{11/2}}$$

input
```
Integrate[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x),x]
```

output
```
(2*Sqrt[e + f*x]*(315*a^4*d^2*f^4*h - 105*a^3*b*d*f^3*(6*c*f*h + d*(3*f*g
+ 4*e*h + f*h*x)) + 21*a^2*b^2*f^2*(15*c^2*f^2*h + 10*c*d*f*(3*f*g + 4*e*h
 + f*h*x) + d^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))) - 3*
a*b^3*f*(-3*d^2*(e + f*x)^2*(-7*f*g + 2*e*h - 5*f*h*x) + 35*c^2*f^2*(3*f*g
 + 4*e*h + f*h*x) + 14*c*d*f*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g +
6*h*x))) + b^4*(18*c*d*f*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h*x) + 21*c^2*f^
2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + d^2*(e + f*x)^2*(
8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)))))/(315*b^5*f^3) +
 (2*(b*c - a*d)^2*(-(b*e) + a*f)^(3/2)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e
+ f*x])/Sqrt[-(b*e) + a*f]])/b^(11/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {170, 27, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx$$
$$\Big\downarrow \text{170}$$
$$\frac{2\int \frac{(c+dx)(e+fx)^{3/2}(9bcfg-4adeh-5acfh-(9adfh-b(9dfg-4deh+4cfh))x)}{2(a+bx)}dx}{9bf}+\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(9bcfg-4adeh-5acfh-(9adfh-b(9dfg-4deh+4cfh))x)}{a+bx}dx}{9bf}+\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

$$\Big\downarrow \; 164$$

$$\frac{\frac{9f(bc-ad)^2(bg-ah)\int \frac{(e+fx)^{3/2}}{a+bx}dx}{b^2}+\frac{2(e+fx)^{5/2}(63a^2d^2f^2h-5bdfx(9adfh-b(4cfh-4deh+9dfg))-9abdf(14cfh-2deh+7dfg)+2b^2(14c^2f^2}{35b^2f^2}}{9bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{9f(bc-ad)^2(bg-ah)\left(\frac{(be-af)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}+\frac{2(e+fx)^{5/2}(63a^2d^2f^2h-5bdfx(9adfh-b(4cfh-4deh+9dfg))-9abdf(14cfh-2deh}{35b^2f^2}}{9bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{9f(bc-ad)^2(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}+2(e+fx)^{5/2}(63a^2d^2f^2h-5bdfx(9adfh-b(4cfh-4deh+}{9bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{9f(bc-ad)^2(bg-ah)\left(\frac{(be-af)\left(\frac{2(be-af)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}+2(e+fx)^{5/2}(63a^2d^2f^2h-5bdfx(9adfh-b(4cf}{9bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2(e+fx)^{5/2}\left(63a^2d^2f^2h-5bdfx(9adfh-b(4cfh-4deh+9dfg))-9abdf(14cfh-2deh+7dfg)+2b^2\left(14c^2f^2h+9cdf(7fg-2eh)+d^2(-e)(9fg-4eh)\right)\right.}{35b^2f^2}}{9bf}$$

$$\frac{2h(c+dx)^2(e+fx)^{5/2}}{9bf}$$

input
```
Int[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x),x]
```

output
```
(2*h*(c + d*x)^2*(e + f*x)^(5/2))/(9*b*f) + ((2*(e + f*x)^(5/2)*(63*a^2*d^
2*f^2*h - 9*a*b*d*f*(7*d*f*g - 2*d*e*h + 14*c*f*h) + 2*b^2*(14*c^2*f^2*h -
 d^2*e*(9*f*g - 4*e*h) + 9*c*d*f*(7*f*g - 2*e*h)) - 5*b*d*f*(9*a*d*f*h - b
*(9*d*f*g - 4*d*e*h + 4*c*f*h))*x))/(35*b^2*f^2) + (9*(b*c - a*d)^2*f*(b*g
 - a*h)*((2*(e + f*x)^(3/2))/(3*b) + ((b*e - a*f)*((2*Sqrt[e + f*x])/b - (
2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2
)))/b))/b^2)/(9*b*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.61

method	result
pseudoelliptic	$-\frac{2\left(f^{3}(af-be)^{2}(ad-bc)^{2}(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)-\left(\left(\left(\frac{x\left(\frac{3x^{2}\left(\frac{7hx}{9}+g\right)d^{2}}{7}+\frac{6xc\left(\frac{5hx}{7}+g\right)d}{5}+c^{2}\left(\frac{3hx}{5}+g\right)\right)b^{4}}{3}-a\right.\right.\right.\right.}{}$
derivativedivides	$\frac{\frac{2\left(2a^{2}b^{2}cde\,f^{3}h\sqrt{fx+e}-2a\,b^{3}cde\,f^{3}g\sqrt{fx+e}+\frac{d^{2}h(fx+e)^{\frac{9}{2}}b^{4}}{9}+\frac{b^{4}d^{2}fg(fx+e)^{\frac{7}{2}}}{7}+\frac{b^{4}c^{2}f^{2}h(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^{4}d^{2}eh(fx+e)^{\frac{7}{2}}}{7}+\frac{b^{4}d^{2}\right.}{}}{}$
default	$\frac{\frac{2\left(2a^{2}b^{2}cde\,f^{3}h\sqrt{fx+e}-2a\,b^{3}cde\,f^{3}g\sqrt{fx+e}+\frac{d^{2}h(fx+e)^{\frac{9}{2}}b^{4}}{9}+\frac{b^{4}d^{2}fg(fx+e)^{\frac{7}{2}}}{7}+\frac{b^{4}c^{2}f^{2}h(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^{4}d^{2}eh(fx+e)^{\frac{7}{2}}}{7}+\frac{b^{4}d^{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*(f^3*(a*f-b*e)^2*(a*d-b*c)^2*(a*h-b*g)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e
)*b)^(1/2))-((1/3*x*(3/7*x^2*(7/9*h*x+g)*d^2+6/5*x*c*(5/7*h*x+g)*d+c^2*(3/
5*h*x+g))*b^4-a*(1/5*x^2*(5/7*h*x+g)*d^2+2/3*x*c*(3/5*h*x+g)*d+c^2*(1/3*h*
x+g))*b^3+a^2*(1/3*x*(3/5*h*x+g)*d^2+2*c*(1/3*h*x+g)*d+h*c^2)*b^2-2*a^3*((
1/6*h*x+1/2*g)*d+c*h)*d*b+a^4*d^2*h)*f^4-4/3*(((-5/42*h*x^3-6/35*g*x^2)*d^
2-3/5*x*c*(4/7*h*x+g)*d-(3/10*h*x+g)*c^2)*b^3+a*(3/10*x*(4/7*h*x+g)*d^2+2*
c*(3/10*h*x+g)*d+h*c^2)*b^2-2*a^2*d*((3/20*h*x+1/2*g)*d+c*h)*b+h*d^2*a^3)*
b*e*f^3+1/5*((1/7*x*(1/3*h*x+g)*d^2+2*c*(1/7*h*x+g)*d+h*c^2)*b^2-2*a*((1/1
4*h*x+1/2*g)*d+c*h)*d*b+a^2*d^2*h)*b^2*e^2*f^2+2/35*(((-2/9*h*x-g)*d-2*c*h
)*b+a*d*h)*d*b^3*e^3*f+8/315*b^4*d^2*e^4*h)*((a*f-b*e)*b)^(1/2)*(f*x+e)^(1
/2))/((a*f-b*e)*b)^(1/2)/f^3/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(271) = 542$.

Time = 0.16 (sec) , antiderivative size = 1630, normalized size of antiderivative = 5.45

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/315*(315*(((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^3 - (a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)*f^4)*g - ((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2
)*e*f^3 - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*f^4)*h)*sqrt((b*e - a*f)/b
)*log((b*f*x + 2*b*e - a*f - 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x +
 a)) + 2*(35*b^4*d^2*f^4*h*x^4 + 5*(9*b^4*d^2*f^4*g + (10*b^4*d^2*e*f^3 +
9*(2*b^4*c*d - a*b^3*d^2)*f^4)*h)*x^3 + 3*(3*(8*b^4*d^2*e*f^3 + 7*(2*b^4*c
*d - a*b^3*d^2)*f^4)*g + (b^4*d^2*e^2*f^2 + 24*(2*b^4*c*d - a*b^3*d^2)*e*f
^3 + 21*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*f^4)*h)*x^2 - 3*(6*b^4*d^2*e
^3*f - 21*(2*b^4*c*d - a*b^3*d^2)*e^2*f^2 - 140*(b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)*e*f^3 + 105*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*f^4)*g + (
8*b^4*d^2*e^4 - 18*(2*b^4*c*d - a*b^3*d^2)*e^3*f + 63*(b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*e^2*f^2 - 420*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*e*
f^3 + 315*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*f^4)*h + (3*(3*b^4*d^2*e^2
*f^2 + 42*(2*b^4*c*d - a*b^3*d^2)*e*f^3 + 35*(b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*f^4)*g - (4*b^4*d^2*e^3*f - 9*(2*b^4*c*d - a*b^3*d^2)*e^2*f^2 - 1
26*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^3 + 105*(a*b^3*c^2 - 2*a^2*b^
2*c*d + a^3*b*d^2)*f^4)*h)*x)*sqrt(f*x + e))/(b^5*f^3), -2/315*(315*(((b^4
*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^3 - (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*f^4)*g - ((a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*e*f^3 - (a^2*b^2
*c^2 - 2*a^3*b*c*d + a^4*d^2)*f^4)*h)*sqrt(-(b*e - a*f)/b)*arctan(-sqrt...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(291) = 582$.

Time = 12.33 (sec) , antiderivative size = 677, normalized size of antiderivative = 2.26

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{9}{2}}}{9bf^2} + \frac{(e+fx)^{\frac{7}{2}}\left(-ad^2fh+2bcdfh-2bd^2eh+bd^2fg\right)}{7b^2f^2} + \frac{(e+fx)^{\frac{5}{2}}\left(a^2d^2f^2h-2abcdf^2h+a\right.}{}\right.}{} \\ e^{\frac{3}{2}}\left(\frac{d^2hx^3}{3b} + \frac{x^2\left(-ad^2h+2bcdh+bd^2g\right)}{2b^2} + \frac{x\left(a^2d^2h-2abcdh-abd^2g+b^2c^2h+2b^2cdg\right)}{b^3}\right. \end{cases}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(3/2)*(h*x+g)/(b*x+a),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(9/2)/(9*b*f**2) + (e + f*x)**(7/2)*(-a*d*
*2*f*h + 2*b*c*d*f*h - 2*b*d**2*e*h + b*d**2*f*g)/(7*b**2*f**2) + (e + f*x
)**(5/2)*(a**2*d**2*f**2*h - 2*a*b*c*d*f**2*h + a*b*d**2*e*f*h - a*b*d**2*
f**2*g + b**2*c**2*f**2*h - 2*b**2*c*d*e*f*h + 2*b**2*c*d*f**2*g + b**2*d*
*2*e**2*h - b**2*d**2*e*f*g)/(5*b**3*f**2) + (e + f*x)**(3/2)*(-a**3*d**2*
f*h + 2*a**2*b*c*d*f*h + a**2*b*d**2*f*g - a*b**2*c**2*f*h - 2*a*b**2*c*d*
f*g + b**3*c**2*f*g)/(3*b**4) + sqrt(e + f*x)*(a**4*d**2*f**2*h - 2*a**3*b
*c*d*f**2*h - a**3*b*d**2*e*f*h - a**3*b*d**2*f**2*g + a**2*b**2*c**2*f**2
*h + 2*a**2*b**2*c*d*e*f*h + 2*a**2*b**2*c*d*f**2*g + a**2*b**2*d**2*e*f*g
 - a*b**3*c**2*e*f*h - a*b**3*c**2*f**2*g - 2*a*b**3*c*d*e*f*g + b**4*c**2
*e*f*g)/b**5 - f*(a*d - b*c)**2*(a*f - b*e)**2*(a*h - b*g)*atan(sqrt(e + f
*x)/sqrt((a*f - b*e)/b))/(b**6*sqrt((a*f - b*e)/b)))/f, Ne(f, 0)), (e**(3/
2)*(d**2*h*x**3/(3*b) + x**2*(-a*d**2*h + 2*b*c*d*h + b*d**2*g)/(2*b**2) +
 x*(a**2*d**2*h - 2*a*b*c*d*h - a*b*d**2*g + b**2*c**2*h + 2*b**2*c*d*g)/b
**3 - (a*d - b*c)**2*(a*h - b*g)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/
b, True))/b**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(271) = 542$.

Time = 0.14 (sec) , antiderivative size = 944, normalized size of antiderivative = 3.16

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x, algorithm="giac")
```

output
```
2*(b^5*c^2*e^2*g - 2*a*b^4*c*d*e^2*g + a^2*b^3*d^2*e^2*g - 2*a*b^4*c^2*e*f
*g + 4*a^2*b^3*c*d*e*f*g - 2*a^3*b^2*d^2*e*f*g + a^2*b^3*c^2*f^2*g - 2*a^3
*b^2*c*d*f^2*g + a^4*b*d^2*f^2*g - a*b^4*c^2*e^2*h + 2*a^2*b^3*c*d*e^2*h -
 a^3*b^2*d^2*e^2*h + 2*a^2*b^3*c^2*e*f*h - 4*a^3*b^2*c*d*e*f*h + 2*a^4*b*d
^2*e*f*h - a^3*b^2*c^2*f^2*h + 2*a^4*b*c*d*f^2*h - a^5*d^2*f^2*h)*arctan(s
qrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^5) + 2/315*(4
5*(f*x + e)^(7/2)*b^8*d^2*f^25*g - 63*(f*x + e)^(5/2)*b^8*d^2*e*f^25*g + 1
26*(f*x + e)^(5/2)*b^8*c*d*f^26*g - 63*(f*x + e)^(5/2)*a*b^7*d^2*f^26*g +
105*(f*x + e)^(3/2)*b^8*c^2*f^27*g - 210*(f*x + e)^(3/2)*a*b^7*c*d*f^27*g
+ 105*(f*x + e)^(3/2)*a^2*b^6*d^2*f^27*g + 315*sqrt(f*x + e)*b^8*c^2*e*f^2
7*g - 630*sqrt(f*x + e)*a*b^7*c*d*e*f^27*g + 315*sqrt(f*x + e)*a^2*b^6*d^2
*e*f^27*g - 315*sqrt(f*x + e)*a*b^7*c^2*f^28*g + 630*sqrt(f*x + e)*a^2*b^6
*c*d*f^28*g - 315*sqrt(f*x + e)*a^3*b^5*d^2*f^28*g + 35*(f*x + e)^(9/2)*b^
8*d^2*f^24*h - 90*(f*x + e)^(7/2)*b^8*d^2*e*f^24*h + 63*(f*x + e)^(5/2)*b^
8*d^2*e^2*f^24*h + 90*(f*x + e)^(7/2)*b^8*c*d*f^25*h - 45*(f*x + e)^(7/2)*
a*b^7*d^2*f^25*h - 126*(f*x + e)^(5/2)*b^8*c*d*e*f^25*h + 63*(f*x + e)^(5/
2)*a*b^7*d^2*e*f^25*h + 63*(f*x + e)^(5/2)*b^8*c^2*f^26*h - 126*(f*x + e)^
(5/2)*a*b^7*c*d*f^26*h + 63*(f*x + e)^(5/2)*a^2*b^6*d^2*f^26*h - 105*(f*x
+ e)^(3/2)*a*b^7*c^2*f^27*h + 210*(f*x + e)^(3/2)*a^2*b^6*c*d*f^27*h - 105
*(f*x + e)^(3/2)*a^3*b^5*d^2*f^27*h - 315*sqrt(f*x + e)*a*b^7*c^2*e*f^2...
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.86

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^2)/(a + b*x),x)
```

output
```
(e + f*x)^(7/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(7*b*f^3) - (2*d^2*h*
(a*f^4 - b*e*f^3))/(7*b^2*f^6)) - (e + f*x)^(3/2)*((2*(c*f - d*e)^2*(e*h -
 f*g))/(3*b*f^3) - (((((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b*f^3) - (2*d^
2*h*(a*f^4 - b*e*f^3))/(b^2*f^6))*(a*f^4 - b*e*f^3))/(b*f^3) - (2*(c*f - d
*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(b*f^3))*(a*f^4 - b*e*f^3))/(3*b*f^3)) -
(e + f*x)^(5/2)*((((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b*f^3) - (2*d^2*h*
(a*f^4 - b*e*f^3))/(b^2*f^6))*(a*f^4 - b*e*f^3))/(5*b*f^3) - (2*(c*f - d*e
)*(c*f*h - 3*d*e*h + 2*d*f*g))/(5*b*f^3)) + (2*d^2*h*(e + f*x)^(9/2))/(9*b
*f^3) + (2*atan((b^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)^2*(a*f - b*e)^(3/2)*(
a*h - b*g))/(b^5*c^2*e^2*g - a^5*d^2*f^2*h - a*b^4*c^2*e^2*h + a^4*b*d^2*f
^2*g + a^2*b^3*c^2*f^2*g + a^2*b^3*d^2*e^2*g - a^3*b^2*c^2*f^2*h - a^3*b^2
*d^2*e^2*h - 2*a*b^4*c*d*e^2*g - 2*a*b^4*c^2*e*f*g + 2*a^4*b*c*d*f^2*h + 2
*a^4*b*d^2*e*f*h + 2*a^2*b^3*c*d*e^2*h - 2*a^3*b^2*c*d*f^2*g + 2*a^2*b^3*c
^2*e*f*h - 2*a^3*b^2*d^2*e*f*g + 4*a^2*b^3*c*d*e*f*g - 4*a^3*b^2*c*d*e*f*h
))*(a*d - b*c)^2*(a*f - b*e)^(3/2)*(a*h - b*g))/b^(11/2) + ((e + f*x)^(1/2
)*(a*f^4 - b*e*f^3)*((2*(c*f - d*e)^2*(e*h - f*g))/(b*f^3) - (((((2*d^2*f*
g - 6*d^2*e*h + 4*c*d*f*h)/(b*f^3) - (2*d^2*h*(a*f^4 - b*e*f^3))/(b^2*f^6)
)*(a*f^4 - b*e*f^3))/(b*f^3) - (2*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))
/(b*f^3))*(a*f^4 - b*e*f^3))/(b*f^3)))/(b*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1635, normalized size of antiderivative = 5.47

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{a+bx}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a),x)
```

output
```
(2*( - 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**4*d**2*f**4*h + 630*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d*f**4*h + 315*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*e*f*
*3*h + 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**3*b*d**2*f**4*g - 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**2*f**4*h - 630*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*
c*d*e*f**3*h - 630*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**2*b**2*c*d*f**4*g - 315*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f**3*g + 31
5*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a*b**3*c**2*e*f**3*h + 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c**2*f**4*g + 630*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c*d*e*f**3*g
- 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*b**4*c**2*e*f**3*g + 315*sqrt(e + f*x)*a**4*b*d**2*f**4*h - 630*sqrt
(e + f*x)*a**3*b**2*c*d*f**4*h - 420*sqrt(e + f*x)*a**3*b**2*d**2*e*f**3*h
 - 315*sqrt(e + f*x)*a**3*b**2*d**2*f**4*g - 105*sqrt(e + f*x)*a**3*b**2*d
**2*f**4*h*x + 315*sqrt(e + f*x)*a**2*b**3*c**2*f**4*h + 840*sqrt(e + f...
```

3.74 $\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 321

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \frac{(bc-ad)\left(9a^2dfh+b^2(4deg+3cfg+2ceh)-ab(7dfg+6deh+5c\right.}{b^5}$$

$$+\frac{2(bc-ad)(2bdg+bch-3adh)(e+fx)^{3/2}}{3b^4}-\frac{(bc-ad)^2(bg-ah)(e+fx)^{3/2}}{b^4(a+bx)}$$

$$-\frac{2d(2adfh-b(dfg-deh+2cfh))(e+fx)^{5/2}}{5b^3f^2}+\frac{2d^2h(e+fx)^{7/2}}{7b^2f^2}$$

$$-\frac{(bc-ad)\sqrt{be-af}\left(9a^2dfh+b^2(4deg+3cfg+2ceh)-ab(7dfg+6deh+5cfh)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{11/2}}$$

output

```
(-a*d+b*c)*(9*a^2*d*f*h+b^2*(2*c*e*h+3*c*f*g+4*d*e*g)-a*b*(5*c*f*h+6*d*e*h
+7*d*f*g))*(f*x+e)^(1/2)/b^5+2/3*(-a*d+b*c)*(-3*a*d*h+b*c*h+2*b*d*g)*(f*x+
e)^(3/2)/b^4-(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(3/2)/b^4/(b*x+a)-2/5*d*(2*a*
d*f*h-b*(2*c*f*h-d*e*h+d*f*g))*(f*x+e)^(5/2)/b^3/f^2+2/7*d^2*h*(f*x+e)^(7/
2)/b^2/f^2-(-a*d+b*c)*(-a*f+b*e)^(1/2)*(9*a^2*d*f*h+b^2*(2*c*e*h+3*c*f*g+4
*d*e*g)-a*b*(5*c*f*h+6*d*e*h+7*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f
+b*e)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \frac{\sqrt{e+fx}(-945a^4d^2f^3h+105a^3bdf^2(14cfh+d(7fg+9eh-6fhx)}{}$$

$$-\frac{(bc-ad)\sqrt{-be+af}(9a^2dfh+b^2(4deg+3cfg+2ceh)-ab(7dfg+6deh+5cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{11/2}}$$

input
```
Integrate[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(-945*a^4*d^2*f^3*h + 105*a^3*b*d*f^2*(14*c*f*h + d*(7*f*g
+ 9*e*h - 6*f*h*x)) + b^4*(6*d^2*x*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h*x) +
 28*c*d*f*x*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + 35*c^2*
f^2*(2*f*x*(3*g + h*x) + e*(-3*g + 8*h*x))) - 7*a^2*b^2*f*(75*c^2*f^2*h +
10*c*d*f*(15*f*g + 19*e*h - 14*f*h*x) + d^2*(12*e^2*h + e*f*(95*g - 96*h*x
) - 2*f^2*x*(35*g + 9*h*x))) - a*b^3*(-35*c^2*f^2*(9*f*g + 11*e*h - 10*f*h
*x) + 2*d^2*(6*e^3*h + f^3*x^2*(49*g + 27*h*x) + 2*e*f^2*x*(119*g + 30*h*x
) + e^2*f*(-21*g + 39*h*x)) + 14*c*d*f*(-6*e^2*h + 2*f^2*x*(25*g + 7*h*x)
+ e*f*(-55*g + 68*h*x)))))/(105*b^5*f^2*(a + b*x)) - ((b*c - a*d)*Sqrt[-(b
*e) + a*f]*(9*a^2*d*f*h + b^2*(4*d*e*g + 3*c*f*g + 2*c*e*h) - a*b*(7*d*f*g
 + 6*d*e*h + 5*c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])
/b^(11/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {166, 27, 25, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{(c+dx)(e+fx)^{3/2}\left(a(4de+5cf)h-2b\left(2deg+\frac{3cfg}{2}+ceh\right)-d(7bfg+2beh-9afh)x\right)}{2(a+bx)}dx}{b(be-af)}-\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{(c+dx)(e+fx)^{3/2}(4bdeg+3bcfg+2bceh-4adeh-5acfh+d(7bfg+2beh-9afh)x)}{a+bx}dx}{2b(be-af)}-\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg+3bcfg+2bceh-4adeh-5acfh+d(7bfg+2beh-9afh)x)}{a+bx}dx}{2b(be-af)}-\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 164$$

$$\frac{\frac{(bc-ad)\left(9a^2dfh-ab(5cfh+6deh+7dfg)+b^2(2ceh+3cfg+4deg)\right)\int \frac{(e+fx)^{3/2}}{a+bx}dx}{b^2}+\frac{2d(e+fx)^{5/2}\left(63a^2df^2h-abf(98cfh+24deh+49dfg)+5bdf\ldots\right.}{35b\ldots}}{2b(be-af)}-\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(bc-ad)\left(9a^2dfh-ab(5cfh+6deh+7dfg)+b^2(2ceh+3cfg+4deg)\right)\left(\frac{(be-af)\int \frac{\sqrt{e+fx}}{a+bx}dx}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}+\frac{2d(e+fx)^{5/2}\left(63a^2df^2h-abf(98cfh+\ldots\right.}{\ldots}}{2b(be-af)}-\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(bc-ad)\left(9a^2dfh-ab(5cfh+6deh+7dfg)+b^2(2ceh+3cfg+4deg)\right)\left(\frac{(be-af)\left(\frac{(be-af)\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{b}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}+\frac{2d(e+fx)^{5}}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{(bc-ad)\left(9a^2dfh-ab(5cfh+6deh+7dfg)+b^2(2ceh+3cfg+4deg)\right)\left(\frac{(be-af)\left(\frac{2(be-af)\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf}+\frac{2\sqrt{e+fx}}{b}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)}{b^2}+\frac{2c}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}}{b}-\frac{2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}}\right)}{b}+\frac{2(e+fx)^{3/2}}{3b}\right)\left(9a^2dfh-ab(5cfh+6deh+7dfg)+b^2(2ceh+3cfg+4deg)\right)}{b^2}+\frac{2d(e}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

input
```
Int[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^2,x]
```

output
```
-(((b*g - a*h)*(c + d*x)^2*(e + f*x)^(5/2))/(b*(b*e - a*f)*(a + b*x))) + (
(2*d*(e + f*x)^(5/2)*(63*a^2*d*f^2*h - a*b*f*(49*d*f*g + 24*d*e*h + 98*c*f
*h) + 2*b^2*(d*e*(7*f*g - 2*e*h) + 7*c*f*(5*f*g + 2*e*h)) + 5*b*d*f*(7*b*f
*g + 2*b*e*h - 9*a*f*h)*x))/(35*b^2*f^2) + ((b*c - a*d)*(9*a^2*d*f*h + b^2
*(4*d*e*g + 3*c*f*g + 2*c*e*h) - a*b*(7*d*f*g + 6*d*e*h + 5*c*f*h))*((2*(e
 + f*x)^(3/2))/(3*b) + ((b*e - a*f)*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a
*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b))/b^2)/(
2*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.54

method	result
pseudoelliptic	$\frac{9(ad-bc)(bx+a)\left(\frac{\left(cfg+\frac{2e(ch+2dg)}{3}\right)b^2}{3}-\frac{5a\left(\left(ch+\frac{7dg}{5}\right)f+\frac{6deh}{5}\right)b}{9}+a^2dfh\right)f^2(af-be)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)-9\sqrt{(af-}}{}$
risch	$-\frac{2\left(-15d^2h\,x^3b^3f^3+42a\,b^2d^2f^3h\,x^2-42b^3cd\,f^3h\,x^2-24b^3d^2e\,f^2h\,x^2-21b^3d^2f^3g\,x^2-105a^2b\,d^2f^3hx+140a\,b^2cd\,f^3h\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{d^2h(fx+e)^{\frac{7}{2}}b^3}{7}+\frac{2a\,b^2d^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3cdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^3d^2eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3d^2fg(fx+e)^{\frac{5}{2}}}{5}-a^2b\,d^2f^2h(fx+e)^{\frac{3}{2}}+\frac{4a\,b}{}\right.}{}$
default	$-\frac{2\left(-\frac{d^2h(fx+e)^{\frac{7}{2}}b^3}{7}+\frac{2a\,b^2d^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{2b^3cdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^3d^2eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3d^2fg(fx+e)^{\frac{5}{2}}}{5}-a^2b\,d^2f^2h(fx+e)^{\frac{3}{2}}+\frac{4a\,b}{}\right.}{}$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
9/((a*f-b*e)*b)^(1/2)*((a*d-b*c)*(b*x+a)*(1/3*(c*f*g+2/3*e*(c*h+2*d*g))*b^
2-5/9*a*((c*h+7/5*d*g)*f+6/5*d*e*h)*b+a^2*d*f*h)*f^2*(a*f-b*e)*arctan(b*(f
*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^(1/2)*(f*x+e)^(1/2)*(1/9*(-
2*x*(1/5*x^2*(5/7*h*x+g)*d^2+2/3*x*c*(3/5*h*x+g)*d+c^2*(1/3*h*x+g))*f^3+e*
(-4/5*x^2*(4/7*h*x+g)*d^2-16/3*(3/10*h*x+g)*x*c*d+c^2*(-8/3*h*x+g))*f^2-4/
5*x*d*(1/2*(1/7*h*x+g)*d+c*h)*e^2*f+4/35*d^2*e^3*h*x)*b^4-11/27*a*(1/11*(-
14/5*x^2*(27/49*h*x+g)*d^2-20*x*c*(7/25*h*x+g)*d+9*c^2*(-10/9*h*x+g))*f^3+
(4/11*(-6/7*h*x^2-17/5*g*x)*d^2+2*c*(-68/55*h*x+g)*d+h*c^2)*e*f^2+12/55*(1
/2*(-13/7*h*x+g)*d+c*h)*d*e^2*f-12/385*d^2*e^3*h)*b^3+5/9*a^2*((-14/15*x*(
9/35*h*x+g)*d^2+2*(-14/15*h*x+g)*c*d+h*c^2)*f^2+38/15*d*e*((-48/95*h*x+1/2
*g)*d+c*h)*f+4/25*d^2*e^2*h)*f*b^2-14/9*a^3*d*(((-3/7*h*x+1/2*g)*d+c*h)*f+
9/14*d*e*h)*f^2*b+a^4*d^2*f^3*h))/f^2/b^5/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(295) = 590$.

Time = 0.15 (sec) , antiderivative size = 1822, normalized size of antiderivative = 5.68

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/210*(105*((4*(a*b^3*c*d - a^2*b^2*d^2)*e*f^2 + (3*a*b^3*c^2 - 10*a^2*b
^2*c*d + 7*a^3*b*d^2)*f^3)*g + (2*(a*b^3*c^2 - 4*a^2*b^2*c*d + 3*a^3*b*d^2
)*e*f^2 - (5*a^2*b^2*c^2 - 14*a^3*b*c*d + 9*a^4*d^2)*f^3)*h + ((4*(b^4*c*d
 - a*b^3*d^2)*e*f^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*f^3)*g +
(2*(b^4*c^2 - 4*a*b^3*c*d + 3*a^2*b^2*d^2)*e*f^2 - (5*a*b^3*c^2 - 14*a^2*b
^2*c*d + 9*a^3*b*d^2)*f^3)*h)*x)*sqrt((b*e - a*f)/b)*log((b*f*x + 2*b*e -
a*f + 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a)) - 2*(30*b^4*d^2*f^
3*h*x^4 + 6*(7*b^4*d^2*f^3*g + (8*b^4*d^2*e*f^2 + (14*b^4*c*d - 9*a*b^3*d^
2)*f^3)*h)*x^3 + 2*(7*(6*b^4*d^2*e*f^2 + (10*b^4*c*d - 7*a*b^3*d^2)*f^3)*g
 + (3*b^4*d^2*e^2*f + 12*(7*b^4*c*d - 5*a*b^3*d^2)*e*f^2 + 7*(5*b^4*c^2 -
14*a*b^3*c*d + 9*a^2*b^2*d^2)*f^3)*h)*x^2 + 7*(6*a*b^3*d^2*e^2*f - 5*(3*b^
4*c^2 - 22*a*b^3*c*d + 19*a^2*b^2*d^2)*e*f^2 + 15*(3*a*b^3*c^2 - 10*a^2*b^
2*c*d + 7*a^3*b*d^2)*f^3)*g - (12*a*b^3*d^2*e^3 - 84*(a*b^3*c*d - a^2*b^2*
d^2)*e^2*f - 35*(11*a*b^3*c^2 - 38*a^2*b^2*c*d + 27*a^3*b*d^2)*e*f^2 + 105
*(5*a^2*b^2*c^2 - 14*a^3*b*c*d + 9*a^4*d^2)*f^3)*h + 2*(7*(3*b^4*d^2*e^2*f
 + 2*(20*b^4*c*d - 17*a*b^3*d^2)*e*f^2 + 5*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a
^2*b^2*d^2)*f^3)*g - (6*b^4*d^2*e^3 - 3*(14*b^4*c*d - 13*a*b^3*d^2)*e^2*f
- 28*(5*b^4*c^2 - 17*a*b^3*c*d + 12*a^2*b^2*d^2)*e*f^2 + 35*(5*a*b^3*c^2 -
 14*a^2*b^2*c*d + 9*a^3*b*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(b^6*f^2*x + a*b^
5*f^2), -1/105*(105*((4*(a*b^3*c*d - a^2*b^2*d^2)*e*f^2 + (3*a*b^3*c^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(295) = 590$.

Time = 0.15 (sec) , antiderivative size = 941, normalized size of antiderivative = 2.93

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(4*b^4*c*d*e^2*g - 4*a*b^3*d^2*e^2*g + 3*b^4*c^2*e*f*g - 14*a*b^3*c*d*e*f*
g + 11*a^2*b^2*d^2*e*f*g - 3*a*b^3*c^2*f^2*g + 10*a^2*b^2*c*d*f^2*g - 7*a^
3*b*d^2*f^2*g + 2*b^4*c^2*e^2*h - 8*a*b^3*c*d*e^2*h + 6*a^2*b^2*d^2*e^2*h
- 7*a*b^3*c^2*e*f*h + 22*a^2*b^2*c*d*e*f*h - 15*a^3*b*d^2*e*f*h + 5*a^2*b^
2*c^2*f^2*h - 14*a^3*b*c*d*f^2*h + 9*a^4*d^2*f^2*h)*arctan(sqrt(f*x + e)*b
/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^5) - (sqrt(f*x + e)*b^4*c^2
*e*f*g - 2*sqrt(f*x + e)*a*b^3*c*d*e*f*g + sqrt(f*x + e)*a^2*b^2*d^2*e*f*g
 - sqrt(f*x + e)*a*b^3*c^2*f^2*g + 2*sqrt(f*x + e)*a^2*b^2*c*d*f^2*g - sqr
t(f*x + e)*a^3*b*d^2*f^2*g - sqrt(f*x + e)*a*b^3*c^2*e*f*h + 2*sqrt(f*x +
e)*a^2*b^2*c*d*e*f*h - sqrt(f*x + e)*a^3*b*d^2*e*f*h + sqrt(f*x + e)*a^2*b
^2*c^2*f^2*h - 2*sqrt(f*x + e)*a^3*b*c*d*f^2*h + sqrt(f*x + e)*a^4*d^2*f^2
*h)/(((f*x + e)*b - b*e + a*f)*b^5) + 2/105*(21*(f*x + e)^(5/2)*b^12*d^2*f
^13*g + 70*(f*x + e)^(3/2)*b^12*c*d*f^14*g - 70*(f*x + e)^(3/2)*a*b^11*d^2
*f^14*g + 210*sqrt(f*x + e)*b^12*c*d*e*f^14*g - 210*sqrt(f*x + e)*a*b^11*d
^2*e*f^14*g + 105*sqrt(f*x + e)*b^12*c^2*f^15*g - 420*sqrt(f*x + e)*a*b^11
*c*d*f^15*g + 315*sqrt(f*x + e)*a^2*b^10*d^2*f^15*g + 15*(f*x + e)^(7/2)*b
^12*d^2*f^12*h - 21*(f*x + e)^(5/2)*b^12*d^2*e*f^12*h + 42*(f*x + e)^(5/2)
*b^12*c*d*f^13*h - 42*(f*x + e)^(5/2)*a*b^11*d^2*f^13*h + 35*(f*x + e)^(3/
2)*b^12*c^2*f^14*h - 140*(f*x + e)^(3/2)*a*b^11*c*d*f^14*h + 105*(f*x + e)
^(3/2)*a^2*b^10*d^2*f^14*h + 105*sqrt(f*x + e)*b^12*c^2*e*f^14*h - 420*...
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.23

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = (e+f\,x)^{5/2}\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{5\,b^2\,f^2}-\frac{4\,d^2\,h\,(a\,f-b\,e)}{5\,b^3\,f^2}\right)$$

$$-(e+f\,x)^{3/2}\left(\frac{2\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b^2\,f^2}-\frac{4\,d^2\,h\,(a\,f-b\,e)}{b^3\,f^2}\right)(a\,f-b\,e)}{3\,b}-\frac{2\,(c\,f-d\,e)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g)}{3\,b^2\,f^2}\right.$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^2,x)
```

output
```
(e + f*x)^(5/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(5*b^2*f^2) - (4*d^2*
h*(a*f - b*e))/(5*b^3*f^2)) - (e + f*x)^(3/2)*((2*((2*d^2*f*g - 6*d^2*e*h
+ 4*c*d*f*h)/(b^2*f^2) - (4*d^2*h*(a*f - b*e))/(b^3*f^2))*(a*f - b*e))/(3*
b) - (2*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(3*b^2*f^2) + (2*d^2*h*(a
*f - b*e)^2)/(3*b^4*f^2)) - (e + f*x)^(1/2)*((((2*d^2*f*g - 6*d^2*e*h + 4*
c*d*f*h)/(b^2*f^2) - (4*d^2*h*(a*f - b*e))/(b^3*f^2))*(a*f - b*e)^2)/b^2 -
 (2*(a*f - b*e)*((2*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b^2*f^2) - (4*d^
2*h*(a*f - b*e))/(b^3*f^2))*(a*f - b*e))/b - (2*(c*f - d*e)*(c*f*h - 3*d*e
*h + 2*d*f*g))/(b^2*f^2) + (2*d^2*h*(a*f - b*e)^2)/(b^4*f^2)))/b + (2*(c*f
 - d*e)^2*(e*h - f*g))/(b^2*f^2)) - ((e + f*x)^(1/2)*(a^4*d^2*f^2*h + b^4*
c^2*e*f*g - a*b^3*c^2*f^2*g - a^3*b*d^2*f^2*g + a^2*b^2*c^2*f^2*h - 2*a^3*
b*c*d*f^2*h - a*b^3*c^2*e*f*h - a^3*b*d^2*e*f*h + 2*a^2*b^2*c*d*f^2*g + a^
2*b^2*d^2*e*f*g - 2*a*b^3*c*d*e*f*g + 2*a^2*b^2*c*d*e*f*h))/(b^6*(e + f*x)
 - b^6*e + a*b^5*f) - (atan((b^(1/2)*(e + f*x)^(1/2)*1i)/(b*e - a*f)^(1/2)
)*(a*d - b*c)*(b*e - a*f)^(1/2)*(2*b^2*c*e*h + 3*b^2*c*f*g + 4*b^2*d*e*g +
 9*a^2*d*f*h - 5*a*b*c*f*h - 6*a*b*d*e*h - 7*a*b*d*f*g)*1i)/b^(11/2) + (2*
d^2*h*(e + f*x)^(7/2))/(7*b^2*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2025, normalized size of antiderivative = 6.31

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x)
```

output
```
(945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**4*d**2*f**3*h - 1470*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d*f**3*h - 630*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*e*f**2*h
 - 735*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*b*d**2*f**3*g + 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*f**3*h*x + 525*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**2*f
**3*h + 840*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**2*b**2*c*d*e*f**2*h + 1050*sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*d*f**3*g - 1470*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2
*b**2*c*d*f**3*h*x + 420*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f**2*g - 630*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f*
*2*h*x - 735*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**2*b**2*d**2*f**3*g*x - 210*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c**2*e*f**2*h - 315*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*
*3*c**2*f**3*g + 525*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq...
```

3.75 $\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 422

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \frac{2(6a^2d^2fh-3abd(dfg+deh+2cfh)+b^2(d^2eg+c^2fh+2cd(fg+}{b^5}$$
$$-\frac{(bc-ad)\left(15a^2dfh+b^2(8deg+3cfg+4ceh)-ab(11dfg+12deh+7cfh)\right)\sqrt{e+fx}}{4b^5(a+bx)}$$
$$+\frac{2d(bdg+2bch-3adh)(e+fx)^{3/2}}{3b^4}-\frac{(bc-ad)^2(bg-ah)(e+fx)^{3/2}}{2b^4(a+bx)^2}+\frac{2d^2h(e+fx)^{5/2}}{5b^3f}$$
$$+\frac{(63a^3d^2f^2h-7a^2bdf(5dfg+12deh+10cfh)-b^3(8d^2e^2g+8cde(3fg+2eh)+3c^2f(fg+4eh))+ab^2(}{4b^{11/2}\sqrt{be-af}}$$

output
```
2*(6*a^2*d^2*f*h-3*a*b*d*(2*c*f*h+d*e*h+d*f*g)+b^2*(d^2*e*g+c^2*f*h+2*c*d*
(e*h+f*g)))*(f*x+e)^(1/2)/b^5-1/4*(-a*d+b*c)*(15*a^2*d*f*h+b^2*(4*c*e*h+3*
c*f*g+8*d*e*g)-a*b*(7*c*f*h+12*d*e*h+11*d*f*g))*(f*x+e)^(1/2)/b^5/(b*x+a)+
2/3*d*(-3*a*d*h+2*b*c*h+b*d*g)*(f*x+e)^(3/2)/b^4-1/2*(-a*d+b*c)^2*(-a*h+b*
g)*(f*x+e)^(3/2)/b^4/(b*x+a)^2+2/5*d^2*h*(f*x+e)^(5/2)/b^3/f+1/4*(63*a^3*d
^2*f^2*h-7*a^2*b*d*f*(10*c*f*h+12*d*e*h+5*d*f*g)-b^3*(8*d^2*e^2*g+8*c*d*e*
(2*e*h+3*f*g)+3*c^2*f*(4*e*h+f*g))+a*b^2*(15*c^2*f^2*h+8*d^2*e*(3*e*h+5*f*
g)+10*c*d*f*(8*e*h+3*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2)
)/b^(11/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \frac{\sqrt{e+fx}(945a^4d^2f^2h - 105a^3bdf(6deh+10cfh+5df(g-3hx)) +}{}$$
$$+\frac{(-63a^3d^2f^2h + 7a^2bdf(5dfg+12deh+10cfh) + b^3(8d^2e^2g + 8cde(3fg+2eh) + 3c^2f(fg+4eh)) - ab^2}{4b^{11/2}\sqrt{-be+af}}$$

input
```
Integrate[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(945*a^4*d^2*f^2*h - 105*a^3*b*d*f*(6*d*e*h + 10*c*f*h + 5*
d*f*(g - 3*h*x)) + b^4*(-15*c^2*f*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x)) +
8*d^2*x^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + 40*c*d*f*
x*(2*f*x*(3*g + h*x) + e*(-3*g + 8*h*x))) - a*b^3*(15*c^2*f*(3*f*g + 2*e*h
 - 25*f*h*x) + 8*d^2*x*(-6*e^2*h + f^2*x*(35*g + 9*h*x) + e*f*(-55*g + 48*
h*x)) + 10*c*d*f*(e*(6*g - 88*h*x) + f*x*(-75*g + 56*h*x))) + a^2*b^2*(225
*c^2*f^2*h + 50*c*d*f*(9*f*g + 10*e*h - 35*f*h*x) + d^2*(24*e^2*h + 2*e*f*
(125*g - 546*h*x) + 7*f^2*x*(-125*g + 72*h*x)))))/(60*b^5*f*(a + b*x)^2) +
 ((-63*a^3*d^2*f^2*h + 7*a^2*b*d*f*(5*d*f*g + 12*d*e*h + 10*c*f*h) + b^3*(
8*d^2*e^2*g + 8*c*d*e*(3*f*g + 2*e*h) + 3*c^2*f*(f*g + 4*e*h)) - a*b^2*(15
*c^2*f^2*h + 8*d^2*e*(5*f*g + 3*e*h) + 10*c*d*f*(3*f*g + 8*e*h)))*ArcTan[(
Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(4*b^(11/2)*Sqrt[-(b*e) + a*f]
)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 163, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx$$

$$\big\downarrow \; 166$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg+bcfg+4bceh-4adeh-5acfh+d(5bfg+4beh-9afh)x)}{2(a+bx)^2}dx}{2b(be-af)} - \frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg+bcfg+4bceh-4adeh-5acfh+d(5bfg+4beh-9afh)x)}{(a+bx)^2}dx}{4b(be-af)} - \frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\big\downarrow \; 163$$

$$\frac{(e+fx)^{5/2}(63a^3d^2f^2h-a^2bdf(70cfh+66deh+35dfg)+ab^2(25c^2f^2h+30cdf(2eh+fg)+2d^2e(4eh+15fg))+2bd^2x(be-af)(-9afh+4beh+5bfg)}{5b^2f(a+bx)(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\big\downarrow \; 60$$

$$\frac{(e+fx)^{5/2}(63a^3d^2f^2h-a^2bdf(70cfh+66deh+35dfg)+ab^2(25c^2f^2h+30cdf(2eh+fg)+2d^2e(4eh+15fg))+2bd^2x(be-af)(-9afh+4beh+5bfg)}{5b^2f(a+bx)(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\big\downarrow \; 60$$

$$\frac{(e+fx)^{5/2}(63a^3d^2f^2h-a^2bdf(70cfh+66deh+35dfg)+ab^2(25c^2f^2h+30cdf(2eh+fg)+2d^2e(4eh+15fg))+2bd^2x(be-af)(-9afh+4beh+5bfg)}{5b^2f(a+bx)(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\big\downarrow \; 73$$

$$\frac{(e+fx)^{5/2}\left(63a^3d^2f^2h-a^2bdf(70cfh+66deh+35dfg)+ab^2\left(25c^2f^2h+30cdf(2eh+fg)+2d^2e(4eh+15fg)\right)+2bd^2x(be-af)(-9afh+4beh+5bfg)\cdots\right.}{5b^2f(a+bx)(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$\big\downarrow$ 221

$$\frac{(e+fx)^{5/2}\left(63a^3d^2f^2h-a^2bdf(70cfh+66deh+35dfg)+ab^2\left(25c^2f^2h+30cdf(2eh+fg)+2d^2e(4eh+15fg)\right)+2bd^2x(be-af)(-9afh+4beh+5bfg)\cdots\right.}{5b^2f(a+bx)(be-af)}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{2b(a+bx)^2(be-af)}$$

input
```
Int[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^3,x]
```

output
```
-1/2*((b*g - a*h)*(c + d*x)^2*(e + f*x)^(5/2))/(b*(b*e - a*f)*(a + b*x)^2)
 + (((e + f*x)^(5/2)*(63*a^3*d^2*f^2*h - 5*b^3*c*f*(4*d*e*g + c*f*g + 4*c*
e*h) - a^2*b*d*f*(35*d*f*g + 66*d*e*h + 70*c*f*h) + a*b^2*(25*c^2*f^2*h +
30*c*d*f*(f*g + 2*e*h) + 2*d^2*e*(15*f*g + 4*e*h)) + 2*b*d^2*(b*e - a*f)*(
5*b*f*g + 4*b*e*h - 9*a*f*h)*x))/(5*b^2*f*(b*e - a*f)*(a + b*x)) - ((63*a^
3*d^2*f^2*h - 7*a^2*b*d*f*(5*d*f*g + 12*d*e*h + 10*c*f*h) - b^3*(8*d^2*e^2
*g + 8*c*d*e*(3*f*g + 2*e*h) + 3*c^2*f*(f*g + 4*e*h)) + a*b^2*(15*c^2*f^2*
h + 8*d^2*e*(5*f*g + 3*e*h) + 10*c*d*f*(3*f*g + 8*e*h)))*((2*(e + f*x)^(3/
2))/(3*b) + ((b*e - a*f)*((2*Sqrt[e + f*x])/b - (2*Sqrt[b*e - a*f]*ArcTanh
[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b))/(2*b^2*(b*e - a*f
)))/(4*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{63\left(\left(\left(\left(-\frac{g f^2 c^2}{21}-\frac{4ce(ch+2dg)f}{21}-\frac{16\left(ch+\frac{dg}{2}\right)d e^2}{63}\right)b^3+\frac{5\left(\left(h c^2+2cdg\right)f^2+\frac{16\left(ch+\frac{dg}{2}\right)def}{3}+\frac{8d^2e^2h}{5}\right)a b^2}{21}-\frac{10a^2\left(\left(ch+\cdots\right.\right.}{}\cdots\right.\right.\right.}{\cdots}$
risch	$\frac{2(3x^2hb^2d^2f^2-15abd^2f^2hx+10b^2cdf^2hx+6b^2d^2efhx+5b^2d^2f^2gx+90a^2d^2f^2h-90abcdf^2h-60abd^2efh-45abd^2\cdots}{15fb^5}$
derivativedivides	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}-abd^2fh(fx+e)^{\frac{3}{2}}+\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+6a^2d^2f^2h\sqrt{fx+e}-6abcdf^2h\sqrt{fx+e}-3abd^2efh\sqrt{f\cdots}\right.}{b^5}$
default	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}-abd^2fh(fx+e)^{\frac{3}{2}}+\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+6a^2d^2f^2h\sqrt{fx+e}-6abcdf^2h\sqrt{fx+e}-3abd^2efh\sqrt{f\cdots}\right.}{b^5}$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-63/4*(((-1/21*g*f^2*c^2-4/21*c*e*(c*h+2*d*g)*f-16/63*(c*h+1/2*d*g)*d*e^2)
*b^3+5/21*((c^2*h+2*c*d*g)*f^2+16/3*(c*h+1/2*d*g)*d*e*f+8/5*d^2*e^2*h)*a*b
^2-10/9*a^2*((c*h+1/2*d*g)*f+6/5*d*e*h)*d*f*b+a^3*d^2*f^2*h)*(b*x+a)^2*f*a
rctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((-5/63*x*(-8/15*x^2*(3/5*h*x+g
)*d^2-16/5*x*c*(1/3*h*x+g)*d+c^2*(-8/5*h*x+g))*f^2-2/63*((-8/5*h*x^3-16/3*
g*x^2)*d^2+4*(-8/3*h*x+g)*x*c*d+c^2*(2*h*x+g))*e*f+8/315*d^2*e^2*h*x^2)*b^
4-2/63*a*((28/3*x^2*(9/35*h*x+g)*d^2-25*(-56/75*h*x+g)*x*c*d+3/2*c^2*(-25/
3*h*x+g))*f^2+((64/5*h*x^2-44/3*g*x)*d^2+2*c*(-44/3*h*x+g)*d+h*c^2)*e*f-8/
5*d^2*e^2*h*x)*b^3+5/21*a^2*(((56/25*h*x^2-35/9*g*x)*d^2+2*c*(-35/9*h*x+g)
*d+h*c^2)*f^2+20/9*d*((-273/125*h*x+1/2*g)*d+c*h)*e*f+8/75*d^2*e^2*h)*b^2-
10/9*a^3*(((-3/2*h*x+1/2*g)*d+c*h)*f+3/5*d*e*h)*d*f*b+a^4*d^2*f^2*h)*((a*f
-b*e)*b)^(1/2)*(f*x+e)^(1/2))/((a*f-b*e)*b)^(1/2)/(b*x+a)^2/b^5/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(392) = 784$.

Time = 0.23 (sec) , antiderivative size = 2866, normalized size of antiderivative = 6.79

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/120*(15*sqrt(b^2*e - a*b*f)*(((8*b^5*d^2*e^2*f + 8*(3*b^5*c*d - 5*a*b^
4*d^2)*e*f^2 + (3*b^5*c^2 - 30*a*b^4*c*d + 35*a^2*b^3*d^2)*f^3)*g + (8*(2*
b^5*c*d - 3*a*b^4*d^2)*e^2*f + 4*(3*b^5*c^2 - 20*a*b^4*c*d + 21*a^2*b^3*d^
2)*e*f^2 - (15*a*b^4*c^2 - 70*a^2*b^3*c*d + 63*a^3*b^2*d^2)*f^3)*h)*x^2 +
(8*a^2*b^3*d^2*e^2*f + 8*(3*a^2*b^3*c*d - 5*a^3*b^2*d^2)*e*f^2 + (3*a^2*b^
3*c^2 - 30*a^3*b^2*c*d + 35*a^4*b*d^2)*f^3)*g + (8*(2*a^2*b^3*c*d - 3*a^3*
b^2*d^2)*e^2*f + 4*(3*a^2*b^3*c^2 - 20*a^3*b^2*c*d + 21*a^4*b*d^2)*e*f^2 -
 (15*a^3*b^2*c^2 - 70*a^4*b*c*d + 63*a^5*d^2)*f^3)*h + 2*((8*a*b^4*d^2*e^2
*f + 8*(3*a*b^4*c*d - 5*a^2*b^3*d^2)*e*f^2 + (3*a*b^4*c^2 - 30*a^2*b^3*c*d
 + 35*a^3*b^2*d^2)*f^3)*g + (8*(2*a*b^4*c*d - 3*a^2*b^3*d^2)*e^2*f + 4*(3*
a*b^4*c^2 - 20*a^2*b^3*c*d + 21*a^3*b^2*d^2)*e*f^2 - (15*a^2*b^3*c^2 - 70*
a^3*b^2*c*d + 63*a^4*b*d^2)*f^3)*h)*x)*log((b*f*x + 2*b*e - a*f + 2*sqrt(b
^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*(24*(b^6*d^2*e*f^2 - a*b^5*d^2
*f^3)*h*x^4 + 8*(5*(b^6*d^2*e*f^2 - a*b^5*d^2*f^3)*g + (6*b^6*d^2*e^2*f +
5*(2*b^6*c*d - 3*a*b^5*d^2)*e*f^2 - (10*a*b^5*c*d - 9*a^2*b^4*d^2)*f^3)*h)
*x^3 + 8*(5*(4*b^6*d^2*e^2*f + (6*b^6*c*d - 11*a*b^5*d^2)*e*f^2 - (6*a*b^5
*c*d - 7*a^2*b^4*d^2)*f^3)*g + (3*b^6*d^2*e^3 + (40*b^6*c*d - 51*a*b^5*d^2
)*e^2*f + (15*b^6*c^2 - 110*a*b^5*c*d + 111*a^2*b^4*d^2)*e*f^2 - (15*a*b^5
*c^2 - 70*a^2*b^4*c*d + 63*a^3*b^3*d^2)*f^3)*h)*x^2 - 5*(2*(3*b^6*c^2 + 6*
a*b^5*c*d - 25*a^2*b^4*d^2)*e^2*f + (3*a*b^5*c^2 - 102*a^2*b^4*c*d + 15...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(392) = 784$.

Time = 0.17 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.46

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g + 24*b^3*c*d*e*f*g - 40*a*b^2*d^2*e*f*g + 3*b^3*c^2*f
^2*g - 30*a*b^2*c*d*f^2*g + 35*a^2*b*d^2*f^2*g + 16*b^3*c*d*e^2*h - 24*a*b
^2*d^2*e^2*h + 12*b^3*c^2*e*f*h - 80*a*b^2*c*d*e*f*h + 84*a^2*b*d^2*e*f*h
- 15*a*b^2*c^2*f^2*h + 70*a^2*b*c*d*f^2*h - 63*a^3*d^2*f^2*h)*arctan(sqrt(
f*x + e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^5) - 1/4*(8*(f*x
+ e)^(3/2)*b^4*c*d*e*f*g - 8*(f*x + e)^(3/2)*a*b^3*d^2*e*f*g - 8*sqrt(f*x
+ e)*b^4*c*d*e^2*f*g + 8*sqrt(f*x + e)*a*b^3*d^2*e^2*f*g + 5*(f*x + e)^(3/
2)*b^4*c^2*f^2*g - 18*(f*x + e)^(3/2)*a*b^3*c*d*f^2*g + 13*(f*x + e)^(3/2)
*a^2*b^2*d^2*f^2*g - 3*sqrt(f*x + e)*b^4*c^2*e*f^2*g + 22*sqrt(f*x + e)*a*
b^3*c*d*e*f^2*g - 19*sqrt(f*x + e)*a^2*b^2*d^2*e*f^2*g + 3*sqrt(f*x + e)*a
*b^3*c^2*f^3*g - 14*sqrt(f*x + e)*a^2*b^2*c*d*f^3*g + 11*sqrt(f*x + e)*a^3
*b*d^2*f^3*g + 4*(f*x + e)^(3/2)*b^4*c^2*e*f*h - 16*(f*x + e)^(3/2)*a*b^3*
c*d*e*f*h + 12*(f*x + e)^(3/2)*a^2*b^2*d^2*e*f*h - 4*sqrt(f*x + e)*b^4*c^2
*e^2*f*h + 16*sqrt(f*x + e)*a*b^3*c*d*e^2*f*h - 12*sqrt(f*x + e)*a^2*b^2*d
^2*e^2*f*h - 9*(f*x + e)^(3/2)*a*b^3*c^2*f^2*h + 26*(f*x + e)^(3/2)*a^2*b^
2*c*d*f^2*h - 17*(f*x + e)^(3/2)*a^3*b*d^2*f^2*h + 11*sqrt(f*x + e)*a*b^3*
c^2*e*f^2*h - 38*sqrt(f*x + e)*a^2*b^2*c*d*e*f^2*h + 27*sqrt(f*x + e)*a^3*
b*d^2*e*f^2*h - 7*sqrt(f*x + e)*a^2*b^2*c^2*f^3*h + 22*sqrt(f*x + e)*a^3*b
*c*d*f^3*h - 15*sqrt(f*x + e)*a^4*d^2*f^3*h)/(((f*x + e)*b - b*e + a*f)^2*
b^5) + 2/15*(5*(f*x + e)^(3/2)*b^12*d^2*f^5*g + 15*sqrt(f*x + e)*b^12*d...
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.03

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^3,x)
```

output
```
(e + f*x)^(3/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(3*b^3*f) - (2*d^2*h*
(a*f - b*e))/(b^4*f)) - (e + f*x)^(1/2)*((3*((2*d^2*f*g - 6*d^2*e*h + 4*c*
d*f*h)/(b^3*f) - (6*d^2*h*(a*f - b*e))/(b^4*f))*(a*f - b*e))/b - (2*(c*f -
 d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(b^3*f) + (6*d^2*h*(a*f - b*e)^2)/(b^5*
f)) - ((e + f*x)^(3/2)*((5*b^4*c^2*f^2*g)/4 + b^4*c^2*e*f*h - (9*a*b^3*c^2
*f^2*h)/4 - (17*a^3*b*d^2*f^2*h)/4 + (13*a^2*b^2*d^2*f^2*g)/4 + 2*b^4*c*d*
e*f*g - (9*a*b^3*c*d*f^2*g)/2 - 2*a*b^3*d^2*e*f*g + (13*a^2*b^2*c*d*f^2*h)
/2 + 3*a^2*b^2*d^2*e*f*h - 4*a*b^3*c*d*e*f*h) - (e + f*x)^(1/2)*((15*a^4*d
^2*f^3*h)/4 - (3*a*b^3*c^2*f^3*g)/4 - (11*a^3*b*d^2*f^3*g)/4 + (3*b^4*c^2*
e*f^2*g)/4 + b^4*c^2*e^2*f*h + (7*a^2*b^2*c^2*f^3*h)/4 + (19*a^2*b^2*d^2*e
*f^2*g)/4 + 3*a^2*b^2*d^2*e^2*f*h - (11*a^3*b*c*d*f^3*h)/2 + 2*b^4*c*d*e^2
*f*g + (7*a^2*b^2*c*d*f^3*g)/2 - (11*a*b^3*c^2*e*f^2*h)/4 - 2*a*b^3*d^2*e^
2*f*g - (27*a^3*b*d^2*e*f^2*h)/4 + (19*a^2*b^2*c*d*e*f^2*h)/2 - (11*a*b^3*
c*d*e*f^2*g)/2 - 4*a*b^3*c*d*e^2*f*h))/(b^7*(e + f*x)^2 - (e + f*x)*(2*b^7
*e - 2*a*b^6*f) + b^7*e^2 + a^2*b^5*f^2 - 2*a*b^6*e*f) + (atan((b^(1/2)*(e
 + f*x)^(1/2))/(a*f - b*e)^(1/2))*(3*b^3*c^2*f^2*g + 8*b^3*d^2*e^2*g - 63*
a^3*d^2*f^2*h + 16*b^3*c*d*e^2*h + 12*b^3*c^2*e*f*h - 15*a*b^2*c^2*f^2*h -
 24*a*b^2*d^2*e^2*h + 35*a^2*b*d^2*f^2*g + 24*b^3*c*d*e*f*g - 30*a*b^2*c*d
*f^2*g + 70*a^2*b*c*d*f^2*h - 40*a*b^2*d^2*e*f*g + 84*a^2*b*d^2*e*f*h - 80
*a*b^2*c*d*e*f*h))/(4*b^(11/2)*(a*f - b*e)^(1/2)) + (2*d^2*h*(e + f*x)^...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3464, normalized size of antiderivative = 8.21

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3,x)
```

output
```
( - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**5*d**2*f**3*h + 1050*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d*f**3*h + 1260*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*e*f**
2*h + 525*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**4*b*d**2*f**3*g - 1890*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*f**3*h*x - 225*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c
**2*f**3*h - 1200*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**3*b**2*c*d*e*f**2*h - 450*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*f**3*g + 2100
*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**3*b**2*c*d*f**3*h*x - 360*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*e**2*f*h - 600*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2
*e*f**2*g + 2520*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**3*b**2*d**2*e*f**2*h*x + 1050*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*f**3*g*x
- 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*b**2*d**2*f**3*h*x**2 + 180*sqrt(b)*sqrt(a*f - b*e)*atan((sq...
```

3.76 $\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4} dx$

Optimal result

Integrand size = 29, antiderivative size = 517

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = -\frac{2d(4adfh-b(dfg+deh+2cfh))\sqrt{e+fx}}{b^5}$$
$$-\frac{(bc-ad)\left(7a^2dfh+b^2(4deg+cfg+2ceh)-ab(5dfg+6deh+3cfh)\right)\sqrt{e+fx}}{4b^5(a+bx)^2}$$
$$+\frac{(55a^3d^2f^2h-a^2bdf(29dfg+78deh+58cfh)-b^3(8d^2e^2g+4cde(5fg+4eh)+c^2f(fg+10eh))+ab^2(}{8b^5(be-af)(a+bx)}$$
$$+\frac{2d^2h(e+fx)^{3/2}}{3b^4}-\frac{(bc-ad)^2(bg-ah)(e+fx)^{3/2}}{3b^4(a+bx)^3}$$
$$+\frac{(105a^3d^2f^3h-35a^2bdf^2(dfg+6deh+2cfh)+b^3(c^2f^2(fg-6eh)-8d^2e^2(3fg+2eh)-12cdef(fg+4}{8b^{11/2}(be-af)^{3/2}}$$

output
```
-2*d*(4*a*d*f*h-b*(2*c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)/b^5-1/4*(-a*d+b*c)*
(7*a^2*d*f*h+b^2*(2*c*e*h+c*f*g+4*d*e*g)-a*b*(3*c*f*h+6*d*e*h+5*d*f*g))*(f
*x+e)^(1/2)/b^5/(b*x+a)^2+1/8*(55*a^3*d^2*f^2*h-a^2*b*d*f*(58*c*f*h+78*d*e
*h+29*d*f*g)-b^3*(8*d^2*e^2*g+4*c*d*e*(4*e*h+5*f*g)+c^2*f*(10*e*h+f*g))+a*
b^2*(11*c^2*f^2*h+12*d^2*e*(2*e*h+3*f*g)+2*c*d*f*(36*e*h+11*f*g)))*(f*x+e)
^(1/2)/b^5/(-a*f+b*e)/(b*x+a)+2/3*d^2*h*(f*x+e)^(3/2)/b^4-1/3*(-a*d+b*c)^2
*(-a*h+b*g)*(f*x+e)^(3/2)/b^4/(b*x+a)^3+1/8*(105*a^3*d^2*f^3*h-35*a^2*b*d*
f^2*(2*c*f*h+6*d*e*h+d*f*g)+b^3*(c^2*f^2*(-6*e*h+f*g)-8*d^2*e^2*(2*e*h+3*f
*g)-12*c*d*e*f*(4*e*h+f*g))+5*a*b^2*f*(c^2*f^2*h+12*d^2*e*(2*e*h+f*g)+2*c*
d*f*(12*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(11/2
)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \frac{\sqrt{e+fx}(315a^5d^2f^2h-105a^4bdf(4deh+2cfh+df(g-8hx))-b^5}{}$$

$$-\frac{(-105a^3d^2f^3h+35a^2bdf^2(dfg+6deh+2cfh)+b^3(8d^2e^2(3fg+2eh)+12cdef(fg+4eh)+c^2f^2(-fg}{8b^{11/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^4,x]
```

output
```
(Sqrt[e + f*x]*(315*a^5*d^2*f^2*h - 105*a^4*b*d*f*(4*d*e*h + 2*c*f*h + d*f
*(g - 8*h*x)) - b^5*(12*c*d*e*x*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x)) - 8*
d^2*e*x^2*(2*f*x*(3*g + h*x) + e*(-3*g + 8*h*x)) + c^2*(3*f^2*g*x^2 + 4*e^
2*(2*g + 3*h*x) + 2*e*f*x*(7*g + 15*h*x))) - a*b^4*(2*c*d*(2*e*f*x*(11*g -
 126*h*x) + 4*e^2*(g + 6*h*x) + 3*f^2*x^2*(-11*g + 16*h*x)) + c^2*(4*e^2*h
 - 2*e*f*(g - 11*h*x) - f^2*x*(8*g + 33*h*x)) + 4*d^2*x*(6*e^2*(g - 11*h*x
) + 4*f^2*x^2*(3*g + h*x) + e*f*x*(-63*g + 52*h*x))) + a^3*b^2*(15*c^2*f^2
*h + 10*c*d*f*(3*f*g + 22*e*h - 56*f*h*x) + d^2*(108*e^2*h + 2*e*f*(55*g -
 567*h*x) + 7*f^2*x*(-40*g + 99*h*x))) + a^2*b^3*(c^2*f*(3*f*g - 8*e*h + 4
0*f*h*x) - 2*c*d*(8*e^2*h + e*f*(8*g - 298*h*x) + f^2*x*(-40*g + 231*h*x))
 + d^2*(2*e*f*x*(149*g - 477*h*x) + 3*f^2*x^2*(-77*g + 48*h*x) + e^2*(-8*g
 + 300*h*x)))))/(24*b^5*(b*e - a*f)*(a + b*x)^3) - ((-105*a^3*d^2*f^3*h +
35*a^2*b*d*f^2*(d*f*g + 6*d*e*h + 2*c*f*h) + b^3*(8*d^2*e^2*(3*f*g + 2*e*h
) + 12*c*d*e*f*(f*g + 4*e*h) + c^2*f^2*(-(f*g) + 6*e*h)) - 5*a*b^2*f*(c^2*
f^2*h + 12*d^2*e*(f*g + 2*e*h) + 2*c*d*f*(f*g + 12*e*h)))*ArcTan[(Sqrt[b]*
Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(8*b^(11/2)*(-(b*e) + a*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 162, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4} \, dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg-bcfg+6bceh-4adeh-5acfh+3d(bfg+2beh-3afh)x)}{2(a+bx)^3}dx}{3b(be-af)} - \frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg-bcfg+6bceh-4adeh-5acfh+3d(bfg+2beh-3afh)x)}{(a+bx)^3}dx}{6b(be-af)} - \frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$\downarrow$ 162

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-3a^3bdf(14cfh+36deh+7dfg)-a^2b^2(5c^2f^2h-6cdf(10eh+fg)-2d^2e(22eh+15fg))+bx(81a^3d^2f^2h-a^2bdf(70cfh+146d}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$\downarrow$ 60

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-3a^3bdf(14cfh+36deh+7dfg)-a^2b^2(5c^2f^2h-6cdf(10eh+fg)-2d^2e(22eh+15fg))+bx(81a^3d^2f^2h-a^2bdf(70cfh+146d}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$\downarrow$ 60

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-3a^3bdf(14cfh+36deh+7dfg)-a^2b^2(5c^2f^2h-6cdf(10eh+fg)-2d^2e(22eh+15fg))+bx(81a^3d^2f^2h-a^2bdf(70cfh+146d}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$\downarrow$ 73

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-3a^3bdf(14cfh+36deh+7dfg)-a^2b^2(5c^2f^2h-6cdf(10eh+fg)-2d^2e(22eh+15fg))+bx(81a^3d^2f^2h-a^2bdf(70cfh+146d}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(e+fx)^{5/2}\left(63a^4d^2f^2h-3a^3bdf(14cfh+36deh+7dfg)-a^2b^2\left(5c^2f^2h-6cdf(10eh+fg)-2d^2e(22eh+15fg)\right)+bx\left(81a^3d^2f^2h-a^2bdf(70cfh+146d\right.\right.}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{3b(a+bx)^3(be-af)}$$

input
```
Int[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^4,x]
```

output
```
-1/3*((b*g - a*h)*(c + d*x)^2*(e + f*x)^(5/2))/(b*(b*e - a*f)*(a + b*x)^3)
 + (((e + f*x)^(5/2)*(63*a^4*d^2*f^2*h - 2*b^4*c*e*(4*d*e*g - c*f*g + 6*c*
e*h) - 3*a^3*b*d*f*(7*d*f*g + 36*d*e*h + 14*c*f*h) - a*b^3*(8*d^2*e^2*g +
16*c*d*e^2*h + c^2*f*(f*g - 16*e*h)) - a^2*b^2*(5*c^2*f^2*h - 6*c*d*f*(f*g
 + 10*e*h) - 2*d^2*e*(15*f*g + 22*e*h)) + b*(81*a^3*d^2*f^2*h - a^2*b*d*f*
(27*d*f*g + 146*d*e*h + 70*c*f*h) - b^3*(16*d^2*e^2*g - c^2*f*(f*g - 6*e*h
) + 12*c*d*e*(f*g + 4*e*h)) + a*b^2*(5*c^2*f^2*h + 10*c*d*f*(f*g + 12*e*h)
 + 4*d^2*e*(11*f*g + 16*e*h)))*x))/(4*b^2*(b*e - a*f)^2*(a + b*x)^2) - (3*
(105*a^3*d^2*f^3*h - 35*a^2*b*d*f^2*(d*f*g + 6*d*e*h + 2*c*f*h) + b^3*(c^2
*f^2*(f*g - 6*e*h) - 8*d^2*e^2*(3*f*g + 2*e*h) - 12*c*d*e*f*(f*g + 4*e*h))
 + 5*a*b^2*f*(c^2*f^2*h + 12*d^2*e*(f*g + 2*e*h) + 2*c*d*f*(f*g + 12*e*h))
)*((2*(e + f*x)^(3/2))/(3*b) + ((b*e - a*f)*((2*Sqrt[e + f*x])/b - (2*Sqrt
[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/b^(3/2)))/b)
)/(8*b^2*(b*e - a*f)^2))/(6*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$\frac{105(bx+a)^3\left(\left(\left(\frac{c^2 g f^3}{105}-\frac{2ce(ch+2dg)f^2}{35}-\frac{16\left(ch+\frac{dg}{2}\right)d e^2 f}{35}-\frac{16d^2e^3h}{105}\right)b^3+\frac{a\left(\left(h c^2+2cdg\right)f^2+24\left(ch+\frac{dg}{2}\right)def+24d^2e^2h\right)f b^2}{21}\right.\right.}{8}$
risch	$-\frac{2d(-hxdbf+12adfh-6bcfh-4hedb-3bdfg)\sqrt{fx+e}}{3b^5}+\frac{-\frac{b^2 f\left(55a^3d^2f^2h-58a^2bcd f^2h-78a^2b d^2efh-29a^2b d^2f^2g+\right.}{}}{}$
derivativedivides	$-\frac{2d\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+4adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^5}+\frac{2\left(-\frac{b^2f\left(55a^3d^2f^2h-58a^2bcd f^2\right.}{}\right.}{}$
default	$-\frac{2d\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+4adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^5}+\frac{2\left(-\frac{b^2f\left(55a^3d^2f^2h-58a^2bcd f^2\right.}{}\right.}{}$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
105/8/((a*f-b*e)*b)^(1/2)*((b*x+a)^3*((1/105*c^2*g*f^3-2/35*c*e*(c*h+2*d*g
)*f^2-16/35*(c*h+1/2*d*g)*d*e^2*f-16/105*d^2*e^3*h)*b^3+1/21*a*((c^2*h+2*c
*d*g)*f^2+24*(c*h+1/2*d*g)*d*e*f+24*d^2*e^2*h)*f*b^2-2/3*a^2*d*((c*h+1/2*d
*g)*f+3*d*e*h)*f^2*b+a^3*d^2*f^3*h)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(
1/2))-((a*f-b*e)*b)^(1/2)*(f*x+e)^(1/2)*((-1/105*c^2*f^2*g*x^2-2/45*x*(-24
/7*x^2*(1/3*h*x+g)*d^2+30/7*x*(-8/5*h*x+g)*c*d+c^2*(15/7*h*x+g))*e*f-8/315
*((-8*h*x^3+3*g*x^2)*d^2+3*c*x*(2*h*x+g)*d+c^2*(3/2*h*x+g))*e^2)*b^5-4/315
*a*(-2*x*((-2*h*x^3-6*g*x^2)*d^2+33/4*x*c*(-16/11*h*x+g)*d+c^2*(33/8*h*x+g
))*f^2-1/2*((-104*h*x^3+126*g*x^2)*d^2-22*x*c*(-126/11*h*x+g)*d+c^2*(-11*h
*x+g))*e*f+((-66*h*x^2+6*g*x)*d^2+2*c*(6*h*x+g)*d+h*c^2)*e^2)*b^4-8/315*a^
2*(((-18*h*x^3+231/8*g*x^2)*d^2-10*x*c*(-231/40*h*x+g)*d-3/8*c^2*(40/3*h*x
+g))*f^2+((477/4*h*x^2-149/4*g*x)*d^2+2*c*(-149/4*h*x+g)*d+h*c^2)*e*f+2*d*
((-75/4*h*x+1/2*g)*d+c*h)*e^2)*b^3+1/21*a^3*((-56/3*(-99/40*h*x+g)*x*d^2+2
*c*(-56/3*h*x+g)*d+h*c^2)*f^2+44/3*d*((-567/110*h*x+1/2*g)*d+c*h)*e*f+36/5
*d^2*e^2*h)*b^2-2/3*a^4*d*(((-4*h*x+1/2*g)*d+c*h)*f+2*d*e*h)*f*b+a^5*d^2*f
^2*h))/(b*x+a)^3/b^5/(a*f-b*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(489) = 978$.

Time = 0.38 (sec) , antiderivative size = 3942, normalized size of antiderivative = 7.62

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1653 vs. $2(489) = 978$.

Time = 0.17 (sec) , antiderivative size = 1653, normalized size of antiderivative = 3.20

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*(24*b^3*d^2*e^2*f*g + 12*b^3*c*d*e*f^2*g - 60*a*b^2*d^2*e*f^2*g - b^3*
c^2*f^3*g - 10*a*b^2*c*d*f^3*g + 35*a^2*b*d^2*f^3*g + 16*b^3*d^2*e^3*h + 4
8*b^3*c*d*e^2*f*h - 120*a*b^2*d^2*e^2*f*h + 6*b^3*c^2*e*f^2*h - 120*a*b^2*
c*d*e*f^2*h + 210*a^2*b*d^2*e*f^2*h - 5*a*b^2*c^2*f^3*h + 70*a^2*b*c*d*f^3
*h - 105*a^3*d^2*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6
*e - a*b^5*f)*sqrt(-b^2*e + a*b*f)) - 1/24*(24*(f*x + e)^(5/2)*b^5*d^2*e^2
*f*g - 48*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + 24*sqrt(f*x + e)*b^5*d^2*e^4*f
*g + 60*(f*x + e)^(5/2)*b^5*c*d*e*f^2*g - 108*(f*x + e)^(5/2)*a*b^4*d^2*e*
f^2*g - 96*(f*x + e)^(3/2)*b^5*c*d*e^2*f^2*g + 240*(f*x + e)^(3/2)*a*b^4*d
^2*e^2*f^2*g + 36*sqrt(f*x + e)*b^5*c*d*e^3*f^2*g - 132*sqrt(f*x + e)*a*b^
4*d^2*e^3*f^2*g + 3*(f*x + e)^(5/2)*b^5*c^2*f^3*g - 66*(f*x + e)^(5/2)*a*b
^4*c*d*f^3*g + 87*(f*x + e)^(5/2)*a^2*b^3*d^2*f^3*g + 8*(f*x + e)^(3/2)*b^
5*c^2*e*f^3*g + 176*(f*x + e)^(3/2)*a*b^4*c*d*e*f^3*g - 328*(f*x + e)^(3/2
)*a^2*b^3*d^2*e*f^3*g - 3*sqrt(f*x + e)*b^5*c^2*e^2*f^3*g - 102*sqrt(f*x +
 e)*a*b^4*c*d*e^2*f^3*g + 249*sqrt(f*x + e)*a^2*b^3*d^2*e^2*f^3*g - 8*(f*x
 + e)^(3/2)*a*b^4*c^2*f^4*g - 80*(f*x + e)^(3/2)*a^2*b^3*c*d*f^4*g + 136*(
f*x + e)^(3/2)*a^3*b^2*d^2*f^4*g + 6*sqrt(f*x + e)*a*b^4*c^2*e*f^4*g + 96*
sqrt(f*x + e)*a^2*b^3*c*d*e*f^4*g - 198*sqrt(f*x + e)*a^3*b^2*d^2*e*f^4*g
- 3*sqrt(f*x + e)*a^2*b^3*c^2*f^5*g - 30*sqrt(f*x + e)*a^3*b^2*c*d*f^5*g +
 57*sqrt(f*x + e)*a^4*b*d^2*f^5*g + 48*(f*x + e)^(5/2)*b^5*c*d*e^2*f*h ...
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1109, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^4,x)
```

output
```
(e + f*x)^(1/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/b^4 + (2*d^2*h*(4*b^4
*e - 4*a*b^3*f))/b^8) - ((e + f*x)^(1/2)*((41*a^4*d^2*f^4*h)/8 + (a*b^3*c^
2*f^4*g)/8 - (19*a^3*b*d^2*f^4*g)/8 - (b^4*c^2*e*f^3*g)/8 + b^4*d^2*e^3*f*
g + (5*a^2*b^2*c^2*f^4*h)/8 + (3*b^4*c^2*e^2*f^2*h)/4 - (9*a*b^3*d^2*e^2*f
^2*g)/2 + (47*a^2*b^2*d^2*e*f^3*g)/8 - (19*a^3*b*c*d*f^4*h)/4 + 2*b^4*c*d*
e^3*f*h + (45*a^2*b^2*d^2*e^2*f^2*h)/4 + (5*a^2*b^2*c*d*f^4*g)/4 - (11*a*b
^3*c^2*e*f^3*h)/8 - 3*a*b^3*d^2*e^3*f*h - (107*a^3*b*d^2*e*f^3*h)/8 + (3*b
^4*c*d*e^2*f^2*g)/2 - 9*a*b^3*c*d*e^2*f^2*h + (47*a^2*b^2*c*d*e*f^3*h)/4 -
 (11*a*b^3*c*d*e*f^3*g)/4) + (e + f*x)^(3/2)*((b^4*c^2*f^3*g)/3 + (5*a*b^3
*c^2*f^3*h)/3 + (35*a^3*b*d^2*f^3*h)/3 - 2*b^4*c^2*e*f^2*h - 2*b^4*d^2*e^2
*f*g - (17*a^2*b^2*d^2*f^3*g)/3 - 18*a^2*b^2*d^2*e*f^2*h + (10*a*b^3*c*d*f
^3*g)/3 - 4*b^4*c*d*e*f^2*g - 4*b^4*c*d*e^2*f*h - (34*a^2*b^2*c*d*f^3*h)/3
 + 8*a*b^3*d^2*e*f^2*g + 6*a*b^3*d^2*e^2*f*h + 16*a*b^3*c*d*e*f^2*h) - ((e
 + f*x)^(5/2)*(b^5*c^2*f^3*g - 11*a*b^4*c^2*f^3*h + 10*b^5*c^2*e*f^2*h + 8
*b^5*d^2*e^2*f*g + 29*a^2*b^3*d^2*f^3*g - 55*a^3*b^2*d^2*f^3*h + 78*a^2*b^
3*d^2*e*f^2*h - 22*a*b^4*c*d*f^3*g + 20*b^5*c*d*e*f^2*g + 16*b^5*c*d*e^2*f
*h + 58*a^2*b^3*c*d*f^3*h - 36*a*b^4*d^2*e*f^2*g - 24*a*b^4*d^2*e^2*f*h -
72*a*b^4*c*d*e*f^2*h))/(8*(a*f - b*e)))/(b^8*(e + f*x)^3 - (e + f*x)^2*(3*
b^8*e - 3*a*b^7*f) + (e + f*x)*(3*b^8*e^2 + 3*a^2*b^6*f^2 - 6*a*b^7*e*f) -
 b^8*e^3 + a^3*b^5*f^3 - 3*a^2*b^6*e*f^2 + 3*a*b^7*e^2*f) + (2*d^2*h*(e...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4947, normalized size of antiderivative = 9.57

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4,x)
```

output
```
(315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**6*d**2*f**3*h - 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c*d*f**3*h - 630*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**2*e*f**2*h -
 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**5*b*d**2*f**3*g + 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**2*f**3*h*x + 15*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c**2*f**
3*h + 360*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**4*b**2*c*d*e*f**2*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*d*f**3*g - 630*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2
*c*d*f**3*h*x + 360*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**4*b**2*d**2*e**2*f*h + 180*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*e*f**2*g
 - 1890*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**4*b**2*d**2*e*f**2*h*x - 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*f**3*g*x + 945*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*
b**2*d**2*f**3*h*x**2 - 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*...
```

3.77 $\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 650

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \frac{2d^2fh\sqrt{e+fx}}{b^5}$$
$$-\frac{(bc-ad)\left(27a^2dfh+b^2(16deg+3cfg+8ceh)-ab(19dfg+24deh+11cfh)\right)\sqrt{e+fx}}{24b^5(a+bx)^3}$$
$$+\frac{(315a^3d^2f^2h-a^2bdf(163dfg+456deh+326cfh)-b^3(48d^2e^2g+16cde(7fg+6eh)+c^2f(3fg+56eh)}{96b^5(be-af)(a+bx)^2}$$
$$+\frac{(325a^3d^2f^3h-3a^2bdf^2(31dfg+232deh+62cfh)+b^3(c^2f^2(3fg-8eh)-16d^2e^2(5fg+4eh)-16cdef}{64b^5(be-af)^2(a+bx}$$
$$-\frac{(bc-ad)^2(bg-ah)(e+fx)^{3/2}}{4b^4(a+bx)^4}$$
$$+\frac{f(315a^3d^2f^3h-35a^2bdf^2(dfg+24deh+2cfh)-b^3(c^2f^2(3fg-8eh)-16cdef(fg-6eh)+48d^2e^2(fg}{64b^{11/2}(be-af)^{5/2}}$$

output
```
2*d^2*f*h*(f*x+e)^(1/2)/b^5-1/24*(-a*d+b*c)*(27*a^2*d*f*h+b^2*(8*c*e*h+3*c
*f*g+16*d*e*g)-a*b*(11*c*f*h+24*d*e*h+19*d*f*g))*(f*x+e)^(1/2)/b^5/(b*x+a)
^3+1/96*(315*a^3*d^2*f^2*h-a^2*b*d*f*(326*c*f*h+456*d*e*h+163*d*f*g)-b^3*(
48*d^2*e^2*g+16*c*d*e*(6*e*h+7*f*g)+c^2*f*(56*e*h+3*f*g))+a*b^2*(59*c^2*f^
2*h+16*d^2*e*(9*e*h+13*f*g)+2*c*d*f*(208*e*h+59*f*g)))*(f*x+e)^(1/2)/b^5/(
-a*f+b*e)/(b*x+a)^2+1/64*(325*a^3*d^2*f^3*h-3*a^2*b*d*f^2*(62*c*f*h+232*d*
e*h+31*d*f*g)+b^3*(c^2*f^2*(-8*e*h+3*f*g)-16*d^2*e^2*(4*e*h+5*f*g)-16*c*d*
e*f*(10*e*h+f*g))+a*b^2*f*(5*c^2*f^2*h+16*d^2*e*(27*e*h+11*f*g)+2*c*d*f*(1
76*e*h+5*f*g)))*(f*x+e)^(1/2)/b^5/(-a*f+b*e)^2/(b*x+a)-1/4*(-a*d+b*c)^2*(-
a*h+b*g)*(f*x+e)^(3/2)/b^4/(b*x+a)^4+1/64*f*(315*a^3*d^2*f^3*h-35*a^2*b*d*
f^2*(2*c*f*h+24*d*e*h+d*f*g)-b^3*(c^2*f^2*(-8*e*h+3*f*g)-16*c*d*e*f*(-6*e*
h+f*g)+48*d^2*e^2*(4*e*h+f*g))-5*a*b^2*f*(c^2*f^2*h+2*c*d*f*(-16*e*h+f*g)-
16*d^2*e*(9*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(
11/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 4.37 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx =$$

$$-\frac{\sqrt{e+fx}(-945a^6d^2f^3h+105a^5bdf^2(18deh+2cfh+df(g-33hx))+b^6(48d^2e^2x^2(fx(5g-8hx)+2e(g$$

$$+\frac{f(-315a^3d^2f^3h+35a^2bdf^2(dfg+24deh+2cfh)+b^3(c^2f^2(3fg-8eh)+48d^2e^2(fg+4eh)+16cdef(-}{64b^{11/2}(-be+af)}$$

input
```
Integrate[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^5,x]
```

output
```
-1/192*(Sqrt[e + f*x]*(-945*a^6*d^2*f^3*h + 105*a^5*b*d*f^2*(18*d*e*h + 2*
c*f*h + d*f*(g - 33*h*x)) + b^6*(48*d^2*e^2*x^2*(f*x*(5*g - 8*h*x) + 2*e*(
g + 2*h*x)) + c^2*(-9*f^3*g*x^3 + 6*e*f^2*x^2*(g + 4*h*x) + 16*e^3*(3*g +
4*h*x) + 8*e^2*f*x*(9*g + 14*h*x)) + 16*c*d*e*x*(3*f^2*g*x^2 + 4*e^2*(2*g
+ 3*h*x) + 2*e*f*x*(7*g + 15*h*x))) + a*b^5*(16*d^2*e*x*(e*f*x*(13*g - 177
*h*x) + 2*e^2*(2*g + 9*h*x) + 3*f^2*x^2*(-11*g + 16*h*x)) + c^2*(16*e^3*h
- 8*e^2*f*(9*g + 5*h*x) - 3*f^3*x^2*(11*g + 5*h*x) - 2*e*f^2*x*(66*g + 79*
h*x)) + 2*c*d*(-15*f^3*g*x^3 + 16*e^3*(g + 4*h*x) + 8*e^2*f*x*(-5*g + 26*h
*x) - 2*e*f^2*x^2*(79*g + 264*h*x))) + a^2*b^4*(c^2*f*(-8*e^2*h + e*f*(6*g
 - 52*h*x) + f^2*x*(33*g + 73*h*x)) + 2*c*d*(16*e^3*h - 8*e^2*f*(g - 19*h*
x) + f^3*x^2*(73*g + 279*h*x) - 2*e*f^2*x*(26*g + 421*h*x)) + d^2*(8*e^2*f
*x*(19*g - 624*h*x) + 3*f^3*x^3*(93*g - 128*h*x) + 16*e^3*(g + 12*h*x) + 2
*e*f^2*x^2*(-421*g + 2580*h*x))) + a^3*b^3*(c^2*f^2*(9*f*g - 14*e*h + 55*f
*h*x) + 2*c*d*f*(40*e^2*h - 2*e*f*(7*g + 314*h*x) + f^2*x*(55*g + 511*h*x)
) + d^2*(48*e^3*h + f^3*x^2*(511*g - 2511*h*x) + 8*e^2*f*(5*g - 459*h*x) +
 2*e*f^2*x*(-314*g + 4665*h*x))) + a^4*b^2*f*(15*c^2*f^2*h + 10*c*d*f*(3*f
*g - 34*e*h + 77*f*h*x) + d^2*(-984*e^2*h + 7*f^2*x*(55*g - 657*h*x) + e*f
*(-170*g + 6972*h*x)))))/(b^5*(b*e - a*f)^2*(a + b*x)^4) + (f*(-315*a^3*d^
2*f^3*h + 35*a^2*b*d*f^2*(d*f*g + 24*d*e*h + 2*c*f*h) + b^3*(c^2*f^2*(3*f*
g - 8*e*h) + 48*d^2*e^2*(f*g + 4*e*h) + 16*c*d*e*f*(-(f*g) + 6*e*h)) - ...
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 162, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg-3bcfg+8bceh-4adeh-5acfh+d(bfg+8beh-9afh)x)}{2(a+bx)^4}dx}{4b(be-af)}-\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(c+dx)(e+fx)^{3/2}(4bdeg-3bcfg+8bceh-4adeh-5acfh+d(bfg+8beh-9afh)x)}{(a+bx)^4}dx}{8b(be-af)} - \frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \text{ 162}$$

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-a^3bdf(14cfh+132deh+7dfg)-a^2b^2(25c^2f^2h+2cdf(fg-12eh)-12d^2e(6eh+fg))+bx(99a^3d^2f^2h-a^2bdf(70cfh+216de}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-a^3bdf(14cfh+132deh+7dfg)-a^2b^2(25c^2f^2h+2cdf(fg-12eh)-12d^2e(6eh+fg))+bx(99a^3d^2f^2h-a^2bdf(70cfh+216de}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-a^3bdf(14cfh+132deh+7dfg)-a^2b^2(25c^2f^2h+2cdf(fg-12eh)-12d^2e(6eh+fg))+bx(99a^3d^2f^2h-a^2bdf(70cfh+216de}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-a^3bdf(14cfh+132deh+7dfg)-a^2b^2(25c^2f^2h+2cdf(fg-12eh)-12d^2e(6eh+fg))+bx(99a^3d^2f^2h-a^2bdf(70cfh+216de}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$\downarrow$ 221

$$\frac{(e+fx)^{5/2}(63a^4d^2f^2h-a^3bdf(14cfh+132deh+7dfg)-a^2b^2(25c^2f^2h+2cdf(fg-12eh)-12d^2e(6eh+fg))+bx(99a^3d^2f^2h-a^2bdf(70cfh+216de}{}$$

$$\frac{(c+dx)^2(e+fx)^{5/2}(bg-ah)}{4b(a+bx)^4(be-af)}$$

input
```
Int[((c + d*x)^2*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^5,x]
```

output
```
-1/4*((b*g - a*h)*(c + d*x)^2*(e + f*x)^(5/2))/(b*(b*e - a*f)*(a + b*x)^4)
 + (((e + f*x)^(5/2)*(63*a^4*d^2*f^2*h - 4*b^4*c*e*(4*d*e*g - 3*c*f*g + 8*
c*e*h) - a^3*b*d*f*(7*d*f*g + 132*d*e*h + 14*c*f*h) - a*b^3*(8*d^2*e^2*g +
 15*c^2*f*(f*g - 4*e*h) - 8*c*d*e*(3*f*g - 2*e*h)) - a^2*b^2*(25*c^2*f^2*h
 + 2*c*d*f*(f*g - 12*e*h) - 12*d^2*e*(f*g + 6*e*h)) + b*(99*a^3*d^2*f^2*h
- a^2*b*d*f*(11*d*f*g + 216*d*e*h + 70*c*f*h) - b^3*(24*d^2*e^2*g + c^2*f*
(3*f*g - 8*e*h) - 16*c*d*e*(f*g - 6*e*h)) - a*b^2*(5*c^2*f^2*h + 10*c*d*f*
(f*g - 16*e*h) - 8*d^2*e*(4*f*g + 15*e*h)))*x))/(12*b^2*(b*e - a*f)^2*(a +
 b*x)^3) - ((315*a^3*d^2*f^3*h - 35*a^2*b*d*f^2*(d*f*g + 24*d*e*h + 2*c*f*
h) - b^3*(c^2*f^2*(3*f*g - 8*e*h) - 16*c*d*e*f*(f*g - 6*e*h) + 48*d^2*e^2*
(f*g + 4*e*h)) - 5*a*b^2*f*(c^2*f^2*h + 2*c*d*f*(f*g - 16*e*h) - 16*d^2*e*
(f*g + 9*e*h)))*(-((e + f*x)^(3/2)/(b*(a + b*x))) + (3*f*((2*Sqrt[e + f*x]
)/b - (2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])
/b^(3/2)))/(2*b)))/(24*b^2*(b*e - a*f)^2))/(8*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 896, normalized size of antiderivative = 1.38

method	result	size
pseudoelliptic	Expression too large to display	896
derivativedivides	Expression too large to display	1240
default	Expression too large to display	1240
risch	Expression too large to display	1240

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-315/64/((a*f-b*e)*b)^(1/2)*((1/105*(-c^2*g*f^3+8/3*c*e*(c*h+2*d*g)*f^2-32
*(c*h+1/2*d*g)*d*e^2*f-64*d^2*e^3*h)*b^3-1/63*a*((c^2*h+2*c*d*g)*f^2-32*(c
*h+1/2*d*g)*d*e*f-144*d^2*e^2*h)*f*b^2-2/9*a^2*d*((c*h+1/2*d*g)*f+12*d*e*h
)*f^2*b+a^3*d^2*f^3*h)*(b*x+a)^4*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1
/2))-((a*f-b*e)*b)^(1/2)*(f*x+e)^(1/2)*(1/105*(c^2*f^3*g*x^3-2/3*x^2*c*(8*
d*g*x+c*(4*h*x+g))*e*f^2-8*x*(10/3*x^2*(-8/5*h*x+g)*d^2+28/9*x*c*(15/7*h*x
+g)*d+c^2*(14/9*h*x+g))*e^2*f-16/3*e^3*(2*(2*h*x^3+g*x^2)*d^2+8/3*x*c*(3/2
*h*x+g)*d+c^2*(4/3*h*x+g)))*b^6-16/945*a*(-33/16*(10/11*d*g*x+c*(5/11*h*x+
g))*x^2*c*f^3-33/4*(4*(-16/11*h*x^3+g*x^2)*d^2+79/33*x*c*(264/79*h*x+g)*d+
c^2*(79/66*h*x+g))*x*e*f^2-9/2*(2/3*(59*h*x^3-13/3*g*x^2)*d^2+10/9*x*c*(-2
6/5*h*x+g)*d+c^2*(5/9*h*x+g))*e^2*f+(2*(9*h*x^2+2*g*x)*d^2+2*c*(4*h*x+g)*d
+h*c^2)*e^3)*b^5+8/945*a^2*(-33/8*x*(93/11*x^2*(-128/93*h*x+g)*d^2+146/33*
x*c*(279/73*h*x+g)*d+c^2*(73/33*h*x+g))*f^3-3/4*((860*h*x^3-421/3*g*x^2)*d
^2-52/3*x*c*(421/26*h*x+g)*d+c^2*(-26/3*h*x+g))*e*f^2+((624*h*x^2-19*g*x)*
d^2+2*c*(-19*h*x+g)*d+h*c^2)*e^2*f-4*((6*h*x+1/2*g)*d+c*h)*d*e^3)*b^4+2/13
5*a^3*((1/2*(2511/7*h*x^3-73*g*x^2)*d^2-55/7*x*c*(511/55*h*x+g)*d-9/14*c^2
*(55/9*h*x+g))*f^3+(1/7*(-4665*h*x^2+314*g*x)*d^2+2*c*(314/7*h*x+g)*d+h*c^
2)*e*f^2-40/7*d*(1/2*(-459/5*h*x+g)*d+c*h)*e^2*f-24/7*d^2*e^3*h)*b^3-1/63*
a^4*f*((77/3*x*(-657/55*h*x+g)*d^2+2*(77/3*h*x+g)*c*d+h*c^2)*f^2-68/3*((-1
743/85*h*x+1/2*g)*d+c*h)*d*e*f-328/5*d^2*e^2*h)*b^2-2/9*((1/2*(-33*h*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2698 vs. $2(622) = 1244$.

Time = 0.57 (sec) , antiderivative size = 5410, normalized size of antiderivative = 8.32

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2444 vs. $2(622) = 1244$.

Time $= 0.19$ (sec) , antiderivative size $= 2444$, normalized size of antiderivative $= 3.76$

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x, algorithm="giac")
```

output
```
1/64*(48*b^3*d^2*e^2*f^2*g - 16*b^3*c*d*e*f^3*g - 80*a*b^2*d^2*e*f^3*g + 3
*b^3*c^2*f^4*g + 10*a*b^2*c*d*f^4*g + 35*a^2*b*d^2*f^4*g + 192*b^3*d^2*e^3
*f*h + 96*b^3*c*d*e^2*f^2*h - 720*a*b^2*d^2*e^2*f^2*h - 8*b^3*c^2*e*f^3*h
- 160*a*b^2*c*d*e*f^3*h + 840*a^2*b*d^2*e*f^3*h + 5*a*b^2*c^2*f^4*h + 70*a
^2*b*c*d*f^4*h - 315*a^3*d^2*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a
*b*f))/((b^7*e^2 - 2*a*b^6*e*f + a^2*b^5*f^2)*sqrt(-b^2*e + a*b*f)) + 2*sq
rt(f*x + e)*d^2*f*h/b^5 - 1/192*(240*(f*x + e)^(7/2)*b^6*d^2*e^2*f^2*g - 6
24*(f*x + e)^(5/2)*b^6*d^2*e^3*f^2*g + 528*(f*x + e)^(3/2)*b^6*d^2*e^4*f^2
*g - 144*sqrt(f*x + e)*b^6*d^2*e^5*f^2*g + 48*(f*x + e)^(7/2)*b^6*c*d*e*f^
3*g - 528*(f*x + e)^(7/2)*a*b^5*d^2*e*f^3*g + 80*(f*x + e)^(5/2)*b^6*c*d*e
^2*f^3*g + 1792*(f*x + e)^(5/2)*a*b^5*d^2*e^2*f^3*g - 176*(f*x + e)^(3/2)*
b^6*c*d*e^3*f^3*g - 1936*(f*x + e)^(3/2)*a*b^5*d^2*e^3*f^3*g + 48*sqrt(f*x
 + e)*b^6*c*d*e^4*f^3*g + 672*sqrt(f*x + e)*a*b^5*d^2*e^4*f^3*g - 9*(f*x +
 e)^(7/2)*b^6*c^2*f^4*g - 30*(f*x + e)^(7/2)*a*b^5*c*d*f^4*g + 279*(f*x +
e)^(7/2)*a^2*b^4*d^2*f^4*g + 33*(f*x + e)^(5/2)*b^6*c^2*e*f^4*g - 226*(f*x
 + e)^(5/2)*a*b^5*c*d*e*f^4*g - 1679*(f*x + e)^(5/2)*a^2*b^4*d^2*e*f^4*g +
 33*(f*x + e)^(3/2)*b^6*c^2*e^2*f^4*g + 462*(f*x + e)^(3/2)*a*b^5*c*d*e^2*
f^4*g + 2673*(f*x + e)^(3/2)*a^2*b^4*d^2*e^2*f^4*g - 9*sqrt(f*x + e)*b^6*c
^2*e^3*f^4*g - 174*sqrt(f*x + e)*a*b^5*c*d*e^3*f^4*g - 1257*sqrt(f*x + e)*
a^2*b^4*d^2*e^3*f^4*g - 33*(f*x + e)^(5/2)*a*b^5*c^2*f^5*g + 146*(f*x +...
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 1797, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^2)/(a + b*x)^5,x)
```

output
```
(2*d^2*f*h*(e + f*x)^(1/2))/b^5 - ((e + f*x)^(3/2)*((11*b^4*c^2*f^4*g)/64
+ (55*a*b^3*c^2*f^4*h)/192 - (643*a^3*b*d^2*f^4*h)/64 - (11*b^4*c^2*e*f^3*
h)/24 + 3*b^4*d^2*e^3*f*h + (385*a^2*b^2*d^2*f^4*g)/192 + (11*b^4*d^2*e^2*
f^2*g)/4 - (69*a*b^3*d^2*e^2*f^2*h)/4 + (193*a^2*b^2*d^2*e*f^3*h)/8 + (55*
a*b^3*c*d*f^4*g)/96 - (11*b^4*c*d*e*f^3*g)/12 + (385*a^2*b^2*c*d*f^4*h)/96
 - (55*a*b^3*d^2*e*f^3*g)/12 + (11*b^4*c*d*e^2*f^2*h)/2 - (55*a*b^3*c*d*e*
f^3*h)/6) + (e + f*x)^(1/2)*((3*a*b^3*c^2*f^5*g)/64 - (187*a^4*d^2*f^5*h)/
64 + (35*a^3*b*d^2*f^5*g)/64 - (3*b^4*c^2*e*f^4*g)/64 - b^4*d^2*e^4*f*h +
(5*a^2*b^2*c^2*f^5*h)/64 + (b^4*c^2*e^2*f^3*h)/8 - (3*b^4*d^2*e^3*f^2*g)/4
 + 2*a*b^3*d^2*e^2*f^3*g - (115*a^2*b^2*d^2*e*f^4*g)/64 + (25*a*b^3*d^2*e^
3*f^2*h)/4 + (35*a^3*b*c*d*f^5*h)/32 - (99*a^2*b^2*d^2*e^2*f^3*h)/8 + (5*a
^2*b^2*c*d*f^5*g)/32 - (13*a*b^3*c^2*e*f^4*h)/64 + (643*a^3*b*d^2*e*f^4*h)
/64 + (b^4*c*d*e^2*f^3*g)/4 - (3*b^4*c*d*e^3*f^2*h)/2 + 4*a*b^3*c*d*e^2*f^
3*h - (115*a^2*b^2*c*d*e*f^4*h)/32 - (13*a*b^3*c*d*e*f^4*g)/32) + ((e + f*
x)^(7/2)*(8*b^6*c^2*e*f^3*h - 5*a*b^5*c^2*f^4*h - 3*b^6*c^2*f^4*g + 64*b^6
*d^2*e^3*f*h + 93*a^2*b^4*d^2*f^4*g - 325*a^3*b^3*d^2*f^4*h + 80*b^6*d^2*e
^2*f^2*g - 432*a*b^5*d^2*e^2*f^2*h + 696*a^2*b^4*d^2*e*f^3*h - 10*a*b^5*c*
d*f^4*g + 16*b^6*c*d*e*f^3*g + 186*a^2*b^4*c*d*f^4*h - 176*a*b^5*d^2*e*f^3
*g + 160*b^6*c*d*e^2*f^2*h - 352*a*b^5*c*d*e*f^3*h))/(64*(a*f - b*e)^2) +
((e + f*x)^(5/2)*(73*a*b^4*c^2*f^4*h - 33*b^5*c^2*f^4*g - 40*b^5*c^2*e*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6644, normalized size of antiderivative = 10.22

$$\int \frac{(c+dx)^2(e+fx)^{3/2}(g+hx)}{(a+bx)^5}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5,x)
```

output
```
( - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**7*d**2*f**4*h + 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d*f**4*h + 2520*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*e*f**3
*h + 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**6*b*d**2*f**4*g - 3780*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*f**4*h*x + 15*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**
2*f**4*h - 480*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**5*b**2*c*d*e*f**3*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d*f**4*g + 840*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5
*b**2*c*d*f**4*h*x - 2160*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*e**2*f**2*h - 240*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*
e*f**3*g + 10080*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**5*b**2*d**2*e*f**3*h*x + 420*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*f**4*g*x -
 5670*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**5*b**2*d**2*f**4*h*x**2 - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqr...
```

3.78 $\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 469

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = -\frac{2(bc-ad)^3(de-cf)(dg-ch)\sqrt{e+fx}}{d^6} - \frac{2(bc-ad)^3(dg-ch)(e+fx)^{3/2}}{3d^5}$$
$$+\frac{2(a^3d^3f^3h+3a^2bd^2f^2(dfg-deh-cfh)+3ab^2df(c^2f^2h-d^2e(fg-eh)-cdf(fg-eh))-b^3(c^3f^3h-}{5d^4f^4}$$
$$+\frac{2b(3a^2d^2f^2h+3abdf(dfg-2deh-cfh)+b^2(c^2f^2h-d^2e(2fg-3eh)-cdf(fg-2eh)))(e+fx)^{7/2}}{7d^3f^4}$$
$$+\frac{2b^2(3adfh+b(dfg-3deh-cfh))(e+fx)^{9/2}}{9d^2f^4}+\frac{2b^3h(e+fx)^{11/2}}{11df^4}$$
$$+\frac{2(bc-ad)^3(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{13/2}}$$

output
```
-2*(-a*d+b*c)^3*(-c*f+d*e)*(-c*h+d*g)*(f*x+e)^(1/2)/d^6-2/3*(-a*d+b*c)^3*(
-c*h+d*g)*(f*x+e)^(3/2)/d^5+2/5*(a^3*d^3*f^3*h+3*a^2*b*d^2*f^2*(-c*f*h-d*e
*h+d*f*g)+3*a*b^2*d*f*(c^2*f^2*h-d^2*e*(-e*h+f*g)-c*d*f*(-e*h+f*g))-b^3*(c
^3*f^3*h-d^3*e^2*(-e*h+f*g)-c*d^2*e*f*(-e*h+f*g)-c^2*d*f^2*(-e*h+f*g)))*(f
*x+e)^(5/2)/d^4/f^4+2/7*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(-c*f*h-2*d*e*h+d*f*g
)+b^2*(c^2*f^2*h-d^2*e*(-3*e*h+2*f*g)-c*d*f*(-2*e*h+f*g)))*(f*x+e)^(7/2)/d
^3/f^4+2/9*b^2*(3*a*d*f*h+b*(-c*f*h-3*d*e*h+d*f*g))*(f*x+e)^(9/2)/d^2/f^4+
2/11*b^3*h*(f*x+e)^(11/2)/d/f^4+2*(-a*d+b*c)^3*(-c*f+d*e)^(3/2)*(-c*h+d*g)
*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(13/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2\sqrt{e+fx}(231a^3d^3f^3(15c^2f^2h-5cdf(3fg+4eh+fhx)+d^2(3e^2h \cdots}{\cdots} + \frac{2(-bc+ad)^3(-de+cf)^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{13/2}}$$

input
```
Integrate[((a + b*x)^3*(e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(231*a^3*d^3*f^3*(15*c^2*f^2*h - 5*c*d*f*(3*f*g + 4*e*h +
 f*h*x) + d^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))) + 99*a
^2*b*d^2*f^2*(-105*c^3*f^3*h - 3*d^3*(e + f*x)^2*(-7*f*g + 2*e*h - 5*f*h*x
) + 35*c^2*d*f^2*(3*f*g + 4*e*h + f*h*x) - 7*c*d^2*f*(3*e^2*h + f^2*x*(5*g
 + 3*h*x) + e*f*(20*g + 6*h*x))) + 33*a*b^2*d*f*(315*c^4*f^4*h - 105*c^3*d
*f^3*(3*f*g + 4*e*h + f*h*x) - 9*c*d^3*f*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*
h*x) + 21*c^2*d^2*f^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))
 + d^4*(e + f*x)^2*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)
)) + b^3*(-3465*c^5*f^5*h + 1155*c^4*d*f^4*(3*f*g + 4*e*h + f*h*x) + 99*c^
2*d^3*f^2*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h*x) - 231*c^3*d^2*f^3*(3*e^2*h
 + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) - 11*c*d^4*f*(e + f*x)^2*(8*e
^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)) - d^5*(e + f*x)^2*(48
*e^3*h - 35*f^3*x^2*(11*g + 9*h*x) - 8*e^2*f*(11*g + 15*h*x) + 10*e*f^2*x*
(22*g + 21*h*x)))))/(3465*d^6*f^4) + (2*(-(b*c) + a*d)^3*(-(d*e) + c*f)^(3
/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(13/
2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}dx$$

$$\Big\downarrow \ 170$$

$$\frac{2\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{2(c+dx)}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \ 27$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(6bceh-af(11dg-5ch)-(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}+\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^3(e+fx)^{5/2}}{11df}-\frac{\int -\frac{(a+bx)^2(e+fx)^{3/2}(11adfg-6bceh-5acfh+(6adfh+b(11dfg-6deh-11cfh))x)}{c+dx}dx}{11df}$$

input
```
Int[((a + b*x)^3*(e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.65

method	result
pseudoelliptic	$\frac{-2 f^{4} (c f-d e)^{2} (a d-b c)^{3} (c h-d g) \arctan\left(\frac{d \sqrt{f x+e}}{\sqrt{(c f-d e) d}}\right)+2 \sqrt{(c f-d e) d}\, \sqrt{f x+e} \left(\left(\frac{x \left(\frac{x^{3} \left(\frac{9 h x}{11}+g\right) b^{3}}{3}+\frac{9 a x^{2} \left(\frac{7 h x}{9}+g\right) b^{2}}{7}\right.}{3}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/((c*f-d*e)*d)^(1/2)*(-f^4*(c*f-d*e)^2*(a*d-b*c)^3*(c*h-d*g)*arctan(d*(f*
x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((1/3*x*
(1/3*x^3*(9/11*h*x+g)*b^3+9/7*a*x^2*(7/9*h*x+g)*b^2+9/5*a^2*x*(5/7*h*x+g)*
b+a^3*(3/5*h*x+g))*d^5-c*(1/7*x^3*(7/9*h*x+g)*b^3+3/5*a*x^2*(5/7*h*x+g)*b^
2+a^2*x*(3/5*h*x+g)*b+a^3*(1/3*h*x+g))*d^4+c^2*(1/5*x^2*(5/7*h*x+g)*b^3+a*
x*(3/5*h*x+g)*b^2+3*a^2*(1/3*h*x+g)*b+h*a^3)*d^3-3*c^3*b*(1/9*x*(3/5*h*x+g
)*b^2+a*(1/3*h*x+g)*b+a^2*h)*d^2+3*c^4*((1/9*h*x+1/3*g)*b+a*h)*b^2*d-c^5*h
*b^3)*f^5-4/3*d*(((-1/11*h*x^4-5/42*g*x^3)*b^3-18/35*a*x^2*(25/36*h*x+g)*b
^2-9/10*a^2*x*(4/7*h*x+g)*b-a^3*(3/10*h*x+g))*d^4+c*((5/42*h*x^3+6/35*g*x^
2)*b^3+9/10*a*x*(4/7*h*x+g)*b^2+3*a^2*(3/10*h*x+g)*b+h*a^3)*d^3-3*c^2*(1/1
0*x*(4/7*h*x+g)*b^2+a*(3/10*h*x+g)*b+a^2*h)*b*d^2+3*c^3*((1/10*h*x+1/3*g)*
b+a*h)*b^2*d-c^4*h*b^3)*e*f^4+1/5*d^2*e^2*((1/21*x^2*(5/11*h*x+g)*b^3+3/7*
a*x*(1/3*h*x+g)*b^2+3*a^2*(1/7*h*x+g)*b+h*a^3)*d^3-3*c*(1/21*x*(1/3*h*x+g)
*b^2+a*(1/7*h*x+g)*b+a^2*h)*b*d^2+3*c^2*((1/21*h*x+1/3*g)*b+a*h)*b^2*d-b^3
*c^3*h)*f^3-6/35*d^3*((2/27*x*(9/22*h*x+g)*b^2+a*(2/9*h*x+g)*b+a^2*h)*d^2-
c*b*((2/27*h*x+1/3*g)*b+a*h)*d+1/3*b^2*c^2*h)*b*e^3*f^2+8/105*(((1/11*h*x+
1/3*g)*b+a*h)*d-1/3*b*c*h)*d^4*b^2*e^4*f-16/1155*b^3*d^5*e^5*h))/f^4/d^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(437) = 874$.

Time = 0.13 (sec) , antiderivative size = 2737, normalized size of antiderivative = 5.84

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/3465*(3465*(((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)
*e*f^4 - (b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*f^5)*
g - ((b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*e*f^4 - (
b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*f^5)*h)*sqrt((d*e
 - c*f)/d)*log((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d
))/(d*x + c)) - 2*(315*b^3*d^5*f^5*h*x^5 + 35*(11*b^3*d^5*f^5*g + (12*b^3*
d^5*e*f^4 - 11*(b^3*c*d^4 - 3*a*b^2*d^5)*f^5)*h)*x^4 + 5*(11*(10*b^3*d^5*e
*f^4 - 9*(b^3*c*d^4 - 3*a*b^2*d^5)*f^5)*g + (3*b^3*d^5*e^2*f^3 - 110*(b^3*
c*d^4 - 3*a*b^2*d^5)*e*f^4 + 99*(b^3*c^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5
)*f^5)*h)*x^3 + 3*(11*(b^3*d^5*e^2*f^3 - 24*(b^3*c*d^4 - 3*a*b^2*d^5)*e*f^
4 + 21*(b^3*c^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*f^5)*g - (6*b^3*d^5*e^3
*f^2 + 11*(b^3*c*d^4 - 3*a*b^2*d^5)*e^2*f^3 - 264*(b^3*c^2*d^3 - 3*a*b^2*c
*d^4 + 3*a^2*b*d^5)*e*f^4 + 231*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c
*d^4 - a^3*d^5)*f^5)*h)*x^2 + 11*(8*b^3*d^5*e^4*f + 18*(b^3*c*d^4 - 3*a*b^
2*d^5)*e^3*f^2 + 63*(b^3*c^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*e^2*f^3 -
420*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*e*f^4 + 315*
(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*f^5)*g - (48*b
^3*d^5*e^5 + 88*(b^3*c*d^4 - 3*a*b^2*d^5)*e^4*f + 198*(b^3*c^2*d^3 - 3*a*b
^2*c*d^4 + 3*a^2*b*d^5)*e^3*f^2 + 693*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a
^2*b*c*d^4 - a^3*d^5)*e^2*f^3 - 4620*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(459) = 918$.

Time = 15.73 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(f*x+e)**(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(b**3*h*(e + f*x)**(11/2)/(11*d*f**3) + (e + f*x)**(9/2)*(3*a
*b**2*d*f*h - b**3*c*f*h - 3*b**3*d*e*h + b**3*d*f*g)/(9*d**2*f**3) + (e +
 f*x)**(7/2)*(3*a**2*b*d**2*f**2*h - 3*a*b**2*c*d*f**2*h - 6*a*b**2*d**2*e
*f*h + 3*a*b**2*d**2*f**2*g + b**3*c**2*f**2*h + 2*b**3*c*d*e*f*h - b**3*c
*d*f**2*g + 3*b**3*d**2*e**2*h - 2*b**3*d**2*e*f*g)/(7*d**3*f**3) + (e + f
*x)**(5/2)*(a**3*d**3*f**3*h - 3*a**2*b*c*d**2*f**3*h - 3*a**2*b*d**3*e*f*
*2*h + 3*a**2*b*d**3*f**3*g + 3*a*b**2*c**2*d*f**3*h + 3*a*b**2*c*d**2*e*f
**2*h - 3*a*b**2*c*d**2*f**3*g + 3*a*b**2*d**3*e**2*f*h - 3*a*b**2*d**3*e*
f**2*g - b**3*c**3*f**3*h - b**3*c**2*d*e*f**2*h + b**3*c**2*d*f**3*g - b*
*3*c*d**2*e**2*f*h + b**3*c*d**2*e*f**2*g - b**3*d**3*e**3*h + b**3*d**3*e
**2*f*g)/(5*d**4*f**3) + (e + f*x)**(3/2)*(-a**3*c*d**3*f*h + a**3*d**4*f*
g + 3*a**2*b*c**2*d**2*f*h - 3*a**2*b*c*d**3*f*g - 3*a*b**2*c**3*d*f*h + 3
*a*b**2*c**2*d**2*f*g + b**3*c**4*f*h - b**3*c**3*d*f*g)/(3*d**5) + sqrt(e
 + f*x)*(a**3*c**2*d**3*f**2*h - a**3*c*d**4*e*f*h - a**3*c*d**4*f**2*g +
a**3*d**5*e*f*g - 3*a**2*b*c**3*d**2*f**2*h + 3*a**2*b*c**2*d**3*e*f*h + 3
*a**2*b*c**2*d**3*f**2*g - 3*a**2*b*c*d**4*e*f*g + 3*a*b**2*c**4*d*f**2*h
- 3*a*b**2*c**3*d**2*e*f*h - 3*a*b**2*c**3*d**2*f**2*g + 3*a*b**2*c**2*d**
3*e*f*g - b**3*c**5*f**2*h + b**3*c**4*d*e*f*h + b**3*c**4*d*f**2*g - b**3
*c**3*d**2*e*f*g)/d**6 - f*(a*d - b*c)**3*(c*f - d*e)**2*(c*h - d*g)*atan(
sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**7*sqrt((c*f - d*e)/d)))/f, Ne(f,...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1557 vs. $2(437) = 874$.

Time = 0.17 (sec) , antiderivative size = 1557, normalized size of antiderivative = 3.32

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b^3*c^3*d^3*e^2*g - 3*a*b^2*c^2*d^4*e^2*g + 3*a^2*b*c*d^5*e^2*g - a^3*
d^6*e^2*g - 2*b^3*c^4*d^2*e*f*g + 6*a*b^2*c^3*d^3*e*f*g - 6*a^2*b*c^2*d^4*
e*f*g + 2*a^3*c*d^5*e*f*g + b^3*c^5*d*f^2*g - 3*a*b^2*c^4*d^2*f^2*g + 3*a^
2*b*c^3*d^3*f^2*g - a^3*c^2*d^4*f^2*g - b^3*c^4*d^2*e^2*h + 3*a*b^2*c^3*d^
3*e^2*h - 3*a^2*b*c^2*d^4*e^2*h + a^3*c*d^5*e^2*h + 2*b^3*c^5*d*e*f*h - 6*
a*b^2*c^4*d^2*e*f*h + 6*a^2*b*c^3*d^3*e*f*h - 2*a^3*c^2*d^4*e*f*h - b^3*c^
6*f^2*h + 3*a*b^2*c^5*d*f^2*h - 3*a^2*b*c^4*d^2*f^2*h + a^3*c^3*d^3*f^2*h)
*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^6) +
 2/3465*(385*(f*x + e)^(9/2)*b^3*d^10*f^41*g - 990*(f*x + e)^(7/2)*b^3*d^1
0*e*f^41*g + 693*(f*x + e)^(5/2)*b^3*d^10*e^2*f^41*g - 495*(f*x + e)^(7/2)
*b^3*c*d^9*f^42*g + 1485*(f*x + e)^(7/2)*a*b^2*d^10*f^42*g + 693*(f*x + e)
^(5/2)*b^3*c*d^9*e*f^42*g - 2079*(f*x + e)^(5/2)*a*b^2*d^10*e*f^42*g + 693
*(f*x + e)^(5/2)*b^3*c^2*d^8*f^43*g - 2079*(f*x + e)^(5/2)*a*b^2*c*d^9*f^4
3*g + 2079*(f*x + e)^(5/2)*a^2*b*d^10*f^43*g - 1155*(f*x + e)^(3/2)*b^3*c^
3*d^7*f^44*g + 3465*(f*x + e)^(3/2)*a*b^2*c^2*d^8*f^44*g - 3465*(f*x + e)^
(3/2)*a^2*b*c*d^9*f^44*g + 1155*(f*x + e)^(3/2)*a^3*d^10*f^44*g - 3465*sqr
t(f*x + e)*b^3*c^3*d^7*e*f^44*g + 10395*sqrt(f*x + e)*a*b^2*c^2*d^8*e*f^44
*g - 10395*sqrt(f*x + e)*a^2*b*c*d^9*e*f^44*g + 3465*sqrt(f*x + e)*a^3*d^1
0*e*f^44*g + 3465*sqrt(f*x + e)*b^3*c^4*d^6*f^45*g - 10395*sqrt(f*x + e)*a
*b^2*c^3*d^7*f^45*g + 10395*sqrt(f*x + e)*a^2*b*c^2*d^8*f^45*g - 3465*s...
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1247, normalized size of antiderivative = 2.66

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^3)/(c + d*x),x)
```

output
```
(e + f*x)^(9/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(9*d*f^4) - (2*b^3*
h*(c*f^5 - d*e*f^4))/(9*d^2*f^8)) - (e + f*x)^(3/2)*((2*(a*f - b*e)^3*(e*h
 - f*g))/(3*d*f^4) + ((c*f^5 - d*e*f^4)*(((c*f^5 - d*e*f^4)*((((2*b^3*f*g
- 8*b^3*e*h + 6*a*b^2*f*h)/(d*f^4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8)
)*(c*f^5 - d*e*f^4))/(d*f^4) - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))
/(d*f^4)))/(d*f^4) + (2*(a*f - b*e)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(d*f^4)
))/(3*d*f^4)) - (e + f*x)^(7/2)*((((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(
d*f^4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5 - d*e*f^4))/(7*d*f^
4) - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(7*d*f^4)) + (e + f*x)^(5
/2)*(((c*f^5 - d*e*f^4)*((((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d*f^4) -
 (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5 - d*e*f^4))/(d*f^4) - (6*b*
(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d*f^4)))/(5*d*f^4) + (2*(a*f - b*e
)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(5*d*f^4)) + (2*atan((d^(1/2)*(e + f*x)^(
1/2)*(a*d - b*c)^3*(c*f - d*e)^(3/2)*(c*h - d*g))/(a^3*d^6*e^2*g + b^3*c^6
*f^2*h - a^3*c*d^5*e^2*h - b^3*c^5*d*f^2*g + a^3*c^2*d^4*f^2*g - b^3*c^3*d
^3*e^2*g - a^3*c^3*d^3*f^2*h + b^3*c^4*d^2*e^2*h + 3*a*b^2*c^2*d^4*e^2*g -
 3*a*b^2*c^3*d^3*e^2*h + 3*a*b^2*c^4*d^2*f^2*g + 3*a^2*b*c^2*d^4*e^2*h - 3
*a^2*b*c^3*d^3*f^2*g + 3*a^2*b*c^4*d^2*f^2*h - 2*a^3*c*d^5*e*f*g - 2*b^3*c
^5*d*e*f*h - 3*a^2*b*c*d^5*e^2*g - 3*a*b^2*c^5*d*f^2*h + 2*a^3*c^2*d^4*e*f
*h + 2*b^3*c^4*d^2*e*f*g - 6*a*b^2*c^3*d^3*e*f*g + 6*a^2*b*c^2*d^4*e*f*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2725, normalized size of antiderivative = 5.81

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{c+dx} \, dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
(2*( - 3465*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c
*f - d*e)))*a**3*c**2*d**3*f**5*h + 3465*sqrt(d)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*e*f**4*h + 3465*sqrt(
d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*
c*d**4*f**5*g - 3465*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a**3*d**5*e*f**4*g + 10395*sqrt(d)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**2*f**5*h -
10395*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a**2*b*c**2*d**3*e*f**4*h - 10395*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*f**5*g + 10395*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*
*2*b*c*d**4*e*f**4*g - 10395*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d
)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c**4*d*f**5*h + 10395*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c**3*d**2
*e*f**4*h + 10395*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a*b**2*c**3*d**2*f**5*g - 10395*sqrt(d)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c**2*d**3*e*f**4*
g + 3465*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
- d*e)))*b**3*c**5*f**5*h - 3465*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**3*c**4*d*e*f**4*h - 3465*sqrt(d)*sq...
```

3.79 $\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 297

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2(bc-ad)^2(de-cf)(dg-ch)\sqrt{e+fx}}{d^5} + \frac{2(bc-ad)^2(dg-ch)(e+fx)^{3/2}}{3d^4} + \frac{2(d(2abdf^2g+a^2df^2h-b^2e(de+2cf)h)-b(de+cf)(2adfh+b(dfg-2deh-cfh)))(e+fx)^{5/2}}{5d^3f^3} + \frac{2b(2adfh+b(dfg-2deh-cfh))(e+fx)^{7/2}}{7d^2f^3} + \frac{2b^2h(e+fx)^{9/2}}{9df^3} - \frac{2(bc-ad)^2(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{11/2}}$$

output
```
2*(-a*d+b*c)^2*(-c*f+d*e)*(-c*h+d*g)*(f*x+e)^(1/2)/d^5+2/3*(-a*d+b*c)^2*(-
c*h+d*g)*(f*x+e)^(3/2)/d^4+2/5*(d*(2*a*b*d*f^2*g+a^2*d*f^2*h-b^2*e*(2*c*f+
d*e)*h)-b*(c*f+d*e)*(2*a*d*f*h+b*(-c*f*h-2*d*e*h+d*f*g)))*(f*x+e)^(5/2)/d^
3/f^3+2/7*b*(2*a*d*f*h+b*(-c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(7/2)/d^2/f^3+2/9
*b^2*h*(f*x+e)^(9/2)/d/f^3-2*(-a*d+b*c)^2*(-c*f+d*e)^(3/2)*(-c*h+d*g)*arct
anh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(11/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2\sqrt{e+fx}(21a^2d^2f^2(15c^2f^2h-5cdf(3fg+4eh+fhx)+d^2(3e^2h+}{}$$
$$+\frac{2(bc-ad)^2(-de+cf)^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{11/2}}$$

input
```
Integrate[((a + b*x)^2*(e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(21*a^2*d^2*f^2*(15*c^2*f^2*h - 5*c*d*f*(3*f*g + 4*e*h +
f*h*x) + d^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))) + 6*a*b
*d*f*(-105*c^3*f^3*h - 3*d^3*(e + f*x)^2*(-7*f*g + 2*e*h - 5*f*h*x) + 35*c
^2*d*f^2*(3*f*g + 4*e*h + f*h*x) - 7*c*d^2*f*(3*e^2*h + f^2*x*(5*g + 3*h*x
) + e*f*(20*g + 6*h*x))) + b^2*(315*c^4*f^4*h - 105*c^3*d*f^3*(3*f*g + 4*e
*h + f*h*x) - 9*c*d^3*f*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h*x) + 21*c^2*d^2
*f^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + d^4*(e + f*x)^
2*(8*e^2*h + 5*f^2*x*(9*g + 7*h*x) - 2*e*f*(9*g + 10*h*x)))))/(315*d^5*f^3
) + (2*(b*c - a*d)^2*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt
[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(11/2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx$$

$$\Big\downarrow \; 170$$

$$\frac{2\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{2(c+dx)}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \; 27$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(4bceh-af(9dg-5ch)-(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}+\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}$$

$$\Big\downarrow \ 25$$

$$\frac{2h(a+bx)^2(e+fx)^{5/2}}{9df}-\frac{\int -\frac{(a+bx)(e+fx)^{3/2}(9adfg-4bceh-5acfh+(4adfh+b(9dfg-4deh-9cfh))x)}{c+dx}dx}{9df}$$

input
```
Int[((a + b*x)^2*(e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.62

method	result
pseudoelliptic	$\frac{-2 f^{3} (cf-de)^{2} (ad-bc)^{2} (ch-dg) \arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2\sqrt{(cf-de)d}\,\sqrt{fx+e}\left(\left(\left(\frac{x\left(\frac{3x^{2}\left(\frac{7hx}{9}+g\right)b^{2}}{7}+\frac{6ax\left(\frac{5hx}{7}+g\right)b}{5}+\cdots\right.}{3}\right.\right.\right.}{\cdots}$
derivativedivides	$\frac{2\left(\frac{2ab\,d^{4} fh(fx+e)^{\frac{7}{2}}}{7}+\frac{h\,b^{2}(fx+e)^{\frac{9}{2}} d^{4}}{9}-2abc\,d^{3} e\,f^{3} g\sqrt{fx+e}+2ab\,c^{2} d^{2} e\,f^{3} h\sqrt{fx+e}-\frac{b^{2} c\,d^{3} fh(fx+e)^{\frac{7}{2}}}{7}+\frac{2ab\,d^{4} f^{2} g(fx+e)^{\frac{5}{2}}}{5}+\cdots\right.}{\cdots}$
default	$\frac{2\left(\frac{2ab\,d^{4} fh(fx+e)^{\frac{7}{2}}}{7}+\frac{h\,b^{2}(fx+e)^{\frac{9}{2}} d^{4}}{9}-2abc\,d^{3} e\,f^{3} g\sqrt{fx+e}+2ab\,c^{2} d^{2} e\,f^{3} h\sqrt{fx+e}-\frac{b^{2} c\,d^{3} fh(fx+e)^{\frac{7}{2}}}{7}+\frac{2ab\,d^{4} f^{2} g(fx+e)^{\frac{5}{2}}}{5}+\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2/((c*f-d*e)*d)^(1/2)*(-f^3*(c*f-d*e)^2*(a*d-b*c)^2*(c*h-d*g)*arctan(d*(f*
x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((1/3*x*
(3/7*x^2*(7/9*h*x+g)*b^2+6/5*a*x*(5/7*h*x+g)*b+a^2*(3/5*h*x+g))*d^4-(1/5*x
^2*(5/7*h*x+g)*b^2+2/3*a*(3/5*h*x+g)*x*b+a^2*(1/3*h*x+g))*c*d^3+c^2*(1/3*x
*(3/5*h*x+g)*b^2+2*a*(1/3*h*x+g)*b+a^2*h)*d^2-2*c^3*((1/6*h*x+1/2*g)*b+a*h
)*b*d+b^2*c^4*h)*f^4-4/3*(((-5/42*h*x^3-6/35*g*x^2)*b^2-3/5*a*x*(4/7*h*x+g
)*b-a^2*(3/10*h*x+g))*d^3+c*(3/10*x*(4/7*h*x+g)*b^2+2*a*(3/10*h*x+g)*b+a^2
*h)*d^2-2*((3/20*h*x+1/2*g)*b+a*h)*c^2*b*d+c^3*h*b^2)*d*e*f^3+1/5*d^2*e^2*
((1/7*x*(1/3*h*x+g)*b^2+2*a*(1/7*h*x+g)*b+a^2*h)*d^2-2*((1/14*h*x+1/2*g)*b
+a*h)*c*b*d+b^2*c^2*h)*f^2-4/35*d^3*(((1/9*h*x+1/2*g)*b+a*h)*d-1/2*b*c*h)*
b*e^3*f+8/315*b^2*d^4*e^4*h))/f^3/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(269) = 538$.

Time = 0.11 (sec) , antiderivative size = 1616, normalized size of antiderivative = 5.44

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/315*(315*(((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*e*f^3 - (b^2*c^3*d - 2
*a*b*c^2*d^2 + a^2*c*d^3)*f^4)*g - ((b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3
)*e*f^3 - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*f^4)*h)*sqrt((d*e - c*f)/d
)*log((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x +
 c)) + 2*(35*b^2*d^4*f^4*h*x^4 + 5*(9*b^2*d^4*f^4*g + (10*b^2*d^4*e*f^3 -
9*(b^2*c*d^3 - 2*a*b*d^4)*f^4)*h)*x^3 + 3*(3*(8*b^2*d^4*e*f^3 - 7*(b^2*c*d
^3 - 2*a*b*d^4)*f^4)*g + (b^2*d^4*e^2*f^2 - 24*(b^2*c*d^3 - 2*a*b*d^4)*e*f
^3 + 21*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*f^4)*h)*x^2 - 3*(6*b^2*d^4*e
^3*f + 21*(b^2*c*d^3 - 2*a*b*d^4)*e^2*f^2 - 140*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*e*f^3 + 105*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*f^4)*g + (
8*b^2*d^4*e^4 + 18*(b^2*c*d^3 - 2*a*b*d^4)*e^3*f + 63*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + a^2*d^4)*e^2*f^2 - 420*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*e*
f^3 + 315*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*f^4)*h + (3*(3*b^2*d^4*e^2
*f^2 - 42*(b^2*c*d^3 - 2*a*b*d^4)*e*f^3 + 35*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*f^4)*g - (4*b^2*d^4*e^3*f + 9*(b^2*c*d^3 - 2*a*b*d^4)*e^2*f^2 - 1
26*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*e*f^3 + 105*(b^2*c^3*d - 2*a*b*c^
2*d^2 + a^2*c*d^3)*f^4)*h)*x)*sqrt(f*x + e))/(d^5*f^3), -2/315*(315*(((b^2
*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*e*f^3 - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2
*c*d^3)*f^4)*g - ((b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*e*f^3 - (b^2*c^4
 - 2*a*b*c^3*d + a^2*c^2*d^2)*f^4)*h)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(289) = 578$.

Time = 11.67 (sec) , antiderivative size = 677, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \begin{cases} \frac{2\left(\frac{b^2h(e+fx)^{\frac{9}{2}}}{9df^2} + \frac{(e+fx)^{\frac{7}{2}}\cdot\left(2abdfh-b^2cfh-2b^2deh+b^2dfg\right)}{7d^2f^2} + \frac{(e+fx)^{\frac{5}{2}}\left(a^2d^2f^2h-2abcdf^2h-2a\cdots\right.}{\cdots}\right.}{\cdots} \\ e^{\frac{3}{2}}\left(\frac{b^2hx^3}{3d} + \frac{x^2\cdot\left(2abdh-b^2ch+b^2dg\right)}{2d^2} + \frac{x\left(a^2d^2h-2abcdh+2abd^2g+b^2c^2h-b^2cdg\right)}{d^3} - \cdots\right. \end{cases}$$

input
```
integrate((b*x+a)**2*(f*x+e)**(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(b**2*h*(e + f*x)**(9/2)/(9*d*f**2) + (e + f*x)**(7/2)*(2*a*b
*d*f*h - b**2*c*f*h - 2*b**2*d*e*h + b**2*d*f*g)/(7*d**2*f**2) + (e + f*x)
**(5/2)*(a**2*d**2*f**2*h - 2*a*b*c*d*f**2*h - 2*a*b*d**2*e*f*h + 2*a*b*d*
*2*f**2*g + b**2*c**2*f**2*h + b**2*c*d*e*f*h - b**2*c*d*f**2*g + b**2*d**
2*e**2*h - b**2*d**2*e*f*g)/(5*d**3*f**2) + (e + f*x)**(3/2)*(-a**2*c*d**2
*f*h + a**2*d**3*f*g + 2*a*b*c**2*d*f*h - 2*a*b*c*d**2*f*g - b**2*c**3*f*h
 + b**2*c**2*d*f*g)/(3*d**4) + sqrt(e + f*x)*(a**2*c**2*d**2*f**2*h - a**2
*c*d**3*e*f*h - a**2*c*d**3*f**2*g + a**2*d**4*e*f*g - 2*a*b*c**3*d*f**2*h
 + 2*a*b*c**2*d**2*e*f*h + 2*a*b*c**2*d**2*f**2*g - 2*a*b*c*d**3*e*f*g + b
**2*c**4*f**2*h - b**2*c**3*d*e*f*h - b**2*c**3*d*f**2*g + b**2*c**2*d**2*
e*f*g)/d**5 - f*(a*d - b*c)**2*(c*f - d*e)**2*(c*h - d*g)*atan(sqrt(e + f*
x)/sqrt((c*f - d*e)/d))/(d**6*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), (e**(3/2
)*(b**2*h*x**3/(3*d) + x**2*(2*a*b*d*h - b**2*c*h + b**2*d*g)/(2*d**2) + x
*(a**2*d**2*h - 2*a*b*c*d*h + 2*a*b*d**2*g + b**2*c**2*h - b**2*c*d*g)/d**
3 - (a*d - b*c)**2*(c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d,
 True))/d**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(269) = 538$.

Time = 0.15 (sec) , antiderivative size = 944, normalized size of antiderivative = 3.18

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^2*c^2*d^3*e^2*g - 2*a*b*c*d^4*e^2*g + a^2*d^5*e^2*g - 2*b^2*c^3*d^2*e
*f*g + 4*a*b*c^2*d^3*e*f*g - 2*a^2*c*d^4*e*f*g + b^2*c^4*d*f^2*g - 2*a*b*c
^3*d^2*f^2*g + a^2*c^2*d^3*f^2*g - b^2*c^3*d^2*e^2*h + 2*a*b*c^2*d^3*e^2*h
 - a^2*c*d^4*e^2*h + 2*b^2*c^4*d*e*f*h - 4*a*b*c^3*d^2*e*f*h + 2*a^2*c^2*d
^3*e*f*h - b^2*c^5*f^2*h + 2*a*b*c^4*d*f^2*h - a^2*c^3*d^2*f^2*h)*arctan(s
qrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^5) + 2/315*(4
5*(f*x + e)^(7/2)*b^2*d^8*f^25*g - 63*(f*x + e)^(5/2)*b^2*d^8*e*f^25*g - 6
3*(f*x + e)^(5/2)*b^2*c*d^7*f^26*g + 126*(f*x + e)^(5/2)*a*b*d^8*f^26*g +
105*(f*x + e)^(3/2)*b^2*c^2*d^6*f^27*g - 210*(f*x + e)^(3/2)*a*b*c*d^7*f^2
7*g + 105*(f*x + e)^(3/2)*a^2*d^8*f^27*g + 315*sqrt(f*x + e)*b^2*c^2*d^6*e
*f^27*g - 630*sqrt(f*x + e)*a*b*c*d^7*e*f^27*g + 315*sqrt(f*x + e)*a^2*d^8
*e*f^27*g - 315*sqrt(f*x + e)*b^2*c^3*d^5*f^28*g + 630*sqrt(f*x + e)*a*b*c
^2*d^6*f^28*g - 315*sqrt(f*x + e)*a^2*c*d^7*f^28*g + 35*(f*x + e)^(9/2)*b^
2*d^8*f^24*h - 90*(f*x + e)^(7/2)*b^2*d^8*e*f^24*h + 63*(f*x + e)^(5/2)*b^
2*d^8*e^2*f^24*h - 45*(f*x + e)^(7/2)*b^2*c*d^7*f^25*h + 90*(f*x + e)^(7/2
)*a*b*d^8*f^25*h + 63*(f*x + e)^(5/2)*b^2*c*d^7*e*f^25*h - 126*(f*x + e)^(
5/2)*a*b*d^8*e*f^25*h + 63*(f*x + e)^(5/2)*b^2*c^2*d^6*f^26*h - 126*(f*x +
 e)^(5/2)*a*b*c*d^7*f^26*h + 63*(f*x + e)^(5/2)*a^2*d^8*f^26*h - 105*(f*x
+ e)^(3/2)*b^2*c^3*d^5*f^27*h + 210*(f*x + e)^(3/2)*a*b*c^2*d^6*f^27*h - 1
05*(f*x + e)^(3/2)*a^2*c*d^7*f^27*h - 315*sqrt(f*x + e)*b^2*c^3*d^5*e*f...
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.88

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx} dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^2)/(c + d*x),x)
```

output
```
(e + f*x)^(7/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(7*d*f^3) - (2*b^2*h*
(c*f^4 - d*e*f^3))/(7*d^2*f^6)) - (e + f*x)^(3/2)*((2*(a*f - b*e)^2*(e*h -
 f*g))/(3*d*f^3) - (((((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d*f^3) - (2*b^
2*h*(c*f^4 - d*e*f^3))/(d^2*f^6))*(c*f^4 - d*e*f^3))/(d*f^3) - (2*(a*f - b
*e)*(a*f*h - 3*b*e*h + 2*b*f*g))/(d*f^3))*(c*f^4 - d*e*f^3))/(3*d*f^3)) -
(e + f*x)^(5/2)*((((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d*f^3) - (2*b^2*h*
(c*f^4 - d*e*f^3))/(d^2*f^6))*(c*f^4 - d*e*f^3))/(5*d*f^3) - (2*(a*f - b*e
)*(a*f*h - 3*b*e*h + 2*b*f*g))/(5*d*f^3)) + (2*b^2*h*(e + f*x)^(9/2))/(9*d
*f^3) + (2*atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)^2*(c*f - d*e)^(3/2)*(
c*h - d*g))/(a^2*d^5*e^2*g - b^2*c^5*f^2*h - a^2*c*d^4*e^2*h + b^2*c^4*d*f
^2*g + a^2*c^2*d^3*f^2*g + b^2*c^2*d^3*e^2*g - a^2*c^3*d^2*f^2*h - b^2*c^3
*d^2*e^2*h - 2*a*b*c*d^4*e^2*g + 2*a*b*c^4*d*f^2*h - 2*a^2*c*d^4*e*f*g + 2
*b^2*c^4*d*e*f*h + 2*a*b*c^2*d^3*e^2*h - 2*a*b*c^3*d^2*f^2*g + 2*a^2*c^2*d
^3*e*f*h - 2*b^2*c^3*d^2*e*f*g + 4*a*b*c^2*d^3*e*f*g - 4*a*b*c^3*d^2*e*f*h
))*(a*d - b*c)^2*(c*f - d*e)^(3/2)*(c*h - d*g))/d^(11/2) + ((e + f*x)^(1/2
)*(c*f^4 - d*e*f^3)*((2*(a*f - b*e)^2*(e*h - f*g))/(d*f^3) - (((((2*b^2*f*
g - 6*b^2*e*h + 4*a*b*f*h)/(d*f^3) - (2*b^2*h*(c*f^4 - d*e*f^3))/(d^2*f^6)
)*(c*f^4 - d*e*f^3))/(d*f^3) - (2*(a*f - b*e)*(a*f*h - 3*b*e*h + 2*b*f*g))
/(d*f^3))*(c*f^4 - d*e*f^3))/(d*f^3)))/(d*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1635, normalized size of antiderivative = 5.51

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{c+dx} dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
(2*( - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a**2*c**2*d**2*f**4*h + 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(
e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*e*f**3*h + 315*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d
**3*f**4*g - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a**2*d**4*e*f**3*g + 630*sqrt(d)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d*f**4*h - 630*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2
*d**2*e*f**3*h - 630*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*f**4*g + 630*sqrt(d)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*e*f**3*g - 315*
sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*
b**2*c**4*f**4*h + 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqr
t(d)*sqrt(c*f - d*e)))*b**2*c**3*d*e*f**3*h + 315*sqrt(d)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**3*d*f**4*g - 315
*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*b**2*c**2*d**2*e*f**3*g + 315*sqrt(e + f*x)*a**2*c**2*d**3*f**4*h - 420*s
qrt(e + f*x)*a**2*c*d**4*e*f**3*h - 315*sqrt(e + f*x)*a**2*c*d**4*f**4*g -
 105*sqrt(e + f*x)*a**2*c*d**4*f**4*h*x + 63*sqrt(e + f*x)*a**2*d**5*e**2*
f**2*h + 420*sqrt(e + f*x)*a**2*d**5*e*f**3*g + 126*sqrt(e + f*x)*a**2*...
```

3.80 $\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 196

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx} \, dx = -\frac{2(bc-ad)(de-cf)(dg-ch)\sqrt{e+fx}}{d^4} - \frac{2(bc-ad)(dg-ch)(e+fx)^{3/2}}{3d^3} - \frac{2(b(de+cf)h-df(bg+ah))(e+fx)^{5/2}}{5d^2f^2} + \frac{2bh(e+fx)^{7/2}}{7df^2} + \frac{2(bc-ad)(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{9/2}}$$

output

```
-2*(-a*d+b*c)*(-c*f+d*e)*(-c*h+d*g)*(f*x+e)^(1/2)/d^4-2/3*(-a*d+b*c)*(-c*h
+d*g)*(f*x+e)^(3/2)/d^3-2/5*(b*(c*f+d*e)*h-d*f*(a*h+b*g))*(f*x+e)^(5/2)/d^
2/f^2+2/7*b*h*(f*x+e)^(7/2)/d/f^2+2*(-a*d+b*c)*(-c*f+d*e)^(3/2)*(-c*h+d*g)
*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2\sqrt{e+fx}(7adf(15c^2f^2h-5cdf(3fg+4eh+fhx)+d^2(3e^2h+f^2x}{}$$
$$+\frac{2(-bc+ad)(-de+cf)^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{9/2}}$$

input
```
Integrate[((a + b*x)*(e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(7*a*d*f*(15*c^2*f^2*h - 5*c*d*f*(3*f*g + 4*e*h + f*h*x)
+ d^2*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))) + b*(-105*c^3*
f^3*h - 3*d^3*(e + f*x)^2*(-7*f*g + 2*e*h - 5*f*h*x) + 35*c^2*d*f^2*(3*f*g
 + 4*e*h + f*h*x) - 7*c*d^2*f*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g +
 6*h*x)))))/(105*d^4*f^2) + (2*(-(b*c) + a*d)*(-(d*e) + c*f)^(3/2)*(d*g -
c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(9/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx$$
$$\Big\downarrow \text{164}$$
$$-\frac{(bc-ad)(dg-ch)\int \frac{(e+fx)^{3/2}}{c+dx}dx}{d^2}-\frac{2(e+fx)^{5/2}(-7df(ah+bg)+7bcfh+2bdeh-5bdfhx)}{35d^2f^2}$$
$$\Big\downarrow \text{60}$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{(de-cf)\int \frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d^2}-\frac{2(e+fx)^{5/2}(-7df(ah+bg)+7bcfh+2bdeh-5bdfhx)}{35d^2f^2}$$

$$\Big\downarrow \; 60$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d^2}-\frac{2(e+fx)^{5/2}(-7df(ah+bg)+7bcfh+2bdeh-5bdfhx)}{35d^2f^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{(de-cf)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d^2}-\frac{2(e+fx)^{5/2}(-7df(ah+bg)+7bcfh+2bdeh-5bdfhx)}{35d^2f^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{(bc-ad)(dg-ch)\left(\frac{(de-cf)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d^2}-\frac{2(e+fx)^{5/2}(-7df(ah+bg)+7bcfh+2bdeh-5bdfhx)}{35d^2f^2}$$

input
```
Int[((a + b*x)*(e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
(-2*(e + f*x)^(5/2)*(2*b*d*e*h + 7*b*c*f*h - 7*d*f*(b*g + a*h) - 5*b*d*f*h
*x))/(35*d^2*f^2) - ((b*c - a*d)*(d*g - c*h)*((2*(e + f*x)^(3/2))/(3*d) +
((d*e - c*f)*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sq
rt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d))/d^2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{(cf-de)d}\,\left(\left(\left(\frac{\left(\frac{5}{7}bfx+af-\frac{2}{7}be\right)(fx+e)^2d^3}{5}-\frac{4c\left(\frac{3bf^2x^2}{20}+\left(\frac{1}{4}af^2+\frac{3}{10}bef\right)x+e\left(af+\frac{3be}{20}\right)\right)fd^2}{3}+c^2f^2\left(\frac{1}{3}bfx+\frac{4}{3}b\right.\right.\right.\right.}{}$
derivativedivides	$\frac{2\left(\frac{hb(fx+e)^{\frac{7}{2}}d^3}{7}+\frac{ad^3fh(fx+e)^{\frac{5}{2}}}{5}-\frac{bcd^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{bd^3eh(fx+e)^{\frac{5}{2}}}{5}+\frac{bd^3fg(fx+e)^{\frac{5}{2}}}{5}-\frac{acd^2f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{ad^3f^2g(fx+e)}{3}\right.}{}$
default	$\frac{2\left(\frac{hb(fx+e)^{\frac{7}{2}}d^3}{7}+\frac{ad^3fh(fx+e)^{\frac{5}{2}}}{5}-\frac{bcd^2fh(fx+e)^{\frac{5}{2}}}{5}-\frac{bd^3eh(fx+e)^{\frac{5}{2}}}{5}+\frac{bd^3fg(fx+e)^{\frac{5}{2}}}{5}-\frac{acd^2f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{ad^3f^2g(fx+e)}{3}\right.}{}$
risch	$\frac{2(15bf^3hd^3x^3+21ad^3f^3hx^2-21bcd^2f^3hx^2+24bd^3ef^2hx^2+21bd^3f^3gx^2-35acd^2f^3hx+42ad^3ef^2hx+35ad^3f^3}{}$

input
```
int((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/((c*f-d*e)*d)^(1/2)*(-((c*f-d*e)*d)^(1/2)*((1/5*(5/7*b*f*x+a*f-2/7*b*e)
*(f*x+e)^2*d^3-4/3*c*(3/20*b*f^2*x^2+(1/4*a*f^2+3/10*b*e*f)*x+e*(a*f+3/20*
b*e))*f*d^2+c^2*f^2*(1/3*b*f*x+4/3*b*e+a*f)*d-b*c^3*f^3)*h-d*g*f*((-1/5*b*
f^2*x^2+(-2/5*b*e*f-1/3*a*f^2)*x-4/3*a*e*f-1/5*b*e^2)*d^2+c*f*(1/3*b*f*x+4
/3*b*e+a*f)*d-b*c^2*f^2))*(f*x+e)^(1/2)+f^2*(c*f-d*e)^2*(c*h-d*g)*(a*d-b*c
)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))/f^2/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(172) = 344.

Time = 0.10 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.35

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/105*(105*(((b*c*d^2 - a*d^3)*e*f^2 - (b*c^2*d - a*c*d^2)*f^3)*g - ((b*
c^2*d - a*c*d^2)*e*f^2 - (b*c^3 - a*c^2*d)*f^3)*h)*sqrt((d*e - c*f)/d)*log
((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c))
- 2*(15*b*d^3*f^3*h*x^3 + 3*(7*b*d^3*f^3*g + (8*b*d^3*e*f^2 - 7*(b*c*d^2 -
 a*d^3)*f^3)*h)*x^2 + 7*(3*b*d^3*e^2*f - 20*(b*c*d^2 - a*d^3)*e*f^2 + 15*(
b*c^2*d - a*c*d^2)*f^3)*g - (6*b*d^3*e^3 + 21*(b*c*d^2 - a*d^3)*e^2*f - 14
0*(b*c^2*d - a*c*d^2)*e*f^2 + 105*(b*c^3 - a*c^2*d)*f^3)*h + (7*(6*b*d^3*e
*f^2 - 5*(b*c*d^2 - a*d^3)*f^3)*g + (3*b*d^3*e^2*f - 42*(b*c*d^2 - a*d^3)*
e*f^2 + 35*(b*c^2*d - a*c*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(d^4*f^2), 2/105*
(105*(((b*c*d^2 - a*d^3)*e*f^2 - (b*c^2*d - a*c*d^2)*f^3)*g - ((b*c^2*d -
a*c*d^2)*e*f^2 - (b*c^3 - a*c^2*d)*f^3)*h)*sqrt(-(d*e - c*f)/d)*arctan(-sq
rt(f*x + e)*d*sqrt(-(d*e - c*f)/d)/(d*e - c*f)) + (15*b*d^3*f^3*h*x^3 + 3*
(7*b*d^3*f^3*g + (8*b*d^3*e*f^2 - 7*(b*c*d^2 - a*d^3)*f^3)*h)*x^2 + 7*(3*b
*d^3*e^2*f - 20*(b*c*d^2 - a*d^3)*e*f^2 + 15*(b*c^2*d - a*c*d^2)*f^3)*g -
(6*b*d^3*e^3 + 21*(b*c*d^2 - a*d^3)*e^2*f - 140*(b*c^2*d - a*c*d^2)*e*f^2
+ 105*(b*c^3 - a*c^2*d)*f^3)*h + (7*(6*b*d^3*e*f^2 - 5*(b*c*d^2 - a*d^3)*f
^3)*g + (3*b*d^3*e^2*f - 42*(b*c*d^2 - a*d^3)*e*f^2 + 35*(b*c^2*d - a*c*d^
2)*f^3)*h)*x)*sqrt(f*x + e))/(d^4*f^2)]
```

Sympy [A] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \begin{cases} \frac{2\left(\frac{bh(e+fx)^{\frac{7}{2}}}{7df} + \frac{(e+fx)^{\frac{5}{2}}(adfh-bcfh-bdeh+bdfg)}{5d^2f} + \frac{(e+fx)^{\frac{3}{2}}\left(-acdfh+ad^2fg+bc^2fh-bcdfg\right)}{3d^3} + \cdots\right)}{\cdots} \\ e^{\frac{3}{2}}\left(\frac{bhx^2}{2d} + \frac{x(adh-bch+bdg)}{d^2} - \frac{(ad-bc)(ch-dg)\left(\begin{cases} \frac{x}{c} & \text{for } d=0 \\ \frac{\log(c+dx)}{d} & \text{otherwise} \end{cases}\right)}{d^2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(f*x+e)**(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(b*h*(e + f*x)**(7/2)/(7*d*f) + (e + f*x)**(5/2)*(a*d*f*h - b
*c*f*h - b*d*e*h + b*d*f*g)/(5*d**2*f) + (e + f*x)**(3/2)*(-a*c*d*f*h + a*
d**2*f*g + b*c**2*f*h - b*c*d*f*g)/(3*d**3) + sqrt(e + f*x)*(a*c**2*d*f**2
*h - a*c*d**2*e*f*h - a*c*d**2*f**2*g + a*d**3*e*f*g - b*c**3*f**2*h + b*c
**2*d*e*f*h + b*c**2*d*f**2*g - b*c*d**2*e*f*g)/d**4 - f*(a*d - b*c)*(c*f
- d*e)**2*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**5*sqrt((
c*f - d*e)/d)))/f, Ne(f, 0)), (e**(3/2)*(b*h*x**2/(2*d) + x*(a*d*h - b*c*h
 + b*d*g)/d**2 - (a*d - b*c)*(c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c
 + d*x)/d, True))/d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(172) = 344$.

Time = 0.13 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx =$$

$$-\frac{2\left(bcd^3e^2g - ad^4e^2g - 2\,bc^2d^2efg + 2\,acd^3efg + bc^3df^2g - ac^2d^2f^2g - bc^2d^2e^2h + acd^3e^2h + 2\,bc^3defh\right.}{\sqrt{-d^2e+cdf}\,d^4}$$

$$+\frac{2\left(21\,(fx+e)^{\frac{5}{2}}bd^6f^{13}g - 35\,(fx+e)^{\frac{3}{2}}bcd^5f^{14}g + 35\,(fx+e)^{\frac{3}{2}}ad^6f^{14}g - 105\,\sqrt{fx+e}bcd^5ef^{14}g + 105\right.}{}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
-2*(b*c*d^3*e^2*g - a*d^4*e^2*g - 2*b*c^2*d^2*e*f*g + 2*a*c*d^3*e*f*g + b*
c^3*d*f^2*g - a*c^2*d^2*f^2*g - b*c^2*d^2*e^2*h + a*c*d^3*e^2*h + 2*b*c^3*
d*e*f*h - 2*a*c^2*d^2*e*f*h - b*c^4*f^2*h + a*c^3*d*f^2*h)*arctan(sqrt(f*x
 + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^4) + 2/105*(21*(f*x
+ e)^(5/2)*b*d^6*f^13*g - 35*(f*x + e)^(3/2)*b*c*d^5*f^14*g + 35*(f*x + e)
^(3/2)*a*d^6*f^14*g - 105*sqrt(f*x + e)*b*c*d^5*e*f^14*g + 105*sqrt(f*x +
e)*a*d^6*e*f^14*g + 105*sqrt(f*x + e)*b*c^2*d^4*f^15*g - 105*sqrt(f*x + e)
*a*c*d^5*f^15*g + 15*(f*x + e)^(7/2)*b*d^6*f^12*h - 21*(f*x + e)^(5/2)*b*d
^6*e*f^12*h - 21*(f*x + e)^(5/2)*b*c*d^5*f^13*h + 21*(f*x + e)^(5/2)*a*d^6
*f^13*h + 35*(f*x + e)^(3/2)*b*c^2*d^4*f^14*h - 35*(f*x + e)^(3/2)*a*c*d^5
*f^14*h + 105*sqrt(f*x + e)*b*c^2*d^4*e*f^14*h - 105*sqrt(f*x + e)*a*c*d^5
*e*f^14*h - 105*sqrt(f*x + e)*b*c^3*d^3*f^15*h + 105*sqrt(f*x + e)*a*c^2*d
^4*f^15*h)/(d^7*f^14)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx=(e+f\,x)^{5/2}\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{5\,d\,f^2}-\frac{2\,b\,h\,\left(c\,f^3-d\,e\,f^2\right)}{5\,d^2\,f^4}\right)$$

$$-(e+f\,x)^{3/2}\left(\frac{2\,\left(a\,f-b\,e\right)\,\left(e\,h-f\,g\right)}{3\,d\,f^2}+\frac{\left(c\,f^3-d\,e\,f^2\right)\,\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{d\,f^2}-\frac{2\,b\,h\,\left(c\,f^3-d\,e\,f^2\right)}{d^2\,f^4}\right)}{3\,d\,f^2}\right)+\frac{2\,b\,h\,(e+}{7\,d}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x))/(c + d*x),x)
```

output
```
(e + f*x)^(5/2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(5*d*f^2) - (2*b*h*(c*f^3 -
 d*e*f^2))/(5*d^2*f^4)) - (e + f*x)^(3/2)*((2*(a*f - b*e)*(e*h - f*g))/(3*
d*f^2) + ((c*f^3 - d*e*f^2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(d*f^2) - (2*b*
h*(c*f^3 - d*e*f^2))/(d^2*f^4)))/(3*d*f^2)) + (2*b*h*(e + f*x)^(7/2))/(7*d
*f^2) + (2*atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)*(c*f - d*e)^(3/2)*(c*
h - d*g))/(a*d^4*e^2*g + b*c^4*f^2*h - a*c*d^3*e^2*h - b*c*d^3*e^2*g - a*c
^3*d*f^2*h - b*c^3*d*f^2*g + a*c^2*d^2*f^2*g + b*c^2*d^2*e^2*h - 2*a*c*d^3
*e*f*g - 2*b*c^3*d*e*f*h + 2*a*c^2*d^2*e*f*h + 2*b*c^2*d^2*e*f*g))*(a*d -
b*c)*(c*f - d*e)^(3/2)*(c*h - d*g))/d^(9/2) + ((e + f*x)^(1/2)*((2*(a*f -
b*e)*(e*h - f*g))/(d*f^2) + ((c*f^3 - d*e*f^2)*((2*a*f*h - 4*b*e*h + 2*b*f
*g)/(d*f^2) - (2*b*h*(c*f^3 - d*e*f^2))/(d^2*f^4)))/(d*f^2))*(c*f^3 - d*e*
f^2))/(d*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 835, normalized size of antiderivative = 4.26

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c),x)
```

output

```
(2*( - 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a*c**2*d*f**3*h + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*e*f**2*h + 105*sqrt(d)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*f**3*g -
 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*
e)))*a*d**3*e*f**2*g + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*f**3*h - 105*sqrt(d)*sqrt(c*f - d*e)*ata
n((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d*e*f**2*h - 105*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c
**2*d*f**3*g + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*b*c*d**2*e*f**2*g + 105*sqrt(e + f*x)*a*c**2*d**2*f**3*
h - 140*sqrt(e + f*x)*a*c*d**3*e*f**2*h - 105*sqrt(e + f*x)*a*c*d**3*f**3*
g - 35*sqrt(e + f*x)*a*c*d**3*f**3*h*x + 21*sqrt(e + f*x)*a*d**4*e**2*f*h
+ 140*sqrt(e + f*x)*a*d**4*e*f**2*g + 42*sqrt(e + f*x)*a*d**4*e*f**2*h*x +
 35*sqrt(e + f*x)*a*d**4*f**3*g*x + 21*sqrt(e + f*x)*a*d**4*f**3*h*x**2 -
105*sqrt(e + f*x)*b*c**3*d*f**3*h + 140*sqrt(e + f*x)*b*c**2*d**2*e*f**2*h
 + 105*sqrt(e + f*x)*b*c**2*d**2*f**3*g + 35*sqrt(e + f*x)*b*c**2*d**2*f**
3*h*x - 21*sqrt(e + f*x)*b*c*d**3*e**2*f*h - 140*sqrt(e + f*x)*b*c*d**3*e*
f**2*g - 42*sqrt(e + f*x)*b*c*d**3*e*f**2*h*x - 35*sqrt(e + f*x)*b*c*d**3*
f**3*g*x - 21*sqrt(e + f*x)*b*c*d**3*f**3*h*x**2 - 6*sqrt(e + f*x)*b*d*...
```

3.81 $\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2(de-cf)(dg-ch)\sqrt{e+fx}}{d^3} + \frac{2(dg-ch)(e+fx)^{3/2}}{3d^2} + \frac{2h(e+fx)^{5/2}}{5df} - \frac{2(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}}$$

output
```
2*(-c*f+d*e)*(-c*h+d*g)*(f*x+e)^(1/2)/d^3+2/3*(-c*h+d*g)*(f*x+e)^(3/2)/d^2
+2/5*h*(f*x+e)^(5/2)/d/f-2*(-c*f+d*e)^(3/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*
x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2\sqrt{e+fx}(15c^2f^2h - 5cdf(3fg+4eh+fhx) + d^2(3e^2h + f^2x(5g+3hx) + e}{15d^3f} + \frac{2(-de+cf)^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
(2*Sqrt[e + f*x]*(15*c^2*f^2*h - 5*c*d*f*(3*f*g + 4*e*h + f*h*x) + d^2*(3*
e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x))))/(15*d^3*f) + (2*(-(d*e
) + c*f)^(3/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*
f]])/d^(7/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(dg-ch)\int \frac{(e+fx)^{3/2}}{c+dx}dx}{d}+\frac{2h(e+fx)^{5/2}}{5df}$$

$$\Big\downarrow \text{60}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\int \frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d}+\frac{2h(e+fx)^{5/2}}{5df}$$

$$\Big\downarrow \text{60}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d}+\frac{2h(e+fx)^{5/2}}{5df}$$

$$\Big\downarrow \text{73}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d}+\frac{2h(e+fx)^{5/2}}{5df}$$

$$\Big\downarrow \text{221}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{d}+\frac{2h(e+fx)^{5/2}}{5df}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/(c + d*x),x]
```

output
```
(2*h*(e + f*x)^(5/2))/(5*d*f) + ((d*g - c*h)*((2*(e + f*x)^(3/2))/(3*d) +
((d*e - c*f)*((2*Sqrt[e + f*x]])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sq
rt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d))/d
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2\left(\left(\left(\frac{(fx+e)^2 h}{5}+\frac{4fg\left(\frac{fx}{4}+e\right)}{3}\right)d^2-\frac{4cf\left(\left(\frac{fx}{4}+e\right)h+\frac{3fg}{4}\right)d}{3}+c^2f^2h\right)\sqrt{(cf-de)d}\,\sqrt{fx+e}-2f(cf-de)^2(ch-dg)\arctan\left(\right.}{f\,d^3\sqrt{(cf-de)d}}$
risch	$\frac{2\left(3f^2d^2h\,x^2-5f^2cdhx+6efxh\,d^2+5f^2d^2gx+15c^2f^2h-20cdehf-15cd\,f^2g+3d^2e^2h+20d^2egf\right)\sqrt{fx+e}}{15f\,d^3}-\frac{2\left(c^3f^2h-\right.}{}$
derivativedivides	$\frac{\frac{2\left(\frac{h(fx+e)^{\frac{5}{2}}d^2}{5}-\frac{cdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{d^2fg(fx+e)^{\frac{3}{2}}}{3}+c^2f^2h\sqrt{fx+e}-cdefh\sqrt{fx+e}-cd\,f^2g\sqrt{fx+e}+d^2efg\sqrt{fx+e}\right)}{d^3}-\frac{2f\left(c^3f^2h-2c^2\right.}{}}{f}$
default	$\frac{\frac{2\left(\frac{h(fx+e)^{\frac{5}{2}}d^2}{5}-\frac{cdfh(fx+e)^{\frac{3}{2}}}{3}+\frac{d^2fg(fx+e)^{\frac{3}{2}}}{3}+c^2f^2h\sqrt{fx+e}-cdefh\sqrt{fx+e}-cd\,f^2g\sqrt{fx+e}+d^2efg\sqrt{fx+e}\right)}{d^3}-\frac{2f\left(c^3f^2h-2c^2\right.}{}}{f}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*(((1/5*(f*x+e)^2*h+4/3*f*g*(1/4*f*x+e))*d^2-4/3*c*f*((1/4*f*x+e)*h+3/4*f
*g)*d+c^2*f^2*h)*((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)-f*(c*f-d*e)^2*(c*h-d*g)
*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))/((c*f-d*e)*d)^(1/2)/f/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.00

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \left[\frac{15\left((d^2ef-cdf^2)g-(cdef-c^2f^2)h\right)\sqrt{\frac{de-cf}{d}}\log\left(\frac{dfx+2\,de-cf-2\,\sqrt{fx+e}d\sqrt{\frac{de-cf}{d}}}{dx+c}\right.\cdots}{} \right.$$

$$-\frac{2\left(15\left((d^2ef-cdf^2)g-(cdef-c^2f^2)h\right)\sqrt{-\frac{de-cf}{d}}\arctan\left(-\frac{\sqrt{fx+e}d\sqrt{-\frac{de-cf}{d}}}{de-cf}\right)-(3\,d^2f^2hx^2+5\,(4\,d^2ef\cdots\right.}{15\,d^3f}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/15*(15*((d^2*e*f - c*d*f^2)*g - (c*d*e*f - c^2*f^2)*h)*sqrt((d*e - c*f)
/d)*log((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x
 + c)) + 2*(3*d^2*f^2*h*x^2 + 5*(4*d^2*e*f - 3*c*d*f^2)*g + (3*d^2*e^2 - 2
0*c*d*e*f + 15*c^2*f^2)*h + (5*d^2*f^2*g + (6*d^2*e*f - 5*c*d*f^2)*h)*x)*s
qrt(f*x + e))/(d^3*f), -2/15*(15*((d^2*e*f - c*d*f^2)*g - (c*d*e*f - c^2*f
^2)*h)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(d*e - c*f)/d)/(
d*e - c*f)) - (3*d^2*f^2*h*x^2 + 5*(4*d^2*e*f - 3*c*d*f^2)*g + (3*d^2*e^2
- 20*c*d*e*f + 15*c^2*f^2)*h + (5*d^2*f^2*g + (6*d^2*e*f - 5*c*d*f^2)*h)*x
)*sqrt(f*x + e))/(d^3*f)]
```

Sympy [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \begin{cases} \frac{2\left(\frac{h(e+fx)^{\frac{5}{2}}}{5d}+\frac{(e+fx)^{\frac{3}{2}}(-cfh+dfg)}{3d^2}+\frac{\sqrt{e+fx}\left(c^2f^2h-cdefh-cdf^2g+d^2efg\right)}{d^3}-\frac{f(cf-de)^2(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^4\sqrt{\frac{cf-de}{d}}}\right)}{f} \\ e^{\frac{3}{2}}\left(\frac{hx}{d}-\frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d}\right) \end{cases}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
Piecewise((2*(h*(e + f*x)**(5/2)/(5*d) + (e + f*x)**(3/2)*(-c*f*h + d*f*g)
/(3*d**2) + sqrt(e + f*x)*(c**2*f**2*h - c*d*e*f*h - c*d*f**2*g + d**2*e*f
*g)/d**3 - f*(c*f - d*e)**2*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e
)/d))/(d**4*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), (e**(3/2)*(h*x/d - (c*h -
d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.74

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2\,(d^3e^2g-2\,cd^2efg+c^2df^2g-cd^2e^2h+2\,c^2defh-c^3f^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cd}}\right.}{\sqrt{-d^2e+cdf}\,d^3}$$
$$+\frac{2\left(5\,(fx+e)^{\frac{3}{2}}d^4f^5g+15\,\sqrt{fx+e}d^4ef^5g-15\,\sqrt{fx+e}cd^3f^6g+3\,(fx+e)^{\frac{5}{2}}d^4f^4h-5\,(fx+e)^{\frac{3}{2}}cd^3f^5h\right.}{15\,d^5f^5}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c),x, algorithm="giac")
```

output
```
2*(d^3*e^2*g - 2*c*d^2*e*f*g + c^2*d*f^2*g - c*d^2*e^2*h + 2*c^2*d*e*f*h -
 c^3*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*
d*f)*d^3) + 2/15*(5*(f*x + e)^(3/2)*d^4*f^5*g + 15*sqrt(f*x + e)*d^4*e*f^5
*g - 15*sqrt(f*x + e)*c*d^3*f^6*g + 3*(f*x + e)^(5/2)*d^4*f^4*h - 5*(f*x +
 e)^(3/2)*c*d^3*f^5*h - 15*sqrt(f*x + e)*c*d^3*e*f^5*h + 15*sqrt(f*x + e)*
c^2*d^2*f^6*h)/(d^5*f^5)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.82

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{2\,h\,(e+f\,x)^{5/2}}{5\,d\,f}$$
$$-(e+f\,x)^{3/2}\left(\frac{2\,e\,h-2\,f\,g}{3\,d\,f}+\frac{2\,h\,(c\,f^2-d\,e\,f)}{3\,d^2\,f^2}\right)+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(c\,f-d\,e)^{3/2}\,(c\,h-d\,g)}{-h\,c^3\,f^2+2\,h\,c^2\,d\,e\,f+g\,c^2\,d\,f^2-h\,c\,d^2\,e^2-2\,g\,c\,d^2\,e\,f+g\,d^3\,e^2}\right)}{d^{7/2}}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/(c + d*x),x)
```

output
```
(2*h*(e + f*x)^(5/2))/(5*d*f) - (e + f*x)^(3/2)*((2*e*h - 2*f*g)/(3*d*f) +
 (2*h*(c*f^2 - d*e*f))/(3*d^2*f^2)) + (2*atan((d^(1/2)*(e + f*x)^(1/2)*(c*
f - d*e)^(3/2)*(c*h - d*g))/(d^3*e^2*g - c^3*f^2*h - c*d^2*e^2*h + c^2*d*f
^2*g - 2*c*d^2*e*f*g + 2*c^2*d*e*f*h))*(c*f - d*e)^(3/2)*(c*h - d*g))/d^(7
/2) + ((e + f*x)^(1/2)*(c*f^2 - d*e*f)*((2*e*h - 2*f*g)/(d*f) + (2*h*(c*f^
2 - d*e*f))/(d^2*f^2)))/(d*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.48

$$\int \frac{(e+fx)^{3/2}(g+hx)}{c+dx}\,dx = \frac{-2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2f^2h+2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cd}{}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(d*x+c),x)
```

output
```
(2*( - 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*c**2*f**2*h + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*c*d*e*f*h + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f**2*g - 15*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*e*f*g + 15*
sqrt(e + f*x)*c**2*d*f**2*h - 20*sqrt(e + f*x)*c*d**2*e*f*h - 15*sqrt(e +
f*x)*c*d**2*f**2*g - 5*sqrt(e + f*x)*c*d**2*f**2*h*x + 3*sqrt(e + f*x)*d**
3*e**2*h + 20*sqrt(e + f*x)*d**3*e*f*g + 6*sqrt(e + f*x)*d**3*e*f*h*x + 5*
sqrt(e + f*x)*d**3*f**2*g*x + 3*sqrt(e + f*x)*d**3*f**2*h*x**2))/(15*d**4*
f)
```

3.82 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = -\frac{2(adfh-b(dfg+deh-cfh))\sqrt{e+fx}}{b^2d^2} + \frac{2h(e+fx)^{3/2}}{3bd} - \frac{2(be-af)^{3/2}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}(bc-ad)} + \frac{2(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}(bc-ad)}$$

output
```
-2*(a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)/b^2/d^2+2/3*h*(f*x+e)^(3
/2)/b/d-2*(-a*f+b*e)^(3/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+
b*e)^(1/2))/b^(5/2)/(-a*d+b*c)+2*(-c*f+d*e)^(3/2)*(-c*h+d*g)*arctanh(d^(1/
2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{2\sqrt{e+fx}(-3bcfh-3adfh+bd(3fg+4eh+fhx))}{3b^2d^2} + \frac{2(-be+af)^{3/2}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{5/2}(bc-ad)} + \frac{2(-de+cf)^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}(-bc+ad)}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(2*Sqrt[e + f*x]*(-3*b*c*f*h - 3*a*d*f*h + b*d*(3*f*g + 4*e*h + f*h*x)))/(
3*b^2*d^2) + (2*(-(b*e) + a*f)^(3/2)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e +
f*x])/Sqrt[-(b*e) + a*f]])/(b^(5/2)*(b*c - a*d)) + (2*(-(d*e) + c*f)^(3/2)
*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(5/2)*
(-(b*c) + a*d))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {171, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{ 171}$$

$$\frac{2\int \frac{3\sqrt{e+fx}(bdeg-acfh-(adfh-b(dfg+deh-cfh))x)}{2(a+bx)(c+dx)}dx}{3bd} + \frac{2h(e+fx)^{3/2}}{3bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{e+fx}(bdeg-acfh-(adfh-b(dfg+deh-cfh))x)}{(a+bx)(c+dx)}dx}{bd}+\frac{2h(e+fx)^{3/2}}{3bd}$$

$$\Big\downarrow \text{171}$$

$$\frac{\frac{2\int \frac{b^2d^2ge^2+a^2cdf^2h+abcf(cfh-d(fg+2eh))+(bdf(bdeg-acfh)-(bde-bcf-adf)(adfh-b(dfg+deh-cfh)))x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd}-\frac{2\sqrt{e+fx}(adfh-b(-cfh+deh+dfg))}{bd}}{bd}+\frac{2h(e+fx)^{3/2}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b^2d^2ge^2+a^2cdf^2h+abcf(cfh-d(fg+2eh))+(bdf(bdeg-acfh)-(bde-bcf-adf)(adfh-b(dfg+deh-cfh)))x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd}-\frac{2\sqrt{e+fx}(adfh-b(-cfh+deh+dfg))}{bd}}{bd}+\frac{2h(e+fx)^{3/2}}{3bd}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{\frac{d^2(be-af)^2(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{b^2(de-cf)^2(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{bd}-\frac{2\sqrt{e+fx}(adfh-b(-cfh+deh+dfg))}{bd}}{bd}+\frac{2h(e+fx)^{3/2}}{3bd}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{2d^2(be-af)^2(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2b^2(de-cf)^2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{bd}-\frac{2\sqrt{e+fx}(adfh-b(-cfh+deh+dfg))}{bd}}{bd}+\frac{2h(e+fx)^{3/2}}{3bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2b^2(de-cf)^{3/2}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)}-\frac{2d^2(be-af)^{3/2}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)}}{bd}-\frac{2\sqrt{e+fx}(adfh-b(-cfh+deh+dfg))}{bd}}{bd}+\frac{2h(e+fx)^{3/2}}{3bd}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(2*h*(e + f*x)^(3/2))/(3*b*d) + ((-2*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))
*Sqrt[e + f*x])/(b*d) + ((-2*d^2*(b*e - a*f)^(3/2)*(b*g - a*h)*ArcTanh[(Sq
rt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)) + (2*b^2*(d*e
 - c*f)^(3/2)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]]
)/(Sqrt[d]*(b*c - a*d)))/(b*d))/(b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{2\left(-\sqrt{(cf-de)d}\,d^{2}(ah-bg)(af-be)^{2}\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\sqrt{(af-be)b}\left(b^{2}(cf-de)^{2}(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right.\right.}{\sqrt{(af-be)b}\,\sqrt{(cf-de)d}\,b^{2}d^{2}(ad-bc)}$
risch	$-\frac{2(-hfdbx+3adfh+3bcfh-4bdeh-3bdfg)\sqrt{fx+e}}{3b^{2}d^{2}}+\frac{2d^{2}\left(f^{2}a^{3}h-2a^{2}befh-a^{2}bf^{2}g+ab^{2}e^{2}h+2ab^{2}efg-b^{3}e^{2}g\right)\arctan}{(ad-bc)\sqrt{(af-be)b}}$
derivativedivides	$-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}+bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^{2}d^{2}}+\frac{2(f^{2}a^{3}h-2a^{2}befh-a^{2}bf^{2}g+ab^{2}e}{b^{2}(ad-}$
default	$-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}+bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^{2}d^{2}}+\frac{2(f^{2}a^{3}h-2a^{2}befh-a^{2}bf^{2}g+ab^{2}e}{b^{2}(ad-}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*(-((c*f-d*e)*d)^(1/2)*d^2*(a*h-
b*g)*(a*f-b*e)^2*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)
^(1/2)*(b^2*(c*f-d*e)^2*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/
2))+((((-1/3*h*x-g)*f-4/3*e*h)*d+c*f*h)*b+a*d*f*h)*(a*d-b*c)*((c*f-d*e)*d)
^(1/2)*(f*x+e)^(1/2)))/b^2/d^2/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 1213, normalized size of antiderivative = 6.35

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/3*(3*((b^2*d^2*e - a*b*d^2*f)*g - (a*b*d^2*e - a^2*d^2*f)*h)*sqrt((b*e
 - a*f)/b)*log((b*f*x + 2*b*e - a*f + 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b
))/(b*x + a)) + 3*((b^2*d^2*e - b^2*c*d*f)*g - (b^2*c*d*e - b^2*c^2*f)*h)*
sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e
 - c*f)/d))/(d*x + c)) - 2*((b^2*c*d - a*b*d^2)*f*h*x + 3*(b^2*c*d - a*b*d
^2)*f*g + (4*(b^2*c*d - a*b*d^2)*e - 3*(b^2*c^2 - a^2*d^2)*f)*h)*sqrt(f*x
+ e))/(b^3*c*d^2 - a*b^2*d^3), -1/3*(6*((b^2*d^2*e - a*b*d^2*f)*g - (a*b*d
^2*e - a^2*d^2*f)*h)*sqrt(-(b*e - a*f)/b)*arctan(-sqrt(f*x + e)*b*sqrt(-(b
*e - a*f)/b)/(b*e - a*f)) + 3*((b^2*d^2*e - b^2*c*d*f)*g - (b^2*c*d*e - b^
2*c^2*f)*h)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)
*d*sqrt((d*e - c*f)/d))/(d*x + c)) - 2*((b^2*c*d - a*b*d^2)*f*h*x + 3*(b^2
*c*d - a*b*d^2)*f*g + (4*(b^2*c*d - a*b*d^2)*e - 3*(b^2*c^2 - a^2*d^2)*f)*
h)*sqrt(f*x + e))/(b^3*c*d^2 - a*b^2*d^3), 1/3*(6*((b^2*d^2*e - b^2*c*d*f)
*g - (b^2*c*d*e - b^2*c^2*f)*h)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)
*d*sqrt(-(d*e - c*f)/d)/(d*e - c*f)) - 3*((b^2*d^2*e - a*b*d^2*f)*g - (a*b
*d^2*e - a^2*d^2*f)*h)*sqrt((b*e - a*f)/b)*log((b*f*x + 2*b*e - a*f + 2*sq
rt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x + a)) + 2*((b^2*c*d - a*b*d^2)*f*h
*x + 3*(b^2*c*d - a*b*d^2)*f*g + (4*(b^2*c*d - a*b*d^2)*e - 3*(b^2*c^2 - a
^2*d^2)*f)*h)*sqrt(f*x + e))/(b^3*c*d^2 - a*b^2*d^3), -2/3*(3*((b^2*d^2*e
- a*b*d^2*f)*g - (a*b*d^2*e - a^2*d^2*f)*h)*sqrt(-(b*e - a*f)/b)*arctan...
```

Sympy [A] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.32

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \begin{cases} \frac{2\left(-\frac{f(cf-de)^2(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^3\sqrt{\frac{cf-de}{d}}(ad-bc)} + \frac{fh(e+fx)^{\frac{3}{2}}}{3bd} + \frac{\sqrt{e+fx}\left(-adf^2h-bcf^2h+bdefh+bdf^2g\right)}{b^2d^2} + \frac{f(af-be)^2 \cdots}{b \cdots}\right)}{f} & \\ e^{\frac{3}{2}}\left(\frac{(ah-bg)\left(\begin{cases} \frac{x}{a} & \text{for}\ b=0 \\ \frac{\log(a+bx)}{b} & \text{otherwise} \end{cases}\right)}{ad-bc} - \frac{(ch-dg)\left(\begin{cases} \frac{x}{c} & \text{for}\ d=0 \\ \frac{\log(c+dx)}{d} & \text{otherwise} \end{cases}\right)}{ad-bc}\right) & \end{cases}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
Piecewise((2*(-f*(c*f - d*e)**2*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f -
 d*e)/d))/(d**3*sqrt((c*f - d*e)/d)*(a*d - b*c)) + f*h*(e + f*x)**(3/2)/(3
*b*d) + sqrt(e + f*x)*(-a*d*f**2*h - b*c*f**2*h + b*d*e*f*h + b*d*f**2*g)/
(b**2*d**2) + f*(a*f - b*e)**2*(a*h - b*g)*atan(sqrt(e + f*x)/sqrt((a*f -
b*e)/b))/(b**3*sqrt((a*f - b*e)/b)*(a*d - b*c)))/f, Ne(f, 0)), (e**(3/2)*(
(a*h - b*g)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/(a*d - b*c)
 - (c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/(a*d - b
*c)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.61

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{2\left(b^3e^2g - 2\,ab^2efg + a^2bf^2g - ab^2e^2h + 2\,a^2befh - a^3f^2h\right)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+ab}}\right.}{(b^3c - ab^2d)\sqrt{-b^2e+abf}}$$
$$- \frac{2\left(d^3e^2g - 2\,cd^2efg + c^2df^2g - cd^2e^2h + 2\,c^2defh - c^3f^2h\right)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(bcd^2 - ad^3)\sqrt{-d^2e+cdf}}$$
$$+ \frac{2\left(3\sqrt{fx+e}b^2d^2fg + (fx+e)^{\frac{3}{2}}b^2d^2h + 3\sqrt{fx+e}b^2d^2eh - 3\sqrt{fx+e}b^2cdfh - 3\sqrt{fx+e}abd^2fh\right)}{3\,b^3d^3}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
2*(b^3*e^2*g - 2*a*b^2*e*f*g + a^2*b*f^2*g - a*b^2*e^2*h + 2*a^2*b*e*f*h -
 a^3*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c - a*b^2*d
)*sqrt(-b^2*e + a*b*f)) - 2*(d^3*e^2*g - 2*c*d^2*e*f*g + c^2*d*f^2*g - c*d
^2*e^2*h + 2*c^2*d*e*f*h - c^3*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e +
 c*d*f))/((b*c*d^2 - a*d^3)*sqrt(-d^2*e + c*d*f)) + 2/3*(3*sqrt(f*x + e)*b
^2*d^2*f*g + (f*x + e)^(3/2)*b^2*d^2*h + 3*sqrt(f*x + e)*b^2*d^2*e*h - 3*s
qrt(f*x + e)*b^2*c*d*f*h - 3*sqrt(f*x + e)*a*b*d^2*f*h)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 13380, normalized size of antiderivative = 70.05

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)),x)
```

output
```
(2*h*(e + f*x)^(3/2))/(3*b*d) - (atan(((((8*(e + f*x)^(1/2)*(a^6*d^6*f^6*h
^2 + b^6*c^6*f^6*h^2 + a^4*b^2*d^6*f^6*g^2 + b^6*c^4*d^2*f^6*g^2 + 2*b^6*d
^6*e^4*f^2*g^2 - 4*a^5*b*d^6*e*f^5*h^2 - 4*b^6*c^5*d*e*f^5*h^2 - 4*a*b^5*d
^6*e^3*f^3*g^2 - 4*a^3*b^3*d^6*e*f^5*g^2 - 4*b^6*c*d^5*e^3*f^3*g^2 - 4*b^6
*c^3*d^3*e*f^5*g^2 - 2*a^5*b*d^6*f^6*g*h - 2*b^6*c^5*d*f^6*g*h + 6*a^2*b^4
*d^6*e^2*f^4*g^2 + a^2*b^4*d^6*e^4*f^2*h^2 - 4*a^3*b^3*d^6*e^3*f^3*h^2 + 6
*a^4*b^2*d^6*e^2*f^4*h^2 + 6*b^6*c^2*d^4*e^2*f^4*g^2 + b^6*c^2*d^4*e^4*f^2
*h^2 - 4*b^6*c^3*d^3*e^3*f^3*h^2 + 6*b^6*c^4*d^2*e^2*f^4*h^2 - 2*a*b^5*d^6
*e^4*f^2*g*h + 8*a^4*b^2*d^6*e*f^5*g*h - 2*b^6*c*d^5*e^4*f^2*g*h + 8*b^6*c
^4*d^2*e*f^5*g*h + 8*a^2*b^4*d^6*e^3*f^3*g*h - 12*a^3*b^3*d^6*e^2*f^4*g*h
+ 8*b^6*c^2*d^4*e^3*f^3*g*h - 12*b^6*c^3*d^3*e^2*f^4*g*h))/(b^3*d^3) + (((
8*(a*b^6*c^3*d^4*f^5*g + a^3*b^4*c*d^6*f^5*g - a*b^6*c^4*d^3*f^5*h - a^4*b
^3*c*d^6*f^5*h - a^3*b^4*d^7*e*f^4*g + a^4*b^3*d^7*e*f^4*h - b^7*c^3*d^4*e
*f^4*g + b^7*c^4*d^3*e*f^4*h - 2*a^2*b^5*c^2*d^5*f^5*g + a^2*b^5*c^3*d^4*f
^5*h + a^3*b^4*c^2*d^5*f^5*h + a^2*b^5*d^7*e^2*f^3*g + a^2*b^5*d^7*e^3*f^2
*h - 2*a^3*b^4*d^7*e^2*f^3*h + b^7*c^2*d^5*e^2*f^3*g + b^7*c^2*d^5*e^3*f^2
*h - 2*b^7*c^3*d^4*e^2*f^3*h - 2*a*b^6*c*d^6*e^2*f^3*g + a*b^6*c^2*d^5*e*f
^4*g + a^2*b^5*c*d^6*e*f^4*g - 2*a*b^6*c*d^6*e^3*f^2*h + a*b^6*c^3*d^4*e*f
^4*h + a^3*b^4*c*d^6*e*f^4*h + 2*a*b^6*c^2*d^5*e^2*f^3*h + 2*a^2*b^5*c*d^6
*e^2*f^3*h - 4*a^2*b^5*c^2*d^5*e*f^4*h))/(b^3*d^3) - (8*(e + f*x)^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.68

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a^2d^3fh-2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)ab\,d}{}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
(2*(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**2*d**3*f*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a*b*d**3*e*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*d**3*f*g + 3*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*d**3*e*g -
3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e))
)*b**3*c**2*f*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*b**3*c*d*e*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
+ f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**3*c*d*f*g - 3*sqrt(d)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**3*d**2*e*g - 3*sq
rt(e + f*x)*a**2*b*d**3*f*h + 4*sqrt(e + f*x)*a*b**2*d**3*e*h + 3*sqrt(e +
 f*x)*a*b**2*d**3*f*g + sqrt(e + f*x)*a*b**2*d**3*f*h*x + 3*sqrt(e + f*x)*
b**3*c**2*d*f*h - 4*sqrt(e + f*x)*b**3*c*d**2*e*h - 3*sqrt(e + f*x)*b**3*c
*d**2*f*g - sqrt(e + f*x)*b**3*c*d**2*f*h*x))/(3*b**3*d**3*(a*d - b*c))
```

3.83 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 247

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \frac{f(bdg+2bch-3adh)\sqrt{e+fx}}{b^2d(bc-ad)} - \frac{(bg-ah)(e+fx)^{3/2}}{b(bc-ad)(a+bx)} - \frac{\sqrt{be-af}\,(3a^2dfh-abf(dg+5ch)-b^2(2deg-3cfg-2ceh))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}(bc-ad)^2} - \frac{2(de-cf)^{3/2}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(bc-ad)^2}$$

output
```
f*(-3*a*d*h+2*b*c*h+b*d*g)*(f*x+e)^(1/2)/b^2/d/(-a*d+b*c)-(-a*h+b*g)*(f*x+
e)^(3/2)/b/(-a*d+b*c)/(b*x+a)-(-a*f+b*e)^(1/2)*(3*a^2*d*f*h-a*b*f*(5*c*h+d
*g)-b^2*(-2*c*e*h-3*c*f*g+2*d*e*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*
e)^(1/2))/b^(5/2)/(-a*d+b*c)^2-2*(-c*f+d*e)^(3/2)*(-c*h+d*g)*arctanh(d^(1/
2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \frac{\frac{(bc-ad)\sqrt{e+fx}\left(-3a^2dfh+b^2(-deg+2cfhx)+ab(deh+2cfh+df(g-2hx))\right)}{b^2d(a+bx)} + \frac{\sqrt{-be+af}\left(-3a^2dfh+abf\right.}{}}{(bc-a}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(-3*a^2*d*f*h + b^2*(-(d*e*g) + 2*c*f*h*x) + a
*b*(d*e*h + 2*c*f*h + d*f*(g - 2*h*x))))/(b^2*d*(a + b*x)) + (Sqrt[-(b*e)
+ a*f]*(-3*a^2*d*f*h + a*b*f*(d*g + 5*c*h) + b^2*(2*d*e*g - 3*c*f*g - 2*c*
e*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/b^(5/2) + (2*(-(
d*e) + c*f)^(3/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) +
 c*f]])/d^(3/2))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx$$

$$\Big\downarrow \text{ 166}$$

$$\frac{\int -\frac{\sqrt{e+fx}\left(3acfh+2b\left(deg-\frac{3cfg}{2}-ceh\right)-f(bdg+2bch-3adh)x\right)}{2(a+bx)(c+dx)}dx}{b(bc-ad)} - \frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sqrt{e+fx}(3acfh+b(2deg-3cfg-2ceh)-f(bdg+2bch-3adh)x)}{(a+bx)(c+dx)}dx}{2b(bc-ad)} - \frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{171}$$

$$-\frac{\frac{2\int \frac{ac(bdg+2bch-3adh)f^2-\left(-\left(\left(2fhc^2-2d(fg+2eh)c+d^2eg\right)b^2\right)-ad(dfg+3deh+2cfh)b+3a^2d^2fh\right)xf+bde(3acfh+b(2deg-3cfg-2ceh))}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd}-\frac{2f\sqrt{e+f}}{}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{ac(bdg+2bch-3adh)f^2-\left(-\left(\left(2fhc^2-2d(fg+2eh)c+d^2eg\right)b^2\right)-ad(dfg+3deh+2cfh)b+3a^2d^2fh\right)xf+bde(3acfh+b(2deg-3cfg-2ceh))}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd}-\frac{2f\sqrt{e+fx}}{}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{174}$$

$$-\frac{\frac{-\frac{d(be-af)\left(3a^2dfh-abf(5ch+dg)-b^2(-2ceh-3cfg+2deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{2b^2(de-cf)^2(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{bd}-\frac{2f\sqrt{e+fx}(-3adh+2bch}{bd}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{-\frac{2d(be-af)\left(3a^2dfh-abf(5ch+dg)-b^2(-2ceh-3cfg+2deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{4b^2(de-cf)^2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{bd}-\frac{2f\sqrt{e+f}}{}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{2d\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^2dfh-abf(5ch+dg)-b^2(-2ceh-3cfg+2deg)\right)}{\sqrt{b}(bc-ad)}+\frac{4b^2(de-cf)^{3/2}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)}}{bd}-\frac{2f\sqrt{e+fx}(-}{}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(bc-ad)}$$

input

```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)),x]
```

output
```
-(((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x))) - ((-2*f*(b*d*g
 + 2*b*c*h - 3*a*d*h)*Sqrt[e + f*x])/(b*d) + ((2*d*Sqrt[b*e - a*f]*(3*a^2*
d*f*h - a*b*f*(d*g + 5*c*h) - b^2*(2*d*e*g - 3*c*f*g - 2*c*e*h))*ArcTanh[(
Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)) + (4*b^2*(d
*e - c*f)^(3/2)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f
]])/(Sqrt[d]*(b*c - a*d)))/(b*d))/(2*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{-3\left(\left(a^{2} f h-\frac{1}{3} b f g a-\frac{2}{3} b^{2} e g\right) d-\frac{5 c\left(\frac{(-2 e h-3 f g) b}{5}+a f h\right) b}{3}\right) d(b x+a)(a f-b e) \sqrt{(c f-d e) d}\, \arctan\left(\frac{b \sqrt{f x+e}}{\sqrt{(a f-b e) b}}\right)+3 \sqrt{(a f}}{d\, b^{2}(b x+}$
derivativedivides	$2 f\left(\frac{h \sqrt{f x+e}}{d\, b^{2}}-\frac{\frac{\left(-\frac{1}{2} a^{3} d\, f^{2} h+\frac{1}{2} a^{2} b c\, f^{2} h+\frac{1}{2} a^{2} b d e f h+\frac{1}{2} a^{2} b d\, f^{2} g-\frac{1}{2} a\, b^{2} c e f h-\frac{1}{2} a\, b^{2} c\, f^{2} g-\frac{1}{2} a\, b^{2} d e f g+\frac{1}{2} b^{3} c e f g\right) \sqrt{f x+e}}{(f x+e) b+a f-b e}}{}\right.$
default	$2 f\left(\frac{h \sqrt{f x+e}}{d\, b^{2}}-\frac{\frac{\left(-\frac{1}{2} a^{3} d\, f^{2} h+\frac{1}{2} a^{2} b c\, f^{2} h+\frac{1}{2} a^{2} b d e f h+\frac{1}{2} a^{2} b d\, f^{2} g-\frac{1}{2} a\, b^{2} c e f h-\frac{1}{2} a\, b^{2} c\, f^{2} g-\frac{1}{2} a\, b^{2} d e f g+\frac{1}{2} b^{3} c e f g\right) \sqrt{f x+e}}{(f x+e) b+a f-b e}}{}\right.$
risch	$\frac{2 h \sqrt{f x+e}\, f}{d\, b^{2}}-\frac{2 f\left(\frac{d\left(\frac{\left(-\frac{1}{2} a^{3} d\, f^{2} h+\frac{1}{2} a^{2} b c\, f^{2} h+\frac{1}{2} a^{2} b d e f h+\frac{1}{2} a^{2} b d\, f^{2} g-\frac{1}{2} a\, b^{2} c e f h-\frac{1}{2} a\, b^{2} c\, f^{2} g-\frac{1}{2} a\, b^{2} d e f g+\frac{1}{2} b^{3} c e f g\right) \sqrt{f}}{(f x+e) b+a f-b e}\right.}{}\right.}{}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3*(-((a^2*f*h-1/3*b*f*g*a-2/3*b^2*e*g)*d-5/3*c*(1/5*(-2*e*h-3*f*g)*b+a*f*h
)*b)*d*(b*x+a)*(a*f-b*e)*((c*f-d*e)*d)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-
b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*(-2/3*b^2*(c*f-d*e)^2*(c*h-d*g)*(b*x+a)
*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((1/3*b^2*e*g-1/3*a
*((-2*h*x+g)*f+e*h)*b+a^2*f*h)*d-2/3*b*c*f*h*(b*x+a))*(f*x+e)^(1/2)*((c*f-
d*e)*d)^(1/2)))/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)/d/b^2/(b*x+a)/(a*d
-b*c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(223) = 446.

Time = 12.39 (sec) , antiderivative size = 2136, normalized size of antiderivative = 8.65

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
[-1/2*(((2*a*b^2*d^2*e - (3*a*b^2*c*d - a^2*b*d^2)*f)*g - (2*a*b^2*c*d*e -
 (5*a^2*b*c*d - 3*a^3*d^2)*f)*h + ((2*b^3*d^2*e - (3*b^3*c*d - a*b^2*d^2)*
f)*g - (2*b^3*c*d*e - (5*a*b^2*c*d - 3*a^2*b*d^2)*f)*h)*x)*sqrt((b*e - a*f
)/b)*log((b*f*x + 2*b*e - a*f - 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*
x + a)) - 2*((a*b^2*d^2*e - a*b^2*c*d*f)*g - (a*b^2*c*d*e - a*b^2*c^2*f)*h
 + ((b^3*d^2*e - b^3*c*d*f)*g - (b^3*c*d*e - b^3*c^2*f)*h)*x)*sqrt((d*e -
c*f)/d)*log((d*f*x + 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/
(d*x + c)) - 2*(2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f*h*x - ((b^3*c*d -
a*b^2*d^2)*e - (a*b^2*c*d - a^2*b*d^2)*f)*g + ((a*b^2*c*d - a^2*b*d^2)*e +
 (2*a*b^2*c^2 - 5*a^2*b*c*d + 3*a^3*d^2)*f)*h)*sqrt(f*x + e))/(a*b^4*c^2*d
 - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3 + (b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^
3)*x), (((2*a*b^2*d^2*e - (3*a*b^2*c*d - a^2*b*d^2)*f)*g - (2*a*b^2*c*d*e
- (5*a^2*b*c*d - 3*a^3*d^2)*f)*h + ((2*b^3*d^2*e - (3*b^3*c*d - a*b^2*d^2)
*f)*g - (2*b^3*c*d*e - (5*a*b^2*c*d - 3*a^2*b*d^2)*f)*h)*x)*sqrt(-(b*e - a
*f)/b)*arctan(-sqrt(f*x + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) + ((a*b^2
*d^2*e - a*b^2*c*d*f)*g - (a*b^2*c*d*e - a*b^2*c^2*f)*h + ((b^3*d^2*e - b^
3*c*d*f)*g - (b^3*c*d*e - b^3*c^2*f)*h)*x)*sqrt((d*e - c*f)/d)*log((d*f*x
+ 2*d*e - c*f - 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)) + (2*(b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f*h*x - ((b^3*c*d - a*b^2*d^2)*e - (a*b^2
*c*d - a^2*b*d^2)*f)*g + ((a*b^2*c*d - a^2*b*d^2)*e + (2*a*b^2*c^2 - 5*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**2/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.62

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx =$$

$$-\frac{(2\,b^3de^2g - 3\,b^3cefg - ab^2defg + 3\,ab^2cf^2g - a^2bdf^2g - 2\,b^3ce^2h + 7\,ab^2cefh - 3\,a^2bdefh - 5\,a^2bcf^2h +}{(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)\sqrt{-b^2e + abf}}$$
$$+\frac{2\,(d^3e^2g - 2\,cd^2efg + c^2df^2g - cd^2e^2h + 2\,c^2defh - c^3f^2h)\arctan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{-d^2e+cdf}}$$
$$+\frac{2\sqrt{fx+e}\,fh}{b^2d} - \frac{\sqrt{fx+e}\,b^2efg - \sqrt{fx+e}\,abf^2g - \sqrt{fx+e}\,abefh + \sqrt{fx+e}\,a^2f^2h}{(b^3c - ab^2d)((fx+e)b - be + af)}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
-(2*b^3*d*e^2*g - 3*b^3*c*e*f*g - a*b^2*d*e*f*g + 3*a*b^2*c*f^2*g - a^2*b*
d*f^2*g - 2*b^3*c*e^2*h + 7*a*b^2*c*e*f*h - 3*a^2*b*d*e*f*h - 5*a^2*b*c*f^
2*h + 3*a^3*d*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(-b^2*e + a*b*f)) + 2*(d^3*e^2*g - 2*c*
d^2*e*f*g + c^2*d*f^2*g - c*d^2*e^2*h + 2*c^2*d*e*f*h - c^3*f^2*h)*arctan(
sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*sqrt(-d^2*e + c*d*f)) + 2*sqrt(f*x + e)*f*h/(b^2*d) - (sqrt(f*x + e)*b^2*
e*f*g - sqrt(f*x + e)*a*b*f^2*g - sqrt(f*x + e)*a*b*e*f*h + sqrt(f*x + e)*
a^2*f^2*h)/((b^3*c - a*b^2*d)*((f*x + e)*b - b*e + a*f))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 18554, normalized size of antiderivative = 75.12

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)),x)
```

output
```
atan(((((2*(2*a*b^8*c^5*d^3*f^5*g + 2*a^5*b^4*c*d^7*f^5*g + 4*a*b^8*c^6*d^
2*f^5*h - 6*a^6*b^3*c*d^7*f^5*h - 2*a^5*b^4*d^8*e*f^4*g + 6*a^6*b^3*d^8*e*
f^4*h - 2*b^9*c^5*d^3*e*f^4*g - 4*b^9*c^6*d^2*e*f^4*h - 8*a^2*b^7*c^4*d^4*
f^5*g + 12*a^3*b^6*c^3*d^5*f^5*g - 8*a^4*b^5*c^2*d^6*f^5*g - 22*a^2*b^7*c^
5*d^3*f^5*h + 48*a^3*b^6*c^4*d^4*f^5*h - 52*a^4*b^5*c^3*d^5*f^5*h + 28*a^5
*b^4*c^2*d^6*f^5*h + 2*a^4*b^5*d^8*e^2*f^3*g - 6*a^5*b^4*d^8*e^2*f^3*h + 2
*b^9*c^4*d^4*e^2*f^3*g + 4*b^9*c^5*d^3*e^2*f^3*h + 12*a^2*b^7*c^2*d^6*e^2*
f^3*g + 48*a^2*b^7*c^3*d^5*e^2*f^3*h - 52*a^3*b^6*c^2*d^6*e^2*f^3*h + 6*a*
b^8*c^4*d^4*e*f^4*g + 6*a^4*b^5*c*d^7*e*f^4*g + 18*a*b^8*c^5*d^3*e*f^4*h -
 22*a^5*b^4*c*d^7*e*f^4*h - 8*a*b^8*c^3*d^5*e^2*f^3*g - 4*a^2*b^7*c^3*d^5*
e*f^4*g - 8*a^3*b^6*c*d^7*e^2*f^3*g - 4*a^3*b^6*c^2*d^6*e*f^4*g - 22*a*b^8
*c^4*d^4*e^2*f^3*h - 26*a^2*b^7*c^4*d^4*e*f^4*h + 4*a^3*b^6*c^3*d^5*e*f^4*
h + 28*a^4*b^5*c*d^7*e^2*f^3*h + 24*a^4*b^5*c^2*d^6*e*f^4*h))/(b^6*c^3*d -
 a^3*b^3*d^4 - 3*a*b^5*c^2*d^2 + 3*a^2*b^4*c*d^3) - (2*(e + f*x)^(1/2)*((4
*b^5*c^2*e^3*h^2 + 4*b^5*d^2*e^3*g^2 - 9*a^5*d^2*f^3*h^2 - 25*a^3*b^2*c^2*
f^3*h^2 - a^3*b^2*d^2*f^3*g^2 - 9*a*b^4*c^2*f^3*g^2 + 9*b^5*c^2*e*f^2*g^2
+ 6*a^2*b^3*c*d*f^3*g^2 - 24*a*b^4*c^2*e^2*f*h^2 + 9*a^4*b*d^2*e*f^2*h^2 +
 30*a^2*b^3*c^2*f^3*g*h - 8*b^5*c*d*e^3*g*h + 45*a^2*b^3*c^2*e*f^2*h^2 - 3
*a^2*b^3*d^2*e*f^2*g^2 + 30*a^4*b*c*d*f^3*h^2 - 12*b^5*c*d*e^2*f*g^2 + 6*a
^4*b*d^2*f^3*g*h + 12*b^5*c^2*e^2*f*g*h + 6*a*b^4*c*d*e*f^2*g^2 - 28*a^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1199, normalized size of antiderivative = 4.85

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c),x)
```

output
```
( - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*d**3*f*h + 5*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**2*b*c*d**2*f*h + sqrt(b)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**3*f*g - 3*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**
3*f*h*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a*b**2*c*d**2*e*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d**2*f*g + 5*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d**2*f*h*
x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a*b**2*d**3*e*g + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a*b**2*d**3*f*g*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c*d**2*e*h*x - 3*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c
*d**2*f*g*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*b**3*d**3*e*g*x - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**3*c**2*f*h + 2*sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**3*c*d*e*h + 2
*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a*b**3*c*d*f*g - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqr...
```

3.84 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 348

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = -\frac{(3a^2dfh+abf(dg-7ch)-b^2(4deg-3cfg-4ceh))\sqrt{e+fx}}{4b^2(bc-ad)^2(a+bx)} - \frac{(bg-ah)(e+fx)^{3/2}}{2b(bc-ad)(a+bx)^2}$$
$$+\frac{(3a^3d^2f^2h+a^2bdf^2(dg-10ch)+ab^2f(4d^2eg+15c^2fh-2cd(3fg+2eh))-b^3(8d^2e^2g-4cde(3fg+2e}{4b^{5/2}(bc-ad)^3\sqrt{be-af}}$$
$$+\frac{2(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)^3}$$

output

```
-1/4*(3*a^2*d*f*h+a*b*f*(-7*c*h+d*g)-b^2*(-4*c*e*h-3*c*f*g+4*d*e*g))*(f*x+
e)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)-1/2*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c
)/(b*x+a)^2+1/4*(3*a^3*d^2*f^2*h+a^2*b*d*f^2*(-10*c*h+d*g)+a*b^2*f*(4*d^2*
e*g+15*c^2*f*h-2*c*d*(2*e*h+3*f*g))-b^3*(8*d^2*e^2*g-4*c*d*e*(2*e*h+3*f*g)
+3*c^2*f*(4*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(
5/2)/(-a*d+b*c)^3/(-a*f+b*e)^(1/2)+2*(-c*f+d*e)^(3/2)*(-c*h+d*g)*arctanh(d
^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \frac{1}{4}\left(-\frac{\sqrt{e+fx}(3a^3dfh+b^3(-4degx+5cfgx+2ce(g+2hx))+a^2b(2deh-}{b^2(bc-ad)^2}\right.$$
$$+\frac{(-3a^3d^2f^2h+a^2bdf^2(-dg+10ch)+ab^2f(-4d^2eg+6cdfg+4cdeh-15c^2fh)+b^3(8d^2e^2g-4cde(3fg}{b^{5/2}(bc-ad)^3\sqrt{-be+af}}$$
$$\left.+\frac{8(-de+cf)^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-bc+ad)^3}\right)$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)),x]
```

output
```
(-((Sqrt[e + f*x]*(3*a^3*d*f*h + b^3*(-4*d*e*g*x + 5*c*f*g*x + 2*c*e*(g +
2*h*x)) + a^2*b*(2*d*e*h - 7*c*f*h + d*f*(g + 5*h*x)) - a*b^2*(d*g*(6*e +
f*x) + c*(-3*f*g - 2*e*h + 9*f*h*x))))/(b^2*(b*c - a*d)^2*(a + b*x)^2)) +
((-3*a^3*d^2*f^2*h + a^2*b*d*f^2*(-(d*g) + 10*c*h) + a*b^2*f*(-4*d^2*e*g +
 6*c*d*f*g + 4*c*d*e*h - 15*c^2*f*h) + b^3*(8*d^2*e^2*g - 4*c*d*e*(3*f*g +
 2*e*h) + 3*c^2*f*(f*g + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*
e) + a*f]])/(b^(5/2)*(b*c - a*d)^3*Sqrt[-(b*e) + a*f]) + (8*(-(d*e) + c*f)
^(3/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sq
rt[d]*(-(b*c) + a*d)^3))/4
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{\sqrt{e+fx}(3acfh+b(4deg-3cfg-4ceh)+f(bdg-4bch+3adh)x)}{2(a+bx)^2(c+dx)}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{e+fx}(3acfh+b(4deg-3cfg-4ceh)+f(bdg-4bch+3adh)x)}{(a+bx)^2(c+dx)}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{166}$$

$$-\frac{\frac{\int -\frac{\left(3f(fg+4eh)c^2-4de(3fg+2eh)c+8d^2e^2g\right)b^2+acf^2(dg-7ch)b+3a^2cdf^2h+f\left(\left(8fhc^2-d(5fg+4eh)c+4d^2eg\right)b^2+adf(dg-7ch)b+3a^2d^2fh\right)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{b(bc-ad)}+\sqrt{e+\cdots}}{4b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\sqrt{e+fx}(3a^2dfh+abf(dg-7ch)-b^2(-4ceh-3cfg+4deg))}{b(a+bx)(bc-ad)}-\frac{\int \frac{\left(3f(fg+4eh)c^2-4de(3fg+2eh)c+8d^2e^2g\right)b^2+acf^2(dg-7ch)b+3a^2cdf^2h+f\left(\left(8fhc^2-\cdots\right.\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{2b(bc-ad)}}{4b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{174}$$

$$-\frac{\frac{\sqrt{e+fx}(3a^2dfh+abf(dg-7ch)-b^2(-4ceh-3cfg+4deg))}{b(a+bx)(bc-ad)}-\frac{-\frac{\left(3a^3d^2f^2h+a^2bdf^2(dg-10ch)+ab^2f\left(15c^2fh-4cdeh-6cdfg+4d^2eg\right)-b^3\left(3c^2f(4eh+\cdots\right.\right.}{bc-ad}}{2b\cdots}}{4b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{\sqrt{e+fx}(3a^2dfh+abf(dg-7ch)-b^2(-4ceh-3cfg+4deg))}{b(a+bx)(bc-ad)}-\frac{-\frac{2\left(3a^3d^2f^2h+a^2bdf^2(dg-10ch)+ab^2f\left(15c^2fh-4cdeh-6cdfg+4d^2eg\right)-b^3\left(3c^2f(4eh+\cdots\right.\right.}{f(bc-ad)}}{\cdots}}{4b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\sqrt{e+fx}\left(3a^2dfh+abf(dg-7ch)-b^2(-4ceh-3cfg+4deg)\right)}{b(a+bx)(bc-ad)}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^3d^2f^2h+a^2bdf^2(dg-10ch)+ab^2f\left(15c^2fh-4cdeh-6cdfg+\right.\right.}{\sqrt{b}(bc-ad)\sqrt{be-af}}}{}}{4b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)),x]
```

output
```
-1/2*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^2) - (((3*a^2*
d*f*h + a*b*f*(d*g - 7*c*h) - b^2*(4*d*e*g - 3*c*f*g - 4*c*e*h))*Sqrt[e +
f*x])/(b*(b*c - a*d)*(a + b*x)) - ((2*(3*a^3*d^2*f^2*h + a^2*b*d*f^2*(d*g
- 10*c*h) + a*b^2*f*(4*d^2*e*g - 6*c*d*f*g - 4*c*d*e*h + 15*c^2*f*h) - b^3
*(8*d^2*e^2*g - 4*c*d*e*(3*f*g + 2*e*h) + 3*c^2*f*(f*g + 4*e*h)))*ArcTanh[
(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e -
a*f]) + (16*b^2*(d*e - c*f)^(3/2)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*
x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(b*c - a*d)))/(2*b*(b*c - a*d)))/(4*b*(b*c
- a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\frac{3\sqrt{(cf-de)d}\,(bx+a)^2\left(\left(\left(-\frac{8d^2e^2g}{3}+4\left(\frac{2}{3}e^2h+efg\right)cd+\left(-4efh-f^2g\right)c^2\right)b^3+5a\left(\frac{4d^2eg}{15}-\frac{4c\left(eh+\frac{3fg}{2}\right)d}{15}+c^2fh\right)fb^2-\frac{10a^2d\left(ch\right.}{3}\right.\right.}{4}}{}$
derivativedivides	$2f^2\left(\frac{-\frac{f\left(5a^3d^2fh-14a^2bcdfh-a^2bd^2fg+9ab^2c^2fh+4ab^2cdeh+6ab^2cdfg-4ab^2d^2eg-4b^3c^2eh-5b^3c^2fg+4b^3cdeg\right)(fx+e)}{8b}}{}\right.$
default	$2f^2\left(\frac{-\frac{f\left(5a^3d^2fh-14a^2bcdfh-a^2bd^2fg+9ab^2c^2fh+4ab^2cdeh+6ab^2cdfg-4ab^2d^2eg-4b^3c^2eh-5b^3c^2fg+4b^3cdeg\right)(fx+e)}{8b}}{}\right.$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/4*(((c*f-d*e)*d)^(1/2)*(b*x+a)^2*((-8/3*d^2*e^2*g+4*(2/3*e^2*h+e*f*g)*c*
d+(-4*e*f*h-f^2*g)*c^2)*b^3+5*a*(4/15*d^2*e*g-4/15*c*(e*h+3/2*f*g)*d+c^2*f
*h)*f*b^2-10/3*a^2*d*(c*h-1/10*d*g)*f^2*b+a^3*d^2*f^2*h)*arctan(b*(f*x+e)^
(1/2)/((a*f-b*e)*b)^(1/2))-(8/3*(c*f-d*e)^2*(b*x+a)^2*(c*h-d*g)*b^2*arctan
(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((c*f-d*e)*d)^(1/2)*(2/3*(
-2*d*e*g*x+c*(5/2*f*g*x+e*(2*h*x+g)))*b^3+2/3*a*(-3*(1/6*f*x+e)*g*d+(3/2*(
-3*h*x+g)*f+e*h)*c)*b^2-7/3*a^2*(1/7*((-5*h*x-g)*f-2*e*h)*d+c*f*h)*b+a^3*d
*f*h)*(f*x+e)^(1/2))*((a*f-b*e)*b)^(1/2))/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d
)^(1/2)/(b*x+a)^2/(a*d-b*c)^3/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1541 vs. $2(318) = 636$.

Time = 31.95 (sec) , antiderivative size = 6207, normalized size of antiderivative = 17.84

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**3/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(318) = 636$.

Time = 0.17 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.97

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \frac{(8\,b^3d^2e^2g - 12\,b^3cdefg - 4\,ab^2d^2efg + 3\,b^3c^2f^2g + 6\,ab^2cdf^2g - a^2bd^2f^2g -}{4\,(b^5c^3 - 3\,ab^4c}$$
$$- \frac{2\,(d^3e^2g - 2\,cd^2efg + c^2df^2g - cd^2e^2h + 2\,c^2defh - c^3f^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{-d^2e+cdf}}$$
$$+ \frac{4\,(fx+e)^{\frac{3}{2}}b^3defg - 4\,\sqrt{fx+e}b^3de^2fg - 5\,(fx+e)^{\frac{3}{2}}b^3cf^2g + (fx+e)^{\frac{3}{2}}ab^2df^2g + 3\,\sqrt{fx+e}b^3cef^2g +}{}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g - 12*b^3*c*d*e*f*g - 4*a*b^2*d^2*e*f*g + 3*b^3*c^2*f^
2*g + 6*a*b^2*c*d*f^2*g - a^2*b*d^2*f^2*g - 8*b^3*c*d*e^2*h + 12*b^3*c^2*e
*f*h + 4*a*b^2*c*d*e*f*h - 15*a*b^2*c^2*f^2*h + 10*a^2*b*c*d*f^2*h - 3*a^3
*d^2*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*c^3 - 3*a*b
^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*e + a*b*f)) - 2*(d^3*e
^2*g - 2*c*d^2*e*f*g + c^2*d*f^2*g - c*d^2*e^2*h + 2*c^2*d*e*f*h - c^3*f^2
*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-d^2*e + c*d*f)) + 1/4*(4*(f*x + e)^(3/2)
*b^3*d*e*f*g - 4*sqrt(f*x + e)*b^3*d*e^2*f*g - 5*(f*x + e)^(3/2)*b^3*c*f^2
*g + (f*x + e)^(3/2)*a*b^2*d*f^2*g + 3*sqrt(f*x + e)*b^3*c*e*f^2*g + 5*sqr
t(f*x + e)*a*b^2*d*e*f^2*g - 3*sqrt(f*x + e)*a*b^2*c*f^3*g - sqrt(f*x + e)
*a^2*b*d*f^3*g - 4*(f*x + e)^(3/2)*b^3*c*e*f*h + 4*sqrt(f*x + e)*b^3*c*e^2
*f*h + 9*(f*x + e)^(3/2)*a*b^2*c*f^2*h - 5*(f*x + e)^(3/2)*a^2*b*d*f^2*h -
 11*sqrt(f*x + e)*a*b^2*c*e*f^2*h + 3*sqrt(f*x + e)*a^2*b*d*e*f^2*h + 7*sq
rt(f*x + e)*a^2*b*c*f^3*h - 3*sqrt(f*x + e)*a^3*d*f^3*h)/((b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*((f*x + e)*b - b*e + a*f)^2)
```

Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 283348, normalized size of antiderivative = 814.22

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)),x)
```

output
```
- atan(((((136*a*b^10*c^7*d^3*f^5*g - 24*b^11*c^8*d^2*f^5*g - 8*a^7*b^4*c*
d^9*f^5*g + 56*a*b^10*c^8*d^2*f^5*h - 24*a^8*b^3*c*d^9*f^5*h + 8*a^7*b^4*d
^10*e*f^4*g + 24*a^8*b^3*d^10*e*f^4*h + 56*b^11*c^7*d^3*e*f^4*g - 32*b^11*
c^8*d^2*e*f^4*h - 312*a^2*b^9*c^6*d^4*f^5*g + 360*a^3*b^8*c^5*d^5*f^5*g -
200*a^4*b^7*c^4*d^6*f^5*g + 24*a^5*b^6*c^3*d^7*f^5*g + 24*a^6*b^5*c^2*d^8*
f^5*g - 360*a^2*b^9*c^7*d^3*f^5*h + 984*a^3*b^8*c^6*d^4*f^5*h - 1480*a^4*b
^7*c^5*d^5*f^5*h + 1320*a^5*b^6*c^4*d^6*f^5*h - 696*a^6*b^5*c^3*d^7*f^5*h
+ 200*a^7*b^4*c^2*d^8*f^5*h - 32*a^6*b^5*d^10*e^2*f^3*g - 32*b^11*c^6*d^4*
e^2*f^3*g + 32*b^11*c^7*d^3*e^2*f^3*h - 480*a^2*b^9*c^4*d^6*e^2*f^3*g + 64
0*a^3*b^8*c^3*d^7*e^2*f^3*g - 480*a^4*b^7*c^2*d^8*e^2*f^3*g + 480*a^2*b^9*
c^5*d^5*e^2*f^3*h - 640*a^3*b^8*c^4*d^6*e^2*f^3*h + 480*a^4*b^7*c^3*d^7*e^
2*f^3*h - 192*a^5*b^6*c^2*d^8*e^2*f^3*h - 328*a*b^10*c^6*d^4*e*f^4*g + 8*a
^6*b^5*c*d^9*e*f^4*g + 136*a*b^10*c^7*d^3*e*f^4*h - 200*a^7*b^4*c*d^9*e*f^
4*h + 192*a*b^10*c^5*d^5*e^2*f^3*g + 792*a^2*b^9*c^5*d^5*e*f^4*g - 1000*a^
3*b^8*c^4*d^6*e*f^4*g + 680*a^4*b^7*c^3*d^7*e*f^4*g + 192*a^5*b^6*c*d^9*e^
2*f^3*g - 216*a^5*b^6*c^2*d^8*e*f^4*g - 192*a*b^10*c^6*d^4*e^2*f^3*h - 120
*a^2*b^9*c^6*d^4*e*f^4*h - 344*a^3*b^8*c^5*d^5*e*f^4*h + 1000*a^4*b^7*c^4*
d^6*e*f^4*h - 1128*a^5*b^6*c^3*d^7*e*f^4*h + 32*a^6*b^5*c*d^9*e^2*f^3*h +
664*a^6*b^5*c^2*d^8*e*f^4*h)/(8*(b^9*c^6 + a^6*b^3*d^6 - 6*a^5*b^4*c*d^5 +
 15*a^2*b^7*c^4*d^2 - 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*d^4 - 6*a*b^8...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4005, normalized size of antiderivative = 11.51

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c),x)
```

output

```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**5*d**3*f**2*h - 10*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**2*f**2*h + sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**3*f**2*g + 6*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
4*b*d**3*f**2*h*x + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*d*f**2*h - 4*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d**2*e*f*h
- 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**3*b**2*c*d**2*f**2*g - 20*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d**2*f**2*h*x + 4*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d
**3*e*f*g + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**3*b**2*d**3*f**2*g*x + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**3*f**2*h*x**2 - 12*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**2*b**3*c**2*d*e*f*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c**2*d*f**2*g + 30*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c**2*d*f
**2*h*x + 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr...
```

3.85 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 579

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx =$$

$$-\frac{(a^2dfh+abf(dg-3ch)-b^2(2deg-cfg-2ceh))\sqrt{e+fx}}{4b^2(bc-ad)^2(a+bx)^2}$$

$$+\frac{(a^3d^2f^2h+a^2bdf^2(dg-4ch)+ab^2f(6d^2eg+11c^2fh-2cd(4fg+3eh))-b^3(8d^2e^2g-2cde(5fg+4eh)}{8b^2(bc-ad)^3(be-af)(a+bx)}$$

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3}$$

$$+\frac{(a^4d^3f^3h+a^3bd^2f^3(dg-5ch)+3a^2b^2df^2(2d^2eg+5c^2fh-cd(3fg+2eh))-ab^3f(24d^3e^2g-5c^3f^2h-}{}$$

$$-\frac{2\sqrt{d}(de-cf)^{3/2}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4}$$

output
```
-1/4*(a^2*d*f*h+a*b*f*(-3*c*h+d*g)-b^2*(-2*c*e*h-c*f*g+2*d*e*g))*(f*x+e)^(
1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^2+1/8*(a^3*d^2*f^2*h+a^2*b*d*f^2*(-4*c*h+d*g
)+a*b^2*f*(6*d^2*e*g+11*c^2*f*h-2*c*d*(3*e*h+4*f*g))-b^3*(8*d^2*e^2*g-2*c*
d*e*(4*e*h+5*f*g)+c^2*f*(10*e*h+f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^3/(-a*
f+b*e)/(b*x+a)-1/3*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)^3+1/8*(a^
4*d^3*f^3*h+a^3*b*d^2*f^3*(-5*c*h+d*g)+3*a^2*b^2*d*f^2*(2*d^2*e*g+5*c^2*f*
h-c*d*(2*e*h+3*f*g))-a*b^3*f*(24*d^3*e^2*g-5*c^3*f^2*h-12*c*d^2*e*(2*e*h+3
*f*g)+9*c^2*d*f*(4*e*h+f*g))+b^4*(16*d^3*e^3*g+c^3*f^2*(-6*e*h+f*g)-8*c*d^
2*e^2*(2*e*h+3*f*g)+6*c^2*d*e*f*(4*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2
)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*d+b*c)^4/(-a*f+b*e)^(3/2)-2*d^(1/2)*(-c*f+
d*e)^(3/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*
d+b*c)^4
```

Mathematica [A] (verified)

Time = 5.91 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.24

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \frac{(bc-ad)\sqrt{e+fx}\left(-3a^5d^2f^2h-a^4bdf(-12cfh+d(3fg+2eh+8fhx))+a^2b^3\left(c^2f(3fg-8eh+40fhx)+d^2g(-\right.\right.}{}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(-3*a^5*d^2*f^2*h - a^4*b*d*f*(-12*c*f*h + d*(
3*f*g + 2*e*h + 8*f*h*x)) + a^2*b^3*(c^2*f*(3*f*g - 8*e*h + 40*f*h*x) + d^
2*g*(-44*e^2 + 46*e*f*x + 3*f^2*x^2) + 2*c*d*(10*e^2*h + e*f*(g - 44*h*x)
- 2*f^2*x*(16*g + 3*h*x))) + a^3*b^2*(15*c^2*f^2*h - 2*c*d*f*(12*f*g + 19*
e*h - 8*f*h*x) + d^2*(8*e^2*h + f^2*x*(8*g + 3*h*x) + 2*e*f*(22*g + 7*h*x)
)) - b^5*(24*d^2*e^2*g*x^2 - 6*c*d*e*x*(5*f*g*x + 2*e*(g + 2*h*x)) + c^2*(
3*f^2*g*x^2 + 4*e^2*(2*g + 3*h*x) + 2*e*f*x*(7*g + 15*h*x))) + a*b^4*(6*d^
2*e*g*x*(-10*e + 3*f*x) + 2*c*d*(-12*f^2*g*x^2 + e*f*x*(32*g - 9*h*x) + 2*
e^2*(7*g + 15*h*x)) + c^2*(-4*e^2*h + 2*e*f*(g - 11*h*x) + f^2*x*(8*g + 33
*h*x)))))/(b^2*(b*e - a*f)*(a + b*x)^3) + (3*(a^4*d^3*f^3*h + a^3*b*d^2*f^
3*(d*g - 5*c*h) + 3*a^2*b^2*d*f^2*(2*d^2*e*g + 5*c^2*f*h - c*d*(3*f*g + 2*
e*h)) + a*b^3*f*(-24*d^3*e^2*g + 5*c^3*f^2*h + 12*c*d^2*e*(3*f*g + 2*e*h)
- 9*c^2*d*f*(f*g + 4*e*h)) + b^4*(16*d^3*e^3*g + c^3*f^2*(f*g - 6*e*h) - 8
*c*d^2*e^2*(3*f*g + 2*e*h) + 6*c^2*d*e*f*(f*g + 4*e*h)))*ArcTan[(Sqrt[b]*S
qrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(5/2)*(-(b*e) + a*f)^(3/2)) + 48*Sqr
t[d]*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[
-(d*e) + c*f]])/(24*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {166, 27, 166, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{3\sqrt{e+fx}(2bdeg+acfh-bc(fg+2eh)+f(bdg-2bch+adh)x)}{2(a+bx)^3(c+dx)}dx}{3b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{e+fx}(2bdeg+acfh-bc(fg+2eh)+f(bdg-2bch+adh)x)}{(a+bx)^3(c+dx)}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 166$$

$$-\frac{\frac{\int -\frac{\left(f(fg+10eh)c^2-2de(5fg+4eh)c+8d^2e^2g\right)b^2+acf^2(dg-3ch)b+a^2cdf^2h+f\left(\left(8fhc^2-d(7fg+6eh)c+6d^2eg\right)b^2+adf(dg-3ch)b+a^2d^2fh\right)x}{2(a+bx)^2(c+dx)\sqrt{e+fx}}dx}{2b(bc-ad)}+\frac{\sqrt{e+f\cdots}}{\cdots}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 27

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{2b(a+bx)^2(bc-ad)}-\frac{\int \frac{\left(f(fg+10eh)c^2-2de(5fg+4eh)c+8d^2e^2g\right)b^2+acf^2(dg-3ch)b+a^2cdf^2h+f\left(\left(8fhc^2-d(7\cdots\right.\right.}{(a+bx)^2(c+dx)\sqrt{e+fx}}\cdots}{4b(bc-ad)}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 168

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{2b(a+bx)^2(bc-ad)}-\frac{\frac{\sqrt{e+fx}\left(a^3d^2f^2h+a^2bdf^2(dg-4ch)+ab^2f\left(11c^2fh-6cdeh-8cdfg+6d^2eg\right)-b^3\left(c^2f(10eh+\cdots\right.\right.}{(a+bx)(bc-ad)(be-af)}\cdots}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 27

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{2b(a+bx)^2(bc-ad)}-\frac{\frac{\int \frac{-\left(\left(f^2(fg-6eh)c^3+6def(fg+4eh)c^2-8d^2e^2(3fg+2eh)c+16d^3e^3g\right)b^3\right)+af\left(-5f^2hc^3+2\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 174

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{2b(a+bx)^2(bc-ad)}-\frac{\frac{\frac{16b^2d(be-af)(de-cf)^2(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(a^4d^3f^3h+a^3bd^2f^3(dg-5ch)+3a^2b\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$\downarrow$ 73

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{2b(a+bx)^2(bc-ad)}-\frac{\frac{\frac{32b^2d(be-af)(de-cf)^2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(a^4d^3f^3h+a^3bd^2f^3(dg-5\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+abf(dg-3ch)-b^2(-2ceh-cfg+2deg)\right)}{2b(a+bx)^2(bc-ad)}-\frac{\frac{\sqrt{e+fx}\left(a^3d^2f^2h+a^2bdf^2(dg-4ch)+ab^2f\left(11c^2fh-6cdeh-8cdfg+6d^2eg\right)-b^3\left(c^2f(10eh+\cdots\right.\right.}{(a+bx)(bc-ad)(be-af)}\cdots}{\cdots}}{\cdots}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)),x]
```

output
```
-1/3*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^3) - (((a^2*d*
f*h + a*b*f*(d*g - 3*c*h) - b^2*(2*d*e*g - c*f*g - 2*c*e*h))*Sqrt[e + f*x]
)/(2*b*(b*c - a*d)*(a + b*x)^2) - (((a^3*d^2*f^2*h + a^2*b*d*f^2*(d*g - 4*
c*h) + a*b^2*f*(6*d^2*e*g - 8*c*d*f*g - 6*c*d*e*h + 11*c^2*f*h) - b^3*(8*d
^2*e^2*g - 2*c*d*e*(5*f*g + 4*e*h) + c^2*f*(f*g + 10*e*h)))*Sqrt[e + f*x])
/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((2*(a^4*d^3*f^3*h + a^3*b*d^2*f^3*
(d*g - 5*c*h) + 3*a^2*b^2*d*f^2*(2*d^2*e*g + 5*c^2*f*h - c*d*(3*f*g + 2*e*
h)) - a*b^3*f*(24*d^3*e^2*g - 5*c^3*f^2*h - 12*c*d^2*e*(3*f*g + 2*e*h) + 9
*c^2*d*f*(f*g + 4*e*h)) + b^4*(16*d^3*e^3*g + c^3*f^2*(f*g - 6*e*h) - 8*c*
d^2*e^2*(3*f*g + 2*e*h) + 6*c^2*d*e*f*(f*g + 4*e*h)))*ArcTanh[(Sqrt[b]*Sqr
t[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f]) - (32*
b^2*Sqrt[d]*(b*e - a*f)*(d*e - c*f)^(3/2)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqr
t[e + f*x])/Sqrt[d*e - c*f]])/(b*c - a*d))/(2*(b*c - a*d)*(b*e - a*f)))/(4
*b*(b*c - a*d)))/(2*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{-\left(\left(a^{4} f^{3} h+a^{3} b f^{3} g+6 a^{2} b^{2} e g f^{2}-24 a b^{3} e^{2} g f+16 b^{4} e^{3} g\right) d^{3}-5 c b\left(\frac{8\left(2 e^{3} h+3 g f e^{2}\right) b^{3}}{5}-\frac{24 a\left(e h+\frac{3 f g}{2}\right) f e b^{2}}{5}+\frac{6 a^{2}\left(e h+\right.}{}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*f-b*e)*b)^(1/2)*(-((a^4*f^3*h+a^3*b*f^3*g+6*a^2*b^2*e*f^2*g-24*a*
b^3*e^2*f*g+16*b^4*e^3*g)*d^3-5*c*b*(8/5*(2*e^3*h+3*e^2*f*g)*b^3-24/5*a*(e
*h+3/2*f*g)*f*e*b^2+6/5*a^2*(e*h+3/2*f*g)*f^2*b+a^3*f^3*h)*d^2+15*(2/5*(4*
e^2*h+e*f*g)*b^2-12/5*a*(e*h+1/4*f*g)*f*b+a^2*f^2*h)*c^2*b^2*f*d+5*(1/5*(-
6*e*h+f*g)*b+a*f*h)*c^3*b^3*f^2)*((c*f-d*e)*d)^(1/2)*(b*x+a)^3*arctan(b*(f
*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+(16*(c*f-d*e)^2*(b*x+a)^3*(a*f-b*e)*(c*h-
d*g)*b^2*d*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*(f*x+e)^(
1/2)*((c*f-d*e)*d)^(1/2)*((8*b^5*e^2*g*x^2+20*a*x*(-3/10*f*x+e)*g*e*b^4+44
/3*a^2*(-3/44*f^2*x^2-23/22*e*f*x+e^2)*g*b^3-8/3*a^3*(x*(3/8*h*x+g)*f^2+11
/2*(7/22*h*x+g)*e*f+e^2*h)*b^2+2/3*a^4*((4*h*x+3/2*g)*f+e*h)*f*b+a^5*f^2*h
)*d^2-4*(x*(5/2*f*g*x+e*(2*h*x+g))*e*b^4+7/3*a*(-6/7*x^2*g*f^2+16/7*x*(-9/
32*h*x+g)*e*f+e^2*(15/7*h*x+g))*b^3+5/3*(1/5*(-3*h*x^2-16*g*x)*f^2+1/10*e*
(-44*h*x+g)*f+e^2*h)*a^2*b^2-19/6*a^3*(4/19*(-2*h*x+3*g)*f+e*h)*f*b+a^4*f^
2*h)*c*b*d-5*c^2*(1/5*(-x^2*g*f^2-14/3*x*(15/7*h*x+g)*e*f-8/3*(3/2*h*x+g)*
e^2)*b^3-4/15*a*((-33/4*h*x^2-2*g*x)*f^2-1/2*e*(-11*h*x+g)*f+e^2*h)*b^2-8/
15*a^2*f*((-5*h*x-3/8*g)*f+e*h)*b+f^2*a^3*h)*b^2))*((a*f-b*e)*b)^(1/2))/((
c*f-d*e)*d)^(1/2)/(b*x+a)^3/(a*f-b*e)/(a*d-b*c)^4/b^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**4/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1853 vs. $2(545) = 1090$.

Time = 0.24 (sec) , antiderivative size = 1853, normalized size of antiderivative = 3.20

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x, algorithm="giac")
```

output

```
-1/8*(16*b^4*d^3*e^3*g - 24*b^4*c*d^2*e^2*f*g - 24*a*b^3*d^3*e^2*f*g + 6*b
^4*c^2*d*e*f^2*g + 36*a*b^3*c*d^2*e*f^2*g + 6*a^2*b^2*d^3*e*f^2*g + b^4*c^
3*f^3*g - 9*a*b^3*c^2*d*f^3*g - 9*a^2*b^2*c*d^2*f^3*g + a^3*b*d^3*f^3*g -
16*b^4*c*d^2*e^3*h + 24*b^4*c^2*d*e^2*f*h + 24*a*b^3*c*d^2*e^2*f*h - 6*b^4
*c^3*e*f^2*h - 36*a*b^3*c^2*d*e*f^2*h - 6*a^2*b^2*c*d^2*e*f^2*h + 5*a*b^3*
c^3*f^3*h + 15*a^2*b^2*c^2*d*f^3*h - 5*a^3*b*c*d^2*f^3*h + a^4*d^3*f^3*h)*
arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^4*e - 4*a*b^6*c^3*d*e
 + 6*a^2*b^5*c^2*d^2*e - 4*a^3*b^4*c*d^3*e + a^4*b^3*d^4*e - a*b^6*c^4*f +
 4*a^2*b^5*c^3*d*f - 6*a^3*b^4*c^2*d^2*f + 4*a^4*b^3*c*d^3*f - a^5*b^2*d^4
*f)*sqrt(-b^2*e + a*b*f)) + 2*(d^4*e^2*g - 2*c*d^3*e*f*g + c^2*d^2*f^2*g -
 c*d^3*e^2*h + 2*c^2*d^2*e*f*h - c^3*d*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(
-d^2*e + c*d*f))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*sqrt(-d^2*e + c*d*f)) - 1/24*(24*(f*x + e)^(5/2)*b^5*d^2*e
^2*f*g - 48*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + 24*sqrt(f*x + e)*b^5*d^2*e^4
*f*g - 30*(f*x + e)^(5/2)*b^5*c*d*e*f^2*g - 18*(f*x + e)^(5/2)*a*b^4*d^2*e
*f^2*g + 48*(f*x + e)^(3/2)*b^5*c*d*e^2*f^2*g + 96*(f*x + e)^(3/2)*a*b^4*d
^2*e^2*f^2*g - 18*sqrt(f*x + e)*b^5*c*d*e^3*f^2*g - 78*sqrt(f*x + e)*a*b^4
*d^2*e^3*f^2*g + 3*(f*x + e)^(5/2)*b^5*c^2*f^3*g + 24*(f*x + e)^(5/2)*a*b^
4*c*d*f^3*g - 3*(f*x + e)^(5/2)*a^2*b^3*d^2*f^3*g + 8*(f*x + e)^(3/2)*b^5*
c^2*e*f^3*g - 112*(f*x + e)^(3/2)*a*b^4*c*d*e*f^3*g - 40*(f*x + e)^(3/2...
```

Mupad [B] (verification not implemented)

Time = 24.80 (sec) , antiderivative size = 544718, normalized size of antiderivative = 940.79

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)),x)
```

output
```
- atan(((((16*a*b^14*c^11*d^2*f^7*g - 16*a^11*b^4*c*d^12*f^7*g - 16*a^12*b
^3*c*d^12*f^7*h + 16*a^11*b^4*d^13*e*f^6*g + 16*a^12*b^3*d^13*e*f^6*h - 16
*b^15*c^11*d^2*e*f^6*g - 256*a^2*b^13*c^10*d^3*f^7*g + 1456*a^3*b^12*c^9*d
^4*f^7*g - 4352*a^4*b^11*c^8*d^5*f^7*g + 7840*a^5*b^10*c^7*d^6*f^7*g - 896
0*a^6*b^9*c^6*d^7*f^7*g + 6496*a^7*b^8*c^5*d^8*f^7*g - 2816*a^8*b^7*c^4*d^
9*f^7*g + 592*a^9*b^6*c^3*d^10*f^7*g + 80*a^2*b^13*c^11*d^2*f^7*h - 576*a^
3*b^12*c^10*d^3*f^7*h + 1712*a^4*b^11*c^9*d^4*f^7*h - 2560*a^5*b^10*c^8*d^
5*f^7*h + 1568*a^6*b^9*c^7*d^6*f^7*h + 896*a^7*b^8*c^6*d^7*f^7*h - 2464*a^
8*b^7*c^5*d^8*f^7*h + 2048*a^9*b^6*c^4*d^9*f^7*h - 880*a^10*b^5*c^3*d^10*f
^7*h + 192*a^11*b^4*c^2*d^11*f^7*h - 128*a^8*b^7*d^13*e^4*f^3*g + 288*a^9*
b^6*d^13*e^3*f^4*g - 176*a^10*b^5*d^13*e^2*f^5*g - 16*a^11*b^4*d^13*e^2*f^
5*h - 128*b^15*c^8*d^5*e^4*f^3*g + 224*b^15*c^9*d^4*e^3*f^4*g - 80*b^15*c^
10*d^3*e^2*f^5*g + 128*b^15*c^9*d^4*e^4*f^3*h - 224*b^15*c^10*d^3*e^3*f^4*
h + 96*b^15*c^11*d^2*e^2*f^5*h - 3584*a^2*b^13*c^6*d^7*e^4*f^3*g + 3968*a^
2*b^13*c^7*d^6*e^3*f^4*g + 1680*a^2*b^13*c^8*d^5*e^2*f^5*g + 7168*a^3*b^12
*c^5*d^8*e^4*f^3*g - 4480*a^3*b^12*c^6*d^7*e^3*f^4*g - 8448*a^3*b^12*c^7*d
^6*e^2*f^5*g - 8960*a^4*b^11*c^4*d^9*e^4*f^3*g - 448*a^4*b^11*c^5*d^8*e^3*
f^4*g + 18144*a^4*b^11*c^6*d^7*e^2*f^5*g + 7168*a^5*b^10*c^3*d^10*e^4*f^3*
g + 7616*a^5*b^10*c^4*d^9*e^3*f^4*g - 21504*a^5*b^10*c^5*d^8*e^2*f^5*g - 3
584*a^6*b^9*c^2*d^11*e^4*f^3*g - 9856*a^6*b^9*c^3*d^10*e^3*f^4*g + 1411...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9592, normalized size of antiderivative = 16.57

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c),x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**7*d**3*f**3*h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**2*f**3*h + 3*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**3*f**3*g + 9*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
**6*b*d**3*f**3*h*x + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*d*f**3*h - 18*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d**2*e*f
**2*h - 27*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**5*b**2*c*d**2*f**3*g - 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d**2*f**3*h*x + 18*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
5*b**2*d**3*e*f**2*g + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**3*f**3*g*x + 9*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**3*f**3*h
*x**2 + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**4*b**3*c**3*f**3*h - 108*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**3*c**2*d*e*f**2*h - 27*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4
*b**3*c**2*d*f**3*g + 135*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b...
```

3.86 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 932

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx =$$
$$-\frac{(3a^2dfh+abf(5dg-11ch)-b^2(8deg-3cfg-8ceh))\sqrt{e+fx}}{24b^2(bc-ad)^2(a+bx)^3}$$
$$+\frac{(3a^3d^2f^2h+a^2bdf^2(5dg-14ch)+ab^2f(40d^2eg+59c^2fh-10cd(5fg+4eh))-b^3(48d^2e^2g-8cde(7fg$$
$$\overline{96b^2(bc-ad)^3(be-af)(a+bx)^2}$$
$$+\frac{(3a^4d^3f^3h+a^3bd^2f^3(5dg-17ch)+a^2b^2df^2(40d^2eg+73c^2fh-5cd(11fg+8eh))+b^4(64d^3e^3g+c^3f^2($$
$$-\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4}$$
$$+\frac{(3a^5d^4f^4h+5a^4bd^3f^4(dg-4ch)+10a^3b^2d^2f^3(4d^2eg+9c^2fh-2cd(3fg+2eh))-30a^2b^3df^2(8d^3e^2g-$$
$$+\frac{2d^{3/2}(de-cf)^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^5}$$

output
```
-1/24*(3*a^2*d*f*h+a*b*f*(-11*c*h+5*d*g)-b^2*(-8*c*e*h-3*c*f*g+8*d*e*g))*(
f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^3+1/96*(3*a^3*d^2*f^2*h+a^2*b*d*f^2*
(-14*c*h+5*d*g)+a*b^2*f*(40*d^2*e*g+59*c^2*f*h-10*c*d*(4*e*h+5*f*g))-b^3*(
48*d^2*e^2*g-8*c*d*e*(6*e*h+7*f*g)+c^2*f*(56*e*h+3*f*g)))*(f*x+e)^(1/2)/b^
2/(-a*d+b*c)^3/(-a*f+b*e)/(b*x+a)^2+1/64*(3*a^4*d^3*f^3*h+a^3*b*d^2*f^3*(-
17*c*h+5*d*g)+a^2*b^2*d*f^2*(40*d^2*e*g+73*c^2*f*h-5*c*d*(8*e*h+11*f*g))+b
^4*(64*d^3*e^3*g+c^3*f^2*(-8*e*h+3*f*g)-16*c*d^2*e^2*(4*e*h+5*f*g)+8*c^2*d
*e*f*(10*e*h+f*g))-a*b^3*f*(112*d^3*e^2*g-5*c^3*f^2*h-16*c*d^2*e*(7*e*h+9*
f*g)+c^2*d*f*(144*e*h+17*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^4/(-a*f+b*e)^
2/(b*x+a)-1/4*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)^4+1/64*(3*a^5*
d^4*f^4*h+5*a^4*b*d^3*f^4*(-4*c*h+d*g)+10*a^3*b^2*d^2*f^3*(4*d^2*e*g+9*c^2
*f*h-2*c*d*(2*e*h+3*f*g))-30*a^2*b^3*d*f^2*(8*d^3*e^2*g-2*c^3*f^2*h-4*c*d^
2*e*(2*e*h+3*f*g)+3*c^2*d*f*(4*e*h+f*g))+5*a*b^4*f*(64*d^4*e^3*g-c^4*f^3*h
+4*c^3*d*f^2*(-6*e*h+f*g)-32*c*d^3*e^2*(2*e*h+3*f*g)+24*c^2*d^2*e*f*(4*e*h
+f*g))-b^5*(128*d^4*e^4*g+c^4*f^3*(-8*e*h+3*f*g)+8*c^3*d*e*f^2*(-6*e*h+f*g
)-64*c*d^3*e^3*(2*e*h+3*f*g)+48*c^2*d^2*e^2*f*(4*e*h+f*g)))*arctanh(b^(1/2
)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*d+b*c)^5/(-a*f+b*e)^(5/2)+2*
d^(3/2)*(-c*f+d*e)^(3/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*
e)^(1/2))/(-a*d+b*c)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5574 vs. $2(932) = 1864$.

Time = 16.29 (sec) , antiderivative size = 5574, normalized size of antiderivative = 5.98

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)} \, dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {166, 27, 166, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{\sqrt{e+fx}(3acfh+b(8deg-3cfg-8ceh)+f(5bdg-8bch+3adh)x)}{2(a+bx)^4(c+dx)}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{e+fx}(3acfh+b(8deg-3cfg-8ceh)+f(5bdg-8bch+3adh)x)}{(a+bx)^4(c+dx)}dx}{8b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \; 166$$

$$-\frac{\frac{\int -\frac{\left(f(3fg+56eh)c^2-8de(7fg+6eh)c+48d^2e^2g\right)b^2+acf^2(5dg-11ch)b+3a^2cdf^2h+f\left(\left(48fhc^2-5d(9fg+8eh)c+40d^2eg\right)b^2+adf(5dg-11ch)b+3a^2d^2fh\right)x}{2(a+bx)^3(c+dx)\sqrt{e+fx}}dx}{3b(bc-ad)}}{8b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\sqrt{e+fx}\left(3a^2dfh+abf(5dg-11ch)-b^2(-8ceh-3cfg+8deg)\right)}{3b(a+bx)^3(bc-ad)}-\frac{\int \frac{\left(f(3fg+56eh)c^2-8de(7fg+6eh)c+48d^2e^2g\right)b^2+acf^2(5dg-11ch)b+3a^2cdf^2h+f((}{(a+bx)^3(c+dx)\sqrt{e+}}}{6b(bc-ad)}}{8b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\sqrt{e+fx}\left(3a^2dfh+abf(5dg-11ch)-b^2(-8ceh-3cfg+8deg)\right)}{3b(a+bx)^3(bc-ad)}-\frac{\frac{\sqrt{e+fx}\left(3a^3d^2f^2h+a^2bdf^2(5dg-14ch)+ab^2f\left(59c^2fh-10cd(4eh+5fg)+40d^2eg\right)-b^3\cdots\right.}{2(a+bx)^2(bc-ad)(be-af)}\cdots}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$\big\downarrow$ 27

$$-\frac{\frac{\sqrt{e+fx}\left(3a^2dfh+abf(5dg-11ch)-b^2(-8ceh-3cfg+8deg)\right)}{3b(a+bx)^3(bc-ad)}-\frac{3\int\frac{-\left(\left(f^2(3fg-8eh)c^3+8def(fg+10eh)c^2-16d^2e^2(5fg+4eh)c+64d^3e^3g\right)b^3\right)+af\left(-\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(bc-ad)}$$

$\big\downarrow$ 168

$$-\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{\left(3dfha^2+bf(5dg-11ch)a-b^2(8deg-3cfg-8ceh)\right)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3+bdf^2(5dg-14ch)a^2+b^2f\left(59fhc^2-10d(5fg+4eh)c+40d^2eg\right)a-b^3\left(f(3\cdots\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}\cdots}{\cdots}}{\cdots}$$

$\big\downarrow$ 27

$$-\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{\left(3dfha^2+bf(5dg-11ch)a-b^2(8deg-3cfg-8ceh)\right)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3+bdf^2(5dg-14ch)a^2+b^2f\left(59fhc^2-10d(5fg+4eh)c+40d^2eg\right)a-b^3\left(f(3\cdots\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}\cdots}{\cdots}}{\cdots}$$

$\big\downarrow$ 174

$$-\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{\left(3dfha^2+bf(5dg-11ch)a-b^2(8deg-3cfg-8ceh)\right)\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3+bdf^2(5dg-14ch)a^2+b^2f\left(59fhc^2-10d(5fg+4eh)c+40d^2eg\right)a-b^3\left(f(3\cdots\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}\cdots}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 73$$

$$-\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{(3dfha^2+bf(5dg-11ch)a-b^2(8deg-3cfg-8ceh))\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3+bdf^2(5dg-14ch)a^2+b^2f\left(59fhc^2-10d(5fg+4eh)c+40d^2eg\right)a-b^3\left(f(3\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}}{}$$

$$\Big\downarrow \; 221$$

$$-\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4}-$$

$$\frac{\frac{(3dfha^2+bf(5dg-11ch)a-b^2(8deg-3cfg-8ceh))\sqrt{e+fx}}{3b(bc-ad)(a+bx)^3}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3+bdf^2(5dg-14ch)a^2+b^2f\left(59fhc^2-10d(5fg+4eh)c+40d^2eg\right)a-b^3\left(f(3\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}}{}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)),x]
```

output
```
-1/4*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^4) - (((3*a^2*
d*f*h + a*b*f*(5*d*g - 11*c*h) - b^2*(8*d*e*g - 3*c*f*g - 8*c*e*h))*Sqrt[e
 + f*x])/(3*b*(b*c - a*d)*(a + b*x)^3) - (((3*a^3*d^2*f^2*h + a^2*b*d*f^2*
(5*d*g - 14*c*h) + a*b^2*f*(40*d^2*e*g + 59*c^2*f*h - 10*c*d*(5*f*g + 4*e*
h)) - b^3*(48*d^2*e^2*g - 8*c*d*e*(7*f*g + 6*e*h) + c^2*f*(3*f*g + 56*e*h)
))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (3*(((3*a^4*d^
3*f^3*h + a^3*b*d^2*f^3*(5*d*g - 17*c*h) + a^2*b^2*d*f^2*(40*d^2*e*g + 73*
c^2*f*h - 5*c*d*(11*f*g + 8*e*h)) + b^4*(64*d^3*e^3*g + c^3*f^2*(3*f*g - 8
*e*h) - 16*c*d^2*e^2*(5*f*g + 4*e*h) + 8*c^2*d*e*f*(f*g + 10*e*h)) - a*b^3
*f*(112*d^3*e^2*g - 5*c^3*f^2*h - 16*c*d^2*e*(9*f*g + 7*e*h) + c^2*d*f*(17
*f*g + 144*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((2
*(3*a^5*d^4*f^4*h + 5*a^4*b*d^3*f^4*(d*g - 4*c*h) + 10*a^3*b^2*d^2*f^3*(4*
d^2*e*g + 9*c^2*f*h - 2*c*d*(3*f*g + 2*e*h)) - 30*a^2*b^3*d*f^2*(8*d^3*e^2
*g - 2*c^3*f^2*h - 4*c*d^2*e*(3*f*g + 2*e*h) + 3*c^2*d*f*(f*g + 4*e*h)) +
5*a*b^4*f*(64*d^4*e^3*g - c^4*f^3*h + 4*c^3*d*f^2*(f*g - 6*e*h) - 32*c*d^3
*e^2*(3*f*g + 2*e*h) + 24*c^2*d^2*e*f*(f*g + 4*e*h)) - b^5*(128*d^4*e^4*g
+ c^4*f^3*(3*f*g - 8*e*h) + 8*c^3*d*e*f^2*(f*g - 6*e*h) - 64*c*d^3*e^3*(3*
f*g + 2*e*h) + 48*c^2*d^2*e^2*f*(f*g + 4*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e +
f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f]) + (256*b^2*d
^(3/2)*(b*e - a*f)^2*(d*e - c*f)^(3/2)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqr...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 22.64 (sec) , antiderivative size = 1526, normalized size of antiderivative = 1.64

method	result	size
pseudoelliptic	Expression too large to display	1526
derivativedivides	Expression too large to display	2601
default	Expression too large to display	2601

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
3/64*((-(a*d-b*c)*((c*f-d*e)*d)^(1/2)*((-b^3*(a*(a^3*d^3-17/3*a^2*b*c*d^2+
73/3*a*b^2*c^2*d+5/3*b^3*c^3)*h+5/3*(a^3*d^3-11*a^2*b*c*d^2-17/5*a*b^2*c^2
*d+3/5*b^3*c^3)*b*g)*x^3-11/3*(a*(a^3*d^3-17/3*a^2*b*c*d^2+73/3*a*b^2*c^2*
d-73/33*b^3*c^3)*h+5/3*(a^3*d^3-11*a^2*b*c*d^2-59/55*a*b^2*c^2*d+3/5*b^3*c
^3)*b*g)*a*b^2*x^2+11/3*a^2*(a*(a^3*d^3-59/33*a^2*b*c*d^2-55/3*a*b^2*c^2*d
+5/3*b^3*c^3)*h-73/33*(a^3*d^3-11*a^2*b*c*d^2+187/73*a*b^2*c^2*d-33/73*b^3
*c^3)*b*g)*b*x+a^3*(a*(-17/3*a^2*b*c*d^2-55/3*a*b^2*c^2*d+5/3*b^3*c^3+a^3*
d^3)*h+5/3*(73/5*a^2*b*c*d^2-17/5*a*b^2*c^2*d+3/5*b^3*c^3+a^3*d^3)*b*g))*f
^3+2/3*(20*(c*h-d*g)*b^4*(a^2*d^2+18/5*a*b*c*d+1/5*b^2*c^2)*x^3+(a*(a^3*d^
3-79/3*b^3*c^3+203/3*a^2*b*c*d^2+737/3*a*b^2*c^2*d)*h-215/3*b*g*(a^3*d^3+7
19/215*a^2*b*c*d^2-67/215*a*b^2*c^2*d-3/215*b^3*c^3))*b^2*x^2-22*a*(a*(a^3
*d^3-269/33*a^2*b*c*d^2-19/3*a*b^2*c^2*d+13/33*b^3*c^3)*h+137/33*(a^3*d^3+
407/137*a^2*b*c*d^2-145/137*a*b^2*c^2*d+33/137*b^3*c^3)*b*g)*b*x+a^2*(a*(1
79/3*a^2*b*c*d^2+113/3*a*b^2*c^2*d+a^3*d^3-7/3*b^3*c^3)*h-191/3*(a^3*d^3+9
5/191*a^2*b*c*d^2+5/191*a*b^2*c^2*d-3/191*b^3*c^3)*b*g))*b*e*f^2-8*(14/3*(
c*h-d*g)*d*(a*d+5/7*b*c)*b^4*x^3+148/9*(c*h-d*g)*b^3*(a^2*d^2+41/74*a*b*c*
d-7/74*b^2*c^2)*x^2-(a*(a^3*d^3-221/9*a^2*b*c*d^2+1/9*a*b^2*c^2*d-5/9*b^3*
c^3)*h+185/9*(a^3*d^3+53/185*a^2*b*c*d^2-31/185*a*b^2*c^2*d+9/185*b^3*c^3)
*b*g)*b*x+a*(a*(61/9*a^2*b*c*d^2+1/9*a*b^2*c^2*d+a^3*d^3+1/9*b^3*c^3)*h-97
/9*(a^3*d^3-53/97*a^2*b*c*d^2+37/97*a*b^2*c^2*d-9/97*b^3*c^3)*b*g))*b^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**5/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4118 vs. 2(894) = 1788.

Time = 0.34 (sec) , antiderivative size = 4118, normalized size of antiderivative = 4.42

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x, algorithm="giac")
```

output
```
1/64*(128*b^5*d^4*e^4*g - 192*b^5*c*d^3*e^3*f*g - 320*a*b^4*d^4*e^3*f*g +
48*b^5*c^2*d^2*e^2*f^2*g + 480*a*b^4*c*d^3*e^2*f^2*g + 240*a^2*b^3*d^4*e^2
*f^2*g + 8*b^5*c^3*d*e*f^3*g - 120*a*b^4*c^2*d^2*e*f^3*g - 360*a^2*b^3*c*d
^3*e*f^3*g - 40*a^3*b^2*d^4*e*f^3*g + 3*b^5*c^4*f^4*g - 20*a*b^4*c^3*d*f^4
*g + 90*a^2*b^3*c^2*d^2*f^4*g + 60*a^3*b^2*c*d^3*f^4*g - 5*a^4*b*d^4*f^4*g
 - 128*b^5*c*d^3*e^4*h + 192*b^5*c^2*d^2*e^3*f*h + 320*a*b^4*c*d^3*e^3*f*h
 - 48*b^5*c^3*d*e^2*f^2*h - 480*a*b^4*c^2*d^2*e^2*f^2*h - 240*a^2*b^3*c*d^
3*e^2*f^2*h - 8*b^5*c^4*e*f^3*h + 120*a*b^4*c^3*d*e*f^3*h + 360*a^2*b^3*c^
2*d^2*e*f^3*h + 40*a^3*b^2*c*d^3*e*f^3*h + 5*a*b^4*c^4*f^4*h - 60*a^2*b^3*
c^3*d*f^4*h - 90*a^3*b^2*c^2*d^2*f^4*h + 20*a^4*b*c*d^3*f^4*h - 3*a^5*d^4*
f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^9*c^5*e^2 - 5*a*b^
8*c^4*d*e^2 + 10*a^2*b^7*c^3*d^2*e^2 - 10*a^3*b^6*c^2*d^3*e^2 + 5*a^4*b^5*
c*d^4*e^2 - a^5*b^4*d^5*e^2 - 2*a*b^8*c^5*e*f + 10*a^2*b^7*c^4*d*e*f - 20*
a^3*b^6*c^3*d^2*e*f + 20*a^4*b^5*c^2*d^3*e*f - 10*a^5*b^4*c*d^4*e*f + 2*a^
6*b^3*d^5*e*f + a^2*b^7*c^5*f^2 - 5*a^3*b^6*c^4*d*f^2 + 10*a^4*b^5*c^3*d^2
*f^2 - 10*a^5*b^4*c^2*d^3*f^2 + 5*a^6*b^3*c*d^4*f^2 - a^7*b^2*d^5*f^2)*sqr
t(-b^2*e + a*b*f)) - 2*(d^5*e^2*g - 2*c*d^4*e*f*g + c^2*d^3*f^2*g - c*d^4*
e^2*h + 2*c^2*d^3*e*f*h - c^3*d^2*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*
e + c*d*f))/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(-d^2*e + c*d*f)) + 1/192*(192*(f*...
```

Mupad [B] (verification not implemented)

Time = 38.91 (sec) , antiderivative size = 894172, normalized size of antiderivative = 959.41

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)),x)
```

output
```
- atan(((((640*a^15*b^4*d^16*e*f^8*g - 384*a^16*b^3*c*d^15*f^9*h - 640*a^1
5*b^4*c*d^15*f^9*g + 384*a^16*b^3*d^16*e*f^8*h - 384*a^2*b^17*c^14*d^2*f^9
*g + 6016*a^3*b^16*c^13*d^3*f^9*g - 48384*a^4*b^15*c^12*d^4*f^9*g + 236800
*a^5*b^14*c^11*d^5*f^9*g - 755840*a^6*b^13*c^10*d^6*f^9*g + 1646208*a^7*b^
12*c^9*d^7*f^9*g - 2514432*a^8*b^11*c^8*d^8*f^9*g + 2723328*a^9*b^10*c^7*d
^9*f^9*g - 2079360*a^10*b^9*c^6*d^10*f^9*g + 1088640*a^11*b^8*c^5*d^11*f^9
*g - 365824*a^12*b^7*c^4*d^12*f^9*g + 66816*a^13*b^6*c^3*d^13*f^9*g - 2944
*a^14*b^5*c^2*d^14*f^9*g - 640*a^3*b^16*c^14*d^2*f^9*h + 13440*a^4*b^15*c^
13*d^3*f^9*h - 97024*a^5*b^14*c^12*d^4*f^9*h + 371456*a^6*b^13*c^11*d^5*f^
9*h - 877440*a^7*b^12*c^10*d^6*f^9*h + 1361280*a^8*b^11*c^9*d^7*f^9*h - 14
05440*a^9*b^10*c^8*d^8*f^9*h + 926208*a^10*b^9*c^7*d^9*f^9*h - 319872*a^11
*b^8*c^6*d^10*f^9*h - 18560*a^12*b^7*c^5*d^11*f^9*h + 72960*a^13*b^6*c^4*d
^12*f^9*h - 32000*a^14*b^5*c^3*d^13*f^9*h + 6016*a^15*b^4*c^2*d^14*f^9*h -
 8192*a^10*b^9*d^16*e^6*f^3*g + 34816*a^11*b^8*d^16*e^5*f^4*g - 56320*a^12
*b^7*d^16*e^4*f^5*g + 41600*a^13*b^6*d^16*e^3*f^6*g - 12544*a^14*b^5*d^16*
e^2*f^7*g + 384*a^14*b^5*d^16*e^3*f^6*h - 768*a^15*b^4*d^16*e^2*f^7*h - 81
92*b^19*c^10*d^6*e^6*f^3*g + 14336*b^19*c^11*d^5*e^5*f^4*g - 5120*b^19*c^1
2*d^4*e^4*f^5*g - 640*b^19*c^13*d^3*e^3*f^6*g - 384*b^19*c^14*d^2*e^2*f^7*
g + 8192*b^19*c^11*d^5*e^6*f^3*h - 14336*b^19*c^12*d^4*e^5*f^4*h + 5120*b^
19*c^13*d^3*e^4*f^5*h + 1024*b^19*c^14*d^2*e^3*f^6*h - 368640*a^2*b^17*...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 18831, normalized size of antiderivative = 20.20

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c),x)
```

output
```
(9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**9*d**4*f**4*h - 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**8*b*c*d**3*f**4*h + 15*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*d**4*f**4*g + 36
*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**8*b*d**4*f**4*h*x + 270*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c**2*d**2*f**4*h - 120*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c*d
**3*e*f**3*h - 180*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**7*b**2*c*d**3*f**4*g - 240*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c*d**3*f**4*h*x
 + 120*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**7*b**2*d**4*e*f**3*g + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*d**4*f**4*g*x + 54*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2
*d**4*f**4*h*x**2 + 180*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**6*b**3*c**3*d*f**4*h - 1080*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b**3*c**2*d**2
*e*f**3*h - 270*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**6*b**3*c**2*d**2*f**4*g + 1080*sqrt(b)*sqrt(a*f - b*...
```

3.87 $\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 424

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{(bc-ad)^2\,(ad(3dfg+2deh-5cfh)+b(6d^2eg+11c^2fh-cd(9fg}{d^6}$$

$$+\frac{2(bc-ad)^2(3bdg-4bch+adh)(e+fx)^{3/2}}{3d^5}+\frac{(bc-ad)^3(dg-ch)(e+fx)^{3/2}}{d^5(c+dx)}$$

$$+\frac{2b(3a^2d^2f^2h+3abdf(dfg-deh-2cfh)+b^2(3c^2f^2h-d^2e(fg-eh)-2cdf(fg-eh)))\,(e+fx)^{5/2}}{5d^4f^3}$$

$$+\frac{2b^2(3adfh+b(dfg-2deh-2cfh))(e+fx)^{7/2}}{7d^3f^3}+\frac{2b^3h(e+fx)^{9/2}}{9d^2f^3}$$

$$-\frac{(bc-ad)^2\sqrt{de-cf}\,(ad(3dfg+2deh-5cfh)+b(6d^2eg+11c^2fh-cd(9fg+8eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right.}{d^{13/2}}$$

output
```
(-a*d+b*c)^2*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)+b*(6*d^2*e*g+11*c^2*f*h-c*d*(
8*e*h+9*f*g)))*(f*x+e)^(1/2)/d^6+2/3*(-a*d+b*c)^2*(a*d*h-4*b*c*h+3*b*d*g)*
(f*x+e)^(3/2)/d^5+(-a*d+b*c)^3*(-c*h+d*g)*(f*x+e)^(3/2)/d^5/(d*x+c)+2/5*b*
(3*a^2*d^2*f^2*h+3*a*b*d*f*(-2*c*f*h-d*e*h+d*f*g)+b^2*(3*c^2*f^2*h-d^2*e*(
-e*h+f*g)-2*c*d*f*(-e*h+f*g)))*(f*x+e)^(5/2)/d^4/f^3+2/7*b^2*(3*a*d*f*h+b*
(-2*c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(7/2)/d^3/f^3+2/9*b^3*h*(f*x+e)^(9/2)/d^
2/f^3-(-a*d+b*c)^2*(-c*f+d*e)^(1/2)*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)+b*(6*d
^2*e*g+11*c^2*f*h-c*d*(8*e*h+9*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+
d*e)^(1/2))/d^(13/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{\sqrt{e+fx}(63a^2bd^2f^2(105c^3f^2h-5c^2df(15fg+19eh-14fhx)+2d}{}$$

$$-\frac{(bc-ad)^2\sqrt{-de+cf}(ad(3dfg+2deh-5cfh)+b(6d^2eg+11c^2fh-cd(9fg+8eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right.}{d^{13/2}}$$

input
```
Integrate[((a + b*x)^3*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(63*a^2*b*d^2*f^2*(105*c^3*f^2*h - 5*c^2*d*f*(15*f*g + 19*e
*h - 14*f*h*x) + 2*d^3*x*(3*e^2*h + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*
x)) + c*d^2*(6*e^2*h + e*f*(55*g - 68*h*x) - 2*f^2*x*(25*g + 7*h*x))) + 10
5*a^3*d^3*f^3*(-15*c^2*f*h + c*d*(9*f*g + 11*e*h - 10*f*h*x) + d^2*(2*f*x*
(3*g + h*x) + e*(-3*g + 8*h*x))) - 9*a*b^2*d*f*(945*c^4*f^3*h - 105*c^3*d*
f^2*(7*f*g + 9*e*h - 6*f*h*x) - 6*d^4*x*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h
*x) + 7*c^2*d^2*f*(12*e^2*h + e*f*(95*g - 96*h*x) - 2*f^2*x*(35*g + 9*h*x)
) + 2*c*d^3*(6*e^3*h + f^3*x^2*(49*g + 27*h*x) + 2*e*f^2*x*(119*g + 30*h*x
) + e^2*f*(-21*g + 39*h*x))) + b^3*(3465*c^5*f^4*h - 105*c^4*d*f^3*(27*f*g
 + 35*e*h - 22*f*h*x) + 2*d^5*x*(e + f*x)^2*(8*e^2*h + 5*f^2*x*(9*g + 7*h*
x) - 2*e*f*(9*g + 10*h*x)) + 21*c^3*d^2*f^2*(18*e^2*h + e*f*(135*g - 124*h
*x) - 2*f^2*x*(45*g + 11*h*x)) + 18*c^2*d^3*f*(4*e^3*h + f^3*x^2*(21*g + 1
1*h*x) + 2*e*f^2*x*(56*g + 13*h*x) + e^2*f*(-14*g + 19*h*x)) + 2*c*d^4*(e
+ f*x)^2*(8*e^2*h - 2*e*f*(9*g - 8*h*x) - f^2*x*(81*g + 55*h*x)))))/(315*d
^6*f^3*(c + d*x)) - ((b*c - a*d)^2*Sqrt[-(d*e) + c*f]*(a*d*(3*d*f*g + 2*d*
e*h - 5*c*f*h) + b*(6*d^2*e*g + 11*c^2*f*h - c*d*(9*f*g + 8*e*h)))*ArcTan[
(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(13/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {166, 27, 170, 27, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}dx$$

$\Big\downarrow$ 166

$$\frac{\int \frac{(a+bx)^2(e+fx)^{3/2}(6be(dg-ch)+a(3dfg+2deh-5cfh)+b(9dfg+2deh-11cfh)x)}{2(c+dx)}dx}{d(de-cf)}-\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^2(e+fx)^{3/2}(6be(dg-ch)+a(3dfg+2deh-5cfh)+b(9dfg+2deh-11cfh)x)}{c+dx}dx}{2d(de-cf)}-\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 170

$$\frac{\frac{2\int -\frac{(a+bx)(e+fx)^{3/2}\left(bc(4be+5af)(9dfg+2deh-11cfh)-9adf(6be(dg-ch)+a(3dfg+2deh-5cfh))-b\left(adf(63dfg+26deh-89cfh)+b\left(2e(9fg-4eh)d^2-cf(81fg\right.\right.\right.}{2(c+dx)}}{9df}}{2d(de-cf)}-\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 27

$$\frac{\frac{2b(a+bx)^2(e+fx)^{5/2}(-11cfh+2deh+9dfg)}{9df}-\frac{\int \frac{(a+bx)(e+fx)^{3/2}\left(bc(4be+5af)(9dfg+2deh-11cfh)-9adf(6be(dg-ch)+a(3dfg+2deh-5cfh))-b\left(adf(6\right.\right.}{c+dx}}{9df}}{2d(de-cf)}-\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 164

$$\frac{\frac{2b(a+bx)^2(e+fx)^{5/2}(-11cfh+2deh+9dfg)}{9df}-\frac{-\frac{9f(bc-ad)^2\left(ad(-5cfh+2deh+3dfg)+b\left(11c^2fh-cd(8eh+9fg)+6d^2eg\right)\right)\int \frac{(e+fx)^{3/2}}{c+dx}dx}{d^2}-\frac{2b(e+fx)^{5/}}{}}{}}{}-\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 60

$$\frac{\frac{2b(a+bx)^2(e+fx)^{5/2}(-11cfh+2deh+9dfg)}{9df}-\frac{-\frac{9f(bc-ad)^2\left(ad(-5cfh+2deh+3dfg)+b\left(11c^2fh-cd(8eh+9fg)+6d^2eg\right)\right)\left(\frac{(de-cf)\int\frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2(e+\cdots}{3\cdots}\cdots\right.}{d^2}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{2b(a+bx)^2(e+fx)^{5/2}(-11cfh+2deh+9dfg)}{9df}-\frac{-\frac{9f(bc-ad)^2\left(ad(-5cfh+2deh+3dfg)+b\left(11c^2fh-cd(8eh+9fg)+6d^2eg\right)\right)\left(\frac{(de-cf)\left(\frac{(de-cf)\int\frac{1}{(c+dx)\cdots}}{d}\cdots\right.}{d}\cdots\right.}{d^2}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2b(a+bx)^2(e+fx)^{5/2}(-11cfh+2deh+9dfg)}{9df}-\frac{-\frac{9f(bc-ad)^2\left(ad(-5cfh+2deh+3dfg)+b\left(11c^2fh-cd(8eh+9fg)+6d^2eg\right)\right)\left(\frac{(de-cf)\left(\frac{2(de-cf)\int\frac{\cdots}{c+\frac{d(e\cdots}{\cdots}}}{d\cdots}\cdots\right.}{\cdots}\cdots\right.}{d^2}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2b(a+bx)^2(e+fx)^{5/2}(-11cfh+2deh+9dfg)}{9df}-\frac{-\frac{2b(e+fx)^{5/2}\left(14a^2d^2f^2(-67cfh+22deh+45dfg)+5bdfx\left(adf(-89cfh+26deh+63dfg)+b\left(99c^2f^2h-cdf(\cdots\right.\right.\right.}{\cdots}\cdots}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[((a + b*x)^3*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
-(((d*g - c*h)*(a + b*x)^3*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x))) + (
(2*b*(9*d*f*g + 2*d*e*h - 11*c*f*h)*(a + b*x)^2*(e + f*x)^(5/2))/(9*d*f) -
 ((-2*b*(e + f*x)^(5/2)*(14*a^2*d^2*f^2*(45*d*f*g + 22*d*e*h - 67*c*f*h) +
 27*a*b*d*f*(63*c^2*f^2*h + 2*d^2*e*(7*f*g - 2*e*h) - c*d*f*(49*f*g + 24*e
*h)) - b^2*(693*c^3*f^3*h + 8*c*d^2*e*f*(27*f*g - 7*e*h) + 4*d^3*e^2*(9*f*
g - 4*e*h) - 9*c^2*d*f^2*(63*f*g + 34*e*h)) + 5*b*d*f*(a*d*f*(63*d*f*g + 2
6*d*e*h - 89*c*f*h) + b*(99*c^2*f^2*h + 2*d^2*e*(9*f*g - 4*e*h) - c*d*f*(8
1*f*g + 28*e*h)))*x))/(35*d^2*f^2) - (9*(b*c - a*d)^2*f*(a*d*(3*d*f*g + 2*
d*e*h - 5*c*f*h) + b*(6*d^2*e*g + 11*c^2*f*h - c*d*(9*f*g + 8*e*h)))*((2*(
e + f*x)^(3/2))/(3*d) + ((d*e - c*f)*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e -
c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d))/d^2)/
(9*d*f))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.81

method	result
pseudoelliptic	$-\frac{5\left(-(ad-bc)^2(cf-de)(xd+c)\left(\frac{(-3afg-2e(ah+3bg))d^2}{5}+c\left(\left(ah+\frac{9bg}{5}\right)f+\frac{8ehb}{5}\right)d-\frac{11b\,c^2fh}{5}\right)f^3\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)\right.}{}$
risch	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-5/((c*f-d*e)*d)^(1/2)*(-(a*d-b*c)^2*(c*f-d*e)*(d*x+c)*(1/5*(-3*a*f*g-2*e*
(a*h+3*b*g))*d^2+c*((a*h+9/5*b*g)*f+8/5*e*h*b)*d-11/5*b*c^2*f*h)*f^3*arcta
n(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(1/5*(-2*x*(1/7*x^3*(7/9*h*x+g)*b^3
+3/5*a*x^2*(5/7*h*x+g)*b^2+a^2*x*(3/5*h*x+g)*b+a^3*(1/3*h*x+g))*f^4+(-16/3
5*x^3*(25/36*h*x+g)*b^3-12/5*a*x^2*(4/7*h*x+g)*b^2-8*a^2*x*(3/10*h*x+g)*b+
a^3*(-8/3*h*x+g))*e*f^3-6/5*(1/21*x*(1/3*h*x+g)*b^2+a*(1/7*h*x+g)*b+a^2*h)
*x*b*e^2*f^2+12/35*(1/3*(2/9*h*x+g)*b+a*h)*x*b^2*e^3*f-16/315*b^3*e^4*h*x)
*d^5-11/15*c*((2/7*(-1/3*h*x^4-27/55*g*x^3)*b^3-42/55*(27/49*h*x+g)*a*x^2*
b^2-30/11*a^2*x*(7/25*h*x+g)*b+9/11*(-10/9*h*x+g)*a^3)*f^4+(-24/77*x^2*(47
/90*h*x+g)*b^3-204/55*(30/119*h*x+g)*a*x*b^2+3*a^2*(-68/55*h*x+g)*b+h*a^3)
*e*f^3+18/55*(-13/21*x*(5/39*h*x+g)*b^2+a*(-13/7*h*x+g)*b+a^2*h)*b*e^2*f^2
-36/385*(1/3*(-16/9*h*x+g)*b+a*h)*b^2*e^3*f+16/1155*b^3*e^4*h)*d^4+c^2*((-
6/25*x^2*(11/21*h*x+g)*b^3-14/5*a*x*(9/35*h*x+g)*b^2+3*a^2*(-14/15*h*x+g)*
b+h*a^3)*f^3+19/5*b*e*(-32/95*x*(13/56*h*x+g)*b^2+a*(-96/95*h*x+g)*b+a^2*h
)*f^2+12/25*(1/3*(-19/14*h*x+g)*b+a*h)*b^2*e^2*f-8/175*b^3*e^3*h)*f*d^3-21
/5*c^3*((-2/7*x*(11/45*h*x+g)*b^2+a*(-6/7*h*x+g)*b+a^2*h)*f^2+9/7*(1/3*(-1
24/135*h*x+g)*b+a*h)*b*e*f+2/35*b^2*e^2*h)*b*f^2*d^2+27/5*c^4*b^2*f^3*((1/
3*(-22/27*h*x+g)*b+a*h)*f+35/81*e*h*b)*d-11/5*b^3*c^5*f^4*h)*((c*f-d*e)*d)
^(1/2)*(f*x+e)^(1/2))/f^3/(d*x+c)/d^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1499 vs. $2(394) = 788$.

Time = 0.22 (sec) , antiderivative size = 3008, normalized size of antiderivative = 7.09

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(f*x+e)**(3/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1485 vs. $2(394) = 788$.

Time = 0.17 (sec) , antiderivative size = 1485, normalized size of antiderivative = 3.50

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(6*b^3*c^2*d^3*e^2*g - 12*a*b^2*c*d^4*e^2*g + 6*a^2*b*d^5*e^2*g - 15*b^3*c
^3*d^2*e*f*g + 33*a*b^2*c^2*d^3*e*f*g - 21*a^2*b*c*d^4*e*f*g + 3*a^3*d^5*e
*f*g + 9*b^3*c^4*d*f^2*g - 21*a*b^2*c^3*d^2*f^2*g + 15*a^2*b*c^2*d^3*f^2*g
 - 3*a^3*c*d^4*f^2*g - 8*b^3*c^3*d^2*e^2*h + 18*a*b^2*c^2*d^3*e^2*h - 12*a
^2*b*c*d^4*e^2*h + 2*a^3*d^5*e^2*h + 19*b^3*c^4*d*e*f*h - 45*a*b^2*c^3*d^2
*e*f*h + 33*a^2*b*c^2*d^3*e*f*h - 7*a^3*c*d^4*e*f*h - 11*b^3*c^5*f^2*h + 2
7*a*b^2*c^4*d*f^2*h - 21*a^2*b*c^3*d^2*f^2*h + 5*a^3*c^2*d^3*f^2*h)*arctan
(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^6) + (sqrt(
f*x + e)*b^3*c^3*d^2*e*f*g - 3*sqrt(f*x + e)*a*b^2*c^2*d^3*e*f*g + 3*sqrt(
f*x + e)*a^2*b*c*d^4*e*f*g - sqrt(f*x + e)*a^3*d^5*e*f*g - sqrt(f*x + e)*b
^3*c^4*d*f^2*g + 3*sqrt(f*x + e)*a*b^2*c^3*d^2*f^2*g - 3*sqrt(f*x + e)*a^2
*b*c^2*d^3*f^2*g + sqrt(f*x + e)*a^3*c*d^4*f^2*g - sqrt(f*x + e)*b^3*c^4*d
*e*f*h + 3*sqrt(f*x + e)*a*b^2*c^3*d^2*e*f*h - 3*sqrt(f*x + e)*a^2*b*c^2*d
^3*e*f*h + sqrt(f*x + e)*a^3*c*d^4*e*f*h + sqrt(f*x + e)*b^3*c^5*f^2*h - 3
*sqrt(f*x + e)*a*b^2*c^4*d*f^2*h + 3*sqrt(f*x + e)*a^2*b*c^3*d^2*f^2*h - s
qrt(f*x + e)*a^3*c^2*d^3*f^2*h)/(((f*x + e)*d - d*e + c*f)*d^6) + 2/315*(4
5*(f*x + e)^(7/2)*b^3*d^16*f^25*g - 63*(f*x + e)^(5/2)*b^3*d^16*e*f^25*g -
 126*(f*x + e)^(5/2)*b^3*c*d^15*f^26*g + 189*(f*x + e)^(5/2)*a*b^2*d^16*f^
26*g + 315*(f*x + e)^(3/2)*b^3*c^2*d^14*f^27*g - 630*(f*x + e)^(3/2)*a*b^2
*c*d^15*f^27*g + 315*(f*x + e)^(3/2)*a^2*b*d^16*f^27*g + 945*sqrt(f*x +...
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1602, normalized size of antiderivative = 3.78

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^3)/(c + d*x)^2,x)
```

output
```
(e + f*x)^(7/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(7*d^2*f^3) - (4*b^
3*h*(c*f - d*e))/(7*d^3*f^3)) - (e + f*x)^(1/2)*((2*(c*f - d*e)*((2*(c*f -
 d*e)*((2*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f
 - d*e))/(d^3*f^3))*(c*f - d*e))/d - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b
*f*g))/(d^2*f^3) + (2*b^3*h*(c*f - d*e)^2)/(d^4*f^3)))/d - (((2*b^3*f*g -
8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d*e))/(d^3*f^3))*(c*f
 - d*e)^2)/d^2 + (2*(a*f - b*e)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(d^2*f^3)))
/d - ((c*f - d*e)^2*((2*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) -
 (4*b^3*h*(c*f - d*e))/(d^3*f^3))*(c*f - d*e))/d - (6*b*(a*f - b*e)*(a*f*h
 - 2*b*e*h + b*f*g))/(d^2*f^3) + (2*b^3*h*(c*f - d*e)^2)/(d^4*f^3)))/d^2 +
 (2*(a*f - b*e)^3*(e*h - f*g))/(d^2*f^3)) + (e + f*x)^(3/2)*((2*(c*f - d*e
)*((2*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d
*e))/(d^3*f^3))*(c*f - d*e))/d - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g
))/(d^2*f^3) + (2*b^3*h*(c*f - d*e)^2)/(d^4*f^3)))/(3*d) - (((2*b^3*f*g -
8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d*e))/(d^3*f^3))*(c*f
 - d*e)^2)/(3*d^2) + (2*(a*f - b*e)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(3*d^2*
f^3)) - (e + f*x)^(5/2)*((2*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^
3) - (4*b^3*h*(c*f - d*e))/(d^3*f^3))*(c*f - d*e))/(5*d) - (6*b*(a*f - b*e
)*(a*f*h - 2*b*e*h + b*f*g))/(5*d^2*f^3) + (2*b^3*h*(c*f - d*e)^2)/(5*d^4*
f^3)) + ((e + f*x)^(1/2)*(b^3*c^5*f^2*h - a^3*d^5*e*f*g + a^3*c*d^4*f^2...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3285, normalized size of antiderivative = 7.75

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x)
```

output
```
(1575*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a**3*c**2*d**3*f**4*h - 630*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*e*f**3*h - 945*sqrt(d)*sqrt(
c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*f
**4*g + 1575*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a**3*c*d**4*f**4*h*x - 630*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**5*e*f**3*h*x - 945*sqrt(d)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d*
*5*f**4*g*x - 6615*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*a**2*b*c**3*d**2*f**4*h + 3780*sqrt(d)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*e*f**3*
h + 4725*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
- d*e)))*a**2*b*c**2*d**3*f**4*g - 6615*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*f**4*h*x - 1890*s
qrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a
**2*b*c*d**4*e*f**3*g + 3780*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d
)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4*e*f**3*h*x + 4725*sqrt(d)*sqrt(
c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4
*f**4*g*x - 1890*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a**2*b*d**5*e*f**3*g*x + 8505*sqrt(d)*sqrt(c*f - d*e)*...
```

3.88 $\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 322

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = -\frac{(bc-ad)\left(ad(3dfg+2deh-5cfh)+b(4d^2eg+9c^2fh-cd(7fg+6eh))\right)\sqrt{e+fx}}{d^5} - \frac{2(bc-ad)(2bdg-3bch+adh)(e+fx)^{3/2}}{3d^4} - \frac{(bc-ad)^2(dg-ch)(e+fx)^{3/2}}{d^4(c+dx)} + \frac{2b(2adfh+b(dfg-deh-2cfh))(e+fx)^{5/2}}{5d^3f^2} + \frac{2b^2h(e+fx)^{7/2}}{7d^2f^2} + \frac{(bc-ad)\sqrt{de-cf}\left(ad(3dfg+2deh-5cfh)+b(4d^2eg+9c^2fh-cd(7fg+6eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{11/2}}$$

output
```
-(-a*d+b*c)*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)+b*(4*d^2*e*g+9*c^2*f*h-c*d*(6*
e*h+7*f*g)))*(f*x+e)^(1/2)/d^5-2/3*(-a*d+b*c)*(a*d*h-3*b*c*h+2*b*d*g)*(f*x
+e)^(3/2)/d^4-(-a*d+b*c)^2*(-c*h+d*g)*(f*x+e)^(3/2)/d^4/(d*x+c)+2/5*b*(2*a
*d*f*h+b*(-2*c*f*h-d*e*h+d*f*g))*(f*x+e)^(5/2)/d^3/f^2+2/7*b^2*h*(f*x+e)^(
7/2)/d^2/f^2+(-a*d+b*c)*(-c*f+d*e)^(1/2)*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)+b
*(4*d^2*e*g+9*c^2*f*h-c*d*(6*e*h+7*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-
c*f+d*e)^(1/2))/d^(11/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{\sqrt{e+fx}(35a^2d^2f^2(-15c^2fh+cd(9fg+11eh-10fhx)+d^2(-3eg\ldots}{\ldots} - \frac{(-bc+ad)\sqrt{-de+cf}(ad(3dfg+2deh-5cfh)+b(4d^2eg+9c^2fh-cd(7fg+6eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{11/2}}$$

input
```
Integrate[((a + b*x)^2*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(35*a^2*d^2*f^2*(-15*c^2*f*h + c*d*(9*f*g + 11*e*h - 10*f*h
*x) + d^2*(-3*e*g + 6*f*g*x + 8*e*h*x + 2*f*h*x^2)) + 14*a*b*d*f*(105*c^3*
f^2*h - 5*c^2*d*f*(15*f*g + 19*e*h - 14*f*h*x) + 2*d^3*x*(3*e^2*h + f^2*x*
(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + c*d^2*(6*e^2*h + e*f*(55*g - 68*h*x)
 - 2*f^2*x*(25*g + 7*h*x))) + b^2*(-945*c^4*f^3*h + 105*c^3*d*f^2*(7*f*g +
 9*e*h - 6*f*h*x) + 6*d^4*x*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h*x) - 2*c*d^
3*(6*e^3*h + f^3*x^2*(49*g + 27*h*x) + 2*e*f^2*x*(119*g + 30*h*x) + e^2*f*
(-21*g + 39*h*x)) + 7*c^2*d^2*f*(-12*e^2*h + 2*f^2*x*(35*g + 9*h*x) + e*f*
(-95*g + 96*h*x)))))/(105*d^5*f^2*(c + d*x)) - ((-(b*c) + a*d)*Sqrt[-(d*e)
 + c*f]*(a*d*(3*d*f*g + 2*d*e*h - 5*c*f*h) + b*(4*d^2*e*g + 9*c^2*f*h - c*
d*(7*f*g + 6*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^
(11/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)(e+fx)^{3/2}(4be(dg-ch)+a(3dfg+2deh-5cfh)+b(7dfg+2deh-9cfh)x)}{2(c+dx)}dx}{d(de-cf)}-\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+bx)(e+fx)^{3/2}(4be(dg-ch)+a(3dfg+2deh-5cfh)+b(7dfg+2deh-9cfh)x)}{c+dx}dx}{2d(de-cf)}-\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 164

$$\frac{\frac{2b(e+fx)^{5/2}(14adf(-7cfh+2deh+5dfg)+b(63c^2f^2h-cdf(24eh+49fg)+2d^2e(7fg-2eh))+5bdfx(-9cfh+2deh+7dfg))}{35d^2f^2}-\frac{(bc-ad)(ad(-5cf}{}}{2d(de-cf)}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 60

$$\frac{\frac{2b(e+fx)^{5/2}(14adf(-7cfh+2deh+5dfg)+b(63c^2f^2h-cdf(24eh+49fg)+2d^2e(7fg-2eh))+5bdfx(-9cfh+2deh+7dfg))}{35d^2f^2}-\frac{(bc-ad)(ad(-5cf}{}}{2d(de-cf)}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 60

$$\frac{\frac{2b(e+fx)^{5/2}(14adf(-7cfh+2deh+5dfg)+b(63c^2f^2h-cdf(24eh+49fg)+2d^2e(7fg-2eh))+5bdfx(-9cfh+2deh+7dfg))}{35d^2f^2}-\frac{(bc-ad)(ad(-5cf}{}}{2d(de-cf)}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2b(e+fx)^{5/2}(14adf(-7cfh+2deh+5dfg)+b(63c^2f^2h-cdf(24eh+49fg)+2d^2e(7fg-2eh))+5bdfx(-9cfh+2deh+7dfg))}{35d^2f^2}-\frac{(bc-ad)(ad(-5cf\ldots}{2d(de-cf)}}{\ldots}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2b(e+fx)^{5/2}(14adf(-7cfh+2deh+5dfg)+b(63c^2f^2h-cdf(24eh+49fg)+2d^2e(7fg-2eh))+5bdfx(-9cfh+2deh+7dfg))}{35d^2f^2}-\frac{(bc-ad)\left(\frac{(de-cf)}{\ldots}\right.}{2d(de-cf)}}{\ldots}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

input

```
Int[((a + b*x)^2*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output

```
-(((d*g - c*h)*(a + b*x)^2*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x))) + (
(2*b*(e + f*x)^(5/2)*(14*a*d*f*(5*d*f*g + 2*d*e*h - 7*c*f*h) + b*(63*c^2*f
^2*h + 2*d^2*e*(7*f*g - 2*e*h) - c*d*f*(49*f*g + 24*e*h)) + 5*b*d*f*(7*d*f
*g + 2*d*e*h - 9*c*f*h)*x))/(35*d^2*f^2) - ((b*c - a*d)*(a*d*(3*d*f*g + 2*
d*e*h - 5*c*f*h) + b*(4*d^2*e*g + 9*c^2*f*h - c*d*(7*f*g + 6*e*h)))*((2*(e
 + f*x)^(3/2))/(3*d) + ((d*e - c*f)*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c
*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d))/d^2)/(
2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.54

method	result
pseudoelliptic	$-\frac{5\left(-(ad-bc)(cf-de)\left(\frac{(-3afg-2e(ah+2bg))d^2}{5}+c\left(\left(ah+\frac{7bg}{5}\right)f+\frac{6ehb}{5}\right)d-\frac{9b\,c^2fh}{5}\right)(xd+c)f^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)\right.}{}$
risch	$-\frac{2(-15b^2h\,x^3d^3f^3-42ab\,d^3f^3h\,x^2+42b^2c\,d^2f^3h\,x^2-24b^2d^3e\,f^2h\,x^2-21b^2d^3f^3g\,x^2-35a^2d^3f^3hx+140abc\,d^2f^3hx}{}$
derivativedivides	$-\frac{2\left(-\frac{h\,b^2(fx+e)^{\frac{7}{2}}d^3}{7}-\frac{2ab\,d^3fh(fx+e)^{\frac{5}{2}}}{5}+\frac{2b^2c\,d^2fh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^2d^3eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^2d^3fg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2d^3f^2h(fx+e)^{\frac{3}{2}}}{3}+4ab\right.}{}$
default	$-\frac{2\left(-\frac{h\,b^2(fx+e)^{\frac{7}{2}}d^3}{7}-\frac{2ab\,d^3fh(fx+e)^{\frac{5}{2}}}{5}+\frac{2b^2c\,d^2fh(fx+e)^{\frac{5}{2}}}{5}+\frac{b^2d^3eh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^2d^3fg(fx+e)^{\frac{5}{2}}}{5}-\frac{a^2d^3f^2h(fx+e)^{\frac{3}{2}}}{3}+4ab\right.}{}$

input
```
int((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/((c*f-d*e)*d)^(1/2)*(-(a*d-b*c)*(c*f-d*e)*(1/5*(-3*a*f*g-2*e*(a*h+2*b*g
))*d^2+c*((a*h+7/5*b*g)*f+6/5*e*h*b)*d-9/5*b*c^2*f*h)*(d*x+c)*f^2*arctan(d
*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*(1/5
*(-2*x*(1/5*x^2*(5/7*h*x+g)*b^2+2/3*a*(3/5*h*x+g)*x*b+a^2*(1/3*h*x+g))*f^3
+(-4/5*x^2*(4/7*h*x+g)*b^2-16/3*a*x*(3/10*h*x+g)*b+a^2*(-8/3*h*x+g))*e*f^2
-4/5*x*(1/2*(1/7*h*x+g)*b+a*h)*b*e^2*f+4/35*b^2*e^3*h*x)*d^4-11/15*c*(1/11
*(-14/5*(27/49*h*x+g)*x^2*b^2-20*a*x*(7/25*h*x+g)*b+9*(-10/9*h*x+g)*a^2)*f
^3+(4/11*(-6/7*h*x^2-17/5*g*x)*b^2+2*a*(-68/55*h*x+g)*b+a^2*h)*e*f^2+12/55
*(1/2*(-13/7*h*x+g)*b+a*h)*b*e^2*f-12/385*b^2*e^3*h)*d^3+c^2*((-14/15*x*(9
/35*h*x+g)*b^2+2*a*(-14/15*h*x+g)*b+a^2*h)*f^2+38/15*((-48/95*h*x+1/2*g)*b
+a*h)*b*e*f+4/25*b^2*e^2*h)*f*d^2-14/5*c^3*(((-3/7*h*x+1/2*g)*b+a*h)*f+9/1
4*e*h*b)*b*f^2*d+9/5*b^2*c^4*f^3*h))/f^2/(d*x+c)/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(296) = 592$.

Time = 0.21 (sec) , antiderivative size = 1833, normalized size of antiderivative = 5.69

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/210*(105*((4*(b^2*c^2*d^2 - a*b*c*d^3)*e*f^2 - (7*b^2*c^3*d - 10*a*b*c^
2*d^2 + 3*a^2*c*d^3)*f^3)*g - (2*(3*b^2*c^3*d - 4*a*b*c^2*d^2 + a^2*c*d^3)
*e*f^2 - (9*b^2*c^4 - 14*a*b*c^3*d + 5*a^2*c^2*d^2)*f^3)*h + ((4*(b^2*c*d^
3 - a*b*d^4)*e*f^2 - (7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*f^3)*g - (
2*(3*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*e*f^2 - (9*b^2*c^3*d - 14*a*b*c^
2*d^2 + 5*a^2*c*d^3)*f^3)*h)*x)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c
*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)) + 2*(30*b^2*d^4*f^3
*h*x^4 + 6*(7*b^2*d^4*f^3*g + (8*b^2*d^4*e*f^2 - (9*b^2*c*d^3 - 14*a*b*d^4
)*f^3)*h)*x^3 + 2*(7*(6*b^2*d^4*e*f^2 - (7*b^2*c*d^3 - 10*a*b*d^4)*f^3)*g
+ (3*b^2*d^4*e^2*f - 12*(5*b^2*c*d^3 - 7*a*b*d^4)*e*f^2 + 7*(9*b^2*c^2*d^2
 - 14*a*b*c*d^3 + 5*a^2*d^4)*f^3)*h)*x^2 + 7*(6*b^2*c*d^3*e^2*f - 5*(19*b^
2*c^2*d^2 - 22*a*b*c*d^3 + 3*a^2*d^4)*e*f^2 + 15*(7*b^2*c^3*d - 10*a*b*c^2
*d^2 + 3*a^2*c*d^3)*f^3)*g - (12*b^2*c*d^3*e^3 + 84*(b^2*c^2*d^2 - a*b*c*d
^3)*e^2*f - 35*(27*b^2*c^3*d - 38*a*b*c^2*d^2 + 11*a^2*c*d^3)*e*f^2 + 105*
(9*b^2*c^4 - 14*a*b*c^3*d + 5*a^2*c^2*d^2)*f^3)*h + 2*(7*(3*b^2*d^4*e^2*f
- 2*(17*b^2*c*d^3 - 20*a*b*d^4)*e*f^2 + 5*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 +
3*a^2*d^4)*f^3)*g - (6*b^2*d^4*e^3 + 3*(13*b^2*c*d^3 - 14*a*b*d^4)*e^2*f -
 28*(12*b^2*c^2*d^2 - 17*a*b*c*d^3 + 5*a^2*d^4)*e*f^2 + 35*(9*b^2*c^3*d -
14*a*b*c^2*d^2 + 5*a^2*c*d^3)*f^3)*h)*x)*sqrt(f*x + e))/(d^6*f^2*x + c*d^5
*f^2), 1/105*(105*((4*(b^2*c^2*d^2 - a*b*c*d^3)*e*f^2 - (7*b^2*c^3*d - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**2*(f*x+e)**(3/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(296) = 592$.

