Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/24-1.1.1.4b

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**158**]. This is test number [24].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.37 (157)	0.63 (1)
Mathematica	96.84 (153)	3.16 (5)
Maple	76.58 (121)	23.42 (37)
Fricas	27.85 (44)	72.15 (114)
Giac	20.25 (32)	79.75 (126)
Reduce	16.46 (26)	83.54 (132)
Mupad	11.39 (18)	88.61 (140)
Maxima	9.49 (15)	90.51 (143)
Sympy	5.06 (8)	94.94 (150)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

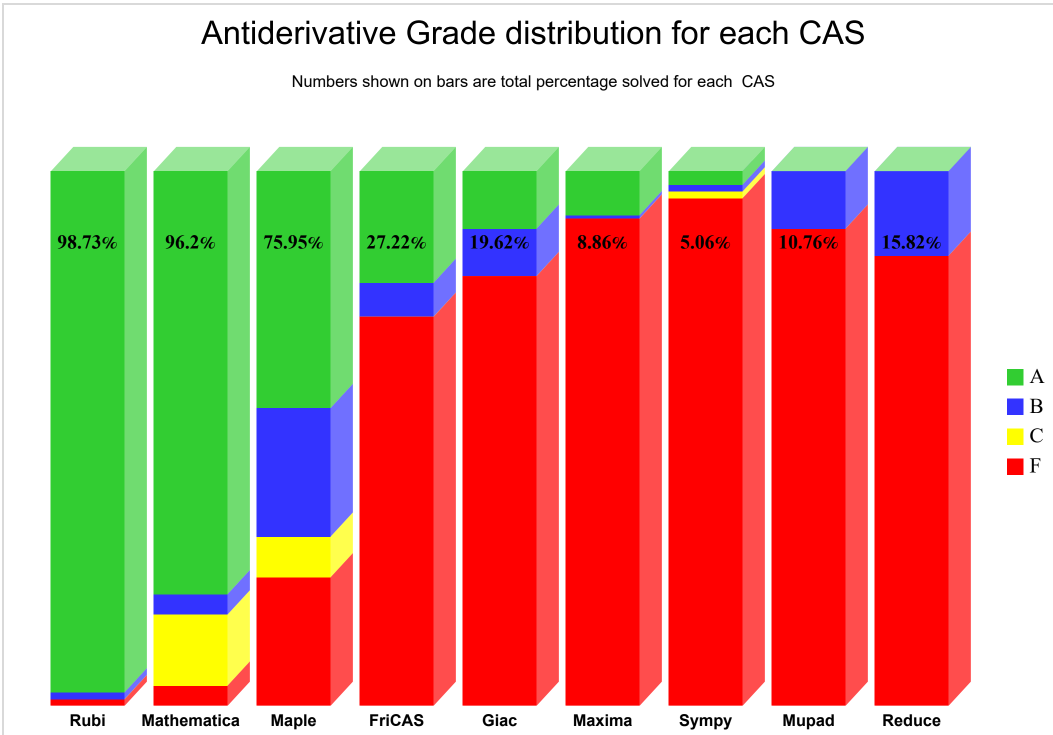
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

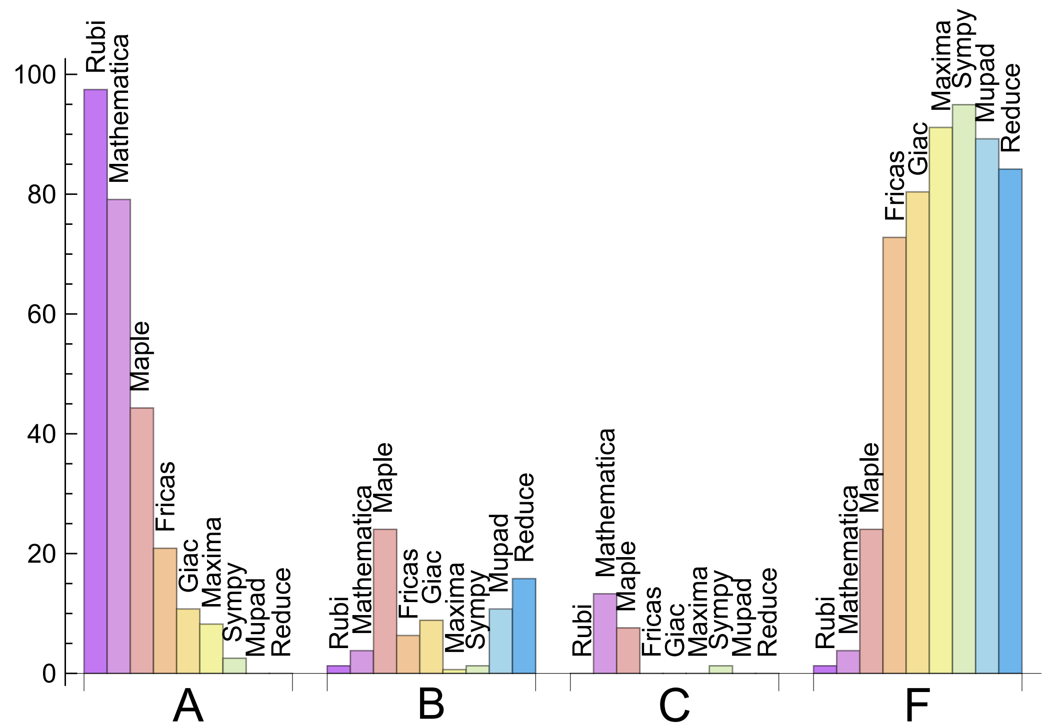
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.468	1.266	0.000	1.266
Mathematica	79.114	3.797	13.291	3.797
Maple	44.304	24.051	7.595	24.051
Fricas	20.886	6.329	0.000	72.785
Giac	10.759	8.861	0.000	80.380
Maxima	8.228	0.633	0.000	91.139
Sympy	2.532	1.266	1.266	94.937
Mupad	0.000	10.759	0.000	89.241
Reduce	0.000	15.823	0.000	84.177

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	5	100.00	0.00	0.00
Maple	37	100.00	0.00	0.00
Fricas	114	71.05	28.95	0.00
Giac	126	89.68	4.76	5.56
Reduce	132	100.00	0.00	0.00
Mupad	140	0.00	100.00	0.00
Maxima	143	98.60	0.00	1.40
Sympy	150	69.33	17.33	13.33

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Rubi	0.52
Maple	2.29
Mupad	3.10
Fricas	6.40
Mathematica	7.59
Reduce	8.45
Sympy	9.36
Giac	19.64

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	104.87	0.96	43.00	0.83
Rubi	275.84	1.17	245.00	1.09
Mathematica	350.92	1.19	198.00	0.95
Mupad	393.50	2.58	110.00	1.21
Fricas	405.55	2.14	61.50	1.03
Reduce	506.69	3.09	67.50	1.23
Giac	650.66	2.81	149.00	1.37
Maple	979.17	3.63	397.00	1.36
Sympy	1163.50	8.17	131.50	2.16

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

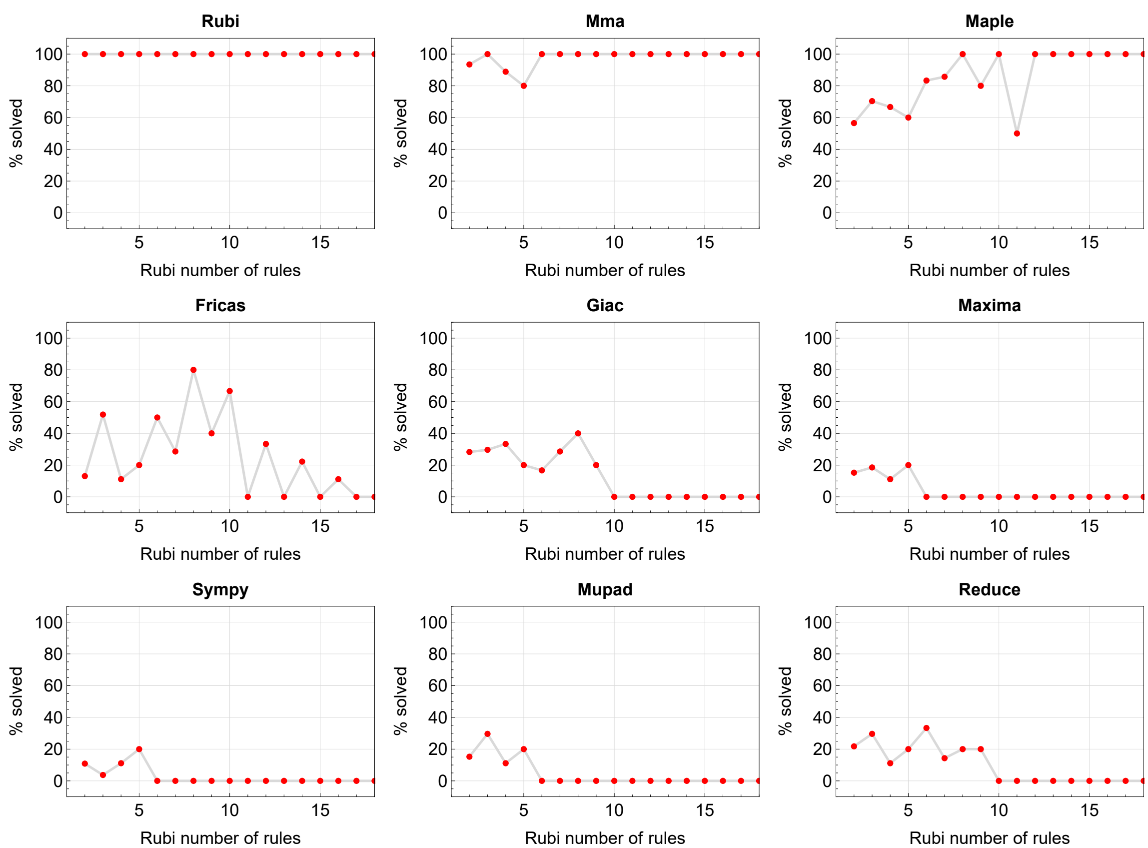


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

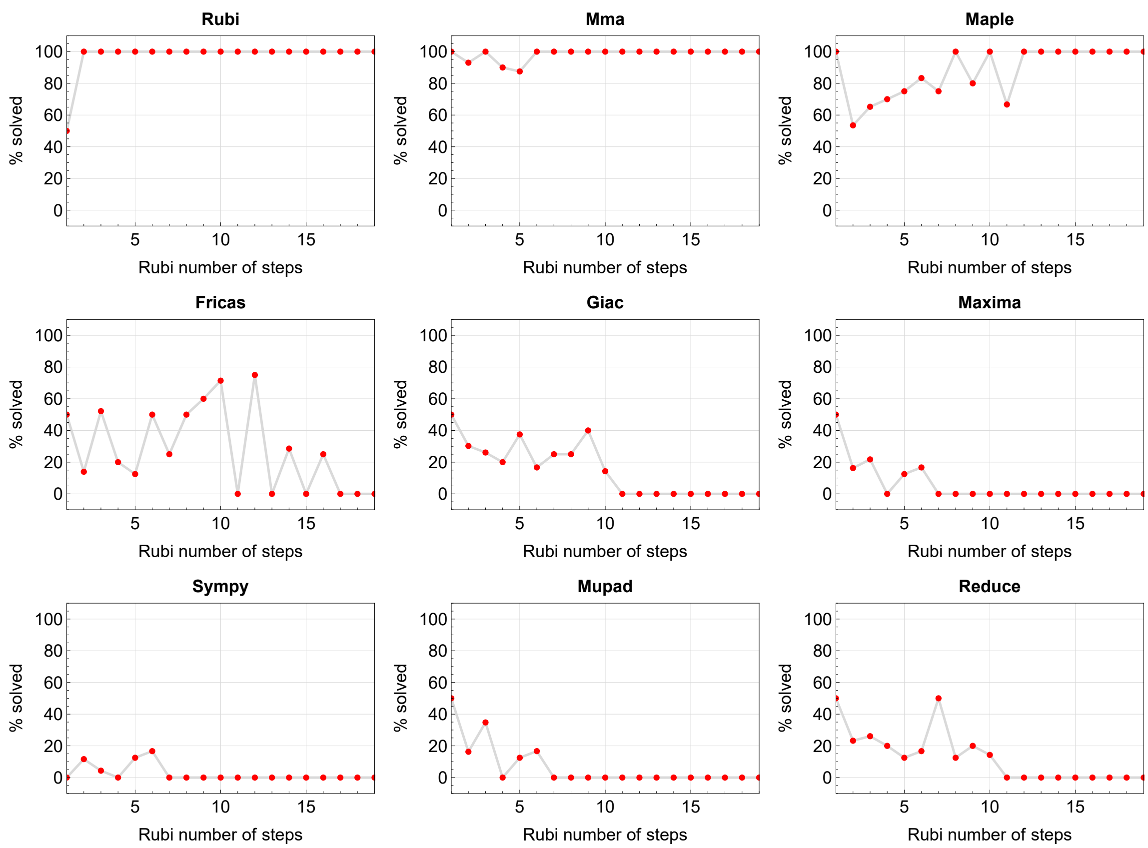


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

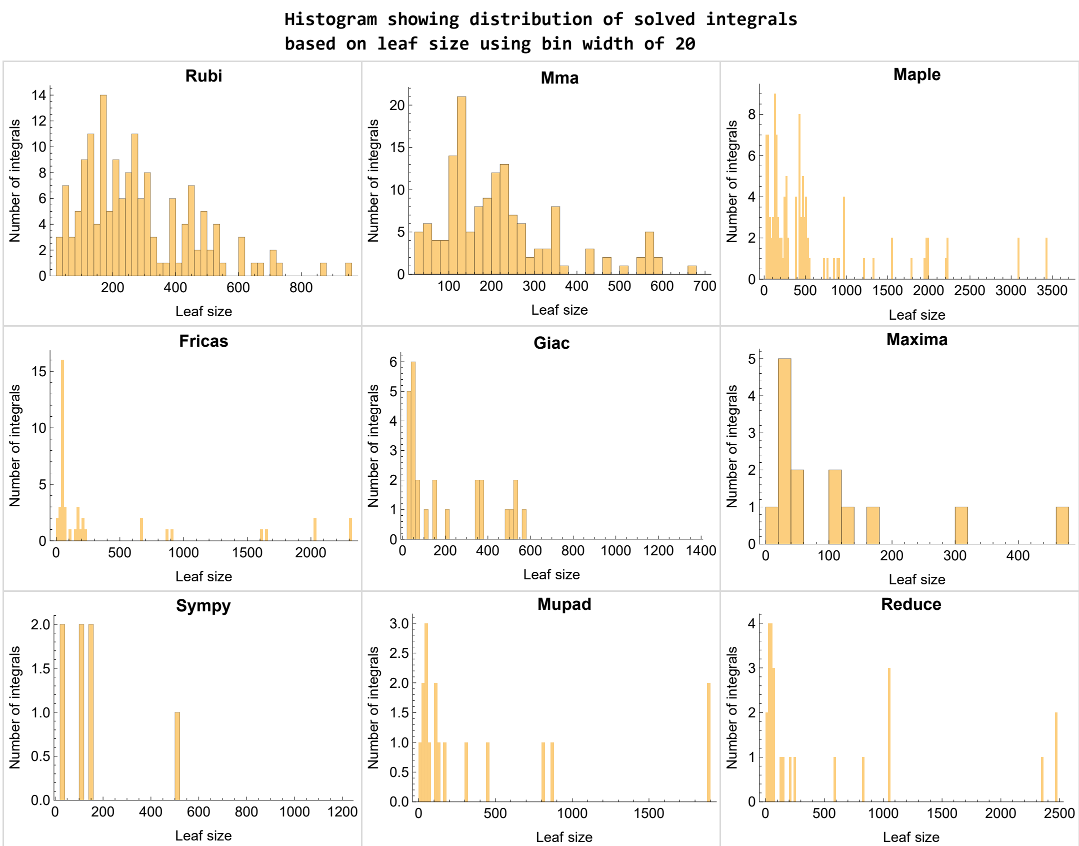


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

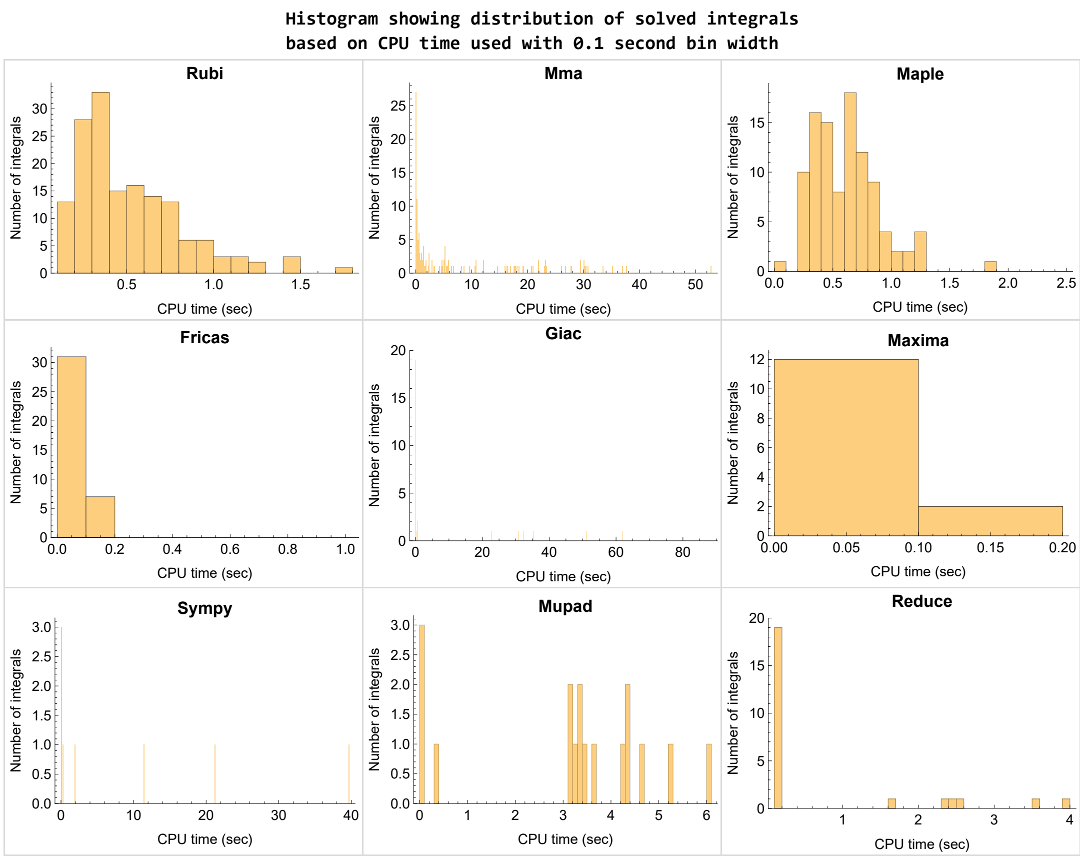


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

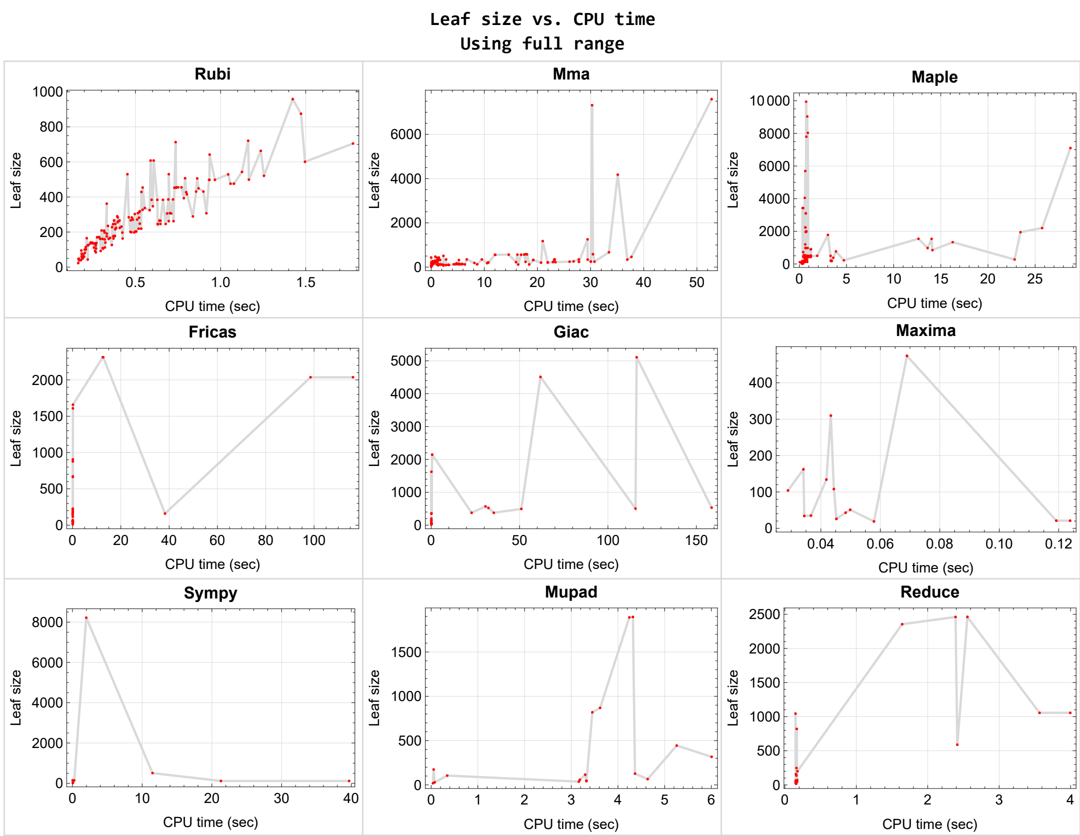


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{158}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {137, 138, 139, 140, 144, 146, 148, 150}

Mathematica {65, 74, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 115, 116, 136, 139, 140, 144, 146, 147, 148, 150, 154}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

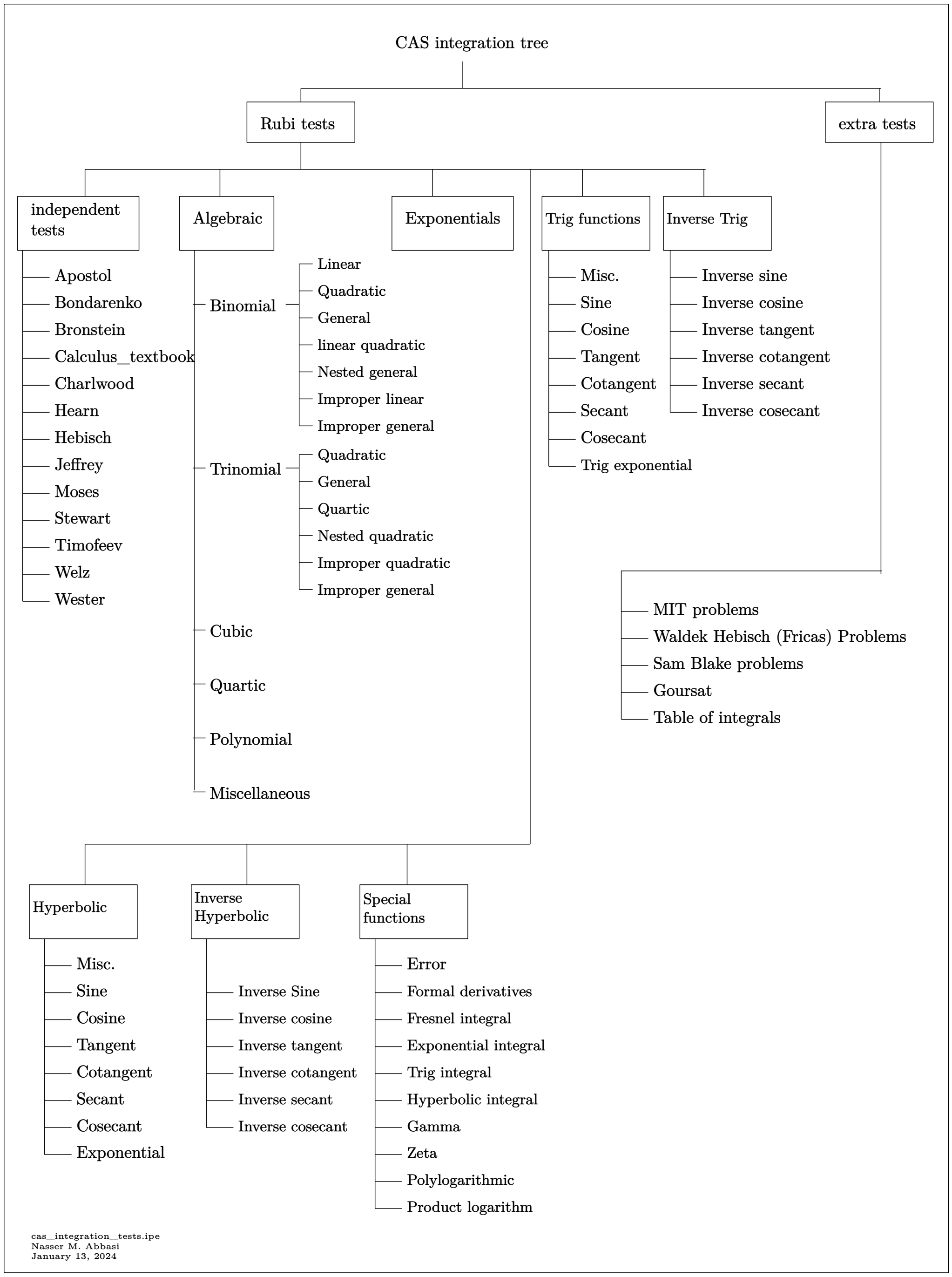
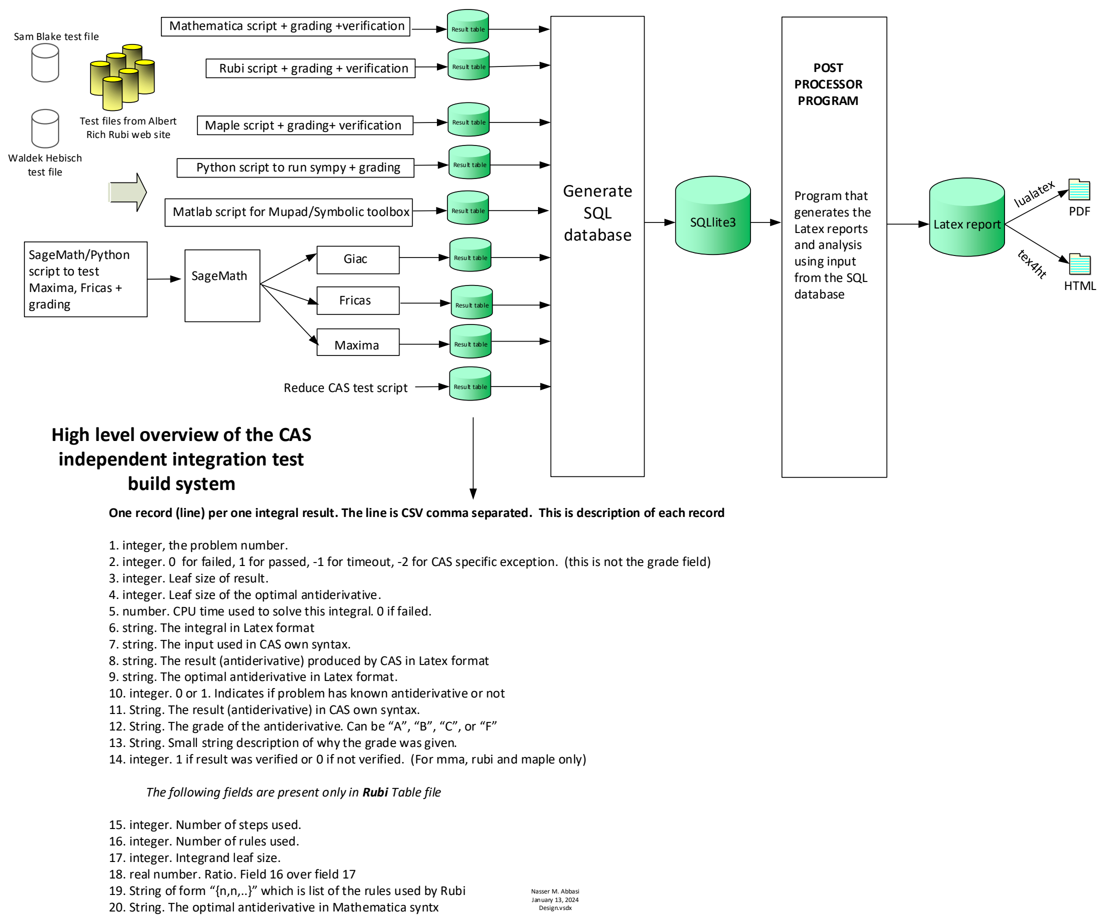


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

B grade { 102, 110 }

C grade { }

F normal fail { 116 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 67, 68, 69, 70, 72, 73, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157 }

B grade { 61, 102, 104, 112, 113, 117 }

C grade { 50, 65, 66, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90 }

F normal fail { 141, 142, 143, 155, 156 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 78, 83, 84, 85, 86, 91, 92, 93, 94, 98, 99, 100, 101, 106, 107, 109, 112, 114 }

B grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 74, 77, 79, 80, 81, 82, 104, 105, 113, 115, 116, 130, 149, 151, 152 }

C grade { 87, 88, 89, 90, 95, 96, 97, 102, 103, 108, 110, 111 }

F normal fail { 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 153, 154, 155, 156, 157 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 58, 59, 60, 61, 67, 68, 69, 70 }

B grade { 10, 11, 14, 24, 74, 75, 130, 149, 151, 152 }

C grade { }

F normal fail { 46, 47, 48, 49, 55, 56, 57, 62, 63, 64, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 153, 154, 155, 156, 157 }

F(-1) timedout fail { 5, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 50, 65, 66, 76, 77, 78, 79, 80, 81, 82, 104, 112, 113 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 33, 34, 35, 36 }

B grade { 130 }

C grade { }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

F(-1) timedout fail { }

F(-2) exception fail { 10, 11 }

Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 33, 34, 35, 36, 37, 38, 39, 40, 41 }

B grade { 5, 10, 11, 14, 15, 20, 21, 24, 25, 26, 27, 29, 30, 130 }

C grade { }

F normal fail { 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

F(-1) timedout fail { 12, 16, 18, 19, 31, 82 }

F(-2) exception fail { 13, 17, 22, 23, 28, 137, 142 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 33, 34, 35, 36, 130, 149, 151, 152 }

C grade { }

F normal fail { }

F(-1) timedout fail { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 153, 154, 155, 156, 157 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 6, 7 }

B grade { 3, 130 }

C grade { 8, 9 }

F normal fail { 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 112, 113, 114, 115, 116, 119, 120, 124, 125, 126, 127, 128, 131, 132 }

F(-1) timedout fail { 4, 5, 16, 21, 22, 28, 88, 89, 90, 96, 97, 98, 106, 111, 122, 129, 136, 137, 138, 141, 142, 143, 155, 156, 157, 158 }

F(-2) exception fail { 117, 118, 121, 123, 133, 134, 135, 139, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 19, 23, 24, 25, 33, 34, 35, 36, 37, 38, 39, 40, 41, 130 }

C grade { }

F normal fail { 10, 11, 12, 15, 16, 17, 18, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	109	108	142	148	142	137	115
N.S.	1	1.00	1.00	0.97	0.96	1.27	1.32	1.27	1.22	1.03
time (sec)	N/A	0.321	0.029	0.052	0.044	0.037	0.022	0.117	0.160	3.299

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	175	162	163	146	200	247	174
N.S.	1	1.00	0.98	1.39	1.29	1.29	1.16	1.59	1.96	1.38
time (sec)	N/A	0.361	0.048	0.286	0.034	0.068	0.259	0.123	0.162	0.053

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	85	102	104	117	507	108	157	105
N.S.	1	1.00	1.01	1.21	1.24	1.39	6.04	1.29	1.87	1.25
time (sec)	N/A	0.267	0.039	0.360	0.029	0.101	11.486	0.116	0.155	0.344

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	102	108	134	160	0	156	200	127
N.S.	1	1.00	0.94	1.00	1.24	1.48	0.00	1.44	1.85	1.18
time (sec)	N/A	0.308	0.044	0.477	0.042	38.296	0.000	0.123	0.176	4.365

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	164	310	0	0	351	821	317
N.S.	1	1.00	1.01	1.01	1.90	0.00	0.00	2.15	5.04	1.94
time (sec)	N/A	0.425	0.106	0.720	0.043	0.000	0.000	0.124	0.166	6.006

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83	0.83
time (sec)	N/A	0.164	0.005	0.271	0.058	0.072	0.053	0.124	0.159	0.039

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	30	34	53	32	31	70	29
N.S.	1	1.00	0.77	0.70	0.79	1.23	0.74	0.72	1.63	0.67
time (sec)	N/A	0.220	0.017	0.270	0.034	0.071	0.067	0.121	0.172	0.075

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	44	21	40	117	43	48	65
N.S.	1	1.00	0.95	1.13	0.54	1.03	3.00	1.10	1.23	1.67
time (sec)	N/A	0.180	0.055	0.313	0.124	0.083	21.295	0.131	0.171	4.636

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	44	21	40	117	43	48	444
N.S.	1	1.00	0.95	1.13	0.54	1.03	3.00	1.10	1.23	11.38
time (sec)	N/A	0.170	0.002	0.283	0.119	0.078	39.684	0.131	0.170	5.258

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	506	433	3427	0	2312	0	2146	29	0
N.S.	1	1.66	1.42	11.27	0.00	7.61	0.00	7.06	0.10	0.00
time (sec)	N/A	0.863	1.447	0.408	0.000	12.708	0.000	0.578	200.018	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	308	433	3427	0	2312	0	2146	55	0
N.S.	1	1.01	1.42	11.27	0.00	7.61	0.00	7.06	0.18	0.00
time (sec)	N/A	0.703	0.010	0.340	0.000	12.539	0.000	0.571	200.017	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	384	265	2239	0	0	0	0	31	0
N.S.	1	1.46	1.01	8.51	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.629	0.977	0.631	0.000	0.000	0.000	0.000	200.031	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	290	209	1200	0	0	0	0	589	0
N.S.	1	1.35	0.97	5.58	0.00	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.393	0.501	0.623	0.000	0.000	0.000	0.000	2.414	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	231	512	0	2036	0	377	1056	0
N.S.	1	1.00	1.43	3.18	0.00	12.65	0.00	2.34	6.56	0.00
time (sec)	N/A	0.293	1.053	0.648	0.000	98.539	0.000	22.883	3.566	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	266	204	1964	0	0	0	494	31	0
N.S.	1	1.30	1.00	9.63	0.00	0.00	0.00	2.42	0.15	0.00
time (sec)	N/A	0.375	0.607	0.714	0.000	0.000	0.000	51.008	200.025	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	454	318	9943	0	0	0	0	31	0
N.S.	1	1.46	1.03	32.07	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.542	1.347	0.718	0.000	0.000	0.000	0.000	200.024	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	705	365	4048	0	0	0	0	31	0
N.S.	1	1.81	0.94	10.38	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.777	1.338	0.576	0.000	0.000	0.000	0.000	200.027	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	384	263	2239	0	0	0	0	31	0
N.S.	1	1.46	1.00	8.51	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.595	0.940	0.651	0.000	0.000	0.000	0.000	200.027	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	306	214	978	0	0	0	0	2461	0
N.S.	1	1.42	1.00	4.55	0.00	0.00	0.00	0.00	11.45	0.00
time (sec)	N/A	0.686	0.526	0.637	0.000	0.000	0.000	0.000	2.556	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	265	207	3098	0	0	0	533	31	0
N.S.	1	1.27	0.99	14.82	0.00	0.00	0.00	2.55	0.15	0.00
time (sec)	N/A	0.647	10.765	0.681	0.000	0.000	0.000	158.670	200.026	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	347	299	7799	0	0	0	4509	31	0
N.S.	1	1.15	0.99	25.74	0.00	0.00	0.00	14.88	0.10	0.00
time (sec)	N/A	0.602	1.032	0.745	0.000	0.000	0.000	61.867	200.033	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	499	269	1782	0	0	0	0	31	0
N.S.	1	1.86	1.00	6.62	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.166	0.600	3.028	0.000	0.000	0.000	0.000	200.026	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	306	214	978	0	0	0	0	2461	0
N.S.	1	1.42	1.00	4.55	0.00	0.00	0.00	0.00	11.45	0.00
time (sec)	N/A	0.710	0.524	0.694	0.000	0.000	0.000	0.000	2.390	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	231	512	0	2036	0	377	1056	0
N.S.	1	1.00	1.43	3.18	0.00	12.65	0.00	2.34	6.56	0.00
time (sec)	N/A	0.306	1.178	0.688	0.000	116.127	0.000	35.437	3.996	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	154	289	0	0	0	359	2355	0
N.S.	1	1.00	0.94	1.77	0.00	0.00	0.00	2.20	14.45	0.00
time (sec)	N/A	0.371	0.406	0.734	0.000	0.000	0.000	0.200	1.642	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	271	220	1998	0	0	0	506	31	0
N.S.	1	1.23	1.00	9.08	0.00	0.00	0.00	2.30	0.14	0.00
time (sec)	N/A	0.510	0.688	0.727	0.000	0.000	0.000	115.590	200.027	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	415	329	8033	0	0	0	5109	31	0
N.S.	1	1.28	1.02	24.79	0.00	0.00	0.00	15.77	0.10	0.00
time (sec)	N/A	0.802	1.684	0.895	0.000	0.000	0.000	116.245	200.031	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	529	269	5696	0	0	0	0	31	0
N.S.	1	1.97	1.00	21.25	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.045	0.847	0.626	0.000	0.000	0.000	0.000	200.024	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	265	207	3098	0	0	0	571	31	0
N.S.	1	1.27	0.99	14.82	0.00	0.00	0.00	2.73	0.15	0.00
time (sec)	N/A	0.641	10.718	0.684	0.000	0.000	0.000	30.737	200.025	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	266	204	1964	0	0	0	523	31	0
N.S.	1	1.30	1.00	9.63	0.00	0.00	0.00	2.56	0.15	0.00
time (sec)	N/A	0.374	0.580	0.665	0.000	0.000	0.000	32.413	200.027	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	271	220	1998	0	0	0	0	31	0
N.S.	1	1.23	1.00	9.08	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.485	0.537	0.745	0.000	0.000	0.000	0.000	200.028	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	427	347	9036	0	0	0	0	31	0
N.S.	1	1.24	1.01	26.27	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.798	2.812	0.839	0.000	0.000	0.000	0.000	200.028	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	81	50	48	51	54	0	48	35	47
N.S.	1	0.46	0.29	0.27	0.29	0.31	0.00	0.27	0.20	0.27
time (sec)	N/A	0.203	0.029	0.296	0.050	0.082	0.000	0.119	0.162	3.323

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	72	45	43	43	49	0	41	30	42
N.S.	1	0.49	0.31	0.29	0.29	0.33	0.00	0.28	0.20	0.29
time (sec)	N/A	0.194	0.027	0.296	0.048	0.086	0.000	0.108	0.159	3.326

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	61	40	38	35	44	0	34	25	37
N.S.	1	0.52	0.34	0.32	0.30	0.38	0.00	0.29	0.21	0.32
time (sec)	N/A	0.185	0.023	0.297	0.037	0.085	0.000	0.109	0.151	3.163

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	50	33	31	26	37	0	25	18	58
N.S.	1	0.57	0.38	0.36	0.30	0.43	0.00	0.29	0.21	0.67
time (sec)	N/A	0.188	0.020	0.292	0.045	0.091	0.000	0.107	0.158	3.185

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	49	43	57	0	176	0	38	39	0
N.S.	1	0.53	0.47	0.62	0.00	1.91	0.00	0.41	0.42	0.00
time (sec)	N/A	0.191	0.028	0.315	0.000	0.092	0.000	0.109	0.157	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	57	48	62	0	188	0	49	46	0
N.S.	1	0.60	0.51	0.65	0.00	1.98	0.00	0.52	0.48	0.00
time (sec)	N/A	0.188	0.043	0.306	0.000	0.096	0.000	0.107	0.150	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	76	58	73	0	207	0	54	55	0
N.S.	1	0.58	0.44	0.55	0.00	1.57	0.00	0.41	0.42	0.00
time (sec)	N/A	0.185	0.043	0.298	0.000	0.094	0.000	0.115	0.153	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	95	63	81	0	217	0	61	60	0
N.S.	1	0.58	0.38	0.49	0.00	1.32	0.00	0.37	0.36	0.00
time (sec)	N/A	0.182	0.044	0.297	0.000	0.092	0.000	0.110	0.151	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	114	68	89	0	227	0	68	65	0
N.S.	1	0.58	0.34	0.45	0.00	1.15	0.00	0.34	0.33	0.00
time (sec)	N/A	0.209	0.052	0.300	0.000	0.091	0.000	0.108	0.152	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	307	135	154	0	69	0	0	194	0
N.S.	1	1.11	0.49	0.56	0.00	0.25	0.00	0.00	0.70	0.00
time (sec)	N/A	0.917	4.697	0.572	0.000	0.086	0.000	0.000	1.164	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	262	130	149	0	64	0	0	171	0
N.S.	1	1.10	0.55	0.63	0.00	0.27	0.00	0.00	0.72	0.00
time (sec)	N/A	0.696	4.670	0.455	0.000	0.076	0.000	0.000	0.978	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	209	125	144	0	59	0	0	148	0
N.S.	1	1.11	0.66	0.77	0.00	0.31	0.00	0.00	0.79	0.00
time (sec)	N/A	0.301	1.376	0.394	0.000	0.077	0.000	0.000	0.818	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	171	120	139	0	54	0	0	125	0
N.S.	1	1.09	0.76	0.89	0.00	0.34	0.00	0.00	0.80	0.00
time (sec)	N/A	0.280	1.583	0.390	0.000	0.077	0.000	0.000	0.602	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	198	139	174	0	0	0	0	31	0
N.S.	1	1.12	0.79	0.99	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.493	5.077	0.595	0.000	0.000	0.000	0.000	200.022	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	203	130	247	0	0	0	0	31	0
N.S.	1	1.12	0.72	1.36	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.505	5.521	0.602	0.000	0.000	0.000	0.000	200.024	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	246	134	273	0	0	0	0	31	0
N.S.	1	1.12	0.61	1.25	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.647	5.698	0.588	0.000	0.000	0.000	0.000	200.026	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	289	139	299	0	0	0	0	31	0
N.S.	1	1.13	0.55	1.17	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.837	5.870	0.578	0.000	0.000	0.000	0.000	200.024	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	601	1254	976	0	0	0	0	31	0
N.S.	1	1.05	2.20	1.71	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.496	29.426	0.879	0.000	0.000	0.000	0.000	200.027	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	262	130	149	0	64	0	0	171	0
N.S.	1	1.10	0.55	0.63	0.00	0.27	0.00	0.00	0.72	0.00
time (sec)	N/A	0.724	5.285	0.470	0.000	0.085	0.000	0.000	1.167	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	219	125	144	0	59	0	0	148	0
N.S.	1	1.10	0.62	0.72	0.00	0.30	0.00	0.00	0.74	0.00
time (sec)	N/A	0.534	4.251	0.413	0.000	0.079	0.000	0.000	0.972	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	171	120	139	0	54	0	0	124	0
N.S.	1	1.09	0.76	0.89	0.00	0.34	0.00	0.00	0.79	0.00
time (sec)	N/A	0.287	1.844	0.382	0.000	0.080	0.000	0.000	0.844	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	115	134	0	49	0	0	104	0
N.S.	1	1.07	0.91	1.06	0.00	0.39	0.00	0.00	0.83	0.00
time (sec)	N/A	0.259	1.075	0.367	0.000	0.080	0.000	0.000	0.627	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	161	95	67	0	0	0	0	31	0
N.S.	1	1.13	0.66	0.47	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.343	2.343	0.395	0.000	0.000	0.000	0.000	200.026	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	203	130	247	0	0	0	0	31	0
N.S.	1	1.12	0.72	1.36	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.500	5.263	0.631	0.000	0.000	0.000	0.000	200.024	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	246	135	273	0	0	0	0	31	0
N.S.	1	1.12	0.62	1.25	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.677	5.046	0.658	0.000	0.000	0.000	0.000	200.025	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	219	125	144	0	59	0	0	148	0
N.S.	1	1.10	0.62	0.72	0.00	0.30	0.00	0.00	0.74	0.00
time (sec)	N/A	0.519	8.664	0.466	0.000	0.077	0.000	0.000	1.013	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	178	120	139	0	54	0	0	125	0
N.S.	1	1.10	0.74	0.86	0.00	0.33	0.00	0.00	0.77	0.00
time (sec)	N/A	0.376	6.343	0.408	0.000	0.087	0.000	0.000	0.853	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	115	134	0	49	0	0	104	0
N.S.	1	1.07	0.91	1.06	0.00	0.39	0.00	0.00	0.83	0.00
time (sec)	N/A	0.236	1.829	0.390	0.000	0.086	0.000	0.000	0.709	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	111	33	0	26	0	0	33	0
N.S.	1	1.00	2.36	0.70	0.00	0.55	0.00	0.00	0.70	0.00
time (sec)	N/A	0.166	2.204	0.319	0.000	0.074	0.000	0.000	0.474	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	111	70	52	0	0	0	0	31	0
N.S.	1	1.13	0.71	0.53	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.257	2.305	0.355	0.000	0.000	0.000	0.000	200.022	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	201	130	247	0	0	0	0	31	0
N.S.	1	1.11	0.72	1.36	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.471	5.454	0.647	0.000	0.000	0.000	0.000	200.025	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	246	142	273	0	0	0	0	31	0
N.S.	1	1.12	0.65	1.25	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.630	5.223	0.696	0.000	0.000	0.000	0.000	200.024	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	325	202	382	0	0	0	0	31	0
N.S.	1	1.11	0.69	1.30	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.584	20.673	3.598	0.000	0.000	0.000	0.000	200.025	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1176	769	0	0	0	0	31	0
N.S.	1	1.00	2.62	1.71	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.871	20.986	3.882	0.000	0.000	0.000	0.000	200.025	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	176	120	139	0	54	0	0	125	0
N.S.	1	1.09	0.74	0.86	0.00	0.33	0.00	0.00	0.77	0.00
time (sec)	N/A	0.370	18.444	0.487	0.000	0.085	0.000	0.000	0.967	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	135	115	134	0	49	0	0	103	0
N.S.	1	1.09	0.93	1.08	0.00	0.40	0.00	0.00	0.83	0.00
time (sec)	N/A	0.254	16.370	0.454	0.000	0.091	0.000	0.000	0.824	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	187	51	0	26	0	0	82	0
N.S.	1	1.06	1.97	0.54	0.00	0.27	0.00	0.00	0.86	0.00
time (sec)	N/A	0.209	8.137	0.339	0.000	0.083	0.000	0.000	0.773	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	79	33	0	11	0	0	40	0
N.S.	1	1.09	1.80	0.75	0.00	0.25	0.00	0.00	0.91	0.00
time (sec)	N/A	0.167	1.331	0.338	0.000	0.083	0.000	0.000	0.599	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	59	109	34	0	0	0	0	31	0
N.S.	1	1.18	2.18	0.68	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.199	3.242	0.415	0.000	0.000	0.000	0.000	200.025	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	199	130	247	0	0	0	0	31	0
N.S.	1	1.10	0.72	1.36	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.476	4.569	0.723	0.000	0.000	0.000	0.000	200.025	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	244	142	273	0	0	0	0	31	0
N.S.	1	1.11	0.65	1.25	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.631	5.849	0.815	0.000	0.000	0.000	0.000	200.026	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	180	500	0	664	0	0	43	0
N.S.	1	1.00	1.31	3.65	0.00	4.85	0.00	0.00	0.31	0.00
time (sec)	N/A	0.244	10.534	1.895	0.000	0.113	0.000	0.000	10.606	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	319	498	0	671	0	0	148	0
N.S.	1	1.00	1.12	1.75	0.00	2.36	0.00	0.00	0.52	0.00
time (sec)	N/A	0.397	19.226	3.309	0.000	0.115	0.000	0.000	20.428	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	197	226	223	0	0	0	0	31	0
N.S.	1	1.19	1.37	1.35	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.423	15.965	4.716	0.000	0.000	0.000	0.000	200.024	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	322	976	0	0	0	0	31	0
N.S.	1	1.00	0.82	2.48	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.784	23.153	13.599	0.000	0.000	0.000	0.000	200.026	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	875	875	4180	1335	0	0	0	0	31	0
N.S.	1	1.00	4.78	1.53	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.472	35.102	16.259	0.000	0.000	0.000	0.000	200.025	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	92	203	181	0	0	0	0	29	0
N.S.	1	1.24	2.74	2.45	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.296	21.936	3.435	0.000	0.000	0.000	0.000	200.018	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	92	203	184	0	0	0	0	84	0
N.S.	1	1.24	2.74	2.49	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.263	0.050	3.327	0.000	0.000	0.000	0.000	79.045	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	106	218	205	0	0	0	0	29	0
N.S.	1	1.23	2.53	2.38	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.310	21.923	3.408	0.000	0.000	0.000	0.000	200.017	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	106	218	212	0	0	0	0	36	0
N.S.	1	1.23	2.53	2.47	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.269	0.050	3.332	0.000	0.000	0.000	0.000	200.023	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	543	565	473	0	0	0	0	240	0
N.S.	1	1.21	1.26	1.05	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.125	16.229	1.240	0.000	0.000	0.000	0.000	8.371	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	498	560	446	0	0	0	0	211	0
N.S.	1	1.21	1.37	1.09	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.934	12.012	0.796	0.000	0.000	0.000	0.000	5.926	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	455	554	421	0	0	0	0	184	0
N.S.	1	1.23	1.50	1.14	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.771	12.051	0.818	0.000	0.000	0.000	0.000	5.435	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	453	564	435	0	0	0	0	382	0
N.S.	1	1.22	1.52	1.17	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.739	14.586	0.421	0.000	0.000	0.000	0.000	5.992	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	498	559	464	0	0	0	0	613	0
N.S.	1	1.50	1.69	1.40	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.932	16.911	0.435	0.000	0.000	0.000	0.000	8.705	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	431	574	493	0	0	0	0	626	0
N.S.	1	1.60	2.13	1.83	0.00	0.00	0.00	0.00	2.33	0.00
time (sec)	N/A	0.856	18.035	0.444	0.000	0.000	0.000	0.000	4.036	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	476	569	522	0	0	0	0	851	0
N.S.	1	1.54	1.84	1.69	0.00	0.00	0.00	0.00	2.75	0.00
time (sec)	N/A	1.058	17.506	0.489	0.000	0.000	0.000	0.000	4.404	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	521	584	551	0	0	0	0	0	0
N.S.	1	1.49	1.67	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.254	17.813	0.510	0.000	0.000	0.000	0.000	4.779	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	498	340	446	0	0	0	0	211	0
N.S.	1	1.21	0.83	1.09	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.934	36.905	0.969	0.000	0.000	0.000	0.000	7.608	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	455	347	421	0	0	0	0	184	0
N.S.	1	1.23	0.94	1.14	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.751	27.781	0.760	0.000	0.000	0.000	0.000	5.597	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	608	318	397	0	0	0	0	39	0
N.S.	1	1.85	0.97	1.21	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.607	5.223	0.566	0.000	0.000	0.000	0.000	4.036	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	282	326	435	0	0	0	0	44	0
N.S.	1	0.85	0.98	1.31	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.521	19.197	0.698	0.000	0.000	0.000	0.000	4.894	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	386	246	464	0	0	0	0	423	0
N.S.	1	1.69	1.07	2.03	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.693	26.041	0.825	0.000	0.000	0.000	0.000	3.648	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	431	251	493	0	0	0	0	626	0
N.S.	1	1.60	0.93	1.83	0.00	0.00	0.00	0.00	2.33	0.00
time (sec)	N/A	0.899	30.049	1.005	0.000	0.000	0.000	0.000	4.055	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	476	258	522	0	0	0	0	851	0
N.S.	1	1.54	0.83	1.69	0.00	0.00	0.00	0.00	2.75	0.00
time (sec)	N/A	1.078	26.790	1.208	0.000	0.000	0.000	0.000	4.409	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	498	340	446	0	0	0	0	211	0
N.S.	1	1.21	0.83	1.09	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.966	29.438	1.066	0.000	0.000	0.000	0.000	8.412	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	455	349	421	0	0	0	0	184	0
N.S.	1	1.23	0.94	1.14	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.740	23.196	0.951	0.000	0.000	0.000	0.000	7.099	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	608	347	397	0	0	0	0	39	0
N.S.	1	1.84	1.05	1.20	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.589	6.702	0.589	0.000	0.000	0.000	0.000	4.028	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	170	129	0	0	0	0	44	0
N.S.	1	1.00	1.68	1.28	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.207	4.785	0.662	0.000	0.000	0.000	0.000	3.990	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	362	237	435	0	0	0	0	49	0
N.S.	1	6.03	3.95	7.25	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.332	27.805	0.806	0.000	0.000	0.000	0.000	4.646	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	386	248	464	0	0	0	0	54	0
N.S.	1	1.70	1.09	2.04	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.720	30.042	0.957	0.000	0.000	0.000	0.000	3.143	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	733	721	7591	1544	0	0	0	0	31	0
N.S.	1	0.98	10.36	2.11	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.162	52.768	12.629	0.000	0.000	0.000	0.000	200.023	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	208	223	1948	0	0	0	0	31	0
N.S.	1	1.29	1.39	12.10	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.316	23.383	23.444	0.000	0.000	0.000	0.000	200.025	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	453	347	421	0	0	0	0	184	0
N.S.	1	1.22	0.93	1.13	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.729	23.249	1.205	0.000	0.000	0.000	0.000	8.074	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	713	347	397	0	0	0	0	94	0
N.S.	1	1.65	0.80	0.92	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.736	9.535	0.729	0.000	0.000	0.000	0.000	6.995	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	157	0	0	0	0	46	0
N.S.	1	1.00	0.95	1.57	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.202	3.345	0.730	0.000	0.000	0.000	0.000	5.331	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	165	90	128	0	0	0	0	51	0
N.S.	1	1.81	0.99	1.41	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.214	2.818	0.837	0.000	0.000	0.000	0.000	4.713	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	530	237	435	0	0	0	0	56	0
N.S.	1	2.80	1.25	2.30	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.453	17.763	0.987	0.000	0.000	0.000	0.000	5.326	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	384	246	464	0	0	0	0	61	0
N.S.	1	1.68	1.07	2.03	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.662	30.725	1.142	0.000	0.000	0.000	0.000	4.104	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	982	958	7319	1541	0	0	0	0	31	0
N.S.	1	0.98	7.45	1.57	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.424	30.276	14.021	0.000	0.000	0.000	0.000	200.024	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	228	583	848	0	0	0	0	31	0
N.S.	1	1.00	2.55	3.70	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.363	30.444	14.112	0.000	0.000	0.000	0.000	200.025	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	198	227	273	0	0	0	0	31	0
N.S.	1	1.23	1.41	1.70	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.319	22.912	22.823	0.000	0.000	0.000	0.000	200.028	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	429	461	2200	0	0	0	0	31	0
N.S.	1	0.98	1.05	5.03	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.535	37.686	25.740	0.000	0.000	0.000	0.000	200.026	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	0	670	7103	0	0	0	0	31	0
N.S.	1	0.00	1.16	12.25	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	33.449	28.744	0.000	0.000	0.000	0.000	200.026	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	248	473	0	0	0	0	0	0	0
N.S.	1	1.09	2.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.525	0.781	0.000	0.000	0.000	0.000	0.000	0.357	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	250	0	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.266	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	0	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.163	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	0	0	0	0	0	30	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.224	0.093	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	193	0	0	0	0	0	60	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.408	0.405	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	314	269	0	0	0	0	0	112	0
N.S.	1	0.95	0.82	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.524	0.833	0.000	0.000	0.000	0.000	0.000	0.325	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	319	285	0	0	0	0	0	0	0
N.S.	1	1.19	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.525	0.777	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	216	174	0	0	0	0	0	0	0
N.S.	1	1.10	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.379	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	153	0	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.152	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	1100	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	8.87	0.00
time (sec)	N/A	0.220	0.103	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	30	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.221	0.092	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	170	0	0	0	0	0	35	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.332	0.231	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	222	177	0	0	0	0	0	37	0
N.S.	1	1.05	0.83	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.387	0.300	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	149	726	474	877	8221	1626	1045	819
N.S.	1	1.00	0.89	4.35	2.84	5.25	49.23	9.74	6.26	4.90
time (sec)	N/A	0.351	0.196	0.405	0.069	0.100	1.981	0.140	0.149	3.451

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	136	120	0	0	0	0	0	1424	0
N.S.	1	0.99	0.88	0.00	0.00	0.00	0.00	0.00	10.39	0.00
time (sec)	N/A	0.266	0.210	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	0	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.040	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	193	0	0	0	0	0	60	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.386	0.094	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	266	195	0	0	0	0	0	0	0
N.S.	1	0.98	0.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.070	0.000	0.000	0.000	0.000	0.000	1.149	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	137	117	0	0	0	0	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	0.064	0.000	0.000	0.000	0.000	0.000	0.304	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	191	265	0	0	0	0	0	29	0
N.S.	1	1.07	1.49	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.332	0.339	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	327	167	0	0	0	0	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	0.246	0.000	0.000	0.000	0.000	0.000	18.483	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	645	663	168	0	0	0	0	0	0	0
N.S.	1	1.03	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.236	0.615	0.000	0.000	0.000	0.000	0.000	7.995	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	589	642	511	0	0	0	0	0	29	0
N.S.	1	1.09	0.87	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.936	2.306	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	267	253	0	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	0.226	0.000	0.000	0.000	0.000	0.000	0.865	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	303	0	0	0	0	0	0	31	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.496	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	389	308	0	0	0	0	0	0	31	0
N.S.	1	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.515	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	912	337	0	0	0	0	0	0	31	0
N.S.	1	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.555	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	245	195	0	0	0	0	0	200	0
N.S.	1	0.98	0.78	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.369	0.271	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	235	189	0	0	0	0	0	94	0
N.S.	1	0.94	0.75	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.338	0.225	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	261	221	0	0	0	0	0	146	0
N.S.	1	1.10	0.93	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.400	0.204	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	205	198	0	0	0	0	0	218	0
N.S.	1	1.19	1.15	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.316	0.211	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	246	237	0	0	0	0	0	290	0
N.S.	1	1.12	1.08	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.381	0.274	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	284	220	894	0	1659	0	0	362	1895
N.S.	1	0.99	0.76	3.10	0.00	5.76	0.00	0.00	1.26	6.58
time (sec)	N/A	0.461	0.364	1.197	0.000	0.177	0.000	0.000	0.159	4.321

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	447	507	422	0	0	0	0	0	558	0
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.791	1.140	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	283	227	906	0	1608	0	0	362	1890
N.S.	1	0.95	0.76	3.03	0.00	5.38	0.00	0.00	1.21	6.32
time (sec)	N/A	0.479	0.364	1.236	0.000	0.150	0.000	0.000	0.149	4.244

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	170	181	509	0	905	0	0	178	869
N.S.	1	0.91	0.97	2.74	0.00	4.87	0.00	0.00	0.96	4.67
time (sec)	N/A	0.275	0.131	0.829	0.000	0.110	0.000	0.000	0.141	3.618

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	104	0	0	0	0	0	33	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.324	0.234	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	231	174	0	0	0	0	0	70	0
N.S.	1	1.23	0.93	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.418	0.166	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	393	530	0	0	0	0	0	0	31	0
N.S.	1	1.35	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.696	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0	29	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.396	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	24	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.267	0.072	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	31	31	0	31	31	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.07	1.07
time (sec)	N/A	0.164	0.744	10.427	0.130	0.106	0.000	0.119	200.019	3.130

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [83] had the largest ratio of [.486485999999999974]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	21	0.095
2	A	2	2	1.00	23	0.087
3	A	2	2	1.00	25	0.080
4	A	2	2	1.00	27	0.074
5	A	2	2	1.00	29	0.069
6	A	2	2	1.00	17	0.118
7	A	3	3	1.00	22	0.136
8	A	5	4	1.00	24	0.167
9	A	6	5	1.00	36	0.139
10	A	8	7	1.66	31	0.226
11	A	9	8	1.01	57	0.140
12	A	8	7	1.46	33	0.212
13	A	7	6	1.35	33	0.182
14	A	4	3	1.00	33	0.091
15	A	5	4	1.30	33	0.121
16	A	8	7	1.46	33	0.212
17	A	2	2	1.81	33	0.061
18	A	8	7	1.46	33	0.212
19	A	2	2	1.42	33	0.061
20	A	2	2	1.27	33	0.061
21	A	2	2	1.15	33	0.061

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.86	33	0.061
23	A	2	2	1.42	33	0.061
24	A	4	3	1.00	33	0.091
25	A	2	2	1.00	33	0.061
26	A	2	2	1.23	33	0.061
27	A	2	2	1.28	33	0.061
28	A	2	2	1.97	33	0.061
29	A	2	2	1.27	33	0.061
30	A	5	4	1.30	33	0.121
31	A	2	2	1.23	33	0.061
32	A	2	2	1.24	33	0.061
33	A	3	3	0.46	30	0.100
34	A	3	3	0.49	30	0.100
35	A	3	3	0.52	28	0.107
36	A	3	3	0.57	27	0.111
37	A	3	3	0.53	30	0.100
38	A	7	6	0.60	30	0.200
39	A	8	7	0.58	30	0.233
40	A	9	8	0.58	30	0.267
41	A	10	9	0.58	30	0.300
42	A	16	16	1.11	35	0.457
43	A	14	14	1.10	35	0.400
44	A	12	12	1.11	33	0.364
45	A	10	10	1.09	28	0.357
46	A	15	14	1.12	35	0.400
47	A	15	14	1.12	35	0.400
48	A	16	15	1.12	35	0.429
49	A	18	17	1.13	35	0.486
50	A	13	12	1.05	35	0.343
51	A	14	14	1.10	35	0.400
52	A	12	12	1.10	35	0.343
53	A	10	10	1.09	33	0.303

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	8	1.07	28	0.286
55	A	13	12	1.13	35	0.343
56	A	15	14	1.12	35	0.400
57	A	17	16	1.12	35	0.457
58	A	12	12	1.10	35	0.343
59	A	10	10	1.10	35	0.286
60	A	8	8	1.07	33	0.242
61	A	3	3	1.00	28	0.107
62	A	10	9	1.13	35	0.257
63	A	15	14	1.11	35	0.400
64	A	17	16	1.12	35	0.457
65	A	9	8	1.11	35	0.229
66	A	2	2	1.00	35	0.057
67	A	10	10	1.09	35	0.286
68	A	9	9	1.09	35	0.257
69	A	6	6	1.06	33	0.182
70	A	3	3	1.09	28	0.107
71	A	6	5	1.18	35	0.143
72	A	14	13	1.10	35	0.371
73	A	16	15	1.11	35	0.429
74	A	3	3	1.00	36	0.083
75	A	6	6	1.00	33	0.182
76	A	5	4	1.19	35	0.114
77	A	2	2	1.00	35	0.057
78	A	2	2	1.00	35	0.057
79	A	5	4	1.24	31	0.129
80	A	4	3	1.24	36	0.083
81	A	5	4	1.23	31	0.129
82	A	4	3	1.23	40	0.075
83	A	19	18	1.21	37	0.486
84	A	16	15	1.21	37	0.405
85	A	15	14	1.23	37	0.378

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	15	14	1.22	37	0.378
87	A	17	16	1.50	37	0.432
88	A	14	13	1.60	37	0.351
89	A	17	16	1.54	37	0.432
90	A	18	17	1.49	37	0.459
91	A	17	16	1.21	37	0.432
92	A	14	13	1.23	37	0.351
93	A	13	12	1.85	37	0.324
94	A	11	10	0.85	37	0.270
95	A	13	12	1.69	37	0.324
96	A	14	13	1.60	37	0.351
97	A	17	16	1.54	37	0.432
98	A	17	16	1.21	37	0.432
99	A	15	14	1.23	37	0.378
100	A	13	12	1.84	37	0.324
101	A	4	3	1.00	37	0.081
102	B	7	6	6.03	37	0.162
103	A	13	12	1.70	37	0.324
104	A	8	7	0.98	37	0.189
105	A	3	2	1.29	37	0.054
106	A	14	13	1.22	37	0.351
107	A	17	16	1.65	37	0.432
108	A	4	3	1.00	37	0.081
109	A	4	3	1.81	37	0.081
110	B	11	10	2.80	37	0.270
111	A	12	11	1.68	37	0.297
112	A	10	9	0.98	37	0.243
113	A	3	2	1.00	37	0.054
114	A	3	2	1.23	37	0.054
115	A	6	5	0.98	37	0.135
116	F	0	0	N/A	0.000	N/A
117	A	2	2	1.09	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
118	A	2	2	1.00	29	0.069
119	A	2	2	1.00	27	0.074
120	A	2	2	1.00	22	0.091
121	A	2	2	1.00	29	0.069
122	A	2	2	0.95	29	0.069
123	A	2	2	1.19	25	0.080
124	A	2	2	1.10	25	0.080
125	A	2	2	1.00	25	0.080
126	A	2	2	1.00	23	0.087
127	A	2	2	1.00	22	0.091
128	A	2	2	1.00	25	0.080
129	A	2	2	1.05	25	0.080
130	A	2	2	1.00	23	0.087
131	A	2	2	0.99	25	0.080
132	A	2	2	1.00	27	0.074
133	A	2	2	1.00	29	0.069
134	A	3	3	0.98	25	0.120
135	A	3	3	1.05	20	0.150
136	A	5	5	1.07	27	0.185
137	A	7	7	1.00	27	0.259
138	A	9	9	1.03	27	0.333
139	A	4	4	1.09	27	0.148
140	A	4	4	1.00	22	0.182
141	A	2	2	1.13	29	0.069
142	A	2	2	0.79	29	0.069
143	A	2	2	0.37	29	0.069
144	A	3	3	0.98	29	0.103
145	A	3	3	0.94	27	0.111
146	A	3	3	1.10	29	0.103
147	A	3	3	1.19	29	0.103
148	A	3	3	1.12	29	0.103
149	A	3	3	0.99	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	11	11	1.13	31	0.355
151	A	3	3	0.95	29	0.103
152	A	3	3	0.91	24	0.125
153	A	2	2	1.00	25	0.080
154	A	6	6	1.23	29	0.207
155	A	5	5	1.35	29	0.172
156	A	4	4	1.00	27	0.148
157	A	3	3	1.00	22	0.136
158	N/A	1	0	1.00	29	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2 +\frac{1}{3}(b(deg+cfg+ceh)+a(dfg+deh+cfh))x^3 +\frac{1}{4}(adfh+b(dfg+deh+cfh))x^4+\frac{1}{5}bdfhx^5$$

output
```
a*c*e*g*x+1/2*(b*c*e*g+a*(c*e*h+c*f*g+d*e*g))*x^2+1/3*(b*(c*e*h+c*f*g+d*e*
g)+a*(c*f*h+d*e*h+d*f*g))*x^3+1/4*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*x^4+1/5*
b*d*f*h*x^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = acegx+\frac{1}{2}(bceg+adeg+acfg+aceh)x^2 +\frac{1}{3}(bdeg+bcfg+adfg+bceh+adeh +acfh)x^3 +\frac{1}{4}(bdfg+bdeh+bcfh+adfh)x^4+\frac{1}{5}bdfhx^5$$

input
```
Integrate[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

output
```
a*c*e*g*x + ((b*c*e*g + a*d*e*g + a*c*f*g + a*c*e*h)*x^2)/2 + ((b*d*e*g +
b*c*f*g + a*d*f*g + b*c*e*h + a*d*e*h + a*c*f*h)*x^3)/3 + ((b*d*f*g + b*d*
e*h + b*c*f*h + a*d*f*h)*x^4)/4 + (b*d*f*h*x^5)/5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(x^3(adfh+b(cfh+deh+dfg))+x^2(a(cfh+deh+dfg)+b(ceh+cfg+deg))+x(a(ceh+cfg+deg)+b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)+acegx+\frac{1}{5}bdfhx^5$$

input
```
Int[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

output
```
a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c
*f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g +
 d*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

method	result
default	$\frac{bdfh\,x^{5}}{5}+\frac{\left(\left(\left(ch+dg\right)b+adh\right)f+bdeh\right)x^{4}}{4}+\frac{\left(\left(bcg+\left(ch+dg\right)a\right)f+\left(\left(ch+dg\right)b+adh\right)e\right)x^{3}}{3}+\frac{\left(acfg+\left(bcg+\left(ch+dg\right)a\right)e\right)x^{2}}{2}$ -
norman	$\frac{bdfh\,x^{5}}{5}+\left(\frac{1}{4}adfh+\frac{1}{4}bcfh+\frac{1}{4}bdeh+\frac{1}{4}bgdf\right)x^{4}+\left(\frac{1}{3}acfh+\frac{1}{3}aedh+\frac{1}{3}adfg+\frac{1}{3}cehb+\frac{1}{3}cg\right.$
orering	$\frac{x\left(12bdfh\,x^{4}+15adfh\,x^{3}+15bcfh\,x^{3}+15bdeh\,x^{3}+15bdfg\,x^{3}+20acfh\,x^{2}+20adeh\,x^{2}+20adfg\,x^{2}+20bceh\,x^{2}+20bcfg\,x^{2}+20b\right.}{60}$
gosper	$\frac{1}{5}bdfh\,x^{5}+\frac{1}{4}x^{4}adfh+\frac{1}{4}x^{4}bcfh+\frac{1}{4}x^{4}bdeh+\frac{1}{4}x^{4}bgdf+\frac{1}{3}x^{3}acfh+\frac{1}{3}x^{3}aedh+\frac{1}{3}x^{3}adfg+$
risch	$\frac{1}{5}bdfh\,x^{5}+\frac{1}{4}x^{4}adfh+\frac{1}{4}x^{4}bcfh+\frac{1}{4}x^{4}bdeh+\frac{1}{4}x^{4}bgdf+\frac{1}{3}x^{3}acfh+\frac{1}{3}x^{3}aedh+\frac{1}{3}x^{3}adfg+$
parallelrisch	$\frac{1}{5}bdfh\,x^{5}+\frac{1}{4}x^{4}adfh+\frac{1}{4}x^{4}bcfh+\frac{1}{4}x^{4}bdeh+\frac{1}{4}x^{4}bgdf+\frac{1}{3}x^{3}acfh+\frac{1}{3}x^{3}aedh+\frac{1}{3}x^{3}adfg+$

input
```
int((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/5*b*d*f*h*x^5+1/4*(((c*h+d*g)*b+a*d*h)*f+b*d*e*h)*x^4+1/3*((b*c*g+(c*h+d
*g)*a)*f+((c*h+d*g)*b+a*d*h)*e)*x^3+1/2*(a*c*f*g+(b*c*g+(c*h+d*g)*a)*e)*x^
2+a*c*e*g*x
```

Fricas [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{1}{5}x^5hfdb + \frac{1}{4}x^4gfdb + \frac{1}{4}x^4hedb + \frac{1}{4}x^4hfcb + \frac{1}{4}x^4hfda + \frac{1}{3}x^3gedb + \frac{1}{3}x^3gfcb + \frac{1}{3}x^3hecb + \frac{1}{3}x^3gfda + \frac{1}{3}x^3heda + \frac{1}{3}x^3hfca + \frac{1}{2}x^2gecb + \frac{1}{2}x^2geda + \frac{1}{2}x^2gfca + \frac{1}{2}x^2heca + xgeca$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
1/5*x^5*h*f*d*b + 1/4*x^4*g*f*d*b + 1/4*x^4*h*e*d*b + 1/4*x^4*h*f*c*b + 1/
4*x^4*h*f*d*a + 1/3*x^3*g*e*d*b + 1/3*x^3*g*f*c*b + 1/3*x^3*h*e*c*b + 1/3*
x^3*g*f*d*a + 1/3*x^3*h*e*d*a + 1/3*x^3*h*f*c*a + 1/2*x^2*g*e*c*b + 1/2*x^
2*g*e*d*a + 1/2*x^2*g*f*c*a + 1/2*x^2*h*e*c*a + x*g*e*c*a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = acegx + \frac{bdfhx^5}{5} + x^4\left(\frac{adfh}{4} + \frac{bcfh}{4} + \frac{bdeh}{4} + \frac{bdfg}{4}\right) + x^3\left(\frac{acfh}{3} + \frac{adeh}{3} + \frac{adfg}{3} + \frac{bceh}{3} + \frac{bcfg}{3} + \frac{bdeg}{3}\right) + x^2\left(\frac{aceh}{2} + \frac{acfg}{2} + \frac{adeg}{2} + \frac{bceg}{2}\right)$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

output
```
a*c*e*g*x + b*d*f*h*x**5/5 + x**4*(a*d*f*h/4 + b*c*f*h/4 + b*d*e*h/4 + b*d
*f*g/4) + x**3*(a*c*f*h/3 + a*d*e*h/3 + a*d*f*g/3 + b*c*e*h/3 + b*c*f*g/3
+ b*d*e*g/3) + x**2*(a*c*e*h/2 + a*c*f*g/2 + a*d*e*g/2 + b*c*e*g/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{1}{5}bdfhx^5 + acegx + \frac{1}{4}(bdfg + (bde + (bc+ad)f)h)x^4 + \frac{1}{3}((bde + (bc+ad)f)g + (acf + (bc+ad)e)h)x^3 + \frac{1}{2}(aceh + (acf + (bc+ad)e)g)x^2$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
1/5*b*d*f*h*x^5 + a*c*e*g*x + 1/4*(b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^
4 + 1/3*((b*d*e + (b*c + a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^3 + 1/2*
(a*c*e*h + (a*c*f + (b*c + a*d)*e)*g)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{1}{5}bdfhx^5 + \frac{1}{4}bdfgx^4 + \frac{1}{4}bdehx^4 + \frac{1}{4}bcfhx^4 + \frac{1}{4}adfhx^4 + \frac{1}{3}bdegx^3 + \frac{1}{3}bcfgx^3 + \frac{1}{3}adfgx^3 + \frac{1}{3}bcehx^3 + \frac{1}{3}adehx^3 + \frac{1}{3}acfhx^3 + \frac{1}{2}bcegx^2 + \frac{1}{2}adegx^2 + \frac{1}{2}acfgx^2 + \frac{1}{2}acehx^2 + acegx$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
1/5*b*d*f*h*x^5 + 1/4*b*d*f*g*x^4 + 1/4*b*d*e*h*x^4 + 1/4*b*c*f*h*x^4 + 1/
4*a*d*f*h*x^4 + 1/3*b*d*e*g*x^3 + 1/3*b*c*f*g*x^3 + 1/3*a*d*f*g*x^3 + 1/3*
b*c*e*h*x^3 + 1/3*a*d*e*h*x^3 + 1/3*a*c*f*h*x^3 + 1/2*b*c*e*g*x^2 + 1/2*a*
d*e*g*x^2 + 1/2*a*c*f*g*x^2 + 1/2*a*c*e*h*x^2 + a*c*e*g*x
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{b\,d\,f\,h\,x^5}{5}+\left(\frac{a\,d\,f\,h}{4}+\frac{b\,c\,f\,h}{4}+\frac{b\,d\,e\,h}{4}+\frac{b\,d\,f\,g}{4}\right)\,x^4 +\left(\frac{a\,c\,f\,h}{3}+\frac{a\,d\,e\,h}{3}+\frac{a\,d\,f\,g}{3}+\frac{b\,c\,e\,h}{3}+\frac{b\,c\,f\,g}{3}+\frac{b\,d\,e\,g}{3}\right)\,x^3 +\left(\frac{a\,c\,e\,h}{2}+\frac{a\,c\,f\,g}{2}+\frac{a\,d\,e\,g}{2}+\frac{b\,c\,e\,g}{2}\right)\,x^2+a\,c\,e\,g\,x$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

output
```
x^3*((a*c*f*h)/3 + (a*d*e*h)/3 + (a*d*f*g)/3 + (b*c*e*h)/3 + (b*c*f*g)/3 +
 (b*d*e*g)/3) + x^2*((a*c*e*h)/2 + (a*c*f*g)/2 + (a*d*e*g)/2 + (b*c*e*g)/2
) + x^4*((a*d*f*h)/4 + (b*c*f*h)/4 + (b*d*e*h)/4 + (b*d*f*g)/4) + a*c*e*g*
x + (b*d*f*h*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.22

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \frac{x(12bdfh\,x^4+15adfh\,x^3+15bcfh\,x^3+15bdeh\,x^3+15bdfg\,x^3+20acfh\,x^2+20adeh\,x^2+20adfg\,x^2+}{60}$$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

output
```
(x*(60*a*c*e*g + 30*a*c*e*h*x + 30*a*c*f*g*x + 20*a*c*f*h*x**2 + 30*a*d*e*
g*x + 20*a*d*e*h*x**2 + 20*a*d*f*g*x**2 + 15*a*d*f*h*x**3 + 30*b*c*e*g*x +
 20*b*c*e*h*x**2 + 20*b*c*f*g*x**2 + 15*b*c*f*h*x**3 + 20*b*d*e*g*x**2 + 1
5*b*d*e*h*x**3 + 15*b*d*f*g*x**3 + 12*b*d*f*h*x**4))/60
```

3.2 $\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx} \, dx = \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3} + \frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2} + \frac{bdfx^3}{3h} - \frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}$$

output
```
(b*(-c*h+d*g)*(-e*h+f*g)-a*h*(-c*f*h-d*e*h+d*f*g))*x/h^3+1/2*(a*d*f*h-b*(-
c*f*h-d*e*h+d*f*g))*x^2/h^2+1/3*b*d*f*x^3/h-(-a*h+b*g)*(-c*h+d*g)*(-e*h+f*
g)*ln(h*x+g)/h^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx} \, dx = \frac{hx(3ah(2cfh+d(-2fg+2eh+fhx))+b(3deh(-2g+hx)+3ch(-2fg+2eh+fhx)+df(6g^2-3g}{6h^4}$$

input
```
Integrate[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

output
```
(h*x*(3*a*h*(2*c*f*h + d*(-2*f*g + 2*e*h + f*h*x)) + b*(3*d*e*h*(-2*g + h*
x) + 3*c*h*(-2*f*g + 2*e*h + f*h*x) + d*f*(6*g^2 - 3*g*h*x + 2*h^2*x^2)))
- 6*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/(6*h^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx} \, dx$$

$$\Big\downarrow \text{ 159}$$

$$\int \left(\frac{(ah-bg)(ch-dg)(eh-fg)}{h^3(g+hx)}+\frac{b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg)}{h^3}+\frac{x(adfh-b(-cfh-deh+$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}+\frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2}+\frac{bdfx^3}{3h}$$

input
```
Int[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

output
```
((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*
f*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g
- a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.39

method	result
norman	$\frac{(acf\,h^2+h^2ead-adfgh+c\,h^2eb-bcfgh-bdegh+bdf\,g^2)x}{h^3}+\frac{(adfh+bcfh+bdeh-bgdf)x^2}{2h^2}+\frac{bdf\,x^3}{3h}+\frac{(ace\,h^3-acfg\,h^2-$
default	$\frac{\frac{1}{3}bdf\,x^3h^2+\frac{1}{2}adf\,h^2x^2+\frac{1}{2}bcf\,h^2x^2+\frac{1}{2}bde\,h^2x^2-\frac{1}{2}bdfgh\,x^2+acf\,h^2x+h^2eadx-adfghx+c\,h^2ebx-bcfghx-bdeghx+bdf\,g^2x}{h^3}$
risch	$\frac{bdf\,x^3}{3h}+\frac{adf\,x^2}{2h}+\frac{bcf\,x^2}{2h}+\frac{bde\,x^2}{2h}-\frac{bdfg\,x^2}{2h^2}+\frac{acfx}{h}+\frac{eadx}{h}-\frac{adfgx}{h^2}+\frac{cebx}{h}-\frac{bcfgx}{h^2}-\frac{bdegx}{h^2}+\frac{bdf\,g^2x}{h^3}$
parallelrisch	$2bdf\,x^3h^3+3x^2adf\,h^3+3x^2bcf\,h^3+3x^2bde\,h^3-3x^2bdfg\,h^2+6\ln(hx+g)ace\,h^3-6\ln(hx+g)acfg\,h^2-6\ln(hx+g)adeg\,h^2+6\ln$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(a*c*f*h^2+a*d*e*h^2-a*d*f*g*h+b*c*e*h^2-b*c*f*g*h-b*d*e*g*h+b*d*f*g^2)/h^
3*x+1/2/h^2*(a*d*f*h+b*c*f*h+b*d*e*h-b*d*f*g)*x^2+1/3*b*d*f*x^3/h+(a*c*e*h
^3-a*c*f*g*h^2-a*d*e*g*h^2+a*d*f*g^2*h-b*c*e*g*h^2+b*c*f*g^2*h+b*d*e*g^2*h
-b*d*f*g^3)/h^4*ln(h*x+g)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$
$$= \frac{2\,bdfh^3x^3 - 3\,(bdfgh^2 - (bde + (bc+ad)f)h^3)x^2 + 6\,(bdfg^2h - (bde + (bc+ad)f)gh^2 + (acf + (bc+a}{6\,h^4}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
1/6*(2*b*d*f*h^3*x^3 - 3*(b*d*f*g*h^2 - (b*d*e + (b*c + a*d)*f)*h^3)*x^2 +
 6*(b*d*f*g^2*h - (b*d*e + (b*c + a*d)*f)*g*h^2 + (a*c*f + (b*c + a*d)*e)*
h^3)*x - 6*(b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f
 + (b*c + a*d)*e)*g*h^2)*log(h*x + g))/h^4
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \frac{bdfx^3}{3h} + x^2\left(\frac{adf}{2h} + \frac{bcf}{2h} + \frac{bde}{2h} - \frac{bdfg}{2h^2}\right) + x\left(\frac{acf}{h} + \frac{ade}{h} - \frac{adfg}{h^2} + \frac{bce}{h} - \frac{bcfg}{h^2} - \frac{bdeg}{h^2} + \frac{bdfg^2}{h^3}\right) + \frac{(ah-bg)\,(ch-dg)\,(eh-fg)\log{(g+hx)}}{h^4}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

output
```
b*d*f*x**3/(3*h) + x**2*(a*d*f/(2*h) + b*c*f/(2*h) + b*d*e/(2*h) - b*d*f*g
/(2*h**2)) + x*(a*c*f/h + a*d*e/h - a*d*f*g/h**2 + b*c*e/h - b*c*f*g/h**2
- b*d*e*g/h**2 + b*d*f*g**2/h**3) + (a*h - b*g)*(c*h - d*g)*(e*h - f*g)*lo
g(g + h*x)/h**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$
$$= \frac{2\,bdfh^2x^3 - 3\,(bdfgh - (bde + (bc+ad)f)h^2)x^2 + 6\,(bdfg^2 - (bde + (bc+ad)f)gh + (acf + (bc+ad)e}{6\,h^3}$$
$$- \frac{(bdfg^3 - aceh^3 - (bde + (bc+ad)f)g^2h + (acf + (bc+ad)e)gh^2)\log(hx+g)}{h^4}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
1/6*(2*b*d*f*h^2*x^3 - 3*(b*d*f*g*h - (b*d*e + (b*c + a*d)*f)*h^2)*x^2 + 6
*(b*d*f*g^2 - (b*d*e + (b*c + a*d)*f)*g*h + (a*c*f + (b*c + a*d)*e)*h^2)*x
)/h^3 - (b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f +
(b*c + a*d)*e)*g*h^2)*log(h*x + g)/h^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$
$$= \frac{2\,bdfh^2x^3 - 3\,bdfghx^2 + 3\,bdeh^2x^2 + 3\,bcfh^2x^2 + 3\,adfh^2x^2 + 6\,bdfg^2x - 6\,bdeghx - 6\,bcfghx - 6\,adfg}{6\,h^3}$$
$$- \frac{(bdfg^3 - bdeg^2h - bcfg^2h - adfg^2h + bcegh^2 + adegh^2 + acfgh^2 - aceh^3)\log(|hx+g|)}{h^4}$$

input
```
integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
1/6*(2*b*d*f*h^2*x^3 - 3*b*d*f*g*h*x^2 + 3*b*d*e*h^2*x^2 + 3*b*c*f*h^2*x^2
 + 3*a*d*f*h^2*x^2 + 6*b*d*f*g^2*x - 6*b*d*e*g*h*x - 6*b*c*f*g*h*x - 6*a*d
*f*g*h*x + 6*b*c*e*h^2*x + 6*a*d*e*h^2*x + 6*a*c*f*h^2*x)/h^3 - (b*d*f*g^3
 - b*d*e*g^2*h - b*c*f*g^2*h - a*d*f*g^2*h + b*c*e*g*h^2 + a*d*e*g*h^2 + a
*c*f*g*h^2 - a*c*e*h^3)*log(abs(h*x + g))/h^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = x\left(\frac{a\,c\,f+a\,d\,e+b\,c\,e}{h}-\frac{g\,\left(\frac{a\,d\,f+b\,c\,f+b\,d\,e}{h}-\frac{b\,d\,f\,g}{h^2}\right)}{h}\right)+x^2\left(\frac{a\,d\,f+b\,c\,f+b\,d\,e}{2\,h}-\frac{b\,d\,f\,g}{2\,h^2}\right)+\frac{\ln\left(g+h\,x\right)\,\left(a\,c\,e\,h^3-b\,d\,f\,g^3-a\,c\,f\,g\,h^2-a\,d\,e\,g\,h^2-b\,c\,e\,g\,h^2+a\,d\,f\,g^2\,h+b\,c\,f\,g^2\,h+b\,d\,e\,g^2\,h\right)}{h^4}+\frac{b\,d\,f\,x^3}{3\,h}$$

input
```
int(((e + f*x)*(a + b*x)*(c + d*x))/(g + h*x),x)
```

output
```
x*((a*c*f + a*d*e + b*c*e)/h - (g*((a*d*f + b*c*f + b*d*e)/h - (b*d*f*g)/h
^2))/h) + x^2*((a*d*f + b*c*f + b*d*e)/(2*h) - (b*d*f*g)/(2*h^2)) + (log(g
 + h*x)*(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e*g*h^2 - b*c*e*g*h^2 +
 a*d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h))/h^4 + (b*d*f*x^3)/(3*h)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx = \frac{6\log(hx+g)\,ace\,h^3-6\log(hx+g)\,acfg\,h^2-6\log(hx+g)\,adeg\,h^2+6\log(hx+g)\,adf\,g^2h-6\log(hx}{}$$

input
```
int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

output
```
(6*log(g + h*x)*a*c*e*h**3 - 6*log(g + h*x)*a*c*f*g*h**2 - 6*log(g + h*x)*
a*d*e*g*h**2 + 6*log(g + h*x)*a*d*f*g**2*h - 6*log(g + h*x)*b*c*e*g*h**2 +
 6*log(g + h*x)*b*c*f*g**2*h + 6*log(g + h*x)*b*d*e*g**2*h - 6*log(g + h*x
)*b*d*f*g**3 + 6*a*c*f*h**3*x + 6*a*d*e*h**3*x - 6*a*d*f*g*h**2*x + 3*a*d*
f*h**3*x**2 + 6*b*c*e*h**3*x - 6*b*c*f*g*h**2*x + 3*b*c*f*h**3*x**2 - 6*b*
d*e*g*h**2*x + 3*b*d*e*h**3*x**2 + 6*b*d*f*g**2*h*x - 3*b*d*f*g*h**2*x**2
+ 2*b*d*f*h**3*x**3)/(6*h**4)
```

3.3 $\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx = \frac{bdx}{fh} + \frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)} - \frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}$$

output
```
b*d*x/f/h+(-a*f+b*e)*(-c*f+d*e)*ln(f*x+e)/f^2/(-e*h+f*g)-(-a*h+b*g)*(-c*h+
d*g)*ln(h*x+g)/h^2/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx = \frac{(be-af)(de-cf)h^2\log(e+fx) + f(bdh(fg-eh)x - f(bg-ah)(dg-ch)\log(g+hx))}{f^2h^2(fg-eh)}$$

input
```
Integrate[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

output
```
((b*e - a*f)*(d*e - c*f)*h^2*Log[e + f*x] + f*(b*d*h*(f*g - e*h)*x - f*(b*
g - a*h)*(d*g - c*h)*Log[g + h*x]))/(f^2*h^2*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{(af-be)(cf-de)}{f(e+fx)(fg-eh)}+\frac{(ah-bg)(ch-dg)}{h(g+hx)(eh-fg)}+\frac{bd}{fh}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}+\frac{bdx}{fh}$$

input
```
Int[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

output
```
(b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
 ((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

method	result
default	$\frac{bdx}{fh}+\frac{\left(ac\,h^{2}-adgh-bcgh+bd\,g^{2}\right)\ln\left(hx+g\right)}{h^{2}\left(eh-fg\right)}+\frac{\left(-ac\,f^{2}+adef+bcef-bd\,e^{2}\right)\ln\left(fx+e\right)}{f^{2}\left(eh-fg\right)}$
norman	$\frac{bdx}{fh}+\frac{\left(ac\,h^{2}-adgh-bcgh+bd\,g^{2}\right)\ln\left(hx+g\right)}{h^{2}\left(eh-fg\right)}-\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)\ln\left(fx+e\right)}{\left(eh-fg\right)f^{2}}$
parallelrisch	$-\frac{\ln\left(fx+e\right)ac\,f^{2}h^{2}-\ln\left(fx+e\right)adef\,h^{2}-\ln\left(fx+e\right)bcef\,h^{2}+\ln\left(fx+e\right)bd\,e^{2}h^{2}-\ln\left(hx+g\right)ac\,f^{2}h^{2}+\ln\left(hx+g\right)ad\,f^{2}gh+\ln(hx+}{f^{2}h^{2}\left(eh-fg\right)}$
risch	$\frac{bdx}{fh}-\frac{\ln\left(fx+e\right)ac}{eh-fg}+\frac{\ln\left(fx+e\right)ade}{\left(eh-fg\right)f}+\frac{\ln\left(fx+e\right)bce}{\left(eh-fg\right)f}-\frac{\ln\left(fx+e\right)bd\,e^{2}}{\left(eh-fg\right)f^{2}}+\frac{\ln\left(-hx-g\right)ac}{eh-fg}-\frac{\ln\left(-hx-g\right)adg}{h\left(eh-fg\right)}-\frac{\ln(-hx}{h(eh}$

input
```
int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
b*d*x/f/h+1/h^2*(a*c*h^2-a*d*g*h-b*c*g*h+b*d*g^2)/(e*h-f*g)*ln(h*x+g)+(-a*
c*f^2+a*d*e*f+b*c*e*f-b*d*e^2)/f^2/(e*h-f*g)*ln(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int\frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx$$
$$=\frac{(bde^2+acf^2-(bc+ad)ef)h^2\log\left(fx+e\right)+(bdf^2gh-bdefh^2)x-(bdf^2g^2+acf^2h^2-(bc+ad)f^2gh)}{f^3gh^2-ef^2h^3}$$

input
```
integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
((b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*h^2*log(f*x + e) + (b*d*f^2*g*h - b
*d*e*f*h^2)*x - (b*d*f^2*g^2 + a*c*f^2*h^2 - (b*c + a*d)*f^2*g*h)*log(h*x
+ g))/(f^3*g*h^2 - e*f^2*h^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(68) = 136$.

Time = 11.49 (sec) , antiderivative size = 507, normalized size of antiderivative = 6.04

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{bdx}{fh} + \frac{(ah-bg)(ch-dg)\log\left(x + \frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2-\frac{e^2fh(ah-bg)(ch-dg)}{eh-fg}+\frac{2ef^2g(ah-bg)(ch-dg}{eh-fg}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}}{h^2(eh-fg)} - \frac{(af-be)(cf-de)\log\left(x + \frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2+\frac{e^2h^3(af-be)(cf-de)}{f(eh-fg)}-\frac{2egh^2(af-be)(cf-de)}{eh-fg}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}}{f^2(eh-fg)}$$

input
```
integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
b*d*x/(f*h) + (a*h - b*g)*(c*h - d*g)*log(x + (a*c*e*f*h**2 + a*c*f**2*g*h
 - 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h + b*d*e*f*g**2 - e**2*f*h*
(a*h - b*g)*(c*h - d*g)/(e*h - f*g) + 2*e*f**2*g*(a*h - b*g)*(c*h - d*g)/(
e*h - f*g) - f**3*g**2*(a*h - b*g)*(c*h - d*g)/(h*(e*h - f*g)))/(2*a*c*f**
2*h**2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**2 - b*c*f**2*g*h + b*d*e
**2*h**2 + b*d*f**2*g**2))/(h**2*(e*h - f*g)) - (a*f - b*e)*(c*f - d*e)*lo
g(x + (a*c*e*f*h**2 + a*c*f**2*g*h - 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e
**2*g*h + b*d*e*f*g**2 + e**2*h**3*(a*f - b*e)*(c*f - d*e)/(f*(e*h - f*g))
 - 2*e*g*h**2*(a*f - b*e)*(c*f - d*e)/(e*h - f*g) + f*g**2*h*(a*f - b*e)*(
c*f - d*e)/(e*h - f*g))/(2*a*c*f**2*h**2 - a*d*e*f*h**2 - a*d*f**2*g*h - b
*c*e*f*h**2 - b*c*f**2*g*h + b*d*e**2*h**2 + b*d*f**2*g**2))/(f**2*(e*h -
f*g))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{bdx}{fh} + \frac{(bde^2+acf^2-(bc+ad)ef)\log(fx+e)}{f^3g-ef^2h} - \frac{(bdg^2+ach^2-(bc+ad)gh)\log(hx+g)}{fgh^2-eh^3}$$

input
```
integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
b*d*x/(f*h) + (b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*log(f*x + e)/(f^3*g -
e*f^2*h) - (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(h*x + g)/(f*g*h^2 - e
*h^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{bdx}{fh} + \frac{(bde^2 - bcef - adef + acf^2)\log(|fx+e|)}{f^3g - ef^2h} - \frac{(bdg^2 - bcgh - adgh + ach^2)\log(|hx+g|)}{fgh^2 - eh^3}$$

input
```
integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
b*d*x/(f*h) + (b*d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)*log(abs(f*x + e))/(f
^3*g - e*f^2*h) - (b*d*g^2 - b*c*g*h - a*d*g*h + a*c*h^2)*log(abs(h*x + g)
)/(f*g*h^2 - e*h^3)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{\ln(e+f\,x)\,\left(a\,c\,f^2 - f\,(a\,d\,e + b\,c\,e) + b\,d\,e^2\right)}{f^3\,g - e\,f^2\,h} + \frac{\ln(g+h\,x)\,\left(a\,c\,h^2 - h\,(a\,d\,g + b\,c\,g) + b\,d\,g^2\right)}{e\,h^3 - f\,g\,h^2} + \frac{b\,d\,x}{f\,h}$$

input
```
int(((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

output
```
(log(e + f*x)*(a*c*f^2 - f*(a*d*e + b*c*e) + b*d*e^2))/(f^3*g - e*f^2*h) +
 (log(g + h*x)*(a*c*h^2 - h*(a*d*g + b*c*g) + b*d*g^2))/(e*h^3 - f*g*h^2)
+ (b*d*x)/(f*h)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.87

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx$$
$$= \frac{-\log(fx+e)\,ac\,f^2h^2+\log(fx+e)\,adef\,h^2+\log(fx+e)\,bcef\,h^2-\log(fx+e)\,bd\,e^2h^2+\log(hx+g)\,\ldots}{f^2h^2\,(eh-\ldots}$$

input
```
int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
( - log(e + f*x)*a*c*f**2*h**2 + log(e + f*x)*a*d*e*f*h**2 + log(e + f*x)*
b*c*e*f*h**2 - log(e + f*x)*b*d*e**2*h**2 + log(g + h*x)*a*c*f**2*h**2 - l
og(g + h*x)*a*d*f**2*g*h - log(g + h*x)*b*c*f**2*g*h + log(g + h*x)*b*d*f*
*2*g**2 + b*d*e*f*h**2*x - b*d*f**2*g*h*x)/(f**2*h**2*(e*h - f*g))
```

3.4 $\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} dx$

Optimal result

Integrand size = 27, antiderivative size = 108

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} dx = -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)} + \frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)} - \frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

output
```
-(-a*d+b*c)*ln(d*x+c)/(-c*f+d*e)/(-c*h+d*g)+(-a*f+b*e)*ln(f*x+e)/(-c*f+d*e
)/(-e*h+f*g)-(-a*h+b*g)*ln(h*x+g)/(-c*h+d*g)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} dx = \frac{(bc-ad)(fg-eh)\log(c+dx)-(be-af)(dg-ch)\log(e+fx)+(de-cf)(bg-ah)\log(g+hx)}{(de-cf)(dg-ch)(-fg+eh)}$$

input
```
Integrate[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
((b*c - a*d)*(f*g - e*h)*Log[c + d*x] - (b*e - a*f)*(d*g - c*h)*Log[e + f*
x] + (d*e - c*f)*(b*g - a*h)*Log[g + h*x])/((d*e - c*f)*(d*g - c*h)*(-(f*g
) + e*h))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)} \, dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{d(ad-bc)}{(c+dx)(de-cf)(dg-ch)}+\frac{f(af-be)}{(e+fx)(de-cf)(eh-fg)}+\frac{h(ah-bg)}{(g+hx)(dg-ch)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

input
```
Int[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
-(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log
[e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g -
c*h)*(f*g - e*h))
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

method	result
default	$\frac{(ad-bc)\ln(xd+c)}{(ch-dg)(cf-de)}+\frac{(ah-bg)\ln(hx+g)}{(eh-fg)(ch-dg)}-\frac{(af-be)\ln(fx+e)}{(cf-de)(eh-fg)}$
norman	$\frac{(ah-bg)\ln(hx+g)}{ce\,h^2-cfgh-degh+df\,g^2}+\frac{(ad-bc)\ln(xd+c)}{(ch-dg)(cf-de)}-\frac{(af-be)\ln(fx+e)}{(cf-de)(eh-fg)}$
parallelrisch	$\frac{\ln(xd+c)adeh-\ln(xd+c)adfg-\ln(xd+c)bceh+\ln(xd+c)bcfg-\ln(fx+e)acfh+\ln(fx+e)adfg+\ln(fx+e)bceh-\ln(fx+e)bdeg}{(ce\,h^2-cfgh-degh+df\,g^2)(cf-de)}$
risch	$\frac{\ln(-hx-g)ah}{ce\,h^2-cfgh-degh+df\,g^2}-\frac{\ln(-hx-g)bg}{ce\,h^2-cfgh-degh+df\,g^2}+\frac{\ln(xd+c)ad}{c^2fh-cdeh-cdfg+d^2eg}-\frac{\ln(xd+c)bc}{c^2fh-cdeh-cdfg+d^2eg}-\frac{}{cefh-}$

input
```
int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(a*d-b*c)/(c*h-d*g)/(c*f-d*e)*ln(d*x+c)+(a*h-b*g)/(e*h-f*g)/(c*h-d*g)*ln(h
*x+g)-(a*f-b*e)/(c*f-d*e)/(e*h-f*g)*ln(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 38.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.48

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{((bc-ad)fg-(bc-ad)eh)\log(dx+c)-((bde-adf)g-(bce-acf)h)\log(fx+e)+((bde-bcf}{(d^2ef-cdf^2)g^2-(d^2e^2-c^2f^2)gh+(cde^2-c^2ef)h^2}$$

input
```
integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
-(((b*c - a*d)*f*g - (b*c - a*d)*e*h)*log(d*x + c) - ((b*d*e - a*d*f)*g -
(b*c*e - a*c*f)*h)*log(f*x + e) + ((b*d*e - b*c*f)*g - (a*d*e - a*c*f)*h)*
log(h*x + g))/((d^2*e*f - c*d*f^2)*g^2 - (d^2*e^2 - c^2*f^2)*g*h + (c*d*e^
2 - c^2*e*f)*h^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.24

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(bc-ad)\log(dx+c)}{(d^2e-cdf)g-(cde-c^2f)h} + \frac{(be-af)\log(fx+e)}{(def-cf^2)g-(de^2-cef)h} - \frac{(bg-ah)\log(hx+g)}{dfg^2+ceh^2-(de+cf)gh}$$

input
```
integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
-(b*c - a*d)*log(d*x + c)/((d^2*e - c*d*f)*g - (c*d*e - c^2*f)*h) + (b*e -
 a*f)*log(f*x + e)/((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h) - (b*g - a*h)*l
og(h*x + g)/(d*f*g^2 + c*e*h^2 - (d*e + c*f)*g*h)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(bcd-ad^2)\log(|dx+c|)}{d^3eg-cd^2fg-cd^2eh+c^2dfh} + \frac{(bef-af^2)\log(|fx+e|)}{def^2g-cf^3g-de^2fh+cef^2h} - \frac{(bgh-ah^2)\log(|hx+g|)}{dfg^2h-degh^2-cfgh^2+ceh^3}$$

input
```
integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
-(b*c*d - a*d^2)*log(abs(d*x + c))/(d^3*e*g - c*d^2*f*g - c*d^2*e*h + c^2*
d*f*h) + (b*e*f - a*f^2)*log(abs(f*x + e))/(d*e*f^2*g - c*f^3*g - d*e^2*f*
h + c*e*f^2*h) - (b*g*h - a*h^2)*log(abs(h*x + g))/(d*f*g^2*h - d*e*g*h^2
- c*f*g*h^2 + c*e*h^3)
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{\ln(e+f\,x)\,(a\,f-b\,e)}{c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g} + \frac{\ln(g+h\,x)\,(a\,h-b\,g)}{c\,e\,h^2+d\,f\,g^2-c\,f\,g\,h-d\,e\,g\,h} + \frac{\ln(c+d\,x)\,(a\,d-b\,c)}{d^2\,e\,g+c^2\,f\,h-c\,d\,e\,h-c\,d\,f\,g}$$

input
```
int((a + b*x)/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

output
```
(log(e + f*x)*(a*f - b*e))/(c*f^2*g + d*e^2*h - c*e*f*h - d*e*f*g) + (log(
g + h*x)*(a*h - b*g))/(c*e*h^2 + d*f*g^2 - c*f*g*h - d*e*g*h) + (log(c + d
*x)*(a*d - b*c))/(d^2*e*g + c^2*f*h - c*d*e*h - c*d*f*g)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.85

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{\log(dx+c)\,adeh-\log(dx+c)\,adfg-\log(dx+c)\,bceh+\log(dx+c)\,bcfg-\log(fx+e)\,acfh+\log(fx}{c^2ef\,h^2-c^2f^2g}$$

input
```
int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
(log(c + d*x)*a*d*e*h - log(c + d*x)*a*d*f*g - log(c + d*x)*b*c*e*h + log(
c + d*x)*b*c*f*g - log(e + f*x)*a*c*f*h + log(e + f*x)*a*d*f*g + log(e + f
*x)*b*c*e*h - log(e + f*x)*b*d*e*g + log(g + h*x)*a*c*f*h - log(g + h*x)*a
*d*e*h - log(g + h*x)*b*c*f*g + log(g + h*x)*b*d*e*g)/(c**2*e*f*h**2 - c**
2*f**2*g*h - c*d*e**2*h**2 + c*d*f**2*g**2 + d**2*e**2*g*h - d**2*e*f*g**2
)
```

3.5 $\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 163

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

output
```
b^2*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)-d^2*ln(d*x+c)/(-a*d+b*c)/(-
c*f+d*e)/(-c*h+d*g)+f^2*ln(f*x+e)/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)-h^2*ln(
h*x+g)/(-a*h+b*g)/(-c*h+d*g)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2\log(c+dx)}{(bc-ad)(-de+cf)(-dg+ch)} - \frac{f^2\log(e+fx)}{(be-af)(de-cf)(-fg+eh)} - \frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
(b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*
x])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) - (f^2*Log[e + f*x])/((b*e
 - a*f)*(d*e - c*f)*(-(f*g) + e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g
 - c*h)*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{b^3}{(a+bx)(bc-ad)(be-af)(bg-ah)} - \frac{d^3}{(c+dx)(bc-ad)(cf-de)(ch-dg)} - \frac{f^3}{(e+fx)(be-af)(de-cf)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
(b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*
x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f
)*(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(
f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

method	result
default	$\frac{d^2\ln(xd+c)}{(ch-dg)(ad-bc)(cf-de)}-\frac{b^2\ln(bx+a)}{(ah-bg)(af-be)(ad-bc)}+\frac{h^2\ln(hx+g)}{(ah-bg)(ch-dg)(eh-fg)}-\frac{f^2\ln(fx+e)}{(af-be)(cf-de)(eh-fg)}$
norman	$\frac{h^2\ln(hx+g)}{ace\,h^3-acfg\,h^2-adeg\,h^2+adf\,g^2h-bceg\,h^2+bcf\,g^2h+bde\,g^2h-bdf\,g^3}+\frac{d^2\ln(xd+c)}{(ch-dg)(ad-bc)(cf-de)}-\frac{f^2\ln(f}{(ac\,f^2-adef-bcef}$
risch	$\frac{d^2\ln(-xd-c)}{a\,c^2dfh-ac\,d^2eh-ac\,d^2fg+a\,d^3eg-c^3hbf+b\,c^2deh+b\,c^2dfg-bc\,d^2eg}+\frac{h^2\ln(hx+g)}{ace\,h^3-acfg\,h^2-adeg\,h^2+adf\,g^2h-bceg\,h^2+bcf\,g}$
parallelrisch	$-\frac{\ln(bx+a)b^2c^2ef\,h^2-\ln(bx+a)b^2c^2f^2gh-\ln(bx+a)b^2cd\,e^2h^2+\ln(bx+a)b^2cd\,f^2g^2+\ln(bx+a)b^2d^2e^2gh-\ln(bx+a)b^2d^2ef\,g}{}$

input
```
int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
d^2/(c*h-d*g)/(a*d-b*c)/(c*f-d*e)*ln(d*x+c)-b^2/(a*h-b*g)/(a*f-b*e)/(a*d-b
*c)*ln(b*x+a)+h^2/(a*h-b*g)/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)-f^2/(a*f-b*e)/(c
*f-d*e)/(e*h-f*g)*ln(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.90

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{b^2\log(bx+a)}{((b^3c-ab^2d)e-(ab^2c-a^2bd)f)g-((ab^2c-a^2bd)e-(a^2bc-a^3d)f)h} - \frac{d^2\log(dx+c)}{((bcd^2-ad^3)e-(bc^2d-acd^2)f)g-((bc^2d-acd^2)e-(bc^3-ac^2d)f)h} + \frac{f^2\log(fx+e)}{(bde^2f+acf^3-(bc+ad)ef^2)g-(bde^3+acef^2-(bc+ad)e^2f)h} - \frac{h^2\log(hx+g)}{bdfg^3-aceh^3-(bde+(bc+ad)f)g^2h+(acf+(bc+ad)e)gh^2}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
b^2*log(b*x + a)/(((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2*b*d)*f)*g - ((a*b^
2*c - a^2*b*d)*e - (a^2*b*c - a^3*d)*f)*h) - d^2*log(d*x + c)/(((b*c*d^2 -
 a*d^3)*e - (b*c^2*d - a*c*d^2)*f)*g - ((b*c^2*d - a*c*d^2)*e - (b*c^3 - a
*c^2*d)*f)*h) + f^2*log(f*x + e)/((b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2
)*g - (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h) - h^2*log(h*x + g)/(b*d
*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*c + a*d)*
e)*g*h^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(163) = 326$.

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \frac{b^3\log(|bx+a|)}{b^4ceg - ab^3deg - ab^3cfg + a^2b^2dfg - ab^3ceh + a^2b^2deh + a^2b^2cfh - a^3bdfh} - \frac{d^3\log(|dx+c|)}{bcd^3eg - ad^4eg - bc^2d^2fg + acd^3fg - bc^2d^2eh + acd^3eh + bc^3dfh - ac^2d^2fh} + \frac{f^3\log(|fx+e|)}{bde^2f^2g - bcef^3g - adef^3g + acf^4g - bde^3fh + bce^2f^2h + ade^2f^2h - acef^3h} - \frac{h^3\log(|hx+g|)}{bdfg^3h - bdeg^2h^2 - bcfg^2h^2 - adfg^2h^2 + bcegh^3 + adegh^3 + acfgh^3 - aceh^4}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
b^3*log(abs(b*x + a))/(b^4*c*e*g - a*b^3*d*e*g - a*b^3*c*f*g + a^2*b^2*d*f
*g - a*b^3*c*e*h + a^2*b^2*d*e*h + a^2*b^2*c*f*h - a^3*b*d*f*h) - d^3*log(
abs(d*x + c))/(b*c*d^3*e*g - a*d^4*e*g - b*c^2*d^2*f*g + a*c*d^3*f*g - b*c
^2*d^2*e*h + a*c*d^3*e*h + b*c^3*d*f*h - a*c^2*d^2*f*h) + f^3*log(abs(f*x
+ e))/(b*d*e^2*f^2*g - b*c*e*f^3*g - a*d*e*f^3*g + a*c*f^4*g - b*d*e^3*f*h
 + b*c*e^2*f^2*h + a*d*e^2*f^2*h - a*c*e*f^3*h) - h^3*log(abs(h*x + g))/(b
*d*f*g^3*h - b*d*e*g^2*h^2 - b*c*f*g^2*h^2 - a*d*f*g^2*h^2 + b*c*e*g*h^3 +
 a*d*e*g*h^3 + a*c*f*g*h^3 - a*c*e*h^4)
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{b^2\ln(a+b\,x)}{b^3\,c\,e\,g-a^3\,d\,f\,h-a\,b^2\,c\,e\,h-a\,b^2\,c\,f\,g-a\,b^2\,d\,e\,g+a^2\,b\,c\,f\,h+a^2\,b\,d\,e\,h+a^2\,b\,d\,f\,g}$$
$$+\frac{d^2\ln(c+d\,x)}{a\,d^3\,e\,g-b\,c^3\,f\,h-a\,c\,d^2\,e\,h-a\,c\,d^2\,f\,g-b\,c\,d^2\,e\,g+a\,c^2\,d\,f\,h+b\,c^2\,d\,e\,h+b\,c^2\,d\,f\,g}$$
$$+\frac{f^2\ln(e+f\,x)}{a\,c\,f^3\,g-b\,d\,e^3\,h-a\,c\,e\,f^2\,h-a\,d\,e\,f^2\,g-b\,c\,e\,f^2\,g+a\,d\,e^2\,f\,h+b\,c\,e^2\,f\,h+b\,d\,e^2\,f\,g}$$
$$+\frac{h^2\ln(g+h\,x)}{a\,c\,e\,h^3-b\,d\,f\,g^3-a\,c\,f\,g\,h^2-a\,d\,e\,g\,h^2-b\,c\,e\,g\,h^2+a\,d\,f\,g^2\,h+b\,c\,f\,g^2\,h+b\,d\,e\,g^2\,h}$$

input
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x)),x)
```

output
```
(b^2*log(a + b*x))/(b^3*c*e*g - a^3*d*f*h - a*b^2*c*e*h - a*b^2*c*f*g - a*
b^2*d*e*g + a^2*b*c*f*h + a^2*b*d*e*h + a^2*b*d*f*g) + (d^2*log(c + d*x))/
(a*d^3*e*g - b*c^3*f*h - a*c*d^2*e*h - a*c*d^2*f*g - b*c*d^2*e*g + a*c^2*d
*f*h + b*c^2*d*e*h + b*c^2*d*f*g) + (f^2*log(e + f*x))/(a*c*f^3*g - b*d*e^
3*h - a*c*e*f^2*h - a*d*e*f^2*g - b*c*e*f^2*g + a*d*e^2*f*h + b*c*e^2*f*h
+ b*d*e^2*f*g) + (h^2*log(g + h*x))/(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 -
 a*d*e*g*h^2 - b*c*e*g*h^2 + a*d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 821, normalized size of antiderivative = 5.04

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
( - log(a + b*x)*b**2*c**2*e*f*h**2 + log(a + b*x)*b**2*c**2*f**2*g*h + lo
g(a + b*x)*b**2*c*d*e**2*h**2 - log(a + b*x)*b**2*c*d*f**2*g**2 - log(a +
b*x)*b**2*d**2*e**2*g*h + log(a + b*x)*b**2*d**2*e*f*g**2 + log(c + d*x)*a
**2*d**2*e*f*h**2 - log(c + d*x)*a**2*d**2*f**2*g*h - log(c + d*x)*a*b*d**
2*e**2*h**2 + log(c + d*x)*a*b*d**2*f**2*g**2 + log(c + d*x)*b**2*d**2*e**
2*g*h - log(c + d*x)*b**2*d**2*e*f*g**2 - log(e + f*x)*a**2*c*d*f**2*h**2
+ log(e + f*x)*a**2*d**2*f**2*g*h + log(e + f*x)*a*b*c**2*f**2*h**2 - log(
e + f*x)*a*b*d**2*f**2*g**2 - log(e + f*x)*b**2*c**2*f**2*g*h + log(e + f*
x)*b**2*c*d*f**2*g**2 + log(g + h*x)*a**2*c*d*f**2*h**2 - log(g + h*x)*a**
2*d**2*e*f*h**2 - log(g + h*x)*a*b*c**2*f**2*h**2 + log(g + h*x)*a*b*d**2*
e**2*h**2 + log(g + h*x)*b**2*c**2*e*f*h**2 - log(g + h*x)*b**2*c*d*e**2*h
**2)/(a**3*c**2*d*e*f**2*h**3 - a**3*c**2*d*f**3*g*h**2 - a**3*c*d**2*e**2
*f*h**3 + a**3*c*d**2*f**3*g**2*h + a**3*d**3*e**2*f*g*h**2 - a**3*d**3*e*
f**2*g**2*h - a**2*b*c**3*e*f**2*h**3 + a**2*b*c**3*f**3*g*h**2 + a**2*b*c
*d**2*e**3*h**3 - a**2*b*c*d**2*f**3*g**3 - a**2*b*d**3*e**3*g*h**2 + a**2
*b*d**3*e*f**2*g**3 + a*b**2*c**3*e**2*f*h**3 - a*b**2*c**3*f**3*g**2*h -
a*b**2*c**2*d*e**3*h**3 + a*b**2*c**2*d*f**3*g**3 + a*b**2*d**3*e**3*g**2*
h - a*b**2*d**3*e**2*f*g**3 - b**3*c**3*e**2*f*g*h**2 + b**3*c**3*e*f**2*g
**2*h + b**3*c**2*d*e**3*g*h**2 - b**3*c**2*d*e*f**2*g**3 - b**3*c*d**2*e*
*3*g**2*h + b**3*c*d**2*e**2*f*g**3)
```

3.6 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{(1+x)(2+x)(3+x)} \, dx = -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x)$$

output
```
-1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(2+x)(3+x)} \, dx = -\frac{1}{2}\log(1+x) + 2\log(2+x) - \frac{3}{2}\log(3+x)$$

input
```
Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

output
```
-1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)(x+2)(x+3)} \, dx$$

$$\Big\downarrow \text{165}$$

$$\int \left(\frac{2}{x+2}-\frac{3}{2(x+3)}-\frac{1}{2(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

input
```
Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

output
```
-1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
norman	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
risch	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20
parallelrisch	$-\frac{\ln(1+x)}{2}+2\ln\left(2+x\right)-\frac{3\ln(3+x)}{2}$	20

input
```
int(x/(1+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
```

output
```
-3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x)
```

output
```
-log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
```

output
```
-3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3}{2}\,\log\left(|x+3|\right)+2\,\log\left(|x+2|\right)-\frac{1}{2}\,\log\left(|x+1|\right)$$

input
```
integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
```

output
```
-3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = 2\,\ln\left(x+2\right)-\frac{\ln\left(x+1\right)}{2}-\frac{3\,\ln\left(x+3\right)}{2}$$

input
```
int(x/((x + 1)*(x + 2)*(x + 3)),x)
```

output
```
2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx = -\frac{3\log(x+3)}{2} + 2\log(x+2) - \frac{\log(x+1)}{2}$$

input
```
int(x/(1+x)/(2+x)/(3+x),x)
```

output
```
( - 3*log(x + 3) + 4*log(x + 2) - log(x + 1))/2
```

3.7 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx = -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}$$

output
```
-12/1375/(3+5*x)^2+201/(45375+75625*x)+20/3993*ln(6-x)+1493/499125*ln(3+5*
x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx = \frac{\frac{99(157+335x)}{(3+5x)^2}+2500\log(-6+x)+1493\log(3+5x)}{499125}$$

input
```
Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

output
```
((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/49
9125
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2027, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x^2}{(x-6)(5x+3)^3}\,dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{(x-1)x^2}{(x-6)(5x+3)^3}dx$$
$$\Big\downarrow \text{165}$$
$$\int \left(\frac{1493}{99825(5x+3)}-\frac{201}{3025(5x+3)^2}+\frac{24}{275(5x+3)^3}+\frac{20}{3993(x-6)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

input
```
Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

output
```
-12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (1
493*Log[3 + 5*x])/499125
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{201x}{3025}+\frac{471}{15125}}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$
norman	$\frac{-\frac{113}{3025}x-\frac{157}{1815}x^2}{(3+5x)^2}+\frac{20\ln(-6+x)}{3993}+\frac{1493\ln(3+5x)}{499125}$
default	$-\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{1493\ln(3+5x)}{499125}+\frac{20\ln(-6+x)}{3993}$
parallelrisch	$\frac{187500\ln(-6+x)x^2+111975\ln\left(x+\frac{3}{5}\right)x^2+225000\ln(-6+x)x+134370\ln\left(x+\frac{3}{5}\right)x-129525x^2+67500\ln(-6+x)+40311\ln(x-}{1497375(3+5x)^2}$

input
```
int((x^3-x^2)/(-6+x)/(3+5*x)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(201/75625*x+471/378125)/(3+5*x)^2+20/3993*ln(-6+x)+1493/499125*ln(3+5*
x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{1493\,(25\,x^2+30\,x+9)\log(5\,x+3)+2500\,(25\,x^2+30\,x+9)\log(x-6)+33165\,x+15543}{499125\,(25\,x^2+30\,x+9)}$$

input
```
integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

output
```
1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)
*log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{1005x+471}{378125x^2+453750x+136125} + \frac{20\log(x-6)}{3993} + \frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

input
```
integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

output
```
(1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 14
93*log(x + 3/5)/499125
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{3(335x+157)}{15125(25x^2+30x+9)} + \frac{1493}{499125}\log(5x+3) + \frac{20}{3993}\log(x-6)$$

input
```
integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

output
```
3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/
3993*log(x - 6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2} + \frac{1493}{499125}\log\left(|5\,x+3|\right) + \frac{20}{3993}\log\left(|x-6|\right)$$

input
```
integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

output
```
3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/399
3*log(abs(x - 6))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \frac{20\ln(x-6)}{3993} + \frac{1493\ln\left(x+\frac{3}{5}\right)}{499125} + \frac{\frac{201\,x}{75625}+\frac{471}{378125}}{x^2+\frac{6\,x}{5}+\frac{9}{25}}$$

input
```
int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

output
```
(20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/3
78125)/((6*x)/5 + x^2 + 9/25)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$$
$$= \frac{74650\log(5x+3)\,x^2+89580\log(5x+3)\,x+26874\log(5x+3)+125000\log(x-6)\,x^2+150000\log(x}{24956250x^2+29947500x+8984250}$$

input
```
int((x^3-x^2)/(-6+x)/(3+5*x)^3,x)
```

output

```
(74650*log(5*x + 3)*x**2 + 89580*log(5*x + 3)*x + 26874*log(5*x + 3) + 125
000*log(x - 6)*x**2 + 150000*log(x - 6)*x + 45000*log(x - 6) - 55275*x**2
+ 11187)/(998250*(25*x**2 + 30*x + 9))
```

3.8 $\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx=-\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+2a\arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

output
```
-(-1+x)^(1/2)*(1+x)^(1/2)/x+2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx=-\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+4a\arctan\left(\sqrt{\frac{-1+x}{1+x}}\right)$$

input
```
Integrate[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 4*a*ArcTan[Sqrt[(-1 + x)/(1 + x)]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {168, 27, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2ax-1}{\sqrt{x-1}x^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{168}$$

$$\int \frac{2a}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{27}$$

$$2a\int \frac{1}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{103}$$

$$2a\int \frac{1}{(x-1)(x+1)+1}d\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{216}$$

$$2a\arctan\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

input
```
Int[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\left(-2 a x \arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right)-\sqrt{x^{2}-1}\right) \sqrt{-1+x}\, \sqrt{1+x}}{x \sqrt{x^{2}-1}}$	44
risch	$-\frac{\sqrt{-1+x}\, \sqrt{1+x}}{x}-\frac{2 a \arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right) \sqrt{\left(1+x\right) \left(-1+x\right)}}{\sqrt{-1+x}\, \sqrt{1+x}}$	47

input
```
int((2*a*x-1)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/
(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = \frac{4\,ax\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

input
```
integrate((2*a*x-1)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
(4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/
x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.00

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\,\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\,\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}} + \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\,\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\,\frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

input
```
integrate((2*a*x-1)/(-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

output
```
-a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
x**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4
, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg
(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(
4*pi**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1
/2, 1, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -2\,a\arcsin\left(\frac{1}{|x|}\right) - \frac{\sqrt{x^2-1}}{x}$$

input
```
integrate((2*a*x-1)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right) - \frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}$$

input
```
integrate((2*a*x-1)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x
- 1))^4 + 4)
```

Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{x-1}\,\sqrt{x+1}}{x} - a\left(\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\right)2\mathrm{i}$$

input
```
int((2*a*x - 1)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
- a*(log(((x - 1)^(1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1) - log(((x - 1)^
(1/2) - 1i)/((x + 1)^(1/2) - 1)))*2i - ((x - 1)^(1/2)*(x + 1)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{-1+2ax}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = \frac{4atan\left(\sqrt{x-1}+\sqrt{x+1}-1\right)ax-4atan\left(\sqrt{x-1}+\sqrt{x+1}+1\right)ax-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

input
```
int((2*a*x-1)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x)
```

output
```
(4*atan(sqrt(x - 1) + sqrt(x + 1) - 1)*a*x - 4*atan(sqrt(x - 1) + sqrt(x +
 1) + 1)*a*x - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

3.9 $\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 39

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+2a\arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

output
```
-(-1+x)^(1/2)*(1+x)^(1/2)/x+2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x}+4a\arctan\left(\sqrt{\frac{-1+x}{1+x}}\right)$$

input
```
Integrate[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 4*a*ArcTan[Sqrt[(-1 + x)/(1 + x)]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {206, 168, 27, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{x-1}x^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{206}$$

$$\int \frac{2ax-1}{\sqrt{x-1}x^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{168}$$

$$\int \frac{2a}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{27}$$

$$2a\int \frac{1}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{103}$$

$$2a\int \frac{1}{(x-1)(x+1)+1}d\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{216}$$

$$2a\arctan\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

input
```
Int[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 206
```
Int[(u_)^(m_.)*(v_)^(n_.)*(w_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[ExpandToS
um[u, x]^m*ExpandToSum[v, x]^n*ExpandToSum[w, x]^p*ExpandToSum[z, x]^q, x]
/; FreeQ[{m, n, p, q}, x] && LinearQ[{u, v, w, z}, x] &&  !LinearMatchQ[{u,
 v, w, z}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}}{x\sqrt{x^2-1}}$	44
risch	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}-\frac{2a\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{-1+x}\,\sqrt{1+x}}$	47

input
```
int((a^2*x^2-(-a*x+1)^2)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
(-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/
(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx=\frac{4\,ax\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

input
```
integrate((a^2*x^2-(-a*x+1)^2)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="
fricas")
```

output
```
(4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/
x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 39.68 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.00

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}} + \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

input
```
integrate((a**2*x**2-(-a*x+1)**2)/(-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

output
```
-a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
x**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4
, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg
(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(
4*pi**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1
/2, 1, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

input
```
integrate((a^2*x^2-(-a*x+1)^2)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="
maxima")
```

output
```
-2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = -4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right) - \frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}$$

input
```
integrate((a^2*x^2-(-a*x+1)^2)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="
giac")
```

output
```
-4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x
- 1))^4 + 4)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 444, normalized size of antiderivative = 11.38

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx$$
$$= a\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)2\mathrm{i} - a^2\operatorname{atan}\left(\frac{1024\,a^6}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}} + \frac{1024\,a^8}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}} - \frac{a^5\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}\right)} - \frac{a^7\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}\right)}\right)4\mathrm{i} - a\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)2\mathrm{i} - \frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)} + a^2\operatorname{acosh}(x) - \frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{4\left(\sqrt{x+1}-1\right)^2}+\frac{1}{4}}{\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}}$$

input
```
int(-((a*x - 1)^2 - a^2*x^2)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
a*log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*2i - a^2*atan((1024*a^6)/(
1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)
 + (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) + (1024*a^8)/(102
4*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) +
(a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) - (a^5*((x - 1)^(1/2
) - 1i)*1024i)/(((x + 1)^(1/2) - 1)*(1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(
1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8*((x - 1)^(1/2) - 1i)*1024i)/(
(x + 1)^(1/2) - 1))) - (a^7*((x - 1)^(1/2) - 1i)*1024i)/(((x + 1)^(1/2) -
1)*(1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2)
- 1) + (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1))))*4i - a*log(
((x - 1)^(1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1)*2i - ((x - 1)^(1/2) - 1i
)/(4*((x + 1)^(1/2) - 1)) + a^2*acosh(x) - ((5*((x - 1)^(1/2) - 1i)^2)/(4*
((x + 1)^(1/2) - 1)^2) + 1/4)/(((x - 1)^(1/2) - 1i)^3/((x + 1)^(1/2) - 1)^
3 + ((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}x^2\sqrt{1+x}}\,dx = \frac{4atan\left(\sqrt{x-1}+\sqrt{x+1}-1\right)ax-4atan\left(\sqrt{x-1}+\sqrt{x+1}+1\right)ax-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

input
```
int((a^2*x^2-(-a*x+1)^2)/(-1+x)^(1/2)/x^2/(1+x)^(1/2),x)
```

output
```
(4*atan(sqrt(x - 1) + sqrt(x + 1) - 1)*a*x - 4*atan(sqrt(x - 1) + sqrt(x +
 1) + 1)*a*x - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

3.10 $\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 304

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = -\frac{2(be-af)(2bfg+beh-3afh)\sqrt{c+dx}}{b^4\sqrt{a+bx}}$$
$$-\frac{f(bcfh+11adfh-4bd(fg+2eh))\sqrt{a+bx}\sqrt{c+dx}}{4b^4d}$$
$$-\frac{2(be-af)^2(bg-ah)(c+dx)^{3/2}}{3b^3(bc-ad)(a+bx)^{3/2}}+\frac{f^2h\sqrt{a+bx}(c+dx)^{3/2}}{2b^3d}$$
$$+\frac{(35a^2d^2f^2h-10abdf(2dfg+4deh+cfh)-b^2(c^2f^2h-8d^2e(2fg+eh)-4cdf(fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\cdots}{\sqrt{b}\cdots}\right.}{4b^{9/2}d^{3/2}}$$

output
```
-2*(-a*f+b*e)*(-3*a*f*h+b*e*h+2*b*f*g)*(d*x+c)^(1/2)/b^4/(b*x+a)^(1/2)-1/4
*f*(b*c*f*h+11*a*d*f*h-4*b*d*(2*e*h+f*g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^4/
d-2/3*(-a*f+b*e)^2*(-a*h+b*g)*(d*x+c)^(3/2)/b^3/(-a*d+b*c)/(b*x+a)^(3/2)+1
/2*f^2*h*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^3/d+1/4*(35*a^2*d^2*f^2*h-10*a*b*d*
f*(c*f*h+4*d*e*h+2*d*f*g)-b^2*(c^2*f^2*h-8*d^2*e*(e*h+2*f*g)-4*c*d*f*(2*e*
h+f*g)))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(9/2)/d^(3
/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{c+dx}(105a^4d^2f^2h-20a^3bdf(5cfh+d(3fg+6eh-7fhx))+2ab^3 \cdots}{\cdots} + \frac{(35a^2d^2f^2h-10abdf(2dfg+4deh+cfh)+b^2(-c^2f^2h+8d^2e(2fg+eh)+4cdf(fg+2eh)))\operatorname{arctanh}\left(\cdots\right.}{4b^{9/2}d^{3/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(e + f*x)^2*(g + h*x))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[c + d*x]*(105*a^4*d^2*f^2*h - 20*a^3*b*d*f*(5*c*f*h + d*(3*f*g + 6*e
*h - 7*f*h*x)) + 2*a*b^3*(3*c^2*f^2*h*x - 4*c*d*(2*e^2*h + 2*e*f*(2*g - 9*
h*x) + 3*f^2*x*(-3*g + h*x)) + d^2*x*(16*e^2*h + 4*e*f*(8*g - 3*h*x) - 3*f
^2*x*(2*g + h*x))) + b^4*(-8*d^2*e^2*g*x + 3*c^2*f^2*h*x^2 + 2*c*d*(12*e*f
*x*(-2*g + h*x) + 3*f^2*x^2*(2*g + h*x) - 4*e^2*(g + 3*h*x))) + a^2*b^2*(3
*c^2*f^2*h + 2*c*d*f*(26*f*g + 52*e*h - 69*f*h*x) + d^2*(24*e^2*h + 16*e*f
*(3*g - 10*h*x) + f^2*x*(-80*g + 21*h*x)))))/(12*b^4*d*(b*c - a*d)*(a + b*
x)^(3/2)) + ((35*a^2*d^2*f^2*h - 10*a*b*d*f*(2*d*f*g + 4*d*e*h + c*f*h) +
b^2*(-(c^2*f^2*h) + 8*d^2*e*(2*f*g + e*h) + 4*c*d*f*(f*g + 2*e*h)))*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(9/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.66, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {167, 27, 167, 27, 164, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 167$$

$$\frac{2\int \frac{(e+fx)^2(bdeg-a(de+6cf)h+3bc(fg+eh)+d(4bfg+3beh-7afh)x)}{2(a+bx)^{3/2}\sqrt{c+dx}}dx}{3b(be-af)} - \frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(e+fx)^2(bdeg-a(de+6cf)h+3bc(fg+eh)+d(4bfg+3beh-7afh)x)}{(a+bx)^{3/2}\sqrt{c+dx}}dx}{3b(be-af)}-\frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

$$\Big\downarrow \; 167$$

$$2\int \frac{(e+fx)\Big(7df(de+4cf)ha^2-b\Big(e(4fg+3eh)d^2+cf(16fg+23eh)d+24c^2f^2h\Big)a+b^2c(12cf(fg+eh)+de(8fg+3eh))+df\Big(35dfha^2-b(20dfg+19deh+31cfh)a+b^2(4d\ldots}{2\sqrt{a+bx}\sqrt{c+dx}} \Big/ b(bc-ad)$$

$$\ldots \Big/ 3b(be-af)$$

$$\frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(e+fx)\Big(7df(de+4cf)ha^2-b\Big(e(4fg+3eh)d^2+cf(16fg+23eh)d+24c^2f^2h\Big)a+b^2c(12cf(fg+eh)+de(8fg+3eh))+df\Big(35dfha^2-b(20dfg+19deh+31cfh)a+b^2(4de\ldots}{\sqrt{a+bx}\sqrt{c+dx}} \Big/ b(bc-ad)$$

$$\ldots \Big/ 3b(be-af)$$

$$\frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

$$\Big\downarrow \; 164$$

$$\frac{3(bc-ad)(be-af)\Big(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\Big(b^2\Big(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\Big)\Big)\Big)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{8b^2d}-\frac{f\sqrt{a+bx}\sqrt{c+dx}\Big(105a^3d^2f^2h-2\ldots}{\ldots}$$

$$\frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

$$\Big\downarrow \; 66$$

$$\frac{3(bc-ad)(be-af)\Big(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\Big(b^2\Big(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\Big)\Big)\Big)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{4b^2d}-\frac{f\sqrt{a+bx}\sqrt{c+dx}\Big(105a^3d^2f^2\ldots}{\ldots}$$

$$\frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{3(bc-ad)(be-af)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)\left(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\left(b^2\left(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\right)\right)\right)}{4b^{5/2}d^{3/2}}-\frac{f\sqrt{a+bx}\sqrt{c+dx}\left(105a^3d^2f\right.}{}}{}$$

$$\frac{2\sqrt{c+dx}(e+fx)^3(bg-ah)}{3b(a+bx)^{3/2}(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*(e + f*x)^2*(g + h*x))/(a + b*x)^(5/2),x]
```

output

```
(-2*(b*g - a*h)*Sqrt[c + d*x]*(e + f*x)^3)/(3*b*(b*e - a*f)*(a + b*x)^(3/2
)) + ((-2*(7*a^2*d*f*h + b^2*(d*e*g + 3*c*(f*g + e*h)) - 2*a*b*(3*c*f*h +
2*d*(f*g + e*h)))*Sqrt[c + d*x]*(e + f*x)^2)/(b*(b*c - a*d)*Sqrt[a + b*x])
 + (-1/4*(f*Sqrt[a + b*x]*Sqrt[c + d*x]*(105*a^3*d^2*f^2*h - 5*a^2*b*d*f*(
12*d*f*g + 45*d*e*h + 20*c*f*h) - b^3*e*(16*d^2*e*g + 3*c^2*f*h + 12*c*d*(
7*f*g + 6*e*h)) + a*b^2*(3*c^2*f^2*h + 4*d^2*e*(27*f*g + 22*e*h) + 4*c*d*f
*(13*f*g + 51*e*h)) - 2*b*d*f*(35*a^2*d*f*h + b^2*(4*d*e*g + 16*c*f*g + 15
*c*e*h) - a*b*(20*d*f*g + 19*d*e*h + 31*c*f*h))*x))/(b^2*d) + (3*(b*c - a*
d)*(b*e - a*f)*(35*a^2*d^2*f^2*h - 10*a*b*d*f*(2*d*f*g + 4*d*e*h + c*f*h)
- b^2*(c^2*f^2*h - 8*d^2*e*(2*f*g + e*h) - 4*c*d*f*(f*g + 2*e*h)))*ArcTanh
[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2)*d^(3/2)))/(b
*(b*c - a*d)))/(3*b*(b*e - a*f))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3426 vs. $2(268) = 536$.

Time = 0.41 (sec) , antiderivative size = 3427, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	3427

input
```
int((d*x+c)^(1/2)*(f*x+e)^2*(h*x+g)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-42*a^2*b^2*d^2*f^2*h*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+24*a*b
^3*d^2*f^2*g*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+16*b^4*d^2*e^2*g*x*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+120*a^3*b*d^2*f^2*g*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)-210*a^4*d^2*f^2*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+3*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
b^5*c^3*f^2*h*x^2-60*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^3*f^2*g+48*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*d^3*e^2*h*x+210*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*
b*d^3*f^2*h*x-120*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*
d+b*c)/(d*b)^(1/2))*a^3*b^2*d^3*f^2*g*x-12*a*b^3*c^2*f^2*h*x*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-64*a*b^3*d^2*e^2*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+48*b^4*c*d*e^2*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+200*a^3*b*c*d
*f^2*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+240*a^3*b*d^2*e*f*h*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)-104*a^2*b^2*c*d*f^2*g*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)-96*a^2*b^2*d^2*e*f*g*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+32*a*b^3
*c*d*e^2*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-135*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c*d^2*f^2*h*x
^2+12*a*b^3*d^2*f^2*h*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-12*b^4*c*d*f
^2*h*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-240*ln(1/2*(2*b*d*x+2*((b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. $2(268) = 536$.

Time = 12.71 (sec) , antiderivative size = 2312, normalized size of antiderivative = 7.61

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^2*(h*x+g)/(b*x+a)^(5/2),x, algorithm="fric
as")
```

output
```
[1/48*(3*((4*(4*(b^5*c*d^2 - a*b^4*d^3)*e*f + (b^5*c^2*d - 6*a*b^4*c*d^2 +
 5*a^2*b^3*d^3)*f^2)*g + (8*(b^5*c*d^2 - a*b^4*d^3)*e^2 + 8*(b^5*c^2*d - 6
*a*b^4*c*d^2 + 5*a^2*b^3*d^3)*e*f - (b^5*c^3 + 9*a*b^4*c^2*d - 45*a^2*b^3*
c*d^2 + 35*a^3*b^2*d^3)*f^2)*h)*x^2 + 4*(4*(a^2*b^3*c*d^2 - a^3*b^2*d^3)*e
*f + (a^2*b^3*c^2*d - 6*a^3*b^2*c*d^2 + 5*a^4*b*d^3)*f^2)*g + (8*(a^2*b^3*
c*d^2 - a^3*b^2*d^3)*e^2 + 8*(a^2*b^3*c^2*d - 6*a^3*b^2*c*d^2 + 5*a^4*b*d^
3)*e*f - (a^2*b^3*c^3 + 9*a^3*b^2*c^2*d - 45*a^4*b*c*d^2 + 35*a^5*d^3)*f^2
)*h + 2*(4*(4*(a*b^4*c*d^2 - a^2*b^3*d^3)*e*f + (a*b^4*c^2*d - 6*a^2*b^3*c
*d^2 + 5*a^3*b^2*d^3)*f^2)*g + (8*(a*b^4*c*d^2 - a^2*b^3*d^3)*e^2 + 8*(a*b
^4*c^2*d - 6*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*e*f - (a*b^4*c^3 + 9*a^2*b^3*c
^2*d - 45*a^3*b^2*c*d^2 + 35*a^4*b*d^3)*f^2)*h)*x)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*s
qrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(6*(b^5*c*d^2 -
a*b^4*d^3)*f^2*h*x^3 + 3*(4*(b^5*c*d^2 - a*b^4*d^3)*f^2*g + (8*(b^5*c*d^2
- a*b^4*d^3)*e*f + (b^5*c^2*d - 8*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*f^2)*h)*x^2
 - 4*(2*b^5*c*d^2*e^2 + 4*(2*a*b^4*c*d^2 - 3*a^2*b^3*d^3)*e*f - (13*a^2*b^
3*c*d^2 - 15*a^3*b^2*d^3)*f^2)*g - (8*(2*a*b^4*c*d^2 - 3*a^2*b^3*d^3)*e^2
- 8*(13*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)*e*f - (3*a^2*b^3*c^2*d - 100*a^3*b
^2*c*d^2 + 105*a^4*b*d^3)*f^2)*h - 2*(4*(b^5*d^3*e^2 + 2*(3*b^5*c*d^2 - 4*
a*b^4*d^3)*e*f - (9*a*b^4*c*d^2 - 10*a^2*b^3*d^3)*f^2)*g + (4*(3*b^5*c*...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{c+dx}\,(e+fx)^2\,(g+hx)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**2*(h*x+g)/(b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)**2*(g + h*x)/(a + b*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^2*(h*x+g)/(b*x+a)^(5/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2146 vs. $2(268) = 536$.

Time = 0.58 (sec) , antiderivative size = 2146, normalized size of antiderivative = 7.06

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^2*(h*x+g)/(b*x+a)^(5/2),x, algorithm="giac
")
```

output
```
1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*f^2*h*a
bs(b)/b^6 + (4*b^12*d^2*f^2*g*abs(b) + 8*b^12*d^2*e*f*h*abs(b) + b^12*c*d*
f^2*h*abs(b) - 13*a*b^11*d^2*f^2*h*abs(b))/(b^17*d^2)) - 1/8*(16*b^2*d^2*e
*f*g*abs(b) + 4*b^2*c*d*f^2*g*abs(b) - 20*a*b*d^2*f^2*g*abs(b) + 8*b^2*d^2
*e^2*h*abs(b) + 8*b^2*c*d*e*f*h*abs(b) - 40*a*b*d^2*e*f*h*abs(b) - b^2*c^2
*f^2*h*abs(b) - 10*a*b*c*d*f^2*h*abs(b) + 35*a^2*d^2*f^2*h*abs(b))*log((sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)
*b^5*d) - 4/3*(b^7*c^2*d^2*e^2*g*abs(b) - 2*a*b^6*c*d^3*e^2*g*abs(b) + a^2
*b^5*d^4*e^2*g*abs(b) + 6*b^7*c^3*d*e*f*g*abs(b) - 20*a*b^6*c^2*d^2*e*f*g*
abs(b) + 22*a^2*b^5*c*d^3*e*f*g*abs(b) - 8*a^3*b^4*d^4*e*f*g*abs(b) - 6*a*
b^6*c^3*d*f^2*g*abs(b) + 19*a^2*b^5*c^2*d^2*f^2*g*abs(b) - 20*a^3*b^4*c*d^
3*f^2*g*abs(b) + 7*a^4*b^3*d^4*f^2*g*abs(b) + 3*b^7*c^3*d*e^2*h*abs(b) - 1
0*a*b^6*c^2*d^2*e^2*h*abs(b) + 11*a^2*b^5*c*d^3*e^2*h*abs(b) - 4*a^3*b^4*d
^4*e^2*h*abs(b) - 12*a*b^6*c^3*d*e*f*h*abs(b) + 38*a^2*b^5*c^2*d^2*e*f*h*a
bs(b) - 40*a^3*b^4*c*d^3*e*f*h*abs(b) + 14*a^4*b^3*d^4*e*f*h*abs(b) + 9*a^
2*b^5*c^3*d*f^2*h*abs(b) - 28*a^3*b^4*c^2*d^2*f^2*h*abs(b) + 29*a^4*b^3*c*
d^3*f^2*h*abs(b) - 10*a^5*b^2*d^4*f^2*h*abs(b) - 12*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c^2*d*e*f*g*abs(b) + 24*(s
qrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^4*c*d^
2*e*f*g*abs(b) - 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \int \frac{(e+f\,x)^2\,(g+h\,x)\,\sqrt{c+d\,x}}{(a+b\,x)^{5/2}}\,dx$$

input
```
int(((e + f*x)^2*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(5/2),x)
```

output
```
int(((e + f*x)^2*(g + h*x)*(c + d*x)^(1/2))/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(e+fx)^2(g+hx)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,(fx+e)^2\,(hx+g)}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^2*(h*x+g)/(b*x+a)^(5/2),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^2*(h*x+g)/(b*x+a)^(5/2),x)
```

3.11 $\int \frac{\sqrt{c+dx}\left(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3\right)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 304

$$\int \frac{\sqrt{c+dx}\left(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3\right)}{(a+bx)^{5/2}}\,dx = -\frac{2(be-af)(2bfg+beh-3afh)\sqrt{c+dx}}{b^4\sqrt{a+bx}} - \frac{f(bcfh+11adfh-4bd(fg+2eh))\sqrt{a+bx}\sqrt{c+dx}}{4b^4d} - \frac{2(be-af)^2(bg-ah)(c+dx)^{3/2}}{3b^3(bc-ad)(a+bx)^{3/2}} + \frac{f^2h\sqrt{a+bx}(c+dx)^{3/2}}{2b^3d} + \frac{(35a^2d^2f^2h-10abdf(2dfg+4deh+cfh)-b^2(c^2f^2h-8d^2e(2fg+eh)-4cdf(fg+2eh)))\operatorname{arctanh}\left(\frac{\sqrt{d}\cdots}{\sqrt{b}\cdots}\right.}{4b^{9/2}d^{3/2}}$$

output
```
-2*(-a*f+b*e)*(-3*a*f*h+b*e*h+2*b*f*g)*(d*x+c)^(1/2)/b^4/(b*x+a)^(1/2)-1/4
*f*(b*c*f*h+11*a*d*f*h-4*b*d*(2*e*h+f*g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^4/
d-2/3*(-a*f+b*e)^2*(-a*h+b*g)*(d*x+c)^(3/2)/b^3/(-a*d+b*c)/(b*x+a)^(3/2)+1
/2*f^2*h*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^3/d+1/4*(35*a^2*d^2*f^2*h-10*a*b*d*
f*(c*f*h+4*d*e*h+2*d*f*g)-b^2*(c^2*f^2*h-8*d^2*e*(e*h+2*f*g)-4*c*d*f*(2*e*
h+f*g)))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(9/2)/d^(3
/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{c+dx}(105a^4d^2f^2h-20a^3bdf(5cfh+$$

$$+\frac{(35a^2d^2f^2h-10abdf(2dfg+4deh+cfh)+b^2(-c^2f^2h+8d^2e(2fg+eh)+4cdf(fg+2eh)))\operatorname{arctanh}\Big(}{4b^{9/2}d^{3/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(e^2*g + e*(2*f*g + e*h)*x + f*(f*g + 2*e*h)*x^2
+ f^2*h*x^3))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[c + d*x]*(105*a^4*d^2*f^2*h - 20*a^3*b*d*f*(5*c*f*h + d*(3*f*g + 6*e
*h - 7*f*h*x)) + 2*a*b^3*(3*c^2*f^2*h*x - 4*c*d*(2*e^2*h + 2*e*f*(2*g - 9*
h*x) + 3*f^2*x*(-3*g + h*x)) + d^2*x*(16*e^2*h + 4*e*f*(8*g - 3*h*x) - 3*f
^2*x*(2*g + h*x))) + b^4*(-8*d^2*e^2*g*x + 3*c^2*f^2*h*x^2 + 2*c*d*(12*e*f
*x*(-2*g + h*x) + 3*f^2*x^2*(2*g + h*x) - 4*e^2*(g + 3*h*x))) + a^2*b^2*(3
*c^2*f^2*h + 2*c*d*f*(26*f*g + 52*e*h - 69*f*h*x) + d^2*(24*e^2*h + 16*e*f
*(3*g - 10*h*x) + f^2*x*(-80*g + 21*h*x)))))/(12*b^4*d*(b*c - a*d)*(a + b*
x)^(3/2)) + ((35*a^2*d^2*f^2*h - 10*a*b*d*f*(2*d*f*g + 4*d*e*h + c*f*h) +
b^2*(-(c^2*f^2*h) + 8*d^2*e*(2*f*g + e*h) + 4*c*d*f*(f*g + 2*e*h)))*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(9/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {2124, 27, 1193, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(e^2g+fx^2(2eh+fg)+ex(eh+2fg)+f^2hx^3\right)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{3\sqrt{c+dx}\left(\frac{(bc-ad)f^2hx^2}{b}+\frac{(bc-ad)f(bfg+2beh-afh)x}{b^2}+\frac{(bc-ad)\left(e(2fg+eh)b^2-af(fg+2eh)b+a^2f^2h\right)}{b^3}\right)}{2(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}\left(\frac{(bc-ad)f^2hx^2}{b}+\frac{(bc-ad)f(bfg+2beh-afh)x}{b^2}+\frac{(bc-ad)\left(e(2fg+eh)b^2-af(fg+2eh)b+a^2f^2h\right)}{b^3}\right)}{(a+bx)^{3/2}}dx}{bc-ad}-\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 1193

$$\frac{-\frac{2\int -\frac{(bc-ad)\sqrt{c+dx}\left((2de(2fg+eh)+cf(fg+2eh))b^2-af(2cfh+5d(fg+2eh))b+(bc-ad)f^2hxb+8a^2df^2h\right)}{2b^3\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{3/2}(be-af)(-3afh+beh+2bf}{b^3\sqrt{a+bx}}}{bc-ad}-\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{c+dx}\left((2de(2fg+eh)+cf(fg+2eh))b^2-af(2cfh+5d(fg+2eh))b+(bc-ad)f^2hxb+8a^2df^2h\right)}{\sqrt{a+bx}}dx}{b^3}-\frac{2(c+dx)^{3/2}(be-af)(-3afh+beh+2bfg)}{b^3\sqrt{a+bx}}}{bc-ad}-\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{\frac{\left(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\left(b^2\left(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\right)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a+bx}}dx}{4d}+\frac{f^2h\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{2d}}{b^3}-\frac{2(c+dx)^{3/2}(be-af}{b^3\sqrt{}}}{bc-ad}-\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\left(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\left(b^2\left(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\right)\right)\right)\left(\frac{(bc-ad)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4d}+\frac{f^2h\sqrt{a+bx}(c+dx)^{3/}}{2d}}{b^3}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{\frac{\left(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\left(b^2\left(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\right)\right)\right)\left(\frac{(bc-ad)\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4d}+\frac{f^2h\sqrt{a+bx}(c+d}{2d}}{b^3}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\frac{\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)\left(35a^2d^2f^2h-10abdf(cfh+4deh+2dfg)-\left(b^2\left(c^2f^2h-4cdf(2eh+fg)-8d^2e(eh+2fg)\right)\right)\right)}{4d}+\frac{f^2h\sqrt{a+bx}(c+}{2}}{b^3}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}(be-af)^2(bg-ah)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

input
```
Int[(Sqrt[c + d*x]*(e^2*g + e*(2*f*g + e*h)*x + f*(f*g + 2*e*h)*x^2 + f^2*
h*x^3))/(a + b*x)^(5/2),x]
```

output
```
(-2*(b*e - a*f)^2*(b*g - a*h)*(c + d*x)^(3/2))/(3*b^3*(b*c - a*d)*(a + b*x
)^(3/2)) + ((-2*(b*e - a*f)*(2*b*f*g + b*e*h - 3*a*f*h)*(c + d*x)^(3/2))/(
b^3*Sqrt[a + b*x]) + (((b*c - a*d)*f^2*h*Sqrt[a + b*x]*(c + d*x)^(3/2))/(2
*d) + ((35*a^2*d^2*f^2*h - 10*a*b*d*f*(2*d*f*g + 4*d*e*h + c*f*h) - b^2*(c
^2*f^2*h - 8*d^2*e*(2*f*g + e*h) - 4*c*d*f*(f*g + 2*e*h)))*((Sqrt[a + b*x]
*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*
Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(4*d))/b^3)/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3426 vs. $2(268) = 536$.

Time = 0.34 (sec) , antiderivative size = 3427, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	3427

input
```
int((d*x+c)^(1/2)*(e^2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x^2+f^2*h*x^3)/(b*x
+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-42*a^2*b^2*d^2*f^2*h*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+24*a*b
^3*d^2*f^2*g*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+16*b^4*d^2*e^2*g*x*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+120*a^3*b*d^2*f^2*g*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)-210*a^4*d^2*f^2*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+3*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
b^5*c^3*f^2*h*x^2-60*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^3*f^2*g+48*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*d^3*e^2*h*x+210*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*
b*d^3*f^2*h*x-120*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*
d+b*c)/(d*b)^(1/2))*a^3*b^2*d^3*f^2*g*x-12*a*b^3*c^2*f^2*h*x*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-64*a*b^3*d^2*e^2*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+48*b^4*c*d*e^2*h*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+200*a^3*b*c*d
*f^2*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+240*a^3*b*d^2*e*f*h*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)-104*a^2*b^2*c*d*f^2*g*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)-96*a^2*b^2*d^2*e*f*g*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+32*a*b^3
*c*d*e^2*h*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-135*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c*d^2*f^2*h*x
^2+12*a*b^3*d^2*f^2*h*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-12*b^4*c*d*f
^2*h*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-240*ln(1/2*(2*b*d*x+2*((b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. $2(268) = 536$.

Time = 12.54 (sec) , antiderivative size = 2312, normalized size of antiderivative = 7.61

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(e^2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x^2+f^2*h*x^3
)/(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((4*(4*(b^5*c*d^2 - a*b^4*d^3)*e*f + (b^5*c^2*d - 6*a*b^4*c*d^2 +
 5*a^2*b^3*d^3)*f^2)*g + (8*(b^5*c*d^2 - a*b^4*d^3)*e^2 + 8*(b^5*c^2*d - 6
*a*b^4*c*d^2 + 5*a^2*b^3*d^3)*e*f - (b^5*c^3 + 9*a*b^4*c^2*d - 45*a^2*b^3*
c*d^2 + 35*a^3*b^2*d^3)*f^2)*h)*x^2 + 4*(4*(a^2*b^3*c*d^2 - a^3*b^2*d^3)*e
*f + (a^2*b^3*c^2*d - 6*a^3*b^2*c*d^2 + 5*a^4*b*d^3)*f^2)*g + (8*(a^2*b^3*
c*d^2 - a^3*b^2*d^3)*e^2 + 8*(a^2*b^3*c^2*d - 6*a^3*b^2*c*d^2 + 5*a^4*b*d^
3)*e*f - (a^2*b^3*c^3 + 9*a^3*b^2*c^2*d - 45*a^4*b*c*d^2 + 35*a^5*d^3)*f^2
)*h + 2*(4*(4*(a*b^4*c*d^2 - a^2*b^3*d^3)*e*f + (a*b^4*c^2*d - 6*a^2*b^3*c
*d^2 + 5*a^3*b^2*d^3)*f^2)*g + (8*(a*b^4*c*d^2 - a^2*b^3*d^3)*e^2 + 8*(a*b
^4*c^2*d - 6*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*e*f - (a*b^4*c^3 + 9*a^2*b^3*c
^2*d - 45*a^3*b^2*c*d^2 + 35*a^4*b*d^3)*f^2)*h)*x)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*s
qrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(6*(b^5*c*d^2 -
a*b^4*d^3)*f^2*h*x^3 + 3*(4*(b^5*c*d^2 - a*b^4*d^3)*f^2*g + (8*(b^5*c*d^2
- a*b^4*d^3)*e*f + (b^5*c^2*d - 8*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*f^2)*h)*x^2
 - 4*(2*b^5*c*d^2*e^2 + 4*(2*a*b^4*c*d^2 - 3*a^2*b^3*d^3)*e*f - (13*a^2*b^
3*c*d^2 - 15*a^3*b^2*d^3)*f^2)*g - (8*(2*a*b^4*c*d^2 - 3*a^2*b^3*d^3)*e^2
- 8*(13*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)*e*f - (3*a^2*b^3*c^2*d - 100*a^3*b
^2*c*d^2 + 105*a^4*b*d^3)*f^2)*h - 2*(4*(b^5*d^3*e^2 + 2*(3*b^5*c*d^2 - 4*
a*b^4*d^3)*e*f - (9*a*b^4*c*d^2 - 10*a^2*b^3*d^3)*f^2)*g + (4*(3*b^5*c*...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx=\int \frac{\sqrt{c+dx}\,(e+fx)^2\,(g+hx)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(e**2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x**2+f**2*h
*x**3)/(b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)*(e + f*x)**2*(g + h*x)/(a + b*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(e^2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x^2+f^2*h*x^3
)/(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2146 vs. $2(268) = 536$.

Time = 0.57 (sec) , antiderivative size = 2146, normalized size of antiderivative = 7.06

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(e^2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x^2+f^2*h*x^3
)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*f^2*h*a
bs(b)/b^6 + (4*b^12*d^2*f^2*g*abs(b) + 8*b^12*d^2*e*f*h*abs(b) + b^12*c*d*
f^2*h*abs(b) - 13*a*b^11*d^2*f^2*h*abs(b))/(b^17*d^2)) - 1/8*(16*b^2*d^2*e
*f*g*abs(b) + 4*b^2*c*d*f^2*g*abs(b) - 20*a*b*d^2*f^2*g*abs(b) + 8*b^2*d^2
*e^2*h*abs(b) + 8*b^2*c*d*e*f*h*abs(b) - 40*a*b*d^2*e*f*h*abs(b) - b^2*c^2
*f^2*h*abs(b) - 10*a*b*c*d*f^2*h*abs(b) + 35*a^2*d^2*f^2*h*abs(b))*log((sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)
*b^5*d) - 4/3*(b^7*c^2*d^2*e^2*g*abs(b) - 2*a*b^6*c*d^3*e^2*g*abs(b) + a^2
*b^5*d^4*e^2*g*abs(b) + 6*b^7*c^3*d*e*f*g*abs(b) - 20*a*b^6*c^2*d^2*e*f*g*
abs(b) + 22*a^2*b^5*c*d^3*e*f*g*abs(b) - 8*a^3*b^4*d^4*e*f*g*abs(b) - 6*a*
b^6*c^3*d*f^2*g*abs(b) + 19*a^2*b^5*c^2*d^2*f^2*g*abs(b) - 20*a^3*b^4*c*d^
3*f^2*g*abs(b) + 7*a^4*b^3*d^4*f^2*g*abs(b) + 3*b^7*c^3*d*e^2*h*abs(b) - 1
0*a*b^6*c^2*d^2*e^2*h*abs(b) + 11*a^2*b^5*c*d^3*e^2*h*abs(b) - 4*a^3*b^4*d
^4*e^2*h*abs(b) - 12*a*b^6*c^3*d*e*f*h*abs(b) + 38*a^2*b^5*c^2*d^2*e*f*h*a
bs(b) - 40*a^3*b^4*c*d^3*e*f*h*abs(b) + 14*a^4*b^3*d^4*e*f*h*abs(b) + 9*a^
2*b^5*c^3*d*f^2*h*abs(b) - 28*a^3*b^4*c^2*d^2*f^2*h*abs(b) + 29*a^4*b^3*c*
d^3*f^2*h*abs(b) - 10*a^5*b^2*d^4*f^2*h*abs(b) - 12*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c^2*d*e*f*g*abs(b) + 24*(s
qrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^4*c*d^
2*e*f*g*abs(b) - 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(e^2\,g+f\,x^2\,(2\,e\,h+f\,g)+f\right.}{(a+b\,x)^{5/2}}$$

input
```
int(((c + d*x)^(1/2)*(e^2*g + f*x^2*(2*e*h + f*g) + f^2*h*x^3 + e*x*(e*h +
 2*f*g)))/(a + b*x)^(5/2),x)
```

output
```
int(((c + d*x)^(1/2)*(e^2*g + f*x^2*(2*e*h + f*g) + f^2*h*x^3 + e*x*(e*h +
 2*f*g)))/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(e^2g+e(2fg+eh)x+f(fg+2eh)x^2+f^2hx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,(e^2g+e(eh+2fg)\,x+f(2e}{(bx+a)^{\frac{5}{2}}}$$

input

```
int((d*x+c)^(1/2)*(e^2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x^2+f^2*h*x^3)/(b*x
+a)^(5/2),x)
```

output

```
int((d*x+c)^(1/2)*(e^2*g+e*(e*h+2*f*g)*x+f*(2*e*h+f*g)*x^2+f^2*h*x^3)/(b*x
+a)^(5/2),x)
```

3.12 $\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)} dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)} dx = \frac{d\sqrt{a+bx}\sqrt{c+dx}}{fh} + \frac{\sqrt{d}(3bcfh+adfh-2bd(fg+eh))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}f^2h^2} - \frac{2\sqrt{be-af}(de-cf)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f^2(fg-eh)} + \frac{2\sqrt{bg-ah}(dg-ch)^{3/2}\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h^2(fg-eh)}$$

output
```
d*(b*x+a)^(1/2)*(d*x+c)^(1/2)/f/h+d^(1/2)*(3*b*c*f*h+a*d*f*h-2*b*d*(e*h+f*
g))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/f^2/h^2-2
*(-a*f+b*e)^(1/2)*(-c*f+d*e)^(3/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/
(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/f^2/(-e*h+f*g)+2*(-a*h+b*g)^(1/2)*(-c*h+d*
g)^(3/2)*arctanh((-c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(
1/2))/h^2/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \frac{d\sqrt{a+bx}\sqrt{c+dx}}{fh} - \frac{2\sqrt{-be+af}(de-cf)^{3/2}\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{f^2(fg-eh)} + \frac{2\sqrt{-bg+ah}(dg-ch)^{3/2}\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{h^2(fg-eh)} - \frac{\sqrt{d}(-3bcfh-adfh+2bd(fg+eh))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}f^2h^2}$$

input
```
Integrate[(Sqrt[a + b*x]*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)),x]
```

output
```
(d*Sqrt[a + b*x]*Sqrt[c + d*x])/(f*h) - (2*Sqrt[-(b*e) + a*f]*(d*e - c*f)^
(3/2)*ArcTan[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c +
d*x])])/(f^2*(f*g - e*h)) + (2*Sqrt[-(b*g) + a*h]*(d*g - c*h)^(3/2)*ArcTan
[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c + d*x])])/(h^2
*(f*g - e*h)) - (Sqrt[d]*(-3*b*c*f*h - a*d*f*h + 2*b*d*(f*g + e*h))*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*f^2*h^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.46, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {196, 113, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}dx$$

$$\Big\downarrow \text{ 196}$$

$$\frac{(bg-ah)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(g+hx)}dx}{fg-eh}-\frac{(be-af)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow 113$$

$$\frac{(bg-ah)\left(\frac{\int -\frac{-2bhc^2+d(bc+ad)g+d(2bdg-3bch+adh)x}{2\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{bh}+\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bh}\right)}{fg-eh}-\frac{(be-af)\left(\frac{\int -\frac{-2bfc^2+d(bc+ad)e+d(2bde-3bcf+adf)x}{2\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{bf}+\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bf}\right)}{fg-eh}$$

$$\Big\downarrow 27$$

$$\frac{(bg-ah)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bh}-\frac{\int \frac{agd^2+(2bdg-3bch+adh)xd+bc(dg-2ch)}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{2bh}\right)}{fg-eh}-\frac{(be-af)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bf}-\frac{\int \frac{aed^2+(2bde-3bcf+adf)xd+bc(de-2cf)}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{2bf}\right)}{fg-eh}$$

$$\Big\downarrow 175$$

$$\frac{(bg-ah)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bh}-\frac{\frac{d(adh-3bch+2bdg)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{h}-\frac{2b(dg-ch)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{h}}{2bh}\right)}{fg-eh}-\frac{(be-af)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bf}-\frac{\frac{d(adf-3bcf+2bde)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}-\frac{2b(de-cf)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{f}}{2bf}\right)}{fg-eh}$$

$$\Big\downarrow 66$$

$$\frac{(bg-ah)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bh}-\frac{\frac{2d(adh-3bch+2bdg)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}-\frac{2b(dg-ch)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{h}}{2bh}\right)}{fg-eh}-\frac{(be-af)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bf}-\frac{\frac{2d(adf-3bcf+2bde)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}-\frac{2b(de-cf)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{f}}{2bf}\right)}{fg-eh}$$

$$\Big\downarrow 104$$

$$\frac{(bg-ah)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bh}-\frac{\frac{2d(adh-3bch+2bdg)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}-\frac{4b(dg-ch)^2\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}}{2bh}\right)}{fg-eh}-\frac{(be-af)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bf}-\frac{\frac{2d(adf-3bcf+2bde)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}-\frac{4b(de-cf)^2\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}}{2bf}\right)}{fg-eh}$$

$$\Big\downarrow \text{221}$$

$$\frac{(bg-ah)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bh}-\frac{\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(adh-3bch+2bdg)}{\sqrt{b}h}-\frac{4b(dg-ch)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h\sqrt{bg-ah}}}{2bh}\right)}{fg-eh}-\frac{(be-af)\left(\frac{d\sqrt{a+bx}\sqrt{c+dx}}{bf}-\frac{\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(adf-3bcf+2bde)}{\sqrt{b}f}-\frac{4b(de-cf)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f\sqrt{be-af}}}{2bf}\right)}{fg-eh}$$

input
```
Int[(Sqrt[a + b*x]*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)),x]
```

output
```
-(((b*e - a*f)*((d*Sqrt[a + b*x]*Sqrt[c + d*x])/(b*f) - ((2*Sqrt[d]*(2*b*d
*e - 3*b*c*f + a*d*f)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/(Sqrt[b]*f) - (4*b*(d*e - c*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a
 + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(f*Sqrt[b*e - a*f]))/(2*b*f)))/
(f*g - e*h)) + ((b*g - a*h)*((d*Sqrt[a + b*x]*Sqrt[c + d*x])/(b*h) - ((2*S
qrt[d]*(2*b*d*g - 3*b*c*h + a*d*h)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(Sqrt[b]*h) - (4*b*(d*g - c*h)^(3/2)*ArcTanh[(Sqrt[d*g
- c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(h*Sqrt[b*g - a*h]
))/(2*b*h)))/(f*g - e*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(223) = 446$.

Time = 0.63 (sec) , antiderivative size = 2239, normalized size of antiderivative = 8.51

method	result	size
default	Expression too large to display	2239

input
```
int((b*x+a)^(1/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*(d*b)^(
1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a
*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*c^2*f^3*h^3-4*((c*h-d
*g)*(a*h-b*g)/h^2)^(1/2)*(d*b)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x
+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/
(f*x+e))*a*c*d*e*f^2*h^3+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*(d*b)^(1/2)*ln(
(a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/
f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*d^2*e^2*f*h^3-2*((c*h-d*g)*(a
*h-b*g)/h^2)^(1/2)*(d*b)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d
*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e
))*b*c^2*e*f^2*h^3+4*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*(d*b)^(1/2)*ln((a*d*f
*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(
1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b*c*d*e^2*f*h^3-2*((c*h-d*g)*(a*h-b*g
)/h^2)^(1/2)*(d*b)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))
^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b*d
^2*e^3*h^3+((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a*d^2*e*f^2*h^3-((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^
2)^(1/2)*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*a*d^2*f^3*g*h^2+3*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{a+bx}(c+dx)^{\frac{3}{2}}}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x)*(c + d*x)**(3/2)/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{bx+a}(dx+c)^{\frac{3}{2}}}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x + a)*(d*x + c)^(3/2)/((f*x + e)*(h*x + g)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{a+b\,x}\,{(c+d\,x)}^{3/2}}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^(1/2)*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^(1/2)*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(1/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.13 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \frac{2\sqrt{b}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{fh} + \frac{2\sqrt{be-af}\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f(fg-eh)} - \frac{2\sqrt{bg-ah}\sqrt{dg-ch}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h(fg-eh)}$$

output
```
2*b^(1/2)*d^(1/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/f/h
+2*(-a*f+b*e)^(1/2)*(-c*f+d*e)^(1/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2
)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/f/(-e*h+f*g)-2*(-a*h+b*g)^(1/2)*(-c*h+d*
g)^(1/2)*arctanh((-c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(
1/2))/h/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \frac{2\left(\sqrt{be-af}\sqrt{-de+cf}\,h\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)-f\sqrt{bg-ah}\sqrt{-dg+ch}\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)+\sqrt{b}\right.}{fh(fg-eh)}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x])/((e + f*x)*(g + h*x)),x]
```

output
```
(2*(Sqrt[b*e - a*f]*Sqrt[-(d*e) + c*f]*h*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c +
d*x])/(Sqrt[-(d*e) + c*f]*Sqrt[a + b*x])] - f*Sqrt[b*g - a*h]*Sqrt[-(d*g)
+ c*h]*ArcTan[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a +
 b*x])] + Sqrt[b]*Sqrt[d]*(f*g - e*h)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[a + b*x])]))/(f*h*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {196, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{196}$$

$$\frac{(bg-ah)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(g+hx)}dx}{fg-eh}-\frac{(be-af)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{140}$$

$$\frac{(bg-ah)\left(\int \frac{c-\frac{dg}{h}}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx+\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{h}\right)}{fg-eh}-\frac{(be-af)\left(\int \frac{c-\frac{de}{f}}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx+\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}\right)}{fg-eh}$$

$$\Big\downarrow \ 27$$

$$\frac{(bg-ah)\left(\left(c-\frac{dg}{h}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx+\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{h}\right)}{fg-eh}-\frac{(be-af)\left(\left(c-\frac{de}{f}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx+\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}\right)}{fg-eh}$$

$$\Big\downarrow \ 66$$

$$\frac{(bg-ah)\left(\left(c-\frac{dg}{h}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx+\frac{2d\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}\right)}{fg-eh}-\frac{(be-af)\left(\left(c-\frac{de}{f}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx+\frac{2d\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}\right)}{fg-eh}$$

$$\Big\downarrow \ 104$$

$$\frac{(bg-ah)\left(2\left(c-\frac{dg}{h}\right)\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}+\frac{2d\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}\right)}{fg-eh}-\frac{(be-af)\left(2\left(c-\frac{de}{f}\right)\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}+\frac{2d\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}\right)}{fg-eh}$$

$$\Big\downarrow \ 221$$

$$\frac{(bg-ah)\left(\frac{2\left(c-\frac{dg}{h}\right)\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}\sqrt{dg-ch}}+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}h}\right)}{fg-eh}-\frac{(be-af)\left(\frac{2\left(c-\frac{de}{f}\right)\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}\sqrt{de-cf}}+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}f}\right)}{fg-eh}$$

input

```
Int[(Sqrt[a + b*x]*Sqrt[c + d*x])/((e + f*x)*(g + h*x)),x]
```

output

```
-(((b*e - a*f)*((2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/(Sqrt[b]*f) + (2*(c - (d*e)/f)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a
+ b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(Sqrt[b*e - a*f]*Sqrt[d*e - c*f]
)))/(f*g - e*h)) + ((b*g - a*h)*((2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x]
)/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*h) + (2*(c - (d*g)/h)*ArcTanh[(Sqrt[d
*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(Sqrt[b*g - a*h
]*Sqrt[d*g - c*h])))/(f*g - e*h)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(179) = 358$.

Time = 0.62 (sec) , antiderivative size = 1200, normalized size of antiderivative = 5.58

method	result	size
default	Expression too large to display	1200

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(1/2)*(d*x+c)^(1/2)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)*
(a*f-b*e)/f^2)^(1/2)*b*d*e*f*h^2-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d
*e)*(a*f-b*e)/f^2)^(1/2)*b*d*f^2*g*h+(d*b)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)
^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*
(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*c*f^2*h^2-(d*b)^(1/
2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a
)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f
*x+e))*a*d*e*f*h^2-(d*b)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*ln((a*d*f*x
+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/
2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b*c*e*f*h^2+(d*b)^(1/2)*((c*h-d*g)*(a*h
-b*g)/h^2)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*(
(c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b*d*e^2*h^2
-(d*b)^(1/2)*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/
2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f
-b*e)/f^2)^(1/2)*a*c*f^2*h^2+(d*b)^(1/2)*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*(
(c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c
*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*a*d*f^2*g*h+(d*b)^(1/2)*ln((a
*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{bx+a}\sqrt{dx+c}}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x + a)*sqrt(d*x + c)/((f*x + e)*(h*x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)*(g + h*x)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \frac{-\sqrt{cf-de}\,\sqrt{af-be}\log\left(-\sqrt{2\sqrt{d}\,\sqrt{b}\,\sqrt{cf-de}\,\sqrt{af-be}+adf+bcf-2bde}+\sqrt{f}\,\sqrt{d}\,\sqrt{bx+a}+\sqrt{}\right.}{}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
( - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f
 - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(
a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*h - sqrt(c*f - d*e)*sqrt(a*f - b
*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b
*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c +
 d*x))*h + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(2*sqrt(d)*sqrt(b)*sqrt(c*f
- d*e)*sqrt(a*f - b*e) + 2*sqrt(d)*sqrt(b)*sqrt(c + d*x)*sqrt(a + b*x)*f +
 2*b*d*e + 2*b*d*f*x)*h + sqrt(c*h - d*g)*sqrt(a*h - b*g)*log( - sqrt(2*sq
rt(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(a*h - b*g) + a*d*h + b*c*h - 2*b*d*g) +
 sqrt(h)*sqrt(d)*sqrt(a + b*x) + sqrt(h)*sqrt(b)*sqrt(c + d*x))*f + sqrt(c
*h - d*g)*sqrt(a*h - b*g)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(
a*h - b*g) + a*d*h + b*c*h - 2*b*d*g) + sqrt(h)*sqrt(d)*sqrt(a + b*x) + sq
rt(h)*sqrt(b)*sqrt(c + d*x))*f - sqrt(c*h - d*g)*sqrt(a*h - b*g)*log(2*sqr
t(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(a*h - b*g) + 2*sqrt(d)*sqrt(b)*sqrt(c +
d*x)*sqrt(a + b*x)*h + 2*b*d*g + 2*b*d*h*x)*f + 2*sqrt(d)*sqrt(b)*log((sqr
t(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*e*h - 2*sqrt(
d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d -
b*c))*f*g)/(f*h*(e*h - f*g))
```

3.14 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = -\frac{2\sqrt{be-af}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{\sqrt{de-cf}(fg-eh)} + \frac{2\sqrt{bg-ah}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{\sqrt{dg-ch}(fg-eh)}$$

output
```
-2*(-a*f+b*e)^(1/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2
)/(d*x+c)^(1/2))/(-c*f+d*e)^(1/2)/(-e*h+f*g)+2*(-a*h+b*g)^(1/2)*arctanh((-
c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-c*h+d*g)^(1
/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \frac{2\sqrt{\frac{b}{d}}\sqrt{d}\left(-\frac{\sqrt{be-af}\arctan\left(\frac{\sqrt{d}\left(-\sqrt{\frac{b}{d}}f\sqrt{a+bx}\sqrt{c+dx}+b(e+fx)\right)}{\sqrt{b}\sqrt{be-af}\sqrt{-de+cf}}\right)}{\sqrt{-de+cf}}+\frac{\sqrt{bg-ah}\arctan\left(\frac{\sqrt{d}\left(-\sqrt{\frac{b}{d}}h\sqrt{a+bx}\sqrt{c+dx}+b(g+hx)\right)}{\sqrt{b}\sqrt{bg-ah}\sqrt{-dg+ch}}\right)}{\sqrt{-dg+ch}}\right)}{\sqrt{b}(-fg+eh)}$$

input
```
Integrate[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(2*Sqrt[b/d]*Sqrt[d]*(-((Sqrt[b*e - a*f]*ArcTan[(Sqrt[d]*(-(Sqrt[b/d]*f*Sq
rt[a + b*x]*Sqrt[c + d*x]) + b*(e + f*x)))/(Sqrt[b]*Sqrt[b*e - a*f]*Sqrt[-
(d*e) + c*f])])/Sqrt[-(d*e) + c*f]) + (Sqrt[b*g - a*h]*ArcTan[(Sqrt[d]*(-(
Sqrt[b/d]*h*Sqrt[a + b*x]*Sqrt[c + d*x]) + b*(g + h*x)))/(Sqrt[b]*Sqrt[b*g
 - a*h]*Sqrt[-(d*g) + c*h])])/Sqrt[-(d*g) + c*h]))/(Sqrt[b]*(-(f*g) + e*h)
)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {196, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{ 196}$$

$$\frac{(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{fg-eh}-\frac{(be-af)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{104}$$

$$\frac{2(bg-ah)\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{fg-eh}-\frac{2(be-af)\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{fg-eh}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{bg-ah}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{dg-ch}(fg-eh)}-\frac{2\sqrt{be-af}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{de-cf}(fg-eh)}$$

input
```
Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*
f]*Sqrt[c + d*x])])/(Sqrt[d*e - c*f]*(f*g - e*h)) + (2*Sqrt[b*g - a*h]*Arc
Tanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(Sq
rt[d*g - c*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(137) = 274$.

Time = 0.65 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.18

method	result
default	$\frac{\sqrt{bx+a}\,\sqrt{xd+c}\left(\ln\left(\frac{adfx+bcfx-2bdex+2\sqrt{(bx+a)(xd+c)}\,\sqrt{\frac{(cf-de)(af-be)}{f^2}}\,f+2acf-ade-bce}{fx+e}\right)\sqrt{\frac{(ch-dg)(ah-bg)}{h^2}}\,afh-\ln\left(\frac{adfx+bcf}{}\right.\right.}{}$

input
```
int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(1/2)*(d*x+c)^(1/2)*(ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x
+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))
*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*a*f*h-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b
*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e
)/(f*x+e))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b*e*h-ln((a*d*h*x+b*c*h*x-2*b*d
*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d
*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*a*f*h+ln((a*d*h*x+b*c
*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h
+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*b*f*g)/((c*
f-d*e)*(a*f-b*e)/f^2)^(1/2)/((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)/(e*h-f*g)/((b*
x+a)*(d*x+c))^(1/2)/h/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(137) = 274$.

Time = 98.54 (sec) , antiderivative size = 2036, normalized size of antiderivative = 12.65

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
[-1/2*(sqrt((b*e - a*f)/(d*e - c*f))*log((8*a^2*c^2*f^2 + (b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*e^2 - 8*(a*b*c^2 + a^2*c*d)*e*f + (8*b^2*d^2*e^2 - 8*(b^2*
c*d + a*b*d^2)*e*f + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*f^2)*x^2 + 4*(2*a*c^2
*f^2 + (b*c*d + a*d^2)*e^2 - (b*c^2 + 3*a*c*d)*e*f + (2*b*d^2*e^2 - (3*b*c
*d + a*d^2)*e*f + (b*c^2 + a*c*d)*f^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt
((b*e - a*f)/(d*e - c*f)) + 2*(4*(b^2*c*d + a*b*d^2)*e^2 - (3*b^2*c^2 + 10
*a*b*c*d + 3*a^2*d^2)*e*f + 4*(a*b*c^2 + a^2*c*d)*f^2)*x)/(f^2*x^2 + 2*e*f
*x + e^2)) + sqrt((b*g - a*h)/(d*g - c*h))*log((8*a^2*c^2*h^2 + (b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*g^2 - 8*(a*b*c^2 + a^2*c*d)*g*h + (8*b^2*d^2*g^2 - 8
*(b^2*c*d + a*b*d^2)*g*h + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*h^2)*x^2 - 4*(2
*a*c^2*h^2 + (b*c*d + a*d^2)*g^2 - (b*c^2 + 3*a*c*d)*g*h + (2*b*d^2*g^2 -
(3*b*c*d + a*d^2)*g*h + (b*c^2 + a*c*d)*h^2)*x)*sqrt(b*x + a)*sqrt(d*x + c
)*sqrt((b*g - a*h)/(d*g - c*h)) + 2*(4*(b^2*c*d + a*b*d^2)*g^2 - (3*b^2*c^
2 + 10*a*b*c*d + 3*a^2*d^2)*g*h + 4*(a*b*c^2 + a^2*c*d)*h^2)*x)/(h^2*x^2 +
 2*g*h*x + g^2)))/(f*g - e*h), -1/2*(2*sqrt(-(b*g - a*h)/(d*g - c*h))*arct
an(-1/2*(2*a*c*h - (b*c + a*d)*g - (2*b*d*g - (b*c + a*d)*h)*x)*sqrt(b*x +
 a)*sqrt(d*x + c)*sqrt(-(b*g - a*h)/(d*g - c*h))/(a*b*c*g - a^2*c*h + (b^2
*d*g - a*b*d*h)*x^2 + ((b^2*c + a*b*d)*g - (a*b*c + a^2*d)*h)*x)) + sqrt((
b*e - a*f)/(d*e - c*f))*log((8*a^2*c^2*f^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^
2)*e^2 - 8*(a*b*c^2 + a^2*c*d)*e*f + (8*b^2*d^2*e^2 - 8*(b^2*c*d + a*b*...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x)/(sqrt(c + d*x)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{bx+a}}{\sqrt{dx+c}(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x + a)/(sqrt(d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(137) = 274$.

Time = 22.88 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \frac{2\left(\sqrt{bd}b^3e-\sqrt{bd}ab^2f\right)\arctan\left(\frac{2\,b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2f}{2\,\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(fg|b|-eh|b|)b} - \frac{2\left(\sqrt{bd}b^3g-\sqrt{bd}ab^2h\right)\arctan\left(\frac{2\,b^2dg-b^2ch-abdh+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2h}{2\,\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}b}\right)}{\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}(fg|b|-eh|b|)b}$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
2*(sqrt(b*d)*b^3*e - sqrt(b*d)*a*b^2*f)*arctan(1/2*(2*b^2*d*e - b^2*c*f -
a*b*d*f + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2*f)/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/(sq
rt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*(f*g*abs(b) - e
*h*abs(b))*b) - 2*(sqrt(b*d)*b^3*g - sqrt(b*d)*a*b^2*h)*arctan(1/2*(2*b^2*
d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*b*
c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)
*(f*g*abs(b) - e*h*abs(b))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Hanged}$$

input
```
int((a + b*x)^(1/2)/((e + f*x)*(g + h*x)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 1056, normalized size of antiderivative = 6.56

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

output

```
( - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f
 - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(
a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*c*h + sqrt(c*f - d*e)*sqrt(a*f -
 b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*
f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqr
t(c + d*x))*d*g - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(
b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sq
rt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*c*h + sqrt(c*f - d*e)
*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e
) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqr
t(b)*sqrt(c + d*x))*d*g + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(2*sqrt(d)*sq
rt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + 2*sqrt(d)*sqrt(b)*sqrt(c + d*x)*sq
rt(a + b*x)*f + 2*b*d*e + 2*b*d*f*x)*c*h - sqrt(c*f - d*e)*sqrt(a*f - b*e)
*log(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + 2*sqrt(d)*sqrt(b)
*sqrt(c + d*x)*sqrt(a + b*x)*f + 2*b*d*e + 2*b*d*f*x)*d*g + sqrt(c*h - d*g
)*sqrt(a*h - b*g)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(a*h -
 b*g) + a*d*h + b*c*h - 2*b*d*g) + sqrt(h)*sqrt(d)*sqrt(a + b*x) + sqrt(h)
*sqrt(b)*sqrt(c + d*x))*c*f - sqrt(c*h - d*g)*sqrt(a*h - b*g)*log( - sqrt(
2*sqrt(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(a*h - b*g) + a*d*h + b*c*h - 2*b*d*
g) + sqrt(h)*sqrt(d)*sqrt(a + b*x) + sqrt(h)*sqrt(b)*sqrt(c + d*x))*d*e...
```

3.15 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2d\sqrt{a+bx}}{(de-cf)(dg-ch)\sqrt{c+dx}} + \frac{2f\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(de-cf)^{3/2}(fg-eh)} - \frac{2h\sqrt{bg-ah}\,\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(dg-ch)^{3/2}(fg-eh)}$$

output
```
-2*d*(b*x+a)^(1/2)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^(1/2)+2*f*(-a*f+b*e)^(1/2
)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(
-c*f+d*e)^(3/2)/(-e*h+f*g)-2*h*(-a*h+b*g)^(1/2)*arctanh((-c*h+d*g)^(1/2)*(
b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-c*h+d*g)^(3/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2d\sqrt{a+bx}}{(de-cf)(dg-ch)\sqrt{c+dx}} + \frac{2f\sqrt{-be+af}\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{(de-cf)^{3/2}(fg-eh)} - \frac{2h\sqrt{-bg+ah}\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{(dg-ch)^{3/2}(fg-eh)}$$

input
```
Integrate[Sqrt[a + b*x]/((c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*d*Sqrt[a + b*x])/((d*e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) + (2*f*Sqrt[-
(b*e) + a*f]*ArcTan[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e) + a*f]*Sq
rt[c + d*x])])/((d*e - c*f)^(3/2)*(f*g - e*h)) - (2*h*Sqrt[-(b*g) + a*h]*A
rcTan[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c + d*x])])
/((d*g - c*h)^(3/2)*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {196, 107, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)} \, dx$$

$$\Big\downarrow \text{196}$$

$$\frac{(bg-ah)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(g+hx)}dx}{fg-eh} - \frac{(be-af)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{107}$$

$$\frac{(bg-ah)\left(\frac{2d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(dg-ch)} - \frac{h\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{dg-ch}\right)}{fg-eh} - \frac{(be-af)\left(\frac{2d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(de-cf)} - \frac{f\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{de-cf}\right)}{fg-eh}$$

$$\Big\downarrow \text{104}$$

$$\frac{(bg-ah)\left(\frac{2d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(dg-ch)}-\frac{2h\int\frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{dg-ch}\right)}{fg-eh}-\frac{(be-af)\left(\frac{2d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(de-cf)}-\frac{2f\int\frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{de-cf}\right)}{fg-eh}$$

$$\Big\downarrow \; 221$$

$$\frac{(bg-ah)\left(\frac{2d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(dg-ch)}-\frac{2h\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}(dg-ch)^{3/2}}\right)}{fg-eh}-\frac{(be-af)\left(\frac{2d\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(de-cf)}-\frac{2f\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}(de-cf)^{3/2}}\right)}{fg-eh}$$

input
```
Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
-(((b*e - a*f)*((2*d*Sqrt[a + b*x])/((b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x]
) - (2*f*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c +
 d*x])])/(Sqrt[b*e - a*f]*(d*e - c*f)^(3/2))))/(f*g - e*h)) + ((b*g - a*h)
*((2*d*Sqrt[a + b*x])/((b*c - a*d)*(d*g - c*h)*Sqrt[c + d*x]) - (2*h*ArcTa
nh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(Sqrt
[b*g - a*h]*(d*g - c*h)^(3/2))))/(f*g - e*h)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(176) = 352$.

Time = 0.71 (sec) , antiderivative size = 1964, normalized size of antiderivative = 9.63

method	result	size
default	Expression too large to display	1964

input
```
int((b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(-ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)
*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*c*d*f*h*x*((c*f-d*e)*(a*
f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2
)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*d^2*e*h*
x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*
g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*
x+g))*b*c*d*f*g*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)-ln((a*d*h*x+b*c*h*x-2*b*
d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-
d*g*a-b*c*g)/(h*x+g))*b*d^2*e*g*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*
f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^
(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*c*d*f*h*x*((c*h-d*g)*(a*h-b*g)/h^2
)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)
*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*d^2*f*g*x*((c*h-d*
g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^
(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b*c*
d*e*h*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((
b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*
e)/(f*x+e))*b*d^2*e*g*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*h*x+b*c*h*
x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*
a*c*h-d*g*a-b*c*g)/(h*x+g))*a*c^2*f*h*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+l...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(1/2)/(d*x+c)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x)/((c + d*x)**(3/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x + a)/((d*x + c)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(176) = 352$.

Time = 51.01 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2\sqrt{bx+a}b^2d}{(d^2eg|b|-cdfg|b|-cdeh|b|+c^2fh|b|)\sqrt{b^2c+(bx+a)bd-abd}} - \frac{2\left(\sqrt{bd}b^3ef-\sqrt{bd}ab^2f^2\right)\arctan\left(\frac{2b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2f}{2\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(defg|b|-cf^2g|b|-de^2h|b|+cefh|b|)b} + \frac{2\left(\sqrt{bd}b^3gh-\sqrt{bd}ab^2h^2\right)\arctan\left(\frac{2b^2dg-b^2ch-abdh+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2h}{2\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}b}\right)}{\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}(dfg^2|b|-degh|b|-cfgh|b|+ceh^2|b|)b}$$

input

```
integrate((b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output

```
-2*sqrt(b*x + a)*b^2*d/((d^2*e*g*abs(b) - c*d*f*g*abs(b) - c*d*e*h*abs(b)
+ c^2*f*h*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - 2*(sqrt(b*d)*b^3*
e*f - sqrt(b*d)*a*b^2*f^2)*arctan(1/2*(2*b^2*d*e - b^2*c*f - a*b*d*f + (sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*f)/(sqrt(-b
^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/(sqrt(-b^2*d^2*e
^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*(d*e*f*g*abs(b) - c*f^2*g*ab
s(b) - d*e^2*h*abs(b) + c*e*f*h*abs(b))*b) + 2*(sqrt(b*d)*b^3*g*h - sqrt(b
*d)*a*b^2*h^2)*arctan(1/2*(2*b^2*d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 +
 b^2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d
*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*(d*f*g^2*abs(b) - d*e*g*h*abs(b) - c*f*g
*h*abs(b) + c*e*h^2*abs(b))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{\sqrt{a+b\,x}}{(e+f\,x)\,\left(g+h\,x\right)\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b*x)^(1/2)/((e + f*x)*(g + h*x)*(c + d*x)^(3/2)),x)
```

output
```
int((a + b*x)^(1/2)/((e + f*x)*(g + h*x)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.16 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 310

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx &= -\frac{2d\sqrt{a+bx}}{3(de-cf)(dg-ch)(c+dx)^{3/2}} \\ &\quad - \frac{2d(3ad(dfg+deh-2cfh)-b(d^2eg-5c^2fh+2cd(fg+eh)))\sqrt{a+bx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} \\ &\quad - \frac{2f^2\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(de-cf)^{5/2}(fg-eh)} + \frac{2h^2\sqrt{bg-ah}\,\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(dg-ch)^{5/2}(fg-eh)}\end{aligned}$$

output
```
-2/3*d*(b*x+a)^(1/2)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^(3/2)-2/3*d*(3*a*d*(-2*
c*f*h+d*e*h+d*f*g)-b*(d^2*e*g-5*c^2*f*h+2*c*d*(e*h+f*g)))*(b*x+a)^(1/2)/(-
a*d+b*c)/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)-2*f^2*(-a*f+b*e)^(1/2)*ar
ctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(-c*f
+d*e)^(5/2)/(-e*h+f*g)+2*h^2*(-a*h+b*g)^(1/2)*arctanh((-c*h+d*g)^(1/2)*(b*
x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-c*h+d*g)^(5/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx =$$
$$-\frac{2d\sqrt{a+bx}(b(-6c^3fh+d^3egx+2cd^2(fg+eh)x+c^2d(3fg+3eh-5fhx))+ad(7c^2fh+d^2(eg-3fg}{3(-bc+ad)(de-cf)^2(dg-ch)^2(c+dx)^{3/2}}$$
$$+\frac{2f^2\sqrt{-be+af}\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{(de-cf)^{5/2}(-fg+eh)}+\frac{2h^2\sqrt{-bg+ah}\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{(dg-ch)^{5/2}(fg-eh)}$$

input
```
Integrate[Sqrt[a + b*x]/((c + d*x)^(5/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*d*Sqrt[a + b*x]*(b*(-6*c^3*f*h + d^3*e*g*x + 2*c*d^2*(f*g + e*h)*x + c
^2*d*(3*f*g + 3*e*h - 5*f*h*x)) + a*d*(7*c^2*f*h + d^2*(e*g - 3*f*g*x - 3*
e*h*x) - 2*c*d*(2*f*g + 2*e*h - 3*f*h*x))))/(3*(-(b*c) + a*d)*(d*e - c*f)^
2*(d*g - c*h)^2*(c + d*x)^(3/2)) + (2*f^2*Sqrt[-(b*e) + a*f]*ArcTan[(Sqrt[
d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((d*e - c*f
)^(5/2)*(-(f*g) + e*h)) + (2*h^2*Sqrt[-(b*g) + a*h]*ArcTan[(Sqrt[d*g - c*h
]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c + d*x])])/((d*g - c*h)^(5/2)*(
f*g - e*h))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.46, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {196, 115, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}dx$$
$$\Big\downarrow \ 196$$
$$\frac{(bg-ah)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(g+hx)}dx}{fg-eh}-\frac{(be-af)\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{115}$$

$$\frac{(bg-ah)\left(\frac{2\int \frac{2bdg-3bch+3adh+2bdhx}{2\sqrt{a+bx}(c+dx)^{3/2}(g+hx)}dx}{3(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)}\right)}{fg-eh}-\frac{(be-af)\left(\frac{2\int \frac{2bde-3bcf+3adf+2bdfx}{2\sqrt{a+bx}(c+dx)^{3/2}(e+fx)}dx}{3(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)}\right)}{fg-eh}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bg-ah)\left(\frac{\int \frac{2bdg-3bch+3adh+2bdhx}{\sqrt{a+bx}(c+dx)^{3/2}(g+hx)}dx}{3(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)}\right)}{fg-eh}-\frac{(be-af)\left(\frac{\int \frac{2bde-3bcf+3adf+2bdfx}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)}dx}{3(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)}\right)}{fg-eh}$$

$$\Big\downarrow \text{169}$$

$$\frac{(bg-ah)\left(\frac{\frac{2\int \frac{3(bc-ad)^2h^2}{2\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}(3adh-5bch+2bdg)}{\sqrt{c+dx}(bc-ad)(dg-ch)}}{3(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)}\right)}{fg-eh}-\frac{(be-af)\left(\frac{\frac{2\int \frac{3(bc-ad)^2f^2}{2\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}(3adf-5bcf+2bde)}{\sqrt{c+dx}(bc-ad)(de-cf)}}{3(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)}\right)}{fg-eh}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bg-ah)\left(\frac{\frac{3h^2(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{dg-ch}+\frac{2d\sqrt{a+bx}(3adh-5bch+2bdg)}{\sqrt{c+dx}(bc-ad)(dg-ch)}}{3(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)}\right)}{fg-eh}-\frac{(be-af)\left(\frac{\frac{3f^2(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{de-cf}+\frac{2d\sqrt{a+bx}(3adf-5bcf+2bde)}{\sqrt{c+dx}(bc-ad)(de-cf)}}{3(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)}\right)}{fg-eh}$$

$$\Big\downarrow \text{104}$$

$$\frac{(bg-ah)\left(\frac{\frac{6h^2(bc-ad)\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{dg-ch}+\frac{2d\sqrt{a+bx}(3adh-5bch+2bdg)}{\sqrt{c+dx}(bc-ad)(dg-ch)}}{3(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)}\right)}{fg-eh}-\frac{(be-af)\left(\frac{\frac{6f^2(bc-ad)\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{de-cf}+\frac{2d\sqrt{a+bx}(3adf-5bcf+2bde)}{\sqrt{c+dx}(bc-ad)(de-cf)}}{3(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)}\right)}{fg-eh}$$

$$\Big\downarrow \; 221$$

$$\frac{(bg-ah)\left(\frac{\frac{6h^2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}(dg-ch)^{3/2}}+\frac{2d\sqrt{a+bx}(3adh-5bch+2bdg)}{\sqrt{c+dx}(bc-ad)(dg-ch)}}{3(bc-ad)(dg-ch)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)}\right)}{fg-eh}-\frac{(be-af)\left(\frac{\frac{6f^2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}(de-cf)^{3/2}}+\frac{2d\sqrt{a+bx}(3adf-5bcf+2bde)}{\sqrt{c+dx}(bc-ad)(de-cf)}}{3(bc-ad)(de-cf)}+\frac{2d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)}\right)}{fg-eh}$$

input
```
Int[Sqrt[a + b*x]/((c + d*x)^(5/2)*(e + f*x)*(g + h*x)),x]
```

output
```
-(((b*e - a*f)*((2*d*Sqrt[a + b*x])/(3*(b*c - a*d)*(d*e - c*f)*(c + d*x)^(
3/2)) + ((2*d*(2*b*d*e - 5*b*c*f + 3*a*d*f)*Sqrt[a + b*x])/((b*c - a*d)*(d
*e - c*f)*Sqrt[c + d*x]) + (6*(b*c - a*d)*f^2*ArcTanh[(Sqrt[d*e - c*f]*Sqr
t[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(Sqrt[b*e - a*f]*(d*e - c*f)
^(3/2)))/(3*(b*c - a*d)*(d*e - c*f))))/(f*g - e*h)) + ((b*g - a*h)*((2*d*S
qrt[a + b*x])/(3*(b*c - a*d)*(d*g - c*h)*(c + d*x)^(3/2)) + ((2*d*(2*b*d*g
 - 5*b*c*h + 3*a*d*h)*Sqrt[a + b*x])/((b*c - a*d)*(d*g - c*h)*Sqrt[c + d*x
]) + (6*(b*c - a*d)*h^2*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g
- a*h]*Sqrt[c + d*x])])/(Sqrt[b*g - a*h]*(d*g - c*h)^(3/2)))/(3*(b*c - a*d
)*(d*g - c*h))))/(f*g - e*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 115
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2
*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9942 vs. $2(274) = 548$.

Time = 0.72 (sec) , antiderivative size = 9943, normalized size of antiderivative = 32.07

method	result	size
default	Expression too large to display	9943

input
```
int((b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(1/2)/(d*x+c)**(5/2)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x + a)/((d*x + c)^(5/2)*(f*x + e)*(h*x + g)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx=\int \frac{\sqrt{a+b\,x}}{(e+f\,x)\,\left(g+h\,x\right)\,\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + b*x)^(1/2)/((e + f*x)*(g + h*x)*(c + d*x)^(5/2)),x)
```

output
```
int((a + b*x)^(1/2)/((e + f*x)*(g + h*x)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx=\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x)
```

3.17 $\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 390

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \frac{(3bcfh+5adfh-4bd(fg+eh))\sqrt{a+bx}\sqrt{c+dx}}{4f^2h^2}$$
$$+\frac{b\sqrt{a+bx}(c+dx)^{3/2}}{2fh}$$
$$+\frac{(3a^2d^2f^2h^2+6abdfh(3cfh-2d(fg+eh))+b^2(3c^2f^2h^2-12cdfh(fg+eh)+8d^2(f^2g^2+efgh+e^2h^2)}{4\sqrt{b}\sqrt{d}f^3h^3}$$
$$+\frac{2(be-af)^{3/2}(de-cf)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f^3(fg-eh)}$$
$$-\frac{2(bg-ah)^{3/2}(dg-ch)^{3/2}\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h^3(fg-eh)}$$

output
```
1/4*(3*b*c*f*h+5*a*d*f*h-4*b*d*(e*h+f*g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/f^2/
h^2+1/2*b*(b*x+a)^(1/2)*(d*x+c)^(3/2)/f/h+1/4*(3*a^2*d^2*f^2*h^2+6*a*b*d*f
*h*(3*c*f*h-2*d*(e*h+f*g))+b^2*(3*c^2*f^2*h^2-12*c*d*f*h*(e*h+f*g)+8*d^2*(
e^2*h^2+e*f*g*h+f^2*g^2)))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(
1/2))/b^(1/2)/d^(1/2)/f^3/h^3+2*(-a*f+b*e)^(3/2)*(-c*f+d*e)^(3/2)*arctanh(
(-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/f^3/(-e*h+f
*g)-2*(-a*h+b*g)^(3/2)*(-c*h+d*g)^(3/2)*arctanh((-c*h+d*g)^(1/2)*(b*x+a)^(
1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/h^3/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \frac{1}{4}\left(\frac{\sqrt{a+bx}\sqrt{c+dx}(5bcfh+5adfh+bd(-4fg-4eh+2fhx))}{f^2h^2} - \frac{8(be-$$

input
```
Integrate[((a + b*x)^(3/2)*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)),x]
```

output
```
((Sqrt[a + b*x]*Sqrt[c + d*x]*(5*b*c*f*h + 5*a*d*f*h + b*d*(-4*f*g - 4*e*h
 + 2*f*h*x)))/(f^2*h^2) - (8*(b*e - a*f)^(3/2)*(-(d*e) + c*f)^(3/2)*ArcTan
[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e) + c*f]*Sqrt[a + b*x])])/(f^3
*(f*g - e*h)) + (8*(b*g - a*h)^(3/2)*(-(d*g) + c*h)^(3/2)*ArcTan[(Sqrt[b*g
 - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])])/(h^3*(f*g - e*
h)) + ((3*a^2*d^2*f^2*h^2 - 6*a*b*d*f*h*(-3*c*f*h + 2*d*(f*g + e*h)) + b^2
*(3*c^2*f^2*h^2 - 12*c*d*f*h*(f*g + e*h) + 8*d^2*(f^2*g^2 + e*f*g*h + e^2*
h^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*
Sqrt[d]*f^3*h^3))/4
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{f(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(fg-eh)} - \frac{h(a+bx)^{3/2}(c+dx)^{3/2}}{(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-adf-bcf+2bde)\left(a^2d^2f^2+2abdf(4de-5cf)-\left(b^2\left(-c^2f^2-8cdef+8d^2e^2\right)\right)\right)}{8b^{3/2}d^{3/2}f^3(fg-eh)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-adh-bch+2bdg)\left(a^2d^2h^2+2abdh(4dg-5ch)-\left(b^2\left(-c^2h^2-8cdgh+8d^2g^2\right)\right)\right)}{8b^{3/2}d^{3/2}h^3(fg-eh)}+\frac{\sqrt{a+bx}\sqrt{c+dx}\left(\frac{a^2d^2f}{b}-2ad(5de-4cf)-b\left(c^2f+6cde-\frac{8d^2e^2}{f}\right)\right)}{8df(fg-eh)}-\frac{\sqrt{a+bx}\sqrt{c+dx}\left(\frac{a^2d^2h}{b}-2ad(5dg-4ch)-b\left(c^2h+6cdg-\frac{8d^2g^2}{h}\right)\right)}{8dh(fg-eh)}+\frac{2(be-af)^{3/2}(de-cf)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f^3(fg-eh)}-\frac{2(bg-ah)^{3/2}(dg-ch)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h^3(fg-eh)}-\frac{\sqrt{a+bx}(c+dx)^{3/2}(-adf-bcf+2bde)}{4df(fg-eh)}+\frac{\sqrt{a+bx}(c+dx)^{3/2}(-adh-bch+2bdg)}{4dh(fg-eh)}$$

input
```
Int[((a + b*x)^(3/2)*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)),x]
```

output
```
(((a^2*d^2*f)/b - 2*a*d*(5*d*e - 4*c*f) - b*(6*c*d*e - (8*d^2*e^2)/f + c^2
*f))*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d*f*(f*g - e*h)) - (((a^2*d^2*h)/b -
2*a*d*(5*d*g - 4*c*h) - b*(6*c*d*g - (8*d^2*g^2)/h + c^2*h))*Sqrt[a + b*x]
*Sqrt[c + d*x])/(8*d*h*(f*g - e*h)) - ((2*b*d*e - b*c*f - a*d*f)*Sqrt[a +
b*x]*(c + d*x)^(3/2))/(4*d*f*(f*g - e*h)) + ((2*b*d*g - b*c*h - a*d*h)*Sqr
t[a + b*x]*(c + d*x)^(3/2))/(4*d*h*(f*g - e*h)) + ((2*b*d*e - b*c*f - a*d*
f)*(a^2*d^2*f^2 + 2*a*b*d*f*(4*d*e - 5*c*f) - b^2*(8*d^2*e^2 - 8*c*d*e*f -
 c^2*f^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^
(3/2)*d^(3/2)*f^3*(f*g - e*h)) - ((2*b*d*g - b*c*h - a*d*h)*(a^2*d^2*h^2 +
 2*a*b*d*h*(4*d*g - 5*c*h) - b^2*(8*d^2*g^2 - 8*c*d*g*h - c^2*h^2))*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(3/2)*d^(3/2)*h^3
*(f*g - e*h)) + (2*(b*e - a*f)^(3/2)*(d*e - c*f)^(3/2)*ArcTanh[(Sqrt[d*e -
 c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(f^3*(f*g - e*h)) -
 (2*(b*g - a*h)^(3/2)*(d*g - c*h)^(3/2)*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a +
b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(h^3*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4047 vs. $2(340) = 680$.

Time = 0.58 (sec) , antiderivative size = 4048, normalized size of antiderivative = 10.38

method	result	size
default	Expression too large to display	4048

input
```
int((b*x+a)^(3/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(3*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*((c*h
-d*g)*(a*h-b*g)/h^2)^(1/2)*a^2*d^2*e*f^3*h^4-4*(d*b)^(1/2)*((b*x+a)*(d*x+c
))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b
*d*f^4*g*h^3*x-32*(d*b)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln((a*d*h*x+
b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2
)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*b*c*d*f^4*g^2*h^2+8*(d*b)^(1/2)*ln((a*
d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2
)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*a^
2*d^2*e^2*f^2*h^4+8*(d*b)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(
d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+
e))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b^2*c^2*e^2*f^2*h^4-8*(d*b)^(1/2)*((c*
f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*
h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*
b^2*c^2*f^4*g^2*h^2-8*(d*b)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln((a*d*
h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^
(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a^2*d^2*f^4*g^2*h^2-3*ln(1/2*(2*b*d*
x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*f-d*e)*(
a*f-b*e)/f^2)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b^2*c^2*f^4*g*h^3-3*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**(3/2)*(c + d*x)**(3/2)/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((b*x + a)^(3/2)*(d*x + c)^(3/2)/((f*x + e)*(h*x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^{3/2}\,(c+d\,x)^{3/2}}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^(3/2)*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^(3/2)*(c + d*x)^(3/2))/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{3}{2}}}{(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(3/2)*(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.18 $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx &= \frac{b\sqrt{a+bx}\sqrt{c+dx}}{fh}\\ &+ \frac{\sqrt{b}(bcfh+3adfh-2bd(fg+eh))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{d}f^2h^2}\\ &- \frac{2(be-af)^{3/2}\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f^2(fg-eh)}\\ &+ \frac{2(bg-ah)^{3/2}\sqrt{dg-ch}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h^2(fg-eh)}\end{aligned}$$

output
```
b*(b*x+a)^(1/2)*(d*x+c)^(1/2)/f/h+b^(1/2)*(b*c*f*h+3*a*d*f*h-2*b*d*(e*h+f*
g))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(1/2)/f^2/h^2-2
*(-a*f+b*e)^(3/2)*(-c*f+d*e)^(1/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/
(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/f^2/(-e*h+f*g)+2*(-a*h+b*g)^(3/2)*(-c*h+d*
g)^(1/2)*arctanh((-c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(
1/2))/h^2/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \frac{b\sqrt{a+bx}\sqrt{c+dx}}{fh} - \frac{2(be-af)^{3/2}\sqrt{-de+cf}\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{f^2(fg-eh)} + \frac{2(bg-ah)^{3/2}\sqrt{-dg+ch}\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{h^2(fg-eh)} + \frac{\sqrt{b}(bcfh+3adfh-2bd(fg+eh))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{d}f^2h^2}$$

input
```
Integrate[((a + b*x)^(3/2)*Sqrt[c + d*x])/((e + f*x)*(g + h*x)),x]
```

output
```
(b*Sqrt[a + b*x]*Sqrt[c + d*x])/(f*h) - (2*(b*e - a*f)^(3/2)*Sqrt[-(d*e) +
 c*f]*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e) + c*f]*Sqrt[a +
b*x])]])/(f^2*(f*g - e*h)) + (2*(b*g - a*h)^(3/2)*Sqrt[-(d*g) + c*h]*ArcTan
[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])]])/(h^2
*(f*g - e*h)) + (Sqrt[b]*(b*c*f*h + 3*a*d*f*h - 2*b*d*(f*g + e*h))*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[d]*f^2*h^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.46, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {196, 113, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{196}$$

$$\frac{(dg-ch)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(g+hx)}dx}{fg-eh}-\frac{(de-cf)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{113}$$

$$\frac{(dg-ch)\left(\frac{\int -\frac{-2dha^2+b(bc+ad)g+b(2bdg+bch-3adh)x}{2\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{dh}+\frac{b\sqrt{a+bx}\sqrt{c+dx}}{dh}\right)}{fg-eh}-\frac{(de-cf)\left(\frac{\int -\frac{-2dfa^2+b(bc+ad)e+b(2bde+bcf-3adf)x}{2\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{df}+\frac{b\sqrt{a+bx}\sqrt{c+dx}}{df}\right)}{fg-eh}$$

$$\Big\downarrow \text{27}$$

$$\frac{(dg-ch)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{dh}-\frac{\int \frac{-2dha^2+b(bc+ad)g+b(2bdg+bch-3adh)x}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{2dh}\right)}{fg-eh}-\frac{(de-cf)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{df}-\frac{\int \frac{-2dfa^2+b(bc+ad)e+b(2bde+bcf-3adf)x}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{2df}\right)}{fg-eh}$$

$$\Big\downarrow \text{175}$$

$$\frac{(dg-ch)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{dh}-\frac{\frac{b(-3adh+bch+2bdg)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{h}-\frac{2d(bg-ah)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{h}}{2dh}\right)}{fg-eh}-\frac{(de-cf)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{df}-\frac{\frac{b(-3adf+bcf+2bde)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}-\frac{2d(be-af)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{f}}{2df}\right)}{fg-eh}$$

$$\Big\downarrow \text{66}$$

$$\frac{(dg-ch)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{dh}-\frac{\frac{2b(-3adh+bch+2bdg)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}-\frac{2d(bg-ah)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{h}}{2dh}\right)}{fg-eh}-\frac{(de-cf)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{df}-\frac{\frac{2b(-3adf+bcf+2bde)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}-\frac{2d(be-af)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{f}}{2df}\right)}{fg-eh}$$

$$\Big\downarrow \text{104}$$

$$\frac{(dg-ch)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{dh}-\frac{\frac{2b(-3adh+bch+2bdg)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}-\frac{4d(bg-ah)^2\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{h}}{2dh}\right)}{fg-eh}-\frac{(de-cf)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{df}-\frac{\frac{2b(-3adf+bcf+2bde)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}-\frac{4d(be-af)^2\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{f}}{2df}\right)}{fg-eh}$$

$$\Big\downarrow \text{221}$$

$$\frac{(dg-ch)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{dh}-\frac{\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-3adh+bch+2bdg)}{\sqrt{d}h}-\frac{4d(bg-ah)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h\sqrt{dg-ch}}}{2dh}\right)}{fg-eh}-\frac{(de-cf)\left(\frac{b\sqrt{a+bx}\sqrt{c+dx}}{df}-\frac{\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-3adf+bcf+2bde)}{\sqrt{d}f}-\frac{4d(be-af)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f\sqrt{de-cf}}}{2df}\right)}{fg-eh}$$

input
```
Int[((a + b*x)^(3/2)*Sqrt[c + d*x])/((e + f*x)*(g + h*x)),x]
```

output
```
-(((d*e - c*f)*((b*Sqrt[a + b*x]*Sqrt[c + d*x])/(d*f) - ((2*Sqrt[b]*(2*b*d
*e + b*c*f - 3*a*d*f)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/(Sqrt[d]*f) - (4*d*(b*e - a*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a
 + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(f*Sqrt[d*e - c*f]))/(2*d*f)))/
(f*g - e*h)) + ((d*g - c*h)*((b*Sqrt[a + b*x]*Sqrt[c + d*x])/(d*h) - ((2*S
qrt[b]*(2*b*d*g + b*c*h - 3*a*d*h)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(Sqrt[d]*h) - (4*d*(b*g - a*h)^(3/2)*ArcTanh[(Sqrt[d*g
- c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(h*Sqrt[d*g - c*h]
))/(2*d*h)))/(f*g - e*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(223) = 446$.

Time = 0.65 (sec) , antiderivative size = 2239, normalized size of antiderivative = 8.51

method	result	size
default	Expression too large to display	2239

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(3*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f
-d*e)*(a*f-b*e)/f^2)^(1/2)*a*b*d*e*f^2*h^3-3*ln(1/2*(2*b*d*x+2*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h-b*g)/h^2)^(
1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*a*b*d*f^3*g*h^2+ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h-b*g
)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*b^2*c*e*f^2*h^3-ln(1/2*(2*b*d
*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*
(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*b^2*c*f^3*g*h^2-2*ln(
1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*(
(c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*b^2*d*e^2*f
*h^3+2*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b
)^(1/2))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*b
^2*d*f^3*g^2*h+2*(d*b)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x
+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))
*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*a^2*c*f^3*h^3-2*(d*b)^(1/2)*ln((a*d*f*x+b
*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)
*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*a^2*d*e*f
^2*h^3-4*(d*b)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/
2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*((c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**(3/2)*sqrt(c + d*x)/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((b*x + a)^(3/2)*sqrt(d*x + c)/((f*x + e)*(h*x + g)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^(3/2)*(c + d*x)^(1/2))/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^(3/2)*(c + d*x)^(1/2))/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}}{(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,\sqrt{dx+c}}{(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

3.19 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \frac{2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{d}fh} + \frac{2(be-af)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f\sqrt{de-cf}(fg-eh)} - \frac{2(bg-ah)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h\sqrt{dg-ch}(fg-eh)}$$

output
```
2*b^(3/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(1/2)/f/h
+2*(-a*f+b*e)^(3/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2
)/(d*x+c)^(1/2))/f/(-c*f+d*e)^(1/2)/(-e*h+f*g)-2*(-a*h+b*g)^(3/2)*arctanh(
(-c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/h/(-c*h+d*g
)^(1/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = 2\left(\frac{(be-af)^{3/2}\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{f\sqrt{-de+cf}(-fg+eh)} + \frac{(bg-ah)^{3/2}\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{h\sqrt{-dg+ch}(fg-eh)} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{d}fh}\right)$$

input
```
Integrate[(a + b*x)^(3/2)/(Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
2*(((b*e - a*f)^(3/2)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e)
+ c*f]*Sqrt[a + b*x])])/(f*Sqrt[-(d*e) + c*f]*(-(f*g) + e*h)) + ((b*g - a*
h)^(3/2)*ArcTan[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])])/(h*Sqrt[-(d*g) + c*h]*(f*g - e*h)) + (b^(3/2)*ArcTanh[(Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[d]*f*h))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{ 198}$$

$$\int \left(\frac{f(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(fg-eh)} - \frac{h(a+bx)^{3/2}}{\sqrt{c+dx}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-3adf+bcf+2bde)}{d^{3/2}f(fg-eh)}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-3adh+bch+2bdg)}{d^{3/2}h(fg-eh)}+\frac{2(be-af)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f\sqrt{de-cf}(fg-eh)}-\frac{2(bg-ah)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h\sqrt{dg-ch}(fg-eh)}$$

input
```
Int[(a + b*x)^(3/2)/(Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
-((Sqrt[b]*(2*b*d*e + b*c*f - 3*a*d*f)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sq
rt[b]*Sqrt[c + d*x])])/(d^(3/2)*f*(f*g - e*h))) + (Sqrt[b]*(2*b*d*g + b*c*
h - 3*a*d*h)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(d^
(3/2)*h*(f*g - e*h)) + (2*(b*e - a*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[
a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(f*Sqrt[d*e - c*f]*(f*g - e*h)
) - (2*(b*g - a*h)^(3/2)*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g
 - a*h]*Sqrt[c + d*x])])/(h*Sqrt[d*g - c*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(179) = 358$.

Time = 0.64 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.55

method	result
default	$\frac{\sqrt{xd+c}\,\sqrt{bx+a}\left(\sqrt{\frac{(ch-dg)(ah-bg)}{h^2}}\ln\left(\frac{adfx+bcfx-2bdex+2\sqrt{(bx+a)(xd+c)}\sqrt{\frac{(cf-de)(af-be)}{f^2}}f+2acf-ade-bce}{fx+e}\right)\sqrt{db}\,a^2f^2h^2-2\sqrt{(c}}{}$

input
```
int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(d*x+c)^(1/2)*(b*x+a)^(1/2)*(((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*ln((a*d*f*x+b
*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)
*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*(d*b)^(1/2)*a^2*f^2*h^2-2*((c*h-d*g)*(a*h
-b*g)/h^2)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*(
(c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*(d*b)^(1/2)
*a*b*e*f*h^2+((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x
+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e
-b*c*e)/(f*x+e))*(d*b)^(1/2)*b^2*e^2*h^2+((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*(
(c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^2*e*f*h^2-((c*h-d*g)*(a*h-b*g)/h^2)^(1/
2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^2*f^2*g*h-ln((a*d*h*x+b*c*h*x-2*b*d
*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d
*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*(d*b)^(1/2)*a^2*f^2*h
^2+2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x
+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^
2)^(1/2)*(d*b)^(1/2)*a*b*f^2*g*h-ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g
)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x
+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*(d*b)^(1/2)*b^2*f^2*g^2)/((c*f-d*e)*(
a*f-b*e)/f^2)^(1/2)/f^2/((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)/h^2/(e*h-f*g)/(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**(3/2)/(sqrt(c + d*x)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^{3/2}}{(e+f\,x)\,(g+h\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((a + b*x)^(3/2)/((e + f*x)*(g + h*x)*(c + d*x)^(1/2)),x)
```

output
```
int((a + b*x)^(3/2)/((e + f*x)*(g + h*x)*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 2461, normalized size of antiderivative = 11.45

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

output

```
( - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f
 - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(
a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*a*c*d*f*h**2 + sqrt(c*f - d*e)*s
qrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*
e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sq
rt(b)*sqrt(c + d*x))*a*d**2*f*g*h + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( -
 sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f -
2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*
b*c*d*e*h**2 - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(
b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sq
rt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b*d**2*e*g*h - sqrt(c
*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(
a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sq
rt(f)*sqrt(b)*sqrt(c + d*x))*a*c*d*f*h**2 + sqrt(c*f - d*e)*sqrt(a*f - b*e
)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c
*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d
*x))*a*d**2*f*g*h + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqr
t(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*
sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b*c*d*e*h**2 - sqrt
(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*...
```

3.20 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2(bc-ad)\sqrt{a+bx}}{(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(de-cf)^{3/2}(fg-eh)} + \frac{2(bg-ah)^{3/2}\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(dg-ch)^{3/2}(fg-eh)}$$

output
```
2*(-a*d+b*c)*(b*x+a)^(1/2)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^(1/2)-2*(-a*f+b*e
)^(3/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1
/2))/(-c*f+d*e)^(3/2)/(-e*h+f*g)+2*(-a*h+b*g)^(3/2)*arctanh((-c*h+d*g)^(1/
2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-c*h+d*g)^(3/2)/(-e*h+f*
g)
```

Mathematica [A] (verified)

Time = 10.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2\left(\frac{(bc-ad)(-fg+eh)\sqrt{a+bx}}{(de-cf)(dg-ch)\sqrt{c+dx}}+\frac{(be-af)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(de-cf)^{3/2}}-\frac{(bg-ah)^{3/2}\text{ar}}{(}\right.}{-fg+eh}$$

input
```
Integrate[(a + b*x)^(3/2)/((c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*(((b*c - a*d)*(-(f*g) + e*h)*Sqrt[a + b*x])/((d*e - c*f)*(d*g - c*h)*Sq
rt[c + d*x]) + ((b*e - a*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/
(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(d*e - c*f)^(3/2) - ((b*g - a*h)^(3/2)*A
rcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(
d*g - c*h)^(3/2)))/(-(f*g) + e*h)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \; 198$$

$$\int \left(\frac{f(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(fg-eh)}-\frac{h(a+bx)^{3/2}}{(c+dx)^{3/2}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2(be-af)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{(de-cf)^{3/2}(fg-eh)}+\frac{2(bg-ah)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{(dg-ch)^{3/2}(fg-eh)}+\frac{2f\sqrt{a+bx}(bc-ad)}{d\sqrt{c+dx}(de-cf)(fg-eh)}-\frac{2h\sqrt{a+bx}(bc-ad)}{d\sqrt{c+dx}(dg-ch)(fg-eh)}$$

input
```
Int[(a + b*x)^(3/2)/((c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*(b*c - a*d)*f*Sqrt[a + b*x])/(d*(d*e - c*f)*(f*g - e*h)*Sqrt[c + d*x])
- (2*(b*c - a*d)*h*Sqrt[a + b*x])/(d*(d*g - c*h)*(f*g - e*h)*Sqrt[c + d*x]
) - (2*(b*e - a*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e
 - a*f]*Sqrt[c + d*x])])/((d*e - c*f)^(3/2)*(f*g - e*h)) + (2*(b*g - a*h)^
(3/2)*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*
x])])/((d*g - c*h)^(3/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3097 vs. $2(181) = 362$.

Time = 0.68 (sec) , antiderivative size = 3098, normalized size of antiderivative = 14.82

method	result	size
default	Expression too large to display	3098

input
```
int((b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(1/2)*(-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((
c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a^2*c*d*f^2*
g*h*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-2*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*
x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)
/(f*x+e))*a*b*c^2*e*f*h^2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+2*ln((a*d*f*x+b*
c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*
f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*d^2*e*f*g*h*x*((c*h-d*g)*(a*h-b*g)/h^2
)^(1/2)+2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*
((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*b*c*d*f^2*g*h*x*
((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)-2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*
g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*
x+g))*a*b*d^2*e*f*g*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+2*ln((a*d*f*x+b*c*
f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+
2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*c*d*e*f*g*h*((c*h-d*g)*(a*h-b*g)/h^2)^(1
/2)-2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*
x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*b*c*d*e*f*g*h*((c*f-
d*e)*(a*f-b*e)/f^2)^(1/2)-2*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+
c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*
a*b*c*d*e*f*h^2*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*
d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**(3/2)/((c + d*x)**(3/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((b*x + a)^(3/2)/((d*x + c)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(181) = 362$.

Time = 158.67 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.55

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2\,(b^3c-ab^2d)\sqrt{bx+a}}{(d^2eg|b|-cdfg|b|-cdeh|b|+c^2fh|b|)\sqrt{b^2c+(bx+a)bd-abd}} + \frac{2\left(\sqrt{bd}b^4e^2-2\sqrt{bd}ab^3ef+\sqrt{bd}a^2b^2f^2\right)\arctan\left(\frac{2\,b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2f}{2\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(defg|b|-cf^2g|b|-de^2h|b|+cefh|b|)b} - \frac{2\left(\sqrt{bd}b^4g^2-2\sqrt{bd}ab^3gh+\sqrt{bd}a^2b^2h^2\right)\arctan\left(\frac{2\,b^2dg-b^2ch-abdh+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2h}{2\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}b}\right)}{\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}(dfg^2|b|-degh|b|-cfgh|b|+ceh^2|b|)b}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
2*(b^3*c - a*b^2*d)*sqrt(b*x + a)/((d^2*e*g*abs(b) - c*d*f*g*abs(b) - c*d*
e*h*abs(b) + c^2*f*h*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 2*(sqr
t(b*d)*b^4*e^2 - 2*sqrt(b*d)*a*b^3*e*f + sqrt(b*d)*a^2*b^2*f^2)*arctan(1/2
*(2*b^2*d*e - b^2*c*f - a*b*d*f + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*f)/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*
f - a*b*c*d*f^2)*b))/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*
c*d*f^2)*(d*e*f*g*abs(b) - c*f^2*g*abs(b) - d*e^2*h*abs(b) + c*e*f*h*abs(b
))*b) - 2*(sqrt(b*d)*b^4*g^2 - 2*sqrt(b*d)*a*b^3*g*h + sqrt(b*d)*a^2*b^2*h
^2)*arctan(1/2*(2*b^2*d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*
h + a*b*d^2*g*h - a*b*c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*
d^2*g*h - a*b*c*d*h^2)*(d*f*g^2*abs(b) - d*e*g*h*abs(b) - c*f*g*h*abs(b) +
 c*e*h^2*abs(b))*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{(a+b\,x)^{3/2}}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + b*x)^(3/2)/((e + f*x)*(g + h*x)*(c + d*x)^(3/2)),x)
```

output
```
int((a + b*x)^(3/2)/((e + f*x)*(g + h*x)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.21 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 303

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \frac{2(bc-ad)\sqrt{a+bx}}{3(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2(3ad(dfg+deh-2cfh)-b(4d^2eg-2c^2fh-cd(fg+eh)))\sqrt{a+bx}}{3(de-cf)^2(dg-ch)^2\sqrt{c+dx}} + \frac{2f(be-af)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(de-cf)^{5/2}(fg-eh)} - \frac{2h(bg-ah)^{3/2}\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(dg-ch)^{5/2}(fg-eh)}$$

output
```
2/3*(-a*d+b*c)*(b*x+a)^(1/2)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^(3/2)+2/3*(3*a*
d*(-2*c*f*h+d*e*h+d*f*g)-b*(4*d^2*e*g-2*c^2*f*h-c*d*(e*h+f*g)))*(b*x+a)^(1
/2)/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)+2*f*(-a*f+b*e)^(3/2)*arctanh((
-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(-c*f+d*e)^(
5/2)/(-e*h+f*g)-2*h*(-a*h+b*g)^(3/2)*arctanh((-c*h+d*g)^(1/2)*(b*x+a)^(1/2
)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-c*h+d*g)^(5/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \frac{2\sqrt{a+bx}(b(3c^3fh-4d^3egx+2c^2dfhx+cd^2(-3eg+fgx+ehx))-}{3(de-cf)^2(dg-} + \frac{2f(be-af)^{3/2}\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{(-de+cf)^{5/2}(-fg+eh)} + \frac{2h(bg-ah)^{3/2}\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{(-dg+ch)^{5/2}(fg-eh)}$$

input
```
Integrate[(a + b*x)^(3/2)/((c + d*x)^(5/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*Sqrt[a + b*x]*(b*(3*c^3*f*h - 4*d^3*e*g*x + 2*c^2*d*f*h*x + c*d^2*(-3*e
*g + f*g*x + e*h*x)) + a*d*(-7*c^2*f*h + d^2*(-(e*g) + 3*f*g*x + 3*e*h*x)
+ 2*c*d*(2*f*g + 2*e*h - 3*f*h*x))))/(3*(d*e - c*f)^2*(d*g - c*h)^2*(c + d
*x)^(3/2)) + (2*f*(b*e - a*f)^(3/2)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])
/(Sqrt[-(d*e) + c*f]*Sqrt[a + b*x])])/((-(d*e) + c*f)^(5/2)*(-(f*g) + e*h)
) + (2*h*(b*g - a*h)^(3/2)*ArcTan[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(
d*g) + c*h]*Sqrt[a + b*x])])/((-(d*g) + c*h)^(5/2)*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{f(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(fg-eh)}-\frac{h(a+bx)^{3/2}}{(c+dx)^{5/2}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2f(be-af)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{(de-cf)^{5/2}(fg-eh)} - \frac{2h(bg-ah)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{(dg-ch)^{5/2}(fg-eh)} - \frac{2f\sqrt{a+bx}(be-af)}{\sqrt{c+dx}(de-cf)^2(fg-eh)} + \frac{2h\sqrt{a+bx}(bg-ah)}{\sqrt{c+dx}(dg-ch)^2(fg-eh)} - \frac{2f(a+bx)^{3/2}}{3(c+dx)^{3/2}(de-cf)(fg-eh)} + \frac{2h(a+bx)^{3/2}}{3(c+dx)^{3/2}(dg-ch)(fg-eh)}$$

input
```
Int[(a + b*x)^(3/2)/((c + d*x)^(5/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*f*(a + b*x)^(3/2))/(3*(d*e - c*f)*(f*g - e*h)*(c + d*x)^(3/2)) + (2*h*
(a + b*x)^(3/2))/(3*(d*g - c*h)*(f*g - e*h)*(c + d*x)^(3/2)) - (2*f*(b*e -
 a*f)*Sqrt[a + b*x])/((d*e - c*f)^2*(f*g - e*h)*Sqrt[c + d*x]) + (2*h*(b*g
 - a*h)*Sqrt[a + b*x])/((d*g - c*h)^2*(f*g - e*h)*Sqrt[c + d*x]) + (2*f*(b
*e - a*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*S
qrt[c + d*x])])/((d*e - c*f)^(5/2)*(f*g - e*h)) - (2*h*(b*g - a*h)^(3/2)*A
rcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(
(d*g - c*h)^(5/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7798 vs. 2(267) = 534.

Time = 0.74 (sec) , antiderivative size = 7799, normalized size of antiderivative = 25.74

method	result	size
default	Expression too large to display	7799

input
```
int((b*x+a)^(3/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(3/2)/(d*x+c)**(5/2)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((b*x + a)^(3/2)/((d*x + c)^(5/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4509 vs. $2(267) = 534$.

Time = 61.87 (sec) , antiderivative size = 4509, normalized size of antiderivative = 14.88

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + a)*((4*b^5*c*d^8*e^3*g^3*abs(b) - 4*a*b^4*d^9*e^3*g^3*abs(
b) - 9*b^5*c^2*d^7*e^2*f*g^3*abs(b) + 6*a*b^4*c*d^8*e^2*f*g^3*abs(b) + 3*a
^2*b^3*d^9*e^2*f*g^3*abs(b) + 6*b^5*c^3*d^6*e*f^2*g^3*abs(b) - 6*a^2*b^3*c
*d^8*e*f^2*g^3*abs(b) - b^5*c^4*d^5*f^3*g^3*abs(b) - 2*a*b^4*c^3*d^6*f^3*g
^3*abs(b) + 3*a^2*b^3*c^2*d^7*f^3*g^3*abs(b) - 9*b^5*c^2*d^7*e^3*g^2*h*abs
(b) + 6*a*b^4*c*d^8*e^3*g^2*h*abs(b) + 3*a^2*b^3*d^9*e^3*g^2*h*abs(b) + 18
*b^5*c^3*d^6*e^2*f*g^2*h*abs(b) - 18*a^2*b^3*c*d^8*e^2*f*g^2*h*abs(b) - 9*
b^5*c^4*d^5*e*f^2*g^2*h*abs(b) - 18*a*b^4*c^3*d^6*e*f^2*g^2*h*abs(b) + 27*
a^2*b^3*c^2*d^7*e*f^2*g^2*h*abs(b) + 12*a*b^4*c^4*d^5*f^3*g^2*h*abs(b) - 1
2*a^2*b^3*c^3*d^6*f^3*g^2*h*abs(b) + 6*b^5*c^3*d^6*e^3*g*h^2*abs(b) - 6*a^
2*b^3*c*d^8*e^3*g*h^2*abs(b) - 9*b^5*c^4*d^5*e^2*f*g*h^2*abs(b) - 18*a*b^4
*c^3*d^6*e^2*f*g*h^2*abs(b) + 27*a^2*b^3*c^2*d^7*e^2*f*g*h^2*abs(b) + 36*a
*b^4*c^4*d^5*e*f^2*g*h^2*abs(b) - 36*a^2*b^3*c^3*d^6*e*f^2*g*h^2*abs(b) +
3*b^5*c^6*d^3*f^3*g*h^2*abs(b) - 18*a*b^4*c^5*d^4*f^3*g*h^2*abs(b) + 15*a^
2*b^3*c^4*d^5*f^3*g*h^2*abs(b) - b^5*c^4*d^5*e^3*h^3*abs(b) - 2*a*b^4*c^3*
d^6*e^3*h^3*abs(b) + 3*a^2*b^3*c^2*d^7*e^3*h^3*abs(b) + 12*a*b^4*c^4*d^5*e
^2*f*h^3*abs(b) - 12*a^2*b^3*c^3*d^6*e^2*f*h^3*abs(b) + 3*b^5*c^6*d^3*e*f^
2*h^3*abs(b) - 18*a*b^4*c^5*d^4*e*f^2*h^3*abs(b) + 15*a^2*b^3*c^4*d^5*e*f^
2*h^3*abs(b) - 2*b^5*c^7*d^2*f^3*h^3*abs(b) + 8*a*b^4*c^6*d^3*f^3*h^3*abs(
b) - 6*a^2*b^3*c^5*d^4*f^3*h^3*abs(b))*(b*x + a)/(b^3*c*d^9*e^4*g^4 - a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx=\int \frac{(a+b\,x)^{3/2}}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((a + b*x)^(3/2)/((e + f*x)*(g + h*x)*(c + d*x)^(5/2)),x)
```

output
```
int((a + b*x)^(3/2)/((e + f*x)*(g + h*x)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^(3/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^(3/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x)
```

3.22 $\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)} dx$

Optimal result

Integrand size = 33, antiderivative size = 269

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)} dx = \frac{d^2\sqrt{a+bx}\sqrt{c+dx}}{bfh} + \frac{d^{3/2}(5bcfh-adfh-2bd(fg+eh))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}f^2h^2} - \frac{2(de-cf)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f^2\sqrt{be-af}(fg-eh)} + \frac{2(dg-ch)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h^2\sqrt{bg-ah}(fg-eh)}$$

output
```
d^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/f/h+d^(3/2)*(5*b*c*f*h-a*d*f*h-2*b*d*(e*
h+f*g))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)/f^2/h
^2-2*(-c*f+d*e)^(5/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1
/2)/(d*x+c)^(1/2))/f^2/(-a*f+b*e)^(1/2)/(-e*h+f*g)+2*(-c*h+d*g)^(5/2)*arct
anh((-c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/h^2/(-a
*h+b*g)^(1/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \frac{d^2\sqrt{a+bx}\sqrt{c+dx}}{bfh} - \frac{2(de-cf)^{5/2}\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{f^2\sqrt{-be+af}(-fg+eh)} - \frac{2(dg-ch)^{5/2}\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{h^2\sqrt{-bg+ah}(fg-eh)} - \frac{d^{3/2}(-5bcfh+adfh+2bd(fg+eh))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}f^2h^2}$$

input
```
Integrate[(c + d*x)^(5/2)/(Sqrt[a + b*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(b*f*h) - (2*(d*e - c*f)^(5/2)*ArcTan[(S
qrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/(f^2*Sq
rt[-(b*e) + a*f]*(-(f*g) + e*h)) - (2*(d*g - c*h)^(5/2)*ArcTan[(Sqrt[d*g -
 c*h]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c + d*x])])/(h^2*Sqrt[-(b*g)
 + a*h]*(f*g - e*h)) - (d^(3/2)*(-5*b*c*f*h + a*d*f*h + 2*b*d*(f*g + e*h))
*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*f^2*h^
2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{f(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(fg-eh)} - \frac{h(c+dx)^{5/2}}{\sqrt{a+bx}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)\left(3a^2d^2f^2+2abdf(2de-5cf)+b^2\left(15c^2f^2-20cdef+8d^2e^2\right)\right)}{4b^{5/2}f^2(fg-eh)}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)\left(3a^2d^2h^2+2abdh(2dg-5ch)+b^2\left(15c^2h^2-20cdgh+8d^2g^2\right)\right)}{4b^{5/2}h^2(fg-eh)}-\frac{2(de-cf)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f^2\sqrt{be-af}(fg-eh)}+\frac{2(dg-ch)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h^2\sqrt{bg-ah}(fg-eh)}-\frac{d\sqrt{a+bx}\sqrt{c+dx}(3adf-7bcf+4bde)}{4b^2f(fg-eh)}+\frac{d\sqrt{a+bx}\sqrt{c+dx}(3adh-7bch+4bdg)}{4b^2h(fg-eh)}$$

input
```
Int[(c + d*x)^(5/2)/(Sqrt[a + b*x]*(e + f*x)*(g + h*x)),x]
```

output
```
-1/4*(d*(4*b*d*e - 7*b*c*f + 3*a*d*f)*Sqrt[a + b*x]*Sqrt[c + d*x])/(b^2*f*
(f*g - e*h)) + (d*(4*b*d*g - 7*b*c*h + 3*a*d*h)*Sqrt[a + b*x]*Sqrt[c + d*x
])/(4*b^2*h*(f*g - e*h)) + (Sqrt[d]*(3*a^2*d^2*f^2 + 2*a*b*d*f*(2*d*e - 5*
c*f) + b^2*(8*d^2*e^2 - 20*c*d*e*f + 15*c^2*f^2))*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2)*f^2*(f*g - e*h)) - (Sqrt[d]*(
3*a^2*d^2*h^2 + 2*a*b*d*h*(2*d*g - 5*c*h) + b^2*(8*d^2*g^2 - 20*c*d*g*h +
15*c^2*h^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*
b^(5/2)*h^2*(f*g - e*h)) - (2*(d*e - c*f)^(5/2)*ArcTanh[(Sqrt[d*e - c*f]*S
qrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(f^2*Sqrt[b*e - a*f]*(f*g
- e*h)) + (2*(d*g - c*h)^(5/2)*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sq
rt[b*g - a*h]*Sqrt[c + d*x])])/(h^2*Sqrt[b*g - a*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1781 vs. $2(229) = 458$.

Time = 3.03 (sec) , antiderivative size = 1782, normalized size of antiderivative = 6.62

method	result	size
default	Expression too large to display	1782

input
```
int((d*x+c)^(5/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2
)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*((c*h-
d*g)*(a*h-b*g)/h^2)^(1/2)*a*d^3*e*f^2*h^3-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2
)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*a*d^3*f^3*g*h^2-5*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*f-d*e)*(a*f-b*e)
/f^2)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b*c*d^2*e*f^2*h^3+5*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*f-d*
e)*(a*f-b*e)/f^2)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b*c*d^2*f^3*g*h^2+
2*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/
2))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*b*d^3*
e^2*f*h^3-2*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1
/2)*b*d^3*f^3*g^2*h+2*(d*b)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln((a*d*
h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^
(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*b*c^3*f^3*h^3-6*(d*b)^(1/2)*((c*f-d*
e)*(a*f-b*e)/f^2)^(1/2)*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*
g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*b*c^
2*d*f^3*g*h^2+6*(d*b)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*ln((a*d*h*x+b*
c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(5/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)/(b*x+a)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{\sqrt{bx+a}(fx+e)(hx+g)}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((d*x + c)^(5/2)/(sqrt(b*x + a)*(f*x + e)*(h*x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(5/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{(c+d\,x)^{5/2}}{(e+f\,x)\,(g+h\,x)\,\sqrt{a+b\,x}}\,dx$$

input
```
int((c + d*x)^(5/2)/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)),x)
```

output
```
int((c + d*x)^(5/2)/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{\sqrt{bx+a}\,(fx+e)\,(hx+g)}dx$$

input
```
int((d*x+c)^(5/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((d*x+c)^(5/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x)
```

3.23 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \frac{2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}fh} + \frac{2(de-cf)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f\sqrt{be-af}(fg-eh)} - \frac{2(dg-ch)^{3/2}\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h\sqrt{bg-ah}(fg-eh)}$$

output
```
2*d^(3/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/f/h
+2*(-c*f+d*e)^(3/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2
)/(d*x+c)^(1/2))/f/(-a*f+b*e)^(1/2)/(-e*h+f*g)-2*(-c*h+d*g)^(3/2)*arctanh(
(-c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/h/(-a*h+b*g
)^(1/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = 2\left(\frac{(de-cf)^{3/2}\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{f\sqrt{-be+af}(-fg+eh)} + \frac{(dg-ch)^{3/2}\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{h\sqrt{-bg+ah}(fg-eh)} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}fh}\right)$$

input
```
Integrate[(c + d*x)^(3/2)/(Sqrt[a + b*x]*(e + f*x)*(g + h*x)),x]
```

output
```
2*(((d*e - c*f)^(3/2)*ArcTan[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e)
+ a*f]*Sqrt[c + d*x])])/(f*Sqrt[-(b*e) + a*f]*(-(f*g) + e*h)) + ((d*g - c*
h)^(3/2)*ArcTan[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c
 + d*x])])/(h*Sqrt[-(b*g) + a*h]*(f*g - e*h)) + (d^(3/2)*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*f*h))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{ 198}$$

$$\int \left(\frac{f(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(fg-eh)} - \frac{h(c+dx)^{3/2}}{\sqrt{a+bx}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(adf-3bcf+2bde)}{b^{3/2}f(fg-eh)}+\frac{\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(adh-3bch+2bdg)}{b^{3/2}h(fg-eh)}+\frac{2(de-cf)^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f\sqrt{be-af}(fg-eh)}-\frac{2(dg-ch)^{3/2}\mathrm{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h\sqrt{bg-ah}(fg-eh)}$$

input
```
Int[(c + d*x)^(3/2)/(Sqrt[a + b*x]*(e + f*x)*(g + h*x)),x]
```

output
```
-((Sqrt[d]*(2*b*d*e - 3*b*c*f + a*d*f)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sq
rt[b]*Sqrt[c + d*x])])/(b^(3/2)*f*(f*g - e*h))) + (Sqrt[d]*(2*b*d*g - 3*b*
c*h + a*d*h)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^
(3/2)*h*(f*g - e*h)) + (2*(d*e - c*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[
a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x])])/(f*Sqrt[b*e - a*f]*(f*g - e*h)
) - (2*(d*g - c*h)^(3/2)*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g
 - a*h]*Sqrt[c + d*x])])/(h*Sqrt[b*g - a*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(179) = 358$.