Time = 0.16 (sec) , antiderivative size = 942, normalized size of antiderivative = 2.93

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(4*b^2*c*d^3*e^2*g - 4*a*b*d^4*e^2*g - 11*b^2*c^2*d^2*e*f*g + 14*a*b*c*d^
3*e*f*g - 3*a^2*d^4*e*f*g + 7*b^2*c^3*d*f^2*g - 10*a*b*c^2*d^2*f^2*g + 3*a
^2*c*d^3*f^2*g - 6*b^2*c^2*d^2*e^2*h + 8*a*b*c*d^3*e^2*h - 2*a^2*d^4*e^2*h
 + 15*b^2*c^3*d*e*f*h - 22*a*b*c^2*d^2*e*f*h + 7*a^2*c*d^3*e*f*h - 9*b^2*c
^4*f^2*h + 14*a*b*c^3*d*f^2*h - 5*a^2*c^2*d^2*f^2*h)*arctan(sqrt(f*x + e)*
d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^5) - (sqrt(f*x + e)*b^2*c^
2*d^2*e*f*g - 2*sqrt(f*x + e)*a*b*c*d^3*e*f*g + sqrt(f*x + e)*a^2*d^4*e*f*
g - sqrt(f*x + e)*b^2*c^3*d*f^2*g + 2*sqrt(f*x + e)*a*b*c^2*d^2*f^2*g - sq
rt(f*x + e)*a^2*c*d^3*f^2*g - sqrt(f*x + e)*b^2*c^3*d*e*f*h + 2*sqrt(f*x +
 e)*a*b*c^2*d^2*e*f*h - sqrt(f*x + e)*a^2*c*d^3*e*f*h + sqrt(f*x + e)*b^2*
c^4*f^2*h - 2*sqrt(f*x + e)*a*b*c^3*d*f^2*h + sqrt(f*x + e)*a^2*c^2*d^2*f^
2*h)/(((f*x + e)*d - d*e + c*f)*d^5) + 2/105*(21*(f*x + e)^(5/2)*b^2*d^12*
f^13*g - 70*(f*x + e)^(3/2)*b^2*c*d^11*f^14*g + 70*(f*x + e)^(3/2)*a*b*d^1
2*f^14*g - 210*sqrt(f*x + e)*b^2*c*d^11*e*f^14*g + 210*sqrt(f*x + e)*a*b*d
^12*e*f^14*g + 315*sqrt(f*x + e)*b^2*c^2*d^10*f^15*g - 420*sqrt(f*x + e)*a
*b*c*d^11*f^15*g + 105*sqrt(f*x + e)*a^2*d^12*f^15*g + 15*(f*x + e)^(7/2)*
b^2*d^12*f^12*h - 21*(f*x + e)^(5/2)*b^2*d^12*e*f^12*h - 42*(f*x + e)^(5/2
)*b^2*c*d^11*f^13*h + 42*(f*x + e)^(5/2)*a*b*d^12*f^13*h + 105*(f*x + e)^(
3/2)*b^2*c^2*d^10*f^14*h - 140*(f*x + e)^(3/2)*a*b*c*d^11*f^14*h + 35*(f*x
 + e)^(3/2)*a^2*d^12*f^14*h + 315*sqrt(f*x + e)*b^2*c^2*d^10*e*f^14*h -...
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = (e+f\,x)^{5/2}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{5\,d^2\,f^2}-\frac{4\,b^2\,h\,(c\,f-d\,e)}{5\,d^3\,f^2}\right)$$

$$-(e+f\,x)^{3/2}\left(\frac{2\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d^2\,f^2}-\frac{4\,b^2\,h\,(c\,f-d\,e)}{d^3\,f^2}\right)\,(c\,f-d\,e)}{3\,d}-\frac{2\,(a\,f-b\,e)\,(a\,f\,h-3\,b\,e\,h+2\,b\,f\,g)}{3\,d^2\,f^2}\right.$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
(e + f*x)^(5/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(5*d^2*f^2) - (4*b^2*
h*(c*f - d*e))/(5*d^3*f^2)) - (e + f*x)^(3/2)*((2*((2*b^2*f*g - 6*b^2*e*h
+ 4*a*b*f*h)/(d^2*f^2) - (4*b^2*h*(c*f - d*e))/(d^3*f^2))*(c*f - d*e))/(3*
d) - (2*(a*f - b*e)*(a*f*h - 3*b*e*h + 2*b*f*g))/(3*d^2*f^2) + (2*b^2*h*(c
*f - d*e)^2)/(3*d^4*f^2)) - (e + f*x)^(1/2)*((((2*b^2*f*g - 6*b^2*e*h + 4*
a*b*f*h)/(d^2*f^2) - (4*b^2*h*(c*f - d*e))/(d^3*f^2))*(c*f - d*e)^2)/d^2 -
 (2*(c*f - d*e)*((2*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d^2*f^2) - (4*b^
2*h*(c*f - d*e))/(d^3*f^2))*(c*f - d*e))/d - (2*(a*f - b*e)*(a*f*h - 3*b*e
*h + 2*b*f*g))/(d^2*f^2) + (2*b^2*h*(c*f - d*e)^2)/(d^4*f^2)))/d + (2*(a*f
 - b*e)^2*(e*h - f*g))/(d^2*f^2)) - ((e + f*x)^(1/2)*(b^2*c^4*f^2*h + a^2*
d^4*e*f*g - a^2*c*d^3*f^2*g - b^2*c^3*d*f^2*g + a^2*c^2*d^2*f^2*h - 2*a*b*
c^3*d*f^2*h - a^2*c*d^3*e*f*h - b^2*c^3*d*e*f*h + 2*a*b*c^2*d^2*f^2*g + b^
2*c^2*d^2*e*f*g - 2*a*b*c*d^3*e*f*g + 2*a*b*c^2*d^2*e*f*h))/(d^6*(e + f*x)
 - d^6*e + c*d^5*f) + (2*b^2*h*(e + f*x)^(7/2))/(7*d^2*f^2) + (atan((d^(1/
2)*(e + f*x)^(1/2)*1i)/(d*e - c*f)^(1/2))*(a*d - b*c)*(d*e - c*f)^(1/2)*(2
*a*d^2*e*h + 3*a*d^2*f*g + 4*b*d^2*e*g + 9*b*c^2*f*h - 5*a*c*d*f*h - 6*b*c
*d*e*h - 7*b*c*d*f*g)*1i)/d^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2025, normalized size of antiderivative = 6.29

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x)
```

output

```
(525*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*
e)))*a**2*c**2*d**2*f**3*h - 210*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*e*f**2*h - 315*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*f*
*3*g + 525*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a**2*c*d**3*f**3*h*x - 210*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4*e*f**2*h*x - 315*sqrt(d)*s
qrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4
*f**3*g*x - 1470*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a*b*c**3*d*f**3*h + 840*sqrt(d)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*e*f**2*h + 1050*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b
*c**2*d**2*f**3*g - 1470*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*f**3*h*x - 420*sqrt(d)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*e*f**2*g
+ 840*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a*b*c*d**3*e*f**2*h*x + 1050*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*f**3*g*x - 420*sqrt(d)*sqrt(
c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*d**4*e*f*
*2*g*x + 945*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sq...
```

3.89 $\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 248

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{(ad(3dfg+2deh-5cfh)+b(2d^2eg+7c^2fh-cd(5fg+4eh)))\sqrt{e+fx}}{d^4} + \frac{2(bdg-2bch+adh)(e+fx)^{3/2}}{3d^3} + \frac{(bc-ad)(dg-ch)(e+fx)^{3/2}}{d^3(c+dx)} + \frac{2bh(e+fx)^{5/2}}{5d^2f} - \frac{\sqrt{de-cf}\,(ad(3dfg+2deh-5cfh)+b(2d^2eg+7c^2fh-cd(5fg+4eh)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{9/2}}$$

output
```
(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)+b*(2*d^2*e*g+7*c^2*f*h-c*d*(4*e*h+5*f*g)))
*(f*x+e)^(1/2)/d^4+2/3*(a*d*h-2*b*c*h+b*d*g)*(f*x+e)^(3/2)/d^3+(-a*d+b*c)*
(-c*h+d*g)*(f*x+e)^(3/2)/d^3/(d*x+c)+2/5*b*h*(f*x+e)^(5/2)/d^2/f-(-c*f+d*e
)^(1/2)*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)+b*(2*d^2*e*g+7*c^2*f*h-c*d*(4*e*h+
5*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{\sqrt{e+fx}(5adf(-15c^2fh+cd(9fg+11eh-10fhx)+d^2(-3eg+6f}{}$$

$$-\frac{\sqrt{-de+cf}(ad(3dfg+2deh-5cfh)+b(2d^2eg+7c^2fh-cd(5fg+4eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{9/2}}$$

input
```
Integrate[((a + b*x)*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(5*a*d*f*(-15*c^2*f*h + c*d*(9*f*g + 11*e*h - 10*f*h*x) + d
^2*(-3*e*g + 6*f*g*x + 8*e*h*x + 2*f*h*x^2)) + b*(105*c^3*f^2*h - 5*c^2*d*
f*(15*f*g + 19*e*h - 14*f*h*x) + 2*d^3*x*(3*e^2*h + f^2*x*(5*g + 3*h*x) +
e*f*(20*g + 6*h*x)) + c*d^2*(6*e^2*h + e*f*(55*g - 68*h*x) - 2*f^2*x*(25*g
 + 7*h*x)))))/(15*d^4*f*(c + d*x)) - (Sqrt[-(d*e) + c*f]*(a*d*(3*d*f*g + 2
*d*e*h - 5*c*f*h) + b*(2*d^2*e*g + 7*c^2*f*h - c*d*(5*f*g + 4*e*h)))*ArcTa
n[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/d^(9/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {163, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{163}$$

$$\frac{\left(ad(-5cfh+2deh+3dfg)+b\left(7c^2fh-cd(4eh+5fg)+2d^2eg\right)\right)\int \frac{(e+fx)^{3/2}}{c+dx}dx}{2d^2(de-cf)}-\frac{(e+fx)^{5/2}(5adf(dg-ch)-bc(-7cfh+2deh+5dfg)-2bdhx(de-cf))}{5d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(ad(-5cfh+2deh+3dfg)+b\left(7c^2fh-cd(4eh+5fg)+2d^2eg\right)\right)\left(\frac{(de-cf)\int \frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{2d^2(de-cf)}-\frac{(e+fx)^{5/2}(5adf(dg-ch)-bc(-7cfh+2deh+5dfg)-2bdhx(de-cf))}{5d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(ad(-5cfh+2deh+3dfg)+b\left(7c^2fh-cd(4eh+5fg)+2d^2eg\right)\right)\left(\frac{(de-cf)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e}{}\right.}{2d^2(de-cf)}$$
$$\frac{(e+fx)^{5/2}(5adf(dg-ch)-bc(-7cfh+2deh+5dfg)-2bdhx(de-cf))}{5d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(ad(-5cfh+2deh+3dfg)+b\left(7c^2fh-cd(4eh+5fg)+2d^2eg\right)\right)\left(\frac{(de-cf)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right.}{d}\right.}{2d^2(de-cf)}$$
$$\frac{(e+fx)^{5/2}(5adf(dg-ch)-bc(-7cfh+2deh+5dfg)-2bdhx(de-cf))}{5d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{(de-cf)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)\left(ad(-5cfh+2deh+3dfg)+b\left(7c^2fh-cd(4eh+5\right.\right.}{2d^2(de-cf)}$$
$$\frac{(e+fx)^{5/2}(5adf(dg-ch)-bc(-7cfh+2deh+5dfg)-2bdhx(de-cf))}{5d^2f(c+dx)(de-cf)}$$

input

```
Int[((a + b*x)*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
-1/5*((e + f*x)^(5/2)*(5*a*d*f*(d*g - c*h) - b*c*(5*d*f*g + 2*d*e*h - 7*c*
f*h) - 2*b*d*(d*e - c*f)*h*x))/(d^2*f*(d*e - c*f)*(c + d*x)) + ((a*d*(3*d*
f*g + 2*d*e*h - 5*c*f*h) + b*(2*d^2*e*g + 7*c^2*f*h - c*d*(5*f*g + 4*e*h))
)*((2*(e + f*x)^(3/2))/(3*d) + ((d*e - c*f)*((2*Sqrt[e + f*x])/d - (2*Sqrt
[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d)
)/(2*d^2*(d*e - c*f))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$-\frac{5\left(-(cf-de)\left(\frac{(-3afg-2(ah+bg)e)d^2}{5}+c\left((ah+bg)f+\frac{4ehb}{5}\right)d-\frac{7bc^2fh}{5}\right)(xd+c)f\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\sqrt{(cf-de)d}\right.}{}$
risch	$-\frac{2(-3bhx^2d^2f^2-5ad^2f^2hx+10bcdf^2hx-6bd^2efhx-5bd^2f^2gx+30acdf^2h-20ad^2efh-15ad^2f^2g-45bc^2f^2h+}{15fd^4}$
derivativedivides	$-\frac{2\left(-\frac{hb(fx+e)^{\frac{5}{2}}d^2}{5}-\frac{ad^2fh(fx+e)^{\frac{3}{2}}}{3}+\frac{2bcdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{bd^2fg(fx+e)^{\frac{3}{2}}}{3}+2acdf^2h\sqrt{fx+e}-ad^2efh\sqrt{fx+e}-ad^2f^2g\sqrt{fx+e}-\right.}{d^4}$
default	$-\frac{2\left(-\frac{hb(fx+e)^{\frac{5}{2}}d^2}{5}-\frac{ad^2fh(fx+e)^{\frac{3}{2}}}{3}+\frac{2bcdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{bd^2fg(fx+e)^{\frac{3}{2}}}{3}+2acdf^2h\sqrt{fx+e}-ad^2efh\sqrt{fx+e}-ad^2f^2g\sqrt{fx+e}-\right.}{d^4}$

input
```
int((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/((c*f-d*e)*d)^(1/2)*(-(c*f-d*e)*(1/5*(-3*a*f*g-2*(a*h+b*g)*e)*d^2+c*((a
*h+b*g)*f+4/5*e*h*b)*d-7/5*b*c^2*f*h)*(d*x+c)*f*arctan(d*(f*x+e)^(1/2)/((c
*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*(1/5*(-2*x*(1/3*(3/5*h
*x+g)*x*b+a*(1/3*h*x+g))*f^2+(4*(-1/5*h*x^2-2/3*g*x)*b+a*(-8/3*h*x+g))*e*f
-2/5*b*e^2*h*x)*d^3-11/15*c*(1/11*(-10*x*(7/25*h*x+g)*b+9*(-10/9*h*x+g)*a)
*f^2+((-68/55*h*x+g)*b+a*h)*e*f+6/55*b*e^2*h)*d^2+c^2*f*(((-14/15*h*x+g)*b
+a*h)*f+19/15*e*h*b)*d-7/5*b*c^3*f^2*h))/f/d^4/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(226) = 452.

Time = 0.14 (sec) , antiderivative size = 944, normalized size of antiderivative = 3.81

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*((2*b*c*d^2*e*f - (5*b*c^2*d - 3*a*c*d^2)*f^2)*g - (2*(2*b*c^2*
d - a*c*d^2)*e*f - (7*b*c^3 - 5*a*c^2*d)*f^2)*h + ((2*b*d^3*e*f - (5*b*c*d
^2 - 3*a*d^3)*f^2)*g - (2*(2*b*c*d^2 - a*d^3)*e*f - (7*b*c^2*d - 5*a*c*d^2
)*f^2)*h)*x)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e
)*d*sqrt((d*e - c*f)/d))/(d*x + c)) - 2*(6*b*d^3*f^2*h*x^3 + 2*(5*b*d^3*f^
2*g + (6*b*d^3*e*f - (7*b*c*d^2 - 5*a*d^3)*f^2)*h)*x^2 + 5*((11*b*c*d^2 -
3*a*d^3)*e*f - 3*(5*b*c^2*d - 3*a*c*d^2)*f^2)*g + (6*b*c*d^2*e^2 - 5*(19*b
*c^2*d - 11*a*c*d^2)*e*f + 15*(7*b*c^3 - 5*a*c^2*d)*f^2)*h + 2*(5*(4*b*d^3
*e*f - (5*b*c*d^2 - 3*a*d^3)*f^2)*g + (3*b*d^3*e^2 - 2*(17*b*c*d^2 - 10*a*
d^3)*e*f + 5*(7*b*c^2*d - 5*a*c*d^2)*f^2)*h)*x)*sqrt(f*x + e))/(d^5*f*x +
c*d^4*f), -1/15*(15*((2*b*c*d^2*e*f - (5*b*c^2*d - 3*a*c*d^2)*f^2)*g - (2*
(2*b*c^2*d - a*c*d^2)*e*f - (7*b*c^3 - 5*a*c^2*d)*f^2)*h + ((2*b*d^3*e*f -
 (5*b*c*d^2 - 3*a*d^3)*f^2)*g - (2*(2*b*c*d^2 - a*d^3)*e*f - (7*b*c^2*d -
5*a*c*d^2)*f^2)*h)*x)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(f*x + e)*d*sqrt(-(
d*e - c*f)/d)/(d*e - c*f)) - (6*b*d^3*f^2*h*x^3 + 2*(5*b*d^3*f^2*g + (6*b*
d^3*e*f - (7*b*c*d^2 - 5*a*d^3)*f^2)*h)*x^2 + 5*((11*b*c*d^2 - 3*a*d^3)*e*
f - 3*(5*b*c^2*d - 3*a*c*d^2)*f^2)*g + (6*b*c*d^2*e^2 - 5*(19*b*c^2*d - 11
*a*c*d^2)*e*f + 15*(7*b*c^3 - 5*a*c^2*d)*f^2)*h + 2*(5*(4*b*d^3*e*f - (5*b
*c*d^2 - 3*a*d^3)*f^2)*g + (3*b*d^3*e^2 - 2*(17*b*c*d^2 - 10*a*d^3)*e*f +
5*(7*b*c^2*d - 5*a*c*d^2)*f^2)*h)*x)*sqrt(f*x + e))/(d^5*f*x + c*d^4*f)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(f*x+e)**(3/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(226) = 452$.

Time = 0.14 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{(2\,bd^3e^2g - 7\,bcd^2efg + 3\,ad^3efg + 5\,bc^2df^2g - 3\,acd^2f^2g - 4\,bcd^2e^2 \cdots}{\cdots}$$
$$+ \frac{\sqrt{fx+e}bcd^2efg - \sqrt{fx+e}ad^3efg - \sqrt{fx+e}bc^2df^2g + \sqrt{fx+e}acd^2f^2g - \sqrt{fx+e}bc^2defh + \sqrt{fx+\cdots}}{((fx+e)d - de + cf)d^4}$$
$$+ \frac{2\left(5\,(fx+e)^{\frac{3}{2}}bd^8f^5g + 15\,\sqrt{fx+e}bd^8ef^5g - 30\,\sqrt{fx+e}bcd^7f^6g + 15\,\sqrt{fx+e}ad^8f^6g + 3\,(fx+e)^{\frac{5}{2}}bd^{\cdots}\right.}{\cdots}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(2*b*d^3*e^2*g - 7*b*c*d^2*e*f*g + 3*a*d^3*e*f*g + 5*b*c^2*d*f^2*g - 3*a*c
*d^2*f^2*g - 4*b*c*d^2*e^2*h + 2*a*d^3*e^2*h + 11*b*c^2*d*e*f*h - 7*a*c*d^
2*e*f*h - 7*b*c^3*f^2*h + 5*a*c^2*d*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^
2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^4) + (sqrt(f*x + e)*b*c*d^2*e*f*g -
sqrt(f*x + e)*a*d^3*e*f*g - sqrt(f*x + e)*b*c^2*d*f^2*g + sqrt(f*x + e)*a*
c*d^2*f^2*g - sqrt(f*x + e)*b*c^2*d*e*f*h + sqrt(f*x + e)*a*c*d^2*e*f*h +
sqrt(f*x + e)*b*c^3*f^2*h - sqrt(f*x + e)*a*c^2*d*f^2*h)/(((f*x + e)*d - d
*e + c*f)*d^4) + 2/15*(5*(f*x + e)^(3/2)*b*d^8*f^5*g + 15*sqrt(f*x + e)*b*
d^8*e*f^5*g - 30*sqrt(f*x + e)*b*c*d^7*f^6*g + 15*sqrt(f*x + e)*a*d^8*f^6*
g + 3*(f*x + e)^(5/2)*b*d^8*f^4*h - 10*(f*x + e)^(3/2)*b*c*d^7*f^5*h + 5*(
f*x + e)^(3/2)*a*d^8*f^5*h - 30*sqrt(f*x + e)*b*c*d^7*e*f^5*h + 15*sqrt(f*
x + e)*a*d^8*e*f^5*h + 45*sqrt(f*x + e)*b*c^2*d^6*f^6*h - 30*sqrt(f*x + e)
*a*c*d^7*f^6*h)/(d^10*f^5)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = (e+f\,x)^{3/2}\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{3\,d^2\,f}-\frac{4\,b\,h\,(c\,f-d\,e)}{3\,d^3\,f}\right)$$
$$-\sqrt{e+f\,x}\left(\frac{2\,(c\,f-d\,e)\,\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{d^2\,f}-\frac{4\,b\,h\,(c\,f-d\,e)}{d^3\,f}\right)}{d}+\frac{2\,(a\,f-b\,e)\,(e\,h-f\,g)}{d^2\,f}+\frac{2\,b\,h\,(c\,f-d\,e)^2}{d^4\,f}\right)$$
$$+\frac{\sqrt{e+f\,x}\,(b\,c^3\,f^2\,h+a\,c\,d^2\,f^2\,g-a\,c^2\,d\,f^2\,h-b\,c^2\,d\,f^2\,g-a\,d^3\,e\,f\,g+a\,c\,d^2\,e\,f\,h+b\,c\,d^2\,e\,f\,g-b\,c^2\,d\,e\,f}{d^5\,(e+f\,x)-d^5\,e+c\,d^4\,f}$$
$$+\frac{2\,b\,h\,(e+f\,x)^{5/2}}{5\,d^2\,f}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,1\mathrm{i}}{\sqrt{d\,e-c\,f}}\right)\,\sqrt{d\,e-c\,f}\,(2\,a\,d^2\,e\,h+3\,a\,d^2\,f\,g+2\,b\,d^2\,e\,g+7\,b\,c^2\,f\,h-5\,a\,c\,d\,f\,h-4\,b\,c\,d\,e\,h-5\,b\,c}{d^{9/2}}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x))/(c + d*x)^2,x)
```

output
```
(e + f*x)^(3/2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(3*d^2*f) - (4*b*h*(c*f - d
*e))/(3*d^3*f)) - (e + f*x)^(1/2)*((2*(c*f - d*e)*((2*a*f*h - 4*b*e*h + 2*
b*f*g)/(d^2*f) - (4*b*h*(c*f - d*e))/(d^3*f)))/d + (2*(a*f - b*e)*(e*h - f
*g))/(d^2*f) + (2*b*h*(c*f - d*e)^2)/(d^4*f)) + ((e + f*x)^(1/2)*(b*c^3*f^
2*h + a*c*d^2*f^2*g - a*c^2*d*f^2*h - b*c^2*d*f^2*g - a*d^3*e*f*g + a*c*d^
2*e*f*h + b*c*d^2*e*f*g - b*c^2*d*e*f*h))/(d^5*(e + f*x) - d^5*e + c*d^4*f
) + (atan((d^(1/2)*(e + f*x)^(1/2)*1i)/(d*e - c*f)^(1/2))*(d*e - c*f)^(1/2
)*(2*a*d^2*e*h + 3*a*d^2*f*g + 2*b*d^2*e*g + 7*b*c^2*f*h - 5*a*c*d*f*h - 4
*b*c*d*e*h - 5*b*c*d*f*g)*1i)/d^(9/2) + (2*b*h*(e + f*x)^(5/2))/(5*d^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.21

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x)
```

output
```
(75*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*a*c**2*d*f**2*h - 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*a*c*d**2*e*f*h - 45*sqrt(d)*sqrt(c*f - d*e)*atan(
(sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*f**2*g + 75*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2
*f**2*h*x - 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*a*d**3*e*f*h*x - 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**3*f**2*g*x - 105*sqrt(d)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*f**2*h + 6
0*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e))
)*b*c**2*d*e*f*h + 75*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt
(d)*sqrt(c*f - d*e)))*b*c**2*d*f**2*g - 105*sqrt(d)*sqrt(c*f - d*e)*atan((
sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d*f**2*h*x - 30*sqrt(d)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**
2*e*f*g + 60*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*b*c*d**2*e*f*h*x + 75*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*f**2*g*x - 30*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*d**3*e*f*g*x -
 75*sqrt(e + f*x)*a*c**2*d**2*f**2*h + 55*sqrt(e + f*x)*a*c*d**3*e*f*h + 4
5*sqrt(e + f*x)*a*c*d**3*f**2*g - 50*sqrt(e + f*x)*a*c*d**3*f**2*h*x - ...
```

3.90 $\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2} \, dx = \frac{2(dfg+deh-2cfh)\sqrt{e+fx}}{d^3} - \frac{(de-cf)(dg-ch)\sqrt{e+fx}}{d^3(c+dx)} + \frac{2h(e+fx)^{3/2}}{3d^2} - \frac{\sqrt{de-cf}(3dfg+2deh-5cfh)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}}$$

output

```
2*(-2*c*f*h+d*e*h+d*f*g)*(f*x+e)^(1/2)/d^3-(-c*f+d*e)*(-c*h+d*g)*(f*x+e)^(
1/2)/d^3/(d*x+c)+2/3*h*(f*x+e)^(3/2)/d^2-(-c*f+d*e)^(1/2)*(-5*c*f*h+2*d*e*
h+3*d*f*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{\sqrt{e+fx}(-15c^2fh+cd(9fg+11eh-10fhx)+d^2(2fx(3g+hx)+e(-3g -}{3d^3(c+dx)}$$
$$-\frac{\sqrt{-de+cf}(3dfg+2deh-5cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
(Sqrt[e + f*x]*(-15*c^2*f*h + c*d*(9*f*g + 11*e*h - 10*f*h*x) + d^2*(2*f*x
*(3*g + h*x) + e*(-3*g + 8*h*x))))/(3*d^3*(c + d*x)) - (Sqrt[-(d*e) + c*f]
*(3*d*f*g + 2*d*e*h - 5*c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e)
+ c*f]])/d^(7/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}dx$$
$$\Big\downarrow \; 87$$
$$\frac{(-5cfh+2deh+3dfg)\int \frac{(e+fx)^{3/2}}{c+dx}dx}{2d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$
$$\Big\downarrow \; 60$$
$$\frac{(-5cfh+2deh+3dfg)\left(\frac{(de-cf)\int \frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{2d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{(-5cfh+2deh+3dfg)\left(\frac{(de-cf)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{2d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{(-5cfh+2deh+3dfg)\left(\frac{(de-cf)\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)}{2d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{(de-cf)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)(-5cfh+2deh+3dfg)}{2d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x]
```

output
```
-(((d*g - c*h)*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x))) + ((3*d*f*g + 2
*d*e*h - 5*c*f*h)*((2*(e + f*x)^(3/2))/(3*d) + ((d*e - c*f)*((2*Sqrt[e + f
*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f
]])/d^(3/2)))/d))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{2(-fdhx+6cfh-4deh-3dfg)\sqrt{fx+e}}{3d^3}+\frac{(2cf-2de)\left(\frac{\left(-\frac{1}{2}cfh+\frac{1}{2}dfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(5cfh-2deh-3dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right.}{2\sqrt{(cf-de)d}}\right.}{d^3}$
pseudoelliptic	$-\frac{5\left(-(cf-de)\left(\frac{(-2eh-3fg)d}{5}+cfh\right)(xd+c)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\left(\frac{\left(-2x\left(\frac{hx}{3}+g\right)f+e\left(-\frac{8hx}{3}+g\right)\right)d^2}{5}-\frac{11c\left(\frac{(-10hx+9}{11}\right.}{15}\right.\right.}{\sqrt{(cf-de)d}\,d^3(xd+c)}$
derivativedivides	$-\frac{2\left(-\frac{h(fx+e)^{\frac{3}{2}}d}{3}+2cfh\sqrt{fx+e}-deh\sqrt{fx+e}-dfg\sqrt{fx+e}\right)}{d^3}+\frac{2\left(-\frac{1}{2}c^2f^2h+\frac{1}{2}cdehf+\frac{1}{2}cdf^2g-\frac{1}{2}d^2egf\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(5c^2}{}$
default	$-\frac{2\left(-\frac{h(fx+e)^{\frac{3}{2}}d}{3}+2cfh\sqrt{fx+e}-deh\sqrt{fx+e}-dfg\sqrt{fx+e}\right)}{d^3}+\frac{2\left(-\frac{1}{2}c^2f^2h+\frac{1}{2}cdehf+\frac{1}{2}cdf^2g-\frac{1}{2}d^2egf\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(5c^2}{}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-d*f*h*x+6*c*f*h-4*d*e*h-3*d*f*g)*(f*x+e)^(1/2)/d^3+1/d^3*(2*c*f-2*d
*e)*((-1/2*c*f*h+1/2*d*f*g)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(5*c*f*h
-2*d*e*h-3*d*f*g)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.63

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \left[-\frac{3\,(3\,cdfg+(2\,cde-5\,c^2f)h+(3\,d^2fg+(2\,d^2e-5\,cdf)h)x)\sqrt{\frac{de-cf}{d}}\log\left(\right.}{}\right.$$

$$-\frac{3\,(3\,cdfg+(2\,cde-5\,c^2f)h+(3\,d^2fg+(2\,d^2e-5\,cdf)h)x)\sqrt{-\frac{de-cf}{d}}\arctan\left(-\frac{\sqrt{fx+e}\,d\sqrt{-\frac{de-cf}{d}}}{de-cf}\right)-(2\,d^2}{3\,(d^4x+cd^3)}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*(3*c*d*f*g + (2*c*d*e - 5*c^2*f)*h + (3*d^2*f*g + (2*d^2*e - 5*c*
d*f)*h)*x)*sqrt((d*e - c*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*
d*sqrt((d*e - c*f)/d))/(d*x + c)) - 2*(2*d^2*f*h*x^2 - 3*(d^2*e - 3*c*d*f)
*g + (11*c*d*e - 15*c^2*f)*h + 2*(3*d^2*f*g + (4*d^2*e - 5*c*d*f)*h)*x)*sq
rt(f*x + e))/(d^4*x + c*d^3), -1/3*(3*(3*c*d*f*g + (2*c*d*e - 5*c^2*f)*h +
 (3*d^2*f*g + (2*d^2*e - 5*c*d*f)*h)*x)*sqrt(-(d*e - c*f)/d)*arctan(-sqrt(
f*x + e)*d*sqrt(-(d*e - c*f)/d)/(d*e - c*f)) - (2*d^2*f*h*x^2 - 3*(d^2*e -
 3*c*d*f)*g + (11*c*d*e - 15*c^2*f)*h + 2*(3*d^2*f*g + (4*d^2*e - 5*c*d*f)
*h)*x)*sqrt(f*x + e))/(d^4*x + c*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.54

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{(3\,d^2efg-3\,cdf^2g+2\,d^2e^2h-7\,cdefh+5\,c^2f^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}d^3}$$
$$-\frac{\sqrt{fx+e}d^2efg-\sqrt{fx+e}cdf^2g-\sqrt{fx+e}cdefh+\sqrt{fx+e}c^2f^2h}{((fx+e)d-de+cf)d^3}$$
$$+\frac{2\left(3\sqrt{fx+e}d^4fg+(fx+e)^{\frac{3}{2}}d^4h+3\sqrt{fx+e}d^4eh-6\sqrt{fx+e}cd^3fh\right)}{3\,d^6}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(3*d^2*e*f*g - 3*c*d*f^2*g + 2*d^2*e^2*h - 7*c*d*e*f*h + 5*c^2*f^2*h)*arct
an(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^3) - (sqr
t(f*x + e)*d^2*e*f*g - sqrt(f*x + e)*c*d*f^2*g - sqrt(f*x + e)*c*d*e*f*h +
 sqrt(f*x + e)*c^2*f^2*h)/(((f*x + e)*d - d*e + c*f)*d^3) + 2/3*(3*sqrt(f*
x + e)*d^4*f*g + (f*x + e)^(3/2)*d^4*h + 3*sqrt(f*x + e)*d^4*e*h - 6*sqrt(
f*x + e)*c*d^3*f*h)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{2\,h\,(e+f\,x)^{3/2}}{3\,d^2}$$
$$-\frac{\sqrt{e+f\,x}\,(h\,c^2\,f^2-g\,c\,d\,f^2-e\,h\,c\,d\,f+e\,g\,d^2\,f)}{d^4\,(e+f\,x)-d^4\,e+c\,d^3\,f}$$
$$-\sqrt{e+f\,x}\left(\frac{2\,e\,h-2\,f\,g}{d^2}-\frac{2\,h\,(2\,d^2\,e-2\,c\,d\,f)}{d^4}\right)$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,1\mathrm{i}}{\sqrt{d\,e-c\,f}}\right)\sqrt{d\,e-c\,f}\,(2\,d\,e\,h-5\,c\,f\,h+3\,d\,f\,g)\,1\mathrm{i}}{d^{7/2}}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/(c + d*x)^2,x)
```

output
```
(2*h*(e + f*x)^(3/2))/(3*d^2) - ((e + f*x)^(1/2)*(c^2*f^2*h - c*d*f^2*g +
d^2*e*f*g - c*d*e*f*h))/(d^4*(e + f*x) - d^4*e + c*d^3*f) - (e + f*x)^(1/2
)*((2*e*h - 2*f*g)/d^2 - (2*h*(2*d^2*e - 2*c*d*f))/d^4) + (atan((d^(1/2)*(
e + f*x)^(1/2)*1i)/(d*e - c*f)^(1/2))*(d*e - c*f)^(1/2)*(2*d*e*h - 5*c*f*h
 + 3*d*f*g)*1i)/d^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.60

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^2}\,dx = \frac{15\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2fh - 6\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cdeh}{}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^2,x)
```

output
```
(15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*c**2*f*h - 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*c*d*e*h - 9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d
)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f*g + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f*h*x - 6*sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*e*h*x - 9*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**
2*f*g*x - 15*sqrt(e + f*x)*c**2*d*f*h + 11*sqrt(e + f*x)*c*d**2*e*h + 9*sq
rt(e + f*x)*c*d**2*f*g - 10*sqrt(e + f*x)*c*d**2*f*h*x - 3*sqrt(e + f*x)*d
**3*e*g + 8*sqrt(e + f*x)*d**3*e*h*x + 6*sqrt(e + f*x)*d**3*f*g*x + 2*sqrt
(e + f*x)*d**3*f*h*x**2)/(3*d**4*(c + d*x))
```

3.91 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 247

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = -\frac{f(bdg-3bch+2adh)\sqrt{e+fx}}{bd^2(bc-ad)} + \frac{(dg-ch)(e+fx)^{3/2}}{d(bc-ad)(c+dx)} - \frac{2(be-af)^{3/2}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(bc-ad)^2} - \frac{\sqrt{de-cf}\,(ad(3dfg+2deh-5cfh)-b(2d^2eg+cdfg-3c^2fh))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}(bc-ad)^2}$$

output
```
-f*(2*a*d*h-3*b*c*h+b*d*g)*(f*x+e)^(1/2)/b/d^2/(-a*d+b*c)+(-c*h+d*g)*(f*x+
e)^(3/2)/d/(-a*d+b*c)/(d*x+c)-2*(-a*f+b*e)^(3/2)*(-a*h+b*g)*arctanh(b^(1/2
)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(3/2)/(-a*d+b*c)^2-(-c*f+d*e)^(1/2)*(a
*d*(-5*c*f*h+2*d*e*h+3*d*f*g)-b*(-3*c^2*f*h+c*d*f*g+2*d^2*e*g))*arctanh(d^
(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \frac{\frac{(bc-ad)\sqrt{e+fx}(-2adfh(c+dx)+b(d^2eg+3c^2fh-cd(fg+eh-2fhx)))}{bd^2(c+dx)} + \frac{2(-be+af)^{3/2}(bg-ah)\arctan(}{b^{3/2}}}{(bc-ad}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)^2),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(-2*a*d*f*h*(c + d*x) + b*(d^2*e*g + 3*c^2*f*h
 - c*d*(f*g + e*h - 2*f*h*x))))/(b*d^2*(c + d*x)) + (2*(-(b*e) + a*f)^(3/2
)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/b^(3/2)
+ (Sqrt[-(d*e) + c*f]*(a*d*(-3*d*f*g - 2*d*e*h + 5*c*f*h) + b*(2*d^2*e*g +
 c*d*f*g - 3*c^2*f*h))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])
/d^(5/2))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)} - \frac{\int -\frac{\sqrt{e+fx}(2bdeg-a(3dfg+2deh-3cfh)-f(bdg-3bch+2adh)x)}{2(a+bx)(c+dx)}dx}{d(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{e+fx}(2bdeg-a(3dfg+2deh-3cfh)-f(bdg-3bch+2adh)x)}{(a+bx)(c+dx)}dx}{2d(bc-ad)} + \frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{171}$$

$$\frac{\frac{2\int \frac{ac(bdg-3bch+2adh)f^2+\left(\left(-3fhc^2+d(fg+3eh)c+d^2eg\right)b^2-2ad(dfg+2deh-cfh)b+2a^2d^2fh\right)xf+bde(2bdeg-a(3dfg+2deh-3cfh))}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd} - \frac{2f\sqrt{e+fx}(2ad}{bd}}{2d(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{ac(bdg-3bch+2adh)f^2+\left(\left(-3fhc^2+d(fg+3eh)c+d^2eg\right)b^2-2ad(dfg+2deh-cfh)b+2a^2d^2fh\right)xf+bde(2bdeg-a(3dfg+2deh-3cfh))}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{bd} - \frac{2f\sqrt{e+fx}(2adh}{bd}}{2d(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{b(de-cf)\left(ad(-5cfh+2deh+3dfg)-b\left(-3c^2fh+cdfg+2d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}+\frac{2d^2(be-af)^2(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}}{bd} - \frac{2f\sqrt{e+fx}(2adh-3bch+b}{bd}}{2d(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2b(de-cf)\left(ad(-5cfh+2deh+3dfg)-b\left(-3c^2fh+cdfg+2d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}+\frac{4d^2(be-af)^2(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{bd} - \frac{2f\sqrt{e+fx}}{}}{2d(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{-\frac{2b\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(-5cfh+2deh+3dfg)-b\left(-3c^2fh+cdfg+2d^2eg\right)\right)}{\sqrt{d}(bc-ad)}-\frac{4d^2(be-af)^{3/2}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)}}{bd} - \frac{2f\sqrt{e+fx}(}{}}{2d(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{d(c+dx)(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)^2),x]
```

output
```
((d*g - c*h)*(e + f*x)^(3/2))/(d*(b*c - a*d)*(c + d*x)) + ((-2*f*(b*d*g -
3*b*c*h + 2*a*d*h)*Sqrt[e + f*x])/(b*d) + ((-4*d^2*(b*e - a*f)^(3/2)*(b*g
- a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a
*d)) - (2*b*Sqrt[d*e - c*f]*(a*d*(3*d*f*g + 2*d*e*h - 5*c*f*h) - b*(2*d^2*
e*g + c*d*f*g - 3*c^2*f*h))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f
]])/(Sqrt[d]*(b*c - a*d)))/(b*d))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{5(cf-de)\sqrt{(af-be)b}\left(\frac{b\left(-3c^2fh+cdfg+2d^2eg\right)}{5}+ad\left(\frac{(-2eh-3fg)d}{5}+cfh\right)\right)(xd+c)b\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2\sqrt{(cf-de}}{b\,d^2(ad-b}$
derivativedivides	$2f\left(\frac{h\sqrt{fx+e}}{b\,d^2}+\frac{\frac{\left(-\frac{1}{2}a\,c^2d\,f^2h+\frac{1}{2}ac\,d^2efh+\frac{1}{2}ac\,d^2f^2g-\frac{1}{2}a\,d^3efg+\frac{1}{2}b\,c^3f^2h-\frac{1}{2}b\,c^2defh-\frac{1}{2}b\,c^2d\,f^2g+\frac{1}{2}bc\,d^2efg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}}{}\right.$
default	$2f\left(\frac{h\sqrt{fx+e}}{b\,d^2}+\frac{\frac{\left(-\frac{1}{2}a\,c^2d\,f^2h+\frac{1}{2}ac\,d^2efh+\frac{1}{2}ac\,d^2f^2g-\frac{1}{2}a\,d^3efg+\frac{1}{2}b\,c^3f^2h-\frac{1}{2}b\,c^2defh-\frac{1}{2}b\,c^2d\,f^2g+\frac{1}{2}bc\,d^2efg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}}{}\right.$
risch	$\frac{2h\sqrt{fx+e}\,f}{b\,d^2}-\frac{2f\left(\frac{d^2\left(f^2a^3h-2a^2befh-a^2b\,f^2g+a\,b^2e^2h+2a\,b^2efg-b^3e^2g\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f(ad-bc)^2\sqrt{(af-be)b}}-\frac{b\left(\left(-\frac{1}{2}a\,c^2d\,f^2h+\frac{1}{2}a\right.\right.}{}\right.}{}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*(5/2*(c*f-d*e)*((a*f-b*e)*b)^(1/
2)*(1/5*b*(-3*c^2*f*h+c*d*f*g+2*d^2*e*g)+a*d*(1/5*(-2*e*h-3*f*g)*d+c*f*h))
*(d*x+c)*b*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)
*(-d^2*(a*f-b*e)^2*(a*h-b*g)*(d*x+c)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^
(1/2))+(a*d-b*c)*((a*f-b*e)*b)^(1/2)*(f*x+e)^(1/2)*(1/2*(-d^2*e*g+c*((-2*h
*x+g)*f+e*h)*d-3*c^2*f*h)*b+a*d*f*h*(d*x+c))))/b/d^2/(a*d-b*c)^2/(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(223) = 446$.

Time = 14.10 (sec) , antiderivative size = 2120, normalized size of antiderivative = 8.58

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*((b^2*c*d^2*e - a*b*c*d^2*f)*g - (a*b*c*d^2*e - a^2*c*d^2*f)*h + (
(b^2*d^3*e - a*b*d^3*f)*g - (a*b*d^3*e - a^2*d^3*f)*h)*x)*sqrt((b*e - a*f)
/b)*log((b*f*x + 2*b*e - a*f - 2*sqrt(f*x + e)*b*sqrt((b*e - a*f)/b))/(b*x
 + a)) + ((2*b^2*c*d^2*e + (b^2*c^2*d - 3*a*b*c*d^2)*f)*g - (2*a*b*c*d^2*e
 + (3*b^2*c^3 - 5*a*b*c^2*d)*f)*h + ((2*b^2*d^3*e + (b^2*c*d^2 - 3*a*b*d^3
)*f)*g - (2*a*b*d^3*e + (3*b^2*c^2*d - 5*a*b*c*d^2)*f)*h)*x)*sqrt((d*e - c
*f)/d)*log((d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(
d*x + c)) + 2*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f*h*x + ((b^2*c*d^2 -
 a*b*d^3)*e - (b^2*c^2*d - a*b*c*d^2)*f)*g - ((b^2*c^2*d - a*b*c*d^2)*e -
(3*b^2*c^3 - 5*a*b*c^2*d + 2*a^2*c*d^2)*f)*h)*sqrt(f*x + e))/(b^3*c^3*d^2
- 2*a*b^2*c^2*d^3 + a^2*b*c*d^4 + (b^3*c^2*d^3 - 2*a*b^2*c*d^4 + a^2*b*d^5
)*x), -1/2*(4*((b^2*c*d^2*e - a*b*c*d^2*f)*g - (a*b*c*d^2*e - a^2*c*d^2*f)
*h + ((b^2*d^3*e - a*b*d^3*f)*g - (a*b*d^3*e - a^2*d^3*f)*h)*x)*sqrt(-(b*e
 - a*f)/b)*arctan(-sqrt(f*x + e)*b*sqrt(-(b*e - a*f)/b)/(b*e - a*f)) - ((2
*b^2*c*d^2*e + (b^2*c^2*d - 3*a*b*c*d^2)*f)*g - (2*a*b*c*d^2*e + (3*b^2*c^
3 - 5*a*b*c^2*d)*f)*h + ((2*b^2*d^3*e + (b^2*c*d^2 - 3*a*b*d^3)*f)*g - (2*
a*b*d^3*e + (3*b^2*c^2*d - 5*a*b*c*d^2)*f)*h)*x)*sqrt((d*e - c*f)/d)*log((
d*f*x + 2*d*e - c*f + 2*sqrt(f*x + e)*d*sqrt((d*e - c*f)/d))/(d*x + c)) -
2*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f*h*x + ((b^2*c*d^2 - a*b*d^3)*e
- (b^2*c^2*d - a*b*c*d^2)*f)*g - ((b^2*c^2*d - a*b*c*d^2)*e - (3*b^2*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.61

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \frac{2\,(b^3e^2g-2\,ab^2efg+a^2bf^2g-ab^2e^2h+2\,a^2befh-a^3f^2h)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+ab}}\right.}{(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{-b^2e+abf}}$$
$$-\frac{(2\,bd^3e^2g-bcd^2efg-3\,ad^3efg-bc^2df^2g+3\,acd^2f^2g-2\,ad^3e^2h-3\,bc^2defh+7\,acd^2efh+3\,bc^3f^2h-}{(b^2c^2d^2-2\,abcd^3+a^2d^4)\sqrt{-d^2e+cdf}}$$
$$+\frac{2\sqrt{fx+e}fh}{bd^2}+\frac{\sqrt{fx+e}d^2efg-\sqrt{fx+e}cdf^2g-\sqrt{fx+e}cdefh+\sqrt{fx+e}c^2f^2h}{(bcd^2-ad^3)((fx+e)d-de+cf)}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
2*(b^3*e^2*g - 2*a*b^2*e*f*g + a^2*b*f^2*g - a*b^2*e^2*h + 2*a^2*b*e*f*h -
 a^3*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)*sqrt(-b^2*e + a*b*f)) - (2*b*d^3*e^2*g - b*c*d^2*e*f*g
 - 3*a*d^3*e*f*g - b*c^2*d*f^2*g + 3*a*c*d^2*f^2*g - 2*a*d^3*e^2*h - 3*b*c
^2*d*e*f*h + 7*a*c*d^2*e*f*h + 3*b*c^3*f^2*h - 5*a*c^2*d*f^2*h)*arctan(sqr
t(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
sqrt(-d^2*e + c*d*f)) + 2*sqrt(f*x + e)*f*h/(b*d^2) + (sqrt(f*x + e)*d^2*e
*f*g - sqrt(f*x + e)*c*d*f^2*g - sqrt(f*x + e)*c*d*e*f*h + sqrt(f*x + e)*c
^2*f^2*h)/((b*c*d^2 - a*d^3)*((f*x + e)*d - d*e + c*f))
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 18557, normalized size of antiderivative = 75.13

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)^2),x)
```

output
```
atan(((((2*(2*a*b^7*c^5*d^4*f^5*g + 2*a^5*b^3*c*d^8*f^5*g - 6*a*b^7*c^6*d^
3*f^5*h + 4*a^6*b^2*c*d^8*f^5*h - 2*a^5*b^3*d^9*e*f^4*g - 4*a^6*b^2*d^9*e*
f^4*h - 2*b^8*c^5*d^4*e*f^4*g + 6*b^8*c^6*d^3*e*f^4*h - 8*a^2*b^6*c^4*d^5*
f^5*g + 12*a^3*b^5*c^3*d^6*f^5*g - 8*a^4*b^4*c^2*d^7*f^5*g + 28*a^2*b^6*c^
5*d^4*f^5*h - 52*a^3*b^5*c^4*d^5*f^5*h + 48*a^4*b^4*c^3*d^6*f^5*h - 22*a^5
*b^3*c^2*d^7*f^5*h + 2*a^4*b^4*d^9*e^2*f^3*g + 4*a^5*b^3*d^9*e^2*f^3*h + 2
*b^8*c^4*d^5*e^2*f^3*g - 6*b^8*c^5*d^4*e^2*f^3*h + 12*a^2*b^6*c^2*d^7*e^2*
f^3*g - 52*a^2*b^6*c^3*d^6*e^2*f^3*h + 48*a^3*b^5*c^2*d^7*e^2*f^3*h + 6*a*
b^7*c^4*d^5*e*f^4*g + 6*a^4*b^4*c*d^8*e*f^4*g - 22*a*b^7*c^5*d^4*e*f^4*h +
 18*a^5*b^3*c*d^8*e*f^4*h - 8*a*b^7*c^3*d^6*e^2*f^3*g - 4*a^2*b^6*c^3*d^6*
e*f^4*g - 8*a^3*b^5*c*d^8*e^2*f^3*g - 4*a^3*b^5*c^2*d^7*e*f^4*g + 28*a*b^7
*c^4*d^5*e^2*f^3*h + 24*a^2*b^6*c^4*d^5*e*f^4*h + 4*a^3*b^5*c^3*d^6*e*f^4*
h - 22*a^4*b^4*c*d^8*e^2*f^3*h - 26*a^4*b^4*c^2*d^7*e*f^4*h))/(a^3*b*d^6 -
 b^4*c^3*d^3 + 3*a*b^3*c^2*d^4 - 3*a^2*b^2*c*d^5) - (2*(e + f*x)^(1/2)*((4
*a^2*d^5*e^3*h^2 + 4*b^2*d^5*e^3*g^2 - 9*b^2*c^5*f^3*h^2 - 25*a^2*c^3*d^2*
f^3*h^2 - b^2*c^3*d^2*f^3*g^2 - 9*a^2*c*d^4*f^3*g^2 + 9*a^2*d^5*e*f^2*g^2
+ 6*a*b*c^2*d^3*f^3*g^2 - 24*a^2*c*d^4*e^2*f*h^2 + 9*b^2*c^4*d*e*f^2*h^2 +
 30*a^2*c^2*d^3*f^3*g*h - 8*a*b*d^5*e^3*g*h + 45*a^2*c^2*d^3*e*f^2*h^2 - 3
*b^2*c^2*d^3*e*f^2*g^2 + 30*a*b*c^4*d*f^3*h^2 - 12*a*b*d^5*e^2*f*g^2 + 6*b
^2*c^4*d*f^3*g*h + 12*a^2*d^5*e^2*f*g*h + 6*a*b*c*d^4*e*f^2*g^2 - 28*a*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1199, normalized size of antiderivative = 4.85

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^2,x)
```

output

```
( - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**2*c*d**3*f*h - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**2*d**4*f*h*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*d**3*e*h + 2*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*d**
3*f*g + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a*b*d**4*e*h*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*d**4*f*g*x - 2*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c*d**3*e*g - 2*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*d
**4*e*g*x + 5*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt
(c*f - d*e)))*a*b**2*c**2*d*f*h - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c*d**2*e*h - 3*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c*d**2*f*
g + 5*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a*b**2*c*d**2*f*h*x - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*d**3*e*h*x - 3*sqrt(d)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*d**3*f*g*x - 3*
sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*
b**3*c**3*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)...
```

3.92 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 315

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = -\frac{(de-cf)(2bdg-bch-adh)\sqrt{e+fx}}{bd(bc-ad)^2(c+dx)}-\frac{(bg-ah)(e+fx)^{3/2}}{b(bc-ad)(a+bx)(c+dx)}$$
$$-\frac{\sqrt{be-af}\left(a^2dfh-b^2(4deg-3cfg-2ceh)+ab(dfg+2deh-5cfh)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(bc-ad)^3}$$
$$+\frac{\sqrt{de-cf}\left(ad(3dfg+2deh-5cfh)-b(4d^2eg-c^2fh-cd(fg+2eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(bc-ad)^3}$$

output

```
-(-c*f+d*e)*(-a*d*h-b*c*h+2*b*d*g)*(f*x+e)^(1/2)/b/d/(-a*d+b*c)^2/(d*x+c)-
(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)/(d*x+c)-(-a*f+b*e)^(1/2)*(a^
2*d*f*h-b^2*(-2*c*e*h-3*c*f*g+4*d*e*g)+a*b*(-5*c*f*h+2*d*e*h+d*f*g))*arcta
nh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(3/2)/(-a*d+b*c)^3+(-c*f+d*e)
^(1/2)*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)-b*(4*d^2*e*g-c^2*f*h-c*d*(2*e*h+f*g
)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx =$$
$$-\frac{\sqrt{e+fx}(a^2dfh(c+dx)+b^2(2d^2egx+c^2fhx+cd(eg-fgx-ehx))+ab(c^2fh-2cd(fg+eh)+d^2(e}{bd(bc-ad)^2(a+bx)(c+dx)}$$
$$-\frac{\sqrt{-be+af}(a^2dfh+b^2(-4deg+3cfg+2ceh)+ab(dfg+2deh-5cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{3/2}(bc-ad)^3}$$
$$+\frac{\sqrt{-de+cf}(ad(-3dfg-2deh+5cfh)+b(4d^2eg-c^2fh-cd(fg+2eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{3/2}(-bc+ad)^3}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^2),x]
```

output
```
-((Sqrt[e + f*x]*(a^2*d*f*h*(c + d*x) + b^2*(2*d^2*e*g*x + c^2*f*h*x + c*d
*(e*g - f*g*x - e*h*x)) + a*b*(c^2*f*h - 2*c*d*(f*g + e*h) + d^2*(e*g - f*
g*x - e*h*x))))/(b*d*(b*c - a*d)^2*(a + b*x)*(c + d*x))) - (Sqrt[-(b*e) +
a*f]*(a^2*d*f*h + b^2*(-4*d*e*g + 3*c*f*g + 2*c*e*h) + a*b*(d*f*g + 2*d*e*
h - 5*c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(3/2)
*(b*c - a*d)^3) + (Sqrt[-(d*e) + c*f]*(a*d*(-3*d*f*g - 2*d*e*h + 5*c*f*h)
+ b*(4*d^2*e*g - c^2*f*h - c*d*(f*g + 2*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*
x])/Sqrt[-(d*e) + c*f]])/(d^(3/2)*(-(b*c) + a*d)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {166, 27, 25, 166, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{\sqrt{e+fx}(4bdeg-3bcfg-2bceh-2adeh+3acfh+f(bdg-2bch+adh)x)}{2(a+bx)(c+dx)^2}dx}{b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{\sqrt{e+fx}(a(2de-3cf)h-b(4deg-3cfg-2ceh)-f(bdg-2bch+adh)x)}{(a+bx)(c+dx)^2}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{e+fx}(a(2de-3cf)h-b(4deg-3cfg-2ceh)-f(bdg-2bch+adh)x)}{(a+bx)(c+dx)^2}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 166$$

$$-\frac{\frac{\int -\frac{-de(4deg-3cfg-2ceh)b^2+a\left(e(3fg+2eh)d^2-2cf(fg+3eh)d+c^2f^2h\right)b+a^2cdf^2h+f\left(-\left(\left(-fhc^2-d(fg+eh)c+2d^2eg\right)b^2\right)+ad(dfg+deh-4cfh)b+a^2d^2fh\right)}{(a+bx)(c+dx)\sqrt{e+fx}}}{d(bc-ad)}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-de(4deg-3cfg-2ceh)b^2+a\left(e(3fg+2eh)d^2-2cf(fg+3eh)d+c^2f^2h\right)b+a^2cdf^2h+f\left(-\left(\left(-fhc^2-d(fg+eh)c+2d^2eg\right)b^2\right)+ad(dfg+deh-4cfh)b+a^2d^2fh\right)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{d(bc-ad)}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{d(be-af)\left(a^2dfh+ab(-5cfh+2deh+dfg)-b^2(-2ceh-3cfg+4deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{b(de-cf)\left(ad(-5cfh+2deh+3dfg)-b\left(c^2(-f)h-cd(2eh+fg)+4d^2eg\right)\right)}{bc-ad}}{d(bc-ad)}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2d(be-af)\left(a^2dfh+ab(-5cfh+2deh+dfg)-b^2(-2ceh-3cfg+4deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2b(de-cf)\left(ad(-5cfh+2deh+3dfg)-b\left(c^2(-f)h-cd(2eh+fg)\right.\right.}{f(bc-ad)}}{d(bc-ad)}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{2b\sqrt{de-cf}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(-5cfh+2deh+3dfg)-b\left(c^2(-f)h-cd(2eh+fg)+4d^2eg\right)\right)}{\sqrt{d}(bc-ad)}-\frac{2d\sqrt{be-af}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(a^2dfh+ab(-5cfh+2de\right.}{\sqrt{b}(bc-ad)}}{d(bc-ad)}}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^2),x]
```

output
```
-(((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)*(c + d*x))) + ((-
2*(d*e - c*f)*(2*b*d*g - b*c*h - a*d*h)*Sqrt[e + f*x])/(d*(b*c - a*d)*(c +
 d*x)) + ((-2*d*Sqrt[b*e - a*f]*(a^2*d*f*h - b^2*(4*d*e*g - 3*c*f*g - 2*c*
e*h) + a*b*(d*f*g + 2*d*e*h - 5*c*f*h))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sq
rt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)) + (2*b*Sqrt[d*e - c*f]*(a*d*(3*d*f*g
 + 2*d*e*h - 5*c*f*h) - b*(4*d^2*e*g - c^2*f*h - c*d*(f*g + 2*e*h)))*ArcTa
nh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(b*c - a*d)))/(d*(b*
c - a*d)))/(2*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.10

method	result
derivativedivides	$2f^2\left(\frac{(cf-de)\left(-\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2d((fx+e)d+cf-de)}+\frac{\left(5acdfh-2a\,d^2eh-3a\,d^2fg-b\,c^2fh-2bcdeh-bcdfg+4b\,d^2eg\right)\arctan}{2d\sqrt{(cf-de)d}}\right.}{f^2(ad-bc)^3}\right.$
default	$2f^2\left(\frac{(cf-de)\left(-\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2d((fx+e)d+cf-de)}+\frac{\left(5acdfh-2a\,d^2eh-3a\,d^2fg-b\,c^2fh-2bcdeh-bcdfg+4b\,d^2eg\right)\arctan}{2d\sqrt{(cf-de)d}}\right.}{f^2(ad-bc)^3}\right.$
pseudoelliptic	$-\frac{-d\sqrt{(cf-de)d}\left((2ceh+3cfg-4deg)b^2-5a\left(\frac{(-2eh-fg)d}{5}+cfh\right)b+a^2dfh\right)(bx+a)(xd+c)(af-be)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)}}\right.}{}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*f^2*((c*f-d*e)/f^2/(a*d-b*c)^3*(-1/2*f*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)
/d*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(5*a*c*d*f*h-2*a*d^2*e*h-3*a*d^2*
f*g-b*c^2*f*h-2*b*c*d*e*h-b*c*d*f*g+4*b*d^2*e*g)/d/((c*f-d*e)*d)^(1/2)*arc
tan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-(a*f-b*e)/f^2/(a*d-b*c)^3*(1/2*f
*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/b*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)-1/2
*(a^2*d*f*h-5*a*b*c*f*h+2*a*b*d*e*h+a*b*d*f*g+2*b^2*c*e*h+3*b^2*c*f*g-4*b^
2*d*e*g)/b/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(291) = 582$.

Time = 13.13 (sec) , antiderivative size = 4238, normalized size of antiderivative = 13.45

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(291) = 582$.

Time = 0.21 (sec) , antiderivative size = 804, normalized size of antiderivative = 2.55

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx =$$
$$-\frac{(4\,b^3de^2g-3\,b^3cefg-5\,ab^2defg+3\,ab^2cf^2g+a^2bdf^2g-2\,b^3ce^2h-2\,ab^2de^2h+7\,ab^2cefh+a^2bdefh}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\sqrt{-b^2e+abf}}$$
$$+\frac{(4\,bd^3e^2g-5\,bcd^2efg-3\,ad^3efg+bc^2df^2g+3\,acd^2f^2g-2\,bcd^2e^2h-2\,ad^3e^2h+bc^2defh+7\,acd^2efh}{(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4)\sqrt{-d^2e+cdf}}$$
$$-\frac{2\,(fx+e)^{\frac{3}{2}}b^2d^2efg-2\,\sqrt{fx+e}b^2d^2e^2fg-(fx+e)^{\frac{3}{2}}b^2cdf^2g-(fx+e)^{\frac{3}{2}}abd^2f^2g+2\,\sqrt{fx+e}b^2cdef^2}{}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(4*b^3*d*e^2*g - 3*b^3*c*e*f*g - 5*a*b^2*d*e*f*g + 3*a*b^2*c*f^2*g + a^2*
b*d*f^2*g - 2*b^3*c*e^2*h - 2*a*b^2*d*e^2*h + 7*a*b^2*c*e*f*h + a^2*b*d*e*
f*h - 5*a^2*b*c*f^2*h + a^3*d*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e +
a*b*f))/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2
*e + a*b*f)) + (4*b*d^3*e^2*g - 5*b*c*d^2*e*f*g - 3*a*d^3*e*f*g + b*c^2*d*
f^2*g + 3*a*c*d^2*f^2*g - 2*b*c*d^2*e^2*h - 2*a*d^3*e^2*h + b*c^2*d*e*f*h
+ 7*a*c*d^2*e*f*h + b*c^3*f^2*h - 5*a*c^2*d*f^2*h)*arctan(sqrt(f*x + e)*d/
sqrt(-d^2*e + c*d*f))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*
d^4)*sqrt(-d^2*e + c*d*f)) - (2*(f*x + e)^(3/2)*b^2*d^2*e*f*g - 2*sqrt(f*x
 + e)*b^2*d^2*e^2*f*g - (f*x + e)^(3/2)*b^2*c*d*f^2*g - (f*x + e)^(3/2)*a*
b*d^2*f^2*g + 2*sqrt(f*x + e)*b^2*c*d*e*f^2*g + 2*sqrt(f*x + e)*a*b*d^2*e*
f^2*g - 2*sqrt(f*x + e)*a*b*c*d*f^3*g - (f*x + e)^(3/2)*b^2*c*d*e*f*h - (f
*x + e)^(3/2)*a*b*d^2*e*f*h + sqrt(f*x + e)*b^2*c*d*e^2*f*h + sqrt(f*x + e
)*a*b*d^2*e^2*f*h + (f*x + e)^(3/2)*b^2*c^2*f^2*h + (f*x + e)^(3/2)*a^2*d^
2*f^2*h - sqrt(f*x + e)*b^2*c^2*e*f^2*h - 2*sqrt(f*x + e)*a*b*c*d*e*f^2*h
- sqrt(f*x + e)*a^2*d^2*e*f^2*h + sqrt(f*x + e)*a*b*c^2*f^3*h + sqrt(f*x +
 e)*a^2*c*d*f^3*h)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*((f*x + e)^2*b
*d - 2*(f*x + e)*b*d*e + b*d*e^2 + (f*x + e)*b*c*f + (f*x + e)*a*d*f - b*c
*e*f - a*d*e*f + a*c*f^2))
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 38955, normalized size of antiderivative = 123.67

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^2),x)
```

output
```
- atan(((((8*a*b^9*c^7*d^3*f^5*g + 8*a^7*b^3*c*d^9*f^5*g - 4*a*b^9*c^8*d^2
*f^5*h - 4*a^8*b^2*c*d^9*f^5*h - 8*a^7*b^3*d^10*e*f^4*g + 4*a^8*b^2*d^10*e
*f^4*h - 8*b^10*c^7*d^3*e*f^4*g + 4*b^10*c^8*d^2*e*f^4*h - 48*a^2*b^8*c^6*
d^4*f^5*g + 120*a^3*b^7*c^5*d^5*f^5*g - 160*a^4*b^6*c^4*d^6*f^5*g + 120*a^
5*b^5*c^3*d^7*f^5*g - 48*a^6*b^4*c^2*d^8*f^5*g + 20*a^2*b^8*c^7*d^3*f^5*h
- 36*a^3*b^7*c^6*d^4*f^5*h + 20*a^4*b^6*c^5*d^5*f^5*h + 20*a^5*b^5*c^4*d^6
*f^5*h - 36*a^6*b^4*c^3*d^7*f^5*h + 20*a^7*b^3*c^2*d^8*f^5*h + 8*a^6*b^4*d
^10*e^2*f^3*g - 4*a^7*b^3*d^10*e^2*f^3*h + 8*b^10*c^6*d^4*e^2*f^3*g - 4*b^
10*c^7*d^3*e^2*f^3*h + 120*a^2*b^8*c^4*d^6*e^2*f^3*g - 160*a^3*b^7*c^3*d^7
*e^2*f^3*g + 120*a^4*b^6*c^2*d^8*e^2*f^3*g - 36*a^2*b^8*c^5*d^5*e^2*f^3*h
+ 20*a^3*b^7*c^4*d^6*e^2*f^3*h + 20*a^4*b^6*c^3*d^7*e^2*f^3*h - 36*a^5*b^5
*c^2*d^8*e^2*f^3*h + 40*a*b^9*c^6*d^4*e*f^4*g + 40*a^6*b^4*c*d^9*e*f^4*g -
 16*a*b^9*c^7*d^3*e*f^4*h - 16*a^7*b^3*c*d^9*e*f^4*h - 48*a*b^9*c^5*d^5*e^
2*f^3*g - 72*a^2*b^8*c^5*d^5*e*f^4*g + 40*a^3*b^7*c^4*d^6*e*f^4*g + 40*a^4
*b^6*c^3*d^7*e*f^4*g - 48*a^5*b^5*c*d^9*e^2*f^3*g - 72*a^5*b^5*c^2*d^8*e*f
^4*g + 20*a*b^9*c^6*d^4*e^2*f^3*h + 16*a^2*b^8*c^6*d^4*e*f^4*h + 16*a^3*b^
7*c^5*d^5*e*f^4*h - 40*a^4*b^6*c^4*d^6*e*f^4*h + 16*a^5*b^5*c^3*d^7*e*f^4*
h + 20*a^6*b^4*c*d^9*e^2*f^3*h + 16*a^6*b^4*c^2*d^8*e*f^4*h)/(a^6*b*d^7 +
b^7*c^6*d - 6*a*b^6*c^5*d^2 - 6*a^5*b^2*c*d^6 + 15*a^2*b^5*c^4*d^3 - 20*a^
3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5) - (2*(e + f*x)^(1/2)*((4*a^2*d^5*e^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2886, normalized size of antiderivative = 9.16

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^2,x)
```

output
```
(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**3*c*d**3*f*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**3*d**4*f*h*x - 5*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**2*f*h + 2*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d
**3*e*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**2*b*c*d**3*f*g - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d**3*f*h*x + 2*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**4*e*h*x
+ sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**2*b*d**4*f*g*x + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**2*b*d**4*f*h*x**2 + 2*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**2*d**2*e*h + 3*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
b**2*c**2*d**2*f*g - 5*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a*b**2*c**2*d**2*f*h*x - 4*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d**3*e*g + 4*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
b**2*c*d**3*e*h*x + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a*b**2*c*d**3*f*g*x - 5*sqrt(b)*sqrt(a*f - b*e)*a...
```

3.93 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 520

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2} \, dx = -\frac{(a^2d^2fh+abd(3dfg+4deh-9cfh)-b^2(12d^2eg+4c^2fh-cd(9fg+8eh)))\sqrt{e+fx}}{4b^2(bc-ad)^3(c+dx)} - \frac{(a^2dfh-b^2(6deg-3cfg-4ceh)+ab(3dfg+2deh-7cfh))\sqrt{e+fx}}{4b^2(bc-ad)^2(a+bx)(c+dx)} - \frac{(bg-ah)(e+fx)^{3/2}}{2b(bc-ad)(a+bx)^2(c+dx)} - \frac{(a^3d^2f^2h+a^2bdf(3dfg+4deh-10cfh)+b^3(24d^2e^2g-8cde(3fg+2eh)+3c^2f(fg+4eh))-ab^2(15c}{4b^{3/2}(bc-ad)^4\sqrt{be-af}} - \frac{\sqrt{de-cf}(ad(3dfg+2deh-5cfh)-b(6d^2eg+c^2fh-cd(3fg+4eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)^4}$$

output
```
-1/4*(a^2*d^2*f*h+a*b*d*(-9*c*f*h+4*d*e*h+3*d*f*g)-b^2*(12*d^2*e*g+4*c^2*f
*h-c*d*(8*e*h+9*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^3/(d*x+c)-1/4*(a^2*d*f
*h-b^2*(-4*c*e*h-3*c*f*g+6*d*e*g)+a*b*(-7*c*f*h+2*d*e*h+3*d*f*g))*(f*x+e)^
(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)/(d*x+c)-1/2*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*
d+b*c)/(b*x+a)^2/(d*x+c)-1/4*(a^3*d^2*f^2*h+a^2*b*d*f*(-10*c*f*h+4*d*e*h+3
*d*f*g)+b^3*(24*d^2*e^2*g-8*c*d*e*(2*e*h+3*f*g)+3*c^2*f*(4*e*h+f*g))-a*b^2
*(15*c^2*f^2*h+8*d^2*e*(e*h+3*f*g)-2*c*d*f*(16*e*h+9*f*g)))*arctanh(b^(1/2
)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(3/2)/(-a*d+b*c)^4/(-a*f+b*e)^(1/2)-(-
c*f+d*e)^(1/2)*(a*d*(-5*c*f*h+2*d*e*h+3*d*f*g)-b*(6*d^2*e*g+c^2*f*h-c*d*(4
*e*h+3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*
d+b*c)^4
```

Mathematica [A] (verified)

Time = 6.08 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = -\frac{(bc-ad)\sqrt{e+fx}\left(-a^3dfh(c+dx)+ab^2\left(d^2x(-18eg+3fgx+4ehx)+c^2(3fg+2eh-17fhx)+cd(-10eg+14\right.\right.}{}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^2),x]
```

output
```
(-(((b*c - a*d)*Sqrt[e + f*x]*(-(a^3*d*f*h*(c + d*x)) + a*b^2*(d^2*x*(-18*
e*g + 3*f*g*x + 4*e*h*x) + c^2*(3*f*g + 2*e*h - 17*f*h*x) + c*d*(-10*e*g +
 14*f*g*x + 14*e*h*x - 9*f*h*x^2)) + a^2*b*(-11*c^2*f*h + c*d*(9*f*g + 10*
e*h - 6*f*h*x) + d^2*(-4*e*g + 5*f*g*x + 6*e*h*x + f*h*x^2)) + b^3*(-12*d^
2*e*g*x^2 + c*d*x*(-6*e*g + 9*f*g*x + 8*e*h*x) + c^2*(f*x*(5*g - 4*h*x) +
2*e*(g + 2*h*x)))))/(b*(a + b*x)^2*(c + d*x))) + ((a^3*d^2*f^2*h + a^2*b*d
*f*(3*d*f*g + 4*d*e*h - 10*c*f*h) + b^3*(24*d^2*e^2*g - 8*c*d*e*(3*f*g + 2
*e*h) + 3*c^2*f*(f*g + 4*e*h)) + a*b^2*(-15*c^2*f^2*h - 8*d^2*e*(3*f*g + e
*h) + 2*c*d*f*(9*f*g + 16*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e
) + a*f]])/(b^(3/2)*Sqrt[-(b*e) + a*f]) + (4*Sqrt[-(d*e) + c*f]*(a*d*(-3*d
*f*g - 2*d*e*h + 5*c*f*h) + b*(6*d^2*e*g + c^2*f*h - c*d*(3*f*g + 4*e*h)))
*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/Sqrt[d])/(4*(b*c - a*
d)^4)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {166, 27, 25, 166, 27, 168, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int -\frac{\sqrt{e+fx}(6bdeg-3bcfg-4bceh-2adeh+3acfh+f(3bdg-4bch+adh)x)}{2(a+bx)^2(c+dx)^2}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int -\frac{\sqrt{e+fx}(a(2de-3cf)h-b(6deg-3cfg-4ceh)-f(3bdg-4bch+adh)x)}{(a+bx)^2(c+dx)^2}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{e+fx}(a(2de-3cf)h-b(6deg-3cfg-4ceh)-f(3bdg-4bch+adh)x)}{(a+bx)^2(c+dx)^2}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 166$$

$$\int \frac{4be(de-cf)(3bdg-2bch-adh)-(2de-cf)\left(dfha^2+b(3dfg+2deh-7cfh)a-b^2(6deg-3cfg-4ceh)\right)-f\left(-\left(\left(8fhc^2-3d(5fg+4eh)c+18d^2eg\right)b^2\right)+ad(3dfg+6deh- \ldots}{2(a+bx)(c+dx)^2\sqrt{e+fx}} \ \Big/ \ b(bc-ad) \ \Big/ \ 4b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\int \frac{4be(de-cf)(3bdg-2bch-adh)-(2de-cf)\left(dfha^2+b(3dfg+2deh-7cfh)a-b^2(6deg-3cfg-4ceh)\right)-f\left(-\left(\left(8fhc^2-3d(5fg+4eh)c+18d^2eg\right)b^2\right)+ad(3dfg+6deh- \ldots}{(a+bx)(c+dx)^2\sqrt{e+fx}} \ \Big/ \ 2b(bc-ad) \ \Big/ \ 4b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\int -\frac{b(de-cf)\left(-\left(\left(3f(fg+4eh)c^2-8de(3fg+2eh)c+24d^2e^2g\right)b^2\right)+a\left(4e(3fg+2eh)d^2-3cf(3fg+8eh)d+11c^2f^2h\right)b+a^2cdf^2h+f\left(-\left(\left(4fhc^2-d(9fg+8eh)c+12d^2\right.\right.\right.}{(a+bx)(c+dx)\sqrt{e+fx}} \Big/ (bc-ad)(de-cf) \Big/ 2b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{b(de-cf)\left(-\left(\left(3f(fg+4eh)c^2-8de(3fg+2eh)c+24d^2e^2g\right)b^2\right)+a\left(4e(3fg+2eh)d^2-3cf(3fg+8eh)d+11c^2f^2h\right)b+a^2cdf^2h+f\left(-\left(\left(4fhc^2-d(9fg+8eh)c+12d^2\right.\right.\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{(bc-ad)(de-cf)} \Big/ 2b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{-\left(\left(3f(fg+4eh)c^2-8de(3fg+2eh)c+24d^2e^2g\right)b^2\right)+a\left(4e(3fg+2eh)d^2-3cf(3fg+8eh)d+11c^2f^2h\right)b+a^2cdf^2h+f\left(-\left(\left(4fhc^2-d(9fg+8eh)c+12d^2eg\right)b^2\right)+\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{bc-ad} \Big/ 2b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 174$$

$$-\frac{b\left(-\frac{\left(a^3d^2f^2h+a^2bdf(-10cfh+4deh+3dfg)-ab^2\left(15c^2f^2h-2cdf(16eh+9fg)+8d^2e(eh+3fg)\right)+b^3\left(3c^2f(4eh+fg)-8cde(2eh+3fg)+24d^2e^2g\right)\right)\int\frac{1}{(a+bx)\sqrt{e-}}}{bc-ad}\right.}{bc-ad}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 73$$

$$-\frac{b\left(-\frac{2\left(a^3d^2f^2h+a^2bdf(-10cfh+4deh+3dfg)-ab^2\left(15c^2f^2h-2cdf(16eh+9fg)+8d^2e(eh+3fg)\right)+b^3\left(3c^2f(4eh+fg)-8cde(2eh+3fg)+24d^2e^2g\right)\right)\int\frac{1}{a+\frac{b(e+f}{f}}}{f(bc-ad)}\right.}{bc-ad}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2\sqrt{e+fx}\left(a^2d^2fh+abd(-9cfh+4deh+3dfg)-\left(b^2\left(4c^2fh-cd(8eh+9fg)+12d^2eg\right)\right)\right)}{(c+dx)(bc-ad)}-\frac{b\left(\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(a^3d^2f^2h+a^2bdf(-10cfh+4deh+3dfg)-\right.}{}\right.}{}}{}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^2),x]
```

output
```
-1/2*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^2*(c + d*x)) +
 (-(((a^2*d*f*h - b^2*(6*d*e*g - 3*c*f*g - 4*c*e*h) + a*b*(3*d*f*g + 2*d*e
*h - 7*c*f*h))*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)*(c + d*x))) + ((-2*
(a^2*d^2*f*h + a*b*d*(3*d*f*g + 4*d*e*h - 9*c*f*h) - b^2*(12*d^2*e*g + 4*c
^2*f*h - c*d*(9*f*g + 8*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)) - (b
*((2*(a^3*d^2*f^2*h + a^2*b*d*f*(3*d*f*g + 4*d*e*h - 10*c*f*h) + b^3*(24*d
^2*e^2*g - 8*c*d*e*(3*f*g + 2*e*h) + 3*c^2*f*(f*g + 4*e*h)) - a*b^2*(15*c^
2*f^2*h + 8*d^2*e*(3*f*g + e*h) - 2*c*d*f*(9*f*g + 16*e*h)))*ArcTanh[(Sqrt
[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f])
 + (8*b*Sqrt[d*e - c*f]*(a*d*(3*d*f*g + 2*d*e*h - 5*c*f*h) - b*(6*d^2*e*g
+ c^2*f*h - c*d*(3*f*g + 4*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e
 - c*f]])/(Sqrt[d]*(b*c - a*d))))/(b*c - a*d))/(2*b*(b*c - a*d)))/(4*b*(b*
c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\sqrt{(cf-de)d}\left(\left(24d^2e^2g-16c\left(eh+\frac{3fg}{2}\right)ed+3(4efh+f^2g)c^2\right)b^3-15a\left(\frac{8d^2e(eh+3fg)}{15}-\frac{32c\left(eh+\frac{9fg}{16}\right)fd}{15}+c^2f^2h\right)b^2-1\right.}{}$
derivativedivides	$2f^3\left(-\frac{\left(-\frac{1}{8}a^3d^2f^2h+\frac{5}{4}a^2bcd\,f^2h-\frac{1}{2}a^2b\,d^2efh-\frac{3}{8}a^2b\,d^2f^2g-\frac{9}{8}a\,b^2c^2f^2h-\frac{1}{4}a\,b^2cd\,f^2g+a\,b^2d^2efg+\frac{1}{2}b^3c^2efh+\frac{5}{8}b^3c^2f^2\right.}{}\right.$
default	$2f^3\left(-\frac{\left(-\frac{1}{8}a^3d^2f^2h+\frac{5}{4}a^2bcd\,f^2h-\frac{1}{2}a^2b\,d^2efh-\frac{3}{8}a^2b\,d^2f^2g-\frac{9}{8}a\,b^2c^2f^2h-\frac{1}{4}a\,b^2cd\,f^2g+a\,b^2d^2efg+\frac{1}{2}b^3c^2efh+\frac{5}{8}b^3c^2f^2\right.}{}\right.$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(((c*f-d*e)*d)^(1/2)*((24*d^2*e^2*g-16*c*(e*h+3/2*f*g)*e*d+3*(4*e*f*h+
f^2*g)*c^2)*b^3-15*a*(8/15*d^2*e*(e*h+3*f*g)-32/15*c*(e*h+9/16*f*g)*f*d+c^
2*f^2*h)*b^2-10*a^2*d*(1/5*(-2*e*h-3/2*f*g)*d+c*f*h)*f*b+a^3*d^2*f^2*h)*(b
*x+a)^2*(d*x+c)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^
(1/2)*(-20*(1/5*(6*d^2*e*g-4*c*(e*h+3/4*f*g)*d+c^2*f*h)*b+a*d*(1/5*(-2*e*h
-3*f*g)*d+c*f*h))*(c*f-d*e)*(b*x+a)^2*(d*x+c)*b*arctan(d*(f*x+e)^(1/2)/((c
*f-d*e)*d)^(1/2))+(a*d-b*c)*((c*f-d*e)*d)^(1/2)*(2*(6*d^2*e*g*x^2+3*x*c*((
-4/3*e*h-3/2*f*g)*x+g*e)*d-(-2*h*f*x^2+(5/2*f*g+2*e*h)*x+g*e)*c^2)*b^3-2*a
*(((2*e*h+3/2*f*g)*x^2-9*e*g*x)*d^2-5*c*(9/10*h*f*x^2+7/5*(-e*h-f*g)*x+g*e
)*d+c^2*(-17/2*f*h*x+3/2*f*g+e*h))*b^2+11*a^2*(1/11*(-h*f*x^2+(-6*e*h-5*f*
g)*x+4*g*e)*d^2-10/11*(-3/5*f*h*x+9/10*f*g+e*h)*c*d+c^2*f*h)*b+a^3*d*f*h*(
d*x+c))*(f*x+e)^(1/2)))/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)/(d*x+c)/(a
*d-b*c)^4/(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3104 vs. $2(486) = 972$.

Time = 23.60 (sec) , antiderivative size = 12461, normalized size of antiderivative = 23.96

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 958, normalized size of antiderivative = 1.84

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(24*b^3*d^2*e^2*g - 24*b^3*c*d*e*f*g - 24*a*b^2*d^2*e*f*g + 3*b^3*c^2*
f^2*g + 18*a*b^2*c*d*f^2*g + 3*a^2*b*d^2*f^2*g - 16*b^3*c*d*e^2*h - 8*a*b^
2*d^2*e^2*h + 12*b^3*c^2*e*f*h + 32*a*b^2*c*d*e*f*h + 4*a^2*b*d^2*e*f*h -
15*a*b^2*c^2*f^2*h - 10*a^2*b*c*d*f^2*h + a^3*d^2*f^2*h)*arctan(sqrt(f*x +
 e)*b/sqrt(-b^2*e + a*b*f))/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*e + a*b*f)) - (6*b*d^3*e^2*g - 9*
b*c*d^2*e*f*g - 3*a*d^3*e*f*g + 3*b*c^2*d*f^2*g + 3*a*c*d^2*f^2*g - 4*b*c*
d^2*e^2*h - 2*a*d^3*e^2*h + 5*b*c^2*d*e*f*h + 7*a*c*d^2*e*f*h - b*c^3*f^2*
h - 5*a*c^2*d*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-d^2
*e + c*d*f)) + (sqrt(f*x + e)*d^2*e*f*g - sqrt(f*x + e)*c*d*f^2*g - sqrt(f
*x + e)*c*d*e*f*h + sqrt(f*x + e)*c^2*f^2*h)/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*((f*x + e)*d - d*e + c*f)) + 1/4*(8*(f*x + e)^(3/2
)*b^3*d*e*f*g - 8*sqrt(f*x + e)*b^3*d*e^2*f*g - 5*(f*x + e)^(3/2)*b^3*c*f^
2*g - 3*(f*x + e)^(3/2)*a*b^2*d*f^2*g + 3*sqrt(f*x + e)*b^3*c*e*f^2*g + 13
*sqrt(f*x + e)*a*b^2*d*e*f^2*g - 3*sqrt(f*x + e)*a*b^2*c*f^3*g - 5*sqrt(f*
x + e)*a^2*b*d*f^3*g - 4*(f*x + e)^(3/2)*b^3*c*e*f*h - 4*(f*x + e)^(3/2)*a
*b^2*d*e*f*h + 4*sqrt(f*x + e)*b^3*c*e^2*f*h + 4*sqrt(f*x + e)*a*b^2*d*e^2
*f*h + 9*(f*x + e)^(3/2)*a*b^2*c*f^2*h - (f*x + e)^(3/2)*a^2*b*d*f^2*h - 1
1*sqrt(f*x + e)*a*b^2*c*e*f^2*h - 5*sqrt(f*x + e)*a^2*b*d*e*f^2*h + 7*s...
```

Mupad [B] (verification not implemented)

Time = 114.84 (sec) , antiderivative size = 97983, normalized size of antiderivative = 188.43

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^2),x)
```

output
```
(log(- (((((b*d^2*f^3*(c*f - d*e)*(8*b^2*c*e*h + 3*b^2*c*f*g - 12*b^2*d*e*
g - a^2*d*f*h - 11*a*b*c*f*h + 4*a*b*d*e*h + 9*a*b*d*f*g))/(a*d - b*c) + (
b^2*d^2*f^2*(e + f*x)^(1/2)*(a*d - b*c)^2*(a*d*f + b*c*f - 2*b*d*e)*(-(2*(
f^6*(a*d - b*c)^24*(16*b^2*c*e*h^2 + a^2*d*f*h^2 + 9*b^2*d*f*g^2 - 16*a*b*
c*f*h^2 + 8*a*b*d*e*h^2 - 24*b^2*d*e*g*h + 6*a*b*d*f*g*h)^2)^(1/2) + 2*a^1
4*d^13*f^4*h^2 + 2304*a^8*b^6*d^13*e^4*g^2 + 256*a^10*b^4*d^13*e^4*h^2 + 1
8*a^12*b^2*d^13*f^4*g^2 + 2304*b^14*c^8*d^5*e^4*g^2 + 1024*b^14*c^10*d^3*e
^4*h^2 + 32*a*b^13*c^13*f^4*h^2 + 18*b^14*c^12*d*f^4*g^2 - 32*b^14*c^13*e*
f^3*h^2 - 56*a^13*b*c*d^12*f^4*h^2 + 16*a^13*b*d^13*e*f^3*h^2 - 1536*a^9*b
^5*d^13*e^4*g*h - 3072*b^14*c^9*d^4*e^4*g*h - 18432*a*b^13*c^7*d^6*e^4*g^2
 - 18432*a^7*b^7*c*d^12*e^4*g^2 - 7168*a*b^13*c^9*d^4*e^4*h^2 + 360*a*b^13
*c^11*d^2*f^4*g^2 - 1024*a^9*b^5*c*d^12*e^4*h^2 + 360*a^11*b^3*c*d^12*f^4*
g^2 + 514*a^2*b^12*c^12*d*f^4*h^2 - 4608*a^9*b^5*d^13*e^3*f*g^2 - 576*a^11
*b^3*d^13*e*f^3*g^2 - 256*a^11*b^3*d^13*e^3*f*h^2 - 4608*b^14*c^9*d^4*e^3*
f*g^2 - 576*b^14*c^11*d^2*e*f^3*g^2 - 1536*b^14*c^11*d^2*e^3*f*h^2 + 576*b
^14*c^12*d*e^2*f^2*h^2 + 12*a^13*b*d^13*f^4*g*h + 64512*a^2*b^12*c^6*d^7*e
^4*g^2 - 129024*a^3*b^11*c^5*d^8*e^4*g^2 + 161280*a^4*b^10*c^4*d^9*e^4*g^2
 - 129024*a^5*b^9*c^3*d^10*e^4*g^2 + 64512*a^6*b^8*c^2*d^11*e^4*g^2 + 2073
6*a^2*b^12*c^8*d^5*e^4*h^2 - 2268*a^2*b^12*c^10*d^3*f^4*g^2 - 30720*a^3*b^
11*c^7*d^6*e^4*h^2 + 3528*a^3*b^11*c^9*d^4*f^4*g^2 + 21504*a^4*b^10*c^6...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8595, normalized size of antiderivative = 16.53

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^2,x)
```

output
```
(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**5*c*d**3*f**2*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**5*d**4*f**2*h*x - 10*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**2*f**2*h + 4*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**4*b*c*d**3*e*f*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**3*f**2*g - 8*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**3*f**2*h*x +
4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**4*b*d**4*e*f*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**4*f**2*g*x + 2*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**4*f**2*h*x**2
 - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*b**2*c**3*d*f**2*h + 32*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*d**2*e*f*h + 18*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**
2*c**2*d**2*f**2*g - 35*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*d**2*f**2*h*x - 8*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d**3*
e**2*h - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr...
```

3.94 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 856

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx =$$

$$-\frac{d(a^3d^2f^2h+a^2bdf(5dfg+6deh-14cfh)+b^3(32d^2e^2g-4cde(7fg+6eh)+c^2f(fg+18eh))-ab^2(19}{8b^2(bc-ad)^4(be-af)(c+dx)}$$

$$-\frac{(a^2dfh-b^2(8deg-3cfg-6ceh)+ab(5dfg+2deh-9cfh))\sqrt{e+fx}}{12b^2(bc-ad)^2(a+bx)^2(c+dx)}$$

$$-\frac{(a^3d^2f^2h+a^2bdf(5dfg+8deh-16cfh)+b^3(48d^2e^2g+3c^2f(fg+10eh)-2cde(23fg+18eh))-ab^2(}{24b^2(bc-ad)^3(be-af)(a+bx)(c+dx)}$$

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)}$$

$$-\frac{(a^4d^3f^3h+a^3bd^2f^2(5dfg+6deh-15cfh)-3a^2b^2df(15c^2f^2h-15cdf(fg+2eh)+4d^2e(5fg+2eh))-}{}$$

$$+\frac{\sqrt{d}\sqrt{de-cf}(ad(3dfg+2deh-5cfh)-b(8d^2eg+3c^2fh-cd(5fg+6eh)))\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^5}$$

output
```
-1/8*d*(a^3*d^2*f^2*h+a^2*b*d*f*(-14*c*f*h+6*d*e*h+5*d*f*g)+b^3*(32*d^2*e^
2*g-4*c*d*e*(6*e*h+7*f*g)+c^2*f*(18*e*h+f*g))-a*b^2*(19*c^2*f^2*h+4*d^2*e*
(2*e*h+9*f*g)-2*c*d*f*(20*e*h+13*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^4/(-a
*f+b*e)/(d*x+c)-1/12*(a^2*d*f*h-b^2*(-6*c*e*h-3*c*f*g+8*d*e*g)+a*b*(-9*c*f
*h+2*d*e*h+5*d*f*g))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^2/(d*x+c)-1/24
*(a^3*d^2*f^2*h+a^2*b*d*f*(-16*c*f*h+8*d*e*h+5*d*f*g)+b^3*(48*d^2*e^2*g+3*
c^2*f*(10*e*h+f*g)-2*c*d*e*(18*e*h+23*f*g))-a*b^2*(33*c^2*f^2*h+2*d^2*e*(6
*e*h+25*f*g)-2*c*d*f*(29*e*h+20*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^3/(-a*
f+b*e)/(b*x+a)/(d*x+c)-1/3*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)^3
/(d*x+c)-1/8*(a^4*d^3*f^3*h+a^3*b*d^2*f^2*(-15*c*f*h+6*d*e*h+5*d*f*g)-3*a^
2*b^2*d*f*(15*c^2*f^2*h-15*c*d*f*(2*e*h+f*g)+4*d^2*e*(2*e*h+5*f*g))-b^4*(6
4*d^3*e^3*g+c^3*f^2*(-6*e*h+f*g)-24*c*d^2*e^2*(2*e*h+3*f*g)+12*c^2*d*e*f*(
4*e*h+f*g))-a*b^3*(5*c^3*f^3*h+120*c*d^2*e*f*(e*h+f*g)-8*d^3*e^2*(2*e*h+15
*f*g)-15*c^2*d*f^2*(6*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^
(1/2))/b^(3/2)/(-a*d+b*c)^5/(-a*f+b*e)^(3/2)+d^(1/2)*(-c*f+d*e)^(1/2)*(a*d
*(-5*c*f*h+2*d*e*h+3*d*f*g)-b*(8*d^2*e*g+3*c^2*f*h-c*d*(6*e*h+5*f*g)))*arc
tanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6288 vs. 2(856) = 1712.

Time = 16.36 (sec) , antiderivative size = 6288, normalized size of antiderivative = 7.35

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {166, 27, 166, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{\sqrt{e+fx}(8bdeg-3bcfg-6bceh-2adeh+3acfh+f(5bdg-6bch+adh)x)}{2(a+bx)^3(c+dx)^2}dx}{3b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{e+fx}(8bdeg-a(2de-3cf)h-3bc(fg+2eh)+f(5bdg-6bch+adh)x)}{(a+bx)^3(c+dx)^2}dx}{6b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 166$$

$$-\frac{\frac{\int -\frac{8be(de-cf)(4bdg-3bch-adh)-(2de-cf)\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2(8deg-3cfg-6ceh)\right)-f\left(-\left(\left(24fhc^2-5d(7fg+6eh)c+40d^2eg\right)b^2\right)-ad(17cfh}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}}}{2b(bc-ad)}}{6b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+ab(-9cfh+2deh+5dfg)-b^2(-6ceh-3cfg+8deg)\right)}{2b(a+bx)^2(c+dx)(bc-ad)}-\frac{\int \frac{8be(de-cf)(4bdg-3bch-adh)-(2de-cf)\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2\right)}{}}{}}{6b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+ab(-9cfh+2deh+5dfg)-b^2(-6ceh-3cfg+8deg)\right)}{2b(a+bx)^2(c+dx)(bc-ad)}-\frac{-\frac{\int\frac{3\left(d^2f^2(2de-cf)ha^3+bdf\left(2e(5fg+6eh)d^2-cf(5fg+32eh)d+12c^2f^2h\right)a^2}{}}{}}{}}{}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$\downarrow$ 27

$$-\frac{\frac{\sqrt{e+fx}\left(a^2dfh+ab(-9cfh+2deh+5dfg)-b^2(-6ceh-3cfg+8deg)\right)}{2b(a+bx)^2(c+dx)(bc-ad)}-\frac{-\frac{3\int\frac{d^2f^2(2de-cf)ha^3+bdf\left(2e(5fg+6eh)d^2-cf(5fg+32eh)d+12c^2f^2h\right)a^2+}{}}{}}{}}{}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)(bc-ad)}$$

$\downarrow$ 168

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)}-$$

$$\frac{\frac{\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2(8deg-3cfg-6ceh)\right)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)}-\frac{-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(5dfg+8deh-16cfh)a^2-b^2\left(2e(25fg+6eh)d^2-2cf(20fg+29\right.\right.}{(bc-ad)(be-af)(a+b}}{}}{}$$

$\downarrow$ 27

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)}-$$

$$\frac{\frac{\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2(8deg-3cfg-6ceh)\right)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)}-\frac{-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(5dfg+8deh-16cfh)a^2-b^2\left(2e(25fg+6eh)d^2-2cf(20fg+29\right.\right.}{(bc-ad)(be-af)(a+b}}{}}{}$$

$\downarrow$ 174

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)}-$$

$$\frac{\frac{\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2(8deg-3cfg-6ceh)\right)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)}-\frac{-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(5dfg+8deh-16cfh)a^2-b^2\left(2e(25fg+6eh)d^2-2cf(20fg+29\right.\right.}{(bc-ad)(be-af)(a+b}}{}}{}$$

$\downarrow$ 73

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)}-$$

$$\frac{\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2(8deg-3cfg-6ceh)\right)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)}-\frac{-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(5dfg+8deh-16cfh)a^2-b^2\left(2e(25fg+6eh)d^2-2cf(20fg+29\right.\right.}{(bc-ad)(be-af)(a+b}}{}$$

$\downarrow$ 221

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)}-$$

$$\frac{\left(dfha^2+b(5dfg+2deh-9cfh)a-b^2(8deg-3cfg-6ceh)\right)\sqrt{e+fx}}{2b(bc-ad)(a+bx)^2(c+dx)}-\frac{-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(5dfg+8deh-16cfh)a^2-b^2\left(2e(25fg+6eh)d^2-2cf(20fg+29\right.\right.}{(bc-ad)(be-af)(a+b}}{}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^2),x]
```

output
```
-1/3*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^3*(c + d*x)) -
 (((a^2*d*f*h - b^2*(8*d*e*g - 3*c*f*g - 6*c*e*h) + a*b*(5*d*f*g + 2*d*e*h
 - 9*c*f*h))*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(a + b*x)^2*(c + d*x)) - (-((
(a^3*d^2*f^2*h + a^2*b*d*f*(5*d*f*g + 8*d*e*h - 16*c*f*h) + b^3*(48*d^2*e^
2*g + 3*c^2*f*(f*g + 10*e*h) - 2*c*d*e*(23*f*g + 18*e*h)) - a*b^2*(33*c^2*
f^2*h + 2*d^2*e*(25*f*g + 6*e*h) - 2*c*d*f*(20*f*g + 29*e*h)))*Sqrt[e + f*
x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x))) - (3*((2*d*(a^3*d^2*f^2
*h + a^2*b*d*f*(5*d*f*g + 6*d*e*h - 14*c*f*h) + b^3*(32*d^2*e^2*g - 4*c*d*
e*(7*f*g + 6*e*h) + c^2*f*(f*g + 18*e*h)) - a*b^2*(19*c^2*f^2*h + 4*d^2*e*
(9*f*g + 2*e*h) - 2*c*d*f*(13*f*g + 20*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*
(c + d*x)) + (b*((2*(a^4*d^3*f^3*h + a^3*b*d^2*f^2*(5*d*f*g + 6*d*e*h - 15
*c*f*h) - 3*a^2*b^2*d*f*(15*c^2*f^2*h - 15*c*d*f*(f*g + 2*e*h) + 4*d^2*e*(
5*f*g + 2*e*h)) - b^4*(64*d^3*e^3*g + c^3*f^2*(f*g - 6*e*h) - 24*c*d^2*e^2
*(3*f*g + 2*e*h) + 12*c^2*d*e*f*(f*g + 4*e*h)) - a*b^3*(5*c^3*f^3*h + 120*
c*d^2*e*f*(f*g + e*h) - 8*d^3*e^2*(15*f*g + 2*e*h) - 15*c^2*d*f^2*(f*g + 6
*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c -
a*d)*Sqrt[b*e - a*f]) - (16*b*Sqrt[d]*(b*e - a*f)*Sqrt[d*e - c*f]*(a*d*(3*
d*f*g + 2*d*e*h - 5*c*f*h) - b*(8*d^2*e*g + 3*c^2*f*h - c*d*(5*f*g + 6*e*h
)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(b*c - a*d)))/(b*c -
 a*d)))/(2*(b*c - a*d)*(b*e - a*f)))/(4*b*(b*c - a*d)))/(6*b*(b*c - a*d...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 14.80 (sec) , antiderivative size = 1136, normalized size of antiderivative = 1.33

method	result	size
pseudoelliptic	Expression too large to display	1136
derivativedivides	Expression too large to display	1576
default	Expression too large to display	1576

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(((c*f-d*e)*d)^(1/2)*((-64*d^3*e^3*g+48*c*(e*h+3/2*f*g)*e^2*d^2-48*c^2
*(e*h+1/4*f*g)*f*e*d+(6*e*f^2*h-f^3*g)*c^3)*b^4-5*((-16/5*e^3*h-24*g*f*e^2
)*d^3+24*c*d^2*e*f*(e*h+f*g)+(-18*e*f^2*h-3*f^3*g)*c^2*d+c^3*f^3*h)*a*b^3-
45*a^2*((8/15*e^2*h+4/3*e*f*g)*d^2+c*(-2*e*f*h-f^2*g)*d+c^2*f^2*h)*d*f*b^2
-15*a^3*((-2/5*e*h-1/3*f*g)*d+c*f*h)*d^2*f^2*b+a^4*d^3*f^3*h)*(b*x+a)^3*(d
*x+c)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-(-40*(c*f-d*e)*d*(d*x+c)
*(b*x+a)^3*b*(a*f-b*e)*((8/5*d^2*e*g+c*(-6/5*e*h-f*g)*d+3/5*c^2*f*h)*b+((-
2/5*e*h-3/5*f*g)*d+c*f*h)*a*d)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))
+(a*d-b*c)*((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((-32*d^3*e^2*g*x^3-16*x^2*c*
((-3/2*e*h-7/4*f*g)*x+g*e)*e*d^2+16/3*x*c^2*((-27/8*e*f*h-3/16*f^2*g)*x^2+
(9/4*e^2*h+23/8*e*f*g)*x+e^2*g)*d-8/3*((15/4*e*f*h+3/8*f^2*g)*x^2+(3/2*e^2
*h+7/4*e*f*g)*x+e^2*g)*c^3)*b^5-4/3*a*(((-6*e^2*h-27*e*f*g)*x^3+60*e^2*g*x
^2)*d^3+32*x*c*((15/16*e*f*h+39/64*f^2*g)*x^2+(-3/2*e^2*h-2*e*f*g)*x+e^2*g
)*d^2-10*(57/40*f^2*h*x^3+(-19/4*e*f*h-23/20*f^2*g)*x^2+(23/10*e^2*h+51/20
*e*f*g)*x+e^2*g)*c^2*d+(-33/4*f^2*h*x^2+(11/2*e*f*h-2*f^2*g)*x+e*(e*h-1/2*
f*g))*c^3)*b^4-8/3*a^2*(22*x*(9/88*(e*h+5/6*f*g)*f*x^2+(-15/44*e^2*h-137/8
8*e*f*g)*x+e^2*g)*d^3+13*c*(-21/52*f^2*h*x^3+(155/52*e*f*h+199/104*f^2*g)*
x^2+(-41/26*e^2*h-141/52*e*f*g)*x+e^2*g)*d^2-4*c^2*(77/16*f^2*h*x^2+(-75/8
*e*f*h-109/32*f^2*g)*x+e*(e*h+1/4*f*g))*d+c^3*f*(-5*f*h*x-3/8*f*g+e*h))*b^
3+5*a^3*((-1/5*f^2*h*x^3+(-46/15*e*f*h-8/3*f^2*g)*x^2+(44/15*e^2*h+206/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**4/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2299 vs. $2(818) = 1636$.

Time $= 0.28$ (sec) , antiderivative size $= 2299$, normalized size of antiderivative $= 2.69$

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(64*b^4*d^3*e^3*g - 72*b^4*c*d^2*e^2*f*g - 120*a*b^3*d^3*e^2*f*g + 12
*b^4*c^2*d*e*f^2*g + 120*a*b^3*c*d^2*e*f^2*g + 60*a^2*b^2*d^3*e*f^2*g + b^
4*c^3*f^3*g - 15*a*b^3*c^2*d*f^3*g - 45*a^2*b^2*c*d^2*f^3*g - 5*a^3*b*d^3*
f^3*g - 48*b^4*c*d^2*e^3*h - 16*a*b^3*d^3*e^3*h + 48*b^4*c^2*d*e^2*f*h + 1
20*a*b^3*c*d^2*e^2*f*h + 24*a^2*b^2*d^3*e^2*f*h - 6*b^4*c^3*e*f^2*h - 90*a
*b^3*c^2*d*e*f^2*h - 90*a^2*b^2*c*d^2*e*f^2*h - 6*a^3*b*d^3*e*f^2*h + 5*a*
b^3*c^3*f^3*h + 45*a^2*b^2*c^2*d*f^3*h + 15*a^3*b*c*d^2*f^3*h - a^4*d^3*f^
3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^5*e - 5*a*b^6*c^
4*d*e + 10*a^2*b^5*c^3*d^2*e - 10*a^3*b^4*c^2*d^3*e + 5*a^4*b^3*c*d^4*e -
a^5*b^2*d^5*e - a*b^6*c^5*f + 5*a^2*b^5*c^4*d*f - 10*a^3*b^4*c^3*d^2*f + 1
0*a^4*b^3*c^2*d^3*f - 5*a^5*b^2*c*d^4*f + a^6*b*d^5*f)*sqrt(-b^2*e + a*b*f
)) + (8*b*d^4*e^2*g - 13*b*c*d^3*e*f*g - 3*a*d^4*e*f*g + 5*b*c^2*d^2*f^2*g
 + 3*a*c*d^3*f^2*g - 6*b*c*d^3*e^2*h - 2*a*d^4*e^2*h + 9*b*c^2*d^2*e*f*h +
 7*a*c*d^3*e*f*h - 3*b*c^3*d*f^2*h - 5*a*c^2*d^2*f^2*h)*arctan(sqrt(f*x +
e)*d/sqrt(-d^2*e + c*d*f))/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2
- 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(-d^2*e + c*d*f)) - (s
qrt(f*x + e)*d^3*e*f*g - sqrt(f*x + e)*c*d^2*f^2*g - sqrt(f*x + e)*c*d^2*e
*f*h + sqrt(f*x + e)*c^2*d*f^2*h)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((f*x + e)*d - d*e + c*f)) - 1/24*(72*(f*
x + e)^(5/2)*b^5*d^2*e^2*f*g - 144*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + 72...
```

Mupad [B] (verification not implemented)

Time = 31.47 (sec) , antiderivative size = 671141, normalized size of antiderivative = 784.04

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^2),x)
```

output
```
atan(((((32*a*b^15*c^13*d^2*f^7*g - 608*a^13*b^3*c*d^14*f^7*g + 32*a^14*b^
2*c*d^14*f^7*h + 608*a^13*b^3*d^15*e*f^6*g - 32*a^14*b^2*d^15*e*f^6*h - 32
*b^16*c^13*d^2*e*f^6*g - 768*a^2*b^14*c^12*d^3*f^7*g + 5312*a^3*b^13*c^11*
d^4*f^7*g - 17920*a^4*b^12*c^10*d^5*f^7*g + 33120*a^5*b^11*c^9*d^6*f^7*g -
 29184*a^6*b^10*c^8*d^7*f^7*g - 8064*a^7*b^9*c^7*d^8*f^7*g + 55296*a^8*b^8
*c^6*d^9*f^7*g - 72480*a^9*b^7*c^5*d^10*f^7*g + 52480*a^10*b^6*c^4*d^11*f^
7*g - 22848*a^11*b^5*c^3*d^12*f^7*g + 5632*a^12*b^4*c^2*d^13*f^7*g + 160*a
^2*b^14*c^13*d^2*f^7*h - 768*a^3*b^13*c^12*d^3*f^7*h - 1088*a^4*b^12*c^11*
d^4*f^7*h + 17920*a^5*b^11*c^10*d^5*f^7*h - 64800*a^6*b^10*c^9*d^6*f^7*h +
 130560*a^7*b^9*c^8*d^7*f^7*h - 169344*a^8*b^8*c^7*d^8*f^7*h + 147456*a^9*
b^7*c^6*d^9*f^7*h - 85920*a^10*b^6*c^5*d^10*f^7*h + 32000*a^11*b^5*c^4*d^1
1*f^7*h - 6720*a^12*b^4*c^3*d^12*f^7*h + 512*a^13*b^3*c^2*d^13*f^7*h - 102
4*a^10*b^6*d^15*e^4*f^3*g + 2688*a^11*b^5*d^15*e^3*f^4*g - 2272*a^12*b^4*d
^15*e^2*f^5*g + 256*a^11*b^5*d^15*e^4*f^3*h - 576*a^12*b^4*d^15*e^3*f^4*h
+ 352*a^13*b^3*d^15*e^2*f^5*h - 1024*b^16*c^10*d^5*e^4*f^3*g + 1408*b^16*c
^11*d^4*e^3*f^4*g - 352*b^16*c^12*d^3*e^2*f^5*g + 768*b^16*c^11*d^4*e^4*f^
3*h - 960*b^16*c^12*d^3*e^3*f^4*h + 192*b^16*c^13*d^2*e^2*f^5*h - 46080*a^
2*b^14*c^8*d^7*e^4*f^3*g + 36480*a^2*b^14*c^9*d^6*e^3*f^4*g + 17088*a^2*b^
14*c^10*d^5*e^2*f^5*g + 122880*a^3*b^13*c^7*d^8*e^4*f^3*g - 48000*a^3*b^13
*c^8*d^7*e^3*f^4*g - 93440*a^3*b^13*c^9*d^6*e^2*f^5*g - 215040*a^4*b^12...
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 18011, normalized size of antiderivative = 21.04

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**7*c*d**3*f**3*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**7*d**4*f**3*h*x - 45*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**2*f**3*h
+ 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**6*b*c*d**3*e*f**2*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**3*f**3*g - 36*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**3*f
**3*h*x + 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**6*b*d**4*e*f**2*h*x + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**4*f**3*g*x + 9*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d
**4*f**3*h*x**2 - 135*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**5*b**2*c**3*d*f**3*h + 270*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*d**2*e*
f**2*h + 135*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**5*b**2*c**2*d**2*f**3*g - 270*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*d**2*f**3*h*x
 - 72*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**5*b**2*c*d**3*e**2*f*h - 180*sqrt(b)*sqrt(a*f - b*e)*atan((sqr...
```

3.95 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1359

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Too large to display}$$

output
```
-1/64*d*(5*a^4*d^3*f^3*h+5*a^3*b*d^2*f^2*(-19*c*f*h+8*d*e*h+7*d*f*g)-b^4*(
320*d^3*e^3*g+c^3*f^2*(-8*e*h+3*f*g)+16*c^2*d*e*f*(14*e*h+f*g)-16*c*d^2*e^
2*(16*e*h+19*f*g))-a*b^3*(5*c^3*f^3*h-16*d^3*e^2*(4*e*h+41*f*g)+48*c*d^2*e
*f*(13*e*h+12*f*g)-5*c^2*d*f^2*(88*e*h+5*f*g))-a^2*b^2*d*f*(225*c^2*f^2*h+
16*d^2*e*(7*e*h+23*f*g)-c*d*f*(472*e*h+263*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+
b*c)^5/(-a*f+b*e)^2/(d*x+c)-1/24*(a^2*d*f*h-b^2*(-8*c*e*h-3*c*f*g+10*d*e*g
)+a*b*(-11*c*f*h+2*d*e*h+7*d*f*g))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^
3/(d*x+c)-1/96*(a^3*d^2*f^2*h+a^2*b*d*f*(-22*c*f*h+12*d*e*h+7*d*f*g)-a*b^2
*(59*c^2*f^2*h+4*d^2*e*(4*e*h+21*f*g)-2*c*d*f*(46*e*h+35*f*g))+b^3*(80*d^2
*e^2*g-4*c*d*e*(16*e*h+19*f*g)+c^2*f*(56*e*h+3*f*g)))*(f*x+e)^(1/2)/b^2/(-
a*d+b*c)^3/(-a*f+b*e)/(b*x+a)^2/(d*x+c)-1/192*(5*a^4*d^3*f^3*h+5*a^3*b*d^2
*f^2*(-21*c*f*h+10*d*e*h+7*d*f*g)-5*a^2*b^2*d*f*(73*c^2*f^2*h-11*c*d*f*(12
*e*h+7*f*g)+2*d^2*e*(16*e*h+49*f*g))-b^4*(480*d^3*e^3*g+3*c^3*f^2*(-8*e*h+
3*f*g)-16*c*d^2*e^2*(24*e*h+31*f*g)+2*c^2*d*e*f*(184*e*h+21*f*g))-a*b^3*(1
5*c^3*f^3*h-16*d^3*e^2*(6*e*h+59*f*g)+4*c*d^2*e*f*(228*e*h+227*f*g)-c^2*d*
f^2*(706*e*h+69*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^4/(-a*f+b*e)^2/(b*x+a)
/(d*x+c)-1/4*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)^4/(d*x+c)-1/64*
(5*a^5*d^4*f^4*h+5*a^4*b*d^3*f^3*(-20*c*f*h+8*d*e*h+7*d*f*g)+b^5*(640*d^4*
e^4*g+c^4*f^3*(-8*e*h+3*f*g)+16*c^3*d*e*f^2*(-6*e*h+f*g)-256*c*d^3*e^3*(2*
e*h+3*f*g)+144*c^2*d^2*e^2*f*(4*e*h+f*g))-10*a^3*b^2*d^2*f^2*(45*c^2*f^...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16895 vs. 2(1359) = 2718.

Time = 16.65 (sec) , antiderivative size = 16895, normalized size of antiderivative = 12.43

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.45 (sec) , antiderivative size = 1443, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {166, 27, 25, 166, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int -\frac{\sqrt{e+fx}(10bdeg-3bcfg-8bceh-2adeh+3acfh+f(7bdg-8bch+adh)x)}{2(a+bx)^4(c+dx)^2}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int -\frac{\sqrt{e+fx}(a(2de-3cf)h-b(10deg-3cfg-8ceh)-f(7bdg-8bch+adh)x)}{(a+bx)^4(c+dx)^2}dx}{8b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{e+fx}(a(2de-3cf)h-b(10deg-3cfg-8ceh)-f(7bdg-8bch+adh)x)}{(a+bx)^4(c+dx)^2}dx}{8b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 166$$

$$\frac{\frac{\int \frac{12be(de-cf)(5bdg-4bch-adh)-(2de-cf)\left(dfha^2+b(7dfg+2deh-11cfh)a-b^2(10deg-3cfg-8ceh)\right)-f\left(-\left(\left(48fhc^2-7d(9fg+8eh)c+70d^2eg\right)b^2\right)-ad(23cfh-}{2(a+bx)^3(c+dx)^2\sqrt{e+fx}}}{3b(bc-ad)}}{8b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{12be(de-cf)(5bdg-4bch-adh)-(2de-cf)\left(dfha^2+b(7dfg+2deh-11cfh)a-b^2(10deg-3cfg-8ceh)\right)-f\left(-\left(\left(48fhc^2-7d(9fg+8eh)c+70d^2eg\right)b^2\right)-ad(23cfh-}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}}{6b(bc-ad)}}{8b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$-\frac{\int \frac{5d^2f^2(2de-cf)ha^3+5bdf\left(2e(7fg+8eh)d^2-cf(7fg+40eh)d+14c^2f^2h\right)a^2+b^2\left(-32e^2(17fg+3eh)d^3+16cef(33fg+37eh)d^2-6c^2f^2(9fg+71eh)d+15c^3f^3h\right)c\cdots}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{5d^2f^2(2de-cf)ha^3+5bdf\left(2e(7fg+8eh)d^2-cf(7fg+40eh)d+14c^2f^2h\right)a^2+b^2\left(-32e^2(17fg+3eh)d^3+16cef(33fg+37eh)d^2-6c^2f^2(9fg+71eh)d+15c^3f^3h\right)c\cdots}{\cdots}}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{(\cdots}{\cdots}}{\cdots}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\sqrt{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\sqrt{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\cdots$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

$$\Big\downarrow \; 174$$

$$-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\cdots$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\cdots$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{e+fx}\left(d^2f^2ha^3+bdf(7dfg+12deh-22cfh)a^2-b^2\left(4e(21fg+4eh)d^2-2cf(35fg+46eh)d+59c^2f^2h\right)a+b^3\left(f(3fg+56eh)c^2-4de(19fg+16eh)c+80d^2e^2g\right)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^2),x]
```

output
```
-1/4*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^4*(c + d*x)) +
 (-1/3*((a^2*d*f*h - b^2*(10*d*e*g - 3*c*f*g - 8*c*e*h) + a*b*(7*d*f*g + 2
*d*e*h - 11*c*f*h))*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^3*(c + d*x)) +
 (-1/2*((a^3*d^2*f^2*h + a^2*b*d*f*(7*d*f*g + 12*d*e*h - 22*c*f*h) - a*b^2
*(59*c^2*f^2*h + 4*d^2*e*(21*f*g + 4*e*h) - 2*c*d*f*(35*f*g + 46*e*h)) + b
^3*(80*d^2*e^2*g - 4*c*d*e*(19*f*g + 16*e*h) + c^2*f*(3*f*g + 56*e*h)))*Sq
rt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)) - (((5*a^4*d^
3*f^3*h + 5*a^3*b*d^2*f^2*(7*d*f*g + 10*d*e*h - 21*c*f*h) - 5*a^2*b^2*d*f*
(73*c^2*f^2*h - 11*c*d*f*(7*f*g + 12*e*h) + 2*d^2*e*(49*f*g + 16*e*h)) - b
^4*(480*d^3*e^3*g + 3*c^3*f^2*(3*f*g - 8*e*h) - 16*c*d^2*e^2*(31*f*g + 24*
e*h) + 2*c^2*d*e*f*(21*f*g + 184*e*h)) - a*b^3*(15*c^3*f^3*h - 16*d^3*e^2*
(59*f*g + 6*e*h) + 4*c*d^2*e*f*(227*f*g + 228*e*h) - c^2*d*f^2*(69*f*g + 7
06*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)) + (
3*((2*d*(5*a^4*d^3*f^3*h + 5*a^3*b*d^2*f^2*(7*d*f*g + 8*d*e*h - 19*c*f*h)
- b^4*(320*d^3*e^3*g + c^3*f^2*(3*f*g - 8*e*h) + 16*c^2*d*e*f*(f*g + 14*e*
h) - 16*c*d^2*e^2*(19*f*g + 16*e*h)) - a*b^3*(5*c^3*f^3*h - 16*d^3*e^2*(41
*f*g + 4*e*h) + 48*c*d^2*e*f*(12*f*g + 13*e*h) - 5*c^2*d*f^2*(5*f*g + 88*e
*h)) - a^2*b^2*d*f*(225*c^2*f^2*h + 16*d^2*e*(23*f*g + 7*e*h) - c*d*f*(263
*f*g + 472*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)) + (b*((2*(5*a^5*d
^4*f^4*h + 5*a^4*b*d^3*f^3*(7*d*f*g + 8*d*e*h - 20*c*f*h) + b^5*(640*d^...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 103.17 (sec) , antiderivative size = 1909, normalized size of antiderivative = 1.40

method	result	size
pseudoelliptic	Expression too large to display	1909
derivativedivides	Expression too large to display	2949
default	Expression too large to display	2949

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
5/64*(((c*f-d*e)*d)^(1/2)*(b*x+a)^4*(d*x+c)*((128*d^4*e^4*g-512/5*c*(e*h+3
/2*f*g)*e^3*d^3+576/5*c^2*(e*h+1/4*f*g)*f*e^2*d^2-96/5*c^3*(e*h-1/6*f*g)*f
^2*e*d-8/5*c^4*(e*h-3/8*f*g)*f^3)*b^5+a*(-128/5*d^4*e^3*(e*h+14*f*g)+1664/
5*c*(e*h+63/52*f*g)*f*e^2*d^3-1584/5*c^2*f^2*e*(e*h+7/33*f*g)*d^2+208/5*(e
*h-7/52*f*g)*c^3*f^3*d+c^4*f^4*h)*b^4-20*a^2*d*((-16/5*e^3*h-84/5*g*f*e^2)
*d^3+96/5*c*(e*h+7/8*f*g)*f*e*d^2-72/5*(e*h+7/48*f*g)*c^2*f^2*d+c^3*f^3*h)
*f*b^3-90*a^3*d^2*((8/15*e^2*h+56/45*e*f*g)*d^2-88/45*(e*h+21/44*f*g)*c*f*
d+c^2*f^2*h)*f^2*b^2-20*a^4*((-2/5*e*h-7/20*f*g)*d+c*f*h)*d^3*f^3*b+a^5*d^
4*f^4*h)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^(1/2)*(
-64*(c*f-d*e)*d^2*((2*d^2*e*g-8/5*c*(e*h+7/8*f*g)*d+c^2*f*h)*b+((-2/5*e*h-
3/5*f*g)*d+c*f*h)*a*d)*(d*x+c)*(b*x+a)^4*b*(a*f-b*e)^2*arctan(d*(f*x+e)^(1
/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((64*d^4*e^3*g*x^4+32*x^3*c*(-19/10*f*g
*x+e*(-8/5*h*x+g))*e^2*d^3-32/3*x^2*c^2*(-3/10*x^2*g*f^2+31/10*x*(-42/31*h
*x+g)*e*f+e^2*(12/5*h*x+g))*e*d^2+16/3*x*c^3*(9/80*f^3*g*x^3+21/40*x^2*e*(
-4/7*h*x+g)*f^2+19/10*x*(46/19*h*x+g)*e^2*f+e^3*(8/5*h*x+g))*d-16/5*c^4*(-
3/16*f^3*g*x^3+1/8*e*x^2*(4*h*x+g)*f^2+3/2*(14/9*h*x+g)*x*e^2*f+e^3*(4/3*h
*x+g)))*b^7-16/15*a*((123*e^2*f*g*x^4-210*x^3*(-2/35*h*x+g)*e^3)*d^4-110*x
^2*c*(54/55*x^2*g*f^2-128/55*x*(-117/256*h*x+g)*e*f+e^2*(-87/55*h*x+g))*e*
d^3+35*x*c^2*(15/112*f^3*g*x^3-529/280*x^2*e*(-660/529*h*x+g)*f^2+183/70*x
*e^2*(-404/183*h*x+g)*f+e^3*(86/35*h*x+g))*d^2-17*c^3*(-21/136*x^3*(-5/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**5/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4781 vs. 2(1317) = 2634.

Time = 0.39 (sec) , antiderivative size = 4781, normalized size of antiderivative = 3.52

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/64*(640*b^5*d^4*e^4*g - 768*b^5*c*d^3*e^3*f*g - 1792*a*b^4*d^4*e^3*f*g +
 144*b^5*c^2*d^2*e^2*f^2*g + 2016*a*b^4*c*d^3*e^2*f^2*g + 1680*a^2*b^3*d^4
*e^2*f^2*g + 16*b^5*c^3*d*e*f^3*g - 336*a*b^4*c^2*d^2*e*f^3*g - 1680*a^2*b
^3*c*d^3*e*f^3*g - 560*a^3*b^2*d^4*e*f^3*g + 3*b^5*c^4*f^4*g - 28*a*b^4*c^
3*d*f^4*g + 210*a^2*b^3*c^2*d^2*f^4*g + 420*a^3*b^2*c*d^3*f^4*g + 35*a^4*b
*d^4*f^4*g - 512*b^5*c*d^3*e^4*h - 128*a*b^4*d^4*e^4*h + 576*b^5*c^2*d^2*e
^3*f*h + 1664*a*b^4*c*d^3*e^3*f*h + 320*a^2*b^3*d^4*e^3*f*h - 96*b^5*c^3*d
*e^2*f^2*h - 1584*a*b^4*c^2*d^2*e^2*f^2*h - 1920*a^2*b^3*c*d^3*e^2*f^2*h -
 240*a^3*b^2*d^4*e^2*f^2*h - 8*b^5*c^4*e*f^3*h + 208*a*b^4*c^3*d*e*f^3*h +
 1440*a^2*b^3*c^2*d^2*e*f^3*h + 880*a^3*b^2*c*d^3*e*f^3*h + 40*a^4*b*d^4*e
*f^3*h + 5*a*b^4*c^4*f^4*h - 100*a^2*b^3*c^3*d*f^4*h - 450*a^3*b^2*c^2*d^2
*f^4*h - 100*a^4*b*c*d^3*f^4*h + 5*a^5*d^4*f^4*h)*arctan(sqrt(f*x + e)*b/s
qrt(-b^2*e + a*b*f))/((b^9*c^6*e^2 - 6*a*b^8*c^5*d*e^2 + 15*a^2*b^7*c^4*d^
2*e^2 - 20*a^3*b^6*c^3*d^3*e^2 + 15*a^4*b^5*c^2*d^4*e^2 - 6*a^5*b^4*c*d^5*
e^2 + a^6*b^3*d^6*e^2 - 2*a*b^8*c^6*e*f + 12*a^2*b^7*c^5*d*e*f - 30*a^3*b^
6*c^4*d^2*e*f + 40*a^4*b^5*c^3*d^3*e*f - 30*a^5*b^4*c^2*d^4*e*f + 12*a^6*b
^3*c*d^5*e*f - 2*a^7*b^2*d^6*e*f + a^2*b^7*c^6*f^2 - 6*a^3*b^6*c^5*d*f^2 +
 15*a^4*b^5*c^4*d^2*f^2 - 20*a^5*b^4*c^3*d^3*f^2 + 15*a^6*b^3*c^2*d^4*f^2
- 6*a^7*b^2*c*d^5*f^2 + a^8*b*d^6*f^2)*sqrt(-b^2*e + a*b*f)) - (10*b*d^5*e
^2*g - 17*b*c*d^4*e*f*g - 3*a*d^5*e*f*g + 7*b*c^2*d^3*f^2*g + 3*a*c*d^4...
```

Mupad [B] (verification not implemented)

Time = 42.89 (sec) , antiderivative size = 1058660, normalized size of antiderivative = 779.00

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^2),x)
```

output
```
(((e + f*x)^(3/2)*(33*b^5*c^4*f^5*g - 15*a^5*d^4*f^5*h + 55*a*b^4*c^4*f^5*
h + 279*a^4*b*d^4*f^5*g - 88*b^5*c^4*e*f^4*h + 3840*b^5*d^4*e^4*f*g + 1834
*a^3*b^2*c*d^3*f^5*g - 1030*a^2*b^3*c^3*d*f^5*h - 10272*a*b^4*d^4*e^3*f^2*
g - 2950*a^3*b^2*d^4*e*f^4*g - 5088*b^5*c*d^3*e^3*f^2*g - 944*b^5*c^3*d*e^
2*f^3*h + 1960*a^2*b^3*c^2*d^2*f^5*g - 2440*a^3*b^2*c^2*d^2*f^5*h + 9136*a
^2*b^3*d^4*e^2*f^3*g + 1824*a^2*b^3*d^4*e^3*f^2*h - 1280*a^3*b^2*d^4*e^2*f
^3*h + 1360*b^5*c^2*d^2*e^2*f^3*g + 3840*b^5*c^2*d^2*e^3*f^2*h - 266*a*b^4
*c^3*d*f^5*g - 410*a^4*b*c*d^3*f^5*h - 768*a*b^4*d^4*e^4*f*h + 206*a^4*b*d
^4*e*f^4*h + 134*b^5*c^3*d*e*f^4*g - 3072*b^5*c*d^3*e^4*f*h + 2106*a*b^4*c
^3*d*e*f^4*h + 12544*a*b^4*c*d^3*e^2*f^3*g - 3122*a*b^4*c^2*d^2*e*f^4*g -
9422*a^2*b^3*c*d^3*e*f^4*g + 9696*a*b^4*c*d^3*e^3*f^2*h + 4686*a^3*b^2*c*d
^3*e*f^4*h - 10048*a*b^4*c^2*d^2*e^2*f^3*h - 10768*a^2*b^3*c*d^3*e^2*f^3*h
 + 8450*a^2*b^3*c^2*d^2*e*f^4*h))/(192*b*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c
^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)) + ((e + f*x)
^(1/2)*(3*a*b^4*c^4*f^6*g - 5*a^5*c*d^3*f^6*h - 3*b^5*c^4*e*f^5*g + 5*a^5*
d^4*e*f^5*h - 320*b^5*d^4*e^5*f*g + 5*a^2*b^3*c^4*f^6*h + 8*b^5*c^4*e^2*f^
4*h - 25*a^2*b^3*c^3*d*f^6*g - 95*a^3*b^2*c^3*d*f^6*h - 225*a^4*b*c^2*d^2*
f^6*h + 1136*a*b^4*d^4*e^4*f^2*g - 93*a^4*b*d^4*e^2*f^4*h + 464*b^5*c*d^3*
e^4*f^2*g - 13*b^5*c^3*d*e^2*f^4*g + 88*b^5*c^3*d*e^3*f^3*h + 185*a^3*b^2*
c^2*d^2*f^6*g - 1472*a^2*b^3*d^4*e^3*f^3*g + 813*a^3*b^2*d^4*e^2*f^4*g ...
```

Reduce [F]

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^2}\,dx = \int \frac{(fx+e)^{\frac{3}{2}}\,(hx+g)}{(bx+a)^5\,(dx+c)^2}dx$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x)
```

output
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^2,x)
```

3.96 $\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 494

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = -\frac{2(bc-ad)\left(a^2d^2fh-abd(8cfh-3d(fg+eh))+b^2(3d^2eg+10c^2fh-6cd(fg+eh))\right)\sqrt{e+fx}}{d^6} - \frac{(bc-ad)^2\left(ad(3dfg+4deh-7cfh)+b(12d^2eg+19c^2fh-cd(15fg+16eh))\right)\sqrt{e+fx}}{4d^6(c+dx)} - \frac{2b(bc-ad)(bdg-2bch+adh)(e+fx)^{3/2}}{d^5} + \frac{(bc-ad)^3(dg-ch)(e+fx)^{3/2}}{2d^5(c+dx)^2} + \frac{2b^2(3adfh+b(dfg-deh-3cfh))(e+fx)^{5/2}}{5d^4f^2} + \frac{2b^3h(e+fx)^{7/2}}{7d^3f^2} + \frac{3(bc-ad)\,(a^2d^2f(dfg+4deh-5cfh)+b^2(8d^3e^2g-33c^3f^2h-4cd^2e(7fg+4eh)+3c^2df(7fg+16eh)}{4d^{13/2}\sqrt{de-cf}}$$

output
```
-2*(-a*d+b*c)*(a^2*d^2*f*h-a*b*d*(8*c*f*h-3*d*(e*h+f*g))+b^2*(3*d^2*e*g+10
*c^2*f*h-6*c*d*(e*h+f*g)))*(f*x+e)^(1/2)/d^6-1/4*(-a*d+b*c)^2*(a*d*(-7*c*f
*h+4*d*e*h+3*d*f*g)+b*(12*d^2*e*g+19*c^2*f*h-c*d*(16*e*h+15*f*g)))*(f*x+e)
^(1/2)/d^6/(d*x+c)-2*b*(-a*d+b*c)*(a*d*h-2*b*c*h+b*d*g)*(f*x+e)^(3/2)/d^5+
1/2*(-a*d+b*c)^3*(-c*h+d*g)*(f*x+e)^(3/2)/d^5/(d*x+c)^2+2/5*b^2*(3*a*d*f*h
+b*(-3*c*f*h-d*e*h+d*f*g))*(f*x+e)^(5/2)/d^4/f^2+2/7*b^3*h*(f*x+e)^(7/2)/d
^3/f^2+3/4*(-a*d+b*c)*(a^2*d^2*f*(-5*c*f*h+4*d*e*h+d*f*g)+b^2*(8*d^3*e^2*g
-33*c^3*f^2*h-4*c*d^2*e*(4*e*h+7*f*g)+3*c^2*d*f*(16*e*h+7*f*g))+2*a*b*d*(1
5*c^2*f^2*h+2*d^2*e*(2*e*h+3*f*g)-c*d*f*(18*e*h+7*f*g)))*arctanh(d^(1/2)*(
f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(13/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{\sqrt{e+fx}(-35a^3d^3f^2(-15c^2fh+cd(3fg+2eh-25fhx)+d^2(fx(5 \cdots}{\cdots}$$

$$+\frac{3(-bc+ad)\left(a^2d^2f(dfg+4deh-5cfh)+b^2(8d^3e^2g-33c^3f^2h-4cd^2e(7fg+4eh)+3c^2df(7fg+16e \cdots\right.}{4d^{13/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)^3*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(-35*a^3*d^3*f^2*(-15*c^2*f*h + c*d*(3*f*g + 2*e*h - 25*f*h
*x) + d^2*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x))) + 7*a*b^2*d*f*(945*c^4*f^
2*h - 105*c^3*d*f*(6*e*h + 5*f*(g - 3*h*x)) + 8*d^4*x^2*(3*e^2*h + f^2*x*(
5*g + 3*h*x) + e*f*(20*g + 6*h*x)) + 8*c*d^3*x*(6*e^2*h + e*f*(55*g - 48*h
*x) - f^2*x*(35*g + 9*h*x)) + c^2*d^2*(24*e^2*h + 2*e*f*(125*g - 546*h*x)
+ 7*f^2*x*(-125*g + 72*h*x))) + b^3*(-3465*c^5*f^3*h + 105*c^4*d*f^2*(21*f
*g + 26*e*h - 55*f*h*x) + 8*d^5*x^2*(e + f*x)^2*(7*f*g - 2*e*h + 5*f*h*x)
- 8*c*d^4*x*(4*e^3*h + f^3*x^2*(21*g + 11*h*x) + 2*e*f^2*x*(56*g + 13*h*x)
 + e^2*f*(-14*g + 19*h*x)) - 4*c^2*d^3*(4*e^3*h + e*f^2*x*(637*g - 408*h*x
) - 6*f^3*x^2*(49*g + 11*h*x) + e^2*f*(-14*g + 82*h*x)) - 21*c^3*d^2*f*(8*
e^2*h + 14*e*f*(5*g - 16*h*x) + f^2*x*(-175*g + 88*h*x))) + 35*a^2*b*d^2*f
^2*(-105*c^3*f*h + 5*c^2*d*(9*f*g + 10*e*h - 35*f*h*x) + 4*d^3*x*(2*f*x*(3
*g + h*x) + e*(-3*g + 8*h*x)) + c*d^2*(f*x*(75*g - 56*h*x) + e*(-6*g + 88*
h*x)))))/(140*d^6*f^2*(c + d*x)^2) + (3*(-(b*c) + a*d)*(a^2*d^2*f*(d*f*g +
 4*d*e*h - 5*c*f*h) + b^2*(8*d^3*e^2*g - 33*c^3*f^2*h - 4*c*d^2*e*(7*f*g +
 4*e*h) + 3*c^2*d*f*(7*f*g + 16*e*h)) + 2*a*b*d*(15*c^2*f^2*h + 2*d^2*e*(3
*f*g + 2*e*h) - c*d*f*(7*f*g + 18*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sq
rt[-(d*e) + c*f]])/(4*d^(13/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {166, 27, 166, 27, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)^2(e+fx)^{3/2}(6be(dg-ch)+a(dfg+4deh-5cfh)+b(7dfg+4deh-11cfh)x)}{2(c+dx)^2}dx}{2d(de-cf)}-\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)^2(e+fx)^{3/2}(6be(dg-ch)+a(dfg+4deh-5cfh)+b(7dfg+4deh-11cfh)x)}{(c+dx)^2}dx}{4d(de-cf)} - \frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 166

$$\frac{\frac{\int \frac{(a+bx)(e+fx)^{3/2}\left(2ad(be-af)(dfg+4deh-5cfh)+(4be+5af)\left(ad(dfg+4deh-5cfh)+b\left(11fhc^2-d(7fg+10eh)c+6d^2eg\right)\right)+b\left(7adf(dfg+4deh-5cfh)+b\left(8e(7fg\right.\right.}{2(c+dx)}}{d(de-cf)}}{4d(de-cf)}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)(e+fx)^{3/2}\left(2ad(be-af)(dfg+4deh-5cfh)+(4be+5af)\left(ad(dfg+4deh-5cfh)+b\left(11fhc^2-d(7fg+10eh)c+6d^2eg\right)\right)+b\left(7adf(dfg+4deh-5cfh)+b\left(8e(7fg\right.\right.}{c+dx}}{2d(de-cf)}}{4d(de-cf)}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 164

$$\frac{2b(e+fx)^{5/2}\left(70a^2d^2f^2(-5cfh+4deh+dfg)+5bdfx\left(7adf(-5cfh+4deh+dfg)+b\left(99c^2f^2h-cdf(100eh+63fg)+8d^2e(eh+7fg)\right)\right)+21abdf\left(63c^2f^2h-cdf(66eh+35\right.\right.}{35d^2f^2}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 60

$$\frac{2b(e+fx)^{5/2}\left(70a^2d^2f^2(-5cfh+4deh+dfg)+5bdfx\left(7adf(-5cfh+4deh+dfg)+b\left(99c^2f^2h-cdf(100eh+63fg)+8d^2e(eh+7fg)\right)\right)+21abdf\left(63c^2f^2h-cdf(66eh+35\right.\right.}{35d^2f^2}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 60

$$\frac{\frac{2b(e+fx)^{5/2}\left(70a^2d^2f^2(-5cfh+4deh+dfg)+5bdfx\left(7adf(-5cfh+4deh+dfg)+b\left(99c^2f^2h-cdf(100eh+63fg)+8d^2e(eh+7fg)\right)\right)+21abdf\left(63c^2f^2h-cdf(66eh+35\right.\right.}{35d^2f^2}}{}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2b(e+fx)^{5/2}\left(70a^2d^2f^2(-5cfh+4deh+dfg)+5bdfx\left(7adf(-5cfh+4deh+dfg)+b\left(99c^2f^2h-cdf(100eh+63fg)+8d^2e(eh+7fg)\right)\right)+21abdf\left(63c^2f^2h-cdf(66eh+35\right.\right.}{35d^2f^2}}{}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2b(e+fx)^{5/2}\left(70a^2d^2f^2(-5cfh+4deh+dfg)+5bdfx\left(7adf(-5cfh+4deh+dfg)+b\left(99c^2f^2h-cdf(100eh+63fg)+8d^2e(eh+7fg)\right)\right)+21abdf\left(63c^2f^2h-cdf(66eh+35\right.\right.}{35d^2f^2}}{}$$

$$\frac{(a+bx)^3(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((a + b*x)^3*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^3*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x)^2)
 + (-(((a*d*(d*f*g + 4*d*e*h - 5*c*f*h) + b*(6*d^2*e*g + 11*c^2*f*h - c*d*
(7*f*g + 10*e*h)))*(a + b*x)^2*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x)))
 + ((2*b*(e + f*x)^(5/2)*(70*a^2*d^2*f^2*(d*f*g + 4*d*e*h - 5*c*f*h) + 21*
a*b*d*f*(63*c^2*f^2*h + 2*d^2*e*(15*f*g + 4*e*h) - c*d*f*(35*f*g + 66*e*h)
) - b^2*(693*c^3*f^3*h - 8*d^3*e^2*(7*f*g - 2*e*h) + 2*c*d^2*e*f*(231*f*g
+ 68*e*h) - 9*c^2*d*f^2*(49*f*g + 90*e*h)) + 5*b*d*f*(7*a*d*f*(d*f*g + 4*d
*e*h - 5*c*f*h) + b*(99*c^2*f^2*h + 8*d^2*e*(7*f*g + e*h) - c*d*f*(63*f*g
+ 100*e*h)))*x))/(35*d^2*f^2) - (3*(b*c - a*d)*(a^2*d^2*f*(d*f*g + 4*d*e*h
 - 5*c*f*h) + b^2*(8*d^3*e^2*g - 33*c^3*f^2*h - 4*c*d^2*e*(7*f*g + 4*e*h)
+ 3*c^2*d*f*(7*f*g + 16*e*h)) + 2*a*b*d*(15*c^2*f^2*h + 2*d^2*e*(3*f*g + 2
*e*h) - c*d*f*(7*f*g + 18*e*h)))*((2*(e + f*x)^(3/2))/(3*d) + ((d*e - c*f)
*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])
/Sqrt[d*e - c*f]])/d^(3/2)))/d))/d^2)/(2*d*(d*e - c*f)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$-\frac{\frac{15(ad-bc)(xd+c)^2 f^2\left(\frac{\left(-a^2 f^2 g-4ae(ah+3bg)f-8b\,e^2(ah+bg)\right)d^3}{5}+c\left(a\left(ah+\frac{14bg}{5}\right)f^2+\frac{36\left(ah+\frac{7bg}{9}\right)bef}{5}+\frac{16b^2e^2h}{5}\right)d^2-6c^2}{4}}{}$
risch	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
15/4/((c*f-d*e)*d)^(1/2)*(-(a*d-b*c)*(d*x+c)^2*f^2*(1/5*(-a^2*f^2*g-4*a*e*
(a*h+3*b*g)*f-8*b*e^2*(a*h+b*g))*d^3+c*(a*(a*h+14/5*b*g)*f^2+36/5*(a*h+7/9
*b*g)*b*e*f+16/5*b^2*e^2*h)*d^2-6*c^2*((a*h+7/10*b*g)*f+8/5*e*h*b)*b*f*d+3
3/5*b^2*c^3*f^2*h)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*
d)^(1/2)*(f*x+e)^(1/2)*((-1/3*(-8/25*x^3*(5/7*h*x+g)*b^3-8/5*a*x^2*(3/5*h*
x+g)*b^2-24/5*a^2*x*(1/3*h*x+g)*b+a^3*(-8/5*h*x+g))*x*f^3-2/15*(-8/5*x^3*(
4/7*h*x+g)*b^3-16*a*x^2*(3/10*h*x+g)*b^2+6*a^2*x*(-8/3*h*x+g)*b+a^3*(2*h*x
+g))*e*f^2+8/25*x^2*(1/3*(1/7*h*x+g)*b+a*h)*b^2*e^2*f-16/525*b^3*e^3*h*x^2
)*d^5-2/15*c*((12/5*x^3*(11/21*h*x+g)*b^3+28*a*x^2*(9/35*h*x+g)*b^2-75/2*(
-56/75*h*x+g)*a^2*x*b+3/2*a^3*(-25/3*h*x+g))*f^3+(64/5*x^2*(13/56*h*x+g)*b
^3-44*a*x*(-48/55*h*x+g)*b^2+3*a^2*(-44/3*h*x+g)*b+h*a^3)*e*f^2-24/5*x*(1/
3*(-19/14*h*x+g)*b+a*h)*b^2*e^2*f+16/35*b^3*e^3*h*x)*d^4+c^2*((56/25*x^2*(
11/49*h*x+g)*b^3-35/3*a*x*(-72/125*h*x+g)*b^2+3*a^2*(-35/9*h*x+g)*b+h*a^3)
*f^3+10/3*b*(-182/125*x*(-408/637*h*x+g)*b^2+a*(-546/125*h*x+g)*b+a^2*h)*e
*f^2+8/25*(1/3*(-41/7*h*x+g)*b+a*h)*b^2*e^2*f-16/525*b^3*e^3*h)*d^3-7*c^3*
(((88/175*h*x^2-g*x)*b^2+a*(-3*h*x+g)*b+a^2*h)*f^2+6/5*b*(1/3*(-16/5*h*x+g
)*b+a*h)*e*f+8/175*b^2*e^2*h)*b*f*d^2+63/5*c^4*((1/3*(-55/21*h*x+g)*b+a*h)
*f+26/63*e*h*b)*b^2*f^2*d-33/5*b^3*c^5*f^3*h))/d^6/(d*x+c)^2/f^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2275 vs. $2(462) = 924$.

Time = 0.30 (sec) , antiderivative size = 4563, normalized size of antiderivative = 9.24

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(f*x+e)**(3/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(462) = 924$.

Time = 0.19 (sec) , antiderivative size = 1617, normalized size of antiderivative = 3.27

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-3/4*(8*b^3*c*d^3*e^2*g - 8*a*b^2*d^4*e^2*g - 28*b^3*c^2*d^2*e*f*g + 40*a*
b^2*c*d^3*e*f*g - 12*a^2*b*d^4*e*f*g + 21*b^3*c^3*d*f^2*g - 35*a*b^2*c^2*d
^2*f^2*g + 15*a^2*b*c*d^3*f^2*g - a^3*d^4*f^2*g - 16*b^3*c^2*d^2*e^2*h + 2
4*a*b^2*c*d^3*e^2*h - 8*a^2*b*d^4*e^2*h + 48*b^3*c^3*d*e*f*h - 84*a*b^2*c^
2*d^2*e*f*h + 40*a^2*b*c*d^3*e*f*h - 4*a^3*d^4*e*f*h - 33*b^3*c^4*f^2*h +
63*a*b^2*c^3*d*f^2*h - 35*a^2*b*c^2*d^2*f^2*h + 5*a^3*c*d^3*f^2*h)*arctan(
sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^6) - 1/4*(12
*(f*x + e)^(3/2)*b^3*c^2*d^3*e*f*g - 24*(f*x + e)^(3/2)*a*b^2*c*d^4*e*f*g
+ 12*(f*x + e)^(3/2)*a^2*b*d^5*e*f*g - 12*sqrt(f*x + e)*b^3*c^2*d^3*e^2*f*
g + 24*sqrt(f*x + e)*a*b^2*c*d^4*e^2*f*g - 12*sqrt(f*x + e)*a^2*b*d^5*e^2*
f*g - 17*(f*x + e)^(3/2)*b^3*c^3*d^2*f^2*g + 39*(f*x + e)^(3/2)*a*b^2*c^2*
d^3*f^2*g - 27*(f*x + e)^(3/2)*a^2*b*c*d^4*f^2*g + 5*(f*x + e)^(3/2)*a^3*d
^5*f^2*g + 27*sqrt(f*x + e)*b^3*c^3*d^2*e*f^2*g - 57*sqrt(f*x + e)*a*b^2*c
^2*d^3*e*f^2*g + 33*sqrt(f*x + e)*a^2*b*c*d^4*e*f^2*g - 3*sqrt(f*x + e)*a^
3*d^5*e*f^2*g - 15*sqrt(f*x + e)*b^3*c^4*d*f^3*g + 33*sqrt(f*x + e)*a*b^2*
c^3*d^2*f^3*g - 21*sqrt(f*x + e)*a^2*b*c^2*d^3*f^3*g + 3*sqrt(f*x + e)*a^3
*c*d^4*f^3*g - 16*(f*x + e)^(3/2)*b^3*c^3*d^2*e*f*h + 36*(f*x + e)^(3/2)*a
*b^2*c^2*d^3*e*f*h - 24*(f*x + e)^(3/2)*a^2*b*c*d^4*e*f*h + 4*(f*x + e)^(3
/2)*a^3*d^5*e*f*h + 16*sqrt(f*x + e)*b^3*c^3*d^2*e^2*f*h - 36*sqrt(f*x + e
)*a*b^2*c^2*d^3*e^2*f*h + 24*sqrt(f*x + e)*a^2*b*c*d^4*e^2*f*h - 4*sqrt...
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 1753, normalized size of antiderivative = 3.55

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^3)/(c + d*x)^3,x)
```

output
```
(e + f*x)^(5/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(5*d^3*f^2) - (6*b^
3*h*(c*f - d*e))/(5*d^4*f^2)) - ((e + f*x)^(3/2)*((5*a^3*d^5*f^2*g)/4 + a^
3*d^5*e*f*h - (9*a^3*c*d^4*f^2*h)/4 + (21*b^3*c^4*d*f^2*h)/4 - (17*b^3*c^3
*d^2*f^2*g)/4 + (39*a*b^2*c^2*d^3*f^2*g)/4 - (51*a*b^2*c^3*d^2*f^2*h)/4 +
(39*a^2*b*c^2*d^3*f^2*h)/4 + 3*a^2*b*d^5*e*f*g - (27*a^2*b*c*d^4*f^2*g)/4
+ 3*b^3*c^2*d^3*e*f*g - 4*b^3*c^3*d^2*e*f*h + 9*a*b^2*c^2*d^3*e*f*h - 6*a*
b^2*c*d^4*e*f*g - 6*a^2*b*c*d^4*e*f*h) - (e + f*x)^(1/2)*((15*b^3*c^4*d*f^
3*g)/4 - (3*a^3*c*d^4*f^3*g)/4 - (19*b^3*c^5*f^3*h)/4 + (3*a^3*d^5*e*f^2*g
)/4 + a^3*d^5*e^2*f*h + (7*a^3*c^2*d^3*f^3*h)/4 - (33*a*b^2*c^3*d^2*f^3*g)
/4 + (21*a^2*b*c^2*d^3*f^3*g)/4 - (33*a^2*b*c^3*d^2*f^3*h)/4 + 3*b^3*c^2*d
^3*e^2*f*g - (27*b^3*c^3*d^2*e*f^2*g)/4 - 4*b^3*c^3*d^2*e^2*f*h + (45*a*b^
2*c^4*d*f^3*h)/4 + 3*a^2*b*d^5*e^2*f*g - (11*a^3*c*d^4*e*f^2*h)/4 + (35*b^
3*c^4*d*e*f^2*h)/4 - 6*a*b^2*c*d^4*e^2*f*g - (33*a^2*b*c*d^4*e*f^2*g)/4 -
6*a^2*b*c*d^4*e^2*f*h + (57*a*b^2*c^2*d^3*e*f^2*g)/4 + 9*a*b^2*c^2*d^3*e^2
*f*h - (81*a*b^2*c^3*d^2*e*f^2*h)/4 + (57*a^2*b*c^2*d^3*e*f^2*h)/4))/(d^8*
(e + f*x)^2 - (e + f*x)*(2*d^8*e - 2*c*d^7*f) + d^8*e^2 + c^2*d^6*f^2 - 2*
c*d^7*e*f) - (e + f*x)^(3/2)*((((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^3
*f^2) - (6*b^3*h*(c*f - d*e))/(d^4*f^2))*(c*f - d*e))/d - (2*b*(a*f - b*e)
*(a*f*h - 2*b*e*h + b*f*g))/(d^3*f^2) + (2*b^3*h*(c*f - d*e)^2)/(d^5*f^2))
 + (e + f*x)^(1/2)*((3*(c*f - d*e)*((3*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^...
```

Reduce [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 5520, normalized size of antiderivative = 11.17

$$\int \frac{(a+bx)^3(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x)
```

output

```
( - 525*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*a**3*c**3*d**3*f**4*h + 420*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*e*f**3*h + 105*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**
2*d**4*f**4*g - 1050*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*f**4*h*x + 840*sqrt(d)*sqrt(c*f - d*e)
*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**5*e*f**3*h*x
+ 210*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*a**3*c*d**5*f**4*g*x - 525*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**5*f**4*h*x**2 + 420*sqrt(d)*sqr
t(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**6*e
*f**3*h*x**2 + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*a**3*d**6*f**4*g*x**2 + 3675*sqrt(d)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**4*d**2*f**4*h -
4200*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*
e)))*a**2*b*c**3*d**3*e*f**3*h - 1575*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**3*f**4*g + 7350*sqrt(
d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*
b*c**3*d**3*f**4*h*x + 840*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**4*e**2*f**2*h + 1260*sqrt(d)*...
```

3.97 $\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 423

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{2(a^2d^2fh+2abd(dfg+deh-3cfh)+b^2(d^2eg+6c^2fh-3cd(fg+}{d^5}$$
$$+\frac{(bc-ad)\,(ad(3dfg+4deh-7cfh)+b(8d^2eg+15c^2fh-cd(11fg+12eh)))\sqrt{e+fx}}{4d^5(c+dx)}$$
$$+\frac{2b(bdg-3bch+2adh)(e+fx)^{3/2}}{3d^4}-\frac{(bc-ad)^2(dg-ch)(e+fx)^{3/2}}{2d^4(c+dx)^2}+\frac{2b^2h(e+fx)^{5/2}}{5d^3f}$$
$$-\frac{(3a^2d^2f(dfg+4deh-5cfh)+2abd(35c^2f^2h+4d^2e(3fg+2eh)-5cdf(3fg+8eh))+b^2(8d^3e^2g-63c}{4d^{11/2}\sqrt{de-cf}}$$

output
```
2*(a^2*d^2*f*h+2*a*b*d*(-3*c*f*h+d*e*h+d*f*g)+b^2*(d^2*e*g+6*c^2*f*h-3*c*d
*(e*h+f*g)))*(f*x+e)^(1/2)/d^5+1/4*(-a*d+b*c)*(a*d*(-7*c*f*h+4*d*e*h+3*d*f
*g)+b*(8*d^2*e*g+15*c^2*f*h-c*d*(12*e*h+11*f*g)))*(f*x+e)^(1/2)/d^5/(d*x+c
)+2/3*b*(2*a*d*h-3*b*c*h+b*d*g)*(f*x+e)^(3/2)/d^4-1/2*(-a*d+b*c)^2*(-c*h+d
*g)*(f*x+e)^(3/2)/d^4/(d*x+c)^2+2/5*b^2*h*(f*x+e)^(5/2)/d^3/f-1/4*(3*a^2*d
^2*f*(-5*c*f*h+4*d*e*h+d*f*g)+2*a*b*d*(35*c^2*f^2*h+4*d^2*e*(2*e*h+3*f*g)-
5*c*d*f*(8*e*h+3*f*g))+b^2*(8*d^3*e^2*g-63*c^3*f^2*h-8*c*d^2*e*(3*e*h+5*f*
g)+7*c^2*d*f*(12*e*h+5*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/
2))/d^(11/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{\sqrt{e+fx}(10abdf(-105c^3fh+5c^2d(9fg+10eh-35fhx)+cd^2(-6\cdots}{\cdots} + \frac{(3a^2d^2f(dfg+4deh-5cfh)+2abd(35c^2f^2h+4d^2e(3fg+2eh)-5cdf(3fg+8eh))+b^2(8d^3e^2g-63c\cdots}{4d^{11/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)^2*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(10*a*b*d*f*(-105*c^3*f*h + 5*c^2*d*(9*f*g + 10*e*h - 35*f*
h*x) + c*d^2*(-6*e*g + 75*f*g*x + 88*e*h*x - 56*f*h*x^2) + 4*d^3*x*(-3*e*g
 + 6*f*g*x + 8*e*h*x + 2*f*h*x^2)) - 15*a^2*d^2*f*(-15*c^2*f*h + c*d*(3*f*
g + 2*e*h - 25*f*h*x) + d^2*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x))) + b^2*(
945*c^4*f^2*h - 105*c^3*d*f*(6*e*h + 5*f*(g - 3*h*x)) + 8*d^4*x^2*(3*e^2*h
 + f^2*x*(5*g + 3*h*x) + e*f*(20*g + 6*h*x)) - 8*c*d^3*x*(-6*e^2*h + f^2*x
*(35*g + 9*h*x) + e*f*(-55*g + 48*h*x)) + c^2*d^2*(24*e^2*h + 2*e*f*(125*g
 - 546*h*x) + 7*f^2*x*(-125*g + 72*h*x)))))/(60*d^5*f*(c + d*x)^2) + ((3*a
^2*d^2*f*(d*f*g + 4*d*e*h - 5*c*f*h) + 2*a*b*d*(35*c^2*f^2*h + 4*d^2*e*(3*
f*g + 2*e*h) - 5*c*d*f*(3*f*g + 8*e*h)) + b^2*(8*d^3*e^2*g - 63*c^3*f^2*h
- 8*c*d^2*e*(5*f*g + 3*e*h) + 7*c^2*d*f*(5*f*g + 12*e*h)))*ArcTan[(Sqrt[d]
*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(4*d^(11/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 163, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)(e+fx)^{3/2}(4be(dg-ch)+a(dfg+4deh-5cfh)+b(5dfg+4deh-9cfh)x)}{2(c+dx)^2}dx}{2d(de-cf)}-\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)(e+fx)^{3/2}(4be(dg-ch)+a(dfg+4deh-5cfh)+b(5dfg+4deh-9cfh)x)}{(c+dx)^2}dx}{4d(de-cf)}-\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 163$$

$$\frac{\frac{(3a^2d^2f(-5cfh+4deh+dfg)+2abd(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg))+b^2(-63c^3f^2h+7c^2df(12eh+5fg)-8cd^2e(3eh+5fg)+8d^3e^2g)}{2d^2(de-cf)}}{}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(3a^2d^2f(-5cfh+4deh+dfg)+2abd(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg))+b^2(-63c^3f^2h+7c^2df(12eh+5fg)-8cd^2e(3eh+5fg)+8d^3e^2g)}{2d^2(de-cf)}}{}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(3a^2d^2f(-5cfh+4deh+dfg)+2abd(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg))+b^2(-63c^3f^2h+7c^2df(12eh+5fg)-8cd^2e(3eh+5fg)+8d^3e^2g)}{2d^2(de-cf)}}{}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{(3a^2d^2f(-5cfh+4deh+dfg)+2abd(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg))+b^2(-63c^3f^2h+7c^2df(12eh+5fg)-8cd^2e(3eh+5fg)+8d^3e^2g)}{2d^2(de-cf)}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\big\downarrow$ 221

$$\frac{\left(\frac{(de-cf)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)(3a^2d^2f(-5cfh+4deh+dfg)+2abd(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh}{2d^2(de-cf)}$$

$$\frac{(a+bx)^2(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((a + b*x)^2*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^2*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x)^2)
 + (-1/5*((e + f*x)^(5/2)*(5*a^2*d^2*f*(d*f*g + 4*d*e*h - 5*c*f*h) + 10*a*
b*d*f*(2*d^2*e*g + 7*c^2*f*h - 3*c*d*(f*g + 2*e*h)) - b^2*c*(63*c^2*f^2*h
+ 2*d^2*e*(15*f*g + 4*e*h) - c*d*f*(35*f*g + 66*e*h)) - 2*b^2*d*(d*e - c*f
)*(5*d*f*g + 4*d*e*h - 9*c*f*h)*x))/(d^2*f*(d*e - c*f)*(c + d*x)) + ((3*a^
2*d^2*f*(d*f*g + 4*d*e*h - 5*c*f*h) + 2*a*b*d*(35*c^2*f^2*h + 4*d^2*e*(3*f
*g + 2*e*h) - 5*c*d*f*(3*f*g + 8*e*h)) + b^2*(8*d^3*e^2*g - 63*c^3*f^2*h -
 8*c*d^2*e*(5*f*g + 3*e*h) + 7*c^2*d*f*(5*f*g + 12*e*h)))*((2*(e + f*x)^(3
/2))/(3*d) + ((d*e - c*f)*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*ArcTan
h[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d))/(2*d^2*(d*e - c*
f)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$-\frac{15(xd+c)^2 f\left(\left(\left(-\frac{a^2f^2g}{5}-\frac{4ae(ah+2bg)f}{5}-\frac{16be^2\left(ah+\frac{bg}{2}\right)}{15}\right)d^3+\left(\left(a^2h+2gab\right)f^2+\frac{16\left(ah+\frac{bg}{2}\right)bef}{3}+\frac{8b^2e^2h}{5}\right)c\,d^2-\frac{14c^2\left(\left(ah\right.\right.}{}\right.\right.}{4}$
risch	$\frac{2\left(3x^2hb^2d^2f^2+10abd^2f^2hx-15b^2cdf^2hx+6b^2d^2efhx+5b^2d^2f^2gx+15a^2d^2f^2h-90abcdf^2h+40abd^2efh+30abd^2\right.}{15fd^5}$
derivativedivides	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+\frac{2abd^2fh(fx+e)^{\frac{3}{2}}}{3}-b^2cdfh(fx+e)^{\frac{3}{2}}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+a^2d^2f^2h\sqrt{fx+e}-6abcdf^2h\sqrt{fx+e}+2abd^2efh\sqrt{fx\right.}{d^5}$
default	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+\frac{2abd^2fh(fx+e)^{\frac{3}{2}}}{3}-b^2cdfh(fx+e)^{\frac{3}{2}}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+a^2d^2f^2h\sqrt{fx+e}-6abcdf^2h\sqrt{fx+e}+2abd^2efh\sqrt{fx\right.}{d^5}$

input
```
int((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
15/4/((c*f-d*e)*d)^(1/2)*(-(d*x+c)^2*f*((-1/5*a^2*f^2*g-4/5*a*e*(a*h+2*b*g
)*f-16/15*b*e^2*(a*h+1/2*b*g))*d^3+((a^2*h+2*a*b*g)*f^2+16/3*(a*h+1/2*b*g)
*b*e*f+8/5*b^2*e^2*h)*c*d^2-14/3*c^2*((a*h+1/2*b*g)*f+6/5*e*h*b)*b*f*d+21/
5*b^2*c^3*f^2*h)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)
^(1/2)*(f*x+e)^(1/2)*((-1/3*x*(-8/15*x^2*(3/5*h*x+g)*b^2-16/5*a*x*(1/3*h*x
+g)*b+a^2*(-8/5*h*x+g))*f^2-2/15*((-8/5*h*x^3-16/3*g*x^2)*b^2+4*a*x*(-8/3*
h*x+g)*b+a^2*(2*h*x+g))*e*f+8/75*b^2*e^2*h*x^2)*d^4-2/15*c*((28/3*x^2*(9/3
5*h*x+g)*b^2-25*(-56/75*h*x+g)*a*x*b+3/2*a^2*(-25/3*h*x+g))*f^2+e*((64/5*h
*x^2-44/3*g*x)*b^2+2*a*(-44/3*h*x+g)*b+a^2*h)*f-8/5*b^2*e^2*h*x)*d^3+(((56
/25*h*x^2-35/9*g*x)*b^2+2*a*(-35/9*h*x+g)*b+a^2*h)*f^2+20/9*((-273/125*h*x
+1/2*g)*b+a*h)*b*e*f+8/75*b^2*e^2*h)*c^2*d^2-14/3*(((-3/2*h*x+1/2*g)*b+a*h
)*f+3/5*e*h*b)*c^3*b*f*d+21/5*b^2*c^4*f^2*h))/d^5/(d*x+c)^2/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(393) = 786$.

Time = 0.26 (sec) , antiderivative size = 2866, normalized size of antiderivative = 6.78

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output

```
[-1/120*(15*sqrt(d^2*e - c*d*f)*(((8*b^2*d^5*e^2*f - 8*(5*b^2*c*d^4 - 3*a*
b*d^5)*e*f^2 + (35*b^2*c^2*d^3 - 30*a*b*c*d^4 + 3*a^2*d^5)*f^3)*g - (8*(3*
b^2*c*d^4 - 2*a*b*d^5)*e^2*f - 4*(21*b^2*c^2*d^3 - 20*a*b*c*d^4 + 3*a^2*d^
5)*e*f^2 + (63*b^2*c^3*d^2 - 70*a*b*c^2*d^3 + 15*a^2*c*d^4)*f^3)*h)*x^2 +
(8*b^2*c^2*d^3*e^2*f - 8*(5*b^2*c^3*d^2 - 3*a*b*c^2*d^3)*e*f^2 + (35*b^2*c
^4*d - 30*a*b*c^3*d^2 + 3*a^2*c^2*d^3)*f^3)*g - (8*(3*b^2*c^3*d^2 - 2*a*b*
c^2*d^3)*e^2*f - 4*(21*b^2*c^4*d - 20*a*b*c^3*d^2 + 3*a^2*c^2*d^3)*e*f^2 +
 (63*b^2*c^5 - 70*a*b*c^4*d + 15*a^2*c^3*d^2)*f^3)*h + 2*((8*b^2*c*d^4*e^2
*f - 8*(5*b^2*c^2*d^3 - 3*a*b*c*d^4)*e*f^2 + (35*b^2*c^3*d^2 - 30*a*b*c^2*
d^3 + 3*a^2*c*d^4)*f^3)*g - (8*(3*b^2*c^2*d^3 - 2*a*b*c*d^4)*e^2*f - 4*(21
*b^2*c^3*d^2 - 20*a*b*c^2*d^3 + 3*a^2*c*d^4)*e*f^2 + (63*b^2*c^4*d - 70*a*
b*c^3*d^2 + 15*a^2*c^2*d^3)*f^3)*h)*x)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d
^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(24*(b^2*d^6*e*f^2 - b^2*c*d^5
*f^3)*h*x^4 + 8*(5*(b^2*d^6*e*f^2 - b^2*c*d^5*f^3)*g + (6*b^2*d^6*e^2*f -
5*(3*b^2*c*d^5 - 2*a*b*d^6)*e*f^2 + (9*b^2*c^2*d^4 - 10*a*b*c*d^5)*f^3)*h)
*x^3 + 8*(5*(4*b^2*d^6*e^2*f - (11*b^2*c*d^5 - 6*a*b*d^6)*e*f^2 + (7*b^2*c
^2*d^4 - 6*a*b*c*d^5)*f^3)*g + (3*b^2*d^6*e^3 - (51*b^2*c*d^5 - 40*a*b*d^6
)*e^2*f + (111*b^2*c^2*d^4 - 110*a*b*c*d^5 + 15*a^2*d^6)*e*f^2 - (63*b^2*c
^3*d^3 - 70*a*b*c^2*d^4 + 15*a^2*c*d^5)*f^3)*h)*x^2 + 5*(2*(25*b^2*c^2*d^4
 - 6*a*b*c*d^5 - 3*a^2*d^6)*e^2*f - (155*b^2*c^3*d^3 - 102*a*b*c^2*d^4 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(f*x+e)**(3/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(393) = 786$.

Time = 0.16 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(8*b^2*d^3*e^2*g - 40*b^2*c*d^2*e*f*g + 24*a*b*d^3*e*f*g + 35*b^2*c^2*
d*f^2*g - 30*a*b*c*d^2*f^2*g + 3*a^2*d^3*f^2*g - 24*b^2*c*d^2*e^2*h + 16*a
*b*d^3*e^2*h + 84*b^2*c^2*d*e*f*h - 80*a*b*c*d^2*e*f*h + 12*a^2*d^3*e*f*h
- 63*b^2*c^3*f^2*h + 70*a*b*c^2*d*f^2*h - 15*a^2*c*d^2*f^2*h)*arctan(sqrt(
f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^5) + 1/4*(8*(f*x
+ e)^(3/2)*b^2*c*d^3*e*f*g - 8*(f*x + e)^(3/2)*a*b*d^4*e*f*g - 8*sqrt(f*x
+ e)*b^2*c*d^3*e^2*f*g + 8*sqrt(f*x + e)*a*b*d^4*e^2*f*g - 13*(f*x + e)^(3
/2)*b^2*c^2*d^2*f^2*g + 18*(f*x + e)^(3/2)*a*b*c*d^3*f^2*g - 5*(f*x + e)^(
3/2)*a^2*d^4*f^2*g + 19*sqrt(f*x + e)*b^2*c^2*d^2*e*f^2*g - 22*sqrt(f*x +
e)*a*b*c*d^3*e*f^2*g + 3*sqrt(f*x + e)*a^2*d^4*e*f^2*g - 11*sqrt(f*x + e)*
b^2*c^3*d*f^3*g + 14*sqrt(f*x + e)*a*b*c^2*d^2*f^3*g - 3*sqrt(f*x + e)*a^2
*c*d^3*f^3*g - 12*(f*x + e)^(3/2)*b^2*c^2*d^2*e*f*h + 16*(f*x + e)^(3/2)*a
*b*c*d^3*e*f*h - 4*(f*x + e)^(3/2)*a^2*d^4*e*f*h + 12*sqrt(f*x + e)*b^2*c^
2*d^2*e^2*f*h - 16*sqrt(f*x + e)*a*b*c*d^3*e^2*f*h + 4*sqrt(f*x + e)*a^2*d
^4*e^2*f*h + 17*(f*x + e)^(3/2)*b^2*c^3*d*f^2*h - 26*(f*x + e)^(3/2)*a*b*c
^2*d^2*f^2*h + 9*(f*x + e)^(3/2)*a^2*c*d^3*f^2*h - 27*sqrt(f*x + e)*b^2*c^
3*d*e*f^2*h + 38*sqrt(f*x + e)*a*b*c^2*d^2*e*f^2*h - 11*sqrt(f*x + e)*a^2*
c*d^3*e*f^2*h + 15*sqrt(f*x + e)*b^2*c^4*f^3*h - 22*sqrt(f*x + e)*a*b*c^3*
d*f^3*h + 7*sqrt(f*x + e)*a^2*c^2*d^2*f^3*h)/(((f*x + e)*d - d*e + c*f)^2*
d^5) + 2/15*(5*(f*x + e)^(3/2)*b^2*d^12*f^5*g + 15*sqrt(f*x + e)*b^2*d^...
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x)^2)/(c + d*x)^3,x)
```

output
```
(e + f*x)^(3/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(3*d^3*f) - (2*b^2*h*
(c*f - d*e))/(d^4*f)) - (e + f*x)^(1/2)*((3*((2*b^2*f*g - 6*b^2*e*h + 4*a*
b*f*h)/(d^3*f) - (6*b^2*h*(c*f - d*e))/(d^4*f))*(c*f - d*e))/d - (2*(a*f -
 b*e)*(a*f*h - 3*b*e*h + 2*b*f*g))/(d^3*f) + (6*b^2*h*(c*f - d*e)^2)/(d^5*
f)) - ((e + f*x)^(3/2)*((5*a^2*d^4*f^2*g)/4 + a^2*d^4*e*f*h - (9*a^2*c*d^3
*f^2*h)/4 - (17*b^2*c^3*d*f^2*h)/4 + (13*b^2*c^2*d^2*f^2*g)/4 + 2*a*b*d^4*
e*f*g - (9*a*b*c*d^3*f^2*g)/2 - 2*b^2*c*d^3*e*f*g + (13*a*b*c^2*d^2*f^2*h)
/2 + 3*b^2*c^2*d^2*e*f*h - 4*a*b*c*d^3*e*f*h) - (e + f*x)^(1/2)*((15*b^2*c
^4*f^3*h)/4 - (3*a^2*c*d^3*f^3*g)/4 - (11*b^2*c^3*d*f^3*g)/4 + (3*a^2*d^4*
e*f^2*g)/4 + a^2*d^4*e^2*f*h + (7*a^2*c^2*d^2*f^3*h)/4 + (19*b^2*c^2*d^2*e
*f^2*g)/4 + 3*b^2*c^2*d^2*e^2*f*h - (11*a*b*c^3*d*f^3*h)/2 + 2*a*b*d^4*e^2
*f*g + (7*a*b*c^2*d^2*f^3*g)/2 - (11*a^2*c*d^3*e*f^2*h)/4 - 2*b^2*c*d^3*e^
2*f*g - (27*b^2*c^3*d*e*f^2*h)/4 + (19*a*b*c^2*d^2*e*f^2*h)/2 - (11*a*b*c*
d^3*e*f^2*g)/2 - 4*a*b*c*d^3*e^2*f*h))/(d^7*(e + f*x)^2 - (e + f*x)*(2*d^7
*e - 2*c*d^6*f) + d^7*e^2 + c^2*d^5*f^2 - 2*c*d^6*e*f) + (atan((d^(1/2)*(e
 + f*x)^(1/2))/(c*f - d*e)^(1/2))*(3*a^2*d^3*f^2*g + 8*b^2*d^3*e^2*g - 63*
b^2*c^3*f^2*h + 16*a*b*d^3*e^2*h + 12*a^2*d^3*e*f*h - 15*a^2*c*d^2*f^2*h -
 24*b^2*c*d^2*e^2*h + 35*b^2*c^2*d*f^2*g + 24*a*b*d^3*e*f*g - 30*a*b*c*d^2
*f^2*g + 70*a*b*c^2*d*f^2*h - 40*b^2*c*d^2*e*f*g + 84*b^2*c^2*d*e*f*h - 80
*a*b*c*d^2*e*f*h))/(4*d^(11/2)*(c*f - d*e)^(1/2)) + (2*b^2*h*(e + f*x)^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3464, normalized size of antiderivative = 8.19

$$\int \frac{(a+bx)^2(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x)
```

output
```
( - 225*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*a**2*c**3*d**2*f**3*h + 180*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*e*f**2*h + 45*sqrt(d)*s
qrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2
*d**3*f**3*g - 450*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*a**2*c**2*d**3*f**3*h*x + 360*sqrt(d)*sqrt(c*f - d*e)*a
tan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*e*f**2*h*x +
90*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a**2*c*d**4*f**3*g*x - 225*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*f**3*h*x**2 + 180*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**5*e*f*
*2*h*x**2 + 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*a**2*d**5*f**3*g*x**2 + 1050*sqrt(d)*sqrt(c*f - d*e)*atan((
sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**4*d*f**3*h - 1200*sqrt(
d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c
**3*d**2*e*f**2*h - 450*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sq
rt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*f**3*g + 2100*sqrt(d)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*f**3*h*x
 + 240*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a*b*c**2*d**3*e**2*f*h + 360*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(...
```

3.98 $\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 279

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = -\frac{2(3bcfh-d(bfg+beh+afh))\sqrt{e+fx}}{d^4} - \frac{(ad(3dfg+4deh-7cfh)+b(4d^2eg+11c^2fh-cd(7fg+8eh)))\sqrt{e+fx}}{4d^4(c+dx)} + \frac{2bh(e+fx)^{3/2}}{3d^3} + \frac{(bc-ad)(dg-ch)(e+fx)^{3/2}}{2d^3(c+dx)^2} - \frac{(3adf(dfg+4deh-5cfh)+b(35c^2f^2h+4d^2e(3fg+2eh)-5cdf(3fg+8eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{9/2}\sqrt{de-cf}}$$

output

```
-2*(3*b*c*f*h-d*(a*f*h+b*e*h+b*f*g))*(f*x+e)^(1/2)/d^4-1/4*(a*d*(-7*c*f*h+
4*d*e*h+3*d*f*g)+b*(4*d^2*e*g+11*c^2*f*h-c*d*(8*e*h+7*f*g)))*(f*x+e)^(1/2)
/d^4/(d*x+c)+2/3*b*h*(f*x+e)^(3/2)/d^3+1/2*(-a*d+b*c)*(-c*h+d*g)*(f*x+e)^(
3/2)/d^3/(d*x+c)^2-1/4*(3*a*d*f*(-5*c*f*h+4*d*e*h+d*f*g)+b*(35*c^2*f^2*h+4
*d^2*e*(2*e*h+3*f*g)-5*c*d*f*(8*e*h+3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)
/(-c*f+d*e)^(1/2))/d^(9/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{\sqrt{e+fx}(b(-105c^3fh+5c^2d(9fg+10eh-35fhx)+cd^2(-6eg+7\ldots}{\ldots}$$
$$+\frac{(3adf(dfg+4deh-5cfh)+b(35c^2f^2h+4d^2e(3fg+2eh)-5cdf(3fg+8eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{4d^{9/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
(Sqrt[e + f*x]*(b*(-105*c^3*f*h + 5*c^2*d*(9*f*g + 10*e*h - 35*f*h*x) + c*
d^2*(-6*e*g + 75*f*g*x + 88*e*h*x - 56*f*h*x^2) + 4*d^3*x*(-3*e*g + 6*f*g*
x + 8*e*h*x + 2*f*h*x^2)) - 3*a*d*(-15*c^2*f*h + c*d*(3*f*g + 2*e*h - 25*f
*h*x) + d^2*(f*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x)))))/(12*d^4*(c + d*x)^2)
+ ((3*a*d*f*(d*f*g + 4*d*e*h - 5*c*f*h) + b*(35*c^2*f^2*h + 4*d^2*e*(3*f*g
 + 2*e*h) - 5*c*d*f*(3*f*g + 8*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[
-(d*e) + c*f]])/(4*d^(9/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {162, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{162}$$
$$\frac{(3adf(-5cfh+4deh+dfg)+b(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg)))\int \frac{(e+fx)^{3/2}}{c+dx}dx}{8d^2(de-cf)^2} - \frac{(e+fx)^{5/2}\left(dx\left(ad(-5cfh+4deh+dfg)+b\left(9c^2fh-cd(8eh+5fg)+4d^2eg\right)\right)+ad\left(-3c^2fh-cd(fg-2eh)\ldots\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$\big\downarrow$ 60

$$\frac{\left(3adf(-5cfh+4deh+dfg)+b\left(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg)\right)\right)\left(\frac{(de-cf)\int\frac{\sqrt{e+fx}}{c+dx}dx}{d}+\frac{2(e+fx)^3}{3d}\right.}{8d^2(de-cf)^2}$$
$$\frac{(e+fx)^{5/2}\left(dx\left(ad(-5cfh+4deh+dfg)+b\left(9c^2fh-cd(8eh+5fg)+4d^2eg\right)\right)+ad\left(-3c^2fh-cd(fg-2eh)\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$\big\downarrow$ 60

$$\frac{\left(3adf(-5cfh+4deh+dfg)+b\left(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg)\right)\right)\left(\frac{(de-cf)\left(\frac{(de-cf)\int\frac{1}{(c+dx)\sqrt{e+fx}}d}{d}\right.}{d}\right.}{8d^2(de-cf)^2}$$
$$\frac{(e+fx)^{5/2}\left(dx\left(ad(-5cfh+4deh+dfg)+b\left(9c^2fh-cd(8eh+5fg)+4d^2eg\right)\right)+ad\left(-3c^2fh-cd(fg-2eh)\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$\big\downarrow$ 73

$$\frac{\left(3adf(-5cfh+4deh+dfg)+b\left(35c^2f^2h-5cdf(8eh+3fg)+4d^2e(2eh+3fg)\right)\right)\left(\frac{(de-cf)\left(\frac{2(de-cf)\int\frac{1}{c+\frac{d(e+fx)}{f}-}}{df}\right.}{d}\right.}{8d^2(de-cf)^2}$$
$$\frac{(e+fx)^{5/2}\left(dx\left(ad(-5cfh+4deh+dfg)+b\left(9c^2fh-cd(8eh+5fg)+4d^2eg\right)\right)+ad\left(-3c^2fh-cd(fg-2eh)\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$\big\downarrow$ 221

$$\frac{\left(\frac{(de-cf)\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{d}+\frac{2(e+fx)^{3/2}}{3d}\right)\left(3adf(-5cfh+4deh+dfg)+b\left(35c^2f^2h-5cdf(8e\right.\right.}{8d^2(de-cf)^2}$$
$$\frac{(e+fx)^{5/2}\left(dx\left(ad(-5cfh+4deh+dfg)+b\left(9c^2fh-cd(8eh+5fg)+4d^2eg\right)\right)+ad\left(-3c^2fh-cd(fg-2eh)\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

input
```
Int[((a + b*x)*(e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/4*((e + f*x)^(5/2)*(a*d*(2*d^2*e*g - 3*c^2*f*h - c*d*(f*g - 2*e*h)) + b
*c*(2*d^2*e*g + 7*c^2*f*h - 3*c*d*(f*g + 2*e*h)) + d*(a*d*(d*f*g + 4*d*e*h
 - 5*c*f*h) + b*(4*d^2*e*g + 9*c^2*f*h - c*d*(5*f*g + 8*e*h)))*x))/(d^2*(d
*e - c*f)^2*(c + d*x)^2) + ((3*a*d*f*(d*f*g + 4*d*e*h - 5*c*f*h) + b*(35*c
^2*f^2*h + 4*d^2*e*(3*f*g + 2*e*h) - 5*c*d*f*(3*f*g + 8*e*h)))*((2*(e + f*
x)^(3/2))/(3*d) + ((d*e - c*f)*((2*Sqrt[e + f*x])/d - (2*Sqrt[d*e - c*f]*A
rcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/d^(3/2)))/d))/(8*d^2*(d*e
 - c*f)^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{15\left(\frac{\left(-g f^2 a-4e(ah+bg)f-\frac{8b e^2 h}{3}\right)d^2}{5}+c\left((ah+bg)f+\frac{8ehb}{3}\right)fd-\frac{7b c^2 f^2 h}{3}\right)(xd+c)^2 \arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{4}+ 15\left(\left(-\left(-\frac{8bhx}{15} \ldots\right.\right.\right.$
risch	$\frac{2(hfdbx+3adfh-9bcfh+4behd+3bdfg)\sqrt{fx+e}}{3d^4}-\frac{2\left(-\frac{9}{8}acd^2f^2h+\frac{1}{2}ad^3efh+\frac{5}{8}ad^3f^2g+\frac{13}{8}bc^2df^2h-bcd^2efh-\frac{9}{8}bcd^2 \ldots\right.}{\ldots}$
derivativedivides	$\frac{\frac{2dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-6bcfh\sqrt{fx+e}+2bdeh\sqrt{fx+e}+2bdfg\sqrt{fx+e}}{d^4}-2\left(\frac{\left(-\frac{9}{8}acd^2f^2h+\frac{1}{2}ad^3efh+\frac{5}{8}ad^3f^2g+ \ldots\right.}{\ldots}\right.$
default	$\frac{\frac{2dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-6bcfh\sqrt{fx+e}+2bdeh\sqrt{fx+e}+2bdfg\sqrt{fx+e}}{d^4}-2\left(\frac{\left(-\frac{9}{8}acd^2f^2h+\frac{1}{2}ad^3efh+\frac{5}{8}ad^3f^2g+ \ldots\right.}{\ldots}\right.$

input
```
int((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
15/4/((c*f-d*e)*d)^(1/2)*(-(1/5*(-g*f^2*a-4*e*(a*h+b*g)*f-8/3*b*e^2*h)*d^2
+c*((a*h+b*g)*f+8/3*e*h*b)*f*d-7/3*b*c^2*f^2*h)*(d*x+c)^2*arctan(d*(f*x+e)
^(1/2)/((c*f-d*e)*d)^(1/2))+(1/3*(-(-8/15*b*h*x^2+8/5*(-a*h-b*g)*x+g*a)*x*
f-2/5*e*(-16/3*b*h*x^2+2*(a*h+b*g)*x+g*a))*d^3-2/15*c*((28/3*b*h*x^2+25/2*
(-a*h-b*g)*x+3/2*g*a)*f+e*(-44/3*b*h*x+a*h+b*g))*d^2+((-35/9*b*h*x+a*h+b*g
)*f+10/9*e*h*b)*c^2*d-7/3*c^3*h*b*f)*(f*x+e)^(1/2)*((c*f-d*e)*d)^(1/2))/(d
*x+c)^2/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(253) = 506$.

Time = 0.18 (sec) , antiderivative size = 1522, normalized size of antiderivative = 5.46

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output

```
[-1/24*(3*sqrt(d^2*e - c*d*f)*((3*(4*b*d^4*e*f - (5*b*c*d^3 - a*d^4)*f^2)*
g + (8*b*d^4*e^2 - 4*(10*b*c*d^3 - 3*a*d^4)*e*f + 5*(7*b*c^2*d^2 - 3*a*c*d
^3)*f^2)*h)*x^2 + 3*(4*b*c^2*d^2*e*f - (5*b*c^3*d - a*c^2*d^2)*f^2)*g + (8
*b*c^2*d^2*e^2 - 4*(10*b*c^3*d - 3*a*c^2*d^2)*e*f + 5*(7*b*c^4 - 3*a*c^3*d
)*f^2)*h + 2*(3*(4*b*c*d^3*e*f - (5*b*c^2*d^2 - a*c*d^3)*f^2)*g + (8*b*c*d
^3*e^2 - 4*(10*b*c^2*d^2 - 3*a*c*d^3)*e*f + 5*(7*b*c^3*d - 3*a*c^2*d^2)*f^
2)*h)*x)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(
d*x + c)) - 2*(8*(b*d^5*e*f - b*c*d^4*f^2)*h*x^3 + 8*(3*(b*d^5*e*f - b*c*d
^4*f^2)*g + (4*b*d^5*e^2 - (11*b*c*d^4 - 3*a*d^5)*e*f + (7*b*c^2*d^3 - 3*a
*c*d^4)*f^2)*h)*x^2 - 3*(2*(b*c*d^4 + a*d^5)*e^2 - (17*b*c^2*d^3 - a*c*d^4
)*e*f + 3*(5*b*c^3*d^2 - a*c^2*d^3)*f^2)*g + (2*(25*b*c^2*d^3 - 3*a*c*d^4)
*e^2 - (155*b*c^3*d^2 - 51*a*c^2*d^3)*e*f + 15*(7*b*c^4*d - 3*a*c^3*d^2)*f
^2)*h - (3*(4*b*d^5*e^2 - (29*b*c*d^4 - 5*a*d^5)*e*f + 5*(5*b*c^2*d^3 - a*
c*d^4)*f^2)*g - (4*(22*b*c*d^4 - 3*a*d^5)*e^2 - (263*b*c^2*d^3 - 87*a*c*d^
4)*e*f + 25*(7*b*c^3*d^2 - 3*a*c^2*d^3)*f^2)*h)*x)*sqrt(f*x + e))/(c^2*d^6
*e - c^3*d^5*f + (d^8*e - c*d^7*f)*x^2 + 2*(c*d^7*e - c^2*d^6*f)*x), 1/12*
(3*sqrt(-d^2*e + c*d*f)*((3*(4*b*d^4*e*f - (5*b*c*d^3 - a*d^4)*f^2)*g + (8
*b*d^4*e^2 - 4*(10*b*c*d^3 - 3*a*d^4)*e*f + 5*(7*b*c^2*d^2 - 3*a*c*d^3)*f^
2)*h)*x^2 + 3*(4*b*c^2*d^2*e*f - (5*b*c^3*d - a*c^2*d^2)*f^2)*g + (8*b*c^2
*d^2*e^2 - 4*(10*b*c^3*d - 3*a*c^2*d^2)*e*f + 5*(7*b*c^4 - 3*a*c^3*d)*f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(f*x+e)**(3/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(253) = 506.

Time = 0.13 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{(12\,bd^2efg - 15\,bcdf^2g + 3\,ad^2f^2g + 8\,bd^2e^2h - 40\,bcdefh + 12\,ad^2e}{4\,\sqrt{-d^2e+cdf}\,d^4}$$
$$- \frac{4\,(fx+e)^{\frac{3}{2}}bd^3efg - 4\,\sqrt{fx+e}bd^3e^2fg - 9\,(fx+e)^{\frac{3}{2}}bcd^2f^2g + 5\,(fx+e)^{\frac{3}{2}}ad^3f^2g + 11\,\sqrt{fx+e}bcd^2e}{}$$
$$+ \frac{2\left(3\,\sqrt{fx+e}bd^6fg + (fx+e)^{\frac{3}{2}}bd^6h + 3\,\sqrt{fx+e}bd^6eh - 9\,\sqrt{fx+e}bcd^5fh + 3\,\sqrt{fx+e}ad^6fh\right)}{3\,d^9}$$

input
```
integrate((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(12*b*d^2*e*f*g - 15*b*c*d*f^2*g + 3*a*d^2*f^2*g + 8*b*d^2*e^2*h - 40*
b*c*d*e*f*h + 12*a*d^2*e*f*h + 35*b*c^2*f^2*h - 15*a*c*d*f^2*h)*arctan(sqr
t(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^4) - 1/4*(4*(f*
x + e)^(3/2)*b*d^3*e*f*g - 4*sqrt(f*x + e)*b*d^3*e^2*f*g - 9*(f*x + e)^(3/
2)*b*c*d^2*f^2*g + 5*(f*x + e)^(3/2)*a*d^3*f^2*g + 11*sqrt(f*x + e)*b*c*d^
2*e*f^2*g - 3*sqrt(f*x + e)*a*d^3*e*f^2*g - 7*sqrt(f*x + e)*b*c^2*d*f^3*g
+ 3*sqrt(f*x + e)*a*c*d^2*f^3*g - 8*(f*x + e)^(3/2)*b*c*d^2*e*f*h + 4*(f*x
 + e)^(3/2)*a*d^3*e*f*h + 8*sqrt(f*x + e)*b*c*d^2*e^2*f*h - 4*sqrt(f*x + e
)*a*d^3*e^2*f*h + 13*(f*x + e)^(3/2)*b*c^2*d*f^2*h - 9*(f*x + e)^(3/2)*a*c
*d^2*f^2*h - 19*sqrt(f*x + e)*b*c^2*d*e*f^2*h + 11*sqrt(f*x + e)*a*c*d^2*e
*f^2*h + 11*sqrt(f*x + e)*b*c^3*f^3*h - 7*sqrt(f*x + e)*a*c^2*d*f^3*h)/(((
f*x + e)*d - d*e + c*f)^2*d^4) + 2/3*(3*sqrt(f*x + e)*b*d^6*f*g + (f*x + e
)^(3/2)*b*d^6*h + 3*sqrt(f*x + e)*b*d^6*e*h - 9*sqrt(f*x + e)*b*c*d^5*f*h
+ 3*sqrt(f*x + e)*a*d^6*f*h)/d^9
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \sqrt{e+f\,x}\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{d^3}\right.$$
$$\left.+\frac{2\,b\,h\,(3\,d^3\,e-3\,c\,d^2\,f)}{d^6}\right)$$
$$+\frac{\sqrt{e+f\,x}\left(\frac{7\,a\,c^2\,d\,f^3\,h}{4}-\frac{3\,a\,c\,d^2\,f^3\,g}{4}-\frac{11\,b\,c^3\,f^3\,h}{4}+\frac{7\,b\,c^2\,d\,f^3\,g}{4}+\frac{3\,a\,d^3\,e\,f^2\,g}{4}+a\,d^3\,e^2\,f\,h+b\,d^3\,e^2\,f\,g-\frac{11\,a\,c\,d^2\,e\,f^2}{4}\right.}{d^6\,(e+f\,x)^2-}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,(3\,a\,d^2\,f^2\,g+35\,b\,c^2\,f^2\,h+8\,b\,d^2\,e^2\,h-15\,a\,c\,d\,f^2\,h-15\,b\,c\,d\,f^2\,g+12\,a\,d^2\,e\,f\,h+12\,b}{4\,d^{9/2}\,\sqrt{c\,f-d\,e}}$$
$$+\frac{2\,b\,h\,(e+f\,x)^{3/2}}{3\,d^3}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(a + b*x))/(c + d*x)^3,x)
```

output
```
(e + f*x)^(1/2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/d^3 + (2*b*h*(3*d^3*e - 3*c
*d^2*f))/d^6) + ((e + f*x)^(1/2)*((7*a*c^2*d*f^3*h)/4 - (3*a*c*d^2*f^3*g)/
4 - (11*b*c^3*f^3*h)/4 + (7*b*c^2*d*f^3*g)/4 + (3*a*d^3*e*f^2*g)/4 + a*d^3
*e^2*f*h + b*d^3*e^2*f*g - (11*a*c*d^2*e*f^2*h)/4 - (11*b*c*d^2*e*f^2*g)/4
 - 2*b*c*d^2*e^2*f*h + (19*b*c^2*d*e*f^2*h)/4) - (e + f*x)^(3/2)*((5*a*d^3
*f^2*g)/4 - (9*a*c*d^2*f^2*h)/4 - (9*b*c*d^2*f^2*g)/4 + (13*b*c^2*d*f^2*h)
/4 + a*d^3*e*f*h + b*d^3*e*f*g - 2*b*c*d^2*e*f*h))/(d^6*(e + f*x)^2 - (e +
 f*x)*(2*d^6*e - 2*c*d^5*f) + d^6*e^2 + c^2*d^4*f^2 - 2*c*d^5*e*f) + (atan
((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))*(3*a*d^2*f^2*g + 35*b*c^2*f^
2*h + 8*b*d^2*e^2*h - 15*a*c*d*f^2*h - 15*b*c*d*f^2*g + 12*a*d^2*e*f*h + 1
2*b*d^2*e*f*g - 40*b*c*d*e*f*h))/(4*d^(9/2)*(c*f - d*e)^(1/2)) + (2*b*h*(e
 + f*x)^(3/2))/(3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1779, normalized size of antiderivative = 6.38

$$\int \frac{(a+bx)(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x)
```

output
```
( - 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a*c**3*d*f**2*h + 36*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)
/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d**2*e*f*h + 9*sqrt(d)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d**2*f**2*g - 90*
sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*
a*c**2*d**2*f**2*h*x + 72*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(
sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*e*f*h*x + 18*sqrt(d)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*f**2*g*x - 45*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c
*d**3*f**2*h*x**2 + 36*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqr
t(d)*sqrt(c*f - d*e)))*a*d**4*e*f*h*x**2 + 9*sqrt(d)*sqrt(c*f - d*e)*atan(
(sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**4*f**2*g*x**2 + 105*sqrt
(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*
*4*f**2*h - 120*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sq
rt(c*f - d*e)))*b*c**3*d*e*f*h - 45*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*d*f**2*g + 210*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*d*f**2*h*
x + 24*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*b*c**2*d**2*e**2*h + 36*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d**2*e*f*g - 240*sqrt(d)*sqrt(c*f...
```

3.99 $\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3} dx = \frac{2fh\sqrt{e+fx}}{d^3} - \frac{(3dfg+4deh-7cfh)\sqrt{e+fx}}{4d^3(c+dx)} - \frac{(dg-ch)(e+fx)^{3/2}}{2d^2(c+dx)^2} - \frac{3f(dfg+4deh-5cfh)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{7/2}\sqrt{de-cf}}$$

output
```
2*f*h*(f*x+e)^(1/2)/d^3-1/4*(-7*c*f*h+4*d*e*h+3*d*f*g)*(f*x+e)^(1/2)/d^3/(
d*x+c)-1/2*(-c*h+d*g)*(f*x+e)^(3/2)/d^2/(d*x+c)^2-3/4*f*(-5*c*f*h+4*d*e*h+
d*f*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)/(-c*f+d*e)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3} dx = -\frac{\sqrt{e+fx}(-15c^2fh+cd(3fg+2eh-25fhx)+d^2(fx(5g-8hx)+2e(g+2hx)))}{4d^3(c+dx)^2} + \frac{3f(dfg+4deh-5cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{4d^{7/2}\sqrt{-de+cf}}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/4*(Sqrt[e + f*x]*(-15*c^2*f*h + c*d*(3*f*g + 2*e*h - 25*f*h*x) + d^2*(f
*x*(5*g - 8*h*x) + 2*e*(g + 2*h*x))))/(d^3*(c + d*x)^2) + (3*f*(d*f*g + 4*
d*e*h - 5*c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(4*d^
(7/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3} dx$$

$$\Big\downarrow \; 87$$

$$\frac{(-5cfh+4deh+dfg)\int \frac{(e+fx)^{3/2}}{(c+dx)^2}dx}{4d(de-cf)} - \frac{(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 51$$

$$\frac{(-5cfh+4deh+dfg)\left(\frac{3f\int \frac{\sqrt{e+fx}}{c+dx}dx}{2d} - \frac{(e+fx)^{3/2}}{d(c+dx)}\right)}{4d(de-cf)} - \frac{(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 60$$

$$\frac{(-5cfh+4deh+dfg)\left(\frac{3f\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d} + \frac{2\sqrt{e+fx}}{d}\right)}{2d} - \frac{(e+fx)^{3/2}}{d(c+dx)}\right)}{4d(de-cf)} - \frac{(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{(-5cfh+4deh+dfg)\left(\frac{3f\left(\frac{2(de-cf)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2\sqrt{e+fx}}{d}\right)}{2d}-\frac{(e+fx)^{3/2}}{d(c+dx)}\right)}{4d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{3f\left(\frac{2\sqrt{e+fx}}{d}-\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}}\right)}{2d}-\frac{(e+fx)^{3/2}}{d(c+dx)}\right)(-5cfh+4deh+dfg)}{4d(de-cf)}-\frac{(e+fx)^{5/2}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x]
```

output
```
-1/2*((d*g - c*h)*(e + f*x)^(5/2))/(d*(d*e - c*f)*(c + d*x)^2) + ((d*f*g +
 4*d*e*h - 5*c*f*h)*(-((e + f*x)^(3/2)/(d*(c + d*x))) + (3*f*((2*Sqrt[e +
f*x])/d - (2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*
f]])/d^(3/2)))/(2*d)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{15\left((xd+c)^2 f\left(\frac{(-4eh-fg)d}{5}+cfh\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)-\sqrt{(cf-de)d}\left(\frac{\left(-\left(-\frac{8hx}{5}+g\right)xf-\frac{2e(2hx+g)}{5}\right)d^2}{3}-\frac{2c\left((-25\right.}{}\right.\right.}{4\sqrt{(cf-de)d}\,d^3(xd+c)^2}$
derivativedivides	$2f\left(\frac{h\sqrt{fx+e}}{d^3}-\frac{\frac{\left(-\frac{9}{8}cdfh+\frac{1}{2}d^2eh+\frac{5}{8}d^2fg\right)(fx+e)^{\frac{3}{2}}+\left(-\frac{7}{8}c^2f^2h+\frac{11}{8}cdehf+\frac{3}{8}cd\,f^2g-\frac{1}{2}d^2e^2h-\frac{3}{8}d^2egf\right)\sqrt{fx+e}}{((fx+e)d+cf-de)^2}+\frac{3(5cf}{}}{d^3}\right.$
default	$2f\left(\frac{h\sqrt{fx+e}}{d^3}-\frac{\frac{\left(-\frac{9}{8}cdfh+\frac{1}{2}d^2eh+\frac{5}{8}d^2fg\right)(fx+e)^{\frac{3}{2}}+\left(-\frac{7}{8}c^2f^2h+\frac{11}{8}cdehf+\frac{3}{8}cd\,f^2g-\frac{1}{2}d^2e^2h-\frac{3}{8}d^2egf\right)\sqrt{fx+e}}{((fx+e)d+cf-de)^2}+\frac{3(5cf}{}}{d^3}\right.$
risch	$\frac{2fh\sqrt{fx+e}}{d^3}-\frac{2f\left(\frac{\left(-\frac{9}{8}cdfh+\frac{1}{2}d^2eh+\frac{5}{8}d^2fg\right)(fx+e)^{\frac{3}{2}}+\left(-\frac{7}{8}c^2f^2h+\frac{11}{8}cdehf+\frac{3}{8}cd\,f^2g-\frac{1}{2}d^2e^2h-\frac{3}{8}d^2egf\right)\sqrt{fx+e}}{((fx+e)d+cf-de)^2}+\frac{3(5cf}{}\right.}{d^3}$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-15/4*((d*x+c)^2*f*(1/5*(-4*e*h-f*g)*d+c*f*h)*arctan(d*(f*x+e)^(1/2)/((c*f
-d*e)*d)^(1/2))-((c*f-d*e)*d)^(1/2)*(1/3*(-(-8/5*h*x+g)*x*f-2/5*e*(2*h*x+g
))*d^2-2/15*c*(1/2*(-25*h*x+3*g)*f+e*h)*d+c^2*f*h)*(f*x+e)^(1/2))/((c*f-d*
e)*d)^(1/2)/d^3/(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(130) = 260$.

Time = 0.18 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.64

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \left[-\frac{3\left(c^2df^2g+\left(d^3f^2g+(4\,d^3ef-5\,cd^2f^2)h\right)x^2+(4\,c^2def-5\,c^3f^2)h+2\,(cd\right.}{}\right.$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*(c^2*d*f^2*g + (d^3*f^2*g + (4*d^3*e*f - 5*c*d^2*f^2)*h)*x^2 + (4
*c^2*d*e*f - 5*c^3*f^2)*h + 2*(c*d^2*f^2*g + (4*c*d^2*e*f - 5*c^2*d*f^2)*h
)*x)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*
sqrt(f*x + e))/(d*x + c)) - 2*(8*(d^4*e*f - c*d^3*f^2)*h*x^2 - (2*d^4*e^2
+ c*d^3*e*f - 3*c^2*d^2*f^2)*g - (2*c*d^3*e^2 - 17*c^2*d^2*e*f + 15*c^3*d*
f^2)*h - (5*(d^4*e*f - c*d^3*f^2)*g + (4*d^4*e^2 - 29*c*d^3*e*f + 25*c^2*d
^2*f^2)*h)*x)*sqrt(f*x + e))/(c^2*d^5*e - c^3*d^4*f + (d^7*e - c*d^6*f)*x^
2 + 2*(c*d^6*e - c^2*d^5*f)*x), 1/4*(3*(c^2*d*f^2*g + (d^3*f^2*g + (4*d^3*
e*f - 5*c*d^2*f^2)*h)*x^2 + (4*c^2*d*e*f - 5*c^3*f^2)*h + 2*(c*d^2*f^2*g +
 (4*c*d^2*e*f - 5*c^2*d*f^2)*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e
 + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) + (8*(d^4*e*f - c*d^3*f^2)*h*x^2 -
(2*d^4*e^2 + c*d^3*e*f - 3*c^2*d^2*f^2)*g - (2*c*d^3*e^2 - 17*c^2*d^2*e*f
+ 15*c^3*d*f^2)*h - (5*(d^4*e*f - c*d^3*f^2)*g + (4*d^4*e^2 - 29*c*d^3*e*f
 + 25*c^2*d^2*f^2)*h)*x)*sqrt(f*x + e))/(c^2*d^5*e - c^3*d^4*f + (d^7*e -
c*d^6*f)*x^2 + 2*(c*d^6*e - c^2*d^5*f)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{2\sqrt{fx+e}fh}{d^3} + \frac{3\,(df^2g+4\,defh-5\,cf^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{4\sqrt{-d^2e+cdf}d^3} - \frac{5\,(fx+e)^{\frac{3}{2}}d^2f^2g-3\sqrt{fx+e}d^2ef^2g+3\sqrt{fx+e}cdf^3g+4\,(fx+e)^{\frac{3}{2}}d^2efh-4\sqrt{fx+e}d^2e^2fh-9\,(f}{4\,((fx+e)d-de+cf)^2d^3}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x, algorithm="giac")
```

output
```
2*sqrt(f*x + e)*f*h/d^3 + 3/4*(d*f^2*g + 4*d*e*f*h - 5*c*f^2*h)*arctan(sqr
t(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^3) - 1/4*(5*(f*
x + e)^(3/2)*d^2*f^2*g - 3*sqrt(f*x + e)*d^2*e*f^2*g + 3*sqrt(f*x + e)*c*d
*f^3*g + 4*(f*x + e)^(3/2)*d^2*e*f*h - 4*sqrt(f*x + e)*d^2*e^2*f*h - 9*(f*
x + e)^(3/2)*c*d*f^2*h + 11*sqrt(f*x + e)*c*d*e*f^2*h - 7*sqrt(f*x + e)*c^
2*f^3*h)/(((f*x + e)*d - d*e + c*f)^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{2\,f\,h\,\sqrt{e+f\,x}}{d^3} - \frac{(e+f\,x)^{3/2}\left(\frac{5\,g\,d^2\,f^2}{4}+e\,h\,d^2\,f-\frac{9\,c\,h\,d\,f^2}{4}\right)-\sqrt{e+f\,x}\left(\frac{7\,h\,c^2\,f^3}{4}-\frac{11\,h\,c\,d\,e\,f^2}{4}-\frac{3\,g\,c\,d\,f^3}{4}+h\,d^2\,e^2\,f+\frac{3\,g\,d^2\,e\,}{4}\right.}{d^5\,(e+f\,x)^2-(e+f\,x)\,(2\,d^5\,e-2\,c\,d^4\,f)+d^5\,e^2+c^2\,d^3\,f^2-2\,c\,d^4\,e\,f} + \frac{3\,f\,\mathrm{atan}\left(\frac{\sqrt{d}\,f\,\sqrt{e+f\,x}\,(4\,d\,e\,h-5\,c\,f\,h+d\,f\,g)}{\sqrt{c\,f-d\,e}\,(d\,f^2\,g-5\,c\,f^2\,h+4\,d\,e\,f\,h)}\right)\,(4\,d\,e\,h-5\,c\,f\,h+d\,f\,g)}{4\,d^{7/2}\,\sqrt{c\,f-d\,e}}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/(c + d*x)^3,x)
```

output
```
(2*f*h*(e + f*x)^(1/2))/d^3 - ((e + f*x)^(3/2)*((5*d^2*f^2*g)/4 - (9*c*d*f
^2*h)/4 + d^2*e*f*h) - (e + f*x)^(1/2)*((7*c^2*f^3*h)/4 - (3*c*d*f^3*g)/4
+ (3*d^2*e*f^2*g)/4 + d^2*e^2*f*h - (11*c*d*e*f^2*h)/4))/(d^5*(e + f*x)^2
- (e + f*x)*(2*d^5*e - 2*c*d^4*f) + d^5*e^2 + c^2*d^3*f^2 - 2*c*d^4*e*f) +
 (3*f*atan((d^(1/2)*f*(e + f*x)^(1/2)*(4*d*e*h - 5*c*f*h + d*f*g))/((c*f -
 d*e)^(1/2)*(d*f^2*g - 5*c*f^2*h + 4*d*e*f*h)))*(4*d*e*h - 5*c*f*h + d*f*g
))/(4*d^(7/2)*(c*f - d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 681, normalized size of antiderivative = 4.48

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(c+dx)^3}\,dx = \frac{-15\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^3f^2h+12\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)}{}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(d*x+c)^3,x)
```

output
```
( - 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*c**3*f**2*h + 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sq
rt(d)*sqrt(c*f - d*e)))*c**2*d*e*f*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*g - 30*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*h*
x + 24*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*c*d**2*e*f*h*x + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(
sqrt(d)*sqrt(c*f - d*e)))*c*d**2*f**2*g*x - 15*sqrt(d)*sqrt(c*f - d*e)*ata
n((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d**2*f**2*h*x**2 + 12*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**
3*e*f*h*x**2 + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*d**3*f**2*g*x**2 + 15*sqrt(e + f*x)*c**3*d*f**2*h - 17*sq
rt(e + f*x)*c**2*d**2*e*f*h - 3*sqrt(e + f*x)*c**2*d**2*f**2*g + 25*sqrt(e
 + f*x)*c**2*d**2*f**2*h*x + 2*sqrt(e + f*x)*c*d**3*e**2*h + sqrt(e + f*x)
*c*d**3*e*f*g - 29*sqrt(e + f*x)*c*d**3*e*f*h*x - 5*sqrt(e + f*x)*c*d**3*f
**2*g*x + 8*sqrt(e + f*x)*c*d**3*f**2*h*x**2 + 2*sqrt(e + f*x)*d**4*e**2*g
 + 4*sqrt(e + f*x)*d**4*e**2*h*x + 5*sqrt(e + f*x)*d**4*e*f*g*x - 8*sqrt(e
 + f*x)*d**4*e*f*h*x**2)/(4*d**4*(c**3*f - c**2*d*e + 2*c**2*d*f*x - 2*c*d
**2*e*x + c*d**2*f*x**2 - d**3*e*x**2))
```

3.100 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 353

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3} dx = -\frac{(ad(3dfg+4deh-7cfh)-b(4d^2eg-cdfg-3c^2fh))\sqrt{e+fx}}{4d^2(bc-ad)^2(c+dx)} + \frac{(dg-ch)(e+fx)^{3/2}}{2d(bc-ad)(c+dx)^2} - \frac{2(be-af)^{3/2}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)^3} + \frac{(3a^2d^2f(dfg+4deh-5cfh)+b^2(8d^3e^2g-4cd^2efg-c^2df^2g-3c^3f^2h)+2abd(5c^2f^2h-2d^2e(3fg+2e}{4d^{5/2}(bc-ad)^3\sqrt{de-cf}}$$

output
```
-1/4*(a*d*(-7*c*f*h+4*d*e*h+3*d*f*g)-b*(-3*c^2*f*h-c*d*f*g+4*d^2*e*g))*(f*
x+e)^(1/2)/d^2/(-a*d+b*c)^2/(d*x+c)+1/2*(-c*h+d*g)*(f*x+e)^(3/2)/d/(-a*d+b
*c)/(d*x+c)^2-2*(-a*f+b*e)^(3/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/
(-a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^3+1/4*(3*a^2*d^2*f*(-5*c*f*h+4*d*e*h+
d*f*g)+b^2*(-3*c^3*f^2*h-c^2*d*f^2*g-4*c*d^2*e*f*g+8*d^3*e^2*g)+2*a*b*d*(5
*c^2*f^2*h-2*d^2*e*(2*e*h+3*f*g)+c*d*f*(2*e*h+3*f*g)))*arctanh(d^(1/2)*(f*
x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-a*d+b*c)^3/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \frac{1}{4}\left(\frac{\sqrt{e+fx}(-ad(-7c^2fh+d^2(2eg+5fgx+4ehx)+cd(3fg+2eh-9fhx}{d^2(bc-ad)^2(}\right.$$

$$+\frac{8(-be+af)^{3/2}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{b}(bc-ad)^3}$$

$$+\frac{(3a^2d^2f(dfg+4deh-5cfh)+b^2(8d^3e^2g-4cd^2efg-c^2df^2g-3c^3f^2h)+2abd(5c^2f^2h-2d^2e(3fg+2e}{d^{5/2}(-bc+ad)^3\sqrt{-de+cf}}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)^3),x]
```

output
```
((Sqrt[e + f*x]*(-(a*d*(-7*c^2*f*h + d^2*(2*e*g + 5*f*g*x + 4*e*h*x) + c*d
*(3*f*g + 2*e*h - 9*f*h*x))) + b*(-3*c^3*f*h + 4*d^3*e*g*x + c*d^2*g*(6*e
+ f*x) - c^2*d*(f*g + 2*e*h + 5*f*h*x))))/(d^2*(b*c - a*d)^2*(c + d*x)^2)
+ (8*(-(b*e) + a*f)^(3/2)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[
-(b*e) + a*f]])/(Sqrt[b]*(b*c - a*d)^3) + ((3*a^2*d^2*f*(d*f*g + 4*d*e*h -
 5*c*f*h) + b^2*(8*d^3*e^2*g - 4*c*d^2*e*f*g - c^2*d*f^2*g - 3*c^3*f^2*h)
+ 2*a*b*d*(5*c^2*f^2*h - 2*d^2*e*(3*f*g + 2*e*h) + c*d*f*(3*f*g + 2*e*h)))
*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(5/2)*(-(b*c) + a*
d)^3*Sqrt[-(d*e) + c*f]))/4
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}-\frac{\int -\frac{\sqrt{e+fx}(4bdeg-a(3dfg+4deh-3cfh)+f(bdg+3bch-4adh)x)}{2(a+bx)(c+dx)^2}dx}{2d(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{e+fx}(4bdeg-a(3dfg+4deh-3cfh)+f(bdg+3bch-4adh)x)}{(a+bx)(c+dx)^2}dx}{4d(bc-ad)}+\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 166$$

$$\frac{-\frac{\int -\frac{-df(7cfh-3d(fg+4eh))a^2+b\left(-4e(3fg+2eh)d^2+cf^2gd+3c^2f^2h\right)a+8b^2d^2e^2g+f\left(\left(3fhc^2+dfgc+4d^2eg\right)b^2-ad(5dfg+4deh+7cfh)b+8a^2d^2fh\right)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{d(bc-ad)}-\cdots}{4d(bc-ad)}+\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-df(7cfh-3d(fg+4eh))a^2+b\left(-4e(3fg+2eh)d^2+cf^2gd+3c^2f^2h\right)a+8b^2d^2e^2g+f\left(\left(3fhc^2+dfgc+4d^2eg\right)b^2-ad(5dfg+4deh+7cfh)b+8a^2d^2fh\right)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2d(bc-ad)}-\frac{\sqrt{e-\cdots}}{\cdots}}{4d(bc-ad)}+\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{8d^2(be-af)^2(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(3a^2d^2f(-5cfh+4deh+dfg)+2abd\left(5c^2f^2h+cdf(2eh+3fg)-2d^2e(2eh+3fg)\right)+b^2\left(-3c^3f^2h-c^2df^2g-4cd^2efg+8d^3\cdots\right.\right.}{bc-ad}}{2d(bc-ad)}\cdots}{4d(bc-ad)}+\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{16d^2(be-af)^2(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(3a^2d^2f(-5cfh+4deh+dfg)+2abd\left(5c^2f^2h+cdf(2eh+3fg)-2d^2e(2eh+3fg)\right)+b^2\left(-3c^3f^2h-c^2df^2g-4c\cdots\right.\right.}{f(bc-ad)}}{2d(bc-ad)}\cdots}{4d(bc-ad)}+\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(3a^2d^2f(-5cfh+4deh+dfg)+2abd\left(5c^2f^2h+cdf(2eh+3fg)-2d^2e(2eh+3fg)\right)+b^2\left(-3c^3f^2h-c^2df^2g-4cd^2efg+8d^3e^2g\right)\right)}{\sqrt{d}(bc-ad)\sqrt{de-cf}}-\frac{16d^2(be}{}}{2d(bc-ad)}}{4d(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(dg-ch)}{2d(c+dx)^2(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)^3),x]
```

output
```
((d*g - c*h)*(e + f*x)^(3/2))/(2*d*(b*c - a*d)*(c + d*x)^2) + (-(((a*d*(3*
d*f*g + 4*d*e*h - 7*c*f*h) - b*(4*d^2*e*g - c*d*f*g - 3*c^2*f*h))*Sqrt[e +
 f*x])/(d*(b*c - a*d)*(c + d*x))) + ((-16*d^2*(b*e - a*f)^(3/2)*(b*g - a*h
)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d))
+ (2*(3*a^2*d^2*f*(d*f*g + 4*d*e*h - 5*c*f*h) + b^2*(8*d^3*e^2*g - 4*c*d^2
*e*f*g - c^2*d*f^2*g - 3*c^3*f^2*h) + 2*a*b*d*(5*c^2*f^2*h - 2*d^2*e*(3*f*
g + 2*e*h) + c*d*f*(3*f*g + 2*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[
d*e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f]))/(2*d*(b*c - a*d)))/(4*
d*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{15\sqrt{(af-be)b}\left(\frac{4\left(-\frac{2b^2e^2g}{3}+\frac{2a\left(eh+\frac{3fg}{2}\right)eb}{3}-a^2\left(eh+\frac{fg}{4}\right)f\right)d^3}{5}+c\left(\frac{4b^2eg}{15}-\frac{4a\left(eh+\frac{3fg}{2}\right)b}{15}+a^2fh\right)fd^2-\frac{2c^2\left(ah-\frac{bg}{10}\right)bf^2}{3}\right.}{4}$
derivativedivides	$2f^2\left(\frac{(af-be)^2(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f^2(ad-bc)^3\sqrt{(af-be)b}}-\frac{-\frac{f\left(9a^2cd^2fh-4a^2d^3eh-5a^2d^3fg-14abc^2dfh+4abcd^2eh+6abcd^2fg\right.}{8d}}{}\right.$
default	$2f^2\left(\frac{(af-be)^2(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f^2(ad-bc)^3\sqrt{(af-be)b}}-\frac{-\frac{f\left(9a^2cd^2fh-4a^2d^3eh-5a^2d^3fg-14abc^2dfh+4abcd^2eh+6abcd^2fg\right.}{8d}}{}\right.$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-15/8*((a*f-b*e)*b)^(1/2)*(4/5*(-2/3*b^2*e^2*g+2/3*a*(e*h+3/2*f*g)*e*b-
a^2*(e*h+1/4*f*g)*f)*d^3+c*(4/15*b^2*e*g-4/15*a*(e*h+3/2*f*g)*b+a^2*f*h)*f
*d^2-2/3*c^2*(a*h-1/10*b*g)*b*f^2*d+1/5*b^2*c^3*f^2*h)*(d*x+c)^2*arctan(d*
(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(d^2*(d*x+c)^2*(a*f
-b*e)^2*(a*h-b*g)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+7/8*((a*f-b*
e)*b)^(1/2)*(a*d-b*c)*(f*x+e)^(1/2)*(2/7*(2*b*e*g*x-a*(5/2*f*g*x+e*(2*h*x+
g)))*d^3-2/7*c*(-3*(1/6*f*x+e)*g*b+a*(3/2*(-3*h*x+g)*f+e*h))*d^2+c^2*(1/7*
((-5*h*x-g)*f-2*e*h)*b+a*f*h)*d-3/7*c^3*h*b*f)))/((a*f-b*e)*b)^(1/2)/((c*f
-d*e)*d)^(1/2)/(a*d-b*c)^3/(d*x+c)^2/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(323) = 646$.

Time = 35.30 (sec) , antiderivative size = 6213, normalized size of antiderivative = 17.60

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(323) = 646$.

Time = 0.16 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.95

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \frac{2\,(b^3e^2g - 2\,ab^2efg + a^2bf^2g - ab^2e^2h + 2\,a^2befh - a^3f^2h)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+ab}}\right.}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{-b^2e+abf}}$$
$$- \frac{(8\,b^2d^3e^2g - 4\,b^2cd^2efg - 12\,abd^3efg - b^2c^2df^2g + 6\,abcd^2f^2g + 3\,a^2d^3f^2g - 8\,abd^3e^2h + 4\,abcd^2efh +}{4\,(b^3c^3d^2 - 3\,ab^2c^2d^3 + 3\,a^2bcd^4 - a^3d^5)\sqrt{}}$$
$$+ \frac{4\,(fx+e)^{\frac{3}{2}}bd^3efg - 4\,\sqrt{fx+e}bd^3e^2fg + (fx+e)^{\frac{3}{2}}bcd^2f^2g - 5\,(fx+e)^{\frac{3}{2}}ad^3f^2g + 5\,\sqrt{fx+e}bcd^2ef^2g}{}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
2*(b^3*e^2*g - 2*a*b^2*e*f*g + a^2*b*f^2*g - a*b^2*e^2*h + 2*a^2*b*e*f*h -
 a^3*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*e + a*b*f)) - 1/4*(8*b^2*d^3
*e^2*g - 4*b^2*c*d^2*e*f*g - 12*a*b*d^3*e*f*g - b^2*c^2*d*f^2*g + 6*a*b*c*
d^2*f^2*g + 3*a^2*d^3*f^2*g - 8*a*b*d^3*e^2*h + 4*a*b*c*d^2*e*f*h + 12*a^2
*d^3*e*f*h - 3*b^2*c^3*f^2*h + 10*a*b*c^2*d*f^2*h - 15*a^2*c*d^2*f^2*h)*ar
ctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3
 + 3*a^2*b*c*d^4 - a^3*d^5)*sqrt(-d^2*e + c*d*f)) + 1/4*(4*(f*x + e)^(3/2)
*b*d^3*e*f*g - 4*sqrt(f*x + e)*b*d^3*e^2*f*g + (f*x + e)^(3/2)*b*c*d^2*f^2
*g - 5*(f*x + e)^(3/2)*a*d^3*f^2*g + 5*sqrt(f*x + e)*b*c*d^2*e*f^2*g + 3*s
qrt(f*x + e)*a*d^3*e*f^2*g - sqrt(f*x + e)*b*c^2*d*f^3*g - 3*sqrt(f*x + e)
*a*c*d^2*f^3*g - 4*(f*x + e)^(3/2)*a*d^3*e*f*h + 4*sqrt(f*x + e)*a*d^3*e^2
*f*h - 5*(f*x + e)^(3/2)*b*c^2*d*f^2*h + 9*(f*x + e)^(3/2)*a*c*d^2*f^2*h +
 3*sqrt(f*x + e)*b*c^2*d*e*f^2*h - 11*sqrt(f*x + e)*a*c*d^2*e*f^2*h - 3*sq
rt(f*x + e)*b*c^3*f^3*h + 7*sqrt(f*x + e)*a*c^2*d*f^3*h)/((b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*((f*x + e)*d - d*e + c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 283348, normalized size of antiderivative = 802.69

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)*(c + d*x)^3),x)
```

output
```
- (((e + f*x)^(3/2)*(5*a*d^2*f^2*g + 5*b*c^2*f^2*h - 9*a*c*d*f^2*h - b*c*d
*f^2*g + 4*a*d^2*e*f*h - 4*b*d^2*e*f*g))/(4*d*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) + ((e + f*x)^(1/2)*(3*b*c^3*f^3*h + 3*a*c*d^2*f^3*g - 7*a*c^2*d*f^3*h
 + b*c^2*d*f^3*g - 3*a*d^3*e*f^2*g - 4*a*d^3*e^2*f*h + 4*b*d^3*e^2*f*g + 1
1*a*c*d^2*e*f^2*h - 5*b*c*d^2*e*f^2*g - 3*b*c^2*d*e*f^2*h))/(4*d^2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)))/(d^2*(e + f*x)^2 - (e + f*x)*(2*d^2*e - 2*c*d*f
) + c^2*f^2 + d^2*e^2 - 2*c*d*e*f) - atan(((((136*a^7*b^3*c*d^10*f^5*g - 8
*a*b^9*c^7*d^4*f^5*g - 24*a^8*b^2*d^11*f^5*g - 24*a*b^9*c^8*d^3*f^5*h + 56
*a^8*b^2*c*d^10*f^5*h + 56*a^7*b^3*d^11*e*f^4*g - 32*a^8*b^2*d^11*e*f^4*h
+ 8*b^10*c^7*d^4*e*f^4*g + 24*b^10*c^8*d^3*e*f^4*h + 24*a^2*b^8*c^6*d^5*f^
5*g + 24*a^3*b^7*c^5*d^6*f^5*g - 200*a^4*b^6*c^4*d^7*f^5*g + 360*a^5*b^5*c
^3*d^8*f^5*g - 312*a^6*b^4*c^2*d^9*f^5*g + 200*a^2*b^8*c^7*d^4*f^5*h - 696
*a^3*b^7*c^6*d^5*f^5*h + 1320*a^4*b^6*c^5*d^6*f^5*h - 1480*a^5*b^5*c^4*d^7
*f^5*h + 984*a^6*b^4*c^3*d^8*f^5*h - 360*a^7*b^3*c^2*d^9*f^5*h - 32*a^6*b^
4*d^11*e^2*f^3*g + 32*a^7*b^3*d^11*e^2*f^3*h - 32*b^10*c^6*d^5*e^2*f^3*g -
 480*a^2*b^8*c^4*d^7*e^2*f^3*g + 640*a^3*b^7*c^3*d^8*e^2*f^3*g - 480*a^4*b
^6*c^2*d^9*e^2*f^3*g - 192*a^2*b^8*c^5*d^6*e^2*f^3*h + 480*a^3*b^7*c^4*d^7
*e^2*f^3*h - 640*a^4*b^6*c^3*d^8*e^2*f^3*h + 480*a^5*b^5*c^2*d^9*e^2*f^3*h
 + 8*a*b^9*c^6*d^5*e*f^4*g - 328*a^6*b^4*c*d^10*e*f^4*g - 200*a*b^9*c^7*d^
4*e*f^4*h + 136*a^7*b^3*c*d^10*e*f^4*h + 192*a*b^9*c^5*d^6*e^2*f^3*g - ...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4007, normalized size of antiderivative = 11.35

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)/(d*x+c)^3,x)
```

output

```
(8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**2*c**3*d**3*f**2*h - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*c**2*d**4*e*f*h + 16*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*c**2*d**4*f**2
*h*x - 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**2*c*d**5*e*f*h*x + 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*c*d**5*f**2*h*x**2 - 8*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*d**6*e*
f*h*x**2 - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a*b*c**3*d**3*e*f*h - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**3*d**3*f**2*g + 8*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**2*d**
4*e**2*h + 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a*b*c**2*d**4*e*f*g - 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**2*d**4*e*f*h*x - 16*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**2*
d**4*f**2*g*x + 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a*b*c*d**5*e**2*h*x + 16*sqrt(b)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*d**5*e*f*g*x - 8*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*...
```

3.101 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 491

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = -\frac{(de-cf)(3bdg-bch-2adh)\sqrt{e+fx}}{2bd(bc-ad)^2(c+dx)^2}$$
$$-\frac{(4a^2d^2fh-abd(9dfg+8deh-9cfh)+b^2(12d^2eg-c^2fh-cd(3fg+4eh)))\sqrt{e+fx}}{4bd(bc-ad)^3(c+dx)}$$
$$-\frac{(bg-ah)(e+fx)^{3/2}}{b(bc-ad)(a+bx)(c+dx)^2}$$
$$+\frac{\sqrt{be-af}\,(a^2dfh+b^2(6deg-3cfg-2ceh)-ab(3dfg+4deh-5cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)^4}$$
$$-\frac{(3a^2d^2f(dfg+4deh-5cfh)+b^2(24d^3e^2g+c^3f^2h-8cd^2e(3fg+eh)+c^2df(3fg+4eh))-2abd(5c^2f}{4d^{3/2}(bc-ad)^4\sqrt{de-cf}}$$

output
```
-1/2*(-c*f+d*e)*(-2*a*d*h-b*c*h+3*b*d*g)*(f*x+e)^(1/2)/b/d/(-a*d+b*c)^2/(d
*x+c)^2-1/4*(4*a^2*d^2*f*h-a*b*d*(-9*c*f*h+8*d*e*h+9*d*f*g)+b^2*(12*d^2*e*
g-c^2*f*h-c*d*(4*e*h+3*f*g)))*(f*x+e)^(1/2)/b/d/(-a*d+b*c)^3/(d*x+c)-(-a*h
+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)/(d*x+c)^2+(-a*f+b*e)^(1/2)*(a^2*d
*f*h+b^2*(-2*c*e*h-3*c*f*g+6*d*e*g)-a*b*(-5*c*f*h+4*d*e*h+3*d*f*g))*arctan
h(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^4-1/4*(3*a^2*
d^2*f*(-5*c*f*h+4*d*e*h+d*f*g)+b^2*(24*d^3*e^2*g+c^3*f^2*h-8*c*d^2*e*(e*h+
3*f*g)+c^2*d*f*(4*e*h+3*f*g))-2*a*b*d*(5*c^2*f^2*h+4*d^2*e*(2*e*h+3*f*g)-c
*d*f*(16*e*h+9*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3
/2)/(-a*d+b*c)^4/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \frac{1}{4}\left(-\frac{\sqrt{e+fx}(ab(-c^3fh+d^3x(-6eg+9fgx+8ehx)+c^2d(9fg+10eh-6\right.$$

$$+\frac{4\sqrt{-be+af}(a^2dfh+b^2(6deg-3cfg-2ceh)+ab(-3dfg-4deh+5cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{b}(bc-ad)^4}$$

$$+\frac{(3a^2d^2f(dfg+4deh-5cfh)+b^2(24d^3e^2g+c^3f^2h-8cd^2e(3fg+eh)+c^2df(3fg+4eh))-2abd(5c^2f^2}{d^{3/2}(bc-ad)^4\sqrt{-de+cf}}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^3),x]
```

output
```
(-((Sqrt[e + f*x]*(a*b*(-(c^3*f*h) + d^3*x*(-6*e*g + 9*f*g*x + 8*e*h*x) +
c^2*d*(9*f*g + 10*e*h - 6*f*h*x) + c*d^2*(-10*e*g + 14*f*g*x + 14*e*h*x -
9*f*h*x^2)) + b^2*(-(c^3*f*h*x) - 12*d^3*e*g*x^2 + c*d^2*x*(-18*e*g + 3*f*
g*x + 4*e*h*x) + c^2*d*(-4*e*g + 5*f*g*x + 6*e*h*x + f*h*x^2)) + a^2*d*(-1
1*c^2*f*h + c*d*(3*f*g + 2*e*h - 17*f*h*x) + d^2*(f*x*(5*g - 4*h*x) + 2*e*
(g + 2*h*x)))))/(d*(-(b*c) + a*d)^3*(a + b*x)*(c + d*x)^2)) + (4*Sqrt[-(b*
e) + a*f]*(a^2*d*f*h + b^2*(6*d*e*g - 3*c*f*g - 2*c*e*h) + a*b*(-3*d*f*g -
 4*d*e*h + 5*c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(
Sqrt[b]*(b*c - a*d)^4) + ((3*a^2*d^2*f*(d*f*g + 4*d*e*h - 5*c*f*h) + b^2*(
24*d^3*e^2*g + c^3*f^2*h - 8*c*d^2*e*(3*f*g + e*h) + c^2*d*f*(3*f*g + 4*e*
h)) - 2*a*b*d*(5*c^2*f^2*h + 4*d^2*e*(3*f*g + 2*e*h) - c*d*f*(9*f*g + 16*e
*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(3/2)*(b*c -
a*d)^4*Sqrt[-(d*e) + c*f]))/4
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {166, 27, 25, 166, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{\sqrt{e+fx}(6bdeg-3bcfg-2bceh-4adeh+3acfh+f(3bdg-2bch-adh)x)}{2(a+bx)(c+dx)^3}dx}{b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{\sqrt{e+fx}(a(4de-3cf)h-b(6deg-3cfg-2ceh)-f(3bdg-2bch-adh)x)}{(a+bx)(c+dx)^3}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{e+fx}(a(4de-3cf)h-b(6deg-3cfg-2ceh)-f(3bdg-2bch-adh)x)}{(a+bx)(c+dx)^3}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 166$$

$$-\frac{\frac{\int \frac{2df(2de-cf)ha^2-b\left(e(9fg+8eh)d^2-cf(3fg+11eh)d+c^2f^2h\right)a+2b^2de(6deg-3cfg-2ceh)+f\left(\left(-fhc^2-3d(fg+eh)c+9d^2eg\right)b^2+2ad(4cfh-3d(fg+eh))b+2\cdots}{(a+bx)(c+dx)^2\sqrt{e+fx}}}{2d(bc-ad)}\cdots}{2b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\frac{\int \frac{b(de-cf)\left(-df(11cfh-3d(fg+4eh))a^2-b\left(8e(3fg+2eh)d^2-3cf(3fg+8eh)d+c^2f^2h\right)a+4b^2de(6deg-3cfg-2ceh)+f\left(\left(-fhc^2-d(3fg+4eh)c+12d^2eg\right)b^2-a\cdots\right.\right.}{2(a+bx)(c+dx)\sqrt{e+fx}}}{(bc-ad)(de-cf)}\cdots}{2d(bc-ad)}\cdots}{2b\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{b\int \frac{-df(11cfh-3d(fg+4eh))a^2-b\left(8e(3fg+2eh)d^2-3cf(3fg+8eh)d+c^2f^2h\right)a+4b^2de(6deg-3cfg-2ceh)+f\left(\left(-fhc^2-d(3fg+4eh)c+12d^2eg\right)b^2-ad(9dfg+8\cdots\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{2(bc-ad)}\cdots}{2d(bc-ad)}\cdots}{2b(bc\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$-\frac{\frac{b\left(\frac{4d(be-af)\left(a^2dfh-ab(-5cfh+4deh+3dfg)+b^2(-2ceh-3cfg+6deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(3a^2d^2f(-5cfh+4deh+dfg)-2abd\left(5c^2f^2h-cdf(16eh+9fg)+\cdots\right.\right.}{\cdots}\right.}{2(bc-ad)}\cdots}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{b\left(\frac{8d(be-af)\left(a^2dfh-ab(-5cfh+4deh+3dfg)+b^2(-2ceh-3cfg+6deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(3a^2d^2f(-5cfh+4deh+dfg)-2abd\left(5c^2f^2h-cdf(16e\cdots\right.\right.}{\cdots}\right.}{2(bc-ad)}\cdots}{\cdots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

$\downarrow$ 221

$$-\frac{b\left(\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(3a^2d^2f(-5cfh+4deh+dfg)-2abd\left(5c^2f^2h-cdf(16eh+9fg)+4d^2e(2eh+3fg)\right)+b^2\left(c^3f^2h+c^2df(4eh+3fg)-8cd^2e(eh+3fg)+2\right.\right.}{\sqrt{d}(bc-ad)\sqrt{de-cf}}}{2(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{b(a+bx)(c+dx)^2(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^3),x]
```

output
```
-(((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)*(c + d*x)^2)) + (
-(((d*e - c*f)*(3*b*d*g - b*c*h - 2*a*d*h)*Sqrt[e + f*x])/(d*(b*c - a*d)*(
c + d*x)^2)) - (((4*a^2*d^2*f*h - a*b*d*(9*d*f*g + 8*d*e*h - 9*c*f*h) + b^
2*(12*d^2*e*g - c^2*f*h - c*d*(3*f*g + 4*e*h)))*Sqrt[e + f*x])/((b*c - a*d
)*(c + d*x)) + (b*((-8*d*Sqrt[b*e - a*f]*(a^2*d*f*h + b^2*(6*d*e*g - 3*c*f
*g - 2*c*e*h) - a*b*(3*d*f*g + 4*d*e*h - 5*c*f*h))*ArcTanh[(Sqrt[b]*Sqrt[e
 + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)) + (2*(3*a^2*d^2*f*(d*f*g
+ 4*d*e*h - 5*c*f*h) + b^2*(24*d^3*e^2*g + c^3*f^2*h - 8*c*d^2*e*(3*f*g +
e*h) + c^2*d*f*(3*f*g + 4*e*h)) - 2*a*b*d*(5*c^2*f^2*h + 4*d^2*e*(3*f*g +
2*e*h) - c*d*f*(9*f*g + 16*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e
 - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f])))/(2*(b*c - a*d)))/(2*d*(b
*c - a*d)))/(2*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{\left(11(ad-bc)\sqrt{(cf-de)d}\left(\left(\left(\frac{\left(c(-xd+c)h-5\left(\frac{3xd}{5}+c\right)dg\right)xcb^2}{11}+\frac{a\left(c(3xd+c)^2h-9dg\left(d^2x^2+\frac{14}{9}cdx+c^2\right)\right)b}{11}+a^2d\left(\left(\frac{17}{11}cdx+\frac{4}{11}d^2x\right.\right.\right.\right.\right.\right.}{}$
derivativedivides	$2f^3\left(-\frac{\left(-\frac{9}{8}a^2cd^2f^2h+\frac{1}{2}a^2d^3efh+\frac{5}{8}a^2d^3f^2g+\frac{5}{4}abc^2df^2h-\frac{1}{4}abcd^2f^2g-abd^3efg-\frac{1}{8}b^2c^3f^2h-\frac{1}{2}b^2c^2defh-\frac{3}{8}b^2c^2df^2g-\right.}{}\right.$
default	$2f^3\left(-\frac{\left(-\frac{9}{8}a^2cd^2f^2h+\frac{1}{2}a^2d^3efh+\frac{5}{8}a^2d^3f^2g+\frac{5}{4}abc^2df^2h-\frac{1}{4}abcd^2f^2g-abd^3efg-\frac{1}{8}b^2c^3f^2h-\frac{1}{2}b^2c^2defh-\frac{3}{8}b^2c^2df^2g-\right.}{}\right.$

input

```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output

```
1/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*(1/4*(11*(a*d-b*c)*((c*f-d*e)*d)
^(1/2)*((1/11*(c*(-d*x+c)*h-5*(3/5*x*d+c)*d*g)*x*c*b^2+1/11*a*(c*(3*d*x+c)
^2*h-9*d*g*(d^2*x^2+14/9*c*d*x+c^2))*b+a^2*d*((17/11*c*d*x+4/11*d^2*x^2+c^
2)*h-3/11*(5/3*x*d+c)*d*g))*f-2/11*d*e*(((2*c*d*x^2+3*c^2*x)*h-2*(3*d^2*x^
2+9/2*c*d*x+c^2)*g)*b^2+5*a*((4/5*d^2*x^2+7/5*c*d*x+c^2)*h-(3/5*x*d+c)*d*g
)*b+a^2*d*((2*d*x+c)*h+d*g)))*(f*x+e)^(1/2)-15*arctan(d*(f*x+e)^(1/2)/((c*
f-d*e)*d)^(1/2))*((-1/15*c^2*(c*h+3*d*g)*b^2+2/3*a*(c*h-9/5*d*g)*c*d*b+a^2
*d^2*(c*h-1/5*d*g))*f^2-4/5*((1/3*h*c^2-2*c*d*g)*b^2+2*a*(4/3*c*d*h-d^2*g)
*b+a^2*d^2*h)*d*e*f+16/15*d^2*(1/2*(c*h-3*d*g)*b+a*d*h)*b*e^2)*(d*x+c)^2*(
b*x+a))*((a*f-b*e)*b)^(1/2)+arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))*((
-3*b^2*c*g+a*(5*c*h-3*d*g)*b+a^2*d*h)*f-4*(1/2*(c*h-3*d*g)*b+a*d*h)*b*e)*d
*((c*f-d*e)*d)^(1/2)*(d*x+c)^2*(b*x+a)*(a*f-b*e))/(d*x+c)^2/(a*d-b*c)^4/(b
*x+a)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3105 vs. $2(459) = 918$.

Time = 23.30 (sec) , antiderivative size = 12465, normalized size of antiderivative = 25.39

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(459) = 918$.

Time = 0.19 (sec) , antiderivative size = 959, normalized size of antiderivative = 1.95

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(6*b^3*d*e^2*g - 3*b^3*c*e*f*g - 9*a*b^2*d*e*f*g + 3*a*b^2*c*f^2*g + 3*a^
2*b*d*f^2*g - 2*b^3*c*e^2*h - 4*a*b^2*d*e^2*h + 7*a*b^2*c*e*f*h + 5*a^2*b*
d*e*f*h - 5*a^2*b*c*f^2*h - a^3*d*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*
e + a*b*f))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b^2*e + a*b*f)) + 1/4*(24*b^2*d^3*e^2*g - 24*b^2*c*d^2*e*
f*g - 24*a*b*d^3*e*f*g + 3*b^2*c^2*d*f^2*g + 18*a*b*c*d^2*f^2*g + 3*a^2*d^
3*f^2*g - 8*b^2*c*d^2*e^2*h - 16*a*b*d^3*e^2*h + 4*b^2*c^2*d*e*f*h + 32*a*
b*c*d^2*e*f*h + 12*a^2*d^3*e*f*h + b^2*c^3*f^2*h - 10*a*b*c^2*d*f^2*h - 15
*a^2*c*d^2*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*sqrt(-d^
2*e + c*d*f)) - (sqrt(f*x + e)*b^2*e*f*g - sqrt(f*x + e)*a*b*f^2*g - sqrt(
f*x + e)*a*b*e*f*h + sqrt(f*x + e)*a^2*f^2*h)/((b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*((f*x + e)*b - b*e + a*f)) - 1/4*(8*(f*x + e)^(3/
2)*b*d^3*e*f*g - 8*sqrt(f*x + e)*b*d^3*e^2*f*g - 3*(f*x + e)^(3/2)*b*c*d^2
*f^2*g - 5*(f*x + e)^(3/2)*a*d^3*f^2*g + 13*sqrt(f*x + e)*b*c*d^2*e*f^2*g
+ 3*sqrt(f*x + e)*a*d^3*e*f^2*g - 5*sqrt(f*x + e)*b*c^2*d*f^3*g - 3*sqrt(f
*x + e)*a*c*d^2*f^3*g - 4*(f*x + e)^(3/2)*b*c*d^2*e*f*h - 4*(f*x + e)^(3/2
)*a*d^3*e*f*h + 4*sqrt(f*x + e)*b*c*d^2*e^2*f*h + 4*sqrt(f*x + e)*a*d^3*e^
2*f*h - (f*x + e)^(3/2)*b*c^2*d*f^2*h + 9*(f*x + e)^(3/2)*a*c*d^2*f^2*h -
5*sqrt(f*x + e)*b*c^2*d*e*f^2*h - 11*sqrt(f*x + e)*a*c*d^2*e*f^2*h + sq...
```

Mupad [B] (verification not implemented)

Time = 116.80 (sec) , antiderivative size = 97984, normalized size of antiderivative = 199.56

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^2*(c + d*x)^3),x)
```

output
```
(log((((((b^2*d*f^3*(a*f - b*e)*(8*a*d^2*e*h + 3*a*d^2*f*g - 12*b*d^2*e*g
- b*c^2*f*h - 11*a*c*d*f*h + 4*b*c*d*e*h + 9*b*c*d*f*g))/(a*d - b*c) - (b^
2*d^2*f^2*(e + f*x)^(1/2)*(a*d - b*c)^2*(a*d*f + b*c*f - 2*b*d*e)*(-(2*(f^
6*(a*d - b*c)^24*(16*a*d^2*e*h^2 + b*c^2*f*h^2 + 9*b*d^2*f*g^2 - 16*a*c*d*
f*h^2 + 8*b*c*d*e*h^2 - 24*b*d^2*e*g*h + 6*b*c*d*f*g*h)^2)^(1/2) + 2*b^13*
c^14*f^4*h^2 + 2304*a^8*b^5*d^14*e^4*g^2 + 1024*a^10*b^3*d^14*e^4*h^2 + 23
04*b^13*c^8*d^6*e^4*g^2 + 256*b^13*c^10*d^4*e^4*h^2 + 18*b^13*c^12*d^2*f^4
*g^2 + 18*a^12*b*d^14*f^4*g^2 + 32*a^13*c*d^13*f^4*h^2 - 32*a^13*d^14*e*f^
3*h^2 - 56*a*b^12*c^13*d*f^4*h^2 - 3072*a^9*b^4*d^14*e^4*g*h + 16*b^13*c^1
3*d*e*f^3*h^2 - 1536*b^13*c^9*d^5*e^4*g*h - 18432*a*b^12*c^7*d^7*e^4*g^2 -
 18432*a^7*b^6*c*d^13*e^4*g^2 - 1024*a*b^12*c^9*d^5*e^4*h^2 + 360*a*b^12*c
^11*d^3*f^4*g^2 - 7168*a^9*b^4*c*d^13*e^4*h^2 + 360*a^11*b^2*c*d^13*f^4*g^
2 + 514*a^12*b*c^2*d^12*f^4*h^2 - 4608*a^9*b^4*d^14*e^3*f*g^2 - 576*a^11*b
^2*d^14*e*f^3*g^2 - 1536*a^11*b^2*d^14*e^3*f*h^2 + 576*a^12*b*d^14*e^2*f^2
*h^2 - 4608*b^13*c^9*d^5*e^3*f*g^2 - 576*b^13*c^11*d^3*e*f^3*g^2 - 256*b^1
3*c^11*d^3*e^3*f*h^2 + 12*b^13*c^13*d*f^4*g*h + 64512*a^2*b^11*c^6*d^8*e^4
*g^2 - 129024*a^3*b^10*c^5*d^9*e^4*g^2 + 161280*a^4*b^9*c^4*d^10*e^4*g^2 -
 129024*a^5*b^8*c^3*d^11*e^4*g^2 + 64512*a^6*b^7*c^2*d^12*e^4*g^2 - 2268*a
^2*b^11*c^10*d^4*f^4*g^2 + 6144*a^3*b^10*c^7*d^7*e^4*h^2 + 3528*a^3*b^10*c
^9*d^5*f^4*g^2 - 10752*a^4*b^9*c^6*d^8*e^4*h^2 + 4302*a^4*b^9*c^8*d^6*f...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8595, normalized size of antiderivative = 17.51

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2/(d*x+c)^3,x)
```

output

```
(4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**3*c**3*d**3*f**2*h - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**2*d**4*e*f*h + 8*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**2*d**4*f**2*
h*x - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**3*c*d**5*e*f*h*x + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c*d**5*f**2*h*x**2 - 4*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*d**6*e*f*
h*x**2 + 20*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**2*b*c**4*d**2*f**2*h - 36*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**3*d**3*e*f*h - 12*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*
b*c**3*d**3*f**2*g + 44*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**2*b*c**3*d**3*f**2*h*x + 16*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**4*e*
*2*h + 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**2*b*c**2*d**4*e*f*g - 76*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**4*e*f*h*x - 24*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b
*c**2*d**4*f**2*g*x + 28*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)...
```

3.102 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 686

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \frac{(a^2d^2fh-abd(5dfg+6deh-9cfh)+b^2(12d^2eg+2c^2fh-cd(7fg+6eh)))}{4b^2(bc-ad)^3(c+dx)^2}$$

$$+\frac{(a^2dfh+b^2(8deg-3cfg-4ceh)-ab(5dfg+4deh-7cfh))\sqrt{e+fx}}{4b^2(bc-ad)^2(a+bx)(c+dx)^2}$$

$$+\frac{3(a^2d^2fh+2abd(3cfh-2d(fg+eh))+b^2(8d^2eg+c^2fh-4cd(fg+eh)))\sqrt{e+fx}}{4b(bc-ad)^4(c+dx)}$$

$$-\frac{(bg-ah)(e+fx)^{3/2}}{2b(bc-ad)(a+bx)^2(c+dx)^2}$$

$$+\frac{3(a^3d^2f^2h-a^2bdf(5dfg+8deh-10cfh)+ab^2(5c^2f^2h-10cdf(fg+2eh)+4d^2e(5fg+2eh))-b^3(16}{4\sqrt{b}(bc-ad)^5\sqrt{be-af}}$$

$$+\frac{3(a^2d^2f(dfg+4deh-5cfh)-2abd(5c^2f^2h-5cdf(fg+2eh)+2d^2e(3fg+2eh))+b^2(16d^3e^2g-c^3f^2}{4\sqrt{d}(bc-ad)^5\sqrt{de-cf}}$$

output
```
1/4*(a^2*d^2*f*h-a*b*d*(-9*c*f*h+6*d*e*h+5*d*f*g)+b^2*(12*d^2*e*g+2*c^2*f*
h-c*d*(6*e*h+7*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^3/(d*x+c)^2+1/4*(a^2*d*
f*h+b^2*(-4*c*e*h-3*c*f*g+8*d*e*g)-a*b*(-7*c*f*h+4*d*e*h+5*d*f*g))*(f*x+e)
^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)/(d*x+c)^2+3/4*(a^2*d^2*f*h+2*a*b*d*(3*c*f*
h-2*d*(e*h+f*g))+b^2*(8*d^2*e*g+c^2*f*h-4*c*d*(e*h+f*g)))*(f*x+e)^(1/2)/b/
(-a*d+b*c)^4/(d*x+c)-1/2*(-a*h+b*g)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)^2/(
d*x+c)^2+3/4*(a^3*d^2*f^2*h-a^2*b*d*f*(-10*c*f*h+8*d*e*h+5*d*f*g)+a*b^2*(5
*c^2*f^2*h-10*c*d*f*(2*e*h+f*g)+4*d^2*e*(2*e*h+5*f*g))-b^3*(16*d^2*e^2*g-4
*c*d*e*(2*e*h+3*f*g)+c^2*f*(4*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a
*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^5/(-a*f+b*e)^(1/2)+3/4*(a^2*d^2*f*(-5*c*
f*h+4*d*e*h+d*f*g)-2*a*b*d*(5*c^2*f^2*h-5*c*d*f*(2*e*h+f*g)+2*d^2*e*(2*e*h
+3*f*g))+b^2*(16*d^3*e^2*g-c^3*f^2*h-4*c*d^2*e*(2*e*h+5*f*g)+c^2*d*f*(8*e*
h+5*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b
*c)^5/(-c*f+d*e)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6339 vs. $2(686) = 1372$.

Time = 16.36 (sec) , antiderivative size = 6339, normalized size of antiderivative = 9.24

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {166, 27, 166, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int \frac{\sqrt{e+fx}(a(4de-3cf)h-b(8deg-3cfg-4ceh)-f(5bdg-4bch-adh)x)}{2(a+bx)^2(c+dx)^3}dx}{2b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{e+fx}(a(4de-3cf)h-b(8deg-3cfg-4ceh)-f(5bdg-4bch-adh)x)}{(a+bx)^2(c+dx)^3}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 166$$

$$\int \frac{8be(de-cf)(2bdg-bch-adh)+(4de-cf)\left(dfha^2-b(5dfg+4deh-7cfh)a+b^2(8deg-3cfg-4ceh)\right)+f\Big(\Big(8fhc^2-5d(5fg+4eh)c+40d^2eg\Big)b^2+ad(29cfh-5d(3fg+4e}{2(a+bx)(c+dx)^3\sqrt{e+fx}}$$

$$b(bc-ad)$$

$$4b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\int \frac{8be(de-cf)(2bdg-bch-adh)+(4de-cf)\left(dfha^2-b(5dfg+4deh-7cfh)a+b^2(8deg-3cfg-4ceh)\right)+f\Big(\Big(8fhc^2-5d(5fg+4eh)c+40d^2eg\Big)b^2+ad(29cfh-5d(3fg+4e}{(a+bx)(c+dx)^3\sqrt{e+fx}}$$

$$2b(bc-ad)$$

$$4b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 168$$

$$\int \frac{6b(de-cf)\Big(df(2de-cf)ha^2-b\Big(8e(fg+eh)d^2-cf(3fg+14eh)d+3c^2f^2h\Big)a+b^2\Big(f(fg+4eh)c^2-4de(3fg+2eh)c+16d^2e^2g\Big)+f\Big(\Big(2fhc^2-d(7fg+6eh)c+12d^2eg\Big)b}{(a+bx)(c+dx)^2\sqrt{e+fx}}$$

$$2(bc-ad)(de-cf)$$

$$2b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$3b\int \frac{df(2de-cf)ha^2-b\left(8e(fg+eh)d^2-cf(3fg+14eh)d+3c^2f^2h\right)a+b^2\left(f(fg+4eh)c^2-4de(3fg+2eh)c+16d^2e^2g\right)+f\left(\left(2fhc^2-d(7fg+6eh)c+12d^2eg\right)b^2-ad(5df \cdots}{(a+bx)(c+dx)^2\sqrt{e+fx}} \Big/ (bc-ad) \cdots \Big/ 2b(bc-ad)$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 168$$

$$3b\left(\frac{\int \frac{b(de-cf)\left(df(dfg+4deh-4cfh)a^2-2b\left(2e(3fg+2eh)d^2-cf(3fg+8eh)d+2c^2f^2h\right)a+b^2\left(f(fg+4eh)c^2-4de(3fg+2eh)c+16d^2e^2g\right)+f\left(\left(fhc^2-4d(fg+eh)c \cdots\right.\right.\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{(bc-ad)(de-cf)} \cdots\right. \Big/ (bc-ad) \cdots$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$3b\left(\frac{b\int \frac{df(dfg+4deh-4cfh)a^2-2b\left(2e(3fg+2eh)d^2-cf(3fg+8eh)d+2c^2f^2h\right)a+b^2\left(f(fg+4eh)c^2-4de(3fg+2eh)c+16d^2e^2g\right)+f\left(\left(fhc^2-4d(fg+eh)c+8d^2eg\right)b \cdots\right.}{(a+bx)(c+dx)\sqrt{e+fx}}}{bc-ad} \cdots\right. \Big/ (bc-ad) \cdots$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$3b\left(b\left(-\frac{\left(a^2d^2f(-5cfh+4deh+dfg)-2abd\left(5c^2f^2h-5cdf(2eh+fg)+2d^2e(2eh+3fg)\right)+b^2\left(c^3\left(-f^2\right)h+c^2df(8eh+5fg)-4cd^2e(2eh+5fg)+16d^3e^2g\right)\right)\int \frac{1}{(c+dx) \cdots}}{bc-ad} \cdots\right.\right. \cdots$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$3b\left(\frac{b\left(-\frac{2\left(a^2d^2f(-5cfh+4deh+dfg)-2abd\left(5c^2f^2h-5cdf(2eh+fg)+2d^2e(2eh+3fg)\right)+b^2\left(c^3\left(-f^2\right)h+c^2df(8eh+5fg)-4cd^2e(2eh+5fg)+16d^3e^2g\right)\right)\int \frac{}{c+\frac{d(e}{}}}{f(bc-ad)}\right.}{}\right.$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{e+fx}\left(a^2dfh-ab(-7cfh+4deh+5dfg)+b^2(-4ceh-3cfg+8deg)\right)}{b(a+bx)(c+dx)^2(bc-ad)}+\frac{2\sqrt{e+fx}\left(a^2d^2fh-abd(-9cfh+6deh+5dfg)+b^2\left(2c^2fh-cd(6eh+7fg)+12d^2eg\right)\right.}{(c+dx)^2(bc-ad)}}{}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{2b(a+bx)^2(c+dx)^2(bc-ad)}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^3),x]
```

output
```
-1/2*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2)
 + (((a^2*d*f*h + b^2*(8*d*e*g - 3*c*f*g - 4*c*e*h) - a*b*(5*d*f*g + 4*d*e
*h - 7*c*f*h))*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)*(c + d*x)^2) + ((2*
(a^2*d^2*f*h - a*b*d*(5*d*f*g + 6*d*e*h - 9*c*f*h) + b^2*(12*d^2*e*g + 2*c
^2*f*h - c*d*(7*f*g + 6*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)^2) +
(3*b*((2*(a^2*d^2*f*h + 2*a*b*d*(3*c*f*h - 2*d*(f*g + e*h)) + b^2*(8*d^2*e
*g + c^2*f*h - 4*c*d*(f*g + e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x))
 + (b*((2*(a^3*d^2*f^2*h - a^2*b*d*f*(5*d*f*g + 8*d*e*h - 10*c*f*h) + a*b^2
*(5*c^2*f^2*h - 10*c*d*f*(f*g + 2*e*h) + 4*d^2*e*(5*f*g + 2*e*h)) - b^3*(1
6*d^2*e^2*g - 4*c*d*e*(3*f*g + 2*e*h) + c^2*f*(f*g + 4*e*h)))*ArcTanh[(Sqr
t[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e - a*f]
) + (2*(a^2*d^2*f*(d*f*g + 4*d*e*h - 5*c*f*h) - 2*a*b*d*(5*c^2*f^2*h - 5*c
*d*f*(f*g + 2*e*h) + 2*d^2*e*(3*f*g + 2*e*h)) + b^2*(16*d^3*e^2*g - c^3*f^
2*h - 4*c*d^2*e*(5*f*g + 2*e*h) + c^2*d*f*(5*f*g + 8*e*h)))*ArcTanh[(Sqrt[
d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f]))
)/(b*c - a*d)))/(b*c - a*d))/(2*b*(b*c - a*d)))/(4*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{3\left(\left(-16d^2e^2g+8c\left(eh+\frac{3fg}{2}\right)ed-4c^2efh-g\,f^2c^2\right)b^3+5a\left(4\left(\frac{2}{5}e^2h+efg\right)d^2+2c\left(-2efh-f^2g\right)d+c^2f^2h\right)b^2+10a^2d\left(\left(-\frac{4eh}{5}-\frac{fg}{2}\right)d\right.}{4}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/((c*f-d*e)*d)^(1/2)*(1/4*((-16*d^2*e^2*g+8*c*(e*h+3/2*f*g)*e*d-4*c^2*e*f
*h-g*f^2*c^2)*b^3+5*a*(4*(2/5*e^2*h+e*f*g)*d^2+2*c*(-2*e*f*h-f^2*g)*d+c^2*
f^2*h)*b^2+10*a^2*d*((-4/5*e*h-1/2*f*g)*d+c*f*h)*f*b+a^3*d^2*f^2*h)*((c*f-
d*e)*d)^(1/2)*(d*x+c)^2*(b*x+a)^2*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/
2))+((a*f-b*e)*b)^(1/2)*(-5/4*((-16/5*d^3*e^2*g+4*c*(2/5*e^2*h+e*f*g)*d^2+
c^2*(-8/5*e*f*h-f^2*g)*d+1/5*c^3*f^2*h)*b^2+2*a*d*(2/5*(2*e^2*h+3*e*f*g)*d
^2+c*(-2*e*f*h-f^2*g)*d+c^2*f^2*h)*b+a^2*d^2*(1/5*(-4*e*h-f*g)*d+c*f*h)*f)
*(d*x+c)^2*(b*x+a)^2*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)
*((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((2*d^3*e*g*x^3+3*x^2*c*(-1/3*f*g*x+e*(
-1/3*h*x+g))*d^2+2/3*x*c^2*(1/8*(3*h*x^2-19*g*x)*f+e*(-9/4*h*x+g))*d-1/6*c
^3*(5/2*x*(-h*x+g)*f+e*(2*h*x+g)))*b^3-1/6*a*(6*(f*g*x^3-3*x^2*(-1/3*h*x+g
)*e)*d^3-28*(-17/28*x*(-9/17*h*x+g)*f+e*(-9/14*h*x+g))*x*c*d^2-7*c^2*(-31/
14*x*(-h*x+g)*f+e*(-16/7*h*x+g))*d+c^3*(1/2*(-19*h*x+3*g)*f+e*h))*b^2+a^2*
(2/3*x*(1/8*(3*h*x^2-19*g*x)*f+e*(-9/4*h*x+g))*d^3+7/6*c*(-31/14*x*(-h*x+g
)*f+e*(-16/7*h*x+g))*d^2-5/3*c^2*(1/10*(-17*h*x+9*g)*f+e*h)*d+c^3*f*h)*b+a
^3*d*(1/6*(-5/2*x*(-h*x+g)*f-e*(2*h*x+g))*d^2-1/6*c*(1/2*(-19*h*x+3*g)*f+e
*h)*d+c^2*f*h))))/((a*f-b*e)*b)^(1/2)/(a*d-b*c)^5/(b*x+a)^2/(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6917 vs. $2(646) = 1292$.

Time = 54.03 (sec) , antiderivative size = 27720, normalized size of antiderivative = 40.41

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2182 vs. $2(646) = 1292$.

Time $= 0.41$ (sec) , antiderivative size $= 2182$, normalized size of antiderivative $= 3.18$

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
3/4*(16*b^3*d^2*e^2*g - 12*b^3*c*d*e*f*g - 20*a*b^2*d^2*e*f*g + b^3*c^2*f^
2*g + 10*a*b^2*c*d*f^2*g + 5*a^2*b*d^2*f^2*g - 8*b^3*c*d*e^2*h - 8*a*b^2*d
^2*e^2*h + 4*b^3*c^2*e*f*h + 20*a*b^2*c*d*e*f*h + 8*a^2*b*d^2*e*f*h - 5*a*
b^2*c^2*f^2*h - 10*a^2*b*c*d*f^2*h - a^3*d^2*f^2*h)*arctan(sqrt(f*x + e)*b
/sqrt(-b^2*e + a*b*f))/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(-b^2*e + a*b*f)) - 3/4*(1
6*b^2*d^3*e^2*g - 20*b^2*c*d^2*e*f*g - 12*a*b*d^3*e*f*g + 5*b^2*c^2*d*f^2*
g + 10*a*b*c*d^2*f^2*g + a^2*d^3*f^2*g - 8*b^2*c*d^2*e^2*h - 8*a*b*d^3*e^2
*h + 8*b^2*c^2*d*e*f*h + 20*a*b*c*d^2*e*f*h + 4*a^2*d^3*e*f*h - b^2*c^3*f^
2*h - 10*a*b*c^2*d*f^2*h - 5*a^2*c*d^2*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(
-d^2*e + c*d*f))/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(-d^2*e + c*d*f)) + 1/4*(24*(f*x
 + e)^(7/2)*b^3*d^3*e*f*g - 72*(f*x + e)^(5/2)*b^3*d^3*e^2*f*g + 72*(f*x +
 e)^(3/2)*b^3*d^3*e^3*f*g - 24*sqrt(f*x + e)*b^3*d^3*e^4*f*g - 12*(f*x + e
)^(7/2)*b^3*c*d^2*f^2*g - 12*(f*x + e)^(7/2)*a*b^2*d^3*f^2*g + 72*(f*x + e
)^(5/2)*b^3*c*d^2*e*f^2*g + 72*(f*x + e)^(5/2)*a*b^2*d^3*e*f^2*g - 108*(f*
x + e)^(3/2)*b^3*c*d^2*e^2*f^2*g - 108*(f*x + e)^(3/2)*a*b^2*d^3*e^2*f^2*g
 + 48*sqrt(f*x + e)*b^3*c*d^2*e^3*f^2*g + 48*sqrt(f*x + e)*a*b^2*d^3*e^3*f
^2*g - 19*(f*x + e)^(5/2)*b^3*c^2*d*f^3*g - 34*(f*x + e)^(5/2)*a*b^2*c*d^2
*f^3*g - 19*(f*x + e)^(5/2)*a^2*b*d^3*f^3*g + 46*(f*x + e)^(3/2)*b^3*c^...
```

Mupad [B] (verification not implemented)

Time = 153.83 (sec) , antiderivative size = 133067, normalized size of antiderivative = 193.98

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^3*(c + d*x)^3),x)
```

output
```
(log((((((3*b^2*d^2*f^3*(a^2*d^2*f^2*g + b^2*c^2*f^2*g + 8*b^2*d^2*e^2*g -
 4*a*b*c^2*f^2*h - 4*a*b*d^2*e^2*h - 4*a^2*c*d*f^2*h - 4*b^2*c*d*e^2*h + 3
*a^2*d^2*e*f*h + 3*b^2*c^2*e*f*h + 6*a*b*c*d*f^2*g - 8*a*b*d^2*e*f*g - 8*b
^2*c*d*e*f*g + 10*a*b*c*d*e*f*h))/(a*d - b*c)^2 - (b^2*d^2*f^2*(e + f*x)^(
1/2)*(a*d - b*c)^2*(a*d*f + b*c*f - 2*b*d*e)*(-(288*(f^8*(a*d - b*c)^30*(a
*d*e*h^2 - a*c*f*h^2 + b*c*e*h^2 + b*d*f*g^2 - 2*b*d*e*g*h)^2)^(1/2) - 147
456*a^10*b^6*d^16*e^5*g^2 - 36864*a^12*b^4*d^16*e^5*h^2 - 147456*b^16*c^10
*d^6*e^5*g^2 - 36864*b^16*c^12*d^4*e^5*h^2 + 288*a*b^15*c^16*f^5*h^2 + 288
*a^15*b*d^16*f^5*g^2 + 288*a^16*c*d^15*f^5*h^2 + 288*b^16*c^15*d*f^5*g^2 -
 288*a^16*d^16*e*f^4*h^2 - 288*b^16*c^16*e*f^4*h^2 + 147456*a^11*b^5*d^16*
e^5*g*h + 147456*b^16*c^11*d^5*e^5*g*h + 1474560*a*b^15*c^9*d^7*e^5*g^2 +
1474560*a^9*b^7*c*d^15*e^5*g^2 + 294912*a*b^15*c^11*d^5*e^5*h^2 + 10080*a*
b^15*c^14*d^2*f^5*g^2 + 294912*a^11*b^5*c*d^15*e^5*h^2 + 10080*a^14*b^2*c*
d^15*f^5*g^2 + 10080*a^2*b^14*c^15*d*f^5*h^2 + 10080*a^15*b*c^2*d^14*f^5*h
^2 + 368640*a^11*b^5*d^16*e^4*f*g^2 - 14400*a^14*b^2*d^16*e*f^4*g^2 + 7372
8*a^13*b^3*d^16*e^4*f*h^2 + 9216*a^15*b*d^16*e^2*f^3*h^2 + 368640*b^16*c^1
1*d^5*e^4*f*g^2 - 14400*b^16*c^14*d^2*e*f^4*g^2 + 73728*b^16*c^13*d^3*e^4*
f*h^2 + 9216*b^16*c^15*d*e^2*f^3*h^2 - 6635520*a^2*b^14*c^8*d^8*e^5*g^2 +
17694720*a^3*b^13*c^7*d^9*e^5*g^2 - 30965760*a^4*b^12*c^6*d^10*e^5*g^2 + 3
7158912*a^5*b^11*c^5*d^11*e^5*g^2 - 30965760*a^6*b^10*c^4*d^12*e^5*g^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 19308, normalized size of antiderivative = 28.15

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^3/(d*x+c)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**5*c**3*d**3*f**3*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**2*d**4*e*f**2*h + 6*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**2*d**4*f*
*3*h*x - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**5*c*d**5*e*f**2*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c*d**5*f**3*h*x**2 - 3*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*d**
6*e*f**2*h*x**2 + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**4*b*c**4*d**2*f**3*h - 54*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**3*d**3*e*f**2*
h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**4*b*c**3*d**3*f**3*g + 66*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**3*d**3*f**3*h*x + 24*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*
c**2*d**4*e**2*f*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**4*e*f**2*g - 114*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**4*e
*f**2*h*x - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**4*b*c**2*d**4*f**3*g*x + 42*sqrt(b)*sqrt(a*f - b*e)*a...
```

3.103 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1108

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \frac{d(5a^3d^2f^2h - 5a^2bdf(7dfg+10deh-14cfh) - b^3(120d^2e^2g - 8cde(11fg+9}{24b^2(bc-}$$

$$+ \frac{(a^2dfh - ab(7dfg+4deh-9cfh) + b^2(10deg - 3c(fg+2eh)))\sqrt{e+fx}}{12b^2(bc-ad)^2(a+bx)^2(c+dx)^2}$$

$$+ \frac{(3a^3d^2f^2h - a^2bdf(21dfg+32deh-44cfh) + ab^2(33c^2f^2h - 14cdf(4fg+7eh) + 2d^2e(49fg+16eh)) -}{24b^2(bc-ad)^3(be-af)(a+bx)(c+dx}$$

$$+ \frac{d(5a^3d^2f^2h - a^2bdf(29dfg+36deh-50cfh) + ab^2(25c^2f^2h - 50cdf(fg+2eh) + 4d^2e(27fg+8eh)) -}{8b(bc-ad)^5(be-af)(c+dx)}$$

$$- \frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}$$

$$+ \frac{(5a^4d^3f^3h - 5a^3bd^2f^2(7dfg+12deh-15cfh) + b^4(160d^3e^3g + c^3f^2(fg-6eh) - 48cd^2e^2(3fg+2eh) +}{}$$

$$- \frac{\sqrt{d}(3a^2d^2f(dfg+4deh-5cfh) + b^2(80d^3e^2g - 15c^3f^2h - 16cd^2e(7fg+3eh) + 5c^2df(7fg+12eh)) -}{4(bc-ad)^6\sqrt{de-cf}}$$

output
```
1/24*d*(5*a^3*d^2*f^2*h-5*a^2*b*d*f*(-14*c*f*h+10*d*e*h+7*d*f*g)-b^3*(120*
d^2*e^2*g-8*c*d*e*(9*e*h+11*f*g)+3*c^2*f*(14*e*h+f*g))+a*b^2*(45*c^2*f^2*h
+8*d^2*e*(6*e*h+19*f*g)-2*c*d*f*(74*e*h+41*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+
b*c)^4/(-a*f+b*e)/(d*x+c)^2+1/12*(a^2*d*f*h-a*b*(-9*c*f*h+4*d*e*h+7*d*f*g)
+b^2*(10*d*e*g-3*c*(2*e*h+f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^2/
(d*x+c)^2+1/24*(3*a^3*d^2*f^2*h-a^2*b*d*f*(-44*c*f*h+32*d*e*h+21*d*f*g)+a*
b^2*(33*c^2*f^2*h-14*c*d*f*(7*e*h+4*f*g)+2*d^2*e*(16*e*h+49*f*g))-b^3*(80*
d^2*e^2*g+3*c^2*f*(10*e*h+f*g)-2*c*d*e*(24*e*h+31*f*g)))*(f*x+e)^(1/2)/b^2
/(-a*d+b*c)^3/(-a*f+b*e)/(b*x+a)/(d*x+c)^2+1/8*d*(5*a^3*d^2*f^2*h-a^2*b*d*
f*(-50*c*f*h+36*d*e*h+29*d*f*g)+a*b^2*(25*c^2*f^2*h-50*c*d*f*(2*e*h+f*g)+4
*d^2*e*(8*e*h+27*f*g))-b^3*(80*d^2*e^2*g-4*c*d*e*(12*e*h+13*f*g)+c^2*f*(24
*e*h+f*g)))*(f*x+e)^(1/2)/b/(-a*d+b*c)^5/(-a*f+b*e)/(d*x+c)-1/3*(-a*h+b*g)
*(f*x+e)^(3/2)/b/(-a*d+b*c)/(b*x+a)^3/(d*x+c)^2+1/8*(5*a^4*d^3*f^3*h-5*a^3
*b*d^2*f^2*(-15*c*f*h+12*d*e*h+7*d*f*g)+b^4*(160*d^3*e^3*g+c^3*f^2*(-6*e*h
+f*g)-48*c*d^2*e^2*(2*e*h+3*f*g)+18*c^2*d*e*f*(4*e*h+f*g))+15*a^2*b^2*d*f*
(5*c^2*f^2*h+2*d^2*e*(4*e*h+7*f*g)-c*d*f*(18*e*h+7*f*g))+a*b^3*(5*c^3*f^3*
h-16*d^3*e^2*(4*e*h+21*f*g)+36*c*d^2*e*f*(8*e*h+7*f*g)-3*c^2*d*f^2*(48*e*h
+7*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*
c)^6/(-a*f+b*e)^(3/2)-1/4*d^(1/2)*(3*a^2*d^2*f*(-5*c*f*h+4*d*e*h+d*f*g)+b^
2*(80*d^3*e^2*g-15*c^3*f^2*h-16*c*d^2*e*(3*e*h+7*f*g)+5*c^2*d*f*(12*e*h...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17142 vs. 2(1108) = 2216.

Time = 16.65 (sec) , antiderivative size = 17142, normalized size of antiderivative = 15.47

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {166, 27, 166, 27, 168, 27, 168, 27, 168, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{\sqrt{e+fx}(10bdeg-3bcfg-6bceh-4adeh+3acfh+f(7bdg-6bch-adh)x)}{2(a+bx)^3(c+dx)^3}dx}{3b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{e+fx}(10bdeg-a(4de-3cf)h-3bc(fg+2eh)+f(7bdg-6bch-adh)x)}{(a+bx)^3(c+dx)^3}dx}{6b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{166}$$

$$-\frac{\frac{\int -\frac{8be(de-cf)(5bdg-3bch-2adh)+(4de-cf)\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3cfg-6ceh)\right)+f\left(\left(24fhc^2-7d(7fg+6eh)c+70d^2eg\right)b^2+ad(43cfh-7}{2(a+bx)^2(c+dx)^3\sqrt{e+fx}}}{2b(bc-ad)}}{6b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{8be(de-cf)(5bdg-3bch-2adh)+(4de-cf)\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)+f\left(\left(24fhc^2-7d(7fg+6eh)c+70d^2eg\right)b^2+ad(43cfh-}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}}{4b(bc-ad)}}{6b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{\sqrt{e+fx}\left(3a^3d^2f^2h-a^2bdf(-44cfh+32deh+21dfg)+ab^2\left(33c^2f^2h-14cdf(7eh+4fg)+2d^2e(16eh+49fg)\right)-b^3\left(3c^2f(10eh+fg)-2cde(24eh+31fg)+80d^2e^2g\right.\right.}{(a+bx)(c+dx)^2(bc-ad)(be-af)}}{}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{5d^2f^2(4de-cf)ha^3-5bdf\left(4e(7fg+10eh)d^2-cf(7fg+64eh)d+12c^2f^2h\right)a^2-b^2\left(-32e^2(19fg+6eh)d^3+2cef(223fg+312eh)d^2-6c^2f^2(8fg+47eh)d+15c^3\right.}{}}{}}{}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{3b(a+bx)^3(c+dx)^2(bc-ad)}$$

$\downarrow$ 168

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\right.\right.}{(bc-ad)(be-af)(}}{}$$

$\downarrow$ 27

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\right.\right.}{(bc-ad)(be-af)(}}{}$$

$\downarrow$ 168

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\right.\right.}{(bc-ad)(be-af)(}}{}$$

$\downarrow$ 25

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\ldots\right.\right.}{(bc-ad)(be-af)(\ldots}}{}$$

$\downarrow$ 27

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\ldots\right.\right.}{(bc-ad)(be-af)(\ldots}}{}$$

$\downarrow$ 174

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\ldots\right.\right.}{(bc-ad)(be-af)(\ldots}}{}$$

$\downarrow$ 73

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\ldots\right.\right.}{(bc-ad)(be-af)(\ldots}}{}$$

$\downarrow$ 221

$$-\frac{(bg-ah)(e+fx)^{3/2}}{3b(bc-ad)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(7dfg+4deh-9cfh)a+b^2(10deg-3c(fg+2eh))\right)}{2b(bc-ad)(a+bx)^2(c+dx)^2}-\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(21dfg+32deh-44cfh)a^2+b^2\left(2e(49fg+16eh)d^2-14cf(\ldots\right.\right.}{(bc-ad)(be-af)(\ldots}}{\ldots}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^3),x]
```

output
```
-1/3*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^3*(c + d*x)^2)
 - (-1/2*((a^2*d*f*h - a*b*(7*d*f*g + 4*d*e*h - 9*c*f*h) + b^2*(10*d*e*g -
 3*c*(f*g + 2*e*h)))*Sqrt[e + f*x])/(b*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2
) - (((3*a^3*d^2*f^2*h - a^2*b*d*f*(21*d*f*g + 32*d*e*h - 44*c*f*h) + a*b^
2*(33*c^2*f^2*h - 14*c*d*f*(4*f*g + 7*e*h) + 2*d^2*e*(49*f*g + 16*e*h)) -
b^3*(80*d^2*e^2*g + 3*c^2*f*(f*g + 10*e*h) - 2*c*d*e*(31*f*g + 24*e*h)))*S
qrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2) + ((2*d*(5*a
^3*d^2*f^2*h - 5*a^2*b*d*f*(7*d*f*g + 10*d*e*h - 14*c*f*h) - b^3*(120*d^2*
e^2*g - 8*c*d*e*(11*f*g + 9*e*h) + 3*c^2*f*(f*g + 14*e*h)) + a*b^2*(45*c^2
*f^2*h + 8*d^2*e*(19*f*g + 6*e*h) - 2*c*d*f*(41*f*g + 74*e*h)))*Sqrt[e + f
*x])/((b*c - a*d)*(c + d*x)^2) + (3*b*((2*d*(5*a^3*d^2*f^2*h - a^2*b*d*f*(
29*d*f*g + 36*d*e*h - 50*c*f*h) + a*b^2*(25*c^2*f^2*h - 50*c*d*f*(f*g + 2*
e*h) + 4*d^2*e*(27*f*g + 8*e*h)) - b^3*(80*d^2*e^2*g - 4*c*d*e*(13*f*g + 1
2*e*h) + c^2*f*(f*g + 24*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)) - (
b*((-2*(5*a^4*d^3*f^3*h - 5*a^3*b*d^2*f^2*(7*d*f*g + 12*d*e*h - 15*c*f*h)
+ b^4*(160*d^3*e^3*g + c^3*f^2*(f*g - 6*e*h) - 48*c*d^2*e^2*(3*f*g + 2*e*h
) + 18*c^2*d*e*f*(f*g + 4*e*h)) + 15*a^2*b^2*d*f*(5*c^2*f^2*h + 2*d^2*e*(7
*f*g + 4*e*h) - c*d*f*(7*f*g + 18*e*h)) + a*b^3*(5*c^3*f^3*h - 16*d^3*e^2*
(21*f*g + 4*e*h) + 36*c*d^2*e*f*(7*f*g + 8*e*h) - 3*c^2*d*f^2*(7*f*g + 48*
e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 52.52 (sec) , antiderivative size = 1520, normalized size of antiderivative = 1.37

method	result	size
pseudoelliptic	Expression too large to display	1520
derivativedivides	Expression too large to display	1973
default	Expression too large to display	1973

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/8*(((c*f-d*e)*d)^(1/2)*(d*x+c)^2*(b*x+a)^3*((32*d^3*e^3*g-96/5*c*(e*h+3/
2*f*g)*e^2*d^2+72/5*c^2*(e*h+1/4*f*g)*f*e*d-6/5*c^3*f^2*(e*h-1/6*f*g))*b^4
+a*((-64/5*e^3*h-336/5*g*f*e^2)*d^3+288/5*c*(e*h+7/8*f*g)*f*e*d^2-144/5*(e
*h+7/48*f*g)*c^2*f^2*d+c^3*f^3*h)*b^3+15*a^2*((8/5*e^2*h+14/5*e*f*g)*d^2-1
8/5*(e*h+7/18*f*g)*c*f*d+c^2*f^2*h)*d*f*b^2+15*a^3*((-4/5*e*h-7/15*f*g)*d+
c*f*h)*d^2*f^2*b+a^4*d^3*f^3*h)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)
)+5*(-6/5*d*(d*x+c)^2*(b*x+a)^3*((-16/3*d^3*e^2*g+16/5*c*(e*h+7/3*f*g)*e*d
^2-4*c^2*f*(e*h+7/12*f*g)*d+c^3*f^2*h)*b^2+10/3*a*d*((16/25*e^2*h+24/25*e*
f*g)*d^2-44/25*(e*h+21/44*f*g)*c*f*d+c^2*f^2*h)*b+a^2*d^2*((-4/5*e*h-1/5*f
*g)*d+c*f*h)*f)*(a*f-b*e)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((-1
6/5*d^4*e^2*g*x^4-24/5*x^3*c*((-2/5*e*h-13/30*f*g)*x+g*e)*e*d^3-16/15*x^2*
c^2*((9/10*e*f*h+3/80*f^2*g)*x^2+(-27/10*e^2*h-61/20*e*f*g)*x+e^2*g)*d^2+4
/15*x*c^3*((-57/10*e*f*h-3/10*f^2*g)*x^2+(12/5*e^2*h+31/10*e*f*g)*x+e^2*g)
*d-8/75*c^4*((15/4*e*f*h+3/8*f^2*g)*x^2+(3/2*e^2*h+7/4*e*f*g)*x+e^2*g))*b^
5-4/75*a*(((-24*e^2*h-81*e*f*g)*x^4+150*e^2*x^3*g)*d^4+230*x^2*c*((15/46*e
*f*h+15/92*f^2*g)*x^2+(-63/115*e^2*h-107/115*e*f*g)*x+e^2*g)*d^3+55*x*c^2*
(-15/44*f^2*h*x^3+(311/110*e*f*h+119/110*f^2*g)*x^2+(-146/55*e^2*h-353/110
*e*f*g)*x+e^2*g)*d^2-13*c^3*(30/13*f^2*h*x^3+(-95/13*e*f*h-17/13*f^2*g)*x^
2+(31/13*e^2*h+35/13*e*f*g)*x+e^2*g)*d+c^4*(-33/4*f^2*h*x^2+(11/2*e*f*h-2*
f^2*g)*x+e*(e*h-1/2*f*g)))*b^4-8/75*a^2*(((27/2*e*f*h+87/8*f^2*g)*x^4+(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**4/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2698 vs. $2(1064) = 2128$.

Time = 0.36 (sec) , antiderivative size = 2698, normalized size of antiderivative = 2.44

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(160*b^4*d^3*e^3*g - 144*b^4*c*d^2*e^2*f*g - 336*a*b^3*d^3*e^2*f*g +
18*b^4*c^2*d*e*f^2*g + 252*a*b^3*c*d^2*e*f^2*g + 210*a^2*b^2*d^3*e*f^2*g +
 b^4*c^3*f^3*g - 21*a*b^3*c^2*d*f^3*g - 105*a^2*b^2*c*d^2*f^3*g - 35*a^3*b
*d^3*f^3*g - 96*b^4*c*d^2*e^3*h - 64*a*b^3*d^3*e^3*h + 72*b^4*c^2*d*e^2*f*
h + 288*a*b^3*c*d^2*e^2*f*h + 120*a^2*b^2*d^3*e^2*f*h - 6*b^4*c^3*e*f^2*h
- 144*a*b^3*c^2*d*e*f^2*h - 270*a^2*b^2*c*d^2*e*f^2*h - 60*a^3*b*d^3*e*f^2
*h + 5*a*b^3*c^3*f^3*h + 75*a^2*b^2*c^2*d*f^3*h + 75*a^3*b*c*d^2*f^3*h + 5
*a^4*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^6*e -
 6*a*b^6*c^5*d*e + 15*a^2*b^5*c^4*d^2*e - 20*a^3*b^4*c^3*d^3*e + 15*a^4*b^
3*c^2*d^4*e - 6*a^5*b^2*c*d^5*e + a^6*b*d^6*e - a*b^6*c^6*f + 6*a^2*b^5*c^
5*d*f - 15*a^3*b^4*c^4*d^2*f + 20*a^4*b^3*c^3*d^3*f - 15*a^5*b^2*c^2*d^4*f
 + 6*a^6*b*c*d^5*f - a^7*d^6*f)*sqrt(-b^2*e + a*b*f)) + 1/4*(80*b^2*d^4*e^
2*g - 112*b^2*c*d^3*e*f*g - 48*a*b*d^4*e*f*g + 35*b^2*c^2*d^2*f^2*g + 42*a
*b*c*d^3*f^2*g + 3*a^2*d^4*f^2*g - 48*b^2*c*d^3*e^2*h - 32*a*b*d^4*e^2*h +
 60*b^2*c^2*d^2*e*f*h + 88*a*b*c*d^3*e*f*h + 12*a^2*d^4*e*f*h - 15*b^2*c^3
*d*f^2*h - 50*a*b*c^2*d^2*f^2*h - 15*a^2*c*d^3*f^2*h)*arctan(sqrt(f*x + e)
*d/sqrt(-d^2*e + c*d*f))/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-d
^2*e + c*d*f)) - 1/4*(16*(f*x + e)^(3/2)*b*d^4*e*f*g - 16*sqrt(f*x + e)*b*
d^4*e^2*f*g - 11*(f*x + e)^(3/2)*b*c*d^3*f^2*g - 5*(f*x + e)^(3/2)*a*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^4*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 98.25 (sec) , antiderivative size = 36506, normalized size of antiderivative = 32.95

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^4(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^4/(d*x+c)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**7*c**3*d**3*f**4*h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**2*d**4*e*f**3*h + 30*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**2*d**4
*f**4*h*x - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**7*c*d**5*e*f**3*h*x + 15*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c*d**5*f**4*h*x**2 - 15*sq
rt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*7*d**6*e*f**3*h*x**2 + 225*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**4*d**2*f**4*h - 405*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**3*d**3
*e*f**3*h - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**6*b*c**3*d**3*f**4*g + 495*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**3*d**3*f**4*h*x +
180*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**6*b*c**2*d**4*e**2*f**2*h + 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**4*e*f**3*g - 855*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
6*b*c**2*d**4*e*f**3*h*x - 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**4*f**4*g*x + 315*sqrt(b)*...
```

3.104 $\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1700

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Too large to display}$$

output
```
1/192*d*(35*a^4*d^3*f^3*h-35*a^3*b*d^2*f^2*(-19*c*f*h+14*d*e*h+9*d*f*g)+b^
4*(1440*d^3*e^3*g+3*c^3*f^2*(-8*e*h+3*f*g)-240*c*d^2*e^2*(4*e*h+5*f*g)+22*
c^2*d*e*f*(32*e*h+3*f*g))+a*b^3*(15*c^3*f^3*h-240*d^3*e^2*(2*e*h+13*f*g)+4
*c*d^2*e*f*(668*e*h+567*f*g)-c^2*d*f^2*(1402*e*h+93*f*g))+a^2*b^2*d*f*(725
*c^2*f^2*h+2*d^2*e*(472*e*h+993*f*g)-c*d*f*(2404*e*h+1041*f*g)))*(f*x+e)^(
1/2)/b^2/(-a*d+b*c)^5/(-a*f+b*e)^2/(d*x+c)^2+1/24*(a^2*d*f*h+b^2*(-8*c*e*h
-3*c*f*g+12*d*e*g)-a*b*(-11*c*f*h+4*d*e*h+9*d*f*g))*(f*x+e)^(1/2)/b^2/(-a*
d+b*c)^2/(b*x+a)^3/(d*x+c)^2+1/96*(3*a^3*d^2*f^2*h-a^2*b*d*f*(-58*c*f*h+40
*d*e*h+27*d*f*g)+a*b^2*(59*c^2*f^2*h+8*d^2*e*(5*e*h+18*f*g)-18*c*d*f*(8*e*
h+5*f*g))-b^3*(120*d^2*e^2*g-16*c*d*e*(5*e*h+6*f*g)+c^2*f*(56*e*h+3*f*g)))
*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^3/(-a*f+b*e)/(b*x+a)^2/(d*x+c)^2+1/192*(21*a
^4*d^3*f^3*h-7*a^3*b*d^2*f^2*(-59*c*f*h+44*d*e*h+27*d*f*g)+b^4*(960*d^3*e^
3*g+3*c^3*f^2*(-8*e*h+3*f*g)-40*c*d^2*e^2*(16*e*h+21*f*g)+4*c^2*d*e*f*(124
*e*h+15*f*g))+7*a^2*b^2*d*f*(73*c^2*f^2*h+4*d^2*e*(22*e*h+45*f*g)-c*d*f*(2
24*e*h+99*f*g))+a*b^3*(15*c^3*f^3*h-40*d^3*e^2*(8*e*h+51*f*g)+104*c*d^2*e*
f*(17*e*h+15*f*g)-c^2*d*f^2*(980*e*h+87*f*g)))*(f*x+e)^(1/2)/b^2/(-a*d+b*c
)^4/(-a*f+b*e)^2/(b*x+a)/(d*x+c)^2+1/64*d*(35*a^4*d^3*f^3*h-a^3*b*d^2*f^2*
(-495*c*f*h+368*d*e*h+267*d*f*g)+b^4*(960*d^3*e^3*g+c^3*f^2*(-8*e*h+3*f*g)
-80*c*d^2*e^2*(8*e*h+9*f*g)+8*c^2*d*e*f*(52*e*h+3*f*g))+a*b^3*(5*c^3*f^3*h
-80*d^3*e^2*(4*e*h+27*f*g)+16*c*d^2*e*f*(113*e*h+87*f*g)-c^2*d*f^2*(832...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29680 vs. 2(1700) = 3400.

Time = 17.18 (sec) , antiderivative size = 29680, normalized size of antiderivative = 17.46

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 1781, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.586, Rules used = {166, 27, 25, 166, 27, 168, 27, 168, 27, 168, 27, 168, 25, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}dx$$

$$\Big\downarrow \textbf{166}$$

$$\frac{\int -\frac{\sqrt{e+fx}(12bdeg-3bcfg-8bceh-4adeh+3acfh+f(9bdg-8bch-adh)x)}{2(a+bx)^4(c+dx)^3}dx}{4b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\int -\frac{\sqrt{e+fx}(a(4de-3cf)h-b(12deg-3cfg-8ceh)-f(9bdg-8bch-adh)x)}{(a+bx)^4(c+dx)^3}dx}{8b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{\sqrt{e+fx}(a(4de-3cf)h-b(12deg-3cfg-8ceh)-f(9bdg-8bch-adh)x)}{(a+bx)^4(c+dx)^3}dx}{8b(bc-ad)}-\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \textbf{166}$$

$$\frac{\frac{\int \frac{24be(de-cf)(3bdg-2bch-adh)+(4de-cf)\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)+3f\left(\left(16fhc^2-3d(9fg+8eh)c+36d^2eg\right)b^2+ad(19cfh-3d(\cdots}{2(a+bx)^3(c+dx)^3\sqrt{e+fx}}}{3b(bc-ad)}\cdots}{8b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\int \frac{24be(de-cf)(3bdg-2bch-adh)+(4de-cf)\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)+3f\left(\left(16fhc^2-3d(9fg+8eh)c+36d^2eg\right)b^2+ad(19cfh-3d(\cdots}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}}{6b(bc-ad)}\cdots}{8b(bc-ad)}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{e+fx}\left(3a^3d^2f^2h-a^2bdf(-58cfh+40deh+27dfg)+ab^2\left(59c^2f^2h-18cdf(8eh+5fg)+8d^2e(5eh+18fg)\right)-b^3\left(c^2f(56eh+3fg)-16cde(5eh+6fg)+120d^2e^2g\right)\right)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)} - \ldots$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{7d^2f^2(4de-cf)ha^3-7bdf\left(12e(3fg+4eh)d^2-cf(9fg+80eh)d+14c^2f^2h\right)a^2-b^2\left(-80e^2(15fg+4eh)d^3+8cef(111fg+151eh)d^2-6c^2f^2(11fg+98eh)d+15c^3f^3h\right) \ldots}{\ldots}$$

$$\frac{(e+fx)^{3/2}(bg-ah)}{4b(a+bx)^4(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)}{3b(bc-ad)(a+bx)^3(c+dx)^2} + \frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg \ldots\right.\right.}{2(bc-ad)(be-af)(a+ \ldots}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)}{3b(bc-ad)(a+bx)^3(c+dx)^2} + \frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg \ldots\right.\right.}{2(bc-ad)(be-af)(a+ \ldots}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\sqrt{e+fx}(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh))}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{e+fx}(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh))}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 168

$$\frac{\frac{\sqrt{e+fx}(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh))}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 25

$$\frac{\frac{\sqrt{e+fx}(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh))}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 174

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 73

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{e+fx}\left(dfha^2-b(9dfg+4deh-11cfh)a+b^2(12deg-3cfg-8ceh)\right)}{3b(bc-ad)(a+bx)^3(c+dx)^2}+\frac{\sqrt{e+fx}\left(3d^2f^2ha^3-bdf(27dfg+40deh-58cfh)a^2+b^2\left(8e(18fg+5eh)d^2-18cf(5fg\right.\right.}{2(bc-ad)(be-af)(a+b}}{}$$

$$\frac{(bg-ah)(e+fx)^{3/2}}{4b(bc-ad)(a+bx)^4(c+dx)^2}$$

input
```
Int[((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^3),x]
```

output
```
-1/4*((b*g - a*h)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(a + b*x)^4*(c + d*x)^2)
 + (((a^2*d*f*h + b^2*(12*d*e*g - 3*c*f*g - 8*c*e*h) - a*b*(9*d*f*g + 4*d*
e*h - 11*c*f*h))*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(a + b*x)^3*(c + d*x)^2)
+ (((3*a^3*d^2*f^2*h - a^2*b*d*f*(27*d*f*g + 40*d*e*h - 58*c*f*h) + a*b^2*
(59*c^2*f^2*h + 8*d^2*e*(18*f*g + 5*e*h) - 18*c*d*f*(5*f*g + 8*e*h)) - b^3
*(120*d^2*e^2*g - 16*c*d*e*(6*f*g + 5*e*h) + c^2*f*(3*f*g + 56*e*h)))*Sqrt
[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)^2) + (((21*a^4
*d^3*f^3*h - 7*a^3*b*d^2*f^2*(27*d*f*g + 44*d*e*h - 59*c*f*h) + b^4*(960*d
^3*e^3*g + 3*c^3*f^2*(3*f*g - 8*e*h) - 40*c*d^2*e^2*(21*f*g + 16*e*h) + 4*
c^2*d*e*f*(15*f*g + 124*e*h)) + 7*a^2*b^2*d*f*(73*c^2*f^2*h + 4*d^2*e*(45*
f*g + 22*e*h) - c*d*f*(99*f*g + 224*e*h)) + a*b^3*(15*c^3*f^3*h - 40*d^3*e
^2*(51*f*g + 8*e*h) + 104*c*d^2*e*f*(15*f*g + 17*e*h) - c^2*d*f^2*(87*f*g
+ 980*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2
) + ((2*d*(35*a^4*d^3*f^3*h - 35*a^3*b*d^2*f^2*(9*d*f*g + 14*d*e*h - 19*c*
f*h) + b^4*(1440*d^3*e^3*g + 3*c^3*f^2*(3*f*g - 8*e*h) - 240*c*d^2*e^2*(5*
f*g + 4*e*h) + 22*c^2*d*e*f*(3*f*g + 32*e*h)) + a*b^3*(15*c^3*f^3*h - 240*
d^3*e^2*(13*f*g + 2*e*h) + 4*c*d^2*e*f*(567*f*g + 668*e*h) - c^2*d*f^2*(93
*f*g + 1402*e*h)) + a^2*b^2*d*f*(725*c^2*f^2*h + 2*d^2*e*(993*f*g + 472*e*
h) - c*d*f*(1041*f*g + 2404*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(c + d*x)^2
) + (3*b*((2*d*(35*a^4*d^3*f^3*h - a^3*b*d^2*f^2*(267*d*f*g + 368*d*e*h...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 252.77 (sec) , antiderivative size = 2761, normalized size of antiderivative = 1.62

method	result	size
pseudoelliptic	Expression too large to display	2761
derivativedivides	Expression too large to display	3386
default	Expression too large to display	3386

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
35/64*(((-384/7*d^4*e^4*g+256/7*c*(e*h+3/2*f*g)*e^3*d^3-1152/35*c^2*(e*h+1
/4*f*g)*f*e^2*d^2+144/35*c^3*(e*h-1/6*f*g)*f^2*e*d+8/35*c^4*(e*h-3/8*f*g)*
f^3)*b^5-1/7*a*((-128*e^4*h-1152*e^3*f*g)*d^4+4736/5*c*(e*h+81/74*f*g)*f*e
^2*d^3-3312/5*c^2*(e*h+9/46*f*g)*f^2*e*d^2+296/5*(e*h-9/74*f*g)*c^3*f^3*d+
c^4*f^4*h)*b^4+4*a^2*d*f*((-64/5*e^3*h-216/5*g*f*e^2)*d^3+228/5*(e*h+27/38
*f*g)*c*f*e*d^2-114/5*c^2*(e*h+9/76*f*g)*f^2*d+c^3*f^3*h)*b^3+30*a^3*d^2*f
^2*((8/5*e^2*h+12/5*e*f*g)*d^2-52/15*(e*h+9/26*f*g)*c*f*d+c^2*f^2*h)*b^2+2
0*a^4*((-4/5*e*h-9/20*f*g)*d+c*f*h)*d^3*f^3*b+a^5*d^4*f^4*h)*((c*f-d*e)*d)
^(1/2)*(d*x+c)^2*(b*x+a)^4*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+41/
7*(-48/41*((-8*d^3*e^2*g+16/3*c*(e*h+9/4*f*g)*e*d^2-112/15*c^2*(e*h+9/16*f
*g)*f*d+7/3*c^3*f^2*h)*b^2+14/3*a*((4/7*e^2*h+6/7*e*f*g)*d^2-58/35*c*(e*h+
27/58*f*g)*f*d+c^2*f^2*h)*d*b+a^2*d^2*((-4/5*e*h-1/5*f*g)*d+c*f*h)*f)*d^2*
(d*x+c)^2*(b*x+a)^4*(a*f-b*e)^2*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)
)+(a*d-b*c)*((c*f-d*e)*d)^(1/2)*((192/41*d^5*e^3*g*x^5+288/41*x^4*c*((-4/9
*e*h-1/2*f*g)*x+g*e)*e^2*d^4+64/41*x^3*c^2*((13/10*e*f*h+3/40*f^2*g)*x^2+(
-3*e^2*h-7/2*e*f*g)*x+e^2*g)*e*d^3-16/41*x^2*c^3*((1/10*h*f^2*e-3/80*g*f^3
)*x^3+(-122/15*h*e^2*f-23/40*e*f^2*g)*x^2+(8/3*e^3*h+7/2*g*f*e^2)*x+e^3*g)
*d^2+32/205*x*c^4*((-1/2*h*f^2*e+3/16*g*f^3)*x^3+(31/6*h*e^2*f+5/8*e*f^2*g
)*x^2+(5/3*e^3*h+2*g*f*e^2)*x+e^3*g)*d-16/205*c^5*((1/2*h*f^2*e-3/16*g*f^3
)*x^3+(7/3*h*e^2*f+1/8*e*f^2*g)*x^2+(4/3*e^3*h+3/2*g*f*e^2)*x+e^3*g))*b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**(3/2)*(h*x+g)/(b*x+a)**5/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5220 vs. 2(1652) = 3304.

Time = 0.48 (sec) , antiderivative size = 5220, normalized size of antiderivative = 3.07

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/64*(1920*b^5*d^4*e^4*g - 1920*b^5*c*d^3*e^3*f*g - 5760*a*b^4*d^4*e^3*f*g
 + 288*b^5*c^2*d^2*e^2*f^2*g + 5184*a*b^4*c*d^3*e^2*f^2*g + 6048*a^2*b^3*d
^4*e^2*f^2*g + 24*b^5*c^3*d*e*f^3*g - 648*a*b^4*c^2*d^2*e*f^3*g - 4536*a^2
*b^3*c*d^3*e*f^3*g - 2520*a^3*b^2*d^4*e*f^3*g + 3*b^5*c^4*f^4*g - 36*a*b^4
*c^3*d*f^4*g + 378*a^2*b^3*c^2*d^2*f^4*g + 1260*a^3*b^2*c*d^3*f^4*g + 315*
a^4*b*d^4*f^4*g - 1280*b^5*c*d^3*e^4*h - 640*a*b^4*d^4*e^4*h + 1152*b^5*c^
2*d^2*e^3*f*h + 4736*a*b^4*c*d^3*e^3*f*h + 1792*a^2*b^3*d^4*e^3*f*h - 144*
b^5*c^3*d*e^2*f^2*h - 3312*a*b^4*c^2*d^2*e^2*f^2*h - 6384*a^2*b^3*c*d^3*e^
2*f^2*h - 1680*a^3*b^2*d^4*e^2*f^2*h - 8*b^5*c^4*e*f^3*h + 296*a*b^4*c^3*d
*e*f^3*h + 3192*a^2*b^3*c^2*d^2*e*f^3*h + 3640*a^3*b^2*c*d^3*e*f^3*h + 560
*a^4*b*d^4*e*f^3*h + 5*a*b^4*c^4*f^4*h - 140*a^2*b^3*c^3*d*f^4*h - 1050*a^
3*b^2*c^2*d^2*f^4*h - 700*a^4*b*c*d^3*f^4*h - 35*a^5*d^4*f^4*h)*arctan(sqr
t(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^9*c^7*e^2 - 7*a*b^8*c^6*d*e^2 + 21*
a^2*b^7*c^5*d^2*e^2 - 35*a^3*b^6*c^4*d^3*e^2 + 35*a^4*b^5*c^3*d^4*e^2 - 21
*a^5*b^4*c^2*d^5*e^2 + 7*a^6*b^3*c*d^6*e^2 - a^7*b^2*d^7*e^2 - 2*a*b^8*c^7
*e*f + 14*a^2*b^7*c^6*d*e*f - 42*a^3*b^6*c^5*d^2*e*f + 70*a^4*b^5*c^4*d^3*
e*f - 70*a^5*b^4*c^3*d^4*e*f + 42*a^6*b^3*c^2*d^5*e*f - 14*a^7*b^2*c*d^6*e
*f + 2*a^8*b*d^7*e*f + a^2*b^7*c^7*f^2 - 7*a^3*b^6*c^6*d*f^2 + 21*a^4*b^5*
c^5*d^2*f^2 - 35*a^5*b^4*c^4*d^3*f^2 + 35*a^6*b^3*c^3*d^4*f^2 - 21*a^7*b^2
*c^2*d^5*f^2 + 7*a^8*b*c*d^6*f^2 - a^9*d^7*f^2)*sqrt(-b^2*e + a*b*f)) -...
```

Mupad [B] (verification not implemented)

Time = 51.16 (sec) , antiderivative size = 1321612, normalized size of antiderivative = 777.42

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(3/2)*(g + h*x))/((a + b*x)^5*(c + d*x)^3),x)
```

output
```
atan(((((49152*a^20*b^2*d^20*f^9*g - 76800*a^19*b^3*c*d^19*f^9*g - 209920*
a^20*b^2*c*d^19*f^9*h - 906240*a^19*b^3*d^20*e*f^8*g + 160768*a^20*b^2*d^2
0*e*f^8*h + 3072*a^2*b^20*c^18*d^2*f^9*g - 76800*a^3*b^19*c^17*d^3*f^9*g +
 1105920*a^4*b^18*c^16*d^4*f^9*g - 8527872*a^5*b^17*c^15*d^5*f^9*g + 39014
400*a^6*b^16*c^14*d^6*f^9*g - 113627136*a^7*b^15*c^13*d^7*f^9*g + 21245952
0*a^8*b^14*c^12*d^8*f^9*g - 225239040*a^9*b^13*c^11*d^9*f^9*g + 11421696*a
^10*b^12*c^10*d^10*f^9*g + 417331200*a^11*b^11*c^9*d^11*f^9*g - 818847744*
a^12*b^10*c^8*d^12*f^9*g + 924917760*a^13*b^9*c^7*d^13*f^9*g - 710062080*a
^14*b^8*c^6*d^14*f^9*g + 381825024*a^15*b^7*c^5*d^15*f^9*g - 140697600*a^1
6*b^6*c^4*d^16*f^9*g + 32759808*a^17*b^5*c^3*d^17*f^9*g - 3732480*a^18*b^4
*c^2*d^18*f^9*g + 5120*a^3*b^19*c^18*d^2*f^9*h - 209920*a^4*b^18*c^17*d^3*
f^9*h + 1761280*a^5*b^17*c^16*d^4*f^9*h - 5693440*a^6*b^16*c^15*d^5*f^9*h
+ 143360*a^7*b^15*c^14*d^6*f^9*h + 65228800*a^8*b^14*c^13*d^7*f^9*h - 2720
97280*a^9*b^13*c^12*d^8*f^9*h + 638443520*a^10*b^12*c^11*d^9*f^9*h - 10118
45120*a^11*b^11*c^10*d^10*f^9*h + 1140705280*a^12*b^10*c^9*d^11*f^9*h - 91
9592960*a^13*b^9*c^8*d^12*f^9*h + 510648320*a^14*b^8*c^7*d^13*f^9*h - 1695
94880*a^15*b^7*c^6*d^14*f^9*h + 10178560*a^16*b^6*c^5*d^15*f^9*h + 2011136
0*a^17*b^5*c^4*d^16*f^9*h - 10280960*a^18*b^4*c^3*d^17*f^9*h + 2298880*a^1
9*b^3*c^2*d^18*f^9*h + 983040*a^14*b^8*d^20*e^6*f^3*g - 4669440*a^15*b^7*d
^20*e^5*f^4*g + 8871936*a^16*b^6*d^20*e^4*f^5*g - 8475648*a^17*b^5*d^20...
```

Reduce [F]

$$\int \frac{(e+fx)^{3/2}(g+hx)}{(a+bx)^5(c+dx)^3}\,dx = \int \frac{(fx+e)^{\frac{3}{2}}\,(hx+g)}{(bx+a)^5\,(dx+c)^3}dx$$

input
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x)
```

output
```
int((f*x+e)^(3/2)*(h*x+g)/(b*x+a)^5/(d*x+c)^3,x)
```

3.105 $\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}} dx$

Optimal result

Integrand size = 27, antiderivative size = 339

$$\begin{aligned}\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}} dx &= \frac{2(be-af)^3(de-cf)(fg-eh)\sqrt{e+fx}}{f^6} \\ &- \frac{2(be-af)^2(bde(4fg-5eh)-bcf(3fg-4eh)-af(dfg-2deh+cfh))(e+fx)^{3/2}}{3f^6} \\ &- \frac{2(be-af)\left(a^2df^2h+abf(3dfg-8deh+3cfh)-b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)(e+fx)^{5/2}}{5f^6} \\ &+ \frac{2b(3a^2df^2h+3abf(dfg-4deh+cfh)-b^2(2de(2fg-5eh)-cf(fg-4eh)))(e+fx)^{7/2}}{7f^6} \\ &+ \frac{2b^2(3adfh+b(dfg-5deh+cfh))(e+fx)^{9/2}}{9f^6}+\frac{2b^3dh(e+fx)^{11/2}}{11f^6}\end{aligned}$$

output
```
2*(-a*f+b*e)^3*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(1/2)/f^6-2/3*(-a*f+b*e)^2*(b
*d*e*(-5*e*h+4*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e
)^(3/2)/f^6-2/5*(-a*f+b*e)*(a^2*d*f^2*h+a*b*f*(3*c*f*h-8*d*e*h+3*d*f*g)-b^
2*(2*d*e*(-5*e*h+3*f*g)-3*c*f*(-2*e*h+f*g)))*(f*x+e)^(5/2)/f^6+2/7*b*(3*a^
2*d*f^2*h+3*a*b*f*(c*f*h-4*d*e*h+d*f*g)-b^2*(2*d*e*(-5*e*h+2*f*g)-c*f*(-4*
e*h+f*g)))*(f*x+e)^(7/2)/f^6+2/9*b^2*(3*a*d*f*h+b*(c*f*h-5*d*e*h+d*f*g))*(
f*x+e)^(9/2)/f^6+2/11*b^3*d*h*(f*x+e)^(11/2)/f^6
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(231a^3f^3(5cf(3fg-2eh+fhx)+d(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx)))+99a^2bf^2(7c}{}$$

input
```
Integrate[((a + b*x)^3*(c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(231*a^3*f^3*(5*c*f*(3*f*g - 2*e*h + f*h*x) + d*(8*e^2*h
- 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x))) + 99*a^2*b*f^2*(7*c*f*(8*e^2
*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d*(-48*e^3*h + 8*e^2*f*(
7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x))) + 33*a
*b^2*f*(3*c*f*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x)
 - 2*e*f^2*x*(14*g + 9*h*x)) + d*(128*e^4*h + 24*e^2*f^2*x*(3*g + 2*h*x) -
 16*e^3*f*(9*g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2*(27*g + 20
*h*x))) + b^3*(11*c*f*(128*e^4*h + 24*e^2*f^2*x*(3*g + 2*h*x) - 16*e^3*f*(
9*g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2*(27*g + 20*h*x)) + d*
(-1280*e^5*h + 128*e^4*f*(11*g + 5*h*x) + 35*f^5*x^4*(11*g + 9*h*x) - 32*e
^3*f^2*x*(22*g + 15*h*x) + 16*e^2*f^3*x^2*(33*g + 25*h*x) - 10*e*f^4*x^3*(
44*g + 35*h*x)))))/(3465*f^6)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \; 159$$

$$\int \left(\frac{b(e+fx)^{5/2}\left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{f^5}+\frac{(e+fx)^{3/2}(\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b(e+fx)^{7/2}\left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{7f^6}-\frac{2(e+fx)^{5/2}(be-af)\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)\right)}{5f^6}+\frac{2b^2(e+fx)^{9/2}(3adfh+b(cfh-5deh+dfg))}{9f^6}-\frac{2(e+fx)^{3/2}(be-af)^2(-af(cfh-2deh+dfg)-bcf(3fg-4eh)+bde(4fg-5eh))}{3f^6}+\frac{2\sqrt{e+fx}(be-af)^3(de-cf)(fg-eh)}{f^6}+\frac{2b^3dh(e+fx)^{11/2}}{11f^6}$$

input
```
Int[((a + b*x)^3*(c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*(b*e - a*f)^3*(d*e - c*f)*(f*g - e*h)*Sqrt[e + f*x])/f^6 - (2*(b*e - a*
f)^2*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(3*f*g - 4*e*h) - a*f*(d*f*g - 2*d*e*h
 + c*f*h))*(e + f*x)^(3/2))/(3*f^6) - (2*(b*e - a*f)*(a^2*d*f^2*h + a*b*f*
(3*d*f*g - 8*d*e*h + 3*c*f*h) - b^2*(2*d*e*(3*f*g - 5*e*h) - 3*c*f*(f*g -
2*e*h)))*(e + f*x)^(5/2))/(5*f^6) + (2*b*(3*a^2*d*f^2*h + 3*a*b*f*(d*f*g -
 4*d*e*h + c*f*h) - b^2*(2*d*e*(2*f*g - 5*e*h) - c*f*(f*g - 4*e*h)))*(e +
f*x)^(7/2))/(7*f^6) + (2*b^2*(3*a*d*f*h + b*(d*f*g - 5*d*e*h + c*f*h))*(e
+ f*x)^(9/2))/(9*f^6) + (2*b^3*d*h*(e + f*x)^(11/2))/(11*f^6)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{2dh\,b^3(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left(3(af-be)b^2d+b^3(cf-de)\right)h+b^3d(-eh+fg)\right)(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(\left(3(af-be)^2bd+3(af-be)b^2(cf-de)\right)h+\left(3(af-\right.\right.}{7}$
default	$\frac{2dh\,b^3(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left(3(af-be)b^2d+b^3(cf-de)\right)h+b^3d(eh-fg)\right)(fx+e)^{\frac{9}{2}}}{9}-\frac{2\left(-\left(3(af-be)^2bd+3(af-be)b^2(cf-de)\right)h+\left(3(af\right.\right.}{7}$
pseudoelliptic	$-\frac{4\sqrt{fx+e}\left(\frac{3\left(-\frac{x^3\left(\frac{7dh\,x^2}{11}+\frac{7(ch+dg)x}{9}+cg\right)b^3}{7}-\frac{3a\,x^2\left(\frac{5dh\,x^2}{9}+\frac{5(ch+dg)x}{7}+cg\right)b^2}{5}-a^2\left(\frac{3dh\,x^2}{7}+\frac{3(ch+dg)x}{5}+cg\right)xb-\left(\right.\right.}{2}\right.}{}$
gosper	$-\frac{2\sqrt{fx+e}\,(-315dh\,b^3x^5f^5-1155a\,b^2d\,f^5h\,x^4-385b^3c\,f^5h\,x^4+350b^3de\,f^4h\,x^4-385b^3d\,f^5g\,x^4-1485a^2bd\,f^5h\,x^3-1}{}$
trager	$-\frac{2\sqrt{fx+e}\,(-315dh\,b^3x^5f^5-1155a\,b^2d\,f^5h\,x^4-385b^3c\,f^5h\,x^4+350b^3de\,f^4h\,x^4-385b^3d\,f^5g\,x^4-1485a^2bd\,f^5h\,x^3-1}{}$
risch	$-\frac{2\sqrt{fx+e}\,(-315dh\,b^3x^5f^5-1155a\,b^2d\,f^5h\,x^4-385b^3c\,f^5h\,x^4+350b^3de\,f^4h\,x^4-385b^3d\,f^5g\,x^4-1485a^2bd\,f^5h\,x^3-1}{}$
orering	$-\frac{2\sqrt{fx+e}\,(-315dh\,b^3x^5f^5-1155a\,b^2d\,f^5h\,x^4-385b^3c\,f^5h\,x^4+350b^3de\,f^4h\,x^4-385b^3d\,f^5g\,x^4-1485a^2bd\,f^5h\,x^3-1}{}$

input
```
int((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^6*(1/11*d*h*b^3*(f*x+e)^(11/2)+1/9*((3*(a*f-b*e)*b^2*d+b^3*(c*f-d*e))*
h+b^3*d*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*((3*(a*f-b*e)^2*b*d+3*(a*f-b*e)*b^2*
(c*f-d*e))*h+(3*(a*f-b*e)*b^2*d+b^3*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(7/2)+1
/5*(((a*f-b*e)^3*d+3*(a*f-b*e)^2*b*(c*f-d*e))*h+(3*(a*f-b*e)^2*b*d+3*(a*f-
b*e)*b^2*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(5/2)+1/3*((a*f-b*e)^3*(c*f-d*e)*h
+((a*f-b*e)^3*d+3*(a*f-b*e)^2*b*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(3/2)+(a*f-
b*e)^3*(c*f-d*e)*(-e*h+f*g)*(f*x+e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\,(315\,b^3df^5hx^5+35\,(11\,b^3df^5g-(10\,b^3def^4-11\,(b^3c+3\,ab^2d)\,f^5)h)x^4-5\,(11\,(8\,b^3def^4-9\,(b^3c+3\,ab$$

input
```
integrate((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*b^3*d*f^5*h*x^5 + 35*(11*b^3*d*f^5*g - (10*b^3*d*e*f^4 - 11*(b
^3*c + 3*a*b^2*d)*f^5)*h)*x^4 - 5*(11*(8*b^3*d*e*f^4 - 9*(b^3*c + 3*a*b^2*
d)*f^5)*g - (80*b^3*d*e^2*f^3 - 88*(b^3*c + 3*a*b^2*d)*e*f^4 + 297*(a*b^2*
c + a^2*b*d)*f^5)*h)*x^3 + 3*(11*(16*b^3*d*e^2*f^3 - 18*(b^3*c + 3*a*b^2*d
)*e*f^4 + 63*(a*b^2*c + a^2*b*d)*f^5)*g - (160*b^3*d*e^3*f^2 - 176*(b^3*c
+ 3*a*b^2*d)*e^2*f^3 + 594*(a*b^2*c + a^2*b*d)*e*f^4 - 231*(3*a^2*b*c + a^
3*d)*f^5)*h)*x^2 + 11*(128*b^3*d*e^4*f + 315*a^3*c*f^5 - 144*(b^3*c + 3*a*
b^2*d)*e^3*f^2 + 504*(a*b^2*c + a^2*b*d)*e^2*f^3 - 210*(3*a^2*b*c + a^3*d)
*e*f^4)*g - 2*(640*b^3*d*e^5 + 1155*a^3*c*e*f^4 - 704*(b^3*c + 3*a*b^2*d)*
e^4*f + 2376*(a*b^2*c + a^2*b*d)*e^3*f^2 - 924*(3*a^2*b*c + a^3*d)*e^2*f^3
)*h - (11*(64*b^3*d*e^3*f^2 - 72*(b^3*c + 3*a*b^2*d)*e^2*f^3 + 252*(a*b^2*
c + a^2*b*d)*e*f^4 - 105*(3*a^2*b*c + a^3*d)*f^5)*g - (640*b^3*d*e^4*f + 1
155*a^3*c*f^5 - 704*(b^3*c + 3*a*b^2*d)*e^3*f^2 + 2376*(a*b^2*c + a^2*b*d)
*e^2*f^3 - 924*(3*a^2*b*c + a^3*d)*e*f^4)*h)*x)*sqrt(f*x + e)/f^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(354) = 708.

Time = 1.63 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.99

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b**3*d*h*(e + f*x)**(11/2)/(11*f**5) + (e + f*x)**(9/2)*(3*a
*b**2*d*f*h + b**3*c*f*h - 5*b**3*d*e*h + b**3*d*f*g)/(9*f**5) + (e + f*x)
**(7/2)*(3*a**2*b*d*f**2*h + 3*a*b**2*c*f**2*h - 12*a*b**2*d*e*f*h + 3*a*b
**2*d*f**2*g - 4*b**3*c*e*f*h + b**3*c*f**2*g + 10*b**3*d*e**2*h - 4*b**3*
d*e*f*g)/(7*f**5) + (e + f*x)**(5/2)*(a**3*d*f**3*h + 3*a**2*b*c*f**3*h -
9*a**2*b*d*e*f**2*h + 3*a**2*b*d*f**3*g - 9*a*b**2*c*e*f**2*h + 3*a*b**2*c
*f**3*g + 18*a*b**2*d*e**2*f*h - 9*a*b**2*d*e*f**2*g + 6*b**3*c*e**2*f*h -
 3*b**3*c*e*f**2*g - 10*b**3*d*e**3*h + 6*b**3*d*e**2*f*g)/(5*f**5) + (e +
 f*x)**(3/2)*(a**3*c*f**4*h - 2*a**3*d*e*f**3*h + a**3*d*f**4*g - 6*a**2*b
*c*e*f**3*h + 3*a**2*b*c*f**4*g + 9*a**2*b*d*e**2*f**2*h - 6*a**2*b*d*e*f*
*3*g + 9*a*b**2*c*e**2*f**2*h - 6*a*b**2*c*e*f**3*g - 12*a*b**2*d*e**3*f*h
 + 9*a*b**2*d*e**2*f**2*g - 4*b**3*c*e**3*f*h + 3*b**3*c*e**2*f**2*g + 5*b
**3*d*e**4*h - 4*b**3*d*e**3*f*g)/(3*f**5) + sqrt(e + f*x)*(-a**3*c*e*f**4
*h + a**3*c*f**5*g + a**3*d*e**2*f**3*h - a**3*d*e*f**4*g + 3*a**2*b*c*e**
2*f**3*h - 3*a**2*b*c*e*f**4*g - 3*a**2*b*d*e**3*f**2*h + 3*a**2*b*d*e**2*
f**3*g - 3*a*b**2*c*e**3*f**2*h + 3*a*b**2*c*e**2*f**3*g + 3*a*b**2*d*e**4
*f*h - 3*a*b**2*d*e**3*f**2*g + b**3*c*e**4*f*h - b**3*c*e**3*f**2*g - b**
3*d*e**5*h + b**3*d*e**4*f*g)/f**5)/f, Ne(f, 0)), ((a**3*c*g*x + b**3*d*h*
x**6/6 + x**5*(3*a*b**2*d*h + b**3*c*h + b**3*d*g)/5 + x**4*(3*a**2*b*d*h
+ 3*a*b**2*c*h + 3*a*b**2*d*g + b**3*c*g)/4 + x**3*(a**3*d*h + 3*a**2*b...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(315\,(fx+e)^{\frac{11}{2}}b^3dh+385\,(b^3dfg-(5\,b^3de-(b^3c+3\,ab^2d)f)h)(fx+e)^{\frac{9}{2}}-495\,((4\,b^3def-(b^3c+3\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(f*x + e)^(11/2)*b^3*d*h + 385*(b^3*d*f*g - (5*b^3*d*e - (b^3*
c + 3*a*b^2*d)*f)*h)*(f*x + e)^(9/2) - 495*((4*b^3*d*e*f - (b^3*c + 3*a*b^
2*d)*f^2)*g - (10*b^3*d*e^2 - 4*(b^3*c + 3*a*b^2*d)*e*f + 3*(a*b^2*c + a^2
*b*d)*f^2)*h)*(f*x + e)^(7/2) + 693*(3*(2*b^3*d*e^2*f - (b^3*c + 3*a*b^2*d
)*e*f^2 + (a*b^2*c + a^2*b*d)*f^3)*g - (10*b^3*d*e^3 - 6*(b^3*c + 3*a*b^2*
d)*e^2*f + 9*(a*b^2*c + a^2*b*d)*e*f^2 - (3*a^2*b*c + a^3*d)*f^3)*h)*(f*x
+ e)^(5/2) - 1155*((4*b^3*d*e^3*f - 3*(b^3*c + 3*a*b^2*d)*e^2*f^2 + 6*(a*b
^2*c + a^2*b*d)*e*f^3 - (3*a^2*b*c + a^3*d)*f^4)*g - (5*b^3*d*e^4 + a^3*c*
f^4 - 4*(b^3*c + 3*a*b^2*d)*e^3*f + 9*(a*b^2*c + a^2*b*d)*e^2*f^2 - 2*(3*a
^2*b*c + a^3*d)*e*f^3)*h)*(f*x + e)^(3/2) + 3465*((b^3*d*e^4*f + a^3*c*f^5
 - (b^3*c + 3*a*b^2*d)*e^3*f^2 + 3*(a*b^2*c + a^2*b*d)*e^2*f^3 - (3*a^2*b*
c + a^3*d)*e*f^4)*g - (b^3*d*e^5 + a^3*c*e*f^4 - (b^3*c + 3*a*b^2*d)*e^4*f
 + 3*(a*b^2*c + a^2*b*d)*e^3*f^2 - (3*a^2*b*c + a^3*d)*e^2*f^3)*h)*sqrt(f*
x + e))/f^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(317) = 634.

Time = 0.13 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output

```
2/3465*(3465*sqrt(f*x + e)*a^3*c*g + 3465*((f*x + e)^(3/2) - 3*sqrt(f*x +
e)*e)*a^2*b*c*g/f + 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*d*g/f +
 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c*h/f + 693*(3*(f*x + e)^(
5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b^2*c*g/f^2 + 693*(3
*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*d*g/
f^2 + 693*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2
)*a^2*b*c*h/f^2 + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(
f*x + e)*e^2)*a^3*d*h/f^2 + 99*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e +
 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^3*c*g/f^3 + 297*(5*(f*x
+ e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x +
 e)*e^3)*a*b^2*d*g/f^3 + 297*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 3
5*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b^2*c*h/f^3 + 297*(5*(f*x
+ e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x +
 e)*e^3)*a^2*b*d*h/f^3 + 11*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e +
378*(f*x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)
*b^3*d*g/f^4 + 11*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*x +
 e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*b^3*c*h/f
^4 + 33*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*x + e)^(5/2)*
e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*a*b^2*d*h/f^4 + 5*(
63*(f*x + e)^(11/2) - 385*(f*x + e)^(9/2)*e + 990*(f*x + e)^(7/2)*e^2 -...
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{(e+f\,x)^{7/2}\,(2\,b^3\,c\,f^2\,g+20\,b^3\,d\,e^2\,h+6\,a\,b^2\,c\,f^2\,h+6\,a\,b^2\,d\,f^2\,g+6\,a^2\,b\,d\,f^2\,h-8\,b^3\,c\,e\,f\,h-8\,b^3\,d\,e\,f\,g}{7\,f^6}$$
$$+\frac{(e+f\,x)^{9/2}\,(2\,b^3\,c\,f\,h-10\,b^3\,d\,e\,h+2\,b^3\,d\,f\,g+6\,a\,b^2\,d\,f\,h)}{9\,f^6}$$
$$+\frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)\,\left(3\,b^2\,c\,f^2\,g+a^2\,d\,f^2\,h+10\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-\right.}{5\,f^6}$$
$$+\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)^2\,\left(a\,c\,f^2\,h+a\,d\,f^2\,g+3\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f\right.}{3\,f^6}$$
$$+\frac{2\,b^3\,d\,h\,(e+f\,x)^{11/2}}{11\,f^6}-\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)^3\,(c\,f-d\,e)\,(e\,h-f\,g)}{f^6}$$

input
```
int(((g + h*x)*(a + b*x)^3*(c + d*x))/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(7/2)*(2*b^3*c*f^2*g + 20*b^3*d*e^2*h + 6*a*b^2*c*f^2*h + 6*a*b
^2*d*f^2*g + 6*a^2*b*d*f^2*h - 8*b^3*c*e*f*h - 8*b^3*d*e*f*g - 24*a*b^2*d*
e*f*h))/(7*f^6) + ((e + f*x)^(9/2)*(2*b^3*c*f*h - 10*b^3*d*e*h + 2*b^3*d*f
*g + 6*a*b^2*d*f*h))/(9*f^6) + (2*(e + f*x)^(5/2)*(a*f - b*e)*(3*b^2*c*f^2
*g + a^2*d*f^2*h + 10*b^2*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 6*b^2*
c*e*f*h - 6*b^2*d*e*f*g - 8*a*b*d*e*f*h))/(5*f^6) + (2*(e + f*x)^(3/2)*(a*
f - b*e)^2*(a*c*f^2*h + a*d*f^2*g + 3*b*c*f^2*g + 5*b*d*e^2*h - 2*a*d*e*f*
h - 4*b*c*e*f*h - 4*b*d*e*f*g))/(3*f^6) + (2*b^3*d*h*(e + f*x)^(11/2))/(11
*f^6) - (2*(e + f*x)^(1/2)*(a*f - b*e)^3*(c*f - d*e)*(e*h - f*g))/f^6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.27

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{fx+e}\,(315b^3d\,f^5h\,x^5+1155a\,b^2d\,f^5h\,x^4+385b^3c\,f^5h\,x^4-350b^3de\,f^4h\,x^4+385b^3d\,f^5g\,x^4+1485a^2}{}$$

input
```
int((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x)
```

output

```
(2*sqrt(e + f*x)*( - 2310*a**3*c*e*f**4*h + 3465*a**3*c*f**5*g + 1155*a**3
*c*f**5*h*x + 1848*a**3*d*e**2*f**3*h - 2310*a**3*d*e*f**4*g - 924*a**3*d*
e*f**4*h*x + 1155*a**3*d*f**5*g*x + 693*a**3*d*f**5*h*x**2 + 5544*a**2*b*c
*e**2*f**3*h - 6930*a**2*b*c*e*f**4*g - 2772*a**2*b*c*e*f**4*h*x + 3465*a*
*2*b*c*f**5*g*x + 2079*a**2*b*c*f**5*h*x**2 - 4752*a**2*b*d*e**3*f**2*h +
5544*a**2*b*d*e**2*f**3*g + 2376*a**2*b*d*e**2*f**3*h*x - 2772*a**2*b*d*e*
f**4*g*x - 1782*a**2*b*d*e*f**4*h*x**2 + 2079*a**2*b*d*f**5*g*x**2 + 1485*
a**2*b*d*f**5*h*x**3 - 4752*a*b**2*c*e**3*f**2*h + 5544*a*b**2*c*e**2*f**3
*g + 2376*a*b**2*c*e**2*f**3*h*x - 2772*a*b**2*c*e*f**4*g*x - 1782*a*b**2*
c*e*f**4*h*x**2 + 2079*a*b**2*c*f**5*g*x**2 + 1485*a*b**2*c*f**5*h*x**3 +
4224*a*b**2*d*e**4*f*h - 4752*a*b**2*d*e**3*f**2*g - 2112*a*b**2*d*e**3*f*
*2*h*x + 2376*a*b**2*d*e**2*f**3*g*x + 1584*a*b**2*d*e**2*f**3*h*x**2 - 17
82*a*b**2*d*e*f**4*g*x**2 - 1320*a*b**2*d*e*f**4*h*x**3 + 1485*a*b**2*d*f*
*5*g*x**3 + 1155*a*b**2*d*f**5*h*x**4 + 1408*b**3*c*e**4*f*h - 1584*b**3*c
*e**3*f**2*g - 704*b**3*c*e**3*f**2*h*x + 792*b**3*c*e**2*f**3*g*x + 528*b
**3*c*e**2*f**3*h*x**2 - 594*b**3*c*e*f**4*g*x**2 - 440*b**3*c*e*f**4*h*x*
*3 + 495*b**3*c*f**5*g*x**3 + 385*b**3*c*f**5*h*x**4 - 1280*b**3*d*e**5*h
+ 1408*b**3*d*e**4*f*g + 640*b**3*d*e**4*f*h*x - 704*b**3*d*e**3*f**2*g*x
- 480*b**3*d*e**3*f**2*h*x**2 + 528*b**3*d*e**2*f**3*g*x**2 + 400*b**3*d*e
**2*f**3*h*x**3 - 440*b**3*d*e*f**4*g*x**3 - 350*b**3*d*e*f**4*h*x**4 +...
```

3.106 $\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 245

$$\begin{aligned}\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx &= -\frac{2(be-af)^2(de-cf)(fg-eh)\sqrt{e+fx}}{f^5} \\ &+ \frac{2(be-af)(bde(3fg-4eh)-bcf(2fg-3eh)-af(dfg-2deh+cfh))(e+fx)^{3/2}}{3f^5} \\ &+ \frac{2\left(a^2df^2h+2abf(dfg-3deh+cfh)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)(e+fx)^{5/2}}{5f^5} \\ &+ \frac{2b(2adfh+b(dfg-4deh+cfh))(e+fx)^{7/2}}{7f^5}+\frac{2b^2dh(e+fx)^{9/2}}{9f^5}\end{aligned}$$

output
```
-2*(-a*f+b*e)^2*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(1/2)/f^5+2/3*(-a*f+b*e)*(b*
d*e*(-4*e*h+3*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)
^(3/2)/f^5+2/5*(a^2*d*f^2*h+2*a*b*f*(c*f*h-3*d*e*h+d*f*g)+b^2*(c*f*(-3*e*h
+f*g)-3*d*e*(-2*e*h+f*g)))*(f*x+e)^(5/2)/f^5+2/7*b*(2*a*d*f*h+b*(c*f*h-4*d
*e*h+d*f*g))*(f*x+e)^(7/2)/f^5+2/9*b^2*d*h*(f*x+e)^(9/2)/f^5
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(21a^2f^2(5cf(3fg-2eh+fhx)+d(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx)))+6abf(7cf(8e$$

input
```
Integrate[((a + b*x)^2*(c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(21*a^2*f^2*(5*c*f*(3*f*g - 2*e*h + f*h*x) + d*(8*e^2*h -
 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x))) + 6*a*b*f*(7*c*f*(8*e^2*h - 2
*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d*(-48*e^3*h + 8*e^2*f*(7*g +
3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x))) + b^2*(3*c*f
*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*
(14*g + 9*h*x)) + d*(128*e^4*h + 24*e^2*f^2*x*(3*g + 2*h*x) - 16*e^3*f*(9*
g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2*(27*g + 20*h*x)))))/(31
5*f^5)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{ 159}$$
$$\int \left(\frac{(e+fx)^{3/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{f^4}+\frac{b(e+fx)^{5/2}(2adf$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2(e+fx)^{5/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{5f^5}+\frac{2b(e+fx)^{7/2}(2adfh+b(cfh-4deh+dfg))}{7f^5}+\frac{2(e+fx)^{3/2}(be-af)(-af(cfh-2deh+dfg)-bcf(2fg-3eh)+bde(3fg-4eh))}{3f^5}-\frac{2\sqrt{e+fx}(be-af)^2(de-cf)(fg-eh)}{f^5}+\frac{2b^2dh(e+fx)^{9/2}}{9f^5}$$

input
```
Int[((a + b*x)^2*(c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(-2*(b*e - a*f)^2*(d*e - c*f)*(f*g - e*h)*Sqrt[e + f*x])/f^5 + (2*(b*e - a
*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(2*f*g - 3*e*h) - a*f*(d*f*g - 2*d*e*h
+ c*f*h))*(e + f*x)^(3/2))/(3*f^5) + (2*(a^2*d*f^2*h + 2*a*b*f*(d*f*g - 3*
d*e*h + c*f*h) + b^2*(c*f*(f*g - 3*e*h) - 3*d*e*(f*g - 2*e*h)))*(e + f*x)^
(5/2))/(5*f^5) + (2*b*(2*a*d*f*h + b*(d*f*g - 4*d*e*h + c*f*h))*(e + f*x)^
(7/2))/(7*f^5) + (2*b^2*d*h*(e + f*x)^(9/2))/(9*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{2dh\,b^2(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(\left(2b(af-be)d+b^2(cf-de)\right)h+d\,b^2(-eh+fg)\right)(fx+e)^{\frac{7}{2}}}{7}+\frac{2\left(\left((af-be)^2d+2b(af-be)(cf-de)\right)h+\left(2b(af-be)d\right.\right.}{5}$
default	$\frac{2dh\,b^2(fx+e)^{\frac{9}{2}}}{9}-\frac{2\left(-\left(2b(af-be)d+b^2(cf-de)\right)h+d\,b^2(eh-fg)\right)(fx+e)^{\frac{7}{2}}}{7}-\frac{2\left(-\left((af-be)^2d+2b(af-be)(cf-de)\right)h+\left(2b(af-be\right.\right.}{5}$
pseudoelliptic	$-\frac{4\sqrt{fx+e}\left(\left(\left(-\frac{3x^2\left(\frac{5dh\,x^2}{9}+\frac{5(ch+dg)x}{7}+cg\right)b^2}{10}-ax\left(\frac{3dh\,x^2}{7}+\frac{3(ch+dg)x}{5}+cg\right)b-\frac{3\left(\frac{dh\,x^2}{5}+\frac{(ch+dg)x}{3}+cg\right)a^2}{2}\right)f^4+\left(\right.\right.\right.}{}$
gosper	$-\frac{2\sqrt{fx+e}\,(-35dh\,b^2x^4f^4-90abd\,f^4h\,x^3-45b^2c\,f^4h\,x^3+40b^2de\,f^3h\,x^3-45b^2d\,f^4g\,x^3-63a^2d\,f^4h\,x^2-126abc\,f^4h\,x^2}{}$
trager	$-\frac{2\sqrt{fx+e}\,(-35dh\,b^2x^4f^4-90abd\,f^4h\,x^3-45b^2c\,f^4h\,x^3+40b^2de\,f^3h\,x^3-45b^2d\,f^4g\,x^3-63a^2d\,f^4h\,x^2-126abc\,f^4h\,x^2}{}$
risch	$-\frac{2\sqrt{fx+e}\,(-35dh\,b^2x^4f^4-90abd\,f^4h\,x^3-45b^2c\,f^4h\,x^3+40b^2de\,f^3h\,x^3-45b^2d\,f^4g\,x^3-63a^2d\,f^4h\,x^2-126abc\,f^4h\,x^2}{}$
orering	$-\frac{2\sqrt{fx+e}\,(-35dh\,b^2x^4f^4-90abd\,f^4h\,x^3-45b^2c\,f^4h\,x^3+40b^2de\,f^3h\,x^3-45b^2d\,f^4g\,x^3-63a^2d\,f^4h\,x^2-126abc\,f^4h\,x^2}{}$

input
```
int((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^5*(1/9*d*h*b^2*(f*x+e)^(9/2)+1/7*((2*b*(a*f-b*e)*d+b^2*(c*f-d*e))*h+d*
b^2*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*(((a*f-b*e)^2*d+2*b*(a*f-b*e)*(c*f-d*e))
*h+(2*b*(a*f-b*e)*d+b^2*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(5/2)+1/3*((a*f-b*e
)^2*(c*f-d*e)*h+((a*f-b*e)^2*d+2*b*(a*f-b*e)*(c*f-d*e))*(-e*h+f*g))*(f*x+e
)^(3/2)+(a*f-b*e)^2*(c*f-d*e)*(-e*h+f*g)*(f*x+e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$=\frac{2\,(35\,b^2df^4hx^4+5\,(9\,b^2df^4g-(8\,b^2def^3-9\,(b^2c+2\,abd)f^4)h)x^3-3\,(3\,(6\,b^2def^3-7\,(b^2c+2\,abd)f^4)g}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^2*d*f^4*h*x^4 + 5*(9*b^2*d*f^4*g - (8*b^2*d*e*f^3 - 9*(b^2*c +
 2*a*b*d)*f^4)*h)*x^3 - 3*(3*(6*b^2*d*e*f^3 - 7*(b^2*c + 2*a*b*d)*f^4)*g -
 (16*b^2*d*e^2*f^2 - 18*(b^2*c + 2*a*b*d)*e*f^3 + 21*(2*a*b*c + a^2*d)*f^4
)*h)*x^2 - 3*(48*b^2*d*e^3*f - 105*a^2*c*f^4 - 56*(b^2*c + 2*a*b*d)*e^2*f^
2 + 70*(2*a*b*c + a^2*d)*e*f^3)*g + 2*(64*b^2*d*e^4 - 105*a^2*c*e*f^3 - 72
*(b^2*c + 2*a*b*d)*e^3*f + 84*(2*a*b*c + a^2*d)*e^2*f^2)*h + (3*(24*b^2*d*
e^2*f^2 - 28*(b^2*c + 2*a*b*d)*e*f^3 + 35*(2*a*b*c + a^2*d)*f^4)*g - (64*b
^2*d*e^3*f - 105*a^2*c*f^4 - 72*(b^2*c + 2*a*b*d)*e^2*f^2 + 84*(2*a*b*c +
a^2*d)*e*f^3)*h)*x)*sqrt(f*x + e)/f^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(260) = 520$.

Time = 1.50 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.53

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{b^2dh(e+fx)^{\frac{9}{2}}}{9f^4} + \frac{(e+fx)^{\frac{7}{2}}\cdot\left(2abdfh+b^2cfh-4b^2deh+b^2dfg\right)}{7f^4} + \frac{(e+fx)^{\frac{5}{2}}\left(a^2df^2h+2abcf^2h-6abdefh+2abdf^2g-3b^2cefh+b^2cf^2g+6b^2de^2h-3b^2defg\right)}{5f^4} \cdots\right.}{\cdots} \\ \frac{a^2cgx+\frac{b^2dhx^5}{5}+\frac{x^4\cdot\left(2abdh+b^2ch+b^2dg\right)}{4}+\frac{x^3\left(a^2dh+2abch+2abdg+b^2cg\right)}{3}+\frac{x^2\left(a^2ch+a^2dg+2abcg\right)}{2}}{\sqrt{e}} \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b**2*d*h*(e + f*x)**(9/2)/(9*f**4) + (e + f*x)**(7/2)*(2*a*b
*d*f*h + b**2*c*f*h - 4*b**2*d*e*h + b**2*d*f*g)/(7*f**4) + (e + f*x)**(5/
2)*(a**2*d*f**2*h + 2*a*b*c*f**2*h - 6*a*b*d*e*f*h + 2*a*b*d*f**2*g - 3*b*
*2*c*e*f*h + b**2*c*f**2*g + 6*b**2*d*e**2*h - 3*b**2*d*e*f*g)/(5*f**4) +
(e + f*x)**(3/2)*(a**2*c*f**3*h - 2*a**2*d*e*f**2*h + a**2*d*f**3*g - 4*a*
b*c*e*f**2*h + 2*a*b*c*f**3*g + 6*a*b*d*e**2*f*h - 4*a*b*d*e*f**2*g + 3*b*
*2*c*e**2*f*h - 2*b**2*c*e*f**2*g - 4*b**2*d*e**3*h + 3*b**2*d*e**2*f*g)/(
3*f**4) + sqrt(e + f*x)*(-a**2*c*e*f**3*h + a**2*c*f**4*g + a**2*d*e**2*f*
*2*h - a**2*d*e*f**3*g + 2*a*b*c*e**2*f**2*h - 2*a*b*c*e*f**3*g - 2*a*b*d*
e**3*f*h + 2*a*b*d*e**2*f**2*g - b**2*c*e**3*f*h + b**2*c*e**2*f**2*g + b*
*2*d*e**4*h - b**2*d*e**3*f*g)/f**4)/f, Ne(f, 0)), ((a**2*c*g*x + b**2*d*h
*x**5/5 + x**4*(2*a*b*d*h + b**2*c*h + b**2*d*g)/4 + x**3*(a**2*d*h + 2*a*
b*c*h + 2*a*b*d*g + b**2*c*g)/3 + x**2*(a**2*c*h + a**2*d*g + 2*a*b*c*g)/2
)/sqrt(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(35\,(fx+e)^{\frac{9}{2}}b^2dh + 45\,(b^2dfg - (4\,b^2de - (b^2c+2\,abd)f)h)(fx+e)^{\frac{7}{2}} - 63\,((3\,b^2def - (b^2c+2\,abd)f\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(f*x + e)^(9/2)*b^2*d*h + 45*(b^2*d*f*g - (4*b^2*d*e - (b^2*c +
2*a*b*d)*f)*h)*(f*x + e)^(7/2) - 63*((3*b^2*d*e*f - (b^2*c + 2*a*b*d)*f^2)
*g - (6*b^2*d*e^2 - 3*(b^2*c + 2*a*b*d)*e*f + (2*a*b*c + a^2*d)*f^2)*h)*(f
*x + e)^(5/2) + 105*((3*b^2*d*e^2*f - 2*(b^2*c + 2*a*b*d)*e*f^2 + (2*a*b*c
 + a^2*d)*f^3)*g - (4*b^2*d*e^3 - a^2*c*f^3 - 3*(b^2*c + 2*a*b*d)*e^2*f +
2*(2*a*b*c + a^2*d)*e*f^2)*h)*(f*x + e)^(3/2) - 315*((b^2*d*e^3*f - a^2*c*
f^4 - (b^2*c + 2*a*b*d)*e^2*f^2 + (2*a*b*c + a^2*d)*e*f^3)*g - (b^2*d*e^4
- a^2*c*e*f^3 - (b^2*c + 2*a*b*d)*e^3*f + (2*a*b*c + a^2*d)*e^2*f^2)*h)*sq
rt(f*x + e))/f^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(229) = 458$.

Time = 0.12 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(315\sqrt{fx+e}a^2cg + \frac{210\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)abcg}{f} + \frac{105\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)a^2dg}{f} + \frac{105\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)a^2ch}{f} + \right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(f*x + e)*a^2*c*g + 210*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*
e)*a*b*c*g/f + 105*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*d*g/f + 105*(
(f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c*h/f + 21*(3*(f*x + e)^(5/2) - 1
0*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b^2*c*g/f^2 + 42*(3*(f*x + e)^
(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*d*g/f^2 + 42*(3*(
f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*c*h/f^2
+ 21*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2
*d*h/f^2 + 9*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2
)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*d*g/f^3 + 9*(5*(f*x + e)^(7/2) - 21*(f*x
 + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*c*h/f^3
 + 18*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 -
 35*sqrt(f*x + e)*e^3)*a*b*d*h/f^3 + (35*(f*x + e)^(9/2) - 180*(f*x + e)^(
7/2)*e + 378*(f*x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x
+ e)*e^4)*b^2*d*h/f^4)/f
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{(e+f\,x)^{7/2}\,(2\,b^2\,c\,f\,h-8\,b^2\,d\,e\,h+2\,b^2\,d\,f\,g+4\,a\,b\,d\,f\,h)}{7\,f^5}$$
$$+\frac{(e+f\,x)^{5/2}\,(2\,b^2\,c\,f^2\,g+2\,a^2\,d\,f^2\,h+12\,b^2\,d\,e^2\,h+4\,a\,b\,c\,f^2\,h+4\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b^2\,d\,e\,f\,g}{5\,f^5}$$
$$+\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)\,(a\,c\,f^2\,h+a\,d\,f^2\,g+2\,b\,c\,f^2\,g+4\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\,h-3\,b\,d\,e\,f}{3\,f^5}$$
$$+\frac{2\,b^2\,d\,h\,(e+f\,x)^{9/2}}{9\,f^5}-\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)^2\,(c\,f-d\,e)\,(e\,h-f\,g)}{f^5}$$

input
```
int(((g + h*x)*(a + b*x)^2*(c + d*x))/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(7/2)*(2*b^2*c*f*h - 8*b^2*d*e*h + 2*b^2*d*f*g + 4*a*b*d*f*h))/
(7*f^5) + ((e + f*x)^(5/2)*(2*b^2*c*f^2*g + 2*a^2*d*f^2*h + 12*b^2*d*e^2*h
 + 4*a*b*c*f^2*h + 4*a*b*d*f^2*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 12*a*b*
d*e*f*h))/(5*f^5) + (2*(e + f*x)^(3/2)*(a*f - b*e)*(a*c*f^2*h + a*d*f^2*g
+ 2*b*c*f^2*g + 4*b*d*e^2*h - 2*a*d*e*f*h - 3*b*c*e*f*h - 3*b*d*e*f*g))/(3
*f^5) + (2*b^2*d*h*(e + f*x)^(9/2))/(9*f^5) - (2*(e + f*x)^(1/2)*(a*f - b*
e)^2*(c*f - d*e)*(e*h - f*g))/f^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{fx+e}\,(35b^2d\,f^4h\,x^4+90abd\,f^4h\,x^3+45b^2c\,f^4h\,x^3-40b^2de\,f^3h\,x^3+45b^2d\,f^4g\,x^3+63a^2d\,f^4h\,x^2+}{}$$

input
```
int((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 210*a**2*c*e*f**3*h + 315*a**2*c*f**4*g + 105*a**2*c*
f**4*h*x + 168*a**2*d*e**2*f**2*h - 210*a**2*d*e*f**3*g - 84*a**2*d*e*f**3
*h*x + 105*a**2*d*f**4*g*x + 63*a**2*d*f**4*h*x**2 + 336*a*b*c*e**2*f**2*h
 - 420*a*b*c*e*f**3*g - 168*a*b*c*e*f**3*h*x + 210*a*b*c*f**4*g*x + 126*a*
b*c*f**4*h*x**2 - 288*a*b*d*e**3*f*h + 336*a*b*d*e**2*f**2*g + 144*a*b*d*e
**2*f**2*h*x - 168*a*b*d*e*f**3*g*x - 108*a*b*d*e*f**3*h*x**2 + 126*a*b*d*
f**4*g*x**2 + 90*a*b*d*f**4*h*x**3 - 144*b**2*c*e**3*f*h + 168*b**2*c*e**2
*f**2*g + 72*b**2*c*e**2*f**2*h*x - 84*b**2*c*e*f**3*g*x - 54*b**2*c*e*f**
3*h*x**2 + 63*b**2*c*f**4*g*x**2 + 45*b**2*c*f**4*h*x**3 + 128*b**2*d*e**4
*h - 144*b**2*d*e**3*f*g - 64*b**2*d*e**3*f*h*x + 72*b**2*d*e**2*f**2*g*x
+ 48*b**2*d*e**2*f**2*h*x**2 - 54*b**2*d*e*f**3*g*x**2 - 40*b**2*d*e*f**3*
h*x**3 + 45*b**2*d*f**4*g*x**3 + 35*b**2*d*f**4*h*x**4))/(315*f**5)
```

3.107 $\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\begin{aligned}\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx &= \frac{2(be-af)(de-cf)(fg-eh)\sqrt{e+fx}}{f^4} \\ &\quad - \frac{2(bde(2fg-3eh)-bcf(fg-2eh)-af(dfg-2deh+cfh))(e+fx)^{3/2}}{3f^4} \\ &\quad + \frac{2(adfh+b(dfg-3deh+cfh))(e+fx)^{5/2}}{5f^4}+\frac{2bdh(e+fx)^{7/2}}{7f^4}\end{aligned}$$

output
```
2*(-a*f+b*e)*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(1/2)/f^4-2/3*(b*d*e*(-3*e*h+2*
f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(3/2)/f^4+2/5*(
a*d*f*h+b*(c*f*h-3*d*e*h+d*f*g))*(f*x+e)^(5/2)/f^4+2/7*b*d*h*(f*x+e)^(7/2)
/f^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(7af(5cf(3fg-2eh+fhx)+d(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx)))+b(7cf(8e^2h-2\ldots}{1\ldots}$$

input
```
Integrate[((a + b*x)*(c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(7*a*f*(5*c*f*(3*f*g - 2*e*h + f*h*x) + d*(8*e^2*h - 2*e*
f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x))) + b*(7*c*f*(8*e^2*h - 2*e*f*(5*g +
 2*h*x) + f^2*x*(5*g + 3*h*x)) + d*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*
f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x)))))/(105*f^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{159}$$
$$\int \left(\frac{(e+fx)^{3/2}(adfh+b(cfh-3deh+dfg))}{f^3}+\frac{\sqrt{e+fx}(af(cfh-2deh+dfg)+bcf(fg-2eh)-bde(2fg-\ldots}{f^3}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{5/2}(adfh+b(cfh-3deh+dfg))}{5f^4}-\frac{2(e+fx)^{3/2}(-af(cfh-2deh+dfg)-bcf(fg-2eh)+bde(2fg-3eh))}{3f^4}+\frac{2\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}{f^4}+\frac{2bdh(e+fx)^{7/2}}{7f^4}$$

input
```
Int[((a + b*x)*(c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*(b*e - a*f)*(d*e - c*f)*(f*g - e*h)*Sqrt[e + f*x])/f^4 - (2*(b*d*e*(2*f
*g - 3*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(d*f*g - 2*d*e*h + c*f*h))*(e + f*
x)^(3/2))/(3*f^4) + (2*(a*d*f*h + b*(d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(
5/2))/(5*f^4) + (2*b*d*h*(e + f*x)^(7/2))/(7*f^4)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{4\left(\frac{\left(-x\left(\frac{3x\left(\frac{5hx}{7}+g\right)d}{5}+c\left(\frac{3hx}{5}+g\right)\right)b-3a\left(\frac{x\left(\frac{3hx}{5}+g\right)d}{3}+c\left(\frac{hx}{3}+g\right)\right)\right)f^3}{2}+\left(\left(\frac{2x\left(\frac{9hx}{14}+g\right)d}{5}+c\left(\frac{2hx}{5}+g\right)\right)b+a\left(\left(\frac{2hx}{5}\right.\right.\right.\right.}{3f^4}$
derivativedivides	$\frac{\frac{2dbh(fx+e)^{\frac{7}{2}}}{7}+\frac{2(((af-be)d+b(cf-de))h+db(-eh+fg))(fx+e)^{\frac{5}{2}}}{5}+\frac{2((af-be)(cf-de)h+((af-be)d+b(cf-de))(-eh+fg))(fx+e}{3}}{f^4}$
default	$\frac{\frac{2dbh(fx+e)^{\frac{7}{2}}}{7}+\frac{2(-((-af+be)d-b(cf-de))h-db(eh-fg))(fx+e)^{\frac{5}{2}}}{5}+\frac{2(-(-af+be)(cf-de)h+((-af+be)d-b(cf-de))(eh-fg))}{3}}{f^4}$
gosper	$-\frac{2\sqrt{fx+e}\,(-15bdh\,x^3f^3-21ad\,f^3h\,x^2-21bc\,f^3h\,x^2+18bde\,f^2h\,x^2-21bd\,f^3g\,x^2-35ac\,f^3hx+28ade\,f^2hx-35ad\,f^3g}{}$
trager	$-\frac{2\sqrt{fx+e}\,(-15bdh\,x^3f^3-21ad\,f^3h\,x^2-21bc\,f^3h\,x^2+18bde\,f^2h\,x^2-21bd\,f^3g\,x^2-35ac\,f^3hx+28ade\,f^2hx-35ad\,f^3g}{}$
risch	$-\frac{2\sqrt{fx+e}\,(-15bdh\,x^3f^3-21ad\,f^3h\,x^2-21bc\,f^3h\,x^2+18bde\,f^2h\,x^2-21bd\,f^3g\,x^2-35ac\,f^3hx+28ade\,f^2hx-35ad\,f^3g}{}$
orering	$-\frac{2\sqrt{fx+e}\,(-15bdh\,x^3f^3-21ad\,f^3h\,x^2-21bc\,f^3h\,x^2+18bde\,f^2h\,x^2-21bd\,f^3g\,x^2-35ac\,f^3hx+28ade\,f^2hx-35ad\,f^3g}{}$

input
```
int((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(1/2*(-x*(3/5*x*(5/7*h*x+g)*d+c*(3/5*h*x+g))*b-3*a*(1/3*x*(3/5*h*x+g)
*d+c*(1/3*h*x+g)))*f^3+((2/5*x*(9/14*h*x+g)*d+c*(2/5*h*x+g))*b+a*((2/5*h*x
+g)*d+c*h))*e*f^2-4/5*(((3/7*h*x+g)*d+c*h)*b+a*d*h)*e^2*f+24/35*b*d*e^3*h)
*(f*x+e)^(1/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$=\frac{2\,(15\,bdf^3hx^3+3\,(7\,bdf^3g-(6\,bdef^2-7\,(bc+ad)f^3)h)x^2+7\,(8\,bde^2f+15\,acf^3-10\,(bc+ad)ef^2)g-}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b*d*f^3*h*x^3 + 3*(7*b*d*f^3*g - (6*b*d*e*f^2 - 7*(b*c + a*d)*f^
3)*h)*x^2 + 7*(8*b*d*e^2*f + 15*a*c*f^3 - 10*(b*c + a*d)*e*f^2)*g - 2*(24*
b*d*e^3 + 35*a*c*e*f^2 - 28*(b*c + a*d)*e^2*f)*h - (7*(4*b*d*e*f^2 - 5*(b*
c + a*d)*f^3)*g - (24*b*d*e^2*f + 35*a*c*f^3 - 28*(b*c + a*d)*e*f^2)*h)*x)
*sqrt(f*x + e)/f^4
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{bdh(e+fx)^{\frac{7}{2}}}{7f^3}+\frac{(e+fx)^{\frac{5}{2}}(adfh+bcfh-3bdeh+bdfg)}{5f^3}+\frac{(e+fx)^{\frac{3}{2}}\left(acf^2h-2adefh+adf^2g-2bcefh+bcf^2g+3bde^2h-2bdefg\right)}{3f^3}+\frac{\sqrt{e+fx}\left(-acef^2h+acf^3g-\right.}{}\right.}{f} \\ \frac{acgx+\frac{bdhx^4}{4}+\frac{x^3(adh+bch+bdg)}{3}+\frac{x^2(ach+adg+bcg)}{2}}{\sqrt{e}} \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b*d*h*(e + f*x)**(7/2)/(7*f**3) + (e + f*x)**(5/2)*(a*d*f*h
+ b*c*f*h - 3*b*d*e*h + b*d*f*g)/(5*f**3) + (e + f*x)**(3/2)*(a*c*f**2*h -
 2*a*d*e*f*h + a*d*f**2*g - 2*b*c*e*f*h + b*c*f**2*g + 3*b*d*e**2*h - 2*b*
d*e*f*g)/(3*f**3) + sqrt(e + f*x)*(-a*c*e*f**2*h + a*c*f**3*g + a*d*e**2*f
*h - a*d*e*f**2*g + b*c*e**2*f*h - b*c*e*f**2*g - b*d*e**3*h + b*d*e**2*f*
g)/f**3)/f, Ne(f, 0)), ((a*c*g*x + b*d*h*x**4/4 + x**3*(a*d*h + b*c*h + b*
d*g)/3 + x**2*(a*c*h + a*d*g + b*c*g)/2)/sqrt(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(15\,(fx+e)^{\frac{7}{2}}bdh+21\,(bdfg-(3\,bde-(bc+ad)f)h)(fx+e)^{\frac{5}{2}}-35\,((2\,bdef-(bc+ad)f^2)g-(3\,bc\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(f*x + e)^(7/2)*b*d*h + 21*(b*d*f*g - (3*b*d*e - (b*c + a*d)*f)*
h)*(f*x + e)^(5/2) - 35*((2*b*d*e*f - (b*c + a*d)*f^2)*g - (3*b*d*e^2 + a*
c*f^2 - 2*(b*c + a*d)*e*f)*h)*(f*x + e)^(3/2) + 105*((b*d*e^2*f + a*c*f^3
- (b*c + a*d)*e*f^2)*g - (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h)*sqrt
(f*x + e))/f^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\left(105\sqrt{fx+e}acg + \frac{35\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)bcg}{f} + \frac{35\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)adg}{f} + \frac{35\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)ach}{f} + \frac{7\left(3(f\right.}{}\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(f*x + e)*a*c*g + 35*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*
b*c*g/f + 35*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*d*g/f + 35*((f*x + e)
^(3/2) - 3*sqrt(f*x + e)*e)*a*c*h/f + 7*(3*(f*x + e)^(5/2) - 10*(f*x + e)^
(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*d*g/f^2 + 7*(3*(f*x + e)^(5/2) - 10*(f*x
 + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*h/f^2 + 7*(3*(f*x + e)^(5/2) - 1
0*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*d*h/f^2 + 3*(5*(f*x + e)^(7/
2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)
*b*d*h/f^3)/f
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{(e+f\,x)^{3/2}\,(2\,a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-4\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f\,g)}{3\,f^4}$$
$$+\frac{(e+f\,x)^{5/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+2\,b\,d\,f\,g)}{5\,f^4}$$
$$-\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)\,(c\,f-d\,e)\,(e\,h-f\,g)}{f^4}+\frac{2\,b\,d\,h\,(e+f\,x)^{7/2}}{7\,f^4}$$

input
```
int(((g + h*x)*(a + b*x)*(c + d*x))/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(3/2)*(2*a*c*f^2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 6*b*d*e^2*h -
4*a*d*e*f*h - 4*b*c*e*f*h - 4*b*d*e*f*g))/(3*f^4) + ((e + f*x)^(5/2)*(2*a*
d*f*h + 2*b*c*f*h - 6*b*d*e*h + 2*b*d*f*g))/(5*f^4) - (2*(e + f*x)^(1/2)*(
a*f - b*e)*(c*f - d*e)*(e*h - f*g))/f^4 + (2*b*d*h*(e + f*x)^(7/2))/(7*f^4
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{fx+e}\,(15bd\,f^3h\,x^3+21ad\,f^3h\,x^2+21bc\,f^3h\,x^2-18bde\,f^2h\,x^2+21bd\,f^3g\,x^2+35ac\,f^3hx-28ade\,}{}$$

input
```
int((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 70*a*c*e*f**2*h + 105*a*c*f**3*g + 35*a*c*f**3*h*x +
56*a*d*e**2*f*h - 70*a*d*e*f**2*g - 28*a*d*e*f**2*h*x + 35*a*d*f**3*g*x +
21*a*d*f**3*h*x**2 + 56*b*c*e**2*f*h - 70*b*c*e*f**2*g - 28*b*c*e*f**2*h*x
 + 35*b*c*f**3*g*x + 21*b*c*f**3*h*x**2 - 48*b*d*e**3*h + 56*b*d*e**2*f*g
+ 24*b*d*e**2*f*h*x - 28*b*d*e*f**2*g*x - 18*b*d*e*f**2*h*x**2 + 21*b*d*f*
*3*g*x**2 + 15*b*d*f**3*h*x**3))/(105*f**4)
```

3.108 $\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = -\frac{2(de-cf)(fg-eh)\sqrt{e+fx}}{f^3} + \frac{2(dfg-2deh+cfh)(e+fx)^{3/2}}{3f^3} + \frac{2dh(e+fx)^{5/2}}{5f^3}$$

output
```
-2*(-c*f+d*e)*(-e*h+f*g)*(f*x+e)^(1/2)/f^3+2/3*(c*f*h-2*d*e*h+d*f*g)*(f*x+
e)^(3/2)/f^3+2/5*d*h*(f*x+e)^(5/2)/f^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\sqrt{e+fx}(5cf(3fg-2eh+fhx)+d(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx)))}{15f^3}$$

input
```
Integrate[((c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(5*c*f*(3*f*g - 2*e*h + f*h*x) + d*(8*e^2*h - 2*e*f*(5*g
+ 2*h*x) + f^2*x*(5*g + 3*h*x))))/(15*f^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{\sqrt{e+fx}(cfh-2deh+dfg)}{f^2}+\frac{(cf-de)(fg-eh)}{f^2\sqrt{e+fx}}+\frac{dh(e+fx)^{3/2}}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{3/2}(cfh-2deh+dfg)}{3f^3}-\frac{2\sqrt{e+fx}(de-cf)(fg-eh)}{f^3}+\frac{2dh(e+fx)^{5/2}}{5f^3}$$

input
```
Int[((c + d*x)*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(-2*(d*e - c*f)*(f*g - e*h)*Sqrt[e + f*x])/f^3 + (2*(d*f*g - 2*d*e*h + c*f
*h)*(e + f*x)^(3/2))/(3*f^3) + (2*d*h*(e + f*x)^(5/2))/(5*f^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{4\left(\frac{\left(-x\left(\frac{3hx}{5}+g\right)d-3c\left(\frac{hx}{3}+g\right)\right)f^{2}}{2}+e\left(\left(\frac{2hx}{5}+g\right)d+ch\right)f-\frac{4d\,e^{2}h}{5}\right)\sqrt{fx+e}}{3f^{3}}$	61
derivativedivides	$\frac{\frac{2dh(fx+e)^{\frac{5}{2}}}{5}+\frac{2((cf-de)h+d(-eh+fg))(fx+e)^{\frac{3}{2}}}{3}+2(cf-de)(-eh+fg)\sqrt{fx+e}}{f^{3}}$	72
gosper	$-\frac{2\sqrt{fx+e}\,\left(-3dh\,x^{2}f^{2}-5c\,f^{2}hx+4defhx-5d\,f^{2}gx+10cefh-15cg\,f^{2}-8d\,e^{2}h+10defg\right)}{15f^{3}}$	73
trager	$-\frac{2\sqrt{fx+e}\,\left(-3dh\,x^{2}f^{2}-5c\,f^{2}hx+4defhx-5d\,f^{2}gx+10cefh-15cg\,f^{2}-8d\,e^{2}h+10defg\right)}{15f^{3}}$	73
risch	$-\frac{2\sqrt{fx+e}\,\left(-3dh\,x^{2}f^{2}-5c\,f^{2}hx+4defhx-5d\,f^{2}gx+10cefh-15cg\,f^{2}-8d\,e^{2}h+10defg\right)}{15f^{3}}$	73
orering	$-\frac{2\sqrt{fx+e}\,\left(-3dh\,x^{2}f^{2}-5c\,f^{2}hx+4defhx-5d\,f^{2}gx+10cefh-15cg\,f^{2}-8d\,e^{2}h+10defg\right)}{15f^{3}}$	73
default	$\frac{\frac{2dh(fx+e)^{\frac{5}{2}}}{5}+\frac{2(-(-cf+de)h-d(eh-fg))(fx+e)^{\frac{3}{2}}}{3}+2(-cf+de)(eh-fg)\sqrt{fx+e}}{f^{3}}$	74

input
```
int((d*x+c)*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(1/2*(-x*(3/5*h*x+g)*d-3*c*(1/3*h*x+g))*f^2+e*((2/5*h*x+g)*d+c*h)*f-4
/5*d*e^2*h)*(f*x+e)^(1/2)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\left(3\,df^2hx^2-5\,(2\,def-3\,cf^2)g+2\,(4\,de^2-5\,cef)h+(5\,df^2g-(4\,def-5\,cf^2)h)x\right)\sqrt{fx+e}}{15\,f^3}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*d*f^2*h*x^2 - 5*(2*d*e*f - 3*c*f^2)*g + 2*(4*d*e^2 - 5*c*e*f)*h +
(5*d*f^2*g - (4*d*e*f - 5*c*f^2)*h)*x)*sqrt(f*x + e)/f^3
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \begin{cases} \frac{2\left(\frac{dh(e+fx)^{\frac{5}{2}}}{5f^2}+\frac{(e+fx)^{\frac{3}{2}}(cfh-2deh+dfg)}{3f^2}+\frac{\sqrt{e+fx}\left(-cefh+cf^2g+de^2h-defg\right)}{f^2}\right)}{f} & \text{for } f\neq 0 \\ \frac{cgx+\frac{dhx^3}{3}+\frac{x^2(ch+dg)}{2}}{\sqrt{e}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(5/2)/(5*f**2) + (e + f*x)**(3/2)*(c*f*h - 2*
d*e*h + d*f*g)/(3*f**2) + sqrt(e + f*x)*(-c*e*f*h + c*f**2*g + d*e**2*h -
d*e*f*g)/f**2)/f, Ne(f, 0)), ((c*g*x + d*h*x**3/3 + x**2*(c*h + d*g)/2)/sq
rt(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\left(3(fx+e)^{\frac{5}{2}}dh+5(dfg-(2de-cf)h)(fx+e)^{\frac{3}{2}}-15((def-cf^2)g-(de^2-cef)h)\sqrt{fx+e}\right)}{15f^3}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*(f*x + e)^(5/2)*d*h + 5*(d*f*g - (2*d*e - c*f)*h)*(f*x + e)^(3/2)
- 15*((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h)*sqrt(f*x + e))/f^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\left(15\sqrt{fx+e}cg+\frac{5\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)dg}{f}+\frac{5\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)ch}{f}+\frac{\left(3(fx+e)^{\frac{5}{2}}-10(fx+e)^{\frac{3}{2}}e+15\sqrt{fx+e}e^2\right)dh}{f^2}\right)}{15f}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(f*x + e)*c*g + 5*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*d*g/f
 + 5*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*c*h/f + (3*(f*x + e)^(5/2) - 10
*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*d*h/f^2)/f
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+f\,x}\,\left(3\,d\,h\,(e+f\,x)^2+15\,c\,f^2\,g+15\,d\,e^2\,h+5\,c\,f\,h\,(e+f\,x)-10\,d\,e\,h\,(e+f\,x)+5\,d\,f\,g\,(e+f}{15\,f^3}$$

input
```
int(((g + h*x)*(c + d*x))/(e + f*x)^(1/2),x)
```

output
```
(2*(e + f*x)^(1/2)*(3*d*h*(e + f*x)^2 + 15*c*f^2*g + 15*d*e^2*h + 5*c*f*h*
(e + f*x) - 10*d*e*h*(e + f*x) + 5*d*f*g*(e + f*x) - 15*c*e*f*h - 15*d*e*f
*g))/(15*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{fx+e}\,(3d\,f^2h\,x^2+5c\,f^2hx-4defhx+5d\,f^2gx-10cefh+15c\,f^2g+8d\,e^2h-10defg)}{15f^3}$$

input
```
int((d*x+c)*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 10*c*e*f*h + 15*c*f**2*g + 5*c*f**2*h*x + 8*d*e**2*h
- 10*d*e*f*g - 4*d*e*f*h*x + 5*d*f**2*g*x + 3*d*f**2*h*x**2))/(15*f**3)
```

3.109 $\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = -\frac{2(d(be+af)h-bf(dg+ch))\sqrt{e+fx}}{b^2f^2}+\frac{2dh(e+fx)^{3/2}}{3bf^2} - \frac{2(bc-ad)(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}\sqrt{be-af}}$$

output
```
-2*(d*(a*f+b*e)*h-b*f*(c*h+d*g))*(f*x+e)^(1/2)/b^2/f^2+2/3*d*h*(f*x+e)^(3/
2)/b/f^2-2*(-a*d+b*c)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^
(1/2))/b^(5/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{2\sqrt{e+fx}(3bcfh-3adfh+bd(3fg-2eh+fhx))}{3b^2f^2} + \frac{2(bc-ad)(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{5/2}\sqrt{-be+af}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*(3*b*c*f*h - 3*a*d*f*h + b*d*(3*f*g - 2*e*h + f*h*x)))/(3
*b^2*f^2) + (2*(b*c - a*d)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt
[-(b*e) + a*f]])/(b^(5/2)*Sqrt[-(b*e) + a*f])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 164$$

$$\frac{(bc-ad)(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b^2}-\frac{2\sqrt{e+fx}(3adfh-3bf(ch+dg)+2bdeh-bdfhx)}{3b^2f^2}$$

$$\Big\downarrow \; 73$$

$$\frac{2(bc-ad)(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b^2f}-\frac{2\sqrt{e+fx}(3adfh-3bf(ch+dg)+2bdeh-bdfhx)}{3b^2f^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{2(bc-ad)(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}\sqrt{be-af}}-\frac{2\sqrt{e+fx}(3adfh-3bf(ch+dg)+2bdeh-bdfhx)}{3b^2f^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
(-2*Sqrt[e + f*x]*(2*b*d*e*h + 3*a*d*f*h - 3*b*f*(d*g + c*h) - b*d*f*h*x))
/(3*b^2*f^2) - (2*(b*c - a*d)*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/
Sqrt[b*e - a*f]])/(b^(5/2)*Sqrt[b*e - a*f])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{2(-hfdbx+3adfh-3bcfh+2bdeh-3bdfg)\sqrt{fx+e}}{3f^2b^2}+\frac{2(a^2dh-abch-abdg+b^2cg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{b^2\sqrt{(af-be)b}}$
pseudoelliptic	$-\frac{2\left(-f^2(ah-bg)(ad-bc)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\sqrt{(af-be)b}\,\sqrt{fx+e}\left(\left(\left(\left(-\frac{hx}{3}-g\right)d-ch\right)f+\frac{2deh}{3}\right)b+adfh\right)\right)}{\sqrt{(af-be)b}\,f^2b^2}$
derivativedivides	$\frac{-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}+bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^2}+\frac{2f^2\left(a^2dh-abch-abdg+b^2cg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{b^2\sqrt{(af-be)b}}}{f^2}$
default	$\frac{-\frac{2\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}+bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^2}+\frac{2f^2\left(a^2dh-abch-abdg+b^2cg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{b^2\sqrt{(af-be)b}}}{f^2}$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*d*f*h*x+3*a*d*f*h-3*b*c*f*h+2*b*d*e*h-3*b*d*f*g)*(f*x+e)^(1/2)/f^
2/b^2+2*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/b^2/((a*f-b*e)*b)^(1/2)*arctan(b
*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.42

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$=\left[\frac{3\left((b^2c-abd)f^2g-(abc-a^2d)f^2h\right)\sqrt{b^2e-abf}\log\left(\frac{bfx+2\,be-af-2\,\sqrt{b^2e-abf}\sqrt{fx+e}}{bx+a}\right)+2\left((b^3def-ab^2df^2\right.}{3\left(b^4ef^2-ab^3f\right.}\right.$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*((b^2*c - a*b*d)*f^2*g - (a*b*c - a^2*d)*f^2*h)*sqrt(b^2*e - a*b*f
)*log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a
)) + 2*((b^3*d*e*f - a*b^2*d*f^2)*h*x + 3*(b^3*d*e*f - a*b^2*d*f^2)*g - (2
*b^3*d*e^2 - (3*b^3*c - a*b^2*d)*e*f + 3*(a*b^2*c - a^2*b*d)*f^2)*h)*sqrt(
f*x + e))/(b^4*e*f^2 - a*b^3*f^3), 2/3*(3*((b^2*c - a*b*d)*f^2*g - (a*b*c
- a^2*d)*f^2*h)*sqrt(-b^2*e + a*b*f)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x
+ e)/(b*f*x + b*e)) + ((b^3*d*e*f - a*b^2*d*f^2)*h*x + 3*(b^3*d*e*f - a*b^
2*d*f^2)*g - (2*b^3*d*e^2 - (3*b^3*c - a*b^2*d)*e*f + 3*(a*b^2*c - a^2*b*d
)*f^2)*h)*sqrt(f*x + e))/(b^4*e*f^2 - a*b^3*f^3)]
```

Sympy [A] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \begin{cases} \frac{2\left(\frac{dh(e+fx)^{\frac{3}{2}}}{3bf}+\frac{\sqrt{e+fx}(-adfh+bcfh-bdeh+bdfg)}{b^2f}+\frac{f(ad-bc)(ah-bg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{af-be}{b}}}\right)}{b^3\sqrt{\frac{af-be}{b}}}\right)}{f} & \text{for } f\neq 0 \\ \frac{\frac{dhx^2}{2b}+\frac{x(-adh+bch+bdg)}{b^2}+\frac{(ad-bc)(ah-bg)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{b^2}}{\sqrt{e}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(3/2)/(3*b*f) + sqrt(e + f*x)*(-a*d*f*h + b*c
*f*h - b*d*e*h + b*d*f*g)/(b**2*f) + f*(a*d - b*c)*(a*h - b*g)*atan(sqrt(e
 + f*x)/sqrt((a*f - b*e)/b))/(b**3*sqrt((a*f - b*e)/b)))/f, Ne(f, 0)), ((d
*h*x**2/(2*b) + x*(-a*d*h + b*c*h + b*d*g)/b**2 + (a*d - b*c)*(a*h - b*g)*
Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/b**2)/sqrt(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{2\left(b^2cg-abdg-abch+a^2dh\right)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{\sqrt{-b^2e+abf}\,b^2} + \frac{2\left(3\sqrt{fx+e}b^2df^5g+(fx+e)^{\frac{3}{2}}b^2df^4h-3\sqrt{fx+e}b^2def^4h+3\sqrt{fx+e}b^2cf^5h-3\sqrt{fx+e}abdf^5h\right)}{3\,b^3f^6}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(b^2*c*g - a*b*d*g - a*b*c*h + a^2*d*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2
*e + a*b*f))/(sqrt(-b^2*e + a*b*f)*b^2) + 2/3*(3*sqrt(f*x + e)*b^2*d*f^5*g
 + (f*x + e)^(3/2)*b^2*d*f^4*h - 3*sqrt(f*x + e)*b^2*d*e*f^4*h + 3*sqrt(f*
x + e)*b^2*c*f^5*h - 3*sqrt(f*x + e)*a*b*d*f^5*h)/(b^3*f^6)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,c\,f\,h-4\,d\,e\,h+2\,d\,f\,g}{b\,f^2}-\frac{2\,d\,h\,\left(a\,f^3-b\,e\,f^2\right)}{b^2\,f^4}\right)+\frac{2\,d\,h\,{\left(e+f\,x\right)}^{3/2}}{3\,b\,f^2}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,\left(a\,d-b\,c\right)\,\left(a\,h-b\,g\right)}{\sqrt{a\,f-b\,e}\,\left(b^2\,c\,g+a^2\,d\,h-a\,b\,c\,h-a\,b\,d\,g\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a\,h-b\,g\right)}{b^{5/2}\,\sqrt{a\,f-b\,e}}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(1/2)*(a + b*x)),x)
```

output
```
(e + f*x)^(1/2)*((2*c*f*h - 4*d*e*h + 2*d*f*g)/(b*f^2) - (2*d*h*(a*f^3 - b
*e*f^2))/(b^2*f^4)) + (2*d*h*(e + f*x)^(3/2))/(3*b*f^2) + (2*atan((b^(1/2)
*(e + f*x)^(1/2)*(a*d - b*c)*(a*h - b*g))/((a*f - b*e)^(1/2)*(b^2*c*g + a^
2*d*h - a*b*c*h - a*b*d*g)))*(a*d - b*c)*(a*h - b*g))/(b^(5/2)*(a*f - b*e)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a^2d\,f^2h-2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)abc\,f^2h-2\sqrt{b}\,\sqrt{af-be}\,atan}{}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x)
```

output

```
(2*(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**2*d*f**2*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a*b*c*f**2*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*d*f**2*g + 3*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c*f**2*g -
3*sqrt(e + f*x)*a**2*b*d*f**2*h + 3*sqrt(e + f*x)*a*b**2*c*f**2*h + sqrt(e
 + f*x)*a*b**2*d*e*f*h + 3*sqrt(e + f*x)*a*b**2*d*f**2*g + sqrt(e + f*x)*a
*b**2*d*f**2*h*x - 3*sqrt(e + f*x)*b**3*c*e*f*h + 2*sqrt(e + f*x)*b**3*d*e
**2*h - 3*sqrt(e + f*x)*b**3*d*e*f*g - sqrt(e + f*x)*b**3*d*e*f*h*x))/(3*b
**3*f**2*(a*f - b*e))
```

3.110 $\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 154

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \frac{2dh\sqrt{e+fx}}{b^2f} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{b^2(be-af)(a+bx)} + \frac{(b^2cfg-a^2dfh-(2be-af)(bdg+bch-2adh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}(be-af)^{3/2}}$$

output