Time = 0.69 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.55

method	result
default	$\frac{\sqrt{bx+a}\,\sqrt{xd+c}\left(\sqrt{db}\,\sqrt{\frac{(ch-dg)(ah-bg)}{h^2}}\,\ln\left(\frac{adfx+bcfx-2bdex+2\sqrt{(bx+a)(xd+c)}\,\sqrt{\frac{(cf-de)(af-be)}{f^2}}\,f+2acf-ade-bce}{fx+e}\right)c^2f^2h^2-2\sqrt{db}\cdots\right.}{\cdots}$

input
```
int((d*x+c)^(3/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(1/2)*(d*x+c)^(1/2)*((d*b)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*l
n((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e
)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*c^2*f^2*h^2-2*(d*b)^(1/2)*((c
*h-d*g)*(a*h-b*g)/h^2)^(1/2)*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x
+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))
*c*d*e*f*h^2+(d*b)^(1/2)*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*ln((a*d*f*x+b*c*f
*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2
*a*c*f-a*d*e-b*c*e)/(f*x+e))*d^2*e^2*h^2-(d*b)^(1/2)*ln((a*d*h*x+b*c*h*x-2
*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c
*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*c^2*f^2*h^2+2*(d*
b)^(1/2)*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e
)/f^2)^(1/2)*c*d*f^2*g*h-(d*b)^(1/2)*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h
-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/
(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*d^2*f^2*g^2+ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h-b*g
)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*d^2*e*f*h^2-ln(1/2*(2*b*d*x+2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*((c*h-d*g)*(a*h
-b*g)/h^2)^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*d^2*f^2*g*h)/((c*f-d*e)*(
a*f-b*e)/f^2)^(1/2)/f^2/((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)/(d*b)^(1/2)/h^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+bx}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(b*x+a)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((c + d*x)**(3/2)/(sqrt(a + b*x)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{bx+a}(fx+e)(hx+g)}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((d*x + c)^(3/2)/(sqrt(b*x + a)*(f*x + e)*(h*x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{(c+d\,x)^{3/2}}{(e+f\,x)\,(g+h\,x)\,\sqrt{a+b\,x}}\,dx$$

input
```
int((c + d*x)^(3/2)/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)),x)
```

output
```
int((c + d*x)^(3/2)/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 2461, normalized size of antiderivative = 11.45

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(3/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x)
```

output

```
( - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f
 - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(
a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*a*b*c*f*h**2 + sqrt(c*f - d*e)*s
qrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*
e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sq
rt(b)*sqrt(c + d*x))*a*b*d*e*h**2 + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( -
 sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f -
2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*
b**2*c*f*g*h - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(
b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sq
rt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b**2*d*e*g*h - sqrt(c
*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(
a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sq
rt(f)*sqrt(b)*sqrt(c + d*x))*a*b*c*f*h**2 + sqrt(c*f - d*e)*sqrt(a*f - b*e
)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c
*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d
*x))*a*b*d*e*h**2 + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqr
t(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*
sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b**2*c*f*g*h - sqrt
(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*...
```

3.24 $\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = -\frac{2\sqrt{de-cf}\,\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{\sqrt{be-af}(fg-eh)} + \frac{2\sqrt{dg-ch}\,\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{\sqrt{bg-ah}(fg-eh)}$$

output
```
-2*(-c*f+d*e)^(1/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2
)/(d*x+c)^(1/2))/(-a*f+b*e)^(1/2)/(-e*h+f*g)+2*(-c*h+d*g)^(1/2)*arctanh((-
c*h+d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(1
/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \frac{2\sqrt{\frac{b}{d}}\sqrt{d}\left(\frac{\sqrt{-de+cf}\arctan\left(\frac{\sqrt{d}\left(-\sqrt{\frac{b}{d}}f\sqrt{a+bx}\sqrt{c+dx}+b(e+fx)\right)}{\sqrt{b}\sqrt{be-af}\sqrt{-de+cf}}\right)}{\sqrt{be-af}}-\frac{\sqrt{-dg+ch}\arctan\left(\frac{\sqrt{d}\left(-\sqrt{\frac{b}{d}}h\sqrt{a+bx}\sqrt{c+dx}+b(g+hx)\right)}{\sqrt{b}\sqrt{bg-ah}\sqrt{-dg+ch}}\right)}{\sqrt{bg-ah}}\right)}{\sqrt{b}(-fg+eh)}$$

input
```
Integrate[Sqrt[c + d*x]/(Sqrt[a + b*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(2*Sqrt[b/d]*Sqrt[d]*((Sqrt[-(d*e) + c*f]*ArcTan[(Sqrt[d]*(-(Sqrt[b/d]*f*S
qrt[a + b*x]*Sqrt[c + d*x]) + b*(e + f*x)))/(Sqrt[b]*Sqrt[b*e - a*f]*Sqrt[
-(d*e) + c*f])])/Sqrt[b*e - a*f] - (Sqrt[-(d*g) + c*h]*ArcTan[(Sqrt[d]*(-(
Sqrt[b/d]*h*Sqrt[a + b*x]*Sqrt[c + d*x]) + b*(g + h*x)))/(Sqrt[b]*Sqrt[b*g
 - a*h]*Sqrt[-(d*g) + c*h])])/Sqrt[b*g - a*h]))/(Sqrt[b]*(-(f*g) + e*h))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {196, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{196}$$

$$\frac{(dg-ch)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{fg-eh}-\frac{(de-cf)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{104}$$

$$\frac{2(dg-ch)\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{fg-eh}-\frac{2(de-cf)\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{fg-eh}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\sqrt{dg-ch}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}(fg-eh)}-\frac{2\sqrt{de-cf}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}(fg-eh)}$$

input
```
Int[Sqrt[c + d*x]/(Sqrt[a + b*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*Sqrt[d*e - c*f]*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*
f]*Sqrt[c + d*x])])/(Sqrt[b*e - a*f]*(f*g - e*h)) + (2*Sqrt[d*g - c*h]*Arc
Tanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(Sq
rt[b*g - a*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(137) = 274$.

Time = 0.69 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.18

method	result
default	$\frac{\sqrt{xd+c}\,\sqrt{bx+a}\left(\ln\left(\frac{adfx+bcfx-2bdex+2\sqrt{(bx+a)(xd+c)}\sqrt{\frac{(cf-de)(af-be)}{f^2}}\,f+2acf-ade-bce}{fx+e}\right)\sqrt{\frac{(ch-dg)(ah-bg)}{h^2}}\,cfh-\ln\left(\frac{adfx+bcf\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(d*x+c)^(1/2)*(b*x+a)^(1/2)*(ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x
+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))
*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*c*f*h-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b
*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e
)/(f*x+e))*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*d*e*h-ln((a*d*h*x+b*c*h*x-2*b*d
*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d
*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*c*f*h+ln((a*d*h*x+b*c
*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h
+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*d*f*g)/((c*
f-d*e)*(a*f-b*e)/f^2)^(1/2)/((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)/(e*h-f*g)/((b*
x+a)*(d*x+c))^(1/2)/h/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(137) = 274$.

Time = 116.13 (sec) , antiderivative size = 2036, normalized size of antiderivative = 12.65

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
[-1/2*(sqrt((d*e - c*f)/(b*e - a*f))*log((8*a^2*c^2*f^2 + (b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*e^2 - 8*(a*b*c^2 + a^2*c*d)*e*f + (8*b^2*d^2*e^2 - 8*(b^2*
c*d + a*b*d^2)*e*f + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*f^2)*x^2 + 4*(2*a^2*c
*f^2 + (b^2*c + a*b*d)*e^2 - (3*a*b*c + a^2*d)*e*f + (2*b^2*d*e^2 - (b^2*c
 + 3*a*b*d)*e*f + (a*b*c + a^2*d)*f^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt
((d*e - c*f)/(b*e - a*f)) + 2*(4*(b^2*c*d + a*b*d^2)*e^2 - (3*b^2*c^2 + 10
*a*b*c*d + 3*a^2*d^2)*e*f + 4*(a*b*c^2 + a^2*c*d)*f^2)*x)/(f^2*x^2 + 2*e*f
*x + e^2)) + sqrt((d*g - c*h)/(b*g - a*h))*log((8*a^2*c^2*h^2 + (b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*g^2 - 8*(a*b*c^2 + a^2*c*d)*g*h + (8*b^2*d^2*g^2 - 8
*(b^2*c*d + a*b*d^2)*g*h + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*h^2)*x^2 - 4*(2
*a^2*c*h^2 + (b^2*c + a*b*d)*g^2 - (3*a*b*c + a^2*d)*g*h + (2*b^2*d*g^2 -
(b^2*c + 3*a*b*d)*g*h + (a*b*c + a^2*d)*h^2)*x)*sqrt(b*x + a)*sqrt(d*x + c
)*sqrt((d*g - c*h)/(b*g - a*h)) + 2*(4*(b^2*c*d + a*b*d^2)*g^2 - (3*b^2*c^
2 + 10*a*b*c*d + 3*a^2*d^2)*g*h + 4*(a*b*c^2 + a^2*c*d)*h^2)*x)/(h^2*x^2 +
 2*g*h*x + g^2)))/(f*g - e*h), 1/2*(2*sqrt(-(d*e - c*f)/(b*e - a*f))*arcta
n(-1/2*(2*a*c*f - (b*c + a*d)*e - (2*b*d*e - (b*c + a*d)*f)*x)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-(d*e - c*f)/(b*e - a*f))/(a*c*d*e - a*c^2*f + (b*d^
2*e - b*c*d*f)*x^2 + ((b*c*d + a*d^2)*e - (b*c^2 + a*c*d)*f)*x)) - sqrt((d
*g - c*h)/(b*g - a*h))*log((8*a^2*c^2*h^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*g^2 - 8*(a*b*c^2 + a^2*c*d)*g*h + (8*b^2*d^2*g^2 - 8*(b^2*c*d + a*b*d...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{c+dx}}{\sqrt{a+bx}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(b*x+a)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(sqrt(c + d*x)/(sqrt(a + b*x)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{dx+c}}{\sqrt{bx+a}(fx+e)(hx+g)}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x + c)/(sqrt(b*x + a)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(137) = 274$.

Time = 35.44 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx$$
$$= \frac{2\left(\frac{\left(\sqrt{bd}b^2de-\sqrt{bd}b^2cf\right)\arctan\left(\frac{2b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2f}{2\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(fg-eh)b} - \frac{\left(\sqrt{bd}b^2dg-\sqrt{bd}b^2ch\right)\arctan\left(\frac{2b^2dg-b^2ch-\cdots}{2\sqrt{\cdots}}\right.}{\sqrt{-b^2d^2g^2+b^2cdgh+ab\cdots}}\right.}{b^2}$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
2*((sqrt(b*d)*b^2*d*e - sqrt(b*d)*b^2*c*f)*arctan(1/2*(2*b^2*d*e - b^2*c*f
 - a*b*d*f + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*f)/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/
(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*(f*g - e*h)*
b) - (sqrt(b*d)*b^2*d*g - sqrt(b*d)*b^2*c*h)*arctan(1/2*(2*b^2*d*g - b^2*c
*h - a*b*d*h + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*b)
)/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*(f*g - e*h
)*b))*abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Hanged}$$

input
```
int((c + d*x)^(1/2)/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 1056, normalized size of antiderivative = 6.56

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)/(h*x+g),x)
```

output

```
( - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f
 - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(
a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*a*h + sqrt(c*f - d*e)*sqrt(a*f -
 b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*
f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqr
t(c + d*x))*b*g - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(
b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sq
rt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*a*h + sqrt(c*f - d*e)
*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e
) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqr
t(b)*sqrt(c + d*x))*b*g + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(2*sqrt(d)*sq
rt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + 2*sqrt(d)*sqrt(b)*sqrt(c + d*x)*sq
rt(a + b*x)*f + 2*b*d*e + 2*b*d*f*x)*a*h - sqrt(c*f - d*e)*sqrt(a*f - b*e)
*log(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + 2*sqrt(d)*sqrt(b)
*sqrt(c + d*x)*sqrt(a + b*x)*f + 2*b*d*e + 2*b*d*f*x)*b*g + sqrt(c*h - d*g
)*sqrt(a*h - b*g)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(a*h -
 b*g) + a*d*h + b*c*h - 2*b*d*g) + sqrt(h)*sqrt(d)*sqrt(a + b*x) + sqrt(h)
*sqrt(b)*sqrt(c + d*x))*a*f - sqrt(c*h - d*g)*sqrt(a*h - b*g)*log( - sqrt(
2*sqrt(d)*sqrt(b)*sqrt(c*h - d*g)*sqrt(a*h - b*g) + a*d*h + b*c*h - 2*b*d*
g) + sqrt(h)*sqrt(d)*sqrt(a + b*x) + sqrt(h)*sqrt(b)*sqrt(c + d*x))*b*e...
```

3.25 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 163

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \frac{2f\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{\sqrt{be-af}\sqrt{de-cf}(fg-eh)} - \frac{2h\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{\sqrt{bg-ah}\sqrt{dg-ch}(fg-eh)}$$

output
```
2*f*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))
/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-e*h+f*g)-2*h*arctanh((-c*h+d*g)^(1/2)
*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(1/2)/(-c*h+d*g)
^(1/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \frac{-\frac{2f\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{\sqrt{be-af}\sqrt{-de+cf}} + \frac{2h\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{\sqrt{bg-ah}\sqrt{-dg+ch}}}{fg-eh}$$

input
```
Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
((-2*f*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e) + c*f]*Sqrt[a +
 b*x])])/(Sqrt[b*e - a*f]*Sqrt[-(d*e) + c*f]) + (2*h*ArcTan[(Sqrt[b*g - a*
h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])])/(Sqrt[b*g - a*h]*Sq
rt[-(d*g) + c*h]))/(f*g - e*h)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{f}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(fg-eh)}-\frac{h}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2f\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}\sqrt{de-cf}(fg-eh)}-\frac{2h\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}\sqrt{dg-ch}(fg-eh)}$$

input
```
Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(2*f*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c + d*x
])])/(Sqrt[b*e - a*f]*Sqrt[d*e - c*f]*(f*g - e*h)) - (2*h*ArcTanh[(Sqrt[d*
g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(Sqrt[b*g - a*h]
*Sqrt[d*g - c*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(139) = 278$.

Time = 0.73 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.77

method	result
default	$\frac{\left(-\ln\left(\frac{adhx+bchx-2bdgx+2\sqrt{\frac{(ch-dg)(ah-bg)}{h^2}}\sqrt{(bx+a)(xd+c)}\,h+2ach-dga-bcg}{hx+g}\right)\sqrt{\frac{(cf-de)(af-be)}{f^2}}+\ln\left(\frac{adfx+bcfx-2bdex+2\sqrt{(bx+c}{}\right.\right.}{\sqrt{(bx+a)(xd+c)}\,(eh-fg)\sqrt{\frac{(ch-dg)(ah-bg)}{h^2}}\sqrt{\frac{(cf-de)(a}{f^2}}}$

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(-ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)
*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*((c*f-d*e)*(a*f-b*e)/f^2)^
(1/2)+ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(
a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*((c*h-d*g)*(a*h-b*g)/h
^2)^(1/2))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(e*h-f*g)/(
(c*h-d*g)*(a*h-b*g)/h^2)^(1/2)/((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}(fx+e)(hx+g)}\,dx$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(139) = 278$.

Time = 0.20 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx =$$

$$-2\sqrt{bd}b^4d\left(\frac{f\arctan\left(\frac{2\,b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2f}{2\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(b^2dfg|b|-b^2deh|b|)b}-\frac{h\arctan\left(\frac{2\,b^2dg-b^2ch-abdh}{2\sqrt{-b^2}}\right.}{\sqrt{-b^2d^2g^2+b^2cdgh+abd}}\right.$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac
")
```

output
```
-2*sqrt(b*d)*b^4*d*(f*arctan(1/2*(2*b^2*d*e - b^2*c*f - a*b*d*f + (sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*f)/(sqrt(-b^2*d^
2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/(sqrt(-b^2*d^2*e^2 +
b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*(b^2*d*f*g*abs(b) - b^2*d*e*h*abs
(b))*b) - h*arctan(1/2*(2*b^2*d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^
2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*
h + a*b*d^2*g*h - a*b*c*d*h^2)*(b^2*d*f*g*abs(b) - b^2*d*e*h*abs(b))*b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Hanged}$$

input
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 2355, normalized size of antiderivative = 14.45

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
( - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f
 - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(
a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*a*c*f*h**2 + sqrt(c*f - d*e)*sqr
t(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e)
 + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt
(b)*sqrt(c + d*x))*a*d*f*g*h + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt
(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d
*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b*c*f
*g*h - sqrt(c*f - d*e)*sqrt(a*f - b*e)*log( - sqrt(2*sqrt(d)*sqrt(b)*sqrt(
c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sq
rt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b*d*f*g**2 - sqrt(c*f - d*e)*
sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e)
 + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt
(b)*sqrt(c + d*x))*a*c*f*h**2 + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2
*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e
) + sqrt(f)*sqrt(d)*sqrt(a + b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*a*d*f*g
*h + sqrt(c*f - d*e)*sqrt(a*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f -
 d*e)*sqrt(a*f - b*e) + a*d*f + b*c*f - 2*b*d*e) + sqrt(f)*sqrt(d)*sqrt(a
+ b*x) + sqrt(f)*sqrt(b)*sqrt(c + d*x))*b*c*f*g*h - sqrt(c*f - d*e)*sqrt(a
*f - b*e)*log(sqrt(2*sqrt(d)*sqrt(b)*sqrt(c*f - d*e)*sqrt(a*f - b*e) + ...
```

3.26 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 220

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2d^2\sqrt{a+bx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2f^2\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{\sqrt{be-af}(de-cf)^{3/2}(fg-eh)} + \frac{2h^2\text{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{\sqrt{bg-ah}(dg-ch)^{3/2}(fg-eh)}$$

output
```
2*d^2*(b*x+a)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^(1/2)-2*f^2*a
rctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(-a*
f+b*e)^(1/2)/(-c*f+d*e)^(3/2)/(-e*h+f*g)+2*h^2*arctanh((-c*h+d*g)^(1/2)*(b
*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(1/2)/(-c*h+d*g)^(3
/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = 2\left(\frac{d^2\sqrt{a+bx}}{(bc-ad)(-de+cf)(-dg+ch)\sqrt{c+dx}} + \frac{f^2\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{\sqrt{-be+af}(de-cf)^{3/2}(fg-eh)} - \frac{h^2\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{\sqrt{-bg+ah}(dg-ch)^{3/2}(fg-eh)}\right)$$

input
```
Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
2*((d^2*Sqrt[a + b*x])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)*Sqrt[c +
 d*x]) + (f^2*ArcTan[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e) + a*f]*S
qrt[c + d*x])])/(Sqrt[-(b*e) + a*f]*(d*e - c*f)^(3/2)*(f*g - e*h)) - (h^2*
ArcTan[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c + d*x])]
)/(Sqrt[-(b*g) + a*h]*(d*g - c*h)^(3/2)*(f*g - e*h)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{f}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(fg-eh)} - \frac{h}{\sqrt{a+bx}(c+dx)^{3/2}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2f^2\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}(de-cf)^{3/2}(fg-eh)}+\frac{2h^2\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}(dg-ch)^{3/2}(fg-eh)}+\frac{2df\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(de-cf)(fg-eh)}-\frac{2dh\sqrt{a+bx}}{\sqrt{c+dx}(bc-ad)(dg-ch)(fg-eh)}$$

input
```
Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*d*f*Sqrt[a + b*x])/((b*c - a*d)*(d*e - c*f)*(f*g - e*h)*Sqrt[c + d*x])
- (2*d*h*Sqrt[a + b*x])/((b*c - a*d)*(d*g - c*h)*(f*g - e*h)*Sqrt[c + d*x]
) - (2*f^2*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])])/(Sqrt[b*e - a*f]*(d*e - c*f)^(3/2)*(f*g - e*h)) + (2*h^2*ArcTan
h[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(Sqrt[
b*g - a*h]*(d*g - c*h)^(3/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(192) = 384$.

Time = 0.73 (sec) , antiderivative size = 1998, normalized size of antiderivative = 9.08

method	result	size
default	Expression too large to display	1998

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)^(1/2)*(ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c
*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*c*d^2*f*h*x
*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*
(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x
+e))*a*d^3*f*g*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d
*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a
*d*e-b*c*e)/(f*x+e))*b*c^2*d*f*h*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+ln((a*d
*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)
^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b*c*d^2*f*g*x*((c*h-d*g)*(a*h-b*g)/
h^2)^(1/2)-ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)
*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*c*d^2*f*h*x*((c
*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a
*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))
*a*d^3*e*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x
+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a
-b*c*g)/(h*x+g))*b*c^2*d*f*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)-ln((a*d*h*x
+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/
2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*b*c*d^2*e*h*x*((c*f-d*e)*(a*f-b*e)/f^2)
^(1/2)+ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*
(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*c^2*d*f*h*((c*h-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x + a)*(d*x + c)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(192) = 384.

Time = 115.59 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2\sqrt{bx+ \cdots}}{(bcd^2eg|b| - ad^3eg|b| - bc^2dfg|b| + acd^2fg|b| - bc^2deh|b| + \cdots}$$
$$+ \frac{2\sqrt{bd}bf^2 \arctan\left(\frac{2b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2 f}{2\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(defg|b| - cf^2g|b| - de^2h|b| + cefh|b|)}$$
$$- \frac{2\sqrt{bd}bh^2 \arctan\left(\frac{2b^2dg-b^2ch-abdh+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2 h}{2\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}b}\right)}{\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}(dfg^2|b| - degh|b| - cfgh|b| + ceh^2|b|)}$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac
")
```

output
```
2*sqrt(b*x + a)*b^2*d^2/((b*c*d^2*e*g*abs(b) - a*d^3*e*g*abs(b) - b*c^2*d*
f*g*abs(b) + a*c*d^2*f*g*abs(b) - b*c^2*d*e*h*abs(b) + a*c*d^2*e*h*abs(b)
+ b*c^3*f*h*abs(b) - a*c^2*d*f*h*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)) + 2*sqrt(b*d)*b*f^2*arctan(1/2*(2*b^2*d*e - b^2*c*f - a*b*d*f + (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*f)/(sqrt(-b^2*
d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/(sqrt(-b^2*d^2*e^2
+ b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*(d*e*f*g*abs(b) - c*f^2*g*abs(b
) - d*e^2*h*abs(b) + c*e*f*h*abs(b))) - 2*sqrt(b*d)*b*h^2*arctan(1/2*(2*b^
2*d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*
b*c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^
2)*(d*f*g^2*abs(b) - d*e*g*h*abs(b) - c*f*g*h*abs(b) + c*e*h^2*abs(b)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{1}{(e+f\,x)\,\left(g+h\,x\right)\,\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.27 $\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 324

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx &= \frac{2d^2\sqrt{a+bx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} \\ &+ \frac{2d^2(3ad(dfg+deh-2cfh)+b(2d^2eg+8c^2fh-5cd(fg+eh)))\sqrt{a+bx}}{3(bc-ad)^2(de-cf)^2(dg-ch)^2\sqrt{c+dx}} \\ &+ \frac{2f^3\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{\sqrt{be-af}(de-cf)^{5/2}(fg-eh)} - \frac{2h^3\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{\sqrt{bg-ah}(dg-ch)^{5/2}(fg-eh)}\end{aligned}$$

output

```
2/3*d^2*(b*x+a)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^(3/2)+2/3*d
^2*(3*a*d*(-2*c*f*h+d*e*h+d*f*g)+b*(2*d^2*e*g+8*c^2*f*h-5*c*d*(e*h+f*g)))*
(b*x+a)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)+2*f^3*a
rctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(-a*
f+b*e)^(1/2)/(-c*f+d*e)^(5/2)/(-e*h+f*g)-2*h^3*arctanh((-c*h+d*g)^(1/2)*(b
*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(1/2)/(-c*h+d*g)^(5
/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \frac{2d^2\sqrt{a+bx}(b(9c^3fh+2d^3egx+cd^2(3eg-5fgx-5ehx)-}{3(bc}$$
$$+\frac{2f^3\arctan\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{\sqrt{-be+af}(de-cf)^{5/2}(-fg+eh)}+\frac{2h^3\arctan\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{-bg+ah}\sqrt{c+dx}}\right)}{\sqrt{-bg+ah}(dg-ch)^{5/2}(fg-eh)}$$

input
```
Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*d^2*Sqrt[a + b*x]*(b*(9*c^3*f*h + 2*d^3*e*g*x + c*d^2*(3*e*g - 5*f*g*x
- 5*e*h*x) - 2*c^2*d*(3*f*g + 3*e*h - 4*f*h*x)) + a*d*(-7*c^2*f*h + d^2*(-
(e*g) + 3*f*g*x + 3*e*h*x) + 2*c*d*(2*f*g + 2*e*h - 3*f*h*x))))/(3*(b*c -
a*d)^2*(d*e - c*f)^2*(d*g - c*h)^2*(c + d*x)^(3/2)) + (2*f^3*ArcTan[(Sqrt[
d*e - c*f]*Sqrt[a + b*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/(Sqrt[-(b*e
) + a*f]*(d*e - c*f)^(5/2)*(-(f*g) + e*h)) + (2*h^3*ArcTan[(Sqrt[d*g - c*h
]*Sqrt[a + b*x])/(Sqrt[-(b*g) + a*h]*Sqrt[c + d*x])])/(Sqrt[-(b*g) + a*h]*
(d*g - c*h)^(5/2)*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(\frac{f}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(fg-eh)}-\frac{h}{\sqrt{a+bx}(c+dx)^{5/2}(g+hx)(fg-eh)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2f^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{\sqrt{be-af}(de-cf)^{5/2}(fg-eh)}-\frac{2h^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{\sqrt{bg-ah}(dg-ch)^{5/2}(fg-eh)}+\frac{2df\sqrt{a+bx}(3adf-5bcf+2bde)}{3\sqrt{c+dx}(bc-ad)^2(de-cf)^2(fg-eh)}+\frac{2df\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(de-cf)(fg-eh)}-\frac{2dh\sqrt{a+bx}(3adh-5bch+2bdg)}{3\sqrt{c+dx}(bc-ad)^2(dg-ch)^2(fg-eh)}-\frac{2dh\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)(dg-ch)(fg-eh)}$$

input
```
Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*d*f*Sqrt[a + b*x])/(3*(b*c - a*d)*(d*e - c*f)*(f*g - e*h)*(c + d*x)^(3/
2)) - (2*d*h*Sqrt[a + b*x])/(3*(b*c - a*d)*(d*g - c*h)*(f*g - e*h)*(c + d*
x)^(3/2)) + (2*d*f*(2*b*d*e - 5*b*c*f + 3*a*d*f)*Sqrt[a + b*x])/(3*(b*c -
a*d)^2*(d*e - c*f)^2*(f*g - e*h)*Sqrt[c + d*x]) - (2*d*h*(2*b*d*g - 5*b*c*
h + 3*a*d*h)*Sqrt[a + b*x])/(3*(b*c - a*d)^2*(d*g - c*h)^2*(f*g - e*h)*Sqr
t[c + d*x]) + (2*f^3*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a
*f]*Sqrt[c + d*x])]])/(Sqrt[b*e - a*f]*(d*e - c*f)^(5/2)*(f*g - e*h)) - (2*
h^3*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x]
)]])/(Sqrt[b*g - a*h]*(d*g - c*h)^(5/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8032 vs. $2(288) = 576$.

Time = 0.90 (sec) , antiderivative size = 8033, normalized size of antiderivative = 24.79

method	result	size
default	Expression too large to display	8033

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{bx+a}(dx+c)^{\frac{5}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5109 vs. $2(288) = 576$.

Time = 116.24 (sec) , antiderivative size = 5109, normalized size of antiderivative = 15.77

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x, algorithm="giac
")
```

output
```
-2*sqrt(b*d)*b*f^3*arctan(1/2*(2*b^2*d*e - b^2*c*f - a*b*d*f + (sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*f)/(sqrt(-b^2*d^2*e
^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/((d^2*e^2*f*g*abs(b) - 2
*c*d*e*f^2*g*abs(b) + c^2*f^3*g*abs(b) - d^2*e^3*h*abs(b) + 2*c*d*e^2*f*h*
abs(b) - c^2*e*f^2*h*abs(b))*sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f
 - a*b*c*d*f^2)) + 2*sqrt(b*d)*b*h^3*arctan(1/2*(2*b^2*d*g - b^2*c*h - a*b
*d*h + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*h
)/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*b))/((d^2*
f*g^3*abs(b) - d^2*e*g^2*h*abs(b) - 2*c*d*f*g^2*h*abs(b) + 2*c*d*e*g*h^2*a
bs(b) + c^2*f*g*h^2*abs(b) - c^2*e*h^3*abs(b))*sqrt(-b^2*d^2*g^2 + b^2*c*d
*g*h + a*b*d^2*g*h - a*b*c*d*h^2)) + 2/3*sqrt(b*x + a)*((2*b^4*d^10*e^3*g^
3*abs(b) - 9*b^4*c*d^9*e^2*f*g^3*abs(b) + 3*a*b^3*d^10*e^2*f*g^3*abs(b) +
12*b^4*c^2*d^8*e*f^2*g^3*abs(b) - 6*a*b^3*c*d^9*e*f^2*g^3*abs(b) - 5*b^4*c
^3*d^7*f^3*g^3*abs(b) + 3*a*b^3*c^2*d^8*f^3*g^3*abs(b) - 9*b^4*c*d^9*e^3*g
^2*h*abs(b) + 3*a*b^3*d^10*e^3*g^2*h*abs(b) + 36*b^4*c^2*d^8*e^2*f*g^2*h*a
bs(b) - 18*a*b^3*c*d^9*e^2*f*g^2*h*abs(b) - 45*b^4*c^3*d^7*e*f^2*g^2*h*abs
(b) + 27*a*b^3*c^2*d^8*e*f^2*g^2*h*abs(b) + 18*b^4*c^4*d^6*f^3*g^2*h*abs(b
) - 12*a*b^3*c^3*d^7*f^3*g^2*h*abs(b) + 12*b^4*c^2*d^8*e^3*g*h^2*abs(b) -
6*a*b^3*c*d^9*e^3*g*h^2*abs(b) - 45*b^4*c^3*d^7*e^2*f*g*h^2*abs(b) + 27*a*
b^3*c^2*d^8*e^2*f*g*h^2*abs(b) + 54*b^4*c^4*d^6*e*f^2*g*h^2*abs(b) - 36...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx=\int \frac{1}{(e+f\,x)\,(g+h\,x)\,\sqrt{a+b\,x}\,{(c+d\,x)}^{5/2}}\,dx$$

input
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x)
```

output
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2)/(f*x+e)/(h*x+g),x)
```

3.28 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 268

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2(bc-ad)^2\sqrt{c+dx}}{b(be-af)(bg-ah)\sqrt{a+bx}} + \frac{2d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}fh} + \frac{2(de-cf)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{f(be-af)^{3/2}(fg-eh)} - \frac{2(dg-ch)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{h(bg-ah)^{3/2}(fg-eh)}$$

output
```
-2*(-a*d+b*c)^2*(d*x+c)^(1/2)/b/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*d^(5
/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)/f/h+2*(-c
*f+d*e)^(5/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x
+c)^(1/2))/f/(-a*f+b*e)^(3/2)/(-e*h+f*g)-2*(-c*h+d*g)^(5/2)*arctanh((-c*h+
d*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/h/(-a*h+b*g)^(3/2
)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = 2\left(-\frac{(bc-ad)^2\sqrt{c+dx}}{b(be-af)(bg-ah)\sqrt{a+bx}} - \frac{(-de+cf)^{5/2}\arctan\left(\frac{\sqrt{be-af}\sqrt{c}}{\sqrt{-de+cf}\sqrt{}}\right.}{f(be-af)^{3/2}(-fg+eh)}\right.$$

input
```
Integrate[(c + d*x)^(5/2)/((a + b*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
2*(-(((b*c - a*d)^2*Sqrt[c + d*x])/(b*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x
])) - ((-(d*e) + c*f)^(5/2)*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-
(d*e) + c*f]*Sqrt[a + b*x])])/(f*(b*e - a*f)^(3/2)*(-(f*g) + e*h)) - ((-(d
*g) + c*h)^(5/2)*ArcTan[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h
]*Sqrt[a + b*x])])/(h*(b*g - a*h)^(3/2)*(f*g - e*h)) + (d^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(b^(3/2)*f*h))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \ 198$$

$$\int \left(\frac{f(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(fg-eh)} - \frac{h(c+dx)^{5/2}}{(a+bx)^{3/2}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(3adf-5bcf+2bde)}{b^{5/2}f(fg-eh)}+\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(3adh-5bch+2bdg)}{b^{5/2}h(fg-eh)}+\frac{2(de-cf)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{f(be-af)^{3/2}(fg-eh)}-\frac{2(dg-ch)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{h(bg-ah)^{3/2}(fg-eh)}+\frac{d\sqrt{a+bx}\sqrt{c+dx}(-3adf+2bcf+bde)}{b^2(be-af)(fg-eh)}-\frac{d\sqrt{a+bx}\sqrt{c+dx}(-3adh+2bch+bdg)}{b^2(bg-ah)(fg-eh)}-\frac{2f(c+dx)^{3/2}(bc-ad)}{b\sqrt{a+bx}(be-af)(fg-eh)}+\frac{2h(c+dx)^{3/2}(bc-ad)}{b\sqrt{a+bx}(bg-ah)(fg-eh)}$$

input
```
Int[(c + d*x)^(5/2)/((a + b*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(d*(b*d*e + 2*b*c*f - 3*a*d*f)*Sqrt[a + b*x]*Sqrt[c + d*x])/(b^2*(b*e - a*
f)*(f*g - e*h)) - (d*(b*d*g + 2*b*c*h - 3*a*d*h)*Sqrt[a + b*x]*Sqrt[c + d*
x])/(b^2*(b*g - a*h)*(f*g - e*h)) - (2*(b*c - a*d)*f*(c + d*x)^(3/2))/(b*(
b*e - a*f)*(f*g - e*h)*Sqrt[a + b*x]) + (2*(b*c - a*d)*h*(c + d*x)^(3/2))/
(b*(b*g - a*h)*(f*g - e*h)*Sqrt[a + b*x]) - (d^(3/2)*(2*b*d*e - 5*b*c*f +
3*a*d*f)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(5/2
)*f*(f*g - e*h)) + (d^(3/2)*(2*b*d*g - 5*b*c*h + 3*a*d*h)*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(5/2)*h*(f*g - e*h)) + (2*(d*
e - c*f)^(5/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sq
rt[c + d*x])])/(f*(b*e - a*f)^(3/2)*(f*g - e*h)) - (2*(d*g - c*h)^(5/2)*Ar
cTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(h
*(b*g - a*h)^(3/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5695 vs. 2(228) = 456.

Time = 0.63 (sec) , antiderivative size = 5696, normalized size of antiderivative = 21.25

method	result	size
default	Expression too large to display	5696

input
```
int((d*x+c)^(5/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(5/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)/(b*x+a)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((d*x+c)^(5/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((d*x + c)^(5/2)/((b*x + a)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(5/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{(c+d\,x)^{5/2}}{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^{3/2}}\,dx$$

input
```
int((c + d*x)^(5/2)/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)),x)
```

output
```
int((c + d*x)^(5/2)/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{3}{2}}(fx+e)(hx+g)}dx$$

input
```
int((d*x+c)^(5/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((d*x+c)^(5/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.29 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2(bc-ad)\sqrt{c+dx}}{(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2(de-cf)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(be-af)^{3/2}(fg-eh)} + \frac{2(dg-ch)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(bg-ah)^{3/2}(fg-eh)}$$

output
```
-2*(-a*d+b*c)*(d*x+c)^(1/2)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)-2*(-c*f+d*
e)^(3/2)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(
1/2))/(-a*f+b*e)^(3/2)/(-e*h+f*g)+2*(-c*h+d*g)^(3/2)*arctanh((-c*h+d*g)^(1
/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(3/2)/(-e*h+f
*g)
```

Mathematica [A] (verified)

Time = 10.72 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2\left(\frac{(bc-ad)(fg-eh)\sqrt{c+dx}}{(be-af)(bg-ah)\sqrt{a+bx}} + \frac{(de-cf)^{3/2}\text{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(be-af)^{3/2}} - \frac{(dg-ch)^{3/2}\text{arct}}{(bg}\right.}{-fg+eh}$$

input
```
Integrate[(c + d*x)^(3/2)/((a + b*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(2*(((b*c - a*d)*(f*g - e*h)*Sqrt[c + d*x])/((b*e - a*f)*(b*g - a*h)*Sqrt[
a + b*x]) + ((d*e - c*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sq
rt[b*e - a*f]*Sqrt[c + d*x])])/(b*e - a*f)^(3/2) - ((d*g - c*h)^(3/2)*ArcT
anh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/(b*g
 - a*h)^(3/2)))/(-(f*g) + e*h)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{ 198}$$

$$\int \left(\frac{f(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(fg-eh)} - \frac{h(c+dx)^{3/2}}{(a+bx)^{3/2}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(de-cf)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{(be-af)^{3/2}(fg-eh)} + \frac{2(dg-ch)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{(bg-ah)^{3/2}(fg-eh)} - \frac{2f\sqrt{c+dx}(bc-ad)}{b\sqrt{a+bx}(be-af)(fg-eh)} + \frac{2h\sqrt{c+dx}(bc-ad)}{b\sqrt{a+bx}(bg-ah)(fg-eh)}$$

input
```
Int[(c + d*x)^(3/2)/((a + b*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*(b*c - a*d)*f*Sqrt[c + d*x])/(b*(b*e - a*f)*(f*g - e*h)*Sqrt[a + b*x])
 + (2*(b*c - a*d)*h*Sqrt[c + d*x])/(b*(b*g - a*h)*(f*g - e*h)*Sqrt[a + b*x
]) - (2*(d*e - c*f)^(3/2)*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*
e - a*f]*Sqrt[c + d*x])])/((b*e - a*f)^(3/2)*(f*g - e*h)) + (2*(d*g - c*h)
^(3/2)*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d
*x])])/((b*g - a*h)^(3/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3097 vs. $2(181) = 362$.

Time = 0.68 (sec) , antiderivative size = 3098, normalized size of antiderivative = 14.82

method	result	size
default	Expression too large to display	3098

input
```
int((d*x+c)^(3/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(d*x+c)^(1/2)*(ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c
*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a^2*d^2*e^2*h
^2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h
-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/
(h*x+g))*a*b*c*d*f^2*g*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+2*ln((a*d*f*x+b
*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)
*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*c*d*e*f*g*h*((c*h-d*g)*(a*h-b*g)/h^2)
^(1/2)-2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*b*c*d*e*f*g*h*((c
*f-d*e)*(a*f-b*e)/f^2)^(1/2)-2*ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d
*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e
))*a*b*c*d*e*f*h^2*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-2*ln((a*d*f*x+b*c*f*x
-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a
*c*f-a*d*e-b*c*e)/(f*x+e))*a^2*c*d*e*f*h^2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)
-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b
*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*c^2*f^2*g*h*((c*h-d*g)*
(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/
2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*d^2
*e^2*g*h*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+2*ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2
*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{3}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(b*x+a)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((c + d*x)**(3/2)/((a + b*x)**(3/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate((d*x + c)^(3/2)/((b*x + a)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(181) = 362$.

Time = 30.74 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.73

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{2\left(\sqrt{bd}d^2e^2|b| - 2\sqrt{bd}cdef|b| + \sqrt{bd}c^2f^2|b|\right)\arctan\left(\frac{2b^2de-b^2cf-abdf-}{2\sqrt{-b^2c}}\right.}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}(befg-af^2g}$$

$$-\frac{2\left(\sqrt{bd}d^2g^2|b| - 2\sqrt{bd}cdgh|b| + \sqrt{bd}c^2h^2|b|\right)\arctan\left(\frac{2b^2dg-b^2ch-abdh+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2h}{2\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}b}\right)}{\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh-abcdh^2}(bfg^2-begh-afgh+aeh^2)b}$$

$$-\frac{4\left(\sqrt{bd}b^2c^2|b| - 2\sqrt{bd}abcd|b| + \sqrt{bd}a^2d^2|b|\right)}{(b^3eg-ab^2fg-ab^2eh+a^2bfh)\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
2*(sqrt(b*d)*d^2*e^2*abs(b) - 2*sqrt(b*d)*c*d*e*f*abs(b) + sqrt(b*d)*c^2*f
^2*abs(b))*arctan(1/2*(2*b^2*d*e - b^2*c*f - a*b*d*f + (sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*f)/(sqrt(-b^2*d^2*e^2 + b^2
*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*b))/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f
 + a*b*d^2*e*f - a*b*c*d*f^2)*(b*e*f*g - a*f^2*g - b*e^2*h + a*e*f*h)*b) -
 2*(sqrt(b*d)*d^2*g^2*abs(b) - 2*sqrt(b*d)*c*d*g*h*abs(b) + sqrt(b*d)*c^2*
h^2*abs(b))*arctan(1/2*(2*b^2*d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^
2*c*d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*
h + a*b*d^2*g*h - a*b*c*d*h^2)*(b*f*g^2 - b*e*g*h - a*f*g*h + a*e*h^2)*b)
- 4*(sqrt(b*d)*b^2*c^2*abs(b) - 2*sqrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2
*d^2*abs(b))/((b^3*e*g - a*b^2*f*g - a*b^2*e*h + a^2*b*f*h)*(b^2*c - a*b*d
 - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(c+d\,x)^{3/2}}{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^{3/2}}\,dx$$

input
```
int((c + d*x)^(3/2)/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)),x)
```

output
```
int((c + d*x)^(3/2)/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int((d*x+c)^(3/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((d*x+c)^(3/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.30 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 204

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2b\sqrt{c+dx}}{(be-af)(bg-ah)\sqrt{a+bx}} + \frac{2f\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(be-af)^{3/2}(fg-eh)} - \frac{2h\sqrt{dg-ch}\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(bg-ah)^{3/2}(fg-eh)}$$

output
```
-2*b*(d*x+c)^(1/2)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*f*(-c*f+d*e)^(1/2
)*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(
-a*f+b*e)^(3/2)/(-e*h+f*g)-2*h*(-c*h+d*g)^(1/2)*arctanh((-c*h+d*g)^(1/2)*(
b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(3/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2b\sqrt{c+dx}}{(be-af)(bg-ah)\sqrt{a+bx}} + \frac{2f\sqrt{-de+cf}\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{(be-af)^{3/2}(fg-eh)} - \frac{2h\sqrt{-dg+ch}\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{(bg-ah)^{3/2}(fg-eh)}$$

input
```
Integrate[Sqrt[c + d*x]/((a + b*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*b*Sqrt[c + d*x])/((b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + (2*f*Sqrt[-
(d*e) + c*f]*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e) + c*f]*Sq
rt[a + b*x])])/((b*e - a*f)^(3/2)*(f*g - e*h)) - (2*h*Sqrt[-(d*g) + c*h]*A
rcTan[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])])
/((b*g - a*h)^(3/2)*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {196, 107, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}dx$$

$$\Big\downarrow \text{196}$$

$$\frac{(dg-ch)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(g+hx)}dx}{fg-eh}-\frac{(de-cf)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)}dx}{fg-eh}$$

$$\Big\downarrow \text{107}$$

$$\frac{(dg-ch)\left(-\frac{h\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(g+hx)}dx}{bg-ah}-\frac{2b\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(bg-ah)}\right)}{fg-eh}-\frac{(de-cf)\left(-\frac{f\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)}dx}{be-af}-\frac{2b\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(be-af)}\right)}{fg-eh}$$

$$\Big\downarrow \text{104}$$

$$\frac{(dg-ch)\left(-\frac{2h\int \frac{1}{bg-ah-\frac{(dg-ch)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{bg-ah}-\frac{2b\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(bg-ah)}\right)}{fg-eh}-\frac{(de-cf)\left(-\frac{2f\int \frac{1}{be-af-\frac{(de-cf)(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{be-af}-\frac{2b\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(be-af)}\right)}{fg-eh}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{(dg-ch)\left(-\frac{2h\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{(bg-ah)^{3/2}\sqrt{dg-ch}}-\frac{2b\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(bg-ah)}\right)}{fg-eh}-\frac{(de-cf)\left(-\frac{2f\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{(be-af)^{3/2}\sqrt{de-cf}}-\frac{2b\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(be-af)}\right)}{fg-eh}$$

input
```
Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
-(((d*e - c*f)*((-2*b*Sqrt[c + d*x])/((b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x
]) - (2*f*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[c
+ d*x])])/((b*e - a*f)^(3/2)*Sqrt[d*e - c*f])))/(f*g - e*h)) + ((d*g - c*h
)*((-2*b*Sqrt[c + d*x])/((b*c - a*d)*(b*g - a*h)*Sqrt[a + b*x]) - (2*h*Arc
Tanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/((b
*g - a*h)^(3/2)*Sqrt[d*g - c*h])))/(f*g - e*h)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 196
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e +
f*x)^(p - 1)*((g + h*x)^q/(a + b*x)), x], x] - Simp[(d*e - c*f)/(b*c - a*d)
   Int[(e + f*x)^(p - 1)*((g + h*x)^q/(c + d*x)), x], x] /; FreeQ[{a, b, c,
 d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(176) = 352$.

Time = 0.66 (sec) , antiderivative size = 1964, normalized size of antiderivative = 9.63

method	result	size
default	Expression too large to display	1964

input
```
int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(-ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)
*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*b*c*f*h*x*((c*f-d*e)*(a*
f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2
)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a*b*d*f*g*
x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*
g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*
x+g))*b^2*c*e*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)-ln((a*d*h*x+b*c*h*x-2*b*
d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-
d*g*a-b*c*g)/(h*x+g))*b^2*d*e*g*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*
f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^
(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*c*f*h*x*((c*h-d*g)*(a*h-b*g)/h^2
)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)
*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a*b*d*e*h*x*((c*h-d*
g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^
(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b^2*
c*f*g*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((
b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*
e)/(f*x+e))*b^2*d*e*g*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*h*x+b*c*h*
x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*
a*c*h-d*g*a-b*c*g)/(h*x+g))*a^2*c*f*h*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+l...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(sqrt(c + d*x)/((a + b*x)**(3/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxima
")
```

output
```
integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(176) = 352.

Time = 32.41 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx =$$

$$-\frac{2\left(\frac{\left(\sqrt{bd}b^2def-\sqrt{bd}b^2cf^2\right)\arctan\left(\frac{2\,b^2de-b^2cf-abdf+\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2f}{2\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}\,b}\right)}{\sqrt{-b^2d^2e^2+b^2cdef+abd^2ef-abcdf^2}\,(befg-af^2g-be^2h+aefh)b} - \frac{\left(\sqrt{bd}b^2dgh-\sqrt{bd}b^2ch^2\right)\arctan\left(\frac{2\,b^2dg-b\cdots}{\cdots}\right.}{\sqrt{-b^2d^2g^2+b^2cdgh+abd^2gh\cdots}}\right.}{b^2\cdots}$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
-2*((sqrt(b*d)*b^2*d*e*f - sqrt(b*d)*b^2*c*f^2)*arctan(1/2*(2*b^2*d*e - b^
2*c*f - a*b*d*f + (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*f)/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)
*b))/(sqrt(-b^2*d^2*e^2 + b^2*c*d*e*f + a*b*d^2*e*f - a*b*c*d*f^2)*(b*e*f*
g - a*f^2*g - b*e^2*h + a*e*f*h)*b) - (sqrt(b*d)*b^2*d*g*h - sqrt(b*d)*b^2
*c*h^2)*arctan(1/2*(2*b^2*d*g - b^2*c*h - a*b*d*h + (sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*h)/(sqrt(-b^2*d^2*g^2 + b^2*c*
d*g*h + a*b*d^2*g*h - a*b*c*d*h^2)*b))/(sqrt(-b^2*d^2*g^2 + b^2*c*d*g*h +
a*b*d^2*g*h - a*b*c*d*h^2)*(b*f*g^2 - b*e*g*h - a*f*g*h + a*e*h^2)*b) + 2*
(sqrt(b*d)*b^3*c - sqrt(b*d)*a*b^2*d)/((b^2*e*g - a*b*f*g - a*b*e*h + a^2*
f*h)*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2)))*abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{\sqrt{c+d\,x}}{(e+f\,x)\,(g+h\,x)\,{(a+b\,x)}^{3/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)),x)
```

output
```
int((c + d*x)^(1/2)/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}(e+fx)(g+hx)}\,dx=\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.31 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 220

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = -\frac{2b^2\sqrt{c+dx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} - \frac{2f^2\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(be-af)^{3/2}\sqrt{de-cf}(fg-eh)} + \frac{2h^2\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(bg-ah)^{3/2}\sqrt{dg-ch}(fg-eh)}$$

output
```
-2*b^2*(d*x+c)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)-2*f^2*
arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2))/(-a
*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-e*h+f*g)+2*h^2*arctanh((-c*h+d*g)^(1/2)*(
b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(3/2)/(-c*h+d*g)^(
1/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = -\frac{2b^2\sqrt{c+dx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}} + \frac{2f^2\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{(be-af)^{3/2}\sqrt{-de+cf}(fg-eh)} - \frac{2h^2\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{(bg-ah)^{3/2}\sqrt{-dg+ch}(fg-eh)}$$

input
```
Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*b^2*Sqrt[c + d*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x])
 + (2*f^2*ArcTan[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e) + c*f]*Sqrt[
a + b*x])])/((b*e - a*f)^(3/2)*Sqrt[-(d*e) + c*f]*(f*g - e*h)) - (2*h^2*Ar
cTan[(Sqrt[b*g - a*h]*Sqrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])])/
((b*g - a*h)^(3/2)*Sqrt[-(d*g) + c*h]*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{f}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(fg-eh)} - \frac{h}{(a+bx)^{3/2}\sqrt{c+dx}(g+hx)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2f^2\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{(be-af)^{3/2}\sqrt{de-cf}(fg-eh)}+\frac{2h^2\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{(bg-ah)^{3/2}\sqrt{dg-ch}(fg-eh)}-\frac{2bf\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(be-af)(fg-eh)}+\frac{2bh\sqrt{c+dx}}{\sqrt{a+bx}(bc-ad)(bg-ah)(fg-eh)}$$

input
```
Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*b*f*Sqrt[c + d*x])/((b*c - a*d)*(b*e - a*f)*(f*g - e*h)*Sqrt[a + b*x])
 + (2*b*h*Sqrt[c + d*x])/((b*c - a*d)*(b*g - a*h)*(f*g - e*h)*Sqrt[a + b*x
]) - (2*f^2*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[
c + d*x])])/((b*e - a*f)^(3/2)*Sqrt[d*e - c*f]*(f*g - e*h)) + (2*h^2*ArcTa
nh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d*x])])/((b*g
 - a*h)^(3/2)*Sqrt[d*g - c*h]*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(192) = 384$.

Time = 0.74 (sec) , antiderivative size = 1998, normalized size of antiderivative = 9.08

method	result	size
default	Expression too large to display	1998

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(d*x+c)^(1/2)*(ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c
*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a^2*b*d*f*h*x
*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*
(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x
+e))*a*b^2*c*f*h*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)-ln((a*d*f*x+b*c*f*x-2*b
*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f
-a*d*e-b*c*e)/(f*x+e))*a*b^2*d*f*g*x*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)+ln((a
*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*(a*f-b*e)/f^
2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*b^3*c*f*g*x*((c*h-d*g)*(a*h-b*g)/
h^2)^(1/2)-ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)
*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*a^2*b*d*f*h*x*((c
*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a
*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))
*a*b^2*c*f*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)+ln((a*d*h*x+b*c*h*x-2*b*d*g
*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*h+2*a*c*h-d*g
*a-b*c*g)/(h*x+g))*a*b^2*d*e*h*x*((c*f-d*e)*(a*f-b*e)/f^2)^(1/2)-ln((a*d*h
*x+b*c*h*x-2*b*d*g*x+2*((c*h-d*g)*(a*h-b*g)/h^2)^(1/2)*((b*x+a)*(d*x+c))^(
1/2)*h+2*a*c*h-d*g*a-b*c*g)/(h*x+g))*b^3*c*e*h*x*((c*f-d*e)*(a*f-b*e)/f^2)
^(1/2)+ln((a*d*f*x+b*c*f*x-2*b*d*e*x+2*((b*x+a)*(d*x+c))^(1/2)*((c*f-d*e)*
(a*f-b*e)/f^2)^(1/2)*f+2*a*c*f-a*d*e-b*c*e)/(f*x+e))*a^3*d*f*h*((c*h-d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}(fx+e)(hx+g)}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{1}{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}(e+fx)(g+hx)}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,(fx+e)\,(hx+g)}dx$$

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

output
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)/(h*x+g),x)
```

3.32 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 344

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = -\frac{2b^2}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}\sqrt{c+dx}} - \frac{2d\left(a^2d^2fh-abd^2(fg+eh)+b^2(2d^2eg+c^2fh-cd(fg+eh))\right)\sqrt{a+bx}}{(bc-ad)^2(be-af)(de-cf)(bg-ah)(dg-ch)\sqrt{c+dx}} + \frac{2f^3\operatorname{arctanh}\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right)}{(be-af)^{3/2}(de-cf)^{3/2}(fg-eh)} - \frac{2h^3\operatorname{arctanh}\left(\frac{\sqrt{dg-ch}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{c+dx}}\right)}{(bg-ah)^{3/2}(dg-ch)^{3/2}(fg-eh)}$$

output
```
-2*b^2/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)/(d*x+c)^(1/2)-2*d*(a
^2*d^2*f*h-a*b*d^2*(e*h+f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g)))*(b*x+a
)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(-a*h+b*g)/(-c*h+d*g)/(d*x+c)^(
1/2)+2*f^3*arctanh((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)
^(1/2))/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(3/2)/(-e*h+f*g)-2*h^3*arctanh((-c*h+d
*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(d*x+c)^(1/2))/(-a*h+b*g)^(3/2)/(
-c*h+d*g)^(3/2)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 2.81 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \frac{-2a^3d^3fh+2ab^2d^3(-eg+fgx+ehx)+2a^2bd^3(eh+f(g}{(bc-ad)^2(be-af)(de}$$
$$+\frac{2f^3\arctan\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{-de+cf}\sqrt{a+bx}}\right)}{(be-af)^{3/2}(-de+cf)^{3/2}(fg-eh)}+\frac{2h^3\arctan\left(\frac{\sqrt{bg-ah}\sqrt{c+dx}}{\sqrt{-dg+ch}\sqrt{a+bx}}\right)}{(bg-ah)^{3/2}(-dg+ch)^{3/2}(-fg+eh)}$$

input
```
Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*a^3*d^3*f*h + 2*a*b^2*d^3*(-(e*g) + f*g*x + e*h*x) + 2*a^2*b*d^3*(e*h
+ f*(g - h*x)) - 2*b^3*(c^3*f*h + 2*d^3*e*g*x + c*d^2*(e*g - f*g*x - e*h*x
) - c^2*d*(f*g + e*h - f*h*x)))/((b*c - a*d)^2*(b*e - a*f)*(d*e - c*f)*(b*
g - a*h)*(d*g - c*h)*Sqrt[a + b*x]*Sqrt[c + d*x]) + (2*f^3*ArcTan[(Sqrt[b*
e - a*f]*Sqrt[c + d*x])/(Sqrt[-(d*e) + c*f]*Sqrt[a + b*x])])/((b*e - a*f)^
(3/2)*(-(d*e) + c*f)^(3/2)*(f*g - e*h)) + (2*h^3*ArcTan[(Sqrt[b*g - a*h]*S
qrt[c + d*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])])/((b*g - a*h)^(3/2)*(-(d
*g) + c*h)^(3/2)*(-(f*g) + e*h))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(\frac{f}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(fg-eh)}-\frac{h}{(a+bx)^{3/2}(c+dx)^{3/2}(g+hx)(fg-eh)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2f^3\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{de-cf}}{\sqrt{c+dx}\sqrt{be-af}}\right)}{(be-af)^{3/2}(de-cf)^{3/2}(fg-eh)}-\frac{2h^3\text{arctanh}\left(\frac{\sqrt{a+bx}\sqrt{dg-ch}}{\sqrt{c+dx}\sqrt{bg-ah}}\right)}{(bg-ah)^{3/2}(dg-ch)^{3/2}(fg-eh)}-\frac{2df\sqrt{a+bx}(-adf-bcf+2bde)}{\sqrt{c+dx}(bc-ad)^2(be-af)(de-cf)(fg-eh)}-\frac{2bf}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)(be-af)(fg-eh)}+\frac{2dh\sqrt{a+bx}(-adh-bch+2bdg)}{\sqrt{c+dx}(bc-ad)^2(bg-ah)(dg-ch)(fg-eh)}+\frac{2bh}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)(bg-ah)(fg-eh)}$$

input
```
Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)*(g + h*x)),x]
```

output
```
(-2*b*f)/((b*c - a*d)*(b*e - a*f)*(f*g - e*h)*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (2*b*h)/((b*c - a*d)*(b*g - a*h)*(f*g - e*h)*Sqrt[a + b*x]*Sqrt[c + d*x
]) - (2*d*f*(2*b*d*e - b*c*f - a*d*f)*Sqrt[a + b*x])/((b*c - a*d)^2*(b*e -
 a*f)*(d*e - c*f)*(f*g - e*h)*Sqrt[c + d*x]) + (2*d*h*(2*b*d*g - b*c*h - a
*d*h)*Sqrt[a + b*x])/((b*c - a*d)^2*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Sq
rt[c + d*x]) + (2*f^3*ArcTanh[(Sqrt[d*e - c*f]*Sqrt[a + b*x])/(Sqrt[b*e -
a*f]*Sqrt[c + d*x])])/((b*e - a*f)^(3/2)*(d*e - c*f)^(3/2)*(f*g - e*h)) -
(2*h^3*ArcTanh[(Sqrt[d*g - c*h]*Sqrt[a + b*x])/(Sqrt[b*g - a*h]*Sqrt[c + d
*x])])/((b*g - a*h)^(3/2)*(d*g - c*h)^(3/2)*(f*g - e*h))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9035 vs. $2(312) = 624$.

Time = 0.84 (sec) , antiderivative size = 9036, normalized size of antiderivative = 26.27

method	result	size
default	Expression too large to display	9036

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\,(e+fx)\,(g+hx)}\,dx$$

input
```
integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)*(e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*(f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(fx+e)(hx+g)}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x, algorithm="giac
")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{(e+f\,x)\,(g+h\,x)\,{(a+b\,x)}^{3/2}\,{(c+d\,x)}^{3/2}}\,dx$$

input
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((e + f*x)*(g + h*x)*(a + b*x)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)(g+hx)}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{3}{2}}\,(fx+e)\,(hx+g)}dx$$

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

output
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)/(h*x+g),x)
```

3.33 $\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}} + \frac{32\sqrt{1+x}\sqrt{a-ax}}{\sqrt{1-x}} - \frac{50(1+x)^{3/2}\sqrt{a-ax}}{3\sqrt{1-x}} + \frac{38(1+x)^{5/2}\sqrt{a-ax}}{5\sqrt{1-x}} - \frac{2(1+x)^{7/2}\sqrt{a-ax}}{\sqrt{1-x}} + \frac{2(1+x)^{9/2}\sqrt{a-ax}}{9\sqrt{1-x}}$$

output
```
8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)+32*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-
x)^(1/2)-50/3*(1+x)^(3/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)+38/5*(1+x)^(5/2)*(-a*
x+a)^(1/2)/(1-x)^(1/2)-2*(1+x)^(7/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)+2/9*(1+x)^
(9/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.29

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a-ax}(656+328x-82x^2+41x^3-20x^4+5x^5)}{45\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*x^3*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[a - a*x]*(656 + 328*x - 82*x^2 + 41*x^3 - 20*x^4 + 5*x^5))/(45*Sqr
t[1 - x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {37, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(x+1)^{3/2}}\,dx$$

$$\Big\downarrow \ 37$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2x^3}{(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \ 99$$

$$\frac{\sqrt{a-ax}\int \left((x+1)^{7/2}-7(x+1)^{5/2}+19(x+1)^{3/2}-25\sqrt{x+1}+\frac{16}{\sqrt{x+1}}-\frac{4}{(x+1)^{3/2}}\right)dx}{\sqrt{1-x}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(\frac{2}{9}(x+1)^{9/2}-2(x+1)^{7/2}+\frac{38}{5}(x+1)^{5/2}-\frac{50}{3}(x+1)^{3/2}+32\sqrt{x+1}+\frac{8}{\sqrt{x+1}}\right)\sqrt{a-ax}}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*x^3*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(Sqrt[a - a*x]*(8/Sqrt[1 + x] + 32*Sqrt[1 + x] - (50*(1 + x)^(3/2))/3 + (3
8*(1 + x)^(5/2))/5 - 2*(1 + x)^(7/2) + (2*(1 + x)^(9/2))/9))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.27

method	result	size
gosper	$\frac{2\sqrt{-ax+a}\,(5x^5-20x^4+41x^3-82x^2+328x+656)}{45\sqrt{1-x}\,\sqrt{1+x}}$	48
default	$\frac{2(5x^5-20x^4+41x^3-82x^2+328x+656)\sqrt{-a(-1+x)}}{45\sqrt{1+x}\,\sqrt{1-x}}$	48
orering	$\frac{2(5x^5-20x^4+41x^3-82x^2+328x+656)(1-x)^{\frac{3}{2}}\sqrt{-ax+a}}{45\sqrt{1+x}\,(-1+x)^2}$	53
risch	$-\frac{2\sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\,(-1+x)a(5x^5-20x^4+41x^3-82x^2+328x+656)}{45\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a(-1+x)}\,\sqrt{a(1+x)}}$	77

input
```
int((1-x)^(3/2)*x^3*(-a*x+a)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/(1-x)^(1/2)/(1+x)^(1/2)*(-a*x+a)^(1/2)*(5*x^5-20*x^4+41*x^3-82*x^2+32
8*x+656)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.31

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = -\frac{2\,(5\,x^5-20\,x^4+41\,x^3-82\,x^2+328\,x+656)\sqrt{-ax+a}\sqrt{x+1}\sqrt{-x+1}}{45\,(x^2-1)}$$

input
```
integrate((1-x)^(3/2)*x^3*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="fricas"
)
```

output
```
-2/45*(5*x^5 - 20*x^4 + 41*x^3 - 82*x^2 + 328*x + 656)*sqrt(-a*x + a)*sqrt
(x + 1)*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \int \frac{x^3\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*x**3*(-a*x+a)**(1/2)/(1+x)**(3/2),x)
```

output
```
Integral(x**3*sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.29

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\left(5\sqrt{a}x^5-20\sqrt{a}x^4+41\sqrt{a}x^3-82\sqrt{a}x^2+328\sqrt{a}x+656\sqrt{a}\right)}{45\sqrt{x+1}}$$

input
```
integrate((1-x)^(3/2)*x^3*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="maxima"
)
```

output
```
2/45*(5*sqrt(a)*x^5 - 20*sqrt(a)*x^4 + 41*sqrt(a)*x^3 - 82*sqrt(a)*x^2 + 3
28*sqrt(a)*x + 656*sqrt(a))/sqrt(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.27

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2}{45}\left(5(x+1)^{\frac{9}{2}}-45(x+1)^{\frac{7}{2}}+171(x+1)^{\frac{5}{2}}-375(x+1)^{\frac{3}{2}}+720\sqrt{x+1}\right.$$

input
```
integrate((1-x)^(3/2)*x^3*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
2/45*(5*(x + 1)^(9/2) - 45*(x + 1)^(7/2) + 171*(x + 1)^(5/2) - 375*(x + 1)
^(3/2) + 720*sqrt(x + 1) + 180/sqrt(x + 1))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.27

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a-a\,x}\,(5\,x^5-20\,x^4+41\,x^3-82\,x^2+328\,x+656)}{45\,\sqrt{1-x}\,\sqrt{x+1}}$$

input
```
int((x^3*(1 - x)^(3/2)*(a - a*x)^(1/2))/(x + 1)^(3/2),x)
```

output
```
(2*(a - a*x)^(1/2)*(328*x - 82*x^2 + 41*x^3 - 20*x^4 + 5*x^5 + 656))/(45*(
1 - x)^(1/2)*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.20

$$\int \frac{(1-x)^{3/2}x^3\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a}\,(5x^5-20x^4+41x^3-82x^2+328x+656)}{45\sqrt{x+1}}$$

input
```
int((1-x)^(3/2)*x^3*(-a*x+a)^(1/2)/(1+x)^(3/2),x)
```

output
```
(2*sqrt(a)*(5*x**5 - 20*x**4 + 41*x**3 - 82*x**2 + 328*x + 656))/(45*sqrt(
x + 1))
```

3.34 $\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = -\frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}}-\frac{24\sqrt{1+x}\sqrt{a-ax}}{\sqrt{1-x}} +\frac{26(1+x)^{3/2}\sqrt{a-ax}}{3\sqrt{1-x}}-\frac{12(1+x)^{5/2}\sqrt{a-ax}}{5\sqrt{1-x}}+\frac{2(1+x)^{7/2}\sqrt{a-ax}}{7\sqrt{1-x}}$$

output
```
-8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-24*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1
-x)^(1/2)+26/3*(1+x)^(3/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)-12/5*(1+x)^(5/2)*(-a
*x+a)^(1/2)/(1-x)^(1/2)+2/7*(1+x)^(7/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.31

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a-ax}(-1336-668x+167x^2-66x^3+15x^4)}{105\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*x^2*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[a - a*x]*(-1336 - 668*x + 167*x^2 - 66*x^3 + 15*x^4))/(105*Sqrt[1
- x^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {37, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2x^2}{(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\sqrt{a-ax}\int \left((x+1)^{5/2}-6(x+1)^{3/2}+13\sqrt{x+1}-\frac{12}{\sqrt{x+1}}+\frac{4}{(x+1)^{3/2}}\right)dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{2}{7}(x+1)^{7/2}-\frac{12}{5}(x+1)^{5/2}+\frac{26}{3}(x+1)^{3/2}-24\sqrt{x+1}-\frac{8}{\sqrt{x+1}}\right)\sqrt{a-ax}}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*x^2*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(Sqrt[a - a*x]*(-8/Sqrt[1 + x] - 24*Sqrt[1 + x] + (26*(1 + x)^(3/2))/3 - (
12*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.29

method	result	size
gosper	$\frac{2\sqrt{-ax+a}\,\left(15x^{4}-66x^{3}+167x^{2}-668x-1336\right)}{105\sqrt{1-x}\,\sqrt{1+x}}$	43
default	$\frac{2\left(15x^{4}-66x^{3}+167x^{2}-668x-1336\right)\sqrt{-a\left(-1+x\right)}}{105\sqrt{1+x}\,\sqrt{1-x}}$	43
orering	$\frac{2\left(15x^{4}-66x^{3}+167x^{2}-668x-1336\right)\left(1-x\right)^{\frac{3}{2}}\sqrt{-ax+a}}{105\sqrt{1+x}\,\left(-1+x\right)^{2}}$	48
risch	$-\frac{2\sqrt{-\frac{\left(1-x\right)\left(1+x\right)a}{-1+x}}\,\left(-1+x\right)a\left(15x^{4}-66x^{3}+167x^{2}-668x-1336\right)}{105\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a\left(-1+x\right)}\,\sqrt{a\left(1+x\right)}}$	72

input
```
int((1-x)^(3/2)*x^2*(-a*x+a)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/(1-x)^(1/2)/(1+x)^(1/2)*(-a*x+a)^(1/2)*(15*x^4-66*x^3+167*x^2-668*x-
1336)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.33

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = -\frac{2\left(15\,x^4-66\,x^3+167\,x^2-668\,x-1336\right)\sqrt{-ax+a}\sqrt{x+1}\sqrt{-x+1}}{105\left(x^2-1\right)}$$

input
```
integrate((1-x)^(3/2)*x^2*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="fricas"
)
```

output
```
-2/105*(15*x^4 - 66*x^3 + 167*x^2 - 668*x - 1336)*sqrt(-a*x + a)*sqrt(x +
1)*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \int \frac{x^2\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*x**2*(-a*x+a)**(1/2)/(1+x)**(3/2),x)
```

output
```
Integral(x**2*sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.29

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\left(15\,\sqrt{a}x^4-66\,\sqrt{a}x^3+167\,\sqrt{a}x^2-668\,\sqrt{a}x-1336\,\sqrt{a}\right)}{105\,\sqrt{x+1}}$$

input
```
integrate((1-x)^(3/2)*x^2*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="maxima"
)
```

output
```
2/105*(15*sqrt(a)*x^4 - 66*sqrt(a)*x^3 + 167*sqrt(a)*x^2 - 668*sqrt(a)*x -
 1336*sqrt(a))/sqrt(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.28

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2}{105}\left(15\,(x+1)^{\frac{7}{2}} - 126\,(x+1)^{\frac{5}{2}} + 455\,(x+1)^{\frac{3}{2}} - 1260\,\sqrt{x+1} - \frac{420}{\sqrt{x+1}}\right)$$

input
```
integrate((1-x)^(3/2)*x^2*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
2/105*(15*(x + 1)^(7/2) - 126*(x + 1)^(5/2) + 455*(x + 1)^(3/2) - 1260*sqr
t(x + 1) - 420/sqrt(x + 1))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = -\frac{2\,\sqrt{a-a\,x}\,\left(-15\,x^4+66\,x^3-167\,x^2+668\,x+1336\right)}{105\,\sqrt{1-x}\,\sqrt{x+1}}$$

input
```
int((x^2*(1 - x)^(3/2)*(a - a*x)^(1/2))/(x + 1)^(3/2),x)
```

output
```
-(2*(a - a*x)^(1/2)*(668*x - 167*x^2 + 66*x^3 - 15*x^4 + 1336))/(105*(1 -
x)^(1/2)*(x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.20

$$\int \frac{(1-x)^{3/2}x^2\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a}\,(15x^4-66x^3+167x^2-668x-1336)}{105\sqrt{x+1}}$$

input
```
int((1-x)^(3/2)*x^2*(-a*x+a)^(1/2)/(1+x)^(3/2),x)
```

output
```
(2*sqrt(a)*(15*x**4 - 66*x**3 + 167*x**2 - 668*x - 1336))/(105*sqrt(x + 1)
)
```

3.35 $\int \frac{(1-x)^{3/2}x\sqrt{a-ax}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 117

$$\int \frac{(1-x)^{3/2}x\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}}+\frac{16\sqrt{1+x}\sqrt{a-ax}}{\sqrt{1-x}} - \frac{10(1+x)^{3/2}\sqrt{a-ax}}{3\sqrt{1-x}}+\frac{2(1+x)^{5/2}\sqrt{a-ax}}{5\sqrt{1-x}}$$

output
```
8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)+16*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-
x)^(1/2)-10/3*(1+x)^(3/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)+2/5*(1+x)^(5/2)*(-a*x
+a)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.34

$$\int \frac{(1-x)^{3/2}x\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a-ax}(158+79x-16x^2+3x^3)}{15\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*x*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[a - a*x]*(158 + 79*x - 16*x^2 + 3*x^3))/(15*Sqrt[1 - x^2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {37, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2} x \sqrt{a-a x}}{(x+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{a-a x} \int \frac{(1-x)^2 x}{(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{86}$$

$$\frac{\sqrt{a-a x} \int \left((x+1)^{3/2}-5 \sqrt{x+1}+\frac{8}{\sqrt{x+1}}-\frac{4}{(x+1)^{3/2}}\right)dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{2}{5} (x+1)^{5/2}-\frac{10}{3} (x+1)^{3/2}+16 \sqrt{x+1}+\frac{8}{\sqrt{x+1}}\right) \sqrt{a-a x}}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*x*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(Sqrt[a - a*x]*(8/Sqrt[1 + x] + 16*Sqrt[1 + x] - (10*(1 + x)^(3/2))/3 + (2
*(1 + x)^(5/2))/5))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2\sqrt{-ax+a}\,(3x^{3}-16x^{2}+79x+158)}{15\sqrt{1-x}\,\sqrt{1+x}}$	38
default	$\frac{2(3x^{3}-16x^{2}+79x+158)\sqrt{-a(-1+x)}}{15\sqrt{1+x}\,\sqrt{1-x}}$	38
orering	$\frac{2(3x^{3}-16x^{2}+79x+158)(1-x)^{\frac{3}{2}}\sqrt{-ax+a}}{15\sqrt{1+x}\,(-1+x)^{2}}$	43
risch	$-\frac{2\sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\,(-1+x)a(3x^{3}-16x^{2}+79x+158)}{15\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a(-1+x)}\,\sqrt{a(1+x)}}$	67

input
```
int((1-x)^(3/2)*x*(-a*x+a)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/(1-x)^(1/2)/(1+x)^(1/2)*(-a*x+a)^(1/2)*(3*x^3-16*x^2+79*x+158)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.38

$$\int \frac{(1-x)^{3/2}x\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = -\frac{2\,(3\,x^3-16\,x^2+79\,x+158)\sqrt{-ax+a}\sqrt{x+1}\sqrt{-x+1}}{15\,(x^2-1)}$$

input
```
integrate((1-x)^(3/2)*x*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*x^3 - 16*x^2 + 79*x + 158)*sqrt(-a*x + a)*sqrt(x + 1)*sqrt(-x + 1
)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}x\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \int \frac{x\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*x*(-a*x+a)**(1/2)/(1+x)**(3/2),x)
```

output
```
Integral(x*sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.30

$$\int \frac{(1-x)^{3/2}x\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\,\left(3\,\sqrt{a}x^3-16\,\sqrt{a}x^2+79\,\sqrt{a}x+158\,\sqrt{a}\right)}{15\,\sqrt{x+1}}$$

input
```
integrate((1-x)^(3/2)*x*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(3*sqrt(a)*x^3 - 16*sqrt(a)*x^2 + 79*sqrt(a)*x + 158*sqrt(a))/sqrt(x
+ 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{(1-x)^{3/2} x \sqrt{a-ax}}{(1+x)^{3/2}} \, dx = \frac{2}{15} \left(3 (x+1)^{\frac{5}{2}} - 25 (x+1)^{\frac{3}{2}} + 120 \sqrt{x+1} + \frac{60}{\sqrt{x+1}}\right) \sqrt{a}$$

input
```
integrate((1-x)^(3/2)*x*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
2/15*(3*(x + 1)^(5/2) - 25*(x + 1)^(3/2) + 120*sqrt(x + 1) + 60/sqrt(x + 1
))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.32

$$\int \frac{(1-x)^{3/2} x \sqrt{a-ax}}{(1+x)^{3/2}} \, dx = \frac{2 \sqrt{a-a\,x} \, (3\,x^3 - 16\,x^2 + 79\,x + 158)}{15 \sqrt{1-x} \sqrt{x+1}}$$

input
```
int((x*(1 - x)^(3/2)*(a - a*x)^(1/2))/(x + 1)^(3/2),x)
```

output
```
(2*(a - a*x)^(1/2)*(79*x - 16*x^2 + 3*x^3 + 158))/(15*(1 - x)^(1/2)*(x + 1
)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.21

$$\int \frac{(1-x)^{3/2} x \sqrt{a-ax}}{(1+x)^{3/2}} \, dx = \frac{2\sqrt{a}\,(3x^3 - 16x^2 + 79x + 158)}{15\sqrt{x+1}}$$

input
```
int((1-x)^(3/2)*x*(-a*x+a)^(1/2)/(1+x)^(3/2),x)
```

output
```
(2*sqrt(a)*(3*x**3 - 16*x**2 + 79*x + 158))/(15*sqrt(x + 1))
```

3.36 $\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx=-\frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}}-\frac{8\sqrt{1+x}\sqrt{a-ax}}{\sqrt{1-x}}+\frac{2(1+x)^{3/2}\sqrt{a-ax}}{3\sqrt{1-x}}$$

output
```
-8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-8*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-
x)^(1/2)+2/3*(1+x)^(3/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.38

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx=\frac{2\sqrt{a-ax}(-23-10x+x^2)}{3\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(2*Sqrt[a - a*x]*(-23 - 10*x + x^2))/(3*Sqrt[1 - x^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2}{(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{a-ax}\int \left(\sqrt{x+1}-\frac{4}{\sqrt{x+1}}+\frac{4}{(x+1)^{3/2}}\right)dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{2}{3}(x+1)^{3/2}-8\sqrt{x+1}-\frac{8}{\sqrt{x+1}}\right)\sqrt{a-ax}}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*Sqrt[a - a*x])/(1 + x)^(3/2),x]
```

output
```
(Sqrt[a - a*x]*(-8/Sqrt[1 + x] - 8*Sqrt[1 + x] + (2*(1 + x)^(3/2))/3))/Sqr
t[1 - x]
```

Defintions of rubi rules used

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.36

method	result	size
gosper	$\frac{2\left(x^{2}-10x-23\right)\sqrt{-ax+a}}{3\sqrt{1-x}\,\sqrt{1+x}}$	31
default	$\frac{2\left(x^{2}-10x-23\right)\sqrt{-a\left(-1+x\right)}}{3\sqrt{1+x}\,\sqrt{1-x}}$	31
orering	$\frac{2\left(x^{2}-10x-23\right)\left(1-x\right)^{\frac{3}{2}}\sqrt{-ax+a}}{3\sqrt{1+x}\,\left(-1+x\right)^{2}}$	36
risch	$-\frac{2\sqrt{-\frac{\left(1-x\right)\left(1+x\right)a}{-1+x}}\,\left(-1+x\right)a\left(x^{2}-10x-23\right)}{3\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a\left(-1+x\right)}\,\sqrt{a\left(1+x\right)}}$	60

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(1-x)^(1/2)/(1+x)^(1/2)*(x^2-10*x-23)*(-a*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.43

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = -\frac{2\sqrt{-ax+a}(x^2-10\,x-23)\sqrt{x+1}\sqrt{-x+1}}{3\,(x^2-1)}$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-a*x + a)*(x^2 - 10*x - 23)*sqrt(x + 1)*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*(-a*x+a)**(1/2)/(1+x)**(3/2),x)
```

output
```
Integral(sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.30

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\left(\sqrt{a}x^2-10\sqrt{a}x-23\sqrt{a}\right)}{3\sqrt{x+1}}$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(sqrt(a)*x^2 - 10*sqrt(a)*x - 23*sqrt(a))/sqrt(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.29

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2}{3}\left((x+1)^{\frac{3}{2}} - 12\sqrt{x+1} - \frac{12}{\sqrt{x+1}}\right)\sqrt{a}$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

output
```
2/3*((x + 1)^(3/2) - 12*sqrt(x + 1) - 12/sqrt(x + 1))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{\sqrt{a-a\,x}\left(\frac{20\,x\,\sqrt{1-x}}{3} + \frac{46\,\sqrt{1-x}}{3} - \frac{2\,x^2\,\sqrt{1-x}}{3}\right)}{x\,\sqrt{x+1} - \sqrt{x+1}}$$

input
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x + 1)^(3/2),x)
```

output
```
((a - a*x)^(1/2)*((20*x*(1 - x)^(1/2))/3 + (46*(1 - x)^(1/2))/3 - (2*x^2*(
1 - x)^(1/2))/3))/(x*(x + 1)^(1/2) - (x + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.21

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{(1+x)^{3/2}}\,dx = \frac{2\sqrt{a}\,(x^2-10x-23)}{3\sqrt{x+1}}$$

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/(1+x)^(3/2),x)
```

output
```
(2*sqrt(a)*(x**2 - 10*x - 23))/(3*sqrt(x + 1))
```

3.37 $\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 92

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}} + \frac{2\sqrt{1+x}\sqrt{a-ax}}{\sqrt{1-x}} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-ax}}{\sqrt{a}\sqrt{1-x^2}}\right)$$

output
```
8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)+2*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-x
)^(1/2)-2*a^(1/2)*arctanh((-a*x+a)^(1/2)/a^(1/2)/(-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \frac{2\sqrt{a-ax}\left(5+x-\sqrt{1+x}\operatorname{arctanh}\left(\sqrt{1+x}\right)\right)}{\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*Sqrt[a - a*x])/(x*(1 + x)^(3/2)),x]
```

output
```
(2*Sqrt[a - a*x]*(5 + x - Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]]))/Sqrt[1 - x^2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {37, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2}{x(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{98}$$

$$\frac{\sqrt{a-ax}\int \left(\frac{1}{x\sqrt{x+1}}+\frac{1}{\sqrt{x+1}}-\frac{4}{(x+1)^{3/2}}\right)dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a-ax}\left(-2\text{arctanh}\left(\sqrt{x+1}\right)+2\sqrt{x+1}+\frac{8}{\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*Sqrt[a - a*x])/(x*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(8/Sqrt[1 + x] + 2*Sqrt[1 + x] - 2*ArcTanh[Sqrt[1 + x]]))/S
qrt[1 - x]
```

Defintions of rubi rules used

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a(1+x)}}{\sqrt{a}}\right)\sqrt{a(1+x)}-ax-5a\right)\sqrt{-a(-1+x)}}{\sqrt{1+x}\,\sqrt{1-x}\,a}$	57
risch	$-\frac{8\sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\,(-1+x)a}{\sqrt{a(1+x)}\,\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a(-1+x)}}-\frac{2\left(\sqrt{ax+a}-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+a}}{\sqrt{a}}\right)\right)\sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\,(-1+x)}{\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a(-1+x)}}$	121

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(1+x)^(1/2)*(a^(1/2)*arctanh((a*(1+x))^(1/2)/a^(1/2))*(a*(1+x))^(1/2)-a
*x-5*a)/(1-x)^(1/2)*(-a*(-1+x))^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.91

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \left[-\frac{2\sqrt{-ax+a}(x+5)\sqrt{x+1}\sqrt{-x+1}-(x^2-1)\sqrt{a}\log\left(-\frac{ax^2+ax+2\sqrt{-ax+a}\sqrt{}}{x^2-}\right.}{x^2-1}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x/(1+x)^(3/2),x, algorithm="fricas")
```

output
```
[-(2*sqrt(-a*x + a)*(x + 5)*sqrt(x + 1)*sqrt(-x + 1) - (x^2 - 1)*sqrt(a)*l
og(-(a*x^2 + a*x + 2*sqrt(-a*x + a)*sqrt(a)*sqrt(x + 1)*sqrt(-x + 1) - 2*a
)/(x^2 - x)))/(x^2 - 1), -2*(sqrt(-a*x + a)*(x + 5)*sqrt(x + 1)*sqrt(-x +
1) + (x^2 - 1)*sqrt(-a)*arctan(sqrt(-a*x + a)*sqrt(-a)*sqrt(x + 1)*sqrt(-x
 + 1)/(a*x - a)))/(x^2 - 1)]
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{x\,(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*(-a*x+a)**(1/2)/x/(1+x)**(3/2),x)
```

output
```
Integral(sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-ax+a}(-x+1)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x/(1+x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*x + a)*(-x + 1)^(3/2)/((x + 1)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.41

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \sqrt{a}\left(2\sqrt{x+1}+\frac{8}{\sqrt{x+1}}-\log\left(\sqrt{x+1}+1\right)+\log\left(\left|\sqrt{x+1}-1\right|\right)\right)$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x/(1+x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(a)*(2*sqrt(x + 1) + 8/sqrt(x + 1) - log(sqrt(x + 1) + 1) + log(abs(sq
rt(x + 1) - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \int \frac{{(1-x)}^{3/2}\,\sqrt{a-a\,x}}{x\,{(x+1)}^{3/2}}\,dx$$

input
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x*(x + 1)^(3/2)),x)
```

output
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x(1+x)^{3/2}}\,dx = \frac{\sqrt{a}\left(\sqrt{x+1}\log\left(\sqrt{x+1}-1\right)-\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)+2x+10\right)}{\sqrt{x+1}}$$

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x/(1+x)^(3/2),x)
```

output
```
(sqrt(a)*(sqrt(x + 1)*log(sqrt(x + 1) - 1) - sqrt(x + 1)*log(sqrt(x + 1) +
 1) + 2*x + 10))/sqrt(x + 1)
```

3.38 $\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = -\frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}} - \frac{\sqrt{1+x}\sqrt{a-ax}}{\sqrt{1-x}x} + 7\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-ax}}{\sqrt{a}\sqrt{1-x^2}}\right)$$

output
```
-8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-x)
^(1/2)/x+7*a^(1/2)*arctanh((-a*x+a)^(1/2)/a^(1/2)/(-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = \frac{\sqrt{a-ax}\left(-1-9x+7x\sqrt{1+x}\text{arctanh}\left(\sqrt{1+x}\right)\right)}{x\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^2*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-1 - 9*x + 7*x*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]]))/(x*Sqrt[
1 - x^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {37, 100, 27, 87, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2}{x^2(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{\sqrt{a-ax}\left(\int -\frac{7-2x}{2x(x+1)^{3/2}}dx-\frac{1}{x\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a-ax}\left(-\frac{1}{2}\int \frac{7-2x}{x(x+1)^{3/2}}dx-\frac{1}{x\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{2}\left(-7\int \frac{1}{x\sqrt{x+1}}dx-\frac{18}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{2}\left(-14\int \frac{1}{x}d\sqrt{x+1}-\frac{18}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{2}\left(14\text{arctanh}\left(\sqrt{x+1}\right)-\frac{18}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^2*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-(1/(x*Sqrt[1 + x])) + (-18/Sqrt[1 + x] + 14*ArcTanh[Sqrt[
1 + x]])/2))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(7 \operatorname{arctanh}\left(\frac{\sqrt{a(1+x)}}{\sqrt{a}}\right) x \sqrt{a(1+x)}-9 \sqrt{a}\, x-\sqrt{a}\right) \sqrt{-a(-1+x)}}{\sqrt{1+x}\, \sqrt{1-x}\, \sqrt{a}\, x}$	62
risch	$\frac{(9x+1) \sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\, (-1+x)a}{x \sqrt{a(1+x)}\, \sqrt{1-x}\, \sqrt{1+x}\, \sqrt{-a(-1+x)}}-\frac{7 \sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{ax+a}}{\sqrt{a}}\right) \sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\, (-1+x)}{\sqrt{1-x}\, \sqrt{1+x}\, \sqrt{-a(-1+x)}}$	118

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^2/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(7*arctanh((a*(1+x))^(1/2)/a^(1/2))*x*(a*(1+x))^(1/2)-9*a^(1/2)*x-a^(1/2))
*(-a*(-1+x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)/a^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.98

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = \left[\frac{2\sqrt{-ax+a}(9\,x+1)\sqrt{x+1}\sqrt{-x+1}+7\,(x^3-x)\sqrt{a}\log\left(-\frac{ax^2+ax-2\sqrt{-ax+a}}{x^2}\right.}{2\,(x^3-x)}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^2/(1+x)^(3/2),x, algorithm="fricas"
)
```

output
```
[1/2*(2*sqrt(-a*x + a)*(9*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 7*(x^3 - x)*sq
rt(a)*log(-(a*x^2 + a*x - 2*sqrt(-a*x + a)*sqrt(a)*sqrt(x + 1)*sqrt(-x + 1
) - 2*a)/(x^2 - x)))/(x^3 - x), (sqrt(-a*x + a)*(9*x + 1)*sqrt(x + 1)*sqrt
(-x + 1) + 7*(x^3 - x)*sqrt(-a)*arctan(sqrt(-a*x + a)*sqrt(-a)*sqrt(x + 1)
*sqrt(-x + 1)/(a*x - a)))/(x^3 - x)]
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{x^2\,(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*(-a*x+a)**(1/2)/x**2/(1+x)**(3/2),x)
```

output
```
Integral(sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x**2*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-ax+a}(-x+1)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^2/(1+x)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*x + a)*(-x + 1)^(3/2)/((x + 1)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = -\frac{1}{2}\sqrt{a}\left(\frac{2\,(9\,x+1)}{(x+1)^{\frac{3}{2}}-\sqrt{x+1}}-7\,\log\left(\sqrt{x+1}+1\right)+7\,\log\left(\left|\sqrt{x+1}-1\right|\right)\right)$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^2/(1+x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(2*(9*x + 1)/((x + 1)^(3/2) - sqrt(x + 1)) - 7*log(sqrt(x + 1
) + 1) + 7*log(abs(sqrt(x + 1) - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{3/2}\,\sqrt{a-a\,x}}{x^2\,(x+1)^{3/2}}\,dx$$

input
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^2*(x + 1)^(3/2)),x)
```

output
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^2*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^2(1+x)^{3/2}}\,dx = \frac{\sqrt{a}\,\left(-7\sqrt{x+1}\,\log\left(\sqrt{x+1}-1\right)x+7\sqrt{x+1}\,\log\left(\sqrt{x+1}+1\right)x-18x-2\right)}{2\sqrt{x+1}\,x}$$

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^2/(1+x)^(3/2),x)
```

output
```
(sqrt(a)*( - 7*sqrt(x + 1)*log(sqrt(x + 1) - 1)*x + 7*sqrt(x + 1)*log(sqrt
(x + 1) + 1)*x - 18*x - 2))/(2*sqrt(x + 1)*x)
```

3.39 $\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 132

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}} - \frac{\sqrt{1+x}\sqrt{a-ax}}{2\sqrt{1-x}x^2} + \frac{15\sqrt{1+x}\sqrt{a-ax}}{4\sqrt{1-x}x} - \frac{47}{4}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-ax}}{\sqrt{a}\sqrt{1-x^2}}\right)$$

output
```
8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-1/2*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1
-x)^(1/2)/x^2+15/4*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)/x-47/4*a^(1/2)*a
rctanh((-a*x+a)^(1/2)/a^(1/2)/(-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \frac{\sqrt{a-ax}\left(-2+13x+47x^2-47x^2\sqrt{1+x}\text{arctanh}\left(\sqrt{1+x}\right)\right)}{4x^2\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^3*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-2 + 13*x + 47*x^2 - 47*x^2*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x
]]))/(4*x^2*Sqrt[1 - x^2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {37, 100, 27, 87, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2}{x^3(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{100}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{2}\int -\frac{13-4x}{2x^2(x+1)^{3/2}}dx-\frac{1}{2x^2\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a-ax}\left(-\frac{1}{4}\int \frac{13-4x}{x^2(x+1)^{3/2}}dx-\frac{1}{2x^2\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{87}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{4}\left(\frac{47}{2}\int \frac{1}{x(x+1)^{3/2}}dx+\frac{13}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{4}\left(\frac{47}{2}\left(\int \frac{1}{x\sqrt{x+1}}dx+\frac{2}{\sqrt{x+1}}\right)+\frac{13}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{4}\left(\frac{47}{2}\left(2\int\frac{1}{x}d\sqrt{x+1}+\frac{2}{\sqrt{x+1}}\right)+\frac{13}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \; 220$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{4}\left(\frac{47}{2}\left(\frac{2}{\sqrt{x+1}}-2\text{arctanh}\left(\sqrt{x+1}\right)\right)+\frac{13}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^3*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-1/2*1/(x^2*Sqrt[1 + x]) + (13/(x*Sqrt[1 + x]) + (47*(2/Sq
rt[1 + x] - 2*ArcTanh[Sqrt[1 + x]]))/2)/4))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{\left(47 \,\operatorname{arctanh}\left(\frac{\sqrt{a \left(1+x \right)}}{\sqrt{a}}\right) x^{2} \sqrt{a \left(1+x \right)}-47 x^{2} \sqrt{a}-13 \sqrt{a}\, x +2 \sqrt{a}\right) \sqrt{-a \left(-1+x \right)}}{4 \sqrt{1+x}\, \sqrt{1-x}\, \sqrt{a}\, x^{2}}$	73
risch	$-\frac{\left(47 x^{2}+13 x -2\right) \sqrt{-\frac{\left(1-x \right) \left(1+x \right) a}{-1+x}}\, \left(-1+x \right) a}{4 x^{2} \sqrt{a \left(1+x \right)}\, \sqrt{1-x}\, \sqrt{1+x}\, \sqrt{-a \left(-1+x \right)}}+\frac{47 \sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{a x +a}}{\sqrt{a}}\right) \sqrt{-\frac{\left(1-x \right) \left(1+x \right) a}{-1+x}}\, \left(-1+x \right)}{4 \sqrt{1-x}\, \sqrt{1+x}\, \sqrt{-a \left(-1+x \right)}}$	124

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^3/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(47*arctanh((a*(1+x))^(1/2)/a^(1/2))*x^2*(a*(1+x))^(1/2)-47*x^2*a^(1/
2)-13*a^(1/2)*x+2*a^(1/2))/(1+x)^(1/2)/(1-x)^(1/2)*(-a*(-1+x))^(1/2)/a^(1/
2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.57

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \left[-\frac{2\sqrt{-ax+a}(47\,x^2+13\,x-2)\sqrt{x+1}\sqrt{-x+1}-47\,(x^4-x^2)\sqrt{a}\log\left(-\frac{ax}{}\right.}{8\,(x^4-x^2)}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^3/(1+x)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/8*(2*sqrt(-a*x + a)*(47*x^2 + 13*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - 47*
(x^4 - x^2)*sqrt(a)*log(-(a*x^2 + a*x + 2*sqrt(-a*x + a)*sqrt(a)*sqrt(x +
1)*sqrt(-x + 1) - 2*a)/(x^2 - x)))/(x^4 - x^2), -1/4*(sqrt(-a*x + a)*(47*x
^2 + 13*x - 2)*sqrt(x + 1)*sqrt(-x + 1) + 47*(x^4 - x^2)*sqrt(-a)*arctan(s
qrt(-a*x + a)*sqrt(-a)*sqrt(x + 1)*sqrt(-x + 1)/(a*x - a)))/(x^4 - x^2)]
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{x^3\,(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*(-a*x+a)**(1/2)/x**3/(1+x)**(3/2),x)
```

output
```
Integral(sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x**3*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-ax+a}(-x+1)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^3/(1+x)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*x + a)*(-x + 1)^(3/2)/((x + 1)^(3/2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \frac{1}{8}\sqrt{a}\left(\frac{64}{\sqrt{x+1}} + \frac{2\left(15\,(x+1)^{\frac{3}{2}} - 17\,\sqrt{x+1}\right)}{x^2} - 47\,\log\left(\sqrt{x+1}+1\right) + 47\,\mathrm{l}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^3/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a)*(64/sqrt(x + 1) + 2*(15*(x + 1)^(3/2) - 17*sqrt(x + 1))/x^2 -
47*log(sqrt(x + 1) + 1) + 47*log(abs(sqrt(x + 1) - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{3/2}\,\sqrt{a-a\,x}}{x^3\,(x+1)^{3/2}}\,dx$$

input
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^3*(x + 1)^(3/2)),x)
```

output
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^3*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^3(1+x)^{3/2}}\,dx = \frac{\sqrt{a}\left(47\sqrt{x+1}\log\left(\sqrt{x+1}-1\right)x^2-47\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)x^2+94x^2+2\right.}{8\sqrt{x+1}\,x^2}$$

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^3/(1+x)^(3/2),x)
```

output
```
(sqrt(a)*(47*sqrt(x + 1)*log(sqrt(x + 1) - 1)*x**2 - 47*sqrt(x + 1)*log(sq
rt(x + 1) + 1)*x**2 + 94*x**2 + 26*x - 4))/(8*sqrt(x + 1)*x**2)
```

3.40 $\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = -\frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}}-\frac{\sqrt{1+x}\sqrt{a-ax}}{3\sqrt{1-x}x^3}+\frac{23\sqrt{1+x}\sqrt{a-ax}}{12\sqrt{1-x}x^2}-\frac{55\sqrt{1+x}\sqrt{a-ax}}{8\sqrt{1-x}x}+\frac{119}{8}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-ax}}{\sqrt{a}\sqrt{1-x^2}}\right)$$

output
```
-8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-1/3*(1+x)^(1/2)*(-a*x+a)^(1/2)/(
1-x)^(1/2)/x^3+23/12*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)/x^2-55/8*(1+x)
^(1/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)/x+119/8*a^(1/2)*arctanh((-a*x+a)^(1/2)/a
^(1/2)/(-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.38

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = \frac{\sqrt{a-ax}\left(-8+38x-119x^2-357x^3+357x^3\sqrt{1+x}\operatorname{arctanh}\left(\sqrt{1+x}\right)\right)}{24x^3\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^4*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-8 + 38*x - 119*x^2 - 357*x^3 + 357*x^3*Sqrt[1 + x]*ArcTan
h[Sqrt[1 + x]]))/(24*x^3*Sqrt[1 - x^2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {37, 100, 27, 87, 52, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2}{x^4(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{3}\int -\frac{19-6x}{2x^3(x+1)^{3/2}}dx-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a-ax}\left(-\frac{1}{6}\int \frac{19-6x}{x^3(x+1)^{3/2}}dx-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{6}\left(\frac{119}{4}\int \frac{1}{x^2(x+1)^{3/2}}dx+\frac{19}{2x^2\sqrt{x+1}}\right)-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3}{2}\int \frac{1}{x(x+1)^{3/2}}dx-\frac{1}{x\sqrt{x+1}}\right)+\frac{19}{2x^2\sqrt{x+1}}\right)-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3}{2}\left(\int \frac{1}{x\sqrt{x+1}}dx+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)+\frac{19}{2x^2\sqrt{x+1}}\right)-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3}{2}\left(2\int \frac{1}{x}d\sqrt{x+1}+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)+\frac{19}{2x^2\sqrt{x+1}}\right)-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{6}\left(\frac{119}{4}\left(-\frac{3}{2}\left(\frac{2}{\sqrt{x+1}}-2\text{arctanh}\left(\sqrt{x+1}\right)\right)-\frac{1}{x\sqrt{x+1}}\right)+\frac{19}{2x^2\sqrt{x+1}}\right)-\frac{1}{3x^3\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^4*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-1/3*1/(x^3*Sqrt[1 + x]) + (19/(2*x^2*Sqrt[1 + x]) + (119*
(-(1/(x*Sqrt[1 + x])) - (3*(2/Sqrt[1 + x] - 2*ArcTanh[Sqrt[1 + x]]))/2))/4
)/6))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(357 \operatorname{arctanh}\left(\frac{\sqrt{a(1+x)}}{\sqrt{a}}\right) x^{3} \sqrt{a(1+x)}-357 x^{3} \sqrt{a}-119 x^{2} \sqrt{a}+38 \sqrt{a}\, x-8 \sqrt{a}\right) \sqrt{-a(-1+x)}}{24 \sqrt{1+x}\, \sqrt{1-x}\, \sqrt{a}\, x^{3}}$	81
risch	$\frac{\left(357 x^{3}+119 x^{2}-38 x+8\right) \sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\, (-1+x) a}{24 x^{3} \sqrt{a(1+x)}\, \sqrt{1-x}\, \sqrt{1+x}\, \sqrt{-a(-1+x)}}-\frac{119 \sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{a x+a}}{\sqrt{a}}\right) \sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\, (-1+x)}{8 \sqrt{1-x}\, \sqrt{1+x}\, \sqrt{-a(-1+x)}}$	129

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^4/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(357*arctanh((a*(1+x))^(1/2)/a^(1/2))*x^3*(a*(1+x))^(1/2)-357*x^3*a^(
1/2)-119*x^2*a^(1/2)+38*a^(1/2)*x-8*a^(1/2))/(1+x)^(1/2)/(1-x)^(1/2)*(-a*(
-1+x))^(1/2)/a^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.32

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = \left[\frac{2\,(357\,x^3+119\,x^2-38\,x+8)\sqrt{-ax+a}\sqrt{x+1}\sqrt{-x+1}+357\,(x^5-x^3)\sqrt{a}}{48\,(x^5-x^3)}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^4/(1+x)^(3/2),x, algorithm="fricas"
)
```

output
```
[1/48*(2*(357*x^3 + 119*x^2 - 38*x + 8)*sqrt(-a*x + a)*sqrt(x + 1)*sqrt(-x
 + 1) + 357*(x^5 - x^3)*sqrt(a)*log(-(a*x^2 + a*x - 2*sqrt(-a*x + a)*sqrt(
a)*sqrt(x + 1)*sqrt(-x + 1) - 2*a)/(x^2 - x)))/(x^5 - x^3), 1/24*((357*x^3
 + 119*x^2 - 38*x + 8)*sqrt(-a*x + a)*sqrt(x + 1)*sqrt(-x + 1) + 357*(x^5
- x^3)*sqrt(-a)*arctan(sqrt(-a*x + a)*sqrt(-a)*sqrt(x + 1)*sqrt(-x + 1)/(a
*x - a)))/(x^5 - x^3)]
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-a\,(x-1)}(1-x)^{\frac{3}{2}}}{x^4\,(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*(-a*x+a)**(1/2)/x**4/(1+x)**(3/2),x)
```

output
```
Integral(sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x**4*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-ax+a}(-x+1)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^4/(1+x)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*x + a)*(-x + 1)^(3/2)/((x + 1)^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx =$$
$$-\frac{1}{48}\sqrt{a}\left(\frac{384}{\sqrt{x+1}}+\frac{2\left(165\,(x+1)^{\frac{5}{2}}-376\,(x+1)^{\frac{3}{2}}+219\,\sqrt{x+1}\right)}{x^3}-357\,\log\left(\sqrt{x+1}+1\right)+357\,\log\left(\right.\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^4/(1+x)^(3/2),x, algorithm="giac")
```

output
```
-1/48*sqrt(a)*(384/sqrt(x + 1) + 2*(165*(x + 1)^(5/2) - 376*(x + 1)^(3/2)
+ 219*sqrt(x + 1))/x^3 - 357*log(sqrt(x + 1) + 1) + 357*log(abs(sqrt(x + 1
) - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{3/2}\,\sqrt{a-a\,x}}{x^4\,(x+1)^{3/2}}\,dx$$

input
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^4*(x + 1)^(3/2)),x)
```

output
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^4*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.36

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^4(1+x)^{3/2}}\,dx = \frac{\sqrt{a}\left(-357\sqrt{x+1}\log\left(\sqrt{x+1}-1\right)x^3+357\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)x^3-714x^3\right.}{48\sqrt{x+1}\,x^3}$$

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^4/(1+x)^(3/2),x)
```

output
```
(sqrt(a)*( - 357*sqrt(x + 1)*log(sqrt(x + 1) - 1)*x**3 + 357*sqrt(x + 1)*l
og(sqrt(x + 1) + 1)*x**3 - 714*x**3 - 238*x**2 + 76*x - 16))/(48*sqrt(x +
1)*x**3)
```

3.41 $\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 198

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \frac{8\sqrt{a-ax}}{\sqrt{1-x}\sqrt{1+x}}-\frac{\sqrt{1+x}\sqrt{a-ax}}{4\sqrt{1-x}x^4}+\frac{31\sqrt{1+x}\sqrt{a-ax}}{24\sqrt{1-x}x^3} -\frac{347\sqrt{1+x}\sqrt{a-ax}}{96\sqrt{1-x}x^2}+\frac{603\sqrt{1+x}\sqrt{a-ax}}{64\sqrt{1-x}x}-\frac{1115}{64}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-ax}}{\sqrt{a}\sqrt{1-x^2}}\right)$$

output
```
8*(-a*x+a)^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-1/4*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1
-x)^(1/2)/x^4+31/24*(1+x)^(1/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)/x^3-347/96*(1+x
)^(1/2)*(-a*x+a)^(1/2)/(1-x)^(1/2)/x^2+603/64*(1+x)^(1/2)*(-a*x+a)^(1/2)/(
1-x)^(1/2)/x-1115/64*a^(1/2)*arctanh((-a*x+a)^(1/2)/a^(1/2)/(-x^2+1)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.34

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \frac{\sqrt{a-ax}\left(-48+200x-446x^2+1115x^3+3345x^4-3345x^4\sqrt{1+x}\operatorname{arctanh}(\sqrt{}\right.}{192x^4\sqrt{1-x^2}}$$

input
```
Integrate[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^5*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-48 + 200*x - 446*x^2 + 1115*x^3 + 3345*x^4 - 3345*x^4*Sqr
t[1 + x]*ArcTanh[Sqrt[1 + x]]))/(192*x^4*Sqrt[1 - x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {37, 100, 27, 87, 52, 52, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(x+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{a-ax}\int \frac{(1-x)^2}{x^5(x+1)^{3/2}}dx}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{4}\int -\frac{25-8x}{2x^4(x+1)^{3/2}}dx-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a-ax}\left(-\frac{1}{8}\int \frac{25-8x}{x^4(x+1)^{3/2}}dx-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{8}\left(\frac{223}{6}\int \frac{1}{x^3(x+1)^{3/2}}dx+\frac{25}{3x^3\sqrt{x+1}}\right)-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5}{4}\int \frac{1}{x^2(x+1)^{3/2}}dx-\frac{1}{2x^2\sqrt{x+1}}\right)+\frac{25}{3x^3\sqrt{x+1}}\right)-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5}{4}\left(-\frac{3}{2}\int\frac{1}{x(x+1)^{3/2}}dx-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)+\frac{25}{3x^3\sqrt{x+1}}\right)-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5}{4}\left(-\frac{3}{2}\left(\int\frac{1}{x\sqrt{x+1}}dx+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)+\frac{25}{3x^3\sqrt{x+1}}\right)-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5}{4}\left(-\frac{3}{2}\left(2\int\frac{1}{x}d\sqrt{x+1}+\frac{2}{\sqrt{x+1}}\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)+\frac{25}{3x^3\sqrt{x+1}}\right)-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

$$\Big\downarrow \; 220$$

$$\frac{\sqrt{a-ax}\left(\frac{1}{8}\left(\frac{223}{6}\left(-\frac{5}{4}\left(-\frac{3}{2}\left(\frac{2}{\sqrt{x+1}}-2\text{arctanh}\left(\sqrt{x+1}\right)\right)-\frac{1}{x\sqrt{x+1}}\right)-\frac{1}{2x^2\sqrt{x+1}}\right)+\frac{25}{3x^3\sqrt{x+1}}\right)-\frac{1}{4x^4\sqrt{x+1}}\right)}{\sqrt{1-x}}$$

input
```
Int[((1 - x)^(3/2)*Sqrt[a - a*x])/(x^5*(1 + x)^(3/2)),x]
```

output
```
(Sqrt[a - a*x]*(-1/4*1/(x^4*Sqrt[1 + x]) + (25/(3*x^3*Sqrt[1 + x]) + (223*
(-1/2*1/(x^2*Sqrt[1 + x]) - (5*(-(1/(x*Sqrt[1 + x])) - (3*(2/Sqrt[1 + x] -
 2*ArcTanh[Sqrt[1 + x]]))/2))/4))/6)/8))/Sqrt[1 - x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.45

method	result	siz
default	$-\frac{\left(3345 \operatorname{arctanh}\left(\frac{\sqrt{a(1+x)}}{\sqrt{a}}\right)x^{4}\sqrt{a(1+x)}-3345x^{4}\sqrt{a}-1115x^{3}\sqrt{a}+446x^{2}\sqrt{a}-200\sqrt{a}\,x+48\sqrt{a}\right)\sqrt{-a(-1+x)}}{192\sqrt{1+x}\,\sqrt{1-x}\,\sqrt{a}\,x^{4}}$	89
risch	$-\frac{(3345x^{4}+1115x^{3}-446x^{2}+200x-48)\sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\,(-1+x)a}{192x^{4}\sqrt{a(1+x)}\,\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a(-1+x)}}+\frac{1115\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{ax+a}}{\sqrt{a}}\right)\sqrt{-\frac{(1-x)(1+x)a}{-1+x}}\,(-1+x)}{64\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-a(-1+x)}}$	13

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^5/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(3345*arctanh((a*(1+x))^(1/2)/a^(1/2))*x^4*(a*(1+x))^(1/2)-3345*x^4
*a^(1/2)-1115*x^3*a^(1/2)+446*x^2*a^(1/2)-200*a^(1/2)*x+48*a^(1/2))/(1+x)^
(1/2)/(1-x)^(1/2)*(-a*(-1+x))^(1/2)/a^(1/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \left[-\frac{2\,(3345\,x^4+1115\,x^3-446\,x^2+200\,x-48)\sqrt{-ax+a}\sqrt{x+1}\sqrt{-x+1}-3}{384\,(x^6-x^4)}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^5/(1+x)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/384*(2*(3345*x^4 + 1115*x^3 - 446*x^2 + 200*x - 48)*sqrt(-a*x + a)*sqr
t(x + 1)*sqrt(-x + 1) - 3345*(x^6 - x^4)*sqrt(a)*log(-(a*x^2 + a*x + 2*sqr
t(-a*x + a)*sqrt(a)*sqrt(x + 1)*sqrt(-x + 1) - 2*a)/(x^2 - x)))/(x^6 - x^4
), -1/192*((3345*x^4 + 1115*x^3 - 446*x^2 + 200*x - 48)*sqrt(-a*x + a)*sqr
t(x + 1)*sqrt(-x + 1) + 3345*(x^6 - x^4)*sqrt(-a)*arctan(sqrt(-a*x + a)*sq
rt(-a)*sqrt(x + 1)*sqrt(-x + 1)/(a*x - a)))/(x^6 - x^4)]
```

Sympy [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-a(x-1)}(1-x)^{\frac{3}{2}}}{x^5(x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-x)**(3/2)*(-a*x+a)**(1/2)/x**5/(1+x)**(3/2),x)
```

output
```
Integral(sqrt(-a*(x - 1))*(1 - x)**(3/2)/(x**5*(x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \int \frac{\sqrt{-ax+a}(-x+1)^{\frac{3}{2}}}{(x+1)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^5/(1+x)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-a*x + a)*(-x + 1)^(3/2)/((x + 1)^(3/2)*x^5), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.34

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \frac{1}{384}\sqrt{a}\left(\frac{3072}{\sqrt{x+1}} + \frac{2\left(1809\,(x+1)^{\frac{7}{2}} - 6121\,(x+1)^{\frac{5}{2}} + 7063\,(x+1)^{\frac{3}{2}} - 2799\right.}{x^4}\right.$$

input
```
integrate((1-x)^(3/2)*(-a*x+a)^(1/2)/x^5/(1+x)^(3/2),x, algorithm="giac")
```

output
```
1/384*sqrt(a)*(3072/sqrt(x + 1) + 2*(1809*(x + 1)^(7/2) - 6121*(x + 1)^(5/
2) + 7063*(x + 1)^(3/2) - 2799*sqrt(x + 1))/x^4 - 3345*log(sqrt(x + 1) + 1
) + 3345*log(abs(sqrt(x + 1) - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \int \frac{(1-x)^{3/2}\,\sqrt{a-a\,x}}{x^5\,(x+1)^{3/2}}\,dx$$

input
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^5*(x + 1)^(3/2)),x)
```

output
```
int(((1 - x)^(3/2)*(a - a*x)^(1/2))/(x^5*(x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.33

$$\int \frac{(1-x)^{3/2}\sqrt{a-ax}}{x^5(1+x)^{3/2}}\,dx = \frac{\sqrt{a}\left(3345\sqrt{x+1}\log\left(\sqrt{x+1}-1\right)x^4 - 3345\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)x^4 + 669\right.}{384\sqrt{x+1}\,x^4}$$

input
```
int((1-x)^(3/2)*(-a*x+a)^(1/2)/x^5/(1+x)^(3/2),x)
```

output
```
(sqrt(a)*(3345*sqrt(x + 1)*log(sqrt(x + 1) - 1)*x**4 - 3345*sqrt(x + 1)*lo
g(sqrt(x + 1) + 1)*x**4 + 6690*x**4 + 2230*x**3 - 892*x**2 + 400*x - 96))/
(384*sqrt(x + 1)*x**4)
```

3.42 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$

Optimal result

Integrand size = 35, antiderivative size = 276

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx &= -\frac{1182926269\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1603800} \\ &\quad -\frac{12243139\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{356400} \\ &\quad -\frac{17561\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}{8910} \\ &\quad -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}{2970} \\ &\quad +\frac{2}{55}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^4 \\ &\quad -\frac{6489123157\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{699840\sqrt{-5+2x}} \\ &\quad +\frac{671693869\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{174960\sqrt{-5+2x}}\end{aligned}$$

output
```
-1182926269/1603800*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-12243139/35
6400*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)-17561/8910*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2-427/2970*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3+2/55*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x
)^(1/2)*(7+5*x)^4-6489123157/1399680*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11
*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+671693869/349920*22^(1/2)*
(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2
)
```

Mathematica [A] (verified)

Time = 4.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$$
$$= \frac{24\sqrt{2-3x}\sqrt{1+4x}(3325071575 - 797747975x - 670058262x^2 - 167736600x^3 + 67338000x^4 + 291600}{}$$

input
```
Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

output
```
(24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(3325071575 - 797747975*x - 670058262*x^2
- 167736600*x^3 + 67338000*x^4 + 29160000*x^5) - 71380354727*Sqrt[66]*Sqrt
[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 57438413230*S
qrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(1
5396480*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {179, 25, 2103, 27, 2103, 27, 2103, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3\,dx$$

$$\Big\downarrow \ 179$$

$$\frac{1}{55}\int -\frac{(5x+7)^3\left(-854x^2+1190x+3\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$$\Big\downarrow \ 25$$

$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4-\frac{1}{55}\int \frac{(5x+7)^3\left(-854x^2+1190x+3\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \ 2103$$

$$\frac{1}{55}\left(\frac{1}{216}\int -\frac{2(5x+7)^2\left(-983416x^2+796645x+193137\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{427}{54}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3\right)+ \frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$$\Big\downarrow \ 27$$

$$\frac{1}{55}\left(-\frac{1}{108}\int \frac{(5x+7)^2\left(-983416x^2+796645x+193137\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{427}{54}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3\right)+ \frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$$\Big\downarrow \ 2103$$

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{168}\int -\frac{56(5x+7)\left(-36729417x^2+11636345x+10149544\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{35122}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5\right.\right. \frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$$\Big\downarrow \ 27$$

$$\frac{1}{55}\left(\frac{1}{108}\left(-\frac{1}{3}\int \frac{(5x+7)\left(-36729417x^2+11636345x+10149544\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{35122}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+\right.\right. \frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$$\Big\downarrow \ 2103$$

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{120}\int -\frac{3(-18926820304x^2-2853602035x+5865927653)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{12243139}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\downarrow$ 27

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(-\frac{1}{40}\int \frac{-18926820304x^2-2853602035x+5865927653}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{12243139}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\downarrow$ 2118

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{1}{108}\int \frac{79860(15398385-53629117x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{4731705076}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)-\frac{1224}{20}\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\downarrow$ 27

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\int \frac{15398385-53629117x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{4731705076}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)-\frac{12243139}{20}\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\downarrow$ 176

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\left(-\frac{237348815}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{53629117}{2}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)-\right.\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\downarrow$ 124

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\left(-\frac{53629117\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{2\sqrt{5-2x}}-\frac{237348815}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right.\right.\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\downarrow$ 123

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\left(-\frac{237348815}{2}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{53629117\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\right.\right.}{2\sqrt{5-2x}}\right.\right.\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\Big\downarrow$ 131

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\left(-\frac{21577165\sqrt{\frac{11}{2}}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{53629117\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\right.}{2\sqrt{5-2x}}\right.\right.\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\Big\downarrow$ 27

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\left(-\frac{237348815\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{2\sqrt{2x-5}}-\frac{53629117\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\right.\right.}{2\sqrt{5-2x}}\right.\right.\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

$\Big\downarrow$ 129

$$\frac{1}{55}\left(\frac{1}{108}\left(\frac{1}{3}\left(\frac{1}{40}\left(-\frac{6655}{9}\left(-\frac{21577165\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{53629117\sqrt{\frac{11}{6}}\sqrt{2}}{}\right.\right.\right.\right.\right.$$
$$\frac{2}{55}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^4$$

input
```
Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^4)/55 + ((-427*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/54 + ((-35122*Sqrt[2
- 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/3 + ((-12243139*Sqrt[2 -
3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/20 + ((-4731705076*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 - (6655*((-53629117*Sqrt[11/6]*Sqrt[-5
 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2*Sqrt[5 - 2*x]
) - (21577165*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1
+ 4*x]], 1/3])/Sqrt[-5 + 2*x]))/9)/40)/3)/108)/55
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.56

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(-8398080000x^{7}-15894144000x^{6}+57788380800x^{5}+29554530236\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{Ellipti}\right.}{}$
risch	$-\frac{\left(14580000x^{4}+70119000x^{3}+91429200x^{2}-106456131x-665014315\right)\left(-2+3x\right)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{\left(2-3x\right)\left(-5+2x\right)\left(1+4x\right)}}{641520\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\,\sqrt{2-3x}}-$
elliptic	$\frac{\sqrt{-\left(-2+3x\right)\left(-5+2x\right)\left(1+4x\right)}\left(-\frac{11828459x\sqrt{-24x^{3}+70x^{2}-21x-10}}{71280}-\frac{133002863\sqrt{-24x^{3}+70x^{2}-21x-10}}{128304}-\frac{1026559\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{1}}{7776\sqrt{-24x^{3}}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/15396480*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-8398080000*x^7-15
894144000*x^6+57788380800*x^5+29554530236*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(
1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-71380354727*(1+
4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(
1/2),3^(1/2))+176080611456*x^4+141293068560*x^3-1085513167176*x^2+36071668
6200*x+159603435600)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.25

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx$$
$$= \frac{1}{641520}\left(14580000\,x^4+70119000\,x^3+91429200\,x^2-106456131\,x-665014315\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3}$$
$$-\frac{32008789087}{5038848}\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$+\frac{6489123157}{699840}\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3,x, algorith
m="fricas")
```

output
```
1/641520*(14580000*x^4 + 70119000*x^3 + 91429200*x^2 - 106456131*x - 66501
4315)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 32008789087/5038848*sqr
t(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 6489123157/6998
40*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/10
8, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx=\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**3,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3, x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx=\int (5x+7)^3\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3,x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \int (5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3,x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^3\,dx$$

input
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3,x)
```

output
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3, x)
```

Reduce [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\,dx = \frac{250\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^4}{11} + \frac{64925\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^3}{594} + \frac{126985\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{891} - \frac{11828459\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{71280} - \frac{2413516597\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{9979200} - \frac{6489123157\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{226800} + \frac{1977413581\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{259200}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3,x)
```

output
```
(453600000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**4 + 2181480000*
sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**3 + 2844464000*sqrt(2*x -
5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2 - 3311968520*sqrt(2*x - 5)*sqrt(4*x
 + 1)*sqrt( - 3*x + 2)*x - 4827033194*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( -
3*x + 2) - 571042837816*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*
x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) + 152260845737*int((sqrt(2*x - 5)
*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/19958
400
```

3.43 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 238

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx &= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200}-\frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700}\\ &\quad-\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\\ &\quad-\frac{17746949\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{29160\sqrt{-5+2x}}\\ &\quad+\frac{12899689\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{58320\sqrt{-5+2x}}\end{aligned}$$

output
```
-5256763/97200*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-8141/2700*(2-3*x
)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)-61/270*(2-3*x)^(1/2)*(-5+2*x)
^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2+2/45*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)*(7+5*x)^3-17746949/58320*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x
)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+12899689/116640*22^(1/2)*(5-2*x)^
(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx$$
$$= \frac{6\sqrt{2-3x}\sqrt{1+4x}(6902575-2933650x-1649952x^2+147600x^3+216000x^4)-35493898\sqrt{66}\sqrt{5-2}}{116640\sqrt{-5}}$$

input
```
Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

output
```
(6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(6902575 - 2933650*x - 1649952*x^2 + 147600
*x^3 + 216000*x^4) - 35493898*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt
[3/11]*Sqrt[1 + 4*x]], 1/3] + 27962495*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[Ar
cSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(116640*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {179, 25, 2103, 27, 2103, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\,dx$$

$$\Big\downarrow \text{179}$$

$$\frac{1}{45}\int -\frac{(5x+7)^2\left(-854x^2+1190x+3\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3-\frac{1}{45}\int \frac{(5x+7)^2\left(-854x^2+1190x+3\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{45}\left(\frac{1}{168}\int -\frac{14(5x+7)\left(-97692x^2+72385x+21419\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{61}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\right)+\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\Big\downarrow$ 27

$$\frac{1}{45}\left(-\frac{1}{12}\int \frac{(5x+7)\left(-97692x^2+72385x+21419\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{61}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\right)+\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\Big\downarrow$ 2103

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{120}\int -\frac{12\left(-10513526x^2+724135x+3510157\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{8141}{5}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)\right)-\frac{61}{6}\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\Big\downarrow$ 27

$$\frac{1}{45}\left(\frac{1}{12}\left(-\frac{1}{10}\int \frac{-10513526x^2+724135x+3510157}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{8141}{5}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)\right)-\frac{61}{6}\sqrt{2-3}\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\Big\downarrow$ 2118

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{1}{108}\int \frac{1815(391335-1173352x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5256763}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)-\frac{8141}{5}\sqrt{2-3x}\sqrt{2x}\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\Big\downarrow$ 27

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\int \frac{391335-1173352x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5256763}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)-\frac{8141}{5}\sqrt{2-3x}\sqrt{2x}\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\Big\downarrow$ 176

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\left(-2542045\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-586676\int\frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)-\frac{5256763}{18}\sqrt{2}\right.\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 124

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\left(-\frac{586676\sqrt{2x-5}\int\frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-2542045\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)-\frac{525676}{18}\right.\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 123

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\left(-2542045\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{293338\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 131

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\left(-\frac{231095\sqrt{22}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{293338\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 27

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\left(-\frac{2542045\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{293338\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.$$
$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 129

$$\frac{1}{45}\left(\frac{1}{12}\left(\frac{1}{10}\left(-\frac{605}{36}\left(-\frac{231095\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{293338\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\text{ar}\right.}{\sqrt{5-}}\right.\right.\right.\right.$$

$$\frac{2}{45}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

input
```
Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/45 + ((-61*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/6 + ((-8141*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/5 + ((-5256763*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 - (605*((-293338*Sqrt[22/3]*Sqrt[-5 + 2*x]
*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x] - (231095
*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3]
)/Sqrt[-5 + 2*x]))/36)/10)/12)/45
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(-15552000x^6-4147200x^5+12899689\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-3549389\right.}{116640(24x^3-7}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{959x\sqrt{-24x^3+70x^2-21x-10}}{540}-\frac{276103\sqrt{-24x^3+70x^2-21x-10}}{3888}-\frac{26089\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{Ellipt}}{2592\sqrt{-24x^3+70x^2-21x}}\right.}{}$
risch	$-\frac{(108000x^3+343800x^2+34524x-1380515)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{19440\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{26089\sqrt{22-33x}\,\sqrt{165-66x}}{7776\sqrt{-}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/116640*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-15552000*x^6-414720
0*x^5+12899689*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Elliptic
F(1/11*(11+44*x)^(1/2),3^(1/2))-35493898*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1
/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+125816544*x^4+16
3495440*x^3-604794324*x^2+171873450*x+82830900)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.27

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx &= \frac{1}{19440}\left(108000\,x^3+343800\,x^2+34524\,x-1380515\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\ &\quad -\frac{163224523}{419904}\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\ &\quad +\frac{17746949}{29160}\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)\end{aligned}$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2,x, algorith
m="fricas")
```

output
```
1/19440*(108000*x^3 + 343800*x^2 + 34524*x - 1380515)*sqrt(4*x + 1)*sqrt(2
*x - 5)*sqrt(-3*x + 2) - 163224523/419904*sqrt(-6)*weierstrassPInverse(847
/108, 6655/2916, x - 35/36) + 17746949/29160*sqrt(-6)*weierstrassZeta(847/
108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**2,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2, x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \int (5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2,x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \int (5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2,x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^2\,dx$$

input
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2,x)
```

output
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2, x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx &= \frac{50\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^3}{9} \\ &+ \frac{955\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{54} \\ &+ \frac{959\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{540} \\ &- \frac{1424903\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{75600} \\ &- \frac{17746949\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{9450} \\ &+ \frac{14475109\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{21600}\end{aligned}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2,x)
```

output
```
(840000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**3 + 2674000*sqrt(2
*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2 + 268520*sqrt(2*x - 5)*sqrt(4*
x + 1)*sqrt( - 3*x + 2)*x - 2849806*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*
x + 2) - 283951184*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2)
/(24*x**3 - 70*x**2 + 21*x + 10),x) + 101325763*int((sqrt(2*x - 5)*sqrt(4*
x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/151200
```

3.44 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx &= -\frac{6017\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1890}-\frac{1363}{630}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\\ &\quad-\frac{5}{21}(2-3x)^{3/2}(-5+2x)^{3/2}\sqrt{1+4x}-\frac{954811\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{22680\sqrt{-5+2x}}\\ &\quad+\frac{66187\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{5670\sqrt{-5+2x}}\end{aligned}$$

output
```
-6017/1890*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-1363/630*(2-3*x)^(3/
2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-5/21*(2-3*x)^(3/2)*(-5+2*x)^(3/2)*(1+4*x)^
(1/2)-954811/45360*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^
(1/2),3^(1/2))/(-5+2*x)^(1/2)+66187/11340*22^(1/2)*(5-2*x)^(1/2)*EllipticF
(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx$$
$$= \frac{24\sqrt{2-3x}\sqrt{1+4x}(48475-37975x-6066x^2+5400x^3)-954811\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4\right.\right.}{45360\sqrt{-5+2x}}$$

input
```
Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

output
```
(24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(48475 - 37975*x - 6066*x^2 + 5400*x^3) -
954811*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3] + 724790*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 +
4*x]], 1/3])/(45360*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {171, 27, 171, 27, 171, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)\,dx$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{28}\int \frac{(1249-2176x)\sqrt{2x-5}\sqrt{4x+1}}{2\sqrt{2-3x}}dx+\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\int \frac{(1249-2176x)\sqrt{2x-5}\sqrt{4x+1}}{\sqrt{2-3x}}dx+\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{1}{30}\int\frac{22(3521-3802x)\sqrt{4x+1}}{\sqrt{2-3x}\sqrt{2x-5}}dx\right)+\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\int\frac{(3521-3802x)\sqrt{4x+1}}{\sqrt{2-3x}\sqrt{2x-5}}dx\right)+\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{3802}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1}{9}\int-\frac{22(3255-7891x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\int\frac{3255-7891x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{3802}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{176}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\left(-\frac{32945}{2}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{7891}{2}\int\frac{\sqrt{2x-}}{\sqrt{2-3x}\sqrt{}}\right.\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{124}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\left(-\frac{7891\sqrt{2x-5}\int\frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{2\sqrt{5-2x}}-\frac{32945}{2}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-}}\right.\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$$\Big\downarrow \text{123}$$

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\left(-\frac{32945}{2}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{7891\sqrt{\frac{11}{6}}\sqrt{2x-5}\right.\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$\big\downarrow$ 131

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\left(-\frac{2995\sqrt{\frac{11}{2}}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{7891\sqrt{\frac{11}{6}}\sqrt{2x}\right.\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$\big\downarrow$ 27

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\left(-\frac{32945\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{2\sqrt{2x-5}}-\frac{7891\sqrt{\frac{11}{6}}\sqrt{2x-5}\right.\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

$\big\downarrow$ 129

$$\frac{1}{56}\left(\frac{1088}{15}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{15}\left(\frac{22}{9}\left(-\frac{2995\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\right.\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}(2x-5)^{3/2}(4x+1)^{3/2}$$

input
```
Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

output
```
(5*Sqrt[2 - 3*x]*(-5 + 2*x)^(3/2)*(1 + 4*x)^(3/2))/28 + ((1088*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/15 - (11*((3802*Sqrt[2 - 3*x]*Sqrt[-5 +
 2*x]*Sqrt[1 + 4*x])/9 + (22*((-7891*Sqrt[11/6]*Sqrt[-5 + 2*x]*EllipticE[A
rcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2*Sqrt[5 - 2*x]) - (2995*Sqrt[11/6
]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5
+ 2*x]))/9))/15)/56
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(-1555200x^5+264748\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-954811\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{\ }\right.}{45360(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{59x\sqrt{-24x^3+70x^2-21x-10}}{30}-\frac{277\sqrt{-24x^3+70x^2-21x-10}}{54}-\frac{31\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \text{EllipticF}\left(\sqrt{\ }\right.}{36\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{\ }}$
risch	$-\frac{(2700x^2+3717x-9695)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{1890\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{31\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \text{EllipticF}\left(2\right.}{108\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x),x,method=_RETURNVER
BOSE)
```

output
```
-1/45360*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(-1555200*x^5+264748*(
1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)
^(1/2),3^(1/2))-954811*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*
EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2395008*x^4+10468080*x^3-18808968*
x^2+3994200*x+2326800)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \frac{1}{1890}\left(2700\,x^2+3717\,x-9695\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} - \frac{549703}{23328}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) + \frac{954811}{22680}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x),x, algorithm=
"fricas")
```

output
```
1/1890*(2700*x^2 + 3717*x - 9695)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x +
2) - 549703/23328*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/
36) + 954811/22680*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstras
sPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\cdot(5x+7)\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7), x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \int (5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x),x, algorithm=
"maxima")
```

output
```
integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx = \int (5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x),x, algorithm=
"giac")
```

output
```
integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx=\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)\,\,dx$$

input
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7),x)
```

output
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7), x)
```

Reduce [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\,dx=\frac{10\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{7}+\frac{59\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{30}-\frac{14907\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{9800}-\frac{954811\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{7350}+\frac{556963\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{8400}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x),x)
```

output
```
(84000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2 + 115640*sqrt(2*x
 - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x - 89442*sqrt(2*x - 5)*sqrt(4*x + 1)
*sqrt( - 3*x + 2) - 7638488*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x +
 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) + 3898741*int((sqrt(2*x - 5)*
sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/58800
```

3.45 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 157

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = -\frac{22}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}(1+4x)^{3/2} - \frac{847\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{270\sqrt{-5+2x}} + \frac{121\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{270\sqrt{-5+2x}}$$

output

```
-22/45*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/10*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(3/2)-847/540*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4
*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+121/540*22^(1/2)*(5-2*x)^(1/2)*
EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx$$
$$=\frac{6\sqrt{2-3x}\sqrt{1+4x}(175-250x+72x^2)-847\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)+605\sqrt{66}\sqrt{5-}}{540\sqrt{-5+2x}}$$

input
```
Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

output
```
(6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(175 - 250*x + 72*x^2) - 847*Sqrt[66]*Sqrt[
5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 605*Sqrt[66]*S
qrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(540*Sqrt[-
5 + 2*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {112, 27, 171, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,dx$$

$$\Big\downarrow \text{112}$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{1}{10}\int \frac{11(9-8x)\sqrt{4x+1}}{2\sqrt{2-3x}\sqrt{2x-5}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\int \frac{(9-8x)\sqrt{4x+1}}{\sqrt{2-3x}\sqrt{2x-5}}dx$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{8}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1}{9}\int-\frac{11(15-28x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)$$

$\downarrow$ 27

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{11}{9}\int\frac{15-28x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{8}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)$$

$\downarrow$ 176

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{11}{9}\left(-55\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-14\int\frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{8}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)$$

$\downarrow$ 124

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{11}{9}\left(-\frac{14\sqrt{2x-5}\int\frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-55\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{8}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)$$

$\downarrow$ 123

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{11}{9}\left(-55\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{7\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{8}{9}\sqrt{2-3x}\sqrt{2x\right.$$

$\downarrow$ 131

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{11}{9}\left(-\frac{5\sqrt{22}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{7\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{8}{9}\sqrt{2-3x\right.$$

$\downarrow$ 27

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-\frac{11}{20}\left(\frac{11}{9}\left(-\frac{55\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{7\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{8}{9}\sqrt{2-3x}\sqrt{2x\right.$$

$$\downarrow 129$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}-$$
$$\frac{11}{20}\left(\frac{11}{9}\left(-\frac{5\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{7\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right.\right.$$

input
```
Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/10 - (11*((8*Sqrt[2 - 3*x]*
Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (11*((-7*Sqrt[22/3]*Sqrt[-5 + 2*x]*Ellip
ticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x] - (5*Sqrt[22/3]
*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 +
 2*x]))/9))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\bigg(121\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-847\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticE}}{540(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\Bigg(\frac{2x\sqrt{-24x^3+70x^2-21x-10}}{5}-\frac{7\sqrt{-24x^3+70x^2-21x-10}}{18}-\frac{\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44}}{11}\right.}{12\sqrt{-24x^3+70x^2-21x-10}}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{(-35+36x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{90\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\Bigg(-\frac{\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{36\sqrt{-24x^3+70x^2-21x-10}}}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/540*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(121*(1+4*x)^(1/2)*(2-3*
x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-84
7*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2))-5184*x^4+20160*x^3-19236*x^2+2250*x+2100)/(24*x^3-70*x^
2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx$$
$$= \frac{1}{90}\,(36\,x-35)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}$$
$$-\frac{1331}{972}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$+\frac{847}{270}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas"
)
```

output
```
1/90*(36*x - 35)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 1331/972*sqr
t(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 847/270*sqrt(-6
)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/29
16, x - 35/36))
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1), x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,dx$$

input
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2),x)
```

output
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,dx = \frac{2\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{5} - \frac{3\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{25} - \frac{242\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{25} + \frac{363\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{50}$$

input

```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x)
```

output

```
(20*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x - 6*sqrt(2*x - 5)*sqrt(
4*x + 1)*sqrt( - 3*x + 2) - 484*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3
*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) + 363*int((sqrt(2*x - 5)*
sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/50
```

3.46 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 176

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx\\
&=\frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{427\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{225\sqrt{-5+2x}}\\
&\quad-\frac{27244\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{1125\sqrt{-5+2x}}\\
&\quad+\frac{4836\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{125\sqrt{-5+2x}}
\end{aligned}$$

output
```
2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/450*22^(1/2)*(5-2*x)^(
1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-27244/1
2375*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))
/(-5+2*x)^(1/2)+4836/1375*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1+4*x)^(
1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \frac{\sqrt{-5+2x}\left(1650\sqrt{2-3x}\sqrt{5-2x}\sqrt{1+4x}-23485\sqrt{11}E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-3759\sqrt{11}\,\text{EllipticF}\left(\right.\right.}{12375\sqrt{5-2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

output
```
(Sqrt[-5 + 2*x]*(1650*Sqrt[2 - 3*x]*Sqrt[5 - 2*x]*Sqrt[1 + 4*x] - 23485*Sq
rt[11]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 3759*Sqrt[11]
*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 24219*Sqrt[11]*Elli
pticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(12375*Sqrt[5 -
 2*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {179, 25, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5x+7}\,dx$$

$$\Big\downarrow \text{179}$$

$$\frac{1}{15}\int -\frac{-854x^2+1190x+3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1}{15}\int \frac{-854x^2+1190x+3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \text{2110}$$

$$\frac{1}{15}\left(\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\int \frac{\frac{11928}{25}-\frac{854x}{5}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 176

$$\frac{1}{15}\left(-\frac{1253}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{427}{5}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 124

$$\frac{1}{15}\left(\frac{427\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{5\sqrt{5-2x}}-\frac{1253}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 123

$$\frac{1}{15}\left(-\frac{1253}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{427\sqrt{\frac{11}{6}}\sqrt{2x-}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 131

$$\frac{1}{15}\left(-\frac{1253\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{427\sqrt{\frac{11}{6}}\sqrt{2}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 27

$$\frac{1}{15}\left(-\frac{1253\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{427\sqrt{\frac{11}{6}}\sqrt{2x-5}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{129}$$

$$\frac{1}{15}\left(\frac{83421}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{186}$$

$$\frac{1}{15}\left(-\frac{166842}{25}\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\right.}{25\sqrt{2x}}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(-\frac{500526}{25}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\right.}{25\sqrt{2x}}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{413}$$

$$\frac{1}{15}\left(-\frac{500526\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{a}\right.}{25\sqrt{2x}}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(-\frac{500526\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{a}\right.}{25\sqrt{2x}}\right.$$
$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{15}\left(-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{427\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}\right.$$

$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/15 + ((427*Sqrt[11/6]*Sqrt[
-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(5*Sqrt[5 - 2*
x]) - (1253*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 +
4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (8073*Sqrt[11 + 2*(2 - 3*x)]*EllipticPi
[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]*Sqrt[-11
- 2*(2 - 3*x)]))/15
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(54488\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+23485\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{Elliptic}}{594000x^3-1732500x^2+}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{-24x^3+70x^2-21x-10}}{15}-\frac{3976\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{15125\sqrt{-24x^3+70x^2-21x-10}}+\frac{854\sqrt{11+44x}\,\sqrt{2}}{}\right.}{\sqrt{2-3x}}$
risch	$-\frac{2(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{15\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{3976\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{45375\sqrt{-24x^3+70x^2-21x-10}}+\frac{854}{}\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x,method=_RETURNVER
BOSE)
```

output
```
1/24750*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(54488*(1+4*x)^(1/2)*(2
-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))
+23485*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(
11+44*x)^(1/2),3^(1/2))-87048*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)
^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))+79200*x^3-231000*x^
2+69300*x+33000)/(24*x^3-70*x^2+21*x+10)
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{5\,x+7}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm=
"fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5x+7}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{5x+7}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{5x+7}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm=
"giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{5\,x+7}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \int \frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}}{7+5x}dx$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x)
```

output
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x)
```

3.47 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx &= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)}+\frac{3\sqrt{22}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{25\sqrt{-5+2x}}\\&\quad+\frac{482\sqrt{\frac{2}{11}}\sqrt{5-2x}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{125\sqrt{-5+2x}}\\&\quad-\frac{17906\sqrt{\frac{2}{11}}\sqrt{5-2x}\operatorname{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{2875\sqrt{-5+2x}}\end{aligned}$$

output

```
-1/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)+3/25*22^(1/2)*(5-2
*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+48
2/1375*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2
))/(-5+2*x)^(1/2)-17906/31625*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1+4*
x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx$$
$$= \frac{\sqrt{-5+2x}\left(-\frac{51150\sqrt{2-3x}\sqrt{1+4x}}{7+5x} + \frac{3\sqrt{11}\left(20460E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+9424\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-26859\,\text{EllipticPi}(\right.}{\sqrt{5-2x}}\right.}{255750}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

output
```
(Sqrt[-5 + 2*x]*((-51150*Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (3*Sqrt[
11]*(20460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 9424*Elli
pticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 26859*EllipticPi[55/124,
 ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/255750
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {178, 25, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^2}\,dx$$

$$\Big\downarrow \text{178}$$

$$\frac{1}{10}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx - \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{10}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx - \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \text{2110}$$

$$\frac{1}{10}\left(-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\int \frac{\frac{72x}{5}-\frac{1204}{25}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \; 176$$

$$\frac{1}{10}\left(\frac{304}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{36}{5}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \; 124$$

$$\frac{1}{10}\left(-\frac{36\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{5\sqrt{5-2x}}+\frac{304}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \; 123$$

$$\frac{1}{10}\left(\frac{304}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{6\sqrt{66}\sqrt{2x-5}E\left(\mathrm{arc}\right.}{5\sqrt{}}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \; 131$$

$$\frac{1}{10}\left(\frac{304\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{6\sqrt{66}\sqrt{2x-5}E}{}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(\frac{304\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{6\sqrt{66}\sqrt{2x-5}E\left(\text{arc}\right.}{5\sqrt{}}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$\downarrow$ 129

$$\frac{1}{10}\left(-\frac{8953}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{304\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}-\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$\downarrow$ 186

$$\frac{1}{10}\left(\frac{17906}{25}\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{304\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{arcs}\right.}{25\sqrt{2x-}}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$\downarrow$ 27

$$\frac{1}{10}\left(\frac{53718}{25}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{304\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{arcs}\right.}{25\sqrt{2x-}}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$\downarrow$ 413

$$\frac{1}{10}\left(\frac{53718\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}+\frac{304\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{arcsin}\right.}{25\sqrt{2x-5}}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$\downarrow$ 27

$$\frac{1}{10}\left(\frac{53718\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}+\frac{304\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{arcsin}\right.}{25\sqrt{2x-5}}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{10}\left(\frac{304\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}-\frac{6\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}+\frac{26}{}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

output
```
-1/5*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x) + ((-6*Sqrt[66
]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(5*Sqrt
[5 - 2*x]) + (304*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqr
t[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (26859*Sqrt[11 + 2*(2 - 3*x)]*Ell
ipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(775*Sqrt[11]*S
qrt[-11 - 2*(2 - 3*x)]))/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{5(7+5x)}+\frac{602\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{15125\sqrt{-24x^3+70x^2-21x-10}}-\frac{36\sqrt{11+44x}\,\sqrt{22\cdots}}{\cdots}\right.}{\sqrt{2-3x}\cdots}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(55430\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x+18975\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{Elli}\cdots\right.}{\cdots}$
risch	$\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{5(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{602\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{45375\sqrt{-24x^3+70x^2-21x-10}}-\frac{12\sqrt{22-33x}\cdots}{\cdots}\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x,method=_RETURNV
ERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-1/5/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+602/15125*(11+44*x)^(1/2)
*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF
(1/11*(11+44*x)^(1/2),3^(1/2))-36/3025*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(11
0-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))-17906/3478
75*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-1
0)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 + 70*x + 49),
x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^2}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorith
m="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{(5x+7)^2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^2}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2,x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = \int \frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}}{\left(7+5x\right)^2}dx$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x)
```

output
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x)
```

3.48 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx &= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2}+\frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} \\ &\quad -\frac{8953\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{1390350\sqrt{-5+2x}} \\ &\quad -\frac{7193\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{112125\sqrt{-5+2x}} \\ &\quad +\frac{14832503\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{79945125\sqrt{22}\sqrt{-5+2x}}\end{aligned}$$

output
```
-1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2+8953*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(3892980+2780700*x)-8953/2780700*22^(1/2)
*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/
2)-7193/1233375*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/
2),3^(1/2))/(-5+2*x)^(1/2)+14832503/1758792750*22^(1/2)*(5-2*x)^(1/2)*Elli
pticPi(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx$$
$$= \frac{\sqrt{-5+2x}\left(\frac{17050\sqrt{2-3x}\sqrt{1+4x}(7057+44765x)}{(7+5x)^2}+\frac{\sqrt{11}\left(-61059460E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+5759676\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right.\right.}{\sqrt{5-2x}}\right.}{9482187000}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

output
```
(Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7057 + 44765*x))/(7 +
 5*x)^2 + (Sqrt[11]*(-61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]
], -1/2] + 5759676*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 4
4497509*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqr
t[5 - 2*x]))/9482187000
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {178, 25, 2107, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^3}\,dx$$
$$\Big\downarrow \text{178}$$
$$\frac{1}{20}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{20}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{20}\left(\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}-\frac{\int \frac{-214872x^2+199200x+106729}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}\right)-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{2110}$$

$$\frac{1}{20}\left(\frac{\frac{14832503}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\int \frac{\frac{2500104}{25}-\frac{214872x}{5}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx}{55614}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}\right)-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{176}$$

$$\frac{1}{20}\left(\frac{\frac{185796}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{107436}{5}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+\frac{14832503}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}+\frac{8953\sqrt{2-}}{27}\right.$$
$$-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{124}$$

$$\frac{1}{20}\left(\frac{\frac{107436\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{5\sqrt{5-2x}}+\frac{185796}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{14832503}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}+\frac{8953\sqrt{}}{}\right.$$
$$-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{123}$$

$$\frac{1}{20}\left(\frac{\frac{185796}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{14832503}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{17906\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}}{55614}\right.$$
$$-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{131}$$

$$\frac{1}{20}\left(\frac{\frac{185796\sqrt{\frac{2}{11}}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{14832503}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{17906\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x\right.\right.}{5\sqrt{5-2x}}}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$\downarrow$ 27

$$\frac{1}{20}\left(\frac{\frac{185796\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{14832503}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{17906\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)\right|}{5\sqrt{5-2x}}}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$\downarrow$ 129

$$\frac{1}{20}\left(\frac{\frac{14832503}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{61932\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{17906\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\right.\right.}{5\sqrt{5-2x}}}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$\downarrow$ 186

$$\frac{1}{20}\left(\frac{-\frac{29665006}{25}\int\frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{61932\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$\downarrow$ 27

$$\frac{1}{20}\left(\frac{-\frac{88995018}{25}\int\frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{61932\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{413}$$

$$\frac{1}{20}\left(\frac{-\frac{88995018\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}+\frac{61932\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}\left(\frac{-\frac{88995018\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}+\frac{61932\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{20}\left(\frac{\frac{61932\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{17906\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}-\frac{44497509\sqrt{2(2-3x)+11}\,\text{Elli}}{775\sqrt{11}}}{55614}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{10(5x+7)^2}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

output
```
-1/10*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2 + ((8953*Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + ((17906*Sqrt
[66]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(5*S
qrt[5 - 2*x]) + (61932*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (44497509*Sqrt[11 + 2*(2 - 3
*x)]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(775*Sq
rt[11]*Sqrt[-11 - 2*(2 - 3*x)]))/55614)/20
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{10(7+5x)^2}+\frac{8953\sqrt{-24x^3+70x^2-21x-10}}{556140(7+5x)}-\frac{104171\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \text{EllipticF}}{140193625\sqrt{-24x^3+70x^2-21x-}}\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(7057+44765x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{556140(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{104171\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \text{EllipticF}\left(\frac{2\sqrt{22-}}{11}\right.}{420580875\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(512860900\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2+283138625\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-}\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x,method=_RETURNV
ERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-1/10/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(1/2)+8953/556140/(7+5*x)*(-
24*x^3+70*x^2-21*x-10)^(1/2)-104171/140193625*(11+44*x)^(1/2)*(22-33*x)^(1
/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*
x)^(1/2),3^(1/2))+8953/28038725*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)
^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/
2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))+14832503/19346720
250*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-
10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^3}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3 + 525*x^2 + 7
35*x + 343), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^3}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^3}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorith
m="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^3}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorith
m="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^3}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3,x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = \int \frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}}{(7+5x)^3}dx$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x)
```

output
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x)
```

3.49 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 255

$$\begin{aligned}
\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx
&= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3}+\frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} \\
&\quad+\frac{16830401\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{30929169960(7+5x)} \\
&\quad-\frac{16830401\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{77322924900\sqrt{-5+2x}} \\
&\quad+\frac{768719\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{3117859875\sqrt{22}\sqrt{-5+2x}} \\
&\quad-\frac{15664616449\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{4446068181750\sqrt{22}\sqrt{-5+2x}}
\end{aligned}$$

output
```
-1/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3+8953/1668420*(2
-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2+16830401*(2-3*x)^(1/2)*
(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(216504189720+154645849800*x)-16830401/154645
849800*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2
))/(-5+2*x)^(1/2)+768719/68592917250*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11
*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-15664616449/97813499998500
*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1
/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \frac{\sqrt{-5+2x}\left(\frac{17050\sqrt{2-3x}\sqrt{1+4x}(-75460017+2007981640x+420760025x^2)}{(7+5x)^3}+\frac{\sqrt{11}\left(-114783334820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+1206932\right.}{}\right.}{527342347818000}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

output
```
(Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-75460017 + 200798164
0*x + 420760025*x^2))/(7 + 5*x)^3 + (Sqrt[11]*(-114783334820*EllipticE[Arc
Sin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 120693246492*EllipticF[ArcSin[(2*
Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 46993849347*EllipticPi[55/124, ArcSin[(2
*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/527342347818000
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.13, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {178, 25, 2107, 2107, 27, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^4}dx$$

$$\Big\downarrow \ 178$$

$$\frac{1}{30}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}dx-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{30}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}dx-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$$\Big\downarrow \ 2107$$

$$\frac{1}{30}\left(\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}-\frac{\int \frac{214872x^2-855020x+401471}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx}{111228}\right)-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$$\Big\downarrow \ 2107$$

$$\frac{1}{30}\left(\frac{\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}-\frac{\int \frac{3\left(-403929624x^2-334343520x+950205793\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}}{111228}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}\right)-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{30}\left(\frac{\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}-\frac{\int \frac{-403929624x^2-334343520x+950205793}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}}{111228}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}\right)-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$$\Big\downarrow \ 2110$$

$$\frac{1}{30}\left(\frac{\frac{-\int \frac{\frac{1155789768}{25}-\frac{403929624x}{5}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}+\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}+\frac{8953\sqrt{2-3x}\sqrt{2}}{55614(5x}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 176

$$\frac{1}{30}\left(\frac{\frac{\frac{3893330532}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{201964812}{5}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}+\frac{16830401\sqrt{2-3x}\sqrt{2x-}}{9269(5x+7)}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 124

$$\frac{1}{30}\left(\frac{\frac{\frac{201964812\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{5\sqrt{5-2x}}+\frac{3893330532}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}+\frac{16830401\sqrt{2-3x}}{9269(5}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 123

$$\frac{1}{30}\left(\frac{\frac{\frac{3893330532}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{33660802\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}}{18538}+16830}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 131

$$\frac{1}{30}\left(\frac{\frac{\frac{3893330532\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{33660802\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\downarrow$ 27

$$\frac{1}{30}\left(\frac{\frac{\frac{3893330532\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{33660802\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}}{18538}+}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\downarrow$ 129

$$\frac{1}{30}\left(\frac{\frac{-\frac{15664616449}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{1297776844\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{33660802\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{\right.\right.}{5\sqrt{5-2x}}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\downarrow$ 186

$$\frac{1}{30}\left(\frac{\frac{\frac{31329232898}{25}\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{1297776844\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{33660802\sqrt{66}\sqrt{2x}}{}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\downarrow$ 27

$$\frac{1}{30}\left(\frac{\frac{\frac{93987698694}{25}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{1297776844\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{33660802\sqrt{66}\sqrt{2x}}{}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 413

$$\frac{1}{30}\left(\frac{\frac{\frac{93987698694\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}+\frac{1297776844\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{336608}{}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 27

$$\frac{1}{30}\left(\frac{\frac{\frac{93987698694\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}+\frac{1297776844\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{336608}{}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

$\big\downarrow$ 412

$$\frac{1}{30}\left(\frac{\frac{\frac{1297776844\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{33660802\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}+\frac{46993849347\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\right.}{775\sqrt{11}\sqrt{-2(2-3}}}{18538}}{111228}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

output
```
-1/15*(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3 + ((8953*Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) + ((16830401
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(9269*(7 + 5*x)) + ((33660802
*Sqrt[66]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(5*Sqrt[5 - 2*x]) + (1297776844*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin
[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (46993849347*Sqrt[
11 + 2*(2 - 3*x)]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(775*Sqrt[11]*Sqrt[-11 - 2*(2 - 3*x)]))/18538)/111228)/30
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{\sqrt{-24x^3+70x^2-21x-10}}{15(7+5x)^3}+\frac{8953\sqrt{-24x^3+70x^2-21x-10}}{1668420(7+5x)^2}+\frac{16830401\sqrt{-24x^3+70x^2-21x-10}}{30929169960(7+5x)}-\frac{48157907\sqrt{}}{}\right.}{}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(420760025x^2+2007981640x-75460017)\sqrt{(2-3x)(-5+2x)(1+4x)}}{30929169960(7+5x)^3\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{48157907\sqrt{22-33x}\,\sqrt{165-6}}{23390184782}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(274048323500\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^3-2661307158125\sqrt{1+4x}\,\sqrt{2-3}\right.}{}$

input

```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x,method=_RETURNV
ERBOSE)
```

output

```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-1/15/(7+5*x)^3*(-24*x^3+70*x^2-21*x-10)^(1/2)+8953/1668420/(7+5*x)^2
*(-24*x^3+70*x^2-21*x-10)^(1/2)+16830401/30929169960/(7+5*x)*(-24*x^3+70*x
^2-21*x-10)^(1/2)-48157907/7796728260750*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(
110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1
/2),3^(1/2))+16830401/1559345652150*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-4
4*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)
^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2)))-15664616449/1
075948499983500*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+
70*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4 + 3500*x^3 +
7350*x^2 + 6860*x + 2401), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^4}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**4,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorith
m="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorith
m="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^4}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4,x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = \int \frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}}{(7+5x)^4}dx$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x)
```

output
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x)
```

3.50 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 570

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$
$$-\frac{2\sqrt{-de+cf}(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{-de+cf}(3a^2dfh^2-3ab(de+cf)h^2-b^2(dg(fg-eh)-ch(fg+2eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{Ellipt}}{3b^3d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2(be-af)\sqrt{-de+cf}(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2/3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b-2/3*(c*f-d*e)^(1/2)*(3*a*d
*f*h-b*(c*f*h+d*e*h+d*f*g))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*Ell
ipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(
1/2))/b^2/d/f^(1/2)/h/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(c*f-
d*e)^(1/2)*(3*a^2*d*f*h^2-3*a*b*(c*f+d*e)*h^2-b^2*(d*g*(-e*h+f*g)-c*h*(2*e
*h+f*g)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*Ellipt
icF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2
))/b^3/d/f^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*f+b*e)*(c*f-d*e)^(1/2
)*(-a*h+b*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*Ell
ipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,(
(-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/b^3/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.43 (sec) , antiderivative size = 1254, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

output
```
(2*Sqrt[c + d*x]*(3*b^2*e*g - 3*a*b*f*g + (b^2*f*g^2)/h - 3*a*b*e*h + (b^2
*e^2*h)/f - (b^2*c^2*f*h)/d^2 + (3*a*b*c*f*h)/d + 2*b^2*f*g*x + 2*b^2*e*h*
x - 3*a*b*f*h*x + (b^2*c*f*h*x)/d + b^2*f*h*x^2 - (b^2*c*e*g)/(c + d*x) -
(3*a*b*d*e*g)/(c + d*x) + (3*a*b*c*f*g)/(c + d*x) + (3*a*b*c*e*h)/(c + d*x
) + (b^2*c^3*f*h)/(d^2*(c + d*x)) - (3*a*b*c^2*f*h)/(d*(c + d*x)) + (b^2*d
*e^2*g)/(c*f + d*f*x) - (b^2*c*e^2*h)/(c*f + d*f*x) + (b^2*d*e*g^2)/(c*h +
 d*h*x) - (b^2*c*f*g^2)/(c*h + d*h*x) - (I*b*Sqrt[-c + (d*e)/f]*(3*a*d*f*h
 - b*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x
))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/
f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/d^2 + (I*b*Sqrt[-c +
(d*e)/f]*(-2*b*f*g - b*e*h + 3*a*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*
(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c
+ (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/d + ((3*I)*b^
2*e*Sqrt[-c + (d*e)/f]*f*g*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]
*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b
*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*
h - c*f*h)])/(d*e - c*f) + ((3*I)*a*b*Sqrt[-c + (d*e)/f]*f^2*g*Sqrt[c + d*
x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*Ell
ipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/S
qrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/(-(d*e) + c*f) + ((3*I...
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {179, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$$

$$\Big\downarrow \text{179}$$

$$\frac{\int \frac{-\left(\left(3adfh-b(dfg+deh+cfh)\right)x^2\right)+2\left(b(deg+cfg+ceh)-a(dfg+deh+cfh)\right)x+3bceg-a(deg+cfg+ceh)}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3b}+\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$$\Big\downarrow \text{2110}$$

$$\frac{\int \frac{\frac{3dfha^2}{b^2}-\frac{3dfga}{b}-\frac{3deha}{b}-\frac{3cfha}{b}+2deg+2cfg+2ceh+\left(dfg+deh+cfh-\frac{3adfh}{b}\right)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{3(bc-ad)(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{b^2}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 176

$$\frac{\frac{\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh+fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{3(bc-ad)(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{b^2}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 124

$$\frac{\frac{\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh+fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{3(bc-ad)(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{b^2}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 123

$$\frac{\frac{\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh+fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{3(bc-ad)(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{b^2}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 131

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh+fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b^2h\sqrt{e+fx}}+\frac{3(bc-ad)(be-af)(bg-ah)\int \frac{}{(a+bx}}{b^2}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 131

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh+fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b^2h\sqrt{e+fx}\sqrt{g+hx}}+\frac{3(bc-ad)(be-}{}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 130

$$\frac{\frac{3(bc-ad)(be-af)(bg-ah)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}+\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh\right.\right.}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}}{3b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 187

$$-\frac{6(bc-ad)(be-af)(bg-ah)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{b^2}+\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(\right.}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 413

$$-\frac{6(bc-ad)(be-af)(bg-ah)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{b^2\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}+\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2\right.}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 413

$$-\frac{6(bc-ad)(be-af)(bg-ah)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{b^2\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}+\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

$\downarrow$ 412

$$\frac{\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3a^2dfh^2-3abh^2(cf+de)-(b^2(dg(fg-eh)-ch(2eh+fg))))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}-\frac{6(be-a}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3b}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

output
```
(2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) + ((2*Sqrt[-(d*e) + c*
f]*(d*e*h + c*f*h + d*f*(g - (3*a*h)/b))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*S
qrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
 ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g +
 h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a^2*d*f*h^2 - 3*a*b*(d*e +
 c*f)*h^2 - b^2*(d*g*(f*g - e*h) - c*h*(f*g + 2*e*h)))*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqr
t[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*d*
Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]) - (6*(b*e - a*f)*Sqrt[-(d*e) + c*f]
*(b*g - a*h)*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c + d*x))/(d
*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2
*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*
x))/d]))/(3*b)
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.71

method	result
elliptic	$\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3b}+\frac{2\left(\frac{a^2dfh-abcfh-abdeh-abdfg+b^2ceh+b^2cfg+b^2d}{b^3}\right.}{\sqrt{df}}\right.$
default	Expression too large to display

input

```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x,method=_RETURNVERB
OSE)
```

output

```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2/3/b*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*
e*g)^(1/2)+2*((a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g
+b^2*d*e*g)/b^3-2/3/b*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(c/d-e/f)*((x+c/d)/
(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*
f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)
*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))+2*(-1/
b^2*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)-2/3/b*(c*f*h+d*e*h+d*f*g))*(c/d-e/f)
*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))
^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*
e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c
/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+
g/h))^(1/2)))-2*(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h
+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^4*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2
)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x
^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-c/d+a/b)*Ell
ipticPi(((x+c/d)/(c/d-e/f))^(1/2),(-c/d+e/f)/(-c/d+a/b),((-c/d+e/f)/(-c/d+
g/h))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(h*x+g)**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bx+a}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="
maxima")
```

output
```
integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bx+a}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="
giac")
```

output
```
integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}{a+b\,x}\,dx$$

input
```
int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x),x)
```

output
```
int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{a+bx}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}{bx+a}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x)
```

3.51 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 238

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx\\
&= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880}+\frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\
&\quad+\frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2+\frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3\\
&\quad+\frac{2629157597\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{163296\sqrt{-5+2x}}\\
&\quad-\frac{1227098543\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{163296\sqrt{-5+2x}}
\end{aligned}$$

output

```
46134551/38880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+26291/540*(2-3*x
)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)+1679/756*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2+1/9*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)*(7+5*x)^3+2629157597/326592*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1
+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-1227098543/326592*22^(1/2)*(5
-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx$$
$$= \frac{6\sqrt{2-3x}\sqrt{1+4x}(-455686385+51484034x+21329208x^2+8614800x^3+1512000x^4)+2629157597\sqrt{}}{326592}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

output
```
(6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-455686385 + 51484034*x + 21329208*x^2 + 8
614800*x^3 + 1512000*x^4) + 2629157597*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[Ar
cSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 2161804579*Sqrt[66]*Sqrt[5 - 2*x]*E
llipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(326592*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {180, 25, 2103, 27, 2103, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}(5x+7)^3}{\sqrt{2x-5}}dx$$

$$\Big\downarrow \ 180$$

$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3-\frac{1}{18}\int -\frac{(5x+7)^2\left(-3358x^2+565x+699\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{18}\int \frac{(5x+7)^2\left(-3358x^2+565x+699\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$$\Big\downarrow \ 2103$$

$$\frac{1}{18}\left(\frac{1679}{42}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{1}{168}\int -\frac{2(5x+7)\left(-4416888x^2-138145x+993625\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\big\downarrow$ 27

$$\frac{1}{18}\left(\frac{1}{84}\int \frac{(5x+7)\left(-4416888x^2-138145x+993625\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1679}{42}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2\right)+$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\big\downarrow$ 2103

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{368074}{5}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{1}{120}\int -\frac{24\left(-322941857x^2-102379055x+80234014\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\big\downarrow$ 27

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\int \frac{-322941857x^2-102379055x+80234014}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{368074}{5}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)\right)+\frac{1679}{42}\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\big\downarrow$ 2118

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{1}{108}\int \frac{165(228338691-956057308x)}{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{322941857}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{368074}{5}\sqrt{2-3}\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\big\downarrow$ 27

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\int \frac{228338691-956057308x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{322941857}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{368074}{5}\sqrt{2-3x}\sqrt{2x}\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\big\downarrow$ 176

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\left(-2161804579\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-478028654\int\frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{3229418}{36}\right.\right.\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 124

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\left(-\frac{478028654\sqrt{2x-5}\int\frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-2161804579\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{322}{}\right.\right.\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 123

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\left(-2161804579\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{239014327\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x}\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 131

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\left(-\frac{196527689\sqrt{22}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{239014327\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{}\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 27

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\left(-\frac{2161804579\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{239014327\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x}\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.\right.$$
$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

$\downarrow$ 129

$$\frac{1}{18}\left(\frac{1}{84}\left(\frac{1}{5}\left(\frac{55}{72}\left(-\frac{196527689\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{239014327\sqrt{\frac{22}{3}}\sqrt{2x-5}E(}{\sqrt{5}}\right.\right.\right.\right.$$

$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/9 + ((1679*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/42 + ((368074*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/5 + ((322941857*Sqrt[2 - 3*x]*
Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 + (55*((-239014327*Sqrt[22/3]*Sqrt[-5 + 2
*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x] - (196
527689*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]]
, 1/3])/Sqrt[-5 + 2*x]))/72)/5)/84)/18
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 180
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[2*(a + b*x)^m*Sqrt[c + d*x]*Sq
rt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Simp[1/(d*(2*m + 3))   Int[
((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*b*c*
e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e*h)*(
2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m + b*(d*
(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(108864000x^6+574905600x^5+1227098543\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-262915\right.}{7838208x^3-22861}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{51901x\sqrt{-24x^3+70x^2-21x-10}}{108}+\frac{13019611\sqrt{-24x^3+70x^2-21x-10}}{7776}+\frac{10873271\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44}}{57024\sqrt{-24x^3+70x}}\right.}{}$
risch	$-\frac{(756000x^3+6197400x^2+26158104x+91137277)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{54432\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{10873271\sqrt{22-33x}\,\sqrt{1}}{17}\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/326592*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(108864000*x^6+5749056
00*x^5+1227098543*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*Ellip
ticF(1/11*(11+44*x)^(1/2),3^(1/2))-2629157597*(1+4*x)^(1/2)*(2-3*x)^(1/2)*
22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+1259114976*
x^4+2963596608*x^3-34609891236*x^2+13052783142*x+5468236620)/(24*x^3-70*x^
2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx$$
$$= \frac{1}{54432}\left(756000\,x^3+6197400\,x^2+26158104\,x+91137277\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} + \frac{4958213249}{419904}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{2629157597}{163296}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2),x, algorith
m="fricas")
```

output
```
1/54432*(756000*x^3 + 6197400*x^2 + 26158104*x + 91137277)*sqrt(4*x + 1)*s
qrt(2*x - 5)*sqrt(-3*x + 2) + 4958213249/419904*sqrt(-6)*weierstrassPInver
se(847/108, 6655/2916, x - 35/36) - 2629157597/163296*sqrt(-6)*weierstrass
Zeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36
))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\,(5x+7)^3}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**3/(-5+2*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**3/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,(5\,x+7)^3}{\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \frac{125\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^3}{9} + \frac{86075\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{756} + \frac{51901\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{108} + \frac{31147061\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{105840} + \frac{2629157597\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{52920} - \frac{259508623\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{30240}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2),x)
```

output
```
(2940000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**3 + 24101000*sqrt
(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2 + 101725960*sqrt(2*x - 5)*sq
rt(4*x + 1)*sqrt( - 3*x + 2)*x + 62294122*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt
( - 3*x + 2) + 10516630388*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x +
2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 1816560361*int((sqrt(2*x - 5
)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/2116
80
```

3.52 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 200

$$\begin{aligned}&\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx\\&=\frac{73207\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1080}+\frac{173}{60}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\&\quad+\frac{1}{7}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\\&\quad+\frac{8198333\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{9072\sqrt{-5+2x}}\\&\quad-\frac{1876633\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{4536\sqrt{-5+2x}}\end{aligned}$$

output

```
73207/1080*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+173/60*(2-3*x)^(1/2)
*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)+1/7*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)*(7+5*x)^2+8198333/18144*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(
1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-1876633/9072*22^(1/2)*(5-2*x
)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 4.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx$$
$$= \frac{12\sqrt{2-3x}\sqrt{1+4x}(-717955+102592x+46836x^2+10800x^3)+8198333\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\right.\right.}{18144\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

output
```
(12*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-717955 + 102592*x + 46836*x^2 + 10800*x^
3) + 8198333*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3] - 6716644*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sq
rt[1 + 4*x]], 1/3])/(18144*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {180, 25, 2103, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}(5x+7)^2}{\sqrt{2x-5}}\,dx$$

$$\Big\downarrow \text{180}$$

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{1}{14}\int -\frac{(5x+7)\left(-2422x^2+175x+543\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{14}\int \frac{(5x+7)\left(-2422x^2+175x+543\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{14}\left(\frac{1211}{30}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{1}{120}\int -\frac{2\left(-2049796x^2-568915x+527177\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{14}\left(\frac{1}{60}\int \frac{-2049796x^2-568915x+527177}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1211}{30}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)\right)+\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 2118$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{1}{108}\int \frac{330(368193-1490606x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{512449}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1211}{30}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x\right.$$
$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\int \frac{368193-1490606x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{512449}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1211}{30}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x\right.$$
$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 176$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\left(-3358322\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-745303\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{512449}{9}\sqrt{2-3x}\sqrt{2\right.\right.$$
$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 124$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\left(-\frac{745303\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-3358322\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{512449}{9}\sqrt{2-3\right.\right.$$
$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 123$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\left(-3358322\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{745303\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)\right.\right.\right.$$

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 131$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\left(-\frac{305302\sqrt{22}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{745303\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right.\right.\right.\right.$$

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\left(-\frac{3358322\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{745303\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)\right.\right.\right.$$

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \; 129$$

$$\frac{1}{14}\left(\frac{1}{60}\left(\frac{55}{18}\left(-\frac{305302\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{745303\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\right.\right.}{\sqrt{5-2x}}\right.\right.\right.\right.$$

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/7 + ((1211*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/30 + ((512449*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (55*((-745303*Sqrt[11/6]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x] - (30530
2*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3
])/Sqrt[-5 + 2*x]))/18)/60)/14
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 180
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[2*(a + b*x)^m*Sqrt[c + d*x]*Sq
rt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Simp[1/(d*(2*m + 3))   Int[
((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*b*c*
e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e*h)*(
2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m + b*(d*
(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(1555200x^5+3753266\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-8198333\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{2}\right.}{435456x^3-1270080x^2+381024x+1}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{293x\sqrt{-24x^3+70x^2-21x-10}}{12}+\frac{20513\sqrt{-24x^3+70x^2-21x-10}}{216}+\frac{17533\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{Ellipti}}{1584\sqrt{-24x^3+70x^2-21x-}}\right.}{\sqrt{2-3}}$
risch	$-\frac{(5400x^2+36918x+143591)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{1512\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{17533\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{Ellipti}}{4752\sqrt{-24x^3+70x^2-21x-}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/18144*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(1555200*x^5+3753266*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^
(1/2),3^(1/2))-8198333*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*
EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+6096384*x^4+11703888*x^3-110665104
*x^2+40615092*x+17230920)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \frac{1}{1512}\left(5400\,x^2+36918\,x+143591\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} + \frac{30577063}{46656}\,\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{8198333}{9072}\,\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2),x, algorith
m="fricas")
```

output
```
1/1512*(5400*x^2 + 36918*x + 143591)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x
 + 2) + 30577063/46656*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x
- 35/36) - 8198333/9072*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weier
strassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\,(5x+7)^2}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**2/(-5+2*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**2/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,(5\,x+7)^2}{\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^2}{\sqrt{-5+2x}}\,dx = \frac{25\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{7} + \frac{293\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{12} + \frac{68631\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{3920} + \frac{8198333\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{2940} - \frac{1767359\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{3360}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2),x)
```

output
```
(84000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2 + 574280*sqrt(2*x
 - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x + 411786*sqrt(2*x - 5)*sqrt(4*x + 1
)*sqrt( - 3*x + 2) + 65586664*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x
 + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 12371513*int((sqrt(2*x -
5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/235
20
```

3.53 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 157

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx &= \frac{223}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} \\ &\quad -\frac{1}{3}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x} \\ &\quad +\frac{1397\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{27\sqrt{-5+2x}} \\ &\quad -\frac{2453\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{108\sqrt{-5+2x}}\end{aligned}$$

output
```
223/36*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-1/3*(2-3*x)^(3/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)+1397/54*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*
x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-2453/216*22^(1/2)*(5-2*x)^(1/2)*
EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{6\sqrt{2-3x}\sqrt{1+4x}(-995+218x+72x^2)+5588\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)-4543\sqrt{66}}{216\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

output
```
(6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-995 + 218*x + 72*x^2) + 5588*Sqrt[66]*Sqr
t[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 4543*Sqrt[66
]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(216*Sqr
t[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {171, 27, 171, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}(5x+7)}{\sqrt{2x-5}}\,dx$$

$$\Big\downarrow \text{ 171}$$

$$\frac{1}{20}\int \frac{5(213-380x)\sqrt{4x+1}}{2\sqrt{2-3x}\sqrt{2x-5}}dx+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \frac{(213-380x)\sqrt{4x+1}}{\sqrt{2-3x}\sqrt{2x-5}}dx+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$$\Big\downarrow \text{ 171}$$

$$\frac{1}{8}\left(\frac{380}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1}{9}\int -\frac{11(537-2032x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{11}{9}\int \frac{537-2032x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{380}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$\downarrow$ 176

$$\frac{1}{8}\left(\frac{11}{9}\left(-4543\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-1016\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{380}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$\downarrow$ 124

$$\frac{1}{8}\left(\frac{11}{9}\left(-\frac{1016\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-4543\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{380}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$\downarrow$ 123

$$\frac{1}{8}\left(\frac{11}{9}\left(-4543\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{508\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{380}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$\downarrow$ 131

$$\frac{1}{8}\left(\frac{11}{9}\left(-\frac{413\sqrt{22}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{508\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{380}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{11}{9}\left(-\frac{4543\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{508\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{380}{9}\sqrt{2-\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

$$\Big\downarrow \; 129$$

$$\frac{1}{8}\left(\frac{11}{9}\left(-\frac{413\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{508\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)\right.}{\sqrt{5-2x}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}(4x+1)^{3/2}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/4 + ((380*Sqrt[2 - 3*x]*Sqr
t[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (11*((-508*Sqrt[22/3]*Sqrt[-5 + 2*x]*Ellipt
icE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x] - (413*Sqrt[22/3
]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5
+ 2*x]))/9)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(2453\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-5588\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticE}\left(\right.\right.}{5184x^3-15120x^2+4536x+2160}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(x\sqrt{-24x^3+70x^2-21x-10}+\frac{199\sqrt{-24x^3+70x^2-21x-10}}{36}+\frac{179\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\right.}{264\sqrt{-24x^3+70x^2-21x-10}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{(199+36x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{179\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right.}{792\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/216*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2453*(1+4*x)^(1/2)*(2-3*
x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55
88*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+4
4*x)^(1/2),3^(1/2))+5184*x^4+13536*x^3-79044*x^2+27234*x+11940)/(24*x^3-70
*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{1}{36}\,(36\,x+199)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} + \frac{142417}{3888}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{1397}{27}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2),x, algorithm=
"fricas")
```

output
```
1/36*(36*x + 199)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 142417/3888
*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) - 1397/27*sqr
t(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 665
5/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\cdot(5x+7)}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)/(-5+2*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,(5\,x+7)}{\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)}{\sqrt{-5+2x}}\,dx = \sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x + \frac{153\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{140} + \frac{5588\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{35} - \frac{1419\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{40}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2),x)
```

output
```
(280*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x + 306*sqrt(2*x - 5)*sq
rt(4*x + 1)*sqrt( - 3*x + 2) + 44704*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt
( - 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 9933*int((sqrt(2*x
 - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/
280
```

3.54 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 126

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{55\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{18\sqrt{-5+2x}} - \frac{11\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{9\sqrt{-5+2x}}$$

output
```
1/3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+55/36*22^(1/2)*(5-2*x)^(1/2
)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-11/18*22^(
1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$$
$$= \frac{12\sqrt{2-3x}(-5+2x)\sqrt{1+4x}+55\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)-44\sqrt{66}\sqrt{5-2x}\,\text{Ellipti}}{36\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

output
```
(12*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 55*Sqrt[66]*Sqrt[5 - 2*x]*Ell
ipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 44*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {112, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}}\,dx$$
$$\Big\downarrow \text{112}$$
$$\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1}{3}\int -\frac{11(3-10x)}{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{11}{6}\int \frac{3-10x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$
$$\Big\downarrow \text{176}$$

$$\frac{11}{6}\left(-22\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-5\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{124}$$

$$\frac{11}{6}\left(-\frac{5\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-22\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{123}$$

$$\frac{11}{6}\left(-22\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{131}$$

$$\frac{11}{6}\left(-\frac{2\sqrt{22}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{6}\left(-\frac{22\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{129}$$

$$\frac{11}{6}\left(-\frac{2\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{1}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3 + (11*((-5*Sqrt[11/6]*Sqrt[
-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x]
- (2*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/Sqrt[-5 + 2*x]))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 112
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*(m + n +
p + 1))), x] - Simp[1/(f*(m + n + p + 1))   Int[(a + b*x)^(m - 1)*(c + d*x)
^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a
*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] &&
GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p
] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,\left(22\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-55\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticE}\left(\frac{\sqrt{11}}{}\right.\right.}{864x^3-2520x^2+756x+360}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{\sqrt{-24x^3+70x^2-21x-10}}{3}+\frac{\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{22\sqrt{-24x^3+70x^2-21x-10}}-\frac{5\sqrt{11+44x}\,\sqrt{22-33x}}{}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{3\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{66\sqrt{-24x^3+70x^2-21x-10}}-\frac{5\sqrt{22-33x}}{}\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/36*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(22*(1+4*x)^(1/2)*(2-3*x)^
(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55*(1
+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)^
(1/2),3^(1/2))+288*x^3-840*x^2+252*x+120)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$$
$$= \frac{1}{3}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}$$
$$+ \frac{1331}{648}\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$- \frac{55}{18}\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas"
)
```

output
```
1/3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 1331/648*sqrt(-6)*weierst
rassPInverse(847/108, 6655/2916, x - 35/36) - 55/18*sqrt(-6)*weierstrassZe
ta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{14} + \frac{66\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{7} - \frac{11\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{4}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x)
```

output
```
(2*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2) + 264*int((sqrt(2*x - 5)*s
qrt(4*x + 1)*sqrt( - 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 7
7*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 +
21*x + 10),x))/28
```

3.55 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 143

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx &= \frac{\sqrt{22}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{5\sqrt{-5+2x}} \\ &\quad + \frac{69\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{25\sqrt{-5+2x}} \\ &\quad - \frac{124\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{25\sqrt{-5+2x}}\end{aligned}$$

output
```
1/5*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/
(-5+2*x)^(1/2)+69/275*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*
11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-124/275*22^(1/2)*(5-2*x)^(1/2)*EllipticPi
(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \frac{\sqrt{5-2x}\left(-110E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+41\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+69\,\text{EllipticPi}\left(\frac{55}{124},\text{arc}\right.\right.}{25\sqrt{-55+22x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

output
```
(Sqrt[5 - 2*x]*(-110*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
 41*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 69*EllipticPi[55
/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(25*Sqrt[-55 + 22*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {181, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)}\,dx$$

$$\Big\downarrow \ 181$$

$$\frac{1}{25}\int \frac{109-60x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{713}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \ 176$$

$$\frac{1}{25}\left(-41\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-30\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)-\frac{713}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \ 124$$

$$\frac{1}{25}\left(-\frac{30\sqrt{2x-5}\int\frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-41\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)-\frac{713}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \; 123$$

$$\frac{1}{25}\left(-41\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{713}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \; 131$$

$$\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{11}}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{713}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{25}\left(-\frac{41\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{713}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \; 129$$

$$\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{713}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \; 186$$

$$\frac{1426}{25}\int\frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{4278}{25}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}+\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)$$

$$\Big\downarrow \textcolor{blue}{413}$$

$$\frac{4278\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}+\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{4278\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}+\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)$$

$$\Big\downarrow \textcolor{blue}{412}$$

$$\frac{69\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{25\sqrt{11}\sqrt{-2(2-3x)-11}}+\frac{1}{25}\left(-\frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

output
```
((-5*Sqrt[66]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/Sqrt[5 - 2*x] - (41*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/
11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x])/25 + (69*Sqrt[11 + 2*(2 - 3*x)]*
EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]
*Sqrt[-11 - 2*(2 - 3*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 181
```
Int[(Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)])/(((a_.) + (b_.)*(x_
))*Sqrt[(c_.) + (d_.)*(x_)]), x_] :> Simp[(b*e - a*f)*((b*g - a*h)/b^2)   I
nt[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[1
/b^2   Int[Simp[b*f*g + b*e*h - a*f*h + b*f*h*x, x]/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.47

method	result
default	$\frac{\left(69\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)+55\,\mathrm{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-124\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{275\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{109\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{3025\sqrt{-24x^{3}+70x^{2}-21x-10}}-\frac{12\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11\,\mathrm{Ellipti}\ldots}{\ldots}\right.}{605\sqrt{-24x\ldots}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+\ldots}}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x),x,method=_RETURNVER
BOSE)
```

output
```
1/275*(69*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+55*EllipticE(1/11*(11+44
*x)^(1/2),3^(1/2))-124*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))*(5
-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x),x, algorithm=
"fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 - 11*x - 35),
x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\cdot(5x+7)}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x),x, algorithm=
"giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \int \frac{\sqrt{-3x+2}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(7+5x)}dx$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x),x)
```

output
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x),x)
```

3.56 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{\sqrt{22}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{195\sqrt{-5+2x}} - \frac{344\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{975\sqrt{-5+2x}} + \frac{12202\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{22425\sqrt{-5+2x}}$$

output
```
(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(273+195*x)-1/195*22^(1/2)*(5-2
*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-34
4/10725*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/
2))/(-5+2*x)^(1/2)+12202/246675*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1+
4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx$$
$$= \frac{\frac{51150\sqrt{2-3x}(-5+2x)\sqrt{1+4x}}{7+5x} + 3\sqrt{55-22x}\left(6820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) + 14508\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right.\right.\right.}{1994850\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

output
```
((51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Elli
pticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 18303*EllipticPi[55/124,
 ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1994850*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {182, 25, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)^2}dx$$

$$\Big\downarrow \; 182$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)} - \frac{1}{78}\int -\frac{24x^2-120x+29}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{78}\int \frac{24x^2-120x+29}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx + \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$$\Big\downarrow \; 2110$$

$$\frac{1}{78}\left(\int \frac{\frac{24x}{5}-\frac{768}{25}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{6101}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\Big\downarrow$ 176

$$\frac{1}{78}\left(-\frac{468}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{12}{5}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+\frac{6101}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\Big\downarrow$ 124

$$\frac{1}{78}\left(\frac{12\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{5\sqrt{5-2x}}-\frac{468}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{6101}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\Big\downarrow$ 123

$$\frac{1}{78}\left(-\frac{468}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{6101}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{2\sqrt{66}\sqrt{2x-5}E\left(\right.}{5}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\Big\downarrow$ 131

$$\frac{1}{78}\left(-\frac{468\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{6101}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{2\sqrt{66}\sqrt{2x-5}}{}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\Big\downarrow$ 27

$$\frac{1}{78}\left(-\frac{468\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{6101}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx+\frac{2\sqrt{66}\sqrt{2x-5}E\left(\mathrm{a}\right.}{5\ldots}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\downarrow$ 129

$$\frac{1}{78}\left(\frac{6101}{25}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\ldots\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\downarrow$ 186

$$\frac{1}{78}\left(-\frac{12202}{25}\int\frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\mathrm{ar}\ldots\right.}{25\sqrt{2x\ldots}}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\downarrow$ 27

$$\frac{1}{78}\left(-\frac{36606}{25}\int\frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\mathrm{ar}\ldots\right.}{25\sqrt{2x\ldots}}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\downarrow$ 413

$$\frac{1}{78}\left(-\frac{36606\sqrt{2(2-3x)+11}\int\frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\mathrm{arcs}\ldots\right.}{25\sqrt{2x-\ldots}}\right.$$
$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$\downarrow$ 27

$$\frac{1}{78}\left(-\frac{36606\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\text{arcs}\right.}{25\sqrt{2x-}}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

$$\Big\downarrow \; 412$$

$$\frac{1}{78}\left(-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{2\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}-\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39(5x+7)}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*(7 + 5*x)) + ((2*Sqrt[66]
*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(5*Sqrt[
5 - 2*x]) - (156*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt
[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (18303*Sqrt[11 + 2*(2 - 3*x)]*Elli
pticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(775*Sqrt[11]*Sq
rt[-11 - 2*(2 - 3*x)]))/78
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 182
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Simp[1/(2*(m +
1)*(b*c - a*d))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)*(f*g
 + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{128\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{39325\sqrt{-24x^3+70x^2-21x-10}}+\frac{4\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11\,\text{Ellip}\cdots}{\cdots}\right.}{7865\sqrt{-2\cdots}}\right.}{\sqrt{2-3x}\cdots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(39560\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x+6325\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{Elliptic}\cdots\right.}{\cdots}$
risch	$-\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{39(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{128\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\text{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{117975\sqrt{-24x^3+70x^2-21x-10}}+\frac{4\sqrt{22-\cdots}}{\cdots}\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^2,x,method=_RETURNV
ERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-128/39325*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+
70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))+4/7865*(11+4
4*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2)
*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+44
*x)^(1/2),3^(1/2)))+1/39/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+12202/2713
425*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-
10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^2,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 + 15*x^2 - 252
*x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^2}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**2,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^2,x, algorith
m="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3x+2}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(7+5x)^2}dx$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^2,x)
```

output
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^2,x)
```

3.57 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} + \frac{361\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{1204970\sqrt{-5+2x}} - \frac{959\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{97175\sqrt{-5+2x}} + \frac{6655867\sqrt{5-2x}\,\mathrm{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{207857325\sqrt{22}\sqrt{-5+2x}}$$

output
```
1/78*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2-361*(2-3*x)^(1/2
)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(3373916+2409940*x)+361/2409940*22^(1/2)*(5
-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-
959/1068925*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3
^(1/2))/(-5+2*x)^(1/2)+6655867/4572861150*22^(1/2)*(5-2*x)^(1/2)*EllipticP
i(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \frac{-\frac{17050\sqrt{2-3x}(-5+2x)\sqrt{1+4x}(-10957+5415x)}{(7+5x)^2} - 3\sqrt{55-22x}\left(2462020E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 9834812\,\text{Ellipt}\right.}{24653686200\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

output
```
((-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(-10957 + 5415*x))/(7 + 5*
x)^2 - 3*Sqrt[55 - 22*x]*(2462020*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[
11]], -1/2] - 9834812*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]
+ 6655867*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(
24653686200*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {182, 25, 2107, 27, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)^3}\,dx$$

$$\Big\downarrow \; 182$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2} - \frac{1}{156}\int -\frac{-24x^2-100x+37}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{156}\int \frac{-24x^2-100x+37}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx + \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{156}\left(\frac{\int \frac{3(-25992x^2-161760x+90715)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}-\frac{1083\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}\right)+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{156}\left(\frac{\int \frac{-25992x^2-161760x+90715}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}-\frac{1083\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}\right)+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{2110}$$

$$\frac{1}{156}\left(\frac{\int \frac{-\frac{25992x}{5}-\frac{626856}{25}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}-\frac{1083\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}\right)+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{176}$$

$$\frac{1}{156}\left(\frac{-\frac{951756}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{12996}{5}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}-\frac{1083\sqrt{2}\cdots}{}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{124}$$

$$\frac{1}{156}\left(\frac{-\frac{12996\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{5\sqrt{5-2x}}-\frac{951756}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}-\frac{1083\cdots}{}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{123}$$

$$\frac{1}{156}\left(\frac{-\frac{951756}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{2166\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right.}{5\sqrt{5-2x}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{131}$$

$$\frac{1}{156}\left(\frac{-\frac{951756\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{2166\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4\right.\right.}{5\sqrt{5-2x}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{156}\left(\frac{-\frac{951756\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{25\sqrt{2x-5}}+\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{2166\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)\right.}{5\sqrt{5-2x}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{129}$$

$$\frac{1}{156}\left(\frac{\frac{6655867}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{317252\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}-\frac{2166\sqrt{66}\sqrt{2x-5}E\left(\arcsin\right.}{5\sqrt{5-2x}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$$\Big\downarrow \text{186}$$

$$\frac{1}{156}\left(\frac{-\frac{13311734}{25}\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{317252\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$\downarrow$ 27

$$\frac{1}{156}\left(\frac{-\frac{39935202}{25}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{317252\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$\downarrow$ 413

$$\frac{1}{156}\left(\frac{-\frac{39935202\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{317252\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$\downarrow$ 27

$$\frac{1}{156}\left(\frac{-\frac{39935202\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{25\sqrt{-2(2-3x)-11}}-\frac{317252\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

$\downarrow$ 412

$$\frac{1}{156}\left(\frac{-\frac{317252\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}-\frac{2166\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{5-2x}}-\frac{19967601\sqrt{2(2-3x)+11}}{775\sqrt{}}}{18538}\right.$$

$$\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(78*(7 + 5*x)^2) + ((-1083*Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(9269*(7 + 5*x)) + ((-2166*Sqrt[
66]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(5*Sq
rt[5 - 2*x]) - (317252*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (19967601*Sqrt[11 + 2*(2 - 3
*x)]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(775*Sq
rt[11]*Sqrt[-11 - 2*(2 - 3*x)]))/18538)/156
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] && !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) && !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) && !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 182
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Simp[1/(2*(m +
1)*(b*c - a*d))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)*(f*g
 + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{26119\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{364503425\sqrt{-24x^3+70x^2-21x-10}}-\frac{1083\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11}{}\right.}{7290068}\right.}{}$
risch	$\frac{(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(-10957+5415x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{1445964(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{26119\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\mathrm{EllipticF}\left(\frac{2\sqrt{22-3}}{11}\right.}{1093510275\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\left(205130100\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2-34249875\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^3,x,method=_RETURNV
ERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-26119/364503425*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-2
4*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-1083/7
2900685*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-2
1*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*Elliptic
F(1/11*(11+44*x)^(1/2),3^(1/2)))+1/78/(7+5*x)^2*(-24*x^3+70*x^2-21*x-10)^(
1/2)-361/481988/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+6655867/50301472650
*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)
^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^3,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4 + 425*x^3 - 1
155*x^2 - 2989*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^3}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**3,x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^3,x, algorith
m="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^3,x, algorith
m="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx=\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)^3}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^3}\,dx=\int \frac{\sqrt{-3x+2}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(7+5x)^3}dx$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^3,x)
```

output
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^3,x)
```

3.58 $\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 200

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} \\ &+ \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\ &+ \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 \\ &+ \frac{15629623\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{9072\sqrt{-5+2x}} \\ &- \frac{13261655\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{18144\sqrt{-5+2x}}\end{aligned}$$

output
```
110743/864*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+121/24*(2-3*x)^(1/2)
*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)+5/28*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)*(7+5*x)^2+15629623/18144*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11
*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-13261655/36288*22^(1/2)*(5
-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.66 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{30\sqrt{2-3x}\sqrt{1+4x}(-1041565+188566x+64224x^2+10800x^3)+31259246\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\right.\right.}{36288\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(30*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-1041565 + 188566*x + 64224*x^2 + 10800*x
^3) + 31259246*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 +
 4*x]], 1/3] - 25260049*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]
*Sqrt[1 + 4*x]], 1/3])/(36288*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {192, 25, 2103, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}(5x+7)^3}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{192}$$

$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{1}{56}\int -\frac{(5x+7)\left(-16940x^2-2667x+7223\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{56}\int \frac{(5x+7)\left(-16940x^2-2667x+7223\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{56}\left(\frac{847}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{1}{120}\int -\frac{20\left(-1550402x^2-458579x+512575\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 27

$$\frac{1}{56}\left(\frac{1}{6}\int \frac{-1550402x^2-458579x+512575}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{847}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)\right)+\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 2118

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{108}\int \frac{3(34731921-125036984x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{775201}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{847}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x\right.$$
$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 27

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\int \frac{34731921-125036984x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{775201}{18}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+\frac{847}{3}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+\right.$$
$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 176

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\left(-277860539\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-62518492\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{775201}{18}\sqrt{2-3\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 124

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\left(-\frac{62518492\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-277860539\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{775201}{18}\sqrt{2\right.\right.$$
$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 123

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\left(-277860539\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{31259246\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right.\right.\right.$$

$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 131

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\left(-\frac{25260049\sqrt{22}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{31259246\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)\right.}{\sqrt{5-2x}}\right.\right.\right.$$

$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 27

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\left(-\frac{277860539\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{31259246\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right.\right.\right.$$

$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

$\downarrow$ 129

$$\frac{1}{56}\left(\frac{1}{6}\left(\frac{1}{36}\left(-\frac{25260049\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{31259246\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\right.}{\sqrt{5-2x}}\right.\right.\right.$$

$$\frac{5}{28}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2$$

input
```
Int[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/28 + ((847*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/3 + ((775201*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 + ((-31259246*Sqrt[22/3]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x] - (25260
049*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/Sqrt[-5 + 2*x])/36)/6)/56
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 192

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b*(a + b*x)^(m - 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Simp[1/(f*h*(2*
m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*(2*m
+ 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) + 2*a*b
*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*d*(f*
g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && In
tegerQ[2*m] && GtQ[m, 1]
```

rule 2103

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(13261655\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-31259246\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \text{I}\right.}{870912x^3-2540160x^2+762048x}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(\frac{905x\sqrt{-24x^3+70x^2-21x-10}}{24}+\frac{148795\sqrt{-24x^3+70x^2-21x-10}}{864}+\frac{1653901\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \text{Ell}}{69696\sqrt{-24x^3+70x^2-21}}\right.}{\sqrt{2}}$
risch	$-\frac{5(5400x^2+45612x+208313)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{6048\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{1653901\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \text{Ell}}{209088\sqrt{-24x^3+70x^2-2}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/36288*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(13261655*(1+4*x)^(1/2)
*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/
2))-31259246*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(
1/11*(11+44*x)^(1/2),3^(1/2))+3888000*x^5+21500640*x^4+57602160*x^3-407101
740*x^2+144920790*x+62493900)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{6048}\left(5400\,x^2+45612\,x+208313\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} + \frac{111640903}{93312}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{15629623}{9072}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="fricas")
```

output
```
5/6048*(5400*x^2 + 45612*x + 208313)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x
 + 2) + 111640903/93312*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x
 - 35/36) - 15629623/9072*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, wei
erstrassPInverse(847/108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}\,(5x+7)^3}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(7+5*x)**3/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*(5*x + 7)**3/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,(5\,x+7)^3}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{125\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{28} + \frac{905\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{24} + \frac{192427\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{7840} + \frac{15629623\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{2940} - \frac{8875763\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{6720}$$

input
```
int((2-3*x)^(1/2)*(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
(210000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2 + 1773800*sqrt(2
*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x + 1154562*sqrt(2*x - 5)*sqrt(4*x
+ 1)*sqrt( - 3*x + 2) + 250073968*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( -
 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 62130341*int((sqrt(2*
x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))
/47040
```

3.59 $\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 162

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{68}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\\ &\quad+\frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)\\ &\quad+\frac{44569\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{432\sqrt{-5+2x}}\\ &\quad-\frac{4015\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{108\sqrt{-5+2x}}\end{aligned}$$

output
```
68/9*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/4*(2-3*x)^(1/2)*(-5+2*x)
^(1/2)*(1+4*x)^(1/2)*(7+5*x)+44569/864*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/
11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-4015/216*22^(1/2)*(5-2*x
)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 6.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{120\sqrt{2-3x}\sqrt{1+4x}(-335+89x+18x^2)+44569\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)-35066\sqrt{}}{864\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-335 + 89*x + 18*x^2) + 44569*Sqrt[66]*S
qrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 35066*Sqrt
[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(864*
Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {192, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}(5x+7)^2}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

$$\Big\downarrow \text{192}$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{1}{40}\int -\frac{5\left(-2176x^2-721x+1031\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{-2176x^2-721x+1031}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{2118}$$

$$\frac{1}{8}\left(\frac{1}{108}\int \frac{12(14991-44569x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{544}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+ \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\left(\frac{1}{9}\int \frac{14991-44569x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{544}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)+ \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{ 176}$$

$$\frac{1}{8}\left(\frac{1}{9}\left(-\frac{192863}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{44569}{2}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{544}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x-\right. \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{ 124}$$

$$\frac{1}{8}\left(\frac{1}{9}\left(-\frac{44569\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{2\sqrt{5-2x}}-\frac{192863}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{544}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4\right. \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{ 123}$$

$$\frac{1}{8}\left(\frac{1}{9}\left(-\frac{192863}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{44569\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}\right)+\frac{544}{9}\sqrt{\right. \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{ 131}$$

$$\frac{1}{8}\left(\frac{1}{9}\left(-\frac{17533\sqrt{\frac{11}{2}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{44569\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}\right)+\frac{544}{9}\right. \frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\left(\frac{1}{9}\left(-\frac{192863\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{2\sqrt{2x-5}}-\frac{44569\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}\right)+\frac{544}{9}\sqrt{2}\right.$$
$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$\downarrow$ 129

$$\frac{1}{8}\left(\frac{1}{9}\left(-\frac{17533\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{44569\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+\right.\right.}{2\sqrt{5-2x}}\right.\right.$$
$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

input
```
Int[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/4 + ((544*Sqrt[2 -
3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + ((-44569*Sqrt[11/6]*Sqrt[-5 + 2*x]*
EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2*Sqrt[5 - 2*x]) - (175
33*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3])/Sqrt[-5 + 2*x])/9)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 192
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b*(a + b*x)^(m - 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Simp[1/(f*h*(2*
m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*(2*m
+ 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) + 2*a*b
*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*d*(f*
g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && In
tegerQ[2*m] && GtQ[m, 1]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
default	Numerator: $\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\Big(16060\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-44569\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \mathrm{Elliptic}$ … ; denominator: $20736x^3-60480x^2+18144x+8640$ …
elliptic	Numerator: $\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\Bigg(\frac{5x\sqrt{-24x^3+70x^2-21x-10}}{4}+\frac{335\sqrt{-24x^3+70x^2-21x-10}}{36}+\frac{4997\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \mathrm{EllipticF}\big(\sqrt{}}{2904\sqrt{-24x^3+70x^2-21x-10}}$ … ; denominator: $\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4}$ …
risch	$-\frac{5(67+9x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-$ Numerator: $\Bigg(\frac{4997\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \mathrm{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{8712\sqrt{-24x^3+70x^2-21x-10}}$ …

input
```
int((2-3*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/864*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(16060*(1+4*x)^(1/2)*(2-3
*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-4
4569*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11
+44*x)^(1/2),3^(1/2))+25920*x^4+117360*x^3-540120*x^2+179640*x+80400)/(24*
x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{5}{36}\,(9\,x+67)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\ &\quad + \frac{1020239}{15552}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\ &\quad - \frac{44569}{432}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x - \frac{35}{36}\right)\right)\end{aligned}$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="fricas")
```

output
```
5/36*(9*x + 67)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 1020239/15552
*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) - 44569/432*s
qrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6
655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}(5x+7)^2}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(7+5*x)**2/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*(5*x + 7)**2/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^2\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^2\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,{(5\,x+7)}^2}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^2}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{5\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{4} \\ &+ \frac{37\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{80} \\ &+ \frac{6367\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{20} \\ &- \frac{18457\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{160}\end{aligned}$$

input
```
int((2-3*x)^(1/2)*(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
(200*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x + 74*sqrt(2*x - 5)*sqr
t(4*x + 1)*sqrt( - 3*x + 2) + 50936*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(
 - 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 18457*int((sqrt(2*x
 - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x))/
160
```

3.60 $\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 126

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{241\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{36\sqrt{-5+2x}} - \frac{55\sqrt{\frac{11}{2}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{36\sqrt{-5+2x}}$$

output
```
5/12*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+241/72*22^(1/2)*(5-2*x)^(1
/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)-55/72*22
^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*
x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{30\sqrt{2-3x}(-5+2x)\sqrt{1+4x}+241\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)-179\sqrt{66}\sqrt{5-2x}\,\text{Ellip}}{72\sqrt{-5+2x}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(30*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 241*Sqrt[66]*Sqrt[5 - 2*x]*El
lipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 179*Sqrt[66]*Sqrt[5 - 2*x
]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(72*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {171, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}(5x+7)}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$
$$\Big\downarrow \text{171}$$
$$\frac{1}{12}\int \frac{441-964x}{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{24}\int \frac{441-964x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$
$$\Big\downarrow \text{176}$$

$$\frac{1}{24}\left(-1969\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-482\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 124

$$\frac{1}{24}\left(-\frac{482\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-1969\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 123

$$\frac{1}{24}\left(-1969\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{241\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 131

$$\frac{1}{24}\left(-\frac{179\sqrt{22}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{241\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 27

$$\frac{1}{24}\left(-\frac{1969\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}-\frac{241\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$\downarrow$ 129

$$\frac{1}{24}\left(-\frac{179\sqrt{\frac{22}{3}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{241\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

input
```
Int[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/12 + ((-241*Sqrt[22/3]*Sqrt
[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 - 2*x]
 - (179*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]
], 1/3])/Sqrt[-5 + 2*x])/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(55\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-241\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \mathrm{EllipticE}\left(\frac{\sqrt{1}\cdots}{}\right.\right.}{1728x^3-5040x^2+1512x+720}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{5\sqrt{-24x^3+70x^2-21x-10}}{12}+\frac{147\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{968\sqrt{-24x^3+70x^2-21x-10}}-\frac{241\sqrt{11+44x}\,\sqrt{22}\cdots}{\cdots}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$-\frac{5(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{12\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{49\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \mathrm{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{968\sqrt{-24x^3+70x^2-21x-10}}-\frac{241\sqrt{22}\cdots}{\cdots}\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/72*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(55*(1+4*x)^(1/2)*(2-3*x)^
(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-241*(
1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(11+44*x)
^(1/2),3^(1/2))+720*x^3-2100*x^2+630*x+300)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= \frac{5}{12}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad+\frac{2233}{648}\sqrt{-6}\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad-\frac{241}{36}\sqrt{-6}\mathrm{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\mathrm{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm=
"fricas")
```

output
```
5/12*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) + 2233/648*sqrt(-6)*weiers
trassPInverse(847/108, 6655/2916, x - 35/36) - 241/36*sqrt(-6)*weierstrass
Zeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x - 35/36
))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}(5x+7)}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(7+5*x)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,(5\,x+7)}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}(7+5x)}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{11\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{70} + \frac{723\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{35} - \frac{247\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{20}$$

input
```
int((2-3*x)^(1/2)*(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output

```
( - 22*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2) + 2892*int((sqrt(2*x -
 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x
) - 1729*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*
x**2 + 21*x + 10),x))/140
```

3.61 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 47

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}$$

output
```
1/4*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/
(-5+2*x)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 111 vs. 2(47) = 94.

Time = 2.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{\frac{2(-5+2x)(-2+3x)}{\sqrt{\frac{1}{2}+2x}} + \sqrt{11}\sqrt{\frac{-5+2x}{1+4x}}\sqrt{\frac{-2+3x}{1+4x}}(1+4x)E\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right)\middle|3\right)}{2\sqrt{2-3x}\sqrt{-10+4x}}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
-1/2*((2*(-5 + 2*x)*(-2 + 3*x))/Sqrt[1/2 + 2*x] + Sqrt[11]*Sqrt[(-5 + 2*x)
/(1 + 4*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*EllipticE[ArcSin[Sqrt[11/
3]/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \ 124$$

$$\frac{\sqrt{5-2x}\int \frac{\sqrt{2}\sqrt{2-3x}}{\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2}\sqrt{2x-5}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{5-2x}\int \frac{\sqrt{2-3x}}{\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}$$

$$\Big\downarrow \ 123$$

$$\frac{\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

input
```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(Sqrt[11/2]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[1 + 4*x]/Sqrt[11]], 3])/(2
*Sqrt[-5 + 2*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result
default	$\frac{\operatorname{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{4\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{2\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{121\sqrt{-24x^{3}+70x^{2}-21x-10}}-\frac{3\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11\operatorname{EllipticE}}{}\right.}{121\sqrt{-24x^{3}+}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{11}{72}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) - \frac{1}{2}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas"
)
```

output
```
11/72*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) - 1/2*sq
rt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 66
55/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{8x^2-18x-5}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(8*x**2 - 18*x - 5),x)
```

3.62 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 98

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\frac{3\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{5\sqrt{-5+2x}} + \frac{124\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{115\sqrt{-5+2x}}$$

output
```
-3/55*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2)
)/(-5+2*x)^(1/2)+124/1265*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1+4*x)^(
1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \frac{3\sqrt{5-2x}\left(\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)\right)}{5\sqrt{-55+22x}}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

output
```
(3*Sqrt[5 - 2*x]*(EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - El
lipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(5*Sqrt[-55 +
 22*x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {193, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)}\,dx$$

$$\Big\downarrow \text{193}$$

$$\frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{3}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{131}$$

$$\frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{3\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{3\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{129}$$

$$\frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{186}$$

$$-\frac{62}{5}\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{186}{5}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{186\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{5\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{186\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{5\sqrt{-2(2-3x)-11}}-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{3\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{-2(2-3x)-11}}$$

input
```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

output
```
-1/5*(Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
 1/3])/Sqrt[-5 + 2*x] - (3*Sqrt[11 + 2*(2 - 3*x)]*EllipticPi[55/124, ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(5*Sqrt[11]*Sqrt[-11 - 2*(2 - 3*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 193
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[d/b   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]), x], x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

method	result
default	$-\frac{\left(69\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-124\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{1265\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(-\frac{3\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{605\sqrt{-24x^3+70x^2-21x-10}}+\frac{124\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticPi}\left(\cdots\right)}{13915\sqrt{-24x^3+70x^2-21x-\cdots}}\cdots\right)}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x,method=_RETURNVER
BOSE)
```

output
```
-1/1265*(69*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-124*EllipticPi(1/11*(1
1+44*x)^(1/2),-55/23,3^(1/2)))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x, algorithm=
"fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 - 34*x^2 - 151
*x - 35), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\cdot(5x+7)}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x),x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{\sqrt{-3x+2}}{\sqrt{2x-5}\,\sqrt{4x+1}\,(7+5x)}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x)
```

output
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x)
```

3.63 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\begin{aligned}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx &= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)}+\frac{\sqrt{22}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{897\sqrt{-5+2x}}\\&\quad-\frac{124\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{4485\sqrt{-5+2x}}\\&\quad+\frac{7142\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{103155\sqrt{-5+2x}}\end{aligned}$$

output
```
-5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(6279+4485*x)+1/897*22^(1/2)
*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/
2)-124/49335*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),
3^(1/2))/(-5+2*x)^(1/2)+7142/1134705*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/1
1*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$
$$= \frac{-\frac{51150\sqrt{2-3x}(-5+2x)\sqrt{1+4x}}{7+5x} - 3\sqrt{55-22x}\left(6820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 14508\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right.\right.\right.}{9176310\sqrt{-5+2x}}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

output
```
((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 2
2*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ell
ipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 10713*EllipticPi[55/124
, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(9176310*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {195, 25, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}\,dx$$
$$\Big\downarrow \text{195}$$
$$-\frac{\int -\frac{-120x^2-336x+479}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{1794} - \frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{-120x^2-336x+479}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{1794} - \frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$
$$\Big\downarrow \text{2110}$$

$$\frac{\int \frac{-24x-\frac{168}{5}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{1794}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 176$$

$$\frac{-\frac{468}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-12\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{1794}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 124$$

$$\frac{-\frac{12\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-\frac{468}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{1794}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 123$$

$$\frac{-\frac{468}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{2\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{1794}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 131$$

$$\frac{-\frac{468\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{5\sqrt{2x-5}}+\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{2\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{1794}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{468\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{5\sqrt{2x-5}}+\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{2\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{1794}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 129$$

$$\frac{\frac{\frac{3571}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{2\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{1794}}{}\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 186$$

$$\frac{-\frac{7142}{5}\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{2\sqrt{66}\sqrt{2x-5}}{}}{1794}\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{21426}{5}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{2\sqrt{66}\sqrt{2x-}}{}}{1794}\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 413$$

$$\frac{-\frac{21426\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{5\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{2\sqrt{66}\sqrt{2x}}{}}{1794}\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{21426\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{5\sqrt{-2(2-3x)-11}}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{2\sqrt{66}\sqrt{2x}}{}}{1794}\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

$$\Big\downarrow \ 412$$

$$\frac{-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{2\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}-\frac{10713\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\text{arc}\right.}{155\sqrt{11}\sqrt{-2(2-3x)-}}}{1794}$$

$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}$$

input
```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

output
```
(-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*(7 + 5*x)) + ((-2*Sqr
t[66]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqr
t[5 - 2*x] - (156*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqr
t[1 + 4*x]], 1/3])/(5*Sqrt[-5 + 2*x]) - (10713*Sqrt[11 + 2*(2 - 3*x)]*Elli
pticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(155*Sqrt[11]*Sq
rt[-11 - 2*(2 - 3*x)]))/1794
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 195
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b*(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h))), x] +
 Simp[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e*g +
c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{28\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{180895\sqrt{-24x^3+70x^2-21x-10}}-\frac{4\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11\,\text{Ellipti}}{}\right.}{36179\sqrt{-24}}\right.}{\sqrt{2-3x}\,\sqrt{}}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(14260\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x-6325\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\text{Elliptic}\right.}{}$
risch	$\frac{5(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{897(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{28\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\text{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{542685\sqrt{-24x^3+70x^2-21x-10}}-\frac{4\sqrt{22-33}}{}\right.}{}$

input

```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x,method=_RETURNV
ERBOSE)
```

output

```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-28/180895*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+
70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-4/36179*(11+
44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^(1/2
)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(11+4
4*x)^(1/2),3^(1/2)))-5/897/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+7142/124
81755*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*
x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx=\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input

```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4 + 110*x^3 - 9
93*x^2 - 1232*x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^2}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**2,x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x, algorith
m="maxima")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{\sqrt{-3x+2}}{\sqrt{2x-5}\,\sqrt{4x+1}\,(7+5x)^2}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x)
```

output
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x)
```

3.64 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$\begin{aligned}\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx &= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} \\ &+ \frac{5365\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{16628586\sqrt{-5+2x}} \\ &- \frac{3571\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{1341015\sqrt{-5+2x}} \\ &+ \frac{5456647\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{318714565\sqrt{22}\sqrt{-5+2x}}\end{aligned}$$

output
```
-5/1794*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2-26825*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(232800204+166285860*x)+5365/33257172*
22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+
2*x)^(1/2)-3571/14751165*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/
2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+5456647/7011720430*22^(1/2)*(5-2*x)^(1
/2)*EllipticPi(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$$
$$= \frac{-17050\sqrt{2-3x}(-5+2x)\sqrt{1+4x}(56093+26825x)-\sqrt{55-22x}(7+5x)^2\left(36589300E\left(\arcsin\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.\right.}{113406956520\sqrt{}}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

output
```
(-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(56093 + 26825*x) - Sqrt[55
 - 22*x]*(7 + 5*x)^2*(36589300*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]
], -1/2] - 64043148*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
49109823*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1
13406956520*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {195, 25, 2107, 27, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}\,dx$$
$$\Big\downarrow \text{195}$$
$$-\frac{\int -\frac{120x^2-1372x+1063}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{120x^2-1372x+1063}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 2107$$

$$\frac{\frac{\int \frac{9\left(-214600x^2-452576x+878339\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{-214600x^2-452576x+878339}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 2110$$

$$\frac{\frac{3\left(\int \frac{-42920x-\frac{152136}{5}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5456647}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)}{18538}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 176$$

$$\frac{\frac{3\left(-\frac{688636}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-21460\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+\frac{5456647}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)}{18538}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 124$$

$$\frac{\frac{3\left(-\frac{21460\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-\frac{688636}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5456647}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)}{18538}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 123$$

$$\frac{\frac{3\left(-\frac{688636}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5456647}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{3588}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \; 131$$

$$\frac{\frac{3\left(-\frac{688636\sqrt{\frac{2}{11}}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{5\sqrt{2x-5}}+\frac{5456647}{5}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\frac{26825\cdots}{\cdots}}{3588}$$

$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(-\frac{688636\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{5\sqrt{2x-5}}+\frac{5456647}{5}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\frac{26825\sqrt{2-\cdots}}{92\cdots}}{3588}$$

$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{129}$$

$$\frac{\frac{3\left(\frac{5456647}{5}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{688636\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\cdots}{3588}$$

$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{186}$$

$$\frac{\frac{3\left(-\frac{10913294}{5}\int\frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{688636\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\cdots\right)}{\sqrt{5-2x}}\cdots\right)}{18538}\cdots}{3588}$$

$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\left(-\frac{32739882}{5}\int\frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{688636\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\cdots\right)}{\sqrt{5-2x}}\cdots\right)}{18538}\cdots}{3588}$$

$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{413}$$

$$\frac{3\left(-\frac{32739882\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{5\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{688636\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-}}{}\right.}{18538}$$
$$3588$$
$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{32739882\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{5\sqrt{-2(2-3x)-11}}-\frac{688636\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-}}{}\right.}{18538}$$
$$3588$$
$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

$$\Big\downarrow \text{412}$$

$$\frac{3\left(-\frac{688636\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{10730\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}-\frac{16369941\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\right.\right.}{155\sqrt{11}\sqrt{-2(2-3x)-11}}\right.}{18538}$$
$$3588$$
$$\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}$$

input
```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

output
```
(-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1794*(7 + 5*x)^2) + ((-26
825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(9269*(7 + 5*x)) + (3*((-1
0730*Sqrt[22/3]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
 1/3])/Sqrt[5 - 2*x] - (688636*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[S
qrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(5*Sqrt[-5 + 2*x]) - (16369941*Sqrt[11 + 2
*(2 - 3*x)]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(155*Sqrt[11]*Sqrt[-11 - 2*(2 - 3*x)])))/18538)/3588
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 186

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 195

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b*(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h))), x] +
 Simp[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e*g +
c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{19017\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{1676715755\sqrt{-24x^3+70x^2-21x-10}}-\frac{5365\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11}{\ldots}\right.}{33534315\ldots}\right.}{\ldots}$
risch	$\frac{5(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(56093+26825x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{33257172(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{6339\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-3\ldots}}{11}\right.}{1676715755\sqrt{-24x^3+70x^2-21x-10}}\right.}{\ldots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(254612300\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2-169668125\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-\ldots}\right.}{\ldots}$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x,method=_RETURNV
ERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-19017/1676715755*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-
24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-5365/
335343151*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2
-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*Ellipt
icF(1/11*(11+44*x)^(1/2),3^(1/2)))-5/1794/(7+5*x)^2*(-24*x^3+70*x^2-21*x-1
0)^(1/2)-26825/33257172/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+5456647/771
28924730*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-
21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx=\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x, algorith
m="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5 + 1950*x^4 -
 4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**3,x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x, algorith
m="maxima")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x, algorith
m="giac")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{\sqrt{-3x+2}}{\sqrt{2x-5}\,\sqrt{4x+1}\,(7+5x)^3}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x)
```

output
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x)
```

3.65 $\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 293

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx &= \frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}\\&\quad-\frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}\end{aligned}$$

output
```
2*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2
)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*
g))^(1/2))/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(c*f-d*e)^(1/2)*(d*(f*x
+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticPi(f^(1/2)*(d*x
+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h
+d*g))^(1/2))/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.67 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2i\sqrt{c+dx}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)-\text{EllipticPi}\left(\frac{b(-de+cf)}{(bc-ad)f},i\text{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)\right)}{b\sqrt{\frac{f(c+dx)}{d(e+fx)}}\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Integrate[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
((-2*I)*Sqrt[c + d*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*(EllipticF[I*ArcSinh
[Sqrt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - Elli
pticPi[(b*(-(d*e) + c*f))/((b*c - a*d)*f), I*ArcSinh[Sqrt[(f*(c + d*x))/(d
*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(b*Sqrt[(f*(c + d*x))/(d*(
e + f*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {193, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 193$$

$$\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{d\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \ 131$$

$$\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{d\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b\sqrt{e+fx}}$$

$$\Big\downarrow \ 131$$

$$\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{d\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \ 130$$

$$\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \ 187$$

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{b}$$

$$\Big\downarrow \ 413$$

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{b\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\Big\downarrow \ 413$$

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{b\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\Big\downarrow \ 412$$

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

input
```
Int[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
- (2*Sqrt[-(d*e) + c*f]*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c
 + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSi
n[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c
*h))])/(b*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c*h)/d + (
h*(c + d*x))/d])
```

Defintions of rubi rules used

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 193
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[d/b   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x]), x], x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.30

method	result
default	$\frac{2\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{\frac{f(xd+c)}{cf-de}}\,\sqrt{-\frac{(hx+g)d}{ch-dg}}\,\sqrt{-\frac{d(fx+e)}{cf-de}}\left(\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)cf-\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(c}{f(}}\cdots\right.\right.}{bf(dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+c\cdots}$
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2d\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\,\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}-\frac{2(ad-bc)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\,\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\cdots}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2\cdots}}\right.}{\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$

input
```
int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b/f*(f/(c*f-d*e)*(d*x+c))^(1/2
)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-d*(f*x+e)/(c*f-d*e))^(1/2)*(EllipticF((f/
(c*f-d*e)*(d*x+c))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*c*f-EllipticF((f
/(c*f-d*e)*(d*x+c))^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*d*e-EllipticPi(
(f/(c*f-d*e)*(d*x+c))^(1/2),-b*(c*f-d*e)/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d
*g))^(1/2))*c*f+EllipticPi((f/(c*f-d*e)*(d*x+c))^(1/2),-b*(c*f-d*e)/f/(a*d
-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*d*e)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2
+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+dx}}{(a+bx)\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)}\,dx$$

input
```
int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
```

output
```
int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.66 $\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 449

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$
$$+\frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{2(bc-ad)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*d*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)*EllipticE(h
^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))/b
/f/h^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)+2*(-a*d+b*c)*(c*f-d
*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*Ellipt
icF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2
))/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*d+b*c)*(c*f-d*e)^(1/2)*(d
*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticPi(f^(1/2)
*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/
(-c*h+d*g))^(1/2))/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.99 (sec) , antiderivative size = 1176, normalized size of antiderivative = 2.62

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b^2*d^2*e^2*f*Sqrt[-e + (c*f)/d]*g - b^2*c*d*e*f^2*Sqrt[-e + (c*f)/d]*
g - a*b*d^2*e*f^2*Sqrt[-e + (c*f)/d]*g + a*b*c*d*f^3*Sqrt[-e + (c*f)/d]*g
- b^2*d^2*e^3*Sqrt[-e + (c*f)/d]*h + b^2*c*d*e^2*f*Sqrt[-e + (c*f)/d]*h +
a*b*d^2*e^2*f*Sqrt[-e + (c*f)/d]*h - a*b*c*d*e*f^2*Sqrt[-e + (c*f)/d]*h -
b^2*d^2*e*f*Sqrt[-e + (c*f)/d]*g*(e + f*x) + a*b*d^2*f^2*Sqrt[-e + (c*f)/d
]*g*(e + f*x) + 2*b^2*d^2*e^2*Sqrt[-e + (c*f)/d]*h*(e + f*x) - b^2*c*d*e*f
*Sqrt[-e + (c*f)/d]*h*(e + f*x) - 2*a*b*d^2*e*f*Sqrt[-e + (c*f)/d]*h*(e +
f*x) + a*b*c*d*f^2*Sqrt[-e + (c*f)/d]*h*(e + f*x) - b^2*d^2*e*Sqrt[-e + (c
*f)/d]*h*(e + f*x)^2 + a*b*d^2*f*Sqrt[-e + (c*f)/d]*h*(e + f*x)^2 + I*b*d*
(b*e - a*f)*(d*e - c*f)*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2
)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticE[I*ArcSinh[Sqrt[-e + (c*f)/d]
/Sqrt[e + f*x]], (d*(-(f*g) + e*h))/((d*e - c*f)*h)] + I*b*(-(b*c) + a*d)*
f*(d*e - c*f)*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*
(g + h*x))/(h*(e + f*x))]*EllipticF[I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt[e +
f*x]], (d*(-(f*g) + e*h))/((d*e - c*f)*h)] - I*b^2*c^2*f^2*h*Sqrt[(f*(c +
d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*Ell
ipticPi[(b*d*e - a*d*f)/(b*d*e - b*c*f), I*ArcSinh[Sqrt[-e + (c*f)/d]/Sqrt
[e + f*x]], (d*(-(f*g) + e*h))/((d*e - c*f)*h)] + (2*I)*a*b*c*d*f^2*h*Sqrt
[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f
*x))]*EllipticPi[(b*d*e - a*d*f)/(b*d*e - b*c*f), I*ArcSinh[Sqrt[-e + (...
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {197, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{197}$$

$$\int \left(\frac{(bc-ad)^2}{b^2(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}+\frac{d(bc-ad)}{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}+\frac{d\sqrt{c+dx}}{b\sqrt{e+fx}\sqrt{g+hx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}+\frac{2d\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

input
```
Int[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*d*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Elli
pticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h)
)/((d*e - c*f)*h))])/(b*f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[
g + h*x]) + (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x
])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqr
t[e + f*x]*Sqrt[g + h*x]) - (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e +
 f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e -
 c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]
], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h
*x])
```

Defintions of rubi rules used

rule 197
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Int[ExpandIntegrand[1/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(-\frac{2d(ad-2bc)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2d^2\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{2\sqrt{xd+c}\sqrt{fx+e}\sqrt{hx+g}\sqrt{\frac{f(xd+c)}{cf-de}}\sqrt{-\frac{(hx+g)d}{ch-dg}}\sqrt{-\frac{d(fx+e)}{cf-de}}\left(\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)acdfh-\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}}\cdots\right.\right.}{\cdots}$

input
```
int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(-2*d*(a*d-2*b*c)/b^2*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g
/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g
*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1
/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))+2*d^2/b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(
1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*
h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)
*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*El
lipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2)))+2*(a^2*d
^2-2*a*b*c*d+b^2*c^2)/b^3*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c
/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d
*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-c/d+a/b)*EllipticPi(((x+c/
d)/(c/d-e/f))^(1/2),(-c/d+e/f)/(-c/d+a/b),((-c/d+e/f)/(-c/d+g/h))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((c + d*x)**(3/2)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
giac")
```

output
```
integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(c+d\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)}\,dx$$

input
```
int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
```

output
```
int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.67 $\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 162

$$\begin{aligned}
&\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx \\
&= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) \\
&\quad - \frac{487585\sqrt{\frac{11}{2}}\sqrt{5-2x}\,E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{1296\sqrt{-5+2x}} \\
&\quad + \frac{128698\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{81\sqrt{-5+2x}}
\end{aligned}$$

output
```
-2135/108*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-5/12*(2-3*x)^(1/2)*(-
5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)-487585/2592*22^(1/2)*(5-2*x)^(1/2)*Elli
pticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+128698/891*22^(1
/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^
(1/2)
```

Mathematica [A] (verified)

Time = 18.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{-6600\sqrt{2-3x}\sqrt{1+4x}(-490+151x+18x^2)-5363435\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)+}{28512\sqrt{-5+2x}}$$

input
```
Integrate[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(-6600*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-490 + 151*x + 18*x^2) - 5363435*Sqrt[
66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 49484
02*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3]
)/(28512*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {185, 27, 2118, 27, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x+7)^3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

$$\Big\downarrow \text{185}$$

$$\frac{1}{120}\int \frac{5(17080x^2+20965x+6997)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{17080x^2+20965x+6997}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \text{2118}$$

$$\frac{1}{24}\left(\frac{1}{108}\int \frac{12(487585x+18138)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{4270}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{24}\left(\frac{1}{9}\int \frac{487585x+18138}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{4270}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\right)-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \ 176$$

$$\frac{1}{24}\left(\frac{1}{9}\left(\frac{2474201}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{487585}{2}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)-\frac{4270}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4\right.$$
$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \ 124$$

$$\frac{1}{24}\left(\frac{1}{9}\left(\frac{487585\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{2\sqrt{5-2x}}+\frac{2474201}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)-\frac{4270}{9}\sqrt{2-3x}\sqrt{2x-5\right.$$
$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \ 123$$

$$\frac{1}{24}\left(\frac{1}{9}\left(\frac{2474201}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{487585\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}\right)-\frac{4270}{9}\right.$$
$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \ 131$$

$$\frac{1}{24}\left(\frac{1}{9}\left(\frac{2474201\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{22}\sqrt{2x-5}}+\frac{487585\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}\right)-\frac{4270}{9}\right.$$
$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{24}\left(\frac{1}{9}\left(\frac{2474201\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{2\sqrt{2x-5}}+\frac{487585\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}\right)-\frac{4270}{9}\right.$$

$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

$$\Big\downarrow \; 129$$

$$\frac{1}{24}\left(\frac{1}{9}\left(\frac{2474201\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{66}\sqrt{2x-5}}+\frac{487585\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right.\right.}{2\sqrt{5-2x}}\right.\right.$$

$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)$$

input
```
Int[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/12 + ((-4270*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + ((487585*Sqrt[11/6]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2*Sqrt[5 - 2*x])
+ (2474201*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(Sqrt[66]*Sqrt[-5 + 2*x]))/9)/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 185

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b^2*(a + b*x)^(m - 2)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Simp[1/(d*f*h
*(2*m - 1))   Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*d*f*h
*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g + c*f*
g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 2118

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(4118336\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \text{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-5363435\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}}{28512(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25x\sqrt{-24x^3+70x^2-21x-10}}{12}-\frac{1225\sqrt{-24x^3+70x^2-21x-10}}{54}+\frac{3023\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \text{Elliptic}}{4356\sqrt{-24x^3+70x^2-21x-}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1-}}$
risch	$\frac{25(98+9x)(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{108\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{3023\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \text{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right.}{13068\sqrt{-24x^3+70x^2-21x-10}}\right.}{}$

input
```
int((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/28512*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(4118336*(1+4*x)^(1/2)
*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/
2))-5363435*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1
/11*(11+44*x)^(1/2),3^(1/2))+1425600*x^4+11365200*x^3-44028600*x^2+1417680
0*x+6468000)/(24*x^3-70*x^2+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$
\begin{aligned}
\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx
&= -\frac{25}{108}\,(9\,x+98)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2} \\
&\quad - \frac{17718443}{46656}\,\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) \\
&\quad + \frac{487585}{1296}\,\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)
\end{aligned}
$$

input
```
integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="fricas")
```

output
```
-25/108*(9*x + 98)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 17718443/4
6656*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 487585/
1296*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/
108, 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral((5*x + 7)**3/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^3}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{12} + \frac{153\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{16} - \frac{13931\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right)}{12} - \frac{40567\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{96}$$

input
```
int((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
( - 200*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x + 918*sqrt(2*x - 5)
*sqrt(4*x + 1)*sqrt( - 3*x + 2) - 111448*int((sqrt(2*x - 5)*sqrt(4*x + 1)*
sqrt( - 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 40567*int((sqr
t(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10)
,x))/96
```

3.68 $\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 124

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{36}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{2135\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{108\sqrt{-5+2x}} + \frac{26089\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{108\sqrt{22}\sqrt{-5+2x}}$$

output
```
-25/36*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-2135/216*22^(1/2)*(5-2*x
)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+2608
9/2376*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2
))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 16.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{1650\sqrt{2-3x}(5-2x)\sqrt{1+4x}-23485\sqrt{66}\sqrt{5-2x}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)+24353\sqrt{66}\sqrt{5-2x}}{2376\sqrt{-5+2x}}$$

input
```
Integrate[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(1650*Sqrt[2 - 3*x]*(5 - 2*x)*Sqrt[1 + 4*x] - 23485*Sqrt[66]*Sqrt[5 - 2*x]
*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 24353*Sqrt[66]*Sqrt[5
- 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2376*Sqrt[-5 + 2
*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {185, 27, 2004, 176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x+7)^2}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{185}$$

$$\frac{1}{72}\int \frac{7(6100x^2+10685x+3003)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{72}\int \frac{6100x^2+10685x+3003}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{2004}$$

$$\frac{7}{72}\int \frac{1220x+429}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{176}$$

$$\frac{7}{72}\left(3479\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+610\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx\right)-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{124}$$

$$\frac{7}{72}\left(\frac{610\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}+3479\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx\right)-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{123}$$

$$\frac{7}{72}\left(3479\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{305\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{131}$$

$$\frac{7}{72}\left(\frac{3479\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}+\frac{305\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{72}\left(\frac{3479\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}+\frac{305\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

$$\Big\downarrow \text{129}$$

$$\frac{7}{72}\left(\frac{3479\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}+\frac{305\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)$$

$$\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

input
```
Int[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 + (7*((305*Sqrt[22/3]*
Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[5 -
2*x] + (3479*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 +
 4*x]], 1/3])/Sqrt[-5 + 2*x]))/72
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 185
```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b^2*(a + b*x)^(m - 2)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Simp[1/(d*f*h
*(2*m - 1))   Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*d*f*h
*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g + c*f*
g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

method	result
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(26089\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-23485\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{Ellip}\right.}{2376(24x^3-70x^2+21x+10)}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\left(-\frac{25\sqrt{-24x^3+70x^2-21x-10}}{36}+\frac{91\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{264\sqrt{-24x^3+70x^2-21x-10}}+\frac{2135\sqrt{11+44x}}{}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{36\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{91\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{792\sqrt{-24x^3+70x^2-21x-10}}-\frac{2135\sqrt{2}}{}\right.}{}$

input
```
int((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2376*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(26089*(1+4*x)^(1/2)*(2
-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))
-23485*(1+4*x)^(1/2)*(2-3*x)^(1/2)*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(
11+44*x)^(1/2),3^(1/2))+39600*x^3-115500*x^2+34650*x+16500)/(24*x^3-70*x^2
+21*x+10)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{36}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2} - \frac{12719}{486}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right) + \frac{2135}{108}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="fricas")
```

output
```
-25/36*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2) - 12719/486*sqrt(-6)*wei
erstrassPInverse(847/108, 6655/2916, x - 35/36) + 2135/108*sqrt(-6)*weiers
trassZeta(847/108, 6655/2916, weierstrassPInverse(847/108, 6655/2916, x -
35/36))
```

Sympy [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^2}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral((5*x + 7)**2/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^2}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^2}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^2}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(7+5x)^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2} - 61\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x^2}{24x^3-70x^2+21x+10}dx\right) - \frac{119\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)}{2}$$

input

```
int((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output

```
(2*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2) - 122*int((sqrt(2*x - 5)*s
qrt(4*x + 1)*sqrt( - 3*x + 2)*x**2)/(24*x**3 - 70*x**2 + 21*x + 10),x) - 1
19*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 +
 21*x + 10),x))/2
```

3.69 $\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{5\sqrt{\frac{11}{2}}\sqrt{5-2x}\,E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{6\sqrt{-5+2x}} + \frac{31\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{3\sqrt{-5+2x}}$$

output
```
-5/12*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2)
)/(-5+2*x)^(1/2)+31/33*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)
*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.97

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= \frac{220\sqrt{1+4x}(10-19x+6x^2)+55\sqrt{66}\sqrt{\frac{-5+2x}{1+4x}}\sqrt{\frac{-2+3x}{1+4x}}(1+4x)^2E\left(\arcsin\left(\frac{\sqrt{11}}{\sqrt{1+4x}}\right)|\frac{1}{3}\right)-78\sqrt{66}\sqrt{\frac{-5+2}{1+4}}}{132\sqrt{2-3x}\sqrt{-5+2x}(1+4x)}$$

input
```
Integrate[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(220*Sqrt[1 + 4*x]*(10 - 19*x + 6*x^2) + 55*Sqrt[66]*Sqrt[(-5 + 2*x)/(1 +
4*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*EllipticE[ArcSin[Sqrt[11]/Sqr
t[1 + 4*x]], 1/3] - 78*Sqrt[66]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*Sqrt[(-2 + 3*x)
/(1 + 4*x)]*(1 + 4*x)^2*EllipticF[ArcSin[Sqrt[11]/Sqrt[1 + 4*x]], 1/3])/(1
32*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {176, 124, 123, 131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x+7}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{176}$$

$$\frac{39}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5}{2}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{124}$$

$$\frac{5\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{2\sqrt{5-2x}}+\frac{39}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{123}$$

$$\frac{39}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}$$

$$\Big\downarrow \text{131}$$

$$\frac{39\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{22}\sqrt{2x-5}}+\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{39\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{2\sqrt{2x-5}}+\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}$$

$$\Big\downarrow \ 129$$

$$\frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}+\frac{5\sqrt{\frac{11}{6}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{2\sqrt{5-2x}}$$

input
```
Int[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(5*Sqrt[11/6]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(2*Sqrt[5 - 2*x]) + (13*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqr
t[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

method	result
default	$\frac{\left(124\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)-55\,\mathrm{EllipticE}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\right)\sqrt{5-2x}\,\sqrt{22}}{132\sqrt{-5+2x}}$
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(\frac{7\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{121\sqrt{-24x^3+70x^2-21x-10}}+\frac{5\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11\,\mathrm{EllipticE}}{}\right.}{121\sqrt{-24x^3+}}\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$

input
```
int((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/132*(124*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-55*EllipticE(1/11*(11+4
4*x)^(1/2),3^(1/2)))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.27

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$
$$= -\frac{427}{216}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$
$$+\frac{5}{6}\sqrt{-6}\text{weierstrassZeta}\left(\frac{847}{108},\frac{6655}{2916},\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)\right)$$

input
```
integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm=
"fricas")
```

output
```
-427/216*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36) + 5/6
*sqrt(-6)*weierstrassZeta(847/108, 6655/2916, weierstrassPInverse(847/108,
 6655/2916, x - 35/36))
```

Sympy [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5x+7}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral((5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5\,x+7}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5\,x+7}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{5\,x+7}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -5\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}\,x}{24x^3-70x^2+21x+10}dx\right) - 7\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)$$

input
```
int((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
 - 5*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2)*x)/(24*x**3 - 70*x*
*2 + 21*x + 10),x) - 7*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/
(24*x**3 - 70*x**2 + 21*x + 10),x)
```

3.70 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 44

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{\sqrt{-5+2x}}$$

output
```
1/11*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))
/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{\sqrt{\frac{-2+3x}{1+4x}}(1+4x)\sqrt{\frac{-10+4x}{11+44x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{1+4x}}\right),3\right)}{\sqrt{2-3x}\sqrt{-5+2x}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
-((Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*Sqrt[(-10 + 4*x)/(11 + 44*x)]*Elli
pticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {131, 27, 129}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{131}$$

$$\frac{\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}$$

$$\Big\downarrow \text{129}$$

$$\frac{\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3]
)/Sqrt[-5 + 2*x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{11\sqrt{-5+2x}}$	33
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{121\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{-24x^{3}+70x^{2}-21x-10}}$	94

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))*(5-2*x)^(1/2)*22^(1/2)/(-5+2*
x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{1}{6}\sqrt{-6}\text{weierstrassPInverse}\left(\frac{847}{108},\frac{6655}{2916},x-\frac{35}{36}\right)$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="frica
s")
```

output
```
-1/6*sqrt(-6)*weierstrassPInverse(847/108, 6655/2916, x - 35/36)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
- int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 +
21*x + 10),x)
```

3.71 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \frac{4\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{23\sqrt{-5+2x}}$$

output
```
4/253*22^(1/2)*(5-2*x)^(1/2)*EllipticPi(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23
,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \frac{3i(-2+3x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}}\right),-\frac{1}{2}\right)-\text{EllipticPi}\left(-\frac{62}{55},i\text{arcsinh}\left(\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}}\right),-\right.\right.}{31\sqrt{1+4x}\sqrt{-55+22x}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

output
```
(((3*I)/31)*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(EllipticF[I*
ArcSinh[Sqrt[11/2]/Sqrt[2 - 3*x]], -1/2] - EllipticPi[-62/55, I*ArcSinh[Sq
rt[11/2]/Sqrt[2 - 3*x]], -1/2]))/(Sqrt[1 + 4*x]*Sqrt[-55 + 22*x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx$$

$$\Big\downarrow \text{186}$$

$$-2\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}$$

$$\Big\downarrow \text{27}$$

$$-6\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}$$

$$\Big\downarrow \text{413}$$

$$-\frac{6\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{\sqrt{11}\sqrt{-2(2-3x)-11}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{6\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{\sqrt{-2(2-3x)-11}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{3\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{-2(2-3x)-11}}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

output
```
(-3*Sqrt[11 + 2*(2 - 3*x)]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqr
t[11]], -1/2])/(31*Sqrt[11]*Sqrt[-11 - 2*(2 - 3*x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{4\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)\sqrt{5-2x}\,\sqrt{22}}{253\sqrt{-5+2x}}$	34
elliptic	$\frac{4\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\mathrm{EllipticPi}\left(\frac{\sqrt{11+44x}}{11},-\frac{55}{23},\sqrt{3}\right)}{2783\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{-24x^{3}+70x^{2}-21x-10}}$	95

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x,method=_RETURNV
ERBOSE)
```

output
```
4/253*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2))*(5-2*x)^(1/2)*22^(1/
2)/(-5+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x, algorith
m="fricas")
```

output
```
integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4 - 182*x^3 -
385*x^2 + 197*x + 70), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\cdot(5x+7)}\,dx$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x),x)
```

output
```
Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x, algorith
m="maxima")
```

output
```
integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{(5\,x+7)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x, algorith
m="giac")
```

output
```
integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = \int \frac{1}{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(7+5x)}dx$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x)
```

output
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x),x)
```

3.72 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\begin{aligned}
&\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx \\
&= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{5\sqrt{22}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{27807\sqrt{-5+2x}} \\
&\quad - \frac{4\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{897\sqrt{-5+2x}} \\
&\quad + \frac{17906\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{639561\sqrt{-5+2x}}
\end{aligned}$$

output

```
-25*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(194649+139035*x)+5/27807*2
2^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2
*x)^(1/2)-4/9867*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+4*x)^(1/2)*11^(1
/2),3^(1/2))/(-5+2*x)^(1/2)+17906/7035171*22^(1/2)*(5-2*x)^(1/2)*EllipticP
i(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$
$$= \frac{-\frac{51150\sqrt{2-3x}(-5+2x)\sqrt{1+4x}}{7+5x} - 3\sqrt{55-22x}\left(6820E\left(\arcsin\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right) - 14508\,\text{EllipticF}\left(\arcsin\left(\frac{2\sqrt{2-3}}{\sqrt{11}}\right.\right.\right.}{56893122\sqrt{-5+2x}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

output
```
((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 2
2*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ell
ipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 26859*EllipticPi[55/124
, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(56893122*Sqrt[-5 + 2*x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {190, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}\,dx$$
$$\Big\downarrow \; 190$$
$$\frac{\int \frac{-600x^2-1680x+7777}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614} - \frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$
$$\Big\downarrow \; 2110$$
$$\frac{\int \frac{-120x-168}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx + 8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614} - \frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$
$$\Big\downarrow \; 176$$

$$\frac{-468\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-60\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$$\Big\downarrow \; 124$$

$$\frac{-\frac{60\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-468\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$$\Big\downarrow \; 123$$

$$\frac{-468\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{10\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{55614}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$$\Big\downarrow \; 131$$

$$\frac{-\frac{468\sqrt{\frac{2}{11}}\sqrt{5-2x}\int \frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}+8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{10\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{55614}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{468\sqrt{5-2x}\int \frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}+8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{10\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{55614}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$$\Big\downarrow \; 129$$

$$\frac{8953\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{10\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}}{55614}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$$\Big\downarrow \; 186$$

$$\frac{-17906\int \frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{10\sqrt{66}\sqrt{2x}}{}}{55614}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$\downarrow$ 27

$$\frac{-53718\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{10\sqrt{66}\sqrt{2x}}{}}{55614}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$\downarrow$ 413

$$\frac{-\frac{53718\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{10\sqrt{66}\sqrt{2}}{}}{55614}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$\downarrow$ 27

$$\frac{-\frac{53718\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{\sqrt{-2(2-3x)-11}}-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{10\sqrt{66}\sqrt{2}}{}}{55614}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

$\downarrow$ 412

$$\frac{-\frac{156\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{10\sqrt{66}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}-\frac{26859\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\text{a}\right.}{31\sqrt{11}\sqrt{-2(2-3x)-}}}{55614}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

output
```
(-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + ((-10
*Sqrt[66]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/Sqrt[5 - 2*x] - (156*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]
*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x] - (26859*Sqrt[11 + 2*(2 - 3*x)]*Elli
pticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(31*Sqrt[11]*Sqr
t[-11 - 2*(2 - 3*x)]))/55614
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 190
```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b^2*(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g -
a*h))), x] - Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[(
(a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a^2*d
*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g
 + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h
))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{28\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\ \mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{1121549\sqrt{-24x^3+70x^2-21x-10}}-\frac{20\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\left(-\frac{11\,\mathrm{Ellip}}{}\right.}{1121549\sqrt{-2}}\right.}{\sqrt{2-3x}}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(14260\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \mathrm{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x-6325\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\ \mathrm{Elliptic}\right.}{}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)}}{27807(7+5x)\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{28\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\ \mathrm{EllipticF}\left(\frac{2\sqrt{22-33x}}{11},\frac{i\sqrt{2}}{2}\right)}{3364647\sqrt{-24x^3+70x^2-21x-10}}+\frac{20\sqrt{22-3}}{}\right.}{}$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x,method=_RETUR
NVERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-28/1121549*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3
+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-20/1121549*
(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70*x^2-21*x-10)^
(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*EllipticF(1/11*(
11+44*x)^(1/2),3^(1/2)))-25/27807/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+1
7906/77386881*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70
*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x, algori
thm="fricas")
```

output
```
integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5 - 70*x^4 - 3
199*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^2}\,dx$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**2,x)
```

output
```
Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x, algori
thm="maxima")
```

output
```
integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{(5\,x+7)^2\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x, algori
thm="giac")
```

output
```
integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^2}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx$$
$$=\int \frac{1}{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(7+5x)^2}dx$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x)
```

output
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^2,x)
```

3.73 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$
\begin{aligned}
&\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx \\
&= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2}-\frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1030972332(7+5x)} \\
&\quad+\frac{44765\sqrt{\frac{11}{2}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{515486166\sqrt{-5+2x}} \\
&\quad-\frac{7159\sqrt{\frac{2}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{8314293\sqrt{-5+2x}} \\
&\quad+\frac{16164435\sqrt{5-2x}\,\text{EllipticPi}\left(-\frac{55}{23},\arcsin\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right),3\right)}{1976030303\sqrt{22}\sqrt{-5+2x}}
\end{aligned}
$$

output

```
-25/55614*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2-223825*(2-3
*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7216806324+5154861660*x)+44765/103
0972332*22^(1/2)*(5-2*x)^(1/2)*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/
2))/(-5+2*x)^(1/2)-7159/91457223*22^(1/2)*(5-2*x)^(1/2)*EllipticF(1/11*(1+
4*x)^(1/2)*11^(1/2),3^(1/2))/(-5+2*x)^(1/2)+16164435/43472666666*22^(1/2)*
(5-2*x)^(1/2)*EllipticPi(1/11*(1+4*x)^(1/2)*11^(1/2),-55/23,3^(1/2))/(-5+2
*x)^(1/2)
```

Mathematica [A] (verified)

Time = 5.85 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \frac{-17050\sqrt{2-3x}(-5+2x)\sqrt{1+4x}(81209+44765x)-\sqrt{55-22x}(7+5x)^2\left(61059460E\left(\arcsin\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.\right.}{703123130424}$$

input

```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

output

```
(-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(81209 + 44765*x) - Sqrt[55
 - 22*x]*(7 + 5*x)^2*(61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]
], -1/2] - 116097852*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
 145479915*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/
(703123130424*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {190, 2107, 27, 2110, 176, 124, 123, 131, 27, 129, 186, 27, 413, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^3}\,dx$$

$$\Big\downarrow \text{190}$$

$$\frac{\int \frac{600x^2-6860x+16079}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2}dx}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \text{2107}$$

$$\frac{\frac{\int \frac{9\left(-1790600x^2-4272160x+13692987\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{55614}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{-1790600x^2-4272160x+13692987}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx}{18538}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \; 2110$$

$$\frac{\frac{3\left(\int \frac{-358120x-353064}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+16164435\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)}{18538}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \; 176$$

$$\frac{\frac{3\left(-1248364\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-179060\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}}dx+16164435\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)}{18538}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \; 124$$

$$\frac{\frac{3\left(-\frac{179060\sqrt{2x-5}\int \frac{\sqrt{5-2x}}{\sqrt{2-3x}\sqrt{4x+1}}dx}{\sqrt{5-2x}}-1248364\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+16164435\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx\right)}{18538}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \; 123$$

$$\frac{\frac{3\left(-1248364\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+16164435\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{9269(5x+7)}}{111228}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$$\Big\downarrow \; 131$$

$$\frac{3\left(-\frac{1248364\sqrt{\frac{2}{11}}\sqrt{5-2x}\int\frac{\sqrt{\frac{11}{2}}}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}+16164435\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\frac{22}{}}{111228}$$

$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\downarrow$ 27

$$\frac{3\left(-\frac{1248364\sqrt{5-2x}\int\frac{1}{\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}}dx}{\sqrt{2x-5}}+16164435\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}-\frac{223825}{}}{111228}$$

$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\downarrow$ 129

$$\frac{3\left(16164435\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)}dx-\frac{1248364\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}\right)}{18538}}{111228}$$

$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\downarrow$ 186

$$\frac{3\left(-32328870\int\frac{3}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{1248364\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\text{arcs}\right.}{\sqrt{5-2}}\right.}{18538}}{111228}$$

$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\downarrow$ 27

$$\frac{3\left(-96986610\int\frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{-2(2-3x)-11}}d\sqrt{2-3x}-\frac{1248364\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\text{arcs}\right.}{\sqrt{5-2}}\right.}{18538}}{111228}$$

$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\downarrow$ 413

$$\frac{3\left(-\frac{96986610\sqrt{2(2-3x)+11}\int \frac{\sqrt{11}}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{\sqrt{11}\sqrt{-2(2-3x)-11}}-\frac{1248364\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x}}{}\right.}{18538}$$
$$\overline{\qquad 111228}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\big\downarrow$ 27

$$\frac{3\left(-\frac{96986610\sqrt{2(2-3x)+11}\int \frac{1}{(31-5(2-3x))\sqrt{11-4(2-3x)}\sqrt{2(2-3x)+11}}d\sqrt{2-3x}}{\sqrt{-2(2-3x)-11}}-\frac{1248364\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x}}{}\right.}{18538}$$
$$\overline{\qquad 111228}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

$\big\downarrow$ 412

$$\frac{3\left(-\frac{1248364\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{89530\sqrt{\frac{22}{3}}\sqrt{2x-5}E\left(\arcsin\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{5-2x}}-\frac{48493305\sqrt{2(2-3x)+11}\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\right.}{31\sqrt{11}\sqrt{-2(2-3x)-11}}\right.}{18538}$$
$$\overline{\qquad 111228}$$
$$\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

output
```
(-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) + ((-
223825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(9269*(7 + 5*x)) + (3*(
(-89530*Sqrt[22/3]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/Sqrt[5 - 2*x] - (1248364*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcS
in[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x] - (48493305*Sqrt[11 + 2
*(2 - 3*x)]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(31*Sqrt[11]*Sqrt[-11 - 2*(2 - 3*x)])))/18538)/111228
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 129
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b, 0] && GtQ
[(b*e - a*f)/b, 0] && PosQ[-b/d] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[(d
*e - c*f)/d, 0] && GtQ[-d/b, 0]) &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[((
-b)*e + a*f)/f, 0] && GtQ[-f/b, 0]) &&  !(SimplerQ[e + f*x, a + b*x] && GtQ
[((-d)*e + c*f)/f, 0] && GtQ[((-b)*e + a*f)/f, 0] && (PosQ[-f/d] || PosQ[-f
/b]))
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 190
```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b^2*(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g -
a*h))), x] - Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[(
(a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a^2*d
*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g
 + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h
))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{-(-2+3x)(-5+2x)(1+4x)}\left(-\frac{44133\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)}{10395637681\sqrt{-24x^3+70x^2-21x-10}}-\frac{44765\sqrt{11+44x}\,\sqrt{22-33x}\,\sqrt{110-44x}\,\left(-\cdots\right.}{1039563\cdots}\right.}{\cdots}$
risch	$\frac{25(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,(81209+44765x)\sqrt{(2-3x)(-5+2x)(1+4x)}}{1030972332(7+5x)^2\sqrt{-(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(\frac{14711\sqrt{22-33x}\,\sqrt{165-66x}\,\sqrt{33+132x}\,\operatorname{EllipticF}\left(\frac{2\sqrt{22-3\cdots}}{11}\right.}{10395637681\sqrt{-24x^3+70x^2-21x-10}}\right.}{\cdots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\left(510436700\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-2x}\,\operatorname{EllipticF}\left(\frac{\sqrt{11+44x}}{11},\sqrt{3}\right)x^2-283138625\sqrt{1+4x}\,\sqrt{2-3x}\,\sqrt{22}\,\sqrt{5-\cdots}\right.}{\cdots}$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x,method=_RETUR
NVERBOSE)
```

output
```
(-(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1
/2)*(-44133/10395637681*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(
-24*x^3+70*x^2-21*x-10)^(1/2)*EllipticF(1/11*(11+44*x)^(1/2),3^(1/2))-4476
5/10395637681*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*x^3+70
*x^2-21*x-10)^(1/2)*(-11/12*EllipticE(1/11*(11+44*x)^(1/2),3^(1/2))+2/3*El
lipticF(1/11*(11+44*x)^(1/2),3^(1/2)))-25/55614/(7+5*x)^2*(-24*x^3+70*x^2-
21*x-10)^(1/2)-223825/1030972332/(7+5*x)*(-24*x^3+70*x^2-21*x-10)^(1/2)+16
164435/478199333326*(11+44*x)^(1/2)*(22-33*x)^(1/2)*(110-44*x)^(1/2)/(-24*
x^3+70*x^2-21*x-10)^(1/2)*EllipticPi(1/11*(11+44*x)^(1/2),-55/23,3^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x, algori
thm="fricas")
```

output
```
integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^6 + 3850*x^5
- 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3}\,dx$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**3,x)
```

output
```
Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x, algori
thm="maxima")
```

output
```
integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x, algori
thm="giac")
```

output
```
integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = \int \frac{1}{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(7+5x)^3}dx$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x)
```

output
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^3,x)
```

3.74 $\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 137

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{-fg+eh}\,i\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

output
```
2*(e*h-f*g)^(1/2)*i*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)*EllipticE(h
^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))/f
/h^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2ii\sqrt{c+dx}\sqrt{g+hx}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)\right)}{h\sqrt{\frac{f(c+dx)}{d(e+fx)}}\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

input
```
Integrate[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
((-2*I)*i*Sqrt[c + d*x]*Sqrt[g + h*x]*(EllipticE[I*ArcSinh[Sqrt[(f*(c + d*
x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - EllipticF[I*ArcSinh[
Sqrt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(h*Sq
rt[(f*(c + d*x))/(d*(e + f*x))]*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*
h)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {35, 124, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{35}$$

$$i\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{124}$$

$$\frac{i\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\int \frac{\sqrt{-\frac{dxf}{de-cf}-\frac{cf}{de-cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}dx}{\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

$$\Big\downarrow \text{123}$$

$$\frac{2i\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{g+hx}\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

input
```
Int[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[-(f*g) + e*h]*i*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Elli
pticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h)
)/((d*e - c*f)*h))])/(f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g
+ h*x])
```

Defintions of rubi rules used

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(120) = 240$.