```
2*d*h*(f*x+e)^(1/2)/b^2/f-(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*
e)/(b*x+a)+(b^2*c*f*g-a^2*d*f*h-(-a*f+2*b*e)*(-2*a*d*h+b*c*h+b*d*g))*arcta
nh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$$
$$=\frac{\sqrt{e+fx}(-3a^2dfh+b^2(-cfg+2dehx)+ab(2deh+cfh+df(g-2hx)))}{b^2f(be-af)(a+bx)}$$
$$-\frac{(3a^2dfh+b^2(2deg-cfg+2ceh)-ab(dfg+4deh+cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{5/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[e + f*x]*(-3*a^2*d*f*h + b^2*(-(c*f*g) + 2*d*e*h*x) + a*b*(2*d*e*h +
 c*f*h + d*f*(g - 2*h*x))))/(b^2*f*(b*e - a*f)*(a + b*x)) - ((3*a^2*d*f*h
+ b^2*(2*d*e*g - c*f*g + 2*c*e*h) - a*b*(d*f*g + 4*d*e*h + c*f*h))*ArcTan[
(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(5/2)*(-(b*e) + a*f)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{163}$$
$$\frac{(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg))\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2b^2(be-af)}-\frac{\sqrt{e+fx}(3a^2dfh-ab(cfh+2deh+dfg)-2bdhx(be-af)+b^2cfg)}{b^2f(a+bx)(be-af)}$$

$$\Big\downarrow 73$$

$$\frac{\left(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b^2f(be-af)}-\frac{\sqrt{e+fx}\left(3a^2dfh-ab(cfh+2deh+dfg)-2bdhx(be-af)+b^2cfg\right)}{b^2f(a+bx)(be-af)}$$

$$\Big\downarrow 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^2dfh-ab(cfh+4deh+dfg)+b^2(2ceh-cfg+2deg)\right)}{b^{5/2}(be-af)^{3/2}}-\frac{\sqrt{e+fx}\left(3a^2dfh-ab(cfh+2deh+dfg)-2bdhx(be-af)+b^2cfg\right)}{b^2f(a+bx)(be-af)}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output
```
-((Sqrt[e + f*x]*(b^2*c*f*g + 3*a^2*d*f*h - a*b*(d*f*g + 2*d*e*h + c*f*h)
- 2*b*d*(b*e - a*f)*h*x))/(b^2*f*(b*e - a*f)*(a + b*x))) - ((3*a^2*d*f*h +
 b^2*(2*d*e*g - c*f*g + 2*c*e*h) - a*b*(d*f*g + 4*d*e*h + c*f*h))*ArcTanh[
(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(5/2)*(b*e - a*f)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-3\left(\frac{(-cfg+2e(ch+dg))b^2}{3}-\frac{a(f(ch+dg)+4deh)b}{3}+a^2dfh\right)(bx+a)f\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+3\left(\frac{(-2dehx+cfg)b^2}{3}-\frac{a(((-2hx-}{}\right.}{b^2(af-be)f(bx+a)\sqrt{(af-be)b}}$
risch	$\frac{2dh\sqrt{fx+e}}{b^2f}-\frac{-\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{(af-be)((fx+e)b+af-be)}+\frac{\left(3a^2dfh-abcfh-4abdeh-abdfg+2b^2ceh-b^2cfg+2b^2deg\right)\arctan\left(\frac{b\sqrt{}}{\sqrt{(a}}\right.}{(af-be)\sqrt{(af-be)b}}}{b^2}$
derivativedivides	$\frac{\frac{2dh\sqrt{fx+e}}{b^2}-\frac{2f\left(-\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(3a^2dfh-abcfh-4abdeh-abdfg+2b^2ceh-b^2cfg+2b^2deg\right)\arctan\left(\frac{b\sqrt{}}{\sqrt{(a}}\right.}{2(af-be)\sqrt{(af-be)b}}\right.}{b^2}}{f}$
default	$\frac{\frac{2dh\sqrt{fx+e}}{b^2}-\frac{2f\left(-\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(3a^2dfh-abcfh-4abdeh-abdfg+2b^2ceh-b^2cfg+2b^2deg\right)\arctan\left(\frac{b\sqrt{}}{\sqrt{(a}}\right.}{2(af-be)\sqrt{(af-be)b}}\right.}{b^2}}{f}$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3*(-(1/3*(-c*f*g+2*e*(c*h+d*g))*b^2-1/3*a*(f*(c*h+d*g)+4*d*e*h)*b+a^2*d*f*
h)*(b*x+a)*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+(1/3*(-2*d*e*h*x+
c*f*g)*b^2-1/3*a*(((-2*h*x+g)*d+c*h)*f+2*d*e*h)*b+a^2*d*f*h)*((a*f-b*e)*b)
^(1/2)*(f*x+e)^(1/2))/((a*f-b*e)*b)^(1/2)/f/b^2/(a*f-b*e)/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(140) = 280$.

Time = 0.12 (sec) , antiderivative size = 793, normalized size of antiderivative = 5.15

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b^2*e - a*b*f)*((2*a*b^2*d*e*f - (a*b^2*c + a^2*b*d)*f^2)*g +
(2*(a*b^2*c - 2*a^2*b*d)*e*f - (a^2*b*c - 3*a^3*d)*f^2)*h + ((2*b^3*d*e*f
- (b^3*c + a*b^2*d)*f^2)*g + (2*(b^3*c - 2*a*b^2*d)*e*f - (a*b^2*c - 3*a^2
*b*d)*f^2)*h)*x)*log((b*f*x + 2*b*e - a*f + 2*sqrt(b^2*e - a*b*f)*sqrt(f*x
 + e))/(b*x + a)) - 2*(2*(b^4*d*e^2 - 2*a*b^3*d*e*f + a^2*b^2*d*f^2)*h*x -
 ((b^4*c - a*b^3*d)*e*f - (a*b^3*c - a^2*b^2*d)*f^2)*g + (2*a*b^3*d*e^2 +
(a*b^3*c - 5*a^2*b^2*d)*e*f - (a^2*b^2*c - 3*a^3*b*d)*f^2)*h)*sqrt(f*x + e
))/(a*b^5*e^2*f - 2*a^2*b^4*e*f^2 + a^3*b^3*f^3 + (b^6*e^2*f - 2*a*b^5*e*f
^2 + a^2*b^4*f^3)*x), (sqrt(-b^2*e + a*b*f)*((2*a*b^2*d*e*f - (a*b^2*c + a
^2*b*d)*f^2)*g + (2*(a*b^2*c - 2*a^2*b*d)*e*f - (a^2*b*c - 3*a^3*d)*f^2)*h
 + ((2*b^3*d*e*f - (b^3*c + a*b^2*d)*f^2)*g + (2*(b^3*c - 2*a*b^2*d)*e*f -
 (a*b^2*c - 3*a^2*b*d)*f^2)*h)*x)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x + e
)/(b*f*x + b*e)) + (2*(b^4*d*e^2 - 2*a*b^3*d*e*f + a^2*b^2*d*f^2)*h*x - ((
b^4*c - a*b^3*d)*e*f - (a*b^3*c - a^2*b^2*d)*f^2)*g + (2*a*b^3*d*e^2 + (a*
b^3*c - 5*a^2*b^2*d)*e*f - (a^2*b^2*c - 3*a^3*b*d)*f^2)*h)*sqrt(f*x + e))/
(a*b^5*e^2*f - 2*a^2*b^4*e*f^2 + a^3*b^3*f^3 + (b^6*e^2*f - 2*a*b^5*e*f^2
+ a^2*b^4*f^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx &= \frac{(2\,b^2deg - b^2cfg - abdfg + 2\,b^2ceh - 4\,abdeh - abcfh + 3\,a^2dfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^3e - ab^2f)\sqrt{-b^2e+abf}} \\ &\quad + \frac{2\sqrt{fx+e}dh}{b^2f} - \frac{\sqrt{fx+e}b^2cfg - \sqrt{fx+e}abdfg - \sqrt{fx+e}abcfh + \sqrt{fx+e}a^2dfh}{(b^3e - ab^2f)((fx+e)b - be + af)}\end{aligned}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
(2*b^2*d*e*g - b^2*c*f*g - a*b*d*f*g + 2*b^2*c*e*h - 4*a*b*d*e*h - a*b*c*f
*h + 3*a^2*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*e - a
*b^2*f)*sqrt(-b^2*e + a*b*f)) + 2*sqrt(f*x + e)*d*h/(b^2*f) - (sqrt(f*x +
e)*b^2*c*f*g - sqrt(f*x + e)*a*b*d*f*g - sqrt(f*x + e)*a*b*c*f*h + sqrt(f*
x + e)*a^2*d*f*h)/((b^3*e - a*b^2*f)*((f*x + e)*b - b*e + a*f))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,\left(b^2\,c\,f\,g-2\,b^2\,c\,e\,h-2\,b^2\,d\,e\,g-3\,a^2\,d\,f\,h+a\,b\,c\,f\,h+4\,a\,b\,d\,e\,h+a\,b\,d\,f\,g\right)}{b^{5/2}\,{\left(a\,f-b\,e\right)}^{3/2}}$$
$$+\frac{\sqrt{e+f\,x}\,\left(b^2\,c\,f\,g+a^2\,d\,f\,h-a\,b\,c\,f\,h-a\,b\,d\,f\,g\right)}{\left(a\,f-b\,e\right)\,\left(b^3\,\left(e+f\,x\right)-b^3\,e+a\,b^2\,f\right)}+\frac{2\,d\,h\,\sqrt{e+f\,x}}{b^2\,f}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(1/2)*(a + b*x)^2),x)
```

output
```
(atan((b^(1/2)*(e + f*x)^(1/2))/(a*f - b*e)^(1/2))*(b^2*c*f*g - 2*b^2*c*e*
h - 2*b^2*d*e*g - 3*a^2*d*f*h + a*b*c*f*h + 4*a*b*d*e*h + a*b*d*f*g))/(b^(
5/2)*(a*f - b*e)^(3/2)) + ((e + f*x)^(1/2)*(b^2*c*f*g + a^2*d*f*h - a*b*c*
f*h - a*b*d*f*g))/((a*f - b*e)*(b^3*(e + f*x) - b^3*e + a*b^2*f)) + (2*d*h
*(e + f*x)^(1/2))/(b^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.84

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*d*f**2*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**2*b*c*f**2*h + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*e*f*h + sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*f**2*g
 - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**2*b*d*f**2*h*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*e*f*h + sqrt(b)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*f**2*g + sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*f**
2*h*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a*b**2*d*e*f*g + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*e*f*h*x + sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*f**2*g*x - 2*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3
*c*e*f*h*x + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*b**3*c*f**2*g*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*d*e*f*g*x + 3*sqrt(e + f*x)*a**3*b*d
*f**2*h - sqrt(e + f*x)*a**2*b**2*c*f**2*h - 5*sqrt(e + f*x)*a**2*b**2*d*e
*f*h - sqrt(e + f*x)*a**2*b**2*d*f**2*g + 2*sqrt(e + f*x)*a**2*b**2*d*f...
```

3.111 $\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 237

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = -\frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{2b^2(be-af)(a+bx)^2} - \frac{(5a^2dfh+b^2(4deg-3cfg+4ceh)-ab(dfg+8deh+cfh))\sqrt{e+fx}}{4b^2(be-af)^2(a+bx)} - \frac{(3a^2df^2h+abf(dfg-8deh+cfh)+b^2(cf(3fg-4eh)-4de(fg-2eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{4b^{5/2}(be-af)^{5/2}}$$

output
```
-1/2*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)/(b*x+a)^2-1/4*(5*a
^2*d*f*h+b^2*(4*c*e*h-3*c*f*g+4*d*e*g)-a*b*(c*f*h+8*d*e*h+d*f*g))*(f*x+e)^
(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)-1/4*(3*a^2*d*f^2*h+a*b*f*(c*f*h-8*d*e*h+d*f
*g)+b^2*(c*f*(-4*e*h+3*f*g)-4*d*e*(-2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(
1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx =$$
$$-\frac{\sqrt{e+fx}(3a^3dfh+b^3(4degx-3cfgx+2ce(g+2hx))+a^2b(-6deh+cfh+df(g+5hx))-ab^2(d(-}{4b^2(be-af)^2(a+bx)^2}$$
$$+\frac{(3a^2df^2h+abf(dfg-8deh+cfh)+b^2(cf(3fg-4eh)+4de(-fg+2eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{4b^{5/2}(-be+af)^{5/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/4*(Sqrt[e + f*x]*(3*a^3*d*f*h + b^3*(4*d*e*g*x - 3*c*f*g*x + 2*c*e*(g +
 2*h*x)) + a^2*b*(-6*d*e*h + c*f*h + d*f*(g + 5*h*x)) - a*b^2*(d*(-2*e*g +
 f*g*x + 8*e*h*x) + c*(5*f*g - 2*e*h + f*h*x))))/(b^2*(b*e - a*f)^2*(a + b
*x)^2) + ((3*a^2*d*f^2*h + a*b*f*(d*f*g - 8*d*e*h + c*f*h) + b^2*(c*f*(3*f
*g - 4*e*h) + 4*d*e*(-(f*g) + 2*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt
[-(b*e) + a*f]])/(4*b^(5/2)*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {162, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{162}$$

$$\frac{\left(3a^2df^2h+abf(cfh-8deh+dfg)+b^2(cf(3fg-4eh)-4de(fg-2eh))\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{8b^2(be-af)^2}-$$
$$\frac{\sqrt{e+fx}\Big(3a^3dfh+bx\big(5a^2dfh-ab(cfh+8deh+dfg)+b^2(4ceh-3cfg+4deg)\big)+a^2b(cfh-6deh+dfg)+}{4b^2(a+bx)^2(be-af)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\left(3a^2df^2h+abf(cfh-8deh+dfg)+b^2(cf(3fg-4eh)-4de(fg-2eh))\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{4b^2f(be-af)^2}-$$
$$\frac{\sqrt{e+fx}\Big(3a^3dfh+bx\big(5a^2dfh-ab(cfh+8deh+dfg)+b^2(4ceh-3cfg+4deg)\big)+a^2b(cfh-6deh+dfg)+}{4b^2(a+bx)^2(be-af)^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^2df^2h+abf(cfh-8deh+dfg)+b^2(cf(3fg-4eh)-4de(fg-2eh))\right)}{4b^{5/2}(be-af)^{5/2}}-$$
$$\frac{\sqrt{e+fx}\Big(3a^3dfh+bx\big(5a^2dfh-ab(cfh+8deh+dfg)+b^2(4ceh-3cfg+4deg)\big)+a^2b(cfh-6deh+dfg)+}{4b^2(a+bx)^2(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/4*(Sqrt[e + f*x]*(2*b^3*c*e*g + 3*a^3*d*f*h + 2*a*b^2*(d*e*g - (5*c*f*g
)/2 + c*e*h) + a^2*b*(d*f*g - 6*d*e*h + c*f*h) + b*(5*a^2*d*f*h + b^2*(4*d
*e*g - 3*c*f*g + 4*c*e*h) - a*b*(d*f*g + 8*d*e*h + c*f*h))*x))/(b^2*(b*e -
 a*f)^2*(a + b*x)^2) - ((3*a^2*d*f^2*h + a*b*f*(d*f*g - 8*d*e*h + c*f*h) +
 b^2*(c*f*(3*f*g - 4*e*h) - 4*d*e*(f*g - 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e
+ f*x])/Sqrt[b*e - a*f]])/(4*b^(5/2)*(b*e - a*f)^(5/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{3 (-((c g f^2 - \frac{4e(ch+dg)f}{3} + \frac{8d e^2 h}{3}) b^2 + \frac{a(f(ch+dg)-8deh)fb}{3} + a^2 d f^2 h) (bx+a)^2 \arctan (\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}) + \sqrt{(af-be)b}}{4\sqrt{(af-be}}$
derivativedivides	$-\frac{2 (\frac{f (5a^2 dfh - abcfh - 8abdeh - abdfg + 4b^2 ceh - 3b^2 cfg + 4b^2 deg) (fx+e)^{\frac{3}{2}}}{8b (a^2 f^2 - 2abfe + b^2 e^2)} + \frac{(3a^2 dfh + abcfh - 8abdeh + abdfg + 4b^2 ceh - 5b^2 cfg}{8b^2 (af-be)}}{((fx+e)b+af-be)^2}$
default	$-\frac{2 (\frac{f (5a^2 dfh - abcfh - 8abdeh - abdfg + 4b^2 ceh - 3b^2 cfg + 4b^2 deg) (fx+e)^{\frac{3}{2}}}{8b (a^2 f^2 - 2abfe + b^2 e^2)} + \frac{(3a^2 dfh + abcfh - 8abdeh + abdfg + 4b^2 ceh - 5b^2 cfg}{8b^2 (af-be)}}{((fx+e)b+af-be)^2}$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-((c*g*f^2-4/3*e*(c*h+d*g)*f+8/3*d*e^2*h)*b^2+1/3*a*(f*(c*h+d*g)-8*d
*e*h)*f*b+a^2*d*f^2*h)*(b*x+a)^2*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2
))+((a*f-b*e)*b)^(1/2)*((-c*f*g*x+2/3*e*(2*(c*h+d*g)*x+c*g))*b^3+2/3*a*(1/
2*((-c*h-d*g)*x-5*c*g)*f+e*(-4*d*h*x+c*h+d*g))*b^2+1/3*a^2*((5*d*h*x+c*h+d
*g)*f-6*d*e*h)*b+a^3*d*f*h)*(f*x+e)^(1/2))/((a*f-b*e)*b)^(1/2)/(a*f-b*e)^2
/b^2/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(219) = 438$.

Time = 0.18 (sec) , antiderivative size = 1432, normalized size of antiderivative = 6.04

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(b^2*e - a*b*f)*(((4*b^4*d*e*f - (3*b^4*c + a*b^3*d)*f^2)*g - (
8*b^4*d*e^2 - 4*(b^4*c + 2*a*b^3*d)*e*f + (a*b^3*c + 3*a^2*b^2*d)*f^2)*h)*
x^2 + (4*a^2*b^2*d*e*f - (3*a^2*b^2*c + a^3*b*d)*f^2)*g - (8*a^2*b^2*d*e^2
 - 4*(a^2*b^2*c + 2*a^3*b*d)*e*f + (a^3*b*c + 3*a^4*d)*f^2)*h + 2*((4*a*b^
3*d*e*f - (3*a*b^3*c + a^2*b^2*d)*f^2)*g - (8*a*b^3*d*e^2 - 4*(a*b^3*c + 2
*a^2*b^2*d)*e*f + (a^2*b^2*c + 3*a^3*b*d)*f^2)*h)*x)*log((b*f*x + 2*b*e -
a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*((2*(b^5*c + a*b
^4*d)*e^2 - (7*a*b^4*c + a^2*b^3*d)*e*f + (5*a^2*b^3*c - a^3*b^2*d)*f^2)*g
 + (2*(a*b^4*c - 3*a^2*b^3*d)*e^2 - (a^2*b^3*c - 9*a^3*b^2*d)*e*f - (a^3*b
^2*c + 3*a^4*b*d)*f^2)*h + ((4*b^5*d*e^2 - (3*b^5*c + 5*a*b^4*d)*e*f + (3*
a*b^4*c + a^2*b^3*d)*f^2)*g + (4*(b^5*c - 2*a*b^4*d)*e^2 - (5*a*b^4*c - 13
*a^2*b^3*d)*e*f + (a^2*b^3*c - 5*a^3*b^2*d)*f^2)*h)*x)*sqrt(f*x + e))/(a^2
*b^6*e^3 - 3*a^3*b^5*e^2*f + 3*a^4*b^4*e*f^2 - a^5*b^3*f^3 + (b^8*e^3 - 3*
a*b^7*e^2*f + 3*a^2*b^6*e*f^2 - a^3*b^5*f^3)*x^2 + 2*(a*b^7*e^3 - 3*a^2*b^
6*e^2*f + 3*a^3*b^5*e*f^2 - a^4*b^4*f^3)*x), -1/4*(sqrt(-b^2*e + a*b*f)*((
(4*b^4*d*e*f - (3*b^4*c + a*b^3*d)*f^2)*g - (8*b^4*d*e^2 - 4*(b^4*c + 2*a*
b^3*d)*e*f + (a*b^3*c + 3*a^2*b^2*d)*f^2)*h)*x^2 + (4*a^2*b^2*d*e*f - (3*a
^2*b^2*c + a^3*b*d)*f^2)*g - (8*a^2*b^2*d*e^2 - 4*(a^2*b^2*c + 2*a^3*b*d)*
e*f + (a^3*b*c + 3*a^4*d)*f^2)*h + 2*((4*a*b^3*d*e*f - (3*a*b^3*c + a^2*b^
2*d)*f^2)*g - (8*a*b^3*d*e^2 - 4*(a*b^3*c + 2*a^2*b^2*d)*e*f + (a^2*b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(219) = 438$.

Time = 0.13 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx =$$

$$-\frac{(4\,b^2defg - 3\,b^2cf^2g - abdf^2g - 8\,b^2de^2h + 4\,b^2cefh + 8\,abdefh - abcf^2h - 3\,a^2df^2h)\arctan\left(\frac{\sqrt{fx-}}{\sqrt{-b^2e}}\right.}{4\,(b^4e^2 - 2\,ab^3ef + a^2b^2f^2)\sqrt{-b^2e+abf}}$$

$$-\frac{4\,(fx+e)^{\frac{3}{2}}b^3defg - 4\,\sqrt{fx+e}b^3de^2fg - 3\,(fx+e)^{\frac{3}{2}}b^3cf^2g - (fx+e)^{\frac{3}{2}}ab^2df^2g + 5\,\sqrt{fx+e}b^3cef^2g}{}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*b^2*d*e*f*g - 3*b^2*c*f^2*g - a*b*d*f^2*g - 8*b^2*d*e^2*h + 4*b^2*
c*e*f*h + 8*a*b*d*e*f*h - a*b*c*f^2*h - 3*a^2*d*f^2*h)*arctan(sqrt(f*x + e
)*b/sqrt(-b^2*e + a*b*f))/((b^4*e^2 - 2*a*b^3*e*f + a^2*b^2*f^2)*sqrt(-b^2
*e + a*b*f)) - 1/4*(4*(f*x + e)^(3/2)*b^3*d*e*f*g - 4*sqrt(f*x + e)*b^3*d*
e^2*f*g - 3*(f*x + e)^(3/2)*b^3*c*f^2*g - (f*x + e)^(3/2)*a*b^2*d*f^2*g +
5*sqrt(f*x + e)*b^3*c*e*f^2*g + 3*sqrt(f*x + e)*a*b^2*d*e*f^2*g - 5*sqrt(f
*x + e)*a*b^2*c*f^3*g + sqrt(f*x + e)*a^2*b*d*f^3*g + 4*(f*x + e)^(3/2)*b^
3*c*e*f*h - 8*(f*x + e)^(3/2)*a*b^2*d*e*f*h - 4*sqrt(f*x + e)*b^3*c*e^2*f*
h + 8*sqrt(f*x + e)*a*b^2*d*e^2*f*h - (f*x + e)^(3/2)*a*b^2*c*f^2*h + 5*(f
*x + e)^(3/2)*a^2*b*d*f^2*h + 3*sqrt(f*x + e)*a*b^2*c*e*f^2*h - 11*sqrt(f*
x + e)*a^2*b*d*e*f^2*h + sqrt(f*x + e)*a^2*b*c*f^3*h + 3*sqrt(f*x + e)*a^3
*d*f^3*h)/((b^4*e^2 - 2*a*b^3*e*f + a^2*b^2*f^2)*((f*x + e)*b - b*e + a*f)
^2)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,\left(3\,b^2\,c\,f^2\,g+3\,a^2\,d\,f^2\,h+8\,b^2\,d\,e^2\,h+a\,b\,c\,f^2\,h+a\,b\,d\,f^2\,g-4\,b^2\,c\,e\,f\,h-4\,b^2\,d\,e\,f\,g-\right.}{4\,b^{5/2}\,{\left(a\,f-b\,e\right)}^{5/2}}$$
$$-\frac{\frac{\sqrt{e+f\,x}\,\left(3\,a^2\,d\,f^2\,h-5\,b^2\,c\,f^2\,g+a\,b\,c\,f^2\,h+a\,b\,d\,f^2\,g+4\,b^2\,c\,e\,f\,h+4\,b^2\,d\,e\,f\,g-8\,a\,b\,d\,e\,f\,h\right)}{4\,b^2\,\left(a\,f-b\,e\right)}-\frac{{\left(e+f\,x\right)}^{3/2}\,\left(3\,b^2\,c\,f^2\,g-5\,a^2\,d\,f^2\,h+a\,b\,c\,f^2\,h\right.}{4\,b\,(a}}{b^2\,{\left(e+f\,x\right)}^2-\left(e+f\,x\right)\,\left(2\,b^2\,e-2\,a\,b\,f\right)+a^2\,f^2+b^2\,e^2-2\,a\,b\,e\,f}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(1/2)*(a + b*x)^3),x)
```

output
```
(atan((b^(1/2)*(e + f*x)^(1/2))/(a*f - b*e)^(1/2))*(3*b^2*c*f^2*g + 3*a^2*
d*f^2*h + 8*b^2*d*e^2*h + a*b*c*f^2*h + a*b*d*f^2*g - 4*b^2*c*e*f*h - 4*b^
2*d*e*f*g - 8*a*b*d*e*f*h))/(4*b^(5/2)*(a*f - b*e)^(5/2)) - (((e + f*x)^(1
/2)*(3*a^2*d*f^2*h - 5*b^2*c*f^2*g + a*b*c*f^2*h + a*b*d*f^2*g + 4*b^2*c*e
*f*h + 4*b^2*d*e*f*g - 8*a*b*d*e*f*h))/(4*b^2*(a*f - b*e)) - ((e + f*x)^(3
/2)*(3*b^2*c*f^2*g - 5*a^2*d*f^2*h + a*b*c*f^2*h + a*b*d*f^2*g - 4*b^2*c*e
*f*h - 4*b^2*d*e*f*g + 8*a*b*d*e*f*h))/(4*b*(a*f - b*e)^2))/(b^2*(e + f*x)
^2 - (e + f*x)*(2*b^2*e - 2*a*b*f) + a^2*f^2 + b^2*e^2 - 2*a*b*e*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1691, normalized size of antiderivative = 7.14

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**4*d*f**2*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**3*b*c*f**2*h - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d*e*f*h + sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d*f**2*g +
6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**3*b*d*f**2*h*x - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*e*f*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*f**2*g + 2*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*
b**2*c*f**2*h*x + 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**2*b**2*d*e**2*h - 4*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*e*f*g - 16*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**
2*d*e*f*h*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**2*b**2*d*f**2*g*x + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*f**2*h*x**2 - 8*sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**
3*c*e*f*h*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a*b**3*c*f**2*g*x + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(...
```

3.112 $\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 329

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = -\frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{3b^2(be-af)(a+bx)^3} - \frac{(7a^2dfh+b^2(6deg-5cfg+6ceh)-ab(dfg+12deh+cfh))\sqrt{e+fx}}{12b^2(be-af)^2(a+bx)^2} - \frac{(a^2df^2h+abf(dfg-4deh+cfh)+b^2(cf(5fg-6eh)-2de(3fg-4eh)))\sqrt{e+fx}}{8b^2(be-af)^3(a+bx)} + \frac{f(a^2df^2h+abf(dfg-4deh+cfh)+b^2(cf(5fg-6eh)-2de(3fg-4eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{8b^{5/2}(be-af)^{7/2}}$$

output
```
-1/3*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)/(b*x+a)^3-1/12*(7*
a^2*d*f*h+b^2*(6*c*e*h-5*c*f*g+6*d*e*g)-a*b*(c*f*h+12*d*e*h+d*f*g))*(f*x+e
)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^2-1/8*(a^2*d*f^2*h+a*b*f*(c*f*h-4*d*e*h+d
*f*g)+b^2*(c*f*(-6*e*h+5*f*g)-2*d*e*(-4*e*h+3*f*g)))*(f*x+e)^(1/2)/b^2/(-a
*f+b*e)^3/(b*x+a)+1/8*f*(a^2*d*f^2*h+a*b*f*(c*f*h-4*d*e*h+d*f*g)+b^2*(c*f*
(-6*e*h+5*f*g)-2*d*e*(-4*e*h+3*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+
b*e)^(1/2))/b^(5/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{e+fx}(-3a^4df^2h - a^3bf(3cfh + d(3fg - 10eh + 8fhx)) + a^2b^2(cf(33fg - 16eh + 8fhx) + d(8e^2h}{}$$
$$+ \frac{f(a^2df^2h + abf(dfg - 4deh + cfh) + b^2(cf(5fg - 6eh) + 2de(-3fg + 4eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{8b^{5/2}(-be+af)^{7/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[e + f*x]*(-3*a^4*d*f^2*h - a^3*b*f*(3*c*f*h + d*(3*f*g - 10*e*h + 8*
f*h*x)) + a^2*b^2*(c*f*(33*f*g - 16*e*h + 8*f*h*x) + d*(8*e^2*h - 2*e*f*(8
*g - 7*h*x) + f^2*x*(8*g + 3*h*x))) + b^4*(6*d*e*x*(-3*f*g*x + 2*e*(g + 2*
h*x)) + c*(15*f^2*g*x^2 + 4*e^2*(2*g + 3*h*x) - 2*e*f*x*(5*g + 9*h*x))) +
a*b^3*(d*(3*f^2*g*x^2 + 4*e^2*(g + 6*h*x) - 2*e*f*x*(25*g + 6*h*x)) + c*(4
*e^2*h + f^2*x*(40*g + 3*h*x) - 2*e*f*(13*g + 25*h*x)))))/(24*b^2*(-(b*e)
+ a*f)^3*(a + b*x)^3) + (f*(a^2*d*f^2*h + a*b*f*(d*f*g - 4*d*e*h + c*f*h)
+ b^2*(c*f*(5*f*g - 6*e*h) + 2*d*e*(-3*f*g + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt
[e + f*x])/Sqrt[-(b*e) + a*f]])/(8*b^(5/2)*(-(b*e) + a*f)^(7/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {162, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{ 162}$$

$$\frac{\left(a^2df^2h+abf(cfh-4deh+dfg)+b^2(cf(5fg-6eh)-2de(3fg-4eh))\right)\int \frac{1}{(a+bx)^2\sqrt{e+fx}}dx}{8b^2(be-af)^2}-\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(7a^2dfh-ab(cfh+12deh+dfg)+b^2(6ceh-5cfg+6deg)\right)+a^2b(3cfh-8deh+3dfg\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\downarrow$ 52

$$\frac{\left(a^2df^2h+abf(cfh-4deh+dfg)+b^2(cf(5fg-6eh)-2de(3fg-4eh))\right)\left(-\frac{f\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2(be-af)}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)}{8b^2(be-af)^2}-\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(7a^2dfh-ab(cfh+12deh+dfg)+b^2(6ceh-5cfg+6deg)\right)+a^2b(3cfh-8deh+3dfg\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\downarrow$ 73

$$\frac{\left(a^2df^2h+abf(cfh-4deh+dfg)+b^2(cf(5fg-6eh)-2de(3fg-4eh))\right)\left(-\frac{\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{be-af}-\frac{\sqrt{e+fx}}{(a+bx)(be-\right.}{8b^2(be-af)^2}-\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(7a^2dfh-ab(cfh+12deh+dfg)+b^2(6ceh-5cfg+6deg)\right)+a^2b(3cfh-8deh+3dfg\right.}{12b^2(a+bx)^3(be-af)^2}$$

$\downarrow$ 221

$$\frac{\left(\frac{f\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)\left(a^2df^2h+abf(cfh-4deh+dfg)+b^2(cf(5fg-6eh)-2de(3fg-4eh)\right.}{8b^2(be-af)^2}-\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(7a^2dfh-ab(cfh+12deh+dfg)+b^2(6ceh-5cfg+6deg)\right)+a^2b(3cfh-8deh+3dfg\right.}{12b^2(a+bx)^3(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
-1/12*(Sqrt[e + f*x]*(4*b^3*c*e*g + 3*a^3*d*f*h + 2*a*b^2*(d*e*g - (9*c*f*
g)/2 + c*e*h) + a^2*b*(3*d*f*g - 8*d*e*h + 3*c*f*h) + b*(7*a^2*d*f*h + b^2
*(6*d*e*g - 5*c*f*g + 6*c*e*h) - a*b*(d*f*g + 12*d*e*h + c*f*h))*x))/(b^2*
(b*e - a*f)^2*(a + b*x)^3) + ((a^2*d*f^2*h + a*b*f*(d*f*g - 4*d*e*h + c*f*
h) + b^2*(c*f*(5*f*g - 6*e*h) - 2*d*e*(3*f*g - 4*e*h)))*(-(Sqrt[e + f*x]/(
(b*e - a*f)*(a + b*x))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*
f]])/(Sqrt[b]*(b*e - a*f)^(3/2))))/(8*b^2*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{\sqrt{(af-be)b}\left(\left(\left(\left(a^2\left(-\frac{bx}{3}+a\right)(3bx+a)f^2-\frac{10ab(2bx+a)e\left(-\frac{3bx}{5}+a\right)f}{3}-\frac{8b^2e^2\left(3b^2x^2+3abx+a^2\right)}{3}\right)h+\left(a\left(\frac{bx}{3}+a\right)(-3b\right.\right.\right.\right.}{}$
derivativedivides	$2f\left(-\frac{-\frac{\left(a^2d\,f^2h+abc\,f^2h-4abdefh+abd\,f^2g-6b^2cefh+5b^2c\,f^2g+8b^2d\,e^2h-6b^2defg\right)(fx+e)^{\frac{5}{2}}}{16\left(a^3f^3-3a^2be\,f^2+3a\,b^2e^2f-b^3e^3\right)}+\frac{\left(a^2d\,f^2h-abc\,f^2h-abd\,f\right.}{}}{}\right.$
default	$2f\left(-\frac{-\frac{\left(a^2d\,f^2h+abc\,f^2h-4abdefh+abd\,f^2g-6b^2cefh+5b^2c\,f^2g+8b^2d\,e^2h-6b^2defg\right)(fx+e)^{\frac{5}{2}}}{16\left(a^3f^3-3a^2be\,f^2+3a\,b^2e^2f-b^3e^3\right)}+\frac{\left(a^2d\,f^2h-abc\,f^2h-abd\,f\right.}{}}{}\right.$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(((a*f-b*e)*b)^(1/2)*(((a^2*(-1/3*b*x+a)*(3*b*x+a)*f^2-10/3*a*b*(2*b*
x+a)*e*(-3/5*b*x+a)*f-8/3*b^2*e^2*(3*b^2*x^2+3*a*b*x+a^2))*h+(a*(1/3*b*x+a
)*(-3*b*x+a)*f^2+16/3*(9/8*b^2*x^2+25/8*a*b*x+a^2)*b*e*f-4/3*b^2*e^2*(3*b*
x+a))*b*g)*d+c*b*((a*(1/3*b*x+a)*(-3*b*x+a)*f^2+16/3*(9/8*b^2*x^2+25/8*a*b
*x+a^2)*b*e*f-4/3*b^2*e^2*(3*b*x+a))*h-11*((5/11*b^2*x^2+40/33*a*b*x+a^2)*
f^2-26/33*(5/13*b*x+a)*b*e*f+8/33*b^2*e^2)*b*g))*(f*x+e)^(1/2)-arctan(b*(f
*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))*(((a^2*f^2-4*a*b*e*f+8*b^2*e^2)*h+b*f*g*(
a*f-6*b*e))*d+c*((a*f-6*b*e)*h+5*b*f*g)*b*f)*(b*x+a)^3*f)/((a*f-b*e)*b)^(1
/2)/(b*x+a)^3/(a*f-b*e)^3/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. 2(309) = 618.

Time = 0.28 (sec) , antiderivative size = 2380, normalized size of antiderivative = 7.23

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(((6*b^5*d*e*f^2 - (5*b^5*c + a*b^4*d)*f^3)*g - (8*b^5*d*e^2*f -
2*(3*b^5*c + 2*a*b^4*d)*e*f^2 + (a*b^4*c + a^2*b^3*d)*f^3)*h)*x^3 + 3*((6*
a*b^4*d*e*f^2 - (5*a*b^4*c + a^2*b^3*d)*f^3)*g - (8*a*b^4*d*e^2*f - 2*(3*a
*b^4*c + 2*a^2*b^3*d)*e*f^2 + (a^2*b^3*c + a^3*b^2*d)*f^3)*h)*x^2 + (6*a^3
*b^2*d*e*f^2 - (5*a^3*b^2*c + a^4*b*d)*f^3)*g - (8*a^3*b^2*d*e^2*f - 2*(3*
a^3*b^2*c + 2*a^4*b*d)*e*f^2 + (a^4*b*c + a^5*d)*f^3)*h + 3*((6*a^2*b^3*d*
e*f^2 - (5*a^2*b^3*c + a^3*b^2*d)*f^3)*g - (8*a^2*b^3*d*e^2*f - 2*(3*a^2*b
^3*c + 2*a^3*b^2*d)*e*f^2 + (a^3*b^2*c + a^4*b*d)*f^3)*h)*x)*sqrt(b^2*e -
a*b*f)*log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*
x + a)) + 2*(3*((6*b^6*d*e^2*f - (5*b^6*c + 7*a*b^5*d)*e*f^2 + (5*a*b^5*c
+ a^2*b^4*d)*f^3)*g - (8*b^6*d*e^3 - 6*(b^6*c + 2*a*b^5*d)*e^2*f + (7*a*b^
5*c + 5*a^2*b^4*d)*e*f^2 - (a^2*b^4*c + a^3*b^3*d)*f^3)*h)*x^2 - (4*(2*b^6
*c + a*b^5*d)*e^3 - 2*(17*a*b^5*c + 10*a^2*b^4*d)*e^2*f + (59*a^2*b^4*c +
13*a^3*b^3*d)*e*f^2 - 3*(11*a^3*b^3*c - a^4*b^2*d)*f^3)*g - (4*(a*b^5*c +
2*a^2*b^4*d)*e^3 - 2*(10*a^2*b^4*c - a^3*b^3*d)*e^2*f + 13*(a^3*b^3*c - a^
4*b^2*d)*e*f^2 + 3*(a^4*b^2*c + a^5*b*d)*f^3)*h - 2*((6*b^6*d*e^3 - (5*b^6
*c + 31*a*b^5*d)*e^2*f + (25*a*b^5*c + 29*a^2*b^4*d)*e*f^2 - 4*(5*a^2*b^4*
c + a^3*b^3*d)*f^3)*g + (6*(b^6*c + 2*a*b^5*d)*e^3 - (31*a*b^5*c + 5*a^2*b
^4*d)*e^2*f + (29*a^2*b^4*c - 11*a^3*b^3*d)*e*f^2 - 4*(a^3*b^3*c - a^4*b^2
*d)*f^3)*h)*x)*sqrt(f*x + e))/(a^3*b^7*e^4 - 4*a^4*b^6*e^3*f + 6*a^5*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(309) = 618$.

Time = 0.14 (sec) , antiderivative size = 922, normalized size of antiderivative = 2.80

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/8*(6*b^2*d*e*f^2*g - 5*b^2*c*f^3*g - a*b*d*f^3*g - 8*b^2*d*e^2*f*h + 6*b
^2*c*e*f^2*h + 4*a*b*d*e*f^2*h - a*b*c*f^3*h - a^2*d*f^3*h)*arctan(sqrt(f*
x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*e^3 - 3*a*b^4*e^2*f + 3*a^2*b^3*e*f^2
 - a^3*b^2*f^3)*sqrt(-b^2*e + a*b*f)) + 1/24*(18*(f*x + e)^(5/2)*b^4*d*e*f
^2*g - 48*(f*x + e)^(3/2)*b^4*d*e^2*f^2*g + 30*sqrt(f*x + e)*b^4*d*e^3*f^2
*g - 15*(f*x + e)^(5/2)*b^4*c*f^3*g - 3*(f*x + e)^(5/2)*a*b^3*d*f^3*g + 40
*(f*x + e)^(3/2)*b^4*c*e*f^3*g + 56*(f*x + e)^(3/2)*a*b^3*d*e*f^3*g - 33*s
qrt(f*x + e)*b^4*c*e^2*f^3*g - 57*sqrt(f*x + e)*a*b^3*d*e^2*f^3*g - 40*(f*
x + e)^(3/2)*a*b^3*c*f^4*g - 8*(f*x + e)^(3/2)*a^2*b^2*d*f^4*g + 66*sqrt(f
*x + e)*a*b^3*c*e*f^4*g + 24*sqrt(f*x + e)*a^2*b^2*d*e*f^4*g - 33*sqrt(f*x
 + e)*a^2*b^2*c*f^5*g + 3*sqrt(f*x + e)*a^3*b*d*f^5*g - 24*(f*x + e)^(5/2)
*b^4*d*e^2*f*h + 48*(f*x + e)^(3/2)*b^4*d*e^3*f*h - 24*sqrt(f*x + e)*b^4*d
*e^4*f*h + 18*(f*x + e)^(5/2)*b^4*c*e*f^2*h + 12*(f*x + e)^(5/2)*a*b^3*d*e
*f^2*h - 48*(f*x + e)^(3/2)*b^4*c*e^2*f^2*h - 48*(f*x + e)^(3/2)*a*b^3*d*e
^2*f^2*h + 30*sqrt(f*x + e)*b^4*c*e^3*f^2*h + 36*sqrt(f*x + e)*a*b^3*d*e^3
*f^2*h - 3*(f*x + e)^(5/2)*a*b^3*c*f^3*h - 3*(f*x + e)^(5/2)*a^2*b^2*d*f^3
*h + 56*(f*x + e)^(3/2)*a*b^3*c*e*f^3*h - 8*(f*x + e)^(3/2)*a^2*b^2*d*e*f^
3*h - 57*sqrt(f*x + e)*a*b^3*c*e^2*f^3*h + 3*sqrt(f*x + e)*a^2*b^2*d*e^2*f
^3*h - 8*(f*x + e)^(3/2)*a^2*b^2*c*f^4*h + 8*(f*x + e)^(3/2)*a^3*b*d*f^4*h
 + 24*sqrt(f*x + e)*a^2*b^2*c*e*f^4*h - 18*sqrt(f*x + e)*a^3*b*d*e*f^4*...
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \frac{\frac{(e+f\,x)^{5/2}\,(5\,b^2\,c\,f^3\,g+a^2\,d\,f^3\,h-6\,b^2\,c\,e\,f^2\,h-6\,b^2\,d\,e\,f^2\,g+8\,b^2\,d\,e^2\,f\,h+a\,b\,c\,f^3\,h+a\,b\,d\,f^3\,g-4\,a\,b\,d\,e\,f^2\,h)}{8\,(a\,f-b\,e)^3}-\frac{\sqrt{e+f\,x}\,(a^2\,d\,f^3\,h-11\,b^2\,c\,f^3}{}}{(e+f\,x)\,(3\,a^2\,b\,f^2-6\,a\,b^2\,e\,f+3\,b^3\,e^2)+b^3\,(e+} + \frac{f\,\mathrm{atan}\left(\frac{\sqrt{b}\,f\,\sqrt{e+f\,x}\,(5\,b^2\,c\,f^2\,g+a^2\,d\,f^2\,h+8\,b^2\,d\,e^2\,h+a\,b\,c\,f^2\,h+a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b^2\,d\,e\,f\,g-4\,a\,b\,d\,e\,f\,h)}{\sqrt{a\,f-b\,e}\,(5\,b^2\,c\,f^3\,g+a^2\,d\,f^3\,h-6\,b^2\,c\,e\,f^2\,h-6\,b^2\,d\,e\,f^2\,g+8\,b^2\,d\,e^2\,f\,h+a\,b\,c\,f^3\,h+a\,b\,d\,f^3\,g-4\,a\,b\,d\,e\,f^2\,h)}\right)\,(5\,b^2\,c\,f^2\,g+}{8\,b^{5/2}\,(a\,f-b\,e)^{7/}}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(1/2)*(a + b*x)^4),x)
```

output
```
(((e + f*x)^(5/2)*(5*b^2*c*f^3*g + a^2*d*f^3*h - 6*b^2*c*e*f^2*h - 6*b^2*d
*e*f^2*g + 8*b^2*d*e^2*f*h + a*b*c*f^3*h + a*b*d*f^3*g - 4*a*b*d*e*f^2*h))
/(8*(a*f - b*e)^3) - ((e + f*x)^(1/2)*(a^2*d*f^3*h - 11*b^2*c*f^3*g + 10*b
^2*c*e*f^2*h + 10*b^2*d*e*f^2*g - 8*b^2*d*e^2*f*h + a*b*c*f^3*h + a*b*d*f^
3*g - 4*a*b*d*e*f^2*h))/(8*b^2*(a*f - b*e)) + ((e + f*x)^(3/2)*(5*b^2*c*f^
3*g - a^2*d*f^3*h - 6*b^2*c*e*f^2*h - 6*b^2*d*e*f^2*g + 6*b^2*d*e^2*f*h +
a*b*c*f^3*h + a*b*d*f^3*g))/(3*b*(a*f - b*e)^2))/((e + f*x)*(3*b^3*e^2 + 3
*a^2*b*f^2 - 6*a*b^2*e*f) + b^3*(e + f*x)^3 - (e + f*x)^2*(3*b^3*e - 3*a*b
^2*f) + a^3*f^3 - b^3*e^3 + 3*a*b^2*e^2*f - 3*a^2*b*e*f^2) + (f*atan((b^(1
/2)*f*(e + f*x)^(1/2)*(5*b^2*c*f^2*g + a^2*d*f^2*h + 8*b^2*d*e^2*h + a*b*c
*f^2*h + a*b*d*f^2*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 4*a*b*d*e*f*h))/((a
*f - b*e)^(1/2)*(5*b^2*c*f^3*g + a^2*d*f^3*h - 6*b^2*c*e*f^2*h - 6*b^2*d*e
*f^2*g + 8*b^2*d*e^2*f*h + a*b*c*f^3*h + a*b*d*f^3*g - 4*a*b*d*e*f^2*h)))*
(5*b^2*c*f^2*g + a^2*d*f^2*h + 8*b^2*d*e^2*h + a*b*c*f^2*h + a*b*d*f^2*g -
 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 4*a*b*d*e*f*h))/(8*b^(5/2)*(a*f - b*e)^(7
/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2680, normalized size of antiderivative = 8.15

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**5*d*f**3*h + 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**4*b*c*f**3*h - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d*e*f**2*h + 3*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d*f
**3*g + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**4*b*d*f**3*h*x - 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*e*f**2*h + 15*sqrt(b)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*f**
3*g + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**3*b**2*c*f**3*h*x + 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e**2*f*h - 18*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e*
f**2*g - 36*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**3*b**2*d*e*f**2*h*x + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*f**3*g*x + 9*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*
d*f**3*h*x**2 - 54*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**2*b**3*c*e*f**2*h*x + 45*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**3*c*f**3*g*x + 9...
```

3.113 $\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 435

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = -\frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{4b^2(be-af)(a+bx)^4}$$
$$-\frac{(9a^2dfh+b^2(8deg-7cfg+8ceh)-ab(dfg+16deh+cfh))\sqrt{e+fx}}{24b^2(be-af)^2(a+bx)^3}$$
$$-\frac{(3a^2df^2h+abf(5dfg-16deh+5cfh)+b^2(5cf(7fg-8eh)-8de(5fg-6eh)))\sqrt{e+fx}}{96b^2(be-af)^3(a+bx)^2}$$
$$+\frac{f(3a^2df^2h+abf(5dfg-16deh+5cfh)+b^2(5cf(7fg-8eh)-8de(5fg-6eh)))\sqrt{e+fx}}{64b^2(be-af)^4(a+bx)}$$
$$-\frac{f^2(3a^2df^2h+abf(5dfg-16deh+5cfh)+b^2(5cf(7fg-8eh)-8de(5fg-6eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{64b^{5/2}(be-af)^{9/2}}$$

output
```
-1/4*(-a*d+b*c)*(-a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)/(b*x+a)^4-1/24*(9*
a^2*d*f*h+b^2*(8*c*e*h-7*c*f*g+8*d*e*g)-a*b*(c*f*h+16*d*e*h+d*f*g))*(f*x+e
)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^3-1/96*(3*a^2*d*f^2*h+a*b*f*(5*c*f*h-16*d
*e*h+5*d*f*g)+b^2*(5*c*f*(-8*e*h+7*f*g)-8*d*e*(-6*e*h+5*f*g)))*(f*x+e)^(1/
2)/b^2/(-a*f+b*e)^3/(b*x+a)^2+1/64*f*(3*a^2*d*f^2*h+a*b*f*(5*c*f*h-16*d*e*
h+5*d*f*g)+b^2*(5*c*f*(-8*e*h+7*f*g)-8*d*e*(-6*e*h+5*f*g)))*(f*x+e)^(1/2)/
b^2/(-a*f+b*e)^4/(b*x+a)-1/64*f^2*(3*a^2*d*f^2*h+a*b*f*(5*c*f*h-16*d*e*h+5
*d*f*g)+b^2*(5*c*f*(-8*e*h+7*f*g)-8*d*e*(-6*e*h+5*f*g)))*arctanh(b^(1/2)*(
f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*f+b*e)^(9/2)
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx$$

$$= \frac{\sqrt{b}\sqrt{e+fx}\left(-9a^5df^3h-3a^4bf^2(5cfh+d(5fg-14eh+11fhx))-b^5\left(8dex\left(15f^2gx^2+4e^2(2g+3hx)-2efx(5g+9hx)\right)+c\left(-105f^3gx^3+16e^3(3g+4h\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^5*Sqrt[e + f*x]),x]
```

output
```
((Sqrt[b]*Sqrt[e + f*x]*(-9*a^5*d*f^3*h - 3*a^4*b*f^2*(5*c*f*h + d*(5*f*g
- 14*e*h + 11*f*h*x)) - b^5*(8*d*e*x*(15*f^2*g*x^2 + 4*e^2*(2*g + 3*h*x) -
 2*e*f*x*(5*g + 9*h*x)) + c*(-105*f^3*g*x^3 + 16*e^3*(3*g + 4*h*x) - 8*e^2
*f*x*(7*g + 10*h*x) + 10*e*f^2*x^2*(7*g + 12*h*x))) + a^3*b^2*f*(c*f*(279*
f*g - 146*e*h + 73*f*h*x) + d*(88*e^2*h + f^2*x*(73*g + 33*h*x) + e*f*(-14
6*g + 52*h*x))) + a*b^4*(c*(-16*e^3*h + 5*f^3*x^2*(77*g + 3*h*x) - 18*e*f^
2*x*(14*g + 25*h*x) + 8*e^2*f*(25*g + 37*h*x)) + d*(15*f^3*g*x^3 - 16*e^3*
(g + 4*h*x) - 6*e*f^2*x^2*(75*g + 8*h*x) + 8*e^2*f*x*(37*g + 70*h*x))) + a
^2*b^3*(d*(-16*e^3*h + 72*e^2*f*(g + 5*h*x) + f^3*x^2*(55*g + 9*h*x) - 2*e
*f^2*x*(310*g + 91*h*x)) + c*f*(72*e^2*h + f^2*x*(511*g + 55*h*x) - 2*e*f*
(163*g + 310*h*x)))))/((b*e - a*f)^4*(a + b*x)^4) + (3*f^2*(3*a^2*d*f^2*h
+ a*b*f*(5*d*f*g - 16*d*e*h + 5*c*f*h) + b^2*(5*c*f*(7*f*g - 8*e*h) + 8*d*
e*(-5*f*g + 6*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(
-(b*e) + a*f)^(9/2))/(192*b^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {162, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx$$

$\downarrow$ 162

$$\frac{\left(3a^2df^2h+abf(5cfh-16deh+5dfg)+b^2(5cf(7fg-8eh)-8de(5fg-6eh))\right)\int \frac{1}{(a+bx)^3\sqrt{e+fx}}dx}{48b^2(be-af)^2}-\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(9a^2dfh-ab(cfh+16deh+dfg)+b^2(8ceh-7cfg+8deg)\right)+5a^2b(cfh-2deh+dfg)\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\downarrow$ 52

$$\frac{\left(3a^2df^2h+abf(5cfh-16deh+5dfg)+b^2(5cf(7fg-8eh)-8de(5fg-6eh))\right)\left(-\frac{3f\int \frac{1}{(a+bx)^2\sqrt{e+fx}}dx}{4(be-af)}-\frac{\sqrt{e}}{2(a+bx)}\right.}{48b^2(be-af)^2}$$
$$\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(9a^2dfh-ab(cfh+16deh+dfg)+b^2(8ceh-7cfg+8deg)\right)+5a^2b(cfh-2deh+dfg)\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\downarrow$ 52

$$\frac{\left(3a^2df^2h+abf(5cfh-16deh+5dfg)+b^2(5cf(7fg-8eh)-8de(5fg-6eh))\right)\left(-\frac{3f\left(-\frac{f\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2(be-af)}-\frac{\sqrt{e}}{(a+bx)}\right.}{4(be-af)}\right.}{48b^2(be-af)^2}$$
$$\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(9a^2dfh-ab(cfh+16deh+dfg)+b^2(8ceh-7cfg+8deg)\right)+5a^2b(cfh-2deh+dfg)\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\downarrow$ 73

$$\frac{\left(3a^2df^2h+abf(5cfh-16deh+5dfg)+b^2(5cf(7fg-8eh)-8de(5fg-6eh))\right)\left(-\frac{3f\left(-\frac{\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{be-af}-\right.}{4(be-af)}\right.}{48b^2(be-af)^2}$$
$$\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(9a^2dfh-ab(cfh+16deh+dfg)+b^2(8ceh-7cfg+8deg)\right)+5a^2b(cfh-2deh+dfg)\right.}{24b^2(a+bx)^4(be-af)^2}$$

$\downarrow$ 221

$$\frac{\left(-\frac{3f\left(\frac{f\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)}{4(be-af)}-\frac{\sqrt{e+fx}}{2(a+bx)^2(be-af)}\right)\left(3a^2df^2h+abf(5cfh-16deh+5dfg)+b^2(5cf(7\right.}{48b^2(be-af)^2}$$

$$\frac{\sqrt{e+fx}\left(3a^3dfh+bx\left(9a^2dfh-ab(cfh+16deh+dfg)+b^2(8ceh-7cfg+8deg)\right)+5a^2b(cfh-2deh+dfg)\right.}{24b^2(a+bx)^4(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^5*Sqrt[e + f*x]),x]
```

output
```
-1/24*(Sqrt[e + f*x]*(6*b^3*c*e*g + 3*a^3*d*f*h + 2*a*b^2*(d*e*g - (13*c*f
*g)/2 + c*e*h) + 5*a^2*b*(d*f*g - 2*d*e*h + c*f*h) + b*(9*a^2*d*f*h + b^2*
(8*d*e*g - 7*c*f*g + 8*c*e*h) - a*b*(d*f*g + 16*d*e*h + c*f*h))*x))/(b^2*(
b*e - a*f)^2*(a + b*x)^4) + ((3*a^2*d*f^2*h + a*b*f*(5*d*f*g - 16*d*e*h +
5*c*f*h) + b^2*(5*c*f*(7*f*g - 8*e*h) - 8*d*e*(5*f*g - 6*e*h)))*(-1/2*Sqrt
[e + f*x]/((b*e - a*f)*(a + b*x)^2) - (3*f*(-(Sqrt[e + f*x]/((b*e - a*f)*(
a + b*x))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]
*(b*e - a*f)^(3/2))))/(4*(b*e - a*f))))/(48*b^2*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{\frac{3(bx+a)^4 f^2\left(\left(\frac{35cg f^2}{3}-\frac{40e(ch+dg)f}{3}+16d e^2h\right)b^2+\frac{5a\left(f(ch+dg)-\frac{16deh}{5}\right)fb}{3}+a^2d f^2h\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{\sqrt{(af-be)b}}-3\left(\frac{\left(-35c f^3\right.}{}\right.}{64}$
derivativedivides	$2f^2\left(-\frac{-\frac{\left(3a^2d f^2h+5abc f^2h-16abdefh+5abd f^2g-40b^2cefh+35b^2c f^2g+48b^2d e^2h-40b^2defg\right)b(fx+e)^{\frac{7}{2}}}{128\left(a^4f^4-4a^3be f^3+6a^2b^2e^2f^2-4a b^3e^3f+b^4e^4\right)}-\frac{11\left(3a^2d f^2h+\right.}{}}{}\right.$
default	$2f^2\left(-\frac{-\frac{\left(3a^2d f^2h+5abc f^2h-16abdefh+5abd f^2g-40b^2cefh+35b^2c f^2g+48b^2d e^2h-40b^2defg\right)b(fx+e)^{\frac{7}{2}}}{128\left(a^4f^4-4a^3be f^3+6a^2b^2e^2f^2-4a b^3e^3f+b^4e^4\right)}-\frac{11\left(3a^2d f^2h+\right.}{}}{}\right.$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/64*((b*x+a)^4*f^2*((35/3*c*g*f^2-40/3*e*(c*h+d*g)*f+16*d*e^2*h)*b^2+5/3*
a*(f*(c*h+d*g)-16/5*d*e*h)*f*b+a^2*d*f^2*h)*arctan(b*(f*x+e)^(1/2)/((a*f-b
*e)*b)^(1/2))-(1/3*(-35*c*f^3*g*x^3+70/3*(12/7*(c*h+d*g)*x+c*g)*x^2*e*f^2-
56/3*x*(18/7*d*h*x^2+10/7*(c*h+d*g)*x+c*g)*e^2*f+16*(2*d*h*x^2+4/3*(c*h+d*
g)*x+c*g)*e^3)*b^5+16/9*a*(-385/16*x^2*(3/77*(c*h+d*g)*x+c*g)*f^3+63/4*x*(
4/21*d*h*x^2+25/14*(c*h+d*g)*x+c*g)*e*f^2-25/2*(14/5*d*h*x^2+37/25*(c*h+d*
g)*x+c*g)*e^2*f+e^3*(4*d*h*x+c*h+d*g))*b^4-8*a^2*(1/8*(d*h*x^3+55/9*(c*h+d
*g)*x^2+511/9*c*g*x)*f^3-163/36*(91/163*d*h*x^2+310/163*(c*h+d*g)*x+c*g)*e
*f^2+e^2*(5*d*h*x+c*h+d*g)*f-2/9*d*e^3*h)*b^3+146/9*a^3*(1/2*(-33/73*d*h*x
^2+(-c*h-d*g)*x-279/73*c*g)*f^2+e*(-26/73*d*h*x+c*h+d*g)*f-44/73*d*e^2*h)*
f*b^2+5/3*a^4*f^2*((11/5*d*h*x+c*h+d*g)*f-14/5*d*e*h)*b+a^5*d*f^3*h)*(f*x+
e)^(1/2)*((a*f-b*e)*b)^(1/2))/((a*f-b*e)*b)^(1/2)/(b*x+a)^4/(a*f-b*e)^4/b^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1767 vs. $2(410) = 820$.

Time = 0.38 (sec) , antiderivative size = 3548, normalized size of antiderivative = 8.16

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**5/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(410) = 820$.

Time = 0.15 (sec) , antiderivative size = 1376, normalized size of antiderivative = 3.16

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/64*(40*b^2*d*e*f^3*g - 35*b^2*c*f^4*g - 5*a*b*d*f^4*g - 48*b^2*d*e^2*f^
2*h + 40*b^2*c*e*f^3*h + 16*a*b*d*e*f^3*h - 5*a*b*c*f^4*h - 3*a^2*d*f^4*h)
*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^4 - 4*a*b^5*e^3*f +
6*a^2*b^4*e^2*f^2 - 4*a^3*b^3*e*f^3 + a^4*b^2*f^4)*sqrt(-b^2*e + a*b*f)) -
 1/192*(120*(f*x + e)^(7/2)*b^5*d*e*f^3*g - 440*(f*x + e)^(5/2)*b^5*d*e^2*
f^3*g + 584*(f*x + e)^(3/2)*b^5*d*e^3*f^3*g - 264*sqrt(f*x + e)*b^5*d*e^4*
f^3*g - 105*(f*x + e)^(7/2)*b^5*c*f^4*g - 15*(f*x + e)^(7/2)*a*b^4*d*f^4*g
 + 385*(f*x + e)^(5/2)*b^5*c*e*f^4*g + 495*(f*x + e)^(5/2)*a*b^4*d*e*f^4*g
 - 511*(f*x + e)^(3/2)*b^5*c*e^2*f^4*g - 1241*(f*x + e)^(3/2)*a*b^4*d*e^2*
f^4*g + 279*sqrt(f*x + e)*b^5*c*e^3*f^4*g + 777*sqrt(f*x + e)*a*b^4*d*e^3*
f^4*g - 385*(f*x + e)^(5/2)*a*b^4*c*f^5*g - 55*(f*x + e)^(5/2)*a^2*b^3*d*f
^5*g + 1022*(f*x + e)^(3/2)*a*b^4*c*e*f^5*g + 730*(f*x + e)^(3/2)*a^2*b^3*
d*e*f^5*g - 837*sqrt(f*x + e)*a*b^4*c*e^2*f^5*g - 747*sqrt(f*x + e)*a^2*b^
3*d*e^2*f^5*g - 511*(f*x + e)^(3/2)*a^2*b^3*c*f^6*g - 73*(f*x + e)^(3/2)*a
^3*b^2*d*f^6*g + 837*sqrt(f*x + e)*a^2*b^3*c*e*f^6*g + 219*sqrt(f*x + e)*a
^3*b^2*d*e*f^6*g - 279*sqrt(f*x + e)*a^3*b^2*c*f^7*g + 15*sqrt(f*x + e)*a^
4*b*d*f^7*g - 144*(f*x + e)^(7/2)*b^5*d*e^2*f^2*h + 528*(f*x + e)^(5/2)*b^
5*d*e^3*f^2*h - 624*(f*x + e)^(3/2)*b^5*d*e^4*f^2*h + 240*sqrt(f*x + e)*b^
5*d*e^5*f^2*h + 120*(f*x + e)^(7/2)*b^5*c*e*f^3*h + 48*(f*x + e)^(7/2)*a*b
^4*d*e*f^3*h - 440*(f*x + e)^(5/2)*b^5*c*e^2*f^3*h - 704*(f*x + e)^(5/2...
```

Mupad [B] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(1/2)*(a + b*x)^5),x)
```

output
```
((11*(e + f*x)^(5/2)*(35*b^2*c*f^4*g + 3*a^2*d*f^4*h - 40*b^2*c*e*f^3*h -
40*b^2*d*e*f^3*g + 48*b^2*d*e^2*f^2*h + 5*a*b*c*f^4*h + 5*a*b*d*f^4*g - 16
*a*b*d*e*f^3*h))/(192*(a*f - b*e)^3) + (b*(e + f*x)^(7/2)*(35*b^2*c*f^4*g
+ 3*a^2*d*f^4*h - 40*b^2*c*e*f^3*h - 40*b^2*d*e*f^3*g + 48*b^2*d*e^2*f^2*h
 + 5*a*b*c*f^4*h + 5*a*b*d*f^4*g - 16*a*b*d*e*f^3*h))/(64*(a*f - b*e)^4) -
 ((e + f*x)^(1/2)*(3*a^2*d*f^4*h - 93*b^2*c*f^4*g + 88*b^2*c*e*f^3*h + 88*
b^2*d*e*f^3*g - 80*b^2*d*e^2*f^2*h + 5*a*b*c*f^4*h + 5*a*b*d*f^4*g - 16*a*
b*d*e*f^3*h))/(64*b^2*(a*f - b*e)) + ((e + f*x)^(3/2)*(511*b^2*c*f^4*g - 3
3*a^2*d*f^4*h - 584*b^2*c*e*f^3*h - 584*b^2*d*e*f^3*g + 624*b^2*d*e^2*f^2*
h + 73*a*b*c*f^4*h + 73*a*b*d*f^4*g - 80*a*b*d*e*f^3*h))/(192*b*(a*f - b*e
)^2))/(b^4*(e + f*x)^4 - (e + f*x)^3*(4*b^4*e - 4*a*b^3*f) - (e + f*x)*(4*
b^4*e^3 - 4*a^3*b*f^3 + 12*a^2*b^2*e*f^2 - 12*a*b^3*e^2*f) + a^4*f^4 + b^4
*e^4 + (e + f*x)^2*(6*b^4*e^2 + 6*a^2*b^2*f^2 - 12*a*b^3*e*f) + 6*a^2*b^2*
e^2*f^2 - 4*a*b^3*e^3*f - 4*a^3*b*e*f^3) + (f^2*atan((b^(1/2)*f^2*(e + f*x
)^(1/2)*(35*b^2*c*f^2*g + 3*a^2*d*f^2*h + 48*b^2*d*e^2*h + 5*a*b*c*f^2*h +
 5*a*b*d*f^2*g - 40*b^2*c*e*f*h - 40*b^2*d*e*f*g - 16*a*b*d*e*f*h))/((a*f
- b*e)^(1/2)*(35*b^2*c*f^4*g + 3*a^2*d*f^4*h - 40*b^2*c*e*f^3*h - 40*b^2*d
*e*f^3*g + 48*b^2*d*e^2*f^2*h + 5*a*b*c*f^4*h + 5*a*b*d*f^4*g - 16*a*b*d*e
*f^3*h)))*(35*b^2*c*f^2*g + 3*a^2*d*f^2*h + 48*b^2*d*e^2*h + 5*a*b*c*f^2*h
 + 5*a*b*d*f^2*g - 40*b^2*c*e*f*h - 40*b^2*d*e*f*g - 16*a*b*d*e*f*h))/(...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3797, normalized size of antiderivative = 8.73

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x)
```

output

```
(9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**6*d*f**4*h + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**5*b*c*f**4*h - 48*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d*e*f**3*h + 15*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d
*f**4*g + 36*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**5*b*d*f**4*h*x - 120*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*e*f**3*h + 105*sqrt(b)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*
c*f**4*g + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**4*b**2*c*f**4*h*x + 144*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e**2*f**2*h - 120*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
4*b**2*d*e*f**3*g - 192*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e*f**3*h*x + 60*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*f**4*g*x +
 54*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**4*b**2*d*f**4*h*x**2 - 480*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**3*c*e*f**3*h*x + 420*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**...
```

3.114 $\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}} dx$

Optimal result

Integrand size = 29, antiderivative size = 577

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}} dx = -\frac{2(be-af)^3(de-cf)^2(fg-eh)\sqrt{e+fx}}{f^7}$$
$$+\frac{2(be-af)^2(de-cf)(bde(5fg-6eh)-bcf(3fg-4eh)-af(2dfg-3deh+cfh))(e+fx)^{3/2}}{3f^7}$$
$$-\frac{2(be-af)\left(a^2df^2(dfg-3deh+2cfh)+abf(3c^2f^2h-d^2e(8fg-15eh)+2cdf(3fg-8eh))-b^2(4c\right.}{5f^7}$$
$$+\frac{2(a^3d^2f^3h+3a^2bdf^2(dfg-4deh+2cfh)+3ab^2f(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh))-b^3(}{7f^7}$$
$$+\frac{2b(3a^2d^2f^2h+3abdf(dfg-5deh+2cfh)+b^2(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)))(e+fx)}{9f^7}$$
$$+\frac{2b^2d(3adfh+b(dfg-6deh+2cfh))(e+fx)^{11/2}}{11f^7}+\frac{2b^3d^2h(e+fx)^{13/2}}{13f^7}$$

output
```
-2*(-a*f+b*e)^3*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(1/2)/f^7+2/3*(-a*f+b*e)^2
*(-c*f+d*e)*(b*d*e*(-6*e*h+5*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-3*d*e*h+
2*d*f*g))*(f*x+e)^(3/2)/f^7-2/5*(-a*f+b*e)*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d*f
*g)+a*b*f*(3*c^2*f^2*h-d^2*e*(-15*e*h+8*f*g)+2*c*d*f*(-8*e*h+3*f*g))-b^2*(
4*c*d*e*f*(-5*e*h+3*f*g)-5*d^2*e^2*(-3*e*h+2*f*g)-3*c^2*f^2*(-2*e*h+f*g)))
*(f*x+e)^(5/2)/f^7+2/7*(a^3*d^2*f^3*h+3*a^2*b*d*f^2*(2*c*f*h-4*d*e*h+d*f*g
)+3*a*b^2*f*(c^2*f^2*h-2*d^2*e*(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g))-b^3*(4
*c*d*e*f*(-5*e*h+2*f*g)-c^2*f^2*(-4*e*h+f*g)-10*d^2*e^2*(-2*e*h+f*g)))*(f*
x+e)^(7/2)/f^7+2/9*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(2*c*f*h-5*d*e*h+d*f*g)+b^
2*(c^2*f^2*h+2*c*d*f*(-5*e*h+f*g)-5*d^2*e*(-3*e*h+f*g)))*(f*x+e)^(9/2)/f^7
+2/11*b^2*d*(3*a*d*f*h+b*(2*c*f*h-6*d*e*h+d*f*g))*(f*x+e)^(11/2)/f^7+2/13*
b^3*d^2*h*(f*x+e)^(13/2)/f^7
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(429a^3f^3(35c^2f^2(3fg-2eh+fhx)+14cdf(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx))+d^2(-\cdots}{\cdots}$$

input
```
Integrate[((a + b*x)^3*(c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(429*a^3*f^3*(35*c^2*f^2*(3*f*g - 2*e*h + f*h*x) + 14*c*d
*f*(8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d^2*(-48*e^3*h
+ 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*
x))) + 429*a^2*b*f^2*(21*c^2*f^2*(8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5
*g + 3*h*x)) + 6*c*d*f*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g
 + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x)) + d^2*(128*e^4*h + 24*e^2*f^2*x*(3*g
 + 2*h*x) - 16*e^3*f*(9*g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2
*(27*g + 20*h*x))) + 39*a*b^2*f*(33*c^2*f^2*(-48*e^3*h + 8*e^2*f*(7*g + 3*
h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x)) + 22*c*d*f*(128
*e^4*h + 24*e^2*f^2*x*(3*g + 2*h*x) - 16*e^3*f*(9*g + 4*h*x) + 5*f^4*x^3*(
9*g + 7*h*x) - 2*e*f^3*x^2*(27*g + 20*h*x)) + d^2*(-1280*e^5*h + 128*e^4*f
*(11*g + 5*h*x) + 35*f^5*x^4*(11*g + 9*h*x) - 32*e^3*f^2*x*(22*g + 15*h*x)
 + 16*e^2*f^3*x^2*(33*g + 25*h*x) - 10*e*f^4*x^3*(44*g + 35*h*x))) + b^3*(
143*c^2*f^2*(128*e^4*h + 24*e^2*f^2*x*(3*g + 2*h*x) - 16*e^3*f*(9*g + 4*h*
x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2*(27*g + 20*h*x)) + 26*c*d*f*(-1
280*e^5*h + 128*e^4*f*(11*g + 5*h*x) + 35*f^5*x^4*(11*g + 9*h*x) - 32*e^3*
f^2*x*(22*g + 15*h*x) + 16*e^2*f^3*x^2*(33*g + 25*h*x) - 10*e*f^4*x^3*(44*
g + 35*h*x)) + 5*d^2*(3072*e^6*h - 256*e^5*f*(13*g + 6*h*x) + 128*e^4*f^2*
x*(13*g + 9*h*x) - 96*e^3*f^3*x^2*(13*g + 10*h*x) + 63*f^6*x^5*(13*g + 11*
h*x) + 40*e^2*f^4*x^3*(26*g + 21*h*x) - 14*e*f^5*x^4*(65*g + 54*h*x)))...
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{(e+fx)^{3/2}(be-af)\left(-a^2df^2(2cfh-3deh+dfg)-abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh\right)\right.}{f^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(e+fx)^{5/2}(be-af)\left(a^2df^2(2cfh-3deh+dfg)+abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh)\right)-\right.}{5f^7}$$

$$\frac{2b(e+fx)^{9/2}\left(3a^2d^2f^2h+3abdf(2cfh-5deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)\right)\right)}{9f^7}+$$

$$\frac{2(e+fx)^{7/2}\left(a^3d^2f^3h+3a^2bdf^2(2cfh-4deh+dfg)+3ab^2f\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)-\right.}{7f^7}$$

$$\frac{2b^2d(e+fx)^{11/2}(3adfh+b(2cfh-6deh+dfg))}{11f^7}+$$

$$\frac{2(e+fx)^{3/2}(be-af)^2(de-cf)(-af(cfh-3deh+2dfg)-bcf(3fg-4eh)+bde(5fg-6eh))}{3f^7}-$$

$$\frac{2\sqrt{e+fx}(be-af)^3(de-cf)^2(fg-eh)}{f^7}+\frac{2b^3d^2h(e+fx)^{13/2}}{13f^7}$$

input
```
Int[((a + b*x)^3*(c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(-2*(b*e - a*f)^3*(d*e - c*f)^2*(f*g - e*h)*Sqrt[e + f*x])/f^7 + (2*(b*e -
 a*f)^2*(d*e - c*f)*(b*d*e*(5*f*g - 6*e*h) - b*c*f*(3*f*g - 4*e*h) - a*f*(
2*d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(3/2))/(3*f^7) - (2*(b*e - a*f)*(a^2
*d*f^2*(d*f*g - 3*d*e*h + 2*c*f*h) + a*b*f*(3*c^2*f^2*h - d^2*e*(8*f*g - 1
5*e*h) + 2*c*d*f*(3*f*g - 8*e*h)) - b^2*(4*c*d*e*f*(3*f*g - 5*e*h) - 5*d^2
*e^2*(2*f*g - 3*e*h) - 3*c^2*f^2*(f*g - 2*e*h)))*(e + f*x)^(5/2))/(5*f^7)
 + (2*(a^3*d^2*f^3*h + 3*a^2*b*d*f^2*(d*f*g - 4*d*e*h + 2*c*f*h) + 3*a*b^2*
f*(c^2*f^2*h - 2*d^2*e*(2*f*g - 5*e*h) + 2*c*d*f*(f*g - 4*e*h)) - b^3*(4*c
*d*e*f*(2*f*g - 5*e*h) - c^2*f^2*(f*g - 4*e*h) - 10*d^2*e^2*(f*g - 2*e*h))
)*(e + f*x)^(7/2))/(7*f^7) + (2*b*(3*a^2*d^2*f^2*h + 3*a*b*d*f*(d*f*g - 5*
d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h + 2*c*d*f*(f*g - 5*e*h) - 5*d^2*e*(f*g -
 3*e*h)))*(e + f*x)^(9/2))/(9*f^7) + (2*b^2*d*(3*a*d*f*h + b*(d*f*g - 6*d*
e*h + 2*c*f*h))*(e + f*x)^(11/2))/(11*f^7) + (2*b^3*d^2*h*(e + f*x)^(13/2)
)/(13*f^7)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{2h\,d^{2}b^{3}(fx+e)^{\frac{13}{2}}}{13}+\frac{2\left(\left(3(af-be)b^{2}d^{2}+2b^{3}d(cf-de)\right)h+b^{3}d^{2}(-eh+fg)\right)(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left(3(af-be)^{2}b\,d^{2}+6(af-be)b^{2}d(cf-de)+\right.\right.}{}$
default	$\frac{2h\,d^{2}b^{3}(fx+e)^{\frac{13}{2}}}{13}-\frac{2\left(-\left(3(af-be)b^{2}d^{2}+2b^{3}d(cf-de)\right)h+b^{3}d^{2}(eh-fg)\right)(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left(3(af-be)^{2}b\,d^{2}+6(af-be)b^{2}d(cf-de)\right.\right.}{}$
pseudoelliptic	$-\frac{4\sqrt{fx+e}\left(\frac{3\left(-\frac{x^{3}\left(7\left(\frac{1}{13}h\,x^{3}+\frac{1}{11}g\,x^{2}\right)d^{2}+\frac{14xc\left(\frac{9hx}{11}+g\right)d}{9}+c^{2}\left(\frac{7hx}{9}+g\right)\right)b^{3}}{7}-\frac{3a\,x^{2}\left(\frac{5x^{2}\left(\frac{9hx}{11}+g\right)d^{2}}{9}+\frac{10xc\left(\frac{7hx}{9}+g\right)d}{7}+\right.}{5}\right.}{}\right.}{}$
gosper	Expression too large to display
trager	Expression too large to display
risch	Expression too large to display
orering	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^7*(1/13*h*d^2*b^3*(f*x+e)^(13/2)+1/11*((3*(a*f-b*e)*b^2*d^2+2*b^3*d*(c
*f-d*e))*h+b^3*d^2*(-e*h+f*g))*(f*x+e)^(11/2)+1/9*((3*(a*f-b*e)^2*b*d^2+6*
(a*f-b*e)*b^2*d*(c*f-d*e)+b^3*(c*f-d*e)^2)*h+(3*(a*f-b*e)*b^2*d^2+2*b^3*d*
(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*(((a*f-b*e)^3*d^2+6*(a*f-b*e)^2*b
*d*(c*f-d*e)+3*(a*f-b*e)*b^2*(c*f-d*e)^2)*h+(3*(a*f-b*e)^2*b*d^2+6*(a*f-b*
e)*b^2*d*(c*f-d*e)+b^3*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*((2*(a*f
-b*e)^3*d*(c*f-d*e)+3*(a*f-b*e)^2*b*(c*f-d*e)^2)*h+((a*f-b*e)^3*d^2+6*(a*f
-b*e)^2*b*d*(c*f-d*e)+3*(a*f-b*e)*b^2*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(5/
2)+1/3*((a*f-b*e)^3*(c*f-d*e)^2*h+(2*(a*f-b*e)^3*d*(c*f-d*e)+3*(a*f-b*e)^2
*b*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(3/2)+(a*f-b*e)^3*(c*f-d*e)^2*(-e*h+f*
g)*(f*x+e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(548) = 1096$.

Time = 0.10 (sec) , antiderivative size = 1174, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3465*b^3*d^2*f^6*h*x^6 + 315*(13*b^3*d^2*f^6*g - (12*b^3*d^2*e*f^
5 - 13*(2*b^3*c*d + 3*a*b^2*d^2)*f^6)*h)*x^5 - 35*(13*(10*b^3*d^2*e*f^5 -
11*(2*b^3*c*d + 3*a*b^2*d^2)*f^6)*g - (120*b^3*d^2*e^2*f^4 - 130*(2*b^3*c*
d + 3*a*b^2*d^2)*e*f^5 + 143*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^6)*h)
*x^4 + 5*(13*(80*b^3*d^2*e^2*f^4 - 88*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^5 + 99
*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^6)*g - (960*b^3*d^2*e^3*f^3 - 104
0*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^4 + 1144*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*
b*d^2)*e*f^5 - 1287*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^6)*h)*x^3 - 3*
(13*(160*b^3*d^2*e^3*f^3 - 176*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^4 + 198*(b^
3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^5 - 231*(3*a*b^2*c^2 + 6*a^2*b*c*d
+ a^3*d^2)*f^6)*g - (1920*b^3*d^2*e^4*f^2 - 2080*(2*b^3*c*d + 3*a*b^2*d^2)
*e^3*f^3 + 2288*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^2*f^4 - 2574*(3*a*
b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^5 + 3003*(3*a^2*b*c^2 + 2*a^3*c*d)*f^
6)*h)*x^2 - 13*(1280*b^3*d^2*e^5*f - 3465*a^3*c^2*f^6 - 1408*(2*b^3*c*d +
3*a*b^2*d^2)*e^4*f^2 + 1584*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^3
- 1848*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2*f^4 + 2310*(3*a^2*b*c^2 +
 2*a^3*c*d)*e*f^5)*g + 2*(7680*b^3*d^2*e^6 - 15015*a^3*c^2*e*f^5 - 8320*(2
*b^3*c*d + 3*a*b^2*d^2)*e^5*f + 9152*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)
*e^4*f^2 - 10296*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^3*f^3 + 12012*(3*
a^2*b*c^2 + 2*a^3*c*d)*e^2*f^4)*h + (13*(640*b^3*d^2*e^4*f^2 - 704*(2*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1907 vs. $2(619) = 1238$.

Time = 2.11 (sec) , antiderivative size = 1907, normalized size of antiderivative = 3.31

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**2*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b**3*d**2*h*(e + f*x)**(13/2)/(13*f**6) + (e + f*x)**(11/2)*
(3*a*b**2*d**2*f*h + 2*b**3*c*d*f*h - 6*b**3*d**2*e*h + b**3*d**2*f*g)/(11
*f**6) + (e + f*x)**(9/2)*(3*a**2*b*d**2*f**2*h + 6*a*b**2*c*d*f**2*h - 15
*a*b**2*d**2*e*f*h + 3*a*b**2*d**2*f**2*g + b**3*c**2*f**2*h - 10*b**3*c*d
*e*f*h + 2*b**3*c*d*f**2*g + 15*b**3*d**2*e**2*h - 5*b**3*d**2*e*f*g)/(9*f
**6) + (e + f*x)**(7/2)*(a**3*d**2*f**3*h + 6*a**2*b*c*d*f**3*h - 12*a**2*
b*d**2*e*f**2*h + 3*a**2*b*d**2*f**3*g + 3*a*b**2*c**2*f**3*h - 24*a*b**2*
c*d*e*f**2*h + 6*a*b**2*c*d*f**3*g + 30*a*b**2*d**2*e**2*f*h - 12*a*b**2*d
**2*e*f**2*g - 4*b**3*c**2*e*f**2*h + b**3*c**2*f**3*g + 20*b**3*c*d*e**2*
f*h - 8*b**3*c*d*e*f**2*g - 20*b**3*d**2*e**3*h + 10*b**3*d**2*e**2*f*g)/(
7*f**6) + (e + f*x)**(5/2)*(2*a**3*c*d*f**4*h - 3*a**3*d**2*e*f**3*h + a**
3*d**2*f**4*g + 3*a**2*b*c**2*f**4*h - 18*a**2*b*c*d*e*f**3*h + 6*a**2*b*c
*d*f**4*g + 18*a**2*b*d**2*e**2*f**2*h - 9*a**2*b*d**2*e*f**3*g - 9*a*b**2
*c**2*e*f**3*h + 3*a*b**2*c**2*f**4*g + 36*a*b**2*c*d*e**2*f**2*h - 18*a*b
**2*c*d*e*f**3*g - 30*a*b**2*d**2*e**3*f*h + 18*a*b**2*d**2*e**2*f**2*g +
6*b**3*c**2*e**2*f**2*h - 3*b**3*c**2*e*f**3*g - 20*b**3*c*d*e**3*f*h + 12
*b**3*c*d*e**2*f**2*g + 15*b**3*d**2*e**4*h - 10*b**3*d**2*e**3*f*g)/(5*f*
*6) + (e + f*x)**(3/2)*(a**3*c**2*f**5*h - 4*a**3*c*d*e*f**4*h + 2*a**3*c*
d*f**5*g + 3*a**3*d**2*e**2*f**3*h - 2*a**3*d**2*e*f**4*g - 6*a**2*b*c**2*
e*f**4*h + 3*a**2*b*c**2*f**5*g + 18*a**2*b*c*d*e**2*f**3*h - 12*a**2*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(548) = 1096$.

Time = 0.04 (sec) , antiderivative size = 1152, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/45045*(3465*(f*x + e)^(13/2)*b^3*d^2*h + 4095*(b^3*d^2*f*g - (6*b^3*d^2*
e - (2*b^3*c*d + 3*a*b^2*d^2)*f)*h)*(f*x + e)^(11/2) - 5005*((5*b^3*d^2*e*
f - (2*b^3*c*d + 3*a*b^2*d^2)*f^2)*g - (15*b^3*d^2*e^2 - 5*(2*b^3*c*d + 3*
a*b^2*d^2)*e*f + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*h)*(f*x + e)^(
9/2) + 6435*((10*b^3*d^2*e^2*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^2 + (b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^3)*g - (20*b^3*d^2*e^3 - 10*(2*b^3*c*d
+ 3*a*b^2*d^2)*e^2*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2 - (3*
a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^3)*h)*(f*x + e)^(7/2) - 9009*((10*b^3
*d^2*e^3*f - 6*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^2 + 3*(b^3*c^2 + 6*a*b^2*c*
d + 3*a^2*b*d^2)*e*f^3 - (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^4)*g - (1
5*b^3*d^2*e^4 - 10*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f + 6*(b^3*c^2 + 6*a*b^2*
c*d + 3*a^2*b*d^2)*e^2*f^2 - 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^3
 + (3*a^2*b*c^2 + 2*a^3*c*d)*f^4)*h)*(f*x + e)^(5/2) + 15015*((5*b^3*d^2*e
^4*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^2 + 3*(b^3*c^2 + 6*a*b^2*c*d + 3*
a^2*b*d^2)*e^2*f^3 - 2*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^4 + (3*a^
2*b*c^2 + 2*a^3*c*d)*f^5)*g - (6*b^3*d^2*e^5 - a^3*c^2*f^5 - 5*(2*b^3*c*d
+ 3*a*b^2*d^2)*e^4*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^2 - 3
*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2*f^3 + 2*(3*a^2*b*c^2 + 2*a^3*c*
d)*e*f^4)*h)*(f*x + e)^(3/2) - 45045*((b^3*d^2*e^5*f - a^3*c^2*f^6 - (2*b^
3*c*d + 3*a*b^2*d^2)*e^4*f^2 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(548) = 1096$.

Time = 0.14 (sec) , antiderivative size = 1344, normalized size of antiderivative = 2.33

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(f*x + e)*a^3*c^2*g + 45045*((f*x + e)^(3/2) - 3*sqrt(f
*x + e)*e)*a^2*b*c^2*g/f + 30030*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3
*c*d*g/f + 15015*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^3*c^2*h/f + 9009*
(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b^2*c^
2*g/f^2 + 18018*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x +
e)*e^2)*a^2*b*c*d*g/f^2 + 3003*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e +
 15*sqrt(f*x + e)*e^2)*a^3*d^2*g/f^2 + 9009*(3*(f*x + e)^(5/2) - 10*(f*x +
 e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*b*c^2*h/f^2 + 6006*(3*(f*x + e)^(5
/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^3*c*d*h/f^2 + 1287*(5
*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt
(f*x + e)*e^3)*b^3*c^2*g/f^3 + 7722*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2
)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b^2*c*d*g/f^3 + 386
1*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*
sqrt(f*x + e)*e^3)*a^2*b*d^2*g/f^3 + 3861*(5*(f*x + e)^(7/2) - 21*(f*x + e
)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b^2*c^2*h/f^3
 + 7722*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2
 - 35*sqrt(f*x + e)*e^3)*a^2*b*c*d*h/f^3 + 1287*(5*(f*x + e)^(7/2) - 21*(f
*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a^3*d^2*h
/f^3 + 286*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*x + e)^(5/
2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*b^3*c*d*g/f^4...
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{(e+f\,x)^{9/2}\,(6\,h\,a^2\,b\,d^2\,f^2+12\,h\,a\,b^2\,c\,d\,f^2-30\,h\,a\,b^2\,d^2\,e\,f+6\,g\,a\,b^2\,d^2\,f^2+2\,h\,b^3\,c^2\,f^2-20\,h\,b^3\,c\,d\,e\,f}{9\,f^7}$$
$$+\frac{(e+f\,x)^{7/2}\,(2\,h\,a^3\,d^2\,f^3+12\,h\,a^2\,b\,c\,d\,f^3-24\,h\,a^2\,b\,d^2\,e\,f^2+6\,g\,a^2\,b\,d^2\,f^3+6\,h\,a\,b^2\,c^2\,f^3-48\,h\,a\,b^2\,c}{}$$
$$+\frac{2\,(e+f\,x)^{5/2}\,(a\,f-b\,e)\,(2\,h\,a^2\,c\,d\,f^3-3\,h\,a^2\,d^2\,e\,f^2+g\,a^2\,d^2\,f^3+3\,h\,a\,b\,c^2\,f^3-16\,h\,a\,b\,c\,d\,e\,f^2+6\,g}{}$$
$$-\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)^3\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{f^7}+\frac{2\,b^3\,d^2\,h\,(e+f\,x)^{13/2}}{13\,f^7}$$
$$+\frac{2\,b^2\,d\,(e+f\,x)^{11/2}\,(3\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+b\,d\,f\,g)}{11\,f^7}$$
$$+\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)^2\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+3\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-4\,b\,c\,e\,f}{3\,f^7}$$

input
```
int(((g + h*x)*(a + b*x)^3*(c + d*x)^2)/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(9/2)*(2*b^3*c^2*f^2*h + 30*b^3*d^2*e^2*h + 4*b^3*c*d*f^2*g - 1
0*b^3*d^2*e*f*g + 6*a*b^2*d^2*f^2*g + 6*a^2*b*d^2*f^2*h - 20*b^3*c*d*e*f*h
 + 12*a*b^2*c*d*f^2*h - 30*a*b^2*d^2*e*f*h))/(9*f^7) + ((e + f*x)^(7/2)*(2
*b^3*c^2*f^3*g + 2*a^3*d^2*f^3*h - 40*b^3*d^2*e^3*h + 6*a*b^2*c^2*f^3*h +
6*a^2*b*d^2*f^3*g - 8*b^3*c^2*e*f^2*h + 20*b^3*d^2*e^2*f*g + 12*a*b^2*c*d*
f^3*g + 12*a^2*b*c*d*f^3*h - 16*b^3*c*d*e*f^2*g + 40*b^3*c*d*e^2*f*h - 24*
a*b^2*d^2*e*f^2*g + 60*a*b^2*d^2*e^2*f*h - 24*a^2*b*d^2*e*f^2*h - 48*a*b^2
*c*d*e*f^2*h))/(7*f^7) + (2*(e + f*x)^(5/2)*(a*f - b*e)*(a^2*d^2*f^3*g + 3
*b^2*c^2*f^3*g - 15*b^2*d^2*e^3*h + 3*a*b*c^2*f^3*h + 2*a^2*c*d*f^3*h - 3*
a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h + 10*b^2*d^2*e^2*f*g + 6*a*b*c*d*f^3*g
 - 8*a*b*d^2*e*f^2*g + 15*a*b*d^2*e^2*f*h - 12*b^2*c*d*e*f^2*g + 20*b^2*c*
d*e^2*f*h - 16*a*b*c*d*e*f^2*h))/(5*f^7) - (2*(e + f*x)^(1/2)*(a*f - b*e)^
3*(c*f - d*e)^2*(e*h - f*g))/f^7 + (2*b^3*d^2*h*(e + f*x)^(13/2))/(13*f^7)
 + (2*b^2*d*(e + f*x)^(11/2)*(3*a*d*f*h + 2*b*c*f*h - 6*b*d*e*h + b*d*f*g)
)/(11*f^7) + (2*(e + f*x)^(3/2)*(a*f - b*e)^2*(c*f - d*e)*(a*c*f^2*h + 2*a
*d*f^2*g + 3*b*c*f^2*g + 6*b*d*e^2*h - 3*a*d*e*f*h - 4*b*c*e*f*h - 5*b*d*e
*f*g))/(3*f^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1509, normalized size of antiderivative = 2.62

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 30030*a**3*c**2*e*f**5*h + 45045*a**3*c**2*f**6*g + 1
5015*a**3*c**2*f**6*h*x + 48048*a**3*c*d*e**2*f**4*h - 60060*a**3*c*d*e*f*
*5*g - 24024*a**3*c*d*e*f**5*h*x + 30030*a**3*c*d*f**6*g*x + 18018*a**3*c*
d*f**6*h*x**2 - 20592*a**3*d**2*e**3*f**3*h + 24024*a**3*d**2*e**2*f**4*g
+ 10296*a**3*d**2*e**2*f**4*h*x - 12012*a**3*d**2*e*f**5*g*x - 7722*a**3*d
**2*e*f**5*h*x**2 + 9009*a**3*d**2*f**6*g*x**2 + 6435*a**3*d**2*f**6*h*x**
3 + 72072*a**2*b*c**2*e**2*f**4*h - 90090*a**2*b*c**2*e*f**5*g - 36036*a**
2*b*c**2*e*f**5*h*x + 45045*a**2*b*c**2*f**6*g*x + 27027*a**2*b*c**2*f**6*
h*x**2 - 123552*a**2*b*c*d*e**3*f**3*h + 144144*a**2*b*c*d*e**2*f**4*g + 6
1776*a**2*b*c*d*e**2*f**4*h*x - 72072*a**2*b*c*d*e*f**5*g*x - 46332*a**2*b
*c*d*e*f**5*h*x**2 + 54054*a**2*b*c*d*f**6*g*x**2 + 38610*a**2*b*c*d*f**6*
h*x**3 + 54912*a**2*b*d**2*e**4*f**2*h - 61776*a**2*b*d**2*e**3*f**3*g - 2
7456*a**2*b*d**2*e**3*f**3*h*x + 30888*a**2*b*d**2*e**2*f**4*g*x + 20592*a
**2*b*d**2*e**2*f**4*h*x**2 - 23166*a**2*b*d**2*e*f**5*g*x**2 - 17160*a**2
*b*d**2*e*f**5*h*x**3 + 19305*a**2*b*d**2*f**6*g*x**3 + 15015*a**2*b*d**2*
f**6*h*x**4 - 61776*a*b**2*c**2*e**3*f**3*h + 72072*a*b**2*c**2*e**2*f**4*
g + 30888*a*b**2*c**2*e**2*f**4*h*x - 36036*a*b**2*c**2*e*f**5*g*x - 23166
*a*b**2*c**2*e*f**5*h*x**2 + 27027*a*b**2*c**2*f**6*g*x**2 + 19305*a*b**2*
c**2*f**6*h*x**3 + 109824*a*b**2*c*d*e**4*f**2*h - 123552*a*b**2*c*d*e**3*
f**3*g - 54912*a*b**2*c*d*e**3*f**3*h*x + 61776*a*b**2*c*d*e**2*f**4*g*...
```

3.115 $\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 409

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2(be-af)^2(de-cf)^2(fg-eh)\sqrt{e+fx}}{f^6}$$
$$-\frac{2(be-af)(de-cf)(bde(4fg-5eh)-bcf(2fg-3eh)-af(2dfg-3deh+cfh))(e+fx)^{3/2}}{3f^6}$$
$$+\frac{2(a^2df^2(dfg-3deh+2cfh)+2abf(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh))+b^2(2d^2e^2(3fg-5e}{5f^6}$$
$$+\frac{2(a^2d^2f^2h+2abdf(dfg-4deh+2cfh)+b^2(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh)))(e+fx)^{7}}{7f^6}$$
$$+\frac{2bd(2adfh+b(dfg-5deh+2cfh))(e+fx)^{9/2}}{9f^6}+\frac{2b^2d^2h(e+fx)^{11/2}}{11f^6}$$

output
```
2*(-a*f+b*e)^2*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(1/2)/f^6-2/3*(-a*f+b*e)*(-
c*f+d*e)*(b*d*e*(-5*e*h+4*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-3*d*e*h+2*d
*f*g))*(f*x+e)^(3/2)/f^6+2/5*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d*f*g)+2*a*b*f*(c
^2*f^2*h+2*c*d*f*(-3*e*h+f*g)-3*d^2*e*(-2*e*h+f*g))+b^2*(2*d^2*e^2*(-5*e*h
+3*f*g)+c^2*f^2*(-3*e*h+f*g)-6*c*d*e*f*(-2*e*h+f*g)))*(f*x+e)^(5/2)/f^6+2/
7*(a^2*d^2*f^2*h+2*a*b*d*f*(2*c*f*h-4*d*e*h+d*f*g)+b^2*(c^2*f^2*h-2*d^2*e*
(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g)))*(f*x+e)^(7/2)/f^6+2/9*b*d*(2*a*d*f*h
+b*(2*c*f*h-5*d*e*h+d*f*g))*(f*x+e)^(9/2)/f^6+2/11*b^2*d^2*h*(f*x+e)^(11/2
)/f^6
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(33a^2f^2(35c^2f^2(3fg-2eh+fhx)+14cdf(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx))+d^2(-4$$

input
```
Integrate[((a + b*x)^2*(c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(33*a^2*f^2*(35*c^2*f^2*(3*f*g - 2*e*h + f*h*x) + 14*c*d*
f*(8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d^2*(-48*e^3*h +
 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x
))) + 22*a*b*f*(21*c^2*f^2*(8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3
*h*x)) + 6*c*d*f*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h
*x) - 2*e*f^2*x*(14*g + 9*h*x)) + d^2*(128*e^4*h + 24*e^2*f^2*x*(3*g + 2*h
*x) - 16*e^3*f*(9*g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2*(27*g
 + 20*h*x))) + b^2*(33*c^2*f^2*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*f^3*
x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x)) + 22*c*d*f*(128*e^4*h + 24*e
^2*f^2*x*(3*g + 2*h*x) - 16*e^3*f*(9*g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x)
- 2*e*f^3*x^2*(27*g + 20*h*x)) + d^2*(-1280*e^5*h + 128*e^4*f*(11*g + 5*h*
x) + 35*f^5*x^4*(11*g + 9*h*x) - 32*e^3*f^2*x*(22*g + 15*h*x) + 16*e^2*f^3
*x^2*(33*g + 25*h*x) - 10*e*f^4*x^3*(44*g + 35*h*x)))))/(3465*f^6)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \; 165$$

$$\int \left(\frac{(e+fx)^{3/2} \left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-\right.\right.}{f^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{5/2} \left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-3e\right.\right.}{5f^6}$$

$$\frac{2(e+fx)^{7/2}\left(a^2d^2f^2h+2abdf(2cfh-4deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)\right)}{7f^6}+$$

$$\frac{2bd(e+fx)^{9/2}(2adfh+b(2cfh-5deh+dfg))}{9f^6}-$$

$$\frac{2(e+fx)^{3/2}(be-af)(de-cf)(-af(cfh-3deh+2dfg)-bcf(2fg-3eh)+bde(4fg-5eh))}{3f^6}+$$

$$\frac{2\sqrt{e+fx}(be-af)^2(de-cf)^2(fg-eh)}{f^6}+\frac{2b^2d^2h(e+fx)^{11/2}}{11f^6}$$

input
```
Int[((a + b*x)^2*(c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*(b*e - a*f)^2*(d*e - c*f)^2*(f*g - e*h)*Sqrt[e + f*x])/f^6 - (2*(b*e -
a*f)*(d*e - c*f)*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(2*f*g - 3*e*h) - a*f*(2*d
*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(3/2))/(3*f^6) + (2*(a^2*d*f^2*(d*f*g -
 3*d*e*h + 2*c*f*h) + 2*a*b*f*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3*d^2*e
*(f*g - 2*e*h)) + b^2*(2*d^2*e^2*(3*f*g - 5*e*h) + c^2*f^2*(f*g - 3*e*h) -
 6*c*d*e*f*(f*g - 2*e*h)))*(e + f*x)^(5/2))/(5*f^6) + (2*(a^2*d^2*f^2*h +
2*a*b*d*f*(d*f*g - 4*d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h - 2*d^2*e*(2*f*g -
5*e*h) + 2*c*d*f*(f*g - 4*e*h)))*(e + f*x)^(7/2))/(7*f^6) + (2*b*d*(2*a*d*
f*h + b*(d*f*g - 5*d*e*h + 2*c*f*h))*(e + f*x)^(9/2))/(9*f^6) + (2*b^2*d^2
*h*(e + f*x)^(11/2))/(11*f^6)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2h\,b^2d^2(fx+e)^{\frac{11}{2}}}{11}+\frac{2\left(\left(2b(af-be)d^2+2b^2d(cf-de)\right)h+b^2d^2(-eh+fg)\right)(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(\left((af-be)^2d^2+4b(af-be)d(cf-de)+b^2(cf-\right.\right.}{}$
default	$\frac{2h\,b^2d^2(fx+e)^{\frac{11}{2}}}{11}-\frac{2\left(-\left(2b(af-be)d^2+2b^2d(cf-de)\right)h+b^2d^2(eh-fg)\right)(fx+e)^{\frac{9}{2}}}{9}-\frac{2\left(-\left((af-be)^2d^2+4b(af-be)d(cf-de)+b^2(c\right.\right.}{}$
pseudoelliptic	$-\frac{4\left(\left(\left(-\frac{3x^2\left(\frac{5x^2\left(\frac{9hx}{11}+g\right)d^2}{9}+\frac{10xc\left(\frac{7hx}{9}+g\right)d}{7}+c^2\left(\frac{5hx}{7}+g\right)\right)b^2}{10}-ax\left(\frac{3x^2\left(\frac{7hx}{9}+g\right)d^2}{7}+\frac{6xc\left(\frac{5hx}{7}+g\right)d}{5}+c^2\left(\frac{3hx}{5}+g\right)\right)b\right.\right.\right.}{}$
gosper	$-\frac{2\sqrt{fx+e}\,(-315h\,b^2d^2x^5f^5-770ab\,d^2f^5h\,x^4-770b^2cd\,f^5h\,x^4+350b^2d^2e\,f^4h\,x^4-385b^2d^2f^5g\,x^4-495a^2d^2f^5h\,x^3-}{}$
trager	$-\frac{2\sqrt{fx+e}\,(-315h\,b^2d^2x^5f^5-770ab\,d^2f^5h\,x^4-770b^2cd\,f^5h\,x^4+350b^2d^2e\,f^4h\,x^4-385b^2d^2f^5g\,x^4-495a^2d^2f^5h\,x^3-}{}$
risch	$-\frac{2\sqrt{fx+e}\,(-315h\,b^2d^2x^5f^5-770ab\,d^2f^5h\,x^4-770b^2cd\,f^5h\,x^4+350b^2d^2e\,f^4h\,x^4-385b^2d^2f^5g\,x^4-495a^2d^2f^5h\,x^3-}{}$
orering	$-\frac{2\sqrt{fx+e}\,(-315h\,b^2d^2x^5f^5-770ab\,d^2f^5h\,x^4-770b^2cd\,f^5h\,x^4+350b^2d^2e\,f^4h\,x^4-385b^2d^2f^5g\,x^4-495a^2d^2f^5h\,x^3-}{}$

input
```
int((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^6*(1/11*h*b^2*d^2*(f*x+e)^(11/2)+1/9*((2*b*(a*f-b*e)*d^2+2*b^2*d*(c*f-
d*e))*h+b^2*d^2*(-e*h+f*g))*(f*x+e)^(9/2)+1/7*(((a*f-b*e)^2*d^2+4*b*(a*f-b
*e)*d*(c*f-d*e)+b^2*(c*f-d*e)^2)*h+(2*b*(a*f-b*e)*d^2+2*b^2*d*(c*f-d*e))*(
-e*h+f*g))*(f*x+e)^(7/2)+1/5*((2*(a*f-b*e)^2*d*(c*f-d*e)+2*b*(a*f-b*e)*(c*
f-d*e)^2)*h+((a*f-b*e)^2*d^2+4*b*(a*f-b*e)*d*(c*f-d*e)+b^2*(c*f-d*e)^2)*(-
e*h+f*g))*(f*x+e)^(5/2)+1/3*((a*f-b*e)^2*(c*f-d*e)^2*h+(2*(a*f-b*e)^2*d*(c
*f-d*e)+2*b*(a*f-b*e)*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e)^(3/2)+(a*f-b*e)^2*(
c*f-d*e)^2*(-e*h+f*g)*(f*x+e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*b^2*d^2*f^5*h*x^5 + 35*(11*b^2*d^2*f^5*g - 2*(5*b^2*d^2*e*f^4
- 11*(b^2*c*d + a*b*d^2)*f^5)*h)*x^4 - 5*(22*(4*b^2*d^2*e*f^4 - 9*(b^2*c*d
 + a*b*d^2)*f^5)*g - (80*b^2*d^2*e^2*f^3 - 176*(b^2*c*d + a*b*d^2)*e*f^4 +
 99*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^5)*h)*x^3 + 3*(11*(16*b^2*d^2*e^2*f^
3 - 36*(b^2*c*d + a*b*d^2)*e*f^4 + 21*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^5)
*g - 2*(80*b^2*d^2*e^3*f^2 - 176*(b^2*c*d + a*b*d^2)*e^2*f^3 + 99*(b^2*c^2
 + 4*a*b*c*d + a^2*d^2)*e*f^4 - 231*(a*b*c^2 + a^2*c*d)*f^5)*h)*x^2 + 11*(
128*b^2*d^2*e^4*f + 315*a^2*c^2*f^5 - 288*(b^2*c*d + a*b*d^2)*e^3*f^2 + 16
8*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^3 - 420*(a*b*c^2 + a^2*c*d)*e*f^4)
*g - 2*(640*b^2*d^2*e^5 + 1155*a^2*c^2*e*f^4 - 1408*(b^2*c*d + a*b*d^2)*e^
4*f + 792*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^2 - 1848*(a*b*c^2 + a^2*c*
d)*e^2*f^3)*h - (22*(32*b^2*d^2*e^3*f^2 - 72*(b^2*c*d + a*b*d^2)*e^2*f^3 +
 42*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e*f^4 - 105*(a*b*c^2 + a^2*c*d)*f^5)*g
 - (640*b^2*d^2*e^4*f + 1155*a^2*c^2*f^5 - 1408*(b^2*c*d + a*b*d^2)*e^3*f^
2 + 792*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^3 - 1848*(a*b*c^2 + a^2*c*d)
*e*f^4)*h)*x)*sqrt(f*x + e)/f^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(444) = 888.

Time = 1.76 (sec) , antiderivative size = 1202, normalized size of antiderivative = 2.94

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2*(d*x+c)**2*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b**2*d**2*h*(e + f*x)**(11/2)/(11*f**5) + (e + f*x)**(9/2)*(
2*a*b*d**2*f*h + 2*b**2*c*d*f*h - 5*b**2*d**2*e*h + b**2*d**2*f*g)/(9*f**5
) + (e + f*x)**(7/2)*(a**2*d**2*f**2*h + 4*a*b*c*d*f**2*h - 8*a*b*d**2*e*f
*h + 2*a*b*d**2*f**2*g + b**2*c**2*f**2*h - 8*b**2*c*d*e*f*h + 2*b**2*c*d*
f**2*g + 10*b**2*d**2*e**2*h - 4*b**2*d**2*e*f*g)/(7*f**5) + (e + f*x)**(5
/2)*(2*a**2*c*d*f**3*h - 3*a**2*d**2*e*f**2*h + a**2*d**2*f**3*g + 2*a*b*c
**2*f**3*h - 12*a*b*c*d*e*f**2*h + 4*a*b*c*d*f**3*g + 12*a*b*d**2*e**2*f*h
 - 6*a*b*d**2*e*f**2*g - 3*b**2*c**2*e*f**2*h + b**2*c**2*f**3*g + 12*b**2
*c*d*e**2*f*h - 6*b**2*c*d*e*f**2*g - 10*b**2*d**2*e**3*h + 6*b**2*d**2*e*
*2*f*g)/(5*f**5) + (e + f*x)**(3/2)*(a**2*c**2*f**4*h - 4*a**2*c*d*e*f**3*
h + 2*a**2*c*d*f**4*g + 3*a**2*d**2*e**2*f**2*h - 2*a**2*d**2*e*f**3*g - 4
*a*b*c**2*e*f**3*h + 2*a*b*c**2*f**4*g + 12*a*b*c*d*e**2*f**2*h - 8*a*b*c*
d*e*f**3*g - 8*a*b*d**2*e**3*f*h + 6*a*b*d**2*e**2*f**2*g + 3*b**2*c**2*e*
*2*f**2*h - 2*b**2*c**2*e*f**3*g - 8*b**2*c*d*e**3*f*h + 6*b**2*c*d*e**2*f
**2*g + 5*b**2*d**2*e**4*h - 4*b**2*d**2*e**3*f*g)/(3*f**5) + sqrt(e + f*x
)*(-a**2*c**2*e*f**4*h + a**2*c**2*f**5*g + 2*a**2*c*d*e**2*f**3*h - 2*a**
2*c*d*e*f**4*g - a**2*d**2*e**3*f**2*h + a**2*d**2*e**2*f**3*g + 2*a*b*c**
2*e**2*f**3*h - 2*a*b*c**2*e*f**4*g - 4*a*b*c*d*e**3*f**2*h + 4*a*b*c*d*e*
*2*f**3*g + 2*a*b*d**2*e**4*f*h - 2*a*b*d**2*e**3*f**2*g - b**2*c**2*e**3*
f**2*h + b**2*c**2*e**2*f**3*g + 2*b**2*c*d*e**4*f*h - 2*b**2*c*d*e**3*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(f*x + e)^(11/2)*b^2*d^2*h + 385*(b^2*d^2*f*g - (5*b^2*d^2*e -
 2*(b^2*c*d + a*b*d^2)*f)*h)*(f*x + e)^(9/2) - 495*(2*(2*b^2*d^2*e*f - (b^
2*c*d + a*b*d^2)*f^2)*g - (10*b^2*d^2*e^2 - 8*(b^2*c*d + a*b*d^2)*e*f + (b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2)*h)*(f*x + e)^(7/2) + 693*((6*b^2*d^2*e^
2*f - 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3)*g
 - (10*b^2*d^2*e^3 - 12*(b^2*c*d + a*b*d^2)*e^2*f + 3*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*e*f^2 - 2*(a*b*c^2 + a^2*c*d)*f^3)*h)*(f*x + e)^(5/2) - 1155*(
2*(2*b^2*d^2*e^3*f - 3*(b^2*c*d + a*b*d^2)*e^2*f^2 + (b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*e*f^3 - (a*b*c^2 + a^2*c*d)*f^4)*g - (5*b^2*d^2*e^4 + a^2*c^2*f
^4 - 8*(b^2*c*d + a*b*d^2)*e^3*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f
^2 - 4*(a*b*c^2 + a^2*c*d)*e*f^3)*h)*(f*x + e)^(3/2) + 3465*((b^2*d^2*e^4*
f + a^2*c^2*f^5 - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*e^2*f^3 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*g - (b^2*d^2*e^5 + a^2*c^2*
e*f^4 - 2*(b^2*c*d + a*b*d^2)*e^4*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*
f^2 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^3)*h)*sqrt(f*x + e))/f^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(387) = 774$.

Time = 0.14 (sec) , antiderivative size = 884, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(f*x + e)*a^2*c^2*g + 2310*((f*x + e)^(3/2) - 3*sqrt(f*x
+ e)*e)*a*b*c^2*g/f + 2310*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c*d*g
/f + 1155*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a^2*c^2*h/f + 231*(3*(f*x
+ e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b^2*c^2*g/f^2 +
924*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*b*
c*d*g/f^2 + 231*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x +
e)*e^2)*a^2*d^2*g/f^2 + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15
*sqrt(f*x + e)*e^2)*a*b*c^2*h/f^2 + 462*(3*(f*x + e)^(5/2) - 10*(f*x + e)^
(3/2)*e + 15*sqrt(f*x + e)*e^2)*a^2*c*d*h/f^2 + 198*(5*(f*x + e)^(7/2) - 2
1*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*c
*d*g/f^3 + 198*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3
/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b*d^2*g/f^3 + 99*(5*(f*x + e)^(7/2) - 21
*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b^2*c^
2*h/f^3 + 396*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/
2)*e^2 - 35*sqrt(f*x + e)*e^3)*a*b*c*d*h/f^3 + 99*(5*(f*x + e)^(7/2) - 21*
(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*a^2*d^2
*h/f^3 + 11*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*x + e)^(5
/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*b^2*d^2*g/f^4 +
 22*(35*(f*x + e)^(9/2) - 180*(f*x + e)^(7/2)*e + 378*(f*x + e)^(5/2)*e^2
- 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x + e)*e^4)*b^2*c*d*h/f^4 + 22*(...
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{(e+fx)^{5/2}\left(4\,h\,a^2\,c\,d\,f^3-6\,h\,a^2\,d^2\,e\,f^2+2\,g\,a^2\,d^2\,f^3+4\,h\,a\,b\,c^2\,f^3-24\,h\,a\,b\,c\,d\,e\,f^2+8\,g\,a\,b\,c\,d\,f^3+2\right.}{}$$
$$+\frac{(e+fx)^{7/2}\left(2\,h\,a^2\,d^2\,f^2+8\,h\,a\,b\,c\,d\,f^2-16\,h\,a\,b\,d^2\,e\,f+4\,g\,a\,b\,d^2\,f^2+2\,h\,b^2\,c^2\,f^2-16\,h\,b^2\,c\,d\,e\,f+4\right.}{7\,f^6}$$
$$-\frac{2\sqrt{e+fx}\,(a\,f-b\,e)^2\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{f^6}+\frac{2\,b^2\,d^2\,h\,(e+fx)^{11/2}}{11\,f^6}$$
$$+\frac{2\,b\,d\,(e+fx)^{9/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-5\,b\,d\,e\,h+b\,d\,f\,g)}{9\,f^6}$$
$$+\frac{2\,(e+fx)^{3/2}\,(a\,f-b\,e)\,(c\,f-d\,e)\,\left(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\right.}{3\,f^6}$$

input
```
int(((g + h*x)*(a + b*x)^2*(c + d*x)^2)/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(5/2)*(2*a^2*d^2*f^3*g + 2*b^2*c^2*f^3*g - 20*b^2*d^2*e^3*h + 4
*a*b*c^2*f^3*h + 4*a^2*c*d*f^3*h - 6*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h +
 12*b^2*d^2*e^2*f*g + 8*a*b*c*d*f^3*g - 12*a*b*d^2*e*f^2*g + 24*a*b*d^2*e^
2*f*h - 12*b^2*c*d*e*f^2*g + 24*b^2*c*d*e^2*f*h - 24*a*b*c*d*e*f^2*h))/(5*
f^6) + ((e + f*x)^(7/2)*(2*a^2*d^2*f^2*h + 2*b^2*c^2*f^2*h + 20*b^2*d^2*e^
2*h + 4*a*b*d^2*f^2*g + 4*b^2*c*d*f^2*g - 8*b^2*d^2*e*f*g + 8*a*b*c*d*f^2*
h - 16*a*b*d^2*e*f*h - 16*b^2*c*d*e*f*h))/(7*f^6) - (2*(e + f*x)^(1/2)*(a*
f - b*e)^2*(c*f - d*e)^2*(e*h - f*g))/f^6 + (2*b^2*d^2*h*(e + f*x)^(11/2))
/(11*f^6) + (2*b*d*(e + f*x)^(9/2)*(2*a*d*f*h + 2*b*c*f*h - 5*b*d*e*h + b*
d*f*g))/(9*f^6) + (2*(e + f*x)^(3/2)*(a*f - b*e)*(c*f - d*e)*(a*c*f^2*h +
2*a*d*f^2*g + 2*b*c*f^2*g + 5*b*d*e^2*h - 3*a*d*e*f*h - 3*b*c*e*f*h - 4*b*
d*e*f*g))/(3*f^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 917, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 2310*a**2*c**2*e*f**4*h + 3465*a**2*c**2*f**5*g + 115
5*a**2*c**2*f**5*h*x + 3696*a**2*c*d*e**2*f**3*h - 4620*a**2*c*d*e*f**4*g
- 1848*a**2*c*d*e*f**4*h*x + 2310*a**2*c*d*f**5*g*x + 1386*a**2*c*d*f**5*h
*x**2 - 1584*a**2*d**2*e**3*f**2*h + 1848*a**2*d**2*e**2*f**3*g + 792*a**2
*d**2*e**2*f**3*h*x - 924*a**2*d**2*e*f**4*g*x - 594*a**2*d**2*e*f**4*h*x*
*2 + 693*a**2*d**2*f**5*g*x**2 + 495*a**2*d**2*f**5*h*x**3 + 3696*a*b*c**2
*e**2*f**3*h - 4620*a*b*c**2*e*f**4*g - 1848*a*b*c**2*e*f**4*h*x + 2310*a*
b*c**2*f**5*g*x + 1386*a*b*c**2*f**5*h*x**2 - 6336*a*b*c*d*e**3*f**2*h + 7
392*a*b*c*d*e**2*f**3*g + 3168*a*b*c*d*e**2*f**3*h*x - 3696*a*b*c*d*e*f**4
*g*x - 2376*a*b*c*d*e*f**4*h*x**2 + 2772*a*b*c*d*f**5*g*x**2 + 1980*a*b*c*
d*f**5*h*x**3 + 2816*a*b*d**2*e**4*f*h - 3168*a*b*d**2*e**3*f**2*g - 1408*
a*b*d**2*e**3*f**2*h*x + 1584*a*b*d**2*e**2*f**3*g*x + 1056*a*b*d**2*e**2*
f**3*h*x**2 - 1188*a*b*d**2*e*f**4*g*x**2 - 880*a*b*d**2*e*f**4*h*x**3 + 9
90*a*b*d**2*f**5*g*x**3 + 770*a*b*d**2*f**5*h*x**4 - 1584*b**2*c**2*e**3*f
**2*h + 1848*b**2*c**2*e**2*f**3*g + 792*b**2*c**2*e**2*f**3*h*x - 924*b**
2*c**2*e*f**4*g*x - 594*b**2*c**2*e*f**4*h*x**2 + 693*b**2*c**2*f**5*g*x**
2 + 495*b**2*c**2*f**5*h*x**3 + 2816*b**2*c*d*e**4*f*h - 3168*b**2*c*d*e**
3*f**2*g - 1408*b**2*c*d*e**3*f**2*h*x + 1584*b**2*c*d*e**2*f**3*g*x + 105
6*b**2*c*d*e**2*f**3*h*x**2 - 1188*b**2*c*d*e*f**4*g*x**2 - 880*b**2*c*d*e
*f**4*h*x**3 + 990*b**2*c*d*f**5*g*x**3 + 770*b**2*c*d*f**5*h*x**4 - 12...
```

3.116 $\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 246

$$\begin{aligned}\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx &= -\frac{2(be-af)(de-cf)^2(fg-eh)\sqrt{e+fx}}{f^5}\\ &+\frac{2(de-cf)(bde(3fg-4eh)-bcf(fg-2eh)-af(2dfg-3deh+cfh))(e+fx)^{3/2}}{3f^5}\\ &+\frac{2(adf(dfg-3deh+2cfh)+b(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)))\,(e+fx)^{5/2}}{5f^5}\\ &+\frac{2d(adfh+b(dfg-4deh+2cfh))(e+fx)^{7/2}}{7f^5}+\frac{2bd^2h(e+fx)^{9/2}}{9f^5}\end{aligned}$$

output
```
-2*(-a*f+b*e)*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(1/2)/f^5+2/3*(-c*f+d*e)*(b*
d*e*(-4*e*h+3*f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-3*d*e*h+2*d*f*g))*(f*x+e)
^(3/2)/f^5+2/5*(a*d*f*(2*c*f*h-3*d*e*h+d*f*g)+b*(c^2*f^2*h+2*c*d*f*(-3*e*h
+f*g)-3*d^2*e*(-2*e*h+f*g)))*(f*x+e)^(5/2)/f^5+2/7*d*(a*d*f*h+b*(2*c*f*h-4
*d*e*h+d*f*g))*(f*x+e)^(7/2)/f^5+2/9*b*d^2*h*(f*x+e)^(9/2)/f^5
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(3af(35c^2f^2(3fg-2eh+fhx)+14cdf(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx))+d^2(-48e^3\ldots}{\ldots}$$

input
```
Integrate[((a + b*x)*(c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(3*a*f*(35*c^2*f^2*(3*f*g - 2*e*h + f*h*x) + 14*c*d*f*(8*
e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d^2*(-48*e^3*h + 8*e^
2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x))) +
 b*(21*c^2*f^2*(8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + 6*c
*d*f*(-48*e^3*h + 8*e^2*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^
2*x*(14*g + 9*h*x)) + d^2*(128*e^4*h + 24*e^2*f^2*x*(3*g + 2*h*x) - 16*e^3
*f*(9*g + 4*h*x) + 5*f^4*x^3*(9*g + 7*h*x) - 2*e*f^3*x^2*(27*g + 20*h*x)))
))/(315*f^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{(e+fx)^{3/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{f^4}+\frac{d(e+fx)^{5/2}(adf\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{5/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{5f^5}+\frac{2d(e+fx)^{7/2}(adfh+b(2cfh-4deh+dfg))}{7f^5}+\frac{2(e+fx)^{3/2}(de-cf)(-af(cfh-3deh+2dfg)-bcf(fg-2eh)+bde(3fg-4eh))}{3f^5}-\frac{2\sqrt{e+fx}(be-af)(de-cf)^2(fg-eh)}{f^5}+\frac{2bd^2h(e+fx)^{9/2}}{9f^5}$$

input
```
Int[((a + b*x)*(c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(-2*(b*e - a*f)*(d*e - c*f)^2*(f*g - e*h)*Sqrt[e + f*x])/f^5 + (2*(d*e - c
*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(2*d*f*g - 3*d*e*h
+ c*f*h))*(e + f*x)^(3/2))/(3*f^5) + (2*(a*d*f*(d*f*g - 3*d*e*h + 2*c*f*h)
 + b*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3*d^2*e*(f*g - 2*e*h)))*(e + f*x
)^(5/2))/(5*f^5) + (2*d*(a*d*f*h + b*(d*f*g - 4*d*e*h + 2*c*f*h))*(e + f*x
)^(7/2))/(7*f^5) + (2*b*d^2*h*(e + f*x)^(9/2))/(9*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2hbd^2(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(\left((af-be)d^2+2bd(cf-de)\right)h+b\,d^2(-eh+fg)\right)(fx+e)^{\frac{7}{2}}}{7}+\frac{2\left(\left(2(af-be)d(cf-de)+b(cf-de)^2\right)h+\left((af-be)d^2+\cdots\right.\right.}{5}$
default	$\frac{2hbd^2(fx+e)^{\frac{9}{2}}}{9}+\frac{2\left(-\left((-af+be)d^2-2bd(cf-de)\right)h-b\,d^2(eh-fg)\right)(fx+e)^{\frac{7}{2}}}{7}+\frac{2\left(-\left(2(-af+be)d(cf-de)-b(cf-de)^2\right)h+\left((-af+\cdots\right.\right.}{5}$
pseudoelliptic	$-\frac{4\sqrt{fx+e}\left(\left(\left(-\frac{3x^2\left(\frac{5bhx^2}{9}+\frac{5(ah+bg)x}{7}+ga\right)d^2}{10}-xc\left(\frac{3bhx^2}{7}+\frac{3(ah+bg)x}{5}+ga\right)d-\frac{3c^2\left(\frac{bhx^2}{5}+\frac{(ah+bg)x}{3}+ga\right)}{2}\right)f^4+e\left(\cdots\right.\right.\right.}{\cdots}$
gosper	$-\frac{2\sqrt{fx+e}\,(-35hbd^2x^4f^4-45ad^2f^4hx^3-90bcdf^4hx^3+40bd^2ef^3hx^3-45bd^2f^4gx^3-126acdf^4hx^2+54ad^2ef^3h\cdots}{\cdots}$
trager	$-\frac{2\sqrt{fx+e}\,(-35hbd^2x^4f^4-45ad^2f^4hx^3-90bcdf^4hx^3+40bd^2ef^3hx^3-45bd^2f^4gx^3-126acdf^4hx^2+54ad^2ef^3h\cdots}{\cdots}$
risch	$-\frac{2\sqrt{fx+e}\,(-35hbd^2x^4f^4-45ad^2f^4hx^3-90bcdf^4hx^3+40bd^2ef^3hx^3-45bd^2f^4gx^3-126acdf^4hx^2+54ad^2ef^3h\cdots}{\cdots}$
orering	$-\frac{2\sqrt{fx+e}\,(-35hbd^2x^4f^4-45ad^2f^4hx^3-90bcdf^4hx^3+40bd^2ef^3hx^3-45bd^2f^4gx^3-126acdf^4hx^2+54ad^2ef^3h\cdots}{\cdots}$

input
```
int((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^5*(1/9*h*b*d^2*(f*x+e)^(9/2)+1/7*(((a*f-b*e)*d^2+2*b*d*(c*f-d*e))*h+b*
d^2*(-e*h+f*g))*(f*x+e)^(7/2)+1/5*((2*(a*f-b*e)*d*(c*f-d*e)+b*(c*f-d*e)^2)
*h+((a*f-b*e)*d^2+2*b*d*(c*f-d*e))*(-e*h+f*g))*(f*x+e)^(5/2)+1/3*((a*f-b*e
)*(c*f-d*e)^2*h+(2*(a*f-b*e)*d*(c*f-d*e)+b*(c*f-d*e)^2)*(-e*h+f*g))*(f*x+e
)^(3/2)+(a*f-b*e)*(c*f-d*e)^2*(-e*h+f*g)*(f*x+e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$=\frac{2\left(35\,bd^2f^4hx^4+5\left(9\,bd^2f^4g-\left(8\,bd^2ef^3-9\left(2\,bcd+ad^2\right)f^4\right)h\right)x^3-3\left(3\left(6\,bd^2ef^3-7\left(2\,bcd+ad^2\right)f^4\right)g\cdots\right.\right.}{\cdots}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b*d^2*f^4*h*x^4 + 5*(9*b*d^2*f^4*g - (8*b*d^2*e*f^3 - 9*(2*b*c*d
 + a*d^2)*f^4)*h)*x^3 - 3*(3*(6*b*d^2*e*f^3 - 7*(2*b*c*d + a*d^2)*f^4)*g -
 (16*b*d^2*e^2*f^2 - 18*(2*b*c*d + a*d^2)*e*f^3 + 21*(b*c^2 + 2*a*c*d)*f^4
)*h)*x^2 - 3*(48*b*d^2*e^3*f - 105*a*c^2*f^4 - 56*(2*b*c*d + a*d^2)*e^2*f^
2 + 70*(b*c^2 + 2*a*c*d)*e*f^3)*g + 2*(64*b*d^2*e^4 - 105*a*c^2*e*f^3 - 72
*(2*b*c*d + a*d^2)*e^3*f + 84*(b*c^2 + 2*a*c*d)*e^2*f^2)*h + (3*(24*b*d^2*
e^2*f^2 - 28*(2*b*c*d + a*d^2)*e*f^3 + 35*(b*c^2 + 2*a*c*d)*f^4)*g - (64*b
*d^2*e^3*f - 105*a*c^2*f^4 - 72*(2*b*c*d + a*d^2)*e^2*f^2 + 84*(b*c^2 + 2*
a*c*d)*e*f^3)*h)*x)*sqrt(f*x + e)/f^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(262) = 524$.

Time = 1.46 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{bd^2h(e+fx)^{\frac{9}{2}}}{9f^4} + \frac{(e+fx)^{\frac{7}{2}}\left(ad^2fh+2bcdfh-4bd^2eh+bd^2fg\right)}{7f^4} + \frac{(e+fx)^{\frac{5}{2}}\cdot\left(2acdf^2h-3ad^2efh+ad^2f^2g+bc^2f^2h-6bcdefh+2bcdf^2g+6bd^2e^2h-3bd^2efg\right)}{5f^4} \right.}{} \\ \frac{ac^2gx+\frac{bd^2hx^5}{5}+\frac{x^4\left(ad^2h+2bcdh+bd^2g\right)}{4}+\frac{x^3\cdot\left(2acdh+ad^2g+bc^2h+2bcdg\right)}{3}+\frac{x^2\left(ac^2h+2acdg+bc^2g\right)}{2}}{\sqrt{e}} \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**2*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b*d**2*h*(e + f*x)**(9/2)/(9*f**4) + (e + f*x)**(7/2)*(a*d**
2*f*h + 2*b*c*d*f*h - 4*b*d**2*e*h + b*d**2*f*g)/(7*f**4) + (e + f*x)**(5/
2)*(2*a*c*d*f**2*h - 3*a*d**2*e*f*h + a*d**2*f**2*g + b*c**2*f**2*h - 6*b*
c*d*e*f*h + 2*b*c*d*f**2*g + 6*b*d**2*e**2*h - 3*b*d**2*e*f*g)/(5*f**4) +
(e + f*x)**(3/2)*(a*c**2*f**3*h - 4*a*c*d*e*f**2*h + 2*a*c*d*f**3*g + 3*a*
d**2*e**2*f*h - 2*a*d**2*e*f**2*g - 2*b*c**2*e*f**2*h + b*c**2*f**3*g + 6*
b*c*d*e**2*f*h - 4*b*c*d*e*f**2*g - 4*b*d**2*e**3*h + 3*b*d**2*e**2*f*g)/(
3*f**4) + sqrt(e + f*x)*(-a*c**2*e*f**3*h + a*c**2*f**4*g + 2*a*c*d*e**2*f
**2*h - 2*a*c*d*e*f**3*g - a*d**2*e**3*f*h + a*d**2*e**2*f**2*g + b*c**2*e
**2*f**2*h - b*c**2*e*f**3*g - 2*b*c*d*e**3*f*h + 2*b*c*d*e**2*f**2*g + b*
d**2*e**4*h - b*d**2*e**3*f*g)/f**4)/f, Ne(f, 0)), ((a*c**2*g*x + b*d**2*h
*x**5/5 + x**4*(a*d**2*h + 2*b*c*d*h + b*d**2*g)/4 + x**3*(2*a*c*d*h + a*d
**2*g + b*c**2*h + 2*b*c*d*g)/3 + x**2*(a*c**2*h + 2*a*c*d*g + b*c**2*g)/2
)/sqrt(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(35\,(fx+e)^{\frac{9}{2}}bd^2h + 45\,(bd^2fg - (4\,bd^2e - (2\,bcd + ad^2)f)h)(fx+e)^{\frac{7}{2}} - 63\,((3\,bd^2ef - (2\,bcd + ad^2)\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(f*x + e)^(9/2)*b*d^2*h + 45*(b*d^2*f*g - (4*b*d^2*e - (2*b*c*d
+ a*d^2)*f)*h)*(f*x + e)^(7/2) - 63*((3*b*d^2*e*f - (2*b*c*d + a*d^2)*f^2)
*g - (6*b*d^2*e^2 - 3*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*h)*(f
*x + e)^(5/2) + 105*((3*b*d^2*e^2*f - 2*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 +
 2*a*c*d)*f^3)*g - (4*b*d^2*e^3 - a*c^2*f^3 - 3*(2*b*c*d + a*d^2)*e^2*f +
2*(b*c^2 + 2*a*c*d)*e*f^2)*h)*(f*x + e)^(3/2) - 315*((b*d^2*e^3*f - a*c^2*
f^4 - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*g - (b*d^2*e^4
- a*c^2*e*f^3 - (2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^2*f^2)*h)*sq
rt(f*x + e))/f^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(228) = 456.

Time = 0.13 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(315\sqrt{fx+e}ac^2g + \frac{105\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)bc^2g}{f} + \frac{210\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)acdg}{f} + \frac{105\left((fx+e)^{\frac{3}{2}}-3\sqrt{fx+e}e\right)ac^2h}{f} + \right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(f*x + e)*a*c^2*g + 105*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*
e)*b*c^2*g/f + 210*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*c*d*g/f + 105*(
(f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*a*c^2*h/f + 42*(3*(f*x + e)^(5/2) - 1
0*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c*d*g/f^2 + 21*(3*(f*x + e)^
(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*d^2*g/f^2 + 21*(3*(
f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*b*c^2*h/f^2
+ 42*(3*(f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*a*c
*d*h/f^2 + 9*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2
)*e^2 - 35*sqrt(f*x + e)*e^3)*b*d^2*g/f^3 + 18*(5*(f*x + e)^(7/2) - 21*(f*
x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*sqrt(f*x + e)*e^3)*b*c*d*h/f^
3 + 9*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 -
 35*sqrt(f*x + e)*e^3)*a*d^2*h/f^3 + (35*(f*x + e)^(9/2) - 180*(f*x + e)^(
7/2)*e + 378*(f*x + e)^(5/2)*e^2 - 420*(f*x + e)^(3/2)*e^3 + 315*sqrt(f*x
+ e)*e^4)*b*d^2*h/f^4)/f
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{(e+f\,x)^{7/2}\,(2\,a\,d^2\,f\,h-8\,b\,d^2\,e\,h+2\,b\,d^2\,f\,g+4\,b\,c\,d\,f\,h)}{7\,f^5}$$
$$+ \frac{(e+f\,x)^{5/2}\,(2\,a\,d^2\,f^2\,g+2\,b\,c^2\,f^2\,h+12\,b\,d^2\,e^2\,h+4\,a\,c\,d\,f^2\,h+4\,b\,c\,d\,f^2\,g-6\,a\,d^2\,e\,f\,h-6\,b\,d^2\,e\,f\,g}{5\,f^5}$$
$$+ \frac{2\,(e+f\,x)^{3/2}\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+b\,c\,f^2\,g+4\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-2\,b\,c\,e\,f\,h-3\,b\,d\,e\,f}{3\,f^5}$$
$$+ \frac{2\,b\,d^2\,h\,(e+f\,x)^{9/2}}{9\,f^5} - \frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{f^5}$$

input
```
int(((g + h*x)*(a + b*x)*(c + d*x)^2)/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(7/2)*(2*a*d^2*f*h - 8*b*d^2*e*h + 2*b*d^2*f*g + 4*b*c*d*f*h))/
(7*f^5) + ((e + f*x)^(5/2)*(2*a*d^2*f^2*g + 2*b*c^2*f^2*h + 12*b*d^2*e^2*h
 + 4*a*c*d*f^2*h + 4*b*c*d*f^2*g - 6*a*d^2*e*f*h - 6*b*d^2*e*f*g - 12*b*c*
d*e*f*h))/(5*f^5) + (2*(e + f*x)^(3/2)*(c*f - d*e)*(a*c*f^2*h + 2*a*d*f^2*
g + b*c*f^2*g + 4*b*d*e^2*h - 3*a*d*e*f*h - 2*b*c*e*f*h - 3*b*d*e*f*g))/(3
*f^5) + (2*b*d^2*h*(e + f*x)^(9/2))/(9*f^5) - (2*(e + f*x)^(1/2)*(a*f - b*
e)*(c*f - d*e)^2*(e*h - f*g))/f^5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{fx+e}\,(35b\,d^2f^4h\,x^4+45a\,d^2f^4h\,x^3+90bcd\,f^4h\,x^3-40b\,d^2e\,f^3h\,x^3+45b\,d^2f^4g\,x^3+126acd\,f^4h\,x^2}{}$$

input
```
int((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 210*a*c**2*e*f**3*h + 315*a*c**2*f**4*g + 105*a*c**2*
f**4*h*x + 336*a*c*d*e**2*f**2*h - 420*a*c*d*e*f**3*g - 168*a*c*d*e*f**3*h
*x + 210*a*c*d*f**4*g*x + 126*a*c*d*f**4*h*x**2 - 144*a*d**2*e**3*f*h + 16
8*a*d**2*e**2*f**2*g + 72*a*d**2*e**2*f**2*h*x - 84*a*d**2*e*f**3*g*x - 54
*a*d**2*e*f**3*h*x**2 + 63*a*d**2*f**4*g*x**2 + 45*a*d**2*f**4*h*x**3 + 16
8*b*c**2*e**2*f**2*h - 210*b*c**2*e*f**3*g - 84*b*c**2*e*f**3*h*x + 105*b*
c**2*f**4*g*x + 63*b*c**2*f**4*h*x**2 - 288*b*c*d*e**3*f*h + 336*b*c*d*e**
2*f**2*g + 144*b*c*d*e**2*f**2*h*x - 168*b*c*d*e*f**3*g*x - 108*b*c*d*e*f*
*3*h*x**2 + 126*b*c*d*f**4*g*x**2 + 90*b*c*d*f**4*h*x**3 + 128*b*d**2*e**4
*h - 144*b*d**2*e**3*f*g - 64*b*d**2*e**3*f*h*x + 72*b*d**2*e**2*f**2*g*x
+ 48*b*d**2*e**2*f**2*h*x**2 - 54*b*d**2*e*f**3*g*x**2 - 40*b*d**2*e*f**3*
h*x**3 + 45*b*d**2*f**4*g*x**3 + 35*b*d**2*f**4*h*x**4))/(315*f**5)
```

3.117 $\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2(de-cf)^2(fg-eh)\sqrt{e+fx}}{f^4} - \frac{2(de-cf)(2dfg-3deh+cfh)(e+fx)^{3/2}}{3f^4} + \frac{2d(dfg-3deh+2cfh)(e+fx)^{5/2}}{5f^4} + \frac{2d^2h(e+fx)^{7/2}}{7f^4}$$

output
```
2*(-c*f+d*e)^2*(-e*h+f*g)*(f*x+e)^(1/2)/f^4-2/3*(-c*f+d*e)*(c*f*h-3*d*e*h+
2*d*f*g)*(f*x+e)^(3/2)/f^4+2/5*d*(2*c*f*h-3*d*e*h+d*f*g)*(f*x+e)^(5/2)/f^4
+2/7*d^2*h*(f*x+e)^(7/2)/f^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\sqrt{e+fx}(35c^2f^2(3fg-2eh+fhx)+14cdf(8e^2h-2ef(5g+2hx)+f^2x(5g+3hx))+d^2(-48e^3h+}{105f^4}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*Sqrt[e + f*x]*(35*c^2*f^2*(3*f*g - 2*e*h + f*h*x) + 14*c*d*f*(8*e^2*h -
 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d^2*(-48*e^3*h + 8*e^2*f*(7*
g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x))))/(105*f^
4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{d(e+fx)^{3/2}(2cfh-3deh+dfg)}{f^3}+\frac{\sqrt{e+fx}(cf-de)(cfh-3deh+2dfg)}{f^3}+\frac{(cf-de)^2(fg-eh)}{f^3\sqrt{e+fx}}+\frac{d^2h(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d(e+fx)^{5/2}(2cfh-3deh+dfg)}{5f^4}-\frac{2(e+fx)^{3/2}(de-cf)(cfh-3deh+2dfg)}{3f^4}+\frac{2\sqrt{e+fx}(de-cf)^2(fg-eh)}{f^4}+\frac{2d^2h(e+fx)^{7/2}}{7f^4}$$

input
```
Int[((c + d*x)^2*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
(2*(d*e - c*f)^2*(f*g - e*h)*Sqrt[e + f*x])/f^4 - (2*(d*e - c*f)*(2*d*f*g
- 3*d*e*h + c*f*h)*(e + f*x)^(3/2))/(3*f^4) + (2*d*(d*f*g - 3*d*e*h + 2*c*
f*h)*(e + f*x)^(5/2))/(5*f^4) + (2*d^2*h*(e + f*x)^(7/2))/(7*f^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{4 ((-\frac{3x^2(\frac{5hx}{7}+g)d^2}{10} - xc(\frac{3hx}{5}+g)d - \frac{3c^2(\frac{hx}{3}+g)}{2}) f^3 + e (\frac{2x(\frac{9hx}{14}+g)d^2}{5} + 2c(\frac{2hx}{5}+g)d + hc^2) f^2 - \frac{8 (\frac{(\frac{3hx}{7}+g)d}{2}}{5}}{3f^4}$
derivativedivides	$\frac{\frac{2d^2h(fx+e)^{\frac{7}{2}}}{7} + \frac{2(2d(cf-de)h+d^2(-eh+fg))(fx+e)^{\frac{5}{2}}}{5} + \frac{2((cf-de)^2h+2d(cf-de)(-eh+fg))(fx+e)^{\frac{3}{2}}}{3} + 2(cf-de)^2(-eh+fg}{f^4}$
default	$\frac{\frac{2d^2h(fx+e)^{\frac{7}{2}}}{7} - \frac{2(-2d(cf-de)h+d^2(eh-fg))(fx+e)^{\frac{5}{2}}}{5} - \frac{2(-(cf-de)^2h+2d(cf-de)(eh-fg))(fx+e)^{\frac{3}{2}}}{3} - 2(cf-de)^2(eh-fg)}{f^4}$
gosper	$-\frac{2\sqrt{fx+e}\,(-15d^2hx^3f^3-42cdf^3hx^2+18d^2ef^2hx^2-21d^2f^3gx^2-35c^2f^3hx+56cdef^2hx-70cdf^3gx-24d^2e^2fhx}{105f^4}$
trager	$-\frac{2\sqrt{fx+e}\,(-15d^2hx^3f^3-42cdf^3hx^2+18d^2ef^2hx^2-21d^2f^3gx^2-35c^2f^3hx+56cdef^2hx-70cdf^3gx-24d^2e^2fhx}{105f^4}$
risch	$-\frac{2\sqrt{fx+e}\,(-15d^2hx^3f^3-42cdf^3hx^2+18d^2ef^2hx^2-21d^2f^3gx^2-35c^2f^3hx+56cdef^2hx-70cdf^3gx-24d^2e^2fhx}{105f^4}$
orering	$-\frac{2\sqrt{fx+e}\,(-15d^2hx^3f^3-42cdf^3hx^2+18d^2ef^2hx^2-21d^2f^3gx^2-35c^2f^3hx+56cdef^2hx-70cdf^3gx-24d^2e^2fhx}{105f^4}$

input
```
int((d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*((-3/10*x^2*(5/7*h*x+g)*d^2-x*c*(3/5*h*x+g)*d-3/2*c^2*(1/3*h*x+g))*f^
3+e*(2/5*x*(9/14*h*x+g)*d^2+2*c*(2/5*h*x+g)*d+h*c^2)*f^2-8/5*(1/2*(3/7*h*x
+g)*d+c*h)*e^2*d*f+24/35*d^2*e^3*h)*(f*x+e)^(1/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\,(15\,d^2f^3hx^3+3\,(7\,d^2f^3g-2\,(3\,d^2ef^2-7\,cdf^3)h)x^2+7\,(8\,d^2e^2f-20\,cdef^2+15\,c^2f^3)g-2\,(24\,d^2e^3-}{105\,f^4}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*d^2*f^3*h*x^3 + 3*(7*d^2*f^3*g - 2*(3*d^2*e*f^2 - 7*c*d*f^3)*h)*
x^2 + 7*(8*d^2*e^2*f - 20*c*d*e*f^2 + 15*c^2*f^3)*g - 2*(24*d^2*e^3 - 56*c
*d*e^2*f + 35*c^2*e*f^2)*h - (14*(2*d^2*e*f^2 - 5*c*d*f^3)*g - (24*d^2*e^2
*f - 56*c*d*e*f^2 + 35*c^2*f^3)*h)*x)*sqrt(f*x + e)/f^4
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{7}{2}}}{7f^3}+\frac{(e+fx)^{\frac{5}{2}}\cdot\left(2cdfh-3d^2eh+d^2fg\right)}{5f^3}+\frac{(e+fx)^{\frac{3}{2}}\left(c^2f^2h-4cdefh+2cdf^2g+3d^2e^2h-2d^2efg\right)}{3f^3}+\frac{\sqrt{e+fx}\left(-c^2ef^2h+c^2f^3g+2cde^2fh-2cdef^2\right.}{f^3}}{f} \\ \frac{c^2gx+\frac{d^2hx^4}{4}+\frac{x^3\cdot\left(2cdh+d^2g\right)}{3}+\frac{x^2\left(c^2h+2cdg\right)}{2}}{\sqrt{e}} \end{cases}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(7/2)/(7*f**3) + (e + f*x)**(5/2)*(2*c*d*f
*h - 3*d**2*e*h + d**2*f*g)/(5*f**3) + (e + f*x)**(3/2)*(c**2*f**2*h - 4*c
*d*e*f*h + 2*c*d*f**2*g + 3*d**2*e**2*h - 2*d**2*e*f*g)/(3*f**3) + sqrt(e
+ f*x)*(-c**2*e*f**2*h + c**2*f**3*g + 2*c*d*e**2*f*h - 2*c*d*e*f**2*g - d
**2*e**3*h + d**2*e**2*f*g)/f**3)/f, Ne(f, 0)), ((c**2*g*x + d**2*h*x**4/4
 + x**3*(2*c*d*h + d**2*g)/3 + x**2*(c**2*h + 2*c*d*g)/2)/sqrt(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(15\,(fx+e)^{\frac{7}{2}}d^2h+21\,(d^2fg-(3\,d^2e-2\,cdf)h)(fx+e)^{\frac{5}{2}}-35\,(2\,(d^2ef-cdf^2)g-(3\,d^2e^2-4\,cdef\right.}{105\,f^4}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(f*x + e)^(7/2)*d^2*h + 21*(d^2*f*g - (3*d^2*e - 2*c*d*f)*h)*(f*
x + e)^(5/2) - 35*(2*(d^2*e*f - c*d*f^2)*g - (3*d^2*e^2 - 4*c*d*e*f + c^2*
f^2)*h)*(f*x + e)^(3/2) + 105*((d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*g - (d^
2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*h)*sqrt(f*x + e))/f^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.66

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(105\,\sqrt{fx+e}c^2g+\frac{70\left((fx+e)^{\frac{3}{2}}-3\,\sqrt{fx+e}e\right)cdg}{f}+\frac{35\left((fx+e)^{\frac{3}{2}}-3\,\sqrt{fx+e}e\right)c^2h}{f}+\frac{7\left(3\,(fx+e)^{\frac{5}{2}}-10\,(fx+e)^{\frac{3}{2}}e+15\,\sqrt{fx+ee}\right.}{f^2}\right.}{105\,f}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(f*x + e)*c^2*g + 70*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*
c*d*g/f + 35*((f*x + e)^(3/2) - 3*sqrt(f*x + e)*e)*c^2*h/f + 7*(3*(f*x + e
)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*d^2*g/f^2 + 14*(3*(
f*x + e)^(5/2) - 10*(f*x + e)^(3/2)*e + 15*sqrt(f*x + e)*e^2)*c*d*h/f^2 +
3*(5*(f*x + e)^(7/2) - 21*(f*x + e)^(5/2)*e + 35*(f*x + e)^(3/2)*e^2 - 35*
sqrt(f*x + e)*e^3)*d^2*h/f^3)/f
```

Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \frac{(e+f\,x)^{5/2}\,(2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h)}{5\,f^4} + \frac{2\,(e+f\,x)^{3/2}\,(c\,f-d\,e)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g)}{3\,f^4} - \frac{2\,\sqrt{e+f\,x}\,(c\,f-d\,e)^2\,(e\,h-f\,g)}{f^4} + \frac{2\,d^2\,h\,(e+f\,x)^{7/2}}{7\,f^4}$$

input
```
int(((g + h*x)*(c + d*x)^2)/(e + f*x)^(1/2),x)
```

output
```
((e + f*x)^(5/2)*(2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h))/(5*f^4) + (2*(e + f*
x)^(3/2)*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(3*f^4) - (2*(e + f*x)^(
1/2)*(c*f - d*e)^2*(e*h - f*g))/f^4 + (2*d^2*h*(e + f*x)^(7/2))/(7*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2(g+hx)}{\sqrt{e+fx}}\,dx = \frac{2\sqrt{fx+e}\,(15d^2f^3h\,x^3+42cd\,f^3h\,x^2-18d^2e\,f^2h\,x^2+21d^2f^3g\,x^2+35c^2f^3hx-56cde\,f^2hx+70cd\,f^3g}{10}$$

input
```
int((d*x+c)^2*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*( - 70*c**2*e*f**2*h + 105*c**2*f**3*g + 35*c**2*f**3*h*x
 + 112*c*d*e**2*f*h - 140*c*d*e*f**2*g - 56*c*d*e*f**2*h*x + 70*c*d*f**3*g
*x + 42*c*d*f**3*h*x**2 - 48*d**2*e**3*h + 56*d**2*e**2*f*g + 24*d**2*e**2
*f*h*x - 28*d**2*e*f**2*g*x - 18*d**2*e*f**2*h*x**2 + 21*d**2*f**3*g*x**2
+ 15*d**2*f**3*h*x**3))/(105*f**4)
```

3.118 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 223

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{2(a^2d^2f^2h-abdf(dfg-deh+2cfh)+b^2(c^2f^2h-d^2e(fg-eh)+2cdf(fg-eh)))\sqrt{e+fx}}{b^3f^3} - \frac{2d(adfh-b(dfg-2deh+2cfh))(e+fx)^{3/2}}{3b^2f^3} + \frac{2d^2h(e+fx)^{5/2}}{5bf^3} - \frac{2(bc-ad)^2(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{7/2}\sqrt{be-af}}$$

output
```
2*(a^2*d^2*f^2*h-a*b*d*f*(2*c*f*h-d*e*h+d*f*g)+b^2*(c^2*f^2*h-d^2*e*(-e*h+
f*g)+2*c*d*f*(-e*h+f*g)))*(f*x+e)^(1/2)/b^3/f^3-2/3*d*(a*d*f*h-b*(2*c*f*h-
2*d*e*h+d*f*g))*(f*x+e)^(3/2)/b^2/f^3+2/5*d^2*h*(f*x+e)^(5/2)/b/f^3-2*(-a*
d+b*c)^2*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(7/2
)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(15a^2d^2f^2h - 5abdf(6cfh + d(3fg-2eh+fhx)) + b^2(15c^2f^2h + 10cdf(3fg-2eh+fhx) +}{15b^3f^3}$$
$$+ \frac{2(bc-ad)^2(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{7/2}\sqrt{-be+af}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*(15*a^2*d^2*f^2*h - 5*a*b*d*f*(6*c*f*h + d*(3*f*g - 2*e*h
 + f*h*x)) + b^2*(15*c^2*f^2*h + 10*c*d*f*(3*f*g - 2*e*h + f*h*x) + d^2*(8
*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)))))/(15*b^3*f^3) + (2*(
b*c - a*d)^2*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]
])/(b^(7/2)*Sqrt[-(b*e) + a*f])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{170}$$
$$\frac{2\int \frac{(c+dx)(5bcfg-a(4de+cf)h-(5adfh-b(5dfg-4deh+4cfh))x)}{2(a+bx)\sqrt{e+fx}}dx}{5bf} + \frac{2h(c+dx)^2\sqrt{e+fx}}{5bf}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)(5bcfg-a(4de+cf)h-(5adfh-b(5dfg-4deh+4cfh))x)}{(a+bx)\sqrt{e+fx}}dx}{5bf}+\frac{2h(c+dx)^2\sqrt{e+fx}}{5bf}$$

$\Big\downarrow$ 164

$$\frac{\frac{5f(bc-ad)^2(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b^2}+\frac{2\sqrt{e+fx}(15a^2d^2f^2h-bdfx(5adfh-b(4cfh-4deh+5dfg))-5abdf(6cfh-2deh+3dfg)+2b^2(6c^2f^2h+}{3b^2f^2}}{5bf}$$
$$\frac{2h(c+dx)^2\sqrt{e+fx}}{5bf}$$

$\Big\downarrow$ 73

$$\frac{\frac{10(bc-ad)^2(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b^2}+\frac{2\sqrt{e+fx}(15a^2d^2f^2h-bdfx(5adfh-b(4cfh-4deh+5dfg))-5abdf(6cfh-2deh+3dfg)+2b^2(6}{3b^2f^2}}{5bf}$$
$$\frac{2h(c+dx)^2\sqrt{e+fx}}{5bf}$$

$\Big\downarrow$ 221

$$\frac{\frac{2\sqrt{e+fx}(15a^2d^2f^2h-bdfx(5adfh-b(4cfh-4deh+5dfg))-5abdf(6cfh-2deh+3dfg)+2b^2(6c^2f^2h+5cdf(3fg-2eh)+d^2(-e)(5fg-4eh)))}{3b^2f^2}-\frac{10}{}}{5bf}$$
$$\frac{2h(c+dx)^2\sqrt{e+fx}}{5bf}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
(2*h*(c + d*x)^2*Sqrt[e + f*x])/(5*b*f) + ((2*Sqrt[e + f*x]*(15*a^2*d^2*f^
2*h - 5*a*b*d*f*(3*d*f*g - 2*d*e*h + 6*c*f*h) + 2*b^2*(6*c^2*f^2*h - d^2*e
*(5*f*g - 4*e*h) + 5*c*d*f*(3*f*g - 2*e*h)) - b*d*f*(5*a*d*f*h - b*(5*d*f*
g - 4*d*e*h + 4*c*f*h))*x))/(3*b^2*f^2) - (10*(b*c - a*d)^2*f*(b*g - a*h)*
ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(5/2)*Sqrt[b*e - a*f]
))/(5*b*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{2\left(f^{3}(ad-bc)^{2}(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)-\left(\left(\left(\left(\frac{x\left(\frac{3hx}{5}+g\right)d^{2}}{3}+2c\left(\frac{hx}{3}+g\right)d+h\,c^{2}\right)b^{2}-2ad\left(\left(\frac{hx}{6}+\frac{g}{2}\right)d+ch\right)b+\right.\right.\right.\right.}{\sqrt{(af-be)b}\,f^{3}b^{3}}$
risch	$\frac{2(3x^{2}h\,b^{2}d^{2}f^{2}-5ab\,d^{2}f^{2}hx+10b^{2}cd\,f^{2}hx-4b^{2}d^{2}efhx+5b^{2}d^{2}f^{2}gx+15a^{2}d^{2}f^{2}h-30abcd\,f^{2}h+10ab\,d^{2}efh-15ab\,d^{2}f}{15f^{3}b^{3}}$
derivativedivides	$\frac{2\left(\frac{d^{2}h(fx+e)^{\frac{5}{2}}b^{2}}{5}-\frac{ab\,d^{2}fh(fx+e)^{\frac{3}{2}}}{3}+\frac{2b^{2}cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^{2}d^{2}eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^{2}d^{2}fg(fx+e)^{\frac{3}{2}}}{3}+a^{2}d^{2}f^{2}h\sqrt{fx+e}-2abcd\,f^{2}h\sqrt{f}\right.}{}$
default	$\frac{2\left(\frac{d^{2}h(fx+e)^{\frac{5}{2}}b^{2}}{5}-\frac{ab\,d^{2}fh(fx+e)^{\frac{3}{2}}}{3}+\frac{2b^{2}cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^{2}d^{2}eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^{2}d^{2}fg(fx+e)^{\frac{3}{2}}}{3}+a^{2}d^{2}f^{2}h\sqrt{fx+e}-2abcd\,f^{2}h\sqrt{f}\right.}{}$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/((a*f-b*e)*b)^(1/2)*(f^3*(a*d-b*c)^2*(a*h-b*g)*arctan(b*(f*x+e)^(1/2)/(
(a*f-b*e)*b)^(1/2))-(((1/3*x*(3/5*h*x+g)*d^2+2*c*(1/3*h*x+g)*d+h*c^2)*b^2-
2*a*d*((1/6*h*x+1/2*g)*d+c*h)*b+a^2*d^2*h)*f^2+2/3*d*(((-2/5*h*x-g)*d-2*c*
h)*b+a*d*h)*b*e*f+8/15*b^2*d^2*e^2*h)*(f*x+e)^(1/2)*((a*f-b*e)*b)^(1/2))/f
^3/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(203) = 406.

Time = 0.12 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.91

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^3*g - (a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2)*f^3*h)*sqrt(b^2*e - a*b*f)*log((b*f*x + 2*b*e - a*f + 2*
sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*(3*(b^4*d^2*e*f^2 - a*b^
3*d^2*f^3)*h*x^2 - 5*(2*b^4*d^2*e^2*f - (6*b^4*c*d - a*b^3*d^2)*e*f^2 + 3*
(2*a*b^3*c*d - a^2*b^2*d^2)*f^3)*g + (8*b^4*d^2*e^3 - 2*(10*b^4*c*d - a*b^
3*d^2)*e^2*f + 5*(3*b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^2 - 15*(a*b^3
*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*f^3)*h + (5*(b^4*d^2*e*f^2 - a*b^3*d^2*f
^3)*g - (4*b^4*d^2*e^2*f - (10*b^4*c*d - a*b^3*d^2)*e*f^2 + 5*(2*a*b^3*c*d
 - a^2*b^2*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(b^5*e*f^3 - a*b^4*f^4), 2/15*(1
5*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^3*g - (a*b^2*c^2 - 2*a^2*b*c*d +
a^3*d^2)*f^3*h)*sqrt(-b^2*e + a*b*f)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x
+ e)/(b*f*x + b*e)) + (3*(b^4*d^2*e*f^2 - a*b^3*d^2*f^3)*h*x^2 - 5*(2*b^4*
d^2*e^2*f - (6*b^4*c*d - a*b^3*d^2)*e*f^2 + 3*(2*a*b^3*c*d - a^2*b^2*d^2)*
f^3)*g + (8*b^4*d^2*e^3 - 2*(10*b^4*c*d - a*b^3*d^2)*e^2*f + 5*(3*b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^2 - 15*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b
*d^2)*f^3)*h + (5*(b^4*d^2*e*f^2 - a*b^3*d^2*f^3)*g - (4*b^4*d^2*e^2*f - (
10*b^4*c*d - a*b^3*d^2)*e*f^2 + 5*(2*a*b^3*c*d - a^2*b^2*d^2)*f^3)*h)*x)*s
qrt(f*x + e))/(b^5*e*f^3 - a*b^4*f^4)]
```

Sympy [A] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{5}{2}}}{5bf^2} + \frac{(e+fx)^{\frac{3}{2}}\left(-ad^2fh+2bcdfh-2bd^2eh+bd^2fg\right)}{3b^2f^2} + \frac{\sqrt{e+fx}\left(a^2d^2f^2h-2abcdf^2h+abd^2efh-abd^2f^2g+b^2c^2f^2h-2b^2cdefh+2b^2cdf^2g+b^2d^2 \cdots\right)}{b^3f^2} \cdots\right)}{f} \\ \frac{\frac{d^2hx^3}{3b} + \frac{x^2\left(-ad^2h+2bcdh+bd^2g\right)}{2b^2} + \frac{x\left(a^2d^2h-2abcdh-abd^2g+b^2c^2h+2b^2cdg\right)}{b^3} - \frac{(ad-bc)^2(ah-bg)\left(\begin{cases} \frac{x}{a} & \text{for}\: b = 0 \\ \frac{\log{(a+bx)}}{b} & \text{otherwise} \end{cases}\right)}{b^3}}{\sqrt{e}} \end{cases}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(5/2)/(5*b*f**2) + (e + f*x)**(3/2)*(-a*d*
*2*f*h + 2*b*c*d*f*h - 2*b*d**2*e*h + b*d**2*f*g)/(3*b**2*f**2) + sqrt(e +
 f*x)*(a**2*d**2*f**2*h - 2*a*b*c*d*f**2*h + a*b*d**2*e*f*h - a*b*d**2*f**
2*g + b**2*c**2*f**2*h - 2*b**2*c*d*e*f*h + 2*b**2*c*d*f**2*g + b**2*d**2*
e**2*h - b**2*d**2*e*f*g)/(b**3*f**2) - f*(a*d - b*c)**2*(a*h - b*g)*atan(
sqrt(e + f*x)/sqrt((a*f - b*e)/b))/(b**4*sqrt((a*f - b*e)/b)))/f, Ne(f, 0)
), ((d**2*h*x**3/(3*b) + x**2*(-a*d**2*h + 2*b*c*d*h + b*d**2*g)/(2*b**2)
+ x*(a**2*d**2*h - 2*a*b*c*d*h - a*b*d**2*g + b**2*c**2*h + 2*b**2*c*d*g)/
b**3 - (a*d - b*c)**2*(a*h - b*g)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)
/b, True))/b**3)/sqrt(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$= \frac{2\left(b^3c^2g - 2\,ab^2cdg + a^2bd^2g - ab^2c^2h + 2\,a^2bcdh - a^3d^2h\right)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{\sqrt{-b^2e+abf}\,b^3}$$

$$+ \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^4d^2f^{13}g - 15\,\sqrt{fx+e}b^4d^2ef^{13}g + 30\,\sqrt{fx+e}b^4cdf^{14}g - 15\,\sqrt{fx+e}ab^3d^2f^{14}g + 3\,(f\right.}{}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(b^3*c^2*g - 2*a*b^2*c*d*g + a^2*b*d^2*g - a*b^2*c^2*h + 2*a^2*b*c*d*h -
 a^3*d^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e + a*
b*f)*b^3) + 2/15*(5*(f*x + e)^(3/2)*b^4*d^2*f^13*g - 15*sqrt(f*x + e)*b^4*
d^2*e*f^13*g + 30*sqrt(f*x + e)*b^4*c*d*f^14*g - 15*sqrt(f*x + e)*a*b^3*d^
2*f^14*g + 3*(f*x + e)^(5/2)*b^4*d^2*f^12*h - 10*(f*x + e)^(3/2)*b^4*d^2*e
*f^12*h + 15*sqrt(f*x + e)*b^4*d^2*e^2*f^12*h + 10*(f*x + e)^(3/2)*b^4*c*d
*f^13*h - 5*(f*x + e)^(3/2)*a*b^3*d^2*f^13*h - 30*sqrt(f*x + e)*b^4*c*d*e*
f^13*h + 15*sqrt(f*x + e)*a*b^3*d^2*e*f^13*h + 15*sqrt(f*x + e)*b^4*c^2*f^
14*h - 30*sqrt(f*x + e)*a*b^3*c*d*f^14*h + 15*sqrt(f*x + e)*a^2*b^2*d^2*f^
14*h)/(b^5*f^15)
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$= (e+f\,x)^{3/2}\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{3\,b\,f^3}-\frac{2\,d^2\,h\,\left(a\,f^4-b\,e\,f^3\right)}{3\,b^2\,f^6}\right)$$
$$-\sqrt{e+f\,x}\left(\frac{\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b\,f^3}-\frac{2\,d^2\,h\,\left(a\,f^4-b\,e\,f^3\right)}{b^2\,f^6}\right)\,\left(a\,f^4-b\,e\,f^3\right)}{b\,f^3}\right.$$
$$\left.-\frac{2\,\left(c\,f-d\,e\right)\,\left(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g\right)}{b\,f^3}\right)+\frac{2\,d^2\,h\,{\left(e+f\,x\right)}^{5/2}}{5\,b\,f^3}$$
$$+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,{\left(a\,d-b\,c\right)}^2\,\left(a\,h-b\,g\right)}{\sqrt{a\,f-b\,e}\,\left(-h\,a^3\,d^2+2\,h\,a^2\,b\,c\,d+g\,a^2\,b\,d^2-h\,a\,b^2\,c^2-2\,g\,a\,b^2\,c\,d+g\,b^3\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(a\,h-b\,g\right)}{b^{7/2}\,\sqrt{a\,f-b\,e}}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(1/2)*(a + b*x)),x)
```

output
```
(e + f*x)^(3/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(3*b*f^3) - (2*d^2*h*
(a*f^4 - b*e*f^3))/(3*b^2*f^6)) - (e + f*x)^(1/2)*((((2*d^2*f*g - 6*d^2*e*
h + 4*c*d*f*h)/(b*f^3) - (2*d^2*h*(a*f^4 - b*e*f^3))/(b^2*f^6))*(a*f^4 - b
*e*f^3))/(b*f^3) - (2*(c*f - d*e)*(c*f*h - 3*d*e*h + 2*d*f*g))/(b*f^3)) +
(2*d^2*h*(e + f*x)^(5/2))/(5*b*f^3) + (2*atan((b^(1/2)*(e + f*x)^(1/2)*(a*
d - b*c)^2*(a*h - b*g))/((a*f - b*e)^(1/2)*(b^3*c^2*g - a^3*d^2*h - a*b^2*
c^2*h + a^2*b*d^2*g - 2*a*b^2*c*d*g + 2*a^2*b*c*d*h)))*(a*d - b*c)^2*(a*h
- b*g))/(b^(7/2)*(a*f - b*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.39

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(1/2),x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**3*d**2*f**3*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d*f**3*h + 15*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**2*f**3*g
- 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a*b**2*c**2*f**3*h - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d*f**3*g + 15*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c**2*f**3*g + 15*
sqrt(e + f*x)*a**3*b*d**2*f**3*h - 30*sqrt(e + f*x)*a**2*b**2*c*d*f**3*h -
 5*sqrt(e + f*x)*a**2*b**2*d**2*e*f**2*h - 15*sqrt(e + f*x)*a**2*b**2*d**2
*f**3*g - 5*sqrt(e + f*x)*a**2*b**2*d**2*f**3*h*x + 15*sqrt(e + f*x)*a*b**
3*c**2*f**3*h + 10*sqrt(e + f*x)*a*b**3*c*d*e*f**2*h + 30*sqrt(e + f*x)*a*
b**3*c*d*f**3*g + 10*sqrt(e + f*x)*a*b**3*c*d*f**3*h*x - 2*sqrt(e + f*x)*a
*b**3*d**2*e**2*f*h + 5*sqrt(e + f*x)*a*b**3*d**2*e*f**2*g + sqrt(e + f*x)
*a*b**3*d**2*e*f**2*h*x + 5*sqrt(e + f*x)*a*b**3*d**2*f**3*g*x + 3*sqrt(e
+ f*x)*a*b**3*d**2*f**3*h*x**2 - 15*sqrt(e + f*x)*b**4*c**2*e*f**2*h + 20*
sqrt(e + f*x)*b**4*c*d*e**2*f*h - 30*sqrt(e + f*x)*b**4*c*d*e*f**2*g - 10*
sqrt(e + f*x)*b**4*c*d*e*f**2*h*x - 8*sqrt(e + f*x)*b**4*d**2*e**3*h + 10*
sqrt(e + f*x)*b**4*d**2*e**2*f*g + 4*sqrt(e + f*x)*b**4*d**2*e**2*f*h*x -
5*sqrt(e + f*x)*b**4*d**2*e*f**2*g*x - 3*sqrt(e + f*x)*b**4*d**2*e*f**2...
```

3.119 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 219

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = -\frac{2d(2adfh-b(dfg-deh+2cfh))\sqrt{e+fx}}{b^3f^2} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{b^3(be-af)(a+bx)} + \frac{2d^2h(e+fx)^{3/2}}{3b^2f^2} - \frac{(bc-ad)\left(5a^2dfh+b^2(4deg-cfg+2ceh)-ab(3dfg+6deh+cfh)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{7/2}(be-af)^{3/2}}$$

output

```
-2*d*(2*a*d*f*h-b*(2*c*f*h-d*e*h+d*f*g))*(f*x+e)^(1/2)/b^3/f^2-(-a*d+b*c)^
2*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/(-a*f+b*e)/(b*x+a)+2/3*d^2*h*(f*x+e)^(3/2)/
b^2/f^2-(-a*d+b*c)*(5*a^2*d*f*h+b^2*(2*c*e*h-c*f*g+4*d*e*g)-a*b*(c*f*h+6*d
*e*h+3*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(7/2)/(-a
*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = -\frac{\sqrt{e+fx}(-15a^3d^2f^2h+a^2bdf(18cfh+d(9fg+8eh-10fhx))+b^3(3c^2f^2g-12cdefhx-2d^2ex(3f}{3b^3f^2(be} - \frac{(bc-ad)\left(5a^2dfh+b^2(4deg-cfg+2ceh)-ab(3dfg+6deh+cfh)\right)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{7/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output
```
-1/3*(Sqrt[e + f*x]*(-15*a^3*d^2*f^2*h + a^2*b*d*f*(18*c*f*h + d*(9*f*g +
8*e*h - 10*f*h*x)) + b^3*(3*c^2*f^2*g - 12*c*d*e*f*h*x - 2*d^2*e*x*(3*f*g
- 2*e*h + f*h*x)) + a*b^2*(-3*c^2*f^2*h - 6*c*d*f*(2*e*h + f*(g - 2*h*x))
+ 2*d^2*(2*e^2*h - 3*e*f*(g - h*x) + f^2*x*(3*g + h*x)))))/(b^3*f^2*(b*e -
 a*f)*(a + b*x)) - ((b*c - a*d)*(5*a^2*d*f*h + b^2*(4*d*e*g - c*f*g + 2*c*
e*h) - a*b*(3*d*f*g + 6*d*e*h + c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqr
t[-(b*e) + a*f]])/(b^(7/2)*(-(b*e) + a*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 25, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(c+dx)((4de+cf)(bg-ah)-2bc(fg-eh)+d(3bfg+2beh-5afh)x)}{2(a+bx)\sqrt{e+fx}}dx}{b(be-af)} - \frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{(c+dx)(a(4de+cf)h-b(4deg-cfg+2ceh)-d(3bfg+2beh-5afh)x)}{(a+bx)\sqrt{e+fx}}dx}{2b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{(c+dx)(a(4de+cf)h-b(4deg-cfg+2ceh)-d(3bfg+2beh-5afh)x)}{(a+bx)\sqrt{e+fx}}dx}{2b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{164}$$

$$-\frac{-\frac{(bc-ad)\left(5a^2dfh-ab(cfh+6deh+3dfg)+b^2(2ceh-cfg+4deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b^2}-\frac{2d\sqrt{e+fx}(15a^2df^2h-abf(18cfh+8deh+9dfg)+bdfx(}{3}}{2b(be-af)}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{2(bc-ad)\left(5a^2dfh-ab(cfh+6deh+3dfg)+b^2(2ceh-cfg+4deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b^2f}-\frac{2d\sqrt{e+fx}(15a^2df^2h-abf(18cfh+8deh+9df}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(5a^2dfh-ab(cfh+6deh+3dfg)+b^2(2ceh-cfg+4deg)\right)}{b^{5/2}\sqrt{be-af}}-\frac{2d\sqrt{e+fx}(15a^2df^2h-abf(18cfh+8deh+9dfg)+bc}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{b(a+bx)(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output
```
-(((b*g - a*h)*(c + d*x)^2*Sqrt[e + f*x])/(b*(b*e - a*f)*(a + b*x))) - ((-
2*d*Sqrt[e + f*x]*(15*a^2*d*f^2*h - a*b*f*(9*d*f*g + 8*d*e*h + 18*c*f*h) +
 2*b^2*(d*e*(3*f*g - 2*e*h) + 3*c*f*(f*g + 2*e*h)) + b*d*f*(3*b*f*g + 2*b*
e*h - 5*a*f*h)*x))/(3*b^2*f^2) + (2*(b*c - a*d)*(5*a^2*d*f*h + b^2*(4*d*e*
g - c*f*g + 2*c*e*h) - a*b*(3*d*f*g + 6*d*e*h + c*f*h))*ArcTanh[(Sqrt[b]*S
qrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(5/2)*Sqrt[b*e - a*f]))/(2*b*(b*e - a*f
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{2d(-hfdbx+6adfh-6bcfh+2bdeh-3bdfg)\sqrt{fx+e}}{3f^2b^3}+\frac{(2ad-2bc)\left(-\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(5a^2dfh-a\cdots\right.}{\cdots}\right.}{\cdots}$
pseudoelliptic	$-\frac{5\left(-\left(\frac{(-cfg+2e(ch+2dg))b^2}{5}-\frac{a((ch+3dg)f+6deh)b}{5}+a^2dfh\right)(ad-bc)(bx+a)f^2\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\left(\frac{\left(-gf^2c^2+4\cdots\right.}{\cdots}\right.\right.}{\cdots}$
derivativedivides	$\frac{-\frac{2d\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}+bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^3}+\frac{2f^2\left(-\frac{f\left(hd^2a^3-2a^2bcdh-a^2bd^2g+ab^2c^2h+\cdots\right.}{2(af-be)((fx+e)b+\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{2d\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}+bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^3}+\frac{2f^2\left(-\frac{f\left(hd^2a^3-2a^2bcdh-a^2bd^2g+ab^2c^2h+\cdots\right.}{2(af-be)((fx+e)b+\cdots}\right.}{\cdots}}{\cdots}$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d*(-b*d*f*h*x+6*a*d*f*h-6*b*c*f*h+2*b*d*e*h-3*b*d*f*g)*(f*x+e)^(1/2)/
f^2/b^3+1/b^3*(2*a*d-2*b*c)*(-1/2*f*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/(a*f
-b*e)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)+1/2*(5*a^2*d*f*h-a*b*c*f*h-6*a*b*d
*e*h-3*a*b*d*f*g+2*b^2*c*e*h-b^2*c*f*g+4*b^2*d*e*g)/(a*f-b*e)/((a*f-b*e)*b
)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(201) = 402$.

Time = 0.13 (sec) , antiderivative size = 1546, normalized size of antiderivative = 7.06

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(b^2*e - a*b*f)*((4*(a*b^3*c*d - a^2*b^2*d^2)*e*f^2 - (a*b^3*c
^2 + 2*a^2*b^2*c*d - 3*a^3*b*d^2)*f^3)*g + (2*(a*b^3*c^2 - 4*a^2*b^2*c*d +
 3*a^3*b*d^2)*e*f^2 - (a^2*b^2*c^2 - 6*a^3*b*c*d + 5*a^4*d^2)*f^3)*h + ((4
*(b^4*c*d - a*b^3*d^2)*e*f^2 - (b^4*c^2 + 2*a*b^3*c*d - 3*a^2*b^2*d^2)*f^3
)*g + (2*(b^4*c^2 - 4*a*b^3*c*d + 3*a^2*b^2*d^2)*e*f^2 - (a*b^3*c^2 - 6*a^
2*b^2*c*d + 5*a^3*b*d^2)*f^3)*h)*x)*log((b*f*x + 2*b*e - a*f - 2*sqrt(b^2*
e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*(2*(b^5*d^2*e^2*f - 2*a*b^4*d^2*e
*f^2 + a^2*b^3*d^2*f^3)*h*x^2 + 3*(2*a*b^4*d^2*e^2*f - (b^5*c^2 - 2*a*b^4*
c*d + 5*a^2*b^3*d^2)*e*f^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + 3*a^3*b^2*d^2)*f
^3)*g - (4*a*b^4*d^2*e^3 - 4*(3*a*b^4*c*d - a^2*b^3*d^2)*e^2*f - (3*a*b^4*
c^2 - 30*a^2*b^3*c*d + 23*a^3*b^2*d^2)*e*f^2 + 3*(a^2*b^3*c^2 - 6*a^3*b^2*
c*d + 5*a^4*b*d^2)*f^3)*h + 2*(3*(b^5*d^2*e^2*f - 2*a*b^4*d^2*e*f^2 + a^2*
b^3*d^2*f^3)*g - (2*b^5*d^2*e^3 - (6*b^5*c*d - a*b^4*d^2)*e^2*f + 4*(3*a*b
^4*c*d - 2*a^2*b^3*d^2)*e*f^2 - (6*a^2*b^3*c*d - 5*a^3*b^2*d^2)*f^3)*h)*x)
*sqrt(f*x + e))/(a*b^6*e^2*f^2 - 2*a^2*b^5*e*f^3 + a^3*b^4*f^4 + (b^7*e^2*
f^2 - 2*a*b^6*e*f^3 + a^2*b^5*f^4)*x), 1/3*(3*sqrt(-b^2*e + a*b*f)*((4*(a*
b^3*c*d - a^2*b^2*d^2)*e*f^2 - (a*b^3*c^2 + 2*a^2*b^2*c*d - 3*a^3*b*d^2)*f
^3)*g + (2*(a*b^3*c^2 - 4*a^2*b^2*c*d + 3*a^3*b*d^2)*e*f^2 - (a^2*b^2*c^2
- 6*a^3*b*c*d + 5*a^4*d^2)*f^3)*h + ((4*(b^4*c*d - a*b^3*d^2)*e*f^2 - (b^4
*c^2 + 2*a*b^3*c*d - 3*a^2*b^2*d^2)*f^3)*g + (2*(b^4*c^2 - 4*a*b^3*c*d ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(201) = 402$.

Time = 0.13 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.86

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{(4\,b^3cdeg - 4\,ab^2d^2eg - b^3c^2fg - 2\,ab^2cdfg + 3\,a^2bd^2fg + 2\,b^3c^2eh - 8\,ab^2cdeh + 6\,a^2bd^2eh - ab^2c^2fh +}{(b^4e - ab^3f)\sqrt{-b^2e + abf}}$$
$$- \frac{\sqrt{fx+e}b^3c^2fg - 2\,\sqrt{fx+e}ab^2cdfg + \sqrt{fx+e}a^2bd^2fg - \sqrt{fx+e}ab^2c^2fh + 2\,\sqrt{fx+e}a^2bcdfh - \sqrt{}}{(b^4e - ab^3f)((fx+e)b - be + af)}$$
$$+ \frac{2\left(3\,\sqrt{fx+e}b^4d^2f^5g + (fx+e)^{\frac{3}{2}}b^4d^2f^4h - 3\,\sqrt{fx+e}b^4d^2ef^4h + 6\,\sqrt{fx+e}b^4cdf^5h - 6\,\sqrt{fx+e}ab^3\right.}{3\,b^6f^6}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
(4*b^3*c*d*e*g - 4*a*b^2*d^2*e*g - b^3*c^2*f*g - 2*a*b^2*c*d*f*g + 3*a^2*b
*d^2*f*g + 2*b^3*c^2*e*h - 8*a*b^2*c*d*e*h + 6*a^2*b*d^2*e*h - a*b^2*c^2*f
*h + 6*a^2*b*c*d*f*h - 5*a^3*d^2*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e +
 a*b*f))/((b^4*e - a*b^3*f)*sqrt(-b^2*e + a*b*f)) - (sqrt(f*x + e)*b^3*c^2
*f*g - 2*sqrt(f*x + e)*a*b^2*c*d*f*g + sqrt(f*x + e)*a^2*b*d^2*f*g - sqrt(
f*x + e)*a*b^2*c^2*f*h + 2*sqrt(f*x + e)*a^2*b*c*d*f*h - sqrt(f*x + e)*a^3
*d^2*f*h)/((b^4*e - a*b^3*f)*((f*x + e)*b - b*e + a*f)) + 2/3*(3*sqrt(f*x
+ e)*b^4*d^2*f^5*g + (f*x + e)^(3/2)*b^4*d^2*f^4*h - 3*sqrt(f*x + e)*b^4*d
^2*e*f^4*h + 6*sqrt(f*x + e)*b^4*c*d*f^5*h - 6*sqrt(f*x + e)*a*b^3*d^2*f^5
*h)/(b^6*f^6)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.05

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,d^2\,f\,g - 6\,d^2\,e\,h + 4\,c\,d\,f\,h}{b^2\,f^2} - \frac{4\,d^2\,h\,(a\,f-b\,e)}{b^3\,f^2}\right)$$
$$+ \frac{\sqrt{e+f\,x}\,(-f\,h\,a^3\,d^2 + 2\,f\,h\,a^2\,b\,c\,d + f\,g\,a^2\,b\,d^2 - f\,h\,a\,b^2\,c^2 - 2\,f\,g\,a\,b^2\,c\,d + f\,g\,b^3\,c^2)}{(a\,f-b\,e)\,\left(b^4\,(e+f\,x) - b^4\,e + a\,b^3\,f\right)}$$
$$+ \frac{2\,d^2\,h\,(e+f\,x)^{3/2}}{3\,b^2\,f^2}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}\,(a\,d-b\,c)\,(b^2\,c\,f\,g - 2\,b^2\,c\,e\,h - 4\,b^2\,d\,e\,g - 5\,a^2\,d\,f\,h + a\,b\,c\,f\,h + 6\,a\,b\,d\,e\,h + 3\,a\,b\,d\,f\,g)}{\sqrt{a\,f-b\,e}\,(b^3\,c^2\,f\,g - 2\,b^3\,c^2\,e\,h + 5\,a^3\,d^2\,f\,h + 4\,a\,b^2\,d^2\,e\,g + a\,b^2\,c^2\,f\,h - 6\,a^2\,b\,d^2\,e\,h - 3\,a^2\,b\,d^2\,f\,g - 4\,b^3\,c\,d\,e\,g + 8\,a\,b^2\,c\,d\,e\,h + 2\,a\,b^2\,c\,d\,f}\right.}{b^{7/2}\,(a\,f}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(1/2)*(a + b*x)^2),x)
```

output
```
(e + f*x)^(1/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b^2*f^2) - (4*d^2*h*
(a*f - b*e))/(b^3*f^2)) + ((e + f*x)^(1/2)*(b^3*c^2*f*g - a^3*d^2*f*h - a*
b^2*c^2*f*h + a^2*b*d^2*f*g - 2*a*b^2*c*d*f*g + 2*a^2*b*c*d*f*h))/((a*f -
b*e)*(b^4*(e + f*x) - b^4*e + a*b^3*f)) + (2*d^2*h*(e + f*x)^(3/2))/(3*b^2
*f^2) + (atan((b^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)*(b^2*c*f*g - 2*b^2*c*e*
h - 4*b^2*d*e*g - 5*a^2*d*f*h + a*b*c*f*h + 6*a*b*d*e*h + 3*a*b*d*f*g))/((
a*f - b*e)^(1/2)*(b^3*c^2*f*g - 2*b^3*c^2*e*h + 5*a^3*d^2*f*h + 4*a*b^2*d^
2*e*g + a*b^2*c^2*f*h - 6*a^2*b*d^2*e*h - 3*a^2*b*d^2*f*g - 4*b^3*c*d*e*g
+ 8*a*b^2*c*d*e*h + 2*a*b^2*c*d*f*g - 6*a^2*b*c*d*f*h)))*(a*d - b*c)*(b^2*
c*f*g - 2*b^2*c*e*h - 4*b^2*d*e*g - 5*a^2*d*f*h + a*b*c*f*h + 6*a*b*d*e*h
+ 3*a*b*d*f*g))/(b^(7/2)*(a*f - b*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1740, normalized size of antiderivative = 7.95

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(1/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**4*d**2*f**3*h - 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d*f**3*h - 18*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*e*f**2*h - 9*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**3*b*d**2*f**3*g + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*f**3*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**2*f**3*h + 2
4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**2*b**2*c*d*e*f**2*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*d*f**3*g - 18*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*d*f**3*
h*x + 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**2*b**2*d**2*e*f**2*g - 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f**2*h*x - 9*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b
**2*d**2*f**3*g*x - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a*b**3*c**2*e*f**2*h + 3*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c**2*f**3*g + 3*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a...
```

3.120 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 329

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}} \, dx = \frac{2d^2h\sqrt{e+fx}}{b^3f} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{2b^3(be-af)(a+bx)^2}$$
$$- \frac{(bc-ad)\left(9a^2dfh + b^2(8deg-3cfg+4ceh) - ab(5dfg+12deh+cfh)\right)\sqrt{e+fx}}{4b^3(be-af)^2(a+bx)}$$
$$- \frac{(4d(be-af)(2be-af)(bdg+2bch-3adh) - f(3a^3d^2fh + a^2bdf(dg+2ch) + b^3c(8deg-3cfg+4c}{4b^{7/2}(be-af)^{5/2}}$$

output
```
2*d^2*h*(f*x+e)^(1/2)/b^3/f-1/2*(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/
(-a*f+b*e)/(b*x+a)^2-1/4*(-a*d+b*c)*(9*a^2*d*f*h+b^2*(4*c*e*h-3*c*f*g+8*d*
e*g)-a*b*(c*f*h+12*d*e*h+5*d*f*g))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^2/(b*x+a)-
1/4*(4*d*(-a*f+b*e)*(-a*f+2*b*e)*(-3*a*d*h+2*b*c*h+b*d*g)-f*(3*a^3*d^2*f*h
+a^2*b*d*f*(2*c*h+d*g)+b^3*c*(4*c*e*h-3*c*f*g+8*d*e*g)-a*b^2*(4*d^2*e*g+c^
2*f*h+2*c*d*(4*e*h+f*g))))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))
/b^(7/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx$$
$$= \frac{\frac{\sqrt{b}\sqrt{e+fx}\left(15a^4d^2f^2h+ab^3\left(2cdf(-2eg+fgx+8ehx)+8d^2ex(fg+2eh-2fhx)+c^2f(5fg-2eh+fhx)\right)+a^3bdf(-6cfh+d(-3fg-26eh+25fhx))\right.}{f(be-af)^2}}{}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
((Sqrt[b]*Sqrt[e + f*x]*(15*a^4*d^2*f^2*h + a*b^3*(2*c*d*f*(-2*e*g + f*g*x
 + 8*e*h*x) + 8*d^2*e*x*(f*g + 2*e*h - 2*f*h*x) + c^2*f*(5*f*g - 2*e*h + f
*h*x)) + a^3*b*d*f*(-6*c*f*h + d*(-3*f*g - 26*e*h + 25*f*h*x)) + b^4*(-8*c
*d*e*f*g*x + 8*d^2*e^2*h*x^2 + c^2*f*(3*f*g*x - 2*e*(g + 2*h*x))) + a^2*b^
2*(-(c^2*f^2*h) - 2*c*d*f*(-6*e*h + f*(g + 5*h*x)) + d^2*(8*e^2*h + e*f*(6
*g - 44*h*x) + f^2*x*(-5*g + 8*h*x)))))/(f*(b*e - a*f)^2*(a + b*x)^2) + ((
-15*a^3*d^2*f^2*h + 3*a^2*b*d*f*(d*f*g + 12*d*e*h + 2*c*f*h) + b^3*(8*d^2*
e^2*g + c^2*f*(3*f*g - 4*e*h) + 8*c*d*e*(-(f*g) + 2*e*h)) + a*b^2*(c^2*f^2
*h + 2*c*d*f*(f*g - 8*e*h) - 8*d^2*e*(f*g + 3*e*h)))*ArcTan[(Sqrt[b]*Sqrt[
e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e) + a*f)^(5/2))/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 25, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \; 166$$
$$\frac{\int \frac{(c+dx)((4de+cf)(bg-ah)-4bc(fg-eh)+d(bfg+4beh-5afh)x)}{2(a+bx)^2\sqrt{e+fx}}dx}{2b(be-af)} - \frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int -\frac{(c+dx)(a(4de+cf)h-b(4deg-3cfg+4ceh)-d(bfg+4beh-5afh)x)}{(a+bx)^2\sqrt{e+fx}}dx}{4b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{(c+dx)(a(4de+cf)h-b(4deg-3cfg+4ceh)-d(bfg+4beh-5afh)x)}{(a+bx)^2\sqrt{e+fx}}dx}{4b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \ 163$$

$$-\frac{\frac{(15a^3d^2f^2h-3a^2bdf(2cfh+12deh+dfg)-ab^2(c^2f^2h+2cdf(fg-8eh)-8d^2e(3eh+fg))-b^3(c^2f(3fg-4eh)-8cde(fg-2eh)+8d^2e^2g))\int \frac{}{(a+bx}}{2b^2(be-af)}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \ 73$$

$$-\frac{\frac{(15a^3d^2f^2h-3a^2bdf(2cfh+12deh+dfg)-ab^2(c^2f^2h+2cdf(fg-8eh)-8d^2e(3eh+fg))-b^3(c^2f(3fg-4eh)-8cde(fg-2eh)+8d^2e^2g))\int \frac{}{a+\frac{b(e}{}}}{b^2f(be-af)}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(be-af)}$$

$$\Big\downarrow \ 221$$

$$-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)(15a^3d^2f^2h-3a^2bdf(2cfh+12deh+dfg)-ab^2(c^2f^2h+2cdf(fg-8eh)-8d^2e(3eh+fg))-b^3(c^2f(3fg-4eh)-8cde(fg-}{b^{5/2}(be-af)^{3/2}}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{2b(a+bx)^2(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((b*g - a*h)*(c + d*x)^2*Sqrt[e + f*x])/(b*(b*e - a*f)*(a + b*x)^2) -
 (-((Sqrt[e + f*x]*(15*a^3*d^2*f^2*h - b^3*c*f*(4*d*e*g - 3*c*f*g + 4*c*e*
h) - a^2*b*d*f*(3*d*f*g + 26*d*e*h + 6*c*f*h) + a*b^2*(c^2*f^2*h - 2*c*d*f
*(f*g - 6*e*h) + 2*d^2*e*(3*f*g + 4*e*h)) + 2*b*d^2*(b*e - a*f)*(b*f*g + 4
*b*e*h - 5*a*f*h)*x))/(b^2*f*(b*e - a*f)*(a + b*x))) - ((15*a^3*d^2*f^2*h
- 3*a^2*b*d*f*(d*f*g + 12*d*e*h + 2*c*f*h) - b^3*(8*d^2*e^2*g + c^2*f*(3*f
*g - 4*e*h) - 8*c*d*e*(f*g - 2*e*h)) - a*b^2*(c^2*f^2*h + 2*c*d*f*(f*g - 8
*e*h) - 8*d^2*e*(f*g + 3*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e -
 a*f]])/(b^(5/2)*(b*e - a*f)^(3/2)))/(4*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$-\frac{15\left(\left(\left(-\frac{gf^2c^2}{5}+\frac{4ce(ch+2dg)f}{15}-\frac{16\left(ch+\frac{dg}{2}\right)de^2}{15}\right)b^3-\frac{\left(\left(hc^2+2cdg\right)f^2+\left(-16cdeh-8d^2eg\right)f-24d^2e^2h\right)ab^2}{15}-\frac{2a^2d\left(\left(ch\cdots\right.\right.}{\cdots}\cdots\right.\right.}{\cdots}$
risch	$\frac{2d^2h\sqrt{fx+e}}{b^3f}-\frac{-\frac{bf\left(9a^3d^2fh-10a^2bcdfh-12a^2bd^2eh-5a^2bd^2fg+ab^2c^2fh+16ab^2cdeh+2ab^2cdfg+8ab^2d^2eg-4b^3c^2eh+3\cdots\right.}{4\left(a^2f^2-2abfe+b^2e^2\right)}\cdots}{\cdots}$
derivativedivides	$\frac{2d^2h\sqrt{fx+e}}{b^3}-\frac{2f\left(-\frac{bf\left(9a^3d^2fh-10a^2bcdfh-12a^2bd^2eh-5a^2bd^2fg+ab^2c^2fh+16ab^2cdeh+2ab^2cdfg+8ab^2d^2eg-4b^3c^2eh+3\cdots\right.}{8\left(a^2f^2-2abfe+b^2e^2\right)}\cdots\right.}{\cdots}$
default	$\frac{2d^2h\sqrt{fx+e}}{b^3}-\frac{2f\left(-\frac{bf\left(9a^3d^2fh-10a^2bcdfh-12a^2bd^2eh-5a^2bd^2fg+ab^2c^2fh+16ab^2cdeh+2ab^2cdfg+8ab^2d^2eg-4b^3c^2eh+3\cdots\right.}{8\left(a^2f^2-2abfe+b^2e^2\right)}\cdots\right.}{\cdots}$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-15/4*(((-1/5*g*f^2*c^2+4/15*c*e*(c*h+2*d*g)*f-16/15*(c*h+1/2*d*g)*d*e^2)*
b^3-1/15*((c^2*h+2*c*d*g)*f^2+(-16*c*d*e*h-8*d^2*e*g)*f-24*d^2*e^2*h)*a*b^
2-2/5*a^2*d*((c*h+1/2*d*g)*f+6*d*e*h)*f*b+a^3*d^2*f^2*h)*(b*x+a)^2*f*arcta
n(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((1/5*c^2*f^2*g*x-2/15*(4*d*g*x+c*(
2*h*x+g))*c*e*f+8/15*d^2*e^2*h*x^2)*b^4-2/15*a*((-c*d*g*x-5/2*c^2*(1/5*h*x
+g))*f^2+((8*h*x^2-4*g*x)*d^2+2*c*(-4*h*x+g)*d+h*c^2)*e*f-8*d^2*e^2*h*x)*b
^3-1/15*a^2*(((-8*h*x^2+5*g*x)*d^2+2*c*(5*h*x+g)*d+h*c^2)*f^2-12*d*((-11/3
*h*x+1/2*g)*d+c*h)*e*f-8*d^2*e^2*h)*b^2-2/5*a^3*d*(((-25/6*h*x+1/2*g)*d+c*
h)*f+13/3*d*e*h)*f*b+a^4*d^2*f^2*h)*(f*x+e)^(1/2)*((a*f-b*e)*b)^(1/2))/((a
*f-b*e)*b)^(1/2)/(a*f-b*e)^2/(b*x+a)^2/b^3/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(307) = 614$.

Time = 0.26 (sec) , antiderivative size = 2500, normalized size of antiderivative = 7.60

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(b^2*e - a*b*f)*(((8*b^5*d^2*e^2*f - 8*(b^5*c*d + a*b^4*d^2)*e*
f^2 + (3*b^5*c^2 + 2*a*b^4*c*d + 3*a^2*b^3*d^2)*f^3)*g + (8*(2*b^5*c*d - 3
*a*b^4*d^2)*e^2*f - 4*(b^5*c^2 + 4*a*b^4*c*d - 9*a^2*b^3*d^2)*e*f^2 + (a*b
^4*c^2 + 6*a^2*b^3*c*d - 15*a^3*b^2*d^2)*f^3)*h)*x^2 + (8*a^2*b^3*d^2*e^2*
f - 8*(a^2*b^3*c*d + a^3*b^2*d^2)*e*f^2 + (3*a^2*b^3*c^2 + 2*a^3*b^2*c*d +
 3*a^4*b*d^2)*f^3)*g + (8*(2*a^2*b^3*c*d - 3*a^3*b^2*d^2)*e^2*f - 4*(a^2*b
^3*c^2 + 4*a^3*b^2*c*d - 9*a^4*b*d^2)*e*f^2 + (a^3*b^2*c^2 + 6*a^4*b*c*d -
 15*a^5*d^2)*f^3)*h + 2*((8*a*b^4*d^2*e^2*f - 8*(a*b^4*c*d + a^2*b^3*d^2)*
e*f^2 + (3*a*b^4*c^2 + 2*a^2*b^3*c*d + 3*a^3*b^2*d^2)*f^3)*g + (8*(2*a*b^4
*c*d - 3*a^2*b^3*d^2)*e^2*f - 4*(a*b^4*c^2 + 4*a^2*b^3*c*d - 9*a^3*b^2*d^2
)*e*f^2 + (a^2*b^3*c^2 + 6*a^3*b^2*c*d - 15*a^4*b*d^2)*f^3)*h)*x)*log((b*f
*x + 2*b*e - a*f + 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*(8*
(b^6*d^2*e^3 - 3*a*b^5*d^2*e^2*f + 3*a^2*b^4*d^2*e*f^2 - a^3*b^3*d^2*f^3)*
h*x^2 - (2*(b^6*c^2 + 2*a*b^5*c*d - 3*a^2*b^4*d^2)*e^2*f - (7*a*b^5*c^2 +
2*a^2*b^4*c*d - 9*a^3*b^3*d^2)*e*f^2 + (5*a^2*b^4*c^2 - 2*a^3*b^3*c*d - 3*
a^4*b^2*d^2)*f^3)*g + (8*a^2*b^4*d^2*e^3 - 2*(a*b^5*c^2 - 6*a^2*b^4*c*d +
17*a^3*b^3*d^2)*e^2*f + (a^2*b^4*c^2 - 18*a^3*b^3*c*d + 41*a^4*b^2*d^2)*e*
f^2 + (a^3*b^3*c^2 + 6*a^4*b^2*c*d - 15*a^5*b*d^2)*f^3)*h - ((8*(b^6*c*d -
 a*b^5*d^2)*e^2*f - (3*b^6*c^2 + 10*a*b^5*c*d - 13*a^2*b^4*d^2)*e*f^2 + (3
*a*b^5*c^2 + 2*a^2*b^4*c*d - 5*a^3*b^3*d^2)*f^3)*g - (16*a*b^5*d^2*e^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(307) = 614$.

Time = 0.15 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.63

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g - 8*b^3*c*d*e*f*g - 8*a*b^2*d^2*e*f*g + 3*b^3*c^2*f^2
*g + 2*a*b^2*c*d*f^2*g + 3*a^2*b*d^2*f^2*g + 16*b^3*c*d*e^2*h - 24*a*b^2*d
^2*e^2*h - 4*b^3*c^2*e*f*h - 16*a*b^2*c*d*e*f*h + 36*a^2*b*d^2*e*f*h + a*b
^2*c^2*f^2*h + 6*a^2*b*c*d*f^2*h - 15*a^3*d^2*f^2*h)*arctan(sqrt(f*x + e)*
b/sqrt(-b^2*e + a*b*f))/((b^5*e^2 - 2*a*b^4*e*f + a^2*b^3*f^2)*sqrt(-b^2*e
 + a*b*f)) + 2*sqrt(f*x + e)*d^2*h/(b^3*f) - 1/4*(8*(f*x + e)^(3/2)*b^4*c*
d*e*f*g - 8*(f*x + e)^(3/2)*a*b^3*d^2*e*f*g - 8*sqrt(f*x + e)*b^4*c*d*e^2*
f*g + 8*sqrt(f*x + e)*a*b^3*d^2*e^2*f*g - 3*(f*x + e)^(3/2)*b^4*c^2*f^2*g
- 2*(f*x + e)^(3/2)*a*b^3*c*d*f^2*g + 5*(f*x + e)^(3/2)*a^2*b^2*d^2*f^2*g
+ 5*sqrt(f*x + e)*b^4*c^2*e*f^2*g + 6*sqrt(f*x + e)*a*b^3*c*d*e*f^2*g - 11
*sqrt(f*x + e)*a^2*b^2*d^2*e*f^2*g - 5*sqrt(f*x + e)*a*b^3*c^2*f^3*g + 2*s
qrt(f*x + e)*a^2*b^2*c*d*f^3*g + 3*sqrt(f*x + e)*a^3*b*d^2*f^3*g + 4*(f*x
+ e)^(3/2)*b^4*c^2*e*f*h - 16*(f*x + e)^(3/2)*a*b^3*c*d*e*f*h + 12*(f*x +
e)^(3/2)*a^2*b^2*d^2*e*f*h - 4*sqrt(f*x + e)*b^4*c^2*e^2*f*h + 16*sqrt(f*x
 + e)*a*b^3*c*d*e^2*f*h - 12*sqrt(f*x + e)*a^2*b^2*d^2*e^2*f*h - (f*x + e)
^(3/2)*a*b^3*c^2*f^2*h + 10*(f*x + e)^(3/2)*a^2*b^2*c*d*f^2*h - 9*(f*x + e
)^(3/2)*a^3*b*d^2*f^2*h + 3*sqrt(f*x + e)*a*b^3*c^2*e*f^2*h - 22*sqrt(f*x
+ e)*a^2*b^2*c*d*e*f^2*h + 19*sqrt(f*x + e)*a^3*b*d^2*e*f^2*h + sqrt(f*x +
 e)*a^2*b^2*c^2*f^3*h + 6*sqrt(f*x + e)*a^3*b*c*d*f^3*h - 7*sqrt(f*x + e)*
a^4*d^2*f^3*h)/((b^5*e^2 - 2*a*b^4*e*f + a^2*b^3*f^2)*((f*x + e)*b - b*...
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx$$

$$= \frac{\frac{(e+f\,x)^{3/2}\,\left(9\,h\,a^3\,b\,d^2\,f^2-10\,h\,a^2\,b^2\,c\,d\,f^2-5\,g\,a^2\,b^2\,d^2\,f^2-12\,e\,h\,a^2\,b^2\,d^2\,f+h\,a\,b^3\,c^2\,f^2+2\,g\,a\,b^3\,c\,d\,f^2+16\,e\,h\,a\,b^3\,c\,d\,f+8\,e\,g\,a\,b^3\,d^2\,f+3\,g\,b^4\,c\cdots}{4\,(a\,f-b\,e)^2}}{b^5\,(e+f\,x)^2\cdots}$$

$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\,\left(-15\,h\,a^3\,d^2\,f^2+6\,h\,a^2\,b\,c\,d\,f^2+36\,h\,a^2\,b\,d^2\,e\,f+3\,g\,a^2\,b\,d^2\,f^2+h\,a\,b^2\,c^2\,f^2-16\,h\,a\cdots}{\cdots}$$

$$+ \frac{2\,d^2\,h\,\sqrt{e+f\,x}}{b^3\,f}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(1/2)*(a + b*x)^3),x)
```

output
```
(((e + f*x)^(3/2)*(3*b^4*c^2*f^2*g - 4*b^4*c^2*e*f*h + a*b^3*c^2*f^2*h + 9
*a^3*b*d^2*f^2*h - 5*a^2*b^2*d^2*f^2*g - 8*b^4*c*d*e*f*g + 2*a*b^3*c*d*f^2
*g + 8*a*b^3*d^2*e*f*g - 10*a^2*b^2*c*d*f^2*h - 12*a^2*b^2*d^2*e*f*h + 16*
a*b^3*c*d*e*f*h))/(4*(a*f - b*e)^2) - ((e + f*x)^(1/2)*(4*b^3*c^2*e*f*h -
7*a^3*d^2*f^2*h - 5*b^3*c^2*f^2*g + a*b^2*c^2*f^2*h + 3*a^2*b*d^2*f^2*g +
8*b^3*c*d*e*f*g + 2*a*b^2*c*d*f^2*g + 6*a^2*b*c*d*f^2*h - 8*a*b^2*d^2*e*f*
g + 12*a^2*b*d^2*e*f*h - 16*a*b^2*c*d*e*f*h))/(4*(a*f - b*e)))/(b^5*(e + f
*x)^2 - (e + f*x)*(2*b^5*e - 2*a*b^4*f) + b^5*e^2 + a^2*b^3*f^2 - 2*a*b^4*
e*f) + (atan((b^(1/2)*(e + f*x)^(1/2))/(a*f - b*e)^(1/2))*(3*b^3*c^2*f^2*g
 + 8*b^3*d^2*e^2*g - 15*a^3*d^2*f^2*h + 16*b^3*c*d*e^2*h - 4*b^3*c^2*e*f*h
 + a*b^2*c^2*f^2*h - 24*a*b^2*d^2*e^2*h + 3*a^2*b*d^2*f^2*g - 8*b^3*c*d*e*
f*g + 2*a*b^2*c*d*f^2*g + 6*a^2*b*c*d*f^2*h - 8*a*b^2*d^2*e*f*g + 36*a^2*b
*d^2*e*f*h - 16*a*b^2*c*d*e*f*h))/(4*b^(7/2)*(a*f - b*e)^(5/2)) + (2*d^2*h
*(e + f*x)^(1/2))/(b^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3133, normalized size of antiderivative = 9.52

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(1/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**5*d**2*f**3*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d*f**3*h + 36*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*e*f**2*h +
3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**4*b*d**2*f**3*g - 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*f**3*h*x + sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*f**3*h - 1
6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**3*b**2*c*d*e*f**2*h + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*f**3*g + 12*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*f**3*
h*x - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**3*b**2*d**2*e**2*f*h - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*e*f**2*g + 72*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**
2*d**2*e*f**2*h*x + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*f**3*g*x - 15*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*f**3*h*x
**2 - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*...
```

3.121 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 475

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = -\frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{3b^3(be-af)(a+bx)^3}$$
$$-\frac{(bc-ad)\left(13a^2dfh+b^2(12deg-5cfg+6ceh)-ab(7dfg+18deh+cfh)\right)\sqrt{e+fx}}{12b^3(be-af)^2(a+bx)^2}$$
$$+\frac{\left(11a^3d^2f^2h-a^2bdf(dfg+30deh+2cfh)-b^3(8d^2e^2g+c^2f(5fg-6eh)-4cde(3fg-4eh))-ab^2(c\right.}{8b^3(be-af)^3(a+bx)}$$
$$+\frac{\left(5a^3d^2f^3h+a^2bdf^2(dfg-18deh+2cfh)+ab^2f(c^2f^2h-4d^2e(fg-6eh)+2cdf(fg-4eh))+b^3(c^2\right.}{8b^{7/2}(be-af)^{7/2}}$$

output
```
-1/3*(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/(-a*f+b*e)/(b*x+a)^3-1/12*(
-a*d+b*c)*(13*a^2*d*f*h+b^2*(6*c*e*h-5*c*f*g+12*d*e*g)-a*b*(c*f*h+18*d*e*h
+7*d*f*g))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^2/(b*x+a)^2+1/8*(11*a^3*d^2*f^2*h-
a^2*b*d*f*(2*c*f*h+30*d*e*h+d*f*g)-b^3*(8*d^2*e^2*g+c^2*f*(-6*e*h+5*f*g)-4
*c*d*e*(-4*e*h+3*f*g))-a*b^2*(c^2*f^2*h+2*c*d*f*(-4*e*h+f*g)-4*d^2*e*(6*e*
h+f*g)))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^3/(b*x+a)+1/8*(5*a^3*d^2*f^3*h+a^2*b
*d*f^2*(2*c*f*h-18*d*e*h+d*f*g)+a*b^2*f*(c^2*f^2*h-4*d^2*e*(-6*e*h+f*g)+2*
c*d*f*(-4*e*h+f*g))+b^3*(c^2*f^2*(-6*e*h+5*f*g)-4*c*d*e*f*(-4*e*h+3*f*g)+8
*d^2*e^2*(-2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^
(7/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{e+fx}(15a^5d^2f^2h+a^4bdf(6cfh+d(3fg-44eh+40fhx))-b^5(24d^2e^2gx^2+12cdex(-3fgx+2e(g$$
$$-\frac{(-5a^3d^2f^3h-a^2bdf^2(dfg-18deh+2cfh)+b^3(8d^2e^2(-fg+2eh)-4cdef(-3fg+4eh)+c^2f^2(-5}{8b^{7/2}(-be+af)^{7/}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[e + f*x]*(15*a^5*d^2*f^2*h + a^4*b*d*f*(6*c*f*h + d*(3*f*g - 44*e*h
+ 40*f*h*x)) - b^5*(24*d^2*e^2*g*x^2 + 12*c*d*e*x*(-3*f*g*x + 2*e*(g + 2*h
*x)) + c^2*(15*f^2*g*x^2 + 4*e^2*(2*g + 3*h*x) - 2*e*f*x*(5*g + 9*h*x))) -
 a^2*b^3*(c^2*f*(33*f*g - 16*e*h + 8*f*h*x) + 2*c*d*(8*e^2*h - 2*e*f*(8*g
- 7*h*x) + f^2*x*(8*g + 3*h*x)) + d^2*(3*f^2*g*x^2 + 4*e^2*(2*g - 27*h*x)
+ 2*e*f*x*(7*g + 45*h*x))) + a*b^4*(12*d^2*e*x*(-2*e*g + f*g*x + 6*e*h*x)
- 2*c*d*(3*f^2*g*x^2 + 4*e^2*(g + 6*h*x) - 2*e*f*x*(25*g + 6*h*x)) + c^2*(
-4*e^2*h - f^2*x*(40*g + 3*h*x) + e*f*(26*g + 50*h*x))) + a^3*b^2*(3*c^2*f
^2*h + 2*c*d*f*(3*f*g - 10*e*h + 8*f*h*x) + d^2*(44*e^2*h + f^2*x*(8*g + 3
3*h*x) - 2*e*f*(5*g + 59*h*x)))))/(24*b^3*(b*e - a*f)^3*(a + b*x)^3) - ((-
5*a^3*d^2*f^3*h - a^2*b*d*f^2*(d*f*g - 18*d*e*h + 2*c*f*h) + b^3*(8*d^2*e^
2*(-(f*g) + 2*e*h) - 4*c*d*e*f*(-3*f*g + 4*e*h) + c^2*f^2*(-5*f*g + 6*e*h)
) - a*b^2*f*(c^2*f^2*h + 2*c*d*f*(f*g - 4*e*h) + 4*d^2*e*(-(f*g) + 6*e*h))
)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(8*b^(7/2)*(-(b*e) +
 a*f)^(7/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {166, 27, 25, 162, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int \frac{(c+dx)((4de+cf)(bg-ah)-6bc(fg-eh)-d(bfg-6beh+5afh)x)}{2(a+bx)^3\sqrt{e+fx}}dx}{3b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int -\frac{(c+dx)(a(4de+cf)h-b(4deg-5cfg+6ceh)+d(bfg-6beh+5afh)x)}{(a+bx)^3\sqrt{e+fx}}dx}{6b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{(c+dx)(a(4de+cf)h-b(4deg-5cfg+6ceh)+d(bfg-6beh+5afh)x)}{(a+bx)^3\sqrt{e+fx}}dx}{6b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \ 162$$

$$-\frac{\frac{3\left(5a^3d^2f^3h+a^2bdf^2(2cfh-18deh+dfg)+ab^2f\left(c^2f^2h+2cdf(fg-4eh)-4d^2e(fg-6eh)\right)+b^3\left(c^2f^2(5fg-6eh)-4cdef(3fg-4eh)+8d^2e^2(fg-2\right.\right.}{8b^2(be-af)^2}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \ 73$$

$$-\frac{\frac{3\left(5a^3d^2f^3h+a^2bdf^2(2cfh-18deh+dfg)+ab^2f\left(c^2f^2h+2cdf(fg-4eh)-4d^2e(fg-6eh)\right)+b^3\left(c^2f^2(5fg-6eh)-4cdef(3fg-4eh)+8d^2e^2(fg-2\right.\right.}{4b^2f(be-af)^2}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(be-af)}$$

$$\Big\downarrow \ 221$$

$$-\frac{-\frac{3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(5a^3d^2f^3h+a^2bdf^2(2cfh-18deh+dfg)+ab^2f\left(c^2f^2h+2cdf(fg-4eh)-4d^2e(fg-6eh)\right)+b^3\left(c^2f^2(5fg-6eh)-4cdef\right.\right.}{4b^{5/2}(be-af)^{5/2}}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{3b(a+bx)^3(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
-1/3*((b*g - a*h)*(c + d*x)^2*Sqrt[e + f*x])/(b*(b*e - a*f)*(a + b*x)^3) -
 (-1/4*(Sqrt[e + f*x]*(15*a^4*d^2*f^2*h - 2*b^4*c*e*(4*d*e*g - 5*c*f*g + 6
*c*e*h) + a^3*b*d*f*(3*d*f*g - 44*d*e*h + 6*c*f*h) - a^2*b^2*(5*c^2*f^2*h
+ 2*d^2*e*(5*f*g - 22*e*h) - 2*c*d*f*(3*f*g - 10*e*h)) - a*b^3*(8*d^2*e^2*
g + c^2*f*(25*f*g - 32*e*h) - 16*c*d*e*(2*f*g - e*h)) + b*(25*a^3*d^2*f^2*
h + a^2*b*d*f*(5*d*f*g - 74*d*e*h - 6*c*f*h) - a*b^2*(3*c^2*f^2*h + 4*d^2*
e*(f*g - 16*e*h) + 6*c*d*f*(f*g - 4*e*h)) - b^3*(16*d^2*e^2*g + 3*c^2*f*(5
*f*g - 6*e*h) - 12*c*d*e*(3*f*g - 4*e*h)))*x))/(b^2*(b*e - a*f)^2*(a + b*x
)^2) - (3*(5*a^3*d^2*f^3*h + a^2*b*d*f^2*(d*f*g - 18*d*e*h + 2*c*f*h) + a*
b^2*f*(c^2*f^2*h - 4*d^2*e*(f*g - 6*e*h) + 2*c*d*f*(f*g - 4*e*h)) + b^3*(c
^2*f^2*(5*f*g - 6*e*h) - 4*c*d*e*f*(3*f*g - 4*e*h) + 8*d^2*e^2*(f*g - 2*e*
h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(4*b^(5/2)*(b*e - a
*f)^(5/2)))/(6*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{\frac{5\left(\left(\left(c^2 g f^3-\frac{6ce(ch+2dg)f^2}{5}+\frac{16\left(ch+\frac{dg}{2}\right)d e^2 f}{5}-\frac{16d^2e^3h}{5}\right)b^3+\frac{a\left(\left(h c^2+2cdg\right)f^2+4\left(-2cdh-d^2g\right)ef+24d^2e^2h\right)f b^2}{5}+\frac{2a^2d\left(\left(\cdots\right.\right.}{\cdots}\right.\right.}{8}}{\cdots}$
derivativedivides	$-\frac{2\left(\frac{f\left(11a^3d^2f^2h-2a^2bcd f^2h-30a^2b d^2efh-a^2b d^2f^2g-a b^2c^2f^2h+8a b^2cdefh-2a b^2cd f^2g+24a b^2d^2e^2h+4a b^2d^2efg+6b^3\cdots\right.}{16b\left(a^3f^3-3a^2be f^2+3a b^2e^2f-b^3e^3\right)}\cdots\right.}{\cdots}$
default	$-\frac{2\left(\frac{f\left(11a^3d^2f^2h-2a^2bcd f^2h-30a^2b d^2efh-a^2b d^2f^2g-a b^2c^2f^2h+8a b^2cdefh-2a b^2cd f^2g+24a b^2d^2e^2h+4a b^2d^2efg+6b^3\cdots\right.}{16b\left(a^3f^3-3a^2be f^2+3a b^2e^2f-b^3e^3\right)}\cdots\right.}{\cdots}$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/8*(((c^2*g*f^3-6/5*c*e*(c*h+2*d*g)*f^2+16/5*(c*h+1/2*d*g)*d*e^2*f-16/5*d
^2*e^3*h)*b^3+1/5*a*((c^2*h+2*c*d*g)*f^2+4*(-2*c*d*h-d^2*g)*e*f+24*d^2*e^2
*h)*f*b^2+2/5*a^2*d*((c*h+1/2*d*g)*f-9*d*e*h)*f^2*b+a^3*d^2*f^3*h)*(b*x+a)
^3*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((-c^2*f^2*g*x^2+2/3*x*c*(1
8/5*d*g*x+c*(9/5*h*x+g))*e*f-8/15*(3*d^2*g*x^2+3*c*x*(2*h*x+g)*d+c^2*(3/2*
h*x+g))*e^2)*b^5-4/15*a*(10*x*c*(3/20*d*g*x+c*(3/40*h*x+g))*f^2-13/2*(6/13
*d^2*g*x^2+50/13*x*c*(6/25*h*x+g)*d+c^2*(25/13*h*x+g))*e*f+e^2*(6*x*(-3*h*
x+g)*d^2+2*c*(6*h*x+g)*d+h*c^2))*b^4+16/15*a^2*((-3/16*d^2*g*x^2-x*c*(3/8*
h*x+g)*d-33/16*c^2*(8/33*h*x+g))*f^2+(-7/8*x*(45/7*h*x+g)*d^2+2*c*(-7/8*h*
x+g)*d+h*c^2)*e*f-d*e^2*(1/2*(-27/2*h*x+g)*d+c*h))*b^3+1/5*a^3*(((11*h*x^2
+8/3*g*x)*d^2+2*c*(8/3*h*x+g)*d+h*c^2)*f^2-20/3*d*e*(1/2*(59/5*h*x+g)*d+c*
h)*f+44/3*d^2*e^2*h)*b^2+2/5*a^4*(((20/3*h*x+1/2*g)*d+c*h)*f-22/3*d*e*h)*d
*f*b+a^5*d^2*f^2*h)*(f*x+e)^(1/2)*((a*f-b*e)*b)^(1/2))/((a*f-b*e)*b)^(1/2)
/(b*x+a)^3/(a*f-b*e)^3/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1897 vs. $2(451) = 902$.

Time = 0.44 (sec) , antiderivative size = 3808, normalized size of antiderivative = 8.02

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(451) = 902$.

Time = 0.16 (sec) , antiderivative size = 1618, normalized size of antiderivative = 3.41

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(8*b^3*d^2*e^2*f*g - 12*b^3*c*d*e*f^2*g - 4*a*b^2*d^2*e*f^2*g + 5*b^3
*c^2*f^3*g + 2*a*b^2*c*d*f^3*g + a^2*b*d^2*f^3*g - 16*b^3*d^2*e^3*h + 16*b
^3*c*d*e^2*f*h + 24*a*b^2*d^2*e^2*f*h - 6*b^3*c^2*e*f^2*h - 8*a*b^2*c*d*e*
f^2*h - 18*a^2*b*d^2*e*f^2*h + a*b^2*c^2*f^3*h + 2*a^2*b*c*d*f^3*h + 5*a^3
*d^2*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^3 - 3*a*b
^5*e^2*f + 3*a^2*b^4*e*f^2 - a^3*b^3*f^3)*sqrt(-b^2*e + a*b*f)) - 1/24*(24
*(f*x + e)^(5/2)*b^5*d^2*e^2*f*g - 48*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + 24
*sqrt(f*x + e)*b^5*d^2*e^4*f*g - 36*(f*x + e)^(5/2)*b^5*c*d*e*f^2*g - 12*(
f*x + e)^(5/2)*a*b^4*d^2*e*f^2*g + 96*(f*x + e)^(3/2)*b^5*c*d*e^2*f^2*g +
48*(f*x + e)^(3/2)*a*b^4*d^2*e^2*f^2*g - 60*sqrt(f*x + e)*b^5*c*d*e^3*f^2*
g - 36*sqrt(f*x + e)*a*b^4*d^2*e^3*f^2*g + 15*(f*x + e)^(5/2)*b^5*c^2*f^3*
g + 6*(f*x + e)^(5/2)*a*b^4*c*d*f^3*g + 3*(f*x + e)^(5/2)*a^2*b^3*d^2*f^3*
g - 40*(f*x + e)^(3/2)*b^5*c^2*e*f^3*g - 112*(f*x + e)^(3/2)*a*b^4*c*d*e*f
^3*g + 8*(f*x + e)^(3/2)*a^2*b^3*d^2*e*f^3*g + 33*sqrt(f*x + e)*b^5*c^2*e^
2*f^3*g + 114*sqrt(f*x + e)*a*b^4*c*d*e^2*f^3*g - 3*sqrt(f*x + e)*a^2*b^3*
d^2*e^2*f^3*g + 40*(f*x + e)^(3/2)*a*b^4*c^2*f^4*g + 16*(f*x + e)^(3/2)*a^
2*b^3*c*d*f^4*g - 8*(f*x + e)^(3/2)*a^3*b^2*d^2*f^4*g - 66*sqrt(f*x + e)*a
*b^4*c^2*e*f^4*g - 48*sqrt(f*x + e)*a^2*b^3*c*d*e*f^4*g + 18*sqrt(f*x + e)
*a^3*b^2*d^2*e*f^4*g + 33*sqrt(f*x + e)*a^2*b^3*c^2*f^5*g - 6*sqrt(f*x + e
)*a^3*b^2*c*d*f^5*g - 3*sqrt(f*x + e)*a^4*b*d^2*f^5*g + 48*(f*x + e)^(5...
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx$$
$$= \frac{\frac{(e+fx)^{5/2}\left(-11\,h\,a^3\,d^2\,f^3+2\,h\,a^2\,b\,c\,d\,f^3+30\,h\,a^2\,b\,d^2\,e\,f^2+g\,a^2\,b\,d^2\,f^3+h\,a\,b^2\,c^2\,f^3-8\,h\,a\,b^2\,c\,d\,e\,f^2+2\,g\,a\,b^2\,c\,d\,f^3-24\,h\,a\,b^2\,d^2\,e^2\,f-4\,g\,a\,b^2\right.}{8\,b\,(a\,f-b\,e)^3}}{}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{e+f\,x}}{\sqrt{a\,f-b\,e}}\right)\left(5\,h\,a^3\,d^2\,f^3+2\,h\,a^2\,b\,c\,d\,f^3-18\,h\,a^2\,b\,d^2\,e\,f^2+g\,a^2\,b\,d^2\,f^3+h\,a\,b^2\,c^2\,f^3-8\,h\,a\,b^2\,c\right.}{}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(1/2)*(a + b*x)^4),x)
```

output
```
(((e + f*x)^(5/2)*(5*b^3*c^2*f^3*g - 11*a^3*d^2*f^3*h + a*b^2*c^2*f^3*h +
a^2*b*d^2*f^3*g - 6*b^3*c^2*e*f^2*h + 8*b^3*d^2*e^2*f*g + 2*a*b^2*c*d*f^3*
g + 2*a^2*b*c*d*f^3*h - 12*b^3*c*d*e*f^2*g + 16*b^3*c*d*e^2*f*h - 4*a*b^2*
d^2*e*f^2*g - 24*a*b^2*d^2*e^2*f*h + 30*a^2*b*d^2*e*f^2*h - 8*a*b^2*c*d*e*
f^2*h))/(8*b*(a*f - b*e)^3) - ((e + f*x)^(1/2)*(5*a^3*d^2*f^3*h - 11*b^3*c
^2*f^3*g + a*b^2*c^2*f^3*h + a^2*b*d^2*f^3*g + 10*b^3*c^2*e*f^2*h - 8*b^3*
d^2*e^2*f*g + 2*a*b^2*c*d*f^3*g + 2*a^2*b*c*d*f^3*h + 20*b^3*c*d*e*f^2*g -
 16*b^3*c*d*e^2*f*h - 4*a*b^2*d^2*e*f^2*g + 24*a*b^2*d^2*e^2*f*h - 18*a^2*
b*d^2*e*f^2*h - 8*a*b^2*c*d*e*f^2*h))/(8*b^3*(a*f - b*e)) + ((e + f*x)^(3/
2)*(5*b^3*c^2*f^3*g - 5*a^3*d^2*f^3*h + a*b^2*c^2*f^3*h - a^2*b*d^2*f^3*g
- 6*b^3*c^2*e*f^2*h + 6*b^3*d^2*e^2*f*g + 2*a*b^2*c*d*f^3*g - 2*a^2*b*c*d*
f^3*h - 12*b^3*c*d*e*f^2*g + 12*b^3*c*d*e^2*f*h - 18*a*b^2*d^2*e^2*f*h + 1
8*a^2*b*d^2*e*f^2*h))/(3*b^2*(a*f - b*e)^2))/((e + f*x)*(3*b^3*e^2 + 3*a^2
*b*f^2 - 6*a*b^2*e*f) + b^3*(e + f*x)^3 - (e + f*x)^2*(3*b^3*e - 3*a*b^2*f
) + a^3*f^3 - b^3*e^3 + 3*a*b^2*e^2*f - 3*a^2*b*e*f^2) + (atan((b^(1/2)*(e
 + f*x)^(1/2))/(a*f - b*e)^(1/2))*(5*b^3*c^2*f^3*g + 5*a^3*d^2*f^3*h - 16*
b^3*d^2*e^3*h + a*b^2*c^2*f^3*h + a^2*b*d^2*f^3*g - 6*b^3*c^2*e*f^2*h + 8*
b^3*d^2*e^2*f*g + 2*a*b^2*c*d*f^3*g + 2*a^2*b*c*d*f^3*h - 12*b^3*c*d*e*f^2
*g + 16*b^3*c*d*e^2*f*h - 4*a*b^2*d^2*e*f^2*g + 24*a*b^2*d^2*e^2*f*h - 18*
a^2*b*d^2*e*f^2*h - 8*a*b^2*c*d*e*f^2*h))/(8*b^(7/2)*(a*f - b*e)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4777, normalized size of antiderivative = 10.06

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(1/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**6*d**2*f**3*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**5*b*c*d*f**3*h - 54*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**2*e*f**2*h + 3*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
**5*b*d**2*f**3*g + 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a**5*b*d**2*f**3*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c**2*f**3*h - 24
*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**4*b**2*c*d*e*f**2*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*d*f**3*g + 18*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*d*f**3*h
*x + 72*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**4*b**2*d**2*e**2*f*h - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*e*f**2*g - 162*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*
*2*d**2*e*f**2*h*x + 9*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*f**3*g*x + 45*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*f**3*h*
x**2 - 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(...
```

3.122 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 653

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = -\frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{4b^3(be-af)(a+bx)^4}$$

$$-\frac{(bc-ad)\left(17a^2dfh+b^2(16deg-7cfg+8ceh)-ab(9dfg+24deh+cfh)\right)\sqrt{e+fx}}{24b^3(be-af)^2(a+bx)^3}$$

$$+\frac{(59a^3d^2f^2h-3a^2bdf(dfg+56deh+2cfh)-b^3(48d^2e^2g+5c^2f(7fg-8eh)-16cde(5fg-6eh))-}{96b^3(be-af)^3(a+bx)^2}$$

$$+\frac{(5a^3d^2f^3h+3a^2bdf^2(dfg-8deh+2cfh)+b^3(5c^2f^2(7fg-8eh)-16cdef(5fg-6eh)+16d^2e^2(3fg}{64b^3(be-af)^4(a+bx)}$$

$$-\frac{f(5a^3d^2f^3h+3a^2bdf^2(dfg-8deh+2cfh)+b^3(5c^2f^2(7fg-8eh)-16cdef(5fg-6eh)+16d^2e^2(3}{64b^{7/2}(be-af)^{9}}$$

output
```
-1/4*(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^3/(-a*f+b*e)/(b*x+a)^4-1/24*(
-a*d+b*c)*(17*a^2*d*f*h+b^2*(8*c*e*h-7*c*f*g+16*d*e*g)-a*b*(c*f*h+24*d*e*h
+9*d*f*g))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^2/(b*x+a)^3+1/96*(59*a^3*d^2*f^2*h
-3*a^2*b*d*f*(2*c*f*h+56*d*e*h+d*f*g)-b^3*(48*d^2*e^2*g+5*c^2*f*(-8*e*h+7*
f*g)-16*c*d*e*(-6*e*h+5*f*g))-a*b^2*(5*c^2*f^2*h+2*c*d*f*(-16*e*h+5*f*g)-1
6*d^2*e*(9*e*h+f*g)))*(f*x+e)^(1/2)/b^3/(-a*f+b*e)^3/(b*x+a)^2+1/64*(5*a^3
*d^2*f^3*h+3*a^2*b*d*f^2*(2*c*f*h-8*d*e*h+d*f*g)+b^3*(5*c^2*f^2*(-8*e*h+7*
f*g)-16*c*d*e*f*(-6*e*h+5*f*g)+16*d^2*e^2*(-4*e*h+3*f*g))+a*b^2*f*(5*c^2*f
^2*h+2*c*d*f*(-16*e*h+5*f*g)-16*d^2*e*(-3*e*h+f*g)))*(f*x+e)^(1/2)/b^3/(-a
*f+b*e)^4/(b*x+a)-1/64*f*(5*a^3*d^2*f^3*h+3*a^2*b*d*f^2*(2*c*f*h-8*d*e*h+d
*f*g)+b^3*(5*c^2*f^2*(-8*e*h+7*f*g)-16*c*d*e*f*(-6*e*h+5*f*g)+16*d^2*e^2*(
-4*e*h+3*f*g))+a*b^2*f*(5*c^2*f^2*h+2*c*d*f*(-16*e*h+5*f*g)-16*d^2*e*(-3*e
*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(7/2)/(-a*f+b*
e)^(9/2)
```

Mathematica [A] (verified)

Time = 5.53 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx =$$

$$-\frac{\sqrt{e+fx}(15a^6d^2f^3h+a^5bdf^2(18cfh+d(9fg-62eh+55fhx))+b^6(48d^2e^2x^2(-3fgx+2e(g+2hx$$

$$+\frac{f(5a^3d^2f^3h+3a^2bdf^2(dfg-8deh+2cfh)+ab^2f(5c^2f^2h+2cdf(5fg-16eh)+16d^2e(-fg+3eh))}{64b^{7/2}(-be+a}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^5*Sqrt[e + f*x]),x]
```

output
```
-1/192*(Sqrt[e + f*x]*(15*a^6*d^2*f^3*h + a^5*b*d*f^2*(18*c*f*h + d*(9*f*g
 - 62*e*h + 55*f*h*x)) + b^6*(48*d^2*e^2*x^2*(-3*f*g*x + 2*e*(g + 2*h*x))
+ 16*c*d*e*x*(15*f^2*g*x^2 + 4*e^2*(2*g + 3*h*x) - 2*e*f*x*(5*g + 9*h*x))
+ c^2*(-105*f^3*g*x^3 + 16*e^3*(3*g + 4*h*x) - 8*e^2*f*x*(7*g + 10*h*x) +
10*e*f^2*x^2*(7*g + 12*h*x))) + a*b^5*(16*d^2*e*x*(3*f^2*g*x^2 + 2*e^2*(2*
g + 9*h*x) - e*f*x*(35*g + 9*h*x)) + c^2*(16*e^3*h - 5*f^3*x^2*(77*g + 3*h
*x) + 18*e*f^2*x*(14*g + 25*h*x) - 8*e^2*f*(25*g + 37*h*x)) + 2*c*d*(-15*f
^3*g*x^3 + 16*e^3*(g + 4*h*x) + 6*e*f^2*x^2*(75*g + 8*h*x) - 8*e^2*f*x*(37
*g + 70*h*x))) + a^4*b^2*f*(15*c^2*f^2*h + 6*c*d*f*(5*f*g - 14*e*h + 11*f*
h*x) + d^2*(104*e^2*h - 6*e*f*(7*g + 38*h*x) + f^2*x*(33*g + 73*h*x))) + a
^3*b^3*(c^2*f^2*(-279*f*g + 146*e*h - 73*f*h*x) - 2*c*d*f*(88*e^2*h + f^2*
x*(73*g + 33*h*x) + e*f*(-146*g + 52*h*x)) + d^2*(48*e^3*h - 3*f^3*x^2*(11
*g + 5*h*x) - 2*e*f^2*x*(26*g + 119*h*x) + e^2*f*(-88*g + 296*h*x))) + a^2
*b^4*(d^2*(-9*f^3*g*x^3 + 24*e^2*f*x*(-15*g + 8*h*x) + 16*e^3*(g + 12*h*x)
 + 2*e*f^2*x^2*(91*g + 36*h*x)) + 2*c*d*(16*e^3*h - 72*e^2*f*(g + 5*h*x) -
 f^3*x^2*(55*g + 9*h*x) + 2*e*f^2*x*(310*g + 91*h*x)) + c^2*f*(-72*e^2*h -
 f^2*x*(511*g + 55*h*x) + e*f*(326*g + 620*h*x)))))/(b^3*(b*e - a*f)^4*(a
+ b*x)^4) + (f*(5*a^3*d^2*f^3*h + 3*a^2*b*d*f^2*(d*f*g - 8*d*e*h + 2*c*f*h
) + a*b^2*f*(5*c^2*f^2*h + 2*c*d*f*(5*f*g - 16*e*h) + 16*d^2*e*(-(f*g) + 3
*e*h)) + b^3*(5*c^2*f^2*(7*f*g - 8*e*h) + 16*d^2*e^2*(3*f*g - 4*e*h) + ...
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 607, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 25, 162, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(c+dx)((4de+cf)(bg-ah)-8bc(fg-eh)-d(3bfg-8beh+5afh)x)}{2(a+bx)^4\sqrt{e+fx}}dx}{4b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{(c+dx)(a(4de+cf)h-b(4deg-7cfg+8ceh)+d(3bfg-8beh+5afh)x)}{(a+bx)^4\sqrt{e+fx}}dx}{8b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{(c+dx)(a(4de+cf)h-b(4deg-7cfg+8ceh)+d(3bfg-8beh+5afh)x)}{(a+bx)^4\sqrt{e+fx}}dx}{8b(be-af)}-\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$\Big\downarrow$ 162

$$-\frac{\frac{(5a^3d^2f^3h+3a^2bdf^2(2cfh-8deh+dfg)+ab^2f(5c^2f^2h+2cdf(5fg-16eh)-16d^2e(fg-3eh))+b^3(5c^2f^2(7fg-8eh)-16cdef(5fg-6eh)+16d^2e^2}{8b^2(be-af)^2}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$\Big\downarrow$ 52

$$-\frac{\frac{(5a^3d^2f^3h+3a^2bdf^2(2cfh-8deh+dfg)+ab^2f(5c^2f^2h+2cdf(5fg-16eh)-16d^2e(fg-3eh))+b^3(5c^2f^2(7fg-8eh)-16cdef(5fg-6eh)+16d^2e^2}{8b^2(be-af)^2}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$\Big\downarrow$ 73

$$-\frac{\frac{(5a^3d^2f^3h+3a^2bdf^2(2cfh-8deh+dfg)+ab^2f(5c^2f^2h+2cdf(5fg-16eh)-16d^2e(fg-3eh))+b^3(5c^2f^2(7fg-8eh)-16cdef(5fg-6eh)+16d^2e^2}{8b^2(be-af)^2}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\left(\frac{f\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)(5a^3d^2f^3h+3a^2bdf^2(2cfh-8deh+dfg)+ab^2f(5c^2f^2h+2cdf(5fg-16eh)-16d^2e(fg-3eh))+b^3(}{8b^2(be-af)^2}}{}$$

$$\frac{(c+dx)^2\sqrt{e+fx}(bg-ah)}{4b(a+bx)^4(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^5*Sqrt[e + f*x]),x]
```

output
```
-1/4*((b*g - a*h)*(c + d*x)^2*Sqrt[e + f*x])/(b*(b*e - a*f)*(a + b*x)^4) -
 (-1/12*(Sqrt[e + f*x]*(15*a^4*d^2*f^2*h - 4*b^4*c*e*(4*d*e*g - 7*c*f*g +
8*c*e*h) + a^3*b*d*f*(9*d*f*g - 52*d*e*h + 18*c*f*h) - 3*a^2*b^2*(3*c^2*f^
2*h - 2*c*d*f*(5*f*g - 12*e*h) + 12*d^2*e*(f*g - 2*e*h)) - a*b^3*(8*d^2*e^
2*g + c^2*f*(63*f*g - 76*e*h) - 8*c*d*e*(7*f*g - 2*e*h)) + b*(35*a^3*d^2*f
^2*h + 3*a^2*b*d*f*(7*d*f*g - 40*d*e*h - 2*c*f*h) - a*b^2*(5*c^2*f^2*h + 2
*c*d*f*(5*f*g - 16*e*h) + 8*d^2*e*(4*f*g - 15*e*h)) - b^3*(24*d^2*e^2*g +
5*c^2*f*(7*f*g - 8*e*h) - 16*c*d*e*(5*f*g - 6*e*h)))*x))/(b^2*(b*e - a*f)^
2*(a + b*x)^3) + ((5*a^3*d^2*f^3*h + 3*a^2*b*d*f^2*(d*f*g - 8*d*e*h + 2*c*
f*h) + b^3*(5*c^2*f^2*(7*f*g - 8*e*h) - 16*c*d*e*f*(5*f*g - 6*e*h) + 16*d^
2*e^2*(3*f*g - 4*e*h)) + a*b^2*f*(5*c^2*f^2*h + 2*c*d*f*(5*f*g - 16*e*h) -
 16*d^2*e*(f*g - 3*e*h)))*(-(Sqrt[e + f*x]/((b*e - a*f)*(a + b*x))) + (f*A
rcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*e - a*f)^(3/2
))))/(8*b^2*(b*e - a*f)^2))/(8*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.32

method	result	size
pseudoelliptic	Expression too large to display	860
derivativedivides	Expression too large to display	1242
default	Expression too large to display	1242

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/64/((a*f-b*e)*b)^(1/2)*(((7*c^2*g*f^3-8*c*e*(c*h+2*d*g)*f^2+96/5*(c*h+1/
2*d*g)*d*e^2*f-64/5*d^2*e^3*h)*b^3+a*((c^2*h+2*c*d*g)*f^2-32/5*(c*h+1/2*d*
g)*d*e*f+48/5*d^2*e^2*h)*f*b^2+6/5*a^2*d*f^2*((c*h+1/2*d*g)*f-4*d*e*h)*b+a
^3*d^2*f^3*h)*(b*x+a)^4*f*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((-7
*c^2*f^3*g*x^3+14/3*x^2*c*(24/7*d*g*x+c*(12/7*h*x+g))*e*f^2-56/15*x*(18/7*
d^2*g*x^2+20/7*(9/5*h*x+g)*x*c*d+c^2*(10/7*h*x+g))*e^2*f+16/5*e^3*(2*(2*h*
x^3+g*x^2)*d^2+8/3*x*c*(3/2*h*x+g)*d+c^2*(4/3*h*x+g)))*b^6+16/15*a*(-385/1
6*x^2*c*(6/77*d*g*x+c*(3/77*h*x+g))*f^3+63/4*x*(4/21*d^2*g*x^2+25/7*x*c*(8
/75*h*x+g)*d+c^2*(25/14*h*x+g))*e*f^2-25/2*(14/5*x^2*(9/35*h*x+g)*d^2+74/2
5*x*c*(70/37*h*x+g)*d+c^2*(37/25*h*x+g))*e^2*f+(2*(9*h*x^2+2*g*x)*d^2+2*c*
(4*h*x+g)*d+h*c^2)*e^3)*b^5-24/5*a^2*(511/72*x*(9/511*d^2*g*x^2+110/511*x*
c*(9/55*h*x+g)*d+c^2*(55/511*h*x+g))*f^3-163/36*(91/163*x^2*(36/91*h*x+g)*
d^2+620/163*x*c*(91/310*h*x+g)*d+c^2*(310/163*h*x+g))*e*f^2+((-8/3*h*x^2+5
*g*x)*d^2+2*c*(5*h*x+g)*d+h*c^2)*e^2*f-4/9*((6*h*x+1/2*g)*d+c*h)*d*e^3)*b^
4+146/15*a^3*((-33/146*(5/11*h*x+g)*x^2*d^2-(33/73*h*x+g)*x*c*d-279/146*(7
3/279*h*x+g)*c^2)*f^3+(-26/73*x*(119/26*h*x+g)*d^2+2*(-26/73*h*x+g)*c*d+h*
c^2)*e*f^2-88/73*(1/2*(-37/11*h*x+g)*d+c*h)*d*e^2*f+24/73*d^2*e^3*h)*b^3+a
^4*f*((11/5*x*(73/33*h*x+g)*d^2+2*c*(11/5*h*x+g)*d+h*c^2)*f^2-28/5*d*e*((1
9/7*h*x+1/2*g)*d+c*h)*f+104/15*d^2*e^2*h)*b^2+6/5*((1/2*(55/9*h*x+g)*d+c*h
)*f-31/9*d*e*h)*a^5*d*f^2*b+a^6*d^2*f^3*h)*(f*x+e)^(1/2)*((a*f-b*e)*b)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2799 vs. $2(625) = 1250$.

Time = 0.72 (sec) , antiderivative size = 5612, normalized size of antiderivative = 8.59

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**5/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2483 vs. $2(625) = 1250$.

Time = 0.18 (sec) , antiderivative size = 2483, normalized size of antiderivative = 3.80

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/64*(48*b^3*d^2*e^2*f^2*g - 80*b^3*c*d*e*f^3*g - 16*a*b^2*d^2*e*f^3*g + 3
5*b^3*c^2*f^4*g + 10*a*b^2*c*d*f^4*g + 3*a^2*b*d^2*f^4*g - 64*b^3*d^2*e^3*
f*h + 96*b^3*c*d*e^2*f^2*h + 48*a*b^2*d^2*e^2*f^2*h - 40*b^3*c^2*e*f^3*h -
 32*a*b^2*c*d*e*f^3*h - 24*a^2*b*d^2*e*f^3*h + 5*a*b^2*c^2*f^4*h + 6*a^2*b
*c*d*f^4*h + 5*a^3*d^2*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))
/((b^7*e^4 - 4*a*b^6*e^3*f + 6*a^2*b^5*e^2*f^2 - 4*a^3*b^4*e*f^3 + a^4*b^3
*f^4)*sqrt(-b^2*e + a*b*f)) + 1/192*(144*(f*x + e)^(7/2)*b^6*d^2*e^2*f^2*g
 - 528*(f*x + e)^(5/2)*b^6*d^2*e^3*f^2*g + 624*(f*x + e)^(3/2)*b^6*d^2*e^4
*f^2*g - 240*sqrt(f*x + e)*b^6*d^2*e^5*f^2*g - 240*(f*x + e)^(7/2)*b^6*c*d
*e*f^3*g - 48*(f*x + e)^(7/2)*a*b^5*d^2*e*f^3*g + 880*(f*x + e)^(5/2)*b^6*
c*d*e^2*f^3*g + 704*(f*x + e)^(5/2)*a*b^5*d^2*e^2*f^3*g - 1168*(f*x + e)^(
3/2)*b^6*c*d*e^3*f^3*g - 1328*(f*x + e)^(3/2)*a*b^5*d^2*e^3*f^3*g + 528*sq
rt(f*x + e)*b^6*c*d*e^4*f^3*g + 672*sqrt(f*x + e)*a*b^5*d^2*e^4*f^3*g + 10
5*(f*x + e)^(7/2)*b^6*c^2*f^4*g + 30*(f*x + e)^(7/2)*a*b^5*c*d*f^4*g + 9*(
f*x + e)^(7/2)*a^2*b^4*d^2*f^4*g - 385*(f*x + e)^(5/2)*b^6*c^2*e*f^4*g - 9
90*(f*x + e)^(5/2)*a*b^5*c*d*e*f^4*g - 209*(f*x + e)^(5/2)*a^2*b^4*d^2*e*f
^4*g + 511*(f*x + e)^(3/2)*b^6*c^2*e^2*f^4*g + 2482*(f*x + e)^(3/2)*a*b^5*
c*d*e^2*f^4*g + 751*(f*x + e)^(3/2)*a^2*b^4*d^2*e^2*f^4*g - 279*sqrt(f*x +
 e)*b^6*c^2*e^3*f^4*g - 1554*sqrt(f*x + e)*a*b^5*c*d*e^3*f^4*g - 567*sqrt(
f*x + e)*a^2*b^4*d^2*e^3*f^4*g + 385*(f*x + e)^(5/2)*a*b^5*c^2*f^5*g + ...
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 1676, normalized size of antiderivative = 2.57

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(1/2)*(a + b*x)^5),x)
```

output
```
(((e + f*x)^(7/2)*(35*b^3*c^2*f^4*g + 5*a^3*d^2*f^4*h + 5*a*b^2*c^2*f^4*h
+ 3*a^2*b*d^2*f^4*g - 40*b^3*c^2*e*f^3*h - 64*b^3*d^2*e^3*f*h + 48*b^3*d^2
*e^2*f^2*g + 48*a*b^2*d^2*e^2*f^2*h + 10*a*b^2*c*d*f^4*g + 6*a^2*b*c*d*f^4
*h - 80*b^3*c*d*e*f^3*g - 16*a*b^2*d^2*e*f^3*g - 24*a^2*b*d^2*e*f^3*h + 96
*b^3*c*d*e^2*f^2*h - 32*a*b^2*c*d*e*f^3*h))/(64*(a*f - b*e)^4) - ((e + f*x
)^(1/2)*(5*a^3*d^2*f^4*h - 93*b^3*c^2*f^4*g + 5*a*b^2*c^2*f^4*h + 3*a^2*b*
d^2*f^4*g + 88*b^3*c^2*e*f^3*h + 64*b^3*d^2*e^3*f*h - 80*b^3*d^2*e^2*f^2*g
 + 48*a*b^2*d^2*e^2*f^2*h + 10*a*b^2*c*d*f^4*g + 6*a^2*b*c*d*f^4*h + 176*b
^3*c*d*e*f^3*g - 16*a*b^2*d^2*e*f^3*g - 24*a^2*b*d^2*e*f^3*h - 160*b^3*c*d
*e^2*f^2*h - 32*a*b^2*c*d*e*f^3*h))/(64*b^3*(a*f - b*e)) + ((e + f*x)^(5/2
)*(385*b^3*c^2*f^4*g - 73*a^3*d^2*f^4*h + 55*a*b^2*c^2*f^4*h + 33*a^2*b*d^
2*f^4*g - 440*b^3*c^2*e*f^3*h - 576*b^3*d^2*e^3*f*h + 528*b^3*d^2*e^2*f^2*
g + 144*a*b^2*d^2*e^2*f^2*h + 110*a*b^2*c*d*f^4*g + 66*a^2*b*c*d*f^4*h - 8
80*b^3*c*d*e*f^3*g - 176*a*b^2*d^2*e*f^3*g + 120*a^2*b*d^2*e*f^3*h + 1056*
b^3*c*d*e^2*f^2*h - 352*a*b^2*c*d*e*f^3*h))/(192*b*(a*f - b*e)^3) - ((e +
f*x)^(3/2)*(55*a^3*d^2*f^4*h - 511*b^3*c^2*f^4*g - 73*a*b^2*c^2*f^4*h + 33
*a^2*b*d^2*f^4*g + 584*b^3*c^2*e*f^3*h + 576*b^3*d^2*e^3*f*h - 624*b^3*d^2
*e^2*f^2*g + 144*a*b^2*d^2*e^2*f^2*h - 146*a*b^2*c*d*f^4*g + 66*a^2*b*c*d*
f^4*h + 1168*b^3*c*d*e*f^3*g + 80*a*b^2*d^2*e*f^3*g - 264*a^2*b*d^2*e*f^3*
h - 1248*b^3*c*d*e^2*f^2*h + 160*a*b^2*c*d*e*f^3*h))/(192*b^2*(a*f - b*...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6708, normalized size of antiderivative = 10.27

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(1/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**7*d**2*f**4*h + 18*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d*f**4*h - 72*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*e*f**3*h + 9*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**6*b*d**2*f**4*g + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*f**4*h*x + 15*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*f**4*h -
96*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**5*b**2*c*d*e*f**3*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d*f**4*g + 72*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d*f**
4*h*x + 144*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**5*b**2*d**2*e**2*f**2*h - 48*sqrt(b)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*e*f**3*g - 288*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**5*b**2*d**2*e*f**3*h*x + 36*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*f**4*g*x + 90*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2
*f**4*h*x**2 - 120*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt...
```

3.123 $\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 393

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \frac{2(a^3d^3f^3h+3a^2bd^2f^2(dfg-deh-cfh)+3ab^2df(c^2f^2h-d^2e(fg-eh)-cdf(fg-eh))-b^3(c^3f^3h-}{d^4f^4} + \frac{2b(3a^2d^2f^2h+3abdf(dfg-2deh-cfh)+b^2(c^2f^2h-d^2e(2fg-3eh)-cdf(fg-2eh)))(e+fx)^{3/2}}{3d^3f^4} + \frac{2b^2(3adfh+b(dfg-3deh-cfh))(e+fx)^{5/2}}{5d^2f^4} + \frac{2b^3h(e+fx)^{7/2}}{7df^4} + \frac{2(bc-ad)^3(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{9/2}\sqrt{de-cf}}$$

output
```
2*(a^3*d^3*f^3*h+3*a^2*b*d^2*f^2*(-c*f*h-d*e*h+d*f*g)+3*a*b^2*d*f*(c^2*f^2
*h-d^2*e*(-e*h+f*g)-c*d*f*(-e*h+f*g))-b^3*(c^3*f^3*h-d^3*e^2*(-e*h+f*g)-c*
d^2*e*f*(-e*h+f*g)-c^2*d*f^2*(-e*h+f*g)))*(f*x+e)^(1/2)/d^4/f^4+2/3*b*(3*a
^2*d^2*f^2*h+3*a*b*d*f*(-c*f*h-2*d*e*h+d*f*g)+b^2*(c^2*f^2*h-d^2*e*(-3*e*h
+2*f*g)-c*d*f*(-2*e*h+f*g)))*(f*x+e)^(3/2)/d^3/f^4+2/5*b^2*(3*a*d*f*h+b*(-
c*f*h-3*d*e*h+d*f*g))*(f*x+e)^(5/2)/d^2/f^4+2/7*b^3*h*(f*x+e)^(7/2)/d/f^4+
2*(-a*d+b*c)^3*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/
d^(9/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(105a^3d^3f^3h+105a^2bd^2f^2(-3cfh+d(3fg-2eh+fhx))+21ab^2df(15c^2f^2h-5cdf(3fg-}{}$$
$$+\frac{2(-bc+ad)^3(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{9/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)^3*(g + h*x))/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*(105*a^3*d^3*f^3*h + 105*a^2*b*d^2*f^2*(-3*c*f*h + d*(3*f
*g - 2*e*h + f*h*x)) + 21*a*b^2*d*f*(15*c^2*f^2*h - 5*c*d*f*(3*f*g - 2*e*h
 + f*h*x) + d^2*(8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x))) + b
^3*(-105*c^3*f^3*h + 35*c^2*d*f^2*(3*f*g - 2*e*h + f*h*x) - 7*c*d^2*f*(8*e
^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + d^3*(-48*e^3*h + 8*e^2
*f*(7*g + 3*h*x) + 3*f^3*x^2*(7*g + 5*h*x) - 2*e*f^2*x*(14*g + 9*h*x)))))/
(105*d^4*f^4) + (2*(-(b*c) + a*d)^3*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f
*x])/Sqrt[-(d*e) + c*f]])/(d^(9/2)*Sqrt[-(d*e) + c*f])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{170}$$
$$\frac{2\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{2(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$
$$\Big\downarrow \text{27}$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)^2(6bceh-af(7dg-ch)-(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}+\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^3\sqrt{e+fx}}{7df}-\frac{\int -\frac{(a+bx)^2(7adfg-6bceh-acfh+(6adfh+b(7dfg-6deh-7cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{7df}$$

input
```
Int[((a + b*x)^3*(g + h*x))/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{-2f^4(ad-bc)^3(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2\left(\left(\left(\left(\frac{x^2\left(\frac{5hx}{7}+g\right)b^3}{5}+ax\left(\frac{3hx}{5}+g\right)b^2+3a^2\left(\frac{hx}{3}+g\right)b+h\,a^3\right)d^3-3c\left(\frac{x}{\cdots}\right.\right.\right.\right.}{\cdots}$
risch	$\frac{2\left(15h\,b^3d^3f^3x^3+63a\,b^2d^3f^3h\,x^2-21b^3c\,d^2f^3h\,x^2-18b^3d^3e\,f^2h\,x^2+21b^3d^3f^3g\,x^2+105a^2b\,d^3f^3hx-105a\,b^2c\,d^2f^3hx\cdots\right.}{\cdots}$
derivativedivides	$\frac{2\left(-a\,b^2c\,d^2f^2h(fx+e)^{\frac{3}{2}}+\frac{3a\,b^2d^3fh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3c\,d^2fh(fx+e)^{\frac{5}{2}}}{5}+\frac{h\,b^3(fx+e)^{\frac{7}{2}}d^3}{7}+3a\,b^2c\,d^2e\,f^2h\sqrt{fx+e}-\frac{3b^3d^3eh(fx+e)^{\frac{5}{2}}}{5}\cdots\right.}{\cdots}$
default	$\frac{2\left(-a\,b^2c\,d^2f^2h(fx+e)^{\frac{3}{2}}+\frac{3a\,b^2d^3fh(fx+e)^{\frac{5}{2}}}{5}-\frac{b^3c\,d^2fh(fx+e)^{\frac{5}{2}}}{5}+\frac{h\,b^3(fx+e)^{\frac{7}{2}}d^3}{7}+3a\,b^2c\,d^2e\,f^2h\sqrt{fx+e}-\frac{3b^3d^3eh(fx+e)^{\frac{5}{2}}}{5}\cdots\right.}{\cdots}$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-f^4*(a*d-b*c)^3*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+
(((1/5*x^2*(5/7*h*x+g)*b^3+a*x*(3/5*h*x+g)*b^2+3*a^2*(1/3*h*x+g)*b+h*a^3)*
d^3-3*c*(1/9*x*(3/5*h*x+g)*b^2+a*(1/3*h*x+g)*b+a^2*h)*b*d^2+3*c^2*(1/3*(1/
3*h*x+g)*b+a*h)*b^2*d-b^3*c^3*h)*f^3-2*d*((2/15*x*(9/14*h*x+g)*b^2+a*(2/5*
h*x+g)*b+a^2*h)*d^2-c*(1/3*(2/5*h*x+g)*b+a*h)*b*d+1/3*b^2*c^2*h)*b*e*f^2+8
/5*d^2*(((1/7*h*x+1/3*g)*b+a*h)*d-1/3*b*c*h)*b^2*e^2*f-16/35*b^3*d^3*e^3*h
)*(f*x+e)^(1/2)*((c*f-d*e)*d)^(1/2))/((c*f-d*e)*d)^(1/2)/f^4/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(369) = 738$.

Time = 0.11 (sec) , antiderivative size = 1561, normalized size of antiderivative = 3.97

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/105*(105*((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*f^4*g
 - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*f^4*h)*sqrt(d^2
*e - c*d*f)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e)
)/(d*x + c)) + 2*(15*(b^3*d^5*e*f^3 - b^3*c*d^4*f^4)*h*x^3 + 3*(7*(b^3*d^5
*e*f^3 - b^3*c*d^4*f^4)*g - (6*b^3*d^5*e^2*f^2 + (b^3*c*d^4 - 21*a*b^2*d^5
)*e*f^3 - 7*(b^3*c^2*d^3 - 3*a*b^2*c*d^4)*f^4)*h)*x^2 + 7*(8*b^3*d^5*e^3*f
 + 2*(b^3*c*d^4 - 15*a*b^2*d^5)*e^2*f^2 + 5*(b^3*c^2*d^3 - 3*a*b^2*c*d^4 +
 9*a^2*b*d^5)*e*f^3 - 15*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4)*f
^4)*g - (48*b^3*d^5*e^4 + 8*(b^3*c*d^4 - 21*a*b^2*d^5)*e^3*f + 14*(b^3*c^2
*d^3 - 3*a*b^2*c*d^4 + 15*a^2*b*d^5)*e^2*f^2 + 35*(b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - 3*a^3*d^5)*e*f^3 - 105*(b^3*c^4*d - 3*a*b^2*c^3*d
^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*f^4)*h - (7*(4*b^3*d^5*e^2*f^2 + (b^3*c*
d^4 - 15*a*b^2*d^5)*e*f^3 - 5*(b^3*c^2*d^3 - 3*a*b^2*c*d^4)*f^4)*g - (24*b
^3*d^5*e^3*f + 4*(b^3*c*d^4 - 21*a*b^2*d^5)*e^2*f^2 + 7*(b^3*c^2*d^3 - 3*a
*b^2*c*d^4 + 15*a^2*b*d^5)*e*f^3 - 35*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a
^2*b*c*d^4)*f^4)*h)*x)*sqrt(f*x + e))/(d^6*e*f^4 - c*d^5*f^5), -2/105*(105
*((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*f^4*g - (b^3*c^4
 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*f^4*h)*sqrt(-d^2*e + c*d*f
)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) - (15*(b^3*d^5*
e*f^3 - b^3*c*d^4*f^4)*h*x^3 + 3*(7*(b^3*d^5*e*f^3 - b^3*c*d^4*f^4)*g -...
```

Sympy [A] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{b^3h(e+fx)^{\frac{7}{2}}}{7df^3}+\frac{(e+fx)^{\frac{5}{2}}\cdot\left(3ab^2dfh-b^3cfh-3b^3deh+b^3dfg\right)}{5d^2f^3}+\frac{(e+fx)^{\frac{3}{2}}\cdot\left(3a^2bd^2f^2h-3ab^2cdf^2h-6ab^2d^2efh+3ab^2d^2f^2g+b^3c^2f^2h+2b^3cdefh-b^3cdf\cdots\right.}{3d^3f^3}\cdots\right.}{\cdots} \\ \frac{\frac{b^3hx^4}{4d}+\frac{x^3\cdot\left(3ab^2dh-b^3ch+b^3dg\right)}{3d^2}+\frac{x^2\cdot\left(3a^2bd^2h-3ab^2cdh+3ab^2d^2g+b^3c^2h-b^3cdg\right)}{2d^3}+\frac{x\left(a^3d^3h-3a^2bcd^2h+3a^2bd^3g+3ab^2c^2dh-3ab^2cd^2g-b^3c^3h+b^3c\cdots\right.}{d^4}\cdots}{\sqrt{e}} \end{cases}$$

input
```
integrate((b*x+a)**3*(h*x+g)/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b**3*h*(e + f*x)**(7/2)/(7*d*f**3) + (e + f*x)**(5/2)*(3*a*b
**2*d*f*h - b**3*c*f*h - 3*b**3*d*e*h + b**3*d*f*g)/(5*d**2*f**3) + (e + f
*x)**(3/2)*(3*a**2*b*d**2*f**2*h - 3*a*b**2*c*d*f**2*h - 6*a*b**2*d**2*e*f
*h + 3*a*b**2*d**2*f**2*g + b**3*c**2*f**2*h + 2*b**3*c*d*e*f*h - b**3*c*d
*f**2*g + 3*b**3*d**2*e**2*h - 2*b**3*d**2*e*f*g)/(3*d**3*f**3) + sqrt(e +
 f*x)*(a**3*d**3*f**3*h - 3*a**2*b*c*d**2*f**3*h - 3*a**2*b*d**3*e*f**2*h
+ 3*a**2*b*d**3*f**3*g + 3*a*b**2*c**2*d*f**3*h + 3*a*b**2*c*d**2*e*f**2*h
 - 3*a*b**2*c*d**2*f**3*g + 3*a*b**2*d**3*e**2*f*h - 3*a*b**2*d**3*e*f**2*
g - b**3*c**3*f**3*h - b**3*c**2*d*e*f**2*h + b**3*c**2*d*f**3*g - b**3*c*
d**2*e**2*f*h + b**3*c*d**2*e*f**2*g - b**3*d**3*e**3*h + b**3*d**3*e**2*f
*g)/(d**4*f**3) - f*(a*d - b*c)**3*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*
f - d*e)/d))/(d**5*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), ((b**3*h*x**4/(4*d)
 + x**3*(3*a*b**2*d*h - b**3*c*h + b**3*d*g)/(3*d**2) + x**2*(3*a**2*b*d**
2*h - 3*a*b**2*c*d*h + 3*a*b**2*d**2*g + b**3*c**2*h - b**3*c*d*g)/(2*d**3
) + x*(a**3*d**3*h - 3*a**2*b*c*d**2*h + 3*a**2*b*d**3*g + 3*a*b**2*c**2*d
*h - 3*a*b**2*c*d**2*g - b**3*c**3*h + b**3*c**2*d*g)/d**4 - (a*d - b*c)**
3*(c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d**4)/sqr
t(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(369) = 738$.

Time = 0.13 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-2*(b^3*c^3*d*g - 3*a*b^2*c^2*d^2*g + 3*a^2*b*c*d^3*g - a^3*d^4*g - b^3*c^
4*h + 3*a*b^2*c^3*d*h - 3*a^2*b*c^2*d^2*h + a^3*c*d^3*h)*arctan(sqrt(f*x +
 e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^4) + 2/105*(21*(f*x +
e)^(5/2)*b^3*d^6*f^25*g - 70*(f*x + e)^(3/2)*b^3*d^6*e*f^25*g + 105*sqrt(f
*x + e)*b^3*d^6*e^2*f^25*g - 35*(f*x + e)^(3/2)*b^3*c*d^5*f^26*g + 105*(f*
x + e)^(3/2)*a*b^2*d^6*f^26*g + 105*sqrt(f*x + e)*b^3*c*d^5*e*f^26*g - 315
*sqrt(f*x + e)*a*b^2*d^6*e*f^26*g + 105*sqrt(f*x + e)*b^3*c^2*d^4*f^27*g -
 315*sqrt(f*x + e)*a*b^2*c*d^5*f^27*g + 315*sqrt(f*x + e)*a^2*b*d^6*f^27*g
 + 15*(f*x + e)^(7/2)*b^3*d^6*f^24*h - 63*(f*x + e)^(5/2)*b^3*d^6*e*f^24*h
 + 105*(f*x + e)^(3/2)*b^3*d^6*e^2*f^24*h - 105*sqrt(f*x + e)*b^3*d^6*e^3*
f^24*h - 21*(f*x + e)^(5/2)*b^3*c*d^5*f^25*h + 63*(f*x + e)^(5/2)*a*b^2*d^
6*f^25*h + 70*(f*x + e)^(3/2)*b^3*c*d^5*e*f^25*h - 210*(f*x + e)^(3/2)*a*b
^2*d^6*e*f^25*h - 105*sqrt(f*x + e)*b^3*c*d^5*e^2*f^25*h + 315*sqrt(f*x +
e)*a*b^2*d^6*e^2*f^25*h + 35*(f*x + e)^(3/2)*b^3*c^2*d^4*f^26*h - 105*(f*x
 + e)^(3/2)*a*b^2*c*d^5*f^26*h + 105*(f*x + e)^(3/2)*a^2*b*d^6*f^26*h - 10
5*sqrt(f*x + e)*b^3*c^2*d^4*e*f^26*h + 315*sqrt(f*x + e)*a*b^2*c*d^5*e*f^2
6*h - 315*sqrt(f*x + e)*a^2*b*d^6*e*f^26*h - 105*sqrt(f*x + e)*b^3*c^3*d^3
*f^27*h + 315*sqrt(f*x + e)*a*b^2*c^2*d^4*f^27*h - 315*sqrt(f*x + e)*a^2*b
*c*d^5*f^27*h + 105*sqrt(f*x + e)*a^3*d^6*f^27*h)/(d^7*f^28)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$

$$= (e+f\,x)^{5/2}\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{5\,d\,f^4}-\frac{2\,b^3\,h\,(c\,f^5-d\,e\,f^4)}{5\,d^2\,f^8}\right)$$

$$-(e+f\,x)^{3/2}\left(\frac{\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{d\,f^4}-\frac{2\,b^3\,h\,(c\,f^5-d\,e\,f^4)}{d^2\,f^8}\right)(c\,f^5-d\,e\,f^4)}{3\,d\,f^4}-\frac{2\,b\,(a\,f-b\,e)\,(a\,f\,h-2\,b\,e\,h$$

$$+\sqrt{e+f\,x}\left(\frac{(c\,f^5-d\,e\,f^4)\left(\frac{\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{d\,f^4}-\frac{2\,b^3\,h\,(c\,f^5-d\,e\,f^4)}{d^2\,f^8}\right)(c\,f^5-d\,e\,f^4)}{d\,f^4}-\frac{6\,b\,(a\,f-b\,e)\,(a\,f\,h-2\,b\,e\,h+b\,f\,g}{d\,f^4}\right.}{d\,f^4}\right.$$

$$+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)^3\,(c\,h-d\,g)}{\sqrt{c\,f-d\,e}\,(-h\,a^3\,c\,d^3+g\,a^3\,d^4+3\,h\,a^2\,b\,c^2\,d^2-3\,g\,a^2\,b\,c\,d^3-3\,h\,a\,b^2\,c^3\,d+3\,g\,a\,b^2\,c^2\,d^2+h\,b^3\,c^4-g\,b^3\,c^3\,d)}\right)(a\,d-b\,c)^3\,(c\,h}{d^{9/2}\,\sqrt{c\,f-d\,e}}$$

$$+\frac{2\,b^3\,h\,(e+f\,x)^{7/2}}{7\,d\,f^4}$$

input
```
int(((g + h*x)*(a + b*x)^3)/((e + f*x)^(1/2)*(c + d*x)),x)
```

output
```
(e + f*x)^(5/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(5*d*f^4) - (2*b^3*
h*(c*f^5 - d*e*f^4))/(5*d^2*f^8)) - (e + f*x)^(3/2)*((((2*b^3*f*g - 8*b^3*
e*h + 6*a*b^2*f*h)/(d*f^4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5
 - d*e*f^4))/(3*d*f^4) - (2*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d*f^
4)) + (e + f*x)^(1/2)*(((c*f^5 - d*e*f^4)*((((2*b^3*f*g - 8*b^3*e*h + 6*a*
b^2*f*h)/(d*f^4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5 - d*e*f^4
))/(d*f^4) - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d*f^4)))/(d*f^4)
 + (2*(a*f - b*e)^2*(a*f*h - 4*b*e*h + 3*b*f*g))/(d*f^4)) + (2*atan((d^(1/
2)*(e + f*x)^(1/2)*(a*d - b*c)^3*(c*h - d*g))/((c*f - d*e)^(1/2)*(a^3*d^4*
g + b^3*c^4*h - a^3*c*d^3*h - b^3*c^3*d*g - 3*a^2*b*c*d^3*g - 3*a*b^2*c^3*
d*h + 3*a*b^2*c^2*d^2*g + 3*a^2*b*c^2*d^2*h)))*(a*d - b*c)^3*(c*h - d*g))/
(d^(9/2)*(c*f - d*e)^(1/2)) + (2*b^3*h*(e + f*x)^(7/2))/(7*d*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1402, normalized size of antiderivative = 3.57

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(2*( - 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a**3*c*d**3*f**4*h + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**4*f**4*g + 315*sqrt(d)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**
2*f**4*h - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*a**2*b*c*d**3*f**4*g - 315*sqrt(d)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c**3*d*f**4*h + 315*sqrt(
d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**
2*c**2*d**2*f**4*g + 105*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*b**3*c**4*f**4*h - 105*sqrt(d)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**3*c**3*d*f**4*g + 105*s
qrt(e + f*x)*a**3*c*d**4*f**4*h - 105*sqrt(e + f*x)*a**3*d**5*e*f**3*h - 3
15*sqrt(e + f*x)*a**2*b*c**2*d**3*f**4*h + 105*sqrt(e + f*x)*a**2*b*c*d**4
*e*f**3*h + 315*sqrt(e + f*x)*a**2*b*c*d**4*f**4*g + 105*sqrt(e + f*x)*a**
2*b*c*d**4*f**4*h*x + 210*sqrt(e + f*x)*a**2*b*d**5*e**2*f**2*h - 315*sqrt
(e + f*x)*a**2*b*d**5*e*f**3*g - 105*sqrt(e + f*x)*a**2*b*d**5*e*f**3*h*x
+ 315*sqrt(e + f*x)*a*b**2*c**3*d**2*f**4*h - 105*sqrt(e + f*x)*a*b**2*c**
2*d**3*e*f**3*h - 315*sqrt(e + f*x)*a*b**2*c**2*d**3*f**4*g - 105*sqrt(e +
 f*x)*a*b**2*c**2*d**3*f**4*h*x - 42*sqrt(e + f*x)*a*b**2*c*d**4*e**2*f**2
*h + 105*sqrt(e + f*x)*a*b**2*c*d**4*e*f**3*g + 21*sqrt(e + f*x)*a*b**2...
```

3.124 $\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 221

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \frac{2(d(2abdf^2g+a^2df^2h-b^2e(de+2cf)h)-b(de+cf)(2adfh+b(dfg-2deh-cfh)))\sqrt{e+fx}}{d^3f^3} + \frac{2b(2adfh+b(dfg-2deh-cfh))(e+fx)^{3/2}}{3d^2f^3} + \frac{2b^2h(e+fx)^{5/2}}{5df^3} - \frac{2(bc-ad)^2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}\sqrt{de-cf}}$$

output
```
2*(d*(2*a*b*d*f^2*g+a^2*d*f^2*h-b^2*e*(2*c*f+d*e)*h)-b*(c*f+d*e)*(2*a*d*f*
h+b*(-c*f*h-2*d*e*h+d*f*g)))*(f*x+e)^(1/2)/d^3/f^3+2/3*b*(2*a*d*f*h+b*(-c*
f*h-2*d*e*h+d*f*g))*(f*x+e)^(3/2)/d^2/f^3+2/5*b^2*h*(f*x+e)^(5/2)/d/f^3-2*
(-a*d+b*c)^2*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^
(7/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$= \frac{2\sqrt{e+fx}(15a^2d^2f^2h + 10abdf(-3cfh + d(3fg - 2eh + fhx)) + b^2(15c^2f^2h - 5cdf(3fg - 2eh + fhx)}{15d^3f^3}$$
$$+ \frac{2(bc-ad)^2(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)^2*(g + h*x))/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*(15*a^2*d^2*f^2*h + 10*a*b*d*f*(-3*c*f*h + d*(3*f*g - 2*e
*h + f*h*x)) + b^2*(15*c^2*f^2*h - 5*c*d*f*(3*f*g - 2*e*h + f*h*x) + d^2*(
8*e^2*h - 2*e*f*(5*g + 2*h*x) + f^2*x*(5*g + 3*h*x)))))/(15*d^3*f^3) + (2*
(b*c - a*d)^2*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f
]])/(d^(7/2)*Sqrt[-(d*e) + c*f])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \text{ 170}$$
$$\frac{2\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{2(c+dx)\sqrt{e+fx}}dx}{5df} + \frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df} - \frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow \; 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\Big\downarrow 25$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\Big\downarrow 25$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\downarrow \text{25}$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\downarrow \text{25}$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\downarrow \text{25}$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\downarrow \text{25}$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\downarrow \text{25}$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\downarrow \text{25}$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

$$\downarrow \text{25}$$

$$\frac{\int -\frac{(a+bx)(4bceh-af(5dg-ch)-(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}+\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}$$

$$\downarrow \text{25}$$

$$\frac{2h(a+bx)^2\sqrt{e+fx}}{5df}-\frac{\int -\frac{(a+bx)(5adfg-4bceh-acfh+(4adfh+b(5dfg-4deh-5cfh))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}$$

input
```
Int[((a + b*x)^2*(g + h*x))/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{-2f^3(ad-bc)^2(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2\left(\left(\left(\left(\frac{x\left(\frac{3hx}{5}+g\right)b^2}{3}+2a\left(\frac{hx}{3}+g\right)b+a^2h\right)d^2-2\left(\left(\frac{hx}{6}+\frac{g}{2}\right)b+ah\right)cbd+b\right.\right.\right.\right.}{f^3d^3\sqrt{(cf-de)d}}$
risch	$\frac{2(3x^2hb^2d^2f^2+10abd^2f^2hx-5b^2cdf^2hx-4b^2d^2efhx+5b^2d^2f^2gx+15a^2d^2f^2h-30abcdf^2h-20abd^2efh+30abd^2f}{15f^3d^3}$
derivativedivides	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+\frac{2abd^2fh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+a^2d^2f^2h\sqrt{fx+e}-2abcdf^2h\sqrt{f}\right.}{}$
default	$\frac{2\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+\frac{2abd^2fh(fx+e)^{\frac{3}{2}}}{3}-\frac{b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+a^2d^2f^2h\sqrt{fx+e}-2abcdf^2h\sqrt{f}\right.}{}$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/((c*f-d*e)*d)^(1/2)*(-f^3*(a*d-b*c)^2*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/(
(c*f-d*e)*d)^(1/2))+(((1/3*x*(3/5*h*x+g)*b^2+2*a*(1/3*h*x+g)*b+a^2*h)*d^2-
2*((1/6*h*x+1/2*g)*b+a*h)*c*b*d+b^2*c^2*h)*f^2-4/3*d*(((1/5*h*x+1/2*g)*b+a
*h)*d-1/2*b*c*h)*b*e*f+8/15*b^2*d^2*e^2*h)*(f*x+e)^(1/2)*((c*f-d*e)*d)^(1/
2))/f^3/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(201) = 402$.

Time = 0.10 (sec) , antiderivative size = 858, normalized size of antiderivative = 3.88

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^3*g - (b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)*f^3*h)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f + 2*
sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(3*(b^2*d^4*e*f^2 - b^2*
c*d^3*f^3)*h*x^2 - 5*(2*b^2*d^4*e^2*f + (b^2*c*d^3 - 6*a*b*d^4)*e*f^2 - 3*
(b^2*c^2*d^2 - 2*a*b*c*d^3)*f^3)*g + (8*b^2*d^4*e^3 + 2*(b^2*c*d^3 - 10*a*
b*d^4)*e^2*f + 5*(b^2*c^2*d^2 - 2*a*b*c*d^3 + 3*a^2*d^4)*e*f^2 - 15*(b^2*c
^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*f^3)*h + (5*(b^2*d^4*e*f^2 - b^2*c*d^3*f
^3)*g - (4*b^2*d^4*e^2*f + (b^2*c*d^3 - 10*a*b*d^4)*e*f^2 - 5*(b^2*c^2*d^2
 - 2*a*b*c*d^3)*f^3)*h)*x)*sqrt(f*x + e))/(d^5*e*f^3 - c*d^4*f^4), 2/15*(1
5*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^3*g - (b^2*c^3 - 2*a*b*c^2*d + a^
2*c*d^2)*f^3*h)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x
+ e)/(d*f*x + d*e)) + (3*(b^2*d^4*e*f^2 - b^2*c*d^3*f^3)*h*x^2 - 5*(2*b^2*
d^4*e^2*f + (b^2*c*d^3 - 6*a*b*d^4)*e*f^2 - 3*(b^2*c^2*d^2 - 2*a*b*c*d^3)*
f^3)*g + (8*b^2*d^4*e^3 + 2*(b^2*c*d^3 - 10*a*b*d^4)*e^2*f + 5*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + 3*a^2*d^4)*e*f^2 - 15*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c
*d^3)*f^3)*h + (5*(b^2*d^4*e*f^2 - b^2*c*d^3*f^3)*g - (4*b^2*d^4*e^2*f + (
b^2*c*d^3 - 10*a*b*d^4)*e*f^2 - 5*(b^2*c^2*d^2 - 2*a*b*c*d^3)*f^3)*h)*x)*s
qrt(f*x + e))/(d^5*e*f^3 - c*d^4*f^4)]
```

Sympy [A] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{b^2h(e+fx)^{\frac{5}{2}}}{5df^2}+\frac{(e+fx)^{\frac{3}{2}}\cdot\left(2abdfh-b^2cfh-2b^2deh+b^2dfg\right)}{3d^2f^2}+\frac{\sqrt{e+fx}\left(a^2d^2f^2h-2abcdf^2h-2abd^2efh+2abd^2f^2g+b^2c^2f^2h+b^2cdefh-b^2cdf^2g+b^2d^2e^2\right.}{d^3f^2}\right.}{f} \\ \frac{\frac{b^2hx^3}{3d}+\frac{x^2\cdot\left(2abdh-b^2ch+b^2dg\right)}{2d^2}+\frac{x\left(a^2d^2h-2abcdh+2abd^2g+b^2c^2h-b^2cdg\right)}{d^3}-\frac{(ad-bc)^2(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d^3}}{\sqrt{e}} \end{cases}$$

input
```
integrate((b*x+a)**2*(h*x+g)/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b**2*h*(e + f*x)**(5/2)/(5*d*f**2) + (e + f*x)**(3/2)*(2*a*b
*d*f*h - b**2*c*f*h - 2*b**2*d*e*h + b**2*d*f*g)/(3*d**2*f**2) + sqrt(e +
f*x)*(a**2*d**2*f**2*h - 2*a*b*c*d*f**2*h - 2*a*b*d**2*e*f*h + 2*a*b*d**2*
f**2*g + b**2*c**2*f**2*h + b**2*c*d*e*f*h - b**2*c*d*f**2*g + b**2*d**2*e
**2*h - b**2*d**2*e*f*g)/(d**3*f**2) - f*(a*d - b*c)**2*(c*h - d*g)*atan(s
qrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**4*sqrt((c*f - d*e)/d)))/f, Ne(f, 0))
, ((b**2*h*x**3/(3*d) + x**2*(2*a*b*d*h - b**2*c*h + b**2*d*g)/(2*d**2) +
x*(a**2*d**2*h - 2*a*b*c*d*h + 2*a*b*d**2*g + b**2*c**2*h - b**2*c*d*g)/d*
*3 - (a*d - b*c)**2*(c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d
, True))/d**3)/sqrt(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(b^2c^2dg-2\,abcd^2g+a^2d^3g-b^2c^3h+2\,abc^2dh-a^2cd^2h\right)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,d^3}$$
$$+\frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^2d^4f^{13}g-15\,\sqrt{fx+e}b^2d^4ef^{13}g-15\,\sqrt{fx+e}b^2cd^3f^{14}g+30\,\sqrt{fx+e}abd^4f^{14}g+3\,(f\right.}{}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(b^2*c^2*d*g - 2*a*b*c*d^2*g + a^2*d^3*g - b^2*c^3*h + 2*a*b*c^2*d*h - a
^2*c*d^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e + c*
d*f)*d^3) + 2/15*(5*(f*x + e)^(3/2)*b^2*d^4*f^13*g - 15*sqrt(f*x + e)*b^2*
d^4*e*f^13*g - 15*sqrt(f*x + e)*b^2*c*d^3*f^14*g + 30*sqrt(f*x + e)*a*b*d^
4*f^14*g + 3*(f*x + e)^(5/2)*b^2*d^4*f^12*h - 10*(f*x + e)^(3/2)*b^2*d^4*e
*f^12*h + 15*sqrt(f*x + e)*b^2*d^4*e^2*f^12*h - 5*(f*x + e)^(3/2)*b^2*c*d^
3*f^13*h + 10*(f*x + e)^(3/2)*a*b*d^4*f^13*h + 15*sqrt(f*x + e)*b^2*c*d^3*
e*f^13*h - 30*sqrt(f*x + e)*a*b*d^4*e*f^13*h + 15*sqrt(f*x + e)*b^2*c^2*d^
2*f^14*h - 30*sqrt(f*x + e)*a*b*c*d^3*f^14*h + 15*sqrt(f*x + e)*a^2*d^4*f^
14*h)/(d^5*f^15)
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = (e+f\,x)^{3/2}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{3\,d\,f^3}-\frac{2\,b^2\,h\,(c\,f^4-d\,e\,f^3)}{3\,d^2\,f^6}\right) - \sqrt{e+f\,x}\left(\frac{\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d\,f^3}-\frac{2\,b^2\,h\,(c\,f^4-d\,e\,f^3)}{d^2\,f^6}\right)(c\,f^4-d\,e\,f^3)}{d\,f^3} - \frac{2\,(a\,f-b\,e)\,(a\,f\,h-3\,b\,e\,h+2\,b\,f\,g)}{d\,f^3}\right) + \frac{2\,b^2\,h\,(e+f\,x)^{5/2}}{5\,d\,f^3} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)^2\,(c\,h-d\,g)}{\sqrt{c\,f-d\,e}\,(-h\,a^2\,c\,d^2+g\,a^2\,d^3+2\,h\,a\,b\,c^2\,d-2\,g\,a\,b\,c\,d^2-h\,b^2\,c^3+g\,b^2\,c^2\,d)}\right)(a\,d-b\,c)^2\,(c\,h-d\,g)}{d^{7/2}\,\sqrt{c\,f-d\,e}}$$

input
```
int(((g + h*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(c + d*x)),x)
```

output
```
(e + f*x)^(3/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(3*d*f^3) - (2*b^2*h*
(c*f^4 - d*e*f^3))/(3*d^2*f^6)) - (e + f*x)^(1/2)*((((2*b^2*f*g - 6*b^2*e*
h + 4*a*b*f*h)/(d*f^3) - (2*b^2*h*(c*f^4 - d*e*f^3))/(d^2*f^6))*(c*f^4 - d
*e*f^3))/(d*f^3) - (2*(a*f - b*e)*(a*f*h - 3*b*e*h + 2*b*f*g))/(d*f^3)) +
(2*b^2*h*(e + f*x)^(5/2))/(5*d*f^3) + (2*atan((d^(1/2)*(e + f*x)^(1/2)*(a*
d - b*c)^2*(c*h - d*g))/((c*f - d*e)^(1/2)*(a^2*d^3*g - b^2*c^3*h - a^2*c*
d^2*h + b^2*c^2*d*g - 2*a*b*c*d^2*g + 2*a*b*c^2*d*h)))*(a*d - b*c)^2*(c*h
- d*g))/(d^(7/2)*(c*f - d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.43

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x)
```

output

```
(2*( - 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*a**2*c*d**2*f**3*h + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**3*f**3*g + 30*sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d*f**3*h
- 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*
e)))*a*b*c*d**2*f**3*g - 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)
/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**3*f**3*h + 15*sqrt(d)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**2*d*f**3*g + 15*
sqrt(e + f*x)*a**2*c*d**3*f**3*h - 15*sqrt(e + f*x)*a**2*d**4*e*f**2*h - 3
0*sqrt(e + f*x)*a*b*c**2*d**2*f**3*h + 10*sqrt(e + f*x)*a*b*c*d**3*e*f**2*
h + 30*sqrt(e + f*x)*a*b*c*d**3*f**3*g + 10*sqrt(e + f*x)*a*b*c*d**3*f**3*
h*x + 20*sqrt(e + f*x)*a*b*d**4*e**2*f*h - 30*sqrt(e + f*x)*a*b*d**4*e*f**
2*g - 10*sqrt(e + f*x)*a*b*d**4*e*f**2*h*x + 15*sqrt(e + f*x)*b**2*c**3*d*
f**3*h - 5*sqrt(e + f*x)*b**2*c**2*d**2*e*f**2*h - 15*sqrt(e + f*x)*b**2*c
**2*d**2*f**3*g - 5*sqrt(e + f*x)*b**2*c**2*d**2*f**3*h*x - 2*sqrt(e + f*x
)*b**2*c*d**3*e**2*f*h + 5*sqrt(e + f*x)*b**2*c*d**3*e*f**2*g + sqrt(e + f
*x)*b**2*c*d**3*e*f**2*h*x + 5*sqrt(e + f*x)*b**2*c*d**3*f**3*g*x + 3*sqrt
(e + f*x)*b**2*c*d**3*f**3*h*x**2 - 8*sqrt(e + f*x)*b**2*d**4*e**3*h + 10*
sqrt(e + f*x)*b**2*d**4*e**2*f*g + 4*sqrt(e + f*x)*b**2*d**4*e**2*f*h*x -
5*sqrt(e + f*x)*b**2*d**4*e*f**2*g*x - 3*sqrt(e + f*x)*b**2*d**4*e*f**2...
```

3.125 $\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}} \, dx = -\frac{2(b(de+cf)h-df(bg+ah))\sqrt{e+fx}}{d^2f^2}+\frac{2bh(e+fx)^{3/2}}{3df^2} + \frac{2(bc-ad)(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}\sqrt{de-cf}}$$

output
```
-2*(b*(c*f+d*e)*h-d*f*(a*h+b*g))*(f*x+e)^(1/2)/d^2/f^2+2/3*b*h*(f*x+e)^(3/
2)/d/f^2+2*(-a*d+b*c)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^
(1/2))/d^(5/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}} \, dx = \frac{2\sqrt{e+fx}(-3bcfh+3adfh+bd(3fg-2eh+fhx))}{3d^2f^2} + \frac{2(-bc+ad)(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}\sqrt{-de+cf}}$$

input
```
Integrate[((a + b*x)*(g + h*x))/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*(-3*b*c*f*h + 3*a*d*f*h + b*d*(3*f*g - 2*e*h + f*h*x)))/(
3*d^2*f^2) + (2*(-(b*c) + a*d)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/
Sqrt[-(d*e) + c*f]])/(d^(5/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{164}$$

$$-\frac{(bc-ad)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d^2}-\frac{2\sqrt{e+fx}(-3df(ah+bg)+3bcfh+2bdeh-bdfhx)}{3d^2f^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2(bc-ad)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2f}-\frac{2\sqrt{e+fx}(-3df(ah+bg)+3bcfh+2bdeh-bdfhx)}{3d^2f^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(bc-ad)(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}\sqrt{de-cf}}-\frac{2\sqrt{e+fx}(-3df(ah+bg)+3bcfh+2bdeh-bdfhx)}{3d^2f^2}$$

input
```
Int[((a + b*x)*(g + h*x))/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(-2*Sqrt[e + f*x]*(2*b*d*e*h + 3*b*c*f*h - 3*d*f*(b*g + a*h) - b*d*f*h*x))
/(3*d^2*f^2) + (2*(b*c - a*d)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/
Sqrt[d*e - c*f]])/(d^(5/2)*Sqrt[d*e - c*f])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2(hbdfx+3adfh-3bcfh-2bdeh+3bgdf)\sqrt{fx+e}}{3f^2d^2}-\frac{2(acdh-a\,d^2g-b\,c^2h+bcdg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d^2\sqrt{(cf-de)d}}$
pseudoelliptic	$\frac{-2f^2(ch-dg)(ad-bc)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2\left(\left(\left(\left(\frac{hx}{3}+g\right)b+ah\right)f-\frac{2ehb}{3}\right)d-bcfh\right)\sqrt{fx+e}\,\sqrt{(cf-de)d}}{f^2d^2\sqrt{(cf-de)d}}$
derivativedivides	$\frac{\frac{2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^2}-\frac{2f^2\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d^2\sqrt{(cf-de)d}}}{f^2}$
default	$\frac{\frac{2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^2}-\frac{2f^2\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d^2\sqrt{(cf-de)d}}}{f^2}$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*d*f*h*x+3*a*d*f*h-3*b*c*f*h-2*b*d*e*h+3*b*d*f*g)*(f*x+e)^(1/2)/f^2/
d^2-2*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/d^2/((c*f-d*e)*d)^(1/2)*arctan(d*(
f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$
$$=\left[\frac{3\left(\left(bcd-ad^2\right)f^2g-\left(bc^2-acd\right)f^2h\right)\sqrt{d^2e-cdf}\log\left(\frac{dfx+2\,de-cf+2\,\sqrt{d^2e-cdf}\sqrt{fx+e}}{dx+c}\right)+2\left(\left(bd^3ef-bcd^2f^2\right.\right.}{3\left(d^4ef^2-cd^3f\right.}\right.$$
$$-\frac{2\left(3\left(\left(bcd-ad^2\right)f^2g-\left(bc^2-acd\right)f^2h\right)\sqrt{-d^2e+cdf}\arctan\left(\frac{\sqrt{-d^2e+cdf}\sqrt{fx+e}}{dfx+de}\right)-\left(\left(bd^3ef-bcd^2f^2\right)h\right.\right.}{3\left(d^4ef^2-cd^3f\right.}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*((b*c*d - a*d^2)*f^2*g - (b*c^2 - a*c*d)*f^2*h)*sqrt(d^2*e - c*d*f
)*log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c
)) + 2*((b*d^3*e*f - b*c*d^2*f^2)*h*x + 3*(b*d^3*e*f - b*c*d^2*f^2)*g - (2
*b*d^3*e^2 + (b*c*d^2 - 3*a*d^3)*e*f - 3*(b*c^2*d - a*c*d^2)*f^2)*h)*sqrt(
f*x + e))/(d^4*e*f^2 - c*d^3*f^3), -2/3*(3*((b*c*d - a*d^2)*f^2*g - (b*c^2
 - a*c*d)*f^2*h)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x
 + e)/(d*f*x + d*e)) - ((b*d^3*e*f - b*c*d^2*f^2)*h*x + 3*(b*d^3*e*f - b*c
*d^2*f^2)*g - (2*b*d^3*e^2 + (b*c*d^2 - 3*a*d^3)*e*f - 3*(b*c^2*d - a*c*d^
2)*f^2)*h)*sqrt(f*x + e))/(d^4*e*f^2 - c*d^3*f^3)]
```

Sympy [A] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{bh(e+fx)^{\frac{3}{2}}}{3df} + \frac{\sqrt{e+fx}(adfh-bcfh-bdeh+bdfg)}{d^2f} - \frac{f(ad-bc)(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^3\sqrt{\frac{cf-de}{d}}}\right)}{f} & \text{for } f \neq 0 \\ \frac{\frac{bhx^2}{2d} + \frac{x(adh-bch+bdg)}{d^2} - \frac{(ad-bc)(ch-dg)\left(\begin{cases} \frac{x}{c} & \text{for } d = 0 \\ \frac{\log(c+dx)}{d} & \text{otherwise} \end{cases}\right)}{d^2}}{\sqrt{e}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(b*h*(e + f*x)**(3/2)/(3*d*f) + sqrt(e + f*x)*(a*d*f*h - b*c*
f*h - b*d*e*h + b*d*f*g)/(d**2*f) - f*(a*d - b*c)*(c*h - d*g)*atan(sqrt(e
+ f*x)/sqrt((c*f - d*e)/d))/(d**3*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), ((b*
h*x**2/(2*d) + x*(a*d*h - b*c*h + b*d*g)/d**2 - (a*d - b*c)*(c*h - d*g)*Pi
ecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d**2)/sqrt(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = -\frac{2\,(bcdg-ad^2g-bc^2h+acdh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,d^2} + \frac{2\left(3\sqrt{fx+e}bd^2f^5g+(fx+e)^{\frac{3}{2}}bd^2f^4h-3\sqrt{fx+e}bd^2ef^4h-3\sqrt{fx+e}bcdf^5h+3\sqrt{fx+e}ad^2f^5h\right.}{3\,d^3f^6}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-2*(b*c*d*g - a*d^2*g - b*c^2*h + a*c*d*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^
2*e + c*d*f))/(sqrt(-d^2*e + c*d*f)*d^2) + 2/3*(3*sqrt(f*x + e)*b*d^2*f^5*
g + (f*x + e)^(3/2)*b*d^2*f^4*h - 3*sqrt(f*x + e)*b*d^2*e*f^4*h - 3*sqrt(f
*x + e)*b*c*d*f^5*h + 3*sqrt(f*x + e)*a*d^2*f^5*h)/(d^3*f^6)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,a\,f\,h-4\,b\,e\,h+2\,b\,f\,g}{d\,f^2}-\frac{2\,b\,h\,(c\,f^3-d\,e\,f^2)}{d^2\,f^4}\right)+\frac{2\,b\,h\,(e+f\,x)^{3/2}}{3\,d\,f^2}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)\,(c\,h-d\,g)}{\sqrt{c\,f-d\,e}\,(a\,d^2\,g+b\,c^2\,h-a\,c\,d\,h-b\,c\,d\,g)}\right)\,(a\,d-b\,c)\,(c\,h-d\,g)}{d^{5/2}\,\sqrt{c\,f-d\,e}}$$

input
```
int(((g + h*x)*(a + b*x))/((e + f*x)^(1/2)*(c + d*x)),x)
```

output
```
(e + f*x)^(1/2)*((2*a*f*h - 4*b*e*h + 2*b*f*g)/(d*f^2) - (2*b*h*(c*f^3 - d
*e*f^2))/(d^2*f^4)) + (2*b*h*(e + f*x)^(3/2))/(3*d*f^2) + (2*atan((d^(1/2)
*(e + f*x)^(1/2)*(a*d - b*c)*(c*h - d*g))/((c*f - d*e)^(1/2)*(a*d^2*g + b*
c^2*h - a*c*d*h - b*c*d*g)))*(a*d - b*c)*(c*h - d*g))/(d^(5/2)*(c*f - d*e)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.76

$$\int \frac{(a+bx)(g+hx)}{(c+dx)\sqrt{e+fx}}\,dx = \frac{-2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)acd\,f^2h+2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)a\,d^2f^2g+2\sqrt{d}\,\sqrt{cf-de}\,a}{}$$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(2*( - 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*a*c*d*f**2*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(
sqrt(d)*sqrt(c*f - d*e)))*a*d**2*f**2*g + 3*sqrt(d)*sqrt(c*f - d*e)*atan((
sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*f**2*h - 3*sqrt(d)*sqrt
(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d*f**2*g
 + 3*sqrt(e + f*x)*a*c*d**2*f**2*h - 3*sqrt(e + f*x)*a*d**3*e*f*h - 3*sqrt
(e + f*x)*b*c**2*d*f**2*h + sqrt(e + f*x)*b*c*d**2*e*f*h + 3*sqrt(e + f*x)
*b*c*d**2*f**2*g + sqrt(e + f*x)*b*c*d**2*f**2*h*x + 2*sqrt(e + f*x)*b*d**
3*e**2*h - 3*sqrt(e + f*x)*b*d**3*e*f*g - sqrt(e + f*x)*b*d**3*e*f*h*x))/(
3*d**3*f**2*(c*f - d*e))
```

3.126 $\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx=\frac{2h\sqrt{e+fx}}{df}-\frac{2(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}\sqrt{de-cf}}$$

output
```
2*h*(f*x+e)^(1/2)/d/f-2*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e
)^(1/2))/d^(3/2)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx=\frac{2h\sqrt{e+fx}}{df}+\frac{2(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{3/2}\sqrt{-de+cf}}$$

input
```
Integrate[(g + h*x)/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*h*Sqrt[e + f*x])/(d*f) + (2*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/
Sqrt[-(d*e) + c*f]])/(d^(3/2)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \ 90$$

$$\frac{(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d}+\frac{2h\sqrt{e+fx}}{df}$$

$$\Big\downarrow \ 73$$

$$\frac{2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df}+\frac{2h\sqrt{e+fx}}{df}$$

$$\Big\downarrow \ 221$$

$$\frac{2h\sqrt{e+fx}}{df}-\frac{2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}\sqrt{de-cf}}$$

input
```
Int[(g + h*x)/((c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*h*Sqrt[e + f*x])/(d*f) - (2*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])
/Sqrt[d*e - c*f]])/(d^(3/2)*Sqrt[d*e - c*f])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{2h\sqrt{fx+e}-\frac{2f(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{df}$	64
risch	$\frac{2h\sqrt{fx+e}}{df}-\frac{2(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d\sqrt{(cf-de)d}}$	65
derivativedivides	$\frac{\frac{2h\sqrt{fx+e}}{d}-\frac{2f(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d\sqrt{(cf-de)d}}}{f}$	67
default	$\frac{\frac{2h\sqrt{fx+e}}{d}-\frac{2f(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d\sqrt{(cf-de)d}}}{f}$	67

input
```
int((h*x+g)/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d*(h*(f*x+e)^(1/2)-f*(c*h-d*g)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1
/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.81

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx = \left[\frac{2\left(d^2e-cdf\right)\sqrt{fx+e}h-\sqrt{d^2e-cdf}(dfg-cfh)\log\left(\frac{dfx+2\,de-cf+2\,\sqrt{d^2e-cdf}\sqrt{fx+e}}{dx+c}\right)}{d^3ef-cd^2f^2}, \frac{2\left(\left(d^2e-cdf\right)\sqrt{f\right.}{}}{}\right.$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[(2*(d^2*e - c*d*f)*sqrt(f*x + e)*h - sqrt(d^2*e - c*d*f)*(d*f*g - c*f*h)*
log((d*f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c))
)/(d^3*e*f - c*d^2*f^2), 2*((d^2*e - c*d*f)*sqrt(f*x + e)*h + sqrt(-d^2*e
+ c*d*f)*(d*f*g - c*f*h)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x
+ d*e)))/(d^3*e*f - c*d^2*f^2)]
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx = \begin{cases} \frac{2\left(\frac{h\sqrt{e+fx}}{d}-\frac{f(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^2\sqrt{\frac{cf-de}{d}}}\right)}{f} & \text{for}\ f\neq 0 \\ \frac{\frac{hx}{d}-\frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for}\ d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d}}{\sqrt{e}} & \text{otherwise} \end{cases}$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(h*sqrt(e + f*x)/d - f*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c
*f - d*e)/d))/(d**2*sqrt((c*f - d*e)/d)))/f, Ne(f, 0)), ((h*x/d - (c*h - d
*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d)/sqrt(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx = \frac{2\,(dg-ch)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,d} + \frac{2\,\sqrt{fx+e}h}{df}$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(d*g - c*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e +
c*d*f)*d) + 2*sqrt(f*x + e)*h/(d*f)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx=\frac{2\,h\,\sqrt{e+f\,x}}{d\,f}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(c\,h-d\,g\right)}{d^{3/2}\,\sqrt{c\,f-d\,e}}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(c + d*x)),x)
```

output
```
(2*h*(e + f*x)^(1/2))/(d*f) - (2*atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e
)^(1/2))*(c*h - d*g))/(d^(3/2)*(c*f - d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.65

$$\int \frac{g+hx}{(c+dx)\sqrt{e+fx}}\,dx$$
$$=\frac{-2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cfh+2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)dfg+2\sqrt{fx+e}\,cdfh-2\sqrt{fx}}{d^2f\,(cf-de)}$$

input
```
int((h*x+g)/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(2*( - sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*c*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*d*f*g + sqrt(e + f*x)*c*d*f*h - sqrt(e + f*x)*d**2*e*h))/(d
**2*f*(c*f - d*e))
```

3.127 $\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 131

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = -\frac{2(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)\sqrt{be-af}} + \frac{2(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)\sqrt{de-cf}}$$

output
```
-2*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(1/2)/(-a*
d+b*c)/(-a*f+b*e)^(1/2)+2*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d
*e)^(1/2))/d^(1/2)/(-a*d+b*c)/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = \frac{2(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{b}(bc-ad)\sqrt{-be+af}} + \frac{2(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-bc+ad)\sqrt{-de+cf}}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(Sqrt[b
]*(b*c - a*d)*Sqrt[-(b*e) + a*f]) + (2*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e
+ f*x])/Sqrt[-(d*e) + c*f]])/(Sqrt[d]*(-(b*c) + a*d)*Sqrt[-(d*e) + c*f])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{174}$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}$$

$$\Big\downarrow \text{73}$$

$$\frac{2(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)\sqrt{de-cf}}-\frac{2(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)\sqrt{be-af}}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(-2*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]
*(b*c - a*d)*Sqrt[b*e - a*f]) + (2*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f
*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{\frac{2(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{\sqrt{(af-be)b}}-\frac{2(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{ad-bc}$	101
derivativedivides	$\frac{2(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)\sqrt{(af-be)b}}+\frac{2(-ch+dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)\sqrt{(cf-de)d}}$	110
default	$\frac{2(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)\sqrt{(af-be)b}}+\frac{2(-ch+dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)\sqrt{(cf-de)d}}$	110

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(a*d-b*c)*((a*h-b*g)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*
e)*b)^(1/2))-(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))/((c*f-d
*e)*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(111) = 222$.

Time = 0.34 (sec) , antiderivative size = 971, normalized size of antiderivative = 7.41

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input

```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output

```
[(sqrt(b^2*e - a*b*f)*((b*d^2*e - b*c*d*f)*g - (a*d^2*e - a*c*d*f)*h)*log(
(b*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + s
qrt(d^2*e - c*d*f)*((b^2*d*e - a*b*d*f)*g - (b^2*c*e - a*b*c*f)*h)*log((d*
f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)))/((b^3
*c*d^2 - a*b^2*d^3)*e^2 - (b^3*c^2*d - a^2*b*d^3)*e*f + (a*b^2*c^2*d - a^2
*b*c*d^2)*f^2), (2*sqrt(-b^2*e + a*b*f)*((b*d^2*e - b*c*d*f)*g - (a*d^2*e
- a*c*d*f)*h)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x + e)/(b*f*x + b*e)) + s
qrt(d^2*e - c*d*f)*((b^2*d*e - a*b*d*f)*g - (b^2*c*e - a*b*c*f)*h)*log((d*
f*x + 2*d*e - c*f + 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)))/((b^3
*c*d^2 - a*b^2*d^3)*e^2 - (b^3*c^2*d - a^2*b*d^3)*e*f + (a*b^2*c^2*d - a^2
*b*c*d^2)*f^2), -(2*sqrt(-d^2*e + c*d*f)*((b^2*d*e - a*b*d*f)*g - (b^2*c*e
 - a*b*c*f)*h)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) -
sqrt(b^2*e - a*b*f)*((b*d^2*e - b*c*d*f)*g - (a*d^2*e - a*c*d*f)*h)*log((b
*f*x + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)))/((b^
3*c*d^2 - a*b^2*d^3)*e^2 - (b^3*c^2*d - a^2*b*d^3)*e*f + (a*b^2*c^2*d - a^
2*b*c*d^2)*f^2), 2*(sqrt(-b^2*e + a*b*f)*((b*d^2*e - b*c*d*f)*g - (a*d^2*e
 - a*c*d*f)*h)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x + e)/(b*f*x + b*e)) -
sqrt(-d^2*e + c*d*f)*((b^2*d*e - a*b*d*f)*g - (b^2*c*e - a*b*c*f)*h)*arcta
n(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)))/((b^3*c*d^2 - a*b^2*d
^3)*e^2 - (b^3*c^2*d - a^2*b*d^3)*e*f + (a*b^2*c^2*d - a^2*b*c*d^2)*f^2...
```

Sympy [A] (verification not implemented)

Time = 20.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.24

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = \begin{cases} \frac{2\left(-\frac{f(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d\sqrt{\frac{cf-de}{d}}(ad-bc)}+\frac{f(ah-bg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{af-be}{b}}}\right)}{b\sqrt{\frac{af-be}{b}}(ad-bc)}\right)}{f} & \text{for } f\neq 0 \\ \frac{\frac{(ah-bg)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{ad-bc}-\frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{ad-bc}}{\sqrt{e}} & \text{otherwise} \end{cases}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Piecewise((2*(-f*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d*sq
rt((c*f - d*e)/d)*(a*d - b*c)) + f*(a*h - b*g)*atan(sqrt(e + f*x)/sqrt((a*
f - b*e)/b))/(b*sqrt((a*f - b*e)/b)*(a*d - b*c)))/f, Ne(f, 0)), (((a*h - b
*g)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/(a*d - b*c) - (c*h
- d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/(a*d - b*c))/sqr
t(e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = \frac{2\,(bg-ah)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{\sqrt{-b^2e+abf}(bc-ad)} - \frac{2\,(dg-ch)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}(bc-ad)}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(b*g - a*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/(sqrt(-b^2*e +
a*b*f)*(b*c - a*d)) - 2*(d*g - c*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c
*d*f))/(sqrt(-d^2*e + c*d*f)*(b*c - a*d))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 3821, normalized size of antiderivative = 29.17

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)),x)
```

output
```
(atan(((((e + f*x)^(1/2)*(16*b^3*d^3*f^2*g^2 + 8*a^2*b*d^3*f^2*h^2 + 8*b^3
*c^2*d*f^2*h^2 - 16*a*b^2*d^3*f^2*g*h - 16*b^3*c*d^2*f^2*g*h) - ((-d*(c*f
- d*e))^(1/2)*(c*h - d*g)*(((e + f*x)^(1/2)*(-d*(c*f - d*e))^(1/2)*(c*h -
d*g)*(8*a*b^4*c^2*d^3*f^3 - 8*b^5*c^3*d^2*f^3 - 8*a^3*b^2*d^5*f^3 + 8*a^2*
b^3*c*d^4*f^3 + 16*a^2*b^3*d^5*e*f^2 + 16*b^5*c^2*d^3*e*f^2 - 32*a*b^4*c*d
^4*e*f^2))/(a*d^3*e - a*c*d^2*f - b*c*d^2*e + b*c^2*d*f) - 8*a^2*b^2*d^4*f
^3*g - 8*b^4*c^2*d^2*f^3*g + 8*a^2*b^2*d^4*e*f^2*h + 8*b^4*c^2*d^2*e*f^2*h
 + 16*a*b^3*c*d^3*f^3*g - 16*a*b^3*c*d^3*e*f^2*h))/(a*d^3*e - a*c*d^2*f -
b*c*d^2*e + b*c^2*d*f))*(-d*(c*f - d*e))^(1/2)*(c*h - d*g)*1i)/(a*d^3*e -
a*c*d^2*f - b*c*d^2*e + b*c^2*d*f) + (((e + f*x)^(1/2)*(16*b^3*d^3*f^2*g^2
 + 8*a^2*b*d^3*f^2*h^2 + 8*b^3*c^2*d*f^2*h^2 - 16*a*b^2*d^3*f^2*g*h - 16*b
^3*c*d^2*f^2*g*h) - ((-d*(c*f - d*e))^(1/2)*(c*h - d*g)*(((e + f*x)^(1/2)*
(-d*(c*f - d*e))^(1/2)*(c*h - d*g)*(8*a*b^4*c^2*d^3*f^3 - 8*b^5*c^3*d^2*f^
3 - 8*a^3*b^2*d^5*f^3 + 8*a^2*b^3*c*d^4*f^3 + 16*a^2*b^3*d^5*e*f^2 + 16*b^
5*c^2*d^3*e*f^2 - 32*a*b^4*c*d^4*e*f^2))/(a*d^3*e - a*c*d^2*f - b*c*d^2*e
+ b*c^2*d*f) + 8*a^2*b^2*d^4*f^3*g + 8*b^4*c^2*d^2*f^3*g - 8*a^2*b^2*d^4*e
*f^2*h - 8*b^4*c^2*d^2*e*f^2*h - 16*a*b^3*c*d^3*f^3*g + 16*a*b^3*c*d^3*e*f
^2*h))/(a*d^3*e - a*c*d^2*f - b*c*d^2*e + b*c^2*d*f))*(-d*(c*f - d*e))^(1/
2)*(c*h - d*g)*1i)/(a*d^3*e - a*c*d^2*f - b*c*d^2*e + b*c^2*d*f))/((((e +
f*x)^(1/2)*(16*b^3*d^3*f^2*g^2 + 8*a^2*b*d^3*f^2*h^2 + 8*b^3*c^2*d*f^2*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.09

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt{e+fx}}\,dx$$

$$= \frac{2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)acdfh - 2\sqrt{b}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a\,d^2eh - 2\sqrt{b}\,\sqrt{af-be}\,atan\left(\cdots\right.}{\cdots}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/2),x)
```

output

```
(2*(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a*c*d*f*h - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a*d**2*e*h - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*b*c*d*f*g + sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*d**2*e*g - sqrt(d)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*f*h + sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*d*f*
g + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*b**2*c*e*h - sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*b**2*d*e*g))/(b*d*(a**2*c*d*f**2 - a**2*d**2*e*f - a*b*c*
*2*f**2 + a*b*d**2*e**2 + b**2*c**2*e*f - b**2*c*d*e**2))
```

3.128 $\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 209

$$\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx = -\frac{(bg-ah)\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)} + \frac{\left(a^2dfh-abf(3dg-ch)+b^2(2deg+cfg-2ceh)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(bc-ad)^2(be-af)^{3/2}} - \frac{2\sqrt{d}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^2\sqrt{de-cf}}$$

output
```
-(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)+(a^2*d*f*h-a*b*f*(
-c*h+3*d*g)+b^2*(-2*c*e*h+c*f*g+2*d*e*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-
a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)-2*d^(1/2)*(-c*h+d*g)
*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^2/(-c*f+d*e)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.93

$$\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx = \frac{\frac{(bc-ad)(-bg+ah)\sqrt{e+fx}}{(be-af)(a+bx)}+\frac{(a^2dfh+abf(-3dg+ch)+b^2(2deg+cfg-2ceh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{b}(-be+af)^{3/2}}+\frac{2\sqrt{d}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{-de+cf}}}{(bc-ad)^2}$$

input
```
Integrate[(g + h*x)/((a + b*x)^2*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(((b*c - a*d)*(-(b*g) + a*h)*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x)) + ((a^
2*d*f*h + a*b*f*(-3*d*g + c*h) + b^2*(2*d*e*g + c*f*g - 2*c*e*h))*ArcTan[(
Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(Sqrt[b]*(-(b*e) + a*f)^(3/2))
 + (2*Sqrt[d]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f
]])/Sqrt[-(d*e) + c*f])/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int -\frac{af(2dg-ch)-b(2deg+cfg-2ceh)-df(bg-ah)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{af(2dg-ch)-b(2deg+cfg-2ceh)-df(bg-ah)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{2d(be-af)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}-\frac{(a^2dfh-abf(3dg-ch)+b^2(-2ceh+cfg+2deg))\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{4d(be-af)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2(a^2dfh-abf(3dg-ch)+b^2(-2ceh+cfg+2deg))\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)(a^2dfh-abf(3dg-ch)+b^2(-2ceh+cfg+2deg))}{\sqrt{b}(bc-ad)\sqrt{be-af}}-\frac{4\sqrt{d}(be-af)(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)\sqrt{de-cf}}}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^2*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
-(((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x))) + ((2*(
a^2*d*f*h - a*b*f*(3*d*g - c*h) + b^2*(2*d*e*g + c*f*g - 2*c*e*h))*ArcTanh
[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c - a*d)*Sqrt[b*e -
 a*f]) - (4*Sqrt[d]*(b*e - a*f)*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x]
)/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/(2*(b*c - a*d)*(b*e - a
*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10

method	result
derivativedivides	$2f\left(-\frac{d(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f(ad-bc)^2\sqrt{(cf-de)d}}+\frac{\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(a^2dfh+abcfh-3abdfg-2b^2ceh+b^2cfg+\ldots\right.}{2(af-be)\sqrt{(af-\ldots}}}{(ad-bc)^2f}\ldots\right.$
default	$2f\left(-\frac{d(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f(ad-bc)^2\sqrt{(cf-de)d}}+\frac{\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(a^2dfh+abcfh-3abdfg-2b^2ceh+b^2cfg+\ldots\right.}{2(af-be)\sqrt{(af-\ldots}}}{(ad-bc)^2f}\ldots\right.$
pseudoelliptic	$\frac{((a^2fh-3bfga+2b^2eg)d+((-2eh+fg)b+afh)cb)(bx+a)\sqrt{(cf-de)d}\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\left(-2d(bx+a)(af-be)(c\ldots\right.}{\sqrt{(af-be)b}\sqrt{(cf-de)d}(af-be)(bx+a)(ad\ldots}$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(-d/f*(c*h-d*g)/(a*d-b*c)^2/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)
/((c*f-d*e)*d)^(1/2))+1/(a*d-b*c)^2/f*(1/2*f*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*
c*g)/(a*f-b*e)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)+1/2*(a^2*d*f*h+a*b*c*f*h-
3*a*b*d*f*g-2*b^2*c*e*h+b^2*c*f*g+2*b^2*d*e*g)/(a*f-b*e)/((a*f-b*e)*b)^(1/
2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(187) = 374$.

Time = 9.62 (sec) , antiderivative size = 2580, normalized size of antiderivative = 12.34

$$\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b^2*e - a*b*f)*((2*a*b^2*d*e + (a*b^2*c - 3*a^2*b*d)*f)*g - (2
*a*b^2*c*e - (a^2*b*c + a^3*d)*f)*h + ((2*b^3*d*e + (b^3*c - 3*a*b^2*d)*f)
*g - (2*b^3*c*e - (a*b^2*c + a^2*b*d)*f)*h)*x)*log((b*f*x + 2*b*e - a*f -
2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*((a*b^3*d*e^2 - 2*a^2*
b^2*d*e*f + a^3*b*d*f^2)*g - (a*b^3*c*e^2 - 2*a^2*b^2*c*e*f + a^3*b*c*f^2)
*h + ((b^4*d*e^2 - 2*a*b^3*d*e*f + a^2*b^2*d*f^2)*g - (b^4*c*e^2 - 2*a*b^3
*c*e*f + a^2*b^2*c*f^2)*h)*x)*sqrt(d/(d*e - c*f))*log((d*f*x + 2*d*e - c*f
 + 2*(d*e - c*f)*sqrt(f*x + e)*sqrt(d/(d*e - c*f)))/(d*x + c)) + 2*(((b^4*
c - a*b^3*d)*e - (a*b^3*c - a^2*b^2*d)*f)*g - ((a*b^3*c - a^2*b^2*d)*e - (
a^2*b^2*c - a^3*b*d)*f)*h)*sqrt(f*x + e))/((a*b^5*c^2 - 2*a^2*b^4*c*d + a^
3*b^3*d^2)*e^2 - 2*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e*f + (a^3*
b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*f^2 + ((b^6*c^2 - 2*a*b^5*c*d + a^2*b
^4*d^2)*e^2 - 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*f + (a^2*b^4*c
^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*f^2)*x), -(sqrt(-b^2*e + a*b*f)*((2*a*b^
2*d*e + (a*b^2*c - 3*a^2*b*d)*f)*g - (2*a*b^2*c*e - (a^2*b*c + a^3*d)*f)*h
 + ((2*b^3*d*e + (b^3*c - 3*a*b^2*d)*f)*g - (2*b^3*c*e - (a*b^2*c + a^2*b*
d)*f)*h)*x)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x + e)/(b*f*x + b*e)) + ((a
*b^3*d*e^2 - 2*a^2*b^2*d*e*f + a^3*b*d*f^2)*g - (a*b^3*c*e^2 - 2*a^2*b^2*c
*e*f + a^3*b*c*f^2)*h + ((b^4*d*e^2 - 2*a*b^3*d*e*f + a^2*b^2*d*f^2)*g - (
b^4*c*e^2 - 2*a*b^3*c*e*f + a^2*b^2*c*f^2)*h)*x)*sqrt(d/(d*e - c*f))*lo...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**2/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.35

$$\begin{aligned}
&\int \frac{g+hx}{(a+bx)^2(c+dx)\sqrt{e+fx}}\,dx \\
&= -\frac{(2\,b^2deg + b^2cfg - 3\,abdfg - 2\,b^2ceh + abcfh + a^2dfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^3c^2e - 2\,ab^2cde + a^2bd^2e - ab^2c^2f + 2\,a^2bcdf - a^3d^2f)\sqrt{-b^2e+abf}} \\
&\quad + \frac{2\,(d^2g - cdh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-d^2e+cdf}} \\
&\quad - \frac{\sqrt{fx+e}bfg - \sqrt{fx+e}afh}{(b^2ce - abde - abcf + a^2df)((fx+e)b - be + af)}
\end{aligned}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-(2*b^2*d*e*g + b^2*c*f*g - 3*a*b*d*f*g - 2*b^2*c*e*h + a*b*c*f*h + a^2*d*
f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^2*e - 2*a*b^2*c*
d*e + a^2*b*d^2*e - a*b^2*c^2*f + 2*a^2*b*c*d*f - a^3*d^2*f)*sqrt(-b^2*e +
 a*b*f)) + 2*(d^2*g - c*d*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)*b*
f*g - sqrt(f*x + e)*a*f*h)/((b^2*c*e - a*b*d*e - a*b*c*f + a^2*d*f)*((f*x
+ e)*b - b*e + a*f))
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 24052, normalized size of antiderivative = 115.08

$$\int \frac{g + hx}{(a + bx)^2(c + dx)\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)),x)
```

output
```
atan(((((2*(4*a^6*b^2*d^7*f^5*g - 2*a*b^7*c^5*d^2*f^5*g - 18*a^5*b^3*c*d^6
*f^5*g - 2*a^6*b^2*c*d^6*f^5*h - 6*a^5*b^3*d^7*e*f^4*g - 2*a^6*b^2*d^7*e*f
^4*h + 2*b^8*c^5*d^2*e*f^4*g + 12*a^2*b^6*c^4*d^3*f^5*g - 28*a^3*b^5*c^3*d
^4*f^5*g + 32*a^4*b^4*c^2*d^5*f^5*g - 2*a^2*b^6*c^5*d^2*f^5*h + 8*a^3*b^5*
c^4*d^3*f^5*h - 12*a^4*b^4*c^3*d^4*f^5*h + 8*a^5*b^3*c^2*d^5*f^5*h + 2*a^4
*b^4*d^7*e^2*f^3*g + 2*a^5*b^3*d^7*e^2*f^3*h + 2*b^8*c^4*d^3*e^2*f^3*g - 4
*b^8*c^5*d^2*e^2*f^3*h + 12*a^2*b^6*c^2*d^5*e^2*f^3*g - 32*a^2*b^6*c^3*d^4
*e^2*f^3*h + 28*a^3*b^5*c^2*d^5*e^2*f^3*h - 14*a*b^7*c^4*d^3*e*f^4*g + 26*
a^4*b^4*c*d^6*e*f^4*g + 6*a*b^7*c^5*d^2*e*f^4*h + 14*a^5*b^3*c*d^6*e*f^4*h
 - 8*a*b^7*c^3*d^4*e^2*f^3*g + 36*a^2*b^6*c^3*d^4*e*f^4*g - 8*a^3*b^5*c*d^
6*e^2*f^3*g - 44*a^3*b^5*c^2*d^5*e*f^4*g + 18*a*b^7*c^4*d^3*e^2*f^3*h - 26
*a^2*b^6*c^4*d^3*e*f^4*h + 44*a^3*b^5*c^3*d^4*e*f^4*h - 12*a^4*b^4*c*d^6*e
^2*f^3*h - 36*a^4*b^4*c^2*d^5*e*f^4*h))/(b^5*c^3*e^2 - a^5*d^3*f^2 + a^2*b
^3*c^3*f^2 - a^3*b^2*d^3*e^2 - 2*a*b^4*c^3*e*f + 2*a^4*b*d^3*e*f - 3*a*b^4
*c^2*d*e^2 + 3*a^4*b*c*d^2*f^2 + 3*a^2*b^3*c*d^2*e^2 - 3*a^3*b^2*c^2*d*f^2
 + 6*a^2*b^3*c^2*d*e*f - 6*a^3*b^2*c*d^2*e*f) - (2*(e + f*x)^(1/2)*((4*b^4
*c^2*e^2*h^2 + b^4*c^2*f^2*g^2 + 4*b^4*d^2*e^2*g^2 + a^4*d^2*f^2*h^2 + a^2
*b^2*c^2*f^2*h^2 + 9*a^2*b^2*d^2*f^2*g^2 + 4*b^4*c*d*e*f*g^2 - 8*b^4*c*d*e
^2*g*h - 4*b^4*c^2*e*f*g*h - 6*a*b^3*c*d*f^2*g^2 + 2*a^3*b*c*d*f^2*h^2 - 4
*a*b^3*c^2*e*f*h^2 - 12*a*b^3*d^2*e*f*g^2 + 2*a*b^3*c^2*f^2*g*h - 6*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2158, normalized size of antiderivative = 10.33

$$\int \frac{g + hx}{(a + bx)^2(c + dx)\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**3*c*d*f**2*h - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**3*d**2*e*f*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*f**2*h - sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d*e*f*h
 - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**2*b*c*d*f**2*g + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d*f**2*h*x + 3*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**2*e*f*g - sqrt(
b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*
b*d**2*e*f*h*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a*b**2*c**2*e*f*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**2*f**2*g + sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**2*f*
*2*h*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a*b**2*c*d*e**2*h + sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d*e*f*g - sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*d*e*f*h*x - 3*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
*b**2*c*d*f**2*g*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(...
```

3.129 $\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 363

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx = -\frac{(bg-ah)\sqrt{e+fx}}{2(bc-ad)(be-af)(a+bx)^2}$$
$$+\frac{\left(3a^2dfh-abf(7dg-ch)+b^2(4deg+3cfg-4ceh)\right)\sqrt{e+fx}}{4(bc-ad)^2(be-af)^2(a+bx)}$$
$$+\frac{(3a^3d^2f^2h-3a^2bdf^2(5dg-2ch)+ab^2f(20d^2eg-c^2fh+10cd(fg-2eh))-b^3(8d^2e^2g+c^2f(3fg-}{4\sqrt{b}(bc-ad)^3(be-af)^{5/2}}$$
$$+\frac{2d^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^3\sqrt{de-cf}}$$

output
```
-1/2*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(3*a^2*d
*f*h-a*b*f*(-c*h+7*d*g)+b^2*(-4*c*e*h+3*c*f*g+4*d*e*g))*(f*x+e)^(1/2)/(-a*
d+b*c)^2/(-a*f+b*e)^2/(b*x+a)+1/4*(3*a^3*d^2*f^2*h-3*a^2*b*d*f^2*(-2*c*h+5
*d*g)+a*b^2*f*(20*d^2*e*g-c^2*f*h+10*c*d*(-2*e*h+f*g))-b^3*(8*d^2*e^2*g+c^
2*f*(-4*e*h+3*f*g)+4*c*d*e*(-2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-
a*f+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^(5/2)+2*d^(3/2)*(-c*h+d*g)
*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^3/(-c*f+d*e)^(
1/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx$$
$$=\frac{1}{4}\left(\frac{\sqrt{e+fx}(5a^3dfh+b^3(4degx+3cfgx-2ce(g+2hx))-a^2b(cfh+d(9fg+2eh-3fhx))+ab^2(dg\right.$$
$$+\frac{(-3a^3d^2f^2h+3a^2bdf^2(5dg-2ch)+ab^2f(-20d^2eg+c^2fh-10cd(fg-2eh))+b^3(8d^2e^2g+c^2f(3f}{\sqrt{b}(bc-ad)^3(-be+af)^{5/2}}$$
$$\left.+\frac{8d^{3/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(-bc+ad)^3\sqrt{-de+cf}}\right)$$

input
```
Integrate[(g + h*x)/((a + b*x)^3*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
((Sqrt[e + f*x]*(5*a^3*d*f*h + b^3*(4*d*e*g*x + 3*c*f*g*x - 2*c*e*(g + 2*h
*x)) - a^2*b*(c*f*h + d*(9*f*g + 2*e*h - 3*f*h*x)) + a*b^2*(d*g*(6*e - 7*f
*x) + c*(5*f*g - 2*e*h + f*h*x))))/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^
2) + ((-3*a^3*d^2*f^2*h + 3*a^2*b*d*f^2*(5*d*g - 2*c*h) + a*b^2*f*(-20*d^2
*e*g + c^2*f*h - 10*c*d*(f*g - 2*e*h)) + b^3*(8*d^2*e^2*g + c^2*f*(3*f*g -
 4*e*h) + 4*c*d*e*(f*g - 2*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*
e) + a*f]])/(Sqrt[b]*(b*c - a*d)^3*(-(b*e) + a*f)^(5/2)) + (8*d^(3/2)*(d*g
 - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/((-(b*c) + a*d
)^3*Sqrt[-(d*e) + c*f]))/4
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int -\frac{af(4dg-ch)-b(4deg+3cfg-4ceh)-3df(bg-ah)x}{2(a+bx)^2(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{af(4dg-ch)-b(4deg+3cfg-4ceh)-3df(bg-ah)x}{(a+bx)^2(c+dx)\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\sqrt{e+fx}\left(3a^2dfh-abf(7dg-ch)+b^2(-4ceh+3cfg+4deg)\right)}{(a+bx)(bc-ad)(be-af)}-\frac{\int -\frac{\left(f(3fg-4eh)c^2+4de(fg-2eh)c+8d^2e^2g\right)b^2-af\left(-fhc^2+d(7fg-16eh)c+16d^2eg\right)b+a^2\cdots}{2(a+bx)(c+dx)\sqrt{e+f\cdots}}}{(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(f(3fg-4eh)c^2+4de(fg-2eh)c+8d^2e^2g\right)b^2-af\left(-fhc^2+7dfgc-16dehc+16d^2eg\right)b+a^2df^2(8dg-5ch)+df\left(3dfha^2-bf(7dg-ch)a+b^2(4deg+3cfg-4ceh)\right)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}\cdots}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{-\frac{\left(3a^3d^2f^2h-3a^2bdf^2(5dg-2ch)+ab^2f\left(c^2(-f)h+10cd(fg-2eh)+20d^2eg\right)-b^3\left(c^2f(3fg-4eh)+4cde(fg-2eh)+8d^2e^2g\right)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{8d^2(be-af)^2(\cdots}{\cdots}}{2(bc-ad)(be-af)}\cdots}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{-\frac{2\left(3a^3d^2f^2h-3a^2bdf^2(5dg-2ch)+ab^2f\left(c^2(-f)h+10cd(fg-2eh)+20d^2eg\right)-b^3\left(c^2f(3fg-4eh)+4cde(fg-2eh)+8d^2e^2g\right)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{16d^2\cdots}{\cdots}}{2(bc-ad)(be-af)}\cdots}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\sqrt{e+fx}\left(3a^2dfh-abf(7dg-ch)+b^2(-4ceh+3cfg+4deg)\right)}{(a+bx)(bc-ad)(be-af)}+\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^3d^2f^2h-3a^2bdf^2(5dg-2ch)+ab^2f\left(c^2(-f)h+10cd(fg-2eh)\right.\right.}{\sqrt{b}(bc-ad)\sqrt{be-af}}}{2}}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^3*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
-1/2*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (
((3*a^2*d*f*h - a*b*f*(7*d*g - c*h) + b^2*(4*d*e*g + 3*c*f*g - 4*c*e*h))*S
qrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((2*(3*a^3*d^2*f^2*h -
 3*a^2*b*d*f^2*(5*d*g - 2*c*h) + a*b^2*f*(20*d^2*e*g - c^2*f*h + 10*c*d*(f
*g - 2*e*h)) - b^3*(8*d^2*e^2*g + c^2*f*(3*f*g - 4*e*h) + 4*c*d*e*(f*g - 2
*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*c -
a*d)*Sqrt[b*e - a*f]) + (16*d^(3/2)*(b*e - a*f)^2*(d*g - c*h)*ArcTanh[(Sqr
t[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/(2*(b
*c - a*d)*(b*e - a*f)))/(4*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{3\sqrt{(cf-de)d}\left(\left(\left(-\frac{8d^2e^2g}{3}+\frac{8c\left(eh-\frac{fg}{2}\right)ed}{3}+\frac{4c^2efh}{3}-g f^2c^2\right)b^3-\frac{a\left(-20d^2eg+10c(2eh-fg)d+c^2fh\right)f b^2}{3}+2a^2\left(ch-\frac{5dg}{2}\right)d f^2b+\cdots}{4}$
derivativedivides	$2f^2\left(-\frac{(ch-dg)d^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f^2(ad-bc)^3\sqrt{(cf-de)d}}+\frac{\frac{fb\left(3a^3d^2fh-2a^2bcdfh-7a^2b d^2fg-a b^2c^2fh-4a b^2cdeh+10a b^2cdfg+4a b^2\cdots\right.}{8a^2f^2-16abfe+8b^2e^2}}{\cdots}\right.$
default	$2f^2\left(-\frac{(ch-dg)d^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{f^2(ad-bc)^3\sqrt{(cf-de)d}}+\frac{\frac{fb\left(3a^3d^2fh-2a^2bcdfh-7a^2b d^2fg-a b^2c^2fh-4a b^2cdeh+10a b^2cdfg+4a b^2\cdots\right.}{8a^2f^2-16abfe+8b^2e^2}}{\cdots}\right.$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/((a*f-b*e)*b)^(1/2)*(((c*f-d*e)*d)^(1/2)*((-8/3*d^2*e^2*g+8/3*c*(e*h-1
/2*f*g)*e*d+4/3*c^2*e*f*h-g*f^2*c^2)*b^3-1/3*a*(-20*d^2*e*g+10*c*(2*e*h-f*
g)*d+c^2*f*h)*f*b^2+2*a^2*(c*h-5/2*d*g)*d*f^2*b+a^3*d^2*f^2*h)*(b*x+a)^2*a
rctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+5/3*(-8/5*d^2*(b*x+a)^2*(a*f-b*
e)^2*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*(f*x+
e)^(1/2)*((c*f-d*e)*d)^(1/2)*(2/5*(2*d*e*g*x-c*(-3/2*f*g*x+e*(2*h*x+g)))*b
^3-2/5*a*(-3*(-7/6*f*x+e)*g*d+(1/2*(-h*x-5*g)*f+e*h)*c)*b^2-1/5*a^2*((3*(-
h*x+3*g)*f+2*e*h)*d+c*f*h)*b+a^3*d*f*h))*((a*f-b*e)*b)^(1/2))/((c*f-d*e)*d
)^(1/2)/(a*f-b*e)^2/(b*x+a)^2/(a*d-b*c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1898 vs. $2(333) = 666$.

Time = 177.67 (sec) , antiderivative size = 7674, normalized size of antiderivative = 21.14

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**3/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(333) = 666$.

Time = 0.16 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.30

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g + 4*b^3*c*d*e*f*g - 20*a*b^2*d^2*e*f*g + 3*b^3*c^2*f^
2*g - 10*a*b^2*c*d*f^2*g + 15*a^2*b*d^2*f^2*g - 8*b^3*c*d*e^2*h - 4*b^3*c^
2*e*f*h + 20*a*b^2*c*d*e*f*h + a*b^2*c^2*f^2*h - 6*a^2*b*c*d*f^2*h - 3*a^3
*d^2*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*c^3*e^2 - 3
*a*b^4*c^2*d*e^2 + 3*a^2*b^3*c*d^2*e^2 - a^3*b^2*d^3*e^2 - 2*a*b^4*c^3*e*f
 + 6*a^2*b^3*c^2*d*e*f - 6*a^3*b^2*c*d^2*e*f + 2*a^4*b*d^3*e*f + a^2*b^3*c
^3*f^2 - 3*a^3*b^2*c^2*d*f^2 + 3*a^4*b*c*d^2*f^2 - a^5*d^3*f^2)*sqrt(-b^2*
e + a*b*f)) - 2*(d^3*g - c*d^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d
*f))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-d^2*e + c*
d*f)) + 1/4*(4*(f*x + e)^(3/2)*b^3*d*e*f*g - 4*sqrt(f*x + e)*b^3*d*e^2*f*g
 + 3*(f*x + e)^(3/2)*b^3*c*f^2*g - 7*(f*x + e)^(3/2)*a*b^2*d*f^2*g - 5*sqr
t(f*x + e)*b^3*c*e*f^2*g + 13*sqrt(f*x + e)*a*b^2*d*e*f^2*g + 5*sqrt(f*x +
 e)*a*b^2*c*f^3*g - 9*sqrt(f*x + e)*a^2*b*d*f^3*g - 4*(f*x + e)^(3/2)*b^3*
c*e*f*h + 4*sqrt(f*x + e)*b^3*c*e^2*f*h + (f*x + e)^(3/2)*a*b^2*c*f^2*h +
3*(f*x + e)^(3/2)*a^2*b*d*f^2*h - 3*sqrt(f*x + e)*a*b^2*c*e*f^2*h - 5*sqrt
(f*x + e)*a^2*b*d*e*f^2*h - sqrt(f*x + e)*a^2*b*c*f^3*h + 5*sqrt(f*x + e)*
a^3*d*f^3*h)/((b^4*c^2*e^2 - 2*a*b^3*c*d*e^2 + a^2*b^2*d^2*e^2 - 2*a*b^3*c
^2*e*f + 4*a^2*b^2*c*d*e*f - 2*a^3*b*d^2*e*f + a^2*b^2*c^2*f^2 - 2*a^3*b*c
*d*f^2 + a^4*d^2*f^2)*((f*x + e)*b - b*e + a*f)^2)
```

Mupad [B] (verification not implemented)

Time = 19.38 (sec) , antiderivative size = 351830, normalized size of antiderivative = 969.23

$$\int \frac{g+hx}{(a+bx)^3(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)),x)
```

output
```
(((e + f*x)^(1/2)*(5*b^2*c*f^2*g + 5*a^2*d*f^2*h - a*b*c*f^2*h - 9*a*b*d*f
^2*g - 4*b^2*c*e*f*h + 4*b^2*d*e*f*g))/(4*(a*f - b*e)*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)) + (b*(e + f*x)^(3/2)*(3*b^2*c*f^2*g + 3*a^2*d*f^2*h + a*b*c*f
^2*h - 7*a*b*d*f^2*g - 4*b^2*c*e*f*h + 4*b^2*d*e*f*g))/(4*(a*f - b*e)^2*(a
^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(b^2*(e + f*x)^2 - (e + f*x)*(2*b^2*e - 2*
a*b*f) + a^2*f^2 + b^2*e^2 - 2*a*b*e*f) - atan(((((64*a^10*b^2*d^10*f^7*g
- 440*a^9*b^3*c*d^9*f^7*g - 40*a^10*b^2*c*d^9*f^7*h - 200*a^9*b^3*d^10*e*f
^6*g - 24*a^10*b^2*d^10*e*f^6*h + 24*a^2*b^10*c^8*d^2*f^7*g - 200*a^3*b^9*
c^7*d^3*f^7*g + 760*a^4*b^8*c^6*d^4*f^7*g - 1704*a^5*b^7*c^5*d^5*f^7*g + 2
440*a^6*b^6*c^4*d^6*f^7*g - 2264*a^7*b^5*c^3*d^7*f^7*g + 1320*a^8*b^4*c^2*
d^8*f^7*g + 8*a^3*b^9*c^8*d^2*f^7*h - 88*a^4*b^8*c^7*d^3*f^7*h + 360*a^5*b
^7*c^6*d^4*f^7*h - 760*a^6*b^6*c^5*d^5*f^7*h + 920*a^7*b^5*c^4*d^6*f^7*h -
 648*a^8*b^4*c^3*d^7*f^7*h + 248*a^9*b^3*c^2*d^8*f^7*h + 32*a^6*b^6*d^10*e
^4*f^3*g - 136*a^7*b^5*d^10*e^3*f^4*g + 240*a^8*b^4*d^10*e^2*f^5*g - 24*a^
8*b^4*d^10*e^3*f^4*h + 48*a^9*b^3*d^10*e^2*f^5*h + 32*b^12*c^6*d^4*e^4*f^3
*g + 8*b^12*c^7*d^3*e^3*f^4*g + 24*b^12*c^8*d^2*e^2*f^5*g - 32*b^12*c^7*d^
3*e^4*f^3*h - 32*b^12*c^8*d^2*e^3*f^4*h + 480*a^2*b^10*c^4*d^6*e^4*f^3*g +
 936*a^2*b^10*c^5*d^5*e^3*f^4*g + 1032*a^2*b^10*c^6*d^4*e^2*f^5*g - 640*a^
3*b^9*c^3*d^7*e^4*f^3*g - 2200*a^3*b^9*c^4*d^6*e^3*f^4*g - 3000*a^3*b^9*c^
5*d^5*e^2*f^5*g + 480*a^4*b^8*c^2*d^8*e^4*f^3*g + 2840*a^4*b^8*c^3*d^7*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6282, normalized size of antiderivative = 17.31

$$\int \frac{g + hx}{(a + bx)^3 (c + dx)\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**5*c*d**2*f**3*h - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**5*d**3*e*f**2*h + 6*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d*f**3*h - 6*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**4*b*c*d**2*e*f**2*h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**2*f**3*g + 6*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**2*f**3*h*
x + 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**4*b*d**3*e*f**2*g - 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**3*e*f**2*h*x - sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**3*f*
*3*h - 19*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**3*b**2*c**2*d*e*f**2*h + 10*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*d*f**3*g + 12*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3
*b**2*c**2*d*f**3*h*x + 20*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d**2*e**2*f*h + 10*sqrt(b)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d**2
*e*f**2*g - 12*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*...
```

3.130 $\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 599

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = -\frac{(bg-ah)\sqrt{e+fx}}{3(bc-ad)(be-af)(a+bx)^3}$$
$$+\frac{(5a^2dfh-abf(11dg-ch)+b^2(6deg+5cfg-6ceh))\sqrt{e+fx}}{12(bc-ad)^2(be-af)^2(a+bx)^2}$$
$$+\frac{(5a^3d^2f^2h-a^2bdf^2(19dg-4ch)+ab^2f(22d^2eg-c^2fh+2cd(8fg-11eh))-b^3(8d^2e^2g+c^2f(5fg-}{8(bc-ad)^3(be-af)^3(a+bx)}$$
$$+\frac{(5a^4d^3f^3h-5a^3bd^2f^3(7dg-3ch)+5a^2b^2df^2(14d^2eg-c^2fh+7cd(fg-2eh))-ab^3f(56d^3e^2g-c^3f}{}$$
$$-\frac{2d^{5/2}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4\sqrt{de-cf}}$$

output
```
-1/3*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^3+1/12*(5*a^2*
d*f*h-a*b*f*(-c*h+11*d*g)+b^2*(-6*c*e*h+5*c*f*g+6*d*e*g))*(f*x+e)^(1/2)/(-
a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2+1/8*(5*a^3*d^2*f^2*h-a^2*b*d*f^2*(-4*c*h
+19*d*g)+a*b^2*f*(22*d^2*e*g-c^2*f*h+2*c*d*(-11*e*h+8*f*g))-b^3*(8*d^2*e^2
*g+c^2*f*(-6*e*h+5*f*g)+2*c*d*e*(-4*e*h+3*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^
3/(-a*f+b*e)^3/(b*x+a)+1/8*(5*a^4*d^3*f^3*h-5*a^3*b*d^2*f^3*(-3*c*h+7*d*g)
+5*a^2*b^2*d*f^2*(14*d^2*e*g-c^2*f*h+7*c*d*(-2*e*h+f*g))-a*b^3*f*(56*d^3*e
^2*g-c^3*f^2*h+7*c^2*d*f*(-4*e*h+3*f*g)+28*c*d^2*e*(-2*e*h+f*g))+b^4*(16*d
^3*e^3*g+c^3*f^2*(-6*e*h+5*f*g)+2*c^2*d*e*f*(-4*e*h+3*f*g)+8*c*d^2*e^2*(-2
*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(1/2)/(-a*d+
b*c)^4/(-a*f+b*e)^(7/2)-2*d^(5/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)
/(-c*f+d*e)^(1/2))/(-a*d+b*c)^4/(-c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 6.69 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.21

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx$$

$$= \frac{(bc-ad)\sqrt{e+fx}\left(33a^5d^2f^2h+a^4bdf(-12cfh+d(-87fg-26eh+40fhx))+b^5\left(-24d^2e^2gx^2+6cdex(-3fgx+2e(g+2hx))+c^2\left(-15f^2gx^2-4e^2(2g\right.\right.\right.}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^4*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(33*a^5*d^2*f^2*h + a^4*b*d*f*(-12*c*f*h + d*(
-87*f*g - 26*e*h + 40*f*h*x)) + b^5*(-24*d^2*e^2*g*x^2 + 6*c*d*e*x*(-3*f*g
*x + 2*e*(g + 2*h*x)) + c^2*(-15*f^2*g*x^2 - 4*e^2*(2*g + 3*h*x) + 2*e*f*x
*(5*g + 9*h*x))) + a^3*b^2*(3*c^2*f^2*h + 2*c*d*f*(-19*e*h + 8*f*(6*g + h*
x)) + d^2*(8*e^2*h + 2*e*f*(58*g - 5*h*x) + f^2*x*(-136*g + 15*h*x))) + a*
b^4*(6*d^2*e*g*x*(-10*e + 11*f*x) + 2*c*d*(24*f^2*g*x^2 + 2*e^2*(7*g + 15*
h*x) - e*f*x*(40*g + 33*h*x)) + c^2*(-4*e^2*h - f^2*x*(40*g + 3*h*x) + e*f
*(26*g + 50*h*x))) + a^2*b^3*(c^2*f*(-33*f*g + 16*e*h - 8*f*h*x) + d^2*g*(
-44*e^2 + 166*e*f*x - 57*f^2*x^2) + 2*c*d*(10*e^2*h + 2*f^2*x*(32*g + 3*h*
x) - e*f*(47*g + 68*h*x)))))/((b*e - a*f)^3*(a + b*x)^3) + (3*(5*a^4*d^3*f
^3*h - 5*a^3*b*d^2*f^3*(7*d*g - 3*c*h) + 5*a^2*b^2*d*f^2*(14*d^2*e*g - c^2
*f*h + 7*c*d*(f*g - 2*e*h)) + b^4*(16*d^3*e^3*g + c^3*f^2*(5*f*g - 6*e*h)
+ 2*c^2*d*e*f*(3*f*g - 4*e*h) + 8*c*d^2*e^2*(f*g - 2*e*h)) + a*b^3*f*(-56*
d^3*e^2*g + c^3*f^2*h - 7*c^2*d*f*(3*f*g - 4*e*h) + 28*c*d^2*e*(-(f*g) + 2
*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(Sqrt[b]*(-(b*
e) + a*f)^(7/2)) + (48*d^(5/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/
Sqrt[-(d*e) + c*f]])/Sqrt[-(d*e) + c*f])/(24*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \ 168$$

$$-\frac{\int -\frac{af(6dg-ch)-b(6deg+5cfg-6ceh)-5df(bg-ah)x}{2(a+bx)^3(c+dx)\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{af(6dg-ch)-b(6deg+5cfg-6ceh)-5df(bg-ah)x}{(a+bx)^3(c+dx)\sqrt{e+fx}}dx}{6(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{\sqrt{e+fx}(5a^2dfh-abf(11dg-ch)+b^2(-6ceh+5cfg+6deg))}{2(a+bx)^2(bc-ad)(be-af)}-\frac{\int -\frac{3\left(\left(f(5fg-6eh)c^2+2de(3fg-4eh)c+8d^2e^2g\right)b^2-af\left(-fhc^2+11dfgc-16dehc+16d^2eg\right.\right.}{2(a+bx)^2(c+dx)}}{2(bc-ad)(be-}}{6(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{\left(f(5fg-6eh)c^2+2de(3fg-4eh)c+8d^2e^2g\right)b^2-af\left(-fhc^2+11dfgc-16dehc+16d^2eg\right)b+a^2df^2(8dg-3ch)+df\left(5dfha^2-bf(11dg-ch)a+b^2(6deg+5cfg-6ceh)\right.}{(a+bx)^2(c+dx)\sqrt{e+fx}}}{4(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 168

$$3\left(\frac{\sqrt{e+fx}\left(5a^3d^2f^2h-a^2bdf^2(19dg-4ch)+ab^2f\left(c^2(-f)h-22cdeh+16cdfg+22d^2eg\right)-b^3\left(c^2f(5fg-6eh)+2cde(3fg-4eh)+8d^2e^2g\right)\right)}{(a+bx)(bc-ad)(be-af)}-\frac{\int -\frac{-\left(\left(f^2(5fg-6eh)c\right.\right.}{}}{}\right.$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$3\left(\frac{\int \frac{-\left(\left(f^2(5fg-6eh)c^3+2def(3fg-4eh)c^2+8d^2e^2(fg-2eh)c+16d^3e^3g\right)b^3\right)+af\left(-f^2hc^3+2df(8fg-11eh)c^2+2d^2e(11fg-24eh)c+48d^3e^2g\right)b^2-a^2df^2\left(-4fh\right.}{}}{}\right.$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 174

$$3\left(\frac{\frac{16d^3(be-af)^3(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(5a^4d^3f^3h-5a^3bd^2f^3(7dg-3ch)+5a^2b^2df^2\left(c^2(-f)h+7cd(fg-2eh)+14d^2eg\right)-ab^3f\left(c^3\left(-f^2\right)h+7c^2df(3fg-\right.\right.}{}}{2(bc-ad)(be-af)}\right.$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \; 73$$

$$3\left(\frac{\frac{32d^3(be-af)^3(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)} - \frac{2\left(5a^4d^3f^3h-5a^3bd^2f^3(7dg-3ch)+5a^2b^2df^2\left(c^2(-f)h+7cd(fg-2eh)+14d^2eg\right)-ab^3f\left(c^3\left(-f^2\right)h+7c \right.\right.}{2(bc-ad)(be-a}\right.$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{e+fx}\left(5a^2dfh-abf(11dg-ch)+b^2(-6ceh+5cfg+6deg)\right)}{2(a+bx)^2(bc-ad)(be-af)} + \frac{3\left(\frac{\sqrt{e+fx}\left(5a^3d^2f^2h-a^2bdf^2(19dg-4ch)+ab^2f\left(c^2(-f)h-22cdeh+16cdfg+22d^2eg\right)-b^3\right.}{(a+bx)(bc-ad)(be-af)}\right.}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(bc-ad)(be-af)}$$

input

```
Int[(g + h*x)/((a + b*x)^4*(c + d*x)*Sqrt[e + f*x]),x]
```

output

```
-1/3*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3) + (
((5*a^2*d*f*h - a*b*f*(11*d*g - c*h) + b^2*(6*d*e*g + 5*c*f*g - 6*c*e*h))*
Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (3*(((5*a^3*d^2*f
^2*h - a^2*b*d*f^2*(19*d*g - 4*c*h) + a*b^2*f*(22*d^2*e*g + 16*c*d*f*g - 2
2*c*d*e*h - c^2*f*h) - b^3*(8*d^2*e^2*g + c^2*f*(5*f*g - 6*e*h) + 2*c*d*e*
(3*f*g - 4*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + ((2
*(5*a^4*d^3*f^3*h - 5*a^3*b*d^2*f^3*(7*d*g - 3*c*h) + 5*a^2*b^2*d*f^2*(14*
d^2*e*g - c^2*f*h + 7*c*d*(f*g - 2*e*h)) - a*b^3*f*(56*d^3*e^2*g - c^3*f^2
*h + 7*c^2*d*f*(3*f*g - 4*e*h) + 28*c*d^2*e*(f*g - 2*e*h)) + b^4*(16*d^3*e
^3*g + c^3*f^2*(5*f*g - 6*e*h) + 2*c^2*d*e*f*(3*f*g - 4*e*h) + 8*c*d^2*e^2
*(f*g - 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b
]*(b*c - a*d)*Sqrt[b*e - a*f]) - (32*d^(5/2)*(b*e - a*f)^3*(d*g - c*h)*Arc
Tanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f
]))/(2*(b*c - a*d)*(b*e - a*f))))/(4*(b*c - a*d)*(b*e - a*f))))/(6*(b*c - a
*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{5\left(\left(\left(\left(a^{4}h-7a^{3}bg\right)f^{3}+14a^{2}b^{2}eg\,f^{2}-\frac{56a\,b^{3}e^{2}gf}{5}+\frac{16b^{4}e^{3}g}{5}\right)d^{3}+3c\left(a^{2}\left(ah+\frac{7bg}{3}\right)f^{3}-\frac{14a\left(ah+\frac{2bg}{5}\right)be\,f^{2}}{3}+\frac{56\left(ah+\frac{bg}{7}\right)b^{2}e^{2}f}{15}-\right.\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
11/8/((a*f-b*e)*b)^(1/2)*(5/11*(((a^4*h-7*a^3*b*g)*f^3+14*a^2*b^2*e*g*f^2-
56/5*a*b^3*e^2*g*f+16/5*b^4*e^3*g)*d^3+3*c*(a^2*(a*h+7/3*b*g)*f^3-14/3*a*(
a*h+2/5*b*g)*b*e*f^2+56/15*(a*h+1/7*b*g)*b^2*e^2*f-16/15*b^3*e^3*h)*b*d^2-
(a*(a*h+21/5*b*g)*f^2-28/5*(a*h+3/14*b*g)*b*e*f+8/5*b^2*e^2*h)*c^2*b^2*f*d
+1/5*c^3*((a*h+5*b*g)*f-6*e*h*b)*b^3*f^2)*((c*f-d*e)*d)^(1/2)*(b*x+a)^3*ar
ctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+(-16/11*d^3*(b*x+a)^3*(a*f-b*e)^
3*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((c*f-d*
e)*d)^(1/2)*(f*x+e)^(1/2)*(((-19/11*b^3*g*x^2-136/33*a*x*(-15/136*h*x+g)*b
^2-29/11*a^2*(-40/87*h*x+g)*b+h*a^3)*a^2*f^2-26/33*(-33/13*b^3*g*x^2-83/13
*a*b^2*g*x-58/13*a^2*(-5/58*h*x+g)*b+h*a^3)*a*b*e*f+8/33*(-3*b^3*g*x^2-15/
2*a*b^2*g*x+h*a^3-11/2*a^2*b*g)*b^2*e^2)*d^2-4/11*c*(a*(-4*b^3*g*x^2-32/3*
a*x*(3/32*h*x+g)*b^2-8*a^2*(1/6*h*x+g)*b+h*a^3)*f^2+19/6*(9/19*b^3*g*x^2+4
0/19*a*x*(33/40*h*x+g)*b^2+47/19*a^2*(68/47*h*x+g)*b+h*a^3)*b*e*f-5/3*b^2*
(3/5*x*(2*h*x+g)*b^2+7/5*a*(15/7*h*x+g)*b+a^2*h)*e^2)*b*d+1/11*((-5*b^3*g*
x^2-40/3*a*x*(3/40*h*x+g)*b^2-11*a^2*(8/33*h*x+g)*b+h*a^3)*f^2+16/3*(5/8*x
*(9/5*h*x+g)*b^2+13/8*a*(25/13*h*x+g)*b+a^2*h)*b*e*f-4/3*((3*h*x+2*g)*b+a*
h)*b^2*e^2)*c^2*b^2))*((a*f-b*e)*b)^(1/2))/(a*f-b*e)^3/((c*f-d*e)*d)^(1/2)
/(b*x+a)^3/(a*d-b*c)^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**4/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. $2(565) = 1130$.