Time = 1.90 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.65

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2ci\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2di\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\cdots}{\cdots}\right.}{\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$\frac{2i\left(\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)c^2fh-\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)cdeh-\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)cdfg+\cdots\right.}{\cdots}$

input
```
int((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2*c*i*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+
e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)
/(-c/d+g/h))^(1/2))+2*d*i*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c
/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d
*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/
d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/
(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(120) = 240.

Time = 0.11 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.85

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = -\frac{2\left(3\sqrt{dfh}dfhi\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-\left(d^2ef+cdf^2\right)gh+\left(d^2e^2-cdef+c^2f^2\right)h^2\right)}{3\,d^2f^2h^2}, -\frac{4\left(2\,d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-3\right.}{}\right.\right.}{}$$

input
```
integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorit
hm="fricas")
```

output
```
-2/3*(3*sqrt(d*f*h)*d*f*h*i*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f +
c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*
d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f
^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3
*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(
2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e
*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c
^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h
))) + (d*f*g + (d*e - 2*c*f)*h)*sqrt(d*f*h)*i*weierstrassPInverse(4/3*(d^2
*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d
^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d
^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (
d*e + c*f)*h)/(d*f*h)))/(d*f^2*h^2)
```

Sympy [F]

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = i\int \frac{\sqrt{c + dx}}{\sqrt{e + fx}\sqrt{g + hx}}\,dx$$

input
```
integrate((d*i*x+c*i)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
i*Integral(sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{dix + ci}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{dix + ci}{\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ci + dix}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{c\,i + d\,i\,x}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,\sqrt{c + d\,x}}\,dx$$

input
```
int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{ci+dix}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{fh\,x^{2}+ehx+fgx+eg}dx\right)i$$

input
```
int((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(e*g + e*h*x + f*g*x + f*h
*x**2),x)*i
```

3.75 $\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\begin{aligned}\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx &= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\ &\quad - \frac{2\sqrt{-de+cf}(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}\end{aligned}$$

output
```
2*b*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d/
f^(1/2)/h/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(c*f-d*e)^(1/2)*(-a
*h+b*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*Elliptic
F(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))
/d/f^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.23 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(-bd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)-ib(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right.\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c+}}$$

input
```
Integrate[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*(-(b*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*b*(d*e - c*f)*h*(
c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
+ d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)] + I*d*(b*e - a*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*
x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[S
qrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*
Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{176}$$

$$\frac{b\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(bg-ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{124}$$

$$\frac{b\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(bg-ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{123}$$

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(bg-ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{131}$$

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Int[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*b*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Elli
pticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/
(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h
)]) - (2*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e +
f*x]*Sqrt[g + h*x])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2a\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2b\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+}{-\frac{c}{d}}}}{\sqrt{df}}\right.}{\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}$
default	$\frac{2\left(\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)acdfh-\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)a\,d^2eh-\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)bcdfg\right.}{}$

input
```
int((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(2*a*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/
f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(
-c/d+g/h))^(1/2))+2*b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g
/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g
*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(
c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d
-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(250) = 500$.

Time = 0.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.36

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(3\sqrt{dfh}bdfh\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-\left(d^2ef+cdf^2\right)gh+\left(d^2e^2-cdef+c^2f^2\right)h^2\right)}{3d^2f^2h^2}, -\frac{4\left(2d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-3\right.}{}\right.\right.}{}$$

input
```
integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
fricas")
```

output
```
-2/3*(3*sqrt(d*f*h)*b*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f +
c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*
d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f
^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3
*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(
2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e
*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c
^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h
))) + (b*d*f*g + (b*d*e + (b*c - 3*a*d)*f)*h)*sqrt(d*f*h)*weierstrassPInve
rse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*
f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*
g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c
*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x
 + d*f*g + (d*e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{bx+a}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{bx+a}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
giac")
```

output
```
integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{a+b\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx \\
&= \left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,x}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx\right)b \\
&\quad + \left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx\right)a
\end{aligned}$$

input
```
int((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output

```
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*x)/(c*e*g + c*e*h*x + c*f*g
*x + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*b + i
nt((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(c*e*g + c*e*h*x + c*f*g*x
+ c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*a
```

3.76 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 165

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
-2*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/
2)*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*
c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(
h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.97 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)-\text{EllipticPi}\left(-\frac{bcf-adf}{bde-bcf},i\text{arcsinh}\right.\right.}{(-bc+ad)\sqrt{-c+\frac{de}{f}}\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(
c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g
- c*f*h)/(d*e*h - c*f*h)] - EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)),
 I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f
*h)]))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$\Big\downarrow \ 187$$
$$-2\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}$$
$$\Big\downarrow \ 413$$
$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

input
```
Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*Sqrt[-(d*e) + c*f]*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c
+ d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin
[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*
h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c
*h)/d + (h*(c + d*x))/d])
```

Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{2\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{\frac{f(xd+c)}{cf-de}}\,\sqrt{-\frac{(hx+g)d}{ch-dg}}\,\sqrt{-\frac{d(fx+e)}{cf-de}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{f(xd+c)}{cf-de}},-\frac{b(cf-de)}{f(ad-bc)},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)(cf-de)}{f(ad-bc)(dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg)}$	223
elliptic	$\frac{2\sqrt{(hx+g)(fx+e)(xd+c)}\,\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\,\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\,\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$	274

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f*(f/(c*f-d*e)*(d*x+c))^(1/2)*
(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-d*(f*x+e)/(c*f-d*e))^(1/2)*EllipticPi((f/(c
*f-d*e)*(d*x+c))^(1/2),-b*(c*f-d*e)/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^
(1/2))*(c*f-d*e)/(a*d-b*c)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*
x+c*f*g*x+d*e*g*x+c*e*g)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.77 $\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 393

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2d\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output

```
2*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)
^(1/2)-2*d*h^(1/2)*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1
/2)*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d
*e)/h)^(1/2))/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(-f*(d*x+c)/(-c*f+d*e))^(1/
2)/(h*x+g)^(1/2)-2*b*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+
g)/(-c*h+d*g))^(1/2)*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(
-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)^2/f^(
1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left((bc-ad)hE\left(i\text{arcsinh}\left(\frac{\sqrt{-c+}}{\sqrt{c+d}}\right.\right.\right.}{}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(
c + d*x))]*((b*c - a*d)*h*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c +
d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + (b*d*g - 2*b*c*h + a*d*h)*Ellipt
icF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h -
c*f*h)] + b*(-(d*g) + c*h)*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)),
I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*
h)]))/((b*c - a*d)^2*Sqrt[-c + (d*e)/f]*(-(d*g) + c*h)*Sqrt[e + f*x]*Sqrt[
g + h*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {197, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{197}$$

$$\int \left(\frac{b}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}-\frac{d}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d\sqrt{h}\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}}-\frac{2b\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2}+\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{\sqrt{c+dx}(bc-ad)(de-cf)(dg-ch)}$$

input
```
Int[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*S
qrt[c + d*x]) - (2*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g +
 h*x))/(f*g - e*h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) +
 e*h]], -((d*(f*g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)*(d*e - c*f)*(d*g
 - c*h)*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*b*Sqrt[-(d*
e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*
EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*
x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*
Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Defintions of rubi rules used

rule 197
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Int[ExpandIntegrand[1/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(353) = 706$.

Time = 13.60 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.48

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(-\frac{2\left(dfh\,x^{2}+dehx+dfgx+deg\right)d}{\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)(ad-bc)\sqrt{\left(x+\frac{c}{d}\right)\left(dfh\,x^{2}+dehx+dfgx+deg\right)}}+\frac{2\left(\frac{d(cfh-deh-dfg)}{\left(c^{2}fh-cdeh-cdfg+d^{2}eg\right)(ad-bc)}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(-2*(d*f*h*x^2+d*e*h*x+d*f*g*x+d*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)*d/
(a*d-b*c)/((x+c/d)*(d*f*h*x^2+d*e*h*x+d*f*g*x+d*e*g))^(1/2)+2*(1/(c^2*f*h-
c*d*e*h-c*d*f*g+d^2*e*g)*d*(c*f*h-d*e*h-d*f*g)/(a*d-b*c)+(d*e*h+d*f*g)/(c^
2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)*d/(a*d-b*c))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^
(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f
*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(
((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))+2/(c^2*f*h-c*d*e*
h-c*d*f*g+d^2*e*g)*f*h*d^2/(a*d-b*c)*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*(
(x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+
d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*Ellip
ticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*Elliptic
F(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2)))-2/(a*d-b*c)*(c
/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/
d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e
*g*x+c*e*g)^(1/2)/(-c/d+a/b)*EllipticPi(((x+c/d)/(c/d-e/f))^(1/2),(-c/d+e/
f)/(-c/d+a/b),((-c/d+e/f)/(-c/d+g/h))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)\,(c+dx)^{\frac{3}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(1/((a + b*x)*(c + d*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)\,(c+d\,x)^{3/2}}\,dx$$

input
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)\,(dx+c)^{\frac{3}{2}}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.78 $\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 875

$$\begin{aligned}
\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx &= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} \\
&+ \frac{2bd^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}} \\
&+ \frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} \\
&- \frac{2bd\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} \\
&- \frac{2\sqrt{f}(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(-de+cf)^{3/2}(dg-ch)\sqrt{e+fx}\sqrt{g+hx}} \\
&- \frac{2b^2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}
\end{aligned}$$

output
```
2/3*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+
c)^(3/2)+2*b*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h
+d*g)/(d*x+c)^(1/2)-4/3*d^2*(-2*c*f*h+d*e*h+d*f*g)*(f*x+e)^(1/2)*(h*x+g)^(
1/2)/(-a*d+b*c)/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)+4/3*d*f^(1/2)*(-2*
c*f*h+d*e*h+d*f*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^
(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a
*d+b*c)/(c*f-d*e)^(3/2)/(-c*h+d*g)^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^
(1/2)-2*b*d*h^(1/2)*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(
1/2)*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+
d*e)/h)^(1/2))/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d*g)/(-f*(d*x+c)/(-c*f+d*e))^
(1/2)/(h*x+g)^(1/2)-2/3*f^(1/2)*(-3*c*f*h+d*e*h+2*d*f*g)*(d*(f*x+e)/(-c*f+
d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(
c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(c*f-d*e)^(3/
2)/(-c*h+d*g)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*b^2*(c*f-d*e)^(1/2)*(d*(f*x+e)
/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticPi(f^(1/2)*(d*x+c)
^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*
g))^(1/2))/(-a*d+b*c)^3/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.10 (sec) , antiderivative size = 4180, normalized size of antiderivative = 4.78

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*((2*d^2)/(3*(b*c - a*d)*(-(d*e)
+ c*f)*(-(d*g) + c*h)*(c + d*x)^2) + (2*d^2*(3*b*d^2*e*g - 5*b*c*d*f*g + 2
*a*d^2*f*g - 5*b*c*d*e*h + 2*a*d^2*e*h + 7*b*c^2*f*h - 4*a*c*d*f*h))/(3*(b
*c - a*d)^2*(-(d*e) + c*f)^2*(-(d*g) + c*h)^2*(c + d*x))) + (2*(c + d*x)^(
3/2)*(-3*b^2*c*d^2*e*Sqrt[-c + (d*e)/f]*f*g*h + 3*a*b*d^3*e*Sqrt[-c + (d*e
)/f]*f*g*h + 5*b^2*c^2*d*Sqrt[-c + (d*e)/f]*f^2*g*h - 7*a*b*c*d^2*Sqrt[-c
+ (d*e)/f]*f^2*g*h + 2*a^2*d^3*Sqrt[-c + (d*e)/f]*f^2*g*h + 5*b^2*c^2*d*e*
Sqrt[-c + (d*e)/f]*f*h^2 - 7*a*b*c*d^2*e*Sqrt[-c + (d*e)/f]*f*h^2 + 2*a^2*
d^3*e*Sqrt[-c + (d*e)/f]*f*h^2 - 7*b^2*c^3*Sqrt[-c + (d*e)/f]*f^2*h^2 + 11
*a*b*c^2*d*Sqrt[-c + (d*e)/f]*f^2*h^2 - 4*a^2*c*d^2*Sqrt[-c + (d*e)/f]*f^2
*h^2 - (3*b^2*c*d^4*e^2*Sqrt[-c + (d*e)/f]*g^2)/(c + d*x)^2 + (3*a*b*d^5*e
^2*Sqrt[-c + (d*e)/f]*g^2)/(c + d*x)^2 + (8*b^2*c^2*d^3*e*Sqrt[-c + (d*e)/
f]*f*g^2)/(c + d*x)^2 - (10*a*b*c*d^4*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c + d*x
)^2 + (2*a^2*d^5*e*Sqrt[-c + (d*e)/f]*f*g^2)/(c + d*x)^2 - (5*b^2*c^3*d^2*
Sqrt[-c + (d*e)/f]*f^2*g^2)/(c + d*x)^2 + (7*a*b*c^2*d^3*Sqrt[-c + (d*e)/f
]*f^2*g^2)/(c + d*x)^2 - (2*a^2*c*d^4*Sqrt[-c + (d*e)/f]*f^2*g^2)/(c + d*x
)^2 + (8*b^2*c^2*d^3*e^2*Sqrt[-c + (d*e)/f]*g*h)/(c + d*x)^2 - (10*a*b*c*d
^4*e^2*Sqrt[-c + (d*e)/f]*g*h)/(c + d*x)^2 + (2*a^2*d^5*e^2*Sqrt[-c + (d*e
)/f]*g*h)/(c + d*x)^2 - (20*b^2*c^3*d^2*e*Sqrt[-c + (d*e)/f]*f*g*h)/(c + d
*x)^2 + (28*a*b*c^2*d^3*e*Sqrt[-c + (d*e)/f]*f*g*h)/(c + d*x)^2 - (8*a^...
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {197, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{197}$$

$$\int \left(\frac{b^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2}-\frac{bd}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2}-\frac{}{(c+dx)^{5/2}\sqrt{e+}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}E\left(\arcsin\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)b}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}+\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}b}{(bc-ad)^2(de-cf)(dg-ch)\sqrt{c+dx}}+\frac{4d\sqrt{f}(dfg+deh-2cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(cf-de)^{3/2}(dg-ch)^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2\sqrt{f}(2dfg+deh-3cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{3(bc-ad)(cf-de)^{3/2}(dg-ch)\sqrt{e+fx}\sqrt{g+hx}}-\frac{4d^2(dfg+deh-2cfh)\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)^2(dg-ch)^2\sqrt{c+dx}}+\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}}$$

input
```
Int[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)*(d*g - c*h)
*(c + d*x)^(3/2)) + (2*b*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(
d*e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) - (4*d^2*(d*f*g + d*e*h - 2*c*f*h)*S
qrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)^2*(d*g - c*h)^2*Sqr
t[c + d*x]) + (4*d*Sqrt[f]*(d*f*g + d*e*h - 2*c*f*h)*Sqrt[(d*(e + f*x))/(d
*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*(b*c - a*d)*(-(d*e) + c*
f)^(3/2)*(d*g - c*h)^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2
*b*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*
h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*
g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*Sqrt[-
((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*Sqrt[f]*(2*d*f*g + d*e*h
 - 3*c*f*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]
*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f
)*h)/(f*(d*g - c*h))])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*Sqr
t[e + f*x]*Sqrt[g + h*x]) - (2*b^2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(
d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/(
(b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
 - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^3*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g +
 h*x])
```

Defintions of rubi rules used

rule 197
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Int[ExpandIntegrand[1/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 16.26 (sec) , antiderivative size = 1335, normalized size of antiderivative = 1.53

method	result	size
elliptic	Expression too large to display	1335
default	Expression too large to display	17329

input
```
int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*
(-2/3/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)/(a*d-b*c)*(d*f*h*x^3+c*f*h*x^2+d*e
*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(x+c/d)^2-2/3*(d*f*h
*x^2+d*e*h*x+d*f*g*x+d*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2*d*(4*a*c*d
*f*h-2*a*d^2*e*h-2*a*d^2*f*g-7*b*c^2*f*h+5*b*c*d*e*h+5*b*c*d*f*g-3*b*d^2*e
*g)/(a*d-b*c)^2/((x+c/d)*(d*f*h*x^2+d*e*h*x+d*f*g*x+d*e*g))^(1/2)+2*(-1/3*
d*f*h/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)/(a*d-b*c)+1/3*d*(c*f*h-d*e*h-d*f*g
)*(4*a*c*d*f*h-2*a*d^2*e*h-2*a*d^2*f*g-7*b*c^2*f*h+5*b*c*d*e*h+5*b*c*d*f*g
-3*b*d^2*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2/(a*d-b*c)^2+1/3*(d*e*h+d
*f*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2*d*(4*a*c*d*f*h-2*a*d^2*e*h-2*a*d
^2*f*g-7*b*c^2*f*h+5*b*c*d*e*h+5*b*c*d*f*g-3*b*d^2*e*g)/(a*d-b*c)^2)*(c/d-
e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e
/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*
x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))
^(1/2))+2/3*d^2*f*h*(4*a*c*d*f*h-2*a*d^2*e*h-2*a*d^2*f*g-7*b*c^2*f*h+5*b*c
*d*e*h+5*b*c*d*f*g-3*b*d^2*e*g)/(c^2*f*h-c*d*e*h-c*d*f*g+d^2*e*g)^2/(a*d-b
*c)^2*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e
/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f
*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),
((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)\,(c+dx)^{\frac{5}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(5/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(1/((a + b*x)*(c + d*x)**(5/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="giac")
```

output
```
integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)),x)
```

output
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(bx+a)\,(dx+c)^{\frac{5}{2}}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.79 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx=-\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

output
```
-2*(f*(d*x+c)/(c*f+d))^(1/2)*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*
f+b),2^(1/2)*(d/(c*f+d))^(1/2))/(a*f+b)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.94 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx$$
$$=\frac{2i(c+dx)\sqrt{\frac{d(-1+fx)}{f(c+dx)}}\sqrt{\frac{d+dfx}{cf+dfx}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right),\frac{-d+cf}{d+cf}\right)-\text{EllipticPi}\left(\frac{bcf-adf}{bd+bcf},i\text{arcsinh}\left(\right.\right.\right.}{(-bc+ad)\sqrt{-\frac{d+cf}{f}}\sqrt{1-f^2x^2}}$$

input
```
Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

output
```
((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f
+ d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d +
c*f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt
[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*S
qrt[-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {730, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-f^2x^2}(a+bx)\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{730}$$

$$\int \frac{1}{\sqrt{1-fx}\sqrt{fx+1}(a+bx)\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{186}$$

$$-2\int \frac{1}{\sqrt{fx+1}(-((1-fx)b)+b+af)\sqrt{c-\frac{d(1-fx)}{f}+\frac{d}{f}}}d\sqrt{1-fx}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\sqrt{1-\frac{d(1-fx)}{cf+d}}\int \frac{1}{\sqrt{fx+1}(-((1-fx)b)+b+af)\sqrt{1-\frac{d(1-fx)}{d+cf}}}d\sqrt{1-fx}}{\sqrt{c-\frac{d(1-fx)}{f}+\frac{d}{f}}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\sqrt{1-\frac{d(1-fx)}{cf+d}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c-\frac{d(1-fx)}{f}+\frac{d}{f}}}$$

input
```
Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

output
```
(-2*Sqrt[1 - (d*(1 - f*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[S
qrt[1 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d/f - (d*(1 -
 f*x))/f])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 730
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/((e + f*x
)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && NegQ[b/a] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(71) = 142.

Time = 3.44 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.45

method	result	size
default	$-\frac{2(cf-d)\,\mathrm{EllipticPi}\left(\sqrt{\frac{f(xd+c)}{cf-d}},-\frac{(cf-d)b}{f(ad-bc)},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{d(fx+1)}{cf-d}}\,\sqrt{-\frac{d(fx-1)}{cf+d}}\,\sqrt{\frac{f(xd+c)}{cf-d}}\,\sqrt{-f^2x^2+1}\,\sqrt{xd+c}}{f(ad-bc)(d\,f^2x^3+c\,f^2x^2-xd-c)}$	181
elliptic	$\frac{2\sqrt{-(xd+c)(f^2x^2-1)}\left(\frac{c}{d}-\frac{1}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x-\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x+\frac{1}{f}}{-\frac{c}{d}+\frac{1}{f}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}},\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\right)}{\sqrt{xd+c}\,\sqrt{-f^2x^2+1}\,b\sqrt{-d\,f^2x^3-c\,f^2x^2+xd+c}\left(-\frac{c}{d}+\frac{a}{b}\right)}$	236

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*f-d)*EllipticPi((f*(d*x+c)/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c
*f-d)/(c*f+d))^(1/2))*(-d*(f*x+1)/(c*f-d))^(1/2)*(-d*(f*x-1)/(c*f+d))^(1/2
)*(f*(d*x+c)/(c*f-d))^(1/2)*(-f^2*x^2+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d-b*c)/(
d*f^2*x^3+c*f^2*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{-(fx-1)\,(fx+1)}\,(a+bx)\,\sqrt{c+dx}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(f*x - 1)*(f*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{-f^2x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{-f^2x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{\sqrt{1-f^2\,x^2}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{(bx+a)\,\sqrt{dx+c}\,\sqrt{-f^2x^2+1}}dx$$

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x)
```

output
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x)
```

3.80 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

output
```
-2*(f*(d*x+c)/(c*f+d))^(1/2)*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*
f+b),2^(1/2)*(d/(c*f+d))^(1/2))/(a*f+b)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.74

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(-1+fx)}{f(c+dx)}}\sqrt{\frac{d+dfx}{cf+dfx}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right),\frac{-d+cf}{d+cf}\right)-\text{EllipticPi}\left(\frac{bcf-adf}{bd+bcf},i\text{arcsinh}\left(\right.\right.\right.}{(-bc+ad)\sqrt{-\frac{d+cf}{f}}\sqrt{1-f^2x^2}}$$

input
```
Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

output
```
((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f
+ d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d +
c*f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt
[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*S
qrt[-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-fx}\sqrt{fx+1}(a+bx)\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{186}$$

$$-2\int \frac{1}{\sqrt{fx+1}(-((1-fx)b)+b+af)\sqrt{c-\frac{d(1-fx)}{f}+\frac{d}{f}}}d\sqrt{1-fx}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\sqrt{1-\frac{d(1-fx)}{cf+d}}\int \frac{1}{\sqrt{fx+1}(-((1-fx)b)+b+af)\sqrt{1-\frac{d(1-fx)}{d+cf}}}d\sqrt{1-fx}}{\sqrt{c-\frac{d(1-fx)}{f}+\frac{d}{f}}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\sqrt{1-\frac{d(1-fx)}{cf+d}}\text{EllipticPi}\left(\frac{2b}{b+af},\arcsin\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right),\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c-\frac{d(1-fx)}{f}+\frac{d}{f}}}$$

input
```
Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

output
```
(-2*Sqrt[1 - (d*(1 - f*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[S
qrt[1 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d/f - (d*(1 -
 f*x))/f])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(71) = 142$.

Time = 3.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.49

method	result	size
default	$-\frac{2(cf-d)\operatorname{EllipticPi}\left(\sqrt{\frac{f(xd+c)}{cf-d}},-\frac{(cf-d)b}{f(ad-bc)},\sqrt{\frac{cf-d}{cf+d}}\right)\sqrt{-\frac{d(fx+1)}{cf-d}}\,\sqrt{-\frac{d(fx-1)}{cf+d}}\,\sqrt{\frac{f(xd+c)}{cf-d}}\,\sqrt{fx+1}\,\sqrt{-fx+1}\,\sqrt{xd+c}}{f(ad-bc)(d\,f^2x^3+c\,f^2x^2-xd-c)}$	184
elliptic	$\frac{2\sqrt{-(xd+c)(f^2x^2-1)}\left(\frac{c}{d}-\frac{1}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x-\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\,\sqrt{\frac{x+\frac{1}{f}}{-\frac{c}{d}+\frac{1}{f}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f}}},\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}+\frac{a}{b}},\sqrt{\frac{-\frac{c}{d}+\frac{1}{f}}{-\frac{c}{d}-\frac{1}{f}}}\right)}{\sqrt{xd+c}\,\sqrt{-fx+1}\,\sqrt{fx+1}\,b\sqrt{-d\,f^2x^3-c\,f^2x^2+xd+c}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$	239

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2*(c*f-d)*EllipticPi((f*(d*x+c)/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c
*f-d)/(c*f+d))^(1/2))*(-d*(f*x+1)/(c*f-d))^(1/2)*(-d*(f*x-1)/(c*f+d))^(1/2
)*(f*(d*x+c)/(c*f-d))^(1/2)*(f*x+1)^(1/2)*(-f*x+1)^(1/2)*(d*x+c)^(1/2)/f/(
a*d-b*c)/(d*f^2*x^3+c*f^2*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{-fx+1}\sqrt{fx+1}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f*x+1)**(1/2)/(f*x+1)**(1/2),x)
```

output
```
Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f*x + 1)*sqrt(f*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorith
m="giac")
```

output
```
integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = \int \frac{1}{\sqrt{1-f\,x}\,\sqrt{f\,x+1}\,(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$$
$$= -\left(\int \frac{\sqrt{dx+c}\,\sqrt{fx+1}\,\sqrt{-fx+1}}{bd\,f^2x^4+ad\,f^2x^3+bc\,f^2x^3+ac\,f^2x^2-bd\,x^2-adx-bcx-ac}dx\right)$$

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x)
```

output
```
 - int((sqrt(c + d*x)*sqrt(f*x + 1)*sqrt( - f*x + 1))/(a*c*f**2*x**2 - a*c
 + a*d*f**2*x**3 - a*d*x + b*c*f**2*x**3 - b*c*x + b*d*f**2*x**4 - b*d*x**
2),x)
```

3.81 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\text{EllipticPi}\left(\frac{2b}{b+af^2},\arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right),\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

output
```
-2*(f^2*(d*x+c)/(c*f^2+d))^(1/2)*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2
*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+d))^(1/2))/(a*f^2+b)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.92 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(-1+f^2x)}{f^2(c+dx)}}\sqrt{\frac{d(1+f^2x)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{-d+cf^2}{d+cf^2}\right)-\text{EllipticPi}\left(\frac{(bc-ad)f^2}{b(d+cf^2)},i\,\text{arcsi}\right.\right.}{(-bc+ad)\sqrt{-c-\frac{d}{f^2}}\sqrt{1-f^4x^2}}$$

input
```
Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

output
```
((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x
))/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]],
(-d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)),
I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-
(b*c) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {730, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-f^4x^2}(a+bx)\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 730}$$

$$\int \frac{1}{\sqrt{1-f^2x}\sqrt{f^2x+1}(a+bx)\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 186}$$

$$-2\int \frac{1}{\sqrt{xf^2+1}\left(af^2+b-b\left(1-f^2x\right)\right)\sqrt{c-\frac{d(1-f^2x)}{f^2}+\frac{d}{f^2}}}d\sqrt{1-f^2x}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{2\sqrt{1-\frac{d(1-f^2x)}{cf^2+d}}\int \frac{1}{\sqrt{xf^2+1}(af^2+b-b(1-f^2x))\sqrt{1-\frac{d\left(1-f^2x\right)}{cf^2+d}}}d\sqrt{1-f^2x}}{\sqrt{c-\frac{d(1-f^2x)}{f^2}+\frac{d}{f^2}}}$$

$$\Big\downarrow \text{ 412}$$

$$-\frac{2\sqrt{1-\frac{d(1-f^2x)}{cf^2+d}}\text{EllipticPi}\left(\frac{2b}{af^2+b},\arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right),\frac{2d}{cf^2+d}\right)}{(af^2+b)\sqrt{c-\frac{d(1-f^2x)}{f^2}+\frac{d}{f^2}}}$$

input
```
Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

output
```
(-2*Sqrt[1 - (d*(1 - f^2*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), Ar
cSin[Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d
/f^2 - (d*(1 - f^2*x))/f^2])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 730
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/((e + f*x
)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && NegQ[b/a] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.

Time = 3.41 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.38

method	result	siz
default	$-\frac{2\left(c\,f^{2}-d\right) \operatorname{EllipticPi}\left(\sqrt{\frac{(xd+c)f^{2}}{c\,f^{2}-d}}, -\frac{\left(c\,f^{2}-d\right)b}{f^{2}(ad-bc)}, \sqrt{\frac{c\,f^{2}-d}{c\,f^{2}+d}}\right)\sqrt{-\frac{d\left(f^{2}x+1\right)}{c\,f^{2}-d}}\,\sqrt{-\frac{d\left(f^{2}x-1\right)}{c\,f^{2}+d}}\,\sqrt{\frac{(xd+c)f^{2}}{c\,f^{2}-d}}\,\sqrt{-f^{4}x^{2}+1}\,\sqrt{xd+c}}{f^{2}(ad-bc)\left(d\,f^{4}x^{3}+c\,f^{4}x^{2}-xd-c\right)}$	20
elliptic	$\frac{2\sqrt{-(xd+c)(f^{4}x^{2}-1)}\left(\frac{c}{d}-\frac{1}{f^{2}}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^{2}}}}\,\sqrt{\frac{x-\frac{1}{f^{2}}}{-\frac{c}{d}-\frac{1}{f^{2}}}}\,\sqrt{\frac{x+\frac{1}{f^{2}}}{-\frac{c}{d}+\frac{1}{f^{2}}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^{2}}}}, \frac{-\frac{c}{d}+\frac{1}{f^{2}}}{-\frac{c}{d}+\frac{a}{b}}, \sqrt{\frac{-\frac{c}{d}+\frac{1}{f^{2}}}{-\frac{c}{d}-\frac{1}{f^{2}}}}\right)}{\sqrt{xd+c}\,\sqrt{-f^{4}x^{2}+1}\,b\sqrt{-d\,f^{4}x^{3}-c\,f^{4}x^{2}+xd+c}\left(-\frac{c}{d}+\frac{a}{b}\right)}$	23

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*
d-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-d*(f^2*x+1)/(c*f^2-d))^(1/2)*(-d*(f^
2*x-1)/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(-f^4*x^2+1)^(1/2)*(
d*x+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{-\left(f^2x-1\right)\left(f^2x+1\right)}\,(a+bx)\,\sqrt{c+dx}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**4*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(f**2*x - 1)*(f**2*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{-f^4x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{-f^4x^2+1}(bx+a)\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{\sqrt{1-f^4\,x^2}\,(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{(bx+a)\,\sqrt{dx+c}\,\sqrt{-f^4x^2+1}}dx$$

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x)
```

output
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x)
```

3.82 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 86

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\text{EllipticPi}\left(\frac{2b}{b+af^2},\arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right),\frac{2d}{d+cf^2}\right)}{(b+af^2)\sqrt{c+dx}}$$

output
```
-2*(f^2*(d*x+c)/(c*f^2+d))^(1/2)*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2
*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+d))^(1/2))/(a*f^2+b)/(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \frac{2i(c+dx)\sqrt{\frac{d(-1+f^2x)}{f^2(c+dx)}}\sqrt{\frac{d(1+f^2x)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{-d+cf^2}{d+cf^2}\right)-\text{EllipticPi}\left(\frac{(bc-ad)f^2}{b(d+cf^2)},i\,\text{arcsi}\right.\right.}{(-bc+ad)\sqrt{-c-\frac{d}{f^2}}\sqrt{1-f^4x^2}}$$

input
```
Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

output
```
((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x
))/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]],
(-d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)),
I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-
(b*c) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-f^2x}\sqrt{f^2x+1}(a+bx)\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 186$$

$$-2\int \frac{1}{\sqrt{xf^2+1}\left(af^2+b-b\left(1-f^2x\right)\right)\sqrt{c-\frac{d(1-f^2x)}{f^2}+\frac{d}{f^2}}}d\sqrt{1-f^2x}$$

$$\Big\downarrow \ 413$$

$$-\frac{2\sqrt{1-\frac{d(1-f^2x)}{cf^2+d}}\int \frac{1}{\sqrt{xf^2+1}(af^2+b-b(1-f^2x))\sqrt{1-\frac{d\left(1-f^2x\right)}{cf^2+d}}}d\sqrt{1-f^2x}}{\sqrt{c-\frac{d(1-f^2x)}{f^2}+\frac{d}{f^2}}}$$

$$\Big\downarrow \ 412$$

$$-\frac{2\sqrt{1-\frac{d(1-f^2x)}{cf^2+d}}\text{EllipticPi}\left(\frac{2b}{af^2+b},\arcsin\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right),\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c-\frac{d(1-f^2x)}{f^2}+\frac{d}{f^2}}}$$

input
```
Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

output
```
(-2*Sqrt[1 - (d*(1 - f^2*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), Ar
cSin[Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d
/f^2 - (d*(1 - f^2*x))/f^2])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(83) = 166$.

Time = 3.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.47

method	result
default	$-\frac{2\left(c\,f^{2}-d\right) \operatorname{EllipticPi}\left(\sqrt{\frac{(xd+c)f^{2}}{c\,f^{2}-d}}, -\frac{\left(c\,f^{2}-d\right)b}{f^{2}(ad-bc)}, \sqrt{\frac{c\,f^{2}-d}{c\,f^{2}+d}}\right)\sqrt{-\frac{d\left(f^{2}x+1\right)}{c\,f^{2}-d}}\,\sqrt{-\frac{d\left(f^{2}x-1\right)}{c\,f^{2}+d}}\,\sqrt{\frac{(xd+c)f^{2}}{c\,f^{2}-d}}\,\sqrt{f^{2}x+1}\,\sqrt{-f^{2}x+1}\,\sqrt{xd+}}{f^{2}(ad-bc)(d\,f^{4}x^{3}+c\,f^{4}x^{2}-xd-c)}$
elliptic	$\frac{2\sqrt{-(xd+c)(f^{4}x^{2}-1)}\,\left(\frac{c}{d}-\frac{1}{f^{2}}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^{2}}}}\,\sqrt{\frac{x-\frac{1}{f^{2}}}{-\frac{c}{d}-\frac{1}{f^{2}}}}\,\sqrt{\frac{x+\frac{1}{f^{2}}}{-\frac{c}{d}+\frac{1}{f^{2}}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{1}{f^{2}}}}, \frac{-\frac{c}{d}+\frac{1}{f^{2}}}{-\frac{c}{d}+\frac{a}{b}}, \sqrt{\frac{-\frac{c}{d}+\frac{1}{f^{2}}}{-\frac{c}{d}-\frac{1}{f^{2}}}}\right)}{\sqrt{xd+c}\,\sqrt{-f^{2}x+1}\,\sqrt{f^{2}x+1}\,b\sqrt{-d\,f^{4}x^{3}-c\,f^{4}x^{2}+xd+c}\,\left(-\frac{c}{d}+\frac{a}{b}\right)}$

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*
d-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-d*(f^2*x+1)/(c*f^2-d))^(1/2)*(-d*(f^
2*x-1)/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(f^2*x+1)^(1/2)*(-f^
2*x+1)^(1/2)*(d*x+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{-f^2x+1}\sqrt{f^2x+1}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x+1)**(1/2)/(f**2*x+1)**(1/2),x)
```

output
```
Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f**2*x + 1)*sqrt(f**2*x + 1)), x
)
```

Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{\sqrt{f^2x+1}\sqrt{-f^2x+1}(bx+a)\sqrt{dx+c}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(1/(sqrt(f^2*x + 1)*sqrt(-f^2*x + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{(a+b\,x)\,\sqrt{1-f^2\,x}\,\sqrt{x\,f^2+1}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = \int \frac{1}{(bx+a)\sqrt{dx+c}\,\sqrt{-f^2x+1}\,\sqrt{f^2x+1}}dx$$

input
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x)
```

output
```
int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x)
```

3.83 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 450

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = -\frac{1471781\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{25600\sqrt{1+4x}}-\frac{267029\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{69120}-\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}{1440}+\frac{1}{20}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}-\frac{1471781\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{102400\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}}+\frac{89446391\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{5529600\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}}+\frac{4681665317\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{5529600\sqrt{2-3x}\sqrt{7+5x}}$$

output
```
-1471781/25600*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)-26
7029/69120*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)-427/14
40*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)+1/20*(2-3*x)^(
1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(5/2)-1471781/102400*253^(1/2)*(
-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1
/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)+
89446391/5529600*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*Elliptic
F(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1
/2)/((7+5*x)/(1+4*x))^(1/2)+4681665317/127180800*253^(1/2)*(-(2-3*x)/(1+4*
x))^(1/2)*(1+4*x)*((7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*
x)^(1/2)/(1+4*x)^(1/2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.23 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.26

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}(-241157+139440x+86400x^2)}{69120} + \frac{\frac{880794698355\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{2-3x}} - \frac{293598232785i\sqrt{253}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{1+4x}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{11}{39}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}} - \frac{35131412470\sqrt{429}\sqrt{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-241157 + 13944
0*x + 86400*x^2))/69120 + ((880794698355*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt
[7 + 5*x])/Sqrt[2 - 3*x] - ((293598232785*I)*Sqrt[253]*Sqrt[(-5 + 2*x)/(-2
 + 3*x)]*Sqrt[1 + 4*x]*EllipticE[I*ArcSinh[(Sqrt[11/39]*Sqrt[7 + 5*x])/Sqr
t[2 - 3*x]], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) - (35131
412470*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticF[ArcSi
n[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqr
t[(1 + 4*x)/(-2 + 3*x)]) - (506348591678*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3
*x)]*Sqrt[1 + 4*x]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/S
qrt[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) + ((57
853855345*I)*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi
[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(S
qrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (276827203510*Sqrt[682]*Sqrt[2 -
 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqr
t[7 + 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4
*x)]))/20427264000
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {179, 25, 2103, 27, 2103, 27, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}\,dx$$

$$\Big\downarrow \text{179}$$

$$\frac{1}{40}\int -\frac{(5x+7)^{3/2}\left(-854x^2+1190x+3\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}-\frac{1}{40}\int \frac{(5x+7)^{3/2}\left(-854x^2+1190x+3\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{40}\left(\frac{1}{144}\int -\frac{2\sqrt{5x+7}(-534058x^2+361720x+128331)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{427}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}\right)+$$
$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{40}\left(-\frac{1}{72}\int \frac{\sqrt{5x+7}(-534058x^2+361720x+128331)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{427}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}\right)+$$
$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\downarrow$ 2103

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{96}\int -\frac{2(-238428522x^2-53274970x+95723929)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{267029}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\right)\right.$$
$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{40}\left(\frac{1}{72}\left(-\frac{1}{48}\int \frac{-238428522x^2-53274970x+95723929}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{267029}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\right)-\frac{42}{3}\right.$$
$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\downarrow$ 2105

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(-\frac{17047639323}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{240}\int -\frac{12(6722787107-4681665317x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right.\right.\right.$$
$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(-\frac{17047639323}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{20}\int \frac{6722787107-4681665317x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\right.\right.\right.$$
$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\downarrow$ 194

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(-\frac{1}{20}\int \frac{6722787107-4681665317x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1549785393\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\ldots}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\big\downarrow$ 27

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(-\frac{1}{20}\int \frac{6722787107-4681665317x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1549785393\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\ldots}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\big\downarrow$ 327

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(-\frac{1}{20}\int \frac{6722787107-4681665317x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{39738087\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4\ldots}}{\sqrt{2x-\ldots}}\right.\right.}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\big\downarrow$ 2101

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{10805030687}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{4681665317}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{\ldots}}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$\big\downarrow$ 183

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{10805030687}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{290263249654(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-}}{3\sqrt{8}}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{10805030687}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{290263249654(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-}}{3}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$$\Big\downarrow \text{188}$$

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{982275517\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{290263249654(2-3x)\sqrt{}}{}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{1964551034\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{290263249654(2-3x)\sqrt{}}{}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{290263249654(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}-\frac{196\ldots}{\ldots}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{40}\left(\frac{1}{72}\left(\frac{1}{48}\left(\frac{1}{20}\left(-\frac{290263249654(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\operatorname{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{15\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}-\frac{196\ldots}{\ldots}\right.\right.\right.\right.$$

$$\frac{1}{20}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}$$

input
```
Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/20 + ((-427*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/36 + ((-267029*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/24 + ((-39738087
*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(10*Sqrt[-5 + 2*x]) + (3973808
7*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt
[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(20*Sqrt[(2 - 3*x)/(5 - 2
*x)]*Sqrt[7 + 5*x]) + ((-1964551034*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*
Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1
+ 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)
/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3
*x))/(2 + (1 + 4*x)/(2 - 3*x))]) - (290263249654*(2 - 3*x)*Sqrt[(5 - 2*x)/
(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11
/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*
Sqrt[1 + 4*x]))/20)/48)/72)/40
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.05

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{581x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{288}-\frac{241157\sqrt{-120x^4+182x^3+385x^2-197x-70}}{69120}-\frac{95723929\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
risch	$-\frac{(86400x^2+139440x-241157)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{69120\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{95723929\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(972452761830\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{89}\cdots}{39}\right.\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(581/288*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(
1/2)-241157/69120*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-95723929/21142
218240*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+
x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2
)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^
(1/2))+5327497/2114221824*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1
/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7
/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+
x))^(1/2),-69/55,1/39*I*897^(1/2)))+4415343/5120*((x+7/5)*(x-5/2)*(x+1/4)-
1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3
+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3
795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795
*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x
+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-
5/2)*(x+1/4))^(1/2)+5/4*x^2*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2))
```

Fricas [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int (5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{\frac{3}{2}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(3/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int (5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int (5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx = \int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{3/2}\,dx$$

input
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2),x)
```

output
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}\,dx &= \frac{5\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^2}{4} \\ &+ \frac{581\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{288} \\ &+ \frac{111205\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{39312} \\ &- \frac{4415343\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{2912} \\ &+ \frac{45982687\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{22464} \\ &+ \frac{5515465\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{78624}\end{aligned}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2),x)
```

output

```
(196560*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2 +
317226*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x + 4448
20*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) - 238428522*
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(120
*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x) + 321878809*int((sqrt(2*x - 5
)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 3
85*x**2 + 197*x + 70),x) + 11030930*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(
5*x + 7)*sqrt( - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x
))/157248
```

3.84 $\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 410

$$\begin{aligned}
\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx
&= -\frac{13027\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{2400\sqrt{1+4x}} + \frac{23}{240}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad - \frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad - \frac{13027\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{9600\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\
&\quad + \frac{100633\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{57600\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\
&\quad + \frac{2120971\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{57600\sqrt{2-3x}\sqrt{7+5x}}
\end{aligned}$$

output
```
-13027/2400*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+23/24
0*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)-1/9*(2-3*x)^(3/
2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)-13027/9600*253^(1/2)*(-(2-3*
x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1
+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)+100633
/57600*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*897
^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*
x)/(1+4*x))^(1/2)+2120971/1324800*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*
x)*((7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x
)^(1/2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.01 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.37

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx$$
$$= \frac{1}{720}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}(-91+240x)$$
$$+ \frac{\frac{7796073285\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{2-3x}} - \frac{2598691095i\sqrt{253}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{1+4x}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{11}{39}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}} - \frac{310954490\sqrt{429}\sqrt{\frac{-5\cdots}{-2\cdots}}}{\cdots}}{\cdots}$$

input
```
Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-91 + 240*x))/7
20 + ((7796073285*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x
] - ((2598691095*I)*Sqrt[253]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*El
lipticE[I*ArcSinh[(Sqrt[11/39]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -39/23])/(Sq
rt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) - (310954490*Sqrt[429]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x
])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) -
(4481783026*Sqrt[429]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]*Sqrt[1 + 4*x]*EllipticPi
[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt
[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]) + ((290533815*I)*Sqrt[682]*Sqrt[2 -
 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[22/23]
*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1
 + 4*x]) - (1958698170*Sqrt[682]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*
EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]], 39/62
])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))/1915056000
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.21, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {179, 2103, 27, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\,dx$$

$$\Big\downarrow \text{179}$$

$$-\frac{1}{18}\int \frac{\sqrt{2-3x}(-138x^2+1042x+617)}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}(2-3x)^{3/2}$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{18}\left(\frac{69}{40}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{1}{160}\int \frac{2(-234486x^2+71770x+85127)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)-\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(\frac{69}{40}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{1}{80}\int \frac{-234486x^2+71770x+85127}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)-\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 2105

$$\frac{1}{18}\left(\frac{1}{80}\left(-\frac{16765749}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{240}\int -\frac{12(6431341-2120971x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{3908}{}\right.\right.$$
$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 27

$$\frac{1}{18}\left(\frac{1}{80}\left(-\frac{16765749}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{20}\int \frac{6431341-2120971x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{39081\surd}{}\right.\right.$$
$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 194

$$\frac{1}{18}\left(\frac{1}{80}\left(-\frac{1}{20}\int \frac{6431341-2120971x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1524159\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\right.\right.$$
$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 27

$$\frac{1}{18}\left(\frac{1}{80}\left(-\frac{1}{20}\int \frac{6431341-2120971x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1524159\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\right.\right.$$
$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 327

$$\frac{1}{18}\left(\frac{1}{80}\left(-\frac{1}{20}\int \frac{6431341-2120971x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{39081\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\right.}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{2101}$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{15052081}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{2120971}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{183}$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{15052081}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{131500202(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-}}}{3\sqrt{897}\sqrt{2x-}}\right.\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{15052081}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{131500202(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-}}}{3\sqrt{2x-5}\sqrt{}}\right.\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{188}$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{1368371\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{131500202(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x}{2-}}}{3}\right.\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{2736742\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{131500202(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x}{2-}}}{3}\right.\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 320$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{131500202(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}-2736742\sqrt{\frac{1}{2}}\right.\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 412$$

$$\frac{1}{18}\left(\frac{1}{80}\left(\frac{1}{20}\left(-\frac{131500202(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{15\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}-2736742\sqrt{\frac{1}{2}}\right.\right.\right.$$

$$\frac{1}{9}(2-3x)^{3/2}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

input
```
Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

output
```
-1/9*((2 - 3*x)^(3/2)*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]) + ((69*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/40 + ((-39081*Sqr
t[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(10*Sqrt[-5 + 2*x]) + (39081*Sqrt[
429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]
*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(20*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sq
rt[7 + 5*x]) + ((-2736742*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5
*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sq
rt[2]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)
]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 +
(1 + 4*x)/(2 - 3*x))]) - (131500202*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sq
rt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 +
 5*x])/Sqrt[2 - 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]
))/20)/80)/18
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{3}-\frac{91\sqrt{-120x^4+182x^3+385x^2-197x-70}}{720}-\frac{85127\sqrt{-\frac{3795\left(x+\right.}{-\frac{2}{3}+x}}}{}\right.}{}$
risch	$-\frac{(-91+240x)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{720\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{85127\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-}{x+}}}{220231440\sqrt{-30\left(\right.}}\right.}{}$
default	$\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(1354687290\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(1/3*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)
-91/720*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-85127/220231440*(-3795*(
x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^
(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2
)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-7177/220
23144*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x
))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)
*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*
897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/3
9*I*897^(1/2)))+13027/160*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/
(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*(
(x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2
),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-
69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx=\int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Sympy [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7), x)
```

Maxima [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Giac [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx = \int \sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx=\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}\,dx$$

input
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2),x)
```

output
```
int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\,dx=\frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{3}+\frac{55\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{117}-\frac{1861\left(\int\frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{13}+\frac{65399\left(\int\frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{234}+\frac{85\left(\int\frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{234}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x)
```

output
```
(78*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x + 110*sqr
t(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) - 33498*int((sqrt(
2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(120*x**4 - 18
2*x**3 - 385*x**2 + 197*x + 70),x) + 65399*int((sqrt(2*x - 5)*sqrt(4*x + 1
)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*x**2 + 197*
x + 70),x) + 85*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x
 + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/234
```

3.85 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 370

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx &= -\frac{427\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{300\sqrt{1+4x}}+\frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\\ &\quad-\frac{427\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)\middle|-\frac{39}{23}\right)}{1200\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}}\\ &\quad+\frac{537\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{800\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}}\\ &\quad-\frac{32543\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{2400\sqrt{2-3x}\sqrt{7+5x}}\end{aligned}$$

output
```
-427/300*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+1/10*(2-
3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)-427/1200*253^(1/2)*(
-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1
/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)+
537/800*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*89
7^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5
*x)/(1+4*x))^(1/2)-32543/55200*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*x)*
((7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(
1/2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.05 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \frac{1}{10}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}$$
$$+ \frac{\frac{85180095\sqrt{1+4x}\sqrt{7+5x}\sqrt{-75+30x}}{\sqrt{2-3x}} - \frac{85180095\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}} + \frac{125222020\sqrt{715}\sqrt{-5+2x}\sqrt{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + ((85180095
*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x])/Sqrt[2 - 3*x] - (85180095*S
qrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[
11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*
Sqrt[1 + 4*x]) + (125222020*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 +
3*x)]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39]
)/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (146904226*Sqrt[715]*Sqrt[-5
 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*
Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1
+ 4*x]) - ((5772195*I)*Sqrt[10230]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)
]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]]
, 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (11544390*Sqrt[10230
]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[
22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3
*x)/(1 + 4*x)]))/(79794000*Sqrt[15])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {179, 25, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{\sqrt{5x+7}}dx$$

$$\Big\downarrow \text{179}$$

$$\frac{1}{20}\int -\frac{-854x^2+1190x+3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{1}{20}\int \frac{-854x^2+1190x+3}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx$$

$$\Big\downarrow \text{2105}$$

$$\frac{1}{20}\left(-\frac{61061}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{240}\int -\frac{4(32543x+51847)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{427\sqrt{2-3x}}{30\sqrt{}}\right.$$
$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow 27$$

$$\frac{1}{20}\left(-\frac{61061}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{60}\int \frac{32543x+51847}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{427\sqrt{2-3x}\sqrt{4x}}{30\sqrt{2x}}\right.$$
$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow 194$$

$$\frac{1}{20}\left(-\frac{1}{60}\int \frac{32543x+51847}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{5551\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{427\sqrt{2-}}{}\right.$$
$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow 27$$

$$\frac{1}{20}\left(-\frac{1}{60}\int \frac{32543x+51847}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{5551\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{427\sqrt{2-}}{}\right.$$
$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow 327$$

$$\frac{1}{20}\left(-\frac{1}{60}\int \frac{32543x+51847}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{427\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\right.$$
$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow 2101$$

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{32543}{3}\int\frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{220627}{3}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)+\frac{427\sqrt{\frac{143}{3}}}{}\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 183$$

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{2017666(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{\sqrt{897}}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{897}\sqrt{2x-5}\sqrt{4x+1}}-\frac{220627}{3}\int\frac{}{\sqrt{2-3x}\sqrt{}}\right.\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{2017666(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}-\frac{220627}{3}\int\frac{}{\sqrt{2-3x}\sqrt{}}\right.\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 188$$

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{2017666(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}-\frac{20057\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5}}{3}\right.\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{2017666(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}-\frac{40114\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5}}{3}\right.\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 320

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{2017666(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}-\frac{40114\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5}}{3}\right.\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 412

$$\frac{1}{20}\left(\frac{1}{60}\left(\frac{2017666(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{15\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}-\frac{40114\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5}}{3}\right.\right.$$

$$\frac{1}{10}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + ((-427*Sqr
t[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(30*Sqrt[-5 + 2*x]) + (427*Sqrt[14
3/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]
*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(20*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sq
rt[7 + 5*x]) + ((-40114*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x
]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt
[2]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*
Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1
 + 4*x)/(2 - 3*x))]) + (2017666*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-
((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x
])/Sqrt[2 - 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]))/6
0)/20
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-120x^4+182x^3+385x^2-197x-70}}{10}-\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\ \mathrm{E}\ldots}{1019590\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\ldots\right.}}\ldots\right.}{\ldots}$
risch	$-\frac{\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{10\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(-\frac{\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\frac{-\frac{2}{3}+x}{x+\frac{1}{4}}}\ldots}{1019590\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x\ldots\right.}}\ldots\right.}{\ldots}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(18158994\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticPi}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\ldots\right.\right.}{\ldots}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(1/10*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)-
1/1019590*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2
/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-
5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*8
97^(1/2))-119/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*(
(x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(
-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x)
)^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),-69/55,1/39*I*897^(1/2)))+427/20*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3
795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*
2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)
/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-
2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3
+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4
))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx=\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{\sqrt{5x+7}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/sqrt(5*x + 7), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{\sqrt{5\,x+7}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \frac{10\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{39} - \frac{488\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{13} + \frac{4669\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{39} - \frac{595\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{39}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x)
```

output
```
(10*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) - 1464*int(
(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(120*x**
4 - 182*x**3 - 385*x**2 + 197*x + 70),x) + 4669*int((sqrt(2*x - 5)*sqrt(4*
x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*x**2 +
 197*x + 70),x) - 595*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt(
 - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/39
```

3.86 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 372

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \\
&-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}}+\frac{12\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{25\sqrt{1+4x}} \\
&+\frac{3\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{25\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\
&-\frac{317\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{50\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\
&+\frac{427\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{50\sqrt{2-3x}\sqrt{7+5x}}
\end{aligned}$$

output
```
-2/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+12/25*(2-3*x
)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+3/25*253^(1/2)*(-(2-3*x
)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+
4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)-317/115
0*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*897^(1/2
)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1
+4*x))^(1/2)+427/1150*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*x)*((7+5*x)/
(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),78/2
3,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.59 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}}$$

$$-\frac{2\left(\frac{9\sqrt{1+4x}\sqrt{7+5x}\sqrt{-75+30x}}{2\sqrt{2-3x}}-\frac{9\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{2\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}}+\frac{86\sqrt{\frac{55}{13}}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}\,\text{EllipticF}\left(\arcsin\right.}{\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

output
```
(-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) - (2*((9
*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x])/(2*Sqrt[2 - 3*x]) - (9*Sqrt
[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/
23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(2*Sqrt[(5 - 2*x)/(2 - 3*x)]*S
qrt[1 + 4*x]) + (86*Sqrt[55/13]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*
EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqr
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (5549*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x
)/(-2 + 3*x)]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
 - 3*x]], -23/39])/(Sqrt[715]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (
(39*I)*Sqrt[165/62]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-2
3/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt
[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (23*Sqrt[165/62]*Sqrt[2 - 3*x]*Sqrt
[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x]
)/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)])))/(2
5*Sqrt[15])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {178, 25, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{3/2}}dx$$

$$\Big\downarrow \text{178}$$

$$\frac{1}{5}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{5}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$$\Big\downarrow \text{2105}$$

$$\frac{1}{5}\left(\frac{1287}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{240}\int \frac{48(427x+258)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6\sqrt{2-3x}\sqrt{4x+1}}{5\sqrt{2x-5}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1287}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{5}\int \frac{427x+258}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6\sqrt{2-3x}\sqrt{4x+1}\sqrt{5}}{5\sqrt{2x-5}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\Big\downarrow$ 194

$$\frac{1}{5}\left(\frac{1}{5}\int \frac{427x+258}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{117\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6\sqrt{2-3x}\sqrt{4x}}{5\sqrt{2x}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{5}\int \frac{427x+258}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{117\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6\sqrt{2-3x}\sqrt{4x}}{5\sqrt{2x}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\Big\downarrow$ 327

$$\frac{1}{5}\left(\frac{1}{5}\int \frac{427x+258}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{3\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6\sqrt{2-}}{}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\Big\downarrow$ 2101

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{1628}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{427}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)-\frac{3\sqrt{429}\sqrt{2-3x}}{}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$$\Big\downarrow \; 183$$

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{1628}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{\sqrt{897}}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)}}{3\sqrt{897}\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{1628}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)}}{3\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$$\Big\downarrow \; 188$$

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{148\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\right.}}{3\sqrt{2x-5}\sqrt{4x+}}\right.\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{296\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\right.}}{3\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\downarrow$ 320

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{296\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-}}{}\right.\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

$\downarrow$ 412

$$\frac{1}{5}\left(\frac{1}{5}\left(\frac{296\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}-\frac{26474(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-}}{}\right.\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

output
```
(-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) + ((6*Sq
rt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (3*Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + ((296*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[2
3 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt
[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 +
 (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/
(2 - 3*x))]) - (26474*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)
/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]))/5)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.17

method	result
elliptic	$\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\left(-120x^3+350x^2-105x-50\right)}{25\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}-\frac{14\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{}}{509795\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+\right.}}\right.$
default	$-\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(146520\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-\right.}{}$

input

```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/25*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-12
0*x^3+350*x^2-105*x-50))^(1/2)-14/509795*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-
2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^
(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(
x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+56/305877*(-3795*(x+7/5)/(-2/3+x)
)^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/
(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*Elliptic
F(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-36/5*((x+7/5)
*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/
2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*E
llipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*Elli
pticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*Elliptic
Pi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+
7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2
+ 70*x + 49), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{3/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = \frac{-10\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}+7320\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{}}{600x^5-70x^4-3199x^3}\right.}{}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x)
```

output
```
( - 10*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 7320*i
nt((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(600*
x**5 - 70*x**4 - 3199*x**3 - 1710*x**2 + 1729*x + 490),x)*x + 10248*int((s
qrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(600*x**5
- 70*x**4 - 3199*x**3 - 1710*x**2 + 1729*x + 490),x) - 23345*int((sqrt(2*x
 - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(600*x**5 - 70*x**4
- 3199*x**3 - 1710*x**2 + 1729*x + 490),x)*x - 32683*int((sqrt(2*x - 5)*sq
rt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(600*x**5 - 70*x**4 - 3199*x
**3 - 1710*x**2 + 1729*x + 490),x) + 2975*int((sqrt(2*x - 5)*sqrt(4*x + 1)
*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(600*x**5 - 70*x**4 - 3199*x**3 - 1710*x*
*2 + 1729*x + 490),x)*x + 4165*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x +
 7)*sqrt( - 3*x + 2))/(600*x**5 - 70*x**4 - 3199*x**3 - 1710*x**2 + 1729*x
 + 490),x))/(11*(5*x + 7))
```

3.87 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 331

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx=-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}-\frac{17906\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{67275\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}+\frac{48706\sqrt{\frac{2}{341}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{14625\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}-\frac{138\sqrt{\frac{2}{341}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{125\sqrt{2-3x}\sqrt{-5+2x}}$$

output
```
-2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-17906/20855
25*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*341^(1/2
)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7+
5*x))^(1/2)+48706/4987125*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*
EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+
2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)-138/42625*682^(1/2)*((2-3*x)/(7+5*x))^(
1/2)*((5-2*x)/(7+5*x))^(1/2)*(7+5*x)*EllipticPi(1/11*341^(1/2)*(1+4*x)^(1/
2)/(7+5*x)^(1/2),55/124,1/62*2418^(1/2))/(2-3*x)^(1/2)/(-5+2*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.91 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(34864+44765x)}{417105(7+5x)^{3/2}} + \frac{\frac{3571978410\sqrt{1+4x}\sqrt{7+5x}\sqrt{-75+30x}}{\sqrt{2-3x}} - \frac{3571978410\sqrt{715}\sqrt{-5+2x}\sqrt{\frac{1+4x}{-2+3x}}E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}} + \frac{5251113560\sqrt{715}\sqrt{-5+2x}\,\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(34864 + 44765*x))/(417105*(
7 + 5*x)^(3/2)) + ((3571978410*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x
])/Sqrt[2 - 3*x] - (3571978410*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2
 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/
39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + (5251113560*Sqrt[715]*Sqr
t[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[
7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x
]) - (6160344428*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Ellip
ticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/
(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - ((344407635*I)*Sqrt[10230]*Sqr
t[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[2
2/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[1 + 4*x]) - (371344545*Sqrt[10230]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 +
4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]],
 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))/(138676984875*Sqrt[1
5])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.50, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {178, 25, 2107, 27, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{5/2}}\,dx$$

$$\Big\downarrow \text{178}$$

$$\frac{1}{15}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{15}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{15}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}-\frac{\int -\frac{2(214872x^2-363155x+20321)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{1}{15}\left(\frac{2\int \frac{214872x^2-363155x+20321}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\big\downarrow$ 2105

$$\frac{1}{15}\left(\frac{2\left(-\frac{3840837}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int \frac{232128(207x+203)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{17906\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}\right)}{27807}+\frac{17906\ldots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{1}{15}\left(\frac{2\left(-\frac{3840837}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{4836}{5}\int \frac{207x+203}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{17906\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}\right)}{27807}+\frac{1790\ldots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\big\downarrow$ 194

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\int \frac{207x+203}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{349167\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{17906\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+\ldots}}{5\sqrt{2x-5}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\int \frac{207x+203}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{349167\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{17906\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\downarrow$ 327

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\int \frac{207x+203}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{8953\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{17906\sqrt{2-3x}\sqrt{4x+1}\sqrt{}}{5\sqrt{2x-5}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\downarrow$ 2101

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(341\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-69\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)+\frac{8953\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{\sqrt{}}\right.\right.}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\downarrow$ 183

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(341\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{62\sqrt{\frac{69}{13}}(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{\sqrt{897}}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}}{\sqrt{2x-5}\sqrt{4x+1}}\right.\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(341\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{4278(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{}}{\sqrt{}}}{\sqrt{2x-5}\sqrt{4x+1}}\right.\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\downarrow$ 188

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(\frac{31\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{4278(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)}}{\sqrt{2x-5}\sqrt{4x+1}}\right.\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(\frac{62\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{4278(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\cdots}}{\sqrt{2x-5}\sqrt{4x+1}}\cdots}{27807\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\Big\downarrow$ 320

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(\frac{62\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}-\frac{4278(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{\cdots}{\sqrt{23-\frac{11(5x\cdots}{2-3\cdots}}}}{\sqrt{2x-5}\sqrt{\cdots}}\cdots}{27\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

$\Big\downarrow$ 412

$$\frac{1}{15}\left(\frac{2\left(-\frac{4836}{5}\left(\frac{62\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}-\frac{1426\sqrt{\frac{3}{143}}(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{Elliptic}\cdots}{5\sqrt{2x-5}\sqrt{4\cdots}}\cdots}{278\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

output
```
(-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^(3/2)) + ((1
7906*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*Sqrt[7 + 5*x]) + (
2*((-17906*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) +
 (8953*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[
(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5
 - 2*x)]*Sqrt[7 + 5*x]) - (4836*((62*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]
*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1
 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*
x))/(2 + (1 + 4*x)/(2 - 3*x))]) - (1426*Sqrt[3/143]*(2 - 3*x)*Sqrt[(5 - 2*
x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt
[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(5*Sqrt[-5 + 2*x]*Sqrt[1 +
 4*x])))/5))/27807)/15
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.40

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{375\left(x+\frac{7}{5}\right)^2}+\frac{-\frac{143248}{139035}x^3+\frac{250684}{83421}x^2-\frac{125342}{139035}x-\frac{35812}{83421}}{\sqrt{\left(x+\frac{7}{5}\right)\left(-120x^3+350x^2-105x-50\right)}}+\frac{81284\sqrt{-\frac{379}{}}}{}\right.}{}$
default	Expression too large to display

input

```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/375*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
)/(x+7/5)^2+17906/2085525*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+3
50*x^2-105*x-50))^(1/2)+81284/127582826085*(-3795*(x+7/5)/(-2/3+x))^(1/2)*
(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x)
)^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795
*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-22348/1962812709*(-3795*(x+7/5)
/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*
((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3
*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*Ell
ipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+7162
4/139035*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(
-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))
^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(
1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2
))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^
(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3
 + 525*x^2 + 735*x + 343), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{5/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = \frac{-10\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}+36600\Big(\int \frac{\sqrt{2x-5}}{3000x^6+3850x^5-16}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x)
```

output

```
( - 10*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 36600*
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(300
0*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430)
,x)*x**2 + 102480*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3
*x + 2)*x**3)/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2
 + 14553*x + 3430),x)*x + 71736*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x
+ 7)*sqrt( - 3*x + 2)*x**3)/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x*
*3 - 3325*x**2 + 14553*x + 3430),x) - 116725*int((sqrt(2*x - 5)*sqrt(4*x +
 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(3000*x**6 + 3850*x**5 - 16485*x**4
- 30943*x**3 - 3325*x**2 + 14553*x + 3430),x)*x**2 - 326830*int((sqrt(2*x
- 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(3000*x**6 + 3850*x**
5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430),x)*x - 228781*in
t((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(3000*x**
6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430),x) +
 14875*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(3
000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 343
0),x)*x**2 + 41650*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( -
3*x + 2))/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 1
4553*x + 3430),x)*x + 29155*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)
*sqrt( - 3*x + 2))/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3...
```

3.88 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 269

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2085525(7+5x)^{3/2}} - \frac{1426348\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{374143185\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} + \frac{4114\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{3253419\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
-2/25*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+17906/20855
25*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-1426348/115984
38735*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*341^(
1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/
(7+5*x))^(1/2)+4114/100855989*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1
/2)*EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/
(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.04 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \frac{2\left(\frac{15\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(59328580+498566971x+89146750x^2)}{(7+5x)^{5/2}}+242\sqrt{15}\left(\frac{8841\sqrt{}}{}\right.\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

output
```
(2*((15*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(59328580 + 498566971*x
 + 89146750*x^2))/(7 + 5*x)^(5/2) + 242*Sqrt[15]*((8841*Sqrt[1 + 4*x]*Sqrt
[7 + 5*x]*Sqrt[-75 + 30*x])/Sqrt[2 - 3*x] - (8841*Sqrt[715]*Sqrt[-5 + 2*x]
*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/S
qrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + (50688
4*Sqrt[55/13]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticF[ArcSin[(
Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(3*Sqrt[(5 - 2*x)/(2 -
 3*x)]*Sqrt[1 + 4*x]) - (32705806*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)
]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -2
3/39])/(3*Sqrt[715]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + ((3203187*I
)*Sqrt[3/3410]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55,
 I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 -
 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (512187*Sqrt[30/341]*Sqrt[2 - 3*x]*Sqrt[
(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])
/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))))/17
3976581025
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.60, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {178, 25, 2107, 27, 2102, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{7/2}}dx$$

$$\Big\downarrow \; 178$$

$$\frac{1}{25}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{25}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 2107$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{\int \frac{1210(210-271x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\int \frac{210-271x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 2102$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{\int \frac{-707280x^2+536354x+630025}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{29470\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 2105$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{2528526\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{322367760}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+11788\sqrt{}}{27807}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{2528526\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+1343199\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+ \text{[illegible]}}{27807}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 188$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{2528526\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{122109\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+}{2-3x}}}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}}{27807}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{2528526\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{244218\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1}{2-3x}}}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}}{27807}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \text{194}$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{\frac{244218\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{229866\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x-\cdots}{5-\cdots}}}{\sqrt{\frac{2-\cdots}{5-\cdots}}}}{27807}\right.}{8342\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(\frac{\frac{244218\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{229866\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+\cdots}{5-2\cdots}}}{\sqrt{\frac{2-\cdots}{5-\cdots}}}}{27807}\right.}{8342\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(-\frac{229866\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{244218\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4\ldots}{2-\ldots}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\ldots}\ldots\right)}{27807\ldots}\ldots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

$\downarrow$ 327

$$\frac{1}{25}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}-\frac{1210\left(-\frac{5894\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{244218\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\ldots}}{\sqrt{2x-5}\sqrt{\frac{5\ldots}{2\ldots}}\ldots}\ldots\right)}{27807\ldots}\ldots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{25(5x+7)^{5/2}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

output
```
(-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(25*(7 + 5*x)^(5/2)) + ((1
7906*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) -
 (1210*((-29470*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*Sqrt[7
+ 5*x]) + ((11788*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x
] - (5894*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcS
in[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(Sqrt[(2 - 3*x)/(
5 - 2*x)]*Sqrt[7 + 5*x]) + (244218*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*S
qrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2
 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x)
)/(2 + (1 + 4*x)/(2 - 3*x))]))/27807))/83421)/25
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{3125\left(x+\frac{7}{5}\right)^3}+\frac{17906\sqrt{-120x^4+182x^3+385x^2-197x-70}}{52138125\left(x+\frac{7}{5}\right)^2}+\frac{-\frac{11410784}{773229249}x}{\sqrt{(}}\right.}{}$
default	$\frac{2\left(170482950\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\ \text{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^4-160464150\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\right.}{}$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/3125*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/
2)/(x+7/5)^3+17906/52138125*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7
/5)^2+1426348/11598438735*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+3
50*x^2-105*x-50))^(1/2)-5544220/64503557180829*(-3795*(x+7/5)/(-2/3+x))^(1
/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/
3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-
3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-1815352/24809060454165*(-37
95*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2
139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^
(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))
-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1
/2)))+5705392/773229249*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-
2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x
+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/
2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69
/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4
 + 3500*x^3 + 7350*x^2 + 6860*x + 2401), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{7/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = \frac{28560\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x-18006\sqrt{2x-5}\,\sqrt{4x+}}{}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x)
```

output
```
(28560*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x - 1800
6*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) - 1360917250*
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(150
00*x**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 49490*x**2
 + 119021*x + 24010),x)*x**3 - 5715852450*int((sqrt(2*x - 5)*sqrt(4*x + 1)
*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(15000*x**7 + 40250*x**6 - 55475*x**
5 - 270110*x**4 - 233226*x**3 + 49490*x**2 + 119021*x + 24010),x)*x**2 - 8
002193430*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*
x**2)/(15000*x**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 +
49490*x**2 + 119021*x + 24010),x)*x - 3734356934*int((sqrt(2*x - 5)*sqrt(4
*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(15000*x**7 + 40250*x**6 - 55
475*x**5 - 270110*x**4 - 233226*x**3 + 49490*x**2 + 119021*x + 24010),x) +
 631786375*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)
)/(15000*x**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 4949
0*x**2 + 119021*x + 24010),x)*x**3 + 2653502775*int((sqrt(2*x - 5)*sqrt(4*
x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(15000*x**7 + 40250*x**6 - 55475*x*
*5 - 270110*x**4 - 233226*x**3 + 49490*x**2 + 119021*x + 24010),x)*x**2 +
3714903885*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)
)/(15000*x**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 4949
0*x**2 + 119021*x + 24010),x)*x + 1733621813*int((sqrt(2*x - 5)*sqrt(4*...
```

3.89 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 309

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23758016\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{57992193675(7+5x)^{3/2}} - \frac{32843987836\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{72826596817065\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} + \frac{80037628\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{633274754931\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
-2/35*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+2558/695175
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+23758016/5799219
3675*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-32843987836/
2257624501329015*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE
(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2
)/((2-3*x)/(7+5*x))^(1/2)+80037628/19631517402861*682^(1/2)*(2-3*x)^(1/2)*
((5-2*x)/(7+5*x))^(1/2)*EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/
2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.51 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \frac{2\left(\frac{90675\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(15395515423270+113490310442229x+54668919175710x^2+}{(7+5x)^{7/2}}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

output
```
(2*((90675*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(15395515423270 + 11
3490310442229*x + 54668919175710*x^2 + 10263746198750*x^3))/(7 + 5*x)^(7/2
) + 11*Sqrt[15]*((27073896336630*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30
*x])/Sqrt[2 - 3*x] - (27073896336630*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*
x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]]
, -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + (39800941623080*Sqr
t[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticF[ArcSin[(Sqrt[11
/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sq
rt[1 + 4*x]) - (46692478872404*Sqrt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2
 + 3*x)]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) + ((3535063529751*
I)*Sqrt[10230]*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55,
 I*ArcSinh[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 -
 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x]) - (4405470235335*Sqrt[10230]*Sqrt[2 - 3*x]
*Sqrt[(-5 + 2*x)/(1 + 4*x)]*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 +
 5*x])/Sqrt[1 + 4*x]], 39/62])/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)])
)))/204710101658008435125
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.54, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {178, 25, 2107, 27, 2107, 27, 2102, 27, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{9/2}}dx$$

$$\Big\downarrow \; 178$$

$$\frac{1}{35}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{1}{35}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow \; 2107$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{\int \frac{2(214872x^2-691065x+274421)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx}{139035}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\int \frac{214872x^2-691065x+274421}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx}{139035}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow \; 2107$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{\int \frac{605(5434995-5812072x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{83153056\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)}{139035}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\downarrow 27$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\int \frac{5434995-5812072x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{83153056\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)}{139035}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\downarrow 2102$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{\int \frac{2\left(-8143137480x^2+6175212589x+8444218475\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{678594790\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-\right)}{139035}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\downarrow 27$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\int\frac{-8143137480x^2+6175212589x+8444218475}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{678594790\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-\cdots\right.}{139035}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$\downarrow$ 2105

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\left(29111716491\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int-\frac{3997251704160}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\cdots\right.}{27807}\cdots\right.}{83421}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$\downarrow$ 27

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\left(29111716491\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+16655215434\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\cdots\right.}{27807}\cdots\right.}{83421}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow \; 188$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}} - \frac{2\left(\frac{605\left(\frac{2\left(29111716491\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx + \frac{1514110494\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{}{\sqrt{\frac{4x+1}{2-3x}+}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}\right.}{27807}\right.}{83}\right.}{}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\left(29111716491\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{3028220988\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{\cdots}{\sqrt{\frac{4x+1}{2-3x}+2\cdots}}\cdots}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\cdots}\cdots\right)}{27807\cdots}\cdots\right)}{83\cdots}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\downarrow \; 194$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\left(\frac{3028220988\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{2646519681\sqrt{\frac{11}{23}\cdots}}{\cdots}\right)}{27807\cdots}\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$\downarrow$ 27

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\left(\frac{3028220988\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{2646519681\sqrt{11}\cdots}{\cdots}\right)}{27807\cdots}\cdots\right)}{\cdots}\cdots}{\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$\downarrow$ 320

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}} - \frac{2\left(\frac{605\left(\frac{2\left(-\frac{2646519681\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}\,d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{3028220988\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\cdots}{\sqrt{2x\cdots}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

$$\Big\downarrow\ 327$$

$$\frac{1}{35}\left(\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{139035(5x+7)^{5/2}}-\frac{2\left(\frac{605\left(\frac{2\left(-\frac{67859479\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{3028220988\sqrt{\frac{11}{23}}\sqrt{\frac{5-\cdots}{2-\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{35(5x+7)^{7/2}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

output
```
(-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(35*(7 + 5*x)^(7/2)) + ((1
7906*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(139035*(7 + 5*x)^(5/2))
- (2*((-83153056*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5
*x)^(3/2)) + (605*((-678594790*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
/(27807*Sqrt[7 + 5*x]) + (2*((135718958*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7
 + 5*x])/Sqrt[-5 + 2*x] - (67859479*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)
/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]],
-23/39])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (3028220988*Sqrt[11/2
3]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3
*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sq
rt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[
(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(2 - 3*x))])))/27807))/8342
1))/139035)/35
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.69

method	result
elliptic	$\dfrac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{2\sqrt{-120x^4+182x^3+385x^2-197x-70}}{21875\left(x+\frac{7}{5}\right)^4}+\frac{2558\sqrt{-120x^4+182x^3+385x^2-197x-70}}{86896875\left(x+\frac{7}{5}\right)^3}+\frac{23758016\sqrt{-12}}{1449}\right.}{}$
default	Expression too large to display

input

```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/21875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1
/2)/(x+7/5)^4+2558/86896875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7
/5)^3+23758016/1449804841875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+
7/5)^2+32843987836/2257624501329015*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(
-120*x^3+350*x^2-105*x-50))^(1/2)-21231177880/179365041452731 2003*(-3795*(
x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^
(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2
)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-59716341
52/689865544048966155*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*
((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*
(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x
))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^
(1/2),-69/55,1/39*I*897^(1/2)))+131375951344/150508300088601*((x+7/5)*(x-5
/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((
x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*Ellipt
icF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE
(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/
69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*
(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3125*x^
5 + 21875*x^4 + 61250*x^3 + 85750*x^2 + 60025*x + 16807), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{9/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x)
```

output

```
(163200*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2 +
1037680*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x - 859
118*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) - 347786821
250*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/
(75000*x**8 + 306250*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4 - 1385
132*x**3 + 941535*x**2 + 953197*x + 168070),x)*x**4 - 1947606199000*int((s
qrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**
8 + 306250*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4 - 1385132*x**3 +
 941535*x**2 + 953197*x + 168070),x)*x**3 - 4089973017900*int((sqrt(2*x -
5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8 + 306250
*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4 - 1385132*x**3 + 941535*x*
*2 + 953197*x + 168070),x)*x**2 - 3817308150040*int((sqrt(2*x - 5)*sqrt(4*
x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8 + 306250*x**7 + 43
75*x**6 - 1738875*x**5 - 3056900*x**4 - 1385132*x**3 + 941535*x**2 + 95319
7*x + 168070),x)*x - 1336057852514*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5
*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8 + 306250*x**7 + 4375*x**6 - 173
8875*x**5 - 3056900*x**4 - 1385132*x**3 + 941535*x**2 + 953197*x + 168070)
,x) + 142225139375*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( -
3*x + 2))/(75000*x**8 + 306250*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x
**4 - 1385132*x**3 + 941535*x**2 + 953197*x + 168070),x)*x**4 + 7964607...
```

3.90 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 349

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{45(7+5x)^{9/2}} + \frac{2558\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1251315(7+5x)^{7/2}} + \frac{4713548\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{19330731225(7+5x)^{5/2}} + \frac{185769084988\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{4837766788562175(7+5x)^{3/2}} - \frac{50143181038544\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{8678953618680541955\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} + \frac{107040362132\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{7546916190156993\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
-2/45*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)+2558/125131
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+4713548/1933073
1225*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+185769084988
/4837766788562175*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)
-50143181038544/2690475621790968004 5*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5
*x))^(1/2)*EllipticE(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^
(1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)+107040362132/233954401894866
783*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticF(1/11*341^(1/
2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7
+5*x))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.81 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = \frac{2\left(\frac{15\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(233513150190136610+1606746337435460287x+127458389781}{(7+5x)^{9/2}}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(11/2),x]
```

output
```
(2*((15*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(233513150190136610 + 1
606746337435460287*x + 1274583897815687760*x^2 + 503323845915526450*x^3 +
78348720372725000*x^4))/(7 + 5*x)^(9/2) + 484*Sqrt[15]*((155402420574*Sqrt
[1 + 4*x]*Sqrt[7 + 5*x]*Sqrt[-75 + 30*x])/Sqrt[2 - 3*x] - (155402420574*Sq
rt[715]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticE[ArcSin[(Sqrt[1
1/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(Sqrt[(5 - 2*x)/(2 - 3*x)]*S
qrt[1 + 4*x]) + (8909738779576*Sqrt[55/13]*Sqrt[-5 + 2*x]*Sqrt[(1 + 4*x)/(
-2 + 3*x)]*EllipticF[ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -2
3/39])/(3*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[1 + 4*x]) - (574885354510084*Sqrt
[-5 + 2*x]*Sqrt[(1 + 4*x)/(-2 + 3*x)]*EllipticPi[-69/55, ArcSin[(Sqrt[11/2
3]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(3*Sqrt[715]*Sqrt[(5 - 2*x)/(2
- 3*x)]*Sqrt[1 + 4*x]) + ((38949314658384*I)*Sqrt[6/1705]*Sqrt[2 - 3*x]*Sq
rt[(1 + 4*x)/(-5 + 2*x)]*EllipticPi[-23/55, I*ArcSinh[(Sqrt[22/23]*Sqrt[7
+ 5*x])/Sqrt[-5 + 2*x]], 23/62])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[1 + 4*x])
 - (16732370454501*Sqrt[15/682]*Sqrt[2 - 3*x]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*E
llipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt[1 + 4*x]], 39/62]
)/(Sqrt[-5 + 2*x]*Sqrt[(-2 + 3*x)/(1 + 4*x)]))))/403571343268645200675
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.459$, Rules used = {178, 25, 2107, 27, 2107, 27, 2107, 27, 2102, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{(5x+7)^{11/2}}dx$$

$$\Big\downarrow \text{178}$$

$$\frac{1}{45}\int -\frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{9/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{45}\int \frac{72x^2-140x+21}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{9/2}}dx-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{\int \frac{14(61392x^2-174025x+60256)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}}dx}{194649}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\int \frac{61392x^2-174025x+60256}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}}dx}{27807}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{\int \frac{3(169687728x^2-767248410x+450955129)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx}{139035}-\frac{7070322\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{46345(5x+7)^{5/2}}\right)}{27807}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{\int \frac{169687728x^2-767248410x+450955129}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx}{46345}-\frac{7070322\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{46345(5x+7)^{5/2}}\right)}{27807}\right)-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{\frac{\int\frac{1210(4457904915-4215897469x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{92884542494\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}}{46345}-\frac{7070322\sqrt{2-3x}\sqrt{2}}{46345(5x+}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$\downarrow$ 27

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{\frac{1210\int\frac{4457904915-4215897469x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{92884542494\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}}{46345}-\frac{7070322\sqrt{2-3}}{46345(}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$\downarrow$ 2102

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{\frac{1210\left(\frac{\int\frac{-124321936459920x^2+9427746848156x+14119652949325}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{518008068580\sqrt{2-3x}\sqrt{2x-5}\sqrt{}}{27807\sqrt{5x+7}}\right.}{83421}}{46345}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$\downarrow$ 2105

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(\frac{44445092284164\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int-\frac{6397307570150640}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+207\cdots}{27807}\right.}{83421}\cdots\right.}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$\downarrow$ 27

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(\frac{44445092284164\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+26655448208961\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+\cdots}}\cdots}{27807}\right.}{83421}\cdots\right.}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$\downarrow$ 188

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(\frac{44445092284164\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2423222564451\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{\cdots}{\sqrt{\frac{4x+\cdots}{2-3\cdots}}\cdots}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\cdots}}{27807\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}} - \frac{2\left(\frac{1210\left(\frac{44445092284164\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{4846445128902\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{}{\sqrt{\frac{4x+1}{2-3x}}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}}{27807}\right.}{}}{}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$\downarrow$ 194

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(\frac{\frac{4846445128902\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{4040462934924\sqrt{\cdots}}{\cdots}}{27807\cdots}\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(\frac{\frac{4846445128902\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{4040462934924\sqrt{\cdots}}{\cdots}}{27807}\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(-\frac{4040462934924\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{4846445128902\sqrt{\frac{11}{23}}\sqrt{\frac{5-2\cdots}{2-3\cdots}}\cdots}{\sqrt{2\cdots}}\cdots\right)}{\cdots}}{\cdots}}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{45}\left(\frac{2558\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807(5x+7)^{7/2}}-\frac{2\left(\frac{1210\left(-\frac{103601613716\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{4846445128902\sqrt{\frac{11}{23}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{45(5x+7)^{9/2}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(11/2),x]
```

output
```
(-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(45*(7 + 5*x)^(9/2)) + ((2
558*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)^(7/2)) -
(2*((-7070322*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(46345*(7 + 5*x)
^(5/2)) + ((-92884542494*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(8342
1*(7 + 5*x)^(3/2)) + (1210*((-518008068580*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sq
rt[1 + 4*x])/(27807*Sqrt[7 + 5*x]) + ((207203227432*Sqrt[2 - 3*x]*Sqrt[1 +
 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x] - (103601613716*Sqrt[429]*Sqrt[2 - 3*x
]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/S
qrt[-5 + 2*x]], -23/39])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (4846
445128902*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23 + (3
1*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3
*x])], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 + (1 + 4*
x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(2 - 3*x
))]))/27807))/83421)/46345))/27807)/45
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 178
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(m + 1))), x] - Simp[1/(2*b*(m + 1))   Int[
((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[d*e*g
+ c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f*h)*x + 3*d*f*h*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.58

method	result	size
elliptic	Expression too large to display	551
default	Expression too large to display	1263

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(-2/140625*(-120*x^4+182*x^3+385*x^2-197*x-70)^(
1/2)/(x+7/5)^5+2558/782071875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x
+7/5)^4+4713548/2416341403125*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x
+7/5)^3+185769084988/120944169714054375*(-120*x^4+182*x^3+385*x^2-197*x-70
)^(1/2)/(x+7/5)^2+50143181038544/26904756217909680045*(-120*x^3+350*x^2-10
5*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2)-248505891908120/149628
11123028289460226 3*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x
-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2
/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2)
,1/39*I*897^(1/2))-63818594049056/575492735501088056162 55*(-3795*(x+7/5)/(
-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((
x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*E
llipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*Ellip
ticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+200572
724154176/1793650414527312003*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7
/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/
2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x
))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^
(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^...
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx=\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(11/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(15625*x
^6 + 131250*x^5 + 459375*x^4 + 857500*x^3 + 900375*x^2 + 504210*x + 117649
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(11/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(11/2),x, alg
orithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{11/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(11/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(11/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{11/2}}\,dx = \text{too large to display}$$

input
```
int((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(11/2),x)
```

output
```
(39168000*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3
+ 303878400*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**
2 + 1072228080*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*
x - 1163025858*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)
- 2524585992293750*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( -
3*x + 2)*x**2)/(375000*x**9 + 2056250*x**8 + 2165625*x**7 - 8663750*x**6 -
 27456625*x**5 - 28323960*x**4 - 4988249*x**3 + 11356730*x**2 + 7512729*x
+ 1176490),x)*x**5 - 17672101946056250*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sq
rt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(375000*x**9 + 2056250*x**8 + 2165625*x
**7 - 8663750*x**6 - 27456625*x**5 - 28323960*x**4 - 4988249*x**3 + 113567
30*x**2 + 7512729*x + 1176490),x)*x**4 - 49481885448957500*int((sqrt(2*x -
 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(375000*x**9 + 2056
250*x**8 + 2165625*x**7 - 8663750*x**6 - 27456625*x**5 - 28323960*x**4 - 4
988249*x**3 + 11356730*x**2 + 7512729*x + 1176490),x)*x**3 - 6927463962854
0500*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)
/(375000*x**9 + 2056250*x**8 + 2165625*x**7 - 8663750*x**6 - 27456625*x**5
 - 28323960*x**4 - 4988249*x**3 + 11356730*x**2 + 7512729*x + 1176490),x)*
x**2 - 48492247739978350*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sq
rt( - 3*x + 2)*x**2)/(375000*x**9 + 2056250*x**8 + 2165625*x**7 - 8663750*
x**6 - 27456625*x**5 - 28323960*x**4 - 4988249*x**3 + 11356730*x**2 + 7...
```

3.91 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 410

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \frac{66377\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{960\sqrt{1+4x}} + \frac{977}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{66377\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{3840\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} - \frac{459563\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{23040\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} - \frac{31069121\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{23040\sqrt{2-3x}\sqrt{7+5x}}$$

output
```
66377/960*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+977/288
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)+1/6*(2-3*x)^(1/2
)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)+66377/3840*253^(1/2)*(-(2-3*x
)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+
4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)-459563/
23040*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*897^
(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x
)/(1+4*x))^(1/2)-31069121/529920*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*x
)*((7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)
^(1/2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 36.90 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = -\frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-37038366\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

output
```
-1/2142720*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-37038366*Sqrt[682]*Sqrt[(-5 - 1
8*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/3
9]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 31389484*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39
]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(
-17232355 - 79187903*x - 38640362*x^2 + 10641080*x^3 + 4555200*x^4 + 11520
00*x^5) + 31069121*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(
-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7
 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {180, 25, 2103, 27, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}(5x+7)^{3/2}}{\sqrt{2x-5}}\,dx$$

$$\Big\downarrow \text{180}$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}-\frac{1}{12}\int -\frac{\sqrt{5x+7}\left(-1954x^2-20x+465\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{12}\int \frac{\sqrt{5x+7}\left(-1954x^2-20x+465\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{12}\left(\frac{977}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{1}{96}\int -\frac{2\left(-1194786x^2-647410x+348709\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)+\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\frac{1}{48}\int \frac{-1194786x^2-647410x+348709}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{977}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\right)+\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{2105}$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{85427199}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{12(31069031-31069121x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{19913}{\right.\right.$$
$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{85427199}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{20}\int \frac{31069031-31069121x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{199131\sqrt{2}}{}\right.\right.$$
$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 194

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\int \frac{31069031-31069121x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{7766109\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{19}{}\right.\right.$$
$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\int \frac{31069031-31069121x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{7766109\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{19}{}\right.\right.$$
$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 327

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\int \frac{31069031-31069121x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{199131\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{2}{3}\right.}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.\right.$$
$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 2101

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{31068851}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{31069121}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)+\right.\right.$$
$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 183

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{31068851}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1926285502(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-}}}{3\sqrt{897}\sqrt{2x-5}}\right.\right.\right.$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow 27$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{31068851}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1926285502(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-}}}{3\sqrt{2x-5}\sqrt{4}}\right.\right.\right.$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow 188$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{2824441\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{1926285502(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+}{2-}}}{3\sqrt{}}\right.\right.\right.$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow 27$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{5648882\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{1926285502(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+}{2-3}}}{3\sqrt{}}\right.\right.\right.$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow 320$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{1926285502(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}+\frac{5648882\sqrt{\frac{11}{23}}\cdots}{}\right.\right.\right.$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \; 412$$

$$\frac{1}{12}\left(\frac{1}{48}\left(\frac{1}{20}\left(\frac{1926285502(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{15\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}+\frac{5648882\sqrt{\frac{1}{2}}\cdots}{}\right.\right.\right.$$

$$\frac{1}{6}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

input

```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

output

```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/6 + ((977*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/24 + ((199131*Sqrt[
2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(10*Sqrt[-5 + 2*x]) - (199131*Sqrt[4
29]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*
Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(20*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqr
t[7 + 5*x]) + ((5648882*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x
]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt
[2]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*
Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1
 + 4*x)/(2 - 3*x))]) + (1926285502*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqr
t[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 +
5*x])/Sqrt[2 - 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
)/20)/48)/12
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 180
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[2*(a + b*x)^m*Sqrt[c + d*x]*Sq
rt[e + f*x]*(Sqrt[g + h*x]/(d*(2*m + 3))), x] - Simp[1/(d*(2*m + 3))   Int[
((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*b*c*
e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e*h)*(
2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m + b*(d*
(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{5x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{6}+\frac{1313\sqrt{-120x^4+182x^3+385x^2-197x-70}}{288}+\frac{348709\sqrt{-\frac{3795}{-\cdots}}\cdots}{\cdots}\right.}{\cdots\left(\frac{348709\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\cdots}{x+\cdots}}\cdots}{88092576\sqrt{-30\left(x\cdots\right.}}\right.}$
risch	$-\frac{(1313+240x)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{288\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\cdots}{\cdots}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-5+2x}\left(2796196590\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{89\cdots}}{39\cdots}\right.\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(5/6*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)
+1313/288*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+348709/88092576*(-3795
*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*213
9^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1
/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-323705
/44046288*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2
/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-
5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/3
9*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55
,1/39*I*897^(1/2)))-66377/64*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/
5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2
)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x)
)^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2
),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}(5x+7)^{\frac{3}{2}}}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(3/2)/(-5+2*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*(5*x + 7)**(3/2)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \int \frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,{(5\,x+7)}^{3/2}}{\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx=\frac{5\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{6}-\frac{2489\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{819}+\frac{331885\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{182}-\frac{843005\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{468}+\frac{234259\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{1638}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2),x)
```

output
```
(2730*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x - 9956*
sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 5973930*int((
sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(120*x**4
 - 182*x**3 - 385*x**2 + 197*x + 70),x) - 5901035*int((sqrt(2*x - 5)*sqrt(
4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*x**2
 + 197*x + 70),x) + 468518*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*
sqrt( - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/3276
```

3.92 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 370

$$
\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx \\
&= \frac{509\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{120\sqrt{1+4x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} \\
&\quad + \frac{509\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{480\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\
&\quad - \frac{399\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{320\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\
&\quad - \frac{70919\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{960\sqrt{2-3x}\sqrt{7+5x}}
\end{aligned}
$$

output
```
509/120*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+1/4*(2-3*
x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)+509/480*253^(1/2)*(-(2
-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1/2)
/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)-399
/320*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*897^(
1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)
/(1+4*x))^(1/2)-70919/22080*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*x)*((7
+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2
),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 27.78 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$$
$$= \frac{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(66960(2-3x) - \frac{3\left(94674\sqrt{682}(2-3x)(7+5x)\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

output
```
(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) - (3*(94674*S
qrt[682]*(2 - 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
cE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 76756*Sqrt[68
2]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-
2 + 3*x)]*(284022*(-35 - 151*x - 34*x^2 + 40*x^3) + 70919*Sqrt[682]*(2 - 3
*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*E
llipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])
))/((2 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(267840*S
qrt[2 - 3*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {179, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}dx$$

$$\Big\downarrow \text{179}$$

$$\frac{1}{8}\int \frac{-1018x^2-410x+309}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{2105}$$

$$\frac{1}{8}\left(\frac{72787}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{4(80129-70919x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{509\sqrt{2-3x}\sqrt{4x}}{30\sqrt{2x}}\right.$$
$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{72787}{20}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{60}\int \frac{80129-70919x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{509\sqrt{2-3x}\sqrt{4x+}}{30\sqrt{2x-}}\right.$$
$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{194}$$

$$\frac{1}{8}\left(\frac{1}{60}\int \frac{80129-70919x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{6617\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{509\sqrt{2-3}}{30}\right.$$
$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{60}\int \frac{80129-70919x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{6617\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{509\sqrt{2-3x}}{30}\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 327

$$\frac{1}{8}\left(\frac{1}{60}\int \frac{80129-70919x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{509\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{20\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 2101

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{98549}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{70919}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)-\frac{509\sqrt{\frac{143}{3}}\sqrt{}}{}\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 183

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{98549}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{4396978(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{\sqrt{89}}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}\right.}}{3\sqrt{897}\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{98549}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{4396978(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7}{2-3x}\right.}}{3\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{188}$$

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{8959\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{4396978(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int}{3\sqrt{2x-5}\sqrt{4x}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{17918\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{4396978(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int}{3\sqrt{2x-5}\sqrt{4}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{4396978(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}+\frac{17918\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x}}{3\sqrt{}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{8}\left(\frac{1}{60}\left(\frac{4396978(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\operatorname{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{15\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}+\frac{17918\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x}}{3\sqrt{}}\right.\right.$$

$$\frac{1}{4}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/4 + ((509*Sqrt[
2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(30*Sqrt[-5 + 2*x]) - (509*Sqrt[143/
3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*S
qrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(20*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt
[7 + 5*x]) + ((17918*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*S
qrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]
*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqr
t[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 +
4*x)/(2 - 3*x))]) + (4396978*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1
 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/
Sqrt[2 - 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]))/60)/
8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 179
```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_] :> Simp[2*(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*(2*m + 5))), x] + Simp[1/(b*(2*m + 5))
Int[((a + b*x)^m/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*b*c*e*
g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*e*g + c*f*g + c*e*h) - a*(d*f*g + d
*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{\sqrt{-120x^4+182x^3+385x^2-197x-70}}{4}+\frac{103\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+\cdots}}}{407836\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\cdots\right.}}\right.}{\cdots}$
risch	$-\frac{\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{4\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{103\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\frac{-\frac{2}{3}+x}{x+\frac{1}{4}}}\cdots}{407836\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\cdots\right.}}\right.}{\cdots}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-5+2x}\left(8869410\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\cdots\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(1/4*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+1
03/407836*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2
/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-
5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*8
97^(1/2))-205/611754*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*(
(x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(
-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x)
)^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),-69/55,1/39*I*897^(1/2)))-509/8*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-37
95*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2
139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/
(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+
x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4)
)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*sqrt(5*x + 7)/sqrt(2*x - 5), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx=\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{5\,x+7}}{\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx=-\frac{59\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{273}+\frac{10180\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{91}-\frac{5000\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{39}+\frac{3979\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{546}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x)
```

output
```
( - 118*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 61080
*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(12
0*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x) - 70000*int((sqrt(2*x - 5)*s
qrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*
x**2 + 197*x + 70),x) + 3979*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7
)*sqrt( - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/546
```

3.93 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 329

$$\begin{aligned}
&\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx \\
&= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{10\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad + \frac{7\sqrt{\frac{11}{39}}\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right),-\frac{23}{39}\right)}{10\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}} \\
&\quad + \frac{41\sqrt{\frac{11}{39}}\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{\frac{7+5x}{5-2x}}\,\text{EllipticPi}\left(\frac{23}{78},\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right),-\frac{23}{39}\right)}{20\sqrt{2-3x}\sqrt{7+5x}}
\end{aligned}$$

output
```
1/5*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-1/10*429^(1/2
)*((2-3*x)/(5-2*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/23*897^(1/2)*(1+4*x)^(
1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2
)+7/390*429^(1/2)*((2-3*x)/(5-2*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/23*897
^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)+41/780*429^(1/2)*((2-3*x)/(5-2*x))^(1/2)*(5-2*x)*((7+5*x
)/(5-2*x))^(1/2)*EllipticPi(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),23
/78,1/39*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.22 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx$$
$$= \frac{\sqrt{2-3x}\left(-3410\sqrt{682}\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{1+4x}{7+5x}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{155-62x}{77+55x}}\right)|\frac{23}{62}\right)+1984\sqrt{682}\sqrt{\frac{5-2x}{7+5x}}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

output
```
(Sqrt[2 - 3*x]*(-3410*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(
7 + 5*x)]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[(155 - 62*x)/(77 + 5
5*x)]], 23/62] + 1984*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(
7 + 5*x)]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 5
5*x)]], 23/62] + Sqrt[(-2 + 3*x)/(7 + 5*x)]*(17050*(10 + 21*x - 70*x^2 + 2
4*x^3) - 1599*Sqrt[682]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^2*Sqrt[(-10 +
19*x - 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, ArcSin[Sqrt[(155 - 62*x)/(77
 + 55*x)]], 23/62])))/(34100*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*((-2 + 3*x)/(7 +
 5*x))^(3/2)*(7 + 5*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {191, 183, 27, 188, 27, 194, 27, 320, 327, 411, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\sqrt{5x+7}}\,dx$$

$$\Big\downarrow \ 191$$

$$\frac{429}{10}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{77}{20}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{41}{20}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \ 183$$

$$\frac{429}{10}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{77}{20}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{\sqrt{713}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{713}\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \ 27$$

$$\frac{429}{10}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{77}{20}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \ 188$$

$$\frac{429}{10}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{7\sqrt{\frac{11}{46}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \; 27$$

$$\frac{429}{10}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{7\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \; 194$$

$$\frac{7\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \; 27$$

$$\frac{7\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{39\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}$$
$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \; 320$$

$$-\frac{39\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}+\frac{7\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+$$
$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \; 327$$

$$\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{10\sqrt{2-3x}\sqrt{4x+1}}-\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{7\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+$$
$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\Big\downarrow \; 411$$

$$\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{11}{78}\int \frac{1}{\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}+\frac{1}{78}\int \frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{10\sqrt{2-3x}\sqrt{4x+1}}-\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{7\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\downarrow \text{320}$$

$$\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{1}{78}\int \frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)\right.}{78\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(5x+7)}{2x-5}+23}}\right.}{10\sqrt{2-3x}\sqrt{4x+1}}$$

$$\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{7\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

$$\downarrow \text{414}$$

$$-\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+$$

$$\frac{7\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+$$

$$\frac{1599\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{78\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(5x+7)}{2x-5}+23}}+\frac{23\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\right.}{390\sqrt{682}\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}}\right.}{10\sqrt{2-3x}\sqrt{4x+1}}$$

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (Sqrt[429
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(10*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) + (7*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23
 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[
2 - 3*x])], -39/23])/(10*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 +
 (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/
(2 - 3*x))]) + (1599*Sqrt[(2 - 3*x)/(5 - 2*x)]*(5 - 2*x)*Sqrt[-((1 + 4*x)/
(5 - 2*x))]*((Sqrt[11/62]*Sqrt[31 + (11*(7 + 5*x))/(-5 + 2*x)]*EllipticF[A
rcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(78*Sqrt[(31 +
(11*(7 + 5*x))/(-5 + 2*x))/(23 + (22*(7 + 5*x))/(-5 + 2*x))]*Sqrt[23 + (22
*(7 + 5*x))/(-5 + 2*x)]) + (23*Sqrt[31 + (11*(7 + 5*x))/(-5 + 2*x)]*Ellipt
icPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(3
90*Sqrt[682]*Sqrt[(31 + (11*(7 + 5*x))/(-5 + 2*x))/(23 + (22*(7 + 5*x))/(-
5 + 2*x))]*Sqrt[23 + (22*(7 + 5*x))/(-5 + 2*x)])))/(10*Sqrt[2 - 3*x]*Sqrt[
1 + 4*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 191
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[Sqrt[a + b*x]*Sqrt[c + d*x]*(
Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Simp[(d*e - c*f)*((f*g - e*h)/(2*f
*h))   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x]), x],
 x] + Simp[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h)   Int[Sqrt[e + f
*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Simp[(d*e - c*f)*
((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{4\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}-\right.}{}$
default	$-\frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{-5+2x}\sqrt{7+5x}\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^2\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(4/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x
)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)
/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5
)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+10/305877*(-3795*(x+7/5)/(-2/3+x))^(1/
2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3
+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/6
9*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(
-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-6*((x+7/5)*(x-5/2)
*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5
/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF
(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/
69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*
(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2
/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2
- 11*x - 35), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\sqrt{5x+7}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*sqrt(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{10x^2-11x-35}dx$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(1/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(10*x**2
- 11*x - 35),x)
```

3.94 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 331

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{1+4x}}{5\sqrt{-5+2x}\sqrt{7+5x}} + \frac{2\sqrt{\frac{11}{39}}\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}\,E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{5\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}} - \frac{2\sqrt{\frac{33}{13}}\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right),-\frac{23}{39}\right)}{5\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{2\sqrt{\frac{33}{13}}\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{\frac{7+5x}{5-2x}}\,\text{EllipticPi}\left(\frac{23}{78},\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right),-\frac{23}{39}\right)}{5\sqrt{2-3x}\sqrt{7+5x}}$$

output
```
-2/5*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(1/2)+2/195*429^(1
/2)*((2-3*x)/(5-2*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/23*897^(1/2)*(1+4*x)
^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(5-2*x))^(1
/2)-2/65*429^(1/2)*((2-3*x)/(5-2*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/23*89
7^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))/(2-3*x)^(1/2)/((7+5
*x)/(5-2*x))^(1/2)+2/65*429^(1/2)*((2-3*x)/(5-2*x))^(1/2)*(5-2*x)*((7+5*x)
/(5-2*x))^(1/2)*EllipticPi(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),23/
78,1/39*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 19.20 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-62\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x
]
```

output
```
(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x
)/(-2 + 3*x)]], 39/62] + 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2
]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 +
 3*x)]], 39/62] - 2*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-961*(-5 - 18*x + 8*x^2) +
 39*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10
*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(
-2 + 3*x)]], 39/62])))/(6045*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-
2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {182, 25, 2004, 2098, 183, 27, 194, 27, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)^{3/2}}dx$$

$$\Big\downarrow \text{182}$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}-\frac{1}{39}\int -\frac{48x^2-130x+25}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{39}\int \frac{48x^2-130x+25}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \text{2004}$$

$$\frac{1}{39}\int \frac{\sqrt{2x-5}(24x-5)}{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \text{2098}$$

$$\frac{1}{39}\left(-\frac{858}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{117}{5}\int \frac{\sqrt{4x+1}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{5x+7}}dx-\frac{4\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+}}{5\sqrt{2x-5}}\right.$$
$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \text{183}$$

$$\frac{1}{39}\left(-\frac{858}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{138\sqrt{\frac{39}{31}}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\int \frac{\sqrt{1209}}{\sqrt{39-\frac{22(5x+7)}{4x+1}}\sqrt{31-\frac{11(5x+7)}{4x+1}}\left(5\right.}}{5\sqrt{2-3x}\sqrt{2x-5}}\right.$$
$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{39}\left(-\frac{858}{5}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{5382\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\int \frac{1}{\sqrt{39-\frac{22(5x+7)}{4x+1}}\sqrt{31-\frac{11(5x+7)}{4x+1}}\left(5-\frac{4(5}{4}\right.}}{5\sqrt{2-3x}\sqrt{2x-5}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \; 194$$

$$\frac{1}{39}\left(\frac{78\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{5382\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\int \frac{1}{\sqrt{39-\frac{22(5x+7)}{4x+1}}\sqrt{31-\frac{11(5x+7)}{4x+1}}}}{5\sqrt{2-3x}\sqrt{2x-5}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{39}\left(\frac{78\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{5382\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\int \frac{1}{\sqrt{39-\frac{22(5x+7)}{4x+1}}\sqrt{31-\frac{11(5x+7)}{4x+1}}}}{5\sqrt{2-3x}\sqrt{2x-5}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{39}\left(-\frac{5382\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\int \frac{1}{\sqrt{39-\frac{22(5x+7)}{4x+1}}\sqrt{31-\frac{11(5x+7)}{4x+1}}\left(5-\frac{4(5x+7)}{4x+1}\right)}d\frac{\sqrt{5x+7}}{\sqrt{4x+1}}}{5\sqrt{2-3x}\sqrt{2x-5}}+\frac{2\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\Big(}{5\sqrt{\frac{2-3}{5-2}}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

$$\Big\downarrow \; 412$$

$$\frac{1}{39}\left(\frac{2\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{2691\sqrt{\frac{2}{341}}\sqrt{-\frac{2-3x}{4x+1}}\sqrt{-\frac{5-2x}{4x+1}}(4x+1)\,\text{EllipticPi}\left(\frac{7}{5}\right.}{25\sqrt{2-3x}\sqrt{2x-5}}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{39\sqrt{5x+7}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*Sqrt[7 + 5*x]) + ((-4*S
qrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) + (2*Sqrt[429
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
 + 5*x]) - (2691*Sqrt[2/341]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[-((5 - 2*x)
/(1 + 4*x))]*(1 + 4*x)*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x]
)/Sqrt[1 + 4*x]], 39/62])/(25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]))/39
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 182
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Simp[1/(2*(m +
1)*(b*c - a*d))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)*(f*g
 + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2098
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*
B*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*Sqrt[a + b*x])), x] + (
-Simp[B*((b*g - a*h)/(2*f*h))   Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d
*x]*Sqrt[g + h*x]), x], x] + Simp[B*(b*e - a*f)*((b*g - a*h)/(2*d*f*h))   I
nt[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{50\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\text{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{11929203\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$\frac{2\sqrt{2-3x}\sqrt{1+4x}\sqrt{-5+2x}\sqrt{7+5x}\left(495\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^{2}\,\text{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-111\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(50/11929203*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/
3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1
/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+
7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-20/917631*(-3795*(x+7/5)/(-2/3+x))^
(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-
2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(
1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/6
9*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+8/13*((x+7/5)*(
x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)
*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*Ell
ipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*Ellipt
icE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi
(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/
5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)+2/195*(-120*x^3+350*x^2-105*x-50)/((x+7
/5)*(-120*x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3
+ 15*x^2 - 252*x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**(3/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{50x^3+15x^2-252x-245}dx$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(3/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(50*x**3
+ 15*x**2 - 252*x - 245),x)
```

3.95 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 229

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} + \frac{1870\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{104949\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} - \frac{44\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{4563\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
2/117*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+1870/325341
9*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*341^(1/2)
*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7+5
*x))^(1/2)-44/141453*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*Ellip
ticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^
(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 26.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}(-23755-122348x-94580x^2+58928x^3)-935\sqrt{682}(-2+3x)(7+5\right.}{3253419\sqrt{2-}}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x
]
```

output
```
(-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-23755 -
122348*x - 94580*x^2 + 58928*x^3) - 935*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*S
qrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5
 + 2*x)/(-2 + 3*x)]], 39/62] + 506*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[(
-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62]))/(3253419*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7 +
 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.69, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {182, 27, 2102, 27, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)^{5/2}}dx$$
$$\Big\downarrow \text{182}$$
$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}-\frac{1}{117}\int -\frac{11(3-10x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx$$
$$\Big\downarrow \text{27}$$

$$\frac{11}{117}\int \frac{3-10x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \text{2102}$$

$$\frac{11}{117}\left(\frac{\int \frac{2(-10200x^2+7735x+3014)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{850\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{117}\left(\frac{2\int \frac{-10200x^2+7735x+3014}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{850\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \text{2105}$$

$$\frac{11}{117}\left(\frac{2\left(36465\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{3191760}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{170\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-\frac{850\sqrt{2}}{}\right.$$
$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{117}\left(\frac{2\left(36465\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+13299\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{170\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-\frac{850\sqrt{2}}{}\right.$$
$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \text{188}$$

$$\frac{11}{117}\left(\frac{2\left(36465\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1209\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{170\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x\cdots}}{\sqrt{2x-5}}\cdots\right.}{27807}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{11}{117}\left(\frac{2\left(36465\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2418\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{170\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x\cdots}}{\sqrt{2x-5}}\cdots\right.}{27807}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$\Big\downarrow$ 194

$$\frac{11}{117}\left(\frac{2\left(\frac{2418\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{3315\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{170\sqrt{2-3x}\cdots}{\sqrt{2\cdots}}\cdots\right.}{27807}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{11}{117}\left(\frac{2\left(\frac{2418\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{3315\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{170\sqrt{2-3x}\sqrt{}}{\sqrt{2}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{11}{117}\left(\frac{2\left(-\frac{3315\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{2418\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right.}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{11}{117}\left(\frac{2\left(-\frac{85\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{2418\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)\right.}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}\right.}{27807}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{117(5x+7)^{3/2}}$$

input

```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(117*(7 + 5*x)^(3/2)) + (11
*((-850*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*Sqrt[7 + 5*x])
+ (2*((170*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x] - (85
*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[
39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(Sqrt[(2 - 3*x)/(5 - 2*x)]
*Sqrt[7 + 5*x]) + (2418*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x
]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt
[2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sq
rt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 +
 4*x)/(2 - 3*x))])))/27807))/117
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 182
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Simp[1/(2*(m +
1)*(b*c - a*d))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)*(f*g
 + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.03

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{12056\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{89}}{39}\right.}{90467822133\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{2\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^3-42075\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-}{-}}\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(12056/90467822133*(-3795*(x+7/5)/(-2/3+x))^(1/2
)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+
x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-37
95*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+2380/6959063241*(-3795*(x+7/5
)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)
*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/
3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*El
lipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-374
00/1084473*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)
*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x
))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897
^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1
/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*89
7^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)+2/2925*(-120*x^4+1
82*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2-1870/3253419*(-120*x^3+350*x^2-10
5*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4
 + 425*x^3 - 1155*x^2 - 2989*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**(5/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*(5*x + 7)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{(5\,x+7)}^{5/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}+50050\Big(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+}}{3000x^6+3850x^5-16485x^4-3094}}{}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(5/2),x)
```

output
```
(2*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 50050*int(
(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(3000*x*
*6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430),x)*
x**2 + 140140*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x +
 2)*x**2)/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 1
4553*x + 3430),x)*x + 98098*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)
*sqrt( - 3*x + 2)*x**2)/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 -
 3325*x**2 + 14553*x + 3430),x) - 11825*int((sqrt(2*x - 5)*sqrt(4*x + 1)*s
qrt(5*x + 7)*sqrt( - 3*x + 2))/(3000*x**6 + 3850*x**5 - 16485*x**4 - 30943
*x**3 - 3325*x**2 + 14553*x + 3430),x)*x**2 - 33110*int((sqrt(2*x - 5)*sqr
t(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(3000*x**6 + 3850*x**5 - 16485*
x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430),x)*x - 23177*int((sqrt(2*x
 - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(3000*x**6 + 3850*x**5
 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430),x))/(238*(25*x**2
 + 70*x + 49))
```

3.96 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 269

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{16267095(7+5x)^{3/2}} + \frac{4092968\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{2918316843\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} - \frac{83314\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{126883341\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output

```
2/195*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3646/162670
95*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+4092968/904678
22133*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*341^(
1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/
(7+5*x))^(1/2)-83314/3933383571*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^
(1/2)*EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2)
)/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 30.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx =$$
$$-\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(31\sqrt{\frac{7+5x}{-2+3x}}(-374624540-2271416114x-2953846743x^2+643813106x^3+37005\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x
]
```

output
```
(-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-37462454
0 - 2271416114*x - 2953846743*x^2 + 643813106*x^3 + 370051256*x^4) - 20464
84*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*
EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 958111
*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*El
lipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(9046782
2133*Sqrt[2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x +
 8*x^2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.60, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {182, 25, 2107, 27, 2102, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)^{7/2}}dx$$
$$\Big\downarrow \text{182}$$
$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}-\frac{1}{195}\int -\frac{-48x^2-90x+41}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{195}\int \frac{-48x^2-90x+41}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \text{2107}$$

$$\frac{1}{195}\left(\frac{\int \frac{110(4449-10111x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{3646\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{195}\left(\frac{110\int \frac{4449-10111x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{3646\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \text{2102}$$

$$\frac{1}{195}\left(\frac{110\left(\frac{\int \frac{-22325280x^2+16930004x+11228239}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{930220\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-\frac{3646\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \text{2105}$$

$$\frac{1}{195}\left(\frac{110\left(\frac{79812876\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{8097495120}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{372088\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\frac{930220\sqrt{2-3x}\sqrt{2x-}}{27807\sqrt{5x+}}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{195}\left(\frac{110\left(\frac{79812876\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+33739563\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{372088\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\frac{930220\sqrt{2-3x}\sqrt{2}}{27807\sqrt{5}}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$\downarrow$ 188

$$\frac{1}{195}\left(\frac{110\left(\frac{79812876\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{3067233\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{372088\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$\downarrow$ 27

$$\frac{1}{195}\left(\frac{110\left(\frac{79812876\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6134466\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{372088\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}\right.}{83421}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$\downarrow$ 194

$$\frac{1}{195}\left(\frac{110\left(\frac{\frac{6134466\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{7255716\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{372088\sqrt{2-3x}\cdots}{\sqrt{2x\cdots}}\cdots}{27807}\cdots\right)}{83421}\cdots\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{195}\left(\frac{110\left(\frac{\frac{6134466\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{7255716\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{372088\sqrt{2-3x}\cdots}{\sqrt{2x\cdots}}\cdots}{27807}\cdots\right)}{83421}\cdots\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{1}{195}\left(\frac{110\left(-\frac{7255716\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6134466\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}\right)}{27807}}{83421}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{195}\left(\frac{110\left(-\frac{186044\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6134466\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\right.}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}\right)}{27807}}{83421}\right)$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{195(5x+7)^{5/2}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(195*(7 + 5*x)^(5/2)) + ((-
3646*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) +
 (110*((-930220*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*Sqrt[7
+ 5*x]) + ((372088*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*
x] - (186044*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[A
rcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(Sqrt[(2 - 3*x
)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (6134466*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x
)]*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x
)/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 -
3*x))/(2 + (1 + 4*x)/(2 - 3*x))]))/27807))/83421)/195
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 182
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Simp[1/(2*(m +
1)*(b*c - a*d))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)*(f*g
 + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{44912956\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\text{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i}{}\right.}{2515638730052331\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{2\left(389302650\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,\text{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^{4}-460458900\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{}\right.}{}$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(44912956/251563873005233 1*(-3795*(x+7/5)/(-2/3+
x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4
)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+5209232/1935106715424
87*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^
(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x
+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897
^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I
*897^(1/2)))-81859360/30155940711*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*
(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139
^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+
x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))
^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(
1/2)+2/24375*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^3-3646/4066
77375*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2-4092968/90467822
133*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2
))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(7/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1250*x^
5 + 3875*x^4 - 2800*x^3 - 23030*x^2 - 29498*x - 12005), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(7/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{(5\,x+7)}^{7/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{7/2}}\,dx = \frac{240\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x+1190\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5}}{}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(7/2),x)
```

output
```
(240*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x + 1190*s
qrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 156395250*int(
(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(15000*x
**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 49490*x**2 + 1
19021*x + 24010),x)*x**3 + 656860050*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt
(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(15000*x**7 + 40250*x**6 - 55475*x**5 - 2
70110*x**4 - 233226*x**3 + 49490*x**2 + 119021*x + 24010),x)*x**2 + 919604
070*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/
(15000*x**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 49490*
x**2 + 119021*x + 24010),x)*x + 429148566*int((sqrt(2*x - 5)*sqrt(4*x + 1)
*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(15000*x**7 + 40250*x**6 - 55475*x**
5 - 270110*x**4 - 233226*x**3 + 49490*x**2 + 119021*x + 24010),x) - 343433
75*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(15000
*x**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 49490*x**2 +
 119021*x + 24010),x)*x**3 - 144242175*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sq
rt(5*x + 7)*sqrt( - 3*x + 2))/(15000*x**7 + 40250*x**6 - 55475*x**5 - 2701
10*x**4 - 233226*x**3 + 49490*x**2 + 119021*x + 24010),x)*x**2 - 201939045
*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(15000*x
**7 + 40250*x**6 - 55475*x**5 - 270110*x**4 - 233226*x**3 + 49490*x**2 + 1
19021*x + 24010),x)*x - 94238221*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(...
```

3.97 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 309

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{50259901185(7+5x)^{3/2}} + \frac{8188888268\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{63116383908123\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} - \frac{136869832\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{2744190604701\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
2/273*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+98/1807455*
(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3217468/502599011
85*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+8188888268/195
6607901151813*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/
11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/(
(2-3*x)/(7+5*x))^(1/2)-136869832/85069908745731*682^(1/2)*(2-3*x)^(1/2)*((
5-2*x)/(7+5*x))^(1/2)*EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)
,1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 26.79 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \frac{2\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(\frac{(-2+3x)(2552362046246+19165803061167x+12313608173580x^2+2559027583750x^3)}{(7+5x)^4}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x
]
```

output
```
(2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(((-2 + 3*x)*(2552362046246
+ 19165803061167*x + 12313608173580*x^2 + 2559027583750*x^3))/(7 + 5*x)^4
- (22*(558333291*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 18611109
7*Sqrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[Arc
Sin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 71545594*Sqrt[682]*
(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/
39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5
 - 18*x + 8*x^2))))/(1956607901151813*Sqrt[2 - 3*x])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.54, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {182, 25, 2107, 27, 2107, 27, 2102, 27, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{4x+1}}{\sqrt{2x-5}(5x+7)^{9/2}}dx$$

$$\Big\downarrow \ 182$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}-\frac{1}{273}\int -\frac{-96x^2-70x+49}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}}dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{273}\int \frac{-96x^2-70x+49}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{7/2}}dx+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 2107$$

$$\frac{1}{273}\left(\frac{\int \frac{18(-2744x^2-126695x+53228)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx}{139035}+\frac{686\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{46345(5x+7)^{5/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{273}\left(\frac{6\int \frac{-2744x^2-126695x+53228}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx}{46345}+\frac{686\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{46345(5x+7)^{5/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 2107$$

$$\frac{1}{273}\left(\frac{6\left(\frac{\int \frac{55(11577207-18317866x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{11261138\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)}{46345}+\frac{686\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{46345(5x+7)^{5/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{273}\left(\frac{6\left(\frac{55\int \frac{11577207-18317866x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{11261138\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)}{46345}+\frac{686\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{46345(5x+7)^{5/2}}\right)+\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 2102$$

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{\int \frac{2\left(-22333331640x^2+16936109827x+16547393786\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{1861110970\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-\frac{11261138\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)}{46345}+\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\int \frac{-22333331640x^2+16936109827x+16547393786}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{1861110970\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-\frac{11261138\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}\right)}{46345}+\frac{6}{\ldots}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\Big\downarrow \; 2105$$

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(79841660613\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{9376040765520}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{372222194\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-\frac{1861110970\sqrt{2-\ldots}}{27807\ldots}\right.}{83421}\right.}{46345}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$\downarrow$ 27

$$\frac{1}{273}\Bigg(\frac{6\Bigg(\frac{55\Bigg(\frac{2\left(79841660613\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+39066836523\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{372222194\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-18611109}{83421}}{46345}$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$\downarrow$ 188

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(79841660613\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{3551530593\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{372222194\sqrt{2-3x}\sqrt{4}}{\sqrt{2x-5}}\right.}{27807}\right.}{83421}\right.}{46345}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$\downarrow$ 27

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(79841660613\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{7103061186\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{372222194\sqrt{2-3x}\sqrt{4x\cdots}}{\sqrt{2x-5}}\cdots\right.}{27807}\cdots\right.}{83421}\cdots\right.}{46345}\cdots\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$$\downarrow 194$$

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(\frac{7103061186\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{7258332783\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+372\cdots}{27807}}{83421}}{46345}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$\downarrow$ 27

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(\frac{7103061186\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{7258332783\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+3722\cdots}{27807}}{83421}}{46345}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$\downarrow$ 320

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(-\frac{7258332783\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{7103061186\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right.\right.}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}\right.}{27807}\right.}{83421}\right.}{46345}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

$\downarrow$ 327

$$\frac{1}{273}\left(\frac{6\left(\frac{55\left(\frac{2\left(-\frac{186111097\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{7103061186\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{}}{\sqrt{2}}\right.\right.}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}\right.}{27807}\right.}{83421}\right.}{46345}\right.$$

$$\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{273(5x+7)^{7/2}}$$

input

```
Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

output
```
(2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(273*(7 + 5*x)^(7/2)) + ((6
86*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(46345*(7 + 5*x)^(5/2)) + (
6*((-11261138*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)
^(3/2)) + (55*((-1861110970*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2
7807*Sqrt[7 + 5*x]) + (2*((372222194*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 +
5*x])/Sqrt[-5 + 2*x] - (186111097*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(
5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -2
3/39])/(Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (7103061186*Sqrt[11/23]
*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x
)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt
[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(2
3 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(2 - 3*x))])))/27807))/83421)
)/46345)/273
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 182
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_] :> Simp[(a + b*x)^(m + 1)*Sqrt[c + d*x
]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d))), x] - Simp[1/(2*(m +
1)*(b*c - a*d))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)*(f*g
 + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.69

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{18911307184\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69}\right.}{7772485129618352013\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	Expression too large to display

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(9/2),x,method=_RET
URNVERBOSE)
```

output

```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(18911307184/7772485129618352013*(-3795*(x+7/5)/
(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*(
(x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*Ellip
ticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+1488888776/5978
83471509104001*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2
)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x
)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2
),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-
69/55,1/39*I*897^(1/2)))-163777765360/652202633717271*((x+7/5)*(x-5/2)*(x+
1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/
(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/6
9*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(
-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-37
95*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x
)*(x-5/2)*(x+1/4))^(1/2)+2/170625*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
)/(x+7/5)^4+98/225931875*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)
^3-3217468/1256497529625*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)
^2-8188888268/1956607901151813*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*
x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\sqrt{2\,x-5}}\,dx$$

input

```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(9/2),x, algo
rithm="fricas")
```

output

```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(6250*x^
6 + 28125*x^5 + 13125*x^4 - 134750*x^3 - 308700*x^2 - 266511*x - 84035), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2)/(7+5*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(9/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{9/2}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2)/(7+5*x)^(9/2),x)
```

output

```
(19200*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2 + 1
22080*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x + 35469
0*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 24422044625
0*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(7
5000*x**8 + 306250*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4 - 138513
2*x**3 + 941535*x**2 + 953197*x + 168070),x)*x**4 + 1367634499000*int((sqr
t(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8
+ 306250*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4 - 1385132*x**3 + 9
41535*x**2 + 953197*x + 168070),x)*x**3 + 2872032447900*int((sqrt(2*x - 5)
*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8 + 306250*x
**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4 - 1385132*x**3 + 941535*x**2
 + 953197*x + 168070),x)*x**2 + 2680563618040*int((sqrt(2*x - 5)*sqrt(4*x
+ 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8 + 306250*x**7 + 4375
*x**6 - 1738875*x**5 - 3056900*x**4 - 1385132*x**3 + 941535*x**2 + 953197*
x + 168070),x)*x + 938197266314*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x
+ 7)*sqrt( - 3*x + 2)*x**2)/(75000*x**8 + 306250*x**7 + 4375*x**6 - 173887
5*x**5 - 3056900*x**4 - 1385132*x**3 + 941535*x**2 + 953197*x + 168070),x)
 - 50635055625*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x
+ 2))/(75000*x**8 + 306250*x**7 + 4375*x**6 - 1738875*x**5 - 3056900*x**4
- 1385132*x**3 + 941535*x**2 + 953197*x + 168070),x)*x**4 - 28355631150...
```

3.98 $\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 410

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{102487\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{768\sqrt{1+4x}} + \frac{6955\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2} + \frac{102487\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{3072\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} - \frac{662837\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{18432\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} - \frac{47673695\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{18432\sqrt{2-3x}\sqrt{7+5x}}$$

output
```
102487/768*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+6955/1
152*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)+5/24*(2-3*x)^
(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(3/2)+102487/3072*253^(1/2)*(-(
2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1/2
)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)-66
2837/18432*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39
*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((
7+5*x)/(1+4*x))^(1/2)-47673695/423936*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(
1+4*x)*((7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1
+4*x)^(1/2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 29.44 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx =$$
$$-\frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-57187746\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+\cdots\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x
]
```

output
```
-1/1714176*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-57187746*Sqrt[682]*Sqrt[(-5 - 1
8*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/3
9]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 46704724*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39
]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(
-27447805 - 124999073*x - 56065622*x^2 + 20626760*x^3 + 6542400*x^4 + 1152
000*x^5) + 47673695*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[
(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*S
qrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(
7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {192, 25, 2103, 27, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}(5x+7)^{5/2}}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{192}$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}-\frac{1}{48}\int -\frac{\sqrt{5x+7}\left(-13910x^2-3136x+6189\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\int \frac{\sqrt{5x+7}\left(-13910x^2-3136x+6189\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx+\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{2103}$$

$$\frac{1}{48}\left(\frac{6955}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{1}{96}\int -\frac{2\left(-9223830x^2-4923686x+3449639\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)+ \frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{48}\int \frac{-9223830x^2-4923686x+3449639}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6955}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\right)+ \frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{2105}$$

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{131900769}{4}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{60(51001337-47673695x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{3074}{}\right.\right. \frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{131900769}{4}\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{4}\int\frac{51001337-47673695x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{307461\sqrt{}}{}\right.\right.$$
$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 194

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\int\frac{51001337-47673695x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{11990979\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{30}{}\right.\right.$$
$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\int\frac{51001337-47673695x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{11990979\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{30}{}\right.\right.$$
$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 327

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\int\frac{51001337-47673695x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{307461\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right.}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}\right.\right.$$
$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 2101

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{57656621}{3}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{47673695}{3}\int\frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)-\right.\right.$$
$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 183

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{57656621}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2955769090(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-\frac{1}{}}}}{3\sqrt{897}\sqrt{2x-5}\sqrt{}}\right.\right.\right.$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{57656621}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2955769090(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-\frac{1}{}}}}{3\sqrt{2x-5}\sqrt{4x}}\right.\right.\right.$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 188

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{5241511\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{2955769090(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+}{2-3x}}}{3\sqrt{2}}\right.\right.\right.$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{10483022\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{2955769090(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+}{2-3}}}{3\sqrt{}}\right.\right.\right.$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$\downarrow$ 320

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{2955769090(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}+\frac{10483022\sqrt{\frac{11}{23}}\right.\right.\right.$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{48}\left(\frac{1}{48}\left(\frac{1}{4}\left(\frac{591153818(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{3\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}+\frac{10483022\sqrt{\frac{11}{23}}\right.\right.\right.$$

$$\frac{5}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}$$

input
```
Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/24 + ((6955
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/24 + ((307461*S
qrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2*Sqrt[-5 + 2*x]) - (307461*Sqr
t[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/2
3]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(4*Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[7 + 5*x]) + ((10483022*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 +
5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(S
qrt[2]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x
)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 +
 (1 + 4*x)/(2 - 3*x))]) + (591153818*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*S
qrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7
+ 5*x])/Sqrt[2 - 3*x]], -23/39])/(3*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]
))/4)/48)/48
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 192
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b*(a + b*x)^(m - 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Simp[1/(f*h*(2*
m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*(2*m
+ 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) + 2*a*b
*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*d*(f*
g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && In
tegerQ[2*m] && GtQ[m, 1]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{25x\sqrt{-120x^4+182x^3+385x^2-197x-70}}{24}+\frac{8635\sqrt{-120x^4+182x^3+385x^2-197x-70}}{1152}+\frac{3449639\sqrt{-\frac{379}{}}}{}\right.}{}$
risch	$-\frac{5(1727+240x)\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{1152\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{3449639\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x}{x}}}{352370304\sqrt{-30}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(1037819178\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{89}}{39}\right.\right.}{}$

input
```
int((2-3*x)^(1/2)*(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(25/24*x*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/
2)+8635/1152*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+3449639/352370304*(
-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2
)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4
))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-2
461843/176185152*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5
/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3
+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1
/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2)
,-69/55,1/39*I*897^(1/2)))-512435/256*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3
795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*
2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)
/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-
2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3
+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4
))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sq
rt(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \text{Timed out}$$

input
```
integrate((2-3*x)**(1/2)*(7+5*x)**(5/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^{\frac{5}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^{\frac{5}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,{(5\,x+7)}^{5/2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x
)
```

Reduce [F]

$$\begin{aligned}\int \frac{\sqrt{2-3x}(7+5x)^{5/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{25\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{24} \\ &- \frac{3355\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{468} \\ &+ \frac{366025\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{104} \\ &- \frac{6568477\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{1872} \\ &- \frac{49411\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{936}\end{aligned}$$

input
```
int((2-3*x)^(1/2)*(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
(1950*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x - 13420
*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 6588450*int(
(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(120*x**
4 - 182*x**3 - 385*x**2 + 197*x + 70),x) - 6568477*int((sqrt(2*x - 5)*sqrt
(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*x**
2 + 197*x + 70),x) - 98822*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*
sqrt( - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/1872
```

3.99 $\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 370

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{785\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{96\sqrt{1+4x}} + \frac{5}{16}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{785\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{384\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} - \frac{491\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{256\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} - \frac{120323\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{768\sqrt{2-3x}\sqrt{7+5x}}$$

output
```
785/96*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+5/16*(2-3*
x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)+785/384*253^(1/2)*(-(2
-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(1/2)
/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)-491
/256*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*897^(
1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)
/(1+4*x))^(1/2)-120323/17664*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*x)*((
7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/
2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 23.20 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(200880+\frac{(2-3x)\left(-\frac{1314090\sqrt{682}(7+5x)\sqrt{\frac{-5-18x}{(2-3}}}{}\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x
]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(200880 + ((2 -
3*x)*((-1314090*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*
EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*
x)^2 + (998820*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*E
llipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x
)^2 + Sqrt[(7 + 5*x)/(-2 + 3*x)]*((3942270*(-35 - 151*x - 34*x^2 + 40*x^3)
)/(-2 + 3*x)^3 + (1082907*Sqrt[682]*((1 + 4*x)/(-2 + 3*x))^(3/2)*Sqrt[(-35
 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[
(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(1 + 4*x))))/(((7 + 5*x)/(-2 + 3*x))^(3/2
)*(5 + 18*x - 8*x^2))))/642816
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {192, 25, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}(5x+7)^{3/2}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

$$\Big\downarrow \; 192$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{1}{32}\int -\frac{-7850x^2-4074x+4121}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{32}\int \frac{-7850x^2-4074x+4121}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 2105$$

$$\frac{1}{32}\left(\frac{112255}{4}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{20(144437-120323x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{785\sqrt{2-3x}\cdots}{6\sqrt{\cdots}}\right.$$
$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{112255}{4}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{1}{12}\int \frac{144437-120323x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{785\sqrt{2-3x}\sqrt{4x\cdots}}{6\sqrt{2x\cdots}}\right.$$
$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 194$$

$$\frac{1}{32}\left(\frac{1}{12}\int \frac{144437-120323x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{10205\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{785\sqrt{2-\cdots}}{\cdots}\right.$$
$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{12}\int \frac{144437-120323x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{10205\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{785\sqrt{2-}\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{32}\left(\frac{1}{12}\int \frac{144437-120323x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{785\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{7\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 2101$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{192665}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{120323}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)-\frac{785\sqrt{\frac{14}{3}}}{}\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 183$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{192665}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{7460026(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5}{2-}\right.}}{3\sqrt{897}\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{192665}{3}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{7460026(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5\cdots}{2-\cdots}\right.}}{3\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{188}$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{17515\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{7460026(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\frac{1\cdots}{}}}}{3\sqrt{2x-5}\sqrt{\cdots}}\right.\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{35030\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{7460026(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\frac{1\cdots}{}}}}{3\sqrt{2x-5}\sqrt{\cdots}}\right.\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{7460026(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{4x+1}}+\frac{35030\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5\cdots}}{3\cdots}\right.\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{32}\left(\frac{1}{12}\left(\frac{7460026(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{15\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}+\frac{35030\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5}}{3}\right.\right.$$

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

input
```
Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/16 + ((785*Sq
rt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(6*Sqrt[-5 + 2*x]) - (785*Sqrt[14
3/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]
*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(4*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqr
t[7 + 5*x]) + ((35030*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*
Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2
]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sq
rt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 +
 4*x)/(2 - 3*x))]) + (7460026*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((
1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])
/Sqrt[2 - 3*x]], -23/39])/(15*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]))/12)
/32
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 192
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b*(a + b*x)^(m - 1)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*(2*m + 1))), x] - Simp[1/(f*h*(2*
m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*(2*m
+ 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) + 2*a*b
*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*d*(f*
g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && In
tegerQ[2*m] && GtQ[m, 1]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{5\sqrt{-120x^4+182x^3+385x^2-197x-70}}{16}+\frac{317\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+}{-\frac{2}{3}}}}{376464\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)(x}}\right.}{}$
risch	$-\frac{5\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{16\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}-\frac{\left(\frac{317\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\frac{-\frac{2}{3}+}{x+\frac{1}{4}}}}{376464\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)(x}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\,\sqrt{7+5x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(17339850\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\right.}{}$

input
```
int((2-3*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(5/16*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)+
317/376464*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-
2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x
-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*
897^(1/2))-679/815672*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*
((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*
(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x
))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^
(1/2),-69/55,1/39*I*897^(1/2)))-3925/32*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(
-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2
)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/
5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/
(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2
/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1
/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx=\int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2
 - 18*x - 5), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}\,(5x+7)^{\frac{3}{2}}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(7+5*x)**(3/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*(5*x + 7)**(3/2)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx=\int \frac{\sqrt{2-3\,x}\,(5\,x+7)^{3/2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}(7+5x)^{3/2}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx=-\frac{160\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{273}+\frac{19625\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{91}-\frac{8527\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{39}-\frac{10994\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{273}$$

input
```
int((2-3*x)^(1/2)*(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
( - 160*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) + 58875
*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(12
0*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x) - 59689*int((sqrt(2*x - 5)*s
qrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*
x**2 + 197*x + 70),x) - 10994*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x +
7)*sqrt( - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/273
```

3.100 $\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 330

$$\begin{aligned}\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx &= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{2\sqrt{1+4x}} \\ &+ \frac{\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\ &- \frac{\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{16\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} \\ &- \frac{179\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\text{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{16\sqrt{2-3x}\sqrt{7+5x}}\end{aligned}$$

output
```
1/2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)+1/8*253^(1/2)
*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^
(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2
)-1/16*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/39*897
^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*
x)/(1+4*x))^(1/2)-179/368*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+4*x)*((7+5
*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),
78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.70 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx =$$
$$-\frac{6820\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5-18x+8x^2)\,E\left(\arcsin\left(\sqrt{\frac{22}{39}}\sqrt{\frac{7+5x}{1+4x}}\right)|\frac{39}{62}\right)-1265\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}}{}$$

input
```
Integrate[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
-1/27280*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 + 4*
x)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4
*x)]], 39/62] - 1265*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(
1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/
(1 + 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83*x
 - 53*x^2 + 30*x^3) + 4117*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^
2*Sqrt[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22
/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/62]))/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x]*S
qrt[(-5 + 2*x)/(1 + 4*x)]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {191, 183, 27, 188, 27, 194, 27, 320, 327, 411, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{191}$$

$$\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{179}{16}\int \frac{\sqrt{2x-5}}{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \text{183}$$

$$\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{\sqrt{713}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{713}\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \text{188}$$

$$\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{39\sqrt{\frac{11}{46}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31\sqrt{\frac{22(5x+7)}{2x-5}+23}}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow 27$$

$$\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{39\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31\sqrt{\frac{22(5x+7)}{2x-5}+23}}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow 194$$

$$-\frac{39\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2\sqrt{\frac{31(4x+1)}{2-3x}+23}}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31\sqrt{\frac{22(5x+7)}{2x-5}+23}}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow 27$$

$$-\frac{39\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{39\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int\frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \ 320$$

$$-\frac{39\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int\frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \ 327$$

$$\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int\frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}}-\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \ 411$$

$$\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{11}{78}\int\frac{1}{\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}+\frac{1}{78}\int\frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{8\sqrt{2-3x}\sqrt{4x+1}}-$$

$$\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-$$

$$\frac{39\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+$$

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \; 320$$

$$\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{1}{78}\int\frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)\right.}{78\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(5x+7)}{2x-5}+23}}\right.}{8\sqrt{2-3x}\sqrt{4x+1}}$$

$$\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-$$

$$\frac{39\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+$$

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

$$\Big\downarrow \; 414$$

$$-\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{78\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(5x+7)}{2x-5}+23}}+\frac{23\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticPi}\left(\frac{78}{55},\arctan\ldots\right.}{390\sqrt{682}\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(\ldots}{2\ldots}}}\right.}{8\sqrt{2-3x}\sqrt{4x+1}}$$

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}$$

input
```
Int[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
 + 5*x]) - (39*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23
 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[
2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 +
(1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(
2 - 3*x))]) + (6981*Sqrt[(2 - 3*x)/(5 - 2*x)]*(5 - 2*x)*Sqrt[-((1 + 4*x)/(
5 - 2*x))]*((Sqrt[11/62]*Sqrt[31 + (11*(7 + 5*x))/(-5 + 2*x)]*EllipticF[Ar
cTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(78*Sqrt[(31 + (
11*(7 + 5*x))/(-5 + 2*x))/(23 + (22*(7 + 5*x))/(-5 + 2*x))]*Sqrt[23 + (22*
(7 + 5*x))/(-5 + 2*x)]) + (23*Sqrt[31 + (11*(7 + 5*x))/(-5 + 2*x)]*Ellipti
cPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(39
0*Sqrt[682]*Sqrt[(31 + (11*(7 + 5*x))/(-5 + 2*x))/(23 + (22*(7 + 5*x))/(-5
 + 2*x))]*Sqrt[23 + (22*(7 + 5*x))/(-5 + 2*x)])))/(8*Sqrt[2 - 3*x]*Sqrt[1
+ 4*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 191
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[Sqrt[a + b*x]*Sqrt[c + d*x]*(
Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Simp[(d*e - c*f)*((f*g - e*h)/(2*f
*h))   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x]), x],
 x] + Simp[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h)   Int[Sqrt[e + f
*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Simp[(d*e - c*f)*
((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{28\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{\sqrt{2-3x}\sqrt{7+5x}\sqrt{-5+2x}\sqrt{1+4x}\left(30690\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^{2}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)+\right.}{}$

input
```
int((2-3*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(28/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+
x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2
)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/
5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-2/27807*(-3795*(x+7/5)/(-2/3+x))^(1/2
)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+
x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69
*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-
3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-15/2*((x+7/5)*(x-5/
2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x
-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*Ellipti
cF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(
1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/6
9*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(
-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 -
 18*x - 5), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((2-3*x)**(1/2)*(7+5*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)*sqrt(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2-3\,x}\,\sqrt{5\,x+7}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{8x^2-18x-5}dx$$

input
```
int((2-3*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(8*x**2 -
 18*x - 5),x)
```

3.101 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 101

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}}$$

output
```
62/2145*(2-3*x)*((5-2*x)/(2-3*x))^(1/2)*(-(1+4*x)/(2-3*x))^(1/2)*EllipticP
i(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I*897^(1/2))*429^
(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.78 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \frac{\sqrt{\frac{1+4x}{7+5x}}(7+5x)^{3/2}\left(-62\sqrt{\frac{5-2x}{7+5x}}\sqrt{\frac{-2+3x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{155-62x}{77+55x}}\right),\frac{23}{62}\right)+117\sqrt{\frac{-10+19x-6x^2}{(7+5x)^2}}\,\text{EllipticP}}{5\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

output
```
(Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^(3/2)*(-62*Sqrt[(5 - 2*x)/(7 + 5*x)]*
Sqrt[(-2 + 3*x)/(7 + 5*x)]*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)]
], 23/62] + 117*Sqrt[(-10 + 19*x - 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62,
ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)]], 23/62]))/(5*Sqrt[682]*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {183, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx$$

$$\Big\downarrow \text{183}$$

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{\sqrt{897}}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{\sqrt{897}\sqrt{2x-5}\sqrt{4x+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{1}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5x+7)}{2-3x}+5\right)\sqrt{\frac{11(5x+7)}{2-3x}+39}}d\frac{\sqrt{5x+7}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{4x+1}}$$

$$\Big\downarrow \text{412}$$

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{5\sqrt{429}\sqrt{2x-5}\sqrt{4x+1}}$$

input
```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

output
```
(62*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*Ellip
ticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/
(5*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

method	result
default	$\frac{62\sqrt{7+5x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,(2-3x)^{\frac{3}{2}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\frac{i\sqrt{897}}{39}\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{-\frac{253(7+5x)}{-2+3x}}}{49335(40x^3-34x^2-151x-35)}$
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{4\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}} \cdots\right.}{\sqrt{2-3x}\,\sqrt{-\cdots}}$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
62/49335*(7+5*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(3/2)/(40*x^3-
34*x^2-151*x-35)*EllipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39
*I*897^(1/2))*((1+4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*
3^(1/2)*13^(1/2)*(-253*(7+5*x)/(-2+3*x))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3
- 34*x^2 - 151*x - 35), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**(1/2),x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{40x^3-34x^2-151x-35}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(40*x**3
- 34*x**2 - 151*x - 35),x)
```

3.102 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 60

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{2\sqrt{\frac{11}{39}}\sqrt{5-2x}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{22}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{62}{39}\right)}{23\sqrt{-5+2x}}$$

output
```
2/897*429^(1/2)*(5-2*x)^(1/2)*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+5*
x)^(1/2),1/39*2418^(1/2))/(-5+2*x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(60) = 120.

Time = 27.80 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\left(-1922\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)+62\sqrt{682}\sqrt{\right.}}{}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

output
```
(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-1922*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
 + 8*x^2) + 62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x
 + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/6
2] - 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x
^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(2
7807*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8
*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 362 vs. $2(60) = 120$.

Time = 0.33 (sec) , antiderivative size = 362, normalized size of antiderivative = 6.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {194, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}\,dx$$

$$\Big\downarrow \text{194}$$

$$\frac{\sqrt{2}\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\int \frac{\sqrt{2}\sqrt{\frac{31(2x-5)}{5x+7}+11}}{\sqrt{\frac{23(2x-5)}{5x+7}+22}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}}{39\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\int \frac{\sqrt{\frac{31(2x-5)}{5x+7}+11}}{\sqrt{\frac{23(2x-5)}{5x+7}+22}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}}{39\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{324}$$

$$\frac{2\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(11\int \frac{1}{\sqrt{\frac{23(2x-5)}{5x+7}+22}\sqrt{\frac{31(2x-5)}{5x+7}+11}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}+31\int \frac{2x-5}{(5x+7)\sqrt{\frac{23(2x-5)}{5x+7}+22}\sqrt{\frac{31(2x-5)}{5x+7}+11}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}\right)}{39\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{2\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(31\int \frac{2x-5}{(5x+7)\sqrt{\frac{23(2x-5)}{5x+7}+22}\sqrt{\frac{31(2x-5)}{5x+7}+11}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}+11}}\right)}{39\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{2\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(31\left(\frac{\sqrt{2x-5}\sqrt{\frac{23(2x-5)}{5x+7}+22}}{23\sqrt{5x+7}\sqrt{\frac{31(2x-5)}{5x+7}+11}}-\frac{11}{23}\int \frac{\sqrt{\frac{23(2x-5)}{5x+7}+22}}{\left(\frac{31(2x-5)}{5x+7}+11\right)^{3/2}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}\right)+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}+11}}\right)}{39\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{2\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}+11}}+31\left(\frac{\sqrt{2x-5}\sqrt{\frac{23(2x-5)}{5x+7}+22}}{23\sqrt{5x+7}\sqrt{\frac{31(2x-5)}{5x+7}+11}}-\frac{\sqrt{\frac{22}{31}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticE}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{23\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}+11}}\right)\right)}{39\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

input

```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

output

```
(2*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(31*((Sqrt[-5 + 2*x]*Sqrt[22 +
(23*(-5 + 2*x))/(7 + 5*x)])/(23*Sqrt[7 + 5*x]*Sqrt[11 + (31*(-5 + 2*x))/(7
 + 5*x)]) - (Sqrt[22/31]*Sqrt[22 + (23*(-5 + 2*x))/(7 + 5*x)]*EllipticE[Ar
cTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x]], 39/62])/(23*Sqrt[(22 + (
23*(-5 + 2*x))/(7 + 5*x))/(11 + (31*(-5 + 2*x))/(7 + 5*x))]*Sqrt[11 + (31*
(-5 + 2*x))/(7 + 5*x)])) + (Sqrt[11/62]*Sqrt[22 + (23*(-5 + 2*x))/(7 + 5*x
)]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x]], 39/62])/(
Sqrt[(22 + (23*(-5 + 2*x))/(7 + 5*x))/(11 + (31*(-5 + 2*x))/(7 + 5*x))]*Sq
rt[11 + (31*(-5 + 2*x))/(7 + 5*x)])))/(39*Sqrt[1 + 4*x]*Sqrt[-((2 - 3*x)/(
7 + 5*x))])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 435, normalized size of antiderivative = 7.25

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{34\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{24942879\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\cdots\right.}{\cdots}$
default	$-\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\left(9\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^{2}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-9\sqrt{\cdots}\right.}{\cdots}$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(34/24942879*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/
3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1
/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+
7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+28/21105513*(-3795*(x+7/5)/(-2/3+x)
)^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/
(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*Elliptic
F(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-40/299*((x+7/
5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(
1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341
*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*El
lipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*Ellipt
icPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(
x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)-2/897*(-120*x^3+350*x^2-105*x-50)/(
(x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4
 + 110*x^3 - 993*x^2 - 1232*x - 245), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**(3/2),x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{3/2}}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{200x^4+110x^3-993x^2-1232x-245}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(200*x**4
 + 110*x**3 - 993*x**2 - 1232*x - 245),x)
```

3.103 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 227

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} + \frac{19666\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{2413827\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} - \frac{8\sqrt{682}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{104949\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
-10/2691*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+19666/74
828637*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*341^
(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*x)
/(7+5*x))^(1/2)-8/104949*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*E
llipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2
*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 30.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(-9833\sqrt{682}(-2+3x)(7+5x)^2\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}E\left(\arcsin\left(\sqrt{\frac{31}{39}}\sqrt{\frac{-5+2x}{-2+3x}}\right)|\frac{39}{62}\right)+31\left(\right.\right.}{74828637}$$

input
```
Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

output
```
(-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-9833*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*S
qrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5
 + 2*x)/(-2 + 3*x)]], 39/62] + 31*(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-389005 - 1
578968*x - 20372*x^2 + 285680*x^3) + 92*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*S
qrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5
 + 2*x)/(-2 + 3*x)]], 39/62])))/(74828637*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sq
rt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.70, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {195, 25, 2102, 27, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 195}$$

$$-\frac{\int -\frac{771-854x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{771-854x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \; 2102$$

$$\frac{\frac{\int \frac{2\left(-1179960x^2+894803x+1190728\right)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{-1179960x^2+894803x+1190728}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \; 2105$$

$$\frac{\frac{2\left(4218357\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{571325040}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{19666\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(4218357\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+2380521\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{19666\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \; 188$$

$$\frac{\frac{2\left(4218357\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{216411\sqrt{\frac{22}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{19666\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}-\frac{98330\sqrt{2}\cdots}{2\cdots}}{2691}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(4218357\int\frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{432822\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{19666\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807} - \frac{98330\sqrt{2-}}{27}$$

$$2691$$

$$\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$\downarrow$ 194

$$\frac{2\left(\frac{432822\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{383487\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{19666\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}$$

$$2691$$

$$\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$\downarrow$ 27

$$\frac{2\left(\frac{432822\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{383487\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{19666\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{27807}$$

$$2691$$

$$\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$\downarrow$ 320

$$\frac{2\left(-\frac{383487\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{432822\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{19666\sqrt{2-3x}}{\sqrt{2}}\right)}{27807}$$

$$2691$$

$$\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

$\downarrow$ 327

$$\frac{\frac{2\left(-\frac{9833\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{432822\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+19666\sqrt{2}\ldots}{27807}}{2691}$$

$$\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}$$

input
```
Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

output
```
(-10*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2691*(7 + 5*x)^(3/2)) +
((-98330*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*Sqrt[7 + 5*x])
 + (2*((19666*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x] -
(9833*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(
Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(Sqrt[(2 - 3*x)/(5 -
2*x)]*Sqrt[7 + 5*x]) + (432822*Sqrt[11/23]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[
7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3
*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2
 + (1 + 4*x)/(2 - 3*x))])))/27807)/2691
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
 + (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 195
```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b*(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*e - a*f)*(b*g - a*h))), x] +
 Simp[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e*g +
c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.04

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{432992\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\text{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{8}}{39}\right.}{2080759909059\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{2\left(499410\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,\text{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^{3}-442485\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\right.}{}$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(432992/2080759909059*(-3795*(x+7/5)/(-2/3+x))^(
1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2
/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(
-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+275324/176064299973*(-379
5*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*21
39^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(
1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-
31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/
2)))-393320/24942879*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3
+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/
4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),
1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/3
9*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55
,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)-2/13455*
(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2-19666/74828637*(-120*x
^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^
5 + 1950*x^4 - 4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

Sympy [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**(5/2),x)
```

output
```
Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Giac [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}}\,dx$$

input
```
integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{5/2}}\,dx$$

input
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

output
```
int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{1000x^5+1950x^4-4195x^3-13111x^2-9849x-1715}dx$$

input
```
int((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x)
```

output
```
int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(1000*x**
5 + 1950*x**4 - 4195*x**3 - 13111*x**2 - 9849*x - 1715),x)
```

3.104 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 733

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$
$$-\frac{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|-\frac{(de-cf)(bg-ah)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$
$$+\frac{(be-af)\sqrt{bg-ah}(dfg+deh-2cfh)\sqrt{c+dx}\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),-\frac{(de-}{(bc-}\right.}{(bc-ad)f^2h\sqrt{fg-eh}\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$
$$+\frac{\sqrt{bg-ah}((bc+ad)fh-bd(fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}(e+fx)\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\operatorname{EllipticPi}\left(\frac{f(bg-ah)}{b(fg-eh)},\text{arcsi}\right.}{bf^2h\sqrt{fg-eh}\sqrt{c+dx}\sqrt{g+hx}}$$

output
```
(b*x+a)^(1/2)*(d*x+c)^(1/2)*(h*x+g)^(1/2)/h/(f*x+e)^(1/2)-(-a*h+b*g)^(1/2)
*(-e*h+f*g)^(1/2)*(d*x+c)^(1/2)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1
/2)*EllipticE((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2
),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/f/h/((-a*f+b*e)*(d
*x+c)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/2)+(-a*f+b*e)*(-a*h+b*g)^(1/2)*
(-2*c*f*h+d*e*h+d*f*g)*(d*x+c)^(1/2)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e
))^(1/2)*EllipticF((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)
^(1/2),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/(-a*d+b*c)/f^
2/h/(-e*h+f*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)
^(1/2)+(-a*h+b*g)^(1/2)*((a*d+b*c)*f*h-b*d*(e*h+f*g))*((-a*f+b*e)*(d*x+c)/
(-a*d+b*c)/(f*x+e))^(1/2)*(f*x+e)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^
(1/2)*EllipticPi((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(
1/2),f*(-a*h+b*g)/b/(-e*h+f*g),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*
g))^(1/2))/b/f^2/h/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7591 vs. 2(733) = 1466.

Time = 52.77 (sec) , antiderivative size = 7591, normalized size of antiderivative = 10.36

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {191, 183, 188, 194, 321, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \ 191$$

$$\frac{(de-cf)(-2afh+beh+bfg)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2f^2h}+\frac{(adfh-b(-cfh+deh+dfg))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}dx}{2f^2h}-\frac{(de-cf)(fg-eh)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}dx}{2fh}+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

$$\Big\downarrow \ 183$$

$$\frac{(de-cf)(-2afh+beh+bfg)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2f^2h}+\frac{(e+fx)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+}{(dg-ch)(e+}}}}{f^2h\sqrt{a+bx}\sqrt{c+dx}}$$

$$\frac{(de-cf)(fg-eh)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}dx}{2fh}+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

$$\Big\downarrow \ 188$$

$$\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{f^2h\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{(e+fx)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+}{(dg-ch)(e+}}}}{f^2h\sqrt{a+bx}\sqrt{c+dx}}$$

$$\frac{(de-cf)(fg-eh)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}dx}{2fh}+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

$$\Big\downarrow \ 194$$

$$\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{f^2h\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{(e+fx)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+}{(dg-ch)(e+}}}}{f^2h\sqrt{a+bx}\sqrt{c+dx}}$$

$$\frac{\sqrt{a+bx}(fg-eh)\sqrt{\frac{(g+hx)(de-cf)}{(e+fx)(dg-ch)}}\int \frac{\sqrt{1-\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}}{\sqrt{1-\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{fh\sqrt{g+hx}\sqrt{-\frac{(a+bx)(de-cf)}{(e+fx)(bc-ad)}}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

$\downarrow$ 321

$$\frac{(e+fx)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+}{(dg-ch)(e+}}}}{f^2h\sqrt{a+bx}\sqrt{c+dx}}$$

$$\frac{\sqrt{a+bx}(fg-eh)\sqrt{\frac{(g+hx)(de-cf)}{(e+fx)(dg-ch)}}\int \frac{\sqrt{1-\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}}{\sqrt{1-\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{fh\sqrt{g+hx}\sqrt{-\frac{(a+bx)(de-cf)}{(e+fx)(bc-ad)}}}+$$

$$\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

$\downarrow$ 327

$$\frac{(e+fx)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+}{(dg-ch)(e+}}}}{f^2h\sqrt{a+bx}\sqrt{c+dx}}$$

$$\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{\frac{(g+hx)(de-cf)}{(e+fx)(dg-ch)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{g+hx}\sqrt{-\frac{(a+bx)(de-cf)}{(e+fx)(bc-ad)}}}+$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

$\downarrow$ 412

$$\frac{(e+fx)\sqrt{bg-ah}\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\,\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{be}}{\sqrt{bg}}\right.\right.}{f^2h^2\sqrt{a+bx}\sqrt{c+dx}\sqrt{be-af}}$$

$$\frac{\sqrt{g+hx}(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{\frac{(g+hx)(de-cf)}{(e+fx)(dg-ch)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{g+hx}\sqrt{-\frac{(a+bx)(de-cf)}{(e+fx)(bc-ad)}}}+$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}}$$

input
```
Int[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[d*g
- c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g -
c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt[d*
g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*
h))])/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqrt[g
 + h*x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a
*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g -
 e*h))/((d*e - c*f)*(b*g - a*h)))])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]
*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) +
 (Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*h)*
(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c
*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcS
in[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e
 - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(f^2*Sqrt[b*e - a*f]*h^2*
Sqrt[a + b*x]*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 191
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[Sqrt[a + b*x]*Sqrt[c + d*x]*(
Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Simp[(d*e - c*f)*((f*g - e*h)/(2*f
*h))   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x]), x],
 x] + Simp[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h)   Int[Sqrt[e + f
*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Simp[(d*e - c*f)*
((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(668) = 1336$.

Time = 12.63 (sec) , antiderivative size = 1544, normalized size of antiderivative = 2.11

method	result	size
elliptic	Expression too large to display	1544
default	Expression too large to display	15529

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*a*c*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/
d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g
/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h
)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(
x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(a*d+b*
c)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+
c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))
^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d
+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+d*b*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)
*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b
)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)*
((c/d*g/h-e/f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c/d+g/h)*EllipticF(((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d
+e/f))^(1/2))+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(
1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))/(-c/d+g/h)+(a*d*f*
h+b*c*f*h+b*d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*EllipticPi(((c/d-e/f)*(x+g...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="giac")
```

output
```
integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{bx+a}\,\sqrt{dx+c}}{\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.105 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 161

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{c+dx}\,E\left(\arctan\left(\frac{\sqrt{-be+af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|\frac{(-bc+ad)(fg-eh)}{(-be+af)(dg-ch)}\right)}{\sqrt{-be+af}\sqrt{bg-ah}\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}$$

output
```
-2*(d*x+c)^(1/2)*EllipticE((a*f-b*e)^(1/2)*(h*x+g)^(1/2)/(-a*h+b*g)^(1/2)/
(f*x+e)^(1/2)/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2),((a*d-b*c)*(-
e*h+f*g)/(a*f-b*e)/(-c*h+d*g))^(1/2))/(a*f-b*e)^(1/2)/(-a*h+b*g)^(1/2)/(b*
x+a)^(1/2)/((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 23.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(fg-eh)\sqrt{a+bx}\sqrt{c+dx}\sqrt{\frac{(-be+af)(bg-ah)(e+fx)(g+hx)}{(fg-eh)^2(a+bx)^2}}E\left(\arcsin\left(\sqrt{\vphantom{x}}\right.\right.}{(be-af)(bg-ah)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{e+fx}\sqrt{\vphantom{x}}}$$

input
```
Integrate[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(f*g - e*h)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[((-(b*e) + a*f)*(b*g - a*h
)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)]*EllipticE[ArcSin[Sqrt[
((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g -
e*h))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*(b*g - a*h)*Sqrt[((b*g - a*
h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {194, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{194}$$

$$-\frac{2\sqrt{c+dx}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\int \frac{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}}{\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{g+hx}(be-af)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

input
```
Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e
*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g -
a*h)))])/((b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e -
 c*f)*(a + b*x))]*Sqrt[g + h*x])
```

Defintions of rubi rules used

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1947 vs. $2(179) = 358$.

Time = 23.44 (sec) , antiderivative size = 1948, normalized size of antiderivative = 12.10

method	result	size
elliptic	Expression too large to display	1948
default	Expression too large to display	2440

input
```
int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(-2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2
+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g)/
((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*
g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(d/b-1/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*
b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g)+(
b*c*e*h+b*c*f*g+b*d*e*g)/(a^2*f*h-a*b*e*h-a*b*f*g+b^2*e*g))*(e/f-g/h)*((c/
d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a
/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-
e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(
((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/
h)/(-c/d+e/f))^(1/2))+2*((a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)/(a^2*f*h-a*b*e*
h-a*b*f*g+b^2*e*g)+(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^2*f*h-a*b*e*h-a*b*f*
g+b^2*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)
^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f
+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)
*(x+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/
2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi
(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+
a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+2*h*d*b*f/(a^2*f*h-a*b*e*h...
```

Fricas [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="fricas")
```

output
```
integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*f*h*
x^4 + a^2*e*g + (b^2*f*g + (b^2*e + 2*a*b*f)*h)*x^3 + ((b^2*e + 2*a*b*f)*g
 + (2*a*b*e + a^2*f)*h)*x^2 + (a^2*e*h + (2*a*b*e + a^2*f)*g)*x), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)/((a + b*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="giac")
```

output
```
integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}}\,dx$$

input
```
int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)),x)
```

output
```
int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.106 $\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 372

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{2135\sqrt{2-3x}\sqrt{-5+2x}\sqrt{7+5x}}{96\sqrt{1+4x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{2135\sqrt{253}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}E\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right)|-\frac{39}{23}\right)}{384\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} + \frac{77101\sqrt{\frac{23}{11}}\sqrt{-\frac{2-3x}{1+4x}}\sqrt{7+5x}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{768\sqrt{2-3x}\sqrt{\frac{7+5x}{1+4x}}} + \frac{553525\sqrt{\frac{11}{23}}\sqrt{-\frac{2-3x}{1+4x}}(1+4x)\sqrt{\frac{7+5x}{1+4x}}\,\mathrm{EllipticPi}\left(\frac{78}{23},\arcsin\left(\frac{\sqrt{\frac{23}{39}}\sqrt{-5+2x}}{\sqrt{1+4x}}\right),-\frac{39}{23}\right)}{768\sqrt{2-3x}\sqrt{7+5x}}$$

output
```
-2135/96*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2)-25/48*(2
-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)-2135/384*253^(1/2)*
(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticE(1/39*897^(1/2)*(-5+2*x)^(
1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/((7+5*x)/(1+4*x))^(1/2)
+77101/8448*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(7+5*x)^(1/2)*EllipticF(1/3
9*897^(1/2)*(-5+2*x)^(1/2)/(1+4*x)^(1/2),1/23*I*897^(1/2))/(2-3*x)^(1/2)/(
(7+5*x)/(1+4*x))^(1/2)+553525/17664*253^(1/2)*(-(2-3*x)/(1+4*x))^(1/2)*(1+
4*x)*((7+5*x)/(1+4*x))^(1/2)*EllipticPi(1/39*897^(1/2)*(-5+2*x)^(1/2)/(1+4
*x)^(1/2),78/23,1/23*I*897^(1/2))/(2-3*x)^(1/2)/(7+5*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 23.25 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.93

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(1227600(-2+3x)+\frac{-13104630\sqrt{682}(-2+3x)}{}\right.}{}$$

input
```
Integrate[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(1227600*(-2 + 3*x) + (-131046
30*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*El
lipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 17113116
*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elli
pticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 385*Sqrt[(
7 + 5*x)/(-2 + 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) - 47445*Sqrt
[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2
 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x
)]], 39/62]))/((2 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2)))
)/(2356992*Sqrt[2 - 3*x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.351, Rules used = {185, 2105, 27, 194, 27, 327, 2101, 183, 27, 188, 27, 320, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x+7)^{5/2}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{ 185}$$

$$\frac{1}{96}\int \frac{64050x^2+89810x+28003}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{ 2105}$$

$$\frac{1}{96}\left(-\frac{915915}{4}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int \frac{60(146323-553525x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{2135\sqrt{2-3x}}{2\sqrt{}}\right.$$
$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{96}\left(-\frac{915915}{4}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{4}\int \frac{146323-553525x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{2135\sqrt{2-3x}\sqrt{}}{2\sqrt{2x}}\right.$$
$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{ 194}$$

$$\frac{1}{96}\left(-\frac{1}{4}\int \frac{146323-553525x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{83265\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{2135\sqrt{2}}{}\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{96}\left(-\frac{1}{4}\int \frac{146323-553525x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{83265\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{2135\sqrt{2}\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{96}\left(-\frac{1}{4}\int \frac{146323-553525x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{2135\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{4\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{2101}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{668081}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{553525}{3}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx\right)+\frac{2135\sqrt{42}}{}\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{183}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{668081}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{34318550(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int \frac{}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5}{2-}\right.}}{3\sqrt{897}\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{668081}{3}\int\frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{34318550(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\frac{11(5x+7)}{2-3x}}\left(\frac{3(5\ldots}{2-\ldots}\right.}}{3\sqrt{2x-5}\sqrt{4x+1}}\right.\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{188}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{29047\sqrt{\frac{46}{11}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{34318550(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\ldots}}}{3\sqrt{2x-5}\sqrt{4\ldots}}\right.\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{1336162\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int\frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{3\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{34318550(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\int\frac{}{\sqrt{23-\frac{11(5\ldots}{2-\ldots}}}}{3\sqrt{2x-5}\sqrt{4x\ldots}}\right.\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{320}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{58094\sqrt{\frac{23}{11}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}-\frac{34318550(2-3x)\sqrt{\frac{5\ldots}{2\ldots}}}{}\right.\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{96}\left(\frac{1}{4}\left(\frac{58094\sqrt{\frac{23}{11}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{3\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}-\frac{6863710(2-3x)\sqrt{\frac{5-}{2-}}\right.\right.$$

$$\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}$$

input
```
Int[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/48 + ((-213
5*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2*Sqrt[-5 + 2*x]) + (2135*Sq
rt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(4*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + ((58094*Sqrt[23/11]*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*
x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqr
t[2]*Sqrt[2 - 3*x])], -39/23])/(3*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]
*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (
1 + 4*x)/(2 - 3*x))]) - (6863710*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[
-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*
x])/Sqrt[2 - 3*x]], -23/39])/(3*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]))/4
)/96
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 185
```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*b^2*(a + b*x)^(m - 2)*Sqrt[c
 + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m - 1))), x] - Simp[1/(d*f*h
*(2*m - 1))   Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*d*f*h
*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g + c*f*
g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 IntegerQ[2*m] && GeQ[m, 2]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(-\frac{25\sqrt{-120x^4+182x^3+385x^2-197x-70}}{48}+\frac{28003\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\cdots}}{14682096\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+\cdots\right.}}\cdots\right.}{\cdots}$
risch	$\frac{25\sqrt{7+5x}\,(-2+3x)\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{(2-3x)(-5+2x)(1+4x)(7+5x)}}{48\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{2-3x}}+\frac{\left(-\frac{28003\sqrt{1705}\sqrt{\frac{x+\frac{7}{5}}{x+\frac{1}{4}}}\left(x+\frac{1}{4}\right)^2\sqrt{1794}\sqrt{\frac{x-\frac{5}{2}}{x+\frac{1}{4}}}\sqrt{2139}\sqrt{\cdots}}{14682096\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(12025458\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\,x^2\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\cdots\right.}{\cdots}$

input
```
int((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(-25/48*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2
)+28003/14682096*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5
/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3
+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1
/39*I*897^(1/2))+44905/7341048*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*8
06^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30
*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5
)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(
-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+10675/32*((x+7/5)*(x-5/2)*(x+1/4)
-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/
3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-
3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-379
5*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(
x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x
-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx=\int \frac{(5\,x+7)^{\frac{5}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*s
qrt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \text{Timed out}$$

input
```
integrate((7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{5}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{5/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{25\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{13} - \frac{7625\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x^3}{120x^4-182x^3-385x^2+197x+70}dx\right)}{13} + \frac{70\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{120x^4-182x^3-385x^2+197x+70}dx\right)}{13} - \frac{13843\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)}{26}$$

input
```
int((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
(50*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2) - 15250*int
((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x**3)/(120*x*
*4 - 182*x**3 - 385*x**2 + 197*x + 70),x) + 140*int((sqrt(2*x - 5)*sqrt(4*
x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(120*x**4 - 182*x**3 - 385*x**2 +
 197*x + 70),x) - 13843*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqr
t( - 3*x + 2))/(120*x**4 - 182*x**3 - 385*x**2 + 197*x + 70),x))/26
```

3.107 $\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 432

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}{8\sqrt{2-3x}} - \frac{5\sqrt{\frac{143}{3}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}\,E\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{8\sqrt{-5+2x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{155\sqrt{\frac{11}{39}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{36\sqrt{-5+2x}\sqrt{-\frac{1+4x}{2-3x}}} - \frac{8959(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{1+4x}{2-3x}}\,\text{EllipticPi}\left(-\frac{69}{55},\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{36\sqrt{429}\sqrt{-5+2x}\sqrt{1+4x}} + \frac{529\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{8\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}}$$

output
```
5/8*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2)-5/24*429^(1/2
)*((5-2*x)/(2-3*x))^(1/2)*(1+4*x)^(1/2)*EllipticE(1/23*253^(1/2)*(7+5*x)^(
1/2)/(2-3*x)^(1/2),1/39*I*897^(1/2))/(-5+2*x)^(1/2)/(-(1+4*x)/(2-3*x))^(1/
2)-155/1404*429^(1/2)*((5-2*x)/(2-3*x))^(1/2)*(1+4*x)^(1/2)*EllipticF(1/23
*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),1/39*I*897^(1/2))/(-5+2*x)^(1/2)/(-
(1+4*x)/(2-3*x))^(1/2)-8959/15444*(2-3*x)*((5-2*x)/(2-3*x))^(1/2)*(-(1+4*x
)/(2-3*x))^(1/2)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69
/55,1/39*I*897^(1/2))*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+529/5456*682^
(1/2)*((2-3*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2)*(7+5*x)*EllipticPi(1
/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/124,1/62*2418^(1/2))/(2-3*x)^
(1/2)/(-5+2*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.53 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.80

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{-5+2x}\left(6820\sqrt{341}\sqrt{\frac{-2+3x}{1+4x}}\sqrt{\frac{7+5x}{1+4x}}(-5-18x+8x^2)\,E\left(\arcsin\left(\sqrt{\right.\right.\right.}$$

input
```
Integrate[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(Sqrt[-5 + 2*x]*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/
(1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)
/(1 + 4*x)]], 39/62] - 6969*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 +
 5*x)/(1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7
+ 5*x)/(1 + 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70
 - 83*x - 53*x^2 + 30*x^3) + 9821*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1
+ 4*x)^2*Sqrt[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[
Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/62])))/(16368*Sqrt[4 - 6*x]*((-
5 + 2*x)/(1 + 4*x))^(3/2)*(1 + 4*x)^(3/2)*Sqrt[7 + 5*x])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.65, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.432, Rules used = {184, 183, 27, 191, 183, 27, 188, 27, 194, 27, 320, 327, 411, 320, 412, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x+7)^{3/2}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{184}$$

$$\frac{31}{3}\int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx-\frac{5}{3}\int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{183}$$

$$\frac{713\sqrt{2}\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{11\sqrt{2}}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{33\sqrt{2-3x}\sqrt{2x-5}}-\frac{5}{3}\int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-\frac{5}{3}\int \frac{\sqrt{2-3x}\sqrt{5x+7}}{\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{191}$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-\frac{5}{3}\left(\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{179}{16}\int \frac{\sqrt{2x}}{\sqrt{2-3x}\sqrt{4x}}\right.$$

$$\Big\downarrow \text{183}$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x}{}\right.$$

$\downarrow$ 27

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{429}{16}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x}{}\right.$$

$\downarrow$ 188

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{39\sqrt{\frac{11}{46}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{6981\sqrt{\frac{2-}{5-}}}{}\right.$$

$\downarrow$ 27

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(\frac{429}{8}\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{39\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{6981\sqrt{\frac{2-}{5-}}}{}\right.$$

$\downarrow$ 194

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(-\frac{39\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+-\frac{6}{}\right.$$

$$\downarrow 27$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}} - \frac{5}{3}\left(-\frac{39\sqrt{11}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}} - \frac{39\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \ldots\right.$$

$$\downarrow 320$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}} - \frac{5}{3}\left(-\frac{39\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}} + \frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\ldots}}}{8\sqrt{2-3x}\sqrt{4x+1}}\right.$$

$$\downarrow 327$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}} - \frac{5}{3}\left(\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\int \frac{1}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}}{8\sqrt{2-3x}\sqrt{4x+1}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\ldots\right.}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x\ldots}}\right.$$

$$\downarrow 411$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}} - \frac{5}{3}\left(\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{11}{78}\int \frac{1}{\sqrt{\frac{11(5x+7)}{2x-5}+31}\sqrt{\frac{22(5x+7)}{2x-5}+23}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}} + \frac{1}{78}\int \frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}\right)}{8\sqrt{2-3x}\sqrt{4x+1}}\right.$$

$$\downarrow 320$$

$$\frac{1426\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{3\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{1}{78}\int \frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x}}{\sqrt{2x-5}}\right.\right.}{78\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(5x+7)}{2x-5}+23}}\right.}{8\sqrt{2-3x}\sqrt{4x+1}}\right.$$

$\downarrow$ 412

$$\frac{713\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\,\text{EllipticPi}\left(\frac{55}{78},\arcsin\left(\frac{\sqrt{\frac{39}{22}}\sqrt{4x+1}}{\sqrt{5x+7}}\right),\frac{62}{39}\right)}{6\sqrt{429}\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(\frac{6981\sqrt{\frac{2-3x}{5-2x}}(5-2x)\sqrt{-\frac{4x+1}{5-2x}}\left(\frac{1}{78}\int \frac{\sqrt{\frac{22(5x+7)}{2x-5}+23}}{\left(5-\frac{2(5x+7)}{2x-5}\right)\sqrt{\frac{11(5x+7)}{2x-5}+31}}d\frac{\sqrt{5x+7}}{\sqrt{2x-5}}+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{11(5x+7)}{2x-5}+31}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x}}{\sqrt{2x-5}}\right.\right.}{78\sqrt{\frac{\frac{11(5x+7)}{2x-5}+31}{\frac{22(5x+7)}{2x-5}+23}}\sqrt{\frac{22(5x+7)}{2x-5}+23}}\right.}{8\sqrt{2-3x}\sqrt{4x+1}}\right.$$

$\downarrow$ 414

$$\frac{713\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\,\text{EllipticPi}\left(\frac{55}{78},\arcsin\left(\frac{\sqrt{\frac{39}{22}}\sqrt{4x+1}}{\sqrt{5x+7}}\right),\frac{62}{39}\right)}{6\sqrt{429}\sqrt{2-3x}\sqrt{2x-5}}-$$

$$\frac{5}{3}\left(-\frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(a\right.}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4}{2-}}}\right.$$

input

```
Int[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(713*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*Ellipti
cPi[55/78, ArcSin[(Sqrt[39/22]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 62/39])/(6*S
qrt[429]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]) - (5*((Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*
Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x
)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]],
 -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (39*Sqrt[11/23]*Sq
rt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*
EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[
-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23
 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(2 - 3*x))]) + (6981*Sqrt[(2 -
 3*x)/(5 - 2*x)]*(5 - 2*x)*Sqrt[-((1 + 4*x)/(5 - 2*x))]*((Sqrt[11/62]*Sqrt
[31 + (11*(7 + 5*x))/(-5 + 2*x)]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*
x])/Sqrt[-5 + 2*x]], 39/62])/(78*Sqrt[(31 + (11*(7 + 5*x))/(-5 + 2*x))/(23
 + (22*(7 + 5*x))/(-5 + 2*x))]*Sqrt[23 + (22*(7 + 5*x))/(-5 + 2*x)]) + (23
*Sqrt[31 + (11*(7 + 5*x))/(-5 + 2*x)]*EllipticPi[78/55, ArcTan[(Sqrt[22/23
]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(390*Sqrt[682]*Sqrt[(31 + (11*(7
 + 5*x))/(-5 + 2*x))/(23 + (22*(7 + 5*x))/(-5 + 2*x))]*Sqrt[23 + (22*(7 +
5*x))/(-5 + 2*x)])))/(8*Sqrt[2 - 3*x]*Sqrt[1 + 4*x])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 184
```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b/d   Int[Sqrt[a + b*x]*(Sqrt
[c + d*x]/(Sqrt[e + f*x]*Sqrt[g + h*x])), x], x] - Simp[(b*c - a*d)/d   Int
[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 191
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[Sqrt[a + b*x]*Sqrt[c + d*x]*(
Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Simp[(d*e - c*f)*((f*g - e*h)/(2*f
*h))   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x]), x],
 x] + Simp[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h)   Int[Sqrt[e + f
*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Simp[(d*e - c*f)*
((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 411
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[-f/(b*e - a*f)   Int[1/(Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] + Simp[b/(b*e - a*f)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqr
t[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ
[f/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.92

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{98\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{\sqrt{7+5x}\,\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\left(107694\sqrt{-\frac{253(7+5x)}{-2+3x}}\,\sqrt{13}\,\sqrt{3}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{1+4x}{-2+3x}}\,x^{2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-\right.}{}$

input

```
int((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(98/305877*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+
x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2
)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795*(x+7/
5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+140/305877*(-3795*(x+7/5)/(-2/3+x))^(
1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2
/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*EllipticF(1
/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*EllipticPi(1/69
*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))+25/2*((x+7/5)*(x
-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*
((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*Elli
pticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-117/62*Ellipti
cE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/55*EllipticPi(
1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5
)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2))
```

Fricas [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x
^3 - 70*x^2 + 21*x + 10), x)
```

Sympy [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5x+7)^{\frac{3}{2}}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral((5*x + 7)**(3/2)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{(5\,x+7)^{3/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \\ &-5\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}\,x}{24x^3-70x^2+21x+10}dx\right) \\ &-7\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)\end{aligned}$$

input
```
int((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output

```
 - 5*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2)*x)/(2
4*x**3 - 70*x**2 + 21*x + 10),x) - 7*int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt
(5*x + 7)*sqrt( - 3*x + 2))/(24*x**3 - 70*x**2 + 21*x + 10),x)
```

3.108 $\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 100

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{23\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\,\text{EllipticPi}\left(\frac{55}{124},\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{2\sqrt{682}\sqrt{2-3x}\sqrt{-5+2x}}$$

output
```
23/1364*682^(1/2)*((2-3*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2)*(7+5*x)*
EllipticPi(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/124,1/62*2418^(1/
2))/(2-3*x)^(1/2)/(-5+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{62\sqrt{1+4x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticPi}\left(-\frac{55}{69},\arcsin\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{7+5x}}\right),-\frac{39}{23}\right)}{3\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{1+4x}{7+5x}}}$$

input
```
Integrate[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(-62*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticPi[-55/69, ArcSin[(Sq
rt[23/11]*Sqrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(3*Sqrt[253]*Sqrt[-5 + 2
*x]*Sqrt[(1 + 4*x)/(7 + 5*x)])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {183, 27, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}dx$$

$$\Big\downarrow \text{183}$$

$$\frac{23\sqrt{2}\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{11\sqrt{2}}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{11\sqrt{2-3x}\sqrt{2x-5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{46\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\int \frac{1}{\sqrt{22-\frac{39(4x+1)}{5x+7}}\sqrt{11-\frac{31(4x+1)}{5x+7}}\left(4-\frac{5(4x+1)}{5x+7}\right)}d\frac{\sqrt{4x+1}}{\sqrt{5x+7}}}{\sqrt{2-3x}\sqrt{2x-5}}$$

$$\Big\downarrow \text{412}$$

$$\frac{23\sqrt{\frac{2-3x}{5x+7}}\sqrt{\frac{5-2x}{5x+7}}(5x+7)\text{EllipticPi}\left(\frac{55}{78},\arcsin\left(\frac{\sqrt{\frac{39}{22}}\sqrt{4x+1}}{\sqrt{5x+7}}\right),\frac{62}{39}\right)}{2\sqrt{429}\sqrt{2-3x}\sqrt{2x-5}}$$

input
```
Int[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

output
```
(23*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*Elliptic
Pi[55/78, ArcSin[(Sqrt[39/22]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 62/39])/(2*Sq
rt[429]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.57

method	result
default	$\frac{62\left(\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-\operatorname{EllipticPi}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},-\frac{69}{55},\frac{i\sqrt{897}}{39}\right)\right)\sqrt{1+4x}\,\sqrt{-5+2x}\,(2-3x)^{\frac{3}{2}}\sqrt{7+5x}\,\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{2}}{29601(40x^3-34x^2-151x-35)}$
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{14\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right)}{305877\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{\sqrt{2-3x}\,\sqrt{}}$

input
```
int((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
62/29601*(EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/2))-E
llipticPi(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),-69/55,1/39*I*897^(1/2)))*(1+
4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(3/2)*(7+5*x)^(1/2)/(40*x^3-34*x^2-151*x
-35)*((1+4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*1
3^(1/2)*(-253*(7+5*x)/(-2+3*x))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx=\int \frac{\sqrt{5\,x+7}}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3
 - 70*x^2 + 21*x + 10), x)
```

Sympy [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}\,dx$$

input
```
integrate((7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

output
```
Integral(sqrt(5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

input
```
integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

input
```
integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \int \frac{\sqrt{5\,x+7}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

input
```
int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

output
```
int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{24x^3-70x^2+21x+10}dx\right)$$

input
```
int((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x)
```

output
```
 - int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(24*x**
*3 - 70*x**2 + 21*x + 10),x)
```

3.109 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 91

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = -\frac{2\sqrt{-5+2x}\sqrt{-\frac{1+4x}{2-3x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{11}{23}}\sqrt{7+5x}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{\sqrt{429}\sqrt{\frac{5-2x}{2-3x}}\sqrt{1+4x}}$$

output
```
-2/429*(-5+2*x)^(1/2)*(-(1+4*x)/(2-3*x))^(1/2)*EllipticF(1/23*253^(1/2)*(7
+5*x)^(1/2)/(2-3*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)/((5-2*x)/(2-3*x))^(1
/2)/(1+4*x)^(1/2)
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = -\frac{2\sqrt{1+4x}\sqrt{\frac{5-2x}{7+5x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{7+5x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{1+4x}{7+5x}}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

output
```
(-2*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticF[ArcSin[(Sqrt[23/11]*
Sqrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(1
+ 4*x)/(7 + 5*x)])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {188, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx$$

$$\Big\downarrow \text{188}$$

$$\frac{\sqrt{\frac{2}{253}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{2\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

output
```
(2*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3
*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(Sq
rt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3
*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(2 - 3*x))])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.41

method	result
default	$\frac{2\sqrt{7+5x}\,\sqrt{1+4x}\,\sqrt{-5+2x}\,(2-3x)^{\frac{3}{2}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)\sqrt{\frac{1+4x}{-2+3x}}\,\sqrt{23}\,\sqrt{\frac{-5+2x}{-2+3x}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{-\frac{253(7+5x)}{-2+3x}}}{9867(40x^3-34x^2-151x-35)}$
elliptic	$\frac{2\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\,\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\,\left(-\frac{2}{3}+x\right)^2\sqrt{806}\,\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\,\sqrt{2139}\,\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\,\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right.}{305877\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/9867*(7+5*x)^(1/2)*(1+4*x)^(1/2)*(-5+2*x)^(1/2)*(2-3*x)^(3/2)/(40*x^3-34
*x^2-151*x-35)*EllipticF(1/23*(-253*(7+5*x)/(-2+3*x))^(1/2),1/39*I*897^(1/
2))*((1+4*x)/(-2+3*x))^(1/2)*23^(1/2)*((-5+2*x)/(-2+3*x))^(1/2)*3^(1/2)*13
^(1/2)*(-253*(7+5*x)/(-2+3*x))^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^
4 - 182*x^3 - 385*x^2 + 197*x + 70), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}\,dx$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**(1/2),x
)
```

output
```
Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx=\int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx=\int \frac{1}{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x, al
gorithm="giac")
```

output
```
integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx=\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx = -\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}dx\right)$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(1/2),x)
```

output
```
 - int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(120*x
**4 - 182*x**3 - 385*x**2 + 197*x + 70),x)
```

3.110 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 189

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{10\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{897\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} + \frac{2\sqrt{\frac{2}{341}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{39\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
10/27807*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*34
1^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-3*
x)/(7+5*x))^(1/2)+2/13299*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*
EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+
2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 17.76 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(1705\sqrt{\frac{7+5x}{-2+3x}}(-5-18x+8x^2)-55\sqrt{682}\sqrt{\frac{-5-18x+8x^2}{(2-3x)^2}}(-14+11x+15x^2)\,E\left(\text{arcs}\right.\right.}{305877\sqrt{2-3x}\sqrt{7+5x}}$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x
]
```

output
```
(-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18
*x + 8*x^2) - 55*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11
*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39
/62] - 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15
*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/
(305877*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
+ 8*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 530 vs. 2(189) = 378.

Time = 0.45 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.80, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {189, 188, 27, 194, 27, 320, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}\,dx$$

$$\Big\downarrow \text{189}$$

$$\frac{5}{31}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx+\frac{3}{31}\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx$$

$$\Big\downarrow \text{ 188}$$

$$\frac{5}{31}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx+\frac{3\sqrt{\frac{2}{253}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{31\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{31}\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx+\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{31\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}$$

$$\Big\downarrow \text{ 194}$$

$$\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{31\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\int \frac{\sqrt{2}\sqrt{\frac{31(2x-5)}{5x+7}+11}}{\sqrt{\frac{23(2x-5)}{5x+7}+22}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{31\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{10\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\int \frac{\sqrt{\frac{31(2x-5)}{5x+7}+11}}{\sqrt{\frac{23(2x-5)}{5x+7}+22}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{10\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\int \frac{\sqrt{\frac{31(2x-5)}{5x+7}+11}}{\sqrt{\frac{23(2x-5)}{5x+7}+22}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}+\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{10\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(11\int\frac{1}{\sqrt{\frac{23(2x-5)}{5x+7}+22}\sqrt{\frac{31(2x-5)}{5x+7}+11}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}+31\int\frac{2x-5}{(5x+7)\sqrt{\frac{23(2x-5)}{5x+7}+22}\sqrt{\frac{31(2x-5)}{5x+7}+11}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}\right)}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}+$$

$$\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}$$

$\downarrow$ 320

$$\frac{10\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(31\int\frac{2x-5}{(5x+7)\sqrt{\frac{23(2x-5)}{5x+7}+22}\sqrt{\frac{31(2x-5)}{5x+7}+11}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}+11}}\right.}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}$$

$\downarrow$ 388

$$\frac{10\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(31\left(\frac{\sqrt{2x-5}\sqrt{\frac{23(2x-5)}{5x+7}+22}}{23\sqrt{5x+7}\sqrt{\frac{31(2x-5)}{5x+7}+11}}-\frac{11}{23}\int\frac{\sqrt{\frac{23(2x-5)}{5x+7}+22}}{\left(\frac{31(2x-5)}{5x+7}+11\right)^{3/2}}d\frac{\sqrt{2x-5}}{\sqrt{5x+7}}\right)+\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}}}\right.}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

$$\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}$$

$\downarrow$ 313

$$\frac{6\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+$$

$$\frac{10\sqrt{2-3x}\sqrt{\frac{4x+1}{5x+7}}\left(\frac{\sqrt{\frac{11}{62}}\sqrt{\frac{23(2x-5)}{5x+7}+22}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right),\frac{39}{62}\right)}{\sqrt{\frac{\frac{23(2x-5)}{5x+7}+22}{\frac{31(2x-5)}{5x+7}+11}}\sqrt{\frac{31(2x-5)}{5x+7}+11}}+31\left(\frac{\sqrt{2x-5}\sqrt{\frac{23(2x-5)}{5x+7}+22}}{23\sqrt{5x+7}\sqrt{\frac{31(2x-5)}{5x+7}+11}}-\frac{\sqrt{\frac{22}{31}}\sqrt{\frac{23(2x-5)}{5x+7}+2\cdots}}{23\sqrt{\frac{23(2x\cdots}{5x\cdots}}{\frac{31(2x\cdots}{5x\cdots}}}\cdots\right.\right.}{1209\sqrt{4x+1}\sqrt{-\frac{2-3x}{5x+7}}}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

output
```
(6*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]*Sqrt[23 + (31*(1 + 4*x))/(2 - 3
*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(31
*Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 - 3*x)]*Sqrt[2 + (1 + 4*x)/(2
- 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/(2 + (1 + 4*x)/(2 - 3*x))]) +
 (10*Sqrt[2 - 3*x]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(31*((Sqrt[-5 + 2*x]*Sqrt[22
+ (23*(-5 + 2*x))/(7 + 5*x)])/(23*Sqrt[7 + 5*x]*Sqrt[11 + (31*(-5 + 2*x))/
(7 + 5*x)]) - (Sqrt[22/31]*Sqrt[22 + (23*(-5 + 2*x))/(7 + 5*x)]*EllipticE[
ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x]], 39/62])/(23*Sqrt[(22 +
 (23*(-5 + 2*x))/(7 + 5*x))/(11 + (31*(-5 + 2*x))/(7 + 5*x))]*Sqrt[11 + (3
1*(-5 + 2*x))/(7 + 5*x)])) + (Sqrt[11/62]*Sqrt[22 + (23*(-5 + 2*x))/(7 + 5
*x)]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x]], 39/62])
/(Sqrt[(22 + (23*(-5 + 2*x))/(7 + 5*x))/(11 + (31*(-5 + 2*x))/(7 + 5*x))]*
Sqrt[11 + (31*(-5 + 2*x))/(7 + 5*x)])))/(1209*Sqrt[1 + 4*x]*Sqrt[-((2 - 3*
x)/(7 + 5*x))])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 189
```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-d/(b*c - a*d)   Int[1/(Sqr
t[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[b/(b*c
 - a*d)   Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.30

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{7252\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{897}}{39}\right.}{8505521739\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}\left(1116\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}x^{2}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)-\right.}{}$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(7252/8505521739*(-3795*(x+7/5)/(-2/3+x))^(1/2)*
(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x)
)^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/69*(-3795
*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+140/654270903*(-3795*(x+7/5)/(-
2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x
+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*(2/3*El
lipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-31/15*Ellipt
icPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))-200/926
9*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)
^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)*
(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))-1
17/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+91/5
5*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*897^(1/2)))
)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)-10/27807*(-120*x^3+350*x^2-
105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^
5 - 70*x^4 - 3199*x^3 - 1710*x^2 + 1729*x + 490), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**(3/2),x
)
```

output
```
Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x, al
gorithm="maxima")
```

output
```
integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)),
x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x, al
gorithm="giac")
```

output
```
integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{3/2}}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x
)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = -\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{600x^5-70x^4-3199x^3-1710x^2+1729x+490}dx\right)$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(3/2),x)
```

output
```
- int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(600*x
**5 - 70*x**4 - 3199*x**3 - 1710*x**2 + 1729*x + 490),x)
```

3.111 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 229

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} + \frac{179060\sqrt{\frac{22}{31}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}E\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{74828637\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}} + \frac{2506\sqrt{\frac{2}{341}}\sqrt{2-3x}\sqrt{\frac{5-2x}{7+5x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{31}{11}}\sqrt{1+4x}}{\sqrt{7+5x}}\right),\frac{39}{62}\right)}{3253419\sqrt{-5+2x}\sqrt{\frac{2-3x}{7+5x}}}$$

output
```
-50/83421*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+179060/
2319687747*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)*EllipticE(1/11*
341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(1/2))/(-5+2*x)^(1/2)/((2-
3*x)/(7+5*x))^(1/2)+2506/1109415879*682^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*
x))^(1/2)*EllipticF(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/62*2418^(
1/2))/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 30.73 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{-5+2x}\sqrt{1+4x}\left(1705\sqrt{\frac{7+5x}{-2+3x}}(-671560-2797991x-294854x^2+608600x^3)-984830\sqrt{682}(-2+\cdots\right.}{25516565\cdots}$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x
]
```

output
```
(-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-671560
 - 2797991*x - 294854*x^2 + 608600*x^3) - 984830*Sqrt[682]*(-2 + 3*x)*(7 +
 5*x)^2*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 28819*Sqrt[682]*(-2 + 3*x)*(7 + 5*
x)^2*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(25516565217*Sqrt[2 - 3*x]*(7 + 5*x)^
(3/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {190, 27, 2102, 2105, 27, 188, 27, 194, 27, 320, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{5/2}}dx$$
$$\Big\downarrow \; 190$$
$$\frac{\int \frac{14(852-305x)}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{14\int \frac{852-305x}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^{3/2}}dx}{83421}-\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$$\Big\downarrow \; 2102$$

$$\frac{14\left(\frac{\int \frac{-1534800x^2+1163890x+2941427}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx}{27807}-\frac{63950\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$$\Big\downarrow \; 2105$$

$$\frac{14\left(\frac{5486910\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx-\frac{1}{240}\int -\frac{1077364080}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{25580\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\frac{63950\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}-$$
$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{14\left(\frac{5486910\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+4489017\int \frac{1}{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{25580\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\frac{63950\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{27807\sqrt{5x+7}}\right)}{83421}$$
$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$$\Big\downarrow \; 188$$

$$\frac{14\left(\frac{5486910\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{4489017\sqrt{\frac{2}{253}}\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{\sqrt{46}}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{25580\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-63950\ldots\right.}{83421}$$
$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{14\left(\frac{5486910\int \frac{\sqrt{2-3x}}{(2x-5)^{3/2}\sqrt{4x+1}\sqrt{5x+7}}dx+\frac{8978034\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}+\frac{25580\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\frac{63950\sqrt{2-3}}{2780}\right.}{83421}$$

$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$\downarrow$ 194

$$\frac{14\left(\frac{\frac{8978034\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{498810\sqrt{\frac{11}{23}}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{23}\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{25580\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\right.}{83421}$$

$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$\downarrow$ 27

$$\frac{14\left(\frac{\frac{8978034\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\int \frac{1}{\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{31(4x+1)}{2-3x}+23}}d\frac{\sqrt{4x+1}}{\sqrt{2-3x}}}{\sqrt{11}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}}-\frac{498810\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int \frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{25580\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{\sqrt{2x-5}}}{27807}-\right.}{83421}$$

$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$\downarrow$ 320

$$\frac{14\left(\frac{-\frac{498810\sqrt{11}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}\int\frac{\sqrt{\frac{4x+1}{2x-5}+1}}{\sqrt{23-\frac{39(4x+1)}{2x-5}}}d\frac{\sqrt{4x+1}}{\sqrt{2x-5}}}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{8978034\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+\frac{25580\sqrt{2-3x}\sqrt{}}{\sqrt{2x}}}{27807}\right.}{83421}$$

$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

$\Big\downarrow$ 327

$$\frac{14\left(\frac{-\frac{12790\sqrt{429}\sqrt{2-3x}\sqrt{\frac{5x+7}{5-2x}}E\left(\arcsin\left(\frac{\sqrt{\frac{39}{23}}\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{\sqrt{\frac{2-3x}{5-2x}}\sqrt{5x+7}}+\frac{8978034\sqrt{\frac{5-2x}{2-3x}}\sqrt{5x+7}\sqrt{\frac{31(4x+1)}{2-3x}+23}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{2-3x}}\sqrt{\frac{4x+1}{2-3x}+2}\sqrt{\frac{\frac{31(4x+1)}{2-3x}+23}{\frac{4x+1}{2-3x}+2}}}+25580\sqrt{2}}{27807}\right.}{83421}$$

$$\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

output
```
(-50*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) +
 (14*((-63950*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*Sqrt[7 +
5*x]) + ((25580*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]
- (12790*Sqrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSi
n[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(Sqrt[(2 - 3*x)/(5
 - 2*x)]*Sqrt[7 + 5*x]) + (8978034*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[7 + 5*x]
*Sqrt[23 + (31*(1 + 4*x))/(2 - 3*x)]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(2 -
 3*x)]*Sqrt[2 + (1 + 4*x)/(2 - 3*x)]*Sqrt[(23 + (31*(1 + 4*x))/(2 - 3*x))/
(2 + (1 + 4*x)/(2 - 3*x))]))/27807))/83421
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 190
```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b^2*(a + b*x)^(m + 1)*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g -
a*h))), x] - Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[(
(a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a^2*d
*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g
 + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h
))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.03

method	result
elliptic	$\frac{\sqrt{-(7+5x)(-2+3x)(-5+2x)(1+4x)}\left(\frac{82359956\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}\left(-\frac{2}{3}+x\right)^{2}\sqrt{806}\sqrt{\frac{x-\frac{5}{2}}{-\frac{2}{3}+x}}\sqrt{2139}\sqrt{\frac{x+\frac{1}{4}}{-\frac{2}{3}+x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{3795\left(x+\frac{7}{5}\right)}{-\frac{2}{3}+x}}}{69},\frac{i\sqrt{}}{}\right)}{709539128989119\sqrt{-30\left(x+\frac{7}{5}\right)\left(-\frac{2}{3}+x\right)\left(x-\frac{5}{2}\right)\left(x+\frac{1}{4}\right)}}\right.}{}$
default	$-\frac{2\left(72514890\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\sqrt{3}\sqrt{\frac{-5+2x}{-2+3x}}\sqrt{23}\sqrt{\frac{1+4x}{-2+3x}}\operatorname{EllipticF}\left(\frac{\sqrt{-\frac{253(7+5x)}{-2+3x}}}{23},\frac{i\sqrt{897}}{39}\right)x^{3}-44317350\sqrt{-\frac{253(7+5x)}{-2+3x}}\sqrt{13}\right.}{}$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
(-(7+5*x)*(-2+3*x)*(-5+2*x)*(1+4*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1
+4*x)^(1/2)/(7+5*x)^(1/2)*(82359956/709539128989119*(-3795*(x+7/5)/(-2/3+x
))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2)*((x+1/4)
/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)*EllipticF(1/
69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(1/2))+2506840/54579932999163
*(-3795*(x+7/5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1
/2)*2139^(1/2)*((x+1/4)/(-2/3+x))^(1/2)/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1
/4))^(1/2)*(2/3*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),1/39*I*897^(
1/2))-31/15*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2),-69/55,1/39*I*8
97^(1/2)))-3581200/773229249*((x+7/5)*(x-5/2)*(x+1/4)-1/80730*(-3795*(x+7/
5)/(-2/3+x))^(1/2)*(-2/3+x)^2*806^(1/2)*((x-5/2)/(-2/3+x))^(1/2)*2139^(1/2
)*((x+1/4)/(-2/3+x))^(1/2)*(181/341*EllipticF(1/69*(-3795*(x+7/5)/(-2/3+x)
)^(1/2),1/39*I*897^(1/2))-117/62*EllipticE(1/69*(-3795*(x+7/5)/(-2/3+x))^(
1/2),1/39*I*897^(1/2))+91/55*EllipticPi(1/69*(-3795*(x+7/5)/(-2/3+x))^(1/2
),-69/55,1/39*I*897^(1/2))))/(-30*(x+7/5)*(-2/3+x)*(x-5/2)*(x+1/4))^(1/2)-
2/83421*(-120*x^4+182*x^3+385*x^2-197*x-70)^(1/2)/(x+7/5)^2-179060/2319687
747*(-120*x^3+350*x^2-105*x-50)/((x+7/5)*(-120*x^3+350*x^2-105*x-50))^(1/2
))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x, al
gorithm="fricas")
```

output
```
integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x
^6 + 3850*x^5 - 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2)/(7+5*x)**(5/2),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x, al
gorithm="maxima")
```

output
```
integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)),
x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

input
```
integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x, al
gorithm="giac")
```

output
```
integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = \int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{5/2}}\,dx$$

input
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

output
```
int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x
)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\left(\int \frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{7+5x}\,\sqrt{-3x+2}}{3000x^6+3850x^5-16485x^4-30943x^3-3325x^2+14553x+3430}dx\right)$$

input
```
int(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)/(7+5*x)^(5/2),x)
```

output
```
 - int((sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)*sqrt( - 3*x + 2))/(3000*
x**6 + 3850*x**5 - 16485*x**4 - 30943*x**3 - 3325*x**2 + 14553*x + 3430),x
)
```

3.112 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 982

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}}$$

$$- \frac{b\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\,E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|-\frac{(de-cf)(bg-ah)}{(bc-ad)(fg-eh)}\right)}{dfh\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

$$+ \frac{b(be-af)\sqrt{bg-ah}(dfg+deh-2cfh)\sqrt{c+dx}\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right), -\frac{(de-c}{(bc-a}\right.}{d(bc-ad)f^2h\sqrt{fg-eh}\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

$$- \frac{2(bc-ad)\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)}, \arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\right.}{d^2\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

$$+ \frac{\sqrt{bg-ah}((bc+ad)fh-bd(fg+eh))\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}(e+fx)\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\,\text{EllipticPi}\left(\frac{f(bg-ah)}{b(fg-eh)}, \arcsin\left(\right.\right.}{df^2h\sqrt{fg-eh}\sqrt{c+dx}\sqrt{g+hx}}$$

output
```
b*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(h*x+g)^(1/2)/d/h/(f*x+e)^(1/2)-b*(-a*h+b*g)
^(1/2)*(-e*h+f*g)^(1/2)*(d*x+c)^(1/2)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+
e))^(1/2)*EllipticE((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e
)^(1/2),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/d/f/h/((-a*f
+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/2)+b*(-a*f+b*e)*(-a*h+b
*g)^(1/2)*(-2*c*f*h+d*e*h+d*f*g)*(d*x+c)^(1/2)*((-a*f+b*e)*(h*x+g)/(-a*h+b
*g)/(f*x+e))^(1/2)*EllipticF((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/
2)/(f*x+e)^(1/2),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/d/(
-a*d+b*c)/f^2/h/(-e*h+f*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^(
1/2)/(h*x+g)^(1/2)-2*(-a*d+b*c)*(-c*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x
+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/
2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2
),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^
(1/2))/d^2/(-a*f+b*e)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)+(-a*h+b*g)^(1/2)*(
(a*d+b*c)*f*h-b*d*(e*h+f*g))*((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^(1/2)
*(f*x+e)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2)*EllipticPi((-e*h+f*
g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/b/(-e*h
+f*g),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/d/f^2/h/(-e*h+
f*g)^(1/2)/(d*x+c)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7319 vs. $2(982) = 1964$.

Time = 30.28 (sec) , antiderivative size = 7319, normalized size of antiderivative = 7.45

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 958, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {184, 183, 191, 183, 188, 194, 321, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{184}$$

$$\frac{b\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{d}$$

$$\Big\downarrow \text{183}$$

$$\frac{b\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}dx}{d}-\frac{2(a+bx)(bc-ad)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{191}$$

$$\frac{b\left(\frac{(de-cf)(-2afh+beh+bfg)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2f^2h}+\frac{(adfh-b(-cfh+deh+dfg))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}dx}{2f^2h}-\frac{(de-cf)(fg-eh)\int}{}\right.}{d}-\frac{2(a+bx)(bc-ad)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{183}$$

$$\frac{b\left(\frac{(de-cf)(-2afh+beh+bfg)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2f^2h}+\frac{(e+fx)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}(adfh-b(-cfh+deh+dfg))\int \frac{}{\left(h-\right.}}{f^2h\sqrt{a+bx}\sqrt{c+dx}}\right.}{d}-\frac{2(a+bx)(bc-ad)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\downarrow 188$$

$$b\left(-\frac{(de-cf)(fg-eh)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}dx}{2fh}+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+}{(fg-eh)(a+}}}}{f^2h(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}\right.$$

$$\frac{2(bc-ad)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\downarrow 194$$

$$b\left(-\frac{(fg-eh)\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\int \frac{\sqrt{1-\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}}{\sqrt{1-\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\int \frac{}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}}}{f^2h(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}\right.$$

$$\frac{2(bc-ad)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\downarrow 321$$

$$b\left(\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{f^2h\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}-\frac{(fg-eh)\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+h}{(dg-ch)(e+f}}}{fh\sqrt{-\frac{(de-cf}{(bc-ad}}}\right.$$

$$\frac{2(bc-ad)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\downarrow 327$$

$$b\left(-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+}}{f^2h\sqrt{bg-ah}\sqrt{fg-}}\right.$$

$$\frac{2(bc-ad)(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{d\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{412}$$

$$b\left(-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2afh)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+}}{f^2h\sqrt{bg-ah}\sqrt{fg-}}\right.$$

$$\frac{2\sqrt{bc-ad}\sqrt{ch-dg}(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)}{(bc-ad)(}\right.}{dh\sqrt{c+dx}\sqrt{e+fx}}$$

input

```
Int[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output

```
(b*((Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[
d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*
g - c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqr
t[d*g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g
 - e*h))]])/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sq
rt[g + h*x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
 - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f
*g - e*h))/((d*e - c*f)*(b*g - a*h)))]])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g -
e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))
]) + (Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e
*h)*(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g
 - c*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h),
ArcSin[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], (
(d*e - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))]])/(f^2*Sqrt[b*e - a*f]*
h^2*Sqrt[a + b*x]*Sqrt[c + d*x])))/d - (2*Sqrt[b*c - a*d]*Sqrt[-(d*g) + c*
h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((
b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h)
)/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) +
c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h...
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 184
```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[b/d   Int[Sqrt[a + b*x]*(Sqrt
[c + d*x]/(Sqrt[e + f*x]*Sqrt[g + h*x])), x], x] - Simp[(b*c - a*d)/d   Int
[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 191
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[Sqrt[a + b*x]*Sqrt[c + d*x]*(
Sqrt[g + h*x]/(h*Sqrt[e + f*x])), x] + (-Simp[(d*e - c*f)*((f*g - e*h)/(2*f
*h))   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x]), x],
 x] + Simp[(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h)   Int[Sqrt[e + f
*x]/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x] + Simp[(d*e - c*f)*
((b*f*g + b*e*h - 2*a*f*h)/(2*f^2*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [A] (verified)

Time = 14.02 (sec) , antiderivative size = 1541, normalized size of antiderivative = 1.57

method	result	size
elliptic	Expression too large to display	1541
default	Expression too large to display	17351

input
```
int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*a^2*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/
d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g
/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h
)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(
x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+4*a*b*(e/
f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*
(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^
(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)
*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*
(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+
g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/
(a/b-g/h)/(-c/d+e/f))^(1/2)))+b^2*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d
-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/
b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)*((c/d*
g/h-e/f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c/d+g/h)*EllipticF(((c/d-e/f)*(x+
g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))
^(1/2))+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(
(-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))/(-c/d+g/h)+(a*d*f*h+b*c*
f*h+b*d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*EllipticPi(((c/d-e/f)*(x+g/h)/(-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.113 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 229

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg}{(be-af)(dg}\right.}{d\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*(-c*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*
(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)
*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((
-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d/(-a*f+b*e)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 583 vs. 2(229) = 458.

Time = 30.44 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.55

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{\frac{(dg-ch)(a+bx)}{(bg-ah)(c+dx)}}(c+dx)^{3/2}\left(\frac{ad\sqrt{\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}(g+hx)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right),\frac{(bc-ad)(-fg+eh)}{(de-cf)(bg-ah)}\right)}{(dg-ch)(c+dx)\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}}+\frac{bc\sqrt{\frac{(dg-c}{(fg-e}}}{}\right.}{}$$

input

```
Integrate[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output

```
(-2*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]*(c + d*x)^(3/2)*
((a*d*Sqrt[((d*g - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*Elli
pticF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((
b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((d*g - c*h)*(c + d
*x)*Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*c*Sqrt[
((d*g - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*EllipticF[ArcSi
n[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*
(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((-(d*g) + c*h)*(c + d*x)*Sqrt
[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*(f*g - e*h)*Sqr
t[((-(d*e) + c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(c + d*x
)^2)]*EllipticPi[(d*(-(f*g) + e*h))/((d*e - c*f)*h), ArcSin[Sqrt[((-(d*e)
+ c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/
((d*e - c*f)*(b*g - a*h))])/((d*e - c*f)*h)))/(d*Sqrt[a + b*x]*Sqrt[e + f*
x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {183, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$\big\downarrow$ 183

$$\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}$$

$\big\downarrow$ 412

$$\frac{2(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Elliptic
Pi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*
x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c -
 a*d)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(210) = 420$.

Time = 14.11 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.70

method	result
elliptic	$\sqrt{(hx+g)(xd+c)(bx+a)(fx+e)}\left(\frac{2a\left(\frac{e}{f}-\frac{g}{h}\right)\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^{2}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{a}{b}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)(}{\left(-\frac{e}{f}+\frac{g}{h}\right)}}\right.}{\left(\frac{c}{d}-\frac{e}{f}\right)\left(-\frac{c}{d}+\frac{g}{h}\right)\sqrt{hdbf\left(x+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}}\right.$
default	Expression too large to display

input
```
int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*a*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d)
)^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h
)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*
(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+
c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*b*(e/f-g/
h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a
/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2
)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c
/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)
/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b
-g/h)/(-c/d+e/f))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algor
ithm="giac")
```

output
```
integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.114 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 161

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{\frac{(fg-eh)(c+dx)}{(dg-ch)(e+fx)}}\sqrt{e+fx}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{fg-eh}\sqrt{c+dx}}$$

output
```
-2*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2)*(f*x+e)^(1/2)*InverseJaco
biAM(arctan((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2))
,((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*f+b*e)^(1/2)/(-e
*h+f*g)^(1/2)/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 22.91 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-bc+ad)(-fg+eh)}{(be-af)(dg-ch)}\right)}{(bg-ah)\sqrt{c+dx}\sqrt{e+fx}\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}$$

input
```
Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sq
rt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Sqrt[g + h*x]*Elliptic
F[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*
c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x
))])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = $\{188, 321\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{188}$$

$$\frac{2\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

input
```
Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Ell
ipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*
x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[b*g -
 a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
 e*h)*(a + b*x)))])
```

Defintions of rubi rules used

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Maple [A] (verified)

Time = 22.82 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.70

method	result
default	$-\frac{2\sqrt{-\frac{(cf-de)(hx+g)}{(eh-fg)(xd+c)}}\,\sqrt{\frac{(ch-dg)(bx+a)}{(ah-bg)(xd+c)}}\,\sqrt{\frac{(ch-dg)(fx+e)}{(eh-fg)(xd+c)}}\,\operatorname{EllipticF}\left(\sqrt{-\frac{(cf-de)(hx+g)}{(eh-fg)(xd+c)}},\sqrt{-\frac{(ad-bc)(eh-fg)}{(ah-bg)(cf-de)}}\right)\left(d^2eh\,x^2-d^2fg\,x^2+2\cdots\right.}{\sqrt{bx+a}\,\sqrt{xd+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,(cf-de)(ch-dg)}$
elliptic	$\frac{2\sqrt{(hx+g)(xd+c)(bx+a)(fx+e)}\left(\frac{e}{f}-\frac{g}{h}\right)\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{a}{b}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\,\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\cdots}{\left(-\frac{e}{f}+\frac{g}{h}\right)\cdots}}\right.}{\sqrt{hx+g}\,\sqrt{xd+c}\,\sqrt{bx+a}\,\sqrt{fx+e}\left(\frac{c}{d}-\frac{e}{f}\right)\left(-\frac{c}{d}+\frac{g}{h}\right)\sqrt{hdbf\left(x+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}}$

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
-2/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(-(c*f-d*e)*(h*
x+g)/(e*h-f*g)/(d*x+c))^(1/2)*((c*h-d*g)*(b*x+a)/(a*h-b*g)/(d*x+c))^(1/2)*
((c*h-d*g)*(f*x+e)/(e*h-f*g)/(d*x+c))^(1/2)*EllipticF((-(c*f-d*e)*(h*x+g)/
(e*h-f*g)/(d*x+c))^(1/2),(-(a*d-b*c)*(e*h-f*g)/(a*h-b*g)/(c*f-d*e))^(1/2))
*(d^2*e*h*x^2-d^2*f*g*x^2+2*c*d*e*h*x-2*c*d*f*g*x+c^2*e*h-c^2*f*g)/(c*f-d*
e)/(c*h-d*g)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*d*f*h*
x^4 + a*c*e*g + (b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^3 + ((b*d*e + (b*c
 + a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^2 + (a*c*e*h + (a*c*f + (b*c +
 a*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.115 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 437

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2b(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{(bc-ad)\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2h\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

output
```
-2*b*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/
2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2)
,((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e
)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a
))^(1/2)/(h*x+g)^(1/2)-2*h*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(
b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b
*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*f+b*e
)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a
))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 37.69 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(-b^2+\frac{bd(fg-eh)(a+bx)\sqrt{-\frac{(de-cf)(dg\cdots}{(fg-\cdots}}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*(-b^2 + (b*d*(f*g - e*h)*(a +
 b*x)*Sqrt[-(((d*e - c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*
(c + d*x)^2))] - b*(d*e - c*f)*(b*g - a*h)*Sqrt[((d*g - c*h)*(a + b*x))/((
b*g - a*h)*(c + d*x))]*EllipticE[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((
f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g -
a*h))] + d*(b*e - a*f)*(b*g - a*h)*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*
h)*(c + d*x))]*EllipticF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*
h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/
((f*g - e*h)*(c + d*x)*Sqrt[((-(d*e) + c*f)*(d*g - c*h)*(e + f*x)*(g + h*x
))/((f*g - e*h)^2*(c + d*x)^2)])))/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sq
rt[a + b*x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {189, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{189}$$

$$\frac{b\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{bc-ad}-\frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{bc-ad}$$

$$\Big\downarrow \text{188}$$

$$\frac{b\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{bc-ad}-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(bc-ad)(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{194}$$

$$-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(bc-ad)(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\int \frac{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}}{\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{g+hx}(bc-ad)(be-af)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{2b\sqrt{c+dx}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\int \frac{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}}{\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{g+hx}(bc-ad)(be-af)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}(bc-ad)\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

input
```
Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
 e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[
f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g
- a*h)))])/((b*c - a*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c +
 d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (2*d*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
 - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f
*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[
f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a +
b*x)))])
```

Defintions of rubi rules used

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 189
```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-d/(b*c - a*d)   Int[1/(Sqr
t[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[b/(b*c
 - a*d)   Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2199 vs. $2(399) = 798$.

Time = 25.74 (sec) , antiderivative size = 2200, normalized size of antiderivative = 5.03

method	result	size
elliptic	Expression too large to display	2200
default	Expression too large to display	8459

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)*b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*
h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/((x+a/b)*(b*d
*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x
+b*c*e*g))^(1/2)+2*((a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2
*c*f*g+b^2*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e
*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b*c*f*g+b*d*e*g)*b/(a^3*d*f
*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g
-b^3*c*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d
)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/
f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b
)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((
-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(-b*(a*d*f*h-b*c*f*h-b*
d*e*h-b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+
a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)*b/(a^3*
d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*
e*g-b^3*c*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+
c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(
-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*...
```

Fricas [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d*f*
h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)*h)*x^4 +
((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2
*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g + (a^2*c*f
+ (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c + a^2*d)*
e)*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x
)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x
)
```

Giac [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.116 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 580

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2b^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}\sqrt{c+dx}} - \frac{2(a^2d^2fh-abd^2(fg+eh)+b^2(2d^2eg+c^2fh-cd(fg+eh)))\sqrt{e+fx}\sqrt{\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right),\frac{(be-af)(dg-ch)}{(de-cf)(bg-ah)}\right)}{(bc-ad)^2(be-af)^{3/2}\sqrt{de-cf}(dg-ch)\sqrt{\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}\sqrt{g+hx}} - \frac{2(adfh+b(dfg-deh-cfh))\sqrt{e+fx}\sqrt{\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{c+dx}}\right),\frac{(be-af)(dg-ch)}{(de-cf)(bg-ah)}\right)}{(bc-ad)(be-af)^{3/2}\sqrt{de-cf}(dg-ch)\sqrt{\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

output
```
-2*b^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a
)^(1/2)/(d*x+c)^(1/2)-2*(a^2*d^2*f*h-a*b*d^2*(e*h+f*g)+b^2*(2*d^2*e*g+c^2*
f*h-c*d*(e*h+f*g)))*(f*x+e)^(1/2)*((-a*d+b*c)*(h*x+g)/(-a*h+b*g)/(d*x+c))^
(1/2)*EllipticE((-c*f+d*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1
/2),((-a*f+b*e)*(-c*h+d*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))/(-a*d+b*c)^2/(-a*
f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-c*h+d*g)/((-a*d+b*c)*(f*x+e)/(-a*f+b*e)/(d
*x+c))^(1/2)/(h*x+g)^(1/2)-2*(a*d*f*h+b*(-c*f*h-d*e*h+d*f*g))*(f*x+e)^(1/2
)*((-a*d+b*c)*(h*x+g)/(-a*h+b*g)/(d*x+c))^(1/2)*EllipticF((-c*f+d*e)^(1/2)
*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(d*x+c)^(1/2),((-a*f+b*e)*(-c*h+d*g)/(-c*f
+d*e)/(-a*h+b*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-c*
h+d*g)/((-a*d+b*c)*(f*x+e)/(-a*f+b*e)/(d*x+c))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 33.45 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\sqrt{c+dx}\left(-b\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)(g+hx)\left(a^3d^3fh-\right.\right.}{}$$

input
```
Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(2*Sqrt[c + d*x]*(-(b*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))
]*(e + f*x)*(g + h*x)*(a^3*d^3*f*h - a*b^2*d^3*(-(e*g) + f*g*x + e*h*x) -
a^2*b*d^3*(e*h + f*(g - h*x)) + b^3*(c^3*f*h + 2*d^3*e*g*x + c*d^2*(e*g -
f*g*x - e*h*x) - c^2*d*(f*g + e*h - f*h*x)))) + (c + d*x)*(b^2*(a^2*d^2*f*
h - a*b*d^2*(f*g + e*h) + b^2*(2*d^2*e*g + c^2*f*h - c*d*(f*g + e*h)))*Sqr
t[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e + f*x)*(g + h*x) + b
*(f*g - e*h)*(a + b*x)*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x)
)/((f*g - e*h)^2*(a + b*x)^2))]*((a^2*d^2*f*h - a*b*d^2*(f*g + e*h) + b^2*
(2*d^2*e*g + c^2*f*h - c*d*(f*g + e*h)))*EllipticE[ArcSin[Sqrt[((-(b*e) +
a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h))/((b*e
 - a*f)*(d*g - c*h))] - 2*b*d*(d*e - c*f)*(b*g - a*h)*EllipticF[ArcSin[Sqr
t[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g
- e*h))/((b*e - a*f)*(d*g - c*h))]))))/(b*(b*c - a*d)^2*(b*e - a*f)*(d*e -
 c*f)*(d*g - c*h)^2*(a + b*x)^(3/2)*(((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x)))^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx$$
$$\Big\downarrow \text{ 200}$$
$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx$$

input
```
Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 200
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_] :> CannotIntegrate[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m,
n, p, q}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7102 vs. $2(534) = 1068$.

Time = 28.74 (sec) , antiderivative size = 7103, normalized size of antiderivative = 12.25

method	result	size
elliptic	Expression too large to display	7103
default	Expression too large to display	18165

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d^2*
f*h*x^6 + a^2*c^2*e*g + (b^2*d^2*f*g + (b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*
f)*h)*x^5 + ((b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)*g + (2*(b^2*c*d + a*b*d
^2)*e + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*h)*x^4 + ((2*(b^2*c*d + a*b*d^2
)*e + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*g + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*e + 2*(a*b*c^2 + a^2*c*d)*f)*h)*x^3 + (((b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*e + 2*(a*b*c^2 + a^2*c*d)*f)*g + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*h
)*x^2 + (a^2*c^2*e*h + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\,\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)*sqrt(e + f*x)*sqrt(g + h*x))
, x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

Giac [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)), x
)
```

Reduce [F]

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{3}{2}}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.117 $\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 227

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = -\frac{d^2(adfh+b(dfg+deh-3cfh))(a+bx)^{1+m}}{b^2f^2h^2(1+m)}+\frac{d^3(a+bx)^{2+m}}{b^2fh(2+m)} - \frac{(de-cf)^3(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{f^2(be-af)(fg-eh)(1+m)} + \frac{(dg-ch)^3(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{h^2(bg-ah)(fg-eh)(1+m)}$$

output
```
-d^2*(a*d*f*h+b*(-3*c*f*h+d*e*h+d*f*g))*(b*x+a)^(1+m)/b^2/f^2/h^2/(1+m)+d^
3*(b*x+a)^(2+m)/b^2/f/h/(2+m)-(-c*f+d*e)^3*(b*x+a)^(1+m)*hypergeom([1, 1+m
],[2+m],-f*(b*x+a)/(-a*f+b*e))/f^2/(-a*f+b*e)/(-e*h+f*g)/(1+m)+(-c*h+d*g)^
3*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+
b*g)/(-e*h+f*g)/(1+m)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 473 vs. $2(227) = 454$.