Time = 0.23 (sec) , antiderivative size = 2114, normalized size of antiderivative = 3.53

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(16*b^4*d^3*e^3*g + 8*b^4*c*d^2*e^2*f*g - 56*a*b^3*d^3*e^2*f*g + 6*b^
4*c^2*d*e*f^2*g - 28*a*b^3*c*d^2*e*f^2*g + 70*a^2*b^2*d^3*e*f^2*g + 5*b^4*
c^3*f^3*g - 21*a*b^3*c^2*d*f^3*g + 35*a^2*b^2*c*d^2*f^3*g - 35*a^3*b*d^3*f
^3*g - 16*b^4*c*d^2*e^3*h - 8*b^4*c^2*d*e^2*f*h + 56*a*b^3*c*d^2*e^2*f*h -
 6*b^4*c^3*e*f^2*h + 28*a*b^3*c^2*d*e*f^2*h - 70*a^2*b^2*c*d^2*e*f^2*h + a
*b^3*c^3*f^3*h - 5*a^2*b^2*c^2*d*f^3*h + 15*a^3*b*c*d^2*f^3*h + 5*a^4*d^3*
f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^4*e^3 - 4*a*b^
6*c^3*d*e^3 + 6*a^2*b^5*c^2*d^2*e^3 - 4*a^3*b^4*c*d^3*e^3 + a^4*b^3*d^4*e^
3 - 3*a*b^6*c^4*e^2*f + 12*a^2*b^5*c^3*d*e^2*f - 18*a^3*b^4*c^2*d^2*e^2*f
+ 12*a^4*b^3*c*d^3*e^2*f - 3*a^5*b^2*d^4*e^2*f + 3*a^2*b^5*c^4*e*f^2 - 12*
a^3*b^4*c^3*d*e*f^2 + 18*a^4*b^3*c^2*d^2*e*f^2 - 12*a^5*b^2*c*d^3*e*f^2 +
3*a^6*b*d^4*e*f^2 - a^3*b^4*c^4*f^3 + 4*a^4*b^3*c^3*d*f^3 - 6*a^5*b^2*c^2*
d^2*f^3 + 4*a^6*b*c*d^3*f^3 - a^7*d^4*f^3)*sqrt(-b^2*e + a*b*f)) + 2*(d^4*
g - c*d^3*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-d^2*e + c*d
*f)) - 1/24*(24*(f*x + e)^(5/2)*b^5*d^2*e^2*f*g - 48*(f*x + e)^(3/2)*b^5*d
^2*e^3*f*g + 24*sqrt(f*x + e)*b^5*d^2*e^4*f*g + 18*(f*x + e)^(5/2)*b^5*c*d
*e*f^2*g - 66*(f*x + e)^(5/2)*a*b^4*d^2*e*f^2*g - 48*(f*x + e)^(3/2)*b^5*c
*d*e^2*f^2*g + 192*(f*x + e)^(3/2)*a*b^4*d^2*e^2*f^2*g + 30*sqrt(f*x + e)*
b^5*c*d*e^3*f^2*g - 126*sqrt(f*x + e)*a*b^4*d^2*e^3*f^2*g + 15*(f*x + e...
```

Mupad [B] (verification not implemented)

Time = 29.18 (sec) , antiderivative size = 634320, normalized size of antiderivative = 1058.96

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)),x)
```

output
```
atan(((((256*a^14*b^2*d^13*f^9*g - 2352*a^13*b^3*c*d^12*f^9*g - 176*a^14*b
^2*c*d^12*f^9*h - 1232*a^13*b^3*d^13*e*f^8*g - 80*a^14*b^2*d^13*e*f^8*h -
80*a^3*b^13*c^11*d^2*f^9*g + 896*a^4*b^12*c^10*d^3*f^9*g - 4592*a^5*b^11*c
^9*d^4*f^9*g + 14336*a^6*b^10*c^8*d^5*f^9*g - 30496*a^7*b^9*c^7*d^6*f^9*g
+ 46592*a^8*b^8*c^6*d^7*f^9*g - 52192*a^9*b^7*c^5*d^8*f^9*g + 42752*a^10*b
^6*c^4*d^9*f^9*g - 24976*a^11*b^5*c^3*d^10*f^9*g + 9856*a^12*b^4*c^2*d^11*
f^9*g - 16*a^4*b^12*c^11*d^2*f^9*h + 192*a^5*b^11*c^10*d^3*f^9*h - 1136*a^
6*b^10*c^9*d^4*f^9*h + 4096*a^7*b^9*c^8*d^5*f^9*h - 9632*a^8*b^8*c^7*d^6*f
^9*h + 15232*a^9*b^7*c^6*d^7*f^9*h - 16352*a^10*b^6*c^5*d^8*f^9*h + 11776*
a^11*b^5*c^4*d^9*f^9*h - 5456*a^12*b^4*c^3*d^10*f^9*h + 1472*a^13*b^3*c^2*
d^11*f^9*h + 128*a^8*b^8*d^13*e^6*f^3*g - 800*a^9*b^7*d^13*e^5*f^4*g + 209
6*a^10*b^6*d^13*e^4*f^5*g - 3024*a^11*b^5*d^13*e^3*f^6*g + 2576*a^12*b^4*d
^13*e^2*f^7*g + 80*a^11*b^5*d^13*e^4*f^5*h - 240*a^12*b^4*d^13*e^3*f^6*h +
 240*a^13*b^3*d^13*e^2*f^7*h + 128*b^16*c^8*d^5*e^6*f^3*g + 32*b^16*c^9*d^
4*e^5*f^4*g + 16*b^16*c^10*d^3*e^4*f^5*g + 80*b^16*c^11*d^2*e^3*f^6*g - 12
8*b^16*c^9*d^4*e^6*f^3*h - 32*b^16*c^10*d^3*e^5*f^4*h - 96*b^16*c^11*d^2*e
^4*f^5*h + 3584*a^2*b^14*c^6*d^7*e^6*f^3*g + 7296*a^2*b^14*c^7*d^6*e^5*f^4
*g + 4080*a^2*b^14*c^8*d^5*e^4*f^5*g + 5360*a^2*b^14*c^9*d^4*e^3*f^6*g + 2
736*a^2*b^14*c^10*d^3*e^2*f^7*g - 7168*a^3*b^13*c^5*d^8*e^6*f^3*g - 24192*
a^3*b^13*c^6*d^7*e^5*f^4*g - 23040*a^3*b^13*c^7*d^6*e^4*f^5*g - 21520*a...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13481, normalized size of antiderivative = 22.51

$$\int \frac{g+hx}{(a+bx)^4(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**7*c*d**3*f**4*h - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**7*d**4*e*f**3*h + 45*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**2*f**4*
h - 45*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**6*b*c*d**3*e*f**3*h - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**3*f**4*g + 45*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**
3*f**4*h*x + 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**6*b*d**4*e*f**3*g - 45*sqrt(b)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**4*e*f**3*h*x - 15*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
5*b**2*c**3*d*f**4*h - 195*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*d**2*e*f**3*h + 105*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*
*2*d**2*f**4*g + 135*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**5*b**2*c**2*d**2*f**4*h*x + 210*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d**3*e
**2*f**2*h + 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**5*b**2*c*d**3*e*f**3*g - 135*sqrt(b)*sqrt(a*f - b*e...
```

3.131 $\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 946

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = -\frac{(bg-ah)\sqrt{e+fx}}{4(bc-ad)(be-af)(a+bx)^4}$$
$$+\frac{\left(7a^2dfh-abf(15dg-ch)+b^2(8deg+7cfg-8ceh)\right)\sqrt{e+fx}}{24(bc-ad)^2(be-af)^2(a+bx)^3}$$
$$+\frac{(35a^3d^2f^2h-3a^2bdf^2(41dg-6ch)+ab^2f(136d^2eg-5c^2fh+2cd(55fg-68eh))-b^3(48d^2e^2g+5c \cdots}{96(bc-ad)^3(be-af)^3(a+bx)^2}$$
$$+\frac{(35a^4d^3f^3h-a^3bd^2f^3(187dg-47ch)+a^2b^2df^2(328d^2eg-23c^2fh+cd(233fg-328eh))-ab^3f(240 \cdots}{\cdots}$$
$$+\frac{(35a^5d^4f^4h-35a^4bd^3f^4(9dg-4ch)+70a^3b^2d^2f^3(12d^2eg-c^2fh+6cd(fg-2eh))-14a^2b^3df^2(72d \cdots}{\cdots}$$
$$+\frac{2d^{7/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^5\sqrt{de-cf}}$$

output
```
-1/4*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^4+1/24*(7*a^2*
d*f*h-a*b*f*(-c*h+15*d*g)+b^2*(-8*c*e*h+7*c*f*g+8*d*e*g))*(f*x+e)^(1/2)/(-
a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^3+1/96*(35*a^3*d^2*f^2*h-3*a^2*b*d*f^2*(-6
*c*h+41*d*g)+a*b^2*f*(136*d^2*e*g-5*c^2*f*h+2*c*d*(-68*e*h+55*f*g))-b^3*(4
8*d^2*e^2*g+5*c^2*f*(-8*e*h+7*f*g)+8*c*d*e*(-6*e*h+5*f*g)))*(f*x+e)^(1/2)/
(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2+1/64*(35*a^4*d^3*f^3*h-a^3*b*d^2*f^3*(
-47*c*h+187*d*g)+a^2*b^2*d*f^2*(328*d^2*e*g-23*c^2*f*h+c*d*(-328*e*h+233*f
*g))-a*b^3*f*(240*d^3*e^2*g-5*c^3*f^2*h+c^2*d*f*(-176*e*h+145*f*g)+16*c*d^
2*e*(-15*e*h+11*f*g))+b^4*(64*d^3*e^3*g+5*c^3*f^2*(-8*e*h+7*f*g)+8*c^2*d*e
*f*(-6*e*h+5*f*g)+16*c*d^2*e^2*(-4*e*h+3*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^4
/(-a*f+b*e)^4/(b*x+a)+1/64*(35*a^5*d^4*f^4*h-35*a^4*b*d^3*f^4*(-4*c*h+9*d*
g)+70*a^3*b^2*d^2*f^3*(12*d^2*e*g-c^2*f*h+6*c*d*(-2*e*h+f*g))-14*a^2*b^3*d
*f^2*(72*d^3*e^2*g-2*c^3*f^2*h+9*c^2*d*f*(-4*e*h+3*f*g)+36*c*d^2*e*(-2*e*h
+f*g))+a*b^4*f*(576*d^4*e^3*g-5*c^4*f^3*h+36*c^3*d*f^2*(-6*e*h+5*f*g)+72*c
^2*d^2*e*f*(-4*e*h+3*f*g)+288*c*d^3*e^2*(-2*e*h+f*g))-b^5*(128*d^4*e^4*g+5
*c^4*f^3*(-8*e*h+7*f*g)+8*c^3*d*e*f^2*(-6*e*h+5*f*g)+16*c^2*d^2*e^2*f*(-4*
e*h+3*f*g)+64*c*d^3*e^3*(-2*e*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f
+b*e)^(1/2))/b^(1/2)/(-a*d+b*c)^5/(-a*f+b*e)^(9/2)+2*d^(7/2)*(-c*h+d*g)*ar
ctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^5/(-c*f+d*e)^(1/2
)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4159 vs. $2(946) = 1892$.

Time = 16.16 (sec) , antiderivative size = 4159, normalized size of antiderivative = 4.40

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^5*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4) - (
-1/3*(((-7*a*d*f*(b*g - a*h))/2 + (b*(-(a*f*(8*d*g - c*h)) + b*(8*d*e*g +
7*c*f*g - 8*c*e*h)))/2)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^
3) - (-1/2*(((-5*a*d*f*(7*a^2*d*f*h - a*b*f*(15*d*g - c*h) + b^2*(8*d*e*g
+ 7*c*f*g - 8*c*e*h)))/4 + (b*(a^2*d*f^2*(48*d*g - 13*c*h) - a*b*f*(96*d^2
*e*g + 75*c*d*f*g - 96*c*d*e*h - 5*c^2*f*h) + b^2*(48*d^2*e^2*g + 5*c^2*f*
(7*f*g - 8*e*h) + 8*c*d*e*(5*f*g - 6*e*h))))/4)*Sqrt[e + f*x])/((b*c - a*d
)*(b*e - a*f)*(a + b*x)^2) - (-((((-3*a*d*f*((-5*a*d*f*(7*a^2*d*f*h - a*b*
f*(15*d*g - c*h) + b^2*(8*d*e*g + 7*c*f*g - 8*c*e*h)))/4 + (b*(a^2*d*f^2*(
48*d*g - 13*c*h) - a*b*f*(96*d^2*e*g + 75*c*d*f*g - 96*c*d*e*h - 5*c^2*f*h
) + b^2*(48*d^2*e^2*g + 5*c^2*f*(7*f*g - 8*e*h) + 8*c*d*e*(5*f*g - 6*e*h))
))/4))/2 + b*(-1/2*(c*f*((-5*a*d*f*(7*a^2*d*f*h - a*b*f*(15*d*g - c*h) + b
^2*(8*d*e*g + 7*c*f*g - 8*c*e*h)))/4 + (b*(a^2*d*f^2*(48*d*g - 13*c*h) - a
*b*f*(96*d^2*e*g + 75*c*d*f*g - 96*c*d*e*h - 5*c^2*f*h) + b^2*(48*d^2*e^2*
g + 5*c^2*f*(7*f*g - 8*e*h) + 8*c*d*e*(5*f*g - 6*e*h))))/4)) - 2*((5*b*c*d
*e*f*(7*a^2*d*f*h - a*b*f*(15*d*g - c*h) + b^2*(8*d*e*g + 7*c*f*g - 8*c*e*
h)))/4 + (a*d*f*(a^2*d*f^2*(48*d*g - 13*c*h) - a*b*f*(96*d^2*e*g + 75*c*d*
f*g - 96*c*d*e*h - 5*c^2*f*h) + b^2*(48*d^2*e^2*g + 5*c^2*f*(7*f*g - 8*e*h
) + 8*c*d*e*(5*f*g - 6*e*h))))/4 - (b*(d*e + c*f)*(a^2*d*f^2*(48*d*g - 13*
c*h) - a*b*f*(96*d^2*e*g + 75*c*d*f*g - 96*c*d*e*h - 5*c^2*f*h) + b^2*(...
```

Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 1051, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{\int -\frac{af(8dg-ch)-b(8deg+7cfg-8ceh)-7df(bg-ah)x}{2(a+bx)^4(c+dx)\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{af(8dg-ch)-b(8deg+7cfg-8ceh)-7df(bg-ah)x}{(a+bx)^4(c+dx)\sqrt{e+fx}}dx}{8(bc-ad)(be-af)} - \frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(bc-ad)(be-af)}$$

$\Big\downarrow$ 168

$$\frac{\frac{\sqrt{e+fx}(7a^2dfh-abf(15dg-ch)+b^2(-8ceh+7cfg+8deg))}{3(a+bx)^3(bc-ad)(be-af)} - \frac{\int -\frac{\left(5f(7fg-8eh)c^2+8de(5fg-6eh)c+48d^2e^2g\right)b^2-af\left(-5fhc^2+75dfgc-96dehc+96d^2e\right.}{2(a+bx)^3(c+dx}}{3(bc-ad)(be}}{8(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(5f(7fg-8eh)c^2+8de(5fg-6eh)c+48d^2e^2g\right)b^2-af\left(-5fhc^2+75dfgc-96dehc+96d^2eg\right)b+a^2df^2(48dg-13ch)+5df\left(7dfha^2-bf(15dg-ch)a+b^2(8deg+7cfg-8\right.}{(a+bx)^3(c+dx)\sqrt{e+fx}}}{6(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(bc-ad)(be-af)}$$

$\Big\downarrow$ 168

$$\frac{\sqrt{e+fx}\left(35a^3d^2f^2h-3a^2bdf^2(41dg-6ch)+ab^2f\left(-5c^2fh+2cd(55fg-68eh)+136d^2eg\right)-b^3\left(5c^2f(7fg-8eh)+8cde(5fg-6eh)+48d^2e^2g\right)\right)}{2(a+bx)^2(bc-ad)(be-af)} - \frac{\int -\frac{3\left(-\left(\left(5f^2(7fg-\right.\right.\right.}{}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$3\int \frac{-\left(\left(5f^2(7fg-8eh)c^3+8def(5fg-6eh)c^2+16d^2e^2(3fg-4eh)c+64d^3e^3g\right)b^3\right)+af\left(-5f^2hc^3+2df(55fg-68eh)c^2+8d^2e(17fg-24eh)c+192d^3e^2g\right)b^2-3a^2df^2(}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(bc-ad)(be-af)}$$

$\Big\downarrow$ 168

$$\frac{\frac{\sqrt{e+fx}\left(7dfha^2-bf(15dg-ch)a+b^2(8deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(35d^2f^2ha^3-3bdf^2(41dg-6ch)a^2+b^2f\left(-5fhc^2+2d(55fg-68eh)c+136d^2eg\right)a-b^3\left(\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4(bc-ad)(be-af)(a+bx)^4}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt{e+fx}\left(7dfha^2-bf(15dg-ch)a+b^2(8deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(35d^2f^2ha^3-3bdf^2(41dg-6ch)a^2+b^2f\left(-5fhc^2+2d(55fg-68eh)c+136d^2eg\right)a-b^3\left(\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4(bc-ad)(be-af)(a+bx)^4}$$

$\downarrow$ 174

$$\frac{\frac{\sqrt{e+fx}\left(7dfha^2-bf(15dg-ch)a+b^2(8deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(35d^2f^2ha^3-3bdf^2(41dg-6ch)a^2+b^2f\left(-5fhc^2+2d(55fg-68eh)c+136d^2eg\right)a-b^3\left(\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4(bc-ad)(be-af)(a+bx)^4}$$

$\downarrow$ 73

$$\frac{\frac{\sqrt{e+fx}\left(7dfha^2-bf(15dg-ch)a+b^2(8deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(35d^2f^2ha^3-3bdf^2(41dg-6ch)a^2+b^2f\left(-5fhc^2+2d(55fg-68eh)c+136d^2eg\right)a-b^3\left(\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4(bc-ad)(be-af)(a+bx)^4}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{e+fx}\left(7dfha^2-bf(15dg-ch)a+b^2(8deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3}+\frac{\sqrt{e+fx}\left(35d^2f^2ha^3-3bdf^2(41dg-6ch)a^2+b^2f\left(-5fhc^2+2d(55fg-68eh)c+136d^2eg\right)a-b^3\left(\right.\right.}{2(bc-ad)(be-af)(a+bx)^2}}{}$$

$$\frac{(bg-ah)\sqrt{e+fx}}{4(bc-ad)(be-af)(a+bx)^4}$$

input
```
Int[(g + h*x)/((a + b*x)^5*(c + d*x)*Sqrt[e + f*x]),x]
```

output
```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4) + (
((7*a^2*d*f*h - a*b*f*(15*d*g - c*h) + b^2*(8*d*e*g + 7*c*f*g - 8*c*e*h))*
Sqrt[e + f*x])/(3*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) + (((35*a^3*d^2*f^2
*h - 3*a^2*b*d*f^2*(41*d*g - 6*c*h) + a*b^2*f*(136*d^2*e*g - 5*c^2*f*h + 2
*c*d*(55*f*g - 68*e*h)) - b^3*(48*d^2*e^2*g + 5*c^2*f*(7*f*g - 8*e*h) + 8*
c*d*e*(5*f*g - 6*e*h)))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x
)^2) + (3*(((35*a^4*d^3*f^3*h - a^3*b*d^2*f^3*(187*d*g - 47*c*h) + a^2*b^2
*d*f^2*(328*d^2*e*g - 23*c^2*f*h + c*d*(233*f*g - 328*e*h)) - a*b^3*f*(240
*d^3*e^2*g - 5*c^3*f^2*h + c^2*d*f*(145*f*g - 176*e*h) + 16*c*d^2*e*(11*f*
g - 15*e*h)) + b^4*(64*d^3*e^3*g + 5*c^3*f^2*(7*f*g - 8*e*h) + 8*c^2*d*e*f
*(5*f*g - 6*e*h) + 16*c*d^2*e^2*(3*f*g - 4*e*h)))*Sqrt[e + f*x])/((b*c - a
*d)*(b*e - a*f)*(a + b*x)) + ((2*(35*a^5*d^4*f^4*h - 35*a^4*b*d^3*f^4*(9*d
*g - 4*c*h) + 70*a^3*b^2*d^2*f^3*(12*d^2*e*g - c^2*f*h + 6*c*d*(f*g - 2*e*
h)) - 14*a^2*b^3*d*f^2*(72*d^3*e^2*g - 2*c^3*f^2*h + 9*c^2*d*f*(3*f*g - 4*
e*h) + 36*c*d^2*e*(f*g - 2*e*h)) + a*b^4*f*(576*d^4*e^3*g - 5*c^4*f^3*h +
36*c^3*d*f^2*(5*f*g - 6*e*h) + 72*c^2*d^2*e*f*(3*f*g - 4*e*h) + 288*c*d^3*
e^2*(f*g - 2*e*h)) - b^5*(128*d^4*e^4*g + 5*c^4*f^3*(7*f*g - 8*e*h) + 8*c^
3*d*e*f^2*(5*f*g - 6*e*h) + 16*c^2*d^2*e^2*f*(3*f*g - 4*e*h) + 64*c*d^3*e^
3*(f*g - 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[
b]*(b*c - a*d)*Sqrt[b*e - a*f]) + (256*d^(7/2)*(b*e - a*f)^4*(d*g - c*h...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 15.54 (sec) , antiderivative size = 1405, normalized size of antiderivative = 1.49

method	result	size
pseudoelliptic	Expression too large to display	1405
derivativedivides	Expression too large to display	2523
default	Expression too large to display	2523

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
93/64/((a*f-b*e)*b)^(1/2)*(35/93*((c*f-d*e)*d)^(1/2)*(b*x+a)^4*(((a^5*h-9*
a^4*b*g)*f^4+24*a^3*b^2*e*f^3*g-144/5*a^2*b^3*e^2*f^2*g+576/35*a*b^4*e^3*f
*g-128/35*b^5*e^4*g)*d^4+4*c*((a^4*h+3*a^3*b*g)*f^4-6*a^2*(a*h+3/5*b*g)*b*
e*f^3+36/5*a*(a*h+2/7*b*g)*b^2*e^2*f^2-144/35*(a*h+1/9*b*g)*b^3*e^3*f+32/3
5*b^4*e^4*h)*b*d^3-2*(a^2*(a*h+27/5*b*g)*f^3-36/5*a*(a*h+3/7*b*g)*b*e*f^2+
144/35*(a*h+1/6*b*g)*b^2*e^2*f-32/35*b^3*e^3*h)*c^2*b^2*f*d^2+4/5*c^3*(a*(
a*h+45/7*b*g)*f^2-54/7*b*(a*h+5/27*b*g)*e*f+12/7*b^2*e^2*h)*b^3*f^2*d-1/7*
c^4*((a*h+7*b*g)*f-8*e*h*b)*b^4*f^3)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^
(1/2))+(-128/93*d^4*(b*x+a)^4*(a*f-b*e)^4*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)
/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((a^3*(-187/93*b^4*g*x^3-643/93*a*x^2*(-35
/643*h*x+g)*b^3-255/31*a^2*(-77/459*h*x+g)*x*b^2-325/93*a^3*(-511/975*h*x+
g)*b+a^4*h)*f^3-326/279*a^2*b*(-492/163*b^4*g*x^3-1735/163*a*b^3*g*x^2-214
6/163*a^2*x*(-35/2146*h*x+g)*b^2-975/163*a^3*(-42/325*h*x+g)*b+a^4*h)*e*f^
2+200/279*a*(-18/5*b^4*g*x^3-316/25*a*b^3*g*x^2-393/25*a^2*b^2*g*x-37/5*a^
3*(-7/185*h*x+g)*b+a^4*h)*b^2*e^2*f-16/93*(-4*b^4*g*x^3-14*a*b^3*g*x^2-52/
3*a^2*b^2*g*x+a^4*h-25/3*a^3*b*g)*b^3*e^3)*d^3-47/93*c*(a^2*(-233/47*b^4*g
*x^3+(-2563/141*g*x^2-h*x^3)*a*b^3-3325/141*a^2*x*(389/3325*h*x+g)*b^2-535
/47*a^3*(251/1605*h*x+g)*b+a^4*h)*f^3+646/141*a*(264/323*b^4*g*x^3+1201/32
3*a*x^2*(492/1201*h*x+g)*b^3+1970/323*a^2*x*(319/394*h*x+g)*b^2+1249/323*a
^3*(1642/1249*h*x+g)*b+a^4*h)*b*e*f^2-680/141*(18/85*b^4*g*x^3+22/17*a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**5/(d*x+c)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4455 vs. $2(908) = 1816$.

Time $= 0.33$ (sec) , antiderivative size $= 4455$, normalized size of antiderivative $= 4.71$

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/64*(128*b^5*d^4*e^4*g + 64*b^5*c*d^3*e^3*f*g - 576*a*b^4*d^4*e^3*f*g + 4
8*b^5*c^2*d^2*e^2*f^2*g - 288*a*b^4*c*d^3*e^2*f^2*g + 1008*a^2*b^3*d^4*e^2
*f^2*g + 40*b^5*c^3*d*e*f^3*g - 216*a*b^4*c^2*d^2*e*f^3*g + 504*a^2*b^3*c*
d^3*e*f^3*g - 840*a^3*b^2*d^4*e*f^3*g + 35*b^5*c^4*f^4*g - 180*a*b^4*c^3*d
*f^4*g + 378*a^2*b^3*c^2*d^2*f^4*g - 420*a^3*b^2*c*d^3*f^4*g + 315*a^4*b*d
^4*f^4*g - 128*b^5*c*d^3*e^4*h - 64*b^5*c^2*d^2*e^3*f*h + 576*a*b^4*c*d^3*
e^3*f*h - 48*b^5*c^3*d*e^2*f^2*h + 288*a*b^4*c^2*d^2*e^2*f^2*h - 1008*a^2*
b^3*c*d^3*e^2*f^2*h - 40*b^5*c^4*e*f^3*h + 216*a*b^4*c^3*d*e*f^3*h - 504*a
^2*b^3*c^2*d^2*e*f^3*h + 840*a^3*b^2*c*d^3*e*f^3*h + 5*a*b^4*c^4*f^4*h - 2
8*a^2*b^3*c^3*d*f^4*h + 70*a^3*b^2*c^2*d^2*f^4*h - 140*a^4*b*c*d^3*f^4*h -
 35*a^5*d^4*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^9*c^5*
e^4 - 5*a*b^8*c^4*d*e^4 + 10*a^2*b^7*c^3*d^2*e^4 - 10*a^3*b^6*c^2*d^3*e^4
+ 5*a^4*b^5*c*d^4*e^4 - a^5*b^4*d^5*e^4 - 4*a*b^8*c^5*e^3*f + 20*a^2*b^7*c
^4*d*e^3*f - 40*a^3*b^6*c^3*d^2*e^3*f + 40*a^4*b^5*c^2*d^3*e^3*f - 20*a^5*
b^4*c*d^4*e^3*f + 4*a^6*b^3*d^5*e^3*f + 6*a^2*b^7*c^5*e^2*f^2 - 30*a^3*b^6
*c^4*d*e^2*f^2 + 60*a^4*b^5*c^3*d^2*e^2*f^2 - 60*a^5*b^4*c^2*d^3*e^2*f^2 +
 30*a^6*b^3*c*d^4*e^2*f^2 - 6*a^7*b^2*d^5*e^2*f^2 - 4*a^3*b^6*c^5*e*f^3 +
20*a^4*b^5*c^4*d*e*f^3 - 40*a^5*b^4*c^3*d^2*e*f^3 + 40*a^6*b^3*c^2*d^3*e*f
^3 - 20*a^7*b^2*c*d^4*e*f^3 + 4*a^8*b*d^5*e*f^3 + a^4*b^5*c^5*f^4 - 5*a^5*
b^4*c^4*d*f^4 + 10*a^6*b^3*c^3*d^2*f^4 - 10*a^7*b^2*c^2*d^3*f^4 + 5*a^8...
```

Mupad [B] (verification not implemented)

Time = 43.57 (sec) , antiderivative size = 998307, normalized size of antiderivative = 1055.29

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^5*(c + d*x)),x)
```

output
```
atan(((((16384*a^18*b^2*d^16*f^11*g - 187776*a^17*b^3*c*d^15*f^11*g - 1190
4*a^18*b^2*c*d^15*f^11*h - 107136*a^17*b^3*d^16*e*f^10*g - 4480*a^18*b^2*d
^16*e*f^10*h + 4480*a^4*b^16*c^14*d^2*f^11*g - 63360*a^5*b^15*c^13*d^3*f^1
1*g + 417024*a^6*b^14*c^12*d^4*f^11*g - 1694976*a^7*b^13*c^11*d^5*f^11*g +
 4765824*a^8*b^12*c^10*d^6*f^11*g - 9846400*a^9*b^11*c^9*d^7*f^11*g + 1549
0560*a^10*b^10*c^8*d^8*f^11*g - 18943488*a^11*b^9*c^7*d^9*f^11*g + 1816435
2*a^12*b^8*c^6*d^10*f^11*g - 13614208*a^13*b^7*c^5*d^11*f^11*g + 7845120*a
^14*b^6*c^4*d^12*f^11*g - 3360000*a^15*b^5*c^3*d^13*f^11*g + 1006464*a^16*
b^4*c^2*d^14*f^11*g + 640*a^5*b^15*c^14*d^2*f^11*h - 9344*a^6*b^14*c^13*d^
3*f^11*h + 64256*a^7*b^13*c^12*d^4*f^11*h - 281344*a^8*b^12*c^11*d^5*f^11*
h + 877440*a^9*b^11*c^10*d^6*f^11*h - 2037120*a^10*b^10*c^9*d^7*f^11*h + 3
568128*a^11*b^9*c^8*d^8*f^11*h - 4710912*a^12*b^8*c^7*d^9*f^11*h + 4645248
*a^13*b^7*c^6*d^10*f^11*h - 3360640*a^14*b^6*c^5*d^11*f^11*h + 1729280*a^1
5*b^5*c^4*d^12*f^11*h - 598784*a^16*b^4*c^3*d^13*f^11*h + 125056*a^17*b^3*
c^2*d^14*f^11*h + 8192*a^10*b^10*d^16*e^8*f^3*g - 67584*a^11*b^9*d^16*e^7*
f^4*g + 244736*a^12*b^8*d^16*e^6*f^5*g - 508544*a^13*b^7*d^16*e^5*f^6*g +
668160*a^14*b^6*d^16*e^4*f^7*g - 575232*a^15*b^5*d^16*e^3*f^8*g + 321024*a
^16*b^4*d^16*e^2*f^9*g - 4480*a^14*b^6*d^16*e^5*f^6*h + 17920*a^15*b^5*d^1
6*e^4*f^7*h - 26880*a^16*b^4*d^16*e^3*f^8*h + 17920*a^17*b^3*d^16*e^2*f^9*
h + 8192*b^20*c^10*d^6*e^8*f^3*g + 2048*b^20*c^11*d^5*e^7*f^4*g + 1024*...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 24455, normalized size of antiderivative = 25.85

$$\int \frac{g+hx}{(a+bx)^5(c+dx)\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(1/2),x)
```

output
```
(105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**9*c*d**4*f**5*h - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*d**5*e*f**4*h + 420*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**2*d**3*f*
*5*h - 420*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**8*b*c*d**4*e*f**4*h - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c*d**4*f**5*g + 420*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*
c*d**4*f**5*h*x + 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**8*b*d**5*e*f**4*g - 420*sqrt(b)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*d**5*e*f**4*h*x -
210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**7*b**2*c**3*d**2*f**5*h - 2310*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c**2*d**3*e*f**4*h + 1260*
sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**7*b**2*c**2*d**3*f**5*g + 1680*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c**2*d**3*f**5*h*x + 2520*sqrt
(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7
*b**2*c*d**4*e**2*f**3*h + 1260*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b**2*c*d**4*e*f**4*g - 1680*sqrt(b...
```

3.132 $\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 316

$$\begin{aligned}\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx &= \frac{2b(3a^2d^2f^2h+3abdf(dfg-deh-2cfh)+b^2(3c^2f^2h-d^2e(fg-eh)-2cdf(fg-eh)))\sqrt{e+fx}}{d^4f^3}\\ &+\frac{(bc-ad)^3(dg-ch)\sqrt{e+fx}}{d^4(de-cf)(c+dx)}\\ &+\frac{2b^2(3adfh+b(dfg-2deh-2cfh))(e+fx)^{3/2}}{3d^3f^3}+\frac{2b^3h(e+fx)^{5/2}}{5d^2f^3}\\ &+\frac{(bc-ad)^2\left(ad(dfg-2deh+cfh)-b(6d^2eg+7c^2fh-cd(5fg+8eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{9/2}(de-cf)^{3/2}}\end{aligned}$$

output
```
2*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(-2*c*f*h-d*e*h+d*f*g)+b^2*(3*c^2*f^2*h-d^2
*e*(-e*h+f*g)-2*c*d*f*(-e*h+f*g)))*(f*x+e)^(1/2)/d^4/f^3+(-a*d+b*c)^3*(-c*
h+d*g)*(f*x+e)^(1/2)/d^4/(-c*f+d*e)/(d*x+c)+2/3*b^2*(3*a*d*f*h+b*(-2*c*f*h
-2*d*e*h+d*f*g))*(f*x+e)^(3/2)/d^3/f^3+2/5*b^3*h*(f*x+e)^(5/2)/d^2/f^3+(-a
*d+b*c)^2*(a*d*(c*f*h-2*d*e*h+d*f*g)-b*(6*d^2*e*g+7*c^2*f*h-c*d*(8*e*h+5*f
*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(9/2)/(-c*f+d*e)^(
3/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{e+fx}(15a^3d^3f^3(-dg+ch)+45a^2bd^2f^2(-3c^2fh+2d^2ehx+cd(fg+2eh-2fhx))+15ab^2df(15c^3}{}$$
$$-\frac{(bc-ad)^2\left(-ad(dfg-2deh+cfh)+b(6d^2eg+7c^2fh-cd(5fg+8eh))\right)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{9/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)^3*(g + h*x))/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[e + f*x]*(15*a^3*d^3*f^3*(-(d*g) + c*h) + 45*a^2*b*d^2*f^2*(-3*c^2*f
*h + 2*d^2*e*h*x + c*d*(f*g + 2*e*h - 2*f*h*x)) + 15*a*b^2*d*f*(15*c^3*f^2
*h + 2*d^3*e*x*(3*f*g - 2*e*h + f*h*x) + c^2*d*f*(-9*f*g - 8*e*h + 10*f*h*
x) - 2*c*d^2*(2*e^2*h - 3*e*f*(g - h*x) + f^2*x*(3*g + h*x))) + b^3*(-105*
c^4*f^3*h + 5*c^3*d*f^2*(15*f*g + 10*e*h - 14*f*h*x) + 2*c*d^3*(e + f*x)*(
-10*e*f*g + 8*e^2*h - f^2*x*(5*g + 3*h*x)) + 2*d^4*e*x*(8*e^2*h - 2*e*f*(5
*g + 2*h*x) + f^2*x*(5*g + 3*h*x)) + 2*c^2*d^2*f*(12*e^2*h + f^2*x*(25*g +
 7*h*x) + e*f*(-20*g + 19*h*x)))))/(15*d^4*f^3*(d*e - c*f)*(c + d*x)) - ((
b*c - a*d)^2*(-(a*d*(d*f*g - 2*d*e*h + c*f*h)) + b*(6*d^2*e*g + 7*c^2*f*h
- c*d*(5*f*g + 8*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]]
)/(d^(9/2)*(-(d*e) + c*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)^2((6be+af)(dg-ch)-2ad(fg-eh)+b(5dfg+2deh-7cfh)x)}{2(c+dx)\sqrt{e+fx}}dx}{d(de-cf)}-\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(dfg-2deh+cfh)+b(5dfg+2deh-7cfh)x)}{(c+dx)\sqrt{e+fx}}dx}{2d(de-cf)}-\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 170$$

$$\frac{\frac{2\int -\frac{(a+bx)\left(bc(4be+af)(5dfg+2deh-7cfh)-5adf(6bdeg-adfg-6bceh+2adeh-acfh)-b\left(3adf(5dfg+6deh-11cfh)+b\left(2e(5fg-4eh)d^2-cf(25fg+12eh)d+35c^2\right.\right.\right.}{2(c+dx)\sqrt{e+fx}}}{5df}}{2d(de-cf)}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2b(a+bx)^2\sqrt{e+fx}(-7cfh+2deh+5dfg)}{5df}-\frac{\int \frac{(a+bx)\left(bc(4be+af)(5dfg+2deh-7cfh)-5adf(6bdeg-adfg-6bceh+2adeh-acfh)-b\left(3adf(5dfg+6deh-11cf\right.\right.}{(c+dx)\sqrt{e+fx}}}{5df}}{2d(de-cf)}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 164$$

$$\frac{2b(a+bx)^2\sqrt{e+fx}(-7cfh+2deh+5dfg)}{5df}-\frac{\frac{5f(bc-ad)^2\left(ad(cfh-2deh+dfg)-b\left(7c^2fh-cd(8eh+5fg)+6d^2eg\right)\right)\int\frac{1}{(c+dx)\sqrt{e+fx}}dx}{d^2}-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(\right.}{}}{}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{2b(a+bx)^2\sqrt{e+fx}(-7cfh+2deh+5dfg)}{5df}-\frac{\frac{10(bc-ad)^2\left(ad(cfh-2deh+dfg)-b\left(7c^2fh-cd(8eh+5fg)+6d^2eg\right)\right)\int\frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2}-\frac{2b\sqrt{e+fx}\left(6\right.}{}}{}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2b(a+bx)^2\sqrt{e+fx}(-7cfh+2deh+5dfg)}{5df} - \frac{-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(-19cfh+14deh+5dfg)+bdfx\left(3adf(-11cfh+6deh+5dfg)+b\left(35c^2f^2h-cdf(12eh+25fg)+\right.\right.\right.}{}}{}}{}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[((a + b*x)^3*(g + h*x))/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-(((d*g - c*h)*(a + b*x)^3*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x))) + ((2
*b*(5*d*f*g + 2*d*e*h - 7*c*f*h)*(a + b*x)^2*Sqrt[e + f*x])/(5*d*f) - ((-2
*b*Sqrt[e + f*x]*(6*a^2*d^2*f^2*(5*d*f*g + 14*d*e*h - 19*c*f*h) - b^2*(105
*c^3*f^3*h + 4*d^3*e^2*(5*f*g - 4*e*h) + 8*c*d^2*e*f*(5*f*g - 3*e*h) - 25*
c^2*d*f^2*(3*f*g + 2*e*h)) + 15*a*b*d*f*(15*c^2*f^2*h + 2*d^2*e*(3*f*g - 2
*e*h) - c*d*f*(9*f*g + 8*e*h)) + b*d*f*(3*a*d*f*(5*d*f*g + 6*d*e*h - 11*c*
f*h) + b*(35*c^2*f^2*h + 2*d^2*e*(5*f*g - 4*e*h) - c*d*f*(25*f*g + 12*e*h)
))*x))/(3*d^2*f^2) - (10*(b*c - a*d)^2*f*(a*d*(d*f*g - 2*d*e*h + c*f*h) -
b*(6*d^2*e*g + 7*c^2*f*h - c*d*(5*f*g + 8*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e +
 f*x])/Sqrt[d*e - c*f]])/(d^(5/2)*Sqrt[d*e - c*f]))/(5*d*f))/(2*d*(d*e - c
*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.17

method	result
risch	$\frac{2b\left(3x^2h\,b^2d^2f^2+15ab\,d^2f^2hx-10b^2cd\,f^2hx-4b^2d^2efhx+5b^2d^2f^2gx+45a^2d^2f^2h-90abcd\,f^2h-30ab\,d^2efh+45ab\,d^2\right.}{15f^3d^4}$
pseudoelliptic	$-\frac{-(ad-bc)^2(xd+c)\left((afg-2e(ah+3bg))d^2+((ah+5bg)f+8ehb)cd-7b\,c^2fh\right)f^3\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\left(\left(-a^3f^3g+\right.\right.}{}$
derivativedivides	$\frac{2b\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+ab\,d^2fh(fx+e)^{\frac{3}{2}}-\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-6abcd\,f^2h\sqrt{\right.}{}$
default	$\frac{2b\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+ab\,d^2fh(fx+e)^{\frac{3}{2}}-\frac{2b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-\frac{2b^2d^2eh(fx+e)^{\frac{3}{2}}}{3}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-6abcd\,f^2h\sqrt{\right.}{}$

input

```
int((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/15*b*(3*b^2*d^2*f^2*h*x^2+15*a*b*d^2*f^2*h*x-10*b^2*c*d*f^2*h*x-4*b^2*d^
2*e*f*h*x+5*b^2*d^2*f^2*g*x+45*a^2*d^2*f^2*h-90*a*b*c*d*f^2*h-30*a*b*d^2*e
*f*h+45*a*b*d^2*f^2*g+45*b^2*c^2*f^2*h+20*b^2*c*d*e*f*h-30*b^2*c*d*f^2*g+8
*b^2*d^2*e^2*h-10*b^2*d^2*e*f*g)*(f*x+e)^(1/2)/f^3/d^4+1/d^4*(2*a^2*d^2-4*
a*b*c*d+2*b^2*c^2)*(-1/2*f*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/(c*f-d*e)*(f*
x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(a*c*d*f*h-2*a*d^2*e*h+a*d^2*f*g-7*b*c^
2*f*h+8*b*c*d*e*h+5*b*c*d*f*g-6*b*d^2*e*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*a
rctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. 2(294) = 588.

Time = 0.20 (sec) , antiderivative size = 2564, normalized size of antiderivative = 8.11

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(d^2*e - c*d*f)*((6*(b^3*c^3*d^2 - 2*a*b^2*c^2*d^3 + a^2*b*c
*d^4)*e*f^3 - (5*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 + a^3*c*d^4
)*f^4)*g - (2*(4*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 6*a^2*b*c^2*d^3 - a^3*c*d^4
)*e*f^3 - (7*b^3*c^5 - 15*a*b^2*c^4*d + 9*a^2*b*c^3*d^2 - a^3*c^2*d^3)*f^4
)*h + ((6*(b^3*c^2*d^3 - 2*a*b^2*c*d^4 + a^2*b*d^5)*e*f^3 - (5*b^3*c^3*d^2
 - 9*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 + a^3*d^5)*f^4)*g - (2*(4*b^3*c^3*d^2 -
 9*a*b^2*c^2*d^3 + 6*a^2*b*c*d^4 - a^3*d^5)*e*f^3 - (7*b^3*c^4*d - 15*a*b^
2*c^3*d^2 + 9*a^2*b*c^2*d^3 - a^3*c*d^4)*f^4)*h)*x)*log((d*f*x + 2*d*e - c
*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*(6*(b^3*d^6*e^2*f
^2 - 2*b^3*c*d^5*e*f^3 + b^3*c^2*d^4*f^4)*h*x^3 + 2*(5*(b^3*d^6*e^2*f^2 -
2*b^3*c*d^5*e*f^3 + b^3*c^2*d^4*f^4)*g - (4*b^3*d^6*e^3*f - (b^3*c*d^5 + 1
5*a*b^2*d^6)*e^2*f^2 - 10*(b^3*c^2*d^4 - 3*a*b^2*c*d^5)*e*f^3 + (7*b^3*c^3
*d^3 - 15*a*b^2*c^2*d^4)*f^4)*h)*x^2 - 5*(4*b^3*c*d^5*e^3*f + 2*(2*b^3*c^2
*d^4 - 9*a*b^2*c*d^5)*e^2*f^2 - (23*b^3*c^3*d^3 - 45*a*b^2*c^2*d^4 + 9*a^2
*b*c*d^5 - 3*a^3*d^6)*e*f^3 + 3*(5*b^3*c^4*d^2 - 9*a*b^2*c^3*d^3 + 3*a^2*b
*c^2*d^4 - a^3*c*d^5)*f^4)*g + (16*b^3*c*d^5*e^4 + 4*(2*b^3*c^2*d^4 - 15*a
*b^2*c*d^5)*e^3*f + 2*(13*b^3*c^3*d^3 - 30*a*b^2*c^2*d^4 + 45*a^2*b*c*d^5)
*e^2*f^2 - 5*(31*b^3*c^4*d^2 - 69*a*b^2*c^3*d^3 + 45*a^2*b*c^2*d^4 - 3*a^3
*c*d^5)*e*f^3 + 15*(7*b^3*c^5*d - 15*a*b^2*c^4*d^2 + 9*a^2*b*c^3*d^3 - a^3
*c^2*d^4)*f^4)*h - 2*(5*(2*b^3*d^6*e^3*f + (b^3*c*d^5 - 9*a*b^2*d^6)*e^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(h*x+g)/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(294) = 588$.

Time = 0.14 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{(6\,b^3c^2d^2eg - 12\,ab^2cd^3eg + 6\,a^2bd^4eg - 5\,b^3c^3dfg + 9\,ab^2c^2d^2fg - 3\,a^2bcd^3fg - a^3d^4fg - 8\,b^3c^3deh + 1}{(d^5e - cd^4}$$
$$+ \frac{\sqrt{fx+e}b^3c^3dfg - 3\,\sqrt{fx+e}ab^2c^2d^2fg + 3\,\sqrt{fx+e}a^2bcd^3fg - \sqrt{fx+e}a^3d^4fg - \sqrt{fx+e}b^3c^4fh +}{(d^5e - cd^4f)((fx+e)d - de + cf)}$$
$$+ \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^3d^8f^{13}g - 15\,\sqrt{fx+e}b^3d^8ef^{13}g - 30\,\sqrt{fx+e}b^3cd^7f^{14}g + 45\,\sqrt{fx+e}ab^2d^8f^{14}g + 3\,(\right.}{}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
(6*b^3*c^2*d^2*e*g - 12*a*b^2*c*d^3*e*g + 6*a^2*b*d^4*e*g - 5*b^3*c^3*d*f*
g + 9*a*b^2*c^2*d^2*f*g - 3*a^2*b*c*d^3*f*g - a^3*d^4*f*g - 8*b^3*c^3*d*e*
h + 18*a*b^2*c^2*d^2*e*h - 12*a^2*b*c*d^3*e*h + 2*a^3*d^4*e*h + 7*b^3*c^4*
f*h - 15*a*b^2*c^3*d*f*h + 9*a^2*b*c^2*d^2*f*h - a^3*c*d^3*f*h)*arctan(sqr
t(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^5*e - c*d^4*f)*sqrt(-d^2*e + c*d*f)
) + (sqrt(f*x + e)*b^3*c^3*d*f*g - 3*sqrt(f*x + e)*a*b^2*c^2*d^2*f*g + 3*s
qrt(f*x + e)*a^2*b*c*d^3*f*g - sqrt(f*x + e)*a^3*d^4*f*g - sqrt(f*x + e)*b
^3*c^4*f*h + 3*sqrt(f*x + e)*a*b^2*c^3*d*f*h - 3*sqrt(f*x + e)*a^2*b*c^2*d
^2*f*h + sqrt(f*x + e)*a^3*c*d^3*f*h)/((d^5*e - c*d^4*f)*((f*x + e)*d - d*
e + c*f)) + 2/15*(5*(f*x + e)^(3/2)*b^3*d^8*f^13*g - 15*sqrt(f*x + e)*b^3*
d^8*e*f^13*g - 30*sqrt(f*x + e)*b^3*c*d^7*f^14*g + 45*sqrt(f*x + e)*a*b^2*
d^8*f^14*g + 3*(f*x + e)^(5/2)*b^3*d^8*f^12*h - 10*(f*x + e)^(3/2)*b^3*d^8
*e*f^12*h + 15*sqrt(f*x + e)*b^3*d^8*e^2*f^12*h - 10*(f*x + e)^(3/2)*b^3*c
*d^7*f^13*h + 15*(f*x + e)^(3/2)*a*b^2*d^8*f^13*h + 30*sqrt(f*x + e)*b^3*c
*d^7*e*f^13*h - 45*sqrt(f*x + e)*a*b^2*d^8*e*f^13*h + 45*sqrt(f*x + e)*b^3
*c^2*d^6*f^14*h - 90*sqrt(f*x + e)*a*b^2*c*d^7*f^14*h + 45*sqrt(f*x + e)*a
^2*b*d^8*f^14*h)/(d^10*f^15)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.15

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$

$$= (e+f\,x)^{3/2}\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{3\,d^2\,f^3}-\frac{4\,b^3\,h\,(c\,f-d\,e)}{3\,d^3\,f^3}\right)$$

$$-\sqrt{e+f\,x}\left(\frac{2\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{d^2\,f^3}-\frac{4\,b^3\,h\,(c\,f-d\,e)}{d^3\,f^3}\right)(c\,f-d\,e)}{d}\right.$$

$$\left.-\frac{6\,b\,(a\,f-b\,e)\,(a\,f\,h-2\,b\,e\,h+b\,f\,g)}{d^2\,f^3}+\frac{2\,b^3\,h\,(c\,f-d\,e)^2}{d^4\,f^3}\right)$$

$$+\frac{\sqrt{e+f\,x}\,(-f\,h\,a^3\,c\,d^3+f\,g\,a^3\,d^4+3\,f\,h\,a^2\,b\,c^2\,d^2-3\,f\,g\,a^2\,b\,c\,d^3-3\,f\,h\,a\,b^2\,c^3\,d+3\,f\,g\,a\,b^2\,c^2\,d^2+}{(c\,f-d\,e)\,(d^5\,(e+f\,x)-d^5\,e+c\,d^4\,f)}$$

$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)^2\,(a\,d^2\,f\,g-2\,a\,d^2\,e\,h-6\,b\,d^2\,e\,g-7\,b\,c^2\,f\,h+a\,c\,d}{\sqrt{c\,f-d\,e}\,(a^3\,d^4\,f\,g-2\,a^3\,d^4\,e\,h-7\,b^3\,c^4\,f\,h-6\,a^2\,b\,d^4\,e\,g+a^3\,c\,d^3\,f\,h+8\,b^3\,c^3\,d\,e\,h+5\,b^3\,c^3\,d\,f\,g-6\,b^3\,c^2\,d^2\,e\,g+12\,a\,b^2\,c\,d^3\,e\,g+12\,a^2}\right.}{}$$

$$+\frac{2\,b^3\,h\,(e+f\,x)^{5/2}}{5\,d^2\,f^3}$$

input
```
int(((g + h*x)*(a + b*x)^3)/((e + f*x)^(1/2)*(c + d*x)^2),x)
```

output
```
(e + f*x)^(3/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(3*d^2*f^3) - (4*b^
3*h*(c*f - d*e))/(3*d^3*f^3)) - (e + f*x)^(1/2)*((2*((2*b^3*f*g - 8*b^3*e*
h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*h*(c*f - d*e))/(d^3*f^3))*(c*f - d*e))
/d - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d^2*f^3) + (2*b^3*h*(c*f
 - d*e)^2)/(d^4*f^3)) + ((e + f*x)^(1/2)*(a^3*d^4*f*g + b^3*c^4*f*h - a^3*
c*d^3*f*h - b^3*c^3*d*f*g - 3*a^2*b*c*d^3*f*g - 3*a*b^2*c^3*d*f*h + 3*a*b^
2*c^2*d^2*f*g + 3*a^2*b*c^2*d^2*f*h))/((c*f - d*e)*(d^5*(e + f*x) - d^5*e
+ c*d^4*f)) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)^2*(a*d^2*f*g - 2*
a*d^2*e*h - 6*b*d^2*e*g - 7*b*c^2*f*h + a*c*d*f*h + 8*b*c*d*e*h + 5*b*c*d*
f*g))/((c*f - d*e)^(1/2)*(a^3*d^4*f*g - 2*a^3*d^4*e*h - 7*b^3*c^4*f*h - 6*
a^2*b*d^4*e*g + a^3*c*d^3*f*h + 8*b^3*c^3*d*e*h + 5*b^3*c^3*d*f*g - 6*b^3*
c^2*d^2*e*g + 12*a*b^2*c*d^3*e*g + 12*a^2*b*c*d^3*e*h + 3*a^2*b*c*d^3*f*g
+ 15*a*b^2*c^3*d*f*h - 18*a*b^2*c^2*d^2*e*h - 9*a*b^2*c^2*d^2*f*g - 9*a^2*
b*c^2*d^2*f*h)))*(a*d - b*c)^2*(a*d^2*f*g - 2*a*d^2*e*h - 6*b*d^2*e*g - 7*
b*c^2*f*h + a*c*d*f*h + 8*b*c*d*e*h + 5*b*c*d*f*g))/(d^(9/2)*(c*f - d*e)^(
3/2)) + (2*b^3*h*(e + f*x)^(5/2))/(5*d^2*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2831, normalized size of antiderivative = 8.96

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output

```
(15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*a**3*c**2*d**3*f**4*h - 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*e*f**3*h + 15*sqrt(d)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*f**4*
g + 15*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**3*c*d**4*f**4*h*x - 30*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**5*e*f**3*h*x + 15*sqrt(d)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**5*f**4*
g*x - 135*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*a**2*b*c**3*d**2*f**4*h + 180*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*e*f**3*h + 45*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**
2*b*c**2*d**3*f**4*g - 135*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*f**4*h*x - 90*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4*e*
f**3*g + 180*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a**2*b*c*d**4*e*f**3*h*x + 45*sqrt(d)*sqrt(c*f - d*e)*atan((s
qrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4*f**4*g*x - 90*sqr
t(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**
2*b*d**5*e*f**3*g*x + 225*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d...
```

3.133 $\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 218

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \frac{2b(2adfh+b(dfg-deh-2cfh))\sqrt{e+fx}}{d^3f^2} - \frac{(bc-ad)^2(dg-ch)\sqrt{e+fx}}{d^3(de-cf)(c+dx)} + \frac{2b^2h(e+fx)^{3/2}}{3d^2f^2} - \frac{(bc-ad)\left(ad(dfg-2deh+cfh)-b(4d^2eg+5c^2fh-3cd(fg+2eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}(de-cf)^{3/2}}$$

output
```
2*b*(2*a*d*f*h+b*(-2*c*f*h-d*e*h+d*f*g))*(f*x+e)^(1/2)/d^3/f^2-(-a*d+b*c)^
2*(-c*h+d*g)*(f*x+e)^(1/2)/d^3/(-c*f+d*e)/(d*x+c)+2/3*b^2*h*(f*x+e)^(3/2)/
d^2/f^2-(-a*d+b*c)*(a*d*(c*f*h-2*d*e*h+d*f*g)-b*(4*d^2*e*g+5*c^2*f*h-3*c*d
*(2*e*h+f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)/(-c
*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx =$$
$$-\frac{\sqrt{e+fx}(3a^2d^2f^2(dg-ch)-6abdf(-3c^2fh+2d^2ehx+cd(fg+2eh-2fhx))+b^2(-15c^3f^2h+c^2d}{3d^3f^2(de-c}$$
$$+\frac{(bc-ad)\left(-ad(dfg-2deh+cfh)+b(4d^2eg+5c^2fh-3cd(fg+2eh))\right)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)^2*(g + h*x))/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-1/3*(Sqrt[e + f*x]*(3*a^2*d^2*f^2*(d*g - c*h) - 6*a*b*d*f*(-3*c^2*f*h + 2
*d^2*e*h*x + c*d*(f*g + 2*e*h - 2*f*h*x)) + b^2*(-15*c^3*f^2*h + c^2*d*f*(
9*f*g + 8*e*h - 10*f*h*x) - 2*d^3*e*x*(3*f*g - 2*e*h + f*h*x) + 2*c*d^2*(2
*e^2*h - 3*e*f*(g - h*x) + f^2*x*(3*g + h*x)))))/(d^3*f^2*(d*e - c*f)*(c +
 d*x)) + ((b*c - a*d)*(-(a*d*(d*f*g - 2*d*e*h + c*f*h)) + b*(4*d^2*e*g + 5
*c^2*f*h - 3*c*d*(f*g + 2*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e
) + c*f]])/(d^(7/2)*(-(d*e) + c*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$
$$\Big\downarrow \ 166$$
$$\frac{\int \frac{(a+bx)((4be+af)(dg-ch)-2ad(fg-eh)+b(3dfg+2deh-5cfh)x)}{2(c+dx)\sqrt{e+fx}}dx}{d(de-cf)}-\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(dfg-2deh+cfh)+b(3dfg+2deh-5cfh)x)}{(c+dx)\sqrt{e+fx}}dx}{2d(de-cf)} - \frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 164$$

$$\frac{\frac{(bc-ad)\left(ad(cfh-2deh+dfg)-b\left(5c^2fh-3cd(2eh+fg)+4d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d^2} + \frac{2b\sqrt{e+fx}\left(6adf(-3cfh+2deh+dfg)+b\left(15c^2f^2h-cdf(8e\right.\right.}{3d^2\ldots}}{2d(de-cf)}$$

$$\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2(bc-ad)\left(ad(cfh-2deh+dfg)-b\left(5c^2fh-3cd(2eh+fg)+4d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2f} + \frac{2b\sqrt{e+fx}\left(6adf(-3cfh+2deh+dfg)+b\left(15c^2f^2\right.\right.}{\ldots}}{2d(de-cf)}$$

$$\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2b\sqrt{e+fx}\left(6adf(-3cfh+2deh+dfg)+b\left(15c^2f^2h-cdf(8eh+9fg)+2d^2e(3fg-2eh)\right)+bdfx(-5cfh+2deh+3dfg)\right)}{3d^2f^2} - \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+}}{\sqrt{de-}}\right.}{\ldots}}{2d(de-cf)}$$

$$\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

input

```
Int[((a + b*x)^2*(g + h*x))/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output

```
-(((d*g - c*h)*(a + b*x)^2*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x))) + ((2
*b*Sqrt[e + f*x]*(6*a*d*f*(d*f*g + 2*d*e*h - 3*c*f*h) + b*(15*c^2*f^2*h +
2*d^2*e*(3*f*g - 2*e*h) - c*d*f*(9*f*g + 8*e*h)) + b*d*f*(3*d*f*g + 2*d*e*
h - 5*c*f*h)*x))/(3*d^2*f^2) - (2*(b*c - a*d)*(a*d*(d*f*g - 2*d*e*h + c*f*
h) - b*(4*d^2*e*g + 5*c^2*f*h - 3*c*d*(f*g + 2*e*h)))*ArcTanh[(Sqrt[d]*Sqr
t[e + f*x])/Sqrt[d*e - c*f]])/(d^(5/2)*Sqrt[d*e - c*f]))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

method	result
risch	$\frac{2b(hbdfx+6adfh-6bcfh-2bdeh+3bgdf)\sqrt{fx+e}}{3f^2d^3}+\frac{(2ad-2bc)\left(-\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2(cf-de)((fx+e)d+cf-de)}+\frac{\left(acdfh-2a\,d^2eh \cdots\right.}{\cdots}\right.}{d^3 \cdots}$
pseudoelliptic	$-\frac{-(ad-bc)(xd+c)\left((afg-2e(ah+2bg))d^2+c((ah+3bg)f+6ehb)d-5b\,c^2fh\right)f^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\left(\left(-a^2f^2g+4 \cdots\right.\right.}{\cdots}$
derivativedivides	$\frac{2b\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^3}-\frac{2f^2\left(\frac{f\left(a^2c\,d^2h-a^2d^3g-2ab\,c^2dh+2abc\,d^2g+c^3h \cdots\right.}{2(cf-de)((fx+e)d+cf-de \cdots}\right.}{\cdots}$
default	$\frac{2b\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^3}-\frac{2f^2\left(\frac{f\left(a^2c\,d^2h-a^2d^3g-2ab\,c^2dh+2abc\,d^2g+c^3h \cdots\right.}{2(cf-de)((fx+e)d+cf-de \cdots}\right.}{\cdots}$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b*(b*d*f*h*x+6*a*d*f*h-6*b*c*f*h-2*b*d*e*h+3*b*d*f*g)*(f*x+e)^(1/2)/f^
2/d^3+1/d^3*(2*a*d-2*b*c)*(-1/2*f*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/(c*f-d
*e)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(a*c*d*f*h-2*a*d^2*e*h+a*d^2*f*g
-5*b*c^2*f*h+6*b*c*d*e*h+3*b*c*d*f*g-4*b*d^2*e*g)/(c*f-d*e)/((c*f-d*e)*d)^
(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(200) = 400.

Time = 0.13 (sec) , antiderivative size = 1547, normalized size of antiderivative = 7.10

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(d^2*e - c*d*f)*((4*(b^2*c^2*d^2 - a*b*c*d^3)*e*f^2 - (3*b^2*
c^3*d - 2*a*b*c^2*d^2 - a^2*c*d^3)*f^3)*g - (2*(3*b^2*c^3*d - 4*a*b*c^2*d^
2 + a^2*c*d^3)*e*f^2 - (5*b^2*c^4 - 6*a*b*c^3*d + a^2*c^2*d^2)*f^3)*h + ((
4*(b^2*c*d^3 - a*b*d^4)*e*f^2 - (3*b^2*c^2*d^2 - 2*a*b*c*d^3 - a^2*d^4)*f^
3)*g - (2*(3*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*e*f^2 - (5*b^2*c^3*d - 6
*a*b*c^2*d^2 + a^2*c*d^3)*f^3)*h)*x)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2
*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(2*(b^2*d^5*e^2*f - 2*b^2*c*d^4*
e*f^2 + b^2*c^2*d^3*f^3)*h*x^2 + 3*(2*b^2*c*d^4*e^2*f - (5*b^2*c^2*d^3 - 2
*a*b*c*d^4 + a^2*d^5)*e*f^2 + (3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*
f^3)*g - (4*b^2*c*d^4*e^3 + 4*(b^2*c^2*d^3 - 3*a*b*c*d^4)*e^2*f - (23*b^2*
c^3*d^2 - 30*a*b*c^2*d^3 + 3*a^2*c*d^4)*e*f^2 + 3*(5*b^2*c^4*d - 6*a*b*c^3
*d^2 + a^2*c^2*d^3)*f^3)*h + 2*(3*(b^2*d^5*e^2*f - 2*b^2*c*d^4*e*f^2 + b^2
*c^2*d^3*f^3)*g - (2*b^2*d^5*e^3 + (b^2*c*d^4 - 6*a*b*d^5)*e^2*f - 4*(2*b^
2*c^2*d^3 - 3*a*b*c*d^4)*e*f^2 + (5*b^2*c^3*d^2 - 6*a*b*c^2*d^3)*f^3)*h)*x
)*sqrt(f*x + e))/(c*d^6*e^2*f^2 - 2*c^2*d^5*e*f^3 + c^3*d^4*f^4 + (d^7*e^2
*f^2 - 2*c*d^6*e*f^3 + c^2*d^5*f^4)*x), -1/3*(3*sqrt(-d^2*e + c*d*f)*((4*(
b^2*c^2*d^2 - a*b*c*d^3)*e*f^2 - (3*b^2*c^3*d - 2*a*b*c^2*d^2 - a^2*c*d^3)
*f^3)*g - (2*(3*b^2*c^3*d - 4*a*b*c^2*d^2 + a^2*c*d^3)*e*f^2 - (5*b^2*c^4
- 6*a*b*c^3*d + a^2*c^2*d^2)*f^3)*h + ((4*(b^2*c*d^3 - a*b*d^4)*e*f^2 - (3
*b^2*c^2*d^2 - 2*a*b*c*d^3 - a^2*d^4)*f^3)*g - (2*(3*b^2*c^2*d^2 - 4*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(h*x+g)/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(200) = 400$.

Time = 0.13 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx =$$

$$-\frac{(4\,b^2cd^2eg - 4\,abd^3eg - 3\,b^2c^2dfg + 2\,abcd^2fg + a^2d^3fg - 6\,b^2c^2deh + 8\,abcd^2eh - 2\,a^2d^3eh + 5\,b^2c^3f}{(d^4e - cd^3f)\sqrt{-d^2e + cdf}}$$

$$-\frac{\sqrt{fx+e}b^2c^2dfg - 2\sqrt{fx+e}abcd^2fg + \sqrt{fx+e}a^2d^3fg - \sqrt{fx+e}b^2c^3fh + 2\sqrt{fx+e}abc^2dfh - \sqrt{}}{(d^4e - cd^3f)((fx+e)d - de + cf)}$$

$$+\frac{2\left(3\sqrt{fx+e}b^2d^4f^5g + (fx+e)^{\frac{3}{2}}b^2d^4f^4h - 3\sqrt{fx+e}b^2d^4ef^4h - 6\sqrt{fx+e}b^2cd^3f^5h + 6\sqrt{fx+e}eab\right.}{3\,d^6f^6}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-(4*b^2*c*d^2*e*g - 4*a*b*d^3*e*g - 3*b^2*c^2*d*f*g + 2*a*b*c*d^2*f*g + a^
2*d^3*f*g - 6*b^2*c^2*d*e*h + 8*a*b*c*d^2*e*h - 2*a^2*d^3*e*h + 5*b^2*c^3*
f*h - 6*a*b*c^2*d*f*h + a^2*c*d^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e
+ c*d*f))/((d^4*e - c*d^3*f)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)*b^2*c^
2*d*f*g - 2*sqrt(f*x + e)*a*b*c*d^2*f*g + sqrt(f*x + e)*a^2*d^3*f*g - sqrt
(f*x + e)*b^2*c^3*f*h + 2*sqrt(f*x + e)*a*b*c^2*d*f*h - sqrt(f*x + e)*a^2*
c*d^2*f*h)/((d^4*e - c*d^3*f)*((f*x + e)*d - d*e + c*f)) + 2/3*(3*sqrt(f*x
 + e)*b^2*d^4*f^5*g + (f*x + e)^(3/2)*b^2*d^4*f^4*h - 3*sqrt(f*x + e)*b^2*
d^4*e*f^4*h - 6*sqrt(f*x + e)*b^2*c*d^3*f^5*h + 6*sqrt(f*x + e)*a*b*d^4*f^
5*h)/(d^6*f^6)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d^2\,f^2}-\frac{4\,b^2\,h\,(c\,f-d\,e)}{d^3\,f^2}\right)$$
$$+\frac{\sqrt{e+f\,x}\,(-f\,h\,a^2\,c\,d^2+f\,g\,a^2\,d^3+2\,f\,h\,a\,b\,c^2\,d-2\,f\,g\,a\,b\,c\,d^2-f\,h\,b^2\,c^3+f\,g\,b^2\,c^2\,d)}{(c\,f-d\,e)\,\left(d^4\,(e+f\,x)-d^4\,e+c\,d^3\,f\right)}$$
$$+\frac{2\,b^2\,h\,(e+f\,x)^{3/2}}{3\,d^2\,f^2}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(a\,d-b\,c)\,\left(a\,d^2\,f\,g-2\,a\,d^2\,e\,h-4\,b\,d^2\,e\,g-5\,b\,c^2\,f\,h+a\,c\,d\,f\,h+6\,b\,c\,d\,e\,h+3\,b\,c\,d\,f\,g\right)}{\sqrt{c\,f-d\,e}\,(a^2\,d^3\,f\,g-2\,a^2\,d^3\,e\,h+5\,b^2\,c^3\,f\,h+4\,b^2\,c\,d^2\,e\,g+a^2\,c\,d^2\,f\,h-6\,b^2\,c^2\,d\,e\,h-3\,b^2\,c^2\,d\,f\,g-4\,a\,b\,d^3\,e\,g+8\,a\,b\,c\,d^2\,e\,h+2\,a\,b\,c\,d^2\,f}\right.}{d^{7/2}\,(c\,f}$$

input
```
int(((g + h*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(c + d*x)^2),x)
```

output
```
(e + f*x)^(1/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d^2*f^2) - (4*b^2*h*
(c*f - d*e))/(d^3*f^2)) + ((e + f*x)^(1/2)*(a^2*d^3*f*g - b^2*c^3*f*h - a^
2*c*d^2*f*h + b^2*c^2*d*f*g - 2*a*b*c*d^2*f*g + 2*a*b*c^2*d*f*h))/((c*f -
d*e)*(d^4*(e + f*x) - d^4*e + c*d^3*f)) + (2*b^2*h*(e + f*x)^(3/2))/(3*d^2
*f^2) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*c)*(a*d^2*f*g - 2*a*d^2*e*
h - 4*b*d^2*e*g - 5*b*c^2*f*h + a*c*d*f*h + 6*b*c*d*e*h + 3*b*c*d*f*g))/((
c*f - d*e)^(1/2)*(a^2*d^3*f*g - 2*a^2*d^3*e*h + 5*b^2*c^3*f*h + 4*b^2*c*d^
2*e*g + a^2*c*d^2*f*h - 6*b^2*c^2*d*e*h - 3*b^2*c^2*d*f*g - 4*a*b*d^3*e*g
+ 8*a*b*c*d^2*e*h + 2*a*b*c*d^2*f*g - 6*a*b*c^2*d*f*h)))*(a*d - b*c)*(a*d^
2*f*g - 2*a*d^2*e*h - 4*b*d^2*e*g - 5*b*c^2*f*h + a*c*d*f*h + 6*b*c*d*e*h
+ 3*b*c*d*f*g))/(d^(7/2)*(c*f - d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1740, normalized size of antiderivative = 7.98

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output
```
(3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a**2*c**2*d**2*f**3*h - 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d
)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*e*f**2*h + 3*sqrt(d)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*f**3*g +
 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a**2*c*d**3*f**3*h*x - 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)
/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4*e*f**2*h*x + 3*sqrt(d)*sqrt(c*f - d*
e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4*f**3*g*x -
18*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a*b*c**3*d*f**3*h + 24*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(
sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*e*f**2*h + 6*sqrt(d)*sqrt(c*f - d*
e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*f**3*g
- 18*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*
e)))*a*b*c**2*d**2*f**3*h*x - 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*e*f**2*g + 24*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*e*f**
2*h*x + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*a*b*c*d**3*f**3*g*x - 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*d**4*e*f**2*g*x + 15*sqrt(d)*sqrt(c
*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**4*f...
```

3.134 $\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 154

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \frac{2bh\sqrt{e+fx}}{d^2f}+\frac{(bc-ad)(dg-ch)\sqrt{e+fx}}{d^2(de-cf)(c+dx)} + \frac{(f(ad^2g-bc^2h)-(2de-cf)(bdg-2bch+adh))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}(de-cf)^{3/2}}$$

output

```
2*b*h*(f*x+e)^(1/2)/d^2/f+(-a*d+b*c)*(-c*h+d*g)*(f*x+e)^(1/2)/d^2/(-c*f+d*
e)/(d*x+c)+(f*(a*d^2*g-b*c^2*h)-(-c*f+2*d*e)*(a*d*h-2*b*c*h+b*d*g))*arctan
h(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \frac{\sqrt{e+fx}(adf(-dg+ch)+b(-3c^2fh+2d^2ehx+cd(fg+2eh-2fhx)))}{d^2f(de-cf)(c+dx)} - \frac{(-ad(dfg-2deh+cfh)+b(2d^2eg+3c^2fh-cd(fg+4eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)*(g + h*x))/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[e + f*x]*(a*d*f*(-(d*g) + c*h) + b*(-3*c^2*f*h + 2*d^2*e*h*x + c*d*(
f*g + 2*e*h - 2*f*h*x))))/(d^2*f*(d*e - c*f)*(c + d*x)) - ((-(a*d*(d*f*g -
 2*d*e*h + c*f*h)) + b*(2*d^2*e*g + 3*c^2*f*h - c*d*(f*g + 4*e*h)))*ArcTan
[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(5/2)*(-(d*e) + c*f)^(3/2
))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 163$$

$$-\frac{\left(ad(cfh-2deh+dfg)-b\left(3c^2fh-cd(4eh+fg)+2d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{2d^2(de-cf)} - \frac{\sqrt{e+fx}(adf(dg-ch)+bc(3cfh-d(2eh+fg))-2bdhx(de-cf))}{d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{\left(ad(cfh-2deh+dfg)-b\left(3c^2fh-cd(4eh+fg)+2d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2f(de-cf)}-\frac{\sqrt{e+fx}(adf(dg-ch)+bc(3cfh-d(2eh+fg))-2bdhx(de-cf))}{d^2f(c+dx)(de-cf)}$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(cfh-2deh+dfg)-b\left(3c^2fh-cd(4eh+fg)+2d^2eg\right)\right)}{d^{5/2}(de-cf)^{3/2}}-\frac{\sqrt{e+fx}(adf(dg-ch)+bc(3cfh-d(2eh+fg))-2bdhx(de-cf))}{d^2f(c+dx)(de-cf)}$$

input
```
Int[((a + b*x)*(g + h*x))/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-((Sqrt[e + f*x]*(a*d*f*(d*g - c*h) + b*c*(3*c*f*h - d*(f*g + 2*e*h)) - 2*
b*d*(d*e - c*f)*h*x))/(d^2*f*(d*e - c*f)*(c + d*x))) + ((a*d*(d*f*g - 2*d*
e*h + c*f*h) - b*(2*d^2*e*g + 3*c^2*f*h - c*d*(f*g + 4*e*h)))*ArcTanh[(Sqr
t[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(5/2)*(d*e - c*f)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.18

method	result
risch	$\frac{2bh\sqrt{fx+e}}{d^2 f} + \frac{-\frac{f\left(acdh-a\,d^2 g-b\,c^2 h+bcdg\right)\sqrt{fx+e}}{(cf-de)((fx+e)d+cf-de)}+\frac{\left(acdfh-2a\,d^2 eh+a\,d^2 fg-3b\,c^2 fh+4bcdeh+bcdfg-2b\,d^2 eg\right)\arctan\Big(\frac{d\cdots}{\sqrt{(\cdots}}}{(cf-de)\sqrt{(cf-de)d}}}{d^2}\cdots$
pseudoelliptic	$\frac{-((2behx-afg)d^2+c(((-2bx+a)h+bg)f+2ehb)d-3b\,c^2 fh)\sqrt{(cf-de)d}\,\sqrt{fx+e}+((afg-2(ah+bg)e)d^2+c((ah+bg)f\cdots}{\sqrt{(cf-de)d}\,f\,d^2(cf-de)(xd+c)}$
derivativedivides	$\frac{\frac{2hb\sqrt{fx+e}}{d^2}-\frac{2f\Big(\frac{f\left(acdh-a\,d^2 g-b\,c^2 h+bcdg\right)\sqrt{fx+e}}{2(cf-de)((fx+e)d+cf-de)}-\frac{\left(acdfh-2a\,d^2 eh+a\,d^2 fg-3b\,c^2 fh+4bcdeh+bcdfg-2b\,d^2 eg\right)\arctan\Big(\frac{d\sqrt{\cdots}}{\sqrt{(c\cdots}}}{2(cf-de)\sqrt{(cf-de)d}}\cdots}{d^2}}{f}$
default	$\frac{\frac{2hb\sqrt{fx+e}}{d^2}-\frac{2f\Big(\frac{f\left(acdh-a\,d^2 g-b\,c^2 h+bcdg\right)\sqrt{fx+e}}{2(cf-de)((fx+e)d+cf-de)}-\frac{\left(acdfh-2a\,d^2 eh+a\,d^2 fg-3b\,c^2 fh+4bcdeh+bcdfg-2b\,d^2 eg\right)\arctan\Big(\frac{d\sqrt{\cdots}}{\sqrt{(c\cdots}}}{2(cf-de)\sqrt{(cf-de)d}}\cdots}{d^2}}{f}$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*h*(f*x+e)^(1/2)/d^2/f+1/d^2*(-f*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/(c*f
-d*e)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+(a*c*d*f*h-2*a*d^2*e*h+a*d^2*f*g-3
*b*c^2*f*h+4*b*c*d*e*h+b*c*d*f*g-2*b*d^2*e*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2
)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(141) = 282$.

Time = 0.11 (sec) , antiderivative size = 807, normalized size of antiderivative = 5.24

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(d^2*e - c*d*f)*((2*b*c*d^2*e*f - (b*c^2*d + a*c*d^2)*f^2)*g - (
2*(2*b*c^2*d - a*c*d^2)*e*f - (3*b*c^3 - a*c^2*d)*f^2)*h + ((2*b*d^3*e*f -
 (b*c*d^2 + a*d^3)*f^2)*g - (2*(2*b*c*d^2 - a*d^3)*e*f - (3*b*c^2*d - a*c*
d^2)*f^2)*h)*x)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x
+ e))/(d*x + c)) + 2*(2*(b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2)*h*x +
((b*c*d^3 - a*d^4)*e*f - (b*c^2*d^2 - a*c*d^3)*f^2)*g + (2*b*c*d^3*e^2 - (
5*b*c^2*d^2 - a*c*d^3)*e*f + (3*b*c^3*d - a*c^2*d^2)*f^2)*h)*sqrt(f*x + e)
)/(c*d^5*e^2*f - 2*c^2*d^4*e*f^2 + c^3*d^3*f^3 + (d^6*e^2*f - 2*c*d^5*e*f^
2 + c^2*d^4*f^3)*x), (sqrt(-d^2*e + c*d*f)*((2*b*c*d^2*e*f - (b*c^2*d + a*
c*d^2)*f^2)*g - (2*(2*b*c^2*d - a*c*d^2)*e*f - (3*b*c^3 - a*c^2*d)*f^2)*h
+ ((2*b*d^3*e*f - (b*c*d^2 + a*d^3)*f^2)*g - (2*(2*b*c*d^2 - a*d^3)*e*f -
(3*b*c^2*d - a*c*d^2)*f^2)*h)*x)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)
/(d*f*x + d*e)) + (2*(b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2)*h*x + ((b
*c*d^3 - a*d^4)*e*f - (b*c^2*d^2 - a*c*d^3)*f^2)*g + (2*b*c*d^3*e^2 - (5*b
*c^2*d^2 - a*c*d^3)*e*f + (3*b*c^3*d - a*c^2*d^2)*f^2)*h)*sqrt(f*x + e))/(
c*d^5*e^2*f - 2*c^2*d^4*e*f^2 + c^3*d^3*f^3 + (d^6*e^2*f - 2*c*d^5*e*f^2 +
 c^2*d^4*f^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.39

$$\begin{aligned}&\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx\\&=\frac{(2\,bd^2eg-bcdfg-ad^2fg-4\,bcdeh+2\,ad^2eh+3\,bc^2fh-acdfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(d^3e-cd^2f)\sqrt{-d^2e+cdf}}\\&\quad+\frac{2\sqrt{fx+e}bh}{d^2f}+\frac{\sqrt{fx+e}bcdfg-\sqrt{fx+e}ad^2fg-\sqrt{fx+e}bc^2fh+\sqrt{fx+e}acdfh}{(d^3e-cd^2f)((fx+e)d-de+cf)}\end{aligned}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
(2*b*d^2*e*g - b*c*d*f*g - a*d^2*f*g - 4*b*c*d*e*h + 2*a*d^2*e*h + 3*b*c^2
*f*h - a*c*d*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^3*e - c
*d^2*f)*sqrt(-d^2*e + c*d*f)) + 2*sqrt(f*x + e)*b*h/(d^2*f) + (sqrt(f*x +
e)*b*c*d*f*g - sqrt(f*x + e)*a*d^2*f*g - sqrt(f*x + e)*b*c^2*f*h + sqrt(f*
x + e)*a*c*d*f*h)/((d^3*e - c*d^2*f)*((f*x + e)*d - d*e + c*f))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(a\,d^2\,f\,g-2\,a\,d^2\,e\,h-2\,b\,d^2\,e\,g-3\,b\,c^2\,f\,h+a\,c\,d\,f\,h+4\,b\,c\,d\,e\,h+b\,c\,d\,f\,g\right)}{d^{5/2}\,{\left(c\,f-d\,e\right)}^{3/2}} + \frac{\sqrt{e+f\,x}\,\left(a\,d^2\,f\,g+b\,c^2\,f\,h-a\,c\,d\,f\,h-b\,c\,d\,f\,g\right)}{\left(c\,f-d\,e\right)\,\left(d^3\,\left(e+f\,x\right)-d^3\,e+c\,d^2\,f\right)} + \frac{2\,b\,h\,\sqrt{e+f\,x}}{d^2\,f}$$

input
```
int(((g + h*x)*(a + b*x))/((e + f*x)^(1/2)*(c + d*x)^2),x)
```

output
```
(atan(((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))*(a*d^2*f*g - 2*a*d^2*e*
h - 2*b*d^2*e*g - 3*b*c^2*f*h + a*c*d*f*h + 4*b*c*d*e*h + b*c*d*f*g))/(d^(
5/2)*(c*f - d*e)^(3/2)) + ((e + f*x)^(1/2)*(a*d^2*f*g + b*c^2*f*h - a*c*d*
f*h - b*c*d*f*g))/((c*f - d*e)*(d^3*(e + f*x) - d^3*e + c*d^2*f)) + (2*b*h
*(e + f*x)^(1/2))/(d^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.84

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a*c**2*d*f**2*h - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a*c*d**2*e*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*f**2*g + sqrt(d)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*f**2*h*x
- 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*a*d**3*e*f*h*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a*d**3*f**2*g*x - 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*f**2*h + 4*sqrt(d)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d*e*f*h
+ sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e))
)*b*c**2*d*f**2*g - 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt
(d)*sqrt(c*f - d*e)))*b*c**2*d*f**2*h*x - 2*sqrt(d)*sqrt(c*f - d*e)*atan((
sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*e*f*g + 4*sqrt(d)*sqr
t(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*e*
f*h*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*b*c*d**2*f**2*g*x - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*b*d**3*e*f*g*x - sqrt(e + f*x)*a*c**2*d**2*f
**2*h + sqrt(e + f*x)*a*c*d**3*e*f*h + sqrt(e + f*x)*a*c*d**3*f**2*g - sqr
t(e + f*x)*a*d**4*e*f*g + 3*sqrt(e + f*x)*b*c**3*d*f**2*h - 5*sqrt(e + ...
```

3.135 $\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = -\frac{(dg-ch)\sqrt{e+fx}}{d(de-cf)(c+dx)}+\frac{(dfg-2deh+cfh)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(de-cf)^{3/2}}$$

output
```
-(-c*h+d*g)*(f*x+e)^(1/2)/d/(-c*f+d*e)/(d*x+c)+(c*f*h-2*d*e*h+d*f*g)*arcta
nh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = \frac{(-dg+ch)\sqrt{e+fx}}{d(de-cf)(c+dx)}+\frac{(dfg-2deh+cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{3/2}(-de+cf)^{3/2}}$$

input
```
Integrate[(g + h*x)/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
((-(d*g) + c*h)*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x)) + ((d*f*g - 2*d*e
*h + c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(3/2)*(
-(d*e) + c*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(cfh-2deh+dfg)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{2d(de-cf)}-\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 73$$

$$-\frac{(cfh-2deh+dfg)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df(de-cf)}-\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)(cfh-2deh+dfg)}{d^{3/2}(de-cf)^{3/2}}-\frac{\sqrt{e+fx}(dg-ch)}{d(c+dx)(de-cf)}$$

input
```
Int[(g + h*x)/((c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-(((d*g - c*h)*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x))) + ((d*f*g - 2*d*e
*h + c*f*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(3/2)*(d*
e - c*f)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$\frac{-\frac{(ch-dg)\sqrt{fx+e}}{xd+c}+\frac{(cfh-2deh+dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{d(cf-de)}$	89
derivativedivides	$-\frac{f(ch-dg)\sqrt{fx+e}}{d(cf-de)((fx+e)d+cf-de)}+\frac{(cfh-2deh+dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d(cf-de)\sqrt{(cf-de)d}}$	112
default	$-\frac{f(ch-dg)\sqrt{fx+e}}{d(cf-de)((fx+e)d+cf-de)}+\frac{(cfh-2deh+dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{d(cf-de)\sqrt{(cf-de)d}}$	112

input
```
int((h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(c*f-d*e)*(-(c*h-d*g)*(f*x+e)^(1/2)/(d*x+c)+(c*f*h-2*d*e*h+d*f*g)/((c*
f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(88) = 176$.

Time = 0.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 4.02

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx$$

$$= \left[-\frac{(cdfg-(2\,cde-c^2f)h+(d^2fg-(2\,d^2e-cdf)h)x)\sqrt{d^2e-cdf}\log\left(\frac{dfx+2\,de-cf-2\,\sqrt{d^2e-cdf}\sqrt{fx+e}}{dx+c}\right)+2}{2\,(cd^4e^2-2\,c^2d^3ef+c^3d^2f^2+(d^5e^2-2\,cd^4ef+c^2d^3f^2)x}\right.$$

$$-\frac{(cdfg-(2\,cde-c^2f)h+(d^2fg-(2\,d^2e-cdf)h)x)\sqrt{-d^2e+cdf}\arctan\left(\frac{\sqrt{-d^2e+cdf}\sqrt{fx+e}}{dfx+de}\right)+((d^3e-}{cd^4e^2-2\,c^2d^3ef+c^3d^2f^2+(d^5e^2-2\,cd^4ef+c^2d^3f^2)x}$$

input
```
integrate((h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((c*d*f*g - (2*c*d*e - c^2*f)*h + (d^2*f*g - (2*d^2*e - c*d*f)*h)*x)
*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt
(f*x + e))/(d*x + c)) + 2*((d^3*e - c*d^2*f)*g - (c*d^2*e - c^2*d*f)*h)*sq
rt(f*x + e))/(c*d^4*e^2 - 2*c^2*d^3*e*f + c^3*d^2*f^2 + (d^5*e^2 - 2*c*d^4
*e*f + c^2*d^3*f^2)*x), -((c*d*f*g - (2*c*d*e - c^2*f)*h + (d^2*f*g - (2*d
^2*e - c*d*f)*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(
f*x + e)/(d*f*x + d*e)) + ((d^3*e - c*d^2*f)*g - (c*d^2*e - c^2*d*f)*h)*sq
rt(f*x + e))/(c*d^4*e^2 - 2*c^2*d^3*e*f + c^3*d^2*f^2 + (d^5*e^2 - 2*c*d^4
*e*f + c^2*d^3*f^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = -\frac{(dfg-2\,deh+cfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(d^2e-cdf)\sqrt{-d^2e+cdf}} - \frac{\sqrt{fx+e}dfg-\sqrt{fx+e}cfh}{(d^2e-cdf)((fx+e)d-de+cf)}$$

input
```
integrate((h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-(d*f*g - 2*d*e*h + c*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((
d^2*e - c*d*f)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)*d*f*g - sqrt(f*x + e
)*c*f*h)/((d^2*e - c*d*f)*((f*x + e)*d - d*e + c*f))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(c\,f\,h-2\,d\,e\,h+d\,f\,g\right)}{d^{3/2}\,\left(c\,f-d\,e\right)^{3/2}} - \frac{\sqrt{e+f\,x}\,\left(c\,f\,h-d\,f\,g\right)}{d\,\left(c\,f-d\,e\right)\,\left(c\,f-d\,e+d\,\left(e+f\,x\right)\right)}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(c + d*x)^2),x)
```

output
```
(atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))*(c*f*h - 2*d*e*h + d*f*
g))/(d^(3/2)*(c*f - d*e)^(3/2)) - ((e + f*x)^(1/2)*(c*f*h - d*f*g))/(d*(c*
f - d*e)*(c*f - d*e + d*(e + f*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.58

$$\int \frac{g+hx}{(c+dx)^2\sqrt{e+fx}}\,dx = \frac{\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2fh-2\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cdeh+\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+}}{\sqrt{d}\,\sqrt{c}}\right.}{}$$

input
```
int((h*x+g)/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*c**2*f*h - 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt
(c*f - d*e)))*c*d*e*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sq
rt(d)*sqrt(c*f - d*e)))*c*d*f*g + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f*h*x - 2*sqrt(d)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*e*h*x + sqrt(d)*sqrt(
c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*f*g*x -
sqrt(e + f*x)*c**2*d*f*h + sqrt(e + f*x)*c*d**2*e*h + sqrt(e + f*x)*c*d**2
*f*g - sqrt(e + f*x)*d**3*e*g)/(d**2*(c**3*f**2 - 2*c**2*d*e*f + c**2*d*f*
*2*x + c*d**2*e**2 - 2*c*d**2*e*f*x + d**3*e**2*x))
```

3.136 $\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 207

$$\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx = \frac{(dg-ch)\sqrt{e+fx}}{(bc-ad)(de-cf)(c+dx)} - \frac{2\sqrt{b}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^2\sqrt{be-af}} + \frac{(ad(dfg-2deh+cfh)+b(2d^2eg-3cdfg+c^2fh))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(bc-ad)^2(de-cf)^{3/2}}$$

output

```
(-c*h+d*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(d*x+c)-2*b^(1/2)*(-a*h+b*g
)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^2/(-a*f+b*e)^
(1/2)+(a*d*(c*f*h-2*d*e*h+d*f*g)+b*(c^2*f*h-3*c*d*f*g+2*d^2*e*g))*arctanh(
d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)^(3
/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95

$$\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx = \frac{\frac{(bc-ad)(-dg+ch)\sqrt{e+fx}}{(-de+cf)(c+dx)}+\frac{2\sqrt{b}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{\sqrt{-be+af}}+\frac{(ad(dfg-2deh+cfh)+b(2d^2eg-3cdfg+c^2fh))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-de+cf)^{3/2}}}{(bc-ad)^2}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
(((b*c - a*d)*(-(d*g) + c*h)*Sqrt[e + f*x])/((-(d*e) + c*f)*(c + d*x)) + (
2*Sqrt[b]*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/
Sqrt[-(b*e) + a*f] + ((a*d*(d*f*g - 2*d*e*h + c*f*h) + b*(2*d^2*e*g - 3*c*
d*f*g + c^2*f*h))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sqr
t[d]*(-(d*e) + c*f)^(3/2)))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{168}$$

$$\frac{\int \frac{2b(de-cf)g+a(dfg-2deh+cfh)+bf(dg-ch)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{(c+dx)(bc-ad)(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2b(de-cf)g+a(dfg-2deh+cfh)+bf(dg-ch)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{(c+dx)(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{2b(bg-ah)(de-cf)\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(ad(cfh-2deh+dfg)+b\left(c^2fh-3cdfg+2d^2eg\right)\right)\int\frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{2(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{(c+dx)(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{4b(bg-ah)(de-cf)\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(ad(cfh-2deh+dfg)+b\left(c^2fh-3cdfg+2d^2eg\right)\right)\int\frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{2(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{(c+dx)(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(cfh-2deh+dfg)+b\left(c^2fh-3cdfg+2d^2eg\right)\right)}{\sqrt{d}(bc-ad)\sqrt{de-cf}}-\frac{4\sqrt{b}(bg-ah)(de-cf)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)\sqrt{be-af}}}{2(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{(c+dx)(bc-ad)(de-cf)}$$

input

```
Int[(g + h*x)/((a + b*x)*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output

```
((d*g - c*h)*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)) + ((-4*Sqr
t[b]*(d*e - c*f)*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*
f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (2*(a*d*(d*f*g - 2*d*e*h + c*f*h) + b
*(2*d^2*e*g - 3*c*d*f*g + c^2*f*h))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d
*e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f]))/(2*(b*c - a*d)*(d*e - c
*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.12

method	result
derivativedivides	$2f\left(-\frac{(ah-bg)b\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f(ad-bc)^2\sqrt{(af-be)b}}-\frac{\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2(cf-de)((fx+e)d+cf-de)}-\frac{\left(acdfh-2a\,d^2eh+a\,d^2fg+b\,c^2fh-3bcdfg\right.}{2(cf-de)\sqrt{(cf}}}{(ad-bc)^2f}\right.$
default	$2f\left(-\frac{(ah-bg)b\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{f(ad-bc)^2\sqrt{(af-be)b}}-\frac{\frac{f\left(acdh-a\,d^2g-b\,c^2h+bcdg\right)\sqrt{fx+e}}{2(cf-de)((fx+e)d+cf-de)}-\frac{\left(acdfh-2a\,d^2eh+a\,d^2fg+b\,c^2fh-3bcdfg\right.}{2(cf-de)\sqrt{(cf}}}{(ad-bc)^2f}\right.$
pseudoelliptic	$-\frac{-(xd+c)\sqrt{(af-be)b}\,\left(b\left(c^2fh-3cdfg+2d^2eg\right)+a((-2eh+fg)d+cfh)d\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\left(2b(xd+c)(cf-de)\right.}{\sqrt{(af-be)b}\,\sqrt{(cf-de)d}\,(ad-bc)^2(cf-de}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(-1/f*(a*h-b*g)*b/(a*d-b*c)^2/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/
2)/((a*f-b*e)*b)^(1/2))-1/(a*d-b*c)^2/f*(1/2*f*(a*c*d*h-a*d^2*g-b*c^2*h+b*
c*d*g)/(c*f-d*e)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)-1/2*(a*c*d*f*h-2*a*d^2*
e*h+a*d^2*f*g+b*c^2*f*h-3*b*c*d*f*g+2*b*d^2*e*g)/(c*f-d*e)/((c*f-d*e)*d)^(
1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(185) = 370.

Time = 9.44 (sec) , antiderivative size = 2596, normalized size of antiderivative = 12.54

$$\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*((b*c*d^3*e^2 - 2*b*c^2*d^2*e*f + b*c^3*d*f^2)*g - (a*c*d^3*e^2 -
 2*a*c^2*d^2*e*f + a*c^3*d*f^2)*h + ((b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^
2*f^2)*g - (a*d^4*e^2 - 2*a*c*d^3*e*f + a*c^2*d^2*f^2)*h)*x)*sqrt(b/(b*e -
 a*f))*log((b*f*x + 2*b*e - a*f + 2*(b*e - a*f)*sqrt(f*x + e)*sqrt(b/(b*e
- a*f)))/(b*x + a)) + sqrt(d^2*e - c*d*f)*((2*b*c*d^2*e - (3*b*c^2*d - a*c
*d^2)*f)*g - (2*a*c*d^2*e - (b*c^3 + a*c^2*d)*f)*h + ((2*b*d^3*e - (3*b*c*
d^2 - a*d^3)*f)*g - (2*a*d^3*e - (b*c^2*d + a*c*d^2)*f)*h)*x)*log((d*f*x +
 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(((b*c*
d^3 - a*d^4)*e - (b*c^2*d^2 - a*c*d^3)*f)*g - ((b*c^2*d^2 - a*c*d^3)*e - (
b*c^3*d - a*c^2*d^2)*f)*h)*sqrt(f*x + e))/((b^2*c^3*d^3 - 2*a*b*c^2*d^4 +
a^2*c*d^5)*e^2 - 2*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e*f + (b^2*
c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*f^2 + ((b^2*c^2*d^4 - 2*a*b*c*d^5 + a
^2*d^6)*e^2 - 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*f + (b^2*c^4*d
^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*f^2)*x), -(sqrt(-d^2*e + c*d*f)*((2*b*c*
d^2*e - (3*b*c^2*d - a*c*d^2)*f)*g - (2*a*c*d^2*e - (b*c^3 + a*c^2*d)*f)*h
 + ((2*b*d^3*e - (3*b*c*d^2 - a*d^3)*f)*g - (2*a*d^3*e - (b*c^2*d + a*c*d^
2)*f)*h)*x)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) + ((b
*c*d^3*e^2 - 2*b*c^2*d^2*e*f + b*c^3*d*f^2)*g - (a*c*d^3*e^2 - 2*a*c^2*d^2
*e*f + a*c^3*d*f^2)*h + ((b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2)*g - (
a*d^4*e^2 - 2*a*c*d^3*e*f + a*c^2*d^2*f^2)*h)*x)*sqrt(b/(b*e - a*f))*lo...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g + hx}{(a + bx)(c + dx)^2 \sqrt{e + fx}}\, dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g + hx}{(a + bx)(c + dx)^2\sqrt{e + fx}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.36

$$\int \frac{g + hx}{(a + bx)(c + dx)^2\sqrt{e + fx}}\, dx = \frac{2\,(b^2g - abh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-b^2e + abf}} - \frac{(2\,bd^2eg - 3\,bcdfg + ad^2fg - 2\,ad^2eh + bc^2fh + acdfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2de - 2\,abcd^2e + a^2d^3e - b^2c^3f + 2\,abc^2df - a^2cd^2f)\sqrt{-d^2e + cdf}} + \frac{\sqrt{fx + e}dfg - \sqrt{fx + e}cfh}{(bcde - ad^2e - bc^2f + acdf)((fx + e)d - de + cf)}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(b^2*g - a*b*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*e + a*b*f)) - (2*b*d^2*e*g - 3*b*c*d*f*g +
 a*d^2*f*g - 2*a*d^2*e*h + b*c^2*f*h + a*c*d*f*h)*arctan(sqrt(f*x + e)*d/s
qrt(-d^2*e + c*d*f))/((b^2*c^2*d*e - 2*a*b*c*d^2*e + a^2*d^3*e - b^2*c^3*f
 + 2*a*b*c^2*d*f - a^2*c*d^2*f)*sqrt(-d^2*e + c*d*f)) + (sqrt(f*x + e)*d*f
*g - sqrt(f*x + e)*c*f*h)/((b*c*d*e - a*d^2*e - b*c^2*f + a*c*d*f)*((f*x +
 e)*d - d*e + c*f))
```

Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 24052, normalized size of antiderivative = 116.19

$$\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^2),x)
```

output
```
atan((((b^3*g^2 + a^2*b*h^2 - 2*a*b^2*g*h)/(b^5*c^4*e - a^5*d^4*f - a*b^4*
c^4*f + a^4*b*d^4*e - 4*a*b^4*c^3*d*e + 4*a^4*b*c*d^3*f - 4*a^3*b^2*c*d^3*
e + 4*a^2*b^3*c^3*d*f + 6*a^2*b^3*c^2*d^2*e - 6*a^3*b^2*c^2*d^2*f))^(1/2)*
(((2*(4*b^7*c^6*d^2*f^5*g - 18*a*b^6*c^5*d^3*f^5*g - 2*a^5*b^2*c*d^7*f^5*g
 - 2*a*b^6*c^6*d^2*f^5*h + 2*a^5*b^2*d^8*e*f^4*g - 6*b^7*c^5*d^3*e*f^4*g -
 2*b^7*c^6*d^2*e*f^4*h + 32*a^2*b^5*c^4*d^4*f^5*g - 28*a^3*b^4*c^3*d^5*f^5
*g + 12*a^4*b^3*c^2*d^6*f^5*g + 8*a^2*b^5*c^5*d^3*f^5*h - 12*a^3*b^4*c^4*d
^4*f^5*h + 8*a^4*b^3*c^3*d^5*f^5*h - 2*a^5*b^2*c^2*d^6*f^5*h + 2*a^4*b^3*d
^8*e^2*f^3*g - 4*a^5*b^2*d^8*e^2*f^3*h + 2*b^7*c^4*d^4*e^2*f^3*g + 2*b^7*c
^5*d^3*e^2*f^3*h + 12*a^2*b^5*c^2*d^6*e^2*f^3*g + 28*a^2*b^5*c^3*d^5*e^2*f
^3*h - 32*a^3*b^4*c^2*d^6*e^2*f^3*h + 26*a*b^6*c^4*d^4*e*f^4*g - 14*a^4*b^
3*c*d^7*e*f^4*g + 14*a*b^6*c^5*d^3*e*f^4*h + 6*a^5*b^2*c*d^7*e*f^4*h - 8*a
*b^6*c^3*d^5*e^2*f^3*g - 44*a^2*b^5*c^3*d^5*e*f^4*g - 8*a^3*b^4*c*d^7*e^2*
f^3*g + 36*a^3*b^4*c^2*d^6*e*f^4*g - 12*a*b^6*c^4*d^4*e^2*f^3*h - 36*a^2*b
^5*c^4*d^4*e*f^4*h + 44*a^3*b^4*c^3*d^5*e*f^4*h + 18*a^4*b^3*c*d^7*e^2*f^3
*h - 26*a^4*b^3*c^2*d^6*e*f^4*h))/(a^3*d^5*e^2 - b^3*c^5*f^2 + a^3*c^2*d^3
*f^2 - b^3*c^3*d^2*e^2 - 2*a^3*c*d^4*e*f + 2*b^3*c^4*d*e*f - 3*a^2*b*c*d^4
*e^2 + 3*a*b^2*c^4*d*f^2 + 3*a*b^2*c^2*d^3*e^2 - 3*a^2*b*c^3*d^2*f^2 - 6*a
*b^2*c^3*d^2*e*f + 6*a^2*b*c^2*d^3*e*f) - (2*(e + f*x)^(1/2)*((b^3*g^2 + a
^2*b*h^2 - 2*a*b^2*g*h)/(b^5*c^4*e - a^5*d^4*f - a*b^4*c^4*f + a^4*b*d^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2158, normalized size of antiderivative = 10.43

$$\int \frac{g+hx}{(a+bx)(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output

```
( - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a*c**3*d*f**2*h + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a*c**2*d**2*e*f*h - 2*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c**2*d**2*f**2*h*x - 2*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
*c*d**3*e**2*h + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a*c*d**3*e*f*h*x - 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*d**4*e**2*h*x + 2*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c**3*d*f**2*
g - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*b*c**2*d**2*e*f*g + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*b*c**2*d**2*f**2*g*x + 2*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c*d**3*e**2*g - 4*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b
*c*d**3*e*f*g*x + 2*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*b*d**4*e**2*g*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d*f**2*h - 2*sqrt(d)*sqrt(
c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**2*e
*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**2*c*d**2*f**2*g + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)...
```

3.137 $\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 347

$$\begin{aligned}
&\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx \\
&= -\frac{d(2bdeg-bc(fg+eh)-a(dfg+deh-2cfh))\sqrt{e+fx}}{(bc-ad)^2(be-af)(de-cf)(c+dx)} \\
&\quad -\frac{(bg-ah)\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)(c+dx)} \\
&\quad +\frac{\sqrt{b}(3a^2dfh+b^2(4deg+cfg-2ceh)-ab(5dfg+2deh-cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^3(be-af)^{3/2}} \\
&\quad -\frac{\sqrt{d}(ad(dfg-2deh+cfh)+b(4d^2eg+3c^2fh-cd(5fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^3(de-cf)^{3/2}}
\end{aligned}$$

output

```
-d*(2*b*d*e*g-b*c*(e*h+f*g)-a*(-2*c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)/(-a*d+
b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x+c)-(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(
-a*f+b*e)/(b*x+a)/(d*x+c)+b^(1/2)*(3*a^2*d*f*h+b^2*(-2*c*e*h+c*f*g+4*d*e*g
)-a*b*(-c*f*h+2*d*e*h+5*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(
1/2))/(-a*d+b*c)^3/(-a*f+b*e)^(3/2)-d^(1/2)*(a*d*(c*f*h-2*d*e*h+d*f*g)+b*(
4*d^2*e*g+3*c^2*f*h-c*d*(2*e*h+5*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*
f+d*e)^(1/2))/(-a*d+b*c)^3/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx =$$
$$-\frac{\sqrt{e+fx}(a^2df(dg-ch)+b^2(c^2fg-2d^2egx+cd(-eg+fgx+ehx))+ab(-c^2fh+2cdh(e-fx)+}{(bc-ad)^2(be-af)(-de+cf)(a+bx)(c+dx)}$$
$$+\frac{\sqrt{b}(3a^2dfh+b^2(4deg+cfg-2ceh)+ab(-5dfg-2deh+cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{(bc-ad)^3(-be+af)^{3/2}}$$
$$+\frac{\sqrt{d}(ad(dfg-2deh+cfh)+b(4d^2eg+3c^2fh-cd(5fg+2eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(-bc+ad)^3(-de+cf)^{3/2}}$$

input
```
Integrate[(g + h*x)/((a + b*x)^2*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-((Sqrt[e + f*x]*(a^2*d*f*(d*g - c*h) + b^2*(c^2*f*g - 2*d^2*e*g*x + c*d*(
-(e*g) + f*g*x + e*h*x)) + a*b*(-(c^2*f*h) + 2*c*d*h*(e - f*x) + d^2*(-(e*
g) + f*g*x + e*h*x))))/((b*c - a*d)^2*(b*e - a*f)*(-(d*e) + c*f)*(a + b*x)
*(c + d*x))) + (Sqrt[b]*(3*a^2*d*f*h + b^2*(4*d*e*g + c*f*g - 2*c*e*h) + a
*b*(-5*d*f*g - 2*d*e*h + c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e
) + a*f]])/((b*c - a*d)^3*(-(b*e) + a*f)^(3/2)) + (Sqrt[d]*(a*d*(d*f*g - 2
*d*e*h + c*f*h) + b*(4*d^2*e*g + 3*c^2*f*h - c*d*(5*f*g + 2*e*h)))*ArcTan[
(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/((-(b*c) + a*d)^3*(-(d*e) + c
*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 168$$

$$-\frac{\int \frac{acfh-2ad(fg+eh)+b(4deg+cfg-2ceh)+3df(bg-ah)x}{2(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{acfh-2ad(fg+eh)+b(4deg+cfg-2ceh)+3df(bg-ah)x}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\int \frac{-df(dfg-2deh+cfh)a^2-b\left(e(3fg+2eh)d^2-2cf(2fg+eh)d+c^2f^2h\right)a+b^2(de-cf)(4deg+cfg-2ceh)+bdf(2bdeg-bc(fg+eh)-a(dfg+deh-2cfh))x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{(bc-ad)(de-cf)}+\frac{2c}{}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 174$$

$$-\frac{\frac{\frac{b(de-cf)\left(3a^2dfh-ab(-cfh+2deh+5dfg)+b^2(-2ceh+cfg+4deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{d(be-af)\left(ad(cfh-2deh+dfg)+b\left(3c^2fh-cd(2eh+5fg)+4d^2eg\right)\right)\int}{bc-ad}}{(bc-ad)(de-cf)}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{\frac{2b(de-cf)\left(3a^2dfh-ab(-cfh+2deh+5dfg)+b^2(-2ceh+cfg+4deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2d(be-af)\left(ad(cfh-2deh+dfg)+b\left(3c^2fh-cd(2eh+5fg)+4\right.\right.}{f(bc-ad)}}{(bc-ad)(de-cf)}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{2\sqrt{d}(be-af)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(cfh-2deh+dfg)+b\left(3c^2fh-cd(2eh+5fg)+4d^2eg\right)\right)}{(bc-ad)\sqrt{de-cf}}-\frac{2\sqrt{b}(de-cf)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^2dfh-ab(-cfh+2d\right.}{(bc-ad)\sqrt{be-af}}}{(bc-ad)(de-cf)}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^2*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-(((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)
)) - ((2*d*(b*(2*d*e*g - c*f*g - c*e*h) - a*(d*f*g + d*e*h - 2*c*f*h))*Sqr
t[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)) + ((-2*Sqrt[b]*(d*e - c*f)
*(3*a^2*d*f*h + b^2*(4*d*e*g + c*f*g - 2*c*e*h) - a*b*(5*d*f*g + 2*d*e*h -
 c*f*h))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sq
rt[b*e - a*f]) + (2*Sqrt[d]*(b*e - a*f)*(a*d*(d*f*g - 2*d*e*h + c*f*h) + b
*(4*d^2*e*g + 3*c^2*f*h - c*d*(5*f*g + 2*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e +
f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/((b*c - a*d)*(d*e -
 c*f)))/(2*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.04

method	result
derivativedivides	$2f^2\left(-\frac{b\left(\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(3a^2dfh+abcfh-2abdeh-5abdfg-2b^2ceh+b^2cfg+4b^2deg\right)\arctan\left(\frac{b\sqrt{fx+}}{\sqrt{(af-b}}\right.}{2(af-be)\sqrt{(af-be)b}}\right.}{f^2(ad-bc)^3}\right.$
default	$2f^2\left(-\frac{b\left(\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2(af-be)((fx+e)b+af-be)}+\frac{\left(3a^2dfh+abcfh-2abdeh-5abdfg-2b^2ceh+b^2cfg+4b^2deg\right)\arctan\left(\frac{b\sqrt{fx+}}{\sqrt{(af-b}}\right.}{2(af-be)\sqrt{(af-be)b}}\right.}{f^2(ad-bc)^3}\right.$
pseudoelliptic	$-\frac{3\left(\frac{2\left(2deg-c\left(eh-\frac{fg}{2}\right)\right)b^2}{3}+\frac{a((-2eh-5fg)d+cfh)b}{3}+a^2dfh\right)(xd+c)(cf-de)(bx+a)b\sqrt{(cf-de)d}\,\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{}$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^2*(-b/f^2/(a*d-b*c)^3*(1/2*f*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/(a*f-b*
e)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)+1/2*(3*a^2*d*f*h+a*b*c*f*h-2*a*b*d*e*
h-5*a*b*d*f*g-2*b^2*c*e*h+b^2*c*f*g+4*b^2*d*e*g)/(a*f-b*e)/((a*f-b*e)*b)^(
1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))-d/f^2/(a*d-b*c)^3*(1/2*f
*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/(c*f-d*e)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-
d*e)-1/2*(a*c*d*f*h-2*a*d^2*e*h+a*d^2*f*g+3*b*c^2*f*h-2*b*c*d*e*h-5*b*c*d*
f*g+4*b*d^2*e*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*
f-d*e)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. $2(323) = 646$.

Time = 172.74 (sec) , antiderivative size = 7176, normalized size of antiderivative = 20.68

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**2/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(323) = 646$.

Time = 0.17 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.42

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-(4*b^3*d*e*g + b^3*c*f*g - 5*a*b^2*d*f*g - 2*b^3*c*e*h - 2*a*b^2*d*e*h +
a*b^2*c*f*h + 3*a^2*b*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/
((b^4*c^3*e - 3*a*b^3*c^2*d*e + 3*a^2*b^2*c*d^2*e - a^3*b*d^3*e - a*b^3*c^
3*f + 3*a^2*b^2*c^2*d*f - 3*a^3*b*c*d^2*f + a^4*d^3*f)*sqrt(-b^2*e + a*b*f
)) + (4*b*d^3*e*g - 5*b*c*d^2*f*g + a*d^3*f*g - 2*b*c*d^2*e*h - 2*a*d^3*e*
h + 3*b*c^2*d*f*h + a*c*d^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*
f))/((b^3*c^3*d*e - 3*a*b^2*c^2*d^2*e + 3*a^2*b*c*d^3*e - a^3*d^4*e - b^3*
c^4*f + 3*a*b^2*c^3*d*f - 3*a^2*b*c^2*d^2*f + a^3*c*d^3*f)*sqrt(-d^2*e + c
*d*f)) - (2*(f*x + e)^(3/2)*b^2*d^2*e*f*g - 2*sqrt(f*x + e)*b^2*d^2*e^2*f*
g - (f*x + e)^(3/2)*b^2*c*d*f^2*g - (f*x + e)^(3/2)*a*b*d^2*f^2*g + 2*sqrt
(f*x + e)*b^2*c*d*e*f^2*g + 2*sqrt(f*x + e)*a*b*d^2*e*f^2*g - sqrt(f*x + e
)*b^2*c^2*f^3*g - sqrt(f*x + e)*a^2*d^2*f^3*g - (f*x + e)^(3/2)*b^2*c*d*e*
f*h - (f*x + e)^(3/2)*a*b*d^2*e*f*h + sqrt(f*x + e)*b^2*c*d*e^2*f*h + sqrt
(f*x + e)*a*b*d^2*e^2*f*h + 2*(f*x + e)^(3/2)*a*b*c*d*f^2*h - 4*sqrt(f*x +
 e)*a*b*c*d*e*f^2*h + sqrt(f*x + e)*a*b*c^2*f^3*h + sqrt(f*x + e)*a^2*c*d*
f^3*h)/((b^3*c^2*d*e^2 - 2*a*b^2*c*d^2*e^2 + a^2*b*d^3*e^2 - b^3*c^3*e*f +
 a*b^2*c^2*d*e*f + a^2*b*c*d^2*e*f - a^3*d^3*e*f + a*b^2*c^3*f^2 - 2*a^2*b
*c^2*d*f^2 + a^3*c*d^2*f^2)*((f*x + e)^2*b*d - 2*(f*x + e)*b*d*e + b*d*e^2
 + (f*x + e)*b*c*f + (f*x + e)*a*d*f - b*c*e*f - a*d*e*f + a*c*f^2))
```

Mupad [B] (verification not implemented)

Time = 19.30 (sec) , antiderivative size = 384543, normalized size of antiderivative = 1108.19

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^2),x)
```

output
```
- atan(((((4*a^9*b^2*d^11*e*f^6*g - 4*a^9*b^2*c*d^10*f^7*g - 4*a*b^10*c^9*
d^2*f^7*g + 4*b^11*c^9*d^2*e*f^6*g + 40*a^2*b^9*c^8*d^3*f^7*g - 160*a^3*b^
8*c^7*d^4*f^7*g + 344*a^4*b^7*c^6*d^5*f^7*g - 440*a^5*b^6*c^5*d^6*f^7*g +
344*a^6*b^5*c^4*d^7*f^7*g - 160*a^7*b^4*c^3*d^8*f^7*g + 40*a^8*b^3*c^2*d^9
*f^7*g - 4*a^2*b^9*c^9*d^2*f^7*h + 20*a^3*b^8*c^8*d^3*f^7*h - 36*a^4*b^7*c
^7*d^4*f^7*h + 20*a^5*b^6*c^6*d^5*f^7*h + 20*a^6*b^5*c^5*d^6*f^7*h - 36*a^
7*b^4*c^4*d^7*f^7*h + 20*a^8*b^3*c^3*d^8*f^7*h - 4*a^9*b^2*c^2*d^9*f^7*h +
 8*a^6*b^5*d^11*e^4*f^3*g - 16*a^7*b^4*d^11*e^3*f^4*g + 4*a^8*b^3*d^11*e^2
*f^5*g - 4*a^7*b^4*d^11*e^4*f^3*h + 12*a^8*b^3*d^11*e^3*f^4*h - 8*a^9*b^2*
d^11*e^2*f^5*h + 8*b^11*c^6*d^5*e^4*f^3*g - 16*b^11*c^7*d^4*e^3*f^4*g + 4*
b^11*c^8*d^3*e^2*f^5*g - 4*b^11*c^7*d^4*e^4*f^3*h + 12*b^11*c^8*d^3*e^3*f^
4*h - 8*b^11*c^9*d^2*e^2*f^5*h + 120*a^2*b^9*c^4*d^7*e^4*f^3*g - 144*a^2*b
^9*c^5*d^6*e^3*f^4*g - 176*a^2*b^9*c^6*d^5*e^2*f^5*g - 160*a^3*b^8*c^3*d^8
*e^4*f^3*g + 80*a^3*b^8*c^4*d^7*e^3*f^4*g + 496*a^3*b^8*c^5*d^6*e^2*f^5*g
+ 120*a^4*b^7*c^2*d^9*e^4*f^3*g + 80*a^4*b^7*c^3*d^8*e^3*f^4*g - 680*a^4*b
^7*c^4*d^7*e^2*f^5*g - 144*a^5*b^6*c^2*d^9*e^3*f^4*g + 496*a^5*b^6*c^3*d^8
*e^2*f^5*g - 176*a^6*b^5*c^2*d^9*e^2*f^5*g - 36*a^2*b^9*c^5*d^6*e^4*f^3*h
+ 144*a^2*b^9*c^6*d^5*e^3*f^4*h - 40*a^2*b^9*c^7*d^4*e^2*f^5*h + 20*a^3*b^
8*c^4*d^7*e^4*f^3*h - 192*a^3*b^8*c^5*d^6*e^3*f^4*h + 8*a^3*b^8*c^6*d^5*e^
2*f^5*h + 20*a^4*b^7*c^3*d^8*e^4*f^3*h + 200*a^4*b^7*c^4*d^7*e^3*f^4*h ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8083, normalized size of antiderivative = 23.29

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output

```
( - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*c**3*d*f**3*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**2*d**2*e*f**2*h - 3*sqrt(b)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c**2*d**2*f*
*3*h*x - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**3*c*d**3*e**2*f*h + 6*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c*d**3*e*f**2*h*x - 3*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*d**4*e
**2*f*h*x - sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**2*b*c**4*f**3*h + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**3*d*e*f**2*h + 5*sqrt(b)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**3*
d*f**3*g - 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**2*b*c**3*d*f**3*h*x - 5*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**2*e**2*f*h - 10*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
2*b*c**2*d**2*e*f**2*g + 10*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d**2*e*f**2*h*x + 5*sqrt(b)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d*
*2*f**3*g*x - 3*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)...
```

3.138 $\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 595

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{d(a^2df(4dfg+7deh-11cfh)+b^2(12d^2e^2g-c^2f(3fg-4eh)-cde(5fg+8eh))-ab(c^2f^2h+d^2e(19f}{4(bc-ad)^3(be-af)^2(de-cf)(c+dx)}$$
$$-\frac{(bg-ah)\sqrt{e+fx}}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}$$
$$+\frac{(5a^2dfh+b^2(6deg+3cfg-4ceh)-ab(9dfg+2deh-cfh))\sqrt{e+fx}}{4(bc-ad)^2(be-af)^2(a+bx)(c+dx)}$$
$$+\frac{\sqrt{b}(15a^3d^2f^2h-5a^2bdf(7dfg+4deh-2cfh)-b^3(24d^2e^2g+c^2f(3fg-4eh)+8cde(fg-2eh))-c}{4(bc-ad)^4(be-af)^{5/2}}$$
$$+\frac{d^{3/2}(ad(dfg-2deh+cfh)+b(6d^2eg+5c^2fh-cd(7fg+4eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4(de-cf)^{3/2}}$$

output
```
1/4*d*(a^2*d*f*(-11*c*f*h+7*d*e*h+4*d*f*g)+b^2*(12*d^2*e^2*g-c^2*f*(-4*e*h
+3*f*g)-c*d*e*(8*e*h+5*f*g))-a*b*(c^2*f^2*h+d^2*e*(4*e*h+19*f*g)-c*d*f*(13
*e*h+11*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x+c)-
1/2*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2/(d*x+c)+1/4*(
5*a^2*d*f*h+b^2*(-4*c*e*h+3*c*f*g+6*d*e*g)-a*b*(-c*f*h+2*d*e*h+9*d*f*g))*(
f*x+e)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)/(d*x+c)+1/4*b^(1/2)*(15*a^3
*d^2*f^2*h-5*a^2*b*d*f*(-2*c*f*h+4*d*e*h+7*d*f*g)-b^3*(24*d^2*e^2*g+c^2*f*
(-4*e*h+3*f*g)+8*c*d*e*(-2*e*h+f*g))-a*b^2*(c^2*f^2*h-2*c*d*f*(-16*e*h+7*f
*g)-8*d^2*e*(e*h+7*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/
(-a*d+b*c)^4/(-a*f+b*e)^(5/2)+d^(3/2)*(a*d*(c*f*h-2*d*e*h+d*f*g)+b*(6*d^2*
e*g+5*c^2*f*h-c*d*(4*e*h+7*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)
^(1/2))/(-a*d+b*c)^4/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 13.01 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.21

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx$$

$$= \frac{(bc-ad)\sqrt{e+fx}\left(4a^4d^2f^2(-dg+ch)+a^3bdf\left(9c^2fh-17cdh(e-fx)+d^2(8eg-8fgx-9ehx)\right)+ab^3\left(d^3ex(-18eg+19fgx+4ehx)+c^3f(5fg-2eh+f\right.\right.}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^3*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(4*a^4*d^2*f^2*(-(d*g) + c*h) + a^3*b*d*f*(9*c
^2*f*h - 17*c*d*h*(e - f*x) + d^2*(8*e*g - 8*f*g*x - 9*e*h*x)) + a*b^3*(d^
3*e*x*(-18*e*g + 19*f*g*x + 4*e*h*x) + c^3*f*(5*f*g - 2*e*h + f*h*x) + c*d
^2*(-11*f^2*g*x^2 + e*f*x*(16*g - 13*h*x) - 2*e^2*(5*g - 7*h*x)) + c^2*d*(
2*e^2*h + e*f*(5*g - 7*h*x) + f^2*x*(-6*g + h*x))) + b^4*(-12*d^3*e^2*g*x^
2 + c*d^2*e*x*(-6*e*g + 5*f*g*x + 8*e*h*x) + c^3*f*(3*f*g*x - 2*e*(g + 2*h
*x)) + c^2*d*(3*f^2*g*x^2 + e*f*x*(3*g - 4*h*x) + 2*e^2*(g + 2*h*x))) - a^
2*b^2*(c^3*f^2*h + c^2*d*f*(13*f*g + 5*e*h - 6*f*h*x) + d^3*(4*f^2*g*x^2 +
 e^2*(4*g - 6*h*x) + e*f*x*(-29*g + 7*h*x)) + c*d^2*(-10*e^2*h + f^2*x*(13
*g - 11*h*x) + e*f*(-13*g + 28*h*x)))))/((b*e - a*f)^2*(-(d*e) + c*f)*(a +
 b*x)^2*(c + d*x)) + (Sqrt[b]*(-15*a^3*d^2*f^2*h + 5*a^2*b*d*f*(7*d*f*g +
4*d*e*h - 2*c*f*h) + b^3*(24*d^2*e^2*g + c^2*f*(3*f*g - 4*e*h) + 8*c*d*e*(
f*g - 2*e*h)) + a*b^2*(c^2*f^2*h - 8*d^2*e*(7*f*g + e*h) + 2*c*d*f*(-7*f*g
 + 16*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e) +
 a*f)^(5/2) + (4*d^(3/2)*(a*d*(d*f*g - 2*d*e*h + c*f*h) + b*(6*d^2*e*g + 5
*c^2*f*h - c*d*(7*f*g + 4*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e
) + c*f]])/(-(d*e) + c*f)^(3/2))/(4*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{\int \frac{acfh-2ad(2fg+eh)+b(6deg+3cfg-4ceh)+5df(bg-ah)x}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{acfh-2ad(2fg+eh)+b(6deg+3cfg-4ceh)+5df(bg-ah)x}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{-\frac{\int \frac{df(8dfg+14deh-7cfh)a^2+b\left(-2e(19fg+4eh)d^2-5cf(fg-4eh)d+c^2f^2h\right)a+b^2\left(f(3fg-4eh)c^2+8de(fg-2eh)c+24d^2e^2g\right)+3df\left(5dfha^2-b(9dfg+2deh-c\cdots\right.}{2(a+bx)(c+dx)^2\sqrt{e+fx}}}{(bc-ad)(be-af)}\cdots}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{df(8dfg+14deh-7cfh)a^2+b\left(-2e(19fg+4eh)d^2-5cf(fg-4eh)d+c^2f^2h\right)a+b^2\left(f(3fg-4eh)c^2+8de(fg-2eh)c+24d^2e^2g\right)+3df\left(5dfha^2-b(9dfg+2deh-c\cdots\right.}{(a+bx)(c+dx)^2\sqrt{e+fx}}}{2(bc-ad)(be-af)}\cdots}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$\int 4d^2f^2(dfg-2deh+cfh)a^3+bdf\left(16e(fg+eh)d^2-cf(24fg+17eh)d+9c^2f^2h\right)a^2-b^2\left(4e^2(11fg+2eh)d^3-cef(37fg+32eh)d^2-c^2f^2(11fg-27eh)d+c^3f^3h\right)\cdots$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 174$$

$$-\frac{b(de-cf)\left(15a^3d^2f^2h-5a^2bdf(-2cfh+4deh+7dfg)-ab^2\left(c^2f^2h-2cdf(7fg-16eh)-8d^2e(eh+7fg)\right)-b^3\left(c^2f(3fg-4eh)+8cde(fg-2eh)+24d^2e^2g\right)\right)\int\cdots}{bc-ad}\cdots \Big/ (bc-ad)(de-cf)$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 73$$

$$-\frac{2b(de-cf)\left(15a^3d^2f^2h-5a^2bdf(-2cfh+4deh+7dfg)-ab^2\left(c^2f^2h-2cdf(7fg-16eh)-8d^2e(eh+7fg)\right)-b^3\left(c^2f(3fg-4eh)+8cde(fg-2eh)+24d^2e^2g\right)\right)\int\cdots}{f(bc-ad)}\cdots \Big/ (bc-ad)(de-cf)$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{\sqrt{e+fx}\left(5a^2dfh-ab(-cfh+2deh+9dfg)+b^2(-4ceh+3cfg+6deg)\right)}{(a+bx)(c+dx)(bc-ad)(be-af)}-\frac{2d\sqrt{e+fx}\left(a^2df(-11cfh+7deh+4dfg)-ab\left(c^2f^2h-cdf(13eh+11fg)+d^2e(4e\right.\right.}{(c+dx)(bc-ad)(de}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^3*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-1/2*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c +
 d*x)) - (-(((5*a^2*d*f*h + b^2*(6*d*e*g + 3*c*f*g - 4*c*e*h) - a*b*(9*d*f
*g + 2*d*e*h - c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(
c + d*x))) - ((2*d*(a^2*d*f*(4*d*f*g + 7*d*e*h - 11*c*f*h) + b^2*(12*d^2*e
^2*g - c^2*f*(3*f*g - 4*e*h) - c*d*e*(5*f*g + 8*e*h)) - a*b*(c^2*f^2*h + d
^2*e*(19*f*g + 4*e*h) - c*d*f*(11*f*g + 13*e*h)))*Sqrt[e + f*x])/((b*c - a
*d)*(d*e - c*f)*(c + d*x)) + ((2*Sqrt[b]*(d*e - c*f)*(15*a^3*d^2*f^2*h - 5
*a^2*b*d*f*(7*d*f*g + 4*d*e*h - 2*c*f*h) - b^3*(24*d^2*e^2*g + c^2*f*(3*f*
g - 4*e*h) + 8*c*d*e*(f*g - 2*e*h)) - a*b^2*(c^2*f^2*h - 2*c*d*f*(7*f*g -
16*e*h) - 8*d^2*e*(7*f*g + e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e
 - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (8*d^(3/2)*(b*e - a*f)^2*(a*d*(d
*f*g - 2*d*e*h + c*f*h) + b*(6*d^2*e*g + 5*c^2*f*h - c*d*(7*f*g + 4*e*h)))
*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e -
 c*f]))/((b*c - a*d)*(d*e - c*f)))/(2*(b*c - a*d)*(b*e - a*f)))/(4*(b*c -
a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.19

method	result
derivativedivides	$2f^3 \left(-\frac{b \left(\frac{bf\left(7a^3d^2fh-6a^2bcdfh-4a^2bd^2eh-11a^2bd^2fg-ab^2c^2fh+14ab^2cdfg+8ab^2d^2eg+4b^3c^2eh-3b^3c^2fg-8b^3cdeg\right)(}{8a^2f^2-16abfe+8b^2e^2} \right.}{(} \right.$
default	$2f^3 \left(-\frac{b \left(\frac{bf\left(7a^3d^2fh-6a^2bcdfh-4a^2bd^2eh-11a^2bd^2fg-ab^2c^2fh+14ab^2cdfg+8ab^2d^2eg+4b^3c^2eh-3b^3c^2fg-8b^3cdeg\right)(}{8a^2f^2-16abfe+8b^2e^2} \right.}{(} \right.$
pseudoelliptic	$-\frac{\frac{15(xd+c)\sqrt{(cf-de)d}\,(cf-de)(bx+a)^2 \left(\frac{4\left(-2d^2e^2g+\frac{4c\left(eh-\frac{fg}{2}\right)ed}{3}+\frac{c^2\left(eh-\frac{3fg}{4}\right)f}{3}\right)b^3}{5} - \frac{a\left(8\left(-e^2h-7efg\right)d^2+2\left(16efh-7\right.\right.}{15} \right.}{4}}{}$

input

```
int((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*f^3*(-b/f^3/(a*d-b*c)^4*((1/8*b*f*(7*a^3*d^2*f*h-6*a^2*b*c*d*f*h-4*a^2*b
*d^2*e*h-11*a^2*b*d^2*f*g-a*b^2*c^2*f*h+14*a*b^2*c*d*f*g+8*a*b^2*d^2*e*g+4
*b^3*c^2*e*h-3*b^3*c^2*f*g-8*b^3*c*d*e*g)/(a^2*f^2-2*a*b*e*f+b^2*e^2)*(f*x
+e)^(3/2)+1/8*(9*a^3*d^2*f*h-10*a^2*b*c*d*f*h-4*a^2*b*d^2*e*h-13*a^2*b*d^2
*f*g+a*b^2*c^2*f*h+18*a*b^2*c*d*f*g+8*a*b^2*d^2*e*g+4*b^3*c^2*e*h-5*b^3*c^
2*f*g-8*b^3*c*d*e*g)*f/(a*f-b*e)*(f*x+e)^(1/2))/((f*x+e)*b+a*f-b*e)^2+1/8*
(15*a^3*d^2*f^2*h+10*a^2*b*c*d*f^2*h-20*a^2*b*d^2*e*f*h-35*a^2*b*d^2*f^2*g
-a*b^2*c^2*f^2*h-32*a*b^2*c*d*e*f*h+14*a*b^2*c*d*f^2*g+8*a*b^2*d^2*e^2*h+5
6*a*b^2*d^2*e*f*g+4*b^3*c^2*e*f*h-3*b^3*c^2*f^2*g+16*b^3*c*d*e^2*h-8*b^3*c
*d*e*f*g-24*b^3*d^2*e^2*g)/(a^2*f^2-2*a*b*e*f+b^2*e^2)/((a*f-b*e)*b)^(1/2)
*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))-d^2/f^3/(a*d-b*c)^4*(1/2*f*(
a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/(c*f-d*e)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*
e)-1/2*(a*c*d*f*h-2*a*d^2*e*h+a*d^2*f*g+5*b*c^2*f*h-4*b*c*d*e*h-7*b*c*d*f*
g+6*b*d^2*e*g)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-
d*e)*d)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**3/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(561) = 1122$.

Time = 0.19 (sec) , antiderivative size = 1273, normalized size of antiderivative = 2.14

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(24*b^4*d^2*e^2*g + 8*b^4*c*d*e*f*g - 56*a*b^3*d^2*e*f*g + 3*b^4*c^2*f
^2*g - 14*a*b^3*c*d*f^2*g + 35*a^2*b^2*d^2*f^2*g - 16*b^4*c*d*e^2*h - 8*a*
b^3*d^2*e^2*h - 4*b^4*c^2*e*f*h + 32*a*b^3*c*d*e*f*h + 20*a^2*b^2*d^2*e*f*
h + a*b^3*c^2*f^2*h - 10*a^2*b^2*c*d*f^2*h - 15*a^3*b*d^2*f^2*h)*arctan(sq
rt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*c^4*e^2 - 4*a*b^5*c^3*d*e^2 + 6*
a^2*b^4*c^2*d^2*e^2 - 4*a^3*b^3*c*d^3*e^2 + a^4*b^2*d^4*e^2 - 2*a*b^5*c^4*
e*f + 8*a^2*b^4*c^3*d*e*f - 12*a^3*b^3*c^2*d^2*e*f + 8*a^4*b^2*c*d^3*e*f -
 2*a^5*b*d^4*e*f + a^2*b^4*c^4*f^2 - 4*a^3*b^3*c^3*d*f^2 + 6*a^4*b^2*c^2*d
^2*f^2 - 4*a^5*b*c*d^3*f^2 + a^6*d^4*f^2)*sqrt(-b^2*e + a*b*f)) - (6*b*d^4
*e*g - 7*b*c*d^3*f*g + a*d^4*f*g - 4*b*c*d^3*e*h - 2*a*d^4*e*h + 5*b*c^2*d
^2*f*h + a*c*d^3*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c
^4*d*e - 4*a*b^3*c^3*d^2*e + 6*a^2*b^2*c^2*d^3*e - 4*a^3*b*c*d^4*e + a^4*d
^5*e - b^4*c^5*f + 4*a*b^3*c^4*d*f - 6*a^2*b^2*c^3*d^2*f + 4*a^3*b*c^2*d^3
*f - a^4*c*d^4*f)*sqrt(-d^2*e + c*d*f)) + (sqrt(f*x + e)*d^3*f*g - sqrt(f*
x + e)*c*d^2*f*h)/((b^3*c^3*d*e - 3*a*b^2*c^2*d^2*e + 3*a^2*b*c*d^3*e - a^
3*d^4*e - b^3*c^4*f + 3*a*b^2*c^3*d*f - 3*a^2*b*c^2*d^2*f + a^3*c*d^3*f)*(
(f*x + e)*d - d*e + c*f)) + 1/4*(8*(f*x + e)^(3/2)*b^4*d*e*f*g - 8*sqrt(f*
x + e)*b^4*d*e^2*f*g + 3*(f*x + e)^(3/2)*b^4*c*f^2*g - 11*(f*x + e)^(3/2)*
a*b^3*d*f^2*g - 5*sqrt(f*x + e)*b^4*c*e*f^2*g + 21*sqrt(f*x + e)*a*b^3*d*e
*f^2*g + 5*sqrt(f*x + e)*a*b^3*c*f^3*g - 13*sqrt(f*x + e)*a^2*b^2*d*f^3...
```

Mupad [B] (verification not implemented)

Time = 33.22 (sec) , antiderivative size = 691756, normalized size of antiderivative = 1162.62

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^2),x)
```

output
```
(((e + f*x)^(1/2)*(4*a^3*d^3*f^4*g - 5*b^3*c^3*f^4*g + a*b^2*c^3*f^4*h - 4
*a^3*c*d^2*f^4*h + 4*b^3*c^3*e*f^3*h - 12*b^3*d^3*e^3*f*g + 25*a*b^2*d^3*e
^2*f^2*g - 9*a^2*b*d^3*e^2*f^2*h + 11*b^3*c*d^2*e^2*f^2*g - 8*b^3*c^2*d*e^
2*f^2*h + 13*a*b^2*c^2*d*f^4*g - 9*a^2*b*c^2*d*f^4*h - 12*a^2*b*d^3*e*f^3*
g + 4*a*b^2*d^3*e^3*f*h + 2*b^3*c^2*d*e*f^3*g + 8*b^3*c*d^2*e^3*f*h - 26*a
*b^2*c*d^2*e*f^3*g + 2*a*b^2*c^2*d*e*f^3*h + 30*a^2*b*c*d^2*e*f^3*h - 19*a
*b^2*c*d^2*e^2*f^2*h))/(4*(a*d - b*c)*(a*f - b*e)*(c*f - d*e)*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)) - (f*(e + f*x)^(5/2)*(4*a*b^3*d^3*e^2*h - 12*b^4*d^3*
e^2*g + 8*b^4*c*d^2*e^2*h + 3*b^4*c^2*d*f^2*g - 4*a^2*b^2*d^3*f^2*g + 11*a
^2*b^2*c*d^2*f^2*h + 19*a*b^3*d^3*e*f*g + 5*b^4*c*d^2*e*f*g - 4*b^4*c^2*d*
e*f*h - 11*a*b^3*c*d^2*f^2*g + a*b^3*c^2*d*f^2*h - 7*a^2*b^2*d^3*e*f*h - 1
3*a*b^3*c*d^2*e*f*h))/(4*(a*f - b*e)^2*(c*f - d*e)*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*(e + f*x)^(3/2)*(8*a^3*d^3*f^4*g - 3*b^
3*c^3*f^4*g - a*b^2*c^3*f^4*h - 17*a^3*c*d^2*f^4*h + 9*a^3*d^3*e*f^3*h + 4
*b^3*c^3*e*f^3*h - 24*b^3*d^3*e^3*f*g + 56*a*b^2*d^3*e^2*f^2*g - 20*a^2*b*
d^3*e^2*f^2*h + 16*b^3*c*d^2*e^2*f^2*g - 12*b^3*c^2*d*e^2*f^2*h + 6*a*b^2*
c^2*d*f^4*g + 13*a^2*b*c*d^2*f^4*g - 6*a^2*b*c^2*d*f^4*h - 37*a^2*b*d^3*e*
f^3*g + 8*a*b^2*d^3*e^3*f*h + 3*b^3*c^2*d*e*f^3*g + 16*b^3*c*d^2*e^3*f*h -
 38*a*b^2*c*d^2*e*f^3*g + 9*a*b^2*c^2*d*e*f^3*h + 50*a^2*b*c*d^2*e*f^3*h -
 40*a*b^2*c*d^2*e^2*f^2*h))/(4*(a*d - b*c)*(a*f - b*e)*(a^2*d^2 + b^2*c...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18732, normalized size of antiderivative = 31.48

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**5*c**3*d**2*f**4*h + 30*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**2*d**3*e*f**3*h - 15*sqrt(b)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c**2*d
**3*f**4*h*x - 15*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**5*c*d**4*e**2*f**2*h + 30*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c*d**4*e*f**3*h*x - 15
*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**5*d**5*e**2*f**2*h*x - 10*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**4*d*f**4*h + 40*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**3*d**2*e
*f**3*h + 35*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**4*b*c**3*d**2*f**4*g - 40*sqrt(b)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**3*d**2*f**4*h*x - 50*s
qrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
**4*b*c**2*d**3*e**2*f**2*h - 70*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**3*e*f**3*g + 100*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*
*2*d**3*e*f**3*h*x + 35*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**3*f**4*g*x - 30*sqrt(b)*sqrt(a*f...
```

3.139 $\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 964

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{d(a^3d^2f^2(8dfg+19deh-27cfh)-b^3(32d^3e^3g-c^2def(7fg-10eh)-c^3f^2(5fg-6eh)-12cd^2e^2(fg\cdots}{\cdots}$$
$$-\frac{(bg-ah)\sqrt{e+fx}}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}$$
$$+\frac{(7a^2dfh+b^2(8deg+5cfg-6ceh)-ab(13dfg+2deh-cfh))\sqrt{e+fx}}{12(bc-ad)^2(be-af)^2(a+bx)^2(c+dx)}$$
$$+\frac{(35a^3d^2f^2h-a^2bdf(89dfg+32deh-16cfh)-b^3(48d^2e^2g+2cde(13fg-18eh)+3c^2f(5fg-6eh)\cdots}{24(bc-ad)^3(be-af)^3(a+bx)(c+\cdots}$$
$$+\frac{\sqrt{b}(35a^4d^3f^3h-35a^3bd^2f^2(3dfg+2deh-cfh)+b^4(64d^3e^3g+c^3f^2(5fg-6eh)+4c^2def(3fg-4e\cdots}{\cdots}$$
$$-\frac{d^{5/2}(ad(dfg-2deh+cfh)+b(8d^2eg+7c^2fh-3cd(3fg+2eh)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^5(de-cf)^{3/2}}$$

output
```
1/8*d*(a^3*d^2*f^2*(-27*c*f*h+19*d*e*h+8*d*f*g)-b^3*(32*d^3*e^3*g-c^2*d*e*
f*(-10*e*h+7*f*g)-c^3*f^2*(-6*e*h+5*f*g)-12*c*d^2*e^2*(2*e*h+f*g))+a*b^2*(
c^3*f^3*h-c^2*d*f^2*(-31*e*h+22*f*g)+4*d^3*e^2*(2*e*h+21*f*g)-2*c*d^2*e*f*
(32*e*h+19*f*g))-a^2*b*d*f*(6*c^2*f^2*h+d^2*e*(22*e*h+65*f*g)-c*d*f*(52*e*
h+41*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^4/(-a*f+b*e)^3/(-c*f+d*e)/(d*x+c)-1/3
*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^3/(d*x+c)+1/12*(7*
a^2*d*f*h+b^2*(-6*c*e*h+5*c*f*g+8*d*e*g)-a*b*(-c*f*h+2*d*e*h+13*d*f*g))*(f
*x+e)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2/(d*x+c)+1/24*(35*a^3*d^2*f
^2*h-a^2*b*d*f*(-16*c*f*h+32*d*e*h+89*d*f*g)-b^3*(48*d^2*e^2*g+2*c*d*e*(-1
8*e*h+13*f*g)+3*c^2*f*(-6*e*h+5*f*g))-a*b^2*(3*c^2*f^2*h-2*c*d*f*(-41*e*h+
28*f*g)-2*d^2*e*(6*e*h+61*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^3/(
b*x+a)/(d*x+c)+1/8*b^(1/2)*(35*a^4*d^3*f^3*h-35*a^3*b*d^2*f^2*(-c*f*h+2*d*
e*h+3*d*f*g)+b^4*(64*d^3*e^3*g+c^3*f^2*(-6*e*h+5*f*g)+4*c^2*d*e*f*(-4*e*h+
3*f*g)+24*c*d^2*e^2*(-2*e*h+f*g))-7*a^2*b^2*d*f*(c^2*f^2*h-c*d*f*(-22*e*h+
9*f*g)-4*d^2*e*(2*e*h+9*f*g))+a*b^3*(c^3*f^3*h-c^2*d*f^2*(-38*e*h+27*f*g)-
8*c*d^2*e*f*(-19*e*h+9*f*g)-8*d^3*e^2*(2*e*h+27*f*g)))*arctanh(b^(1/2)*(f*
x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^5/(-a*f+b*e)^(7/2)-d^(5/2)*(a*d*(c
*f*h-2*d*e*h+d*f*g)+b*(8*d^2*e*g+7*c^2*f*h-3*c*d*(2*e*h+3*f*g)))*arctanh(d
^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^5/(-c*f+d*e)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5396 vs. $2(964) = 1928$.

Time = 16.21 (sec) , antiderivative size = 5396, normalized size of antiderivative = 5.60

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^4*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {168, 27, 168, 27, 168, 27, 168, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 168$$

$$-\frac{\int \frac{acfh-2ad(3fg+eh)+b(8deg+5cfg-6ceh)+7df(bg-ah)x}{2(a+bx)^3(c+dx)^2\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{acfh-2ad(3fg+eh)+b(8deg+5cfg-6ceh)+7df(bg-ah)x}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}dx}{6(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\int \frac{df(24dfg+22deh-11cfh)a^2+b\left(-2e(41fg+6eh)d^2-cf(31fg-52eh)d+3c^2f^2h\right)a+b^2\left(3f(5fg-6eh)c^2+2de(13fg-18eh)c+48d^2e^2g\right)+5df\left(7dfha^2-b(13df\right.}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}}}{2(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{df(24dfg+22deh-11cfh)a^2+b\left(-2e(41fg+6eh)d^2-cf(31fg-52eh)d+3c^2f^2h\right)a+b^2\left(3f(5fg-6eh)c^2+2de(13fg-18eh)c+48d^2e^2g\right)+5df\left(7dfha^2-b(13df\right.}{(a+bx)^2(c+dx)^2\sqrt{e+fx}}}{4(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\sqrt{e+fx}\left(35a^3d^2f^2h-a^2bdf(-16cfh+32deh+89dfg)-ab^2\left(3c^2f^2h-2cdf(28fg-41eh)-2d^2e(6eh+61fg)\right)-b^3\left(3c^2f(5fg-6eh)+2cde(13fg-18eh)+48d^2e^2g\right.}{(a+bx)(c+dx)(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)(bc-ad)(be-af)}$$

$\downarrow$ 27

$$-\frac{-\frac{\sqrt{e+fx}\left(35a^3d^2f^2h-a^2bdf(-16cfh+32deh+89dfg)-ab^2\left(3c^2f^2h-2cdf(28fg-41eh)-2d^2e(6eh+61fg)\right)-b^3\left(3c^2f(5fg-6eh)+2cde(13fg-18eh)+48d^2e^2g\right.}{(a+bx)(c+dx)(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)(bc-ad)(be-af)}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}(7dfha^2-b(13dfg+2deh-cfh)a+b^2(8deg+5cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\left(35d^2f^2ha^3-bdf(89dfg+32deh-16cfh)a^2-b^2\left(-2e(61fg+6eh)d^2-2cf(28fg-4\right.\right.}{(bc-ad)(be-af)(}}{}$$

$\downarrow$ 25

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}(7dfha^2-b(13dfg+2deh-cfh)a+b^2(8deg+5cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\left(35d^2f^2ha^3-bdf(89dfg+32deh-16cfh)a^2-b^2\left(-2e(61fg+6eh)d^2-2cf(28fg-4\right.\right.}{(bc-ad)(be-af)(}}{}$$

$\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}(7dfha^2-b(13dfg+2deh-cfh)a+b^2(8deg+5cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\left(35d^2f^2ha^3-bdf(89dfg+32deh-16cfh)a^2-b^2\left(-2e(61fg+6eh)d^2-2cf(28fg-4\right.\right.}{(bc-ad)(be-af)(}}{}$$

$\downarrow$ 73

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-$$

$$-\frac{\sqrt{e+fx}\left(7dfha^2-b(13dfg+2deh-cfh)a+b^2(8deg+5cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\left(35d^2f^2ha^3-bdf(89dfg+32deh-16cfh)a^2-b^2\left(-2e(61fg+6eh)d^2-2cf(28fg-4\right.\right.}{(bc-ad)(be-af)(}$$

$\downarrow$ 221

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-$$

$$-\frac{\sqrt{e+fx}\left(7dfha^2-b(13dfg+2deh-cfh)a+b^2(8deg+5cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)}-\frac{\left(35d^2f^2ha^3-bdf(89dfg+32deh-16cfh)a^2-b^2\left(-2e(61fg+6eh)d^2-2cf(28fg-4\right.\right.}{(bc-ad)(be-af)(}$$

input
```
Int[(g + h*x)/((a + b*x)^4*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-1/3*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c +
 d*x)) - (-1/2*((7*a^2*d*f*h + b^2*(8*d*e*g + 5*c*f*g - 6*c*e*h) - a*b*(13
*d*f*g + 2*d*e*h - c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*
x)^2*(c + d*x)) - (((35*a^3*d^2*f^2*h - a^2*b*d*f*(89*d*f*g + 32*d*e*h - 1
6*c*f*h) - b^3*(48*d^2*e^2*g + 2*c*d*e*(13*f*g - 18*e*h) + 3*c^2*f*(5*f*g
- 6*e*h)) - a*b^2*(3*c^2*f^2*h - 2*c*d*f*(28*f*g - 41*e*h) - 2*d^2*e*(61*f
*g + 6*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x))
 - (3*((-2*d*(a^3*d^2*f^2*(8*d*f*g + 19*d*e*h - 27*c*f*h) - b^3*(32*d^3*e^
3*g - c^2*d*e*f*(7*f*g - 10*e*h) - c^3*f^2*(5*f*g - 6*e*h) - 12*c*d^2*e^2*
(f*g + 2*e*h)) + a*b^2*(c^3*f^3*h - c^2*d*f^2*(22*f*g - 31*e*h) + 4*d^3*e^
2*(21*f*g + 2*e*h) - 2*c*d^2*e*f*(19*f*g + 32*e*h)) - a^2*b*d*f*(6*c^2*f^2
*h + d^2*e*(65*f*g + 22*e*h) - c*d*f*(41*f*g + 52*e*h)))*Sqrt[e + f*x])/((
b*c - a*d)*(d*e - c*f)*(c + d*x)) - ((2*Sqrt[b]*(d*e - c*f)*(35*a^4*d^3*f^
3*h - 35*a^3*b*d^2*f^2*(3*d*f*g + 2*d*e*h - c*f*h) + b^4*(64*d^3*e^3*g + c
^3*f^2*(5*f*g - 6*e*h) + 4*c^2*d*e*f*(3*f*g - 4*e*h) + 24*c*d^2*e^2*(f*g -
 2*e*h)) - 7*a^2*b^2*d*f*(c^2*f^2*h - c*d*f*(9*f*g - 22*e*h) - 4*d^2*e*(9*
f*g + 2*e*h)) + a*b^3*(c^3*f^3*h - c^2*d*f^2*(27*f*g - 38*e*h) - 8*c*d^2*e
*f*(9*f*g - 19*e*h) - 8*d^3*e^2*(27*f*g + 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e
 + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) - (16*d^(5/2)*(b*
e - a*f)^3*(a*d*(d*f*g - 2*d*e*h + c*f*h) + b*(8*d^2*e*g + 7*c^2*f*h - ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 50.12 (sec) , antiderivative size = 1477, normalized size of antiderivative = 1.53

method	result	size
pseudoelliptic	Expression too large to display	1477
derivativedivides	Expression too large to display	1531
default	Expression too large to display	1531

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/((a*f-b*e)*b)^(1/2)*(35/8*(d*x+c)*((c*f-d*e)*d)^(1/2)*(c*f-d*e)*(b*x+a)
^3*b*((64/35*d^3*e^3*g-48/35*(e*h-1/2*f*g)*c*e^2*d^2-16/35*(e*h-3/4*f*g)*c
^2*f*e*d-6/35*(e*h-5/6*f*g)*c^3*f^2)*b^4+1/35*a*((-16*e^3*h-216*e^2*f*g)*d
^3+(152*c*e^2*f*h-72*c*e*f^2*g)*d^2+(38*c^2*e*f^2*h-27*c^2*f^3*g)*d+c^3*f^
3*h)*b^3-1/5*a^2*d*f*((-8*e^2*h-36*e*f*g)*d^2+(22*c*e*f*h-9*c*f^2*g)*d+c^2
*f^2*h)*b^2+a^3*((-2*e*h-3*f*g)*d+c*f*h)*d^2*f^2*b+a^4*d^3*f^3*h)*arctan(b
*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+(-d^3*(d*x+c)*((8*d^2*e*g+(-6*c*e*h-9*
c*f*g)*d+7*c^2*f*h)*b+a*((-2*e*h+f*g)*d+c*f*h)*d)*(b*x+a)^3*(a*f-b*e)^3*ar
ctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*(f*x+e)^(1/2)*((c*f-d*
e)*d)^(1/2)*((4*d^4*e^3*g*x^3+2*x^2*c*(-3/4*f*g*x+e*(-3/2*h*x+g))*e^2*d^3-
2/3*x*c^2*e*(21/16*x^2*g*f^2+11/8*x*e*(-15/11*h*x+g)*f+e^2*(9/4*h*x+g))*d^
2+1/3*(-15/8*f^3*g*x^3-11/8*x^2*(-18/11*h*x+g)*e*f^2+3/4*e^2*x*(3*h*x+g)*f
+e^3*(3/2*h*x+g))*c^3*d-1/3*c^4*f*(15/8*x^2*g*f^2-5/4*x*(9/5*h*x+g)*e*f+(3
/2*h*x+g)*e^2))*b^6-1/6*a*((63*e^2*f*g*x^3-60*x^2*(-1/10*h*x+g)*e^3)*d^4-3
2*x*c*(57/64*x^2*g*f^2-13/8*(-12/13*h*x+g)*x*e*f+e^2*(-3/2*h*x+g))*e*d^3+1
0*c^2*(-33/20*f^3*g*x^3-7/20*x^2*(-93/14*h*x+g)*e*f^2+7/20*x*(-83/7*h*x+g)
*e^2*f+e^3*(23/10*h*x+g))*d^2-c^3*((-3/4*h*x^3+13/2*g*x^2)*f^3-37/2*x*(43/
74*h*x+g)*e*f^2+7/2*e^2*(3*h*x+g)*f+e^3*h)*d+c^4*((3/4*h*x^2+10*g*x)*f^2-1
3/2*(25/13*h*x+g)*e*f+e^2*h)*f)*b^5+2/3*a^2*(11*x*e*(195/176*x^2*g*f^2-317
/88*x*(-33/317*h*x+g)*e*f+e^2*(-15/44*h*x+g))*d^4+13/2*c*(-123/104*f^3*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input

```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**4/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2757 vs. $2(926) = 1852$.

Time = 0.28 (sec) , antiderivative size = 2757, normalized size of antiderivative = 2.86

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(64*b^5*d^3*e^3*g + 24*b^5*c*d^2*e^2*f*g - 216*a*b^4*d^3*e^2*f*g + 12
*b^5*c^2*d*e*f^2*g - 72*a*b^4*c*d^2*e*f^2*g + 252*a^2*b^3*d^3*e*f^2*g + 5*
b^5*c^3*f^3*g - 27*a*b^4*c^2*d*f^3*g + 63*a^2*b^3*c*d^2*f^3*g - 105*a^3*b^
2*d^3*f^3*g - 48*b^5*c*d^2*e^3*h - 16*a*b^4*d^3*e^3*h - 16*b^5*c^2*d*e^2*f
*h + 152*a*b^4*c*d^2*e^2*f*h + 56*a^2*b^3*d^3*e^2*f*h - 6*b^5*c^3*e*f^2*h
+ 38*a*b^4*c^2*d*e*f^2*h - 154*a^2*b^3*c*d^2*e*f^2*h - 70*a^3*b^2*d^3*e*f^
2*h + a*b^4*c^3*f^3*h - 7*a^2*b^3*c^2*d*f^3*h + 35*a^3*b^2*c*d^2*f^3*h + 3
5*a^4*b*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^8*c^5*
e^3 - 5*a*b^7*c^4*d*e^3 + 10*a^2*b^6*c^3*d^2*e^3 - 10*a^3*b^5*c^2*d^3*e^3
+ 5*a^4*b^4*c*d^4*e^3 - a^5*b^3*d^5*e^3 - 3*a*b^7*c^5*e^2*f + 15*a^2*b^6*c
^4*d*e^2*f - 30*a^3*b^5*c^3*d^2*e^2*f + 30*a^4*b^4*c^2*d^3*e^2*f - 15*a^5*
b^3*c*d^4*e^2*f + 3*a^6*b^2*d^5*e^2*f + 3*a^2*b^6*c^5*e*f^2 - 15*a^3*b^5*c
^4*d*e*f^2 + 30*a^4*b^4*c^3*d^2*e*f^2 - 30*a^5*b^3*c^2*d^3*e*f^2 + 15*a^6*
b^2*c*d^4*e*f^2 - 3*a^7*b*d^5*e*f^2 - a^3*b^5*c^5*f^3 + 5*a^4*b^4*c^4*d*f^
3 - 10*a^5*b^3*c^3*d^2*f^3 + 10*a^6*b^2*c^2*d^3*f^3 - 5*a^7*b*c*d^4*f^3 +
a^8*d^5*f^3)*sqrt(-b^2*e + a*b*f)) + (8*b*d^5*e*g - 9*b*c*d^4*f*g + a*d^5*
f*g - 6*b*c*d^4*e*h - 2*a*d^5*e*h + 7*b*c^2*d^3*f*h + a*c*d^4*f*h)*arctan(
sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^5*c^5*d*e - 5*a*b^4*c^4*d^2*e +
10*a^2*b^3*c^3*d^3*e - 10*a^3*b^2*c^2*d^4*e + 5*a^4*b*c*d^5*e - a^5*d^6*e
- b^5*c^6*f + 5*a*b^4*c^5*d*f - 10*a^2*b^3*c^4*d^2*f + 10*a^3*b^2*c^3*d...
```

Mupad [B] (verification not implemented)

Time = 44.96 (sec) , antiderivative size = 1088917, normalized size of antiderivative = 1129.58

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^2),x)
```

output
```
- atan(((((256*a^17*b^2*d^17*e*f^10*g - 256*a^17*b^2*c*d^16*f^11*g - 160*a
^3*b^16*c^15*d^2*f^11*g + 2304*a^4*b^15*c^14*d^3*f^11*g - 15552*a^5*b^14*c
^13*d^4*f^11*g + 66048*a^6*b^13*c^12*d^5*f^11*g - 197856*a^7*b^12*c^11*d^6
*f^11*g + 440320*a^8*b^11*c^10*d^7*f^11*g - 743808*a^9*b^10*c^9*d^8*f^11*g
 + 958464*a^10*b^9*c^8*d^9*f^11*g - 937056*a^11*b^8*c^7*d^10*f^11*g + 6853
12*a^12*b^7*c^6*d^11*f^11*g - 365760*a^13*b^6*c^5*d^12*f^11*g + 136704*a^1
4*b^5*c^4*d^13*f^11*g - 33312*a^15*b^4*c^3*d^14*f^11*g + 4608*a^16*b^3*c^2
*d^15*f^11*g - 32*a^4*b^15*c^15*d^2*f^11*h + 512*a^5*b^14*c^14*d^3*f^11*h
- 4288*a^6*b^13*c^13*d^4*f^11*h + 21504*a^7*b^12*c^12*d^5*f^11*h - 68960*a
^8*b^11*c^11*d^6*f^11*h + 148224*a^9*b^10*c^10*d^7*f^11*h - 219264*a^10*b^
9*c^9*d^8*f^11*h + 224256*a^11*b^8*c^8*d^9*f^11*h - 154848*a^12*b^7*c^7*d^
10*f^11*h + 66560*a^13*b^6*c^6*d^11*f^11*h - 12992*a^14*b^5*c^5*d^12*f^11*
h - 2048*a^15*b^4*c^4*d^13*f^11*h + 1632*a^16*b^3*c^3*d^14*f^11*h - 256*a^
17*b^2*c^2*d^15*f^11*h + 1024*a^10*b^9*d^17*e^8*f^3*g - 6272*a^11*b^8*d^17
*e^7*f^4*g + 15968*a^12*b^7*d^17*e^6*f^5*g - 21536*a^13*b^6*d^17*e^5*f^6*g
 + 15904*a^14*b^5*d^17*e^4*f^7*g - 5600*a^15*b^4*d^17*e^3*f^8*g + 256*a^16
*b^3*d^17*e^2*f^9*g - 256*a^11*b^8*d^17*e^8*f^3*h + 1600*a^12*b^7*d^17*e^7
*f^4*h - 4192*a^13*b^6*d^17*e^6*f^5*h + 6048*a^14*b^5*d^17*e^5*f^6*h - 515
2*a^15*b^4*d^17*e^4*f^7*h + 2464*a^16*b^3*d^17*e^3*f^8*h - 512*a^17*b^2*d^
17*e^2*f^9*h + 1024*b^19*c^10*d^7*e^8*f^3*g - 1920*b^19*c^11*d^6*e^7*f^...
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 34058, normalized size of antiderivative = 35.33

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output
```
( - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**7*c**3*d**3*f**5*h + 210*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**2*d**4*e*f**4*h - 105*sqrt(b)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c**
2*d**4*f**5*h*x - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**7*c*d**5*e**2*f**3*h + 210*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c*d**5*e*f**4*h*x
 - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**7*d**6*e**2*f**3*h*x - 105*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**4*d**2*f**5*h + 420*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*
c**3*d**3*e*f**4*h + 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**6*b*c**3*d**3*f**5*g - 420*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**3*d**3*f*
*5*h*x - 525*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**6*b*c**2*d**4*e**2*f**3*h - 630*sqrt(b)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**4*e*f**4*g
+ 1050*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**6*b*c**2*d**4*e*f**4*h*x + 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**4*f**5*g*x - 31...
```

3.140 $\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1482

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

output
```
1/64*d*(a^4*d^3*f^3*(-251*c*f*h+187*d*e*h+64*d*f*g)+b^4*(320*d^4*e^4*g-c^3
*d*e*f^2*(-56*e*h+45*f*g)-5*c^4*f^3*(-8*e*h+7*f*g)-32*c^2*d^2*e^2*f*(-3*e*
h+2*f*g)-16*c*d^3*e^3*(16*e*h+7*f*g))-a*b^3*(5*c^4*f^4*h-c^2*d^2*e*f^2*(-3
92*e*h+263*f*g)-c^3*d*f^3*(-227*e*h+185*f*g)+16*d^4*e^3*(4*e*h+73*f*g)-16*
c*d^3*e^2*f*(59*e*h+29*f*g))-a^3*b*d^2*f^2*(95*c^2*f^2*h+d^2*e*(328*e*h+77
1*f*g)-c*d*f*(679*e*h+515*f*g))+a^2*b^2*d*f*(31*c^3*f^3*h-c^2*d*f^2*(-601*
e*h+409*f*g)+80*d^3*e^2*(3*e*h+19*f*g)-c*d^2*e*f*(1256*e*h+727*f*g)))*(f*x
+e)^(1/2)/(-a*d+b*c)^5/(-a*f+b*e)^4/(-c*f+d*e)/(d*x+c)-1/4*(-a*h+b*g)*(f*x
+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^4/(d*x+c)+1/24*(9*a^2*d*f*h+b^2*(-
8*c*e*h+7*c*f*g+10*d*e*g)-a*b*(-c*f*h+2*d*e*h+17*d*f*g))*(f*x+e)^(1/2)/(-a
*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^3/(d*x+c)+1/96*(63*a^3*d^2*f^2*h-a^2*b*d*f*
(-22*c*f*h+44*d*e*h+167*d*f*g)-b^3*(80*d^2*e^2*g+4*c*d*e*(-16*e*h+13*f*g)+
5*c^2*f*(-8*e*h+7*f*g))-a*b^2*(5*c^2*f^2*h-2*c*d*f*(-78*e*h+61*f*g)-4*d^2*
e*(4*e*h+53*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2/(d*x+
c)+1/192*(315*a^4*d^3*f^3*h-a^3*b*d^2*f^2*(-233*c*f*h+466*d*e*h+1027*d*f*g
)+b^4*(480*d^3*e^3*g+2*c^2*d*e*f*(-104*e*h+85*f*g)+16*c*d^2*e^2*(-24*e*h+1
7*f*g)+15*c^3*f^2*(-8*e*h+7*f*g))+a*b^3*(15*c^3*f^3*h-c^2*d*f^2*(-606*e*h+
485*f*g)-4*c*d^2*e*f*(-324*e*h+221*f*g)-16*d^3*e^2*(6*e*h+107*f*g))-a^2*b^
2*d*f*(83*c^2*f^2*h-c*d*f*(-1460*e*h+927*f*g)-2*d^2*e*(176*e*h+1077*f*g)))
*(f*x+e)^(1/2)/(-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)/(d*x+c)+1/64*b^(1/2)*(...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9480 vs. $2(1482) = 2964$.

Time = 16.34 (sec) , antiderivative size = 9480, normalized size of antiderivative = 6.40

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^5*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.82 (sec) , antiderivative size = 1606, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 168$$

$$-\frac{\int \frac{acfh-2ad(4fg+eh)+b(10deg+7cfg-8ceh)+9df(bg-ah)x}{2(a+bx)^4(c+dx)^2\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{acfh-2ad(4fg+eh)+b(10deg+7cfg-8ceh)+9df(bg-ah)x}{(a+bx)^4(c+dx)^2\sqrt{e+fx}}dx}{8(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\int \frac{-3df(5cfh-2d(8fg+5eh))a^2+b(-2e(71fg+8eh)d^2-cf(73fg-100eh)d+5c^2f^2h)a+b^2(5f(7fg-8eh)c^2+4de(13fg-16eh)c+80d^2e^2g)+7df(9dfha^2-b(1}{2(a+bx)^3(c+dx)^2\sqrt{e+fx}}}{3(bc-ad)(be-af)}}{8(bc-ad)(be-af}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{-3df(5cfh-2d(8fg+5eh))a^2+b(-2e(71fg+8eh)d^2-cf(73fg-100eh)d+5c^2f^2h)a+b^2(5f(7fg-8eh)c^2+4de(13fg-16eh)c+80d^2e^2g)+7df(9dfha^2-b(1}{(a+bx)^3(c+dx)^2\sqrt{e+fx}}}{6(bc-ad)(be-af)}}{8(bc-ad)(be-af}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\sqrt{e+fx}\left(63a^3d^2f^2h-a^2bdf(-22cfh+44deh+167dfg)-ab^2\left(5c^2f^2h-2cdf(61fg-78eh)-4d^2e(4eh+53fg)\right)-b^3\left(5c^2f(7fg-8eh)+4cde(13fg-16eh)+80d^2e^2\right.}{2(a+bx)^2(c+dx)(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)(bc-ad)(be-af)}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{3d^2f^2(64dfg+82deh-41cfh)a^3+bdf\left(-2e(547fg+136eh)d^2-cf(317fg-680eh)d+58c^2f^2h\right)a^2-b^2\left(-32e^2(41fg+3eh)d^3-16cef(39fg-61eh)d^2-2c^2f^2(\right.}{}}{}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)(bc-ad)(be-af)}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}(9dfha^2-b(17dfg+2deh-cfh)a+b^2(10deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(63d^2f^2ha^3-bdf(167dfg+44deh-22cfh)a^2-b^2\left(-4e(53fg+4eh)d^2-2c\right.\right.}{2(bc-ad)(be-af}}{}$$

$\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}(9dfha^2-b(17dfg+2deh-cfh)a+b^2(10deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(63d^2f^2ha^3-bdf(167dfg+44deh-22cfh)a^2-b^2\left(-4e(53fg+4eh)d^2-2c\right.\right.}{2(bc-ad)(be-af}}{}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}(9dfha^2-b(17dfg+2deh-cfh)a+b^2(10deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\sqrt{e+fx}\left(63d^2f^2ha^3-bdf(167dfg+44deh-22cfh)a^2-b^2\left(-4e(53fg+4eh)d^2-2c\right.\right.}{2(bc-ad)(be-af}}{}$$

$\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(9dfha^2-b(17dfg+2deh-cfh)a+b^2(10deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\frac{\sqrt{e+fx}\left(63d^2f^2ha^3-bdf(167dfg+44deh-22cfh)a^2-b^2\left(-4e(53fg+4eh)d^2-2c\right.\right.}{2(bc-ad)(be-af}}{}}{}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(9dfha^2-b(17dfg+2deh-cfh)a+b^2(10deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\frac{\sqrt{e+fx}\left(63d^2f^2ha^3-bdf(167dfg+44deh-22cfh)a^2-b^2\left(-4e(53fg+4eh)d^2-2c\right.\right.}{2(bc-ad)(be-af}}{}}{}$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(9dfha^2-b(17dfg+2deh-cfh)a+b^2(10deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)}-\frac{\frac{\sqrt{e+fx}\left(63d^2f^2ha^3-bdf(167dfg+44deh-22cfh)a^2-b^2\left(-4e(53fg+4eh)d^2-2c\right.\right.}{2(bc-ad)(be-af}}{}}{}$$

input
```
Int[(g + h*x)/((a + b*x)^5*(c + d*x)^2*Sqrt[e + f*x]),x]
```

output
```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4*(c +
 d*x)) - (-1/3*((9*a^2*d*f*h + b^2*(10*d*e*g + 7*c*f*g - 8*c*e*h) - a*b*(1
7*d*f*g + 2*d*e*h - c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b
*x)^3*(c + d*x)) - (((63*a^3*d^2*f^2*h - a^2*b*d*f*(167*d*f*g + 44*d*e*h -
 22*c*f*h) - b^3*(80*d^2*e^2*g + 4*c*d*e*(13*f*g - 16*e*h) + 5*c^2*f*(7*f*
g - 8*e*h)) - a*b^2*(5*c^2*f^2*h - 2*c*d*f*(61*f*g - 78*e*h) - 4*d^2*e*(53
*f*g + 4*e*h)))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c +
 d*x)) + (((315*a^4*d^3*f^3*h - a^3*b*d^2*f^2*(1027*d*f*g + 466*d*e*h - 23
3*c*f*h) + b^4*(480*d^3*e^3*g + 2*c^2*d*e*f*(85*f*g - 104*e*h) + 16*c*d^2*
e^2*(17*f*g - 24*e*h) + 15*c^3*f^2*(7*f*g - 8*e*h)) + a*b^3*(15*c^3*f^3*h
- c^2*d*f^2*(485*f*g - 606*e*h) - 4*c*d^2*e*f*(221*f*g - 324*e*h) - 16*d^3
*e^2*(107*f*g + 6*e*h)) - a^2*b^2*d*f*(83*c^2*f^2*h - c*d*f*(927*f*g - 146
0*e*h) - 2*d^2*e*(1077*f*g + 176*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e -
 a*f)*(a + b*x)*(c + d*x)) + (3*((2*d*(a^4*d^3*f^3*(64*d*f*g + 187*d*e*h -
 251*c*f*h) + b^4*(320*d^4*e^4*g - c^3*d*e*f^2*(45*f*g - 56*e*h) - 5*c^4*f
^3*(7*f*g - 8*e*h) - 32*c^2*d^2*e^2*f*(2*f*g - 3*e*h) - 16*c*d^3*e^3*(7*f*
g + 16*e*h)) - a*b^3*(5*c^4*f^4*h - c^2*d^2*e*f^2*(263*f*g - 392*e*h) - c^
3*d*f^3*(185*f*g - 227*e*h) + 16*d^4*e^3*(73*f*g + 4*e*h) - 16*c*d^3*e^2*f
*(29*f*g + 59*e*h)) - a^3*b*d^2*f^2*(95*c^2*f^2*h + d^2*e*(771*f*g + 328*e
*h) - c*d*f*(515*f*g + 679*e*h)) + a^2*b^2*d*f*(31*c^3*f^3*h - c^2*d*f^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 213.83 (sec) , antiderivative size = 2496, normalized size of antiderivative = 1.68

method	result	size
pseudoelliptic	Expression too large to display	2496
derivativedivides	Expression too large to display	2885
default	Expression too large to display	2885

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*f-b*e)*b)^(1/2)/(a*f-b*e)^4*(315/64*((c*f-d*e)*d)^(1/2)*((a^4*(-11/
3*b*g+a*h)*f^4-8/3*a^3*(a*h-22/5*b*g)*b*e*f^3+16/5*a^2*(a*h-33/7*b*g)*b^2*
e^2*f^2-64/35*a*(a*h-44/9*b*g)*b^3*e^3*f+128/315*b^4*e^4*(a*h-5*b*g))*d^4+
4/3*c*(a^3*(a*h+11/5*b*g)*f^4-28/5*a^2*(a*h+33/49*b*g)*b*e*f^3+288/35*a*(a
*h+11/36*b*g)*b^2*e^2*f^2-544/105*b^3*e^3*(a*h+2/17*b*g)*f+128/105*b^4*e^4
*h)*b*d^3-2/5*(a^2*(a*h+33/7*b*g)*f^3-48/7*a*(a*h+11/18*b*g)*b*e*f^2+40/7*
b^2*e^2*(a*h+1/5*b*g)*f-32/21*b^3*e^3*h)*c^2*b^2*f*d^2+4/35*c^3*b^3*f^2*(a
*(a*h+55/9*b*g)*f^2-68/9*(a*h+5/17*b*g)*b*e*f+8/3*b^2*e^2*h)*d-1/63*c^4*((
a*h+7*b*g)*f-8*e*h*b)*b^4*f^3)*(d*x+c)*(c*f-d*e)*(b*x+a)^4*b*arctan(b*(f*x
+e)^(1/2)/((a*f-b*e)*b)^(1/2))+(-((a*f*g-2*e*(a*h-5*b*g))*d^2+c*((a*h-11*b
*g)*f-8*e*h*b)*d+9*b*c^2*f*h)*d^4*(d*x+c)*(b*x+a)^4*(a*f-b*e)^4*arctan(d*(
f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(a*d-b*c)*((c*f-d*e)*d)^(1/2)*((-a^4*g*(
b*x+a)^4*f^4+4*a^3*(771/256*b^4*g*x^4+8225/768*a*x^3*(-561/8225*h*x+g)*b^3
+10463/768*a^2*x^2*(-1929/10463*h*x+g)*b^2+1789/256*(-765/1789*h*x+g)*a^3*
x*b+a^4*(-325/256*h*x+g))*b*e*f^3-6*a^2*(95/24*b^4*g*x^4+2681/192*a*x^3*(-
164/2681*h*x+g)*b^3+5035/288*(-347/2014*h*x+g)*a^2*x^2*b^2+4955/576*a^3*x*
(-2146/4955*h*x+g)*b+a^4*(-325/192*h*x+g))*b^2*e^2*f^2+4*a*(73/16*b^4*g*x^
4+769/48*a*x^3*(-45/769*h*x+g)*b^3+1913/96*a^2*x^2*(-316/1913*h*x+g)*b^2+3
09/32*(-131/309*h*x+g)*a^3*x*b+a^4*(-185/96*h*x+g))*b^3*e^3*f-b^4*e^4*(5*b
^4*g*x^4+35/2*a*x^3*(-2/35*h*x+g)*b^3+65/3*a^2*x^2*(-21/130*h*x+g)*b^2+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g + hx}{(a + bx)^5 (c + dx)^2 \sqrt{e + fx}}\, dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**5/(d*x+c)**2/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g + hx}{(a + bx)^5 (c + dx)^2 \sqrt{e + fx}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5347 vs. 2(1440) = 2880.

Time = 0.40 (sec) , antiderivative size = 5347, normalized size of antiderivative = 3.61

$$\int \frac{g + hx}{(a + bx)^5 (c + dx)^2 \sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/64*(640*b^6*d^4*e^4*g + 256*b^6*c*d^3*e^3*f*g - 2816*a*b^5*d^4*e^3*f*g +
 144*b^6*c^2*d^2*e^2*f^2*g - 1056*a*b^5*c*d^3*e^2*f^2*g + 4752*a^2*b^4*d^4
*e^2*f^2*g + 80*b^6*c^3*d*e*f^3*g - 528*a*b^5*c^2*d^2*e*f^3*g + 1584*a^2*b
^4*c*d^3*e*f^3*g - 3696*a^3*b^3*d^4*e*f^3*g + 35*b^6*c^4*f^4*g - 220*a*b^5
*c^3*d*f^4*g + 594*a^2*b^4*c^2*d^2*f^4*g - 924*a^3*b^3*c*d^3*f^4*g + 1155*
a^4*b^2*d^4*f^4*g - 512*b^6*c*d^3*e^4*h - 128*a*b^5*d^4*e^4*h - 192*b^6*c^
2*d^2*e^3*f*h + 2176*a*b^5*c*d^3*e^3*f*h + 576*a^2*b^4*d^4*e^3*f*h - 96*b^
6*c^3*d*e^2*f^2*h + 720*a*b^5*c^2*d^2*e^2*f^2*h - 3456*a^2*b^4*c*d^3*e^2*f
^2*h - 1008*a^3*b^3*d^4*e^2*f^2*h - 40*b^6*c^4*e*f^3*h + 272*a*b^5*c^3*d*e
*f^3*h - 864*a^2*b^4*c^2*d^2*e*f^3*h + 2352*a^3*b^3*c*d^3*e*f^3*h + 840*a^
4*b^2*d^4*e*f^3*h + 5*a*b^5*c^4*f^4*h - 36*a^2*b^4*c^3*d*f^4*h + 126*a^3*b
^3*c^2*d^2*f^4*h - 420*a^4*b^2*c*d^3*f^4*h - 315*a^5*b*d^4*f^4*h)*arctan(s
qrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^10*c^6*e^4 - 6*a*b^9*c^5*d*e^4 +
15*a^2*b^8*c^4*d^2*e^4 - 20*a^3*b^7*c^3*d^3*e^4 + 15*a^4*b^6*c^2*d^4*e^4 -
 6*a^5*b^5*c*d^5*e^4 + a^6*b^4*d^6*e^4 - 4*a*b^9*c^6*e^3*f + 24*a^2*b^8*c^
5*d*e^3*f - 60*a^3*b^7*c^4*d^2*e^3*f + 80*a^4*b^6*c^3*d^3*e^3*f - 60*a^5*b
^5*c^2*d^4*e^3*f + 24*a^6*b^4*c*d^5*e^3*f - 4*a^7*b^3*d^6*e^3*f + 6*a^2*b^
8*c^6*e^2*f^2 - 36*a^3*b^7*c^5*d*e^2*f^2 + 90*a^4*b^6*c^4*d^2*e^2*f^2 - 12
0*a^5*b^5*c^3*d^3*e^2*f^2 + 90*a^6*b^4*c^2*d^4*e^2*f^2 - 36*a^7*b^3*c*d^5*
e^2*f^2 + 6*a^8*b^2*d^6*e^2*f^2 - 4*a^3*b^7*c^6*e*f^3 + 24*a^4*b^6*c^5*...
```

Mupad [B] (verification not implemented)

Time = 58.38 (sec) , antiderivative size = 1563828, normalized size of antiderivative = 1055.21

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^5*(c + d*x)^2),x)
```

output
```
atan(((((16384*a^21*b^2*c*d^19*f^13*g - 16384*a^21*b^2*d^20*e*f^12*g - 896
0*a^4*b^19*c^18*d^2*f^13*g + 154880*a^5*b^18*c^17*d^3*f^13*g - 1264384*a^6
*b^17*c^16*d^4*f^13*g + 6485248*a^7*b^16*c^15*d^5*f^13*g - 23510784*a^8*b^
15*c^14*d^6*f^13*g + 64258304*a^9*b^14*c^13*d^7*f^13*g - 137631488*a^10*b^
13*c^12*d^8*f^13*g + 236169472*a^11*b^12*c^11*d^9*f^13*g - 327813376*a^12*
b^11*c^10*d^10*f^13*g + 368031488*a^13*b^10*c^9*d^11*f^13*g - 331620608*a^
14*b^9*c^8*d^12*f^13*g + 236429056*a^15*b^8*c^7*d^13*f^13*g - 130569472*a^
16*b^7*c^6*d^14*f^13*g + 54160128*a^17*b^6*c^5*d^15*f^13*g - 16104704*a^18
*b^5*c^4*d^16*f^13*g + 3179264*a^19*b^4*c^3*d^17*f^13*g - 360448*a^20*b^3*
c^2*d^18*f^13*g - 1280*a^5*b^18*c^18*d^2*f^13*h + 23296*a^6*b^17*c^17*d^3*
f^13*h - 204032*a^7*b^16*c^16*d^4*f^13*h + 1180416*a^8*b^15*c^15*d^5*f^13*
h - 4966656*a^9*b^14*c^14*d^6*f^13*h + 15587072*a^10*b^13*c^13*d^7*f^13*h
- 36729088*a^11*b^12*c^12*d^8*f^13*h + 65187584*a^12*b^11*c^11*d^9*f^13*h
- 87174912*a^13*b^10*c^10*d^10*f^13*h + 87372032*a^14*b^9*c^9*d^11*f^13*h
- 64624384*a^15*b^8*c^8*d^12*f^13*h + 34097408*a^16*b^7*c^7*d^13*f^13*h -
11895552*a^17*b^6*c^6*d^14*f^13*h + 2186496*a^18*b^5*c^5*d^15*f^13*h + 586
24*a^19*b^4*c^4*d^16*f^13*h - 113408*a^20*b^3*c^3*d^17*f^13*h + 16384*a^21
*b^2*c^2*d^18*f^13*h - 81920*a^12*b^11*d^20*e^10*f^3*g + 667648*a^13*b^10*
d^20*e^9*f^4*g - 2379776*a^14*b^9*d^20*e^8*f^5*g + 4840704*a^15*b^8*d^20*e
^7*f^6*g - 6132736*a^16*b^7*d^20*e^6*f^7*g + 4922880*a^17*b^6*d^20*e^5*...
```

Reduce [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 55226, normalized size of antiderivative = 37.26

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(1/2),x)
```

output

```
( - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**9*c**3*d**4*f**6*h + 1890*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*c**2*d**5*e*f**5*h - 945*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*c*
*2*d**5*f**6*h*x - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a**9*c*d**6*e**2*f**4*h + 1890*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*c*d**6*e*f**5*h
*x - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**9*d**7*e**2*f**4*h*x - 1260*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**4*d**3*f**6*h + 5040*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
8*b*c**3*d**4*e*f**5*h + 3465*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**3*d**4*f**6*g - 5040*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**3*d
**4*f**6*h*x - 6300*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**8*b*c**2*d**5*e**2*f**4*h - 6930*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**2*d**5*e
*f**5*g + 12600*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**8*b*c**2*d**5*e*f**5*h*x + 3465*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**2*d**5*f**...
```

3.141 $\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 390

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{2b^2(3adfh+b(dfg-deh-3cfh))\sqrt{e+fx}}{d^4f^2} + \frac{(bc-ad)^3(dg-ch)\sqrt{e+fx}}{2d^4(de-cf)(c+dx)^2} + \frac{(bc-ad)^2\,(ad(3dfg-4deh+cfh)-b(12d^2eg+13c^2fh-cd(9fg+16eh)))\,\sqrt{e+fx}}{4d^4(de-cf)^2(c+dx)} + \frac{2b^3h(e+fx)^{3/2}}{3d^3f^2} + \frac{(bc-ad)\,(a^2d^2f(3dfg-4deh+cfh)+2abd(5c^2f^2h+cdf(3fg-14eh)-6d^2e(fg-2eh))+b^2(24d}{4d^{9/2}(de-cf)^{5/2}}$$

output

```
2*b^2*(3*a*d*f*h+b*(-3*c*f*h-d*e*h+d*f*g))*(f*x+e)^(1/2)/d^4/f^2+1/2*(-a*d
+b*c)^3*(-c*h+d*g)*(f*x+e)^(1/2)/d^4/(-c*f+d*e)/(d*x+c)^2+1/4*(-a*d+b*c)^2
*(a*d*(c*f*h-4*d*e*h+3*d*f*g)-b*(12*d^2*e*g+13*c^2*f*h-c*d*(16*e*h+9*f*g))
)*(f*x+e)^(1/2)/d^4/(-c*f+d*e)^2/(d*x+c)+2/3*b^3*h*(f*x+e)^(3/2)/d^3/f^2+1
/4*(-a*d+b*c)*(a^2*d^2*f*(c*f*h-4*d*e*h+3*d*f*g)+2*a*b*d*(5*c^2*f^2*h+c*d*
f*(-14*e*h+3*f*g)-6*d^2*e*(-2*e*h+f*g))+b^2*(24*d^3*e^2*g-35*c^3*f^2*h-12*
c*d^2*e*(4*e*h+3*f*g)+5*c^2*d*f*(16*e*h+3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(
1/2)/(-c*f+d*e)^(1/2))/d^(9/2)/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx =$$

$$-\frac{\sqrt{e+fx}(3a^3d^3f^2(c^2fh+d^2(2eg-3fgx+4ehx)-cd(5fg-2eh+fhx))+9a^2bd^2f^2(3c^3fh+4d^3e}{}$$

$$+\frac{(-bc+ad)\left(a^2d^2f(3dfg-4deh+cfh)+2abd(5c^2f^2h+cdf(3fg-14eh)+6d^2e(-fg+2eh))+b^2(}{4d^{9/2}(-de+cf)^{5/2}}$$

input
```
Integrate[((a + b*x)^3*(g + h*x))/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/12*(Sqrt[e + f*x]*(3*a^3*d^3*f^2*(c^2*f*h + d^2*(2*e*g - 3*f*g*x + 4*e*
h*x) - c*d*(5*f*g - 2*e*h + f*h*x)) + 9*a^2*b*d^2*f^2*(3*c^3*f*h + 4*d^3*e
*g*x + c*d^2*(2*e*g - f*g*x - 8*e*h*x) + c^2*d*(f*g - 6*e*h + 5*f*h*x)) -
9*a*b^2*d*f*(15*c^4*f^2*h + 8*d^4*e^2*h*x^2 + c^3*d*f*(-3*f*g - 26*e*h + 2
5*f*h*x) + 8*c*d^3*e*x*(2*e*h + f*(g - 2*h*x)) + c^2*d^2*(8*e^2*h + e*f*(6
*g - 44*h*x) + f^2*x*(-5*g + 8*h*x))) + b^3*(105*c^5*f^3*h - 5*c^4*d*f^2*(
9*f*g + 34*e*h - 35*f*h*x) - 8*d^5*e^2*x^2*(3*f*g - 2*e*h + f*h*x) + 4*c^2
*d^3*(4*e^3*h + 3*e*f^2*x*(11*g - 8*h*x) - 6*e^2*f*(g - 3*h*x) - 2*f^3*x^2
*(3*g + h*x)) + 8*c*d^4*e*x*(4*e^2*h + 2*f^2*x*(3*g + h*x) + e*f*(-6*g + 3
*h*x)) + c^3*d^2*f*(40*e^2*h + 6*e*f*(13*g - 48*h*x) + f^2*x*(-75*g + 56*h
*x)))))/(d^4*f^2*(d*e - c*f)^2*(c + d*x)^2) + ((-(b*c) + a*d)*(a^2*d^2*f*(
3*d*f*g - 4*d*e*h + c*f*h) + 2*a*b*d*(5*c^2*f^2*h + c*d*f*(3*f*g - 14*e*h)
 + 6*d^2*e*(-(f*g) + 2*e*h)) + b^2*(24*d^3*e^2*g - 35*c^3*f^2*h - 12*c*d^2
*e*(3*f*g + 4*e*h) + 5*c^2*d*f*(3*f*g + 16*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e +
 f*x])/Sqrt[-(d*e) + c*f]])/(4*d^(9/2)*(-(d*e) + c*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 166, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$$

$\big\downarrow$ 166

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(3dfg-4deh+cfh)+b(3dfg+4deh-7cfh)x)}{2(c+dx)^2\sqrt{e+fx}}dx}{2d(de-cf)}-\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(3dfg-4deh+cfh)+b(3dfg+4deh-7cfh)x)}{(c+dx)^2\sqrt{e+fx}}dx}{4d(de-cf)}-\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\big\downarrow$ 166

$$\frac{\frac{\int -\frac{(a+bx)\left(2ad(be-af)(3dfg-4deh+cfh)+(4be+af)\left(ad(3dfg-4deh+cfh)-b\left(7fhc^2-d(3fg+10eh)c+6d^2eg\right)\right)+b\left(3adf(3dfg-4deh+cfh)-b\left(8e(3fg+eh)d^2-c\ldots\right.\right.\right.}{2(c+dx)\sqrt{e+fx}}}{d(de-cf)}\ldots}{4d(de-cf)}-$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\big\downarrow$ 27

$$\frac{\frac{(a+bx)^2\sqrt{e+fx}\left(ad(cfh-4deh+3dfg)-b\left(7c^2fh-cd(10eh+3fg)+6d^2eg\right)\right)}{d(c+dx)(de-cf)}-\frac{\int \frac{(a+bx)\left(2ad(be-af)(3dfg-4deh+cfh)+(4be+af)\left(ad(3dfg-4deh+cfh\ldots\right.\right.}{\ldots}}{\ldots}}{4d(de-cf)}-$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\big\downarrow$ 164

$$\frac{(a+bx)^2\sqrt{e+fx}\left(ad(cfh-4deh+3dfg)-b\left(7c^2fh-cd(10eh+3fg)+6d^2eg\right)\right)}{d(c+dx)(de-cf)}-\frac{\frac{(bc-ad)\left(a^2d^2f(cfh-4deh+3dfg)+2abd\left(5c^2f^2h+cdf(3fg-14eh)-6d^2e\ldots\right.\right.}{\ldots}}{\ldots}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\big\downarrow$ 73

$$\frac{(a+bx)^2\sqrt{e+fx}\left(ad(cfh-4deh+3dfg)-b\left(7c^2fh-cd(10eh+3fg)+6d^2eg\right)\right)}{d(c+dx)(de-cf)}-\frac{\frac{2(bc-ad)\left(a^2d^2f(cfh-4deh+3dfg)+2abd\left(5c^2f^2h+cdf(3fg-14eh)-6d^2\ldots\right.\right.}{\ldots}}{\ldots}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$\downarrow$ 221

$$\frac{\frac{(a+bx)^2\sqrt{e+fx}\left(ad(cfh-4deh+3dfg)-b\left(7c^2fh-cd(10eh+3fg)+6d^2eg\right)\right)}{d(c+dx)(de-cf)}-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(cfh-4deh+3dfg)+bdfx\left(3adf(cfh-4deh+3dfg)-b\left(\ldots\right.\right.\right.}{\ldots}}{\ldots}$$

$$\frac{(a+bx)^3\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((a + b*x)^3*(g + h*x))/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^3*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x)^2) +
 (((a*d*(3*d*f*g - 4*d*e*h + c*f*h) - b*(6*d^2*e*g + 7*c^2*f*h - c*d*(3*f*
g + 10*e*h)))*(a + b*x)^2*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x)) - ((2*b
*Sqrt[e + f*x]*(6*a^2*d^2*f^2*(3*d*f*g - 4*d*e*h + c*f*h) - 9*a*b*d*f*(15*
c^2*f^2*h + 2*d^2*e*(3*f*g + 4*e*h) - c*d*f*(3*f*g + 26*e*h)) + b^2*(105*c
^3*f^3*h - 8*d^3*e^2*(3*f*g - 2*e*h) + 2*c*d^2*e*f*(39*f*g + 20*e*h) - 5*c
^2*d*f^2*(9*f*g + 34*e*h)) + b*d*f*(3*a*d*f*(3*d*f*g - 4*d*e*h + c*f*h) -
b*(35*c^2*f^2*h + 8*d^2*e*(3*f*g + e*h) - c*d*f*(15*f*g + 52*e*h)))*x))/(3
*d^2*f^2) - (2*(b*c - a*d)*(a^2*d^2*f*(3*d*f*g - 4*d*e*h + c*f*h) + 2*a*b*
d*(5*c^2*f^2*h + c*d*f*(3*f*g - 14*e*h) - 6*d^2*e*(f*g - 2*e*h)) + b^2*(24
*d^3*e^2*g - 35*c^3*f^2*h - 12*c*d^2*e*(3*f*g + 4*e*h) + 5*c^2*d*f*(3*f*g
+ 16*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(5/2)*Sqr
t[d*e - c*f]))/(2*d*(d*e - c*f)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.51

method	result
risch	$\frac{2b^2(hbdfx+9adfh-9bcfh-2bdeh+3bgdf)\sqrt{fx+e}}{3f^2d^4} + \frac{(2ad-2bc)\left(\frac{df\left(a^2c\,d^2fh-4a^2d^3eh+3a^2d^3fg-14ab\,c^2dfh+20abc\,d^2\right.}{}\right.}{}$
pseudoelliptic	$-\frac{-(ad-bc)(xd+c)^2f^2\left(\left(3a^2f^2g-4ae(ah+3bg)f+24b\,e^2(ah+bg)\right)d^3+c\left(\left(a^2h+6gab\right)f^2-28\left(ah+\frac{9bg}{7}\right)bef-48b^2e^2h\right.\right.}{}$
derivativedivides	$\frac{2b^2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+3adfh\sqrt{fx+e}-3bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^4}-\frac{2f^2\left(-\frac{df\left(a^3c\,d^3fh-4a^3d^4eh+3a^3d^4fg-15a^2b\right.}{}\right.}{}$
default	$\frac{2b^2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+3adfh\sqrt{fx+e}-3bcfh\sqrt{fx+e}-bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^4}-\frac{2f^2\left(-\frac{df\left(a^3c\,d^3fh-4a^3d^4eh+3a^3d^4fg-15a^2b\right.}{}\right.}{}$

input

```
int((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*b^2*(b*d*f*h*x+9*a*d*f*h-9*b*c*f*h-2*b*d*e*h+3*b*d*f*g)*(f*x+e)^(1/2)/
f^2/d^4+1/d^4*(2*a*d-2*b*c)*((1/8*d*f*(a^2*c*d^2*f*h-4*a^2*d^3*e*h+3*a^2*d
^3*f*g-14*a*b*c^2*d*f*h+20*a*b*c*d^2*e*h+6*a*b*c*d^2*f*g-12*a*b*d^3*e*g+13
*b^2*c^3*f*h-16*b^2*c^2*d*e*h-9*b^2*c^2*d*f*g+12*b^2*c*d^2*e*g)/(c^2*f^2-2
*c*d*e*f+d^2*e^2)*(f*x+e)^(3/2)-1/8*(a^2*c*d^2*f*h+4*a^2*d^3*e*h-5*a^2*d^3
*f*g+10*a*b*c^2*d*f*h-20*a*b*c*d^2*e*h-2*a*b*c*d^2*f*g+12*a*b*d^3*e*g-11*b
^2*c^3*f*h+16*b^2*c^2*d*e*h+7*b^2*c^2*d*f*g-12*b^2*c*d^2*e*g)*f/(c*f-d*e)*
(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2+1/8*(a^2*c*d^2*f^2*h-4*a^2*d^3*e*f*h+
3*a^2*d^3*f^2*g+10*a*b*c^2*d*f^2*h-28*a*b*c*d^2*e*f*h+6*a*b*c*d^2*f^2*g+24
*a*b*d^3*e^2*h-12*a*b*d^3*e*f*g-35*b^2*c^3*f^2*h+80*b^2*c^2*d*e*f*h+15*b^2
*c^2*d*f^2*g-48*b^2*c*d^2*e^2*h-36*b^2*c*d^2*e*f*g+24*b^2*d^3*e^2*g)/(c^2*
f^2-2*c*d*e*f+d^2*e^2)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*
e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1983 vs. $2(364) = 728$.

Time = 0.34 (sec) , antiderivative size = 3979, normalized size of antiderivative = 10.20

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(h*x+g)/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1306 vs. $2(364) = 728$.

Time = 0.15 (sec) , antiderivative size = 1306, normalized size of antiderivative = 3.35

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(24*b^3*c*d^3*e^2*g - 24*a*b^2*d^4*e^2*g - 36*b^3*c^2*d^2*e*f*g + 24*
a*b^2*c*d^3*e*f*g + 12*a^2*b*d^4*e*f*g + 15*b^3*c^3*d*f^2*g - 9*a*b^2*c^2*
d^2*f^2*g - 3*a^2*b*c*d^3*f^2*g - 3*a^3*d^4*f^2*g - 48*b^3*c^2*d^2*e^2*h +
 72*a*b^2*c*d^3*e^2*h - 24*a^2*b*d^4*e^2*h + 80*b^3*c^3*d*e*f*h - 108*a*b^
2*c^2*d^2*e*f*h + 24*a^2*b*c*d^3*e*f*h + 4*a^3*d^4*e*f*h - 35*b^3*c^4*f^2*
h + 45*a*b^2*c^3*d*f^2*h - 9*a^2*b*c^2*d^2*f^2*h - a^3*c*d^3*f^2*h)*arctan
(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^6*e^2 - 2*c*d^5*e*f + c^2*d^4*f
^2)*sqrt(-d^2*e + c*d*f)) - 1/4*(12*(f*x + e)^(3/2)*b^3*c^2*d^3*e*f*g - 24
*(f*x + e)^(3/2)*a*b^2*c*d^4*e*f*g + 12*(f*x + e)^(3/2)*a^2*b*d^5*e*f*g -
12*sqrt(f*x + e)*b^3*c^2*d^3*e^2*f*g + 24*sqrt(f*x + e)*a*b^2*c*d^4*e^2*f*
g - 12*sqrt(f*x + e)*a^2*b*d^5*e^2*f*g - 9*(f*x + e)^(3/2)*b^3*c^3*d^2*f^2
*g + 15*(f*x + e)^(3/2)*a*b^2*c^2*d^3*f^2*g - 3*(f*x + e)^(3/2)*a^2*b*c*d^
4*f^2*g - 3*(f*x + e)^(3/2)*a^3*d^5*f^2*g + 19*sqrt(f*x + e)*b^3*c^3*d^2*e
*f^2*g - 33*sqrt(f*x + e)*a*b^2*c^2*d^3*e*f^2*g + 9*sqrt(f*x + e)*a^2*b*c*
d^4*e*f^2*g + 5*sqrt(f*x + e)*a^3*d^5*e*f^2*g - 7*sqrt(f*x + e)*b^3*c^4*d*
f^3*g + 9*sqrt(f*x + e)*a*b^2*c^3*d^2*f^3*g + 3*sqrt(f*x + e)*a^2*b*c^2*d^
3*f^3*g - 5*sqrt(f*x + e)*a^3*c*d^4*f^3*g - 16*(f*x + e)^(3/2)*b^3*c^3*d^2
*e*f*h + 36*(f*x + e)^(3/2)*a*b^2*c^2*d^3*e*f*h - 24*(f*x + e)^(3/2)*a^2*b
*c*d^4*e*f*h + 4*(f*x + e)^(3/2)*a^3*d^5*e*f*h + 16*sqrt(f*x + e)*b^3*c^3*
d^2*e^2*f*h - 36*sqrt(f*x + e)*a*b^2*c^2*d^3*e^2*f*h + 24*sqrt(f*x + e)...
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1242, normalized size of antiderivative = 3.18

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(a + b*x)^3)/((e + f*x)^(1/2)*(c + d*x)^3),x)
```

output
```
(e + f*x)^(1/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^3*f^2) - (6*b^3*
h*(c*f - d*e))/(d^4*f^2)) - (((e + f*x)^(1/2)*(11*b^3*c^4*f^2*h - 5*a^3*d^
4*f^2*g + 4*a^3*d^4*e*f*h + a^3*c*d^3*f^2*h - 7*b^3*c^3*d*f^2*g + 9*a*b^2*
c^2*d^2*f^2*g + 9*a^2*b*c^2*d^2*f^2*h + 12*a^2*b*d^4*e*f*g - 16*b^3*c^3*d*
e*f*h + 3*a^2*b*c*d^3*f^2*g - 21*a*b^2*c^3*d*f^2*h + 12*b^3*c^2*d^2*e*f*g
+ 36*a*b^2*c^2*d^2*e*f*h - 24*a*b^2*c*d^3*e*f*g - 24*a^2*b*c*d^3*e*f*h))/(
4*(c*f - d*e)) - ((e + f*x)^(3/2)*(3*a^3*d^5*f^2*g - 4*a^3*d^5*e*f*h + a^3
*c*d^4*f^2*h - 13*b^3*c^4*d*f^2*h + 9*b^3*c^3*d^2*f^2*g - 15*a*b^2*c^2*d^3
*f^2*g + 27*a*b^2*c^3*d^2*f^2*h - 15*a^2*b*c^2*d^3*f^2*h - 12*a^2*b*d^5*e*
f*g + 3*a^2*b*c*d^4*f^2*g - 12*b^3*c^2*d^3*e*f*g + 16*b^3*c^3*d^2*e*f*h -
36*a*b^2*c^2*d^3*e*f*h + 24*a*b^2*c*d^4*e*f*g + 24*a^2*b*c*d^4*e*f*h))/(4*
(c*f - d*e)^2))/(d^6*(e + f*x)^2 - (e + f*x)*(2*d^6*e - 2*c*d^5*f) + d^6*e
^2 + c^2*d^4*f^2 - 2*c*d^5*e*f) + (atan((d^(1/2)*(e + f*x)^(1/2)*(a*d - b*
c)*(3*a^2*d^3*f^2*g + 24*b^2*d^3*e^2*g - 35*b^2*c^3*f^2*h + 24*a*b*d^3*e^2
*h - 4*a^2*d^3*e*f*h + a^2*c*d^2*f^2*h - 48*b^2*c*d^2*e^2*h + 15*b^2*c^2*d
*f^2*g - 12*a*b*d^3*e*f*g + 6*a*b*c*d^2*f^2*g + 10*a*b*c^2*d*f^2*h - 36*b^
2*c*d^2*e*f*g + 80*b^2*c^2*d*e*f*h - 28*a*b*c*d^2*e*f*h))/((c*f - d*e)^(1/
2)*(3*a^3*d^4*f^2*g + 35*b^3*c^4*f^2*h - 4*a^3*d^4*e*f*h + 24*a*b^2*d^4*e^
2*g + 24*a^2*b*d^4*e^2*h - 24*b^3*c*d^3*e^2*g + a^3*c*d^3*f^2*h - 15*b^3*c
^3*d*f^2*g + 48*b^3*c^2*d^2*e^2*h + 9*a*b^2*c^2*d^2*f^2*g + 9*a^2*b*c^2...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4932, normalized size of antiderivative = 12.65

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output

```
(3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*a**3*c**3*d**3*f**4*h - 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*e*f**3*h + 9*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*f
**4*g + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
 - d*e)))*a**3*c**2*d**4*f**4*h*x - 24*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(
e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**5*e*f**3*h*x + 18*sqrt(d)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*
d**5*f**4*g*x + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a**3*c*d**5*f**4*h*x**2 - 12*sqrt(d)*sqrt(c*f - d*e)*ata
n((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**6*e*f**3*h*x**2 + 9
*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a**3*d**6*f**4*g*x**2 + 27*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)
/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**4*d**2*f**4*h - 72*sqrt(d)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**3*
e*f**3*h + 9*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(
c*f - d*e)))*a**2*b*c**3*d**3*f**4*g + 54*sqrt(d)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**3*f**4*h*x + 72*s
qrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a
**2*b*c**2*d**4*e**2*f**2*h - 36*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +...
```

3.142 $\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 324

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{2b^2h\sqrt{e+fx}}{d^3f} - \frac{(bc-ad)^2(dg-ch)\sqrt{e+fx}}{2d^3(de-cf)(c+dx)^2}$$
$$- \frac{(bc-ad)\,(ad(3dfg-4deh+cfh)-b(8d^2eg+9c^2fh-cd(5fg+12eh)))\,\sqrt{e+fx}}{4d^3(de-cf)^2(c+dx)}$$
$$+ \frac{(2abdf(4de-cf)(dg-ch)-4b(de-cf)(2de-cf)(bdg-3bch+2adh)-a^2d^2f(3dfg-4deh+cfh}{4d^{7/2}(de-cf)^{5/2}}$$

output
```
2*b^2*h*(f*x+e)^(1/2)/d^3/f-1/2*(-a*d+b*c)^2*(-c*h+d*g)*(f*x+e)^(1/2)/d^3/
(-c*f+d*e)/(d*x+c)^2-1/4*(-a*d+b*c)*(a*d*(c*f*h-4*d*e*h+3*d*f*g)-b*(8*d^2*
e*g+9*c^2*f*h-c*d*(12*e*h+5*f*g)))*(f*x+e)^(1/2)/d^3/(-c*f+d*e)^2/(d*x+c)+
1/4*(2*a*b*d*f*(-c*f+4*d*e)*(-c*h+d*g)-4*b*(-c*f+d*e)*(-c*f+2*d*e)*(2*a*d*
h-3*b*c*h+b*d*g)-a^2*d^2*f*(c*f*h-4*d*e*h+3*d*f*g)-b^2*c*f*(-3*c^2*f*h-c*d
*f*g+4*d^2*e*g))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)/(
-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$$

$$= \frac{\frac{\sqrt{d}\sqrt{e+fx}\left(-2abdf\left(3c^3fh+4d^3egx+cd^2(2eg-fgx-8ehx)+c^2d(fg-6eh+5fhx)\right)+a^2d^2f\left(-c^2fh+cd(5fg-2eh+fhx)+d^2(3fgx-2e(g+2hx))\right)}{f(de-cf)^2(c+dx}}{}$$

input
```
Integrate[((a + b*x)^2*(g + h*x))/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
((Sqrt[d]*Sqrt[e + f*x]*(-2*a*b*d*f*(3*c^3*f*h + 4*d^3*e*g*x + c*d^2*(2*e*
g - f*g*x - 8*e*h*x) + c^2*d*(f*g - 6*e*h + 5*f*h*x)) + a^2*d^2*f*(-(c^2*f
*h) + c*d*(5*f*g - 2*e*h + f*h*x) + d^2*(3*f*g*x - 2*e*(g + 2*h*x))) + b^2
*(15*c^4*f^2*h + 8*d^4*e^2*h*x^2 + c^3*d*f*(-3*f*g - 26*e*h + 25*f*h*x) +
8*c*d^3*e*x*(2*e*h + f*(g - 2*h*x)) + c^2*d^2*(8*e^2*h + e*f*(6*g - 44*h*x
) + f^2*x*(-5*g + 8*h*x)))))/(f*(d*e - c*f)^2*(c + d*x)^2) + ((a^2*d^2*f*(
3*d*f*g - 4*d*e*h + c*f*h) + 2*a*b*d*(3*c^2*f^2*h + c*d*f*(f*g - 8*e*h) +
4*d^2*e*(-(f*g) + 2*e*h)) + b^2*(8*d^3*e^2*g - 15*c^3*f^2*h - 8*c*d^2*e*(f
*g + 3*e*h) + 3*c^2*d*f*(f*g + 12*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sq
rt[-(d*e) + c*f]])/(-(d*e) + c*f)^(5/2))/(4*d^(7/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(3dfg-4deh+cfh)+b(dfg+4deh-5cfh)x)}{2(c+dx)^2\sqrt{e+fx}}dx}{2d(de-cf)}-\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(3dfg-4deh+cfh)+b(dfg+4deh-5cfh)x)}{(c+dx)^2\sqrt{e+fx}}dx}{4d(de-cf)}-\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 163$$

$$\frac{(a^2d^2f(cfh-4deh+3dfg)+2abd(3c^2f^2h+cdf(fg-8eh)-4d^2e(fg-2eh))+b^2(-15c^3f^2h+3c^2df(12eh+fg)-8cd^2e(3eh+fg)+8d^3e^2g))\int \frac{}{(c+dx)}}{2d^2(de-cf)}$$

$$\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{(a^2d^2f(cfh-4deh+3dfg)+2abd(3c^2f^2h+cdf(fg-8eh)-4d^2e(fg-2eh))+b^2(-15c^3f^2h+3c^2df(12eh+fg)-8cd^2e(3eh+fg)+8d^3e^2g))\int \frac{}{c+\frac{d(e}{}}}{d^2f(de-cf)}$$

$$\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{e+fx}(a^2d^2f(cfh-4deh+3dfg)-2abdf(3c^2fh+cd(fg-6eh)+2d^2eg)+b^2c(15c^2f^2h-cdf(26eh+3fg)+2d^2e(4eh+3fg))+2b^2dx(de-cf)(-5cf}{d^2f(c+dx)(de-cf)}$$

$$\frac{(a+bx)^2\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[((a + b*x)^2*(g + h*x))/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^2*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x)^2) +
 ((Sqrt[e + f*x]*(a^2*d^2*f*(3*d*f*g - 4*d*e*h + c*f*h) - 2*a*b*d*f*(2*d^2
*e*g + 3*c^2*f*h + c*d*(f*g - 6*e*h)) + b^2*c*(15*c^2*f^2*h + 2*d^2*e*(3*f
*g + 4*e*h) - c*d*f*(3*f*g + 26*e*h)) + 2*b^2*d*(d*e - c*f)*(d*f*g + 4*d*e
*h - 5*c*f*h)*x))/(d^2*f*(d*e - c*f)*(c + d*x)) - ((a^2*d^2*f*(3*d*f*g - 4
*d*e*h + c*f*h) + 2*a*b*d*(3*c^2*f^2*h + c*d*f*(f*g - 8*e*h) - 4*d^2*e*(f*
g - 2*e*h)) + b^2*(8*d^3*e^2*g - 15*c^3*f^2*h - 8*c*d^2*e*(f*g + 3*e*h) +
3*c^2*d*f*(f*g + 12*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]
])/(d^(5/2)*(d*e - c*f)^(3/2)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{-(xd+c)^2 f\left(\left(3a^2f^2g-4ae(ah+2bg)f+16b\,e^2ha+8b^2e^2g\right)d^3+\left(\left(a^2h+2gab\right)f^2+\left(-16ehba-8b^2eg\right)f-24b^2e^2h\right)c\,d^2 \cdots}{\cdots}$
risch	$\frac{2b^2h\sqrt{fx+e}}{d^3f}+\frac{\frac{2df\left(a^2c\,d^2fh-4a^2d^3eh+3a^2d^3fg-10ab\,c^2dfh+16abc\,d^2eh+2abc\,d^2fg-8ab\,d^3eg+9b^2c^3fh-12b^2c^2deh-5b^2c^2 \cdots\right)}{8c^2f^2-16cdef+8d^2e^2} \cdots}{\cdots}$
derivativedivides	$\frac{\frac{2h\,b^2\sqrt{fx+e}}{d^3}-\frac{2f\left(-\frac{df\left(a^2c\,d^2fh-4a^2d^3eh+3a^2d^3fg-10ab\,c^2dfh+16abc\,d^2eh+2abc\,d^2fg-8ab\,d^3eg+9b^2c^3fh-12b^2c^2deh-5b^2 \cdots\right)}{8\left(c^2f^2-2cdef+d^2e^2\right)} \cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{2h\,b^2\sqrt{fx+e}}{d^3}-\frac{2f\left(-\frac{df\left(a^2c\,d^2fh-4a^2d^3eh+3a^2d^3fg-10ab\,c^2dfh+16abc\,d^2eh+2abc\,d^2fg-8ab\,d^3eg+9b^2c^3fh-12b^2c^2deh-5b^2 \cdots\right)}{8\left(c^2f^2-2cdef+d^2e^2\right)} \cdots\right)}{\cdots}}{\cdots}$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(d*x+c)^2*f*((3*a^2*f^2*g-4*a*e*(a*h+2*b*g)*f+16*b*e^2*h*a+8*b^2*e^
2*g)*d^3+((a^2*h+2*a*b*g)*f^2+(-16*a*b*e*h-8*b^2*e*g)*f-24*b^2*e^2*h)*c*d^
2+6*((a*h+1/2*b*g)*f+6*e*h*b)*c^2*b*f*d-15*b^2*c^3*f^2*h)*arctan(d*(f*x+e)
^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((-3*a^2*f^2
*g*x+2*a*(4*x*g*b+a*(2*h*x+g))*e*f-8*b^2*e^2*h*x^2)*d^4+2*((-a*b*g*x-5/2*a
^2*(1/5*h*x+g))*f^2+((8*h*x^2-4*g*x)*b^2+2*a*(-4*h*x+g)*b+a^2*h)*e*f-8*b^2
*e^2*h*x)*c*d^3+c^2*(((-8*h*x^2+5*g*x)*b^2+2*a*(5*h*x+g)*b+a^2*h)*f^2-12*b
*((-11/3*h*x+1/2*g)*b+a*h)*e*f-8*b^2*e^2*h)*d^2+6*(((-25/6*h*x+1/2*g)*b+a*
h)*f+13/3*e*h*b)*c^3*b*f*d-15*b^2*c^4*f^2*h))/((c*f-d*e)*d)^(1/2)/(c*f-d*e
)^2/(d*x+c)^2/d^3/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. $2(302) = 604$.

Time = 0.25 (sec) , antiderivative size = 2518, normalized size of antiderivative = 7.77

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(d^2*e - c*d*f)*(((8*b^2*d^5*e^2*f - 8*(b^2*c*d^4 + a*b*d^5)*e*f
^2 + (3*b^2*c^2*d^3 + 2*a*b*c*d^4 + 3*a^2*d^5)*f^3)*g - (8*(3*b^2*c*d^4 -
2*a*b*d^5)*e^2*f - 4*(9*b^2*c^2*d^3 - 4*a*b*c*d^4 - a^2*d^5)*e*f^2 + (15*b
^2*c^3*d^2 - 6*a*b*c^2*d^3 - a^2*c*d^4)*f^3)*h)*x^2 + (8*b^2*c^2*d^3*e^2*f
 - 8*(b^2*c^3*d^2 + a*b*c^2*d^3)*e*f^2 + (3*b^2*c^4*d + 2*a*b*c^3*d^2 + 3*
a^2*c^2*d^3)*f^3)*g - (8*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3)*e^2*f - 4*(9*b^2*
c^4*d - 4*a*b*c^3*d^2 - a^2*c^2*d^3)*e*f^2 + (15*b^2*c^5 - 6*a*b*c^4*d - a
^2*c^3*d^2)*f^3)*h + 2*((8*b^2*c*d^4*e^2*f - 8*(b^2*c^2*d^3 + a*b*c*d^4)*e
*f^2 + (3*b^2*c^3*d^2 + 2*a*b*c^2*d^3 + 3*a^2*c*d^4)*f^3)*g - (8*(3*b^2*c^
2*d^3 - 2*a*b*c*d^4)*e^2*f - 4*(9*b^2*c^3*d^2 - 4*a*b*c^2*d^3 - a^2*c*d^4)
*e*f^2 + (15*b^2*c^4*d - 6*a*b*c^3*d^2 - a^2*c^2*d^3)*f^3)*h)*x)*log((d*f*
x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*(8*(
b^2*d^6*e^3 - 3*b^2*c*d^5*e^2*f + 3*b^2*c^2*d^4*e*f^2 - b^2*c^3*d^3*f^3)*h
*x^2 + (2*(3*b^2*c^2*d^4 - 2*a*b*c*d^5 - a^2*d^6)*e^2*f - (9*b^2*c^3*d^3 -
 2*a*b*c^2*d^4 - 7*a^2*c*d^5)*e*f^2 + (3*b^2*c^4*d^2 + 2*a*b*c^3*d^3 - 5*a
^2*c^2*d^4)*f^3)*g + (8*b^2*c^2*d^4*e^3 - 2*(17*b^2*c^3*d^3 - 6*a*b*c^2*d^
4 + a^2*c*d^5)*e^2*f + (41*b^2*c^4*d^2 - 18*a*b*c^3*d^3 + a^2*c^2*d^4)*e*f
^2 - (15*b^2*c^5*d - 6*a*b*c^4*d^2 - a^2*c^3*d^3)*f^3)*h + ((8*(b^2*c*d^5
- a*b*d^6)*e^2*f - (13*b^2*c^2*d^4 - 10*a*b*c*d^5 - 3*a^2*d^6)*e*f^2 + (5*
b^2*c^3*d^3 - 2*a*b*c^2*d^4 - 3*a^2*c*d^5)*f^3)*g + (16*b^2*c*d^5*e^3 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(h*x+g)/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(302) = 604$.

Time = 0.14 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.67

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*b^2*d^3*e^2*g - 8*b^2*c*d^2*e*f*g - 8*a*b*d^3*e*f*g + 3*b^2*c^2*d*f
^2*g + 2*a*b*c*d^2*f^2*g + 3*a^2*d^3*f^2*g - 24*b^2*c*d^2*e^2*h + 16*a*b*d
^3*e^2*h + 36*b^2*c^2*d*e*f*h - 16*a*b*c*d^2*e*f*h - 4*a^2*d^3*e*f*h - 15*
b^2*c^3*f^2*h + 6*a*b*c^2*d*f^2*h + a^2*c*d^2*f^2*h)*arctan(sqrt(f*x + e)*
d/sqrt(-d^2*e + c*d*f))/((d^5*e^2 - 2*c*d^4*e*f + c^2*d^3*f^2)*sqrt(-d^2*e
 + c*d*f)) + 2*sqrt(f*x + e)*b^2*h/(d^3*f) + 1/4*(8*(f*x + e)^(3/2)*b^2*c*
d^3*e*f*g - 8*(f*x + e)^(3/2)*a*b*d^4*e*f*g - 8*sqrt(f*x + e)*b^2*c*d^3*e^
2*f*g + 8*sqrt(f*x + e)*a*b*d^4*e^2*f*g - 5*(f*x + e)^(3/2)*b^2*c^2*d^2*f^
2*g + 2*(f*x + e)^(3/2)*a*b*c*d^3*f^2*g + 3*(f*x + e)^(3/2)*a^2*d^4*f^2*g
+ 11*sqrt(f*x + e)*b^2*c^2*d^2*e*f^2*g - 6*sqrt(f*x + e)*a*b*c*d^3*e*f^2*g
 - 5*sqrt(f*x + e)*a^2*d^4*e*f^2*g - 3*sqrt(f*x + e)*b^2*c^3*d*f^3*g - 2*s
qrt(f*x + e)*a*b*c^2*d^2*f^3*g + 5*sqrt(f*x + e)*a^2*c*d^3*f^3*g - 12*(f*x
 + e)^(3/2)*b^2*c^2*d^2*e*f*h + 16*(f*x + e)^(3/2)*a*b*c*d^3*e*f*h - 4*(f*
x + e)^(3/2)*a^2*d^4*e*f*h + 12*sqrt(f*x + e)*b^2*c^2*d^2*e^2*f*h - 16*sqr
t(f*x + e)*a*b*c*d^3*e^2*f*h + 4*sqrt(f*x + e)*a^2*d^4*e^2*f*h + 9*(f*x +
e)^(3/2)*b^2*c^3*d*f^2*h - 10*(f*x + e)^(3/2)*a*b*c^2*d^2*f^2*h + (f*x + e
)^(3/2)*a^2*c*d^3*f^2*h - 19*sqrt(f*x + e)*b^2*c^3*d*e*f^2*h + 22*sqrt(f*x
 + e)*a*b*c^2*d^2*e*f^2*h - 3*sqrt(f*x + e)*a^2*c*d^3*e*f^2*h + 7*sqrt(f*x
 + e)*b^2*c^4*f^3*h - 6*sqrt(f*x + e)*a*b*c^3*d*f^3*h - sqrt(f*x + e)*a^2*
c^2*d^2*f^3*h)/((d^5*e^2 - 2*c*d^4*e*f + c^2*d^3*f^2)*((f*x + e)*d - d*...
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$$
$$= \frac{\frac{(e+f\,x)^{3/2}\,\left(h\,a^2\,c\,d^3\,f^2+3\,g\,a^2\,d^4\,f^2-4\,e\,h\,a^2\,d^4\,f-10\,h\,a\,b\,c^2\,d^2\,f^2+2\,g\,a\,b\,c\,d^3\,f^2+16\,e\,h\,a\,b\,c\,d^3\,f-8\,e\,g\,a\,b\,d^4\,f+9\,h\,b^2\,c^3\,d\,f^2-5\,g\,b^2\,c^2\,d^2\,f^2}{4\,(c\,f-d\,e)^2}}{d^5\,(e+f\,x)^2 -}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(h\,a^2\,c\,d^2\,f^2-4\,h\,a^2\,d^3\,e\,f+3\,g\,a^2\,d^3\,f^2+6\,h\,a\,b\,c^2\,d\,f^2-16\,h\,a\,b\,c\,d^2\,e\,f+2\,g\,a\,b\,c\,d}{}$$
$$+ \frac{2\,b^2\,h\,\sqrt{e+f\,x}}{d^3\,f}$$

input
```
int(((g + h*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(c + d*x)^3),x)
```

output
```
(((e + f*x)^(3/2)*(3*a^2*d^4*f^2*g - 4*a^2*d^4*e*f*h + a^2*c*d^3*f^2*h + 9
*b^2*c^3*d*f^2*h - 5*b^2*c^2*d^2*f^2*g - 8*a*b*d^4*e*f*g + 2*a*b*c*d^3*f^2
*g + 8*b^2*c*d^3*e*f*g - 10*a*b*c^2*d^2*f^2*h - 12*b^2*c^2*d^2*e*f*h + 16*
a*b*c*d^3*e*f*h))/(4*(c*f - d*e)^2) - ((e + f*x)^(1/2)*(4*a^2*d^3*e*f*h -
7*b^2*c^3*f^2*h - 5*a^2*d^3*f^2*g + a^2*c*d^2*f^2*h + 3*b^2*c^2*d*f^2*g +
8*a*b*d^3*e*f*g + 2*a*b*c*d^2*f^2*g + 6*a*b*c^2*d*f^2*h - 8*b^2*c*d^2*e*f*
g + 12*b^2*c^2*d*e*f*h - 16*a*b*c*d^2*e*f*h))/(4*(c*f - d*e)))/(d^5*(e + f
*x)^2 - (e + f*x)*(2*d^5*e - 2*c*d^4*f) + d^5*e^2 + c^2*d^3*f^2 - 2*c*d^4*
e*f) + (atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))*(3*a^2*d^3*f^2*g
 + 8*b^2*d^3*e^2*g - 15*b^2*c^3*f^2*h + 16*a*b*d^3*e^2*h - 4*a^2*d^3*e*f*h
 + a^2*c*d^2*f^2*h - 24*b^2*c*d^2*e^2*h + 3*b^2*c^2*d*f^2*g - 8*a*b*d^3*e*
f*g + 2*a*b*c*d^2*f^2*g + 6*a*b*c^2*d*f^2*h - 8*b^2*c*d^2*e*f*g + 36*b^2*c
^2*d*e*f*h - 16*a*b*c*d^2*e*f*h))/(4*d^(7/2)*(c*f - d*e)^(5/2)) + (2*b^2*h
*(e + f*x)^(1/2))/(d^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3133, normalized size of antiderivative = 9.67

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a**2*c**3*d**2*f**3*h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*e*f**2*h + 3*sqrt(d)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*f**3
*g + 2*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**2*c**2*d**3*f**3*h*x - 8*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*e*f**2*h*x + 6*sqrt(d)*sqrt
(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*
f**3*g*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*
f - d*e)))*a**2*c*d**4*f**3*h*x**2 - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(
e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**5*e*f**2*h*x**2 + 3*sqrt(d)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d*
*5*f**3*g*x**2 + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*a*b*c**4*d*f**3*h - 16*sqrt(d)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*e*f**2*h + 2*sqrt(
d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c
**3*d**2*f**3*g + 12*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*f**3*h*x + 16*sqrt(d)*sqrt(c*f - d*e)*a
tan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**3*e**2*f*h -
8*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*...
```

3.143 $\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 239

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{(bc-ad)(dg-ch)\sqrt{e+fx}}{2d^2(de-cf)(c+dx)^2} + \frac{(ad(3dfg-4deh+cfh)-b(4d^2eg+5c^2fh-cd(fg+8eh)))\sqrt{e+fx}}{4d^2(de-cf)^2(c+dx)} - \frac{(adf(3dfg-4deh+cfh)+b(3c^2f^2h+cdf(fg-8eh)-4d^2e(fg-2eh)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{5/2}(de-cf)^{5/2}}$$

output

```
1/2*(-a*d+b*c)*(-c*h+d*g)*(f*x+e)^(1/2)/d^2/(-c*f+d*e)/(d*x+c)^2+1/4*(a*d*
(c*f*h-4*d*e*h+3*d*f*g)-b*(4*d^2*e*g+5*c^2*f*h-c*d*(8*e*h+f*g)))*(f*x+e)^(
1/2)/d^2/(-c*f+d*e)^2/(d*x+c)-1/4*(a*d*f*(c*f*h-4*d*e*h+3*d*f*g)+b*(3*c^2*
f^2*h+c*d*f*(-8*e*h+f*g)-4*d^2*e*(-2*e*h+f*g)))*arctanh(d^(1/2)*(f*x+e)^(1
/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = -\frac{\sqrt{e+fx}(ad(c^2fh+d^2(2eg-3fgx+4ehx)-cd(5fg-2eh+fhx))+b(3c^3fh+4d^3egx+cd^2(2eg}{4d^2(de-cf)^2(c+dx)^2} + \frac{(adf(3dfg-4deh+cfh)+b(3c^2f^2h+cdf(fg-8eh)+4d^2e(-fg+2eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{4d^{5/2}(-de+cf)^{5/2}}$$

input
```
Integrate[((a + b*x)*(g + h*x))/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/4*(Sqrt[e + f*x]*(a*d*(c^2*f*h + d^2*(2*e*g - 3*f*g*x + 4*e*h*x) - c*d*
(5*f*g - 2*e*h + f*h*x)) + b*(3*c^3*f*h + 4*d^3*e*g*x + c*d^2*(2*e*g - f*g
*x - 8*e*h*x) + c^2*d*(f*g - 6*e*h + 5*f*h*x))))/(d^2*(d*e - c*f)^2*(c + d
*x)^2) + ((a*d*f*(3*d*f*g - 4*d*e*h + c*f*h) + b*(3*c^2*f^2*h + c*d*f*(f*g
 - 8*e*h) + 4*d^2*e*(-(f*g) + 2*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt
[-(d*e) + c*f]])/(4*d^(5/2)*(-(d*e) + c*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {162, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{162}$$

$$\frac{(adf(cfh-4deh+3dfg)+b(3c^2f^2h+cdf(fg-8eh)-4d^2e(fg-2eh)))\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{8d^2(de-cf)^2} - \frac{\sqrt{e+fx}(-dx(ad(cfh-4deh+3dfg)-b(5c^2fh-cd(8eh+fg)+4d^2eg))+ad(c^2fh+2cdeh-5cdfg+2d^2e}{4d^2(c+dx)^2(de-cf)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(adf(cfh-4deh+3dfg)+b\left(3c^2f^2h+cdf(fg-8eh)-4d^2e(fg-2eh)\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{4d^2f(de-cf)^2}-\frac{\sqrt{e+fx}\left(-dx\left(ad(cfh-4deh+3dfg)-b\left(5c^2fh-cd(8eh+fg)+4d^2eg\right)\right)+ad\left(c^2fh+2cdeh-5cdfg+2d^2e\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(adf(cfh-4deh+3dfg)+b\left(3c^2f^2h+cdf(fg-8eh)-4d^2e(fg-2eh)\right)\right)}{4d^{5/2}(de-cf)^{5/2}}-\frac{\sqrt{e+fx}\left(-dx\left(ad(cfh-4deh+3dfg)-b\left(5c^2fh-cd(8eh+fg)+4d^2eg\right)\right)+ad\left(c^2fh+2cdeh-5cdfg+2d^2e\right.\right.}{4d^2(c+dx)^2(de-cf)^2}$$

input

```
Int[((a + b*x)*(g + h*x))/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output

```
-1/4*(Sqrt[e + f*x]*(a*d*(2*d^2*e*g - 5*c*d*f*g + 2*c*d*e*h + c^2*f*h) + b
*c*(2*d^2*e*g + 3*c^2*f*h + c*d*(f*g - 6*e*h)) - d*(a*d*(3*d*f*g - 4*d*e*h
 + c*f*h) - b*(4*d^2*e*g + 5*c^2*f*h - c*d*(f*g + 8*e*h)))*x))/(d^2*(d*e -
 c*f)^2*(c + d*x)^2) - ((a*d*f*(3*d*f*g - 4*d*e*h + c*f*h) + b*(3*c^2*f^2*
h + c*d*f*(f*g - 8*e*h) - 4*d^2*e*(f*g - 2*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e
+ f*x])/Sqrt[d*e - c*f]])/(4*d^(5/2)*(d*e - c*f)^(5/2))
```

Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-(xd+c)^{2}\left(\left(3g\,f^{2}a-4e(ah+bg)f+8b\,e^{2}h\right)d^{2}+c\left((ah+bg)f-8ehb\right)fd+3b\,c^{2}f^{2}h\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\sqrt{(cf-de)d}}{4\sqrt{(cf-}}$
derivativedivides	$-\frac{2\left(-\frac{f\left(acdfh-4a\,d^{2}eh+3a\,d^{2}fg-5b\,c^{2}fh+8bcdeh+bcdfg-4b\,d^{2}eg\right)(fx+e)^{\frac{3}{2}}}{8d\left(c^{2}f^{2}-2cdef+d^{2}e^{2}\right)}+\frac{\left(acdfh+4a\,d^{2}eh-5a\,d^{2}fg+3b\,c^{2}fh-8bcdeh+b\right.}{8d^{2}(cf-de)}\right.}{((fx+e)d+cf-de)^{2}}$
default	$-\frac{2\left(-\frac{f\left(acdfh-4a\,d^{2}eh+3a\,d^{2}fg-5b\,c^{2}fh+8bcdeh+bcdfg-4b\,d^{2}eg\right)(fx+e)^{\frac{3}{2}}}{8d\left(c^{2}f^{2}-2cdef+d^{2}e^{2}\right)}+\frac{\left(acdfh+4a\,d^{2}eh-5a\,d^{2}fg+3b\,c^{2}fh-8bcdeh+b\right.}{8d^{2}(cf-de)}\right.}{((fx+e)d+cf-de)^{2}}$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(d*x+c)^2*((3*g*f^2*a-4*e*(a*h+b*g)*f+8*b*e^2*h)*d^2+c*((a*h+b*g)*f
-8*e*h*b)*f*d+3*b*c^2*f^2*h)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+(
(c*f-d*e)*d)^(1/2)*(f*x+e)^(1/2)*((-3*x*a*f*g+2*e*(2*(a*h+b*g)*x+g*a))*d^3
+2*c*(1/2*((-a*h-b*g)*x-5*g*a)*f+e*(-4*b*h*x+a*h+b*g))*d^2+c^2*((5*b*h*x+a
*h+b*g)*f-6*e*h*b)*d+3*c^3*h*b*f))/((c*f-d*e)*d)^(1/2)/(c*f-d*e)^2/d^2/(d*
x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(219) = 438$.

Time = 0.15 (sec) , antiderivative size = 1442, normalized size of antiderivative = 6.03

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(d^2*e - c*d*f)*(((4*b*d^4*e*f - (b*c*d^3 + 3*a*d^4)*f^2)*g - (
8*b*d^4*e^2 - 4*(2*b*c*d^3 + a*d^4)*e*f + (3*b*c^2*d^2 + a*c*d^3)*f^2)*h)*
x^2 + (4*b*c^2*d^2*e*f - (b*c^3*d + 3*a*c^2*d^2)*f^2)*g - (8*b*c^2*d^2*e^2
 - 4*(2*b*c^3*d + a*c^2*d^2)*e*f + (3*b*c^4 + a*c^3*d)*f^2)*h + 2*((4*b*c*
d^3*e*f - (b*c^2*d^2 + 3*a*c*d^3)*f^2)*g - (8*b*c*d^3*e^2 - 4*(2*b*c^2*d^2
 + a*c*d^3)*e*f + (3*b*c^3*d + a*c^2*d^2)*f^2)*h)*x)*log((d*f*x + 2*d*e -
c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*((2*(b*c*d^4 + a
*d^5)*e^2 - (b*c^2*d^3 + 7*a*c*d^4)*e*f - (b*c^3*d^2 - 5*a*c^2*d^3)*f^2)*g
 - (2*(3*b*c^2*d^3 - a*c*d^4)*e^2 - (9*b*c^3*d^2 - a*c^2*d^3)*e*f + (3*b*c
^4*d + a*c^3*d^2)*f^2)*h + ((4*b*d^5*e^2 - (5*b*c*d^4 + 3*a*d^5)*e*f + (b*
c^2*d^3 + 3*a*c*d^4)*f^2)*g - (4*(2*b*c*d^4 - a*d^5)*e^2 - (13*b*c^2*d^3 -
 5*a*c*d^4)*e*f + (5*b*c^3*d^2 - a*c^2*d^3)*f^2)*h)*x)*sqrt(f*x + e))/(c^2
*d^6*e^3 - 3*c^3*d^5*e^2*f + 3*c^4*d^4*e*f^2 - c^5*d^3*f^3 + (d^8*e^3 - 3*
c*d^7*e^2*f + 3*c^2*d^6*e*f^2 - c^3*d^5*f^3)*x^2 + 2*(c*d^7*e^3 - 3*c^2*d^
6*e^2*f + 3*c^3*d^5*e*f^2 - c^4*d^4*f^3)*x), -1/4*(sqrt(-d^2*e + c*d*f)*((
(4*b*d^4*e*f - (b*c*d^3 + 3*a*d^4)*f^2)*g - (8*b*d^4*e^2 - 4*(2*b*c*d^3 +
a*d^4)*e*f + (3*b*c^2*d^2 + a*c*d^3)*f^2)*h)*x^2 + (4*b*c^2*d^2*e*f - (b*c
^3*d + 3*a*c^2*d^2)*f^2)*g - (8*b*c^2*d^2*e^2 - 4*(2*b*c^3*d + a*c^2*d^2)*
e*f + (3*b*c^4 + a*c^3*d)*f^2)*h + 2*((4*b*c*d^3*e*f - (b*c^2*d^2 + 3*a*c*
d^3)*f^2)*g - (8*b*c*d^3*e^2 - 4*(2*b*c^2*d^2 + a*c*d^3)*e*f + (3*b*c^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(219) = 438.

Time = 0.13 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx =$$

$$-\frac{(4\,bd^2efg - bcdf^2g - 3\,ad^2f^2g - 8\,bd^2e^2h + 8\,bcdefh + 4\,ad^2efh - 3\,bc^2f^2h - acdf^2h)\arctan\left(\frac{\sqrt{fx}}{\sqrt{-d^2e}}\right.}{4\,(d^4e^2 - 2\,cd^3ef + c^2d^2f^2)\sqrt{-d^2e+cdf}}$$

$$-\frac{4\,(fx+e)^{\frac{3}{2}}bd^3efg - 4\,\sqrt{fx+e}bd^3e^2fg - (fx+e)^{\frac{3}{2}}bcd^2f^2g - 3\,(fx+e)^{\frac{3}{2}}ad^3f^2g + 3\,\sqrt{fx+e}bcd^2ef}{}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*b*d^2*e*f*g - b*c*d*f^2*g - 3*a*d^2*f^2*g - 8*b*d^2*e^2*h + 8*b*c*
d*e*f*h + 4*a*d^2*e*f*h - 3*b*c^2*f^2*h - a*c*d*f^2*h)*arctan(sqrt(f*x + e
)*d/sqrt(-d^2*e + c*d*f))/((d^4*e^2 - 2*c*d^3*e*f + c^2*d^2*f^2)*sqrt(-d^2
*e + c*d*f)) - 1/4*(4*(f*x + e)^(3/2)*b*d^3*e*f*g - 4*sqrt(f*x + e)*b*d^3*
e^2*f*g - (f*x + e)^(3/2)*b*c*d^2*f^2*g - 3*(f*x + e)^(3/2)*a*d^3*f^2*g +
3*sqrt(f*x + e)*b*c*d^2*e*f^2*g + 5*sqrt(f*x + e)*a*d^3*e*f^2*g + sqrt(f*x
 + e)*b*c^2*d*f^3*g - 5*sqrt(f*x + e)*a*c*d^2*f^3*g - 8*(f*x + e)^(3/2)*b*
c*d^2*e*f*h + 4*(f*x + e)^(3/2)*a*d^3*e*f*h + 8*sqrt(f*x + e)*b*c*d^2*e^2*
f*h - 4*sqrt(f*x + e)*a*d^3*e^2*f*h + 5*(f*x + e)^(3/2)*b*c^2*d*f^2*h - (f
*x + e)^(3/2)*a*c*d^2*f^2*h - 11*sqrt(f*x + e)*b*c^2*d*e*f^2*h + 3*sqrt(f*
x + e)*a*c*d^2*e*f^2*h + 3*sqrt(f*x + e)*b*c^3*f^3*h + sqrt(f*x + e)*a*c^2
*d*f^3*h)/((d^4*e^2 - 2*c*d^3*e*f + c^2*d^2*f^2)*((f*x + e)*d - d*e + c*f)
^2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}}{\sqrt{c\,f-d\,e}}\right)\,\left(3\,a\,d^2\,f^2\,g+3\,b\,c^2\,f^2\,h+8\,b\,d^2\,e^2\,h+a\,c\,d\,f^2\,h+b\,c\,d\,f^2\,g-4\,a\,d^2\,e\,f\,h-4\,b\,d^2\,e\,f\,g-\right.}{4\,d^{5/2}\,{\left(c\,f-d\,e\right)}^{5/2}} - \frac{\frac{\sqrt{e+f\,x}\,\left(3\,b\,c^2\,f^2\,h-5\,a\,d^2\,f^2\,g+a\,c\,d\,f^2\,h+b\,c\,d\,f^2\,g+4\,a\,d^2\,e\,f\,h+4\,b\,d^2\,e\,f\,g-8\,b\,c\,d\,e\,f\,h\right)}{4\,d^2\,\left(c\,f-d\,e\right)} - \frac{{\left(e+f\,x\right)}^{3/2}\,\left(3\,a\,d^2\,f^2\,g-5\,b\,c^2\,f^2\,h+a\,c\,d\,f^2\,\right.}{4\,d\,(}}{d^2\,{\left(e+f\,x\right)}^2-\left(e+f\,x\right)\,\left(2\,d^2\,e-2\,c\,d\,f\right)+c^2\,f^2+d^2\,e^2-2\,c\,d\,e\,}$$

input
```
int(((g + h*x)*(a + b*x))/((e + f*x)^(1/2)*(c + d*x)^3),x)
```

output
```
(atan((d^(1/2)*(e + f*x)^(1/2))/(c*f - d*e)^(1/2))*(3*a*d^2*f^2*g + 3*b*c^
2*f^2*h + 8*b*d^2*e^2*h + a*c*d*f^2*h + b*c*d*f^2*g - 4*a*d^2*e*f*h - 4*b*
d^2*e*f*g - 8*b*c*d*e*f*h))/(4*d^(5/2)*(c*f - d*e)^(5/2)) - (((e + f*x)^(1
/2)*(3*b*c^2*f^2*h - 5*a*d^2*f^2*g + a*c*d*f^2*h + b*c*d*f^2*g + 4*a*d^2*e
*f*h + 4*b*d^2*e*f*g - 8*b*c*d*e*f*h))/(4*d^2*(c*f - d*e)) - ((e + f*x)^(3
/2)*(3*a*d^2*f^2*g - 5*b*c^2*f^2*h + a*c*d*f^2*h + b*c*d*f^2*g - 4*a*d^2*e
*f*h - 4*b*d^2*e*f*g + 8*b*c*d*e*f*h))/(4*d*(c*f - d*e)^2))/(d^2*(e + f*x)
^2 - (e + f*x)*(2*d^2*e - 2*c*d*f) + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1691, normalized size of antiderivative = 7.08

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*a*c**3*d*f**2*h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a*c**2*d**2*e*f*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((s
qrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d**2*f**2*g + 2*sqrt(d)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d
**2*f**2*h*x - 8*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a*c*d**3*e*f*h*x + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*f**2*g*x + sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*f**2*h*
x**2 - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
- d*e)))*a*d**4*e*f*h*x**2 + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**4*f**2*g*x**2 + 3*sqrt(d)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**4*f**2*h - 8*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*
c**3*d*e*f*h + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*b*c**3*d*f**2*g + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*d*f**2*h*x + 8*sqrt(d)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d**2*e**2*
h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*b*c**2*d**2*e*f*g - 16*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x...
```

3.144 $\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = -\frac{(dg-ch)\sqrt{e+fx}}{2d(de-cf)(c+dx)^2}+\frac{(3dfg-4deh+cfh)\sqrt{e+fx}}{4d(de-cf)^2(c+dx)} - \frac{f(3dfg-4deh+cfh)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{3/2}(de-cf)^{5/2}}$$

output
```
-1/2*(-c*h+d*g)*(f*x+e)^(1/2)/d/(-c*f+d*e)/(d*x+c)^2+1/4*(c*f*h-4*d*e*h+3*
d*f*g)*(f*x+e)^(1/2)/d/(-c*f+d*e)^2/(d*x+c)-1/4*f*(c*f*h-4*d*e*h+3*d*f*g)*
arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(3/2)/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{\frac{\sqrt{d}\sqrt{e+fx}(-c^2fh+cd(5fg-2eh+fhx)+d^2(3fgx-2e(g+2hx)))}{(de-cf)^2(c+dx)^2}+\frac{f(3dfg-4deh+cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(-de+cf)^{5/2}}}{4d^{3/2}}$$

input
```
Integrate[(g + h*x)/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
((Sqrt[d]*Sqrt[e + f*x]*(-(c^2*f*h) + c*d*(5*f*g - 2*e*h + f*h*x) + d^2*(3
*f*g*x - 2*e*(g + 2*h*x))))/((d*e - c*f)^2*(c + d*x)^2) + (f*(3*d*f*g - 4*
d*e*h + c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(-(d*e)
 + c*f)^(5/2))/(4*d^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(cfh-4deh+3dfg)\int \frac{1}{(c+dx)^2\sqrt{e+fx}}dx}{4d(de-cf)}-\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(cfh-4deh+3dfg)\left(-\frac{f\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{2(de-cf)}-\frac{\sqrt{e+fx}}{(c+dx)(de-cf)}\right)}{4d(de-cf)}-\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(cfh-4deh+3dfg)\left(-\frac{\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{de-cf}-\frac{\sqrt{e+fx}}{(c+dx)(de-cf)}\right)}{4d(de-cf)}-\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(\frac{f\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(de-cf)^{3/2}}-\frac{\sqrt{e+fx}}{(c+dx)(de-cf)}\right)(cfh-4deh+3dfg)}{4d(de-cf)}-\frac{\sqrt{e+fx}(dg-ch)}{2d(c+dx)^2(de-cf)}$$

input
```
Int[(g + h*x)/((c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((d*g - c*h)*Sqrt[e + f*x])/(d*(d*e - c*f)*(c + d*x)^2) - ((3*d*f*g -
 4*d*e*h + c*f*h)*(-(Sqrt[e + f*x]/((d*e - c*f)*(c + d*x))) + (f*ArcTanh[(
Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(d*e - c*f)^(3/2))))/(4*
d*(d*e - c*f))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{-f(xd+c)^2((-4eh+3fg)d+cfh)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+\sqrt{(cf-de)d}\left((-3fgx+2e(2hx+g))d^2+2c\left(\frac{(-hx-5g)f}{2}+eh\right.\right.}{4(cf-de)^2\sqrt{(cf-de)d}\,d(xd+c)^2}$
derivativedivides	$2f\left(-\frac{-\frac{(cfh-4deh+3dfg)(fx+e)^{\frac{3}{2}}}{8(c^2f^2-2cdef+d^2e^2)}+\frac{(cfh+4deh-5dfg)\sqrt{fx+e}}{8(cf-de)d}}{((fx+e)d+cf-de)^2}+\frac{(cfh-4deh+3dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{8(c^2f^2-2cdef+d^2e^2)d\sqrt{(cf-de)d}}\right)$
default	$2f\left(-\frac{-\frac{(cfh-4deh+3dfg)(fx+e)^{\frac{3}{2}}}{8(c^2f^2-2cdef+d^2e^2)}+\frac{(cfh+4deh-5dfg)\sqrt{fx+e}}{8(cf-de)d}}{((fx+e)d+cf-de)^2}+\frac{(cfh-4deh+3dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{8(c^2f^2-2cdef+d^2e^2)d\sqrt{(cf-de)d}}\right)$

input
```
int((h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(c*f-d*e)^2/((c*f-d*e)*d)^(1/2)*(-f*(d*x+c)^2*((-4*e*h+3*f*g)*d+c*f*h
)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*((-3*f*g
*x+2*e*(2*h*x+g))*d^2+2*c*(1/2*(-h*x-5*g)*f+e*h)*d+c^2*f*h)*(f*x+e)^(1/2))
/d/(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(135) = 270.

Time = 0.11 (sec) , antiderivative size = 839, normalized size of antiderivative = 5.41

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((3*c^2*d*f^2*g + (3*d^3*f^2*g - (4*d^3*e*f - c*d^2*f^2)*h)*x^2 - (4*
c^2*d*e*f - c^3*f^2)*h + 2*(3*c*d^2*f^2*g - (4*c*d^2*e*f - c^2*d*f^2)*h)*x
)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqr
t(f*x + e))/(d*x + c)) - 2*((2*d^4*e^2 - 7*c*d^3*e*f + 5*c^2*d^2*f^2)*g +
(2*c*d^3*e^2 - c^2*d^2*e*f - c^3*d*f^2)*h - (3*(d^4*e*f - c*d^3*f^2)*g - (
4*d^4*e^2 - 5*c*d^3*e*f + c^2*d^2*f^2)*h)*x)*sqrt(f*x + e))/(c^2*d^5*e^3 -
 3*c^3*d^4*e^2*f + 3*c^4*d^3*e*f^2 - c^5*d^2*f^3 + (d^7*e^3 - 3*c*d^6*e^2*
f + 3*c^2*d^5*e*f^2 - c^3*d^4*f^3)*x^2 + 2*(c*d^6*e^3 - 3*c^2*d^5*e^2*f +
3*c^3*d^4*e*f^2 - c^4*d^3*f^3)*x), 1/4*((3*c^2*d*f^2*g + (3*d^3*f^2*g - (4
*d^3*e*f - c*d^2*f^2)*h)*x^2 - (4*c^2*d*e*f - c^3*f^2)*h + 2*(3*c*d^2*f^2*
g - (4*c*d^2*e*f - c^2*d*f^2)*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*
e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) - ((2*d^4*e^2 - 7*c*d^3*e*f + 5*c^
2*d^2*f^2)*g + (2*c*d^3*e^2 - c^2*d^2*e*f - c^3*d*f^2)*h - (3*(d^4*e*f - c
*d^3*f^2)*g - (4*d^4*e^2 - 5*c*d^3*e*f + c^2*d^2*f^2)*h)*x)*sqrt(f*x + e))
/(c^2*d^5*e^3 - 3*c^3*d^4*e^2*f + 3*c^4*d^3*e*f^2 - c^5*d^2*f^3 + (d^7*e^3
 - 3*c*d^6*e^2*f + 3*c^2*d^5*e*f^2 - c^3*d^4*f^3)*x^2 + 2*(c*d^6*e^3 - 3*c
^2*d^5*e^2*f + 3*c^3*d^4*e*f^2 - c^4*d^3*f^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.66

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{(3\,df^2g-4\,defh+cf^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{4\,(d^3e^2-2\,cd^2ef+c^2df^2)\sqrt{-d^2e+cdf}} + \frac{3\,(fx+e)^{\frac{3}{2}}d^2f^2g-5\,\sqrt{fx+e}d^2ef^2g+5\,\sqrt{fx+e}cdf^3g-4\,(fx+e)^{\frac{3}{2}}d^2efh+4\,\sqrt{fx+e}d^2e^2fh+(}{4\,(d^3e^2-2\,cd^2ef+c^2df^2)((fx+e)d-de+cf)^2}$$

input
```
integrate((h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*d*f^2*g - 4*d*e*f*h + c*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e +
 c*d*f))/((d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sqrt(-d^2*e + c*d*f)) + 1/4*
(3*(f*x + e)^(3/2)*d^2*f^2*g - 5*sqrt(f*x + e)*d^2*e*f^2*g + 5*sqrt(f*x +
e)*c*d*f^3*g - 4*(f*x + e)^(3/2)*d^2*e*f*h + 4*sqrt(f*x + e)*d^2*e^2*f*h +
 (f*x + e)^(3/2)*c*d*f^2*h - 3*sqrt(f*x + e)*c*d*e*f^2*h - sqrt(f*x + e)*c
^2*f^3*h)/((d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*((f*x + e)*d - d*e + c*f)^2
)
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.47

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx = \frac{\frac{(e+f\,x)^{3/2}\,(c\,f^2\,h+3\,d\,f^2\,g-4\,d\,e\,f\,h)}{4\,(c\,f-d\,e)^2}-\frac{\sqrt{e+f\,x}\,(c\,f^2\,h-5\,d\,f^2\,g+4\,d\,e\,f\,h)}{4\,d\,(c\,f-d\,e)}}{d^2\,(e+f\,x)^2-(e+f\,x)\,\left(2\,d^2\,e-2\,c\,d\,f\right)+c^2\,f^2+d^2\,e^2-2\,c\,d\,e\,f} + \frac{f\,\mathrm{atan}\left(\frac{\sqrt{d}\,f\,\sqrt{e+f\,x}\,(c\,f\,h-4\,d\,e\,h+3\,d\,f\,g)}{\sqrt{c\,f-d\,e}\,(c\,f^2\,h+3\,d\,f^2\,g-4\,d\,e\,f\,h)}\right)\,(c\,f\,h-4\,d\,e\,h+3\,d\,f\,g)}{4\,d^{3/2}\,(c\,f-d\,e)^{5/2}}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(c + d*x)^3),x)
```

output
```
(((e + f*x)^(3/2)*(c*f^2*h + 3*d*f^2*g - 4*d*e*f*h))/(4*(c*f - d*e)^2) - (
(e + f*x)^(1/2)*(c*f^2*h - 5*d*f^2*g + 4*d*e*f*h))/(4*d*(c*f - d*e)))/(d^2
*(e + f*x)^2 - (e + f*x)*(2*d^2*e - 2*c*d*f) + c^2*f^2 + d^2*e^2 - 2*c*d*e
*f) + (f*atan((d^(1/2)*f*(e + f*x)^(1/2)*(c*f*h - 4*d*e*h + 3*d*f*g))/((c*
f - d*e)^(1/2)*(c*f^2*h + 3*d*f^2*g - 4*d*e*f*h)))*(c*f*h - 4*d*e*h + 3*d*
f*g))/(4*d^(3/2)*(c*f - d*e)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 735, normalized size of antiderivative = 4.74

$$\int \frac{g+hx}{(c+dx)^3\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^3f^2h-4\sqrt{d}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2defh+3\sqrt{d}\,\sqrt{cf-de}\,atan\left(\right.}{}$$

input
```
int((h*x+g)/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*c**3*f**2*h - 4*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*c**2*d*e*f*h + 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*g + 2*sqrt(d)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*h*x - 8*sq
rt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*
d**2*e*f*h*x + 6*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*c*d**2*f**2*g*x + sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d**2*f**2*h*x**2 - 4*sqrt(d)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**3*e*f*h*x**2
+ 3*sqrt(d)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*d**3*f**2*g*x**2 - sqrt(e + f*x)*c**3*d*f**2*h - sqrt(e + f*x)*c**2*d*
*2*e*f*h + 5*sqrt(e + f*x)*c**2*d**2*f**2*g + sqrt(e + f*x)*c**2*d**2*f**2
*h*x + 2*sqrt(e + f*x)*c*d**3*e**2*h - 7*sqrt(e + f*x)*c*d**3*e*f*g - 5*sq
rt(e + f*x)*c*d**3*e*f*h*x + 3*sqrt(e + f*x)*c*d**3*f**2*g*x + 2*sqrt(e +
f*x)*d**4*e**2*g + 4*sqrt(e + f*x)*d**4*e**2*h*x - 3*sqrt(e + f*x)*d**4*e*
f*g*x)/(4*d**2*(c**5*f**3 - 3*c**4*d*e*f**2 + 2*c**4*d*f**3*x + 3*c**3*d**
2*e**2*f - 6*c**3*d**2*e*f**2*x + c**3*d**2*f**3*x**2 - c**2*d**3*e**3 + 6
*c**2*d**3*e**2*f*x - 3*c**2*d**3*e*f**2*x**2 - 2*c*d**4*e**3*x + 3*c*d**4
*e**2*f*x**2 - d**5*e**3*x**2))
```

3.145 $\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 363

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx = \frac{(dg-ch)\sqrt{e+fx}}{2(bc-ad)(de-cf)(c+dx)^2}$$
$$+\frac{(ad(3dfg-4deh+cfh)+b(4d^2eg-7cdfg+3c^2fh))\sqrt{e+fx}}{4(bc-ad)^2(de-cf)^2(c+dx)}$$
$$-\frac{2b^{3/2}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^3\sqrt{be-af}}$$
$$+\frac{(a^2d^2f(3dfg-4deh+cfh)+b^2(8d^3e^2g-20cd^2efg+15c^2df^2g-3c^3f^2h)-2abd(3c^2f^2h-2d^2e(fg}{4\sqrt{d}(bc-ad)^3(de-cf)^{5/2}}$$

output
```
1/2*(-c*h+d*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(d*x+c)^2+1/4*(a*d*(c*f
*h-4*d*e*h+3*d*f*g)+b*(3*c^2*f*h-7*c*d*f*g+4*d^2*e*g))*(f*x+e)^(1/2)/(-a*d
+b*c)^2/(-c*f+d*e)^2/(d*x+c)-2*b^(3/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^
(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^3/(-a*f+b*e)^(1/2)+1/4*(a^2*d^2*f*(c*f*
h-4*d*e*h+3*d*f*g)+b^2*(-3*c^3*f^2*h+15*c^2*d*f^2*g-20*c*d^2*e*f*g+8*d^3*e
^2*g)-2*a*b*d*(3*c^2*f^2*h-2*d^2*e*(-2*e*h+f*g)+5*c*d*f*(-2*e*h+f*g)))*arc
tanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-a*d+b*c)^3/(-c*f+d*
e)^(5/2)
```

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.02

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx$$
$$=\frac{1}{4}\left(\frac{\sqrt{e+fx}(b(5c^3fh+4d^3egx+cd^2g(6e-7fx)+c^2d(-9fg-2eh+3fhx))+ad(-c^2fh+cd(5fg-\right.$$
$$+\frac{8b^{3/2}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{(bc-ad)^3\sqrt{-be+af}}$$
$$+\frac{(a^2d^2f(3dfg-4deh+cfh)+b^2(8d^3e^2g-20cd^2efg+15c^2df^2g-3c^3f^2h)-2abd(3c^2f^2h+5cdf(fg}{\sqrt{d}(-bc+ad)^3(-de+cf)^{5/2}}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
((Sqrt[e + f*x]*(b*(5*c^3*f*h + 4*d^3*e*g*x + c*d^2*g*(6*e - 7*f*x) + c^2*
d*(-9*f*g - 2*e*h + 3*f*h*x)) + a*d*(-(c^2*f*h) + c*d*(5*f*g - 2*e*h + f*h
*x) + d^2*(3*f*g*x - 2*e*(g + 2*h*x)))))/((b*c - a*d)^2*(d*e - c*f)^2*(c +
 d*x)^2) + (8*b^(3/2)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*
e) + a*f]])/((b*c - a*d)^3*Sqrt[-(b*e) + a*f]) + ((a^2*d^2*f*(3*d*f*g - 4*
d*e*h + c*f*h) + b^2*(8*d^3*e^2*g - 20*c*d^2*e*f*g + 15*c^2*d*f^2*g - 3*c^
3*f^2*h) - 2*a*b*d*(3*c^2*f^2*h + 5*c*d*f*(f*g - 2*e*h) + 2*d^2*e*(-(f*g)
+ 2*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sqrt[d]*(-
(b*c) + a*d)^3*(-(d*e) + c*f)^(5/2)))/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {168, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}dx$$

$$\Big\downarrow \ \color{blue}{168}$$

$$\frac{\int \frac{4b(de-cf)g+a(3dfg-4deh+cfh)+3bf(dg-ch)x}{2(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{\int \frac{4b(de-cf)g+a(3dfg-4deh+cfh)+3bf(dg-ch)x}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{4(bc-ad)(de-cf)}+\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

$$\Big\downarrow \ \color{blue}{168}$$

$$\frac{\frac{\int \frac{df(3dfg-4deh+cfh)a^2-b\left(-4e(fg-2eh)d^2+cf(7fg-16eh)d+5c^2f^2h\right)a+8b^2(de-cf)^2g+bf\left(ad(3dfg-4deh+cfh)+b\left(3fhc^2-7dfgc+4d^2eg\right)\right)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{(bc-ad)(de-cf)}+\frac{\sqrt{e+f}}{}}{4(bc-ad)(de-cf)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{\frac{\int \frac{df(3dfg-4deh+cfh)a^2-b\left(-4e(fg-2eh)d^2+cf(7fg-16eh)d+5c^2f^2h\right)a+8b^2(de-cf)^2g+bf\left(ad(3dfg-4deh+cfh)+b\left(3fhc^2-7dfgc+4d^2eg\right)\right)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}+\frac{\sqrt{e+f}}{}}{4(bc-ad)(de-cf)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

$$\Big\downarrow \ \color{blue}{174}$$

$$\frac{\frac{\frac{8b^2(bg-ah)(de-cf)^2\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{\left(a^2d^2f(cfh-4deh+3dfg)-2abd\left(3c^2f^2h+5cdf(fg-2eh)-2d^2e(fg-2eh)\right)+b^2\left(-3c^3f^2h+15c^2df^2g-20cd^2efg+8d^3\right.}{bc-ad}}{2(bc-ad)(de-cf)}}{4(bc-ad)(de-cf)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

$$\Big\downarrow \ \color{blue}{73}$$

$$\frac{\frac{\frac{16b^2(bg-ah)(de-cf)^2\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2\left(a^2d^2f(cfh-4deh+3dfg)-2abd\left(3c^2f^2h+5cdf(fg-2eh)-2d^2e(fg-2eh)\right)+b^2\left(-3c^3f^2h+15c^2df^2g-20cd\right.\right.}{f(bc-ad)}}{2(bc-ad)(de-cf)}}{4(bc-ad)(de-cf)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(a^2d^2f(cfh-4deh+3dfg)-2abd\left(3c^2f^2h+5cdf(fg-2eh)-2d^2e(fg-2eh)\right)+b^2\left(-3c^3f^2h+15c^2df^2g-20cd^2efg+8d^3e^2g\right)\right)}{\sqrt{d}(bc-ad)\sqrt{de-cf}}-\frac{16b^{3/2}(b}{}}{2(bc-ad)(de-cf)}}{4(bc-ad)(de-cf)}$$

$$\frac{\sqrt{e+fx}(dg-ch)}{2(c+dx)^2(bc-ad)(de-cf)}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
((d*g - c*h)*Sqrt[e + f*x])/(2*(b*c - a*d)*(d*e - c*f)*(c + d*x)^2) + (((a
*d*(3*d*f*g - 4*d*e*h + c*f*h) + b*(4*d^2*e*g - 7*c*d*f*g + 3*c^2*f*h))*Sq
rt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)) + ((-16*b^(3/2)*(d*e - c*
f)^2*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c -
 a*d)*Sqrt[b*e - a*f]) + (2*(a^2*d^2*f*(3*d*f*g - 4*d*e*h + c*f*h) + b^2*(
8*d^3*e^2*g - 20*c*d^2*e*f*g + 15*c^2*d*f^2*g - 3*c^3*f^2*h) - 2*a*b*d*(3*
c^2*f^2*h - 2*d^2*e*(f*g - 2*e*h) + 5*c*d*f*(f*g - 2*e*h)))*ArcTanh[(Sqrt[
d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(b*c - a*d)*Sqrt[d*e - c*f]))
/(2*(b*c - a*d)*(d*e - c*f)))/(4*(b*c - a*d)*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{\left(-\left(\left(\left(7cgx\,d^{2}+9c^{2}\left(-\frac{hx}{3}+g\right)d-5c^{3}h\right)b+ad\left(-3d^{2}gx-5c\left(\frac{hx}{5}+g\right)d+h\,c^{2}\right)\right)f+2d\left(\left(-2d^{2}gx+h\,c^{2}-3cdg\right)b+a\left((2hx\cdots\right.\right.\right.}{\cdots}$
derivativedivides	$2f^{2}\left(-\frac{-\frac{fd\left(a^{2}c\,d^{2}fh-4a^{2}d^{3}eh+3a^{2}d^{3}fg+2ab\,c^{2}dfh+4abc\,d^{2}eh-10abc\,d^{2}fg+4ab\,d^{3}eg-3b^{2}c^{3}fh+7b^{2}c^{2}dfg-4b^{2}c\,d^{2}eg\right)(fx\cdots}{8\left(c^{2}f^{2}-2cdef+d^{2}e^{2}\right)}}{((fx\cdots}\right.$
default	$2f^{2}\left(-\frac{-\frac{fd\left(a^{2}c\,d^{2}fh-4a^{2}d^{3}eh+3a^{2}d^{3}fg+2ab\,c^{2}dfh+4abc\,d^{2}eh-10abc\,d^{2}fg+4ab\,d^{3}eg-3b^{2}c^{3}fh+7b^{2}c^{2}dfg-4b^{2}c\,d^{2}eg\right)(fx\cdots}{8\left(c^{2}f^{2}-2cdef+d^{2}e^{2}\right)}}{((fx\cdots}\right.$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*f-b*e)*b)^(1/2)/((c*f-d*e)*d)^(1/2)*((-(((7*c*g*x*d^2+9*c^2*(-1/3*
h*x+g)*d-5*c^3*h)*b+a*d*(-3*d^2*g*x-5*c*(1/5*h*x+g)*d+h*c^2))*f+2*d*((-2*d
^2*g*x+c^2*h-3*c*d*g)*b+a*((2*h*x+g)*d+c*h)*d)*e)*(a*d-b*c)*((c*f-d*e)*d)^
(1/2)*(f*x+e)^(1/2)+arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))*(d*x+c)^2*
(((-3*c^3*h+15*c^2*d*g)*b^2-6*a*c*d*(c*h+5/3*d*g)*b+a^2*d^2*(c*h+3*d*g))*f
^2-4*d^2*e*(a*h-b*g)*(a*d-5*b*c)*f-8*b*d^3*e^2*(a*h-b*g)))*((a*f-b*e)*b)^(
1/2)+8*b^2*(d*x+c)^2*(c*f-d*e)^2*(a*h-b*g)*arctan(b*(f*x+e)^(1/2)/((a*f-b*
e)*b)^(1/2))*((c*f-d*e)*d)^(1/2))/(c*f-d*e)^2/(d*x+c)^2/(a*d-b*c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. $2(333) = 666$.

Time = 172.56 (sec) , antiderivative size = 7703, normalized size of antiderivative = 21.22

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(333) = 666$.

Time = 0.16 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.30

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
2*(b^3*g - a*b^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*e + a*b*f)) - 1/4*(8
*b^2*d^3*e^2*g - 20*b^2*c*d^2*e*f*g + 4*a*b*d^3*e*f*g + 15*b^2*c^2*d*f^2*g
 - 10*a*b*c*d^2*f^2*g + 3*a^2*d^3*f^2*g - 8*a*b*d^3*e^2*h + 20*a*b*c*d^2*e
*f*h - 4*a^2*d^3*e*f*h - 3*b^2*c^3*f^2*h - 6*a*b*c^2*d*f^2*h + a^2*c*d^2*f
^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^3*d^2*e^2 - 3*a
*b^2*c^2*d^3*e^2 + 3*a^2*b*c*d^4*e^2 - a^3*d^5*e^2 - 2*b^3*c^4*d*e*f + 6*a
*b^2*c^3*d^2*e*f - 6*a^2*b*c^2*d^3*e*f + 2*a^3*c*d^4*e*f + b^3*c^5*f^2 - 3
*a*b^2*c^4*d*f^2 + 3*a^2*b*c^3*d^2*f^2 - a^3*c^2*d^3*f^2)*sqrt(-d^2*e + c*
d*f)) + 1/4*(4*(f*x + e)^(3/2)*b*d^3*e*f*g - 4*sqrt(f*x + e)*b*d^3*e^2*f*g
 - 7*(f*x + e)^(3/2)*b*c*d^2*f^2*g + 3*(f*x + e)^(3/2)*a*d^3*f^2*g + 13*sq
rt(f*x + e)*b*c*d^2*e*f^2*g - 5*sqrt(f*x + e)*a*d^3*e*f^2*g - 9*sqrt(f*x +
 e)*b*c^2*d*f^3*g + 5*sqrt(f*x + e)*a*c*d^2*f^3*g - 4*(f*x + e)^(3/2)*a*d^
3*e*f*h + 4*sqrt(f*x + e)*a*d^3*e^2*f*h + 3*(f*x + e)^(3/2)*b*c^2*d*f^2*h
+ (f*x + e)^(3/2)*a*c*d^2*f^2*h - 5*sqrt(f*x + e)*b*c^2*d*e*f^2*h - 3*sqrt
(f*x + e)*a*c*d^2*e*f^2*h + 5*sqrt(f*x + e)*b*c^3*f^3*h - sqrt(f*x + e)*a*
c^2*d*f^3*h)/((b^2*c^2*d^2*e^2 - 2*a*b*c*d^3*e^2 + a^2*d^4*e^2 - 2*b^2*c^3
*d*e*f + 4*a*b*c^2*d^2*e*f - 2*a^2*c*d^3*e*f + b^2*c^4*f^2 - 2*a*b*c^3*d*f
^2 + a^2*c^2*d^2*f^2)*((f*x + e)*d - d*e + c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 351830, normalized size of antiderivative = 969.23

$$\int \frac{g+hx}{(a+bx)(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^3),x)
```

output
```
(((e + f*x)^(1/2)*(5*a*d^2*f^2*g + 5*b*c^2*f^2*h - a*c*d*f^2*h - 9*b*c*d*f
^2*g - 4*a*d^2*e*f*h + 4*b*d^2*e*f*g))/(4*(c*f - d*e)*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)) + (d*(e + f*x)^(3/2)*(3*a*d^2*f^2*g + 3*b*c^2*f^2*h + a*c*d*f
^2*h - 7*b*c*d*f^2*g - 4*a*d^2*e*f*h + 4*b*d^2*e*f*g))/(4*(c*f - d*e)^2*(a
^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(d^2*(e + f*x)^2 - (e + f*x)*(2*d^2*e - 2*
c*d*f) + c^2*f^2 + d^2*e^2 - 2*c*d*e*f) - atan(((((64*b^10*c^10*d^2*f^7*g
- 440*a*b^9*c^9*d^3*f^7*g - 40*a*b^9*c^10*d^2*f^7*h - 200*b^10*c^9*d^3*e*f
^6*g - 24*b^10*c^10*d^2*e*f^6*h + 1320*a^2*b^8*c^8*d^4*f^7*g - 2264*a^3*b^
7*c^7*d^5*f^7*g + 2440*a^4*b^6*c^6*d^6*f^7*g - 1704*a^5*b^5*c^5*d^7*f^7*g
+ 760*a^6*b^4*c^4*d^8*f^7*g - 200*a^7*b^3*c^3*d^9*f^7*g + 24*a^8*b^2*c^2*d
^10*f^7*g + 248*a^2*b^8*c^9*d^3*f^7*h - 648*a^3*b^7*c^8*d^4*f^7*h + 920*a^
4*b^6*c^7*d^5*f^7*h - 760*a^5*b^5*c^6*d^6*f^7*h + 360*a^6*b^4*c^5*d^7*f^7*
h - 88*a^7*b^3*c^4*d^8*f^7*h + 8*a^8*b^2*c^3*d^9*f^7*h + 32*a^6*b^4*d^12*e
^4*f^3*g + 8*a^7*b^3*d^12*e^3*f^4*g + 24*a^8*b^2*d^12*e^2*f^5*g - 32*a^7*b
^3*d^12*e^4*f^3*h - 32*a^8*b^2*d^12*e^3*f^4*h + 32*b^10*c^6*d^6*e^4*f^3*g
- 136*b^10*c^7*d^5*e^3*f^4*g + 240*b^10*c^8*d^4*e^2*f^5*g - 24*b^10*c^8*d^
4*e^3*f^4*h + 48*b^10*c^9*d^3*e^2*f^5*h + 480*a^2*b^8*c^4*d^8*e^4*f^3*g -
2088*a^2*b^8*c^5*d^7*e^3*f^4*g + 4056*a^2*b^8*c^6*d^6*e^2*f^5*g - 640*a^3*
b^7*c^3*d^9*e^4*f^3*g + 2840*a^3*b^7*c^4*d^8*e^3*f^4*g - 6024*a^3*b^7*c^5*
d^7*e^2*f^5*g + 480*a^4*b^6*c^2*d^10*e^4*f^3*g - 2200*a^4*b^6*c^3*d^9*e...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6278, normalized size of antiderivative = 17.29

$$\int \frac{g + hx}{(a + bx)(c + dx)^3\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output

```
(8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a*b*c**5*d*f**3*h - 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a*b*c**4*d**2*e*f**2*h + 16*sqrt(b)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**4*d**2*f**3*h
*x + 24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a*b*c**3*d**3*e**2*f*h - 48*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**3*d**3*e*f**2*h*x + 8*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**3*
d**3*f**3*h*x**2 - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a*b*c**2*d**4*e**3*h + 48*sqrt(b)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**2*d**4*e**2*f*h*x -
24*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a*b*c**2*d**4*e*f**2*h*x**2 - 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*d**5*e**3*h*x + 24*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*d**5*e*
*2*f*h*x**2 - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a*b*d**6*e**3*h*x**2 - 8*sqrt(b)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**5*d*f**3*g + 24*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**4
*d**2*e*f**2*g - 16*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr...
```

3.146 $\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 582

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx$$
$$= -\frac{d(3bdeg-bc(2fg+eh)-a(dfg+2deh-3cfh))\sqrt{e+fx}}{2(bc-ad)^2(be-af)(de-cf)(c+dx)^2}$$
$$-\frac{(bg-ah)\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)(c+dx)^2}$$
$$+\frac{d(a^2df(3dfg-4deh+cfh)-b^2(12d^2e^2g-cde(19fg+4eh)+c^2f(4fg+7eh))+ab(11c^2f^2h+d^2e(}{4(bc-ad)^3(be-af)(de-cf)^2(c+dx)}$$
$$+\frac{b^{3/2}(5a^2dfh+b^2(6deg+cfg-2ceh)-ab(7dfg+4deh-cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^4(be-af)^{3/2}}$$
$$-\frac{\sqrt{d}(a^2d^2f(3dfg-4deh+cfh)-2abd(5c^2f^2h+cdf(7fg-16eh)-4d^2e(fg-2eh))+b^2(24d^3e^2g-}{4(bc-ad)^4(de-cf)^{5/2}}$$

output
```
-1/2*d*(3*b*d*e*g-b*c*(e*h+2*f*g)-a*(-3*c*f*h+2*d*e*h+d*f*g))*(f*x+e)^(1/2
)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x+c)^2-(-a*h+b*g)*(f*x+e)^(1/2)/(-
a*d+b*c)/(-a*f+b*e)/(b*x+a)/(d*x+c)^2+1/4*d*(a^2*d*f*(c*f*h-4*d*e*h+3*d*f*
g)-b^2*(12*d^2*e^2*g-c*d*e*(4*e*h+19*f*g)+c^2*f*(7*e*h+4*f*g))+a*b*(11*c^2
*f^2*h+d^2*e*(8*e*h+5*f*g)-c*d*f*(13*e*h+11*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c
)^3/(-a*f+b*e)/(-c*f+d*e)^2/(d*x+c)+b^(3/2)*(5*a^2*d*f*h+b^2*(-2*c*e*h+c*f
*g+6*d*e*g)-a*b*(-c*f*h+4*d*e*h+7*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-
a*f+b*e)^(1/2))/(-a*d+b*c)^4/(-a*f+b*e)^(3/2)-1/4*d^(1/2)*(a^2*d^2*f*(c*f*
h-4*d*e*h+3*d*f*g)-2*a*b*d*(5*c^2*f^2*h+c*d*f*(-16*e*h+7*f*g)-4*d^2*e*(-2*
e*h+f*g))+b^2*(24*d^3*e^2*g-15*c^3*f^2*h-8*c*d^2*e*(e*h+7*f*g)+5*c^2*d*f*(
4*e*h+7*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^
4/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 12.23 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.25

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx$$

$$= \frac{(bc-ad)\sqrt{e+fx}\left(a^3d^2f\left(-c^2fh+cd(5fg-2eh+fhx)+d^2(3fgx-2e(g+2hx))\right)+a^2bd\left(9c^3f^2h+c^2df(-13fg-5eh+6fhx)+cd^2\left(2e^2h+ef(5g-7hx\right.\right.\right.}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^2*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
(((b*c - a*d)*Sqrt[e + f*x]*(a^3*d^2*f*(-(c^2*f*h) + c*d*(5*f*g - 2*e*h +
f*h*x) + d^2*(3*f*g*x - 2*e*(g + 2*h*x))) + a^2*b*d*(9*c^3*f^2*h + c^2*d*f
*(-13*f*g - 5*e*h + 6*f*h*x) + c*d^2*(2*e^2*h + e*f*(5*g - 7*h*x) + f^2*x*
(-6*g + h*x)) + d^3*(3*f^2*g*x^2 + e*f*x*(3*g - 4*h*x) + 2*e^2*(g + 2*h*x)
)) - b^3*(4*c^4*f^2*g + 12*d^4*e^2*g*x^2 + c*d^3*e*x*(18*e*g - 19*f*g*x -
4*e*h*x) + c^3*d*f*(-8*e*g + 8*f*g*x + 9*e*h*x) + c^2*d^2*(4*f^2*g*x^2 + e
^2*(4*g - 6*h*x) + e*f*x*(-29*g + 7*h*x))) + a*b^2*(4*c^4*f^2*h + 17*c^3*d
*f*h*(-e + f*x) + d^4*e*x*(-6*e*g + 5*f*g*x + 8*e*h*x) + c*d^3*(-11*f^2*g*
x^2 + e*f*x*(16*g - 13*h*x) - 2*e^2*(5*g - 7*h*x)) + c^2*d^2*(10*e^2*h + e
*f*(13*g - 28*h*x) + f^2*x*(-13*g + 11*h*x)))))/((b*e - a*f)*(d*e - c*f)^2
*(a + b*x)*(c + d*x)^2) + (4*b^(3/2)*(5*a^2*d*f*h + b^2*(6*d*e*g + c*f*g -
 2*c*e*h) + a*b*(-7*d*f*g - 4*d*e*h + c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x
])/Sqrt[-(b*e) + a*f]])/(-(b*e) + a*f)^(3/2) + (Sqrt[d]*(a^2*d^2*f*(3*d*f*
g - 4*d*e*h + c*f*h) - 2*a*b*d*(5*c^2*f^2*h + c*d*f*(7*f*g - 16*e*h) + 4*d
^2*e*(-(f*g) + 2*e*h)) + b^2*(24*d^3*e^2*g - 15*c^3*f^2*h - 8*c*d^2*e*(7*f
*g + e*h) + 5*c^2*d*f*(7*f*g + 4*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqr
t[-(d*e) + c*f]])/(-(d*e) + c*f)^(5/2))/(4*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {168, 27, 168, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{acfh-2ad(fg+2eh)+b(6deg+cfg-2ceh)+5df(bg-ah)x}{2(a+bx)(c+dx)^3\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{acfh-2ad(fg+2eh)+b(6deg+cfg-2ceh)+5df(bg-ah)x}{(a+bx)(c+dx)^3\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{\frac{\int \frac{-df(3dfg-4deh+cfh)a^2-b\left(e(5fg+8eh)d^2-cf(8fg+7eh)d+2c^2f^2h\right)a+2b^2(de-cf)(6deg+cfg-2ceh)+3bdf(3bdeg-bc(2fg+eh)-a(dfg+2deh-3cfh))x}{(a+bx)(c+dx)^2\sqrt{e+fx}}dx}{2(bc-ad)(de-cf)}}{2(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 168

$$-\frac{\frac{\int \frac{-d^2f^2(3dfg-4deh+cfh)a^3+bdf\left(-e(5fg-12eh)d^2+cf(11fg-27eh)d+9c^2f^2h\right)a^2+b^2\left(-16e^2(fg+eh)d^3+cef(37fg+32eh)d^2-c^2f^2(24fg+17eh)d+4c^3f^3h\right. \ldots}{2(a+b \ldots}}{(bc \ldots}}{\ldots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$-\frac{\frac{\int \frac{-d^2f^2(3dfg-4deh+cfh)a^3+bdf\left(-e(5fg-12eh)d^2+cf(11fg-27eh)d+9c^2f^2h\right)a^2+b^2\left(-16e^2(fg+eh)d^3+cef(37fg+32eh)d^2-c^2f^2(24fg+17eh)d+4c^3f^3h\right. \ldots}{(a+bx \ldots}}{2(b \ldots}}{\ldots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 174

$$-\frac{\frac{\frac{4b^2(de-cf)^2\left(5a^2dfh-ab(-cfh+4deh+7dfg)+b^2(-2ceh+cfg+6deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{d(be-af)\left(a^2d^2f(cfh-4deh+3dfg)-2abd\left(5c^2f^2h+cdf(7fg-16 \ldots\right.\right.}{\ldots}}{2(bc-ad)(de-cf)}}{\ldots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 73

$$-\frac{\frac{\frac{8b^2(de-cf)^2\left(5a^2dfh-ab(-cfh+4deh+7dfg)+b^2(-2ceh+cfg+6deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2d(be-af)\left(a^2d^2f(cfh-4deh+3dfg)-2abd\left(5c^2f^2h+cdf \ldots\right.\right.}{\ldots}}{2(bc-ad)(de-cf)}}{\ldots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 221

$$\frac{\frac{\frac{2\sqrt{d}(be-af)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(a^2d^2f(cfh-4deh+3dfg)-2abd\left(5c^2f^2h+cdf(7fg-16eh)-4d^2e(fg-2eh)\right)+b^2\left(-15c^3f^2h+5c^2df(4eh+7fg)-8cd^2e(eh\right.\right.}{(bc-ad)\sqrt{de-cf}}}{2(bc-ad)(de-cf)}}{} - \frac{\sqrt{e+fx}(bg-ah)}{(a+bx)(c+dx)^2(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^2*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-(((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)
^2)) - ((d*(b*(3*d*e*g - 2*c*f*g - c*e*h) - a*(d*f*g + 2*d*e*h - 3*c*f*h))
*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)^2) + (-((d*(a^2*d*f*(3*
d*f*g - 4*d*e*h + c*f*h) - b^2*(12*d^2*e^2*g - c*d*e*(19*f*g + 4*e*h) + c^
2*f*(4*f*g + 7*e*h)) + a*b*(11*c^2*f^2*h + d^2*e*(5*f*g + 8*e*h) - c*d*f*(
11*f*g + 13*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x))) + (
(-8*b^(3/2)*(d*e - c*f)^2*(5*a^2*d*f*h + b^2*(6*d*e*g + c*f*g - 2*c*e*h) -
 a*b*(7*d*f*g + 4*d*e*h - c*f*h))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e
 - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (2*Sqrt[d]*(b*e - a*f)*(a^2*d^2*
f*(3*d*f*g - 4*d*e*h + c*f*h) - 2*a*b*d*(5*c^2*f^2*h + c*d*f*(7*f*g - 16*e
*h) - 4*d^2*e*(f*g - 2*e*h)) + b^2*(24*d^3*e^2*g - 15*c^3*f^2*h - 8*c*d^2*
e*(7*f*g + e*h) + 5*c^2*d*f*(7*f*g + 4*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*
x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/(2*(b*c - a*d)*(d*e -
 c*f)))/(2*(b*c - a*d)*(d*e - c*f)))/(2*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.21

method	result
derivativedivides	$2f^3\left(-\frac{d\left(-\frac{df\left(a^2c\,d^2fh-4a^2d^3eh+3a^2d^3fg+6ab\,c^2dfh-14abc\,d^2fg+8ab\,d^3eg-7b^2c^3fh+4b^2c^2deh+11b^2c^2dfg-8b^2c\,d^2eg\right.}{8\left(c^2f^2-2cdef+d^2e^2\right)}\right.}{}\right.$
default	$2f^3\left(-\frac{d\left(-\frac{df\left(a^2c\,d^2fh-4a^2d^3eh+3a^2d^3fg+6ab\,c^2dfh-14abc\,d^2fg+8ab\,d^3eg-7b^2c^3fh+4b^2c^2deh+11b^2c^2dfg-8b^2c\,d^2eg\right.}{8\left(c^2f^2-2cdef+d^2e^2\right)}\right.}{}\right.$
pseudoelliptic	$-\frac{-\sqrt{(af-be)b}\,\left(\left(\left(24b^2e^2g+8a\left(-2e^2h+efg\right)b+\left(-4efh+3f^2g\right)a^2\right)d^3+c\left(8\left(-e^2h-7efg\right)b^2+2\left(16efh-7f^2g\right)ab+a^2f\right.\right.\right.}{}$

input

```
int((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*f^3*(-d/f^3/(a*d-b*c)^4*((-1/8*d*f*(a^2*c*d^2*f*h-4*a^2*d^3*e*h+3*a^2*d^
3*f*g+6*a*b*c^2*d*f*h-14*a*b*c*d^2*f*g+8*a*b*d^3*e*g-7*b^2*c^3*f*h+4*b^2*c
^2*d*e*h+11*b^2*c^2*d*f*g-8*b^2*c*d^2*e*g)/(c^2*f^2-2*c*d*e*f+d^2*e^2)*(f*
x+e)^(3/2)+1/8*(a^2*c*d^2*f*h+4*a^2*d^3*e*h-5*a^2*d^3*f*g-10*a*b*c^2*d*f*h
+18*a*b*c*d^2*f*g-8*a*b*d^3*e*g+9*b^2*c^3*f*h-4*b^2*c^2*d*e*h-13*b^2*c^2*d
*f*g+8*b^2*c*d^2*e*g)*f/(c*f-d*e)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2-1/8
*(a^2*c*d^2*f^2*h-4*a^2*d^3*e*f*h+3*a^2*d^3*f^2*g-10*a*b*c^2*d*f^2*h+32*a*
b*c*d^2*e*f*h-14*a*b*c*d^2*f^2*g-16*a*b*d^3*e^2*h+8*a*b*d^3*e*f*g-15*b^2*c
^3*f^2*h+20*b^2*c^2*d*e*f*h+35*b^2*c^2*d*f^2*g-8*b^2*c*d^2*e^2*h-56*b^2*c*
d^2*e*f*g+24*b^2*d^3*e^2*g)/(c^2*f^2-2*c*d*e*f+d^2*e^2)/((c*f-d*e)*d)^(1/2
)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))+b^2/f^3/(a*d-b*c)^4*(1/2*f*
(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/(a*f-b*e)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b
*e)+1/2*(5*a^2*d*f*h+a*b*c*f*h-4*a*b*d*e*h-7*a*b*d*f*g-2*b^2*c*e*h+b^2*c*f
*g+6*b^2*d*e*g)/(a*f-b*e)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f
-b*e)*b)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**2/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1274 vs. $2(550) = 1100$.

Time = 0.19 (sec) , antiderivative size = 1274, normalized size of antiderivative = 2.19

$$\int \frac{g + hx}{(a + bx)^2 (c + dx)^3 \sqrt{e + fx}} \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-(6*b^4*d*e*g + b^4*c*f*g - 7*a*b^3*d*f*g - 2*b^4*c*e*h - 4*a*b^3*d*e*h +
a*b^3*c*f*h + 5*a^2*b^2*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f)
)/((b^5*c^4*e - 4*a*b^4*c^3*d*e + 6*a^2*b^3*c^2*d^2*e - 4*a^3*b^2*c*d^3*e
+ a^4*b*d^4*e - a*b^4*c^4*f + 4*a^2*b^3*c^3*d*f - 6*a^3*b^2*c^2*d^2*f + 4*
a^4*b*c*d^3*f - a^5*d^4*f)*sqrt(-b^2*e + a*b*f)) + 1/4*(24*b^2*d^4*e^2*g -
 56*b^2*c*d^3*e*f*g + 8*a*b*d^4*e*f*g + 35*b^2*c^2*d^2*f^2*g - 14*a*b*c*d^
3*f^2*g + 3*a^2*d^4*f^2*g - 8*b^2*c*d^3*e^2*h - 16*a*b*d^4*e^2*h + 20*b^2*
c^2*d^2*e*f*h + 32*a*b*c*d^3*e*f*h - 4*a^2*d^4*e*f*h - 15*b^2*c^3*d*f^2*h
- 10*a*b*c^2*d^2*f^2*h + a^2*c*d^3*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2
*e + c*d*f))/((b^4*c^4*d^2*e^2 - 4*a*b^3*c^3*d^3*e^2 + 6*a^2*b^2*c^2*d^4*e
^2 - 4*a^3*b*c*d^5*e^2 + a^4*d^6*e^2 - 2*b^4*c^5*d*e*f + 8*a*b^3*c^4*d^2*e
*f - 12*a^2*b^2*c^3*d^3*e*f + 8*a^3*b*c^2*d^4*e*f - 2*a^4*c*d^5*e*f + b^4*
c^6*f^2 - 4*a*b^3*c^5*d*f^2 + 6*a^2*b^2*c^4*d^2*f^2 - 4*a^3*b*c^3*d^3*f^2
+ a^4*c^2*d^4*f^2)*sqrt(-d^2*e + c*d*f)) - (sqrt(f*x + e)*b^3*f*g - sqrt(f
*x + e)*a*b^2*f*h)/((b^4*c^3*e - 3*a*b^3*c^2*d*e + 3*a^2*b^2*c*d^2*e - a^3
*b*d^3*e - a*b^3*c^3*f + 3*a^2*b^2*c^2*d*f - 3*a^3*b*c*d^2*f + a^4*d^3*f)*
((f*x + e)*b - b*e + a*f)) - 1/4*(8*(f*x + e)^(3/2)*b*d^4*e*f*g - 8*sqrt(f
*x + e)*b*d^4*e^2*f*g - 11*(f*x + e)^(3/2)*b*c*d^3*f^2*g + 3*(f*x + e)^(3/
2)*a*d^4*f^2*g + 21*sqrt(f*x + e)*b*c*d^3*e*f^2*g - 5*sqrt(f*x + e)*a*d^4*
e*f^2*g - 13*sqrt(f*x + e)*b*c^2*d^2*f^3*g + 5*sqrt(f*x + e)*a*c*d^3*f^...
```

Mupad [B] (verification not implemented)

Time = 33.23 (sec) , antiderivative size = 691757, normalized size of antiderivative = 1188.59

$$\int \frac{g + hx}{(a + bx)^2(c + dx)^3\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^3),x)
```

output
```
- (((e + f*x)^(1/2)*(4*b^3*c^3*f^4*g - 5*a^3*d^3*f^4*g - 4*a*b^2*c^3*f^4*h
 + a^3*c*d^2*f^4*h + 4*a^3*d^3*e*f^3*h - 12*b^3*d^3*e^3*f*g + 11*a*b^2*d^3
*e^2*f^2*g - 8*a^2*b*d^3*e^2*f^2*h + 25*b^3*c*d^2*e^2*f^2*g - 9*b^3*c^2*d*
e^2*f^2*h + 13*a^2*b*c*d^2*f^4*g - 9*a^2*b*c^2*d*f^4*h + 2*a^2*b*d^3*e*f^3
*g + 8*a*b^2*d^3*e^3*f*h - 12*b^3*c^2*d*e*f^3*g + 4*b^3*c*d^2*e^3*f*h - 26
*a*b^2*c*d^2*e*f^3*g + 30*a*b^2*c^2*d*e*f^3*h + 2*a^2*b*c*d^2*e*f^3*h - 19
*a*b^2*c*d^2*e^2*f^2*h))/(4*(a*d - b*c)*(a*f - b*e)*(c*f - d*e)*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)) - (f*(e + f*x)^(5/2)*(8*a*b^2*d^4*e^2*h - 12*b^3*d^
4*e^2*g + 3*a^2*b*d^4*f^2*g + 4*b^3*c*d^3*e^2*h - 4*b^3*c^2*d^2*f^2*g + 11
*a*b^2*c^2*d^2*f^2*h + 5*a*b^2*d^4*e*f*g - 4*a^2*b*d^4*e*f*h + 19*b^3*c*d^
3*e*f*g - 11*a*b^2*c*d^3*f^2*g + a^2*b*c*d^3*f^2*h - 7*b^3*c^2*d^2*e*f*h -
 13*a*b^2*c*d^3*e*f*h))/(4*(a*f - b*e)*(c*f - d*e)^2*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*(e + f*x)^(3/2)*(8*b^3*c^3*f^4*g - 3*
a^3*d^3*f^4*g - 17*a*b^2*c^3*f^4*h - a^3*c*d^2*f^4*h + 4*a^3*d^3*e*f^3*h +
 9*b^3*c^3*e*f^3*h - 24*b^3*d^3*e^3*f*g + 16*a*b^2*d^3*e^2*f^2*g - 12*a^2*
b*d^3*e^2*f^2*h + 56*b^3*c*d^2*e^2*f^2*g - 20*b^3*c^2*d*e^2*f^2*h + 13*a*b
^2*c^2*d*f^4*g + 6*a^2*b*c*d^2*f^4*g - 6*a^2*b*c^2*d*f^4*h + 3*a^2*b*d^3*e
*f^3*g + 16*a*b^2*d^3*e^3*f*h - 37*b^3*c^2*d*e*f^3*g + 8*b^3*c*d^2*e^3*f*h
 - 38*a*b^2*c*d^2*e*f^3*g + 50*a*b^2*c^2*d*e*f^3*h + 9*a^2*b*c*d^2*e*f^3*h
 - 40*a*b^2*c*d^2*e^2*f^2*h))/(4*(a*d - b*c)*(c*f - d*e)*(a^2*d^2 + b^2...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18732, normalized size of antiderivative = 32.19

$$\int \frac{g + hx}{(a + bx)^2(c + dx)^3\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
(20*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**3*b*c**5*d*f**4*h - 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**4*d**2*e*f**3*h + 40*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**4*d
**2*f**4*h*x + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**3*b*c**3*d**3*e**2*f**2*h - 120*sqrt(b)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**3*d**3*e*f*
*3*h*x + 20*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**3*b*c**3*d**3*f**4*h*x**2 - 20*sqrt(b)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**2*d**4*e**3*f*h +
120*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**3*b*c**2*d**4*e**2*f**2*h*x - 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**2*d**4*e*f**3*h*x**2 - 4
0*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**3*b*c*d**5*e**3*f*h*x + 60*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d**5*e**2*f**2*h*x**2 - 20*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b
*d**6*e**3*f*h*x**2 + 4*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**6*f**4*h - 28*sqrt(b)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**5*d*e*f...
```

3.147 $\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 922

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx$$

$$= \frac{d(a^2df(2dfg+9deh-11cfh)+b^2(12d^2e^2g-c^2f(3fg-4eh)-cde(7fg+6eh))-ab(c^2f^2h+d^2e(17f}{4(bc-ad)^3(be-af)^2(de-cf)(c+dx)^2}$$

$$-\frac{(bg-ah)\sqrt{e+fx}}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}$$

$$+\frac{(7a^2dfh+b^2(8deg+3cfg-4ceh)-ab(11dfg+4deh-cfh))\sqrt{e+fx}}{4(bc-ad)^2(be-af)^2(a+bx)(c+dx)^2}$$

$$+\frac{d(a^3d^2f^2(3dfg-4deh+cfh)+b^3(24d^3e^3g+c^3f^2(3fg-4eh)-12cd^2e^2(3fg+eh)+c^2def(6fg+}{}$$

$$+\frac{b^{3/2}(35a^3d^2f^2h-7a^2bdf(9dfg+8deh-2cfh)-b^3(48d^2e^2g+c^2f(3fg-4eh)+12cde(fg-2eh))-}{4(bc-ad)^5(be-af)^{5/2}}$$

$$+\frac{d^{3/2}(a^2d^2f(3dfg-4deh+cfh)-2abd(7c^2f^2h+cdf(9fg-22eh)-6d^2e(fg-2eh))+b^2(48d^3e^2g-}{4(bc-ad)^5(de-cf)^{5/2}}$$

output
```
1/4*d*(a^2*d*f*(-11*c*f*h+9*d*e*h+2*d*f*g)+b^2*(12*d^2*e^2*g-c^2*f*(-4*e*h
+3*f*g)-c*d*e*(6*e*h+7*f*g))-a*b*(c^2*f^2*h+d^2*e*(6*e*h+17*f*g)-c*d*f*(11
*e*h+13*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x+c)^
2-1/2*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2/(d*x+c)^2+1
/4*(7*a^2*d*f*h+b^2*(-4*c*e*h+3*c*f*g+8*d*e*g)-a*b*(-c*f*h+4*d*e*h+11*d*f*
g))*(f*x+e)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)/(d*x+c)^2+1/4*d*(a^3*d
^2*f^2*(c*f*h-4*d*e*h+3*d*f*g)+b^3*(24*d^3*e^3*g+c^3*f^2*(-4*e*h+3*f*g)-12
*c*d^2*e^2*(e*h+3*f*g)+c^2*d*e*f*(19*e*h+6*f*g))+a^2*b*d*f*(22*c^2*f^2*h+d
^2*e*(19*e*h+6*f*g)-c*d*f*(32*e*h+15*f*g))+a*b^2*(c^3*f^3*h-12*d^3*e^2*(e*
h+3*f*g)+2*c*d^2*e*f*(17*e*h+30*f*g)-c^2*d*f^2*(32*e*h+15*f*g)))*(f*x+e)^(
1/2)/(-a*d+b*c)^4/(-a*f+b*e)^2/(-c*f+d*e)^2/(d*x+c)+1/4*b^(3/2)*(35*a^3*d^
2*f^2*h-7*a^2*b*d*f*(-2*c*f*h+8*d*e*h+9*d*f*g)-b^3*(48*d^2*e^2*g+c^2*f*(-4
*e*h+3*f*g)+12*c*d*e*(-2*e*h+f*g))-a*b^2*(c^2*f^2*h-2*c*d*f*(-22*e*h+9*f*g
)-12*d^2*e*(2*e*h+9*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))
/(-a*d+b*c)^5/(-a*f+b*e)^(5/2)+1/4*d^(3/2)*(a^2*d^2*f*(c*f*h-4*d*e*h+3*d*f
*g)-2*a*b*d*(7*c^2*f^2*h+c*d*f*(-22*e*h+9*f*g)-6*d^2*e*(-2*e*h+f*g))+b^2*(
48*d^3*e^2*g-35*c^3*f^2*h-12*c*d^2*e*(2*e*h+9*f*g)+7*c^2*d*f*(8*e*h+9*f*g)
))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^5/(-c*f+d*e)
^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5488 vs. $2(922) = 1844$.

Time = 16.20 (sec) , antiderivative size = 5488, normalized size of antiderivative = 5.95

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^3*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {168, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 168$$

$$-\frac{\int \frac{acfh-4ad(fg+eh)+b(8deg+3cfg-4ceh)+7df(bg-ah)x}{2(a+bx)^2(c+dx)^3\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{acfh-4ad(fg+eh)+b(8deg+3cfg-4ceh)+7df(bg-ah)x}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\int \frac{df(8dfg+36deh-9cfh)a^2+b\left(-4e(17fg+6eh)d^2-3cf(fg-8eh)d+c^2f^2h\right)a+b^2\left(f(3fg-4eh)c^2+12de(fg-2eh)c+48d^2e^2g\right)+5df\left(7dfha^2-b(11dfg+4deh- \right.\right.}{2(a+bx)(c+dx)^3\sqrt{e+fx}}}{(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{df(8dfg+36deh-9cfh)a^2+b\left(-4e(17fg+6eh)d^2-3cf(fg-8eh)d+c^2f^2h\right)a+b^2\left(f(3fg-4eh)c^2+12de(fg-2eh)c+48d^2e^2g\right)+5df\left(7dfha^2-b(11dfg+4deh- \right.\right.}{(a+bx)(c+dx)^3\sqrt{e+fx}}}{2(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\int \frac{2\left(2d^2f^2(3dfg-4deh+cfh)a^3+bdf\left(2e(6fg+19eh)d^2-cf(24fg+37eh)d+11c^2f^2h\right)a^2-b^2\left(24e^2(3fg+eh)d^3-cef(69fg+50eh)d^2-c^2f^2(9fg-31eh)d+c^3f\right.}{}}{}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{2d^2f^2(3dfg-4deh+cfh)a^3+bdf\left(2e(6fg+19eh)d^2-cf(24fg+37eh)d+11c^2f^2h\right)a^2-b^2\left(24e^2(3fg+eh)d^3-cef(69fg+50eh)d^2-c^2f^2(9fg-31eh)d+c^3f^3h\right.}{}}{}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}(7dfha^2-b(11dfg+4deh-cfh)a+b^2(8deg+3cfg-4ceh))}{(bc-ad)(be-af)(a+bx)(c+dx)^2}-\frac{\frac{2d\sqrt{e+fx}\left(df(2dfg+9deh-11cfh)a^2-b\left(e(17fg+6eh)d^2-cf(13fg+11eh)d+c^2f^2\right.\right.}{(bc-ad)(de-cf)(c+dx)^2}}{}}{}$$

$\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}(7dfha^2-b(11dfg+4deh-cfh)a+b^2(8deg+3cfg-4ceh))}{(bc-ad)(be-af)(a+bx)(c+dx)^2}-\frac{\frac{2d\sqrt{e+fx}\left(df(2dfg+9deh-11cfh)a^2-b\left(e(17fg+6eh)d^2-cf(13fg+11eh)d+c^2f^2\right.\right.}{(bc-ad)(de-cf)(c+dx)^2}}{}}{}$$

$\downarrow$ 73

$$-\frac{\sqrt{e+fx}(bg-ah)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}(7dfha^2-b(11dfg+4deh-cfh)a+b^2(8deg+3cfg-4ceh))}{(bc-ad)(be-af)(a+bx)(c+dx)^2}-\frac{\frac{2d\sqrt{e+fx}\left(df(2dfg+9deh-11cfh)a^2-b\left(e(17fg+6eh)d^2-cf(13fg+11eh)d+c^2f^2\right.\right.}{(bc-ad)(de-cf)(c+dx)^2}}{}}{}$$

$\downarrow$ 221

$$-\frac{\sqrt{e+fx}(bg-ah)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-$$

$$-\frac{\frac{\sqrt{e+fx}(7dfha^2-b(11dfg+4deh-cfh)a+b^2(8deg+3cfg-4ceh))}{(bc-ad)(be-af)(a+bx)(c+dx)^2}-\frac{\frac{2d\sqrt{e+fx}\left(df(2dfg+9deh-11cfh)a^2-b\left(e(17fg+6eh)d^2-cf(13fg+11eh)d+c^2f^2\right.\right.}{(bc-ad)(de-cf)(c+dx)^2}}{}}{}$$

input
```
Int[(g + h*x)/((a + b*x)^3*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c +
 d*x)^2) - (-(((7*a^2*d*f*h + b^2*(8*d*e*g + 3*c*f*g - 4*c*e*h) - a*b*(11*
d*f*g + 4*d*e*h - c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x
)*(c + d*x)^2)) - ((2*d*(a^2*d*f*(2*d*f*g + 9*d*e*h - 11*c*f*h) + b^2*(12*
d^2*e^2*g - c^2*f*(3*f*g - 4*e*h) - c*d*e*(7*f*g + 6*e*h)) - a*b*(c^2*f^2*
h + d^2*e*(17*f*g + 6*e*h) - c*d*f*(13*f*g + 11*e*h)))*Sqrt[e + f*x])/((b*
c - a*d)*(d*e - c*f)*(c + d*x)^2) + ((2*d*(a^3*d^2*f^2*(3*d*f*g - 4*d*e*h
+ c*f*h) + b^3*(24*d^3*e^3*g + c^3*f^2*(3*f*g - 4*e*h) - 12*c*d^2*e^2*(3*f
*g + e*h) + c^2*d*e*f*(6*f*g + 19*e*h)) + a^2*b*d*f*(22*c^2*f^2*h + d^2*e*
(6*f*g + 19*e*h) - c*d*f*(15*f*g + 32*e*h)) + a*b^2*(c^3*f^3*h - 12*d^3*e^
2*(3*f*g + e*h) + 2*c*d^2*e*f*(30*f*g + 17*e*h) - c^2*d*f^2*(15*f*g + 32*e
*h)))*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)) + ((2*b^(3/2)*(d*
e - c*f)^2*(35*a^3*d^2*f^2*h - 7*a^2*b*d*f*(9*d*f*g + 8*d*e*h - 2*c*f*h) -
 b^3*(48*d^2*e^2*g + c^2*f*(3*f*g - 4*e*h) + 12*c*d*e*(f*g - 2*e*h)) - a*b
^2*(c^2*f^2*h - 2*c*d*f*(9*f*g - 22*e*h) - 12*d^2*e*(9*f*g + 2*e*h)))*ArcT
anh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]
) + (2*d^(3/2)*(b*e - a*f)^2*(a^2*d^2*f*(3*d*f*g - 4*d*e*h + c*f*h) - 2*a*
b*d*(7*c^2*f^2*h + c*d*f*(9*f*g - 22*e*h) - 6*d^2*e*(f*g - 2*e*h)) + b^2*(
48*d^3*e^2*g - 35*c^3*f^2*h - 12*c*d^2*e*(9*f*g + 2*e*h) + 7*c^2*d*f*(9*f*
g + 8*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 59.86 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	Expression too large to display	1094
default	Expression too large to display	1094
pseudoelliptic	Expression too large to display	1488

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^4*(b^2/f^4/(a*d-b*c)^5*((1/8*b*f*(11*a^3*d^2*f*h-10*a^2*b*c*d*f*h-8*a^
2*b*d^2*e*h-15*a^2*b*d^2*f*g-a*b^2*c^2*f*h+4*a*b^2*c*d*e*h+18*a*b^2*c*d*f*
g+12*a*b^2*d^2*e*g+4*b^3*c^2*e*h-3*b^3*c^2*f*g-12*b^3*c*d*e*g)/(a^2*f^2-2*
a*b*e*f+b^2*e^2)*(f*x+e)^(3/2)+1/8*(13*a^3*d^2*f*h-14*a^2*b*c*d*f*h-8*a^2*
b*d^2*e*h-17*a^2*b*d^2*f*g+a*b^2*c^2*f*h+4*a*b^2*c*d*e*h+22*a*b^2*c*d*f*g+
12*a*b^2*d^2*e*g+4*b^3*c^2*e*h-5*b^3*c^2*f*g-12*b^3*c*d*e*g)*f/(a*f-b*e)*(
f*x+e)^(1/2))/((f*x+e)*b+a*f-b*e)^2+1/8*(35*a^3*d^2*f^2*h+14*a^2*b*c*d*f^2
*h-56*a^2*b*d^2*e*f*h-63*a^2*b*d^2*f^2*g-a*b^2*c^2*f^2*h-44*a*b^2*c*d*e*f*
h+18*a*b^2*c*d*f^2*g+24*a*b^2*d^2*e^2*h+108*a*b^2*d^2*e*f*g+4*b^3*c^2*e*f*
h-3*b^3*c^2*f^2*g+24*b^3*c*d*e^2*h-12*b^3*c*d*e*f*g-48*b^3*d^2*e^2*g)/(a^2
*f^2-2*a*b*e*f+b^2*e^2)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b
*e)*b)^(1/2)))-d^2/f^4/(a*d-b*c)^5*((-1/8*d*f*(a^2*c*d^2*f*h-4*a^2*d^3*e*h
+3*a^2*d^3*f*g+10*a*b*c^2*d*f*h-4*a*b*c*d^2*e*h-18*a*b*c*d^2*f*g+12*a*b*d^
3*e*g-11*b^2*c^3*f*h+8*b^2*c^2*d*e*h+15*b^2*c^2*d*f*g-12*b^2*c*d^2*e*g)/(c
^2*f^2-2*c*d*e*f+d^2*e^2)*(f*x+e)^(3/2)+1/8*(a^2*c*d^2*f*h+4*a^2*d^3*e*h-5
*a^2*d^3*f*g-14*a*b*c^2*d*f*h+4*a*b*c*d^2*e*h+22*a*b*c*d^2*f*g-12*a*b*d^3*
e*g+13*b^2*c^3*f*h-8*b^2*c^2*d*e*h-17*b^2*c^2*d*f*g+12*b^2*c*d^2*e*g)*f/(c
*f-d*e)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2-1/8*(a^2*c*d^2*f^2*h-4*a^2*d^
3*e*f*h+3*a^2*d^3*f^2*g-14*a*b*c^2*d*f^2*h+44*a*b*c*d^2*e*f*h-18*a*b*c*d^2
*f^2*g-24*a*b*d^3*e^2*h+12*a*b*d^3*e*f*g-35*b^2*c^3*f^2*h+56*b^2*c^2*d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**3/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5069 vs. $2(882) = 1764$.

Time = 0.47 (sec) , antiderivative size = 5069, normalized size of antiderivative = 5.50

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(48*b^5*d^2*e^2*g + 12*b^5*c*d*e*f*g - 108*a*b^4*d^2*e*f*g + 3*b^5*c^2
*f^2*g - 18*a*b^4*c*d*f^2*g + 63*a^2*b^3*d^2*f^2*g - 24*b^5*c*d*e^2*h - 24
*a*b^4*d^2*e^2*h - 4*b^5*c^2*e*f*h + 44*a*b^4*c*d*e*f*h + 56*a^2*b^3*d^2*e
*f*h + a*b^4*c^2*f^2*h - 14*a^2*b^3*c*d*f^2*h - 35*a^3*b^2*d^2*f^2*h)*arct
an(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^5*e^2 - 5*a*b^6*c^4*d*e^2
 + 10*a^2*b^5*c^3*d^2*e^2 - 10*a^3*b^4*c^2*d^3*e^2 + 5*a^4*b^3*c*d^4*e^2 -
 a^5*b^2*d^5*e^2 - 2*a*b^6*c^5*e*f + 10*a^2*b^5*c^4*d*e*f - 20*a^3*b^4*c^3
*d^2*e*f + 20*a^4*b^3*c^2*d^3*e*f - 10*a^5*b^2*c*d^4*e*f + 2*a^6*b*d^5*e*f
 + a^2*b^5*c^5*f^2 - 5*a^3*b^4*c^4*d*f^2 + 10*a^4*b^3*c^3*d^2*f^2 - 10*a^5
*b^2*c^2*d^3*f^2 + 5*a^6*b*c*d^4*f^2 - a^7*d^5*f^2)*sqrt(-b^2*e + a*b*f))
- 1/4*(48*b^2*d^5*e^2*g - 108*b^2*c*d^4*e*f*g + 12*a*b*d^5*e*f*g + 63*b^2*
c^2*d^3*f^2*g - 18*a*b*c*d^4*f^2*g + 3*a^2*d^5*f^2*g - 24*b^2*c*d^4*e^2*h
- 24*a*b*d^5*e^2*h + 56*b^2*c^2*d^3*e*f*h + 44*a*b*c*d^4*e*f*h - 4*a^2*d^5
*e*f*h - 35*b^2*c^3*d^2*f^2*h - 14*a*b*c^2*d^3*f^2*h + a^2*c*d^4*f^2*h)*ar
ctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^5*c^5*d^2*e^2 - 5*a*b^4*c^4
*d^3*e^2 + 10*a^2*b^3*c^3*d^4*e^2 - 10*a^3*b^2*c^2*d^5*e^2 + 5*a^4*b*c*d^6
*e^2 - a^5*d^7*e^2 - 2*b^5*c^6*d*e*f + 10*a*b^4*c^5*d^2*e*f - 20*a^2*b^3*c
^4*d^3*e*f + 20*a^3*b^2*c^3*d^4*e*f - 10*a^4*b*c^2*d^5*e*f + 2*a^5*c*d^6*e
*f + b^5*c^7*f^2 - 5*a*b^4*c^6*d*f^2 + 10*a^2*b^3*c^5*d^2*f^2 - 10*a^3*b^2
*c^4*d^3*f^2 + 5*a^4*b*c^3*d^4*f^2 - a^5*c^2*d^5*f^2)*sqrt(-d^2*e + c*d...
```

Mupad [B] (verification not implemented)

Time = 44.43 (sec) , antiderivative size = 1098262, normalized size of antiderivative = 1191.17

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^3),x)
```

output
```
(((e + f*x)^(5/2)*(6*a^4*b*d^5*f^5*g + 6*b^5*c^4*d*f^5*g - 72*b^5*d^5*e^4*
f*g - 25*a*b^4*c^3*d^2*f^5*g - 25*a^3*b^2*c*d^4*f^5*g + 144*a*b^4*d^5*e^3*
f^2*g + a^3*b^2*d^5*e*f^4*g + 144*b^5*c*d^4*e^3*f^2*g + b^5*c^3*d^2*e*f^4*
g - 34*a^2*b^3*c^2*d^3*f^5*g + 34*a^2*b^3*c^3*d^2*f^5*h + 34*a^3*b^2*c^2*d
^3*f^5*h - 73*a^2*b^3*d^5*e^2*f^3*g - 75*a^2*b^3*d^5*e^3*f^2*h + 41*a^3*b^
2*d^5*e^2*f^3*h - 73*b^5*c^2*d^3*e^2*f^3*g - 75*b^5*c^2*d^3*e^3*f^2*h + 41
*b^5*c^3*d^2*e^2*f^3*h + 2*a*b^4*c^4*d*f^5*h + 2*a^4*b*c*d^4*f^5*h + 36*a*
b^4*d^5*e^4*f*h - 8*a^4*b*d^5*e*f^4*h + 36*b^5*c*d^4*e^4*f*h - 8*b^5*c^4*d
*e*f^4*h - 286*a*b^4*c*d^4*e^2*f^3*g + 143*a*b^4*c^2*d^3*e*f^4*g + 143*a^2
*b^3*c*d^4*e*f^4*g - 138*a*b^4*c*d^4*e^3*f^2*h - 51*a*b^4*c^3*d^2*e*f^4*h
- 51*a^3*b^2*c*d^4*e*f^4*h + 175*a*b^4*c^2*d^3*e^2*f^3*h + 175*a^2*b^3*c*d
^4*e^2*f^3*h - 170*a^2*b^3*c^2*d^3*e*f^4*h))/(4*(a*c*f^2 + b*d*e^2 - a*d*e
*f - b*c*e*f)^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)) - ((e + f*x)^(1/2)*(a*b^3*c^4*f^5*h - 5*b^4*c^4*f^5*g - 5*a
^4*d^4*f^5*g + a^4*c*d^3*f^5*h + 4*a^4*d^4*e*f^4*h + 4*b^4*c^4*e*f^4*h + 2
4*b^4*d^4*e^4*f*g - 13*a^2*b^2*c^3*d*f^5*h - 13*a^3*b*c^2*d^2*f^5*h - 48*a
*b^3*d^4*e^3*f^2*g - 12*a^3*b*d^4*e^2*f^3*h - 48*b^4*c*d^3*e^3*f^2*g - 12*
b^4*c^3*d*e^2*f^3*h + 21*a^2*b^2*d^4*e^2*f^3*g + 25*a^2*b^2*d^4*e^3*f^2*h
+ 21*b^4*c^2*d^2*e^2*f^3*g + 25*b^4*c^2*d^2*e^3*f^2*h + 17*a*b^3*c^3*d*f^5
*g + 17*a^3*b*c*d^3*f^5*g + 3*a^3*b*d^4*e*f^4*g - 12*a*b^3*d^4*e^4*f*h ...
```

Reduce [F]

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}\,dx=\int \frac{hx+g}{\left(bx+a\right)^{3}\left(dx+c\right)^{3}\sqrt{fx+e}}dx$$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(1/2),x)
```

3.148 $\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1417

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

output
```
1/24*d*(a^3*d^2*f^2*(-101*c*f*h+89*d*e*h+12*d*f*g)-b^3*(120*d^3*e^3*g-c^2*
d*e*f*(-42*e*h+29*f*g)-3*c^3*f^2*(-6*e*h+5*f*g)-8*c*d^2*e^2*(9*e*h+8*f*g))
+a*b^2*(3*c^3*f^3*h-c^2*d*f^2*(-109*e*h+74*f*g)+8*d^3*e^2*(6*e*h+37*f*g)-2
*c*d^2*e*f*(98*e*h+93*f*g))-a^2*b*d*f*(22*c^2*f^2*h+d^2*e*(122*e*h+203*f*g
)-c*d*f*(180*e*h+167*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^4/(-a*f+b*e)^3/(-c*f+
d*e)/(d*x+c)^2-1/3*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^
3/(d*x+c)^2+1/12*(9*a^2*d*f*h+b^2*(-6*c*e*h+5*c*f*g+10*d*e*g)-a*b*(-c*f*h+
4*d*e*h+15*d*f*g))*(f*x+e)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2/(d*x+
c)^2+1/24*(63*a^3*d^2*f^2*h-a^2*b*d*f*(-20*c*f*h+80*d*e*h+129*d*f*g)-b^3*(
80*d^2*e^2*g+2*c*d*e*(-24*e*h+17*f*g)+3*c^2*f*(-6*e*h+5*f*g))-a*b^2*(3*c^2
*f^2*h-2*c*d*f*(-49*e*h+32*f*g)-2*d^2*e*(16*e*h+97*f*g)))*(f*x+e)^(1/2)/(-
a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)/(d*x+c)^2+1/8*d*(2*a^4*d^3*f^3*(c*f*h-4*d*
e*h+3*d*f*g)+a^3*b*d^2*f^2*(71*c^2*f^2*h-2*c*d*f*(56*e*h+19*f*g)+d^2*e*(65
*e*h+14*f*g))-b^4*(80*d^4*e^4*g+c^4*f^3*(-6*e*h+5*f*g)+4*c^3*d*e*f^2*(-3*e
*h+2*f*g)-4*c*d^3*e^3*(12*e*h+29*f*g)+c^2*d^2*e^2*f*(72*e*h+17*f*g))+a^2*b
^2*d*f*(8*c^3*f^3*h-d^3*e^2*(84*e*h+149*f*g)+2*c*d^2*e*f*(103*e*h+128*f*g)
-c^2*d*f^2*(166*e*h+71*f*g))-a*b^3*(c^4*f^4*h-4*c^3*d*f^3*(-10*e*h+7*f*g)-
4*d^4*e^3*(8*e*h+51*f*g)+2*c*d^3*e^2*f*(82*e*h+157*f*g)-c^2*d^2*e*f^2*(197
*e*h+58*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^5/(-a*f+b*e)^3/(-c*f+d*e)^2/(d*x+c
)+1/8*b^(3/2)*(105*a^4*d^3*f^3*h-21*a^3*b*d^2*f^2*(-3*c*f*h+12*d*e*h+11...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 9721 vs. 2(1417) = 2834.

Time = 16.34 (sec) , antiderivative size = 9721, normalized size of antiderivative = 6.86

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^4*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.65 (sec) , antiderivative size = 1526, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 168$$

$$-\frac{\int \frac{b(10deg+5cfg-6ceh)-a(6dfg+4deh-cfh)+9df(bg-ah)x}{2(a+bx)^3(c+dx)^3\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{b(10deg+5cfg-6ceh)-a(6dfg+4deh-cfh)+9df(bg-ah)x}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}dx}{6(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\int \frac{df(24dfg+52deh-13cfh)a^2+b\left(-4e(31fg+8eh)d^2-cf(29fg-56eh)d+3c^2f^2h\right)a+b^2\left(3f(5fg-6eh)c^2+2de(17fg-24eh)c+80d^2e^2g\right)+7df\left(9dfha^2-b(15df\cdots\right.}{2(a+bx)^2(c+dx)^3\sqrt{e+fx}}}{2(bc-ad)(be-af)}\cdots}{6(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{\int \frac{df(24dfg+52deh-13cfh)a^2+b\left(-4e(31fg+8eh)d^2-cf(29fg-56eh)d+3c^2f^2h\right)a+b^2\left(3f(5fg-6eh)c^2+2de(17fg-24eh)c+80d^2e^2g\right)+7df\left(9dfha^2-b(15df\cdots\right.}{(a+bx)^2(c+dx)^3\sqrt{e+fx}}}{4(bc-ad)(be-af)}\cdots}{6(bc-ad)(be-af)}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{-\frac{\sqrt{e+fx}\left(63a^3d^2f^2h-a^2bdf(-20cfh+80deh+129dfg)-ab^2\left(3c^2f^2h-2cdf(32fg-49eh)-2d^2e(16eh+97fg)\right)-b^3\left(3c^2f(5fg-6eh)+2cde(17fg-24eh)+80d^2e\right.}{(a+bx)(c+dx)^2(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$-\frac{-\frac{\sqrt{e+fx}\left(63a^3d^2f^2h-a^2bdf(-20cfh+80deh+129dfg)-ab^2\left(3c^2f^2h-2cdf(32fg-49eh)-2d^2e(16eh+97fg)\right)-b^3\left(3c^2f(5fg-6eh)+2cde(17fg-24eh)+80d^2e\right.}{(a+bx)(c+dx)^2(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{3(a+bx)^3(c+dx)^2(bc-ad)(be-af)}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(9dfha^2-b(15dfg+4deh-cfh)a+b^2(10deg+5cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-\frac{\left(63d^2f^2ha^3-bdf(129dfg+80deh-20cfh)a^2-b^2\left(-2e(97fg+16eh)d^2-2cf(32fg\right.\right.}{(bc-ad)(be-af}}{}$$

$\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(9dfha^2-b(15dfg+4deh-cfh)a+b^2(10deg+5cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-\frac{\left(63d^2f^2ha^3-bdf(129dfg+80deh-20cfh)a^2-b^2\left(-2e(97fg+16eh)d^2-2cf(32fg\right.\right.}{(bc-ad)(be-af}}{}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(9dfha^2-b(15dfg+4deh-cfh)a+b^2(10deg+5cfg-6ceh)\right)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-\frac{\left(63d^2f^2ha^3-bdf(129dfg+80deh-20cfh)a^2-b^2\left(-2e(97fg+16eh)d^2-2cf(32fg\right.\right.}{(bc-ad)(be-af}}{}$$

$\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(9dfha^2-b(15dfg+4deh-cfh)a+b^2(10deg+5cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-\frac{\left(63d^2f^2ha^3-bdf(129dfg+80deh-20cfh)a^2-b^2\left(-2e(97fg+16eh)d^2-2cf(32fg\right.\right.}{(bc-ad)(be-af}$$

$\downarrow$ 73

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(9dfha^2-b(15dfg+4deh-cfh)a+b^2(10deg+5cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-\frac{\left(63d^2f^2ha^3-bdf(129dfg+80deh-20cfh)a^2-b^2\left(-2e(97fg+16eh)d^2-2cf(32fg\right.\right.}{(bc-ad)(be-af}$$

$\downarrow$ 221

$$-\frac{\sqrt{e+fx}(bg-ah)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(9dfha^2-b(15dfg+4deh-cfh)a+b^2(10deg+5cfg-6ceh))}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}-\frac{\left(63d^2f^2ha^3-bdf(129dfg+80deh-20cfh)a^2-b^2\left(-2e(97fg+16eh)d^2-2cf(32fg\right.\right.}{(bc-ad)(be-af}$$

input
```
Int[(g + h*x)/((a + b*x)^4*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/3*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c +
 d*x)^2) - (-1/2*((9*a^2*d*f*h + b^2*(10*d*e*g + 5*c*f*g - 6*c*e*h) - a*b*
(15*d*f*g + 4*d*e*h - c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a +
 b*x)^2*(c + d*x)^2) - (((63*a^3*d^2*f^2*h - a^2*b*d*f*(129*d*f*g + 80*d*e
*h - 20*c*f*h) - b^3*(80*d^2*e^2*g + 2*c*d*e*(17*f*g - 24*e*h) + 3*c^2*f*(
5*f*g - 6*e*h)) - a*b^2*(3*c^2*f^2*h - 2*c*d*f*(32*f*g - 49*e*h) - 2*d^2*e
*(97*f*g + 16*e*h)))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c
+ d*x)^2) - ((-2*d*(a^3*d^2*f^2*(12*d*f*g + 89*d*e*h - 101*c*f*h) - b^3*(1
20*d^3*e^3*g - c^2*d*e*f*(29*f*g - 42*e*h) - 3*c^3*f^2*(5*f*g - 6*e*h) - 8
*c*d^2*e^2*(8*f*g + 9*e*h)) + a*b^2*(3*c^3*f^3*h - c^2*d*f^2*(74*f*g - 109
*e*h) + 8*d^3*e^2*(37*f*g + 6*e*h) - 2*c*d^2*e*f*(93*f*g + 98*e*h)) - a^2*
b*d*f*(22*c^2*f^2*h + d^2*e*(203*f*g + 122*e*h) - c*d*f*(167*f*g + 180*e*h
)))*Sqrt[e + f*x])/((b*c - a*d)*(d*e - c*f)*(c + d*x)^2) - (3*((2*d*(2*a^4
*d^3*f^3*(3*d*f*g - 4*d*e*h + c*f*h) + a^3*b*d^2*f^2*(71*c^2*f^2*h - 2*c*d
*f*(19*f*g + 56*e*h) + d^2*e*(14*f*g + 65*e*h)) - b^4*(80*d^4*e^4*g + c^4*
f^3*(5*f*g - 6*e*h) + 4*c^3*d*e*f^2*(2*f*g - 3*e*h) - 4*c*d^3*e^3*(29*f*g
+ 12*e*h) + c^2*d^2*e^2*f*(17*f*g + 72*e*h)) + a^2*b^2*d*f*(8*c^3*f^3*h -
d^3*e^2*(149*f*g + 84*e*h) + 2*c*d^2*e*f*(128*f*g + 103*e*h) - c^2*d*f^2*(
71*f*g + 166*e*h)) - a*b^3*(c^4*f^4*h - 4*c^3*d*f^3*(7*f*g - 10*e*h) - 4*d
^4*e^3*(51*f*g + 8*e*h) + 2*c*d^3*e^2*f*(157*f*g + 82*e*h) - c^2*d^2*e*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 293.42 (sec) , antiderivative size = 1854, normalized size of antiderivative = 1.31

method	result	size
derivativedivides	Expression too large to display	1854
default	Expression too large to display	1854
pseudoelliptic	Expression too large to display	2594

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^5*(b^2/f^5/(a*d-b*c)^6*((1/16*b^2*f*(41*a^4*d^3*f^2*h-33*a^3*b*c*d^2*f
^2*h-60*a^3*b*d^3*e*f*h-71*a^3*b*d^3*f^2*g-9*a^2*b^2*c^2*d*f^2*h+18*a^2*b^
2*c*d^2*e*f*h+99*a^2*b^2*c*d^2*f^2*g+24*a^2*b^2*d^3*e^2*h+114*a^2*b^2*d^3*
e*f*g+a*b^3*c^3*f^2*h+48*a*b^3*c^2*d*e*f*h-33*a*b^3*c^2*d*f^2*g-132*a*b^3*
c*d^2*e*f*g-48*a*b^3*d^3*e^2*g-6*b^4*c^3*e*f*h+5*b^4*c^3*f^2*g-24*b^4*c^2*
d*e^2*h+18*b^4*c^2*d*e*f*g+48*b^4*c*d^2*e^2*g)/(a^3*f^3-3*a^2*b*e*f^2+3*a*
b^2*e^2*f-b^3*e^3)*(f*x+e)^(5/2)+1/6*(35*a^4*d^3*f^2*h-33*a^3*b*c*d^2*f^2*
h-48*a^3*b*d^3*e*f*h-59*a^3*b*d^3*f^2*g-3*a^2*b^2*c^2*d*f^2*h+18*a^2*b^2*c
*d^2*e*f*h+87*a^2*b^2*c*d^2*f^2*g+18*a^2*b^2*d^3*e^2*h+90*a^2*b^2*d^3*e*f*
g+a*b^3*c^3*f^2*h+36*a*b^3*c^2*d*e*f*h-33*a*b^3*c^2*d*f^2*g-108*a*b^3*c*d^
2*e*f*g-36*a*b^3*d^3*e^2*g-6*b^4*c^3*e*f*h+5*b^4*c^3*f^2*g-18*b^4*c^2*d*e^
2*h+18*b^4*c^2*d*e*f*g+36*b^4*c*d^2*e^2*g)*b*f/(a^2*f^2-2*a*b*e*f+b^2*e^2)
*(f*x+e)^(3/2)+1/16*(55*a^4*d^3*f^2*h-63*a^3*b*c*d^2*f^2*h-68*a^3*b*d^3*e*
f*h-89*a^3*b*d^3*f^2*g+9*a^2*b^2*c^2*d*f^2*h+30*a^2*b^2*c*d^2*e*f*h+141*a^
2*b^2*c*d^2*f^2*g+24*a^2*b^2*d^3*e^2*h+126*a^2*b^2*d^3*e*f*g-a*b^3*c^3*f^2
*h+48*a*b^3*c^2*d*e*f*h-63*a*b^3*c^2*d*f^2*g-156*a*b^3*c*d^2*e*f*g-48*a*b^
3*d^3*e^2*g-10*b^4*c^3*e*f*h+11*b^4*c^3*f^2*g-24*b^4*c^2*d*e^2*h+30*b^4*c^
2*d*e*f*g+48*b^4*c*d^2*e^2*g)*f/(a*f-b*e)*(f*x+e)^(1/2))/((f*x+e)*b+a*f-b*
e)^3+1/16*(105*a^4*d^3*f^3*h+63*a^3*b*c*d^2*f^3*h-252*a^3*b*d^3*e*f^2*h-23
1*a^3*b*d^3*f^3*g-9*a^2*b^2*c^2*d*f^3*h-270*a^2*b^2*c*d^2*e*f^2*h+99*a^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**4/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3617 vs. 2(1373) = 2746.

Time = 0.40 (sec) , antiderivative size = 3617, normalized size of antiderivative = 2.55

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(160*b^6*d^3*e^3*g + 48*b^6*c*d^2*e^2*f*g - 528*a*b^5*d^3*e^2*f*g + 1
8*b^6*c^2*d*e*f^2*g - 132*a*b^5*c*d^2*e*f^2*g + 594*a^2*b^4*d^3*e*f^2*g +
5*b^6*c^3*f^3*g - 33*a*b^5*c^2*d*f^3*g + 99*a^2*b^4*c*d^2*f^3*g - 231*a^3*
b^3*d^3*f^3*g - 96*b^6*c*d^2*e^3*h - 64*a*b^5*d^3*e^3*h - 24*b^6*c^2*d*e^2
*f*h + 288*a*b^5*c*d^2*e^2*f*h + 216*a^2*b^4*d^3*e^2*f*h - 6*b^6*c^3*e*f^2
*h + 48*a*b^5*c^2*d*e*f^2*h - 270*a^2*b^4*c*d^2*e*f^2*h - 252*a^3*b^3*d^3*
e*f^2*h + a*b^5*c^3*f^3*h - 9*a^2*b^4*c^2*d*f^3*h + 63*a^3*b^3*c*d^2*f^3*h
 + 105*a^4*b^2*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b
^9*c^6*e^3 - 6*a*b^8*c^5*d*e^3 + 15*a^2*b^7*c^4*d^2*e^3 - 20*a^3*b^6*c^3*d
^3*e^3 + 15*a^4*b^5*c^2*d^4*e^3 - 6*a^5*b^4*c*d^5*e^3 + a^6*b^3*d^6*e^3 -
3*a*b^8*c^6*e^2*f + 18*a^2*b^7*c^5*d*e^2*f - 45*a^3*b^6*c^4*d^2*e^2*f + 60
*a^4*b^5*c^3*d^3*e^2*f - 45*a^5*b^4*c^2*d^4*e^2*f + 18*a^6*b^3*c*d^5*e^2*f
 - 3*a^7*b^2*d^6*e^2*f + 3*a^2*b^7*c^6*e*f^2 - 18*a^3*b^6*c^5*d*e*f^2 + 45
*a^4*b^5*c^4*d^2*e*f^2 - 60*a^5*b^4*c^3*d^3*e*f^2 + 45*a^6*b^3*c^2*d^4*e*f
^2 - 18*a^7*b^2*c*d^5*e*f^2 + 3*a^8*b*d^6*e*f^2 - a^3*b^6*c^6*f^3 + 6*a^4*
b^5*c^5*d*f^3 - 15*a^5*b^4*c^4*d^2*f^3 + 20*a^6*b^3*c^3*d^3*f^3 - 15*a^7*b
^2*c^2*d^4*f^3 + 6*a^8*b*c*d^5*f^3 - a^9*d^6*f^3)*sqrt(-b^2*e + a*b*f)) +
1/4*(80*b^2*d^6*e^2*g - 176*b^2*c*d^5*e*f*g + 16*a*b*d^6*e*f*g + 99*b^2*c^
2*d^4*f^2*g - 22*a*b*c*d^5*f^2*g + 3*a^2*d^6*f^2*g - 48*b^2*c*d^5*e^2*h -
32*a*b*d^6*e^2*h + 108*b^2*c^2*d^4*e*f*h + 56*a*b*c*d^5*e*f*h - 4*a^2*d...
```

Mupad [B] (verification not implemented)

Time = 56.15 (sec) , antiderivative size = 1590452, normalized size of antiderivative = 1122.41

$$\int \frac{g + hx}{(a + bx)^4(c + dx)^3\sqrt{e + fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^3),x)
```

output
```
- atan(((((11264*a^4*b^18*c^18*d^3*f^13*g - 640*a^3*b^19*c^19*d^2*f^13*g -
 94336*a^5*b^17*c^17*d^4*f^13*g + 506880*a^6*b^16*c^16*d^5*f^13*g - 195571
2*a^7*b^15*c^15*d^6*f^13*g + 5691136*a^8*b^14*c^14*d^7*f^13*g - 12764288*a
^9*b^13*c^13*d^8*f^13*g + 22274560*a^10*b^12*c^12*d^9*f^13*g - 30349440*a^
11*b^11*c^11*d^10*f^13*g + 32268544*a^12*b^10*c^10*d^11*f^13*g - 26652032*
a^13*b^9*c^9*d^12*f^13*g + 16943104*a^14*b^8*c^8*d^13*f^13*g - 8164992*a^1
5*b^7*c^7*d^14*f^13*g + 2914560*a^16*b^6*c^6*d^15*f^13*g - 744832*a^17*b^5
*c^5*d^16*f^13*g + 129536*a^18*b^4*c^4*d^17*f^13*g - 14080*a^19*b^3*c^3*d^
18*f^13*g + 768*a^20*b^2*c^2*d^19*f^13*g - 128*a^4*b^18*c^19*d^2*f^13*h +
2560*a^5*b^17*c^18*d^3*f^13*h - 27776*a^6*b^16*c^17*d^4*f^13*h + 175872*a^
7*b^15*c^16*d^5*f^13*h - 700800*a^8*b^14*c^15*d^6*f^13*h + 1866752*a^9*b^1
3*c^14*d^7*f^13*h - 3439744*a^10*b^12*c^13*d^8*f^13*h + 4412672*a^11*b^11*
c^12*d^9*f^13*h - 3805824*a^12*b^10*c^11*d^10*f^13*h + 1886720*a^13*b^9*c^
10*d^11*f^13*h - 35200*a^14*b^8*c^9*d^12*f^13*h - 739072*a^15*b^7*c^8*d^13
*f^13*h + 607104*a^16*b^6*c^7*d^14*f^13*h - 258048*a^17*b^5*c^6*d^15*f^13*
h + 62080*a^18*b^4*c^5*d^16*f^13*h - 7424*a^19*b^3*c^4*d^17*f^13*h + 256*a
^20*b^2*c^3*d^18*f^13*h + 10240*a^12*b^10*d^21*e^10*f^3*g - 61952*a^13*b^9
*d^21*e^9*f^4*g + 155264*a^14*b^8*d^21*e^8*f^5*g - 205184*a^15*b^7*d^21*e^
7*f^6*g + 148864*a^16*b^6*d^21*e^6*f^7*g - 54656*a^17*b^5*d^21*e^5*f^8*g +
 7936*a^18*b^4*d^21*e^4*f^9*g - 1280*a^19*b^3*d^21*e^3*f^10*g + 768*a^2...
```

Reduce [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 60838, normalized size of antiderivative = 42.93

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
(315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**7*b*c**5*d**3*f**6*h - 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**4*d**4*e*f**5*h + 630*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*
c**4*d**4*f**6*h*x + 945*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**7*b*c**3*d**5*e**2*f**4*h - 1890*sqrt(b)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**3*d
**5*e*f**5*h*x + 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**7*b*c**3*d**5*f**6*h*x**2 - 315*sqrt(b)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**2*d**6*e
**3*f**3*h + 1890*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**7*b*c**2*d**6*e**2*f**4*h*x - 945*sqrt(b)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**2*d**6*e*
f**5*h*x**2 - 630*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**7*b*c*d**7*e**3*f**3*h*x + 945*sqrt(b)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c*d**7*e**2*f**
4*h*x**2 - 315*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**7*b*d**8*e**3*f**3*h*x**2 + 189*sqrt(b)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b**2*c**6*d**2*f**6
*h - 1323*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(...
```

3.149 $\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 2077

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

output
```
1/192*d*(a^4*d^3*f^3*(-1123*c*f*h+1027*d*e*h+96*d*f*g)+b^4*(1440*d^4*e^4*g
-2*c^2*d^2*e^2*f*(-256*e*h+167*f*g)-c^3*d*e*f^2*(-232*e*h+185*f*g)-15*c^4*
f^3*(-8*e*h+7*f*g)-240*c*d^3*e^3*(4*e*h+3*f*g))-a*b^3*(15*c^4*f^4*h-c^2*d^
2*e*f^2*(-1898*e*h+1223*f*g)-11*c^3*d*f^3*(-69*e*h+55*f*g)+240*d^4*e^3*(2*
e*h+21*f*g)-4*c*d^3*e^2*f*(884*e*h+707*f*g))-a^3*b*d^2*f^2*(409*c^2*f^2*h+
d^2*e*(2154*e*h+2747*f*g)-c*d*f*(2947*e*h+2363*f*g))+a^2*b^2*d*f*(107*c^3*
f^3*h-c^2*d*f^2*(-2425*e*h+1519*f*g)+2*d^3*e^2*(856*e*h+3073*f*g)-c*d^2*e*
f*(4820*e*h+4051*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^5/(-a*f+b*e)^4/(-c*f+d*e)
/(d*x+c)^2-1/4*(-a*h+b*g)*(f*x+e)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^4/(d
*x+c)^2+1/24*(11*a^2*d*f*h+b^2*(-8*c*e*h+7*c*f*g+12*d*e*g)-a*b*(-c*f*h+4*d
*e*h+19*d*f*g))*(f*x+e)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^3/(d*x+c)^
2+1/96*(99*a^3*d^2*f^2*h-a^2*b*d*f*(-26*c*f*h+104*d*e*h+219*d*f*g)-b^3*(12
0*d^2*e^2*g+5*c^2*f*(-8*e*h+7*f*g)+16*c*d*e*(-5*e*h+4*f*g))-a*b^2*(5*c^2*f
^2*h-2*c*d*f*(-88*e*h+67*f*g)-8*d^2*e*(5*e*h+38*f*g)))*(f*x+e)^(1/2)/(-a*d
+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2/(d*x+c)^2+1/192*(693*a^4*d^3*f^3*h-a^3*b*d^
2*f^2*(-349*c*f*h+1396*d*e*h+1725*d*f*g)+b^4*(960*d^3*e^3*g+4*c^2*d*e*f*(-
68*e*h+55*f*g)+40*c*d^2*e^2*(-16*e*h+11*f*g)+15*c^3*f^2*(-8*e*h+7*f*g))+a*
b^3*(15*c^3*f^3*h-c^2*d*f^2*(-684*e*h+535*f*g)-88*c*d^2*e*f*(-23*e*h+15*f*
g)-40*d^3*e^2*(8*e*h+83*f*g))-a^2*b^2*d*f*(97*c^2*f^2*h-c*d*f*(-2048*e*h+1
195*f*g)-4*d^2*e*(282*e*h+995*f*g)))*(f*x+e)^(1/2)/(-a*d+b*c)^4/(-a*f+b...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14739 vs. 2(2077) = 4154.

Time = 16.58 (sec) , antiderivative size = 14739, normalized size of antiderivative = 7.10

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^5*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 5.54 (sec) , antiderivative size = 2212, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 168, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{\int \frac{acfh-4ad(2fg+eh)+b(12deg+7cfg-8ceh)+11df(bg-ah)x}{2(a+bx)^4(c+dx)^3\sqrt{e+fx}}dx}{4(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{acfh-4ad(2fg+eh)+b(12deg+7cfg-8ceh)+11df(bg-ah)x}{(a+bx)^4(c+dx)^3\sqrt{e+fx}}dx}{8(bc-ad)(be-af)}-\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{-\frac{\int \frac{df(48dfg+68deh-17cfh)a^2+b\left(-4e(49fg+10eh)d^2-cf(71fg-104eh)d+5c^2f^2h\right)a+b^2\left(5f(7fg-8eh)c^2+16de(4fg-5eh)c+120d^2e^2g\right)+9df\left(11dfha^2-b(1\cdots\right)}{2(a+bx)^3(c+dx)^3\sqrt{e+fx}}}{3(bc-ad)(be-af)}\cdots}{8(bc-ad)(be-af\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{\int \frac{df(48dfg+68deh-17cfh)a^2+b\left(-4e(49fg+10eh)d^2-cf(71fg-104eh)d+5c^2f^2h\right)a+b^2\left(5f(7fg-8eh)c^2+16de(4fg-5eh)c+120d^2e^2g\right)+9df\left(11dfha^2-b(1\cdots\right)}{(a+bx)^3(c+dx)^3\sqrt{e+fx}}}{6(bc-ad)(be-af)}\cdots}{8(bc-ad)(be-af\cdots}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{-\frac{\sqrt{e+fx}\left(99a^3d^2f^2h-a^2bdf(-26cfh+104deh+219dfg)-ab^2\left(5c^2f^2h-2cdf(67fg-88eh)-8d^2e(5eh+38fg)\right)-b^3\left(5c^2f(7fg-8eh)+16cde(4fg-5eh)+120d^2e\right.}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)^2(bc-ad)(be-af)}$$

$\downarrow$ 27

$$-\frac{-\frac{\sqrt{e+fx}\left(99a^3d^2f^2h-a^2bdf(-26cfh+104deh+219dfg)-ab^2\left(5c^2f^2h-2cdf(67fg-88eh)-8d^2e(5eh+38fg)\right)-b^3\left(5c^2f(7fg-8eh)+16cde(4fg-5eh)+120d^2e\right.}{2(a+bx)^2(c+dx)^2(bc-ad)(be-af)}}{}$$

$$\frac{\sqrt{e+fx}(bg-ah)}{4(a+bx)^4(c+dx)^2(bc-ad)(be-af)}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}}{}$$

$\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}}{}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$\frac{-\frac{\sqrt{e+fx}\left(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}}{}$$

$\downarrow$ 27

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}$$

$\downarrow$ 168

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}$$

$\downarrow$ 174

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}$$

$\downarrow$ 73

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh))}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}$$

$\downarrow$ 221

$$-\frac{\sqrt{e+fx}(bg-ah)}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2}-$$

$$-\frac{\sqrt{e+fx}\left(11dfha^2-b(19dfg+4deh-cfh)a+b^2(12deg+7cfg-8ceh)\right)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2}-\frac{\left(99d^2f^2ha^3-bdf(219dfg+104deh-26cfh)a^2-b^2\left(-8e(38fg+5eh)d^2-2cf(67f\right.\right.}{2(bc-ad)(be-a}$$

input
```
Int[(g + h*x)/((a + b*x)^5*(c + d*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/4*((b*g - a*h)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4*(c +
 d*x)^2) - (-1/3*((11*a^2*d*f*h + b^2*(12*d*e*g + 7*c*f*g - 8*c*e*h) - a*b
*(19*d*f*g + 4*d*e*h - c*f*h))*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a
+ b*x)^3*(c + d*x)^2) - (((99*a^3*d^2*f^2*h - a^2*b*d*f*(219*d*f*g + 104*d
*e*h - 26*c*f*h) - b^3*(120*d^2*e^2*g + 5*c^2*f*(7*f*g - 8*e*h) + 16*c*d*e
*(4*f*g - 5*e*h)) - a*b^2*(5*c^2*f^2*h - 2*c*d*f*(67*f*g - 88*e*h) - 8*d^2
*e*(38*f*g + 5*e*h)))*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^
2*(c + d*x)^2) - (-(((693*a^4*d^3*f^3*h - a^3*b*d^2*f^2*(1725*d*f*g + 1396
*d*e*h - 349*c*f*h) + b^4*(960*d^3*e^3*g + 4*c^2*d*e*f*(55*f*g - 68*e*h) +
 40*c*d^2*e^2*(11*f*g - 16*e*h) + 15*c^3*f^2*(7*f*g - 8*e*h)) + a*b^3*(15*
c^3*f^3*h - c^2*d*f^2*(535*f*g - 684*e*h) - 88*c*d^2*e*f*(15*f*g - 23*e*h)
 - 40*d^3*e^2*(83*f*g + 8*e*h)) - a^2*b^2*d*f*(97*c^2*f^2*h - c*d*f*(1195*
f*g - 2048*e*h) - 4*d^2*e*(995*f*g + 282*e*h)))*Sqrt[e + f*x])/((b*c - a*d
)*(b*e - a*f)*(a + b*x)*(c + d*x)^2)) - ((2*d*(a^4*d^3*f^3*(96*d*f*g + 102
7*d*e*h - 1123*c*f*h) + b^4*(1440*d^4*e^4*g - 2*c^2*d^2*e^2*f*(167*f*g - 2
56*e*h) - c^3*d*e*f^2*(185*f*g - 232*e*h) - 15*c^4*f^3*(7*f*g - 8*e*h) - 2
40*c*d^3*e^3*(3*f*g + 4*e*h)) - a*b^3*(15*c^4*f^4*h - c^2*d^2*e*f^2*(1223*
f*g - 1898*e*h) - 11*c^3*d*f^3*(55*f*g - 69*e*h) + 240*d^4*e^3*(21*f*g + 2
*e*h) - 4*c*d^3*e^2*f*(707*f*g + 884*e*h)) - a^3*b*d^2*f^2*(409*c^2*f^2*h
+ d^2*e*(2747*f*g + 2154*e*h) - c*d*f*(2363*f*g + 2947*e*h)) + a^2*b^2*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 3250, normalized size of antiderivative = 1.56

output too large to display

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
2*f^6*(-d^4/f^6/(a*d-b*c)^7*((-1/8*d*f*(a^2*c*d^2*f*h-4*a^2*d^3*e*h+3*a^2*
d^3*f*g+18*a*b*c^2*d*f*h-12*a*b*c*d^2*e*h-26*a*b*c*d^2*f*g+20*a*b*d^3*e*g-
19*b^2*c^3*f*h+16*b^2*c^2*d*e*h+23*b^2*c^2*d*f*g-20*b^2*c*d^2*e*g)/(c^2*f^
2-2*c*d*e*f+d^2*e^2)*(f*x+e)^(3/2)+1/8*(a^2*c*d^2*f*h+4*a^2*d^3*e*h-5*a^2*
d^3*f*g-22*a*b*c^2*d*f*h+12*a*b*c*d^2*e*h+30*a*b*c*d^2*f*g-20*a*b*d^3*e*g+
21*b^2*c^3*f*h-16*b^2*c^2*d*e*h-25*b^2*c^2*d*f*g+20*b^2*c*d^2*e*g)*f/(c*f-
d*e)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2-1/8*(a^2*c*d^2*f^2*h-4*a^2*d^3*e
*f*h+3*a^2*d^3*f^2*g-22*a*b*c^2*d*f^2*h+68*a*b*c*d^2*e*f*h-26*a*b*c*d^2*f^
2*g-40*a*b*d^3*e^2*h+20*a*b*d^3*e*f*g-99*b^2*c^3*f^2*h+176*b^2*c^2*d*e*f*h
+143*b^2*c^2*d*f^2*g-80*b^2*c*d^2*e^2*h-260*b^2*c*d^2*e*f*g+120*b^2*d^3*e^
2*g)/(c^2*f^2-2*c*d*e*f+d^2*e^2)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2
)/((c*f-d*e)*d)^(1/2)))+b^2/f^6/(a*d-b*c)^7*((1/128*b^3*f*(515*a^5*d^4*f^3
*h-356*a^4*b*c*d^3*f^3*h-1136*a^4*b*d^4*e*f^2*h-1083*a^4*b*d^4*f^3*g-198*a
^3*b^2*c^2*d^2*f^3*h+104*a^3*b^2*c*d^3*e*f^2*h+1716*a^3*b^2*c*d^3*f^3*g+91
2*a^3*b^2*d^4*e^2*f*h+2616*a^3*b^2*d^4*e*f^2*g+44*a^2*b^3*c^3*d*f^3*h+1320
*a^2*b^3*c^2*d^2*e*f^2*h-858*a^2*b^3*c^2*d^2*f^3*g+240*a^2*b^3*c*d^3*e^2*f
*h-3432*a^2*b^3*c*d^3*e*f^2*g-256*a^2*b^3*d^4*e^3*h-2208*a^2*b^3*d^4*e^2*f
*g-5*a*b^4*c^4*f^3*h-328*a*b^4*c^3*d*e*f^2*h+260*a*b^4*c^3*d*f^3*g-1296*a*
b^4*c^2*d^2*e^2*f*h+936*a*b^4*c^2*d^2*e*f^2*g-128*a*b^4*c*d^3*e^3*h+2496*a
*b^4*c*d^3*e^2*f*g+640*a*b^4*d^4*e^3*g+40*b^5*c^4*e*f^2*h-35*b^5*c^4*f^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**5/(d*x+c)**3/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6287 vs. 2(2029) = 4058.

Time = 0.53 (sec) , antiderivative size = 6287, normalized size of antiderivative = 3.03

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/64*(1920*b^7*d^4*e^4*g + 640*b^7*c*d^3*e^3*f*g - 8320*a*b^6*d^4*e^3*f*g
+ 288*b^7*c^2*d^2*e^2*f^2*g - 2496*a*b^6*c*d^3*e^2*f^2*g + 13728*a^2*b^5*d
^4*e^2*f^2*g + 120*b^7*c^3*d*e*f^3*g - 936*a*b^6*c^2*d^2*e*f^3*g + 3432*a^
2*b^5*c*d^3*e*f^3*g - 10296*a^3*b^4*d^4*e*f^3*g + 35*b^7*c^4*f^4*g - 260*a
*b^6*c^3*d*f^4*g + 858*a^2*b^5*c^2*d^2*f^4*g - 1716*a^3*b^4*c*d^3*f^4*g +
3003*a^4*b^3*d^4*f^4*g - 1280*b^7*c*d^3*e^4*h - 640*a*b^6*d^4*e^4*h - 384*
b^7*c^2*d^2*e^3*f*h + 5248*a*b^6*c*d^3*e^3*f*h + 2816*a^2*b^5*d^4*e^3*f*h
- 144*b^7*c^3*d*e^2*f^2*h + 1296*a*b^6*c^2*d^2*e^2*f^2*h - 7920*a^2*b^5*c*
d^3*e^2*f^2*h - 4752*a^3*b^4*d^4*e^2*f^2*h - 40*b^7*c^4*e*f^3*h + 328*a*b^
6*c^3*d*e*f^3*h - 1320*a^2*b^5*c^2*d^2*e*f^3*h + 5016*a^3*b^4*c*d^3*e*f^3*
h + 3696*a^4*b^3*d^4*e*f^3*h + 5*a*b^6*c^4*f^4*h - 44*a^2*b^5*c^3*d*f^4*h
+ 198*a^3*b^4*c^2*d^2*f^4*h - 924*a^4*b^3*c*d^3*f^4*h - 1155*a^5*b^2*d^4*f
^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^11*c^7*e^4 - 7*a*b^
10*c^6*d*e^4 + 21*a^2*b^9*c^5*d^2*e^4 - 35*a^3*b^8*c^4*d^3*e^4 + 35*a^4*b^
7*c^3*d^4*e^4 - 21*a^5*b^6*c^2*d^5*e^4 + 7*a^6*b^5*c*d^6*e^4 - a^7*b^4*d^7
*e^4 - 4*a*b^10*c^7*e^3*f + 28*a^2*b^9*c^6*d*e^3*f - 84*a^3*b^8*c^5*d^2*e^
3*f + 140*a^4*b^7*c^4*d^3*e^3*f - 140*a^5*b^6*c^3*d^4*e^3*f + 84*a^6*b^5*c
^2*d^5*e^3*f - 28*a^7*b^4*c*d^6*e^3*f + 4*a^8*b^3*d^7*e^3*f + 6*a^2*b^9*c^
7*e^2*f^2 - 42*a^3*b^8*c^6*d*e^2*f^2 + 126*a^4*b^7*c^5*d^2*e^2*f^2 - 210*a
^5*b^6*c^4*d^3*e^2*f^2 + 210*a^6*b^5*c^3*d^4*e^2*f^2 - 126*a^7*b^4*c^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(1/2)*(a + b*x)^5*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 92781, normalized size of antiderivative = 44.67

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(1/2),x)
```

output
```
(3465*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**9*b*c**5*d**4*f**7*h - 10395*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e
 + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c**4*d**5*e*f**6*h + 6930*sqr
t(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
9*b*c**4*d**5*f**7*h*x + 10395*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c**3*d**6*e**2*f**5*h - 20790*sqrt(b
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b
*c**3*d**6*e*f**6*h*x + 3465*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c**3*d**6*f**7*h*x**2 - 3465*sqrt(b)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c*
*2*d**7*e**3*f**4*h + 20790*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c**2*d**7*e**2*f**5*h*x - 10395*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*
c**2*d**7*e*f**6*h*x**2 - 6930*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c*d**8*e**3*f**4*h*x + 10395*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*
c*d**8*e**2*f**5*h*x**2 - 3465*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*d**9*e**3*f**4*h*x**2 + 2772*sqrt(b)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*
*2*c**6*d**3*f**7*h - 19404*sqrt(b)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)...
```

3.150 $\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 337

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = -\frac{2(be-af)^3(de-cf)(fg-eh)}{f^6\sqrt{e+fx}} - \frac{2(be-af)^2(bde(4fg-5eh)-bcf(3fg-4eh)-af(dfg-2deh+cfh))\sqrt{e+fx}}{f^6} - \frac{2(be-af)\left(a^2df^2h+abf(3dfg-8deh+3cfh)-b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)(e+fx)^{3/2}}{3f^6} + \frac{2b(3a^2df^2h+3abf(dfg-4deh+cfh)-b^2(2de(2fg-5eh)-cf(fg-4eh)))(e+fx)^{5/2}}{5f^6} + \frac{2b^2(3adfh+b(dfg-5deh+cfh))(e+fx)^{7/2}}{7f^6} + \frac{2b^3dh(e+fx)^{9/2}}{9f^6}$$

output
```
-2*(-a*f+b*e)^3*(-c*f+d*e)*(-e*h+f*g)/f^6/(f*x+e)^(1/2)-2*(-a*f+b*e)^2*(b*
d*e*(-5*e*h+4*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)
^(1/2)/f^6-2/3*(-a*f+b*e)*(a^2*d*f^2*h+a*b*f*(3*c*f*h-8*d*e*h+3*d*f*g)-b^2
*(2*d*e*(-5*e*h+3*f*g)-3*c*f*(-2*e*h+f*g)))*(f*x+e)^(3/2)/f^6+2/5*b*(3*a^2
*d*f^2*h+3*a*b*f*(c*f*h-4*d*e*h+d*f*g)-b^2*(2*d*e*(-5*e*h+2*f*g)-c*f*(-4*e
*h+f*g)))*(f*x+e)^(5/2)/f^6+2/7*b^2*(3*a*d*f*h+b*(c*f*h-5*d*e*h+d*f*g))*(f
*x+e)^(7/2)/f^6+2/9*b^3*d*h*(f*x+e)^(9/2)/f^6
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(105a^3f^3(3cf(-fg+2eh+fhx)+d(-8e^2h+ef(6g-4hx)+f^2x(3$$

input
```
Integrate[((a + b*x)^3*(c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(105*a^3*f^3*(3*c*f*(-(f*g) + 2*e*h + f*h*x) + d*(-8*e^2*h + e*f*(6*g -
 4*h*x) + f^2*x*(3*g + h*x))) + 63*a^2*b*f^2*(5*c*f*(-8*e^2*h + e*f*(6*g -
 4*h*x) + f^2*x*(3*g + h*x)) + d*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x
^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x))) + 9*a*b^2*f*(7*c*f*(48*e^3*h
 - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x
)) - 3*d*(128*e^4*h - 16*e^3*f*(7*g - 4*h*x) - 8*e^2*f^2*x*(7*g + 2*h*x) +
 2*e*f^3*x^2*(7*g + 4*h*x) - f^4*x^3*(7*g + 5*h*x))) + b^3*(9*c*f*(-128*e^
4*h + 16*e^3*f*(7*g - 4*h*x) + 8*e^2*f^2*x*(7*g + 2*h*x) - 2*e*f^3*x^2*(7*
g + 4*h*x) + f^4*x^3*(7*g + 5*h*x)) + d*(1280*e^5*h - 128*e^4*f*(9*g - 5*h
*x) + 16*e^2*f^3*x^2*(9*g + 5*h*x) - 32*e^3*f^2*x*(18*g + 5*h*x) + 5*f^5*x
^4*(9*g + 7*h*x) - 2*e*f^4*x^3*(36*g + 25*h*x)))))/(315*f^6*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{b(e+fx)^{3/2}\left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh)\right)\right)\right)}{f^5}+\frac{\sqrt{e+fx}(be$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b(e+fx)^{5/2}\left(3a^2df^2h+3abf(cfh-4deh+dfg)-\left(b^2(2de(2fg-5eh)-cf(fg-4eh))\right)\right)}{5f^6}-\frac{2(e+fx)^{3/2}(be-af)\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(2de(3fg-5eh)-3cf(fg-2eh))\right)\right)}{3f^6}+\frac{2b^2(e+fx)^{7/2}(3adfh+b(cfh-5deh+dfg))}{7f^6}-\frac{2\sqrt{e+fx}(be-af)^2(-af(cfh-2deh+dfg)-bcf(3fg-4eh)+bde(4fg-5eh))}{f^6}-\frac{2(be-af)^3(de-cf)(fg-eh)}{f^6\sqrt{e+fx}}+\frac{2b^3dh(e+fx)^{9/2}}{9f^6}$$

input
```
Int[((a + b*x)^3*(c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(-2*(b*e - a*f)^3*(d*e - c*f)*(f*g - e*h))/(f^6*Sqrt[e + f*x]) - (2*(b*e -
 a*f)^2*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(3*f*g - 4*e*h) - a*f*(d*f*g - 2*d*
e*h + c*f*h))*Sqrt[e + f*x])/f^6 - (2*(b*e - a*f)*(a^2*d*f^2*h + a*b*f*(3*
d*f*g - 8*d*e*h + 3*c*f*h) - b^2*(2*d*e*(3*f*g - 5*e*h) - 3*c*f*(f*g - 2*e
*h)))*(e + f*x)^(3/2))/(3*f^6) + (2*b*(3*a^2*d*f^2*h + 3*a*b*f*(d*f*g - 4*
d*e*h + c*f*h) - b^2*(2*d*e*(2*f*g - 5*e*h) - c*f*(f*g - 4*e*h)))*(e + f*x
)^(5/2))/(5*f^6) + (2*b^2*(3*a*d*f*h + b*(d*f*g - 5*d*e*h + c*f*h))*(e + f
*x)^(7/2))/(7*f^6) + (2*b^3*d*h*(e + f*x)^(9/2))/(9*f^6)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$2\left(\frac{x^3\left(\frac{5dh\,x^2}{9}+\frac{5(ch+dg)x}{7}+cg\right)b^3}{5}+a\,x^2\left(\frac{3dh\,x^2}{7}+\frac{3(ch+dg)x}{5}+cg\right)b^2+3a^2x\left(\frac{dh\,x^2}{5}+\frac{(ch+dg)x}{3}+cg\right)b-a^3\left(-\frac{dh\,x^2}{3}+(-c\right.\right.$
risch	$2(35dh\,b^3x^4f^4+135a\,b^2d\,f^4h\,x^3+45b^3c\,f^4h\,x^3-85b^3de\,f^3h\,x^3+45b^3d\,f^4g\,x^3+189a^2bd\,f^4h\,x^2+189a\,b^2c\,f^4h\,x^2-351$
gosper	$4a^3de\,f^4g+\frac{512}{63}b^3d\,e^5h-\frac{12}{5}a\,b^2de\,f^4g\,x^2+\frac{48}{5}a^2bd\,e^2f^3hx+\frac{48}{5}a\,b^2c\,e^2f^3hx-8a^2bde\,f^4gx-8a\,b^2ce\,f^4gx-\frac{384}{35}a\,b^2d\,e^3$
trager	$4a^3de\,f^4g+\frac{512}{63}b^3d\,e^5h-\frac{12}{5}a\,b^2de\,f^4g\,x^2+\frac{48}{5}a^2bd\,e^2f^3hx+\frac{48}{5}a\,b^2c\,e^2f^3hx-8a^2bde\,f^4gx-8a\,b^2ce\,f^4gx-\frac{384}{35}a\,b^2d\,e^3$
orering	$4a^3de\,f^4g+\frac{512}{63}b^3d\,e^5h-\frac{12}{5}a\,b^2de\,f^4g\,x^2+\frac{48}{5}a^2bd\,e^2f^3hx+\frac{48}{5}a\,b^2c\,e^2f^3hx-8a^2bde\,f^4gx-8a\,b^2ce\,f^4gx-\frac{384}{35}a\,b^2d\,e^3$
derivativedivides	$\frac{2dh\,b^3(fx+e)^{\frac{9}{2}}}{9}+12a\,b^2d\,e^2fh(fx+e)^{\frac{3}{2}}-6a\,b^2de\,f^2g(fx+e)^{\frac{3}{2}}-6a\,b^2ce\,f^2h(fx+e)^{\frac{3}{2}}-24a\,b^2d\,e^3fh\sqrt{fx+e}+18a^2bd\,e^2$
default	$\frac{2dh\,b^3(fx+e)^{\frac{9}{2}}}{9}+12a\,b^2d\,e^2fh(fx+e)^{\frac{3}{2}}-6a\,b^2de\,f^2g(fx+e)^{\frac{3}{2}}-6a\,b^2ce\,f^2h(fx+e)^{\frac{3}{2}}-24a\,b^2d\,e^3fh\sqrt{fx+e}+18a^2bd\,e^2$

input
```
int((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/(f*x+e)^(1/2)*(1/2*(1/5*x^3*(5/9*d*h*x^2+5/7*(c*h+d*g)*x+c*g)*b^3+a*x^2*
(3/7*d*h*x^2+3/5*(c*h+d*g)*x+c*g)*b^2+3*a^2*x*(1/5*d*h*x^2+1/3*(c*h+d*g)*x
+c*g)*b-a^3*(-1/3*d*h*x^2+(-c*h-d*g)*x+c*g))*f^5+(-1/5*x^2*(25/63*d*h*x^2+
4/7*(c*h+d*g)*x+c*g)*b^3-2*a*x*(6/35*d*h*x^2+3/10*(c*h+d*g)*x+c*g)*b^2+3*a
^2*(-1/5*d*h*x^2+2/3*(-c*h-d*g)*x+c*g)*b+a^3*(c*h+d*g-2/3*d*h*x))*e*f^4-4/
3*(-3/5*x*(10/63*d*h*x^2+2/7*(c*h+d*g)*x+c*g)*b^3+3*a*(-6/35*d*h*x^2+3/5*(
-c*h-d*g)*x+c*g)*b^2+3*a^2*(-3/5*d*h*x+c*h+d*g)*b+d*h*a^3)*e^2*f^3+24/5*(1
/3*(-10/63*d*h*x^2+4/7*(-c*h-d*g)*x+c*g)*b^2+a*(-4/7*d*h*x+c*h+d*g)*b+a^2*
d*h)*b*e^3*f^2-192/35*b^2*e^4*(1/3*(-5/9*d*h*x+c*h+d*g)*b+a*d*h)*f+128/63*
b^3*d*e^5*h)/f^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(317) = 634$.

Time = 0.13 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\,(35\,b^3df^5hx^5+5\,(9\,b^3df^5g-(10\,b^3def^4-9\,(b^3c+3\,ab^2d)f^5)h)x^4-(\cdots}{\cdots}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^3*d*f^5*h*x^5 + 5*(9*b^3*d*f^5*g - (10*b^3*d*e*f^4 - 9*(b^3*c
+ 3*a*b^2*d)*f^5)*h)*x^4 - (9*(8*b^3*d*e*f^4 - 7*(b^3*c + 3*a*b^2*d)*f^5)*
g - (80*b^3*d*e^2*f^3 - 72*(b^3*c + 3*a*b^2*d)*e*f^4 + 189*(a*b^2*c + a^2*
b*d)*f^5)*h)*x^3 + (9*(16*b^3*d*e^2*f^3 - 14*(b^3*c + 3*a*b^2*d)*e*f^4 + 3
5*(a*b^2*c + a^2*b*d)*f^5)*g - (160*b^3*d*e^3*f^2 - 144*(b^3*c + 3*a*b^2*d
)*e^2*f^3 + 378*(a*b^2*c + a^2*b*d)*e*f^4 - 105*(3*a^2*b*c + a^3*d)*f^5)*h
)*x^2 - 9*(128*b^3*d*e^4*f + 35*a^3*c*f^5 - 112*(b^3*c + 3*a*b^2*d)*e^3*f^
2 + 280*(a*b^2*c + a^2*b*d)*e^2*f^3 - 70*(3*a^2*b*c + a^3*d)*e*f^4)*g + 2*
(640*b^3*d*e^5 + 315*a^3*c*e*f^4 - 576*(b^3*c + 3*a*b^2*d)*e^4*f + 1512*(a
*b^2*c + a^2*b*d)*e^3*f^2 - 420*(3*a^2*b*c + a^3*d)*e^2*f^3)*h - (9*(64*b^
3*d*e^3*f^2 - 56*(b^3*c + 3*a*b^2*d)*e^2*f^3 + 140*(a*b^2*c + a^2*b*d)*e*f
^4 - 35*(3*a^2*b*c + a^3*d)*f^5)*g - (640*b^3*d*e^4*f + 315*a^3*c*f^5 - 57
6*(b^3*c + 3*a*b^2*d)*e^3*f^2 + 1512*(a*b^2*c + a^2*b*d)*e^2*f^3 - 420*(3*
a^2*b*c + a^3*d)*e*f^4)*h)*x)*sqrt(f*x + e)/(f^7*x + e*f^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(352) = 704$.

Time = 62.60 (sec) , antiderivative size = 804, normalized size of antiderivative = 2.39

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b^3dh(e+fx)^{\frac{9}{2}}}{9f^5}+\frac{(e+fx)^{\frac{7}{2}}\cdot\left(3ab^2dfh+b^3cfh-5b^3deh+b^3dfg\right)}{7f^5}+\frac{(e+fx)^{\frac{5}{2}}\cdot\left(3a^2bdf^2h+3ab^2cf^2h-12\cdots\right.}{\cdots}\cdots\right.}{\cdots} & \cdots \\ \frac{a^3cgx+\frac{b^3dhx^6}{6}+\frac{x^5\cdot\left(3ab^2dh+b^3ch+b^3dg\right)}{5}+\frac{x^4\cdot\left(3a^2bdh+3ab^2ch+3ab^2dg+b^3cg\right)}{4}+\frac{x^3\left(a^3dh+3a^2bch\cdots\right.}{\cdots}\cdots}{e^{\frac{3}{2}}} & \cdots \end{cases}$$

input
```
integrate((b*x+a)**3*(d*x+c)*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b**3*d*h*(e + f*x)**(9/2)/(9*f**5) + (e + f*x)**(7/2)*(3*a*b
**2*d*f*h + b**3*c*f*h - 5*b**3*d*e*h + b**3*d*f*g)/(7*f**5) + (e + f*x)**
(5/2)*(3*a**2*b*d*f**2*h + 3*a*b**2*c*f**2*h - 12*a*b**2*d*e*f*h + 3*a*b**
2*d*f**2*g - 4*b**3*c*e*f*h + b**3*c*f**2*g + 10*b**3*d*e**2*h - 4*b**3*d*
e*f*g)/(5*f**5) + (e + f*x)**(3/2)*(a**3*d*f**3*h + 3*a**2*b*c*f**3*h - 9*
a**2*b*d*e*f**2*h + 3*a**2*b*d*f**3*g - 9*a*b**2*c*e*f**2*h + 3*a*b**2*c*f
**3*g + 18*a*b**2*d*e**2*f*h - 9*a*b**2*d*e*f**2*g + 6*b**3*c*e**2*f*h - 3
*b**3*c*e*f**2*g - 10*b**3*d*e**3*h + 6*b**3*d*e**2*f*g)/(3*f**5) + sqrt(e
 + f*x)*(a**3*c*f**4*h - 2*a**3*d*e*f**3*h + a**3*d*f**4*g - 6*a**2*b*c*e*
f**3*h + 3*a**2*b*c*f**4*g + 9*a**2*b*d*e**2*f**2*h - 6*a**2*b*d*e*f**3*g
+ 9*a*b**2*c*e**2*f**2*h - 6*a*b**2*c*e*f**3*g - 12*a*b**2*d*e**3*f*h + 9*
a*b**2*d*e**2*f**2*g - 4*b**3*c*e**3*f*h + 3*b**3*c*e**2*f**2*g + 5*b**3*d
*e**4*h - 4*b**3*d*e**3*f*g)/f**5 + (a*f - b*e)**3*(c*f - d*e)*(e*h - f*g)
/(f**5*sqrt(e + f*x)))/f, Ne(f, 0)), ((a**3*c*g*x + b**3*d*h*x**6/6 + x**5
*(3*a*b**2*d*h + b**3*c*h + b**3*d*g)/5 + x**4*(3*a**2*b*d*h + 3*a*b**2*c*
h + 3*a*b**2*d*g + b**3*c*g)/4 + x**3*(a**3*d*h + 3*a**2*b*c*h + 3*a**2*b*
d*g + 3*a*b**2*c*g)/3 + x**2*(a**3*c*h + a**3*d*g + 3*a**2*b*c*g)/2)/e**(3
/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(fx+e)^{\frac{9}{2}}b^3dh+45\,(b^3dfg-(5\,b^3de-(b^3c+3\,ab^2d)\,f)\,h)\,(fx+e)^{\frac{7}{2}}-63\,((4\,b^3def-(b^3c+3\,ab^2 \cdots}{\cdots}\right)}{\cdots}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(f*x + e)^(9/2)*b^3*d*h + 45*(b^3*d*f*g - (5*b^3*d*e - (b^3*c +
 3*a*b^2*d)*f)*h)*(f*x + e)^(7/2) - 63*((4*b^3*d*e*f - (b^3*c + 3*a*b^2*d)
*f^2)*g - (10*b^3*d*e^2 - 4*(b^3*c + 3*a*b^2*d)*e*f + 3*(a*b^2*c + a^2*b*d
)*f^2)*h)*(f*x + e)^(5/2) + 105*(3*(2*b^3*d*e^2*f - (b^3*c + 3*a*b^2*d)*e*
f^2 + (a*b^2*c + a^2*b*d)*f^3)*g - (10*b^3*d*e^3 - 6*(b^3*c + 3*a*b^2*d)*e
^2*f + 9*(a*b^2*c + a^2*b*d)*e*f^2 - (3*a^2*b*c + a^3*d)*f^3)*h)*(f*x + e)
^(3/2) - 315*((4*b^3*d*e^3*f - 3*(b^3*c + 3*a*b^2*d)*e^2*f^2 + 6*(a*b^2*c
+ a^2*b*d)*e*f^3 - (3*a^2*b*c + a^3*d)*f^4)*g - (5*b^3*d*e^4 + a^3*c*f^4 -
 4*(b^3*c + 3*a*b^2*d)*e^3*f + 9*(a*b^2*c + a^2*b*d)*e^2*f^2 - 2*(3*a^2*b*
c + a^3*d)*e*f^3)*h)*sqrt(f*x + e))/f^5 - 315*((b^3*d*e^4*f + a^3*c*f^5 -
(b^3*c + 3*a*b^2*d)*e^3*f^2 + 3*(a*b^2*c + a^2*b*d)*e^2*f^3 - (3*a^2*b*c +
 a^3*d)*e*f^4)*g - (b^3*d*e^5 + a^3*c*e*f^4 - (b^3*c + 3*a*b^2*d)*e^4*f +
3*(a*b^2*c + a^2*b*d)*e^3*f^2 - (3*a^2*b*c + a^3*d)*e^2*f^3)*h)/(sqrt(f*x
+ e)*f^5))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(317) = 634$.

Time = 0.15 (sec) , antiderivative size = 965, normalized size of antiderivative = 2.86

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-2*(b^3*d*e^4*f*g - b^3*c*e^3*f^2*g - 3*a*b^2*d*e^3*f^2*g + 3*a*b^2*c*e^2*
f^3*g + 3*a^2*b*d*e^2*f^3*g - 3*a^2*b*c*e*f^4*g - a^3*d*e*f^4*g + a^3*c*f^
5*g - b^3*d*e^5*h + b^3*c*e^4*f*h + 3*a*b^2*d*e^4*f*h - 3*a*b^2*c*e^3*f^2*
h - 3*a^2*b*d*e^3*f^2*h + 3*a^2*b*c*e^2*f^3*h + a^3*d*e^2*f^3*h - a^3*c*e*
f^4*h)/(sqrt(f*x + e)*f^6) + 2/315*(45*(f*x + e)^(7/2)*b^3*d*f^49*g - 252*
(f*x + e)^(5/2)*b^3*d*e*f^49*g + 630*(f*x + e)^(3/2)*b^3*d*e^2*f^49*g - 12
60*sqrt(f*x + e)*b^3*d*e^3*f^49*g + 63*(f*x + e)^(5/2)*b^3*c*f^50*g + 189*
(f*x + e)^(5/2)*a*b^2*d*f^50*g - 315*(f*x + e)^(3/2)*b^3*c*e*f^50*g - 945*
(f*x + e)^(3/2)*a*b^2*d*e*f^50*g + 945*sqrt(f*x + e)*b^3*c*e^2*f^50*g + 28
35*sqrt(f*x + e)*a*b^2*d*e^2*f^50*g + 315*(f*x + e)^(3/2)*a*b^2*c*f^51*g +
 315*(f*x + e)^(3/2)*a^2*b*d*f^51*g - 1890*sqrt(f*x + e)*a*b^2*c*e*f^51*g
- 1890*sqrt(f*x + e)*a^2*b*d*e*f^51*g + 945*sqrt(f*x + e)*a^2*b*c*f^52*g +
 315*sqrt(f*x + e)*a^3*d*f^52*g + 35*(f*x + e)^(9/2)*b^3*d*f^48*h - 225*(f
*x + e)^(7/2)*b^3*d*e*f^48*h + 630*(f*x + e)^(5/2)*b^3*d*e^2*f^48*h - 1050
*(f*x + e)^(3/2)*b^3*d*e^3*f^48*h + 1575*sqrt(f*x + e)*b^3*d*e^4*f^48*h +
45*(f*x + e)^(7/2)*b^3*c*f^49*h + 135*(f*x + e)^(7/2)*a*b^2*d*f^49*h - 252
*(f*x + e)^(5/2)*b^3*c*e*f^49*h - 756*(f*x + e)^(5/2)*a*b^2*d*e*f^49*h + 6
30*(f*x + e)^(3/2)*b^3*c*e^2*f^49*h + 1890*(f*x + e)^(3/2)*a*b^2*d*e^2*f^4
9*h - 1260*sqrt(f*x + e)*b^3*c*e^3*f^49*h - 3780*sqrt(f*x + e)*a*b^2*d*e^3
*f^49*h + 189*(f*x + e)^(5/2)*a*b^2*c*f^50*h + 189*(f*x + e)^(5/2)*a^2*...
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{(e+f\,x)^{5/2}\,(2\,b^3\,c\,f^2\,g+20\,b^3\,d\,e^2\,h+6\,a\,b^2\,c\,f^2\,h+6\,a\,b^2\,d\,f^2\,g+6\,a^2}{5\,f^6}$$
$$-\frac{2\,a^3\,c\,f^5\,g-2\,b^3\,d\,e^5\,h-2\,a^3\,c\,e\,f^4\,h-2\,a^3\,d\,e\,f^4\,g+2\,b^3\,c\,e^4\,f\,h+2\,b^3\,d\,e^4\,f\,g-2\,b^3\,c\,e^3\,f^2\,g+2\,a^3\,d\,e^2}{}$$
$$+\frac{(e+f\,x)^{7/2}\,(2\,b^3\,c\,f\,h-10\,b^3\,d\,e\,h+2\,b^3\,d\,f\,g+6\,a\,b^2\,d\,f\,h)}{7\,f^6}$$
$$+\frac{2\,(e+f\,x)^{3/2}\,(a\,f-b\,e)\,\left(3\,b^2\,c\,f^2\,g+a^2\,d\,f^2\,h+10\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b\right.}{3\,f^6}$$
$$+\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)^2\,(a\,c\,f^2\,h+a\,d\,f^2\,g+3\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-4\,b\,c\,e\,f\,h-4\,b\,d\,e\,f\,g)}{f^6}$$
$$+\frac{2\,b^3\,d\,h\,(e+f\,x)^{9/2}}{9\,f^6}$$

input
```
int(((g + h*x)*(a + b*x)^3*(c + d*x))/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(5/2)*(2*b^3*c*f^2*g + 20*b^3*d*e^2*h + 6*a*b^2*c*f^2*h + 6*a*b
^2*d*f^2*g + 6*a^2*b*d*f^2*h - 8*b^3*c*e*f*h - 8*b^3*d*e*f*g - 24*a*b^2*d*
e*f*h))/(5*f^6) - (2*a^3*c*f^5*g - 2*b^3*d*e^5*h - 2*a^3*c*e*f^4*h - 2*a^3
*d*e*f^4*g + 2*b^3*c*e^4*f*h + 2*b^3*d*e^4*f*g - 2*b^3*c*e^3*f^2*g + 2*a^3
*d*e^2*f^3*h - 6*a^2*b*c*e*f^4*g + 6*a*b^2*d*e^4*f*h + 6*a*b^2*c*e^2*f^3*g
 - 6*a*b^2*c*e^3*f^2*h - 6*a*b^2*d*e^3*f^2*g + 6*a^2*b*c*e^2*f^3*h + 6*a^2
*b*d*e^2*f^3*g - 6*a^2*b*d*e^3*f^2*h)/(f^6*(e + f*x)^(1/2)) + ((e + f*x)^(
7/2)*(2*b^3*c*f*h - 10*b^3*d*e*h + 2*b^3*d*f*g + 6*a*b^2*d*f*h))/(7*f^6) +
 (2*(e + f*x)^(3/2)*(a*f - b*e)*(3*b^2*c*f^2*g + a^2*d*f^2*h + 10*b^2*d*e^
2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^2*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 8*a*
b*d*e*f*h))/(3*f^6) + (2*(e + f*x)^(1/2)*(a*f - b*e)^2*(a*c*f^2*h + a*d*f^
2*g + 3*b*c*f^2*g + 5*b*d*e^2*h - 2*a*d*e*f*h - 4*b*c*e*f*h - 4*b*d*e*f*g)
)/f^6 + (2*b^3*d*h*(e + f*x)^(9/2))/(9*f^6)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.29

$$\int \frac{(a+bx)^3(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(630*a**3*c*e*f**4*h - 315*a**3*c*f**5*g + 315*a**3*c*f**5*h*x - 840*a*
*3*d*e**2*f**3*h + 630*a**3*d*e*f**4*g - 420*a**3*d*e*f**4*h*x + 315*a**3*
d*f**5*g*x + 105*a**3*d*f**5*h*x**2 - 2520*a**2*b*c*e**2*f**3*h + 1890*a**
2*b*c*e*f**4*g - 1260*a**2*b*c*e*f**4*h*x + 945*a**2*b*c*f**5*g*x + 315*a*
*2*b*c*f**5*h*x**2 + 3024*a**2*b*d*e**3*f**2*h - 2520*a**2*b*d*e**2*f**3*g
 + 1512*a**2*b*d*e**2*f**3*h*x - 1260*a**2*b*d*e*f**4*g*x - 378*a**2*b*d*e
*f**4*h*x**2 + 315*a**2*b*d*f**5*g*x**2 + 189*a**2*b*d*f**5*h*x**3 + 3024*
a*b**2*c*e**3*f**2*h - 2520*a*b**2*c*e**2*f**3*g + 1512*a*b**2*c*e**2*f**3
*h*x - 1260*a*b**2*c*e*f**4*g*x - 378*a*b**2*c*e*f**4*h*x**2 + 315*a*b**2*
c*f**5*g*x**2 + 189*a*b**2*c*f**5*h*x**3 - 3456*a*b**2*d*e**4*f*h + 3024*a
*b**2*d*e**3*f**2*g - 1728*a*b**2*d*e**3*f**2*h*x + 1512*a*b**2*d*e**2*f**
3*g*x + 432*a*b**2*d*e**2*f**3*h*x**2 - 378*a*b**2*d*e*f**4*g*x**2 - 216*a
*b**2*d*e*f**4*h*x**3 + 189*a*b**2*d*f**5*g*x**3 + 135*a*b**2*d*f**5*h*x**
4 - 1152*b**3*c*e**4*f*h + 1008*b**3*c*e**3*f**2*g - 576*b**3*c*e**3*f**2*
h*x + 504*b**3*c*e**2*f**3*g*x + 144*b**3*c*e**2*f**3*h*x**2 - 126*b**3*c*
e*f**4*g*x**2 - 72*b**3*c*e*f**4*h*x**3 + 63*b**3*c*f**5*g*x**3 + 45*b**3*
c*f**5*h*x**4 + 1280*b**3*d*e**5*h - 1152*b**3*d*e**4*f*g + 640*b**3*d*e**
4*f*h*x - 576*b**3*d*e**3*f**2*g*x - 160*b**3*d*e**3*f**2*h*x**2 + 144*b**
3*d*e**2*f**3*g*x**2 + 80*b**3*d*e**2*f**3*h*x**3 - 72*b**3*d*e*f**4*g*x**
3 - 50*b**3*d*e*f**4*h*x**4 + 45*b**3*d*f**5*g*x**4 + 35*b**3*d*f**5*h*...
```

3.151 $\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 243

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(be-af)^2(de-cf)(fg-eh)}{f^5\sqrt{e+fx}} + \frac{2(be-af)(bde(3fg-4eh)-bcf(2fg-3eh)-af(dfg-2deh+cfh))\sqrt{e+fx}}{f^5} + \frac{2(a^2df^2h+2abf(dfg-3deh+cfh)+b^2(cf(fg-3eh)-3de(fg-2eh)))(e+fx)^{3/2}}{3f^5} + \frac{2b(2adfh+b(dfg-4deh+cfh))(e+fx)^{5/2}}{5f^5} + \frac{2b^2dh(e+fx)^{7/2}}{7f^5}$$

output

```
2*(-a*f+b*e)^2*(-c*f+d*e)*(-e*h+f*g)/f^5/(f*x+e)^(1/2)+2*(-a*f+b*e)*(b*d*e
*(-4*e*h+3*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(1
/2)/f^5+2/3*(a^2*d*f^2*h+2*a*b*f*(c*f*h-3*d*e*h+d*f*g)+b^2*(c*f*(-3*e*h+f*
g)-3*d*e*(-2*e*h+f*g)))*(f*x+e)^(3/2)/f^5+2/5*b*(2*a*d*f*h+b*(c*f*h-4*d*e*
h+d*f*g))*(f*x+e)^(5/2)/f^5+2/7*b^2*d*h*(f*x+e)^(7/2)/f^5
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{70a^2f^2(3cf(-fg+2eh+fhx)+d(-8e^2h+ef(6g-4hx)+f^2x(3g-$$

input
```
Integrate[((a + b*x)^2*(c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(70*a^2*f^2*(3*c*f*(-(f*g) + 2*e*h + f*h*x) + d*(-8*e^2*h + e*f*(6*g - 4*h
*x) + f^2*x*(3*g + h*x))) + 28*a*b*f*(5*c*f*(-8*e^2*h + e*f*(6*g - 4*h*x)
+ f^2*x*(3*g + h*x)) + d*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g
+ 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x))) + b^2*(14*c*f*(48*e^3*h - 8*e^2*f*(5
*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x)) - 6*d*(128
*e^4*h - 16*e^3*f*(7*g - 4*h*x) - 8*e^2*f^2*x*(7*g + 2*h*x) + 2*e*f^3*x^2*
(7*g + 4*h*x) - f^4*x^3*(7*g + 5*h*x))))/(105*f^5*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 159}$$

$$\int \left(\frac{\sqrt{e+fx}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{f^4}+\frac{b(e+fx)^{3/2}(2adfh+$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(e+fx)^{3/2}\left(a^2df^2h+2abf(cfh-3deh+dfg)+b^2(cf(fg-3eh)-3de(fg-2eh))\right)}{3f^5}+\frac{2b(e+fx)^{5/2}(2adfh+b(cfh-4deh+dfg))}{5f^5}+\frac{2\sqrt{e+fx}(be-af)(-af(cfh-2deh+dfg)-bcf(2fg-3eh)+bde(3fg-4eh))}{f^5}+\frac{2(be-af)^2(de-cf)(fg-eh)}{f^5\sqrt{e+fx}}+\frac{2b^2dh(e+fx)^{7/2}}{7f^5}$$

input
```
Int[((a + b*x)^2*(c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(b*e - a*f)^2*(d*e - c*f)*(f*g - e*h))/(f^5*Sqrt[e + f*x]) + (2*(b*e -
a*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(2*f*g - 3*e*h) - a*f*(d*f*g - 2*d*e*h
 + c*f*h))*Sqrt[e + f*x])/f^5 + (2*(a^2*d*f^2*h + 2*a*b*f*(d*f*g - 3*d*e*h
 + c*f*h) + b^2*(c*f*(f*g - 3*e*h) - 3*d*e*(f*g - 2*e*h)))*(e + f*x)^(3/2)
)/(3*f^5) + (2*b*(2*a*d*f*h + b*(d*f*g - 4*d*e*h + c*f*h))*(e + f*x)^(5/2)
)/(5*f^5) + (2*b^2*d*h*(e + f*x)^(7/2))/(7*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$4\left(\frac{x^2\left(\frac{3dh\,x^2}{7}+\frac{3(ch+dg)x}{5}+cg\right)b^2}{6}+ax\left(\frac{dh\,x^2}{5}+\frac{(ch+dg)x}{3}+cg\right)b-\frac{\left(-\frac{dh\,x^2}{3}+(-ch-dg)x+cg\right)a^2}{2}\right)f^4+4\left(\left(-\frac{4dh\,x^3}{35}+\frac{(-c\cdots}{}\right.\right.$
risch	$2(15dh\,b^2x^3f^3+42abd\,f^3h\,x^2+21b^2c\,f^3h\,x^2-39b^2de\,f^2h\,x^2+21b^2d\,f^3g\,x^2+35a^2d\,f^3hx+70abc\,f^3hx-126abde\,f^2hx\cdots$
gosper	$8abce\,f^3g+\frac{2}{7}dh\,b^2x^4f^4-\frac{8}{3}a^2de\,f^3hx+\frac{32}{35}b^2d\,e^2f^2h\,x^2-\frac{4}{5}b^2de\,f^3g\,x^2-\frac{16}{3}a^2d\,e^2f^2h-2cg\,a^2f^4-\frac{8}{5}abde\,f^3h\,x^2+\frac{32}{5}ab\cdots$
trager	$8abce\,f^3g+\frac{2}{7}dh\,b^2x^4f^4-\frac{8}{3}a^2de\,f^3hx+\frac{32}{35}b^2d\,e^2f^2h\,x^2-\frac{4}{5}b^2de\,f^3g\,x^2-\frac{16}{3}a^2d\,e^2f^2h-2cg\,a^2f^4-\frac{8}{5}abde\,f^3h\,x^2+\frac{32}{5}ab\cdots$
orering	$8abce\,f^3g+\frac{2}{7}dh\,b^2x^4f^4-\frac{8}{3}a^2de\,f^3hx+\frac{32}{35}b^2d\,e^2f^2h\,x^2-\frac{4}{5}b^2de\,f^3g\,x^2-\frac{16}{3}a^2d\,e^2f^2h-2cg\,a^2f^4-\frac{8}{5}abde\,f^3h\,x^2+\frac{32}{5}ab\cdots$
derivativedivides	$-2b^2defg(fx+e)^{\frac{3}{2}}+\frac{4abdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{4abc\,f^2h(fx+e)^{\frac{3}{2}}}{3}-4a^2de\,f^2h\sqrt{fx+e}-8abce\,f^2h\sqrt{fx+e}+12abd\,e^2fh\sqrt{fx+e}-8\cdots$
default	$-2b^2defg(fx+e)^{\frac{3}{2}}+\frac{4abdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{4abc\,f^2h(fx+e)^{\frac{3}{2}}}{3}-4a^2de\,f^2h\sqrt{fx+e}-8abce\,f^2h\sqrt{fx+e}+12abd\,e^2fh\sqrt{fx+e}-8\cdots$

input
```
int((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*((1/6*x^2*(3/7*d*h*x^2+3/5*(c*h+d*g)*x+c*g)*b^2+a*x*(1/5*d*h*x^2+1/3*(c*
h+d*g)*x+c*g)*b-1/2*(-1/3*d*h*x^2+(-c*h-d*g)*x+c*g)*a^2)*f^4+((-4/35*d*h*x
^3+1/5*(-c*h-d*g)*x^2-2/3*c*g*x)*b^2+2*a*(-1/5*d*h*x^2+2/3*(-c*h-d*g)*x+c*
g)*b+a^2*(c*h+d*g-2/3*d*h*x))*e*f^3-4/3*((-6/35*d*h*x^2+3/5*(-c*h-d*g)*x+c
*g)*b^2+2*a*(-3/5*d*h*x+c*h+d*g)*b+a^2*d*h)*e^2*f^2+16/5*((-2/7*d*h*x+1/2*
c*h+1/2*d*g)*b+a*d*h)*b*e^3*f-64/35*b^2*d*e^4*h)/(f*x+e)^(1/2)/f^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\,(15\,b^2df^4hx^4+3\,(7\,b^2df^4g-(8\,b^2def^3-7\,(b^2c+2\,abd)f^4)h)x^3-(7\,\cdots}{\cdots}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^2*d*f^4*h*x^4 + 3*(7*b^2*d*f^4*g - (8*b^2*d*e*f^3 - 7*(b^2*c +
 2*a*b*d)*f^4)*h)*x^3 - (7*(6*b^2*d*e*f^3 - 5*(b^2*c + 2*a*b*d)*f^4)*g - (
48*b^2*d*e^2*f^2 - 42*(b^2*c + 2*a*b*d)*e*f^3 + 35*(2*a*b*c + a^2*d)*f^4)*
h)*x^2 + 7*(48*b^2*d*e^3*f - 15*a^2*c*f^4 - 40*(b^2*c + 2*a*b*d)*e^2*f^2 +
 30*(2*a*b*c + a^2*d)*e*f^3)*g - 2*(192*b^2*d*e^4 - 105*a^2*c*e*f^3 - 168*
(b^2*c + 2*a*b*d)*e^3*f + 140*(2*a*b*c + a^2*d)*e^2*f^2)*h + (7*(24*b^2*d*
e^2*f^2 - 20*(b^2*c + 2*a*b*d)*e*f^3 + 15*(2*a*b*c + a^2*d)*f^4)*g - (192*
b^2*d*e^3*f - 105*a^2*c*f^4 - 168*(b^2*c + 2*a*b*d)*e^2*f^2 + 140*(2*a*b*c
 + a^2*d)*e*f^3)*h)*x)*sqrt(f*x + e)/(f^6*x + e*f^5)
```

Sympy [A] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b^2dh(e+fx)^{\frac{7}{2}}}{7f^4} + \frac{(e+fx)^{\frac{5}{2}}\cdot\left(2abdfh+b^2cfh-4b^2deh+b^2dfg\right)}{5f^4} + \frac{(e+fx)^{\frac{3}{2}}\left(a^2df^2h+2abcf^2h-6abdefh\right.}{}\right.}{} \\ \frac{a^2cgx+\frac{b^2dhx^5}{5}+\frac{x^4\cdot\left(2abdh+b^2ch+b^2dg\right)}{4}+\frac{x^3\left(a^2dh+2abch+2abdg+b^2cg\right)}{3}+\frac{x^2\left(a^2ch+a^2dg+2abcg\right)}{2}}{e^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b**2*d*h*(e + f*x)**(7/2)/(7*f**4) + (e + f*x)**(5/2)*(2*a*b
*d*f*h + b**2*c*f*h - 4*b**2*d*e*h + b**2*d*f*g)/(5*f**4) + (e + f*x)**(3/
2)*(a**2*d*f**2*h + 2*a*b*c*f**2*h - 6*a*b*d*e*f*h + 2*a*b*d*f**2*g - 3*b*
*2*c*e*f*h + b**2*c*f**2*g + 6*b**2*d*e**2*h - 3*b**2*d*e*f*g)/(3*f**4) +
sqrt(e + f*x)*(a**2*c*f**3*h - 2*a**2*d*e*f**2*h + a**2*d*f**3*g - 4*a*b*c
*e*f**2*h + 2*a*b*c*f**3*g + 6*a*b*d*e**2*f*h - 4*a*b*d*e*f**2*g + 3*b**2*
c*e**2*f*h - 2*b**2*c*e*f**2*g - 4*b**2*d*e**3*h + 3*b**2*d*e**2*f*g)/f**4
 + (a*f - b*e)**2*(c*f - d*e)*(e*h - f*g)/(f**4*sqrt(e + f*x)))/f, Ne(f, 0
)), ((a**2*c*g*x + b**2*d*h*x**5/5 + x**4*(2*a*b*d*h + b**2*c*h + b**2*d*g
)/4 + x**3*(a**2*d*h + 2*a*b*c*h + 2*a*b*d*g + b**2*c*g)/3 + x**2*(a**2*c*
h + a**2*d*g + 2*a*b*c*g)/2)/e**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(fx+e)^{\frac{7}{2}}b^2dh+21\,(b^2dfg-(4\,b^2de-(b^2c+2\,abd)\,f)\,h)\,(fx+e)^{\frac{5}{2}}-35\,((3\,b^2def-(b^2c+2\,abd}{}\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(f*x + e)^(7/2)*b^2*d*h + 21*(b^2*d*f*g - (4*b^2*d*e - (b^2*c +
 2*a*b*d)*f)*h)*(f*x + e)^(5/2) - 35*((3*b^2*d*e*f - (b^2*c + 2*a*b*d)*f^2
)*g - (6*b^2*d*e^2 - 3*(b^2*c + 2*a*b*d)*e*f + (2*a*b*c + a^2*d)*f^2)*h)*(
f*x + e)^(3/2) + 105*((3*b^2*d*e^2*f - 2*(b^2*c + 2*a*b*d)*e*f^2 + (2*a*b*
c + a^2*d)*f^3)*g - (4*b^2*d*e^3 - a^2*c*f^3 - 3*(b^2*c + 2*a*b*d)*e^2*f +
 2*(2*a*b*c + a^2*d)*e*f^2)*h)*sqrt(f*x + e))/f^4 + 105*((b^2*d*e^3*f - a^
2*c*f^4 - (b^2*c + 2*a*b*d)*e^2*f^2 + (2*a*b*c + a^2*d)*e*f^3)*g - (b^2*d*
e^4 - a^2*c*e*f^3 - (b^2*c + 2*a*b*d)*e^3*f + (2*a*b*c + a^2*d)*e^2*f^2)*h
)/(sqrt(f*x + e)*f^4))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(229) = 458.

Time = 0.14 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\,(b^2de^3fg-b^2ce^2f^2g-2\,abde^2f^2g+2\,abcef^3g+a^2def^3g-a^2cf^4g-}{\sqrt{fx+}} + \frac{2\left(21\,(fx+e)^{\frac{5}{2}}b^2df^{31}g-105\,(fx+e)^{\frac{3}{2}}b^2def^{31}g+315\,\sqrt{fx+e}b^2de^2f^{31}g+35\,(fx+e)^{\frac{3}{2}}b^2cf^{32}g+70\,(f\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^2*d*e^3*f*g - b^2*c*e^2*f^2*g - 2*a*b*d*e^2*f^2*g + 2*a*b*c*e*f^3*g +
 a^2*d*e*f^3*g - a^2*c*f^4*g - b^2*d*e^4*h + b^2*c*e^3*f*h + 2*a*b*d*e^3*f
*h - 2*a*b*c*e^2*f^2*h - a^2*d*e^2*f^2*h + a^2*c*e*f^3*h)/(sqrt(f*x + e)*f
^5) + 2/105*(21*(f*x + e)^(5/2)*b^2*d*f^31*g - 105*(f*x + e)^(3/2)*b^2*d*e
*f^31*g + 315*sqrt(f*x + e)*b^2*d*e^2*f^31*g + 35*(f*x + e)^(3/2)*b^2*c*f^
32*g + 70*(f*x + e)^(3/2)*a*b*d*f^32*g - 210*sqrt(f*x + e)*b^2*c*e*f^32*g
- 420*sqrt(f*x + e)*a*b*d*e*f^32*g + 210*sqrt(f*x + e)*a*b*c*f^33*g + 105*
sqrt(f*x + e)*a^2*d*f^33*g + 15*(f*x + e)^(7/2)*b^2*d*f^30*h - 84*(f*x + e
)^(5/2)*b^2*d*e*f^30*h + 210*(f*x + e)^(3/2)*b^2*d*e^2*f^30*h - 420*sqrt(f
*x + e)*b^2*d*e^3*f^30*h + 21*(f*x + e)^(5/2)*b^2*c*f^31*h + 42*(f*x + e)^
(5/2)*a*b*d*f^31*h - 105*(f*x + e)^(3/2)*b^2*c*e*f^31*h - 210*(f*x + e)^(3
/2)*a*b*d*e*f^31*h + 315*sqrt(f*x + e)*b^2*c*e^2*f^31*h + 630*sqrt(f*x + e
)*a*b*d*e^2*f^31*h + 70*(f*x + e)^(3/2)*a*b*c*f^32*h + 35*(f*x + e)^(3/2)*
a^2*d*f^32*h - 420*sqrt(f*x + e)*a*b*c*e*f^32*h - 210*sqrt(f*x + e)*a^2*d*
e*f^32*h + 105*sqrt(f*x + e)*a^2*c*f^33*h)/f^35
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{(e+f\,x)^{5/2}\,(2\,b^2\,c\,f\,h-8\,b^2\,d\,e\,h+2\,b^2\,d\,f\,g+4\,a\,b\,d\,f\,h)}{5\,f^5}$$
$$+\frac{(e+f\,x)^{3/2}\,(2\,b^2\,c\,f^2\,g+2\,a^2\,d\,f^2\,h+12\,b^2\,d\,e^2\,h+4\,a\,b\,c\,f^2\,h+4\,a\,b\,d\,f^2\,g-6\,b^2\,c\,e\,f\,h-6\,b^2\,d\,e\,f\,g-}{3\,f^5}$$
$$-\frac{2\,a^2\,c\,f^4\,g+2\,b^2\,d\,e^4\,h-2\,a^2\,c\,e\,f^3\,h-2\,a^2\,d\,e\,f^3\,g-2\,b^2\,c\,e^3\,f\,h-2\,b^2\,d\,e^3\,f\,g+2\,b^2\,c\,e^2\,f^2\,g+2\,a^2\,d\,e^2}{f^5\,\sqrt{e+f\,x}}$$
$$+\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)\,\left(a\,c\,f^2\,h+a\,d\,f^2\,g+2\,b\,c\,f^2\,g+4\,b\,d\,e^2\,h-2\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\,h-3\,b\,d\,e\,f\,g\right)}{f^5}$$
$$+\frac{2\,b^2\,d\,h\,(e+f\,x)^{7/2}}{7\,f^5}$$

input
```
int(((g + h*x)*(a + b*x)^2*(c + d*x))/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(5/2)*(2*b^2*c*f*h - 8*b^2*d*e*h + 2*b^2*d*f*g + 4*a*b*d*f*h))/
(5*f^5) + ((e + f*x)^(3/2)*(2*b^2*c*f^2*g + 2*a^2*d*f^2*h + 12*b^2*d*e^2*h
 + 4*a*b*c*f^2*h + 4*a*b*d*f^2*g - 6*b^2*c*e*f*h - 6*b^2*d*e*f*g - 12*a*b*
d*e*f*h))/(3*f^5) - (2*a^2*c*f^4*g + 2*b^2*d*e^4*h - 2*a^2*c*e*f^3*h - 2*a
^2*d*e*f^3*g - 2*b^2*c*e^3*f*h - 2*b^2*d*e^3*f*g + 2*b^2*c*e^2*f^2*g + 2*a
^2*d*e^2*f^2*h - 4*a*b*c*e*f^3*g - 4*a*b*d*e^3*f*h + 4*a*b*c*e^2*f^2*h + 4
*a*b*d*e^2*f^2*g)/(f^5*(e + f*x)^(1/2)) + (2*(e + f*x)^(1/2)*(a*f - b*e)*(
a*c*f^2*h + a*d*f^2*g + 2*b*c*f^2*g + 4*b*d*e^2*h - 2*a*d*e*f*h - 3*b*c*e*
f*h - 3*b*d*e*f*g))/f^5 + (2*b^2*d*h*(e + f*x)^(7/2))/(7*f^5)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^2(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{-\frac{8}{3}a^2de\,f^3hx - \frac{32}{3}abc\,e^2f^2h + 8abce\,f^3g + 4abc\,f^4gx - 2a^2c\,f^4g - \frac{256}{35}b}{}$$

input
```
int((b*x+a)^2*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(210*a**2*c*e*f**3*h - 105*a**2*c*f**4*g + 105*a**2*c*f**4*h*x - 280*a*
*2*d*e**2*f**2*h + 210*a**2*d*e*f**3*g - 140*a**2*d*e*f**3*h*x + 105*a**2*
d*f**4*g*x + 35*a**2*d*f**4*h*x**2 - 560*a*b*c*e**2*f**2*h + 420*a*b*c*e*f
**3*g - 280*a*b*c*e*f**3*h*x + 210*a*b*c*f**4*g*x + 70*a*b*c*f**4*h*x**2 +
 672*a*b*d*e**3*f*h - 560*a*b*d*e**2*f**2*g + 336*a*b*d*e**2*f**2*h*x - 28
0*a*b*d*e*f**3*g*x - 84*a*b*d*e*f**3*h*x**2 + 70*a*b*d*f**4*g*x**2 + 42*a*
b*d*f**4*h*x**3 + 336*b**2*c*e**3*f*h - 280*b**2*c*e**2*f**2*g + 168*b**2*
c*e**2*f**2*h*x - 140*b**2*c*e*f**3*g*x - 42*b**2*c*e*f**3*h*x**2 + 35*b**
2*c*f**4*g*x**2 + 21*b**2*c*f**4*h*x**3 - 384*b**2*d*e**4*h + 336*b**2*d*e
**3*f*g - 192*b**2*d*e**3*f*h*x + 168*b**2*d*e**2*f**2*g*x + 48*b**2*d*e**
2*f**2*h*x**2 - 42*b**2*d*e*f**3*g*x**2 - 24*b**2*d*e*f**3*h*x**3 + 21*b**
2*d*f**4*g*x**3 + 15*b**2*d*f**4*h*x**4))/(105*sqrt(e + f*x)*f**5)
```

3.152 $\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = -\frac{2(be-af)(de-cf)(fg-eh)}{f^4\sqrt{e+fx}} - \frac{2(bde(2fg-3eh)-bcf(fg-2eh)-af(dfg-2deh+cfh))\sqrt{e+fx}}{f^4} + \frac{2(adfh+b(dfg-3deh+cfh))(e+fx)^{3/2}}{3f^4} + \frac{2bdh(e+fx)^{5/2}}{5f^4}$$

output
```
-2*(-a*f+b*e)*(-c*f+d*e)*(-e*h+f*g)/f^4/(f*x+e)^(1/2)-2*(b*d*e*(-3*e*h+2*f
*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(1/2)/f^4+2/3*(a
*d*f*h+b*(c*f*h-3*d*e*h+d*f*g))*(f*x+e)^(3/2)/f^4+2/5*b*d*h*(f*x+e)^(5/2)/
f^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(5af(3cf(-fg+2eh+fhx)+d(-8e^2h+ef(6g-4hx)+f^2x(3g+$$

input
```
Integrate[((a + b*x)*(c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(5*a*f*(3*c*f*(-(f*g) + 2*e*h + f*h*x) + d*(-8*e^2*h + e*f*(6*g - 4*h*x
) + f^2*x*(3*g + h*x))) + b*(5*c*f*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*x*(
3*g + h*x)) + d*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x)
- 2*e*f^2*x*(10*g + 3*h*x)))))/(15*f^4*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{\sqrt{e+fx}(adfh+b(cfh-3deh+dfg))}{f^3}+\frac{af(cfh-2deh+dfg)+bcf(fg-2eh)-bde(2fg-3eh)}{f^3\sqrt{e+fx}}+\frac{(af$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{3/2}(adfh+b(cfh-3deh+dfg))}{3f^4}-\frac{2\sqrt{e+fx}(-af(cfh-2deh+dfg)-bcf(fg-2eh)+bde(2fg-3eh))}{f^4}-\frac{2(be-af)(de-cf)(fg-eh)}{f^4\sqrt{e+fx}}+\frac{2bdh(e+fx)^{5/2}}{5f^4}$$

input
```
Int[((a + b*x)*(c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(-2*(b*e - a*f)*(d*e - c*f)*(f*g - e*h))/(f^4*Sqrt[e + f*x]) - (2*(b*d*e*(
2*f*g - 3*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(d*f*g - 2*d*e*h + c*f*h))*Sqrt
[e + f*x])/f^4 + (2*(a*d*f*h + b*(d*f*g - 3*d*e*h + c*f*h))*(e + f*x)^(3/2
))/(3*f^4) + (2*b*d*h*(e + f*x)^(5/2))/(5*f^4)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\left(\left(\left(6h\,x^{3}+10g\,x^{2}\right)d+30x\left(\frac{hx}{3}+g\right)c\right)b-30a\left(\left(-\frac{1}{3}h\,x^{2}-gx\right)d+c\left(-hx+g\right)\right)\right)f^{3}+60\left(\left(\left(-\frac{1}{5}h\,x^{2}-\frac{2}{3}gx\right)d+c\left(-\frac{2hx}{3}+g\right)\right)\right.}{15\sqrt{fx+e}\,f^{4}}$
risch	$\frac{2\left(3f^{2}bdh\,x^{2}+5f^{2}adhx+5f^{2}bchx-9efbdhx+5f^{2}bdgx+15ach\,f^{2}-25adefh+15adg\,f^{2}-25bcefh+15bc\,f^{2}g+33bd\,e^{2}h\right.}{15f^{4}}$
gosper	$\frac{\frac{2}{5}bdh\,x^{3}f^{3}+\frac{2}{3}ad\,f^{3}h\,x^{2}+\frac{2}{3}bc\,f^{3}h\,x^{2}-\frac{4}{5}bde\,f^{2}h\,x^{2}+\frac{2}{3}bd\,f^{3}g\,x^{2}+2ac\,f^{3}hx-\frac{8}{3}ade\,f^{2}hx+2ad\,f^{3}gx-\frac{8}{3}bce\,f^{2}hx+2bc\,f}{\sqrt{fx}}$
trager	$\frac{\frac{2}{5}bdh\,x^{3}f^{3}+\frac{2}{3}ad\,f^{3}h\,x^{2}+\frac{2}{3}bc\,f^{3}h\,x^{2}-\frac{4}{5}bde\,f^{2}h\,x^{2}+\frac{2}{3}bd\,f^{3}g\,x^{2}+2ac\,f^{3}hx-\frac{8}{3}ade\,f^{2}hx+2ad\,f^{3}gx-\frac{8}{3}bce\,f^{2}hx+2bc\,f}{\sqrt{fx}}$
orering	$\frac{\frac{2}{5}bdh\,x^{3}f^{3}+\frac{2}{3}ad\,f^{3}h\,x^{2}+\frac{2}{3}bc\,f^{3}h\,x^{2}-\frac{4}{5}bde\,f^{2}h\,x^{2}+\frac{2}{3}bd\,f^{3}g\,x^{2}+2ac\,f^{3}hx-\frac{8}{3}ade\,f^{2}hx+2ad\,f^{3}gx-\frac{8}{3}bce\,f^{2}hx+2bc\,f}{\sqrt{fx}}$
derivativedivides	$\frac{\frac{2dbh(fx+e)^{\frac{5}{2}}}{5}+\frac{2adfh(fx+e)^{\frac{3}{2}}}{3}+\frac{2bcfh(fx+e)^{\frac{3}{2}}}{3}-2bdeh(fx+e)^{\frac{3}{2}}+\frac{2bdfg(fx+e)^{\frac{3}{2}}}{3}+2ac\,f^{2}h\sqrt{fx+e}-4adefh\sqrt{fx+e}+2a}{}$
default	$\frac{\frac{2dbh(fx+e)^{\frac{5}{2}}}{5}+\frac{2adfh(fx+e)^{\frac{3}{2}}}{3}+\frac{2bcfh(fx+e)^{\frac{3}{2}}}{3}-2bdeh(fx+e)^{\frac{3}{2}}+\frac{2bdfg(fx+e)^{\frac{3}{2}}}{3}+2ac\,f^{2}h\sqrt{fx+e}-4adefh\sqrt{fx+e}+2a}{}$

input
```
int((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((((6*h*x^3+10*g*x^2)*d+30*x*(1/3*h*x+g)*c)*b-30*a*((-1/3*h*x^2-g*x)*
d+c*(-h*x+g)))*f^3+60*(((-1/5*h*x^2-2/3*g*x)*d+c*(-2/3*h*x+g))*b+a*((-2/3*
h*x+g)*d+c*h))*e*f^2-80*(((-3/5*h*x+g)*d+c*h)*b+a*d*h)*e^2*f+96*b*d*e^3*h)
/(f*x+e)^(1/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\,(3\,bdf^3hx^3 + (5\,bdf^3g - (6\,bdef^2 - 5\,(bc+ad)f^3)h)x^2 - 5\,(8\,bde^2f + 3}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*b*d*f^3*h*x^3 + (5*b*d*f^3*g - (6*b*d*e*f^2 - 5*(b*c + a*d)*f^3)*h
)*x^2 - 5*(8*b*d*e^2*f + 3*a*c*f^3 - 6*(b*c + a*d)*e*f^2)*g + 2*(24*b*d*e^
3 + 15*a*c*e*f^2 - 20*(b*c + a*d)*e^2*f)*h - (5*(4*b*d*e*f^2 - 3*(b*c + a*
d)*f^3)*g - (24*b*d*e^2*f + 15*a*c*f^3 - 20*(b*c + a*d)*e*f^2)*h)*x)*sqrt(
f*x + e)/(f^5*x + e*f^4)
```

Sympy [A] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{bdh(e+fx)^{\frac{5}{2}}}{5f^3} + \frac{(e+fx)^{\frac{3}{2}}(adfh+bcfh-3bdeh+bdfg)}{3f^3} + \frac{\sqrt{e+fx}\left(acf^2h-2adefh+adf^2g-2bcefh+bcf^2g-\right.}{f^3}\right.}{f} \\ \frac{acgx+\frac{bdhx^4}{4}+\frac{x^3(adh+bch+bdg)}{3}+\frac{x^2(ach+adg+bcg)}{2}}{e^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b*d*h*(e + f*x)**(5/2)/(5*f**3) + (e + f*x)**(3/2)*(a*d*f*h
+ b*c*f*h - 3*b*d*e*h + b*d*f*g)/(3*f**3) + sqrt(e + f*x)*(a*c*f**2*h - 2*
a*d*e*f*h + a*d*f**2*g - 2*b*c*e*f*h + b*c*f**2*g + 3*b*d*e**2*h - 2*b*d*e
*f*g)/f**3 + (a*f - b*e)*(c*f - d*e)*(e*h - f*g)/(f**3*sqrt(e + f*x)))/f,
Ne(f, 0)), ((a*c*g*x + b*d*h*x**4/4 + x**3*(a*d*h + b*c*h + b*d*g)/3 + x**
2*(a*c*h + a*d*g + b*c*g)/2)/e**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(fx+e)^{\frac{5}{2}}bdh+5\,(bdfg-(3\,bde-(bc+ad)f)h)(fx+e)^{\frac{3}{2}}-15\,((2\,bdef-(bc+ad)f^2)g-(3\,bde^2+a}{f^3}\right.}{1}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(f*x + e)^(5/2)*b*d*h + 5*(b*d*f*g - (3*b*d*e - (b*c + a*d)*f)*h)
*(f*x + e)^(3/2) - 15*((2*b*d*e*f - (b*c + a*d)*f^2)*g - (3*b*d*e^2 + a*c*
f^2 - 2*(b*c + a*d)*e*f)*h)*sqrt(f*x + e))/f^3 - 15*((b*d*e^2*f + a*c*f^3
- (b*c + a*d)*e*f^2)*g - (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h)/(sqr
t(f*x + e)*f^3))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx =$$
$$-\frac{2\,(bde^2fg-bcef^2g-adef^2g+acf^3g-bde^3h+bce^2fh+ade^2fh-acef^2h)}{\sqrt{fx+e}\,f^4}$$
$$+\frac{2\left(5\,(fx+e)^{\frac{3}{2}}bdf^{17}g-30\,\sqrt{fx+e}bdef^{17}g+15\,\sqrt{fx+e}bcf^{18}g+15\,\sqrt{fx+e}adf^{18}g+3\,(fx+e)^{\frac{5}{2}}bdf^{16}\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-2*(b*d*e^2*f*g - b*c*e*f^2*g - a*d*e*f^2*g + a*c*f^3*g - b*d*e^3*h + b*c*
e^2*f*h + a*d*e^2*f*h - a*c*e*f^2*h)/(sqrt(f*x + e)*f^4) + 2/15*(5*(f*x +
e)^(3/2)*b*d*f^17*g - 30*sqrt(f*x + e)*b*d*e*f^17*g + 15*sqrt(f*x + e)*b*c
*f^18*g + 15*sqrt(f*x + e)*a*d*f^18*g + 3*(f*x + e)^(5/2)*b*d*f^16*h - 15*
(f*x + e)^(3/2)*b*d*e*f^16*h + 45*sqrt(f*x + e)*b*d*e^2*f^16*h + 5*(f*x +
e)^(3/2)*b*c*f^17*h + 5*(f*x + e)^(3/2)*a*d*f^17*h - 30*sqrt(f*x + e)*b*c*
e*f^17*h - 30*sqrt(f*x + e)*a*d*e*f^17*h + 15*sqrt(f*x + e)*a*c*f^18*h)/f^
20
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{\sqrt{e+f\,x}\,(2\,a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+6\,b\,d\,e^2\,h-4\,a\,d\,e\,f\,h-4\,b}{f^4}$$
$$+\frac{(e+f\,x)^{3/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-6\,b\,d\,e\,h+2\,b\,d\,f\,g)}{3\,f^4}$$
$$-\frac{2\,a\,c\,f^3\,g-2\,b\,d\,e^3\,h-2\,a\,c\,e\,f^2\,h-2\,a\,d\,e\,f^2\,g-2\,b\,c\,e\,f^2\,g+2\,a\,d\,e^2\,f\,h+2\,b\,c\,e^2\,f\,h+2\,b\,d\,e^2\,f\,g}{f^4\,\sqrt{e+f\,x}}$$
$$+\frac{2\,b\,d\,h\,(e+f\,x)^{5/2}}{5\,f^4}$$

input
```
int(((g + h*x)*(a + b*x)*(c + d*x))/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(1/2)*(2*a*c*f^2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 6*b*d*e^2*h -
4*a*d*e*f*h - 4*b*c*e*f*h - 4*b*d*e*f*g))/f^4 + ((e + f*x)^(3/2)*(2*a*d*f*
h + 2*b*c*f*h - 6*b*d*e*h + 2*b*d*f*g))/(3*f^4) - (2*a*c*f^3*g - 2*b*d*e^3
*h - 2*a*c*e*f^2*h - 2*a*d*e*f^2*g - 2*b*c*e*f^2*g + 2*a*d*e^2*f*h + 2*b*c
*e^2*f*h + 2*b*d*e^2*f*g)/(f^4*(e + f*x)^(1/2)) + (2*b*d*h*(e + f*x)^(5/2)
)/(5*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{\frac{2}{5}bd\,f^3h\,x^3+\frac{2}{3}ad\,f^3h\,x^2+\frac{2}{3}bc\,f^3h\,x^2-\frac{4}{5}bde\,f^2h\,x^2+\frac{2}{3}bd\,f^3g\,x^2+2ac\,}{}$$

input
```
int((b*x+a)*(d*x+c)*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(30*a*c*e*f**2*h - 15*a*c*f**3*g + 15*a*c*f**3*h*x - 40*a*d*e**2*f*h +
30*a*d*e*f**2*g - 20*a*d*e*f**2*h*x + 15*a*d*f**3*g*x + 5*a*d*f**3*h*x**2
- 40*b*c*e**2*f*h + 30*b*c*e*f**2*g - 20*b*c*e*f**2*h*x + 15*b*c*f**3*g*x
+ 5*b*c*f**3*h*x**2 + 48*b*d*e**3*h - 40*b*d*e**2*f*g + 24*b*d*e**2*f*h*x
- 20*b*d*e*f**2*g*x - 6*b*d*e*f**2*h*x**2 + 5*b*d*f**3*g*x**2 + 3*b*d*f**3
*h*x**3))/(15*sqrt(e + f*x)*f**4)
```

3.153 $\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(de-cf)(fg-eh)}{f^3\sqrt{e+fx}} + \frac{2(dfg-2deh+cfh)\sqrt{e+fx}}{f^3} + \frac{2dh(e+fx)^{3/2}}{3f^3}$$

output
```
2*(-c*f+d*e)*(-e*h+f*g)/f^3/(f*x+e)^(1/2)+2*(c*f*h-2*d*e*h+d*f*g)*(f*x+e)^
(1/2)/f^3+2/3*d*h*(f*x+e)^(3/2)/f^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{6cf(-fg+2eh+fhx)+2d(-8e^2h+ef(6g-4hx)+f^2x(3g+hx))}{3f^3\sqrt{e+fx}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(6*c*f*(-(f*g) + 2*e*h + f*h*x) + 2*d*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*
x*(3*g + h*x)))/(3*f^3*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}} \, dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{cfh-2deh+dfg}{f^2\sqrt{e+fx}}+\frac{(cf-de)(fg-eh)}{f^2(e+fx)^{3/2}}+\frac{dh\sqrt{e+fx}}{f^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2\sqrt{e+fx}(cfh-2deh+dfg)}{f^3}+\frac{2(de-cf)(fg-eh)}{f^3\sqrt{e+fx}}+\frac{2dh(e+fx)^{3/2}}{3f^3}$$

input
```
Int[((c + d*x)*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(d*e - c*f)*(f*g - e*h))/(f^3*Sqrt[e + f*x]) + (2*(d*f*g - 2*d*e*h + c*
f*h)*Sqrt[e + f*x])/f^3 + (2*d*h*(e + f*x)^(3/2))/(3*f^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$\frac{((2h\,x^2+6gx)d-6c(-hx+g))f^2+12e\left(\left(-\frac{2hx}{3}+g\right)d+ch\right)f-16d\,e^2h}{3\sqrt{fx+e}\,f^3}$	64
gosper	$\frac{\frac{2}{3}dh\,x^2f^2+2c\,f^2hx-\frac{8}{3}defhx+2d\,f^2gx+4cefh-2cg\,f^2-\frac{16}{3}d\,e^2h+4defg}{\sqrt{fx+e}\,f^3}$	72
trager	$\frac{\frac{2}{3}dh\,x^2f^2+2c\,f^2hx-\frac{8}{3}defhx+2d\,f^2gx+4cefh-2cg\,f^2-\frac{16}{3}d\,e^2h+4defg}{\sqrt{fx+e}\,f^3}$	72
risch	$\frac{2(fdhx+3cfh-5deh+3dfg)\sqrt{fx+e}}{3f^3}+\frac{2cefh-2cg\,f^2-2d\,e^2h+2defg}{\sqrt{fx+e}\,f^3}$	72
orering	$\frac{\frac{2}{3}dh\,x^2f^2+2c\,f^2hx-\frac{8}{3}defhx+2d\,f^2gx+4cefh-2cg\,f^2-\frac{16}{3}d\,e^2h+4defg}{\sqrt{fx+e}\,f^3}$	72
derivativedivides	$\frac{\frac{2h(fx+e)^{\frac{3}{2}}d}{3}+2cfh\sqrt{fx+e}-4deh\sqrt{fx+e}+2dfg\sqrt{fx+e}-\frac{2\left(-cefh+cg\,f^2+d\,e^2h-defg\right)}{\sqrt{fx+e}}}{f^3}$	86
default	$\frac{\frac{2h(fx+e)^{\frac{3}{2}}d}{3}+2cfh\sqrt{fx+e}-4deh\sqrt{fx+e}+2dfg\sqrt{fx+e}-\frac{2\left(-cefh+cg\,f^2+d\,e^2h-defg\right)}{\sqrt{fx+e}}}{f^3}$	86

input
```
int((d*x+c)*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(((2*h*x^2+6*g*x)*d-6*c*(-h*x+g))*f^2+12*e*((-2/3*h*x+g)*d+c*h)*f-16*d
*e^2*h)/(f*x+e)^(1/2)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(df^2hx^2+3\left(2\,def-cf^2\right)g-2\left(4\,de^2-3\,cef\right)h+\left(3\,df^2g-\left(4\,def-3\,cf^2\right)h\right)x}{3\left(f^4x+ef^3\right)}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(d*f^2*h*x^2 + 3*(2*d*e*f - c*f^2)*g - 2*(4*d*e^2 - 3*c*e*f)*h + (3*d*
f^2*g - (4*d*e*f - 3*c*f^2)*h)*x)*sqrt(f*x + e)/(f^4*x + e*f^3)
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{dh(e+fx)^{\frac{3}{2}}}{3f^2}+\frac{\sqrt{e+fx}(cfh-2deh+dfg)}{f^2}+\frac{(cf-de)(eh-fg)}{f^2\sqrt{e+fx}}\right)}{f} & \text{for } f\neq 0 \\ \frac{cgx+\frac{dhx^3}{3}+\frac{x^2(ch+dg)}{2}}{e^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(d*h*(e + f*x)**(3/2)/(3*f**2) + sqrt(e + f*x)*(c*f*h - 2*d*e
*h + d*f*g)/f**2 + (c*f - d*e)*(e*h - f*g)/(f**2*sqrt(e + f*x)))/f, Ne(f,
0)), ((c*g*x + d*h*x**3/3 + x**2*(c*h + d*g)/2)/e**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(\frac{(fx+e)^{\frac{3}{2}}dh+3\,(dfg-(2\,de-cf)h)\sqrt{fx+e}}{f^2}+\frac{3\,((def-cf^2)g-(de^2-cef)h)}{\sqrt{fx+e}f^2}\right)}{3\,f}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((f*x + e)^(3/2)*d*h + 3*(d*f*g - (2*d*e - c*f)*h)*sqrt(f*x + e))/f^2
 + 3*((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h)/(sqrt(f*x + e)*f^2))/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(defg - cf^2g - de^2h + cefh\right)}{\sqrt{fx+e}f^3} + \frac{2\left(3\sqrt{fx+e}df^7g + (fx+e)^{\frac{3}{2}}df^6h - 6\sqrt{fx+e}def^6h + 3\sqrt{fx+e}cf^7h\right)}{3f^9}$$

input
```
integrate((d*x+c)*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(d*e*f*g - c*f^2*g - d*e^2*h + c*e*f*h)/(sqrt(f*x + e)*f^3) + 2/3*(3*sqr
t(f*x + e)*d*f^7*g + (f*x + e)^(3/2)*d*f^6*h - 6*sqrt(f*x + e)*d*e*f^6*h +
 3*sqrt(f*x + e)*c*f^7*h)/f^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{\frac{2\,d\,h\,(e+f\,x)^2}{3} - 2\,c\,f^2\,g - 2\,d\,e^2\,h + 2\,c\,f\,h\,(e+f\,x) - 4\,d\,e\,h\,(e+f\,x) + 2\,d\,f\,g\,(e}{f^3\,\sqrt{e+f\,x}}$$

input
```
int(((g + h*x)*(c + d*x))/(e + f*x)^(3/2),x)
```

output
```
((2*d*h*(e + f*x)^2)/3 - 2*c*f^2*g - 2*d*e^2*h + 2*c*f*h*(e + f*x) - 4*d*e
*h*(e + f*x) + 2*d*f*g*(e + f*x) + 2*c*e*f*h + 2*d*e*f*g)/(f^3*(e + f*x)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{\frac{2}{3}d\,f^2h\,x^2+2c\,f^2hx-\frac{8}{3}defhx+2d\,f^2gx+4cefh-2c\,f^2g-\frac{16}{3}d\,e^2h+4defg}{\sqrt{fx+e}\,f^3}$$

input
```
int((d*x+c)*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(6*c*e*f*h - 3*c*f**2*g + 3*c*f**2*h*x - 8*d*e**2*h + 6*d*e*f*g - 4*d*e
*f*h*x + 3*d*f**2*g*x + d*f**2*h*x**2))/(3*sqrt(e + f*x)*f**3)
```

3.154 $\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = -\frac{2(de-cf)(fg-eh)}{f^2(be-af)\sqrt{e+fx}} + \frac{2dh\sqrt{e+fx}}{bf^2} - \frac{2(bc-ad)(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(be-af)^{3/2}}$$

output
```
-2*(-c*f+d*e)*(-e*h+f*g)/f^2/(-a*f+b*e)/(f*x+e)^(1/2)+2*d*h*(f*x+e)^(1/2)/
b/f^2-2*(-a*d+b*c)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/
2))/b^(3/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{2(bcf(-fg+eh)+adfh(e+fx)+bde(-2eh+f(g-hx)))}{bf^2(-be+af)\sqrt{e+fx}} - \frac{2(bc-ad)(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{3/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(b*c*f*(-(f*g) + e*h) + a*d*f*h*(e + f*x) + b*d*e*(-2*e*h + f*(g - h*x)
)))/(b*f^2*(-(b*e) + a*f)*Sqrt[e + f*x]) - (2*(b*c - a*d)*(b*g - a*h)*ArcT
an[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(3/2)*(-(b*e) + a*f)^(3
/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{163}$$

$$\frac{(bc-ad)(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b(be-af)}+\frac{2\left(-ef(adh+bch+bdg)+dfhx(be-af)+bcf^2g+2bde^2h\right)}{bf^2\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \text{73}$$

$$\frac{2(bc-ad)(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf(be-af)}+\frac{2\left(-ef(adh+bch+bdg)+dfhx(be-af)+bcf^2g+2bde^2h\right)}{bf^2\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(-ef(adh+bch+bdg)+dfhx(be-af)+bcf^2g+2bde^2h\right)}{bf^2\sqrt{e+fx}(be-af)}-\frac{2(bc-ad)(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(be-af)^{3/2}}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(b*c*f^2*g + 2*b*d*e^2*h - e*f*(b*d*g + b*c*h + a*d*h) + d*f*(b*e - a*f
)*h*x))/(b*f^2*(b*e - a*f)*Sqrt[e + f*x]) - (2*(b*c - a*d)*(b*g - a*h)*Arc
Tanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(3/2)*(b*e - a*f)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{2dh\sqrt{fx+e}}{b}-\frac{2\left(-cefh+cg\,f^{2}+d\,e^{2}h-defg\right)}{(af-be)\sqrt{fx+e}}-\frac{2f^{2}\left(a^{2}dh-abch-abdg+b^{2}cg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(af-be)b\sqrt{(af-be)b}}}{f^{2}}$
default	$\frac{\frac{2dh\sqrt{fx+e}}{b}-\frac{2\left(-cefh+cg\,f^{2}+d\,e^{2}h-defg\right)}{(af-be)\sqrt{fx+e}}-\frac{2f^{2}\left(a^{2}dh-abch-abdg+b^{2}cg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(af-be)b\sqrt{(af-be)b}}}{f^{2}}$
pseudoelliptic	$\frac{-2\sqrt{fx+e}\,f^{2}(ah-bg)(ad-bc)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+2\sqrt{(af-be)b}\,\left(\left(-cg\,f^{2}+e((-hx+g)d+ch)f-2d\,e^{2}h\right)b+adfh(fx\right.}{f^{2}b(af-be)\sqrt{fx+e}\,\sqrt{(af-be)b}}$
risch	$\frac{2dh\sqrt{fx+e}}{b\,f^{2}}-\frac{2\left(-\frac{\left(cefh-cg\,f^{2}-d\,e^{2}h+defg\right)b}{(af-be)\sqrt{fx+e}}+\frac{f^{2}\left(a^{2}dh-abch-abdg+b^{2}cg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(af-be)\sqrt{(af-be)b}}\right)}{b\,f^{2}}$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^2*(d*h/b*(f*x+e)^(1/2)-(-c*e*f*h+c*f^2*g+d*e^2*h-d*e*f*g)/(a*f-b*e)/(f
*x+e)^(1/2)-f^2*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/(a*f-b*e)/b/((a*f-b*e)*b
)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(109) = 218.

Time = 0.10 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.33

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \left[-\frac{((b^2c-abd)ef^2g-(abc-a^2d)ef^2h+((b^2c-abd)f^3g-(abc-a^2d)f^3h)}{}\right.$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-(((b^2*c - a*b*d)*e*f^2*g - (a*b*c - a^2*d)*e*f^2*h + ((b^2*c - a*b*d)*f
^3*g - (a*b*c - a^2*d)*f^3*h)*x)*sqrt(b^2*e - a*b*f)*log((b*f*x + 2*b*e -
a*f + 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*((b^3*d*e^2*f -
2*a*b^2*d*e*f^2 + a^2*b*d*f^3)*h*x - (b^3*d*e^2*f + a*b^2*c*f^3 - (b^3*c +
 a*b^2*d)*e*f^2)*g + (2*b^3*d*e^3 - (b^3*c + 3*a*b^2*d)*e^2*f + (a*b^2*c +
 a^2*b*d)*e*f^2)*h)*sqrt(f*x + e))/(b^4*e^3*f^2 - 2*a*b^3*e^2*f^3 + a^2*b^
2*e*f^4 + (b^4*e^2*f^3 - 2*a*b^3*e*f^4 + a^2*b^2*f^5)*x), 2*(((b^2*c - a*b
*d)*e*f^2*g - (a*b*c - a^2*d)*e*f^2*h + ((b^2*c - a*b*d)*f^3*g - (a*b*c -
a^2*d)*f^3*h)*x)*sqrt(-b^2*e + a*b*f)*arctan(sqrt(-b^2*e + a*b*f)*sqrt(f*x
 + e)/(b*f*x + b*e)) + ((b^3*d*e^2*f - 2*a*b^2*d*e*f^2 + a^2*b*d*f^3)*h*x
- (b^3*d*e^2*f + a*b^2*c*f^3 - (b^3*c + a*b^2*d)*e*f^2)*g + (2*b^3*d*e^3 -
 (b^3*c + 3*a*b^2*d)*e^2*f + (a*b^2*c + a^2*b*d)*e*f^2)*h)*sqrt(f*x + e))/
(b^4*e^3*f^2 - 2*a*b^3*e^2*f^3 + a^2*b^2*e*f^4 + (b^4*e^2*f^3 - 2*a*b^3*e*
f^4 + a^2*b^2*f^5)*x)]
```

Sympy [A] (verification not implemented)

Time = 8.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{(cf-de)(eh-fg)}{f\sqrt{e+fx}(af-be)} + \frac{dh\sqrt{e+fx}}{bf} - \frac{f(ad-bc)(ah-bg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{af-be}{b}}}\right)}{b^2\sqrt{\frac{af-be}{b}}(af-be)}\right)}{f} & \text{for } f \neq 0 \\ \frac{\frac{dhx^2}{2b} + \frac{x(-adh+bch+bdg)}{b^2} + \frac{(ad-bc)(ah-bg)\left(\begin{cases} \frac{x}{a} & \text{for } b=0 \\ \frac{\log(a+bx)}{b} & \text{otherwise} \end{cases}\right)}{b^2}}{e^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*((c*f - d*e)*(e*h - f*g)/(f*sqrt(e + f*x)*(a*f - b*e)) + d*h*
sqrt(e + f*x)/(b*f) - f*(a*d - b*c)*(a*h - b*g)*atan(sqrt(e + f*x)/sqrt((a
*f - b*e)/b))/(b**2*sqrt((a*f - b*e)/b)*(a*f - b*e)))/f, Ne(f, 0)), ((d*h*
x**2/(2*b) + x*(-a*d*h + b*c*h + b*d*g)/b**2 + (a*d - b*c)*(a*h - b*g)*Pie
cewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/b**2)/e**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{2\,(b^2cg-abdg-abch+a^2dh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^2e-abf)\sqrt{-b^2e+abf}} - \frac{2\,(defg-cf^2g-de^2h+cefh)}{(bef^2-af^3)\sqrt{fx+e}} + \frac{2\,\sqrt{fx+e}dh}{bf^2}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^2*c*g - a*b*d*g - a*b*c*h + a^2*d*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2
*e + a*b*f))/((b^2*e - a*b*f)*sqrt(-b^2*e + a*b*f)) - 2*(d*e*f*g - c*f^2*g
 - d*e^2*h + c*e*f*h)/((b*e*f^2 - a*f^3)*sqrt(f*x + e)) + 2*sqrt(f*x + e)*
d*h/(b*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{2\,d\,h\,\sqrt{e+f\,x}}{b\,f^2} - \frac{2\,(b\,c\,f^2\,g+b\,d\,e^2\,h-b\,c\,e\,f\,h-b\,d\,e\,f\,g)}{b\,f^2\,\sqrt{e+f\,x}\,(a\,f-b\,e)} + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{e+f\,x}\,(b^2\,e-a\,b\,f)\,(a\,d-b\,c)\,(a\,h-b\,g)}{\sqrt{b}\,(a\,f-b\,e)^{3/2}\,(2\,b^2\,c\,g+2\,a^2\,d\,h-2\,a\,b\,c\,h-2\,a\,b\,d\,g)}\right)\,(a\,d-b\,c)\,(a\,h-b\,g)}{b^{3/2}\,(a\,f-b\,e)^{3/2}}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(3/2)*(a + b*x)),x)
```

output
```
(2*d*h*(e + f*x)^(1/2))/(b*f^2) - (2*(b*c*f^2*g + b*d*e^2*h - b*c*e*f*h -
b*d*e*f*g))/(b*f^2*(e + f*x)^(1/2)*(a*f - b*e)) + (2*atan((2*(e + f*x)^(1/
2)*(b^2*e - a*b*f)*(a*d - b*c)*(a*h - b*g))/(b^(1/2)*(a*f - b*e)^(3/2)*(2*
b^2*c*g + 2*a^2*d*h - 2*a*b*c*h - 2*a*b*d*g)))*(a*d - b*c)*(a*h - b*g))/(b
^(3/2)*(a*f - b*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.03

$$\int \frac{(c+dx)(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{-2\sqrt{b}\,\sqrt{fx+e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a^2d\,f^2h+2\sqrt{b}\,\sqrt{fx+e}\,\sqrt{af-be}}{}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x)
```

output
```
(2*( - sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**2*d*f**2*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*f**2*h + sqrt(b)*
sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a*b*d*f**2*g - sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c*f**2*g + a**2*b*d*e*f**2*h + a**
2*b*d*f**3*h*x + a*b**2*c*e*f**2*h - a*b**2*c*f**3*g - 3*a*b**2*d*e**2*f*h
 + a*b**2*d*e*f**2*g - 2*a*b**2*d*e*f**2*h*x - b**3*c*e**2*f*h + b**3*c*e*
f**2*g + 2*b**3*d*e**3*h - b**3*d*e**2*f*g + b**3*d*e**2*f*h*x))/(sqrt(e +
 f*x)*b**2*f**2*(a**2*f**2 - 2*a*b*e*f + b**2*e**2))
```

3.155 $\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 180

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}} \, dx = \frac{2(de-cf)(fg-eh)}{f(be-af)^2\sqrt{e+fx}} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{b(be-af)^2(a+bx)} - \frac{(a^2dfh+b^2(2deg-3cfg+2ceh)+ab(dfg-4deh+cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{3/2}(be-af)^{5/2}}$$

output
```
2*(-c*f+d*e)*(-e*h+f*g)/f/(-a*f+b*e)^2/(f*x+e)^(1/2)-(-a*d+b*c)*(-a*h+b*g)
*(f*x+e)^(1/2)/b/(-a*f+b*e)^2/(b*x+a)-(a^2*d*f*h+b^2*(2*c*e*h-3*c*f*g+2*d*
e*g)+a*b*(c*f*h-4*d*e*h+d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(
1/2))/b^(3/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}} \, dx = \frac{-a^2dfh(e+fx)-b^2(2de(-fg+eh)x+cf(eg+3fgx-2ehx))+ab(d(3ef}{bf(be-af)^2(a+bx)\sqrt{e+fx}} + \frac{(a^2dfh+b^2(2deg-3cfg+2ceh)+ab(dfg-4deh+cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{3/2}(-be+af)^{5/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(-(a^2*d*f*h*(e + f*x)) - b^2*(2*d*e*(-(f*g) + e*h)*x + c*f*(e*g + 3*f*g*x
 - 2*e*h*x)) + a*b*(d*(3*e*f*g - 2*e^2*h + f^2*g*x) + c*f*(-2*f*g + 3*e*h
+ f*h*x)))/(b*f*(b*e - a*f)^2*(a + b*x)*Sqrt[e + f*x]) + ((a^2*d*f*h + b^2
*(2*d*e*g - 3*c*f*g + 2*c*e*h) + a*b*(d*f*g - 4*d*e*h + c*f*h))*ArcTan[(Sq
rt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(3/2)*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {161, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 161$$

$$\frac{\left(a^2dfh+ab(cfh-4deh+dfg)+b^2(2ceh-3cfg+2deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2b(be-af)^2}-\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(3fg-2eh)-2de(fg-eh))\right)+a^2defh+2ab\left(-\frac{3}{2}ef(ch+dg)+cf^2g+de^2\right.}{bf(a+bx)\sqrt{e+fx}(be-af)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(a^2dfh+ab(cfh-4deh+dfg)+b^2(2ceh-3cfg+2deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf(be-af)^2}-\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(3fg-2eh)-2de(fg-eh))\right)+a^2defh+2ab\left(-\frac{3}{2}ef(ch+dg)+cf^2g+de^2\right.}{bf(a+bx)\sqrt{e+fx}(be-af)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(a^2dfh+ab(cfh-4deh+dfg)+b^2(2ceh-3cfg+2deg)\right)}{b^{3/2}(be-af)^{5/2}}-\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(3fg-2eh)-2de(fg-eh))\right)+a^2defh+2ab\left(-\frac{3}{2}ef(ch+dg)+cf^2g+de^2\right.}{bf(a+bx)\sqrt{e+fx}(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-((b^2*c*e*f*g + a^2*d*e*f*h + 2*a*b*(c*f^2*g + d*e^2*h - (3*e*f*(d*g + c*
h))/2) + (a^2*d*f^2*h - a*b*f^2*(d*g + c*h) + b^2*(c*f*(3*f*g - 2*e*h) - 2
*d*e*(f*g - e*h)))*x)/(b*f*(b*e - a*f)^2*(a + b*x)*Sqrt[e + f*x])) - ((a^2
*d*f*h + b^2*(2*d*e*g - 3*c*f*g + 2*c*e*h) + a*b*(d*f*g - 4*d*e*h + c*f*h)
)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(b^(3/2)*(b*e - a*f)^(
5/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{2f\left(-\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2b((fx+e)b+af-be)}+\frac{\left(a^2dfh+abcfh-4abdeh+abdfg+2b^2ceh-3b^2cfg+2b^2deg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{2b\sqrt{(af-be)b}}\right)}{(af-be)^2}-}{f}$
default	$\frac{\frac{2f\left(-\frac{f\left(a^2dh-abch-abdg+b^2cg\right)\sqrt{fx+e}}{2b((fx+e)b+af-be)}+\frac{\left(a^2dfh+abcfh-4abdeh+abdfg+2b^2ceh-3b^2cfg+2b^2deg\right)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{2b\sqrt{(af-be)b}}\right)}{(af-be)^2}-}{f}$
pseudoelliptic	$-\frac{-((-3cfg+2e(ch+dg))b^2+a(f(ch+dg)-4deh)b+a^2dfh)(bx+a)\sqrt{fx+e}\,f\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+\sqrt{(af-be)b}\,\left((3a\right.}{\sqrt{fx+e}\,\sqrt{(af-be)b}}$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(f/(a*f-b*e)^2*(-1/2*f*(a^2*d*h-a*b*c*h-a*b*d*g+b^2*c*g)/b*(f*x+e)^(1/
2)/((f*x+e)*b+a*f-b*e)+1/2*(a^2*d*f*h+a*b*c*f*h-4*a*b*d*e*h+a*b*d*f*g+2*b^
2*c*e*h-3*b^2*c*f*g+2*b^2*d*e*g)/b/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1
/2)/((a*f-b*e)*b)^(1/2)))-(-c*e*f*h+c*f^2*g+d*e^2*h-d*e*f*g)/(a*f-b*e)^2/(
f*x+e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(166) = 332$.

Time = 0.14 (sec) , antiderivative size = 1437, normalized size of antiderivative = 7.98

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2*e - a*b*f)*(((2*b^3*d*e*f^2 - (3*b^3*c - a*b^2*d)*f^3)*g +
(2*(b^3*c - 2*a*b^2*d)*e*f^2 + (a*b^2*c + a^2*b*d)*f^3)*h)*x^2 + (2*a*b^2*
d*e^2*f - (3*a*b^2*c - a^2*b*d)*e*f^2)*g + (2*(a*b^2*c - 2*a^2*b*d)*e^2*f
+ (a^2*b*c + a^3*d)*e*f^2)*h + ((2*b^3*d*e^2*f - 3*(b^3*c - a*b^2*d)*e*f^2
 - (3*a*b^2*c - a^2*b*d)*f^3)*g + (2*(b^3*c - 2*a*b^2*d)*e^2*f + 3*(a*b^2*
c - a^2*b*d)*e*f^2 + (a^2*b*c + a^3*d)*f^3)*h)*x)*log((b*f*x + 2*b*e - a*f
 - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*((2*a^2*b^2*c*f^3 -
 (b^4*c - 3*a*b^3*d)*e^2*f - (a*b^3*c + 3*a^2*b^2*d)*e*f^2)*g - (2*a*b^3*d
*e^3 - (3*a*b^3*c + a^2*b^2*d)*e^2*f + (3*a^2*b^2*c - a^3*b*d)*e*f^2)*h +
((2*b^4*d*e^2*f - (3*b^4*c + a*b^3*d)*e*f^2 + (3*a*b^3*c - a^2*b^2*d)*f^3)
*g - (2*b^4*d*e^3 - 2*(b^4*c + a*b^3*d)*e^2*f + (a*b^3*c + a^2*b^2*d)*e*f^
2 + (a^2*b^2*c - a^3*b*d)*f^3)*h)*x)*sqrt(f*x + e))/(a*b^5*e^4*f - 3*a^2*b
^4*e^3*f^2 + 3*a^3*b^3*e^2*f^3 - a^4*b^2*e*f^4 + (b^6*e^3*f^2 - 3*a*b^5*e^
2*f^3 + 3*a^2*b^4*e*f^4 - a^3*b^3*f^5)*x^2 + (b^6*e^4*f - 2*a*b^5*e^3*f^2
+ 2*a^3*b^3*e*f^4 - a^4*b^2*f^5)*x), (sqrt(-b^2*e + a*b*f)*(((2*b^3*d*e*f^
2 - (3*b^3*c - a*b^2*d)*f^3)*g + (2*(b^3*c - 2*a*b^2*d)*e*f^2 + (a*b^2*c +
 a^2*b*d)*f^3)*h)*x^2 + (2*a*b^2*d*e^2*f - (3*a*b^2*c - a^2*b*d)*e*f^2)*g
+ (2*(a*b^2*c - 2*a^2*b*d)*e^2*f + (a^2*b*c + a^3*d)*e*f^2)*h + ((2*b^3*d*
e^2*f - 3*(b^3*c - a*b^2*d)*e*f^2 - (3*a*b^2*c - a^2*b*d)*f^3)*g + (2*(b^3
*c - 2*a*b^2*d)*e^2*f + 3*(a*b^2*c - a^2*b*d)*e*f^2 + (a^2*b*c + a^3*d)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(166) = 332.

Time = 0.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \frac{(2\,b^2deg-3\,b^2cfg+abdfg+2\,b^2ceh-4\,abdeh+abcfh+a^2dfh)\arctan\left(\frac{}{\sqrt{}}\right.}{(b^3e^2-2\,ab^2ef+a^2bf^2)\sqrt{-b^2e+abf}} + \frac{2\,(fx+e)b^2defg-2\,b^2de^2fg-3\,(fx+e)b^2cf^2g+(fx+e)abdf^2g+2\,b^2cef^2g+2\,abdef^2g-2\,abcf^3g}{(b^3e^2f-2\,ab^2ef^2+a^2b}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
(2*b^2*d*e*g - 3*b^2*c*f*g + a*b*d*f*g + 2*b^2*c*e*h - 4*a*b*d*e*h + a*b*c
*f*h + a^2*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*e^2 -
 2*a*b^2*e*f + a^2*b*f^2)*sqrt(-b^2*e + a*b*f)) + (2*(f*x + e)*b^2*d*e*f*g
 - 2*b^2*d*e^2*f*g - 3*(f*x + e)*b^2*c*f^2*g + (f*x + e)*a*b*d*f^2*g + 2*b
^2*c*e*f^2*g + 2*a*b*d*e*f^2*g - 2*a*b*c*f^3*g - 2*(f*x + e)*b^2*d*e^2*h +
 2*b^2*d*e^3*h + 2*(f*x + e)*b^2*c*e*f*h - 2*b^2*c*e^2*f*h - 2*a*b*d*e^2*f
*h + (f*x + e)*a*b*c*f^2*h - (f*x + e)*a^2*d*f^2*h + 2*a*b*c*e*f^2*h)/((b^
3*e^2*f - 2*a*b^2*e*f^2 + a^2*b*f^3)*((f*x + e)^(3/2)*b - sqrt(f*x + e)*b*
e + sqrt(f*x + e)*a*f))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,\left(a^2\,b\,f^2-2\,a\,b^2\,e\,f+b^3\,e^2\right)}{\sqrt{b}\,{\left(a\,f-b\,e\right)}^{5/2}}\right)\,\left(2\,b^2\,c\,e\,h-3\,b^2\,c\,f\,g+2\,b^2\,d\,e\,g+a^2\,d\,f\,h\right.}{b^{3/2}\,{\left(a\,f-b\,e\right)}^{5/2}} - \frac{\frac{2\,\left(c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g\right)}{a\,f-b\,e}-\frac{\left(e+f\,x\right)\,\left(a\,b\,c\,f^2\,h-a^2\,d\,f^2\,h-2\,b^2\,d\,e^2\,h-3\,b^2\,c\,f^2\,g+a\,b\,d\,f^2\,g+2\,b^2\,c\,e\,f\,h+2\,b^2\,d\,e\,f\,g\right)}{b\,{\left(a\,f-b\,e\right)}^2}}{\sqrt{e+f\,x}\,\left(a\,f^2-b\,e\,f\right)+b\,f\,{\left(e+f\,x\right)}^{3/2}}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(3/2)*(a + b*x)^2),x)
```

output
```
(atan(((e + f*x)^(1/2)*(b^3*e^2 + a^2*b*f^2 - 2*a*b^2*e*f))/(b^(1/2)*(a*f
- b*e)^(5/2)))*(2*b^2*c*e*h - 3*b^2*c*f*g + 2*b^2*d*e*g + a^2*d*f*h + a*b*
c*f*h - 4*a*b*d*e*h + a*b*d*f*g))/(b^(3/2)*(a*f - b*e)^(5/2)) - ((2*(c*f^2
*g + d*e^2*h - c*e*f*h - d*e*f*g))/(a*f - b*e) - ((e + f*x)*(a*b*c*f^2*h -
 a^2*d*f^2*h - 2*b^2*d*e^2*h - 3*b^2*c*f^2*g + a*b*d*f^2*g + 2*b^2*c*e*f*h
 + 2*b^2*d*e*f*g))/(b*(a*f - b*e)^2))/((e + f*x)^(1/2)*(a*f^2 - b*e*f) + b
*f*(e + f*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1089, normalized size of antiderivative = 6.05

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x)
```

output

```
(sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**3*d*f**2*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan(
(sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*f**2*h - 4*sqrt(b)*s
qrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**2*b*d*e*f*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
 f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d*f**2*g + sqrt(b)*sqrt(e + f*x
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b
*d*f**2*h*x + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*e*f*h - 3*sqrt(b)*sqrt(e + f*x)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c*f*
*2*g + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a*b**2*c*f**2*h*x + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*e*f*g -
4*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a*b**2*d*e*f*h*x + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*d*f**2*g*x + 2*sqr
t(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*b**3*c*e*f*h*x - 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c*f**2*g*x + 2*sqrt(b)*sq
rt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - ...
```

3.156 $\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 278

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = -\frac{2(de-cf)(fg-eh)}{(be-af)^3\sqrt{e+fx}} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{2b(be-af)^2(a+bx)^2}$$
$$-\frac{(a^2dfh+b^2(4deg-7cfg+4ceh)+ab(3dfg-8deh+3cfh))\sqrt{e+fx}}{4b(be-af)^3(a+bx)}$$
$$+\frac{(a^2df^2h+abf(3dfg-8deh+3cfh)-b^2(3cf(5fg-4eh)-4de(3fg-2eh)))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{4b^{3/2}(be-af)^{7/2}}$$

output
```
-2*(-c*f+d*e)*(-e*h+f*g)/(-a*f+b*e)^3/(f*x+e)^(1/2)-1/2*(-a*d+b*c)*(-a*h+b
*g)*(f*x+e)^(1/2)/b/(-a*f+b*e)^2/(b*x+a)^2-1/4*(a^2*d*f*h+b^2*(4*c*e*h-7*c
*f*g+4*d*e*g)+a*b*(3*c*f*h-8*d*e*h+3*d*f*g))*(f*x+e)^(1/2)/b/(-a*f+b*e)^3/
(b*x+a)+1/4*(a^2*d*f^2*h+a*b*f*(3*c*f*h-8*d*e*h+3*d*f*g)-b^2*(3*c*f*(-4*e*
h+5*f*g)-4*d*e*(-2*e*h+3*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(
1/2))/b^(3/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{-a^3dfh(e+fx)+a^2b(cf(-8fg+13eh+5fhx)+d(-14e^2h+ef(13g-5hx)+f^2x(5g+hx)))+b^3(}{}$$

$$-\frac{(-a^2df^2h+abf(-3dfg+8deh-3cfh)+b^2(3cf(5fg-4eh)+4de(-3fg+2eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{4b^{3/2}(-be+af)^{7/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/4*(-(a^3*d*f*h*(e + f*x)) + a^2*b*(c*f*(-8*f*g + 13*e*h + 5*f*h*x) + d*
(-14*e^2*h + e*f*(13*g - 5*h*x) + f^2*x*(5*g + h*x))) + b^3*(4*d*e*x*(3*f*
g*x + e*(g - 2*h*x)) + c*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*x) + e*f*x*(-5*g
+ 12*h*x))) + a*b^2*(d*(3*f^2*g*x^2 + 2*e^2*(g - 12*h*x) + e*f*x*(21*g - 8
*h*x)) + c*(2*e^2*h + f^2*x*(-25*g + 3*h*x) + e*f*(-9*g + 21*h*x))))/(b*(b
*e - a*f)^3*(a + b*x)^2*Sqrt[e + f*x]) - ((-(a^2*d*f^2*h) + a*b*f*(-3*d*f*
g + 8*d*e*h - 3*c*f*h) + b^2*(3*c*f*(5*f*g - 4*e*h) + 4*d*e*(-3*f*g + 2*e*
h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(4*b^(3/2)*(-(b*e
) + a*f)^(7/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {161, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 161$$

$$\frac{\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(3cf(5fg-4eh)-4de(3fg-2eh))\right)\right)\int\frac{1}{(a+bx)^2\sqrt{e+fx}}dx}{4bf(be-af)^2}-\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(5fg-4eh)-4de(fg-eh))\right)+a^2defh+2ab\left(-\frac{5}{2}ef(ch+dg)+2cf^2g+2d\right.}{2bf(a+bx)^2\sqrt{e+fx}(be-af)^2}$$

$\downarrow$ 52

$$\frac{\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(3cf(5fg-4eh)-4de(3fg-2eh))\right)\right)\left(-\frac{f\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{2(be-af)}-\frac{\sqrt{e+f}}{(a+bx)(be}\right.}{4bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(5fg-4eh)-4de(fg-eh))\right)+a^2defh+2ab\left(-\frac{5}{2}ef(ch+dg)+2cf^2g+2d\right.}{2bf(a+bx)^2\sqrt{e+fx}(be-af)^2}$$

$\downarrow$ 73

$$\frac{\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(3cf(5fg-4eh)-4de(3fg-2eh))\right)\right)\left(-\frac{\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{be-af}-\frac{}{(a+}\right.}{4bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(5fg-4eh)-4de(fg-eh))\right)+a^2defh+2ab\left(-\frac{5}{2}ef(ch+dg)+2cf^2g+2d\right.}{2bf(a+bx)^2\sqrt{e+fx}(be-af)^2}$$

$\downarrow$ 221

$$\frac{\left(\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)\left(a^2df^2h+abf(3cfh-8deh+3dfg)-\left(b^2(3cf(5fg-4eh)-4de(3fg-\right.\right.}{4bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(5fg-4eh)-4de(fg-eh))\right)+a^2defh+2ab\left(-\frac{5}{2}ef(ch+dg)+2cf^2g+2d\right.}{2bf(a+bx)^2\sqrt{e+fx}(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*(b^2*c*e*f*g + a^2*d*e*f*h + 2*a*b*(2*c*f^2*g + 2*d*e^2*h - (5*e*f*(d
*g + c*h))/2) + (a^2*d*f^2*h - a*b*f^2*(d*g + c*h) + b^2*(c*f*(5*f*g - 4*e
*h) - 4*d*e*(f*g - e*h)))*x)/(b*f*(b*e - a*f)^2*(a + b*x)^2*Sqrt[e + f*x])
 + ((a^2*d*f^2*h + a*b*f*(3*d*f*g - 8*d*e*h + 3*c*f*h) - b^2*(3*c*f*(5*f*g
 - 4*e*h) - 4*d*e*(3*f*g - 2*e*h)))*(-(Sqrt[e + f*x]/((b*e - a*f)*(a + b*x
))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*e -
 a*f)^(3/2))))/(4*b*f*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$-\frac{-\left(\left(-15cg\,f^{2}+12e(ch+dg)f-8d\,e^{2}h\right)b^{2}+3a\left(f(ch+dg)-\frac{8deh}{3}\right)fb+a^{2}d\,f^{2}h\right)(bx+a)^{2}\sqrt{fx+e}\,\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{}$
derivativedivides	$-\frac{2\left(-cefh+cg\,f^{2}+d\,e^{2}h-defg\right)}{(af-be)^{3}\sqrt{fx+e}}+\frac{2\left(\left(\frac{1}{8}a^{2}d\,f^{2}h+\frac{3}{8}abc\,f^{2}h-abdefh+\frac{3}{8}abd\,f^{2}g+\frac{1}{2}cefh\,b^{2}-\frac{7}{8}b^{2}c\,f^{2}g+\frac{1}{2}b^{2}defg\right)(fx+e)\right.}{}$
default	$-\frac{2\left(-cefh+cg\,f^{2}+d\,e^{2}h-defg\right)}{(af-be)^{3}\sqrt{fx+e}}+\frac{2\left(\left(\frac{1}{8}a^{2}d\,f^{2}h+\frac{3}{8}abc\,f^{2}h-abdefh+\frac{3}{8}abd\,f^{2}g+\frac{1}{2}cefh\,b^{2}-\frac{7}{8}b^{2}c\,f^{2}g+\frac{1}{2}b^{2}defg\right)(fx+e)\right.}{}$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)*(-((-15*c*g*f^2+12*e*(c*h+d*g)*f-8*
d*e^2*h)*b^2+3*a*(f*(c*h+d*g)-8/3*d*e*h)*f*b+a^2*d*f^2*h)*(b*x+a)^2*(f*x+e
)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*((
15*c*f^2*g*x^2+5*x*e*(12/5*(-c*h-d*g)*x+c*g)*f-2*(-4*d*h*x^2+2*(c*h+d*g)*x
+c*g)*e^2)*b^3-2*a*(1/2*(3*(c*h+d*g)*x^2-25*c*g*x)*f^2-9/2*(8/9*d*h*x^2+7/
3*(-c*h-d*g)*x+c*g)*e*f+e^2*(-12*d*h*x+c*h+d*g))*b^2-13*a^2*(1/13*(d*h*x^2
+5*(c*h+d*g)*x-8*c*g)*f^2+e*(-5/13*d*h*x+c*h+d*g)*f-14/13*d*e^2*h)*b+a^3*d
*f*h*(f*x+e)))/(a*f-b*e)^3/(b*x+a)^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. 2(255) = 510.

Time = 0.25 (sec) , antiderivative size = 2506, normalized size of antiderivative = 9.01

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((3*(4*b^4*d*e*f^2 - (5*b^4*c - a*b^3*d)*f^3)*g - (8*b^4*d*e^2*f -
4*(3*b^4*c - 2*a*b^3*d)*e*f^2 - (3*a*b^3*c + a^2*b^2*d)*f^3)*h)*x^3 + (3*(
4*b^4*d*e^2*f - (5*b^4*c - 9*a*b^3*d)*e*f^2 - 2*(5*a*b^3*c - a^2*b^2*d)*f^
3)*g - (8*b^4*d*e^3 - 12*(b^4*c - 2*a*b^3*d)*e^2*f - 3*(9*a*b^3*c - 5*a^2*
b^2*d)*e*f^2 - 2*(3*a^2*b^2*c + a^3*b*d)*f^3)*h)*x^2 + 3*(4*a^2*b^2*d*e^2*
f - (5*a^2*b^2*c - a^3*b*d)*e*f^2)*g - (8*a^2*b^2*d*e^3 - 4*(3*a^2*b^2*c -
 2*a^3*b*d)*e^2*f - (3*a^3*b*c + a^4*d)*e*f^2)*h + (3*(8*a*b^3*d*e^2*f - 2
*(5*a*b^3*c - 3*a^2*b^2*d)*e*f^2 - (5*a^2*b^2*c - a^3*b*d)*f^3)*g - (16*a*
b^3*d*e^3 - 24*(a*b^3*c - a^2*b^2*d)*e^2*f - 6*(3*a^2*b^2*c - a^3*b*d)*e*f
^2 - (3*a^3*b*c + a^4*d)*f^3)*h)*x)*sqrt(b^2*e - a*b*f)*log((b*f*x + 2*b*e
 - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*((3*(4*b^5*d*
e^2*f - (5*b^5*c + 3*a*b^4*d)*e*f^2 + (5*a*b^4*c - a^2*b^3*d)*f^3)*g - (8*
b^5*d*e^3 - 12*b^5*c*e^2*f + 9*(a*b^4*c - a^2*b^3*d)*e*f^2 + (3*a^2*b^3*c
+ a^3*b^2*d)*f^3)*h)*x^2 + (8*a^3*b^2*c*f^3 + 2*(b^5*c + a*b^4*d)*e^3 - 11
*(a*b^4*c - a^2*b^3*d)*e^2*f + (a^2*b^3*c - 13*a^3*b^2*d)*e*f^2)*g + (2*(a
*b^4*c - 7*a^2*b^3*d)*e^3 + (11*a^2*b^3*c + 13*a^3*b^2*d)*e^2*f - (13*a^3*
b^2*c - a^4*b*d)*e*f^2)*h + ((4*b^5*d*e^3 - (5*b^5*c - 17*a*b^4*d)*e^2*f -
 4*(5*a*b^4*c + 4*a^2*b^3*d)*e*f^2 + 5*(5*a^2*b^3*c - a^3*b^2*d)*f^3)*g +
(4*(b^5*c - 6*a*b^4*d)*e^3 + (17*a*b^4*c + 19*a^2*b^3*d)*e^2*f - 4*(4*a^2*
b^3*c - a^3*b^2*d)*e*f^2 - (5*a^3*b^2*c - a^4*b*d)*f^3)*h)*x)*sqrt(f*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(255) = 510$.

Time = 0.15 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = -\frac{(12\,b^2defg - 15\,b^2cf^2g + 3\,abdf^2g - 8\,b^2de^2h + 12\,b^2cefh - 8\,abdefh + 3\,abcf^2h + a^2df^2h)\arctan\left(\frac{\sqrt{\ }}{\sqrt{-}}\right.}{4\,(b^4e^3 - 3\,ab^3e^2f + 3\,a^2b^2ef^2 - a^3bf^3)\sqrt{-b^2e + abf}} - \frac{2\,(defg - cf^2g - de^2h + cefh)}{(b^3e^3 - 3\,ab^2e^2f + 3\,a^2bef^2 - a^3f^3)\sqrt{fx+e}} - \frac{4\,(fx+e)^{\frac{3}{2}}b^3defg - 4\,\sqrt{fx+e}b^3de^2fg - 7\,(fx+e)^{\frac{3}{2}}b^3cf^2g + 3\,(fx+e)^{\frac{3}{2}}ab^2df^2g + 9\,\sqrt{fx+e}b^3cef^2g}{}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(12*b^2*d*e*f*g - 15*b^2*c*f^2*g + 3*a*b*d*f^2*g - 8*b^2*d*e^2*h + 12
*b^2*c*e*f*h - 8*a*b*d*e*f*h + 3*a*b*c*f^2*h + a^2*d*f^2*h)*arctan(sqrt(f*
x + e)*b/sqrt(-b^2*e + a*b*f))/((b^4*e^3 - 3*a*b^3*e^2*f + 3*a^2*b^2*e*f^2
 - a^3*b*f^3)*sqrt(-b^2*e + a*b*f)) - 2*(d*e*f*g - c*f^2*g - d*e^2*h + c*e
*f*h)/((b^3*e^3 - 3*a*b^2*e^2*f + 3*a^2*b*e*f^2 - a^3*f^3)*sqrt(f*x + e))
 - 1/4*(4*(f*x + e)^(3/2)*b^3*d*e*f*g - 4*sqrt(f*x + e)*b^3*d*e^2*f*g - 7*(
f*x + e)^(3/2)*b^3*c*f^2*g + 3*(f*x + e)^(3/2)*a*b^2*d*f^2*g + 9*sqrt(f*x
 + e)*b^3*c*e*f^2*g - sqrt(f*x + e)*a*b^2*d*e*f^2*g - 9*sqrt(f*x + e)*a*b^2
*c*f^3*g + 5*sqrt(f*x + e)*a^2*b*d*f^3*g + 4*(f*x + e)^(3/2)*b^3*c*e*f*h -
 8*(f*x + e)^(3/2)*a*b^2*d*e*f*h - 4*sqrt(f*x + e)*b^3*c*e^2*f*h + 8*sqrt(
f*x + e)*a*b^2*d*e^2*f*h + 3*(f*x + e)^(3/2)*a*b^2*c*f^2*h + (f*x + e)^(3/
2)*a^2*b*d*f^2*h - sqrt(f*x + e)*a*b^2*c*e*f^2*h - 7*sqrt(f*x + e)*a^2*b*d
*e*f^2*h + 5*sqrt(f*x + e)*a^2*b*c*f^3*h - sqrt(f*x + e)*a^3*d*f^3*h)/((b^
4*e^3 - 3*a*b^3*e^2*f + 3*a^2*b^2*e*f^2 - a^3*b*f^3)*((f*x + e)*b - b*e +
a*f)^2)
```

Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{\frac{2\,(c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g)}{a\,f-b\,e}-\frac{(e+f\,x)^2\,(a^2\,d\,f^2\,h-15\,b^2\,c\,f^2\,g-8\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f^2\,g+12\,b^2\,c\,e\,f\,h+12\,b^2\,d\,e\,f\,g-8\,a\,b\,d\,e\,f\,}{4\,(a\,f-b\,e)^3}}{b^2\,(e+f\,x)^{5/2}-(e+f\,x)^{3/2}\,(2\,b^2\,e-2\,a\,b\,f)+\sqrt{e}}$$

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,(-a^3\,b\,f^3+3\,a^2\,b^2\,e\,f^2-3\,a\,b^3\,e^2\,f+b^4\,e^3)}{\sqrt{b}\,(a\,f-b\,e)^{7/2}}\right)\,(a^2\,d\,f^2\,h-15\,b^2\,c\,f^2\,g-8\,b^2\,d\,e^2\,h+3\,a\,b\,c\,f^2\,h+3\,a\,b\,d\,f}{4\,b^{3/2}\,(a\,f-b\,e)^{7/2}}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(3/2)*(a + b*x)^3),x)
```

output
```
- ((2*(c*f^2*g + d*e^2*h - c*e*f*h - d*e*f*g))/(a*f - b*e) - ((e + f*x)^2*
(a^2*d*f^2*h - 15*b^2*c*f^2*g - 8*b^2*d*e^2*h + 3*a*b*c*f^2*h + 3*a*b*d*f^
2*g + 12*b^2*c*e*f*h + 12*b^2*d*e*f*g - 8*a*b*d*e*f*h))/(4*(a*f - b*e)^3)
+ ((e + f*x)*(25*b^2*c*f^2*g + a^2*d*f^2*h + 16*b^2*d*e^2*h - 5*a*b*c*f^2*
h - 5*a*b*d*f^2*g - 20*b^2*c*e*f*h - 20*b^2*d*e*f*g + 8*a*b*d*e*f*h))/(4*b
*(a*f - b*e)^2))/(b^2*(e + f*x)^(5/2) - (e + f*x)^(3/2)*(2*b^2*e - 2*a*b*f
) + (e + f*x)^(1/2)*(a^2*f^2 + b^2*e^2 - 2*a*b*e*f)) - (atan(((e + f*x)^(1
/2)*(b^4*e^3 - a^3*b*f^3 + 3*a^2*b^2*e*f^2 - 3*a*b^3*e^2*f))/(b^(1/2)*(a*f
 - b*e)^(7/2)))*(a^2*d*f^2*h - 15*b^2*c*f^2*g - 8*b^2*d*e^2*h + 3*a*b*c*f^
2*h + 3*a*b*d*f^2*g + 12*b^2*c*e*f*h + 12*b^2*d*e*f*g - 8*a*b*d*e*f*h))/(4
*b^(3/2)*(a*f - b*e)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2007, normalized size of antiderivative = 7.22

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**4*d*f**2*h + 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*f**2*h - 8*sqrt(b)
*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**3*b*d*e*f*h + 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d*f**2*g + 2*sqrt(b)*sqrt(e
 + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**3*b*d*f**2*h*x + 12*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*e*f*h - 15*sqrt(b)*sqrt(e
 + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**2*b**2*c*f**2*g + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*f**2*h*x - 8*sqrt(b)*sqrt
(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**2*b**2*d*e**2*h + 12*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt
(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*e*f*g - 16*sqrt(b)*sqr
t(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**2*b**2*d*e*f*h*x + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqr
t(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d*f**2*g*x + sqrt(b)*sq
rt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**2*b**2*d*f**2*h*x**2 + 24*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*...
```

3.157 $\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 374

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \frac{2f(de-cf)(fg-eh)}{(be-af)^4\sqrt{e+fx}} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{3b(be-af)^2(a+bx)^3}$$
$$-\frac{(a^2dfh+b^2(6deg-11cfg+6ceh)+ab(5dfg-12deh+5cfh))\sqrt{e+fx}}{12b(be-af)^3(a+bx)^2}$$
$$+\frac{(a^2df^2h+abf(5dfg-12deh+5cfh)-b^2(cf(19fg-14eh)-2de(7fg-4eh)))\sqrt{e+fx}}{8b(be-af)^4(a+bx)}$$
$$-\frac{f(a^2df^2h+abf(5dfg-12deh+5cfh)-b^2(5cf(7fg-6eh)-6de(5fg-4eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{8b^{3/2}(be-af)^{9/2}}$$

output
```
2*f*(-c*f+d*e)*(-e*h+f*g)/(-a*f+b*e)^4/(f*x+e)^(1/2)-1/3*(-a*d+b*c)*(-a*h+
b*g)*(f*x+e)^(1/2)/b/(-a*f+b*e)^2/(b*x+a)^3-1/12*(a^2*d*f*h+b^2*(6*c*e*h-1
1*c*f*g+6*d*e*g)+a*b*(5*c*f*h-12*d*e*h+5*d*f*g))*(f*x+e)^(1/2)/b/(-a*f+b*e
)^3/(b*x+a)^2+1/8*(a^2*d*f^2*h+a*b*f*(5*c*f*h-12*d*e*h+5*d*f*g)-b^2*(c*f*(
-14*e*h+19*f*g)-2*d*e*(-4*e*h+7*f*g)))*(f*x+e)^(1/2)/b/(-a*f+b*e)^4/(b*x+a
)-1/8*f*(a^2*d*f^2*h+a*b*f*(5*c*f*h-12*d*e*h+5*d*f*g)-b^2*(5*c*f*(-6*e*h+7
*f*g)-6*d*e*(-4*e*h+5*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2
))/b^(3/2)/(-a*f+b*e)^(9/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \frac{-3a^4df^2h(e+fx)+a^3bf(3cf(-16fg+27eh+11fhx)+d(-94e^2h+ef(8$$

$$-\frac{f(-a^2df^2h+abf(-5dfg+12deh-5cfh)+b^2(5cf(7fg-6eh)+6de(-5fg+4eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right.}{8b^{3/2}(-be+af)^{9/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^4*(e + f*x)^(3/2)),x]
```

output
```
(-3*a^4*d*f^2*h*(e + f*x) + a^3*b*f*(3*c*f*(-16*f*g + 27*e*h + 11*f*h*x) +
 d*(-94*e^2*h + e*f*(81*g - 38*h*x) + f^2*x*(33*g + 8*h*x))) - b^4*(6*d*e*
x*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*x) + e*f*x*(-5*g + 12*h*x)) + c*(105*f^3
*g*x^3 + 5*e*f^2*x^2*(7*g - 18*h*x) + 4*e^3*(2*g + 3*h*x) - 2*e^2*f*x*(7*g
 + 15*h*x))) + a*b^3*(d*(15*f^3*g*x^3 + 2*e^2*f*x*(41*g - 102*h*x) + e*f^2
*x^2*(245*g - 36*h*x) - 4*e^3*(g + 6*h*x)) + c*(-4*e^3*h + 5*f^3*x^2*(-56*
g + 3*h*x) + 49*e*f^2*x*(-2*g + 5*h*x) + e^2*f*(38*g + 82*h*x))) + a^2*b^2
*(d*(-8*e^3*h + e^2*f*(28*g - 250*h*x) + e*f^2*x*(212*g - 95*h*x) + f^3*x^
2*(40*g + 3*h*x)) + c*f*(28*e^2*h + f^2*x*(-231*g + 40*h*x) + e*f*(-87*g +
 212*h*x))))/(24*b*(b*e - a*f)^4*(a + b*x)^3*Sqrt[e + f*x]) - (f*(-(a^2*d*
f^2*h) + a*b*f*(-5*d*f*g + 12*d*e*h - 5*c*f*h) + b^2*(5*c*f*(7*f*g - 6*e*h
) + 6*d*e*(-5*f*g + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) +
a*f]])/(8*b^(3/2)*(-(b*e) + a*f)^(9/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {161, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx$$

$\Big\downarrow$ 161

$$\frac{\left(a^2df^2h+abf(5cfh-12deh+5dfg)-\left(b^2(5cf(7fg-6eh)-6de(5fg-4eh))\right)\right)\int \frac{1}{(a+bx)^3\sqrt{e+fx}}dx}{6bf(be-af)^2}-\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(7fg-6eh)-6de(fg-eh))\right)+a^2defh+2ab\left(-\frac{7}{2}ef(ch+dg)+3cf^2g+3d\right.}{3bf(a+bx)^3\sqrt{e+fx}(be-af)^2}$$

$\Big\downarrow$ 52

$$\frac{\left(a^2df^2h+abf(5cfh-12deh+5dfg)-\left(b^2(5cf(7fg-6eh)-6de(5fg-4eh))\right)\right)\left(-\frac{3f\int \frac{1}{(a+bx)^2\sqrt{e+fx}}dx}{4(be-af)}-\frac{\sqrt{}}{2(a+b}\right.}{6bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(7fg-6eh)-6de(fg-eh))\right)+a^2defh+2ab\left(-\frac{7}{2}ef(ch+dg)+3cf^2g+3d\right.}{3bf(a+bx)^3\sqrt{e+fx}(be-af)^2}$$

$\Big\downarrow$ 52

$$\frac{\left(a^2df^2h+abf(5cfh-12deh+5dfg)-\left(b^2(5cf(7fg-6eh)-6de(5fg-4eh))\right)\right)\left(-\frac{3f\left(-\frac{f\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{2(be-af)}-\frac{\sqrt{}}{(a+b}\right.}{4(be-af)}\right.}{6bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(7fg-6eh)-6de(fg-eh))\right)+a^2defh+2ab\left(-\frac{7}{2}ef(ch+dg)+3cf^2g+3d\right.}{3bf(a+bx)^3\sqrt{e+fx}(be-af)^2}$$

$\Big\downarrow$ 73

$$\frac{\left(a^2df^2h+abf(5cfh-12deh+5dfg)-\left(b^2(5cf(7fg-6eh)-6de(5fg-4eh))\right)\right)\left(-\frac{3f\left(-\frac{\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{be-af}-\right.}{4(be-af)}\right.}{6bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(7fg-6eh)-6de(fg-eh))\right)+a^2defh+2ab\left(-\frac{7}{2}ef(ch+dg)+3cf^2g+3d\right.}{3bf(a+bx)^3\sqrt{e+fx}(be-af)^2}$$

$\Big\downarrow$ 221

$$\frac{\left(-\frac{3f\left(\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)}{4(be-af)}-\frac{\sqrt{e+fx}}{2(a+bx)^2(be-af)}\right)(a^2df^2h+abf(5cfh-12deh+5dfg)-(b^2(5cf(7f$$
$$\overline{6bf(be-af)^2}$$
$$\frac{x(a^2df^2h-abf^2(ch+dg)+b^2(cf(7fg-6eh)-6de(fg-eh)))+a^2defh+2ab(-\frac{7}{2}ef(ch+dg)+3cf^2g+3d}{3bf(a+bx)^3\sqrt{e+fx}(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^4*(e + f*x)^(3/2)),x]
```

output
```
-1/3*(b^2*c*e*f*g + a^2*d*e*f*h + 2*a*b*(3*c*f^2*g + 3*d*e^2*h - (7*e*f*(d
*g + c*h))/2) + (a^2*d*f^2*h - a*b*f^2*(d*g + c*h) + b^2*(c*f*(7*f*g - 6*e
*h) - 6*d*e*(f*g - e*h)))*x)/(b*f*(b*e - a*f)^2*(a + b*x)^3*Sqrt[e + f*x])
 + ((a^2*d*f^2*h + a*b*f*(5*d*f*g - 12*d*e*h + 5*c*f*h) - b^2*(5*c*f*(7*f*
g - 6*e*h) - 6*d*e*(5*f*g - 4*e*h)))*(-1/2*Sqrt[e + f*x]/((b*e - a*f)*(a +
 b*x)^2) - (3*f*(-(Sqrt[e + f*x]/((b*e - a*f)*(a + b*x))) + (f*ArcTanh[(Sq
rt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/(Sqrt[b]*(b*e - a*f)^(3/2))))/(4*(b
*e - a*f))))/(6*b*f*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{\left(\left(-35cg\,f^{2}+30e(ch+dg)f-24d\,e^{2}h\right)b^{2}+5a\left(f(ch+dg)-\frac{12deh}{5}\right)fb+a^{2}d\,f^{2}h\right)(bx+a)^{3}\sqrt{fx+e}\,f\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{}$
derivativedivides	$2f\left(-\frac{-cefh+cg\,f^{2}+d\,e^{2}h-defg}{(af-be)^{4}\sqrt{fx+e}}+\frac{\left(\frac{7}{8}b^{3}cefh-\frac{19}{16}b^{3}c\,f^{2}g-\frac{1}{2}b^{3}d\,e^{2}h+\frac{7}{8}b^{3}defg+\frac{1}{16}a^{2}bd\,f^{2}h+\frac{5}{16}a\,b^{2}c\,f^{2}h-\frac{3}{4}a\,b^{2}def\right.}{}\right.$
default	$2f\left(-\frac{-cefh+cg\,f^{2}+d\,e^{2}h-defg}{(af-be)^{4}\sqrt{fx+e}}+\frac{\left(\frac{7}{8}b^{3}cefh-\frac{19}{16}b^{3}c\,f^{2}g-\frac{1}{2}b^{3}d\,e^{2}h+\frac{7}{8}b^{3}defg+\frac{1}{16}a^{2}bd\,f^{2}h+\frac{5}{16}a\,b^{2}c\,f^{2}h-\frac{3}{4}a\,b^{2}def\right.}{}\right.$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(((-35*c*g*f^2+30*e*(c*h+d*g)*f-24*d*e^2*h)*b^2+5*a*(f*(c*h+d*g)-12/5*
d*e*h)*f*b+a^2*d*f^2*h)*(b*x+a)^3*(f*x+e)^(1/2)*f*arctan(b*(f*x+e)^(1/2)/(
(a*f-b*e)*b)^(1/2))-((35*c*f^3*g*x^3+35/3*x^2*(18/7*(-c*h-d*g)*x+c*g)*e*f^
2-14/3*x*(-36/7*d*h*x^2+15/7*(c*h+d*g)*x+c*g)*e^2*f+8/3*e^3*(3*d*h*x^2+3/2
*(c*h+d*g)*x+c*g))*b^4+4/3*a*(5*(3/4*(-c*h-d*g)*x^3+14*c*g*x^2)*f^3+49/2*x
*e*(18/49*d*h*x^2+5/2*(-c*h-d*g)*x+c*g)*f^2-19/2*(-102/19*d*h*x^2+41/19*(c
*h+d*g)*x+c*g)*e^2*f+e^3*(6*d*h*x+c*h+d*g))*b^3-28/3*a^2*((3/28*d*h*x^3+10
/7*(c*h+d*g)*x^2-33/4*c*g*x)*f^3-87/28*e*(95/87*d*h*x^2+212/87*(-c*h-d*g)*
x+c*g)*f^2+e^2*(-125/14*d*h*x+c*h+d*g)*f-2/7*d*e^3*h)*b^2-27*a^3*(1/27*(8/
3*d*h*x^2+11*(c*h+d*g)*x-16*c*g)*f^2+e*(-38/81*d*h*x+c*h+d*g)*f-94/81*d*e^
2*h)*f*b+a^4*d*f^2*h*(f*x+e))*((a*f-b*e)*b)^(1/2))/(f*x+e)^(1/2)/((a*f-b*e
)*b)^(1/2)/(b*x+a)^3/(a*f-b*e)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(347) = 694$.

Time = 0.49 (sec) , antiderivative size = 3796, normalized size of antiderivative = 10.15

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**4/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. 2(347) = 694.

Time = 0.15 (sec) , antiderivative size = 1033, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/8*(30*b^2*d*e*f^2*g - 35*b^2*c*f^3*g + 5*a*b*d*f^3*g - 24*b^2*d*e^2*f*h
+ 30*b^2*c*e*f^2*h - 12*a*b*d*e*f^2*h + 5*a*b*c*f^3*h + a^2*d*f^3*h)*arcta
n(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^5*e^4 - 4*a*b^4*e^3*f + 6*a^2*
b^3*e^2*f^2 - 4*a^3*b^2*e*f^3 + a^4*b*f^4)*sqrt(-b^2*e + a*b*f)) + 2*(d*e*
f^2*g - c*f^3*g - d*e^2*f*h + c*e*f^2*h)/((b^4*e^4 - 4*a*b^3*e^3*f + 6*a^2
*b^2*e^2*f^2 - 4*a^3*b*e*f^3 + a^4*f^4)*sqrt(f*x + e)) + 1/24*(42*(f*x + e
)^(5/2)*b^4*d*e*f^2*g - 96*(f*x + e)^(3/2)*b^4*d*e^2*f^2*g + 54*sqrt(f*x +
 e)*b^4*d*e^3*f^2*g - 57*(f*x + e)^(5/2)*b^4*c*f^3*g + 15*(f*x + e)^(5/2)*
a*b^3*d*f^3*g + 136*(f*x + e)^(3/2)*b^4*c*e*f^3*g + 56*(f*x + e)^(3/2)*a*b
^3*d*e*f^3*g - 87*sqrt(f*x + e)*b^4*c*e^2*f^3*g - 75*sqrt(f*x + e)*a*b^3*d
*e^2*f^3*g - 136*(f*x + e)^(3/2)*a*b^3*c*f^4*g + 40*(f*x + e)^(3/2)*a^2*b^
2*d*f^4*g + 174*sqrt(f*x + e)*a*b^3*c*e*f^4*g - 12*sqrt(f*x + e)*a^2*b^2*d
*e*f^4*g - 87*sqrt(f*x + e)*a^2*b^2*c*f^5*g + 33*sqrt(f*x + e)*a^3*b*d*f^5
*g - 24*(f*x + e)^(5/2)*b^4*d*e^2*f*h + 48*(f*x + e)^(3/2)*b^4*d*e^3*f*h -
 24*sqrt(f*x + e)*b^4*d*e^4*f*h + 42*(f*x + e)^(5/2)*b^4*c*e*f^2*h - 36*(f
*x + e)^(5/2)*a*b^3*d*e*f^2*h - 96*(f*x + e)^(3/2)*b^4*c*e^2*f^2*h + 48*(f
*x + e)^(3/2)*a*b^3*d*e^2*f^2*h + 54*sqrt(f*x + e)*b^4*c*e^3*f^2*h - 12*sq
rt(f*x + e)*a*b^3*d*e^3*f^2*h + 15*(f*x + e)^(5/2)*a*b^3*c*f^3*h + 3*(f*x
+ e)^(5/2)*a^2*b^2*d*f^3*h + 56*(f*x + e)^(3/2)*a*b^3*c*e*f^3*h - 104*(f*x
 + e)^(3/2)*a^2*b^2*d*e*f^3*h - 75*sqrt(f*x + e)*a*b^3*c*e^2*f^3*h + 93...
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \frac{f\,\mathrm{atan}\left(\frac{f\,\sqrt{e+f\,x}\,\left(a^4\,b\,f^4-4\,a^3\,b^2\,e\,f^3+6\,a^2\,b^3\,e^2\,f^2-4\,a\,b^4\,e^3\,f+b^5\,e^4\right)\,\left(a^2\,d\,f^2\,h-35\,b^2\,c\,f^2\,g-24\,b^2\,\cdots\right.}{\sqrt{b}\,\left(a\,f-b\,e\right)^{9/2}\,\left(a^2\,d\,f^3\,h-35\,b^2\,c\,f^3\,g+30\,b^2\,c\,e\,f^2\,h+30\,b^2\,d\,e\,f^2\,g-24\,b\cdots\right.}\right.}{\cdots} - \frac{\frac{2\,\left(c\,f^3\,g-c\,e\,f^2\,h-d\,e\,f^2\,g+d\,e^2\,f\,h\right)}{a\,f-b\,e}-\frac{{\left(e+f\,x\right)}^2\,\left(a^2\,d\,f^3\,h-35\,b^2\,c\,f^3\,g+30\,b^2\,c\,e\,f^2\,h+30\,b^2\,d\,e\,f^2\,g-24\,b^2\,d\,e^2\,f\,h+5\,a\,b\,c\,f^3\,h+5\,a\,b\,d\,f^3\,g-\cdots\right.}{3\,{\left(a\,f-b\,e\right)}^3}}{\sqrt{e+f\,x}\,\left(a^3\,f^3-3\,a^2\,b\,e\,f^2+\cdots\right.}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(3/2)*(a + b*x)^4),x)
```

output
```
(f*atan((f*(e + f*x)^(1/2)*(b^5*e^4 + a^4*b*f^4 - 4*a^3*b^2*e*f^3 + 6*a^2*
b^3*e^2*f^2 - 4*a*b^4*e^3*f)*(a^2*d*f^2*h - 35*b^2*c*f^2*g - 24*b^2*d*e^2*
h + 5*a*b*c*f^2*h + 5*a*b*d*f^2*g + 30*b^2*c*e*f*h + 30*b^2*d*e*f*g - 12*a
*b*d*e*f*h))/(b^(1/2)*(a*f - b*e)^(9/2)*(a^2*d*f^3*h - 35*b^2*c*f^3*g + 30
*b^2*c*e*f^2*h + 30*b^2*d*e*f^2*g - 24*b^2*d*e^2*f*h + 5*a*b*c*f^3*h + 5*a
*b*d*f^3*g - 12*a*b*d*e*f^2*h)))*(a^2*d*f^2*h - 35*b^2*c*f^2*g - 24*b^2*d*
e^2*h + 5*a*b*c*f^2*h + 5*a*b*d*f^2*g + 30*b^2*c*e*f*h + 30*b^2*d*e*f*g -
12*a*b*d*e*f*h))/(8*b^(3/2)*(a*f - b*e)^(9/2)) - ((2*(c*f^3*g - c*e*f^2*h
- d*e*f^2*g + d*e^2*f*h))/(a*f - b*e) - ((e + f*x)^2*(a^2*d*f^3*h - 35*b^2
*c*f^3*g + 30*b^2*c*e*f^2*h + 30*b^2*d*e*f^2*g - 24*b^2*d*e^2*f*h + 5*a*b*
c*f^3*h + 5*a*b*d*f^3*g - 12*a*b*d*e*f^2*h))/(3*(a*f - b*e)^3) - (b*(e + f
*x)^3*(a^2*d*f^3*h - 35*b^2*c*f^3*g + 30*b^2*c*e*f^2*h + 30*b^2*d*e*f^2*g
- 24*b^2*d*e^2*f*h + 5*a*b*c*f^3*h + 5*a*b*d*f^3*g - 12*a*b*d*e*f^2*h))/(8
*(a*f - b*e)^4) + ((e + f*x)*(77*b^2*c*f^3*g + a^2*d*f^3*h - 66*b^2*c*e*f^
2*h - 66*b^2*d*e*f^2*g + 56*b^2*d*e^2*f*h - 11*a*b*c*f^3*h - 11*a*b*d*f^3*
g + 20*a*b*d*e*f^2*h))/(8*b*(a*f - b*e)^2))/((e + f*x)^(1/2)*(a^3*f^3 - b^
3*e^3 + 3*a*b^2*e^2*f - 3*a^2*b*e*f^2) + b^3*(e + f*x)^(7/2) - (e + f*x)^(
5/2)*(3*b^3*e - 3*a*b^2*f) + (e + f*x)^(3/2)*(3*b^3*e^2 + 3*a^2*b*f^2 - 6*
a*b^2*e*f))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3040, normalized size of antiderivative = 8.13

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x)
```

output
```
(3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**5*d*f**3*h + 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*f**3*h - 36*sqr
t(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*
f - b*e)))*a**4*b*d*e*f**2*h + 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*at
an((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d*f**3*g + 9*sqrt(b
)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**4*b*d*f**3*h*x + 90*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan(
(sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*e*f**2*h - 105*sq
rt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**3*b**2*c*f**3*g + 45*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*f**3*h*x - 7
2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**3*b**2*d*e**2*f*h + 90*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e*f**2
*g - 108*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*a**3*b**2*d*e*f**2*h*x + 45*sqrt(b)*sqrt(e + f*x)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2
*d*f**3*g*x + 9*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d*f**3*h*x**2 + 270*sqrt(b)*sqr...
```

3.158 $\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 477

$$\begin{aligned}\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx &= -\frac{2f^2(de-cf)(fg-eh)}{(be-af)^5\sqrt{e+fx}} - \frac{(bc-ad)(bg-ah)\sqrt{e+fx}}{4b(be-af)^2(a+bx)^4} \\ &- \frac{(a^2dfh+b^2(8deg-15cfg+8ceh)+ab(7dfg-16deh+7cfh))\sqrt{e+fx}}{24b(be-af)^3(a+bx)^3} \\ &+ \frac{(5a^2df^2h+5abf(7dfg-16deh+7cfh)-b^2(cf(123fg-88eh)-8de(11fg-6eh)))\sqrt{e+fx}}{96b(be-af)^4(a+bx)^2} \\ &- \frac{f(5a^2df^2h+5abf(7dfg-16deh+7cfh)-b^2(cf(187fg-152eh)-8de(19fg-14eh)))\sqrt{e+fx}}{64b(be-af)^5(a+bx)} \\ &+ \frac{5f^2(a^2df^2h+abf(7dfg-16deh+7cfh)-b^2(7cf(9fg-8eh)-8de(7fg-6eh)))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{64b^{3/2}(be-af)^{11/2}}\end{aligned}$$

output
```
-2*f^2*(-c*f+d*e)*(-e*h+f*g)/(-a*f+b*e)^5/(f*x+e)^(1/2)-1/4*(-a*d+b*c)*(-a
*h+b*g)*(f*x+e)^(1/2)/b/(-a*f+b*e)^2/(b*x+a)^4-1/24*(a^2*d*f*h+b^2*(8*c*e*
h-15*c*f*g+8*d*e*g)+a*b*(7*c*f*h-16*d*e*h+7*d*f*g))*(f*x+e)^(1/2)/b/(-a*f+
b*e)^3/(b*x+a)^3+1/96*(5*a^2*d*f^2*h+5*a*b*f*(7*c*f*h-16*d*e*h+7*d*f*g)-b^
2*(c*f*(-88*e*h+123*f*g)-8*d*e*(-6*e*h+11*f*g)))*(f*x+e)^(1/2)/b/(-a*f+b*e
)^4/(b*x+a)^2-1/64*f*(5*a^2*d*f^2*h+5*a*b*f*(7*c*f*h-16*d*e*h+7*d*f*g)-b^2
*(c*f*(-152*e*h+187*f*g)-8*d*e*(-14*e*h+19*f*g)))*(f*x+e)^(1/2)/b/(-a*f+b*
e)^5/(b*x+a)+5/64*f^2*(a^2*d*f^2*h+a*b*f*(7*c*f*h-16*d*e*h+7*d*f*g)-b^2*(7
*c*f*(-8*e*h+9*f*g)-8*d*e*(-6*e*h+7*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(
-a*f+b*e)^(1/2))/b^(3/2)/(-a*f+b*e)^(11/2)
```

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \frac{-15a^5df^3h(e+fx)+b^5(8dex(105f^3gx^3+5ef^2x^2(7g-18hx)+4e^3(2g+3\cdots}{\cdots} - \frac{5f^2(-a^2df^2h+abf(-7dfg+16deh-7cfh)+b^2(7cf(9fg-8eh)+8de(-7fg+6eh)))\arctan\left(\frac{\sqrt{b}\sqrt{e+\cdots}}{\sqrt{-be+\cdots}}\right)}{64b^{3/2}(-be+af)^{11/2}}$$

input
```
Integrate[((c + d*x)*(g + h*x))/((a + b*x)^5*(e + f*x)^(3/2)),x]
```

output

```
(-15*a^5*d*f^3*h*(e + f*x) + b^5*(8*d*e*x*(105*f^3*g*x^3 + 5*e*f^2*x^2*(7*
g - 18*h*x) + 4*e^3*(2*g + 3*h*x) - 2*e^2*f*x*(7*g + 15*h*x)) + c*(-945*f^
4*g*x^4 + 16*e^4*(3*g + 4*h*x) + 105*e*f^3*x^3*(-3*g + 8*h*x) - 8*e^3*f*x*
(9*g + 14*h*x) + 14*e^2*f^2*x^2*(9*g + 20*h*x))) + a^4*b*f^2*(3*c*f*(-128*
f*g + 221*e*h + 93*f*h*x) + d*(-794*e^2*h + e*f*(663*g - 337*h*x) + f^2*x*
(279*g + 73*h*x))) + a*b^4*(d*(105*f^4*g*x^4 + 10*e^2*f^2*x^2*(105*g - 272
*h*x) + 5*e*f^3*x^3*(623*g - 48*h*x) + 16*e^4*(g + 4*h*x) - 8*e^3*f*x*(51*
g + 110*h*x)) + c*(16*e^4*h + 105*f^4*x^3*(-33*g + h*x) - 24*e^3*f*(11*g +
 17*h*x) + 6*e^2*f^2*x*(78*g + 175*h*x) + 7*e*f^3*x^2*(-171*g + 445*h*x)))
 + a^2*b^3*(d*(16*e^4*h + 6*e^2*f^2*x*(238*g - 635*h*x) + 7*e*f^3*x^2*(603
*g - 125*h*x) + 5*f^4*x^3*(77*g + 3*h*x) - 8*e^3*f*(13*g + 75*h*x)) + c*f*
(-104*e^3*h + 42*e^2*f*(15*g + 34*h*x) + 7*f^3*x^2*(-657*g + 55*h*x) + 9*e
*f^2*x*(-185*g + 469*h*x))) + a^3*b^2*f*(d*(-152*e^3*h + e^2*f*(370*g - 28
92*h*x) + e*f^2*x*(2417*g - 1149*h*x) + f^3*x^2*(511*g + 55*h*x)) + c*f*(3
70*e^2*h + f^2*x*(-2511*g + 511*h*x) + e*f*(-975*g + 2417*h*x))))/(192*b*(
-(b*e) + a*f)^5*(a + b*x)^4*Sqrt[e + f*x]) - (5*f^2*(-(a^2*d*f^2*h) + a*b*
f*(-7*d*f*g + 16*d*e*h - 7*c*f*h) + b^2*(7*c*f*(9*f*g - 8*e*h) + 8*d*e*(-7
*f*g + 6*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(64*b^
(3/2)*(-(b*e) + a*f)^(11/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {161, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{161}$$

$$\frac{\left(a^2df^2h+abf(7cfh-16deh+7dfg)-\left(b^2(7cf(9fg-8eh)-8de(7fg-6eh))\right)\right)\int \frac{1}{(a+bx)^4\sqrt{e+fx}}dx}{8bf(be-af)^2}-\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(9fg-8eh)-8de(fg-eh))\right)+a^2defh+2ab\left(-\frac{9}{2}ef(ch+dg)+4cf^2g+4d\right.}{4bf(a+bx)^4\sqrt{e+fx}(be-af)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{\left(a^2df^2h+abf(7cfh-16deh+7dfg)-\left(b^2(7cf(9fg-8eh)-8de(7fg-6eh))\right)\right)\left(-\frac{5f\int\frac{1}{(a+bx)^3\sqrt{e+fx}}dx}{6(be-af)}-\frac{\sqrt{}}{3(a+b}\right.}{8bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(9fg-8eh)-8de(fg-eh))\right)+a^2defh+2ab\left(-\frac{9}{2}ef(ch+dg)+4cf^2g+4d\right.}{4bf(a+bx)^4\sqrt{e+fx}(be-af)^2}$$

$\downarrow$ 52

$$\frac{\left(a^2df^2h+abf(7cfh-16deh+7dfg)-\left(b^2(7cf(9fg-8eh)-8de(7fg-6eh))\right)\right)\left(-\frac{5f\left(-\frac{3f\int\frac{1}{(a+bx)^2\sqrt{e+fx}}dx}{4(be-af)}-\frac{}{2(a}\right.}{6(be-af)}\right.}{8bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(9fg-8eh)-8de(fg-eh))\right)+a^2defh+2ab\left(-\frac{9}{2}ef(ch+dg)+4cf^2g+4d\right.}{4bf(a+bx)^4\sqrt{e+fx}(be-af)^2}$$

$\downarrow$ 52

$$\frac{\left(a^2df^2h+abf(7cfh-16deh+7dfg)-\left(b^2(7cf(9fg-8eh)-8de(7fg-6eh))\right)\right)\left(-\frac{5f\left(-\frac{3f\left(-\frac{f\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{2(be-af)}-\right.}{4(be-af)}\right.}{6(\right.}{8bf(be-af)^2}$$
$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(9fg-8eh)-8de(fg-eh))\right)+a^2defh+2ab\left(-\frac{9}{2}ef(ch+dg)+4cf^2g+4d\right.}{4bf(a+bx)^4\sqrt{e+fx}(be-af)^2}$$

$\downarrow$ 73

$$\frac{\left(a^2df^2h+abf(7cfh-16deh+7dfg)-\left(b^2(7cf(9fg-8eh)-8de(7fg-6eh))\right)\right)\left(-\frac{5f\left(-\frac{3f\left(-\frac{\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{\cdots}}{be-af}\cdots\right)}{4(be-\cdots}\cdots\right)}{\cdots}\cdots\right)}{8bf(be-af)^2}$$

$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(9fg-8eh)-8de(fg-eh))\right)+a^2defh+2ab\left(-\frac{9}{2}ef(ch+dg)+4cf^2g+4d\cdots\right.}{4bf(a+bx)^4\sqrt{e+fx}(be-af)^2}$$

$\Big\downarrow$ 221

$$\frac{\left(-\frac{5f\left(-\frac{3f\left(\frac{f\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)}{4(be-af)}-\frac{\sqrt{e+fx}}{2(a+bx)^2(be-af)}\right)}{6(be-af)}-\frac{\sqrt{e+fx}}{3(a+bx)^3(be-af)}\right)\left(a^2df^2h+abf(7cfh-16deh\cdots\right.}{8bf(be-af)^2}$$

$$\frac{x\left(a^2df^2h-abf^2(ch+dg)+b^2(cf(9fg-8eh)-8de(fg-eh))\right)+a^2defh+2ab\left(-\frac{9}{2}ef(ch+dg)+4cf^2g+4d\cdots\right.}{4bf(a+bx)^4\sqrt{e+fx}(be-af)^2}$$

input
```
Int[((c + d*x)*(g + h*x))/((a + b*x)^5*(e + f*x)^(3/2)),x]
```

output
```
-1/4*(b^2*c*e*f*g + a^2*d*e*f*h + 2*a*b*(4*c*f^2*g + 4*d*e^2*h - (9*e*f*(d
*g + c*h))/2) + (a^2*d*f^2*h - a*b*f^2*(d*g + c*h) + b^2*(c*f*(9*f*g - 8*e
*h) - 8*d*e*(f*g - e*h)))*x)/(b*f*(b*e - a*f)^2*(a + b*x)^4*Sqrt[e + f*x])
 + ((a^2*d*f^2*h + a*b*f*(7*d*f*g - 16*d*e*h + 7*c*f*h) - b^2*(7*c*f*(9*f*
g - 8*e*h) - 8*d*e*(7*f*g - 6*e*h)))*(-1/3*Sqrt[e + f*x]/((b*e - a*f)*(a +
 b*x)^3) - (5*f*(-1/2*Sqrt[e + f*x]/((b*e - a*f)*(a + b*x)^2) - (3*f*(-(Sq
rt[e + f*x]/((b*e - a*f)*(a + b*x))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/
Sqrt[b*e - a*f]])/(Sqrt[b]*(b*e - a*f)^(3/2))))/(4*(b*e - a*f))))/(6*(b*e
- a*f))))/(8*b*f*(b*e - a*f)^2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.49

method	result
pseudoelliptic	$5\sqrt{(af-$ ... $\frac{\frac{5(bx+a)^4\sqrt{fx+e}\left(\left(-63cg\,f^2+56e(ch+dg)f-48d\,e^2h\right)b^2+7a\left(f(ch+dg)-\frac{16deh}{7}\right)fb+a^2d\,f^2h\right)f^2\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{64}-\cdots}{\cdots}$
derivativedivides	$2f^2\left(-\frac{-cefh+cg\,f^2+d\,e^2h-defg}{(af-be)^5\sqrt{fx+e}}+\frac{\left(\frac{19}{16}b^4cefh-\frac{187}{128}b^4c\,f^2g-\frac{7}{8}b^4d\,e^2h+\frac{19}{16}b^4defg+\frac{5}{128}a^2b^2d\,f^2h+\frac{35}{128}a\,b^3c\,f^2h-\frac{5}{8}\cdots\right.}{\cdots}\right.$
default	$2f^2\left(-\frac{-cefh+cg\,f^2+d\,e^2h-defg}{(af-be)^5\sqrt{fx+e}}+\frac{\left(\frac{19}{16}b^4cefh-\frac{187}{128}b^4c\,f^2g-\frac{7}{8}b^4d\,e^2h+\frac{19}{16}b^4defg+\frac{5}{128}a^2b^2d\,f^2h+\frac{35}{128}a\,b^3c\,f^2h-\frac{5}{8}\cdots\right.}{\cdots}\right.$

input

```
int((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
5/64*((b*x+a)^4*(f*x+e)^(1/2)*((-63*c*g*f^2+56*e*(c*h+d*g)*f-48*d*e^2*h)*b
^2+7*a*(f*(c*h+d*g)-16/7*d*e*h)*f*b+a^2*d*f^2*h)*f^2*arctan(b*(f*x+e)^(1/2
)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^(1/2)*((63*c*f^4*g*x^4+21*x^3*(8/3*(-
c*h-d*g)*x+c*g)*e*f^3-42/5*x^2*(-40/7*d*h*x^2+20/9*(c*h+d*g)*x+c*g)*e^2*f^
2+24/5*x*(10/3*d*h*x^2+14/9*(c*h+d*g)*x+c*g)*e^3*f-16/5*(2*d*h*x^2+4/3*(c*
h+d*g)*x+c*g)*e^4)*b^5-16/15*a*(-3465/16*x^3*(1/33*(-c*h-d*g)*x+c*g)*f^4-1
197/16*x^2*(80/399*d*h*x^2+445/171*(-c*h-d*g)*x+c*g)*e*f^3+117/4*x*(-680/1
17*d*h*x^2+175/78*(c*h+d*g)*x+c*g)*e^2*f^2-33/2*e^3*(10/3*d*h*x^2+17/11*(c
*h+d*g)*x+c*g)*f+e^4*(4*d*h*x+c*h+d*g))*b^4+104/15*a^2*(4599/104*x^2*(-5/1
533*d*h*x^2+55/657*(-c*h-d*g)*x+c*g)*f^4+1665/104*x*(175/333*d*h*x^2+469/1
85*(-c*h-d*g)*x+c*g)*e*f^3-315/52*(-127/21*d*h*x^2+34/15*(c*h+d*g)*x+c*g)*
e^2*f^2+e^3*(75/13*d*h*x+c*h+d*g)*f-2/13*d*e^4*h)*b^3-74/3*a^3*(1/74*(11*d
*h*x^3+511/5*(c*h+d*g)*x^2-2511/5*c*g*x)*f^3-195/74*(383/325*d*h*x^2+2417/
975*(-c*h-d*g)*x+c*g)*e*f^2+e^2*(-1446/185*d*h*x+c*h+d*g)*f-76/185*d*e^3*h
)*f*b^2-221/5*a^4*(1/221*(73/3*d*h*x^2+93*(c*h+d*g)*x-128*c*g)*f^2+e*(-337
/663*d*h*x+c*h+d*g)*f-794/663*d*e^2*h)*f^2*b+a^5*d*f^3*h*(f*x+e)))/(f*x+e)
^(1/2)/((a*f-b*e)*b)^(1/2)/(b*x+a)^4/(a*f-b*e)^5/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2620 vs. $2(446) = 892$.

Time = 1.10 (sec) , antiderivative size = 5254, normalized size of antiderivative = 11.01

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)**5/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1508 vs. $2(446) = 892$.

Time = 0.18 (sec) , antiderivative size = 1508, normalized size of antiderivative = 3.16

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-5/64*(56*b^2*d*e*f^3*g - 63*b^2*c*f^4*g + 7*a*b*d*f^4*g - 48*b^2*d*e^2*f^
2*h + 56*b^2*c*e*f^3*h - 16*a*b*d*e*f^3*h + 7*a*b*c*f^4*h + a^2*d*f^4*h)*a
rctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^5 - 5*a*b^5*e^4*f + 10
*a^2*b^4*e^3*f^2 - 10*a^3*b^3*e^2*f^3 + 5*a^4*b^2*e*f^4 - a^5*b*f^5)*sqrt(
-b^2*e + a*b*f)) - 2*(d*e*f^3*g - c*f^4*g - d*e^2*f^2*h + c*e*f^3*h)/((b^5
*e^5 - 5*a*b^4*e^4*f + 10*a^2*b^3*e^3*f^2 - 10*a^3*b^2*e^2*f^3 + 5*a^4*b*e
*f^4 - a^5*f^5)*sqrt(f*x + e)) - 1/192*(456*(f*x + e)^(7/2)*b^5*d*e*f^3*g
- 1544*(f*x + e)^(5/2)*b^5*d*e^2*f^3*g + 1784*(f*x + e)^(3/2)*b^5*d*e^3*f^
3*g - 696*sqrt(f*x + e)*b^5*d*e^4*f^3*g - 561*(f*x + e)^(7/2)*b^5*c*f^4*g
+ 105*(f*x + e)^(7/2)*a*b^4*d*f^4*g + 1929*(f*x + e)^(5/2)*b^5*c*e*f^4*g +
 1159*(f*x + e)^(5/2)*a*b^4*d*e*f^4*g - 2295*(f*x + e)^(3/2)*b^5*c*e^2*f^4
*g - 3057*(f*x + e)^(3/2)*a*b^4*d*e^2*f^4*g + 975*sqrt(f*x + e)*b^5*c*e^3*
f^4*g + 1809*sqrt(f*x + e)*a*b^4*d*e^3*f^4*g - 1929*(f*x + e)^(5/2)*a*b^4*
c*f^5*g + 385*(f*x + e)^(5/2)*a^2*b^3*d*f^5*g + 4590*(f*x + e)^(3/2)*a*b^4
*c*e*f^5*g + 762*(f*x + e)^(3/2)*a^2*b^3*d*e*f^5*g - 2925*sqrt(f*x + e)*a*
b^4*c*e^2*f^5*g - 1251*sqrt(f*x + e)*a^2*b^3*d*e^2*f^5*g - 2295*(f*x + e)^
(3/2)*a^2*b^3*c*f^6*g + 511*(f*x + e)^(3/2)*a^3*b^2*d*f^6*g + 2925*sqrt(f*
x + e)*a^2*b^3*c*e*f^6*g - 141*sqrt(f*x + e)*a^3*b^2*d*e*f^6*g - 975*sqrt(
f*x + e)*a^3*b^2*c*f^7*g + 279*sqrt(f*x + e)*a^4*b*d*f^7*g - 336*(f*x + e)
^(7/2)*b^5*d*e^2*f^2*h + 1104*(f*x + e)^(5/2)*b^5*d*e^3*f^2*h - 1200*(f...
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 991, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(c + d*x))/((e + f*x)^(3/2)*(a + b*x)^5),x)
```

output
```
((55*(e + f*x)^3*(7*a*b^2*c*f^4*h - 63*b^3*c*f^4*g + 7*a*b^2*d*f^4*g + a^2
*b*d*f^4*h + 56*b^3*c*e*f^3*h + 56*b^3*d*e*f^3*g - 48*b^3*d*e^2*f^2*h - 16
*a*b^2*d*e*f^3*h))/(192*(a*f - b*e)^4) - (2*(c*f^4*g - c*e*f^3*h - d*e*f^3
*g + d*e^2*f^2*h))/(a*f - b*e) + (73*(e + f*x)^2*(a^2*d*f^4*h - 63*b^2*c*f
^4*g + 56*b^2*c*e*f^3*h + 56*b^2*d*e*f^3*g - 48*b^2*d*e^2*f^2*h + 7*a*b*c*
f^4*h + 7*a*b*d*f^4*g - 16*a*b*d*e*f^3*h))/(192*(a*f - b*e)^3) - ((e + f*x
)*(837*b^2*c*f^4*g + 5*a^2*d*f^4*h - 744*b^2*c*e*f^3*h - 744*b^2*d*e*f^3*g
 + 656*b^2*d*e^2*f^2*h - 93*a*b*c*f^4*h - 93*a*b*d*f^4*g + 176*a*b*d*e*f^3
*h))/(64*b*(a*f - b*e)^2) + (5*b^2*(e + f*x)^4*(a^2*d*f^4*h - 63*b^2*c*f^4
*g + 56*b^2*c*e*f^3*h + 56*b^2*d*e*f^3*g - 48*b^2*d*e^2*f^2*h + 7*a*b*c*f^
4*h + 7*a*b*d*f^4*g - 16*a*b*d*e*f^3*h))/(64*(a*f - b*e)^5))/(b^4*(e + f*x
)^(9/2) - (e + f*x)^(3/2)*(4*b^4*e^3 - 4*a^3*b*f^3 + 12*a^2*b^2*e*f^2 - 12
*a*b^3*e^2*f) - (e + f*x)^(7/2)*(4*b^4*e - 4*a*b^3*f) + (e + f*x)^(1/2)*(a
^4*f^4 + b^4*e^4 + 6*a^2*b^2*e^2*f^2 - 4*a*b^3*e^3*f - 4*a^3*b*e*f^3) + (e
 + f*x)^(5/2)*(6*b^4*e^2 + 6*a^2*b^2*f^2 - 12*a*b^3*e*f)) - (5*f^2*atan((5
*f^2*(e + f*x)^(1/2)*(b^6*e^5 - a^5*b*f^5 + 5*a^4*b^2*e*f^4 + 10*a^2*b^4*e
^3*f^2 - 10*a^3*b^3*e^2*f^3 - 5*a*b^5*e^4*f)*(a^2*d*f^2*h - 63*b^2*c*f^2*g
 - 48*b^2*d*e^2*h + 7*a*b*c*f^2*h + 7*a*b*d*f^2*g + 56*b^2*c*e*f*h + 56*b^
2*d*e*f*g - 16*a*b*d*e*f*h))/(b^(1/2)*(a*f - b*e)^(11/2)*(5*a^2*d*f^4*h -
315*b^2*c*f^4*g + 280*b^2*c*e*f^3*h + 280*b^2*d*e*f^3*g - 240*b^2*d*e^2...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 4152, normalized size of antiderivative = 8.70

$$\int \frac{(c+dx)(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x)
```

output

```
(15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**6*d*f**4*h + 105*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c*f**4*h - 240*
sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**5*b*d*e*f**3*h + 105*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d*f**4*g + 60*s
qrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**5*b*d*f**4*h*x + 840*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*e*f**3*h -
945*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**4*b**2*c*f**4*g + 420*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*f**4*
h*x - 720*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e**2*f**2*h + 840*sqrt(b)*sqrt(e + f*x
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b
**2*d*e*f**3*g - 960*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*e*f**3*h*x + 420*sqrt(b)*sq
rt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e
)))*a**4*b**2*d*f**4*g*x + 90*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d*f**4*h*x**2 + 3...
```

3.159 $\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 575

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(be-af)^3(de-cf)^2(fg-eh)}{f^7\sqrt{e+fx}}$$
$$+\frac{2(be-af)^2(de-cf)(bde(5fg-6eh)-bcf(3fg-4eh)-af(2dfg-3deh+cfh))\sqrt{e+fx}}{f^7}$$
$$-\frac{2(be-af)\left(a^2df^2(dfg-3deh+2cfh)+abf(3c^2f^2h-d^2e(8fg-15eh)+2cdf(3fg-8eh))-b^2(4cde\cdots\right.}{3f^7}$$
$$+\frac{2(a^3d^2f^3h+3a^2bdf^2(dfg-4deh+2cfh)+3ab^2f(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh))-b^3(4c\cdots}{5f^7}$$
$$+\frac{2b(3a^2d^2f^2h+3abdf(dfg-5deh+2cfh)+b^2(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)))\,(e+fx)^{7/2}}{7f^7}$$
$$+\frac{2b^2d(3adfh+b(dfg-6deh+2cfh))(e+fx)^{9/2}}{9f^7}+\frac{2b^3d^2h(e+fx)^{11/2}}{11f^7}$$

output
```
2*(-a*f+b*e)^3*(-c*f+d*e)^2*(-e*h+f*g)/f^7/(f*x+e)^(1/2)+2*(-a*f+b*e)^2*(-
c*f+d*e)*(b*d*e*(-6*e*h+5*f*g)-b*c*f*(-4*e*h+3*f*g)-a*f*(c*f*h-3*d*e*h+2*d
*f*g))*(f*x+e)^(1/2)/f^7-2/3*(-a*f+b*e)*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d*f*g)
+a*b*f*(3*c^2*f^2*h-d^2*e*(-15*e*h+8*f*g)+2*c*d*f*(-8*e*h+3*f*g))-b^2*(4*c
*d*e*f*(-5*e*h+3*f*g)-5*d^2*e^2*(-3*e*h+2*f*g)-3*c^2*f^2*(-2*e*h+f*g)))*(f
*x+e)^(3/2)/f^7+2/5*(a^3*d^2*f^3*h+3*a^2*b*d*f^2*(2*c*f*h-4*d*e*h+d*f*g)+3
*a*b^2*f*(c^2*f^2*h-2*d^2*e*(-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g))-b^3*(4*c*
d*e*f*(-5*e*h+2*f*g)-c^2*f^2*(-4*e*h+f*g)-10*d^2*e^2*(-2*e*h+f*g)))*(f*x+e
)^(5/2)/f^7+2/7*b*(3*a^2*d^2*f^2*h+3*a*b*d*f*(2*c*f*h-5*d*e*h+d*f*g)+b^2*(
c^2*f^2*h+2*c*d*f*(-5*e*h+f*g)-5*d^2*e*(-3*e*h+f*g)))*(f*x+e)^(7/2)/f^7+2/
9*b^2*d*(3*a*d*f*h+b*(2*c*f*h-6*d*e*h+d*f*g))*(f*x+e)^(9/2)/f^7+2/11*b^3*d
^2*h*(f*x+e)^(11/2)/f^7
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(231a^3f^3(15c^2f^2(-fg+2eh+fhx)+10cdf(-8e^2h+ef(6g-4hx)$$

input
```
Integrate[((a + b*x)^3*(c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(231*a^3*f^3*(15*c^2*f^2*(-(f*g) + 2*e*h + f*h*x) + 10*c*d*f*(-8*e^2*h
+ e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x)) + d^2*(48*e^3*h - 8*e^2*f*(5*g -
3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x))) + 99*a^2*b*f^2
*(35*c^2*f^2*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x)) + 14*c*d*f
*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10
*g + 3*h*x)) - 3*d^2*(128*e^4*h - 16*e^3*f*(7*g - 4*h*x) - 8*e^2*f^2*x*(7*
g + 2*h*x) + 2*e*f^3*x^2*(7*g + 4*h*x) - f^4*x^3*(7*g + 5*h*x))) + 33*a*b^
2*f*(21*c^2*f^2*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x)
- 2*e*f^2*x*(10*g + 3*h*x)) + 18*c*d*f*(-128*e^4*h + 16*e^3*f*(7*g - 4*h*x
) + 8*e^2*f^2*x*(7*g + 2*h*x) - 2*e*f^3*x^2*(7*g + 4*h*x) + f^4*x^3*(7*g +
 5*h*x)) + d^2*(1280*e^5*h - 128*e^4*f*(9*g - 5*h*x) + 16*e^2*f^3*x^2*(9*g
 + 5*h*x) - 32*e^3*f^2*x*(18*g + 5*h*x) + 5*f^5*x^4*(9*g + 7*h*x) - 2*e*f^
4*x^3*(36*g + 25*h*x))) + b^3*(99*c^2*f^2*(-128*e^4*h + 16*e^3*f*(7*g - 4*
h*x) + 8*e^2*f^2*x*(7*g + 2*h*x) - 2*e*f^3*x^2*(7*g + 4*h*x) + f^4*x^3*(7*
g + 5*h*x)) + 22*c*d*f*(1280*e^5*h - 128*e^4*f*(9*g - 5*h*x) + 16*e^2*f^3*
x^2*(9*g + 5*h*x) - 32*e^3*f^2*x*(18*g + 5*h*x) + 5*f^5*x^4*(9*g + 7*h*x)
- 2*e*f^4*x^3*(36*g + 25*h*x)) - 5*d^2*(3072*e^6*h - 256*e^5*f*(11*g - 6*h
*x) - 128*e^4*f^2*x*(11*g + 3*h*x) + 32*e^3*f^3*x^2*(11*g + 6*h*x) - 7*f^6
*x^5*(11*g + 9*h*x) - 8*e^2*f^4*x^3*(22*g + 15*h*x) + 2*e*f^5*x^4*(55*g +
42*h*x)))))/(3465*f^7*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{\sqrt{e+fx}(be-af)\left(-a^2df^2(2cfh-3deh+dfg)-abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh)\right)-\right.}{f^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(e+fx)^{3/2}(be-af)\left(a^2df^2(2cfh-3deh+dfg)+abf\left(3c^2f^2h+2cdf(3fg-8eh)+d^2(-e)(8fg-15eh)\right)-\right.}{3f^7}$$

$$\frac{2b(e+fx)^{7/2}\left(3a^2d^2f^2h+3abdf(2cfh-5deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-5eh)-5d^2e(fg-3eh)\right)\right)}{7f^7}+$$

$$\frac{2(e+fx)^{5/2}\left(a^3d^2f^3h+3a^2bdf^2(2cfh-4deh+dfg)+3ab^2f\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)-\right.}{5f^7}$$

$$\frac{2b^2d(e+fx)^{9/2}(3adfh+b(2cfh-6deh+dfg))}{9f^7}+$$

$$\frac{2\sqrt{e+fx}(be-af)^2(de-cf)(-af(cfh-3deh+2dfg)-bcf(3fg-4eh)+bde(5fg-6eh))}{f^7}+$$

$$\frac{2(be-af)^3(de-cf)^2(fg-eh)}{f^7\sqrt{e+fx}}+\frac{2b^3d^2h(e+fx)^{11/2}}{11f^7}$$

input

```
Int[((a + b*x)^3*(c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output

```
(2*(b*e - a*f)^3*(d*e - c*f)^2*(f*g - e*h))/(f^7*Sqrt[e + f*x]) + (2*(b*e
- a*f)^2*(d*e - c*f)*(b*d*e*(5*f*g - 6*e*h) - b*c*f*(3*f*g - 4*e*h) - a*f*
(2*d*f*g - 3*d*e*h + c*f*h))*Sqrt[e + f*x])/f^7 - (2*(b*e - a*f)*(a^2*d*f^
2*(d*f*g - 3*d*e*h + 2*c*f*h) + a*b*f*(3*c^2*f^2*h - d^2*e*(8*f*g - 15*e*h
) + 2*c*d*f*(3*f*g - 8*e*h)) - b^2*(4*c*d*e*f*(3*f*g - 5*e*h) - 5*d^2*e^2*
(2*f*g - 3*e*h) - 3*c^2*f^2*(f*g - 2*e*h)))*(e + f*x)^(3/2))/(3*f^7) + (2*
(a^3*d^2*f^3*h + 3*a^2*b*d*f^2*(d*f*g - 4*d*e*h + 2*c*f*h) + 3*a*b^2*f*(c^
2*f^2*h - 2*d^2*e*(2*f*g - 5*e*h) + 2*c*d*f*(f*g - 4*e*h)) - b^3*(4*c*d*e*
f*(2*f*g - 5*e*h) - c^2*f^2*(f*g - 4*e*h) - 10*d^2*e^2*(f*g - 2*e*h)))*(e
+ f*x)^(5/2))/(5*f^7) + (2*b*(3*a^2*d^2*f^2*h + 3*a*b*d*f*(d*f*g - 5*d*e*h
 + 2*c*f*h) + b^2*(c^2*f^2*h + 2*c*d*f*(f*g - 5*e*h) - 5*d^2*e*(f*g - 3*e*
h)))*(e + f*x)^(7/2))/(7*f^7) + (2*b^2*d*(3*a*d*f*h + b*(d*f*g - 6*d*e*h +
 2*c*f*h))*(e + f*x)^(9/2))/(9*f^7) + (2*b^3*d^2*h*(e + f*x)^(11/2))/(11*f
^7)
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$2\left(\frac{\left(\frac{5x^2\left(\frac{9hx}{11}+g\right)d^2}{9}+\frac{10xc\left(\frac{7hx}{9}+g\right)d}{7}+c^2\left(\frac{5hx}{7}+g\right)\right)x^3b^3}{5}+a\,x^2\left(\frac{3x^2\left(\frac{7hx}{9}+g\right)d^2}{7}+\frac{6xc\left(\frac{5hx}{7}+g\right)d}{5}+c^2\left(\frac{3hx}{5}+g\right)\right)b^2+3a$
risch	Expression too large to display
gosper	Expression too large to display
trager	Expression too large to display
orering	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/(f*x+e)^(1/2)*(1/2*(1/5*(5/9*x^2*(9/11*h*x+g)*d^2+10/7*x*c*(7/9*h*x+g)*d
+c^2*(5/7*h*x+g))*x^3*b^3+a*x^2*(3/7*x^2*(7/9*h*x+g)*d^2+6/5*x*c*(5/7*h*x+
g)*d+c^2*(3/5*h*x+g))*b^2+3*a^2*x*(1/5*x^2*(5/7*h*x+g)*d^2+2/3*x*c*(3/5*h*
x+g)*d+c^2*(1/3*h*x+g))*b-a^3*(-1/3*x^2*(3/5*h*x+g)*d^2-2*x*c*(1/3*h*x+g)*
d+c^2*(-h*x+g)))*f^6+(-1/5*(25/63*(42/55*h*x+g)*x^2*d^2+8/7*x*c*(25/36*h*x
+g)*d+c^2*(4/7*h*x+g))*x^2*b^3-2*a*x*(1/7*(5/6*h*x^3+6/5*g*x^2)*d^2+3/5*x*
c*(4/7*h*x+g)*d+(3/10*h*x+g)*c^2)*b^2+3*a^2*(-1/5*x^2*(4/7*h*x+g)*d^2-4/3*
(3/10*h*x+g)*x*c*d+c^2*(-2/3*h*x+g))*b+a^3*((-1/5*h*x^2-2/3*g*x)*d^2+2*c*(
-2/3*h*x+g)*d+h*c^2))*e*f^5-8/3*(-3/10*(10/63*x^2*(15/22*h*x+g)*d^2+4/7*x*
c*(5/9*h*x+g)*d+c^2*(2/7*h*x+g))*x*b^3+3/2*a*(-6/35*x^2*(5/9*h*x+g)*d^2-6/
5*x*c*(2/7*h*x+g)*d+c^2*(-3/5*h*x+g))*b^2+3/2*a^2*(-3/5*x*(2/7*h*x+g)*d^2+
2*c*(-3/5*h*x+g)*d+h*c^2)*b+a^3*d*(1/2*(-3/5*h*x+g)*d+c*h))*e^2*f^4+8/5*e^
3*((-10/63*x^2*(6/11*h*x+g)*d^2-8/7*x*c*(5/18*h*x+g)*d+c^2*(-4/7*h*x+g))*b
^3+3*a*(-4/7*x*(5/18*h*x+g)*d^2+2*c*(-4/7*h*x+g)*d+h*c^2)*b^2+6*a^2*d*((-2
/7*h*x+1/2*g)*d+c*h)*b+h*d^2*a^3)*f^3-192/35*(1/3*(-5/9*x*(3/11*h*x+g)*d^2
+2*(-5/9*h*x+g)*c*d+h*c^2)*b^2+2*a*d*(1/2*(-5/9*h*x+g)*d+c*h)*b+a^2*d^2*h)
*b*e^4*f^2+128/21*d*(((-2/11*h*x+1/3*g)*d+2/3*c*h)*b+a*d*h)*b^2*e^5*f-512/
231*b^3*d^2*e^6*h)/f^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. 2(548) = 1096.

Time = 0.10 (sec) , antiderivative size = 1183, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/3465*(315*b^3*d^2*f^6*h*x^6 + 35*(11*b^3*d^2*f^6*g - (12*b^3*d^2*e*f^5 -
 11*(2*b^3*c*d + 3*a*b^2*d^2)*f^6)*h)*x^5 - 5*(11*(10*b^3*d^2*e*f^5 - 9*(2
*b^3*c*d + 3*a*b^2*d^2)*f^6)*g - (120*b^3*d^2*e^2*f^4 - 110*(2*b^3*c*d + 3
*a*b^2*d^2)*e*f^5 + 99*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^6)*h)*x^4 +
 (11*(80*b^3*d^2*e^2*f^4 - 72*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^5 + 63*(b^3*c^
2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^6)*g - (960*b^3*d^2*e^3*f^3 - 880*(2*b^3*
c*d + 3*a*b^2*d^2)*e^2*f^4 + 792*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f
^5 - 693*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^6)*h)*x^3 - (11*(160*b^3*
d^2*e^3*f^3 - 144*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^4 + 126*(b^3*c^2 + 6*a*b
^2*c*d + 3*a^2*b*d^2)*e*f^5 - 105*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^
6)*g - (1920*b^3*d^2*e^4*f^2 - 1760*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^3 + 15
84*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^2*f^4 - 1386*(3*a*b^2*c^2 + 6*a
^2*b*c*d + a^3*d^2)*e*f^5 + 1155*(3*a^2*b*c^2 + 2*a^3*c*d)*f^6)*h)*x^2 + 1
1*(1280*b^3*d^2*e^5*f - 315*a^3*c^2*f^6 - 1152*(2*b^3*c*d + 3*a*b^2*d^2)*e
^4*f^2 + 1008*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^3 - 840*(3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2*f^4 + 630*(3*a^2*b*c^2 + 2*a^3*c*d)*e*f^
5)*g - 2*(7680*b^3*d^2*e^6 - 3465*a^3*c^2*e*f^5 - 7040*(2*b^3*c*d + 3*a*b^
2*d^2)*e^5*f + 6336*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^4*f^2 - 5544*(
3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^3*f^3 + 4620*(3*a^2*b*c^2 + 2*a^3*c
*d)*e^2*f^4)*h + (11*(640*b^3*d^2*e^4*f^2 - 576*(2*b^3*c*d + 3*a*b^2*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(d*x+c)**2*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(548) = 1096$.

Time = 0.04 (sec) , antiderivative size = 1160, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/3465*((315*(f*x + e)^(11/2)*b^3*d^2*h + 385*(b^3*d^2*f*g - (6*b^3*d^2*e
- (2*b^3*c*d + 3*a*b^2*d^2)*f)*h)*(f*x + e)^(9/2) - 495*((5*b^3*d^2*e*f -
(2*b^3*c*d + 3*a*b^2*d^2)*f^2)*g - (15*b^3*d^2*e^2 - 5*(2*b^3*c*d + 3*a*b^
2*d^2)*e*f + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*h)*(f*x + e)^(7/2)
 + 693*((10*b^3*d^2*e^2*f - 4*(2*b^3*c*d + 3*a*b^2*d^2)*e*f^2 + (b^3*c^2 +
 6*a*b^2*c*d + 3*a^2*b*d^2)*f^3)*g - (20*b^3*d^2*e^3 - 10*(2*b^3*c*d + 3*a
*b^2*d^2)*e^2*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e*f^2 - (3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^3)*h)*(f*x + e)^(5/2) - 1155*((10*b^3*d^2*
e^3*f - 6*(2*b^3*c*d + 3*a*b^2*d^2)*e^2*f^2 + 3*(b^3*c^2 + 6*a*b^2*c*d + 3
*a^2*b*d^2)*e*f^3 - (3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*f^4)*g - (15*b^3
*d^2*e^4 - 10*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f + 6*(b^3*c^2 + 6*a*b^2*c*d +
 3*a^2*b*d^2)*e^2*f^2 - 3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^3 + (3
*a^2*b*c^2 + 2*a^3*c*d)*f^4)*h)*(f*x + e)^(3/2) + 3465*((5*b^3*d^2*e^4*f -
 4*(2*b^3*c*d + 3*a*b^2*d^2)*e^3*f^2 + 3*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*
d^2)*e^2*f^3 - 2*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e*f^4 + (3*a^2*b*c^
2 + 2*a^3*c*d)*f^5)*g - (6*b^3*d^2*e^5 - a^3*c^2*f^5 - 5*(2*b^3*c*d + 3*a*
b^2*d^2)*e^4*f + 4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^2 - 3*(3*a*
b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*e^2*f^3 + 2*(3*a^2*b*c^2 + 2*a^3*c*d)*e*f
^4)*h)*sqrt(f*x + e))/f^6 + 3465*((b^3*d^2*e^5*f - a^3*c^2*f^6 - (2*b^3*c*
d + 3*a*b^2*d^2)*e^4*f^2 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*e^3*f^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. 2(548) = 1096.

Time = 0.17 (sec) , antiderivative size = 1849, normalized size of antiderivative = 3.22

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^3*d^2*e^5*f*g - 2*b^3*c*d*e^4*f^2*g - 3*a*b^2*d^2*e^4*f^2*g + b^3*c^2
*e^3*f^3*g + 6*a*b^2*c*d*e^3*f^3*g + 3*a^2*b*d^2*e^3*f^3*g - 3*a*b^2*c^2*e
^2*f^4*g - 6*a^2*b*c*d*e^2*f^4*g - a^3*d^2*e^2*f^4*g + 3*a^2*b*c^2*e*f^5*g
 + 2*a^3*c*d*e*f^5*g - a^3*c^2*f^6*g - b^3*d^2*e^6*h + 2*b^3*c*d*e^5*f*h +
 3*a*b^2*d^2*e^5*f*h - b^3*c^2*e^4*f^2*h - 6*a*b^2*c*d*e^4*f^2*h - 3*a^2*b
*d^2*e^4*f^2*h + 3*a*b^2*c^2*e^3*f^3*h + 6*a^2*b*c*d*e^3*f^3*h + a^3*d^2*e
^3*f^3*h - 3*a^2*b*c^2*e^2*f^4*h - 2*a^3*c*d*e^2*f^4*h + a^3*c^2*e*f^5*h)/
(sqrt(f*x + e)*f^7) + 2/3465*(385*(f*x + e)^(9/2)*b^3*d^2*f^71*g - 2475*(f
*x + e)^(7/2)*b^3*d^2*e*f^71*g + 6930*(f*x + e)^(5/2)*b^3*d^2*e^2*f^71*g -
 11550*(f*x + e)^(3/2)*b^3*d^2*e^3*f^71*g + 17325*sqrt(f*x + e)*b^3*d^2*e^
4*f^71*g + 990*(f*x + e)^(7/2)*b^3*c*d*f^72*g + 1485*(f*x + e)^(7/2)*a*b^2
*d^2*f^72*g - 5544*(f*x + e)^(5/2)*b^3*c*d*e*f^72*g - 8316*(f*x + e)^(5/2)
*a*b^2*d^2*e*f^72*g + 13860*(f*x + e)^(3/2)*b^3*c*d*e^2*f^72*g + 20790*(f*
x + e)^(3/2)*a*b^2*d^2*e^2*f^72*g - 27720*sqrt(f*x + e)*b^3*c*d*e^3*f^72*g
 - 41580*sqrt(f*x + e)*a*b^2*d^2*e^3*f^72*g + 693*(f*x + e)^(5/2)*b^3*c^2*
f^73*g + 4158*(f*x + e)^(5/2)*a*b^2*c*d*f^73*g + 2079*(f*x + e)^(5/2)*a^2*
b*d^2*f^73*g - 3465*(f*x + e)^(3/2)*b^3*c^2*e*f^73*g - 20790*(f*x + e)^(3/
2)*a*b^2*c*d*e*f^73*g - 10395*(f*x + e)^(3/2)*a^2*b*d^2*e*f^73*g + 10395*s
qrt(f*x + e)*b^3*c^2*e^2*f^73*g + 62370*sqrt(f*x + e)*a*b^2*c*d*e^2*f^73*g
 + 31185*sqrt(f*x + e)*a^2*b*d^2*e^2*f^73*g + 3465*(f*x + e)^(3/2)*a*b^...
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1011, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(a + b*x)^3*(c + d*x)^2)/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(7/2)*(2*b^3*c^2*f^2*h + 30*b^3*d^2*e^2*h + 4*b^3*c*d*f^2*g - 1
0*b^3*d^2*e*f*g + 6*a*b^2*d^2*f^2*g + 6*a^2*b*d^2*f^2*h - 20*b^3*c*d*e*f*h
 + 12*a*b^2*c*d*f^2*h - 30*a*b^2*d^2*e*f*h))/(7*f^7) + ((e + f*x)^(5/2)*(2
*b^3*c^2*f^3*g + 2*a^3*d^2*f^3*h - 40*b^3*d^2*e^3*h + 6*a*b^2*c^2*f^3*h +
6*a^2*b*d^2*f^3*g - 8*b^3*c^2*e*f^2*h + 20*b^3*d^2*e^2*f*g + 12*a*b^2*c*d*
f^3*g + 12*a^2*b*c*d*f^3*h - 16*b^3*c*d*e*f^2*g + 40*b^3*c*d*e^2*f*h - 24*
a*b^2*d^2*e*f^2*g + 60*a*b^2*d^2*e^2*f*h - 24*a^2*b*d^2*e*f^2*h - 48*a*b^2
*c*d*e*f^2*h))/(5*f^7) - (2*a^3*c^2*f^6*g + 2*b^3*d^2*e^6*h - 2*a^3*c^2*e*
f^5*h - 2*b^3*d^2*e^5*f*g + 2*a^3*d^2*e^2*f^4*g - 2*b^3*c^2*e^3*f^3*g - 2*
a^3*d^2*e^3*f^3*h + 2*b^3*c^2*e^4*f^2*h + 6*a*b^2*c^2*e^2*f^4*g - 6*a*b^2*
c^2*e^3*f^3*h + 6*a*b^2*d^2*e^4*f^2*g + 6*a^2*b*c^2*e^2*f^4*h - 6*a^2*b*d^
2*e^3*f^3*g + 6*a^2*b*d^2*e^4*f^2*h - 4*a^3*c*d*e*f^5*g - 4*b^3*c*d*e^5*f*
h - 6*a^2*b*c^2*e*f^5*g - 6*a*b^2*d^2*e^5*f*h + 4*a^3*c*d*e^2*f^4*h + 4*b^
3*c*d*e^4*f^2*g - 12*a*b^2*c*d*e^3*f^3*g + 12*a^2*b*c*d*e^2*f^4*g + 12*a*b
^2*c*d*e^4*f^2*h - 12*a^2*b*c*d*e^3*f^3*h)/(f^7*(e + f*x)^(1/2)) + (2*(e +
 f*x)^(3/2)*(a*f - b*e)*(a^2*d^2*f^3*g + 3*b^2*c^2*f^3*g - 15*b^2*d^2*e^3*
h + 3*a*b*c^2*f^3*h + 2*a^2*c*d*f^3*h - 3*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^
2*h + 10*b^2*d^2*e^2*f*g + 6*a*b*c*d*f^3*g - 8*a*b*d^2*e*f^2*g + 15*a*b*d^
2*e^2*f*h - 12*b^2*c*d*e*f^2*g + 20*b^2*c*d*e^2*f*h - 16*a*b*c*d*e*f^2*h))
/(3*f^7) + (2*b^3*d^2*h*(e + f*x)^(11/2))/(11*f^7) + (2*b^2*d*(e + f*x)...
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1511, normalized size of antiderivative = 2.63

$$\int \frac{(a+bx)^3(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(6930*a**3*c**2*e*f**5*h - 3465*a**3*c**2*f**6*g + 3465*a**3*c**2*f**6*
h*x - 18480*a**3*c*d*e**2*f**4*h + 13860*a**3*c*d*e*f**5*g - 9240*a**3*c*d
*e*f**5*h*x + 6930*a**3*c*d*f**6*g*x + 2310*a**3*c*d*f**6*h*x**2 + 11088*a
**3*d**2*e**3*f**3*h - 9240*a**3*d**2*e**2*f**4*g + 5544*a**3*d**2*e**2*f*
*4*h*x - 4620*a**3*d**2*e*f**5*g*x - 1386*a**3*d**2*e*f**5*h*x**2 + 1155*a
**3*d**2*f**6*g*x**2 + 693*a**3*d**2*f**6*h*x**3 - 27720*a**2*b*c**2*e**2*
f**4*h + 20790*a**2*b*c**2*e*f**5*g - 13860*a**2*b*c**2*e*f**5*h*x + 10395
*a**2*b*c**2*f**6*g*x + 3465*a**2*b*c**2*f**6*h*x**2 + 66528*a**2*b*c*d*e*
*3*f**3*h - 55440*a**2*b*c*d*e**2*f**4*g + 33264*a**2*b*c*d*e**2*f**4*h*x
- 27720*a**2*b*c*d*e*f**5*g*x - 8316*a**2*b*c*d*e*f**5*h*x**2 + 6930*a**2*
b*c*d*f**6*g*x**2 + 4158*a**2*b*c*d*f**6*h*x**3 - 38016*a**2*b*d**2*e**4*f
**2*h + 33264*a**2*b*d**2*e**3*f**3*g - 19008*a**2*b*d**2*e**3*f**3*h*x +
16632*a**2*b*d**2*e**2*f**4*g*x + 4752*a**2*b*d**2*e**2*f**4*h*x**2 - 4158
*a**2*b*d**2*e*f**5*g*x**2 - 2376*a**2*b*d**2*e*f**5*h*x**3 + 2079*a**2*b*
d**2*f**6*g*x**3 + 1485*a**2*b*d**2*f**6*h*x**4 + 33264*a*b**2*c**2*e**3*f
**3*h - 27720*a*b**2*c**2*e**2*f**4*g + 16632*a*b**2*c**2*e**2*f**4*h*x -
13860*a*b**2*c**2*e*f**5*g*x - 4158*a*b**2*c**2*e*f**5*h*x**2 + 3465*a*b**
2*c**2*f**6*g*x**2 + 2079*a*b**2*c**2*f**6*h*x**3 - 76032*a*b**2*c*d*e**4*
f**2*h + 66528*a*b**2*c*d*e**3*f**3*g - 38016*a*b**2*c*d*e**3*f**3*h*x + 3
3264*a*b**2*c*d*e**2*f**4*g*x + 9504*a*b**2*c*d*e**2*f**4*h*x**2 - 8316...
```

3.160 $\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 407

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = -\frac{2(be-af)^2(de-cf)^2(fg-eh)}{f^6\sqrt{e+fx}}$$
$$-\frac{2(be-af)(de-cf)(bde(4fg-5eh)-bcf(2fg-3eh)-af(2dfg-3deh+cfh))\sqrt{e+fx}}{f^6}$$
$$+\frac{2(a^2df^2(dfg-3deh+2cfh)+2abf(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh))+b^2(2d^2e^2(3fg-5eh)}{3f^6}$$
$$+\frac{2(a^2d^2f^2h+2abdf(dfg-4deh+2cfh)+b^2(c^2f^2h-2d^2e(2fg-5eh)+2cdf(fg-4eh)))\,(e+fx)^{5/2}}{5f^6}$$
$$+\frac{2bd(2adfh+b(dfg-5deh+2cfh))(e+fx)^{7/2}}{7f^6}+\frac{2b^2d^2h(e+fx)^{9/2}}{9f^6}$$

output

```
-2*(-a*f+b*e)^2*(-c*f+d*e)^2*(-e*h+f*g)/f^6/(f*x+e)^(1/2)-2*(-a*f+b*e)*(-c
*f+d*e)*(b*d*e*(-5*e*h+4*f*g)-b*c*f*(-3*e*h+2*f*g)-a*f*(c*f*h-3*d*e*h+2*d*
f*g))*(f*x+e)^(1/2)/f^6+2/3*(a^2*d*f^2*(2*c*f*h-3*d*e*h+d*f*g)+2*a*b*f*(c^
2*f^2*h+2*c*d*f*(-3*e*h+f*g)-3*d^2*e*(-2*e*h+f*g))+b^2*(2*d^2*e^2*(-5*e*h+
3*f*g)+c^2*f^2*(-3*e*h+f*g)-6*c*d*e*f*(-2*e*h+f*g)))*(f*x+e)^(3/2)/f^6+2/5
*(a^2*d^2*f^2*h+2*a*b*d*f*(2*c*f*h-4*d*e*h+d*f*g)+b^2*(c^2*f^2*h-2*d^2*e*(
-5*e*h+2*f*g)+2*c*d*f*(-4*e*h+f*g)))*(f*x+e)^(5/2)/f^6+2/7*b*d*(2*a*d*f*h+
b*(2*c*f*h-5*d*e*h+d*f*g))*(f*x+e)^(7/2)/f^6+2/9*b^2*d^2*h*(f*x+e)^(9/2)/f
^6
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(21a^2f^2(15c^2f^2(-fg+2eh+fhx)+10cdf(-8e^2h+ef(6g-4hx)}{}$$

input
```
Integrate[((a + b*x)^2*(c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(21*a^2*f^2*(15*c^2*f^2*(-(f*g) + 2*e*h + f*h*x) + 10*c*d*f*(-8*e^2*h +
 e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x)) + d^2*(48*e^3*h - 8*e^2*f*(5*g - 3
*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x))) + 6*a*b*f*(35*c
^2*f^2*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x)) + 14*c*d*f*(48*e
^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3
*h*x)) - 3*d^2*(128*e^4*h - 16*e^3*f*(7*g - 4*h*x) - 8*e^2*f^2*x*(7*g + 2*
h*x) + 2*e*f^3*x^2*(7*g + 4*h*x) - f^4*x^3*(7*g + 5*h*x))) + b^2*(21*c^2*f
^2*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(
10*g + 3*h*x)) + 18*c*d*f*(-128*e^4*h + 16*e^3*f*(7*g - 4*h*x) + 8*e^2*f^2
*x*(7*g + 2*h*x) - 2*e*f^3*x^2*(7*g + 4*h*x) + f^4*x^3*(7*g + 5*h*x)) + d^
2*(1280*e^5*h - 128*e^4*f*(9*g - 5*h*x) + 16*e^2*f^3*x^2*(9*g + 5*h*x) - 3
2*e^3*f^2*x*(18*g + 5*h*x) + 5*f^5*x^4*(9*g + 7*h*x) - 2*e*f^4*x^3*(36*g +
 25*h*x)))))/(315*f^6*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 165$$

$$\int \left(\frac{\sqrt{e+fx}\left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-3e\right.\right.}{f^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(e+fx)^{3/2}\left(a^2df^2(2cfh-3deh+dfg)+2abf\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)+b^2\left(c^2f^2(fg-3e\right.\right.}{3f^6}$$

$$\frac{2(e+fx)^{5/2}\left(a^2d^2f^2h+2abdf(2cfh-4deh+dfg)+b^2\left(c^2f^2h+2cdf(fg-4eh)-2d^2e(2fg-5eh)\right)\right)}{5f^6}+$$

$$\frac{2bd(e+fx)^{7/2}(2adfh+b(2cfh-5deh+dfg))}{7f^6}-$$

$$\frac{2\sqrt{e+fx}(be-af)(de-cf)(-af(cfh-3deh+2dfg)-bcf(2fg-3eh)+bde(4fg-5eh))}{f^6}-$$

$$\frac{2(be-af)^2(de-cf)^2(fg-eh)}{f^6\sqrt{e+fx}}+\frac{2b^2d^2h(e+fx)^{9/2}}{9f^6}$$

input

```
Int[((a + b*x)^2*(c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output

```
(-2*(b*e - a*f)^2*(d*e - c*f)^2*(f*g - e*h))/(f^6*Sqrt[e + f*x]) - (2*(b*e
 - a*f)*(d*e - c*f)*(b*d*e*(4*f*g - 5*e*h) - b*c*f*(2*f*g - 3*e*h) - a*f*(
2*d*f*g - 3*d*e*h + c*f*h))*Sqrt[e + f*x])/f^6 + (2*(a^2*d*f^2*(d*f*g - 3*
d*e*h + 2*c*f*h) + 2*a*b*f*(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3*d^2*e*(f
*g - 2*e*h)) + b^2*(2*d^2*e^2*(3*f*g - 5*e*h) + c^2*f^2*(f*g - 3*e*h) - 6*
c*d*e*f*(f*g - 2*e*h)))*(e + f*x)^(3/2))/(3*f^6) + (2*(a^2*d^2*f^2*h + 2*a
*b*d*f*(d*f*g - 4*d*e*h + 2*c*f*h) + b^2*(c^2*f^2*h - 2*d^2*e*(2*f*g - 5*e
*h) + 2*c*d*f*(f*g - 4*e*h)))*(e + f*x)^(5/2))/(5*f^6) + (2*b*d*(2*a*d*f*h
 + b*(d*f*g - 5*d*e*h + 2*c*f*h))*(e + f*x)^(7/2))/(7*f^6) + (2*b^2*d^2*h*
(e + f*x)^(9/2))/(9*f^6)
```

Defintions of rubi rules used

rule 165

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{4\left(\frac{x^2\left(\frac{3x^2\left(\frac{7hx}{9}+g\right)d^2}{7}+\frac{6xc\left(\frac{5hx}{7}+g\right)d}{5}+c^2\left(\frac{3hx}{5}+g\right)\right)b^2}{6}+ax\left(\frac{x^2\left(\frac{5hx}{7}+g\right)d^2}{5}+\frac{2xc\left(\frac{3hx}{5}+g\right)d}{3}+c^2\left(\frac{hx}{3}+g\right)\right)b-\frac{a^2\left(-\frac{x^2}{\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{2(35h\,b^2d^2x^4f^4+90ab\,d^2f^4h\,x^3+90b^2cd\,f^4h\,x^3-85b^2d^2e\,f^3h\,x^3+45b^2d^2f^4g\,x^3+63a^2d^2f^4h\,x^2+252abcd\,f^4h\,x^2-23\cdots}{\cdots}$
gosper	$\frac{-\frac{16}{5}abcde\,f^4h\,x^2+\frac{64}{5}abcd\,e^2f^3hx-\frac{32}{3}abcde\,f^4gx-2g\,a^2c^2f^5+\frac{32}{5}b^2c^2e^3f^2h-\frac{16}{3}b^2c^2e^2f^3g-\frac{16}{3}a^2d^2e^2f^3g+\frac{128}{5}abcd\cdots}{\cdots}$
trager	$\frac{-\frac{16}{5}abcde\,f^4h\,x^2+\frac{64}{5}abcd\,e^2f^3hx-\frac{32}{3}abcde\,f^4gx-2g\,a^2c^2f^5+\frac{32}{5}b^2c^2e^3f^2h-\frac{16}{3}b^2c^2e^2f^3g-\frac{16}{3}a^2d^2e^2f^3g+\frac{128}{5}abcd\cdots}{\cdots}$
orering	$\frac{-\frac{16}{5}abcde\,f^4h\,x^2+\frac{64}{5}abcd\,e^2f^3hx-\frac{32}{3}abcde\,f^4gx-2g\,a^2c^2f^5+\frac{32}{5}b^2c^2e^3f^2h-\frac{16}{3}b^2c^2e^2f^3g-\frac{16}{3}a^2d^2e^2f^3g+\frac{128}{5}abcd\cdots}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/(f*x+e)^(1/2)*((1/6*x^2*(3/7*x^2*(7/9*h*x+g)*d^2+6/5*x*c*(5/7*h*x+g)*d+c
^2*(3/5*h*x+g))*b^2+a*x*(1/5*x^2*(5/7*h*x+g)*d^2+2/3*x*c*(3/5*h*x+g)*d+c^2
*(1/3*h*x+g))*b-1/2*a^2*(-1/3*x^2*(3/5*h*x+g)*d^2-2*x*c*(1/3*h*x+g)*d+c^2*
(-h*x+g)))*f^5+(-2/3*x*((5/42*h*x^3+6/35*g*x^2)*d^2+3/5*x*c*(4/7*h*x+g)*d+
(3/10*h*x+g)*c^2)*b^2+2*a*(-1/5*x^2*(4/7*h*x+g)*d^2-4/3*(3/10*h*x+g)*x*c*d
+c^2*(-2/3*h*x+g))*b+a^2*((-1/5*h*x^2-2/3*g*x)*d^2+2*c*(-2/3*h*x+g)*d+h*c^
2))*e*f^4-8/3*((-3/35*x^2*(5/9*h*x+g)*d^2-3/5*x*c*(2/7*h*x+g)*d+1/2*c^2*(-
3/5*h*x+g))*b^2+a*(-3/5*x*(2/7*h*x+g)*d^2+2*c*(-3/5*h*x+g)*d+h*c^2)*b+a^2*
d*((-3/10*h*x+1/2*g)*d+c*h))*e^2*f^3+8/5*((-4/7*x*(5/18*h*x+g)*d^2+2*c*(-4
/7*h*x+g)*d+h*c^2)*b^2+4*a*((-2/7*h*x+1/2*g)*d+c*h)*d*b+a^2*d^2*h)*e^3*f^2
-128/35*d*(((-5/18*h*x+1/2*g)*d+c*h)*b+a*d*h)*b*e^4*f+128/63*b^2*d^2*e^5*h
)/f^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^2*d^2*f^5*h*x^5 + 5*(9*b^2*d^2*f^5*g - 2*(5*b^2*d^2*e*f^4 - 9*
(b^2*c*d + a*b*d^2)*f^5)*h)*x^4 - (18*(4*b^2*d^2*e*f^4 - 7*(b^2*c*d + a*b*
d^2)*f^5)*g - (80*b^2*d^2*e^2*f^3 - 144*(b^2*c*d + a*b*d^2)*e*f^4 + 63*(b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*f^5)*h)*x^3 + (3*(48*b^2*d^2*e^2*f^3 - 84*(b^
2*c*d + a*b*d^2)*e*f^4 + 35*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^5)*g - 2*(80
*b^2*d^2*e^3*f^2 - 144*(b^2*c*d + a*b*d^2)*e^2*f^3 + 63*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*e*f^4 - 105*(a*b*c^2 + a^2*c*d)*f^5)*h)*x^2 - 3*(384*b^2*d^2
*e^4*f + 105*a^2*c^2*f^5 - 672*(b^2*c*d + a*b*d^2)*e^3*f^2 + 280*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*e^2*f^3 - 420*(a*b*c^2 + a^2*c*d)*e*f^4)*g + 2*(640
*b^2*d^2*e^5 + 315*a^2*c^2*e*f^4 - 1152*(b^2*c*d + a*b*d^2)*e^4*f + 504*(b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^2 - 840*(a*b*c^2 + a^2*c*d)*e^2*f^3)*h
 - (6*(96*b^2*d^2*e^3*f^2 - 168*(b^2*c*d + a*b*d^2)*e^2*f^3 + 70*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*e*f^4 - 105*(a*b*c^2 + a^2*c*d)*f^5)*g - (640*b^2*d
^2*e^4*f + 315*a^2*c^2*f^5 - 1152*(b^2*c*d + a*b*d^2)*e^3*f^2 + 504*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^3 - 840*(a*b*c^2 + a^2*c*d)*e*f^4)*h)*x)*s
qrt(f*x + e)/(f^7*x + e*f^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(440) = 880$.

Time = 88.52 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.33

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b^2d^2h(e+fx)^{\frac{9}{2}}}{9f^5} + \frac{(e+fx)^{\frac{7}{2}}\cdot\left(2abd^2fh+2b^2cdfh-5b^2d^2eh+b^2d^2fg\right)}{7f^5} + \frac{(e+fx)^{\frac{5}{2}}\left(a^2d^2f^2h+4abcdf^2\right.}{} \right.}{} \\ \frac{a^2c^2gx+\frac{b^2d^2hx^6}{6}+\frac{x^5\cdot\left(2abd^2h+2b^2cdh+b^2d^2g\right)}{5}+\frac{x^4\left(a^2d^2h+4abcdh+2abd^2g+b^2c^2h+2b^2cdg\right)}{4}+}{e^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)**2*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b**2*d**2*h*(e + f*x)**(9/2)/(9*f**5) + (e + f*x)**(7/2)*(2*
a*b*d**2*f*h + 2*b**2*c*d*f*h - 5*b**2*d**2*e*h + b**2*d**2*f*g)/(7*f**5)
+ (e + f*x)**(5/2)*(a**2*d**2*f**2*h + 4*a*b*c*d*f**2*h - 8*a*b*d**2*e*f*h
 + 2*a*b*d**2*f**2*g + b**2*c**2*f**2*h - 8*b**2*c*d*e*f*h + 2*b**2*c*d*f*
*2*g + 10*b**2*d**2*e**2*h - 4*b**2*d**2*e*f*g)/(5*f**5) + (e + f*x)**(3/2
)*(2*a**2*c*d*f**3*h - 3*a**2*d**2*e*f**2*h + a**2*d**2*f**3*g + 2*a*b*c**
2*f**3*h - 12*a*b*c*d*e*f**2*h + 4*a*b*c*d*f**3*g + 12*a*b*d**2*e**2*f*h -
 6*a*b*d**2*e*f**2*g - 3*b**2*c**2*e*f**2*h + b**2*c**2*f**3*g + 12*b**2*c
*d*e**2*f*h - 6*b**2*c*d*e*f**2*g - 10*b**2*d**2*e**3*h + 6*b**2*d**2*e**2
*f*g)/(3*f**5) + sqrt(e + f*x)*(a**2*c**2*f**4*h - 4*a**2*c*d*e*f**3*h + 2
*a**2*c*d*f**4*g + 3*a**2*d**2*e**2*f**2*h - 2*a**2*d**2*e*f**3*g - 4*a*b*
c**2*e*f**3*h + 2*a*b*c**2*f**4*g + 12*a*b*c*d*e**2*f**2*h - 8*a*b*c*d*e*f
**3*g - 8*a*b*d**2*e**3*f*h + 6*a*b*d**2*e**2*f**2*g + 3*b**2*c**2*e**2*f*
*2*h - 2*b**2*c**2*e*f**3*g - 8*b**2*c*d*e**3*f*h + 6*b**2*c*d*e**2*f**2*g
 + 5*b**2*d**2*e**4*h - 4*b**2*d**2*e**3*f*g)/f**5 + (a*f - b*e)**2*(c*f -
 d*e)**2*(e*h - f*g)/(f**5*sqrt(e + f*x)))/f, Ne(f, 0)), ((a**2*c**2*g*x +
 b**2*d**2*h*x**6/6 + x**5*(2*a*b*d**2*h + 2*b**2*c*d*h + b**2*d**2*g)/5 +
 x**4*(a**2*d**2*h + 4*a*b*c*d*h + 2*a*b*d**2*g + b**2*c**2*h + 2*b**2*c*d
*g)/4 + x**3*(2*a**2*c*d*h + a**2*d**2*g + 2*a*b*c**2*h + 4*a*b*c*d*g + b*
*2*c**2*g)/3 + x**2*(a**2*c**2*h + 2*a**2*c*d*g + 2*a*b*c**2*g)/2)/e**(...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(f*x + e)^(9/2)*b^2*d^2*h + 45*(b^2*d^2*f*g - (5*b^2*d^2*e - 2*
(b^2*c*d + a*b*d^2)*f)*h)*(f*x + e)^(7/2) - 63*(2*(2*b^2*d^2*e*f - (b^2*c*
d + a*b*d^2)*f^2)*g - (10*b^2*d^2*e^2 - 8*(b^2*c*d + a*b*d^2)*e*f + (b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*f^2)*h)*(f*x + e)^(5/2) + 105*((6*b^2*d^2*e^2*f
- 6*(b^2*c*d + a*b*d^2)*e*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3)*g - (
10*b^2*d^2*e^3 - 12*(b^2*c*d + a*b*d^2)*e^2*f + 3*(b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*e*f^2 - 2*(a*b*c^2 + a^2*c*d)*f^3)*h)*(f*x + e)^(3/2) - 315*(2*(2*
b^2*d^2*e^3*f - 3*(b^2*c*d + a*b*d^2)*e^2*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*e*f^3 - (a*b*c^2 + a^2*c*d)*f^4)*g - (5*b^2*d^2*e^4 + a^2*c^2*f^4 -
8*(b^2*c*d + a*b*d^2)*e^3*f + 3*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2*f^2 -
4*(a*b*c^2 + a^2*c*d)*e*f^3)*h)*sqrt(f*x + e))/f^5 - 315*((b^2*d^2*e^4*f +
 a^2*c^2*f^5 - 2*(b^2*c*d + a*b*d^2)*e^3*f^2 + (b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*e^2*f^3 - 2*(a*b*c^2 + a^2*c*d)*e*f^4)*g - (b^2*d^2*e^5 + a^2*c^2*e*f
^4 - 2*(b^2*c*d + a*b*d^2)*e^4*f + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^3*f^2
 - 2*(a*b*c^2 + a^2*c*d)*e^2*f^3)*h)/(sqrt(f*x + e)*f^5))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(387) = 774$.

Time = 0.15 (sec) , antiderivative size = 1137, normalized size of antiderivative = 2.79

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-2*(b^2*d^2*e^4*f*g - 2*b^2*c*d*e^3*f^2*g - 2*a*b*d^2*e^3*f^2*g + b^2*c^2*
e^2*f^3*g + 4*a*b*c*d*e^2*f^3*g + a^2*d^2*e^2*f^3*g - 2*a*b*c^2*e*f^4*g -
2*a^2*c*d*e*f^4*g + a^2*c^2*f^5*g - b^2*d^2*e^5*h + 2*b^2*c*d*e^4*f*h + 2*
a*b*d^2*e^4*f*h - b^2*c^2*e^3*f^2*h - 4*a*b*c*d*e^3*f^2*h - a^2*d^2*e^3*f^
2*h + 2*a*b*c^2*e^2*f^3*h + 2*a^2*c*d*e^2*f^3*h - a^2*c^2*e*f^4*h)/(sqrt(f
*x + e)*f^6) + 2/315*(45*(f*x + e)^(7/2)*b^2*d^2*f^49*g - 252*(f*x + e)^(5
/2)*b^2*d^2*e*f^49*g + 630*(f*x + e)^(3/2)*b^2*d^2*e^2*f^49*g - 1260*sqrt(
f*x + e)*b^2*d^2*e^3*f^49*g + 126*(f*x + e)^(5/2)*b^2*c*d*f^50*g + 126*(f*
x + e)^(5/2)*a*b*d^2*f^50*g - 630*(f*x + e)^(3/2)*b^2*c*d*e*f^50*g - 630*(
f*x + e)^(3/2)*a*b*d^2*e*f^50*g + 1890*sqrt(f*x + e)*b^2*c*d*e^2*f^50*g +
1890*sqrt(f*x + e)*a*b*d^2*e^2*f^50*g + 105*(f*x + e)^(3/2)*b^2*c^2*f^51*g
 + 420*(f*x + e)^(3/2)*a*b*c*d*f^51*g + 105*(f*x + e)^(3/2)*a^2*d^2*f^51*g
 - 630*sqrt(f*x + e)*b^2*c^2*e*f^51*g - 2520*sqrt(f*x + e)*a*b*c*d*e*f^51*
g - 630*sqrt(f*x + e)*a^2*d^2*e*f^51*g + 630*sqrt(f*x + e)*a*b*c^2*f^52*g
+ 630*sqrt(f*x + e)*a^2*c*d*f^52*g + 35*(f*x + e)^(9/2)*b^2*d^2*f^48*h - 2
25*(f*x + e)^(7/2)*b^2*d^2*e*f^48*h + 630*(f*x + e)^(5/2)*b^2*d^2*e^2*f^48
*h - 1050*(f*x + e)^(3/2)*b^2*d^2*e^3*f^48*h + 1575*sqrt(f*x + e)*b^2*d^2*
e^4*f^48*h + 90*(f*x + e)^(7/2)*b^2*c*d*f^49*h + 90*(f*x + e)^(7/2)*a*b*d^
2*f^49*h - 504*(f*x + e)^(5/2)*b^2*c*d*e*f^49*h - 504*(f*x + e)^(5/2)*a*b*
d^2*e*f^49*h + 1260*(f*x + e)^(3/2)*b^2*c*d*e^2*f^49*h + 1260*(f*x + e)...
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{(e+f\,x)^{3/2}\,(4\,h\,a^2\,c\,d\,f^3-6\,h\,a^2\,d^2\,e\,f^2+2\,g\,a^2\,d^2\,f^3+4\,h\,a\,b\,c^2\,f^3-}{}$$
$$-\frac{-2\,h\,a^2\,c^2\,e\,f^4+2\,g\,a^2\,c^2\,f^5+4\,h\,a^2\,c\,d\,e^2\,f^3-4\,g\,a^2\,c\,d\,e\,f^4-2\,h\,a^2\,d^2\,e^3\,f^2+2\,g\,a^2\,d^2\,e^2\,f^3+4\,h\,a\,b\,c^2}{}$$
$$+\frac{(e+f\,x)^{5/2}\,(2\,h\,a^2\,d^2\,f^2+8\,h\,a\,b\,c\,d\,f^2-16\,h\,a\,b\,d^2\,e\,f+4\,g\,a\,b\,d^2\,f^2+2\,h\,b^2\,c^2\,f^2-16\,h\,b^2\,c\,d\,e\,f+4\,g}{5\,f^6}$$
$$+\frac{2\,b^2\,d^2\,h\,(e+f\,x)^{9/2}}{9\,f^6}+\frac{2\,b\,d\,(e+f\,x)^{7/2}\,(2\,a\,d\,f\,h+2\,b\,c\,f\,h-5\,b\,d\,e\,h+b\,d\,f\,g)}{7\,f^6}$$
$$+\frac{2\,\sqrt{e+f\,x}\,(a\,f-b\,e)\,(c\,f-d\,e)\,(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+2\,b\,c\,f^2\,g+5\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-3\,b\,c\,e\,f\,h-4}{f^6}$$

input
```
int(((g + h*x)*(a + b*x)^2*(c + d*x)^2)/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(3/2)*(2*a^2*d^2*f^3*g + 2*b^2*c^2*f^3*g - 20*b^2*d^2*e^3*h + 4
*a*b*c^2*f^3*h + 4*a^2*c*d*f^3*h - 6*a^2*d^2*e*f^2*h - 6*b^2*c^2*e*f^2*h +
 12*b^2*d^2*e^2*f*g + 8*a*b*c*d*f^3*g - 12*a*b*d^2*e*f^2*g + 24*a*b*d^2*e^
2*f*h - 12*b^2*c*d*e*f^2*g + 24*b^2*c*d*e^2*f*h - 24*a*b*c*d*e*f^2*h))/(3*
f^6) - (2*a^2*c^2*f^5*g - 2*b^2*d^2*e^5*h - 2*a^2*c^2*e*f^4*h + 2*b^2*d^2*
e^4*f*g + 2*a^2*d^2*e^2*f^3*g + 2*b^2*c^2*e^2*f^3*g - 2*a^2*d^2*e^3*f^2*h
- 2*b^2*c^2*e^3*f^2*h - 4*a*b*c^2*e*f^4*g + 4*a*b*d^2*e^4*f*h - 4*a^2*c*d*
e*f^4*g + 4*b^2*c*d*e^4*f*h + 4*a*b*c^2*e^2*f^3*h - 4*a*b*d^2*e^3*f^2*g +
4*a^2*c*d*e^2*f^3*h - 4*b^2*c*d*e^3*f^2*g + 8*a*b*c*d*e^2*f^3*g - 8*a*b*c*
d*e^3*f^2*h)/(f^6*(e + f*x)^(1/2)) + ((e + f*x)^(5/2)*(2*a^2*d^2*f^2*h + 2
*b^2*c^2*f^2*h + 20*b^2*d^2*e^2*h + 4*a*b*d^2*f^2*g + 4*b^2*c*d*f^2*g - 8*
b^2*d^2*e*f*g + 8*a*b*c*d*f^2*h - 16*a*b*d^2*e*f*h - 16*b^2*c*d*e*f*h))/(5
*f^6) + (2*b^2*d^2*h*(e + f*x)^(9/2))/(9*f^6) + (2*b*d*(e + f*x)^(7/2)*(2*
a*d*f*h + 2*b*c*f*h - 5*b*d*e*h + b*d*f*g))/(7*f^6) + (2*(e + f*x)^(1/2)*(
a*f - b*e)*(c*f - d*e)*(a*c*f^2*h + 2*a*d*f^2*g + 2*b*c*f^2*g + 5*b*d*e^2*
h - 3*a*d*e*f*h - 3*b*c*e*f*h - 4*b*d*e*f*g))/f^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 919, normalized size of antiderivative = 2.26

$$\int \frac{(a+bx)^2(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x)
```

output

```
(2*(630*a**2*c**2*e*f**4*h - 315*a**2*c**2*f**5*g + 315*a**2*c**2*f**5*h*x
 - 1680*a**2*c*d*e**2*f**3*h + 1260*a**2*c*d*e*f**4*g - 840*a**2*c*d*e*f**
4*h*x + 630*a**2*c*d*f**5*g*x + 210*a**2*c*d*f**5*h*x**2 + 1008*a**2*d**2*
e**3*f**2*h - 840*a**2*d**2*e**2*f**3*g + 504*a**2*d**2*e**2*f**3*h*x - 42
0*a**2*d**2*e*f**4*g*x - 126*a**2*d**2*e*f**4*h*x**2 + 105*a**2*d**2*f**5*
g*x**2 + 63*a**2*d**2*f**5*h*x**3 - 1680*a*b*c**2*e**2*f**3*h + 1260*a*b*c
**2*e*f**4*g - 840*a*b*c**2*e*f**4*h*x + 630*a*b*c**2*f**5*g*x + 210*a*b*c
**2*f**5*h*x**2 + 4032*a*b*c*d*e**3*f**2*h - 3360*a*b*c*d*e**2*f**3*g + 20
16*a*b*c*d*e**2*f**3*h*x - 1680*a*b*c*d*e*f**4*g*x - 504*a*b*c*d*e*f**4*h*
x**2 + 420*a*b*c*d*f**5*g*x**2 + 252*a*b*c*d*f**5*h*x**3 - 2304*a*b*d**2*e
**4*f*h + 2016*a*b*d**2*e**3*f**2*g - 1152*a*b*d**2*e**3*f**2*h*x + 1008*a
*b*d**2*e**2*f**3*g*x + 288*a*b*d**2*e**2*f**3*h*x**2 - 252*a*b*d**2*e*f**
4*g*x**2 - 144*a*b*d**2*e*f**4*h*x**3 + 126*a*b*d**2*f**5*g*x**3 + 90*a*b*
d**2*f**5*h*x**4 + 1008*b**2*c**2*e**3*f**2*h - 840*b**2*c**2*e**2*f**3*g
+ 504*b**2*c**2*e**2*f**3*h*x - 420*b**2*c**2*e*f**4*g*x - 126*b**2*c**2*e
*f**4*h*x**2 + 105*b**2*c**2*f**5*g*x**2 + 63*b**2*c**2*f**5*h*x**3 - 2304
*b**2*c*d*e**4*f*h + 2016*b**2*c*d*e**3*f**2*g - 1152*b**2*c*d*e**3*f**2*h
*x + 1008*b**2*c*d*e**2*f**3*g*x + 288*b**2*c*d*e**2*f**3*h*x**2 - 252*b**
2*c*d*e*f**4*g*x**2 - 144*b**2*c*d*e*f**4*h*x**3 + 126*b**2*c*d*f**5*g*x**
3 + 90*b**2*c*d*f**5*h*x**4 + 1280*b**2*d**2*e**5*h - 1152*b**2*d**2*e*...
```

3.161 $\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 244

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(be-af)(de-cf)^2(fg-eh)}{f^5\sqrt{e+fx}} + \frac{2(de-cf)(bde(3fg-4eh)-bcf(fg-2eh)-af(2dfg-3deh+cfh))\sqrt{e+fx}}{f^5} + \frac{2(adf(dfg-3deh+2cfh)+b(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)))\,(e+fx)^{3/2}}{3f^5} + \frac{2d(adfh+b(dfg-4deh+2cfh))(e+fx)^{5/2}}{5f^5} + \frac{2bd^2h(e+fx)^{7/2}}{7f^5}$$

output
```
2*(-a*f+b*e)*(-c*f+d*e)^2*(-e*h+f*g)/f^5/(f*x+e)^(1/2)+2*(-c*f+d*e)*(b*d*e
*(-4*e*h+3*f*g)-b*c*f*(-2*e*h+f*g)-a*f*(c*f*h-3*d*e*h+2*d*f*g))*(f*x+e)^(1
/2)/f^5+2/3*(a*d*f*(2*c*f*h-3*d*e*h+d*f*g)+b*(c^2*f^2*h+2*c*d*f*(-3*e*h+f*
g)-3*d^2*e*(-2*e*h+f*g)))*(f*x+e)^(3/2)/f^5+2/5*d*(a*d*f*h+b*(2*c*f*h-4*d*
e*h+d*f*g))*(f*x+e)^(5/2)/f^5+2/7*b*d^2*h*(f*x+e)^(7/2)/f^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2(7af(15c^2f^2(-fg+2eh+fhx)+10cdf(-8e^2h+ef(6g-4hx)+f^2}{}$$

input
```
Integrate[((a + b*x)*(c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(7*a*f*(15*c^2*f^2*(-(f*g) + 2*e*h + f*h*x) + 10*c*d*f*(-8*e^2*h + e*f*
(6*g - 4*h*x) + f^2*x*(3*g + h*x)) + d^2*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x)
 + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x))) + b*(35*c^2*f^2*(-8*
e^2*h + e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x)) + 14*c*d*f*(48*e^3*h - 8*e^
2*f*(5*g - 3*h*x) + f^3*x^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x)) - 3*
d^2*(128*e^4*h - 16*e^3*f*(7*g - 4*h*x) - 8*e^2*f^2*x*(7*g + 2*h*x) + 2*e*
f^3*x^2*(7*g + 4*h*x) - f^4*x^3*(7*g + 5*h*x)))))/(105*f^5*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 159$$

$$\int \left(\frac{\sqrt{e+fx}\,(adf(2cfh-3deh+dfg)+b(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)))}{f^4}+\frac{d(e+fx)^{3/2}(adfh+}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2(e+fx)^{3/2}\left(adf(2cfh-3deh+dfg)+b\left(c^2f^2h+2cdf(fg-3eh)-3d^2e(fg-2eh)\right)\right)}{3f^5}+\frac{2d(e+fx)^{5/2}(adfh+b(2cfh-4deh+dfg))}{5f^5}+\frac{2\sqrt{e+fx}(de-cf)(-af(cfh-3deh+2dfg)-bcf(fg-2eh)+bde(3fg-4eh))}{f^5}+\frac{2(be-af)(de-cf)^2(fg-eh)}{f^5\sqrt{e+fx}}+\frac{2bd^2h(e+fx)^{7/2}}{7f^5}$$

input
```
Int[((a + b*x)*(c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(2*(b*e - a*f)*(d*e - c*f)^2*(f*g - e*h))/(f^5*Sqrt[e + f*x]) + (2*(d*e -
c*f)*(b*d*e*(3*f*g - 4*e*h) - b*c*f*(f*g - 2*e*h) - a*f*(2*d*f*g - 3*d*e*h
 + c*f*h))*Sqrt[e + f*x])/f^5 + (2*(a*d*f*(d*f*g - 3*d*e*h + 2*c*f*h) + b*
(c^2*f^2*h + 2*c*d*f*(f*g - 3*e*h) - 3*d^2*e*(f*g - 2*e*h)))*(e + f*x)^(3/
2))/(3*f^5) + (2*d*(a*d*f*h + b*(d*f*g - 4*d*e*h + 2*c*f*h))*(e + f*x)^(5/
2))/(5*f^5) + (2*b*d^2*h*(e + f*x)^(7/2))/(7*f^5)
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$4\left(\frac{x^2\left(\frac{3bh\,x^2}{7}+\frac{3(ah+bg)x}{5}+ga\right)d^2}{6}+xc\left(\frac{bh\,x^2}{5}+\frac{(ah+bg)x}{3}+ga\right)d-\frac{c^2\left(-\frac{bh\,x^2}{3}+(-ah-bg)x+ga\right)}{2}\right)f^4+4\left(\left(-\frac{4bh\,x^3}{35}+\frac{(-a}{}\right.\right.$
risch	$2(15hb\,d^2x^3f^3+21a\,d^2f^3h\,x^2+42bcd\,f^3h\,x^2-39b\,d^2e\,f^2h\,x^2+21b\,d^2f^3g\,x^2+70acd\,f^3hx-63a\,d^2e\,f^2hx+35a\,d^2f^3gx-$
gosper	$\frac{2}{5}a\,d^2f^4h\,x^3-\frac{8}{5}bcde\,f^3h\,x^2-\frac{16}{3}acde\,f^3hx+\frac{32}{5}bcd\,e^2f^2hx-\frac{16}{3}bcde\,f^3gx-\frac{32}{3}bcd\,e^2f^2g+\frac{32}{5}b\,d^2e^3fg+\frac{32}{5}a\,d^2e^3fh-\frac{16}{3}$
trager	$\frac{2}{5}a\,d^2f^4h\,x^3-\frac{8}{5}bcde\,f^3h\,x^2-\frac{16}{3}acde\,f^3hx+\frac{32}{5}bcd\,e^2f^2hx-\frac{16}{3}bcde\,f^3gx-\frac{32}{3}bcd\,e^2f^2g+\frac{32}{5}b\,d^2e^3fg+\frac{32}{5}a\,d^2e^3fh-\frac{16}{3}$
orering	$\frac{2}{5}a\,d^2f^4h\,x^3-\frac{8}{5}bcde\,f^3h\,x^2-\frac{16}{3}acde\,f^3hx+\frac{32}{5}bcd\,e^2f^2hx-\frac{16}{3}bcde\,f^3gx-\frac{32}{3}bcd\,e^2f^2g+\frac{32}{5}b\,d^2e^3fg+\frac{32}{5}a\,d^2e^3fh-\frac{16}{3}$
derivativedivides	$\frac{4bcdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{4acd\,f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{2hb\,d^2(fx+e)^{\frac{7}{2}}}{7}-2b\,d^2efg(fx+e)^{\frac{3}{2}}-4a\,d^2e\,f^2g\sqrt{fx+e}-4bcdefh(fx+e)^{\frac{3}{2}}+2b\,c^2f$
default	$\frac{4bcdfh(fx+e)^{\frac{5}{2}}}{5}+\frac{4acd\,f^2h(fx+e)^{\frac{3}{2}}}{3}+\frac{2hb\,d^2(fx+e)^{\frac{7}{2}}}{7}-2b\,d^2efg(fx+e)^{\frac{3}{2}}-4a\,d^2e\,f^2g\sqrt{fx+e}-4bcdefh(fx+e)^{\frac{3}{2}}+2b\,c^2f$

input
```
int((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/(f*x+e)^(1/2)*((1/6*x^2*(3/7*b*h*x^2+3/5*(a*h+b*g)*x+g*a)*d^2+x*c*(1/5*b
*h*x^2+1/3*(a*h+b*g)*x+g*a)*d-1/2*c^2*(-1/3*b*h*x^2+(-a*h-b*g)*x+g*a))*f^4
+((-4/35*b*h*x^3+1/5*(-a*h-b*g)*x^2-2/3*a*g*x)*d^2+2*c*(-1/5*b*h*x^2+2/3*(
-a*h-b*g)*x+g*a)*d+c^2*(a*h+b*g-2/3*b*h*x))*e*f^3-8/3*((-3/35*b*h*x^2+3/10
*(-a*h-b*g)*x+1/2*g*a)*d^2+c*(-3/5*b*h*x+a*h+b*g)*d+1/2*b*c^2*h)*e^2*f^2+8
/5*d*((-4/7*b*h*x+a*h+b*g)*d+2*b*c*h)*e^3*f-64/35*b*d^2*e^4*h)/f^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(15\,bd^2f^4hx^4+3\left(7\,bd^2f^4g-\left(8\,bd^2ef^3-7\left(2\,bcd+ad^2\right)f^4\right)h\right)x^3-(7\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*b*d^2*f^4*h*x^4 + 3*(7*b*d^2*f^4*g - (8*b*d^2*e*f^3 - 7*(2*b*c*d
 + a*d^2)*f^4)*h)*x^3 - (7*(6*b*d^2*e*f^3 - 5*(2*b*c*d + a*d^2)*f^4)*g - (
48*b*d^2*e^2*f^2 - 42*(2*b*c*d + a*d^2)*e*f^3 + 35*(b*c^2 + 2*a*c*d)*f^4)*
h)*x^2 + 7*(48*b*d^2*e^3*f - 15*a*c^2*f^4 - 40*(2*b*c*d + a*d^2)*e^2*f^2 +
 30*(b*c^2 + 2*a*c*d)*e*f^3)*g - 2*(192*b*d^2*e^4 - 105*a*c^2*e*f^3 - 168*
(2*b*c*d + a*d^2)*e^3*f + 140*(b*c^2 + 2*a*c*d)*e^2*f^2)*h + (7*(24*b*d^2*
e^2*f^2 - 20*(2*b*c*d + a*d^2)*e*f^3 + 15*(b*c^2 + 2*a*c*d)*f^4)*g - (192*
b*d^2*e^3*f - 105*a*c^2*f^4 - 168*(2*b*c*d + a*d^2)*e^2*f^2 + 140*(b*c^2 +
 2*a*c*d)*e*f^3)*h)*x)*sqrt(f*x + e)/(f^6*x + e*f^5)
```

Sympy [A] (verification not implemented)

Time = 23.10 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{bd^2h(e+fx)^{\frac{7}{2}}}{7f^4} + \frac{(e+fx)^{\frac{5}{2}}\left(ad^2fh+2bcdfh-4bd^2eh+bd^2fg\right)}{5f^4} + \frac{(e+fx)^{\frac{3}{2}}\cdot\left(2acdf^2h-3ad^2efh+ad^2f^2g\right.}{} \right.}{} \\ \frac{ac^2gx + \frac{bd^2hx^5}{5} + \frac{x^4\left(ad^2h+2bcdh+bd^2g\right)}{4} + \frac{x^3\cdot\left(2acdh+ad^2g+bc^2h+2bcdg\right)}{3} + \frac{x^2\left(ac^2h+2acdg+bc^2g\right)}{2}}{e^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**2*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b*d**2*h*(e + f*x)**(7/2)/(7*f**4) + (e + f*x)**(5/2)*(a*d**
2*f*h + 2*b*c*d*f*h - 4*b*d**2*e*h + b*d**2*f*g)/(5*f**4) + (e + f*x)**(3/
2)*(2*a*c*d*f**2*h - 3*a*d**2*e*f*h + a*d**2*f**2*g + b*c**2*f**2*h - 6*b*
c*d*e*f*h + 2*b*c*d*f**2*g + 6*b*d**2*e**2*h - 3*b*d**2*e*f*g)/(3*f**4) +
sqrt(e + f*x)*(a*c**2*f**3*h - 4*a*c*d*e*f**2*h + 2*a*c*d*f**3*g + 3*a*d**
2*e**2*f*h - 2*a*d**2*e*f**2*g - 2*b*c**2*e*f**2*h + b*c**2*f**3*g + 6*b*c
*d*e**2*f*h - 4*b*c*d*e*f**2*g - 4*b*d**2*e**3*h + 3*b*d**2*e**2*f*g)/f**4
 + (a*f - b*e)*(c*f - d*e)**2*(e*h - f*g)/(f**4*sqrt(e + f*x)))/f, Ne(f, 0
)), ((a*c**2*g*x + b*d**2*h*x**5/5 + x**4*(a*d**2*h + 2*b*c*d*h + b*d**2*g
)/4 + x**3*(2*a*c*d*h + a*d**2*g + b*c**2*h + 2*b*c*d*g)/3 + x**2*(a*c**2*
h + 2*a*c*d*g + b*c**2*g)/2)/e**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(fx+e)^{\frac{7}{2}}bd^2h+21\,(bd^2fg-(4\,bd^2e-(2\,bcd+ad^2)\,f)\,h)\,(fx+e)^{\frac{5}{2}}-35\,((3\,bd^2ef-(2\,bcd+ad^2}{}\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(f*x + e)^(7/2)*b*d^2*h + 21*(b*d^2*f*g - (4*b*d^2*e - (2*b*c*d
 + a*d^2)*f)*h)*(f*x + e)^(5/2) - 35*((3*b*d^2*e*f - (2*b*c*d + a*d^2)*f^2
)*g - (6*b*d^2*e^2 - 3*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*h)*(
f*x + e)^(3/2) + 105*((3*b*d^2*e^2*f - 2*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2
+ 2*a*c*d)*f^3)*g - (4*b*d^2*e^3 - a*c^2*f^3 - 3*(2*b*c*d + a*d^2)*e^2*f +
 2*(b*c^2 + 2*a*c*d)*e*f^2)*h)*sqrt(f*x + e))/f^4 + 105*((b*d^2*e^3*f - a*
c^2*f^4 - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*g - (b*d^2*
e^4 - a*c^2*e*f^3 - (2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^2*f^2)*h
)/(sqrt(f*x + e)*f^4))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(228) = 456.

Time = 0.14 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.35

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\,(bd^2e^3fg-2\,bcde^2f^2g-ad^2e^2f^2g+bc^2ef^3g+2\,acdef^3g-ac^2f^4g-}{\sqrt{fx+}}$$
$$+\frac{2\left(21\,(fx+e)^{\frac{5}{2}}bd^2f^{31}g-105\,(fx+e)^{\frac{3}{2}}bd^2ef^{31}g+315\,\sqrt{fx+e}bd^2e^2f^{31}g+70\,(fx+e)^{\frac{3}{2}}bcdf^{32}g+35\,(\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b*d^2*e^3*f*g - 2*b*c*d*e^2*f^2*g - a*d^2*e^2*f^2*g + b*c^2*e*f^3*g + 2
*a*c*d*e*f^3*g - a*c^2*f^4*g - b*d^2*e^4*h + 2*b*c*d*e^3*f*h + a*d^2*e^3*f
*h - b*c^2*e^2*f^2*h - 2*a*c*d*e^2*f^2*h + a*c^2*e*f^3*h)/(sqrt(f*x + e)*f
^5) + 2/105*(21*(f*x + e)^(5/2)*b*d^2*f^31*g - 105*(f*x + e)^(3/2)*b*d^2*e
*f^31*g + 315*sqrt(f*x + e)*b*d^2*e^2*f^31*g + 70*(f*x + e)^(3/2)*b*c*d*f^
32*g + 35*(f*x + e)^(3/2)*a*d^2*f^32*g - 420*sqrt(f*x + e)*b*c*d*e*f^32*g
- 210*sqrt(f*x + e)*a*d^2*e*f^32*g + 105*sqrt(f*x + e)*b*c^2*f^33*g + 210*
sqrt(f*x + e)*a*c*d*f^33*g + 15*(f*x + e)^(7/2)*b*d^2*f^30*h - 84*(f*x + e
)^(5/2)*b*d^2*e*f^30*h + 210*(f*x + e)^(3/2)*b*d^2*e^2*f^30*h - 420*sqrt(f
*x + e)*b*d^2*e^3*f^30*h + 42*(f*x + e)^(5/2)*b*c*d*f^31*h + 21*(f*x + e)^
(5/2)*a*d^2*f^31*h - 210*(f*x + e)^(3/2)*b*c*d*e*f^31*h - 105*(f*x + e)^(3
/2)*a*d^2*e*f^31*h + 630*sqrt(f*x + e)*b*c*d*e^2*f^31*h + 315*sqrt(f*x + e
)*a*d^2*e^2*f^31*h + 35*(f*x + e)^(3/2)*b*c^2*f^32*h + 70*(f*x + e)^(3/2)*
a*c*d*f^32*h - 210*sqrt(f*x + e)*b*c^2*e*f^32*h - 420*sqrt(f*x + e)*a*c*d*
e*f^32*h + 105*sqrt(f*x + e)*a*c^2*f^33*h)/f^35
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{(e+f\,x)^{5/2}\,(2\,a\,d^2\,f\,h-8\,b\,d^2\,e\,h+2\,b\,d^2\,f\,g+4\,b\,c\,d\,f\,h)}{5\,f^5}$$
$$+\frac{(e+f\,x)^{3/2}\,(2\,a\,d^2\,f^2\,g+2\,b\,c^2\,f^2\,h+12\,b\,d^2\,e^2\,h+4\,a\,c\,d\,f^2\,h+4\,b\,c\,d\,f^2\,g-6\,a\,d^2\,e\,f\,h-6\,b\,d^2\,e\,f\,g-}{3\,f^5}$$
$$-\frac{2\,a\,c^2\,f^4\,g+2\,b\,d^2\,e^4\,h-2\,a\,c^2\,e\,f^3\,h-2\,b\,c^2\,e\,f^3\,g-2\,a\,d^2\,e^3\,f\,h-2\,b\,d^2\,e^3\,f\,g+2\,a\,d^2\,e^2\,f^2\,g+2\,b\,c^2\,e^2}{f^5\,\sqrt{e+f\,x}}$$
$$+\frac{2\,\sqrt{e+f\,x}\,(c\,f-d\,e)\,\left(a\,c\,f^2\,h+2\,a\,d\,f^2\,g+b\,c\,f^2\,g+4\,b\,d\,e^2\,h-3\,a\,d\,e\,f\,h-2\,b\,c\,e\,f\,h-3\,b\,d\,e\,f\,g\right)}{f^5}$$
$$+\frac{2\,b\,d^2\,h\,(e+f\,x)^{7/2}}{7\,f^5}$$

input
```
int(((g + h*x)*(a + b*x)*(c + d*x)^2)/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(5/2)*(2*a*d^2*f*h - 8*b*d^2*e*h + 2*b*d^2*f*g + 4*b*c*d*f*h))/
(5*f^5) + ((e + f*x)^(3/2)*(2*a*d^2*f^2*g + 2*b*c^2*f^2*h + 12*b*d^2*e^2*h
 + 4*a*c*d*f^2*h + 4*b*c*d*f^2*g - 6*a*d^2*e*f*h - 6*b*d^2*e*f*g - 12*b*c*
d*e*f*h))/(3*f^5) - (2*a*c^2*f^4*g + 2*b*d^2*e^4*h - 2*a*c^2*e*f^3*h - 2*b
*c^2*e*f^3*g - 2*a*d^2*e^3*f*h - 2*b*d^2*e^3*f*g + 2*a*d^2*e^2*f^2*g + 2*b
*c^2*e^2*f^2*h - 4*a*c*d*e*f^3*g - 4*b*c*d*e^3*f*h + 4*a*c*d*e^2*f^2*h + 4
*b*c*d*e^2*f^2*g)/(f^5*(e + f*x)^(1/2)) + (2*(e + f*x)^(1/2)*(c*f - d*e)*(
a*c*f^2*h + 2*a*d*f^2*g + b*c*f^2*g + 4*b*d*e^2*h - 3*a*d*e*f*h - 2*b*c*e*
f*h - 3*b*d*e*f*g))/f^5 + (2*b*d^2*h*(e + f*x)^(7/2))/(7*f^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx)(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{-\frac{16}{3}acde\,f^3hx + \frac{32}{5}bcd\,e^2f^2hx - \frac{16}{3}bcde\,f^3gx - \frac{8}{5}bcde\,f^3h\,x^2 + \frac{2}{3}a\,d^2f^4$$

input
```
int((b*x+a)*(d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(210*a*c**2*e*f**3*h - 105*a*c**2*f**4*g + 105*a*c**2*f**4*h*x - 560*a*
c*d*e**2*f**2*h + 420*a*c*d*e*f**3*g - 280*a*c*d*e*f**3*h*x + 210*a*c*d*f*
*4*g*x + 70*a*c*d*f**4*h*x**2 + 336*a*d**2*e**3*f*h - 280*a*d**2*e**2*f**2
*g + 168*a*d**2*e**2*f**2*h*x - 140*a*d**2*e*f**3*g*x - 42*a*d**2*e*f**3*h
*x**2 + 35*a*d**2*f**4*g*x**2 + 21*a*d**2*f**4*h*x**3 - 280*b*c**2*e**2*f*
*2*h + 210*b*c**2*e*f**3*g - 140*b*c**2*e*f**3*h*x + 105*b*c**2*f**4*g*x +
 35*b*c**2*f**4*h*x**2 + 672*b*c*d*e**3*f*h - 560*b*c*d*e**2*f**2*g + 336*
b*c*d*e**2*f**2*h*x - 280*b*c*d*e*f**3*g*x - 84*b*c*d*e*f**3*h*x**2 + 70*b
*c*d*f**4*g*x**2 + 42*b*c*d*f**4*h*x**3 - 384*b*d**2*e**4*h + 336*b*d**2*e
**3*f*g - 192*b*d**2*e**3*f*h*x + 168*b*d**2*e**2*f**2*g*x + 48*b*d**2*e**
2*f**2*h*x**2 - 42*b*d**2*e*f**3*g*x**2 - 24*b*d**2*e*f**3*h*x**3 + 21*b*d
**2*f**4*g*x**3 + 15*b*d**2*f**4*h*x**4))/(105*sqrt(e + f*x)*f**5)
```

3.162 $\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = -\frac{2(de-cf)^2(fg-eh)}{f^4\sqrt{e+fx}} - \frac{2(de-cf)(2dfg-3deh+cfh)\sqrt{e+fx}}{f^4} + \frac{2d(dfg-3deh+2cfh)(e+fx)^{3/2}}{3f^4} + \frac{2d^2h(e+fx)^{5/2}}{5f^4}$$

output
```
-2*(-c*f+d*e)^2*(-e*h+f*g)/f^4/(f*x+e)^(1/2)-2*(-c*f+d*e)*(c*f*h-3*d*e*h+2
*d*f*g)*(f*x+e)^(1/2)/f^4+2/3*d*(2*c*f*h-3*d*e*h+d*f*g)*(f*x+e)^(3/2)/f^4+
2/5*d^2*h*(f*x+e)^(5/2)/f^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{30c^2f^2(-fg+2eh+fhx)+20cdf(-8e^2h+ef(6g-4hx)+f^2x(3g+hx))+}{15f^4\sqrt{e+}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(30*c^2*f^2*(-(f*g) + 2*e*h + f*h*x) + 20*c*d*f*(-8*e^2*h + e*f*(6*g - 4*h
*x) + f^2*x*(3*g + h*x)) + 2*d^2*(48*e^3*h - 8*e^2*f*(5*g - 3*h*x) + f^3*x
^2*(5*g + 3*h*x) - 2*e*f^2*x*(10*g + 3*h*x)))/(15*f^4*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{d\sqrt{e+fx}(2cfh-3deh+dfg)}{f^3}+\frac{(cf-de)(cfh-3deh+2dfg)}{f^3\sqrt{e+fx}}+\frac{(cf-de)^2(fg-eh)}{f^3(e+fx)^{3/2}}+\frac{d^2h(e+fx)^{3/2}}{f^3}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{2d(e+fx)^{3/2}(2cfh-3deh+dfg)}{3f^4}-\frac{2\sqrt{e+fx}(de-cf)(cfh-3deh+2dfg)}{f^4}-\frac{2(de-cf)^2(fg-eh)}{f^4\sqrt{e+fx}}+\frac{2d^2h(e+fx)^{5/2}}{5f^4}$$

input
```
Int[((c + d*x)^2*(g + h*x))/(e + f*x)^(3/2),x]
```

output
```
(-2*(d*e - c*f)^2*(f*g - e*h))/(f^4*Sqrt[e + f*x]) - (2*(d*e - c*f)*(2*d*f
*g - 3*d*e*h + c*f*h)*Sqrt[e + f*x])/f^4 + (2*d*(d*f*g - 3*d*e*h + 2*c*f*h
)*(e + f*x)^(3/2))/(3*f^4) + (2*d^2*h*(e + f*x)^(5/2))/(5*f^4)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\left(\left(6h\,x^{3}+10g\,x^{2}\right)d^{2}+60xc\left(\frac{hx}{3}+g\right)d-30c^{2}\left(-hx+g\right)\right)f^{3}+60e\left(\left(-\frac{1}{5}h\,x^{2}-\frac{2}{3}gx\right)d^{2}+2c\left(-\frac{2hx}{3}+g\right)d+h\,c^{2}\right)f^{2}-160\left(\left(-\cdots\right.\right.}{15\sqrt{fx+e}\,f^{4}}$
risch	$\frac{2\left(3f^{2}d^{2}h\,x^{2}+10f^{2}cdhx-9efxh\,d^{2}+5f^{2}d^{2}gx+15c^{2}f^{2}h-50cdehf+30cd\,f^{2}g+33d^{2}e^{2}h-25d^{2}egf\right)\sqrt{fx+e}}{15f^{4}}+\frac{2c^{2}e\,f^{2}\cdots}{}$
gosper	$\frac{\frac{2}{5}d^{2}h\,x^{3}f^{3}+\frac{4}{3}cd\,f^{3}h\,x^{2}-\frac{4}{5}d^{2}e\,f^{2}h\,x^{2}+\frac{2}{3}d^{2}f^{3}g\,x^{2}+2c^{2}f^{3}hx-\frac{16}{3}cde\,f^{2}hx+4cd\,f^{3}gx+\frac{16}{5}d^{2}e^{2}fhx-\frac{8}{3}d^{2}e\,f^{2}gx+4c^{2}e\cdots}{\sqrt{fx+e}\,f^{4}}$
trager	$\frac{\frac{2}{5}d^{2}h\,x^{3}f^{3}+\frac{4}{3}cd\,f^{3}h\,x^{2}-\frac{4}{5}d^{2}e\,f^{2}h\,x^{2}+\frac{2}{3}d^{2}f^{3}g\,x^{2}+2c^{2}f^{3}hx-\frac{16}{3}cde\,f^{2}hx+4cd\,f^{3}gx+\frac{16}{5}d^{2}e^{2}fhx-\frac{8}{3}d^{2}e\,f^{2}gx+4c^{2}e\cdots}{\sqrt{fx+e}\,f^{4}}$
orering	$\frac{\frac{2}{5}d^{2}h\,x^{3}f^{3}+\frac{4}{3}cd\,f^{3}h\,x^{2}-\frac{4}{5}d^{2}e\,f^{2}h\,x^{2}+\frac{2}{3}d^{2}f^{3}g\,x^{2}+2c^{2}f^{3}hx-\frac{16}{3}cde\,f^{2}hx+4cd\,f^{3}gx+\frac{16}{5}d^{2}e^{2}fhx-\frac{8}{3}d^{2}e\,f^{2}gx+4c^{2}e\cdots}{\sqrt{fx+e}\,f^{4}}$
derivativedivides	$\frac{\frac{2h(fx+e)^{\frac{5}{2}}d^{2}}{5}+\frac{4cdfh(fx+e)^{\frac{3}{2}}}{3}-2d^{2}eh(fx+e)^{\frac{3}{2}}+\frac{2d^{2}fg(fx+e)^{\frac{3}{2}}}{3}+2c^{2}f^{2}h\sqrt{fx+e}-8cdefh\sqrt{fx+e}+4cd\,f^{2}g\sqrt{fx+e}+6d\cdots}{f^{4}}$
default	$\frac{\frac{2h(fx+e)^{\frac{5}{2}}d^{2}}{5}+\frac{4cdfh(fx+e)^{\frac{3}{2}}}{3}-2d^{2}eh(fx+e)^{\frac{3}{2}}+\frac{2d^{2}fg(fx+e)^{\frac{3}{2}}}{3}+2c^{2}f^{2}h\sqrt{fx+e}-8cdefh\sqrt{fx+e}+4cd\,f^{2}g\sqrt{fx+e}+6d\cdots}{f^{4}}$

input
```
int((d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(((6*h*x^3+10*g*x^2)*d^2+60*x*c*(1/3*h*x+g)*d-30*c^2*(-h*x+g))*f^3+60
*e*((-1/5*h*x^2-2/3*g*x)*d^2+2*c*(-2/3*h*x+g)*d+h*c^2)*f^2-160*((-3/10*h*x
+1/2*g)*d+c*h)*e^2*d*f+96*d^2*e^3*h)/(f*x+e)^(1/2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\,(3\,d^2f^3hx^3 + (5\,d^2f^3g - 2\,(3\,d^2ef^2 - 5\,cdf^3)h)x^2 - 5\,(8\,d^2e^2f - 12\,cdef^2 + 3}{}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*d^2*f^3*h*x^3 + (5*d^2*f^3*g - 2*(3*d^2*e*f^2 - 5*c*d*f^3)*h)*x^2
- 5*(8*d^2*e^2*f - 12*c*d*e*f^2 + 3*c^2*f^3)*g + 2*(24*d^2*e^3 - 40*c*d*e^
2*f + 15*c^2*e*f^2)*h - (10*(2*d^2*e*f^2 - 3*c*d*f^3)*g - (24*d^2*e^2*f -
40*c*d*e*f^2 + 15*c^2*f^3)*h)*x)*sqrt(f*x + e)/(f^5*x + e*f^4)
```

Sympy [A] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d^2h(e+fx)^{\frac{5}{2}}}{5f^3} + \frac{(e+fx)^{\frac{3}{2}}\cdot\left(2cdfh-3d^2eh+d^2fg\right)}{3f^3} + \frac{\sqrt{e+fx}\left(c^2f^2h-4cdefh+2cdf^2g+3d^2e^2h-2d^2efg\right)}{f^3} + \frac{(cf-de)}{f^3\sqrt{}}\right.}{f} \\ \frac{c^2gx+\frac{d^2hx^4}{4}+\frac{x^3\cdot\left(2cdh+d^2g\right)}{3}+\frac{x^2\left(c^2h+2cdg\right)}{2}}{e^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(d**2*h*(e + f*x)**(5/2)/(5*f**3) + (e + f*x)**(3/2)*(2*c*d*f
*h - 3*d**2*e*h + d**2*f*g)/(3*f**3) + sqrt(e + f*x)*(c**2*f**2*h - 4*c*d*
e*f*h + 2*c*d*f**2*g + 3*d**2*e**2*h - 2*d**2*e*f*g)/f**3 + (c*f - d*e)**2
*(e*h - f*g)/(f**3*sqrt(e + f*x)))/f, Ne(f, 0)), ((c**2*g*x + d**2*h*x**4/
4 + x**3*(2*c*d*h + d**2*g)/3 + x**2*(c**2*h + 2*c*d*g)/2)/e**(3/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(fx+e)^{\frac{5}{2}}d^2h+5\,(d^2fg-(3\,d^2e-2\,cdf)h)(fx+e)^{\frac{3}{2}}-15\,(2\,(d^2ef-cdf^2)g-(3\,d^2e^2-4\,cdef+c^2f^2)h)\sqrt{f\cdots}}{f^3}\cdots\right.}{15\,f}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(f*x + e)^(5/2)*d^2*h + 5*(d^2*f*g - (3*d^2*e - 2*c*d*f)*h)*(f*x
+ e)^(3/2) - 15*(2*(d^2*e*f - c*d*f^2)*g - (3*d^2*e^2 - 4*c*d*e*f + c^2*f^
2)*h)*sqrt(f*x + e))/f^3 - 15*((d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*g - (d^
2*e^3 - 2*c*d*e^2*f + c^2*e*f^2)*h)/(sqrt(f*x + e)*f^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(110) = 220.

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.81

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = -\frac{2\,(d^2e^2fg-2\,cdef^2g+c^2f^3g-d^2e^3h+2\,cde^2fh-c^2ef^2h)}{\sqrt{fx+e}f^4} + \frac{2\left(5\,(fx+e)^{\frac{3}{2}}d^2f^{17}g-30\,\sqrt{fx+e}d^2ef^{17}g+30\,\sqrt{fx+e}cdf^{18}g+3\,(fx+e)^{\frac{5}{2}}d^2f^{16}h-15\,(fx+e)^{\frac{3}{2}}d^2e\cdots\right.}{15\,f^{20}}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-2*(d^2*e^2*f*g - 2*c*d*e*f^2*g + c^2*f^3*g - d^2*e^3*h + 2*c*d*e^2*f*h -
c^2*e*f^2*h)/(sqrt(f*x + e)*f^4) + 2/15*(5*(f*x + e)^(3/2)*d^2*f^17*g - 30
*sqrt(f*x + e)*d^2*e*f^17*g + 30*sqrt(f*x + e)*c*d*f^18*g + 3*(f*x + e)^(5
/2)*d^2*f^16*h - 15*(f*x + e)^(3/2)*d^2*e*f^16*h + 45*sqrt(f*x + e)*d^2*e^
2*f^16*h + 10*(f*x + e)^(3/2)*c*d*f^17*h - 60*sqrt(f*x + e)*c*d*e*f^17*h +
 15*sqrt(f*x + e)*c^2*f^18*h)/f^20
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{(e+f\,x)^{3/2}\,(2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h)}{3\,f^4} - \frac{-2\,h\,c^2\,e\,f^2+2\,g\,c^2\,f^3+4\,h\,c\,d\,e^2\,f-4\,g\,c\,d\,e\,f^2-2\,h\,d^2\,e^3+2\,g\,d^2\,e^2\,f}{f^4\,\sqrt{e+f\,x}} + \frac{2\,\sqrt{e+f\,x}\,(c\,f-d\,e)\,\left(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g\right)}{f^4} + \frac{2\,d^2\,h\,(e+f\,x)^{5/2}}{5\,f^4}$$

input
```
int(((g + h*x)*(c + d*x)^2)/(e + f*x)^(3/2),x)
```

output
```
((e + f*x)^(3/2)*(2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h))/(3*f^4) - (2*c^2*f^3
*g - 2*d^2*e^3*h - 2*c^2*e*f^2*h + 2*d^2*e^2*f*g - 4*c*d*e*f^2*g + 4*c*d*e
^2*f*h)/(f^4*(e + f*x)^(1/2)) + (2*(e + f*x)^(1/2)*(c*f - d*e)*(c*f*h - 3*
d*e*h + 2*d*f*g))/f^4 + (2*d^2*h*(e + f*x)^(5/2))/(5*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2(g+hx)}{(e+fx)^{3/2}}\,dx = \frac{\frac{2}{5}d^2f^3h\,x^3+\frac{4}{3}cd\,f^3h\,x^2-\frac{4}{5}d^2e\,f^2h\,x^2+\frac{2}{3}d^2f^3g\,x^2+2c^2f^3hx-\frac{16}{3}cde\,f^2hx+4}{}$$

input
```
int((d*x+c)^2*(h*x+g)/(f*x+e)^(3/2),x)
```

output
```
(2*(30*c**2*e*f**2*h - 15*c**2*f**3*g + 15*c**2*f**3*h*x - 80*c*d*e**2*f*h
 + 60*c*d*e*f**2*g - 40*c*d*e*f**2*h*x + 30*c*d*f**3*g*x + 10*c*d*f**3*h*x
**2 + 48*d**2*e**3*h - 40*d**2*e**2*f*g + 24*d**2*e**2*f*h*x - 20*d**2*e*f
**2*g*x - 6*d**2*e*f**2*h*x**2 + 5*d**2*f**3*g*x**2 + 3*d**2*f**3*h*x**3))
/(15*sqrt(e + f*x)*f**4)
```

3.163 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 173

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{2(de-cf)^2(fg-eh)}{f^3(be-af)\sqrt{e+fx}} - \frac{2d(adfh-b(dfg-2deh+2cfh))\sqrt{e+fx}}{b^2f^3} + \frac{2d^2h(e+fx)^{3/2}}{3bf^3} - \frac{2(bc-ad)^2(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}(be-af)^{3/2}}$$

output
```
2*(-c*f+d*e)^2*(-e*h+f*g)/f^3/(-a*f+b*e)/(f*x+e)^(1/2)-2*d*(a*d*f*h-b*(2*c
*f*h-2*d*e*h+d*f*g))*(f*x+e)^(1/2)/b^2/f^3+2/3*d^2*h*(f*x+e)^(3/2)/b/f^3-2
*(-a*d+b*c)^2*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b
^(5/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{-6a^2d^2f^2h(e+fx)+2abdf(e+fx)(6cfh+d(3fg-2eh+fhx))-2b^2(3c^2}{3b^2f^3(}$$
$$-\frac{2(bc-ad)^2(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{5/2}(-be+af)^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)*(e + f*x)^(3/2)),x]
```

output
```
(-6*a^2*d^2*f^2*h*(e + f*x) + 2*a*b*d*f*(e + f*x)*(6*c*f*h + d*(3*f*g - 2*
e*h + f*h*x)) - 2*b^2*(3*c^2*f^2*(f*g - e*h) + 6*c*d*e*f*(-(f*g) + 2*e*h +
 f*h*x) + d^2*e*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x))))/(3*b^
2*f^3*(-(b*e) + a*f)*Sqrt[e + f*x]) - (2*(b*c - a*d)^2*(b*g - a*h)*ArcTan[
(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(b^(5/2)*(-(b*e) + a*f)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {167, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 167}$$

$$\frac{2(c+dx)^2(fg-eh)}{f\sqrt{e+fx}(be-af)}-\frac{2\int -\frac{(c+dx)(bcfg-a(4dfg-4deh+cfh)-d(3bfg-4beh+afh)x)}{2(a+bx)\sqrt{e+fx}}dx}{f(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(c+dx)(bcfg-a(4dfg-4deh+cfh)-d(3bfg-4beh+afh)x)}{(a+bx)\sqrt{e+fx}}dx}{f(be-af)}+\frac{2(c+dx)^2(fg-eh)}{f\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 164

$$\frac{\frac{f(bc-ad)^2(bg-ah)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b^2}+\frac{2d\sqrt{e+fx}(3a^2df^2h-abf(6cfh-2deh+3dfg)-bdfx(afh-4beh+3bfg)+2b^2(de(3fg-4eh)-3cf(fg-2e\cdots}{3b^2f^2}}{f(be-af)}$$
$$\frac{2(c+dx)^2(fg-eh)}{f\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2(bc-ad)^2(bg-ah)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{b^2}+\frac{2d\sqrt{e+fx}(3a^2df^2h-abf(6cfh-2deh+3dfg)-bdfx(afh-4beh+3bfg)+2b^2(de(3fg-4eh)-3c\cdots}{3b^2f^2}}{f(be-af)}$$
$$\frac{2(c+dx)^2(fg-eh)}{f\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2d\sqrt{e+fx}(3a^2df^2h-abf(6cfh-2deh+3dfg)-bdfx(afh-4beh+3bfg)+2b^2(de(3fg-4eh)-3cf(fg-2eh)))}{3b^2f^2}-\frac{2f(bc-ad)^2(bg-ah)\text{arctanh}\left(\cdots\right)}{b^{5/2}\sqrt{be-af}}}{f(be-af)}$$
$$\frac{2(c+dx)^2(fg-eh)}{f\sqrt{e+fx}(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(f*g - e*h)*(c + d*x)^2)/(f*(b*e - a*f)*Sqrt[e + f*x]) + ((2*d*Sqrt[e +
 f*x]*(3*a^2*d*f^2*h - a*b*f*(3*d*f*g - 2*d*e*h + 6*c*f*h) + 2*b^2*(d*e*(3
*f*g - 4*e*h) - 3*c*f*(f*g - 2*e*h)) - b*d*f*(3*b*f*g - 4*b*e*h + a*f*h)*x
))/(3*b^2*f^2) - (2*(b*c - a*d)^2*f*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e +
f*x])/Sqrt[b*e - a*f]])/(b^(5/2)*Sqrt[b*e - a*f]))/(f*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-\frac{2d\sqrt{fx+e}\,(-hbdfx+3adfh-6bcfh+5bdeh-3bgdf)}{3b^{2}}+\frac{2\left(c^{2}e\,f^{2}h-c^{2}g\,f^{3}-2cd\,e^{2}fh+2cde\,f^{2}g+d^{2}e^{3}h-d^{2}e^{2}fg\right)}{(af-be)\sqrt{fx+e}}+\frac{2\left(h\,d^{2}a^{3}-2a^{2}b\cdots\right.}{\cdots}}{f^{3}}$
risch	$-\frac{2d(-hbdfx+3adfh-6bcfh+5bdeh-3bgdf)\sqrt{fx+e}}{3f^{3}b^{2}}+\frac{\frac{2\left(c^{2}e\,f^{2}h-c^{2}g\,f^{3}-2cd\,e^{2}fh+2cde\,f^{2}g+d^{2}e^{3}h-d^{2}e^{2}fg\right)b^{2}}{(af-be)\sqrt{fx+e}}+\frac{2f^{3}\cdots}{\cdots}}{\cdots}$
derivativedivides	$\frac{-\frac{2d\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}+2bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^{2}}-\frac{2\left(-c^{2}e\,f^{2}h+c^{2}g\,f^{3}+2cd\,e^{2}fh-2cde\,f^{2}g-d^{2}e^{3}\cdots\right.}{(af-be)\sqrt{fx+e}}\cdots}{f^{3}}$
default	$\frac{-\frac{2d\left(-\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}+2bdeh\sqrt{fx+e}-bdfg\sqrt{fx+e}\right)}{b^{2}}-\frac{2\left(-c^{2}e\,f^{2}h+c^{2}g\,f^{3}+2cd\,e^{2}fh-2cde\,f^{2}g-d^{2}e^{3}\cdots\right.}{(af-be)\sqrt{fx+e}}\cdots}{f^{3}}$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^3*(-1/3*d*(f*x+e)^(1/2)*(-b*d*f*h*x+3*a*d*f*h-6*b*c*f*h+5*b*d*e*h-3*b*
d*f*g)/b^2+(c^2*e*f^2*h-c^2*f^3*g-2*c*d*e^2*f*h+2*c*d*e*f^2*g+d^2*e^3*h-d^
2*e^2*f*g)/(a*f-b*e)/(f*x+e)^(1/2)+(a^3*d^2*h-2*a^2*b*c*d*h-a^2*b*d^2*g+a*
b^2*c^2*h+2*a*b^2*c*d*g-b^3*c^2*g)/b^2/(a*f-b*e)*f^3/((a*f-b*e)*b)^(1/2)*a
rctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(155) = 310$.

Time = 0.13 (sec) , antiderivative size = 1266, normalized size of antiderivative = 7.32

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*e*f^3*g - (a*b^2*c^2 - 2*a^2*
b*c*d + a^3*d^2)*e*f^3*h + ((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^4*g - (a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*f^4*h)*x)*sqrt(b^2*e - a*b*f)*log((b*f*x
 + 2*b*e - a*f - 2*sqrt(b^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) + 2*((b^4
*d^2*e^2*f^2 - 2*a*b^3*d^2*e*f^3 + a^2*b^2*d^2*f^4)*h*x^2 + 3*(2*b^4*d^2*e
^3*f - a*b^3*c^2*f^4 - (2*b^4*c*d + 3*a*b^3*d^2)*e^2*f^2 + (b^4*c^2 + 2*a*
b^3*c*d + a^2*b^2*d^2)*e*f^3)*g - (8*b^4*d^2*e^4 - 2*(6*b^4*c*d + 5*a*b^3*
d^2)*e^3*f + (3*b^4*c^2 + 18*a*b^3*c*d - a^2*b^2*d^2)*e^2*f^2 - 3*(a*b^3*c
^2 + 2*a^2*b^2*c*d - a^3*b*d^2)*e*f^3)*h + (3*(b^4*d^2*e^2*f^2 - 2*a*b^3*d
^2*e*f^3 + a^2*b^2*d^2*f^4)*g - (4*b^4*d^2*e^3*f - (6*b^4*c*d + 5*a*b^3*d^
2)*e^2*f^2 + 2*(6*a*b^3*c*d - a^2*b^2*d^2)*e*f^3 - 3*(2*a^2*b^2*c*d - a^3*
b*d^2)*f^4)*h)*x)*sqrt(f*x + e))/(b^5*e^3*f^3 - 2*a*b^4*e^2*f^4 + a^2*b^3*
e*f^5 + (b^5*e^2*f^4 - 2*a*b^4*e*f^5 + a^2*b^3*f^6)*x), 2/3*(3*((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*e*f^3*g - (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*e*
f^3*h + ((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f^4*g - (a*b^2*c^2 - 2*a^2*b*
c*d + a^3*d^2)*f^4*h)*x)*sqrt(-b^2*e + a*b*f)*arctan(sqrt(-b^2*e + a*b*f)*
sqrt(f*x + e)/(b*f*x + b*e)) + ((b^4*d^2*e^2*f^2 - 2*a*b^3*d^2*e*f^3 + a^2
*b^2*d^2*f^4)*h*x^2 + 3*(2*b^4*d^2*e^3*f - a*b^3*c^2*f^4 - (2*b^4*c*d + 3*
a*b^3*d^2)*e^2*f^2 + (b^4*c^2 + 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^3)*g - (8*b
^4*d^2*e^4 - 2*(6*b^4*c*d + 5*a*b^3*d^2)*e^3*f + (3*b^4*c^2 + 18*a*b^3*...
```

Sympy [A] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.76

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{(cf-de)^2(eh-fg)}{f^2\sqrt{e+fx}(af-be)}+\frac{d^2h(e+fx)^{\frac{3}{2}}}{3bf^2}+\frac{\sqrt{e+fx}\left(-ad^2fh+2bcdfh-2bd^2eh+bd^2fg\right)}{b^2f^2}+\frac{f(ad-bc)^2(ah-bg)\operatorname{atan}\left(\cdots\right)}{b^3\sqrt{\frac{af-be}{b}}(af-be)\cdots}\cdots\right)}{f} & \cdots \\ \frac{\frac{d^2hx^3}{3b}+\frac{x^2\left(-ad^2h+2bcdh+bd^2g\right)}{2b^2}+\frac{x\left(a^2d^2h-2abcdh-abd^2g+b^2c^2h+2b^2cdg\right)}{b^3}-\frac{(ad-bc)^2(ah-bg)\left(\begin{cases}\frac{x}{a} & \cdots \\ \frac{\log(a+b\cdots}{b} & \cdots\end{cases}\cdots\right)}{b^3}\cdots}{e^{\frac{3}{2}}} & \cdots \end{cases}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*((c*f - d*e)**2*(e*h - f*g)/(f**2*sqrt(e + f*x)*(a*f - b*e))
+ d**2*h*(e + f*x)**(3/2)/(3*b*f**2) + sqrt(e + f*x)*(-a*d**2*f*h + 2*b*c*
d*f*h - 2*b*d**2*e*h + b*d**2*f*g)/(b**2*f**2) + f*(a*d - b*c)**2*(a*h - b
*g)*atan(sqrt(e + f*x)/sqrt((a*f - b*e)/b))/(b**3*sqrt((a*f - b*e)/b)*(a*f
 - b*e)))/f, Ne(f, 0)), ((d**2*h*x**3/(3*b) + x**2*(-a*d**2*h + 2*b*c*d*h
+ b*d**2*g)/(2*b**2) + x*(a**2*d**2*h - 2*a*b*c*d*h - a*b*d**2*g + b**2*c*
*2*h + 2*b**2*c*d*g)/b**3 - (a*d - b*c)**2*(a*h - b*g)*Piecewise((x/a, Eq(
b, 0)), (log(a + b*x)/b, True))/b**3)/e**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{2\,(b^3c^2g - 2\,ab^2cdg + a^2bd^2g - ab^2c^2h + 2\,a^2bcdh - a^3d^2h)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right.}{(b^3e - ab^2f)\sqrt{-b^2e+abf}}$$
$$+ \frac{2\,(d^2e^2fg - 2\,cdef^2g + c^2f^3g - d^2e^3h + 2\,cde^2fh - c^2ef^2h)}{(bef^3 - af^4)\sqrt{fx+e}}$$
$$+ \frac{2\left(3\,\sqrt{fx+e}b^2d^2f^7g + (fx+e)^{\frac{3}{2}}b^2d^2f^6h - 6\,\sqrt{fx+e}b^2d^2ef^6h + 6\,\sqrt{fx+e}b^2cdf^7h - 3\,\sqrt{fx+e}abd^2f\right.}{3\,b^3f^9}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^3*c^2*g - 2*a*b^2*c*d*g + a^2*b*d^2*g - a*b^2*c^2*h + 2*a^2*b*c*d*h -
 a^3*d^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*e - a*b^2*f
)*sqrt(-b^2*e + a*b*f)) + 2*(d^2*e^2*f*g - 2*c*d*e*f^2*g + c^2*f^3*g - d^2
*e^3*h + 2*c*d*e^2*f*h - c^2*e*f^2*h)/((b*e*f^3 - a*f^4)*sqrt(f*x + e)) +
2/3*(3*sqrt(f*x + e)*b^2*d^2*f^7*g + (f*x + e)^(3/2)*b^2*d^2*f^6*h - 6*sqr
t(f*x + e)*b^2*d^2*e*f^6*h + 6*sqrt(f*x + e)*b^2*c*d*f^7*h - 3*sqrt(f*x +
e)*a*b*d^2*f^7*h)/(b^3*f^9)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.88

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,d^2\,f\,g-6\,d^2\,e\,h+4\,c\,d\,f\,h}{b\,f^3} - \frac{2\,d^2\,h\,(a\,f^4-b\,e\,f^3)}{b^2\,f^6}\right) + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{e+f\,x}\,(a\,d-b\,c)^2\,(a\,h-b\,g)\,(b^3\,e-a\,b^2\,f)}{b^{3/2}\,(a\,f-b\,e)^{3/2}\,(-2\,h\,a^3\,d^2+4\,h\,a^2\,b\,c\,d+2\,g\,a^2\,b\,d^2-2\,h\,a\,b^2\,c^2-4\,g\,a\,b^2\,c\,d+2\,g\,b^3\,c^2)}\right)\,(a\,d-b\,c)^2\,(a\,h-b\,g)}{b^{5/2}\,(a\,f-b\,e)^{3/2}} + \frac{2\,d^2\,h\,(e+f\,x)^{3/2}}{3\,b\,f^3} - \frac{2\,(-h\,b^2\,c^2\,e\,f^2+g\,b^2\,c^2\,f^3+2\,h\,b^2\,c\,d\,e^2\,f-2\,g\,b^2\,c\,d\,e\,f^2-h\,b^2\,d^2\,e^3+g\,b^2\,d^2\,e^2\,f)}{b^2\,f^3\,\sqrt{e+f\,x}\,(a\,f-b\,e)}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(3/2)*(a + b*x)),x)
```

output
```
(e + f*x)^(1/2)*((2*d^2*f*g - 6*d^2*e*h + 4*c*d*f*h)/(b*f^3) - (2*d^2*h*(a
*f^4 - b*e*f^3))/(b^2*f^6)) + (2*atan((2*(e + f*x)^(1/2)*(a*d - b*c)^2*(a*
h - b*g)*(b^3*e - a*b^2*f))/(b^(3/2)*(a*f - b*e)^(3/2)*(2*b^3*c^2*g - 2*a^
3*d^2*h - 2*a*b^2*c^2*h + 2*a^2*b*d^2*g - 4*a*b^2*c*d*g + 4*a^2*b*c*d*h)))
*(a*d - b*c)^2*(a*h - b*g))/(b^(5/2)*(a*f - b*e)^(3/2)) + (2*d^2*h*(e + f*
x)^(3/2))/(3*b*f^3) - (2*(b^2*c^2*f^3*g - b^2*d^2*e^3*h - b^2*c^2*e*f^2*h
+ b^2*d^2*e^2*f*g - 2*b^2*c*d*e*f^2*g + 2*b^2*c*d*e^2*f*h))/(b^2*f^3*(e +
f*x)^(1/2)*(a*f - b*e))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 790, normalized size of antiderivative = 4.57

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)(e+fx)^{3/2}}\,dx = \frac{-2\sqrt{b}\,\sqrt{fx+e}\,\sqrt{af-be}\,atan\left(\frac{\sqrt{fx+e}\,b}{\sqrt{b}\,\sqrt{af-be}}\right)a^2b\,d^2f^3g+2\sqrt{b}\,\sqrt{fx+e}\,\sqrt{af-b}}{}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)/(f*x+e)^(3/2),x)
```

output
```
(2*(3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**3*d**2*f**3*h - 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d*f**3*h -
 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**2*b*d**2*f**3*g + 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**2*f**3*h +
 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a*b**2*c*d*f**3*g - 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**3*c**2*f**3*g - 3*
a**3*b*d**2*e*f**3*h - 3*a**3*b*d**2*f**4*h*x + 6*a**2*b**2*c*d*e*f**3*h +
 6*a**2*b**2*c*d*f**4*h*x + a**2*b**2*d**2*e**2*f**2*h + 3*a**2*b**2*d**2*
e*f**3*g + 2*a**2*b**2*d**2*e*f**3*h*x + 3*a**2*b**2*d**2*f**4*g*x + a**2*
b**2*d**2*f**4*h*x**2 + 3*a*b**3*c**2*e*f**3*h - 3*a*b**3*c**2*f**4*g - 18
*a*b**3*c*d*e**2*f**2*h + 6*a*b**3*c*d*e*f**3*g - 12*a*b**3*c*d*e*f**3*h*x
 + 10*a*b**3*d**2*e**3*f*h - 9*a*b**3*d**2*e**2*f**2*g + 5*a*b**3*d**2*e**
2*f**2*h*x - 6*a*b**3*d**2*e*f**3*g*x - 2*a*b**3*d**2*e*f**3*h*x**2 - 3*b*
*4*c**2*e**2*f**2*h + 3*b**4*c**2*e*f**3*g + 12*b**4*c*d*e**3*f*h - 6*b**4
*c*d*e**2*f**2*g + 6*b**4*c*d*e**2*f**2*h*x - 8*b**4*d**2*e**4*h + 6*b**4*
d**2*e**3*f*g - 4*b**4*d**2*e**3*f*h*x + 3*b**4*d**2*e**2*f**2*g*x + b**4*
d**2*e**2*f**2*h*x**2))/(3*sqrt(e + f*x)*b**3*f**3*(a**2*f**2 - 2*a*b*e...
```

3.164 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 216

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = -\frac{2(de-cf)^2(fg-eh)}{f^2(be-af)^2\sqrt{e+fx}} + \frac{2d^2h\sqrt{e+fx}}{b^2f^2} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{b^2(be-af)^2(a+bx)} - \frac{(bc-ad)\left(3a^2dfh+b^2(4deg-3cfg+2ceh)+ab(cfh-d(fg+6eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{b^{5/2}(be-af)^{5/2}}$$

output
```
-2*(-c*f+d*e)^2*(-e*h+f*g)/f^2/(-a*f+b*e)^2/(f*x+e)^(1/2)+2*d^2*h*(f*x+e)^
(1/2)/b^2/f^2-(-a*d+b*c)^2*(-a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+
a)-(-a*d+b*c)*(3*a^2*d*f*h+b^2*(2*c*e*h-3*c*f*g+4*d*e*g)+a*b*(c*f*h-d*(6*e
*h+f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*f+b*
e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \frac{3a^3d^2f^2h(e+fx)-a^2bdf(e+fx)(4deh+2cfh+df(g-2hx))+b^3(4cdef}{}$$
$$+\frac{(bc-ad)\left(3a^2dfh+b^2(4deg-3cfg+2ceh)+ab(cfh-d(fg+6eh))\right)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{b^{5/2}(-be+af)^{5/2}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(3*a^3*d^2*f^2*h*(e + f*x) - a^2*b*d*f*(e + f*x)*(4*d*e*h + 2*c*f*h + d*f*
(g - 2*h*x)) + b^3*(4*c*d*e*f*(f*g - e*h)*x + 2*d^2*e^2*x*(-(f*g) + 2*e*h
+ f*h*x) - c^2*f^2*(3*f*g*x + e*(g - 2*h*x))) + a*b^2*(2*c*d*f*(3*e*f*g -
2*e^2*h + f^2*g*x) + c^2*f^2*(-2*f*g + 3*e*h + f*h*x) + 2*d^2*e*(2*e^2*h -
 2*f^2*h*x^2 - e*f*(g + h*x))))/(b^2*f^2*(b*e - a*f)^2*(a + b*x)*Sqrt[e +
f*x]) + ((b*c - a*d)*(3*a^2*d*f*h + b^2*(4*d*e*g - 3*c*f*g + 2*c*e*h) + a*
b*(c*f*h - d*(f*g + 6*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) +
a*f]])/(b^(5/2)*(-(b*e) + a*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.51, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx$$
$$\Big\downarrow \text{166}$$
$$\frac{\int -\frac{(c+dx)(a(4de-cf)h-b(4deg-3cfg+2ceh)-d(bfg+2beh-3afh)x)}{2(a+bx)(e+fx)^{3/2}}dx}{b(be-af)}-\frac{(c+dx)^2(bg-ah)}{b(a+bx)\sqrt{e+fx}(be-af)}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(c+dx)(a(4de-cf)h-b(4deg-3cfg+2ceh)-d(bfg+2beh-3afh)x)}{(a+bx)(e+fx)^{3/2}}dx}{2b(be-af)}-\frac{(c+dx)^2(bg-ah)}{b(a+bx)\sqrt{e+fx}(be-af)}$$

$\big\downarrow$ 163

$$-\frac{-\frac{(bc-ad)(3a^2dfh+ab(cfh-d(6eh+fg))+b^2(2ceh-3cfg+4deg))\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{b(be-af)}-\frac{2(3a^2d^2ef^2h-abf(-c^2f^2h+2cdefh+d^2e(4eh+fg))}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(bg-ah)}{b(a+bx)\sqrt{e+fx}(be-af)}$$

$\big\downarrow$ 73

$$-\frac{-\frac{2(bc-ad)(3a^2dfh+ab(cfh-d(6eh+fg))+b^2(2ceh-3cfg+4deg))\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{bf(be-af)}-\frac{2(3a^2d^2ef^2h-abf(-c^2f^2h+2cdefh+d^2e(}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(bg-ah)}{b(a+bx)\sqrt{e+fx}(be-af)}$$

$\big\downarrow$ 221

$$-\frac{\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)(3a^2dfh+ab(cfh-d(6eh+fg))+b^2(2ceh-3cfg+4deg))}{b^{3/2}(be-af)^{3/2}}-\frac{2(3a^2d^2ef^2h-abf(-c^2f^2h+2cdefh+d^2e(4eh+f}{}}{2b(be-af)}$$

$$\frac{(c+dx)^2(bg-ah)}{b(a+bx)\sqrt{e+fx}(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-(((b*g - a*h)*(c + d*x)^2)/(b*(b*e - a*f)*(a + b*x)*Sqrt[e + f*x])) - ((-
2*(3*a^2*d^2*e*f^2*h - b^2*(2*d^2*e^2*(f*g - 2*e*h) - 2*c*d*e*f*(3*f*g - 2
*e*h) + c^2*f^2*(3*f*g - 2*e*h)) - a*b*f*(2*c*d*e*f*h - c^2*f^2*h + d^2*e*
(f*g + 4*e*h)) + d^2*f*(b*e - a*f)*(b*f*g + 2*b*e*h - 3*a*f*h)*x))/(b*f^2*
(b*e - a*f)*Sqrt[e + f*x]) + (2*(b*c - a*d)*(3*a^2*d*f*h + b^2*(4*d*e*g -
3*c*f*g + 2*c*e*h) + a*b*(c*f*h - d*(f*g + 6*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[
e + f*x])/Sqrt[b*e - a*f]])/(b^(3/2)*(b*e - a*f)^(3/2)))/(2*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{2d^2h\sqrt{fx+e}}{b^2}-\frac{2f^2\left(\frac{\left(-\frac{1}{2}a^3d^2fh+a^2bcdfh+\frac{1}{2}a^2b\,d^2fg-\frac{1}{2}a\,b^2c^2fh-a\,b^2cdfg+\frac{1}{2}b^3c^2fg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{(3a^3d^2fh-2a^2bcdfh-6a^2b\,d\cdots}{\cdots}\right.\cdots}{(af-b\cdots}$
default	$\frac{2d^2h\sqrt{fx+e}}{b^2}-\frac{2f^2\left(\frac{\left(-\frac{1}{2}a^3d^2fh+a^2bcdfh+\frac{1}{2}a^2b\,d^2fg-\frac{1}{2}a\,b^2c^2fh-a\,b^2cdfg+\frac{1}{2}b^3c^2fg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{(3a^3d^2fh-2a^2bcdfh-6a^2b\,d\cdots}{\cdots}\right.\cdots}{(af-b\cdots}$
pseudoelliptic	$\frac{-3\left(\left(-cfg+\frac{2e(ch+2dg)}{3}\right)b^2+\frac{a((ch-dg)f-6deh)b}{3}+a^2dfh\right)(bx+a)\sqrt{fx+e}\,f^2(ad-bc)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+3\sqrt{(af-be\cdots}}{\cdots}$
risch	$\frac{2d^2h\sqrt{fx+e}}{b^2f^2}-\frac{2\left(f^2\left(\frac{\left(-\frac{1}{2}a^3d^2fh+a^2bcdfh+\frac{1}{2}a^2b\,d^2fg-\frac{1}{2}a\,b^2c^2fh-a\,b^2cdfg+\frac{1}{2}b^3c^2fg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{(3a^3d^2fh-2a^2bcdfh-\cdots}{\cdots}\right.\cdots\right.}{\cdots}$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^2*(d^2*h/b^2*(f*x+e)^(1/2)-f^2/(a*f-b*e)^2/b^2*((-1/2*a^3*d^2*f*h+a^2*
b*c*d*f*h+1/2*a^2*b*d^2*f*g-1/2*a*b^2*c^2*f*h-a*b^2*c*d*f*g+1/2*b^3*c^2*f*
g)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)+1/2*(3*a^3*d^2*f*h-2*a^2*b*c*d*f*h-6*
a^2*b*d^2*e*h-a^2*b*d^2*f*g-a*b^2*c^2*f*h+8*a*b^2*c*d*e*h-2*a*b^2*c*d*f*g+
4*a*b^2*d^2*e*g-2*b^3*c^2*e*h+3*b^3*c^2*f*g-4*b^3*c*d*e*g)/((a*f-b*e)*b)^(
1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))-(-c^2*e*f^2*h+c^2*f^3*g+
2*c*d*e^2*f*h-2*c*d*e*f^2*g-d^2*e^3*h+d^2*e^2*f*g)/(a*f-b*e)^2/(f*x+e)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(200) = 400$.

Time = 0.18 (sec) , antiderivative size = 2405, normalized size of antiderivative = 11.13

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b^2*e - a*b*f)*(((4*(b^4*c*d - a*b^3*d^2)*e*f^3 - (3*b^4*c^2 -
 2*a*b^3*c*d - a^2*b^2*d^2)*f^4)*g + (2*(b^4*c^2 - 4*a*b^3*c*d + 3*a^2*b^2
*d^2)*e*f^3 + (a*b^3*c^2 + 2*a^2*b^2*c*d - 3*a^3*b*d^2)*f^4)*h)*x^2 + (4*(
a*b^3*c*d - a^2*b^2*d^2)*e^2*f^2 - (3*a*b^3*c^2 - 2*a^2*b^2*c*d - a^3*b*d^
2)*e*f^3)*g + (2*(a*b^3*c^2 - 4*a^2*b^2*c*d + 3*a^3*b*d^2)*e^2*f^2 + (a^2*
b^2*c^2 + 2*a^3*b*c*d - 3*a^4*d^2)*e*f^3)*h + ((4*(b^4*c*d - a*b^3*d^2)*e^
2*f^2 - 3*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*e*f^3 - (3*a*b^3*c^2 - 2*a
^2*b^2*c*d - a^3*b*d^2)*f^4)*g + (2*(b^4*c^2 - 4*a*b^3*c*d + 3*a^2*b^2*d^2
)*e^2*f^2 + 3*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*e*f^3 + (a^2*b^2*c^2
 + 2*a^3*b*c*d - 3*a^4*d^2)*f^4)*h)*x)*log((b*f*x + 2*b*e - a*f + 2*sqrt(b
^2*e - a*b*f)*sqrt(f*x + e))/(b*x + a)) - 2*(2*(b^5*d^2*e^3*f - 3*a*b^4*d^
2*e^2*f^2 + 3*a^2*b^3*d^2*e*f^3 - a^3*b^2*d^2*f^4)*h*x^2 - (2*a*b^4*d^2*e^
3*f - 2*a^2*b^3*c^2*f^4 + (b^5*c^2 - 6*a*b^4*c*d - a^2*b^3*d^2)*e^2*f^2 +
(a*b^4*c^2 + 6*a^2*b^3*c*d - a^3*b^2*d^2)*e*f^3)*g + (4*a*b^4*d^2*e^4 - 4*
(a*b^4*c*d + 2*a^2*b^3*d^2)*e^3*f + (3*a*b^4*c^2 + 2*a^2*b^3*c*d + 7*a^3*b
^2*d^2)*e^2*f^2 - (3*a^2*b^3*c^2 - 2*a^3*b^2*c*d + 3*a^4*b*d^2)*e*f^3)*h -
 ((2*b^5*d^2*e^3*f - 2*(2*b^5*c*d + a*b^4*d^2)*e^2*f^2 + (3*b^5*c^2 + 2*a*
b^4*c*d + a^2*b^3*d^2)*e*f^3 - (3*a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)
*f^4)*g - (4*b^5*d^2*e^4 - 2*(2*b^5*c*d + 3*a*b^4*d^2)*e^3*f + 2*(b^5*c^2
+ 2*a*b^4*c*d)*e^2*f^2 - (a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*e*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(200) = 400.

Time = 0.14 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.87

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \frac{(4\,b^3cdeg - 4\,ab^2d^2eg - 3\,b^3c^2fg + 2\,ab^2cdfg + a^2bd^2fg + 2\,b^3c^2eh - 8\,ab^2cd}{(b^4e^2 - 2\,ab^3ef + a^2b^2}$$
$$+ \frac{2\sqrt{fx+e}d^2h}{b^2f^2}$$
$$- \frac{2\,(fx+e)b^3d^2e^2fg - 2\,b^3d^2e^3fg - 4\,(fx+e)b^3cdef^2g + 4\,b^3cde^2f^2g + 2\,ab^2d^2e^2f^2g + 3\,(fx+e)b^3c^2f^3}{}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
(4*b^3*c*d*e*g - 4*a*b^2*d^2*e*g - 3*b^3*c^2*f*g + 2*a*b^2*c*d*f*g + a^2*b
*d^2*f*g + 2*b^3*c^2*e*h - 8*a*b^2*c*d*e*h + 6*a^2*b*d^2*e*h + a*b^2*c^2*f
*h + 2*a^2*b*c*d*f*h - 3*a^3*d^2*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e +
 a*b*f))/((b^4*e^2 - 2*a*b^3*e*f + a^2*b^2*f^2)*sqrt(-b^2*e + a*b*f)) + 2*
sqrt(f*x + e)*d^2*h/(b^2*f^2) - (2*(f*x + e)*b^3*d^2*e^2*f*g - 2*b^3*d^2*e
^3*f*g - 4*(f*x + e)*b^3*c*d*e*f^2*g + 4*b^3*c*d*e^2*f^2*g + 2*a*b^2*d^2*e
^2*f^2*g + 3*(f*x + e)*b^3*c^2*f^3*g - 2*(f*x + e)*a*b^2*c*d*f^3*g + (f*x
+ e)*a^2*b*d^2*f^3*g - 2*b^3*c^2*e*f^3*g - 4*a*b^2*c*d*e*f^3*g + 2*a*b^2*c
^2*f^4*g - 2*(f*x + e)*b^3*d^2*e^3*h + 2*b^3*d^2*e^4*h + 4*(f*x + e)*b^3*c
*d*e^2*f*h - 4*b^3*c*d*e^3*f*h - 2*a*b^2*d^2*e^3*f*h - 2*(f*x + e)*b^3*c^2
*e*f^2*h + 2*b^3*c^2*e^2*f^2*h + 4*a*b^2*c*d*e^2*f^2*h - (f*x + e)*a*b^2*c
^2*f^3*h + 2*(f*x + e)*a^2*b*c*d*f^3*h - (f*x + e)*a^3*d^2*f^3*h - 2*a*b^2
*c^2*e*f^3*h)/((b^4*e^2*f^2 - 2*a*b^3*e*f^3 + a^2*b^2*f^4)*((f*x + e)^(3/2
)*b - sqrt(f*x + e)*b*e + sqrt(f*x + e)*a*f))
```

Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.78

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \frac{2\,d^2\,h\,\sqrt{e+f\,x}}{b^2\,f^2}$$
$$-\frac{\frac{2\,\left(-h\,b^2\,c^2\,e\,f^2+g\,b^2\,c^2\,f^3+2\,h\,b^2\,c\,d\,e^2\,f-2\,g\,b^2\,c\,d\,e\,f^2-h\,b^2\,d^2\,e^3+g\,b^2\,d^2\,e^2\,f\right)}{a\,f-b\,e}-\frac{(e+f\,x)\,\left(h\,a^3\,d^2\,f^3-2\,h\,a^2\,b\,c\,d\,f^3-g\,a^2\,b\,d^2\,f^3+h\,a\,b^2\,c^2}{}}{\sqrt{e+f\,x}\,\left(a\,b^2\,f^3-b^3\,e\,f^2\right)+b^3\,f^2\,(e}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,(a\,d-b\,c)\,\left(a^2\,b^2\,f^2-2\,a\,b^3\,e\,f+b^4\,e^2\right)\,\left(2\,b^2\,c\,e\,h-3\,b^2\,c\,f\,g+4\,b^2\,d\,e\,g+3\,a^2\,d\,f\,h+a\,b\,c\,f\,h-6\,a\,b\,d\,e\,h-a\,b\,d\,f\,g\right)}{b^{3/2}\,(a\,f-b\,e)^{5/2}\,\left(2\,b^3\,c^2\,e\,h-3\,b^3\,c^2\,f\,g-3\,a^3\,d^2\,f\,h-4\,a\,b^2\,d^2\,e\,g+a\,b^2\,c^2\,f\,h+6\,a^2\,b\,d^2\,e\,h+a^2\,b\,d^2\,f\,g+4\,b^3\,c\,d\,e\,g-8\,a\,b^2\,c\,d\,e\,h+2\,a\,b^2\,c}\right.}{b^{5/2}\,(a\,f}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(3/2)*(a + b*x)^2),x)
```

output
```
(2*d^2*h*(e + f*x)^(1/2))/(b^2*f^2) - ((2*(b^2*c^2*f^3*g - b^2*d^2*e^3*h -
 b^2*c^2*e*f^2*h + b^2*d^2*e^2*f*g - 2*b^2*c*d*e*f^2*g + 2*b^2*c*d*e^2*f*h
))/(a*f - b*e) - ((e + f*x)*(a^3*d^2*f^3*h - 3*b^3*c^2*f^3*g + 2*b^3*d^2*e
^3*h + a*b^2*c^2*f^3*h - a^2*b*d^2*f^3*g + 2*b^3*c^2*e*f^2*h - 2*b^3*d^2*e
^2*f*g + 2*a*b^2*c*d*f^3*g - 2*a^2*b*c*d*f^3*h + 4*b^3*c*d*e*f^2*g - 4*b^3
*c*d*e^2*f*h))/(a*f - b*e)^2)/((e + f*x)^(1/2)*(a*b^2*f^3 - b^3*e*f^2) + b
^3*f^2*(e + f*x)^(3/2)) + (atan(((e + f*x)^(1/2)*(a*d - b*c)*(b^4*e^2 + a^
2*b^2*f^2 - 2*a*b^3*e*f)*(2*b^2*c*e*h - 3*b^2*c*f*g + 4*b^2*d*e*g + 3*a^2*
d*f*h + a*b*c*f*h - 6*a*b*d*e*h - a*b*d*f*g))/(b^(3/2)*(a*f - b*e)^(5/2)*(
2*b^3*c^2*e*h - 3*b^3*c^2*f*g - 3*a^3*d^2*f*h - 4*a*b^2*d^2*e*g + a*b^2*c^
2*f*h + 6*a^2*b*d^2*e*h + a^2*b*d^2*f*g + 4*b^3*c*d*e*g - 8*a*b^2*c*d*e*h
+ 2*a*b^2*c*d*f*g + 2*a^2*b*c*d*f*h)))*(a*d - b*c)*(2*b^2*c*e*h - 3*b^2*c*
f*g + 4*b^2*d*e*g + 3*a^2*d*f*h + a*b*c*f*h - 6*a*b*d*e*h - a*b*d*f*g))/(b
^(5/2)*(a*f - b*e)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1935, normalized size of antiderivative = 8.96

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^2/(f*x+e)^(3/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**4*d**2*f**3*h + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d*f**3*h +
 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**3*b*d**2*e*f**2*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**2*f**3*g -
 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**3*b*d**2*f**3*h*x + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**2*f**3*
h - 8*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**2*b**2*c*d*e*f**2*h + 2*sqrt(b)*sqrt(e + f*x)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*d
*f**3*g + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c*d*f**3*h*x - 4*sqrt(b)*sqrt(e + f*x)
*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*
*2*d**2*e*f**2*g + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*d**2*e*f**2*h*x + sqrt(b)*sqr
t(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**2*b**2*d**2*f**3*g*x + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan(
(sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**3*c**2*e*f**2*h - 3*s...
```

3.165 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 354

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \frac{2(de-cf)^2(fg-eh)}{f(be-af)^3\sqrt{e+fx}} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{2b^2(be-af)^2(a+bx)^2}$$
$$-\frac{(bc-ad)\left(5a^2dfh+b^2(8deg-7cfg+4ceh)-ab(dfg+12deh-3cfh)\right)\sqrt{e+fx}}{4b^2(be-af)^3(a+bx)}$$
$$+\frac{(3a^3d^2f^2h+a^2bdf(dfg-12deh+2cfh)+ab^2(3c^2f^2h+2cdf(3fg-8eh)-8d^2e(fg-3eh))-b^3(8d^2e^2\ldots}{4b^{5/2}(be-af)^{7/2}}$$

output
```
2*(-c*f+d*e)^2*(-e*h+f*g)/f/(-a*f+b*e)^3/(f*x+e)^(1/2)-1/2*(-a*d+b*c)^2*(-
a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^2-1/4*(-a*d+b*c)*(5*a^2*d*
f*h+b^2*(4*c*e*h-7*c*f*g+8*d*e*g)-a*b*(-3*c*f*h+12*d*e*h+d*f*g))*(f*x+e)^(
1/2)/b^2/(-a*f+b*e)^3/(b*x+a)+1/4*(3*a^3*d^2*f^2*h+a^2*b*d*f*(2*c*f*h-12*d
*e*h+d*f*g)+a*b^2*(3*c^2*f^2*h+2*c*d*f*(-8*e*h+3*f*g)-8*d^2*e*(-3*e*h+f*g)
)-b^3*(8*d^2*e^2*g+3*c^2*f*(-4*e*h+5*f*g)-8*c*d*e*(-2*e*h+3*f*g)))*arctanh
(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \frac{\sqrt{b}(-3a^4d^2f^2h(e+fx)-a^3bdf(e+fx)(-10deh+2cfh+df(g+5hx))+b^4(8d^2e^2(-fg+eh)x^2+8cdefx(3fg}{}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((Sqrt[b]*(-3*a^4*d^2*f^2*h*(e + f*x) - a^3*b*d*f*(e + f*x)*(-10*d*e*h + 2
*c*f*h + d*f*(g + 5*h*x)) + b^4*(8*d^2*e^2*(-(f*g) + e*h)*x^2 + 8*c*d*e*f*
x*(3*f*g*x + e*(g - 2*h*x)) + c^2*f*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*x) + e
*f*x*(-5*g + 12*h*x))) + a^2*b^2*(c^2*f^2*(-8*f*g + 13*e*h + 5*f*h*x) + 2*
c*d*f*(-14*e^2*h + e*f*(13*g - 5*h*x) + f^2*x*(5*g + h*x)) + d^2*(8*e^3*h
+ f^3*g*x^2 - 2*e^2*f*(7*g - 6*h*x) + e*f^2*x*(-5*g + 12*h*x))) + a*b^3*(-
8*d^2*e*x*(3*e*f*g - 2*e^2*h + f^2*g*x) + 2*c*d*f*(3*f^2*g*x^2 + 2*e^2*(g
- 12*h*x) + e*f*x*(21*g - 8*h*x)) + c^2*f*(2*e^2*h + f^2*x*(-25*g + 3*h*x)
 + e*f*(-9*g + 21*h*x)))))/(f*(-(b*e) + a*f)^3*(a + b*x)^2*Sqrt[e + f*x])
- ((-3*a^3*d^2*f^2*h - a^2*b*d*f*(d*f*g - 12*d*e*h + 2*c*f*h) + b^3*(8*d^2
*e^2*g + 3*c^2*f*(5*f*g - 4*e*h) + 8*c*d*e*(-3*f*g + 2*e*h)) + a*b^2*(-3*c
^2*f^2*h + 8*d^2*e*(f*g - 3*e*h) + 2*c*d*f*(-3*f*g + 8*e*h)))*ArcTan[(Sqrt
[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e) + a*f)^(7/2))/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 161, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{(c+dx)(a(4de-cf)h-b(4deg-5cfg+4ceh)+d(bfg-4beh+3afh)x)}{2(a+bx)^2(e+fx)^{3/2}}dx}{2b(be-af)} - \frac{(c+dx)^2(bg-ah)}{2b(a+bx)^2\sqrt{e+fx}(be-af)}$$

$\downarrow$ 27

$$-\frac{\int \frac{(c+dx)(a(4de-cf)h-b(4deg-5cfg+4ceh)+d(bfg-4beh+3afh)x)}{(a+bx)^2(e+fx)^{3/2}}dx}{4b(be-af)} - \frac{(c+dx)^2(bg-ah)}{2b(a+bx)^2\sqrt{e+fx}(be-af)}$$

$\downarrow$ 161

$$-\frac{\frac{(3a^3d^2f^2h+a^2bdf(2cfh-12deh+dfg)+ab^2(3c^2f^2h+2cdf(3fg-8eh)-8d^2e(fg-3eh))-b^3(3c^2f(5fg-4eh)-8cde(3fg-2eh)+8d^2e^2g))\int \frac{}{(a+}}{2b(be-af)^2}}{} \frac{(c+dx)^2(bg-ah)}{2b(a+bx)^2\sqrt{e+fx}(be-af)}$$

$\downarrow$ 73

$$-\frac{\frac{(3a^3d^2f^2h+a^2bdf(2cfh-12deh+dfg)+ab^2(3c^2f^2h+2cdf(3fg-8eh)-8d^2e(fg-3eh))-b^3(3c^2f(5fg-4eh)-8cde(3fg-2eh)+8d^2e^2g))\int \frac{}{a+}}{bf(be-af)^2}}{} \frac{(c+dx)^2(bg-ah)}{2b(a+bx)^2\sqrt{e+fx}(be-af)}$$

$\downarrow$ 221

$$-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)(3a^3d^2f^2h+a^2bdf(2cfh-12deh+dfg)+ab^2(3c^2f^2h+2cdf(3fg-8eh)-8d^2e(fg-3eh))-b^3(3c^2f(5fg-4eh)-8cde(3f}{b^{3/2}(be-af)^{5/2}}}{} \frac{(c+dx)^2(bg-ah)}{2b(a+bx)^2\sqrt{e+fx}(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*((b*g - a*h)*(c + d*x)^2)/(b*(b*e - a*f)*(a + b*x)^2*Sqrt[e + f*x]) -
 (-((3*a^3*d^2*e*f^2*h - b^3*c*e*f*(4*d*e*g - 5*c*f*g + 4*c*e*h) + a^2*b*f
*(2*c*d*e*f*h - 2*c^2*f^2*h + d^2*e*(f*g - 10*e*h)) - a*b^2*(2*c*d*e*f*(13
*f*g - 14*e*h) - c^2*f^2*(10*f*g - 9*e*h) - 2*d^2*e^2*(7*f*g - 4*e*h)) + (
3*a^3*d^2*f^3*h + a^2*b*d*f^2*(d*f*g - 8*d*e*h - 2*c*f*h) + b^3*(2*d^2*e^2
*(5*f*g - 4*e*h) + 3*c^2*f^2*(5*f*g - 4*e*h) - 8*c*d*e*f*(3*f*g - 2*e*h))
- a*b^2*f*(3*c^2*f^2*h + 2*c*d*f*(3*f*g - 8*e*h) - 2*d^2*e*(2*f*g - e*h)))
*x)/(b*f*(b*e - a*f)^2*(a + b*x)*Sqrt[e + f*x])) - ((3*a^3*d^2*f^2*h + a^2
*b*d*f*(d*f*g - 12*d*e*h + 2*c*f*h) + a*b^2*(3*c^2*f^2*h + 2*c*d*f*(3*f*g
- 8*e*h) - 8*d^2*e*(f*g - 3*e*h)) - b^3*(8*d^2*e^2*g + 3*c^2*f*(5*f*g - 4*
e*h) - 8*c*d*e*(3*f*g - 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e
- a*f]])/(b^(3/2)*(b*e - a*f)^(5/2)))/(4*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.64

method	result
pseudoelliptic	$\frac{3\left(\left(a\left(a^2f^2-4abfe+8b^2e^2\right)h+\frac{bg\left(a^2f^2-8abfe-8b^2e^2\right)}{3}\right)d^2+\frac{2cb\left(\left(a^2f^2-8abfe-8b^2e^2\right)h+3bfg(af+4be)\right)d}{3}+c^2b^2f((af+4be)\right.}{4}$
derivativedivides	$2f\left(-\frac{f\left(5a^3d^2fh-2a^2bcdfh-12a^2b\,d^2eh-a^2b\,d^2fg-3a\,b^2c^2fh+16a\,b^2cdeh-6a\,b^2cdfg+8a\,b^2d^2eg-4b^3c^2eh+7b^3c^2fg-8b^3cdeg\right.}{8b}\right.$
default	$2f\left(-\frac{f\left(5a^3d^2fh-2a^2bcdfh-12a^2b\,d^2eh-a^2b\,d^2fg-3a\,b^2c^2fh+16a\,b^2cdeh-6a\,b^2cdfg+8a\,b^2d^2eg-4b^3c^2eh+7b^3c^2fg-8b^3cdeg\right.}{8b}\right.$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/(f*x+e)^(1/2)*(((a*(a^2*f^2-4*a*b*e*f+8*b^2*e^2)*h+1/3*b*g*(a^2*f^2-8*
a*b*e*f-8*b^2*e^2))*d^2+2/3*c*b*((a^2*f^2-8*a*b*e*f-8*b^2*e^2)*h+3*b*f*g*(
a*f+4*b*e))*d+c^2*b^2*f*((a*f+4*b*e)*h-5*b*f*g))*arctan(b*(f*x+e)^(1/2)/((
a*f-b*e)*b)^(1/2))*(b*x+a)^2*f*(f*x+e)^(1/2)-((a*f-b*e)*b)^(1/2)*(((a^3*x*
(a+5/3*b*x)*f^3+(-3*b*x+a)*a^2*(4/3*b*x+a)*e*f^2-10/3*a^2*(6/5*b*x+a)*b*e^
2*f-8/3*b^2*e^3*(b*x+a)^2)*h+1/3*(a^2*x*(-b*x+a)*f^2+a*e*(8*b^2*x^2+5*a*b*
x+a^2)*f+14*(4/7*b^2*x^2+12/7*a*b*x+a^2)*b*e^2)*b*g*f)*d^2+2/3*((a^2*x*(-b
*x+a)*f^2+a*e*(8*b^2*x^2+5*a*b*x+a^2)*f+14*(4/7*b^2*x^2+12/7*a*b*x+a^2)*b*
e^2)*h-13*b*g*(5/13*(3/5*b*x+a)*a*x*f^2+e*(12/13*b^2*x^2+21/13*a*b*x+a^2)*
f+2/13*b*e^2*(2*b*x+a)))*c*b*f*d-13/3*c^2*((5/13*(3/5*b*x+a)*a*x*f^2+e*(12
/13*b^2*x^2+21/13*a*b*x+a^2)*f+2/13*b*e^2*(2*b*x+a))*h-8/13*((15/8*b^2*x^2
+25/8*a*b*x+a^2)*f^2+9/8*(5/9*b*x+a)*b*e*f-1/4*b^2*e^2)*g)*b^2*f))/((a*f-b
*e)*b)^(1/2)/(b*x+a)^2/(a*f-b*e)^3/b^2/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1992 vs. $2(332) = 664$.

Time = 0.39 (sec) , antiderivative size = 3998, normalized size of antiderivative = 11.29

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(332) = 664$.

Time = 0.15 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.77

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*b^3*d^2*e^2*g - 24*b^3*c*d*e*f*g + 8*a*b^2*d^2*e*f*g + 15*b^3*c^2*f
^2*g - 6*a*b^2*c*d*f^2*g - a^2*b*d^2*f^2*g + 16*b^3*c*d*e^2*h - 24*a*b^2*d
^2*e^2*h - 12*b^3*c^2*e*f*h + 16*a*b^2*c*d*e*f*h + 12*a^2*b*d^2*e*f*h - 3*
a*b^2*c^2*f^2*h - 2*a^2*b*c*d*f^2*h - 3*a^3*d^2*f^2*h)*arctan(sqrt(f*x + e
)*b/sqrt(-b^2*e + a*b*f))/((b^5*e^3 - 3*a*b^4*e^2*f + 3*a^2*b^3*e*f^2 - a^
3*b^2*f^3)*sqrt(-b^2*e + a*b*f)) + 2*(d^2*e^2*f*g - 2*c*d*e*f^2*g + c^2*f^
3*g - d^2*e^3*h + 2*c*d*e^2*f*h - c^2*e*f^2*h)/((b^3*e^3*f - 3*a*b^2*e^2*f
^2 + 3*a^2*b*e*f^3 - a^3*f^4)*sqrt(f*x + e)) - 1/4*(8*(f*x + e)^(3/2)*b^4*
c*d*e*f*g - 8*(f*x + e)^(3/2)*a*b^3*d^2*e*f*g - 8*sqrt(f*x + e)*b^4*c*d*e^
2*f*g + 8*sqrt(f*x + e)*a*b^3*d^2*e^2*f*g - 7*(f*x + e)^(3/2)*b^4*c^2*f^2*
g + 6*(f*x + e)^(3/2)*a*b^3*c*d*f^2*g + (f*x + e)^(3/2)*a^2*b^2*d^2*f^2*g
+ 9*sqrt(f*x + e)*b^4*c^2*e*f^2*g - 2*sqrt(f*x + e)*a*b^3*c*d*e*f^2*g - 7*
sqrt(f*x + e)*a^2*b^2*d^2*e*f^2*g - 9*sqrt(f*x + e)*a*b^3*c^2*f^3*g + 10*s
qrt(f*x + e)*a^2*b^2*c*d*f^3*g - sqrt(f*x + e)*a^3*b*d^2*f^3*g + 4*(f*x +
e)^(3/2)*b^4*c^2*e*f*h - 16*(f*x + e)^(3/2)*a*b^3*c*d*e*f*h + 12*(f*x + e)
^(3/2)*a^2*b^2*d^2*e*f*h - 4*sqrt(f*x + e)*b^4*c^2*e^2*f*h + 16*sqrt(f*x +
 e)*a*b^3*c*d*e^2*f*h - 12*sqrt(f*x + e)*a^2*b^2*d^2*e^2*f*h + 3*(f*x + e)
^(3/2)*a*b^3*c^2*f^2*h + 2*(f*x + e)^(3/2)*a^2*b^2*c*d*f^2*h - 5*(f*x + e)
^(3/2)*a^3*b*d^2*f^2*h - sqrt(f*x + e)*a*b^3*c^2*e*f^2*h - 14*sqrt(f*x + e
)*a^2*b^2*c*d*e*f^2*h + 15*sqrt(f*x + e)*a^3*b*d^2*e*f^2*h + 5*sqrt(f*x...
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.21

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{\frac{2\left(-h\,c^2\,e\,f^2+g\,c^2\,f^3+2\,h\,c\,d\,e^2\,f-2\,g\,c\,d\,e\,f^2-h\,d^2\,e^3+g\,d^2\,e^2\,f\right)}{a\,f-b\,e}-\frac{(e+f\,x)^2\,\left(-5\,h\,a^3\,d^2\,f^3+2\,h\,a^2\,b\,c\,d\,f^3+12\,h\,a^2\,b\,d^2\,e\,f^2+g\,a^2\,b\,d^2\,f^3+3\,h\right.}{}}{}$$

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{e+f\,x}\,\left(-a^3\,b^2\,f^3+3\,a^2\,b^3\,e\,f^2-3\,a\,b^4\,e^2\,f+b^5\,e^3\right)}{b^{3/2}\,{\left(a\,f-b\,e\right)}^{7/2}}\right)\,\left(3\,h\,a^3\,d^2\,f^2+2\,h\,a^2\,b\,c\,d\,f^2-12\,h\,a^2\,b\,d^2\,e\,f+g\,a^2\,b\,d^2\,f^2\right.}{}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(3/2)*(a + b*x)^3),x)
```

output
```
- ((2*(c^2*f^3*g - d^2*e^3*h - c^2*e*f^2*h + d^2*e^2*f*g - 2*c*d*e*f^2*g +
 2*c*d*e^2*f*h))/(a*f - b*e) - ((e + f*x)^2*(8*b^3*d^2*e^3*h - 5*a^3*d^2*f
^3*h - 15*b^3*c^2*f^3*g + 3*a*b^2*c^2*f^3*h + a^2*b*d^2*f^3*g + 12*b^3*c^2
*e*f^2*h - 8*b^3*d^2*e^2*f*g + 6*a*b^2*c*d*f^3*g + 2*a^2*b*c*d*f^3*h + 24*
b^3*c*d*e*f^2*g - 16*b^3*c*d*e^2*f*h - 8*a*b^2*d^2*e*f^2*g + 12*a^2*b*d^2*
e*f^2*h - 16*a*b^2*c*d*e*f^2*h))/(4*b*(a*f - b*e)^3) + ((e + f*x)*(25*b^3*
c^2*f^3*g + 3*a^3*d^2*f^3*h - 16*b^3*d^2*e^3*h - 5*a*b^2*c^2*f^3*h + a^2*b
*d^2*f^3*g - 20*b^3*c^2*e*f^2*h + 16*b^3*d^2*e^2*f*g - 10*a*b^2*c*d*f^3*g
+ 2*a^2*b*c*d*f^3*h - 40*b^3*c*d*e*f^2*g + 32*b^3*c*d*e^2*f*h + 8*a*b^2*d^
2*e*f^2*g - 12*a^2*b*d^2*e*f^2*h + 16*a*b^2*c*d*e*f^2*h))/(4*b^2*(a*f - b*
e)^2))/((e + f*x)^(1/2)*(a^2*f^3 + b^2*e^2*f - 2*a*b*e*f^2) + (e + f*x)^(3
/2)*(2*a*b*f^2 - 2*b^2*e*f) + b^2*f*(e + f*x)^(5/2)) - (atan(((e + f*x)^(1
/2)*(b^5*e^3 - a^3*b^2*f^3 + 3*a^2*b^3*e*f^2 - 3*a*b^4*e^2*f))/(b^(3/2)*(a
*f - b*e)^(7/2)))*(3*a^3*d^2*f^2*h - 8*b^3*d^2*e^2*g - 15*b^3*c^2*f^2*g -
16*b^3*c*d*e^2*h + 12*b^3*c^2*e*f*h + 3*a*b^2*c^2*f^2*h + 24*a*b^2*d^2*e^2
*h + a^2*b*d^2*f^2*g + 24*b^3*c*d*e*f*g + 6*a*b^2*c*d*f^2*g + 2*a^2*b*c*d*
f^2*h - 8*a*b^2*d^2*e*f*g - 12*a^2*b*d^2*e*f*h - 16*a*b^2*c*d*e*f*h))/(4*b
^(5/2)*(a*f - b*e)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3593, normalized size of antiderivative = 10.15

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^3/(f*x+e)^(3/2),x)
```

output
```
(3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**5*d**2*f**3*h + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d*f**3*h - 12
*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**4*b*d**2*e*f**2*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**2*f**3*g + 6
*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**4*b*d**2*f**3*h*x + 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*f**3*
h - 16*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*e*f**2*h + 6*sqrt(b)*sqrt(e + f*x)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*
d*f**3*g + 4*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c*d*f**3*h*x + 24*sqrt(b)*sqrt(e + f*
x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*
b**2*d**2*e**2*f*h - 8*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*e*f**2*g - 24*sqrt(b)*
sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*b**2*d**2*e*f**2*h*x + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*d**2*f**3*g...
```

3.166 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 517

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = -\frac{2(de-cf)^2(fg-eh)}{(be-af)^4\sqrt{e+fx}} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{3b^2(be-af)^2(a+bx)^3}$$
$$-\frac{(bc-ad)\left(7a^2dfh+b^2(12deg-11cfg+6ceh)-ab(dfg+18deh-5cfh)\right)\sqrt{e+fx}}{12b^2(be-af)^3(a+bx)^2}$$
$$+\frac{(a^3d^2f^2h+a^2bdf(dfg-6deh+2cfh)-b^3(8d^2e^2g+c^2f(19fg-14eh)-4cde(7fg-4eh))+ab^2(5c^2f^2 \ldots}{8b^2(be-af)^4(a+bx)}$$
$$-\frac{(a^3d^2f^3h+a^2bdf^2(dfg-6deh+2cfh)+ab^2f(5c^2f^2h+2cdf(5fg-12eh)-12d^2e(fg-2eh))-b^3(5c \ldots}{8b^{5/2}(be-af)^{9/2}}$$

output
```
-2*(-c*f+d*e)^2*(-e*h+f*g)/(-a*f+b*e)^4/(f*x+e)^(1/2)-1/3*(-a*d+b*c)^2*(-a
*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^3-1/12*(-a*d+b*c)*(7*a^2*d*
f*h+b^2*(6*c*e*h-11*c*f*g+12*d*e*g)-a*b*(-5*c*f*h+18*d*e*h+d*f*g))*(f*x+e)
^(1/2)/b^2/(-a*f+b*e)^3/(b*x+a)^2+1/8*(a^3*d^2*f^2*h+a^2*b*d*f*(2*c*f*h-6*
d*e*h+d*f*g)-b^3*(8*d^2*e^2*g+c^2*f*(-14*e*h+19*f*g)-4*c*d*e*(-4*e*h+7*f*g
))+a*b^2*(5*c^2*f^2*h+2*c*d*f*(-12*e*h+5*f*g)-12*d^2*e*(-2*e*h+f*g)))*(f*x
+e)^(1/2)/b^2/(-a*f+b*e)^4/(b*x+a)-1/8*(a^3*d^2*f^3*h+a^2*b*d*f^2*(2*c*f*h
-6*d*e*h+d*f*g)+a*b^2*f*(5*c^2*f^2*h+2*c*d*f*(-12*e*h+5*f*g)-12*d^2*e*(-2*
e*h+f*g))-b^3*(5*c^2*f^2*(-6*e*h+7*f*g)-12*c*d*e*f*(-4*e*h+5*f*g)+8*d^2*e^
2*(-2*e*h+3*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)
/(-a*f+b*e)^(9/2)
```

Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \frac{-3a^5d^2f^2h(e+fx)-a^4bdf(e+fx)(6cfh+d(3fg-16eh+8fhx))-b^5(2\cdots}{\cdots} + \frac{(a^3d^2f^3h+a^2bdf^2(dfg-6deh+2cfh)+ab^2f(5c^2f^2h+2cdf(5fg-12eh)+12d^2e(-fg+2eh))+b^3(1\cdots}{8b^{5/2}(-be+af)^{9/2}}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^4*(e + f*x)^(3/2)),x]
```

output
```
(-3*a^5*d^2*f^2*h*(e + f*x) - a^4*b*d*f*(e + f*x)*(6*c*f*h + d*(3*f*g - 16
*e*h + 8*f*h*x)) - b^5*(24*d^2*e^2*x^2*(3*f*g*x + e*(g - 2*h*x)) + 12*c*d*
e*x*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*x) + e*f*x*(-5*g + 12*h*x)) + c^2*(105
*f^3*g*x^3 + 5*e*f^2*x^2*(7*g - 18*h*x) + 4*e^3*(2*g + 3*h*x) - 2*e^2*f*x*
(7*g + 15*h*x))) + a^3*b^2*(3*c^2*f^2*(-16*f*g + 27*e*h + 11*f*h*x) + 2*c*
d*f*(-94*e^2*h + e*f*(81*g - 38*h*x) + f^2*x*(33*g + 8*h*x)) + d^2*(92*e^3
*h + f^3*x^2*(8*g + 3*h*x) + e*f^2*x*(-38*g + 17*h*x) + e^2*f*(-94*g + 58*
h*x))) + a*b^4*(-12*d^2*e*x*(3*f^2*g*x^2 + 2*e^2*(g - 9*h*x) + e*f*x*(17*g
 - 6*h*x)) + c^2*(-4*e^3*h + 5*f^3*x^2*(-56*g + 3*h*x) + 49*e*f^2*x*(-2*g
+ 5*h*x) + e^2*f*(38*g + 82*h*x)) - 2*c*d*(-15*f^3*g*x^3 + 4*e^3*(g + 6*h*
x) + e*f^2*x^2*(-245*g + 36*h*x) + 2*e^2*f*x*(-41*g + 102*h*x))) + a^2*b^3
*(2*c*d*(-8*e^3*h + e^2*f*(28*g - 250*h*x) + e*f^2*x*(212*g - 95*h*x) + f^
3*x^2*(40*g + 3*h*x)) + c^2*f*(28*e^2*h + f^2*x*(-231*g + 40*h*x) + e*f*(-
87*g + 212*h*x)) + d^2*(3*f^3*g*x^3 + 10*e^2*f*x*(-25*g + 9*h*x) - e*f^2*x
^2*(95*g + 18*h*x) + e^3*(-8*g + 252*h*x))))/(24*b^2*(b*e - a*f)^4*(a + b*
x)^3*Sqrt[e + f*x]) + ((a^3*d^2*f^3*h + a^2*b*d*f^2*(d*f*g - 6*d*e*h + 2*c
*f*h) + a*b^2*f*(5*c^2*f^2*h + 2*c*d*f*(5*f*g - 12*e*h) + 12*d^2*e*(-(f*g)
 + 2*e*h)) + b^3*(12*c*d*e*f*(5*f*g - 4*e*h) + 8*d^2*e^2*(-3*f*g + 2*e*h)
+ 5*c^2*f^2*(-7*f*g + 6*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e)
+ a*f]])/(8*b^(5/2)*(-(b*e) + a*f)^(9/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {166, 27, 161, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{(c+dx)(a(4de-cf)h-b(4deg-7cfg+6ceh)+3d(bfg-2beh+afh)x)}{2(a+bx)^3(e+fx)^{3/2}}dx}{3b(be-af)}-\frac{(c+dx)^2(bg-ah)}{3b(a+bx)^3\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(c+dx)(a(4de-cf)h-b(4deg-7cfg+6ceh)+3d(bfg-2beh+afh)x)}{(a+bx)^3(e+fx)^{3/2}}dx}{6b(be-af)}-\frac{(c+dx)^2(bg-ah)}{3b(a+bx)^3\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \; 161$$

$$-\frac{\frac{3(a^3d^2f^3h+a^2bdf^2(2cfh-6deh+dfg)+ab^2f(5c^2f^2h+2cdf(5fg-12eh)-12d^2e(fg-2eh))-(b^3(5c^2f^2(7fg-6eh)-12cdef(5fg-4eh)+8d^2e^2(}{4bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{3b(a+bx)^3\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \; 52$$

$$-\frac{\frac{3(a^3d^2f^3h+a^2bdf^2(2cfh-6deh+dfg)+ab^2f(5c^2f^2h+2cdf(5fg-12eh)-12d^2e(fg-2eh))-(b^3(5c^2f^2(7fg-6eh)-12cdef(5fg-4eh)+8d^2e^2(}{4bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{3b(a+bx)^3\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{3\left(a^3d^2f^3h+a^2bdf^2(2cfh-6deh+dfg)+ab^2f\left(5c^2f^2h+2cdf(5fg-12eh)-12d^2e(fg-2eh)\right)-\left(b^3\left(5c^2f^2(7fg-6eh)-12cdef(5fg-4eh)+8d^2e^2(\right.\right.\right.}{4bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{3b(a+bx)^3\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{3\left(\frac{f\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)\left(a^3d^2f^3h+a^2bdf^2(2cfh-6deh+dfg)+ab^2f\left(5c^2f^2h+2cdf(5fg-12eh)-12d^2e(fg-2eh)\right)-\left(b^3\left(\right.\right.\right.}{4bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{3b(a+bx)^3\sqrt{e+fx}(be-af)}$$

input
```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^4*(e + f*x)^(3/2)),x]
```

output
```
-1/3*((b*g - a*h)*(c + d*x)^2)/(b*(b*e - a*f)*(a + b*x)^3*Sqrt[e + f*x]) -
 (-1/2*(3*a^3*d^2*e*f^2*h - b^3*c*e*f*(4*d*e*g - 7*c*f*g + 6*c*e*h) + a^2*
b*f*(6*c*d*e*f*h - 4*c^2*f^2*h + d^2*e*(3*f*g - 14*e*h)) - a*b^2*(2*c*d*e*
f*(33*f*g - 32*e*h) - c^2*f^2*(28*f*g - 25*e*h) - 8*d^2*e^2*(4*f*g - 3*e*h
)) + (3*a^3*d^2*f^3*h + a^2*b*d*f^2*(3*d*f*g - 10*d*e*h - 2*c*f*h) + b^3*(
4*d^2*e^2*(7*f*g - 6*e*h) + 5*c^2*f^2*(7*f*g - 6*e*h) - 12*c*d*e*f*(5*f*g
- 4*e*h)) - a*b^2*f*(5*c^2*f^2*h + 2*c*d*f*(5*f*g - 12*e*h) - 4*d^2*e*(f*g
 - e*h)))*x)/(b*f*(b*e - a*f)^2*(a + b*x)^2*Sqrt[e + f*x]) + (3*(a^3*d^2*f
^3*h + a^2*b*d*f^2*(d*f*g - 6*d*e*h + 2*c*f*h) + a*b^2*f*(5*c^2*f^2*h + 2*
c*d*f*(5*f*g - 12*e*h) - 12*d^2*e*(f*g - 2*e*h)) - b^3*(5*c^2*f^2*(7*f*g -
 6*e*h) - 12*c*d*e*f*(5*f*g - 4*e*h) + 8*d^2*e^2*(3*f*g - 2*e*h)))*(-(Sqrt
[e + f*x]/((b*e - a*f)*(a + b*x))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sq
rt[b*e - a*f]])/(Sqrt[b]*(b*e - a*f)^(3/2))))/(4*b*f*(b*e - a*f)^2))/(6*b*
(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$\frac{(bx+a)^3\sqrt{fx+e}\left(\left(-35c^2g\,f^3+30ce(ch+2dg)f^2-48\left(ch+\frac{dg}{2}\right)d\,e^2f+16d^2e^3h\right)b^3+5a\left(\left(h\,c^2+2cdg\right)f^2-\frac{24\left(ch+\frac{dg}{2}\right)d}{5}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((b*x+a)^3*(f*x+e)^(1/2)*((-35*c^2*g*f^3+30*c*e*(c*h+2*d*g)*f^2-48*(c*
h+1/2*d*g)*d*e^2*f+16*d^2*e^3*h)*b^3+5*a*((c^2*h+2*c*d*g)*f^2-24/5*(c*h+1/
2*d*g)*d*e*f+24/5*d^2*e^2*h)*f*b^2+2*a^2*d*((c*h+1/2*d*g)*f-3*d*e*h)*f^2*b
+a^3*d^2*f^3*h)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^
(1/2)*((35*c^2*f^3*g*x^3+35/3*x^2*c*(18/7*(-c*h-2*d*g)*x+c*g)*e*f^2-14/3*x
*(36/7*(-2*c*d*h-d^2*g)*x^2+15/7*c*(c*h+2*d*g)*x+c^2*g)*e^2*f+8/3*(-6*d^2*
h*x^3+3*(2*c*d*h+d^2*g)*x^2+3*(1/2*h*c^2+c*d*g)*x+c^2*g)*e^3)*b^5+4/3*a*(5
*(-3/4*c*(c*h+2*d*g)*x^3+14*c^2*g*x^2)*f^3+49/2*(18/49*(2*c*d*h+d^2*g)*x^2
+5*(-c*d*g-1/2*h*c^2)*x+c^2*g)*x*e*f^2-19/2*(36/19*d^2*h*x^3-204/19*(c*h+1
/2*d*g)*d*x^2+41/19*c*(c*h+2*d*g)*x+c^2*g)*e^2*f+(-54*d^2*h*x^2+6*(2*c*d*h
+d^2*g)*x+h*c^2+2*c*d*g)*e^3)*b^4-28/3*a^2*(-33/4*x*(1/77*(-2*c*d*h-d^2*g)
*x^2-40/231*c*(c*h+2*d*g)*x+c^2*g)*f^3-87/28*(6/29*d^2*h*x^3+95/87*(2*c*d*
h+d^2*g)*x^2-212/87*c*(c*h+2*d*g)*x+c^2*g)*e*f^2+(45/14*d^2*h*x^2-125/7*(c
*h+1/2*d*g)*d*x+h*c^2+2*c*d*g)*e^2*f-4/7*(-63/4*d*h*x+c*h+1/2*d*g)*d*e^3)*
b^3-27*a^3*(1/27*(d^2*h*x^3+8/3*(2*c*d*h+d^2*g)*x^2+11*c*(c*h+2*d*g)*x-16*
c^2*g)*f^3+(17/81*d^2*h*x^2+38/81*(-2*c*d*h-d^2*g)*x+h*c^2+2*c*d*g)*e*f^2-
188/81*d*(-29/94*d*h*x+c*h+1/2*d*g)*e^2*f+92/81*d^2*e^3*h)*b^2+2*a^4*d*((4
/3*d*h*x+c*h+1/2*d*g)*f-8/3*d*e*h)*(f*x+e)*f*b+a^5*d^2*f^2*h*(f*x+e)))/(f*
x+e)^(1/2)/((a*f-b*e)*b)^(1/2)/(b*x+a)^3/(a*f-b*e)^4/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2940 vs. $2(491) = 982$.