Time = 0.78 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx$$
$$= \frac{(a+bx)^{1+m}\left(-\left((bc-ad)(3+m)\left((bc-ad)(2+m)\left((de-cf)(bg-ah)\operatorname{Hypergeometric2F1}\left(1,1+\right.\right.\right.\right.\right.}{}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^3)/((e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*(-((b*c - a*d)*(3 + m)*((b*c - a*d)*(2 + m)*((d*e - c*f
)*(b*g - a*h)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a
*f)] - (b*e - a*f)*(d*g - c*h)*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a +
b*x))/(-(b*g) + a*h)]) + d*(1 + m)*(a + b*x)*((d*e - c*f)*(b*g - a*h)*Hype
rgeometric2F1[1, 2 + m, 3 + m, (f*(a + b*x))/(-(b*e) + a*f)] - (b*e - a*f)
*(d*g - c*h)*Hypergeometric2F1[1, 2 + m, 3 + m, (h*(a + b*x))/(-(b*g) + a*
h)]))) - d*(1 + m)*(a + b*x)*((b*c - a*d)*(3 + m)*((d*e - c*f)*(b*g - a*h)
*Hypergeometric2F1[1, 2 + m, 3 + m, (f*(a + b*x))/(-(b*e) + a*f)] - (b*e -
 a*f)*(d*g - c*h)*Hypergeometric2F1[1, 2 + m, 3 + m, (h*(a + b*x))/(-(b*g)
 + a*h)]) + d*(2 + m)*(a + b*x)*((d*e - c*f)*(b*g - a*h)*Hypergeometric2F1
[1, 3 + m, 4 + m, (f*(a + b*x))/(-(b*e) + a*f)] - (b*e - a*f)*(d*g - c*h)*
Hypergeometric2F1[1, 3 + m, 4 + m, (h*(a + b*x))/(-(b*g) + a*h)]))))/(b^2*
(b*e - a*f)*(b*g - a*h)*(f*g - e*h)*(1 + m)*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3(a+bx)^m}{(e+fx)(g+hx)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(-\frac{d^2(a+bx)^m(-3cfh+deh+dfg)}{f^2h^2}+\frac{(a+bx)^m(cf-de)^3}{f^2(e+fx)(fg-eh)}+\frac{(a+bx)^m(ch-dg)^3}{h^2(g+hx)(eh-fg)}+\frac{d^3x(a+bx)^m}{fh}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ad^3(a+bx)^{m+1}}{b^2fh(m+1)}+\frac{d^3(a+bx)^{m+2}}{b^2fh(m+2)}-\frac{d^2(a+bx)^{m+1}(-3cfh+deh+dfg)}{bf^2h^2(m+1)}-\frac{(a+bx)^{m+1}(de-cf)^3\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{f^2(m+1)(be-af)(fg-eh)}+\frac{(a+bx)^{m+1}(dg-ch)^3\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)(fg-eh)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^3)/((e + f*x)*(g + h*x)),x]
```

output
```
-((a*d^3*(a + b*x)^(1 + m))/(b^2*f*h*(1 + m))) - (d^2*(d*f*g + d*e*h - 3*c
*f*h)*(a + b*x)^(1 + m))/(b*f^2*h^2*(1 + m)) + (d^3*(a + b*x)^(2 + m))/(b^
2*f*h*(2 + m)) - ((d*e - c*f)^3*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 +
 m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])/(f^2*(b*e - a*f)*(f*g - e*h)*(1
+ m)) + ((d*g - c*h)^3*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((h*(a + b*x))/(b*g - a*h))])/(h^2*(b*g - a*h)*(f*g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^3}{(fx+e)(hx+g)}dx$$

input
```
int((b*x+a)^m*(d*x+c)^3/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^3/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^3(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^3/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x + a)^m/(f*h*x^2 +
e*g + (f*g + e*h)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**3/(f*x+e)/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^3(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^3/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^3(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^3/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m\,(c+d\,x)^3}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^m*(c + d*x)^3)/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^m*(c + d*x)^3)/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^3}{(e+fx)(g+hx)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^3/(f*x+e)/(h*x+g),x)
```

output
```
( - (a + b*x)**m*a**2*d**3*e*f*h**2*m - (a + b*x)**m*a**2*d**3*f**2*g*h*m
+ 3*(a + b*x)**m*a*b*c**2*d*f**2*h**2*m**2 + 9*(a + b*x)**m*a*b*c**2*d*f**
2*h**2*m + 6*(a + b*x)**m*a*b*c**2*d*f**2*h**2 - 3*(a + b*x)**m*a*b*c*d**2
*e*f*h**2*m - 6*(a + b*x)**m*a*b*c*d**2*e*f*h**2 - 3*(a + b*x)**m*a*b*c*d*
*2*f**2*g*h*m - 6*(a + b*x)**m*a*b*c*d**2*f**2*g*h + (a + b*x)**m*a*b*d**3
*e**2*h**2*m + 2*(a + b*x)**m*a*b*d**3*e**2*h**2 - (a + b*x)**m*a*b*d**3*e
*f*g*h*m**2 - (a + b*x)**m*a*b*d**3*e*f*g*h*m + 2*(a + b*x)**m*a*b*d**3*e*
f*g*h + (a + b*x)**m*a*b*d**3*e*f*h**2*m**2*x + (a + b*x)**m*a*b*d**3*f**2
*g**2*m + 2*(a + b*x)**m*a*b*d**3*f**2*g**2 + (a + b*x)**m*a*b*d**3*f**2*g
*h*m**2*x + (a + b*x)**m*b**2*c**3*f**2*h**2*m**2 + 3*(a + b*x)**m*b**2*c*
*3*f**2*h**2*m + 2*(a + b*x)**m*b**2*c**3*f**2*h**2 - 3*(a + b*x)**m*b**2*
c*d**2*e*f*g*h*m**2 - 9*(a + b*x)**m*b**2*c*d**2*e*f*g*h*m - 6*(a + b*x)**
m*b**2*c*d**2*e*f*g*h + 3*(a + b*x)**m*b**2*c*d**2*e*f*h**2*m**2*x + 6*(a
+ b*x)**m*b**2*c*d**2*e*f*h**2*m*x + 3*(a + b*x)**m*b**2*c*d**2*f**2*g*h*m
**2*x + 6*(a + b*x)**m*b**2*c*d**2*f**2*g*h*m*x + (a + b*x)**m*b**2*d**3*e
**2*g*h*m**2 + 3*(a + b*x)**m*b**2*d**3*e**2*g*h*m + 2*(a + b*x)**m*b**2*d
**3*e**2*g*h - (a + b*x)**m*b**2*d**3*e**2*h**2*m**2*x - 2*(a + b*x)**m*b*
*2*d**3*e**2*h**2*m*x + (a + b*x)**m*b**2*d**3*e*f*g**2*m**2 + 3*(a + b*x)
**m*b**2*d**3*e*f*g**2*m + 2*(a + b*x)**m*b**2*d**3*e*f*g**2 - 2*(a + b*x)
**m*b**2*d**3*e*f*g*h*m**2*x - 4*(a + b*x)**m*b**2*d**3*e*f*g*h*m*x + (...
```

3.118 $\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 177

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \frac{d^2(a+bx)^{1+m}}{bfh(1+m)} + \frac{(de-cf)^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{f(be-af)(fg-eh)(1+m)} - \frac{(dg-ch)^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{h(bg-ah)(fg-eh)(1+m)}$$

output

```
d^2*(b*x+a)^(1+m)/b/f/h/(1+m)+(-c*f+d*e)^2*(b*x+a)^(1+m)*hypergeom([1, 1+m
],[2+m],-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b*e)/(-e*h+f*g)/(1+m)-(-c*h+d*g)^2*
(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/h/(-a*h+b*g)
/(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx$$
$$= \frac{(a+bx)^{1+m}\left(-\left((bc-ad)(2+m)\left((de-cf)(bg-ah)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)\right.\right.\right.}{}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^2)/((e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*(-((b*c - a*d)*(2 + m)*((d*e - c*f)*(b*g - a*h)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a*f)] - (b*e - a*f)*(d
*g - c*h)*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)]
)) - d*(1 + m)*(a + b*x)*((d*e - c*f)*(b*g - a*h)*Hypergeometric2F1[1, 2 +
 m, 3 + m, (f*(a + b*x))/(-(b*e) + a*f)] - (b*e - a*f)*(d*g - c*h)*Hyperge
ometric2F1[1, 2 + m, 3 + m, (h*(a + b*x))/(-(b*g) + a*h)])))/(b*(b*e - a*f
)*(b*g - a*h)*(f*g - e*h)*(1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(a+bx)^m}{(e+fx)(g+hx)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(\frac{(a+bx)^m(cf-de)^2}{f(e+fx)(fg-eh)}+\frac{(a+bx)^m(ch-dg)^2}{h(g+hx)(eh-fg)}+\frac{d^2(a+bx)^m}{fh}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^{m+1}(de-cf)^2\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)(fg-eh)}-\frac{(a+bx)^{m+1}(dg-ch)^2\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{h(m+1)(bg-ah)(fg-eh)}+\frac{d^2(a+bx)^{m+1}}{bfh(m+1)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^2)/((e + f*x)*(g + h*x)),x]
```

output
```
(d^2*(a + b*x)^(1 + m))/(b*f*h*(1 + m)) + ((d*e - c*f)^2*(a + b*x)^(1 + m)
*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e
 - a*f)*(f*g - e*h)*(1 + m)) - ((d*g - c*h)^2*(a + b*x)^(1 + m)*Hypergeome
tric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/(h*(b*g - a*h)*(f*
g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m\,(xd+c)^2}{(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^m*(d*x+c)^2/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^2/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^2(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^2/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h
)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**2/(f*x+e)/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^2(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^2/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)^2(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^2/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m\,(c+d\,x)^2}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^m*(c + d*x)^2)/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^m*(c + d*x)^2)/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^2}{(e+fx)(g+hx)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^2/(f*x+e)/(h*x+g),x)
```

output

```
(2*(a + b*x)**m*a*c*d*f*h*m + 2*(a + b*x)**m*a*c*d*f*h - (a + b*x)**m*a*d*
*2*e*h - (a + b*x)**m*a*d**2*f*g + (a + b*x)**m*b*c**2*f*h*m + (a + b*x)**
m*b*c**2*f*h - (a + b*x)**m*b*d**2*e*g*m - (a + b*x)**m*b*d**2*e*g + (a +
b*x)**m*b*d**2*e*h*m*x + (a + b*x)**m*b*d**2*f*g*m*x + int((a + b*x)**m/(a
*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 +
 a*f**2*g**2*x + a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f
*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g
*h*x**3),x)*a*b*c**2*e**2*f*h**3*m**2 + int((a + b*x)**m/(a*e**2*g*h + a*e
**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x
+ a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e
*f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b
*c**2*e**2*f*h**3*m + 2*int((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e
*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**
2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*
e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*c**2*e*f**2*g*h
**2*m**2 + 2*int((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2
*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 + b*e**2*
g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*h**2*x*
*3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*c**2*e*f**2*g*h**2*m + int
((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x ...
```

3.119 $\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = -\frac{(de-cf)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)} + \frac{(dg-ch)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(fg-eh)(1+m)}$$

output
```
-(-c*f+d*e)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*x+a)/(-a*f+b*e))/
(-a*f+b*e)/(-e*h+f*g)/(1+m)+(-c*h+d*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2
+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{1+m}\left(-\frac{(de-cf)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{be-af}+\frac{(dg-ch)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{h(a+bx)}{-bg+ah}\right)}{bg-ah}\right)}{(fg-eh)(1+m)}$$

input
```
Integrate[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*(-(((d*e - c*f)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(
a + b*x))/(-(b*e) + a*f)])/(b*e - a*f)) + ((d*g - c*h)*Hypergeometric2F1[1
, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/(b*g - a*h)))/((f*g - e*h)*
(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {174, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(a+bx)^m}{(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \ 174$$

$$\frac{(dg-ch)\int \frac{(a+bx)^m}{g+hx}dx}{fg-eh}-\frac{(de-cf)\int \frac{(a+bx)^m}{e+fx}dx}{fg-eh}$$

$$\Big\downarrow \ 78$$

$$\frac{(a+bx)^{m+1}(dg-ch)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

input
```
Int[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

output
```
-(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(
a + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)
*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g
 - a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)}{(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h)*x), x)
```

Sympy [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^m\,(c+dx)}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**m*(c + d*x)/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m\,(c+d\,x)}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output

```
((a + b*x)**m*a*d + (a + b*x)**m*b*c + int((a + b*x)**m/(a*e**2*g*h + a*e*
*2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x +
 a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*
f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*
c*e**2*h**2*m + 2*int((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**
2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 + b*
e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*h*
*2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*c*e*f*g*h*m + int((a
+ b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f
*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*
x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**
2 + b*f**2*g*h*x**3),x)*a*b*c*f**2*g**2*m - int((a + b*x)**m/(a*e**2*g*h +
 a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**
2*x + a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2
*b*e*f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)
*a*b*d*e**2*g*h*m - int((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g
**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 +
b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*
h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*d*e*f*g**2*m - int(
(a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x +...
```

3.120 $\int \frac{(a+bx)^m}{(e+fx)(g+hx)} dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)} dx = \frac{f(a+bx)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)} - \frac{h(a+bx)^{1+m} \text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(fg-eh)(1+m)}$$

output
```
f*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*x+a)/(-a*f+b*e))/(-a*f+b*e)
/(-e*h+f*g)/(1+m)-h*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*
h+b*g))/(-a*h+b*g)/(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{1+m}\left((-bfg+afh)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)+(be-af)h\,\text{Hypergeomet}\right.}{(be-af)(bg-ah)(-fg+eh)(1+m)}$$

input
```
Integrate[(a + b*x)^m/((e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*((-(b*f*g) + a*f*h)*Hypergeometric2F1[1, 1 + m, 2 + m,
(f*(a + b*x))/(-(b*e) + a*f)] + (b*e - a*f)*h*Hypergeometric2F1[1, 1 + m,
2 + m, (h*(a + b*x))/(-(b*g) + a*h)]))/((b*e - a*f)*(b*g - a*h)*(-(f*g) +
e*h)*(1 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {97, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \; 97$$

$$\frac{f\int \frac{(a+bx)^m}{e+fx}dx}{fg-eh}-\frac{h\int \frac{(a+bx)^m}{g+hx}dx}{fg-eh}$$

$$\Big\downarrow \; 78$$

$$\frac{f(a+bx)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)} - \frac{h(a+bx)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}$$

input
```
Int[(a + b*x)^m/((e + f*x)*(g + h*x)),x]
```

output
```
(f*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b
*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m)) - (h*(a + b*x)^(1 + m)*Hype
rgeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)
*(f*g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(bx+a)^m}{(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^m/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx=\int \frac{(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h)*x), x)
```

Sympy [F]

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx=\int \frac{(a+bx)^m}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**m/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**m/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx=\int \frac{(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int((a + b*x)^m/((e + f*x)*(g + h*x)),x)
```

output
```
int((a + b*x)^m/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m}{(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{fh\,x^2+ehx+fgx+eg}dx$$

input
```
int((b*x+a)^m/(f*x+e)/(h*x+g),x)
```

output
```
int((a + b*x)**m/(e*g + e*h*x + f*g*x + f*h*x**2),x)
```

3.121 $\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 224

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{d^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)} - \frac{f^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(de-cf)(fg-eh)(1+m)} + \frac{h^2(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(dg-ch)(fg-eh)(1+m)}$$

output
```
d^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*
c)/(-c*f+d*e)/(-c*h+d*g)/(1+m)-f^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],
-f*(b*x+a)/(-a*f+b*e))/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)/(1+m)+h^2*(b*x+a)^
(1+m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-c*h+d*g
)/(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{1+m}\left(\frac{d^2 \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)(-de+cf)(-dg+ch)}+\frac{f^2 \text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{(be-af)(de-cf)(-fg+eh)}+\frac{h^2 \text{Hypergeometric}}{(bg-ah}\right.}{1+m}$$

input
```
Integrate[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*((d^2*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/
(-(b*c) + a*d)])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) + (f^2*Hyperg
eometric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a*f)])/((b*e - a*f)*(
d*e - c*f)*(-(f*g) + e*h)) + (h^2*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a
 + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(d*g - c*h)*(f*g - e*h))))/(1 + m)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{d^2(a+bx)^m}{(c+dx)(de-cf)(dg-ch)}+\frac{f^2(a+bx)^m}{(e+fx)(de-cf)(eh-fg)}+\frac{h^2(a+bx)^m}{(g+hx)(dg-ch)(fg-eh)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(a+bx)^{m+1}\operatorname{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\operatorname{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\operatorname{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(dg-ch)(fg-eh)}$$

input
```
Int[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
(d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/
(b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b
*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f)
)])/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)
*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g -
 a*h)*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m}{(xd+c)\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx=\int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*x + a)^m/(d*f*h*x^3 + c*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c
*e*h + (d*e + c*f)*g)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx=\int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)}\,dx$$

input
```
int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

output
```
int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \int \frac{(bx+a)^m}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx$$

input
```
int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
int((a + b*x)**m/(c*e*g + c*e*h*x + c*f*g*x + c*f*h*x**2 + d*e*g*x + d*e*h
*x**2 + d*f*g*x**2 + d*f*h*x**3),x)
```

3.122 $\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 330

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \frac{d^2(a+bx)^{1+m}}{(bc-ad)(de-cf)(dg-ch)(c+dx)} + \frac{d^2(ad(dfg+deh-2cfh)-b(cd(fg+eh)(1-m)-c^2fh(2-m)+d^2egm))\,(a+bx)^{1+m}\,\text{Hypergeom}}{(bc-ad)^2(de-cf)^2(dg-ch)^2(1+m)} + \frac{f^3(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(de-cf)^2(fg-eh)(1+m)} - \frac{h^3(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(dg-ch)^2(fg-eh)(1+m)}$$

output

```
d^2*(b*x+a)^(1+m)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)+d^2*(a*d*(-2*c*
f*h+d*e*h+d*f*g)-b*(c*d*(e*h+f*g)*(1-m)-c^2*f*h*(2-m)+d^2*e*g*m))*(b*x+a)^
(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(-c*f+d
*e)^2/(-c*h+d*g)^2/(1+m)+f^3*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*
x+a)/(-a*f+b*e))/(-a*f+b*e)/(-c*f+d*e)^2/(-e*h+f*g)/(1+m)-h^3*(b*x+a)^(1+m
)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-c*h+d*g)^2/
(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \frac{(a+bx)^{1+m}\left(-\frac{d^2(dfg+deh-2cfh)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)(de-cf)^2(dg-ch)^2}+\frac{f^3\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{(be-af)(de-cf)^2(fg-eh)}+\right.}{1+m}$$

input
```
Integrate[(a + b*x)^m/((c + d*x)^2*(e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*(-((d^2*(d*f*g + d*e*h - 2*c*f*h)*Hypergeometric2F1[1,
1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)*(d*e - c*f)^2*(d
*g - c*h)^2)) + (f^3*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b
*e) + a*f)])/((b*e - a*f)*(d*e - c*f)^2*(f*g - e*h)) + (-((h^3*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(f*g -
 e*h))) + (b*d^2*(-(d*g) + c*h)*Hypergeometric2F1[2, 1 + m, 2 + m, (d*(a +
 b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(-(d*e) + c*f)))/(d*g - c*h)^2))/(1
 + m)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \; 198$$

$$\int \left(-\frac{d^2(a+bx)^m(-2cfh+deh+dfg)}{(c+dx)(de-cf)^2(dg-ch)^2}+\frac{d^2(a+bx)^m}{(c+dx)^2(de-cf)(dg-ch)}-\frac{f^3(a+bx)^m}{(e+fx)(de-cf)^2(eh-fg)}-\frac{}{(g+}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+bx)^{m+1}(-2cfh+deh+dfg)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)^2(dg-ch)^2}+\frac{bd^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(2,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^2(de-cf)(dg-ch)}+\frac{f^3(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)^2(fg-eh)}-\frac{h^3(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(dg-ch)^2(fg-eh)}$$

input
```
Int[(a + b*x)^m/((c + d*x)^2*(e + f*x)*(g + h*x)),x]
```

output
```
-((d^2*(d*f*g + d*e*h - 2*c*f*h)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1
+ m, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(d*e - c*f)^2*(d*g
 - c*h)^2*(1 + m))) + (f^3*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, -((f*(a + b*x))/(b*e - a*f))])/((b*e - a*f)*(d*e - c*f)^2*(f*g - e*h
)*(1 + m)) - (h^3*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((
h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)*(d*g - c*h)^2*(f*g - e*h)*(1 + m)
) + (b*d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a +
b*x))/(b*c - a*d))])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*(1 + m))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m}{(xd+c)^2\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^m/(d*x+c)^2/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m/(d*x+c)^2/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)^2(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)^2/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*x + a)^m/(d^2*f*h*x^4 + c^2*e*g + (d^2*f*g + (d^2*e + 2*c*d*f)
*h)*x^3 + ((d^2*e + 2*c*d*f)*g + (2*c*d*e + c^2*f)*h)*x^2 + (c^2*e*h + (2*
c*d*e + c^2*f)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m/(d*x+c)**2/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)^2(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)^2/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m/((d*x + c)^2*(f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)^2(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)^2/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m/((d*x + c)^2*(f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)^2}\,dx$$

input
```
int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)^2),x)
```

output
```
int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)(g+hx)}\,dx$$
$$=\int \frac{(bx+a)^m}{d^2fh\,x^4+2cdfh\,x^3+d^2eh\,x^3+d^2fg\,x^3+c^2fh\,x^2+2cdeh\,x^2+2cdfg\,x^2+d^2eg\,x^2+c^2ehx+c^2fgx$$

input
```
int((b*x+a)^m/(d*x+c)^2/(f*x+e)/(h*x+g),x)
```

output
```
int((a + b*x)**m/(c**2*e*g + c**2*e*h*x + c**2*f*g*x + c**2*f*h*x**2 + 2*c
*d*e*g*x + 2*c*d*e*h*x**2 + 2*c*d*f*g*x**2 + 2*c*d*f*h*x**3 + d**2*e*g*x**
2 + d**2*e*h*x**3 + d**2*f*g*x**3 + d**2*f*h*x**4),x)
```

3.123 $\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 268

$$\begin{aligned}\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx &= \frac{(a^2d^2f^2+abdf(de+cf)+b^2(d^2e^2+cdef+c^2f^2))\,(e+fx)^{1+n}}{b^3d^3f^3(1+n)} \\ &\quad - \frac{(2bde+bcf+adf)(e+fx)^{2+n}}{b^2d^2f^3(2+n)} + \frac{(e+fx)^{3+n}}{bdf^3(3+n)} \\ &\quad - \frac{a^4(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)} \\ &\quad + \frac{c^4(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{d^3(bc-ad)(de-cf)(1+n)}\end{aligned}$$

output
```
(a^2*d^2*f^2+a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+c*d*e*f+d^2*e^2))*(f*x+e)^(1+n
)/b^3/d^3/f^3/(1+n)-(a*d*f+b*c*f+2*b*d*e)*(f*x+e)^(2+n)/b^2/d^2/f^3/(2+n)+
(f*x+e)^(3+n)/b/d/f^3/(3+n)-a^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(
f*x+e)/(-a*f+b*e))/b^3/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^4*(f*x+e)^(1+n)*hyper
geom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d^3/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(-\frac{a^4d^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)}+\frac{-((bc-ad)(-de+cf)(a^2d^2f^2(6+5n+n^2)+abdf(3+n)(cf(2+n)+d(}{}\right.}{}$$

input
```
Integrate[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
((e + f*x)^(1 + n)*(-((a^4*d^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e +
f*x))/(b*e - a*f)])/((b*c - a*d)*(b*e - a*f))) + (-((b*c - a*d)*(-(d*e) +
c*f)*(a^2*d^2*f^2*(6 + 5*n + n^2) + a*b*d*f*(3 + n)*(c*f*(2 + n) + d*(e -
f*(1 + n)*x)) + b^2*(c^2*f^2*(6 + 5*n + n^2) + c*d*f*(3 + n)*(e - f*(1 + n
)*x) + d^2*(2*e^2 - 2*e*f*(1 + n)*x + f^2*(2 + 3*n + n^2)*x^2)))) + b^3*c^
4*f^3*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*
e - c*f)])/((-(b*c) + a*d)*f^3*(-(d*e) + c*f)*(2 + n)*(3 + n))))/(b^3*d^3*
(1 + n))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx$$
$$\Big\downarrow \; 198$$
$$\int \left(\frac{a^4(e+fx)^n}{b^3(a+bx)(bc-ad)}+\frac{(a^2d^2+abcd+b^2c^2)\,(e+fx)^n}{b^3d^3}-\frac{x(ad+bc)(e+fx)^n}{b^2d^2}+\frac{c^4(e+fx)^n}{d^3(c+dx)(ad-bc)}+\frac{x^2(}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)}+\frac{(a^2d^2+abcd+b^2c^2)(e+fx)^{n+1}}{b^3d^3f(n+1)}+\frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)}-\frac{(ad+bc)(e+fx)^{n+2}}{b^2d^2f^2(n+2)}+\frac{c^4(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{d^3(n+1)(bc-ad)(de-cf)}+\frac{e^2(e+fx)^{n+1}}{bdf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bdf^3(n+2)}+\frac{(e+fx)^{n+3}}{bdf^3(n+3)}$$

input
```
Int[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
(e^2*(e + f*x)^(1 + n))/(b*d*f^3*(1 + n)) + ((b*c + a*d)*e*(e + f*x)^(1 +
n))/(b^2*d^2*f^2*(1 + n)) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(e + f*x)^(1 +
n))/(b^3*d^3*f*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*d*f^3*(2 + n)) - ((b*
c + a*d)*(e + f*x)^(2 + n))/(b^2*d^2*f^2*(2 + n)) + (e + f*x)^(3 + n)/(b*d
*f^3*(3 + n)) - (a^4*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
(b*(e + f*x))/(b*e - a*f)])/(b^3*(b*c - a*d)*(b*e - a*f)*(1 + n)) + (c^4*(
e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c
*f)])/(d^3*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^4(fx+e)^n}{(bx+a)\,(xd+c)}dx$$

input
```
int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^4/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**4*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^4}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^4\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{too large to display}$$

input
```
int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
( - (e + f*x)**n*a**3*c*d**2*f**3*n**3 - 6*(e + f*x)**n*a**3*c*d**2*f**3*n
**2 - 11*(e + f*x)**n*a**3*c*d**2*f**3*n - 6*(e + f*x)**n*a**3*c*d**2*f**3
 - (e + f*x)**n*a**3*d**3*e*f**2*n**2 - 5*(e + f*x)**n*a**3*d**3*e*f**2*n
- 6*(e + f*x)**n*a**3*d**3*e*f**2 + (e + f*x)**n*a**3*d**3*f**3*n**3*x + 5
*(e + f*x)**n*a**3*d**3*f**3*n**2*x + 6*(e + f*x)**n*a**3*d**3*f**3*n*x -
(e + f*x)**n*a**2*b*c**2*d*f**3*n**3 - 6*(e + f*x)**n*a**2*b*c**2*d*f**3*n
**2 - 11*(e + f*x)**n*a**2*b*c**2*d*f**3*n - 6*(e + f*x)**n*a**2*b*c**2*d*
f**3 + (e + f*x)**n*a**2*b*c*d**2*e*f**2*n**3 + 4*(e + f*x)**n*a**2*b*c*d*
*2*e*f**2*n**2 + (e + f*x)**n*a**2*b*c*d**2*e*f**2*n - 6*(e + f*x)**n*a**2
*b*c*d**2*e*f**2 + 2*(e + f*x)**n*a**2*b*c*d**2*f**3*n**3*x + 10*(e + f*x)
**n*a**2*b*c*d**2*f**3*n**2*x + 12*(e + f*x)**n*a**2*b*c*d**2*f**3*n*x + (
e + f*x)**n*a**2*b*d**3*e**2*f*n**2 + 3*(e + f*x)**n*a**2*b*d**3*e**2*f*n
- (e + f*x)**n*a**2*b*d**3*e*f**2*n**3*x - 3*(e + f*x)**n*a**2*b*d**3*e*f*
*2*n**2*x - (e + f*x)**n*a**2*b*d**3*f**3*n**3*x**2 - 4*(e + f*x)**n*a**2*
b*d**3*f**3*n**2*x**2 - 3*(e + f*x)**n*a**2*b*d**3*f**3*n*x**2 - (e + f*x)
**n*a*b**2*c**3*f**3*n**3 - 6*(e + f*x)**n*a*b**2*c**3*f**3*n**2 - 11*(e +
 f*x)**n*a*b**2*c**3*f**3*n - 6*(e + f*x)**n*a*b**2*c**3*f**3 + (e + f*x)*
*n*a*b**2*c**2*d*e*f**2*n**3 + 4*(e + f*x)**n*a*b**2*c**2*d*e*f**2*n**2 +
(e + f*x)**n*a*b**2*c**2*d*e*f**2*n - 6*(e + f*x)**n*a*b**2*c**2*d*e*f**2
+ 2*(e + f*x)**n*a*b**2*c**2*d*f**3*n**3*x + 10*(e + f*x)**n*a*b**2*c**...
```

3.124 $\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = -\frac{(bde+bcf+adf)(e+fx)^{1+n}}{b^2d^2f^2(1+n)}+\frac{(e+fx)^{2+n}}{bdf^2(2+n)} + \frac{a^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} - \frac{c^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{d^2(bc-ad)(de-cf)(1+n)}$$

output
```
-(a*d*f+b*c*f+b*d*e)*(f*x+e)^(1+n)/b^2/d^2/f^2/(1+n)+(f*x+e)^(2+n)/b/d/f^2
/(2+n)+a^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/b^
2/(-a*d+b*c)/(-a*f+b*e)/(1+n)-c^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d
*(f*x+e)/(-c*f+d*e))/d^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.89

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(\frac{a^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{be-af}+\frac{(bc-ad)(-de+cf)(bcf(2+n)+adf(2+n)+bd(e-f(1+n)x))-b^2c^3f^2(2+n}{d^2f^2(de-cf)(2+n)}\right.}{b^2(bc-ad)(1+n)}$$

input
```
Integrate[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
((e + f*x)^(1 + n)*((a^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/
(b*e - a*f)])/(b*e - a*f) + ((b*c - a*d)*(-(d*e) + c*f)*(b*c*f*(2 + n) + a
*d*f*(2 + n) + b*d*(e - f*(1 + n)*x)) - b^2*c^3*f^2*(2 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*f^2*(d*e - c*f)*(2 +
 n))))/(b^2*(b*c - a*d)*(1 + n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \ 198$$

$$\int \left(-\frac{a^3(e+fx)^n}{b^2(a+bx)(bc-ad)}+\frac{(-ad-bc)(e+fx)^n}{b^2d^2}-\frac{c^3(e+fx)^n}{d^2(c+dx)(ad-bc)}+\frac{x(e+fx)^n}{bd}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)}-\frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)}-\frac{c^3(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)}-\frac{e(e+fx)^{n+1}}{bdf^2(n+1)}+\frac{(e+fx)^{n+2}}{bdf^2(n+2)}$$

input
```
Int[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
-((e*(e + f*x)^(1 + n))/(b*d*f^2*(1 + n))) - ((b*c + a*d)*(e + f*x)^(1 + n
))/(b^2*d^2*f*(1 + n)) + (e + f*x)^(2 + n)/(b*d*f^2*(2 + n)) + (a^3*(e + f
*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])
/(b^2*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (c^3*(e + f*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*(b*c - a*d)*(d*e
 - c*f)*(1 + n))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^3(fx+e)^n}{(bx+a)\,(xd+c)}dx$$

input
```
int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^3/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^3(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate(x**3*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

output
```
Integral(x**3*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^3\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{too large to display}$$

input
```
int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
((e + f*x)**n*a**2*c*d*f**2*n**2 + 3*(e + f*x)**n*a**2*c*d*f**2*n + 2*(e +
 f*x)**n*a**2*c*d*f**2 + (e + f*x)**n*a**2*d**2*e*f*n + 2*(e + f*x)**n*a**
2*d**2*e*f - (e + f*x)**n*a**2*d**2*f**2*n**2*x - 2*(e + f*x)**n*a**2*d**2
*f**2*n*x + (e + f*x)**n*a*b*c**2*f**2*n**2 + 3*(e + f*x)**n*a*b*c**2*f**2
*n + 2*(e + f*x)**n*a*b*c**2*f**2 - (e + f*x)**n*a*b*c*d*e*f*n**2 - (e + f
*x)**n*a*b*c*d*e*f*n + 2*(e + f*x)**n*a*b*c*d*e*f - 2*(e + f*x)**n*a*b*c*d
*f**2*n**2*x - 4*(e + f*x)**n*a*b*c*d*f**2*n*x - (e + f*x)**n*a*b*d**2*e**
2*n + (e + f*x)**n*a*b*d**2*e*f*n**2*x + (e + f*x)**n*a*b*d**2*f**2*n**2*x
**2 + (e + f*x)**n*a*b*d**2*f**2*n*x**2 + (e + f*x)**n*b**2*c**2*e*f*n + 2
*(e + f*x)**n*b**2*c**2*e*f - (e + f*x)**n*b**2*c**2*f**2*n**2*x - 2*(e +
f*x)**n*b**2*c**2*f**2*n*x - (e + f*x)**n*b**2*c*d*e**2*n + (e + f*x)**n*b
**2*c*d*e*f*n**2*x + (e + f*x)**n*b**2*c*d*f**2*n**2*x**2 + (e + f*x)**n*b
**2*c*d*f**2*n*x**2 - int((e + f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**2*d
**2*e*x + a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2
*a*b*c*d*f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2
*c**2*f*x**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a**4*c**2*d**2*f**3*n
**3 - 3*int((e + f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x + a**2
*d**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**
2 + a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**2*f*x**2 +
 b**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a**4*c**2*d**2*f**3*n**2 - 2*int...
```

3.125 $\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{(e+fx)^{1+n}}{bdf(1+n)} - \frac{a^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)} + \frac{c^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{d(bc-ad)(de-cf)(1+n)}$$

output
```
(f*x+e)^(1+n)/b/d/f/(1+n)-a^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*
x+e)/(-a*f+b*e))/b/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^2*(f*x+e)^(1+n)*hypergeom
([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{(e+fx)^{1+n}\left(a^2df(-de+cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+(be-af)\left(-((bc-ad)(-\right.\right.}{bd(bc-ad)f(be-af)(de-cf)(1+n}$$

input
```
Integrate[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
((e + f*x)^(1 + n)*(a^2*d*f*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 +
 n, (b*(e + f*x))/(b*e - a*f)] + (b*e - a*f)*(-((b*c - a*d)*(-(d*e) + c*f)
) + b*c^2*f*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])
))/(b*d*(b*c - a*d)*f*(b*e - a*f)*(d*e - c*f)*(1 + n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \; 198$$

$$\int \left(\frac{a^2(e+fx)^n}{b(a+bx)(bc-ad)}+\frac{c^2(e+fx)^n}{d(c+dx)(ad-bc)}+\frac{(e+fx)^n}{bd}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)}+\frac{c^2(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)}+\frac{(e+fx)^{n+1}}{bdf(n+1)}$$

input
```
Int[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
(e + f*x)^(1 + n)/(b*d*f*(1 + n)) - (a^2*(e + f*x)^(1 + n)*Hypergeometric2
F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b*(b*c - a*d)*(b*e - a*f)
*(1 + n)) + (c^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(
e + f*x))/(d*e - c*f)])/(d*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^2(fx+e)^n}{(bx+a)(xd+c)}dx$$

input
```
int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)} dx = \int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)} dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^2/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)} dx = \int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)} dx$$

input
```
integrate(x**2*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

output
```
Integral(x**2*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)} dx = \int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)} dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^2}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^2\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{too large to display}$$

input
```
int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
( - (e + f*x)**n*a*c*f*n - (e + f*x)**n*a*c*f - (e + f*x)**n*a*d*e + (e +
f*x)**n*a*d*f*n*x - (e + f*x)**n*b*c*e + (e + f*x)**n*b*c*f*n*x + int((e +
 f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d**2*f*x**2 + a
*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**2 + a*b*d**2*e*x
**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**2*f*x**2 + b**2*c*d*e*x**2
 + b**2*c*d*f*x**3),x)*a**3*c**2*d*f**2*n**2 + int((e + f*x)**n/(a**2*c*d*
e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**
2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x*
*3 + b**2*c**2*e*x + b**2*c**2*f*x**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3)
,x)*a**3*c**2*d*f**2*n + int((e + f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**
2*d**2*e*x + a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x
+ 2*a*b*c*d*f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b
**2*c**2*f*x**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a**2*b*c**3*f**2*n
**2 + int((e + f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d
**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**2
+ a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**2*f*x**2 + b
**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a**2*b*c**3*f**2*n - int(((e + f*x)**
n*x**2)/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d**2*f*x**2 + a*
b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**2 + a*b*d**2*e*x*
*2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**2*f*x**2 + b**2*c*d*e*x*...
```

3.126 $\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{a(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} - \frac{c(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

output
```
a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/
(-a*f+b*e)/(1+n)-c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+
d*e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{(e+fx)^{1+n}\left(a(-de+cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+c(be-af)\,\text{Hypergeometric2}\right.}{(bc-ad)(be-af)(-de+cf)(1+n)}$$

input
```
Integrate[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
((e + f*x)^(1 + n)*(a*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b
*(e + f*x))/(b*e - a*f)] + c*(b*e - a*f)*Hypergeometric2F1[1, 1 + n, 2 + n
, (d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1
+ n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {174, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \text{174}$$

$$\frac{c\int \frac{(e+fx)^n}{c+dx}dx}{bc-ad}-\frac{a\int \frac{(e+fx)^n}{a+bx}dx}{bc-ad}$$

$$\Big\downarrow \text{78}$$

$$\frac{a(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}-\frac{c(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

input
```
Int[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
(a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e
 - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n)) - (c*(e + f*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e
- c*f)*(1 + n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Maple [F]

$$\int \frac{x(fx+e)^n}{(bx+a)\,(xd+c)}dx$$

input
```
int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate(x*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

output
```
Integral(x*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
((e + f*x)**n*e - int((e + f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*
e*x + a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b
*c*d*f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**
2*f*x**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a**2*c*d*e*f*n - int((e +
 f*x)**n/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d**2*f*x**2 + a
*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**2 + a*b*d**2*e*x
**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**2*f*x**2 + b**2*c*d*e*x**2
 + b**2*c*d*f*x**3),x)*a*b*c**2*e*f*n + int(((e + f*x)**n*x**2)/(a**2*c*d*
e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**
2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x*
*3 + b**2*c**2*e*x + b**2*c**2*f*x**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3)
,x)*a**2*d**2*f**2*n + 2*int(((e + f*x)**n*x**2)/(a**2*c*d*e + a**2*c*d*f*
x + a**2*d**2*e*x + a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c
*d*e*x + 2*a*b*c*d*f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*
e*x + b**2*c**2*f*x**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a*b*c*d*f**
2*n - int(((e + f*x)**n*x**2)/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x +
 a**2*d**2*f*x**2 + a*b*c**2*e + a*b*c**2*f*x + 2*a*b*c*d*e*x + 2*a*b*c*d*
f*x**2 + a*b*d**2*e*x**2 + a*b*d**2*f*x**3 + b**2*c**2*e*x + b**2*c**2*f*x
**2 + b**2*c*d*e*x**2 + b**2*c*d*f*x**3),x)*a*b*d**2*e*f*n + int(((e + f*x
)**n*x**2)/(a**2*c*d*e + a**2*c*d*f*x + a**2*d**2*e*x + a**2*d**2*f*x**...
```

3.127 $\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = -\frac{b(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)} + \frac{d(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}$$

output
```
-b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)
/(-a*f+b*e)/(1+n)+d*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f
+d*e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{(e+fx)^{1+n}\left(b(de-cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+d(-be+af)\,\text{Hypergeometric2}\right.}{(bc-ad)(be-af)(-de+cf)(1+n)}$$

input
```
Integrate[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

output
```
((e + f*x)^(1 + n)*(b*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
 + f*x))/(b*e - a*f)] + d*(-(b*e) + a*f)*Hypergeometric2F1[1, 1 + n, 2 + n
, (d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1
+ n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {97, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)} dx$$

$$\Big\downarrow \text{ 97}$$

$$\frac{b\int \frac{(e+fx)^n}{a+bx}dx}{bc-ad}-\frac{d\int \frac{(e+fx)^n}{c+dx}dx}{bc-ad}$$

$$\Big\downarrow \text{ 78}$$

$$\frac{d(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}-\frac{b(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

input
```
Int[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

output
```
-((b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b
*e - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n))) + (d*(e + f*x)^(1 + n)*Hype
rgeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d
*e - c*f)*(1 + n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(fx+e)^n}{(bx+a)(xd+c)}dx$$

input
```
int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(e+fx)^n}{(a+bx)\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**n/(b*x+a)/(d*x+c),x)
```

output
```
Integral((e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx=\int \frac{(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((e + f*x)^n/((a + b*x)*(c + d*x)),x)
```

output
```
int((e + f*x)^n/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx=\int \frac{(fx+e)^n}{bd\,x^2+adx+bcx+ac}dx$$

input
```
int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int((e + f*x)**n/(a*c + a*d*x + b*c*x + b*d*x**2),x)
```

3.128 $\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$
\begin{aligned}
\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx &= \frac{b^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)} \\
&\quad - \frac{d^2(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{c(bc-ad)(de-cf)(1+n)} \\
&\quad - \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ace(1+n)}
\end{aligned}
$$

output
```
b^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a/(-a*d+b
*c)/(-a*f+b*e)/(1+n)-d^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/
(-c*f+d*e))/c/(-a*d+b*c)/(-c*f+d*e)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n]
,[2+n],1+f*x/e)/a/c/e/(1+n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = -\frac{(e+fx)^{1+n}\left(b^2ce(de-cf)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)+(-be+af)\left(ad^2e\,\text{Hyperg}\right.\right.}{ac(-bc+ad)e(-}$$

input
```
Integrate[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

output
```
-(((e + f*x)^(1 + n)*(b^2*c*e*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 +
n, (b*(e + f*x))/(b*e - a*f)] + (-(b*e) + a*f)*(a*d^2*e*Hypergeometric2F1[
1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)] - (b*c - a*d)*(-(d*e) + c*f)*H
ypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])))/(a*c*(-(b*c) + a*d)*e*(-
(b*e) + a*f)*(-(d*e) + c*f)*(1 + n)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \ 198$$

$$\int \left(\frac{b^2(e+fx)^n}{a(a+bx)(ad-bc)}+\frac{d^2(e+fx)^n}{c(c+dx)(bc-ad)}+\frac{(e+fx)^n}{acx}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{b^2(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)}-\frac{d^2(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)}-\frac{(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ace(n+1)}$$

input
```
Int[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

output
```
(b^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b
*e - a*f)])/(a*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (d^2*(e + f*x)^(1 + n)*H
ypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c*(b*c - a*
d)*(d*e - c*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, 1 + (f*x)/e])/(a*c*e*(1 + n))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(fx+e)^n}{x\,(bx+a)\,(xd+c)}dx$$

input
```
int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

output
```
int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(e+fx)^n}{x\,(a+bx)\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)**n/x/(b*x+a)/(d*x+c),x)
```

output
```
Integral((e + f*x)**n/(x*(a + b*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(e+f\,x)^n}{x\,(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((e + f*x)^n/(x*(a + b*x)*(c + d*x)),x)
```

output
```
int((e + f*x)^n/(x*(a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{bd\,x^3+ad\,x^2+bc\,x^2+acx}dx$$

input
```
int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

output
```
int((e + f*x)**n/(a*c*x + a*d*x**2 + b*c*x**2 + b*d*x**3),x)
```

3.129 $\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 212

$$\begin{aligned}\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx &= -\frac{(e+fx)^{1+n}}{acex} - \frac{b^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)} \\ &\quad + \frac{d^3(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{c^2(bc-ad)(de-cf)(1+n)} \\ &\quad + \frac{(bce+ade-acfn)(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{a^2c^2e^2(1+n)}\end{aligned}$$

output
```
-(f*x+e)^(1+n)/a/c/e/x-b^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e
)/(-a*f+b*e))/a^2/(-a*d+b*c)/(-a*f+b*e)/(1+n)+d^3*(f*x+e)^(1+n)*hypergeom(
[1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/c^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)+(-a*c
*f*n+a*d*e+b*c*e)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a^2/c^2/
e^2/(1+n)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(-\frac{b^3 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)}+\frac{-\frac{d^3 \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(-de+cf)}+\frac{(bc+ad)e \text{Hypergeometric2}}{}}{c^2}\right.}{1+n}$$

input
```
Integrate[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

output
```
((e + f*x)^(1 + n)*(-((b^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x)
)/(b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f))) + (-((d^3*Hypergeometric2F1
[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(-(d*e) + c*f))
) + ((b*c + a*d)*e*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e] + a*c*f
*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*e^2))/c^2))/(1 + n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(-\frac{b^3(e+fx)^n}{a^2(a+bx)(ad-bc)}+\frac{(-ad-bc)(e+fx)^n}{a^2c^2x}-\frac{d^3(e+fx)^n}{c^2(c+dx)(bc-ad)}+\frac{(e+fx)^n}{acx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)}+\frac{(ad+bc)(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{a^2c^2e(n+1)}+\frac{d^3(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{d(e+fx)}{de-cf}\right)}{c^2(n+1)(bc-ad)(de-cf)}+\frac{f(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ace^2(n+1)}$$

input
```
Int[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

output
```
-((b^3*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/
(b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (d^3*(e + f*x)^(1 +
 n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c^2*(b
*c - a*d)*(d*e - c*f)*(1 + n)) + ((b*c + a*d)*(e + f*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*c^2*e*(1 + n)) + (f*(e + f*x)^
(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*c*e^2*(1 + n))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(fx+e)^n}{x^2\,(bx+a)\,(xd+c)}dx$$

input
```
int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

output
```
int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*d*x^4 + a*c*x^2 + (b*c + a*d)*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**n/x**2/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx=\int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx=\int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx=\int \frac{(e+f\,x)^n}{x^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x)
```

output
```
int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n}{bd\,x^4+ad\,x^3+bc\,x^3+ac\,x^2}dx$$

input
```
int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

output
```
int((e + f*x)**n/(a*c*x**2 + a*d*x**3 + b*c*x**3 + b*d*x**4),x)
```

3.130 $\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 167

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)} + \frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))\,(a+bx)^{2+m}}{b^4(2+m)} - \frac{(3adfh-b(dfg+deh+cfh))(a+bx)^{3+m}}{b^4(3+m)} + \frac{dfh(a+bx)^{4+m}}{b^4(4+m)}$$

output
```
(-a*d+b*c)*(-a*f+b*e)*(-a*h+b*g)*(b*x+a)^(1+m)/b^4/(1+m)+(3*a^2*d*f*h+b^2*
(c*e*h+c*f*g+d*e*g)-2*a*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(2+m)/b^4/(2+m)-(3*
a*d*f*h-b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(3+m)/b^4/(3+m)+d*f*h*(b*x+a)^(4+m)
/b^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}\left(\frac{(bc-ad)(be-af)(bg-ah)}{1+m}+\frac{(3a^2dfh+b^2(deg+cfg+ceh)-2ab(dfg+deh+cfh))(a+bx)}{2+m}+\frac{(-3adfh+b(dfg+deh+cfh))(a\ldots}{3+m}\right.}{b^4}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(((b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(1 + m) + ((3*a^
2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(a
+ b*x))/(2 + m) + ((-3*a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^2)/(
3 + m) + (d*f*h*(a + b*x)^3)/(4 + m)))/b^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(e+fx)(g+hx)(a+bx)^m\,dx$$

$$\Big\downarrow \text{159}$$

$$\int \left(\frac{(a+bx)^{m+1}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^3}+\frac{(bc-ad)(be-af)(bg-ah)(a+b\ldots}{b^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-\frac{(a+bx)^{m+3}(3adfh-b(cfh+deh+dfg))}{b^4(m+3)}+\frac{dfh(a+bx)^{m+4}}{b^4(m+4)}$$

input
```
Int[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

output
```
((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((
3*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))
*(a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h
))*(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 +
m))
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(167) = 334$.

Time = 0.40 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.35

method	result
gosper	$-\frac{(bx+a)^{1+m}(-b^3dfh\,m^3x^3-b^3cfh\,m^3x^2-b^3deh\,m^3x^2-b^3dfg\,m^3x^2-6b^3dfh\,m^2x^3+3a\,b^2dfh\,m^2x^2-b^3ceh\,m^3x-b^3cfg\,m}{}$
orering	$-\frac{(bx+a)^{m}(-b^3dfh\,m^3x^3-b^3cfh\,m^3x^2-b^3deh\,m^3x^2-b^3dfg\,m^3x^2-6b^3dfh\,m^2x^3+3a\,b^2dfh\,m^2x^2-b^3ceh\,m^3x-b^3cfg\,m^3x}{}$
norman	$\frac{(adfhm+bcfhm+bdehm+bdfgm+4bcfh+4bdeh+4bgdf)x^3\mathrm{e}^{m\ln(bx+a)}}{b(m^2+7m+12)}+\frac{(a\,b^2ceh\,m^3+a\,b^2cfg\,m^3+a\,b^2deg\,m^3+b^3ceg\,m^3}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(b*x+a)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(-b^3*d*f*h*m^3*x^3-b^3*c
*f*h*m^3*x^2-b^3*d*e*h*m^3*x^2-b^3*d*f*g*m^3*x^2-6*b^3*d*f*h*m^2*x^3+3*a*b
^2*d*f*h*m^2*x^2-b^3*c*e*h*m^3*x-b^3*c*f*g*m^3*x-7*b^3*c*f*h*m^2*x^2-b^3*d
*e*g*m^3*x-7*b^3*d*e*h*m^2*x^2-7*b^3*d*f*g*m^2*x^2-11*b^3*d*f*h*m*x^3+2*a*
b^2*c*f*h*m^2*x+2*a*b^2*d*e*h*m^2*x+2*a*b^2*d*f*g*m^2*x+9*a*b^2*d*f*h*m*x^
2-b^3*c*e*g*m^3-8*b^3*c*e*h*m^2*x-8*b^3*c*f*g*m^2*x-14*b^3*c*f*h*m*x^2-8*b
^3*d*e*g*m^2*x-14*b^3*d*e*h*m*x^2-14*b^3*d*f*g*m*x^2-6*b^3*d*f*h*x^3-6*a^2
*b*d*f*h*m*x+a*b^2*c*e*h*m^2+a*b^2*c*f*g*m^2+10*a*b^2*c*f*h*m*x+a*b^2*d*e*
g*m^2+10*a*b^2*d*e*h*m*x+10*a*b^2*d*f*g*m*x+6*a*b^2*d*f*h*x^2-9*b^3*c*e*g*
m^2-19*b^3*c*e*h*m*x-19*b^3*c*f*g*m*x-8*b^3*c*f*h*x^2-19*b^3*d*e*g*m*x-8*b
^3*d*e*h*x^2-8*b^3*d*f*g*x^2-2*a^2*b*c*f*h*m-2*a^2*b*d*e*h*m-2*a^2*b*d*f*g
*m-6*a^2*b*d*f*h*x+7*a*b^2*c*e*h*m+7*a*b^2*c*f*g*m+8*a*b^2*c*f*h*x+7*a*b^2
*d*e*g*m+8*a*b^2*d*e*h*x+8*a*b^2*d*f*g*x-26*b^3*c*e*g*m-12*b^3*c*e*h*x-12*
b^3*c*f*g*x-12*b^3*d*e*g*x+6*a^3*d*f*h-8*a^2*b*c*f*h-8*a^2*b*d*e*h-8*a^2*b
*d*f*g+12*a*b^2*c*e*h+12*a*b^2*c*f*g+12*a*b^2*d*e*g-24*b^3*c*e*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(167) = 334$.

Time = 0.10 (sec) , antiderivative size = 877, normalized size of antiderivative = 5.25

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
(a*b^3*c*e*g*m^3 + (b^4*d*f*h*m^3 + 6*b^4*d*f*h*m^2 + 11*b^4*d*f*h*m + 6*b
^4*d*f*h)*x^4 + (8*b^4*d*f*g + (b^4*d*f*g + (b^4*d*e + (b^4*c + a*b^3*d)*f
)*h)*m^3 + (7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + 3*a*b^3*d)*f)*h)*m^2 + 8
*(b^4*d*e + b^4*c*f)*h + 2*(7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + a*b^3*d)
*f)*h)*m)*x^3 - (a^2*b^2*c*e*h + (a^2*b^2*c*f - (9*a*b^3*c - a^2*b^2*d)*e)
*g)*m^2 + (12*b^4*c*e*h + ((b^4*d*e + (b^4*c + a*b^3*d)*f)*g + (a*b^3*c*f
+ (b^4*c + a*b^3*d)*e)*h)*m^3 + ((8*b^4*d*e + (8*b^4*c + 5*a*b^3*d)*f)*g +
 ((8*b^4*c + 5*a*b^3*d)*e + (5*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m^2 + 12*(b^4*
d*e + b^4*c*f)*g + ((19*b^4*d*e + (19*b^4*c + 4*a*b^3*d)*f)*g + ((19*b^4*c
 + 4*a*b^3*d)*e + (4*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m)*x^2 + 4*(3*(2*a*b^3*c
 - a^2*b^2*d)*e - (3*a^2*b^2*c - 2*a^3*b*d)*f)*g - 2*(2*(3*a^2*b^2*c - 2*a
^3*b*d)*e - (4*a^3*b*c - 3*a^4*d)*f)*h + (((26*a*b^3*c - 7*a^2*b^2*d)*e -
(7*a^2*b^2*c - 2*a^3*b*d)*f)*g + (2*a^3*b*c*f - (7*a^2*b^2*c - 2*a^3*b*d)*
e)*h)*m + (24*b^4*c*e*g + (a*b^3*c*e*h + (a*b^3*c*f + (b^4*c + a*b^3*d)*e)
*g)*m^3 + (((9*b^4*c + 7*a*b^3*d)*e + (7*a*b^3*c - 2*a^2*b^2*d)*f)*g - (2*
a^2*b^2*c*f - (7*a*b^3*c - 2*a^2*b^2*d)*e)*h)*m^2 + 2*(((13*b^4*c + 6*a*b^
3*d)*e + 2*(3*a*b^3*c - 2*a^2*b^2*d)*f)*g + (2*(3*a*b^3*c - 2*a^2*b^2*d)*e
 - (4*a^2*b^2*c - 3*a^3*b*d)*f)*h)*m)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3
 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8221 vs. $2(160) = 320$.

Time = 1.98 (sec) , antiderivative size = 8221, normalized size of antiderivative = 49.23

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)**m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

output

```
Piecewise((a**m*(c*e*g*x + c*e*h*x**2/2 + c*f*g*x**2/2 + c*f*h*x**3/3 + d*
e*g*x**2/2 + d*e*h*x**3/3 + d*f*g*x**3/3 + d*f*h*x**4/4), Eq(b, 0)), (6*a*
*3*d*f*h*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) + 11*a**3*d*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) - 2*a**2*b*c*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x
**2 + 6*b**7*x**3) - 2*a**2*b*d*e*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b
**6*x**2 + 6*b**7*x**3) - 2*a**2*b*d*f*g/(6*a**3*b**4 + 18*a**2*b**5*x + 1
8*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*f*h*x*log(a/b + x)/(6*a**3*b**4
 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*f*h*x/(6*a
**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*e*h/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*f*
g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2
*c*f*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a
*b**2*d*e*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
- 6*a*b**2*d*e*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) - 6*a*b**2*d*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b
**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2/(6*a**3*b**4
+ 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c*e*g/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*e*h*x/(6...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(167) = 334$.

Time = 0.07 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.84

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$$
$$= \frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m deg}{(m^2+3\,m+2)b^2} + \frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m cfg}{(m^2+3\,m+2)b^2}$$
$$+ \frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m ceh}{(m^2+3\,m+2)b^2} + \frac{(bx+a)^{m+1}ceg}{b(m+1)}$$
$$+ \frac{((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m dfg}{(m^3+6\,m^2+11\,m+6)b^3}$$
$$+ \frac{((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m deh}{(m^3+6\,m^2+11\,m+6)b^3}$$
$$+ \frac{((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m cfh}{(m^3+6\,m^2+11\,m+6)b^3}$$
$$+ \frac{((m^3+6\,m^2+11\,m+6)b^4x^4+(m^3+3\,m^2+2\,m)ab^3x^3-3\,(m^2+m)a^2b^2x^2+6\,a^3bmx-6\,a^4)(bx+}{(m^4+10\,m^3+35\,m^2+50\,m+24)b^4}$$

input
```
integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d*e*g/((m^2 + 3*m + 2)*b^2)
+ (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*f*g/((m^2 + 3*m + 2)*b^2
) + (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*e*h/((m^2 + 3*m + 2)*b
^2) + (b*x + a)^(m + 1)*c*e*g/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^
2 + m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d*f*g/((m^3 + 6*m^2 +
11*m + 6)*b^3) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*
m*x + 2*a^3)*(b*x + a)^m*d*e*h/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^2 + 3*
m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*
f*h/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (
m^3 + 3*m^2 + 2*m)*a*b^3*x^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a
^4)*(b*x + a)^m*d*f*h/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1626 vs. $2(167) = 334$.

Time = 0.14 (sec) , antiderivative size = 1626, normalized size of antiderivative = 9.74

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
((b*x + a)^m*b^4*d*f*h*m^3*x^4 + (b*x + a)^m*b^4*d*f*g*m^3*x^3 + (b*x + a)
^m*b^4*d*e*h*m^3*x^3 + (b*x + a)^m*b^4*c*f*h*m^3*x^3 + (b*x + a)^m*a*b^3*d
*f*h*m^3*x^3 + 6*(b*x + a)^m*b^4*d*f*h*m^2*x^4 + (b*x + a)^m*b^4*d*e*g*m^3
*x^2 + (b*x + a)^m*b^4*c*f*g*m^3*x^2 + (b*x + a)^m*a*b^3*d*f*g*m^3*x^2 + (
b*x + a)^m*b^4*c*e*h*m^3*x^2 + (b*x + a)^m*a*b^3*d*e*h*m^3*x^2 + (b*x + a)
^m*a*b^3*c*f*h*m^3*x^2 + 7*(b*x + a)^m*b^4*d*f*g*m^2*x^3 + 7*(b*x + a)^m*b
^4*d*e*h*m^2*x^3 + 7*(b*x + a)^m*b^4*c*f*h*m^2*x^3 + 3*(b*x + a)^m*a*b^3*d
*f*h*m^2*x^3 + 11*(b*x + a)^m*b^4*d*f*h*m*x^4 + (b*x + a)^m*b^4*c*e*g*m^3*
x + (b*x + a)^m*a*b^3*d*e*g*m^3*x + (b*x + a)^m*a*b^3*c*f*g*m^3*x + (b*x +
 a)^m*a*b^3*c*e*h*m^3*x + 8*(b*x + a)^m*b^4*d*e*g*m^2*x^2 + 8*(b*x + a)^m*
b^4*c*f*g*m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*f*g*m^2*x^2 + 8*(b*x + a)^m*b^4*
c*e*h*m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*e*h*m^2*x^2 + 5*(b*x + a)^m*a*b^3*c*
f*h*m^2*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m^2*x^2 + 14*(b*x + a)^m*b^4*d*f
*g*m*x^3 + 14*(b*x + a)^m*b^4*d*e*h*m*x^3 + 14*(b*x + a)^m*b^4*c*f*h*m*x^3
 + 2*(b*x + a)^m*a*b^3*d*f*h*m*x^3 + 6*(b*x + a)^m*b^4*d*f*h*x^4 + (b*x +
a)^m*a*b^3*c*e*g*m^3 + 9*(b*x + a)^m*b^4*c*e*g*m^2*x + 7*(b*x + a)^m*a*b^3
*d*e*g*m^2*x + 7*(b*x + a)^m*a*b^3*c*f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*f
*g*m^2*x + 7*(b*x + a)^m*a*b^3*c*e*h*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*e*h*m
^2*x - 2*(b*x + a)^m*a^2*b^2*c*f*h*m^2*x + 19*(b*x + a)^m*b^4*d*e*g*m*x^2
+ 19*(b*x + a)^m*b^4*c*f*g*m*x^2 + 4*(b*x + a)^m*a*b^3*d*f*g*m*x^2 + 19...
```

Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 819, normalized size of antiderivative = 4.90

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$$

$$= \frac{x\,(a+b\,x)^m\,(24\,b^4\,c\,e\,g+9\,b^4\,c\,e\,g\,m^2+b^4\,c\,e\,g\,m^3+26\,b^4\,c\,e\,g\,m+12\,a\,b^3\,c\,e\,h\,m+12\,a\,b^3\,c\,f\,g\,m+12\,a}{}$$

$$-\frac{(a+b\,x)^m\,(6\,a^4\,d\,f\,h+12\,a^2\,b^2\,c\,e\,h+12\,a^2\,b^2\,c\,f\,g+12\,a^2\,b^2\,d\,e\,g-24\,a\,b^3\,c\,e\,g-8\,a^3\,b\,c\,f\,h-8\,a^3\,b}{}$$

$$+\frac{x^3\,(a+b\,x)^m\,(m^2+3\,m+2)\,(4\,b\,c\,f\,h+4\,b\,d\,e\,h+4\,b\,d\,f\,g+a\,d\,f\,h\,m+b\,c\,f\,h\,m+b\,d\,e\,h\,m+b\,d\,f}{b\,(m^4+10\,m^3+35\,m^2+50\,m+24)}$$

$$+\frac{x^2\,(m+1)\,(a+b\,x)^m\,(12\,b^2\,c\,e\,h+12\,b^2\,c\,f\,g+12\,b^2\,d\,e\,g+b^2\,c\,e\,h\,m^2+b^2\,c\,f\,g\,m^2+b^2\,d\,e\,g\,m^2+7\,b}{b}$$

$$+\frac{d\,f\,h\,x^4\,(a+b\,x)^m\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x),x)
```

output
```
(x*(a + b*x)^m*(24*b^4*c*e*g + 9*b^4*c*e*g*m^2 + b^4*c*e*g*m^3 + 26*b^4*c*
e*g*m + 12*a*b^3*c*e*h*m + 12*a*b^3*c*f*g*m + 12*a*b^3*d*e*g*m + 6*a^3*b*d
*f*h*m + 7*a*b^3*c*e*h*m^2 + 7*a*b^3*c*f*g*m^2 + 7*a*b^3*d*e*g*m^2 + a*b^3
*c*e*h*m^3 + a*b^3*c*f*g*m^3 + a*b^3*d*e*g*m^3 - 8*a^2*b^2*c*f*h*m - 8*a^2
*b^2*d*e*h*m - 8*a^2*b^2*d*f*g*m - 2*a^2*b^2*c*f*h*m^2 - 2*a^2*b^2*d*e*h*m
^2 - 2*a^2*b^2*d*f*g*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) - ((a
 + b*x)^m*(6*a^4*d*f*h + 12*a^2*b^2*c*e*h + 12*a^2*b^2*c*f*g + 12*a^2*b^2*
d*e*g - 24*a*b^3*c*e*g - 8*a^3*b*c*f*h - 8*a^3*b*d*e*h - 8*a^3*b*d*f*g - 2
6*a*b^3*c*e*g*m - 2*a^3*b*c*f*h*m - 2*a^3*b*d*e*h*m - 2*a^3*b*d*f*g*m - 9*
a*b^3*c*e*g*m^2 - a*b^3*c*e*g*m^3 + 7*a^2*b^2*c*e*h*m + 7*a^2*b^2*c*f*g*m
+ 7*a^2*b^2*d*e*g*m + a^2*b^2*c*e*h*m^2 + a^2*b^2*c*f*g*m^2 + a^2*b^2*d*e*
g*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(3*m
+ m^2 + 2)*(4*b*c*f*h + 4*b*d*e*h + 4*b*d*f*g + a*d*f*h*m + b*c*f*h*m + b*
d*e*h*m + b*d*f*g*m))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^2*(m +
1)*(a + b*x)^m*(12*b^2*c*e*h + 12*b^2*c*f*g + 12*b^2*d*e*g + b^2*c*e*h*m^2
 + b^2*c*f*g*m^2 + b^2*d*e*g*m^2 + 7*b^2*c*e*h*m + 7*b^2*c*f*g*m + 7*b^2*d
*e*g*m - 3*a^2*d*f*h*m + a*b*c*f*h*m^2 + a*b*d*e*h*m^2 + a*b*d*f*g*m^2 + 4
*a*b*c*f*h*m + 4*a*b*d*e*h*m + 4*a*b*d*f*g*m))/(b^2*(50*m + 35*m^2 + 10*m^
3 + m^4 + 24)) + (d*f*h*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m +
35*m^2 + 10*m^3 + m^4 + 24)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1045, normalized size of antiderivative = 6.26

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

output
```
((a + b*x)**m*( - 6*a**4*d*f*h + 2*a**3*b*c*f*h*m + 8*a**3*b*c*f*h + 2*a**
3*b*d*e*h*m + 8*a**3*b*d*e*h + 2*a**3*b*d*f*g*m + 8*a**3*b*d*f*g + 6*a**3*
b*d*f*h*m*x - a**2*b**2*c*e*h*m**2 - 7*a**2*b**2*c*e*h*m - 12*a**2*b**2*c*
e*h - a**2*b**2*c*f*g*m**2 - 7*a**2*b**2*c*f*g*m - 12*a**2*b**2*c*f*g - 2*
a**2*b**2*c*f*h*m**2*x - 8*a**2*b**2*c*f*h*m*x - a**2*b**2*d*e*g*m**2 - 7*
a**2*b**2*d*e*g*m - 12*a**2*b**2*d*e*g - 2*a**2*b**2*d*e*h*m**2*x - 8*a**2
*b**2*d*e*h*m*x - 2*a**2*b**2*d*f*g*m**2*x - 8*a**2*b**2*d*f*g*m*x - 3*a**
2*b**2*d*f*h*m**2*x**2 - 3*a**2*b**2*d*f*h*m*x**2 + a*b**3*c*e*g*m**3 + 9*
a*b**3*c*e*g*m**2 + 26*a*b**3*c*e*g*m + 24*a*b**3*c*e*g + a*b**3*c*e*h*m**
3*x + 7*a*b**3*c*e*h*m**2*x + 12*a*b**3*c*e*h*m*x + a*b**3*c*f*g*m**3*x +
7*a*b**3*c*f*g*m**2*x + 12*a*b**3*c*f*g*m*x + a*b**3*c*f*h*m**3*x**2 + 5*a
*b**3*c*f*h*m**2*x**2 + 4*a*b**3*c*f*h*m*x**2 + a*b**3*d*e*g*m**3*x + 7*a*
b**3*d*e*g*m**2*x + 12*a*b**3*d*e*g*m*x + a*b**3*d*e*h*m**3*x**2 + 5*a*b**
3*d*e*h*m**2*x**2 + 4*a*b**3*d*e*h*m*x**2 + a*b**3*d*f*g*m**3*x**2 + 5*a*b
**3*d*f*g*m**2*x**2 + 4*a*b**3*d*f*g*m*x**2 + a*b**3*d*f*h*m**3*x**3 + 3*a
*b**3*d*f*h*m**2*x**3 + 2*a*b**3*d*f*h*m*x**3 + b**4*c*e*g*m**3*x + 9*b**4
*c*e*g*m**2*x + 26*b**4*c*e*g*m*x + 24*b**4*c*e*g*x + b**4*c*e*h*m**3*x**2
 + 8*b**4*c*e*h*m**2*x**2 + 19*b**4*c*e*h*m*x**2 + 12*b**4*c*e*h*x**2 + b*
*4*c*f*g*m**3*x**2 + 8*b**4*c*f*g*m**2*x**2 + 19*b**4*c*f*g*m*x**2 + 12*b*
*4*c*f*g*x**2 + b**4*c*f*h*m**3*x**3 + 7*b**4*c*f*h*m**2*x**3 + 14*b**4...
```

3.131 $\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \frac{(b(de+cf)h-df(bg+ah))(a+bx)^{1+m}}{b^2h^2(1+m)}+\frac{df(a+bx)^{2+m}}{b^2h(2+m)} + \frac{(dg-ch)(fg-eh)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{h^2(bg-ah)(1+m)}$$

output
```
(b*(c*f+d*e)*h-d*f*(a*h+b*g))*(b*x+a)^(1+m)/b^2/h^2/(1+m)+d*f*(b*x+a)^(2+m
)/b^2/h/(2+m)+(-c*h+d*g)*(-e*h+f*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m]
,-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+b*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \frac{(a+bx)^{1+m}\left(\frac{-adfh+b(-dfg+deh+cfh)}{b^2(1+m)}+\frac{dfh(a+bx)}{b^2(2+m)}+\frac{(dg-ch)(fg-eh)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{h(a+bx)}{-bg+ah}\right)}{(bg-ah)(1+m)}\right)}{h^2}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*((-(a*d*f*h) + b*(-(d*f*g) + d*e*h + c*f*h))/(b^2*(1 +
m)) + (d*f*h*(a + b*x))/(b^2*(2 + m)) + ((d*g - c*h)*(f*g - e*h)*Hypergeom
etric2F1[1, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(1 +
 m))))/h^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{164, 78\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)(a+bx)^m}{g+hx}\,dx$$

$$\Big\downarrow \; 164$$

$$\frac{(dg-ch)(fg-eh)\int \frac{(a+bx)^m}{g+hx}dx}{h^2} - \frac{(a+bx)^{m+1}(adfh-bh(m+2)(cf+de)+bdfg(m+2)-bdfh(m+1)x)}{b^2h^2(m+1)(m+2)}$$

$$\Big\downarrow \; 78$$

$$\frac{(a+bx)^{m+1}(dg-ch)(fg-eh)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)} - \frac{(a+bx)^{m+1}(adfh-bh(m+2)(cf+de)+bdfg(m+2)-bdfh(m+1)x)}{b^2h^2(m+1)(m+2)}$$

input
```
Int[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

output
```
-(((a + b*x)^(1 + m)*(a*d*f*h + b*d*f*g*(2 + m) - b*(d*e + c*f)*h*(2 + m)
- b*d*f*h*(1 + m)*x))/(b^2*h^2*(1 + m)*(2 + m))) + ((d*g - c*h)*(f*g - e*h
)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*
g - a*h))])/(h^2*(b*g - a*h)*(1 + m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)(fx+e)}{hx+g}dx$$

input
```
int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((d*f*x^2 + c*e + (d*e + c*f)*x)*(b*x + a)^m/(h*x + g), x)
```

Sympy [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(a+bx)^m\,(c+dx)\,(e+fx)}{g+hx}\,dx$$

input
```
integrate((b*x+a)**m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**m*(c + d*x)*(e + f*x)/(g + h*x), x)
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(dx+c)(fx+e)(bx+a)^m}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \int \frac{(e+f\,x)\,(a+b\,x)^m\,(c+d\,x)}{g+h\,x}\,dx$$

input
```
int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x),x)
```

output
```
int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

output
```
( - (a + b*x)**m*a**2*d*f*g*h*m + (a + b*x)**m*a*b*c*e*h**2*m**2 + 3*(a +
b*x)**m*a*b*c*e*h**2*m + 2*(a + b*x)**m*a*b*c*e*h**2 - (a + b*x)**m*a*b*c*
f*g*h*m - 2*(a + b*x)**m*a*b*c*f*g*h - (a + b*x)**m*a*b*d*e*g*h*m - 2*(a +
 b*x)**m*a*b*d*e*g*h + (a + b*x)**m*a*b*d*f*g**2*m + 2*(a + b*x)**m*a*b*d*
f*g**2 + (a + b*x)**m*a*b*d*f*g*h*m**2*x + (a + b*x)**m*b**2*c*f*g*h*m**2*
x + 2*(a + b*x)**m*b**2*c*f*g*h*m*x + (a + b*x)**m*b**2*d*e*g*h*m**2*x + 2
*(a + b*x)**m*b**2*d*e*g*h*m*x - (a + b*x)**m*b**2*d*f*g**2*m**2*x - 2*(a
+ b*x)**m*b**2*d*f*g**2*m*x + (a + b*x)**m*b**2*d*f*g*h*m**2*x**2 + (a + b
*x)**m*b**2*d*f*g*h*m*x**2 - int(((a + b*x)**m*x)/(a*g + a*h*x + b*g*x + b
*h*x**2),x)*a*b**2*c*e*h**3*m**3 - 3*int(((a + b*x)**m*x)/(a*g + a*h*x + b
*g*x + b*h*x**2),x)*a*b**2*c*e*h**3*m**2 - 2*int(((a + b*x)**m*x)/(a*g + a
*h*x + b*g*x + b*h*x**2),x)*a*b**2*c*e*h**3*m + int(((a + b*x)**m*x)/(a*g
+ a*h*x + b*g*x + b*h*x**2),x)*a*b**2*c*f*g*h**2*m**3 + 3*int(((a + b*x)**
m*x)/(a*g + a*h*x + b*g*x + b*h*x**2),x)*a*b**2*c*f*g*h**2*m**2 + 2*int(((
a + b*x)**m*x)/(a*g + a*h*x + b*g*x + b*h*x**2),x)*a*b**2*c*f*g*h**2*m + i
nt(((a + b*x)**m*x)/(a*g + a*h*x + b*g*x + b*h*x**2),x)*a*b**2*d*e*g*h**2*
m**3 + 3*int(((a + b*x)**m*x)/(a*g + a*h*x + b*g*x + b*h*x**2),x)*a*b**2*d
*e*g*h**2*m**2 + 2*int(((a + b*x)**m*x)/(a*g + a*h*x + b*g*x + b*h*x**2),x
)*a*b**2*d*e*g*h**2*m - int(((a + b*x)**m*x)/(a*g + a*h*x + b*g*x + b*h*x*
*2),x)*a*b**2*d*f*g**2*h*m**3 - 3*int(((a + b*x)**m*x)/(a*g + a*h*x + b...
```

3.132 $\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = -\frac{(de-cf)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)} + \frac{(dg-ch)(a+bx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(fg-eh)(1+m)}$$

output
```
-(-c*f+d*e)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*x+a)/(-a*f+b*e))/
(-a*f+b*e)/(-e*h+f*g)/(1+m)+(-c*h+d*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2
+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx$$
$$=\frac{(a+bx)^{1+m}\left(-\frac{(de-cf)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{be-af}+\frac{(dg-ch)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{h(a+bx)}{-bg+ah}\right)}{bg-ah}\right)}{(fg-eh)(1+m)}$$

input
```
Integrate[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*(-(((d*e - c*f)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(
a + b*x))/(-(b*e) + a*f)])/(b*e - a*f)) + ((d*g - c*h)*Hypergeometric2F1[1
, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/(b*g - a*h)))/((f*g - e*h)*
(1 + m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {174, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(a+bx)^m}{(e+fx)(g+hx)}\,dx$$
$$\Big\downarrow \; 174$$
$$\frac{(dg-ch)\int \frac{(a+bx)^m}{g+hx}dx}{fg-eh}-\frac{(de-cf)\int \frac{(a+bx)^m}{e+fx}dx}{fg-eh}$$
$$\Big\downarrow \; 78$$

$$\frac{(a+bx)^{m+1}(dg-ch)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

input
```
Int[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

output
```
-(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(
a + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)
*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g
 - a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)}{(fx+e)(hx+g)}dx$$

input
```
int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h)*x), x)
```

Sympy [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(a+bx)^m\,(c+dx)}{(e+fx)\,(g+hx)}\,dx$$

input
```
integrate((b*x+a)**m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Integral((a + b*x)**m*(c + d*x)/((e + f*x)*(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(dx+c)(bx+a)^m}{(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m\,(c+d\,x)}{(e+f\,x)\,(g+h\,x)}\,dx$$

input
```
int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

output
```
int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

output

```
((a + b*x)**m*a*d + (a + b*x)**m*b*c + int((a + b*x)**m/(a*e**2*g*h + a*e*
*2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x +
 a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*
f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*
c*e**2*h**2*m + 2*int((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**
2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 + b*
e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*h*
*2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*c*e*f*g*h*m + int((a
+ b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f
*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*
x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**
2 + b*f**2*g*h*x**3),x)*a*b*c*f**2*g**2*m - int((a + b*x)**m/(a*e**2*g*h +
 a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**
2*x + a*f**2*g*h*x**2 + b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2
*b*e*f*g*h*x**2 + b*e*f*h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)
*a*b*d*e**2*g*h*m - int((a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g
**2 + 2*a*e*f*g*h*x + a*e*f*h**2*x**2 + a*f**2*g**2*x + a*f**2*g*h*x**2 +
b*e**2*g*h*x + b*e**2*h**2*x**2 + b*e*f*g**2*x + 2*b*e*f*g*h*x**2 + b*e*f*
h**2*x**3 + b*f**2*g**2*x**2 + b*f**2*g*h*x**3),x)*a*b*d*e*f*g**2*m - int(
(a + b*x)**m/(a*e**2*g*h + a*e**2*h**2*x + a*e*f*g**2 + 2*a*e*f*g*h*x +...
```

3.133 $\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 224

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \frac{d^2(a+bx)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)} - \frac{f^2(a+bx)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{f(a+bx)}{be-af}\right)}{(be-af)(de-cf)(fg-eh)(1+m)} + \frac{h^2(a+bx)^{1+m}\operatorname{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{h(a+bx)}{bg-ah}\right)}{(bg-ah)(dg-ch)(fg-eh)(1+m)}$$

output
```
d^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*
c)/(-c*f+d*e)/(-c*h+d*g)/(1+m)-f^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],
-f*(b*x+a)/(-a*f+b*e))/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)/(1+m)+h^2*(b*x+a)^
(1+m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-c*h+d*g
)/(-e*h+f*g)/(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{(a+bx)^{1+m}\left(\frac{d^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)(-de+cf)(-dg+ch)}+\frac{f^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{f(a+bx)}{-be+af}\right)}{(be-af)(de-cf)(-fg+eh)}+\frac{h^2\,\text{Hypergeometric}}{(bg-ah)}\right.}{1+m}$$

input
```
Integrate[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
((a + b*x)^(1 + m)*((d^2*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/
(-(b*c) + a*d)])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) + (f^2*Hyperg
eometric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a*f)])/((b*e - a*f)*(
d*e - c*f)*(-(f*g) + e*h)) + (h^2*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a
 + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(d*g - c*h)*(f*g - e*h))))/(1 + m)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(\frac{d^2(a+bx)^m}{(c+dx)(de-cf)(dg-ch)}+\frac{f^2(a+bx)^m}{(e+fx)(de-cf)(eh-fg)}+\frac{h^2(a+bx)^m}{(g+hx)(dg-ch)(fg-eh)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{d^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(dg-ch)(fg-eh)}$$

input
```
Int[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
(d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/
(b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b
*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f)
)])/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)
*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g -
 a*h)*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m}{(xd+c)\,(fx+e)\,(hx+g)}dx$$

input
```
int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*x + a)^m/(d*f*h*x^3 + c*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c
*e*h + (d*e + c*f)*g)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{(dx+c)(fx+e)(hx+g)}\,dx$$

input
```
integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(a+b\,x)^m}{(e+f\,x)\,(g+h\,x)\,(c+d\,x)}\,dx$$

input
```
int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

output
```
int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx = \int \frac{(bx+a)^m}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx$$

input
```
int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
int((a + b*x)**m/(c*e*g + c*e*h*x + c*f*g*x + c*f*h*x**2 + d*e*g*x + d*e*h
*x**2 + d*f*g*x**2 + d*f*h*x**3),x)
```

3.134 $\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 272

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = -\frac{(bcfh(2+m)-bd(fg+eh)(3+m+n)+adfh(4+m+2n))(a+bx)^{1+m}(c+dx)^{1+n}}{b^2d^2(2+m+n)(3+m+n)} + \frac{fh(a+bx)^{2+m}(c+dx)^{1+n}}{b^2d(3+m+n)} + \frac{((bc(1+m)+ad(1+n))(bcfh(2+m)-bd(fg+eh)(3+m+n)+adfh(4+m+2n))+d(2+m-}{b^2d^2(bc}$$

output

```
-(b*c*f*h*(2+m)-b*d*(e*h+f*g)*(3+m+n)+a*d*f*h*(4+m+2*n))*(b*x+a)^(1+m)*(d*
x+c)^(1+n)/b^2/d^2/(2+m+n)/(3+m+n)+f*h*(b*x+a)^(2+m)*(d*x+c)^(1+n)/b^2/d/(
3+m+n)+((b*c*(1+m)+a*d*(1+n))*(b*c*f*h*(2+m)-b*d*(e*h+f*g)*(3+m+n)+a*d*f*h
*(4+m+2*n))+d*(2+m+n)*(b^2*d*e*g*(3+m+n)-a*f*h*(b*c*(2+m)+a*d*(1+n))))*(b*
x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))
/b^2/d^2/(-a*d+b*c)/(1+m)/(2+m+n)/(3+m+n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.72

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{d(a+bx)}{-bc+ad}\right)+\right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^2*f*h*Hypergeometric2F1[1 + m,
 -2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h
- d*(f*g + e*h))*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(
b*c) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[1 + m, -n, 2 +
 m, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c
- a*d))^n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m(c+dx)^n\,dx$$
$$\Big\downarrow \text{164}$$
$$\frac{\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d(m+n+3)(eh+fg))+b^2\left(c^2fh(m+1)(m+2)-cd(m\right.\right.}{b^2d^2(m+n+2)(m+n+3}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$
$$\Big\downarrow \text{80}$$

$$\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m+1)-d(m+n+3)(eh+fg))+b^2(c^2fh(\right.}{b^2d^2(m+n}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

$\Big\downarrow$ 79

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+\right.}{b}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(adfh(n+2)+bcfh(m+2)-bd(m+n+3)(eh+fg)-bdfhx(m+n+2))}{b^2d^2(m+n+2)(m+n+3)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n)
 - b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m +
n)*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*
(1 + m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*
(f*g + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b
*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x)
)/(b*c - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(
b*c - a*d))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e) (hx+g) dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx) dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^n dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx=\int (e+f\,x)\,\left(g+h\,x\right)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n\,dx$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx=\text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

output
```
((c + d*x)**n*(a + b*x)**m*a**3*c*d**2*f*h*m*n + 2*(c + d*x)**n*(a + b*x)*
*m*a**3*c*d**2*f*h*m - (c + d*x)**n*(a + b*x)**m*a**3*d**3*f*h*m*n**2*x -
2*(c + d*x)**n*(a + b*x)**m*a**3*d**3*f*h*m*n*x - 2*(c + d*x)**n*(a + b*x)
**m*a**2*b*c**2*d*f*h*m*n - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*e*h*m*
*2 - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*e*h*m*n - 3*(c + d*x)**n*(a +
 b*x)**m*a**2*b*c*d**2*e*h*m - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*g
*m**2 - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*g*m*n - 3*(c + d*x)**n*(
a + b*x)**m*a**2*b*c*d**2*f*g*m - (c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*
f*h*m**2*n*x - 2*(c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*h*m**2*x + 2*(c
 + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f*h*m*n**2*x + (c + d*x)**n*(a + b*x
)**m*a**2*b*d**3*e*h*m**2*n*x + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*h*
m*n**2*x + 3*(c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*h*m*n*x + (c + d*x)**
n*(a + b*x)**m*a**2*b*d**3*f*g*m**2*n*x + (c + d*x)**n*(a + b*x)**m*a**2*b
*d**3*f*g*m*n**2*x + 3*(c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f*g*m*n*x + (
c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f*h*m**2*n*x**2 + (c + d*x)**n*(a + b
*x)**m*a**2*b*d**3*f*h*m*n**2*x**2 + (c + d*x)**n*(a + b*x)**m*a**2*b*d**3
*f*h*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*a*b**2*c**3*f*h*m*n + 2*(c + d*x
)**n*(a + b*x)**m*a*b**2*c**3*f*h*n - (c + d*x)**n*(a + b*x)**m*a*b**2*c**
2*d*e*h*m*n - (c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*e*h*n**2 - 3*(c + d*
x)**n*(a + b*x)**m*a*b**2*c**2*d*e*h*n - (c + d*x)**n*(a + b*x)**m*a*b*...
```

3.135 $\int (a+bx)^m (c+dx)^n (e+fx)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 131

$$\int (a+bx)^m (c+dx)^n (e+fx)\,dx = \frac{f(a+bx)^{1+m}(c+dx)^{1+n}}{bd(2+m+n)} - \frac{(bcf(1+m)+adf(1+n)-bde(2+m+n))(a+bx)^{1+m}(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,2+m+\right.}{bd(bc-ad)(1+m)(2+m+n)}$$

output
```
f*(b*x+a)^(1+m)*(d*x+c)^(1+n)/b/d/(2+m+n)-(b*c*f*(1+m)+a*d*f*(1+n)-b*d*e*(
2+m+n))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/
(-a*d+b*c))/b/d/(-a*d+b*c)/(1+m)/(2+m+n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int (a+bx)^m (c+dx)^n (e+fx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^n \left(bf(c+dx) + \frac{(-bcf(1+m)-adf(1+n)+bde(2+m+n))\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(1+m,-n,2+m\right.}{1+m}\right.}{b^2d(2+m+n)}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(b*f*(c + d*x) + ((-(b*c*f*(1 + m)) - a*d*f
*(1 + n) + b*d*e*(2 + m + n))*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a +
b*x))/(-(b*c) + a*d)])/((1 + m)*((b*(c + d*x))/(b*c - a*d))^n)))/(b^2*d*(2
 + m + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(a+bx)^m(c+dx)^n\,dx$$

$$\Big\downarrow \text{90}$$

$$\left(e-\frac{f(ad(n+1)+bc(m+1))}{bd(m+n+2)}\right)\int (a+bx)^m(c+dx)^n dx+\frac{f(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+2)}$$

$$\Big\downarrow \text{80}$$

$$(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(e-\frac{f(ad(n+1)+bc(m+1))}{bd(m+n+2)}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx+\frac{f(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+2)}$$

$$\Big\downarrow \text{79}$$

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(e-\frac{f(ad(n+1)+bc(m+1))}{bd(m+n+2)}\right)\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{b(m+1)}+\frac{f(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+2)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x),x]
```

output
```
(f*(a + b*x)^(1 + m)*(c + d*x)^(1 + n))/(b*d*(2 + m + n)) + ((e - (f*(b*c*
(1 + m) + a*d*(1 + n)))/(b*d*(2 + m + n)))*(a + b*x)^(1 + m)*(c + d*x)^n*H
ypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 +
m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

Maple [F]

$$\int (bx + a)^m (xd + c)^n (fx + e)\, dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)\,dx = \int (fx+e)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e),x, algorithm="fricas")
```

output
```
integral((f*x + e)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n(e+fx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)\,dx = \int (fx+e)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)\,dx=\int (fx+e)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)\,dx=\int (e+f\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n\,dx$$

input
```
int((e + f*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)\,dx=\text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e),x)
```

output
```
( - (c + d*x)**n*(a + b*x)**m*a**2*c*d*f*m + (c + d*x)**n*(a + b*x)**m*a**
2*d**2*f*m*n*x - (c + d*x)**n*(a + b*x)**m*a*b*c**2*f*n + (c + d*x)**n*(a
+ b*x)**m*a*b*c*d*e*m**2 + 2*(c + d*x)**n*(a + b*x)**m*a*b*c*d*e*m*n + 2*(
c + d*x)**n*(a + b*x)**m*a*b*c*d*e*m + (c + d*x)**n*(a + b*x)**m*a*b*c*d*e
*n**2 + 2*(c + d*x)**n*(a + b*x)**m*a*b*c*d*e*n + (c + d*x)**n*(a + b*x)**
m*a*b*c*d*f*m**2*x + (c + d*x)**n*(a + b*x)**m*a*b*c*d*f*n**2*x + (c + d*x
)**n*(a + b*x)**m*a*b*d**2*e*m*n*x + (c + d*x)**n*(a + b*x)**m*a*b*d**2*e*
n**2*x + 2*(c + d*x)**n*(a + b*x)**m*a*b*d**2*e*n*x + (c + d*x)**n*(a + b*
x)**m*a*b*d**2*f*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*a*b*d**2*f*n**2*x**2
 + (c + d*x)**n*(a + b*x)**m*a*b*d**2*f*n*x**2 + (c + d*x)**n*(a + b*x)**m
*b**2*c**2*f*m*n*x + (c + d*x)**n*(a + b*x)**m*b**2*c*d*e*m**2*x + (c + d*
x)**n*(a + b*x)**m*b**2*c*d*e*m*n*x + 2*(c + d*x)**n*(a + b*x)**m*b**2*c*d
*e*m*x + (c + d*x)**n*(a + b*x)**m*b**2*c*d*f*m**2*x**2 + (c + d*x)**n*(a
+ b*x)**m*b**2*c*d*f*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*b**2*c*d*f*m*x**
2 - int(((c + d*x)**n*(a + b*x)**m*x)/(a**2*c*d*m**2*n + 2*a**2*c*d*m*n**2
 + 3*a**2*c*d*m*n + a**2*c*d*n**3 + 3*a**2*c*d*n**2 + 2*a**2*c*d*n + a**2*
d**2*m**2*n*x + 2*a**2*d**2*m*n**2*x + 3*a**2*d**2*m*n*x + a**2*d**2*n**3*
x + 3*a**2*d**2*n**2*x + 2*a**2*d**2*n*x + a*b*c**2*m**3 + 2*a*b*c**2*m**2
*n + 3*a*b*c**2*m**2 + a*b*c**2*m*n**2 + 3*a*b*c**2*m*n + 2*a*b*c**2*m + a
*b*c*d*m**3*x + 3*a*b*c*d*m**2*n*x + 3*a*b*c*d*m**2*x + 3*a*b*c*d*m*n**...
```

3.136 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 178

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx = -\frac{(fg-eh)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h(bg-ah)(1+m)} + \frac{f(a+bx)^{1+m}(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,2+m+n,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)h(1+m)}$$

output
```
-(-e*h+f*g)*(b*x+a)^(1+m)*(d*x+c)^n*AppellF1(1+m,-n,1,2+m,-d*(b*x+a)/(-a*d
+b*c),-h*(b*x+a)/(-a*h+b*g))/h/(-a*h+b*g)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)
+f*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d
+b*c))/(-a*d+b*c)/h/(1+m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx$$
$$= \frac{(a+bx)^m(c+dx)^n\left(-dfg(1+n)(a+bx)\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,\frac{d(a+bx)}{-bc+ad},\frac{h(a+bx)}{-bg+ah}\right)\right.}{}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x))/(g + h*x),x]
```

output
```
((a + b*x)^m*(c + d*x)^n*(-((d*f*g*(1 + n)*(a + b*x)*AppellF1[1 + m, -n, 1
, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (h*(a + b*x))/(-(b*g) + a*h)])/((b*
(c + d*x))/(b*c - a*d))^n) + (d*e*h*(1 + n)*(a + b*x)*AppellF1[1 + m, -n,
1, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (h*(a + b*x))/(-(b*g) + a*h)])/((b
*(c + d*x))/(b*c - a*d))^n + (f*(b*g - a*h)*(1 + m)*(c + d*x)*Hypergeometr
ic2F1[-m, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/((d*(a + b*x))/(-(b*c)
 + a*d))^m))/(d*h*(b*g - a*h)*(1 + m)*(1 + n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {175, 80, 79, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)(a+bx)^m(c+dx)^n}{g+hx}\,dx$$

$$\Big\downarrow \text{175}$$

$$\frac{f\int (a+bx)^m(c+dx)^n dx}{h} - \frac{(fg-eh)\int \frac{(a+bx)^m(c+dx)^n}{g+hx}dx}{h}$$

$$\Big\downarrow \text{80}$$

$$\frac{f(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx}{h}-\frac{(fg-eh)\int\frac{(a+bx)^m(c+dx)^n}{g+hx}dx}{h}$$

$$\Big\downarrow \; 79$$

$$\frac{f(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{bh(m+1)}-\frac{(fg-eh)\int\frac{(a+bx)^m(c+dx)^n}{g+hx}dx}{h}$$

$$\Big\downarrow \; 154$$

$$\frac{f(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{bh(m+1)}-\frac{(c+dx)^n(fg-eh)\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int\frac{(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n}{g+hx}dx}{h}$$

$$\Big\downarrow \; 153$$

$$\frac{f(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{bh(m+1)}-\frac{(a+bx)^{m+1}(c+dx)^n(fg-eh)\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h(m+1)(bg-ah)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x))/(g + h*x),x]
```

output
```
-(((f*g - e*h)*(a + b*x)^(1 + m)*(c + d*x)^n*AppellF1[1 + m, -n, 1, 2 + m,
 -((d*(a + b*x))/(b*c - a*d)), -((h*(a + b*x))/(b*g - a*h))])/(h*(b*g - a*
h)*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)) + (f*(a + b*x)^(1 + m)*(c + d*x
)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*
h*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)}{hx+g} dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx} dx = \int \frac{(fx+e)(bx+a)^m(dx+c)^n}{hx+g} dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h*x + g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx} dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx=\int \frac{(fx+e)(bx+a)^m(dx+c)^n}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h*x + g), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx=\int \frac{(fx+e)(bx+a)^m(dx+c)^n}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx=\int \frac{(e+f\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{g+h\,x}\,dx$$

input
```
int(((e + f*x)*(a + b*x)^m*(c + d*x)^n)/(g + h*x),x)
```

output
```
int(((e + f*x)*(a + b*x)^m*(c + d*x)^n)/(g + h*x), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{g+hx}\,dx = \int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)}{hx+g}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g),x)
```

3.137 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \frac{(fg-eh)(a+bx)^{1+m}(c+dx)^{1+n}}{(bg-ah)(dg-ch)(g+hx)}$$

$$+\frac{(ah(cfh+dehn-dfg(1+n))+b(ch(ehm-fg(1+m))-dg(eh(m+n)-fg(1+m+n))))(a+b}{h(bg-ah)^2(dg-ch)(1+}$$

$$-\frac{bd(fg-eh)(1+m+n)(a+bx)^{1+m}(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,2+m+n,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)h(bg-ah)(dg-ch)(1+m)}$$

output
```
(-e*h+f*g)*(b*x+a)^(1+m)*(d*x+c)^(1+n)/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)+(a*h*
(c*f*h+d*e*h*n-d*f*g*(1+n))+b*(c*h*(e*h*m-f*g*(1+m))-d*g*(e*h*(m+n)-f*g*(1
+m+n))))*(b*x+a)^(1+m)*(d*x+c)^n*AppellF1(1+m,-n,1,2+m,-d*(b*x+a)/(-a*d+b*
c),-h*(b*x+a)/(-a*h+b*g))/h/(-a*h+b*g)^2/(-c*h+d*g)/(1+m)/((b*(d*x+c)/(-a*
d+b*c))^n)-b*d*(-e*h+f*g)*(1+m+n)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1
, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/h/(-a*h+b*g)/(-c*h+d*g)/(
1+m)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(f(bg-ah)\operatorname{AppellF1}\left(1+m,-n,1,2+m,\frac{d(a+bx)}{-bc+ad},\frac{h(a+bx)}{-bg+ah}\right)+b(-f\right.}{h(bg-ah)^2(1+m)}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x))/(g + h*x)^2,x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(f*(b*g - a*h)*AppellF1[1 + m, -n, 1, 2 + m
, (d*(a + b*x))/(-(b*c) + a*d), (h*(a + b*x))/(-(b*g) + a*h)] + b*(-(f*g)
+ e*h)*AppellF1[1 + m, -n, 2, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (h*(a +
 b*x))/(-(b*g) + a*h)]))/(h*(b*g - a*h)^2*(1 + m)*((b*(c + d*x))/(b*c - a*
d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {168, 25, 175, 80, 79, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)(a+bx)^m(c+dx)^n}{(g+hx)^2}\,dx$$
$$\Big\downarrow \; 168$$
$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}-\frac{\int -\frac{(a+bx)^m(c+dx)^n(bdeg-(bc+ad)eh+acfh-(fg-eh)(bc(m+1)+ad(n+1))-bd(fg-eh)(m+n+1)x)}{g+hx}dx}{(bg-ah)(dg-ch)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(a+bx)^m(c+dx)^n(bdeg-(bc+ad)eh+acfh-(fg-eh)(bc(m+1)+ad(n+1))-bd(fg-eh)(m+n+1)x)}{g+hx}dx}{(bg-ah)(dg-ch)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}$$

$\Big\downarrow$ 175

$$\frac{\frac{(ah(cfh+dehn-dfg(n+1))+b(ch(ehm-fg(m+1))-dg(eh(m+n)-fg(m+n+1))))\int \frac{(a+bx)^m(c+dx)^n}{g+hx}dx}{h}-\frac{bd(m+n+1)(fg-eh)\int(a+bx)^m(c}{h}}{(bg-ah)(dg-ch)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}$$

$\Big\downarrow$ 80

$$\frac{\frac{(ah(cfh+dehn-dfg(n+1))+b(ch(ehm-fg(m+1))-dg(eh(m+n)-fg(m+n+1))))\int \frac{(a+bx)^m(c+dx)^n}{g+hx}dx}{h}-\frac{bd(m+n+1)(c+dx)^n(fg-eh)\left(\frac{b(c}{bc}\right.}{}}{(bg-ah)(dg-ch)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}$$

$\Big\downarrow$ 79

$$\frac{\frac{(ah(cfh+dehn-dfg(n+1))+b(ch(ehm-fg(m+1))-dg(eh(m+n)-fg(m+n+1))))\int \frac{(a+bx)^m(c+dx)^n}{g+hx}dx}{h}-\frac{d(m+n+1)(a+bx)^{m+1}(c+dx)^n(f}{}}{(bg-ah)(dg-ch)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}$$

$\Big\downarrow$ 154

$$\frac{\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(ah(cfh+dehn-dfg(n+1))+b(ch(ehm-fg(m+1))-dg(eh(m+n)-fg(m+n+1))))\int \frac{(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n}{g+hx}dx}{h}-\frac{d(}{}}{(bg-ah)(dg-ch)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}$$

$\Big\downarrow$ 153

$$\frac{\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)(ah(cfh+dehn-dfg(n+1))+b(ch(ehm-fg(m+1))-dg(eh}{h(m+1)(bg-ah)}}{(bg-ah)(dg-}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{(g+hx)(bg-ah)(dg-ch)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x))/(g + h*x)^2,x]
```

output
```
((f*g - e*h)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n))/((b*g - a*h)*(d*g - c*h)
*(g + h*x)) + (((a*h*(c*f*h + d*e*h*n - d*f*g*(1 + n)) + b*(c*h*(e*h*m - f
*g*(1 + m)) - d*g*(e*h*(m + n) - f*g*(1 + m + n))))*(a + b*x)^(1 + m)*(c +
 d*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((h*(
a + b*x))/(b*g - a*h))])/(h*(b*g - a*h)*(1 + m)*((b*(c + d*x))/(b*c - a*d)
)^n) - (d*(f*g - e*h)*(1 + m + n)*(a + b*x)^(1 + m)*(c + d*x)^n*Hypergeome
tric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(h*(1 + m)*((b*(c
 + d*x))/(b*c - a*d))^n))/((b*g - a*h)*(d*g - c*h))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)}{(hx+g)^2} dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^2,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \int \frac{(fx+e)(bx+a)^m(dx+c)^n}{(hx+g)^2}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h^2*x^2 + 2*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \int \frac{(fx+e)(bx+a)^m(dx+c)^n}{(hx+g)^2}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h*x + g)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,1,1,1,0,0]%%%}+%%%{1,[0,1,1,0,0,1,1]%%%} / %%%{1,[
0,0,0,0,0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \int \frac{(e+f\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(g+h\,x)^2}\,dx$$

input
```
int(((e + f*x)*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^2,x)
```

output
```
int(((e + f*x)*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^2}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^2,x)
```

output
```
((c + d*x)**n*(a + b*x)**m*a*c*f*h*m + (c + d*x)**n*(a + b*x)**m*a*c*f*h*n
 + (c + d*x)**n*(a + b*x)**m*a*d*e*h*m + (c + d*x)**n*(a + b*x)**m*a*d*e*h
*n - (c + d*x)**n*(a + b*x)**m*a*d*f*g*n - (c + d*x)**n*(a + b*x)**m*a*d*f
*g + (c + d*x)**n*(a + b*x)**m*a*d*f*h*n*x - (c + d*x)**n*(a + b*x)**m*a*d
*f*h*x + (c + d*x)**n*(a + b*x)**m*b*c*e*h*m + (c + d*x)**n*(a + b*x)**m*b
*c*e*h*n - (c + d*x)**n*(a + b*x)**m*b*c*f*g*m - (c + d*x)**n*(a + b*x)**m
*b*c*f*g + (c + d*x)**n*(a + b*x)**m*b*c*f*h*m*x - (c + d*x)**n*(a + b*x)*
*m*b*c*f*h*x + (c + d*x)**n*(a + b*x)**m*b*d*f*g*m*x + (c + d*x)**n*(a + b
*x)**m*b*d*f*g*n*x + int(((c + d*x)**n*(a + b*x)**m*x**2)/(a**2*c*d*g**2*h
*m*n - a**2*c*d*g**2*h*m + a**2*c*d*g**2*h*n**2 - a**2*c*d*g**2*h*n + 2*a*
*2*c*d*g*h**2*m*n*x - 2*a**2*c*d*g*h**2*m*x + 2*a**2*c*d*g*h**2*n**2*x - 2
*a**2*c*d*g*h**2*n*x + a**2*c*d*h**3*m*n*x**2 - a**2*c*d*h**3*m*x**2 + a**
2*c*d*h**3*n**2*x**2 - a**2*c*d*h**3*n*x**2 + a**2*d**2*g**2*h*m*n*x - a**
2*d**2*g**2*h*m*x + a**2*d**2*g**2*h*n**2*x - a**2*d**2*g**2*h*n*x + 2*a**
2*d**2*g*h**2*m*n*x**2 - 2*a**2*d**2*g*h**2*m*x**2 + 2*a**2*d**2*g*h**2*n*
*2*x**2 - 2*a**2*d**2*g*h**2*n*x**2 + a**2*d**2*h**3*m*n*x**3 - a**2*d**2*
h**3*m*x**3 + a**2*d**2*h**3*n**2*x**3 - a**2*d**2*h**3*n*x**3 + a*b*c**2*
g**2*h*m**2 + a*b*c**2*g**2*h*m*n - a*b*c**2*g**2*h*m - a*b*c**2*g**2*h*n
+ 2*a*b*c**2*g*h**2*m**2*x + 2*a*b*c**2*g*h**2*m*n*x - 2*a*b*c**2*g*h**2*m
*x - 2*a*b*c**2*g*h**2*n*x + a*b*c**2*h**3*m**2*x**2 + a*b*c**2*h**3*m*...
```

3.138 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 645

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx = \frac{(fg-eh)(a+bx)^{1+m}(c+dx)^{1+n}}{2(bg-ah)(dg-ch)(g+hx)^2}$$

$$-\frac{(ah(2cfh-deh(1-n)-dfg(1+n))-b(ch(eh(1-m)+fg(1+m))-dg(eh(2-m-n)+fg(m}{2(bg-ah)^2(dg-ch)^2(g+hx)}$$

$$+\frac{(a^2dh^2n(2cfh-deh(1-n)-dfg(1+n))-b^2(c^2h^2m(eh(1-m)+fg(1+m))-2cdgh(ehm(1-n}{}$$

$$+\frac{bd(1+m+n)(ah(2cfh-deh(1-n)-dfg(1+n))-b(ch(eh(1-m)+fg(1+m))-dg(eh(2-m}{2(bc-ad)h(bg-ah)}$$

output
```
1/2*(-e*h+f*g)*(b*x+a)^(1+m)*(d*x+c)^(1+n)/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^2
-1/2*(a*h*(2*c*f*h-d*e*h*(1-n)-d*f*g*(1+n))-b*(c*h*(e*h*(1-m)+f*g*(1+m))-d
*g*(e*h*(2-m-n)+f*g*(m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)/(-a*h+b*g)^2/(-c*
h+d*g)^2/(h*x+g)+1/2*(a^2*d*h^2*n*(2*c*f*h-d*e*h*(1-n)-d*f*g*(1+n))-b^2*(c
^2*h^2*m*(e*h*(1-m)+f*g*(1+m))-2*c*d*g*h*(e*h*m*(1-m-n)+f*g*(1+m)*(m+n))+d
^2*g^2*(m+n)*(e*h*(1-m-n)+f*g*(1+m+n)))+2*a*b*h*(c^2*f*h^2*m+d^2*g*(e*h*(1
-m-n)*n+f*g*(1+n)*(m+n))+c*d*h*(e*h*m*n-f*g*(2*n+m*(2+n)))))*(b*x+a)^(1+m)
*(d*x+c)^n*AppellF1(1+m,-n,1,2+m,-d*(b*x+a)/(-a*d+b*c),-h*(b*x+a)/(-a*h+b*
g))/h/(-a*h+b*g)^3/(-c*h+d*g)^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)+1/2*b*d*(
1+m+n)*(a*h*(2*c*f*h-d*e*h*(1-n)-d*f*g*(1+n))-b*(c*h*(e*h*(1-m)+f*g*(1+m))
-d*g*(e*h*(2-m-n)+f*g*(m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2
+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/h/(-a*h+b*g)^2/(-c*h+d*g)^2/
(1+m)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.26

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx$$
$$= \frac{b(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(f(bg-ah)\operatorname{AppellF1}\left(1+m,-n,2,2+m,\frac{d(a+bx)}{-bc+ad},\frac{h(a+bx)}{-bg+ah}\right)+b(-\right.}{h(bg-ah)^3(1+m)}$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x))/(g + h*x)^3,x]
```

output
```
(b*(a + b*x)^(1 + m)*(c + d*x)^n*(f*(b*g - a*h)*AppellF1[1 + m, -n, 2, 2 +
 m, (d*(a + b*x))/(-(b*c) + a*d), (h*(a + b*x))/(-(b*g) + a*h)] + b*(-(f*g
) + e*h)*AppellF1[1 + m, -n, 3, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (h*(a
 + b*x))/(-(b*g) + a*h)]))/(h*(b*g - a*h)^3*(1 + m)*((b*(c + d*x))/(b*c -
a*d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {168, 25, 168, 25, 175, 80, 79, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)(a+bx)^m(c+dx)^n}{(g+hx)^3}\,dx$$

$$\Big\downarrow \ 168$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}-\frac{\int -\frac{(a+bx)^m(c+dx)^n(2(bdeg-(bc+ad)eh+acfh)-(fg-eh)(bc(m+1)+ad(n+1))-bd(fg-eh)(m+n)x)}{(g+hx)^2}dx}{2(bg-ah)(dg-ch)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(a+bx)^m(c+dx)^n(b(2deg-cf(m+1)g-ceh(1-m))+a(2cfh-de(1-n)h-dfg(n+1))-bd(fg-eh)(m+n)x)}{(g+hx)^2}dx}{2(bg-ah)(dg-ch)}+\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$$\Big\downarrow \ 168$$

$$-\int -\frac{(a+bx)^m(c+dx)^n\Big(dhn(2cfh-de(1-n)h-dfg(n+1))a^2+b\Big(-g\Big(fg(-m-n+1)(n+1)+eh\Big((n-1)^2+m(n+1)\Big)\Big)d^2+2ch(ehmn+fg(-nm-m-n+1))d+2c^2f$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$$\Big\downarrow \ 25$$

$$\int \frac{(a+bx)^m(c+dx)^n\Big(dhn(2cfh-de(1-n)h-dfg(n+1))a^2+b\Big(-g\Big(fg(-m-n+1)(n+1)+eh\Big((n-1)^2+m(n+1)\Big)\Big)d^2+2ch(ehmn+fg(-nm-m-n+1))d+2c^2fh^2m$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$$\Big\downarrow \ 175$$

$$\frac{\frac{\left(a^2dh^2n(2cfh-deh(1-n)-dfg(n+1))+2abh\left(c^2fh^2m+cdh(ehmn-fg(m(n+2)+2n))+d^2g(ehn(-m-n+1)+fg(n+1)(m+n))\right)-\left(b^2\left(c^2h^2m(eh(1-m)+fg(m+\right.\right.}{h}}{}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$\downarrow$ 80

$$\frac{\frac{\left(a^2dh^2n(2cfh-deh(1-n)-dfg(n+1))+2abh\left(c^2fh^2m+cdh(ehmn-fg(m(n+2)+2n))+d^2g(ehn(-m-n+1)+fg(n+1)(m+n))\right)-\left(b^2\left(c^2h^2m(eh(1-m)+fg(m+\right.\right.}{h}}{}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$\downarrow$ 79

$$\frac{\frac{\left(a^2dh^2n(2cfh-deh(1-n)-dfg(n+1))+2abh\left(c^2fh^2m+cdh(ehmn-fg(m(n+2)+2n))+d^2g(ehn(-m-n+1)+fg(n+1)(m+n))\right)-\left(b^2\left(c^2h^2m(eh(1-m)+fg(m+\right.\right.}{h}}{}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$\downarrow$ 154

$$\frac{\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(a^2dh^2n(2cfh-deh(1-n)-dfg(n+1))+2abh\left(c^2fh^2m+cdh(ehmn-fg(m(n+2)+2n))+d^2g(ehn(-m-n+1)+fg(n+1)(m+n))\right)-\left(b^2\left(c^2\right.\right.}{h}}{}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

$\downarrow$ 153

$$\frac{\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)\left(a^2dh^2n(2cfh-deh(1-n)-dfg(n+1))+2abh\left(c^2fh^2m+cdh(ehmn-fg(\right.\right.}{}}{}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{n+1}(fg-eh)}{2(g+hx)^2(bg-ah)(dg-ch)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x))/(g + h*x)^3,x]
```

output
```
((f*g - e*h)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n))/(2*(b*g - a*h)*(d*g - c*
h)*(g + h*x)^2) + (-(((a*h*(2*c*f*h - d*e*h*(1 - n) - d*f*g*(1 + n)) - b*(
c*h*(e*h*(1 - m) + f*g*(1 + m)) - d*g*(e*h*(2 - m - n) + f*g*(m + n))))*(a
 + b*x)^(1 + m)*(c + d*x)^(1 + n))/((b*g - a*h)*(d*g - c*h)*(g + h*x))) +
(((a^2*d*h^2*n*(2*c*f*h - d*e*h*(1 - n) - d*f*g*(1 + n)) - b^2*(c^2*h^2*m*
(e*h*(1 - m) + f*g*(1 + m)) - 2*c*d*g*h*(e*h*m*(1 - m - n) + f*g*(1 + m)*(
m + n)) + d^2*g^2*(m + n)*(e*h*(1 - m - n) + f*g*(1 + m + n))) + 2*a*b*h*(
c^2*f*h^2*m + d^2*g*(e*h*(1 - m - n)*n + f*g*(1 + n)*(m + n)) + c*d*h*(e*h
*m*n - f*g*(2*n + m*(2 + n)))))*(a + b*x)^(1 + m)*(c + d*x)^n*AppellF1[1 +
 m, -n, 1, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((h*(a + b*x))/(b*g - a*h
))])/(h*(b*g - a*h)*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n) + (d*(1 + m + n
)*(a*h*(2*c*f*h - d*e*h*(1 - n) - d*f*g*(1 + n)) - b*(c*h*(e*h*(1 - m) + f
*g*(1 + m)) - d*g*(e*h*(2 - m - n) + f*g*(m + n))))*(a + b*x)^(1 + m)*(c +
 d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])
/(h*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n))/((b*g - a*h)*(d*g - c*h)))/(2*
(b*g - a*h)*(d*g - c*h))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)}{(hx+g)^3}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^3,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx=\int \frac{(fx+e)(bx+a)^m(dx+c)^n}{(hx+g)^3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^3,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*
h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx=\int \frac{(fx+e)(bx+a)^m(dx+c)^n}{(hx+g)^3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h*x + g)^3, x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx = \int \frac{(fx+e)(bx+a)^m(dx+c)^n}{(hx+g)^3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*x + a)^m*(d*x + c)^n/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx = \int \frac{(e+f\,x)\,(a+b\,x)^m\,(c+d\,x)^n}{(g+h\,x)^3}\,dx$$

input
```
int(((e + f*x)*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^3,x)
```

output
```
int(((e + f*x)*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)}{(g+hx)^3}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)/(h*x+g)^3,x)
```

output
```
((c + d*x)**n*(a + b*x)**m*a*c*f*h*m + (c + d*x)**n*(a + b*x)**m*a*c*f*h*n
 + (c + d*x)**n*(a + b*x)**m*a*d*e*h*m + (c + d*x)**n*(a + b*x)**m*a*d*e*h
*n - (c + d*x)**n*(a + b*x)**m*a*d*e*h - (c + d*x)**n*(a + b*x)**m*a*d*f*g
*n - (c + d*x)**n*(a + b*x)**m*a*d*f*g + (c + d*x)**n*(a + b*x)**m*a*d*f*h
*n*x - 2*(c + d*x)**n*(a + b*x)**m*a*d*f*h*x + (c + d*x)**n*(a + b*x)**m*b
*c*e*h*m + (c + d*x)**n*(a + b*x)**m*b*c*e*h*n - (c + d*x)**n*(a + b*x)**m
*b*c*e*h - (c + d*x)**n*(a + b*x)**m*b*c*f*g*m - (c + d*x)**n*(a + b*x)**m
*b*c*f*g + (c + d*x)**n*(a + b*x)**m*b*c*f*h*m*x - 2*(c + d*x)**n*(a + b*x
)**m*b*c*f*h*x + (c + d*x)**n*(a + b*x)**m*b*d*f*g*m*x + (c + d*x)**n*(a +
 b*x)**m*b*d*f*g*n*x + int(((c + d*x)**n*(a + b*x)**m*x**2)/(a**2*c*d*g**3
*h*m*n - 2*a**2*c*d*g**3*h*m + a**2*c*d*g**3*h*n**2 - 3*a**2*c*d*g**3*h*n
+ 2*a**2*c*d*g**3*h + 3*a**2*c*d*g**2*h**2*m*n*x - 6*a**2*c*d*g**2*h**2*m*
x + 3*a**2*c*d*g**2*h**2*n**2*x - 9*a**2*c*d*g**2*h**2*n*x + 6*a**2*c*d*g*
*2*h**2*x + 3*a**2*c*d*g*h**3*m*n*x**2 - 6*a**2*c*d*g*h**3*m*x**2 + 3*a**2
*c*d*g*h**3*n**2*x**2 - 9*a**2*c*d*g*h**3*n*x**2 + 6*a**2*c*d*g*h**3*x**2
+ a**2*c*d*h**4*m*n*x**3 - 2*a**2*c*d*h**4*m*x**3 + a**2*c*d*h**4*n**2*x**
3 - 3*a**2*c*d*h**4*n*x**3 + 2*a**2*c*d*h**4*x**3 + a**2*d**2*g**3*h*m*n*x
 - 2*a**2*d**2*g**3*h*m*x + a**2*d**2*g**3*h*n**2*x - 3*a**2*d**2*g**3*h*n
*x + 2*a**2*d**2*g**3*h*x + 3*a**2*d**2*g**2*h**2*m*n*x**2 - 6*a**2*d**2*g
**2*h**2*m*x**2 + 3*a**2*d**2*g**2*h**2*n**2*x**2 - 9*a**2*d**2*g**2*h*...
```

3.139 $\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 589

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx =$$

$$-\frac{(d(3+m+n)\left(2abcf^2h(3+m)-b^2de(2fg+eh)(4+m+n)+a^2df^2h(6+m+3n)\right)-f(bc(2+m}{b^3d^3(2+m+n}$$

$$-\frac{f(bcfh(3+m)-bd(fg+2eh)(4+m+n)+adfh(9+2m+3n))(a+bx)^{2+m}(c+dx)^{1+n}}{b^3d^2(3+m+n)(4+m+n)}$$

$$+\frac{f^2h(a+bx)^{3+m}(c+dx)^{1+n}}{b^3d(4+m+n)}$$

$$-\frac{(d(2+m+n)\left(d(3+m+n)\left(a^2bcf^2h(3+m)+a^3df^2h(1+n)-b^3de^2g(4+m+n)\right)-af(bc(2+m}{}$$

output
```
-(d*(3+m+n)*(2*a*b*c*f^2*h*(3+m)-b^2*d*e*(e*h+2*f*g)*(4+m+n)+a^2*d*f^2*h*(
6+m+3*n))-f*(b*c*(2+m)+a*d*(4+m+2*n))*(b*c*f*h*(3+m)-b*d*(2*e*h+f*g)*(4+m+
n)+a*d*f*h*(9+2*m+3*n)))*(b*x+a)^(1+m)*(d*x+c)^(1+n)/b^3/d^3/(2+m+n)/(3+m+
n)/(4+m+n)-f*(b*c*f*h*(3+m)-b*d*(2*e*h+f*g)*(4+m+n)+a*d*f*h*(9+2*m+3*n))*(
b*x+a)^(2+m)*(d*x+c)^(1+n)/b^3/d^2/(3+m+n)/(4+m+n)+f^2*h*(b*x+a)^(3+m)*(d*
x+c)^(1+n)/b^3/d/(4+m+n)-(d*(2+m+n)*(d*(3+m+n)*(a^2*b*c*f^2*h*(3+m)+a^3*d*
f^2*h*(1+n)-b^3*d*e^2*g*(4+m+n))-a*f*(b*c*(2+m)+a*d*(1+n))*(b*c*f*h*(3+m)-
b*d*(2*e*h+f*g)*(4+m+n)+a*d*f*h*(9+2*m+3*n)))-(b*c*(1+m)+a*d*(1+n))*(d*(3+
m+n)*(2*a*b*c*f^2*h*(3+m)-b^2*d*e*(e*h+2*f*g)*(4+m+n)+a^2*d*f^2*h*(6+m+3*n
))-f*(b*c*(2+m)+a*d*(4+m+2*n))*(b*c*f*h*(3+m)-b*d*(2*e*h+f*g)*(4+m+n)+a*d*
f*h*(9+2*m+3*n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],
-d*(b*x+a)/(-a*d+b*c))/b^3/d^3/(-a*d+b*c)/(1+m)/(2+m+n)/(3+m+n)/(4+m+n)
```

Mathematica [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.87

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx$$

$$= \frac{(a+bx)^m(c+dx)^n\left(\frac{bh(c+dx)\left(d^2f(1+n)(bde(5+m+n)-f(bc(3+m)+ad(2+n)))(a+bx)^2+bd^3f(1+n)(3+m+n)(a+bx)^2(e+fx)+(-\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^2*(g + h*x),x]
```

output
```
((a + b*x)^m*(c + d*x)^n*((b*h*(c + d*x)*(d^2*f*(1 + n)*(b*d*e*(5 + m + n)
 - f*(b*c*(3 + m) + a*d*(2 + n)))*(a + b*x)^2 + b*d^3*f*(1 + n)*(3 + m + n
)*(a + b*x)^2*(e + f*x) + ((-(b*c) + a*d)*(a^2*d^2*f^2*(2 + 3*n + n^2) - 2
*a*b*d*f*(1 + n)*(-(c*f*(2 + m)) + d*e*(4 + m + n)) + b^2*(c^2*f^2*(6 + 5*
m + m^2) - 2*c*d*e*f*(2 + m)*(4 + m + n) + d^2*e^2*(12 + m^2 + 7*n + n^2 +
 m*(7 + 2*n))))*Hypergeometric2F1[-1 - m, 1 + n, 2 + n, (b*(c + d*x))/(b*c
 - a*d)])/((d*(a + b*x))/(-(b*c) + a*d))^m))/((1 + n)*(4 + m + n)) + (d^2*
(b*g - a*h)*(a + b*x)*(b*f*(1 + m)*(b*d*e*(4 + m + n) - f*(b*c*(2 + m) + a
*d*(2 + n)))*(c + d*x) + b^2*d*f*(1 + m)*(2 + m + n)*(c + d*x)*(e + f*x) +
 ((a^2*d^2*f^2*(2 + 3*n + n^2) - 2*a*b*d*f*(1 + n)*(-(c*f*(1 + m)) + d*e*(
3 + m + n)) + b^2*(c^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(1 + m)*(3 + m + n)
 + d^2*e^2*(6 + m^2 + 5*n + n^2 + m*(5 + 2*n))))*Hypergeometric2F1[1 + m,
-n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/((b*(c + d*x))/(b*c - a*d))^n))/
((1 + m)*(2 + m + n))))/(b^4*d^4*(3 + m + n))
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {170, 164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2(g+hx)(a+bx)^m(c+dx)^n\,dx$$

$$\Big\downarrow \text{170}$$

$$\frac{\int (a+bx)^m(c+dx)^n(e+fx)(bdeg(m+n+4)-h(2acf+bce(m+1)+ade(n+1))+(2bdeh-f(bc(m+3)-}{bd(m+n+4)}$$

$$\frac{h(e+fx)^2(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+4)}$$

$$\Big\downarrow \text{164}$$

$$\frac{\frac{(a^2d^2f(n+1)(n+2)(-fh(ad(n+3)+bc(m+3))+2bdeh+bdfg(m+n+4))+b^2(c^2f(m+1)(m+2)(-fh(ad(n+3)+bc(m+3))+2bdeh+bdfg(m+n+4}{}}{}$$

$$\frac{h(e+fx)^2(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+4)}$$

$\Big\downarrow$ 80

$$\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(a^2d^2f(n+1)(n+2)(-fh(ad(n+3)+bc(m+3))+2bdeh+bdfg(m+n+4))+b^2(c^2f(m+1)(m+2)(-fh(ad(n+3)+bc(m+3))-}{}$$

$$\frac{h(e+fx)^2(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+4)}$$

$\Big\downarrow$ 79

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)(a^2d^2f(n+1)(n+2)(-fh(ad(n+3)+bc(m+3))+2bdeh+bdfg(}{}$$

$$\frac{h(e+fx)^2(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+4)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^2*(g + h*x),x]
```

output
```
(h*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^2)/(b*d*(4 + m + n)) + (-
(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*(2 + m)*(2*b*d*e*h + b*d*f*g*
(4 + m + n) - f*h*(b*c*(3 + m) + a*d*(3 + n))) + a*d*f*(2 + n)*(2*b*d*e*h
+ b*d*f*g*(4 + m + n) - f*h*(b*c*(3 + m) + a*d*(3 + n))) + 2*b*d*(3 + m +
n)*(a*f*h*(c*f + d*e*(2 + n)) - b*e*(d*e*h - c*f*h*(2 + m) + d*f*g*(4 + m
+ n))) - b*d*f*(2 + m + n)*(2*b*d*e*h + b*d*f*g*(4 + m + n) - f*h*(b*c*(3
+ m) + a*d*(3 + n)))*x))/(b^2*d^2*(2 + m + n)*(3 + m + n))) + ((a^2*d^2*f*
(1 + n)*(2 + n)*(2*b*d*e*h + b*d*f*g*(4 + m + n) - f*h*(b*c*(3 + m) + a*d*
(3 + n))) + a*b*d*(1 + n)*(2*c*f*(1 + m)*(2*b*d*e*h + b*d*f*g*(4 + m + n)
- f*h*(b*c*(3 + m) + a*d*(3 + n))) + 2*d*(3 + m + n)*(a*f*h*(c*f + d*e*(2
+ n)) - b*e*(d*e*h - c*f*h*(2 + m) + d*f*g*(4 + m + n)))) + b^2*(d^2*e*(2
+ m + n)*(3 + m + n)*(b*d*e*g*(4 + m + n) - h*(2*a*c*f + b*c*e*(1 + m) + a
*d*e*(1 + n))) + c^2*f*(1 + m)*(2 + m)*(2*b*d*e*h + b*d*f*g*(4 + m + n) -
f*h*(b*c*(3 + m) + a*d*(3 + n))) + 2*c*d*(1 + m)*(3 + m + n)*(a*f*h*(c*f +
 d*e*(2 + n)) - b*e*(d*e*h - c*f*h*(2 + m) + d*f*g*(4 + m + n)))))*(a + b*
x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b
*c - a*d))^n))/(b*d*(4 + m + n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e)^2 (hx+g)\, dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\, dx = \int (fx+e)^2(hx+g)(bx+a)^m(dx+c)^n\, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f^2*h*x^3 + e^2*g + (f^2*g + 2*e*f*h)*x^2 + (2*e*f*g + e^2*h)*x)
*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**2*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx = \int (fx+e)^2(hx+g)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx = \int (fx+e)^2(hx+g)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx = \int (e+f\,x)^2\,(g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

input
```
int((e + f*x)^2*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)^2*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2(g+hx)\,dx = \int (bx+a)^m\,(dx+c)^n\,(fx+e)^2\,(hx+g)\,dx$$

input

```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x)
```

output

```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2*(h*x+g),x)
```

3.140 $\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 266

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx$$
$$= -\frac{f(bcf(2+m)-2bde(3+m+n)+adf(4+m+2n))(a+bx)^{1+m}(c+dx)^{1+n}}{b^2d^2(2+m+n)(3+m+n)}$$
$$+\frac{f^2(a+bx)^{2+m}(c+dx)^{1+n}}{b^2d(3+m+n)}$$
$$-\frac{(d(2+m+n)\left(abcf^2(2+m)+a^2df^2(1+n)-b^2de^2(3+m+n)\right)-f(bc(1+m)+ad(1+n))(bcf(2\ldots}{b^2d^2(bc-a\ldots}$$

output
```
-f*(b*c*f*(2+m)-2*b*d*e*(3+m+n)+a*d*f*(4+m+2*n))*(b*x+a)^(1+m)*(d*x+c)^(1+
n)/b^2/d^2/(2+m+n)/(3+m+n)+f^2*(b*x+a)^(2+m)*(d*x+c)^(1+n)/b^2/d/(3+m+n)-(
d*(2+m+n)*(a*b*c*f^2*(2+m)+a^2*d*f^2*(1+n)-b^2*d*e^2*(3+m+n))-f*(b*c*(1+m)
+a*d*(1+n))*(b*c*f*(2+m)-2*b*d*e*(3+m+n)+a*d*f*(4+m+2*n)))*(b*x+a)^(1+m)*(
d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a
*d+b*c)/(1+m)/(2+m+n)/(3+m+n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.95

$$\int (a+bx)^m (c+dx)^n (e+fx)^2 \, dx$$

$$= \frac{(a+bx)^{1+m}(c+dx)^n \left(\frac{f(bde(4+m+n)-f(bc(2+m)+ad(2+n)))(c+dx)}{bd(2+m+n)} + f(c+dx)(e+fx) + \frac{(a^2d^2f^2(2+3n+n^2)-2abd}{} \right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^2,x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*((f*(b*d*e*(4 + m + n) - f*(b*c*(2 + m) + a
*d*(2 + n)))*(c + d*x))/(b*d*(2 + m + n)) + f*(c + d*x)*(e + f*x) + ((a^2*
d^2*f^2*(2 + 3*n + n^2) - 2*a*b*d*f*(1 + n)*(-(c*f*(1 + m)) + d*e*(3 + m +
 n)) + b^2*(c^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(1 + m)*(3 + m + n) + d^2*
e^2*(6 + m^2 + 5*n + n^2 + m*(5 + 2*n))))*Hypergeometric2F1[1 + m, -n, 2 +
 m, (d*(a + b*x))/(-(b*c) + a*d)])/(b^2*d*(1 + m)*(2 + m + n)*((b*(c + d*x
))/(b*c - a*d))^n)))/(b*d*(3 + m + n))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {101, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 (a+bx)^m (c+dx)^n \, dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int (a+bx)^m (c+dx)^n \left(bd(m+n+3)e^2 - f(acf+bce(m+1)+ade(n+1)) + f(bde(m+n+4) - f(bc(m+2\right.}{bd(m+n+3)}$$

$$\frac{f(e+fx)(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+3)}$$

$$\Big\downarrow 90$$

$$\frac{\left(-f(acf+ade(n+1)+bce(m+1))+\frac{f(ad(n+1)+bc(m+1))(adf(n+2)+bcf(m+2)-bde(m+n+4))}{bd(m+n+2)}+bde^2(m+n+3)\right)\int}{bd(m+n+3)}$$

$$\frac{f(e+fx)(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+3)}$$

$$\Big\downarrow 80$$

$$\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(-f(acf+ade(n+1)+bce(m+1))+\frac{f(ad(n+1)+bc(m+1))(adf(n+2)+bcf(m+2)-bde(m+n+4))}{bd(m+n+2)}\right.}{bd(m+n}$$

$$\frac{f(e+fx)(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+3)}$$

$$\Big\downarrow 79$$

$$\frac{\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(-f(acf+ade(n+1)+bce(m+1))+\frac{f(ad(n+1)+bc(m+1))(adf(n+2)+bcf(m+2)-bde(m+n+4))}{bd(m+n+2)}+bde^2(m+n+3)\right)\mathrm{H}}{b(m+1)}}{bd(m+n+3)}$$

$$\frac{f(e+fx)(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+3)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^2,x]
```

output
```
(f*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x))/(b*d*(3 + m + n)) + ((f*
(b*d*e*(4 + m + n) - f*(b*c*(2 + m) + a*d*(2 + n)))*(a + b*x)^(1 + m)*(c +
 d*x)^(1 + n))/(b*d*(2 + m + n)) + ((b*d*e^2*(3 + m + n) - f*(a*c*f + b*c*
e*(1 + m) + a*d*e*(1 + n)) + (f*(b*c*(1 + m) + a*d*(1 + n))*(b*c*f*(2 + m)
 + a*d*f*(2 + n) - b*d*e*(4 + m + n)))/(b*d*(2 + m + n)))*(a + b*x)^(1 + m
)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a
*d))])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n))/(b*d*(3 + m + n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e)^2 \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2,x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx=\int (fx+e)^2(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx=\int (fx+e)^2(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx=\int (fx+e)^2(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx=\int (e+f\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

input
```
int((e + f*x)^2*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)^2*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^2\,dx=\text{too large to display}$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2,x)
```

output
```
((c + d*x)**n*(a + b*x)**m*a**3*c*d**2*f**2*m*n + 2*(c + d*x)**n*(a + b*x)
**m*a**3*c*d**2*f**2*m - (c + d*x)**n*(a + b*x)**m*a**3*d**3*f**2*m*n**2*x
 - 2*(c + d*x)**n*(a + b*x)**m*a**3*d**3*f**2*m*n*x - 2*(c + d*x)**n*(a +
b*x)**m*a**2*b*c**2*d*f**2*m*n - 2*(c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2
*e*f*m**2 - 2*(c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*e*f*m*n - 6*(c + d*x
)**n*(a + b*x)**m*a**2*b*c*d**2*e*f*m - (c + d*x)**n*(a + b*x)**m*a**2*b*c
*d**2*f**2*m**2*n*x - 2*(c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f**2*m**2*
x + 2*(c + d*x)**n*(a + b*x)**m*a**2*b*c*d**2*f**2*m*n**2*x + 2*(c + d*x)*
*n*(a + b*x)**m*a**2*b*d**3*e*f*m**2*n*x + 2*(c + d*x)**n*(a + b*x)**m*a**
2*b*d**3*e*f*m*n**2*x + 6*(c + d*x)**n*(a + b*x)**m*a**2*b*d**3*e*f*m*n*x
+ (c + d*x)**n*(a + b*x)**m*a**2*b*d**3*f**2*m**2*n*x**2 + (c + d*x)**n*(a
 + b*x)**m*a**2*b*d**3*f**2*m*n**2*x**2 + (c + d*x)**n*(a + b*x)**m*a**2*b
*d**3*f**2*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*a*b**2*c**3*f**2*m*n + 2*(
c + d*x)**n*(a + b*x)**m*a*b**2*c**3*f**2*n - 2*(c + d*x)**n*(a + b*x)**m*
a*b**2*c**2*d*e*f*m*n - 2*(c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*e*f*n**2
 - 6*(c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*e*f*n + 2*(c + d*x)**n*(a + b
*x)**m*a*b**2*c**2*d*f**2*m**2*n*x - (c + d*x)**n*(a + b*x)**m*a*b**2*c**2
*d*f**2*m*n**2*x - 2*(c + d*x)**n*(a + b*x)**m*a*b**2*c**2*d*f**2*n**2*x +
 (c + d*x)**n*(a + b*x)**m*a*b**2*c*d**2*e**2*m**3 + 3*(c + d*x)**n*(a + b
*x)**m*a*b**2*c*d**2*e**2*m**2*n + 5*(c + d*x)**n*(a + b*x)**m*a*b**2*c...
```

3.141 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 267

$$\begin{aligned}
\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx
&= \frac{(fg-eh)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(bg-ah)(1+m)} \\
&\quad - \frac{f(dfg-2deh+cfh)(a+bx)^{1+m}(c+dx)^{1+n}\operatorname{Hypergeometric2F1}\left(1,2+m+n,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{d(bc-ad)h^2(1+m)} \\
&\quad + \frac{f^2(a+bx)^{1+m}(c+dx)^{2+n}\operatorname{Hypergeometric2F1}\left(1,3+m+n,2+m,-\frac{d(a+bx)}{bc-ad}\right)}{d(bc-ad)h(1+m)}
\end{aligned}$$

output
```
(-e*h+f*g)^2*(b*x+a)^(1+m)*(d*x+c)^n*AppellF1(1+m,-n,1,2+m,-d*(b*x+a)/(-a*
d+b*c),-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+b*g)/(1+m)/((b*(d*x+c)/(-a*d+b*c))
^n)-f*(c*f*h-2*d*e*h+d*f*g)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+
n],[2+m],-d*(b*x+a)/(-a*d+b*c))/d/(-a*d+b*c)/h^2/(1+m)+f^2*(b*x+a)^(1+m)*(
d*x+c)^(2+n)*hypergeom([1, 3+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/d/(-a*d+b*c
)/h/(1+m)
```

Mathematica [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx=\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x),x]
```

output
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x), x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2(a+bx)^m(c+dx)^n}{g+hx}\,dx$$
$$\Big\downarrow \text{198}$$
$$\int \left(-\frac{f(a+bx)^m(c+dx)^n(cfh-2deh+dfg)}{dh^2}+\frac{(a+bx)^m(c+dx)^n(eh-fg)^2}{h^2(g+hx)}+\frac{f^2(a+bx)^m(c+dx)^{n+1}}{dh}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(a+bx)^{m+1}(c+dx)^n(fg-eh)^2\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}+\frac{f^2(bc-ad)(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n-1,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{b^2dh(m+1)}-\frac{f(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(cfh-2deh+dfg)\,\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{bdh^2(m+1)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x),x]
```

output
```
((f*g - e*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*AppellF1[1 + m, -n, 1, 2 + m,
 -((d*(a + b*x))/(b*c - a*d)), -((h*(a + b*x))/(b*g - a*h))])/(h^2*(b*g -
a*h)*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n) + ((b*c - a*d)*f^2*(a + b*x)^(
1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -1 - n, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(b^2*d*h*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n) - (f*(d*f*
g - 2*d*e*h + c*f*h)*(a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m
, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*d*h^2*(1 + m)*((b*(c + d*x)
)/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)^2}{hx+g}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)*(b*x + a)^m*(d*x + c)^n/(h*x + g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**2/(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n/(h*x + g), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx = \int \frac{(e+f\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n}{g+h\,x}\,dx$$

input
```
int(((e + f*x)^2*(a + b*x)^m*(c + d*x)^n)/(g + h*x),x)
```

output
```
int(((e + f*x)^2*(a + b*x)^m*(c + d*x)^n)/(g + h*x), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{g+hx}\,dx = \int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)^2}{hx+g}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g),x)
```