Time $= 0.72$ (sec) , antiderivative size $= 5894$, normalized size of antiderivative $= 11.40$

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**4/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1761 vs. $2(491) = 982$.

Time = 0.17 (sec) , antiderivative size = 1761, normalized size of antiderivative = 3.41

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(24*b^3*d^2*e^2*f*g - 60*b^3*c*d*e*f^2*g + 12*a*b^2*d^2*e*f^2*g + 35*
b^3*c^2*f^3*g - 10*a*b^2*c*d*f^3*g - a^2*b*d^2*f^3*g - 16*b^3*d^2*e^3*h +
48*b^3*c*d*e^2*f*h - 24*a*b^2*d^2*e^2*f*h - 30*b^3*c^2*e*f^2*h + 24*a*b^2*
c*d*e*f^2*h + 6*a^2*b*d^2*e*f^2*h - 5*a*b^2*c^2*f^3*h - 2*a^2*b*c*d*f^3*h
- a^3*d^2*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^6*e^4 -
4*a*b^5*e^3*f + 6*a^2*b^4*e^2*f^2 - 4*a^3*b^3*e*f^3 + a^4*b^2*f^4)*sqrt(-b
^2*e + a*b*f)) - 2*(d^2*e^2*f*g - 2*c*d*e*f^2*g + c^2*f^3*g - d^2*e^3*h +
2*c*d*e^2*f*h - c^2*e*f^2*h)/((b^4*e^4 - 4*a*b^3*e^3*f + 6*a^2*b^2*e^2*f^2
 - 4*a^3*b*e*f^3 + a^4*f^4)*sqrt(f*x + e)) - 1/24*(24*(f*x + e)^(5/2)*b^5*
d^2*e^2*f*g - 48*(f*x + e)^(3/2)*b^5*d^2*e^3*f*g + 24*sqrt(f*x + e)*b^5*d^
2*e^4*f*g - 84*(f*x + e)^(5/2)*b^5*c*d*e*f^2*g + 36*(f*x + e)^(5/2)*a*b^4*
d^2*e*f^2*g + 192*(f*x + e)^(3/2)*b^5*c*d*e^2*f^2*g - 48*(f*x + e)^(3/2)*a
*b^4*d^2*e^2*f^2*g - 108*sqrt(f*x + e)*b^5*c*d*e^3*f^2*g + 12*sqrt(f*x + e
)*a*b^4*d^2*e^3*f^2*g + 57*(f*x + e)^(5/2)*b^5*c^2*f^3*g - 30*(f*x + e)^(5
/2)*a*b^4*c*d*f^3*g - 3*(f*x + e)^(5/2)*a^2*b^3*d^2*f^3*g - 136*(f*x + e)^
(3/2)*b^5*c^2*e*f^3*g - 112*(f*x + e)^(3/2)*a*b^4*c*d*e*f^3*g + 104*(f*x +
 e)^(3/2)*a^2*b^3*d^2*e*f^3*g + 87*sqrt(f*x + e)*b^5*c^2*e^2*f^3*g + 150*s
qrt(f*x + e)*a*b^4*c*d*e^2*f^3*g - 93*sqrt(f*x + e)*a^2*b^3*d^2*e^2*f^3*g
+ 136*(f*x + e)^(3/2)*a*b^4*c^2*f^4*g - 80*(f*x + e)^(3/2)*a^2*b^3*c*d*f^4
*g - 8*(f*x + e)^(3/2)*a^3*b^2*d^2*f^4*g - 174*sqrt(f*x + e)*a*b^4*c^2*...
```

Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 1085, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(3/2)*(a + b*x)^4),x)
```

output
```
(atan(((e + f*x)^(1/2)*(b^6*e^4 + a^4*b^2*f^4 - 4*a^3*b^3*e*f^3 + 6*a^2*b^
4*e^2*f^2 - 4*a*b^5*e^3*f))/(b^(3/2)*(a*f - b*e)^(9/2)))*(a^3*d^2*f^3*h -
35*b^3*c^2*f^3*g + 16*b^3*d^2*e^3*h + 5*a*b^2*c^2*f^3*h + a^2*b*d^2*f^3*g
+ 30*b^3*c^2*e*f^2*h - 24*b^3*d^2*e^2*f*g + 10*a*b^2*c*d*f^3*g + 2*a^2*b*c
*d*f^3*h + 60*b^3*c*d*e*f^2*g - 48*b^3*c*d*e^2*f*h - 12*a*b^2*d^2*e*f^2*g
+ 24*a*b^2*d^2*e^2*f*h - 6*a^2*b*d^2*e*f^2*h - 24*a*b^2*c*d*e*f^2*h))/(8*b
^(5/2)*(a*f - b*e)^(9/2)) - ((2*(c^2*f^3*g - d^2*e^3*h - c^2*e*f^2*h + d^2
*e^2*f*g - 2*c*d*e*f^2*g + 2*c*d*e^2*f*h))/(a*f - b*e) - ((e + f*x)^3*(a^3
*d^2*f^3*h - 35*b^3*c^2*f^3*g + 16*b^3*d^2*e^3*h + 5*a*b^2*c^2*f^3*h + a^2
*b*d^2*f^3*g + 30*b^3*c^2*e*f^2*h - 24*b^3*d^2*e^2*f*g + 10*a*b^2*c*d*f^3*
g + 2*a^2*b*c*d*f^3*h + 60*b^3*c*d*e*f^2*g - 48*b^3*c*d*e^2*f*h - 12*a*b^2
*d^2*e*f^2*g + 24*a*b^2*d^2*e^2*f*h - 6*a^2*b*d^2*e*f^2*h - 24*a*b^2*c*d*e
*f^2*h))/(8*(a*f - b*e)^4) - ((e + f*x)^2*(18*b^3*d^2*e^3*h - a^3*d^2*f^3*
h - 35*b^3*c^2*f^3*g + 5*a*b^2*c^2*f^3*h + a^2*b*d^2*f^3*g + 30*b^3*c^2*e*
f^2*h - 24*b^3*d^2*e^2*f*g + 10*a*b^2*c*d*f^3*g + 2*a^2*b*c*d*f^3*h + 60*b
^3*c*d*e*f^2*g - 48*b^3*c*d*e^2*f*h - 12*a*b^2*d^2*e*f^2*g + 18*a*b^2*d^2*
e^2*f*h - 24*a*b^2*c*d*e*f^2*h))/(3*b*(a*f - b*e)^3) + ((e + f*x)*(77*b^3*
c^2*f^3*g + a^3*d^2*f^3*h - 48*b^3*d^2*e^3*h - 11*a*b^2*c^2*f^3*h + a^2*b*
d^2*f^3*g - 66*b^3*c^2*e*f^2*h + 56*b^3*d^2*e^2*f*g - 22*a*b^2*c*d*f^3*g +
 2*a^2*b*c*d*f^3*h - 132*b^3*c*d*e*f^2*g + 112*b^3*c*d*e^2*f*h + 20*a*b...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 5440, normalized size of antiderivative = 10.52

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^4(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^4/(f*x+e)^(3/2),x)
```

output
```
(3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**6*d**2*f**3*h + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c*d*f**3*h - 18
*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**5*b*d**2*e*f**2*h + 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**2*f**3*g +
 9*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**5*b*d**2*f**3*h*x + 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c**2*f*
*3*h - 72*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sq
rt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*d*e*f**2*h + 30*sqrt(b)*sqrt(e + f*x)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**
2*c*d*f**3*g + 18*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c*d*f**3*h*x + 72*sqrt(b)*sqrt(e
 + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**4*b**2*d**2*e**2*f*h - 36*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*e*f**2*g - 54*sq
rt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a
*f - b*e)))*a**4*b**2*d**2*e*f**2*h*x + 9*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*d**2*...
```

3.167 $\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 695

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \frac{2f(de-cf)^2(fg-eh)}{(be-af)^5\sqrt{e+fx}} - \frac{(bc-ad)^2(bg-ah)\sqrt{e+fx}}{4b^2(be-af)^2(a+bx)^4}$$
$$- \frac{(bc-ad)\left(9a^2dfh+b^2(16deg-15cfg+8ceh)-ab(dfg+24deh-7cfh)\right)\sqrt{e+fx}}{24b^2(be-af)^3(a+bx)^3}$$
$$+ \frac{(3a^3d^2f^2h+a^2bdf(5dfg-24deh+10cfh)+ab^2(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh))-}{96b^2(be-af)^4(a+bx)^2}$$
$$- \frac{(3a^3d^2f^3h+a^2bdf^2(5dfg-24deh+10cfh)+ab^2f(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh)}{64b^2(be-af)^5(a+b}$$
$$+ \frac{f(3a^3d^2f^3h+a^2bdf^2(5dfg-24deh+10cfh)+ab^2f(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh)}{64b^{5/2}(be-af)^1}$$

output
```
2*f*(-c*f+d*e)^2*(-e*h+f*g)/(-a*f+b*e)^5/(f*x+e)^(1/2)-1/4*(-a*d+b*c)^2*(-
a*h+b*g)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^4-1/24*(-a*d+b*c)*(9*a^2*d
*f*h+b^2*(8*c*e*h-15*c*f*g+16*d*e*g)-a*b*(-7*c*f*h+24*d*e*h+d*f*g))*(f*x+e
)^(1/2)/b^2/(-a*f+b*e)^3/(b*x+a)^3+1/96*(3*a^3*d^2*f^2*h+a^2*b*d*f*(10*c*f
*h-24*d*e*h+5*d*f*g)+a*b^2*(35*c^2*f^2*h+10*c*d*f*(-16*e*h+7*f*g)-16*d^2*e
*(-9*e*h+5*f*g))-b^3*(48*d^2*e^2*g+c^2*f*(-88*e*h+123*f*g)-16*c*d*e*(-6*e*
h+11*f*g)))*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^4/(b*x+a)^2-1/64*(3*a^3*d^2*f^3*h
+a^2*b*d*f^2*(10*c*f*h-24*d*e*h+5*d*f*g)+a*b^2*f*(35*c^2*f^2*h+10*c*d*f*(-
16*e*h+7*f*g)-16*d^2*e*(-9*e*h+5*f*g))-b^3*(c^2*f^2*(-152*e*h+187*f*g)-16*
c*d*e*f*(-14*e*h+19*f*g)+16*d^2*e^2*(-4*e*h+7*f*g)))*(f*x+e)^(1/2)/b^2/(-a
*f+b*e)^5/(b*x+a)+1/64*f*(3*a^3*d^2*f^3*h+a^2*b*d*f^2*(10*c*f*h-24*d*e*h+5
*d*f*g)+a*b^2*f*(35*c^2*f^2*h+10*c*d*f*(-16*e*h+7*f*g)-16*d^2*e*(-9*e*h+5*
f*g))-b^3*(35*c^2*f^2*(-8*e*h+9*f*g)-80*c*d*e*f*(-6*e*h+7*f*g)+48*d^2*e^2*
(-4*e*h+5*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/b^(5/2)/(
-a*f+b*e)^(11/2)
```

Mathematica [A] (verified)

Time = 9.84 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + d*x)^2*(g + h*x))/((a + b*x)^5*(e + f*x)^(3/2)),x]
```

output
```
((Sqrt[b]*(-9*a^6*d^2*f^3*h*(e + f*x) - 3*a^5*b*d*f^2*(e + f*x)*(10*c*f*h
+ d*(5*f*g - 22*e*h + 11*f*h*x)) + b^6*(48*d^2*e^2*x^2*(-15*f^2*g*x^2 + 2*
e^2*(g + 2*h*x) + e*f*x*(-5*g + 12*h*x)) + 16*c*d*e*x*(105*f^3*g*x^3 + 5*e
*f^2*x^2*(7*g - 18*h*x) + 4*e^3*(2*g + 3*h*x) - 2*e^2*f*x*(7*g + 15*h*x))
+ c^2*(-945*f^4*g*x^4 + 16*e^4*(3*g + 4*h*x) + 105*e*f^3*x^3*(-3*g + 8*h*x
) - 8*e^3*f*x*(9*g + 14*h*x) + 14*e^2*f^2*x^2*(9*g + 20*h*x))) + a*b^5*(2*
c*d*(105*f^4*g*x^4 + 10*e^2*f^2*x^2*(105*g - 272*h*x) + 5*e*f^3*x^3*(623*g
 - 48*h*x) + 16*e^4*(g + 4*h*x) - 8*e^3*f*x*(51*g + 110*h*x)) + 16*d^2*e*x
*(-15*f^3*g*x^3 + 2*e^3*(2*g + 9*h*x) + e*f^2*x^2*(-170*g + 27*h*x) + e^2*
f*x*(-55*g + 141*h*x)) + c^2*(16*e^4*h + 105*f^4*x^3*(-33*g + h*x) - 24*e^
3*f*(11*g + 17*h*x) + 6*e^2*f^2*x*(78*g + 175*h*x) + 7*e*f^3*x^2*(-171*g +
 445*h*x))) + a^4*b^2*f*(3*c^2*f^2*(-128*f*g + 221*e*h + 93*f*h*x) + 2*c*d
*f*(-794*e^2*h + e*f*(663*g - 337*h*x) + f^2*x*(279*g + 73*h*x)) + d^2*(84
0*e^3*h + f^3*x^2*(73*g + 33*h*x) + e*f^2*x*(-337*g + 45*h*x) + e^2*f*(-79
4*g + 468*h*x))) + a^2*b^4*(2*c*d*(16*e^4*h + 6*e^2*f^2*x*(238*g - 635*h*x
) + 7*e*f^3*x^2*(603*g - 125*h*x) + 5*f^4*x^3*(77*g + 3*h*x) - 8*e^3*f*(13
*g + 75*h*x)) + d^2*(15*f^4*g*x^4 + 16*e^4*(g + 12*h*x) + 30*e^2*f^2*x^2*(
-127*g + 52*h*x) - e*f^3*x^3*(875*g + 72*h*x) + 24*e^3*f*x*(-25*g + 172*h*
x)) + c^2*f*(-104*e^3*h + 42*e^2*f*(15*g + 34*h*x) + 7*f^3*x^2*(-657*g + 5
5*h*x) + 9*e*f^2*x*(-185*g + 469*h*x))) + a^3*b^3*(2*c*d*f*(-152*e^3*h ...
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 654, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 161, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int -\frac{(c+dx)(a(4de-cf)h-b(4deg-9cfg+8ceh)+d(5bfg-8beh+3afh)x)}{2(a+bx)^4(e+fx)^{3/2}}dx}{4b(be-af)}-\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(c+dx)(a(4de-cf)h-b(4deg-9cfg+8ceh)+d(5bfg-8beh+3afh)x)}{(a+bx)^4(e+fx)^{3/2}}dx}{8b(be-af)}-\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 161

$$-\frac{\frac{(3a^3d^2f^3h+a^2bdf^2(10cfh-24deh+5dfg)+ab^2f(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh))-(b^3(35c^2f^2(9fg-8eh)-80cdef(7fg-6eh)}{6bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 52

$$-\frac{\frac{(3a^3d^2f^3h+a^2bdf^2(10cfh-24deh+5dfg)+ab^2f(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh))-(b^3(35c^2f^2(9fg-8eh)-80cdef(7fg-6eh)}{6bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 52

$$-\frac{\frac{(3a^3d^2f^3h+a^2bdf^2(10cfh-24deh+5dfg)+ab^2f(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh))-(b^3(35c^2f^2(9fg-8eh)-80cdef(7fg-6eh)}{6bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 73

$$-\frac{\frac{(3a^3d^2f^3h+a^2bdf^2(10cfh-24deh+5dfg)+ab^2f(35c^2f^2h+10cdf(7fg-16eh)-16d^2e(5fg-9eh))-(b^3(35c^2f^2(9fg-8eh)-80cdef(7fg-6eh)}{6bf(be-af)^2}}{}$$

$$\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

$\Big\downarrow$ 221

$$-\frac{\left(-\frac{3f\left(\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{\sqrt{b}(be-af)^{3/2}}-\frac{\sqrt{e+fx}}{(a+bx)(be-af)}\right)}{4(be-af)}-\frac{\sqrt{e+fx}}{2(a+bx)^2(be-af)}\right)(3a^3d^2f^3h+a^2bdf^2(10cfh-24deh+5dfg)+ab^2f(35c^2f^2h+10cdf(7fg\cdots}{6bf(be-af)^2}$$

$$\frac{(c+dx)^2(bg-ah)}{4b(a+bx)^4\sqrt{e+fx}(be-af)}$$

input

```
Int[((c + d*x)^2*(g + h*x))/((a + b*x)^5*(e + f*x)^(3/2)),x]
```

output

```
-1/4*((b*g - a*h)*(c + d*x)^2)/(b*(b*e - a*f)*(a + b*x)^4*Sqrt[e + f*x]) -
 (-1/3*(3*a^3*d^2*e*f^2*h - b^3*c*e*f*(4*d*e*g - 9*c*f*g + 8*c*e*h) - a*b^
2*(2*c*d*e*f*(61*f*g - 58*e*h) - c^2*f^2*(54*f*g - 49*e*h) - 2*d^2*e^2*(29
*f*g - 24*e*h)) + a^2*b*f*(10*c*d*e*f*h - 6*c^2*f^2*h + d^2*e*(5*f*g - 18*
e*h)) + (3*a^3*d^2*f^3*h + a^2*b*d*f^2*(5*d*f*g - 12*d*e*h - 2*c*f*h) + b^
3*(6*d^2*e^2*(9*f*g - 8*e*h) + 7*c^2*f^2*(9*f*g - 8*e*h) - 16*c*d*e*f*(7*f
*g - 6*e*h)) - a*b^2*f*(7*c^2*f^2*h + 2*c*d*f*(7*f*g - 16*e*h) - 2*d^2*e*(
2*f*g - 3*e*h)))*x)/(b*f*(b*e - a*f)^2*(a + b*x)^3*Sqrt[e + f*x]) + ((3*a^
3*d^2*f^3*h + a^2*b*d*f^2*(5*d*f*g - 24*d*e*h + 10*c*f*h) + a*b^2*f*(35*c^
2*f^2*h + 10*c*d*f*(7*f*g - 16*e*h) - 16*d^2*e*(5*f*g - 9*e*h)) - b^3*(35*
c^2*f^2*(9*f*g - 8*e*h) - 80*c*d*e*f*(7*f*g - 6*e*h) + 48*d^2*e^2*(5*f*g -
 4*e*h)))*(-1/2*Sqrt[e + f*x]/((b*e - a*f)*(a + b*x)^2) - (3*f*(-(Sqrt[e +
 f*x]/((b*e - a*f)*(a + b*x))) + (f*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b
*e - a*f]])/(Sqrt[b]*(b*e - a*f)^(3/2))))/(4*(b*e - a*f))))/(6*b*f*(b*e -
a*f)^2))/(8*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 166

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 1179, normalized size of antiderivative = 1.70

method	result	size
pseudoelliptic	Expression too large to display	1179
derivativedivides	Expression too large to display	1780
default	Expression too large to display	1780

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/64/(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)*((b*x+a)^4*(f*x+e)^(1/2)*((-105*c^2
*g*f^3+280/3*c*e*(c*h+2*d*g)*f^2-160*(c*h+1/2*d*g)*d*e^2*f+64*d^2*e^3*h)*b
^3+35/3*a*((c^2*h+2*c*d*g)*f^2-32/7*(c*h+1/2*d*g)*d*e*f+144/35*d^2*e^2*h)*
f*b^2+10/3*a^2*((c*h+1/2*d*g)*f-12/5*d*e*h)*d*f^2*b+a^3*d^2*f^3*h)*f*arcta
n(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-((a*f-b*e)*b)^(1/2)*((105*c^2*f^4*g
*x^4+35*x^3*c*(8/3*(-c*h-2*d*g)*x+c*g)*e*f^3-14*x^2*e^2*(40/7*(-2*c*d*h-d^
2*g)*x^2+20/9*c*(c*h+2*d*g)*x+c^2*g)*f^2+8*x*(-8*d^2*h*x^3+10/3*(2*c*d*h+d
^2*g)*x^2+14/9*c*(c*h+2*d*g)*x+c^2*g)*e^3*f-16/3*(4*d^2*h*x^3+2*(2*c*d*h+d
^2*g)*x^2+4/3*c*(c*h+2*d*g)*x+c^2*g)*e^4)*b^6-16/9*a*(-3465/16*x^3*c*(1/33
*(-c*h-2*d*g)*x+c*g)*f^4-1197/16*x^2*e*(160/399*(c*h+1/2*d*g)*d*x^2-445/17
1*c*(c*h+2*d*g)*x+c^2*g)*f^3+117/4*x*(12/13*d^2*h*x^3-1360/117*(c*h+1/2*d*
g)*d*x^2+175/78*c*(c*h+2*d*g)*x+c^2*g)*e^2*f^2-33/2*(-94/11*d^2*h*x^3+10/3
*(2*c*d*h+d^2*g)*x^2+17/11*c*(c*h+2*d*g)*x+c^2*g)*e^3*f+(18*d^2*h*x^2+4*(2
*c*d*h+d^2*g)*x+h*c^2+2*c*d*g)*e^4)*b^5+104/9*a^2*(4599/104*x^2*(-10/1533*
(c*h+1/2*d*g)*d*x^2-55/657*c*(c*h+2*d*g)*x+c^2*g)*f^4+1665/104*x*(8/185*d^
2*h*x^3+350/333*(c*h+1/2*d*g)*d*x^2-469/185*c*(c*h+2*d*g)*x+c^2*g)*e*f^3-3
15/52*(52/21*d^2*h*x^3-254/21*(c*h+1/2*d*g)*d*x^2+34/15*c*(c*h+2*d*g)*x+c^
2*g)*e^2*f^2+(-516/13*d^2*h*x^2+75/13*(2*c*d*h+d^2*g)*x+h*c^2+2*c*d*g)*e^3
*f-4/13*d*e^4*(6*d*h*x+c*h+1/2*d*g))*b^4-370/9*a^3*(-2511/370*x*(-1/279*d^
2*h*x^3-110/2511*(c*h+1/2*d*g)*d*x^2-511/2511*c*(c*h+2*d*g)*x+c^2*g)*f^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4078 vs. $2(665) = 1330$.

Time = 1.24 (sec) , antiderivative size = 8170, normalized size of antiderivative = 11.76

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(h*x+g)/(b*x+a)**5/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2648 vs. $2(665) = 1330$.

Time = 0.20 (sec) , antiderivative size = 2648, normalized size of antiderivative = 3.81

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/64*(240*b^3*d^2*e^2*f^2*g - 560*b^3*c*d*e*f^3*g + 80*a*b^2*d^2*e*f^3*g +
 315*b^3*c^2*f^4*g - 70*a*b^2*c*d*f^4*g - 5*a^2*b*d^2*f^4*g - 192*b^3*d^2*
e^3*f*h + 480*b^3*c*d*e^2*f^2*h - 144*a*b^2*d^2*e^2*f^2*h - 280*b^3*c^2*e*
f^3*h + 160*a*b^2*c*d*e*f^3*h + 24*a^2*b*d^2*e*f^3*h - 35*a*b^2*c^2*f^4*h
- 10*a^2*b*c*d*f^4*h - 3*a^3*d^2*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e
 + a*b*f))/((b^7*e^5 - 5*a*b^6*e^4*f + 10*a^2*b^5*e^3*f^2 - 10*a^3*b^4*e^2
*f^3 + 5*a^4*b^3*e*f^4 - a^5*b^2*f^5)*sqrt(-b^2*e + a*b*f)) + 2*(d^2*e^2*f
^2*g - 2*c*d*e*f^3*g + c^2*f^4*g - d^2*e^3*f*h + 2*c*d*e^2*f^2*h - c^2*e*f
^3*h)/((b^5*e^5 - 5*a*b^4*e^4*f + 10*a^2*b^3*e^3*f^2 - 10*a^3*b^2*e^2*f^3
+ 5*a^4*b*e*f^4 - a^5*f^5)*sqrt(f*x + e)) + 1/192*(336*(f*x + e)^(7/2)*b^6
*d^2*e^2*f^2*g - 1104*(f*x + e)^(5/2)*b^6*d^2*e^3*f^2*g + 1200*(f*x + e)^(
3/2)*b^6*d^2*e^4*f^2*g - 432*sqrt(f*x + e)*b^6*d^2*e^5*f^2*g - 912*(f*x +
e)^(7/2)*b^6*c*d*e*f^3*g + 240*(f*x + e)^(7/2)*a*b^5*d^2*e*f^3*g + 3088*(f
*x + e)^(5/2)*b^6*c*d*e^2*f^3*g + 224*(f*x + e)^(5/2)*a*b^5*d^2*e^2*f^3*g
- 3568*(f*x + e)^(3/2)*b^6*c*d*e^3*f^3*g - 1232*(f*x + e)^(3/2)*a*b^5*d^2*
e^3*f^3*g + 1392*sqrt(f*x + e)*b^6*c*d*e^4*f^3*g + 768*sqrt(f*x + e)*a*b^5
*d^2*e^4*f^3*g + 561*(f*x + e)^(7/2)*b^6*c^2*f^4*g - 210*(f*x + e)^(7/2)*a
*b^5*c*d*f^4*g - 15*(f*x + e)^(7/2)*a^2*b^4*d^2*f^4*g - 1929*(f*x + e)^(5/
2)*b^6*c^2*e*f^4*g - 2318*(f*x + e)^(5/2)*a*b^5*c*d*e*f^4*g + 935*(f*x + e
)^(5/2)*a^2*b^4*d^2*e*f^4*g + 2295*(f*x + e)^(3/2)*b^6*c^2*e^2*f^4*g + ...
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 1824, normalized size of antiderivative = 2.62

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(c + d*x)^2)/((e + f*x)^(3/2)*(a + b*x)^5),x)
```

output
```
((11*(e + f*x)^3*(3*a^3*d^2*f^4*h - 315*b^3*c^2*f^4*g + 35*a*b^2*c^2*f^4*h
 + 5*a^2*b*d^2*f^4*g + 280*b^3*c^2*e*f^3*h + 192*b^3*d^2*e^3*f*h - 240*b^3
*d^2*e^2*f^2*g + 144*a*b^2*d^2*e^2*f^2*h + 70*a*b^2*c*d*f^4*g + 10*a^2*b*c
*d*f^4*h + 560*b^3*c*d*e*f^3*g - 80*a*b^2*d^2*e*f^3*g - 24*a^2*b*d^2*e*f^3
*h - 480*b^3*c*d*e^2*f^2*h - 160*a*b^2*c*d*e*f^3*h))/(192*(a*f - b*e)^4) -
 (2*(c^2*f^4*g + d^2*e^2*f^2*g - c^2*e*f^3*h - d^2*e^3*f*h + 2*c*d*e^2*f^2
*h - 2*c*d*e*f^3*g))/(a*f - b*e) + (b*(e + f*x)^4*(3*a^3*d^2*f^4*h - 315*b
^3*c^2*f^4*g + 35*a*b^2*c^2*f^4*h + 5*a^2*b*d^2*f^4*g + 280*b^3*c^2*e*f^3*
h + 192*b^3*d^2*e^3*f*h - 240*b^3*d^2*e^2*f^2*g + 144*a*b^2*d^2*e^2*f^2*h
+ 70*a*b^2*c*d*f^4*g + 10*a^2*b*c*d*f^4*h + 560*b^3*c*d*e*f^3*g - 80*a*b^2
*d^2*e*f^3*g - 24*a^2*b*d^2*e*f^3*h - 480*b^3*c*d*e^2*f^2*h - 160*a*b^2*c*
d*e*f^3*h))/(64*(a*f - b*e)^5) - ((e + f*x)*(837*b^3*c^2*f^4*g + 3*a^3*d^2
*f^4*h - 93*a*b^2*c^2*f^4*h + 5*a^2*b*d^2*f^4*g - 744*b^3*c^2*e*f^3*h - 57
6*b^3*d^2*e^3*f*h + 656*b^3*d^2*e^2*f^2*g - 240*a*b^2*d^2*e^2*f^2*h - 186*
a*b^2*c*d*f^4*g + 10*a^2*b*c*d*f^4*h - 1488*b^3*c*d*e*f^3*g + 176*a*b^2*d^
2*e*f^3*g - 24*a^2*b*d^2*e*f^3*h + 1312*b^3*c*d*e^2*f^2*h + 352*a*b^2*c*d*
e*f^3*h))/(64*b^2*(a*f - b*e)^2) + ((e + f*x)^2*(511*a*b^2*c^2*f^4*h - 33*
a^3*d^2*f^4*h - 4599*b^3*c^2*f^4*g + 73*a^2*b*d^2*f^4*g + 4088*b^3*c^2*e*f
^3*h + 2880*b^3*d^2*e^3*f*h - 3504*b^3*d^2*e^2*f^2*g + 1872*a*b^2*d^2*e^2*
f^2*h + 1022*a*b^2*c*d*f^4*g + 146*a^2*b*c*d*f^4*h + 8176*b^3*c*d*e*f^3...
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 7371, normalized size of antiderivative = 10.61

$$\int \frac{(c+dx)^2(g+hx)}{(a+bx)^5(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(h*x+g)/(b*x+a)^5/(f*x+e)^(3/2),x)
```

output
```
(9*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**7*d**2*f**4*h + 30*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d*f**4*h - 7
2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sq
rt(a*f - b*e)))*a**6*b*d**2*e*f**3*h + 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**2*f**4*g
 + 36*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**6*b*d**2*f**4*h*x + 105*sqrt(b)*sqrt(e + f*x)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c**
2*f**4*h - 480*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b
)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d*e*f**3*h + 210*sqrt(b)*sqrt(e +
 f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*5*b**2*c*d*f**4*g + 120*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*c*d*f**4*h*x + 432*sqrt(b
)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**5*b**2*d**2*e**2*f**2*h - 240*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*e*f*
*3*g - 288*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a**5*b**2*d**2*e*f**3*h*x + 60*sqrt(b)*sqrt(e + f
*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a...
```

3.168 $\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 270

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = -\frac{2(be-af)^3(fg-eh)}{f^4(de-cf)\sqrt{e+fx}} + \frac{2b(3a^2d^2f^2h+3abdf(dfg-2deh-cfh)+b^2(c^2f^2h-d^2e(2fg-3eh)-cdf(fg-2eh)))\sqrt{e+fx}}{d^3f^4} + \frac{2b^2(3adfh+b(dfg-3deh-cfh))(e+fx)^{3/2}}{3d^2f^4} + \frac{2b^3h(e+fx)^{5/2}}{5df^4} + \frac{2(bc-ad)^3(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}(de-cf)^{3/2}}$$

output
```
-2*(-a*f+b*e)^3*(-e*h+f*g)/f^4/(-c*f+d*e)/(f*x+e)^(1/2)+2*b*(3*a^2*d^2*f^2
*h+3*a*b*d*f*(-c*f*h-2*d*e*h+d*f*g)+b^2*(c^2*f^2*h-d^2*e*(-3*e*h+2*f*g)-c*
d*f*(-2*e*h+f*g)))*(f*x+e)^(1/2)/d^3/f^4+2/3*b^2*(3*a*d*f*h+b*(-c*f*h-3*d*
e*h+d*f*g))*(f*x+e)^(3/2)/d^2/f^4+2/5*b^3*h*(f*x+e)^(5/2)/d/f^4+2*(-a*d+b*
c)^3*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)/(-
c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{30a^3d^3f^3(-fg+eh)+90a^2bd^2f^2(cfh(e+fx)+de(-2eh+f(g-hx)))-3}{}$$
$$+\frac{2(bc-ad)^3(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}(-de+cf)^{3/2}}$$

input

```
Integrate[((a + b*x)^3*(g + h*x))/((c + d*x)*(e + f*x)^(3/2)),x]
```

output

```
(30*a^3*d^3*f^3*(-(f*g) + e*h) + 90*a^2*b*d^2*f^2*(c*f*h*(e + f*x) + d*e*(
-2*e*h + f*(g - h*x))) - 30*a*b^2*d*f*(3*c^2*f^2*h*(e + f*x) - c*d*f*(e +
f*x)*(3*f*g - 2*e*h + f*h*x) + d^2*e*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*x
*(3*g + h*x))) + 2*b^3*(15*c^3*f^3*h*(e + f*x) - 5*c^2*d*f^2*(e + f*x)*(3*
f*g - 2*e*h + f*h*x) + c*d^2*f*(e + f*x)*(8*e^2*h - 2*e*f*(5*g + 2*h*x) +
f^2*x*(5*g + 3*h*x)) + d^3*e*(-48*e^3*h + 8*e^2*f*(5*g - 3*h*x) - f^3*x^2*
(5*g + 3*h*x) + 2*e*f^2*x*(10*g + 3*h*x))))/(15*d^3*f^4*(-(d*e) + c*f)*Sqr
t[e + f*x]) + (2*(b*c - a*d)^3*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/
Sqrt[-(d*e) + c*f]])/(d^(7/2)*(-(d*e) + c*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {167, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 167$$

$$\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)} - \frac{2\int -\frac{(a+bx)^2(af(dg-ch)-6bc(fg-eh)-b(5dfg-6deh+cfh)x)}{2(c+dx)\sqrt{e+fx}}dx}{f(de-cf)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^2(af(dg-ch)-bc(6fg-6eh)-b(5dfg-6deh+cfh)x)}{(c+dx)\sqrt{e+fx}}dx}{f(de-cf)}+\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$\big\downarrow$ 170

$$\frac{\frac{2\int \frac{(a+bx)(bc(4be+af)(5dfg-6deh+cfh)+5adf(af(dg-ch)-bc(6fg-6eh))+b((4bde+5bcf-4adf)(5dfg-6deh+cfh)+5df(af(dg-ch)-bc(6fg-6eh)))x)}{2(c+dx)\sqrt{e+fx}}dx}{5df}-2}{f(de-cf)}$$

$$\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)(bc(4be+af)(5dfg-6deh+cfh)+5adf(af(dg-ch)-bc(6fg-6eh))+b((4bde+5bcf-4adf)(5dfg-6deh+cfh)+5df(af(dg-ch)-bc(6fg-6eh)))x)}{(c+dx)\sqrt{e+fx}}dx}{5df}-2b}{f(de-cf)}$$

$$\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$\big\downarrow$ 164

$$\frac{-\frac{5f^2(bc-ad)^3(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d^2}-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(7cfh-12deh+5dfg)-15abdf\left(3c^2f^2h-cdf(3fg-2eh)+2d^2e(3fg-4eh)\right)-bdfx((-4adf+5bcf+}{5df}}{}$$

$$\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$\big\downarrow$ 73

$$\frac{-\frac{10f(bc-ad)^3(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2}-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(7cfh-12deh+5dfg)-15abdf\left(3c^2f^2h-cdf(3fg-2eh)+2d^2e(3fg-4eh)\right)-bdfx((-4adf}{5df}}{}$$

$$\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$\big\downarrow$ 221

$$\frac{\frac{10f^2(bc-ad)^3(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}\sqrt{de-cf}}-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(7cfh-12deh+5dfg)-15abdf\left(3c^2f^2h-cdf(3fg-2eh)+2d^2e(3fg-4eh)\right)-bdfx((-4adf+5b\cdots}{\cdots}}{5df}$$

$$\frac{2(a+bx)^3(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

input
```
Int[((a + b*x)^3*(g + h*x))/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(f*g - e*h)*(a + b*x)^3)/(f*(d*e - c*f)*Sqrt[e + f*x]) + ((-2*b*(5*d*f*
g - 6*d*e*h + c*f*h)*(a + b*x)^2*Sqrt[e + f*x])/(5*d*f) + ((-2*b*Sqrt[e +
f*x]*(6*a^2*d^2*f^2*(5*d*f*g - 12*d*e*h + 7*c*f*h) - 15*a*b*d*f*(3*c^2*f^2
*h + 2*d^2*e*(3*f*g - 4*e*h) - c*d*f*(3*f*g - 2*e*h)) + b^2*(15*c^3*f^3*h
+ 8*d^3*e^2*(5*f*g - 6*e*h) - 2*c*d^2*e*f*(5*f*g - 4*e*h) - 5*c^2*d*f^2*(3
*f*g - 2*e*h)) - b*d*f*((4*b*d*e + 5*b*c*f - 4*a*d*f)*(5*d*f*g - 6*d*e*h +
 c*f*h) + 5*d*f*(a*f*(d*g - c*h) - b*c*(6*f*g - 6*e*h)))*x))/(3*d^2*f^2) +
 (10*(b*c - a*d)^3*f^2*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*
e - c*f]])/(d^(5/2)*Sqrt[d*e - c*f]))/(5*d*f))/(f*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.55

method	result
pseudoelliptic	$\frac{2\sqrt{fx+e}\,f^4(ad-bc)^3(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2\left(\left(-a^3d^3g+3xcb\left(\frac{x\left(\frac{3hx}{5}+g\right)b^2}{9}+a\left(\frac{hx}{3}+g\right)b+a^2h\right)d^2-3x\,c^2b\right.\right.}{}$
risch	$\frac{2b\left(3x^2h\,b^2d^2f^2+15ab\,d^2f^2hx-5b^2cd\,f^2hx-9b^2d^2efhx+5b^2d^2f^2gx+45a^2d^2f^2h-45abcd\,f^2h-75ab\,d^2efh+45ab\,d^2\right.}{15f^4d^3}$
derivativedivides	$\frac{2b\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+ab\,d^2fh(fx+e)^{\frac{3}{2}}-\frac{b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-b^2d^2eh(fx+e)^{\frac{3}{2}}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-3abcd\,f^2h\sqrt{fx}\right.}{}$
default	$\frac{2b\left(\frac{d^2h(fx+e)^{\frac{5}{2}}b^2}{5}+ab\,d^2fh(fx+e)^{\frac{3}{2}}-\frac{b^2cdfh(fx+e)^{\frac{3}{2}}}{3}-b^2d^2eh(fx+e)^{\frac{3}{2}}+\frac{b^2d^2fg(fx+e)^{\frac{3}{2}}}{3}+3a^2d^2f^2h\sqrt{fx+e}-3abcd\,f^2h\sqrt{fx}\right.}{}$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/(f*x+e)^(1/2)*((f*x+e)^(1/2)*f^4*(a*d-b*c)^3*(c*h-d*g)*arctan(d*(f*x+e)^
(1/2)/((c*f-d*e)*d)^(1/2))+((-a^3*d^3*g+3*x*c*b*(1/9*x*(3/5*h*x+g)*b^2+a*(
1/3*h*x+g)*b+a^2*h)*d^2-3*x*c^2*b^2*(1/3*(1/3*h*x+g)*b+a*h)*d+b^3*c^3*h*x)
*f^4+((-1/3*x^2*(3/5*h*x+g)*b^3-3*a*x*(1/3*h*x+g)*b^2+3*a^2*(-h*x+g)*b+h*a
^3)*d^3+3*c*(-1/9*x*(1/5*h*x+g)*b^2+a*(-1/3*h*x+g)*b+a^2*h)*b*d^2-3*c^2*(1
/3*(-1/3*h*x+g)*b+a*h)*b^2*d+b^3*c^3*h)*e*f^3-6*d*((-2/9*x*(3/10*h*x+g)*b^
2+a*(-2/3*h*x+g)*b+a^2*h)*d^2+1/3*(1/3*(-2/5*h*x+g)*b+a*h)*c*b*d-1/9*b^2*c
^2*h)*b*e^2*f^2+8*d^2*(((-1/5*h*x+1/3*g)*b+a*h)*d+1/15*b*c*h)*b^2*e^3*f-16
/5*b^3*d^3*e^4*h)*((c*f-d*e)*d)^(1/2))/((c*f-d*e)*d)^(1/2)/f^4/d^3/(c*f-d*
e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. $2(248) = 496$.

Time = 0.16 (sec) , antiderivative size = 2179, normalized size of antiderivative = 8.07

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*e*f^4*
g - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*e*f^4*h + ((b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*f^5*g - (b^3*c^4 - 3*
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*f^5*h)*x)*sqrt(d^2*e - c*d*f)*l
og((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c))
- 2*(3*(b^3*d^5*e^2*f^3 - 2*b^3*c*d^4*e*f^4 + b^3*c^2*d^3*f^5)*h*x^3 + (5*
(b^3*d^5*e^2*f^3 - 2*b^3*c*d^4*e*f^4 + b^3*c^2*d^3*f^5)*g - (6*b^3*d^5*e^3
*f^2 - (7*b^3*c*d^4 + 15*a*b^2*d^5)*e^2*f^3 - 2*(2*b^3*c^2*d^3 - 15*a*b^2*
c*d^4)*e*f^4 + 5*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3)*f^5)*h)*x^2 - 5*(8*b^3*d^
5*e^4*f + 3*a^3*c*d^4*f^5 - 2*(5*b^3*c*d^4 + 9*a*b^2*d^5)*e^3*f^2 - (b^3*c
^2*d^3 - 27*a*b^2*c*d^4 - 9*a^2*b*d^5)*e^2*f^3 + 3*(b^3*c^3*d^2 - 3*a*b^2*
c^2*d^3 - 3*a^2*b*c*d^4 - a^3*d^5)*e*f^4)*g + (48*b^3*d^5*e^5 - 8*(7*b^3*c
*d^4 + 15*a*b^2*d^5)*e^4*f - 2*(b^3*c^2*d^3 - 75*a*b^2*c*d^4 - 45*a^2*b*d^
5)*e^3*f^2 - 5*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 27*a^2*b*c*d^4 + 3*a^3*d^5
)*e^2*f^3 + 15*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 + a^3*c*d^4)
*e*f^4)*h - (5*(4*b^3*d^5*e^3*f^2 - (5*b^3*c*d^4 + 9*a*b^2*d^5)*e^2*f^3 -
2*(b^3*c^2*d^3 - 9*a*b^2*c*d^4)*e*f^4 + 3*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3)*
f^5)*g - (24*b^3*d^5*e^4*f - 4*(7*b^3*c*d^4 + 15*a*b^2*d^5)*e^3*f^2 - (b^3
*c^2*d^3 - 75*a*b^2*c*d^4 - 45*a^2*b*d^5)*e^2*f^3 - 10*(b^3*c^3*d^2 - 3*a*
b^2*c^2*d^3 + 9*a^2*b*c*d^4)*e*f^4 + 15*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(262) = 524$.

Time = 25.13 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b^3h(e+fx)^{\frac{5}{2}}}{5df^3} + \frac{(af-be)^3(eh-fg)}{f^3\sqrt{e+fx}(cf-de)} + \frac{(e+fx)^{\frac{3}{2}}\cdot\left(3ab^2dfh-b^3cfh-3b^3deh+b^3dfg\right)}{3d^2f^3} + \frac{\sqrt{e+fx}\left(3a^2bd^2f^2h-3ab^2c\cdots\right.}{\cdots}\right.}{\cdots} \\ \frac{\frac{b^3hx^4}{4d} + \frac{x^3\cdot\left(3ab^2dh-b^3ch+b^3dg\right)}{3d^2} + \frac{x^2\cdot\left(3a^2bd^2h-3ab^2cdh+3ab^2d^2g+b^3c^2h-b^3cdg\right)}{2d^3} + \frac{x\left(a^3d^3h-3a^2bcd^2h+3a^2\cdots\right.}{\cdots}}{e^{\frac{3}{2}}} \end{cases}$$

input

```
integrate((b*x+a)**3*(h*x+g)/(d*x+c)/(f*x+e)**(3/2),x)
```

output

```
Piecewise((2*(b**3*h*(e + f*x)**(5/2)/(5*d*f**3) + (a*f - b*e)**3*(e*h - f
*g)/(f**3*sqrt(e + f*x)*(c*f - d*e)) + (e + f*x)**(3/2)*(3*a*b**2*d*f*h -
b**3*c*f*h - 3*b**3*d*e*h + b**3*d*f*g)/(3*d**2*f**3) + sqrt(e + f*x)*(3*a
**2*b*d**2*f**2*h - 3*a*b**2*c*d*f**2*h - 6*a*b**2*d**2*e*f*h + 3*a*b**2*d
**2*f**2*g + b**3*c**2*f**2*h + 2*b**3*c*d*e*f*h - b**3*c*d*f**2*g + 3*b**
3*d**2*e**2*h - 2*b**3*d**2*e*f*g)/(d**3*f**3) + f*(a*d - b*c)**3*(c*h - d
*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**4*sqrt((c*f - d*e)/d)*(c*f
 - d*e)))/f, Ne(f, 0)), ((b**3*h*x**4/(4*d) + x**3*(3*a*b**2*d*h - b**3*c*
h + b**3*d*g)/(3*d**2) + x**2*(3*a**2*b*d**2*h - 3*a*b**2*c*d*h + 3*a*b**2
*d**2*g + b**3*c**2*h - b**3*c*d*g)/(2*d**3) + x*(a**3*d**3*h - 3*a**2*b*c
*d**2*h + 3*a**2*b*d**3*g + 3*a*b**2*c**2*d*h - 3*a*b**2*c*d**2*g - b**3*c
**3*h + b**3*c**2*d*g)/d**4 - (a*d - b*c)**3*(c*h - d*g)*Piecewise((x/c, E
q(d, 0)), (log(c + d*x)/d, True))/d**4)/e**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(248) = 496$.

Time = 0.15 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = -\frac{2\,(b^3c^3dg - 3\,ab^2c^2d^2g + 3\,a^2bcd^3g - a^3d^4g - b^3c^4h + 3\,ab^2c^3dh - 3\,a^2bc^2d^2h + a^3cd^3h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cd}}\right.}{(d^4e - cd^3f)\sqrt{-d^2e+cdf}} - \frac{2\,(b^3e^3fg - 3\,ab^2e^2f^2g + 3\,a^2bef^3g - a^3f^4g - b^3e^4h + 3\,ab^2e^3fh - 3\,a^2be^2f^2h + a^3ef^3h)}{(def^4 - cf^5)\sqrt{fx+e}} + \frac{2\left(5\,(fx+e)^{\frac{3}{2}}b^3d^4f^{17}g - 30\,\sqrt{fx+e}b^3d^4ef^{17}g - 15\,\sqrt{fx+e}b^3cd^3f^{18}g + 45\,\sqrt{fx+e}ab^2d^4f^{18}g + 3\,(fx\right.}{}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-2*(b^3*c^3*d*g - 3*a*b^2*c^2*d^2*g + 3*a^2*b*c*d^3*g - a^3*d^4*g - b^3*c^
4*h + 3*a*b^2*c^3*d*h - 3*a^2*b*c^2*d^2*h + a^3*c*d^3*h)*arctan(sqrt(f*x +
 e)*d/sqrt(-d^2*e + c*d*f))/((d^4*e - c*d^3*f)*sqrt(-d^2*e + c*d*f)) - 2*(
b^3*e^3*f*g - 3*a*b^2*e^2*f^2*g + 3*a^2*b*e*f^3*g - a^3*f^4*g - b^3*e^4*h
+ 3*a*b^2*e^3*f*h - 3*a^2*b*e^2*f^2*h + a^3*e*f^3*h)/((d*e*f^4 - c*f^5)*sq
rt(f*x + e)) + 2/15*(5*(f*x + e)^(3/2)*b^3*d^4*f^17*g - 30*sqrt(f*x + e)*b
^3*d^4*e*f^17*g - 15*sqrt(f*x + e)*b^3*c*d^3*f^18*g + 45*sqrt(f*x + e)*a*b
^2*d^4*f^18*g + 3*(f*x + e)^(5/2)*b^3*d^4*f^16*h - 15*(f*x + e)^(3/2)*b^3*
d^4*e*f^16*h + 45*sqrt(f*x + e)*b^3*d^4*e^2*f^16*h - 5*(f*x + e)^(3/2)*b^3
*c*d^3*f^17*h + 15*(f*x + e)^(3/2)*a*b^2*d^4*f^17*h + 30*sqrt(f*x + e)*b^3
*c*d^3*e*f^17*h - 90*sqrt(f*x + e)*a*b^2*d^4*e*f^17*h + 15*sqrt(f*x + e)*b
^3*c^2*d^2*f^18*h - 45*sqrt(f*x + e)*a*b^2*c*d^3*f^18*h + 45*sqrt(f*x + e)
*a^2*b*d^4*f^18*h)/(d^5*f^20)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = (e+f\,x)^{3/2}\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{3\,d\,f^4}-\frac{2\,b^3\,h\,(c\,f^5-d\,e\,f^4)}{3\,d^2\,f^8}\right)$$
$$-\sqrt{e+f\,x}\left(\frac{\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{d\,f^4}-\frac{2\,b^3\,h\,(c\,f^5-d\,e\,f^4)}{d^2\,f^8}\right)(c\,f^5-d\,e\,f^4)}{d\,f^4}-\frac{6\,b\,(a\,f-b\,e)\,(a\,f\,h-2\,b\,e\,h+b\,f}{d\,f^4}\right.$$
$$+\frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{e+f\,x}\,(a\,d-b\,c)^3\,(c\,h-d\,g)\,(d^4\,e-c\,d^3\,f)}{d^{5/2}\,(c\,f-d\,e)^{3/2}\,(-2\,h\,a^3\,c\,d^3+2\,g\,a^3\,d^4+6\,h\,a^2\,b\,c^2\,d^2-6\,g\,a^2\,b\,c\,d^3-6\,h\,a\,b^2\,c^3\,d+6\,g\,a\,b^2\,c^2\,d^2+2\,h\,b^3\,c^4-2\,g\,b^3\,c^3\,d)}\right)(a\,d-b\,c}{d^{7/2}\,(c\,f-d\,e)^{3/2}}$$
$$+\frac{2\,b^3\,h\,(e+f\,x)^{5/2}}{5\,d\,f^4}$$
$$-\frac{2\,(-h\,a^3\,d^3\,e\,f^3+g\,a^3\,d^3\,f^4+3\,h\,a^2\,b\,d^3\,e^2\,f^2-3\,g\,a^2\,b\,d^3\,e\,f^3-3\,h\,a\,b^2\,d^3\,e^3\,f+3\,g\,a\,b^2\,d^3\,e^2\,f^2+h\,b^3\,d}{d^3\,f^4\,\sqrt{e+f\,x}\,(c\,f-d\,e)}$$

input
```
int(((g + h*x)*(a + b*x)^3)/((e + f*x)^(3/2)*(c + d*x)),x)
```

output
```
(e + f*x)^(3/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(3*d*f^4) - (2*b^3*
h*(c*f^5 - d*e*f^4))/(3*d^2*f^8)) - (e + f*x)^(1/2)*((((2*b^3*f*g - 8*b^3*
e*h + 6*a*b^2*f*h)/(d*f^4) - (2*b^3*h*(c*f^5 - d*e*f^4))/(d^2*f^8))*(c*f^5
 - d*e*f^4))/(d*f^4) - (6*b*(a*f - b*e)*(a*f*h - 2*b*e*h + b*f*g))/(d*f^4)
) + (2*atan((2*(e + f*x)^(1/2)*(a*d - b*c)^3*(c*h - d*g)*(d^4*e - c*d^3*f)
)/(d^(5/2)*(c*f - d*e)^(3/2)*(2*a^3*d^4*g + 2*b^3*c^4*h - 2*a^3*c*d^3*h -
2*b^3*c^3*d*g - 6*a^2*b*c*d^3*g - 6*a*b^2*c^3*d*h + 6*a*b^2*c^2*d^2*g + 6*
a^2*b*c^2*d^2*h)))*(a*d - b*c)^3*(c*h - d*g))/(d^(7/2)*(c*f - d*e)^(3/2))
+ (2*b^3*h*(e + f*x)^(5/2))/(5*d*f^4) - (2*(a^3*d^3*f^4*g + b^3*d^3*e^4*h
- a^3*d^3*e*f^3*h - b^3*d^3*e^3*f*g + 3*a*b^2*d^3*e^2*f^2*g + 3*a^2*b*d^3*
e^2*f^2*h - 3*a^2*b*d^3*e*f^3*g - 3*a*b^2*d^3*e^3*f*h))/(d^3*f^4*(e + f*x)
^(1/2)*(c*f - d*e))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1423, normalized size of antiderivative = 5.27

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x)
```

output
```
(2*(15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a**3*c*d**3*f**4*h - 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**4*f**4*
g - 45*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**2*f**4*h + 45*sqrt(d)*sqrt(e + f*x)*sq
rt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d
**3*f**4*g + 45*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*
d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b**2*c**3*d*f**4*h - 45*sqrt(d)*sqrt(e + f
*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*
*2*c**2*d**2*f**4*g - 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(
e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**3*c**4*f**4*h + 15*sqrt(d)*sqrt(
e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*b**3*c**3*d*f**4*g + 15*a**3*c*d**4*e*f**4*h - 15*a**3*c*d**4*f**5*g - 15
*a**3*d**5*e**2*f**3*h + 15*a**3*d**5*e*f**4*g + 45*a**2*b*c**2*d**3*e*f**
4*h + 45*a**2*b*c**2*d**3*f**5*h*x - 135*a**2*b*c*d**4*e**2*f**3*h + 45*a*
*2*b*c*d**4*e*f**4*g - 90*a**2*b*c*d**4*e*f**4*h*x + 90*a**2*b*d**5*e**3*f
**2*h - 45*a**2*b*d**5*e**2*f**3*g + 45*a**2*b*d**5*e**2*f**3*h*x - 45*a*b
**2*c**3*d**2*e*f**4*h - 45*a*b**2*c**3*d**2*f**5*h*x + 15*a*b**2*c**2*d**
3*e**2*f**3*h + 45*a*b**2*c**2*d**3*e*f**4*g + 30*a*b**2*c**2*d**3*e*f**4*
h*x + 45*a*b**2*c**2*d**3*f**5*g*x + 15*a*b**2*c**2*d**3*f**5*h*x**2 + ...
```

3.169 $\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 173

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2(be-af)^2(fg-eh)}{f^3(de-cf)\sqrt{e+fx}} + \frac{2b(2adfh+b(dfg-2deh-cfh))\sqrt{e+fx}}{d^2f^3} + \frac{2b^2h(e+fx)^{3/2}}{3df^3} - \frac{2(bc-ad)^2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}(de-cf)^{3/2}}$$

output
```
2*(-a*f+b*e)^2*(-e*h+f*g)/f^3/(-c*f+d*e)/(f*x+e)^(1/2)+2*b*(2*a*d*f*h+b*(-
c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(1/2)/d^2/f^3+2/3*b^2*h*(f*x+e)^(3/2)/d/f^3-
2*(-a*d+b*c)^2*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/
d^(5/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2(3a^2d^2f^2(-fg+eh)+6abdf(cfh(e+fx)+de(fg-2eh-fhx))+b^2(-3c}{3d^2} + \frac{2(bc-ad)^2(-dg+ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)^2*(g + h*x))/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(3*a^2*d^2*f^2*(-(f*g) + e*h) + 6*a*b*d*f*(c*f*h*(e + f*x) + d*e*(f*g -
 2*e*h - f*h*x)) + b^2*(-3*c^2*f^2*h*(e + f*x) + c*d*f*(e + f*x)*(3*f*g -
2*e*h + f*h*x) + d^2*e*(8*e^2*h - f^2*x*(3*g + h*x) + e*f*(-6*g + 4*h*x)))
))/(3*d^2*f^3*(-(d*e) + c*f)*Sqrt[e + f*x]) + (2*(b*c - a*d)^2*(-(d*g) + c
*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(5/2)*(-(d*e) +
 c*f)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {167, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{167}$$

$$\frac{2(a+bx)^2(fg-eh)}{f\sqrt{e+fx}(de-cf)} - \frac{2\int -\frac{(a+bx)(af(dg-ch)-4bc(fg-eh)-b(3dfg-4deh+cfh)x)}{2(c+dx)\sqrt{e+fx}}dx}{f(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)(af(dg-ch)-bc(4fg-4eh)-b(3dfg-4deh+cfh)x)}{(c+dx)\sqrt{e+fx}}dx}{f(de-cf)}+\frac{2(a+bx)^2(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{164}$$

$$\frac{\frac{f(bc-ad)^2(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d^2}-\frac{2b\sqrt{e+fx}(6adf(cfh-2deh+dfg)-b(3c^2f^2h-cdf(3fg-2eh)+2d^2e(3fg-4eh))+bdfx(cfh-4deh+3dfg}{3d^2f^2}}{f(de-cf)}$$
$$\frac{2(a+bx)^2(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(bc-ad)^2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2}-\frac{2b\sqrt{e+fx}(6adf(cfh-2deh+dfg)-b(3c^2f^2h-cdf(3fg-2eh)+2d^2e(3fg-4eh))+bdfx(cfh-4d}{3d^2f^2}}{f(de-cf)}$$
$$\frac{2(a+bx)^2(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2f(bc-ad)^2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}\sqrt{de-cf}}-\frac{2b\sqrt{e+fx}(6adf(cfh-2deh+dfg)-b(3c^2f^2h-cdf(3fg-2eh)+2d^2e(3fg-4eh))+bdfx(cfh-4d}{3d^2f^2}}{f(de-cf)}$$
$$\frac{2(a+bx)^2(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

input
```
Int[((a + b*x)^2*(g + h*x))/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(f*g - e*h)*(a + b*x)^2)/(f*(d*e - c*f)*Sqrt[e + f*x]) + ((-2*b*Sqrt[e
+ f*x]*(6*a*d*f*(d*f*g - 2*d*e*h + c*f*h) - b*(3*c^2*f^2*h + 2*d^2*e*(3*f*
g - 4*e*h) - c*d*f*(3*f*g - 2*e*h)) + b*d*f*(3*d*f*g - 4*d*e*h + c*f*h)*x)
)/(3*d^2*f^2) - (2*(b*c - a*d)^2*f*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f
*x])/Sqrt[d*e - c*f]])/(d^(5/2)*Sqrt[d*e - c*f]))/(f*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$\frac{\frac{2b\sqrt{fx+e}\,(hbdfx+6adfh-3bcfh-5bdeh+3bgdf)}{3d^{2}}+\frac{2\left(a^{2}e\,f^{2}h-a^{2}f^{3}g-2ab\,e^{2}fh+2abe\,f^{2}g+b^{2}e^{3}h-b^{2}e^{2}fg\right)}{(cf-de)\sqrt{fx+e}}+\frac{2f^{3}\left(a^{2}c\,d^{2}h-a^{2}d^{3}\cdots\right.}{\cdots}}{f^{3}}$
risch	$\frac{2b(hbdfx+6adfh-3bcfh-5bdeh+3bgdf)\sqrt{fx+e}}{3f^{3}d^{2}}+\frac{\frac{2\left(a^{2}e\,f^{2}h-a^{2}f^{3}g-2ab\,e^{2}fh+2abe\,f^{2}g+b^{2}e^{3}h-b^{2}e^{2}fg\right)d^{2}}{(cf-de)\sqrt{fx+e}}+\frac{2f^{3}\left(a^{2}c\cdots\right.}{\cdots}}{f\cdots}$
derivativedivides	$\frac{\frac{2b\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-2bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^{2}}-\frac{2\left(-a^{2}e\,f^{2}h+a^{2}f^{3}g+2ab\,e^{2}fh-2abe\,f^{2}g-b^{2}e^{3}h+\cdots\right.}{(cf-de)\sqrt{fx+e}}\cdots}{f^{3}}$
default	$\frac{\frac{2b\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+2adfh\sqrt{fx+e}-bcfh\sqrt{fx+e}-2bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^{2}}-\frac{2\left(-a^{2}e\,f^{2}h+a^{2}f^{3}g+2ab\,e^{2}fh-2abe\,f^{2}g-b^{2}e^{3}h+\cdots\right.}{(cf-de)\sqrt{fx+e}}\cdots}{f^{3}}$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^3*(1/3*b*(f*x+e)^(1/2)*(b*d*f*h*x+6*a*d*f*h-3*b*c*f*h-5*b*d*e*h+3*b*d*
f*g)/d^2+(a^2*e*f^2*h-a^2*f^3*g-2*a*b*e^2*f*h+2*a*b*e*f^2*g+b^2*e^3*h-b^2*
e^2*f*g)/(c*f-d*e)/(f*x+e)^(1/2)+f^3*(a^2*c*d^2*h-a^2*d^3*g-2*a*b*c^2*d*h+
2*a*b*c*d^2*g+b^2*c^3*h-b^2*c^2*d*g)/(c*f-d*e)/d^2/((c*f-d*e)*d)^(1/2)*arc
tan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(155) = 310.

Time = 0.11 (sec) , antiderivative size = 1262, normalized size of antiderivative = 7.29

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*e*f^3*g - (b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*e*f^3*h + ((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^4*g - (b
^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*f^4*h)*x)*sqrt(d^2*e - c*d*f)*log((d*f*x
 + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*((b^2
*d^4*e^2*f^2 - 2*b^2*c*d^3*e*f^3 + b^2*c^2*d^2*f^4)*h*x^2 + 3*(2*b^2*d^4*e
^3*f - a^2*c*d^3*f^4 - (3*b^2*c*d^3 + 2*a*b*d^4)*e^2*f^2 + (b^2*c^2*d^2 +
2*a*b*c*d^3 + a^2*d^4)*e*f^3)*g - (8*b^2*d^4*e^4 - 2*(5*b^2*c*d^3 + 6*a*b*
d^4)*e^3*f - (b^2*c^2*d^2 - 18*a*b*c*d^3 - 3*a^2*d^4)*e^2*f^2 + 3*(b^2*c^3
*d - 2*a*b*c^2*d^2 - a^2*c*d^3)*e*f^3)*h + (3*(b^2*d^4*e^2*f^2 - 2*b^2*c*d
^3*e*f^3 + b^2*c^2*d^2*f^4)*g - (4*b^2*d^4*e^3*f - (5*b^2*c*d^3 + 6*a*b*d^
4)*e^2*f^2 - 2*(b^2*c^2*d^2 - 6*a*b*c*d^3)*e*f^3 + 3*(b^2*c^3*d - 2*a*b*c^
2*d^2)*f^4)*h)*x)*sqrt(f*x + e))/(d^5*e^3*f^3 - 2*c*d^4*e^2*f^4 + c^2*d^3*
e*f^5 + (d^5*e^2*f^4 - 2*c*d^4*e*f^5 + c^2*d^3*f^6)*x), 2/3*(3*((b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*e*f^3*g - (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*e*
f^3*h + ((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f^4*g - (b^2*c^3 - 2*a*b*c^2*
d + a^2*c*d^2)*f^4*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)*
sqrt(f*x + e)/(d*f*x + d*e)) + ((b^2*d^4*e^2*f^2 - 2*b^2*c*d^3*e*f^3 + b^2
*c^2*d^2*f^4)*h*x^2 + 3*(2*b^2*d^4*e^3*f - a^2*c*d^3*f^4 - (3*b^2*c*d^3 +
2*a*b*d^4)*e^2*f^2 + (b^2*c^2*d^2 + 2*a*b*c*d^3 + a^2*d^4)*e*f^3)*g - (8*b
^2*d^4*e^4 - 2*(5*b^2*c*d^3 + 6*a*b*d^4)*e^3*f - (b^2*c^2*d^2 - 18*a*b*...
```

Sympy [A] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b^2h(e+fx)^{\frac{3}{2}}}{3df^2} + \frac{(af-be)^2(eh-fg)}{f^2\sqrt{e+fx}(cf-de)} + \frac{\sqrt{e+fx}\left(2abdfh-b^2cfh-2b^2deh+b^2dfg\right)}{d^2f^2} + \frac{f(ad-bc)^2(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+\cdots}}{\sqrt{cf\cdots}}\right.}{d^3\sqrt{\frac{cf-de}{d}}(cf-de)\cdots}\right.}{f} \\ \frac{\frac{b^2hx^3}{3d} + \frac{x^2\cdot\left(2abdh-b^2ch+b^2dg\right)}{2d^2} + \frac{x\left(a^2d^2h-2abcdh+2abd^2g+b^2c^2h-b^2cdg\right)}{d^3} - \frac{(ad-bc)^2(ch-dg)\left(\left(\begin{cases}\frac{x}{c} \\ \frac{\log(c+dx\cdots}{d}\end{cases}\right.\right.}{d^3\cdots}}{e^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(h*x+g)/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b**2*h*(e + f*x)**(3/2)/(3*d*f**2) + (a*f - b*e)**2*(e*h - f
*g)/(f**2*sqrt(e + f*x)*(c*f - d*e)) + sqrt(e + f*x)*(2*a*b*d*f*h - b**2*c
*f*h - 2*b**2*d*e*h + b**2*d*f*g)/(d**2*f**2) + f*(a*d - b*c)**2*(c*h - d*
g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d**3*sqrt((c*f - d*e)/d)*(c*f
- d*e)))/f, Ne(f, 0)), ((b**2*h*x**3/(3*d) + x**2*(2*a*b*d*h - b**2*c*h +
b**2*d*g)/(2*d**2) + x*(a**2*d**2*h - 2*a*b*c*d*h + 2*a*b*d**2*g + b**2*c*
*2*h - b**2*c*d*g)/d**3 - (a*d - b*c)**2*(c*h - d*g)*Piecewise((x/c, Eq(d,
 0)), (log(c + d*x)/d, True))/d**3)/e**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\,(b^2c^2dg-2\,abcd^2g+a^2d^3g-b^2c^3h+2\,abc^2dh-a^2cd^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(d^3e-cd^2f)\sqrt{-d^2e+cdf}} + \frac{2\,(b^2e^2fg-2\,abef^2g+a^2f^3g-b^2e^3h+2\,abe^2fh-a^2ef^2h)}{(def^3-cf^4)\sqrt{fx+e}} + \frac{2\left(3\,\sqrt{fx+e}b^2d^2f^7g+(fx+e)^{\frac{3}{2}}b^2d^2f^6h-6\,\sqrt{fx+e}b^2d^2ef^6h-3\,\sqrt{fx+e}b^2cdf^7h+6\,\sqrt{fx+e}abd^2f\right.}{3\,d^3f^9}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^2*c^2*d*g - 2*a*b*c*d^2*g + a^2*d^3*g - b^2*c^3*h + 2*a*b*c^2*d*h - a
^2*c*d^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^3*e - c*d^2*f
)*sqrt(-d^2*e + c*d*f)) + 2*(b^2*e^2*f*g - 2*a*b*e*f^2*g + a^2*f^3*g - b^2
*e^3*h + 2*a*b*e^2*f*h - a^2*e*f^2*h)/((d*e*f^3 - c*f^4)*sqrt(f*x + e)) +
2/3*(3*sqrt(f*x + e)*b^2*d^2*f^7*g + (f*x + e)^(3/2)*b^2*d^2*f^6*h - 6*sqr
t(f*x + e)*b^2*d^2*e*f^6*h - 3*sqrt(f*x + e)*b^2*c*d*f^7*h + 6*sqrt(f*x +
e)*a*b*d^2*f^7*h)/(d^3*f^9)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,b^2\,f\,g-6\,b^2\,e\,h+4\,a\,b\,f\,h}{d\,f^3} - \frac{2\,b^2\,h\,(c\,f^4-d\,e\,f^3)}{d^2\,f^6}\right) + \frac{2\,b^2\,h\,(e+f\,x)^{3/2}}{3\,d\,f^3} + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{e+f\,x}\,(a\,d-b\,c)^2\,(c\,h-d\,g)\,(d^3\,e-c\,d^2\,f)}{d^{3/2}\,(c\,f-d\,e)^{3/2}\,(-2\,h\,a^2\,c\,d^2+2\,g\,a^2\,d^3+4\,h\,a\,b\,c^2\,d-4\,g\,a\,b\,c\,d^2-2\,h\,b^2\,c^3+2\,g\,b^2\,c^2\,d)}\right)\,(a\,d-b\,c)^2\,(c\,h-d\,g)}{d^{5/2}\,(c\,f-d\,e)^{3/2}} - \frac{2\,(-h\,a^2\,d^2\,e\,f^2+g\,a^2\,d^2\,f^3+2\,h\,a\,b\,d^2\,e^2\,f-2\,g\,a\,b\,d^2\,e\,f^2-h\,b^2\,d^2\,e^3+g\,b^2\,d^2\,e^2\,f)}{d^2\,f^3\,\sqrt{e+f\,x}\,(c\,f-d\,e)}$$

input
```
int(((g + h*x)*(a + b*x)^2)/((e + f*x)^(3/2)*(c + d*x)),x)
```

output
```
(e + f*x)^(1/2)*((2*b^2*f*g - 6*b^2*e*h + 4*a*b*f*h)/(d*f^3) - (2*b^2*h*(c
*f^4 - d*e*f^3))/(d^2*f^6)) + (2*b^2*h*(e + f*x)^(3/2))/(3*d*f^3) + (2*ata
n((2*(e + f*x)^(1/2)*(a*d - b*c)^2*(c*h - d*g)*(d^3*e - c*d^2*f))/(d^(3/2)
*(c*f - d*e)^(3/2)*(2*a^2*d^3*g - 2*b^2*c^3*h - 2*a^2*c*d^2*h + 2*b^2*c^2*
d*g - 4*a*b*c*d^2*g + 4*a*b*c^2*d*h)))*(a*d - b*c)^2*(c*h - d*g))/(d^(5/2)
*(c*f - d*e)^(3/2)) - (2*(a^2*d^2*f^3*g - b^2*d^2*e^3*h - a^2*d^2*e*f^2*h
+ b^2*d^2*e^2*f*g - 2*a*b*d^2*e*f^2*g + 2*a*b*d^2*e^2*f*h))/(d^2*f^3*(e +
f*x)^(1/2)*(c*f - d*e))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 790, normalized size of antiderivative = 4.57

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2a^2c\,d^3e\,f^3h + 8ab\,d^4e^3fh - 4ab\,d^4e^2f^2g - 2b^2c^3de\,f^3h - 2b^2c^3d\,f^4hx + 2b^2c^2}{}$$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x)
```

output
```
(2*(3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a**2*c*d**2*f**3*h - 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**3*f**3*g
- 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a*b*c**2*d*f**3*h + 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**2*f**3*g +
3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sq
rt(c*f - d*e)))*b**2*c**3*f**3*h - 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)
*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*c**2*d*f**3*g + 3*
a**2*c*d**3*e*f**3*h - 3*a**2*c*d**3*f**4*g - 3*a**2*d**4*e**2*f**2*h + 3*
a**2*d**4*e*f**3*g + 6*a*b*c**2*d**2*e*f**3*h + 6*a*b*c**2*d**2*f**4*h*x -
 18*a*b*c*d**3*e**2*f**2*h + 6*a*b*c*d**3*e*f**3*g - 12*a*b*c*d**3*e*f**3*
h*x + 12*a*b*d**4*e**3*f*h - 6*a*b*d**4*e**2*f**2*g + 6*a*b*d**4*e**2*f**2
*h*x - 3*b**2*c**3*d*e*f**3*h - 3*b**2*c**3*d*f**4*h*x + b**2*c**2*d**2*e*
*2*f**2*h + 3*b**2*c**2*d**2*e*f**3*g + 2*b**2*c**2*d**2*e*f**3*h*x + 3*b*
*2*c**2*d**2*f**4*g*x + b**2*c**2*d**2*f**4*h*x**2 + 10*b**2*c*d**3*e**3*f
*h - 9*b**2*c*d**3*e**2*f**2*g + 5*b**2*c*d**3*e**2*f**2*h*x - 6*b**2*c*d*
*3*e*f**3*g*x - 2*b**2*c*d**3*e*f**3*h*x**2 - 8*b**2*d**4*e**4*h + 6*b**2*
d**4*e**3*f*g - 4*b**2*d**4*e**3*f*h*x + 3*b**2*d**4*e**2*f**2*g*x + b**2*
d**4*e**2*f**2*h*x**2))/(3*sqrt(e + f*x)*d**3*f**3*(c**2*f**2 - 2*c*d*e...
```

3.170 $\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 123

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = -\frac{2(be-af)(fg-eh)}{f^2(de-cf)\sqrt{e+fx}} + \frac{2bh\sqrt{e+fx}}{df^2} + \frac{2(bc-ad)(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(de-cf)^{3/2}}$$

output
```
-2*(-a*f+b*e)*(-e*h+f*g)/f^2/(-c*f+d*e)/(f*x+e)^(1/2)+2*b*h*(f*x+e)^(1/2)/
d/f^2+2*(-a*d+b*c)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/
2))/d^(3/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2(adf(-fg+eh)+bcfh(e+fx)+bde(-2eh+f(g-hx)))}{df^2(-de+cf)\sqrt{e+fx}} - \frac{2(-bc+ad)(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{3/2}(-de+cf)^{3/2}}$$

input
```
Integrate[((a + b*x)*(g + h*x))/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(a*d*f*(-(f*g) + e*h) + b*c*f*h*(e + f*x) + b*d*e*(-2*e*h + f*(g - h*x)
)))/(d*f^2*(-(d*e) + c*f)*Sqrt[e + f*x]) - (2*(-(b*c) + a*d)*(d*g - c*h)*A
rcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(3/2)*(-(d*e) + c*f)
^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{ 163}$$

$$\frac{2\left(-ef(adh+bch+bdg)+adf^2g+bfhx(de-cf)+2bde^2h\right)}{df^2\sqrt{e+fx}(de-cf)}-\frac{(bc-ad)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d(de-cf)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\left(-ef(adh+bch+bdg)+adf^2g+bfhx(de-cf)+2bde^2h\right)}{df^2\sqrt{e+fx}(de-cf)}-\frac{2(bc-ad)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df(de-cf)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2(bc-ad)(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(de-cf)^{3/2}}+\frac{2\left(-ef(adh+bch+bdg)+adf^2g+bfhx(de-cf)+2bde^2h\right)}{df^2\sqrt{e+fx}(de-cf)}$$

input
```
Int[((a + b*x)*(g + h*x))/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(a*d*f^2*g + 2*b*d*e^2*h - e*f*(b*d*g + b*c*h + a*d*h) + b*f*(d*e - c*f
)*h*x))/(d*f^2*(d*e - c*f)*Sqrt[e + f*x]) + (2*(b*c - a*d)*(d*g - c*h)*Arc
Tanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(3/2)*(d*e - c*f)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{2hb\sqrt{fx+e}}{d}-\frac{2\left(-afhe+g\,f^{2}a+b\,e^{2}h-bfge\right)}{(cf-de)\sqrt{fx+e}}+\frac{2f^{2}\left(acdh-a\,d^{2}g-b\,c^{2}h+bcdg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(cf-de)d\sqrt{(cf-de)d}}}{f^{2}}$
default	$\frac{\frac{2hb\sqrt{fx+e}}{d}-\frac{2\left(-afhe+g\,f^{2}a+b\,e^{2}h-bfge\right)}{(cf-de)\sqrt{fx+e}}+\frac{2f^{2}\left(acdh-a\,d^{2}g-b\,c^{2}h+bcdg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(cf-de)d\sqrt{(cf-de)d}}}{f^{2}}$
pseudoelliptic	$\frac{2\sqrt{fx+e}\,f^{2}(ch-dg)(ad-bc)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+2((-g\,f^{2}a+((-hx+g)b+ah)ef-2b\,e^{2}h)d+bcfh(fx+e))\sqrt{(cf-d}}{f^{2}d(cf-de)\sqrt{fx+e}\,\sqrt{(cf-de)d}}$
risch	$\frac{2bh\sqrt{fx+e}}{d\,f^{2}}+\frac{\frac{2\left(afhe-g\,f^{2}a-b\,e^{2}h+bfge\right)d}{(cf-de)\sqrt{fx+e}}+\frac{2f^{2}\left(acdh-a\,d^{2}g-b\,c^{2}h+bcdg\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(cf-de)\sqrt{(cf-de)d}}}{f^{2}d}$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^2*(h*b/d*(f*x+e)^(1/2)-(-a*e*f*h+a*f^2*g+b*e^2*h-b*e*f*g)/(c*f-d*e)/(f
*x+e)^(1/2)+f^2*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/(c*f-d*e)/d/((c*f-d*e)*d
)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(109) = 218.

Time = 0.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 5.34

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \left[-\frac{((bcd-ad^2)ef^2g-(bc^2-acd)ef^2h+((bcd-ad^2)f^3g-(bc^2-acd)f^3h)}{}\right.$$

$$-\frac{2\left(((bcd-ad^2)ef^2g-(bc^2-acd)ef^2h+((bcd-ad^2)f^3g-(bc^2-acd)f^3h)x\right)\sqrt{-d^2e+cdf}\arctan\left(\frac{\sqrt{-}}{}\right.}{d^4e^3f}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-((((b*c*d - a*d^2)*e*f^2*g - (b*c^2 - a*c*d)*e*f^2*h + ((b*c*d - a*d^2)*f
^3*g - (b*c^2 - a*c*d)*f^3*h)*x)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e -
c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*((b*d^3*e^2*f -
2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*x - (b*d^3*e^2*f + a*c*d^2*f^3 - (b*c*d^2
 + a*d^3)*e*f^2)*g + (2*b*d^3*e^3 - (3*b*c*d^2 + a*d^3)*e^2*f + (b*c^2*d +
 a*c*d^2)*e*f^2)*h)*sqrt(f*x + e))/(d^4*e^3*f^2 - 2*c*d^3*e^2*f^3 + c^2*d^
2*e*f^4 + (d^4*e^2*f^3 - 2*c*d^3*e*f^4 + c^2*d^2*f^5)*x), -2*((((b*c*d - a*
d^2)*e*f^2*g - (b*c^2 - a*c*d)*e*f^2*h + ((b*c*d - a*d^2)*f^3*g - (b*c^2 -
 a*c*d)*f^3*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)*sqrt(f*
x + e)/(d*f*x + d*e)) - ((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*x
 - (b*d^3*e^2*f + a*c*d^2*f^3 - (b*c*d^2 + a*d^3)*e*f^2)*g + (2*b*d^3*e^3
- (3*b*c*d^2 + a*d^3)*e^2*f + (b*c^2*d + a*c*d^2)*e*f^2)*h)*sqrt(f*x + e))
/(d^4*e^3*f^2 - 2*c*d^3*e^2*f^3 + c^2*d^2*e*f^4 + (d^4*e^2*f^3 - 2*c*d^3*e
*f^4 + c^2*d^2*f^5)*x)]
```

Sympy [A] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{bh\sqrt{e+fx}}{df}+\frac{(af-be)(eh-fg)}{f\sqrt{e+fx}(cf-de)}+\frac{f(ad-bc)(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d^2\sqrt{\frac{cf-de}{d}}(cf-de)}\right)}{f} & \text{for } f\neq 0 \\ \frac{\frac{bhx^2}{2d}+\frac{x(adh-bch+bdg)}{d^2}-\frac{(ad-bc)(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d^2}}{e^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(b*h*sqrt(e + f*x)/(d*f) + (a*f - b*e)*(e*h - f*g)/(f*sqrt(e
+ f*x)*(c*f - d*e)) + f*(a*d - b*c)*(c*h - d*g)*atan(sqrt(e + f*x)/sqrt((c
*f - d*e)/d))/(d**2*sqrt((c*f - d*e)/d)*(c*f - d*e)))/f, Ne(f, 0)), ((b*h*
x**2/(2*d) + x*(a*d*h - b*c*h + b*d*g)/d**2 - (a*d - b*c)*(c*h - d*g)*Piec
ewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/d**2)/e**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = -\frac{2\,(bcdg-ad^2g-bc^2h+acdh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(d^2e-cdf)\sqrt{-d^2e+cdf}} - \frac{2\,(befg-af^2g-be^2h+aefh)}{(def^2-cf^3)\sqrt{fx+e}} + \frac{2\,\sqrt{fx+e}bh}{df^2}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-2*(b*c*d*g - a*d^2*g - b*c^2*h + a*c*d*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^
2*e + c*d*f))/((d^2*e - c*d*f)*sqrt(-d^2*e + c*d*f)) - 2*(b*e*f*g - a*f^2*
g - b*e^2*h + a*e*f*h)/((d*e*f^2 - c*f^3)*sqrt(f*x + e)) + 2*sqrt(f*x + e)
*b*h/(d*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\,b\,h\,\sqrt{e+f\,x}}{d\,f^2} - \frac{2\,(a\,d\,f^2\,g+b\,d\,e^2\,h-a\,d\,e\,f\,h-b\,d\,e\,f\,g)}{d\,f^2\,\sqrt{e+f\,x}\,(c\,f-d\,e)} + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{e+f\,x}\,(d^2\,e-c\,d\,f)\,(a\,d-b\,c)\,(c\,h-d\,g)}{\sqrt{d}\,(c\,f-d\,e)^{3/2}\,(2\,a\,d^2\,g+2\,b\,c^2\,h-2\,a\,c\,d\,h-2\,b\,c\,d\,g)}\right)\,(a\,d-b\,c)\,(c\,h-d\,g)}{d^{3/2}\,(c\,f-d\,e)^{3/2}}$$

input
```
int(((g + h*x)*(a + b*x))/((e + f*x)^(3/2)*(c + d*x)),x)
```

output
```
(2*b*h*(e + f*x)^(1/2))/(d*f^2) - (2*(a*d*f^2*g + b*d*e^2*h - a*d*e*f*h -
b*d*e*f*g))/(d*f^2*(e + f*x)^(1/2)*(c*f - d*e)) + (2*atan((2*(e + f*x)^(1/
2)*(d^2*e - c*d*f)*(a*d - b*c)*(c*h - d*g))/(d^(1/2)*(c*f - d*e)^(3/2)*(2*
a*d^2*g + 2*b*c^2*h - 2*a*c*d*h - 2*b*c*d*g)))*(a*d - b*c)*(c*h - d*g))/(d
^(3/2)*(c*f - d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.03

$$\int \frac{(a+bx)(g+hx)}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{fx+e}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)acd\,f^2h-2\sqrt{d}\,\sqrt{fx+e}\,\sqrt{cf-de}\,a}{}$$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)/(f*x+e)^(3/2),x)
```

output

```
(2*(sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a*c*d*f**2*h - sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*ata
n((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**2*f**2*g - sqrt(d)*sqr
t(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)
))*b*c**2*f**2*h + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d*f**2*g + a*c*d**2*e*f**2*h - a*c*d*
*2*f**3*g - a*d**3*e**2*f*h + a*d**3*e*f**2*g + b*c**2*d*e*f**2*h + b*c**2
*d*f**3*h*x - 3*b*c*d**2*e**2*f*h + b*c*d**2*e*f**2*g - 2*b*c*d**2*e*f**2*
h*x + 2*b*d**3*e**3*h - b*d**3*e**2*f*g + b*d**3*e**2*f*h*x))/(sqrt(e + f*
x)*d**2*f**2*(c**2*f**2 - 2*c*d*e*f + d**2*e**2))
```

3.171 $\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2(fg-eh)}{f(de-cf)\sqrt{e+fx}} - \frac{2(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(de-cf)^{3/2}}$$

output
```
2*(-e*h+f*g)/f/(-c*f+d*e)/(f*x+e)^(1/2)-2*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+
e)^(1/2)/(-c*f+d*e)^(1/2))/d^(1/2)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{-2fg+2eh}{f(-de+cf)\sqrt{e+fx}} - \frac{2(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-de+cf)^{3/2}}$$

input
```
Integrate[(g + h*x)/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(-2*f*g + 2*e*h)/(f*(-(d*e) + c*f)*Sqrt[e + f*x]) - (2*(d*g - c*h)*ArcTan[
(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sqrt[d]*(-(d*e) + c*f)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 87$$

$$\frac{(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{de-cf}+\frac{2(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{2(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(de-cf)}+\frac{2(fg-eh)}{f\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{2(fg-eh)}{f\sqrt{e+fx}(de-cf)}-\frac{2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(de-cf)^{3/2}}$$

input
```
Int[(g + h*x)/((c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(2*(f*g - e*h))/(f*(d*e - c*f)*Sqrt[e + f*x]) - (2*(d*g - c*h)*ArcTanh[(Sq
rt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(d*e - c*f)^(3/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{\frac{2(eh-fg)}{\sqrt{fx+e}}+\frac{2f(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{(cf-de)f}$	77
derivativedivides	$\frac{-\frac{2(-eh+fg)}{(cf-de)\sqrt{fx+e}}+\frac{2f(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(cf-de)\sqrt{(cf-de)d}}}{f}$	88
default	$\frac{-\frac{2(-eh+fg)}{(cf-de)\sqrt{fx+e}}+\frac{2f(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(cf-de)\sqrt{(cf-de)d}}}{f}$	88

input
```
int((h*x+g)/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/(c*f-d*e)*((e*h-f*g)/(f*x+e)^(1/2)+f*(c*h-d*g)/((c*f-d*e)*d)^(1/2)*arc
tan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.28

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \left[\frac{(defg-cefh+(df^2g-cf^2h)x)\sqrt{d^2e-cdf}\log\left(\frac{dfx+2\,de-cf-2\,\sqrt{d^2e-cdf}\sqrt{fx+e}}{dx+c}\right.}{d^3e^3f-2\,cd^2e^2f^2+c^2def^3+(d^3e^2f^2-2\,c\right.$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[((d*e*f*g - c*e*f*h + (d*f^2*g - c*f^2*h)*x)*sqrt(d^2*e - c*d*f)*log((d*f
*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*((d
^2*e*f - c*d*f^2)*g - (d^2*e^2 - c*d*e*f)*h)*sqrt(f*x + e))/(d^3*e^3*f - 2
*c*d^2*e^2*f^2 + c^2*d*e*f^3 + (d^3*e^2*f^2 - 2*c*d^2*e*f^3 + c^2*d*f^4)*x
), 2*((d*e*f*g - c*e*f*h + (d*f^2*g - c*f^2*h)*x)*sqrt(-d^2*e + c*d*f)*arc
tan(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) + ((d^2*e*f - c*d*f^
2)*g - (d^2*e^2 - c*d*e*f)*h)*sqrt(f*x + e))/(d^3*e^3*f - 2*c*d^2*e^2*f^2
+ c^2*d*e*f^3 + (d^3*e^2*f^2 - 2*c*d^2*e*f^3 + c^2*d*f^4)*x)]
```

Sympy [A] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{eh-fg}{\sqrt{e+fx}(cf-de)}+\frac{f(ch-dg)\,\mathrm{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{d\sqrt{\frac{cf-de}{d}}(cf-de)}\right)}{f} & \text{for } f\neq 0 \\ \frac{\frac{hx}{d}-\frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{d}}{e^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*((e*h - f*g)/(sqrt(e + f*x)*(c*f - d*e)) + f*(c*h - d*g)*atan
(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(d*sqrt((c*f - d*e)/d)*(c*f - d*e)))/f
, Ne(f, 0)), ((h*x/d - (c*h - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x
)/d, True)))/d)/e**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\,(dg-ch)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{\sqrt{-d^2e+cdf}\,(de-cf)} + \frac{2\,(fg-eh)}{(def-cf^2)\sqrt{fx+e}}$$

input
```
integrate((h*x+g)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(d*g - c*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/(sqrt(-d^2*e +
c*d*f)*(d*e - c*f)) + 2*(f*g - e*h)/((d*e*f - c*f^2)*sqrt(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\,(e\,h-f\,g)}{f\,\sqrt{e+f\,x}\,\left(c\,f-d\,e\right)}+\frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{d}\,\sqrt{e+f\,x}\,(c\,h-d\,g)}{\sqrt{c\,f-d\,e}\,(2\,c\,h-2\,d\,g)}\right)\,(c\,h-d\,g)}{\sqrt{d}\,{\left(c\,f-d\,e\right)}^{3/2}}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(c + d*x)),x)
```

output
```
(2*(e*h - f*g))/(f*(e + f*x)^(1/2)*(c*f - d*e)) + (2*atan((2*d^(1/2)*(e +
f*x)^(1/2)*(c*h - d*g))/((c*f - d*e)^(1/2)*(2*c*h - 2*d*g)))*(c*h - d*g))/
(d^(1/2)*(c*f - d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.83

$$\int \frac{g+hx}{(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{fx+e}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)cfh-2\sqrt{d}\,\sqrt{fx+e}\,\sqrt{cf-de}\,atan\left(\right.}{\sqrt{fx+e}\,df\,(c^2f^2-2cdef+}$$

input
```
int((h*x+g)/(d*x+c)/(f*x+e)^(3/2),x)
```

output
```
(2*(sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*c*f*h - sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d*f*g + c*d*e*f*h - c*d*f**2*g - d
**2*e**2*h + d**2*e*f*g))/(sqrt(e + f*x)*d*f*(c**2*f**2 - 2*c*d*e*f + d**2
*e**2))
```

3.172 $\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 170

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = -\frac{2(fg-eh)}{(be-af)(de-cf)\sqrt{e+fx}} - \frac{2\sqrt{b}(bg-ah)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)(be-af)^{3/2}} + \frac{2\sqrt{d}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)(de-cf)^{3/2}}$$

output
```
(2*e*h-2*f*g)/(-a*f+b*e)/(-c*f+d*e)/(f*x+e)^(1/2)-2*b^(1/2)*(-a*h+b*g)*arc
tanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(3/2)+2
*d^(1/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+
b*c)/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = 2\left(\frac{-fg+eh}{(be-af)(de-cf)\sqrt{e+fx}} + \frac{\sqrt{b}(bg-ah)\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{(-bc+ad)(-be+af)^{3/2}} + \frac{\sqrt{d}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(bc-ad)(-de+cf)^{3/2}}\right)$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
2*((-(f*g) + e*h)/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) + (Sqrt[b]*(b*g
- a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/((-(b*c) + a*d)
*(-(b*e) + a*f)^(3/2)) + (Sqrt[d]*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x
])/Sqrt[-(d*e) + c*f]])/((b*c - a*d)*(-(d*e) + c*f)^(3/2)))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}} \, dx$$

$$\Big\downarrow \text{169}$$

$$-\frac{2\int -\frac{bdeg-bcfg-adfg+acfh-bd(fg-eh)x}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{(be-af)(de-cf)}-\frac{2(fg-eh)}{\sqrt{e+fx}(be-af)(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(de-cf)g-af(dg-ch)-bd(fg-eh)x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{(be-af)(de-cf)}-\frac{2(fg-eh)}{\sqrt{e+fx}(be-af)(de-cf)}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{b(bg-ah)(de-cf)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{d(be-af)(dg-ch)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{(be-af)(de-cf)}-\frac{2(fg-eh)}{\sqrt{e+fx}(be-af)(de-cf)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2b(bg-ah)(de-cf)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2d(be-af)(dg-ch)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{(be-af)(de-cf)}-\frac{2(fg-eh)}{\sqrt{e+fx}(be-af)(de-cf)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\sqrt{d}(be-af)(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)\sqrt{de-cf}}-\frac{2\sqrt{b}(bg-ah)(de-cf)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)\sqrt{be-af}}}{(be-af)(de-cf)}-\frac{2(fg-eh)}{\sqrt{e+fx}(be-af)(de-cf)}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(-2*(f*g - e*h))/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) + ((-2*Sqrt[b]*(d
*e - c*f)*(b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((
b*c - a*d)*Sqrt[b*e - a*f]) + (2*Sqrt[d]*(b*e - a*f)*(d*g - c*h)*ArcTanh[(
Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/((
b*e - a*f)*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$-\frac{2b(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)(af-be)\sqrt{(af-be)b}}-\frac{2(-eh+fg)}{(af-be)(cf-de)\sqrt{fx+e}}+\frac{2d(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)(cf-de)\sqrt{(cf-de)d}}$	169
default	$-\frac{2b(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)(af-be)\sqrt{(af-be)b}}-\frac{2(-eh+fg)}{(af-be)(cf-de)\sqrt{fx+e}}+\frac{2d(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)(cf-de)\sqrt{(cf-de)d}}$	169
pseudoelliptic	$-\frac{2b(ah-bg)\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)}{(ad-bc)(af-be)\sqrt{(af-be)b}}+\frac{2eh-2fg}{(af-be)(cf-de)\sqrt{fx+e}}+\frac{2d(ch-dg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)(cf-de)\sqrt{(cf-de)d}}$	169

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(a*h-b*g)/(a*d-b*c)/(a*f-b*e)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1
/2)/((a*f-b*e)*b)^(1/2))-2*(-e*h+f*g)/(a*f-b*e)/(c*f-d*e)/(f*x+e)^(1/2)+2*
d*(c*h-d*g)/(a*d-b*c)/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)
/((c*f-d*e)*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(148) = 296$.

Time = 20.21 (sec) , antiderivative size = 1604, normalized size of antiderivative = 9.44

$$\int \frac{g + hx}{(a + bx)(c + dx)(e + fx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-((((b*d*e^2 - b*c*e*f)*g - (a*d*e^2 - a*c*e*f)*h + ((b*d*e*f - b*c*f^2)*g
 - (a*d*e*f - a*c*f^2)*h)*x)*sqrt(b/(b*e - a*f))*log((b*f*x + 2*b*e - a*f
+ 2*(b*e - a*f)*sqrt(f*x + e)*sqrt(b/(b*e - a*f)))/(b*x + a)) + ((b*d*e^2
- a*d*e*f)*g - (b*c*e^2 - a*c*e*f)*h + ((b*d*e*f - a*d*f^2)*g - (b*c*e*f -
 a*c*f^2)*h)*x)*sqrt(d/(d*e - c*f))*log((d*f*x + 2*d*e - c*f - 2*(d*e - c*
f)*sqrt(f*x + e)*sqrt(d/(d*e - c*f)))/(d*x + c)) + 2*((b*c - a*d)*f*g - (b
*c - a*d)*e*h)*sqrt(f*x + e))/((b^2*c*d - a*b*d^2)*e^3 - (b^2*c^2 - a^2*d^
2)*e^2*f + (a*b*c^2 - a^2*c*d)*e*f^2 + ((b^2*c*d - a*b*d^2)*e^2*f - (b^2*c
^2 - a^2*d^2)*e*f^2 + (a*b*c^2 - a^2*c*d)*f^3)*x), -(2*((b*d*e^2 - a*d*e*f
)*g - (b*c*e^2 - a*c*e*f)*h + ((b*d*e*f - a*d*f^2)*g - (b*c*e*f - a*c*f^2)
*h)*x)*sqrt(-d/(d*e - c*f))*arctan(sqrt(f*x + e)*sqrt(-d/(d*e - c*f))) + (
(b*d*e^2 - b*c*e*f)*g - (a*d*e^2 - a*c*e*f)*h + ((b*d*e*f - b*c*f^2)*g - (
a*d*e*f - a*c*f^2)*h)*x)*sqrt(b/(b*e - a*f))*log((b*f*x + 2*b*e - a*f + 2*
(b*e - a*f)*sqrt(f*x + e)*sqrt(b/(b*e - a*f)))/(b*x + a)) + 2*((b*c - a*d)
*f*g - (b*c - a*d)*e*h)*sqrt(f*x + e))/((b^2*c*d - a*b*d^2)*e^3 - (b^2*c^2
 - a^2*d^2)*e^2*f + (a*b*c^2 - a^2*c*d)*e*f^2 + ((b^2*c*d - a*b*d^2)*e^2*f
 - (b^2*c^2 - a^2*d^2)*e*f^2 + (a*b*c^2 - a^2*c*d)*f^3)*x), (2*((b*d*e^2 -
 b*c*e*f)*g - (a*d*e^2 - a*c*e*f)*h + ((b*d*e*f - b*c*f^2)*g - (a*d*e*f -
a*c*f^2)*h)*x)*sqrt(-b/(b*e - a*f))*arctan(sqrt(f*x + e)*sqrt(-b/(b*e - a*
f))) - ((b*d*e^2 - a*d*e*f)*g - (b*c*e^2 - a*c*e*f)*h + ((b*d*e*f - a*d...
```

Sympy [A] (verification not implemented)

Time = 18.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{f(eh-fg)}{\sqrt{e+fx}(af-be)(cf-de)} + \frac{f(ch-dg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{cf-de}{d}}}\right)}{\sqrt{\frac{cf-de}{d}}(ad-bc)(cf-de)} - \frac{f(ah-bg)\operatorname{atan}\left(\frac{\sqrt{e+fx}}{\sqrt{\frac{af-be}{b}}}\right)}{\sqrt{\frac{af-be}{b}}(ad-bc)(af-be)}\right)}{f} & \text{for} \\ \frac{\frac{(ah-bg)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{ad-bc} - \frac{(ch-dg)\left(\begin{cases}\frac{x}{c} & \text{for } d=0\\ \frac{\log(c+dx)}{d} & \text{otherwise}\end{cases}\right)}{ad-bc}}{e^{\frac{3}{2}}} & \text{oth} \end{cases}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Piecewise((2*(f*(e*h - f*g)/(sqrt(e + f*x)*(a*f - b*e)*(c*f - d*e)) + f*(c
*h - d*g)*atan(sqrt(e + f*x)/sqrt((c*f - d*e)/d))/(sqrt((c*f - d*e)/d)*(a*
d - b*c)*(c*f - d*e)) - f*(a*h - b*g)*atan(sqrt(e + f*x)/sqrt((a*f - b*e)/
b))/(sqrt((a*f - b*e)/b)*(a*d - b*c)*(a*f - b*e)))/f, Ne(f, 0)), (((a*h -
b*g)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/(a*d - b*c) - (c*h
 - d*g)*Piecewise((x/c, Eq(d, 0)), (log(c + d*x)/d, True))/(a*d - b*c))/e*
*(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = \frac{2\,(b^2g-abh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^2ce-abde-abcf+a^2df)\sqrt{-b^2e+abf}} - \frac{2\,(d^2g-cdh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(bcde-ad^2e-bc^2f+acdf)\sqrt{-d^2e+cdf}} - \frac{2\,(fg-eh)}{(bde^2-bcef-adef+acf^2)\sqrt{fx+e}}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^2*g - a*b*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^2*c*e -
 a*b*d*e - a*b*c*f + a^2*d*f)*sqrt(-b^2*e + a*b*f)) - 2*(d^2*g - c*d*h)*ar
ctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b*c*d*e - a*d^2*e - b*c^2*f +
 a*c*d*f)*sqrt(-d^2*e + c*d*f)) - 2*(f*g - e*h)/((b*d*e^2 - b*c*e*f - a*d*
e*f + a*c*f^2)*sqrt(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 30821, normalized size of antiderivative = 181.30

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)*(c + d*x)),x)
```

output
```
atan((a^2*b^5*d^5*e^7*(e + f*x)^(1/2)*((b^3*g^2 + a^2*b*h^2 - 2*a*b^2*g*h)
/(b^5*c^2*e^3 - a^5*d^2*f^3 + a^2*b^3*d^2*e^3 - a^3*b^2*c^2*f^3 - 2*a*b^4*
c*d*e^3 + 2*a^4*b*c*d*f^3 - 3*a*b^4*c^2*e^2*f + 3*a^4*b*d^2*e*f^2 + 3*a^2*
b^3*c^2*e*f^2 - 3*a^3*b^2*d^2*e^2*f + 6*a^2*b^3*c*d*e^2*f - 6*a^3*b^2*c*d*
e*f^2))^(3/2)*2i - a^4*b^3*c^5*f^7*(e + f*x)^(1/2)*((b^3*g^2 + a^2*b*h^2 -
 2*a*b^2*g*h)/(b^5*c^2*e^3 - a^5*d^2*f^3 + a^2*b^3*d^2*e^3 - a^3*b^2*c^2*f
^3 - 2*a*b^4*c*d*e^3 + 2*a^4*b*c*d*f^3 - 3*a*b^4*c^2*e^2*f + 3*a^4*b*d^2*e
*f^2 + 3*a^2*b^3*c^2*e*f^2 - 3*a^3*b^2*d^2*e^2*f + 6*a^2*b^3*c*d*e^2*f - 6
*a^3*b^2*c*d*e*f^2))^(3/2)*1i - a^7*c^2*d^3*f^7*(e + f*x)^(1/2)*((b^3*g^2
+ a^2*b*h^2 - 2*a*b^2*g*h)/(b^5*c^2*e^3 - a^5*d^2*f^3 + a^2*b^3*d^2*e^3 -
a^3*b^2*c^2*f^3 - 2*a*b^4*c*d*e^3 + 2*a^4*b*c*d*f^3 - 3*a*b^4*c^2*e^2*f +
3*a^4*b*d^2*e*f^2 + 3*a^2*b^3*c^2*e*f^2 - 3*a^3*b^2*d^2*e^2*f + 6*a^2*b^3*
c*d*e^2*f - 6*a^3*b^2*c*d*e*f^2))^(3/2)*1i + b^7*c^2*d^3*e^7*(e + f*x)^(1/
2)*((b^3*g^2 + a^2*b*h^2 - 2*a*b^2*g*h)/(b^5*c^2*e^3 - a^5*d^2*f^3 + a^2*b
^3*d^2*e^3 - a^3*b^2*c^2*f^3 - 2*a*b^4*c*d*e^3 + 2*a^4*b*c*d*f^3 - 3*a*b^4
*c^2*e^2*f + 3*a^4*b*d^2*e*f^2 + 3*a^2*b^3*c^2*e*f^2 - 3*a^3*b^2*d^2*e^2*f
 + 6*a^2*b^3*c*d*e^2*f - 6*a^3*b^2*c*d*e*f^2))^(3/2)*2i - a^7*d^5*e^2*f^5*
(e + f*x)^(1/2)*((b^3*g^2 + a^2*b*h^2 - 2*a*b^2*g*h)/(b^5*c^2*e^3 - a^5*d^
2*f^3 + a^2*b^3*d^2*e^3 - a^3*b^2*c^2*f^3 - 2*a*b^4*c*d*e^3 + 2*a^4*b*c*d*
f^3 - 3*a*b^4*c^2*e^2*f + 3*a^4*b*d^2*e*f^2 + 3*a^2*b^3*c^2*e*f^2 - 3*a...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 909, normalized size of antiderivative = 5.35

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(3/2),x)
```

output
```
(2*( - sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a*c**2*f**2*h + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*c*d*e*f*h - sqrt(b)
*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a*d**2*e**2*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c**2*f**2*g - 2*sqrt(b)*sqrt(e + f*
x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b*c*d
*e*f*g + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqr
t(b)*sqrt(a*f - b*e)))*b*d**2*e**2*g + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*
e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*f**2*h - sqrt(
d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
- d*e)))*a**2*d*f**2*g - 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqr
t(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*e*f*h + 2*sqrt(d)*sqrt(e +
f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b
*d*e*f*g + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*b**2*c*e**2*h - sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b**2*d*e**2*g + a**
2*c*d*e*f**2*h - a**2*c*d*f**3*g - a**2*d**2*e**2*f*h + a**2*d**2*e*f**2*g
 - a*b*c**2*e*f**2*h + a*b*c**2*f**3*g + a*b*d**2*e**3*h - a*b*d**2*e**2*f
*g + b**2*c**2*e**2*f*h - b**2*c**2*e*f**2*g - b**2*c*d*e**3*h + b**2*c...
```

3.173 $\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 287

$$\begin{aligned}\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = & -\frac{f(b(deg-3cfg+2ceh)+a(2dfg-3deh+cfh))}{(bc-ad)(be-af)^2(de-cf)\sqrt{e+fx}} \\ & -\frac{bg-ah}{(bc-ad)(be-af)(a+bx)\sqrt{e+fx}} \\ & +\frac{\sqrt{b}(3a^2dfh-abf(5dg+ch)+b^2(2deg+3cfg-2ceh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^2(be-af)^{5/2}} \\ & -\frac{2d^{3/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^2(de-cf)^{3/2}}\end{aligned}$$

output
```
-f*(b*(2*c*e*h-3*c*f*g+d*e*g)+a*(c*f*h-3*d*e*h+2*d*f*g))/(-a*d+b*c)/(-a*f+
b*e)^2/(-c*f+d*e)/(f*x+e)^(1/2)-(-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)/(
f*x+e)^(1/2)+b^(1/2)*(3*a^2*d*f*h-a*b*f*(c*h+5*d*g)+b^2*(-2*c*e*h+3*c*f*g+
2*d*e*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^2/(-a
*f+b*e)^(5/2)-2*d^(3/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e
)^(1/2))/(-a*d+b*c)^2/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = \frac{\frac{(bc-ad)(2a^2df(-fg+eh)-b^2deg(e+fx)+abcf(2fg-3eh-fhx)+abd(e^2h-2f^2gx+3efhx)+b}{(be-af)^2(de-cf)(a+bx)\sqrt{e+fx}}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^2*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(((b*c - a*d)*(2*a^2*d*f*(-(f*g) + e*h) - b^2*d*e*g*(e + f*x) + a*b*c*f*(2
*f*g - 3*e*h - f*h*x) + a*b*d*(e^2*h - 2*f^2*g*x + 3*e*f*h*x) + b^2*c*f*(3
*f*g*x + e*(g - 2*h*x))))/((b*e - a*f)^2*(d*e - c*f)*(a + b*x)*Sqrt[e + f*
x]) + (Sqrt[b]*(-3*a^2*d*f*h + a*b*f*(5*d*g + c*h) + b^2*(-2*d*e*g - 3*c*f
*g + 2*c*e*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e)
 + a*f)^(5/2) - (2*d^(3/2)*(d*g - c*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt
[-(d*e) + c*f]])/(-(d*e) + c*f)^(3/2))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int -\frac{af(2dg+ch)-2b\left(deg+\frac{3cfg}{2}-ceh\right)-3df(bg-ah)x}{2(a+bx)(c+dx)(e+fx)^{3/2}}dx}{(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{af(2dg+ch)-b(2deg+3cfg-2ceh)-3df(bg-ah)x}{(a+bx)(c+dx)(e+fx)^{3/2}}dx}{2(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 169

$$-\frac{\frac{2\int \frac{(de-cf)(2deg+3cfg-2ceh)b^2-af\left(-fhc^2-d(2fg+eh)c+4d^2eg\right)b+df(b(deg-3cfg+2ceh)+a(2dfg-3deh+cfh))xb+2a^2df^2(dg-ch)}{2(a+bx)(c+dx)\sqrt{e+fx}}dx}{(be-af)(de-cf)}-\frac{2f(a(cfh-3\ldots}{\sqrt{\ldots}}}{2(bc-ad)(be-af)}$$

$$\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{(de-cf)(2deg+3cfg-2ceh)b^2-af\left(-fhc^2-d(2fg+eh)c+4d^2eg\right)b+df(b(deg-3cfg+2ceh)+a(2dfg-3deh+cfh))xb+2a^2df^2(dg-ch)}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{(be-af)(de-cf)}-\frac{2f(a(cfh-3de\ldots}{\sqrt{e-\ldots}}}{2(bc-ad)(be-af)}$$

$$\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 174

$$-\frac{\frac{\frac{b(de-cf)\left(3a^2dfh-abf(ch+5dg)+b^2(-2ceh+3cfg+2deg)\right)\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{2d^2(be-af)^2(dg-ch)\int\frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{(be-af)(de-cf)}-\frac{2f(a(cfh-3deh+2dfg)+b\ldots}{\sqrt{e+fx}(be-af)\ldots}}{2(bc-ad)(be-af)}$$

$$\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 73

$$-\frac{\frac{\frac{2b(de-cf)\left(3a^2dfh-abf(ch+5dg)+b^2(-2ceh+3cfg+2deg)\right)\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{4d^2(be-af)^2(dg-ch)\int\frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{(be-af)(de-cf)}-\frac{2f(a(cfh-\ldots}{\ldots}}{2(bc-ad)(be-af)}$$

$$\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{4d^{3/2}(be-af)^2(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)\sqrt{de-cf}}-\frac{2\sqrt{b}(de-cf)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(3a^2dfh-abf(ch+5dg)+b^2(-2ceh+3cfg+2deg)\right)}{(bc-ad)\sqrt{be-af}}}{(be-af)(de-cf)}-\frac{2f(a(cfh-3\ldots}{\sqrt{\ldots}}}{2(bc-ad)(be-af)}$$

$$\frac{bg-ah}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^2*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
-((b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)*Sqrt[e + f*x])) + ((-2*f*
(b*(d*e*g - 3*c*f*g + 2*c*e*h) + a*(2*d*f*g - 3*d*e*h + c*f*h)))/((b*e - a
*f)*(d*e - c*f)*Sqrt[e + f*x]) - ((-2*Sqrt[b]*(d*e - c*f)*(3*a^2*d*f*h - a
*b*f*(5*d*g + c*h) + b^2*(2*d*e*g + 3*c*f*g - 2*c*e*h))*ArcTanh[(Sqrt[b]*S
qrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (4*d^(3/2)
*(b*e - a*f)^2*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]
])/((b*c - a*d)*Sqrt[d*e - c*f]))/((b*e - a*f)*(d*e - c*f)))/(2*(b*c - a*d
)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.97

method	result
derivativedivides	$2f\left(\frac{(ch-dg)d^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)^2 f(cf-de)\sqrt{(cf-de)d}}-\frac{b\left(\frac{\left(\frac{1}{2}a^2dfh-\frac{1}{2}abcfh-\frac{1}{2}abdfg+\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{(3a^2dfh-abcfh-5abdfg-2b^2}{}\right.}{(ad-bc)^2(af-be)^2 f}\right.$
default	$2f\left(\frac{(ch-dg)d^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)^2 f(cf-de)\sqrt{(cf-de)d}}-\frac{b\left(\frac{\left(\frac{1}{2}a^2dfh-\frac{1}{2}abcfh-\frac{1}{2}abdfg+\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{(3a^2dfh-abcfh-5abdfg-2b^2}{}\right.}{(ad-bc)^2(af-be)^2 f}\right.$
pseudoelliptic	$\frac{-3\left(\left(a\left(ah-\frac{5bg}{3}\right)f+\frac{2b^2eg}{3}\right)d-\frac{cb((ah-3bg)f+2ehb)}{3}\right)\sqrt{(cf-de)d}\,(bx+a)(cf-de)\sqrt{fx+e}\,b\arctan\left(\frac{b\sqrt{fx+e}}{\sqrt{(af-be)b}}\right)+2\left(\right.}{}$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*((c*h-d*g)*d^2/(a*d-b*c)^2/f/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d*(f
*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))-b/(a*d-b*c)^2/(a*f-b*e)^2/f*((1/2*a^2*d*f
*h-1/2*a*b*c*f*h-1/2*a*b*d*f*g+1/2*b^2*c*f*g)*(f*x+e)^(1/2)/((f*x+e)*b+a*f
-b*e)+1/2*(3*a^2*d*f*h-a*b*c*f*h-5*a*b*d*f*g-2*b^2*c*e*h+3*b^2*c*f*g+2*b^2
*d*e*g)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))-(
-e*h+f*g)/(c*f-d*e)/(a*f-b*e)^2/(f*x+e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**2/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(263) = 526$.

Time = 0.16 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.17

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx =$$

$$-\frac{(2\,b^3deg+3\,b^3cfg-5\,ab^2dfg-2\,b^3ceh-ab^2cfh+3\,a^2bdfh)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^4c^2e^2-2\,ab^3cde^2+a^2b^2d^2e^2-2\,ab^3c^2ef+4\,a^2b^2cdef-2\,a^3bd^2ef+a^2b^2c^2f^2-2\,a^3bcdf^2+a^4d^2f^2)\sqrt{}}$$

$$+\frac{2\,(d^3g-cd^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2de-2\,abcd^2e+a^2d^3e-b^2c^3f+2\,abc^2df-a^2cd^2f)\sqrt{-d^2e+cdf}}$$

$$-\frac{(fx+e)b^2defg-3\,(fx+e)b^2cf^2g+2\,(fx+e)abdf^2g+2\,b^2cef^2g-2\,abdef^2g-2\,abcf^3g+2\,a^2df^3g+}{(b^3cde^3-ab^2d^2e^3-b^3c^2e^2f-ab^2cde^2f+2\,a^2bd^2e^2f+2\,ab^2c^2ef^2-a^2bc}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-(2*b^3*d*e*g + 3*b^3*c*f*g - 5*a*b^2*d*f*g - 2*b^3*c*e*h - a*b^2*c*f*h +
3*a^2*b*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^4*c^2*e^2
- 2*a*b^3*c*d*e^2 + a^2*b^2*d^2*e^2 - 2*a*b^3*c^2*e*f + 4*a^2*b^2*c*d*e*f
- 2*a^3*b*d^2*e*f + a^2*b^2*c^2*f^2 - 2*a^3*b*c*d*f^2 + a^4*d^2*f^2)*sqrt(
-b^2*e + a*b*f)) + 2*(d^3*g - c*d^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e
+ c*d*f))/((b^2*c^2*d*e - 2*a*b*c*d^2*e + a^2*d^3*e - b^2*c^3*f + 2*a*b*c^
2*d*f - a^2*c*d^2*f)*sqrt(-d^2*e + c*d*f)) - ((f*x + e)*b^2*d*e*f*g - 3*(f
*x + e)*b^2*c*f^2*g + 2*(f*x + e)*a*b*d*f^2*g + 2*b^2*c*e*f^2*g - 2*a*b*d*
e*f^2*g - 2*a*b*c*f^3*g + 2*a^2*d*f^3*g + 2*(f*x + e)*b^2*c*e*f*h - 3*(f*x
 + e)*a*b*d*e*f*h - 2*b^2*c*e^2*f*h + 2*a*b*d*e^2*f*h + (f*x + e)*a*b*c*f^
2*h + 2*a*b*c*e*f^2*h - 2*a^2*d*e*f^2*h)/((b^3*c*d*e^3 - a*b^2*d^2*e^3 - b
^3*c^2*e^2*f - a*b^2*c*d*e^2*f + 2*a^2*b*d^2*e^2*f + 2*a*b^2*c^2*e*f^2 - a
^2*b*c*d*e*f^2 - a^3*d^2*e*f^2 - a^2*b*c^2*f^3 + a^3*c*d*f^3)*((f*x + e)^(
3/2)*b - sqrt(f*x + e)*b*e + sqrt(f*x + e)*a*f))
```

Mupad [B] (verification not implemented)

Time = 27.36 (sec) , antiderivative size = 404752, normalized size of antiderivative = 1410.29

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^2*(c + d*x)),x)
```

output
```
- atan((((e + f*x)^(1/2)*(18*a^6*b^10*c^10*d^3*f^15*g^2 - 114*a^7*b^9*c^9*
d^4*f^15*g^2 + 284*a^8*b^8*c^8*d^5*f^15*g^2 - 348*a^9*b^7*c^7*d^6*f^15*g^2
 + 218*a^10*b^6*c^6*d^7*f^15*g^2 - 74*a^11*b^5*c^5*d^8*f^15*g^2 + 24*a^12*
b^4*c^4*d^9*f^15*g^2 - 8*a^13*b^3*c^3*d^10*f^15*g^2 + 2*a^8*b^8*c^10*d^3*f
^15*h^2 - 18*a^9*b^7*c^9*d^4*f^15*h^2 + 68*a^10*b^6*c^8*d^5*f^15*h^2 - 116
*a^11*b^5*c^7*d^6*f^15*h^2 + 90*a^12*b^4*c^6*d^7*f^15*h^2 - 26*a^13*b^3*c^
5*d^8*f^15*h^2 + 16*a^3*b^13*d^13*e^13*f^2*g^2 - 168*a^4*b^12*d^13*e^12*f^
3*g^2 + 770*a^5*b^11*d^13*e^11*f^4*g^2 - 2020*a^6*b^10*d^13*e^10*f^5*g^2 +
 3350*a^7*b^9*d^13*e^9*f^6*g^2 - 3664*a^8*b^8*d^13*e^8*f^7*g^2 + 2678*a^9*
b^7*d^13*e^7*f^8*g^2 - 1300*a^10*b^6*d^13*e^6*f^9*g^2 + 410*a^11*b^5*d^13*
e^5*f^10*g^2 - 80*a^12*b^4*d^13*e^4*f^11*g^2 + 8*a^13*b^3*d^13*e^3*f^12*g^
2 + 18*a^7*b^9*d^13*e^11*f^4*h^2 - 108*a^8*b^8*d^13*e^10*f^5*h^2 + 270*a^9
*b^7*d^13*e^9*f^6*h^2 - 360*a^10*b^6*d^13*e^8*f^7*h^2 + 270*a^11*b^5*d^13*
e^7*f^8*h^2 - 108*a^12*b^4*d^13*e^6*f^9*h^2 + 18*a^13*b^3*d^13*e^5*f^10*h^
2 - 16*b^16*c^3*d^10*e^13*f^2*g^2 + 40*b^16*c^4*d^9*e^12*f^3*g^2 - 2*b^16*
c^5*d^8*e^11*f^4*g^2 - 62*b^16*c^6*d^7*e^10*f^5*g^2 + 20*b^16*c^7*d^6*e^9*
f^6*g^2 + 68*b^16*c^8*d^5*e^8*f^7*g^2 - 66*b^16*c^9*d^4*e^7*f^8*g^2 + 18*b
^16*c^10*d^3*e^6*f^9*g^2 - 16*b^16*c^5*d^8*e^13*f^2*h^2 + 64*b^16*c^6*d^7*
e^12*f^3*h^2 - 104*b^16*c^7*d^6*e^11*f^4*h^2 + 88*b^16*c^8*d^5*e^10*f^5*h^
2 - 40*b^16*c^9*d^4*e^9*f^6*h^2 + 8*b^16*c^10*d^3*e^8*f^7*h^2 + 48*a*b^...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 4163, normalized size of antiderivative = 14.51

$$\int \frac{g+hx}{(a+bx)^2(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)/(f*x+e)^(3/2),x)
```

output

```
( - 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**3*c**2*d*f**3*h + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*c*d**2*e*f**
2*h - 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**3*d**3*e**2*f*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**3*f**3*
h - 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**2*b*c**2*d*e*f**2*h + 5*sqrt(b)*sqrt(e + f*x)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d
*f**3*g - 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c**2*d*f**3*h*x + sqrt(b)*sqrt(e + f*x)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*
d**2*e**2*f*h - 10*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*c*d**2*e*f**2*g + 6*sqrt(b)*sqrt(e
 + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a**2*b*c*d**2*e*f**2*h*x + 5*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b*d**3*e**2*f*g - 3*sqrt(b
)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
 b*e)))*a**2*b*d**3*e**2*f*h*x + 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**3*e*f**2*h -...
```

3.174 $\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 506

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx =$$
$$-\frac{f(a^2df(8dfg-15deh+7cfh)+abf(11d^2eg-27cdfg+19cdeh-3c^2fh)-b^2(4d^2e^2g+cde(3fg-4eh$$
$$\overline{\qquad 4(bc-ad)^2(be-af)^3(de-cf)\sqrt{e+fx}}$$
$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}$$
$$+\frac{5a^2dfh-abf(9dg+ch)+b^2(4deg+5cfg-4ceh)}{4(bc-ad)^2(be-af)^2(a+bx)\sqrt{e+fx}}$$
$$+\frac{\sqrt{b}(15a^3d^2f^2h-5a^2bdf^2(7dg+2ch)+ab^2f(28d^2eg+3c^2fh+14cd(3fg-2eh))-b^3(8d^2e^2g+3c^2f(5$$
$$\overline{\qquad 4(bc-ad)^3(be-af)^{7/2}}$$
$$+\frac{2d^{5/2}(dg-ch)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^3(de-cf)^{3/2}}$$

output
```
-1/4*f*(a^2*d*f*(7*c*f*h-15*d*e*h+8*d*f*g)+a*b*f*(-3*c^2*f*h+19*c*d*e*h-27
*c*d*f*g+11*d^2*e*g)-b^2*(4*d^2*e^2*g+c*d*e*(-4*e*h+3*f*g)-3*c^2*f*(-4*e*h
+5*f*g)))/(-a*d+b*c)^2/(-a*f+b*e)^3/(-c*f+d*e)/(f*x+e)^(1/2)-1/2*(-a*h+b*g
)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2/(f*x+e)^(1/2)+1/4*(5*a^2*d*f*h-a*b*f*(c*
h+9*d*g)+b^2*(-4*c*e*h+5*c*f*g+4*d*e*g))/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)
/(f*x+e)^(1/2)+1/4*b^(1/2)*(15*a^3*d^2*f^2*h-5*a^2*b*d*f^2*(2*c*h+7*d*g)+a
*b^2*f*(28*d^2*e*g+3*c^2*f*h+14*c*d*(-2*e*h+3*f*g))-b^3*(8*d^2*e^2*g+3*c^2
*f*(-4*e*h+5*f*g)+4*c*d*e*(-2*e*h+3*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(
-a*f+b*e)^(1/2))/(-a*d+b*c)^3/(-a*f+b*e)^(7/2)+2*d^(5/2)*(-c*h+d*g)*arctan
h(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^3/(-c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 6.50 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.30

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \frac{1}{4}\left(\frac{8a^4d^2f^2(-fg+eh)+a^3bdf(cf(16fg-25eh-9fhx)+d(9e^2h\right.$$

$$+\frac{\sqrt{b}(15a^3d^2f^2h-5a^2bdf^2(7dg+2ch)+ab^2f(28d^2eg+3c^2fh+14cd(3fg-2eh))+b^3(-8d^2e^2g+4cde(}{(bc-ad)^3(-be+af)^{7/2}}$$

$$\left.-\frac{8d^{5/2}(dg-ch)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(-bc+ad)^3(-de+cf)^{3/2}}\right)$$

input
```
Integrate[(g + h*x)/((a + b*x)^3*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
((8*a^4*d^2*f^2*(-(f*g) + e*h) + a^3*b*d*f*(c*f*(16*f*g - 25*e*h - 9*f*h*x
) + d*(9*e^2*h - 16*f^2*g*x + 25*e*f*h*x)) + b^4*(4*d^2*e^2*g*x*(e + f*x)
- c*d*e*(e + f*x)*(-3*f*g*x + 2*e*(g + 2*h*x)) + c^2*f*(-15*f^2*g*x^2 + 2*
e^2*(g + 2*h*x) + e*f*x*(-5*g + 12*h*x))) + a*b^3*(d^2*e*g*(6*e^2 - 5*e*f*
x - 11*f^2*x^2) + c*d*(-2*e^3*h + 27*f^3*g*x^2 + e*f^2*x*(14*g - 19*h*x) +
 e^2*f*(3*g - 5*h*x)) + c^2*f*(2*e^2*h + f^2*x*(-25*g + 3*h*x) + e*f*(-9*g
 + 21*h*x))) - a^2*b^2*(c^2*f^2*(8*f*g - 13*e*h - 5*f*h*x) + d^2*(2*e^3*h
+ 8*f^3*g*x^2 + e*f^2*x*(13*g - 15*h*x) + e^2*f*(13*g - 5*h*x)) + c*d*f*(3
*e^2*h + f^2*x*(-45*g + 7*h*x) + e*f*(-13*g + 42*h*x))))/((b*c - a*d)^2*(b
*e - a*f)^3*(d*e - c*f)*(a + b*x)^2*Sqrt[e + f*x]) + (Sqrt[b]*(15*a^3*d^2*
f^2*h - 5*a^2*b*d*f^2*(7*d*g + 2*c*h) + a*b^2*f*(28*d^2*e*g + 3*c^2*f*h +
14*c*d*(3*f*g - 2*e*h)) + b^3*(-8*d^2*e^2*g + 4*c*d*e*(-3*f*g + 2*e*h) + 3
*c^2*f*(-5*f*g + 4*e*h)))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f
]])/((b*c - a*d)^3*(-(b*e) + a*f)^(7/2)) - (8*d^(5/2)*(d*g - c*h)*ArcTan[(
Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/((-(b*c) + a*d)^3*(-(d*e) + c*
f)^(3/2)))/4
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int -\frac{af(4dg+ch)-b(4deg+5cfg-4ceh)-5df(bg-ah)x}{2(a+bx)^2(c+dx)(e+fx)^{3/2}}dx}{2(bc-ad)(be-af)}-\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{af(4dg+ch)-b(4deg+5cfg-4ceh)-5df(bg-ah)x}{(a+bx)^2(c+dx)(e+fx)^{3/2}}dx}{4(bc-ad)(be-af)}-\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{5a^2dfh-abf(ch+9dg)+b^2(-4ceh+5cfg+4deg)}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int -\frac{\left(3f(5fg-4eh)c^2+4de(3fg-2eh)c+8d^2e^2g\right)b^2-af\left(3fhc^2+d(27fg-16eh)c+16d^2eg\right)b+a^2df^2(8dg}{2(a+bx)(c+dx)(e+fx)^{3/2}}}{(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(3f(5fg-4eh)c^2+4de(3fg-2eh)c+8d^2e^2g\right)b^2-af\left(3fhc^2+d(27fg-16eh)c+16d^2eg\right)b+a^2df^2(8dg+7ch)+3df\left(5dfha^2-bf(9dg+ch)a+b^2(4deg+5cfg-4ceh)\right)}{(a+bx)(c+dx)(e+fx)^{3/2}}}{2(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 169

$$-\frac{\frac{2\int \frac{-\left((de-cf)\left(3f(5fg-4eh)c^2+4de(3fg-2eh)c+8d^2e^2g\right)b^3\right)+af\left(-3f^2hc^3-df(27fg-19eh)c^2+d^2e(11fg-24eh)c+24d^3e^2g\right)b^2-a^2df^2\left(-7fhc^2-d(8fg+9e\right.}{2(a+bx)(}}{(be-af}}{}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$-\frac{\frac{\int \frac{-\left((de-cf)\left(3f(5fg-4eh)c^2+4de(3fg-2eh)c+8d^2e^2g\right)b^3\right)+af\left(-3f^2hc^3-df(27fg-19eh)c^2+d^2e(11fg-24eh)c+24d^3e^2g\right)b^2-a^2df^2\left(-7fhc^2-d(8fg+9eh\right.}{(a+bx)(c+}}{(be-af}}{}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 174

$$-\frac{\frac{\frac{b(de-cf)\left(15a^3d^2f^2h-5a^2bdf^2(2ch+7dg)+ab^2f\left(3c^2fh+14cd(3fg-2eh)+28d^2eg\right)-b^3\left(3c^2f(5fg-4eh)+4cde(3fg-2eh)+8d^2e^2g\right)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}+\frac{8d}{}}{(be-af)(de-cf)}}{2(bc-ad}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 73

$$-\frac{\frac{\frac{2b(de-cf)\left(15a^3d^2f^2h-5a^2bdf^2(2ch+7dg)+ab^2f\left(3c^2fh+14cd(3fg-2eh)+28d^2eg\right)-b^3\left(3c^2f(5fg-4eh)+4cde(3fg-2eh)+8d^2e^2g\right)\right)\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e}}{f(bc-ad)}}{(be-af)(de-cf)}}{}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{5a^2dfh-abf(ch+9dg)+b^2(-4ceh+5cfg+4deg)}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}+\frac{-\frac{2f\left(a^2df(7cfh-15deh+8dfg)+abf\left(-3c^2fh+19cdeh-27cdfg+11d^2eg\right)-b^2\left(-3c^2f(5fg-4eh)+cde(3\right.\right.}{\sqrt{e+fx}(be-af)(de-cf)}}{}}{}$$

$$\frac{bg-ah}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^3*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
-1/2*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*Sqrt[e + f*x]) + ((5
*a^2*d*f*h - a*b*f*(9*d*g + c*h) + b^2*(4*d*e*g + 5*c*f*g - 4*c*e*h))/((b*
c - a*d)*(b*e - a*f)*(a + b*x)*Sqrt[e + f*x]) + ((-2*f*(a^2*d*f*(8*d*f*g -
 15*d*e*h + 7*c*f*h) + a*b*f*(11*d^2*e*g - 27*c*d*f*g + 19*c*d*e*h - 3*c^2
*f*h) - b^2*(4*d^2*e^2*g + c*d*e*(3*f*g - 4*e*h) - 3*c^2*f*(5*f*g - 4*e*h)
)))/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) - ((-2*Sqrt[b]*(d*e - c*f)*(15
*a^3*d^2*f^2*h - 5*a^2*b*d*f^2*(7*d*g + 2*c*h) + a*b^2*f*(28*d^2*e*g + 3*c
^2*f*h + 14*c*d*(3*f*g - 2*e*h)) - b^3*(8*d^2*e^2*g + 3*c^2*f*(5*f*g - 4*e
*h) + 4*c*d*e*(3*f*g - 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e -
 a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) - (16*d^(5/2)*(b*e - a*f)^3*(d*g - c
*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*
e - c*f]))/((b*e - a*f)*(d*e - c*f)))/(2*(b*c - a*d)*(b*e - a*f)))/(4*(b*c
 - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.34

method	result
derivativedivides	$2f^2\left(-\frac{-eh+fg}{(cf-de)(af-be)^3\sqrt{fx+e}}+\frac{(ch-dg)d^3\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)^3f^2(cf-de)\sqrt{(cf-de)d}}-\frac{b\left(\left(\frac{7}{8}a^3b\,d^2f^2h-\frac{5}{4}a^2b^2cd\,f^2h-\frac{11}{8}a^2b^2d^2\right.\right.}{}\right.$
default	$2f^2\left(-\frac{-eh+fg}{(cf-de)(af-be)^3\sqrt{fx+e}}+\frac{(ch-dg)d^3\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{(ad-bc)^3f^2(cf-de)\sqrt{(cf-de)d}}-\frac{b\left(\left(\frac{7}{8}a^3b\,d^2f^2h-\frac{5}{4}a^2b^2cd\,f^2h-\frac{11}{8}a^2b^2d^2\right.\right.}{}\right.$
pseudoelliptic	$-\frac{\frac{15(bx+a)^2(cf-de)\sqrt{fx+e}\left(\left(\left(a^2\left(ah-\frac{7bg}{3}\right)d^2-\frac{2ac\left(ah-\frac{21bg}{5}\right)bd}{3}+\frac{b^2c^2(ah-5bg)}{5}\right)f^2-\frac{28\left(ad-\frac{3bc}{7}\right)(ch-dg)b^2ef}{15}+\frac{8b^3d\,e^2(c}{15}\right.\right.}{4}}{}$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^2*(-(-e*h+f*g)/(c*f-d*e)/(a*f-b*e)^3/(f*x+e)^(1/2)+(c*h-d*g)*d^3/(a*d-
b*c)^3/f^2/(c*f-d*e)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)
*d)^(1/2))-b/(a*d-b*c)^3/(a*f-b*e)^3/f^2*(((7/8*a^3*b*d^2*f^2*h-5/4*a^2*b^
2*c*d*f^2*h-11/8*a^2*b^2*d^2*f^2*g+3/8*a*b^3*c^2*f^2*h-1/2*a*b^3*c*d*e*f*h
+9/4*a*b^3*c*d*f^2*g+1/2*a*b^3*d^2*e*f*g+1/2*b^4*c^2*e*f*h-7/8*b^4*c^2*f^2
*g-1/2*b^4*c*d*e*f*g)*(f*x+e)^(3/2)+1/8*f*(9*a^4*d^2*f^2*h-14*a^3*b*c*d*f^
2*h-9*a^3*b*d^2*e*f*h-13*a^3*b*d^2*f^2*g+5*a^2*b^2*c^2*f^2*h+10*a^2*b^2*c*
d*e*f*h+22*a^2*b^2*c*d*f^2*g+17*a^2*b^2*d^2*e*f*g-a*b^3*c^2*e*f*h-9*a*b^3*
c^2*f^2*g+4*a*b^3*c*d*e^2*h-26*a*b^3*c*d*e*f*g-4*a*b^3*d^2*e^2*g-4*b^4*c^2
*e^2*h+9*b^4*c^2*e*f*g+4*b^4*c*d*e^2*g)*(f*x+e)^(1/2))/((f*x+e)*b+a*f-b*e)
^2+1/8*(15*a^3*d^2*f^2*h-10*a^2*b*c*d*f^2*h-35*a^2*b*d^2*f^2*g+3*a*b^2*c^2
*f^2*h-28*a*b^2*c*d*e*f*h+42*a*b^2*c*d*f^2*g+28*a*b^2*d^2*e*f*g+12*b^3*c^2
*e*f*h-15*b^3*c^2*f^2*g+8*b^3*c*d*e^2*h-12*b^3*c*d*e*f*g-8*b^3*d^2*e^2*g)/
((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**3/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(472) = 944$.

Time = 0.23 (sec) , antiderivative size = 1130, normalized size of antiderivative = 2.23

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*b^4*d^2*e^2*g + 12*b^4*c*d*e*f*g - 28*a*b^3*d^2*e*f*g + 15*b^4*c^2*
f^2*g - 42*a*b^3*c*d*f^2*g + 35*a^2*b^2*d^2*f^2*g - 8*b^4*c*d*e^2*h - 12*b
^4*c^2*e*f*h + 28*a*b^3*c*d*e*f*h - 3*a*b^3*c^2*f^2*h + 10*a^2*b^2*c*d*f^2
*h - 15*a^3*b*d^2*f^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^
6*c^3*e^3 - 3*a*b^5*c^2*d*e^3 + 3*a^2*b^4*c*d^2*e^3 - a^3*b^3*d^3*e^3 - 3*
a*b^5*c^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a^3*b^3*c*d^2*e^2*f + 3*a^4*b^
2*d^3*e^2*f + 3*a^2*b^4*c^3*e*f^2 - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^
2*e*f^2 - 3*a^5*b*d^3*e*f^2 - a^3*b^3*c^3*f^3 + 3*a^4*b^2*c^2*d*f^3 - 3*a^
5*b*c*d^2*f^3 + a^6*d^3*f^3)*sqrt(-b^2*e + a*b*f)) - 2*(d^4*g - c*d^3*h)*a
rctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^3*d*e - 3*a*b^2*c^2*d^
2*e + 3*a^2*b*c*d^3*e - a^3*d^4*e - b^3*c^4*f + 3*a*b^2*c^3*d*f - 3*a^2*b*
c^2*d^2*f + a^3*c*d^3*f)*sqrt(-d^2*e + c*d*f)) - 2*(f^3*g - e*f^2*h)/((b^3
*d*e^4 - b^3*c*e^3*f - 3*a*b^2*d*e^3*f + 3*a*b^2*c*e^2*f^2 + 3*a^2*b*d*e^2
*f^2 - 3*a^2*b*c*e*f^3 - a^3*d*e*f^3 + a^3*c*f^4)*sqrt(f*x + e)) + 1/4*(4*
(f*x + e)^(3/2)*b^4*d*e*f*g - 4*sqrt(f*x + e)*b^4*d*e^2*f*g + 7*(f*x + e)^
(3/2)*b^4*c*f^2*g - 11*(f*x + e)^(3/2)*a*b^3*d*f^2*g - 9*sqrt(f*x + e)*b^4
*c*e*f^2*g + 17*sqrt(f*x + e)*a*b^3*d*e*f^2*g + 9*sqrt(f*x + e)*a*b^3*c*f^
3*g - 13*sqrt(f*x + e)*a^2*b^2*d*f^3*g - 4*(f*x + e)^(3/2)*b^4*c*e*f*h + 4
*sqrt(f*x + e)*b^4*c*e^2*f*h - 3*(f*x + e)^(3/2)*a*b^3*c*f^2*h + 7*(f*x +
e)^(3/2)*a^2*b^2*d*f^2*h + sqrt(f*x + e)*a*b^3*c*e*f^2*h - 9*sqrt(f*x +...
```

Mupad [B] (verification not implemented)

Time = 79.02 (sec) , antiderivative size = 802142, normalized size of antiderivative = 1585.26

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^3*(c + d*x)),x)
```

output
```
atan(-(((e + f*x)^(1/2)*(28800*a^9*b^15*c^15*d^3*f^20*g^2 - 334080*a^10*b^
14*c^14*d^4*f^20*g^2 + 1759872*a^11*b^13*c^13*d^5*f^20*g^2 - 5532672*a^12*
b^12*c^12*d^6*f^20*g^2 + 11475200*a^13*b^11*c^11*d^7*f^20*g^2 - 16381440*a
^14*b^10*c^10*d^8*f^20*g^2 + 16285952*a^15*b^9*c^9*d^9*f^20*g^2 - 11152384
*a^16*b^8*c^8*d^10*f^20*g^2 + 5092992*a^17*b^7*c^7*d^11*f^20*g^2 - 1480960
*a^18*b^6*c^6*d^12*f^20*g^2 + 279680*a^19*b^5*c^5*d^13*f^20*g^2 - 49152*a^
20*b^4*c^4*d^14*f^20*g^2 + 8192*a^21*b^3*c^3*d^15*f^20*g^2 + 1152*a^11*b^1
3*c^15*d^3*f^20*h^2 - 14592*a^12*b^12*c^14*d^4*f^20*h^2 + 87680*a^13*b^11*
c^13*d^5*f^20*h^2 - 322560*a^14*b^10*c^12*d^6*f^20*h^2 + 803072*a^15*b^9*c
^11*d^7*f^20*h^2 - 1406464*a^16*b^8*c^10*d^8*f^20*h^2 + 1734912*a^17*b^7*c
^9*d^9*f^20*h^2 - 1469440*a^18*b^6*c^8*d^10*f^20*h^2 + 809600*a^19*b^5*c^7
*d^11*f^20*h^2 - 260352*a^20*b^4*c^6*d^12*f^20*h^2 + 36992*a^21*b^3*c^5*d^
13*f^20*h^2 + 16384*a^6*b^18*d^18*e^18*f^2*g^2 - 253952*a^7*b^17*d^18*e^17
*f^3*g^2 + 1843200*a^8*b^16*d^18*e^16*f^4*g^2 - 8279040*a^9*b^15*d^18*e^15
*f^5*g^2 + 25639040*a^10*b^14*d^18*e^14*f^6*g^2 - 57751680*a^11*b^13*d^18*
e^13*f^7*g^2 + 97309184*a^12*b^12*d^18*e^12*f^8*g^2 - 124285440*a^13*b^11*
d^18*e^11*f^9*g^2 + 120672000*a^14*b^10*d^18*e^10*f^10*g^2 - 88549120*a^15
*b^9*d^18*e^9*f^11*g^2 + 48409088*a^16*b^8*d^18*e^8*f^12*g^2 - 19256832*a^
17*b^7*d^18*e^7*f^13*g^2 + 5389440*a^18*b^6*d^18*e^6*f^14*g^2 - 1016960*a^
19*b^5*d^18*e^5*f^15*g^2 + 122880*a^20*b^4*d^18*e^4*f^16*g^2 - 8192*a^2...
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 10467, normalized size of antiderivative = 20.69

$$\int \frac{g+hx}{(a+bx)^3(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)/(f*x+e)^(3/2),x)
```

output

```
( - 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**5*c**2*d**2*f**4*h + 30*sqrt(b)*sqrt(e + f*x)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*c*d**3*
e*f**3*h - 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)
/(sqrt(b)*sqrt(a*f - b*e)))*a**5*d**4*e**2*f**2*h + 10*sqrt(b)*sqrt(e + f*
x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*
b*c**3*d*f**4*h - 20*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**2*e*f**3*h + 35*sqrt(b)*
sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**4*b*c**2*d**2*f**4*g - 30*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*a
tan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c**2*d**2*f**4*h*x
 + 10*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**4*b*c*d**3*e**2*f**2*h - 70*sqrt(b)*sqrt(e + f*x)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*
d**3*e*f**3*g + 60*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*c*d**3*e*f**3*h*x + 35*sqrt(b)*sqr
t(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**4*b*d**4*e**2*f**2*g - 30*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b*d**4*e**2*f**2*h*x -
3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)...
```

3.175 $\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 831

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx =$$

$$-\frac{f(a^3d^2f^2(16dfg-35deh+19cfh)+a^2bdf^2(41d^2eg-16c^2fh-cd(89fg-64eh))-ab^2f(30d^3e^2g-5c\ldots}{8(bc-\ldots}$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}$$

$$+\frac{7a^2dfh-abf(13dg+ch)+b^2(6deg+7cfg-6ceh)}{12(bc-ad)^2(be-af)^2(a+bx)^2\sqrt{e+fx}}$$

$$+\frac{35a^3d^2f^2h-a^2bdf^2(89dg+16ch)+ab^2f(78d^2eg+5c^2fh+2cd(50fg-39eh))-b^3(24d^2e^2g+5c^2f(7f\ldots}{24(bc-ad)^3(be-af)^3(a+bx)\sqrt{e+fx}}$$

$$+\frac{\sqrt{b}(35a^4d^3f^3h-35a^3bd^2f^3(3dg+ch)+21a^2b^2df^2(6d^2eg+c^2fh+cd(9fg-6eh))-ab^3f(72d^3e^2g+5c\ldots}{\ldots}$$

$$-\frac{2d^{7/2}(dg-ch)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4(de-cf)^{3/2}}$$

output
```
-1/8*f*(a^3*d^2*f^2*(19*c*f*h-35*d*e*h+16*d*f*g)+a^2*b*d*f^2*(41*d^2*e*g-1
6*c^2*f*h-c*d*(-64*e*h+89*f*g))-a*b^2*f*(30*d^3*e^2*g-5*c^3*f^2*h-c^2*d*f*
(-83*e*h+100*f*g)+2*c*d^2*e*(-15*e*h+11*f*g))+b^3*(8*d^3*e^3*g+c^2*d*e*f*(
-6*e*h+5*f*g)-5*c^3*f^2*(-6*e*h+7*f*g)+2*c*d^2*e^2*(-4*e*h+3*f*g)))/(-a*d+
b*c)^3/(-a*f+b*e)^4/(-c*f+d*e)/(f*x+e)^(1/2)-1/3*(-a*h+b*g)/(-a*d+b*c)/(-a
*f+b*e)/(b*x+a)^3/(f*x+e)^(1/2)+1/12*(7*a^2*d*f*h-a*b*f*(c*h+13*d*g)+b^2*(
-6*c*e*h+7*c*f*g+6*d*e*g))/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2/(f*x+e)^(1/
2)+1/24*(35*a^3*d^2*f^2*h-a^2*b*d*f^2*(16*c*h+89*d*g)+a*b^2*f*(78*d^2*e*g+
5*c^2*f*h+2*c*d*(-39*e*h+50*f*g))-b^3*(24*d^2*e^2*g+5*c^2*f*(-6*e*h+7*f*g)
+6*c*d*e*(-4*e*h+5*f*g)))/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)/(f*x+e)^(1/2)+
1/8*b^(1/2)*(35*a^4*d^3*f^3*h-35*a^3*b*d^2*f^3*(c*h+3*d*g)+21*a^2*b^2*d*f^
2*(6*d^2*e*g+c^2*f*h+c*d*(-6*e*h+9*f*g))-a*b^3*f*(72*d^3*e^2*g+5*c^3*f^2*h
+27*c^2*d*f*(-4*e*h+5*f*g)+36*c*d^2*e*(-2*e*h+3*f*g))+b^4*(16*d^3*e^3*g+5*
c^3*f^2*(-6*e*h+7*f*g)+6*c^2*d*e*f*(-4*e*h+5*f*g)+8*c*d^2*e^2*(-2*e*h+3*f*
g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^4/(-a*f+b*
e)^(9/2)-2*d^(7/2)*(-c*h+d*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/
2))/(-a*d+b*c)^4/(-c*f+d*e)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.66 (sec) , antiderivative size = 565, normalized size of antiderivative = 0.68

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \frac{-\frac{8(bc-ad)(be-af)(bg-ah)}{(a+bx)^3}+\frac{2\left(7a^2dfh-abf(13dg+ch)+b^2(6deg+7cfg-6ceh)\right)}{(a+bx)^2}+\frac{35a^3 \cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(g + h*x)/((a + b*x)^4*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
((-8*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(a + b*x)^3 + (2*(7*a^2*d*f*h -
a*b*f*(13*d*g + c*h) + b^2*(6*d*e*g + 7*c*f*g - 6*c*e*h)))/(a + b*x)^2 + (
35*a^3*d^2*f^2*h - a^2*b*d*f^2*(89*d*g + 16*c*h) + a*b^2*f*(78*d^2*e*g + 5
*c^2*f*h + 2*c*d*(50*f*g - 39*e*h)) + b^3*(-24*d^2*e^2*g + 6*c*d*e*(-5*f*g
 + 4*e*h) + 5*c^2*f*(-7*f*g + 6*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x))
 + (3*(d*e - c*f)*(35*a^4*d^3*f^3*h - 35*a^3*b*d^2*f^3*(3*d*g + c*h) + 21*
a^2*b^2*d*f^2*(6*d^2*e*g + 9*c*d*f*g - 6*c*d*e*h + c^2*f*h) - a*b^3*f*(72*
d^3*e^2*g + 5*c^3*f^2*h + 27*c^2*d*f*(5*f*g - 4*e*h) - 36*c*d^2*e*(-3*f*g
+ 2*e*h)) + b^4*(16*d^3*e^3*g + 5*c^3*f^2*(7*f*g - 6*e*h) - 8*c*d^2*e^2*(-
3*f*g + 2*e*h) - 6*c^2*d*e*f*(-5*f*g + 4*e*h)))*Hypergeometric2F1[-1/2, 1,
 1/2, (b*(e + f*x))/(b*e - a*f)] - 48*d^3*(b*e - a*f)^4*(d*g - c*h)*Hyperg
eometric2F1[-1/2, 1, 1/2, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)^2*(b*e
- a*f)^2*(-(d*e) + c*f)))/(24*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int -\frac{af(6dg+ch)-b(6deg+7cfg-6ceh)-7df(bg-ah)x}{2(a+bx)^3(c+dx)(e+fx)^{3/2}}dx}{3(bc-ad)(be-af)}-\frac{bg-ah}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{af(6dg+ch)-b(6deg+7cfg-6ceh)-7df(bg-ah)x}{(a+bx)^3(c+dx)(e+fx)^{3/2}}dx}{6(bc-ad)(be-af)}-\frac{bg-ah}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{7a^2dfh-abf(ch+13dg)+b^2(-6ceh+7cfg+6deg)}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int -\frac{\left(5f(7fg-6eh)c^2+6de(5fg-4eh)c+24d^2e^2g\right)b^2-af\left(5fhc^2+d(65fg-48eh)c+48d^2eg\right)b+a^2df^2(2}{2(a+bx)^2(c+dx)(e+fx)^{3/2}}}{2(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{bg-ah}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(5f(7fg-6eh)c^2+6de(5fg-4eh)c+24d^2e^2g\right)b^2-af\left(5fhc^2+d(65fg-48eh)c+48d^2eg\right)b+a^2df^2(24dg+11ch)+5df\left(7dfha^2-bf(13dg+ch)a+b^2(6deg+7cfg-6ce}{(a+bx)^2(c+dx)(e+fx)^{3/2}}}{4(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{bg-ah}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 168

$$\frac{\frac{35a^3d^2f^2h-a^2bdf^2(16ch+89dg)+ab^2f\left(5c^2fh+2cd(50fg-39eh)+78d^2eg\right)-b^3\left(5c^2f(7fg-6eh)+6cde(5fg-4eh)+24d^2e^2g\right)}{(a+bx)\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int -\frac{3\left(-\left(\left(5f^2(7fg-6eh)c^3+6de\right.\right.\right.}{}}{}}{}$$

$$\frac{bg-ah}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{3\int \frac{-\left(\left(5f^2(7fg-6eh)c^3+6def(5fg-4eh)c^2+8d^2e^2(3fg-2eh)c+16d^3e^3g\right)b^3\right)+af\left(5f^2hc^3+2df(50fg-39eh)c^2+6d^2e(13fg-8eh)c+48d^3e^2g\right)b^2-a^2df^2\left(16fhc\right.}{}}{}$$

$$\frac{bg-ah}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 169

$$\frac{\frac{7dfha^2-bf(13dg+ch)a+b^2(6deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}+\frac{\frac{35d^2f^2ha^3-bdf^2(89dg+16ch)a^2+b^2f\left(5fhc^2+2d(50fg-39eh)c+78d^2eg\right)a-b^3\left(5f(7fg-6eh)c^2+6de(5\right.}{(bc-ad)(be-af)(a+bx)\sqrt{e+fx}}}{}}{}$$

$$\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}$$

$\big\downarrow$ 27

$$\frac{\frac{7dfha^2-bf(13dg+ch)a+b^2(6deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}+\frac{\frac{35d^2f^2ha^3-bdf^2(89dg+16ch)a^2+b^2f\left(5fhc^2+2d(50fg-39eh)c+78d^2eg\right)a-b^3\left(5f(7fg-6eh)c^2+6de(5\right.}{(bc-ad)(be-af)(a+bx)\sqrt{e+fx}}}{}}{}$$

$$\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}$$

$\Big\downarrow$ 174

$$\frac{\frac{7dfha^2-bf(13dg+ch)a+b^2(6deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}+\frac{\frac{35d^2f^2ha^3-bdf^2(89dg+16ch)a^2+b^2f\left(5fhc^2+2d(50fg-39eh)c+78d^2eg\right)a-b^3\left(5f(7fg-6eh)c^2+6de(5\right.}{(bc-ad)(be-af)(a+bx)\sqrt{e+fx}}}{}}{}$$

$$\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}$$

$\Big\downarrow$ 73

$$\frac{\frac{7dfha^2-bf(13dg+ch)a+b^2(6deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}+\frac{\frac{35d^2f^2ha^3-bdf^2(89dg+16ch)a^2+b^2f\left(5fhc^2+2d(50fg-39eh)c+78d^2eg\right)a-b^3\left(5f(7fg-6eh)c^2+6de(5\right.}{(bc-ad)(be-af)(a+bx)\sqrt{e+fx}}}{}}{}$$

$$\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}$$

$\Big\downarrow$ 221

$$\frac{\frac{7dfha^2-bf(13dg+ch)a+b^2(6deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}+\frac{\frac{35d^2f^2ha^3-bdf^2(89dg+16ch)a^2+b^2f\left(5fhc^2+2d(50fg-39eh)c+78d^2eg\right)a-b^3\left(5f(7fg-6eh)c^2+6de(5\right.}{(bc-ad)(be-af)(a+bx)\sqrt{e+fx}}}{}}{}$$

$$\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}$$

input
```
Int[(g + h*x)/((a + b*x)^4*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
-1/3*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*Sqrt[e + f*x]) + ((7
*a^2*d*f*h - a*b*f*(13*d*g + c*h) + b^2*(6*d*e*g + 7*c*f*g - 6*c*e*h))/(2*
(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*Sqrt[e + f*x]) + ((35*a^3*d^2*f^2*h -
a^2*b*d*f^2*(89*d*g + 16*c*h) + a*b^2*f*(78*d^2*e*g + 5*c^2*f*h + 2*c*d*(5
0*f*g - 39*e*h)) - b^3*(24*d^2*e^2*g + 5*c^2*f*(7*f*g - 6*e*h) + 6*c*d*e*(
5*f*g - 4*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x)*Sqrt[e + f*x]) + (3*((
-2*f*(a^3*d^2*f^2*(16*d*f*g - 35*d*e*h + 19*c*f*h) + a^2*b*d*f^2*(41*d^2*e
*g - 89*c*d*f*g + 64*c*d*e*h - 16*c^2*f*h) - a*b^2*f*(30*d^3*e^2*g - 5*c^3
*f^2*h - c^2*d*f*(100*f*g - 83*e*h) + 2*c*d^2*e*(11*f*g - 15*e*h)) + b^3*(
8*d^3*e^3*g + c^2*d*e*f*(5*f*g - 6*e*h) - 5*c^3*f^2*(7*f*g - 6*e*h) + 2*c*
d^2*e^2*(3*f*g - 4*e*h))))/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) - ((-2*
Sqrt[b]*(d*e - c*f)*(35*a^4*d^3*f^3*h - 35*a^3*b*d^2*f^3*(3*d*g + c*h) + 2
1*a^2*b^2*d*f^2*(6*d^2*e*g + 9*c*d*f*g - 6*c*d*e*h + c^2*f*h) - a*b^3*f*(7
2*d^3*e^2*g + 5*c^3*f^2*h + 27*c^2*d*f*(5*f*g - 4*e*h) + 36*c*d^2*e*(3*f*g
 - 2*e*h)) + b^4*(16*d^3*e^3*g + 5*c^3*f^2*(7*f*g - 6*e*h) + 6*c^2*d*e*f*(
5*f*g - 4*e*h) + 8*c*d^2*e^2*(3*f*g - 2*e*h)))*ArcTanh[(Sqrt[b]*Sqrt[e + f
*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (32*d^(7/2)*(b*e -
a*f)^4*(d*g - c*h)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c
 - a*d)*Sqrt[d*e - c*f]))/((b*e - a*f)*(d*e - c*f))))/(2*(b*c - a*d)*(b*e
- a*f)))/(4*(b*c - a*d)*(b*e - a*f)))/(6*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 30.70 (sec) , antiderivative size = 1318, normalized size of antiderivative = 1.59

method	result	size
pseudoelliptic	Expression too large to display	1318
derivativedivides	Expression too large to display	1698
default	Expression too large to display	1698

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-35/16*((c*f-d*e)*d)^(1/2)*(b*x+a)^3*(c*f-d*e)*(f*x+e)^(1/2)*b*(((a^4*h
-3*a^3*b*g)*d^3-a^2*(a*h-27/5*b*g)*c*b*d^2+3/5*a*c^2*(a*h-45/7*b*g)*b^2*d-
1/7*b^3*c^3*(a*h-7*b*g))*f^3-18/5*(a^2*d^2-6/7*a*b*c*d+5/21*b^2*c^2)*(c*h-
d*g)*b^2*e*f^2+72/35*d*(c*h-d*g)*(a*d-1/3*b*c)*b^3*e^2*f-16/35*b^4*d^2*e^3
*(c*h-d*g))*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+((f*x+e)^(1/2)*d^4
*(b*x+a)^3*(a*f-b*e)^4*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2
))+((-a^3*g*(b*x+a)^3*d^3+3*a^2*c*(89/48*b^3*g*x^3+89/18*a*x^2*(-57/712*h*
x+g)*b^2+199/48*a^2*x*(-136/597*h*x+g)*b+a^3*(-29/48*h*x+g))*b*d^2-3*a*c^2
*(25/12*b^3*g*x^3+50/9*a*x^2*(-3/50*h*x+g)*b^2+55/12*(-32/165*h*x+g)*a^2*x
*b+a^3*(-2/3*h*x+g))*b^2*d+c^3*(35/16*b^3*g*x^3+35/6*a*x^2*(-3/56*h*x+g)*b
^2+77/16*a^2*x*(-40/231*h*x+g)*b+a^3*(g-11/16*h*x))*b^3)*f^4+(a^2*(-41/16*
b^4*g*x^3-35/6*a*x^2*(-3/8*h*x+g)*b^3-55/16*a^2*x*(-56/33*h*x+g)*b^2+77/16
*a^3*b*h*x+a^4*h)*d^3-77/16*a*(-2/7*b^4*g*x^3-265/231*a*x^2*(-192/265*h*x+
g)*b^3-356/231*a^2*x*(-595/356*h*x+g)*b^2-5/7*a^3*(-626/165*h*x+g)*b+a^4*h
)*c*b*d^2+5*(-1/16*b^4*g*x^3-7/12*a*x^2*(-249/140*h*x+g)*b^3-313/240*a^2*x
*(-680/313*h*x+g)*b^2-19/20*a^3*(-631/228*h*x+g)*b+a^4*h)*c^2*b^2*d-27/16*
c^3*(-35/81*x^2*(-18/7*h*x+g)*b^3-98/81*a*x*(-5/2*h*x+g)*b^2-29/27*(-212/8
7*h*x+g)*a^2*b+h*a^3)*b^3)*e*f^3+29/16*(a*(30/29*b^4*g*x^3+103/87*a*b^3*g*
x^2-128/87*(-35/128*h*x+g)*a^2*x*b^2-55/29*a^3*(-98/165*h*x+g)*b+a^4*h)*d^
3-2/3*c*(9/29*b^4*g*x^3+13/29*a*x^2*(45/13*h*x+g)*b^3-22/29*a^2*x*(-52/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**4/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2492 vs. $2(794) = 1588$.

Time $= 0.44$ (sec) , antiderivative size $= 2492$, normalized size of antiderivative $= 3.00$

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(16*b^5*d^3*e^3*g + 24*b^5*c*d^2*e^2*f*g - 72*a*b^4*d^3*e^2*f*g + 30*
b^5*c^2*d*e*f^2*g - 108*a*b^4*c*d^2*e*f^2*g + 126*a^2*b^3*d^3*e*f^2*g + 35
*b^5*c^3*f^3*g - 135*a*b^4*c^2*d*f^3*g + 189*a^2*b^3*c*d^2*f^3*g - 105*a^3
*b^2*d^3*f^3*g - 16*b^5*c*d^2*e^3*h - 24*b^5*c^2*d*e^2*f*h + 72*a*b^4*c*d^
2*e^2*f*h - 30*b^5*c^3*e*f^2*h + 108*a*b^4*c^2*d*e*f^2*h - 126*a^2*b^3*c*d
^2*e*f^2*h - 5*a*b^4*c^3*f^3*h + 21*a^2*b^3*c^2*d*f^3*h - 35*a^3*b^2*c*d^2
*f^3*h + 35*a^4*b*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/
((b^8*c^4*e^4 - 4*a*b^7*c^3*d*e^4 + 6*a^2*b^6*c^2*d^2*e^4 - 4*a^3*b^5*c*d^
3*e^4 + a^4*b^4*d^4*e^4 - 4*a*b^7*c^4*e^3*f + 16*a^2*b^6*c^3*d*e^3*f - 24*
a^3*b^5*c^2*d^2*e^3*f + 16*a^4*b^4*c*d^3*e^3*f - 4*a^5*b^3*d^4*e^3*f + 6*a
^2*b^6*c^4*e^2*f^2 - 24*a^3*b^5*c^3*d*e^2*f^2 + 36*a^4*b^4*c^2*d^2*e^2*f^2
 - 24*a^5*b^3*c*d^3*e^2*f^2 + 6*a^6*b^2*d^4*e^2*f^2 - 4*a^3*b^5*c^4*e*f^3
+ 16*a^4*b^4*c^3*d*e*f^3 - 24*a^5*b^3*c^2*d^2*e*f^3 + 16*a^6*b^2*c*d^3*e*f
^3 - 4*a^7*b*d^4*e*f^3 + a^4*b^4*c^4*f^4 - 4*a^5*b^3*c^3*d*f^4 + 6*a^6*b^2
*c^2*d^2*f^4 - 4*a^7*b*c*d^3*f^4 + a^8*d^4*f^4)*sqrt(-b^2*e + a*b*f)) + 2*
(d^5*g - c*d^4*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^4*d
*e - 4*a*b^3*c^3*d^2*e + 6*a^2*b^2*c^2*d^3*e - 4*a^3*b*c*d^4*e + a^4*d^5*e
 - b^4*c^5*f + 4*a*b^3*c^4*d*f - 6*a^2*b^2*c^3*d^2*f + 4*a^3*b*c^2*d^3*f -
 a^4*c*d^4*f)*sqrt(-d^2*e + c*d*f)) + 2*(f^4*g - e*f^3*h)/((b^4*d*e^5 - b^
4*c*e^4*f - 4*a*b^3*d*e^4*f + 4*a*b^3*c*e^3*f^2 + 6*a^2*b^2*d*e^3*f^2 -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^4*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 20501, normalized size of antiderivative = 24.67

$$\int \frac{g+hx}{(a+bx)^4(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((h*x+g)/(b*x+a)^4/(d*x+c)/(f*x+e)^(3/2),x)
```

output

```
( - 105*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**7*c**2*d**3*f**5*h + 210*sqrt(b)*sqrt(e + f*x)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*c*d**
4*e*f**4*h - 105*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*d**5*e**2*f**3*h + 105*sqrt(b)*sqrt(e
+ f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
**6*b*c**3*d**2*f**5*h - 210*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((s
qrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**3*e*f**4*h + 315
*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a**6*b*c**2*d**3*f**5*g - 315*sqrt(b)*sqrt(e + f*x)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c**2*d**
3*f**5*h*x + 105*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**4*e**2*f**3*h - 630*sqrt(b)*sqr
t(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)
))*a**6*b*c*d**4*e*f**4*g + 630*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*c*d**4*e*f**4*h*x + 3
15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**6*b*d**5*e**2*f**3*g - 315*sqrt(b)*sqrt(e + f*x)*sqrt(
a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b*d**5*e
**2*f**3*h*x - 63*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + ...
```

3.176 $\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1290

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-1/64*f*(a^4*d^3*f^3*(187*c*f*h-315*d*e*h+128*d*f*g)+a^3*b*d^2*f^3*(-233*c
^2*f*h+745*c*d*e*h-1027*c*d*f*g+515*d^2*e*g)-a^2*b^2*d*f^2*(568*d^3*e^2*g-
145*c^3*f^2*h-c^2*d*f*(-1481*e*h+1745*f*g)+c*d^2*e*(-568*e*h+409*f*g))+a*b
^3*f*(304*d^4*e^3*g-35*c^4*f^3*h+c^2*d^2*e*f*(-224*e*h+185*f*g)-25*c^3*d*f
^2*(-43*e*h+49*f*g)+16*c*d^3*e^2*(-19*e*h+14*f*g))-b^4*(64*d^4*e^4*g+5*c^3
*d*e*f^2*(-8*e*h+7*f*g)-35*c^4*f^3*(-8*e*h+9*f*g)+8*c^2*d^2*e^2*f*(-6*e*h+
5*f*g)+16*c*d^3*e^3*(-4*e*h+3*f*g)))/(-a*d+b*c)^4/(-a*f+b*e)^5/(-c*f+d*e)/
(f*x+e)^(1/2)-1/4*(-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^4/(f*x+e)^(1/2)
+1/24*(9*a^2*d*f*h-a*b*f*(c*h+17*d*g)+b^2*(-8*c*e*h+9*c*f*g+8*d*e*g))/(-a*
d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^3/(f*x+e)^(1/2)+1/96*(63*a^3*d^2*f^2*h-a^2*b
*d*f^2*(22*c*h+167*d*g)+a*b^2*f*(152*d^2*e*g+7*c^2*f*h+2*c*d*(-76*e*h+91*f
*g))-b^3*(48*d^2*e^2*g+7*c^2*f*(-8*e*h+9*f*g)+8*c*d*e*(-6*e*h+7*f*g)))/(-a
*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2/(f*x+e)^(1/2)+1/192*(315*a^4*d^3*f^3*h-a^
3*b*d^2*f^3*(233*c*h+1027*d*g)+a^2*b^2*d*f^2*(1336*d^2*e*g+145*c^2*f*h+c*d
*(-1336*e*h+1745*f*g))-a*b^3*f*(816*d^3*e^2*g+35*c^3*f^2*h+5*c^2*d*f*(-208
*e*h+245*f*g)+16*c*d^2*e*(-51*e*h+65*f*g))+b^4*(192*d^3*e^3*g+35*c^3*f^2*(
-8*e*h+9*f*g)+40*c^2*d*e*f*(-6*e*h+7*f*g)+48*c*d^2*e^2*(-4*e*h+5*f*g)))/(-
a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)/(f*x+e)^(1/2)+1/64*b^(1/2)*(315*a^5*d^4*f^
4*h-105*a^4*b*d^3*f^4*(4*c*h+11*d*g)+42*a^3*b^2*d^2*f^3*(44*d^2*e*g+9*c^2*
f*h+22*c*d*(-2*e*h+3*f*g))-18*a^2*b^3*d*f^2*(88*d^3*e^2*g+10*c^3*f^2*h+...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.75 (sec) , antiderivative size = 918, normalized size of antiderivative = 0.71

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \frac{48(-bg+ah)+\frac{8(9a^2dfh-abf(17dg+ch)+b^2(8deg+9cfg-8ceh))(a+bx)}{(bc-ad)(be-af)}+\frac{(a+bx)^2}{}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^5*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
(48*(-(b*g) + a*h) + (8*(9*a^2*d*f*h - a*b*f*(17*d*g + c*h) + b^2*(8*d*e*g
 + 9*c*f*g - 8*c*e*h))*(a + b*x))/((b*c - a*d)*(b*e - a*f)) + ((a + b*x)^2
*(2*(b*c - a*d)^2*(b*e - a*f)^2*(63*a^3*d^2*f^2*h - a^2*b*d*f^2*(167*d*g +
 22*c*h) + a*b^2*f*(152*d^2*e*g + 7*c^2*f*h + 2*c*d*(91*f*g - 76*e*h)) + b
^3*(-48*d^2*e^2*g + 8*c*d*e*(-7*f*g + 6*e*h) + 7*c^2*f*(-9*f*g + 8*e*h)))
- ((a + b*x)*(-((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(-315*a^4*d^3*f^3*h
 + a^3*b*d^2*f^3*(1027*d*g + 233*c*h) - a^2*b^2*d*f^2*(1336*d^2*e*g + 145*
c^2*f*h + c*d*(1745*f*g - 1336*e*h)) + b^4*(-192*d^3*e^3*g + 48*c*d^2*e^2*
(-5*f*g + 4*e*h) + 40*c^2*d*e*f*(-7*f*g + 6*e*h) + 35*c^3*f^2*(-9*f*g + 8*
e*h)) + a*b^3*f*(816*d^3*e^2*g + 35*c^3*f^2*h + 5*c^2*d*f*(245*f*g - 208*e
*h) - 16*c*d^2*e*(-65*f*g + 51*e*h)))) - 3*(a + b*x)*((d*e - c*f)*(-315*a^
5*d^4*f^4*h + 105*a^4*b*d^3*f^4*(11*d*g + 4*c*h) - 42*a^3*b^2*d^2*f^3*(44*
d^2*e*g + 9*c^2*f*h + 22*c*d*(3*f*g - 2*e*h)) + 18*a^2*b^3*d*f^2*(88*d^3*e
^2*g + 10*c^3*f^2*h + 33*c^2*d*f*(5*f*g - 4*e*h) - 44*c*d^2*e*(-3*f*g + 2*
e*h)) - a*b^4*f*(704*d^4*e^3*g + 35*c^4*f^3*h + 220*c^3*d*f^2*(7*f*g - 6*e
*h) - 352*c*d^3*e^2*(-3*f*g + 2*e*h) - 264*c^2*d^2*e*f*(-5*f*g + 4*e*h)) +
 b^5*(128*d^4*e^4*g + 35*c^4*f^3*(9*f*g - 8*e*h) - 64*c*d^3*e^3*(-3*f*g +
2*e*h) - 48*c^2*d^2*e^2*f*(-5*f*g + 4*e*h) - 40*c^3*d*e*f^2*(-7*f*g + 6*e*
h)))*Hypergeometric2F1[-1/2, 1, 1/2, (b*(e + f*x))/(b*e - a*f)] - 128*d^4*
(b*e - a*f)^5*(d*g - c*h)*Hypergeometric2F1[-1/2, 1, 1/2, (d*(e + f*x))...
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 1414, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int -\frac{af(8dg+ch)-b(8deg+9cfg-8ceh)-9df(bg-ah)x}{2(a+bx)^4(c+dx)(e+fx)^{3/2}}dx}{4(bc-ad)(be-af)}-\frac{bg-ah}{4(a+bx)^4\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{af(8dg+ch)-b(8deg+9cfg-8ceh)-9df(bg-ah)x}{(a+bx)^4(c+dx)(e+fx)^{3/2}}dx}{8(bc-ad)(be-af)} - \frac{bg-ah}{4(a+bx)^4\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{9a^2dfh-abf(ch+17dg)+b^2(-8ceh+9cfg+8deg)}{3(a+bx)^3\sqrt{e+fx}(bc-ad)(be-af)} - \frac{\int -\frac{\left(7f(9fg-8eh)c^2+8de(7fg-6eh)c+48d^2e^2g\right)b^2-af\left(7fhc^2+d(119fg-96eh)c+96d^2eg\right)b+3a^2df^2}{2(a+bx)^3(c+dx)(e+fx)^{3/2}}}{3(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{bg-ah}{4(a+bx)^4\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(7f(9fg-8eh)c^2+8de(7fg-6eh)c+48d^2e^2g\right)b^2-af\left(7fhc^2+d(119fg-96eh)c+96d^2eg\right)b+3a^2df^2(16dg+5ch)+7df\left(9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8c\right.}{(a+bx)^3(c+dx)(e+fx)^{3/2}}}{6(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{bg-ah}{4(a+bx)^4\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$\frac{63a^3d^2f^2h-a^2bdf^2(22ch+167dg)+ab^2f\left(7c^2fh+2cd(91fg-76eh)+152d^2eg\right)-b^3\left(7c^2f(9fg-8eh)+8cde(7fg-6eh)+48d^2e^2g\right)}{2(a+bx)^2\sqrt{e+fx}(bc-ad)(be-af)} - \int -\frac{-\left(\left(35f^2(9fg-8eh)c^3+40\right.\right.}{}$$

$$\frac{bg-ah}{4(a+bx)^4\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\int \frac{-\left(\left(35f^2(9fg-8eh)c^3+40def(7fg-6eh)c^2+48d^2e^2(5fg-4eh)c+192d^3e^3g\right)b^3\right)+af\left(35f^2hc^3+10df(91fg-76eh)c^2+8d^2e(95fg-72eh)c+576d^3e^2g\right)b^2-a^2df^2}{}$$

$$\frac{bg-ah}{4(a+bx)^4\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8de\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

$\downarrow$ 27

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8de\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

$\downarrow$ 169

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8de\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

$\downarrow$ 27

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8de\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

$\downarrow$ 174

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8d\cdots\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{\cdots}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

$\big\downarrow$ 73

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8d\cdots\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{\cdots}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

$\big\downarrow$ 221

$$\frac{\frac{9dfha^2-bf(17dg+ch)a+b^2(8deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3\sqrt{e+fx}}+\frac{63d^2f^2ha^3-bdf^2(167dg+22ch)a^2+b^2f\left(7fhc^2+2d(91fg-76eh)c+152d^2eg\right)a-b^3\left(7f(9fg-8eh)c^2+8d\cdots\right.}{2(bc-ad)(be-af)(a+bx)^2\sqrt{e+fx}}}{\cdots}$$

$$\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4\sqrt{e+fx}}$$

input

```
Int[(g + h*x)/((a + b*x)^5*(c + d*x)*(e + f*x)^(3/2)),x]
```

output
```
-1/4*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4*Sqrt[e + f*x]) + ((9
*a^2*d*f*h - a*b*f*(17*d*g + c*h) + b^2*(8*d*e*g + 9*c*f*g - 8*c*e*h))/(3*
(b*c - a*d)*(b*e - a*f)*(a + b*x)^3*Sqrt[e + f*x]) + ((63*a^3*d^2*f^2*h -
a^2*b*d*f^2*(167*d*g + 22*c*h) + a*b^2*f*(152*d^2*e*g + 7*c^2*f*h + 2*c*d*
(91*f*g - 76*e*h)) - b^3*(48*d^2*e^2*g + 7*c^2*f*(9*f*g - 8*e*h) + 8*c*d*e
*(7*f*g - 6*e*h)))/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*Sqrt[e + f*x]) +
 ((315*a^4*d^3*f^3*h - a^3*b*d^2*f^3*(1027*d*g + 233*c*h) + a^2*b^2*d*f^2*
(1336*d^2*e*g + 145*c^2*f*h + c*d*(1745*f*g - 1336*e*h)) - a*b^3*f*(816*d^
3*e^2*g + 35*c^3*f^2*h + 5*c^2*d*f*(245*f*g - 208*e*h) + 16*c*d^2*e*(65*f*
g - 51*e*h)) + b^4*(192*d^3*e^3*g + 35*c^3*f^2*(9*f*g - 8*e*h) + 40*c^2*d*
e*f*(7*f*g - 6*e*h) + 48*c*d^2*e^2*(5*f*g - 4*e*h)))/((b*c - a*d)*(b*e - a
*f)*(a + b*x)*Sqrt[e + f*x]) + (3*((-2*f*(a^4*d^3*f^3*(128*d*f*g - 315*d*e
*h + 187*c*f*h) + a^3*b*d^2*f^3*(515*d^2*e*g - 1027*c*d*f*g + 745*c*d*e*h
- 233*c^2*f*h) - a^2*b^2*d*f^2*(568*d^3*e^2*g - 145*c^3*f^2*h - c^2*d*f*(1
745*f*g - 1481*e*h) + c*d^2*e*(409*f*g - 568*e*h)) + a*b^3*f*(304*d^4*e^3*
g - 35*c^4*f^3*h + c^2*d^2*e*f*(185*f*g - 224*e*h) - 25*c^3*d*f^2*(49*f*g
- 43*e*h) + 16*c*d^3*e^2*(14*f*g - 19*e*h)) - b^4*(64*d^4*e^4*g + 5*c^3*d*
e*f^2*(7*f*g - 8*e*h) - 35*c^4*f^3*(9*f*g - 8*e*h) + 8*c^2*d^2*e^2*f*(5*f*
g - 6*e*h) + 16*c*d^3*e^3*(3*f*g - 4*e*h))))/((b*e - a*f)*(d*e - c*f)*Sqrt
[e + f*x]) - ((-2*Sqrt[b]*(d*e - c*f)*(315*a^5*d^4*f^4*h - 105*a^4*b*d^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 207.01 (sec) , antiderivative size = 2320, normalized size of antiderivative = 1.80

method	result	size
pseudoelliptic	Expression too large to display	2320
derivativedivides	Expression too large to display	3576
default	Expression too large to display	3576

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-315/128*((c*f-d*e)*d)^(1/2)*(b*x+a)^4*(c*f-d*e)*(f*x+e)^(1/2)*((a^4*(-
11/3*b*g+a*h)*d^4-4/3*a^3*c*(a*h-33/5*b*g)*b*d^3+6/5*a^2*c^2*(a*h-55/7*b*g
)*b^2*d^2-4/7*a*c^3*(a*h-77/9*b*g)*b^3*d+1/9*b^4*c^4*(a*h-9*b*g))*f^4-88/1
5*(a^3*d^3-9/7*a^2*b*c*d^2+5/7*a*b^2*c^2*d-5/33*b^3*c^3)*(c*h-d*g)*b^2*e*f
^3+176/35*d*(c*h-d*g)*(a^2*d^2-2/3*a*b*c*d+5/33*b^2*c^2)*b^3*e^2*f^2-704/3
15*d^2*(c*h-d*g)*(a*d-3/11*b*c)*b^4*e^3*f+128/315*b^5*d^3*e^4*(c*h-d*g))*b
*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))+((a*f-b*e)*b)^(1/2)*((f*x+e)^
(1/2)*d^5*(b*x+a)^4*(a*f-b*e)^5*(c*h-d*g)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e
)*d)^(1/2))+((c*f-d*e)*d)^(1/2)*((-a^4*g*(b*x+a)^4*d^4+4*a^3*c*b*(1027/512
*b^4*g*x^4+11297/1536*a*x^3*(-51/1027*h*x+g)*b^3+15071/1536*a^2*x^2*(-1929
/15071*h*x+g)*b^2+2813/512*a^3*x*(-765/2813*h*x+g)*b+a^4*(-325/512*h*x+g))
*d^3-6*a^2*c^2*(1745/768*b^4*g*x^4+19195/2304*a*x^3*(-699/19195*h*x+g)*b^3
+25477/2304*a^2*x^2*(-2563/25477*h*x+g)*b^2+4655/768*a^3*x*(-5/21*h*x+g)*b
+a^4*(-535/768*h*x+g))*b^2*d^2+4*a*c^3*(1225/512*b^4*g*x^4+13475/1536*a*(-
87/2695*h*x+g)*x^3*b^3+17885/1536*a^2*x^2*(-319/3577*h*x+g)*b^2+3255/512*a
^3*(-2117/9765*h*x+g)*x*b+a^4*(-367/512*h*x+g))*b^3*d-c^4*b^4*(315/128*b^4
*g*x^4+1155/128*a*x^3*(-1/33*h*x+g)*b^3+1533/128*a^2*x^2*(-55/657*h*x+g)*b
^2+837/128*a^3*x*(-511/2511*h*x+g)*b+a^4*(-93/128*h*x+g)))*f^5+(a^3*(-515/
128*b^5*g*x^4-5153/384*a*x^3*(-945/5153*h*x+g)*b^4-5855/384*a^2*x^2*(-693/
1171*h*x+g)*b^3-765/128*a^3*x*(-511/255*h*x+g)*b^2+837/128*a^4*b*h*x+a^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**5/(d*x+c)/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4921 vs. $2(1248) = 2496$.

Time = 1.00 (sec) , antiderivative size = 4921, normalized size of antiderivative = 3.81

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/64*(128*b^6*d^4*e^4*g + 192*b^6*c*d^3*e^3*f*g - 704*a*b^5*d^4*e^3*f*g +
240*b^6*c^2*d^2*e^2*f^2*g - 1056*a*b^5*c*d^3*e^2*f^2*g + 1584*a^2*b^4*d^4*
e^2*f^2*g + 280*b^6*c^3*d*e*f^3*g - 1320*a*b^5*c^2*d^2*e*f^3*g + 2376*a^2*
b^4*c*d^3*e*f^3*g - 1848*a^3*b^3*d^4*e*f^3*g + 315*b^6*c^4*f^4*g - 1540*a*
b^5*c^3*d*f^4*g + 2970*a^2*b^4*c^2*d^2*f^4*g - 2772*a^3*b^3*c*d^3*f^4*g +
1155*a^4*b^2*d^4*f^4*g - 128*b^6*c*d^3*e^4*h - 192*b^6*c^2*d^2*e^3*f*h + 7
04*a*b^5*c*d^3*e^3*f*h - 240*b^6*c^3*d*e^2*f^2*h + 1056*a*b^5*c^2*d^2*e^2*
f^2*h - 1584*a^2*b^4*c*d^3*e^2*f^2*h - 280*b^6*c^4*e*f^3*h + 1320*a*b^5*c^
3*d*e*f^3*h - 2376*a^2*b^4*c^2*d^2*e*f^3*h + 1848*a^3*b^3*c*d^3*e*f^3*h -
35*a*b^5*c^4*f^4*h + 180*a^2*b^4*c^3*d*f^4*h - 378*a^3*b^3*c^2*d^2*f^4*h +
 420*a^4*b^2*c*d^3*f^4*h - 315*a^5*b*d^4*f^4*h)*arctan(sqrt(f*x + e)*b/sqr
t(-b^2*e + a*b*f))/((b^10*c^5*e^5 - 5*a*b^9*c^4*d*e^5 + 10*a^2*b^8*c^3*d^2
*e^5 - 10*a^3*b^7*c^2*d^3*e^5 + 5*a^4*b^6*c*d^4*e^5 - a^5*b^5*d^5*e^5 - 5*
a*b^9*c^5*e^4*f + 25*a^2*b^8*c^4*d*e^4*f - 50*a^3*b^7*c^3*d^2*e^4*f + 50*a
^4*b^6*c^2*d^3*e^4*f - 25*a^5*b^5*c*d^4*e^4*f + 5*a^6*b^4*d^5*e^4*f + 10*a
^2*b^8*c^5*e^3*f^2 - 50*a^3*b^7*c^4*d*e^3*f^2 + 100*a^4*b^6*c^3*d^2*e^3*f^
2 - 100*a^5*b^5*c^2*d^3*e^3*f^2 + 50*a^6*b^4*c*d^4*e^3*f^2 - 10*a^7*b^3*d^
5*e^3*f^2 - 10*a^3*b^7*c^5*e^2*f^3 + 50*a^4*b^6*c^4*d*e^2*f^3 - 100*a^5*b^
5*c^3*d^2*e^2*f^3 + 100*a^6*b^4*c^2*d^3*e^2*f^3 - 50*a^7*b^3*c*d^4*e^2*f^3
 + 10*a^8*b^2*d^5*e^2*f^3 + 5*a^4*b^6*c^5*e*f^4 - 25*a^5*b^5*c^4*d*e*f^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^5*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 34973, normalized size of antiderivative = 27.11

$$\int \frac{g+hx}{(a+bx)^5(c+dx)(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)/(f*x+e)^(3/2),x)
```

output
```
( - 945*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**9*c**2*d**4*f**6*h + 1890*sqrt(b)*sqrt(e + f*x)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*c*d*
*5*e*f**5*h - 945*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*d**6*e**2*f**4*h + 1260*sqrt(b)*sqrt(
e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))
*a**8*b*c**3*d**3*f**6*h - 2520*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**2*d**4*e*f**5*h +
3465*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**8*b*c**2*d**4*f**6*g - 3780*sqrt(b)*sqrt(e + f*x)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c**
2*d**4*f**6*h*x + 1260*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e
+ f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c*d**5*e**2*f**4*h - 6930*sqrt
(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
 - b*e)))*a**8*b*c*d**5*e*f**5*g + 7560*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b*c*d**5*e*f**5
*h*x + 3465*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a**8*b*d**6*e**2*f**4*g - 3780*sqrt(b)*sqrt(e +
f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
8*b*d**6*e**2*f**4*h*x - 1134*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*ata...
```

3.177 $\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 265

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{2(be-af)^3(fg-eh)}{f^3(de-cf)^2\sqrt{e+fx}} + \frac{2b^2(3adfh+b(dfg-2deh-2cfh))\sqrt{e+fx}}{d^3f^3} + \frac{(bc-ad)^3(dg-ch)\sqrt{e+fx}}{d^3(de-cf)^2(c+dx)} + \frac{2b^3h(e+fx)^{3/2}}{3d^2f^3} + \frac{(bc-ad)^2\left(ad(3dfg-2deh-cfh)-b(6d^2eg+5c^2fh-cd(3fg+8eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{7/2}(de-cf)^{5/2}}$$

output

```
2*(-a*f+b*e)^3*(-e*h+f*g)/f^3/(-c*f+d*e)^2/(f*x+e)^(1/2)+2*b^2*(3*a*d*f*h+
b*(-2*c*f*h-2*d*e*h+d*f*g))*(f*x+e)^(1/2)/d^3/f^3+(-a*d+b*c)^3*(-c*h+d*g)*
(f*x+e)^(1/2)/d^3/(-c*f+d*e)^2/(d*x+c)+2/3*b^3*h*(f*x+e)^(3/2)/d^2/f^3+(-a
*d+b*c)^2*(a*d*(-c*f*h-2*d*e*h+3*d*f*g)-b*(6*d^2*e*g+5*c^2*f*h-c*d*(8*e*h+
3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(7/2)/(-c*f+d*e
)^(5/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{-9a^2bd^2f^2(2d^2e(-fg+eh)x+c^2fh(e+fx)+cd(-3efg+2e^2h-f^2gx))+}{}$$
$$+\frac{(bc-ad)^2\left(ad(-3dfg+2deh+cfh)+b(6d^2eg+5c^2fh-cd(3fg+8eh))\right)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{7/2}(-de+cf)^{5/2}}$$

input
```
Integrate[((a + b*x)^3*(g + h*x))/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(-9*a^2*b*d^2*f^2*(2*d^2*e*(-(f*g) + e*h)*x + c^2*f*h*(e + f*x) + c*d*(-3*
e*f*g + 2*e^2*h - f^2*g*x)) + 3*a^3*d^3*f^3*(c*(-2*f*g + 3*e*h + f*h*x) -
d*(3*f*g*x + e*(g - 2*h*x))) - 9*a*b^2*d*f*(-3*c^3*f^2*h*(e + f*x) - 2*d^3
*e^2*x*(-(f*g) + 2*e*h + f*h*x) + c^2*d*f*(e + f*x)*(4*e*h + f*(g - 2*h*x)
) + 2*c*d^2*e*(-2*e^2*h + 2*f^2*h*x^2 + e*f*(g + h*x))) + b^3*(-15*c^4*f^3
*h*(e + f*x) + c^3*d*f^2*(e + f*x)*(9*f*g + 14*e*h - 10*f*h*x) + 2*d^4*e^2
*x*(-8*e^2*h + e*f*(6*g - 4*h*x) + f^2*x*(3*g + h*x)) - 2*c*d^3*e*(-6*e^2*
f*g + 8*e^3*h + 3*e*f^2*x*(g - h*x) + 2*f^3*x^2*(3*g + h*x)) + 2*c^2*d^2*f
*(e + f*x)*(4*e^2*h + f^2*x*(3*g + h*x) + e*f*(-6*g + 5*h*x))))/(3*d^3*f^3
*(d*e - c*f)^2*(c + d*x)*Sqrt[e + f*x]) + ((b*c - a*d)^2*(a*d*(-3*d*f*g +
2*d*e*h + c*f*h) + b*(6*d^2*e*g + 5*c^2*f*h - c*d*(3*f*g + 8*e*h)))*ArcTan
[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(7/2)*(-(d*e) + c*f)^(5/2
))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 167, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(3dfg-2deh-cfh)+b(3dfg+2deh-5cfh)x)}{2(c+dx)(e+fx)^{3/2}}dx}{d(de-cf)}-\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(3dfg-2deh-cfh)+b(3dfg+2deh-5cfh)x)}{(c+dx)(e+fx)^{3/2}}dx}{2d(de-cf)}-\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 167$$

$$\frac{\frac{2(a+bx)^2(be-af)(-cfh-2deh+3dfg)}{f\sqrt{e+fx}(de-cf)}-\frac{2\int \frac{(a+bx)\left(df(3dfg-2deh-cfh)a^2-bf\left(fhc^2+d(9fg-16eh)c+6d^2eg\right)a+4b^2ce(3dfg-2deh-cfh)-b\left(3adf(3dfg-\cdots\right.\right.}{2(c+dx)\sqrt{e+fx}}\cdots}{f(de-cf)}}{2d(de-cf)}-\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2(a+bx)^2(be-af)(-cfh-2deh+3dfg)}{f\sqrt{e+fx}(de-cf)}-\frac{\int \frac{(a+bx)\left(df(3dfg-2deh-cfh)a^2-bf\left(fhc^2+d(9fg-16eh)c+6d^2eg\right)a+4b^2ce(3dfg-2deh-cfh)-b\left(3adf(3dfg-2\cdots\right.\right.}{(c+dx)\sqrt{e+fx}}\cdots}{f(de-cf)}}{2d(de-cf)}-\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 164$$

$$\frac{\frac{2(a+bx)^2(be-af)(-cfh-2deh+3dfg)}{f\sqrt{e+fx}(de-cf)}-\frac{\frac{f(bc-ad)^2\left(ad(-cfh-2deh+3dfg)-b\left(5c^2fh-cd(8eh+3fg)+6d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d^2}-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(\cdots\right.}{\cdots}}{\cdots}}{\cdots}-\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2(a+bx)^2(be-af)(-cfh-2deh+3dfg)}{f\sqrt{e+fx}(de-cf)}-\frac{\frac{2(bc-ad)^2\left(ad(-cfh-2deh+3dfg)-b\left(5c^2fh-cd(8eh+3fg)+6d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{d^2}-\frac{2b\sqrt{e+fx}\left(6\cdots\right.}{\cdots}}{\cdots}}{\cdots}-\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2(a+bx)^2(be-af)(-cfh-2deh+3dfg)}{f\sqrt{e+fx}(de-cf)} - \frac{-\frac{2b\sqrt{e+fx}\left(6a^2d^2f^2(-cfh-2deh+3dfg)+bdfx\left(3adf(-cfh-2deh+3dfg)+b\left(5c^2f^2h-cdf(4eh+3fg)-2d^2e(3fg\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(a+bx)^3(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

input
```
Int[((a + b*x)^3*(g + h*x))/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-(((d*g - c*h)*(a + b*x)^3)/(d*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x])) + ((2
*(b*e - a*f)*(3*d*f*g - 2*d*e*h - c*f*h)*(a + b*x)^2)/(f*(d*e - c*f)*Sqrt[
e + f*x]) - ((-2*b*Sqrt[e + f*x]*(6*a^2*d^2*f^2*(3*d*f*g - 2*d*e*h - c*f*h
) + 9*a*b*d*f*(3*c^2*f^2*h - 2*d^2*e*(f*g - 2*e*h) - c*d*f*(f*g + 4*e*h))
- b^2*(15*c^3*f^3*h - 4*d^3*e^2*(3*f*g - 4*e*h) + 4*c*d^2*e*f*(3*f*g - 2*e
*h) - c^2*d*f^2*(9*f*g + 14*e*h)) + b*d*f*(3*a*d*f*(3*d*f*g - 2*d*e*h - c*
f*h) + b*(5*c^2*f^2*h - 2*d^2*e*(3*f*g - 4*e*h) - c*d*f*(3*f*g + 4*e*h)))*
x))/(3*d^2*f^2) - (2*(b*c - a*d)^2*f*(a*d*(3*d*f*g - 2*d*e*h - c*f*h) - b*
(6*d^2*e*g + 5*c^2*f*h - c*d*(3*f*g + 8*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f
*x])/Sqrt[d*e - c*f]])/(d^(5/2)*Sqrt[d*e - c*f]))/(f*(d*e - c*f)))/(2*d*(d
*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(245) = 490$.

Time = 0.73 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.94

method	result
risch	$\frac{2b^2(hbdfx+9adfh-6bcfh-5bdeh+3bgdf)\sqrt{fx+e}}{3f^3d^3}+\frac{2d^3\left(a^3e\,f^3h-a^3f^4g-3a^2b\,e^2f^2h+3a^2be\,f^3g+3a\,b^2e^3fh-3a\,b^2e^2f^2 \cdots\right.}{(cf-de)^2\sqrt{fx+e}}$
derivativedivides	$\frac{2b^2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+3adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}-2bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^3}-\frac{2\left(-a^3e\,f^3h+a^3f^4g+3a^2b\,e^2f^2h-3a^2be\,f^3g-3 \cdots\right.}{(cf-de)^2\sqrt{f \cdots}}$
default	$\frac{2b^2\left(\frac{dh(fx+e)^{\frac{3}{2}}b}{3}+3adfh\sqrt{fx+e}-2bcfh\sqrt{fx+e}-2bdeh\sqrt{fx+e}+bdfg\sqrt{fx+e}\right)}{d^3}-\frac{2\left(-a^3e\,f^3h+a^3f^4g+3a^2b\,e^2f^2h-3a^2be\,f^3g-3 \cdots\right.}{(cf-de)^2\sqrt{f \cdots}}$
pseudoelliptic	$(xd+c)\sqrt{fx+e}\,((-3afg+2e(ah+3bg))d^2+c((ah-3bg)f-8ehb)d+5b\,c^2fh)f^3(ad-bc)^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\sqrt{(cf \cdots}$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2*(b*d*f*h*x+9*a*d*f*h-6*b*c*f*h-5*b*d*e*h+3*b*d*f*g)*(f*x+e)^(1/2)/
f^3/d^3+2/d^3/f^3*(d^3*(a^3*e*f^3*h-a^3*f^4*g-3*a^2*b*e^2*f^2*h+3*a^2*b*e*
f^3*g+3*a*b^2*e^3*f*h-3*a*b^2*e^2*f^2*g-b^3*e^4*h+b^3*e^3*f*g)/(c*f-d*e)^2
/(f*x+e)^(1/2)+f^3/(c*f-d*e)^2*((1/2*a^3*c*d^3*f*h-1/2*a^3*d^4*f*g-3/2*a^2
*b*c^2*d^2*f*h+3/2*a^2*b*c*d^3*f*g+3/2*a*b^2*c^3*d*f*h-3/2*a*b^2*c^2*d^2*f
*g-1/2*c^4*f*h*b^3+1/2*b^3*c^3*d*f*g)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/
2*(a^3*c*d^3*f*h+2*a^3*d^4*e*h-3*a^3*d^4*f*g+3*a^2*b*c^2*d^2*f*h-12*a^2*b*
c*d^3*e*h+3*a^2*b*c*d^3*f*g+6*a^2*b*d^4*e*g-9*a*b^2*c^3*d*f*h+18*a*b^2*c^2
*d^2*e*h+3*a*b^2*c^2*d^2*f*g-12*a*b^2*c*d^3*e*g+5*b^3*c^4*f*h-8*b^3*c^3*d*
e*h-3*b^3*c^3*d*f*g+6*b^3*c^2*d^2*e*g)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e
)^(1/2)/((c*f-d*e)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(245) = 490$.

Time = 0.28 (sec) , antiderivative size = 3748, normalized size of antiderivative = 14.14

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(h*x+g)/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(245) = 490$.

Time = 0.15 (sec) , antiderivative size = 936, normalized size of antiderivative = 3.53

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
(6*b^3*c^2*d^2*e*g - 12*a*b^2*c*d^3*e*g + 6*a^2*b*d^4*e*g - 3*b^3*c^3*d*f*
g + 3*a*b^2*c^2*d^2*f*g + 3*a^2*b*c*d^3*f*g - 3*a^3*d^4*f*g - 8*b^3*c^3*d*
e*h + 18*a*b^2*c^2*d^2*e*h - 12*a^2*b*c*d^3*e*h + 2*a^3*d^4*e*h + 5*b^3*c^
4*f*h - 9*a*b^2*c^3*d*f*h + 3*a^2*b*c^2*d^2*f*h + a^3*c*d^3*f*h)*arctan(sq
rt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^5*e^2 - 2*c*d^4*e*f + c^2*d^3*f^2)
*sqrt(-d^2*e + c*d*f)) + (2*(f*x + e)*b^3*d^4*e^3*f*g - 2*b^3*d^4*e^4*f*g
- 6*(f*x + e)*a*b^2*d^4*e^2*f^2*g + 2*b^3*c*d^3*e^3*f^2*g + 6*a*b^2*d^4*e^
3*f^2*g + 6*(f*x + e)*a^2*b*d^4*e*f^3*g - 6*a*b^2*c*d^3*e^2*f^3*g - 6*a^2*
b*d^4*e^2*f^3*g + (f*x + e)*b^3*c^3*d*f^4*g - 3*(f*x + e)*a*b^2*c^2*d^2*f^
4*g + 3*(f*x + e)*a^2*b*c*d^3*f^4*g - 3*(f*x + e)*a^3*d^4*f^4*g + 6*a^2*b*
c*d^3*e*f^4*g + 2*a^3*d^4*e*f^4*g - 2*a^3*c*d^3*f^5*g - 2*(f*x + e)*b^3*d^
4*e^4*h + 2*b^3*d^4*e^5*h + 6*(f*x + e)*a*b^2*d^4*e^3*f*h - 2*b^3*c*d^3*e^
4*f*h - 6*a*b^2*d^4*e^4*f*h - 6*(f*x + e)*a^2*b*d^4*e^2*f^2*h + 6*a*b^2*c*
d^3*e^3*f^2*h + 6*a^2*b*d^4*e^3*f^2*h + 2*(f*x + e)*a^3*d^4*e*f^3*h - 6*a^
2*b*c*d^3*e^2*f^3*h - 2*a^3*d^4*e^2*f^3*h - (f*x + e)*b^3*c^4*f^4*h + 3*(f
*x + e)*a*b^2*c^3*d*f^4*h - 3*(f*x + e)*a^2*b*c^2*d^2*f^4*h + (f*x + e)*a^
3*c*d^3*f^4*h + 2*a^3*c*d^3*e*f^4*h)/((d^5*e^2*f^3 - 2*c*d^4*e*f^4 + c^2*d
^3*f^5)*((f*x + e)^(3/2)*d - sqrt(f*x + e)*d*e + sqrt(f*x + e)*c*f)) + 2/3
*(3*sqrt(f*x + e)*b^3*d^4*f^7*g + (f*x + e)^(3/2)*b^3*d^4*f^6*h - 6*sqrt(f
*x + e)*b^3*d^4*e*f^6*h - 6*sqrt(f*x + e)*b^3*c*d^3*f^7*h + 9*sqrt(f*x ...
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.16

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \sqrt{e+f\,x}\left(\frac{2\,b^3\,f\,g-8\,b^3\,e\,h+6\,a\,b^2\,f\,h}{d^2\,f^3} - \frac{4\,b^3\,h\,(c\,f-d\,e)}{d^3\,f^3}\right)$$
$$- \frac{\frac{2\,\left(-h\,a^3\,d^3\,e\,f^3+g\,a^3\,d^3\,f^4+3\,h\,a^2\,b\,d^3\,e^2\,f^2-3\,g\,a^2\,b\,d^3\,e\,f^3-3\,h\,a\,b^2\,d^3\,e^3\,f+3\,g\,a\,b^2\,d^3\,e^2\,f^2+h\,b^3\,d^3\,e^4-g\,b^3\,d^3\,e^3\,f\right)}{c\,f-d\,e} - \frac{(e+f\,x)\,\left(h\,a^3\,c\,d^3\,f \cdots\right.}{\cdots}}{\cdots}$$
$$+ \frac{2\,b^3\,h\,(e+f\,x)^{3/2}}{3\,d^2\,f^3}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,(a\,d-b\,c)^2\,\left(c^2\,d^3\,f^2-2\,c\,d^4\,e\,f+d^5\,e^2\right)\,\left(2\,a\,d^2\,e\,h-3\,a\,d^2\,f\,g+6\,b\,d^2\,e\,g+5\,b \cdots\right.}{d^{5/2}\,(c\,f-d\,e)^{5/2}\,\left(2\,a^3\,d^4\,e\,h-3\,a^3\,d^4\,f\,g+5\,b^3\,c^4\,f\,h+6\,a^2\,b\,d^4\,e\,g+a^3\,c\,d^3\,f\,h-8\,b^3\,c^3\,d\,e\,h-3\,b^3\,c^3\,d\,f\,g+6\,b^3\,c^2\,d^2\,e\,g-12\,a\,b^2\,c\,d^3\,e\,g-1 \cdots\right.}\right.}{\cdots}$$

input
```
int(((g + h*x)*(a + b*x)^3)/((e + f*x)^(3/2)*(c + d*x)^2),x)
```

output
```
(e + f*x)^(1/2)*((2*b^3*f*g - 8*b^3*e*h + 6*a*b^2*f*h)/(d^2*f^3) - (4*b^3*
h*(c*f - d*e))/(d^3*f^3)) - ((2*(a^3*d^3*f^4*g + b^3*d^3*e^4*h - a^3*d^3*e
*f^3*h - b^3*d^3*e^3*f*g + 3*a*b^2*d^3*e^2*f^2*g + 3*a^2*b*d^3*e^2*f^2*h -
 3*a^2*b*d^3*e*f^3*g - 3*a*b^2*d^3*e^3*f*h))/(c*f - d*e) - ((e + f*x)*(a^3
*c*d^3*f^4*h - b^3*c^4*f^4*h - 2*b^3*d^4*e^4*h - 3*a^3*d^4*f^4*g + b^3*c^3
*d*f^4*g + 2*a^3*d^4*e*f^3*h + 2*b^3*d^4*e^3*f*g - 3*a*b^2*c^2*d^2*f^4*g -
 3*a^2*b*c^2*d^2*f^4*h - 6*a*b^2*d^4*e^2*f^2*g - 6*a^2*b*d^4*e^2*f^2*h + 3
*a^2*b*c*d^3*f^4*g + 3*a*b^2*c^3*d*f^4*h + 6*a^2*b*d^4*e*f^3*g + 6*a*b^2*d
^4*e^3*f*h))/(c*f - d*e)^2)/((e + f*x)^(1/2)*(c*d^3*f^4 - d^4*e*f^3) + d^4
*f^3*(e + f*x)^(3/2)) + (2*b^3*h*(e + f*x)^(3/2))/(3*d^2*f^3) + (atan(((e
+ f*x)^(1/2)*(a*d - b*c)^2*(d^5*e^2 + c^2*d^3*f^2 - 2*c*d^4*e*f)*(2*a*d^2*
e*h - 3*a*d^2*f*g + 6*b*d^2*e*g + 5*b*c^2*f*h + a*c*d*f*h - 8*b*c*d*e*h -
3*b*c*d*f*g))/(d^(5/2)*(c*f - d*e)^(5/2)*(2*a^3*d^4*e*h - 3*a^3*d^4*f*g +
5*b^3*c^4*f*h + 6*a^2*b*d^4*e*g + a^3*c*d^3*f*h - 8*b^3*c^3*d*e*h - 3*b^3*
c^3*d*f*g + 6*b^3*c^2*d^2*e*g - 12*a*b^2*c*d^3*e*g - 12*a^2*b*c*d^3*e*h +
3*a^2*b*c*d^3*f*g - 9*a*b^2*c^3*d*f*h + 18*a*b^2*c^2*d^2*e*h + 3*a*b^2*c^2
*d^2*f*g + 3*a^2*b*c^2*d^2*f*h)))*(a*d - b*c)^2*(2*a*d^2*e*h - 3*a*d^2*f*g
 + 6*b*d^2*e*g + 5*b*c^2*f*h + a*c*d*f*h - 8*b*c*d*e*h - 3*b*c*d*f*g))/(d^
(7/2)*(c*f - d*e)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 3035, normalized size of antiderivative = 11.45

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
(3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a**3*c**2*d**3*f**4*h + 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f
- d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*e*f**
3*h - 9*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt
(d)*sqrt(c*f - d*e)))*a**3*c*d**4*f**4*g + 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**4*f**
4*h*x + 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sq
rt(d)*sqrt(c*f - d*e)))*a**3*d**5*e*f**3*h*x - 9*sqrt(d)*sqrt(e + f*x)*sqr
t(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**5*f
**4*g*x + 9*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(
sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**2*f**4*h - 36*sqrt(d)*sqrt(e + f*
x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*
b*c**2*d**3*e*f**3*h + 9*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(
e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**2*d**3*f**4*g + 9*sqrt(d)
*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
d*e)))*a**2*b*c**2*d**3*f**4*h*x + 18*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c*d**4*e*f**3*g
 - 36*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a**2*b*c*d**4*e*f**3*h*x + 9*sqrt(d)*sqrt(e + f*x)*sqr
t(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c...
```

3.178 $\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 218

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = -\frac{2(be-af)^2(fg-eh)}{f^2(de-cf)^2\sqrt{e+fx}} + \frac{2b^2h\sqrt{e+fx}}{d^2f^2} - \frac{(bc-ad)^2(dg-ch)\sqrt{e+fx}}{d^2(de-cf)^2(c+dx)} - \frac{(bc-ad)\left(ad(3dfg-2deh-cfh)-b(4d^2eg+3c^2fh-cd(fg+6eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{5/2}(de-cf)^{5/2}}$$

output

```
-2*(-a*f+b*e)^2*(-e*h+f*g)/f^2/(-c*f+d*e)^2/(f*x+e)^(1/2)+2*b^2*h*(f*x+e)^
(1/2)/d^2/f^2-(-a*d+b*c)^2*(-c*h+d*g)*(f*x+e)^(1/2)/d^2/(-c*f+d*e)^2/(d*x+
c)-(-a*d+b*c)*(a*d*(-c*f*h-2*d*e*h+3*d*f*g)-b*(4*d^2*e*g+3*c^2*f*h-c*d*(6*
e*h+f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-c*f+d
*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{-2abdf(2d^2e(-fg+eh)x+c^2fh(e+fx)+cd(-3efg+2e^2h-f^2gx))+a}{}$$

$$+\frac{(-bc+ad)\,(ad(-3dfg+2deh+cfh)+b(4d^2eg+3c^2fh-cd(fg+6eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{5/2}(-de+cf)^{5/2}}$$

input
```
Integrate[((a + b*x)^2*(g + h*x))/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(-2*a*b*d*f*(2*d^2*e*(-(f*g) + e*h)*x + c^2*f*h*(e + f*x) + c*d*(-3*e*f*g
+ 2*e^2*h - f^2*g*x)) + a^2*d^2*f^2*(-(d*(e*g + 3*f*g*x - 2*e*h*x)) + c*(-
2*f*g + 3*e*h + f*h*x)) + b^2*(3*c^3*f^2*h*(e + f*x) + 2*d^3*e^2*x*(-(f*g)
 + 2*e*h + f*h*x) + c^2*d*f*(e + f*x)*(-(f*g) - 4*e*h + 2*f*h*x) + 2*c*d^2
*e*(2*e^2*h - 2*f^2*h*x^2 - e*f*(g + h*x))))/(d^2*f^2*(d*e - c*f)^2*(c + d
*x)*Sqrt[e + f*x]) + ((-(b*c) + a*d)*(a*d*(-3*d*f*g + 2*d*e*h + c*f*h) + b
*(4*d^2*e*g + 3*c^2*f*h - c*d*(f*g + 6*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x
])/Sqrt[-(d*e) + c*f]])/(d^(5/2)*(-(d*e) + c*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(3dfg-2deh-cfh)+b(dfg+2deh-3cfh)x)}{2(c+dx)(e+fx)^{3/2}}dx}{d(de-cf)} - \frac{(a+bx)^2(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(3dfg-2deh-cfh)+b(dfg+2deh-3cfh)x)}{(c+dx)(e+fx)^{3/2}}dx}{2d(de-cf)} - \frac{(a+bx)^2(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{ 163}$$

$$\frac{\frac{(bc-ad)\left(ad(-cfh-2deh+3dfg)-b\left(3c^2fh-cd(6eh+fg)+4d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{d(de-cf)} + \frac{2\left(-a^2df^2(-cfh-2deh+3dfg)+2abdef(-cfh-2deh+3\right.}{}}{2d(de-cf)}$$

$$\frac{(a+bx)^2(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{2(bc-ad)\left(ad(-cfh-2deh+3dfg)-b\left(3c^2fh-cd(6eh+fg)+4d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df(de-cf)} + \frac{2\left(-a^2df^2(-cfh-2deh+3dfg)+2abdef(-cfh\right.}{}}{2d(de-cf)}$$

$$\frac{(a+bx)^2(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2\left(-a^2df^2(-cfh-2deh+3dfg)+2abdef(-cfh-2deh+3dfg)+b^2e\left(3c^2f^2h-cdf(4eh+fg)-2d^2e(fg-2eh)\right)+b^2fx(de-cf)(-3cfh+2deh+dfg)\right)}{df^2\sqrt{e+fx}(de-cf)} -}{2d(de-cf)}$$

$$\frac{(a+bx)^2(dg-ch)}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

input
```
Int[((a + b*x)^2*(g + h*x))/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-(((d*g - c*h)*(a + b*x)^2)/(d*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x])) + ((2
*(2*a*b*d*e*f*(3*d*f*g - 2*d*e*h - c*f*h) - a^2*d*f^2*(3*d*f*g - 2*d*e*h -
 c*f*h) + b^2*e*(3*c^2*f^2*h - 2*d^2*e*(f*g - 2*e*h) - c*d*f*(f*g + 4*e*h)
) + b^2*f*(d*e - c*f)*(d*f*g + 2*d*e*h - 3*c*f*h)*x))/(d*f^2*(d*e - c*f)*S
qrt[e + f*x]) - (2*(b*c - a*d)*(a*d*(3*d*f*g - 2*d*e*h - c*f*h) - b*(4*d^2
*e*g + 3*c^2*f*h - c*d*(f*g + 6*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqr
t[d*e - c*f]])/(d^(3/2)*(d*e - c*f)^(3/2)))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{\frac{2h\,b^2\sqrt{fx+e}}{d^2}-\frac{2\left(-a^2e\,f^2h+a^2f^3g+2ab\,e^2fh-2abe\,f^2g-b^2e^3h+b^2e^2fg\right)}{(cf-de)^2\sqrt{fx+e}}+\frac{2f^2\left(\frac{\left(\frac{1}{2}a^2c\,d^2fh-\frac{1}{2}a^2d^3fg-ab\,c^2dfh+abc\,d^2fg+\right.}{(fx+e)d+cf-de}\right.}{}}{}$
default	$\frac{\frac{2h\,b^2\sqrt{fx+e}}{d^2}-\frac{2\left(-a^2e\,f^2h+a^2f^3g+2ab\,e^2fh-2abe\,f^2g-b^2e^3h+b^2e^2fg\right)}{(cf-de)^2\sqrt{fx+e}}+\frac{2f^2\left(\frac{\left(\frac{1}{2}a^2c\,d^2fh-\frac{1}{2}a^2d^3fg-ab\,c^2dfh+abc\,d^2fg+\right.}{(fx+e)d+cf-de}\right.}{}}{}$
pseudoelliptic	$\frac{(xd+c)\sqrt{fx+e}\,((-3afg+2e(ah+2bg))d^2+((ah-bg)f-6ehb)cd+3b\,c^2fh)\,f^2(ad-bc)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\sqrt{(cf-}}{}$
risch	$\frac{2b^2h\sqrt{fx+e}}{d^2f^2}+\frac{2d^2\left(a^2e\,f^2h-a^2f^3g-2ab\,e^2fh+2abe\,f^2g+b^2e^3h-b^2e^2fg\right)}{(cf-de)^2\sqrt{fx+e}}+\frac{2f^2\left(\frac{\left(\frac{1}{2}a^2c\,d^2fh-\frac{1}{2}a^2d^3fg-ab\,c^2dfh+abc\,d^2\right.}{(fx+e)d+cf}\right.}{}$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f^2*(h*b^2/d^2*(f*x+e)^(1/2)+f^2/(c*f-d*e)^2/d^2*((1/2*a^2*c*d^2*f*h-1/2
*a^2*d^3*f*g-a*b*c^2*d*f*h+a*b*c*d^2*f*g+1/2*b^2*c^3*f*h-1/2*b^2*c^2*d*f*g
)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(a^2*c*d^2*f*h+2*a^2*d^3*e*h-3*a^2
*d^3*f*g+2*a*b*c^2*d*f*h-8*a*b*c*d^2*e*h+2*a*b*c*d^2*f*g+4*a*b*d^3*e*g-3*b
^2*c^3*f*h+6*b^2*c^2*d*e*h+b^2*c^2*d*f*g-4*b^2*c*d^2*e*g)/((c*f-d*e)*d)^(1
/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-(-a^2*e*f^2*h+a^2*f^3*g+2
*a*b*e^2*f*h-2*a*b*e*f^2*g-b^2*e^3*h+b^2*e^2*f*g)/(c*f-d*e)^2/(f*x+e)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(202) = 404$.

Time = 0.18 (sec) , antiderivative size = 2423, normalized size of antiderivative = 11.11

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output

```
[-1/2*(sqrt(d^2*e - c*d*f)*(((4*(b^2*c*d^3 - a*b*d^4)*e*f^3 - (b^2*c^2*d^2
 + 2*a*b*c*d^3 - 3*a^2*d^4)*f^4)*g - (2*(3*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2
*d^4)*e*f^3 - (3*b^2*c^3*d - 2*a*b*c^2*d^2 - a^2*c*d^3)*f^4)*h)*x^2 + (4*(
b^2*c^2*d^2 - a*b*c*d^3)*e^2*f^2 - (b^2*c^3*d + 2*a*b*c^2*d^2 - 3*a^2*c*d^
3)*e*f^3)*g - (2*(3*b^2*c^3*d - 4*a*b*c^2*d^2 + a^2*c*d^3)*e^2*f^2 - (3*b^
2*c^4 - 2*a*b*c^3*d - a^2*c^2*d^2)*e*f^3)*h + ((4*(b^2*c*d^3 - a*b*d^4)*e^
2*f^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*e*f^3 - (b^2*c^3*d + 2*a*b
*c^2*d^2 - 3*a^2*c*d^3)*f^4)*g - (2*(3*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4
)*e^2*f^2 + 3*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*e*f^3 - (3*b^2*c^4 -
 2*a*b*c^3*d - a^2*c^2*d^2)*f^4)*h)*x)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d
^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) - 2*(2*(b^2*d^5*e^3*f - 3*b^2*c*d^
4*e^2*f^2 + 3*b^2*c^2*d^3*e*f^3 - b^2*c^3*d^2*f^4)*h*x^2 - (2*b^2*c*d^4*e^
3*f - 2*a^2*c^2*d^3*f^4 - (b^2*c^2*d^3 + 6*a*b*c*d^4 - a^2*d^5)*e^2*f^2 -
(b^2*c^3*d^2 - 6*a*b*c^2*d^3 - a^2*c*d^4)*e*f^3)*g + (4*b^2*c*d^4*e^4 - 4*
(2*b^2*c^2*d^3 + a*b*c*d^4)*e^3*f + (7*b^2*c^3*d^2 + 2*a*b*c^2*d^3 + 3*a^2
*c*d^4)*e^2*f^2 - (3*b^2*c^4*d - 2*a*b*c^3*d^2 + 3*a^2*c^2*d^3)*e*f^3)*h -
 ((2*b^2*d^5*e^3*f - 2*(b^2*c*d^4 + 2*a*b*d^5)*e^2*f^2 + (b^2*c^2*d^3 + 2*
a*b*c*d^4 + 3*a^2*d^5)*e*f^3 - (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + 3*a^2*c*d^4)
*f^4)*g - (4*b^2*d^5*e^4 - 2*(3*b^2*c*d^4 + 2*a*b*d^5)*e^3*f + 2*(2*a*b*c*
d^4 + a^2*d^5)*e^2*f^2 + (5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*e*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(h*x+g)/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(202) = 404$.

Time = 0.14 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.86

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx =$$

$$-\frac{(4\,b^2cd^2eg-4\,abd^3eg-b^2c^2dfg-2\,abcd^2fg+3\,a^2d^3fg-6\,b^2c^2deh+8\,abcd^2eh-2\,a^2d^3eh+3\,b^2c^3fh-}{(d^4e^2-2\,cd^3ef+c^2d^2f^2)\sqrt{-d^2e+cdf}}$$
$$+\frac{2\sqrt{fx+e}b^2h}{d^2f^2}$$
$$-\frac{2\,(fx+e)b^2d^3e^2fg-2\,b^2d^3e^3fg-4\,(fx+e)abd^3ef^2g+2\,b^2cd^2e^2f^2g+4\,abd^3e^2f^2g+(fx+e)b^2c^2df^3g}{}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-(4*b^2*c*d^2*e*g - 4*a*b*d^3*e*g - b^2*c^2*d*f*g - 2*a*b*c*d^2*f*g + 3*a^
2*d^3*f*g - 6*b^2*c^2*d*e*h + 8*a*b*c*d^2*e*h - 2*a^2*d^3*e*h + 3*b^2*c^3*
f*h - 2*a*b*c^2*d*f*h - a^2*c*d^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e
+ c*d*f))/((d^4*e^2 - 2*c*d^3*e*f + c^2*d^2*f^2)*sqrt(-d^2*e + c*d*f)) + 2
*sqrt(f*x + e)*b^2*h/(d^2*f^2) - (2*(f*x + e)*b^2*d^3*e^2*f*g - 2*b^2*d^3*
e^3*f*g - 4*(f*x + e)*a*b*d^3*e*f^2*g + 2*b^2*c*d^2*e^2*f^2*g + 4*a*b*d^3*
e^2*f^2*g + (f*x + e)*b^2*c^2*d*f^3*g - 2*(f*x + e)*a*b*c*d^2*f^3*g + 3*(f
*x + e)*a^2*d^3*f^3*g - 4*a*b*c*d^2*e*f^3*g - 2*a^2*d^3*e*f^3*g + 2*a^2*c*
d^2*f^4*g - 2*(f*x + e)*b^2*d^3*e^3*h + 2*b^2*d^3*e^4*h + 4*(f*x + e)*a*b*
d^3*e^2*f*h - 2*b^2*c*d^2*e^3*f*h - 4*a*b*d^3*e^3*f*h - 2*(f*x + e)*a^2*d^
3*e*f^2*h + 4*a*b*c*d^2*e^2*f^2*h + 2*a^2*d^3*e^2*f^2*h - (f*x + e)*b^2*c^
3*f^3*h + 2*(f*x + e)*a*b*c^2*d*f^3*h - (f*x + e)*a^2*c*d^2*f^3*h - 2*a^2*
c*d^2*e*f^3*h)/((d^4*e^2*f^2 - 2*c*d^3*e*f^3 + c^2*d^2*f^4)*((f*x + e)^(3/
2)*d - sqrt(f*x + e)*d*e + sqrt(f*x + e)*c*f))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.76

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,(a\,d-b\,c)\,\left(c^2\,d^2\,f^2-2\,c\,d^3\,e\,f+d^4\,e^2\right)\,\left(2\,a\,d^2\,e\,h-3\,a\,d^2\,f\,g+4\,b\,d^2\,e\,g+\right.}{d^{3/2}\,(c\,f-d\,e)^{5/2}\,\left(2\,a^2\,d^3\,e\,h-3\,a^2\,d^3\,f\,g-3\,b^2\,c^3\,f\,h-4\,b^2\,c\,d^2\,e\,g+a^2\,c\,d^2\,f\,h+6\,b^2\,c^2\,d\,e\,h+b^2\,c^2\right.}}{}$$

$$-\frac{\frac{2\,\left(-h\,a^2\,d^2\,e\,f^2+g\,a^2\,d^2\,f^3+2\,h\,a\,b\,d^2\,e^2\,f-2\,g\,a\,b\,d^2\,e\,f^2-h\,b^2\,d^2\,e^3+g\,b^2\,d^2\,e^2\,f\right)}{c\,f-d\,e}-\frac{(e+f\,x)\,\left(h\,a^2\,c\,d^2\,f^3+2\,h\,a^2\,d^3\,e\,f^2-3\,g\,a^2\,d^3\,f^3-2\,h\,a\,b\right.}{}}{\sqrt{e+f\,x}\,\left(c\,d^2\,f^3-d^3\,e\,f^2\right)+d^3\,f^2\,(e}$$

$$+\frac{2\,b^2\,h\,\sqrt{e+f\,x}}{d^2\,f^2}$$

input
```
int(((g + h*x)*(a + b*x)^2)/((e + f*x)^(3/2)*(c + d*x)^2),x)
```

output
```
(atan(((e + f*x)^(1/2)*(a*d - b*c)*(d^4*e^2 + c^2*d^2*f^2 - 2*c*d^3*e*f)*(
2*a*d^2*e*h - 3*a*d^2*f*g + 4*b*d^2*e*g + 3*b*c^2*f*h + a*c*d*f*h - 6*b*c*
d*e*h - b*c*d*f*g))/(d^(3/2)*(c*f - d*e)^(5/2)*(2*a^2*d^3*e*h - 3*a^2*d^3*
f*g - 3*b^2*c^3*f*h - 4*b^2*c*d^2*e*g + a^2*c*d^2*f*h + 6*b^2*c^2*d*e*h +
b^2*c^2*d*f*g + 4*a*b*d^3*e*g - 8*a*b*c*d^2*e*h + 2*a*b*c*d^2*f*g + 2*a*b*
c^2*d*f*h))*(a*d - b*c)*(2*a*d^2*e*h - 3*a*d^2*f*g + 4*b*d^2*e*g + 3*b*c^
2*f*h + a*c*d*f*h - 6*b*c*d*e*h - b*c*d*f*g))/(d^(5/2)*(c*f - d*e)^(5/2))
 - ((2*(a^2*d^2*f^3*g - b^2*d^2*e^3*h - a^2*d^2*e*f^2*h + b^2*d^2*e^2*f*g -
 2*a*b*d^2*e*f^2*g + 2*a*b*d^2*e^2*f*h))/(c*f - d*e) - ((e + f*x)*(b^2*c^3
*f^3*h - 3*a^2*d^3*f^3*g + 2*b^2*d^3*e^3*h + a^2*c*d^2*f^3*h - b^2*c^2*d*f
^3*g + 2*a^2*d^3*e*f^2*h - 2*b^2*d^3*e^2*f*g + 2*a*b*c*d^2*f^3*g - 2*a*b*c
^2*d*f^3*h + 4*a*b*d^3*e*f^2*g - 4*a*b*d^3*e^2*f*h))/(c*f - d*e)^2)/((e +
f*x)^(1/2)*(c*d^2*f^3 - d^3*e*f^2) + d^3*f^2*(e + f*x)^(3/2)) + (2*b^2*h*(
e + f*x)^(1/2))/(d^2*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1935, normalized size of antiderivative = 8.88

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
(sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*a**2*c**2*d**2*f**3*h + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*e*f**2*
h - 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a**2*c*d**3*f**3*g + sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**3*f**3*h*
x + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d
)*sqrt(c*f - d*e)))*a**2*d**4*e*f**2*h*x - 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*
f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**4*f**3*
g*x + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt
(d)*sqrt(c*f - d*e)))*a*b*c**3*d*f**3*h - 8*sqrt(d)*sqrt(e + f*x)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*e*
f**2*h + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d**2*f**3*g + 2*sqrt(d)*sqrt(e + f*x)*sq
rt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**2*d
**2*f**3*h*x + 4*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*e*f**2*g - 8*sqrt(d)*sqrt(e + f*
x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c
*d**3*e*f**2*h*x + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c*d**3*f**3*g*x + 4*sqrt(d)*sqrt...
```

3.179 $\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 181

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{2(be-af)(fg-eh)}{f(de-cf)^2\sqrt{e+fx}} + \frac{(bc-ad)(dg-ch)\sqrt{e+fx}}{d(de-cf)^2(c+dx)} + \frac{(ad(3dfg-2deh-cfh)-b(2d^2eg+c^2fh+cd(fg-4eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{d^{3/2}(de-cf)^{5/2}}$$

output
```
2*(-a*f+b*e)*(-e*h+f*g)/f/(-c*f+d*e)^2/(f*x+e)^(1/2)+(-a*d+b*c)*(-c*h+d*g)
*(f*x+e)^(1/2)/d/(-c*f+d*e)^2/(d*x+c)+(a*d*(-c*f*h-2*d*e*h+3*d*f*g)-b*(2*d
^2*e*g+c^2*f*h+c*d*(-4*e*h+f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)
^(1/2))/d^(3/2)/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{-b(2d^2e(-fg+eh)x+c^2fh(e+fx)+cd(-3efg+2e^2h-f^2gx))+adf(-}{df(de-cf)^2(c+dx)\sqrt{e+fx}} + \frac{(ad(-3dfg+2deh+cfh)+b(2d^2eg+c^2fh+cd(fg-4eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{d^{3/2}(-de+cf)^{5/2}}$$

input
```
Integrate[((a + b*x)*(g + h*x))/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(-(b*(2*d^2*e*(-(f*g) + e*h)*x + c^2*f*h*(e + f*x) + c*d*(-3*e*f*g + 2*e^2
*h - f^2*g*x))) + a*d*f*(-(d*(e*g + 3*f*g*x - 2*e*h*x)) + c*(-2*f*g + 3*e*
h + f*h*x)))/(d*f*(d*e - c*f)^2*(c + d*x)*Sqrt[e + f*x]) + ((a*d*(-3*d*f*g
 + 2*d*e*h + c*f*h) + b*(2*d^2*e*g + c^2*f*h + c*d*(f*g - 4*e*h)))*ArcTan[
(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(d^(3/2)*(-(d*e) + c*f)^(5/2)
)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {161, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{161}$$

$$-\frac{\left(ad(-cfh-2deh+3dfg)-b\left(c^2fh+cd(fg-4eh)+2d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{2d(de-cf)^2}-\frac{x\left(adf(-cfh-2deh+3dfg)-b\left(-c^2f^2h+cdf^2g+2d^2e(fg-eh)\right)\right)+adf(-3ceh+2cfg+deg)-bce(-cfh-}{df(c+dx)\sqrt{e+fx}(de-cf)^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\left(ad(-cfh-2deh+3dfg)-b\left(c^2fh+cd(fg-4eh)+2d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{df(de-cf)^2}-\frac{x\left(adf(-cfh-2deh+3dfg)-b\left(-c^2f^2h+cdf^2g+2d^2e(fg-eh)\right)\right)+adf(-3ceh+2cfg+deg)-bce(-cfh-}{df(c+dx)\sqrt{e+fx}(de-cf)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(-cfh-2deh+3dfg)-b(c^2fh+cd(fg-4eh)+2d^2eg)\right)}{d^{3/2}(de-cf)^{5/2}}-$$
$$\frac{x\left(adf(-cfh-2deh+3dfg)-b\left(-c^2f^2h+cdf^2g+2d^2e(fg-eh)\right)\right)+adf(-3ceh+2cfg+deg)-bce(-cfh-}{df(c+dx)\sqrt{e+fx}(de-cf)^2}$$

input
```
Int[((a + b*x)*(g + h*x))/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-((a*d*f*(d*e*g + 2*c*f*g - 3*c*e*h) - b*c*e*(3*d*f*g - 2*d*e*h - c*f*h) +
 (a*d*f*(3*d*f*g - 2*d*e*h - c*f*h) - b*(c*d*f^2*g - c^2*f^2*h + 2*d^2*e*(
f*g - e*h)))*x)/(d*f*(d*e - c*f)^2*(c + d*x)*Sqrt[e + f*x])) + ((a*d*(3*d*
f*g - 2*d*e*h - c*f*h) - b*(2*d^2*e*g + c^2*f*h + c*d*(f*g - 4*e*h)))*ArcT
anh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(3/2)*(d*e - c*f)^(5/2))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{2\left(-afhe+g f^2 a+b e^2 h-bfge\right)}{(cf-de)^2\sqrt{fx+e}}+\frac{2f\left(\frac{f\left(acdh-a d^2 g-b c^2 h+bcdg\right)\sqrt{fx+e}}{2d((fx+e)d+cf-de)}+\frac{\left(acdfh+2a d^2 eh-3a d^2 fg+b c^2 fh-4bcdeh+bcdfg+\cdots\right.}{2d\sqrt{(cf-de)d}\cdots}\right.}{(cf-de)^2}}{f}$
default	$\frac{-\frac{2\left(-afhe+g f^2 a+b e^2 h-bfge\right)}{(cf-de)^2\sqrt{fx+e}}+\frac{2f\left(\frac{f\left(acdh-a d^2 g-b c^2 h+bcdg\right)\sqrt{fx+e}}{2d((fx+e)d+cf-de)}+\frac{\left(acdfh+2a d^2 eh-3a d^2 fg+b c^2 fh-4bcdeh+bcdfg+\cdots\right.}{2d\sqrt{(cf-de)d}\cdots}\right.}{(cf-de)^2}}{f}$
pseudoelliptic	$\frac{((-3afg+2(ah+bg)e)d^2+c((ah+bg)f-4ehb)d+b c^2 fh)(xd+c)\sqrt{fx+e}\, f \arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\left(\left(-a f^2 gx-\frac{(-2xg\cdots}{}\right.\right.}{\sqrt{fx+e}\,\sqrt{(cf-de)d}\, f(c\cdots}$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(-(-a*e*f*h+a*f^2*g+b*e^2*h-b*e*f*g)/(c*f-d*e)^2/(f*x+e)^(1/2)+f/(c*f-
d*e)^2*(1/2*f*(a*c*d*h-a*d^2*g-b*c^2*h+b*c*d*g)/d*(f*x+e)^(1/2)/((f*x+e)*d
+c*f-d*e)+1/2*(a*c*d*f*h+2*a*d^2*e*h-3*a*d^2*f*g+b*c^2*f*h-4*b*c*d*e*h+b*c
*d*f*g+2*b*d^2*e*g)/d/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e
)*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(167) = 334$.

Time = 0.14 (sec) , antiderivative size = 1445, normalized size of antiderivative = 7.98

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(d^2*e - c*d*f)*(((2*b*d^3*e*f^2 + (b*c*d^2 - 3*a*d^3)*f^3)*g -
(2*(2*b*c*d^2 - a*d^3)*e*f^2 - (b*c^2*d + a*c*d^2)*f^3)*h)*x^2 + (2*b*c*d^
2*e^2*f + (b*c^2*d - 3*a*c*d^2)*e*f^2)*g - (2*(2*b*c^2*d - a*c*d^2)*e^2*f
- (b*c^3 + a*c^2*d)*e*f^2)*h + ((2*b*d^3*e^2*f + 3*(b*c*d^2 - a*d^3)*e*f^2
 + (b*c^2*d - 3*a*c*d^2)*f^3)*g - (2*(2*b*c*d^2 - a*d^3)*e^2*f + 3*(b*c^2*
d - a*c*d^2)*e*f^2 - (b*c^3 + a*c^2*d)*f^3)*h)*x)*log((d*f*x + 2*d*e - c*f
 - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*((2*a*c^2*d^2*f^3 +
 (3*b*c*d^3 - a*d^4)*e^2*f - (3*b*c^2*d^2 + a*c*d^3)*e*f^2)*g - (2*b*c*d^3
*e^3 - (b*c^2*d^2 + 3*a*c*d^3)*e^2*f - (b*c^3*d - 3*a*c^2*d^2)*e*f^2)*h +
((2*b*d^4*e^2*f - (b*c*d^3 + 3*a*d^4)*e*f^2 - (b*c^2*d^2 - 3*a*c*d^3)*f^3)
*g - (2*b*d^4*e^3 - 2*(b*c*d^3 + a*d^4)*e^2*f + (b*c^2*d^2 + a*c*d^3)*e*f^
2 - (b*c^3*d - a*c^2*d^2)*f^3)*h)*x)*sqrt(f*x + e))/(c*d^5*e^4*f - 3*c^2*d
^4*e^3*f^2 + 3*c^3*d^3*e^2*f^3 - c^4*d^2*e*f^4 + (d^6*e^3*f^2 - 3*c*d^5*e^
2*f^3 + 3*c^2*d^4*e*f^4 - c^3*d^3*f^5)*x^2 + (d^6*e^4*f - 2*c*d^5*e^3*f^2
+ 2*c^3*d^3*e*f^4 - c^4*d^2*f^5)*x), (sqrt(-d^2*e + c*d*f)*(((2*b*d^3*e*f^
2 + (b*c*d^2 - 3*a*d^3)*f^3)*g - (2*(2*b*c*d^2 - a*d^3)*e*f^2 - (b*c^2*d +
 a*c*d^2)*f^3)*h)*x^2 + (2*b*c*d^2*e^2*f + (b*c^2*d - 3*a*c*d^2)*e*f^2)*g
- (2*(2*b*c^2*d - a*c*d^2)*e^2*f - (b*c^3 + a*c^2*d)*e*f^2)*h + ((2*b*d^3*
e^2*f + 3*(b*c*d^2 - a*d^3)*e*f^2 + (b*c^2*d - 3*a*c*d^2)*f^3)*g - (2*(2*b
*c*d^2 - a*d^3)*e^2*f + 3*(b*c^2*d - a*c*d^2)*e*f^2 - (b*c^3 + a*c^2*d)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(167) = 334.

Time = 0.13 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{(2\,bd^2eg+bcdfg-3\,ad^2fg-4\,bcdeh+2\,ad^2eh+bc^2fh+acdfh)\arctan\left(\frac{-}{\sqrt{}}\right.}{(d^3e^2-2\,cd^2ef+c^2df^2)\sqrt{-d^2e+cdf}} + \frac{2\,(fx+e)bd^2efg-2\,bd^2e^2fg+(fx+e)bcdf^2g-3\,(fx+e)ad^2f^2g+2\,bcdef^2g+2\,ad^2ef^2g-2\,acdf^3g}{(d^3e^2f-2\,cd^2ef^2+c^2d}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
(2*b*d^2*e*g + b*c*d*f*g - 3*a*d^2*f*g - 4*b*c*d*e*h + 2*a*d^2*e*h + b*c^2
*f*h + a*c*d*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((d^3*e^2 -
 2*c*d^2*e*f + c^2*d*f^2)*sqrt(-d^2*e + c*d*f)) + (2*(f*x + e)*b*d^2*e*f*g
 - 2*b*d^2*e^2*f*g + (f*x + e)*b*c*d*f^2*g - 3*(f*x + e)*a*d^2*f^2*g + 2*b
*c*d*e*f^2*g + 2*a*d^2*e*f^2*g - 2*a*c*d*f^3*g - 2*(f*x + e)*b*d^2*e^2*h +
 2*b*d^2*e^3*h + 2*(f*x + e)*a*d^2*e*f*h - 2*b*c*d*e^2*f*h - 2*a*d^2*e^2*f
*h - (f*x + e)*b*c^2*f^2*h + (f*x + e)*a*c*d*f^2*h + 2*a*c*d*e*f^2*h)/((d^
3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*((f*x + e)^(3/2)*d - sqrt(f*x + e)*d*
e + sqrt(f*x + e)*c*f))
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{e+f\,x}\,\left(c^2\,d\,f^2-2\,c\,d^2\,e\,f+d^3\,e^2\right)}{\sqrt{d}\,{\left(c\,f-d\,e\right)}^{5/2}}\right)\,\left(2\,a\,d^2\,e\,h-3\,a\,d^2\,f\,g+2\,b\,d^2\,e\,g+b\,c^2\,f\,h\right.}{d^{3/2}\,{\left(c\,f-d\,e\right)}^{5/2}}$$
$$-\frac{\frac{2\,\left(a\,f^2\,g+b\,e^2\,h-a\,e\,f\,h-b\,e\,f\,g\right)}{c\,f-d\,e}-\frac{\left(e+f\,x\right)\,\left(a\,c\,d\,f^2\,h-b\,c^2\,f^2\,h-2\,b\,d^2\,e^2\,h-3\,a\,d^2\,f^2\,g+b\,c\,d\,f^2\,g+2\,a\,d^2\,e\,f\,h+2\,b\,d^2\,e\,f\,g\right)}{d\,{\left(c\,f-d\,e\right)}^2}}{\sqrt{e+f\,x}\,\left(c\,f^2-d\,e\,f\right)+d\,f\,{\left(e+f\,x\right)}^{3/2}}$$

input
```
int(((g + h*x)*(a + b*x))/((e + f*x)^(3/2)*(c + d*x)^2),x)
```

output
```
(atan(((e + f*x)^(1/2)*(d^3*e^2 + c^2*d*f^2 - 2*c*d^2*e*f))/(d^(1/2)*(c*f
- d*e)^(5/2)))*(2*a*d^2*e*h - 3*a*d^2*f*g + 2*b*d^2*e*g + b*c^2*f*h + a*c*
d*f*h - 4*b*c*d*e*h + b*c*d*f*g))/(d^(3/2)*(c*f - d*e)^(5/2)) - ((2*(a*f^2
*g + b*e^2*h - a*e*f*h - b*e*f*g))/(c*f - d*e) - ((e + f*x)*(a*c*d*f^2*h -
 b*c^2*f^2*h - 2*b*d^2*e^2*h - 3*a*d^2*f^2*g + b*c*d*f^2*g + 2*a*d^2*e*f*h
 + 2*b*d^2*e*f*g))/(d*(c*f - d*e)^2))/((e + f*x)^(1/2)*(c*f^2 - d*e*f) + d
*f*(e + f*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1089, normalized size of antiderivative = 6.02

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
(sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*a*c**2*d*f**2*h + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*a
tan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*e*f*h - 3*sqrt(d
)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f -
 d*e)))*a*c*d**2*f**2*g + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt
(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**2*f**2*h*x + 2*sqrt(d)*sqrt
(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e))
)*a*d**3*e*f*h*x - 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e +
f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**3*f**2*g*x + sqrt(d)*sqrt(e + f*x)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*
f**2*h - 4*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*b*c**2*d*e*f*h + sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**2*d*f**2*g +
sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt
(c*f - d*e)))*b*c**2*d*f**2*h*x + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*
atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*e*f*g - 4*sqrt(
d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f
- d*e)))*b*c*d**2*e*f*h*x + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c*d**2*f**2*g*x + 2*sqrt(d)*sq
rt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - ...
```

3.180 $\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = -\frac{2(fg-eh)}{(de-cf)^2\sqrt{e+fx}} - \frac{(dg-ch)\sqrt{e+fx}}{(de-cf)^2(c+dx)} + \frac{(3dfg-2deh-cfh)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(de-cf)^{5/2}}$$

output

```
(2*e*h-2*f*g)/(-c*f+d*e)^2/(f*x+e)^(1/2)-(-c*h+d*g)*(f*x+e)^(1/2)/(-c*f+d*
e)^2/(d*x+c)+(-c*f*h-2*d*e*h+3*d*f*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+
d*e)^(1/2))/d^(1/2)/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{c(-2fg+3eh+fhx)-d(3fgx+e(g-2hx))}{(de-cf)^2(c+dx)\sqrt{e+fx}} + \frac{(-3dfg+2deh+cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-de+cf)^{5/2}}$$

input
```
Integrate[(g + h*x)/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(c*(-2*f*g + 3*e*h + f*h*x) - d*(3*f*g*x + e*(g - 2*h*x)))/((d*e - c*f)^2*
(c + d*x)*Sqrt[e + f*x]) + ((-3*d*f*g + 2*d*e*h + c*f*h)*ArcTan[(Sqrt[d]*S
qrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(Sqrt[d]*(-(d*e) + c*f)^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(-cfh-2deh+3dfg)\int \frac{1}{(c+dx)(e+fx)^{3/2}}dx}{2d(de-cf)}-\frac{dg-ch}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \ 61$$

$$-\frac{(-cfh-2deh+3dfg)\left(\frac{d\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{de-cf}+\frac{2}{\sqrt{e+fx}(de-cf)}\right)}{2d(de-cf)}-\frac{dg-ch}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \ 73$$

$$-\frac{(-cfh-2deh+3dfg)\left(\frac{2d\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(de-cf)}+\frac{2}{\sqrt{e+fx}(de-cf)}\right)}{2d(de-cf)}-\frac{dg-ch}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \ 221$$

$$-\frac{\left(\frac{2}{\sqrt{e+fx}(de-cf)}-\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(de-cf)^{3/2}}\right)(-cfh-2deh+3dfg)}{2d(de-cf)}-\frac{dg-ch}{d(c+dx)\sqrt{e+fx}(de-cf)}$$

input
```
Int[(g + h*x)/((c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-((d*g - c*h)/(d*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x])) - ((3*d*f*g - 2*d*e
*h - c*f*h)*(2/((d*e - c*f)*Sqrt[e + f*x]) - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*S
qrt[e + f*x])/Sqrt[d*e - c*f]])/(d*e - c*f)^(3/2)))/(2*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2(-eh+fg)}{(cf-de)^2\sqrt{fx+e}}+\frac{\frac{2\left(\frac{1}{2}cfh-\frac{1}{2}dfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(cfh+2deh-3dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{(cf-de)^2}$
default	$-\frac{2(-eh+fg)}{(cf-de)^2\sqrt{fx+e}}+\frac{\frac{2\left(\frac{1}{2}cfh-\frac{1}{2}dfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{(cfh+2deh-3dfg)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{\sqrt{(cf-de)d}}}{(cf-de)^2}$
pseudoelliptic	$\frac{\sqrt{fx+e}\,(xd+c)((2eh-3fg)d+cfh)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\left(\left(-fgx-\frac{e(-2hx+g)}{3}\right)d+c\left(\frac{(hx-2g)f}{3}+eh\right)\right)\sqrt{(cf-de)d}}{\sqrt{fx+e}\,\sqrt{(cf-de)d}\,(cf-de)^2(xd+c)}$

input
```
int((h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-e*h+f*g)/(c*f-d*e)^2/(f*x+e)^(1/2)+2/(c*f-d*e)^2*((1/2*c*f*h-1/2*d*f*
g)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(c*f*h+2*d*e*h-3*d*f*g)/((c*f-d*e
)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(114) = 228$.

Time = 0.10 (sec) , antiderivative size = 796, normalized size of antiderivative = 6.22

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*c*d*e*f*g + (3*d^2*f^2*g - (2*d^2*e*f + c*d*f^2)*h)*x^2 - (2*c*d
*e^2 + c^2*e*f)*h + (3*(d^2*e*f + c*d*f^2)*g - (2*d^2*e^2 + 3*c*d*e*f + c^
2*f^2)*h)*x)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e -
 c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*((d^3*e^2 + c*d^2*e*f - 2*c^2*d*f^2)
*g - 3*(c*d^2*e^2 - c^2*d*e*f)*h + (3*(d^3*e*f - c*d^2*f^2)*g - (2*d^3*e^2
 - c*d^2*e*f - c^2*d*f^2)*h)*x)*sqrt(f*x + e))/(c*d^4*e^4 - 3*c^2*d^3*e^3*
f + 3*c^3*d^2*e^2*f^2 - c^4*d*e*f^3 + (d^5*e^3*f - 3*c*d^4*e^2*f^2 + 3*c^2
*d^3*e*f^3 - c^3*d^2*f^4)*x^2 + (d^5*e^4 - 2*c*d^4*e^3*f + 2*c^3*d^2*e*f^3
 - c^4*d*f^4)*x), -((3*c*d*e*f*g + (3*d^2*f^2*g - (2*d^2*e*f + c*d*f^2)*h)
*x^2 - (2*c*d*e^2 + c^2*e*f)*h + (3*(d^2*e*f + c*d*f^2)*g - (2*d^2*e^2 + 3
*c*d*e*f + c^2*f^2)*h)*x)*sqrt(-d^2*e + c*d*f)*arctan(sqrt(-d^2*e + c*d*f)
*sqrt(f*x + e)/(d*f*x + d*e)) + ((d^3*e^2 + c*d^2*e*f - 2*c^2*d*f^2)*g - 3
*(c*d^2*e^2 - c^2*d*e*f)*h + (3*(d^3*e*f - c*d^2*f^2)*g - (2*d^3*e^2 - c*d
^2*e*f - c^2*d*f^2)*h)*x)*sqrt(f*x + e))/(c*d^4*e^4 - 3*c^2*d^3*e^3*f + 3*
c^3*d^2*e^2*f^2 - c^4*d*e*f^3 + (d^5*e^3*f - 3*c*d^4*e^2*f^2 + 3*c^2*d^3*e
*f^3 - c^3*d^2*f^4)*x^2 + (d^5*e^4 - 2*c*d^4*e^3*f + 2*c^3*d^2*e*f^3 - c^4
*d*f^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.51

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = -\frac{(3\,dfg-2\,deh-cfh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(d^2e^2-2\,cdef+c^2f^2)\sqrt{-d^2e+cdf}} - \frac{3\,(fx+e)dfg-2\,defg+2\,cf^2g-2\,(fx+e)deh+2\,de^2h-(fx+e)cfh-2\,cefh}{(d^2e^2-2\,cdef+c^2f^2)\left((fx+e)^{\frac{3}{2}}d-\sqrt{fx+e}de+\sqrt{fx+e}cf\right)}$$

input
```
integrate((h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-(3*d*f*g - 2*d*e*h - c*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/
((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sqrt(-d^2*e + c*d*f)) - (3*(f*x + e)*d*f*
g - 2*d*e*f*g + 2*c*f^2*g - 2*(f*x + e)*d*e*h + 2*d*e^2*h - (f*x + e)*c*f*
h - 2*c*e*f*h)/((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*((f*x + e)^(3/2)*d - sqrt(
f*x + e)*d*e + sqrt(f*x + e)*c*f))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{\frac{2\,(e\,h-f\,g)}{c\,f-d\,e}+\frac{(e+f\,x)\,(c\,f\,h+2\,d\,e\,h-3\,d\,f\,g)}{{(c\,f-d\,e)}^2}}{d\,{(e+f\,x)}^{3/2}+\sqrt{e+f\,x}\,(c\,f-d\,e)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e+f\,x}\,(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2)}{{(c\,f-d\,e)}^{5/2}}\right)\,(c\,f\,h+2\,d\,e\,h-3\,d\,f\,g)}{\sqrt{d}\,{(c\,f-d\,e)}^{5/2}}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(c + d*x)^2),x)
```

output
```
((2*(e*h - f*g))/(c*f - d*e) + ((e + f*x)*(c*f*h + 2*d*e*h - 3*d*f*g))/(c*
f - d*e)^2)/(d*(e + f*x)^(3/2) + (e + f*x)^(1/2)*(c*f - d*e)) + (atan((d^(
1/2)*(e + f*x)^(1/2)*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f))/(c*f - d*e)^(5/2))*(
c*f*h + 2*d*e*h - 3*d*f*g))/(d^(1/2)*(c*f - d*e)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.72

$$\int \frac{g+hx}{(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{fx+e}\,\sqrt{cf-de}\,atan\left(\frac{\sqrt{fx+e}\,d}{\sqrt{d}\,\sqrt{cf-de}}\right)c^2fh+2\sqrt{d}\,\sqrt{fx+e}\,\sqrt{cf-de}\,atan}{}$$

input
```
int((h*x+g)/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
(sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*c**2*f*h + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sq
rt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*e*h - 3*sqrt(d)*sqrt(e + f*x
)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d*f*
g + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*c*d*f*h*x + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan
((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**2*e*h*x - 3*sqrt(d)*sqrt(
e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))
*d**2*f*g*x + 3*c**2*d*e*f*h - 2*c**2*d*f**2*g + c**2*d*f**2*h*x - 3*c*d**
2*e**2*h + c*d**2*e*f*g + c*d**2*e*f*h*x - 3*c*d**2*f**2*g*x + d**3*e**2*g
 - 2*d**3*e**2*h*x + 3*d**3*e*f*g*x)/(sqrt(e + f*x)*d*(c**4*f**3 - 3*c**3*
d*e*f**2 + c**3*d*f**3*x + 3*c**2*d**2*e**2*f - 3*c**2*d**2*e*f**2*x - c*d
**3*e**3 + 3*c*d**3*e**2*f*x - d**4*e**3*x))
```

3.181 $\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 288

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx=\frac{f(b(deg+2cfg-3ceh)-a(3dfg-2deh-cfh))}{(bc-ad)(be-af)(de-cf)^2\sqrt{e+fx}}+\frac{dg-ch}{(bc-ad)(de-cf)(c+dx)\sqrt{e+fx}}-\frac{2b^{3/2}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^2(be-af)^{3/2}}+\frac{\sqrt{d}(ad(3dfg-2deh-cfh)+b(2d^2eg-5cdfg+3c^2fh))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^2(de-cf)^{5/2}}$$

output
```
f*(b*(-3*c*e*h+2*c*f*g+d*e*g)-a*(-c*f*h-2*d*e*h+3*d*f*g))/(-a*d+b*c)/(-a*f
+b*e)/(-c*f+d*e)^2/(f*x+e)^(1/2)+(-c*h+d*g)/(-a*d+b*c)/(-c*f+d*e)/(d*x+c)/
(f*x+e)^(1/2)-2*b^(3/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e
)^(1/2))/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)+d^(1/2)*(a*d*(-c*f*h-2*d*e*h+3*d*f*
g)+b*(3*c^2*f*h-5*c*d*f*g+2*d^2*e*g))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+
d*e)^(1/2))/(-a*d+b*c)^2/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{\frac{(bc-ad)\left(adf(-d(eg+3fgx-2ehx)+c(-2fg+3eh+fhx))+b\left(2c^2f(fg-eh)+d^2eg(e+fx)-cd(\right.\right.}{(be-af)(de-cf)^2(c+dx)\sqrt{e+fx}}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(((b*c - a*d)*(a*d*f*(-(d*(e*g + 3*f*g*x - 2*e*h*x)) + c*(-2*f*g + 3*e*h +
 f*h*x)) + b*(2*c^2*f*(f*g - e*h) + d^2*e*g*(e + f*x) - c*d*(e^2*h - 2*f^2
*g*x + 3*e*f*h*x))))/((b*e - a*f)*(d*e - c*f)^2*(c + d*x)*Sqrt[e + f*x]) -
 (2*b^(3/2)*(b*g - a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]]
)/(-(b*e) + a*f)^(3/2) + (Sqrt[d]*(a*d*(-3*d*f*g + 2*d*e*h + c*f*h) + b*(-
2*d^2*e*g + 5*c*d*f*g - 3*c^2*f*h))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(
d*e) + c*f]])/(-(d*e) + c*f)^(5/2))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{168}$$

$$\frac{\int \frac{2b(de-cf)g+a(3dfg-2deh-cfh)+3bf(dg-ch)x}{2(a+bx)(c+dx)(e+fx)^{3/2}}dx}{(bc-ad)(de-cf)}+\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2b(de-cf)g+a(3dfg-2deh-cfh)+3bf(dg-ch)x}{(a+bx)(c+dx)(e+fx)^{3/2}}dx}{2(bc-ad)(de-cf)}+\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{2f(b(-3ceh+2cfg+deg)-a(-cfh-2deh+3dfg))}{\sqrt{e+fx}(be-af)(de-cf)}-\frac{2\int -\frac{-df(3dfg-2deh-cfh)a^2-b\left(-e(fg-2eh)d^2-cf(2fg+eh)d+2c^2f^2h\right)a+2b^2(de-cf)^2g+bdf(b(\cdots}{2(a+bx)(c+dx)\sqrt{e+fx}}\cdots}{(be-af)(de-cf)}}{2(bc-ad)(de-cf)}\cdots$$

$$\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-df(3dfg-2deh-cfh)a^2-b\left(-e(fg-2eh)d^2-cf(2fg+eh)d+2c^2f^2h\right)a+2b^2(de-cf)^2g+bdf(b(deg+2cfg-3ceh)-a(3dfg-2deh-cfh))x}{(a+bx)(c+dx)\sqrt{e+fx}}dx}{(be-af)(de-cf)}+\frac{2f(b(-3ceh+\cdots}{\sqrt{\cdots}}}{2(bc-ad)(de-cf)}\cdots$$

$$\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{2b^2(bg-ah)(de-cf)^2\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{d(be-af)\left(ad(-cfh-2deh+3dfg)+b\left(3c^2fh-5cdfg+2d^2eg\right)\right)\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{bc-ad}}{(be-af)(de-cf)}+\frac{2f(b(-3ceh+2cfg+deg)-\cdots}{\sqrt{e+fx}(be-af\cdots}}{2(bc-ad)(de-cf)}\cdots$$

$$\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{4b^2(bg-ah)(de-cf)^2\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2d(be-af)\left(ad(-cfh-2deh+3dfg)+b\left(3c^2fh-5cdfg+2d^2eg\right)\right)\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(bc-ad)}}{(be-af)(de-cf)}+\frac{2f(b(-3ce\cdots}{\cdots}}{2(bc-ad)(de-cf)}\cdots$$

$$\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2\sqrt{d}(be-af)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(-cfh-2deh+3dfg)+b\left(3c^2fh-5cdfg+2d^2eg\right)\right)}{(bc-ad)\sqrt{de-cf}}-\frac{4b^{3/2}(bg-ah)(de-cf)^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)\sqrt{be-af}}}{(be-af)(de-cf)}+\frac{2f(b(-3ceh\cdots}{\cdots}}{2(bc-ad)(de-cf)}\cdots$$

$$\frac{dg-ch}{(c+dx)\sqrt{e+fx}(bc-ad)(de-cf)}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(d*g - c*h)/((b*c - a*d)*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x]) + ((2*f*(b*(
d*e*g + 2*c*f*g - 3*c*e*h) - a*(3*d*f*g - 2*d*e*h - c*f*h)))/((b*e - a*f)*
(d*e - c*f)*Sqrt[e + f*x]) + ((-4*b^(3/2)*(d*e - c*f)^2*(b*g - a*h)*ArcTan
h[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f])
+ (2*Sqrt[d]*(b*e - a*f)*(a*d*(3*d*f*g - 2*d*e*h - c*f*h) + b*(2*d^2*e*g -
 5*c*d*f*g + 3*c^2*f*h))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])
/((b*c - a*d)*Sqrt[d*e - c*f]))/((b*e - a*f)*(d*e - c*f)))/(2*(b*c - a*d)*
(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

method	result
derivativedivides	$2f\left(\frac{d\left(\frac{\left(\frac{1}{2}acdfh-\frac{1}{2}a\,d^{2}fg-\frac{1}{2}b\,c^{2}fh+\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{\left(acdfh+2a\,d^{2}eh-3a\,d^{2}fg-3b\,c^{2}fh+5bcdfg-2b\,d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx}}{\sqrt{(cf-}}\right.}{2\sqrt{(cf-de)d}}\right.}{(ad-bc)^{2}(cf-de)^{2}f}\right.$
default	$2f\left(\frac{d\left(\frac{\left(\frac{1}{2}acdfh-\frac{1}{2}a\,d^{2}fg-\frac{1}{2}b\,c^{2}fh+\frac{1}{2}bcdfg\right)\sqrt{fx+e}}{(fx+e)d+cf-de}+\frac{\left(acdfh+2a\,d^{2}eh-3a\,d^{2}fg-3b\,c^{2}fh+5bcdfg-2b\,d^{2}eg\right)\arctan\left(\frac{d\sqrt{fx}}{\sqrt{(cf-}}\right.}{2\sqrt{(cf-de)d}}\right.}{(ad-bc)^{2}(cf-de)^{2}f}\right.$
pseudoelliptic	$\frac{(xd+c)\sqrt{(af-be)b}\,d\sqrt{fx+e}\,(af-be)\left(\left(\left(-3h\,c^{2}+5cdg\right)f-2d^{2}eg\right)b+ad\left((ch-3dg)f+2deh\right)\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)+3\left(\right.}{}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(d/(a*d-b*c)^2/(c*f-d*e)^2/f*((1/2*a*c*d*f*h-1/2*a*d^2*f*g-1/2*b*c^2*f
*h+1/2*b*c*d*f*g)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(a*c*d*f*h+2*a*d^2
*e*h-3*a*d^2*f*g-3*b*c^2*f*h+5*b*c*d*f*g-2*b*d^2*e*g)/((c*f-d*e)*d)^(1/2)*
arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))+(a*h-b*g)*b^2/f/(a*d-b*c)^2/(
a*f-b*e)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))-(
-e*h+f*g)/(c*f-d*e)^2/(a*f-b*e)/(f*x+e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(265) = 530.

Time = 0.16 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.15

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{2\,(b^3g-ab^2h)\arctan\left(\frac{\sqrt{fx+e}b}{\sqrt{-b^2e+abf}}\right)}{(b^3c^2e-2\,ab^2cde+a^2bd^2e-ab^2c^2f+2\,a^2bcdf-a^3d^2f)\sqrt{-b^2e+a}}$$
$$-\frac{(2\,bd^3eg-5\,bcd^2fg+3\,ad^3fg-2\,ad^3eh+3\,bc^2dfh-acd^2fh)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{(b^2c^2d^2e^2-2\,abcd^3e^2+a^2d^4e^2-2\,b^2c^3def+4\,abc^2d^2ef-2\,a^2cd^3ef+b^2c^4f^2-2\,abc^3df^2+a^2c^2d^2f^2)\sqrt{}}$$
$$+\frac{(fx+e)bd^2efg+2\,(fx+e)bcdf^2g-3\,(fx+e)ad^2f^2g-2\,bcdef^2g+2\,ad^2ef^2g+2\,bc^2f^3g-2\,acdf^3g-}{(b^2cd^2e^3-abd^3e^3-2\,b^2c^2de^2f+abcd^2e^2f+a^2d^3e^2f+b^2c^3ef^2+abc^2def}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^3*g - a*b^2*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^3*c^2
*e - 2*a*b^2*c*d*e + a^2*b*d^2*e - a*b^2*c^2*f + 2*a^2*b*c*d*f - a^3*d^2*f
)*sqrt(-b^2*e + a*b*f)) - (2*b*d^3*e*g - 5*b*c*d^2*f*g + 3*a*d^3*f*g - 2*a
*d^3*e*h + 3*b*c^2*d*f*h - a*c*d^2*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e
 + c*d*f))/((b^2*c^2*d^2*e^2 - 2*a*b*c*d^3*e^2 + a^2*d^4*e^2 - 2*b^2*c^3*d
*e*f + 4*a*b*c^2*d^2*e*f - 2*a^2*c*d^3*e*f + b^2*c^4*f^2 - 2*a*b*c^3*d*f^2
 + a^2*c^2*d^2*f^2)*sqrt(-d^2*e + c*d*f)) + ((f*x + e)*b*d^2*e*f*g + 2*(f*
x + e)*b*c*d*f^2*g - 3*(f*x + e)*a*d^2*f^2*g - 2*b*c*d*e*f^2*g + 2*a*d^2*e
*f^2*g + 2*b*c^2*f^3*g - 2*a*c*d*f^3*g - 3*(f*x + e)*b*c*d*e*f*h + 2*(f*x
+ e)*a*d^2*e*f*h + 2*b*c*d*e^2*f*h - 2*a*d^2*e^2*f*h + (f*x + e)*a*c*d*f^2
*h - 2*b*c^2*e*f^2*h + 2*a*c*d*e*f^2*h)/((b^2*c*d^2*e^3 - a*b*d^3*e^3 - 2*
b^2*c^2*d*e^2*f + a*b*c*d^2*e^2*f + a^2*d^3*e^2*f + b^2*c^3*e*f^2 + a*b*c^
2*d*e*f^2 - 2*a^2*c*d^2*e*f^2 - a*b*c^3*f^3 + a^2*c^2*d*f^3)*((f*x + e)^(3
/2)*d - sqrt(f*x + e)*d*e + sqrt(f*x + e)*c*f))
```

Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 404752, normalized size of antiderivative = 1405.39

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)*(c + d*x)^2),x)
```

output
```
atan((((-(((8*a^4*b^5*d^9*e^5*g^2 - 4*b^9*c^9*f^5*g^2 - 9*a^9*d^9*f^5*g^2
- 4*a^2*b^7*c^9*f^5*h^2 + 8*a^6*b^3*d^9*e^5*h^2 + 8*b^9*c^4*d^5*e^5*g^2 -
a^9*c^2*d^7*f^5*h^2 - 4*a^9*d^9*e^2*f^3*h^2 + 16*a*b^8*c^8*d*f^5*g^2 + 66*
a^8*b*c*d^8*f^5*g^2 + 15*a^8*b*d^9*e*f^4*g^2 - 16*a^5*b^4*d^9*e^5*g*h - 4*
a^9*c*d^8*e*f^4*h^2 + 20*b^9*c^8*d*e*f^4*g^2 - 32*a*b^8*c^3*d^6*e^5*g^2 -
32*a^3*b^6*c*d^8*e^5*g^2 - 32*a^5*b^4*c*d^8*e^5*h^2 + 7*a^3*b^6*c^8*d*f^5*
h^2 + 10*a^8*b*c^3*d^6*f^5*h^2 - 12*a^7*b^2*d^9*e^4*f*h^2 + 12*a^8*b*d^9*e
^3*f^2*h^2 - 40*b^9*c^5*d^4*e^4*f*g^2 + 9*b^9*c^8*d*e^3*f^2*h^2 + 8*a*b^8*
c^9*f^5*g*h + 6*a^9*c*d^8*f^5*g*h + 12*a^9*d^9*e*f^4*g*h + 48*a^2*b^7*c^2*
d^7*e^5*g^2 + 8*a^2*b^7*c^4*d^5*e^5*h^2 - 24*a^2*b^7*c^7*d^2*f^5*g^2 - 32*
a^3*b^6*c^3*d^6*e^5*h^2 - 9*a^3*b^6*c^6*d^3*f^5*g^2 + 48*a^4*b^5*c^2*d^7*e
^5*h^2 + 126*a^4*b^5*c^5*d^4*f^5*g^2 - 279*a^5*b^4*c^4*d^5*f^5*g^2 + 316*a
^6*b^3*c^3*d^6*f^5*g^2 - 199*a^7*b^2*c^2*d^7*f^5*g^2 + 18*a^4*b^5*c^7*d^2*
f^5*h^2 - 63*a^5*b^4*c^6*d^3*f^5*h^2 + 72*a^6*b^3*c^5*d^4*f^5*h^2 - 39*a^7
*b^2*c^4*d^5*f^5*h^2 - 15*a^6*b^3*d^9*e^3*f^2*g^2 + 5*a^7*b^2*d^9*e^2*f^3*
g^2 + 65*b^9*c^6*d^3*e^3*f^2*g^2 - 40*b^9*c^7*d^2*e^2*f^3*g^2 - 230*a*b^8*
c^5*d^4*e^3*f^2*g^2 + 85*a*b^8*c^6*d^3*e^2*f^3*g^2 - 240*a^2*b^7*c^3*d^6*e
^4*f*g^2 + 195*a^2*b^7*c^6*d^3*e*f^4*g^2 + 160*a^3*b^6*c^2*d^7*e^4*f*g^2 -
 450*a^3*b^6*c^5*d^4*e*f^4*g^2 + 765*a^4*b^5*c^4*d^5*e*f^4*g^2 + 90*a^5*b^
4*c*d^8*e^3*f^2*g^2 - 780*a^5*b^4*c^3*d^6*e*f^4*g^2 + 10*a^6*b^3*c*d^8*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4163, normalized size of antiderivative = 14.45

$$\int \frac{g+hx}{(a+bx)(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output

```
(2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a*b*c**4*f**3*h - 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)
*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**3*d*e*f**2*h + 2
*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqr
t(a*f - b*e)))*a*b*c**3*d*f**3*h*x + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*
e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c**2*d**2*e**2*f*
h - 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a*b*c**2*d**2*e*f**2*h*x - 2*sqrt(b)*sqrt(e + f*x)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*c*d**3*
e**3*h + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(s
qrt(b)*sqrt(a*f - b*e)))*a*b*c*d**3*e**2*f*h*x - 2*sqrt(b)*sqrt(e + f*x)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b*d**4*
e**3*h*x - 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**4*f**3*g + 6*sqrt(b)*sqrt(e + f*x)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**3*d*
e*f**2*g - 2*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**3*d*f**3*g*x - 6*sqrt(b)*sqrt(e + f*x)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**
2*d**2*e**2*f*g + 6*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*b**2*c**2*d**2*e*f**2*g*x + 2*sqrt(b)...
```

3.182 $\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 494

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx =$$

$$-\frac{f(a^2df(3dfg-2deh-cfh)+b^2(2d^2e^2g+c^2f(3fg-2eh)-cde(2fg+eh))-ab(c^2f^2h+4cdf(fg-2}{(bc-ad)^2(be-af)^2(de-cf)^2\sqrt{e+fx}}$$

$$-\frac{d(2bdeg-bc(fg+eh)-a(dfg+deh-2cfh))}{(bc-ad)^2(be-af)(de-cf)(c+dx)\sqrt{e+fx}}$$

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+fx}}$$

$$+\frac{b^{3/2}(5a^2dfh+b^2(4deg+3cfg-2ceh)-ab(7dfg+2deh+cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^3(be-af)^{5/2}}$$

$$-\frac{d^{3/2}(ad(3dfg-2deh-cfh)+b(4d^2eg+5c^2fh-cd(7fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^3(de-cf)^{5/2}}$$

output
```
-f*(a^2*d*f*(-c*f*h-2*d*e*h+3*d*f*g)+b^2*(2*d^2*e^2*g+c^2*f*(-2*e*h+3*f*g)
-c*d*e*(e*h+2*f*g))-a*b*(c^2*f^2*h+4*c*d*f*(-2*e*h+f*g)+d^2*e*(e*h+2*f*g))
)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)^(1/2)-d*(2*b*d*e*g-b*c*(e
*h+f*g)-a*(-2*c*f*h+d*e*h+d*f*g))/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(d*x+
c)/(f*x+e)^(1/2)-(-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)/(d*x+c)/(f*x+e)^
(1/2)+b^(3/2)*(5*a^2*d*f*h+b^2*(-2*c*e*h+3*c*f*g+4*d*e*g)-a*b*(c*f*h+2*d*e
*h+7*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))/(-a*d+b*c)^3/
(-a*f+b*e)^(5/2)-d^(3/2)*(a*d*(-c*f*h-2*d*e*h+3*d*f*g)+b*(4*d^2*e*g+5*c^2*
f*h-c*d*(2*e*h+7*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-
a*d+b*c)^3/(-c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 10.95 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.23

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{a^3d^2f^2(-d(eg+3fgx-2ehx)+c(-2fg+3eh+fhx))+ab^2(d^3e$$

$$+\frac{b^{3/2}(-5a^2dfh+b^2(-4deg-3cfg+2ceh)+ab(7dfg+2deh+cfh))\arctan\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{-be+af}}\right)}{(bc-ad)^3(-be+af)^{5/2}}$$

$$-\frac{d^{3/2}(ad(3dfg-2deh-cfh)+b(4d^2eg+5c^2fh-cd(7fg+2eh)))\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{(-bc+ad)^3(-de+cf)^{5/2}}$$

input
```
Integrate[(g + h*x)/((a + b*x)^2*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
(a^3*d^2*f^2*(-(d*(e*g + 3*f*g*x - 2*e*h*x)) + c*(-2*f*g + 3*e*h + f*h*x))
 + a*b^2*(d^3*e*(e + f*x)*(-(e*g) + 2*f*g*x + e*h*x) + c^3*f^2*(-2*f*g + 3
*e*h + f*h*x) + 2*c*d^2*(e^3*h - e^2*f*h*x + 2*f^3*g*x^2 - 4*e*f^2*h*x^2)
+ c^2*d*f*(-2*e^2*h - 3*e*f*h*x + f^2*x*(2*g + h*x))) + a^2*b*d*f*(4*c^2*f
*(f*g - e*h) + c*d*(-2*e^2*h - 3*e*f*h*x + f^2*x*(2*g + h*x)) + d^2*(2*e^2
*g - 3*f^2*g*x^2 + e*f*x*(g + 2*h*x))) + b^3*(-2*d^3*e^2*g*x*(e + f*x) + c
*d^2*e*(e + f*x)*(-(e*g) + 2*f*g*x + e*h*x) - c^3*f^2*(3*f*g*x + e*(g - 2*
h*x)) + c^2*d*f*(2*e^2*g - 3*f^2*g*x^2 + e*f*x*(g + 2*h*x))))/((b*c - a*d)
^2*(b*e - a*f)^2*(d*e - c*f)^2*(a + b*x)*(c + d*x)*Sqrt[e + f*x]) + (b^(3/
2)*(-5*a^2*d*f*h + b^2*(-4*d*e*g - 3*c*f*g + 2*c*e*h) + a*b*(7*d*f*g + 2*d
*e*h + c*f*h))*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/((b*c -
 a*d)^3*(-(b*e) + a*f)^(5/2)) - (d^(3/2)*(a*d*(3*d*f*g - 2*d*e*h - c*f*h)
+ b*(4*d^2*e*g + 5*c^2*f*h - c*d*(7*f*g + 2*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e
+ f*x])/Sqrt[-(d*e) + c*f]])/((-(b*c) + a*d)^3*(-(d*e) + c*f)^(5/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {168, 27, 168, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}dx$$

$$\Big\downarrow \; 168$$

$$-\frac{\int \frac{b(4deg+3cfg-2ceh)-a(cfh+2d(fg+eh))+5df(bg-ah)x}{2(a+bx)(c+dx)^2(e+fx)^{3/2}}dx}{(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{b(4deg+3cfg-2ceh)-a(cfh+2d(fg+eh))+5df(bg-ah)x}{(a+bx)(c+dx)^2(e+fx)^{3/2}}dx}{2(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{\frac{\int \frac{-df(3dfg-2deh-cfh)a^2+b\left(-e(fg+2eh)d^2+2cf(2fg-eh)d+c^2f^2h\right)a+b^2(de-cf)(4deg+3cfg-2ceh)+3bdf(2bdeg-bc(fg+eh)-a(dfg+deh-2cfh))x}{(a+bx)(c+dx)(e+fx)^{3/2}}dx}{(bc-ad)(de-cf)}+\cdots}{2(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 169$$

$$-\frac{\frac{2f\left(a^2df(-cfh-2deh+3dfg)-ab\left(c^2f^2h+4cdf(fg-2eh)+d^2e(eh+2fg)\right)+b^2\left(c^2f(3fg-2eh)-cde(eh+2fg)+2d^2e^2g\right)\right)}{\sqrt{e+fx}(be-af)(de-cf)}-\frac{2\int -\frac{d^2f^2(3dfg-2deh-cfh)a^3+2bd\cdots}{\cdots}}{\cdots}}{\cdots}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{d^2f^2(3dfg-2deh-cfh)a^3+2bdf\left(-e(fg-2eh)d^2-cf(2fg+eh)d+2c^2f^2h\right)a^2-b^2\left(e^2(5fg+2eh)d^3-2cef(6fg+eh)d^2+2c^2f^2(2fg+eh)d+c^3f^3h\right)a+b^3(de-c\cdots}{(a+bx)(c+dx)\sqrt{e\cdots}}}{(be-af)(de-c\cdots}}{\cdots}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 174$$

$$-\frac{\frac{\frac{b^2(de-cf)^2\left(5a^2dfh-ab(cfh+2deh+7dfg)+b^2(-2ceh+3cfg+4deg)\right)\int \frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{d^2(be-af)^2\left(ad(-cfh-2deh+3dfg)+b\left(5c^2fh-cd(2eh+7fg)+4d^2\cdots\right.\right.}{bc-ad}}{(be-af)(de-cf)}}{(bc-ad)(de-\cdots}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{\frac{2b^2(de-cf)^2\left(5a^2dfh-ab(cfh+2deh+7dfg)+b^2(-2ceh+3cfg+4deg)\right)\int \frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2d^2(be-af)^2\left(ad(-cfh-2deh+3dfg)+b\left(5c^2fh-cd(2eh+\cdots\right.\right.}{f(bc-ad)}}{(be-af)(de-cf)}}{(bc-\cdots}-\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{2d^{3/2}(be-af)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(ad(-cfh-2deh+3dfg)+b\left(5c^2fh-cd(2eh+7fg)+4d^2eg\right)\right)}{(bc-ad)\sqrt{de-cf}}-\frac{2b^{3/2}(de-cf)^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)\left(5a^2dfh-ab(c\right.}{(bc-ad)\sqrt{be-\ldots}}}{(be-af)(de-cf)}}{(bc-\ldots}$$

$$\frac{bg-ah}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

input
```
Int[(g + h*x)/((a + b*x)^2*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-((b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)*Sqrt[e + f*x]))
 - ((2*d*(b*(2*d*e*g - c*f*g - c*e*h) - a*(d*f*g + d*e*h - 2*c*f*h)))/((b*
c - a*d)*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x]) + ((2*f*(a^2*d*f*(3*d*f*g -
2*d*e*h - c*f*h) + b^2*(2*d^2*e^2*g + c^2*f*(3*f*g - 2*e*h) - c*d*e*(2*f*g
 + e*h)) - a*b*(c^2*f^2*h + 4*c*d*f*(f*g - 2*e*h) + d^2*e*(2*f*g + e*h))))
/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) + ((-2*b^(3/2)*(d*e - c*f)^2*(5*a
^2*d*f*h + b^2*(4*d*e*g + 3*c*f*g - 2*c*e*h) - a*b*(7*d*f*g + 2*d*e*h + c*
f*h))*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[
b*e - a*f]) + (2*d^(3/2)*(b*e - a*f)^2*(a*d*(3*d*f*g - 2*d*e*h - c*f*h) +
b*(4*d^2*e*g + 5*c^2*f*h - c*d*(7*f*g + 2*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e +
 f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/((b*e - a*f)*(d*e
- c*f)))/((b*c - a*d)*(d*e - c*f)))/(2*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.79

method	result
derivativedivides	$2f^2\left(\frac{b^2\left(\frac{\left(\frac{1}{2}a^2dfh-\frac{1}{2}abcfh-\frac{1}{2}abdfg+\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{\left(5a^2dfh-abcfh-2abdeh-7abdfg-2b^2ceh+3b^2cfg+4b^2deg\right)\arctan}{2\sqrt{(af-be)b}}\right.}{(ad-bc)^3f^2(af-be)^2}\right.$
default	$2f^2\left(\frac{b^2\left(\frac{\left(\frac{1}{2}a^2dfh-\frac{1}{2}abcfh-\frac{1}{2}abdfg+\frac{1}{2}b^2cfg\right)\sqrt{fx+e}}{(fx+e)b+af-be}+\frac{\left(5a^2dfh-abcfh-2abdeh-7abdfg-2b^2ceh+3b^2cfg+4b^2deg\right)\arctan}{2\sqrt{(af-be)b}}\right.}{(ad-bc)^3f^2(af-be)^2}\right.$
pseudoelliptic	$\frac{5(xd+c)(cf-de)^2\sqrt{fx+e}\,\sqrt{(cf-de)d}\left(\left(\left(\frac{3b^2cg}{5}-\frac{a(ch+7dg)b}{5}+a^2dh\right)f-\frac{2((ch-2dg)b+adh)be}{5}\right)b^2(bx+a)\arctan\left(\frac{b\sqrt{f}}{\sqrt{(af}}\right.\right.}{}$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^2*(b^2/(a*d-b*c)^3/f^2/(a*f-b*e)^2*((1/2*a^2*d*f*h-1/2*a*b*c*f*h-1/2*a
*b*d*f*g+1/2*b^2*c*f*g)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)+1/2*(5*a^2*d*f*h
-a*b*c*f*h-2*a*b*d*e*h-7*a*b*d*f*g-2*b^2*c*e*h+3*b^2*c*f*g+4*b^2*d*e*g)/((
a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))-(-e*h+f*g)/
(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)^(1/2)+d^2/(a*d-b*c)^3/f^2/(c*f-d*e)^2*((1/
2*a*c*d*f*h-1/2*a*d^2*f*g-1/2*b*c^2*f*h+1/2*b*c*d*f*g)*(f*x+e)^(1/2)/((f*x
+e)*d+c*f-d*e)+1/2*(a*c*d*f*h+2*a*d^2*e*h-3*a*d^2*f*g-5*b*c^2*f*h+2*b*c*d*
e*h+7*b*c*d*f*g-4*b*d^2*e*g)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((
c*f-d*e)*d)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**2/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. 2(468) = 936.

Time = 0.25 (sec) , antiderivative size = 1867, normalized size of antiderivative = 3.78

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-(4*b^4*d*e*g + 3*b^4*c*f*g - 7*a*b^3*d*f*g - 2*b^4*c*e*h - 2*a*b^3*d*e*h
- a*b^3*c*f*h + 5*a^2*b^2*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*
f))/((b^5*c^3*e^2 - 3*a*b^4*c^2*d*e^2 + 3*a^2*b^3*c*d^2*e^2 - a^3*b^2*d^3*
e^2 - 2*a*b^4*c^3*e*f + 6*a^2*b^3*c^2*d*e*f - 6*a^3*b^2*c*d^2*e*f + 2*a^4*
b*d^3*e*f + a^2*b^3*c^3*f^2 - 3*a^3*b^2*c^2*d*f^2 + 3*a^4*b*c*d^2*f^2 - a^
5*d^3*f^2)*sqrt(-b^2*e + a*b*f)) + (4*b*d^4*e*g - 7*b*c*d^3*f*g + 3*a*d^4*
f*g - 2*b*c*d^3*e*h - 2*a*d^4*e*h + 5*b*c^2*d^2*f*h - a*c*d^3*f*h)*arctan(
sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^3*d^2*e^2 - 3*a*b^2*c^2*d^3*
e^2 + 3*a^2*b*c*d^4*e^2 - a^3*d^5*e^2 - 2*b^3*c^4*d*e*f + 6*a*b^2*c^3*d^2*
e*f - 6*a^2*b*c^2*d^3*e*f + 2*a^3*c*d^4*e*f + b^3*c^5*f^2 - 3*a*b^2*c^4*d*
f^2 + 3*a^2*b*c^3*d^2*f^2 - a^3*c^2*d^3*f^2)*sqrt(-d^2*e + c*d*f)) - (2*(f
*x + e)^2*b^3*d^3*e^2*f*g - 2*(f*x + e)*b^3*d^3*e^3*f*g - 2*(f*x + e)^2*b^
3*c*d^2*e*f^2*g - 2*(f*x + e)^2*a*b^2*d^3*e*f^2*g + 3*(f*x + e)*b^3*c*d^2*
e^2*f^2*g + 3*(f*x + e)*a*b^2*d^3*e^2*f^2*g + 3*(f*x + e)^2*b^3*c^2*d*f^3*
g - 4*(f*x + e)^2*a*b^2*c*d^2*f^3*g + 3*(f*x + e)^2*a^2*b*d^3*f^3*g - 7*(f
*x + e)*b^3*c^2*d*e*f^3*g + 8*(f*x + e)*a*b^2*c*d^2*e*f^3*g - 7*(f*x + e)*
a^2*b*d^3*e*f^3*g + 2*b^3*c^2*d*e^2*f^3*g - 4*a*b^2*c*d^2*e^2*f^3*g + 2*a^
2*b*d^3*e^2*f^3*g + 3*(f*x + e)*b^3*c^3*f^4*g - 2*(f*x + e)*a*b^2*c^2*d*f^
4*g - 2*(f*x + e)*a^2*b*c*d^2*f^4*g + 3*(f*x + e)*a^3*d^3*f^4*g - 2*b^3*c^
3*e*f^4*g + 2*a*b^2*c^2*d*e*f^4*g + 2*a^2*b*c*d^2*e*f^4*g - 2*a^3*d^3*e...
```

Mupad [B] (verification not implemented)

Time = 99.21 (sec) , antiderivative size = 853229, normalized size of antiderivative = 1727.18

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^2*(c + d*x)^2),x)
```

output
```
- atan((((e + f*x)^(1/2)*(18*a^6*b^15*c^18*d^3*f^20*g^2 - 192*a^7*b^14*c^1
7*d^4*f^20*g^2 + 872*a^8*b^13*c^16*d^5*f^20*g^2 - 2208*a^9*b^12*c^15*d^6*f
^20*g^2 + 3518*a^10*b^11*c^14*d^7*f^20*g^2 - 4000*a^11*b^10*c^13*d^8*f^20*
g^2 + 3984*a^12*b^9*c^12*d^9*f^20*g^2 - 4000*a^13*b^8*c^11*d^10*f^20*g^2 +
 3518*a^14*b^7*c^10*d^11*f^20*g^2 - 2208*a^15*b^6*c^9*d^12*f^20*g^2 + 872*
a^16*b^5*c^8*d^13*f^20*g^2 - 192*a^17*b^4*c^7*d^14*f^20*g^2 + 18*a^18*b^3*
c^6*d^15*f^20*g^2 + 2*a^8*b^13*c^18*d^3*f^20*h^2 - 32*a^9*b^12*c^17*d^4*f^
20*h^2 + 250*a^10*b^11*c^16*d^5*f^20*h^2 - 960*a^11*b^10*c^15*d^6*f^20*h^2
 + 2052*a^12*b^9*c^14*d^7*f^20*h^2 - 2624*a^13*b^8*c^13*d^8*f^20*h^2 + 205
2*a^14*b^7*c^12*d^9*f^20*h^2 - 960*a^15*b^6*c^11*d^10*f^20*h^2 + 250*a^16*
b^5*c^10*d^11*f^20*h^2 - 32*a^17*b^4*c^9*d^12*f^20*h^2 + 2*a^18*b^3*c^8*d^
13*f^20*h^2 + 64*a^6*b^15*d^21*e^18*f^2*g^2 - 576*a^7*b^14*d^21*e^17*f^3*g
^2 + 2228*a^8*b^13*d^21*e^16*f^4*g^2 - 4768*a^9*b^12*d^21*e^15*f^5*g^2 + 5
960*a^10*b^11*d^21*e^14*f^6*g^2 - 3976*a^11*b^10*d^21*e^13*f^7*g^2 + 578*a
^12*b^9*d^21*e^12*f^8*g^2 + 1004*a^13*b^8*d^21*e^11*f^9*g^2 - 442*a^14*b^7
*d^21*e^10*f^10*g^2 - 320*a^15*b^6*d^21*e^9*f^11*g^2 + 362*a^16*b^5*d^21*e
^8*f^12*g^2 - 132*a^17*b^4*d^21*e^7*f^13*g^2 + 18*a^18*b^3*d^21*e^6*f^14*g
^2 + 16*a^8*b^13*d^21*e^18*f^2*h^2 - 168*a^9*b^12*d^21*e^17*f^3*h^2 + 770*
a^10*b^11*d^21*e^16*f^4*h^2 - 2020*a^11*b^10*d^21*e^15*f^5*h^2 + 3350*a^12
*b^9*d^21*e^14*f^6*h^2 - 3664*a^13*b^8*d^21*e^13*f^7*h^2 + 2678*a^14*b^...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12668, normalized size of antiderivative = 25.64

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output

```
(5*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**3*b*c**4*d*f**4*h - 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
- b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**3*d**2*
e*f**3*h + 5*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**3*d**2*f**4*h*x + 15*sqrt(b)*sqrt(e +
 f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*3*b*c**2*d**3*e**2*f**2*h - 15*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c**2*d**3*e*f**3*h*x
- 5*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**3*b*c*d**4*e**3*f*h + 15*sqrt(b)*sqrt(e + f*x)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*c*d**4*
e**2*f**2*h*x - 5*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x
)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b*d**5*e**3*f*h*x - sqrt(b)*sqrt(e +
f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**
2*b**2*c**5*f**4*h + sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**4*d*e*f**3*h - 7*sqrt(b)*s
qrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**2*b**2*c**4*d*f**4*g + 4*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*ata
n((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**2*b**2*c**4*d*f**4*h*x +
 3*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b...
```

3.183 $\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 848

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx =$$

$$-\frac{f(4a^3d^2f^2(3dfg-2deh-cfh)-b^3(12d^3e^3g-2c^2def(3fg-4eh)+3c^3f^2(5fg-4eh)-cd^2e^2(9fg+8\ldots}{}$$

$$+\frac{d(a^2df(4dfg+9deh-13cfh)+b^2(12d^2e^2g-c^2f(5fg-4eh)-cde(3fg+8eh))+ab(c^2f^2h-d^2e(21f\ldots}{4(bc-ad)^3(be-af)^2(de-cf)(c+dx)\sqrt{e+fx}}$$

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}$$

$$+\frac{7a^2dfh+b^2(6deg+5cfg-4ceh)-ab(11dfg+2deh+cfh)}{4(bc-ad)^2(be-af)^2(a+bx)(c+dx)\sqrt{e+fx}}$$

$$+\frac{b^{3/2}(35a^3d^2f^2h-7a^2bdf(9dfg+4deh+2cfh)-b^3(24d^2e^2g+3c^2f(5fg-4eh)+8cde(3fg-2eh))+a\ldots}{4(bc-ad)^4(be-af)^{7/2}}$$

$$+\frac{d^{5/2}(ad(3dfg-2deh-cfh)+b(6d^2eg+7c^2fh-cd(9fg+4eh)))\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(bc-ad)^4(de-cf)^{5/2}}$$

output
```
-1/4*f*(4*a^3*d^2*f^2*(-c*f*h-2*d*e*h+3*d*f*g)-b^3*(12*d^3*e^3*g-2*c^2*d*e
*f*(-4*e*h+3*f*g)+3*c^3*f^2*(-4*e*h+5*f*g)-c*d^2*e^2*(8*e*h+9*f*g))-a^2*b*
d*f*(11*c^2*f^2*h+2*c*d*f*(-29*e*h+12*f*g)+d^2*e*(11*e*h+12*f*g))+a*b^2*(3
*c^3*f^3*h+13*c^2*d*f^2*(-2*e*h+3*f*g)+d^3*e^2*(4*e*h+27*f*g)-c*d^2*e*f*(1
7*e*h+30*f*g)))/(-a*d+b*c)^3/(-a*f+b*e)^3/(-c*f+d*e)^2/(f*x+e)^(1/2)+1/4*d
*(a^2*d*f*(-13*c*f*h+9*d*e*h+4*d*f*g)+b^2*(12*d^2*e^2*g-c^2*f*(-4*e*h+5*f*
g)-c*d*e*(8*e*h+3*f*g))+a*b*(c^2*f^2*h-d^2*e*(4*e*h+21*f*g)+c*d*f*(11*e*h+
13*f*g)))/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x+c)/(f*x+e)^(1/2)-1/2*(
-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2/(d*x+c)/(f*x+e)^(1/2)+1/4*(7*a^2
*d*f*h+b^2*(-4*c*e*h+5*c*f*g+6*d*e*g)-a*b*(c*f*h+2*d*e*h+11*d*f*g))/(-a*d+
b*c)^2/(-a*f+b*e)^2/(b*x+a)/(d*x+c)/(f*x+e)^(1/2)+1/4*b^(3/2)*(35*a^3*d^2*
f^2*h-7*a^2*b*d*f*(2*c*f*h+4*d*e*h+9*d*f*g)-b^3*(24*d^2*e^2*g+3*c^2*f*(-4*
e*h+5*f*g)+8*c*d*e*(-2*e*h+3*f*g))+a*b^2*(3*c^2*f^2*h+2*c*d*f*(-16*e*h+27*
f*g)+8*d^2*e*(e*h+9*f*g)))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2))
/(-a*d+b*c)^4/(-a*f+b*e)^(7/2)+d^(5/2)*(a*d*(-c*f*h-2*d*e*h+3*d*f*g)+b*(6*
d^2*e*g+7*c^2*f*h-c*d*(4*e*h+9*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+
d*e)^(1/2))/(-a*d+b*c)^4/(-c*f+d*e)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.89 (sec) , antiderivative size = 561, normalized size of antiderivative = 0.66

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{\frac{-bg+ah}{(a+bx)^2}+\frac{7a^2dfh+b^2(6deg+5cfg-4ceh)-ab(11dfg+2deh+cfh)}{2(bc-ad)(be-af)(a+bx)}-\frac{d(bc-ad)(be-af)(-}{}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^3*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
((-(b*g) + a*h)/(a + b*x)^2 + (7*a^2*d*f*h + b^2*(6*d*e*g + 5*c*f*g - 4*c*
e*h) - a*b*(11*d*f*g + 2*d*e*h + c*f*h))/(2*(b*c - a*d)*(b*e - a*f)*(a + b
*x)) - (d*(b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(a^2*d*f*(4*d*f*g + 9*d*e
*h - 13*c*f*h) + b^2*(12*d^2*e^2*g + c^2*f*(-5*f*g + 4*e*h) - c*d*e*(3*f*g
 + 8*e*h)) + a*b*(c^2*f^2*h - d^2*e*(21*f*g + 4*e*h) + c*d*f*(13*f*g + 11*
e*h))) + (c + d*x)*(-(b*(d*e - c*f)^2*(-35*a^3*d^2*f^2*h + 7*a^2*b*d*f*(9*
d*f*g + 4*d*e*h + 2*c*f*h) - a*b^2*(3*c^2*f^2*h + 2*c*d*f*(27*f*g - 16*e*h
) + 8*d^2*e*(9*f*g + e*h)) + b^3*(24*d^2*e^2*g + 3*c^2*f*(5*f*g - 4*e*h) -
 8*c*d*e*(-3*f*g + 2*e*h)))*Hypergeometric2F1[-1/2, 1, 1/2, (b*(e + f*x))/
(b*e - a*f)]) + 4*d^2*(b*e - a*f)^3*(a*d*(3*d*f*g - 2*d*e*h - c*f*h) + b*(
6*d^2*e*g + 7*c^2*f*h - c*d*(9*f*g + 4*e*h)))*Hypergeometric2F1[-1/2, 1, 1
/2, (d*(e + f*x))/(d*e - c*f)]))/(2*(b*c - a*d)^3*(b*e - a*f)^2*(d*e - c*f
)^2))/(2*(b*c - a*d)*(b*e - a*f)*(c + d*x)*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 936, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {168, 27, 168, 27, 168, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{\int \frac{b(6deg+5cfg-4ceh)-a(4dfg+2deh+cfh)+7df(bg-ah)x}{2(a+bx)^2(c+dx)^2(e+fx)^{3/2}}dx}{2(bc-ad)(be-af)}-\frac{bg-ah}{2(a+bx)^2(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{b(6deg+5cfg-4ceh)-a(4dfg+2deh+cfh)+7df(bg-ah)x}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}dx}{4(bc-ad)(be-af)}-\frac{bg-ah}{2(a+bx)^2(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{-\frac{\int \frac{df(8dfg+18deh+9cfh)a^2-b(2e(21fg+4eh)d^2+cf(29fg-12eh)d+3c^2f^2h)a+b^2(3f(5fg-4eh)c^2+8de(3fg-2eh)c+24d^2e^2g)+5df(7dfha^2-b(11dfg+2de}{2(a+bx)(c+dx)^2(e+fx)^{3/2}}}{(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\int \frac{df(8dfg+18deh+9cfh)a^2-b(2e(21fg+4eh)d^2+cf(29fg-12eh)d+3c^2f^2h)a+b^2(3f(5fg-4eh)c^2+8de(3fg-2eh)c+24d^2e^2g)+5df(7dfha^2-b(11dfg+2de}{(a+bx)(c+dx)^2(e+fx)^{3/2}}}{2(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 168

$$-\frac{-\frac{\int \frac{4d^2f^2(3dfg-2deh-cfh)a^3+bdf(16d^2he^2-11c^2f^2h-cdf(24fg-19eh))a^2+b^2(-4e^2(9fg+2eh)d^3+cef(9fg+16eh)d^2+c^2f^2(39fg-23eh)d+3c^3f^3h)a+b}{}}{}}{}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$\big\downarrow$ 169

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{7dfha^2-b(11dfg+2deh+cfh)a+b^2(6deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+fx}}-\frac{\frac{2d(df(4dfg+9deh-13cfh)a^2+b(-e(21fg+4eh)d^2+cf(13fg+11eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)\sqrt{e+fx}}}{}}{}$$

$\big\downarrow$ 27

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{7dfha^2-b(11dfg+2deh+cfh)a+b^2(6deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+fx}}-\frac{\frac{2d(df(4dfg+9deh-13cfh)a^2+b(-e(21fg+4eh)d^2+cf(13fg+11eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)\sqrt{e+fx}}}{}}{}$$

$\big\downarrow$ 174

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-$$

$$-\frac{7dfha^2-b(11dfg+2deh+cfh)a+b^2(6deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+fx}}-\frac{\frac{2d\left(df(4dfg+9deh-13cfh)a^2+b\left(-e(21fg+4eh)d^2+cf(13fg+11eh)d+c^2f^2h\right)a+b^2\left(-f(5fg\right.\right.}{(bc-ad)(de-cf)(c+dx)\sqrt{e+fx}}}{}$$

$\downarrow$ 73

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-$$

$$-\frac{7dfha^2-b(11dfg+2deh+cfh)a+b^2(6deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+fx}}-\frac{\frac{2d\left(df(4dfg+9deh-13cfh)a^2+b\left(-e(21fg+4eh)d^2+cf(13fg+11eh)d+c^2f^2h\right)a+b^2\left(-f(5fg\right.\right.}{(bc-ad)(de-cf)(c+dx)\sqrt{e+fx}}}{}$$

$\downarrow$ 221

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-$$

$$-\frac{7dfha^2-b(11dfg+2deh+cfh)a+b^2(6deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+fx}}-\frac{\frac{2d\left(df(4dfg+9deh-13cfh)a^2+b\left(-e(21fg+4eh)d^2+cf(13fg+11eh)d+c^2f^2h\right)a+b^2\left(-f(5fg\right.\right.}{(bc-ad)(de-cf)(c+dx)\sqrt{e+fx}}}{}$$

input

```
Int[(g + h*x)/((a + b*x)^3*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-1/2*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)*Sqrt[e + f
*x]) - (-((7*a^2*d*f*h + b^2*(6*d*e*g + 5*c*f*g - 4*c*e*h) - a*b*(11*d*f*g
 + 2*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)*Sqrt[e +
 f*x])) - ((2*d*(a^2*d*f*(4*d*f*g + 9*d*e*h - 13*c*f*h) + b^2*(12*d^2*e^2*
g - c^2*f*(5*f*g - 4*e*h) - c*d*e*(3*f*g + 8*e*h)) + a*b*(c^2*f^2*h - d^2*
e*(21*f*g + 4*e*h) + c*d*f*(13*f*g + 11*e*h))))/((b*c - a*d)*(d*e - c*f)*(
c + d*x)*Sqrt[e + f*x]) + ((-2*f*(4*a^3*d^2*f^2*(3*d*f*g - 2*d*e*h - c*f*h
) - b^3*(12*d^3*e^3*g - 2*c^2*d*e*f*(3*f*g - 4*e*h) + 3*c^3*f^2*(5*f*g - 4
*e*h) - c*d^2*e^2*(9*f*g + 8*e*h)) - a^2*b*d*f*(11*c^2*f^2*h + 2*c*d*f*(12
*f*g - 29*e*h) + d^2*e*(12*f*g + 11*e*h)) + a*b^2*(3*c^3*f^3*h + 13*c^2*d*
f^2*(3*f*g - 2*e*h) + d^3*e^2*(27*f*g + 4*e*h) - c*d^2*e*f*(30*f*g + 17*e*
h))))/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) - ((-2*b^(3/2)*(d*e - c*f)^2
*(35*a^3*d^2*f^2*h - 7*a^2*b*d*f*(9*d*f*g + 4*d*e*h + 2*c*f*h) - b^3*(24*d
^2*e^2*g + 3*c^2*f*(5*f*g - 4*e*h) + 8*c*d*e*(3*f*g - 2*e*h)) + a*b^2*(3*c
^2*f^2*h + 2*c*d*f*(27*f*g - 16*e*h) + 8*d^2*e*(9*f*g + e*h)))*ArcTanh[(Sq
rt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) - (8*
d^(5/2)*(b*e - a*f)^3*(a*d*(3*d*f*g - 2*d*e*h - c*f*h) + b*(6*d^2*e*g + 7*
c^2*f*h - c*d*(9*f*g + 4*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e -
 c*f]])/((b*c - a*d)*Sqrt[d*e - c*f]))/((b*e - a*f)*(d*e - c*f)))/((b*c -
a*d)*(d*e - c*f)))/(2*(b*c - a*d)*(b*e - a*f)))/(4*(b*c - a*d)*(b*e - a...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 99.40 (sec) , antiderivative size = 815, normalized size of antiderivative = 0.96

method	result
derivativedivides	$2f^3\left(-\frac{-eh+fg}{(cf-de)^2(af-be)^3\sqrt{fx+e}}+\frac{b^2\left(\left(\frac{11}{8}a^3b\,d^2f^2h-\frac{7}{4}a^2b^2cd\,f^2h-\frac{1}{2}a^2b^2d^2efh-\frac{15}{8}a^2b^2d^2f^2g+\frac{3}{8}a\,b^3c^2f^2h+\frac{11}{4}\right.\right.}{}\right.$
default	$2f^3\left(-\frac{-eh+fg}{(cf-de)^2(af-be)^3\sqrt{fx+e}}+\frac{b^2\left(\left(\frac{11}{8}a^3b\,d^2f^2h-\frac{7}{4}a^2b^2cd\,f^2h-\frac{1}{2}a^2b^2d^2efh-\frac{15}{8}a^2b^2d^2f^2g+\frac{3}{8}a\,b^3c^2f^2h+\frac{11}{4}\right.\right.}{}\right.$
pseudoelliptic	Expression too large to display

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^3*(-(-e*h+f*g)/(c*f-d*e)^2/(a*f-b*e)^3/(f*x+e)^(1/2)+b^2/(a*d-b*c)^4/f
^3/(a*f-b*e)^3*(((11/8*a^3*b*d^2*f^2*h-7/4*a^2*b^2*c*d*f^2*h-1/2*a^2*b^2*d
^2*e*f*h-15/8*a^2*b^2*d^2*f^2*g+3/8*a*b^3*c^2*f^2*h+11/4*a*b^3*c*d*f^2*g+a
*b^3*d^2*e*f*g+1/2*b^4*c^2*e*f*h-7/8*b^4*c^2*f^2*g-b^4*c*d*e*f*g)*(f*x+e)^
(3/2)+1/8*f*(13*a^4*d^2*f^2*h-18*a^3*b*c*d*f^2*h-17*a^3*b*d^2*e*f*h-17*a^3
*b*d^2*f^2*g+5*a^2*b^2*c^2*f^2*h+18*a^2*b^2*c*d*e*f*h+26*a^2*b^2*c*d*f^2*g
+4*a^2*b^2*d^2*e^2*h+25*a^2*b^2*d^2*e*f*g-a*b^3*c^2*e*f*h-9*a*b^3*c^2*f^2*
g-34*a*b^3*c*d*e*f*g-8*a*b^3*d^2*e^2*g-4*b^4*c^2*e^2*h+9*b^4*c^2*e*f*g+8*b
^4*c*d*e^2*g)*(f*x+e)^(1/2))/((f*x+e)*b+a*f-b*e)^2+1/8*(35*a^3*d^2*f^2*h-1
4*a^2*b*c*d*f^2*h-28*a^2*b*d^2*e*f*h-63*a^2*b*d^2*f^2*g+3*a*b^2*c^2*f^2*h-
32*a*b^2*c*d*e*f*h+54*a*b^2*c*d*f^2*g+8*a*b^2*d^2*e^2*h+72*a*b^2*d^2*e*f*g
+12*b^3*c^2*e*f*h-15*b^3*c^2*f^2*g+16*b^3*c*d*e^2*h-24*b^3*c*d*e*f*g-24*b^
3*d^2*e^2*g)/((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2
)))+d^3/(a*d-b*c)^4/f^3/(c*f-d*e)^2*((1/2*a*c*d*f*h-1/2*a*d^2*f*g-1/2*b*c^
2*f*h+1/2*b*c*d*f*g)*(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(a*c*d*f*h+2*a*
d^2*e*h-3*a*d^2*f*g-7*b*c^2*f*h+4*b*c*d*e*h+9*b*c*d*f*g-6*b*d^2*e*g)/((c*f
-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**3/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2417 vs. $2(810) = 1620$.

Time = 0.32 (sec) , antiderivative size = 2417, normalized size of antiderivative = 2.85

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/4*(24*b^5*d^2*e^2*g + 24*b^5*c*d*e*f*g - 72*a*b^4*d^2*e*f*g + 15*b^5*c^2
*f^2*g - 54*a*b^4*c*d*f^2*g + 63*a^2*b^3*d^2*f^2*g - 16*b^5*c*d*e^2*h - 8*
a*b^4*d^2*e^2*h - 12*b^5*c^2*e*f*h + 32*a*b^4*c*d*e*f*h + 28*a^2*b^3*d^2*e
*f*h - 3*a*b^4*c^2*f^2*h + 14*a^2*b^3*c*d*f^2*h - 35*a^3*b^2*d^2*f^2*h)*ar
ctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^7*c^4*e^3 - 4*a*b^6*c^3*d*e
^3 + 6*a^2*b^5*c^2*d^2*e^3 - 4*a^3*b^4*c*d^3*e^3 + a^4*b^3*d^4*e^3 - 3*a*b
^6*c^4*e^2*f + 12*a^2*b^5*c^3*d*e^2*f - 18*a^3*b^4*c^2*d^2*e^2*f + 12*a^4*
b^3*c*d^3*e^2*f - 3*a^5*b^2*d^4*e^2*f + 3*a^2*b^5*c^4*e*f^2 - 12*a^3*b^4*c
^3*d*e*f^2 + 18*a^4*b^3*c^2*d^2*e*f^2 - 12*a^5*b^2*c*d^3*e*f^2 + 3*a^6*b*d
^4*e*f^2 - a^3*b^4*c^4*f^3 + 4*a^4*b^3*c^3*d*f^3 - 6*a^5*b^2*c^2*d^2*f^3 +
 4*a^6*b*c*d^3*f^3 - a^7*d^4*f^3)*sqrt(-b^2*e + a*b*f)) - (6*b*d^5*e*g - 9
*b*c*d^4*f*g + 3*a*d^5*f*g - 4*b*c*d^4*e*h - 2*a*d^5*e*h + 7*b*c^2*d^3*f*h
 - a*c*d^4*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^4*c^4*d^2
*e^2 - 4*a*b^3*c^3*d^3*e^2 + 6*a^2*b^2*c^2*d^4*e^2 - 4*a^3*b*c*d^5*e^2 + a
^4*d^6*e^2 - 2*b^4*c^5*d*e*f + 8*a*b^3*c^4*d^2*e*f - 12*a^2*b^2*c^3*d^3*e*
f + 8*a^3*b*c^2*d^4*e*f - 2*a^4*c*d^5*e*f + b^4*c^6*f^2 - 4*a*b^3*c^5*d*f^
2 + 6*a^2*b^2*c^4*d^2*f^2 - 4*a^3*b*c^3*d^3*f^2 + a^4*c^2*d^4*f^2)*sqrt(-d
^2*e + c*d*f)) + ((f*x + e)*b^3*d^4*e^3*f*g - 3*(f*x + e)*a*b^2*d^4*e^2*f^
2*g + 3*(f*x + e)*a^2*b*d^4*e*f^3*g + 2*(f*x + e)*b^3*c^3*d*f^4*g - 6*(f*x
 + e)*a*b^2*c^2*d^2*f^4*g + 6*(f*x + e)*a^2*b*c*d^3*f^4*g - 3*(f*x + e)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^3*(c + d*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 26977, normalized size of antiderivative = 31.81

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
(35*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*
sqrt(a*f - b*e)))*a**5*b*c**4*d**2*f**5*h - 105*sqrt(b)*sqrt(e + f*x)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c**3*
d**3*e*f**4*h + 35*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c**3*d**3*f**5*h*x + 105*sqrt(b)*s
qrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*
e)))*a**5*b*c**2*d**4*e**2*f**3*h - 105*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b
*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c**2*d**4*e*f
**4*h*x - 35*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*c*d**5*e**3*f**2*h + 105*sqrt(b)*sqrt(e
+ f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a
**5*b*c*d**5*e**2*f**3*h*x - 35*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan
((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**5*b*d**6*e**3*f**2*h*x -
14*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**4*b**2*c**5*d*f**5*h + 14*sqrt(b)*sqrt(e + f*x)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c**4
*d**2*e*f**4*h - 63*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**4*b**2*c**4*d**2*f**5*g + 56*sqrt(b)*
sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**4*b**2*c**4*d**2*f**5*h*x + 42*sqrt(b)*sqrt(e + f*x)*sqrt(a*f ...
```

3.184 $\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1340

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-1/8*f*(8*a^4*d^3*f^3*(-c*f*h-2*d*e*h+3*d*f*g)+b^4*(32*d^4*e^4*g-c^2*d^2*e
^2*f*(-18*e*h+13*f*g)-2*c^3*d*e*f^2*(-6*e*h+5*f*g)+5*c^4*f^3*(-6*e*h+7*f*g
)-4*c*d^3*e^3*(6*e*h+5*f*g))-a*b^3*(5*c^4*f^4*h+2*c^3*d*f^3*(-53*e*h+65*f*
g)-7*c^2*d^2*e*f^2*(-11*e*h+8*f*g)+4*d^4*e^3*(2*e*h+27*f*g)-2*c*d^3*e^2*f*
(40*e*h+43*f*g))-a^3*b*d^2*f^2*(41*c^2*f^2*h+2*c*d*f*(-89*e*h+32*f*g)+d^2*
e*(41*e*h+32*f*g))+a^2*b^2*d*f*(22*c^3*f^3*h+c^2*d*f^2*(-110*e*h+167*f*g)+
d^3*e^2*(30*e*h+119*f*g)-2*c*d^2*e*f*(43*e*h+71*f*g)))/(-a*d+b*c)^4/(-a*f+
b*e)^4/(-c*f+d*e)^2/(f*x+e)^(1/2)+1/24*d*(a^3*d^2*f^2*(-113*c*f*h+89*d*e*h
+24*d*f*g)-b^3*(96*d^3*e^3*g-c^2*d*e*f*(-18*e*h+25*f*g)-5*c^3*f^2*(-6*e*h+
7*f*g)-12*c*d^2*e^2*(6*e*h+f*g))-a*b^2*(5*c^3*f^3*h+c^2*d*f^2*(-101*e*h+13
0*f*g)-12*d^3*e^2*(2*e*h+23*f*g)+2*c*d^2*e*f*(96*e*h+37*f*g))+a^2*b*d*f*(2
2*c^2*f^2*h-d^2*e*(78*e*h+239*f*g)+c*d*f*(128*e*h+167*f*g)))/(-a*d+b*c)^4/
(-a*f+b*e)^3/(-c*f+d*e)/(d*x+c)/(f*x+e)^(1/2)-1/3*(-a*h+b*g)/(-a*d+b*c)/(-
a*f+b*e)/(b*x+a)^3/(d*x+c)/(f*x+e)^(1/2)+1/12*(9*a^2*d*f*h+b^2*(-6*c*e*h+7
*c*f*g+8*d*e*g)-a*b*(c*f*h+2*d*e*h+15*d*f*g))/(-a*d+b*c)^2/(-a*f+b*e)^2/(b
*x+a)^2/(d*x+c)/(f*x+e)^(1/2)+1/24*(63*a^3*d^2*f^2*h-a^2*b*d*f*(20*c*f*h+4
0*d*e*h+129*d*f*g)-b^3*(48*d^2*e^2*g+2*c*d*e*(-18*e*h+23*f*g)+5*c^2*f*(-6*
e*h+7*f*g))+a*b^2*(5*c^2*f^2*h+2*c*d*f*(-43*e*h+58*f*g)+2*d^2*e*(6*e*h+71*
f*g)))/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)/(d*x+c)/(f*x+e)^(1/2)+1/8*b^(3/2)
*(105*a^4*d^3*f^3*h-21*a^3*b*d^2*f^2*(3*c*f*h+6*d*e*h+11*d*f*g)+b^4*(64...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.65 (sec) , antiderivative size = 902, normalized size of antiderivative = 0.67

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \frac{-\frac{8(bc-ad)(be-af)(bg-ah)}{(a+bx)^3}+\frac{2\left(9a^2dfh+b^2(8deg+7cfg-6ceh)-ab(15dfg+2deh+cfh)\right)}{(a+bx)^2}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^4*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
((-8*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(a + b*x)^3 + (2*(9*a^2*d*f*h +
b^2*(8*d*e*g + 7*c*f*g - 6*c*e*h) - a*b*(15*d*f*g + 2*d*e*h + c*f*h)))/(a
+ b*x)^2 + (63*a^3*d^2*f^2*h - a^2*b*d*f*(129*d*f*g + 40*d*e*h + 20*c*f*h)
 + a*b^2*(5*c^2*f^2*h + 2*c*d*f*(58*f*g - 43*e*h) + 2*d^2*e*(71*f*g + 6*e*
h)) + b^3*(-48*d^2*e^2*g + 5*c^2*f*(-7*f*g + 6*e*h) + 2*c*d*e*(-23*f*g + 1
8*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + (d*(b*c - a*d)*(b*e - a*f)*
(-(d*e) + c*f)*(a^3*d^2*f^2*(-24*d*f*g - 89*d*e*h + 113*c*f*h) + b^3*(96*d
^3*e^3*g + 5*c^3*f^2*(-7*f*g + 6*e*h) - 12*c*d^2*e^2*(f*g + 6*e*h) + c^2*d
*e*f*(-25*f*g + 18*e*h)) + a*b^2*(5*c^3*f^3*h + c^2*d*f^2*(130*f*g - 101*e
*h) - 12*d^3*e^2*(23*f*g + 2*e*h) + 2*c*d^2*e*f*(37*f*g + 96*e*h)) + a^2*b
*d*f*(-22*c^2*f^2*h + d^2*e*(239*f*g + 78*e*h) - c*d*f*(167*f*g + 128*e*h)
)) - 3*(c + d*x)*(b*(d*e - c*f)^2*(105*a^4*d^3*f^3*h - 21*a^3*b*d^2*f^2*(1
1*d*f*g + 6*d*e*h + 3*c*f*h) + b^4*(64*d^3*e^3*g + 5*c^3*f^2*(7*f*g - 6*e*
h) + 12*c^2*d*e*f*(5*f*g - 4*e*h) + 24*c*d^2*e^2*(3*f*g - 2*e*h)) + 9*a^2*
b^2*d*f*(3*c^2*f^2*h + 3*c*d*f*(11*f*g - 6*e*h) + 4*d^2*e*(11*f*g + 2*e*h)
) - a*b^3*(5*c^3*f^3*h + 3*c^2*d*f^2*(55*f*g - 42*e*h) + 8*d^3*e^2*(33*f*g
 + 2*e*h) - 24*c*d^2*e*f*(-11*f*g + 7*e*h)))*Hypergeometric2F1[-1/2, 1, 1/
2, (b*(e + f*x))/(b*e - a*f)] - 8*d^3*(b*e - a*f)^4*(a*d*(3*d*f*g - 2*d*e*
h - c*f*h) + b*(8*d^2*e*g + 9*c^2*f*h - c*d*(11*f*g + 6*e*h)))*Hypergeomet
ric2F1[-1/2, 1, 1/2, (d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)^3*(b*e -...
```

Rubi [A] (verified)

Time = 3.78 (sec) , antiderivative size = 1452, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{b(8deg+7cfg-6ceh)-a(6dfg+2deh+cfh)+9df(bg-ah)x}{2(a+bx)^3(c+dx)^2(e+fx)^{3/2}}dx}{3(bc-ad)(be-af)}-\frac{bg-ah}{3(a+bx)^3(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{b(8deg+7cfg-6ceh)-a(6dfg+2deh+cfh)+9df(bg-ah)x}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}dx}{6(bc-ad)(be-af)}-\frac{bg-ah}{3(a+bx)^3(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{\int \frac{df(24dfg+26deh+13cfh)a^2-b\left(2e(43fg+6eh)d^2+cf(67fg-44eh)d+5c^2f^2h\right)a+b^2\left(5f(7fg-6eh)c^2+2de(23fg-18eh)c+48d^2e^2g\right)+7df\left(9dfha^2-b(15dfg\right.}{2(a+bx)^2(c+dx)^2(e+fx)^{3/2}}}{2(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{df(24dfg+26deh+13cfh)a^2-b\left(2e(43fg+6eh)d^2+cf(67fg-44eh)d+5c^2f^2h\right)a+b^2\left(5f(7fg-6eh)c^2+2de(23fg-18eh)c+48d^2e^2g\right)+7df\left(9dfha^2-b(15dfg\right.}{(a+bx)^2(c+dx)^2(e+fx)^{3/2}}}{4(bc-ad)(be-af)}}{6(bc-ad)(be-af)}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{63a^3d^2f^2h-a^2bdf(20cfh+40deh+129dfg)+ab^2\left(5c^2f^2h+2cdf(58fg-43eh)+2d^2e(6eh+71fg)\right)-b^3\left(5c^2f(7fg-6eh)+2cde(23fg-18eh)+48d^2e^2g\right)}{(a+bx)(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int -\frac{d}{}}{}}{}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{d^2f^2(48dfg+178deh+89cfh)a^3-bdf\left(2e(239fg+78eh)d^2+cf(311fg-56eh)d+56c^2f^2h\right)a^2+b^2\left(24e^2(23fg+2eh)d^3+2cef(281fg-162eh)d^2+4c^2f^2(80fg\right.}{}}{}}{}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\cdots\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f\cdots}}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\cdots\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f\cdots}}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 169$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\cdots\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f\cdots}}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\cdots\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f\cdots}}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 174$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$\frac{-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\cdots\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f\cdots}}}{\cdots}}{\cdots}$$

$$\Big\downarrow 73$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f}}}{}$$

$$\Big\downarrow 221$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(15dfg+2deh+cfh)a+b^2(8deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{e+fx}}-\frac{\frac{63d^2f^2ha^3-bdf(129dfg+40deh+20cfh)a^2+b^2\left(2e(71fg+6eh)d^2+2cf(58fg-43eh)d+5c^2f^2\right.}{(bc-ad)(be-af)(a+bx)(c+dx)\sqrt{e+f}}}{}$$

input
```
Int[(g + h*x)/((a + b*x)^4*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-1/3*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c + d*x)*Sqrt[e + f
*x]) - (-1/2*(9*a^2*d*f*h + b^2*(8*d*e*g + 7*c*f*g - 6*c*e*h) - a*b*(15*d*
f*g + 2*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)*Sqr
t[e + f*x]) - ((63*a^3*d^2*f^2*h - a^2*b*d*f*(129*d*f*g + 40*d*e*h + 20*c*
f*h) - b^3*(48*d^2*e^2*g + 2*c*d*e*(23*f*g - 18*e*h) + 5*c^2*f*(7*f*g - 6*
e*h)) + a*b^2*(5*c^2*f^2*h + 2*c*d*f*(58*f*g - 43*e*h) + 2*d^2*e*(71*f*g +
 6*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)*Sqrt[e + f*x]) + ((
2*d*(a^3*d^2*f^2*(24*d*f*g + 89*d*e*h - 113*c*f*h) - b^3*(96*d^3*e^3*g - c
^2*d*e*f*(25*f*g - 18*e*h) - 5*c^3*f^2*(7*f*g - 6*e*h) - 12*c*d^2*e^2*(f*g
 + 6*e*h)) - a*b^2*(5*c^3*f^3*h + c^2*d*f^2*(130*f*g - 101*e*h) - 12*d^3*e
^2*(23*f*g + 2*e*h) + 2*c*d^2*e*f*(37*f*g + 96*e*h)) + a^2*b*d*f*(22*c^2*f
^2*h - d^2*e*(239*f*g + 78*e*h) + c*d*f*(167*f*g + 128*e*h))))/((b*c - a*d
)*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x]) + (3*((-2*f*(8*a^4*d^3*f^3*(3*d*f*g
 - 2*d*e*h - c*f*h) + b^4*(32*d^4*e^4*g - c^2*d^2*e^2*f*(13*f*g - 18*e*h)
- 2*c^3*d*e*f^2*(5*f*g - 6*e*h) + 5*c^4*f^3*(7*f*g - 6*e*h) - 4*c*d^3*e^3*
(5*f*g + 6*e*h)) - a*b^3*(5*c^4*f^4*h + 2*c^3*d*f^3*(65*f*g - 53*e*h) - 7*
c^2*d^2*e*f^2*(8*f*g - 11*e*h) + 4*d^4*e^3*(27*f*g + 2*e*h) - 2*c*d^3*e^2*
f*(43*f*g + 40*e*h)) - a^3*b*d^2*f^2*(41*c^2*f^2*h + 2*c*d*f*(32*f*g - 89*
e*h) + d^2*e*(32*f*g + 41*e*h)) + a^2*b^2*d*f*(22*c^3*f^3*h + c^2*d*f^2*(1
67*f*g - 110*e*h) + d^3*e^2*(119*f*g + 30*e*h) - 2*c*d^2*e*f*(71*f*g + ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 398.57 (sec) , antiderivative size = 1943, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	Expression too large to display	1943
default	Expression too large to display	1943
pseudoelliptic	Expression too large to display	2422

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^4*(-(-e*h+f*g)/(c*f-d*e)^2/(a*f-b*e)^4/(f*x+e)^(1/2)+b^2/(a*d-b*c)^5/f
^4/(a*f-b*e)^4*(((19/16*b^6*c^3*f^3*g-15/8*a^3*b^3*d^3*e*f^2*h+137/16*a^2*
b^4*c*d^2*f^3*g+1/2*a^2*b^4*d^3*e^2*f*h+19/4*a^2*b^4*d^3*e*f^2*g-85/16*a*b
^5*c^2*d*f^3*g-3/2*a*b^5*d^3*e^2*f*g-b^6*c^2*d*e^2*f*h+7/4*b^6*c^2*d*e*f^2
*g+3/2*b^6*c*d^2*e^2*f*g-63/16*b^3*a^3*c*d^2*f^3*h+27/16*b^4*a^2*c^2*d*f^3
*h-1/8*a^2*b^4*c*d^2*e*f^2*h+23/8*a*b^5*c^2*d*e*f^2*h+1/2*a*b^5*c*d^2*e^2*
f*h-13/2*a*b^5*c*d^2*e*f^2*g+41/16*a^4*b^2*d^3*f^3*h-71/16*a^3*d^3*f^3*g*b
^3-7/8*b^6*c^3*e*f^2*h-5/16*b^5*a*c^3*f^3*h)*(f*x+e)^(5/2)+1/6*b*f*(35*a^5
*d^3*f^3*h-57*a^4*b*c*d^2*f^3*h-59*a^4*b*d^3*e*f^2*h-59*a^4*b*d^3*f^3*g+27
*a^3*b^2*c^2*d*f^3*h+57*a^3*b^2*c*d^2*e*f^2*h+117*a^3*b^2*c*d^2*f^3*g+30*a
^3*b^2*d^3*e^2*f*h+119*a^3*b^2*d^3*e*f^2*g-5*a^2*b^3*c^3*f^3*h+9*a^2*b^3*c
^2*d*e*f^2*h-75*a^2*b^3*c^2*d*f^3*g+6*a^2*b^3*c*d^2*e^2*f*h-201*a^2*b^3*c*
d^2*e*f^2*g-6*a^2*b^3*d^3*e^3*h-78*a^2*b^3*d^3*e^2*f*g-7*a*b^4*c^3*e*f^2*h
+17*a*b^4*c^3*f^3*g-48*a*b^4*c^2*d*e^2*f*h+99*a*b^4*c^2*d*e*f^2*g-6*a*b^4*
c*d^2*e^3*h+102*a*b^4*c*d^2*e^2*f*g+18*a*b^4*d^3*e^3*g+12*b^5*c^3*e^2*f*h-
17*b^5*c^3*e*f^2*g+12*b^5*c^2*d*e^3*h-24*b^5*c^2*d*e^2*f*g-18*b^5*c*d^2*e^
3*g)*(f*x+e)^(3/2)+(-97/16*a^5*b*c*d^2*f^5*h-9*a^5*b*d^3*e*f^4*h+53/16*a^4
*b^2*c^2*d*f^5*h+183/16*a^4*b^2*c*d^2*f^5*g+131/16*a^4*b^2*d^3*e^2*f^3*h+1
31/8*a^4*b^2*d^3*e*f^4*g-123/16*a^3*b^3*c^2*d*f^5*g-281/16*a^3*b^3*d^3*e^2
*f^3*g+1/4*a^2*b^4*c^3*e*f^4*h+33/4*a^2*b^4*d^3*e^3*f^2*g+25/16*a*b^5*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**4/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4339 vs. 2(1298) = 2596.