3.142 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 389

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = -\frac{(fg-eh)^2(a+bx)^{1+m}(c+dx)^{1+n}}{h(bg-ah)(dg-ch)(g+hx)}$$
$$-\frac{(fg-eh)(ah(2cfh+dehn-dfg(2+n))+b(ch(ehm-fg(2+m))-dg(eh(m+n)-fg(2+m+}{h^2(bg-ah)^2(dg-c}$$
$$-\frac{(bcf^2gh+af^2h(dg-ch)+bd(2efgh(1+m+n)-e^2h^2(1+m+n)-f^2g^2(2+m+n)))\,(a+bx)^{1-}}{(bc-ad)h^2(bg-ah)(dg-ch)(1+}$$

output
```
-(-e*h+f*g)^2*(b*x+a)^(1+m)*(d*x+c)^(1+n)/h/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)-
(-e*h+f*g)*(a*h*(2*c*f*h+d*e*h*n-d*f*g*(2+n))+b*(c*h*(e*h*m-f*g*(2+m))-d*g
*(e*h*(m+n)-f*g*(2+m+n))))*(b*x+a)^(1+m)*(d*x+c)^n*AppellF1(1+m,-n,1,2+m,-
d*(b*x+a)/(-a*d+b*c),-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+b*g)^2/(-c*h+d*g)/(1
+m)/((b*(d*x+c)/(-a*d+b*c))^n)-(b*c*f^2*g*h+a*f^2*h*(-c*h+d*g)+b*d*(2*e*f*
g*h*(1+m+n)-e^2*h^2*(1+m+n)-f^2*g^2*(2+m+n)))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*
hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/h^2/(-a*h+b*g
)/(-c*h+d*g)/(1+m)
```

Mathematica [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x)^2,x]
```

output
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x)^2, x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2(a+bx)^m(c+dx)^n}{(g+hx)^2}\,dx$$

$$\Big\downarrow \; 198$$

$$\int \left(-\frac{2f(a+bx)^m(c+dx)^n(fg-eh)}{h^2(g+hx)}+\frac{(a+bx)^m(c+dx)^n(eh-fg)^2}{h^2(g+hx)^2}+\frac{f^2(a+bx)^m(c+dx)^n}{h^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2f(a+bx)^{m+1}(c+dx)^n(fg-eh)\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}+\frac{b(a+bx)^{m+1}(c+dx)^n(fg-eh)^2\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,2,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)^2}+\frac{f^2(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{bh^2(m+1)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x)^2,x]
```

output
```
(-2*f*(f*g - e*h)*(a + b*x)^(1 + m)*(c + d*x)^n*AppellF1[1 + m, -n, 1, 2 +
 m, -((d*(a + b*x))/(b*c - a*d)), -((h*(a + b*x))/(b*g - a*h))])/(h^2*(b*g
 - a*h)*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n) + (b*(f*g - e*h)^2*(a + b*x
)^(1 + m)*(c + d*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((d*(a + b*x))/(b*c -
 a*d)), -((h*(a + b*x))/(b*g - a*h))])/(h^2*(b*g - a*h)^2*(1 + m)*((b*(c +
 d*x))/(b*c - a*d))^n) + (f^2*(a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric
2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*h^2*(1 + m)*((b*(c
 + d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)^2}{(hx+g)^2} dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{(hx+g)^2}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)*(b*x + a)^m*(d*x + c)^n/(h^2*x^2 + 2*g*
h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**2/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{(hx+g)^2}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n/(h*x + g)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,2,2,0,0]%%%}+%%%{-2,[0,1,1,1,1,1,1]%%%}+%%%{1,[0,
1,1,0,0,2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \int \frac{(e+f\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n}{(g+h\,x)^2}\,dx$$

input
```
int(((e + f*x)^2*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^2,x)
```

output
```
int(((e + f*x)^2*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^2}\,dx = \int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)^2}{(hx+g)^2}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^2,x)
```

3.143 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 912

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx = -\frac{(fg-eh)^2(a+bx)^{1+m}(c+dx)^{1+n}}{2h(bg-ah)(dg-ch)(g+hx)^2}$$

$$+\frac{(fg-eh)(ah(4cfh-deh(1-n)-dfg(3+n))-b(ch(eh(1-m)+fg(3+m))-dg(eh(2-m-n$$

$$2h(bg-ah)^2(dg-ch)^2(g+hx)$$

$$+\frac{\left(b^2(2d^2e^2g^2h-c^2h(e^2h^2(1-m)m+2efghm(1+m)-f^2g^2(2+3m+m^2))-cdg(2efgh(1+m)(1\right.}{}$$

$$-\frac{bd(fg-eh)(1+m+n)(ah(4cfh-deh(1-n)-dfg(3+n))-b(ch(eh(1-m)+fg(3+m))-dg}{2(bc-ad)h^2(b}$$

output
```
-1/2*(-e*h+f*g)^2*(b*x+a)^(1+m)*(d*x+c)^(1+n)/h/(-a*h+b*g)/(-c*h+d*g)/(h*x
+g)^2+1/2*(-e*h+f*g)*(a*h*(4*c*f*h-d*e*h*(1-n)-d*f*g*(3+n))-b*(c*h*(e*h*(1
-m)+f*g*(3+m))-d*g*(e*h*(2-m-n)+f*g*(2+m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)
/h/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)+1/2*(b^2*(2*d^2*e^2*g^2*h-c^2*h*(e^2*
h^2*(1-m)*m+2*e*f*g*h*m*(1+m)-f^2*g^2*(m^2+3*m+2))-c*d*g*(2*e*f*g*h*(1+m)*
(1-m-n)+f^2*g^2*(1+m)*(1+m+n)+e^2*h^2*(1+m^2-m*(2-n)+n)))+a^2*h*(2*c^2*f^2
*h^2+4*c*d*f*h*(e*h*n-f*g*(1+n))-d^2*(e^2*h^2*(1-n)*n+2*e*f*g*h*n*(1+n)-f^
2*g^2*(n^2+3*n+2)))+a*b*(4*c^2*f*h^2*(e*h*m-f*g*(1+m))-d^2*g*(2*e*f*g*h*(1
-m-n)*(1+n)+f^2*g^2*(1+n)*(1+m+n)+e^2*h^2*((-1+n)^2+m*(1+n)))+2*c*d*h*(e^2
*h^2*m*n+2*e*f*g*h*(-m*n-m-n+1)+f^2*g^2*(2+2*n+m*(2+n))))+b*d*g*(-e*h+f*g)
*(1+m+n)*(a*h*(4*c*f*h-d*e*h*(1-n)-d*f*g*(3+n))-b*(c*h*(e*h*(1-m)+f*g*(3+m
))-d*g*(e*h*(2-m-n)+f*g*(2+m+n))))/h)*(b*x+a)^(1+m)*(d*x+c)^n*AppellF1(1+m
,-n,1,2+m,-d*(b*x+a)/(-a*d+b*c),-h*(b*x+a)/(-a*h+b*g))/h/(-a*h+b*g)^3/(-c*
h+d*g)^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)-1/2*b*d*(-e*h+f*g)*(1+m+n)*(a*h*
(4*c*f*h-d*e*h*(1-n)-d*f*g*(3+n))-b*(c*h*(e*h*(1-m)+f*g*(3+m))-d*g*(e*h*(2
-m-n)+f*g*(2+m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m
],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/h^2/(-a*h+b*g)^2/(-c*h+d*g)^2/(1+m)
```

Mathematica [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x)^3,x]
```

output
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x)^3, x]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2(a+bx)^m(c+dx)^n}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(-\frac{2f(a+bx)^m(c+dx)^n(fg-eh)}{h^2(g+hx)^2}+\frac{(a+bx)^m(c+dx)^n(eh-fg)^2}{h^2(g+hx)^3}+\frac{f^2(a+bx)^m(c+dx)^n}{h^2(g+hx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(a+bx)^{m+1}(c+dx)^n(fg-eh)^2\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,3,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)^3}-\frac{2bf(a+bx)^{m+1}(c+dx)^n(fg-eh)\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,2,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)^2}+\frac{f^2(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^2)/(g + h*x)^3,x]
```

output
```
(f^2*(a + b*x)^(1 + m)*(c + d*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a +
 b*x))/(b*c - a*d)), -((h*(a + b*x))/(b*g - a*h))])/(h^2*(b*g - a*h)*(1 +
m)*((b*(c + d*x))/(b*c - a*d))^n) - (2*b*f*(f*g - e*h)*(a + b*x)^(1 + m)*(
c + d*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((
h*(a + b*x))/(b*g - a*h))])/(h^2*(b*g - a*h)^2*(1 + m)*((b*(c + d*x))/(b*c
 - a*d))^n) + (b^2*(f*g - e*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*AppellF1[1
+ m, -n, 3, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((h*(a + b*x))/(b*g - a*
h))])/(h^2*(b*g - a*h)^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)^2}{(hx+g)^3} dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3} dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{(hx+g)^3} dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)*(b*x + a)^m*(d*x + c)^n/(h^3*x^3 + 3*g*
h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3} dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**2/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{(hx+g)^3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n/(h*x + g)^3, x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx = \int \frac{(fx+e)^2(bx+a)^m(dx+c)^n}{(hx+g)^3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*x + a)^m*(d*x + c)^n/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx = \int \frac{(e+f\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n}{(g+h\,x)^3}\,dx$$

input
```
int(((e + f*x)^2*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^3,x)
```

output
```
int(((e + f*x)^2*(a + b*x)^m*(c + d*x)^n)/(g + h*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^2}{(g+hx)^3}\,dx = \int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)^2}{(hx+g)^3}dx$$

input

```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x)
```

output

```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^2/(h*x+g)^3,x)
```

3.144 $\int (a+bx)^m (c+dx)^{1-m} (e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 251

$$\int (a+bx)^m (c+dx)^{1-m} (e+fx)(g+hx)\,dx$$
$$= \frac{(4bd(fg+eh) - adfh(6-m) - bcfh(2+m))(a+bx)^{1+m}(c+dx)^{2-m}}{12b^2d^2}$$
$$+ \frac{fh(a+bx)^{2+m}(c+dx)^{2-m}}{4b^2d}$$
$$+ \frac{(a^2d^2fh(6-5m+m^2) - 2abd(2-m)(2d(fg+eh) - cfh(1+m)) + b^2(12d^2eg - 4cd(fg+eh)(1+}{12b^2d^2(bc-ac}$$

output
```
1/12*(4*b*d*(e*h+f*g)-a*d*f*h*(6-m)-b*c*f*h*(2+m))*(b*x+a)^(1+m)*(d*x+c)^(
2-m)/b^2/d^2+1/4*f*h*(b*x+a)^(2+m)*(d*x+c)^(2-m)/b^2/d+1/12*(a^2*d^2*f*h*(
m^2-5*m+6)-2*a*b*d*(2-m)*(2*d*(e*h+f*g)-c*f*h*(1+m))+b^2*(12*d^2*e*g-4*c*d
*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(2-m)*hyperge
om([1, 3],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(1+m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.78

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{-1+m}\left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(-3+m,1+m,2+m,\frac{d(a+}{-bc-}\right.\right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*((b*(c + d*x))/(b*c - a*d))^(-1 + m)*
((b*c - a*d)^2*f*h*Hypergeometric2F1[-3 + m, 1 + m, 2 + m, (d*(a + b*x))/(
-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric
2F1[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(
d*g - c*h)*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) +
 a*d)])))/(b^3*d^2*(1 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m(c+dx)^{1-m}\,dx$$
$$\Big\downarrow \text{164}$$
$$\frac{(a^2d^2fh(m^2-5m+6)-2abd(2-m)(2d(eh+fg)-cfh(m+1))+b^2(c^2fh(m^2+3m+2)-4cd(m+1)(eh}{12b^2d^2}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{2-m}(-adfh(3-m)-bcfh(m+2)+4bd(eh+fg)+3bdfhx)}{12b^2d^2}$$
$$\Big\downarrow \text{80}$$

$$\frac{(bc-ad)(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(a^2d^2fh(m^2-5m+6)-2abd(2-m)(2d(eh+fg)-cfh(m+1))+b^2(c^2fh\right.}{12b^3d^2}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{2-m}(-adfh(3-m)-bcfh(m+2)+4bd(eh+fg)+3bdfhx)}{12b^2d^2}$$

$$\Big\downarrow \text{79}$$

$$\frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\text{Hypergeometric2F1}\left(m-1,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(m^2\right.}{12b^4}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{2-m}(-adfh(3-m)-bcfh(m+2)+4bd(eh+fg)+3bdfhx)}{12b^2d^2}$$

input
```
Int[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(2 - m)*(4*b*d*(f*g + e*h) - a*d*f*h*(3 - m)
- b*c*f*h*(2 + m) + 3*b*d*f*h*x))/(12*b^2*d^2) + ((b*c - a*d)*(a^2*d^2*f*h
*(6 - 5*m + m^2) - 2*a*b*d*(2 - m)*(2*d*(f*g + e*h) - c*f*h*(1 + m)) + b^2
*(12*d^2*e*g - 4*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a +
b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[-1 + m, 1 + m
, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(12*b^4*d^2*(1 + m)*(c + d*x)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^{-m+1} (fx+e) (hx+g) \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-m+1)*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^(-m+1)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx) \, dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1} \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m + 1), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(1-m)*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m+1}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$$
$$=\int (e+f\,x)\,\,(g+h\,x)\,\,(a+b\,x)^m\,(c+d\,x)^{1-m}\,dx$$

input
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m),x)
```

output
```
int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m), x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$$
$$=\left(\int \frac{(bx+a)^m}{(dx+c)^m}dx\right)ceg+\left(\int \frac{(bx+a)^m\,x^3}{(dx+c)^m}dx\right)dfh+\left(\int \frac{(bx+a)^m\,x^2}{(dx+c)^m}dx\right)cfh$$
$$+\left(\int \frac{(bx+a)^m\,x^2}{(dx+c)^m}dx\right)deh+\left(\int \frac{(bx+a)^m\,x^2}{(dx+c)^m}dx\right)dfg$$
$$+\left(\int \frac{(bx+a)^m\,x}{(dx+c)^m}dx\right)ceh+\left(\int \frac{(bx+a)^m\,x}{(dx+c)^m}dx\right)cfg+\left(\int \frac{(bx+a)^m\,x}{(dx+c)^m}dx\right)deg$$

input
```
int((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((a + b*x)**m/(c + d*x)**m,x)*c*e*g + int(((a + b*x)**m*x**3)/(c + d*x)
**m,x)*d*f*h + int(((a + b*x)**m*x**2)/(c + d*x)**m,x)*c*f*h + int(((a + b
*x)**m*x**2)/(c + d*x)**m,x)*d*e*h + int(((a + b*x)**m*x**2)/(c + d*x)**m,
x)*d*f*g + int(((a + b*x)**m*x)/(c + d*x)**m,x)*c*e*h + int(((a + b*x)**m*
x)/(c + d*x)**m,x)*c*f*g + int(((a + b*x)**m*x)/(c + d*x)**m,x)*d*e*g
```

3.145 $\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 251

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(3bd(fg+eh)-adfh(4-m)-bcfh(2+m))(a+bx)^{1+m}(c+dx)^{1-m}}{6b^2d^2}$$
$$+ \frac{fh(a+bx)^{2+m}(c+dx)^{1-m}}{3b^2d}$$
$$+ \frac{(a^2d^2fh(2-3m+m^2)-abd(1-m)(3d(fg+eh)-2cfh(1+m))+b^2(6d^2eg-3cd(fg+eh)(1+n}{6b^2d^2(bc-ad)}$$

output
```
1/6*(3*b*d*(e*h+f*g)-a*d*f*h*(4-m)-b*c*f*h*(2+m))*(b*x+a)^(1+m)*(d*x+c)^(1
-m)/b^2/d^2+1/3*f*h*(b*x+a)^(2+m)*(d*x+c)^(1-m)/b^2/d+1/6*(a^2*d^2*f*h*(m^
2-3*m+2)-a*b*d*(1-m)*(3*d*(e*h+f*g)-2*c*f*h*(1+m))+b^2*(6*d^2*e*g-3*c*d*(e
*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(1-m)*hypergeom(
[1, 2],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(1+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.75

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\left((bc-ad)^2fh\,\text{Hypergeometric2F1}\left(-2+m,1+m,2+m,\frac{d(a+bx)}{-bc+ad}\right)\right.}{}$$

input
```
Integrate[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

output
```
((a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*((b*c - a*d)^2*f*h*Hyperg
eometric2F1[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*
c - a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1[-1 + m, 1 + m, 2 + m,
 (d*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric
2F1[m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*(c
+ d*x)^m)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m(c+dx)^{-m}\,dx$$

$$\Big\downarrow \text{164}$$

$$\frac{\left(a^2d^2fh(m^2-3m+2)-abd(1-m)(3d(eh+fg)-2cfh(m+1))+b^2(c^2fh(m^2+3m+2)-3cd(m+1)(eh\right.}{6b^2d^2}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{1-m}(-adfh(2-m)-bcfh(m+2)+3bd(eh+fg)+2bdfhx)}{6b^2d^2}$$

$$\Big\downarrow \text{80}$$

$$\frac{(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(a^2d^2fh(m^2-3m+2)-abd(1-m)(3d(eh+fg)-2cfh(m+1))+b^2(c^2fh(m^2+3m\right.}{6b^2d^2}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{1-m}(-adfh(2-m)-bcfh(m+2)+3bd(eh+fg)+2bdfhx)}{6b^2d^2}$$

$$\Big\downarrow \; 79$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(m^2-3m+2)-\right.}{6b^3d^2(m+}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{1-m}(-adfh(2-m)-bcfh(m+2)+3bd(eh+fg)+2bdfhx)}{6b^2d^2}$$

input
```
Int[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*(3*b*d*(f*g + e*h) - a*d*f*h*(2 - m)
- b*c*f*h*(2 + m) + 2*b*d*f*h*x))/(6*b^2*d^2) + ((a^2*d^2*f*h*(2 - 3*m + m
^2) - a*b*d*(1 - m)*(3*d*(f*g + e*h) - 2*c*f*h*(1 + m)) + b^2*(6*d^2*e*g -
 3*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(
(b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a +
b*x))/(b*c - a*d))])/(6*b^3*d^2*(1 + m)*(c + d*x)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int (bx+a)^m (fx+e) (hx+g) (xd+c)^{-m} \, dx$$

input
```
int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

output
```
int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

input
```
integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m/(d*x + c)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(f*x+e)*(h*x+g)/((d*x+c)**m),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

input
```
integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

input
```
integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m (c+dx)^{-m} (e+fx)(g+hx)\,dx = \int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^m}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m,x)
```

output
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m, x)
```

Reduce [F]

$$\begin{aligned}&\int (a+bx)^m (c+dx)^{-m} (e+fx)(g+hx)\,dx \\ &= \left(\int \frac{(bx+a)^m}{(dx+c)^m}dx\right) eg + \left(\int \frac{(bx+a)^m x^2}{(dx+c)^m}dx\right) fh \\ &\quad + \left(\int \frac{(bx+a)^m x}{(dx+c)^m}dx\right) eh + \left(\int \frac{(bx+a)^m x}{(dx+c)^m}dx\right) fg\end{aligned}$$

input
```
int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

output
```
int((a + b*x)**m/(c + d*x)**m,x)*e*g + int(((a + b*x)**m*x**2)/(c + d*x)**
m,x)*f*h + int(((a + b*x)**m*x)/(c + d*x)**m,x)*e*h + int(((a + b*x)**m*x)
/(c + d*x)**m,x)*f*g
```

3.146 $\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 237

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \frac{fh(a+bx)^{1+m}(c+dx)^{1-m}}{2bd^2} + \frac{(de-cf)(dg-ch)(a+bx)^{1+m}(c+dx)^{-m}}{d^2(bc-ad)m} + \frac{(a^2d^2fh(1-m)m - 2abdm(d(fg+eh)-cfh(1+m)) - b^2(2d^2eg - 2cd(fg+eh)(1+m) + c^2fh(2}{2bd^2(bc-ad)^2m(1+}$$

output

```
1/2*f*h*(b*x+a)^(1+m)*(d*x+c)^(1-m)/b/d^2+(-c*f+d*e)*(-c*h+d*g)*(b*x+a)^(1
+m)/d^2/(-a*d+b*c)/m/((d*x+c)^m)+1/2*(a^2*d^2*f*h*(1-m)*m-2*a*b*d*m*(d*(e*
h+f*g)-c*f*h*(1+m))-b^2*(2*d^2*e*g-2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+
2)))*(b*x+a)^(1+m)*(d*x+c)^(1-m)*hypergeom([1, 2],[2+m],-d*(b*x+a)/(-a*d+b
*c))/b/d^2/(-a*d+b*c)^2/m/(1+m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.93

$$\int (a+bx)^m (c+dx)^{-1-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(b(adfhm(c+dx)-b(2d^2eg+c^2fh(2+m)+cd(-2fg-2eh+fhmx)))+\right.}{2}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(b*(a*d*f*h*m*(c + d*x) - b*(2*d^2*e*g + c^2*f*h*(2 + m
) + c*d*(-2*f*g - 2*e*h + f*h*m*x))) + ((a^2*d^2*f*h*(-1 + m)*m + 2*a*b*d*
m*(d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(2*d^2*e*g - 2*c*d*(f*g + e*h)*(1
+ m) + c^2*f*h*(2 + 3*m + m^2)))*((b*(c + d*x))/(b*c - a*d))^m*Hypergeomet
ric2F1[m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(1 + m)))/(2*b^2*d^
2*(-(b*c) + a*d)*m*(c + d*x)^m)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {163, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m (c+dx)^{-m-1}\,dx$$

$$\Big\downarrow \text{163}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-$$
$$\frac{\left(d^2\left(a^2(-f)h(1-m)m+2abm(eh+fg)+2b^2eg\right)-2bcd(m+1)(afhm+beh+bfg)+b^2c^2fh(m+1)(m+2)\right.}{2bd^2m(bc-ad)}$$

$$\Big\downarrow \text{80}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-$$
$$\frac{(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(d^2\left(a^2(-f)h(1-m)m+2abm(eh+fg)+2b^2eg\right)-2bcd(m+1)(afhm+beh+bfg)+\right.}{2bd^2m(bc-ad)}$$

$\downarrow$ 79

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(d^2\left(a^2(-f)h(1-m)m+\right.\right.}{2b^2d^2m(m+1)(bc-ad)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(2*b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*(2*b*(f*g + e*h)
 + a*f*h*m) + d*(b*c - a*d)*f*h*m*x))/(2*b*d^2*(b*c - a*d)*m*(c + d*x)^m)
- ((b^2*c^2*f*h*(1 + m)*(2 + m) - 2*b*c*d*(1 + m)*(b*f*g + b*e*h + a*f*h*m
) + d^2*(2*b^2*e*g + 2*a*b*(f*g + e*h)*m - a^2*f*h*(1 - m)*m))*(a + b*x)^(
1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -(
(d*(a + b*x))/(b*c - a*d))])/(2*b^2*d^2*(b*c - a*d)*m*(1 + m)*(c + d*x)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^{-1-m} (fx+e) (hx+g) \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-1}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 1), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(-1-m)*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-1}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-1}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx=\int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+1}}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1),x)
```

output
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1), x)
```

Reduce [F]

$$\begin{aligned}&\int (a+bx)^m(c+dx)^{-1-m}(e+fx)(g+hx)\,dx\\&=\left(\int \frac{(bx+a)^m}{(dx+c)^m c+(dx+c)^m dx}dx\right)eg+\left(\int \frac{(bx+a)^m x^2}{(dx+c)^m c+(dx+c)^m dx}dx\right)fh\\&\quad+\left(\int \frac{(bx+a)^m x}{(dx+c)^m c+(dx+c)^m dx}dx\right)eh+\left(\int \frac{(bx+a)^m x}{(dx+c)^m c+(dx+c)^m dx}dx\right)fg\end{aligned}$$

input
```
int((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((a + b*x)**m/((c + d*x)**m*c + (c + d*x)**m*d*x),x)*e*g + int(((a + b*
x)**m*x**2)/((c + d*x)**m*c + (c + d*x)**m*d*x),x)*f*h + int(((a + b*x)**m
*x)/((c + d*x)**m*c + (c + d*x)**m*d*x),x)*e*h + int(((a + b*x)**m*x)/((c
+ d*x)**m*c + (c + d*x)**m*d*x),x)*f*g
```

3.147 $\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 172

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \frac{(de-cf)(dg-ch)(a+bx)^{1+m}(c+dx)^{-1-m}}{d^2(bc-ad)(1+m)} + \frac{fh(a+bx)^{1+m}(c+dx)^{-m}}{bd^2} + \frac{(bd(fg+eh)+adfhm-bcfh(2+m))(a+bx)^{1+m}(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(1,1,1-m,\frac{b(c+d}{bc-d}\right.}{bd^2(bc-ad)m}$$

output
```
(-c*f+d*e)*(-c*h+d*g)*(b*x+a)^(1+m)*(d*x+c)^(-1-m)/d^2/(-a*d+b*c)/(1+m)+f*
h*(b*x+a)^(1+m)/b/d^2/((d*x+c)^m)+(b*d*(e*h+f*g)+a*d*f*h*m-b*c*f*h*(2+m))*
(b*x+a)^(1+m)*hypergeom([1, 1],[1-m],b*(d*x+c)/(-a*d+b*c))/b/d^2/(-a*d+b*c
)/m/((d*x+c)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.15

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx$$
$$= \frac{(a+bx)^m(c+dx)^{-m}\left(-\frac{d(a+bx)\left(adfh(1+m)(c+dx)-b\left(d^2eg+c^2fh(2+m)+cd(-fg-eh+fh(1+m)x)\right)\right)}{c+dx}+\frac{(bc-ad)(1+m)(-bd(f\ldots}{\ldots}\right.}{bd^3(bc-ad)(1+m)}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^m*(-((d*(a + b*x)*(a*d*f*h*(1 + m)*(c + d*x) - b*(d^2*e*g + c^2
*f*h*(2 + m) + c*d*(-(f*g) - e*h + f*h*(1 + m)*x))))/(c + d*x)) + ((b*c -
a*d)*(1 + m)*(-(b*d*(f*g + e*h)) - a*d*f*h*m + b*c*f*h*(2 + m))*Hypergeome
tric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(m*((d*(a + b*x))/(-(b*
c) + a*d))^m)))/(b*d^3*(b*c - a*d)*(1 + m)*(c + d*x)^m)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {160, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m(c+dx)^{-m-2}\,dx$$

$$\Big\downarrow \text{160}$$

$$\frac{(adfhm-bcfh(m+2)+bd(eh+fg))\int (a+bx)^m(c+dx)^{-m-1}dx}{bd^2} - \frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b\left(c^2fh(m+2)-cd(eh+fg)+d^2eg\right)\right)}{bd^2(m+1)(bc-ad)}$$

$$\Big\downarrow \text{80}$$

$$\frac{(a+bx)^m\left(-\frac{d(a+bx)}{bc-ad}\right)^{-m}(adfhm-bcfh(m+2)+bd(eh+fg))\int(c+dx)^{-m-1}\left(-\frac{bxd}{bc-ad}-\frac{ad}{bc-ad}\right)^m dx}{bd^2}-$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b\left(c^2fh(m+2)-cd(eh+fg)+d^2eg\right)\right)}{bd^2(m+1)(bc-ad)}$$

$$\Big\downarrow \text{79}$$

$$-\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b\left(c^2fh(m+2)-cd(eh+fg)+d^2eg\right)\right)}{bd^2(m+1)(bc-ad)}$$
$$\frac{(a+bx)^m(c+dx)^{-m}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{b(c+dx)}{bc-ad}\right)(adfhm-bcfh(m+2)+b}{bd^3m}$$

input
```
Int[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*(a*c*d*f*h*(1 + m) - b*(d^2*e*g -
c*d*(f*g + e*h) + c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(1 + m)*x))/(b*d^2*
(b*c - a*d)*(1 + m))) - ((b*d*(f*g + e*h) + a*d*f*h*m - b*c*f*h*(2 + m))*(
a + b*x)^m*Hypergeometric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(b
*d^3*m*(-((d*(a + b*x))/(b*c - a*d)))^m*(c + d*x)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 160
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d*(f*g
+ e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Simp[(a*d*f*h*m + b*(d*
(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d)   Int[(a + b*x)^(m + 1)*(c + d*x)^n,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0] &&
NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Maple [F]

$$\int (bx+a)^m (xd+c)^{-2-m} (fx+e) (hx+g) \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx) \, dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-2} \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 2), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(-2-m)*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-2}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Giac [F]

$$\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-2}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx = \int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+2}}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2),x)
```

output
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2), x)
```

Reduce [F]

$$\begin{aligned}\int (a+bx)^m(c+dx)^{-2-m}(e+fx)(g+hx)\,dx &= \left(\int \frac{(bx+a)^m}{(dx+c)^m c^2+2\,(dx+c)^m cdx+(dx+c)^m d^2x^2}dx\right)eg \\ &\quad+\left(\int \frac{(bx+a)^m x^2}{(dx+c)^m c^2+2\,(dx+c)^m cdx+(dx+c)^m d^2x^2}dx\right)fh \\ &\quad+\left(\int \frac{(bx+a)^m x}{(dx+c)^m c^2+2\,(dx+c)^m cdx+(dx+c)^m d^2x^2}dx\right)eh \\ &\quad+\left(\int \frac{(bx+a)^m x}{(dx+c)^m c^2+2\,(dx+c)^m cdx+(dx+c)^m d^2x^2}dx\right)fg\end{aligned}$$

input
```
int((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((a + b*x)**m/((c + d*x)**m*c**2 + 2*(c + d*x)**m*c*d*x + (c + d*x)**m*
d**2*x**2),x)*e*g + int(((a + b*x)**m*x**2)/((c + d*x)**m*c**2 + 2*(c + d*
x)**m*c*d*x + (c + d*x)**m*d**2*x**2),x)*f*h + int(((a + b*x)**m*x)/((c +
d*x)**m*c**2 + 2*(c + d*x)**m*c*d*x + (c + d*x)**m*d**2*x**2),x)*e*h + int
(((a + b*x)**m*x)/((c + d*x)**m*c**2 + 2*(c + d*x)**m*c*d*x + (c + d*x)**m
*d**2*x**2),x)*f*g
```

3.148 $\int (a+bx)^m (c+dx)^{-3-m} (e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 219

$$\int (a+bx)^m (c+dx)^{-3-m} (e+fx)(g+hx)\,dx = \frac{(de-cf)(dg-ch)(a+bx)^{1+m}(c+dx)^{-2-m}}{d^2(bc-ad)(2+m)} - \frac{(ad(dfg+deh-2cfh)(2+m)-b(d^2eg+cd(fg+eh)(1+m)-c^2fh(3+2m)))\,(a+bx)^{1+m}(c+d}{d^2(bc-ad)^2(1+m)(2+m)} + \frac{fh(a+bx)^{1+m}(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(1,1,1-m,\frac{b(c+dx)}{bc-ad}\right)}{d^2(bc-ad)m}$$

output
```
(-c*f+d*e)*(-c*h+d*g)*(b*x+a)^(1+m)*(d*x+c)^(-2-m)/d^2/(-a*d+b*c)/(2+m)-(a
*d*(-2*c*f*h+d*e*h+d*f*g)*(2+m)-b*(d^2*e*g+c*d*(e*h+f*g)*(1+m)-c^2*f*h*(3+
2*m)))*(b*x+a)^(1+m)*(d*x+c)^(-1-m)/d^2/(-a*d+b*c)^2/(1+m)/(2+m)+f*h*(b*x+
a)^(1+m)*hypergeom([1, 1],[1-m],b*(d*x+c)/(-a*d+b*c))/d^2/(-a*d+b*c)/m/((d
*x+c)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int (a+bx)^m (c+dx)^{-3-m} (e+fx)(g+hx)\, dx =$$

$$-\frac{(a+bx)^m (c+dx)^{-2-m} \left(d^3(a+bx)\left(-a^3dfh(1+m)+a^2bfh(cm-d(3+2m)x)+ab^2(ceh+deg(1\right.\right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

output
```
-(((a + b*x)^m*(c + d*x)^(-2 - m)*(d^3*(a + b*x)*(-(a^3*d*f*h*(1 + m)) + a
^2*b*f*h*(c*m - d*(3 + 2*m)*x) + a*b^2*(c*e*h + d*e*g*(1 + m) + d*f*g*(2 +
 m)*x + d*e*h*(2 + m)*x + c*f*(g + 2*h*(1 + m)*x)) - b^3*(d*e*g*x + c*(e*g
*(2 + m) + f*g*(1 + m)*x + e*h*(1 + m)*x))) + ((b*c - a*d)^4*f*h*(1 + m)*H
ypergeometric2F1[-2 - m, -2 - m, -1 - m, (b*(c + d*x))/(b*c - a*d)])/((d*(
a + b*x))/(-(b*c) + a*d))^m))/(b^2*d^3*(b*c - a*d)^2*(1 + m)*(2 + m)))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {162, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m (c+dx)^{-m-3}\, dx$$

$$\Big\downarrow \text{162}$$

$$\frac{fh\int (a+bx)^{m+2}(c+dx)^{-m-3}dx}{b^2} - \frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^3(-d)fh(m+1)-bx\left(a^2dfh(2m+3)-ab(2cfh(m+1)+d(m+2)(eh+fg))+b\right.\right.}{b^2(m+1)(m+2)(bc-}$$

$$\Big\downarrow \text{80}$$

$$\frac{bfh(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\int(a+bx)^{m+2}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-m-3}dx}{(bc-ad)^3}-$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^3(-d)fh(m+1)-bx\left(a^2dfh(2m+3)-ab(2cfh(m+1)+d(m+2)(eh+fg))+b\right.\right.}{b^2(m+1)(m+2)(bc-}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\text{Hypergeometric2F1}\left(m+3,m+3,m+4,-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}-$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^3(-d)fh(m+1)-bx\left(a^2dfh(2m+3)-ab(2cfh(m+1)+d(m+2)(eh+fg))+b\right.\right.}{b^2(m+1)(m+2)(bc-}$$

input
```
Int[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

output
```
-(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m)*(a^2*b*c*f*h*m - a^3*d*f*h*(1 + m)
 - b^3*c*e*g*(2 + m) + a*b^2*(c*(f*g + e*h) + d*e*g*(1 + m)) - b*(a^2*d*f*
h*(3 + 2*m) + b^2*(d*e*g + c*(f*g + e*h)*(1 + m)) - a*b*(2*c*f*h*(1 + m) +
 d*(f*g + e*h)*(2 + m)))*x))/(b^2*(b*c - a*d)^2*(1 + m)*(2 + m))) + (f*h*(
a + b*x)^(3 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[3 + m, 3
+ m, 4 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(3 + m)*(c + d*x
)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

Maple [F]

$$\int (bx+a)^m (xd+c)^{-3-m} (fx+e) (hx+g)\, dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\, dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 3), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(-3-m)*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m (c+dx)^{-3-m}(e+fx)(g+hx)\,dx = \int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+3}}\,dx$$

input
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3),x)
```

output
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3), x)
```

Reduce [F]

$$\begin{aligned}\int (a+bx)^m (c+dx)^{-3-m}(e+fx)(g+hx)\,dx &= \left(\int \frac{(bx+a)^m}{(dx+c)^m c^3+3(dx+c)^m c^2dx+3(dx+c)^m c\,d^2x^2+(dx+c)^m d^3x^3}dx\right) eg \\ &+ \left(\int \frac{(bx+a)^m x^2}{(dx+c)^m c^3+3(dx+c)^m c^2dx+3(dx+c)^m c\,d^2x^2+(dx+c)^m d^3x^3}dx\right) fh \\ &+ \left(\int \frac{(bx+a)^m x}{(dx+c)^m c^3+3(dx+c)^m c^2dx+3(dx+c)^m c\,d^2x^2+(dx+c)^m d^3x^3}dx\right) eh \\ &+ \left(\int \frac{(bx+a)^m x}{(dx+c)^m c^3+3(dx+c)^m c^2dx+3(dx+c)^m c\,d^2x^2+(dx+c)^m d^3x^3}dx\right) fg\end{aligned}$$

input
```
int((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((a + b*x)**m/((c + d*x)**m*c**3 + 3*(c + d*x)**m*c**2*d*x + 3*(c + d*x
)**m*c*d**2*x**2 + (c + d*x)**m*d**3*x**3),x)*e*g + int(((a + b*x)**m*x**2
)/((c + d*x)**m*c**3 + 3*(c + d*x)**m*c**2*d*x + 3*(c + d*x)**m*c*d**2*x**
2 + (c + d*x)**m*d**3*x**3),x)*f*h + int(((a + b*x)**m*x)/((c + d*x)**m*c*
*3 + 3*(c + d*x)**m*c**2*d*x + 3*(c + d*x)**m*c*d**2*x**2 + (c + d*x)**m*d
**3*x**3),x)*e*h + int(((a + b*x)**m*x)/((c + d*x)**m*c**3 + 3*(c + d*x)**
m*c**2*d*x + 3*(c + d*x)**m*c*d**2*x**2 + (c + d*x)**m*d**3*x**3),x)*f*g
```

3.149 $\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 288

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx$$
$$= \frac{(de-cf)(dg-ch)(a+bx)^{1+m}(c+dx)^{-3-m}}{d^2(bc-ad)(3+m)}$$
$$- \frac{(ad(dfg+deh-2cfh)(3+m) - b(2d^2eg+cd(fg+eh)(1+m)-2c^2fh(2+m)))\,(a+bx)^{1+m}(c+}{d^2(bc-ad)^2(2+m)(3+m)}$$
$$+ \frac{(a^2d^2fh(6+5m+m^2) - abd(3+m)(d(fg+eh)+2cfh(1+m)) + b^2(2d^2eg+cd(fg+eh)(1+m)}{d^2(bc-ad)^3(1+m)(2+m)(3+m)}$$

output

```
(-c*f+d*e)*(-c*h+d*g)*(b*x+a)^(1+m)*(d*x+c)^(-3-m)/d^2/(-a*d+b*c)/(3+m)-(a
*d*(-2*c*f*h+d*e*h+d*f*g)*(3+m)-b*(2*d^2*e*g+c*d*(e*h+f*g)*(1+m)-2*c^2*f*h
*(2+m)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)/d^2/(-a*d+b*c)^2/(2+m)/(3+m)+(a^2*d^
2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2*e*g+c
*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-1-m)/d^2/
(-a*d+b*c)^3/(1+m)/(2+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.76

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx$$
$$=\frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(adfh(3+m)(c+dx)+\frac{(a^2d^2fh(6+5m+m^2)-abd(3+m)(d(fg+eh)+2cfh(1+m))+b^2(2d^2eg-}{(bc-ad)}\right.}{bd^2(b}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*d*f*h*(3 + m)*(c + d*x) + ((a^2*d
^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 + m)) +
 b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c +
 d*x)*(-(a*d*(1 + m)) + b*c*(2 + m) + b*d*x))/((b*c - a*d)^2*(1 + m)*(2 +
m)) + b*(d^2*e*g - c^2*f*h*(2 + m) - c*d*(e*h + f*(g + h*(3 + m)*x)))))/(b
*d^2*(b*c - a*d)*(3 + m))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {163, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)(g+hx)(a+bx)^m(c+dx)^{-m-4}\,dx$$

$$\Big\downarrow \text{163}$$

$$\frac{(a^2d^2fh(m^2+5m+6)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2(c^2fh(m^2+3m+2)+cd(m+1)(eh+}{bd^2(m+3)(bc-ad)}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(-dfh(m+3)x(bc-ad)+acdfh(m+3)+b(c^2(-f)h(m+2)-cd(eh+fg)+d^2eg\right.}{bd^2(m+3)(bc-ad)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+cd(m+1)(eh+\right.\right.}{bd^2(m+3)(bc-ad)}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(-dfh(m+3)x(bc-ad)+acdfh(m+3)+b(c^2(-f)h(m+2)-cd(eh+fg)+d^2eg\right.}{bd^2(m+3)(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\left(\frac{(a+bx)^{m+1}(c+dx)^{-m-2}}{(m+2)(bc-ad)}+\frac{b(a+bx)^{m+1}(c+dx)^{-m-1}}{(m+1)(m+2)(bc-ad)^2}\right)\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg\right.}{bd^2(m+3)(bc-ad)}$$
$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(-dfh(m+3)x(bc-ad)+acdfh(m+3)+b(c^2(-f)h(m+2)-cd(eh+fg)+d^2eg\right.}{bd^2(m+3)(bc-ad)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*(d^2*e*g - c*
d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x))/(b*d^2*(b
*c - a*d)*(3 + m)) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g
 + e*h) + 2*c*f*h*(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^
2*f*h*(2 + 3*m + m^2)))*(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/((b*c - a*
d)*(2 + m)) + (b*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/((b*c - a*d)^2*(1 +
 m)*(2 + m))))/(b*d^2*(b*c - a*d)*(3 + m))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(288) = 576$.

Time = 1.20 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.10

method	result
gosper	$-\frac{(bx+a)^{1+m}(xd+c)^{-3-m}\left(a^2d^2fh\,m^2x^2-2abcdfh\,m^2x^2+b^2c^2fh\,m^2x^2+a^2d^2eh\,m^2x+a^2d^2fg\,m^2x+5a^2d^2fhm\,x^2-2abc\right.}{}$
orering	$-\frac{(bx+a)(xd+c)\left(a^2d^2fh\,m^2x^2-2abcdfh\,m^2x^2+b^2c^2fh\,m^2x^2+a^2d^2eh\,m^2x+a^2d^2fg\,m^2x+5a^2d^2fhm\,x^2-2abcdeh\,m^2x-\right.}{}$
parallelrisch	Expression too large to display

input
```
int((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+a)^(1+m)*(d*x+c)^(-3-m)/(a^3*d^3*m^3-3*a^2*b*c*d^2*m^3+3*a*b^2*c^2*d
*m^3-b^3*c^3*m^3+6*a^3*d^3*m^2-18*a^2*b*c*d^2*m^2+18*a*b^2*c^2*d*m^2-6*b^3
*c^3*m^2+11*a^3*d^3*m-33*a^2*b*c*d^2*m+33*a*b^2*c^2*d*m-11*b^3*c^3*m+6*a^3
*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)*(a^2*d^2*f*h*m^2*x^2-2*a*b*c
*d*f*h*m^2*x^2+b^2*c^2*f*h*m^2*x^2+a^2*d^2*e*h*m^2*x+a^2*d^2*f*g*m^2*x+5*a
^2*d^2*f*h*m*x^2-2*a*b*c*d*e*h*m^2*x-2*a*b*c*d*f*g*m^2*x-8*a*b*c*d*f*h*m*x
^2-a*b*d^2*e*h*m*x^2-a*b*d^2*f*g*m*x^2+b^2*c^2*e*h*m^2*x+b^2*c^2*f*g*m^2*x
+3*b^2*c^2*f*h*m*x^2+b^2*c*d*e*h*m*x^2+b^2*c*d*f*g*m*x^2+2*a^2*c*d*f*h*m*x
+a^2*d^2*e*g*m^2+4*a^2*d^2*e*h*m*x+4*a^2*d^2*f*g*m*x+6*a^2*d^2*f*h*x^2-2*a
*b*c^2*f*h*m*x-2*a*b*c*d*e*g*m^2-8*a*b*c*d*e*h*m*x-8*a*b*c*d*f*g*m*x-6*a*b
*c*d*f*h*x^2-2*a*b*d^2*e*g*m*x-3*a*b*d^2*e*h*x^2-3*a*b*d^2*f*g*x^2+b^2*c^2
*e*g*m^2+4*b^2*c^2*e*h*m*x+4*b^2*c^2*f*g*m*x+2*b^2*c^2*f*h*x^2+2*b^2*c*d*e
*g*m*x+b^2*c*d*e*h*x^2+b^2*c*d*f*g*x^2+2*b^2*d^2*e*g*x^2+a^2*c*d*e*h*m+a^2
*c*d*f*g*m+6*a^2*c*d*f*h*x+3*a^2*d^2*e*g*m+3*a^2*d^2*e*h*x+3*a^2*d^2*f*g*x
-a*b*c^2*e*h*m-a*b*c^2*f*g*m-2*a*b*c^2*f*h*x-8*a*b*c*d*e*g*m-10*a*b*c*d*e*
h*x-10*a*b*c*d*f*g*x-2*a*b*d^2*e*g*x+5*b^2*c^2*e*g*m+3*b^2*c^2*e*h*x+3*b^2
*c^2*f*g*x+6*b^2*c*d*e*g*x+2*a^2*c^2*f*h+a^2*c*d*e*h+a^2*c*d*f*g+2*a^2*d^2
*e*g-3*a*b*c^2*e*h-3*a*b*c^2*f*g-6*a*b*c*d*e*g+6*b^2*c^2*e*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(288) = 576$.

Time = 0.18 (sec) , antiderivative size = 1659, normalized size of antiderivative = 5.76

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

output
```
((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*g*m^2 + ((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*f*h*m^2 + (2*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g
+ ((b^3*c*d^2 - 3*a*b^2*d^3)*e + 2*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^
3)*f)*h + ((b^3*c*d^2 - a*b^2*d^3)*f*g + ((b^3*c*d^2 - a*b^2*d^3)*e + (3*b
^3*c^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*f)*h)*m)*x^4 + (((b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*f*g + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*e +
(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*h)*m^2 + 4*(2*b^3*c*d^2
*e + (b^3*c^2*d - 3*a*b^2*c*d^2)*f)*g + 2*(2*(b^3*c^2*d - 3*a*b^2*c*d^2)*e
 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 3*a^3*d^3)*f)*h + ((2*(b^3*c
*d^2 - a*b^2*d^3)*e + (5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*f)*g + (
(5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*e + (3*b^3*c^3 - 7*a*b^2*c^2*d
 - a^2*b*c*d^2 + 5*a^3*d^3)*f)*h)*m)*x^3 + ((((b^3*c^2*d - 2*a*b^2*c*d^2 +
 a^2*b*d^3)*e + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + ((b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2*b*c^2
*d + a^3*c*d^2)*f)*h)*m^2 + 3*(4*b^3*c^2*d*e + (b^3*c^3 - 3*a*b^2*c^2*d -
3*a^2*b*c*d^2 + a^3*d^3)*f)*g + 3*(4*a^3*c*d^2*f + (b^3*c^3 - 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + a^3*d^3)*e)*h + (((7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b
*d^3)*e + 4*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + (4*(b^3
*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 8*a^2*b*c^2*d
 + 7*a^3*c*d^2)*f)*h)*m)*x^2 + (2*(3*a*b^2*c^3 - 3*a^2*b*c^2*d + a^3*c*...
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(-4-m)*(f*x+e)*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-4}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Giac [F]

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx = \int (fx+e)(hx+g)(bx+a)^m (dx+c)^{-m-4}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 1895, normalized size of antiderivative = 6.58

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 4),x)
```

output
```
- ((a + b*x)^m*(2*a^3*c^3*f*h + 6*a*b^2*c^3*e*g - 3*a^2*b*c^3*e*h - 3*a^2*
b*c^3*f*g + 2*a^3*c*d^2*e*g + a^3*c^2*d*e*h + a^3*c^2*d*f*g - 6*a^2*b*c^2*
d*e*g + 5*a*b^2*c^3*e*g*m - a^2*b*c^3*e*h*m - a^2*b*c^3*f*g*m + 3*a^3*c*d^
2*e*g*m + a^3*c^2*d*e*h*m + a^3*c^2*d*f*g*m + a*b^2*c^3*e*g*m^2 + a^3*c*d^
2*e*g*m^2 - 2*a^2*b*c^2*d*e*g*m^2 - 8*a^2*b*c^2*d*e*g*m))/((a*d - b*c)^3*(
c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x^3*(a + b*x)^m*(6*a^3*d^3*f
*h + 2*b^3*c^3*f*h + 8*b^3*c*d^2*e*g + 4*b^3*c^2*d*e*h + 4*b^3*c^2*d*f*g +
 5*a^3*d^3*f*h*m + 3*b^3*c^3*f*h*m + a^3*d^3*f*h*m^2 + b^3*c^3*f*h*m^2 - 1
2*a*b^2*c*d^2*e*h - 12*a*b^2*c*d^2*f*g - 6*a*b^2*c^2*d*f*h + 6*a^2*b*c*d^2
*f*h - 2*a*b^2*d^3*e*g*m + 3*a^2*b*d^3*e*h*m + 3*a^2*b*d^3*f*g*m + 2*b^3*c
*d^2*e*g*m + 5*b^3*c^2*d*e*h*m + 5*b^3*c^2*d*f*g*m + a^2*b*d^3*e*h*m^2 + a
^2*b*d^3*f*g*m^2 + b^3*c^2*d*e*h*m^2 + b^3*c^2*d*f*g*m^2 - 2*a*b^2*c*d^2*e
*h*m^2 - 2*a*b^2*c*d^2*f*g*m^2 - a*b^2*c^2*d*f*h*m^2 - a^2*b*c*d^2*f*h*m^2
 - 8*a*b^2*c*d^2*e*h*m - 8*a*b^2*c*d^2*f*g*m - 7*a*b^2*c^2*d*f*h*m - a^2*b
*c*d^2*f*h*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
- (x*(a + b*x)^m*(2*a^3*d^3*e*g + 6*b^3*c^3*e*g + 4*a^3*c*d^2*e*h + 4*a^3*
c*d^2*f*g + 8*a^3*c^2*d*f*h + 3*a^3*d^3*e*g*m + 5*b^3*c^3*e*g*m + a^3*d^3*
e*g*m^2 + b^3*c^3*e*g*m^2 + 6*a*b^2*c^2*d*e*g - 6*a^2*b*c*d^2*e*g - 12*a^2
*b*c^2*d*e*h - 12*a^2*b*c^2*d*f*g + 3*a*b^2*c^3*e*h*m + 3*a*b^2*c^3*f*g*m
- 2*a^2*b*c^3*f*h*m + 5*a^3*c*d^2*e*h*m + 5*a^3*c*d^2*f*g*m + 2*a^3*c^2...
```

Reduce [F]

$$
\begin{aligned}
&\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx \\
&= \left(\int \frac{(bx+a)^m}{(dx+c)^m c^4+4(dx+c)^m c^3dx+6(dx+c)^m c^2d^2x^2+4(dx+c)^m c\,d^3x^3+(dx+c)^m d^4x^4}dx\right) eg \\
&\quad+\left(\int \frac{(bx+a)^m x^2}{(dx+c)^m c^4+4(dx+c)^m c^3dx+6(dx+c)^m c^2d^2x^2+4(dx+c)^m c\,d^3x^3+(dx+c)^m d^4x^4}dx\right) \\
&\quad+\left(\int \frac{(bx+a)^m x}{(dx+c)^m c^4+4(dx+c)^m c^3dx+6(dx+c)^m c^2d^2x^2+4(dx+c)^m c\,d^3x^3+(dx+c)^m d^4x^4}dx\right) \\
&\quad+\left(\int \frac{(bx+a)^m x}{(dx+c)^m c^4+4(dx+c)^m c^3dx+6(dx+c)^m c^2d^2x^2+4(dx+c)^m c\,d^3x^3+(dx+c)^m d^4x^4}dx\right)
\end{aligned}
$$

input
```
int((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x)
```

output
```
int((a + b*x)**m/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x
)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4)
,x)*e*g + int(((a + b*x)**m*x**2)/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3
*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d
*x)**m*d**4*x**4),x)*f*h + int(((a + b*x)**m*x)/((c + d*x)**m*c**4 + 4*(c
+ d*x)**m*c**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3
*x**3 + (c + d*x)**m*d**4*x**4),x)*e*h + int(((a + b*x)**m*x)/((c + d*x)**
m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c +
d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4),x)*f*g
```

3.150 $\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 447

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$

$$= -\frac{(bc-ad)^2(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^3(de-cf)(3+m)}$$

$$-\frac{(bc-ad)\left(ad(2dfg+cfh(1+m)-deh(3+m))-b(2d^2eg(3+m)+c^2fh(7+3m)-cd(2fg(2+m}{d^3(de-cf)^2(2+m)(3+m)}$$

$$+\frac{(b^2c(de-cf)^2h(2+m)(3+m)+b(de-cf)(bdg-2bch+2adh)(3+m)(cf(1+m)-de(2+m))+}{}$$

$$-\frac{b^2h(c+dx)^{-m}(e+fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,1,1-m,-\frac{f(c+dx)}{de-cf}\right)}{d^3(de-cf)m}$$

output
```
-(-a*d+b*c)^2*(-c*h+d*g)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)/(3+m)
-(-a*d+b*c)*(a*d*(2*d*f*g+c*f*h*(1+m)-d*e*h*(3+m))-b*(2*d^2*e*g*(3+m)+c^2*
f*h*(7+3*m)-c*d*(2*f*g*(2+m)+3*e*h*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d
^3/(-c*f+d*e)^2/(2+m)/(3+m)+(b^2*c*(-c*f+d*e)^2*h*(2+m)*(3+m)+b*(-c*f+d*e)
*(2*a*d*h-2*b*c*h+b*d*g)*(3+m)*(c*f*(1+m)-d*e*(2+m))+f*(b^2*c*(-c*h+d*g)*(
c*f*(1+m)-d*e*(3+m))-2*a*b*d*(-c*h+d*g)*(c*f*(1+m)-d*e*(3+m))-a^2*d^2*(2*d
*f*g+c*f*h*(1+m)-d*e*h*(3+m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e
)^3/(1+m)/(2+m)/(3+m)-b^2*h*(f*x+e)^(1+m)*hypergeom([1, 1],[1-m],-f*(d*x+c
)/(-c*f+d*e))/d^3/(-c*f+d*e)/m/((d*x+c)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 1.14 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.94

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(c+dx)^{-3-m}(e+fx)^m\left(-d(dg-ch)(e+fx)\left(-((bc-ad)(de-cf)^2(1+m)(2+m)(adf+bcf(2+m\right.\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

output
```
((c + d*x)^(-3 - m)*(e + f*x)^m*(-(d*(d*g - c*h)*(e + f*x)*(-((b*c - a*d)*
(d*e - c*f)^2*(1 + m)*(2 + m)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))) + b
*d*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*(a + b*x) + (b^2*(d*e - c*f)*(2 +
 m)*(c*f*(1 + m) - d*e*(3 + m)) + 2*d*f*(-(a^2*d*f) - b*(b*c*e + a*c*f*(1
+ m) - a*d*e*(3 + m))))*(c + d*x)*(-(c*f*(2 + m)) + d*(e + e*m - f*x)))) -
 (d*e - c*f)*h*(3 + m)*(c + d*x)*(d*(b*c - a*d)^2*f*(d*e - c*f)*(1 + m)*(e
 + f*x) - (c + d*x)*(d*(a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(2 + m)
) + b^2*(-(c^2*f^2*(1 + m)) + d^2*e^2*(2 + m)))*(e + f*x) - (b^2*(d*e - c*
f)^3*(2 + m)*Hypergeometric2F1[-1 - m, -1 - m, -m, (f*(c + d*x))/(-(d*e) +
 c*f)])/((d*(e + f*x))/(d*e - c*f))^m))))/(d^4*f*(d*e - c*f)^3*(1 + m)*(2
+ m)*(3 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {177, 100, 25, 88, 80, 79, 101, 25, 88, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(g+hx)(c+dx)^{-m-4}(e+fx)^m\,dx$$

$$\Big\downarrow \text{177}$$

$$\frac{(dg-ch)\int (a+bx)^2(c+dx)^{-m-4}(e+fx)^m dx}{d}+\frac{h\int (a+bx)^2(c+dx)^{-m-3}(e+fx)^m dx}{d}$$

$$\Big\downarrow \text{100}$$

$$\frac{h\left(\frac{\int -(c+dx)^{-m-2}(e+fx)^m\left(-c(cf(m+1)-de(m+2))b^2-d(de-cf)(m+2)xb^2+2ad(cf(m+1)-de(m+2))b+a^2d^2f\right)dx}{d^2(m+2)(de-cf)}-\frac{(bc-ad)^2(c+dx)^{-m\ldots}}{d^2(m+2)(\ldots}\ldots\right.}{d}$$
$$\frac{(dg-ch)\int (a+bx)^2(c+dx)^{-m-4}(e+fx)^m dx}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{h\left(-\frac{\int (c+dx)^{-m-2}(e+fx)^m\left(-c(cf(m+1)-de(m+2))b^2-d(de-cf)(m+2)xb^2+2ad(cf(m+1)-de(m+2))b+a^2d^2f\right)dx}{d^2(m+2)(de-cf)}-\frac{(bc-ad)^2(c+dx)^{-m\ldots}}{d^2(m+2)(\ldots}\ldots\right.}{d}$$
$$\frac{(dg-ch)\int (a+bx)^2(c+dx)^{-m-4}(e+fx)^m dx}{d}$$

$$\Big\downarrow \text{88}$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}-b^2(m+2)(de-cf)\int (c+dx)^{-m-1}(e+fx)^m dx}{d^2(m+2)(de-cf)}-\frac{(bc-ad)^2(c+dx)^{-m-2}(e+\ldots}{d^2(m+2)(de-cf)}\ldots\right.}{d}$$
$$\frac{(dg-ch)\int (a+bx)^2(c+dx)^{-m-4}(e+fx)^m dx}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}-b^2(m+2)(de-cf)(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\int(c+dx)^{-m-1}\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^m}{d^2(m+2)(de-cf)}\right.}{d}$$

$$\frac{(dg-ch)\int(a+bx)^2(c+dx)^{-m-4}(e+fx)^m dx}{d}$$

$\big\downarrow$ 79

$$\frac{(dg-ch)\int(a+bx)^2(c+dx)^{-m-4}(e+fx)^m dx}{d}+$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}+\frac{b^2(m+2)(de-cf)(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,\right.}{dm}}{d^2(m+2)(de-cf)}\right.}{d}$$

$\big\downarrow$ 101

$$\frac{(dg-ch)\left(-\frac{\int-(c+dx)^{-m-4}(e+fx)^m\left(dfa^2+b(bce-ad(m+3)e+acf(m+1))-b^2(de-cf)(m+2)x\right)dx}{df}-\frac{b(a+bx)(c+dx)^{-m-3}(e+fx)^{m+1}}{df}\right.}{d}$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}+\frac{b^2(m+2)(de-cf)(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,\right.}{dm}}{d^2(m+2)(de-cf)}\right.}{d}$$

$\big\downarrow$ 25

$$\frac{(dg-ch)\left(\frac{\int(c+dx)^{-m-4}(e+fx)^m\left(dfa^2+b(bce-ad(m+3)e+acf(m+1))-b^2(de-cf)(m+2)x\right)dx}{df}-\frac{b(a+bx)(c+dx)^{-m-3}(e+fx)^{m+1}}{df}\right)}{d}+$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}+\frac{b^2(m+2)(de-cf)(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,\right.}{dm}}{d^2(m+2)(de-cf)}\right.}{d}$$

$\big\downarrow$ 88

$$\frac{(dg-ch)\left(\frac{\frac{\left(\frac{b^2(m+2)(cf(m+1)-de(m+3))}{d}-\frac{2f\left(a^2df+b(acf(m+1)-ade(m+3)+bce)\right)}{de-cf}\right)\int (c+dx)^{-m-3}(e+fx)^m dx}{m+3}+\frac{(bc-ad)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(d}}{df}}{d}$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}+\frac{b^2(m+2)(de-cf)(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,\right.}{dm}}{d^2(m+2)(de-cf)}\right.}{d}$$

$$\Big\downarrow 55$$

$$\frac{(dg-ch)\left(\frac{\frac{\left(\frac{b^2(m+2)(cf(m+1)-de(m+3))}{d}-\frac{2f\left(a^2df+b(acf(m+1)-ade(m+3)+bce)\right)}{de-cf}\right)\left(-\frac{f\int (c+dx)^{-m-2}(e+fx)^m dx}{(m+2)(de-cf)}-\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(m+2)(de-cf)}\right)}{m+3}}{df}\right.}{d}$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}+\frac{b^2(m+2)(de-cf)(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,\right.}{dm}}{d^2(m+2)(de-cf)}\right.}{d}$$

$$\Big\downarrow 48$$

$$\frac{(dg-ch)\left(\frac{\frac{\left(\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}}{(m+1)(m+2)(de-cf)^2}-\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(m+2)(de-cf)}\right)\left(\frac{b^2(m+2)(cf(m+1)-de(m+3))}{d}-\frac{2f\left(a^2df+b(acf(m+1)-ade(m+3)+bce)\right)}{de-cf}\right)}{m+3}+}{df}\right.}{d}$$

$$\frac{h\left(-\frac{\frac{(bc-ad)(c+dx)^{-m-1}(e+fx)^{m+1}(adf+bcf(2m+3)-2bde(m+2))}{(m+1)(de-cf)}+\frac{b^2(m+2)(de-cf)(c+dx)^{-m}(e+fx)^m\left(\frac{d(e+fx)}{de-cf}\right)^{-m}\text{Hypergeometric2F1}\left(-m,-m,\right.}{dm}}{d^2(m+2)(de-cf)}\right.}{d}$$

input
```
Int[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

output
```
((d*g - c*h)*(-((b*(a + b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*f))
+ (((b*c - a*d)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c + d*x)^(-3 - m)
*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)) + (((b^2*(2 + m)*(c*f*(1 + m)
- d*e*(3 + m)))/d - (2*f*(a^2*d*f + b*(b*c*e + a*c*f*(1 + m) - a*d*e*(3 +
m))))/(d*e - c*f))*(-(((c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/((d*e - c*f)*
(2 + m))) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/((d*e - c*f)^2*(1 + m
)*(2 + m))))/(3 + m))/(d*f)))/d + (h*(-(((b*c - a*d)^2*(c + d*x)^(-2 - m)*
(e + f*x)^(1 + m))/(d^2*(d*e - c*f)*(2 + m))) - (((b*c - a*d)*(a*d*f - 2*b
*d*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/((d*
e - c*f)*(1 + m)) + (b^2*(d*e - c*f)*(2 + m)*(e + f*x)^m*Hypergeometric2F1
[-m, -m, 1 - m, -((f*(c + d*x))/(d*e - c*f))])/(d*m*(c + d*x)^m*((d*(e + f
*x))/(d*e - c*f))^m))/(d^2*(d*e - c*f)*(2 + m))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int (bx+a)^2 (xd+c)^{-4-m} (fx+e)^m (hx+g)\,dx$$

input
```
int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

output
```
int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas"
)
```

output
```
integral((b^2*h*x^3 + a^2*g + (b^2*g + 2*a*b*h)*x^2 + (2*a*b*g + a^2*h)*x)
*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**2*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima"
)
```

output
```
integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)^2(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int \frac{(e+f\,x)^m\,(g+h\,x)\,(a+b\,x)^2}{(c+d\,x)^{m+4}}\,dx$$

input
```
int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4),x)
```

output
```
int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4), x)
```

Reduce [F]

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$

$$= \left(\int \frac{(fx+e)^m}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right) a^2$$

$$+\left(\int \frac{(fx+e)^m x^3}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$

$$+2\left(\int \frac{(fx+e)^m x^2}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$

$$+\left(\int \frac{(fx+e)^m x^2}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$

$$+\left(\int \frac{(fx+e)^m x}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$

$$+2\left(\int \frac{(fx+e)^m x}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$

input

```
int((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

output

```
int((e + f*x)**m/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x
)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4)
,x)*a**2*g + int(((e + f*x)**m*x**3)/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c
**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c
+ d*x)**m*d**4*x**4),x)*b**2*h + 2*int(((e + f*x)**m*x**2)/((c + d*x)**m*c
**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x
)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4),x)*a*b*h + int(((e + f*x)**m*x*
*2)/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x)**m*c**2*d**
2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4),x)*b**2*g +
int(((e + f*x)**m*x)/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c +
 d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x
**4),x)*a**2*h + 2*int(((e + f*x)**m*x)/((c + d*x)**m*c**4 + 4*(c + d*x)**
m*c**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 +
(c + d*x)**m*d**4*x**4),x)*a*b*g
```

3.151 $\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 299

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(bc-ad)(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2(de-cf)(3+m)}$$
$$+ \frac{(ad(2dfg+cfh(1+m)-deh(3+m))-b(2c^2fh(2+m)+d^2eg(3+m)-cd(fg(1+m)+2eh(3+}{d^2(de-cf)^2(2+m)(3+m)}$$
$$- \frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)-d^2e(3+m)(fg-eh(2+m))+c}{d^2(de-cf)^3(1+m)(2+m)(3+m)}$$

output
```
(-a*d+b*c)*(-c*h+d*g)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^2/(-c*f+d*e)/(3+m)+(a
*d*(2*d*f*g+c*f*h*(1+m)-d*e*h*(3+m))-b*(2*c^2*f*h*(2+m)+d^2*e*g*(3+m)-c*d*
(f*g*(1+m)+2*e*h*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^2/(-c*f+d*e)^2/(2
+m)/(3+m)-(a*d*f*(2*d*f*g+c*f*h*(1+m)-d*e*h*(3+m))+b*(c^2*f^2*h*(m^2+3*m+2
)-d^2*e*(3+m)*(f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*h*(3+m))))*(d*x+c)^(-1-
m)*(f*x+e)^(1+m)/d^2/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.76

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{(c+dx)^{-3-m}(e+fx)^{1+m}\left(adf(dg-ch)+\frac{(adf(2dfg+cfh(1+m)-deh(3+m))+b(c^2f^2h(2+3m+m^2)+d^2e(3+m)(-fg+eh(}{(de-cf)^2(1+m)(}\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

output
```
((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) + ((a*d*f*(2*d*f*
g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m + m^2) + d^2*e*
(3 + m)*(-(f*g) + e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))))*(c
+ d*x)*(c*f*(2 + m) - d*(e + e*m - f*x)))/((d*e - c*f)^2*(1 + m)*(2 + m))
- b*(c^2*f*h*(2 + m) - d^2*e*h*(3 + m)*x + c*d*(-(e*h*(3 + m)) + f*(g + h*
(3 + m)*x)))))/(d^2*f*(-(d*e) + c*f)*(3 + m))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {163, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(g+hx)(c+dx)^{-m-4}(e+fx)^m\,dx$$

$$\Big\downarrow \; 163$$

$$-\frac{(adf(cfh(m+1)-deh(m+3)+2dfg)+b(c^2f^2h(m^2+3m+2)+cdf(m+1)(fg-2eh(m+3))+d^2(-e)(}{d^2f(m+3)(de-cf)}$$
$$\frac{(c+dx)^{-m-3}(e+fx)^{m+1}(adf(dg-ch)-bc(cfh(m+2)-deh(m+3)+dfg)+bdh(m+3)x(de-cf))}{d^2f(m+3)(de-cf)}$$

$$\Big\downarrow \; 55$$

$$-\frac{\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h(m^2+3m+2)+cdf(m+1)(fg-2eh(m+3))+d^2(-e)(\right.\right.}{d^2f(m+3)(de-cf)}$$
$$\frac{(c+dx)^{-m-3}(e+fx)^{m+1}(adf(dg-ch)-bc(cfh(m+2)-deh(m+3)+dfg)+bdh(m+3)x(de-cf))}{d^2f(m+3)(de-cf)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}}{(m+1)(m+2)(de-cf)^2}-\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(m+2)(de-cf)}\right)\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h(m^2+3m\right.\right.}{d^2f(m+3)(de-cf)}$$
$$\frac{(c+dx)^{-m-3}(e+fx)^{m+1}(adf(dg-ch)-bc(cfh(m+2)-deh(m+3)+dfg)+bdh(m+3)x(de-cf))}{d^2f(m+3)(de-cf)}$$

input
```
Int[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

output
```
-(((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) - b*c*(d*f*g +
c*f*h*(2 + m) - d*e*h*(3 + m)) + b*d*(d*e - c*f)*h*(3 + m)*x))/(d^2*f*(d*e
 - c*f)*(3 + m))) - ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*
(c^2*f^2*h*(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1
+ m)*(f*g - 2*e*h*(3 + m))))*(-(((c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/((d
*e - c*f)*(2 + m))) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/((d*e - c*f
)^2*(1 + m)*(2 + m))))/(d^2*f*(d*e - c*f)*(3 + m))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(299) = 598$.

Time = 1.24 (sec) , antiderivative size = 906, normalized size of antiderivative = 3.03

method	result
gosper	$-\frac{(xd+c)^{-3-m}(fx+e)^{1+m}(-b c^2 f^2 h m^2 x^2+2bcdefh m^2 x^2-b d^2 e^2 h m^2 x^2-a c^2 f^2 h m^2 x+2acdefh m^2 x-acd f^2 hm x^2-a}{}$
orering	$-\frac{(fx+e)(xd+c)(-b c^2 f^2 h m^2 x^2+2bcdefh m^2 x^2-b d^2 e^2 h m^2 x^2-a c^2 f^2 h m^2 x+2acdefh m^2 x-acd f^2 hm x^2-a d^2 e^2 h m^2 x}{}$
parallelrisch	Expression too large to display

input
```
int((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-(d*x+c)^(-3-m)*(f*x+e)^(1+m)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f
*m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3
*e^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3
*f^3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)*(-b*c^2*f^2*h*m^2*x^2+2*b*c*
d*e*f*h*m^2*x^2-b*d^2*e^2*h*m^2*x^2-a*c^2*f^2*h*m^2*x+2*a*c*d*e*f*h*m^2*x-
a*c*d*f^2*h*m*x^2-a*d^2*e^2*h*m^2*x+a*d^2*e*f*h*m*x^2-b*c^2*f^2*g*m^2*x-3*
b*c^2*f^2*h*m*x^2+2*b*c*d*e*f*g*m^2*x+8*b*c*d*e*f*h*m*x^2-b*c*d*f^2*g*m*x^
2-b*d^2*e^2*g*m^2*x-5*b*d^2*e^2*h*m*x^2+b*d^2*e*f*g*m*x^2-a*c^2*f^2*g*m^2-
4*a*c^2*f^2*h*m*x+2*a*c*d*e*f*g*m^2+8*a*c*d*e*f*h*m*x-2*a*c*d*f^2*g*m*x-a*
c*d*f^2*h*x^2-a*d^2*e^2*g*m^2-4*a*d^2*e^2*h*m*x+2*a*d^2*e*f*g*m*x+3*a*d^2*
e*f*h*x^2-2*a*d^2*f^2*g*x^2+2*b*c^2*e*f*h*m*x-4*b*c^2*f^2*g*m*x-2*b*c^2*f^
2*h*x^2-2*b*c*d*e^2*h*m*x+8*b*c*d*e*f*g*m*x+6*b*c*d*e*f*h*x^2-b*c*d*f^2*g*
x^2-4*b*d^2*e^2*g*m*x-6*b*d^2*e^2*h*x^2+3*b*d^2*e*f*g*x^2+a*c^2*e*f*h*m-5*
a*c^2*f^2*g*m-3*a*c^2*f^2*h*x-a*c*d*e^2*h*m+8*a*c*d*e*f*g*m+10*a*c*d*e*f*h
*x-6*a*c*d*f^2*g*x-3*a*d^2*e^2*g*m-3*a*d^2*e^2*h*x+2*a*d^2*e*f*g*x+b*c^2*e
*f*g*m+2*b*c^2*e*f*h*x-3*b*c^2*f^2*g*x-b*c*d*e^2*g*m-6*b*c*d*e^2*h*x+10*b*
c*d*e*f*g*x-3*b*d^2*e^2*g*x+3*a*c^2*e*f*h-6*a*c^2*f^2*g-a*c*d*e^2*h+6*a*c*
d*e*f*g-2*a*d^2*e^2*g-2*b*c^2*e^2*h+3*b*c^2*e*f*g-b*c*d*e^2*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(300) = 600$.

Time = 0.15 (sec) , antiderivative size = 1608, normalized size of antiderivative = 5.38

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

output
```
-(((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*m^2 - (3*b*d^3*e*f^2 -
(b*c*d^2 + 2*a*d^3)*f^3)*g + (6*b*d^3*e^2*f - 3*(2*b*c*d^2 + a*d^3)*e*f^2
+ (2*b*c^2*d + a*c*d^2)*f^3)*h - ((b*d^3*e*f^2 - b*c*d^2*f^3)*g - (5*b*d^3
*e^2*f - (8*b*c*d^2 + a*d^3)*e*f^2 + (3*b*c^2*d + a*c*d^2)*f^3)*h)*m)*x^4
+ (a*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3*e*f^2)*g*m^2 + (((b*d^3*e^2*f - 2
*b*c*d^2*e*f^2 + b*c^2*d*f^3)*g + (b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (
b*c^2*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*h)*m^2 - 4*(3*b*c*d^2*
e*f^2 - (b*c^2*d + 2*a*c*d^2)*f^3)*g + 2*(3*b*d^3*e^3 + 3*b*c*d^2*e^2*f -
3*(b*c^2*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + 2*a*c^2*d)*f^3)*h + ((3*b*d^3*e^2
*f - 2*(4*b*c*d^2 + a*d^3)*e*f^2 + (5*b*c^2*d + 2*a*c*d^2)*f^3)*g + (5*b*d
^3*e^3 - (b*c*d^2 - 3*a*d^3)*e^2*f - (7*b*c^2*d + 8*a*c*d^2)*e*f^2 + (3*b*
c^3 + 5*a*c^2*d)*f^3)*h)*m)*x^3 + (((b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f -
 (b*c^2*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*g + (a*c^3*f^3 + (b*
c*d^2 + a*d^3)*e^3 - (2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - a*c^2*d)*e*f^2
)*h)*m^2 + 3*(b*d^3*e^3 - 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + (b*c^3 + 4*a
*c^2*d)*f^3)*g - 3*(3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3 - (4*b*c
*d^2 + a*d^3)*e^3)*h + ((4*b*d^3*e^3 - (4*b*c*d^2 - a*d^3)*e^2*f - 4*(b*c^
2*d + 2*a*c*d^2)*e*f^2 + (4*b*c^3 + 7*a*c^2*d)*f^3)*g + (4*a*c^3*f^3 + (7*
b*c*d^2 + 4*a*d^3)*e^3 - 4*(2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - 4*a*c^2*
d)*e*f^2)*h)*m)*x^2 + (6*a*c^3*e*f^2 + (b*c^2*d + 2*a*c*d^2)*e^3 - 3*(b...
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (bx+a)(hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 1890, normalized size of antiderivative = 6.32

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^m*(g + h*x)*(a + b*x))/(c + d*x)^(m + 4),x)
```

output
```
((e + f*x)^m*(2*b*c^3*e^3*h + 2*a*c*d^2*e^3*g + a*c^2*d*e^3*h + b*c^2*d*e^
3*g + 6*a*c^3*e*f^2*g - 3*a*c^3*e^2*f*h - 3*b*c^3*e^2*f*g - 6*a*c^2*d*e^2*
f*g + 3*a*c*d^2*e^3*g*m + a*c^2*d*e^3*h*m + b*c^2*d*e^3*g*m + 5*a*c^3*e*f^
2*g*m - a*c^3*e^2*f*h*m - b*c^3*e^2*f*g*m + a*c*d^2*e^3*g*m^2 + a*c^3*e*f^
2*g*m^2 - 2*a*c^2*d*e^2*f*g*m^2 - 8*a*c^2*d*e^2*f*g*m))/((c*f - d*e)^3*(c
+ d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*a*c^3*f^3*g +
 2*a*d^3*e^3*g + 4*a*c*d^2*e^3*h + 4*b*c*d^2*e^3*g + 8*b*c^2*d*e^3*h + 5*a
*c^3*f^3*g*m + 3*a*d^3*e^3*g*m + a*c^3*f^3*g*m^2 + a*d^3*e^3*g*m^2 - 6*a*c
*d^2*e^2*f*g + 6*a*c^2*d*e*f^2*g - 12*a*c^2*d*e^2*f*h - 12*b*c^2*d*e^2*f*g
 + 5*a*c*d^2*e^3*h*m + 5*b*c*d^2*e^3*g*m + 2*b*c^2*d*e^3*h*m + 3*a*c^3*e*f
^2*h*m + 3*b*c^3*e*f^2*g*m - 2*b*c^3*e^2*f*h*m + a*c*d^2*e^3*h*m^2 + b*c*d
^2*e^3*g*m^2 + a*c^3*e*f^2*h*m^2 + b*c^3*e*f^2*g*m^2 - a*c*d^2*e^2*f*g*m^2
 - a*c^2*d*e*f^2*g*m^2 - 2*a*c^2*d*e^2*f*h*m^2 - 2*b*c^2*d*e^2*f*g*m^2 - 7
*a*c*d^2*e^2*f*g*m - a*c^2*d*e*f^2*g*m - 8*a*c^2*d*e^2*f*h*m - 8*b*c^2*d*e
^2*f*g*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x
^4*(e + f*x)^m*(2*a*d^3*f^3*g + a*c*d^2*f^3*h + b*c*d^2*f^3*g + 2*b*c^2*d*
f^3*h - 3*a*d^3*e*f^2*h - 3*b*d^3*e*f^2*g + 6*b*d^3*e^2*f*h - 6*b*c*d^2*e*
f^2*h + a*c*d^2*f^3*h*m + b*c*d^2*f^3*g*m + 3*b*c^2*d*f^3*h*m - a*d^3*e*f^
2*h*m - b*d^3*e*f^2*g*m + 5*b*d^3*e^2*f*h*m + b*c^2*d*f^3*h*m^2 + b*d^3*e^
2*f*h*m^2 - 2*b*c*d^2*e*f^2*h*m^2 - 8*b*c*d^2*e*f^2*h*m))/((c*f - d*e)^...
```

Reduce [F]

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \left(\int \frac{(fx+e)^m}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right) ag$$
$$+ \left(\int \frac{(fx+e)^m x^2}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$
$$+ \left(\int \frac{(fx+e)^m x}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$
$$+ \left(\int \frac{(fx+e)^m x}{(dx+c)^m c^4 + 4(dx+c)^m c^3dx + 6(dx+c)^m c^2d^2x^2 + 4(dx+c)^m c\,d^3x^3 + (dx+c)^m d^4x^4}dx\right)$$

input
```
int((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

output
```
int((e + f*x)**m/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x
)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4)
,x)*a*g + int(((e + f*x)**m*x**2)/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3
*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d
*x)**m*d**4*x**4),x)*b*h + int(((e + f*x)**m*x)/((c + d*x)**m*c**4 + 4*(c
+ d*x)**m*c**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3
*x**3 + (c + d*x)**m*d**4*x**4),x)*a*h + int(((e + f*x)**m*x)/((c + d*x)**
m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c +
d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4),x)*b*g
```

3.152 $\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$\begin{aligned}\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx\\ &= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)}\\ &\quad+\frac{(2dfg+cfh(1+m)-deh(3+m))(c+dx)^{-2-m}(e+fx)^{1+m}}{d(de-cf)^2(2+m)(3+m)}\\ &\quad-\frac{f(2dfg+cfh(1+m)-deh(3+m))(c+dx)^{-1-m}(e+fx)^{1+m}}{d(de-cf)^3(1+m)(2+m)(3+m)}\end{aligned}$$

output

```
-(-c*h+d*g)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)/(3+m)+(2*d*f*g+c*f*h
*(1+m)-d*e*h*(3+m))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)^2/(2+m)/(3+m
)-f*(2*d*f*g+c*f*h*(1+m)-d*e*h*(3+m))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d/(-c*f
+d*e)^3/(1+m)/(2+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(-3-m)} - \frac{(-2dfg-h(de(-3-m)+cf(1+m)))\left(\frac{(c+dx)^{-2-m}(e+fx)^{1+m}}{(de-cf)(-2-m)}+\frac{f(c+dx)^{-1-m}(e+fx)^{1+m}}{(de-cf)^2(-2-m)(-1-m)}\right)}{d(de-cf)(-3-m)}$$

input
```
Integrate[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

output
```
((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(-3 - m)
) - ((-2*d*f*g - h*(d*e*(-3 - m) + c*f*(1 + m)))*(((c + d*x)^(-2 - m)*(e +
 f*x)^(1 + m))/((d*e - c*f)*(-2 - m)) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1
 + m))/((d*e - c*f)^2*(-2 - m)*(-1 - m))))/(d*(d*e - c*f)*(-3 - m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {88, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)(c+dx)^{-m-4}(e+fx)^m\,dx$$

$$\Big\downarrow \text{ 88}$$

$$-\frac{(cfh(m+1)-deh(m+3)+2dfg)\int (c+dx)^{-m-3}(e+fx)^m dx}{d(m+3)(de-cf)}-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}$$

$$\Big\downarrow \text{ 55}$$

$$-\frac{(cfh(m+1)-deh(m+3)+2dfg)\left(-\frac{f\int (c+dx)^{-m-2}(e+fx)^m dx}{(m+2)(de-cf)}-\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(m+2)(de-cf)}\right)}{d(m+3)(de-cf)}-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}-\frac{\left(\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}}{(m+1)(m+2)(de-cf)^2}-\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(m+2)(de-cf)}\right)(cfh(m+1)-deh(m+3)+2dfg)}{d(m+3)(de-cf)}$$

input

```
Int[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

output

```
-(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m
))) - ((2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m))*(-(((c + d*x)^(-2 - m)*(e
 + f*x)^(1 + m))/((d*e - c*f)*(2 + m))) + (f*(c + d*x)^(-1 - m)*(e + f*x)^
(1 + m))/((d*e - c*f)^2*(1 + m)*(2 + m))))/(d*(d*e - c*f)*(3 + m))
```

Defintions of rubi rules used

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(186) = 372$.

Time = 0.83 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.74

method	result
gosper	$-\frac{(xd+c)^{-3-m}(fx+e)^{1+m}(-c^2f^2hm^2x+2cdefhm^2x-cdf^2hmx^2-d^2e^2hm^2x+d^2efhmx^2-c^2f^2gm^2-4c^2f^2hmx+2cc}{c^3f^3m^3-3c^2def^2m^3+3c}$
orering	$-\frac{(fx+e)(xd+c)(-c^2f^2hm^2x+2cdefhm^2x-cdf^2hmx^2-d^2e^2hm^2x+d^2efhmx^2-c^2f^2gm^2-4c^2f^2hmx+2cdefgm^2+8}{c^3f^3m^3-3c^2def^2r}$
parallelrisch	Expression too large to display

input
```
int((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
-(d*x+c)^(-3-m)*(f*x+e)^(1+m)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f
*m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3
*e^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3
*f^3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)*(-c^2*f^2*h*m^2*x+2*c*d*e*f*
h*m^2*x-c*d*f^2*h*m*x^2-d^2*e^2*h*m^2*x+d^2*e*f*h*m*x^2-c^2*f^2*g*m^2-4*c^
2*f^2*h*m*x+2*c*d*e*f*g*m^2+8*c*d*e*f*h*m*x-2*c*d*f^2*g*m*x-c*d*f^2*h*x^2-
d^2*e^2*g*m^2-4*d^2*e^2*h*m*x+2*d^2*e*f*g*m*x+3*d^2*e*f*h*x^2-2*d^2*f^2*g*
x^2+c^2*e*f*h*m-5*c^2*f^2*g*m-3*c^2*f^2*h*x-c*d*e^2*h*m+8*c*d*e*f*g*m+10*c
*d*e*f*h*x-6*c*d*f^2*g*x-3*d^2*e^2*g*m-3*d^2*e^2*h*x+2*d^2*e*f*g*x+3*c^2*e
*f*h-6*c^2*f^2*g-c*d*e^2*h+6*c*d*e*f*g-2*d^2*e^2*g)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(186) = 372$.

Time = 0.11 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.87

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

output
```
-((2*d^3*f^3*g - (d^3*e*f^2 - c*d^2*f^3)*h*m - (3*d^3*e*f^2 - c*d^2*f^3)*h
)*x^4 + (c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*g*m^2 + (8*c*d^2*f^3*g + (
d^3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*h*m^2 - 4*(3*c*d^2*e*f^2 - c^2*d*f^
3)*h - (2*(d^3*e*f^2 - c*d^2*f^3)*g - (3*d^3*e^2*f - 8*c*d^2*e*f^2 + 5*c^2
*d*f^3)*h)*m)*x^3 + (12*c^2*d*f^3*g + ((d^3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*
f^3)*g + (d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m^2 + 3*(d^3*e
^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*h + ((d^3*e^2*f - 8*c*d^2*e*
f^2 + 7*c^2*d*f^3)*g + 4*(d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h
)*m)*x^2 + 2*(c*d^2*e^3 - 3*c^2*d*e^2*f + 3*c^3*e*f^2)*g + (c^2*d*e^3 - 3*
c^3*e^2*f)*h + ((3*c*d^2*e^3 - 8*c^2*d*e^2*f + 5*c^3*e*f^2)*g + (c^2*d*e^3
 - c^3*e^2*f)*h)*m + (((d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*g +
 (c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*h)*m^2 + 2*(d^3*e^3 - 3*c*d^2*e^2
*f + 3*c^2*d*e*f^2 + 3*c^3*f^3)*g + 4*(c*d^2*e^3 - 3*c^2*d*e^2*f)*h + ((3*
d^3*e^3 - 7*c*d^2*e^2*f - c^2*d*e*f^2 + 5*c^3*f^3)*g + (5*c*d^2*e^3 - 8*c^
2*d*e^2*f + 3*c^3*e*f^2)*h)*m)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m/(6*d^3*e^
3 - 18*c*d^2*e^2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f
 + 3*c^2*d*e*f^2 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f
^2 - c^3*f^3)*m^2 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)
*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Giac [F]

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \int (hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

input
```
integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

output
```
integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 869, normalized size of antiderivative = 4.67

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$= \frac{x^2\,(e+f\,x)^m\,(h\,c^3\,f^3\,m^2+4\,h\,c^3\,f^3\,m+3\,h\,c^3\,f^3-h\,c^2\,d\,e\,f^2\,m^2-4\,h\,c^2\,d\,e\,f^2\,m-9\,h\,c^2\,d\,e\,f^2+g\,c^2\,d}{}$$
$$+\frac{x\,(e+f\,x)^m\,(h\,c^3\,e\,f^2\,m^2+3\,h\,c^3\,e\,f^2\,m+g\,c^3\,f^3\,m^2+5\,g\,c^3\,f^3\,m+6\,g\,c^3\,f^3-2\,h\,c^2\,d\,e^2\,f\,m^2-8\,h\,c}{}$$
$$+\frac{c\,e\,(e+f\,x)^m\,(-h\,c^2\,e\,f\,m-3\,h\,c^2\,e\,f+g\,c^2\,f^2\,m^2+5\,g\,c^2\,f^2\,m+6\,g\,c^2\,f^2+h\,c\,d\,e^2\,m+h\,c\,d\,e^2-2}{(c\,f-d\,e)^3\,(c+d\,x)^{m+4}\,(m^3+6\,m^2}$$
$$+\frac{d^2\,f^2\,x^4\,(e+f\,x)^m\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g+c\,f\,h\,m-d\,e\,h\,m)}{(c\,f-d\,e)^3\,(c+d\,x)^{m+4}\,(m^3+6\,m^2+11\,m+6)}$$
$$+\frac{d\,f\,x^3\,(e+f\,x)^m\,(4\,c\,f+c\,f\,m-d\,e\,m)\,(c\,f\,h-3\,d\,e\,h+2\,d\,f\,g+c\,f\,h\,m-d\,e\,h\,m)}{(c\,f-d\,e)^3\,(c+d\,x)^{m+4}\,(m^3+6\,m^2+11\,m+6)}$$

input
```
int(((e + f*x)^m*(g + h*x))/(c + d*x)^(m + 4),x)
```

output
```
(x^2*(e + f*x)^m*(3*c^3*f^3*h + 3*d^3*e^3*h + c^3*f^3*h*m^2 + d^3*e^3*h*m^
2 + 12*c^2*d*f^3*g + 4*c^3*f^3*h*m + 4*d^3*e^3*h*m - 9*c*d^2*e^2*f*h - 9*c
^2*d*e*f^2*h + 7*c^2*d*f^3*g*m + d^3*e^2*f*g*m + c^2*d*f^3*g*m^2 + d^3*e^2
*f*g*m^2 - 8*c*d^2*e*f^2*g*m - 4*c*d^2*e^2*f*h*m - 4*c^2*d*e*f^2*h*m - 2*c
*d^2*e*f^2*g*m^2 - c*d^2*e^2*f*h*m^2 - c^2*d*e*f^2*h*m^2))/((c*f - d*e)^3*
(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*c^3*f^3*g
+ 2*d^3*e^3*g + c^3*f^3*g*m^2 + d^3*e^3*g*m^2 + 4*c*d^2*e^3*h + 5*c^3*f^3*
g*m + 3*d^3*e^3*g*m - 6*c*d^2*e^2*f*g + 6*c^2*d*e*f^2*g - 12*c^2*d*e^2*f*h
 + 5*c*d^2*e^3*h*m + 3*c^3*e*f^2*h*m + c*d^2*e^3*h*m^2 + c^3*e*f^2*h*m^2 -
 7*c*d^2*e^2*f*g*m - c^2*d*e*f^2*g*m - 8*c^2*d*e^2*f*h*m - c*d^2*e^2*f*g*m
^2 - c^2*d*e*f^2*g*m^2 - 2*c^2*d*e^2*f*h*m^2))/((c*f - d*e)^3*(c + d*x)^(m
 + 4)*(11*m + 6*m^2 + m^3 + 6)) + (c*e*(e + f*x)^m*(6*c^2*f^2*g + 2*d^2*e^
2*g + c^2*f^2*g*m^2 + d^2*e^2*g*m^2 + c*d*e^2*h - 3*c^2*e*f*h + 5*c^2*f^2*
g*m + 3*d^2*e^2*g*m - 6*c*d*e*f*g + c*d*e^2*h*m - c^2*e*f*h*m - 2*c*d*e*f*
g*m^2 - 8*c*d*e*f*g*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m
^3 + 6)) + (d^2*f^2*x^4*(e + f*x)^m*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m -
 d*e*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d
*f*x^3*(e + f*x)^m*(4*c*f + c*f*m - d*e*m)*(c*f*h - 3*d*e*h + 2*d*f*g + c*
f*h*m - d*e*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6
))
```

Reduce [F]

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$
$$=\left(\int \frac{(fx+e)^m}{(dx+c)^m c^4+4(dx+c)^m c^3dx+6(dx+c)^m c^2d^2x^2+4(dx+c)^m c\,d^3x^3+(dx+c)^m d^4x^4}dx\right)g$$
$$+\left(\int \frac{(fx+e)^m x}{(dx+c)^m c^4+4(dx+c)^m c^3dx+6(dx+c)^m c^2d^2x^2+4(dx+c)^m c\,d^3x^3+(dx+c)^m d^4x^4}dx\right)h$$

input
```
int((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x)
```

output
```
int((e + f*x)**m/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x + 6*(c + d*x
)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**m*d**4*x**4)
,x)*g + int(((e + f*x)**m*x)/((c + d*x)**m*c**4 + 4*(c + d*x)**m*c**3*d*x
+ 6*(c + d*x)**m*c**2*d**2*x**2 + 4*(c + d*x)**m*c*d**3*x**3 + (c + d*x)**
m*d**4*x**4),x)*h
```

3.153 $\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{dx}{c}\right)}{c(bc-ad)(1+m)}$$

output
```
b*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b*x/a,-f*x/e)/a/(-a*d+b*c)/(1+m
)/((1+f*x/e)^n)-d*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-d*x/c,-f*x/e)/c
/(-a*d+b*c)/(1+m)/((1+f*x/e)^n)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\left(-bc\,\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{bx}{a}\right)+ad\,\text{AppellF1}\left(1+m,-n,\right.\right.}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
(x^(1 + m)*(e + f*x)^n*(-(b*c*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -(
(b*x)/a)]) + a*d*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)]))/(
a*c*(-(b*c) + a*d)*(1 + m)*(1 + (f*x)/e)^n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \text{198}$$

$$\int \left(\frac{bx^m(e+fx)^n}{(a+bx)(bc-ad)}-\frac{dx^m(e+fx)^n}{(c+dx)(bc-ad)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{dx}{c}\right)}{c(m+1)(bc-ad)}$$

input
```
Int[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

output
```
(b*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b*x)
/a)])/(a*(b*c - a*d)*(1 + m)*(1 + (f*x)/e)^n) - (d*x^(1 + m)*(e + f*x)^n*A
ppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)])/(c*(b*c - a*d)*(1 +
m)*(1 + (f*x)/e)^n)
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^m (fx+e)^n}{(bx+a)\,(xd+c)}dx$$

input
```
int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^m/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**m*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{(fx+e)^n x^m}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^m\,(e+f\,x)^n}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

output
```
int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \frac{x^m(fx+e)^n}{bd\,x^2+adx+bcx+ac}dx$$

input
```
int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

output
```
int((x**m*(e + f*x)**n)/(a*c + a*d*x + b*c*x + b*d*x**2),x)
```

3.154 $\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 188

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \frac{B(a+bx)^{1+m}(c+dx)^{-m}}{bf} + \frac{(aBdfm-b(Bde-Adf+Bcfm))(a+bx)^{1+m}(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(1,1,1-m,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)f^2m} - \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,\text{Hypergeometric2F1}\left(1,-m,1-m,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}$$

output

```
B*(b*x+a)^(1+m)/b/f/((d*x+c)^m)+(a*B*d*f*m-b*(B*c*f*m-A*d*f+B*d*e))*(b*x+a
)^(1+m)*hypergeom([1, 1],[1-m],b*(d*x+c)/(-a*d+b*c))/b/(-a*d+b*c)/f^2/m/((
d*x+c)^m)-(-A*f+B*e)*(b*x+a)^m*hypergeom([1, -m],[1-m],(-a*f+b*e)*(d*x+c)/
(-c*f+d*e)/(b*x+a))/f^2/m/((d*x+c)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$$
$$= \frac{(a+bx)^m(c+dx)^{-m}\left(b(Be-Af)(1+m)\,\text{Hypergeometric2F1}\left(1,m,1+m,\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)+\left(\frac{b(c+dx)}{bc-ad}\right)\right.}{}$$

input
```
Integrate[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

output
```
((a + b*x)^m*(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[1, m, 1 + m, ((d*e -
 c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))] + ((b*(c + d*x))/(b*c - a*d))^m*
(-(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[m, m, 1 + m, (d*(a + b*x))/(-(b
*c) + a*d)]) + B*f*m*(a + b*x)*Hypergeometric2F1[m, 1 + m, 2 + m, (d*(a +
b*x))/(-(b*c) + a*d)])))/(b*f^2*m*(1 + m)*(c + d*x)^m)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {173, 25, 88, 80, 79, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(a+bx)^m(c+dx)^{-m}}{e+fx}\,dx$$
$$\Big\downarrow \text{ 173}$$
$$\frac{(Be-Af)(de-cf)\int \frac{(a+bx)^m(c+dx)^{-m-1}}{e+fx}dx}{f^2}+\frac{\int -(a+bx)^m(c+dx)^{-m-1}(Bde-Bcf-Adf-Bdfx)dx}{f^2}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{(Be-Af)(de-cf)\int \frac{(a+bx)^m(c+dx)^{-m-1}}{e+fx}dx}{f^2}-\frac{\int (a+bx)^m(c+dx)^{-m-1}(Bde-Bcf-Adf-Bdfx)dx}{f^2}$$

$$\Big\downarrow \; 88$$

$$\frac{(Be-Af)(de-cf)\int \frac{(a+bx)^m(c+dx)^{-m-1}}{e+fx}dx}{f^2}-\frac{\frac{(aBdfm-b(-Adf+Bcfm+Bde))\int (a+bx)^m(c+dx)^{-m}dx}{m(bc-ad)}+\frac{d(a+bx)^{m+1}(Be-Af)(c+dx)^{-m}}{m(bc-ad)}}{f^2}$$

$$\Big\downarrow \; 80$$

$$\frac{(Be-Af)(de-cf)\int \frac{(a+bx)^m(c+dx)^{-m-1}}{e+fx}dx}{f^2}-\frac{\frac{(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m(aBdfm-b(-Adf+Bcfm+Bde))\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-m}dx}{m(bc-ad)}+\frac{d(a+bx)^{m+1}(Be-Af)(c+dx)^{-m}}{m(bc-ad)}}{f^2}$$

$$\Big\downarrow \; 79$$

$$\frac{(Be-Af)(de-cf)\int \frac{(a+bx)^m(c+dx)^{-m-1}}{e+fx}dx}{f^2}-\frac{\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)(aBdfm-b(-Adf+Bcfm+Bde))}{bm(m+1)(bc-ad)}+\frac{d(a+bx)^{m+1}(Be-Af)}{m(bc-ad)}}{f^2}$$

$$\Big\downarrow \; 141$$

$$-\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\text{Hypergeometric2F1}\left(1,-m,1-m,\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}-\frac{\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left(m,m+1,m+2,-\frac{d(a+bx)}{bc-ad}\right)(aBdfm-b(-Adf+Bcfm+Bde))}{bm(m+1)(bc-ad)}+\frac{d(a+bx)^{m+1}(Be-Af)}{m(bc-ad)}}{f^2}$$

input
```
Int[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

output
```
-(((B*e - A*f)*(a + b*x)^m*Hypergeometric2F1[1, -m, 1 - m, ((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))])/(f^2*m*(c + d*x)^m)) - ((d*(B*e - A*f)*
(a + b*x)^(1 + m))/((b*c - a*d)*m*(c + d*x)^m) + ((a*B*d*f*m - b*(B*d*e -
A*d*f + B*c*f*m))*(a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeo
metric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*m
*(1 + m)*(c + d*x)^m))/f^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 173
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((g_.) + (h_.)*(x_
)))/((e_.) + (f_.)*(x_)), x_] :> Simp[(f*g - e*h)*((c*f - d*e)^(m + n + 1)/
f^(m + n + 2))   Int[(a + b*x)^m/((c + d*x)^(m + 1)*(e + f*x)), x], x] + Si
mp[1/f^(m + n + 2)   Int[((a + b*x)^m/(c + d*x)^(m + 1))*ExpandToSum[(f^(m
+ n + 2)*(c + d*x)^(m + n + 1)*(g + h*x) - (f*g - e*h)*(c*f - d*e)^(m + n +
 1))/(e + f*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[m
+ n + 1, 0] && (LtQ[m, 0] || SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Maple [F]

$$\int \frac{(bx+a)^m (Bx+A) (xd+c)^{-m}}{fx+e} dx$$

input
```
int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

output
```
int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx} dx = \int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m} dx$$

input
```
integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(B*x+A)/((d*x+c)**m)/(f*x+e),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx=\int \frac{(A+B\,x)\,\left(a+b\,x\right)^m}{\left(e+f\,x\right)\,\left(c+d\,x\right)^m}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m),x)
```

output
```
int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx=\left(\int \frac{(bx+a)^m}{(dx+c)^m e+(dx+c)^m fx}dx\right)a+\left(\int \frac{(bx+a)^m x}{(dx+c)^m e+(dx+c)^m fx}dx\right)b$$

input
```
int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

output
```
int((a + b*x)**m/((c + d*x)**m*e + (c + d*x)**m*f*x),x)*a + int(((a + b*x)
**m*x)/((c + d*x)**m*e + (c + d*x)**m*f*x),x)*b
```

3.155 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 393

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$
$$= \frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(1+m)}$$
$$+\frac{2h(bg-ah)(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(2+m,-n,-p,3+m,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(2+m)}$$
$$+\frac{h^2(a+bx)^{3+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(3+m,-n,-p,4+m,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(3+m)}$$

output
```
(-a*h+b*g)^2*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(
b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/(1+m)/((b*(d*x+c)/(-a*d+b*c))
^n)/((b*(f*x+e)/(-a*f+b*e))^p)+2*h*(-a*h+b*g)*(b*x+a)^(2+m)*(d*x+c)^n*(f*x
+e)^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/
b^3/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+h^2*(b*x+a
)^(3+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a*d+b*c),-
f*(b*x+a)/(-a*f+b*e))/b^3/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a
*f+b*e))^p)
```

Mathematica [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

output
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2, x]
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {199, 177, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2(a+bx)^m(c+dx)^n(e+fx)^p\,dx$$

$$\Big\downarrow \text{199}$$

$$\frac{(bg-ah)\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)dx}{b}+\frac{h\int (a+bx)^{m+1}(c+dx)^n(e+fx)^p(g+hx)dx}{b}$$

$$\Big\downarrow \text{177}$$

$$\frac{(bg-ah)\left(\frac{(bg-ah)\int (a+bx)^m(c+dx)^n(e+fx)^pdx}{b}+\frac{h\int (a+bx)^{m+1}(c+dx)^n(e+fx)^pdx}{b}\right)}{b}+\frac{h\left(\frac{(bg-ah)\int (a+bx)^{m+1}(c+dx)^n(e+fx)^pdx}{b}+\frac{h\int (a+bx)^{m+2}(c+dx)^n(e+fx)^pdx}{b}\right)}{b}$$

$$\Big\downarrow \text{157}$$

$$\frac{(bg-ah)\left(\frac{(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^pdx}{b}+\frac{h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right.}{b}\right.}{b}$$

$$\frac{h\left(\frac{(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^pdx}{b}+\frac{h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int(a+bx)^{m+2}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+f}{b}\right.}{b}$$

$$\Big\downarrow \; 156$$

$$\frac{(bg-ah)\left(\frac{(bg-ah)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^pdx}{b}+\frac{h(c+dx)^n(e+fx)^p\left(\right.}{}\right.}{b}$$

$$\frac{h\left(\frac{(bg-ah)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\int(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^pdx}{b}+\frac{h(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right.}{}\right.}{b}$$

$$\Big\downarrow \; 155$$

$$\frac{(bg-ah)\left(\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{h(a+bx}{}\right.}{b}$$

$$\frac{h\left(\frac{(bg-ah)(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+2,-n,-p,m+3,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+2)}+\frac{h(a+bx)^{m+3}(c+d}{}\right.}{b}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

output
```
((b*g - a*h)*(((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*Appel
lF1[1 + m, -n, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b
*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*
e - a*f))^p) + (h*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m
, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f)
)])/(b^2*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))
^p)))/b + (h*(((b*g - a*h)*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*Appel
lF1[2 + m, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b
*e - a*f))])/(b^2*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*
e - a*f))^p) + (h*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3 + m
, -n, -p, 4 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f)
)])/(b^2*(3 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))
^p)))/b
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

rule 199
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Simp[(b*g - a*h)/b   Int
[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1] || (
 !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e)^p (hx+g)^2 \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx=\int (hx+g)^2(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral((h^2*x^2 + 2*g*h*x + g^2)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx=\int (hx+g)^2(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (hx+g)^2(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (e+f\,x)^p\,(g+h\,x)^2\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

input
```
int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,(hx+g)^2\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

3.156 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$$
$$= \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(}{b}\right.}{b^2(1+m)}$$
$$+\frac{h(a+bx)^{2+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\operatorname{AppellF1}\left(2+m,-n,-p,3+m,-\frac{d(a+bx)}{bc-ad},\right.}{b^2(2+m)}$$

output
```
(-a*h+b*g)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*
x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+h*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(
2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(
d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Mathematica [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```

output
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {177, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)(a+bx)^m(c+dx)^n(e+fx)^p\,dx$$

$$\Big\downarrow \ 177$$

$$\frac{(bg-ah)\int (a+bx)^m(c+dx)^n(e+fx)^pdx}{b}+\frac{h\int (a+bx)^{m+1}(c+dx)^n(e+fx)^pdx}{b}$$

$$\Big\downarrow \ 157$$

$$\frac{(bg-ah)(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^pdx}{b}+\frac{h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int (a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^pdx}{b}$$

$$\Big\downarrow \ 156$$

$$\frac{(bg-ah)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\int(a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p dx}{b}+\frac{h(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\int(a+bx)^{m+1}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^p dx}{b}$$

$$\Big\downarrow \text{155}$$

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(a+bx)}{bc-ad},-\right.}{b^2(m+1)}$$

$$\frac{h(a+bx)^{m+2}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+2,-n,-p,m+3,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right.}{b^2(m+2)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```

output
```
((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n,
 -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
 (h*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m, -n, -p, 3 +
m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 +
m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 177
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h/b   Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p, x], x] + Simp[(b*g - a*h)/b   Int[(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] && (Su
mSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e)^p (hx+g)\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="fricas")
```

output
```
integral((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="giac")
```

output
```
integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (e+f\,x)^p\,(g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

input
```
int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,(hx+g)\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

3.157 $\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$$
$$= \frac{(a+bx)^{1+m} (c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^p \left(\frac{b(e+fx)}{be-af}\right)^{-p} \operatorname{AppellF1}\left(1+m,-n,-p,2+m,-\frac{d(a+bx)}{bc-ad},-\frac{f(\ldots}{b\ldots}\right.}{b(1+m)}$$

output
```
(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x+a)/(-a*d+
b*c),-f*(b*x+a)/(-a*f+b*e))/b/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)
/(-a*f+b*e))^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$$
$$= \frac{(a+bx)^{1+m} (c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} (e+fx)^p \left(\frac{b(e+fx)}{be-af}\right)^{-p} \operatorname{AppellF1}\left(1+m,-n,-p,2+m,\frac{d(a+bx)}{-bc+ad},\frac{f(a+b\ldots}{-be+a\ldots}\right.}{b(1+m)}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m,
(d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)])/(b*(1 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx$$

$$\Big\downarrow \text{157}$$

$$(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^pdx$$

$$\Big\downarrow \text{156}$$

$$(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\left(\frac{be}{be-af}+\frac{bfx}{be-af}\right)^pdx$$

$$\Big\downarrow \text{155}$$

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\text{AppellF1}\left(m+1,-n,-p,m+2,-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(1 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n (fx+e)^p \, dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

Fricas [F]

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx = \int (bx+a)^m (dx+c)^n (fx+e)^p \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="fricas")
```

output
```
integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx = \int (bx+a)^m (dx+c)^n (fx+e)^p \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx=\int (bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="giac")
```

output
```
integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx=\int (e+f\,x)^p\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

input
```
int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx=\int (bx+a)^m\,(dx+c)^n\,(fx+e)^p\,dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

3.158 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} \, dx = \operatorname{Int}\left(\frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx},x\right)$$

output
```
Defer(Int)((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} \, dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} \, dx$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
```

output
```
Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {200}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} \, dx$$
$$\Big\downarrow \; 200$$
$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}dx$$

input
```
Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 200
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_] :> CannotIntegrate[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m,
n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 10.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(bx+a)^m (xd+c)^n (fx+e)^p}{hx+g} dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx = \int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g} dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="fricas")
```

output
```
integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx} dx = \text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p/(h*x+g),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(bx+a)^m(dx+c)^n(fx+e)^p}{hx+g}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

Mupad [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{{\left(e+f\,x\right)}^p\,{\left(a+b\,x\right)}^m\,{\left(c+d\,x\right)}^n}{g+h\,x} \,d x$$

input
```
int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x),x)
```

output
```
int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(bx+a)^m\,(dx+c)^n\,(fx+e)^p}{hx+g}dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file