Time = 0.49 (sec) , antiderivative size = 4339, normalized size of antiderivative = 3.24

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(64*b^6*d^3*e^3*g + 72*b^6*c*d^2*e^2*f*g - 264*a*b^5*d^3*e^2*f*g + 60
*b^6*c^2*d*e*f^2*g - 264*a*b^5*c*d^2*e*f^2*g + 396*a^2*b^4*d^3*e*f^2*g + 3
5*b^6*c^3*f^3*g - 165*a*b^5*c^2*d*f^3*g + 297*a^2*b^4*c*d^2*f^3*g - 231*a^
3*b^3*d^3*f^3*g - 48*b^6*c*d^2*e^3*h - 16*a*b^5*d^3*e^3*h - 48*b^6*c^2*d*e
^2*f*h + 168*a*b^5*c*d^2*e^2*f*h + 72*a^2*b^4*d^3*e^2*f*h - 30*b^6*c^3*e*f
^2*h + 126*a*b^5*c^2*d*e*f^2*h - 162*a^2*b^4*c*d^2*e*f^2*h - 126*a^3*b^3*d
^3*e*f^2*h - 5*a*b^5*c^3*f^3*h + 27*a^2*b^4*c^2*d*f^3*h - 63*a^3*b^3*c*d^2
*f^3*h + 105*a^4*b^2*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f
))/((b^9*c^5*e^4 - 5*a*b^8*c^4*d*e^4 + 10*a^2*b^7*c^3*d^2*e^4 - 10*a^3*b^6
*c^2*d^3*e^4 + 5*a^4*b^5*c*d^4*e^4 - a^5*b^4*d^5*e^4 - 4*a*b^8*c^5*e^3*f +
 20*a^2*b^7*c^4*d*e^3*f - 40*a^3*b^6*c^3*d^2*e^3*f + 40*a^4*b^5*c^2*d^3*e^
3*f - 20*a^5*b^4*c*d^4*e^3*f + 4*a^6*b^3*d^5*e^3*f + 6*a^2*b^7*c^5*e^2*f^2
 - 30*a^3*b^6*c^4*d*e^2*f^2 + 60*a^4*b^5*c^3*d^2*e^2*f^2 - 60*a^5*b^4*c^2*
d^3*e^2*f^2 + 30*a^6*b^3*c*d^4*e^2*f^2 - 6*a^7*b^2*d^5*e^2*f^2 - 4*a^3*b^6
*c^5*e*f^3 + 20*a^4*b^5*c^4*d*e*f^3 - 40*a^5*b^4*c^3*d^2*e*f^3 + 40*a^6*b^
3*c^2*d^3*e*f^3 - 20*a^7*b^2*c*d^4*e*f^3 + 4*a^8*b*d^5*e*f^3 + a^4*b^5*c^5
*f^4 - 5*a^5*b^4*c^4*d*f^4 + 10*a^6*b^3*c^3*d^2*f^4 - 10*a^7*b^2*c^2*d^3*f
^4 + 5*a^8*b*c*d^4*f^4 - a^9*d^5*f^4)*sqrt(-b^2*e + a*b*f)) + (8*b*d^6*e*g
 - 11*b*c*d^5*f*g + 3*a*d^6*f*g - 6*b*c*d^5*e*h - 2*a*d^6*e*h + 9*b*c^2*d^
4*f*h - a*c*d^5*f*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g + hx}{(a + bx)^4 (c + dx)^2 (e + fx)^{3/2}}\, dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^4*(c + d*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 46641, normalized size of antiderivative = 34.81

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((h*x+g)/(b*x+a)^4/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output

```
(315*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)
*sqrt(a*f - b*e)))*a**7*b*c**4*d**3*f**6*h - 945*sqrt(b)*sqrt(e + f*x)*sqr
t(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**3
*d**4*e*f**5*h + 315*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**3*d**4*f**6*h*x + 945*sqrt(b)
*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f -
b*e)))*a**7*b*c**2*d**5*e**2*f**4*h - 945*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -
 b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c**2*d**5*e
*f**5*h*x - 315*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*
b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*c*d**6*e**3*f**3*h + 945*sqrt(b)*sqrt
(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e))
)*a**7*b*c*d**6*e**2*f**4*h*x - 315*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*
atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**7*b*d**7*e**3*f**3*h*
x - 189*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt
(b)*sqrt(a*f - b*e)))*a**6*b**2*c**5*d**2*f**6*h + 189*sqrt(b)*sqrt(e + f*
x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*
b**2*c**4*d**3*e*f**5*h - 693*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((
sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**6*b**2*c**4*d**3*f**6*g + 7
56*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*s
qrt(a*f - b*e)))*a**6*b**2*c**4*d**3*f**6*h*x + 567*sqrt(b)*sqrt(e + f*...
```

3.185 $\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1991

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-1/64*f*(64*a^5*d^4*f^4*(-c*f*h-2*d*e*h+3*d*f*g)-b^5*(320*d^5*e^5*g-c^3*d^
2*e^2*f^2*(-104*e*h+85*f*g)-16*c^2*d^3*e^3*f*(-10*e*h+7*f*g)-10*c^4*d*e*f^
3*(-8*e*h+7*f*g)+35*c^5*f^4*(-8*e*h+9*f*g)-16*c*d^4*e^4*(16*e*h+11*f*g))+a
*b^4*(35*c^5*f^5*h-c^3*d^2*e*f^3*(-547*e*h+450*f*g)-3*c^2*d^3*e^2*f^2*(-28
0*e*h+197*f*g)+5*c^4*d*f^4*(-262*e*h+301*f*g)+16*d^5*e^4*(4*e*h+89*f*g)-16
*c*d^4*e^3*f*(71*e*h+58*f*g))-5*a^4*b*d^3*f^3*(103*c^2*f^2*h+2*c*d*f*(-199
*e*h+64*f*g)+d^2*e*(103*e*h+64*f*g))-a^2*b^3*d*f*(185*c^4*f^4*h+c^3*d*f^3*
(-2298*e*h+2785*f*g)-3*c^2*d^2*e*f^2*(-595*e*h+422*f*g)+16*d^4*e^3*(19*e*h
+149*f*g)-3*c*d^3*e^2*f*(632*e*h+661*f*g))+a^3*b^2*d^2*f^2*(409*c^3*f^3*h+
17*c^2*d*f^2*(-90*e*h+139*f*g)+d^3*e^2*(568*e*h+1723*f*g)-c*d^2*e*f*(1367*
e*h+2166*f*g)))/(-a*d+b*c)^5/(-a*f+b*e)^5/(-c*f+d*e)^2/(f*x+e)^(1/2)+1/192
*d*(a^4*d^3*f^3*(-1219*c*f*h+1027*d*e*h+192*d*f*g)+b^4*(960*d^4*e^4*g-5*c^
3*d*e*f^2*(-40*e*h+49*f*g)-35*c^4*f^3*(-8*e*h+9*f*g)-32*c^2*d^2*e^2*f*(-3*
e*h+5*f*g)-48*c*d^3*e^3*(16*e*h+f*g))+a*b^3*(35*c^4*f^4*h+c^2*d^2*e*f^2*(-
728*e*h+1055*f*g)+5*c^3*d*f^3*(-255*e*h+301*f*g)-48*d^4*e^3*(4*e*h+79*f*g)
+16*c*d^3*e^2*f*(183*e*h+29*f*g))+a^3*b*d^2*f^2*(409*c^2*f^2*h-d^2*e*(1336
*e*h+3131*f*g)+c*d*f*(1695*e*h+2363*f*g))-a^2*b^2*d*f*(185*c^3*f^3*h+c^2*d
*f^2*(-2113*e*h+2785*f*g)-16*d^3*e^2*(51*e*h+341*f*g)+c*d^2*e*f*(3896*e*h+
1519*f*g)))/(-a*d+b*c)^5/(-a*f+b*e)^4/(-c*f+d*e)/(d*x+c)/(f*x+e)^(1/2)-1/4
*(-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^4/(d*x+c)/(f*x+e)^(1/2)+1/24*...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 13.79 (sec) , antiderivative size = 1410, normalized size of antiderivative = 0.71

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(g + h*x)/((a + b*x)^5*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output

```
(48*(-(b*g) + a*h) - (8*(-11*a^2*d*f*h + b^2*(-10*d*e*g - 9*c*f*g + 8*c*e*
h) + a*b*(19*d*f*g + 2*d*e*h + c*f*h))*(a + b*x))/((b*c - a*d)*(b*e - a*f)
) + ((a + b*x)^2*(2*(b*c - a*d)^3*(b*e - a*f)^2*(99*a^3*d^2*f^2*h - a^2*b*
d*f*(219*d*f*g + 52*d*e*h + 26*c*f*h) + a*b^2*(7*c^2*f^2*h + 2*c*d*f*(101*
f*g - 82*e*h) + 4*d^2*e*(59*f*g + 4*e*h)) + b^3*(-80*d^2*e^2*g + 7*c^2*f*(
-9*f*g + 8*e*h) + 4*c*d*e*(-19*f*g + 16*e*h))) - ((a + b*x)*((b*c - a*d)^2
*(b*e - a*f)*(d*e - c*f)^2*(-693*a^4*d^3*f^3*h + a^3*b*d^2*f^2*(1725*d*f*g
 + 698*d*e*h + 349*c*f*h) + a*b^3*(35*c^3*f^3*h + c^2*d*f^2*(1379*f*g - 11
42*e*h) + 12*c*d^2*e*f*(155*f*g - 116*e*h) + 16*d^3*e^2*(121*f*g + 6*e*h))
 + b^4*(-480*d^3*e^3*g + 35*c^3*f^2*(-9*f*g + 8*e*h) + 16*c*d^2*e^2*(-31*f
*g + 24*e*h) + 2*c^2*d*e*f*(-217*f*g + 184*e*h)) - a^2*b^2*d*f*(171*c^2*f^
2*h + c*d*f*(2309*f*g - 1604*e*h) + 2*d^2*e*(1433*f*g + 208*e*h))) + (a +
b*x)*(d*(b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(a^4*d^3*f^3*(192*d*f*g + 1
027*d*e*h - 1219*c*f*h) + b^4*(960*d^4*e^4*g + 32*c^2*d^2*e^2*f*(-5*f*g +
3*e*h) + 35*c^4*f^3*(-9*f*g + 8*e*h) - 48*c*d^3*e^3*(f*g + 16*e*h) + 5*c^3
*d*e*f^2*(-49*f*g + 40*e*h)) + a*b^3*(35*c^4*f^4*h + c^2*d^2*e*f^2*(1055*f
*g - 728*e*h) + 5*c^3*d*f^3*(301*f*g - 255*e*h) - 48*d^4*e^3*(79*f*g + 4*e
*h) + 16*c*d^3*e^2*f*(29*f*g + 183*e*h)) + a^3*b*d^2*f^2*(409*c^2*f^2*h -
d^2*e*(3131*f*g + 1336*e*h) + c*d*f*(2363*f*g + 1695*e*h)) + a^2*b^2*d*f*(
-185*c^3*f^3*h + 16*d^3*e^2*(341*f*g + 51*e*h) + c^2*d*f^2*(-2785*f*g +...
```

Rubi [A] (verified)

Time = 6.05 (sec) , antiderivative size = 2129, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{2b\left(5deg+\frac{9cfg}{2}-4ceh\right)-a(8dfg+2deh+cfh)+11df(bg-ah)x}{2(a+bx)^4(c+dx)^2(e+fx)^{3/2}}dx}{4(bc-ad)(be-af)}-\frac{bg-ah}{4(a+bx)^4(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{b(10deg+9cfg-8ceh)-a(8dfg+2deh+cfh)+11df(bg-ah)x}{(a+bx)^4(c+dx)^2(e+fx)^{3/2}}dx}{8(bc-ad)(be-af)}-\frac{bg-ah}{4(a+bx)^4(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{\int \frac{df(48dfg+34deh+17cfh)a^2-b\left(2e(73fg+8eh)d^2+cf(121fg-92eh)d+7c^2f^2h\right)a+b^2\left(7f(9fg-8eh)c^2+4de(19fg-16eh)c+80d^2e^2g\right)+9df\left(11dfha^2-b(19d\right.}{2(a+bx)^3(c+dx)^2(e+fx)^{3/2}}}{3(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{df(48dfg+34deh+17cfh)a^2-b\left(2e(73fg+8eh)d^2+cf(121fg-92eh)d+7c^2f^2h\right)a+b^2\left(7f(9fg-8eh)c^2+4de(19fg-16eh)c+80d^2e^2g\right)+9df\left(11dfha^2-b(19d\right.}{(a+bx)^3(c+dx)^2(e+fx)^{3/2}}}{6(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{99a^3d^2f^2h-a^2bdf(26cfh+52deh+219dfg)+ab^2\left(7c^2f^2h+2cdf(101fg-82eh)+4d^2e(4eh+59fg)\right)-b^3\left(7c^2f(9fg-8eh)+4cde(19fg-16eh)+80d^2e^2g\right)}{2(a+bx)^2(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int -}{}}{}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{d^2f^2(192dfg+334deh+167cfh)a^3-bdf\left(2e(607fg+152eh)d^2+cf(895fg-456eh)d+122c^2f^2h\right)a^2+b^2\left(32e^2(43fg+3eh)d^3+16cef(83fg-59eh)d^2+2c^2f^2(4\right.}{}}{}}{}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$\downarrow$ 27

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$\downarrow$ 168

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$\downarrow$ 27

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$\downarrow$ 169

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$$\Big\downarrow 27$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$$\Big\downarrow 174$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$$\Big\downarrow 73$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

$$\Big\downarrow 221$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(19dfg+2deh+cfh)a+b^2(10deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)\sqrt{e+fx}}-\frac{99d^2f^2ha^3-bdf(219dfg+52deh+26cfh)a^2+b^2\left(4e(59fg+4eh)d^2+2cf(101fg-82eh)d+7c\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)\sqrt{}}$$

input
```
Int[(g + h*x)/((a + b*x)^5*(c + d*x)^2*(e + f*x)^(3/2)),x]
```

output
```
-1/4*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4*(c + d*x)*Sqrt[e + f
*x]) - (-1/3*(11*a^2*d*f*h + b^2*(10*d*e*g + 9*c*f*g - 8*c*e*h) - a*b*(19*
d*f*g + 2*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c + d*x)*S
qrt[e + f*x]) - ((99*a^3*d^2*f^2*h - a^2*b*d*f*(219*d*f*g + 52*d*e*h + 26*
c*f*h) - b^3*(80*d^2*e^2*g + 4*c*d*e*(19*f*g - 16*e*h) + 7*c^2*f*(9*f*g -
8*e*h)) + a*b^2*(7*c^2*f^2*h + 2*c*d*f*(101*f*g - 82*e*h) + 4*d^2*e*(59*f*
g + 4*e*h)))/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)*Sqrt[e + f*x
]) + ((693*a^4*d^3*f^3*h - a^3*b*d^2*f^2*(1725*d*f*g + 698*d*e*h + 349*c*f
*h) + b^4*(480*d^3*e^3*g + 2*c^2*d*e*f*(217*f*g - 184*e*h) + 16*c*d^2*e^2*
(31*f*g - 24*e*h) + 35*c^3*f^2*(9*f*g - 8*e*h)) - a*b^3*(35*c^3*f^3*h + c^
2*d*f^2*(1379*f*g - 1142*e*h) + 12*c*d^2*e*f*(155*f*g - 116*e*h) + 16*d^3*
e^2*(121*f*g + 6*e*h)) + a^2*b^2*d*f*(171*c^2*f^2*h + c*d*f*(2309*f*g - 16
04*e*h) + 2*d^2*e*(1433*f*g + 208*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x
)*(c + d*x)*Sqrt[e + f*x]) + ((2*d*(a^4*d^3*f^3*(192*d*f*g + 1027*d*e*h -
1219*c*f*h) + b^4*(960*d^4*e^4*g - 5*c^3*d*e*f^2*(49*f*g - 40*e*h) - 35*c^
4*f^3*(9*f*g - 8*e*h) - 32*c^2*d^2*e^2*f*(5*f*g - 3*e*h) - 48*c*d^3*e^3*(f
*g + 16*e*h)) + a*b^3*(35*c^4*f^4*h + c^2*d^2*e*f^2*(1055*f*g - 728*e*h) +
 5*c^3*d*f^3*(301*f*g - 255*e*h) - 48*d^4*e^3*(79*f*g + 4*e*h) + 16*c*d^3*
e^2*f*(29*f*g + 183*e*h)) + a^3*b*d^2*f^2*(409*c^2*f^2*h - d^2*e*(3131*f*g
 + 1336*e*h) + c*d*f*(2363*f*g + 1695*e*h)) - a^2*b^2*d*f*(185*c^3*f^3*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3943 vs. 2(1945) = 3890.

Time = 0.08 (sec) , antiderivative size = 3944, normalized size of antiderivative = 1.98

output too large to display

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
2*f^5*(-(-e*h+f*g)/(c*f-d*e)^2/(a*f-b*e)^5/(f*x+e)^(1/2)+d^5/(a*d-b*c)^6/f
^5/(c*f-d*e)^2*((1/2*a*c*d*f*h-1/2*a*d^2*f*g-1/2*b*c^2*f*h+1/2*b*c*d*f*g)*
(f*x+e)^(1/2)/((f*x+e)*d+c*f-d*e)+1/2*(a*c*d*f*h+2*a*d^2*e*h-3*a*d^2*f*g-1
1*b*c^2*f*h+8*b*c*d*e*h+13*b*c*d*f*g-10*b*d^2*e*g)/((c*f-d*e)*d)^(1/2)*arc
tan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))+b^2/(a*d-b*c)^6/f^5/(a*f-b*e)^5*
(((297/64*b^5*a^3*c^2*d^2*f^4*h-55/32*b^6*a^2*c^3*d*f^4*h-2*b^8*c*d^3*e^3*
f*g-231/32*b^4*a^4*c*d^3*f^4*h-5/8*a^3*b^5*c*d^3*e*f^3*h+39/4*a^2*b^6*c^2*
d^2*e*f^3*h+3*a^2*b^6*c*d^3*e^2*f^2*h-189/8*a^2*b^6*c*d^3*e*f^3*g-47/8*a*b
^7*c^3*d*e*f^3*h-57/8*a*b^7*c^2*d^2*e^2*f^2*h+99/8*a*b^7*c^2*d^2*e*f^3*g-a
*b^7*c*d^3*e^3*f*h+45/4*a*b^7*c*d^3*e^2*f^2*g-187/128*b^8*c^4*f^4*g+647/32
*a^3*b^5*c*d^3*f^4*g+109/8*a^3*b^5*d^4*e*f^3*g-1185/64*a^2*b^6*c^2*d^2*f^4
*g-69/8*a^2*b^6*d^4*e^2*f^2*g+263/32*a*b^7*c^3*d*f^4*g+2*a*b^7*d^4*e^3*f*g
+7/4*b^8*c^3*d*e^2*f^2*h-19/8*b^8*c^3*d*e*f^3*g+3/2*b^8*c^2*d^2*e^3*f*h-21
/8*b^8*c^2*d^2*e^2*f^2*g+19/8*a^3*b^5*d^4*e^2*f^2*h-1/2*a^2*b^6*d^4*e^3*f*
h-71/16*a^4*b^4*d^4*e*f^3*h+515/128*a^5*b^3*d^4*f^4*h+19/16*b^8*c^4*e*f^3*
h+35/128*b^7*a*c^4*f^4*h-1083/128*a^4*d^4*f^4*g*b^4)*(f*x+e)^(7/2)+1/384*f
*b^2*(5153*a^6*d^4*f^4*h-9652*a^5*b*c*d^3*f^4*h-10633*a^5*b*d^4*e*f^3*h-10
633*a^5*b*d^4*f^4*g+6534*a^4*b^2*c^2*d^2*f^4*h+9284*a^4*b^2*c*d^3*e*f^3*h+
25908*a^4*b^2*c*d^3*f^4*g+8312*a^4*b^2*d^4*e^2*f^2*h+27257*a^4*b^2*d^4*e*f
^3*g-2420*a^3*b^3*c^3*d*f^4*h+5274*a^3*b^3*c^2*d^2*e*f^3*h-24150*a^3*b^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**5/(d*x+c)**2/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7412 vs. 2(1945) = 3890.

Time = 0.85 (sec) , antiderivative size = 7412, normalized size of antiderivative = 3.72

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/64*(640*b^7*d^4*e^4*g + 768*b^7*c*d^3*e^3*f*g - 3328*a*b^6*d^4*e^3*f*g +
 720*b^7*c^2*d^2*e^2*f^2*g - 3744*a*b^6*c*d^3*e^2*f^2*g + 6864*a^2*b^5*d^4
*e^2*f^2*g + 560*b^7*c^3*d*e*f^3*g - 3120*a*b^6*c^2*d^2*e*f^3*g + 6864*a^2
*b^5*c*d^3*e*f^3*g - 6864*a^3*b^4*d^4*e*f^3*g + 315*b^7*c^4*f^4*g - 1820*a
*b^6*c^3*d*f^4*g + 4290*a^2*b^5*c^2*d^2*f^4*g - 5148*a^3*b^4*c*d^3*f^4*g +
 3003*a^4*b^3*d^4*f^4*g - 512*b^7*c*d^3*e^4*h - 128*a*b^6*d^4*e^4*h - 576*
b^7*c^2*d^2*e^3*f*h + 2432*a*b^6*c*d^3*e^3*f*h + 704*a^2*b^5*d^4*e^3*f*h -
 480*b^7*c^3*d*e^2*f^2*h + 2448*a*b^6*c^2*d^2*e^2*f^2*h - 4224*a^2*b^5*c*d
^3*e^2*f^2*h - 1584*a^3*b^4*d^4*e^2*f^2*h - 280*b^7*c^4*e*f^3*h + 1520*a*b
^6*c^3*d*e*f^3*h - 3168*a^2*b^5*c^2*d^2*e*f^3*h + 2640*a^3*b^4*c*d^3*e*f^3
*h + 1848*a^4*b^3*d^4*e*f^3*h - 35*a*b^6*c^4*f^4*h + 220*a^2*b^5*c^3*d*f^4
*h - 594*a^3*b^4*c^2*d^2*f^4*h + 924*a^4*b^3*c*d^3*f^4*h - 1155*a^5*b^2*d^
4*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^11*c^6*e^5 - 6*a
*b^10*c^5*d*e^5 + 15*a^2*b^9*c^4*d^2*e^5 - 20*a^3*b^8*c^3*d^3*e^5 + 15*a^4
*b^7*c^2*d^4*e^5 - 6*a^5*b^6*c*d^5*e^5 + a^6*b^5*d^6*e^5 - 5*a*b^10*c^6*e^
4*f + 30*a^2*b^9*c^5*d*e^4*f - 75*a^3*b^8*c^4*d^2*e^4*f + 100*a^4*b^7*c^3*
d^3*e^4*f - 75*a^5*b^6*c^2*d^4*e^4*f + 30*a^6*b^5*c*d^5*e^4*f - 5*a^7*b^4*
d^6*e^4*f + 10*a^2*b^9*c^6*e^3*f^2 - 60*a^3*b^8*c^5*d*e^3*f^2 + 150*a^4*b^
7*c^4*d^2*e^3*f^2 - 200*a^5*b^6*c^3*d^3*e^3*f^2 + 150*a^6*b^5*c^2*d^4*e^3*
f^2 - 60*a^7*b^4*c*d^5*e^3*f^2 + 10*a^8*b^3*d^6*e^3*f^2 - 10*a^3*b^8*c^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx=\text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^5*(c + d*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 72551, normalized size of antiderivative = 36.44

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^2(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^2/(f*x+e)^(3/2),x)
```

output
```
(3465*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**9*b*c**4*d**4*f**7*h - 10395*sqrt(b)*sqrt(e + f*x)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c
**3*d**5*e*f**6*h + 3465*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c**3*d**5*f**7*h*x + 10395*s
qrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**9*b*c**2*d**6*e**2*f**5*h - 10395*sqrt(b)*sqrt(e + f*x)*sq
rt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c**
2*d**6*e*f**6*h*x - 3465*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*c*d**7*e**3*f**4*h + 10395*s
qrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(
a*f - b*e)))*a**9*b*c*d**7*e**2*f**5*h*x - 3465*sqrt(b)*sqrt(e + f*x)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**9*b*d**8*
e**3*f**4*h*x - 2772*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e +
f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b**2*c**5*d**3*f**7*h + 2772*sqrt(
b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a**8*b**2*c**4*d**4*e*f**6*h - 9009*sqrt(b)*sqrt(e + f*x)*sqrt(a*
f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b**2*c**4*
d**4*f**7*g + 11088*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f
*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**8*b**2*c**4*d**4*f**7*h*x + 8316*s...
```

3.186 $\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 385

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = -\frac{2(be-af)^3(fg-eh)}{f^2(de-cf)^3\sqrt{e+fx}} + \frac{2b^3h\sqrt{e+fx}}{d^3f^2} + \frac{(bc-ad)^3(dg-ch)\sqrt{e+fx}}{2d^3(de-cf)^2(c+dx)^2} + \frac{(bc-ad)^2\left(ad(7dfg-4deh-3cfh)-b(12d^2eg+9c^2fh-cd(5fg+16eh))\right)\sqrt{e+fx}}{4d^3(de-cf)^3(c+dx)} + \frac{3(bc-ad)\left(a^2d^2f(5dfg-4deh-cfh)-2abd(c^2f^2h+2d^2e(3fg-2eh)-cdf(fg+2eh))+b^2(8d^3e^2g\right.}{4d^{7/2}(de-cf)^{7/2}}$$

output
```
-2*(-a*f+b*e)^3*(-e*h+f*g)/f^2/(-c*f+d*e)^3/(f*x+e)^(1/2)+2*b^3*h*(f*x+e)^
(1/2)/d^3/f^2+1/2*(-a*d+b*c)^3*(-c*h+d*g)*(f*x+e)^(1/2)/d^3/(-c*f+d*e)^2/(
d*x+c)^2+1/4*(-a*d+b*c)^2*(a*d*(-3*c*f*h-4*d*e*h+7*d*f*g)-b*(12*d^2*e*g+9*
c^2*f*h-c*d*(16*e*h+5*f*g)))*(f*x+e)^(1/2)/d^3/(-c*f+d*e)^3/(d*x+c)+3/4*(-
a*d+b*c)*(a^2*d^2*f*(-c*f*h-4*d*e*h+5*d*f*g)-2*a*b*d*(c^2*f^2*h+2*d^2*e*(-
2*e*h+3*f*g)-c*d*f*(2*e*h+f*g))+b^2*(8*d^3*e^2*g-5*c^3*f^2*h-4*c*d^2*e*(4*
e*h+f*g)+c^2*d*f*(16*e*h+f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(
1/2))/d^(7/2)/(-c*f+d*e)^(7/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 786 vs. $2(385) = 770$.

Time = 2.24 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.04

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{-3a^2bd^2f^2(c^3fh(e+fx)+4d^3ex(-eg-3fgx+2ehx)+c^2d(14e^2h-f^2x(\ldots}{\ldots}$$

$$-\frac{3(bc-ad)\left(a^2d^2f(-5dfg+4deh+cfh)-2abd(-c^2f^2h+2d^2e(-3fg+2eh)+cdf(fg+2eh))+b^2(-\ldots\right.}{4d^{7/2}(-de+cf)^{7/2}}$$

input
```
Integrate[((a + b*x)^3*(g + h*x))/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
(-3*a^2*b*d^2*f^2*(c^3*f*h*(e + f*x) + 4*d^3*e*x*(-(e*g) - 3*f*g*x + 2*e*h
*x) + c^2*d*(14*e^2*h - f^2*x*(5*g + h*x) + e*f*(-13*g + 5*h*x)) + c*d^2*(
-3*f^2*g*x^2 - 2*e^2*(g - 12*h*x) + e*f*x*(-21*g + 8*h*x))) + b^3*(15*c^5*
f^3*h*(e + f*x) - 8*d^5*e^3*x^2*(-(f*g) + 2*e*h + f*h*x) + c^4*d*f^2*(e +
f*x)*(-3*f*g - 38*e*h + 25*f*h*x) + 8*c*d^4*e^2*x*(-4*e^2*h + 3*f^2*h*x^2
+ e*f*(2*g + h*x)) - 4*c^2*d^3*e*(4*e^3*h + 3*f^3*x^2*(-g + 2*h*x) - 3*e*f
^2*x*(g + 2*h*x) - 2*e^2*f*(g + 5*h*x)) + c^3*d^2*f*(e + f*x)*(24*e^2*h +
2*e*f*(5*g - 32*h*x) + f^2*x*(-5*g + 8*h*x))) - 3*a*b^2*d*f*(8*d^4*e^2*(f*
g - e*h)*x^2 + 3*c^4*f^2*h*(e + f*x) + 8*c*d^3*e*x*(3*e*f*g - 2*e^2*h + f^
2*g*x) + c^3*d*f*(e + f*x)*(-10*e*h + f*(g + 5*h*x)) - c^2*d^2*(8*e^3*h +
f^3*g*x^2 - 2*e^2*f*(7*g - 6*h*x) + e*f^2*x*(-5*g + 12*h*x))) + a^3*d^3*f^
2*(c^2*f*(-8*f*g + 13*e*h + 5*f*h*x) + d^2*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h
*x) + e*f*x*(-5*g + 12*h*x)) + c*d*(2*e^2*h + f^2*x*(-25*g + 3*h*x) + e*f*
(-9*g + 21*h*x))))/(4*d^3*f^2*(-(d*e) + c*f)^3*(c + d*x)^2*Sqrt[e + f*x])
- (3*(b*c - a*d)*(a^2*d^2*f*(-5*d*f*g + 4*d*e*h + c*f*h) - 2*a*b*d*(-(c^2*
f^2*h) + 2*d^2*e*(-3*f*g + 2*e*h) + c*d*f*(f*g + 2*e*h)) + b^2*(-8*d^3*e^2
*g + 5*c^3*f^2*h + 4*c*d^2*e*(f*g + 4*e*h) - c^2*d*f*(f*g + 16*e*h)))*ArcT
an[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(4*d^(7/2)*(-(d*e) + c*f)^
(7/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.66, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 166, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(5dfg-4deh-cfh)+b(dfg+4deh-5cfh)x)}{2(c+dx)^2(e+fx)^{3/2}}dx}{2d(de-cf)}-\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)^2(6be(dg-ch)-a(5dfg-4deh-cfh)+b(dfg+4deh-5cfh)x)}{(c+dx)^2(e+fx)^{3/2}}dx}{4d(de-cf)}-\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{166}$$

$$\frac{\frac{\int -\frac{(a+bx)\left(2ad(be-af)(5dfg-4deh-cfh)+(4be-af)\left(ad(5dfg-4deh-cfh)-b\left(5fhc^2-d(fg+10eh)c+6d^2eg\right)\right)+b\left(adf(5dfg-4deh-cfh)-b\left(8e(fg+eh)d^2-cf(3\ldots\right.\right.\right.}{2(c+dx)(e+fx)^{3/2}}}{d(de-cf)}\ldots}{4d(de-cf)}\ldots$$

$$\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(a+bx)^2\left(ad(-cfh-4deh+5dfg)-b\left(5c^2fh-cd(10eh+fg)+6d^2eg\right)\right)}{d(c+dx)\sqrt{e+fx}(de-cf)}-\frac{\int \frac{(a+bx)\left(2ad(be-af)(5dfg-4deh-cfh)+(4be-af)\left(ad(5dfg-4deh-cfh)-b\left(5f\ldots\right.\right.\right.}{\ldots}}{\ldots}}{4d(de-cf)}\ldots$$

$$\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{163}$$

$$\frac{\frac{(a+bx)^2\left(ad(-cfh-4deh+5dfg)-b\left(5c^2fh-cd(10eh+fg)+6d^2eg\right)\right)}{d(c+dx)\sqrt{e+fx}(de-cf)}-\frac{3(bc-ad)\left(a^2d^2f(-cfh-4deh+5dfg)-2abd\left(c^2f^2h-cdf(2eh+fg)+2d^2e(3fg-2e\right.\right.}{d(c}}{\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(a+bx)^2\left(ad(-cfh-4deh+5dfg)-b\left(5c^2fh-cd(10eh+fg)+6d^2eg\right)\right)}{d(c+dx)\sqrt{e+fx}(de-cf)}-\frac{6(bc-ad)\left(a^2d^2f(-cfh-4deh+5dfg)-2abd\left(c^2f^2h-cdf(2eh+fg)+2d^2e(3fg-2e\right.\right.}{}}{\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(a+bx)^2\left(ad(-cfh-4deh+5dfg)-b\left(5c^2fh-cd(10eh+fg)+6d^2eg\right)\right)}{d(c+dx)\sqrt{e+fx}(de-cf)}-\frac{-\frac{6(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(a^2d^2f(-cfh-4deh+5dfg)-2abd\left(c^2f^2h-\right.\right.}{d^3}}{}}{\frac{(a+bx)^3(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}}$$

input
```
Int[((a + b*x)^3*(g + h*x))/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^3)/(d*(d*e - c*f)*(c + d*x)^2*Sqrt[e + f*x]) +
 (((a*d*(5*d*f*g - 4*d*e*h - c*f*h) - b*(6*d^2*e*g + 5*c^2*f*h - c*d*(f*g
+ 10*e*h)))*(a + b*x)^2)/(d*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x]) - ((-2*(3
*a^3*d^2*f^3*(5*d*f*g - 4*d*e*h - c*f*h) + 3*a*b^2*d*e*f*(3*c^2*f^2*h + c*
d*f*(f*g - 10*e*h) + 2*d^2*e*(7*f*g - 4*e*h)) - a^2*b*d*f^2*(5*c^2*f^2*h +
 2*d^2*e*(23*f*g - 16*e*h) - c*d*f*(f*g + 18*e*h)) - b^3*e*(15*c^3*f^3*h +
 8*d^3*e^2*(f*g - 2*e*h) + 2*c*d^2*e*f*(5*f*g + 12*e*h) - c^2*d*f^2*(3*f*g
 + 38*e*h)) - b^2*f*(d*e - c*f)*(a*d*f*(5*d*f*g - 4*d*e*h - c*f*h) - b*(15
*c^2*f^2*h + 8*d^2*e*(f*g + e*h) - c*d*f*(3*f*g + 28*e*h)))*x))/(d*f^2*(d*
e - c*f)*Sqrt[e + f*x]) - (6*(b*c - a*d)*(a^2*d^2*f*(5*d*f*g - 4*d*e*h - c
*f*h) - 2*a*b*d*(c^2*f^2*h + 2*d^2*e*(3*f*g - 2*e*h) - c*d*f*(f*g + 2*e*h)
) + b^2*(8*d^3*e^2*g - 5*c^3*f^2*h - 4*c*d^2*e*(f*g + 4*e*h) + c^2*d*f*(f*
g + 16*e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(3/2)*(
d*e - c*f)^(3/2)))/(2*d*(d*e - c*f)))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(361) = 722.

Time = 1.11 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.03

method	result
pseudoelliptic	$\frac{3(xd+c)^2\sqrt{fx+e}\,f^2(ad-bc)\left(\left(-5a^2f^2g+4ae(ah+3bg)f-8b\,e^2(ah+bg)\right)d^3+\left(\left(a^2h-2gab\right)f^2-4be(ah-bg)f+16b^2e^2h\right)c\,d^2+2\left(\left(a\right.\right.\right.}{4}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*((d*x+c)^2*(f*x+e)^(1/2)*f^2*(a*d-b*c)*((-5*a^2*f^2*g+4*a*e*(a*h+3*b*g
)*f-8*b*e^2*(a*h+b*g))*d^3+((a^2*h-2*a*b*g)*f^2-4*b*e*(a*h-b*g)*f+16*b^2*e
^2*h)*c*d^2+2*((a*h-1/2*b*g)*f-8*e*h*b)*c^2*b*f*d+5*b^2*c^3*f^2*h)*arctan(
d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2))+13/3*((c*f-d*e)*d)^(1/2)*(1/13*(-15*a
^3*f^4*g*x^2-5*a^2*x*(-36/5*x*g*b+a*(-12/5*h*x+g))*e*f^3+2*a*(-12*b^2*g*x^
2+6*a*x*(-2*h*x+g)*b+a^2*(2*h*x+g))*e^2*f^2+24*x^2*b^2*e^3*(1/3*(-h*x+g)*b
+a*h)*f-16*b^3*e^4*h*x^2)*d^5+2/13*c*(-25/2*a^2*x*(-9/25*x*g*b+a*(-3/25*h*
x+g))*f^4-9/2*a*(8/3*b^2*g*x^2-7*a*x*(-8/21*h*x+g)*b+a^2*(-7/3*h*x+g))*e*f
^3+(12*x^3*h*b^3-36*a*b^2*g*x+3*a^2*(-12*h*x+g)*b+h*a^3)*e^2*f^2+24*(1/3*(
1/2*h*x+g)*b+a*h)*x*b^2*e^3*f-16*b^3*e^4*h*x)*d^4+c^2*(-8/13*a*(-3/8*b^2*g
*x^2-15/8*a*x*(1/5*h*x+g)*b+a^2*(-5/8*h*x+g))*f^4+(12/13*x^2*(-2*h*x+g)*b^
3-15/13*a*x*(-12/5*h*x+g)*b^2+3*a^2*(-5/13*h*x+g)*b+h*a^3)*e*f^3-42/13*(-2
/7*x*(2*h*x+g)*b^2+a*(-6/7*h*x+g)*b+a^2*h)*b*e^2*f^2+24/13*b^2*e^3*(1/3*(5
*h*x+g)*b+a*h)*f-16/13*b^3*e^4*h)*d^3-3/13*c^3*((5/3*(-8/5*h*x+g)*x*b^2+a*
(5*h*x+g)*b+a^2*h)*f^2-10*(1/3*(-32/5*h*x+g)*b+a*h)*b*e*f-8*b^2*e^2*h)*(f*
x+e)*b*f*d^2-9/13*c^4*(f*x+e)*((1/3*(-25/3*h*x+g)*b+a*h)*f+38/9*e*h*b)*b^2
*f^2*d+15/13*b^3*c^5*f^3*h*(f*x+e)))/(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)/(d*
x+c)^2/(c*f-d*e)^3/d^3/f^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2893 vs. $2(361) = 722$.

Time = 0.51 (sec) , antiderivative size = 5799, normalized size of antiderivative = 15.06

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(h*x+g)/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(361) = 722$.

Time = 0.17 (sec) , antiderivative size = 1383, normalized size of antiderivative = 3.59

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-3/4*(8*b^3*c*d^3*e^2*g - 8*a*b^2*d^4*e^2*g - 4*b^3*c^2*d^2*e*f*g - 8*a*b^
2*c*d^3*e*f*g + 12*a^2*b*d^4*e*f*g + b^3*c^3*d*f^2*g + a*b^2*c^2*d^2*f^2*g
 + 3*a^2*b*c*d^3*f^2*g - 5*a^3*d^4*f^2*g - 16*b^3*c^2*d^2*e^2*h + 24*a*b^2
*c*d^3*e^2*h - 8*a^2*b*d^4*e^2*h + 16*b^3*c^3*d*e*f*h - 12*a*b^2*c^2*d^2*e
*f*h - 8*a^2*b*c*d^3*e*f*h + 4*a^3*d^4*e*f*h - 5*b^3*c^4*f^2*h + 3*a*b^2*c
^3*d*f^2*h + a^2*b*c^2*d^2*f^2*h + a^3*c*d^3*f^2*h)*arctan(sqrt(f*x + e)*d
/sqrt(-d^2*e + c*d*f))/((d^6*e^3 - 3*c*d^5*e^2*f + 3*c^2*d^4*e*f^2 - c^3*d
^3*f^3)*sqrt(-d^2*e + c*d*f)) - 2*(b^3*e^3*f*g - 3*a*b^2*e^2*f^2*g + 3*a^2
*b*e*f^3*g - a^3*f^4*g - b^3*e^4*h + 3*a*b^2*e^3*f*h - 3*a^2*b*e^2*f^2*h +
 a^3*e*f^3*h)/((d^3*e^3*f^2 - 3*c*d^2*e^2*f^3 + 3*c^2*d*e*f^4 - c^3*f^5)*s
qrt(f*x + e)) + 2*sqrt(f*x + e)*b^3*h/(d^3*f^2) - 1/4*(12*(f*x + e)^(3/2)*
b^3*c^2*d^3*e*f*g - 24*(f*x + e)^(3/2)*a*b^2*c*d^4*e*f*g + 12*(f*x + e)^(3
/2)*a^2*b*d^5*e*f*g - 12*sqrt(f*x + e)*b^3*c^2*d^3*e^2*f*g + 24*sqrt(f*x +
 e)*a*b^2*c*d^4*e^2*f*g - 12*sqrt(f*x + e)*a^2*b*d^5*e^2*f*g - 5*(f*x + e)
^(3/2)*b^3*c^3*d^2*f^2*g + 3*(f*x + e)^(3/2)*a*b^2*c^2*d^3*f^2*g + 9*(f*x
+ e)^(3/2)*a^2*b*c*d^4*f^2*g - 7*(f*x + e)^(3/2)*a^3*d^5*f^2*g + 15*sqrt(f
*x + e)*b^3*c^3*d^2*e*f^2*g - 21*sqrt(f*x + e)*a*b^2*c^2*d^3*e*f^2*g - 3*s
qrt(f*x + e)*a^2*b*c*d^4*e*f^2*g + 9*sqrt(f*x + e)*a^3*d^5*e*f^2*g - 3*sqr
t(f*x + e)*b^3*c^4*d*f^3*g - 3*sqrt(f*x + e)*a*b^2*c^3*d^2*f^3*g + 15*sqrt
(f*x + e)*a^2*b*c^2*d^3*f^3*g - 9*sqrt(f*x + e)*a^3*c*d^4*f^3*g - 16*(f...
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.00

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(((g + h*x)*(a + b*x)^3)/((e + f*x)^(3/2)*(c + d*x)^3),x)
```

output
```
(2*b^3*h*(e + f*x)^(1/2))/(d^3*f^2) - ((2*(a^3*d^3*f^4*g + b^3*d^3*e^4*h -
 a^3*d^3*e*f^3*h - b^3*d^3*e^3*f*g + 3*a*b^2*d^3*e^2*f^2*g + 3*a^2*b*d^3*e
^2*f^2*h - 3*a^2*b*d^3*e*f^3*g - 3*a*b^2*d^3*e^3*f*h))/(c*f - d*e) - ((e +
 f*x)^2*(3*a^3*c*d^4*f^4*h - 8*b^3*d^5*e^4*h - 15*a^3*d^5*f^4*g + 9*b^3*c^
4*d*f^4*h + 12*a^3*d^5*e*f^3*h + 8*b^3*d^5*e^3*f*g - 5*b^3*c^3*d^2*f^4*g +
 3*a*b^2*c^2*d^3*f^4*g - 15*a*b^2*c^3*d^2*f^4*h + 3*a^2*b*c^2*d^3*f^4*h -
24*a*b^2*d^5*e^2*f^2*g - 24*a^2*b*d^5*e^2*f^2*h + 12*b^3*c^2*d^3*e*f^3*g -
 16*b^3*c^3*d^2*e*f^3*h + 9*a^2*b*c*d^4*f^4*g + 36*a^2*b*d^5*e*f^3*g + 24*
a*b^2*d^5*e^3*f*h - 24*a*b^2*c*d^4*e*f^3*g - 24*a^2*b*c*d^4*e*f^3*h + 36*a
*b^2*c^2*d^3*e*f^3*h))/(4*(c*f - d*e)^3) + ((e + f*x)*(25*a^3*d^4*f^4*g -
7*b^3*c^4*f^4*h + 16*b^3*d^4*e^4*h - 5*a^3*c*d^3*f^4*h + 3*b^3*c^3*d*f^4*g
 - 20*a^3*d^4*e*f^3*h - 16*b^3*d^4*e^3*f*g + 3*a*b^2*c^2*d^2*f^4*g + 3*a^2
*b*c^2*d^2*f^4*h + 48*a*b^2*d^4*e^2*f^2*g + 48*a^2*b*d^4*e^2*f^2*h - 12*b^
3*c^2*d^2*e*f^3*g - 15*a^2*b*c*d^3*f^4*g + 9*a*b^2*c^3*d*f^4*h - 60*a^2*b*
d^4*e*f^3*g - 48*a*b^2*d^4*e^3*f*h + 16*b^3*c^3*d*e*f^3*h + 24*a*b^2*c*d^3
*e*f^3*g + 24*a^2*b*c*d^3*e*f^3*h - 36*a*b^2*c^2*d^2*e*f^3*h))/(4*(c*f - d
*e)^2))/((e + f*x)^(1/2)*(c^2*d^3*f^4 + d^5*e^2*f^2 - 2*c*d^4*e*f^3) + (e
+ f*x)^(3/2)*(2*c*d^4*f^3 - 2*d^5*e*f^2) + d^5*f^2*(e + f*x)^(5/2)) - (3*a
tan((3*(e + f*x)^(1/2)*(a*d - b*c)*(d^6*e^3 - c^3*d^3*f^3 + 3*c^2*d^4*e*f^
2 - 3*c*d^5*e^2*f)*(5*a^2*d^3*f^2*g + 8*b^2*d^3*e^2*g - 5*b^2*c^3*f^2*h...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5447, normalized size of antiderivative = 14.15

$$\int \frac{(a+bx)^3(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output

```
(3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a**3*c**3*d**3*f**4*h + 12*sqrt(d)*sqrt(e + f*x)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*e
*f**3*h - 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*a**3*c**2*d**4*f**4*g + 6*sqrt(d)*sqrt(e + f*x)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*
*2*d**4*f**4*h*x + 24*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**5*e*f**3*h*x - 30*sqrt(d)*sq
rt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*a**3*c*d**5*f**4*g*x + 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((s
qrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*c*d**5*f**4*h*x**2 + 12*sq
rt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c
*f - d*e)))*a**3*d**6*e*f**3*h*x**2 - 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**3*d**6*f**4*g*x*
*2 + 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a**2*b*c**4*d**2*f**4*h - 24*sqrt(d)*sqrt(e + f*x)*sq
rt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**
3*d**3*e*f**3*h + 9*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f
*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*b*c**3*d**3*f**4*g + 6*sqrt(d)*sqrt
(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*...
```

3.187 $\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 355

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{2(be-af)^2(fg-eh)}{f(de-cf)^3\sqrt{e+fx}} - \frac{(bc-ad)^2(dg-ch)\sqrt{e+fx}}{2d^2(de-cf)^2(c+dx)^2}$$
$$- \frac{(bc-ad)\left(ad(7dfg-4deh-3cfh)-b(8d^2eg+5c^2fh-cd(fg+12eh))\right)\sqrt{e+fx}}{4d^2(de-cf)^3(c+dx)}$$
$$- \frac{(3a^2d^2f(5dfg-4deh-cfh)+b^2(8d^3e^2g-3c^3f^2h-c^2df(fg-12eh)+8cd^2e(fg-3eh))-2abd(c^2f^2}{4d^{5/2}(de-cf)^{7/2}}$$

output
```
2*(-a*f+b*e)^2*(-e*h+f*g)/f/(-c*f+d*e)^3/(f*x+e)^(1/2)-1/2*(-a*d+b*c)^2*(-
c*h+d*g)*(f*x+e)^(1/2)/d^2/(-c*f+d*e)^2/(d*x+c)^2-1/4*(-a*d+b*c)*(a*d*(-3*
c*f*h-4*d*e*h+7*d*f*g)-b*(8*d^2*e*g+5*c^2*f*h-c*d*(12*e*h+f*g)))*(f*x+e)^(
1/2)/d^2/(-c*f+d*e)^3/(d*x+c)-1/4*(3*a^2*d^2*f*(-c*f*h-4*d*e*h+5*d*f*g)+b^
2*(8*d^3*e^2*g-3*c^3*f^2*h-c^2*d*f*(-12*e*h+f*g)+8*c*d^2*e*(-3*e*h+f*g))-2
*a*b*d*(c^2*f^2*h+c*d*f*(-8*e*h+3*f*g)+4*d^2*e*(-2*e*h+3*f*g)))*arctanh(d^
(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/d^(5/2)/(-c*f+d*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{\frac{\sqrt{d}\left(-2abdf\left(c^3fh(e+fx)+4d^3ex(-eg-3fgx+2ehx)+c^2d\left(14e^2h-f^2x(5g+hx)+ef(-13g+5hx)\right)+cd^2\left(-\right.\right.\right.}{}}{}$$

input
```
Integrate[((a + b*x)^2*(g + h*x))/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((Sqrt[d]*(-2*a*b*d*f*(c^3*f*h*(e + f*x) + 4*d^3*e*x*(-(e*g) - 3*f*g*x + 2
*e*h*x) + c^2*d*(14*e^2*h - f^2*x*(5*g + h*x) + e*f*(-13*g + 5*h*x)) + c*d
^2*(-3*f^2*g*x^2 - 2*e^2*(g - 12*h*x) + e*f*x*(-21*g + 8*h*x))) - b^2*(8*d
^4*e^2*(f*g - e*h)*x^2 + 3*c^4*f^2*h*(e + f*x) + 8*c*d^3*e*x*(3*e*f*g - 2*
e^2*h + f^2*g*x) + c^3*d*f*(e + f*x)*(-10*e*h + f*(g + 5*h*x)) - c^2*d^2*(
8*e^3*h + f^3*g*x^2 - 2*e^2*f*(7*g - 6*h*x) + e*f^2*x*(-5*g + 12*h*x))) +
a^2*d^2*f*(c^2*f*(-8*f*g + 13*e*h + 5*f*h*x) + d^2*(-15*f^2*g*x^2 + 2*e^2*
(g + 2*h*x) + e*f*x*(-5*g + 12*h*x)) + c*d*(2*e^2*h + f^2*x*(-25*g + 3*h*x
) + e*f*(-9*g + 21*h*x)))))/(f*(-(d*e) + c*f)^3*(c + d*x)^2*Sqrt[e + f*x])
 - ((-3*a^2*d^2*f*(-5*d*f*g + 4*d*e*h + c*f*h) + 2*a*b*d*(-(c^2*f^2*h) + 4
*d^2*e*(-3*f*g + 2*e*h) + c*d*f*(-3*f*g + 8*e*h)) + b^2*(8*d^3*e^2*g - 3*c
^3*f^2*h + 8*c*d^2*e*(f*g - 3*e*h) + c^2*d*f*(-(f*g) + 12*e*h)))*ArcTan[(S
qrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(-(d*e) + c*f)^(7/2))/(4*d^(5/2
))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {166, 27, 161, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(5dfg-4deh-cfh)-b(dfg-4deh+3cfh)x)}{2(c+dx)^2(e+fx)^{3/2}}dx}{2d(de-cf)}-\frac{(a+bx)^2(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+bx)(4be(dg-ch)-a(5dfg-4deh-cfh)-b(dfg-4deh+3cfh)x)}{(c+dx)^2(e+fx)^{3/2}}dx}{4d(de-cf)}-\frac{(a+bx)^2(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$\Big\downarrow$ 161

$$\frac{(3a^2d^2f(-cfh-4deh+5dfg)-2abd(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh))+b^2(-3c^3f^2h-c^2df(fg-12eh)+8cd^2e(fg-3eh)+8d^3e^2g))\int \frac{}{(c+d}}{2d(de-cf)^2}$$

$$\frac{(a+bx)^2(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$\Big\downarrow$ 73

$$\frac{(3a^2d^2f(-cfh-4deh+5dfg)-2abd(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh))+b^2(-3c^3f^2h-c^2df(fg-12eh)+8cd^2e(fg-3eh)+8d^3e^2g))\int \frac{}{c+\frac{d(}{}}}{df(de-cf)^2}$$

$$\frac{(a+bx)^2(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$\Big\downarrow$ 221

$$\frac{x(3a^2d^2f^2(-cfh-4deh+5dfg)-2abdf(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh))+b^2(3c^3f^3h+c^2df^2(fg-8eh)+2cd^2ef(2fg-eh)+2d^3e^2(5fg-4}{df(c+dx)\sqrt{e+fx}}$$

$$\frac{(a+bx)^2(dg-ch)}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

input

```
Int[((a + b*x)^2*(g + h*x))/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*((d*g - c*h)*(a + b*x)^2)/(d*(d*e - c*f)*(c + d*x)^2*Sqrt[e + f*x]) +
 ((a^2*d*f*(d*e + 2*c*f)*(5*d*f*g - 4*d*e*h - c*f*h) - 2*a*b*d*e*f*(2*d^2*
e*g + 13*c*d*f*g - 14*c*d*e*h - c^2*f*h) + b^2*c*e*(3*c^2*f^2*h + c*d*f*(f
*g - 10*e*h) + 2*d^2*e*(7*f*g - 4*e*h)) + (3*a^2*d^2*f^2*(5*d*f*g - 4*d*e*
h - c*f*h) - 2*a*b*d*f*(c^2*f^2*h + c*d*f*(3*f*g - 8*e*h) + 4*d^2*e*(3*f*g
 - 2*e*h)) + b^2*(3*c^3*f^3*h + c^2*d*f^2*(f*g - 8*e*h) + 2*d^3*e^2*(5*f*g
 - 4*e*h) + 2*c*d^2*e*f*(2*f*g - e*h)))*x)/(d*f*(d*e - c*f)^2*(c + d*x)*Sq
rt[e + f*x]) - ((3*a^2*d^2*f*(5*d*f*g - 4*d*e*h - c*f*h) + b^2*(8*d^3*e^2*
g - 3*c^3*f^2*h - c^2*d*f*(f*g - 12*e*h) + 8*c*d^2*e*(f*g - 3*e*h)) - 2*a*
b*d*(c^2*f^2*h + c*d*f*(3*f*g - 8*e*h) + 4*d^2*e*(3*f*g - 2*e*h)))*ArcTanh
[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(d^(3/2)*(d*e - c*f)^(5/2)))/(4
*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.64

method	result
pseudoelliptic	$\frac{3(xd+c)^2\left(\left(c\left(c^2f^2-4cdef+8d^2e^2\right)h+\frac{dg\left(c^2f^2-8cdef-8d^2e^2\right)}{3}\right)b^2+\frac{2ad\left(\left(c^2f^2-8cdef-8d^2e^2\right)h+3dfg(cf+4de)\right)b}{3}+a^2d^2((cf}{4}$
derivativedivides	$-\frac{2\left(-a^2ef^2h+a^2f^3g+2abe^2fh-2abef^2g-b^2e^3h+b^2e^2fg\right)}{(cf-de)^3\sqrt{fx+e}}+\frac{2f\left(\frac{f\left(3a^2cd^2fh+4a^2d^3eh-7a^2d^3fg+2abc^2dfh-16abcd^2eh+6\right.}{}\right.}{}$
default	$-\frac{2\left(-a^2ef^2h+a^2f^3g+2abe^2fh-2abef^2g-b^2e^3h+b^2e^2fg\right)}{(cf-de)^3\sqrt{fx+e}}+\frac{2f\left(\frac{f\left(3a^2cd^2fh+4a^2d^3eh-7a^2d^3fg+2abc^2dfh-16abcd^2eh+6\right.}{}\right.}{}$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*((d*x+c)^2*((c*(c^2*f^2-4*c*d*e*f+8*d^2*e^2)*h+1/3*d*g*(c^2*f^2-8*c*d*
e*f-8*d^2*e^2))*b^2+2/3*a*d*((c^2*f^2-8*c*d*e*f-8*d^2*e^2)*h+3*d*f*g*(c*f+
4*d*e))*b+a^2*d^2*((c*f+4*d*e)*h-5*d*f*g)*f)*arctan(d*(f*x+e)^(1/2)/((c*f-
d*e)*d)^(1/2))*f*(f*x+e)^(1/2)+13/3*(1/13*((-3*c^3*x*(5/3*x*d+c)*f^3-3*c^2
*(4/3*x*d+c)*e*(-3*d*x+c)*f^2+10*c^2*d*(6/5*x*d+c)*e^2*f+8*d^2*e^3*(d*x+c)
^2)*h-d*(x*c^2*(-d*x+c)*f^2+c*e*(8*d^2*x^2+5*c*d*x+c^2)*f+14*d*e^2*(4/7*d^
2*x^2+12/7*c*d*x+c^2))*g*f)*b^2-2/13*a*d*((x*c^2*(-d*x+c)*f^2+c*e*(8*d^2*x
^2+5*c*d*x+c^2)*f+14*d*e^2*(4/7*d^2*x^2+12/7*c*d*x+c^2))*h-13*d*(5/13*x*c*
(3/5*x*d+c)*f^2+e*(12/13*d^2*x^2+21/13*c*d*x+c^2)*f+2/13*d*e^2*(2*d*x+c))*
g)*f*b+a^2*d^2*((5/13*x*c*(3/5*x*d+c)*f^2+e*(12/13*d^2*x^2+21/13*c*d*x+c^2
)*f+2/13*d*e^2*(2*d*x+c))*h-8/13*((15/8*d^2*x^2+25/8*c*d*x+c^2)*f^2+9/8*(5
/9*x*d+c)*d*e*f-1/4*d^2*e^2)*g)*f)*((c*f-d*e)*d)^(1/2))/(f*x+e)^(1/2)/((c*
f-d*e)*d)^(1/2)/(d*x+c)^2/(c*f-d*e)^3/d^2/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(333) = 666$.

Time = 0.40 (sec) , antiderivative size = 3988, normalized size of antiderivative = 11.23

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2*(h*x+g)/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(333) = 666$.

Time = 0.15 (sec) , antiderivative size = 980, normalized size of antiderivative = 2.76

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*b^2*d^3*e^2*g + 8*b^2*c*d^2*e*f*g - 24*a*b*d^3*e*f*g - b^2*c^2*d*f^
2*g - 6*a*b*c*d^2*f^2*g + 15*a^2*d^3*f^2*g - 24*b^2*c*d^2*e^2*h + 16*a*b*d
^3*e^2*h + 12*b^2*c^2*d*e*f*h + 16*a*b*c*d^2*e*f*h - 12*a^2*d^3*e*f*h - 3*
b^2*c^3*f^2*h - 2*a*b*c^2*d*f^2*h - 3*a^2*c*d^2*f^2*h)*arctan(sqrt(f*x + e
)*d/sqrt(-d^2*e + c*d*f))/((d^5*e^3 - 3*c*d^4*e^2*f + 3*c^2*d^3*e*f^2 - c^
3*d^2*f^3)*sqrt(-d^2*e + c*d*f)) + 2*(b^2*e^2*f*g - 2*a*b*e*f^2*g + a^2*f^
3*g - b^2*e^3*h + 2*a*b*e^2*f*h - a^2*e*f^2*h)/((d^3*e^3*f - 3*c*d^2*e^2*f
^2 + 3*c^2*d*e*f^3 - c^3*f^4)*sqrt(f*x + e)) + 1/4*(8*(f*x + e)^(3/2)*b^2*
c*d^3*e*f*g - 8*(f*x + e)^(3/2)*a*b*d^4*e*f*g - 8*sqrt(f*x + e)*b^2*c*d^3*
e^2*f*g + 8*sqrt(f*x + e)*a*b*d^4*e^2*f*g - (f*x + e)^(3/2)*b^2*c^2*d^2*f^
2*g - 6*(f*x + e)^(3/2)*a*b*c*d^3*f^2*g + 7*(f*x + e)^(3/2)*a^2*d^4*f^2*g
+ 7*sqrt(f*x + e)*b^2*c^2*d^2*e*f^2*g + 2*sqrt(f*x + e)*a*b*c*d^3*e*f^2*g
- 9*sqrt(f*x + e)*a^2*d^4*e*f^2*g + sqrt(f*x + e)*b^2*c^3*d*f^3*g - 10*sqr
t(f*x + e)*a*b*c^2*d^2*f^3*g + 9*sqrt(f*x + e)*a^2*c*d^3*f^3*g - 12*(f*x +
 e)^(3/2)*b^2*c^2*d^2*e*f*h + 16*(f*x + e)^(3/2)*a*b*c*d^3*e*f*h - 4*(f*x
+ e)^(3/2)*a^2*d^4*e*f*h + 12*sqrt(f*x + e)*b^2*c^2*d^2*e^2*f*h - 16*sqrt(
f*x + e)*a*b*c*d^3*e^2*f*h + 4*sqrt(f*x + e)*a^2*d^4*e^2*f*h + 5*(f*x + e)
^(3/2)*b^2*c^3*d*f^2*h - 2*(f*x + e)^(3/2)*a*b*c^2*d^2*f^2*h - 3*(f*x + e)
^(3/2)*a^2*c*d^3*f^2*h - 15*sqrt(f*x + e)*b^2*c^3*d*e*f^2*h + 14*sqrt(f*x
+ e)*a*b*c^2*d^2*e*f^2*h + sqrt(f*x + e)*a^2*c*d^3*e*f^2*h + 3*sqrt(f*x...
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.20

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{\frac{2\left(-h\,a^2\,e\,f^2+g\,a^2\,f^3+2\,h\,a\,b\,e^2\,f-2\,g\,a\,b\,e\,f^2-h\,b^2\,e^3+g\,b^2\,e^2\,f\right)}{c\,f-d\,e}+\frac{(e+f\,x)\left(-5\,h\,a^2\,c\,d^2\,f^3-20\,h\,a^2\,d^3\,e\,f^2+25\,g\,a^2\,d^3\,f^3+2\,h\,a\,b\,c^2\,d\,f^3+16\cdots\right.}{\cdots}}{\cdots}$$

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,\left(-c^3\,d^2\,f^3+3\,c^2\,d^3\,e\,f^2-3\,c\,d^4\,e^2\,f+d^5\,e^3\right)}{d^{3/2}\,{\left(c\,f-d\,e\right)}^{7/2}}\right)\,\left(3\,h\,a^2\,c\,d^2\,f^2+12\,h\,a^2\,d^3\,e\,f-15\,g\,a^2\,d^3\,f^2+2\,h\,a\,b\,c^2\,d\,f\cdots\right.}{\cdots}$$

input
```
int(((g + h*x)*(a + b*x)^2)/((e + f*x)^(3/2)*(c + d*x)^3),x)
```

output
```
- ((2*(a^2*f^3*g - b^2*e^3*h - a^2*e*f^2*h + b^2*e^2*f*g - 2*a*b*e*f^2*g +
 2*a*b*e^2*f*h))/(c*f - d*e) + ((e + f*x)*(25*a^2*d^3*f^3*g + 3*b^2*c^3*f^
3*h - 16*b^2*d^3*e^3*h - 5*a^2*c*d^2*f^3*h + b^2*c^2*d*f^3*g - 20*a^2*d^3*
e*f^2*h + 16*b^2*d^3*e^2*f*g - 10*a*b*c*d^2*f^3*g + 2*a*b*c^2*d*f^3*h - 40
*a*b*d^3*e*f^2*g + 32*a*b*d^3*e^2*f*h + 8*b^2*c*d^2*e*f^2*g - 12*b^2*c^2*d
*e*f^2*h + 16*a*b*c*d^2*e*f^2*h))/(4*d^2*(c*f - d*e)^2) - ((e + f*x)^2*(8*
b^2*d^3*e^3*h - 5*b^2*c^3*f^3*h - 15*a^2*d^3*f^3*g + 3*a^2*c*d^2*f^3*h + b
^2*c^2*d*f^3*g + 12*a^2*d^3*e*f^2*h - 8*b^2*d^3*e^2*f*g + 6*a*b*c*d^2*f^3*
g + 2*a*b*c^2*d*f^3*h + 24*a*b*d^3*e*f^2*g - 16*a*b*d^3*e^2*f*h - 8*b^2*c*
d^2*e*f^2*g + 12*b^2*c^2*d*e*f^2*h - 16*a*b*c*d^2*e*f^2*h))/(4*d*(c*f - d*
e)^3))/((e + f*x)^(1/2)*(c^2*f^3 + d^2*e^2*f - 2*c*d*e*f^2) + (e + f*x)^(3
/2)*(2*c*d*f^2 - 2*d^2*e*f) + d^2*f*(e + f*x)^(5/2)) - (atan(((e + f*x)^(1
/2)*(d^5*e^3 - c^3*d^2*f^3 + 3*c^2*d^3*e*f^2 - 3*c*d^4*e^2*f))/(d^(3/2)*(c
*f - d*e)^(7/2)))*(3*b^2*c^3*f^2*h - 8*b^2*d^3*e^2*g - 15*a^2*d^3*f^2*g -
16*a*b*d^3*e^2*h + 12*a^2*d^3*e*f*h + 3*a^2*c*d^2*f^2*h + 24*b^2*c*d^2*e^2
*h + b^2*c^2*d*f^2*g + 24*a*b*d^3*e*f*g + 6*a*b*c*d^2*f^2*g + 2*a*b*c^2*d*
f^2*h - 8*b^2*c*d^2*e*f*g - 12*b^2*c^2*d*e*f*h - 16*a*b*c*d^2*e*f*h))/(4*d
^(5/2)*(c*f - d*e)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3593, normalized size of antiderivative = 10.12

$$\int \frac{(a+bx)^2(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output

```
(3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a**2*c**3*d**2*f**3*h + 12*sqrt(d)*sqrt(e + f*x)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*e
*f**2*h - 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/
(sqrt(d)*sqrt(c*f - d*e)))*a**2*c**2*d**3*f**3*g + 6*sqrt(d)*sqrt(e + f*x)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*
*2*d**3*f**3*h*x + 24*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e +
 f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*e*f**2*h*x - 30*sqrt(d)*sq
rt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e
)))*a**2*c*d**4*f**3*g*x + 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((s
qrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*c*d**4*f**3*h*x**2 + 12*sq
rt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c
*f - d*e)))*a**2*d**5*e*f**2*h*x**2 - 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a**2*d**5*f**3*g*x*
*2 + 2*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(
d)*sqrt(c*f - d*e)))*a*b*c**4*d*f**3*h - 16*sqrt(d)*sqrt(e + f*x)*sqrt(c*f
 - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*e*
f**2*h + 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(s
qrt(d)*sqrt(c*f - d*e)))*a*b*c**3*d**2*f**3*g + 4*sqrt(d)*sqrt(e + f*x)*sq
rt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*b*c**...
```

3.188 $\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 280

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = -\frac{2(be-af)(fg-eh)}{(de-cf)^3\sqrt{e+fx}}+\frac{(bc-ad)(dg-ch)\sqrt{e+fx}}{2d(de-cf)^2(c+dx)^2}$$
$$+\frac{(ad(7dfg-4deh-3cfh)-b(4d^2eg+c^2fh+cd(3fg-8eh)))\sqrt{e+fx}}{4d(de-cf)^3(c+dx)}$$
$$-\frac{(3adf(5dfg-4deh-cfh)-b(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4d^{3/2}(de-cf)^{7/2}}$$

output
```
-2*(-a*f+b*e)*(-e*h+f*g)/(-c*f+d*e)^3/(f*x+e)^(1/2)+1/2*(-a*d+b*c)*(-c*h+d
*g)*(f*x+e)^(1/2)/d/(-c*f+d*e)^2/(d*x+c)^2+1/4*(a*d*(-3*c*f*h-4*d*e*h+7*d*
f*g)-b*(4*d^2*e*g+c^2*f*h+c*d*(-8*e*h+3*f*g)))*(f*x+e)^(1/2)/d/(-c*f+d*e)^
3/(d*x+c)-1/4*(3*a*d*f*(-c*f*h-4*d*e*h+5*d*f*g)-b*(c^2*f^2*h+c*d*f*(-8*e*h
+3*f*g)+4*d^2*e*(-2*e*h+3*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^
(1/2))/d^(3/2)/(-c*f+d*e)^(7/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{\sqrt{d}\left(b\left(c^3fh(e+fx)+4d^3ex(-eg-3fgx+2ehx)+c^2d\left(14e^2h-f^2x(5g+hx)+ef(-13g+5hx)\right)+cd^2\left(-3f^2gx\right.\right.\right.}{}$$

input
```
Integrate[((a + b*x)*(g + h*x))/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((Sqrt[d]*(b*(c^3*f*h*(e + f*x) + 4*d^3*e*x*(-(e*g) - 3*f*g*x + 2*e*h*x) +
 c^2*d*(14*e^2*h - f^2*x*(5*g + h*x) + e*f*(-13*g + 5*h*x)) + c*d^2*(-3*f^
2*g*x^2 - 2*e^2*(g - 12*h*x) + e*f*x*(-21*g + 8*h*x))) - a*d*(c^2*f*(-8*f*
g + 13*e*h + 5*f*h*x) + d^2*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*x) + e*f*x*(-5
*g + 12*h*x)) + c*d*(2*e^2*h + f^2*x*(-25*g + 3*h*x) + e*f*(-9*g + 21*h*x)
))))/((d*e - c*f)^3*(c + d*x)^2*Sqrt[e + f*x]) - ((-3*a*d*f*(-5*d*f*g + 4*
d*e*h + c*f*h) + b*(-(c^2*f^2*h) + 4*d^2*e*(-3*f*g + 2*e*h) + c*d*f*(-3*f*
g + 8*e*h)))*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]])/(-(d*e) +
 c*f)^(7/2))/(4*d^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {161, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{161}$$

$$-\frac{\left(3adf(-cfh-4deh+5dfg)-b\left(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh)\right)\right)\int \frac{1}{(c+dx)^2\sqrt{e+fx}}dx}{4df(de-cf)^2}-\frac{x\left(adf(-cfh-4deh+5dfg)-b\left(-c^2f^2h+cdf^2g+4d^2e(fg-eh)\right)\right)+adf(-5ceh+4cfg+deg)-bce(-cfh-}{2df(c+dx)^2\sqrt{e+fx}(de-cf)^2}$$

$$\Big\downarrow \; 52$$

$$-\frac{\left(3adf(-cfh-4deh+5dfg)-b\left(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh)\right)\right)\left(-\frac{f\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{2(de-cf)}-\frac{\sqrt{e+f}}{(c+dx)(de}\right.}{4df(de-cf)^2}$$
$$\frac{x\left(adf(-cfh-4deh+5dfg)-b\left(-c^2f^2h+cdf^2g+4d^2e(fg-eh)\right)\right)+adf(-5ceh+4cfg+deg)-bce(-cfh-}{2df(c+dx)^2\sqrt{e+fx}(de-cf)^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{\left(3adf(-cfh-4deh+5dfg)-b\left(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-2eh)\right)\right)\left(-\frac{\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{de-cf}-\frac{}{(c+}\right.}{4df(de-cf)^2}$$
$$\frac{x\left(adf(-cfh-4deh+5dfg)-b\left(-c^2f^2h+cdf^2g+4d^2e(fg-eh)\right)\right)+adf(-5ceh+4cfg+deg)-bce(-cfh-}{2df(c+dx)^2\sqrt{e+fx}(de-cf)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(\frac{f\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{\sqrt{d}(de-cf)^{3/2}}-\frac{\sqrt{e+fx}}{(c+dx)(de-cf)}\right)\left(3adf(-cfh-4deh+5dfg)-b\left(c^2f^2h+cdf(3fg-8eh)+4d^2e(3fg-\right.\right.}{4df(de-cf)^2}$$
$$\frac{x\left(adf(-cfh-4deh+5dfg)-b\left(-c^2f^2h+cdf^2g+4d^2e(fg-eh)\right)\right)+adf(-5ceh+4cfg+deg)-bce(-cfh-}{2df(c+dx)^2\sqrt{e+fx}(de-cf)^2}$$

input
```
Int[((a + b*x)*(g + h*x))/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*(a*d*f*(d*e*g + 4*c*f*g - 5*c*e*h) - b*c*e*(5*d*f*g - 4*d*e*h - c*f*h
) + (a*d*f*(5*d*f*g - 4*d*e*h - c*f*h) - b*(c*d*f^2*g - c^2*f^2*h + 4*d^2*
e*(f*g - e*h)))*x)/(d*f*(d*e - c*f)^2*(c + d*x)^2*Sqrt[e + f*x]) - ((3*a*d
*f*(5*d*f*g - 4*d*e*h - c*f*h) - b*(c^2*f^2*h + c*d*f*(3*f*g - 8*e*h) + 4*
d^2*e*(3*f*g - 2*e*h)))*(-(Sqrt[e + f*x]/((d*e - c*f)*(c + d*x))) + (f*Arc
Tanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/(Sqrt[d]*(d*e - c*f)^(3/2))
))/(4*d*f*(d*e - c*f)^2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$\frac{3(xd+c)^2\sqrt{fx+e}\left(\left(-5g\,f^2a+4e(ah+bg)f-\frac{8b\,e^2h}{3}\right)d^2+c\left((ah+bg)f-\frac{8ehb}{3}\right)fd+\frac{b\,c^2f^2h}{3}\right)\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{4}+\frac{13\sqrt{(cf-de)d}}{}$
derivativedivides	$\frac{2\left(\left(\frac{3}{8}acd\,f^2h+\frac{1}{2}a\,d^2efh-\frac{7}{8}a\,d^2f^2g+\frac{1}{8}b\,c^2f^2h-bcdefh+\frac{3}{8}bcd\,f^2g+\frac{1}{2}b\,d^2efg\right)(fx+e)^{\frac{3}{2}}+\frac{f\left(5a\,c^2d\,f^2h-ac\,d^2efh-9ac\,d^2f^2g-\right.}{((fx+e)d+cf-de)^2}\right.}{}$
default	$\frac{2\left(\left(\frac{3}{8}acd\,f^2h+\frac{1}{2}a\,d^2efh-\frac{7}{8}a\,d^2f^2g+\frac{1}{8}b\,c^2f^2h-bcdefh+\frac{3}{8}bcd\,f^2g+\frac{1}{2}b\,d^2efg\right)(fx+e)^{\frac{3}{2}}+\frac{f\left(5a\,c^2d\,f^2h-ac\,d^2efh-9ac\,d^2f^2g-\right.}{((fx+e)d+cf-de)^2}\right.}{}$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*((d*x+c)^2*(f*x+e)^(1/2)*((-5*g*f^2*a+4*e*(a*h+b*g)*f-8/3*b*e^2*h)*d^2
+c*((a*h+b*g)*f-8/3*e*h*b)*f*d+1/3*b*c^2*f^2*h)*arctan(d*(f*x+e)^(1/2)/((c
*f-d*e)*d)^(1/2))+13/3*((c*f-d*e)*d)^(1/2)*(1/13*(-15*a*f^2*g*x^2-5*x*(12/
5*(-a*h-b*g)*x+g*a)*e*f+2*(-4*b*h*x^2+2*(a*h+b*g)*x+g*a)*e^2)*d^3+2/13*c*(
1/2*(3*(a*h+b*g)*x^2-25*a*g*x)*f^2-9/2*(8/9*b*h*x^2+7/3*(-a*h-b*g)*x+g*a)*
e*f+e^2*(-12*b*h*x+a*h+b*g))*d^2+(1/13*(b*h*x^2+5*(a*h+b*g)*x-8*g*a)*f^2+e
*(a*h+b*g-5/13*b*h*x)*f-14/13*b*e^2*h)*c^2*d-1/13*b*c^3*f*h*(f*x+e)))/(f*x
+e)^(1/2)/((c*f-d*e)*d)^(1/2)/(d*x+c)^2/(c*f-d*e)^3/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(258) = 516$.

Time = 0.24 (sec) , antiderivative size = 2496, normalized size of antiderivative = 8.91

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((3*(4*b*d^4*e*f^2 + (b*c*d^3 - 5*a*d^4)*f^3)*g - (8*b*d^4*e^2*f +
4*(2*b*c*d^3 - 3*a*d^4)*e*f^2 - (b*c^2*d^2 + 3*a*c*d^3)*f^3)*h)*x^3 + (3*(
4*b*d^4*e^2*f + (9*b*c*d^3 - 5*a*d^4)*e*f^2 + 2*(b*c^2*d^2 - 5*a*c*d^3)*f^
3)*g - (8*b*d^4*e^3 + 12*(2*b*c*d^3 - a*d^4)*e^2*f + 3*(5*b*c^2*d^2 - 9*a*
c*d^3)*e*f^2 - 2*(b*c^3*d + 3*a*c^2*d^2)*f^3)*h)*x^2 + 3*(4*b*c^2*d^2*e^2*
f + (b*c^3*d - 5*a*c^2*d^2)*e*f^2)*g - (8*b*c^2*d^2*e^3 + 4*(2*b*c^3*d - 3
*a*c^2*d^2)*e^2*f - (b*c^4 + 3*a*c^3*d)*e*f^2)*h + (3*(8*b*c*d^3*e^2*f + 2
*(3*b*c^2*d^2 - 5*a*c*d^3)*e*f^2 + (b*c^3*d - 5*a*c^2*d^2)*f^3)*g - (16*b*
c*d^3*e^3 + 24*(b*c^2*d^2 - a*c*d^3)*e^2*f + 6*(b*c^3*d - 3*a*c^2*d^2)*e*f
^2 - (b*c^4 + 3*a*c^3*d)*f^3)*h)*x)*sqrt(d^2*e - c*d*f)*log((d*f*x + 2*d*e
 - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c)) + 2*((3*(4*b*d^5*
e^2*f - (3*b*c*d^4 + 5*a*d^5)*e*f^2 - (b*c^2*d^3 - 5*a*c*d^4)*f^3)*g - (8*
b*d^5*e^3 - 12*a*d^5*e^2*f - 9*(b*c^2*d^3 - a*c*d^4)*e*f^2 + (b*c^3*d^2 +
3*a*c^2*d^3)*f^3)*h)*x^2 + (8*a*c^3*d^2*f^3 + 2*(b*c*d^4 + a*d^5)*e^3 + 11
*(b*c^2*d^3 - a*c*d^4)*e^2*f - (13*b*c^3*d^2 - a*c^2*d^3)*e*f^2)*g - (2*(7
*b*c^2*d^3 - a*c*d^4)*e^3 - (13*b*c^3*d^2 + 11*a*c^2*d^3)*e^2*f - (b*c^4*d
 - 13*a*c^3*d^2)*e*f^2)*h + ((4*b*d^5*e^3 + (17*b*c*d^4 - 5*a*d^5)*e^2*f -
 4*(4*b*c^2*d^3 + 5*a*c*d^4)*e*f^2 - 5*(b*c^3*d^2 - 5*a*c^2*d^3)*f^3)*g -
(4*(6*b*c*d^4 - a*d^5)*e^3 - (19*b*c^2*d^3 + 17*a*c*d^4)*e^2*f - 4*(b*c^3*
d^2 - 4*a*c^2*d^3)*e*f^2 - (b*c^4*d - 5*a*c^3*d^2)*f^3)*h)*x)*sqrt(f*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(258) = 516$.

Time = 0.15 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.17

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{(12\,bd^2efg + 3\,bcdf^2g - 15\,ad^2f^2g - 8\,bd^2e^2h - 8\,bcdefh + 12\,ad^2efh + bc^2f^2h + 3\,acdf^2h)\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{-\cdots}}\right.}{4\,(d^4e^3 - 3\,cd^3e^2f + 3\,c^2d^2ef^2 - c^3df^3)\sqrt{-d^2e + cdf}}$$

$$-\frac{2\,(befg - af^2g - be^2h + aefh)}{(d^3e^3 - 3\,cd^2e^2f + 3\,c^2def^2 - c^3f^3)\sqrt{fx+e}}$$

$$-\frac{4\,(fx+e)^{\frac{3}{2}}bd^3efg - 4\,\sqrt{fx+e}bd^3e^2fg + 3\,(fx+e)^{\frac{3}{2}}bcd^2f^2g - 7\,(fx+e)^{\frac{3}{2}}ad^3f^2g - \sqrt{fx+e}bcd^2ef^2g\cdots}{\cdots}$$

input
```
integrate((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(12*b*d^2*e*f*g + 3*b*c*d*f^2*g - 15*a*d^2*f^2*g - 8*b*d^2*e^2*h - 8*
b*c*d*e*f*h + 12*a*d^2*e*f*h + b*c^2*f^2*h + 3*a*c*d*f^2*h)*arctan(sqrt(f*
x + e)*d/sqrt(-d^2*e + c*d*f))/((d^4*e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2
 - c^3*d*f^3)*sqrt(-d^2*e + c*d*f)) - 2*(b*e*f*g - a*f^2*g - b*e^2*h + a*e
*f*h)/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sqrt(f*x + e))
 - 1/4*(4*(f*x + e)^(3/2)*b*d^3*e*f*g - 4*sqrt(f*x + e)*b*d^3*e^2*f*g + 3*(
f*x + e)^(3/2)*b*c*d^2*f^2*g - 7*(f*x + e)^(3/2)*a*d^3*f^2*g - sqrt(f*x +
e)*b*c*d^2*e*f^2*g + 9*sqrt(f*x + e)*a*d^3*e*f^2*g + 5*sqrt(f*x + e)*b*c^2
*d*f^3*g - 9*sqrt(f*x + e)*a*c*d^2*f^3*g - 8*(f*x + e)^(3/2)*b*c*d^2*e*f*h
 + 4*(f*x + e)^(3/2)*a*d^3*e*f*h + 8*sqrt(f*x + e)*b*c*d^2*e^2*f*h - 4*sqr
t(f*x + e)*a*d^3*e^2*f*h + (f*x + e)^(3/2)*b*c^2*d*f^2*h + 3*(f*x + e)^(3/
2)*a*c*d^2*f^2*h - 7*sqrt(f*x + e)*b*c^2*d*e*f^2*h - sqrt(f*x + e)*a*c*d^2
*e*f^2*h - sqrt(f*x + e)*b*c^3*f^3*h + 5*sqrt(f*x + e)*a*c^2*d*f^3*h)/((d^
4*e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2 - c^3*d*f^3)*((f*x + e)*d - d*e +
c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{\frac{2\,\left(a\,f^2\,g+b\,e^2\,h-a\,e\,f\,h-b\,e\,f\,g\right)}{c\,f-d\,e}-\frac{{\left(e+f\,x\right)}^2\,\left(b\,c^2\,f^2\,h-15\,a\,d^2\,f^2\,g-8\,b\,d^2\,e^2\,h+3\,a\,c\,d\,f^2\,h+3\,b\,c\,d\,f^2\,g+12\,a\,d^2\,e\,f\,h+12\,b\,d^2\,e\,f\,g-8\,b\,c\,d\,e\,f\,\ldots}{4\,{\left(c\,f-d\,e\right)}^3}}{d^2\,{\left(e+f\,x\right)}^{5/2}-{\left(e+f\,x\right)}^{3/2}\,\left(2\,d^2\,e-2\,c\,d\,f\right)+\sqrt{e}\ldots}$$

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{e+f\,x}\,\left(-c^3\,d\,f^3+3\,c^2\,d^2\,e\,f^2-3\,c\,d^3\,e^2\,f+d^4\,e^3\right)}{\sqrt{d}\,{\left(c\,f-d\,e\right)}^{7/2}}\right)\,\left(b\,c^2\,f^2\,h-15\,a\,d^2\,f^2\,g-8\,b\,d^2\,e^2\,h+3\,a\,c\,d\,f^2\,h+3\,b\,c\,d\,f\ldots}{4\,d^{3/2}\,{\left(c\,f-d\,e\right)}^{7/2}}$$

input
```
int(((g + h*x)*(a + b*x))/((e + f*x)^(3/2)*(c + d*x)^3),x)
```

output
```
- ((2*(a*f^2*g + b*e^2*h - a*e*f*h - b*e*f*g))/(c*f - d*e) - ((e + f*x)^2*
(b*c^2*f^2*h - 15*a*d^2*f^2*g - 8*b*d^2*e^2*h + 3*a*c*d*f^2*h + 3*b*c*d*f^
2*g + 12*a*d^2*e*f*h + 12*b*d^2*e*f*g - 8*b*c*d*e*f*h))/(4*(c*f - d*e)^3)
+ ((e + f*x)*(25*a*d^2*f^2*g + b*c^2*f^2*h + 16*b*d^2*e^2*h - 5*a*c*d*f^2*
h - 5*b*c*d*f^2*g - 20*a*d^2*e*f*h - 20*b*d^2*e*f*g + 8*b*c*d*e*f*h))/(4*d
*(c*f - d*e)^2))/(d^2*(e + f*x)^(5/2) - (e + f*x)^(3/2)*(2*d^2*e - 2*c*d*f
) + (e + f*x)^(1/2)*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (atan(((e + f*x)^(1
/2)*(d^4*e^3 - c^3*d*f^3 + 3*c^2*d^2*e*f^2 - 3*c*d^3*e^2*f))/(d^(1/2)*(c*f
 - d*e)^(7/2)))*(b*c^2*f^2*h - 15*a*d^2*f^2*g - 8*b*d^2*e^2*h + 3*a*c*d*f^
2*h + 3*b*c*d*f^2*g + 12*a*d^2*e*f*h + 12*b*d^2*e*f*g - 8*b*c*d*e*f*h))/(4
*d^(3/2)*(c*f - d*e)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2007, normalized size of antiderivative = 7.17

$$\int \frac{(a+bx)(g+hx)}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
(3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*a*c**3*d*f**2*h + 12*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d**2*e*f*h - 15
*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqr
t(c*f - d*e)))*a*c**2*d**2*f**2*g + 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e
)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c**2*d**2*f**2*h*x +
 24*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*
sqrt(c*f - d*e)))*a*c*d**3*e*f*h*x - 30*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*c*d**3*f**2*g*x +
3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sq
rt(c*f - d*e)))*a*c*d**3*f**2*h*x**2 + 12*sqrt(d)*sqrt(e + f*x)*sqrt(c*f -
 d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*a*d**4*e*f*h*x**2
- 15*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)
*sqrt(c*f - d*e)))*a*d**4*f**2*g*x**2 + sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d
*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**4*f**2*h - 8*sq
rt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c
*f - d*e)))*b*c**3*d*e*f*h + 3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan(
(sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*b*c**3*d*f**2*g + 2*sqrt(d)*s
qrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*
e)))*b*c**3*d*f**2*h*x - 8*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((...
```

3.189 $\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{2f(fg-eh)}{(de-cf)^3\sqrt{e+fx}} - \frac{(dg-ch)\sqrt{e+fx}}{2(de-cf)^2(c+dx)^2} + \frac{(7dfg-4deh-3cfh)\sqrt{e+fx}}{4(de-cf)^3(c+dx)} - \frac{3f(5dfg-4deh-cfh)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{4\sqrt{d}(de-cf)^{7/2}}$$

output
```
2*f*(-e*h+f*g)/(-c*f+d*e)^3/(f*x+e)^(1/2)-1/2*(-c*h+d*g)*(f*x+e)^(1/2)/(-c
*f+d*e)^2/(d*x+c)^2+1/4*(-3*c*f*h-4*d*e*h+7*d*f*g)*(f*x+e)^(1/2)/(-c*f+d*e
)^3/(d*x+c)-3/4*f*(-c*f*h-4*d*e*h+5*d*f*g)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(
-c*f+d*e)^(1/2))/d^(1/2)/(-c*f+d*e)^(7/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{1}{4}\left(\frac{c^2f(-8fg+13eh+5fhx)+d^2(-15f^2gx^2+2e^2(g+2hx)+efx(-5g+}{(-de+cf)^3(c+dx)^2} + \frac{3f(-5dfg+4deh+cfh)\arctan\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{-de+cf}}\right)}{\sqrt{d}(-de+cf)^{7/2}}\right)$$

input
```
Integrate[(g + h*x)/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((c^2*f*(-8*f*g + 13*e*h + 5*f*h*x) + d^2*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*
x) + e*f*x*(-5*g + 12*h*x)) + c*d*(2*e^2*h + f^2*x*(-25*g + 3*h*x) + e*f*(
-9*g + 21*h*x)))/((-(d*e) + c*f)^3*(c + d*x)^2*Sqrt[e + f*x]) + (3*f*(-5*d
*f*g + 4*d*e*h + c*f*h)*ArcTan[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[-(d*e) + c*f]]
)/(Sqrt[d]*(-(d*e) + c*f)^(7/2)))/4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(-cfh-4deh+5dfg)\int \frac{1}{(c+dx)^2(e+fx)^{3/2}}dx}{4d(de-cf)}-\frac{dg-ch}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{(-cfh-4deh+5dfg)\left(-\frac{3f\int \frac{1}{(c+dx)(e+fx)^{3/2}}dx}{2(de-cf)}-\frac{1}{(c+dx)\sqrt{e+fx}(de-cf)}\right)}{4d(de-cf)}-\frac{dg-ch}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(-cfh-4deh+5dfg)\left(-\frac{3f\left(\frac{d\int \frac{1}{(c+dx)\sqrt{e+fx}}dx}{de-cf}+\frac{2}{\sqrt{e+fx}(de-cf)}\right)}{2(de-cf)}-\frac{1}{(c+dx)\sqrt{e+fx}(de-cf)}\right)}{4d(de-cf)}-\frac{dg-ch}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow 73$$

$$-\frac{(-cfh-4deh+5dfg)\left(-\frac{3f\left(\frac{2d\int \frac{1}{c+\frac{d(e+fx)}{f}-\frac{de}{f}}d\sqrt{e+fx}}{f(de-cf)}+\frac{2}{\sqrt{e+fx}(de-cf)}\right)}{2(de-cf)}-\frac{1}{(c+dx)\sqrt{e+fx}(de-cf)}\right)}{4d(de-cf)}-\frac{dg-ch}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

$$\Big\downarrow 221$$

$$-\frac{\left(-\frac{3f\left(\frac{2}{\sqrt{e+fx}(de-cf)}-\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)}{(de-cf)^{3/2}}\right)}{2(de-cf)}-\frac{1}{(c+dx)\sqrt{e+fx}(de-cf)}\right)(-cfh-4deh+5dfg)}{4d(de-cf)}-\frac{dg-ch}{2d(c+dx)^2\sqrt{e+fx}(de-cf)}$$

input
```
Int[(g + h*x)/((c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*(d*g - c*h)/(d*(d*e - c*f)*(c + d*x)^2*Sqrt[e + f*x]) - ((5*d*f*g - 4
*d*e*h - c*f*h)*(-(1/((d*e - c*f)*(c + d*x)*Sqrt[e + f*x])) - (3*f*(2/((d*
e - c*f)*Sqrt[e + f*x]) - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[
d*e - c*f]])/(d*e - c*f)^(3/2)))/(2*(d*e - c*f))))/(4*d*(d*e - c*f))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07

method	result
derivativedivides	$2f\left(\frac{\frac{\left(\frac{3}{8}cdfh+\frac{1}{2}d^2eh-\frac{7}{8}d^2fg\right)(fx+e)^{\frac{3}{2}}+\left(\frac{5}{8}c^2f^2h-\frac{1}{8}cdehf-\frac{9}{8}cdf^2g-\frac{1}{2}d^2e^2h+\frac{9}{8}d^2egf\right)\sqrt{fx+e}}{((fx+e)d+cf-de)^2}+\frac{3(cfh+4deh-5dfg)\arctan\cdots}{8\sqrt{(cf-de)\cdots}}}{(cf-de)^3}\cdots\right.$
default	$2f\left(\frac{\frac{\left(\frac{3}{8}cdfh+\frac{1}{2}d^2eh-\frac{7}{8}d^2fg\right)(fx+e)^{\frac{3}{2}}+\left(\frac{5}{8}c^2f^2h-\frac{1}{8}cdehf-\frac{9}{8}cdf^2g-\frac{1}{2}d^2e^2h+\frac{9}{8}d^2egf\right)\sqrt{fx+e}}{((fx+e)d+cf-de)^2}+\frac{3(cfh+4deh-5dfg)\arctan\cdots}{8\sqrt{(cf-de)\cdots}}}{(cf-de)^3}\cdots\right.$
pseudoelliptic	$\frac{\frac{3\sqrt{fx+e}\,f((4eh-5fg)d+cfh)(xd+c)^2\arctan\left(\frac{d\sqrt{fx+e}}{\sqrt{(cf-de)d}}\right)}{4}+13\left(\frac{\left(-15x^2gf^2-5x\left(-\frac{12hx}{5}+g\right)ef+2e^2(2hx+g)\right)d^2}{13}+\frac{2c\left(\left(3hx^2\cdots\right.\right.}{\cdots}\cdots\right.}{(cf-de)^3\sqrt{fx+e}\,(xd+c)^2\sqrt{(cf-de)d}}$

input
```
int((h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(1/(c*f-d*e)^3*(((3/8*c*d*f*h+1/2*d^2*e*h-7/8*d^2*f*g)*(f*x+e)^(3/2)+(
5/8*c^2*f^2*h-1/8*c*d*e*h*f-9/8*c*d*f^2*g-1/2*d^2*e^2*h+9/8*d^2*e*g*f)*(f*
x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2+3/8*(c*f*h+4*d*e*h-5*d*f*g)/((c*f-d*e)*d
)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-(-e*h+f*g)/(c*f-d*e)^
3/(f*x+e)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(159) = 318$.

Time = 0.14 (sec) , antiderivative size = 1468, normalized size of antiderivative = 8.11

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(5*c^2*d*e*f^2*g + (5*d^3*f^3*g - (4*d^3*e*f^2 + c*d^2*f^3)*h)*x^3
 + (5*(d^3*e*f^2 + 2*c*d^2*f^3)*g - (4*d^3*e^2*f + 9*c*d^2*e*f^2 + 2*c^2*d
*f^3)*h)*x^2 - (4*c^2*d*e^2*f + c^3*e*f^2)*h + (5*(2*c*d^2*e*f^2 + c^2*d*f
^3)*g - (8*c*d^2*e^2*f + 6*c^2*d*e*f^2 + c^3*f^3)*h)*x)*sqrt(d^2*e - c*d*f
)*log((d*f*x + 2*d*e - c*f - 2*sqrt(d^2*e - c*d*f)*sqrt(f*x + e))/(d*x + c
)) + 2*(3*(5*(d^4*e*f^2 - c*d^3*f^3)*g - (4*d^4*e^2*f - 3*c*d^3*e*f^2 - c^
2*d^2*f^3)*h)*x^2 - (2*d^4*e^3 - 11*c*d^3*e^2*f + c^2*d^2*e*f^2 + 8*c^3*d*
f^3)*g - (2*c*d^3*e^3 + 11*c^2*d^2*e^2*f - 13*c^3*d*e*f^2)*h + (5*(d^4*e^2
*f + 4*c*d^3*e*f^2 - 5*c^2*d^2*f^3)*g - (4*d^4*e^3 + 17*c*d^3*e^2*f - 16*c
^2*d^2*e*f^2 - 5*c^3*d*f^3)*h)*x)*sqrt(f*x + e))/(c^2*d^5*e^5 - 4*c^3*d^4*
e^4*f + 6*c^4*d^3*e^3*f^2 - 4*c^5*d^2*e^2*f^3 + c^6*d*e*f^4 + (d^7*e^4*f -
 4*c*d^6*e^3*f^2 + 6*c^2*d^5*e^2*f^3 - 4*c^3*d^4*e*f^4 + c^4*d^3*f^5)*x^3
+ (d^7*e^5 - 2*c*d^6*e^4*f - 2*c^2*d^5*e^3*f^2 + 8*c^3*d^4*e^2*f^3 - 7*c^4
*d^3*e*f^4 + 2*c^5*d^2*f^5)*x^2 + (2*c*d^6*e^5 - 7*c^2*d^5*e^4*f + 8*c^3*d
^4*e^3*f^2 - 2*c^4*d^3*e^2*f^3 - 2*c^5*d^2*e*f^4 + c^6*d*f^5)*x), 1/4*(3*(
5*c^2*d*e*f^2*g + (5*d^3*f^3*g - (4*d^3*e*f^2 + c*d^2*f^3)*h)*x^3 + (5*(d^
3*e*f^2 + 2*c*d^2*f^3)*g - (4*d^3*e^2*f + 9*c*d^2*e*f^2 + 2*c^2*d*f^3)*h)*
x^2 - (4*c^2*d*e^2*f + c^3*e*f^2)*h + (5*(2*c*d^2*e*f^2 + c^2*d*f^3)*g - (
8*c*d^2*e^2*f + 6*c^2*d*e*f^2 + c^3*f^3)*h)*x)*sqrt(-d^2*e + c*d*f)*arctan
(sqrt(-d^2*e + c*d*f)*sqrt(f*x + e)/(d*f*x + d*e)) + (3*(5*(d^4*e*f^2 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(159) = 318.

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.88

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{3\,(5\,df^2g-4\,defh-cf^2h)\arctan\left(\frac{\sqrt{fx+e}d}{\sqrt{-d^2e+cdf}}\right)}{4\,(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3)\sqrt{-d^2e+cdf}} + \frac{2\,(f^2g-efh)}{(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3)\sqrt{fx+e}} + \frac{7\,(fx+e)^{\frac{3}{2}}d^2f^2g-9\,\sqrt{fx+e}d^2ef^2g+9\,\sqrt{fx+e}cdf^3g-4\,(fx+e)^{\frac{3}{2}}d^2efh+4\,\sqrt{fx+e}d^2e^2fh-3\,(fx}{4\,(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3)((fx+e)d-de+c}$$

input
```
integrate((h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
3/4*(5*d*f^2*g - 4*d*e*f*h - c*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e +
 c*d*f))/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*sqrt(-d^2*e
+ c*d*f)) + 2*(f^2*g - e*f*h)/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 -
c^3*f^3)*sqrt(f*x + e)) + 1/4*(7*(f*x + e)^(3/2)*d^2*f^2*g - 9*sqrt(f*x +
e)*d^2*e*f^2*g + 9*sqrt(f*x + e)*c*d*f^3*g - 4*(f*x + e)^(3/2)*d^2*e*f*h +
 4*sqrt(f*x + e)*d^2*e^2*f*h - 3*(f*x + e)^(3/2)*c*d*f^2*h + sqrt(f*x + e)
*c*d*e*f^2*h - 5*sqrt(f*x + e)*c^2*f^3*h)/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^
2*d*e*f^2 - c^3*f^3)*((f*x + e)*d - d*e + c*f)^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.64

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{\frac{5(e+fx)(cf^2h-5df^2g+4defh)}{4(cf-de)^2} - \frac{2(f^2g-efh)}{cf-de} + \frac{3d(e+fx)^2(cf^2h-5df^2g+4defh}{4(cf-de)^3}}{d^2(e+fx)^{5/2} - (e+fx)^{3/2}(2d^2e-2cdf) + \sqrt{e+fx}\,(c^2f^2-2cdef+} + \frac{3f\,\mathrm{atan}\left(\frac{3\sqrt{d}\,f\sqrt{e+fx}\,(cfh+4deh-5dfg)(c^3f^3-3c^2def^2+3cd^2e^2f-d^3e^3)}{(cf-de)^{7/2}(3cf^2h-15df^2g+12defh)}\right)(cfh+4deh-5dfg)}{4\sqrt{d}\,(cf-de)^{7/2}}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(c + d*x)^3),x)
```

output
```
((5*(e + f*x)*(c*f^2*h - 5*d*f^2*g + 4*d*e*f*h))/(4*(c*f - d*e)^2) - (2*(f
^2*g - e*f*h))/(c*f - d*e) + (3*d*(e + f*x)^2*(c*f^2*h - 5*d*f^2*g + 4*d*e
*f*h))/(4*(c*f - d*e)^3))/(d^2*(e + f*x)^(5/2) - (e + f*x)^(3/2)*(2*d^2*e
- 2*c*d*f) + (e + f*x)^(1/2)*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) + (3*f*atan(
(3*d^(1/2)*f*(e + f*x)^(1/2)*(c*f*h + 4*d*e*h - 5*d*f*g)*(c^3*f^3 - d^3*e^
3 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2))/((c*f - d*e)^(7/2)*(3*c*f^2*h - 15*d*f
^2*g + 12*d*e*f*h)))*(c*f*h + 4*d*e*h - 5*d*f*g))/(4*d^(1/2)*(c*f - d*e)^(
7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 892, normalized size of antiderivative = 4.93

$$\int \frac{g+hx}{(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
(3*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*s
qrt(c*f - d*e)))*c**3*f**2*h + 12*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*at
an((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*e*f*h - 15*sqrt(d)*
sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d
*e)))*c**2*d*f**2*g + 6*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e
 + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c**2*d*f**2*h*x + 24*sqrt(d)*sqrt(e
+ f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c
*d**2*e*f*h*x - 30*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*
x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d**2*f**2*g*x + 3*sqrt(d)*sqrt(e + f*x)
*sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*c*d**2*
f**2*h*x**2 + 12*sqrt(d)*sqrt(e + f*x)*sqrt(c*f - d*e)*atan((sqrt(e + f*x)
*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**3*e*f*h*x**2 - 15*sqrt(d)*sqrt(e + f*x)*
sqrt(c*f - d*e)*atan((sqrt(e + f*x)*d)/(sqrt(d)*sqrt(c*f - d*e)))*d**3*f**
2*g*x**2 + 13*c**3*d*e*f**2*h - 8*c**3*d*f**3*g + 5*c**3*d*f**3*h*x - 11*c
**2*d**2*e**2*f*h - c**2*d**2*e*f**2*g + 16*c**2*d**2*e*f**2*h*x - 25*c**2
*d**2*f**3*g*x + 3*c**2*d**2*f**3*h*x**2 - 2*c*d**3*e**3*h + 11*c*d**3*e**
2*f*g - 17*c*d**3*e**2*f*h*x + 20*c*d**3*e*f**2*g*x + 9*c*d**3*e*f**2*h*x*
*2 - 15*c*d**3*f**3*g*x**2 - 2*d**4*e**3*g - 4*d**4*e**3*h*x + 5*d**4*e**2
*f*g*x - 12*d**4*e**2*f*h*x**2 + 15*d**4*e*f**2*g*x**2)/(4*sqrt(e + f*x)*d
*(c**6*f**4 - 4*c**5*d*e*f**3 + 2*c**5*d*f**4*x + 6*c**4*d**2*e**2*f**2...
```

3.190 $\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 516

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = -\frac{f(3a^2df(5dfg-4deh-cfh)-b^2(4d^2e^2g-11cdefg-c^2f(8fg-15eh))+ab(7c^2f^2h-cdf(27fg-19}{4(bc-ad)^2(be-af)(de-cf)^3\sqrt{e+fx}}$$
$$+\frac{dg-ch}{2(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}$$
$$+\frac{ad(5dfg-4deh-cfh)+b(4d^2eg-9cdfg+5c^2fh)}{4(bc-ad)^2(de-cf)^2(c+dx)\sqrt{e+fx}}$$
$$-\frac{2b^{5/2}(bg-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^3(be-af)^{3/2}}$$
$$+\frac{\sqrt{d}(3a^2d^2f(5dfg-4deh-cfh)+b^2(8d^3e^2g-28cd^2efg+35c^2df^2g-15c^3f^2h)+2abd(5c^2f^2h-7cdf(}{4(bc-ad)^3(de-cf)^{7/2}}$$

output
```
-1/4*f*(3*a^2*d*f*(-c*f*h-4*d*e*h+5*d*f*g)-b^2*(4*d^2*e^2*g-11*c*d*e*f*g-c
^2*f*(-15*e*h+8*f*g))+a*b*(7*c^2*f^2*h-c*d*f*(-19*e*h+27*f*g)-d^2*e*(-4*e*
h+3*f*g)))/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)^3/(f*x+e)^(1/2)+1/2*(-c*h+d*
g)/(-a*d+b*c)/(-c*f+d*e)/(d*x+c)^2/(f*x+e)^(1/2)+1/4*(a*d*(-c*f*h-4*d*e*h+
5*d*f*g)+b*(5*c^2*f*h-9*c*d*f*g+4*d^2*e*g))/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x
+c)/(f*x+e)^(1/2)-2*b^(5/2)*(-a*h+b*g)*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f
+b*e)^(1/2))/(-a*d+b*c)^3/(-a*f+b*e)^(3/2)+1/4*d^(1/2)*(3*a^2*d^2*f*(-c*f*
h-4*d*e*h+5*d*f*g)+b^2*(-15*c^3*f^2*h+35*c^2*d*f^2*g-28*c*d^2*e*f*g+8*d^3*
e^2*g)+2*a*b*d*(5*c^2*f^2*h-7*c*d*f*(-2*e*h+3*f*g)+d^2*(-4*e^2*h+6*e*f*g))
)*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c)^3/(-c*f+d*e)^
(7/2)
```

Mathematica [A] (verified)

Time = 5.70 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.26

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{(bc-ad)\left(b^2\left(8c^4f^2(-fg+eh)+4d^4e^2gx(e+fx)+c^3df\left(9e^2h-16f^2gx+25efhx\right)+cd^3eg\left(6e^2-5\cdots\right.\right.\right.}{\cdots}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
(((b*c - a*d)*(b^2*(8*c^4*f^2*(-(f*g) + e*h) + 4*d^4*e^2*g*x*(e + f*x) + c
^3*d*f*(9*e^2*h - 16*f^2*g*x + 25*e*f*h*x) + c*d^3*e*g*(6*e^2 - 5*e*f*x -
11*f^2*x^2) - c^2*d^2*(2*e^3*h + 8*f^3*g*x^2 + e*f^2*x*(13*g - 15*h*x) + e
^2*f*(13*g - 5*h*x))) + a^2*d^2*f*(c^2*f*(-8*f*g + 13*e*h + 5*f*h*x) + d^2
*(-15*f^2*g*x^2 + 2*e^2*(g + 2*h*x) + e*f*x*(-5*g + 12*h*x)) + c*d*(2*e^2*
h + f^2*x*(-25*g + 3*h*x) + e*f*(-9*g + 21*h*x))) - a*b*d*(c^3*f^2*(-16*f*
g + 25*e*h + 9*f*h*x) + d^3*e*(e + f*x)*(-3*f*g*x + 2*e*(g + 2*h*x)) + c*d
^2*(2*e^3*h - 27*f^3*g*x^2 + e^2*f*(-3*g + 5*h*x) + e*f^2*x*(-14*g + 19*h*
x)) + c^2*d*f*(3*e^2*h + f^2*x*(-45*g + 7*h*x) + e*f*(-13*g + 42*h*x)))))/
((b*e - a*f)*(d*e - c*f)^3*(c + d*x)^2*Sqrt[e + f*x]) - (8*b^(5/2)*(b*g -
a*h)*ArcTan[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[-(b*e) + a*f]])/(-(b*e) + a*f)^(3
/2) + (Sqrt[d]*(-3*a^2*d^2*f*(-5*d*f*g + 4*d*e*h + c*f*h) + b^2*(8*d^3*e^2
*g - 28*c*d^2*e*f*g + 35*c^2*d*f^2*g - 15*c^3*f^2*h) + 2*a*b*d*(5*c^2*f^2*
h - 7*c*d*f*(3*f*g - 2*e*h) + d^2*(6*e*f*g - 4*e^2*h)))*ArcTan[(Sqrt[d]*Sq
rt[e + f*x]])/Sqrt[-(d*e) + c*f]])/(-(d*e) + c*f)^(7/2))/(4*(b*c - a*d)^3)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \ 168$$

$$\frac{\int \frac{4b(de-cf)g+a(5dfg-4deh-cfh)+5bf(dg-ch)x}{2(a+bx)(c+dx)^2(e+fx)^{3/2}}dx}{2(bc-ad)(de-cf)}+\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4b(de-cf)g+a(5dfg-4deh-cfh)+5bf(dg-ch)x}{(a+bx)(c+dx)^2(e+fx)^{3/2}}dx}{4(bc-ad)(de-cf)}+\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{\int \frac{3df(5dfg-4deh-cfh)a^2+b\left(4e(3fg-2eh)d^2-cf(27fg-16eh)d+7c^2f^2h\right)a+8b^2(de-cf)^2g+3bf\left(ad(5dfg-4deh-cfh)+b\left(5fhc^2-9dfgc+4d^2eg\right)\right)x}{2(a+bx)(c+dx)(e+fx)^{3/2}}dx}{(bc-ad)(de-cf)}+\frac{ad(}{}}{4(bc-ad)(de-cf)}$$
$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3df(5dfg-4deh-cfh)a^2+b\left(4e(3fg-2eh)d^2-cf(27fg-16eh)d+7c^2f^2h\right)a+8b^2(de-cf)^2g+3bf\left(ad(5dfg-4deh-cfh)+b\left(5fhc^2-9dfgc+4d^2eg\right)\right)x}{(a+bx)(c+dx)(e+fx)^{3/2}}dx}{2(bc-ad)(de-cf)}+\frac{ad(}{}}{4(bc-ad)(de-cf)}$$
$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$$\Big\downarrow \ 169$$

$$-\frac{2\int -\frac{-3d^2f^2(5dfg-4deh-cfh)a^3-bdf\left(-e(3fg-4eh)d^2-cf(27fg-19eh)d+7c^2f^2h\right)a^2+b^2\left(4e^2(fg-2eh)d^3-cef(11fg-24eh)d^2-c^2f^2(8fg+9eh)d+8c^3f^3h\right)\cdots}{2(a+bx)(c+dx)\cdots}}{(be-af)(de-\cdots}$$

$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$\downarrow$ 27

$$\int \frac{\frac{-3d^2f^2(5dfg-4deh-cfh)a^3-bdf\left(-e(3fg-4eh)d^2-cf(27fg-19eh)d+7c^2f^2h\right)a^2+b^2\left(4e^2(fg-2eh)d^3-cef(11fg-24eh)d^2-c^2f^2(8fg+9eh)d+8c^3f^3h\right)a+8\cdots}{(a+bx)(c+dx)\sqrt{e+f\cdots}}}{(be-af)(de-cf)}$$

$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$\downarrow$ 174

$$\frac{\frac{8b^3(bg-ah)(de-cf)^3\int\frac{1}{(a+bx)\sqrt{e+fx}}dx}{bc-ad}-\frac{d(be-af)\left(3a^2d^2f(-cfh-4deh+5dfg)+2abd\left(5c^2f^2h-7cdf(3fg-2eh)+d^2\left(6efg-4e^2h\right)\right)+b^2\left(-15c^3f^2h+35c^2df^2g\cdots\right.\right.}{bc-ad}}{(be-af)(de-cf)} \cdots \frac{}{2(b\cdots}$$

$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$\downarrow$ 73

$$\frac{\frac{16b^3(bg-ah)(de-cf)^3\int\frac{1}{a+\frac{b(e+fx)}{f}-\frac{be}{f}}d\sqrt{e+fx}}{f(bc-ad)}-\frac{2d(be-af)\left(3a^2d^2f(-cfh-4deh+5dfg)+2abd\left(5c^2f^2h-7cdf(3fg-2eh)+d^2\left(6efg-4e^2h\right)\right)+b^2\left(-15c^3f^2h\cdots\right.\right.}{f(bc-ad)}}{(be-af)(de-cf)}$$

$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

$\downarrow$ 221

$$\frac{\frac{2\sqrt{d}(be-af)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{de-cf}}\right)\left(3a^2d^2f(-cfh-4deh+5dfg)+2abd\left(5c^2f^2h-7cdf(3fg-2eh)+d^2\left(6efg-4e^2h\right)\right)+b^2\left(-15c^3f^2h+35c^2df^2g-28cd^2efg+8d\cdots\right.\right.}{(bc-ad)\sqrt{de-cf}}}{(be-af)(de-cf)}$$

$$\frac{dg-ch}{2(c+dx)^2\sqrt{e+fx}(bc-ad)(de-cf)}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
(d*g - c*h)/(2*(b*c - a*d)*(d*e - c*f)*(c + d*x)^2*Sqrt[e + f*x]) + ((a*d*
(5*d*f*g - 4*d*e*h - c*f*h) + b*(4*d^2*e*g - 9*c*d*f*g + 5*c^2*f*h))/((b*c
 - a*d)*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x]) + ((-2*f*(3*a^2*d*f*(5*d*f*g
- 4*d*e*h - c*f*h) - b^2*(4*d^2*e^2*g - 11*c*d*e*f*g - c^2*f*(8*f*g - 15*e
*h)) + a*b*(7*c^2*f^2*h - c*d*f*(27*f*g - 19*e*h) - d^2*e*(3*f*g - 4*e*h))
))/((b*e - a*f)*(d*e - c*f)*Sqrt[e + f*x]) + ((-16*b^(5/2)*(d*e - c*f)^3*(
b*g - a*h)*ArcTanh[(Sqrt[b]*Sqrt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*
Sqrt[b*e - a*f]) + (2*Sqrt[d]*(b*e - a*f)*(3*a^2*d^2*f*(5*d*f*g - 4*d*e*h
- c*f*h) + b^2*(8*d^3*e^2*g - 28*c*d^2*e*f*g + 35*c^2*d*f^2*g - 15*c^3*f^2
*h) + 2*a*b*d*(5*c^2*f^2*h - 7*c*d*f*(3*f*g - 2*e*h) + d^2*(6*e*f*g - 4*e^
2*h)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt
[d*e - c*f]))/((b*e - a*f)*(d*e - c*f)))/(2*(b*c - a*d)*(d*e - c*f)))/(4*(
b*c - a*d)*(d*e - c*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.32

method	result
derivativedivides	$2f^2\left(\frac{d\left(\frac{\left(\frac{3}{8}a^2 c\, d^3 f^2 h+\frac{1}{2}a^2 d^4 e f h-\frac{7}{8}a^2 d^4 f^2 g-\frac{5}{4}ab\, c^2 d^2 f^2 h-\frac{1}{2}abc\, d^3 e f h+\frac{9}{4}abc\, d^3 f^2 g-\frac{1}{2}ab\, d^4 e f g+\frac{7}{8}b^2 c^3 d\, f^2 h-\frac{11}{8}b^2 c^2 d}{}\right.}{}\right.$
default	$2f^2\left(\frac{d\left(\frac{\left(\frac{3}{8}a^2 c\, d^3 f^2 h+\frac{1}{2}a^2 d^4 e f h-\frac{7}{8}a^2 d^4 f^2 g-\frac{5}{4}ab\, c^2 d^2 f^2 h-\frac{1}{2}abc\, d^3 e f h+\frac{9}{4}abc\, d^3 f^2 g-\frac{1}{2}ab\, d^4 e f g+\frac{7}{8}b^2 c^3 d\, f^2 h-\frac{11}{8}b^2 c^2 d}{}\right.}{}\right.$
pseudoelliptic	$\frac{\frac{3\left(\left(5\left(c^3 h-\frac{7}{3}c^2 d g\right)b^2-\frac{10acd\left(ch-\frac{21dg}{5}\right)b}{3}+a^2 d^2(ch-5dg)\right)f^2+4(ah-bg)d^2 e\left(ad-\frac{7bc}{3}\right)f+\frac{8b\,d^3 e^2(ah-bg)}{3}\right)(xd+c)^2\sqrt{(af-be}{}}{4}$

input

```
int((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2*f^2*(d/(a*d-b*c)^3/(c*f-d*e)^3/f^2*(((3/8*a^2*c*d^3*f^2*h+1/2*a^2*d^4*e*
f*h-7/8*a^2*d^4*f^2*g-5/4*a*b*c^2*d^2*f^2*h-1/2*a*b*c*d^3*e*f*h+9/4*a*b*c*
d^3*f^2*g-1/2*a*b*d^4*e*f*g+7/8*b^2*c^3*d*f^2*h-11/8*b^2*c^2*d^2*f^2*g+1/2
*b^2*c*d^3*e*f*g)*(f*x+e)^(3/2)+1/8*f*(5*a^2*c^2*d^2*f^2*h-a^2*c*d^3*e*f*h
-9*a^2*c*d^3*f^2*g-4*a^2*d^4*e^2*h+9*a^2*d^4*e*f*g-14*a*b*c^3*d*f^2*h+10*a
*b*c^2*d^2*e*f*h+22*a*b*c^2*d^2*f^2*g+4*a*b*c*d^3*e^2*h-26*a*b*c*d^3*e*f*g
+4*a*b*d^4*e^2*g+9*b^2*c^4*f^2*h-9*b^2*c^3*d*e*f*h-13*b^2*c^3*d*f^2*g+17*b
^2*c^2*d^2*e*f*g-4*b^2*c*d^3*e^2*g)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2+1
/8*(3*a^2*c*d^2*f^2*h+12*a^2*d^3*e*f*h-15*a^2*d^3*f^2*g-10*a*b*c^2*d*f^2*h
-28*a*b*c*d^2*e*f*h+42*a*b*c*d^2*f^2*g+8*a*b*d^3*e^2*h-12*a*b*d^3*e*f*g+15
*b^2*c^3*f^2*h-35*b^2*c^2*d*f^2*g+28*b^2*c*d^2*e*f*g-8*b^2*d^3*e^2*g)/((c*
f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2)))-(-e*h+f*g)/(c
*f-d*e)^3/(a*f-b*e)/(f*x+e)^(1/2)-(a*h-b*g)*b^3/f^2/(a*d-b*c)^3/(a*f-b*e)/
((a*f-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(483) = 966$.

Time = 0.23 (sec) , antiderivative size = 1130, normalized size of antiderivative = 2.19

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
2*(b^4*g - a*b^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^4*c^3
*e - 3*a*b^3*c^2*d*e + 3*a^2*b^2*c*d^2*e - a^3*b*d^3*e - a*b^3*c^3*f + 3*a
^2*b^2*c^2*d*f - 3*a^3*b*c*d^2*f + a^4*d^3*f)*sqrt(-b^2*e + a*b*f)) - 1/4*
(8*b^2*d^4*e^2*g - 28*b^2*c*d^3*e*f*g + 12*a*b*d^4*e*f*g + 35*b^2*c^2*d^2*
f^2*g - 42*a*b*c*d^3*f^2*g + 15*a^2*d^4*f^2*g - 8*a*b*d^4*e^2*h + 28*a*b*c
*d^3*e*f*h - 12*a^2*d^4*e*f*h - 15*b^2*c^3*d*f^2*h + 10*a*b*c^2*d^2*f^2*h
- 3*a^2*c*d^3*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^3*c^
3*d^3*e^3 - 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c*d^5*e^3 - a^3*d^6*e^3 - 3*b^3*
c^4*d^2*e^2*f + 9*a*b^2*c^3*d^3*e^2*f - 9*a^2*b*c^2*d^4*e^2*f + 3*a^3*c*d^
5*e^2*f + 3*b^3*c^5*d*e*f^2 - 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^3*d^3*e*f^
2 - 3*a^3*c^2*d^4*e*f^2 - b^3*c^6*f^3 + 3*a*b^2*c^5*d*f^3 - 3*a^2*b*c^4*d^
2*f^3 + a^3*c^3*d^3*f^3)*sqrt(-d^2*e + c*d*f)) - 2*(f^3*g - e*f^2*h)/((b*d
^3*e^4 - 3*b*c*d^2*e^3*f - a*d^3*e^3*f + 3*b*c^2*d*e^2*f^2 + 3*a*c*d^2*e^2
*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 + a*c^3*f^4)*sqrt(f*x + e)) + 1/4*(4*
(f*x + e)^(3/2)*b*d^4*e*f*g - 4*sqrt(f*x + e)*b*d^4*e^2*f*g - 11*(f*x + e)
^(3/2)*b*c*d^3*f^2*g + 7*(f*x + e)^(3/2)*a*d^4*f^2*g + 17*sqrt(f*x + e)*b*
c*d^3*e*f^2*g - 9*sqrt(f*x + e)*a*d^4*e*f^2*g - 13*sqrt(f*x + e)*b*c^2*d^2
*f^3*g + 9*sqrt(f*x + e)*a*c*d^3*f^3*g - 4*(f*x + e)^(3/2)*a*d^4*e*f*h + 4
*sqrt(f*x + e)*a*d^4*e^2*f*h + 7*(f*x + e)^(3/2)*b*c^2*d^2*f^2*h - 3*(f*x
+ e)^(3/2)*a*c*d^3*f^2*h - 9*sqrt(f*x + e)*b*c^2*d^2*e*f^2*h + sqrt(f*x...
```

Mupad [B] (verification not implemented)

Time = 84.56 (sec) , antiderivative size = 802140, normalized size of antiderivative = 1554.53

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)*(c + d*x)^3),x)
```

output
```
- ((2*(f^3*g - e*f^2*h))/(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f) + ((e + f
*x)^2*(15*a^2*d^4*f^3*g - 3*a^2*c*d^3*f^3*h - 12*a^2*d^4*e*f^2*h - 4*b^2*d
^4*e^2*f*g + 8*b^2*c^2*d^2*f^3*g - 15*b^2*c^2*d^2*e*f^2*h - 27*a*b*c*d^3*f
^3*g - 3*a*b*d^4*e*f^2*g + 4*a*b*d^4*e^2*f*h + 7*a*b*c^2*d^2*f^3*h + 11*b^
2*c*d^3*e*f^2*g + 19*a*b*c*d^3*e*f^2*h))/(4*(c*f - d*e)^2*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d)*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)) + ((e + f*x)*(25*
a^2*d^3*f^3*g - 5*a^2*c*d^2*f^3*h + 16*b^2*c^2*d*f^3*g - 20*a^2*d^3*e*f^2*
h - 4*b^2*d^3*e^2*f*g - 45*a*b*c*d^2*f^3*g + 9*a*b*c^2*d*f^3*h - 5*a*b*d^3
*e*f^2*g + 4*a*b*d^3*e^2*f*h + 13*b^2*c*d^2*e*f^2*g - 25*b^2*c^2*d*e*f^2*h
 + 37*a*b*c*d^2*e*f^2*h))/(4*(c*f - d*e)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(
a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)))/(d^2*(e + f*x)^(5/2) - (e + f*x)^
(3/2)*(2*d^2*e - 2*c*d*f) + (e + f*x)^(1/2)*(c^2*f^2 + d^2*e^2 - 2*c*d*e*f
)) - atan((((e + f*x)^(1/2)*(8192*a^3*b^15*c^21*d^3*f^20*g^2 - 49152*a^4*b
^14*c^20*d^4*f^20*g^2 + 279680*a^5*b^13*c^19*d^5*f^20*g^2 - 1480960*a^6*b^
12*c^18*d^6*f^20*g^2 + 5092992*a^7*b^11*c^17*d^7*f^20*g^2 - 11152384*a^8*b
^10*c^16*d^8*f^20*g^2 + 16285952*a^9*b^9*c^15*d^9*f^20*g^2 - 16381440*a^10
*b^8*c^14*d^10*f^20*g^2 + 11475200*a^11*b^7*c^13*d^11*f^20*g^2 - 5532672*a
^12*b^6*c^12*d^12*f^20*g^2 + 1759872*a^13*b^5*c^11*d^13*f^20*g^2 - 334080*
a^14*b^4*c^10*d^14*f^20*g^2 + 28800*a^15*b^3*c^9*d^15*f^20*g^2 + 36992*a^5
*b^13*c^21*d^3*f^20*h^2 - 260352*a^6*b^12*c^20*d^4*f^20*h^2 + 809600*a^...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10467, normalized size of antiderivative = 20.28

$$\int \frac{g+hx}{(a+bx)(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
( - 8*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a*b**2*c**6*f**4*h + 32*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**5*d*e*
f**3*h - 16*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(
sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**5*d*f**4*h*x - 48*sqrt(b)*sqrt(e + f*x
)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2
*c**4*d**2*e**2*f**2*h + 64*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**4*d**2*e*f**3*h*x - 8*
sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a*b**2*c**4*d**2*f**4*h*x**2 + 32*sqrt(b)*sqrt(e + f*x)*sqrt
(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**3*
d**3*e**3*f*h - 96*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*
x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**3*d**3*e**2*f**2*h*x + 32*sqrt(
b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f
- b*e)))*a*b**2*c**3*d**3*e*f**3*h*x**2 - 8*sqrt(b)*sqrt(e + f*x)*sqrt(a*f
 - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**2*d**4
*e**4*h + 64*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/
(sqrt(b)*sqrt(a*f - b*e)))*a*b**2*c**2*d**4*e**3*f*h*x - 48*sqrt(b)*sqrt(e
 + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*
a*b**2*c**2*d**4*e**2*f**2*h*x**2 - 16*sqrt(b)*sqrt(e + f*x)*sqrt(a*f -...
```

3.191 $\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 838

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx =$$

$$-\frac{f(3a^3d^2f^2(5dfg-4deh-cfh)+b^3(12d^3e^3g-4c^3f^2(3fg-2eh)-cd^2e^2(27fg+4eh)+c^2def(12fg+\cdots}{\cdots}$$

$$-\frac{d(3bdeg-bc(2fg+eh)-a(dfg+2deh-3cfh))}{2(bc-ad)^2(be-af)(de-cf)(c+dx)^2\sqrt{e+fx}}$$

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}$$

$$+\frac{d(a^2df(5dfg-4deh-cfh)-b^2(12d^2e^2g-cde(21fg+4eh)+c^2f(4fg+9eh))+ab(13c^2f^2h+d^2e(3f\cdots}{4(bc-ad)^3(be-af)(de-cf)^2(c+dx)\sqrt{e+fx}}$$

$$+\frac{b^{5/2}(7a^2dfh+b^2(6deg+3cfg-2ceh)-ab(9dfg+4deh+cfh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e+fx}}{\sqrt{be-af}}\right)}{(bc-ad)^4(be-af)^{5/2}}$$

$$-\frac{d^{3/2}(3a^2d^2f(5dfg-4deh-cfh)+2abd(7c^2f^2h-cdf(27fg-16eh)+4d^2e(3fg-2eh))+b^2(24d^3e^2g\cdots}{4(bc-ad)^4(de-cf)^{7/2}}$$

output
```
-1/4*f*(3*a^3*d^2*f^2*(-c*f*h-4*d*e*h+5*d*f*g)+b^3*(12*d^3*e^3*g-4*c^3*f^2
*(-2*e*h+3*f*g)-c*d^2*e^2*(4*e*h+27*f*g)+c^2*d*e*f*(11*e*h+12*f*g))+a*b^2*
(4*c^3*f^3*h+2*c^2*d*f^2*(-29*e*h+12*f*g)-d^3*e^2*(8*e*h+9*f*g)+c*d^2*e*f*
(17*e*h+30*f*g))+a^2*b*d*f*(11*c^2*f^2*h-13*c*d*f*(-2*e*h+3*f*g)-d^2*(-8*e
^2*h+6*e*f*g)))/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)^3/(f*x+e)^(1/2)-1/2*d
*(3*b*d*e*g-b*c*(e*h+2*f*g)-a*(-3*c*f*h+2*d*e*h+d*f*g))/(-a*d+b*c)^2/(-a*f
+b*e)/(-c*f+d*e)/(d*x+c)^2/(f*x+e)^(1/2)-(-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2)+1/4*d*(a^2*d*f*(-c*f*h-4*d*e*h+5*d*f*g)-b^
2*(12*d^2*e^2*g-c*d*e*(4*e*h+21*f*g)+c^2*f*(9*e*h+4*f*g))+a*b*(13*c^2*f^2*
h+d^2*e*(8*e*h+3*f*g)-c*d*f*(11*e*h+13*f*g)))/(-a*d+b*c)^3/(-a*f+b*e)/(-c*
f+d*e)^2/(d*x+c)/(f*x+e)^(1/2)+b^(5/2)*(7*a^2*d*f*h+b^2*(-2*c*e*h+3*c*f*g+
6*d*e*g)-a*b*(c*f*h+4*d*e*h+9*d*f*g))*arctanh(b^(1/2)*(f*x+e)^(1/2)/(-a*f+
b*e)^(1/2))/(-a*d+b*c)^4/(-a*f+b*e)^(5/2)-1/4*d^(3/2)*(3*a^2*d^2*f*(-c*f*h
-4*d*e*h+5*d*f*g)+2*a*b*d*(7*c^2*f^2*h-c*d*f*(-16*e*h+27*f*g)+4*d^2*e*(-2*
e*h+3*f*g))+b^2*(24*d^3*e^2*g-35*c^3*f^2*h-8*c*d^2*e*(e*h+9*f*g)+7*c^2*d*f
*(4*e*h+9*f*g)))*arctanh(d^(1/2)*(f*x+e)^(1/2)/(-c*f+d*e)^(1/2))/(-a*d+b*c
)^4/(-c*f+d*e)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.12 (sec) , antiderivative size = 544, normalized size of antiderivative = 0.65

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{\frac{d(3bdeg-bc(2fg+eh)-a(dfg+2deh-3cfh))}{(bc-ad)(-de+cf)} + \frac{2(-bg+ah)}{a+bx} + \frac{(c+dx)\Big(d(bc-ad)(be-af)(}{}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^2*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((d*(3*b*d*e*g - b*c*(2*f*g + e*h) - a*(d*f*g + 2*d*e*h - 3*c*f*h)))/((b*c
 - a*d)*(-(d*e) + c*f)) + (2*(-(b*g) + a*h))/(a + b*x) + ((c + d*x)*(d*(b*
c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(a^2*d*f*(-5*d*f*g + 4*d*e*h + c*f*h)
+ b^2*(12*d^2*e^2*g - c*d*e*(21*f*g + 4*e*h) + c^2*f*(4*f*g + 9*e*h)) + a*
b*(-13*c^2*f^2*h - d^2*e*(3*f*g + 8*e*h) + c*d*f*(13*f*g + 11*e*h))) - (c
+ d*x)*(4*b^2*(d*e - c*f)^3*(7*a^2*d*f*h + b^2*(6*d*e*g + 3*c*f*g - 2*c*e*
h) - a*b*(9*d*f*g + 4*d*e*h + c*f*h))*Hypergeometric2F1[-1/2, 1, 1/2, (b*(
e + f*x))/(b*e - a*f)] - d*(b*e - a*f)^2*(-3*a^2*d^2*f*(-5*d*f*g + 4*d*e*h
 + c*f*h) + b^2*(24*d^3*e^2*g - 35*c^3*f^2*h - 8*c*d^2*e*(9*f*g + e*h) + 7
*c^2*d*f*(9*f*g + 4*e*h)) + 2*a*b*d*(7*c^2*f^2*h + 4*d^2*e*(3*f*g - 2*e*h)
 + c*d*f*(-27*f*g + 16*e*h)))*Hypergeometric2F1[-1/2, 1, 1/2, (d*(e + f*x)
)/(d*e - c*f)])))/(2*(b*c - a*d)^3*(b*e - a*f)*(d*e - c*f)^3))/(2*(b*c - a
*d)*(b*e - a*f)*(c + d*x)^2*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 915, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {168, 27, 168, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{b(6deg+3cfg-2ceh)-a(2dfg+4deh+cfh)+7df(bg-ah)x}{2(a+bx)(c+dx)^3(e+fx)^{3/2}}dx}{(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{b(6deg+3cfg-2ceh)-a(2dfg+4deh+cfh)+7df(bg-ah)x}{(a+bx)(c+dx)^3(e+fx)^{3/2}}dx}{2(bc-ad)(be-af)}-\frac{bg-ah}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{\frac{\int \frac{-df(5dfg-4deh-cfh)a^2+b\left(-e(3fg+8eh)d^2+cf(8fg+eh)d+2c^2f^2h\right)a+2b^2(de-cf)(6deg+3cfg-2ceh)+5bdf(3bdeg-bc(2fg+eh)-a(dfg+2deh-3cfh))x}{(a+bx)(c+dx)^2(e+fx)^{3/2}}d}{2(bc-ad)(de-cf)}}{2(bc-ad)(be-af)}$$

$$\frac{bg-ah}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 168$$

$$-\frac{\frac{\int \frac{-3d^2f^2(5dfg-4deh-cfh)a^3-bdf\left(e(9fg-4eh)d^2-cf(39fg-23eh)d+11c^2f^2h\right)a^2-b^2\left(16d^3he^3-cd^2f(9fg+16eh)e+4c^3f^3h+c^2df^2(24fg-19eh)\right)a+4b^3(d}{2(a+bx)(c+}}{(bc-ad}}{}$$

$$\frac{bg-ah}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{-3d^2f^2(5dfg-4deh-cfh)a^3-bdf\left(e(9fg-4eh)d^2-cf(39fg-23eh)d+11c^2f^2h\right)a^2-b^2\left(16d^3he^3-cd^2f(9fg+16eh)e+4c^3f^3h+c^2df^2(24fg-19eh)\right)a+4b^3(d}{(a+bx)(c+d}}{2(bc-a}}{}$$

$$\frac{bg-ah}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 169$$

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-$$

$$\frac{\frac{d(b(3deg-2cfg-ceh)-a(dfg+2deh-3cfh))}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}+\frac{\frac{2f\left(3d^2f^2(5dfg-4deh-cfh)a^3+bdf\left(-2e(3fg-4eh)d^2-13cf(3fg-2eh)d+11c^2f^2h\right)a^2+b^2\left(-e^2(9fg+8eh\right.\right.}{}}{}}{}$$

$$\Big\downarrow \ 27$$

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-$$

$$\frac{\frac{d(b(3deg-2cfg-ceh)-a(dfg+2deh-3cfh))}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}+\frac{\frac{2f\left(3d^2f^2(5dfg-4deh-cfh)a^3+bdf\left(-2e(3fg-4eh)d^2-13cf(3fg-2eh)d+11c^2f^2h\right)a^2+b^2\left(-e^2(9fg+8eh\right.\right.}{}}{}}{}$$

$$\Big\downarrow \ 174$$

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-$$

$$\frac{\frac{d(b(3deg-2cfg-ceh)-a(dfg+2deh-3cfh))}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}+\frac{\frac{2f\left(3d^2f^2(5dfg-4deh-cfh)a^3+bdf\left(-2e(3fg-4eh)d^2-13cf(3fg-2eh)d+11c^2f^2h\right)a^2+b^2\left(-e^2(9fg+8e\right.}{}}{}}{}$$

$\downarrow$ 73

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-$$

$$\frac{\frac{d(b(3deg-2cfg-ceh)-a(dfg+2deh-3cfh))}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}+\frac{\frac{2f\left(3d^2f^2(5dfg-4deh-cfh)a^3+bdf\left(-2e(3fg-4eh)d^2-13cf(3fg-2eh)d+11c^2f^2h\right)a^2+b^2\left(-e^2(9fg+8e\right.}{}}{}}{}$$

$\downarrow$ 221

$$-\frac{bg-ah}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-$$

$$\frac{\frac{d(b(3deg-2cfg-ceh)-a(dfg+2deh-3cfh))}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}+\frac{\frac{2f\left(3d^2f^2(5dfg-4deh-cfh)a^3+bdf\left(-2e(3fg-4eh)d^2-13cf(3fg-2eh)d+11c^2f^2h\right)a^2+b^2\left(-e^2(9fg+8e\right.}{}}{}}{}$$

input
```
Int[(g + h*x)/((a + b*x)^2*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-((b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2*Sqrt[e + f*x]
)) - ((d*(b*(3*d*e*g - 2*c*f*g - c*e*h) - a*(d*f*g + 2*d*e*h - 3*c*f*h)))/
((b*c - a*d)*(d*e - c*f)*(c + d*x)^2*Sqrt[e + f*x]) + (-((d*(a^2*d*f*(5*d*
f*g - 4*d*e*h - c*f*h) - b^2*(12*d^2*e^2*g - c*d*e*(21*f*g + 4*e*h) + c^2*
f*(4*f*g + 9*e*h)) + a*b*(13*c^2*f^2*h + d^2*e*(3*f*g + 8*e*h) - c*d*f*(13
*f*g + 11*e*h))))/((b*c - a*d)*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x])) + ((2
*f*(3*a^3*d^2*f^2*(5*d*f*g - 4*d*e*h - c*f*h) + a^2*b*d*f*(11*c^2*f^2*h -
2*d^2*e*(3*f*g - 4*e*h) - 13*c*d*f*(3*f*g - 2*e*h)) + b^3*(12*d^3*e^3*g -
4*c^3*f^2*(3*f*g - 2*e*h) - c*d^2*e^2*(27*f*g + 4*e*h) + c^2*d*e*f*(12*f*g
 + 11*e*h)) + a*b^2*(4*c^3*f^3*h + 2*c^2*d*f^2*(12*f*g - 29*e*h) - d^3*e^2
*(9*f*g + 8*e*h) + c*d^2*e*f*(30*f*g + 17*e*h))))/((b*e - a*f)*(d*e - c*f)
*Sqrt[e + f*x]) + ((-8*b^(5/2)*(d*e - c*f)^3*(7*a^2*d*f*h + b^2*(6*d*e*g +
 3*c*f*g - 2*c*e*h) - a*b*(9*d*f*g + 4*d*e*h + c*f*h))*ArcTanh[(Sqrt[b]*Sq
rt[e + f*x])/Sqrt[b*e - a*f]])/((b*c - a*d)*Sqrt[b*e - a*f]) + (2*d^(3/2)*
(b*e - a*f)^2*(3*a^2*d^2*f*(5*d*f*g - 4*d*e*h - c*f*h) + 2*a*b*d*(7*c^2*f^
2*h - c*d*f*(27*f*g - 16*e*h) + 4*d^2*e*(3*f*g - 2*e*h)) + b^2*(24*d^3*e^2
*g - 35*c^3*f^2*h - 8*c*d^2*e*(9*f*g + e*h) + 7*c^2*d*f*(9*f*g + 4*e*h)))*
ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/Sqrt[d*e - c*f]])/((b*c - a*d)*Sqrt[d*e -
c*f]))/((b*e - a*f)*(d*e - c*f)))/(2*(b*c - a*d)*(d*e - c*f)))/(2*(b*c - a
*d)*(d*e - c*f)))/(2*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 126.09 (sec) , antiderivative size = 817, normalized size of antiderivative = 0.97

method	result
derivativedivides	$2f^3\left(-\frac{-eh+fg}{(cf-de)^3(af-be)^2\sqrt{fx+e}}+\frac{d^2\left(\left(\frac{3}{8}a^2c\,d^3f^2h+\frac{1}{2}a^2d^4efh-\frac{7}{8}a^2d^4f^2g-\frac{7}{4}ab\,c^2d^2f^2h+\frac{11}{4}abc\,d^3f^2g-ab\,d^4efg\right.\right.}{}\right.$
default	$2f^3\left(-\frac{-eh+fg}{(cf-de)^3(af-be)^2\sqrt{fx+e}}+\frac{d^2\left(\left(\frac{3}{8}a^2c\,d^3f^2h+\frac{1}{2}a^2d^4efh-\frac{7}{8}a^2d^4f^2g-\frac{7}{4}ab\,c^2d^2f^2h+\frac{11}{4}abc\,d^3f^2g-ab\,d^4efg\right.\right.}{}\right.$
pseudoelliptic	Expression too large to display

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^3*(-(-e*h+f*g)/(c*f-d*e)^3/(a*f-b*e)^2/(f*x+e)^(1/2)+d^2/(a*d-b*c)^4/f
^3/(c*f-d*e)^3*(((3/8*a^2*c*d^3*f^2*h+1/2*a^2*d^4*e*f*h-7/8*a^2*d^4*f^2*g-
7/4*a*b*c^2*d^2*f^2*h+11/4*a*b*c*d^3*f^2*g-a*b*d^4*e*f*g+11/8*b^2*c^3*d*f^
2*h-1/2*b^2*c^2*d^2*e*f*h-15/8*b^2*c^2*d^2*f^2*g+b^2*c*d^3*e*f*g)*(f*x+e)^
(3/2)+1/8*f*(5*a^2*c^2*d^2*f^2*h-a^2*c*d^3*e*f*h-9*a^2*c*d^3*f^2*g-4*a^2*d
^4*e^2*h+9*a^2*d^4*e*f*g-18*a*b*c^3*d*f^2*h+18*a*b*c^2*d^2*e*f*h+26*a*b*c^
2*d^2*f^2*g-34*a*b*c*d^3*e*f*g+8*a*b*d^4*e^2*g+13*b^2*c^4*f^2*h-17*b^2*c^3
*d*e*f*h-17*b^2*c^3*d*f^2*g+4*b^2*c^2*d^2*e^2*h+25*b^2*c^2*d^2*e*f*g-8*b^2
*c*d^3*e^2*g)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2+1/8*(3*a^2*c*d^2*f^2*h+
12*a^2*d^3*e*f*h-15*a^2*d^3*f^2*g-14*a*b*c^2*d*f^2*h-32*a*b*c*d^2*e*f*h+54
*a*b*c*d^2*f^2*g+16*a*b*d^3*e^2*h-24*a*b*d^3*e*f*g+35*b^2*c^3*f^2*h-28*b^2
*c^2*d*e*f*h-63*b^2*c^2*d*f^2*g+8*b^2*c*d^2*e^2*h+72*b^2*c*d^2*e*f*g-24*b^
2*d^3*e^2*g)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)^(1/2
)))-b^3/(a*d-b*c)^4/f^3/(a*f-b*e)^2*((1/2*a^2*d*f*h-1/2*a*b*c*f*h-1/2*a*b*
d*f*g+1/2*b^2*c*f*g)*(f*x+e)^(1/2)/((f*x+e)*b+a*f-b*e)+1/2*(7*a^2*d*f*h-a*
b*c*f*h-4*a*b*d*e*h-9*a*b*d*f*g-2*b^2*c*e*h+3*b^2*c*f*g+6*b^2*d*e*g)/((a*f
-b*e)*b)^(1/2)*arctan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**2/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(802) = 1604$.

Time = 0.37 (sec) , antiderivative size = 2420, normalized size of antiderivative = 2.89

$$\int \frac{g + hx}{(a + bx)^2(c + dx)^3(e + fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-(6*b^5*d*e*g + 3*b^5*c*f*g - 9*a*b^4*d*f*g - 2*b^5*c*e*h - 4*a*b^4*d*e*h
- a*b^4*c*f*h + 7*a^2*b^3*d*f*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*
f))/((b^6*c^4*e^2 - 4*a*b^5*c^3*d*e^2 + 6*a^2*b^4*c^2*d^2*e^2 - 4*a^3*b^3*
c*d^3*e^2 + a^4*b^2*d^4*e^2 - 2*a*b^5*c^4*e*f + 8*a^2*b^4*c^3*d*e*f - 12*a
^3*b^3*c^2*d^2*e*f + 8*a^4*b^2*c*d^3*e*f - 2*a^5*b*d^4*e*f + a^2*b^4*c^4*f
^2 - 4*a^3*b^3*c^3*d*f^2 + 6*a^4*b^2*c^2*d^2*f^2 - 4*a^5*b*c*d^3*f^2 + a^6
*d^4*f^2)*sqrt(-b^2*e + a*b*f)) + 1/4*(24*b^2*d^5*e^2*g - 72*b^2*c*d^4*e*f
*g + 24*a*b*d^5*e*f*g + 63*b^2*c^2*d^3*f^2*g - 54*a*b*c*d^4*f^2*g + 15*a^2
*d^5*f^2*g - 8*b^2*c*d^4*e^2*h - 16*a*b*d^5*e^2*h + 28*b^2*c^2*d^3*e*f*h +
 32*a*b*c*d^4*e*f*h - 12*a^2*d^5*e*f*h - 35*b^2*c^3*d^2*f^2*h + 14*a*b*c^2
*d^3*f^2*h - 3*a^2*c*d^4*f^2*h)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f
))/((b^4*c^4*d^3*e^3 - 4*a*b^3*c^3*d^4*e^3 + 6*a^2*b^2*c^2*d^5*e^3 - 4*a^3
*b*c*d^6*e^3 + a^4*d^7*e^3 - 3*b^4*c^5*d^2*e^2*f + 12*a*b^3*c^4*d^3*e^2*f
- 18*a^2*b^2*c^3*d^4*e^2*f + 12*a^3*b*c^2*d^5*e^2*f - 3*a^4*c*d^6*e^2*f +
3*b^4*c^6*d*e*f^2 - 12*a*b^3*c^5*d^2*e*f^2 + 18*a^2*b^2*c^4*d^3*e*f^2 - 12
*a^3*b*c^3*d^4*e*f^2 + 3*a^4*c^2*d^5*e*f^2 - b^4*c^7*f^3 + 4*a*b^3*c^6*d*f
^3 - 6*a^2*b^2*c^5*d^2*f^3 + 4*a^3*b*c^4*d^3*f^3 - a^4*c^3*d^4*f^3)*sqrt(-
d^2*e + c*d*f)) - ((f*x + e)*b^4*d^3*e^3*f*g - 3*(f*x + e)*b^4*c*d^2*e^2*f
^2*g + 3*(f*x + e)*b^4*c^2*d*e*f^3*g - 3*(f*x + e)*b^4*c^3*f^4*g + 6*(f*x
+ e)*a*b^3*c^2*d*f^4*g - 6*(f*x + e)*a^2*b^2*c*d^2*f^4*g + 2*(f*x + e)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^2*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 26977, normalized size of antiderivative = 32.19

$$\int \frac{g+hx}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)^2/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
( - 28*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(
b)*sqrt(a*f - b*e)))*a**3*b**2*c**6*d*f**5*h + 112*sqrt(b)*sqrt(e + f*x)*s
qrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2
*c**5*d**2*e*f**4*h - 56*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(
e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**5*d**2*f**5*h*x - 168*
sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt
(a*f - b*e)))*a**3*b**2*c**4*d**3*e**2*f**3*h + 224*sqrt(b)*sqrt(e + f*x)*
sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**
2*c**4*d**3*e*f**4*h*x - 28*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sq
rt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**4*d**3*f**5*h*x**2
+ 112*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b
)*sqrt(a*f - b*e)))*a**3*b**2*c**3*d**4*e**3*f**2*h - 336*sqrt(b)*sqrt(e +
 f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a*
*3*b**2*c**3*d**4*e**2*f**3*h*x + 112*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e
)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**3*d**4*e*
f**4*h*x**2 - 28*sqrt(b)*sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)
*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c**2*d**5*e**4*f*h + 224*sqrt(b)*
sqrt(e + f*x)*sqrt(a*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b
*e)))*a**3*b**2*c**2*d**5*e**3*f**2*h*x - 168*sqrt(b)*sqrt(e + f*x)*sqrt(a
*f - b*e)*atan((sqrt(e + f*x)*b)/(sqrt(b)*sqrt(a*f - b*e)))*a**3*b**2*c...
```

3.192 $\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1292

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-3/4*f*(a^4*d^3*f^3*(-c*f*h-4*d*e*h+5*d*f*g)+a^3*b*d^2*f^2*(5*c^2*f^2*h-c*
d*f*(-11*e*h+17*f*g)-d^2*e*(-4*e*h+3*f*g))-b^4*(8*d^4*e^4*g+c^3*d*e*f^2*(-
4*e*h+3*f*g)-c^4*f^3*(-4*e*h+5*f*g)-4*c*d^3*e^3*(e*h+4*f*g)+c^2*d^2*e^2*f*
(9*e*h+5*f*g))+a^2*b^2*d*f*(5*c^3*f^3*h+2*c^2*d*f^2*(-23*e*h+8*f*g)-d^3*e^
2*(9*e*h+5*f*g)+c*d^2*e*f*(20*e*h+19*f*g))-a*b^3*(c^4*f^4*h+c^3*d*f^3*(-11
*e*h+17*f*g)-4*d^4*e^3*(e*h+4*f*g)+2*c*d^3*e^2*f*(7*e*h+19*f*g)-c^2*d^2*e*
f^2*(20*e*h+19*f*g)))/(-a*d+b*c)^4/(-a*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^(1/2)
+1/4*d*(a^2*d*f*(-13*c*f*h+11*d*e*h+2*d*f*g)+b^2*(12*d^2*e^2*g-c^2*f*(-4*e
*h+5*f*g)-c*d*e*(6*e*h+5*f*g))+a*b*(c^2*f^2*h+3*c*d*f*(3*e*h+5*f*g)-d^2*e*
(6*e*h+19*f*g)))/(-a*d+b*c)^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x+c)^2/(f*x+e)^(1
/2)-1/2*(-a*h+b*g)/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2)
+1/4*(9*a^2*d*f*h+b^2*(-4*c*e*h+5*c*f*g+8*d*e*g)-a*b*(c*f*h+4*d*e*h+13*d*f
*g))/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)/(d*x+c)^2/(f*x+e)^(1/2)+1/4*d*(a^3*
d^2*f^2*(-c*f*h-4*d*e*h+5*d*f*g)+b^3*(24*d^3*e^3*g+c^3*f^2*(-4*e*h+5*f*g)-
12*c*d^2*e^2*(e*h+3*f*g)+c^2*d*e*f*(21*e*h+2*f*g))+a^2*b*d*f*(26*c^2*f^2*h
+d^2*e*(21*e*h+2*f*g)-c*d*f*(32*e*h+17*f*g))-a*b^2*(c^3*f^3*h+12*d^3*e^2*(
e*h+3*f*g)-2*c*d^2*e*f*(15*e*h+34*f*g)+c^2*d*f^2*(32*e*h+17*f*g)))/(-a*d+b
*c)^4/(-a*f+b*e)^2/(-c*f+d*e)^2/(d*x+c)/(f*x+e)^(1/2)+3/4*b^(5/2)*(21*a^3*
d^2*f^2*h-3*a^2*b*d*f*(2*c*f*h+8*d*e*h+11*d*f*g)-b^3*(16*d^2*e^2*g+c^2*f*(
-4*e*h+5*f*g)+4*c*d*e*(-2*e*h+3*f*g))+a*b^2*(c^2*f^2*h+2*c*d*f*(-6*e*h+...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.72 (sec) , antiderivative size = 871, normalized size of antiderivative = 0.67

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \frac{\frac{-2bg+2ah}{(a+bx)^2} + \frac{9a^2dfh+b^2(8deg+5cfg-4ceh)-ab(13dfg+4deh+cfh)}{(bc-ad)(be-af)(a+bx)} - \frac{-d(bc-ad)^2(be-}{}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)^3*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((-2*b*g + 2*a*h)/(a + b*x)^2 + (9*a^2*d*f*h + b^2*(8*d*e*g + 5*c*f*g - 4*
c*e*h) - a*b*(13*d*f*g + 4*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b
*x)) - (-(d*(b*c - a*d)^2*(b*e - a*f)*(d*e - c*f)^2*(a^2*d*f*(2*d*f*g + 11
*d*e*h - 13*c*f*h) + b^2*(12*d^2*e^2*g + c^2*f*(-5*f*g + 4*e*h) - c*d*e*(5
*f*g + 6*e*h)) + a*b*(c^2*f^2*h + 3*c*d*f*(5*f*g + 3*e*h) - d^2*e*(19*f*g
+ 6*e*h)))) + (c + d*x)*(-(d*(b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(a^3*d
^2*f^2*(-5*d*f*g + 4*d*e*h + c*f*h) + b^3*(-24*d^3*e^3*g + 12*c*d^2*e^2*(3
*f*g + e*h) + c^3*f^2*(-5*f*g + 4*e*h) - c^2*d*e*f*(2*f*g + 21*e*h)) + a^2
*b*d*f*(-26*c^2*f^2*h - d^2*e*(2*f*g + 21*e*h) + c*d*f*(17*f*g + 32*e*h))
+ a*b^2*(c^3*f^3*h + 12*d^3*e^2*(3*f*g + e*h) - 2*c*d^2*e*f*(34*f*g + 15*e
*h) + c^2*d*f^2*(17*f*g + 32*e*h)))) - 3*(c + d*x)*(b^2*(d*e - c*f)^3*(-21
*a^3*d^2*f^2*h + 3*a^2*b*d*f*(11*d*f*g + 8*d*e*h + 2*c*f*h) + b^3*(16*d^2*
e^2*g + c^2*f*(5*f*g - 4*e*h) + 4*c*d*e*(3*f*g - 2*e*h)) - a*b^2*(c^2*f^2*
h + 2*c*d*f*(11*f*g - 6*e*h) + 4*d^2*e*(11*f*g + 2*e*h)))*Hypergeometric2F
1[-1/2, 1, 1/2, (b*(e + f*x))/(b*e - a*f)] - d^2*(b*e - a*f)^3*(a^2*d^2*f*
(5*d*f*g - 4*d*e*h - c*f*h) + b^2*(16*d^3*e^2*g - 21*c^3*f^2*h - 4*c*d^2*e
*(11*f*g + 2*e*h) + 3*c^2*d*f*(11*f*g + 8*e*h)) + 2*a*b*d*(3*c^2*f^2*h + c
*d*f*(-11*f*g + 6*e*h) + d^2*(6*e*f*g - 4*e^2*h)))*Hypergeometric2F1[-1/2,
 1, 1/2, (d*(e + f*x))/(d*e - c*f)])))/((b*c - a*d)^4*(b*e - a*f)^2*(d*e -
 c*f)^3))/(4*(b*c - a*d)*(b*e - a*f)*(c + d*x)^2*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 3.62 (sec) , antiderivative size = 1388, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{b(8deg+5cfg-4ceh)-a(cfh+4d(fg+eh))+9df(bg-ah)x}{2(a+bx)^2(c+dx)^3(e+fx)^{3/2}}dx}{2(bc-ad)(be-af)}-\frac{bg-ah}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{b(8deg+5cfg-4ceh)-a(cfh+4d(fg+eh))+9df(bg-ah)x}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}dx}{4(bc-ad)(be-af)}-\frac{bg-ah}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{\int \frac{df(8dfg+44deh+11cfh)a^2-b\left(4e(19fg+6eh)d^2+cf(31fg-8eh)d+3c^2f^2h\right)a+3b^2\left(f(5fg-4eh)c^2+4de(3fg-2eh)c+16d^2e^2g\right)+7df\left(9dfha^2-b(13dfg+4de\right.}{2(a+bx)(c+dx)^3(e+fx)^{3/2}}}{(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{df(8dfg+44deh+11cfh)a^2-b\left(4e(19fg+6eh)d^2+cf(31fg-8eh)d+3c^2f^2h\right)a+3b^2\left(f(5fg-4eh)c^2+4de(3fg-2eh)c+16d^2e^2g\right)+7df\left(9dfha^2-b(13dfg+4de\right.}{(a+bx)(c+dx)^3(e+fx)^{3/2}}}{2(bc-ad)(be-af)}}{4(bc-ad)(be-af)}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{\int \frac{2\left(2d^2f^2(5dfg-4deh-cfh)a^3-bdf\left(-2e(2fg+21eh)d^2+3cf(8fg+3eh)d+13c^2f^2h\right)a^2+b^2\left(-24e^2(3fg+eh)d^3+cef(41fg+30eh)d^2+c^2f^2(41fg-19eh)d+\right.\right.}{}}{}}{}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{2d^2f^2(5dfg-4deh-cfh)a^3-bdf\left(-2e(2fg+21eh)d^2+3cf(8fg+3eh)d+13c^2f^2h\right)a^2+b^2\left(-24e^2(3fg+eh)d^3+cef(41fg+30eh)d^2+c^2f^2(41fg-19eh)d+3c\right.}{}}{}}{}$$

$$\frac{bg-ah}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{2d(df(2dfg+11deh-13cfh)a^2+b(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}$$

$\Big\downarrow$ 27

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{2d(df(2dfg+11deh-13cfh)a^2+b(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}$$

$\Big\downarrow$ 169

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{2d(df(2dfg+11deh-13cfh)a^2+b(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}$$

$\Big\downarrow$ 27

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{2d(df(2dfg+11deh-13cfh)a^2+b(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}$$

$\Big\downarrow$ 174

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{2d(df(2dfg+11deh-13cfh)a^2+b(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h)a+b^2(-f(5fg}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{\frac{2d\left(df(2dfg+11deh-13cfh)a^2+b\left(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h\right)a+b^2\left(-f(5fg\right.\right.}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}}{}$$

$$\Big\downarrow \text{221}$$

$$-\frac{bg-ah}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{9dfha^2-b(13dfg+4deh+cfh)a+b^2(8deg+5cfg-4ceh)}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e+fx}}-\frac{\frac{2d\left(df(2dfg+11deh-13cfh)a^2+b\left(-e(19fg+6eh)d^2+3cf(5fg+3eh)d+c^2f^2h\right)a+b^2\left(-f(5fg\right.\right.}{(bc-ad)(de-cf)(c+dx)^2\sqrt{e+fx}}}{}$$

input
```
Int[(g + h*x)/((a + b*x)^3*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/2*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)^2*Sqrt[e +
 f*x]) - (-((9*a^2*d*f*h + b^2*(8*d*e*g + 5*c*f*g - 4*c*e*h) - a*b*(13*d*f
*g + 4*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2*Sqrt
[e + f*x])) - ((2*d*(a^2*d*f*(2*d*f*g + 11*d*e*h - 13*c*f*h) + b^2*(12*d^2
*e^2*g - c^2*f*(5*f*g - 4*e*h) - c*d*e*(5*f*g + 6*e*h)) + a*b*(c^2*f^2*h +
 3*c*d*f*(5*f*g + 3*e*h) - d^2*e*(19*f*g + 6*e*h))))/((b*c - a*d)*(d*e - c
*f)*(c + d*x)^2*Sqrt[e + f*x]) + ((2*d*(a^3*d^2*f^2*(5*d*f*g - 4*d*e*h - c
*f*h) + b^3*(24*d^3*e^3*g + c^3*f^2*(5*f*g - 4*e*h) - 12*c*d^2*e^2*(3*f*g
+ e*h) + c^2*d*e*f*(2*f*g + 21*e*h)) + a^2*b*d*f*(26*c^2*f^2*h + d^2*e*(2*
f*g + 21*e*h) - c*d*f*(17*f*g + 32*e*h)) - a*b^2*(c^3*f^3*h + 12*d^3*e^2*(
3*f*g + e*h) - 2*c*d^2*e*f*(34*f*g + 15*e*h) + c^2*d*f^2*(17*f*g + 32*e*h)
)))/((b*c - a*d)*(d*e - c*f)*(c + d*x)*Sqrt[e + f*x]) + (3*((-2*f*(a^4*d^3
*f^3*(5*d*f*g - 4*d*e*h - c*f*h) + a^3*b*d^2*f^2*(5*c^2*f^2*h - c*d*f*(17*
f*g - 11*e*h) - d^2*e*(3*f*g - 4*e*h)) - b^4*(8*d^4*e^4*g + c^3*d*e*f^2*(3
*f*g - 4*e*h) - c^4*f^3*(5*f*g - 4*e*h) - 4*c*d^3*e^3*(4*f*g + e*h) + c^2*
d^2*e^2*f*(5*f*g + 9*e*h)) + a^2*b^2*d*f*(5*c^3*f^3*h + 2*c^2*d*f^2*(8*f*g
 - 23*e*h) - d^3*e^2*(5*f*g + 9*e*h) + c*d^2*e*f*(19*f*g + 20*e*h)) - a*b^
3*(c^4*f^4*h + c^3*d*f^3*(17*f*g - 11*e*h) - 4*d^4*e^3*(4*f*g + e*h) + 2*c
*d^3*e^2*f*(19*f*g + 7*e*h) - c^2*d^2*e*f^2*(19*f*g + 20*e*h))))/((b*e - a
*f)*(d*e - c*f)*Sqrt[e + f*x]) - ((-2*b^(5/2)*(d*e - c*f)^3*(21*a^3*d^2...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 539.70 (sec) , antiderivative size = 1287, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	Expression too large to display	1287
default	Expression too large to display	1287
pseudoelliptic	Expression too large to display	2473

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f^4*(-(-e*h+f*g)/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^(1/2)-b^3/(a*d-b*c)^5/f
^4/(a*f-b*e)^3*(((15/8*a^3*b*d^2*f^2*h-9/4*a^2*b^2*c*d*f^2*h-a^2*b^2*d^2*e
*f*h-19/8*a^2*b^2*d^2*f^2*g+3/8*a*b^3*c^2*f^2*h+1/2*a*b^3*c*d*e*f*h+13/4*a
*b^3*c*d*f^2*g+3/2*a*b^3*d^2*e*f*g+1/2*b^4*c^2*e*f*h-7/8*b^4*c^2*f^2*g-3/2
*b^4*c*d*e*f*g)*(f*x+e)^(3/2)+1/8*f*(17*a^4*d^2*f^2*h-22*a^3*b*c*d*f^2*h-2
5*a^3*b*d^2*e*f*h-21*a^3*b*d^2*f^2*g+5*a^2*b^2*c^2*f^2*h+26*a^2*b^2*c*d*e*
f*h+30*a^2*b^2*c*d*f^2*g+8*a^2*b^2*d^2*e^2*h+33*a^2*b^2*d^2*e*f*g-a*b^3*c^
2*e*f*h-9*a*b^3*c^2*f^2*g-4*a*b^3*c*d*e^2*h-42*a*b^3*c*d*e*f*g-12*a*b^3*d^
2*e^2*g-4*b^4*c^2*e^2*h+9*b^4*c^2*e*f*g+12*b^4*c*d*e^2*g)*(f*x+e)^(1/2))/(
(f*x+e)*b+a*f-b*e)^2+3/8*(21*a^3*d^2*f^2*h-6*a^2*b*c*d*f^2*h-24*a^2*b*d^2*
e*f*h-33*a^2*b*d^2*f^2*g+a*b^2*c^2*f^2*h-12*a*b^2*c*d*e*f*h+22*a*b^2*c*d*f
^2*g+8*a*b^2*d^2*e^2*h+44*a*b^2*d^2*e*f*g+4*b^3*c^2*e*f*h-5*b^3*c^2*f^2*g+
8*b^3*c*d*e^2*h-12*b^3*c*d*e*f*g-16*b^3*d^2*e^2*g)/((a*f-b*e)*b)^(1/2)*arc
tan(b*(f*x+e)^(1/2)/((a*f-b*e)*b)^(1/2)))+d^3/(a*d-b*c)^5/f^4/(c*f-d*e)^3*
(((3/8*a^2*c*d^3*f^2*h+1/2*a^2*d^4*e*f*h-7/8*a^2*d^4*f^2*g-9/4*a*b*c^2*d^2
*f^2*h+1/2*a*b*c*d^3*e*f*h+13/4*a*b*c*d^3*f^2*g-3/2*a*b*d^4*e*f*g+15/8*b^2
*c^3*d*f^2*h-b^2*c^2*d^2*e*f*h-19/8*b^2*c^2*d^2*f^2*g+3/2*b^2*c*d^3*e*f*g)
*(f*x+e)^(3/2)+1/8*f*(5*a^2*c^2*d^2*f^2*h-a^2*c*d^3*e*f*h-9*a^2*c*d^3*f^2*
g-4*a^2*d^4*e^2*h+9*a^2*d^4*e*f*g-22*a*b*c^3*d*f^2*h+26*a*b*c^2*d^2*e*f*h+
30*a*b*c^2*d^2*f^2*g-4*a*b*c*d^3*e^2*h-42*a*b*c*d^3*e*f*g+12*a*b*d^4*e^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**3/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7132 vs. 2(1249) = 2498.

Time = 6.87 (sec) , antiderivative size = 7132, normalized size of antiderivative = 5.52

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
3/4*(16*b^6*d^2*e^2*g + 12*b^6*c*d*e*f*g - 44*a*b^5*d^2*e*f*g + 5*b^6*c^2*
f^2*g - 22*a*b^5*c*d*f^2*g + 33*a^2*b^4*d^2*f^2*g - 8*b^6*c*d*e^2*h - 8*a*
b^5*d^2*e^2*h - 4*b^6*c^2*e*f*h + 12*a*b^5*c*d*e*f*h + 24*a^2*b^4*d^2*e*f*
h - a*b^5*c^2*f^2*h + 6*a^2*b^4*c*d*f^2*h - 21*a^3*b^3*d^2*f^2*h)*arctan(s
qrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^8*c^5*e^3 - 5*a*b^7*c^4*d*e^3 + 1
0*a^2*b^6*c^3*d^2*e^3 - 10*a^3*b^5*c^2*d^3*e^3 + 5*a^4*b^4*c*d^4*e^3 - a^5
*b^3*d^5*e^3 - 3*a*b^7*c^5*e^2*f + 15*a^2*b^6*c^4*d*e^2*f - 30*a^3*b^5*c^3
*d^2*e^2*f + 30*a^4*b^4*c^2*d^3*e^2*f - 15*a^5*b^3*c*d^4*e^2*f + 3*a^6*b^2
*d^5*e^2*f + 3*a^2*b^6*c^5*e*f^2 - 15*a^3*b^5*c^4*d*e*f^2 + 30*a^4*b^4*c^3
*d^2*e*f^2 - 30*a^5*b^3*c^2*d^3*e*f^2 + 15*a^6*b^2*c*d^4*e*f^2 - 3*a^7*b*d
^5*e*f^2 - a^3*b^5*c^5*f^3 + 5*a^4*b^4*c^4*d*f^3 - 10*a^5*b^3*c^3*d^2*f^3
+ 10*a^6*b^2*c^2*d^3*f^3 - 5*a^7*b*c*d^4*f^3 + a^8*d^5*f^3)*sqrt(-b^2*e +
a*b*f)) - 3/4*(16*b^2*d^6*e^2*g - 44*b^2*c*d^5*e*f*g + 12*a*b*d^6*e*f*g +
33*b^2*c^2*d^4*f^2*g - 22*a*b*c*d^5*f^2*g + 5*a^2*d^6*f^2*g - 8*b^2*c*d^5*
e^2*h - 8*a*b*d^6*e^2*h + 24*b^2*c^2*d^4*e*f*h + 12*a*b*c*d^5*e*f*h - 4*a^
2*d^6*e*f*h - 21*b^2*c^3*d^3*f^2*h + 6*a*b*c^2*d^4*f^2*h - a^2*c*d^5*f^2*h
)*arctan(sqrt(f*x + e)*d/sqrt(-d^2*e + c*d*f))/((b^5*c^5*d^3*e^3 - 5*a*b^4
*c^4*d^4*e^3 + 10*a^2*b^3*c^3*d^5*e^3 - 10*a^3*b^2*c^2*d^6*e^3 + 5*a^4*b*c
*d^7*e^3 - a^5*d^8*e^3 - 3*b^5*c^6*d^2*e^2*f + 15*a*b^4*c^5*d^3*e^2*f - 30
*a^2*b^3*c^4*d^4*e^2*f + 30*a^3*b^2*c^3*d^5*e^2*f - 15*a^4*b*c^2*d^6*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^3*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{g+hx}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}\,dx = \int \frac{hx+g}{(bx+a)^3\,(dx+c)^3\,(fx+e)^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
int((h*x+g)/(b*x+a)^3/(d*x+c)^3/(f*x+e)^(3/2),x)
```

3.193 $\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1935

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-1/8*f*(6*a^5*d^4*f^4*(-c*f*h-4*d*e*h+5*d*f*g)+2*a^4*b*d^3*f^3*(19*c^2*f^2
*h-c*d*f*(-40*e*h+63*f*g)-4*d^2*e*(-4*e*h+3*f*g))+b^5*(80*d^5*e^5*g+3*c^3*
d^2*e^2*f^2*(-10*e*h+7*f*g)+3*c^4*d*e*f^3*(-6*e*h+5*f*g)-5*c^5*f^4*(-6*e*h
+7*f*g)-4*c*d^4*e^4*(12*e*h+37*f*g)+c^2*d^3*e^3*f*(96*e*h+37*f*g))+a^3*b^2
*d^2*f^2*(71*c^3*f^3*h+5*c^2*d*f^2*(-105*e*h+32*f*g)-d^3*e^2*(119*e*h+44*f
*g)+c*d^2*e*f*(273*e*h+184*f*g))-a^2*b^3*d*f*(28*c^4*f^4*h+c^3*d*f^3*(-168
*e*h+269*f*g)-d^4*e^3*(108*e*h+245*f*g)-3*c^2*d^2*e*f^2*(124*e*h+109*f*g)+
c*d^3*e^2*f*(320*e*h+603*f*g))+a*b^4*(5*c^5*f^5*h+c^4*d*f^4*(-129*e*h+160*
f*g)-3*c^3*d^2*e*f^3*(-47*e*h+34*f*g)-4*d^5*e^4*(8*e*h+63*f*g)+14*c*d^4*e^
3*f*(14*e*h+37*f*g)-c^2*d^3*e^2*f^2*(331*e*h+174*f*g)))/(-a*d+b*c)^5/(-a*f
+b*e)^4/(-c*f+d*e)^3/(f*x+e)^(1/2)+1/24*d*(3*a^3*d^2*f^2*(-47*c*f*h+43*d*e
*h+4*d*f*g)-b^3*(120*d^3*e^3*g-c^2*d*e*f*(-30*e*h+41*f*g)-5*c^3*f^2*(-6*e*
h+7*f*g)-8*c*d^2*e^2*(9*e*h+4*f*g))-a*b^2*(5*c^3*f^3*h+c^2*d*f^2*(-109*e*h
+146*f*g)-8*d^3*e^2*(6*e*h+41*f*g)+2*c*d^2*e*f*(94*e*h+73*f*g))+a^2*b*d*f*
(26*c^2*f^2*h-d^2*e*(142*e*h+255*f*g)+c*d*f*(152*e*h+219*f*g)))/(-a*d+b*c)
^4/(-a*f+b*e)^3/(-c*f+d*e)/(d*x+c)^2/(f*x+e)^(1/2)-1/3*(-a*h+b*g)/(-a*d+b*
c)/(-a*f+b*e)/(b*x+a)^3/(d*x+c)^2/(f*x+e)^(1/2)+1/12*(11*a^2*d*f*h+b^2*(-6
*c*e*h+7*c*f*g+10*d*e*g)-a*b*(c*f*h+4*d*e*h+17*d*f*g))/(-a*d+b*c)^2/(-a*f+
b*e)^2/(b*x+a)^2/(d*x+c)^2/(f*x+e)^(1/2)+1/24*(99*a^3*d^2*f^2*h-3*a^2*b*d*
f*(8*c*f*h+32*d*e*h+59*d*f*g)-b^3*(80*d^2*e^2*g+2*c*d*e*(-24*e*h+31*f*g...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 12.76 (sec) , antiderivative size = 1338, normalized size of antiderivative = 0.69

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(g + h*x)/((a + b*x)^4*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
((-8*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(a + b*x)^3 + (22*a^2*d*f*h + 2*
b^2*(10*d*e*g + 7*c*f*g - 6*c*e*h) - 2*a*b*(17*d*f*g + 4*d*e*h + c*f*h))/(
a + b*x)^2 + (99*a^3*d^2*f^2*h - 3*a^2*b*d*f*(59*d*f*g + 32*d*e*h + 8*c*f*
h) + a*b^2*(5*c^2*f^2*h + 2*c*d*f*(66*f*g - 47*e*h) + 2*d^2*e*(111*f*g + 1
6*e*h)) + b^3*(-80*d^2*e^2*g + 5*c^2*f*(-7*f*g + 6*e*h) + 2*c*d*e*(-31*f*g
 + 24*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + (-(d*(b*c - a*d)^2*(b*e
 - a*f)*(d*e - c*f)^2*(-3*a^3*d^2*f^2*(4*d*f*g + 43*d*e*h - 47*c*f*h) + b^
3*(120*d^3*e^3*g + 5*c^3*f^2*(-7*f*g + 6*e*h) - 8*c*d^2*e^2*(4*f*g + 9*e*h
) + c^2*d*e*f*(-41*f*g + 30*e*h)) + a*b^2*(5*c^3*f^3*h + c^2*d*f^2*(146*f*
g - 109*e*h) - 8*d^3*e^2*(41*f*g + 6*e*h) + 2*c*d^2*e*f*(73*f*g + 94*e*h))
 + a^2*b*d*f*(-26*c^2*f^2*h + d^2*e*(255*f*g + 142*e*h) - c*d*f*(219*f*g +
 152*e*h)))) + (c + d*x)*(-(d*(b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(-6*a
^4*d^3*f^3*(-5*d*f*g + 4*d*e*h + c*f*h) + b^4*(-240*d^4*e^4*g + c^2*d^2*e^
2*f*(f*g - 216*e*h) + 4*c^3*d*e*f^2*(-5*f*g + 3*e*h) + 36*c*d^3*e^3*(9*f*g
 + 4*e*h) + 5*c^4*f^3*(-7*f*g + 6*e*h)) + a^3*b*d^2*f^2*(269*c^2*f^2*h - 2
*c*d*f*(63*f*g + 194*e*h) + d^2*e*(6*f*g + 239*e*h)) - a^2*b^2*d*f*(28*c^3
*f^3*h + d^3*e^2*(467*f*g + 276*e*h) - 2*c*d^2*e*f*(458*f*g + 289*e*h) + c
^2*d*f^2*(269*f*g + 454*e*h)) + a*b^3*(5*c^4*f^4*h + 4*c^3*d*f^3*(40*f*g -
 31*e*h) + 12*d^4*e^3*(53*f*g + 8*e*h) - 2*c*d^3*e^2*f*(487*f*g + 234*e*h)
 + c^2*d^2*e*f^2*(58*f*g + 611*e*h))) - 3*(c + d*x)*(b^2*(d*e - c*f)^3...
```

Rubi [A] (verified)

Time = 5.80 (sec) , antiderivative size = 2056, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{2b\left(5deg+\frac{7cfg}{2}-3ceh\right)-a(6dfg+4deh+cfh)+11df(bg-ah)x}{2(a+bx)^3(c+dx)^3(e+fx)^{3/2}}dx}{3(bc-ad)(be-af)}-\frac{bg-ah}{3(a+bx)^3(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{b(10deg+7cfg-6ceh)-a(6dfg+4deh+cfh)+11df(bg-ah)x}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}dx}{6(bc-ad)(be-af)}-\frac{bg-ah}{3(a+bx)^3(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{\int \frac{3df(8dfg+20deh+5cfh)a^2-b\left(4e(33fg+8eh)d^2+cf(69fg-40eh)d+5c^2f^2h\right)a+b^2\left(5f(7fg-6eh)c^2+2de(31fg-24eh)c+80d^2e^2g\right)+9df\left(11dfha^2-b(17dfg\cdots\right.}{2(a+bx)^2(c+dx)^3(e+fx)^{3/2}}}{2(bc-ad)(be-af)}\cdots}{6(bc-ad)(be-af)}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{3df(8dfg+20deh+5cfh)a^2-b\left(4e(33fg+8eh)d^2+cf(69fg-40eh)d+5c^2f^2h\right)a+b^2\left(5f(7fg-6eh)c^2+2de(31fg-24eh)c+80d^2e^2g\right)+9df\left(11dfha^2-b(17dfg\cdots\right.}{(a+bx)^2(c+dx)^3(e+fx)^{3/2}}}{4(bc-ad)(be-af)}\cdots}{6(bc-ad)(be-af)}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{-\frac{99a^3d^2f^2h-3a^2bdf(8cfh+32deh+59dfg)+ab^2\left(5c^2f^2h+2cdf(66fg-47eh)+2d^2e(16eh+111fg)\right)-b^3\left(5c^2f(7fg-6eh)+2cde(31fg-24eh)+80d^2e^2g\right)}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int \cdots}{\cdots}}{\cdots}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{99a^3d^2f^2h-3a^2bdf(8cfh+32deh+59dfg)+ab^2\left(5c^2f^2h+2cdf(66fg-47eh)+2d^2e(16eh+111fg)\right)-b^3\left(5c^2f(7fg-6eh)+2cde(31fg-24eh)+80d^2e^2g\right)}{(a+bx)(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}-\frac{\int \cdots}{\cdots}}{\cdots}$$

$$\frac{bg-ah}{3(a+bx)^3(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 168$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$\Big\downarrow$ 27

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$\Big\downarrow$ 168

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$\Big\downarrow$ 27

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$\Big\downarrow$ 169

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$$\Big\downarrow 27$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$$\Big\downarrow 174$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$$\Big\downarrow 73$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

$$\Big\downarrow 221$$

$$-\frac{bg-ah}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{11dfha^2-b(17dfg+4deh+cfh)a+b^2(10deg+7cfg-6ceh)}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{e+fx}}-\frac{99d^2f^2ha^3-3bdf(59dfg+32deh+8cfh)a^2+b^2\left(2e(111fg+16eh)d^2+2cf(66fg-47eh)d+5c\right.}{(bc-ad)(be-af)(a+bx)(c+dx)^2\sqrt{e}}$$

input
```
Int[(g + h*x)/((a + b*x)^4*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/3*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c + d*x)^2*Sqrt[e +
 f*x]) - (-1/2*(11*a^2*d*f*h + b^2*(10*d*e*g + 7*c*f*g - 6*c*e*h) - a*b*(1
7*d*f*g + 4*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)
^2*Sqrt[e + f*x]) - ((99*a^3*d^2*f^2*h - 3*a^2*b*d*f*(59*d*f*g + 32*d*e*h
+ 8*c*f*h) - b^3*(80*d^2*e^2*g + 2*c*d*e*(31*f*g - 24*e*h) + 5*c^2*f*(7*f*
g - 6*e*h)) + a*b^2*(5*c^2*f^2*h + 2*c*d*f*(66*f*g - 47*e*h) + 2*d^2*e*(11
1*f*g + 16*e*h)))/((b*c - a*d)*(b*e - a*f)*(a + b*x)*(c + d*x)^2*Sqrt[e +
f*x]) - ((-2*d*(3*a^3*d^2*f^2*(4*d*f*g + 43*d*e*h - 47*c*f*h) - b^3*(120*d
^3*e^3*g - c^2*d*e*f*(41*f*g - 30*e*h) - 5*c^3*f^2*(7*f*g - 6*e*h) - 8*c*d
^2*e^2*(4*f*g + 9*e*h)) - a*b^2*(5*c^3*f^3*h + c^2*d*f^2*(146*f*g - 109*e*
h) - 8*d^3*e^2*(41*f*g + 6*e*h) + 2*c*d^2*e*f*(73*f*g + 94*e*h)) + a^2*b*d
*f*(26*c^2*f^2*h - d^2*e*(255*f*g + 142*e*h) + c*d*f*(219*f*g + 152*e*h)))
)/((b*c - a*d)*(d*e - c*f)*(c + d*x)^2*Sqrt[e + f*x]) - ((2*d*(6*a^4*d^3*f
^3*(5*d*f*g - 4*d*e*h - c*f*h) - b^4*(240*d^4*e^4*g - c^2*d^2*e^2*f*(f*g -
 216*e*h) + 5*c^4*f^3*(7*f*g - 6*e*h) + 4*c^3*d*e*f^2*(5*f*g - 3*e*h) - 36
*c*d^3*e^3*(9*f*g + 4*e*h)) + a^3*b*d^2*f^2*(269*c^2*f^2*h - 2*c*d*f*(63*f
*g + 194*e*h) + d^2*e*(6*f*g + 239*e*h)) - a^2*b^2*d*f*(28*c^3*f^3*h + d^3
*e^2*(467*f*g + 276*e*h) - 2*c*d^2*e*f*(458*f*g + 289*e*h) + c^2*d*f^2*(26
9*f*g + 454*e*h)) + a*b^3*(5*c^4*f^4*h + 4*c^3*d*f^3*(40*f*g - 31*e*h) + 1
2*d^4*e^3*(53*f*g + 8*e*h) - 2*c*d^3*e^2*f*(487*f*g + 234*e*h) + c^2*d^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2348, normalized size of antiderivative = 1.21

Expression too large to display

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
2*f^5*(-b^3/(a*d-b*c)^6/f^5/(a*f-b*e)^4*(((-19/4*a^3*b^3*d^3*e*f^2*h+189/1
6*a^2*b^4*c*d^2*f^3*g+3/2*a^2*b^4*d^3*e^2*f*h+69/8*a^2*b^4*d^3*e*f^2*g-99/
16*a*b^5*c^2*d*f^3*g-3*a*b^5*d^3*e^2*f*g-3/2*b^6*c^2*d*e^2*f*h+21/8*b^6*c^
2*d*e*f^2*g+3*b^6*c*d^2*e^2*f*g-99/16*b^3*a^3*c*d^2*f^3*h+33/16*b^4*a^2*c^
2*d*f^3*h+19/16*b^6*c^3*f^3*g+71/16*a^4*b^2*d^3*f^3*h-109/16*a^3*d^3*f^3*g
*b^3-7/8*b^6*c^3*e*f^2*h-5/16*b^5*a*c^3*f^3*h+21/8*a^2*b^4*c*d^2*e*f^2*h+3
*a*b^5*c^2*d*e*f^2*h-45/4*a*b^5*c*d^2*e*f^2*g)*(f*x+e)^(5/2)+1/6*b*f*(59*a
^5*d^3*f^3*h-87*a^4*b*c*d^2*f^3*h-119*a^4*b*d^3*e*f^2*h-89*a^4*b*d^3*f^3*g
+33*a^3*b^2*c^2*d*f^3*h+123*a^3*b^2*c*d^2*e*f^2*h+159*a^3*b^2*c*d^2*f^3*g+
78*a^3*b^2*d^3*e^2*f*h+197*a^3*b^2*d^3*e*f^2*g-5*a^2*b^3*c^3*f^3*h+3*a^2*b
^3*c^2*d*e*f^2*h-87*a^2*b^3*c^2*d*f^3*g-36*a^2*b^3*c*d^2*e^2*f*h-303*a^2*b
^3*c*d^2*e*f^2*g-18*a^2*b^3*d^3*e^3*h-144*a^2*b^3*d^3*e^2*f*g-7*a*b^4*c^3*
e*f^2*h+17*a*b^4*c^3*f^3*g-54*a*b^4*c^2*d*e^2*f*h+123*a*b^4*c^2*d*e*f^2*g+
180*a*b^4*c*d^2*e^2*f*g+36*a*b^4*d^3*e^3*g+12*b^5*c^3*e^2*f*h-17*b^5*c^3*e
*f^2*g+18*b^5*c^2*d*e^3*h-36*b^5*c^2*d*e^2*f*g-36*b^5*c*d^2*e^3*g)*(f*x+e)
^(3/2)+(-141/16*a^5*b*c*d^2*f^5*h-131/8*a^5*b*d^3*e*f^4*h+63/16*a^4*b^2*c^
2*d*f^5*h+243/16*a^4*b^2*c*d^2*f^5*g+281/16*a^4*b^2*d^3*e^2*f^3*h+103/4*a^
4*b^2*d^3*e*f^4*g-141/16*a^3*b^3*c^2*d*f^5*g-479/16*a^3*b^3*d^3*e^2*f^3*g+
1/4*a^2*b^4*c^3*e*f^4*h+123/8*a^2*b^4*d^3*e^3*f^2*g+25/16*a*b^5*c^3*e^2*f^
3*h+29/16*b^6*c^3*e^2*f^3*g-131/16*a^5*b*d^3*f^5*g-11/16*a^3*b^3*c^3*f^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**4/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4418 vs. 2(1887) = 3774.

Time = 1.00 (sec) , antiderivative size = 4418, normalized size of antiderivative = 2.28

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(160*b^7*d^3*e^3*g + 144*b^7*c*d^2*e^2*f*g - 624*a*b^6*d^3*e^2*f*g +
90*b^7*c^2*d*e*f^2*g - 468*a*b^6*c*d^2*e*f^2*g + 858*a^2*b^5*d^3*e*f^2*g +
 35*b^7*c^3*f^3*g - 195*a*b^6*c^2*d*f^3*g + 429*a^2*b^5*c*d^2*f^3*g - 429*
a^3*b^4*d^3*f^3*g - 96*b^7*c*d^2*e^3*h - 64*a*b^6*d^3*e^3*h - 72*b^7*c^2*d
*e^2*f*h + 288*a*b^6*c*d^2*e^2*f*h + 264*a^2*b^5*d^3*e^2*f*h - 30*b^7*c^3*
e*f^2*h + 144*a*b^6*c^2*d*e*f^2*h - 198*a^2*b^5*c*d^2*e*f^2*h - 396*a^3*b^
4*d^3*e*f^2*h - 5*a*b^6*c^3*f^3*h + 33*a^2*b^5*c^2*d*f^3*h - 99*a^3*b^4*c*
d^2*f^3*h + 231*a^4*b^3*d^3*f^3*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*
b*f))/((b^10*c^6*e^4 - 6*a*b^9*c^5*d*e^4 + 15*a^2*b^8*c^4*d^2*e^4 - 20*a^3
*b^7*c^3*d^3*e^4 + 15*a^4*b^6*c^2*d^4*e^4 - 6*a^5*b^5*c*d^5*e^4 + a^6*b^4*
d^6*e^4 - 4*a*b^9*c^6*e^3*f + 24*a^2*b^8*c^5*d*e^3*f - 60*a^3*b^7*c^4*d^2*
e^3*f + 80*a^4*b^6*c^3*d^3*e^3*f - 60*a^5*b^5*c^2*d^4*e^3*f + 24*a^6*b^4*c
*d^5*e^3*f - 4*a^7*b^3*d^6*e^3*f + 6*a^2*b^8*c^6*e^2*f^2 - 36*a^3*b^7*c^5*
d*e^2*f^2 + 90*a^4*b^6*c^4*d^2*e^2*f^2 - 120*a^5*b^5*c^3*d^3*e^2*f^2 + 90*
a^6*b^4*c^2*d^4*e^2*f^2 - 36*a^7*b^3*c*d^5*e^2*f^2 + 6*a^8*b^2*d^6*e^2*f^2
 - 4*a^3*b^7*c^6*e*f^3 + 24*a^4*b^6*c^5*d*e*f^3 - 60*a^5*b^5*c^4*d^2*e*f^3
 + 80*a^6*b^4*c^3*d^3*e*f^3 - 60*a^7*b^3*c^2*d^4*e*f^3 + 24*a^8*b^2*c*d^5*
e*f^3 - 4*a^9*b*d^6*e*f^3 + a^4*b^6*c^6*f^4 - 6*a^5*b^5*c^5*d*f^4 + 15*a^6
*b^4*c^4*d^2*f^4 - 20*a^7*b^3*c^3*d^3*f^4 + 15*a^8*b^2*c^2*d^4*f^4 - 6*a^9
*b*c*d^5*f^4 + a^10*d^6*f^4)*sqrt(-b^2*e + a*b*f)) + 1/4*(80*b^2*d^7*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^4*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{g + hx}{(a + bx)^4 (c + dx)^3 (e + fx)^{3/2}} \, dx = \int \frac{hx + g}{(bx + a)^4 (dx + c)^3 (fx + e)^{\frac{3}{2}}} dx$$

input
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
int((h*x+g)/(b*x+a)^4/(d*x+c)^3/(f*x+e)^(3/2),x)
```

3.194 $\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}} dx$

Optimal result

Integrand size = 29, antiderivative size = 2754

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}} dx = \text{Too large to display}$$

output
```
-1/64*f*(48*a^6*d^5*f^5*(-c*f*h-4*d*e*h+5*d*f*g)+16*a^5*b*d^4*f^4*(23*c^2*
f^2*h-c*d*f*(-47*e*h+75*f*g)-5*d^2*e*(-4*e*h+3*f*g))-b^6*(960*d^6*e^6*g+c^
3*d^3*e^3*f^2*(-296*e*h+195*f*g)+3*c^4*d^2*e^2*f^3*(-56*e*h+45*f*g)+15*c^5
*d*e*f^4*(-8*e*h+7*f*g)-35*c^6*f^5*(-8*e*h+9*f*g)-80*c*d^5*e^5*(8*e*h+21*f
*g)+8*c^2*d^4*e^4*f*(148*e*h+45*f*g))+a^2*b^4*d*f*(225*c^5*f^5*h-3*c^3*d^2
*e*f^3*(-1289*e*h+905*f*g)+3*c^4*d*f^4*(-1097*e*h+1355*f*g)-8*d^5*e^4*(178
*e*h+795*f*g)+7*c*d^4*e^3*f*(904*e*h+1905*f*g)-5*c^2*d^3*e^2*f^2*(1861*e*h
+945*f*g))+a^4*b^2*d^3*f^3*(1083*c^3*f^3*h+c^2*d*f^2*(-7009*e*h+1920*f*g)-
d^3*e^2*(1723*e*h+480*f*g)+c*d^2*e*f*(4049*e*h+2160*f*g))-a*b^5*(35*c^6*f^
6*h-c^3*d^3*e^2*f^3*(-1661*e*h+1125*f*g)-3*c^4*d^2*e*f^4*(-323*e*h+265*f*g
)+15*c^5*d*f^5*(-103*e*h+119*f*g)-80*d^6*e^5*(4*e*h+51*f*g)+16*c*d^5*e^4*f
*(197*e*h+480*f*g)-c^2*d^4*e^3*f^2*(5392*e*h+2025*f*g))-a^3*b^3*d^2*f^2*(6
33*c^4*f^4*h+15*c^3*d*f^3*(-181*e*h+301*f*g)-d^4*e^3*(2384*e*h+4035*f*g)+c
*d^3*e^2*f*(6679*e*h+10185*f*g)-c^2*d^2*e*f^2*(7013*e*h+5865*f*g)))/(-a*d+
b*c)^6/(-a*f+b*e)^5/(-c*f+d*e)^3/(f*x+e)^(1/2)+1/192*d*(3*a^4*d^3*f^3*(-60
7*c*f*h+575*d*e*h+32*d*f*g)+b^4*(1440*d^4*e^4*g-c^3*d*e*f^2*(-328*e*h+399*
f*g)-2*c^2*d^2*e^2*f*(-128*e*h+195*f*g)-35*c^4*f^3*(-8*e*h+9*f*g)-240*c*d^
3*e^3*(4*e*h+f*g))+a*b^3*(35*c^4*f^4*h+3*c^3*d*f^3*(-459*e*h+553*f*g)+3*c^
2*d^2*e*f^2*(-454*e*h+659*f*g)-240*d^4*e^3*(2*e*h+23*f*g)+4*c*d^3*e^2*f*(8
92*e*h+375*f*g))-a^2*b^2*d*f*(211*c^3*f^3*h+3*c^2*d*f^2*(-813*e*h+1159*...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 16.57 (sec) , antiderivative size = 14143, normalized size of antiderivative = 5.14

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(g + h*x)/((a + b*x)^5*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 8.83 (sec) , antiderivative size = 2899, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.586, Rules used = {168, 27, 168, 27, 168, 27, 168, 27, 168, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}dx$$

$$\Big\downarrow \text{168}$$

$$-\frac{\int \frac{2b\left(6deg+\frac{9cfg}{2}-4ceh\right)-a(8dfg+4deh+cfh)+13df(bg-ah)x}{2(a+bx)^4(c+dx)^3(e+fx)^{3/2}}dx}{4(bc-ad)(be-af)}-\frac{bg-ah}{4(a+bx)^4(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{b(12deg+9cfg-8ceh)-a(8dfg+4deh+cfh)+13df(bg-ah)x}{(a+bx)^4(c+dx)^3(e+fx)^{3/2}}dx}{8(bc-ad)(be-af)}-\frac{bg-ah}{4(a+bx)^4(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{-\frac{\int \frac{df(48dfg+76deh+19cfh)a^2-b\left(4e(51fg+10eh)d^2+cf(123fg-88eh)d+7c^2f^2h\right)a+b^2\left(7f(9fg-8eh)c^2+16de(6fg-5eh)c+120d^2e^2g\right)+11df\left(13dfha^2-b(2\right.}{2(a+bx)^3(c+dx)^3(e+fx)^{3/2}}}{3(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{df(48dfg+76deh+19cfh)a^2-b\left(4e(51fg+10eh)d^2+cf(123fg-88eh)d+7c^2f^2h\right)a+b^2\left(7f(9fg-8eh)c^2+16de(6fg-5eh)c+120d^2e^2g\right)+11df\left(13dfha^2-b(2\right.}{(a+bx)^3(c+dx)^3(e+fx)^{3/2}}}{6(bc-ad)(be-af)}}{8(bc-ad)(be-af)}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$-\frac{143a^3d^2f^2h-3a^2bdf(10cfh+40deh+93dfg)+ab^2\left(7c^2f^2h+2cdf(111fg-88eh)+8d^2e(5eh+42fg)\right)-b^3\left(7c^2f(9fg-8eh)+16cde(6fg-5eh)+120d^2e^2g\right)}{2(a+bx)^2(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)} - \frac{\int \cdots}{\cdots}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\int \frac{3d^2f^2(64dfg+292deh+73cfh)a^3-bdf\left(236e(9fg+4eh)d^2+cf(963fg-304eh)d+134c^2f^2h\right)a^2+b^2\left(80e^2(33fg+4eh)d^3+8cef(249fg-169eh)d^2+2c^2f^2(483\cdots\right.}{\cdots}$$

$$\frac{bg-ah}{4(a+bx)^4(c+dx)^2\sqrt{e+fx}(bc-ad)(be-af)}$$

$\downarrow$ 168

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\cdots\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\cdots}$$

$\downarrow$ 27

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\cdots\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\cdots}$$

$\downarrow$ 168

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\cdots\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\cdots}$$

$\downarrow$ 27

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{}}}{}$$

$\downarrow$ 168

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{}}}{}$$

$\downarrow$ 27

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{}}}{}$$

$\downarrow$ 169

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2\sqrt{}}}{}$$

$\downarrow$ 27

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}$$

$$\Big\downarrow \; 174$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{bg-ah}{4(bc-ad)(be-af)(a+bx)^4(c+dx)^2\sqrt{e+fx}}-$$

$$-\frac{13dfha^2-b(21dfg+4deh+cfh)a+b^2(12deg+9cfg-8ceh)}{3(bc-ad)(be-af)(a+bx)^3(c+dx)^2\sqrt{e+fx}}-\frac{143d^2f^2ha^3-3bdf(93dfg+40deh+10cfh)a^2+b^2\left(8e(42fg+5eh)d^2+2cf(111fg-88eh)d+7\right.}{2(bc-ad)(be-af)(a+bx)^2(c+dx)^2}$$

input
```
Int[(g + h*x)/((a + b*x)^5*(c + d*x)^3*(e + f*x)^(3/2)),x]
```

output
```
-1/4*(b*g - a*h)/((b*c - a*d)*(b*e - a*f)*(a + b*x)^4*(c + d*x)^2*Sqrt[e +
 f*x]) - (-1/3*(13*a^2*d*f*h + b^2*(12*d*e*g + 9*c*f*g - 8*c*e*h) - a*b*(2
1*d*f*g + 4*d*e*h + c*f*h))/((b*c - a*d)*(b*e - a*f)*(a + b*x)^3*(c + d*x)
^2*Sqrt[e + f*x]) - ((143*a^3*d^2*f^2*h - 3*a^2*b*d*f*(93*d*f*g + 40*d*e*h
 + 10*c*f*h) - b^3*(120*d^2*e^2*g + 7*c^2*f*(9*f*g - 8*e*h) + 16*c*d*e*(6*
f*g - 5*e*h)) + a*b^2*(7*c^2*f^2*h + 2*c*d*f*(111*f*g - 88*e*h) + 8*d^2*e*
(42*f*g + 5*e*h)))/(2*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2*(c + d*x)^2*Sqrt
[e + f*x]) + ((1287*a^4*d^3*f^3*h - 3*a^3*b*d^2*f^2*(901*d*f*g + 652*d*e*h
 + 163*c*f*h) + b^4*(960*d^3*e^3*g + 4*c^2*d*e*f*(147*f*g - 124*e*h) + 40*
c*d^2*e^2*(21*f*g - 16*e*h) + 35*c^3*f^2*(9*f*g - 8*e*h)) - a*b^3*(35*c^3*
f^3*h + c^2*d*f^2*(1533*f*g - 1244*e*h) + 56*c*d^2*e*f*(51*f*g - 37*e*h) +
 40*d^3*e^2*(93*f*g + 8*e*h)) + a^2*b^2*d*f*(197*c^2*f^2*h + c*d*f*(2961*f
*g - 1888*e*h) + 4*d^2*e*(1287*f*g + 326*e*h)))/((b*c - a*d)*(b*e - a*f)*(
a + b*x)*(c + d*x)^2*Sqrt[e + f*x]) + ((2*d*(3*a^4*d^3*f^3*(32*d*f*g + 575
*d*e*h - 607*c*f*h) + b^4*(1440*d^4*e^4*g - c^3*d*e*f^2*(399*f*g - 328*e*h
) - 2*c^2*d^2*e^2*f*(195*f*g - 128*e*h) - 35*c^4*f^3*(9*f*g - 8*e*h) - 240
*c*d^3*e^3*(f*g + 4*e*h)) + a*b^3*(35*c^4*f^4*h + 3*c^3*d*f^3*(553*f*g - 4
59*e*h) + 3*c^2*d^2*e*f^2*(659*f*g - 454*e*h) - 240*d^4*e^3*(23*f*g + 2*e*
h) + 4*c*d^3*e^2*f*(375*f*g + 892*e*h)) - a^2*b^2*d*f*(211*c^3*f^3*h + 3*c
^2*d*f^2*(1159*f*g - 813*e*h) - 2*d^3*e^2*(3765*f*g + 968*e*h) + 3*c*d^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 4394, normalized size of antiderivative = 1.60

output too large to display

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
2*f^6*(d^5/(a*d-b*c)^7/f^6/(c*f-d*e)^3*(((3/8*a^2*c*d^3*f^2*h+1/2*a^2*d^4*
e*f*h-7/8*a^2*d^4*f^2*g-13/4*a*b*c^2*d^2*f^2*h+3/2*a*b*c*d^3*e*f*h+17/4*a*
b*c*d^3*f^2*g-5/2*a*b*d^4*e*f*g+23/8*b^2*c^3*d*f^2*h-2*b^2*c^2*d^2*e*f*h-2
7/8*b^2*c^2*d^2*f^2*g+5/2*b^2*c*d^3*e*f*g)*(f*x+e)^(3/2)+1/8*f*(5*a^2*c^2*
d^2*f^2*h-a^2*c*d^3*e*f*h-9*a^2*c*d^3*f^2*g-4*a^2*d^4*e^2*h+9*a^2*d^4*e*f*
g-30*a*b*c^3*d*f^2*h+42*a*b*c^2*d^2*e*f*h+38*a*b*c^2*d^2*f^2*g-12*a*b*c*d^
3*e^2*h-58*a*b*c*d^3*e*f*g+20*a*b*d^4*e^2*g+25*b^2*c^4*f^2*h-41*b^2*c^3*d*
e*f*h-29*b^2*c^3*d*f^2*g+16*b^2*c^2*d^2*e^2*h+49*b^2*c^2*d^2*e*f*g-20*b^2*
c*d^3*e^2*g)*(f*x+e)^(1/2))/((f*x+e)*d+c*f-d*e)^2+1/8*(3*a^2*c*d^2*f^2*h+1
2*a^2*d^3*e*f*h-15*a^2*d^3*f^2*g-26*a*b*c^2*d*f^2*h-44*a*b*c*d^2*e*f*h+90*
a*b*c*d^2*f^2*g+40*a*b*d^3*e^2*h-60*a*b*d^3*e*f*g+143*b^2*c^3*f^2*h-208*b^
2*c^2*d*e*f*h-195*b^2*c^2*d*f^2*g+80*b^2*c*d^2*e^2*h+300*b^2*c*d^2*e*f*g-1
20*b^2*d^3*e^2*g)/((c*f-d*e)*d)^(1/2)*arctan(d*(f*x+e)^(1/2)/((c*f-d*e)*d)
^(1/2)))-(-e*h+f*g)/(c*f-d*e)^3/(a*f-b*e)^5/(f*x+e)^(1/2)-b^3/(a*d-b*c)^7/
f^6/(a*f-b*e)^5*(((429/64*b^5*a^3*c^2*d^2*f^4*h-65/32*b^6*a^2*c^3*d*f^4*h-
5*b^8*c*d^3*e^3*f*g-429/32*b^4*a^4*c*d^3*f^4*h+131/16*a^3*b^5*c*d^3*e*f^3*
h+171/16*a^2*b^6*c^2*d^2*e*f^3*h+3/8*a^2*b^6*c*d^3*e^2*f^2*h-747/16*a^2*b^
6*c*d^3*e*f^3*g-103/16*a*b^7*c^3*d*e*f^3*h-93/8*a*b^7*c^2*d^2*e^2*f^2*h+33
9/16*a*b^7*c^2*d^2*e*f^3*g-a*b^7*c*d^3*e^3*f*h+51/2*a*b^7*c*d^3*e^2*f^2*g-
187/128*b^8*c^4*f^4*g+1025/32*a^3*b^5*c*d^3*f^4*g+465/16*a^3*b^5*d^4*e*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)**5/(d*x+c)**3/(f*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7241 vs. 2(2702) = 5404.

Time = 1.54 (sec) , antiderivative size = 7241, normalized size of antiderivative = 2.63

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
1/64*(1920*b^8*d^4*e^4*g + 1920*b^8*c*d^3*e^3*f*g - 9600*a*b^7*d^4*e^3*f*g
 + 1440*b^8*c^2*d^2*e^2*f^2*g - 8640*a*b^7*c*d^3*e^2*f^2*g + 18720*a^2*b^6
*d^4*e^2*f^2*g + 840*b^8*c^3*d*e*f^3*g - 5400*a*b^7*c^2*d^2*e*f^3*g + 1404
0*a^2*b^6*c*d^3*e*f^3*g - 17160*a^3*b^5*d^4*e*f^3*g + 315*b^8*c^4*f^4*g -
2100*a*b^7*c^3*d*f^4*g + 5850*a^2*b^6*c^2*d^2*f^4*g - 8580*a^3*b^5*c*d^3*f
^4*g + 6435*a^4*b^4*d^4*f^4*g - 1280*b^8*c*d^3*e^4*h - 640*a*b^7*d^4*e^4*h
 - 1152*b^8*c^2*d^2*e^3*f*h + 5504*a*b^7*c*d^3*e^3*f*h + 3328*a^2*b^6*d^4*
e^3*f*h - 720*b^8*c^3*d*e^2*f^2*h + 4176*a*b^7*c^2*d^2*e^2*f^2*h - 8112*a^
2*b^6*c*d^3*e^2*f^2*h - 6864*a^3*b^5*d^4*e^2*f^2*h - 280*b^8*c^4*e*f^3*h +
 1720*a*b^7*c^3*d*e*f^3*h - 4056*a^2*b^6*c^2*d^2*e*f^3*h + 3432*a^3*b^5*c*
d^3*e*f^3*h + 6864*a^4*b^4*d^4*e*f^3*h - 35*a*b^7*c^4*f^4*h + 260*a^2*b^6*
c^3*d*f^4*h - 858*a^3*b^5*c^2*d^2*f^4*h + 1716*a^4*b^4*c*d^3*f^4*h - 3003*
a^5*b^3*d^4*f^4*h)*arctan(sqrt(f*x + e)*b/sqrt(-b^2*e + a*b*f))/((b^12*c^7
*e^5 - 7*a*b^11*c^6*d*e^5 + 21*a^2*b^10*c^5*d^2*e^5 - 35*a^3*b^9*c^4*d^3*e
^5 + 35*a^4*b^8*c^3*d^4*e^5 - 21*a^5*b^7*c^2*d^5*e^5 + 7*a^6*b^6*c*d^6*e^5
 - a^7*b^5*d^7*e^5 - 5*a*b^11*c^7*e^4*f + 35*a^2*b^10*c^6*d*e^4*f - 105*a^
3*b^9*c^5*d^2*e^4*f + 175*a^4*b^8*c^4*d^3*e^4*f - 175*a^5*b^7*c^3*d^4*e^4*
f + 105*a^6*b^6*c^2*d^5*e^4*f - 35*a^7*b^5*c*d^6*e^4*f + 5*a^8*b^4*d^7*e^4
*f + 10*a^2*b^10*c^7*e^3*f^2 - 70*a^3*b^9*c^6*d*e^3*f^2 + 210*a^4*b^8*c^5*
d^2*e^3*f^2 - 350*a^5*b^7*c^4*d^3*e^3*f^2 + 350*a^6*b^6*c^3*d^4*e^3*f^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx=\text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(3/2)*(a + b*x)^5*(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{g+hx}{(a+bx)^5(c+dx)^3(e+fx)^{3/2}}\,dx=\int \frac{hx+g}{(bx+a)^5\,(dx+c)^3\,(fx+e)^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(3/2),x)
```

output
```
int((h*x+g)/(b*x+a)^5/(d*x+c)^3/(f*x+e)^(3/2),x)
```

3.195 $\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 395

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = -\frac{(12adfh-b(8dfg+deh+3cfh))\sqrt{c+dx}\sqrt{e+fx}}{4b^3f} + \frac{(2bdg+bch-3adh)(c+dx)^{3/2}\sqrt{e+fx}}{2b^2(bc-ad)} - \frac{(bg-ah)(c+dx)^{5/2}\sqrt{e+fx}}{b(bc-ad)(a+bx)} + \frac{(24a^2d^2f^2h-8abdf(2dfg+deh+3cfh)+b^2(3c^2f^2h+d^2e(4fg-eh)+6cdf(2fg+eh)))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{}}{\sqrt{d}\sqrt{}}\right.}{4b^4\sqrt{d}f^{3/2}} - \frac{\sqrt{bc-ad}(6a^2dfh+b^2(3deg+cfg+2ceh)-ab(4dfg+5deh+3cfh))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4\sqrt{be-af}}$$

output

```
-1/4*(12*a*d*f*h-b*(3*c*f*h+d*e*h+8*d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^
3/f+1/2*(-3*a*d*h+b*c*h+2*b*d*g)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c
)-(-a*h+b*g)*(d*x+c)^(5/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(b*x+a)+1/4*(24*a^2*
d^2*f^2*h-8*a*b*d*f*(3*c*f*h+d*e*h+2*d*f*g)+b^2*(3*c^2*f^2*h+d^2*e*(-e*h+4
*f*g)+6*c*d*f*(e*h+2*f*g)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^
(1/2))/b^4/d^(1/2)/f^(3/2)-(-a*d+b*c)^(1/2)*(6*a^2*d*f*h+b^2*(2*c*e*h+c*f*
g+3*d*e*g)-a*b*(3*c*f*h+5*d*e*h+4*d*f*g))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)
^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(-12a^2dfh+ab(9cfh+d(8fg+eh-6fhx))+b^2(cf(-4g+5hx)+dx(4fg+eh+2f}{f(a+bx)}}{}$$

input
```
Integrate[((c + d*x)^(3/2)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^2,x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(-12*a^2*d*f*h + a*b*(9*c*f*h + d*(8*f*g +
 e*h - 6*f*h*x)) + b^2*(c*f*(-4*g + 5*h*x) + d*x*(4*f*g + e*h + 2*f*h*x)))
)/(f*(a + b*x)) + (4*Sqrt[b*c - a*d]*(6*a^2*d*f*h + b^2*(3*d*e*g + c*f*g +
 2*c*e*h) - a*b*(4*d*f*g + 5*d*e*h + 3*c*f*h))*ArcTan[(Sqrt[b*c - a*d]*Sqr
t[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])]])/Sqrt[-(b*e) + a*f] + ((24
*a^2*d^2*f^2*h - 8*a*b*d*f*(2*d*f*g + d*e*h + 3*c*f*h) + b^2*(3*c^2*f^2*h
+ d^2*e*(4*f*g - e*h) + 6*c*d*f*(2*f*g + e*h)))*ArcTanh[(Sqrt[d]*Sqrt[e +
f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(Sqrt[d]*f^(3/2)))/(4*b^4)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {166, 27, 171, 25, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \ 166$$

$$\frac{\int -\frac{\sqrt{c+dx}\sqrt{e+fx}(3a(de+cf)h-b(3deg+cfg+2ceh)-2d(2bfg+beh-3afh)x)}{2(a+bx)}dx}{b(be-af)} - \frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}(3a(de+cf)h-b(3deg+cfg+2ceh)-2d(2bfg+beh-3afh)x)}{a+bx}dx}{2b(be-af)}-$$
$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 171

$$-\frac{\frac{\int -\frac{\sqrt{e+fx}\left(ad(de+3cf)(3afh-b(2fg+eh))-2bcf(3a(de+cf)h-b(3deg+cfg+2ceh))+d\left((de(4fg-eh)+cf(4fg+5eh))b^2-af(8dfg+7deh+9cfh)b+12a^2df^2h\right)\cdots\right.}{(a+bx)\sqrt{c+dx}}}{2bf}\cdots}{2b(be-af)}\cdots$$
$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 25

$$-\frac{-\frac{\int \frac{\sqrt{e+fx}\left(ad(de+3cf)(3afh-b(2fg+eh))-2bcf(3a(de+cf)h-b(3deg+cfg+2ceh))+d\left((de(4fg-eh)+cf(4fg+5eh))b^2-af(8dfg+7deh+9cfh)b+12a^2df^2h\right)\cdots\right.}{(a+bx)\sqrt{c+dx}}}{2bf}\cdots}{2b(be-af)}\cdots$$
$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 171

$$-\frac{-\frac{\frac{\int \frac{d(be-af)\left(12df(de+cf)ha^2-b\left(e(8fg+eh)d^2+2cf(4fg+11eh)d+9c^2f^2h\right)a+4b^2cf(3deg+cfg+2ceh)+\left(\left(e(4fg-eh)d^2+6cf(2fg+eh)d+3c^2f^2h\right)b^2-8adf\cdots\right.\right.}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{bd}\cdots}{2bf}\cdots}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{(be-af)\int \frac{12df(de+cf)ha^2-b\left(e(8fg+eh)d^2+2cf(4fg+11eh)d+9c^2f^2h\right)a+4b^2cf(3deg+cfg+2ceh)+\left(\left(e(4fg-eh)d^2+6cf(2fg+eh)d+3c^2f^2h\right)b^2-8adf(2\cdots\right.}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2b}\cdots}{2bf}\cdots}{2\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 175

$$-\frac{-\frac{(be-af)\left(\frac{\left(24a^2d^2f^2h-8abdf(3cfh+deh+2dfg)+b^2\left(3c^2f^2h+6cdf(eh+2fg)+d^2e(4fg-eh)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}+\frac{4f(bc-ad)\left(6a^2dfh-ab(3cfh+5deh+4\cdots}{\cdots}\right.}{2b}\cdots}{2bf\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\big\downarrow$ 66

$$-\frac{-\frac{(be-af)\left(\frac{2\left(24a^2d^2f^2h-8abdf(3cfh+deh+2dfg)+b^2\left(3c^2f^2h+6cdf(eh+2fg)+d^2e(4fg-eh)\right)\right)\int\frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}+\frac{4f(bc-ad)\left(6a^2dfh-ab(3cfh+5\cdots}{\cdots}\right.}{2b}\cdots}{2bf\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\big\downarrow$ 104

$$-\frac{-\frac{(be-af)\left(\frac{2\left(24a^2d^2f^2h-8abdf(3cfh+deh+2dfg)+b^2\left(3c^2f^2h+6cdf(eh+2fg)+d^2e(4fg-eh)\right)\right)\int\frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}+\frac{8f(bc-ad)\left(6a^2dfh-ab(3cfh+5\cdots}{\cdots}\right.}{2b}\cdots}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\big\downarrow$ 221

$$-\frac{-\frac{(be-af)\left(\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(24a^2d^2f^2h-8abdf(3cfh+deh+2dfg)+b^2\left(3c^2f^2h+6cdf(eh+2fg)+d^2e(4fg-eh)\right)\right)}{b\sqrt{d}\sqrt{f}}-\frac{8f\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{e+fx}}\cdots\right.}{\cdots}\right.}{2b}\cdots}{2b\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}(bg-ah)}{b(a+bx)(be-af)}$$

input
```
Int[((c + d*x)^(3/2)*Sqrt[e + f*x]*(g + h*x))/(a + b*x)^2,x]
```

output
```
-(((b*g - a*h)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x)))
 - (-((d*(2*b*f*g + b*e*h - 3*a*f*h)*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b*f))
 - (((12*a^2*d*f^2*h - a*b*f*(8*d*f*g + 7*d*e*h + 9*c*f*h) + b^2*(d*e*(4*f
*g - e*h) + c*f*(4*f*g + 5*e*h)))*Sqrt[c + d*x]*Sqrt[e + f*x])/b + ((b*e -
 a*f)*((2*(24*a^2*d^2*f^2*h - 8*a*b*d*f*(2*d*f*g + d*e*h + 3*c*f*h) + b^2*
(3*c^2*f^2*h + d^2*e*(4*f*g - e*h) + 6*c*d*f*(2*f*g + e*h)))*ArcTanh[(Sqrt
[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) - (8*Sqrt
[b*c - a*d]*f*(6*a^2*d*f*h + b^2*(3*d*e*g + c*f*g + 2*c*e*h) - a*b*(4*d*f*
g + 5*d*e*h + 3*c*f*h))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c
- a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*e - a*f])))/(2*b))/(2*b*f))/(2*b*(b*e -
a*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4510 vs. $2(353) = 706$.

Time = 0.44 (sec) , antiderivative size = 4511, normalized size of antiderivative = 11.42

method	result	size
default	Expression too large to display	4511

input
```
int((d*x+c)^(3/2)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-8*b^4*c*f*g*((a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/b^2)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-4*ln((-2*a*d*f*x+b*c
*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c^2*f^2*g*x*(d*f)^(1/2)+3*l
n(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b^4*c^2*f^2*h*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-ln(1/2*(2*d
*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^2*e
^2*h*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-16*ln((-2*a*d*f*x+b*c
*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^2*f^2*g*(d*f)^(1/2)+12*
ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c^2*f
^2*h*(d*f)^(1/2)-4*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2
)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*a*b^3*c^2*f^2*g*(d*f)^(1/2)+24*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d^2*f^2*h*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)+28*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d
*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2
*b*c*e)/(b*x+a))*a*b^3*c*d*e*f*h*x*(d*f)^(1/2)-16*ln(1/2*(2*d*f*x+2*((f*x+
e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d^2*f^2*g*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(3/2)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx=\int \frac{(c+dx)^{\frac{3}{2}}\sqrt{e+fx}\,(g+hx)}{(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(f*x+e)**(1/2)*(h*x+g)/(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sqrt(e + f*x)*(g + h*x)/(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(353) = 706$.

Time = 1.30 (sec) , antiderivative size = 1652, normalized size of antiderivative = 4.18

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x, algorithm="giac
")
```

output
```
1/4*sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*h*abs(d
)/(b^2*d^2) + (4*b^7*d^2*f^2*g*abs(d) + b^7*d^2*e*f*h*abs(d) + 3*b^7*c*d*f
^2*h*abs(d) - 8*a*b^6*d^2*f^2*h*abs(d))/(b^9*d^3*f^2)) - (3*sqrt(d*f)*b^3*
c*d*e*g*abs(d) - 3*sqrt(d*f)*a*b^2*d^2*e*g*abs(d) + sqrt(d*f)*b^3*c^2*f*g*
abs(d) - 5*sqrt(d*f)*a*b^2*c*d*f*g*abs(d) + 4*sqrt(d*f)*a^2*b*d^2*f*g*abs(
d) + 2*sqrt(d*f)*b^3*c^2*e*h*abs(d) - 7*sqrt(d*f)*a*b^2*c*d*e*h*abs(d) + 5
*sqrt(d*f)*a^2*b*d^2*e*h*abs(d) - 3*sqrt(d*f)*a*b^2*c^2*f*h*abs(d) + 9*sqr
t(d*f)*a^2*b*c*d*f*h*abs(d) - 6*sqrt(d*f)*a^3*d^2*f*h*abs(d))*arctan(-1/2*
(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2
 - a^2*d^2*f^2)*d))/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d
^2*f^2)*b^4*d) - 2*(sqrt(d*f)*b^3*c*d^3*e^2*g*abs(d) - sqrt(d*f)*a*b^2*d^4
*e^2*g*abs(d) - 2*sqrt(d*f)*b^3*c^2*d^2*e*f*g*abs(d) + 2*sqrt(d*f)*a*b^2*c
*d^3*e*f*g*abs(d) + sqrt(d*f)*b^3*c^3*d*f^2*g*abs(d) - sqrt(d*f)*a*b^2*c^2
*d^2*f^2*g*abs(d) - sqrt(d*f)*a*b^2*c*d^3*e^2*h*abs(d) + sqrt(d*f)*a^2*b*d
^4*e^2*h*abs(d) + 2*sqrt(d*f)*a*b^2*c^2*d^2*e*f*h*abs(d) - 2*sqrt(d*f)*a^2
*b*c*d^3*e*f*h*abs(d) - sqrt(d*f)*a*b^2*c^3*d*f^2*h*abs(d) + sqrt(d*f)*a^2
*b*c^2*d^2*f^2*h*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*b^3*c*d*e*g*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*a*b^2*d^2*e*g*abs(d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \int \frac{\sqrt{e+f\,x}\,(g+h\,x)\,{(c+d\,x)}^{3/2}}{{(a+b\,x)}^2}\,dx$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^(3/2))/(a + b*x)^2,x)
```

output
```
int(((e + f*x)^(1/2)*(g + h*x)*(c + d*x)^(3/2))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}\sqrt{e+fx}(g+hx)}{(a+bx)^2}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\,(hx+g)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^(3/2)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x)
```

output
```
int((d*x+c)^(3/2)*(f*x+e)^(1/2)*(h*x+g)/(b*x+a)^2,x)
```

3.196 $\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 396

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2} \, dx = -\frac{(12adfh-b(8dfg+3deh+cfh))\sqrt{c+dx}\sqrt{e+fx}}{4b^3d} + \frac{(2bfg+beh-3afh)\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(be-af)} - \frac{(bg-ah)\sqrt{c+dx}(e+fx)^{5/2}}{b(be-af)(a+bx)} + \frac{(24a^2d^2f^2h-8abdf(2dfg+3deh+cfh)-b^2(c^2f^2h-3d^2e(4fg+eh)-2cdf(2fg+3eh)))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4b^4d^{3/2}\sqrt{f}} - \frac{\sqrt{be-af}(6a^2dfh+b^2(deg+3cfg+2ceh)-ab(4dfg+3deh+5cfh))\operatorname{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4\sqrt{bc-ad}}$$

output
```
-1/4*(12*a*d*f*h-b*(c*f*h+3*d*e*h+8*d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^
3/d+1/2*(-3*a*f*h+b*e*h+2*b*f*g)*(d*x+c)^(1/2)*(f*x+e)^(3/2)/b^2/(-a*f+b*e
)-(-a*h+b*g)*(d*x+c)^(1/2)*(f*x+e)^(5/2)/b/(-a*f+b*e)/(b*x+a)+1/4*(24*a^2*
d^2*f^2*h-8*a*b*d*f*(c*f*h+3*d*e*h+2*d*f*g)-b^2*(c^2*f^2*h-3*d^2*e*(e*h+4*
f*g)-2*c*d*f*(3*e*h+2*f*g)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)
^(1/2))/b^4/d^(3/2)/f^(1/2)-(-a*f+b*e)^(1/2)*(6*a^2*d*f*h+b^2*(2*c*e*h+3*c
*f*g+d*e*g)-a*b*(5*c*f*h+3*d*e*h+4*d*f*g))*arctanh((-a*f+b*e)^(1/2)*(d*x+c
)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(-12a^2dfh+b^2(cfhx+2dfx(2g+hx)+de(-4g+5hx))+ab(cfh+d(8fg+9eh-6}{d(a+bx)}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^2,x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(-12*a^2*d*f*h + b^2*(c*f*h*x + 2*d*f*x*(2
*g + h*x) + d*e*(-4*g + 5*h*x)) + a*b*(c*f*h + d*(8*f*g + 9*e*h - 6*f*h*x)
)))/(d*(a + b*x)) + (4*Sqrt[b*e - a*f]*(6*a^2*d*f*h + b^2*(d*e*g + 3*c*f*g
 + 2*c*e*h) - a*b*(4*d*f*g + 3*d*e*h + 5*c*f*h))*ArcTan[(Sqrt[b*e - a*f]*S
qrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*c) + a*d] + ((
24*a^2*d^2*f^2*h - 8*a*b*d*f*(2*d*f*g + 3*d*e*h + c*f*h) + b^2*(-(c^2*f^2*
h) + 3*d^2*e*(4*f*g + e*h) + 2*c*d*f*(2*f*g + 3*e*h)))*ArcTanh[(Sqrt[f]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(d^(3/2)*Sqrt[f]))/(4*b^4)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {166, 27, 171, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{166}$$

$$\frac{\int -\frac{(e+fx)^{3/2}(a(de+5cf)h-b(deg+3cfg+2ceh)-2d(2bfg+beh-3afh)x)}{2(a+bx)\sqrt{c+dx}}dx}{b(be-af)} - \frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(e+fx)^{3/2}(a(de+5cf)h-b(deg+3cfg+2ceh)-2d(2bfg+beh-3afh)x)}{(a+bx)\sqrt{c+dx}}dx}{2b(be-af)}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{ 171}$$

$$-\frac{\frac{\int \frac{d(be-af)\sqrt{e+fx}(3a(de+3cf)h-2b(deg+3cfg+2ceh)+(12adfh-b(8dfg+3deh+cfh))x)}{(a+bx)\sqrt{c+dx}}dx}{2bd}-\frac{\sqrt{c+dx}(e+fx)^{3/2}(-3afh+beh+2bfg)}{b}}{2b(be-af)}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{(be-af)\int \frac{\sqrt{e+fx}(3a(de+3cf)h-2b(deg+3cfg+2ceh)+(12adfh-b(8dfg+3deh+cfh))x)}{(a+bx)\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}(e+fx)^{3/2}(-3afh+beh+2bfg)}{b}}{2b(be-af)}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{ 171}$$

$$-\frac{\frac{(be-af)\left(\frac{\int \frac{2bde(3a(de+3cf)h-2b(deg+3cfg+2ceh))-a(de+cf)(12adfh-b(8dfg+3deh+cfh))-\left(-\left(\left(-3e(4fg+eh)d^2-2cf(2fg+3eh)d+c^2f^2h\right)b^2\right)-8adf(2\cdots}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{bd}\cdots\right.}{2b}\cdots}{2b(be-af)}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{(be-af)\left(\frac{\int \frac{2bde(3a(de+3cf)h-2b(deg+3cfg+2ceh))-a(de+cf)(12adfh-b(8dfg+3deh+cfh))-\left(-\left(\left(-3e(4fg+eh)d^2-2cf(2fg+3eh)d+c^2f^2h\right)b^2\right)-8adf(2\cdots}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2bd}\cdots\right.}{2b}\cdots}{2b(be-af)}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$$\Big\downarrow \text{ 175}$$

$$-\frac{(be-af)\left(-\frac{\frac{\left(24a^2d^2f^2h-8abdf(cfh+3deh+2dfg)-\left(b^2\left(c^2f^2h-2cdf(3eh+2fg)-3d^2e(eh+4fg)\right)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}}{2bd}-\frac{4d(be-af)\left(6a^2dfh-ab(5cfh+3de\ldots\right.}{\ldots}\right.}{2b}\ldots}{2b(be\ldots}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 66

$$-\frac{(be-af)\left(-\frac{\frac{2\left(24a^2d^2f^2h-8abdf(cfh+3deh+2dfg)-\left(b^2\left(c^2f^2h-2cdf(3eh+2fg)-3d^2e(eh+4fg)\right)\right)\right)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}}{2bd}-\frac{4d(be-af)\left(6a^2dfh-ab(5cf\ldots\right.}{\ldots}\right.}{2b}\ldots}{2b(b\ldots}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 104

$$-\frac{(be-af)\left(-\frac{\frac{2\left(24a^2d^2f^2h-8abdf(cfh+3deh+2dfg)-\left(b^2\left(c^2f^2h-2cdf(3eh+2fg)-3d^2e(eh+4fg)\right)\right)\right)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}}{2bd}-\frac{8d(be-af)\left(6a^2dfh-ab(5cf\ldots\right.}{\ldots}\right.}{2b}\ldots}{\ldots}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

$\downarrow$ 221

$$-\frac{(be-af)\left(\frac{\frac{8d\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(6a^2dfh-ab(5cfh+3deh+4dfg)+b^2(2ceh+3cfg+deg)\right)}{b\sqrt{bc-ad}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(24a^2d^2f^2h-8abdf\ldots\right.}{\ldots}}{2bd}\ldots\right.}{2b}\ldots}{2b(\ldots}-\frac{\sqrt{c+dx}(e+fx)^{5/2}(bg-ah)}{b(a+bx)(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*(e + f*x)^(3/2)*(g + h*x))/(a + b*x)^2,x]
```

output
```
-(((b*g - a*h)*Sqrt[c + d*x]*(e + f*x)^(5/2))/(b*(b*e - a*f)*(a + b*x))) -
 (-(((2*b*f*g + b*e*h - 3*a*f*h)*Sqrt[c + d*x]*(e + f*x)^(3/2))/b) + ((b*e
 - a*f)*(((12*a*d*f*h - b*(8*d*f*g + 3*d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[
e + f*x])/(b*d) + ((-2*(24*a^2*d^2*f^2*h - 8*a*b*d*f*(2*d*f*g + 3*d*e*h +
c*f*h) - b^2*(c^2*f^2*h - 3*d^2*e*(4*f*g + e*h) - 2*c*d*f*(2*f*g + 3*e*h))
)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqr
t[f]) + (8*d*Sqrt[b*e - a*f]*(6*a^2*d*f*h + b^2*(d*e*g + 3*c*f*g + 2*c*e*h
) - a*b*(4*d*f*g + 3*d*e*h + 5*c*f*h))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d
*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]))/(2*b*d)))/(2*b
))/(2*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4510 vs. $2(354) = 708$.

Time = 0.42 (sec) , antiderivative size = 4511, normalized size of antiderivative = 11.39

method	result	size
default	Expression too large to display	4511

input
```
int((d*x+c)^(1/2)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^2*f^2*h*x*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)+3*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^2*e^2*h*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)-16*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2
)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*a^3*b*d^2*f^2*g*(d*f)^(1/2)+24*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d^2*f^2*h*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)-8*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*
x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*
b*c*e)/(b*x+a))*a*b^3*c*d*e^2*h*(d*f)^(1/2)+12*ln((-2*a*d*f*x+b*c*f*x+b*d*
e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d^2*e^2*h*x*(d*f)^(1/2)-8*ln((-2*a
*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d*e^2*h*x*(d*f)
^(1/2)+28*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b
^3*c*d*e*f*h*x*(d*f)^(1/2)-16*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d^2*f^2*g*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)-ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2} \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)^{\frac{3}{2}}\,(g+hx)}{(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(3/2)*(h*x+g)/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)**(3/2)*(g + h*x)/(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1657 vs. $2(354) = 708$.

Time = 1.34 (sec) , antiderivative size = 1657, normalized size of antiderivative = 4.18

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x, algorithm="giac
")
```

output
```
1/4*sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*f*h*abs
(d)/(b^2*d^3) + (4*b^7*d^4*f^3*g*abs(d) + 5*b^7*d^4*e*f^2*h*abs(d) - b^7*c
*d^3*f^3*h*abs(d) - 8*a*b^6*d^4*f^3*h*abs(d))/(b^9*d^6*f^2)) - (sqrt(d*f)*
b^3*d*e^2*g*abs(d) + 3*sqrt(d*f)*b^3*c*e*f*g*abs(d) - 5*sqrt(d*f)*a*b^2*d*
e*f*g*abs(d) - 3*sqrt(d*f)*a*b^2*c*f^2*g*abs(d) + 4*sqrt(d*f)*a^2*b*d*f^2*
g*abs(d) + 2*sqrt(d*f)*b^3*c*e^2*h*abs(d) - 3*sqrt(d*f)*a*b^2*d*e^2*h*abs(
d) - 7*sqrt(d*f)*a*b^2*c*e*f*h*abs(d) + 9*sqrt(d*f)*a^2*b*d*e*f*h*abs(d) +
 5*sqrt(d*f)*a^2*b*c*f^2*h*abs(d) - 6*sqrt(d*f)*a^3*d*f^2*h*abs(d))*arctan
(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c
*d*f^2 - a^2*d^2*f^2)*d))/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 -
 a^2*d^2*f^2)*b^4*d) - 2*(sqrt(d*f)*b^3*d^3*e^3*g*abs(d) - 2*sqrt(d*f)*b^3
*c*d^2*e^2*f*g*abs(d) - sqrt(d*f)*a*b^2*d^3*e^2*f*g*abs(d) + sqrt(d*f)*b^3
*c^2*d*e*f^2*g*abs(d) + 2*sqrt(d*f)*a*b^2*c*d^2*e*f^2*g*abs(d) - sqrt(d*f)
*a*b^2*c^2*d*f^3*g*abs(d) - sqrt(d*f)*a*b^2*d^3*e^3*h*abs(d) + 2*sqrt(d*f)
*a*b^2*c*d^2*e^2*f*h*abs(d) + sqrt(d*f)*a^2*b*d^3*e^2*f*h*abs(d) - sqrt(d*
f)*a*b^2*c^2*d*e*f^2*h*abs(d) - 2*sqrt(d*f)*a^2*b*c*d^2*e*f^2*h*abs(d) + s
qrt(d*f)*a^2*b*c^2*d*f^3*h*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b^3*d*e^2*g*abs(d) - sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b^3*c*e*f*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \int \frac{(e+f\,x)^{3/2}\,(g+h\,x)\,\sqrt{c+d\,x}}{(a+b\,x)^2}\,dx$$

input
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^2,x)
```

output
```
int(((e + f*x)^(3/2)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(e+fx)^{3/2}(g+hx)}{(a+bx)^2}\,dx = \int \frac{\sqrt{dx+c}\,(fx+e)^{\frac{3}{2}}\,(hx+g)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(3/2)*(h*x+g)/(b*x+a)^2,x)
```

3.197 $\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 479

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx =$$

$$-\frac{(bc-ad)^2\left(3a^2d^2fh-6abd(dfg+deh-cfh)+b^2(16d^2eg+7c^2fh-10cd(fg+eh))\right)\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^4}$$

$$+\frac{(bc-ad)\left(3a^2d^2fh-6abd(dfg+deh-cfh)+b^2(16d^2eg+7c^2fh-10cd(fg+eh))\right)(a+bx)^{3/2}\sqrt{c+d}}{192b^3d^3}$$

$$+\frac{\left(3a^2d^2fh-6abd(dfg+deh-cfh)+b^2(16d^2eg+7c^2fh-10cd(fg+eh))\right)(a+bx)^{5/2}\sqrt{c+dx}}{48b^3d^2}$$

$$-\frac{(7bcfh+13adfh-10bd(fg+eh))(a+bx)^{5/2}(c+dx)^{3/2}}{40b^2d^2}+\frac{fh(a+bx)^{7/2}(c+dx)^{3/2}}{5b^2d}$$

$$+\frac{(bc-ad)^3\left(3a^2d^2fh-6abd(dfg+deh-cfh)+b^2(16d^2eg+7c^2fh-10cd(fg+eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right.}{128b^{7/2}d^{9/2}}$$

output
```
-1/128*(-a*d+b*c)^2*(3*a^2*d^2*f*h-6*a*b*d*(-c*f*h+d*e*h+d*f*g)+b^2*(16*d^
2*e*g+7*c^2*f*h-10*c*d*(e*h+f*g)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d^4+1/1
92*(-a*d+b*c)*(3*a^2*d^2*f*h-6*a*b*d*(-c*f*h+d*e*h+d*f*g)+b^2*(16*d^2*e*g+
7*c^2*f*h-10*c*d*(e*h+f*g)))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d^3+1/48*(3*a
^2*d^2*f*h-6*a*b*d*(-c*f*h+d*e*h+d*f*g)+b^2*(16*d^2*e*g+7*c^2*f*h-10*c*d*(
e*h+f*g)))*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3/d^2-1/40*(7*b*c*f*h+13*a*d*f*h-
10*b*d*(e*h+f*g))*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b^2/d^2+1/5*f*h*(b*x+a)^(7/2
)*(d*x+c)^(3/2)/b^2/d+1/128*(-a*d+b*c)^3*(3*a^2*d^2*f*h-6*a*b*d*(-c*f*h+d*
e*h+d*f*g)+b^2*(16*d^2*e*g+7*c^2*f*h-10*c*d*(e*h+f*g)))*arctanh(d^(1/2)*(b
*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.96

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}(45a^4d^4fh-30a^3bd^3(cfh+d(3fg+3eh+fhx))+2ab^3d(95c^3fh-c^2d(155f\cdots}{\cdots} + \frac{(bc-ad)^3\left(3a^2d^2fh-6abd(dfg+deh-cfh)+b^2(16d^2eg+7c^2fh-10cd(fg+eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{128b^{7/2}d^{9/2}}$$

input
```
Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x]*(e + f*x)*(g + h*x),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(45*a^4*d^4*f*h - 30*a^3*b*d^3*(c*f*h + d*(3*
f*g + 3*e*h + f*h*x)) + 2*a*b^3*d*(95*c^3*f*h - c^2*d*(155*f*g + 155*e*h +
 61*f*h*x) + 4*c*d^2*(80*e*g + 25*f*g*x + 25*e*h*x + 12*f*h*x^2) + 8*d^3*x
*(70*e*g + 45*f*g*x + 45*e*h*x + 33*f*h*x^2)) + 6*a^2*b^2*d^2*(-6*c^2*f*h
+ 3*c*d*(5*f*g + 5*e*h + f*h*x) + 2*d^2*(5*e*(4*g + h*x) + f*x*(5*g + 2*h*
x))) + b^4*(-105*c^4*f*h + 10*c^3*d*(15*f*g + 15*e*h + 7*f*h*x) + 16*c*d^3
*x*(5*e*(2*g + h*x) + f*x*(5*g + 3*h*x)) + 32*d^4*x^2*(5*e*(4*g + 3*h*x) +
 3*f*x*(5*g + 4*h*x)) - 4*c^2*d^2*(5*e*(12*g + 5*h*x) + f*x*(25*g + 14*h*x
)))))/(1920*b^3*d^4) + ((b*c - a*d)^3*(3*a^2*d^2*f*h - 6*a*b*d*(d*f*g + d*
e*h - c*f*h) + b^2*(16*d^2*e*g + 7*c^2*f*h - 10*c*d*(f*g + e*h)))*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(7/2)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.59, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {164, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx$$

$$\Big\downarrow \text{164}$$

$$\frac{\left(3a^2d^2fh-6abd(-cfh+deh+dfg)+b^2\left(7c^2fh-10cd(eh+fg)+16d^2eg\right)\right)\int (a+bx)^{3/2}\sqrt{c+dx}dx}{16b^2d^2}-\frac{(a+bx)^{5/2}(c+dx)^{3/2}(5adfh+7bcfh-10bd(eh+fg)-8bdfhx)}{40b^2d^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(3a^2d^2fh-6abd(-cfh+deh+dfg)+b^2\left(7c^2fh-10cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{6b}+\frac{(a+bx)^{5/2}\sqrt{\ldots}}{3b}\ldots\right.}{16b^2d^2}$$
$$\frac{(a+bx)^{5/2}(c+dx)^{3/2}(5adfh+7bcfh-10bd(eh+fg)-8bdfhx)}{40b^2d^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(3a^2d^2fh-6abd(-cfh+deh+dfg)+b^2\left(7c^2fh-10cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\int \ldots}{4\ldots}\right.}{6b}\ldots\right.}{16b^2d^2}$$
$$\frac{(a+bx)^{5/2}(c+dx)^{3/2}(5adfh+7bcfh-10bd(eh+fg)-8bdfhx)}{40b^2d^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(3a^2d^2fh-6abd(-cfh+deh+dfg)+b^2\left(7c^2fh-10cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{16b^2d^2}$$

$$\frac{(a+bx)^{5/2}(c+dx)^{3/2}(5adfh+7bcfh-10bd(eh+fg)-8bdfhx)}{40b^2d^2}$$

$\Big\downarrow$ 66

$$\frac{\left(3a^2d^2fh-6abd(-cfh+deh+dfg)+b^2\left(7c^2fh-10cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{16b^2d^2}$$

$$\frac{(a+bx)^{5/2}(c+dx)^{3/2}(5adfh+7bcfh-10bd(eh+fg)-8bdfhx)}{40b^2d^2}$$

$\Big\downarrow$ 221

$$\frac{\left(\frac{(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6b}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}\right)\left(3a^2d^2fh-6abd(-c\cdots\right.}{16b^2d^2}$$

$$\frac{(a+bx)^{5/2}(c+dx)^{3/2}(5adfh+7bcfh-10bd(eh+fg)-8bdfhx)}{40b^2d^2}$$

input
```
Int[(a + b*x)^(3/2)*Sqrt[c + d*x]*(e + f*x)*(g + h*x),x]
```

output
```
-1/40*((a + b*x)^(5/2)*(c + d*x)^(3/2)*(7*b*c*f*h + 5*a*d*f*h - 10*b*d*(f*
g + e*h) - 8*b*d*f*h*x))/(b^2*d^2) + ((3*a^2*d^2*f*h - 6*a*b*d*(d*f*g + d*
e*h - c*f*h) + b^2*(16*d^2*e*g + 7*c^2*f*h - 10*c*d*(f*g + e*h)))*(((a + b
*x)^(5/2)*Sqrt[c + d*x])/(3*b) + ((b*c - a*d)*(((a + b*x)^(3/2)*Sqrt[c + d
*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d
)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/
2))))/(4*d)))/(6*b)))/(16*b^2*d^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2382 vs. $2(435) = 870$.

Time = 0.32 (sec) , antiderivative size = 2383, normalized size of antiderivative = 4.97

method	result	size
default	Expression too large to display	2383

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-1/3840*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(-45*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*c*d^4*f*h-30*ln(1/2*(2*b*
d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c^
2*d^3*f*h+120*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*
c)/(d*b)^(1/2))*a^3*b^2*c*d^4*e*h+180*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*
a^3*b*d^4*f*g+60*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^3*b*d^4*f*h*x-120*(
d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*d^4*e*h*x-120*(d*b)^(1/2)*((b*x
+a)*(d*x+c))^(1/2)*a^2*b^2*d^4*f*g*x-2240*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1
/2)*a*b^3*d^4*e*g*x-140*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^4*c^3*d*f*h*
x+200*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^4*c^2*d^2*e*h*x+200*(d*b)^(1/2
)*((b*x+a)*(d*x+c))^(1/2)*b^4*c^2*d^2*f*g*x-320*(d*b)^(1/2)*((b*x+a)*(d*x+
c))^(1/2)*b^4*c*d^3*e*g*x+60*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^3*b*c*d
^3*f*h+72*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c^2*d^2*f*h-180*(d*b
)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c*d^3*e*h-180*(d*b)^(1/2)*((b*x+a)
*(d*x+c))^(1/2)*a^2*b^2*c*d^3*f*g-380*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*
a*b^3*c^3*d*f*h+620*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2*e*h+
620*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2*f*g-768*b^4*d^4*f*h*
x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-960*b^4*d^4*e*h*x^3*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)-960*b^4*d^4*f*g*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)-90*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(...
```

Fricas [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 1746, normalized size of antiderivative = 3.65

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x, algorithm="fricas
")
```

output
```
[-1/7680*(15*sqrt(b*d)*(2*(8*(b^5*c^3*d^2 - 3*a*b^4*c^2*d^3 + 3*a^2*b^3*c*
d^4 - a^3*b^2*d^5)*e - (5*b^5*c^4*d - 12*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3
 + 4*a^3*b^2*c*d^4 - 3*a^4*b*d^5)*f)*g - (2*(5*b^5*c^4*d - 12*a*b^4*c^3*d^
2 + 6*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 - 3*a^4*b*d^5)*e - (7*b^5*c^5 - 15
*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - 3*a
^5*d^5)*f)*h)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d
*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d
^2)*x) - 4*(384*b^5*d^5*f*h*x^4 + 48*(10*b^5*d^5*f*g + (10*b^5*d^5*e + (b^
5*c*d^4 + 11*a*b^4*d^5)*f)*h)*x^3 + 8*(10*(8*b^5*d^5*e + (b^5*c*d^4 + 9*a*
b^4*d^5)*f)*g + (10*(b^5*c*d^4 + 9*a*b^4*d^5)*e - (7*b^5*c^2*d^3 - 12*a*b^
4*c*d^4 - 3*a^2*b^3*d^5)*f)*h)*x^2 - 10*(8*(3*b^5*c^2*d^3 - 8*a*b^4*c*d^4
- 3*a^2*b^3*d^5)*e - (15*b^5*c^3*d^2 - 31*a*b^4*c^2*d^3 + 9*a^2*b^3*c*d^4
- 9*a^3*b^2*d^5)*f)*g + (10*(15*b^5*c^3*d^2 - 31*a*b^4*c^2*d^3 + 9*a^2*b^3
*c*d^4 - 9*a^3*b^2*d^5)*e - (105*b^5*c^4*d - 190*a*b^4*c^3*d^2 + 36*a^2*b^
3*c^2*d^3 + 30*a^3*b^2*c*d^4 - 45*a^4*b*d^5)*f)*h + 2*(10*(8*(b^5*c*d^4 +
7*a*b^4*d^5)*e - (5*b^5*c^2*d^3 - 10*a*b^4*c*d^4 - 3*a^2*b^3*d^5)*f)*g - (
10*(5*b^5*c^2*d^3 - 10*a*b^4*c*d^4 - 3*a^2*b^3*d^5)*e - (35*b^5*c^3*d^2 -
61*a*b^4*c^2*d^3 + 9*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*f)*h)*x)*sqrt(b*x + a
)*sqrt(d*x + c))/(b^4*d^5), -1/3840*(15*sqrt(-b*d)*(2*(8*(b^5*c^3*d^2 - 3*
a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*e - (5*b^5*c^4*d - 12*a*...
```

Sympy [F]

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \int (a+bx)^{\frac{3}{2}}\sqrt{c+dx}(e+fx)\,(g+hx)\,\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**(1/2)*(f*x+e)*(h*x+g),x)
```

output
```
Integral((a + b*x)**(3/2)*sqrt(c + d*x)*(e + f*x)*(g + h*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2485 vs. $2(435) = 870$.

Time = 0.46 (sec) , antiderivative size = 2485, normalized size of antiderivative = 5.19

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
-1/1920*(1920*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2
*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*sqrt(b*x + a))*a^2*e*g*abs(b)/b^2 - 10*(sqrt(b^2*c + (b*x + a)*b*d - a
*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11
*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/
(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 9
3*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*f*g*
abs(b) - 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)
*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2
*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqr
t(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c
*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*e*h*abs(b) - 20*(sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5
 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^
2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^
12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + ...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \int (e+f\,x)\,\left(g+h\,x\right)\,\left(a+b\,x\right)^{3/2}\,\sqrt{c+d\,x}\,dx$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)^(3/2)*(c + d*x)^(1/2),x)
```

output
```
int((e + f*x)*(g + h*x)*(a + b*x)^(3/2)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int (a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)\,dx=\int (bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,(fx+e)\,(hx+g)\,dx$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x)
```

3.198 $\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 382

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \frac{(bc-ad)\left(5a^2d^2fh+2abd(3cfh-4d(fg+eh))+b^2(16d^2eg+5c^2fh-8cd(fg+eh))\right)\sqrt{a+bx}\sqrt{c+d}}{64b^3d^3} + \frac{\left(6acfh+\frac{5a^2dfh}{b}-8ad(fg+eh)+b\left(16deg+\frac{5c^2fh}{d}-8c(fg+eh)\right)\right)(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} - \frac{(5bcfh+11adfh-8bd(fg+eh))(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2d^2} + \frac{fh(a+bx)^{5/2}(c+dx)^{3/2}}{4b^2d} - \frac{(bc-ad)^2\left(5a^2d^2fh+2abd(3cfh-4d(fg+eh))+b^2(16d^2eg+5c^2fh-8cd(fg+eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{b}}\right.}{64b^{7/2}d^{7/2}}$$

output

```
1/64*(-a*d+b*c)*(5*a^2*d^2*f*h+2*a*b*d*(3*c*f*h-4*d*(e*h+f*g))+b^2*(16*d^2
*e*g+5*c^2*f*h-8*c*d*(e*h+f*g)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d^3+1/32*
(6*a*c*f*h+5*a^2*d*f*h/b-8*a*d*(e*h+f*g)+b*(16*d*e*g+5*c^2*f*h/d-8*c*(e*h+
f*g)))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^2/d-1/24*(5*b*c*f*h+11*a*d*f*h-8*b*d*
(e*h+f*g))*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^2/d^2+1/4*f*h*(b*x+a)^(5/2)*(d*x+
c)^(3/2)/b^2/d-1/64*(-a*d+b*c)^2*(5*a^2*d^2*f*h+2*a*b*d*(3*c*f*h-4*d*(e*h+
f*g))+b^2*(16*d^2*e*g+5*c^2*f*h-8*c*d*(e*h+f*g)))*arctanh(d^(1/2)*(b*x+a)^
(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx$$
$$= \frac{\sqrt{a+bx}\sqrt{c+dx}(15a^3d^3fh - a^2bd^2(7cfh+2d(12fg+12eh+5fhx)) + ab^2d(-7c^2fh+4cd(4fg+4eh$$
$$-\frac{(bc-ad)^2\left(5a^2d^2fh-2abd(-3cfh+4d(fg+eh))+b^2(16d^2eg+5c^2fh-8cd(fg+eh))\right)\operatorname{arctanh}\left(\right.}{64b^{7/2}d^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)*(g + h*x),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(15*a^3*d^3*f*h - a^2*b*d^2*(7*c*f*h + 2*d*(1
2*f*g + 12*e*h + 5*f*h*x)) + a*b^2*d*(-7*c^2*f*h + 4*c*d*(4*f*g + 4*e*h +
f*h*x) + 8*d^2*(6*e*g + 2*f*g*x + 2*e*h*x + f*h*x^2)) + b^3*(15*c^3*f*h -
2*c^2*d*(12*f*g + 12*e*h + 5*f*h*x) + 8*c*d^2*(6*e*g + 2*f*g*x + 2*e*h*x +
 f*h*x^2) + 16*d^3*x*(6*e*g + 4*f*g*x + 4*e*h*x + 3*f*h*x^2))))/(192*b^3*d
^3) - ((b*c - a*d)^2*(5*a^2*d^2*f*h - 2*a*b*d*(-3*c*f*h + 4*d*(f*g + e*h))
 + b^2*(16*d^2*e*g + 5*c^2*f*h - 8*c*d*(f*g + e*h)))*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*b^(7/2)*d^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {164, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx$$
$$\Big\downarrow \; 164$$

$$\frac{\left(5a^2d^2fh+2abd(3cfh-4d(eh+fg))+b^2\left(5c^2fh-8cd(eh+fg)+16d^2eg\right)\right)\int\sqrt{a+bx}\sqrt{c+dx}dx}{16b^2d^2}-\frac{(a+bx)^{3/2}(c+dx)^{3/2}(5adfh+5bcfh-8bd(eh+fg)-6bdfhx)}{24b^2d^2}$$

$\big\downarrow$ 60

$$\frac{\left(5a^2d^2fh+2abd(3cfh-4d(eh+fg))+b^2\left(5c^2fh-8cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\int\frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{4b}+\frac{(a+bx)^{3/2}\sqrt{c+\cdots}}{2b}\cdots\right.}{16b^2d^2}-\frac{(a+bx)^{3/2}(c+dx)^{3/2}(5adfh+5bcfh-8bd(eh+fg)-6bdfhx)}{24b^2d^2}$$

$\big\downarrow$ 60

$$\frac{\left(5a^2d^2fh+2abd(3cfh-4d(eh+fg))+b^2\left(5c^2fh-8cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{\cdots}{\sqrt{a+\cdots}}}{2d\cdots}\cdots\right.}{4b}\cdots\right.}{16b^2d^2}-\frac{(a+bx)^{3/2}(c+dx)^{3/2}(5adfh+5bcfh-8bd(eh+fg)-6bdfhx)}{24b^2d^2}$$

$\big\downarrow$ 66

$$\frac{\left(5a^2d^2fh+2abd(3cfh-4d(eh+fg))+b^2\left(5c^2fh-8cd(eh+fg)+16d^2eg\right)\right)\left(\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{\cdots}{b-\frac{d\cdots}{\cdots}}}{\cdots}\cdots\right.}{4b}\cdots\right.}{16b^2d^2}-\frac{(a+bx)^{3/2}(c+dx)^{3/2}(5adfh+5bcfh-8bd(eh+fg)-6bdfhx)}{24b^2d^2}$$

$\big\downarrow$ 221

$$\frac{\left(\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4b}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}\right)\left(5a^2d^2fh+2abd(3cfh-4d(eh+fg))+b^2(\cdots\right.}{16b^2d^2}-\frac{(a+bx)^{3/2}(c+dx)^{3/2}(5adfh+5bcfh-8bd(eh+fg)-6bdfhx)}{24b^2d^2}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)*(g + h*x),x]
```

output
```
-1/24*((a + b*x)^(3/2)*(c + d*x)^(3/2)*(5*b*c*f*h + 5*a*d*f*h - 8*b*d*(f*g
 + e*h) - 6*b*d*f*h*x))/(b^2*d^2) + ((5*a^2*d^2*f*h + 2*a*b*d*(3*c*f*h - 4
*d*(f*g + e*h)) + b^2*(16*d^2*e*g + 5*c^2*f*h - 8*c*d*(f*g + e*h)))*(((a +
 b*x)^(3/2)*Sqrt[c + d*x])/(2*b) + ((b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d
*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/(Sqrt[b]*d^(3/2))))/(4*b)))/(16*b^2*d^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1626 vs. $2(344) = 688$.

Time = 0.33 (sec) , antiderivative size = 1627, normalized size of antiderivative = 4.26

method	result	size
default	Expression too large to display	1627

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(-96*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c*d^3*e*g+48*(d*b)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*b^3*c^2*d*f*g+24*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^2*d^2*e*h+24*ln(1/2*(2*b
*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^2
*d^2*f*g+24*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*a^2*b^2*c*d^3*f*g-12*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d*f*h-32*(d*b)^(1/2)*((b*x+
a)*(d*x+c))^(1/2)*a*b^2*d^3*e*h*x-32*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a
*b^2*d^3*f*g*x+20*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^3*c^2*d*f*h*x-32*(
d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^3*c*d^2*e*h*x-32*(d*b)^(1/2)*((b*x+a)
*(d*x+c))^(1/2)*b^3*c*d^2*f*g*x+14*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2
*b*c*d^2*f*h+14*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^2*c^2*d*f*h-32*(d*
b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^2*c*d^2*e*h-32*(d*b)^(1/2)*((b*x+a)*(
d*x+c))^(1/2)*a*b^2*c*d^2*f*g+20*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b
*d^3*f*h*x+48*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b*d^3*f*g-96*(d*b)^(
1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^2*d^3*e*g+48*(d*b)^(1/2)*((b*x+a)*(d*x+c)
)^(1/2)*b^3*c^2*d*e*h-16*b^3*c*d^2*f*h*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)-16*a*b^2*d^3*f*h*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-30*(d*b)^(1/
2)*((b*x+a)*(d*x+c))^(1/2)*a^3*d^3*f*h-30*(d*b)^(1/2)*((b*x+a)*(d*x+c))...
```

Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1190, normalized size of antiderivative = 3.12

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x, algorithm="fricas
")
```

output
```
[1/768*(3*sqrt(b*d)*(8*(2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e -
(b^4*c^3*d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*f)*g - (8*(b^4*c^3
*d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*e - (5*b^4*c^4 - 4*a*b^3*c
^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4*d^4)*f)*h)*log(8*b^2*d^2*
x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sq
rt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(48*b^4*d^4*f*h*x
^3 + 8*(8*b^4*d^4*f*g + (8*b^4*d^4*e + (b^4*c*d^3 + a*b^3*d^4)*f)*h)*x^2 +
 8*(6*(b^4*c*d^3 + a*b^3*d^4)*e - (3*b^4*c^2*d^2 - 2*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f)*g - (8*(3*b^4*c^2*d^2 - 2*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*e - (15*
b^4*c^3*d - 7*a*b^3*c^2*d^2 - 7*a^2*b^2*c*d^3 + 15*a^3*b*d^4)*f)*h + 2*(8*
(6*b^4*d^4*e + (b^4*c*d^3 + a*b^3*d^4)*f)*g + (8*(b^4*c*d^3 + a*b^3*d^4)*e
 - (5*b^4*c^2*d^2 - 2*a*b^3*c*d^3 + 5*a^2*b^2*d^4)*f)*h)*x)*sqrt(b*x + a)*
sqrt(d*x + c))/(b^4*d^4), 1/384*(3*sqrt(-b*d)*(8*(2*(b^4*c^2*d^2 - 2*a*b^3
*c*d^3 + a^2*b^2*d^4)*e - (b^4*c^3*d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3
*b*d^4)*f)*g - (8*(b^4*c^3*d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*
e - (5*b^4*c^4 - 4*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 5*a^4
*d^4)*f)*h)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt
(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48*b^4*d^4
*f*h*x^3 + 8*(8*b^4*d^4*f*g + (8*b^4*d^4*e + (b^4*c*d^3 + a*b^3*d^4)*f)*h)
*x^2 + 8*(6*(b^4*c*d^3 + a*b^3*d^4)*e - (3*b^4*c^2*d^2 - 2*a*b^3*c*d^3 ...
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \int \sqrt{a+bx}\sqrt{c+dx}(e+fx)\left(g+hx\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)*(f*x+e)*(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)*(e + f*x)*(g + h*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. 2(344) = 688.

Time = 0.32 (sec) , antiderivative size = 1366, normalized size of antiderivative = 3.58

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
-1/192*(192*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*sqrt(b*x + a))*a*e*g*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(
2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(
b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d
^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b
^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*f*h*abs(b)/
b - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^
2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d)
)*e*g*abs(b)/b^2 - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
(b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*d))*a*f*g*abs(b)/b^3 - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2
*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a*e*h*abs(b)/b^3 - 8*(sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d)*(2*(4*b*x + 4*a + (b*c*d^3 - 13*a*d^4)/d^4)*(b*x + a)...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Hanged}$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 1398, normalized size of antiderivative = 3.66

$$\int \sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)*(h*x+g),x)
```

output
```
(15*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b*d**4*f*h - 7*sqrt(c + d*x)*sqrt(a +
 b*x)*a**2*b**2*c*d**3*f*h - 24*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**2*d**4
*e*h - 24*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**2*d**4*f*g - 10*sqrt(c + d*x
)*sqrt(a + b*x)*a**2*b**2*d**4*f*h*x - 7*sqrt(c + d*x)*sqrt(a + b*x)*a*b**
3*c**2*d**2*f*h + 16*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*c*d**3*e*h + 16*sq
rt(c + d*x)*sqrt(a + b*x)*a*b**3*c*d**3*f*g + 4*sqrt(c + d*x)*sqrt(a + b*x
)*a*b**3*c*d**3*f*h*x + 48*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*d**4*e*g + 1
6*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*d**4*e*h*x + 16*sqrt(c + d*x)*sqrt(a
+ b*x)*a*b**3*d**4*f*g*x + 8*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*d**4*f*h*x
**2 + 15*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c**3*d*f*h - 24*sqrt(c + d*x)*sq
rt(a + b*x)*b**4*c**2*d**2*e*h - 24*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c**2*
d**2*f*g - 10*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c**2*d**2*f*h*x + 48*sqrt(c
 + d*x)*sqrt(a + b*x)*b**4*c*d**3*e*g + 16*sqrt(c + d*x)*sqrt(a + b*x)*b**
4*c*d**3*e*h*x + 16*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c*d**3*f*g*x + 8*sqrt
(c + d*x)*sqrt(a + b*x)*b**4*c*d**3*f*h*x**2 + 96*sqrt(c + d*x)*sqrt(a + b
*x)*b**4*d**4*e*g*x + 64*sqrt(c + d*x)*sqrt(a + b*x)*b**4*d**4*e*h*x**2 +
64*sqrt(c + d*x)*sqrt(a + b*x)*b**4*d**4*f*g*x**2 + 48*sqrt(c + d*x)*sqrt(
a + b*x)*b**4*d**4*f*h*x**3 - 15*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x
) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**4*d**4*f*h + 12*sqrt(d)*sqr
t(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c...
```

3.199 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 278

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx$$
$$= \frac{\left(8bdeg+2acfh+\frac{bc^2fh}{d}+\frac{5a^2dfh}{b}-2bc(fg+eh)-6ad(fg+eh)\right)\sqrt{a+bx}\sqrt{c+dx}}{8b^2d}$$
$$-\frac{(bcfh+3adfh-2bd(fg+eh))\sqrt{a+bx}(c+dx)^{3/2}}{4b^2d^2}+\frac{fh(a+bx)^{3/2}(c+dx)^{3/2}}{3b^2d}$$
$$+\frac{(bc-ad)\left(5a^2d^2fh+2abd(cfh-3d(fg+eh))+b^2(8d^2eg+c^2fh-2cd(fg+eh))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}d^{5/2}}$$

output
```
1/8*(8*b*d*e*g+2*a*c*f*h+b*c^2*f*h/d+5*a^2*d*f*h/b-2*b*c*(e*h+f*g)-6*a*d*(
e*h+f*g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d-1/4*(b*c*f*h+3*a*d*f*h-2*b*d*(
e*h+f*g))*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^2/d^2+1/3*f*h*(b*x+a)^(3/2)*(d*x+c
)^(3/2)/b^2/d+1/8*(-a*d+b*c)*(5*a^2*d^2*f*h+2*a*b*d*(c*f*h-3*d*(e*h+f*g))+
b^2*(8*d^2*e*g+c^2*f*h-2*c*d*(e*h+f*g)))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(
1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx$$
$$= \frac{\sqrt{c+dx}\left(\sqrt{d}\sqrt{a+bx}(15a^2d^2fh-2abd(2cfh+d(9fg+9eh+5fhx))+b^2(-3c^2fh+2cd(3fg+3eh\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[c + d*x]*(Sqrt[d]*Sqrt[a + b*x]*(15*a^2*d^2*f*h - 2*a*b*d*(2*c*f*h +
 d*(9*f*g + 9*e*h + 5*f*h*x)) + b^2*(-3*c^2*f*h + 2*c*d*(3*f*g + 3*e*h + f
*h*x) + 4*d^2*(6*e*g + 3*f*g*x + 3*e*h*x + 2*f*h*x^2))) + (3*Sqrt[b*c - a*
d]*(5*a^2*d^2*f*h + 2*a*b*d*(c*f*h - 3*d*(f*g + e*h)) + b^2*(8*d^2*e*g + c
^2*f*h - 2*c*d*(f*g + e*h)))*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*
d]])/Sqrt[(b*(c + d*x))/(b*c - a*d)]))/(24*b^3*d^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {164, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \; 164$$
$$\frac{\left(5a^2d^2fh+2abd(cfh-3d(eh+fg))+b^2\left(c^2fh-2cd(eh+fg)+8d^2eg\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}dx}{8b^2d^2}-\frac{\sqrt{a+bx}(c+dx)^{3/2}(5adfh+3bcfh-6bd(eh+fg)-4bdfhx)}{12b^2d^2}$$
$$\Big\downarrow \; 60$$

$$\frac{\left(5a^2d^2fh+2abd(cfh-3d(eh+fg))+b^2\left(c^2fh-2cd(eh+fg)+8d^2eg\right)\right)\left(\frac{(bc-ad)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{a+bx}\sqrt{c+d}}{b}\right.}{8b^2d^2}$$
$$\frac{\sqrt{a+bx}(c+dx)^{3/2}(5adfh+3bcfh-6bd(eh+fg)-4bdfhx)}{12b^2d^2}$$

$\Big\downarrow$ 66

$$\frac{\left(5a^2d^2fh+2abd(cfh-3d(eh+fg))+b^2\left(c^2fh-2cd(eh+fg)+8d^2eg\right)\right)\left(\frac{(bc-ad)\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}+\frac{\sqrt{a+bx}\sqrt{}}{b}\right.}{8b^2d^2}$$
$$\frac{\sqrt{a+bx}(c+dx)^{3/2}(5adfh+3bcfh-6bd(eh+fg)-4bdfhx)}{12b^2d^2}$$

$\Big\downarrow$ 221

$$\frac{\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)\left(5a^2d^2fh+2abd(cfh-3d(eh+fg))+b^2\left(c^2fh-2cd(eh+fg)+8\right.\right.}{8b^2d^2}$$
$$\frac{\sqrt{a+bx}(c+dx)^{3/2}(5adfh+3bcfh-6bd(eh+fg)-4bdfhx)}{12b^2d^2}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/Sqrt[a + b*x],x]
```

output
```
-1/12*(Sqrt[a + b*x]*(c + d*x)^(3/2)*(3*b*c*f*h + 5*a*d*f*h - 6*b*d*(f*g +
 e*h) - 4*b*d*f*h*x))/(b^2*d^2) + ((5*a^2*d^2*f*h + 2*a*b*d*(c*f*h - 3*d*(
f*g + e*h)) + b^2*(8*d^2*e*g + c^2*f*h - 2*c*d*(f*g + e*h)))*((Sqrt[a + b*
x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(8*b^2*d^2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(246) = 492$.

Time = 0.33 (sec) , antiderivative size = 1019, normalized size of antiderivative = 3.67

method	result	size
default	Expression too large to display	1019

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x+c)^(1/2)*(b*x+a)^(1/2)*(20*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*
a*b*d^2*f*h*x-4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^2*c*d*f*h*x+8*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b*c*d*f*h+6*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^2*d*e*h+6*ln(1/2*(2*b*
d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^2*d*
f*g-9*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)
^(1/2))*a^2*b*c*d^2*f*h+12*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^2*e*h+12*ln(1/2*(2*b*d*x+2*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^2*f*g-3*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*
c^3*f*h+15*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*a^3*d^3*f*h+6*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^2*c^2*f*h
-12*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^2*c*d*e*h-12*((b*x+a)*(d*x+c))^(
1/2)*(d*b)^(1/2)*b^2*c*d*f*g-24*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^2*d^
2*e*h*x-24*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^2*d^2*f*g*x-3*ln(1/2*(2*b
*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^2
*d*f*h-18*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(
d*b)^(1/2))*a^2*b*d^3*e*h-18*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*d^3*f*g+24*ln(1/2*(2*b*d*x+2*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*d^3*e*g-24*ln(1...
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 774, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/96*(3*sqrt(b*d)*(2*(4*(b^3*c*d^2 - a*b^2*d^3)*e - (b^3*c^2*d + 2*a*b^2
*c*d^2 - 3*a^2*b*d^3)*f)*g - (2*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*
e - (b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*f)*h)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d^3*f*h*
x^2 + 6*(4*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g + (6*(b^3*c*d^2 - 3*
a*b^2*d^3)*e - (3*b^3*c^2*d + 4*a*b^2*c*d^2 - 15*a^2*b*d^3)*f)*h + 2*(6*b^
3*d^3*f*g + (6*b^3*d^3*e + (b^3*c*d^2 - 5*a*b^2*d^3)*f)*h)*x)*sqrt(b*x + a
)*sqrt(d*x + c))/(b^4*d^3), -1/48*(3*sqrt(-b*d)*(2*(4*(b^3*c*d^2 - a*b^2*d
^3)*e - (b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*f)*g - (2*(b^3*c^2*d + 2
*a*b^2*c*d^2 - 3*a^2*b*d^3)*e - (b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5
*a^3*d^3)*f)*h)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*
sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(8*b^3*
d^3*f*h*x^2 + 6*(4*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g + (6*(b^3*c*
d^2 - 3*a*b^2*d^3)*e - (3*b^3*c^2*d + 4*a*b^2*c*d^2 - 15*a^2*b*d^3)*f)*h +
 2*(6*b^3*d^3*f*g + (6*b^3*d^3*e + (b^3*c*d^2 - 5*a*b^2*d^3)*f)*h)*x)*sqrt
(b*x + a)*sqrt(d*x + c))/(b^4*d^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)\,(g+hx)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/sqrt(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(246) = 492$.

Time = 0.21 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.01

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx = -\frac{24\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)eg|b|}{b^2} - \frac{6\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*
sqrt(b*x + a))*e*g*abs(b)/b^2 - 6*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*
b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
- 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)))/(sqrt(b*d)*d))*f*g*abs(b)/b^3 - 6*(sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2
+ 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*e*h*abs(b)/b^3 - (sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)*(2*(4*b*x + 4*a + (b*c*d^3 - 13*a*d^4)/d^4)*(b*x +
a) - 3*(b^2*c^2*d^2 + 2*a*b*c*d^3 - 11*a^2*d^4)/d^4)*sqrt(b*x + a) - 3*(b^
4*c^3 + a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*log(abs(-sqrt(b*d)*sq
rt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2))*f*h*a
bs(b)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 60.63 (sec) , antiderivative size = 2493, normalized size of antiderivative = 8.97

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(1/2),x)
```

output
```
(((2*a*d*e*g + 2*b*c*e*g)*((a + b*x)^(1/2) - a^(1/2)))/(d^2*((c + d*x)^(1/
2) - c^(1/2))) + ((2*a*d*e*g + 2*b*c*e*g)*((a + b*x)^(1/2) - a^(1/2))^3)/(
b*d*((c + d*x)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*e*g*((a + b*x)^(1/
2) - a^(1/2))^2)/(d*((c + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^
(1/2))^4/((c + d*x)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) -
 a^(1/2))^2)/(d*((c + d*x)^(1/2) - c^(1/2))^2)) - ((((a + b*x)^(1/2) - a^(
1/2))*((b^5*c^3*f*h)/4 - (5*a^3*b^2*d^3*f*h)/4 + (a*b^4*c^2*d*f*h)/4 + (3*
a^2*b^3*c*d^2*f*h)/4))/(d^8*((c + d*x)^(1/2) - c^(1/2))) - (((a + b*x)^(1/
2) - a^(1/2))^3*((17*b^4*c^3*f*h)/12 - (85*a^3*b*d^3*f*h)/12 + (91*a*b^3*c
^2*d*f*h)/4 + (17*a^2*b^2*c*d^2*f*h)/4))/(d^7*((c + d*x)^(1/2) - c^(1/2))^
3) - (((a + b*x)^(1/2) - a^(1/2))^5*((33*a^3*d^3*f*h)/2 + (19*b^3*c^3*f*h)
/2 + (275*a*b^2*c^2*d*f*h)/2 + (313*a^2*b*c*d^2*f*h)/2))/(d^6*((c + d*x)^(
1/2) - c^(1/2))^5) + (((a + b*x)^(1/2) - a^(1/2))^11*((b^3*c^3*f*h)/4 - (5
*a^3*d^3*f*h)/4 + (a*b^2*c^2*d*f*h)/4 + (3*a^2*b*c*d^2*f*h)/4))/(b^3*d^3*(
(c + d*x)^(1/2) - c^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^9*((17*b^3*c
^3*f*h)/12 - (85*a^3*d^3*f*h)/12 + (91*a*b^2*c^2*d*f*h)/4 + (17*a^2*b*c*d^
2*f*h)/4))/(b^2*d^4*((c + d*x)^(1/2) - c^(1/2))^9) - (((a + b*x)^(1/2) - a
^(1/2))^7*((33*a^3*d^3*f*h)/2 + (19*b^3*c^3*f*h)/2 + (275*a*b^2*c^2*d*f*h)
/2 + (313*a^2*b*c*d^2*f*h)/2))/(b*d^5*((c + d*x)^(1/2) - c^(1/2))^7) + (a^
(1/2)*c^(1/2)*(32*b*c^2*f*h + 96*a*c*d*f*h)*((a + b*x)^(1/2) - a^(1/2))...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 878, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(1/2),x)
```

output

```
(15*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b*d**3*f*h - 4*sqrt(c + d*x)*sqrt(a +
 b*x)*a*b**2*c*d**2*f*h - 18*sqrt(c + d*x)*sqrt(a + b*x)*a*b**2*d**3*e*h -
 18*sqrt(c + d*x)*sqrt(a + b*x)*a*b**2*d**3*f*g - 10*sqrt(c + d*x)*sqrt(a
+ b*x)*a*b**2*d**3*f*h*x - 3*sqrt(c + d*x)*sqrt(a + b*x)*b**3*c**2*d*f*h +
 6*sqrt(c + d*x)*sqrt(a + b*x)*b**3*c*d**2*e*h + 6*sqrt(c + d*x)*sqrt(a +
b*x)*b**3*c*d**2*f*g + 2*sqrt(c + d*x)*sqrt(a + b*x)*b**3*c*d**2*f*h*x + 2
4*sqrt(c + d*x)*sqrt(a + b*x)*b**3*d**3*e*g + 12*sqrt(c + d*x)*sqrt(a + b*
x)*b**3*d**3*e*h*x + 12*sqrt(c + d*x)*sqrt(a + b*x)*b**3*d**3*f*g*x + 8*sq
rt(c + d*x)*sqrt(a + b*x)*b**3*d**3*f*h*x**2 - 15*sqrt(d)*sqrt(b)*log((sqr
t(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*d**3*f*h
 + 9*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/s
qrt(a*d - b*c))*a**2*b*c*d**2*f*h + 18*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a
 + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b*d**3*e*h + 18*sqr
t(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d
- b*c))*a**2*b*d**3*f*g + 3*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + s
qrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c**2*d*f*h - 12*sqrt(d)*sqrt
(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a
*b**2*c*d**2*e*h - 12*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)
*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c*d**2*f*g - 24*sqrt(d)*sqrt(b)*lo
g((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b*...
```

3.200 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 223

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx = -\frac{2(be-af)(bg-ah)\sqrt{c+dx}}{b^3\sqrt{a+bx}} + \frac{(bcfh-9adfh+4bd(fg+eh))\sqrt{a+bx}\sqrt{c+dx}}{4b^3d} + \frac{fh(a+bx)^{3/2}\sqrt{c+dx}}{2b^3} + \frac{(15a^2d^2fh-6abd(cfh+2d(fg+eh))+b^2(8d^2eg-c^2fh+4cd(fg+eh)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{7/2}d^{3/2}}$$

output

```
-2*(-a*f+b*e)*(-a*h+b*g)*(d*x+c)^(1/2)/b^3/(b*x+a)^(1/2)+1/4*(b*c*f*h-9*a*
d*f*h+4*b*d*(e*h+f*g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d+1/2*f*h*(b*x+a)^(
3/2)*(d*x+c)^(1/2)/b^3+1/4*(15*a^2*d^2*f*h-6*a*b*d*(c*f*h+2*d*(e*h+f*g))+b
^2*(8*d^2*e*g-c^2*f*h+4*c*d*(e*h+f*g)))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1
/2)/(d*x+c)^(1/2))/b^(7/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx = \frac{\sqrt{c+dx}(-15a^2dfh+ab(cfh+d(12fg+12eh-5fhx))+b^2(cfhx+}{4b^3d\sqrt{a+bx}}$$
$$+\frac{(15a^2d^2fh-6abd(cfh+2d(fg+eh))+b^2(8d^2eg-c^2fh+4cd(fg+eh)))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{7/2}d^{3/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[c + d*x]*(-15*a^2*d*f*h + a*b*(c*f*h + d*(12*f*g + 12*e*h - 5*f*h*x)
) + b^2*(c*f*h*x + d*(-8*e*g + 4*f*g*x + 4*e*h*x + 2*f*h*x^2))))/(4*b^3*d*
Sqrt[a + b*x]) + ((15*a^2*d^2*f*h - 6*a*b*d*(c*f*h + 2*d*(f*g + e*h)) + b^
2*(8*d^2*e*g - c^2*f*h + 4*c*d*(f*g + e*h)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(7/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {163, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx$$
$$\Big\downarrow \; 163$$
$$\frac{(15a^2d^2fh-6abd(cfh+2d(eh+fg))+b^2\left(c^2(-f)h+4cd(eh+fg)+8d^2eg\right))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}dx}{4b^2d(bc-ad)}-\frac{(c+dx)^{3/2}\left(5a^2dfh-ab(cfh+4d(eh+fg))-bfhx(bc-ad)+4b^2deg\right)}{2b^2d\sqrt{a+bx}(bc-ad)}$$
$$\Big\downarrow \; 60$$

$$\frac{\left(15a^2d^2fh-6abd(cfh+2d(eh+fg))+b^2\left(c^2(-f)h+4cd(eh+fg)+8d^2eg\right)\right)\left(\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2b}+\sqrt{a+bx}\right.}{4b^2d(bc-ad)}$$
$$\frac{(c+dx)^{3/2}\left(5a^2dfh-ab(cfh+4d(eh+fg))-bfhx(bc-ad)+4b^2deg\right)}{2b^2d\sqrt{a+bx}(bc-ad)}$$

$\Big\downarrow$ 66

$$\frac{\left(15a^2d^2fh-6abd(cfh+2d(eh+fg))+b^2\left(c^2(-f)h+4cd(eh+fg)+8d^2eg\right)\right)\left(\frac{(bc-ad)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}+\sqrt{a}\right.}{4b^2d(bc-ad)}$$
$$\frac{(c+dx)^{3/2}\left(5a^2dfh-ab(cfh+4d(eh+fg))-bfhx(bc-ad)+4b^2deg\right)}{2b^2d\sqrt{a+bx}(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)\left(15a^2d^2fh-6abd(cfh+2d(eh+fg))+b^2\left(c^2(-f)h+4cd(eh+fg\right.\right.}{4b^2d(bc-ad)}$$
$$\frac{(c+dx)^{3/2}\left(5a^2dfh-ab(cfh+4d(eh+fg))-bfhx(bc-ad)+4b^2deg\right)}{2b^2d\sqrt{a+bx}(bc-ad)}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(3/2),x]
```

output
```
-1/2*((c + d*x)^(3/2)*(4*b^2*d*e*g + 5*a^2*d*f*h - a*b*(c*f*h + 4*d*(f*g +
 e*h)) - b*(b*c - a*d)*f*h*x))/(b^2*d*(b*c - a*d)*Sqrt[a + b*x]) + ((15*a^
2*d^2*f*h - 6*a*b*d*(c*f*h + 2*d*(f*g + e*h)) + b^2*(8*d^2*e*g - c^2*f*h +
 4*c*d*(f*g + e*h)))*((Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTa
nh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(
4*b^2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1117 vs. $2(193) = 386$.

Time = 0.33 (sec) , antiderivative size = 1118, normalized size of antiderivative = 5.01

method	result	size
default	Expression too large to display	1118

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x+c)^(1/2)*(-10*a*b*d*f*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-6*l
n(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))
*a*b^2*c*d*f*h*x+8*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a
*d+b*c)/(d*b)^(1/2))*b^3*d^2*e*g*x-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^2*f*h*x+24*a*b*d*f*g*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)-30*a^2*d*f*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
-16*b^2*d*e*g*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+15*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*d^2*f*h*x-12*
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2)
)*a*b^2*d^2*e*h*x-12*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a*b^2*d^2*f*g*x+4*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c*d*e*h*x+4*ln(1/2*(2*b*d*x
+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c*d*f*g*x
-6*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1
/2))*a^2*b*c*d*f*h+4*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d*e*h+8*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*d^2*e*g-12*ln(1/2*(2*b*d*x+
2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*d^2*e*h-
12*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1
/2))*a^2*b*d^2*f*g-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(193) = 386$.

Time = 3.84 (sec) , antiderivative size = 802, normalized size of antiderivative = 3.60

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/16*(sqrt(b*d)*(4*(2*a*b^2*d^2*e + (a*b^2*c*d - 3*a^2*b*d^2)*f)*g + (4*(
a*b^2*c*d - 3*a^2*b*d^2)*e - (a*b^2*c^2 + 6*a^2*b*c*d - 15*a^3*d^2)*f)*h +
 (4*(2*b^3*d^2*e + (b^3*c*d - 3*a*b^2*d^2)*f)*g + (4*(b^3*c*d - 3*a*b^2*d^
2)*e - (b^3*c^2 + 6*a*b^2*c*d - 15*a^2*b*d^2)*f)*h)*x)*log(8*b^2*d^2*x^2 +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*
x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^3*d^2*f*h*x^2 - 4
*(2*b^3*d^2*e - 3*a*b^2*d^2*f)*g + (12*a*b^2*d^2*e + (a*b^2*c*d - 15*a^2*b
*d^2)*f)*h + (4*b^3*d^2*f*g + (4*b^3*d^2*e + (b^3*c*d - 5*a*b^2*d^2)*f)*h)
*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*d^2*x + a*b^4*d^2), -1/8*(sqrt(-b*d)
*(4*(2*a*b^2*d^2*e + (a*b^2*c*d - 3*a^2*b*d^2)*f)*g + (4*(a*b^2*c*d - 3*a^
2*b*d^2)*e - (a*b^2*c^2 + 6*a^2*b*c*d - 15*a^3*d^2)*f)*h + (4*(2*b^3*d^2*e
 + (b^3*c*d - 3*a*b^2*d^2)*f)*g + (4*(b^3*c*d - 3*a*b^2*d^2)*e - (b^3*c^2
+ 6*a*b^2*c*d - 15*a^2*b*d^2)*f)*h)*x)*arctan(1/2*(2*b*d*x + b*c + a*d)*sq
rt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a
*b*d^2)*x)) - 2*(2*b^3*d^2*f*h*x^2 - 4*(2*b^3*d^2*e - 3*a*b^2*d^2*f)*g + (
12*a*b^2*d^2*e + (a*b^2*c*d - 15*a^2*b*d^2)*f)*h + (4*b^3*d^2*f*g + (4*b^3
*d^2*e + (b^3*c*d - 5*a*b^2*d^2)*f)*h)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^
5*d^2*x + a*b^4*d^2)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx=\int \frac{\sqrt{c+dx}\,(e+fx)\,(g+hx)}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/(a + b*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(193) = 386$.

Time = 0.29 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx = \frac{1}{4}\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)fh|b|}{b^5} + \frac{4\,b^{10}d^2fg|b|+4}{}\right.$$
$$-\frac{4\,(b^3cdeg|b| - ab^2d^2eg|b| - ab^2cdfg|b| + a^2bd^2fg|b| - ab^2cdeh|b| + a^2bd^2eh|b| + a^2bcdfh|b| - a^3d^2fh|b|)}{\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)\sqrt{bd}b^3}$$
$$-\frac{(8\,b^2d^2eg|b| + 4\,b^2cdfg|b| - 12\,abd^2fg|b| + 4\,b^2cdeh|b| - 12\,abd^2eh|b| - b^2c^2fh|b| - 6\,abcdfh|b| + 15\,a^2d^2}{8\,\sqrt{bd}b^4d}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*f*h*abs
(b)/b^5 + (4*b^10*d^2*f*g*abs(b) + 4*b^10*d^2*e*h*abs(b) + b^10*c*d*f*h*ab
s(b) - 9*a*b^9*d^2*f*h*abs(b))/(b^14*d^2)) - 4*(b^3*c*d*e*g*abs(b) - a*b^2
*d^2*e*g*abs(b) - a*b^2*c*d*f*g*abs(b) + a^2*b*d^2*f*g*abs(b) - a*b^2*c*d*
e*h*abs(b) + a^2*b*d^2*e*h*abs(b) + a^2*b*c*d*f*h*abs(b) - a^3*d^2*f*h*abs
(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2)*sqrt(b*d)*b^3) - 1/8*(8*b^2*d^2*e*g*abs(b) + 4*b^2*c*d*f*g
*abs(b) - 12*a*b*d^2*f*g*abs(b) + 4*b^2*c*d*e*h*abs(b) - 12*a*b*d^2*e*h*ab
s(b) - b^2*c^2*f*h*abs(b) - 6*a*b*c*d*f*h*abs(b) + 15*a^2*d^2*f*h*abs(b))*
log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sq
rt(b*d)*b^4*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx=\int \frac{(e+f\,x)\,(g+h\,x)\,\sqrt{c+d\,x}}{(a+b\,x)^{3/2}}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(3/2),x)
```

output
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(3/2),x)
```

output
```
(15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqr
t(c + d*x))/sqrt(a*d - b*c))*a**2*d**2*f*h - 6*sqrt(d)*sqrt(b)*sqrt(a + b*
x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*
b*c*d*f*h - 12*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) +
sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b*d**2*e*h - 12*sqrt(d)*sqrt(b)*
sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d
 - b*c))*a*b*d**2*f*g - sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a
+ b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**2*c**2*f*h + 4*sqrt(d)
*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))
/sqrt(a*d - b*c))*b**2*c*d*e*h + 4*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt
(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**2*c*d*f*g +
 8*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt
(c + d*x))/sqrt(a*d - b*c))*b**2*d**2*e*g - 10*sqrt(d)*sqrt(b)*sqrt(a + b*
x)*a**2*d**2*f*h + 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*b*c*d*f*h + 9*sqrt(d)
*sqrt(b)*sqrt(a + b*x)*a*b*d**2*e*h + 9*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*b*
d**2*f*g - sqrt(d)*sqrt(b)*sqrt(a + b*x)*b**2*c*d*e*h - sqrt(d)*sqrt(b)*sq
rt(a + b*x)*b**2*c*d*f*g - 8*sqrt(d)*sqrt(b)*sqrt(a + b*x)*b**2*d**2*e*g -
 15*sqrt(c + d*x)*a**2*b*d**2*f*h + sqrt(c + d*x)*a*b**2*c*d*f*h + 12*sqrt
(c + d*x)*a*b**2*d**2*e*h + 12*sqrt(c + d*x)*a*b**2*d**2*f*g - 5*sqrt(c +
d*x)*a*b**2*d**2*f*h*x + sqrt(c + d*x)*b**3*c*d*f*h*x - 8*sqrt(c + d*x)...
```

3.201 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 177

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = -\frac{2(bfg+beh-2afh)\sqrt{c+dx}}{b^3\sqrt{a+bx}} + \frac{fh\sqrt{a+bx}\sqrt{c+dx}}{b^3} - \frac{2(be-af)(bg-ah)(c+dx)^{3/2}}{3b^2(bc-ad)(a+bx)^{3/2}} + \frac{(bcfh-5adfh+2bd(fg+eh))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}\sqrt{d}}$$

output

```
-2*(-2*a*f*h+b*e*h+b*f*g)*(d*x+c)^(1/2)/b^3/(b*x+a)^(1/2)+f*h*(b*x+a)^(1/2
)*(d*x+c)^(1/2)/b^3-2/3*(-a*f+b*e)*(-a*h+b*g)*(d*x+c)^(3/2)/b^2/(-a*d+b*c)
/(b*x+a)^(3/2)+(b*c*f*h-5*a*d*f*h+2*b*d*(e*h+f*g))*arctanh(d^(1/2)*(b*x+a)
^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{c+dx}(-15a^3dfh-b^3(2degx-3cfx(-2g+hx)+2ce(g+3hx))+\cdots}{\cdots} + \frac{(bcfh-5adfh+2bd(fg+eh))\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{7/2}\sqrt{d}}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[c + d*x]*(-15*a^3*d*f*h - b^3*(2*d*e*g*x - 3*c*f*x*(-2*g + h*x) + 2*
c*e*(g + 3*h*x)) + a^2*b*(13*c*f*h + d*(6*f*g + 6*e*h - 20*f*h*x)) + a*b^2
*(-2*c*(2*f*g + 2*e*h - 9*f*h*x) + d*x*(8*f*g + 8*e*h - 3*f*h*x))))/(3*b^3
*(b*c - a*d)*(a + b*x)^(3/2)) + ((b*c*f*h - 5*a*d*f*h + 2*b*d*(f*g + e*h))
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(b^(7/2)*Sqrt[d
])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {160, 57, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 160$$

$$\frac{(-5adfh+bcfh+2bd(eh+fg))\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}dx}{2b^2d} - \frac{(c+dx)^{3/2}\left(5a^2dfh-2ab\left(\frac{3cfh}{2}+deh+dfg\right)-3bfhx(bc-ad)+2b^2deg\right)}{3b^2d(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 57$$

$$\frac{(-5adfh+bcfh+2bd(eh+fg))\left(\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{2b^2d}-\frac{(c+dx)^{3/2}\left(5a^2dfh-2ab\left(\frac{3cfh}{2}+deh+dfg\right)-3bfhx(bc-ad)+2b^2deg\right)}{3b^2d(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 66$$

$$\frac{(-5adfh+bcfh+2bd(eh+fg))\left(\frac{2d\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{2b^2d}-\frac{(c+dx)^{3/2}\left(5a^2dfh-2ab\left(\frac{3cfh}{2}+deh+dfg\right)-3bfhx(bc-ad)+2b^2deg\right)}{3b^2d(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 221$$

$$\frac{\left(\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}\right)(-5adfh+bcfh+2bd(eh+fg))}{2b^2d}-\frac{(c+dx)^{3/2}\left(5a^2dfh-2ab\left(\frac{3cfh}{2}+deh+dfg\right)-3bfhx(bc-ad)+2b^2deg\right)}{3b^2d(a+bx)^{3/2}(bc-ad)}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(5/2),x]
```

output
```
-1/3*((c + d*x)^(3/2)*(2*b^2*d*e*g + 5*a^2*d*f*h - 2*a*b*(d*f*g + d*e*h +
(3*c*f*h)/2) - 3*b*(b*c - a*d)*f*h*x))/(b^2*d*(b*c - a*d)*(a + b*x)^(3/2))
 + ((b*c*f*h - 5*a*d*f*h + 2*b*d*(f*g + e*h))*((-2*Sqrt[c + d*x])/(b*Sqrt[
a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d
*x])])/b^(3/2)))/(2*b^2*d)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 160
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d*(f*g
+ e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Simp[(a*d*f*h*m + b*(d*
(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d)   Int[(a + b*x)^(m + 1)*(c + d*x)^n,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0] &&
NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1587 vs. $2(151) = 302$.

Time = 0.34 (sec) , antiderivative size = 1588, normalized size of antiderivative = 8.97

method	result	size
default	Expression too large to display	1588

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-12*b^3*c*e*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-12*b^3*c*f*g*x*(
(b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-4*b^3*d*e*g*x*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+16*a*b^2*d*f*g*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-18*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3
*c*d*f*h*x^2-36*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^2*b^2*c*d*f*h*x+12*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c*d*e*h*x+12*ln(1/2*(2*b*d*
x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c*d*f*
g*x-30*a^3*d*f*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-4*b^3*c*e*g*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)-6*a*b^2*d*f*h*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)-40*a^2*b*d*f*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+6*ln(1/2*(2*b*
d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c*d*f*
g*x^2+30*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*a^3*b*d^2*f*h*x-12*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^2*e*h*x-12*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^2*f*g*x+6
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a*b^3*c^2*f*h*x-18*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)+a*d+b*c)/(d*b)^(1/2))*a^3*b*c*d*f*h+6*b^3*c*f*h*x^2*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)-8*a*b^2*c*f*g*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+15*ln...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(151) = 302$.

Time = 9.91 (sec) , antiderivative size = 1178, normalized size of antiderivative = 6.66

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(5/2),x, algorithm="fricas
")
```

output
```
[-1/12*(3*(2*(a^2*b^2*c*d - a^3*b*d^2)*f*g + (2*(b^4*c*d - a*b^3*d^2)*f*g
+ (2*(b^4*c*d - a*b^3*d^2)*e + (b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*f)*
h)*x^2 + (2*(a^2*b^2*c*d - a^3*b*d^2)*e + (a^2*b^2*c^2 - 6*a^3*b*c*d + 5*a
^4*d^2)*f)*h + 2*(2*(a*b^3*c*d - a^2*b^2*d^2)*f*g + (2*(a*b^3*c*d - a^2*b^
2*d^2)*e + (a*b^3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^2)*f)*h)*x)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*
sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(3*(b
^4*c*d - a*b^3*d^2)*f*h*x^2 - 2*(b^4*c*d*e + (2*a*b^3*c*d - 3*a^2*b^2*d^2)
*f)*g - (2*(2*a*b^3*c*d - 3*a^2*b^2*d^2)*e - (13*a^2*b^2*c*d - 15*a^3*b*d^
2)*f)*h - 2*((b^4*d^2*e + (3*b^4*c*d - 4*a*b^3*d^2)*f)*g + ((3*b^4*c*d - 4
*a*b^3*d^2)*e - (9*a*b^3*c*d - 10*a^2*b^2*d^2)*f)*h)*x)*sqrt(b*x + a)*sqrt
(d*x + c))/(a^2*b^5*c*d - a^3*b^4*d^2 + (b^7*c*d - a*b^6*d^2)*x^2 + 2*(a*b
^6*c*d - a^2*b^5*d^2)*x), -1/6*(3*(2*(a^2*b^2*c*d - a^3*b*d^2)*f*g + (2*(b
^4*c*d - a*b^3*d^2)*f*g + (2*(b^4*c*d - a*b^3*d^2)*e + (b^4*c^2 - 6*a*b^3*
c*d + 5*a^2*b^2*d^2)*f)*h)*x^2 + (2*(a^2*b^2*c*d - a^3*b*d^2)*e + (a^2*b^2
*c^2 - 6*a^3*b*c*d + 5*a^4*d^2)*f)*h + 2*(2*(a*b^3*c*d - a^2*b^2*d^2)*f*g
+ (2*(a*b^3*c*d - a^2*b^2*d^2)*e + (a*b^3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^
2)*f)*h)*x)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*
x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*
(3*(b^4*c*d - a*b^3*d^2)*f*h*x^2 - 2*(b^4*c*d*e + (2*a*b^3*c*d - 3*a^2*...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)\,(g+hx)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/(a + b*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. 2(151) = 302.

Time = 0.37 (sec) , antiderivative size = 1238, normalized size of antiderivative = 6.99

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*f*h*abs(b)/b^5 - 1/2*(2*
b*d*f*g*abs(b) + 2*b*d*e*h*abs(b) + b*c*f*h*abs(b) - 5*a*d*f*h*abs(b))*log
((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(
b*d)*b^4) - 4/3*(b^6*c^2*d^2*e*g*abs(b) - 2*a*b^5*c*d^3*e*g*abs(b) + a^2*b
^4*d^4*e*g*abs(b) + 3*b^6*c^3*d*f*g*abs(b) - 10*a*b^5*c^2*d^2*f*g*abs(b) +
 11*a^2*b^4*c*d^3*f*g*abs(b) - 4*a^3*b^3*d^4*f*g*abs(b) + 3*b^6*c^3*d*e*h*
abs(b) - 10*a*b^5*c^2*d^2*e*h*abs(b) + 11*a^2*b^4*c*d^3*e*h*abs(b) - 4*a^3
*b^3*d^4*e*h*abs(b) - 6*a*b^5*c^3*d*f*h*abs(b) + 19*a^2*b^4*c^2*d^2*f*h*ab
s(b) - 20*a^3*b^3*c*d^3*f*h*abs(b) + 7*a^4*b^2*d^4*f*h*abs(b) - 6*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2*d*f*g*ab
s(b) + 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2*a*b^3*c*d^2*f*g*abs(b) - 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^2*a^2*b^2*d^3*f*g*abs(b) - 6*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2*d*e*h*abs(b) + 12*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^3*c*d^2*e*h*
abs(b) - 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^2*a^2*b^2*d^3*e*h*abs(b) + 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^2*a*b^3*c^2*d*f*h*abs(b) - 24*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*c*d^2*f*h*abs(b) + 12*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx=\int \frac{(e+f\,x)\,(g+h\,x)\,\sqrt{c+d\,x}}{{(a+b\,x)}^{5/2}}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(5/2),x)
```

output
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1308, normalized size of antiderivative = 7.39

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(5/2),x)
```

output
```
( - 30*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*
sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*d**2*f*h + 36*sqrt(d)*sqrt(b)*sqrt(a
+ b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c)
)*a**2*b*c*d*f*h + 12*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a +
b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b*d**2*e*h + 12*sqrt(d
)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x)
)/sqrt(a*d - b*c))*a**2*b*d**2*f*g - 30*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log(
(sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b*d*
*2*f*h*x - 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sq
rt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c**2*f*h - 12*sqrt(d)*sqrt(b)
*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*
d - b*c))*a*b**2*c*d*e*h - 12*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*s
qrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c*d*f*g + 36
*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c
 + d*x))/sqrt(a*d - b*c))*a*b**2*c*d*f*h*x + 12*sqrt(d)*sqrt(b)*sqrt(a + b
*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a
*b**2*d**2*e*h*x + 12*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a +
b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*d**2*f*g*x - 6*sqrt(
d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x
))/sqrt(a*d - b*c))*b**3*c**2*f*h*x - 12*sqrt(d)*sqrt(b)*sqrt(a + b*x)*...
```

3.202 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}} dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}} dx = -\frac{2fh\sqrt{c+dx}}{b^3\sqrt{a+bx}} - \frac{2(be-af)(bg-ah)(c+dx)^{3/2}}{5b^2(bc-ad)(a+bx)^{5/2}} - \frac{2(8a^2dfh-b^2(2deg-5c(fg+eh))-ab(10cfh+3d(fg+eh)))(c+dx)^{3/2}}{15b^2(bc-ad)^2(a+bx)^{3/2}} + \frac{2\sqrt{d}fh\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}$$

output
```
-2*f*h*(d*x+c)^(1/2)/b^3/(b*x+a)^(1/2)-2/5*(-a*f+b*e)*(-a*h+b*g)*(d*x+c)^(
3/2)/b^2/(-a*d+b*c)/(b*x+a)^(5/2)-2/15*(8*a^2*d*f*h-b^2*(2*d*e*g-5*c*(e*h+
f*g))-a*b*(10*c*f*h+3*d*(e*h+f*g)))*(d*x+c)^(3/2)/b^2/(-a*d+b*c)^2/(b*x+a)
^(3/2)+2*d^(1/2)*f*h*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/
b^(7/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = -\frac{2\sqrt{c+dx}(15a^4d^2fh+5a^3bdfh(-5c+7dx)+a^2b^2fh(8c^2-59cdx+23d^2x^2)+b^4(-2d^2egx^2+cdx(5fg$$

$$+\frac{2\sqrt{d}fh\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{7/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(7/2),x]
```

output
```
(-2*Sqrt[c + d*x]*(15*a^4*d^2*f*h + 5*a^3*b*d*f*h*(-5*c + 7*d*x) + a^2*b^2
*f*h*(8*c^2 - 59*c*d*x + 23*d^2*x^2) + b^4*(-2*d^2*e*g*x^2 + c*d*x*(5*f*g*
x + e*(g + 5*h*x)) + c^2*(5*f*x*(g + 3*h*x) + e*(3*g + 5*h*x))) + a*b^3*(-
(d^2*x*(5*e*g + 3*f*g*x + 3*e*h*x)) + 2*c^2*(e*h + f*(g + 10*h*x)) - c*d*(
e*(5*g + h*x) + f*x*(g + 40*h*x)))))/(15*b^3*(b*c - a*d)^2*(a + b*x)^(5/2)
) + (2*Sqrt[d]*f*h*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])
])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {162, 57, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx$$

$$\Big\downarrow \; 162$$

$$\frac{fh\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}dx}{b^2}-\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(8a^2dfh-ab(10cfh+3d(eh+fg))-b^2(2deg-5c(eh+fg))\right)-7a^2bcfh-ab^2(5deg\right.}{15b^2(a+bx)^{5/2}(bc-ad)^2}$$

$\Big\downarrow$ 57

$$\frac{fh\left(\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{b^2}-\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(8a^2dfh-ab(10cfh+3d(eh+fg))-b^2(2deg-5c(eh+fg))\right)-7a^2bcfh-ab^2(5deg\right.}{15b^2(a+bx)^{5/2}(bc-ad)^2}$$

$\Big\downarrow$ 66

$$\frac{fh\left(\frac{2d\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{b^2}-\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(8a^2dfh-ab(10cfh+3d(eh+fg))-b^2(2deg-5c(eh+fg))\right)-7a^2bcfh-ab^2(5deg\right.}{15b^2(a+bx)^{5/2}(bc-ad)^2}$$

$\Big\downarrow$ 221

$$\frac{fh\left(\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}\right)}{b^2}-\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(8a^2dfh-ab(10cfh+3d(eh+fg))-b^2(2deg-5c(eh+fg))\right)-7a^2bcfh-ab^2(5deg\right.}{15b^2(a+bx)^{5/2}(bc-ad)^2}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(7/2),x]
```

output
```
(-2*(c + d*x)^(3/2)*(3*b^3*c*e*g - 7*a^2*b*c*f*h + 5*a^3*d*f*h - a*b^2*(5*
d*e*g - 2*c*(f*g + e*h)) + b*(8*a^2*d*f*h - b^2*(2*d*e*g - 5*c*(f*g + e*h)
) - a*b*(10*c*f*h + 3*d*(f*g + e*h)))*x))/(15*b^2*(b*c - a*d)^2*(a + b*x)^
(5/2)) + (f*h*((-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)))/b^2
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(178) = 356$.

Time = 0.33 (sec) , antiderivative size = 1417, normalized size of antiderivative = 6.88

method	result	size
default	Expression too large to display	1417

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-16*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^2*c^2*f*h+4*(d*b)^(1/2
)*((b*x+a)*(d*x+c))^(1/2)*b^4*d^2*e*g*x^2+45*ln(1/2*(2*b*d*x+2*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^3*f*h*x-70*(d*b)^(1
/2)*((b*x+a)*(d*x+c))^(1/2)*a^3*b*d^2*f*h*x+10*(d*b)^(1/2)*((b*x+a)*(d*x+c
))^(1/2)*a*b^3*d^2*e*g*x+50*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^3*b*c*d*
f*h+6*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*d^2*e*h*x^2+6*(d*b)^(1/2)*
((b*x+a)*(d*x+c))^(1/2)*a*b^3*d^2*f*g*x^2-10*(d*b)^(1/2)*((b*x+a)*(d*x+c))
^(1/2)*b^4*c*d*e*h*x^2-10*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^4*c*d*f*g*
x^2-90*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b
)^(1/2))*a^3*b^2*c*d^2*f*h*x+45*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^2*d*f*h*x-90*ln(1/2*(2*b*d*x+2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c*d^2*f
*h*x^2+45*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(
d*b)^(1/2))*a*b^4*c^2*d*f*h*x^2-46*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2
*b^2*d^2*f*h*x^2+80*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c*d*f*h*x^2+
118*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c*d*f*h*x+2*(d*b)^(1/2)*((
b*x+a)*(d*x+c))^(1/2)*a*b^3*c*d*e*h*x-40*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/
2)*a*b^3*c^2*f*h*x+10*a*b^3*c*d*e*g*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-2*
b^4*c*d*e*g*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-30*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*c*d^2*f*h+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(178) = 356$.

Time = 29.93 (sec) , antiderivative size = 1267, normalized size of antiderivative = 6.15

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(7/2),x, algorithm="fricas
")
```

output
```
[1/30*(15*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*f*h*x^3 + 3*(a*b^4*c^2 -
2*a^2*b^3*c*d + a^3*b^2*d^2)*f*h*x^2 + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2)*f*h*x + (a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*f*h)*sqrt(d/b)*log(
8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b
*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(
((2*b^4*d^2*e - (5*b^4*c*d - 3*a*b^3*d^2)*f)*g - ((5*b^4*c*d - 3*a*b^3*d^2
)*e + (15*b^4*c^2 - 40*a*b^3*c*d + 23*a^2*b^2*d^2)*f)*h)*x^2 - (2*a*b^3*c^
2*f + (3*b^4*c^2 - 5*a*b^3*c*d)*e)*g - (2*a*b^3*c^2*e + (8*a^2*b^2*c^2 - 2
5*a^3*b*c*d + 15*a^4*d^2)*f)*h - (((b^4*c*d - 5*a*b^3*d^2)*e + (5*b^4*c^2
- a*b^3*c*d)*f)*g + ((5*b^4*c^2 - a*b^3*c*d)*e + (20*a*b^3*c^2 - 59*a^2*b^
2*c*d + 35*a^3*b*d^2)*f)*h)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(a^3*b^5*c^2 -
 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*x^3 +
 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*
b^5*c*d + a^4*b^4*d^2)*x), -1/15*(15*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*f*h*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*f*h*x^2 + 3*(a^2*b
^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*f*h*x + (a^3*b^2*c^2 - 2*a^4*b*c*d + a
^5*d^2)*f*h)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqr
t(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(((2*b^
4*d^2*e - (5*b^4*c*d - 3*a*b^3*d^2)*f)*g - ((5*b^4*c*d - 3*a*b^3*d^2)*e +
(15*b^4*c^2 - 40*a*b^3*c*d + 23*a^2*b^2*d^2)*f)*h)*x^2 - (2*a*b^3*c^2*f...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)\,(g+hx)}{(a+bx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(7/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/(a + b*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(7/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(178) = 356$.

Time = 0.51 (sec) , antiderivative size = 2589, normalized size of antiderivative = 12.57

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
-sqrt(b*d)*f*h*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2)/b^5 + 4/15*(2*sqrt(b*d)*b^9*c^3*d^2*e*g*abs(b) - 6*sqrt
(b*d)*a*b^8*c^2*d^3*e*g*abs(b) + 6*sqrt(b*d)*a^2*b^7*c*d^4*e*g*abs(b) - 2*
sqrt(b*d)*a^3*b^6*d^5*e*g*abs(b) - 5*sqrt(b*d)*b^9*c^4*d*f*g*abs(b) + 18*s
qrt(b*d)*a*b^8*c^3*d^2*f*g*abs(b) - 24*sqrt(b*d)*a^2*b^7*c^2*d^3*f*g*abs(b
) + 14*sqrt(b*d)*a^3*b^6*c*d^4*f*g*abs(b) - 3*sqrt(b*d)*a^4*b^5*d^5*f*g*ab
s(b) - 5*sqrt(b*d)*b^9*c^4*d*e*h*abs(b) + 18*sqrt(b*d)*a*b^8*c^3*d^2*e*h*a
bs(b) - 24*sqrt(b*d)*a^2*b^7*c^2*d^3*e*h*abs(b) + 14*sqrt(b*d)*a^3*b^6*c*d
^4*e*h*abs(b) - 3*sqrt(b*d)*a^4*b^5*d^5*e*h*abs(b) - 15*sqrt(b*d)*b^9*c^5*
f*h*abs(b) + 85*sqrt(b*d)*a*b^8*c^4*d*f*h*abs(b) - 188*sqrt(b*d)*a^2*b^7*c
^3*d^2*f*h*abs(b) + 204*sqrt(b*d)*a^3*b^6*c^2*d^3*f*h*abs(b) - 109*sqrt(b*
d)*a^4*b^5*c*d^4*f*h*abs(b) + 23*sqrt(b*d)*a^5*b^4*d^5*f*h*abs(b) - 10*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b
^7*c^2*d^2*e*g*abs(b) + 20*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a*b^6*c*d^3*e*g*abs(b) - 10*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^5*d^4*e*g
*abs(b) + 10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*b^7*c^3*d*f*g*abs(b) - 20*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^6*c^2*d^2*f*g*abs(b) + 10*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = \int \frac{(e+f\,x)\,(g+h\,x)\,\sqrt{c+d\,x}}{{(a+b\,x)}^{7/2}}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(7/2),x)
```

output
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1567, normalized size of antiderivative = 7.61

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(7/2),x)
```

output
```
(2*(15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*
sqrt(c + d*x))/sqrt(a*d - b*c))*a**4*d**2*f*h - 30*sqrt(d)*sqrt(b)*sqrt(a
+ b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c)
)*a**3*b*c*d*f*h + 30*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a +
b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*b*d**2*f*h*x + 15*sqrt
(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*
x))/sqrt(a*d - b*c))*a**2*b**2*c**2*f*h - 60*sqrt(d)*sqrt(b)*sqrt(a + b*x)
*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2
*b**2*c*d*f*h*x + 15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b
*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b**2*d**2*f*h*x**2 + 30
*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c
 + d*x))/sqrt(a*d - b*c))*a*b**3*c**2*f*h*x - 30*sqrt(d)*sqrt(b)*sqrt(a +
b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*
a*b**3*c*d*f*h*x**2 + 15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a
 + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**4*c**2*f*h*x**2 + 5*s
qrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*d**2*f*h - 16*sqrt(d)*sqrt(b)*sqrt(a + b
*x)*a**3*b*c*d*f*h + 3*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b*d**2*e*h + 3*s
qrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b*d**2*f*g + 10*sqrt(d)*sqrt(b)*sqrt(a +
 b*x)*a**3*b*d**2*f*h*x + 9*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**2*c**2*f
*h - sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**2*c*d*e*h - sqrt(d)*sqrt(b)*...
```

3.203 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 232

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = -\frac{2(be-af)(bg-ah)(c+dx)^{3/2}}{7b^2(bc-ad)(a+bx)^{7/2}} - \frac{2(10a^2dfh-b^2(4deg-7c(fg+eh))-ab(14cfh+3d(fg+eh)))\,(c+dx)^{3/2}}{35b^2(bc-ad)^2(a+bx)^{5/2}} - \frac{2(15a^2d^2fh+6abd(dfg+deh-7cfh)+b^2(8d^2eg+35c^2fh-14cd(fg+eh)))\,(c+dx)^{3/2}}{105b^2(bc-ad)^3(a+bx)^{3/2}}$$

output
```
-2/7*(-a*f+b*e)*(-a*h+b*g)*(d*x+c)^(3/2)/b^2/(-a*d+b*c)/(b*x+a)^(7/2)-2/35
*(10*a^2*d*f*h-b^2*(4*d*e*g-7*c*(e*h+f*g))-a*b*(14*c*f*h+3*d*(e*h+f*g)))*(
d*x+c)^(3/2)/b^2/(-a*d+b*c)^2/(b*x+a)^(5/2)-2/105*(15*a^2*d^2*f*h+6*a*b*d*
(-7*c*f*h+d*e*h+d*f*g)+b^2*(8*d^2*e*g+35*c^2*f*h-14*c*d*(e*h+f*g)))*(d*x+c
)^(3/2)/b^2/(-a*d+b*c)^3/(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx =$$
$$-\frac{2(c+dx)^{3/2}\left(2ab(d^2x(14eg+3fgx+3ehx)+c^2(3fg+3eh+14fhx)-cd(21eg+29fgx+29ehx+21\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(9/2),x]
```

output
```
(-2*(c + d*x)^(3/2)*(2*a*b*(d^2*x*(14*e*g + 3*f*g*x + 3*e*h*x) + c^2*(3*f*
g + 3*e*h + 14*f*h*x) - c*d*(21*e*g + 29*f*g*x + 29*e*h*x + 21*f*h*x^2)) +
 a^2*(8*c^2*f*h - 2*c*d*(7*f*g + 7*e*h + 6*f*h*x) + d^2*(7*e*(5*g + 3*h*x)
 + 3*f*x*(7*g + 5*h*x))) + b^2*(8*d^2*e*g*x^2 - 2*c*d*x*(6*e*g + 7*f*g*x +
 7*e*h*x) + c^2*(7*f*x*(3*g + 5*h*x) + 3*e*(5*g + 7*h*x)))))/(105*(b*c - a
*d)^3*(a + b*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {162, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx$$
$$\Big\downarrow \; 162$$
$$\frac{\left(15a^2d^2fh+6abd(-7cfh+deh+dfg)+b^2\left(35c^2fh-14cd(eh+fg)+8d^2eg\right)\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}dx}{35b^2(bc-ad)^2}-$$
$$\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(10a^2dfh-ab(14cfh+3d(eh+fg))-b^2(4deg-7c(eh+fg))\right)+2a^2b\left(-\frac{9cfh}{2}+de\right.\right.}{35b^2(a+bx)^{7/2}(bc-ad)^2}$$

$\downarrow$ 48

$$-\frac{2(c+dx)^{3/2}\left(15a^2d^2fh+6abd(-7cfh+deh+dfg)+b^2\left(35c^2fh-14cd(eh+fg)+8d^2eg\right)\right)}{105b^2(a+bx)^{3/2}(bc-ad)^3}-\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(10a^2dfh-ab(14cfh+3d(eh+fg))-b^2(4deg-7c(eh+fg))\right)+2a^2b\left(-\frac{9cfh}{2}+deh\right.\right.}{35b^2(a+bx)^{7/2}(bc-ad)^2}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(9/2),x]
```

output
```
(-2*(15*a^2*d^2*f*h + 6*a*b*d*(d*f*g + d*e*h - 7*c*f*h) + b^2*(8*d^2*e*g +
 35*c^2*f*h - 14*c*d*(f*g + e*h)))*(c + d*x)^(3/2))/(105*b^2*(b*c - a*d)^3
*(a + b*x)^(3/2)) - (2*(c + d*x)^(3/2)*(5*b^3*c*e*g + 5*a^3*d*f*h - 2*a*b^
2*((9*d*e*g)/2 - c*f*g - c*e*h) + 2*a^2*b*(d*f*g + d*e*h - (9*c*f*h)/2) +
b*(10*a^2*d*f*h - b^2*(4*d*e*g - 7*c*(f*g + e*h)) - a*b*(14*c*f*h + 3*d*(f
*g + e*h)))*x))/(35*b^2*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.30

method	result
default	$\frac{2(xd+c)^{\frac{3}{2}}\left(15a^2d^2fh\,x^2-42abcdfh\,x^2+6ab\,d^2eh\,x^2+6ab\,d^2fg\,x^2+35b^2c^2fh\,x^2-14b^2cdeh\,x^2-14b^2cdfg\,x^2+8b^2d^2eg\,x^2-12a^2cdfh\right.}{}$
gosper	$\frac{2(xd+c)^{\frac{3}{2}}\left(15a^2d^2fh\,x^2-42abcdfh\,x^2+6ab\,d^2eh\,x^2+6ab\,d^2fg\,x^2+35b^2c^2fh\,x^2-14b^2cdeh\,x^2-14b^2cdfg\,x^2+8b^2d^2eg\,x^2-12a^2cdfh\right.}{}$
orering	$\frac{2(xd+c)^{\frac{3}{2}}\left(15a^2d^2fh\,x^2-42abcdfh\,x^2+6ab\,d^2eh\,x^2+6ab\,d^2fg\,x^2+35b^2c^2fh\,x^2-14b^2cdeh\,x^2-14b^2cdfg\,x^2+8b^2d^2eg\,x^2-12a^2cdfh\right.}{}$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(d*x+c)^(3/2)/(b*x+a)^(7/2)*(15*a^2*d^2*f*h*x^2-42*a*b*c*d*f*h*x^2+6
*a*b*d^2*e*h*x^2+6*a*b*d^2*f*g*x^2+35*b^2*c^2*f*h*x^2-14*b^2*c*d*e*h*x^2-1
4*b^2*c*d*f*g*x^2+8*b^2*d^2*e*g*x^2-12*a^2*c*d*f*h*x+21*a^2*d^2*e*h*x+21*a
^2*d^2*f*g*x+28*a*b*c^2*f*h*x-58*a*b*c*d*e*h*x-58*a*b*c*d*f*g*x+28*a*b*d^2
*e*g*x+21*b^2*c^2*e*h*x+21*b^2*c^2*f*g*x-12*b^2*c*d*e*g*x+8*a^2*c^2*f*h-14
*a^2*c*d*e*h-14*a^2*c*d*f*g+35*a^2*d^2*e*g+6*a*b*c^2*e*h+6*a*b*c^2*f*g-42*
a*b*c*d*e*g+15*b^2*c^2*e*g)/(a*d-b*c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(214) = 428$.

Time = 74.33 (sec) , antiderivative size = 656, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = -\frac{2\left(\left(2\left(4\,b^2d^3e-\left(7\,b^2cd^2-3\,abd^3\right)f\right)g-\left(2\left(7\,b^2cd^2-3\,abd^3\right)e-\left(35\,b^2c^2d-42\,abcd^2+15\,a^2d^3\right)f\right)h\right)x^3-\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(9/2),x, algorithm="fricas
")
```

output
```
-2/105*((2*(4*b^2*d^3*e - (7*b^2*c*d^2 - 3*a*b*d^3)*f)*g - (2*(7*b^2*c*d^2
 - 3*a*b*d^3)*e - (35*b^2*c^2*d - 42*a*b*c*d^2 + 15*a^2*d^3)*f)*h)*x^3 - (
(4*(b^2*c*d^2 - 7*a*b*d^3)*e - (7*b^2*c^2*d - 52*a*b*c*d^2 + 21*a^2*d^3)*f
)*g - ((7*b^2*c^2*d - 52*a*b*c*d^2 + 21*a^2*d^3)*e + (35*b^2*c^3 - 14*a*b*
c^2*d + 3*a^2*c*d^2)*f)*h)*x^2 + ((15*b^2*c^3 - 42*a*b*c^2*d + 35*a^2*c*d^
2)*e + 2*(3*a*b*c^3 - 7*a^2*c^2*d)*f)*g + 2*(4*a^2*c^3*f + (3*a*b*c^3 - 7*
a^2*c^2*d)*e)*h + (((3*b^2*c^2*d - 14*a*b*c*d^2 + 35*a^2*d^3)*e + (21*b^2*
c^3 - 52*a*b*c^2*d + 7*a^2*c*d^2)*f)*g + ((21*b^2*c^3 - 52*a*b*c^2*d + 7*a
^2*c*d^2)*e + 4*(7*a*b*c^3 - a^2*c^2*d)*f)*h)*x)*sqrt(b*x + a)*sqrt(d*x +
c)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3
*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5*c^3 - 3*a^3*b^4
*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^3 - 3*a^4*b^3*c
^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)\,(g+hx)}{(a+bx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(9/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/(a + b*x)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(9/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3978 vs. $2(214) = 428$.

Time = 0.53 (sec) , antiderivative size = 3978, normalized size of antiderivative = 17.15

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
-4/105*(8*sqrt(b*d)*b^12*c^4*d^3*e*g*abs(b) - 32*sqrt(b*d)*a*b^11*c^3*d^4*
e*g*abs(b) + 48*sqrt(b*d)*a^2*b^10*c^2*d^5*e*g*abs(b) - 32*sqrt(b*d)*a^3*b
^9*c*d^6*e*g*abs(b) + 8*sqrt(b*d)*a^4*b^8*d^7*e*g*abs(b) - 14*sqrt(b*d)*b^
12*c^5*d^2*f*g*abs(b) + 62*sqrt(b*d)*a*b^11*c^4*d^3*f*g*abs(b) - 108*sqrt(
b*d)*a^2*b^10*c^3*d^4*f*g*abs(b) + 92*sqrt(b*d)*a^3*b^9*c^2*d^5*f*g*abs(b)
 - 38*sqrt(b*d)*a^4*b^8*c*d^6*f*g*abs(b) + 6*sqrt(b*d)*a^5*b^7*d^7*f*g*abs
(b) - 14*sqrt(b*d)*b^12*c^5*d^2*e*h*abs(b) + 62*sqrt(b*d)*a*b^11*c^4*d^3*e
*h*abs(b) - 108*sqrt(b*d)*a^2*b^10*c^3*d^4*e*h*abs(b) + 92*sqrt(b*d)*a^3*b
^9*c^2*d^5*e*h*abs(b) - 38*sqrt(b*d)*a^4*b^8*c*d^6*e*h*abs(b) + 6*sqrt(b*d
)*a^5*b^7*d^7*e*h*abs(b) + 35*sqrt(b*d)*b^12*c^6*d*f*h*abs(b) - 182*sqrt(b
*d)*a*b^11*c^5*d^2*f*h*abs(b) + 393*sqrt(b*d)*a^2*b^10*c^4*d^3*f*h*abs(b)
- 452*sqrt(b*d)*a^3*b^9*c^3*d^4*f*h*abs(b) + 293*sqrt(b*d)*a^4*b^8*c^2*d^5
*f*h*abs(b) - 102*sqrt(b*d)*a^5*b^7*c*d^6*f*h*abs(b) + 15*sqrt(b*d)*a^6*b^
6*d^7*f*h*abs(b) - 56*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*b^10*c^3*d^3*e*g*abs(b) + 168*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^9*c^2*d^4*e*g*
abs(b) - 168*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^2*b^8*c*d^5*e*g*abs(b) + 56*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^7*d^6*e*g*abs(b) + 98
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*...
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = \frac{\sqrt{c+d\,x}\,\left(\frac{30\,b^2\,c^3\,e\,g+16\,a^2\,c^3\,f\,h+70\,a^2\,c\,d^2\,e\,g-28\,a^2\,c^2\,d\,e\,h-28\,a^2\,c^2\,d\,f\,g+12\,a\,b\,c^3\,e\,h+1}{105\,b^3\,(a\,d-b\,c)^3}\right.}{}$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(9/2),x)
```

output
```
((c + d*x)^(1/2)*((30*b^2*c^3*e*g + 16*a^2*c^3*f*h + 70*a^2*c*d^2*e*g - 28
*a^2*c^2*d*e*h - 28*a^2*c^2*d*f*g + 12*a*b*c^3*e*h + 12*a*b*c^3*f*g - 84*a
*b*c^2*d*e*g)/(105*b^3*(a*d - b*c)^3) + (x*(70*a^2*d^3*e*g + 42*b^2*c^3*e*
h + 42*b^2*c^3*f*g + 14*a^2*c*d^2*e*h + 14*a^2*c*d^2*f*g + 6*b^2*c^2*d*e*g
 - 8*a^2*c^2*d*f*h + 56*a*b*c^3*f*h - 28*a*b*c*d^2*e*g - 104*a*b*c^2*d*e*h
 - 104*a*b*c^2*d*f*g))/(105*b^3*(a*d - b*c)^3) + (x^3*(16*b^2*d^3*e*g + 30
*a^2*d^3*f*h - 28*b^2*c*d^2*e*h - 28*b^2*c*d^2*f*g + 70*b^2*c^2*d*f*h + 12
*a*b*d^3*e*h + 12*a*b*d^3*f*g - 84*a*b*c*d^2*f*h))/(105*b^3*(a*d - b*c)^3)
 + (x^2*(42*a^2*d^3*e*h + 42*a^2*d^3*f*g + 70*b^2*c^3*f*h - 8*b^2*c*d^2*e*
g + 6*a^2*c*d^2*f*h + 14*b^2*c^2*d*e*h + 14*b^2*c^2*d*f*g + 56*a*b*d^3*e*g
 - 104*a*b*c*d^2*e*h - 104*a*b*c*d^2*f*g - 28*a*b*c^2*d*f*h))/(105*b^3*(a*
d - b*c)^3)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (3*a*x^2*
(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

Reduce [B] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 1732, normalized size of antiderivative = 7.47

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(9/2),x)
```

output

```
(2*(15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*d**3*f*h - 18*sqrt(d)*sqrt(b)*sq
rt(a + b*x)*a**4*b*c*d**2*f*h - 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b*d**
3*e*h - 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b*d**3*f*g + 45*sqrt(d)*sqrt(
b)*sqrt(a + b*x)*a**4*b*d**3*f*h*x - 5*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*
b**2*c**2*d*f*h + 14*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**2*c*d**2*e*h +
14*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**2*c*d**2*f*g - 54*sqrt(d)*sqrt(b)
*sqrt(a + b*x)*a**3*b**2*c*d**2*f*h*x - 8*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*
*3*b**2*d**3*e*g - 18*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**2*d**3*e*h*x -
 18*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**2*d**3*f*g*x + 45*sqrt(d)*sqrt(b
)*sqrt(a + b*x)*a**3*b**2*d**3*f*h*x**2 - 15*sqrt(d)*sqrt(b)*sqrt(a + b*x)
*a**2*b**3*c**2*d*f*h*x + 42*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**3*c*d**
2*e*h*x + 42*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**3*c*d**2*f*g*x - 54*sqr
t(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**3*c*d**2*f*h*x**2 - 24*sqrt(d)*sqrt(b)*
sqrt(a + b*x)*a**2*b**3*d**3*e*g*x - 18*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2
*b**3*d**3*e*h*x**2 - 18*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**3*d**3*f*g*
x**2 + 15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**3*d**3*f*h*x**3 - 15*sqrt(
d)*sqrt(b)*sqrt(a + b*x)*a*b**4*c**2*d*f*h*x**2 + 42*sqrt(d)*sqrt(b)*sqrt(
a + b*x)*a*b**4*c*d**2*e*h*x**2 + 42*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*b**4*
c*d**2*f*g*x**2 - 18*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*b**4*c*d**2*f*h*x**3
- 24*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*b**4*d**3*e*g*x**2 - 6*sqrt(d)*sqr...
```

3.204 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 330

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = -\frac{2(be-af)(bg-ah)(c+dx)^{3/2}}{9b^2(bc-ad)(a+bx)^{9/2}} - \frac{2(4a^2dfh-ab(dfg+deh+6cfh)-b^2(2deg-3c(fg+eh)))(c+dx)^{3/2}}{21b^2(bc-ad)^2(a+bx)^{7/2}} - \frac{2(5a^2d^2fh-2abd(9cfh-2d(fg+eh))+b^2(8d^2eg+21c^2fh-12cd(fg+eh)))(c+dx)^{3/2}}{105b^2(bc-ad)^3(a+bx)^{5/2}} + \frac{4d(5a^2d^2fh-2abd(9cfh-2d(fg+eh))+b^2(8d^2eg+21c^2fh-12cd(fg+eh)))(c+dx)^{3/2}}{315b^2(bc-ad)^4(a+bx)^{3/2}}$$

output
```
-2/9*(-a*f+b*e)*(-a*h+b*g)*(d*x+c)^(3/2)/b^2/(-a*d+b*c)/(b*x+a)^(9/2)-2/21
*(4*a^2*d*f*h-a*b*(6*c*f*h+d*e*h+d*f*g)-b^2*(2*d*e*g-3*c*(e*h+f*g)))*(d*x+
c)^(3/2)/b^2/(-a*d+b*c)^2/(b*x+a)^(7/2)-2/105*(5*a^2*d^2*f*h-2*a*b*d*(9*c*
f*h-2*d*(e*h+f*g))+b^2*(8*d^2*e*g+21*c^2*f*h-12*c*d*(e*h+f*g)))*(d*x+c)^(3
/2)/b^2/(-a*d+b*c)^3/(b*x+a)^(5/2)+4/315*d*(5*a^2*d^2*f*h-2*a*b*d*(9*c*f*h
-2*d*(e*h+f*g))+b^2*(8*d^2*e*g+21*c^2*f*h-12*c*d*(e*h+f*g)))*(d*x+c)^(3/2)
/b^2/(-a*d+b*c)^4/(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = \frac{2(c+dx)^{3/2}\left(ab^2(8d^3x^2(9eg+fgx+ehx)-2c^3(5fg+5eh+18fhx)\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(11/2),x]
```

output
```
(2*(c + d*x)^(3/2)*(a*b^2*(8*d^3*x^2*(9*e*g + f*g*x + e*h*x) - 2*c^3*(5*f*
g + 5*e*h + 18*f*h*x) - 12*c*d^2*x*(9*e*g + 10*f*g*x + 10*e*h*x + 3*f*h*x^
2) + 3*c^2*d*(45*e*g + 59*f*g*x + 59*e*h*x + 81*f*h*x^2)) + 3*a^3*d*(8*c^2
*f*h - 2*c*d*(7*f*g + 7*e*h + 6*f*h*x) + d^2*(7*e*(5*g + 3*h*x) + 3*f*x*(7
*g + 5*h*x))) - b^3*(-16*d^3*e*g*x^3 + 24*c*d^2*x^2*(f*g*x + e*(g + h*x))
- 6*c^2*d*x*(e*(5*g + 6*h*x) + f*x*(6*g + 7*h*x)) + c^3*(9*f*x*(5*g + 7*h*
x) + 5*e*(7*g + 9*h*x))) + a^2*b*(-8*c^3*f*h + 12*c^2*d*(3*f*g + 3*e*h + 1
0*f*h*x) + 2*d^3*x*(9*e*(7*g + 2*h*x) + f*x*(18*g + 5*h*x)) - 3*c*d^2*(9*e
*(7*g + 9*h*x) + f*x*(81*g + 59*h*x)))))/(315*(b*c - a*d)^4*(a + b*x)^(9/2
))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {162, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx$$

$$\Big\downarrow \; 162$$

$$\frac{\left(5a^2d^2fh-2abd(9cfh-2d(eh+fg))+b^2\left(21c^2fh-12cd(eh+fg)+8d^2eg\right)\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}dx}{21b^2(bc-ad)^2} - \frac{2(c+dx)^{3/2}\left(5a^3dfh+3bx\left(4a^2dfh-ab(6cfh+deh+dfg)-b^2(2deg-3c(eh+fg))\right)-a^2b(11cfh-4d(eh+\right.}{63b^2(a+bx)^{9/2}(bc-ad)^2}$$

$$\Big\downarrow \text{55}$$

$$\frac{\left(5a^2d^2fh-2abd(9cfh-2d(eh+fg))+b^2\left(21c^2fh-12cd(eh+fg)+8d^2eg\right)\right)\left(-\frac{2d\int\frac{\sqrt{c+dx}}{(a+bx)^{5/2}}dx}{5(bc-ad)}-\frac{2(c+dx)^3}{5(a+bx)^{5/2}(b}\right.}{21b^2(bc-ad)^2}$$
$$\frac{2(c+dx)^{3/2}\left(5a^3dfh+3bx\left(4a^2dfh-ab(6cfh+deh+dfg)-b^2(2deg-3c(eh+fg))\right)-a^2b(11cfh-4d(eh+\right.}{63b^2(a+bx)^{9/2}(bc-ad)^2}$$

$$\Big\downarrow \text{48}$$

$$\frac{\left(\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}\right)\left(5a^2d^2fh-2abd(9cfh-2d(eh+fg))+b^2\left(21c^2fh-12cd(eh+fg)+\right.\right.}{21b^2(bc-ad)^2}$$
$$\frac{2(c+dx)^{3/2}\left(5a^3dfh+3bx\left(4a^2dfh-ab(6cfh+deh+dfg)-b^2(2deg-3c(eh+fg))\right)-a^2b(11cfh-4d(eh+\right.}{63b^2(a+bx)^{9/2}(bc-ad)^2}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(11/2),x]
```

output
```
(-2*(c + d*x)^(3/2)*(7*b^3*c*e*g + 5*a^3*d*f*h - a*b^2*(13*d*e*g - 2*c*(f*
g + e*h)) - a^2*b*(11*c*f*h - 4*d*(f*g + e*h)) + 3*b*(4*a^2*d*f*h - a*b*(d
*f*g + d*e*h + 6*c*f*h) - b^2*(2*d*e*g - 3*c*(f*g + e*h)))*x))/(63*b^2*(b*
c - a*d)^2*(a + b*x)^(9/2)) + ((5*a^2*d^2*f*h - 2*a*b*d*(9*c*f*h - 2*d*(f*
g + e*h)) + b^2*(8*d^2*e*g + 21*c^2*f*h - 12*c*d*(f*g + e*h)))*((-2*(c + d
*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2))/(15*(b*
c - a*d)^2*(a + b*x)^(3/2))))/(21*b^2*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(306) = 612.

Time = 0.35 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.86

method	result
default	$\frac{2(xd+c)^{\frac{3}{2}}\left(10a^2b\,d^3fh\,x^3-36a\,b^2c\,d^2fh\,x^3+8a\,b^2d^3eh\,x^3+8a\,b^2d^3fg\,x^3+42b^3c^2dfh\,x^3-24b^3c\,d^2eh\,x^3-24b^3c\,d^2fg\,x^3+16b^3d^3eg\,x\right.}{}$
gosper	$\frac{2(xd+c)^{\frac{3}{2}}\left(10a^2b\,d^3fh\,x^3-36a\,b^2c\,d^2fh\,x^3+8a\,b^2d^3eh\,x^3+8a\,b^2d^3fg\,x^3+42b^3c^2dfh\,x^3-24b^3c\,d^2eh\,x^3-24b^3c\,d^2fg\,x^3+16b^3d^3eg\,x\right.}{}$
orering	$\frac{2(xd+c)^{\frac{3}{2}}\left(10a^2b\,d^3fh\,x^3-36a\,b^2c\,d^2fh\,x^3+8a\,b^2d^3eh\,x^3+8a\,b^2d^3fg\,x^3+42b^3c^2dfh\,x^3-24b^3c\,d^2eh\,x^3-24b^3c\,d^2fg\,x^3+16b^3d^3eg\,x\right.}{}$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(d*x+c)^(3/2)/(b*x+a)^(9/2)*(10*a^2*b*d^3*f*h*x^3-36*a*b^2*c*d^2*f*h
*x^3+8*a*b^2*d^3*e*h*x^3+8*a*b^2*d^3*f*g*x^3+42*b^3*c^2*d*f*h*x^3-24*b^3*c
*d^2*e*h*x^3-24*b^3*c*d^2*f*g*x^3+16*b^3*d^3*e*g*x^3+45*a^3*d^3*f*h*x^2-17
7*a^2*b*c*d^2*f*h*x^2+36*a^2*b*d^3*e*h*x^2+36*a^2*b*d^3*f*g*x^2+243*a*b^2*
c^2*d*f*h*x^2-120*a*b^2*c*d^2*e*h*x^2-120*a*b^2*c*d^2*f*g*x^2+72*a*b^2*d^3
*e*g*x^2-63*b^3*c^3*f*h*x^2+36*b^3*c^2*d*e*h*x^2+36*b^3*c^2*d*f*g*x^2-24*b
^3*c*d^2*e*g*x^2-36*a^3*c*d^2*f*h*x+63*a^3*d^3*e*h*x+63*a^3*d^3*f*g*x+120*
a^2*b*c^2*d*f*h*x-243*a^2*b*c*d^2*e*h*x-243*a^2*b*c*d^2*f*g*x+126*a^2*b*d^
3*e*g*x-36*a*b^2*c^3*f*h*x+177*a*b^2*c^2*d*e*h*x+177*a*b^2*c^2*d*f*g*x-108
*a*b^2*c*d^2*e*g*x-45*b^3*c^3*e*h*x-45*b^3*c^3*f*g*x+30*b^3*c^2*d*e*g*x+24
*a^3*c^2*d*f*h-42*a^3*c*d^2*e*h-42*a^3*c*d^2*f*g+105*a^3*d^3*e*g-8*a^2*b*c
^3*f*h+36*a^2*b*c^2*d*e*h+36*a^2*b*c^2*d*f*g-189*a^2*b*c*d^2*e*g-10*a*b^2*
c^3*e*h-10*a*b^2*c^3*f*g+135*a*b^2*c^2*d*e*g-35*b^3*c^3*e*g)/(a*d-b*c)^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(11/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)\,(g+hx)}{(a+bx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(11/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/(a + b*x)**(11/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(11/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5705 vs. 2(306) = 612.

Time = 0.69 (sec) , antiderivative size = 5705, normalized size of antiderivative = 17.29

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(11/2),x, algorithm="giac"
)
```

output
```
8/315*(8*sqrt(b*d)*b^14*c^5*d^4*e*g*abs(b) - 40*sqrt(b*d)*a*b^13*c^4*d^5*e
*g*abs(b) + 80*sqrt(b*d)*a^2*b^12*c^3*d^6*e*g*abs(b) - 80*sqrt(b*d)*a^3*b^
11*c^2*d^7*e*g*abs(b) + 40*sqrt(b*d)*a^4*b^10*c*d^8*e*g*abs(b) - 8*sqrt(b*
d)*a^5*b^9*d^9*e*g*abs(b) - 12*sqrt(b*d)*b^14*c^6*d^3*f*g*abs(b) + 64*sqrt
(b*d)*a*b^13*c^5*d^4*f*g*abs(b) - 140*sqrt(b*d)*a^2*b^12*c^4*d^5*f*g*abs(b
) + 160*sqrt(b*d)*a^3*b^11*c^3*d^6*f*g*abs(b) - 100*sqrt(b*d)*a^4*b^10*c^2
*d^7*f*g*abs(b) + 32*sqrt(b*d)*a^5*b^9*c*d^8*f*g*abs(b) - 4*sqrt(b*d)*a^6*
b^8*d^9*f*g*abs(b) - 12*sqrt(b*d)*b^14*c^6*d^3*e*h*abs(b) + 64*sqrt(b*d)*a
*b^13*c^5*d^4*e*h*abs(b) - 140*sqrt(b*d)*a^2*b^12*c^4*d^5*e*h*abs(b) + 160
*sqrt(b*d)*a^3*b^11*c^3*d^6*e*h*abs(b) - 100*sqrt(b*d)*a^4*b^10*c^2*d^7*e*
h*abs(b) + 32*sqrt(b*d)*a^5*b^9*c*d^8*e*h*abs(b) - 4*sqrt(b*d)*a^6*b^8*d^9
*e*h*abs(b) + 21*sqrt(b*d)*b^14*c^7*d^2*f*h*abs(b) - 123*sqrt(b*d)*a*b^13*
c^6*d^3*f*h*abs(b) + 305*sqrt(b*d)*a^2*b^12*c^5*d^4*f*h*abs(b) - 415*sqrt(
b*d)*a^3*b^11*c^4*d^5*f*h*abs(b) + 335*sqrt(b*d)*a^4*b^10*c^3*d^6*f*h*abs(
b) - 161*sqrt(b*d)*a^5*b^9*c^2*d^7*f*h*abs(b) + 43*sqrt(b*d)*a^6*b^8*c*d^8
*f*h*abs(b) - 5*sqrt(b*d)*a^7*b^7*d^9*f*h*abs(b) - 72*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^12*c^4*d^4*e*g*a
bs(b) + 288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*a*b^11*c^3*d^5*e*g*abs(b) - 432*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^2*d^6*e*g*ab...
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx=\frac{\sqrt{c+d\,x}\,\left(\frac{x^4\,\left(32\,b^3\,d^4\,e\,g+16\,a\,b^2\,d^4\,e\,h+16\,a\,b^2\,d^4\,f\,g+20\,a^2\,b\,d^4\,f\,h-48\,b^3\,c\,d^3\,e\,h-48\,b^3\,c\,d^3}{315\,b^4\,(a\,d-b\,c)^4}\right.}{}$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(11/2),x)
```

output
```
((c + d*x)^(1/2)*((x^4*(32*b^3*d^4*e*g + 16*a*b^2*d^4*e*h + 16*a*b^2*d^4*f
*g + 20*a^2*b*d^4*f*h - 48*b^3*c*d^3*e*h - 48*b^3*c*d^3*f*g + 84*b^3*c^2*d
^2*f*h - 72*a*b^2*c*d^3*f*h))/(315*b^4*(a*d - b*c)^4) - (x*(90*b^3*c^4*e*h
 - 210*a^3*d^4*e*g + 90*b^3*c^4*f*g + 72*a*b^2*c^4*f*h - 42*a^3*c*d^3*e*h
- 42*a^3*c*d^3*f*g + 10*b^3*c^3*d*e*g + 24*a^3*c^2*d^2*f*h + 126*a^2*b*c*d
^3*e*g - 334*a*b^2*c^3*d*e*h - 334*a*b^2*c^3*d*f*g - 224*a^2*b*c^3*d*f*h -
 54*a*b^2*c^2*d^2*e*g + 414*a^2*b*c^2*d^2*e*h + 414*a^2*b*c^2*d^2*f*g))/(3
15*b^4*(a*d - b*c)^4) - (70*b^3*c^4*e*g + 20*a*b^2*c^4*e*h + 20*a*b^2*c^4*
f*g + 16*a^2*b*c^4*f*h - 210*a^3*c*d^3*e*g - 48*a^3*c^3*d*f*h + 84*a^3*c^2
*d^2*e*h + 84*a^3*c^2*d^2*f*g - 270*a*b^2*c^3*d*e*g - 72*a^2*b*c^3*d*e*h -
 72*a^2*b*c^3*d*f*g + 378*a^2*b*c^2*d^2*e*g)/(315*b^4*(a*d - b*c)^4) + (x^
2*(126*a^3*d^4*e*h + 126*a^3*d^4*f*g - 126*b^3*c^4*f*h + 252*a^2*b*d^4*e*g
 + 18*a^3*c*d^3*f*h - 18*b^3*c^3*d*e*h - 18*b^3*c^3*d*f*g + 12*b^3*c^2*d^2
*e*g - 72*a*b^2*c*d^3*e*g - 414*a^2*b*c*d^3*e*h - 414*a^2*b*c*d^3*f*g + 41
4*a*b^2*c^3*d*f*h + 114*a*b^2*c^2*d^2*e*h + 114*a*b^2*c^2*d^2*f*g - 114*a^
2*b*c^2*d^2*f*h))/(315*b^4*(a*d - b*c)^4) + (2*d*x^3*(9*a*d - b*c)*(8*b^2*
d^2*e*g + 5*a^2*d^2*f*h + 21*b^2*c^2*f*h + 4*a*b*d^2*e*h + 4*a*b*d^2*f*g -
 12*b^2*c*d*e*h - 12*b^2*c*d*f*g - 18*a*b*c*d*f*h))/(315*b^4*(a*d - b*c)^4
)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x
)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*x)^(1/2))/...
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 2628, normalized size of antiderivative = 7.96

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(11/2),x)
```

output

```
(2*( - 10*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**6*d**4*f*h + 36*sqrt(d)*sqrt(b)
*sqrt(a + b*x)*a**5*b*c*d**3*f*h - 8*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b*
d**4*e*h - 8*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b*d**4*f*g - 40*sqrt(d)*sq
rt(b)*sqrt(a + b*x)*a**5*b*d**4*f*h*x - 42*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a
**4*b**2*c**2*d**2*f*h + 24*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**2*c*d**3
*e*h + 24*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**2*c*d**3*f*g + 144*sqrt(d)
*sqrt(b)*sqrt(a + b*x)*a**4*b**2*c*d**3*f*h*x - 16*sqrt(d)*sqrt(b)*sqrt(a
+ b*x)*a**4*b**2*d**4*e*g - 32*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**2*d**
4*e*h*x - 32*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**2*d**4*f*g*x - 60*sqrt(
d)*sqrt(b)*sqrt(a + b*x)*a**4*b**2*d**4*f*h*x**2 - 168*sqrt(d)*sqrt(b)*sqr
t(a + b*x)*a**3*b**3*c**2*d**2*f*h*x + 96*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*
*3*b**3*c*d**3*e*h*x + 96*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**3*c*d**3*f
*g*x + 216*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**3*c*d**3*f*h*x**2 - 64*sq
rt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**3*d**4*e*g*x - 48*sqrt(d)*sqrt(b)*sqrt
(a + b*x)*a**3*b**3*d**4*e*h*x**2 - 48*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*
b**3*d**4*f*g*x**2 - 40*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**3*d**4*f*h*x
**3 - 252*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**4*c**2*d**2*f*h*x**2 + 144
*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b**4*c*d**3*e*h*x**2 + 144*sqrt(d)*sqr
t(b)*sqrt(a + b*x)*a**2*b**4*c*d**3*f*g*x**2 + 144*sqrt(d)*sqrt(b)*sqrt(a
+ b*x)*a**2*b**4*c*d**3*f*h*x**3 - 96*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**...
```

3.205 $\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 432

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx=-\frac{2(be-af)(bg-ah)(c+dx)^{3/2}}{11b^2(bc-ad)(a+bx)^{11/2}}$$
$$-\frac{2(14a^2dfh-b^2(8deg-11c(fg+eh))-ab(22cfh+3d(fg+eh)))\,(c+dx)^{3/2}}{99b^2(bc-ad)^2(a+bx)^{9/2}}$$
$$-\frac{2(5a^2d^2fh-2abd(11cfh-3d(fg+eh))+b^2(16d^2eg+33c^2fh-22cd(fg+eh)))\,(c+dx)^{3/2}}{231b^2(bc-ad)^3(a+bx)^{7/2}}$$
$$+\frac{8d(5a^2d^2fh-2abd(11cfh-3d(fg+eh))+b^2(16d^2eg+33c^2fh-22cd(fg+eh)))\,(c+dx)^{3/2}}{1155b^2(bc-ad)^4(a+bx)^{5/2}}$$
$$-\frac{16d^2(5a^2d^2fh-2abd(11cfh-3d(fg+eh))+b^2(16d^2eg+33c^2fh-22cd(fg+eh)))\,(c+dx)^{3/2}}{3465b^2(bc-ad)^5(a+bx)^{3/2}}$$

output
```
-2/11*(-a*f+b*e)*(-a*h+b*g)*(d*x+c)^(3/2)/b^2/(-a*d+b*c)/(b*x+a)^(11/2)-2/
99*(14*a^2*d*f*h-b^2*(8*d*e*g-11*c*(e*h+f*g))-a*b*(22*c*f*h+3*d*(e*h+f*g))
)*(d*x+c)^(3/2)/b^2/(-a*d+b*c)^2/(b*x+a)^(9/2)-2/231*(5*a^2*d^2*f*h-2*a*b*
d*(11*c*f*h-3*d*(e*h+f*g))+b^2*(16*d^2*e*g+33*c^2*f*h-22*c*d*(e*h+f*g)))*(
d*x+c)^(3/2)/b^2/(-a*d+b*c)^3/(b*x+a)^(7/2)+8/1155*d*(5*a^2*d^2*f*h-2*a*b*
d*(11*c*f*h-3*d*(e*h+f*g))+b^2*(16*d^2*e*g+33*c^2*f*h-22*c*d*(e*h+f*g)))*(
d*x+c)^(3/2)/b^2/(-a*d+b*c)^4/(b*x+a)^(5/2)-16/3465*d^2*(5*a^2*d^2*f*h-2*a
*b*d*(11*c*f*h-3*d*(e*h+f*g))+b^2*(16*d^2*e*g+33*c^2*f*h-22*c*d*(e*h+f*g))
)*(d*x+c)^(3/2)/b^2/(-a*d+b*c)^5/(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx =$$
$$-\frac{2(c+dx)^{3/2}\left(22a^3bd(-8c^3fh+3c^2d(9fg+9eh+26fhx)+d^3x(84eg+27fgx+27ehx+10fhx^2)-6\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(13/2),x]
```

output
```
(-2*(c + d*x)^(3/2)*(22*a^3*b*d*(-8*c^3*f*h + 3*c^2*d*(9*f*g + 9*e*h + 26*
f*h*x) + d^3*x*(84*e*g + 27*f*g*x + 27*e*h*x + 10*f*h*x^2) - 6*c*d^2*(21*e
*g + 26*f*g*x + 26*e*h*x + 19*f*h*x^2)) + 2*a*b^3*(8*d^4*x^3*(44*e*g + 3*f
*g*x + 3*e*h*x) + 5*c^4*(7*f*g + 7*e*h + 22*f*h*x) - 8*c*d^3*x^2*(66*e*g +
 65*f*g*x + 65*e*h*x + 11*f*h*x^2) + 3*c^2*d^2*x*(220*e*g + 257*f*g*x + 25
7*e*h*x + 286*f*h*x^2) - 2*c^3*d*(385*e*g + 480*f*g*x + 480*e*h*x + 627*f*
h*x^2)) + 33*a^4*d^2*(8*c^2*f*h - 2*c*d*(7*f*g + 7*e*h + 6*f*h*x) + d^2*(7
*e*(5*g + 3*h*x) + 3*f*x*(7*g + 5*h*x))) + b^4*(128*d^4*e*g*x^4 - 16*c*d^3
*x^3*(12*e*g + 11*f*g*x + 11*e*h*x) + 5*c^4*(11*f*x*(7*g + 9*h*x) + 7*e*(9
*g + 11*h*x)) + 24*c^2*d^2*x^2*(11*f*x*(g + h*x) + e*(10*g + 11*h*x)) - 2*
c^3*d*x*(33*f*x*(5*g + 6*h*x) + 5*e*(28*g + 33*h*x))) + 2*a^2*b^2*(20*c^4*
f*h - c^3*d*(165*f*g + 165*e*h + 514*f*h*x) + 4*d^4*x^2*(33*e*(6*g + h*x)
+ f*x*(33*g + 5*h*x)) - c*d^3*x*(99*e*(12*g + 13*h*x) + f*x*(1287*g + 514*
h*x)) + 3*c^2*d^2*(33*e*(15*g + 19*h*x) + f*x*(627*g + 799*h*x)))))/(3465*
(b*c - a*d)^5*(a + b*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {162, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx$$

$$\Big\downarrow \; 162$$

$$\frac{\left(5a^2d^2fh-2abd(11cfh-3d(eh+fg))+b^2\left(33c^2fh-22cd(eh+fg)+16d^2eg\right)\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}dx}{33b^2(bc-ad)^2}-\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(14a^2dfh-ab(22cfh+3d(eh+fg))-b^2(8deg-11c(eh+fg))\right)-a^2b(13cfh-6d(e\right.}{99b^2(a+bx)^{11/2}(bc-ad)^2}$$

$$\Big\downarrow \; 55$$

$$\frac{\left(5a^2d^2fh-2abd(11cfh-3d(eh+fg))+b^2\left(33c^2fh-22cd(eh+fg)+16d^2eg\right)\right)\left(-\frac{4d\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}dx}{7(bc-ad)}-\frac{2(c+d}{7(a+bx)^{7/}}\right.}{33b^2(bc-ad)^2}$$
$$\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(14a^2dfh-ab(22cfh+3d(eh+fg))-b^2(8deg-11c(eh+fg))\right)-a^2b(13cfh-6d(e\right.}{99b^2(a+bx)^{11/2}(bc-ad)^2}$$

$$\Big\downarrow \; 55$$

$$\frac{\left(5a^2d^2fh-2abd(11cfh-3d(eh+fg))+b^2\left(33c^2fh-22cd(eh+fg)+16d^2eg\right)\right)\left(-\frac{4d\left(-\frac{2d\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}dx}{5(bc-ad)}-\frac{2(c}{5(a+bx}\right.}{7(bc-ad)}\right.}{33b^2(bc-ad)^2}$$
$$\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(14a^2dfh-ab(22cfh+3d(eh+fg))-b^2(8deg-11c(eh+fg))\right)-a^2b(13cfh-6d(e\right.}{99b^2(a+bx)^{11/2}(bc-ad)^2}$$

$$\Big\downarrow \; 48$$

$$\frac{\left(-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}-\frac{4d\left(\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}\right)}{7(bc-ad)}\right)\left(5a^2d^2fh-2abd(11cfh-3d(eh+fg))+b^2\left(33c^2\right.\right.}{33b^2(bc-ad)^2}$$
$$\frac{2(c+dx)^{3/2}\left(5a^3dfh+bx\left(14a^2dfh-ab(22cfh+3d(eh+fg))-b^2(8deg-11c(eh+fg))\right)-a^2b(13cfh-6d(e\right.}{99b^2(a+bx)^{11/2}(bc-ad)^2}$$

input
```
Int[(Sqrt[c + d*x]*(e + f*x)*(g + h*x))/(a + b*x)^(13/2),x]
```

output
```
(-2*(c + d*x)^(3/2)*(9*b^3*c*e*g + 5*a^3*d*f*h - a*b^2*(17*d*e*g - 2*c*(f*
g + e*h)) - a^2*b*(13*c*f*h - 6*d*(f*g + e*h)) + b*(14*a^2*d*f*h - b^2*(8*
d*e*g - 11*c*(f*g + e*h)) - a*b*(22*c*f*h + 3*d*(f*g + e*h)))*x))/(99*b^2*
(b*c - a*d)^2*(a + b*x)^(11/2)) + ((5*a^2*d^2*f*h - 2*a*b*d*(11*c*f*h - 3*
d*(f*g + e*h)) + b^2*(16*d^2*e*g + 33*c^2*f*h - 22*c*d*(f*g + e*h)))*((-2*
(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) - (4*d*((-2*(c + d*x)^(3/
2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2))/(15*(b*c - a*d
)^2*(a + b*x)^(3/2))))/(7*(b*c - a*d))))/(33*b^2*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(402) = 804$.

Time = 0.34 (sec) , antiderivative size = 1010, normalized size of antiderivative = 2.34

method	result	size
default	Expression too large to display	1010
gosper	Expression too large to display	1066
orering	Expression too large to display	1066

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*(d*x+c)^(3/2)/(b*x+a)^(11/2)*(40*a^2*b^2*d^4*f*h*x^4-176*a*b^3*c*d^
3*f*h*x^4+48*a*b^3*d^4*e*h*x^4+48*a*b^3*d^4*f*g*x^4+264*b^4*c^2*d^2*f*h*x^
4-176*b^4*c*d^3*e*h*x^4-176*b^4*c*d^3*f*g*x^4+128*b^4*d^4*e*g*x^4+220*a^3*
b*d^4*f*h*x^3-1028*a^2*b^2*c*d^3*f*h*x^3+264*a^2*b^2*d^4*e*h*x^3+264*a^2*b
^2*d^4*f*g*x^3+1716*a*b^3*c^2*d^2*f*h*x^3-1040*a*b^3*c*d^3*e*h*x^3-1040*a*
b^3*c*d^3*f*g*x^3+704*a*b^3*d^4*e*g*x^3-396*b^4*c^3*d*f*h*x^3+264*b^4*c^2*
d^2*e*h*x^3+264*b^4*c^2*d^2*f*g*x^3-192*b^4*c*d^3*e*g*x^3+495*a^4*d^4*f*h*
x^2-2508*a^3*b*c*d^3*f*h*x^2+594*a^3*b*d^4*e*h*x^2+594*a^3*b*d^4*f*g*x^2+4
794*a^2*b^2*c^2*d^2*f*h*x^2-2574*a^2*b^2*c*d^3*e*h*x^2-2574*a^2*b^2*c*d^3*
f*g*x^2+1584*a^2*b^2*d^4*e*g*x^2-2508*a*b^3*c^3*d*f*h*x^2+1542*a*b^3*c^2*d
^2*e*h*x^2+1542*a*b^3*c^2*d^2*f*g*x^2-1056*a*b^3*c*d^3*e*g*x^2+495*b^4*c^4
*f*h*x^2-330*b^4*c^3*d*e*h*x^2-330*b^4*c^3*d*f*g*x^2+240*b^4*c^2*d^2*e*g*x
^2-396*a^4*c*d^3*f*h*x+693*a^4*d^4*e*h*x+693*a^4*d^4*f*g*x+1716*a^3*b*c^2*
d^2*f*h*x-3432*a^3*b*c*d^3*e*h*x-3432*a^3*b*c*d^3*f*g*x+1848*a^3*b*d^4*e*g
*x-1028*a^2*b^2*c^3*d*f*h*x+3762*a^2*b^2*c^2*d^2*e*h*x+3762*a^2*b^2*c^2*d^
2*f*g*x-2376*a^2*b^2*c*d^3*e*g*x+220*a*b^3*c^4*f*h*x-1920*a*b^3*c^3*d*e*h*
x-1920*a*b^3*c^3*d*f*g*x+1320*a*b^3*c^2*d^2*e*g*x+385*b^4*c^4*e*h*x+385*b^
4*c^4*f*g*x-280*b^4*c^3*d*e*g*x+264*a^4*c^2*d^2*f*h-462*a^4*c*d^3*e*h-462*
a^4*c*d^3*f*g+1155*a^4*d^4*e*g-176*a^3*b*c^3*d*f*h+594*a^3*b*c^2*d^2*e*h+5
94*a^3*b*c^2*d^2*f*g-2772*a^3*b*c*d^3*e*g+40*a^2*b^2*c^4*f*h-330*a^2*b^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(13/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx = \int \frac{\sqrt{c+dx}(e+fx)\,(g+hx)}{(a+bx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)*(h*x+g)/(b*x+a)**(13/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)*(g + h*x)/(a + b*x)**(13/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(13/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7493 vs. $2(402) = 804$.

Time = 0.91 (sec) , antiderivative size = 7493, normalized size of antiderivative = 17.34

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(13/2),x, algorithm="giac"
)
```

output
```
-32/3465*(16*sqrt(b*d)*b^16*c^6*d^5*e*g*abs(b) - 96*sqrt(b*d)*a*b^15*c^5*d
^6*e*g*abs(b) + 240*sqrt(b*d)*a^2*b^14*c^4*d^7*e*g*abs(b) - 320*sqrt(b*d)*
a^3*b^13*c^3*d^8*e*g*abs(b) + 240*sqrt(b*d)*a^4*b^12*c^2*d^9*e*g*abs(b) -
96*sqrt(b*d)*a^5*b^11*c*d^10*e*g*abs(b) + 16*sqrt(b*d)*a^6*b^10*d^11*e*g*a
bs(b) - 22*sqrt(b*d)*b^16*c^7*d^4*f*g*abs(b) + 138*sqrt(b*d)*a*b^15*c^6*d^
5*f*g*abs(b) - 366*sqrt(b*d)*a^2*b^14*c^5*d^6*f*g*abs(b) + 530*sqrt(b*d)*a
^3*b^13*c^4*d^7*f*g*abs(b) - 450*sqrt(b*d)*a^4*b^12*c^3*d^8*f*g*abs(b) + 2
22*sqrt(b*d)*a^5*b^11*c^2*d^9*f*g*abs(b) - 58*sqrt(b*d)*a^6*b^10*c*d^10*f*
g*abs(b) + 6*sqrt(b*d)*a^7*b^9*d^11*f*g*abs(b) - 22*sqrt(b*d)*b^16*c^7*d^4
*e*h*abs(b) + 138*sqrt(b*d)*a*b^15*c^6*d^5*e*h*abs(b) - 366*sqrt(b*d)*a^2*
b^14*c^5*d^6*e*h*abs(b) + 530*sqrt(b*d)*a^3*b^13*c^4*d^7*e*h*abs(b) - 450*
sqrt(b*d)*a^4*b^12*c^3*d^8*e*h*abs(b) + 222*sqrt(b*d)*a^5*b^11*c^2*d^9*e*h
*abs(b) - 58*sqrt(b*d)*a^6*b^10*c*d^10*e*h*abs(b) + 6*sqrt(b*d)*a^7*b^9*d^
11*e*h*abs(b) + 33*sqrt(b*d)*b^16*c^8*d^3*f*h*abs(b) - 220*sqrt(b*d)*a*b^1
5*c^7*d^4*f*h*abs(b) + 632*sqrt(b*d)*a^2*b^14*c^6*d^5*f*h*abs(b) - 1020*sq
rt(b*d)*a^3*b^13*c^5*d^6*f*h*abs(b) + 1010*sqrt(b*d)*a^4*b^12*c^4*d^7*f*h*
abs(b) - 628*sqrt(b*d)*a^5*b^11*c^3*d^8*f*h*abs(b) + 240*sqrt(b*d)*a^6*b^1
0*c^2*d^9*f*h*abs(b) - 52*sqrt(b*d)*a^7*b^9*c*d^10*f*h*abs(b) + 5*sqrt(b*d
)*a^8*b^8*d^11*f*h*abs(b) - 176*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14*c^5*d^5*e*g*abs(b) + 880*sqrt(b*...
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(13/2),x)
```

output
```
((c + d*x)^(1/2)*((630*b^4*c^5*e*g + 140*a*b^3*c^5*e*h + 140*a*b^3*c^5*f*g
 + 2310*a^4*c*d^4*e*g + 80*a^2*b^2*c^5*f*h - 924*a^4*c^2*d^3*e*h - 924*a^4
*c^2*d^3*f*g + 528*a^4*c^3*d^2*f*h + 5940*a^2*b^2*c^3*d^2*e*g - 3080*a*b^3
*c^4*d*e*g - 352*a^3*b*c^4*d*f*h - 5544*a^3*b*c^2*d^3*e*g - 660*a^2*b^2*c^
4*d*e*h - 660*a^2*b^2*c^4*d*f*g + 1188*a^3*b*c^3*d^2*e*h + 1188*a^3*b*c^3*
d^2*f*g)/(3465*b^5*(a*d - b*c)^5) + (x^2*(1386*a^4*d^5*e*h + 1386*a^4*d^5*
f*g + 990*b^4*c^5*f*h + 3696*a^3*b*d^5*e*g + 198*a^4*c*d^4*f*h + 110*b^4*c
^4*d*e*h + 110*b^4*c^4*d*f*g - 80*b^4*c^3*d^2*e*g + 2376*a^2*b^2*c^2*d^3*e
*h + 2376*a^2*b^2*c^2*d^3*f*g + 7532*a^2*b^2*c^3*d^2*f*h - 5676*a^3*b*c*d^
4*e*h - 5676*a^3*b*c*d^4*f*g - 4576*a*b^3*c^4*d*f*h + 528*a*b^3*c^2*d^3*e*
g - 1584*a^2*b^2*c*d^4*e*g - 756*a*b^3*c^3*d^2*e*h - 756*a*b^3*c^3*d^2*f*g
 - 1584*a^3*b*c^2*d^3*f*h))/(3465*b^5*(a*d - b*c)^5) + (x^5*(256*b^4*d^5*e
*g + 96*a*b^3*d^5*e*h + 96*a*b^3*d^5*f*g - 352*b^4*c*d^4*e*h - 352*b^4*c*d
^4*f*g + 80*a^2*b^2*d^5*f*h + 528*b^4*c^2*d^3*f*h - 352*a*b^3*c*d^4*f*h))/
(3465*b^5*(a*d - b*c)^5) + (x*(2310*a^4*d^5*e*g + 770*b^4*c^5*e*h + 770*b^
4*c^5*f*g + 440*a*b^3*c^5*f*h + 462*a^4*c*d^4*e*h + 462*a^4*c*d^4*f*g + 70
*b^4*c^4*d*e*g - 264*a^4*c^2*d^3*f*h + 1188*a^2*b^2*c^2*d^3*e*g + 6864*a^2
*b^2*c^3*d^2*e*h + 6864*a^2*b^2*c^3*d^2*f*g - 1848*a^3*b*c*d^4*e*g - 3700*
a*b^3*c^4*d*e*h - 3700*a*b^3*c^4*d*f*g - 440*a*b^3*c^3*d^2*e*g - 5676*a^3*
b*c^2*d^3*e*h - 5676*a^3*b*c^2*d^3*f*g - 1976*a^2*b^2*c^4*d*f*h + 3080*...
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 3656, normalized size of antiderivative = 8.46

$$\int \frac{\sqrt{c+dx}(e+fx)(g+hx)}{(a+bx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(f*x+e)*(h*x+g)/(b*x+a)^(13/2),x)
```

output
```
(2*( - 40*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**7*d**5*f*h + 176*sqrt(d)*sqrt(b
)*sqrt(a + b*x)*a**6*b*c*d**4*f*h - 48*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**6*
b*d**5*e*h - 48*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**6*b*d**5*f*g - 200*sqrt(d
)*sqrt(b)*sqrt(a + b*x)*a**6*b*d**5*f*h*x - 264*sqrt(d)*sqrt(b)*sqrt(a + b
*x)*a**5*b**2*c**2*d**3*f*h + 176*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b**2*
c*d**4*e*h + 176*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b**2*c*d**4*f*g + 880*
sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b**2*c*d**4*f*h*x - 128*sqrt(d)*sqrt(b)
*sqrt(a + b*x)*a**5*b**2*d**5*e*g - 240*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5
*b**2*d**5*e*h*x - 240*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b**2*d**5*f*g*x
- 400*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**5*b**2*d**5*f*h*x**2 - 1320*sqrt(d)
*sqrt(b)*sqrt(a + b*x)*a**4*b**3*c**2*d**3*f*h*x + 880*sqrt(d)*sqrt(b)*sqr
t(a + b*x)*a**4*b**3*c*d**4*e*h*x + 880*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4
*b**3*c*d**4*f*g*x + 1760*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**3*c*d**4*f
*h*x**2 - 640*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**3*d**5*e*g*x - 480*sqr
t(d)*sqrt(b)*sqrt(a + b*x)*a**4*b**3*d**5*e*h*x**2 - 480*sqrt(d)*sqrt(b)*s
qrt(a + b*x)*a**4*b**3*d**5*f*g*x**2 - 400*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a
**4*b**3*d**5*f*h*x**3 - 2640*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**4*c**2
*d**3*f*h*x**2 + 1760*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**4*c*d**4*e*h*x
**2 + 1760*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**4*c*d**4*f*g*x**2 + 1760*
sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*b**4*c*d**4*f*h*x**3 - 1280*sqrt(d)*...
```

3.206 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2B\sqrt{-bc+ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}\,E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2\sqrt{-bc+ad}(Be-Af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2*B*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d
^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b/
d^(1/2)/f/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2*(a*d-b*c)^(1/2)*(-A
*f+B*e)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*Elliptic
F(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))
/b/d^(1/2)/f/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.16 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\left(-b^2B\sqrt{-a+\frac{bc}{d}}(c+dx)(e+fx)-iB(bc-ad)f(a+bx)^{3/2}\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{b(e+fx)}{f(a+bx)}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-a+}}{\sqrt{a+b}}\right.\right.\right.}{b^2\sqrt{-a+\frac{bc}{d}}df\sqrt{a+}}$$

input
```
Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(-2*(-(b^2*B*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)) - I*B*(b*c - a*d)*f*(
a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a
+ b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a
*d*f)/(b*c*f - a*d*f)] + I*b*(B*c - A*d)*f*(a + b*x)^(3/2)*Sqrt[(b*(c + d*
x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[S
qrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(b^2*
Sqrt[-a + (b*c)/d]*d*f*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 176$$

$$\frac{B\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}-\frac{(Be-Af)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}$$

$$\Big\downarrow \text{124}$$

$$\frac{B\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{(Be-Af)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}$$

$$\Big\downarrow \text{123}$$

$$\frac{2B\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{(Be-Af)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}$$

$$\Big\downarrow \text{131}$$

$$\frac{2B\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{(Be-Af)\sqrt{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2B\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{(Be-Af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{130}$$

$$\frac{2B\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}-\frac{2\sqrt{ad-bc}(Be-Af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}$$

input
```
Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/
(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f
)]) - (2*Sqrt[-(b*c) + a*d]*(B*e - A*f)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sq
rt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c +
d*x]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !L
tQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(fx+e)(bx+a)(xd+c)}\left(\frac{2A\left(\frac{c}{d}-\frac{a}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{a}{b}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{a}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{a}{b}}{-\frac{c}{d}+\frac{e}{f}}}\right)}{\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}+\frac{2B\left(\frac{c}{d}-\frac{a}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{a}{b}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\sqrt{\frac{x+}{-\frac{c}{d}}}}{\sqrt{bd}}\right.}{\sqrt{fx+e}\,\sqrt{bx+a}\,\sqrt{xd+c}}$
default	$-\frac{2\left(A\operatorname{EllipticF}\left(\sqrt{-\frac{b(xd+c)}{ad-bc}},\sqrt{-\frac{(ad-bc)f}{b(cf-de)}}\right)a\,d^2f-A\operatorname{EllipticF}\left(\sqrt{-\frac{b(xd+c)}{ad-bc}},\sqrt{-\frac{(ad-bc)f}{b(cf-de)}}\right)bcdf-B\operatorname{EllipticF}\left(\sqrt{-\frac{b(xd+c)}{ad-bc}},\sqrt{-\right.\right.}{}$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output

```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(f*x+e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*
(2*A*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/
b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*
e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(
-c/d+e/f))^(1/2))+2*B*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e
/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e
*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/f)*EllipticE(((x+c/d)/(
c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))-e/f*EllipticF(((x+c/d)/(c/d
-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(250) = 500$.

Time = 0.13 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(3\sqrt{bdf}\,Bbdf\,\mathrm{weierstrassZeta}\left(\frac{4\left(b^2d^2e^2-\left(b^2cd+abd^2\right)ef+\left(b^2c^2-abcd+a^2d^2\right)f^2\right)}{3\,b^2d^2f^2},-\frac{4\left(2\,b^3d^3e^3-3\left(b^3cd^2+ab^2d^3\right)e^2f-3\left(b\right.\right.}{}\right.\right.}{}$$

input

```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
-2/3*(3*sqrt(b*d*f)*B*b*d*f*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d +
a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*
b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d
^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3
*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d
+ a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(
2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c
*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a
^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f
))) + (B*b*d*e + (B*b*c + (B*a - 3*A*b)*d)*f)*sqrt(b*d*f)*weierstrassPInve
rse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*
d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*
e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x
 + b*d*e + (b*c + a*d)*f)/(b*d*f)))/(b^2*d^2*f^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{fx+e}\,\sqrt{dx+c}\,\sqrt{bx+a}}{df\,x^2+cfx+dex+ce}dx$$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x))/(c*e + c*f*x + d*e*x + d*f
*x**2),x)
```

3.207 $\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 229

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{2B\sqrt{ex}\sqrt{c+dx}}{de\sqrt{a+bx}} - \frac{2\sqrt{a}B\sqrt{c+dx}E\left(\arctan\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{\frac{a(c+dx)}{c(a+bx)}}} + \frac{2\sqrt{a}A\sqrt{c+dx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{e}\sqrt{a+bx}\sqrt{\frac{a(c+dx)}{c(a+bx)}}}$$

output
```
2*B*(e*x)^(1/2)*(d*x+c)^(1/2)/d/e/(b*x+a)^(1/2)-2*a^(1/2)*B*(d*x+c)^(1/2)*
EllipticE(b^(1/2)*(e*x)^(1/2)/a^(1/2)/e^(1/2)/(1+b*x/a)^(1/2),(1-a*d/b/c)^
(1/2))/b^(1/2)/d/e^(1/2)/(b*x+a)^(1/2)/(a*(d*x+c)/c/(b*x+a))^(1/2)+2*a^(1/
2)*A*(d*x+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*(e*x)^(1/2)/a^(1/2)/e^(1
/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^(1/2)/(b*x+a)^(1/2)/(a*(d*x+c)/c/(b*x+
a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx$$
$$= \frac{\frac{2aB(a+bx)(c+dx)}{b}+2ia\sqrt{\frac{a}{b}}Bd\sqrt{1+\frac{a}{bx}}\sqrt{1+\frac{c}{dx}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{a}{b}}}{\sqrt{x}}\right)|\frac{bc}{ad}\right)-2i\sqrt{\frac{a}{b}}(-Ab+aB)d\sqrt{1+\frac{a}{bx}}}{ad\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

output
```
((2*a*B*(a + b*x)*(c + d*x))/b + (2*I)*a*Sqrt[a/b]*B*d*Sqrt[1 + a/(b*x)]*S
qrt[1 + c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*
d)] - (2*I)*Sqrt[a/b]*(-(A*b) + a*B)*d*Sqrt[1 + a/(b*x)]*Sqrt[1 + c/(d*x)]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], (b*c)/(a*d)])/(a*d*Sqrt[e
*x]*Sqrt[a + b*x]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 176}$$

$$\frac{B\int \frac{\sqrt{c+dx}}{\sqrt{ex}\sqrt{a+bx}}dx}{d}-\frac{(Bc-Ad)\int \frac{1}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{ 122}$$

$$\frac{B\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{ex}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{(Bc-Ad)\int \frac{1}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{120}$$

$$\frac{2\sqrt{-a}B\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{(Bc-Ad)\int \frac{1}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{127}$$

$$\frac{2\sqrt{-a}B\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(Bc-Ad)\int \frac{1}{\sqrt{ex}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}}dx}{d\sqrt{a+bx}\sqrt{c+dx}}$$

$$\Big\downarrow \text{126}$$

$$\frac{2\sqrt{-a}B\sqrt{\frac{bx}{a}+1}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{\frac{dx}{c}+1}}-\frac{2\sqrt{-a}\sqrt{\frac{bx}{a}+1}\sqrt{\frac{dx}{c}+1}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{c+dx}}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

output
```
(2*Sqrt[-a]*B*Sqrt[1 + (b*x)/a]*Sqrt[c + d*x]*EllipticE[ArcSin[(Sqrt[b]*Sq
rt[e*x])/(Sqrt[-a]*Sqrt[e])], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[e]*Sqrt[a + b*
x]*Sqrt[1 + (d*x)/c]) - (2*Sqrt[-a]*(B*c - A*d)*Sqrt[1 + (b*x)/a]*Sqrt[1 +
 (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqrt[e*x])/(Sqrt[-a]*Sqrt[e])], (a*d)/
(b*c)])/(Sqrt[b]*d*Sqrt[e]*Sqrt[a + b*x]*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{xd+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)bd-B\,\mathrm{EllipticF}\left(\sqrt{\frac{xd+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)ad+B\,\mathrm{EllipticE}\left(\sqrt{\frac{xd+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)ad-B\,\mathrm{EllipticE}\left(\right.}{b\,d^2(bd\,x^2+adx+bcx+ac)\sqrt{ex}}$
elliptic	$\frac{\sqrt{ex(bx+a)(xd+c)}\left(\frac{2Ac\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{c}{d}+\frac{a}{b}\right)}}\right)}{d\sqrt{bde\,x^3+ade\,x^2+bce\,x^2+acex}}+\frac{2Bc\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{c}{d}+\frac{a}{b}}}\sqrt{-\frac{xd}{c}}\left(\left(-\frac{c}{d}+\right.\right.}{}\right.}{\sqrt{ex}\,\sqrt{bx+a}\,\sqrt{xd+c}}$

input
```
int((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2*(A*EllipticF(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*b*d-B*EllipticF((
(d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*d+B*EllipticE(((d*x+c)/c)^(1/2)
,(-b*c/(a*d-b*c))^(1/2))*a*d-B*EllipticE(((d*x+c)/c)^(1/2),(-b*c/(a*d-b*c)
)^(1/2))*b*c)*c*(-1/c*x*d)^(1/2)*(d*(b*x+a)/(a*d-b*c))^(1/2)*((d*x+c)/c)^(
1/2)*(d*x+c)^(1/2)*(b*x+a)^(1/2)/b/d^2/(b*d*x^2+a*d*x+b*c*x+a*c)/(e*x)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.35

$$\int\frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx=$$
$$-\frac{2\left(3\sqrt{bde}Bbd\mathrm{weierstrassZeta}\left(\frac{4\left(b^2c^2-abcd+a^2d^2\right)}{3\,b^2d^2},-\frac{4\left(2\,b^3c^3-3\,ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3\right)}{27\,b^3d^3},\mathrm{weierstrassPInverse}\left(\right.\right.\right.}{}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-2/3*(3*sqrt(b*d*e)*B*b*d*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2
)/(b^2*d^2), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)
/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2
), -4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3)
, 1/3*(3*b*d*x + b*c + a*d)/(b*d))) + (B*b*c + (B*a - 3*A*b)*d)*sqrt(b*d*e
)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*(
2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b
*d*x + b*c + a*d)/(b*d)))/(b^2*d^2*e)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx = \int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a + b*x)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx = \int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,c+\sqrt{x}\,dx}dx\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

output
```
(sqrt(e)*int((sqrt(c + d*x)*sqrt(a + b*x))/(sqrt(x)*c + sqrt(x)*d*x),x))/e
```

3.208 $\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx = \frac{2B\sqrt{c}\sqrt{a+bx}\sqrt{1-\frac{dx}{c}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{c}\sqrt{e}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{e}\sqrt{1+\frac{bx}{a}}\sqrt{c-dx}} + \frac{2(Ab-aB)\sqrt{c}\sqrt{1+\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{c}\sqrt{e}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{e}\sqrt{a+bx}\sqrt{c-dx}}$$

output
```
2*B*c^(1/2)*(b*x+a)^(1/2)*(1-d*x/c)^(1/2)*EllipticE(d^(1/2)*(e*x)^(1/2)/c^
(1/2)/e^(1/2),(-b*c/a/d)^(1/2))/b/d^(1/2)/e^(1/2)/(1+b*x/a)^(1/2)/(-d*x+c)
^(1/2)+2*(A*b-B*a)*c^(1/2)*(1+b*x/a)^(1/2)*(1-d*x/c)^(1/2)*EllipticF(d^(1/
2)*(e*x)^(1/2)/c^(1/2)/e^(1/2),(-b*c/a/d)^(1/2))/b/d^(1/2)/e^(1/2)/(b*x+a)
^(1/2)/(-d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.19 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx$$
$$= \frac{\frac{2aB(a+bx)(-c+dx)}{b} + 2ia\sqrt{\frac{a}{b}}Bd\sqrt{1+\frac{a}{bx}}\sqrt{1-\frac{c}{dx}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{a}{b}}}{\sqrt{x}}\right)|-\frac{bc}{ad}\right) - 2i\sqrt{\frac{a}{b}}(-Ab+aB)d\sqrt{1+}}{ad\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*Sqrt[a + b*x]*Sqrt[c - d*x]),x]
```

output
```
((2*a*B*(a + b*x)*(-c + d*x))/b + (2*I)*a*Sqrt[a/b]*B*d*Sqrt[1 + a/(b*x)]*
Sqrt[1 - c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], -((b*c)/
(a*d))] - (2*I)*Sqrt[a/b]*(-(A*b) + a*B)*d*Sqrt[1 + a/(b*x)]*Sqrt[1 - c/(d
*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[a/b]/Sqrt[x]], -((b*c)/(a*d))])/(a*d
*Sqrt[e*x]*Sqrt[a + b*x]*Sqrt[c - d*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx$$

$$\Big\downarrow \text{ 176}$$

$$\frac{(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}dx}{d} - \frac{B\int \frac{\sqrt{c-dx}}{\sqrt{ex}\sqrt{a+bx}}dx}{d}$$

$$\Big\downarrow \text{ 122}$$

$$\frac{(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}dx}{d}-\frac{B\sqrt{\frac{bx}{a}+1}\sqrt{c-dx}\int \frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{ex}\sqrt{\frac{bx}{a}+1}}dx}{d\sqrt{a+bx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{120}$$

$$\frac{(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}dx}{d}-\frac{2\sqrt{-a}B\sqrt{\frac{bx}{a}+1}\sqrt{c-dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{127}$$

$$\frac{\sqrt{\frac{bx}{a}+1}\sqrt{1-\frac{dx}{c}}(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{\frac{bx}{a}+1}\sqrt{1-\frac{dx}{c}}}dx}{d\sqrt{a+bx}\sqrt{c-dx}}-\frac{2\sqrt{-a}B\sqrt{\frac{bx}{a}+1}\sqrt{c-dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{126}$$

$$\frac{2\sqrt{c}\sqrt{\frac{bx}{a}+1}\sqrt{1-\frac{dx}{c}}(Ad+Bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{c}\sqrt{e}}\right),-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{e}\sqrt{a+bx}\sqrt{c-dx}}-\frac{2\sqrt{-a}B\sqrt{\frac{bx}{a}+1}\sqrt{c-dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{-a}\sqrt{e}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a+bx}\sqrt{1-\frac{dx}{c}}}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*Sqrt[a + b*x]*Sqrt[c - d*x]),x]
```

output
```
(-2*Sqrt[-a]*B*Sqrt[1 + (b*x)/a]*Sqrt[c - d*x]*EllipticE[ArcSin[(Sqrt[b]*S
qrt[e*x])/(Sqrt[-a]*Sqrt[e])], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[e]*Sqrt[a
+ b*x]*Sqrt[1 - (d*x)/c]) + (2*Sqrt[c]*(B*c + A*d)*Sqrt[1 + (b*x)/a]*Sqrt[
1 - (d*x)/c]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[e*x])/(Sqrt[c]*Sqrt[e])], -((b
*c)/(a*d))])/(d^(3/2)*Sqrt[e]*Sqrt[a + b*x]*Sqrt[c - d*x])
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+a}{a}},\sqrt{\frac{ad}{ad+bc}}\right)bd+B\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+a}{a}},\sqrt{\frac{ad}{ad+bc}}\right)bc-B\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+a}{a}},\sqrt{\frac{ad}{ad+bc}}\right)ad-B\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+}{a}}\right.}{b^2d(-bd\,x^2-adx+bcx+ac)\sqrt{ex}}$
elliptic	$\frac{\sqrt{(-xd+c)(bx+a)ex}\left(\frac{2Aa\sqrt{\frac{\left(x+\frac{a}{b}\right)b}{a}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{a}{b}-\frac{c}{d}}}\sqrt{-\frac{bx}{a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{a}{b}\right)b}{a}},\sqrt{-\frac{a}{b\left(-\frac{a}{b}-\frac{c}{d}\right)}}\right)}{b\sqrt{-bde\,x^3-ade\,x^2+bce\,x^2+acex}}+\frac{2Ba\sqrt{\frac{\left(x+\frac{a}{b}\right)b}{a}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{a}{b}-\frac{c}{d}}}\sqrt{-\frac{bx}{a}}\left(\left(-\frac{a}{b}\right.\right.}{}\right.}{\sqrt{ex}\,\sqrt{bx+a}\,\sqrt{-xd+c}}$

input
```
int((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(-d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2*(A*EllipticF(((b*x+a)/a)^(1/2),(a*d/(a*d+b*c))^(1/2))*b*d+B*EllipticF(((
b*x+a)/a)^(1/2),(a*d/(a*d+b*c))^(1/2))*b*c-B*EllipticE(((b*x+a)/a)^(1/2),(
a*d/(a*d+b*c))^(1/2))*a*d-B*EllipticE(((b*x+a)/a)^(1/2),(a*d/(a*d+b*c))^(1
/2))*b*c)*a*(-b*x/a)^(1/2)*((-d*x+c)*b/(a*d+b*c))^(1/2)*((b*x+a)/a)^(1/2)*
(-d*x+c)^(1/2)*(b*x+a)^(1/2)/b^2/d/(-b*d*x^2-a*d*x+b*c*x+a*c)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx = \frac{2\left(3\sqrt{-bde}Bbd\mathrm{weierstrassZeta}\left(\frac{4\left(b^2c^2+abcd+a^2d^2\right)}{3b^2d^2},\frac{4\left(2b^3c^3+3ab^2c^2d-3a^2bcd^2-2a^3d^3\right)}{27b^3d^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(b\right.\right.\right.}{}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(-d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
2/3*(3*sqrt(-b*d*e)*B*b*d*weierstrassZeta(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2
)/(b^2*d^2), 4/27*(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/
(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2)/(b^2*d^2)
, 4/27*(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/(b^3*d^3),
1/3*(3*b*d*x - b*c + a*d)/(b*d))) - (B*b*c - (B*a - 3*A*b)*d)*sqrt(-b*d*e)
*weierstrassPInverse(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2)/(b^2*d^2), 4/27*(2*
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d
*x - b*c + a*d)/(b*d)))/(b^2*d^2*e)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx = \int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(b*x+a)**(1/2)/(-d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a + b*x)*sqrt(c - d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{-dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(-d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(-d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{-dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(-d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(-d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx=\int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{a+b\,x}\,\sqrt{c-d\,x}}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x)^(1/2)*(c - d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x)^(1/2)*(c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx}\sqrt{c-dx}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{-dx+c}\,\sqrt{bx+a}}{\sqrt{x}\,c-\sqrt{x}\,dx}dx\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(b*x+a)^(1/2)/(-d*x+c)^(1/2),x)
```

output
```
(sqrt(e)*int((sqrt(c - d*x)*sqrt(a + b*x))/(sqrt(x)*c - sqrt(x)*d*x),x))/e
```

3.209 $\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx = \frac{2\sqrt{a}B\sqrt{1-\frac{bx}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a-bx}\sqrt{1+\frac{dx}{c}}} - \frac{2\sqrt{a}(Bc-Ad)\sqrt{1-\frac{bx}{a}}\sqrt{1+\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a-bx}\sqrt{c+dx}}$$

output
```
2*a^(1/2)*B*(1-b*x/a)^(1/2)*(d*x+c)^(1/2)*EllipticE(b^(1/2)*(e*x)^(1/2)/a^
(1/2)/e^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/d/e^(1/2)/(-b*x+a)^(1/2)/(1+d*x/c)
^(1/2)-2*a^(1/2)*(-A*d+B*c)*(1-b*x/a)^(1/2)*(1+d*x/c)^(1/2)*EllipticF(b^(1
/2)*(e*x)^(1/2)/a^(1/2)/e^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/d/e^(1/2)/(-b*x+
a)^(1/2)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx$$
$$= \frac{-2\sqrt{-\frac{a}{b}}B(a-bx)(c+dx)-2iaBd\sqrt{1-\frac{a}{bx}}\sqrt{1+\frac{c}{dx}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{a}{b}}}{\sqrt{x}}\right)|-\frac{bc}{ad}\right)+2i(Ab+aB)d\sqrt{}}{\sqrt{-\frac{a}{b}}bd\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

output
```
(-2*Sqrt[-(a/b)]*B*(a - b*x)*(c + d*x) - (2*I)*a*B*d*Sqrt[1 - a/(b*x)]*Sqr
t[1 + c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[-(a/b)]/Sqrt[x]], -((b*c)/
(a*d))] + (2*I)*(A*b + a*B)*d*Sqrt[1 - a/(b*x)]*Sqrt[1 + c/(d*x)]*x^(3/2)*
EllipticF[I*ArcSinh[Sqrt[-(a/b)]/Sqrt[x]], -((b*c)/(a*d))])/(Sqrt[-(a/b)]*
b*d*Sqrt[e*x]*Sqrt[a - b*x]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{176}$$
$$\frac{(aB+Ab)\int \frac{1}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}dx}{b}-\frac{B\int \frac{\sqrt{a-bx}}{\sqrt{ex}\sqrt{c+dx}}dx}{b}$$
$$\Big\downarrow \text{122}$$

$$\frac{(aB+Ab)\int \frac{1}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}dx}{b}-\frac{B\sqrt{a-bx}\sqrt{\frac{dx}{c}+1}\int \frac{\sqrt{1-\frac{bx}{a}}}{\sqrt{ex}\sqrt{\frac{dx}{c}+1}}dx}{b\sqrt{1-\frac{bx}{a}}\sqrt{c+dx}}$$

$$\Big\downarrow \; 120$$

$$\frac{(aB+Ab)\int \frac{1}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}dx}{b}-\frac{2B\sqrt{-c}\sqrt{a-bx}\sqrt{\frac{dx}{c}+1}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{-c}\sqrt{e}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{e}\sqrt{1-\frac{bx}{a}}\sqrt{c+dx}}$$

$$\Big\downarrow \; 127$$

$$\frac{\sqrt{1-\frac{bx}{a}}\sqrt{\frac{dx}{c}+1}(aB+Ab)\int \frac{1}{\sqrt{ex}\sqrt{1-\frac{bx}{a}}\sqrt{\frac{dx}{c}+1}}dx}{b\sqrt{a-bx}\sqrt{c+dx}}-\frac{2B\sqrt{-c}\sqrt{a-bx}\sqrt{\frac{dx}{c}+1}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{-c}\sqrt{e}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{e}\sqrt{1-\frac{bx}{a}}\sqrt{c+dx}}$$

$$\Big\downarrow \; 126$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx}{a}}\sqrt{\frac{dx}{c}+1}(aB+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right),-\frac{ad}{bc}\right)}{b^{3/2}\sqrt{e}\sqrt{a-bx}\sqrt{c+dx}}-\frac{2B\sqrt{-c}\sqrt{a-bx}\sqrt{\frac{dx}{c}+1}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{-c}\sqrt{e}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{e}\sqrt{1-\frac{bx}{a}}\sqrt{c+dx}}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

output
```
(-2*B*Sqrt[-c]*Sqrt[a - b*x]*Sqrt[1 + (d*x)/c]*EllipticE[ArcSin[(Sqrt[d]*S
qrt[e*x])/(Sqrt[-c]*Sqrt[e])], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[e]*Sqrt[1
- (b*x)/a]*Sqrt[c + d*x]) + (2*Sqrt[a]*(A*b + a*B)*Sqrt[1 - (b*x)/a]*Sqrt[
1 + (d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqrt[e*x])/(Sqrt[a]*Sqrt[e])], -((a
*d)/(b*c))])/(b^(3/2)*Sqrt[e]*Sqrt[a - b*x]*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{xd+c}{c}},\sqrt{\frac{bc}{ad+bc}}\right)bd+B\,\mathrm{EllipticF}\left(\sqrt{\frac{xd+c}{c}},\sqrt{\frac{bc}{ad+bc}}\right)ad-B\,\mathrm{EllipticE}\left(\sqrt{\frac{xd+c}{c}},\sqrt{\frac{bc}{ad+bc}}\right)ad-B\,\mathrm{EllipticE}\left(\sqrt{\frac{xd+c}{c}}\right.\right.}{b\,d^{2}\left(-bd\,x^{2}+adx-bcx+ac\right)\sqrt{ex}}$
elliptic	$\frac{\sqrt{(xd+c)(-bx+a)ex}\left(\frac{2Ac\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x-\frac{a}{b}}{-\frac{a}{b}-\frac{c}{d}}}\sqrt{-\frac{xd}{c}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}},\sqrt{-\frac{c}{d\left(-\frac{a}{b}-\frac{c}{d}\right)}}\right)}{d\sqrt{-bde\,x^{3}+ade\,x^{2}-bce\,x^{2}+acex}}+\frac{2Bc\sqrt{\frac{\left(x+\frac{c}{d}\right)d}{c}}\sqrt{\frac{x-\frac{a}{b}}{-\frac{a}{b}-\frac{c}{d}}}\sqrt{-\frac{xd}{c}}\left(\left(-\frac{a}{b}\right.\right.}{}\right.}{\sqrt{ex}\,\sqrt{-bx+a}\,\sqrt{xd+c}}$

input
```
int((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2*(A*EllipticF(((d*x+c)/c)^(1/2),(b*c/(a*d+b*c))^(1/2))*b*d+B*EllipticF(((
d*x+c)/c)^(1/2),(b*c/(a*d+b*c))^(1/2))*a*d-B*EllipticE(((d*x+c)/c)^(1/2),(
b*c/(a*d+b*c))^(1/2))*a*d-B*EllipticE(((d*x+c)/c)^(1/2),(b*c/(a*d+b*c))^(1
/2))*b*c)*c*(-1/c*x*d)^(1/2)*((-b*x+a)*d/(a*d+b*c))^(1/2)*((d*x+c)/c)^(1/2
)*(d*x+c)^(1/2)*(-b*x+a)^(1/2)/b/d^2/(-b*d*x^2+a*d*x-b*c*x+a*c)/(e*x)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx = \frac{2\left(3\sqrt{-bde}Bbd\,\mathrm{weierstrassZeta}\left(\frac{4\left(b^{2}c^{2}+abcd+a^{2}d^{2}\right)}{3b^{2}d^{2}},-\frac{4\left(2b^{3}c^{3}+3ab^{2}c^{2}d-3a^{2}bcd^{2}-2a^{3}d^{3}\right)}{27b^{3}d^{3}},\mathrm{weierstrassPInverse}\left(\frac{4}{}\right.\right.\right.}{}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
2/3*(3*sqrt(-b*d*e)*B*b*d*weierstrassZeta(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2
)/(b^2*d^2), -4/27*(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)
/(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2)/(b^2*d^2
), -4/27*(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/(b^3*d^3)
, 1/3*(3*b*d*x + b*c - a*d)/(b*d))) + (B*b*c - (B*a + 3*A*b)*d)*sqrt(-b*d*
e)*weierstrassPInverse(4/3*(b^2*c^2 + a*b*c*d + a^2*d^2)/(b^2*d^2), -4/27*
(2*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 2*a^3*d^3)/(b^3*d^3), 1/3*(3*
b*d*x + b*c - a*d)/(b*d)))/(b^2*d^2*e)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx=\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(-b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a - b*x)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx=\int \frac{Bx+A}{\sqrt{-bx+a}\sqrt{dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((B*x + A)/(sqrt(-b*x + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx = \int \frac{Bx+A}{\sqrt{-bx+a}\sqrt{dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((B*x + A)/(sqrt(-b*x + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx = \int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{a-b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a - b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a - b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c+dx}}\,dx = \frac{\sqrt{e}\left(\left(\int \frac{\sqrt{x}\,\sqrt{dx+c}\,\sqrt{-bx+a}}{-bd\,x^3+ad\,x^2-bc\,x^2+acx}dx\right)a+\left(\int \frac{\sqrt{x}\,\sqrt{dx+c}\,\sqrt{-bx+a}}{-bd\,x^2+adx-bcx+ac}dx\right)b\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

output
```
(sqrt(e)*(int((sqrt(x)*sqrt(c + d*x)*sqrt(a - b*x))/(a*c*x + a*d*x**2 - b*
c*x**2 - b*d*x**3),x)*a + int((sqrt(x)*sqrt(c + d*x)*sqrt(a - b*x))/(a*c +
 a*d*x - b*c*x - b*d*x**2),x)*b))/e
```

3.210 $\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 206

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx &= -\frac{2B\sqrt{c}\sqrt{a-bx}\sqrt{1-\frac{dx}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{ex}}{\sqrt{c}\sqrt{e}}\right)|\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{e}\sqrt{1-\frac{bx}{a}}\sqrt{c-dx}} \\ &\quad+\frac{2\sqrt{a}(Ab+aB)\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right),\frac{ad}{bc}\right)}{b^{3/2}\sqrt{e}\sqrt{a-bx}\sqrt{c-dx}}\end{aligned}$$

output
```
-2*B*c^(1/2)*(-b*x+a)^(1/2)*(1-d*x/c)^(1/2)*EllipticE(d^(1/2)*(e*x)^(1/2)/
c^(1/2)/e^(1/2),(b*c/a/d)^(1/2))/b/d^(1/2)/e^(1/2)/(1-b*x/a)^(1/2)/(-d*x+c
)^(1/2)+2*a^(1/2)*(A*b+B*a)*(1-b*x/a)^(1/2)*(1-d*x/c)^(1/2)*EllipticF(b^(1
/2)*(e*x)^(1/2)/a^(1/2)/e^(1/2),(a*d/b/c)^(1/2))/b^(3/2)/e^(1/2)/(-b*x+a)^
(1/2)/(-d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx = \frac{2\sqrt{-\frac{a}{b}}B(a-bx)(c-dx)-2iaBd\sqrt{1-\frac{a}{bx}}\sqrt{1-\frac{c}{dx}}x^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{a}{b}}}{\sqrt{x}}\right)|\frac{bc}{ad}\right)+2i(Ab+aB)d\sqrt{1-}}{\sqrt{-\frac{a}{b}}bd\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*Sqrt[a - b*x]*Sqrt[c - d*x]),x]
```

output
```
(2*Sqrt[-(a/b)]*B*(a - b*x)*(c - d*x) - (2*I)*a*B*d*Sqrt[1 - a/(b*x)]*Sqrt
[1 - c/(d*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[-(a/b)]/Sqrt[x]], (b*c)/(a*
d)] + (2*I)*(A*b + a*B)*d*Sqrt[1 - a/(b*x)]*Sqrt[1 - c/(d*x)]*x^(3/2)*Elli
pticF[I*ArcSinh[Sqrt[-(a/b)]/Sqrt[x]], (b*c)/(a*d)])/(Sqrt[-(a/b)]*b*d*Sqr
t[e*x]*Sqrt[a - b*x]*Sqrt[c - d*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {176, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx$$

$$\Big\downarrow \; 176$$

$$\frac{(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}dx}{d}-\frac{B\int \frac{\sqrt{c-dx}}{\sqrt{ex}\sqrt{a-bx}}dx}{d}$$

$$\Big\downarrow \; 122$$

$$\frac{(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}dx}{d}-\frac{B\sqrt{1-\frac{bx}{a}}\sqrt{c-dx}\int \frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{ex}\sqrt{1-\frac{bx}{a}}}dx}{d\sqrt{a-bx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{120}$$

$$\frac{(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}dx}{d}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx}{a}}\sqrt{c-dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a-bx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{127}$$

$$\frac{\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}(Ad+Bc)\int \frac{1}{\sqrt{ex}\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}}dx}{d\sqrt{a-bx}\sqrt{c-dx}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx}{a}}\sqrt{c-dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a-bx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{126}$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx}{a}}\sqrt{1-\frac{dx}{c}}(Ad+Bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right),\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a-bx}\sqrt{c-dx}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx}{a}}\sqrt{c-dx}E\left(\arcsin\left(\frac{\sqrt{b}\sqrt{ex}}{\sqrt{a}\sqrt{e}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{e}\sqrt{a-bx}\sqrt{1-\frac{dx}{c}}}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*Sqrt[a - b*x]*Sqrt[c - d*x]),x]
```

output
```
(-2*Sqrt[a]*B*Sqrt[1 - (b*x)/a]*Sqrt[c - d*x]*EllipticE[ArcSin[(Sqrt[b]*Sq
rt[e*x])/(Sqrt[a]*Sqrt[e])], (a*d)/(b*c)])/(Sqrt[b]*d*Sqrt[e]*Sqrt[a - b*x
]*Sqrt[1 - (d*x)/c]) + (2*Sqrt[a]*(B*c + A*d)*Sqrt[1 - (b*x)/a]*Sqrt[1 - (
d*x)/c]*EllipticF[ArcSin[(Sqrt[b]*Sqrt[e*x])/(Sqrt[a]*Sqrt[e])], (a*d)/(b*
c)])/(Sqrt[b]*d*Sqrt[e]*Sqrt[a - b*x]*Sqrt[c - d*x])
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.10

method	result
default	$-\frac{2\left(A\,\text{EllipticF}\left(\sqrt{\frac{-xd+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)bd+B\,\text{EllipticF}\left(\sqrt{\frac{-xd+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)ad-B\,\text{EllipticE}\left(\sqrt{\frac{-xd+c}{c}},\sqrt{-\frac{bc}{ad-bc}}\right)ad+B\,\text{Elli}\cdots\right.}{b\,d^2(bd\,x^2-adx-bcx+ac)\sqrt{ex}}$
elliptic	$\frac{\sqrt{(-xd+c)(-bx+a)ex}\left(-\frac{2Ac\sqrt{-\frac{\left(x-\frac{c}{d}\right)d}{c}}\sqrt{\frac{x-\frac{a}{b}}{\frac{c}{d}-\frac{a}{b}}}\sqrt{\frac{xd}{c}}\,\text{EllipticF}\left(\sqrt{-\frac{\left(x-\frac{c}{d}\right)d}{c}},\sqrt{\frac{c}{d\left(\frac{c}{d}-\frac{a}{b}\right)}}\right)}{d\sqrt{bde\,x^3-ade\,x^2-bce\,x^2+acex}}-\frac{2Bc\sqrt{-\frac{\left(x-\frac{c}{d}\right)d}{c}}\sqrt{\frac{x-\frac{a}{b}}{\frac{c}{d}-\frac{a}{b}}}\sqrt{\frac{xd}{c}}\left(\left(\frac{c}{d}-\cdots\right.\right.}{\cdots}\right.}{\sqrt{ex}\,\sqrt{-bx+a}\,\sqrt{-xd+c}}$

input
```
int((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(-d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2*(A*EllipticF(((-d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*b*d+B*EllipticF
(((-d*x+c)/c)^(1/2),(-b*c/(a*d-b*c))^(1/2))*a*d-B*EllipticE(((-d*x+c)/c)^(
1/2),(-b*c/(a*d-b*c))^(1/2))*a*d+B*EllipticE(((-d*x+c)/c)^(1/2),(-b*c/(a*d
-b*c))^(1/2))*b*c)*c*(1/c*x*d)^(1/2)*((-b*x+a)*d/(a*d-b*c))^(1/2)*(((-d*x+c
)/c)^(1/2)*(-d*x+c)^(1/2)*(-b*x+a)^(1/2)/b/d^2/(b*d*x^2-a*d*x-b*c*x+a*c)/(
e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx =$$
$$-\frac{2\left(3\sqrt{bde}Bbd\text{weierstrassZeta}\left(\frac{4\left(b^2c^2-abcd+a^2d^2\right)}{3\,b^2d^2},\frac{4\left(2\,b^3c^3-3\,ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3\right)}{27\,b^3d^3},\text{weierstrassPInverse}\left(\frac{4\,(\cdots}{\cdots}\right.\right.\right.}{\cdots}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(-d*x+c)^(1/2),x, algorithm="
fricas")
```

output
```
-2/3*(3*sqrt(b*d*e)*B*b*d*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2
)/(b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/
(b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2)
, 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3),
1/3*(3*b*d*x - b*c - a*d)/(b*d))) - (B*b*c + (B*a + 3*A*b)*d)*sqrt(b*d*e)*
weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2)/(b^2*d^2), 4/27*(2*b
^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)/(b^3*d^3), 1/3*(3*b*d*
x - b*c - a*d)/(b*d)))/(b^2*d^2*e)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx=\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(-b*x+a)**(1/2)/(-d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a - b*x)*sqrt(c - d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx=\int \frac{Bx+A}{\sqrt{-bx+a}\sqrt{-dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(-d*x+c)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((B*x + A)/(sqrt(-b*x + a)*sqrt(-d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx = \int \frac{Bx+A}{\sqrt{-bx+a}\sqrt{-dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(-d*x+c)^(1/2),x, algorithm="
giac")
```

output
```
integrate((B*x + A)/(sqrt(-b*x + a)*sqrt(-d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx = \int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{a-b\,x}\,\sqrt{c-d\,x}}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a - b*x)^(1/2)*(c - d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a - b*x)^(1/2)*(c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx}\sqrt{c-dx}}\,dx = \frac{\sqrt{e}\left(\left(\int \frac{\sqrt{x}\,\sqrt{-dx+c}\,\sqrt{-bx+a}}{bd\,x^3-ad\,x^2-bc\,x^2+acx}dx\right)a+\left(\int \frac{\sqrt{x}\,\sqrt{-dx+c}\,\sqrt{-bx+a}}{bd\,x^2-adx-bcx+ac}dx\right)b\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(-b*x+a)^(1/2)/(-d*x+c)^(1/2),x)
```

output
```
(sqrt(e)*(int((sqrt(x)*sqrt(c - d*x)*sqrt(a - b*x))/(a*c*x - a*d*x**2 - b*
c*x**2 + b*d*x**3),x)*a + int((sqrt(x)*sqrt(c - d*x)*sqrt(a - b*x))/(a*c -
 a*d*x - b*c*x + b*d*x**2),x)*b))/e
```

3.211 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 145

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{2a^{3/2}BE\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)} + \frac{2\sqrt{a}(aBe+A(b-be))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right),\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

output
```
-2*a^(3/2)*B*EllipticE((1-c)^(1/2)*(b*x+a)^(1/2)/a^(1/2),((1-e)/(1-c))^(1/
2))/b^2/(1-c)^(1/2)/(1-e)+2*a^(1/2)*(a*B*e+A*(-b*e+b))*EllipticF((1-c)^(1/
2)*(b*x+a)^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2))/b^2/(1-c)^(1/2)/(1-e)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{2\sqrt{\frac{a}{-1+c}}(a+bx)^{3/2}\left(-B\sqrt{\frac{a}{-1+c}}\left(-1+c+\frac{a}{a+bx}\right)\left(-1+e+\frac{a}{a+bx}\right)-\frac{iaB(-1+e)\sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}}\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}E}{\sqrt{a+bx}}\right.}{ab^2(-1+e)\sqrt{c+\frac{b(}{}}}$$

input
```
Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*
(-1 + e)*x)/a]),x]
```

output
```
(-2*Sqrt[a/(-1 + c)]*(a + b*x)^(3/2)*(-(B*Sqrt[a/(-1 + c)]*(-1 + c + a/(a
+ b*x))*(-1 + e + a/(a + b*x))) - (I*a*B*(-1 + e)*Sqrt[(-1 + c + a/(a + b*
x))/(-1 + c)]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sq
rt[a/(-1 + c)]/Sqrt[a + b*x]], (-1 + c)/(-1 + e)])/Sqrt[a + b*x] + (I*(a*B
*c + A*(b - b*c))*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x))/(-1 + c)]*Sqrt[(-1
+ e + a/(a + b*x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + c)]/Sqrt[a +
 b*x]], (-1 + c)/(-1 + e)])/Sqrt[a + b*x]))/(a*b^2*(-1 + e)*Sqrt[c + (b*(-
1 + c)*x)/a]*Sqrt[e + (b*(-1 + e)*x)/a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {176, 123, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}dx$$

$$\Big\downarrow \text{176}$$

$$\left(\frac{aBe}{b-be}+A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c-\frac{b(1-c)x}{a}}\sqrt{e-\frac{b(1-e)x}{a}}}dx-\frac{aB\int \frac{\sqrt{e-\frac{b(1-e)x}{a}}}{\sqrt{a+bx}\sqrt{c-\frac{b(1-c)x}{a}}}dx}{b(1-e)}$$

$$\Big\downarrow \text{123}$$

$$\left(\frac{aBe}{b-be}+A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c-\frac{b(1-c)x}{a}}\sqrt{e-\frac{b(1-e)x}{a}}}dx-\frac{2a^{3/2}BE\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

$$\Big\downarrow \text{129}$$

$$\frac{2\sqrt{a}\left(\frac{aBe}{b-be}+A\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right),\frac{1-e}{1-c}\right)}{b\sqrt{1-c}}-\frac{2a^{3/2}BE\left(\arcsin\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)}$$

input
```
Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(-1 +
e)*x)/a]),x]
```

output
```
(-2*a^(3/2)*B*EllipticE[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 -
e)/(1 - c)])/(b^2*Sqrt[1 - c]*(1 - e)) + (2*Sqrt[a]*(A + (a*B*e)/(b - b*e)
)*EllipticF[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e)/(1 - c)])
/(b*Sqrt[1 - c])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(127) = 254$.

Time = 12.84 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.17

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{(c-1)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{c-1}}\right)bce-A\,\mathrm{EllipticF}\left(\sqrt{\frac{(c-1)(bex+ae-bx)}{a(c-e)}},\sqrt{\frac{c-e}{c-1}}\right)b\,e^{2}-B\,\mathrm{EllipticF}\left(\sqrt{\frac{(c-1)(bex+ae-bx)}{a(c-e)}},\right.\right.}{}$
elliptic	$\frac{\sqrt{\frac{(bx+a)(bcx+ac-bx)(bex+ae-bx)}{a^{2}}}\left(\frac{2A\left(\frac{ae}{(-1+e)b}-\frac{ac}{b(c-1)}\right)\sqrt{\frac{x+\frac{ae}{(-1+e)b}}{\frac{ae}{(-1+e)b}-\frac{ac}{b(c-1)}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{ae}{(-1+e)b}+\frac{a}{b}}}\sqrt{\frac{x+\frac{ac}{b(c-1)}}{-\frac{ae}{(-1+e)b}+\frac{ac}{b(c-1)}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{\sqrt{\frac{b^{3}ce\,x^{3}}{a^{2}}+\frac{3b^{2}ce\,x^{2}}{a}-\frac{b^{3}c\,x^{3}}{a^{2}}-\frac{b^{3}e\,x^{3}}{a^{2}}+3bcex-\frac{2b^{2}c\,x^{2}}{a}-\frac{2b^{2}e\,x^{2}}{a}+\frac{b^{3}x^{3}}{a^{2}}+ace-}}\right.}{}$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(c+b*(c-1)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
2*(A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(1/2))*
b*c*e-A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(1/2
))*b*e^2-B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1))^(
1/2))*a*c*e+B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c-1)
)^(1/2))*a*e^2-A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(c
-1))^(1/2))*b*c+A*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/(
c-1))^(1/2))*b*e+B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)/
(c-1))^(1/2))*a*c-B*EllipticF(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e)
/(c-1))^(1/2))*a*e-B*EllipticE(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-e
)/(c-1))^(1/2))*a*c+B*EllipticE(((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2),((c-
e)/(c-1))^(1/2))*a*e)*(-(-1+e)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2)*(-(b*x+a)*(-
1+e)/a)^(1/2)*((c-1)*(b*e*x+a*e-b*x)/a/(c-e))^(1/2)*a/(b*x+a)^(1/2)/((b*c*
x+a*c-b*x)/a)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)/(-1+e)^2/(c-1)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(117) = 234$.

Time = 0.10 (sec) , antiderivative size = 1228, normalized size of antiderivative = 8.47

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1
/2),x, algorithm="fricas")
```

output
```
-2/3*((B*a^3 - 3*A*a^2*b - (2*B*a^3 - 3*A*a^2*b)*c - (2*B*a^3 - 3*A*a^2*b
- 3*(B*a^3 - A*a^2*b)*c)*e)*sqrt(-(b^3*c - b^3 - (b^3*c - b^3)*e)/a^2)*wei
erstrassPInverse(4/3*(a^2*c^2 + a^2*e^2 - a^2*c + a^2 - (a^2*c + a^2)*e)/(
b^2*c^2 - 2*b^2*c + (b^2*c^2 - 2*b^2*c + b^2)*e^2 + b^2 - 2*(b^2*c^2 - 2*b
^2*c + b^2)*e), 4/27*(2*a^3*c^3 + 2*a^3*e^3 - 3*a^3*c^2 - 3*a^3*c + 2*a^3
- 3*(a^3*c + a^3)*e^2 - 3*(a^3*c^2 - 4*a^3*c + a^3)*e)/(b^3*c^3 - 3*b^3*c^
2 + 3*b^3*c - (b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^3 - b^3 + 3*(b^3*c^3
 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^2 - 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3
)*e), 1/3*(2*a*c - (3*a*c - 2*a)*e + 3*(b*c - (b*c - b)*e - b)*x - a)/(b*c
 - (b*c - b)*e - b)) - 3*(B*a^2*b*c - B*a^2*b - (B*a^2*b*c - B*a^2*b)*e)*s
qrt(-(b^3*c - b^3 - (b^3*c - b^3)*e)/a^2)*weierstrassZeta(4/3*(a^2*c^2 + a
^2*e^2 - a^2*c + a^2 - (a^2*c + a^2)*e)/(b^2*c^2 - 2*b^2*c + (b^2*c^2 - 2*
b^2*c + b^2)*e^2 + b^2 - 2*(b^2*c^2 - 2*b^2*c + b^2)*e), 4/27*(2*a^3*c^3 +
 2*a^3*e^3 - 3*a^3*c^2 - 3*a^3*c + 2*a^3 - 3*(a^3*c + a^3)*e^2 - 3*(a^3*c^
2 - 4*a^3*c + a^3)*e)/(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - (b^3*c^3 - 3*b^3*c^
2 + 3*b^3*c - b^3)*e^3 - b^3 + 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e^2
 - 3*(b^3*c^3 - 3*b^3*c^2 + 3*b^3*c - b^3)*e), weierstrassPInverse(4/3*(a^
2*c^2 + a^2*e^2 - a^2*c + a^2 - (a^2*c + a^2)*e)/(b^2*c^2 - 2*b^2*c + (b^2
*c^2 - 2*b^2*c + b^2)*e^2 + b^2 - 2*(b^2*c^2 - 2*b^2*c + b^2)*e), 4/27*(2*
a^3*c^3 + 2*a^3*e^3 - 3*a^3*c^2 - 3*a^3*c + 2*a^3 - 3*(a^3*c + a^3)*e^2...
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{bcx}{a}-\frac{bx}{a}}\sqrt{e+\frac{bex}{a}-\frac{bx}{a}}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(1/2)/(c+b*(-1+c)*x/a)**(1/2)/(e+b*(-1+e)*x/a)*
*(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + b*c*x/a - b*x/a)*sqrt(e + b*e*x
/a - b*x/a)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*
x/a + e)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1
/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*
x/a + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+B\,x}{\sqrt{c+\frac{b\,x\,(c-1)}{a}}\,\sqrt{e+\frac{b\,x\,(e-1)}{a}}\,\sqrt{a+b\,x}}\,dx$$

input
```
int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a
+ b*x)^(1/2)),x)
```

output
```
int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a
+ b*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$$
$$=\left(\int \frac{\sqrt{bx+a}\,\sqrt{bex+ae-bx}\,\sqrt{bcx+ac-bx}}{b^2ce\,x^2+2abcex-b^2c\,x^2-b^2e\,x^2+a^2ce-abcx-abex+b^2x^2}dx\right)a$$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x)
```

output
```
int((sqrt(a + b*x)*sqrt(a*e + b*e*x - b*x)*sqrt(a*c + b*c*x - b*x))/(a**2*
c*e + 2*a*b*c*e*x - a*b*c*x - a*b*e*x + b**2*c*e*x**2 - b**2*c*x**2 - b**2
*e*x**2 + b**2*x**2),x)*a
```

3.212 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 208

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx &= \frac{2\sqrt{a}B\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right)|-\frac{ad}{(bc-ad)(1-e)}\right)}{bd\sqrt{1-e}\sqrt{\frac{b(c+dx)}{bc-ad}}} \\ &\quad - \frac{2\sqrt{a}(Bc-Ad)\sqrt{\frac{b(c+dx)}{bc-ad}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right),-\frac{ad}{(bc-ad)(1-e)}\right)}{bd\sqrt{1-e}\sqrt{c+dx}}\end{aligned}$$

output
```
2*a^(1/2)*B*(d*x+c)^(1/2)*EllipticE((1-e)^(1/2)*(b*x+a)^(1/2)/a^(1/2),(-a*
d/(-a*d+b*c)/(1-e))^(1/2))/b/d/(1-e)^(1/2)/(b*(d*x+c)/(-a*d+b*c))^(1/2)-2*
a^(1/2)*(-A*d+B*c)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*EllipticF((1-e)^(1/2)*(b*x
+a)^(1/2)/a^(1/2),(-a*d/(-a*d+b*c)/(1-e))^(1/2))/b/d/(1-e)^(1/2)/(d*x+c)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.37 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{2\sqrt{\frac{a}{-1+e}}(a+bx)^{3/2}\left(-\frac{bB\sqrt{\frac{a}{-1+e}}(c+dx)(ae+b(-1+e)x)}{(a+bx)^2}-\frac{iaBd\sqrt{\frac{b(c+dx)}{d(a+bx)}}\sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{a}{-1+e}}}{\sqrt{a+bx}}\right)|\frac{(bc-ad)(-}{ad}\right.}{\sqrt{a+bx}}\right.}{ab^2d\sqrt{c+dx}\sqrt{e+\frac{b(-1+}{a}}}$$

input
```
Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a
]),x]
```

output
```
(-2*Sqrt[a/(-1 + e)]*(a + b*x)^(3/2)*(-((b*B*Sqrt[a/(-1 + e)]*(c + d*x)*(a
*e + b*(-1 + e)*x))/(a + b*x)^2) - (I*a*B*d*Sqrt[(b*(c + d*x))/(d*(a + b*x
))]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[a/(-1 +
 e)]/Sqrt[a + b*x]], ((b*c - a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x] + (I*d*(
a*B*e + A*(b - b*e))*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(-1 + e + a/(a
 + b*x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + e)]/Sqrt[a + b*x]], ((
b*c - a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x]))/(a*b^2*d*Sqrt[c + d*x]*Sqrt[e
 + (b*(-1 + e)*x)/a])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {176, 124, 123, 131, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

$$\Big\downarrow \text{176}$$

$$\left(\frac{aBe}{b-be}+A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e-\frac{b(1-e)x}{a}}}dx-\frac{aB\int \frac{\sqrt{e-\frac{b(1-e)x}{a}}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b(1-e)}$$

$$\Big\downarrow \text{124}$$

$$\left(\frac{aBe}{b-be}+A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e-\frac{b(1-e)x}{a}}}dx-\frac{aB\sqrt{\frac{b(c+dx)}{bc-ad}}\int \frac{\sqrt{e-\frac{b(1-e)x}{a}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{b(1-e)\sqrt{c+dx}}$$

$$\Big\downarrow \text{123}$$

$$\left(\frac{aBe}{b-be}+A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e-\frac{b(1-e)x}{a}}}dx-\frac{2aB\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{\left(\frac{aBe}{b-be}+A\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e-\frac{b(1-e)x}{a}}}dx}{\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

$$\Big\downarrow \text{129}$$

$$\frac{2\sqrt{a}\left(\frac{aBe}{b-be}+A\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-e}\sqrt{a+bx}}{\sqrt{a}}\right),-\frac{ad}{(bc-ad)(1-e)}\right)}{b\sqrt{1-e}\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}(1-e)\sqrt{c+dx}}$$

input
```
Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]),x]
```

output
```
(-2*a*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticE[ArcSi
n[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], -(((b*c - a*d)*(1 - e))/(a*
d))])/(b^2*Sqrt[d]*(1 - e)*Sqrt[c + d*x]) + (2*Sqrt[a]*(A + (a*B*e)/(b - b
*e))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticF[ArcSin[(Sqrt[1 - e]*Sqrt[a
+ b*x])/Sqrt[a]], -((a*d)/((b*c - a*d)*(1 - e)))])/(b*Sqrt[1 - e]*Sqrt[c +
 d*x])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(182) = 364$.

Time = 4.54 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.50

method	result
elliptic	$\frac{\sqrt{\frac{(bx+a)(xd+c)(bex+ae-bx)}{a}}\left(\frac{2A\left(\frac{ae}{(-1+e)b}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{ae}{(-1+e)b}}{\frac{ae}{(-1+e)b}-\frac{c}{d}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{ae}{(-1+e)b}+\frac{a}{b}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{ae}{(-1+e)b}+\frac{c}{d}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{ae}{(-1+e)b}}{\frac{ae}{(-1+e)b}-\frac{c}{d}}},\sqrt{\frac{-\frac{ae}{(-1+e)}}{-\frac{ae}{(-1+e)}}}\right.}{\sqrt{\frac{b^2dex^3}{a}+2bdex^2+\frac{b^2cex^2}{a}-\frac{x^3db^2}{a}+adex+2bcex-bdx^2-\frac{b^2cx^2}{a}+ace-bcx}}\right.}{}$
default	$\frac{2\sqrt{bx+a}\sqrt{xd+c}\sqrt{\frac{d(bex+ae-bx)}{ade-bce+bc}}\sqrt{-\frac{(bx+a)(-1+e)}{a}}\sqrt{-\frac{(xd+c)b(-1+e)}{ade-bce+bc}}\left(A\operatorname{EllipticF}\left(\sqrt{\frac{d(bex+ae-bx)}{ade-bce+bc}},\sqrt{\frac{ade-bce+bc}{da}}\right)abde^2-A\operatorname{El}\right.}{}$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)*((b*x+a)*(d*x+c)*(
b*e*x+a*e-b*x)/a)^(1/2)*(2*A*(a*e/(-1+e)/b-c/d)*((x+a*e/(-1+e)/b)/(a*e/(-1
+e)/b-c/d))^(1/2)*((x+a/b)/(-a*e/(-1+e)/b+a/b))^(1/2)*((x+c/d)/(-a*e/(-1+e
)/b+c/d))^(1/2)/(1/a*b^2*d*e*x^3+2*b*d*e*x^2+1/a*b^2*c*e*x^2-1/a*x^3*d*b^2
+a*d*e*x+2*b*c*e*x-b*d*x^2-1/a*b^2*c*x^2+a*c*e-b*c*x)^(1/2)*EllipticF(((x+
a*e/(-1+e)/b)/(a*e/(-1+e)/b-c/d))^(1/2),((-a*e/(-1+e)/b+c/d)/(-a*e/(-1+e)/
b+a/b))^(1/2))+2*B*(a*e/(-1+e)/b-c/d)*((x+a*e/(-1+e)/b)/(a*e/(-1+e)/b-c/d)
)^(1/2)*((x+a/b)/(-a*e/(-1+e)/b+a/b))^(1/2)*((x+c/d)/(-a*e/(-1+e)/b+c/d))^
(1/2)/(1/a*b^2*d*e*x^3+2*b*d*e*x^2+1/a*b^2*c*e*x^2-1/a*x^3*d*b^2+a*d*e*x+2
*b*c*e*x-b*d*x^2-1/a*b^2*c*x^2+a*c*e-b*c*x)^(1/2)*((-a*e/(-1+e)/b+a/b)*Ell
ipticE(((x+a*e/(-1+e)/b)/(a*e/(-1+e)/b-c/d))^(1/2),((-a*e/(-1+e)/b+c/d)/(-
a*e/(-1+e)/b+a/b))^(1/2))-a/b*EllipticF(((x+a*e/(-1+e)/b)/(a*e/(-1+e)/b-c/
d))^(1/2),((-a*e/(-1+e)/b+c/d)/(-a*e/(-1+e)/b+a/b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(176) = 352$.

Time $= 0.11$ (sec) , antiderivative size $= 1126$, normalized size of antiderivative $= 5.41$

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, al
gorithm="fricas")
```

output
```
2/3*((B*a*b*c + (B*a^2 - 3*A*a*b)*d - (B*a*b*c + (2*B*a^2 - 3*A*a*b)*d)*e)
*sqrt((b^2*d*e - b^2*d)/a)*weierstrassPInverse(4/3*(b^2*c^2 - a*b*c*d + a^
2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2
*d^2)*e)/(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d
^3)*e^2 - 3*(2*b^3*c^3 - 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^
3*e^3 - 3*b^3*d^3*e^2 + 3*b^3*d^3*e - b^3*d^3), -1/3*(b*c + a*d - (b*c + 2
*a*d)*e - 3*(b*d*e - b*d)*x)/(b*d*e - b*d)) - 3*(B*a*b*d*e - B*a*b*d)*sqrt
((b^2*d*e - b^2*d)/a)*weierstrassZeta(4/3*(b^2*c^2 - a*b*c*d + a^2*d^2 + (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*e)/
(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^3 - 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*e^2 -
 3*(2*b^3*c^3 - 4*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*e)/(b^3*d^3*e^3 - 3
*b^3*d^3*e^2 + 3*b^3*d^3*e - b^3*d^3), weierstrassPInverse(4/3*(b^2*c^2 -
a*b*c*d + a^2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e^2 - (2*b^2*c^2 - 3*a
*b*c*d + a^2*d^2)*e)/(b^2*d^2*e^2 - 2*b^2*d^2*e + b^2*d^2), 4/27*(2*b^3*c^
3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3 - 2*(b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*e^3 + 3*(2*b^3*c^3 - 5*a*b^2*c^2*d + 4*a^2*...
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{bex}{a}-\frac{bx}{a}}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(e+b*(-1+e)*x/a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + b*e*x/a - b*x/a))
, x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
 x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=\int \frac{A+B\,x}{\sqrt{e+\frac{b\,x\,(e-1)}{a}}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bex+ae-bx}}{bde\,x^2+adex+bcex-bd\,x^2+ace-bcx}dx\right)$$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(c + d*x)*sqrt(a + b*x)*sqrt(a*e + b*e*x - b*x))/(a*c*e +
 a*d*e*x + b*c*e*x - b*c*x + b*d*e*x**2 - b*d*x**2),x)
```

3.213 $\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 92

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx = -\frac{2a\sqrt{bc+2d}\sqrt{\frac{b(c+dx)}{bc+2d}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{2-bx}}{\sqrt{bc+2d}}\right)|\frac{1}{4}\left(2+\frac{bc}{d}\right)\right)}{b\sqrt{d}\sqrt{c+dx}}$$

output
```
-2*a*(b*c+2*d)^(1/2)*(b*(d*x+c)/(b*c+2*d))^(1/2)*EllipticE(d^(1/2)*(-b*x+2
)^(1/2)/(b*c+2*d)^(1/2),1/2*(2+b*c/d)^(1/2))/b/d^(1/2)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.67 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.14

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx = -\frac{a\left(d^2\sqrt{-c-\frac{2d}{b}}(4-b^2x^2)+ib(bc+2d)\sqrt{\frac{d(-2+bx)}{b(c+dx)}}\sqrt{\frac{d(2+bx)}{b(c+dx)}}(c+dx)^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c-\frac{2d}{b}}}{\sqrt{c+dx}}\right)|\frac{bc-2d}{bc+2d}\right)\right.}{bd^2\sqrt{-c-\frac{2d}{b}}\sqrt{c+dx}\sqrt{4-}}$$

input
```
Integrate[(a + (a*b*x)/2)/(Sqrt[2 - b*x]*Sqrt[2 + b*x]*Sqrt[c + d*x]),x]
```

output
```
-((a*(d^2*Sqrt[-c - (2*d)/b]*(4 - b^2*x^2) + I*b*(b*c + 2*d)*Sqrt[(d*(-2 +
 b*x))/(b*(c + d*x))]*Sqrt[(d*(2 + b*x))/(b*(c + d*x))]*(c + d*x)^(3/2)*El
lipticE[I*ArcSinh[Sqrt[-c - (2*d)/b]/Sqrt[c + d*x]], (b*c - 2*d)/(b*c + 2*
d)] - (4*I)*b*d*Sqrt[(d*(-2 + b*x))/(b*(c + d*x))]*Sqrt[(d*(2 + b*x))/(b*(
c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (2*d)/b]/Sqrt[c +
 d*x]], (b*c - 2*d)/(b*c + 2*d)]))/(b*d^2*Sqrt[-c - (2*d)/b]*Sqrt[c + d*x]
*Sqrt[4 - b^2*x^2]))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {35, 124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{abx}{2}+a}{\sqrt{2-bx}\sqrt{bx+2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 35}$$

$$\frac{1}{2}a\int \frac{\sqrt{bx+2}}{\sqrt{2-bx}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 124}$$

$$\frac{a\sqrt{\frac{b(c+dx)}{bc+2d}}\int \frac{\sqrt{bx+2}}{2\sqrt{2-bx}\sqrt{\frac{bc}{bc+2d}+\frac{bdx}{bc+2d}}}dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\sqrt{\frac{b(c+dx)}{bc+2d}}\int \frac{\sqrt{bx+2}}{\sqrt{2-bx}\sqrt{\frac{bc}{bc+2d}+\frac{bdx}{bc+2d}}}dx}{2\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 123}$$

$$-\frac{2a\sqrt{bc+2d}\sqrt{\frac{b(c+dx)}{bc+2d}}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{2-bx}}{\sqrt{bc+2d}}\right)|\frac{1}{4}\left(\frac{bc}{d}+2\right)\right)}{b\sqrt{d}\sqrt{c+dx}}$$

input
```
Int[(a + (a*b*x)/2)/(Sqrt[2 - b*x]*Sqrt[2 + b*x]*Sqrt[c + d*x]),x]
```

output
```
(-2*a*Sqrt[b*c + 2*d]*Sqrt[(b*(c + d*x))/(b*c + 2*d)]*EllipticE[ArcSin[(Sq
rt[d]*Sqrt[2 - b*x])/Sqrt[b*c + 2*d]], (2 + (b*c)/d)/4])/(b*Sqrt[d]*Sqrt[c
 + d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(77) = 154$.

Time = 1.45 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.13

method	result
default	$\frac{a\left(\text{EllipticE}\left(\sqrt{\frac{(xd+c)b}{bc-2d}},\sqrt{\frac{bc-2d}{bc+2d}}\right)b^2c^2-4\,\text{EllipticF}\left(\sqrt{\frac{(xd+c)b}{bc-2d}},\sqrt{\frac{bc-2d}{bc+2d}}\right)bcd-4\,\text{EllipticE}\left(\sqrt{\frac{(xd+c)b}{bc-2d}},\sqrt{\frac{bc-2d}{bc+2d}}\right)d^2+8\,\text{EllipticF}\left(\right.}{b\,d^2(x^3d\,b^2+b^2c\,x^2-4xd-4c)}$
elliptic	$\frac{\sqrt{-(xd+c)(b^2x^2-4)}\left(\frac{2a\left(\frac{c}{d}-\frac{2}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{2}{b}}}\sqrt{\frac{x-\frac{2}{b}}{-\frac{c}{d}-\frac{2}{b}}}\sqrt{\frac{x+\frac{2}{b}}{-\frac{c}{d}+\frac{2}{b}}}\,\text{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{2}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{2}{b}}{-\frac{c}{d}-\frac{2}{b}}}\right)}{\sqrt{-x^3d\,b^2-b^2c\,x^2+4xd+4c}}+\frac{ab\left(\frac{c}{d}-\frac{2}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{2}{b}}}\sqrt{\frac{x-\frac{2}{b}}{-\frac{c}{d}-\frac{2}{b}}}\sqrt{\frac{x+\frac{2}{b}}{-\frac{c}{d}+\frac{2}{b}}}\left(\right.}{}\right.}{\sqrt{bx+2}\,\sqrt{-bx+2}\,\sqrt{xd+c}}$

input
```
int((a+1/2*a*b*x)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
a*(EllipticE(((d*x+c)*b/(b*c-2*d))^(1/2),((b*c-2*d)/(b*c+2*d))^(1/2))*b^2*
c^2-4*EllipticF(((d*x+c)*b/(b*c-2*d))^(1/2),((b*c-2*d)/(b*c+2*d))^(1/2))*b
*c*d-4*EllipticE(((d*x+c)*b/(b*c-2*d))^(1/2),((b*c-2*d)/(b*c+2*d))^(1/2))*
d^2+8*EllipticF(((d*x+c)*b/(b*c-2*d))^(1/2),((b*c-2*d)/(b*c+2*d))^(1/2))*d
^2)*(-d*(b*x+2)/(b*c-2*d))^(1/2)*(-d*(b*x-2)/(b*c+2*d))^(1/2)*((d*x+c)*b/(
b*c-2*d))^(1/2)*(-b*x+2)^(1/2)*(b*x+2)^(1/2)*(d*x+c)^(1/2)/b/d^2/(b^2*d*x^
3+b^2*c*x^2-4*d*x-4*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(77) = 154$.

Time = 0.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx$$
$$= \frac{3\sqrt{-b^2d}abd\text{weierstrassZeta}\left(\frac{4\,(b^2c^2+12\,d^2)}{3\,b^2d^2}, -\frac{8\,(b^2c^3-36\,cd^2)}{27\,b^2d^3}, \text{weierstrassPInverse}\left(\frac{4\,(b^2c^2+12\,d^2)}{3\,b^2d^2}, -\frac{8\,(b^2c^3-36\,cd^2}{27\,b^2d^3}\right.\right.}{3\,b^2d^2}$$

input
```
integrate((a+1/2*a*b*x)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*(3*sqrt(-b^2*d)*a*b*d*weierstrassZeta(4/3*(b^2*c^2 + 12*d^2)/(b^2*d^2)
, -8/27*(b^2*c^3 - 36*c*d^2)/(b^2*d^3), weierstrassPInverse(4/3*(b^2*c^2 +
 12*d^2)/(b^2*d^2), -8/27*(b^2*c^3 - 36*c*d^2)/(b^2*d^3), 1/3*(3*d*x + c)/
d)) + (a*b*c - 6*a*d)*sqrt(-b^2*d)*weierstrassPInverse(4/3*(b^2*c^2 + 12*d
^2)/(b^2*d^2), -8/27*(b^2*c^3 - 36*c*d^2)/(b^2*d^3), 1/3*(3*d*x + c)/d))/(
b^2*d^2)
```

Sympy [F]

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx = \frac{a\int \frac{\sqrt{bx+2}}{\sqrt{c+dx}\sqrt{-bx+2}}\,dx}{2}$$

input
```
integrate((a+1/2*a*b*x)/(-b*x+2)**(1/2)/(b*x+2)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
a*Integral(sqrt(b*x + 2)/(sqrt(c + d*x)*sqrt(-b*x + 2)), x)/2
```

Maxima [F]

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx = \int \frac{abx+2\,a}{2\,\sqrt{bx+2}\sqrt{-bx+2}\sqrt{dx+c}}\,dx$$

input
```
integrate((a+1/2*a*b*x)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/2*integrate((a*b*x + 2*a)/(sqrt(b*x + 2)*sqrt(-b*x + 2)*sqrt(d*x + c)),
x)
```

Giac [F]

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx=\int \frac{abx+2\,a}{2\,\sqrt{bx+2}\sqrt{-bx+2}\sqrt{dx+c}}\,dx$$

input
```
integrate((a+1/2*a*b*x)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(1/2*(a*b*x + 2*a)/(sqrt(b*x + 2)*sqrt(-b*x + 2)*sqrt(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx=\int \frac{a+\frac{a\,b\,x}{2}}{\sqrt{2-b\,x}\,\sqrt{b\,x+2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((a + (a*b*x)/2)/((2 - b*x)^(1/2)*(b*x + 2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((a + (a*b*x)/2)/((2 - b*x)^(1/2)*(b*x + 2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+\frac{abx}{2}}{\sqrt{2-bx}\sqrt{2+bx}\sqrt{c+dx}}\,dx=-\frac{\left(\int \frac{\sqrt{dx+c}\,\sqrt{bx+2}\,\sqrt{-bx+2}}{bd\,x^2+bcx-2dx-2c}dx\right)a}{2}$$

input
```
int((a+1/2*a*b*x)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(d*x+c)^(1/2),x)
```

output
```
( - int((sqrt(c + d*x)*sqrt(b*x + 2)*sqrt( - b*x + 2))/(b*c*x + b*d*x**2 -
 2*c - 2*d*x),x)*a)/2
```

3.214 $\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)} dx$

Optimal result

Integrand size = 29, antiderivative size = 448

$$\begin{aligned}\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx &= -\frac{3(adfh-b(dfg+deh-cfh))(e+fx)^{2/3}}{2b^2d^2} \\ &+ \frac{3h(e+fx)^{5/3}}{5bd} + \frac{\sqrt{3}(be-af)^{5/3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{b^{8/3}(bc-ad)} \\ &- \frac{\sqrt{3}(de-cf)^{5/3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{d^{8/3}(bc-ad)} \\ &- \frac{(be-af)^{5/3}(bg-ah)\log(a+bx)}{2b^{8/3}(bc-ad)} + \frac{(de-cf)^{5/3}(dg-ch)\log(c+dx)}{2d^{8/3}(bc-ad)} \\ &+ \frac{3(be-af)^{5/3}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{8/3}(bc-ad)} \\ &- \frac{3(de-cf)^{5/3}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{8/3}(bc-ad)}\end{aligned}$$

output
```
-3/2*(a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)^(2/3)/b^2/d^2+3/5*h*(f*x+e)^
(5/3)/b/d+3^(1/2)*(-a*f+b*e)^(5/3)*(-a*h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x
+e)^(1/3)/(-a*f+b*e)^(1/3))*3^(1/2))/b^(8/3)/(-a*d+b*c)-3^(1/2)*(-c*f+d*e)
^(5/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*
3^(1/2))/d^(8/3)/(-a*d+b*c)-1/2*(-a*f+b*e)^(5/3)*(-a*h+b*g)*ln(b*x+a)/b^(8
/3)/(-a*d+b*c)+1/2*(-c*f+d*e)^(5/3)*(-c*h+d*g)*ln(d*x+c)/d^(8/3)/(-a*d+b*c
)+3/2*(-a*f+b*e)^(5/3)*(-a*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3
))/b^(8/3)/(-a*d+b*c)-3/2*(-c*f+d*e)^(5/3)*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-
d^(1/3)*(f*x+e)^(1/3))/d^(8/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.12

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{3b^{2/3}d^{2/3}(bc-ad)(e+fx)^{2/3}(-5bcfh-5adfh+bd(5fg+7eh+2fhx))-1}{}$$

input
```
Integrate[((e + f*x)^(5/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(3*b^(2/3)*d^(2/3)*(b*c - a*d)*(e + f*x)^(2/3)*(-5*b*c*f*h - 5*a*d*f*h + b
*d*(5*f*g + 7*e*h + 2*f*h*x)) - 10*Sqrt[3]*d^(8/3)*(-(b*e) + a*f)^(5/3)*(b
*g - a*h)*ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e) + a*f)^(1/3))/Sq
rt[3]] + 10*Sqrt[3]*b^(8/3)*(-(d*e) + c*f)^(5/3)*(d*g - c*h)*ArcTan[(1 - (
2*d^(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))/Sqrt[3]] - 10*d^(8/3)*(-(
b*e) + a*f)^(5/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(1/3) + b^(1/3)*(e + f*x)
^(1/3)] + 10*b^(8/3)*(-(d*e) + c*f)^(5/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(
1/3) + d^(1/3)*(e + f*x)^(1/3)] + 5*d^(8/3)*(-(b*e) + a*f)^(5/3)*(b*g - a*
h)*Log[(-(b*e) + a*f)^(2/3) - b^(1/3)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3)
 + b^(2/3)*(e + f*x)^(2/3)] - 5*b^(8/3)*(-(d*e) + c*f)^(5/3)*(d*g - c*h)*L
og[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f*x)^(1/3) + d
^(2/3)*(e + f*x)^(2/3)])/(10*b^(8/3)*d^(8/3)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {174, 60, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{174}$$

$$\frac{(bg-ah)\int \frac{(e+fx)^{5/3}}{a+bx}dx}{bc-ad}-\frac{(dg-ch)\int \frac{(e+fx)^{5/3}}{c+dx}dx}{bc-ad}$$

$$\Big\downarrow \text{60}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\int \frac{(e+fx)^{2/3}}{a+bx}dx}{b}+\frac{3(e+fx)^{5/3}}{5b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\int \frac{(e+fx)^{2/3}}{c+dx}dx}{d}+\frac{3(e+fx)^{5/3}}{5d}\right)}{bc-ad}$$

$$\Big\downarrow \text{60}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt[3]{e+fx}}dx}{b}+\frac{3(e+fx)^{2/3}}{2b}\right)}{b}+\frac{3(e+fx)^{5/3}}{5b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{e+fx}}dx}{d}+\frac{3(e+fx)^{2/3}}{2d}\right)}{d}+\frac{3(e+fx)^{5/3}}{5d}\right)}{bc-ad}$$

$$\Big\downarrow \text{67}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e}}{2b}\right.}{b}}{b}}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e}}{2d}\right.}{d}}{d}}{bc-ad}$$

$\downarrow$ 16

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}\cdots}{2b^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}\cdots}{2d^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)}{bc-ad}$$

$$\downarrow 1082$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{b}}{b}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{d}}{d}\right.}{bc-ad}$$

$$\downarrow 217$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{b}+\frac{3(e+fx)^{2/}}{2b}\cdots}{b}\cdots\right.}{bc-ad}$$

$$(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{d}+\frac{3(e+fx)^{2}}{2d}\cdots}{d}\cdots\right.$$

input
```
Int[((e + f*x)^(5/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
((b*g - a*h)*((3*(e + f*x)^(5/3))/(5*b) + ((b*e - a*f)*((3*(e + f*x)^(2/3)
)/(2*b) + ((b*e - a*f)*((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(
b*e - a*f)^(1/3))/Sqrt[3]])/(b^(2/3)*(b*e - a*f)^(1/3)) - Log[a + b*x]/(2*
b^(2/3)*(b*e - a*f)^(1/3)) + (3*Log[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^
(1/3)])/(2*b^(2/3)*(b*e - a*f)^(1/3))))/b))/b))/(b*c - a*d) - ((d*g - c*h)
*((3*(e + f*x)^(5/3))/(5*d) + ((d*e - c*f)*((3*(e + f*x)^(2/3))/(2*d) + ((
d*e - c*f)*((Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(
1/3))/Sqrt[3]])/(d^(2/3)*(d*e - c*f)^(1/3)) - Log[c + d*x]/(2*d^(2/3)*(d*e
 - c*f)^(1/3)) + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d
^(2/3)*(d*e - c*f)^(1/3))))/d))/d))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{\left(-3(fx+e)^{\frac{2}{3}}\left(\left(\left(\left(-\frac{2hx}{5}-g\right)f-\frac{7eh}{5}\right)b+afh\right)d+bcfh\right)b(ad-bc)\left(\frac{af-be}{b}\right)^{\frac{1}{3}}+(af-be)^{2}d^{2}\left(2\arctan\left(\frac{\sqrt{3}\left(2(fx+e)^{\frac{1}{3}}\cdots\right.}{3\left(\frac{af-\cdots}{b}\cdots\right.}\cdots\right.\right.\right.}{\cdots}$
derivativedivides	$-\frac{3\left(-\frac{h(fx+e)^{\frac{5}{3}}bd}{5}+\frac{(adfh+bcfh-bdeh-bgdf)(fx+e)^{\frac{2}{3}}}{2}\right)}{b^{2}d^{2}}+\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\cdots\right.}{6b\left(\frac{af-\cdots}{b}\cdots\right.}\cdots\right.}{\cdots}$
default	$-\frac{3\left(-\frac{h(fx+e)^{\frac{5}{3}}bd}{5}+\frac{(adfh+bcfh-bdeh-bgdf)(fx+e)^{\frac{2}{3}}}{2}\right)}{b^{2}d^{2}}+\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\cdots\right.}{6b\left(\frac{af-\cdots}{b}\cdots\right.}\cdots\right.}{\cdots}$
risch	$-\frac{3(-2hbdfx+5adfh+5bcfh-7bdeh-5bgdf)(fx+e)^{\frac{2}{3}}}{10b^{2}d^{2}}+\frac{\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(f\cdots\right.}{6b\left(\frac{af-be}{b}\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$

input
```
int((f*x+e)^(5/3)*(h*x+g)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/((c*f-d*e)/d)^(1/3)*((-3*(f*x+e)^(2/3)*((((-2/5*h*x-g)*f-7/5*e*h)*b+a*
f*h)*d+b*c*f*h)*b*(a*d-b*c)*((a*f-b*e)/b)^(1/3)+(a*f-b*e)^2*d^2*(2*arctan(
1/3*3^(1/2)*(2*(f*x+e)^(1/3)-((a*f-b*e)/b)^(1/3))/((a*f-b*e)/b)^(1/3))*3^(
1/2)+ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/3
))-2*ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3)))*(a*h-b*g))*d*((c*f-d*e)/d)^(1/
3)-(2*arctan(1/3*3^(1/2)*(2*(f*x+e)^(1/3)-((c*f-d*e)/d)^(1/3))/((c*f-d*e)/
d)^(1/3))*3^(1/2)+ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*f
-d*e)/d)^(2/3))-2*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3)))*(c*f-d*e)^2*((a*f
-b*e)/b)^(1/3)*(c*h-d*g)*b^3)/((a*f-b*e)/b)^(1/3)/b^3/d^3/(a*d-b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^(5/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**(5/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(5/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(378) = 756$.

Time = 0.42 (sec) , antiderivative size = 1026, normalized size of antiderivative = 2.29

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(5/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
(b^3*e^2*g*((b*e - a*f)/b)^(1/3) - 2*a*b^2*e*f*g*((b*e - a*f)/b)^(1/3) + a
^2*b*f^2*g*((b*e - a*f)/b)^(1/3) - a*b^2*e^2*h*((b*e - a*f)/b)^(1/3) + 2*a
^2*b*e*f*h*((b*e - a*f)/b)^(1/3) - a^3*f^2*h*((b*e - a*f)/b)^(1/3))*((b*e
- a*f)/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)^(1/3)))/(b^4*c*e
 - a*b^3*d*e - a*b^3*c*f + a^2*b^2*d*f) - (d^3*e^2*g*((d*e - c*f)/d)^(1/3)
 - 2*c*d^2*e*f*g*((d*e - c*f)/d)^(1/3) + c^2*d*f^2*g*((d*e - c*f)/d)^(1/3)
 - c*d^2*e^2*h*((d*e - c*f)/d)^(1/3) + 2*c^2*d*e*f*h*((d*e - c*f)/d)^(1/3)
 - c^3*f^2*h*((d*e - c*f)/d)^(1/3))*((d*e - c*f)/d)^(1/3)*log(abs((f*x + e
)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b*c*d^3*e - a*d^4*e - b*c^2*d^2*f + a*c
*d^3*f) + ((b^3*e - a*b^2*f)^(2/3)*(sqrt(3)*b^2*e - sqrt(3)*a*b*f)*g - (b^
3*e - a*b^2*f)^(2/3)*(sqrt(3)*a*b*e - sqrt(3)*a^2*f)*h)*arctan(1/3*sqrt(3)
*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e - a*f)/b)^(1/3))/(b^5*c
 - a*b^4*d) - ((d^3*e - c*d^2*f)^(2/3)*(sqrt(3)*d^2*e - sqrt(3)*c*d*f)*g -
 (d^3*e - c*d^2*f)^(2/3)*(sqrt(3)*c*d*e - sqrt(3)*c^2*f)*h)*arctan(1/3*sqr
t(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3))/((d*e - c*f)/d)^(1/3))/(b
*c*d^4 - a*d^5) - 1/2*((b^3*e - a*b^2*f)^(2/3)*(b^2*e - a*b*f)*g - (b^3*e
- a*b^2*f)^(2/3)*(a*b*e - a^2*f)*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*
((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/(b^5*c - a*b^4*d) + 1/2*((d
^3*e - c*d^2*f)^(2/3)*(d^2*e - c*d*f)*g - (d^3*e - c*d^2*f)^(2/3)*(c*d*e -
 c^2*f)*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((d*e - c*f)/d)^(1/3) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(5/3)*(g + h*x))/((a + b*x)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 4185, normalized size of antiderivative = 9.34

$$\int \frac{(e+fx)^{5/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input

```
int((f*x+e)^(5/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output

```
( - 10*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/
6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a
**3*d**2*f**2*h + 20*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a
*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b
*e)**(1/6)))*a**2*b*d**2*e*f*h + 10*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*at
an((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b*
*(1/6)*(a*f - b*e)**(1/6)))*a**2*b*d**2*f**2*g - 10*d**(2/3)*(c*f - d*e)**
(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e +
f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*d**2*e**2*h - 20*d**(2/
3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) -
2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*d**2*e*
f*g + 10*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(
1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))
*b**3*d**2*e**2*g - 10*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*
(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f -
 b*e)**(1/6)))*a**3*d**2*f**2*h + 20*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*a
tan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b
**(1/6)*(a*f - b*e)**(1/6)))*a**2*b*d**2*e*f*h + 10*d**(2/3)*(c*f - d*e)**
(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e +
f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a**2*b*d**2*f**2*g - 10*d**...
```

3.215 $\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 406

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{3h(e+fx)^{2/3}}{2bd} + \frac{\sqrt{3}(be-af)^{2/3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{b^{5/3}(bc-ad)} - \frac{\sqrt{3}(de-cf)^{2/3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{d^{5/3}(bc-ad)} - \frac{(be-af)^{2/3}(bg-ah)\log(a+bx)}{2b^{5/3}(bc-ad)} + \frac{(de-cf)^{2/3}(dg-ch)\log(c+dx)}{2d^{5/3}(bc-ad)} + \frac{3(be-af)^{2/3}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{5/3}(bc-ad)} - \frac{3(de-cf)^{2/3}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{5/3}(bc-ad)}$$

output
```
3/2*h*(f*x+e)^(2/3)/b/d+3^(1/2)*(-a*f+b*e)^(2/3)*(-a*h+b*g)*arctan(1/3*(1+
2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1/3))*3^(1/2))/b^(5/3)/(-a*d+b*c)-3^(1
/2)*(-c*f+d*e)^(2/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*
f+d*e)^(1/3))*3^(1/2))/d^(5/3)/(-a*d+b*c)-1/2*(-a*f+b*e)^(2/3)*(-a*h+b*g)*
ln(b*x+a)/b^(5/3)/(-a*d+b*c)+1/2*(-c*f+d*e)^(2/3)*(-c*h+d*g)*ln(d*x+c)/d^(
5/3)/(-a*d+b*c)+3/2*(-a*f+b*e)^(2/3)*(-a*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3
)*(f*x+e)^(1/3))/b^(5/3)/(-a*d+b*c)-3/2*(-c*f+d*e)^(2/3)*(-c*h+d*g)*ln((-c
*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/d^(5/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.16

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{3b^{2/3}d^{2/3}(bc-ad)h(e+fx)^{2/3}+2\sqrt{3}d^{5/3}(-be+af)^{2/3}(bg-ah)\arctan\left(\frac{1-}{}\right.}{}$$

input
```
Integrate[((e + f*x)^(2/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(3*b^(2/3)*d^(2/3)*(b*c - a*d)*h*(e + f*x)^(2/3) + 2*Sqrt[3]*d^(5/3)*(-(b*
e) + a*f)^(2/3)*(b*g - a*h)*ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e
) + a*f)^(1/3))/Sqrt[3]] - 2*Sqrt[3]*b^(5/3)*(-(d*e) + c*f)^(2/3)*(d*g - c
*h)*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))/Sqrt[3]]
 + 2*d^(5/3)*(-(b*e) + a*f)^(2/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(1/3) + b
^(1/3)*(e + f*x)^(1/3)] - 2*b^(5/3)*(-(d*e) + c*f)^(2/3)*(d*g - c*h)*Log[(
-(d*e) + c*f)^(1/3) + d^(1/3)*(e + f*x)^(1/3)] - d^(5/3)*(-(b*e) + a*f)^(2
/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(2/3) - b^(1/3)*(-(b*e) + a*f)^(1/3)*(e
 + f*x)^(1/3) + b^(2/3)*(e + f*x)^(2/3)] + b^(5/3)*(-(d*e) + c*f)^(2/3)*(d
*g - c*h)*Log[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f*x
)^(1/3) + d^(2/3)*(e + f*x)^(2/3)])/(2*b^(5/3)*d^(5/3)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {174, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{174}$$

$$\frac{(bg-ah)\int \frac{(e+fx)^{2/3}}{a+bx}dx}{bc-ad}-\frac{(dg-ch)\int \frac{(e+fx)^{2/3}}{c+dx}dx}{bc-ad}$$

$$\Big\downarrow \text{60}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\int \frac{1}{(a+bx)\sqrt[3]{e+fx}}dx}{b}+\frac{3(e+fx)^{2/3}}{2b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{e+fx}}dx}{d}+\frac{3(e+fx)^{2/3}}{2d}\right)}{bc-ad}$$

$$\Big\downarrow \text{67}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\right.}{b}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\right.}{d}\right.}{bc-ad}$$

$$\downarrow 16$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{3\int\frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{2b^{2/3}\sqrt[3]{be-a\cdots}}\cdots\right)}{b}\cdots\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{3\int\frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{\cdots}\right.}{2d^{2/3}\sqrt[3]{de-c\cdots}}\cdots\right)}{d}\cdots\right)}{bc-ad}$$

$\downarrow$ 1082

$$\frac{(bg-ah)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{b}+\frac{3(e+\cdots}{\cdots}\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{d}+\frac{3(e\cdots}{\cdots}\right)}{bc-ad}$$

$\downarrow$ 217

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{b}+\frac{3(e+fx)^{2/3}}{2b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{d}+\frac{3(e+fx)^{2/3}}{2d}\right)}{bc-ad}$$

input
```
Int[((e + f*x)^(2/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
((b*g - a*h)*((3*(e + f*x)^(2/3))/(2*b) + ((b*e - a*f)*((Sqrt[3]*ArcTan[(1
 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*f)^(1/3))/Sqrt[3]])/(b^(2/3)*(b*e
- a*f)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*e - a*f)^(1/3)) + (3*Log[(b*e -
 a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])/(2*b^(2/3)*(b*e - a*f)^(1/3))))/b)
)/(b*c - a*d) - ((d*g - c*h)*((3*(e + f*x)^(2/3))/(2*d) + ((d*e - c*f)*((S
qrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3))/Sqrt[3]]
)/(d^(2/3)*(d*e - c*f)^(1/3)) - Log[c + d*x]/(2*d^(2/3)*(d*e - c*f)^(1/3))
 + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(2/3)*(d*e -
c*f)^(1/3))))/d))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{b\left(3(fx+e)^{\frac{2}{3}}\left(\frac{cf-de}{d}\right)^{\frac{1}{3}} dh(ad-bc)+\left(2\arctan\left(\frac{2\sqrt{3}\,(fx+e)^{\frac{1}{3}}}{3\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-\cdots}{d}\cdots\right.\right.\right.\right.}{\cdots}$
derivativedivides	$\frac{3h(fx+e)^{\frac{2}{3}}}{2bd}-\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(\cdots}{\left(\frac{a\cdots}{}\right)}\cdots\right)}{\cdots}\right)}{3b\left(\frac{af-be}{b}\cdots\right)}\cdots\right)}{(ad-bc)b}$
default	$\frac{3h(fx+e)^{\frac{2}{3}}}{2bd}-\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(\cdots}{\left(\frac{a\cdots}{}\right)}\cdots\right)}{\cdots}\right)}{3b\left(\frac{af-be}{b}\cdots\right)}\cdots\right)}{(ad-bc)b}$
risch	$\frac{3h(fx+e)^{\frac{2}{3}}}{2bd}-\frac{\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(f\cdots}{\left(\frac{af\cdots}{}\right)}\cdots\right)}{\cdots}\right)}{3b\left(\frac{af-be}{b}\cdots\right)}\cdots\right)}{ad-bc}}{\cdots}$

input
```
int((f*x+e)^(2/3)*(h*x+g)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/((c*f-d*e)/d)^(1/3)*(b*(3*(f*x+e)^(2/3)*((c*f-d*e)/d)^(1/3)*d*h*(a*d-b
*c)+(2*arctan(2/3*3^(1/2)/((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1/3*3^(1/2))*3
^(1/2)+ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*f-d*e)/d)^(2
/3))-2*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3)))*(c*f-d*e)*(c*h-d*g)*b)*((a*f
-b*e)/b)^(1/3)-2*((c*f-d*e)/d)^(1/3)*(a*f-b*e)*d^2*(arctan(2/3*3^(1/2)/((a
*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)-1/3*3^(1/2))*3^(1/2)-ln((f*x+e)^(1/3)+((a*f
-b*e)/b)^(1/3))+1/2*ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a
*f-b*e)/b)^(2/3)))*(a*h-b*g))/((a*f-b*e)/b)^(1/3)/b^2/d^2/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(340) = 680$.

Time = 57.63 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.71

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(2/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*(b*d*g - a*d*h)*((b^2*e^2 - 2*a*b*e*f + a^2*f^2)/b^2)^(1/3
)*arctan(-1/3*(2*sqrt(3)*(f*x + e)^(1/3)*b*((b^2*e^2 - 2*a*b*e*f + a^2*f^2
)/b^2)^(1/3) + sqrt(3)*(b*e - a*f))/(b*e - a*f)) - 2*sqrt(3)*(b*d*g - b*c*
h)*(-(d^2*e^2 - 2*c*d*e*f + c^2*f^2)/d^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*(f*
x + e)^(1/3)*d*(-(d^2*e^2 - 2*c*d*e*f + c^2*f^2)/d^2)^(1/3) - sqrt(3)*(d*e
 - c*f))/(d*e - c*f)) - 3*(b*c - a*d)*(f*x + e)^(2/3)*h + (b*d*g - a*d*h)*
((b^2*e^2 - 2*a*b*e*f + a^2*f^2)/b^2)^(1/3)*log(-(f*x + e)^(1/3)*b*((b^2*e
^2 - 2*a*b*e*f + a^2*f^2)/b^2)^(2/3) - (b*e - a*f)*(f*x + e)^(2/3) - (b*e
- a*f)*((b^2*e^2 - 2*a*b*e*f + a^2*f^2)/b^2)^(1/3)) + (b*d*g - b*c*h)*(-(d
^2*e^2 - 2*c*d*e*f + c^2*f^2)/d^2)^(1/3)*log(-(f*x + e)^(1/3)*d*(-(d^2*e^2
 - 2*c*d*e*f + c^2*f^2)/d^2)^(2/3) - (d*e - c*f)*(f*x + e)^(2/3) + (d*e -
c*f)*(-(d^2*e^2 - 2*c*d*e*f + c^2*f^2)/d^2)^(1/3)) - 2*(b*d*g - a*d*h)*((b
^2*e^2 - 2*a*b*e*f + a^2*f^2)/b^2)^(1/3)*log(b*((b^2*e^2 - 2*a*b*e*f + a^2
*f^2)/b^2)^(2/3) - (b*e - a*f)*(f*x + e)^(1/3)) - 2*(b*d*g - b*c*h)*(-(d^2
*e^2 - 2*c*d*e*f + c^2*f^2)/d^2)^(1/3)*log(d*(-(d^2*e^2 - 2*c*d*e*f + c^2*
f^2)/d^2)^(2/3) - (d*e - c*f)*(f*x + e)^(1/3)))/(b^2*c*d - a*b*d^2)
```

Sympy [F]

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \int \frac{(e+fx)^{\frac{2}{3}}\,(g+hx)}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**(2/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
Integral((e + f*x)**(2/3)*(g + h*x)/((a + b*x)*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(2/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(340) = 680$.

Time = 0.27 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.81

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(2/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
(b^2*e*g*((b*e - a*f)/b)^(1/3) - a*b*f*g*((b*e - a*f)/b)^(1/3) - a*b*e*h*(
(b*e - a*f)/b)^(1/3) + a^2*f*h*((b*e - a*f)/b)^(1/3))*((b*e - a*f)/b)^(1/3
)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)^(1/3)))/(b^3*c*e - a*b^2*d*e -
 a*b^2*c*f + a^2*b*d*f) - (d^2*e*g*((d*e - c*f)/d)^(1/3) - c*d*f*g*((d*e -
 c*f)/d)^(1/3) - c*d*e*h*((d*e - c*f)/d)^(1/3) + c^2*f*h*((d*e - c*f)/d)^(
1/3))*((d*e - c*f)/d)^(1/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3
)))/(b*c*d^2*e - a*d^3*e - b*c^2*d*f + a*c*d^2*f) + (sqrt(3)*(b^3*e - a*b^
2*f)^(2/3)*b*g - sqrt(3)*(b^3*e - a*b^2*f)^(2/3)*a*h)*arctan(1/3*sqrt(3)*(
2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e - a*f)/b)^(1/3))/(b^4*c -
 a*b^3*d) - (sqrt(3)*(d^3*e - c*d^2*f)^(2/3)*d*g - sqrt(3)*(d^3*e - c*d^2*
f)^(2/3)*c*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3
))/((d*e - c*f)/d)^(1/3))/(b*c*d^3 - a*d^4) - 1/2*((b^3*e - a*b^2*f)^(2/3)
*b*g - (b^3*e - a*b^2*f)^(2/3)*a*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*
((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/(b^4*c - a*b^3*d) + 1/2*((d
^3*e - c*d^2*f)^(2/3)*d*g - (d^3*e - c*d^2*f)^(2/3)*c*h)*log((f*x + e)^(2/
3) + (f*x + e)^(1/3)*((d*e - c*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/(b*c*d
^3 - a*d^4) + 3/2*(f*x + e)^(2/3)*h/(b*d)
```

Mupad [B] (verification not implemented)

Time = 40.97 (sec) , antiderivative size = 9140, normalized size of antiderivative = 22.51

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(2/3)*(g + h*x))/((a + b*x)*(c + d*x)),x)
```

output
```
log((243*f^4*(e + f*x)^(1/3)*(a*f - b*e)^2*(a*h - b*g)^2*(c*f - d*e)^2*(c*
h - d*g)^2*(a^2*d^2*f^2*h + b^2*c^2*f^2*h + b^2*d^2*e^2*h - a*b*d^2*f^2*g
- b^2*c*d*f^2*g + 2*b^2*d^2*e*f*g + a*b*c*d*f^2*h - 2*a*b*d^2*e*f*h - 2*b^
2*c*d*e*f*h))/(b^2*d^2) - ((243*b*d*f^4*(e + f*x)^(1/3)*(a*d - b*c)^2*(a*f
 - b*e)^2*(c*f - d*e)^2*(a^2*d^2*h^2 + b^2*c^2*h^2 + 2*b^2*d^2*g^2 - 2*a*b
*d^2*g*h - 2*b^2*c*d*g*h) + 243*b^3*d^3*f^4*(a*d - b*c)^4*(a*f - b*e)*(c*f
 - d*e)*(a*d*f + b*c*f - 2*b*d*e)*(-((c*f - d*e)^2*(c*h - d*g)^3)/(d^5*(a*
d - b*c)^3))^(2/3))*(-((c*f - d*e)^2*(c*h - d*g)^3)/(d^5*(a*d - b*c)^3))^(
1/3) - (243*f^4*(a*d - b*c)^2*(a*f - b*e)*(c*f - d*e)*(a^5*d^5*f^3*h^3 + b
^5*c^5*f^3*h^3 - a^2*b^3*d^5*e^3*h^3 - a^2*b^3*d^5*f^3*g^3 - b^5*c^2*d^3*e
^3*h^3 - b^5*c^2*d^3*f^3*g^3 - 3*b^5*d^5*e^2*f*g^3 - 3*b^5*d^5*e^3*g^2*h -
 a*b^4*c*d^4*e^3*h^3 - a*b^4*c*d^4*f^3*g^3 + a*b^4*c^4*d*f^3*h^3 + a^4*b*c
*d^4*f^3*h^3 + 3*a*b^4*d^5*e*f^2*g^3 - 3*a^4*b*d^5*e*f^2*h^3 + 3*a*b^4*d^5
*e^3*g*h^2 + 3*b^5*c*d^4*e*f^2*g^3 - 3*a^4*b*d^5*f^3*g*h^2 - 3*b^5*c^4*d*e
*f^2*h^3 + 3*b^5*c*d^4*e^3*g*h^2 - 3*b^5*c^4*d*f^3*g*h^2 + 3*a^3*b^2*d^5*e
^2*f*h^3 + 3*a^3*b^2*d^5*f^3*g^2*h + 3*b^5*c^3*d^2*e^2*f*h^3 + 3*b^5*c^3*d
^2*f^3*g^2*h + a^2*b^3*c^3*d^2*f^3*h^3 + a^3*b^2*c^2*d^3*f^3*h^3 - 3*a^2*b
^3*c^2*d^3*e*f^2*h^3 - 3*a^2*b^3*c^2*d^3*f^3*g*h^2 + 9*a*b^4*d^5*e^2*f*g^2
*h + 9*b^5*c*d^4*e^2*f*g^2*h + 3*a*b^4*c^2*d^3*e^2*f*h^3 - 3*a*b^4*c^3*d^2
*e*f^2*h^3 + 3*a^2*b^3*c*d^4*e^2*f*h^3 - 3*a^3*b^2*c*d^4*e*f^2*h^3 + 3*...
```

Reduce [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 2549, normalized size of antiderivative = 6.28

$$\int \frac{(e+fx)^{2/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(2/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
(2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*s
qrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a**2*
d*f*h - 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**
(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6))
)*a*b*d*e*h - 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f -
b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**
(1/6)))*a*b*d*f*g + 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(
a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f -
b*e)**(1/6)))*b**2*d*e*g + 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**
(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*
(a*f - b*e)**(1/6)))*a**2*d*f*h - 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*at
an((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b*
*(1/6)*(a*f - b*e)**(1/6)))*a*b*d*e*h - 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt
(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6
))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*d*f*g + 2*d**(2/3)*(c*f - d*e)**(1/3
)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)
**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b**2*d*e*g - 2*b**(2/3)*(a*f - b*e
)**(1/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) - 2*d**(1/3)*(e
 + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*b*c**2*f*h + 2*b**(2/3)*(a*
f - b*e)**(1/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) - 2*d...
```

3.216 $\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 386

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \frac{\sqrt{3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{b^{2/3}(bc-ad)\sqrt[3]{be-af}} - \frac{\sqrt{3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{d^{2/3}(bc-ad)\sqrt[3]{de-cf}} - \frac{(bg-ah)\log(a+bx)}{2b^{2/3}(bc-ad)\sqrt[3]{be-af}} + \frac{(dg-ch)\log(c+dx)}{2d^{2/3}(bc-ad)\sqrt[3]{de-cf}} + \frac{3(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}(bc-ad)\sqrt[3]{be-af}} - \frac{3(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}(bc-ad)\sqrt[3]{de-cf}}$$

output
```
3^(1/2)*(-a*h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1/3))
*3^(1/2))/b^(2/3)/(-a*d+b*c)/(-a*f+b*e)^(1/3)-3^(1/2)*(-c*h+d*g)*arctan(1/
3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^(1/2))/d^(2/3)/(-a*d+b*c)
/(-c*f+d*e)^(1/3)-1/2*(-a*h+b*g)*ln(b*x+a)/b^(2/3)/(-a*d+b*c)/(-a*f+b*e)^(
1/3)+1/2*(-c*h+d*g)*ln(d*x+c)/d^(2/3)/(-a*d+b*c)/(-c*f+d*e)^(1/3)+3/2*(-a*
h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/b^(2/3)/(-a*d+b*c)/(-a*f
+b*e)^(1/3)-3/2*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/d^(2
/3)/(-a*d+b*c)/(-c*f+d*e)^(1/3)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.21

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx$$

$$= \frac{-2\sqrt{3}d^{2/3}\sqrt[3]{-de+cf}(bg-ah)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{-be+af}}}{\sqrt{3}}\right)+2\sqrt{3}b^{2/3}\sqrt[3]{-be+af}(dg-ch)\arctan\left(\frac{1-\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(1/3)),x]
```

output
```
(-2*Sqrt[3]*d^(2/3)*(-(d*e) + c*f)^(1/3)*(b*g - a*h)*ArcTan[(1 - (2*b^(1/3
)*(e + f*x)^(1/3))/(-(b*e) + a*f)^(1/3))/Sqrt[3]] + 2*Sqrt[3]*b^(2/3)*(-(b
*e) + a*f)^(1/3)*(d*g - c*h)*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*
e) + c*f)^(1/3))/Sqrt[3]] - 2*d^(2/3)*(-(d*e) + c*f)^(1/3)*(b*g - a*h)*Log
[(-(b*e) + a*f)^(1/3) + b^(1/3)*(e + f*x)^(1/3)] + 2*b^(2/3)*(-(b*e) + a*f
)^(1/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(1/3) + d^(1/3)*(e + f*x)^(1/3)] +
d^(2/3)*(-(d*e) + c*f)^(1/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(2/3) - b^(1/3
)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3) + b^(2/3)*(e + f*x)^(2/3)] - b^(2/3
)*(-(b*e) + a*f)^(1/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d
*e) + c*f)^(1/3)*(e + f*x)^(1/3) + d^(2/3)*(e + f*x)^(2/3)])/(2*b^(2/3)*d^
(2/3)*(b*c - a*d)*(-(b*e) + a*f)^(1/3)*(-(d*e) + c*f)^(1/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {174, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}dx$$

$$\Big\downarrow \text{174}$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)\sqrt[3]{e+fx}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)\sqrt[3]{e+fx}}dx}{bc-ad}$$

$$\Big\downarrow \text{67}$$

$$\frac{(bg-ah)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\ldots\right)}{bc-ad}$$
$$\frac{(dg-ch)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\ldots\right)}{bc-ad}$$

$$\Big\downarrow \text{16}$$

$$\frac{(bg-ah)\left(\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+\right.}{2b^{2/3}\sqrt[3]{be-af}}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+\right.}{2d^{2/3}\sqrt[3]{de-cf}}\right.}{bc-ad}$$

$\downarrow$ 1082

$$\frac{(bg-ah)\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{bc-ad}-$$

$$\frac{(dg-ch)\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{bc-ad}$$

$\downarrow$ 217

$$\frac{(bg-ah)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{bc-ad}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(1/3)),x]
```

output
```
((b*g - a*h)*((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*f)
^(1/3))/Sqrt[3]])/(b^(2/3)*(b*e - a*f)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b
*e - a*f)^(1/3)) + (3*Log[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])/(2
*b^(2/3)*(b*e - a*f)^(1/3))))/(b*c - a*d) - ((d*g - c*h)*((Sqrt[3]*ArcTan[
(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3))/Sqrt[3]])/(d^(2/3)*(d*
e - c*f)^(1/3)) - Log[c + d*x]/(2*d^(2/3)*(d*e - c*f)^(1/3)) + (3*Log[(d*e
 - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(2/3)*(d*e - c*f)^(1/3))))/
(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}\cdots\right)}{ad-bc}$
default	$-\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}\cdots\right)}{ad-bc}$
pseudoelliptic	$\frac{\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)(ah-bg)d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}{2}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)(ch-\cdots}{2}}{\cdots}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3*(-1/3/d/((c*f-d*e)/d)^(1/3)*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))+1/6/d
/((c*f-d*e)/d)^(1/3)*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((
c*f-d*e)/d)^(2/3))+1/3*3^(1/2)/d/((c*f-d*e)/d)^(1/3)*arctan(1/3*3^(1/2)*(2
/((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1)))*(c*h-d*g)/(a*d-b*c)+3*(-1/3/b/((a*
f-b*e)/b)^(1/3)*ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))+1/6/b/((a*f-b*e)/b)^
(1/3)*ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/
3))+1/3*3^(1/2)/b/((a*f-b*e)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*f-b*e)/b)^
(1/3)*(f*x+e)^(1/3)-1)))*(a*h-b*g)/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. 2(324) = 648.

Time = 5.25 (sec) , antiderivative size = 3716, normalized size of antiderivative = 9.63

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \int \frac{g+hx}{(a+bx)\,(c+dx)\,\sqrt[3]{e+fx}}\,dx$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(1/3),x)
```

output
```
Integral((g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)**(1/3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.55

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/3),x, algorithm="giac")
```

output
```
(b*g*((b*e - a*f)/b)^(1/3) - a*h*((b*e - a*f)/b)^(1/3))*((b*e - a*f)/b)^(1
/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)^(1/3)))/(b^2*c*e - a*b*d*e -
 a*b*c*f + a^2*d*f) - (d*g*((d*e - c*f)/d)^(1/3) - c*h*((d*e - c*f)/d)^(1/
3))*((d*e - c*f)/d)^(1/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3))
)/(b*c*d*e - a*d^2*e - b*c^2*f + a*c*d*f) + (sqrt(3)*b*g - sqrt(3)*a*h)*ar
ctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e - a*f)/
b)^(1/3))/((b^3*e - a*b^2*f)^(1/3)*b*c - (b^3*e - a*b^2*f)^(1/3)*a*d) - (s
qrt(3)*d*g - sqrt(3)*c*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e -
c*f)/d)^(1/3))/((d*e - c*f)/d)^(1/3))/((d^3*e - c*d^2*f)^(1/3)*b*c - (d^3*
e - c*d^2*f)^(1/3)*a*d) - 1/2*(b*g - a*h)*log((f*x + e)^(2/3) + (f*x + e)^
(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/((b^3*e - a*b^2*f)^(1
/3)*b*c - (b^3*e - a*b^2*f)^(1/3)*a*d) + 1/2*(d*g - c*h)*log((f*x + e)^(2/
3) + (f*x + e)^(1/3)*((d*e - c*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/((d^3*
e - c*d^2*f)^(1/3)*b*c - (d^3*e - c*d^2*f)^(1/3)*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 16501, normalized size of antiderivative = 42.75

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(1/3)*(a + b*x)*(c + d*x)),x)
```

output
```
log((e + f*x)^(1/3)*(243*b^4*d^4*f^5*g^5 - 729*a*b^3*d^4*f^5*g^4*h - 729*b
^4*c*d^3*f^5*g^4*h + 243*b^4*d^4*e*f^4*g^4*h - 243*a^2*b^2*c^3*d*f^5*h^5 -
 243*a^3*b*c^2*d^2*f^5*h^5 - 243*a^3*b*d^4*f^5*g^2*h^3 - 243*b^4*c^3*d*f^5
*g^2*h^3 + 729*a^2*b^2*d^4*f^5*g^3*h^2 + 729*b^4*c^2*d^2*f^5*g^3*h^2 + 243
*a^2*b^2*c^2*d^2*e*f^4*h^5 - 1701*a*b^3*c^2*d^2*f^5*g^2*h^3 - 1701*a^2*b^2
*c*d^3*f^5*g^2*h^3 + 1215*a^2*b^2*c^2*d^2*f^5*g*h^4 + 243*a^2*b^2*d^4*e*f^
4*g^2*h^3 + 243*b^4*c^2*d^2*e*f^4*g^2*h^3 + 486*a*b^3*c^3*d*f^5*g*h^4 + 48
6*a^3*b*c*d^3*f^5*g*h^4 + 1944*a*b^3*c*d^3*f^5*g^3*h^2 - 486*a*b^3*d^4*e*f
^4*g^3*h^2 - 486*b^4*c*d^3*e*f^4*g^3*h^2 + 972*a*b^3*c*d^3*e*f^4*g^2*h^3 -
 486*a*b^3*c^2*d^2*e*f^4*g*h^4 - 486*a^2*b^2*c*d^3*e*f^4*g*h^4) - (-(a^3*h
^3 - b^3*g^3 + 3*a*b^2*g^2*h - 3*a^2*b*g*h^2)/(b^6*c^3*e - a^3*b^3*d^3*e +
 a^4*b^2*d^3*f - a*b^5*c^3*f - 3*a*b^5*c^2*d*e + 3*a^2*b^4*c*d^2*e + 3*a^2
*b^4*c^2*d*f - 3*a^3*b^3*c*d^2*f))^(2/3)*((-(a^3*h^3 - b^3*g^3 + 3*a*b^2*g
^2*h - 3*a^2*b*g*h^2)/(b^6*c^3*e - a^3*b^3*d^3*e + a^4*b^2*d^3*f - a*b^5*c
^3*f - 3*a*b^5*c^2*d*e + 3*a^2*b^4*c*d^2*e + 3*a^2*b^4*c^2*d*f - 3*a^3*b^3
*c*d^2*f))^(1/3)*((-(a^3*h^3 - b^3*g^3 + 3*a*b^2*g^2*h - 3*a^2*b*g*h^2)/(b
^6*c^3*e - a^3*b^3*d^3*e + a^4*b^2*d^3*f - a*b^5*c^3*f - 3*a*b^5*c^2*d*e +
 3*a^2*b^4*c*d^2*e + 3*a^2*b^4*c^2*d*f - 3*a^3*b^3*c*d^2*f))^(2/3)*(486*a^
3*b^6*c^4*d^5*f^7 - 729*a^2*b^7*c^5*d^4*f^7 + 486*a^4*b^5*c^3*d^6*f^7 - 72
9*a^5*b^4*c^2*d^7*f^7 - 486*a^4*b^5*d^9*e^3*f^4 + 729*a^5*b^4*d^9*e^2*f...
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.13

$$\int \frac{g+hx}{(a+bx)(c+dx)\sqrt[3]{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(1/3),x)
```

output

```
( - 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
h + 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b*
g - 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
h + 2*d**(2/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b*
g + 2*b**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) - 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*c*
h - 2*b**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) - 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*d*
g + 2*b**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) + 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*c*
h - 2*b**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) + 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*d*
g + 2*b**(2/3)*(a*f - b*e)**(1/3)*log((c*f - d*e)**(1/3) + d**(1/3)*(e + f
*x)**(1/3))*c*h - 2*b**(2/3)*(a*f - b*e)**(1/3)*log((c*f - d*e)**(1/3) + d
**(1/3)*(e + f*x)**(1/3))*d*g - b**(2/3)*(a*f - b*e)**(1/3)*log( - d**(1/6
)*(e + f*x)**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) + (c*f - d*e)**(1/3) + d*...
```

3.217 $\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 425

$$\begin{aligned}\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx =& -\frac{3(fg-eh)}{(be-af)(de-cf)\sqrt[3]{e+fx}} \\ &+ \frac{\sqrt{3}\sqrt[3]{b}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{(bc-ad)(be-af)^{4/3}} \\ &- \frac{\sqrt{3}\sqrt[3]{d}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{(bc-ad)(de-cf)^{4/3}} - \frac{\sqrt[3]{b}(bg-ah)\log(a+bx)}{2(bc-ad)(be-af)^{4/3}} \\ &+ \frac{\sqrt[3]{d}(dg-ch)\log(c+dx)}{2(bc-ad)(de-cf)^{4/3}} + \frac{3\sqrt[3]{b}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2(bc-ad)(be-af)^{4/3}} \\ &- \frac{3\sqrt[3]{d}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2(bc-ad)(de-cf)^{4/3}}\end{aligned}$$

output
```
(3*e*h-3*f*g)/(-a*f+b*e)/(-c*f+d*e)/(f*x+e)^(1/3)+3^(1/2)*b^(1/3)*(-a*h+b*
g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1/3))*3^(1/2))/(-a*d+
b*c)/(-a*f+b*e)^(4/3)-3^(1/2)*d^(1/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/3)*(
f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^(1/2))/(-a*d+b*c)/(-c*f+d*e)^(4/3)-1/2*b^
(1/3)*(-a*h+b*g)*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)^(4/3)+1/2*d^(1/3)*(-c*h+d
*g)*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)^(4/3)+3/2*b^(1/3)*(-a*h+b*g)*ln((-a*f+
b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/(-a*d+b*c)/(-a*f+b*e)^(4/3)-3/2*d^(1/3)*
(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/(-a*d+b*c)/(-c*f+d*e
)^(4/3)
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.22

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \frac{3(-fg+eh)}{(be-af)(de-cf)\sqrt[3]{e+fx}} + \frac{\sqrt{3}\sqrt[3]{b}(bg-ah)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{-be+af}}}{\sqrt{3}}\right)}{(bc-ad)(-be+af)^{4/3}} - \frac{\sqrt{3}\sqrt[3]{d}(dg-ch)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{-de+cf}}}{\sqrt{3}}\right)}{(bc-ad)(-de+cf)^{4/3}} + \frac{\sqrt[3]{b}(bg-ah)\log\left(\sqrt[3]{-be+af}+\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{(bc-ad)(-be+af)^{4/3}} + \frac{\sqrt[3]{d}(dg-ch)\log\left(\sqrt[3]{-de+cf}+\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{(-bc+ad)(-de+cf)^{4/3}} - \frac{\sqrt[3]{b}(bg-ah)\log\left((-be+af)^{2/3}-\sqrt[3]{b}\sqrt[3]{-be+af}\sqrt[3]{e+fx}+b^{2/3}(e+fx)^{2/3}\right)}{2(bc-ad)(-be+af)^{4/3}} - \frac{\sqrt[3]{d}(dg-ch)\log\left((-de+cf)^{2/3}-\sqrt[3]{d}\sqrt[3]{-de+cf}\sqrt[3]{e+fx}+d^{2/3}(e+fx)^{2/3}\right)}{2(-bc+ad)(-de+cf)^{4/3}}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(4/3)),x]
```

output
```
(3*(-(f*g) + e*h))/((b*e - a*f)*(d*e - c*f)*(e + f*x)^(1/3)) + (Sqrt[3]*b^
(1/3)*(b*g - a*h)*ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e) + a*f)^(
1/3))/Sqrt[3]])/((b*c - a*d)*(-(b*e) + a*f)^(4/3)) - (Sqrt[3]*d^(1/3)*(d*g
 - c*h)*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))/Sqrt
[3]])/((b*c - a*d)*(-(d*e) + c*f)^(4/3)) + (b^(1/3)*(b*g - a*h)*Log[(-(b*e
) + a*f)^(1/3) + b^(1/3)*(e + f*x)^(1/3)])/((b*c - a*d)*(-(b*e) + a*f)^(4/
3)) + (d^(1/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(1/3) + d^(1/3)*(e + f*x)^(1
/3)])/((-(b*c) + a*d)*(-(d*e) + c*f)^(4/3)) - (b^(1/3)*(b*g - a*h)*Log[(-(
b*e) + a*f)^(2/3) - b^(1/3)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3) + b^(2/3)
*(e + f*x)^(2/3)])/(2*(b*c - a*d)*(-(b*e) + a*f)^(4/3)) - (d^(1/3)*(d*g -
c*h)*Log[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f*x)^(1/
3) + d^(2/3)*(e + f*x)^(2/3)])/(2*(-(b*c) + a*d)*(-(d*e) + c*f)^(4/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {174, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}dx$$

$$\Big\downarrow \text{174}$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)(e+fx)^{4/3}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)(e+fx)^{4/3}}dx}{bc-ad}$$

$$\Big\downarrow \text{61}$$

$$\frac{(bg-ah)\left(\frac{b\int \frac{1}{(a+bx)\sqrt[3]{e+fx}}dx}{be-af}+\frac{3}{\sqrt[3]{e+fx}(be-af)}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{d\int \frac{1}{(c+dx)\sqrt[3]{e+fx}}dx}{de-cf}+\frac{3}{\sqrt[3]{e+fx}(de-cf)}\right)}{bc-ad}$$

$$\Big\downarrow \; 67$$

$$\frac{(bg-ah)\left(\frac{b\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\log}{2b^{2/3}\sqrt[3]{}}\cdots\right)}{be-af}\cdots\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\log}{2d^{2/3}\sqrt[3]{}}\cdots\right)}{de-cf}\cdots\right)}{bc-ad}$$

$$\Big\downarrow \; 16$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+f}\right.}{2b^{2/3}\sqrt[3]{be-af}}\right.}{be-af}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+}\right.}{2d^{2/3}\sqrt[3]{de-cf}}\right.}{de-cf}\right.}{bc-ad}$$

$$\Big\downarrow \; 1082$$

$$\frac{(bg-ah)\left(\frac{b\left(-\frac{3\int\frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{be-af}+\frac{3}{\sqrt[3]{e+fx}}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(-\frac{3\int\frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{de-cf}+\frac{3}{\sqrt[3]{e+fx}}\right.}{bc-ad}$$

$\downarrow$ 217

$$\frac{(bg-ah)\left(\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{be-af}+\frac{3}{\sqrt[3]{e+fx}(be-af)}\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{de-cf}+\frac{3}{\sqrt[3]{e+fx}(de-cf)}\right)}{bc-ad}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(4/3)),x]
```

output
```
((b*g - a*h)*(3/((b*e - a*f)*(e + f*x)^(1/3)) + (b*((Sqrt[3]*ArcTan[(1 + (
2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*f)^(1/3))/Sqrt[3]])/(b^(2/3)*(b*e - a*
f)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*e - a*f)^(1/3)) + (3*Log[(b*e - a*f
)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])/(2*b^(2/3)*(b*e - a*f)^(1/3))))/(b*e -
 a*f)))/(b*c - a*d) - ((d*g - c*h)*(3/((d*e - c*f)*(e + f*x)^(1/3)) + (d*(
(Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3))/Sqrt[3
]])/(d^(2/3)*(d*e - c*f)^(1/3)) - Log[c + d*x]/(2*d^(2/3)*(d*e - c*f)^(1/3
)) + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(2/3)*(d*e
- c*f)^(1/3))))/(d*e - c*f)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}\right)}{(ad-bc)(cf-de)}$
default	$\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}\right)}{(ad-bc)(cf-de)}$
pseudoelliptic	$-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)(ch-dg)}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(ad-bc)(cf-de)}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)(ch-dg)}{2\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(ad-bc)(cf-de)}+\frac{\sqrt{3}\arctan}{\left(cf\right.}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3*(-1/3/d/((c*f-d*e)/d)^(1/3)*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))+1/6/d/
((c*f-d*e)/d)^(1/3)*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c
*f-d*e)/d)^(2/3))+1/3*3^(1/2)/d/((c*f-d*e)/d)^(1/3)*arctan(1/3*3^(1/2)*(2/
((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1)))*d*(c*h-d*g)/(a*d-b*c)/(c*f-d*e)-3*(
-e*h+f*g)/(c*f-d*e)/(a*f-b*e)/(f*x+e)^(1/3)-3*(-1/3/b/((a*f-b*e)/b)^(1/3)*
ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))+1/6/b/((a*f-b*e)/b)^(1/3)*ln((f*x+e)
^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/3))+1/3*3^(1/2)/
b/((a*f-b*e)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*f-b*e)/b)^(1/3)*(f*x+e)^(1
/3)-1)))*b*(a*h-b*g)/(a*d-b*c)/(a*f-b*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \int \frac{g+hx}{(a+bx)\,(c+dx)\,(e+fx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(4/3),x)
```

output
```
Integral((g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)**(4/3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(4/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(361) = 722$.

Time = 0.80 (sec) , antiderivative size = 912, normalized size of antiderivative = 2.15

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(4/3),x, algorithm="giac")
```

output
```
(b^2*g*((b*e - a*f)/b)^(1/3) - a*b*h*((b*e - a*f)/b)^(1/3))*((b*e - a*f)/b
)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)^(1/3)))/(b^3*c*e^2 - a*b
^2*d*e^2 - 2*a*b^2*c*e*f + 2*a^2*b*d*e*f + a^2*b*c*f^2 - a^3*d*f^2) - (d^2
*g*((d*e - c*f)/d)^(1/3) - c*d*h*((d*e - c*f)/d)^(1/3))*((d*e - c*f)/d)^(1
/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b*c*d^2*e^2 - a*d^3
*e^2 - 2*b*c^2*d*e*f + 2*a*c*d^2*e*f + b*c^3*f^2 - a*c^2*d*f^2) + (sqrt(3)
*(b^3*e - a*b^2*f)^(2/3)*b*g - sqrt(3)*(b^3*e - a*b^2*f)^(2/3)*a*h)*arctan
(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e - a*f)/b)^(
1/3))/((b^4*e^2 - 2*a*b^3*e*f + a^2*b^2*f^2)*c - (a*b^3*e^2 - 2*a^2*b^2*e*
f + a^3*b*f^2)*d) + (sqrt(3)*(d^3*e - c*d^2*f)^(2/3)*d*g - sqrt(3)*(d^3*e
- c*d^2*f)^(2/3)*c*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)
/d)^(1/3))/((d*e - c*f)/d)^(1/3))/((d^4*e^2 - 2*c*d^3*e*f + c^2*d^2*f^2)*a
 - (c*d^3*e^2 - 2*c^2*d^2*e*f + c^3*d*f^2)*b) - 1/2*((b^3*e - a*b^2*f)^(2/
3)*b*g - (b^3*e - a*b^2*f)^(2/3)*a*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3
)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/((b^4*e^2 - 2*a*b^3*e*f +
 a^2*b^2*f^2)*c - (a*b^3*e^2 - 2*a^2*b^2*e*f + a^3*b*f^2)*d) - 1/2*((d^3*e
 - c*d^2*f)^(2/3)*d*g - (d^3*e - c*d^2*f)^(2/3)*c*h)*log((f*x + e)^(2/3) +
 (f*x + e)^(1/3)*((d*e - c*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/((d^4*e^2
- 2*c*d^3*e*f + c^2*d^2*f^2)*a - (c*d^3*e^2 - 2*c^2*d^2*e*f + c^3*d*f^2)*b
) - 3*(f*g - e*h)/((b*d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)*(f*x + e)^(1...
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 373297, normalized size of antiderivative = 878.35

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(4/3)*(a + b*x)*(c + d*x)),x)
```

output
```
log((((4*(b^4*d^4*g^6 + 3*a^2*b^2*d^4*g^4*h^2 + 3*b^4*c^2*d^2*g^4*h^2 + a^
3*b*c^3*d*h^6 - 3*a*b^3*d^4*g^5*h - 3*b^4*c*d^3*g^5*h - a^3*b*d^4*g^3*h^3
- b^4*c^3*d*g^3*h^3 + 9*a*b^3*c*d^3*g^4*h^2 + 3*a*b^3*c^3*d*g^2*h^4 - 3*a^
2*b^2*c^3*d*g*h^5 + 3*a^3*b*c*d^3*g^2*h^4 - 3*a^3*b*c^2*d^2*g*h^5 - 9*a*b^
3*c^2*d^2*g^3*h^3 - 9*a^2*b^2*c*d^3*g^3*h^3 + 9*a^2*b^2*c^2*d^2*g^2*h^4)*(
a^4*b^6*c^10*f^8 + a^6*b^4*d^10*e^8 + a^10*c^4*d^6*f^8 + b^10*c^6*d^4*e^8
+ a^10*d^10*e^4*f^4 + b^10*c^10*e^4*f^4 + 15*a^2*b^8*c^4*d^6*e^8 - 20*a^3*
b^7*c^3*d^7*e^8 + 15*a^4*b^6*c^2*d^8*e^8 + 15*a^6*b^4*c^8*d^2*f^8 - 20*a^7
*b^3*c^7*d^3*f^8 + 15*a^8*b^2*c^6*d^4*f^8 + 6*a^2*b^8*c^10*e^2*f^6 + 6*a^8
*b^2*d^10*e^6*f^2 + 6*a^10*c^2*d^8*e^2*f^6 + 6*b^10*c^8*d^2*e^6*f^2 - 6*a*
b^9*c^5*d^5*e^8 - 6*a^5*b^5*c*d^9*e^8 - 6*a^5*b^5*c^9*d*f^8 - 6*a^9*b*c^5*
d^5*f^8 - 4*a*b^9*c^10*e^3*f^5 - 4*a^3*b^7*c^10*e*f^7 - 4*a^7*b^3*d^10*e^7
*f - 4*a^9*b*d^10*e^5*f^3 - 4*a^10*c*d^9*e^3*f^5 - 4*a^10*c^3*d^7*e*f^7 -
4*b^10*c^7*d^3*e^7*f - 4*b^10*c^9*d*e^5*f^3 + 20*a*b^9*c^6*d^4*e^7*f + 10*
a*b^9*c^9*d*e^4*f^4 + 20*a^4*b^6*c^9*d*e*f^7 + 20*a^6*b^4*c*d^9*e^7*f + 10
*a^9*b*c*d^9*e^4*f^4 + 20*a^9*b*c^4*d^6*e*f^7 - 20*a*b^9*c^7*d^3*e^6*f^2 -
 36*a^2*b^8*c^5*d^5*e^7*f + 20*a^3*b^7*c^4*d^6*e^7*f - 20*a^3*b^7*c^9*d*e^
2*f^6 + 20*a^4*b^6*c^3*d^7*e^7*f - 36*a^5*b^5*c^2*d^8*e^7*f - 36*a^5*b^5*c
^8*d^2*e*f^7 + 20*a^6*b^4*c^7*d^3*e*f^7 - 20*a^7*b^3*c*d^9*e^6*f^2 + 20*a^
7*b^3*c^6*d^4*e*f^7 - 36*a^8*b^2*c^5*d^5*e*f^7 - 20*a^9*b*c^3*d^7*e^2*f...
```

Reduce [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 2870, normalized size of antiderivative = 6.75

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(4/3),x)
```

output
```
(2*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*
f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*
e)**(1/6)))*a*c*f*h - 2*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(
3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6)
)/(b**(1/6)*(a*f - b*e)**(1/6)))*a*d*e*h - 2*b**(1/3)*(e + f*x)**(1/3)*(c*
f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(
1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b*c*f*g + 2*b**(1/3)
*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(
1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))
*b*d*e*g + 2*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b*
*(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)
*(a*f - b*e)**(1/6)))*a*c*f*h - 2*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(
1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f
*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*d*e*h - 2*b**(1/3)*(e + f*x)*
*(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3
) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b*c*f*g +
2*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f
 - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e
)**(1/6)))*b*d*e*g - 2*d**(1/3)*(e + f*x)**(1/3)*(a*f - b*e)**(1/3)*sqrt(3
)*atan((d**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) - 2*d**(1/3)*(e + f*x)**(1/...
```

3.218 $\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 493

$$\begin{aligned}
\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx &= -\frac{3(fg-eh)}{4(be-af)(de-cf)(e+fx)^{4/3}} \\
&+ \frac{3(bcf^2g+af^2(dg-ch)-bde(2fg-eh))}{(be-af)^2(de-cf)^2\sqrt[3]{e+fx}} \\
&+ \frac{\sqrt{3}b^{4/3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{(bc-ad)(be-af)^{7/3}} \\
&- \frac{\sqrt{3}d^{4/3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{(bc-ad)(de-cf)^{7/3}} - \frac{b^{4/3}(bg-ah)\log(a+bx)}{2(bc-ad)(be-af)^{7/3}} \\
&+ \frac{d^{4/3}(dg-ch)\log(c+dx)}{2(bc-ad)(de-cf)^{7/3}} + \frac{3b^{4/3}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2(bc-ad)(be-af)^{7/3}} \\
&- \frac{3d^{4/3}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2(bc-ad)(de-cf)^{7/3}}
\end{aligned}$$

output
```
1/4*(3*e*h-3*f*g)/(-a*f+b*e)/(-c*f+d*e)/(f*x+e)^(4/3)+3*(b*c*f^2*g+a*f^2*(
-c*h+d*g)-b*d*e*(-e*h+2*f*g))/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)^(1/3)+3^(1
/2)*b^(4/3)*(-a*h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1
/3))*3^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(7/3)-3^(1/2)*d^(4/3)*(-c*h+d*g)*arcta
n(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^(1/2))/(-a*d+b*c)/(-c
*f+d*e)^(7/3)-1/2*b^(4/3)*(-a*h+b*g)*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)^(7/3)
+1/2*d^(4/3)*(-c*h+d*g)*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)^(7/3)+3/2*b^(4/3)*
(-a*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/(-a*d+b*c)/(-a*f+b*e
)^(7/3)-3/2*d^(4/3)*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/
(-a*d+b*c)/(-c*f+d*e)^(7/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx = \frac{3\left(\frac{(bg-ah)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,-\frac{1}{3},\frac{b(e+fx)}{be-af}\right)}{be-af}+\frac{(-dg+ch)\,\text{Hypergeometric2F1}\left(-\frac{4}{3}\right.}{de-cf}\right.}{4(bc-ad)(e+fx)^{4/3}}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(7/3)),x]
```

output
```
(3*(((b*g - a*h)*Hypergeometric2F1[-4/3, 1, -1/3, (b*(e + f*x))/(b*e - a*f
)])/(b*e - a*f) + ((-(d*g) + c*h)*Hypergeometric2F1[-4/3, 1, -1/3, (d*(e +
 f*x))/(d*e - c*f)])/(d*e - c*f)))/(4*(b*c - a*d)*(e + f*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {174, 61, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx$$

$$\Big\downarrow \; 174$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)(e+fx)^{7/3}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)(e+fx)^{7/3}}dx}{bc-ad}$$

$$\Big\downarrow \; 61$$

$$\frac{(bg-ah)\left(\frac{b\int \frac{1}{(a+bx)(e+fx)^{4/3}}dx}{be-af}+\frac{3}{4(e+fx)^{4/3}(be-af)}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{d\int \frac{1}{(c+dx)(e+fx)^{4/3}}dx}{de-cf}+\frac{3}{4(e+fx)^{4/3}(de-cf)}\right)}{bc-ad}$$

$$\Big\downarrow \; 61$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\int \frac{1}{(a+bx)\sqrt[3]{e+fx}}dx}{be-af}+\frac{3}{\sqrt[3]{e+fx}(be-af)}\right)}{be-af}+\frac{3}{4(e+fx)^{4/3}(be-af)}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{d\left(\frac{d\int \frac{1}{(c+dx)\sqrt[3]{e+fx}}dx}{de-cf}+\frac{3}{\sqrt[3]{e+fx}(de-cf)}\right)}{de-cf}+\frac{3}{4(e+fx)^{4/3}(de-cf)}\right)}{bc-ad}$$

$$\Big\downarrow \; 67$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\cdots}{2b^{2/3}\cdots}\cdots\right.}{be-af}\cdots}{be-af}\cdots\right.}{bc-ad}\cdots$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{d\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\cdots}{2d^{2/3}\cdots}\cdots\right.}{de-cf}\cdots}{de-cf}\cdots\right.}{bc-ad}\cdots$$

$$\Big\downarrow \text{16}$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\left(\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+\cdots}\right)}{2b^{2/3}\sqrt[3]{be-af}}\cdots\right)}{be-af}\cdots\right)}{be-af}\cdots\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{d\left(\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+\cdots}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\cdots\right)}{de-cf}\cdots\right)}{de-cf}\cdots\right)}{bc-ad}$$

$$\Big\downarrow 1082$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{be-af}+\frac{3}{\sqrt[3]{e+fx}}}{be-af}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{d\left(-\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{de-cf}+\frac{3}{\sqrt[3]{e+fx}}}{de-cf}\right.}{bc-ad}$$

$$\Big\downarrow 217$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{be-af}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{2/3}\sqrt[3]{be-af}}\right)}{be-af}+\frac{3}{\sqrt[3]{e+fx}(be-af)}\right)}{be-af}\right)}{bc-ad}$$

$$(dg-ch)\left(\frac{d\left(\frac{d\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2d^{2/3}\sqrt[3]{de-cf}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{2/3}\sqrt[3]{de-cf}}\right)}{de-cf}+\frac{3}{\sqrt[3]{e+fx}(de-cf)}\right)}{de-cf}\right.$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(7/3)),x]
```

output
```
((b*g - a*h)*(3/(4*(b*e - a*f)*(e + f*x)^(4/3)) + (b*(3/((b*e - a*f)*(e +
f*x)^(1/3)) + (b*((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*e -
a*f)^(1/3))/Sqrt[3]])/(b^(2/3)*(b*e - a*f)^(1/3)) - Log[a + b*x]/(2*b^(2/3
)*(b*e - a*f)^(1/3)) + (3*Log[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)]
)/(2*b^(2/3)*(b*e - a*f)^(1/3))))/(b*e - a*f)))/(b*e - a*f)))/(b*c - a*d)
- ((d*g - c*h)*(3/(4*(d*e - c*f)*(e + f*x)^(4/3)) + (d*(3/((d*e - c*f)*(e
+ f*x)^(1/3)) + (d*((Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e
- c*f)^(1/3))/Sqrt[3]])/(d^(2/3)*(d*e - c*f)^(1/3)) - Log[c + d*x]/(2*d^(2
/3)*(d*e - c*f)^(1/3)) + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3
)])/(2*d^(2/3)*(d*e - c*f)^(1/3))))/(d*e - c*f)))/(d*e - c*f)))/(b*c - a*d
)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}\right)}{(ad-bc)(af-be)^2}$
default	$\frac{3\left(-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}\right)}{(ad-bc)(af-be)^2}$
pseudoelliptic	$-\frac{b\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)(ah-bg)}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(ad-bc)(af-be)^2}+\frac{b\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)(ah-bg)}{2\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(ad-bc)(af-be)^2}+\frac{\sqrt{3}\,b\,\mathrm{ar}}{(}$

input

```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(7/3),x,method=_RETURNVERBOSE)
```

output

```
3*(-1/3/b/((a*f-b*e)/b)^(1/3)*ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))+1/6/b/
((a*f-b*e)/b)^(1/3)*ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a
*f-b*e)/b)^(2/3))+1/3*3^(1/2)/b/((a*f-b*e)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)-1)))*(a*h-b*g)*b^2/(a*d-b*c)/(a*f-b*e)^2
-3/4*(-e*h+f*g)/(c*f-d*e)/(a*f-b*e)/(f*x+e)^(4/3)-3*(a*c*f^2*h-a*d*f^2*g-b
*c*f^2*g-b*d*e^2*h+2*b*d*e*f*g)/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)^(1/3)-3*(-
1/3/d/((c*f-d*e)/d)^(1/3)*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))+1/6/d/((c*
f-d*e)/d)^(1/3)*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*f-d
*e)/d)^(2/3))+1/3*3^(1/2)/d/((c*f-d*e)/d)^(1/3)*arctan(1/3*3^(1/2)*(2/((c*
f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1)))*(c*h-d*g)*d^2/(a*d-b*c)/(c*f-d*e)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(7/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(7/3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(7/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(425) = 850$.

Time = 1.54 (sec) , antiderivative size = 1277, normalized size of antiderivative = 2.59

$$\int \frac{g + hx}{(a + bx)(c + dx)(e + fx)^{7/3}} \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(7/3),x, algorithm="giac")
```

output
```
(b^3*g*((b*e - a*f)/b)^(1/3) - a*b^2*h*((b*e - a*f)/b)^(1/3))*((b*e - a*f)
/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)^(1/3)))/(b^4*c*e^3 - a
*b^3*d*e^3 - 3*a*b^3*c*e^2*f + 3*a^2*b^2*d*e^2*f + 3*a^2*b^2*c*e*f^2 - 3*a
^3*b*d*e*f^2 - a^3*b*c*f^3 + a^4*d*f^3) - (d^3*g*((d*e - c*f)/d)^(1/3) - c
*d^2*h*((d*e - c*f)/d)^(1/3))*((d*e - c*f)/d)^(1/3)*log(abs((f*x + e)^(1/3
) - ((d*e - c*f)/d)^(1/3)))/(b*c*d^3*e^3 - a*d^4*e^3 - 3*b*c^2*d^2*e^2*f +
 3*a*c*d^3*e^2*f + 3*b*c^3*d*e*f^2 - 3*a*c^2*d^2*e*f^2 - b*c^4*f^3 + a*c^3
*d*f^3) + (sqrt(3)*(b^3*e - a*b^2*f)^(2/3)*b*g - sqrt(3)*(b^3*e - a*b^2*f)
^(2/3)*a*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))
/((b*e - a*f)/b)^(1/3))/((b^4*e^3 - 3*a*b^3*e^2*f + 3*a^2*b^2*e*f^2 - a^3*
b*f^3)*c - (a*b^3*e^3 - 3*a^2*b^2*e^2*f + 3*a^3*b*e*f^2 - a^4*f^3)*d) + (s
qrt(3)*(d^3*e - c*d^2*f)^(2/3)*d*g - sqrt(3)*(d^3*e - c*d^2*f)^(2/3)*c*h)*
arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3))/((d*e - c*f
)/d)^(1/3))/((d^4*e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2 - c^3*d*f^3)*a - (
c*d^3*e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*b) - 1/2*((b^3*e -
a*b^2*f)^(2/3)*b*g - (b^3*e - a*b^2*f)^(2/3)*a*h)*log((f*x + e)^(2/3) + (f
*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/((b^4*e^3 - 3
*a*b^3*e^2*f + 3*a^2*b^2*e*f^2 - a^3*b*f^3)*c - (a*b^3*e^3 - 3*a^2*b^2*e^2
*f + 3*a^3*b*e*f^2 - a^4*f^3)*d) - 1/2*((d^3*e - c*d^2*f)^(2/3)*d*g - (d^3
*e - c*d^2*f)^(2/3)*c*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((d*e - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(7/3)*(a + b*x)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 9649, normalized size of antiderivative = 19.57

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{7/3}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(7/3),x)
```

output

```
( - 4*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*
(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f -
 b*e)**(1/6)))*a*b*c**2*e*f**2*h - 4*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)
**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e
+ f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*c**2*f**3*h*x + 8*b**(1/
3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)*
*(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)
))*a*b*c*d*e**2*f*h + 8*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(
3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6)
)/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*c*d*e*f**2*h*x - 4*b**(1/3)*(e + f*x)
**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(
3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*d**2*
e**3*h - 4*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(
1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(
a*f - b*e)**(1/6)))*a*b*d**2*e**2*f*h*x + 4*b**(1/3)*(e + f*x)**(1/3)*(c*f
 - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1
/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b**2*c**2*e*f**2*g +
4*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f
 - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e
)**(1/6)))*b**2*c**2*f**3*g*x - 8*b**(1/3)*(e + f*x)**(1/3)*(c*f - d*e)...
```

3.219 $\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 446

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = -\frac{3(adfh-b(dfg+deh-cfh))\sqrt[3]{e+fx}}{b^2d^2} + \frac{3h(e+fx)^{4/3}}{4bd} - \frac{\sqrt{3}(be-af)^{4/3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{b^{7/3}(bc-ad)} + \frac{\sqrt{3}(de-cf)^{4/3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{d^{7/3}(bc-ad)} - \frac{(be-af)^{4/3}(bg-ah)\log(a+bx)}{2b^{7/3}(bc-ad)} + \frac{(de-cf)^{4/3}(dg-ch)\log(c+dx)}{2d^{7/3}(bc-ad)} + \frac{3(be-af)^{4/3}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{7/3}(bc-ad)} - \frac{3(de-cf)^{4/3}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{7/3}(bc-ad)}$$

output
```
-3*(a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/3)/b^2/d^2+3/4*h*(f*x+e)^(4
/3)/b/d-3^(1/2)*(-a*f+b*e)^(4/3)*(-a*h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e
)^(1/3)/(-a*f+b*e)^(1/3))*3^(1/2))/b^(7/3)/(-a*d+b*c)+3^(1/2)*(-c*f+d*e)^(
4/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^
(1/2))/d^(7/3)/(-a*d+b*c)-1/2*(-a*f+b*e)^(4/3)*(-a*h+b*g)*ln(b*x+a)/b^(7/3
)/(-a*d+b*c)+1/2*(-c*f+d*e)^(4/3)*(-c*h+d*g)*ln(d*x+c)/d^(7/3)/(-a*d+b*c)+
3/2*(-a*f+b*e)^(4/3)*(-a*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))
/b^(7/3)/(-a*d+b*c)-3/2*(-c*f+d*e)^(4/3)*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^
(1/3)*(f*x+e)^(1/3))/d^(7/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.12

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{3\sqrt[3]{b}\sqrt[3]{d}(bc-ad)\sqrt[3]{e+fx}(-4bcfh-4adfh+bd(4fg+5eh+fhx))-4\sqrt{3}d}{}$$

input
```
Integrate[((e + f*x)^(4/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(3*b^(1/3)*d^(1/3)*(b*c - a*d)*(e + f*x)^(1/3)*(-4*b*c*f*h - 4*a*d*f*h + b
*d*(4*f*g + 5*e*h + f*h*x)) - 4*Sqrt[3]*d^(7/3)*(-(b*e) + a*f)^(4/3)*(b*g
- a*h)*ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e) + a*f)^(1/3))/Sqrt[
3]] + 4*Sqrt[3]*b^(7/3)*(-(d*e) + c*f)^(4/3)*(d*g - c*h)*ArcTan[(1 - (2*d^
(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))/Sqrt[3]] + 4*d^(7/3)*(-(b*e)
+ a*f)^(4/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(1/3) + b^(1/3)*(e + f*x)^(1/3
)] - 4*b^(7/3)*(-(d*e) + c*f)^(4/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(1/3) +
 d^(1/3)*(e + f*x)^(1/3)] - 2*d^(7/3)*(-(b*e) + a*f)^(4/3)*(b*g - a*h)*Log
[(-(b*e) + a*f)^(2/3) - b^(1/3)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3) + b^(
2/3)*(e + f*x)^(2/3)] + 2*b^(7/3)*(-(d*e) + c*f)^(4/3)*(d*g - c*h)*Log[(-(
d*e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f*x)^(1/3) + d^(2/3)
*(e + f*x)^(2/3)])/(4*b^(7/3)*d^(7/3)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {174, 60, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{174}$$

$$\frac{(bg-ah)\int \frac{(e+fx)^{4/3}}{a+bx}dx}{bc-ad}-\frac{(dg-ch)\int \frac{(e+fx)^{4/3}}{c+dx}dx}{bc-ad}$$

$$\Big\downarrow \text{60}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\int \frac{\sqrt[3]{e+fx}}{a+bx}dx}{b}+\frac{3(e+fx)^{4/3}}{4b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\int \frac{\sqrt[3]{e+fx}}{c+dx}dx}{d}+\frac{3(e+fx)^{4/3}}{4d}\right)}{bc-ad}$$

$$\Big\downarrow \text{60}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\int \frac{1}{(a+bx)(e+fx)^{2/3}}dx}{b}+\frac{3\sqrt[3]{e+fx}}{b}\right)}{b}+\frac{3(e+fx)^{4/3}}{4b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\int \frac{1}{(c+dx)(e+fx)^{2/3}}dx}{d}+\frac{3\sqrt[3]{e+fx}}{d}\right)}{d}+\frac{3(e+fx)^{4/3}}{4d}\right)}{bc-ad}$$

$$\Big\downarrow \text{69}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e\cdots}}{2\sqrt[3]{b}(be-af)^{2/3}}\cdots\right.}{b}\cdots}{b}\cdots\right.}{bc-ad}\cdots$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e\cdots}}{2\sqrt[3]{d}(de-cf)^{2/3}}\cdots\right.}{d}\cdots}{d}\cdots\right.}{bc-ad}\cdots$$

$$\Big\downarrow \text{16}$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}\right.}{2\sqrt[3]{b}(be}\right.}{b}}{b}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}\right.}{2\sqrt[3]{d}(de}\right.}{d}}{d}\right.}{bc-ad}$$

$\downarrow$ 1082

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{b}+\frac{3\sqrt[3]{\cdots}}{\cdots}\cdots}{b}\cdots\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{d}+\frac{3\cdots}{\cdots}\cdots}{d}\cdots\right)}{bc-ad}$$

$\downarrow$ 217

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{(be-af)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{b}+\frac{3\sqrt[3]{e+\cdots}}{b}\cdots\right)}{b}\cdots\right)}{bc-ad}$$

$$(dg-ch)\left(\frac{(de-cf)\left(\frac{(de-cf)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{d}+\frac{3\sqrt[3]{e+\cdots}}{d}\cdots\right)}{d}\cdots\right.$$

input
```
Int[((e + f*x)^(4/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
((b*g - a*h)*((3*(e + f*x)^(4/3))/(4*b) + ((b*e - a*f)*((3*(e + f*x)^(1/3)
)/b + ((b*e - a*f)*(-((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*
e - a*f)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*e - a*f)^(2/3))) - Log[a + b*x]/(2*b
^(1/3)*(b*e - a*f)^(2/3)) + (3*Log[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^(
1/3)])/(2*b^(1/3)*(b*e - a*f)^(2/3))))/b))/b))/(b*c - a*d) - ((d*g - c*h)*
((3*(e + f*x)^(4/3))/(4*d) + ((d*e - c*f)*((3*(e + f*x)^(1/3))/d + ((d*e -
 c*f)*(-((Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3
))/Sqrt[3]])/(d^(1/3)*(d*e - c*f)^(2/3))) - Log[c + d*x]/(2*d^(1/3)*(d*e -
 c*f)^(2/3)) + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(
1/3)*(d*e - c*f)^(2/3))))/d))/d))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{-\left(-6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\left(\left(\left(\left(-\frac{hx}{4}-g\right)f-\frac{5eh}{4}\right)b+afh\right)d+bcfh\right)(ad-bc)(fx+e)^{\frac{1}{3}}+\left(-2\arctan\left(\frac{\sqrt{3}\left(2(fx+e)^{\frac{1}{3}}-\left(\frac{cf-}{d}\right.\right.}{3\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}\right.\right.\right.}{}$
derivativedivides	$-\frac{3\left(-\frac{h(fx+e)^{\frac{4}{3}}bd}{4}+adfh(fx+e)^{\frac{1}{3}}+bcfh(fx+e)^{\frac{1}{3}}-bdeh(fx+e)^{\frac{1}{3}}-bdfg(fx+e)^{\frac{1}{3}}\right)}{b^2d^2}+\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}\right.}{}$
default	$-\frac{3\left(-\frac{h(fx+e)^{\frac{4}{3}}bd}{4}+adfh(fx+e)^{\frac{1}{3}}+bcfh(fx+e)^{\frac{1}{3}}-bdeh(fx+e)^{\frac{1}{3}}-bdfg(fx+e)^{\frac{1}{3}}\right)}{b^2d^2}+\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}\right.}{}$

input
```
int((f*x+e)^(4/3)*(h*x+g)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((c*f-d*e)/d)^(2/3)*(-(-6*d*((c*f-d*e)/d)^(2/3)*((((-1/4*h*x-g)*f-5/4
*e*h)*b+a*f*h)*d+b*c*f*h)*(a*d-b*c)*(f*x+e)^(1/3)+(-2*arctan(1/3*3^(1/2)*(
2*(f*x+e)^(1/3)-((c*f-d*e)/d)^(1/3))/((c*f-d*e)/d)^(1/3))*3^(1/2)+ln((f*x+
e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*f-d*e)/d)^(2/3))-2*ln((f*x+
e)^(1/3)+((c*f-d*e)/d)^(1/3)))*(c*f-d*e)^2*(c*h-d*g)*b^2)*b*((a*f-b*e)/b)^
(2/3)+(a*f-b*e)^2*d^3*(a*h-b*g)*((c*f-d*e)/d)^(2/3)*(-2*arctan(1/3*3^(1/2)
*(2*(f*x+e)^(1/3)-((a*f-b*e)/b)^(1/3))/((a*f-b*e)/b)^(1/3))*3^(1/2)+ln((f*
x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/3))-2*ln((f*
x+e)^(1/3)+((a*f-b*e)/b)^(1/3))))/((a*f-b*e)/b)^(2/3)/b^3/d^3/(a*d-b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^(4/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \int \frac{(e+fx)^{\frac{4}{3}}\,(g+hx)}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**(4/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
Integral((e + f*x)**(4/3)*(g + h*x)/((a + b*x)*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(4/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(378) = 756$.

Time = 0.40 (sec) , antiderivative size = 857, normalized size of antiderivative = 1.92

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(4/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
(b^3*e^2*g - 2*a*b^2*e*f*g + a^2*b*f^2*g - a*b^2*e^2*h + 2*a^2*b*e*f*h - a
^3*f^2*h)*((b*e - a*f)/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)^
(1/3)))/(b^4*c*e - a*b^3*d*e - a*b^3*c*f + a^2*b^2*d*f) - (d^3*e^2*g - 2*c
*d^2*e*f*g + c^2*d*f^2*g - c*d^2*e^2*h + 2*c^2*d*e*f*h - c^3*f^2*h)*((d*e
- c*f)/d)^(1/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b*c*d^3
*e - a*d^4*e - b*c^2*d^2*f + a*c*d^3*f) - ((b^3*e - a*b^2*f)^(1/3)*(sqrt(3
)*b^2*e - sqrt(3)*a*b*f)*g - (b^3*e - a*b^2*f)^(1/3)*(sqrt(3)*a*b*e - sqrt
(3)*a^2*f)*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3
))/((b*e - a*f)/b)^(1/3))/(b^4*c - a*b^3*d) + ((d^3*e - c*d^2*f)^(1/3)*(sq
rt(3)*d^2*e - sqrt(3)*c*d*f)*g - (d^3*e - c*d^2*f)^(1/3)*(sqrt(3)*c*d*e -
sqrt(3)*c^2*f)*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^
(1/3))/((d*e - c*f)/d)^(1/3))/(b*c*d^3 - a*d^4) - 1/2*((b^3*e - a*b^2*f)^(
1/3)*(b^2*e - a*b*f)*g - (b^3*e - a*b^2*f)^(1/3)*(a*b*e - a^2*f)*h)*log((f
*x + e)^(2/3) + (f*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2
/3))/(b^4*c - a*b^3*d) + 1/2*((d^3*e - c*d^2*f)^(1/3)*(d^2*e - c*d*f)*g -
(d^3*e - c*d^2*f)^(1/3)*(c*d*e - c^2*f)*h)*log((f*x + e)^(2/3) + (f*x + e)
^(1/3)*((d*e - c*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/(b*c*d^3 - a*d^4) +
3/4*(4*(f*x + e)^(1/3)*b^3*d^3*f*g + (f*x + e)^(4/3)*b^3*d^3*h + 4*(f*x +
e)^(1/3)*b^3*d^3*e*h - 4*(f*x + e)^(1/3)*b^3*c*d^2*f*h - 4*(f*x + e)^(1/3)
*a*b^2*d^3*f*h)/(b^4*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(4/3)*(g + h*x))/((a + b*x)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 4185, normalized size of antiderivative = 9.38

$$\int \frac{(e+fx)^{4/3}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(4/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output

```
( - 4*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
*3*d**2*f**2*h + 8*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f
 - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e
)**(1/6)))*a**2*b*d**2*e*f*h + 4*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan(
(b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1
/6)*(a*f - b*e)**(1/6)))*a**2*b*d**2*f**2*g - 4*d**(1/3)*(c*f - d*e)**(2/3
)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)
**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*d**2*e**2*h - 8*d**(1/3)*(c
*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**
(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*d**2*e*f*g +
 4*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*s
qrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b**3*
d**2*e**2*g - 4*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f -
b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**
(1/6)))*a**3*d**2*f**2*h + 8*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**
(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*
(a*f - b*e)**(1/6)))*a**2*b*d**2*e*f*h + 4*d**(1/3)*(c*f - d*e)**(2/3)*sqr
t(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/
6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a**2*b*d**2*f**2*g - 4*d**(1/3)*(c*f...
```

3.220 $\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 404

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{3h\sqrt[3]{e+fx}}{bd} - \frac{\sqrt{3}\sqrt[3]{be-af}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{b^{4/3}(bc-ad)} + \frac{\sqrt{3}\sqrt[3]{de-cf}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{d^{4/3}(bc-ad)} - \frac{\sqrt[3]{be-af}(bg-ah)\log(a+bx)}{2b^{4/3}(bc-ad)} + \frac{\sqrt[3]{de-cf}(dg-ch)\log(c+dx)}{2d^{4/3}(bc-ad)} + \frac{3\sqrt[3]{be-af}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{4/3}(bc-ad)} - \frac{3\sqrt[3]{de-cf}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2d^{4/3}(bc-ad)}$$

output
```
3*h*(f*x+e)^(1/3)/b/d-3^(1/2)*(-a*f+b*e)^(1/3)*(-a*h+b*g)*arctan(1/3*(1+2*
b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1/3))*3^(1/2))/b^(4/3)/(-a*d+b*c)+3^(1/2
)*(-c*f+d*e)^(1/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+
d*e)^(1/3))*3^(1/2))/d^(4/3)/(-a*d+b*c)-1/2*(-a*f+b*e)^(1/3)*(-a*h+b*g)*ln
(b*x+a)/b^(4/3)/(-a*d+b*c)+1/2*(-c*f+d*e)^(1/3)*(-c*h+d*g)*ln(d*x+c)/d^(4/
3)/(-a*d+b*c)+3/2*(-a*f+b*e)^(1/3)*(-a*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*
(f*x+e)^(1/3))/b^(4/3)/(-a*d+b*c)-3/2*(-c*f+d*e)^(1/3)*(-c*h+d*g)*ln((-c*f
+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/d^(4/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx$$

$$= \frac{6\sqrt[3]{b}\sqrt[3]{d}(bc-ad)h\sqrt[3]{e+fx}+2\sqrt{3}d^{4/3}\sqrt[3]{-be+af}(bg-ah)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{-be+af}}}{\sqrt{3}}\right)-2\sqrt{3}b^{4/3}\sqrt[3]{-}}{}$$

input
```
Integrate[((e + f*x)^(1/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
(6*b^(1/3)*d^(1/3)*(b*c - a*d)*h*(e + f*x)^(1/3) + 2*Sqrt[3]*d^(4/3)*(-(b*
e) + a*f)^(1/3)*(b*g - a*h)*ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e
) + a*f)^(1/3))/Sqrt[3]] - 2*Sqrt[3]*b^(4/3)*(-(d*e) + c*f)^(1/3)*(d*g - c
*h)*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))/Sqrt[3]]
 - 2*d^(4/3)*(-(b*e) + a*f)^(1/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(1/3) + b
^(1/3)*(e + f*x)^(1/3)] + 2*b^(4/3)*(-(d*e) + c*f)^(1/3)*(d*g - c*h)*Log[(
-(d*e) + c*f)^(1/3) + d^(1/3)*(e + f*x)^(1/3)] + d^(4/3)*(-(b*e) + a*f)^(1
/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(2/3) - b^(1/3)*(-(b*e) + a*f)^(1/3)*(e
 + f*x)^(1/3) + b^(2/3)*(e + f*x)^(2/3)] - b^(4/3)*(-(d*e) + c*f)^(1/3)*(d
*g - c*h)*Log[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f*x
)^(1/3) + d^(2/3)*(e + f*x)^(2/3)])/(2*b^(4/3)*d^(4/3)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {174, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \; 174$$

$$\frac{(bg-ah)\int \frac{\sqrt[3]{e+fx}}{a+bx}dx}{bc-ad}-\frac{(dg-ch)\int \frac{\sqrt[3]{e+fx}}{c+dx}dx}{bc-ad}$$

$$\Big\downarrow \; 60$$

$$\frac{(bg-ah)\left(\frac{(be-af)\int \frac{1}{(a+bx)(e+fx)^{2/3}}dx}{b}+\frac{3\sqrt[3]{e+fx}}{b}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{(de-cf)\int \frac{1}{(c+dx)(e+fx)^{2/3}}dx}{d}+\frac{3\sqrt[3]{e+fx}}{d}\right)}{bc-ad}$$

$$\Big\downarrow \; 69$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{b}(be-af)^{2/3}}-\cdots\right.}{b}\cdots\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{d}(de-cf)^{2/3}}-\cdots\right.}{d}\cdots\right.}{bc-ad}$$

$$\Big\downarrow \; 16$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{2\sqrt[3]{b}(be-af)^{2/3}}\cdots\right.}{b}\cdots\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{\cdots}\right.}{2\sqrt[3]{d}(de-cf)^{2/3}}\cdots\right.}{d}\cdots\right.}{bc-ad}$$

$\downarrow$ 1082

$$\frac{(bg-ah)\left(\frac{(be-af)\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{b}+\frac{3\sqrt[3]{e+}}{b}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{d}+\frac{3\sqrt[3]{e+}}{d}\right.}{bc-ad}$$

$$\Big\downarrow \ 217$$

$$\frac{(bg-ah)\left(\frac{(be-af)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{b}+\frac{3\sqrt[3]{e+fx}}{b}\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{(de-cf)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{d}+\frac{3\sqrt[3]{e+fx}}{d}\right)}{bc-ad}$$

input
```
Int[((e + f*x)^(1/3)*(g + h*x))/((a + b*x)*(c + d*x)),x]
```

output
```
((b*g - a*h)*((3*(e + f*x)^(1/3))/b + ((b*e - a*f)*(-((Sqrt[3]*ArcTan[(1 +
 (2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*f)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*e -
a*f)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*e - a*f)^(2/3)) + (3*Log[(b*e -
a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])/(2*b^(1/3)*(b*e - a*f)^(2/3))))/b))
/(b*c - a*d) - ((d*g - c*h)*((3*(e + f*x)^(1/3))/d + ((d*e - c*f)*(-((Sqrt
[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3))/Sqrt[3]])/(
d^(1/3)*(d*e - c*f)^(2/3))) - Log[c + d*x]/(2*d^(1/3)*(d*e - c*f)^(2/3)) +
 (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(1/3)*(d*e - c*
f)^(2/3))))/d))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{\left(3(fx+e)^{\frac{1}{3}}\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}dh(ad-bc)+\left(\arctan\left(\frac{2\sqrt{3}\,(fx+e)^{\frac{1}{3}}}{3\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\right.\right.}{}\right.\right.}{}$
derivativedivides	$\frac{3h(fx+e)^{\frac{1}{3}}}{bd}+\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\sqrt{3}\left(\frac{2(fx}{\left(\frac{cf-}{d}\right.}\right.\right.}{3d\left(\frac{cf-de}{d}\right)}\right.}{d(ad-bc)}$
default	$\frac{3h(fx+e)^{\frac{1}{3}}}{bd}+\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\sqrt{3}\left(\frac{2(fx}{\left(\frac{cf-}{d}\right.}\right.\right.}{3d\left(\frac{cf-de}{d}\right)}\right.}{d(ad-bc)}$

input
```
int((f*x+e)^(1/3)*(h*x+g)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/((c*f-d*e)/d)^(2/3)*((3*(f*x+e)^(1/3)*((c*f-d*e)/d)^(2/3)*d*h*(a*d-b*c)+
(arctan(2/3*3^(1/2)/((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1/3*3^(1/2))*3^(1/2)
+ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))-1/2*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^
(1/3)*(f*x+e)^(1/3)+((c*f-d*e)/d)^(2/3)))*(c*f-d*e)*(c*h-d*g)*b)*b*((a*f-b
*e)/b)^(2/3)+1/2*(a*f-b*e)*d^2*(a*h-b*g)*(-2*arctan(2/3*3^(1/2)/((a*f-b*e)
/b)^(1/3)*(f*x+e)^(1/3)-1/3*3^(1/2))*3^(1/2)+ln((f*x+e)^(2/3)-((a*f-b*e)/b
)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/3))-2*ln((f*x+e)^(1/3)+((a*f-b*e)/b
)^(1/3)))*((c*f-d*e)/d)^(2/3))/((a*f-b*e)/b)^(2/3)/(a*d-b*c)/b^2/d^2
```

Fricas [A] (verification not implemented)

Time = 35.70 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \frac{2\sqrt{3}(bdg-adh)\left(\frac{be-af}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(fx+e)^{\frac{1}{3}}b\left(\frac{be-af}{b}\right)^{\frac{2}{3}}+\sqrt{3}(be-af)}{3(be-af)}\right)+2\sqrt{3}(bdg-bch)\left(-\frac{de-cf}{d}\right)^{\frac{1}{3}}\arcta$$

input
```
integrate((f*x+e)^(1/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(b*d*g - a*d*h)*((b*e - a*f)/b)^(1/3)*arctan(-1/3*(2*sqrt(3
)*(f*x + e)^(1/3)*b*((b*e - a*f)/b)^(2/3) + sqrt(3)*(b*e - a*f))/(b*e - a*
f)) + 2*sqrt(3)*(b*d*g - b*c*h)*(-(d*e - c*f)/d)^(1/3)*arctan(-1/3*(2*sqrt
(3)*(f*x + e)^(1/3)*d*(-(d*e - c*f)/d)^(2/3) + sqrt(3)*(d*e - c*f))/(d*e -
 c*f)) + 6*(b*c - a*d)*(f*x + e)^(1/3)*h - (b*d*g - a*d*h)*((b*e - a*f)/b)
^(1/3)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e
 - a*f)/b)^(2/3)) - (b*d*g - b*c*h)*(-(d*e - c*f)/d)^(1/3)*log((f*x + e)^(
2/3) - (f*x + e)^(1/3)*(-(d*e - c*f)/d)^(1/3) + (-(d*e - c*f)/d)^(2/3)) +
2*(b*d*g - a*d*h)*((b*e - a*f)/b)^(1/3)*log((f*x + e)^(1/3) - ((b*e - a*f)
/b)^(1/3)) + 2*(b*d*g - b*c*h)*(-(d*e - c*f)/d)^(1/3)*log((f*x + e)^(1/3)
+ (-(d*e - c*f)/d)^(1/3)))/(b^2*c*d - a*b*d^2)
```

Sympy [F]

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**(1/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
Integral((e + f*x)**(1/3)*(g + h*x)/((a + b*x)*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/3)*(h*x+g)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
(b^2*e*g - a*b*f*g - a*b*e*h + a^2*f*h)*((b*e - a*f)/b)^(1/3)*log(abs((f*x
 + e)^(1/3) - ((b*e - a*f)/b)^(1/3)))/(b^3*c*e - a*b^2*d*e - a*b^2*c*f + a
^2*b*d*f) - (d^2*e*g - c*d*f*g - c*d*e*h + c^2*f*h)*((d*e - c*f)/d)^(1/3)*
log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b*c*d^2*e - a*d^3*e - b
*c^2*d*f + a*c*d^2*f) - (sqrt(3)*(b^3*e - a*b^2*f)^(1/3)*b*g - sqrt(3)*(b^
3*e - a*b^2*f)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e -
a*f)/b)^(1/3))/((b*e - a*f)/b)^(1/3))/(b^3*c - a*b^2*d) + (sqrt(3)*(d^3*e
- c*d^2*f)^(1/3)*d*g - sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*c*h)*arctan(1/3*sqr
t(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3))/((d*e - c*f)/d)^(1/3))/(b
*c*d^2 - a*d^3) - 1/2*((b^3*e - a*b^2*f)^(1/3)*b*g - (b^3*e - a*b^2*f)^(1/
3)*a*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*
e - a*f)/b)^(2/3))/(b^3*c - a*b^2*d) + 1/2*((d^3*e - c*d^2*f)^(1/3)*d*g -
(d^3*e - c*d^2*f)^(1/3)*c*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((d*e -
 c*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/(b*c*d^2 - a*d^3) + 3*(f*x + e)^(1
/3)*h/(b*d)
```

Mupad [B] (verification not implemented)

Time = 25.16 (sec) , antiderivative size = 7103, normalized size of antiderivative = 17.58

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^(1/3)*(g + h*x))/((a + b*x)*(c + d*x)),x)
```

output
```
log(- ((243*b^3*d^3*f^4*(e + f*x)^(1/3)*(a*d - b*c)^4*(a*f - b*e)*(c*f - d
*e)*(e*h - f*g) + 243*b^3*d^3*f^4*(a*d - b*c)^4*(a*f - b*e)*(c*f - d*e)*(a
*d*f + b*c*f - 2*b*d*e)*(((c*f - d*e)*(c*h - d*g)^3)/(d^4*(a*d - b*c)^3))^
(1/3))*(((c*f - d*e)*(c*h - d*g)^3)/(d^4*(a*d - b*c)^3))^(2/3) + (243*f^4*
(a*d - b*c)^2*(a*f - b*e)*(c*f - d*e)*(a^4*d^4*f^2*h^3 + b^4*c^4*f^2*h^3 +
 a^2*b^2*d^4*e^2*h^3 + b^4*c^2*d^2*e^2*h^3 + 2*b^4*d^4*e*f*g^3 - a*b^3*d^4
*f^2*g^3 - b^4*c*d^3*f^2*g^3 + 3*b^4*d^4*e^2*g^2*h + a*b^3*c*d^3*e^2*h^3 +
 a*b^3*c^3*d*f^2*h^3 + a^3*b*c*d^3*f^2*h^3 - 3*a*b^3*d^4*e^2*g*h^2 - 3*a^3
*b*d^4*f^2*g*h^2 - 3*b^4*c*d^3*e^2*g*h^2 - 3*b^4*c^3*d*f^2*g*h^2 + 3*a^2*b
^2*d^4*f^2*g^2*h + 3*b^4*c^2*d^2*f^2*g^2*h - 2*a^3*b*d^4*e*f*h^3 - 2*b^4*c
^3*d*e*f*h^3 + a^2*b^2*c^2*d^2*f^2*h^3 - 6*a*b^3*d^4*e*f*g^2*h - 6*b^4*c*d
^3*e*f*g^2*h - 2*a*b^3*c^2*d^2*e*f*h^3 - 2*a^2*b^2*c*d^3*e*f*h^3 + 3*a*b^3
*c*d^3*f^2*g^2*h + 6*a^2*b^2*d^4*e*f*g*h^2 + 6*b^4*c^2*d^2*e*f*g*h^2 - 3*a
*b^3*c^2*d^2*f^2*g*h^2 - 3*a^2*b^2*c*d^3*f^2*g*h^2 + 6*a*b^3*c*d^3*e*f*g*h
^2))/(b*d))*(((c*f - d*e)*(c*h - d*g)^3)/(d^4*(a*d - b*c)^3))^(1/3) - (243
*f^4*(e + f*x)^(1/3)*(a*f - b*e)*(a*h - b*g)*(c*f - d*e)*(c*h - d*g)*(2*b^
4*d^4*e^2*g^2 + a^4*d^4*f^2*h^2 + b^4*c^4*f^2*h^2 + a^2*b^2*d^4*e^2*h^2 +
a^2*b^2*d^4*f^2*g^2 + b^4*c^2*d^2*e^2*h^2 + b^4*c^2*d^2*f^2*g^2 - 2*a*b^3*
d^4*e*f*g^2 - 2*a^3*b*d^4*e*f*h^2 - 2*a*b^3*d^4*e^2*g*h - 2*b^4*c*d^3*e*f*
g^2 - 2*a^3*b*d^4*f^2*g*h - 2*b^4*c^3*d*e*f*h^2 - 2*b^4*c*d^3*e^2*g*h -...
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 2549, normalized size of antiderivative = 6.31

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/3)*(h*x+g)/(b*x+a)/(d*x+c),x)
```

output
```
(2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*s
qrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a**2*
d*f*h - 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**
(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6))
)*a*b*d*e*h - 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f -
b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**
(1/6)))*a*b*d*f*g + 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(
a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f -
b*e)**(1/6)))*b**2*d*e*g + 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**
(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*
(a*f - b*e)**(1/6)))*a**2*d*f*h - 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*at
an((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b*
*(1/6)*(a*f - b*e)**(1/6)))*a*b*d*e*h - 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt
(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6
))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*d*f*g + 2*d**(1/3)*(c*f - d*e)**(2/3
)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)
**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b**2*d*e*g - 2*b**(1/3)*(a*f - b*e
)**(2/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) - 2*d**(1/3)*(e
 + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*b*c**2*f*h + 2*b**(1/3)*(a*
f - b*e)**(2/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) - 2*d...
```

3.221 $\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 386

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = -\frac{\sqrt{3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)(be-af)^{2/3}} + \frac{\sqrt{3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)(de-cf)^{2/3}} - \frac{(bg-ah)\log(a+bx)}{2\sqrt[3]{b}(bc-ad)(be-af)^{2/3}} + \frac{(dg-ch)\log(c+dx)}{2\sqrt[3]{d}(bc-ad)(de-cf)^{2/3}} + \frac{3(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(bc-ad)(be-af)^{2/3}} - \frac{3(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(bc-ad)(de-cf)^{2/3}}$$

output
```
-3^(1/2)*(-a*h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1/3)
)*3^(1/2))/b^(1/3)/(-a*d+b*c)/(-a*f+b*e)^(2/3)+3^(1/2)*(-c*h+d*g)*arctan(1
/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^(1/2))/d^(1/3)/(-a*d+b*c
)/(-c*f+d*e)^(2/3)-1/2*(-a*h+b*g)*ln(b*x+a)/b^(1/3)/(-a*d+b*c)/(-a*f+b*e)^
(2/3)+1/2*(-c*h+d*g)*ln(d*x+c)/d^(1/3)/(-a*d+b*c)/(-c*f+d*e)^(2/3)+3/2*(-a
*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/b^(1/3)/(-a*d+b*c)/(-a*
f+b*e)^(2/3)-3/2*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/d^(
1/3)/(-a*d+b*c)/(-c*f+d*e)^(2/3)
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.21

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \frac{-2\sqrt{3}\sqrt[3]{d}(-de+cf)^{2/3}(bg-ah)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{-be+af}}}{\sqrt{3}}\right)+2\sqrt{3}}{}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(2/3)),x]
```

output
```
(-2*Sqrt[3]*d^(1/3)*(-(d*e) + c*f)^(2/3)*(b*g - a*h)*ArcTan[(1 - (2*b^(1/3
)*(e + f*x)^(1/3))/(-(b*e) + a*f)^(1/3))/Sqrt[3]] + 2*Sqrt[3]*b^(1/3)*(-(b
*e) + a*f)^(2/3)*(d*g - c*h)*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*
e) + c*f)^(1/3))/Sqrt[3]] + 2*d^(1/3)*(-(d*e) + c*f)^(2/3)*(b*g - a*h)*Log
[(-(b*e) + a*f)^(1/3) + b^(1/3)*(e + f*x)^(1/3)] - 2*b^(1/3)*(-(b*e) + a*f
)^(2/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(1/3) + d^(1/3)*(e + f*x)^(1/3)] -
d^(1/3)*(-(d*e) + c*f)^(2/3)*(b*g - a*h)*Log[(-(b*e) + a*f)^(2/3) - b^(1/3
)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3) + b^(2/3)*(e + f*x)^(2/3)] + b^(1/3
)*(-(b*e) + a*f)^(2/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d
*e) + c*f)^(1/3)*(e + f*x)^(1/3) + d^(2/3)*(e + f*x)^(2/3)])/(2*b^(1/3)*d^
(1/3)*(b*c - a*d)*(-(b*e) + a*f)^(2/3)*(-(d*e) + c*f)^(2/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {174, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 174}$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)(e+fx)^{2/3}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)(e+fx)^{2/3}}dx}{bc-ad}$$

$$\Big\downarrow \text{ 69}$$

$$\frac{(bg-ah)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{b}(be-af)^{2/3}}-\frac{\text{lo}\cdots}{2\sqrt[3]{\cdots}}\cdots\right)}{bc-ad}$$
$$\frac{(dg-ch)\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\text{lo}\cdots}{2\sqrt[3]{\cdots}}\cdots\right)}{bc-ad}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(bg-ah)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+\right.}}{2\sqrt[3]{b}(be-af)^{2/3}}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e-\right.}}{2\sqrt[3]{d}(de-cf)^{2/3}}\right.}{bc-ad}$$

$\downarrow$ 1082

$$\frac{(bg-ah)\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{bc-ad}-$$

$$\frac{(dg-ch)\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{bc-ad}$$

$\downarrow$ 217

$$\frac{(bg-ah)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{bc-ad}-\frac{(dg-ch)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{bc-ad}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(2/3)),x]
```

output
```
((b*g - a*h)*(-((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*
f)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*e - a*f)^(2/3))) - Log[a + b*x]/(2*b^(1/3)
*(b*e - a*f)^(2/3)) + (3*Log[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])
/(2*b^(1/3)*(b*e - a*f)^(2/3))))/(b*c - a*d) - ((d*g - c*h)*(-((Sqrt[3]*Ar
cTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3))/Sqrt[3]])/(d^(1/3
)*(d*e - c*f)^(2/3))) - Log[c + d*x]/(2*d^(1/3)*(d*e - c*f)^(2/3)) + (3*Lo
g[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(1/3)*(d*e - c*f)^(2/
3))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}\right)(-}{ad-bc}$
default	$\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}\right)(-}{ad-bc}$
pseudoelliptic	$\frac{-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)(ah-bg)d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}{2}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)(c}{2}}{}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*(1/3/d/((c*f-d*e)/d)^(2/3)*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))-1/6/d/(
(c*f-d*e)/d)^(2/3)*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*
f-d*e)/d)^(2/3))+1/3/d/((c*f-d*e)/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
(c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1)))*(-c*h+d*g)/(a*d-b*c)+3*(1/3/b/((a*f-
b*e)/b)^(2/3)*ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))-1/6/b/((a*f-b*e)/b)^(2
/3)*ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/3)
)+1/3/b/((a*f-b*e)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*f-b*e)/b)^(1
/3)*(f*x+e)^(1/3)-1)))*(a*h-b*g)/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1455 vs. $2(324) = 648$.

Time = 49.49 (sec) , antiderivative size = 6174, normalized size of antiderivative = 15.99

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(2/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \int \frac{g+hx}{(a+bx)\,(c+dx)\,(e+fx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(2/3),x)
```

output
```
Integral((g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)**(2/3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(2/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.67

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(2/3),x, algorithm="giac")
```

output
```
(b*g - a*h)*((b*e - a*f)/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b
)^(1/3)))/(b^2*c*e - a*b*d*e - a*b*c*f + a^2*d*f) - (d*g - c*h)*((d*e - c*
f)/d)^(1/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b*c*d*e - a
*d^2*e - b*c^2*f + a*c*d*f) - (sqrt(3)*(b^3*e - a*b^2*f)^(1/3)*b*g - sqrt(
3)*(b^3*e - a*b^2*f)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((
b*e - a*f)/b)^(1/3))/((b*e - a*f)/b)^(1/3))/((b^3*e - a*b^2*f)*c - (a*b^2*
e - a^2*b*f)*d) - (sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*d*g - sqrt(3)*(d^3*e -
c*d^2*f)^(1/3)*c*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d
)^(1/3))/((d*e - c*f)/d)^(1/3))/((d^3*e - c*d^2*f)*a - (c*d^2*e - c^2*d*f)
*b) - 1/2*((b^3*e - a*b^2*f)^(1/3)*b*g - (b^3*e - a*b^2*f)^(1/3)*a*h)*log(
(f*x + e)^(2/3) + (f*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^
(2/3))/((b^3*e - a*b^2*f)*c - (a*b^2*e - a^2*b*f)*d) - 1/2*((d^3*e - c*d^2
*f)^(1/3)*d*g - (d^3*e - c*d^2*f)^(1/3)*c*h)*log((f*x + e)^(2/3) + (f*x +
e)^(1/3)*((d*e - c*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/((d^3*e - c*d^2*f)
*a - (c*d^2*e - c^2*d*f)*b)
```

Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 13287, normalized size of antiderivative = 34.42

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(2/3)*(a + b*x)*(c + d*x)),x)
```

output
```
log((-(c^3*h^3 - d^3*g^3 + 3*c*d^2*g^2*h - 3*c^2*d*g*h^2)/(a^3*d^6*e^2 - b
^3*c^5*d*f^2 + a^3*c^2*d^4*f^2 - b^3*c^3*d^3*e^2 - 2*a^3*c*d^5*e*f - 3*a^2
*b*c*d^5*e^2 + 2*b^3*c^4*d^2*e*f + 3*a*b^2*c^2*d^4*e^2 + 3*a*b^2*c^4*d^2*f
^2 - 3*a^2*b*c^3*d^3*f^2 - 6*a*b^2*c^3*d^3*e*f + 6*a^2*b*c^2*d^4*e*f))^(1/
3)*((-(c^3*h^3 - d^3*g^3 + 3*c*d^2*g^2*h - 3*c^2*d*g*h^2)/(a^3*d^6*e^2 - b
^3*c^5*d*f^2 + a^3*c^2*d^4*f^2 - b^3*c^3*d^3*e^2 - 2*a^3*c*d^5*e*f - 3*a^2
*b*c*d^5*e^2 + 2*b^3*c^4*d^2*e*f + 3*a*b^2*c^2*d^4*e^2 + 3*a*b^2*c^4*d^2*f
^2 - 3*a^2*b*c^3*d^3*f^2 - 6*a*b^2*c^3*d^3*e*f + 6*a^2*b*c^2*d^4*e*f))^(2/
3)*((e + f*x)^(1/3)*(243*a^5*b^3*d^8*f^6*g + 243*b^8*c^5*d^3*f^6*g - 729*a
*b^7*c^4*d^4*f^6*g - 729*a^4*b^4*c*d^7*f^6*g - 243*a*b^7*c^5*d^3*f^6*h - 2
43*a^5*b^3*c*d^7*f^6*h - 486*a^4*b^4*d^8*e*f^5*g - 486*b^8*c^4*d^4*e*f^5*g
 + 486*a^2*b^6*c^3*d^5*f^6*g + 486*a^3*b^5*c^2*d^6*f^6*g + 972*a^2*b^6*c^4
*d^4*f^6*h - 1458*a^3*b^5*c^3*d^5*f^6*h + 972*a^4*b^4*c^2*d^6*f^6*h + 243*
a^4*b^4*d^8*e^2*f^4*h + 243*b^8*c^4*d^4*e^2*f^4*h + 1458*a^2*b^6*c^2*d^6*e
^2*f^4*h + 1944*a*b^7*c^3*d^5*e*f^5*g + 1944*a^3*b^5*c*d^7*e*f^5*g - 2916*
a^2*b^6*c^2*d^6*e*f^5*g - 972*a*b^7*c^3*d^5*e^2*f^4*h - 972*a^3*b^5*c*d^7*
e^2*f^4*h) + (-(c^3*h^3 - d^3*g^3 + 3*c*d^2*g^2*h - 3*c^2*d*g*h^2)/(a^3*d^
6*e^2 - b^3*c^5*d*f^2 + a^3*c^2*d^4*f^2 - b^3*c^3*d^3*e^2 - 2*a^3*c*d^5*e*
f - 3*a^2*b*c*d^5*e^2 + 2*b^3*c^4*d^2*e*f + 3*a*b^2*c^2*d^4*e^2 + 3*a*b^2*
c^4*d^2*f^2 - 3*a^2*b*c^3*d^3*f^2 - 6*a*b^2*c^3*d^3*e*f + 6*a^2*b*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.13

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(2/3),x)
```

output

```
( - 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
h + 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b*
g - 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
h + 2*d**(1/3)*(c*f - d*e)**(2/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b*
g + 2*b**(1/3)*(a*f - b*e)**(2/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) - 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*c*
h - 2*b**(1/3)*(a*f - b*e)**(2/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) - 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*d*
g + 2*b**(1/3)*(a*f - b*e)**(2/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) + 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*c*
h - 2*b**(1/3)*(a*f - b*e)**(2/3)*sqrt(3)*atan((d**(1/6)*(c*f - d*e)**(1/6
)*sqrt(3) + 2*d**(1/3)*(e + f*x)**(1/6))/(d**(1/6)*(c*f - d*e)**(1/6)))*d*
g - 2*b**(1/3)*(a*f - b*e)**(2/3)*log((c*f - d*e)**(1/3) + d**(1/3)*(e + f
*x)**(1/3))*c*h + 2*b**(1/3)*(a*f - b*e)**(2/3)*log((c*f - d*e)**(1/3) + d
**(1/3)*(e + f*x)**(1/3))*d*g + b**(1/3)*(a*f - b*e)**(2/3)*log( - d**(1/6
)*(e + f*x)**(1/6)*(c*f - d*e)**(1/6)*sqrt(3) + (c*f - d*e)**(1/3) + d*...
```

3.222 $\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 427

$$\begin{aligned}\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx &= -\frac{3(fg-eh)}{2(be-af)(de-cf)(e+fx)^{2/3}} \\ &- \frac{\sqrt{3}b^{2/3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{(bc-ad)(be-af)^{5/3}} \\ &+ \frac{\sqrt{3}d^{2/3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{(bc-ad)(de-cf)^{5/3}} - \frac{b^{2/3}(bg-ah)\log(a+bx)}{2(bc-ad)(be-af)^{5/3}} \\ &+ \frac{d^{2/3}(dg-ch)\log(c+dx)}{2(bc-ad)(de-cf)^{5/3}} + \frac{3b^{2/3}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2(bc-ad)(be-af)^{5/3}} \\ &- \frac{3d^{2/3}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2(bc-ad)(de-cf)^{5/3}}\end{aligned}$$

output
```
1/2*(3*e*h-3*f*g)/(-a*f+b*e)/(-c*f+d*e)/(f*x+e)^(2/3)-3^(1/2)*b^(2/3)*(-a*
h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^(1/3))*3^(1/2))/(-
a*d+b*c)/(-a*f+b*e)^(5/3)+3^(1/2)*d^(2/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/
3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^(1/2))/(-a*d+b*c)/(-c*f+d*e)^(5/3)-1/
2*b^(2/3)*(-a*h+b*g)*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)^(5/3)+1/2*d^(2/3)*(-c
*h+d*g)*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)^(5/3)+3/2*b^(2/3)*(-a*h+b*g)*ln((-
a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/(-a*d+b*c)/(-a*f+b*e)^(5/3)-3/2*d^(2
/3)*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3))/(-a*d+b*c)/(-c*f
+d*e)^(5/3)
```

Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.22

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \frac{1}{2}\left(\frac{3(-fg+eh)}{(be-af)(de-cf)(e+fx)^{2/3}}\right.$$
$$+\frac{2\sqrt{3}b^{2/3}(bg-ah)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{-be+af}}}{\sqrt{3}}\right)}{(bc-ad)(-be+af)^{5/3}}$$
$$-\frac{2\sqrt{3}d^{2/3}(dg-ch)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{-de+cf}}}{\sqrt{3}}\right)}{(bc-ad)(-de+cf)^{5/3}}$$
$$+\frac{2b^{2/3}(bg-ah)\log\left(\sqrt[3]{-be+af}+\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{(-bc+ad)(-be+af)^{5/3}}$$
$$+\frac{2d^{2/3}(dg-ch)\log\left(\sqrt[3]{-de+cf}+\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{(bc-ad)(-de+cf)^{5/3}}$$
$$+\frac{b^{2/3}(bg-ah)\log\left((-be+af)^{2/3}-\sqrt[3]{b}\sqrt[3]{-be+af}\sqrt[3]{e+fx}+b^{2/3}(e+fx)^{2/3}\right)}{(bc-ad)(-be+af)^{5/3}}$$
$$\left.+\frac{d^{2/3}(dg-ch)\log\left((-de+cf)^{2/3}-\sqrt[3]{d}\sqrt[3]{-de+cf}\sqrt[3]{e+fx}+d^{2/3}(e+fx)^{2/3}\right)}{(-bc+ad)(-de+cf)^{5/3}}\right)$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(5/3)),x]
```

output
```
((3*(-(f*g) + e*h))/((b*e - a*f)*(d*e - c*f)*(e + f*x)^(2/3)) + (2*Sqrt[3]
*b^(2/3)*(b*g - a*h)*ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e) + a*f
)^(1/3))/Sqrt[3]])/((b*c - a*d)*(-(b*e) + a*f)^(5/3)) - (2*Sqrt[3]*d^(2/3)
*(d*g - c*h)*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))
/Sqrt[3]])/((b*c - a*d)*(-(d*e) + c*f)^(5/3)) + (2*b^(2/3)*(b*g - a*h)*Log
[(-(b*e) + a*f)^(1/3) + b^(1/3)*(e + f*x)^(1/3)])/((-(b*c) + a*d)*(-(b*e)
+ a*f)^(5/3)) + (2*d^(2/3)*(d*g - c*h)*Log[(-(d*e) + c*f)^(1/3) + d^(1/3)*
(e + f*x)^(1/3)])/((b*c - a*d)*(-(d*e) + c*f)^(5/3)) + (b^(2/3)*(b*g - a*h
)*Log[(-(b*e) + a*f)^(2/3) - b^(1/3)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3)
+ b^(2/3)*(e + f*x)^(2/3)])/((b*c - a*d)*(-(b*e) + a*f)^(5/3)) + (d^(2/3)*
(d*g - c*h)*Log[(-(d*e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f
*x)^(1/3) + d^(2/3)*(e + f*x)^(2/3)])/((-(b*c) + a*d)*(-(d*e) + c*f)^(5/3)
))/2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {174, 61, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}} \, dx$$

$$\Big\downarrow \text{ 174}$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)(e+fx)^{5/3}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)(e+fx)^{5/3}}dx}{bc-ad}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(bg-ah)\left(\frac{b\int \frac{1}{(a+bx)(e+fx)^{2/3}}dx}{be-af}+\frac{3}{2(e+fx)^{2/3}(be-af)}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{d\int \frac{1}{(c+dx)(e+fx)^{2/3}}dx}{de-cf}+\frac{3}{2(e+fx)^{2/3}(de-cf)}\right)}{bc-ad}$$

$$\downarrow 69$$

$$\frac{(bg-ah)\left(\frac{b\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(\cdots}{2\sqrt[3]{b}(\cdots}\cdots\right)}{be-af}\cdots\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log\cdots}{2\sqrt[3]{d}(\cdots}\cdots\right)}{de-cf}\cdots\right)}{bc-ad}$$

$$\downarrow 16$$

$$\frac{(bg-ah)\left(\frac{b\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+f}\right.}{2\sqrt[3]{b}(be-af)^{2/3}}\right.}{be-af}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+f}\right.}{2\sqrt[3]{d}(de-cf)^{2/3}}\right.}{de-cf}\right.}{bc-ad}$$

$\downarrow$ 1082

$$\frac{(bg-ah)\left(\frac{b\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{be-af}+\frac{3}{2(e+fx)^{2/3}(be-\cdots}\right.}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{3\int \frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{de-cf}+\frac{3}{2(e+fx)^{2/3}(de\cdots}\right.}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{(bg-ah)\left(\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{be-af}+\frac{3}{2(e+fx)^{2/3}(be-af)}\right)}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{de-cf}+\frac{3}{2(e+fx)^{2/3}(de-cf)}\right)}{bc-ad}$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(5/3)),x]
```

output
```
((b*g - a*h)*(3/(2*(b*e - a*f)*(e + f*x)^(2/3)) + (b*(-((Sqrt[3]*ArcTan[(1
 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*f)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*e
- a*f)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*e - a*f)^(2/3)) + (3*Log[(b*e
- a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])/(2*b^(1/3)*(b*e - a*f)^(2/3))))/(
b*e - a*f)))/(b*c - a*d) - ((d*g - c*h)*(3/(2*(d*e - c*f)*(e + f*x)^(2/3))
 + (d*(-((Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3
))/Sqrt[3]])/(d^(1/3)*(d*e - c*f)^(2/3))) - Log[c + d*x]/(2*d^(1/3)*(d*e -
 c*f)^(2/3)) + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(
1/3)*(d*e - c*f)^(2/3))))/(d*e - c*f)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}\right)}{(ad-bc)(af-be)}$
default	$-\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}\right)}{(ad-bc)(af-be)}$
pseudoelliptic	$-\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)(ah-bg)}{\left(\frac{af-be}{b}\right)^{\frac{2}{3}}(af-be)(ad-bc)}+\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)(ah-bg)}{2\left(\frac{af-be}{b}\right)^{\frac{2}{3}}(af-be)(ad-bc)}-\frac{\sqrt{3}\arctan}{\left(a\right.}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-3*(1/3/b/((a*f-b*e)/b)^(2/3)*ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))-1/6/b/
((a*f-b*e)/b)^(2/3)*ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a
*f-b*e)/b)^(2/3))+1/3/b/((a*f-b*e)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)-1)))*b*(a*h-b*g)/(a*d-b*c)/(a*f-b*e)+3*(
1/3/d/((c*f-d*e)/d)^(2/3)*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))-1/6/d/((c*
f-d*e)/d)^(2/3)*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*f-d
*e)/d)^(2/3))+1/3/d/((c*f-d*e)/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((c*
f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1)))*d*(c*h-d*g)/(a*d-b*c)/(c*f-d*e)-3/2*(-e
*h+f*g)/(c*f-d*e)/(a*f-b*e)/(f*x+e)^(2/3)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \int \frac{g+hx}{(a+bx)\,(c+dx)\,(e+fx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(5/3),x)
```

output
```
Integral((g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)**(5/3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(5/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. $2(361) = 722$.

Time = 0.79 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.96

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(5/3),x, algorithm="giac")
```

output
```
(b^2*g - a*b*h)*((b*e - a*f)/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*
f)/b)^(1/3)))/(b^3*c*e^2 - a*b^2*d*e^2 - 2*a*b^2*c*e*f + 2*a^2*b*d*e*f + a
^2*b*c*f^2 - a^3*d*f^2) - (d^2*g - c*d*h)*((d*e - c*f)/d)^(1/3)*log(abs((f
*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b*c*d^2*e^2 - a*d^3*e^2 - 2*b*c^2
*d*e*f + 2*a*c*d^2*e*f + b*c^3*f^2 - a*c^2*d*f^2) - (sqrt(3)*(b^3*e - a*b^
2*f)^(1/3)*b*g - sqrt(3)*(b^3*e - a*b^2*f)^(1/3)*a*h)*arctan(1/3*sqrt(3)*(
2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e - a*f)/b)^(1/3))/((b^3*e^
2 - 2*a*b^2*e*f + a^2*b*f^2)*c - (a*b^2*e^2 - 2*a^2*b*e*f + a^3*f^2)*d) -
(sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*d*g - sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*c*h
)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3))/((d*e - c
*f)/d)^(1/3))/((d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*a - (c*d^2*e^2 - 2*c^2*
d*e*f + c^3*f^2)*b) - 1/2*((b^3*e - a*b^2*f)^(1/3)*b*g - (b^3*e - a*b^2*f)
^(1/3)*a*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) +
((b*e - a*f)/b)^(2/3))/((b^3*e^2 - 2*a*b^2*e*f + a^2*b*f^2)*c - (a*b^2*e^2
 - 2*a^2*b*e*f + a^3*f^2)*d) - 1/2*((d^3*e - c*d^2*f)^(1/3)*d*g - (d^3*e -
 c*d^2*f)^(1/3)*c*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((d*e - c*f)/d)
^(1/3) + ((d*e - c*f)/d)^(2/3))/((d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*a - (
c*d^2*e^2 - 2*c^2*d*e*f + c^3*f^2)*b) - 3/2*(f*g - e*h)/((b*d*e^2 - b*c*e*
f - a*d*e*f + a*c*f^2)*(f*x + e)^(2/3))
```

Mupad [B] (verification not implemented)

Time = 18.76 (sec) , antiderivative size = 413671, normalized size of antiderivative = 968.78

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \text{Too large to display}$$

input
```
int((g + h*x)/((e + f*x)^(5/3)*(a + b*x)*(c + d*x)),x)
```

output
```
log((e + f*x)^(1/3)*(486*b^14*d^14*e^14*f^4*g^4 - 3402*a*b^13*d^14*e^13*f^
5*g^4 - 3402*b^14*c*d^13*e^13*f^5*g^4 + 243*a^6*b^8*c^8*d^6*f^18*g^4 + 243
*a^8*b^6*c^6*d^8*f^18*g^4 + 486*a^9*b^5*c^9*d^5*f^18*h^4 + 10449*a^2*b^12*
d^14*e^12*f^6*g^4 - 18468*a^3*b^11*d^14*e^11*f^7*g^4 + 20655*a^4*b^10*d^14
*e^10*f^8*g^4 - 15066*a^5*b^9*d^14*e^9*f^9*g^4 + 7047*a^6*b^8*d^14*e^8*f^1
0*g^4 - 1944*a^7*b^7*d^14*e^7*f^11*g^4 + 243*a^8*b^6*d^14*e^6*f^12*g^4 + 1
0449*b^14*c^2*d^12*e^12*f^6*g^4 - 18468*b^14*c^3*d^11*e^11*f^7*g^4 + 20655
*b^14*c^4*d^10*e^10*f^8*g^4 - 15066*b^14*c^5*d^9*e^9*f^9*g^4 + 7047*b^14*c
^6*d^8*e^8*f^10*g^4 - 1944*b^14*c^7*d^7*e^7*f^11*g^4 + 243*b^14*c^8*d^6*e^
6*f^12*g^4 - 69984*a*b^13*c^2*d^12*e^11*f^7*g^4 + 120528*a*b^13*c^3*d^11*e
^10*f^8*g^4 - 131220*a*b^13*c^4*d^10*e^9*f^9*g^4 + 93312*a*b^13*c^5*d^9*e^
8*f^10*g^4 - 42768*a*b^13*c^6*d^8*e^7*f^11*g^4 + 11664*a*b^13*c^7*d^7*e^6*
f^12*g^4 - 1458*a*b^13*c^8*d^6*e^5*f^13*g^4 - 69984*a^2*b^12*c*d^13*e^11*f
^7*g^4 + 120528*a^3*b^11*c*d^13*e^10*f^8*g^4 - 131220*a^4*b^10*c*d^13*e^9*
f^9*g^4 + 93312*a^5*b^9*c*d^13*e^8*f^10*g^4 - 1458*a^5*b^9*c^8*d^6*e*f^17*
g^4 - 42768*a^6*b^8*c*d^13*e^7*f^11*g^4 - 1944*a^6*b^8*c^7*d^7*e*f^17*g^4
+ 11664*a^7*b^7*c*d^13*e^6*f^12*g^4 - 1944*a^7*b^7*c^6*d^8*e*f^17*g^4 - 14
58*a^8*b^6*c*d^13*e^5*f^13*g^4 - 1458*a^8*b^6*c^5*d^9*e*f^17*g^4 + 243*a*b
^13*c^3*d^11*e^14*f^4*h^4 - 1458*a*b^13*c^4*d^10*e^13*f^5*h^4 + 3645*a*b^1
3*c^5*d^9*e^12*f^6*h^4 - 4860*a*b^13*c^6*d^8*e^11*f^7*h^4 + 3645*a*b^13...
```

Reduce [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 4618, normalized size of antiderivative = 10.81

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{5/3}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(5/3),x)
```

output

```
(2*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*
f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*
e)**(1/6)))*a*b*c**2*f**2*h - 4*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/
3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x
)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*c*d*e*f*h + 2*d**(1/3)*(e + f
*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sq
rt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*d*
*2*e**2*h - 2*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b
**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6
)*(a*f - b*e)**(1/6)))*b**2*c**2*f**2*g + 4*d**(1/3)*(e + f*x)**(2/3)*(a*f
 - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1
/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*b**2*c*d*e*f*g - 2*d*
*(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b
*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(
1/6)))*b**2*d**2*e**2*g + 2*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*s
qrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) + 2*b**(1/3)*(e + f*x)**(
1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*c**2*f**2*h - 4*d**(1/3)*(e + f*x
)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt
(3) + 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b*c*d*
e*f*h + 2*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b*...
```

3.223 $\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 495

$$\begin{aligned}\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx &= -\frac{3(fg-eh)}{5(be-af)(de-cf)(e+fx)^{5/3}} \\
&+ \frac{3(bcf^2g+af^2(dg-ch)-bde(2fg-eh))}{2(be-af)^2(de-cf)^2(e+fx)^{2/3}} \\
&- \frac{\sqrt{3}b^{5/3}(bg-ah)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{(bc-ad)(be-af)^{8/3}} \\
&+ \frac{\sqrt{3}d^{5/3}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{(bc-ad)(de-cf)^{8/3}} - \frac{b^{5/3}(bg-ah)\log(a+bx)}{2(bc-ad)(be-af)^{8/3}} \\
&+ \frac{d^{5/3}(dg-ch)\log(c+dx)}{2(bc-ad)(de-cf)^{8/3}} + \frac{3b^{5/3}(bg-ah)\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2(bc-ad)(be-af)^{8/3}} \\
&- \frac{3d^{5/3}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2(bc-ad)(de-cf)^{8/3}}\end{aligned}$$

output
```
1/5*(3*e*h-3*f*g)/(-a*f+b*e)/(-c*f+d*e)/(f*x+e)^(5/3)+3/2*(b*c*f^2*g+a*f^2
*(-c*h+d*g)-b*d*e*(-e*h+2*f*g))/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)^(2/3)-3^
(1/2)*b^(5/3)*(-a*h+b*g)*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/3)/(-a*f+b*e)^
(1/3))*3^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(8/3)+3^(1/2)*d^(5/3)*(-c*h+d*g)*arc
tan(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3)/(-c*f+d*e)^(1/3))*3^(1/2))/(-a*d+b*c)/(
-c*f+d*e)^(8/3)-1/2*b^(5/3)*(-a*h+b*g)*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)^(8/
3)+1/2*d^(5/3)*(-c*h+d*g)*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)^(8/3)+3/2*b^(5/3
)*(-a*h+b*g)*ln((-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/(-a*d+b*c)/(-a*f+b
*e)^(8/3)-3/2*d^(5/3)*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1/3)
)/(-a*d+b*c)/(-c*f+d*e)^(8/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx = \frac{3\left(\frac{(bg-ah)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},1,-\frac{2}{3},\frac{b(e+fx)}{be-af}\right)}{be-af}+\frac{(-dg+ch)\,\text{Hypergeometric2F1}\left(-\frac{5}{3}\right.}{de-cf}\right.}{5(bc-ad)(e+fx)^{5/3}}$$

input
```
Integrate[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(8/3)),x]
```

output
```
(3*(((b*g - a*h)*Hypergeometric2F1[-5/3, 1, -2/3, (b*(e + f*x))/(b*e - a*f
)])/(b*e - a*f) + ((-(d*g) + c*h)*Hypergeometric2F1[-5/3, 1, -2/3, (d*(e +
 f*x))/(d*e - c*f)])/(d*e - c*f)))/(5*(b*c - a*d)*(e + f*x)^(5/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {174, 61, 61, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx$$

$$\Big\downarrow \; 174$$

$$\frac{(bg-ah)\int \frac{1}{(a+bx)(e+fx)^{8/3}}dx}{bc-ad}-\frac{(dg-ch)\int \frac{1}{(c+dx)(e+fx)^{8/3}}dx}{bc-ad}$$

$$\Big\downarrow \; 61$$

$$\frac{(bg-ah)\left(\frac{b\int \frac{1}{(a+bx)(e+fx)^{5/3}}dx}{be-af}+\frac{3}{5(e+fx)^{5/3}(be-af)}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{d\int \frac{1}{(c+dx)(e+fx)^{5/3}}dx}{de-cf}+\frac{3}{5(e+fx)^{5/3}(de-cf)}\right)}{bc-ad}$$

$$\Big\downarrow \; 61$$

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\int \frac{1}{(a+bx)(e+fx)^{2/3}}dx}{be-af}+\frac{3}{2(e+fx)^{2/3}(be-af)}\right)}{be-af}+\frac{3}{5(e+fx)^{5/3}(be-af)}\right)}{bc-ad}-\frac{(dg-ch)\left(\frac{d\left(\frac{d\int \frac{1}{(c+dx)(e+fx)^{2/3}}dx}{de-cf}+\frac{3}{2(e+fx)^{2/3}(de-cf)}\right)}{de-cf}+\frac{3}{5(e+fx)^{5/3}(de-cf)}\right)}{bc-ad}$$

$$\Big\downarrow \; 69$$

$$(bg-ah)\left(\frac{b\left(\frac{b\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{b}(be-af)^{2/3}}-\frac{\cdots}{2\sqrt[3]{\cdots}}\cdots\right)}{be-af}\cdots\right)}{be-af}\cdots\right)}{bc-ad}\cdots$$

$$(dg-ch)\left(\frac{d\left(\frac{d\left(-\frac{3\int \frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{3\int \frac{1}{\frac{\sqrt[3]{de-cf}}{\sqrt[3]{d}}-\sqrt[3]{e+fx}}d\sqrt[3]{e+fx}}{2\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\cdots}{2\sqrt[3]{\cdots}}\cdots\right)}{de-cf}\cdots\right)}{de-cf}\cdots\right)}{bc-ad}\cdots$$

↓ 16

$$(bg-ah)\left(\frac{b\left(\frac{b\left(-\frac{3\int\frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+\right.}{2\sqrt[3]{b}(be-af)^{2/3}}}{be-af}}{be-af}}{bc-ad}$$

$$(dg-ch)\left(\frac{d\left(\frac{d\left(-\frac{3\int\frac{1}{\frac{(de-cf)^{2/3}}{d^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{de-cf}}{\sqrt[3]{d}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2d^{2/3}\sqrt[3]{de-cf}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+\right.}{2\sqrt[3]{d}(de-cf)^{2/3}}}{de-cf}}{de-cf}}{bc-ad}$$

$\Big\downarrow$ 1082

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\left(\frac{3\int\frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{be-af}+\frac{3}{2(e+fx)^{2/3}(be-a}}{be-af}}{bc-ad}$$

$$\frac{(dg-ch)\left(\frac{d\left(\frac{d\left(\frac{3\int\frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{de-cf}+\frac{3}{2(e+fx)^{2/3}(de-}}{de-cf}}{bc-ad}$$

$\Big\downarrow$ 217

$$\frac{(bg-ah)\left(\frac{b\left(\frac{b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)}{be-af}+\frac{3}{2(e+fx)^{2/3}(be-af)}\right)}{be-af}\right)}{bc-ad}+$$

$$(dg-ch)\left(\frac{d\left(\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}(de-cf)^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{d}(de-cf)^{2/3}}+\frac{3\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(de-cf)^{2/3}}\right)}{de-cf}+\frac{3}{2(e+fx)^{2/3}(de-cf)}\right)}{de-cf}\right)+$$

input
```
Int[(g + h*x)/((a + b*x)*(c + d*x)*(e + f*x)^(8/3)),x]
```

output
```
((b*g - a*h)*(3/(5*(b*e - a*f)*(e + f*x)^(5/3)) + (b*(3/(2*(b*e - a*f)*(e
+ f*x)^(2/3)) + (b*(-((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*
e - a*f)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*e - a*f)^(2/3))) - Log[a + b*x]/(2*b
^(1/3)*(b*e - a*f)^(2/3)) + (3*Log[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^(
1/3)])/(2*b^(1/3)*(b*e - a*f)^(2/3))))/(b*e - a*f)))/(b*e - a*f)))/(b*c -
a*d) - ((d*g - c*h)*(3/(5*(d*e - c*f)*(e + f*x)^(5/3)) + (d*(3/(2*(d*e - c
*f)*(e + f*x)^(2/3)) + (d*(-((Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(e + f*x)^(1/
3))/(d*e - c*f)^(1/3))/Sqrt[3]])/(d^(1/3)*(d*e - c*f)^(2/3))) - Log[c + d*
x]/(2*d^(1/3)*(d*e - c*f)^(2/3)) + (3*Log[(d*e - c*f)^(1/3) - d^(1/3)*(e +
 f*x)^(1/3)])/(2*d^(1/3)*(d*e - c*f)^(2/3))))/(d*e - c*f)))/(d*e - c*f)))/
(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}\right)(a}{(ad-bc)(af-be)^2}$
default	$\frac{3\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{af-be}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{af-be}{b}\right)^{\frac{2}{3}}}\right)(a}{(ad-bc)(af-be)^2}$
pseudoelliptic	$\frac{b\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{1}{3}}\right)(ah-bg)}{\left(\frac{af-be}{b}\right)^{\frac{2}{3}}(ad-bc)(af-be)^2}-\frac{b\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{af-be}{b}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{af-be}{b}\right)^{\frac{2}{3}}\right)(ah-bg)}{2\left(\frac{af-be}{b}\right)^{\frac{2}{3}}(ad-bc)(af-be)^2}+\frac{b\sqrt{3}\,\mathrm{arcta}}{\left(a\right.}$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(8/3),x,method=_RETURNVERBOSE)
```

output
```
3*(1/3/b/((a*f-b*e)/b)^(2/3)*ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))-1/6/b/(
(a*f-b*e)/b)^(2/3)*ln((f*x+e)^(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*
f-b*e)/b)^(2/3))+1/3/b/((a*f-b*e)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
(a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)-1)))*(a*h-b*g)*b^2/(a*d-b*c)/(a*f-b*e)^2-
3/5*(-e*h+f*g)/(c*f-d*e)/(a*f-b*e)/(f*x+e)^(5/3)-3/2*(a*c*f^2*h-a*d*f^2*g-
b*c*f^2*g-b*d*e^2*h+2*b*d*e*f*g)/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)^(2/3)-3*(
1/3/d/((c*f-d*e)/d)^(2/3)*ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))-1/6/d/((c*
f-d*e)/d)^(2/3)*ln((f*x+e)^(2/3)-((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)+((c*f-d
*e)/d)^(2/3))+1/3/d/((c*f-d*e)/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((c*
f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1)))*(c*h-d*g)*d^2/(a*d-b*c)/(c*f-d*e)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(8/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)**(8/3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(8/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(425) = 850$.

Time = 3.49 (sec) , antiderivative size = 1239, normalized size of antiderivative = 2.50

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(8/3),x, algorithm="giac")
```

output
```
(b^3*g - a*b^2*h)*((b*e - a*f)/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e -
a*f)/b)^(1/3)))/(b^4*c*e^3 - a*b^3*d*e^3 - 3*a*b^3*c*e^2*f + 3*a^2*b^2*d*e
^2*f + 3*a^2*b^2*c*e*f^2 - 3*a^3*b*d*e*f^2 - a^3*b*c*f^3 + a^4*d*f^3) - (d
^3*g - c*d^2*h)*((d*e - c*f)/d)^(1/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*
f)/d)^(1/3)))/(b*c*d^3*e^3 - a*d^4*e^3 - 3*b*c^2*d^2*e^2*f + 3*a*c*d^3*e^2
*f + 3*b*c^3*d*e*f^2 - 3*a*c^2*d^2*e*f^2 - b*c^4*f^3 + a*c^3*d*f^3) - (sqr
t(3)*(b^3*e - a*b^2*f)^(1/3)*b^2*g - sqrt(3)*(b^3*e - a*b^2*f)^(1/3)*a*b*h
)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e - a
*f)/b)^(1/3))/((b^4*e^3 - 3*a*b^3*e^2*f + 3*a^2*b^2*e*f^2 - a^3*b*f^3)*c -
 (a*b^3*e^3 - 3*a^2*b^2*e^2*f + 3*a^3*b*e*f^2 - a^4*f^3)*d) - (sqrt(3)*(d^
3*e - c*d^2*f)^(1/3)*d^2*g - sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*c*d*h)*arctan
(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3))/((d*e - c*f)/d)^(
1/3))/((d^4*e^3 - 3*c*d^3*e^2*f + 3*c^2*d^2*e*f^2 - c^3*d*f^3)*a - (c*d^3*
e^3 - 3*c^2*d^2*e^2*f + 3*c^3*d*e*f^2 - c^4*f^3)*b) - 1/2*((b^3*e - a*b^2*
f)^(1/3)*b^2*g - (b^3*e - a*b^2*f)^(1/3)*a*b*h)*log((f*x + e)^(2/3) + (f*x
 + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/((b^4*e^3 - 3*a
*b^3*e^2*f + 3*a^2*b^2*e*f^2 - a^3*b*f^3)*c - (a*b^3*e^3 - 3*a^2*b^2*e^2*f
 + 3*a^3*b*e*f^2 - a^4*f^3)*d) - 1/2*((d^3*e - c*d^2*f)^(1/3)*d^2*g - (d^3
*e - c*d^2*f)^(1/3)*c*d*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((d*e - c
*f)/d)^(1/3) + ((d*e - c*f)/d)^(2/3))/((d^4*e^3 - 3*c*d^3*e^2*f + 3*c^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx = \text{Hanged}$$

input
```
int((g + h*x)/((e + f*x)^(8/3)*(a + b*x)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 13315, normalized size of antiderivative = 26.90

$$\int \frac{g+hx}{(a+bx)(c+dx)(e+fx)^{8/3}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)/(b*x+a)/(d*x+c)/(f*x+e)^(8/3),x)
```

output
```
( - 10*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)
*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f
- b*e)**(1/6)))*a*b**2*c**3*e*f**3*h - 10*d**(1/3)*(e + f*x)**(2/3)*(a*f -
 b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3
)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*c**3*f**4*h*x +
30*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*
f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*
e)**(1/6)))*a*b**2*c**2*d*e**2*f**2*h + 30*d**(1/3)*(e + f*x)**(2/3)*(a*f
- b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/
3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*c**2*d*e*f**3*h
*x - 30*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6
)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f
 - b*e)**(1/6)))*a*b**2*c*d**2*e**3*f*h - 30*d**(1/3)*(e + f*x)**(2/3)*(a*
f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(
1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*c*d**2*e**2*f
**2*h*x + 10*d**(1/3)*(e + f*x)**(2/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b*
*(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)
*(a*f - b*e)**(1/6)))*a*b**2*d**3*e**4*h + 10*d**(1/3)*(e + f*x)**(2/3)*(a
*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**
(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*d**3*e**3...
```

3.224 $\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 528

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = -\frac{(bg-ah)\sqrt[3]{e+fx}}{b(bc-ad)(a+bx)}$$
$$-\frac{(a^2dfh+2abf(dg-2ch)-b^2(3deg-cfg-3ceh))\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}(bc-ad)^2(be-af)^{2/3}}$$
$$-\frac{\sqrt{3}\sqrt[3]{de-cf}(dg-ch)\arctan\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{e+fx}}{\sqrt[3]{de-cf}}}{\sqrt{3}}\right)}{\sqrt[3]{d}(bc-ad)^2}$$
$$-\frac{(a^2dfh+2abf(dg-2ch)-b^2(3deg-cfg-3ceh))\log(a+bx)}{6b^{4/3}(bc-ad)^2(be-af)^{2/3}}$$
$$-\frac{\sqrt[3]{de-cf}(dg-ch)\log(c+dx)}{2\sqrt[3]{d}(bc-ad)^2}$$
$$+\frac{(a^2dfh+2abf(dg-2ch)-b^2(3deg-cfg-3ceh))\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2b^{4/3}(bc-ad)^2(be-af)^{2/3}}$$
$$+\frac{3\sqrt[3]{de-cf}(dg-ch)\log\left(\sqrt[3]{de-cf}-\sqrt[3]{d}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{d}(bc-ad)^2}$$

output
```
-(-a*h+b*g)*(f*x+e)^(1/3)/b/(-a*d+b*c)/(b*x+a)-1/3*(a^2*d*f*h+2*a*b*f*(-2*
c*h+d*g)-b^2*(-3*c*e*h-c*f*g+3*d*e*g))*arctan(1/3*(1+2*b^(1/3)*(f*x+e)^(1/
3)/(-a*f+b*e)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/(-a*d+b*c)^2/(-a*f+b*e)^(2/3
)-3^(1/2)*(-c*f+d*e)^(1/3)*(-c*h+d*g)*arctan(1/3*(1+2*d^(1/3)*(f*x+e)^(1/3
)/(-c*f+d*e)^(1/3))*3^(1/2))/d^(1/3)/(-a*d+b*c)^2-1/6*(a^2*d*f*h+2*a*b*f*(
-2*c*h+d*g)-b^2*(-3*c*e*h-c*f*g+3*d*e*g))*ln(b*x+a)/b^(4/3)/(-a*d+b*c)^2/(
-a*f+b*e)^(2/3)-1/2*(-c*f+d*e)^(1/3)*(-c*h+d*g)*ln(d*x+c)/d^(1/3)/(-a*d+b*
c)^2+1/2*(a^2*d*f*h+2*a*b*f*(-2*c*h+d*g)-b^2*(-3*c*e*h-c*f*g+3*d*e*g))*ln(
(-a*f+b*e)^(1/3)-b^(1/3)*(f*x+e)^(1/3))/b^(4/3)/(-a*d+b*c)^2/(-a*f+b*e)^(2
/3)+3/2*(-c*f+d*e)^(1/3)*(-c*h+d*g)*ln((-c*f+d*e)^(1/3)-d^(1/3)*(f*x+e)^(1
/3))/d^(1/3)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx =$$

$$-\frac{\frac{6(bc-ad)(bg-ah)\sqrt[3]{e+fx}}{b(a+bx)}+\frac{2\sqrt{3}\left(a^2dfh+2abf(dg-2ch)+b^2(-3deg+cfg+3ceh)\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{-be+af}}}{\sqrt{3}}\right)}{b^{4/3}(-be+af)^{2/3}}-\frac{6\sqrt{3}\sqrt[3]{-d}}{}}{}$$

input
```
Integrate[((e + f*x)^(1/3)*(g + h*x))/((a + b*x)^2*(c + d*x)),x]
```

output
```
-1/6*((6*(b*c - a*d)*(b*g - a*h)*(e + f*x)^(1/3))/(b*(a + b*x)) + (2*Sqrt[
3]*(a^2*d*f*h + 2*a*b*f*(d*g - 2*c*h) + b^2*(-3*d*e*g + c*f*g + 3*c*e*h))*
ArcTan[(1 - (2*b^(1/3)*(e + f*x)^(1/3))/(-(b*e) + a*f)^(1/3))/Sqrt[3]])/(b
^(4/3)*(-(b*e) + a*f)^(2/3)) - (6*Sqrt[3]*(-(d*e) + c*f)^(1/3)*(d*g - c*h)
*ArcTan[(1 - (2*d^(1/3)*(e + f*x)^(1/3))/(-(d*e) + c*f)^(1/3))/Sqrt[3]])/d
^(1/3) - (2*(a^2*d*f*h + 2*a*b*f*(d*g - 2*c*h) + b^2*(-3*d*e*g + c*f*g + 3
*c*e*h))*Log[(-(b*e) + a*f)^(1/3) + b^(1/3)*(e + f*x)^(1/3)])/(b^(4/3)*(-(
b*e) + a*f)^(2/3)) + (6*(-(d*e) + c*f)^(1/3)*(d*g - c*h)*Log[(-(d*e) + c*f
)^(1/3) + d^(1/3)*(e + f*x)^(1/3)])/d^(1/3) + ((a^2*d*f*h + 2*a*b*f*(d*g -
 2*c*h) + b^2*(-3*d*e*g + c*f*g + 3*c*e*h))*Log[(-(b*e) + a*f)^(2/3) - b^(
1/3)*(-(b*e) + a*f)^(1/3)*(e + f*x)^(1/3) + b^(2/3)*(e + f*x)^(2/3)])/(b^(
4/3)*(-(b*e) + a*f)^(2/3)) - (3*(-(d*e) + c*f)^(1/3)*(d*g - c*h)*Log[(-(d*
e) + c*f)^(2/3) - d^(1/3)*(-(d*e) + c*f)^(1/3)*(e + f*x)^(1/3) + d^(2/3)*(
e + f*x)^(2/3)])/d^(1/3))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {166, 27, 174, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}dx$$

$$\Big\downarrow \; 166$$

$$\frac{\int -\frac{acfh+b(3deg-cfg-3ceh)+f(2bdg-3bch+adh)x}{3(a+bx)(c+dx)(e+fx)^{2/3}}dx}{b(bc-ad)}-\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{3bdeg+acfh-bc(fg+3eh)+f(2bdg-3bch+adh)x}{(a+bx)(c+dx)(e+fx)^{2/3}}dx}{3b(bc-ad)}-\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 174$$

$$-\frac{-\frac{\left(a^2dfh+2abf(dg-2ch)-b^2(-3ceh-cfg+3deg)\right)\int \frac{1}{(a+bx)(e+fx)^{2/3}}dx}{bc-ad}-\frac{3b(de-cf)(dg-ch)\int \frac{1}{(c+dx)(e+fx)^{2/3}}dx}{bc-ad}}{3b(bc-ad)}-\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 69$$

$$-\frac{-\frac{\left(a^2dfh+2abf(dg-2ch)-b^2(-3ceh-cfg+3deg)\right)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{3\int \frac{\frac{\sqrt[3]{be-af}}{\sqrt[3]{b}}\cdots}{\cdots}}{2\sqrt[3]{\cdots}}\cdots\right.}{bc-ad}\cdots}{\cdots}-$$

$$\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 16$$

$$-\frac{-\frac{\left(a^2dfh+2abf(dg-2ch)-b^2(-3ceh-cfg+3deg)\right)\left(-\frac{3\int \frac{1}{\frac{(be-af)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{e+fx}\sqrt[3]{be-af}}{\sqrt[3]{b}}+(e+fx)^{2/3}}d\sqrt[3]{e+fx}}{2b^{2/3}\sqrt[3]{be-af}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}\cdots\right.}{bc-ad}\cdots}{\cdots}-$$

$$\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$\big\downarrow$ 1082

$$-\frac{-\frac{(a^2dfh+2abf(dg-2ch)-b^2(-3ceh-cfg+3deg))\left(\frac{3\int\frac{1}{-(e+fx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\ldots\right.}{2\sqrt[3]{b}(be-af)^{2\ldots}}\right.}{bc-ad}}{3b(b\ldots}$$

$$\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

$\big\downarrow$ 217

$$-\frac{-\frac{\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{e+fx}}{\sqrt[3]{be-af}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(be-af)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(be-af)^{2/3}}+\frac{3\log\left(\sqrt[3]{be-af}-\sqrt[3]{b}\sqrt[3]{e+fx}\right)}{2\sqrt[3]{b}(be-af)^{2/3}}\right)(a^2dfh+2abf(dg-2ch)-b^2(-3ceh-cfg\ldots}{bc-ad}}{3b(bc-a\ldots}$$

$$\frac{\sqrt[3]{e+fx}(bg-ah)}{b(a+bx)(bc-ad)}$$

input

```
Int[((e + f*x)^(1/3)*(g + h*x))/((a + b*x)^2*(c + d*x)),x]
```

output

```
-(((b*g - a*h)*(e + f*x)^(1/3))/(b*(b*c - a*d)*(a + b*x))) - (-(((a^2*d*f*
h + 2*a*b*f*(d*g - 2*c*h) - b^2*(3*d*e*g - c*f*g - 3*c*e*h))*(-((Sqrt[3]*A
rcTan[(1 + (2*b^(1/3)*(e + f*x)^(1/3))/(b*e - a*f)^(1/3))/Sqrt[3]])/(b^(1/
3)*(b*e - a*f)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*e - a*f)^(2/3)) + (3*L
og[(b*e - a*f)^(1/3) - b^(1/3)*(e + f*x)^(1/3)])/(2*b^(1/3)*(b*e - a*f)^(2
/3))))/(b*c - a*d)) - (3*b*(d*e - c*f)*(d*g - c*h)*(-((Sqrt[3]*ArcTan[(1 +
 (2*d^(1/3)*(e + f*x)^(1/3))/(d*e - c*f)^(1/3))/Sqrt[3]])/(d^(1/3)*(d*e -
c*f)^(2/3))) - Log[c + d*x]/(2*d^(1/3)*(d*e - c*f)^(2/3)) + (3*Log[(d*e -
c*f)^(1/3) - d^(1/3)*(e + f*x)^(1/3)])/(2*d^(1/3)*(d*e - c*f)^(2/3))))/(b*
c - a*d))/(3*b*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-3\left((fx+e)^{\frac{1}{3}}\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}d(ah-bg)(ad-bc)-\left(\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)\sqrt{3}+\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(\right.}{}\right.\right.}{}$
derivativedivides	$3f\left(\frac{\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}\right.}{f(ad-bc)^{2}}\right.$
default	$3f\left(\frac{\left(\frac{\ln\left((fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((fx+e)^{\frac{2}{3}}-\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}(fx+e)^{\frac{1}{3}}+\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(fx+e)^{\frac{1}{3}}}{\left(\frac{cf-de}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{cf-de}{d}\right)^{\frac{2}{3}}}\right.}{f(ad-bc)^{2}}\right.$

input
```
int((f*x+e)^(1/3)*(h*x+g)/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((c*f-d*e)/d)^(2/3)*(-3*((f*x+e)^(1/3)*((c*f-d*e)/d)^(2/3)*d*(a*h-b*g)
*(a*d-b*c)-(arctan(1/3*3^(1/2)*(2/((c*f-d*e)/d)^(1/3)*(f*x+e)^(1/3)-1))*3^
(1/2)+ln((f*x+e)^(1/3)+((c*f-d*e)/d)^(1/3))-1/2*ln((f*x+e)^(2/3)-((c*f-d*e
)/d)^(1/3)*(f*x+e)^(1/3)+((c*f-d*e)/d)^(2/3)))*(c*f-d*e)*(b*x+a)*(c*h-d*g)
*b)*b*((a*f-b*e)/b)^(2/3)+(arctan(1/3*3^(1/2)*(2/((a*f-b*e)/b)^(1/3)*(f*x+
e)^(1/3)-1))*3^(1/2)+ln((f*x+e)^(1/3)+((a*f-b*e)/b)^(1/3))-1/2*ln((f*x+e)^
(2/3)-((a*f-b*e)/b)^(1/3)*(f*x+e)^(1/3)+((a*f-b*e)/b)^(2/3)))*((a^2*f*h+2*
a*b*f*g-3*b^2*e*g)*d-4*(1/4*(-3*e*h-f*g)*b+a*f*h)*c*b)*d*(b*x+a)*((c*f-d*e
)/d)^(2/3))/((a*f-b*e)/b)^(2/3)/d/(a*d-b*c)^2/b^2/(b*x+a)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)^(1/3)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx=\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**(1/3)*(h*x+g)/(b*x+a)**2/(d*x+c),x)
```

output
```
Integral((e + f*x)**(1/3)*(g + h*x)/((a + b*x)**2*(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)^(1/3)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^(1/3)*(h*x+g)/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
-1/3*(3*b^2*d*e*g - b^2*c*f*g - 2*a*b*d*f*g - 3*b^2*c*e*h + 4*a*b*c*f*h -
a^2*d*f*h)*((b*e - a*f)/b)^(1/3)*log(abs((f*x + e)^(1/3) - ((b*e - a*f)/b)
^(1/3)))/(b^4*c^2*e - 2*a*b^3*c*d*e + a^2*b^2*d^2*e - a*b^3*c^2*f + 2*a^2*
b^2*c*d*f - a^3*b*d^2*f) + (d^2*e*g - c*d*f*g - c*d*e*h + c^2*f*h)*((d*e -
 c*f)/d)^(1/3)*log(abs((f*x + e)^(1/3) - ((d*e - c*f)/d)^(1/3)))/(b^2*c^2*
d*e - 2*a*b*c*d^2*e + a^2*d^3*e - b^2*c^3*f + 2*a*b*c^2*d*f - a^2*c*d^2*f)
 - (b^2*c*f*g + a^2*d*f*h - (3*b^2*e - 2*a*b*f)*d*g + (3*b^2*e - 4*a*b*f)*
c*h)*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((b*e - a*f)/b)^(1/3))/((b*e
- a*f)/b)^(1/3))/(sqrt(3)*(b^3*e - a*b^2*f)^(2/3)*b^2*c^2 - 2*sqrt(3)*(b^3
*e - a*b^2*f)^(2/3)*a*b*c*d + sqrt(3)*(b^3*e - a*b^2*f)^(2/3)*a^2*d^2) - (
sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*d*g - sqrt(3)*(d^3*e - c*d^2*f)^(1/3)*c*h)
*arctan(1/3*sqrt(3)*(2*(f*x + e)^(1/3) + ((d*e - c*f)/d)^(1/3))/((d*e - c*
f)/d)^(1/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/6*(b^2*c*f*g + a^2*d*
f*h - (3*b^2*e - 2*a*b*f)*d*g + (3*b^2*e - 4*a*b*f)*c*h)*log((f*x + e)^(2/
3) + (f*x + e)^(1/3)*((b*e - a*f)/b)^(1/3) + ((b*e - a*f)/b)^(2/3))/((b^3*
e - a*b^2*f)^(2/3)*b^2*c^2 - 2*(b^3*e - a*b^2*f)^(2/3)*a*b*c*d + (b^3*e -
a*b^2*f)^(2/3)*a^2*d^2) - 1/2*((d^3*e - c*d^2*f)^(1/3)*d*g - (d^3*e - c*d^
2*f)^(1/3)*c*h)*log((f*x + e)^(2/3) + (f*x + e)^(1/3)*((d*e - c*f)/d)^(1/3
) + ((d*e - c*f)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - ((f*x + e
)^(1/3)*b*f*g - (f*x + e)^(1/3)*a*f*h)/((b^2*c - a*b*d)*((f*x + e)*b - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/3)*(g + h*x))/((a + b*x)^2*(c + d*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 6541, normalized size of antiderivative = 12.39

$$\int \frac{\sqrt[3]{e+fx}(g+hx)}{(a+bx)^2(c+dx)} \, dx = \text{Too large to display}$$

input
```
int((f*x+e)^(1/3)*(h*x+g)/(b*x+a)^2/(d*x+c),x)
```

output
```
( - 2*d**(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
*3*d*f*h + 8*d**(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e
)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/
6)))*a**2*b*c*f*h - 4*d**(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(
a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f -
b*e)**(1/6)))*a**2*b*d*f*g - 2*d**(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b
**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6
)*(a*f - b*e)**(1/6)))*a**2*b*d*f*h*x - 6*d**(1/3)*(a*f - b*e)**(1/3)*sqrt
(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6
))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*c*e*h - 2*d**(1/3)*(a*f - b*e)**(
1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/3)*(e + f
*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*c*f*g + 8*d**(1/3)*(a*f
- b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3) - 2*b**(1/
3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*c*f*h*x + 6*d**
(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6)*sqrt(3)
 - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*b**2*d*e*
g - 4*d**(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*f - b*e)**(1/6
)*sqrt(3) - 2*b**(1/3)*(e + f*x)**(1/6))/(b**(1/6)*(a*f - b*e)**(1/6)))*a*
b**2*d*f*g*x - 6*d**(1/3)*(a*f - b*e)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*...
```

3.225 $\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right) dx = \frac{d(ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{ae(1+m)} + \frac{c(ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{a}\right)}{ae(1+m)}$$

output
```
d*(e*x)^(1+m)*hypergeom([1, 1+m],[2+m],-b*x/a)/a/e/(1+m)+c*(e*x)^(1+m)*hyp
ergeom([1, 1+m],[2+m],b*x/a)/a/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right) dx = \frac{x(ex)^m \left(d \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)+c \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{a}\right)\right)}{a(1+m)}$$

input
```
Integrate[(c*(e*x)^m)/(a - b*x) + (d*(e*x)^m)/(a + b*x),x]
```

output
```
(x*(e*x)^m*(d*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)] + c*Hypergeom
etric2F1[1, 1 + m, 2 + m, (b*x)/a]))/(a*(1 + m))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{c(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{a}\right)}{ae(m+1)}+\frac{d(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{ae(m+1)}$$

input
```
Int[(c*(e*x)^m)/(a - b*x) + (d*(e*x)^m)/(a + b*x),x]
```

output
```
(d*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*e*(1 +
 m)) + (c*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/a])/(a*e*
(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{c(ex)^m}{-bx+a}+\frac{d(ex)^m}{bx+a}\right)dx$$

input
```
int(c*(e*x)^m/(-b*x+a)+d*(e*x)^m/(b*x+a),x)
```

output
```
int(c*(e*x)^m/(-b*x+a)+d*(e*x)^m/(b*x+a),x)
```

Fricas [F]

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right)dx=\int -\frac{(ex)^m c}{bx-a}+\frac{(ex)^m d}{bx+a}\,dx$$

input
```
integrate(c*(e*x)^m/(-b*x+a)+d*(e*x)^m/(b*x+a),x, algorithm="fricas")
```

output
```
integral(-(a*c + a*d + (b*c - b*d)*x)*(e*x)^m/(b^2*x^2 - a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right)dx=c\left(\frac{e^m m x^{m+1}\Phi\left(\frac{bxe^{2i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)}+\frac{e^m x^{m+1}\Phi\left(\frac{bxe^{2i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)}\right)+d\left(\frac{e^m m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)}+\frac{e^m x^{m+1}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a\Gamma(m+2)}\right)$$

input
```
integrate(c*(e*x)**m/(-b*x+a)+d*(e*x)**m/(b*x+a),x)
```

output
```
c*(e**m*m*x**(m + 1)*lerchphi(b*x*exp_polar(2*I*pi)/a, 1, m + 1)*gamma(m +
 1)/(a*gamma(m + 2)) + e**m*x**(m + 1)*lerchphi(b*x*exp_polar(2*I*pi)/a, 1
, m + 1)*gamma(m + 1)/(a*gamma(m + 2))) + d*(e**m*m*x**(m + 1)*lerchphi(b*
x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*gamma(m + 2)) + e**m*x**(m
+ 1)*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*gamma(m + 2
)))
```

Maxima [F]

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right)\,dx=\int -\frac{(ex)^m\,c}{bx-a}+\frac{(ex)^m\,d}{bx+a}\,dx$$

input
```
integrate(c*(e*x)^m/(-b*x+a)+d*(e*x)^m/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(-(e*x)^m*c/(b*x - a) + (e*x)^m*d/(b*x + a), x)
```

Giac [F]

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right)\,dx=\int -\frac{(ex)^m\,c}{bx-a}+\frac{(ex)^m\,d}{bx+a}\,dx$$

input
```
integrate(c*(e*x)^m/(-b*x+a)+d*(e*x)^m/(b*x+a),x, algorithm="giac")
```

output
```
integrate(-(e*x)^m*c/(b*x - a) + (e*x)^m*d/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right)\,dx=\int \frac{c\,{\left(e\,x\right)}^m}{a-b\,x}+\frac{d\,{\left(e\,x\right)}^m}{a+b\,x}\,dx$$

input
```
int((c*(e*x)^m)/(a - b*x) + (d*(e*x)^m)/(a + b*x),x)
```

output
```
int((c*(e*x)^m)/(a - b*x) + (d*(e*x)^m)/(a + b*x), x)
```

Reduce [F]

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right)\,dx$$
$$=\frac{e^m\left(-x^mc+x^md+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abcm+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abdm+\left(\int \frac{x^m}{-b^2x^3+a^2x}dx\right)a^2cm-\left(\int \frac{x^m}{-b^2x^3+a^2x}\right.\right.}{bm}$$

input
```
int(c*(e*x)^m/(-b*x+a)+d*(e*x)^m/(b*x+a),x)
```

output
```
(e**m*( - x**m*c + x**m*d + int(x**m/(a**2 - b**2*x**2),x)*a*b*c*m + int(x
**m/(a**2 - b**2*x**2),x)*a*b*d*m + int(x**m/(a**2*x - b**2*x**3),x)*a**2*
c*m - int(x**m/(a**2*x - b**2*x**3),x)*a**2*d*m))/(b*m)
```

3.226 $\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 70

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right) \, dx = \frac{d(ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)}{ae(1+m)} + \frac{c(ex)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{a}\right)}{ae(1+m)}$$

output
```
d*(e*x)^(1+m)*hypergeom([1, 1+m],[2+m],-b*x/a)/a/e/(1+m)+c*(e*x)^(1+m)*hyp
ergeom([1, 1+m],[2+m],b*x/a)/a/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right) \, dx = \frac{x(ex)^m \left(d \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right) + c \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{a}\right)\right)}{a(1+m)}$$

input
```
Integrate[(e*x)^m*(c/(a - b*x) + d/(a + b*x)),x]
```

output
```
(x*(e*x)^m*(d*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)] + c*Hypergeom
etric2F1[1, 1 + m, 2 + m, (b*x)/a]))/(a*(1 + m))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right) dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{c(ex)^m}{a-bx}+\frac{d(ex)^m}{a+bx}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{a}\right)}{ae(m+1)}+\frac{d(ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{ae(m+1)}$$

input
```
Int[(e*x)^m*(c/(a - b*x) + d/(a + b*x)),x]
```

output
```
(d*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*e*(1 +
 m)) + (c*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/a])/(a*e*
(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [F]

$$\int (ex)^m \left(\frac{c}{-bx+a}+\frac{d}{bx+a}\right) dx$$

input
```
int((e*x)^m*(c/(-b*x+a)+d/(b*x+a)),x)
```

output
```
int((e*x)^m*(c/(-b*x+a)+d/(b*x+a)),x)
```

Fricas [F]

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right) dx = \int -(ex)^m \left(\frac{c}{bx-a}-\frac{d}{bx+a}\right) dx$$

input
```
integrate((e*x)^m*(c/(-b*x+a)+d/(b*x+a)),x, algorithm="fricas")
```

output
```
integral(-(a*c + a*d + (b*c - b*d)*x)*(e*x)^m/(b^2*x^2 - a^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(51) = 102$.

Time = 2.87 (sec) , antiderivative size = 379, normalized size of antiderivative = 5.41

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right)\,dx = \frac{ce^m m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{ce^m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{de^m m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{de^m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{bce^m m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a^2\Gamma\left(\frac{m}{2}+2\right)} + \frac{bce^m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a^2\Gamma\left(\frac{m}{2}+2\right)} - \frac{bde^m m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a^2\Gamma\left(\frac{m}{2}+2\right)} - \frac{bde^m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a^2\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((e*x)**m*(c/(-b*x+a)+d/(b*x+a)),x)
```

output
```
c*e**m*m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + c*e**m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + d*e**m*m*x**(m + 1)*le
rchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2
)) + d*e**m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + b*c*e**m*m*x**(m + 2)*lerchphi(b**2*x**2/a**
2, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a**2*gamma(m/2 + 2)) + b*c*e**m*x**(m + 2
)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a**2*gamma(m/2 +
2)) - b*d*e**m*m*x**(m + 2)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a**2*gamma(m/2 + 2)) - b*d*e**m*x**(m + 2)*lerchphi(b**2*x**2/a**
2, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a**2*gamma(m/2 + 2))
```

Maxima [F]

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right)\,dx=\int -(ex)^m\left(\frac{c}{bx-a}-\frac{d}{bx+a}\right)\,dx$$

input
```
integrate((e*x)^m*(c/(-b*x+a)+d/(b*x+a)),x, algorithm="maxima")
```

output
```
-integrate((e*x)^m*(c/(b*x - a) - d/(b*x + a)), x)
```

Giac [F]

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right)\,dx=\int -(ex)^m\left(\frac{c}{bx-a}-\frac{d}{bx+a}\right)\,dx$$

input
```
integrate((e*x)^m*(c/(-b*x+a)+d/(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-(e*x)^m*(c/(b*x - a) - d/(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right)\,dx=\int (e\,x)^m\,\left(\frac{c}{a-b\,x}+\frac{d}{a+b\,x}\right)\,dx$$

input
```
int((e*x)^m*(c/(a - b*x) + d/(a + b*x)),x)
```

output
```
int((e*x)^m*(c/(a - b*x) + d/(a + b*x)), x)
```

Reduce [F]

$$\int (ex)^m \left(\frac{c}{a-bx}+\frac{d}{a+bx}\right)\,dx$$
$$=\frac{e^m\left(-x^m c+x^m d+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abcm+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abdm+\left(\int \frac{x^m}{-b^2x^3+a^2x}dx\right)a^2cm-\left(\int \frac{x^m}{-b^2x^3+a^2x}\right.}{bm}$$

input
```
int((e*x)^m*(c/(-b*x+a)+d/(b*x+a)),x)
```

output
```
(e**m*( - x**m*c + x**m*d + int(x**m/(a**2 - b**2*x**2),x)*a*b*c*m + int(x
**m/(a**2 - b**2*x**2),x)*a*b*d*m + int(x**m/(a**2*x - b**2*x**3),x)*a**2*
c*m - int(x**m/(a**2*x - b**2*x**3),x)*a**2*d*m))/(b*m)
```

3.227 $\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)} dx$

Optimal result

Integrand size = 35, antiderivative size = 100

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)} dx = \frac{(c+d)(ex)^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \frac{b^2x^2}{a^2}\right)}{ae(1+m)} + \frac{b(c-d)(ex)^{2+m} \text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, \frac{b^2x^2}{a^2}\right)}{a^2e^2(2+m)}$$

output
```
(c+d)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],b^2*x^2/a^2)/a/e/(1
+m)+b*(c-d)*(e*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],b^2*x^2/a^2)/a^2/
e^2/(2+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx = \frac{x(ex)^m \left(d\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)+c\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{bx}{a}\right)\right)}{a(1+m)}$$

input
```
Integrate[((e*x)^m*(a*(c + d) + b*(c - d)*x))/((a - b*x)*(a + b*x)),x]
```

output
```
(x*(e*x)^m*(d*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)] + c*Hypergeom
etric2F1[1, 1 + m, 2 + m, (b*x)/a]))/(a*(1 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {174, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m(a(c+d)+bx(c-d))}{(a-bx)(a+bx)}\,dx$$

$$\Big\downarrow \text{174}$$

$$c\int \frac{(ex)^m}{a-bx}dx+d\int \frac{(ex)^m}{a+bx}dx$$

$$\Big\downarrow \text{74}$$

$$\frac{c(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{bx}{a}\right)}{ae(m+1)}+\frac{d(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{ae(m+1)}$$

input
```
Int[((e*x)^m*(a*(c + d) + b*(c - d)*x))/((a - b*x)*(a + b*x)),x]
```

output
```
(d*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*e*(1 +
 m)) + (c*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/a])/(a*e*
(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Maple [F]

$$\int \frac{(ex)^m (a(c+d)+b(c-d)x)}{(-bx+a)(bx+a)}dx$$

input
```
int((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x)
```

output
```
int((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx = \int -\frac{(b(c-d)x+a(c+d))(ex)^m}{(bx+a)(bx-a)}\,dx$$

input
```
integrate((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x, algorithm="frica
s")
```

output
```
integral(-(a*c + a*d + (b*c - b*d)*x)*(e*x)^m/(b^2*x^2 - a^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(78) = 156.

Time = 2.98 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.79

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx = \frac{ce^m m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{ce^m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{de^m m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{de^m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{bce^m m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a^2\Gamma\left(\frac{m}{2}+2\right)} + \frac{bce^m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a^2\Gamma\left(\frac{m}{2}+2\right)} - \frac{bde^m m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a^2\Gamma\left(\frac{m}{2}+2\right)} - \frac{bde^m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a^2\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((e*x)**m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x)
```

output
```
c*e**m*m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + c*e**m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + d*e**m*m*x**(m + 1)*le
rchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2
)) + d*e**m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + b*c*e**m*m*x**(m + 2)*lerchphi(b**2*x**2/a**
2, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a**2*gamma(m/2 + 2)) + b*c*e**m*x**(m + 2
)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a**2*gamma(m/2 +
2)) - b*d*e**m*m*x**(m + 2)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a**2*gamma(m/2 + 2)) - b*d*e**m*x**(m + 2)*lerchphi(b**2*x**2/a**
2, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a**2*gamma(m/2 + 2))
```

Maxima [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx = \int -\frac{(b(c-d)x+a(c+d))(ex)^m}{(bx+a)(bx-a)}\,dx$$

input
```
integrate((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x, algorithm="maxim
a")
```

output
```
-integrate((b*(c - d)*x + a*(c + d))*(e*x)^m/((b*x + a)*(b*x - a)), x)
```

Giac [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx = \int -\frac{(b(c-d)x+a(c+d))(ex)^m}{(bx+a)(bx-a)}\,dx$$

input
```
integrate((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x, algorithm="giac"
)
```

output
```
integrate(-(b*(c - d)*x + a*(c + d))*(e*x)^m/((b*x + a)*(b*x - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx=\int \frac{(a\,(c+d)+b\,x\,(c-d))\,\left(e\,x\right)^m}{(a+b\,x)\,\left(a-b\,x\right)}\,dx$$

input
```
int(((a*(c + d) + b*x*(c - d))*(e*x)^m)/((a + b*x)*(a - b*x)),x)
```

output
```
int(((a*(c + d) + b*x*(c - d))*(e*x)^m)/((a + b*x)*(a - b*x)), x)
```

Reduce [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{(a-bx)(a+bx)}\,dx$$
$$=\frac{e^m\left(-x^mc+x^md+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abcm+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abdm+\left(\int \frac{x^m}{-b^2x^3+a^2x}dx\right)a^2cm-\left(\int \frac{x^m}{-b^2x^3+a^2x}\right.\right.}{bm}$$

input
```
int((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b*x+a)/(b*x+a),x)
```

output
```
(e**m*( - x**m*c + x**m*d + int(x**m/(a**2 - b**2*x**2),x)*a*b*c*m + int(x
**m/(a**2 - b**2*x**2),x)*a*b*d*m + int(x**m/(a**2*x - b**2*x**3),x)*a**2*
c*m - int(x**m/(a**2*x - b**2*x**3),x)*a**2*d*m))/(b*m)
```

3.228 $\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 100

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx = \frac{(c+d)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\frac{b^2x^2}{a^2}\right)}{ae(1+m)} + \frac{b(c-d)(ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\frac{b^2x^2}{a^2}\right)}{a^2e^2(2+m)}$$

output
```
(c+d)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],b^2*x^2/a^2)/a/e/(1
+m)+b*(c-d)*(e*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],b^2*x^2/a^2)/a^2/
e^2/(2+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx = \frac{x(ex)^m\left(\frac{b(c-d)x\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},\frac{b^2x^2}{a^2}\right)}{2+m}+\frac{a(c+d)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\frac{b^2x^2}{a^2}\right)}{1+m}\right)}{a^2}$$

input
```
Integrate[((e*x)^m*(a*(c + d) + b*(c - d)*x))/(a^2 - b^2*x^2),x]
```

output
```
(x*(e*x)^m*((b*(c - d)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, (b^2*x^2)/
a^2])/(2 + m) + (a*(c + d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (b^2
*x^2)/a^2])/(1 + m)))/a^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m(a(c+d)+bx(c-d))}{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \; 557$$

$$a(c+d)\int \frac{(ex)^m}{a^2-b^2x^2}dx+\frac{b(c-d)\int \frac{(ex)^{m+1}}{a^2-b^2x^2}dx}{e}$$

$$\Big\downarrow \; 278$$

$$\frac{b(c-d)(ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\frac{b^2x^2}{a^2}\right)}{a^2e^2(m+2)}+\frac{(c+d)(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\frac{b^2x^2}{a^2}\right)}{ae(m+1)}$$

input
```
Int[((e*x)^m*(a*(c + d) + b*(c - d)*x))/(a^2 - b^2*x^2),x]
```

output
```
((c + d)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (b^2*x^2
)/a^2])/(a*e*(1 + m)) + (b*(c - d)*(e*x)^(2 + m)*Hypergeometric2F1[1, (2 +
 m)/2, (4 + m)/2, (b^2*x^2)/a^2])/(a^2*e^2*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

Maple [F]

$$\int \frac{(ex)^m (a(c+d)+b(c-d)x)}{-b^2x^2+a^2}dx$$

input
```
int((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b^2*x^2+a^2),x)
```

output
```
int((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b^2*x^2+a^2),x)
```

Fricas [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx = \int -\frac{(b(c-d)x+a(c+d))(ex)^m}{b^2x^2-a^2}\,dx$$

input
```
integrate((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b^2*x^2+a^2),x, algorithm="fricas"
)
```

output
```
integral(-(a*c + a*d + (b*c - b*d)*x)*(e*x)^m/(b^2*x^2 - a^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(78) = 156.

Time = 2.77 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.79

$$\begin{aligned}\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx &= \frac{ce^m m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} \\ &+ \frac{ce^m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} \\ &+ \frac{de^m m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} \\ &+ \frac{de^m x^{m+1}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} \\ &+ \frac{bce^m m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a^2\Gamma\left(\frac{m}{2}+2\right)} \\ &+ \frac{bce^m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a^2\Gamma\left(\frac{m}{2}+2\right)} \\ &- \frac{bde^m m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a^2\Gamma\left(\frac{m}{2}+2\right)} \\ &- \frac{bde^m x^{m+2}\Phi\left(\frac{b^2x^2}{a^2},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a^2\Gamma\left(\frac{m}{2}+2\right)}\end{aligned}$$

input
```
integrate((e*x)**m*(a*(c+d)+b*(c-d)*x)/(-b**2*x**2+a**2),x)
```

output
```
c*e**m*m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 + 1/2
)/(4*a*gamma(m/2 + 3/2)) + c*e**m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + d*e**m*m*x**(m + 1)*le
rchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2
)) + d*e**m*x**(m + 1)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + b*c*e**m*m*x**(m + 2)*lerchphi(b**2*x**2/a**
2, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a**2*gamma(m/2 + 2)) + b*c*e**m*x**(m + 2
)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a**2*gamma(m/2 +
2)) - b*d*e**m*m*x**(m + 2)*lerchphi(b**2*x**2/a**2, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a**2*gamma(m/2 + 2)) - b*d*e**m*x**(m + 2)*lerchphi(b**2*x**2/a**
2, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a**2*gamma(m/2 + 2))
```

Maxima [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx=\int -\frac{(b(c-d)x+a(c+d))(ex)^m}{b^2x^2-a^2}\,dx$$

input
```
integrate((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b^2*x^2+a^2),x, algorithm="maxima"
)
```

output
```
-integrate((b*(c - d)*x + a*(c + d))*(e*x)^m/(b^2*x^2 - a^2), x)
```

Giac [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx=\int -\frac{(b(c-d)x+a(c+d))(ex)^m}{b^2x^2-a^2}\,dx$$

input
```
integrate((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
integrate(-(b*(c - d)*x + a*(c + d))*(e*x)^m/(b^2*x^2 - a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx=\int \frac{(a\,(c+d)+b\,x\,(c-d))\,\left(e\,x\right)^m}{a^2-b^2\,x^2}\,dx$$

input
```
int(((a*(c + d) + b*x*(c - d))*(e*x)^m)/(a^2 - b^2*x^2),x)
```

output
```
int(((a*(c + d) + b*x*(c - d))*(e*x)^m)/(a^2 - b^2*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(a(c+d)+b(c-d)x)}{a^2-b^2x^2}\,dx$$
$$=\frac{e^m\left(-x^m c+x^m d+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abcm+\left(\int \frac{x^m}{-b^2x^2+a^2}dx\right)abdm+\left(\int \frac{x^m}{-b^2x^3+a^2x}dx\right)a^2cm-\left(\int \frac{x^m}{-b^2x^3+a^2x}\right.\right.}{bm}$$

input
```
int((e*x)^m*(a*(c+d)+b*(c-d)*x)/(-b^2*x^2+a^2),x)
```

output
```
(e**m*( - x**m*c + x**m*d + int(x**m/(a**2 - b**2*x**2),x)*a*b*c*m + int(x
**m/(a**2 - b**2*x**2),x)*a*b*d*m + int(x**m/(a**2*x - b**2*x**3),x)*a**2*
c*m - int(x**m/(a**2*x - b**2*x**3),x)*a**2*d*m))/(b*m)
```

3.229 $\int x(a+bx)^m(c+dx)^n(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 242

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx = \frac{(bde(3+m+n)-f(bc(2+m)+ad(4+m+2n)))(a+bx)^{1+m}(c+dx)^{1+n}}{b^2d^2(2+m+n)(3+m+n)} + \frac{f(a+bx)^{2+m}(c+dx)^{1+n}}{b^2d(3+m+n)} - \frac{(adf(2+m+n)(bc(2+m)+ad(1+n))+(bc(1+m)+ad(1+n))(bde(3+m+n)-f(bc(2+m)}{b^2d^2(bc-ad)(1+m)(2}$$

output
```
(b*d*e*(3+m+n)-f*(b*c*(2+m)+a*d*(4+m+2*n)))*(b*x+a)^(1+m)*(d*x+c)^(1+n)/b^
2/d^2/(2+m+n)/(3+m+n)+f*(b*x+a)^(2+m)*(d*x+c)^(1+n)/b^2/d/(3+m+n)-(a*d*f*(
2+m+n)*(b*c*(2+m)+a*d*(1+n))+(b*c*(1+m)+a*d*(1+n))*(b*d*e*(3+m+n)-f*(b*c*(
2+m)+a*d*(4+m+2*n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+
m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(1+m)/(2+m+n)/(3+m+n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.74

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left((bc-ad)^2f\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{d(a+bx)}{-bc+ad}\right)+\right.}{}$$

input
```
Integrate[x*(a + b*x)^m*(c + d*x)^n*(e + f*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^2*f*Hypergeometric2F1[1 + m, -
2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(-(d*e) + 2
*c*f)*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)
]) + b*c*(-(d*e) + c*f)*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a + b*x))/
(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(e+fx)(a+bx)^m(c+dx)^n\,dx$$

$$\Big\downarrow \text{164}$$

$$\frac{(a^2d^2f(n+1)(n+2)+abd(n+1)(2cf(m+1)-de(m+n+3))+b^2c(m+1)(cf(m+2)-de(m+n+3)))\int}{b^2d^2(m+n+2)(m+n+3)}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adf(n+2)+bcf(m+2)-bde(m+n+3)-bdfx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

$$\Big\downarrow \text{80}$$

$$\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(a^2d^2f(n+1)(n+2)+abd(n+1)(2cf(m+1)-de(m+n+3))+b^2c(m+1)(cf(m+\right.}{b^2d^2(m+n+2)(m+n+3)}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adf(n+2)+bcf(m+2)-bde(m+n+3)-bdfx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

$$\Big\downarrow \text{79}$$

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(a^2d^2f(n+1)(n+2)+abd(n+1)(2cf(m+1)-de(m+n+3))+b^2c(m+\right.}{b^3d^2(m+1)(m+n+2)(m+n}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adf(n+2)+bcf(m+2)-bde(m+n+3)-bdfx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

input
```
Int[x*(a + b*x)^m*(c + d*x)^n*(e + f*x),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*(2 + m) + a*d*f*(2 + n) - b*
d*e*(3 + m + n) - b*d*f*(2 + m + n)*x))/(b^2*d^2*(2 + m + n)*(3 + m + n)))
 + ((a^2*d^2*f*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*(1 + m) - d*e*(3 + m
 + n)) + b^2*c*(1 + m)*(c*f*(2 + m) - d*e*(3 + m + n)))*(a + b*x)^(1 + m)*
(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d
))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b*c - a*d))^
n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int x(bx+a)^m\,(xd+c)^n\,(fx+e)\,dx$$

input
```
int(x*(b*x+a)^m*(d*x+c)^n*(f*x+e),x)
```

output
```
int(x*(b*x+a)^m*(d*x+c)^n*(f*x+e),x)
```

Fricas [F]

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx = \int (fx+e)(bx+a)^m(dx+c)^n x\,dx$$

input
```
integrate(x*(b*x+a)^m*(d*x+c)^n*(f*x+e),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x*(b*x+a)**m*(d*x+c)**n*(f*x+e),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx = \int (fx+e)(bx+a)^m(dx+c)^n x\,dx$$

input
```
integrate(x*(b*x+a)^m*(d*x+c)^n*(f*x+e),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n*x, x)
```

Giac [F]

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx = \int (fx+e)(bx+a)^m(dx+c)^n x\,dx$$

input
```
integrate(x*(b*x+a)^m*(d*x+c)^n*(f*x+e),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx=\int x\,(e+f\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n\,dx$$

input
```
int(x*(e + f*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int(x*(e + f*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int x(a+bx)^m(c+dx)^n(e+fx)\,dx=\text{too large to display}$$

input
```
int(x*(b*x+a)^m*(d*x+c)^n*(f*x+e),x)
```

output

```
((c + d*x)**n*(a + b*x)**m*a**3*c*d**2*f*m*n + 2*(c + d*x)**n*(a + b*x)**m
*a**3*c*d**2*f*m - (c + d*x)**n*(a + b*x)**m*a**3*d**3*f*m*n**2*x - 2*(c +
 d*x)**n*(a + b*x)**m*a**3*d**3*f*m*n*x - 2*(c + d*x)**n*(a + b*x)**m*a**2
*b*c**2*d*f*m*n - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*e*m**2 - (c + d*
x)**n*(a + b*x)**m*a**2*b*c*d**2*e*m*n - 3*(c + d*x)**n*(a + b*x)**m*a**2*
b*c*d**2*e*m - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*m**2*n*x - 2*(c +
 d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*m**2*x + 2*(c + d*x)**n*(a + b*x)**m
*a**2*b*c*d**2*f*m*n**2*x + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*m**2*n
*x + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*m*n**2*x + 3*(c + d*x)**n*(a
+ b*x)**m*a**2*b*d**3*e*m*n*x + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f*m*
*2*n*x**2 + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f*m*n**2*x**2 + (c + d*x
)**n*(a + b*x)**m*a**2*b*d**3*f*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*a*b**
2*c**3*f*m*n + 2*(c + d*x)**n*(a + b*x)**m*a*b**2*c**3*f*n - (c + d*x)**n*
(a + b*x)**m*a*b**2*c**2*d*e*m*n - (c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d
*e*n**2 - 3*(c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*e*n + 2*(c + d*x)**n*(
a + b*x)**m*a*b**2*c**2*d*f*m**2*n*x - (c + d*x)**n*(a + b*x)**m*a*b**2*c*
*2*d*f*m*n**2*x - 2*(c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*f*n**2*x + (c
+ d*x)**n*(a + b*x)**m*a*b**2*c*d**2*e*m**3*x + (c + d*x)**n*(a + b*x)**m*
a*b**2*c*d**2*e*m**2*n*x + 3*(c + d*x)**n*(a + b*x)**m*a*b**2*c*d**2*e*m**
2*x + (c + d*x)**n*(a + b*x)**m*a*b**2*c*d**2*e*m*n**2*x + (c + d*x)**n...
```

3.230 $\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 272

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = -\frac{(bcfh(2+m)-bd(fg+eh)(3+m+n)+adfh(4+m+2n))(a+bx)^{1+m}(c+dx)^{1+n}}{b^2d^2(2+m+n)(3+m+n)} + \frac{fh(a+bx)^{2+m}(c+dx)^{1+n}}{b^2d(3+m+n)} + \frac{((bc(1+m)+ad(1+n))(bcfh(2+m)-bd(fg+eh)(3+m+n)+adfh(4+m+2n))+d(2+m-}{b^2d^2(bc}$$

output
```
-(b*c*f*h*(2+m)-b*d*(e*h+f*g)*(3+m+n)+a*d*f*h*(4+m+2*n))*(b*x+a)^(1+m)*(d*
x+c)^(1+n)/b^2/d^2/(2+m+n)/(3+m+n)+f*h*(b*x+a)^(2+m)*(d*x+c)^(1+n)/b^2/d/(
3+m+n)+((b*c*(1+m)+a*d*(1+n))*(b*c*f*h*(2+m)-b*d*(e*h+f*g)*(3+m+n)+a*d*f*h
*(4+m+2*n))+d*(2+m+n)*(b^2*d*e*g*(3+m+n)-a*f*h*(b*c*(2+m)+a*d*(1+n))))*(b*
x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))
/b^2/d^2/(-a*d+b*c)/(1+m)/(2+m+n)/(3+m+n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{d(a+bx)}{-bc+ad}\right)+\right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^2*f*h*Hypergeometric2F1[1 + m,
 -2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h
- d*(f*g + e*h))*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(
b*c) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[1 + m, -n, 2 +
 m, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c
- a*d))^n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m(c+dx)^n\,dx$$

$$\Big\downarrow \text{164}$$

$$\frac{\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d(m+n+3)(eh+fg))+b^2\left(c^2fh(m+1)(m+2)-cd(m\right.\right.}{b^2d^2(m+n+2)(m+n+3}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

$$\Big\downarrow \text{80}$$

$$\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d(m+n+3)(eh+fg))+b^2(c^2fh(\right.}{b^2d^2(m+n}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

$\downarrow$ 79

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+\right.}{b}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n)
- b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m +
n)*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*
(1 + m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*
(f*g + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b
*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x)
)/(b*c - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(
b*c - a*d))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e) (hx+g) \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx) \, dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^n \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \int (e+f\,x)\;(g+h\,x)\;(a+b\,x)^m\;(c+d\,x)^n\,dx$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

output

```
((c + d*x)**n*(a + b*x)**m*a**3*c*d**2*f*h*m*n + 2*(c + d*x)**n*(a + b*x)*
*m*a**3*c*d**2*f*h*m - (c + d*x)**n*(a + b*x)**m*a**3*d**3*f*h*m*n**2*x -
2*(c + d*x)**n*(a + b*x)**m*a**3*d**3*f*h*m*n*x - 2*(c + d*x)**n*(a + b*x)
**m*a**2*b*c**2*d*f*h*m*n - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*e*h*m*
*2 - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*e*h*m*n - 3*(c + d*x)**n*(a +
 b*x)**m*a**2*b*c*d**2*e*h*m - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*g
*m**2 - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*g*m*n - 3*(c + d*x)**n*(
a + b*x)**m*a**2*b*c*d**2*f*g*m - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*
f*h*m**2*n*x - 2*(c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*h*m**2*x + 2*(c
 + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*h*m*n**2*x + (c + d*x)**n*(a + b*x
)**m*a**2*b*d**3*e*h*m**2*n*x + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*h*
m*n**2*x + 3*(c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*h*m*n*x + (c + d*x)**
n*(a + b*x)**m*a**2*b*d**3*f*g*m**2*n*x + (c + d*x)**n*(a + b*x)**m*a**2*b
*d**3*f*g*m*n**2*x + 3*(c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f*g*m*n*x + (
c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f*h*m**2*n*x**2 + (c + d*x)**n*(a + b
*x)**m*a**2*b*d**3*f*h*m*n**2*x**2 + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3
*f*h*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*a*b**2*c**3*f*h*m*n + 2*(c + d*x
)**n*(a + b*x)**m*a*b**2*c**3*f*h*n - (c + d*x)**n*(a + b*x)**m*a*b**2*c**
2*d*e*h*m*n - (c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*e*h*n**2 - 3*(c + d*
x)**n*(a + b*x)**m*a*b**2*c**2*d*e*h*n - (c + d*x)**n*(a + b*x)**m*a*b*...
```

3.231 $\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 264

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}} \, dx = -\frac{2(be-af)h(a+bx)^m\left(-\frac{f(a+bx)}{be-af}\right)^{-m}(c+dx)^n\left(-\frac{f(c+dx)}{de-cf}\right)^{-n}\sqrt{e+fx}\,\text{AppellF1}\left(\frac{1}{2},-1-m,-n,\frac{3}{2},\frac{b}{}\right.}{bf^2} + \frac{2(bg-ah)(a+bx)^m\left(-\frac{f(a+bx)}{be-af}\right)^{-m}(c+dx)^n\left(-\frac{f(c+dx)}{de-cf}\right)^{-n}\sqrt{e+fx}\,\text{AppellF1}\left(\frac{1}{2},-m,-n,\frac{3}{2},\frac{b(e+fx)}{be-af}\right.}{bf}$$

output
```
-2*(-a*f+b*e)*h*(b*x+a)^m*(d*x+c)^n*(f*x+e)^(1/2)*AppellF1(1/2,-1-m,-n,3/2
,b*(f*x+e)/(-a*f+b*e),d*(f*x+e)/(-c*f+d*e))/b/f^2/((-f*(b*x+a)/(-a*f+b*e))
^m)/((-f*(d*x+c)/(-c*f+d*e))^n)+2*(-a*h+b*g)*(b*x+a)^m*(d*x+c)^n*(f*x+e)^(
1/2)*AppellF1(1/2,-m,-n,3/2,b*(f*x+e)/(-a*f+b*e),d*(f*x+e)/(-c*f+d*e))/b/f
/((-f*(b*x+a)/(-a*f+b*e))^m)/((-f*(d*x+c)/(-c*f+d*e))^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx =$$
$$-\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\sqrt{e+fx}\Big((be-af)h\,\text{AppellF1}\left(1+m,-n,-\frac{1}{2},2+m,\frac{d(a+bx)}{-bc+ad},\frac{f(a-}{-be}\right.}{bf(-be+af)(1+m)\sqrt{\frac{b(e+fx)}{be-af}}}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^n*Sqrt[e + f*x]*((b*e - a*f)*h*AppellF1[1 +
 m, -n, -1/2, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) +
 a*f)] + b*(f*g - e*h)*AppellF1[1 + m, -n, 1/2, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)]))/(b*f*(-(b*e) + a*f)*(1 + m)*((b
*(c + d*x))/(b*c - a*d))^n*Sqrt[(b*(e + f*x))/(b*e - a*f)]))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {177, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)(a+bx)^m(c+dx)^n}{\sqrt{e+fx}}dx$$
$$\Big\downarrow \; 177$$
$$\frac{(bg-ah)\int \frac{(a+bx)^m(c+dx)^n}{\sqrt{e+fx}}dx}{b}+\frac{h\int \frac{(a+bx)^{m+1}(c+dx)^n}{\sqrt{e+fx}}dx}{b}$$
$$\Big\downarrow \; 157$$

$$\frac{(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int\frac{(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n}{\sqrt{e+fx}}dx}{b}+\frac{h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int\frac{(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n}{\sqrt{e+fx}}dx}{b}$$

$$\Big\downarrow \text{156}$$

$$\frac{(bg-ah)(c+dx)^n\sqrt{\frac{b(e+fx)}{be-af}}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int\frac{(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n}{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{b\sqrt{e+fx}}+\frac{h(c+dx)^n\sqrt{\frac{b(e+fx)}{be-af}}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int\frac{(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n}{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{b\sqrt{e+fx}}$$

$$\Big\downarrow \text{155}$$

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n\sqrt{\frac{b(e+fx)}{be-af}}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,\frac{1}{2},m+2,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)\sqrt{e+fx}}+\frac{h(a+bx)^{m+2}(c+dx)^n\sqrt{\frac{b(e+fx)}{be-af}}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+2,-n,\frac{1}{2},m+3,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+2)\sqrt{e+fx}}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(g + h*x))/Sqrt[e + f*x],x]
```

output
```
((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*Sqrt[(b*(e + f*x))/(b*e - a*f)]
*AppellF1[1 + m, -n, 1/2, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b
*x))/(b*e - a*f))]])/(b^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*Sqrt[e + f*
x]) + (h*(a + b*x)^(2 + m)*(c + d*x)^n*Sqrt[(b*(e + f*x))/(b*e - a*f)]*App
ellF1[2 + m, -n, 1/2, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))
/(b*e - a*f))]])/(b^2*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (hx+g)}{\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx = \int \frac{(hx+g)(bx+a)^m(dx+c)^n}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
integral((h*x + g)*(b*x + a)^m*(d*x + c)^n/sqrt(f*x + e), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(h*x+g)/(f*x+e)**(1/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx=\int \frac{(hx+g)(bx+a)^m(dx+c)^n}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n/sqrt(f*x + e), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx=\int \frac{(hx+g)(bx+a)^m(dx+c)^n}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n/sqrt(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx=\int \frac{(g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{\sqrt{e+f\,x}}\,dx$$

input
```
int(((g + h*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(1/2),x)
```

output
```
int(((g + h*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(g+hx)}{\sqrt{e+fx}}\,dx = \int \frac{(bx+a)^m\,(dx+c)^n\,(hx+g)}{\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(h*x+g)/(f*x+e)^(1/2),x)
```

3.232 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-m-n}\, dx$

Optimal result

Integrand size = 33, antiderivative size = 268

$$\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-m-n}\, dx = \frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^{-m-n} \left(\frac{b(e+fx)}{be-af}\right)^{m+n} \operatorname{AppellF1}\left(1+m,-n,m+n,\right.}{b^2(1+m)} + \frac{B(a+bx)^{2+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^{-m-n} \left(\frac{b(e+fx)}{be-af}\right)^{m+n} \operatorname{AppellF1}\left(2+m,-n,m+n,3+m,\right.}{b^2(2+m)}$$

output
```
(A*b-B*a)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m
+n)*AppellF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b
^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)+B*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^(-m-
n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(2+m,-n,m+n,3+m,-d*(b*x+a)/(-a*d+b
*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$
$$=\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$

input
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

output
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n), x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {177, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^{-m-n}\,dx$$

$$\Big\downarrow \text{177}$$

$$\frac{(Ab-aB)\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}dx}{b}+\frac{B\int (a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}dx}{b}$$

$$\Big\downarrow \text{157}$$

$$\frac{(Ab-aB)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}dx}{b}+\frac{B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int (a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}dx}{b}$$

$$\Big\downarrow \text{156}$$

$$\frac{(Ab-aB)(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^{-m}}{b}$$
$$\frac{B(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\int(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^{-m-n}dx}{b}$$

$$\Big\downarrow \textbf{155}$$

$$\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+1,-n,m+n,m+2,\right.}{b^2(m+1)}$$
$$\frac{B(a+bx)^{m+2}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+2,-n,m+n,m+3,-\frac{d(a+bx)}{bc-ad}\right.}{b^2(m+2)}$$

input
```
Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

output
```
((A*b - a*B)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x
))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/
(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(
b*c - a*d))^n) + (B*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(
e + f*x))/(b*e - a*f))^(m + n)*AppellF1[2 + m, -n, m + n, 3 + m, -((d*(a +
 b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((b*(c +
d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (xd+c)^n (fx+e)^{-m-n}\,dx$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
```

output
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="fricas"
)
```

output
```
integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-m-n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="maxima"
)
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$
$$=\int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx=\int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n),x)
```

output
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n), x)
```

Reduce [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$
$$=\left(\int \frac{(dx+c)^n\,(bx+a)^m\,x}{(fx+e)^{m+n}}dx\right)b+\left(\int \frac{(dx+c)^n\,(bx+a)^m}{(fx+e)^{m+n}}dx\right)a$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
```

output
```
int(((c + d*x)**n*(a + b*x)**m*x)/(e + f*x)**(m + n),x)*b + int(((c + d*x)
**n*(a + b*x)**m)/(e + f*x)**(m + n),x)*a
```

3.233 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-1-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 283

$$\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-1-m-n}\,dx$$
$$= \frac{B(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^{-m-n} \left(\frac{b(e+fx)}{be-af}\right)^{m+n} \operatorname{AppellF1}\left(1+m,-n,m+n,2+m,-\right.}{bf(1+m)}$$
$$- \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^{-m-n} \left(\frac{b(e+fx)}{be-af}\right)^{m+n} \operatorname{AppellF1}\left(1+m,-n,1+\right.}{f(be-af)(1+m)}$$

output
```
B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appe
llF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b/f/(1+m)
/((b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m
-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*
d+b*c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.73

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{1-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{-1+m+n}\left(B(be-af)\,\text{AppellF1}\left(1+m,-n,\right.\right.}{f(be-a}$$

input
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(1 - m - n)*((b*(e + f*x))/(b*e -
 a*f))^(-1 + m + n)*(B*(b*e - a*f)*AppellF1[1 + m, -n, m + n, 2 + m, (d*(a
 + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + b*(-(B*e) + A*f)*
AppellF1[1 + m, -n, 1 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a
+ b*x))/(-(b*e) + a*f)]))/(f*(b*e - a*f)^2*(1 + m)*((b*(c + d*x))/(b*c - a
*d))^n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {177, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^{-m-n-1}\,dx$$

$$\Big\downarrow \; 177$$

$$\frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}dx}{f}-\frac{(Be-Af)\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n-1}dx}{f}$$

$$\Big\downarrow \; 157$$

$$\frac{B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n}dx}{f}-\frac{(Be-Af)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n-1}dx}{f}$$

$\Big\downarrow$ 156

$$\frac{B(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^{-m-n}dx}{f}-\frac{b(Be-Af)(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^{-}}{f(be-af)}$$

$\Big\downarrow$ 155

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+1,-n,m+n,m+2,-\frac{d(a+bx)}{bc-ad}\right.}{bf(m+1)}-\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+1,-n,m+n+1,m\right.}{f(m+1)(be-af)}$$

input
```
Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

output
```
(B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e -
a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b*c - a*d
)), -((f*(a + b*x))/(b*e - a*f))])/(b*f*(1 + m)*((b*(c + d*x))/(b*c - a*d)
)^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(
e + f*x))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*
(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1
+ m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (xd+c)^n (fx+e)^{-1-m-n} dx$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

output
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n} dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-1} dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="frica
s")
```

output
```
integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n} dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-1-m-n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-1}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-1}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+1}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1),x)
```

output
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1), x)
```

Reduce [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx$$
$$= \left(\int \frac{(dx+c)^n\,(bx+a)^m\,x}{(fx+e)^{m+n}\,e+(fx+e)^{m+n}\,fx}dx\right)b + \left(\int \frac{(dx+c)^n\,(bx+a)^m}{(fx+e)^{m+n}\,e+(fx+e)^{m+n}\,fx}dx\right)a$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x)
```

output
```
int(((c + d*x)**n*(a + b*x)**m*x)/((e + f*x)**(m + n)*e + (e + f*x)**(m +
n)*f*x),x)*b + int(((c + d*x)**n*(a + b*x)**m)/((e + f*x)**(m + n)*e + (e
+ f*x)**(m + n)*f*x),x)*a
```

3.234 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 277

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(1+m,-n,1+m+n,2+\right.}{f(be-af)(1+m)}$$
$$-\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1-m-n}\text{Hypergeometric2F1}\left(1+m,-n,\right.}{f(be-af)(1+m)}$$

output
```
B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appe
llF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f
+b*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*
(f*x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/
(f*x+e))/f/(-a*f+b*e)/(1+m)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.78

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx =$$
$$-\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-1-m-n}\left(\frac{b(e+fx)}{be-af}\right)^n\left(B(e+fx)\left(\frac{b(e+fx)}{be-af}\right)^m\text{AppellF1}\left(1+\right.\right.}{f(-}$$

input
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*((b*(e + f*x))/(b*
e - a*f))^n*(B*(e + f*x)*((b*(e + f*x))/(b*e - a*f))^m*AppellF1[1 + m, -n,
 1 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a
*f)] + (-(B*e) + A*f)*Hypergeometric2F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*
(a + b*x))/((b*c - a*d)*(e + f*x))]))/(f*(-(b*e) + a*f)*(1 + m)*((b*(c + d
*x))/(b*c - a*d))^n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {177, 142, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^{-m-n-2}\,dx$$

$$\Big\downarrow \text{177}$$

$$\frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n-1}dx}{f}-\frac{(Be-Af)\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n-2}dx}{f}$$

$$\Big\downarrow \text{142}$$

$$\frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n-1}dx}{f}-$$
$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{(\right.}{f(m+1)(be-af)}$$

$\Big\downarrow$ 157

$$\frac{B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-m-n-1}dx}{f}-$$
$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{(\right.}{f(m+1)(be-af)}$$

$\Big\downarrow$ 156

$$\frac{bB(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^{-m-n-1}dx}{f(be-af)}$$
$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{(\right.}{f(m+1)(be-af)}$$

$\Big\downarrow$ 155

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\text{AppellF1}\left(m+1,-n,m+n+1,m+2,-\frac{d(a}{bc}\right.}{f(m+1)(be-af)}$$
$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{(\right.}{f(m+1)(be-af)}$$

input
```
Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

output
```
(B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e -
a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a + b*x))/(b*c -
 a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*((b*(c + d*x
))/(b*c - a*d))^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^
(-1 - m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))
/((b*c - a*d)*(e + f*x)))])/(f*(b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x)
)/((b*c - a*d)*(e + f*x)))^n)
```

Defintions of rubi rules used

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] && !IntegerQ[n]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (xd+c)^n (fx+e)^{-2-m-n} dx$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

output
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n} dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-2} dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="frica
s")
```

output
```
integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-2-m-n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-2}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-2}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2),x)
```

output
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2), x)
```

Reduce [F]

$$\begin{aligned}&\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx\\&=\left(\int \frac{(dx+c)^n\,(bx+a)^m\,x}{(fx+e)^{m+n}\,e^2+2\,(fx+e)^{m+n}\,efx+(fx+e)^{m+n}\,f^2x^2}dx\right)b\\&\quad+\left(\int \frac{(dx+c)^n\,(bx+a)^m}{(fx+e)^{m+n}\,e^2+2\,(fx+e)^{m+n}\,efx+(fx+e)^{m+n}\,f^2x^2}dx\right)a\end{aligned}$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x)
```

output
```
int(((c + d*x)**n*(a + b*x)**m*x)/((e + f*x)**(m + n)*e**2 + 2*(e + f*x)**
(m + n)*e*f*x + (e + f*x)**(m + n)*f**2*x**2),x)*b + int(((c + d*x)**n*(a
+ b*x)**m)/((e + f*x)**(m + n)*e**2 + 2*(e + f*x)**(m + n)*e*f*x + (e + f*
x)**(m + n)*f**2*x**2),x)*a
```

3.235 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 261

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \frac{(b(Bce(1+m)+Acf(1+n)-Ade(2+m+n))+a(Adf(1+m)+Bde(1+n)-Bcf(2+m+n))}{(be-af)^2(de}$$

output
```
(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m-n)/(-a*f+b*e)/(-c*f+d
*e)/(2+m+n)-(b*(B*c*e*(1+m)+A*c*f*(1+n)-A*d*e*(2+m+n))+a*(A*d*f*(1+m)+B*d*
e*(1+n)-B*c*f*(2+m+n)))*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom
([-n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^2/(-c*
f+d*e)/(1+m)/(2+m+n)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.85

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx =$$

$$-\frac{(a+bx)^{1+m}(c+dx)^n(e+fx)^{-2-m-n}\left((-Be+Af)(c+dx)+\frac{(b(Bce(1+m)+Acf(1+n)-Ade(2+m+n))+a(Adf}{}\right.}{(be-af)(de-cf)(2}$$

input
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-2 - m - n)*((-(B*e) + A*f)*(c
 + d*x) + ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) - A*d*e*(2 + m + n)) + a*(A*d
*f*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))*(e + f*x)*Hypergeometric2
F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/
((b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n))
)/((b*e - a*f)*(d*e - c*f)*(2 + m + n)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {172, 27, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^{-m-n-3}\,dx$$

$$\Big\downarrow \text{ 172}$$

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{\int (b(Bce(m+1)+Acf(n+1)-Ade(m+n+2))+a(Adf(m+1)+Bde(n+1)-Bcf(m+n+2)))(a+bx)}{(m+n+2)(be-af)(de-cf)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a(Adf(m+1)-Bcf(m+n+2)+Bde(n+1))+b(Acf(n+1)-Ade(m+n+2)+Bce(m+1)))\int(a+bx)}{(m+n+2)(be-af)(de-cf)}$$

$\big\downarrow$ 142

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a+bx)^{m+1}(c+dx)^{n}(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1)-Bcf(m+n+2)+Bde(n+1))+b(}{(m+1)(m+n+2)(be}$$

input
```
Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

output
```
((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/(
(b*e - a*f)*(d*e - c*f)*(2 + m + n)) - ((b*(B*c*e*(1 + m) + A*c*f*(1 + n)
- A*d*e*(2 + m + n)) + a*(A*d*f*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n
)))*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1
[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((
b*e - a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*
c - a*d)*(e + f*x)))^n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (xd+c)^n (fx+e)^{-3-m-n} dx$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

output
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n} dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-3} dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="frica
s")
```

output
```
integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-3-m-n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-3}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$$
$$= \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-3}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+3}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3),x)
```

output
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3), x)
```

Reduce [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx$$
$$= \left(\int \frac{(dx+c)^n\,(bx+a)^m\,x}{(fx+e)^{m+n}\,e^3+3\,(fx+e)^{m+n}\,e^2fx+3\,(fx+e)^{m+n}\,e\,f^2x^2+(fx+e)^{m+n}\,f^3x^3}dx\right)b + \left(\int \frac{(dx+c)^n\,(bx+a)^m}{(fx+e)^{m+n}\,e^3+3\,(fx+e)^{m+n}\,e^2fx+3\,(fx+e)^{m+n}\,e\,f^2x^2+(fx+e)^{m+n}\,f^3x^3}dx\right)a$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x)
```

output
```
int(((c + d*x)**n*(a + b*x)**m*x)/((e + f*x)**(m + n)*e**3 + 3*(e + f*x)**
(m + n)*e**2*f*x + 3*(e + f*x)**(m + n)*e*f**2*x**2 + (e + f*x)**(m + n)*f
**3*x**3),x)*b + int(((c + d*x)**n*(a + b*x)**m)/((e + f*x)**(m + n)*e**3
+ 3*(e + f*x)**(m + n)*e**2*f*x + 3*(e + f*x)**(m + n)*e*f**2*x**2 + (e +
f*x)**(m + n)*f**3*x**3),x)*a
```

3.236 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-4-m-n}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 558

$$\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-4-m-n}\,dx$$
$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)}$$
$$+ \frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(Be(de+cf(1+m))+Af(cf(2+n)-de(4}{(be-af)^2(de-cf)^2(2+m+n)(3+m+n)}$$
$$+ \frac{((2+m+n)(abcdf(Be-Af)-bde(b(Bce(1+m)+Acf(2+n)-Ade(3+m+n))+a(Adf(2+}{}$$

output
```
(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-3-m-n)/(-a*f+b*e)/(-c*f+d
*e)/(3+m+n)+(a*f*(A*d*f*(2+m)+B*d*e*(1+n)-B*c*f*(3+m+n))+b*(B*e*(d*e+c*f*(
1+m))+A*f*(c*f*(2+n)-d*e*(4+m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-
2-m-n)/(-a*f+b*e)^2/(-c*f+d*e)^2/(2+m+n)/(3+m+n)+((2+m+n)*(a*b*c*d*f*(-A*f
+B*e)-b*d*e*(b*(B*c*e*(1+m)+A*c*f*(2+n)-A*d*e*(3+m+n))+a*(A*d*f*(2+m)+B*d*
e*(1+n)-B*c*f*(3+m+n)))+(a*d+b*c)*f*(b*(B*c*e*(1+m)+A*c*f*(2+n)-A*d*e*(3+m
+n))+a*(A*d*f*(2+m)+B*d*e*(1+n)-B*c*f*(3+m+n))))-(b*c*(1+m)+a*d*(1+n))*(a*
f*(A*d*f*(2+m)+B*d*e*(1+n)-B*c*f*(3+m+n))+b*(B*e*(d*e+c*f*(1+m))+A*f*(c*f*
(2+n)-d*e*(4+m+n)))))*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([
-n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^3/(-c*f+
d*e)^2/(1+m)/(2+m+n)/(3+m+n)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.91

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx =$$

$$-\frac{(a+bx)^{1+m}(c+dx)^n(e+fx)^{-3-m-n}\left(-((Be-Af)(c+dx))-\frac{(af(Adf(2+m)+Bde(1+n)-Bcf(3+m+n))+b(\cdots}{(be-\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-3 - m - n)*(-((B*e - A*f)*(c
+ d*x)) - ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B
*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n))))*(c + d*x)*(
e + f*x))/((b*e - a*f)*(d*e - c*f)*(2 + m + n)) - (((2 + m + n)*(a*b*c*d*f
*(B*e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)
) + a*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d)*f
*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 + m)
 + B*d*e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f
*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(
1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(e + f*x)^2*Hypergeometri
c2F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))]
)/((b*e - a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/
((b*c - a*d)*(e + f*x)))^n)))/((b*e - a*f)*(d*e - c*f)*(3 + m + n)))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {172, 172, 27, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(a+bx)^m(c+dx)^n(e+fx)^{-m-n-4}\,dx$$

$\Big\downarrow$ 172

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-3}}{(m+n+3)(be-af)(de-cf)} -$$
$$\frac{\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n-3}(b(Bce(m+1)+Acf(n+2)-Ade(m+n+3))+a(Adf(m+2)+Bde(}{(m+n+3)(be-af)(de-cf)}$$

$\Big\downarrow$ 172

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-3}}{(m+n+3)(be-af)(de-cf)} -$$
$$-\frac{\int ((m+n+2)(abcdf(Be-Af)-bde(b(Bce(m+1)+Acf(n+2)-Ade(m+n+3))+a(Adf(m+2)+Bde(n+1)-Bcf(m+n+3)))+(bc+ad)f(b(Bce(}{}$$

$\Big\downarrow$ 27

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-3}}{(m+n+3)(be-af)(de-cf)} -$$
$$-\frac{((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Acf(n+2)-Ade(m+n+3)+Bce(m+1)))+f(ad+bc)(a(Adf(m+2)-Bcf(m+}{}$$

$\Big\downarrow$ 142

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-3}}{(m+n+3)(be-af)(de-cf)} -$$
$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Acf(n+2)-Ade(m-}{}$$

input
```
Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

output
```
((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-3 - m - n))/(
(b*e - a*f)*(d*e - c*f)*(3 + m + n)) - (-(((a*f*(A*d*f*(2 + m) + B*d*e*(1
+ n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n)
- d*e*(4 + m + n))))*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m
 - n))/((b*e - a*f)*(d*e - c*f)*(2 + m + n))) - (((2 + m + n)*(a*b*c*d*f*(
B*e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n))
+ a*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d)*f*(
b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 + m) +
 B*d*e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f*(
A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1
+ m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(a + b*x)^(1 + m)*(c + d*x)
^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*
f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)^2*(d*e - c*f)*(1 + m
)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n))/((b*e
- a*f)*(d*e - c*f)*(3 + m + n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (xd+c)^n (fx+e)^{-4-m-n} dx$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

output
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n} dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-4} dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="frica
s")
```

output
```
integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n} dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-4-m-n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-4}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n(fx+e)^{-m-n-4}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(e+f\,x)^{m+n+4}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4),x)
```

output
```
int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4), x)
```

Reduce [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx$$
$$= \left(\int \frac{(dx+c)^n\,(bx+a)^m\,x}{(fx+e)^{m+n}\,e^4+4\,(fx+e)^{m+n}\,e^3fx+6\,(fx+e)^{m+n}\,e^2f^2x^2+4\,(fx+e)^{m+n}\,e\,f^3x^3+(fx+e)^{m+}}\right.$$
$$+\left(\int \frac{(dx+c)^n\,(bx+a)^m}{(fx+e)^{m+n}\,e^4+4\,(fx+e)^{m+n}\,e^3fx+6\,(fx+e)^{m+n}\,e^2f^2x^2+4\,(fx+e)^{m+n}\,e\,f^3x^3+(fx+e)}\right.$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x)
```

output
```
int(((c + d*x)**n*(a + b*x)**m*x)/((e + f*x)**(m + n)*e**4 + 4*(e + f*x)**
(m + n)*e**3*f*x + 6*(e + f*x)**(m + n)*e**2*f**2*x**2 + 4*(e + f*x)**(m +
 n)*e*f**3*x**3 + (e + f*x)**(m + n)*f**4*x**4),x)*b + int(((c + d*x)**n*(
a + b*x)**m)/((e + f*x)**(m + n)*e**4 + 4*(e + f*x)**(m + n)*e**3*f*x + 6*
(e + f*x)**(m + n)*e**2*f**2*x**2 + 4*(e + f*x)**(m + n)*e*f**3*x**3 + (e
+ f*x)**(m + n)*f**4*x**4),x)*a
```

3.237 $\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 174

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx = -\frac{a(2+p+q)x^p(a+bx)^p(a+2bx)^{1+q}\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{1+q}{2},\frac{3+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{2b(1+q)} + \frac{(3+2p+q)x^p(a+bx)^p(a+2bx)^{2+q}\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{2+q}{2},\frac{4+q}{2},\frac{(a+2bx)^2}{a^2}\right)}{2b(2+q)}$$

output
```
-1/2*a*(2+p+q)*x^p*(b*x+a)^p*(2*b*x+a)^(1+q)*hypergeom([-p, 1/2+1/2*q],[3/
2+1/2*q],(2*b*x+a)^2/a^2)/b/(1+q)/((1-(2*b*x+a)^2/a^2)^p)+1/2*(3+2*p+q)*x^
p*(b*x+a)^p*(2*b*x+a)^(2+q)*hypergeom([-p, 1+1/2*q],[2+1/2*q],(2*b*x+a)^2/
a^2)/b/(2+q)/((1-(2*b*x+a)^2/a^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx$$
$$=\frac{x^{1+p}(a+bx)^p(a+2bx)^q\left(1+\frac{bx}{a}\right)^{-p}\left(1+\frac{2bx}{a}\right)^{-q}\left(a(2+p)\operatorname{AppellF1}\left(1+p,-p,-q,2+p,-\frac{bx}{a},-\frac{2bx}{a}\right)+\right.}{2+p}$$

input
```
Integrate[x^p*(a + b*x)^p*(a + 2*b*x)^q*(a*(1 + p) + 2*b*(3 + 2*p + q)*x),
x]
```

output
```
(x^(1 + p)*(a + b*x)^p*(a + 2*b*x)^q*(a*(2 + p)*AppellF1[1 + p, -p, -q, 2
+ p, -((b*x)/a), (-2*b*x)/a] + 2*b*(3 + 2*p + q)*x*AppellF1[2 + p, -p, -q,
 3 + p, -((b*x)/a), (-2*b*x)/a]))/((2 + p)*(1 + (b*x)/a)^p*(1 + (2*b*x)/a)
^q)
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {177, 146, 152, 152, 150, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p(a+bx)^p(a+2bx)^q(a(p+1)+2bx(2p+q+3))\,dx$$

$$\Big\downarrow \text{177}$$

$$2b(2p+q+3)\int x^{p+1}(a+bx)^p(a+2bx)^q dx+a(p+1)\int x^p(a+bx)^p(a+2bx)^q dx$$

$$\Big\downarrow \text{146}$$

$$\frac{a(p+1)x^p(a+bx)^p\left(ax+bx^2\right)^{-p}\int (a+2bx)^q\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{2b}+2b(2p+q+3)\int x^{p+1}(a+bx)^p(a+2bx)^q dx$$

$\Big\downarrow$ 152

$$\frac{a(p+1)x^p(a+bx)^p\left(ax+bx^2\right)^{-p}\int (a+2bx)^q\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{2b}+2b(2p+q+3)(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\int x^{p+1}(a+2bx)^q\left(\frac{bx}{a}+1\right)^p dx$$

$\Big\downarrow$ 152

$$\frac{a(p+1)x^p(a+bx)^p\left(ax+bx^2\right)^{-p}\int (a+2bx)^q\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{2b}+2b(2p+q+3)(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\int x^{p+1}\left(\frac{bx}{a}+1\right)^p\left(\frac{2bx}{a}+1\right)^q dx$$

$\Big\downarrow$ 150

$$\frac{a(p+1)x^p(a+bx)^p\left(ax+bx^2\right)^{-p}\int (a+2bx)^q\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p d(a+2bx)}{2b}+\frac{2b(2p+q+3)x^{p+2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\text{AppellF1}\left(p+2,-p,-q,p+3,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p+2}$$

$\Big\downarrow$ 279

$$\frac{a(p+1)x^p(a+bx)^p\left(ax+bx^2\right)^{-p}\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p\int (a+2bx)^q\left(1-\frac{(a+2bx)^2}{a^2}\right)^p d(a+2bx)}{2b}+\frac{2b(2p+q+3)x^{p+2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\text{AppellF1}\left(p+2,-p,-q,p+3,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p+2}$$

$\Big\downarrow$ 278

$$\frac{a(p+1)x^p(a+bx)^p\left(ax+bx^2\right)^{-p}\left(1-\frac{(a+2bx)^2}{a^2}\right)^{-p}\left(\frac{(a+2bx)^2}{4b}-\frac{a^2}{4b}\right)^p(a+2bx)^{q+1}\text{Hypergeometric2F1}\left(-p,\frac{q+1}{2}\right.}{2b(q+1)}$$
$$\frac{2b(2p+q+3)x^{p+2}(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}(a+2bx)^q\left(\frac{2bx}{a}+1\right)^{-q}\text{AppellF1}\left(p+2,-p,-q,p+3,-\frac{bx}{a},-\frac{2bx}{a}\right)}{p+2}$$

input
```
Int[x^p*(a + b*x)^p*(a + 2*b*x)^q*(a*(1 + p) + 2*b*(3 + 2*p + q)*x),x]
```

output
```
(2*b*(3 + 2*p + q)*x^(2 + p)*(a + b*x)^p*(a + 2*b*x)^q*AppellF1[2 + p, -p,
 -q, 3 + p, -((b*x)/a), (-2*b*x)/a])/((2 + p)*(1 + (b*x)/a)^p*(1 + (2*b*x)
/a)^q) + (a*(1 + p)*x^p*(a + b*x)^p*(a + 2*b*x)^(1 + q)*(-1/4*a^2/b + (a +
 2*b*x)^2/(4*b))^p*Hypergeometric2F1[-p, (1 + q)/2, (3 + q)/2, (a + 2*b*x)
^2/a^2])/(2*b*(1 + q)*(a*x + b*x^2)^p*(1 - (a + 2*b*x)^2/a^2)^p)
```

Defintions of rubi rules used

rule 146
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(c + d*x)^n*((e + f*x)^p/(b*(c*e + (d*e + c*f)*x + d*f*
x^2)^n))   Subst[Int[x^m*(c*e - (d*e + c*f)^2/(4*d*f) + d*f*(x^2/b^2))^n, x
], x, a + b*x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[p, n] &&
 EqQ[b*d*e + b*c*f - 2*a*d*f, 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^p (bx+a)^p (2bx+a)^q (a(p+1)+2b(3+2p+q)x)\,dx$$

input
```
int(x^p*(b*x+a)^p*(2*b*x+a)^q*(a*(p+1)+2*b*(3+2*p+q)*x),x)
```

output
```
int(x^p*(b*x+a)^p*(2*b*x+a)^q*(a*(p+1)+2*b*(3+2*p+q)*x),x)
```

Fricas [F]

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx$$
$$= \int (2\,b(2\,p+q+3)x+a(p+1))(2\,bx+a)^q(bx+a)^p x^p\,dx$$

input
```
integrate(x^p*(b*x+a)^p*(2*b*x+a)^q*(a*(p+1)+2*b*(3+2*p+q)*x),x, algorithm
="fricas")
```

output
```
integral((a*p + 2*(2*b*p + b*q + 3*b)*x + a)*(2*b*x + a)^q*(b*x + a)^p*x^p
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.66 (sec) , antiderivative size = 1647, normalized size of antiderivative = 9.47

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx = \text{Too large to display}$$

input
```
integrate(x**p*(b*x+a)**p*(2*b*x+a)**q*(a*(p+1)+2*b*(3+2*p+q)*x),x)
```

output
```
a**(2*p + 1)*a**(q + 1)*b**q*b**(-p - q - 1)*p*meijerg(((-p/2 - q/2, -p/2
- q/2 + 1/2, 1), (1/2 - q/2, -p - q/2, -p - q/2 + 1/2)), ((-p - q/2 - 1/2,
 -p - q/2, -p/2 - q/2, -p - q/2 + 1/2, -p/2 - q/2 + 1/2), (0,)), a**2/(4*b
**2*(a/(2*b) + x)**2))/(8*2**(2*p)*pi*gamma(-p)) - a**(2*p + 1)*a**(q + 1)
*b**q*b**(-p - q - 1)*p*meijerg(((-p/2 - q/2 - 1/2, -p/2 - q/2, 1), (-q/2,
 -p - q/2 - 1/2, -p - q/2)), ((-p - q/2 - 1/2, -p/2 - q/2 - 1/2, -p - q/2,
 -p/2 - q/2, -p - q/2 - 1), (0,)), a**2/(4*b**2*(a/(2*b) + x)**2))/(4*2**(
2*p)*pi*gamma(-p)) - a**(2*p + 1)*a**(q + 1)*b**q*b**(-p - q - 1)*p*meijer
g(((-q/2 - 1, -q/2 - 1/2, -q/2, -p/2 - q/2 - 1/2, -p/2 - q/2 - 1, 1), ()),
 ((-p/2 - q/2 - 1/2, -p/2 - q/2 - 1), (-q/2 - 1, -q/2 - 1/2, -p - q/2 - 1,
 0)), a**2*exp_polar(2*I*pi)/(4*b**2*(a/(2*b) + x)**2))*exp(I*pi*p)*exp(I*
pi*q)/(4*2**(2*p)*pi*gamma(-p)) - a**(2*p + 1)*a**(q + 1)*b**q*b**(-p - q
- 1)*p*meijerg(((-q/2 - 1/2, -q/2, 1/2 - q/2, -p/2 - q/2 - 1/2, -p/2 - q/2
, 1), ()), ((-p/2 - q/2 - 1/2, -p/2 - q/2), (-q/2 - 1/2, -q/2, -p - q/2 -
1/2, 0)), a**2*exp_polar(2*I*pi)/(4*b**2*(a/(2*b) + x)**2))*exp(I*pi*p)*ex
p(I*pi*q)/(8*2**(2*p)*pi*gamma(-p)) + a**(2*p + 1)*a**(q + 1)*b**q*b**(-p
- q - 1)*q*meijerg(((-p/2 - q/2, -p/2 - q/2 + 1/2, 1), (1/2 - q/2, -p - q/
2, -p - q/2 + 1/2)), ((-p - q/2 - 1/2, -p - q/2, -p/2 - q/2, -p - q/2 + 1/
2, -p/2 - q/2 + 1/2), (0,)), a**2/(4*b**2*(a/(2*b) + x)**2))/(8*2**(2*p)*p
i*gamma(-p)) - a**(2*p + 1)*a**(q + 1)*b**q*b**(-p - q - 1)*q*meijerg((...
```

Maxima [F]

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx$$
$$=\int (2\,b(2\,p+q+3)x+a(p+1))(2\,bx+a)^q(bx+a)^p x^p\,dx$$

input
```
integrate(x^p*(b*x+a)^p*(2*b*x+a)^q*(a*(p+1)+2*b*(3+2*p+q)*x),x, algorithm
="maxima")
```

output
```
integrate((2*b*(2*p + q + 3)*x + a*(p + 1))*(2*b*x + a)^q*(b*x + a)^p*x^p,
 x)
```

Giac [F]

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx$$
$$=\int (2\,b(2\,p+q+3)x+a(p+1))(2\,bx+a)^q(bx+a)^p x^p\,dx$$

input
```
integrate(x^p*(b*x+a)^p*(2*b*x+a)^q*(a*(p+1)+2*b*(3+2*p+q)*x),x, algorithm
="giac")
```

output
```
integrate((2*b*(2*p + q + 3)*x + a*(p + 1))*(2*b*x + a)^q*(b*x + a)^p*x^p,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx$$
$$=\int x^p\,(a\,(p+1)+2\,b\,x\,(2\,p+q+3))\,(a+b\,x)^p\,(a+2\,b\,x)^q\,dx$$

input
```
int(x^p*(a*(p + 1) + 2*b*x*(2*p + q + 3))*(a + b*x)^p*(a + 2*b*x)^q,x)
```

output
```
int(x^p*(a*(p + 1) + 2*b*x*(2*p + q + 3))*(a + b*x)^p*(a + 2*b*x)^q, x)
```

Reduce [F]

$$\int x^p(a+bx)^p(a+2bx)^q(a(1+p)+2b(3+2p+q)x)\,dx=\text{too large to display}$$

input
```
int(x^p*(b*x+a)^p*(2*b*x+a)^q*(a*(p+1)+2*b*(3+2*p+q)*x),x)
```

output

```
( - 2*x**p*(a + 2*b*x)**q*(a + b*x)**p*a**2*p**3 - x**p*(a + 2*b*x)**q*(a
+ b*x)**p*a**2*p**2*q - 6*x**p*(a + 2*b*x)**q*(a + b*x)**p*a**2*p**2 - 2*x
**p*(a + 2*b*x)**q*(a + b*x)**p*a**2*p*q - 4*x**p*(a + 2*b*x)**q*(a + b*x)
**p*a**2*p - x**p*(a + 2*b*x)**q*(a + b*x)**p*a**2*q + 24*x**p*(a + 2*b*x)
**q*(a + b*x)**p*a*b*p**3*x + 32*x**p*(a + 2*b*x)**q*(a + b*x)**p*a*b*p**2
*q*x + 40*x**p*(a + 2*b*x)**q*(a + b*x)**p*a*b*p**2*x + 14*x**p*(a + 2*b*x
)**q*(a + b*x)**p*a*b*p*q**2*x + 36*x**p*(a + 2*b*x)**q*(a + b*x)**p*a*b*p
*q*x + 8*x**p*(a + 2*b*x)**q*(a + b*x)**p*a*b*p*x + 2*x**p*(a + 2*b*x)**q*
(a + b*x)**p*a*b*q**3*x + 8*x**p*(a + 2*b*x)**q*(a + b*x)**p*a*b*q**2*x +
4*x**p*(a + 2*b*x)**q*(a + b*x)**p*a*b*q*x + 32*x**p*(a + 2*b*x)**q*(a + b
*x)**p*b**2*p**3*x**2 + 48*x**p*(a + 2*b*x)**q*(a + b*x)**p*b**2*p**2*q*x*
*2 + 64*x**p*(a + 2*b*x)**q*(a + b*x)**p*b**2*p**2*x**2 + 24*x**p*(a + 2*b
*x)**q*(a + b*x)**p*b**2*p*q**2*x**2 + 64*x**p*(a + 2*b*x)**q*(a + b*x)**p
*b**2*p*q*x**2 + 24*x**p*(a + 2*b*x)**q*(a + b*x)**p*b**2*p*x**2 + 4*x**p*
(a + 2*b*x)**q*(a + b*x)**p*b**2*q**3*x**2 + 16*x**p*(a + 2*b*x)**q*(a + b
*x)**p*b**2*q**2*x**2 + 12*x**p*(a + 2*b*x)**q*(a + b*x)**p*b**2*q*x**2 -
64*int((x**p*(a + 2*b*x)**q*(a + b*x)**p*x)/(8*a**2*p**3 + 12*a**2*p**2*q
+ 12*a**2*p**2 + 6*a**2*p*q**2 + 12*a**2*p*q + 4*a**2*p + a**2*q**3 + 3*a*
*2*q**2 + 2*a**2*q + 24*a*b*p**3*x + 36*a*b*p**2*q*x + 36*a*b*p**2*x + 18*
a*b*p*q**2*x + 36*a*b*p*q*x + 12*a*b*p*x + 3*a*b*q**3*x + 9*a*b*q**2*x ...
```

3.238 $\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 368

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx$$
$$= \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n(-((bc-2ad)f)+bdfx)^n\left(-c(bc-2ad)f+2ad^2fx+bd^2fx^2\right)^{-n}\left(1-\right.}{b^2(1+m)}$$
$$+\frac{h(a+bx)^{2+m}(c+dx)^n(-((bc-2ad)f)+bdfx)^n\left(-c(bc-2ad)f+2ad^2fx+bd^2fx^2\right)^{-n}\left(1-\frac{d^2(a+bx)}{(bc-ad)}\right.}{b^2(2+m)}$$

output
```
(-a*h+b*g)*(b*x+a)^(1+m)*(d*x+c)^n*(-(-2*a*d+b*c)*f+b*d*f*x)^n*(-(-a*d+b*c
)^2*f/b+d^2*f*(b*x+a)^2/b)^n*hypergeom([-n, 1/2+1/2*m],[3/2+1/2*m],d^2*(b*
x+a)^2/(-a*d+b*c)^2)/b^2/(1+m)/((-c*(-2*a*d+b*c)*f+2*a*d^2*f*x+b*d^2*f*x^2
)^n)/((1-d^2*(b*x+a)^2/(-a*d+b*c)^2)^n)+h*(b*x+a)^(2+m)*(d*x+c)^n*(-(-2*a*
d+b*c)*f+b*d*f*x)^n*(-(-a*d+b*c)^2*f/b+d^2*f*(b*x+a)^2/b)^n*hypergeom([-n,
 1+1/2*m],[2+1/2*m],d^2*(b*x+a)^2/(-a*d+b*c)^2)/b^2/(2+m)/((-c*(-2*a*d+b*c
)*f+2*a*d^2*f*x+b*d^2*f*x^2)^n)/((1-d^2*(b*x+a)^2/(-a*d+b*c)^2)^n)
```

Mathematica [F]

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx$$
$$=\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(-(b*c*f) + 2*a*d*f + b*d*f*x)^n*(g + h*
x),x]
```

output
```
Integrate[(a + b*x)^m*(c + d*x)^n*(-(b*c*f) + 2*a*d*f + b*d*f*x)^n*(g + h*
x), x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {177, 146, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)(a+bx)^m(c+dx)^n(2adf-bcf+bdfx)^n\,dx$$

$$\Big\downarrow \text{177}$$

$$\frac{(bg-ah)\int (a+bx)^m(c+dx)^n(bdfx-(bc-2ad)f)^ndx}{b}+\frac{h\int (a+bx)^{m+1}(c+dx)^n(bdfx-(bc-2ad)f)^ndx}{b}$$

$$\Big\downarrow \text{146}$$

$$\frac{(bg-ah)(c+dx)^n(bdfx-f(bc-2ad))^n\left(-cf(bc-2ad)+2ad^2fx+bd^2fx^2\right)^{-n}\int (a+bx)^m\left(\frac{d^2f(a+bx)^2}{b}-\frac{(bc-}{}\right.}{b^2}$$
$$\frac{h(c+dx)^n(bdfx-f(bc-2ad))^n\left(-cf(bc-2ad)+2ad^2fx+bd^2fx^2\right)^{-n}\int (a+bx)^{m+1}\left(\frac{d^2f(a+bx)^2}{b}-\frac{(bc-ad)^2f}{b}\right)}{b^2}$$

$$\Big\downarrow \; 279$$

$$\frac{(bg-ah)(c+dx)^n \left(1-\frac{d^2(a+bx)^2}{(bc-ad)^2}\right)^{-n} (bdfx-f(bc-2ad))^n \left(-cf(bc-2ad)+2ad^2fx+bd^2fx^2\right)^{-n} \Big(\frac{d^2f(a+bx)^2}{b}}{b^2}$$

$$\frac{h(c+dx)^n \left(1-\frac{d^2(a+bx)^2}{(bc-ad)^2}\right)^{-n} (bdfx-f(bc-2ad))^n \left(-cf(bc-2ad)+2ad^2fx+bd^2fx^2\right)^{-n} \Big(\frac{d^2f(a+bx)^2}{b}-\frac{f(bc-}{b}}{b^2}$$

$$\Big\downarrow \; 278$$

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n \left(1-\frac{d^2(a+bx)^2}{(bc-ad)^2}\right)^{-n} (bdfx-f(bc-2ad))^n \left(-cf(bc-2ad)+2ad^2fx+bd^2fx^2\right)^{-}}{b^2(m+1)}$$

$$\frac{h(a+bx)^{m+2}(c+dx)^n \left(1-\frac{d^2(a+bx)^2}{(bc-ad)^2}\right)^{-n} (bdfx-f(bc-2ad))^n \left(-cf(bc-2ad)+2ad^2fx+bd^2fx^2\right)^{-n} \Big(\frac{d^2f(a-}{b}}{b^2(m+2)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(-(b*c*f) + 2*a*d*f + b*d*f*x)^n*(g + h*x),x]
```

output
```
((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(-((b*c - 2*a*d)*f) + b*d*f*x)^
n*(-(((b*c - a*d)^2*f)/b) + (d^2*f*(a + b*x)^2)/b)^n*Hypergeometric2F1[(1
+ m)/2, -n, (3 + m)/2, (d^2*(a + b*x)^2)/(b*c - a*d)^2])/(b^2*(1 + m)*(-(c
*(b*c - 2*a*d)*f) + 2*a*d^2*f*x + b*d^2*f*x^2)^n*(1 - (d^2*(a + b*x)^2)/(b
*c - a*d)^2)^n) + (h*(a + b*x)^(2 + m)*(c + d*x)^n*(-((b*c - 2*a*d)*f) + b
*d*f*x)^n*(-(((b*c - a*d)^2*f)/b) + (d^2*f*(a + b*x)^2)/b)^n*Hypergeometri
c2F1[(2 + m)/2, -n, (4 + m)/2, (d^2*(a + b*x)^2)/(b*c - a*d)^2])/(b^2*(2 +
 m)*(-(c*(b*c - 2*a*d)*f) + 2*a*d^2*f*x + b*d^2*f*x^2)^n*(1 - (d^2*(a + b*
x)^2)/(b*c - a*d)^2)^n)
```

Defintions of rubi rules used

rule 146
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(c + d*x)^n*((e + f*x)^p/(b*(c*e + (d*e + c*f)*x + d*f*
x^2)^n))   Subst[Int[x^m*(c*e - (d*e + c*f)^2/(4*d*f) + d*f*(x^2/b^2))^n, x
], x, a + b*x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[p, n] &&
 EqQ[b*d*e + b*c*f - 2*a*d*f, 0]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fbdx+2adf-bcf)^n (hx+g)\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(b*d*f*x+2*a*d*f-b*c*f)^n*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(b*d*f*x+2*a*d*f-b*c*f)^n*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx$$
$$=\int (hx+g)(bdfx-bcf+2\,adf)^n(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(b*d*f*x+2*a*d*f-b*c*f)^n*(h*x+g),x, algorit
hm="fricas")
```

output
```
integral((h*x + g)*(b*d*f*x - (b*c - 2*a*d)*f)^n*(b*x + a)^m*(d*x + c)^n,
x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(b*d*f*x+2*a*d*f-b*c*f)**n*(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx$$
$$= \int (hx+g)(bdfx-bcf+2\,adf)^n(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(b*d*f*x+2*a*d*f-b*c*f)^n*(h*x+g),x, algorit
hm="maxima")
```

output
```
integrate((h*x + g)*(b*d*f*x - b*c*f + 2*a*d*f)^n*(b*x + a)^m*(d*x + c)^n,
 x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx = \int (hx+g)(bdfx-bcf+2\,adf)^n(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(b*d*f*x+2*a*d*f-b*c*f)^n*(h*x+g),x, algorit
hm="giac")
```

output
```
integrate((h*x + g)*(b*d*f*x - b*c*f + 2*a*d*f)^n*(b*x + a)^m*(d*x + c)^n,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx = \int (g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^n\,(2\,a\,d\,f-b\,c\,f+b\,d\,f\,x)^n\,dx$$

input
```
int((g + h*x)*(a + b*x)^m*(c + d*x)^n*(2*a*d*f - b*c*f + b*d*f*x)^n,x)
```

output
```
int((g + h*x)*(a + b*x)^m*(c + d*x)^n*(2*a*d*f - b*c*f + b*d*f*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(-bcf+2adf+bdfx)^n(g+hx)\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(b*d*f*x+2*a*d*f-b*c*f)^n*(h*x+g),x)
```

output
```
( - (c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a**3*d**2*h*m
*n - (c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a**3*d**2*h*
m - 2*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a**3*d**2*h
*n**2 - 2*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a**3*d*
*2*h*n + 2*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a**2*b
*c*d*h*m*n + 4*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a*
*2*b*c*d*h*n**2 + (c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n
*a**2*b*d**2*g*m**2 + 3*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f
*x)**n*a**2*b*d**2*g*m*n + 2*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f +
b*d*f*x)**n*a**2*b*d**2*g*m + 2*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f
 + b*d*f*x)**n*a**2*b*d**2*g*n**2 + 2*(c + d*x)**n*(a + b*x)**m*(2*a*d*f -
 b*c*f + b*d*f*x)**n*a**2*b*d**2*g*n + (c + d*x)**n*(a + b*x)**m*(2*a*d*f
- b*c*f + b*d*f*x)**n*a**2*b*d**2*h*m**2*x + 4*(c + d*x)**n*(a + b*x)**m*(
2*a*d*f - b*c*f + b*d*f*x)**n*a**2*b*d**2*h*m*n*x + 4*(c + d*x)**n*(a + b*
x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a**2*b*d**2*h*n**2*x - (c + d*x)**n*(
a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a*b**2*c**2*h*m*n - 2*(c + d*x)
**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a*b**2*c**2*h*n**2 + 2*(c
+ d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a*b**2*c*d*g*m*n + 4
*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a*b**2*c*d*g*n**
2 + 4*(c + d*x)**n*(a + b*x)**m*(2*a*d*f - b*c*f + b*d*f*x)**n*a*b**2*c...
```

3.239 $\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 368

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$$
$$= \frac{(dg-ch)(a+bx)^m(c+dx)^{1+n}((2bc-ad)f+bdfx)^m\left(a(2bc-ad)f+2b^2cfx+b^2dfx^2\right)^{-m}\left(1-\frac{b^2(c+dx)^2}{(bc-ad)^2}\right)^{-m}\cdots}{d^2(1+n)}$$
$$+\frac{h(a+bx)^m(c+dx)^{2+n}((2bc-ad)f+bdfx)^m\left(a(2bc-ad)f+2b^2cfx+b^2dfx^2\right)^{-m}\left(1-\frac{b^2(c+dx)^2}{(bc-ad)^2}\right)^{-m}\cdots}{d^2(2+n)}$$

output
```
(-c*h+d*g)*(b*x+a)^m*(d*x+c)^(1+n)*((-a*d+2*b*c)*f+b*d*f*x)^m*(-(-a*d+b*c)
^2*f/d+b^2*f*(d*x+c)^2/d)^m*hypergeom([-m, 1/2+1/2*n],[3/2+1/2*n],b^2*(d*x
+c)^2/(-a*d+b*c)^2)/d^2/(1+n)/((a*(-a*d+2*b*c)*f+2*b^2*c*f*x+b^2*d*f*x^2)^
m)/((1-b^2*(d*x+c)^2/(-a*d+b*c)^2)^m)+h*(b*x+a)^m*(d*x+c)^(2+n)*((-a*d+2*b
*c)*f+b*d*f*x)^m*(-(-a*d+b*c)^2*f/d+b^2*f*(d*x+c)^2/d)^m*hypergeom([-m, 1+
1/2*n],[2+1/2*n],b^2*(d*x+c)^2/(-a*d+b*c)^2)/d^2/(2+n)/((a*(-a*d+2*b*c)*f+
2*b^2*c*f*x+b^2*d*f*x^2)^m)/((1-b^2*(d*x+c)^2/(-a*d+b*c)^2)^m)
```

Mathematica [F]

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx = \int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(2*b*c*f - a*d*f + b*d*f*x)^m*(g + h*x),
x]
```

output
```
Integrate[(a + b*x)^m*(c + d*x)^n*(2*b*c*f - a*d*f + b*d*f*x)^m*(g + h*x),
 x]
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.51 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {177, 146, 157, 156, 155, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)(a+bx)^m(c+dx)^n(-adf+2bcf+bdfx)^m\,dx$$

$$\Big\downarrow \text{177}$$

$$\frac{(bg-ah)\int (a+bx)^m(c+dx)^n((2bc-ad)f+bdxf)^m dx}{b}+\frac{h\int (a+bx)^{m+1}(c+dx)^n((2bc-ad)f+bdxf)^m dx}{b}$$

$$\Big\downarrow \text{146}$$

$$\frac{(bg-ah)(a+bx)^m(f(2bc-ad)+bdfx)^m\left(af(2bc-ad)+2b^2cfx+b^2dfx^2\right)^{-m}\int(c+dx)^n\left(\frac{b^2f(c+dx)^2}{d}-\frac{(bc-ad}{d}\right.}{bd}$$
$$\frac{h\int(a+bx)^{m+1}(c+dx)^n((2bc-ad)f+bdxf)^mdx}{b}$$

$\big\downarrow$ 157

$$\frac{(bg-ah)(a+bx)^m(f(2bc-ad)+bdfx)^m\left(af(2bc-ad)+2b^2cfx+b^2dfx^2\right)^{-m}\int(c+dx)^n\left(\frac{b^2f(c+dx)^2}{d}-\frac{(bc-ad}{d}\right.}{bd}$$
$$\frac{h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n((2bc-ad)f+bdxf)^mdx}{b}$$

$\big\downarrow$ 156

$$\frac{(bg-ah)(a+bx)^m(f(2bc-ad)+bdfx)^m\left(af(2bc-ad)+2b^2cfx+b^2dfx^2\right)^{-m}\int(c+dx)^n\left(\frac{b^2f(c+dx)^2}{d}-\frac{(bc-ad}{d}\right.}{bd}$$
$$\frac{h2^m(c+dx)^n\left(\frac{-ad+2bc+bdx}{bc-ad}\right)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(f(2bc-ad)+bdfx)^m\int(a+bx)^{m+1}\left(\frac{2bc-ad}{2(bc-ad)}+\frac{bdx}{2(bc-ad)}\right)^m\left(\frac{bc}{bc-ad}\right.}{b}$$

$\big\downarrow$ 155

$$\frac{(bg-ah)(a+bx)^m(f(2bc-ad)+bdfx)^m\left(af(2bc-ad)+2b^2cfx+b^2dfx^2\right)^{-m}\int(c+dx)^n\left(\frac{b^2f(c+dx)^2}{d}-\frac{(bc-ad}{d}\right.}{bd}$$
$$\frac{h2^m(a+bx)^{m+2}(c+dx)^n\left(\frac{-ad+2bc+bdx}{bc-ad}\right)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(f(2bc-ad)+bdfx)^m\operatorname{AppellF1}\left(m+2,-m,-n,m-\right.}{b^2(m+2)}$$

$\big\downarrow$ 279

$$\frac{(bg-ah)(a+bx)^m\left(1-\frac{b^2(c+dx)^2}{(bc-ad)^2}\right)^{-m}(f(2bc-ad)+bdfx)^m\left(af(2bc-ad)+2b^2cfx+b^2dfx^2\right)^{-m}\left(\frac{b^2f(c+dx)^2}{d}\right.}{bd}$$
$$\frac{h2^m(a+bx)^{m+2}(c+dx)^n\left(\frac{-ad+2bc+bdx}{bc-ad}\right)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(f(2bc-ad)+bdfx)^m\operatorname{AppellF1}\left(m+2,-m,-n,m-\right.}{b^2(m+2)}$$

$\big\downarrow$ 278

$$\frac{h2^m(a+bx)^{m+2}(c+dx)^n\left(\frac{-ad+2bc+bdx}{bc-ad}\right)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(f(2bc-ad)+bdfx)^m\operatorname{AppellF1}\left(m+2,-m,-n,m-\right.}{b^2(m+2)}$$
$$\frac{(bg-ah)(a+bx)^m(c+dx)^{n+1}\left(1-\frac{b^2(c+dx)^2}{(bc-ad)^2}\right)^{-m}(f(2bc-ad)+bdfx)^m\left(af(2bc-ad)+2b^2cfx+b^2dfx^2\right)^{-m}}{bd(n+1)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(2*b*c*f - a*d*f + b*d*f*x)^m*(g + h*x),x]
```

output
```
(2^m*h*(a + b*x)^(2 + m)*(c + d*x)^n*((2*b*c - a*d)*f + b*d*f*x)^m*AppellF
1[2 + m, -m, -n, 3 + m, -1/2*(d*(a + b*x))/(b*c - a*d), -((d*(a + b*x))/(b
*c - a*d))])/(b^2*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((2*b*c - a*d + b*
d*x)/(b*c - a*d))^m) + ((b*g - a*h)*(a + b*x)^m*(c + d*x)^(1 + n)*((2*b*c
- a*d)*f + b*d*f*x)^m*(-(((b*c - a*d)^2*f)/d) + (b^2*f*(c + d*x)^2)/d)^m*H
ypergeometric2F1[-m, (1 + n)/2, (3 + n)/2, (b^2*(c + d*x)^2)/(b*c - a*d)^2
])/(b*d*(1 + n)*(a*(2*b*c - a*d)*f + 2*b^2*c*f*x + b^2*d*f*x^2)^m*(1 - (b^
2*(c + d*x)^2)/(b*c - a*d)^2)^m)
```

Defintions of rubi rules used

rule 146
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(c + d*x)^n*((e + f*x)^p/(b*(c*e + (d*e + c*f)*x + d*f*
x^2)^n))   Subst[Int[x^m*(c*e - (d*e + c*f)^2/(4*d*f) + d*f*(x^2/b^2))^n, x
], x, a + b*x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[p, n] &&
 EqQ[b*d*e + b*c*f - 2*a*d*f, 0]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fbdx-adf+2bcf)^m (hx+g)\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(b*d*f*x-a*d*f+2*b*c*f)^m*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(b*d*f*x-a*d*f+2*b*c*f)^m*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$$
$$=\int (hx+g)(bdfx+2\,bcf-adf)^m(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(b*d*f*x-a*d*f+2*b*c*f)^m*(h*x+g),x, algorit
hm="fricas")
```

output
```
integral((h*x + g)*(b*d*f*x + (2*b*c - a*d)*f)^m*(b*x + a)^m*(d*x + c)^n,
x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(b*d*f*x-a*d*f+2*b*c*f)**m*(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$$
$$=\int (hx+g)(bdfx+2\,bcf-adf)^m(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(b*d*f*x-a*d*f+2*b*c*f)^m*(h*x+g),x, algorit
hm="maxima")
```

output
```
integrate((h*x + g)*(b*d*f*x + 2*b*c*f - a*d*f)^m*(b*x + a)^m*(d*x + c)^n,
 x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$$
$$=\int (hx+g)(bdfx+2\,bcf-adf)^m(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(b*d*f*x-a*d*f+2*b*c*f)^m*(h*x+g),x, algorit
hm="giac")
```

output
```
integrate((h*x + g)*(b*d*f*x + 2*b*c*f - a*d*f)^m*(b*x + a)^m*(d*x + c)^n,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx$$
$$=\int (g+h\,x)\;(a+b\,x)^m\;(c+d\,x)^n\;(2\,b\,c\,f-a\,d\,f+b\,d\,f\,x)^m\,dx$$

input
```
int((g + h*x)*(a + b*x)^m*(c + d*x)^n*(2*b*c*f - a*d*f + b*d*f*x)^m,x)
```

output
```
int((g + h*x)*(a + b*x)^m*(c + d*x)^n*(2*b*c*f - a*d*f + b*d*f*x)^m, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(2bcf-adf+bdfx)^m(g+hx)\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(b*d*f*x-a*d*f+2*b*c*f)^m*(h*x+g),x)
```

output
```
( - 2*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*a**2*c*d
**2*h*m**2 - (c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*a
**2*c*d**2*h*m*n - 2*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f
*x)**m*a**2*d**3*g*m**2 - (c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f +
b*d*f*x)**m*a**2*d**3*g*m*n - 2*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*
c*f + b*d*f*x)**m*a**2*d**3*g*m + 4*(c + d*x)**n*(a + b*x)**m*( - a*d*f +
2*b*c*f + b*d*f*x)**m*a*b*c**2*d*h*m**2 + 2*(c + d*x)**n*(a + b*x)**m*( -
a*d*f + 2*b*c*f + b*d*f*x)**m*a*b*c**2*d*h*m*n + 4*(c + d*x)**n*(a + b*x)*
*m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*a*b*c*d**2*g*m**2 + 2*(c + d*x)**n*(a
 + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*a*b*c*d**2*g*m*n + 4*(c + d*x
)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*a*b*c*d**2*g*m - 2*(c
+ d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*b**2*c**3*h*m**2
- (c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*b**2*c**3*h*
m*n - 2*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*b**2*c
**3*h*m - (c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*b**2
*c**3*h*n + 2*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d*f*x)**m*
b**2*c**2*d*g*m**2 + 3*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c*f + b*d
*f*x)**m*b**2*c**2*d*g*m*n + 2*(c + d*x)**n*(a + b*x)**m*( - a*d*f + 2*b*c
*f + b*d*f*x)**m*b**2*c**2*d*g*m + (c + d*x)**n*(a + b*x)**m*( - a*d*f + 2
*b*c*f + b*d*f*x)**m*b**2*c**2*d*g*n**2 + 2*(c + d*x)**n*(a + b*x)**m*(...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file